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Online Student Discussions in a Blended Learning Classroom: Reconciling Conflicts Between a Flipped Instruction Model and Reform-Based Mathematics

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ABSTRACT

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Two ideas are prevalent in teacher professional development today. Teachers are finding new and innovative ways to incorporate technology into their classroom. The use of video and social media is increasing. One type of pedagogy that has emerged among the blended learning pedagogies is flipped instruction, where students participate in some of the instruction outside of the classroom. Another prevalent idea is the focus on inquiry learning and reform-based mathematics instruction. This pedagogy adheres to the idea that students can use their problem solving skills to understand complex mathematics.

This qualitative content analysis outlines how one researcher sought to find a balance between the two ideas. The two ideas conflicted at times, but the researcher ultimately found innovative ways to reconcile those conflicts. The study describes how one fourth-grade class used a website to engage in mathematics conversations in a blended learning environment. This blended learning environment maintained the values of a reform-based mathematics classroom.

The researcher found that students engaged in conversation online contained instances where students formed theories, questioned one another's theories, built on the thinking of other students, used precise and formal language, and used evidence to support student thinking. Teachers that implement blended learning or flipped instruction should seek out methods that adhere to an inquiry approach to teaching mathematics. The researcher also found that the development of a student in a particular conceptual understanding may have an impact on the depth of conversation they engage in.

Keywords: blended learning, mathematics education, technology uses in education, creative thinking

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1: Introduction

In the winter of 2012 I had a conversation with my principal about the future of education. We had both seen a recent article on CNN.com (Green, 2012) that talked about the benefit of a new pedagogical model called flipped instruction. Flipped instruction is a form of blended learning where students participate in a portion of instruction outside of the classroom instead of inside. This is usually facilitated through digital, web-based media. Students then spend a greater amount of time applying concepts in class and less time in lecture-based instruction. Online instruction, outside of the classroom, is typically distributed in the form of video. During the conversation with my principal we talked about the logistics of implementing such a program at our school. He and I share a common desire to give our students the best possible resources; we saw the potential benefits of exporting a portion of meaningful classroom experiences outside the traditional classroom. Through flipped instruction we felt we could give the students a meaningful learning experience anywhere beyond the classroom. The future of education, we felt, would use technology and social media to implement different types of blended learning, such as flipped instruction (e.g., Horn, Staker, Hernandez, Hassel, & Albeidinger, 2011; Staker & Horn, 2012).

Blended learning refers to instructional models that combine online and traditional face-to-face instruction (Bonk & Graham, 2006). Blended learning does not require that a specific pedagogical approach be used. Flipped instruction is one form of blended learning. While it does not imply a specific pedagogy, many of the popular flipped instruction models follow a direct instruction approach, focusing on procedure rather than understanding. Personally, I wanted to keep my classroom relevant, give my students opportunities to learn outside of my classroom, and engage parents in meaningful ways while their children are learning. I valued the opportunities that a flipped classroom would provide to meet these goals. The flipped model would increase my ability to custom tailor instruction to my students. I began creating videos and began to flip my classroom. At that point in time, I did not anticipate the conflicts between practice and theory that would surface from doing so.

Shortly into my first attempt at flipping my classroom, I began to see a conflict between the flipped model I had employed and how I believed math should be taught. While flipped instruction does not necessarily require a particular pedagogy, many of the models follow a pedagogy described as instrumental understanding by Skemp (1976). Instrumental understanding refers to a pedagogy that focuses on procedures. For example, Khan Academy is an online resource that has an enormous library of videos explicitly explaining how to do math procedures (Thompson, 2011).

In contrast to *instrumental understanding*, Skemp (1976) refers to a pedagogy called *relational understanding*, which focuses on understanding of mathematical concepts and principles. A relational understanding pedagogy would support an inquiry approach to instruction where the students construct and talk about their understanding through problem solving. A pedagogical model called reform-based mathematics instruction stresses this type of understanding. *Reform-based mathematics* focuses on students constructing their own understanding, and then talking about that understanding as it relates to previous knowledge and their peers.

A pedagogy based on an instrumental understanding would require a teacher to explain to students how to solve a certain problem, and then give them similar problems to practice the procedure. My original flipped videos catered to students' instrumental understanding and focused mainly on procedure. In contrast, my regular classroom instruction focused less on procedure and more on helping students develop relational understanding. Thus, in a typical classroom lesson, students would talk about various ways to solve a problem, and focus on the "why" instead of the "how." I soon realized the videos I had produced were instrumental in nature, and conflicted with the relational nature of my regular classroom instruction.

A typical pattern for instruction when I started using a flipped classroom approach began with an assignment for students to go home and watch a video about how to solve a problem or perform a procedure. In class the next day, I would introduce a task and ask them to use their own reasoning to solve it. It soon became apparent that when they came to school and participated in a task, because they had already learned a traditional algorithm, they had developed a response set toward an instrumental approach to solving mathematical tasks. They were not able to imagine the task and find creative ways to solve the problem on their own, or they appeared to be limited to the procedures or strategies presented in the video. My students seemed incapable of engaging the patient problem solving I had seen earlier and that they continued to exhibit in other classroom situations.

I found the flipped model to be effective in some ways and ineffective in others. Among other benefits, flipping helped me communicate with parents, structure stronger interventions, and cater my instruction to students' specific needs. Despite these benefits, I saw pedagogical flaws in the model. Students would learn one strategy as outlined in a video, and then be limited to applying only that strategy in problem solving. I saw that in some ways the videos were squashing the students' creativity in problem solving.

The first flipped lesson I taught required students to come back to school with an understanding of the topic I was prepared to teach. While I had a few students that were unable to access the videos at home because of technical difficulties or a lack of Internet access, I was able to make accommodations for that small population through the use of school machines. The benefits from the first lesson taught using the flipped model made me feel like the lesson was a success, so I decided to start making more videos. The first edition of my videos typically explained a few strategies for solving a type of problem. The videos employed direct instruction focused on procedure and instrumental understanding (Skemp, 1976). I would present a problem, and then I would show one or two ways to solve the problem. I modeled many of the videos after videos I had seen on Khan Academy (https://www.khanacademy.org). The videos were to be watched before the lessons (Sams & Bergmann, 2013). Students were to go home, watch the video, and then in the next few days a lesson would be centered on the information from the video. My goal in those first videos was for students to effectively use the procedures and strategies I presented in problem solving a task in a following lesson. The main idea of the videos was that if the students watched me solve the problem and heard me explain it, they would be able to mimic my strategy and solve similar problems. Resources exist that I could have used without taking the time to create my own productions. However, I found that my own voice was the most powerful tool I could use in the videos I gave my students. Students were familiar with my voice and my cadence, and they tended to be more engaged when the videos came from me rather than from a third party.

While flipped instruction has many benefits, I noticed that my teaching was deviating from my beliefs about mathematics instruction. During my regular classroom instruction I value students' ability to communicate with one another as they learn material (NCTM, 2000). I want students to be able to talk about their learning and share evidences that support their own claims, as well as the claims of others. The videos did not provide much in the way of producing discussions. If students had good questions or comments as they watched the videos, they were unable to share those questions or comments with their peers. By the time they got back to class they might remember the content in the video, but it was unlikely they would remember their questions or comments. I wanted to create a different type of flipped instruction in which students still had an experience outside of class that we could build on during class. From these experiences I wanted this new model to allow students to explore and talk about their understanding in a way that matched more closely with the relational understanding pedagogy I valued and was trying to enact in my classroom instruction. I wanted a model that would place value both on the benefits of a flipped instruction model, but also of the benefits of reform-based mathematics, which follow a relational understanding pedagogy (Skemp, 1976).

As a result of the conflict between flipped instruction as I was enacting it and reform-based mathematics, I developed an instructional strategy that combined a reform-based mathematics approach but also followed a blended learning model that is different from the typical flipped classroom. For me such a model would lead students to develop both problem solving skills and fluency in the discourse of mathematics. In this study, I wanted to examine the efficacy of this model and explore student conversations both inside the classroom and outside the classroom in order to address the following two questions:

1. What does reform-based mathematics instruction look like in a fourth-grade blended learning classroom model?
2. How do fourth-grade students demonstrate elements of successful reform-based mathematics when participating in technology-mediated online math discussions?

Chapter 2: Review of Literature

This section will outline the background behind reform-based mathematics instruction, an inquiry-based teaching model that allows students to form their own conclusions and eventually be guided to the mathematics they have unearthed. It will also discuss flipped instruction, a blended learning instructional model that frequently uses technology to disseminate information in a non-traditional format. It will then discuss how the two models conflict with one another, and provide a possible reconciliation of the two models through the use of student discussions. Next I will outline the importance of reasoning and proof in math discussions, and finally conclude with a rationale for this research. The literature guides the project to answer the following two questions: (a) what does reform-based mathematics instruction look like in a fourth-grade blended learning classroom model; and (b) how do fourth-grade students demonstrate elements of successful reform-based mathematics when participating in technology-mediated online math discussions?

Reform-Based Mathematics Instruction

The topic of mathematics instruction has been highly researched over the past few decades (Beatty & Geiger, 2010). The research essentially presents two philosophical ideas regarding math instruction. While it is not quite that simple, most ideas can be put into one of two categories outlined by Skemp (1976). Skemp argued that instruction is based either on instrumental understanding or relational understanding. Instruction aimed at instrumental understanding focuses on the procedure of the mathematics being performed. This instructional approach would focus on rules and procedures, and would lack an emphasis on the underlying relationships of the mathematical concepts. Relational understanding, on the other hand, focuses on those underlying relationships within the mathematical concepts. Instructional strategies following a relational understanding orientation would focus on students developing multiple access points to problem solving. This study will assume a relational understanding orientation to mathematics instruction. Other researchers (e.g., Cobb, wood, Yackel, & McNeal, 1992; Hendrickson, Hilton, & Bahr, 2008; Thompson, 1996) often refer to this orientation to mathematics instruction as reform-based mathematics instruction.

Instruction that is based on the theoretical ideas of relational understanding is also often inquiry-based instruction. Researchers within this orientation come from multiple and divergent theoretical underpinnings. Those that subscribe to the idea that problems can be solved from multiple access points appropriately enough come from different backgrounds, approach the same questions, and answer them differently. For example, some researchers (e.g., Thompson, 1996) subscribe to a constructivist view that students develop their ability to prove and articulate their understanding. Others take up the sociocultural views of math education (e.g., Bauersfeld, 1980; Cobb, 1994; Krummheuer, 1995; Saxe, 1991) subscribing to the idea that students use context and social situations to teach and learn. Both constructivists and socioculturalists place a high value on students' ability to articulate proof and reasoning of their mathematical understanding. Thompson (1996) said "I am unable to separate matters of learning from matters of reasoning" (p. 267).

