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OPTIMIZED HYBRID FUZZY FED PID CONTROL OF NONLINEAR SYSTEMS

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This Thesis is Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Electrical Engineering

Islamic University of Gaza, Palestine

1428هـ – 2007 م

To my parents who have been
a constant source of motivation, and support.
To my fiancée.

ACKNOWLEDGMENTS

First and foremost, I would like to thank my lovely university and my professors for their support, outstanding guidance and encouragement through out my study.

I would like to express my gratitude and appreciation to Dr. Basil Hamed for all the help and guidance he provided throughout my graduate program, and to the other my committee, Dr. Hatem El Aydi and Dr. Mahir Sabra.

I would like to thank my family, especially my parents, for their encouragement, patience, and assistance over all years. I am forever indebted to my parents, who have always kept me in their prayers.

Abstract

The design of controllers for nonlinear systems in industry is a complex and difficult task. The development of nonlinear control techniques has been approached in many different ways with varied results. One approach which has shown promise for solving nonlinear control problems is the use of fuzzy logic control. This thesis proposes a new method utilizing proportional–integral-derivative (PID) control as a hybrid fuzzy PID controller for nonlinear system. The salient feature of the proposed approach is that it combines the fuzzy gain scheduling method and a fuzzy Fed PID controller to solve the nonlinear control problem. The resultant fuzzy rule base of the proposed controller contains one part for a non optimized controller. This single part of the rules uses the Takagi-Sugeno method for solving the nonlinear problem and compares it to the mamdani method. The number of fuzzy rules are minimized using a method of series reduction fuzzy rule base. The simulation results of a nonlinear system show that the performance of a Fed PID Hybrid Takagi-Sugeno fuzzy controller is better than that of the conventional fuzzy PID controller or Hybrid Mamdani fuzzy Fed PID controller, especially using the reduction of the number of fired rules.

ملخص البحث

تصميم أنظمة التحكم للأنظمة غير الخطية في الصناعة يعتبر مهمة معقدة. أحد الطرق التي أظهرت حلول لمشاكل الأنظمة غير الخطية هي استخدام التحكم المنطقي الضبابي. هذه الرسالة تقترح طريقة جديدة لاستخدام التحكم التناسبي- التكاملي- التفاضلي (بي أي دي) وهي أنظمة التحكم الضبابية الهجينة (بي أي دي) للأنظمة غير الخطية. السمة البارزة للنهج المقترح هو انه يجمع بين طريقة الكسب المجدولة الضبابية وبين التحكم المغذى (بي أي دي) لحل مشكلة التحكم غير الخطية. القواعد الناتجة للمتحكم المقترح تحتوي على قسم غير مثالي. الجزء الأحادي من القواعد يستخدم طريقة التكاجي- سوجينو لحل المشكلة غير الخطية ويقارنها مع طريقة المامداني. لقد تم تقليل عدد القواعد الضبابية باستخدام طريقة القواعد الضبابية التسلسلية. النتائج المأخوذة على جهاز الحاسوب توضح أن كفاءة الطريقة الجديدة وهي طريقة التحكم الضبابي المهجن بي أي دي المغذى بطريقة التكاجي – سوجينو تعطي أفضل نتائج من أي طريقة تقليدية خصوصا إذا استخدمت طريقة المامداني، وبالتالي ستظهر أفضل نتائج مع تقليل عدد القواعد الضبابية.

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CHAPTER 1
Introduction

1.1 Introduction:

Fuzzy control is considered as one of the most important sciences in our industrial revolution nowadays. The progress in the automation fields makes fast steps by using robotics and controllable machines.

A scientific research in the area of Hybrid Fuzzy Fed PID Control will be proposed to increase the knowledge in this field, so the best efforts will be held to collect data from books and internet to support this research.

PID control is widely used in industrial applications because of its simplicity. Stability of PID controller can be guaranteed theoretically, and zero steady-state tracking error can be achieved for linear plant in steady-state phase. Computer simulations of PID control algorithm have revealed that the tracking error is quite often oscillatory, however, with large amplitudes during the transient phase. To improve the performance of the PID controllers, several strategies have been proposed, such as adaptive and supervising techniques.

Fuzzy control methodology is considered as an effective method to deal with disturbances and uncertainties in terms of ignorance and ambiguity. Fuzzy PID controller combining fuzzy technology with traditional PID control algorithm has become the most effective domain in artificial intelligence control [1],[2].

The most common problem resulted early depending on the complexity of Fuzzy Logic Control (FLC) is the tuning problem. It is hard to design and tune FLCs manually for most machine problems especially nonlinear industrial systems. For alleviation of difficulties in constructing the fuzzy rule base, there is the conventional nonlinear design method which was inherited in the fuzzy control area, such as fuzzy sliding mode control, fuzzy gain scheduling [3],[4], and adaptive fuzzy control [5],[6]. The error signal for most control systems is available to the controller if the reference input is continuous. The analytical calculations present two-inputs for FLC to employ which are proportional error signal and velocity error signal. PID controller is the most common controller used in industries, most of development of fuzzy controllers revolve around fuzzy PID controllers to insure the existence of conventional controllers in the overall control structure, simply called Hybrid Fuzzy Controllers [7],[8].

The key idea of the proposed method is as follows: First, the fuzzy gain scheduling method is applied to linearize the nonlinear system at *frozen times*. A fuzzy Fed PID controller is designed for each frozen system by replacing the conventional PID controller by an incremental FLC, the integral part of the PID controller is fed by a differentiated feedback gain, this Fed PID controller is the new method used in this thesis and it gives the best results anyway. Second, fuzzification of the reference input is performed for the system, while the control space of error signals is linearly partitioned after normalization. Third, the fuzzy rule base is constructed in a recursive way to obtain better nonlinear control as well as to guarantee closed-loop stability of the frozen system.

It should be emphasized that because the proposed approach utilizes some modern control theorems, tuning the hybrid fuzzy controller is much easier than tuning a conventional fuzzy logic controller.

The gain scheduling method is introduced as an effective nonlinear control method for nonlinear systems. Finally a novel fuzzy Fed PID controller is proposed. We show that recursive design of the fuzzy rule base can guarantee stability of local closed-loop systems. Then, control of a pole-balancing robot illustrates how the proposed design method can be easily applied to a nonlinear robotics system.

1.2 Motivation:

The main stimulus of choosing this thesis is nowadays progress in automation fields that use robotics and controllable machines which deal with nonlinear science. A huge knowledge in control research, intelligent control, mechanical and electrical engineering would be achieved. It is interesting science to be applied and developed where it is proposed for the industries in Gaza to apply the new technique in their working machines. The Hybrid Fuzzy Fed PID Control gives us more experience in control systems, Intelligent systems and Mechatronics fields that help us to understand the concepts of systems stability, designing digital and analog controllers, and controlling industrial instruments.

The main point which makes the decision of selecting this topic is that the subject of hybrid fuzzy control is very hot topic, the experience in this field is weak, at least in the Gaza Strip. So the work in the field of Hybrid Fuzzy Control of Mechatronics Systems will increase the knowledge of intelligent systems and develop the control systems which used in factories in Gaza.

1.3 Literature Review:

Fuzzy logic has been around since 1965, when L.A. Zadeh laid the foundation of the linguistic model [9]. Fuzzy sets theory provides a systematic frame work for dealing with different types of uncertainty with a single conceptual framework.

The work of Mamdani and Asilian in 1975 showed the first practical application of fuzzy control that implemented Zadeh's fuzzy sets theory [10]. The other method in use is the Sugeno model, which is a nonlinear model consisting of a number of rule-based linear models and membership functions which determine the degrees of confidence of the rules.

In 1993, Zhen-Yu Zhao, Masayoshi Tomizuka, and Satoru Isaka described the development of a fuzzy gain scheduling scheme of PID controllers for control process [4].

A simple technique to design a generalized Sugeno-type controller (GSC) is proposed by Ch. Clifton, A. Homaifar and M. Bikdash in 1996 [11]. A hybrid fuzzy-PID controller is approximated using recursive least squares by a Sugeno-type controller.

In 1998, Wei Li presented approaches to the design of a hybrid fuzzy logic proportional plus conventional integral derivative (fuzzy P+ID) controller in an incremental form [7]. The controller is constructed by using an incremental fuzzy logic controller in place of the proportional term in a conventional PID controller.