Research has shown the importance of reasoning and proof in classroom instruction (e.g., Cobb et al., 1992). Cobb and associates (1992) looked at lessons taught in multiple classrooms through which they compared different classroom pedagogies. They looked at classrooms following a traditional direct instruction model, which mirrored Skemp's (1976) instrumental understanding. They contrasted those classrooms with those that followed an inquiry model of instruction, which mirrored Skemp's relational understanding. They described a direct instruction model classroom where the only answers deemed right were those that were procedurally correct. Learning in these classrooms was limited to the students producing the processes of the mathematics. In contrast another classroom where an inquiry pedagogy was employed allowed students to demonstrate relational understanding rather than procedural understanding. The inquiry-based pedagogy included problem solving and task-based student engagement. The study done by Cobb and associates (1992) showed that reasoning and proof surface naturally when students are engaged in an inquiry instruction model.

Smith and Stein (2011) outline guidelines for creating a culture that allows for a reform-based mathematics instruction. They focus on building meaningful classroom discussions. As part of that focus instructors create a culture where students are able to restate other students' thinking, connect to their own strategies, as questions, and build on one another's thinking. Smith and Stein also suggest that part of reform-based mathematics is continuously assessing student learning while students go through the problem-solving process. They encourage instructors to solve problems prior to teaching a lesson, and attempt to anticipate misconceptions and questions students may have. They also encourage teachers to keep a record and assess students as they problem solve.

Reasoning and Proof

NCTM (2000) lists reasoning and proof as one of the principles to be taught in grades 3 through 5. NCTM lists the following four standards that instruction should facilitate for students within this grade band. Students should

1. Recognize reasoning and proof as fundamental aspects of mathematics;
2. Make and investigate mathematical conjectures;

3. Develop and evaluate mathematical arguments and proofs; and
4. Select and use various types of reasoning and methods of proof (NCTM, 2000, p. 188).

Reasoning and Proof in the Classroom

NCTM (2000) also says that students in this age group need to be able to make conjectures. From these conjectures, students can make broad generalizations. Teachers can facilitate this by asking questions including, "What if I gave you twenty more problems like this to do— would they all work the same way? How do you know?" (NCTM, 2000, p. 190) When students answer these questions they begin to form arguments. Students learn what makes a convincing argument and what does not. Often students may make conjectures that seem to be correct and mathematically sound but are not. According to NCTM, when they learn about convincing arguments and mathematically sound conjectures from their failures as they create false arguments and find the flaws in faulty conjectures, they strengthen their learning. When the theoretical modeling and real world do not match up, the students are forced to ask deeper, more meaningful questions to answer complex questions (Meyer, 2010). Or, as NCTM (2000) puts it, "these 'wrong' ideas often are opportunities for important mathematical discussions and discoveries" (p. 190).

Teachers' Responsibilities in Fostering Reasoning and Proof

A teacher can do four specific things to make sure that reasoning and proof are present in a classroom. First, teachers establish a safe mathematical community. Second, they make students responsible for articulating their own reasoning and for working hard to understand the reasoning of others. They also remind students of conjectures and arguments they have developed for application on further work. Finally, teachers make decisions about which conjectures are mathematically significant for students to pursue. (NCTM, 2000).

Blended Learning

When students use video or online media at home before they are taught a concept at school they are participating in a blended model of instruction. There are many types of blended learning. One form of blended learning is called flipped instruction (Sams & Bergmann, 2013; Horn et al., 2011; Staker & Horn, 2012). The flipped instruction model also does not imply one particular pedagogy. However, typically teachers that flip their classrooms will give the students homework to watch a video, read a passage, or another activity where they learn something at home. The model is based on an idea that there are two parts of any lesson: the part where the teacher explains a concept or a topic, and the part where students apply the concept. This particular model of a flipped classroom where students receive instruction outside of class and then practice the concept in class is based in an instrumental understanding model rather than a relational understanding model. Historically, in classrooms the teacher spends the bulk of time explaining things in class and the students practice at home. By flipping the instruction and practice, students receive the explanation at home and can receive help from their teacher while they practice at school (Sams & Bergmann, 2013).

The flipped model allows for teachers to allocate their time differently. They are able to spend less class time engaging in direct instruction and more time helping groups or individuals as they practice the concepts they have learned from the instruction. (Horn et al., 2011; Staker & Horn, 2012; Staker and Horn 2012), Jon Sipe, an executive at the publishing company Houghton Mifflin Harcourt said in an interview about his company's blended learning software that the purpose of the software.

> So teachers don't have to 'waste their time' on some of these things that they've always had to do. They can spend much more time on individualized learning, identifying

> specific student needs. Let students cover the basics, if you will, on their own, and let teachers delve into enrichment and individualized learning. (Barseghian, 2011, p. 1)

The software programs Sipe refers to contain instructional videos demonstrating to students the procedure of how to solve certain types of problems. For example, khanacademy.org, another blended learning digital resource, contains an enormous library of instructional videos in a large range of topics including mathematics. The videos are constructed from what Skemp (1976) would classify as primarily an instrumental orientation. In other words, they are how-to videos that focus narrowly on teaching procedure.

Conflict Between Reform-Based Math Instruction and Flipped Models

A recent tweet by popular reform-based math instruction blogger Dan Meyer (2013) identified, “The blended learning lie: Students must learn math basic skills before they can do anything interesting with them.” While blended learning does not directly refer to one pedagogy, in this tweet, Meyer assumes that it does. The particular model of blended learning Meyer is referring to is one that places emphasis on an instrumental orientation. Meyer’s tweet represents the conflict between these two models. Many flipped instruction models that follow an instrumental orientation have the mathematics serving the conversation. Instead of talking about a real-world problem, the conversation jumps right into difficult math terminology. In this flipped model, the teacher records a video about the mathematics and has the students learn the procedure in an abstract mathematical format before learning how it applies to the real world. In a reform-based mathematics model, or in a relational understanding orientation, the conversation would serve the mathematics. The students would naturally identify a problem and then discover through problem solving the mathematical concepts surrounding the topic. Instead of showing students how to solve a problem with the use of examples and formulas, a real life context would allow for students to solve and discuss math using their own problem solving skills, ability to reason, and support claims with proof. Thus, the two models can be combined not so much by moving away from using a blended approach, but by using a pedagogical approach that targets the development of relational understanding. From this stance, the instruction that would typically be received through digital media is changed, and task-based problem solving is combined with student discussions that build on their discussions in class.

Rationale for this Study

Yackel and Hanna (2003) point out that there is an emphasis on instructional strategies that highlight reasoning and proof in mathematics education research. Among other purposes, they argue that reasoning and proof can provide a systematic approach for students to explain, discover, verify, construct theory, explore a definition, and to find the consequences of assumptions. This emphasis on reasoning and proof as a focal point in classroom instruction has roots in both constructivist and sociocultural theories. Skemp (1976) produced research that supported this idea a few decades ago. Others built on Skemp's work in the 1990s (e.g., Cobb et al., 1992; Thompson, 1996) and through the 2000s (e.g., Yackel & Hanna, 2003), providing evidence that inquiry-based mathematics instruction can naturally bring mathematical concepts to the surface and improve student learning. Today Beatty and Geiger (2010) call for research in mathematics instruction that directly applies to today's digital technologies. Increasing research regarding these digital technologies and learning makes sense, as online formats are becoming more common (Horn et al., 2011; Staker & Horn, 2012). Indeed, some researchers have done research examined how math instruction can benefit from the implementation of digital technologies in elementary schools (e.g., Moss & Beatty, 2006; Nason & Woodruff, 2003). What is absent from this research is evidence that digital technologies can impact the learning of my students sharing the same classroom (e.g., Moss & Beatty, 2006; Nason & Woodruff, 2003) while maintaining a reform-based mathematics instructional framework (e.g., Cobb et al., 1992; Hendrickson et al., 2008; Thompson, 1996).

Beatty and Geiger (2010) describe the theoretical impact of having students collaborate digitally. According to these researchers, there is an increased interest among researchers in the mathematics education in theoretical perspectives associated with socioculturalism. Like Beatty and Geiger, a number of researchers are taking up research that "include(s) references to sociocultural theory, collaboration, learning communities and classroom discourse" (p. 260). Their purpose is to demonstrate the value of these tools in developing students' mathematics skills. Such research, based on Vygotskian ideas, is designed to demonstrate how students learn from their social context through collaboration and involvement in mathematics discourse.

Moss and Beatty (2006) performed a study where students used Knowledge Forum, a software that allowed for students to collaborate with one another digitally. The results of this study demonstrated that students who are forced to limit their communication to a text-based format were still able to collaborate and benefit from discussing mathematics. They were able to question each other, make conjectures, show evidence, and demonstrate precise, formalized language. The students were from separate schools and for the purposes of the study interacted in only 12 lessons electronically. The study provided evidence that even when students are strangers, they can engage proficiently in this kind of discourse. The study left unanswered what mathematics discourse and reasoning might develop if students are already known to each other and have face-to-face as well as text based communications. The Moss and Beatty (2006) study is relevant to the current study because it showed that students in fourth-grade had the cognitive and social skills to negotiate a community of practice and to discuss mathematics at a high level.

These researchers stressed the importance of setting up an environment where students can develop these skills. Moss and Beatty created a virtual safe community of practice that cut across geographic boundaries, yet still supported students in developing these skills. However, they left open the question of what impact creating this environment in a regular school classroom might have on students' mathematical reasoning and discourse.

Beatty and Geiger (2010) pointed out that technology in mathematics instruction can serve two purposes. The first purpose is for it to be used as a procedural tool. An example of a procedural tool may be a calculator or a spreadsheet. The second purpose is for technology within mathematics instruction to facilitate collaboration. Technologies such as chat rooms, Google Docs, email, forums, and what Gee and Hayes (2011) refer to as *passionate affinity spaces* may be used to facilitate collaboration purposes. Passionate affinity spaces are places where learners collaborate and share ideas about the any given topic, such as mathematics. Some technologies are specifically designed to fulfill both these purposes. An example of a tool that fulfills both purposes is Knowledge Forum, which is Computer Supported Collaborative Learning software (Nason & Woodruff, 2003). Some collaborative technologies were not specifically designed for math instruction, but have been repurposed to be used as math instruction technologies. Beatty and Geiger (2010) call for further research to find how collaborative tools can build on the sociocultural perspectives of learning. Specifically, they call for research to be done in four areas: technology designed for both "learning mathematics and collaboration," technology designed for "learning mathematics but not specifically for collaboration," technology designed for "collaboration but not necessarily learning mathematics," and technology designed for "neither learning mathematics nor collaboration" (p. 263). Beatty and Geiger outline the studies already done in these four areas, and call for further research in each one. A particular area they identified is one where research about technology designed for student collaboration focuses not so much on learning mathematics, but rather on the ways in which digital and face-to face collaboration and interaction might impact students' engagement in mathematical reasoning and discourse. As new technologies and pedagogical strategies emerge it is important to study and describe them because it is the "divergent uses of technology by students and teachers that provide the most exciting outcomes" (p. 277).