Meng Joo Er, and Ya Lei Sun presented a new approach toward optimal design of a hybrid PID controller in 2001 which is applicable for controlling linear as well as nonlinear systems using genetic algorithms [8]. This method is difficult in math and there are clear ripples in the step response, also the overshoot is found.

In 2004, Ya Lei Sun and Meng Joo Er showed a new approach towards optimal design of a hybrid fuzzy controller for robotics systems [12]. The feature of their proposed approach combines the fuzzy gain scheduling method and a fuzzy PID controller to solve the nonlinear control problem, but the minimum values of the overshoot and the steady state error are not satisfied compared with other controllers.

1.4 Contribution:

The focus of this thesis is to establish design techniques for hybrid fuzzy PID controllers for nonlinear systems. This thesis therefore represents a new contribution approach for PID controller to the development of fuzzy PID control system methodology by describing the details, design, and a comparison between the two most widely used fuzzy controllers (The Mamdani and Sugeno Models) in fuzzy control. It also presents an example of design fuzzy controllers for nonlinear control problem (The Inverted Pendulum) which can be characterized by two or four variables, that is, two or four dimensional systems. The simulation of the inverted pendulum is applied using Simulink MATLAB program which shows the step response of each model. The important step which will give a new method is the technique which combines the PID controller, the Fuzzy Logic, and the gain scheduling method.

1.5 Outline of The Thesis:

This thesis contains six chapters, the first one talk about introduction and motivation of the project. The second chapter presents the PID controller definition, tasks, types and tuning. Third chapter is Fuzzy control history, fuzzy control design, and implementation. Chapter four is nonlinear systems, phenomenon, common nonlinearities and nonlinear control problem design. Fifth chapter is the core one which is the hybrid fuzzy control problem design, Fed PID control and Takagi-Sugeno Fed PID control of the hybrid fuzzy control design, comparison and results, this chapter contains the salient feature of thesis name which presents the new technique. The final chapter is a conclusion chapter.

CHAPTER 2

PID Controller

Most industries use variety control systems to perform various tasks. The PID controller is the most widely used strategy in those industries, around 95%, so it is used for various control problems such as automated systems and plants [13]. The PID controller consists of three basic elements or terms, Proportional, Integral, and Derivative controller.

The PID controller is implemented to meet various design specifications for the system. That specifications can include the rising and settling time, overshoot, the accuracy of the system step response and the steady state error. These specifications will appear clearly in the results and the illustrative examples that used in this thesis.

There are many types of PID controllers, each of them has a specified function to do. The P, PI, PD or PID are the basic types of PID controller. Also the PID controller can be divided into parallel PID and series PID controller. The new idea illustrated in this research is called Fed PID controller as shown in Figure 2.1 [14].

2.1 PID Controllers Tasks:

The operation of a PID controller can be understood by separating the three terms individually:

The proportional control is a pure gain adjustment acting on the signal of error to provide the driving input to do the process. The *speed* of the system can be adjusted using the proportional control. Proportional control amplifies the error to motivate the plant towards the desired response. That controller can reduce the steady state error but cannot eliminate it. At the same time the proportional controller produces an excessive overshoot and oscillation [15].

The integral control is used to provide the required accuracy for the control system. It is used to eliminate the steady state error. Usually the small integral gain is used to avoid noise and destabilize the closed loop system.

To increase the damping in the system, derivative control action is normally introduced. The derivative element amplifies the existing noise which can cause problems including instability. The control acts on the error slope, thereby it minimize the overshoot. High derivative gain can increase the rising time and the settling time.

Effects of each controller P, I, and D on a closed-loop system are summarized in the Table 2.1 shown below.

Table 2.1: Closed loop response of the PID controller terms

C.L. Response	***Rising Time***	***Overshoot***	***Settling Time***	***S.S. Error***
P	Decrease	Increase	Small Change	Decrease
I	Decrease	Increase	Increase	Eliminate
D	Small Change	Decrease	Decrease	Small Change

The PID controller can be described showing a new idea is called Fed PID controller. The Fed PID controller provides the higher performance of the control specifications especially the steady state error and the overshoot which makes the minimum steady state error and the minimum overshoot among the other PID controllers [13],[16].

2.2 PID Controllers Types:

Not all manufactures produce PID controllers that conform the ideal structure. So before tuning it is important to know the configuration of the PID algorithm. The majority of tuning rules are only valid for the ideal architecture. If the algorithm is different then the controller parameters suggested by a particular tuning methodology will have to be modified.

Ideal PID

One disadvantage of this ideal configuration is that a sudden change in set point and hence error will cause the derivative term to become very large, where the mathematical representation of this algorithm is shown below:

$$T(s) = K\left(T_P + \frac{1}{T_i s} + T_D s\right) \tag{2.1}$$

Where K is the gain and T_p, T_i, T_D is proportional, integral and derivative time respectively, and the value for each of which are shown in Table 2.2 and Table 2.3.

Series PID

The mathematical representation of this algorithm is:

$$T(s) = \left(K_P + \frac{K_I}{s}\right)(K_D s + 1) \tag{2.2}$$

As with the ideal implementation the series mode can include either derivative on the error or derivative on the measurement.

Parallel PID

The mathematical description is,

$$T(s) = K_P + \frac{K_I}{s} + K_D s \tag{2.3}$$

The proportional gain only acts on the error, whereas with the ideal algorithm it acts on the integral and derivative modes as well [17].

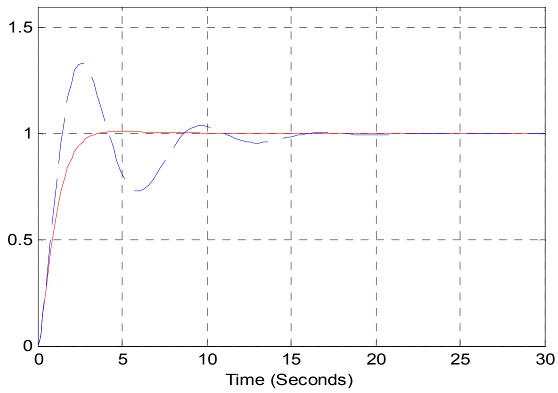
Fed PID Controller

This type of PID controller is considered the first contribution in this research. The name of Fed is quoted from the feedback, from the output to the input of the integrator. The feedback of the integrator is a differential feedback. This technique decreases the overshoot and the steady state error for any PID controller design. One can ask if the differential feedback acts on proportional or the derivative terms of the PID controller. Under experiments on the MATLAB simulation the step response shows only the positive response (minimum overshoot & steady state error) for the differential feedback that acts on the integrator. More details will be described for this type of PID controller in Chapter 5. Figure 2.1 shows the step response of the proportional, derivative, and integral deferential feedback versus the step response of the conventional PID controller for second order system. The continuous line for the conventional PID controller.

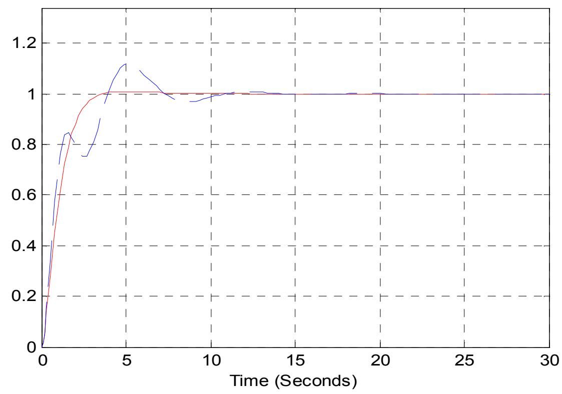
Now, the new method of designing PID controller inserted her is called Fed PID controller. The mathematical description of the Fed PID control is shown in equation (2.4),

$$T(s) = K_P + \left(\frac{K_I + 1}{K_I} \right) \frac{1}{s} + K_D s \tag{2.4}$$

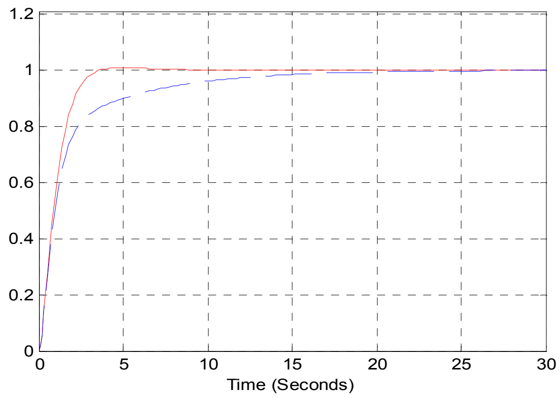
This type of PID controller is used to decrease the steady state error and the overshoot as well, which gives the better results among other controllers.