In order to fulfill the need for a research study that these researchers have outlined, I have designed a study that will address my questions of describing a blended learning classroom that is prescribing to a reform-based mathematics pedagogy, as well as identify those aspects of such a program that may support or limit the implementation of the two in a fourth-grade classroom.

Chapter 3: Method

This qualitative content analysis (Marshall & Rossman, 2011) is a study describing a form of blended learning classroom where reform-based mathematics is employed, and the elements of reform-based mathematics found in the online math discussions of fourth-grade students. I developed a blended learning model in my classroom, and at the same time implemented reform-based mathematics instruction focused on relational understanding and an inquiry approach. In this chapter I will outline the norms of reform-based math instruction, how my specific type of a blended learning classroom was set up, how I collected data, and how I analyzed those data.

There are researchers in the field who believe that the solution to many problems faced in education today can be addressed by flipped instruction (e.g., Horn et al., 2011; Staker & Horn, 2012). Many flipped instruction models would seem to be ideal if you believe that students need to know and be able to execute mathematical procedures and algorithms before they can problem solve. By allowing students to listen to a lecture, or learn a procedure before spending time with the instructor, students potentially have a head start in being able to demonstrate the skills they have learned. Instead of spending time teaching concepts, teachers can spend time practicing, assessing, and intervening for those that are struggling.

However, teaching a procedure first is a practice that directly conflicts with the basis of reform-based math instruction. Reform-based math instruction is grounded in the constructivist idea that students can problem solve and naturally discover complex mathematical concepts. To those who subscribe to the reform-based mathematics idea that students do not need to be taught a procedure before they can understand concepts and problem solve, flipped instruction that shows students how to solve a problem seems to run counter to the approach that would build students' relational understanding, a notion on which the NCTM Standards are built.

Historically there has been a push for students to attend to schoolwork outside of the classroom (e.g., Cooper, 1989; Corno, 1996; Epstein & Van Voorhis, 2001), and the push today is no different (Corno, 2000). Administrative policy and pressure from parents make homework an expectation in most, if not all, classrooms today. I wanted to make the homework I sent home as effective as possible. While I first embraced a flipped instruction model as a way to meet these pressures, over time I saw the conflict between the particular model of flipped instruction I had adopted and the reform-based mathematics approach I already had in place in my classroom. Using a flipped model that was instrumental in nature resulted in a reduction of discourse and engagement in problem solving in my students.

The school in which this study was conducted was an ideal setting for establishing both a blended learning or flipped instruction style environment coupled with instruction heavily influenced by the reform-based mathematics background. Blended learning contains a digital component coupled with face-to-face instruction. Interaction on wikispaces is one way I had students receive learning experiences digitally. The types of activities they were asked to do on the wikispace were directly tied to a reform-based mathematics approach. I had students participate several times per week in a flipped classroom blended learning activity at home for homework. Students visited the wikispace and responded to prompts or tasks depending on the assignment. Assignments were chosen based on students' need, the depth of learning I had decided was needed, and the order of learning objectives that I had previously decided upon. I used these interactions as a way to assess student learning, and was constantly looking for ways to support what I do in my mathematics instruction. For these reasons, my classroom was perfectly suited for this study. My research questions helped me describe the implementation of blended learning causing me to seek out how it had benefitted or hindered my mathematics instruction. I needed to know if it was worthwhile to combine blended learning and reform-based mathematics, or if the two ideas could coexist.

Participants

The school in which this research was conducted reported 580 students for the school year of 2012-2013, the year before the research was conducted. Of the 580 students, one student was Native American, four students were Asian, seven students were black, 19 were Hispanic, six were Pacific Islander, 12 were mixed race, and 531 (92%) were white. The year the research was conducted was the third year in the school's history.

The fourth-grade class in which the research was conducted contained 32 students. The class had 14 boys and 17 girls. The class was predominately white, with two students that were of Asian descent. The students had been using computers in class at a ratio of 1:1 for part of the day for a few weeks at the time that data were collected.

Design

This study was a content analysis that contained aspects of both a descriptive study as well as an explanatory study (Marshall & Rossman, 2011). The study was labeled a content analysis because it employed a priori coding strategies applied to text produced by students. Answers to both the first and second research question were descriptive in nature, as its primary purpose is to describe the pedagogies in my classroom, and show how students used elements of reform-based mathematics in a blended learning environment.

Background and Procedures

Before I started to formally investigate the matter, I tried several different methods of flipping my classroom. From my experiments and first attempts at flipped instruction, I found that some of the most useful artifacts for me as a teacher were products students created through discussions and dialogue. Students had an experience outside of the classroom and wrote about their experience online. I found it particularly useful when I could see the students' discussions unfold each evening from my own computer. There were a number of benefits that made this pedagogy attractive to me. I found that when they wrote their thinking online, I as the instructor came to class more prepared because I could see where both good reasoning and misconceptions were developing. The students grew because they were able to articulate their learning with their peers in a safe environment.

I valued this portion of the particular flipped instruction approach I had adopted. Some flipped instruction approaches I have observed would require the students to participate in instruction through video, reading, or another experience that would teach them the subject. This would free the teacher up during what would otherwise be used in lecture to help students practice what they had already learned through the recorded instruction. I soon recognized that my classroom did not allow for that. I do not spend a great deal of my time lecturing. Having students participate in mathematics dialogue for homework allowed me to flip my instruction, but did not require me to match the model of flipped instruction where teachers lecture first followed by students subsequently working independently.

Before flipping my classroom, much of class time was allotted for students to explore and talk about their discoveries. The lecture is usually last and often takes a smaller portion of time than exploring and practicing concepts. During the lecture portion of my lessons, students share their thinking. There are times when I direct their thinking, but much of the talking comes from students.

I believe that students can problem-solve before knowing specific procedures. Those researchers who support reform-based mathematics instruction would agree (e.g., Fosnot & Dolk, 2001; Hendrickson, et al., 2008; Smith & Stein, 2011). After attempting to implement a flipped instruction model in previous years, I found value in switching from the traditional homework of worksheets, to activities with social and interactive components. I also found that using digital technologies facilitated these components effectively, since the technology allowed me to capture a written record of problem solving and reasoning. I based my units of instruction on a reform-based mathematics instruction model. Lessons were designed to help students move from developing understanding of a topic, to solidifying that topic, and then practicing the procedure of the topic. These lessons were designed and arranged in that order.

In order to set up a model that facilitates online discussion and mathematical explorations I established an online environment where students could do just that. On the first day of school I introduced my class to a wikispaces website where they could complete homework one to three times per week. This website consisted of pages where problems were posed for students requiring them to explore a mathematical task, problem-solve, and then share their findings. Students would go to the website and would be met with a picture or prompt. They would be given a question and asked to respond by sharing their thinking. After students completed their assignment to share their thinking, they were invited and encouraged to return to the website and respond to their peers' comments, conjectures, questions, and solutions.

I worked to establish a culture on the wiki that gave me the benefit of collaborative homework. That homework gave me the same positive impacts a flipped instruction model can provide, while encouraging students to develop their own problem-solving skills and articulate them to their peers. The tasks typically contained mathematics from an upcoming lesson, so students were rarely familiar with the context or the specific mathematics involved in the task. Giving students tasks ahead of what we cover in class without giving them the procedure to solve such problems helped strike a balance between the theories behind both flipped instruction and reform-based mathematics. Finding a balance between these two orientations satisfied the conflict between the flipped instruction models I have used with the theoretical underpinnings of the reform-based mathematics instruction movement.

The unit in which I collected data was titled *Solving Addition and Subtraction Word Problems Involving Fractions and Mixed Numbers*. Students had a previous understanding of how to add fractions with like denominators. The unit was designed to have students move from the conceptual understanding of adding and subtracting fractions and generating equivalent fractions to solve problems involving fractions and mixed numbers. In order to problem solve some of these tasks, students had to return to their previous understanding about fractions to compose and decompose whole numbers into fractional quantities. Three vignettes will be used to show how students develop, solidify, and practice their conceptual understanding of fractions.

This program contained a few key points from reform-based mathematics. Students were able to practice their problem solving skills, and explore a concept prior to delving into the procedural aspect of an idea. Students were also able to discuss things with each other, which is one of those key points. Students were given contexts and enough information to solve problems, but were not given any direction on how precisely to solve the problems. They were required to solve the problem using whatever prior knowledge they had. The problem solving aspect of the task forced students to form theories even if they were not sure, firm, or even correct. Then students questioned one another's theories, elaborated on the theories of others, and eventually described the procedure of how they solved problems using precise and accurate language.

There were a few drawbacks to the system. Some students did not have full access to the Internet at home. Those students did not get to spend as much time on the wiki outside of class, and were given time in class to complete assignments. There were a few other isolated instances of parents who were not fully supportive of non-traditional homework. In other cases parents did not feel comfortable with students spending as much time on the wiki as may be required. Even though students averaged about six minutes per homework assignment on the website, a few parents raised questions about the value of that time. On the other hand, some parents shared that they felt that homework should take more time than what the website required.

Students visited our class wiki regularly for homework assignments. In mathematics the homework assignments were problem-solving tasks, and designed to foster discussions. I sometimes took part in the discussions by asking probing questions and provide feedback. However, during data collection I did not engage in student conversations.

Figure 1 shows an example task from the beginning of the year. Students explored the arrangement of a six-by-eight array, as well as concepts of doubling and halving factors of a multiple. They were asked to justify their response to the problem with evidence and proof. This figure gives an example of a task on the classroom wiki given to students at the beginning of the year. The task was accessible to students, but only through their wikispaces account. They were able to log in, and then comment on the task. On this particular task, they were to explain how doubling and halving helps to solve multiplication problems.

After students explored the task, they were asked to enter text to justify their answer. Figure 2 shows the discussion box where students were able to enter text to complete the assignment. Each night when this homework was assigned, students were to engage with each other through this website in a threaded discussion. The discussions that students had on this website were the primary data for this study. At the bottom of each task, students were able to enter their comments in a threaded discussion format.