(a) Differential P



(b) Differential D



(c) Differential I

Figure 2.1: Step response of differential feedback applied to (a) P, (b) D, (c) I.

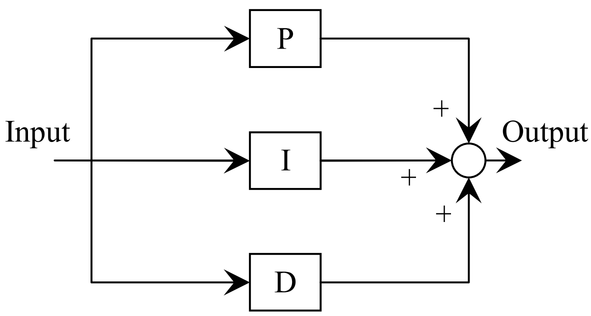
General steps for PID controller design:

Obtain an open-loop response and determine what needs to be improved

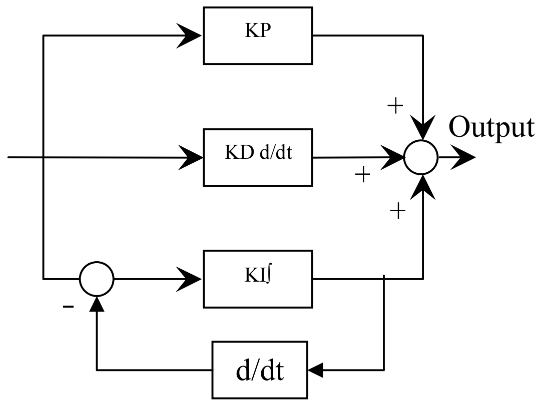
Add a proportional control to improve the rise time

Add a derivative control to improve the overshoot

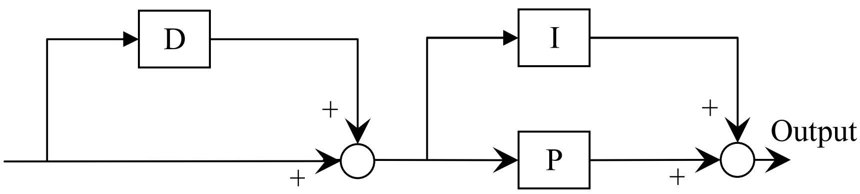
Add an integral control to eliminate the steady-state error



(a) Parallel PID



(b) FED PID



(c) Series PID

Figure 2.2: PID Controllers Types (a) Parallel PID, (b) FED PID, (c) Series PID.

2.3 PID Tuning:

A huge number of methods for control design can be applied to PID control. There are number of special methods that are made by training for PID control have also been developed, these methods are often called tuning methods.

The most well known tuning methods are Ziegler and Nichols methods. They have had a major influence on the practice of PID control for more than half a century. The methods are based on characterization of dynamic process by a few parameters and simple equations for the controller parameters. It is surprising that the methods are so widely referenced because they give moderately good tuning only in restricted situations [15].

The Step Response Method:

One tuning method presented by Ziegler and Nichols is based on a process information in the form of the open loop step response. This method can be viewed as a traditional method based on modeling and control where a very simple process model is used. The step response is characterized by only two parameters a and L, as shown in Figure 2.2.

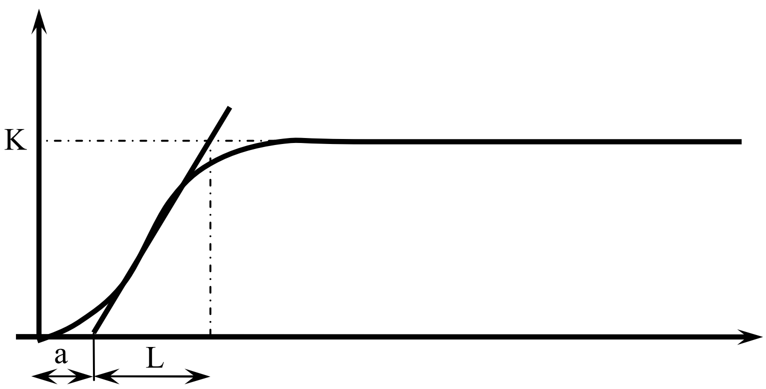


Figure2.3: Characteristic of the step response in the Ziegler-Nichols method

The point where the slope of the step response has the maximum value is first determined, and the tangent at that maximum point is drawn. The intersections between the tangent and the coordinate axes give the parameters a and L. The a and L lengths determine the controller parameters which is obtained from Table 2.2.

Table2.2: PID controller parameters obtained for the Ziegler-Nichols step response method.

Controller	***K***	***Ti***	***Td***	***Tp***
P	1/a	-	-	4L
PI	0.9/a	3L	-	5.7L
PID	1.2/a	2L	L/2	3.4L

The Frequency Response Method:

Ziegler and Nichols develop a second method which is based on a simple characterization of the frequency response of the process dynamics. This design is based on knowledge of only one point on the Nyquist curve of the process transfer function, in a desired point where the Nyquist curve intersects the negative real axis as shown in Figure 2.3 [13],[18].

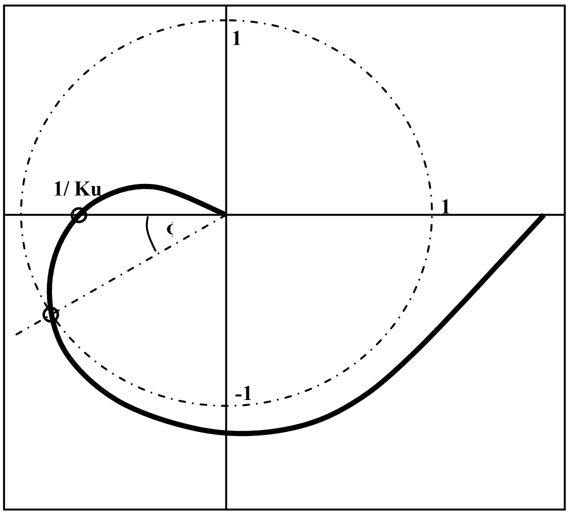


Figure2.4: Characteristic of the frequency response in the Ziegler-Nichols method

The gain when the oscillation occurs is K_u and the period of the oscillation is T_u. Gain of oscillation and frequency of oscillation can be obtained from the Nyquist diagram where K_u is the inverse value when the curve intersects the x-axis. The frequency of oscillation T_u is obtained from the angle where the Nyquist curve intersect the unit circle The parameters of the controller are then given by Table 2.3. An estimate of the period T_p of the dominant dynamics of the closed loop system is also given in Table 2.3.

Table2.3: PID controller parameters for the Ziegler-Nichols frequency response method.

Controller	***K***	***Ti***	***Td***	***Tp***
P	0.5Ku	-	-	Tu
PI	0.4Ku	0.8Tu	-	1.4Tu
PID	0.6Ku	0.5Tu	0.125Tu	0.85Tu

The most salient problem in the PID controller is the tuning of the proportional, integral and derivative gain. The tuning problem wastes time, money and efforts. So, the fuzzy control is used as a good approach to solve the problem of tuning that is done by supervisory fuzzy PID control. Also a salient feature can be done using hybrid fuzzy PID control by adding the gain scheduling method as illustrated in the research.

CHAPTER 3

Fuzzy Logic Control

Fuzzy logic is the logic that deals with fuzzy sets. The concept of fuzzy sets and fuzzy logic are used in fuzzy control. A fuzzy set is a set that does not have sharp or crisp boundaries. In other words, there is a softness associated with the membership of elements in fuzzy set. Fuzzy set theory has been developed over the past years as means for describing sets whose boundaries are vague or imprecise.

3.1 Fuzzy Logic History:

The first paper published in Fuzzy set theory was by Lotfi Zadeh. He was one of the leading authorities in control theory in early 1960’s. Lot of questions couldn't be explained with true or false logic. Since Boolean logic couldn't answer some of these questions with a simple yes or no, then fuzzy logic should be used. If you were to humanize a computer, you will have to use fuzzy logic to imitate the way a human brain works, and attempt to turn artificial intelligence into real intelligence. The way that humans think uses fuzzy logic.

Japanese were the first to use fuzzy logic in application in 1980's. Japanese and Korean companies are using fuzzy logic to enhance things like computers, air conditioners, automobile parts, cameras, televisions, washing machines, and robotics. In October of 1993, at the Tokyo Motor show, Mitsubishi had a computer which imitates the information processing in a driver's brain. The computer studies the driver's normal driving habits and selects a response from different situations that could happen. If a built in radar system detects an object in the road, then the fuzzy logic system would decide whether or not the driver was aware of the obstacle based on previous driving patterns. If the driver does not respond as the computer predicted, then the computer can automatically take control of the brakes to avoid a collision [19].

3.2 Fuzzy Logic & Fuzzy Control:

Fuzzy logic has become a common well known word in machine control. However, the term itself provides certain skepticism, sounding equivalent to half-baked logic or ambiguous logic. Fuzzy logic is a way of interfacing analog processes that move through a continuous range of values, to a digital computer, that seems to be well-defined discrete numeric values [20].

For example, consider an antilock braking system, directed by a microcontroller. The microcontroller has to make decisions based on brake temperature, speed, and other variables in the system. The variable temperature in this system can be divided into a range of states, such as: cold, cool, moderate, warm, hot, and very hot. An arbitrary threshold might be set to divide warm from hot, but this would result in a discontinuous change when the input value passed over that threshold.

The way around this is to make the states fuzzy, that is, allow them to change gradually from one state to the next. You could define the input temperature states using membership functions as shown in Figure 3.1:

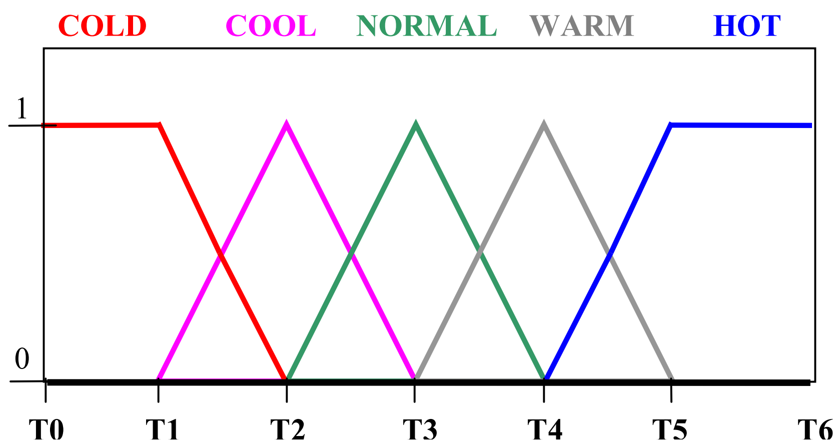


Figure 3.1 Membership function

With this scheme, the input variable's state no longer jumps abruptly from one state to the next. Instead, as the temperature changes, it loses value in one membership function while gaining value in the next. At any one time, the truth value of the brake temperature will almost always be in some degree part of two membership functions: 0.7 normal and 0.3 warm, or 0.8 normal and 0.2 cool, and so on.

A fuzzy set is represented by a membership function which gives a degree of membership within the set of any element of the universe of discourse. The membership function maps the elements of the universe onto numerical values in the interval [0,1].

Basic operations on sets in crisp set theory are the set complement, set intersection, and set union. Fuzzy set operations are very important because they can describe intersections between variables

For a given element x of the universe, the following function theoretic operations for the set theoretic operations of union, intersection, and complement are defined [21]:

Intersection (AND):

Consider two fuzzy sets A and B, as shown in Figure 3.2, in the same universe X. A∩B

$\mu_{A\cap B}(x)=\mu_A(x)\wedge\mu_B(x)=\min[\mu_A(x).\mu_B(x)] \qquad \forall\, x \in X$

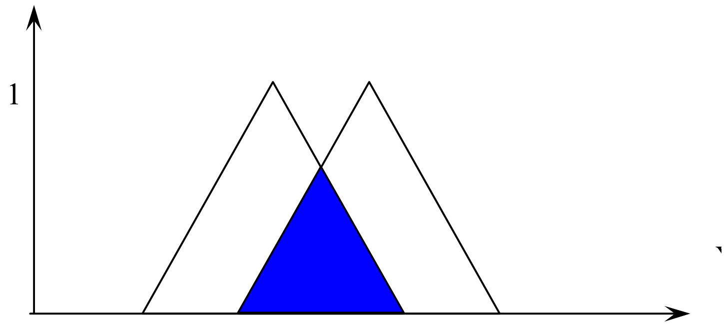


Figure 3.2 Intersection of fuzzy sets A and B

Union (OR):

Consider two fuzzy sets A and B in the same universe X. AU B is the whole area covered by the sets as shown in Figure 3.3.

$\mu_{AUB}(x)=\mu_A(x)\wedge\mu_B(x)=\max[\mu_A(x).\mu_B(x)] \qquad \forall\, x \in X$

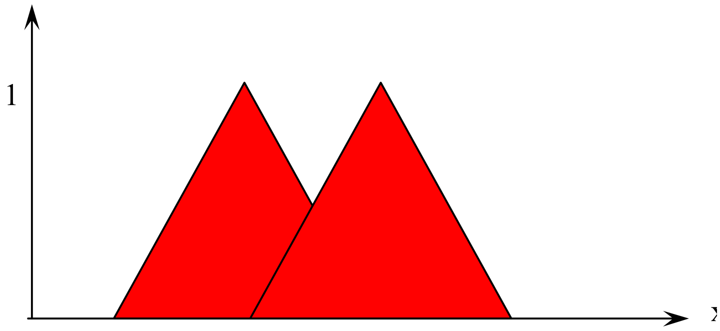


Figure 3.3 Union of fuzzy sets A and B

Complement (NOT):

Consider a fuzzy set A in universe X. Its complement $\overline{A}$ as displayed in Figure 3.4.

$\mu_{\overline{A}}(x) = 1 - \mu_A(x) \qquad \forall\, x \in X$

The connective operator used mostly in fuzzy logic control is the minimum (AND)

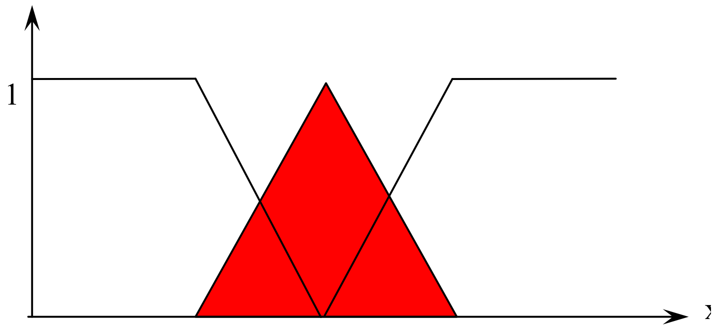


Figure 3.4 Complement of fuzzy sets A

The most popular choices for the shape of the membership function are the trapezoid-shaped, the bell shaped, the triangle-shaped and Gaussian function as shown in Figure 3.5.

The concept of a linguistic variable, which is naturally ambiguous, is basic to the understanding of fuzzy logic control. In particular, it can employ fuzzy sets to represent linguistic variables. A linguistic variable can be regarded either a variable whose value is fuzzy number or as a variable whose values are defined in linguistic terms.