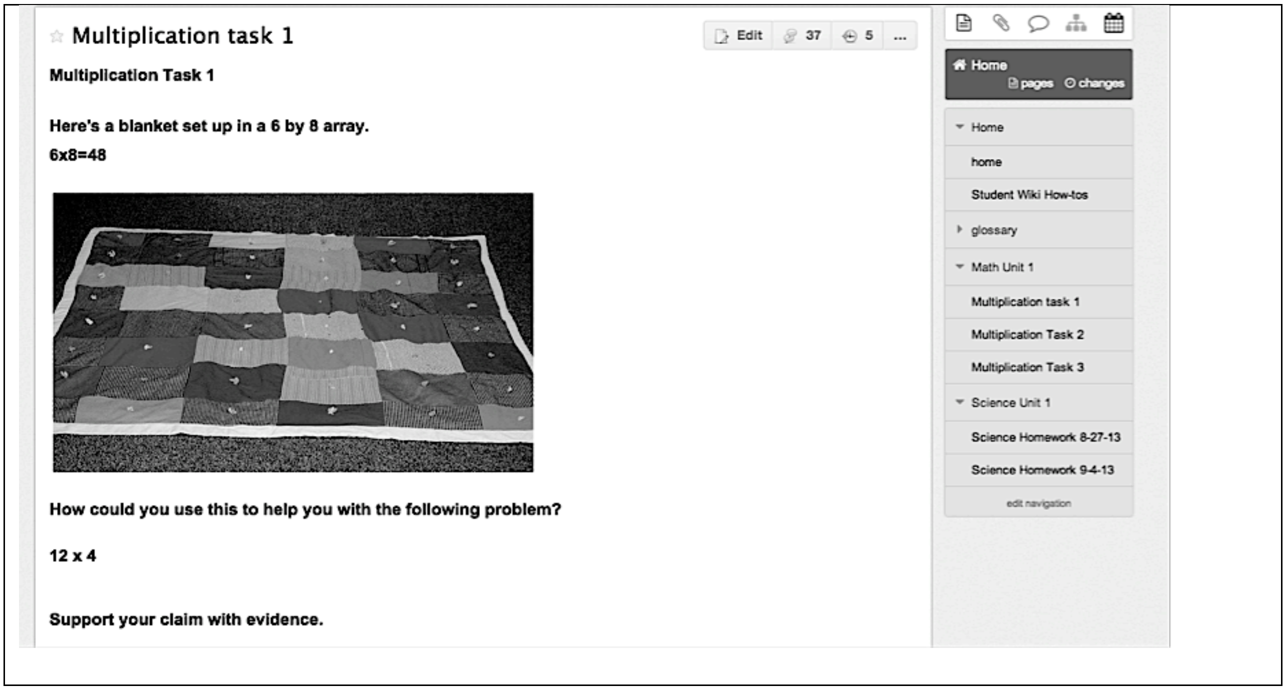

Figure 1. Multiplication Task 1. Screen capture of the task titled *Multiplication Task 1*.

Data Sources

I collected data from one instructional unit. I collected data from the student wiki and I journal of my own experiences of teaching. I collected data during Unit Nine, which focused on solving addition and subtraction word problems with mixed numbers. The unit correlated to the Common Core State Standards from Numbers and Operations—Fraction clusters three and four. This unit dealt with representing and problem-solving with fractions, as well as performing addition, subtraction and multiplication with some fractions. Students were required to plot fractions on a number line, and were to be fluent in vocabulary such as numerator, denominator, equivalency, factor, and multiple.

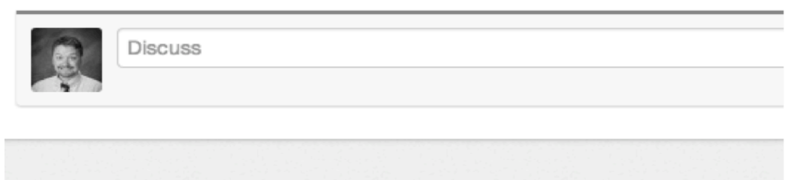

Figure 2. Discussion box.

During the unit, I planned four different tasks to challenge the students to add and compare differences of fractions with like denominators. These tasks dealt with fractions that when added, have a value greater than one. The Rock Task was given at the beginning of the unit, when students had little experience in working with mixed numbers and improper fractions. I gave students the task to plot on a number line the values of the measurement of a few rocks. The Long Jump Task challenged students to compare data from a long jump competition and assign gold, silver, and bronze medals. The Add Mixed Numbers Task required students to find a flaw in how an example was reported. This task was assigned as my students were beginning to further understand the math concepts and procedures found in the unit objectives. In the Pizza Task students took orders of pizza to see how much pizza needed to be prepared. The tasks were intentionally designed and ordered to build on one another. Students were to move from developing understanding about the topic, to solidifying that understanding, and ultimately practicing that understanding. The unit was designed as a model for reform-based mathematics. The entire unit was included instances where students *developed*, *solidified*, and then *practiced* their understanding. During *develop* lessons and tasks, students explored the mathematics and use their problem solving abilities to discover complicated mathematics. During *solidify* lessons and tasks students began to explore the mathematics they had already uncovered as they went deeper into the conceptual understanding behind the math. *Practice* lessons and tasks allowed students to develop fluency in working with algorithms and proven strategies that supported the concepts they had learned. In the following sections, I will outline each task. The tasks were designed according to the *develop, solidify,* and *practice* framework.

Rock task. The Rock Task was the first task given to students for homework during Unit 9. I took a picture of three different possible ways to plot the lengths of rocks on a number line. The math task described a situation where three fictional students, Al, Billy, and Chuck put three identical rocks on a number line. Two of the fictional students did it correctly, although at first glance, it appears they did it differently. Al did have enough understanding to use fractions that were greater than one. The rock that was listed as $1\frac{1}{4}$ inches was incorrectly placed on the $\frac{1}{4}$ inch line. The other two set their number lines up correctly. However, Billy wrote the correct mixed number on the number line, whereas Chuck wrote improper fractions on his. Both Billy and Chuck plotted their rock lengths on the appropriate spaces on the number line. Billy plotted the rock measuring $1\frac{1}{4}$ inches on the correct line. Chuck plotted the rock measuring $1\frac{1}{4}$ inches on the $\frac{5}{4}$ inches line. In this document, references to this task will be listed as *Rock Task.* The Rock Task can be found in Appendix A.

Long jump task. The task the next night required students to compare and combine mixed numbers. The task was designed to get students to develop strategies and the conceptual understanding of adding fractions that have a value larger than one. The task gave students a table of data from five athletes competing in the long jump. Each athlete had three jumps. The directions did not specify for the students to add the athletes' jumps together. They added the individual jumps together to come up with a final score for each athlete and then awarded gold, silver, and bronze for the three best athletes. In this document, references to data collected during this task will be listed as *Long Jump Task.* The Long Jump Task can be found in Appendix B.

Add mixed numbers task. The Add Mixed Numbers Task was designed to give students an opportunity to solidify and practice some of the concepts we had been exploring. The task was given nearly a week after the other two tasks previously discussed. The task gave the following equation: $4\frac{3}{4} + 5\frac{3}{4} = 9\frac{6}{4}$, and posed the following questions: "Is this the right solution to this problem? What would you do differently? How would you solve it? Would your answer be the same or different, and why?" The timing of this task was designed to give students an opportunity to solidify concepts that they had explored thus far into the unit. Students needed to be able add mixed numbers to solve this task. Students had developed the beginning of a conceptual understanding about how to convert from mixed numbers to improper fractions. This task was designed to give students an opportunity to solidify some of these skills, and to talk about them. In this document, references to data collected during this task will be listed as *Add Mixed Numbers Task.* The Add Mixed Numbers Task can be found in Appendix C.

Pizza task. The Pizza Task was given at the very end of the unit, a few days after the Add Mixed Numbers Task. It required students to add mixed numbers that were part of a large pizza order, to find out how many pizzas were ordered. At this point students had multiple strategies to convert back and forth between mixed numbers and improper fractions. They also had multiple strategies to add mixed numbers. This task gave students six different fractions they would need to convert and then add. This gave them the repetition they need in order to practice the procedure that they have spent the past two weeks developing the conceptual understanding for. The unit was designed to give these tasks at this point in the unit so that students had time to solidify their understanding at this point in the unit. In this document, references to data collected during this task will be listed as *Pizza Task.* The Pizza Task can be found in Appendix D.

Journaling. I kept a journal of my learning throughout this process. My journal contained the dates that assignments were given as well as my own decision making process as the unit progressed. As I returned to write the vignettes, I consulted the journal to keep track of my thoughts, ideas, and reasons for making decisions. My journaling took the form of emails to myself and notes during lessons.

Data Analysis

My first reading of the data occurred as the data initially came in. I was not reading the students' responses as a researcher, but as their teacher. I focused on identifying students whose discourse suggested they were struggling with the mathematics involved in the task. I did not pay specific attention to the a priori codes that I had previously chosen. After reading through the data a second time, some themes began to emerge. I highlighted five different copies of the data looking through each of the four tasks five times; once for every *a priori* code. During data analysis, I returned three more times considering each code to find specific examples.

After students discussed the mathematics online, I printed each threaded discussion five times, once for each of five traits Moss and Beatty (2006) described in fourth-grade conversations around similar math tasks. I analyzed the data using Moss and Beatty's five traits of quality mathematics discussion. Moss and Beatty showed that fourth-grade students demonstrated the following five traits: (a) using precise and formal language, (b) forming theories, (c) questioning one another's theories, (d) elaborating on thinking and comparing ideas, and (e) offering evidence and justification. I used these five traits as my *a priori* codes during data analysis. In addition to Moss and Beatty, NCTM standards suggest that students within this grade band should have the skills to meet the following four standards: (a) recognize reasoning and proof as fundamental aspects of mathematics, (b) make and investigate mathematical conjectures, (c) develop and evaluate mathematical arguments and proofs, and (d) select and use various types of reasoning and methods of proof (p. 188).

Each of these standards from NCTM support the traits described in the findings of Moss and Beatty (2006). I noticed how the traits that Moss and Beatty describe were directly tied to the NCTM Standards I was already teaching. The traits that Moss and Beatty found are evidences of the NCTM Standards in the classroom. I knew that if I could find the traits that Moss and Beatty described, then I would also be finding evidences of the NCTM Standards, and focusing on a relational understanding at the core of reform-based mathematics.

The traits described by Moss and Beatty are practical examples of the theoretical ideas outlined under NCTM. If my students showed examples of Moss and Beatty's traits, then they also demonstrated examples of these NCTM standards.

Use precise and formal language. I began my data analysis by coding for examples of students using precise and formal language. This code identified incidents where students describe and used language that matched the way a mathematician would write. I was looking for words that included but were not limited to: numerator, denominator, add, subtract, reduce, simplify, equivalent, whole, equal, improper fraction, and mixed number. I chose not to include number sentences or equations, or instances where students explained things using only numbers, because it was not always clear what they were trying to convey. I also sought out phrases where students used correct mathematic language even if it did not include the exact vocabulary listed above.