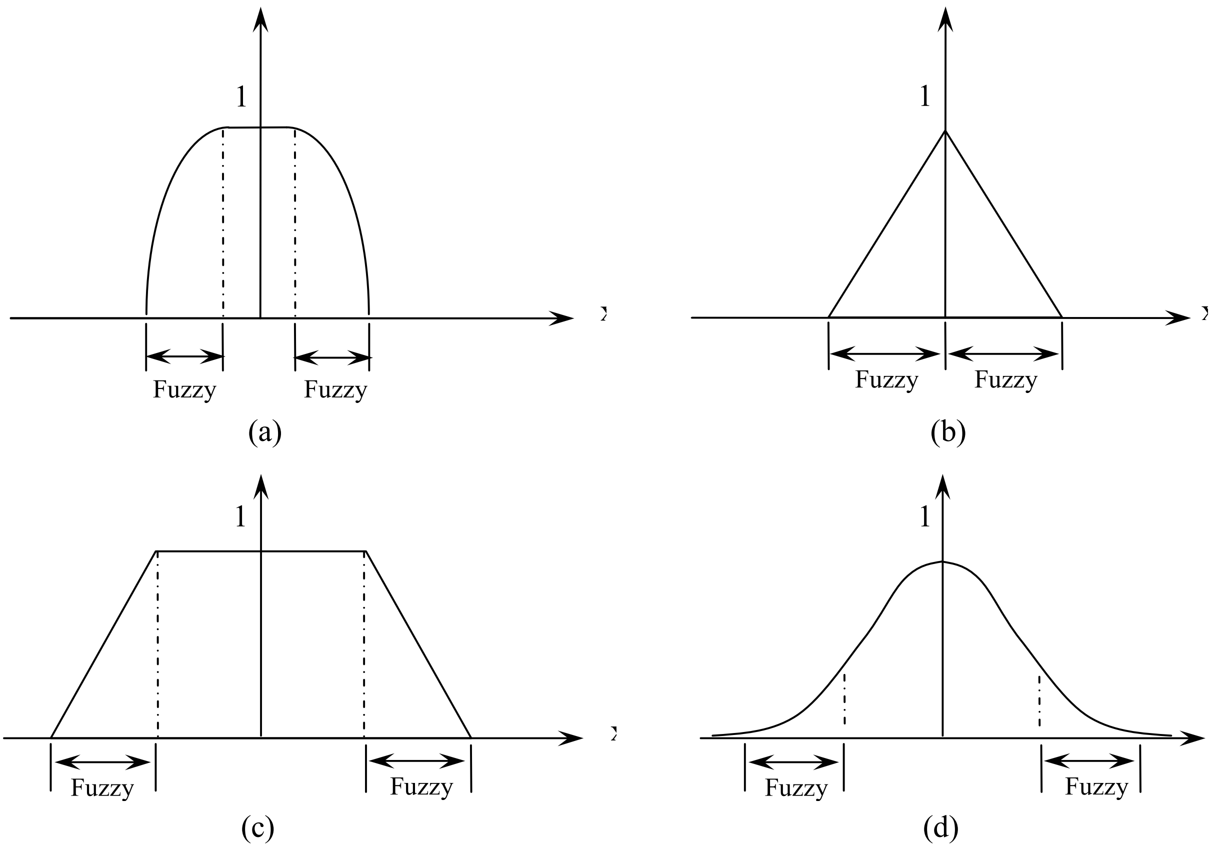


Figure 3.5 Membership Functions (a) Bell-Shaped (b) Triangular (c) Trapezoidal (d) Gaussian.

Assume the two input variables are (brake temperature) and (speed) that have values defined as fuzzy sets. The output variable, (brake pressure), is defined by a fuzzy set that can have values like (static), (slightly increased), (slightly decreased), and so on.

This rule is very puzzling since it looks like it could be used without bothering with fuzzy logic, but the decision based on these results are combined to give a specific (crisp) answer, the actual brake pressure, a procedure known as (Defuzzification). The combination of fuzzy operations and rule-based describes a (fuzzy expert system).

Traditional control systems are based on mathematical models in which the control system is described using one or more differential equations that define the system response to its inputs. Such systems are often implemented as "proportional-integral-derivative (PID)" controllers. They are the products of decades of development and theoretical analysis, and are highly effective.

If PID and other traditional control systems are so developed, it has problems versus the fuzzy control. Fuzzy control has some advantages. In many cases, the mathematical model of the control process may not exist, or may be too expensive in terms of computer processing power and memory, and a system based on experimental rules may be more effective.

Furthermore, fuzzy logic is well suited to low-cost implementations. Such systems can be easily upgraded by adding new rules to improve performance or add new features. In many cases, fuzzy control can be used to improve existing traditional controller systems by adding an extra layer of intelligence to the current control method [22].

3.3 Fuzzy Control Implementation:

Fuzzy controllers' concepts are very simple. They consist of three main stages as shown in Figure 3.6, an input stage, a processing stage, and an output stage.

The input stage maps sensor or other inputs, such as switches to the appropriate membership functions and truth values. The processing stage enables each rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value.

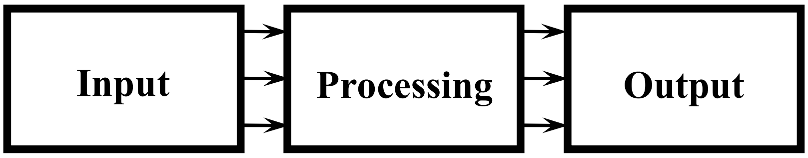


Figure 3.6: Fuzzy Control System

As discussed previously, the most common shapes of membership functions used are triangular, trapezoidal, Gaussian and bell curves, but the shape is generally less important than the number of curves and their placement.

The major components to design the fuzzy logic control shown in Figure 3.7 are the Fuzzification, knowledge base, decision making logic, and Defuzzification [20],[23].

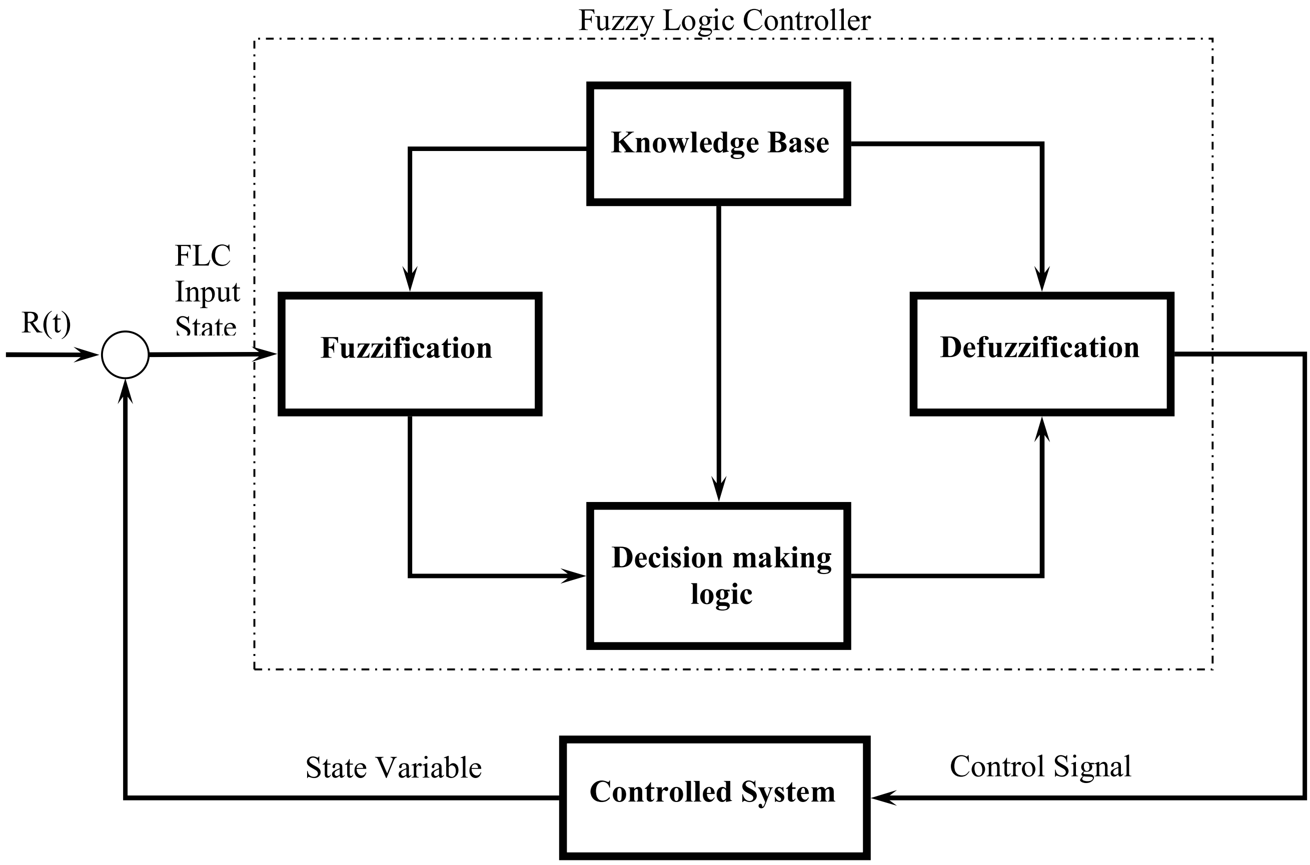


Figure 3.7: Fuzzy Logic Controller Configuration

3.3.1 Fuzzification:

The Fuzzification comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. The membership function is used to associate a grade to each linguistic term. Fuzzification plays an important role in dealing with uncertain information which might be objective in nature.

3.3.2 Knowledge Base:

The knowledge base of Fuzzy Logic Controller (FLC) is comprised of two parts: a data base and a fuzzy control rule base. Some issues will be discussed relating to the data base in this part and the rule base in the next part [21].

In the data base part, there are four principal design parameters for an FLC: discretization and normalization of universe of discourse, fuzzy partition of input and output spaces, and membership function of primary fuzzy set.

A fuzzy system is characterized by a set of linguistic statements usually represented in the form of "if-then" rules. In this section, we examine several topics related to fuzzy control rules:

1- Source of fuzzy control rules

There are two principal approaches to the derivation of fuzzy control rules. The first is a heuristic method in which rules are formed by analyzing the behavior of a controlled process. The derivation relies on the qualitative knowledge of process behavior. The second approach is basically a deterministic method which can systematically determine the linguistic structure of rules.

We can use four modes of derivation of fuzzy control rules. These four modes are not mutually exclusive, and it is necessary to combine them to obtain an effective system.

- Expert experience and control engineering knowledge: operating manual and questionnaire.
- Based on operators' control actions: observation of human controller's actions in terms of input-output operating data.
- Based on the fuzzy model of a process: linguistic description of the dynamic characteristics of a process.
- Based on learning: ability to modify control rules such as self-organizing controller.

2- Types of fuzzy control rules

There are two types of control rules: state evaluation control rules and object evaluation fuzzy control rules.

a) State evaluation fuzzy control rules [22]: State variables are in the antecedent part of rules and control variables are in the consequent part. In the case of MISO (multiple input single output), they are characterized as a collection of rules of the form.

$$R_1\text{: if } x \text{ is } A_1, \dots \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } C_1$$
$$R_2\text{: if } x \text{ is } A_2, \dots \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } C_2 \qquad (3.1)$$
$$\dots$$
$$R_n\text{: if } x \text{ is } A_n, \dots \text{ and } y \text{ is } B_n \text{ then } z \text{ is } C_n$$

where x, … y and z are linguistic variables representing the process state variable and the control variable. A_i, … B_i and C_i are linguistic values of the variables x, … y and z in the universe of discourse U, … V and W, respectively i = 1, 2, … , n. That is,

$$\begin{aligned} & x \in U, A_i \subset U \\ & \dots \\ & y \in V, B_i \subset V \\ & z \in W, C_i \subset W \end{aligned} \qquad (3.2)$$

In a more general version, the consequent part is represented as a function of the state variable x, … y.

R_i: if x is A_i, … and y is B_i then $z = f_i(x, \dots y)$ (3.3)

The state evaluation rules evaluate the process state at time t and compute a fuzzy control action at time t.

b) Object evaluation fuzzy control rules: It is also called predictive fuzzy control. They predict present and future control actions, and evaluate control objectives. A typical rule is described as

$$R_1\text{: if } (z \text{ is } C_1 \rightarrow (x \text{ is } A_1 \text{ and } y \text{ is } B_1)) \text{ then } z \text{ is } C_1.$$
$$R_2\text{: if } (z \text{ is } C_2 \rightarrow (x \text{ is } A_2 \text{ and } y \text{ is } B_2)) \text{ then } z \text{ is } C_2. \qquad (3.4)$$
$$\dots$$
$$R_n\text{: if } (z \text{ is } C_n \rightarrow (x \text{ is } A_n \text{ and } y \text{ is } B_n)) \text{ then } z \text{ is } C_n.$$

A control action is determined by an objective evaluation that satisfies the desired states and objectives. Note x and y are performance indices for the evaluation and z is control command.

In linguistic terms, the rule is interpreted as: if the performance index x is A_i and index y is B_i when a control command z_i is C_i, then this rule is selected, and the control command C_i is taken to be the output of the controller.

3.3.3 Decision Making:

In decision making logic part, there are lot of inference methods described like: Mamdani method, Larsen method, Tsukamoto method, and Takagi-Sugeno-Kang (TSK) method. The most important and widely used in the environment of fuzzy control are the Mamdani and Takagi-Sugeno methods [24].

3.3.4 Mamdani Method:

In 1974, Mamdani published the first paper for fuzzy applications [1]. Mamdani method was proposed as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators.
The source of knowledge of fuzzy logic to construct the control algorithm comes from the control protocol of the human operator. This protocol consists of a set of conditional (If-Then) statements, where the first part of each contains a so-called condition (antecedent) while the second (consequent) part deals with an action (control) that has to be taken. Therefore, it mimics the human strategy which control is to be realized when a certain state of the process controlled is observed. IF(a set of conditions are satisfied) THEN (a set of consequences can be inferred). In classical (crisp) logic, a proposition P is either true or false. In fuzzy logic, a proposition P is assigned a degree of truth or falsity (P can be any value on the interval [0,1]) with the fuzzy set involved. A fuzzy logic proposition, P, it can be expressed as "IF x is A THEN y is B," implying that P induces a possibility distribution of y given x. Basically, fuzzy control rules provide a convenient way for expressing control policy and domain knowledge and have the form (Mamdani):

$$\begin{array}{l} R_1\text{: if } x \text{ is } A_1, \ldots \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } C_1 \\ R_2\text{: if } x \text{ is } A_2, \ldots \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } C_2 \\ \ldots.. \\ R_n\text{: if } x \text{ is } A_n, \ldots \text{ and } y \text{ is } B_n \text{ then } z \text{ is } C_n \end{array} \tag{3.5}$$

Fuzzy Implication Functions:

Fuzzy control rule is a fuzzy relation which is expressed as a fuzzy implication. It is also described as how a variety of fuzzy implication relations or Zadeh extension principle might be used to derive the fuzzy relational equations. A fuzzy control rule, “IF x is A THEN y is B”, is represented by fuzzy implication function and is denoted by A→B, where A and B are fuzzy sets in universes U and V with membership function μ_A and μ_B, respectively. Several methods for obtaining the solution of fuzzy implication function are described here [20]:

(1) Mini-operation rule of fuzzy implication [Mamdani]:

$$A \to B = A \times B = \int_{U \times V} \frac{\mu_A(u) \wedge \mu_B(v)}{(u,v)} \tag{3.6}$$

(2) Product-operation rule of fuzzy implication [Larsen]:

$$A \to B = A \times B = \int_{U \times V} \frac{\mu_A(u)\mu_B(v)}{(u,v)} \tag{3.7}$$

(3) Material implication:

$$A \to B = (NOT\ A)\, OR\, B \tag{3.8}$$

(4) Propositional calculus:

$$A \to B = (NOT\ A)\, OR\, (A\ AND\ B) \tag{3.9}$$

(5) Extended propositional calculus:

$$A \to B = (NOT\ A \times NOT\ B)\, OR\, B \tag{3.10}$$

(6) Generalization of modus ponens:

$$A \to B = \sup\{c \in [0,1], A \times B \le B\} \tag{3.11}$$

(7) Generalization of modus tollens:

$$A \to B = \inf\{t \in [0,1], A + t \le A\} \tag{3.12}$$

The rules contain the condition as well as the action part of the linguistic terms that reflect the operator knowledge of the process. Also, they give a clear impression of the level of precision at which one has to perform computations in order to mimic a human behavior. The linguistic terms suggest work with sets rather than with single numerical quantities.

While building the linguistic variables for a process, some aspects can be taken into account as stated in [23]:

- Characteristics of human control behavior.
- Development of process control skills.
- Individual differences between process operators.
- Task factor affecting performance.
- Organization of the operator control behavior.