Form theories. In the next round of coding I looked for instances where students formed theories, I identified student statements where they asserted something. For example LW's assertion from the Add Mixed Numbers Task, “my answer would be different.” In this statement, LW is forming a theory without yet explaining his thinking.

Question one another’s theories. The nature of the conversations on the wikispaces website was organized such that students could post a level one comment, where students gave their answer or explain their thinking, as well as level two comments, which were replies to original comments on the wiki. Students left response posts that agreed with the original poster, disagreed with the original poster, asked the original poster a question, connected to the original poster, or they posted something that did not fit into one of these four categories. This category included posts where students questioned the assertion of the original poster. This did not always come in the form of a question.

Elaborate on the thinking of others. I coded examples of this trait when students responded with the intent to build on another’s comments, or when students used other students’ theories brought forward in class. When CD had an explanation and strategy show up in multiple students’ explanations, students specifically give credit to CD’s rule as a way that helped them solve the problem, posts were included in this category.

Offer evidence and justification. In this category, I coded words and phrases that explained why and how students solved problems. Through a few of the tasks students used the word *because* to begin their explanation or their evidence. I coded words and phrases that answered how a student solved a problem.

As I read through each assignment, I searched coded examples of Moss and Beatty’s traits. Statements representative of each of the *a priori* code are listed in Table 1. In the following section I will explain what I looked for as I coded for each code. In order to ensure trustworthiness, I gave the explanations of each code to another teacher and had this teacher code

14% of my total data set. Using examples such as those found in Table 1 and the other explanations of each code provided in the previous section, I compared the other teacher's coding to my own coding. We were able to achieve 83% inter-rater reliability.

Table 1

Student Samples of Each a Priori Code.

"A priori" Codes				
Forming Theories	Use Precise and Formal Language	Question one Another's Theories	Elaborate on the Thinking of Others	Offer Evidence and Justification
i think mary is first in [sic] second is freida and thed [sic] is malaika (MH, Rock Task)	It is wrong because you have a whole number and an improper fraction, which wrong because we talked about it at school, so that is like an improper mixed fraction number? How bizarre is that? (SW, Add Mixed Numbers Task,)	Actually, I do think a part of it is wrong. 1. When the numerator is bigger than the denominator, it's just called in improper fraction (KH, Rock Task).	So, if you use the CD rule and multiply the whole number by the denominator, then add the numerator, you should get the answer (SW, Unit 9, Add Mixed Numbers Task).	I think that billy is right because Al's line plot put chuck and billy in the wrong places (EC, Rock Task)

The unit of analysis for this analysis is an entry made by a student. Replies were also considered. If one comment or reply contained aspects of the trait I was searching for, I counted the entire comment or reply as being evidence of the trait.

Journaling. As I planned the unit I used for collecting data, I maintained a journal in the form of emails to myself. These emails contained a plan of what order the lessons were to go in, as well as how the unit was structured. I returned to these notes or journal as I pieced back together a timeline of events throughout the unit.

Trustworthiness

I made efforts to maintain an attention to trustworthiness during data analysis. By having another teacher triangulate and check my work, I was able to discuss my findings and analysis to ensure my analysis was grounded in the data. Another teacher checked my timelines and journaling to ensure that my explanation of the planning and implementation of the unit was firmly based on the data. While I was analyzing the data, I came across posts from students that did not fit into any of my *a priori* codes. I paid close attention to these examples by returning to them frequently to make sure they were not being overlooked. I was able to train another teacher according to the codes I had set up. We worked through coding together until we both came to a consensus about each code. We were able to come to a consensus without major disagreements. Through these repeated readings, the use of a codebook, triangulation with another teacher, and attending to negative examples, I have made efforts to maintain an attention to trustworthiness.

Chapter 4: Findings and Discussion

I wanted to find a balance between a flipped instruction model that combined the benefits of blended learning with the pedagogies of relational understanding and conceptual understanding, which are found in reform-based mathematics. In seeking the balance between these two ideas I found it necessary to describe what my classroom looks like as I engage in reform-based mathematics principles through a blended learning environment. I also found it helpful to the body of research behind these two ideas to describe how students engage in a blended learning environment that focuses on reform-based mathematics. In the following two sections, I will address how my findings describe reform-based mathematics instruction in a fourth-grade blended learning classroom as well as show how students use elements of reform-based mathematics while engaging in a blended learning classroom.

In the first section I will use three vignettes to describe reform-based mathematics instruction in my fourth-grade blended learning classroom. Each vignette details how specific tasks follow the framework outlined by Hendrickson, Hilton, and Bahr (2008). Each task was classified according to this framework as *develop*, *solidify*, or *practice.* The unit begins as students are developing their mathematical understanding through explorations. Then, students solidify the mathematics behind their explorations. Finally, students practice the strategies and algorithms they have discovered and discussed. The vignettes outline student discussions from class and draws heavily from text they submitted on the wikispace.

Vignette 1: Develop

I was starting a unit on adding mixed numbers. My students already had some experience with fractions, and we had briefly talked in passing about the idea of fractions with a value greater than one, but I did not expect the entire class to remember that earlier conversation, nor have internalized that concept. Before I started any in-class instruction, I assigned the Rock Task as a homework assignment designed to have students begin to think about mixed numbers, improper fractions, and the concept of fractions with a value larger than one. The following day we began the math lesson by discussing the previous night's homework. Between our discussion about the homework and our work that day, we came up with definitions for mixed numbers and improper fractions.

This unit was designed based on the reform-based mathematics idea that students will move from developing understanding to solidifying understanding and then practicing their understanding. The first few assignments were designed to have students develop their understanding. The Rock Task was designed to get students to begin to think about the relationship between mixed numbers and improper fractions. The Rock Task was assigned the night before the Cookie Task, which was assigned in class. Up to this point the majority of the work we had done with fractions had dealt with fractions with a value less than one. The idea of an improper fraction or mixed number had surfaced naturally and unintentionally in other contexts. Many students already knew about improper fractions and mixed numbers, at least that they existed and had names. Only one student (KH) used the term "improper fraction" in the writing about this task, so while the idea may have surfaced in previous lessons, they were not yet fluent with the term.

This homework task was followed the next day by a lesson further developing and beginning to solidify the terminologies and concepts dealing with improper fractions and mixed numbers. The day following the Rock Task, students performed a task called the Cookie Task. This task required the students to combine orders of cookies. The cookies could be ordered in fractions, and students had to problem solve the number of cookies that would need to be baked to solve the problem. Each order required students to combine fractions to a sum that was greater than one whole. In class, I had students share their thinking and talk about their process of problem solving. Students shared a few different strategies, and our conversation bounced between the cookie task, and how it related to the concepts that we had explored during the previous night's homework, the Rock Task. From our conversations two ideas began to emerge, and these ideas continued throughout the unit. The first idea was that fractions larger than one whole could be expressed with a numerator that was larger than a denominator. When this idea was brought forward, we gave it the proper mathematical term, *improper fraction.* The other idea was that numbers larger than one whole could be expressed with a whole number and a fraction. When this idea was brought forward, we gave it the proper mathematical term, *mixed number*.

Some of the conversations from the homework served as a launching point for the discussions on the day we engaged in the Cookie Task. The conversation from the Rock Task had multiple aspects because there was more than one correct answer. Because students had worked through questions on the Rock Task, some were ready to conceptualize mixed numbers the next day. For example, KH distinguished the difference between improper fractions and fractions that were not improper. He said, "Billy just wrote proper fractions. Chuck wrote improper fractions." I identified KH as someone that could articulate these two ideas, and so during the Cookie Task the following day, I chose to have that particular student lead the discussions as we developed a more firm understanding of the two ideas of *improper fractions* and *mixed numbers*.

The task allowed for students to take differing viewpoints and still be correct. Students could take the stance that Billy was correct. They could take the stance that Chuck was correct.

Lastly, they could also take the correct stance that both Billy and Chuck were correct. None of the three viewpoints was wrong, and you can use evidence to back up all three. Eighteen students took the stance that Billy was correct, and five took the stance that Billy and Chuck were both correct. Of the five that took the stance that both Billy and Chuck were correct, they stated that they still liked Billy's solution better. For example EG said, "Chuck and Billy's number lines both seem right. But I like Billy's better because he writes out the whole number instead of just fractions" (Rock Task).

Between the discussions in class and the discourse of the wikispace, it was obvious that students were deepening their understanding of mixed numbers and improper fractions at the beginning of this unit. The students' vocabulary had not yet become precise and formal, but their discussions showed that they were thinking and applying their new knowledge. An example of this was when KH, writing online referred to a mixed number as a "proper fraction" (Rock Task). In mathematics we do not use the term *proper fraction*. KH might not have known this, nor was that distinction particularly important to him at that point. Perhaps he used the term *proper fraction* because he knew what an improper fraction was, and he might have thought that *proper* is the opposite of *improper.* He might have concluded that if the opposite of *improper* is *proper,* then if a fraction is not an *improper fraction* it must be a *proper fraction.* This line of thinking shows that at this point in the unit students were beginning to apply their previous knowledge of fractions to a new and previously unknown concept. The conversations in class and on the wikispace showed that during this portion of the unit students were deepening their understanding of mixed numbers and improper fractions.

Vignette 2: Solidify

Between the two tasks from earlier in the unit, students now had begun to combine mixed numbers. One week after the Rock and Cookie tasks we began to more deeply explore combining fractions that were larger than one. The focus point of this instructional unit was to have student solidify their understanding of the mathematics involved in adding two mixed numbers. Students needed to be able to decompose a mixed number or improper fraction and combine the values. At this point students were using the terms *mixed number* and *improper fraction* correctly. Students were required to combine fractions within the already familiar context of cookies. On the day that Add Mixed Numbers was assigned, we returned to the Cookie Task from before and talked about the mathematics behind adding numbers like $4\frac{3}{4} + 3\frac{3}{4}$. During class that day two ideas emerged and surfaced again that night on the homework.

During class, TT argued that the easiest way to add these numbers would be to combine the whole numbers, and then the fractions. After TT had added the fractions, he saw that he would need to convert an improper fraction to a mixed number, and then add again (See Figure 3). The two strategies listed in Table 3 are strategies that were discussed openly in class. The class coined the strategy as *TT's Rule, and CD's Rule.* Taking ownership of a particular strategy is a common feature of reform-based mathematics. A student will solve a problem using logic, reasoning, and proof. After the problem is solved and information is shared with other students, labels such as *TT's Rule* are given by the students. When other students solve a problem similar to the way outlined by the first student to share, the other students connect their strategies, and use the first students' terminology.