3.3.5 Takagi-Sugeno Method:

In 1985, Takagi and Sugeno published the paper of fuzzy systems [25]. The fuzzy inference system proposed by Takagi and Sugeno, known as the T-S model in fuzzy system literature provides a powerful tool for modeling complex nonlinear systems. The basic idea of the T-S model is the fact that an arbitrary complex system is a combination of mutually inter-linked subsystems. Schematic representation of a Takagi-Sugeno fuzzy system is shown in Figure 3.4. Given properly defined input variables and membership functions, the Takagi-Sugeno fuzzy rules for a system considered herein are in the form of

$$R_i\text{: IF } x_1 \text{ is } A_{i1} \text{ and } \dots \text{ and } x_m \text{ is } A_{im} \text{ THEN} \tag{3.13}$$

$$Y_i = a_{i1}x_1 + \dots + a_{im}x_m + a_{i0}$$

Where $R_i(i=1,2,\dots,c)$ denotes the ith fuzzy rule, xj $(j=1,2,\dots m)$ are the input (antecedent) variables, y_i are the rule output variables, $A_{i1}, \dots A_{im}$ are fuzzy sets defined in the antecedent space, and $a_{i1},\dots a_{im}$, a_{i0} are the model consequent parameters that have to be identified in a given input crisp vector $x=(x_1,\dots x_m)^T$, the inferred global output of the Takagi-Sugeno model is computed by taking the weighted average of individual rules' contributions

$$\hat{y} = \frac{\sum_{i=1}^{c} \tau_i(x).y_i}{\sum_{i=1}^{c} \tau_i(x)} \tag{3.14}$$

Where $\tau_i(x)$ is the degree of fulfillment of the i^{th} fuzzy rule, defined by

$$\tau_i(x) = \mathrm{Min}\{\mu_{Ai1}(x_1) \ldots \mu_{Aim}(x_m)\} \text{ or} \quad (3.15)$$

$$\tau_i(x) = \mu_{Ai1}(x_1).\ \mu_{Ai2}(x_2) \ldots \mu_{Aim}(x_m)\ i=1,2,\ldots,c$$

for the minimum and product conjunction operators, respectively. $\mu_{Aij} : R \rightarrow [0,1]$ is the membership function of the antecedent fuzzy set A_{ij} [11],[12]. That represented clearly in Figure 3.8 which shows the schematic representation of Takagi-Sugeno model.

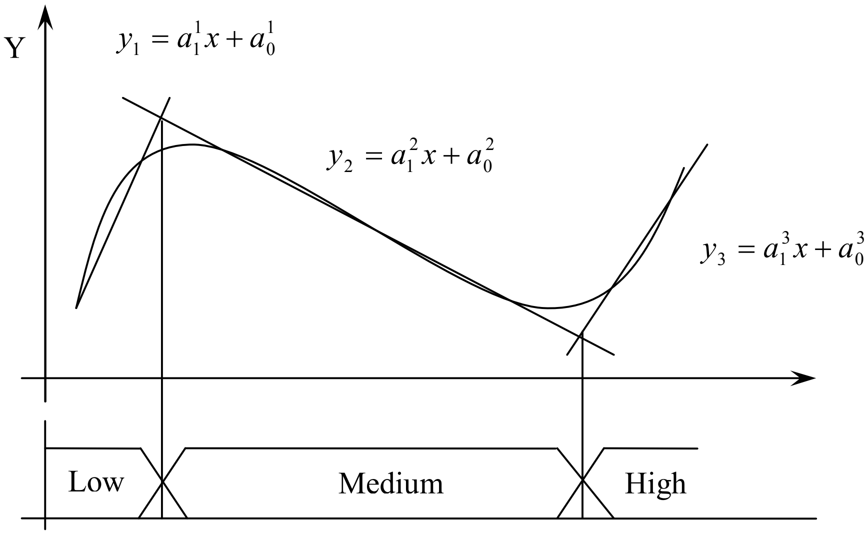


Figure 3.8: Schematic representation of Takagi-Sugeno model

3.3.6 Mamdani versus Takagi-Sugeno:

A brief description of the most popular methods used in fuzzy logic control is represented. The first one, Mamdani type fuzzy models are systems based on fuzzy (If-Then) rules with linguistic fuzzy sets in both antecedents and consequents. This type of fuzzy systems is named after E.H. Mamdani, who was the first researcher to use fuzzy logic.

The T-S and later the Sugeno and Kang, this type is similar to the Mamdani type model in the sense that they are both described by (If-Then) rules and that their antecedents have linguistic fuzzy sets. However T-S models differ in the consequents which are represented by analytic dynamical or algebraic equations. The system dynamics are written as a set of fuzzy implications which characterize local models in the state space. The main feature of

T-S fuzzy model expresses the local dynamics of each fuzzy rule by a linear dynamical model. The overall fuzzy model is achieved by blending of these rules. This type of modeling was able to approximate nonlinear systems efficiently. Since the local models are linear, the linear control methodology can be used to design the local controllers, so we obtain a global controller for the system.

The advantages of Mamdani Model:

1- It is better suited to human input.
2- It has gained widespread acceptance.
3- It is widely used for second order systems with both linear and nonlinear characteristics.
4- It is easy to implement.

The advantages of Takagi-Sugeno Model:

1- Better suited to mathematical analysis.
2- Computationally efficient.
3- Works well with optimization and adaptive technique.
4- Work better in multi dimensional systems than Mamdani model.
5- Provides a more systematic approach to the design of Fuzzy Logic Controller.
6- It is uses less fuzzy variables than the Mamdani model, since the low numbers of fuzzy variables ill reduces the number of implications.
7- It is the only fuzzy model that allows a stability analysis using Lyapunov's direct method (for nonlinear systems).

3.3.7 Defuzzification:

The output decision of a fuzzy logic controller is a fuzzy value and is represented by a membership function to precise quantity. A defuzzification process is aimed at producing a non-fuzzy control action that represents the possibility distribution of an inferred fuzzy control action. There are several methods available for defuzzification of fuzzy control inference [23]:

1. Center of Area: this method is the best known method for defuzzification. In this method:

$$Control\ \ Action = \frac{\sum_{i=1}^{l} \mu_f(w_i).w_i}{\sum_{i=1}^{l} \mu_f(w_i)} \tag{3.16}$$

where l is the number of quantification levels of the output, w_i is the amount of control action at the quantification level i, and $\mu_f(w_i)$ is the membership grade of w_i in f. This method determines the center of area below which is the combination of membership functions.

2. Centroid Method: In this method, the weighted average of the membership function or the center of the gravity of the area bounded by the membership function curve is computed to be most typically crisp value of the fuzzy quantity. This method is the most widely used. This method yields

$$Control\ \ Action = \frac{\sum_{i=1}^{n} w_i.z_i}{\sum_{i=1}^{n} w_i} \tag{3.17}$$

where n is the number of rules with firing weight w_i and z_i is the amount of control action recommended by rule i.

3. Center of Largest Area: This method is applied when the overall output fuzzy set is nonconvex, that is, consists of at minimum two convex subsets. This method then finds the convex fuzzy subset with the largest area and defines the crisp output value to be the center of area particular fuzzy subset.

4. Mean of Maxima (MOM): this method can be divided to three methods:

a. Minimum of Maxima

b. Middle of Maxima
c. Maximum of Maxima

These three methods are used when the membership grade has a unique peak point. The crisp value corresponding to the peak of the membership is taken as the best value of the fuzzy quantity.

$$Control\ \ Action = \sum_{i=1}^{n} \frac{w_i}{l} \tag{3.18}$$

where l is the number of quantified w values which reach their maximum membership.

5. Height: This is a method that uses the individual clipped or scaled control outputs. Height method takes the peak value of each clipped and builds the weighted sum of these peak values. It is the only method that will be used, when the output is singleton

$$Control\ \ Action = \frac{\sum_{i=1}^{n} p^{(i)}.h_i}{\sum_{i=1}^{l} h_i} \tag{3.19}$$

where $p^{(i)}$ is the peak value of the output, hi is the height of clipped fuzzy sets, and n is the number of rules in the system.

Example 3.1 and 3.2 illustrate the Mamdani versus Takagi-Sugeno methods as shown:

<u>Example 3.1:</u> Consider a Mamdani FLC consisting of two rules and let the inputs be.

$\dot{x}_1 = 13 \ \ and \ \ \dot{x}_2 = 16$

R_1: IF x_1 is ***Medium Positive*** and x_2 is ***Small Positive*** THEN y is ***Medium Positive***.