As we worked towards solidifying the concept of combining mixed numbers, I allowed and encouraged students to label these strategies. The other strategy that immerged was coined as *CD's Rule*. CD had talked in earlier lessons about how to convert mixed numbers into improper fractions. He had had success with that previous algorithm, and understood the concept that you could rewrite every mixed number as an equivalent improper fraction. During the Cookie Task he and his group had been leaders in the class, showing how and why you multiply the whole number by the denominator, and then add the numerator in order to identify the numerator for the improper fraction. The label *CD's Rule* applied more to converting mixed numbers to improper fractions.

TT's Rule		CD's Rule	
$4\frac{3}{4}+3\frac{3}{4}$		$4\frac{3}{4}+3\frac{3}{4}$	
$4+3=7$	Add the whole numbers	$4\frac{3}{4}=\frac{19}{4}$	Convert mixed to improper
$\frac{3}{4}+\frac{3}{4}=\frac{6}{4}$	Add fractions	$3\frac{3}{4}=\frac{15}{4}$	Convert mixed to improper
$\frac{6}{4}=1\frac{2}{4}$	TT converts improper to mixed	$4\frac{3}{4}+3\frac{3}{4}=\frac{19}{4}+\frac{15}{4}$	Equivalent equations
$1\frac{2}{4}+7=8\frac{2}{4}$	Add mixed number to whole	$\frac{19}{4}+\frac{15}{4}=\frac{34}{4}$	Add improper fractions
$4\frac{3}{4}+3\frac{3}{4}=8\frac{2}{4}$	Final solution	$\frac{34}{4}=8\frac{2}{4}$	Convert improper to mixed
		$4\frac{3}{4}+3\frac{3}{4}=8\frac{2}{4}$	Final solution

Figure 3. Two students' rules for adding mixed numbers.

While TT's Rule applied directly to combining two mixed numbers, CD's Rule only applied to converting improper fractions to mixed numbers. It was members of the class that later applied CD's Rule to adding mixed numbers. This application did not occur in class, but occurred during homework as part of the Add Mixed Numbers Task. RK explained, "Let's use

CD's Rule," In addition to RK, seven other students cited CD's Rule as a way that they solved the same problem. (Add Mixed Numbers Task)

Another example of rich discussion in the Add Mixed Numbers Task occurred when VP had the following exchange online with ZM, CC, and KR.

VP: I would get the same number because I add the top numbers then thw [sic] whole numbers and it would be 9 6/4.

(Reply) ZM: You need to add the numerator

(Reply) CC: I disagree because whole numbers and improper fractions don't go together.

(Reply) KR: No it would be 10 2/4 or 10 1/2

This exchange highlights one of the functions of the online-mediated conversations. Three different students disagreed with VP, and gave guidance, help, and correction. Even though the conversation was riddled with misconceptions, this was a rich example of online discussions that I used as a launching point for discussion the following day. From this exchange I knew that ZM and CC thought that VP was wrong. However, both of them had their own misconceptions. ZM stated that she didn't add the numerator, but it is clear that VP did. CC incorrectly stated, "whole numbers and improper fractions don't go together." KR, on the other hand, got it, and I was able to invite him to explain the misconceptions that VP, ZM, and CC had the following day in class. (Add Mixed Numbers)

A prominent feature in reform-based mathematics is the idea that students share knowledge to build on their own experiences. Students built on CD's work, and borrowed TT's strategy to solve these problems. This evidences students' conceptual understanding taking place. The students were explaining how they solved the problem instead of simply answering the problem. In order to explain how they solved the problem however, they had to the have language to do so. They were naturally labeling their thinking to the strategies they have seen in class. Instead of having students solidify their understanding through the use of practice worksheets, the blended learning atmosphere allowed these students to continue the collaborative nature of the reform-based mathematics classroom during their online homework. As they worked to solidify their understanding they relied on the thinking of others, but this was only available during homework because of the blended learning nature of their homework. Blended learning and reform-based mathematics appeared to be working harmoniously and encouraging deep, rich, and meaningful conversations where students demonstrate their reasoning with proof. I later found that while blended learning facilitated these rich conversations as students were developing and solidifying their conceptual understanding, rich conversations were not always the norm.

Vignette 3: Practice

As previously noted, a classroom that follows a reform-based mathematics approach will move from the development of a concept, to solidify, and eventually challenge students to practice their understanding. At that point in the learning process students are required to demonstrate the procedural knowledge they have come to understand. My students had completed three homework tasks over two weeks and were becoming fluent in adding and subtracting mixed numbers. They had developed meaningful strategies through the tasks, and had solidified their understanding through evaluating the mistakes of others. As we moved away from a task structure in class to a lesson that gave students frequent opportunities to solve problems, these problems were less based in a context, and more frequently stood alone. Students were able to apply one of the two prominent strategies to problems. The final homework task of this unit was to see if the students could apply their learning to a new scenario. Students had added and subtracted mixed numbers and improper fractions, but they had yet to compare mixed numbers and improper fractions. The final online homework assignment, the Pizza Task, had them add mixed numbers, compare, and then subtract to describe how much larger one fraction was than another.

From my observations both in class and from conversations on the wiki, I perceived my class to generally understand the concepts within this unit. I expected student conversations to continue to contain the rich, meaningful discussions. During the Add Mixed Numbers Task when VP made a mistake, three students pointed out that VP had made a mistake, and came up with their own theories on how to fix the problem. While some of the theories presented by VP's peers still contained misconceptions, at least the conversation was started. During the Pizza Task, KH made an error similar to VP's error in the Add Mixed Numbers Task. The comment was posted at a similar time of day as VP's was during the Add Mixed Numbers Task. Both comments were neither first, nor last. Both comments presented incorrect solutions. The solution to the problem was that $23\frac{1}{4}$ pizzas were ordered. Most of the class was able to correctly problem solve and come up with this solution.

> KH: All together they ordered 91/4. 91÷4-22 R3/4. So all together they ate 22 ¾ of pizza. (Pizza Task)

KH gave an incorrect solution, and no one responded, corrected, or gave input. So why did they not respond? Why was it that earlier in the unit it was expected that students would give input and correction to incorrect answers, but then stay silent when a peer made a mistake at the end of the unit? It seems that students had shifted their focus from the conceptual to the procedural. Perhaps students no longer spent time to read their peers' work because they were too focused on how they were going to answer the question. The examples of rich conversation when someone made a mistake during the Add Mixed Numbers Task were virtually non-existent during the final Pizza Task.

Through this unit I had seen theories brought forward by the class, students building on the thinking of others, questioning one another, and students using formalized language. As they reached the end of the unit, I expected the conversation to be at least as rich as they were in the beginning. The conversation the students engaged in during the Pizza Task did not contain the amount of comments that contained theories, questions, and connections that I had expected. Students came to the wiki, read the problem, solved the problem, and left their answer. Their answers were concise, and to the point, and generated almost no replies. This was contrary to what I expected.

At this point students were fluent in the algorithm. Students knew how to solve the problem, and understood the concept, but also the procedure. When everyone understands the procedure, their need to defend their own procedure, question each other's procedure, or build on their explanation of others is diminished. The only time a student questioned someone else's work was when one of the two got an incorrect answer. For example, VP did not regroup his final answer, and explained to CC that while CC's answer looked different, their two answers had the same value. He said, "I got your first answer 20 13/4 but I didn't do the whole regrouping thing so I ended up with 20 13/4" (Add Mixed Numbers Task). VP was not discussing the difference between two varying strategies. In fact, VP was not questioning CC's strategy at all. VP was pointing out the flaw in his own strategy, and how VP was one step behind CC. This discussion was one of only two online conversations that occurred for this homework. Everyone else in the class posted their answer and went on their way.

So why did students not engage in conversation at the end of the unit when their understanding of the concept was arguably deeper than it had been at any other point? Perhaps students did not feel a need to defend a strategy that had already been proven valid for two weeks. Some may have forgotten a step. Others may have solved the problem incorrectly. Due to the fact that the vast majority of the students understood the procedure on the conceptual level, there was little need to discuss it further. The blended learning aspect told me much less about my students as they were practicing this concept than it did when they were developing their understanding and solidifying those concepts.

A blended learning classroom can still uphold the values of reform-based mathematics. The two ideas can, with planning, coexist. The challenge lies within the design of the activities that students engage in outside of class. When blended learning involves students learning procedural understanding prior to the development and solidifying of conceptual understanding, there is conflict. A blended learning classroom planned based on a reform-based mathematics approach contains opportunities for students to explore learning outside of class, and in a collaborative experience, share their learning with their peers.

Elements of Reform-Based Mathematics

In the following section I will outline how this particular type of blended learning enabled students to demonstrate elements of successful reform-based mathematics while participating in online math discussions. I will outline each of the elements of quality reform-based mathematics instruction that Moss and Beatty (2006) described. They asserted that in such settings students: (a) formed theories, (b) offered evidence and justification, (c) questioned one another's theories, (d) elaborated on thinking and comparing ideas, and (e) Used precise and formal language. I will show how each a priori category was found in student writing, and the conditions under which each category was found. The timing of when each trait surfaces helps demonstrate the process students go through as they learned these mathematical ideas. These traits demonstrated that in order for students to accomplish the tasks that I had prepared, students engaged in higher-level thinking. Over time the traits manifested differently for different tasks.

Table 2 shows how frequently evidences of the five discussion traits (Beatty & Moss, 2006) appeared in the student data. By the end of the homework tasks during this unit, most of the five traits became less common. This was contrary to what I anticipated. If by the end of the unit student understanding improved, I wondered why evidence supporting students' theories, and evidence of students questioning one another's theories decrease?

Table 2

Number of Student Comments Evidencing Traits in Each Task

Traits	Tasks			
	Rock Task	Long Jump Task	Mixed Numbers	Pizza Task
Form Theories	27	30	14	8
Offer evidence and justification	31	2	14	8
Question one another's theories	5	16	9	0
Elaborate on thinking of others	6	4	12	2
Use precise and formal language	15	0	26	21

These data provide evidence of reform-base mathematics learning at its core. Students were engaging in conversations where they developed and solidified theories towards the beginning of the unit. As time went on however, the amount of theorizing and questioning diminished. The underlying idea behind the reform-based mathematics movement is that student learning is deepened while students are developing and solidifying concepts. The process to develop conceptual understanding or relational understanding is the meaningful learning that is most important for students. The first three tasks were designed to help students develop and solidify their learning, and these were the tasks where students most frequently demonstrated the five traits that were important indicators of deep learning. By the time the students got to the final task, they were so focused on the procedural understanding that only a few of the students showed evidence of four of the five traits. The one trait most of the students did demonstrate was the use of precise and formal language. In the following section, I will show that each trait demonstrates a different portion of the students' progression in problem-solving new mathematics concepts.

Form theories. Students formed theories as they were developing the understanding of the concepts. Students used the term "I think" 24 times during the Rock Task. Many times students led off with that phrase. As time went on, as they began to solidify their understanding in mixed numbers and improper fractions, they became more assertive in their language. By the end of the unit, during the Pizza Task, the phrase "I think" appeared only five times. Instead of leading off with "I think," students use language such as "First I added…" or "Well what I did was…" (Pizza Task). The language the students used is evidence of the progression of their thinking.

As the students approached a previously unknown concept, they had to problem-solve using their previous understanding to come up with a reasonable solution. They describe their understanding timidly, or begin with the phrase "I think." They are guarded about their idea, and they wanted credit so long as their solution was correct. As time went on, their use of the phrase “I think” dropped. It dropped perhaps because they were no longer theorizing about how these concepts worked. They had developed beyond theorizing, and were ready to just practice the procedure.

Offer evidence and justification for student thinking. In three of the four tasks included in this study, students consistently used evidence to support their thinking. Examples of students justifying their thinking would often follow the word *because*. In KL’s statement during the Rock Task, he supported his claim that Billy is the only correct answer. KL said, “I think that Billy plotted his line plot correctly because the denominator is bigger than the numerator like it is supposed to be” (Rock Task). KL both formed a theory and provided evidence for the students’ thinking. His theory was that Billy plotted the rock correctly. He supports that theory with the misconception that a numerator is always supposed to be smaller than the denominator. Even though the fictional student from the homework task, Chuck, had a correct answer, KL explained why Chuck’s answer violates KL’s justification, and therefore does not match his theory. KL says, “Chuck's line plot on the other hand is wrong because the numerators start to get bigger than the denominators which isn't supposed to happen” (Rock Task).

KL uses the word “because” to denote that the justification for his theory was developing. He felt the need to justify his thinking, and not just state it as fact. It seems that when students had moved to a procedural understanding they no longer used the word “because,” they just answer the question. KL was not alone in using the word “because” during this task. In fact, throughout the unit many students used the word “because” to preface evidence and justification for their thinking (See Table 3). The word “because” was prevalent in the Rock Task, and present in the Add Mixed Numbers Task, but effectively absent in the Long Jump and Pizza Tasks. It is notable that evidence or justification for student thinking was nearly absent during the Long Jump task, with students using the word "because" once during the Long Jump Task.

Table 3 shows the number of times the phrases "I think" and the word because" shows up in each task. Many students used the words "I think" and "because" fewer times across time, which suggests that their conceptual understanding became solidified and moved towards procedural. Students did not need to justify their answer during the final task, because they had already formed those theories, Further, students did not use the word "because" and they did not spend much time having two-sided conversations during the Pizza Task either. Late in the unit they were ready to move from theory to fact. During the Long Jump Task instances of the word "because" dropped considerably. This may have had to do with the task, or the students' understanding of the mathematics involved.

Table 3

Frequency of Use of "I think," and "Because."

Indicator Texts	Tasks			
	Rock Task	Long Jump Task	Mixed Numbers	Pizza Task
Instances of the phrase "I think"	24	11	11	5
Instances of the word "because"	40	1	30	5

Question one another's theories. The two tasks focused on solidifying student understanding were the second and third tasks in the sequence of tasks: the Long Jump Task and the Add Mixed Numbers Task. Because these tasks were focused on solidifying student understanding, students naturally moved towards both questioning one another's theories and elaborating on the thinking of others during these tasks. As students questioned one another's theories, they found their flawed logic, and discovered which theories held true. This is important for students to discover so that they know which theories will eventually help them repeatedly, quickly, and consistently find the correct answers. During the Rock Task, SW and KH had the following exchange online:

SW: I think it is Billy's. I think this because he has the fractions going 1/4, 2/4, 3/4, 4/4. Then he goes on with 1 1/4, 1 2/4, 1 3/4, 1 4/4. Al's is wrong because his answers say that rock A was smallest, but instead he put rock C as smallest because he ignored the 1 in 1 1/4. Billy's has them correctly. Chuck's is wrong because from 4/4, he kept on going to 5/4, 6/4, 7/4, and 8/4. That is incorrect, so I think the answer is Billy's.

(Reply) KH: Actually, I do think a part of it is wrong. 1. When the numerator is bigger than the denominator, it`s just called in improper fraction.

(Reply) SW: Well, true, but still, um, well, ttrruuee, thanks for the point, but I still think Billy's is right.

(Reply) KH: It is, but Chuck`s is right too.

In this exchange, KH respectfully pointed out to SW that he was wrong, or at least only partially correct. SW was confident that he was not wrong, but he can saw the bit of truth in what KH has said. SW knew a little about improper fractions, and revealed the wheels spinning in his head. "Well, true, but still, um, well, ttrruuee…" He separated each word with a comma, signaling he was pausing and thinking about KH's questioning of his thinking. It is evident in the way he types the word "ttrruuee…" He thanked the poster for his comment, and then returned to his original theory that only Billy was correct. KH jumped in with “It is, but Chuck’s is right too.”

In order to decide which strategies have merit or value, students needed to sort out the strategies that could not help them find the correct answers repeatedly, quickly, and consistently. They needed to sift through the many examples from their peers to find the strategies and statements that had value. My job was to help direct the students to those strategies and statements that would be most helpful for them. Between their conversations and my guidance students were able to identify two specific examples that helped them add mixed numbers and improper fractions. These two strategies were referred to as *CD’s Rule,* and *TT’s Rule.* Before they accepted these two strategies as valid strategies, students questioned theses to strategies during class and eventually began to use them to solve problems. In the following section I will further explain these two strategies and how students elaborated on them.

Elaborate on the thinking of others. While students did not elaborate on the thinking of others often on the website, there were two examples that many students referred to frequently, which have already been discussed, that is, CD’s method to convert a mixed number into an improper fraction, known by the students as *CD’s Rule,* and TT’s strategy to add mixed numbers, known as *TT’s Rule*. The term *rule* comes from the label described by the class. Although the students typically referred to the ideas as methods and strategies, occasionally a student would describe an idea as a rule. CD explained the conceptual understanding behind converting a mixed number to an improper fraction. The strategy he explained was the standard algorithm. He explained how to multiply the whole number by the denominator, and then add the numerator in order to identify the numerator for an improper fraction. He drew, modeled, and explained the algorithm to the class. Even though the algorithm was a traditional algorithm, the class assigned ownership of the idea and began to call it CD's Rule. When students began to use CD's Rule not just for converting mixed numbers into improper fractions, but as a step on their way to add mixed numbers, the title *CD's Rule* transferred to the new application (See Figure 3).

Elaborating on the thinking of others came at a time when students had found theories that had merit, have been shown to work, and made sense to the students. The two strategies discussed previously were not the only two methods or strategies that were presented. As the class came to their own conclusions about how to add mixed numbers, they attached their own strategies to one of these two strategies. By naming the strategies the students now had come up with a procedure for solving problems, moving from the conceptual to the procedural. Students explained how they solved a problem by starting with "Let's use CD's rule" (Add Mixed Numbers Task). They began to describe the procedure that they are using to solve the problem. They already understood the concept behind the procedure, and were solidifying that conceptual understanding as they explored the procedure.

Use precise and formal language. As students started this unit they grasped for anything they already knew. One example of this is when the students insisted that having a numerator larger than a denominator was an incorrect way of writing a fraction. As they formed theories during the Rock Task, a *develop* task, they relied on their vocabulary from previous learning experiences to orient themselves in a new experience dealing with numbers greater than one, but less than two. They correctly used words like "numerator" and "denominator" (KH, Rock Task). As they developed, questioned, and elaborated on those theories, however, the amount of formal language peaked during the Add Mixed numbers task, a *solidify* task. The amount of formal language slightly decreased during the Pizza Task, a *practice* task. In contrast, during the Long Jump Task, for example, there were no instances where a student used precise and formal language. Instead, they stayed within the context of the task. In the Long Jump Task, students were to decide which long jumper got first place. MB said, "Mary is in 1st place, her total score was 25 1/12ft." (Long Jump Task). MB did not explain how he came to the solution, nor did he use sophisticated formal language. The rest of the responses during this task are similar. They lack the formal language found in other tasks.

The lack of formal language in this specific task may be evidence of the impact of the type of task having on the type of language students used. As previously discussed, they also did not justify their theories during this task. In other tasks students used more precise and formal language to justify their thinking, but the formal language surfaced as they tried to explain their thinking. In the Long Jump task they gave answers, but no justification. As they solidified their understanding and the tasks moved to *practicing*, such as in the Add Mixed Numbers Task and Pizza Task, precise and formal language increased.

Summary of Findings

Technology-mediated online math discussions gave me an insight into how progression in problem solving within a single math concept was revealed in the discourse of the students. In other words, analysis revealed the process as my students went through as they learned how to add and subtract mixed numbers. Within a blended learning model, I wanted to formatively assess students on their understanding of the concepts we were studying. I wanted to give students an inquiry experience before entering the classroom so they had some problem solving experience with the contexts and concepts we would be studying. Such inquiry experiences are critical in reform-based mathematics.

I was able to explore how a blended learning model can help demonstrate how students describe their problem solving in math. Blended learning appeared to support a relational understanding approach to learning when it rooted in principles that are in harmony with the principles of reform-based mathematics. The online and text-based aspect of these conversations revealed student thinking and allowed me to respond directly to students that needed help. I had a good idea of the conversations going on in the home. I had a small window into how homework was being done, and the collaboration occurring. I got to see what I had never seen before, which is homework being done as it was being completed.

Chapter 5: Conclusion

As I started this research I was faced with two ideas. These two ideas did not seem to contradict each other at first, but it soon became obvious that both ideas could not be sustainable simultaneously. The first came from my beliefs about the way mathematics instruction should happen. I believed that students should discover their learning through their experiences, and social interactions. I wanted to build a classroom that fostered student engagement in math tasks that helped students conceptualize mathematics prior to focusing on the procedure. The other idea was that by sending meaningful homework home, through a typical flipped instruction model I could monitor, instruct, and assess student learning outside of my classroom. It did not take me long to realize that the two ideas could not exist at the same time without alterations to my approach.

I started to create a blended learning classroom by sending home videos that gave instruction to students prior to learning the material in class. This contradicted directly with my mathematics pedagogy. Students engaged in procedural learning but did not fully understand the mathematics; they just understood a procedure. Certainly, my students learned how to do the multiplication procedure, but they were no longer interested in the conceptual understanding. Once they procedurally solved the problem they were no longer interested in revisiting problems to create the conceptual understanding behind the procedure that they had partially mastered. Thus, full mastery was never reached. I wanted my students to do more than what a five-function calculator can do. I wanted my students to reason. I wanted my students to form theories, and justify their answers with evidence.

I also valued meaningful mathematics homework. So I decided to try something different than the usual math homework. I decided to design tasks based on how I believed math should be taught, but I designed them to take place at home, and for student discussion to occur online. Because discussion plays a large part in my philosophy regarding mathematics instruction, I naturally wanted to put it at the center of my instruction. Through discussion captured online through the wikispaces website the process of their understanding mathematics were made visible. Students came to the website I had created to share their thinking, commented on each other's thoughts, formed theories, questioned one another's answers, and used formalized language that demonstrated their learning. Through adjusting the kinds of tasks I assigned in a blended learning environment, I created a place that bridged the gap between the pedagogically sound principles of reform-based mathematics, and the innovation and power of a blended learning model where learning was not confined to my classroom.

Analysis of the data revealed that these two ideas could be implemented so long as the particular blended learning model implemented prompted students to engage in various types of problem solving. I also realized how blended learning could be used to document a student's growth throughout a unit. This study showed that student understanding moved from discovering concepts and strategies, to practicing strategies that have been proven by peers.

Through this study, I have observed aspects of technology-mediated discussions that embody elements of a successful reform-based mathematics. According to Hendrickson, and associates (2008), effective implementation of reform-based mathematics will contain a social aspect. Students must be able to share their thinking, and discuss their findings with other students. These social situations help students build their knowledge. NCTM (2000) encourages students to develop theories and support or prove those claims with evidence. Threaded conversations in a technology-mediated discussion provided opportunities for students to share their learning with each other. In the first online task of this unit, there were 60 comments, and

32 students logged into the website 102 times (Rock Task). That means students returned to the website to share, read, and engage in an online community. Technology-mediated discussions provided students the venue to share their thinking and read their peers' work.

Another key component to reform-based mathematics instruction is collecting data on students as they work towards solving problems (Smith & Stein, 2011). While students solve complex problems in class, I can monitor their work and make anecdotal notes as they work. In the past I have not had the ability to assess their work before the following math lesson. Through the use of the wiki, I was able to identify misconceptions as they arose, and respond accordingly. Technology-mediated online discussions supported my ability to assess student learning, and helped me discover the elements of reform-based mathematics that they were engaging in through their discussions.

This study shows that when a blended learning classroom adopts a reform-based mathematics approach, students engage in mathematics discussions online to explore mathematics. Students form theories, question one another's theories, elaborate on the thinking of others, and use formal language. Students use the conversations from the classroom and apply the strategies and statements made by their peers to solve problems at home. The features of technology that a blended learning environment provide give the teacher unprecedented access to evidence of student thinking. Technology can give teachers the opportunity to quickly collect and assess statements made by students in order to more fully respond to the needs of students.

Moss and Beatty (2006) found that fourth graders were able to work online in a collaborative environment. They found that students formed theories, questioned one another's theories, elaborated on the thinking of others, offered evidence and justification, and used precise and formal language. Similarly, this study found that fourth graders who interacted face-to-face as well as online were also able to demonstrate those traits. Beatty and Geiger (2010) called for research that demonstrates the value of technology resources and tools for developing students' math skills. This study did not set out to show that this particular program developed students' math skills; however, the study definitely demonstrated that student progression was occurring. Beatty and Geiger discuss the difference between a procedural tool, and a collaborative tool. My study describes the blended learning program I implemented as a collaborative tool that gave students an outlet to demonstrate their math skills.

When there is a focus on students who are forming theories and supporting their theories with evidence, students have naturally shown their reasoning, and supported their evidence with proof. Yackel and Hanna (2003) discussed the importance of reasoning and proof in math instruction. By designing instruction focused on reasoning and proof, and moved away from a narrow focus on procedure, I was able to implement a blended learning program that supported the inquiry nature that is key to reform-based mathematics (Hendrickson, et al., 2008). This project gave the benefits of students receiving help at home (Bergman, 2012), while maintaining the focus on conceptual understanding on which reform-based mathematics is founded (e.g., Hendrickson, et al., 2008; Cobb et al., 1992). The collaborative nature at home also allowed me to give students a social experience where they learn from each other (e.g., Bauersfeld, 1980; Krummheuer, 1995; Cobb, 1994; Saxe, 1991). At first look, finding a balance between blended learning and reform-based mathematics seemed impossible. It is now clear that a balance between the two is possible.

Implications

Teachers often look at the end of the unit as a time to assess student learning. The end of the unit is a time for summative assessments, unit tests, and quizzes that identify whether student has learned a subject or not. Using blended instruction tools, I was able to explore how students can form theories, justify their answers with evidence, and use formalized language. I also came to realize that the end of the unit might not be the best time to look for evidence of those traits.

Prior to this study, the end of my units of instruction students were so focused on the procedures involved in solving the problems, and the answers that they came up with, that they were not apt to justify their answers with evidence, and they appeared to work from the procedure given to them rather than act on the theories they had formed. The time students typically form theories is towards the beginning of their learning. Thus, teachers need to look beyond the end of unit test for evidence of student learning, and capture traces of learning while it is actually occurring.

Limitations

The generalizability of this study was limited by its scope. Because this study was only one classroom, it is difficult to say the results are applicable to any fourth-grade class. While the findings may be accurate for this particular class, it may not be generalized to other classes. This study was limited to only one unit of instruction in one curricular subject and to one instructor. If other teachers teaching another unit could replicate the findings this study remains to be seen.

Future Research

Teachers in a blended learning classroom can adopt reform-based mathematics instruction. Through blended learning opportunities, students can engage in meaningful mathematics discussion. The technology components of such a system can give an instructor insight into student learning and thinking. Further research is needed to further understand the phenomenon that occurred during this unit of instruction. For example further work might explore why students appear no longer engage in building theories once their understanding has moved from conceptual to procedural processes, new studies are needed.

In this study some students posted more than others. Five of my students posted considerably fewer times, and five students posted considerably more when compared to the rest of the class. Students posted an average of eight total posts during the unit. It may be interesting to further study those students that posted one standard deviation more than the mean of the class. What was the value of their comments? Was there quality to their comments, or just a high quantity? It would also be interesting to look at the five students that posted one standard deviation below the average of the class. What was the quality of their posts? What were the contextual factors that led to their absence on the wiki? In this study, it appeared that students who followed certain problem-solving strategies may have been influenced by some students. To definitively identify whether their pattern of problem solving continues from one unit to the next and in other disciplines, more data are needed. Further research is needed to document these students' situation and what impacted their participation. It would also be interesting to look at posts over an entire year, and if one particular unit is an accurate measure for the amount a student comments.

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Appendix A
Rock Task

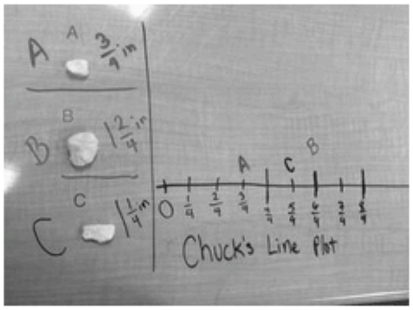
☆ Unit 9 Rock Task

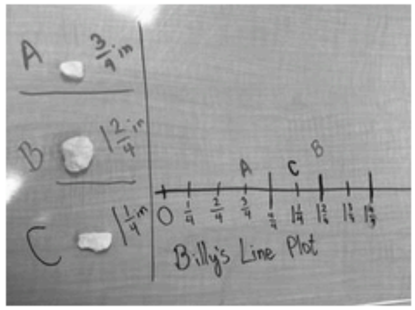
Edit | 60 | 1 | ...

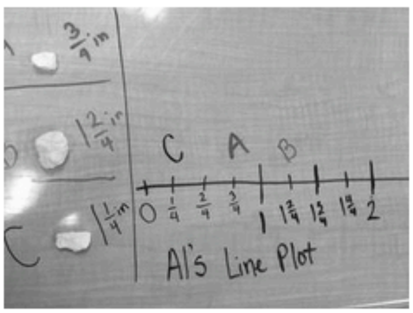
A line plot is a way to show fractions on a number line.

Students measured some rocks, and then plotted their length on a line plot. Here we have three students' line plot.

1. Which student do you think plotted their measurement correctly?
2. What evidence do you have to support your claim?







Appendix B
Long Jump Task

Long Jump Task

Here is some data from a long jump competition.

Name	1st Jump	2nd Jump	3rd Jump	Total Score
Kim	$7\frac{3}{12}$ feet	$6\frac{11}{12}$ feet	$6\frac{5}{12}$ feet	
Amanda	$5\frac{9}{12}$ feet	$6\frac{1}{12}$ feet	$6\frac{4}{12}$ feet	
Malaika	$7\frac{9}{12}$ feet	$6\frac{2}{12}$ feet	$7\frac{11}{12}$ feet	
Mary	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	
Freida	$7\frac{10}{12}$ feet	$7\frac{10}{12}$ feet	$8\frac{2}{12}$ feet	

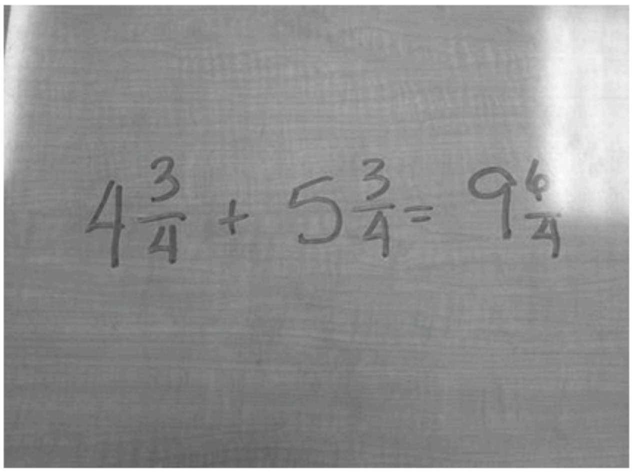
Who won Bronze (3rd place), Silver (2nd place), and Gold (1st place)?

By how much?

Appendix C
Add Mixed Numbers Task

☆ Unit 9 Add Mixed Numbers Task

Edit | 113 | 1 | ...



Is this the right solution to this problem? What would you do differently? How would you solve it? Would your answer be the same or different, and why?

Appendix D
Pizza Task

Pizza Task

Order	Cheese	Ham	Pepperoni	Hawaiian
Mr. Young	$2\frac{1}{4}$	$1\ \frac{2}{4}$	$4\ \frac{3}{4}$	
Mr. Beckstrand	$6|\ \frac{3}{4}$	$5\frac{2}{4}$		$2\ \frac{2}{4}$

Ignore the line after that six… I don't know what its doing there.

Here is a pizza order for Mr. Becklestrund and Mr. Young.

How much total pizza was ordered between the two of them?

What did you have to do before you could solve this problem?