R_2: IF x_1 is ***Small Positive*** and x_2 is ***Medium Positive*** THEN y is ***Small Positive***.

x_1 and x_2 are the inputs of the system and y is the output. From Figure 3.9 you can obtain:

R_1: $\mu_{MP}(\dot{x}_1) = 0.25$ $\mu_{SP}(\dot{x}_2) = 0.7$

R_2: $\mu_{SP}(\dot{x}_1) = 0.85$ $\mu_{MP}(\dot{x}_2) = 0.5$

The weight of rule 1, and rule 2 are calculated by:

$w_1 = \min(\mu_{MP}(\dot{x}_1), \mu_{SP}(\dot{x}_2)) = \min(0.25, 0.7) = 0.25$

$w_2 = \min(\mu_{SP}(\dot{x}_1), \mu_{MP}(\dot{x}_2)) = \min(0.85, 0.5) = 0.5$

The control actions will be obtained by using the Centroid Method:

$$Control\ \ Action = \frac{\sum_{i=1}^{l} \mu_f(w_i).w_i}{\sum_{i=1}^{l} \mu_f(w_i)} = (0.25*20+0.5*10) / (0.25+0..5) = 13.333$$

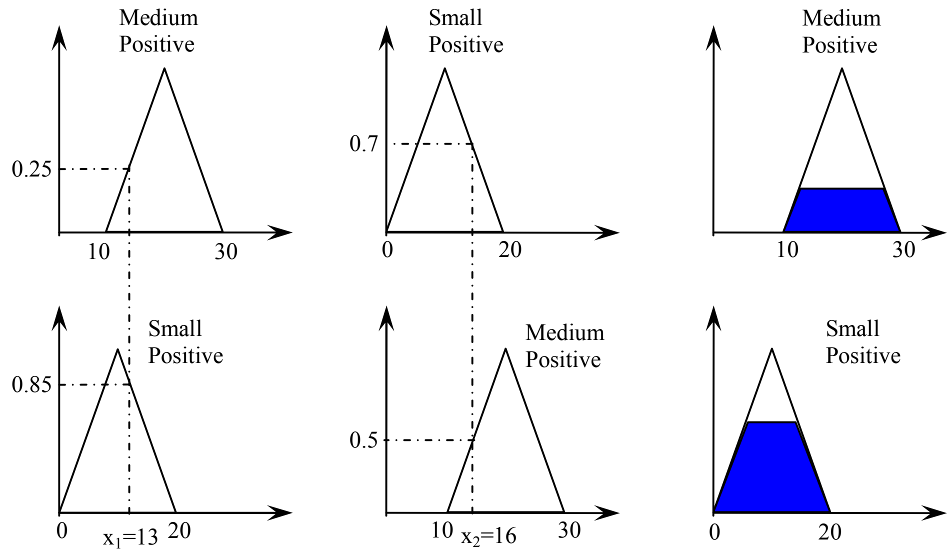


Figure 3.9: Defuzzification of the combined rules

So the control action effected on the output function is 13.33.

<u>Example 3.2:</u> Consider a Sugeno FLC consisting of two rules and let the inputs be

$\dot{x}_1 = 9 \;\; and \;\; \dot{x}_2 = 27$

R_1: IF x_1 is ***SMALL*** and x_2 is ***BIG*** THEN $\boldsymbol{u_1=x_1+4x_2+1}$.

R_2: IF x_1 is ***BIG*** and x2 is MEDIUM THEN $\boldsymbol{u_2=2x_1-3x_2+2}$.

From Figure 3.10 we can obtain:

$\mu_{SMALL}(\dot{x}_1) = 0.6$ $\qquad$ $\mu_{MEDIUM}(\dot{x}_2) = 0.7$

$\mu_{BIG}(\dot{x}_1) = 0.4$ $\mu_{BIG}(\dot{x}_2) = 0.45$

The rule weights of R_1 and R_2 we obtain:

min(0.6, 0.45)=0.45 min(0.4, 0.7)=0.4

The output of rule R_1 and R_2 we have:

u_1= 9 + 4*27 + 1 = 118

u_2= 2*9 - 3*27 + 2 = - 61

So the two pairs corresponding to each rule are (0.45, 118) and (0.4, -61) thus by taking the weighted normalized sum we get:

u = (0.45*118+0.4*(-61)) / (0.45+0.4) = 33.765

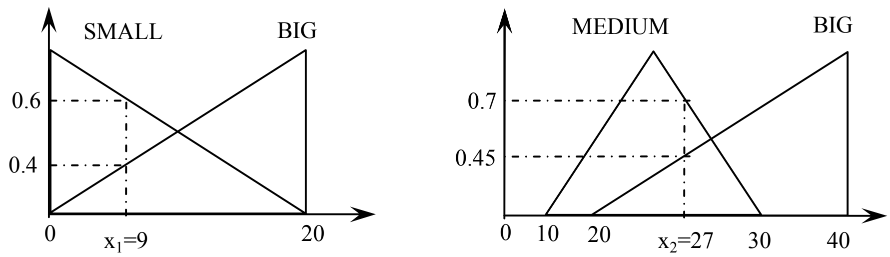


Figure 3.10: Fuzzification Procedure

The control action acted on the output function is 33.765.

The previous two examples show a brief description for Mamdani and Takagi-Sugeno. Where, Mamdani deals only with linguistic output, but the Sugeno deals with nonlinear functions and algebraic equations as shown above that grants more flexibility in real life where most of real dynamical systems are nonlinear systems.

CHAPTER 4

Nonlinear Systems

The analysis and design of nonlinear dynamical systems in electrical circuits, mechanical systems, control systems and other engineering fields needs a wide range of nonlinear systems tools. In this work, we introduce some of these tools which used in the design of nonlinear systems illustrated in this thesis.

4.1 What is Nonlinear?

Extensive theoretical techniques for the analysis and design of linear control systems have been developed over the last 50 years. Unfortunately, practically, all systems exhibit nonlinear behavior and the use of linear analysis only, may not provide an adequate description of the behavior. Linear systems have the important property that they satisfy the superposition principle. This leads to many important advantages in methods for their analysis. For example, in a simple feedback loop with both set point and disturbance inputs, they effect on the output when they are applied simultaneously that is the same as the sum of their individual effects when applied separately. This would not be the case if the system were nonlinear. Thus, mathematically a linear system may be defined as one which with input x(t) and output y(t) satisfies the property that the output for an input $ax_1(t)+bx_2(t)$ is $ay_1(t)+by_2(t)$, if $y_1(t)$ and $y_2(t)$ are the outputs in response to the inputs $x_1(t)$ and $x_2(t)$, respectively, a and b are constants. A nonlinear system is defined as one which does not satisfy the superposition property. The simplest form of nonlinear system is the static nonlinearity where the output depends only on the current value of input but in a nonlinear manner, for example the mathematical relationship

$$y(t) = ax(t) + bx^3(t) \tag{4.1}$$

where the output is a linear plus cubed function of the input.

Moreover, the relationship could involve both nonlinearity and dynamics so it might be described by the nonlinear differential equation

$$\frac{\partial^2 y}{\partial t^2} + a\left[\frac{\partial y}{\partial t}\right]^3 + by(t) = x(t) \tag{4.2}$$

From an engineering viewpoint it may be desirable to think of this equation in terms of a block diagram consisting of linear dynamic elements and a static nonlinearity, which in this case is a cubic with input $\partial y/\partial t$ and output $a(\partial y/\partial t)^3$. A major point about nonlinear systems is that their response is amplitude dependent so that if a particular form of response, or measurement of it, occurs for one input magnitude it may not result for another input magnitude. This means that in a feedback control system with a nonlinear plant, if the designed controller does not produce a linear system then, to adequately describe the system behavior, one need to investigate the total allowable range of the system variables. For a linear system one can claim that a system has an optimum response, assuming optimum is precisely defined, for example minimization of the integral squared error, using results obtained for single input amplitude. On the other hand for a nonlinear system the response to all input amplitudes must be investigated and the optimum choice of parameters to minimize the criterion will be amplitude dependent. Perhaps the most interesting aspect of nonlinear systems is that they exhibit forms of behavior that aren't possible in linear systems and more details will be discussed later [6].

4.2 Nonlinear Phenomenon:

There are a lot of nonlinear phenomena that will be described here to ensure the criterion of the nonlinear systems that can take place only in the presence of nonlinearity, and hence they cannot be described by linear models. So the following examples describe the nonlinear phenomena:

Finite escape time:

The state of an unstable linear system goes to infinity as time approaches infinity, a nonlinear systems' state, however, can go to infinity in finite time as shown Figure 4.1.

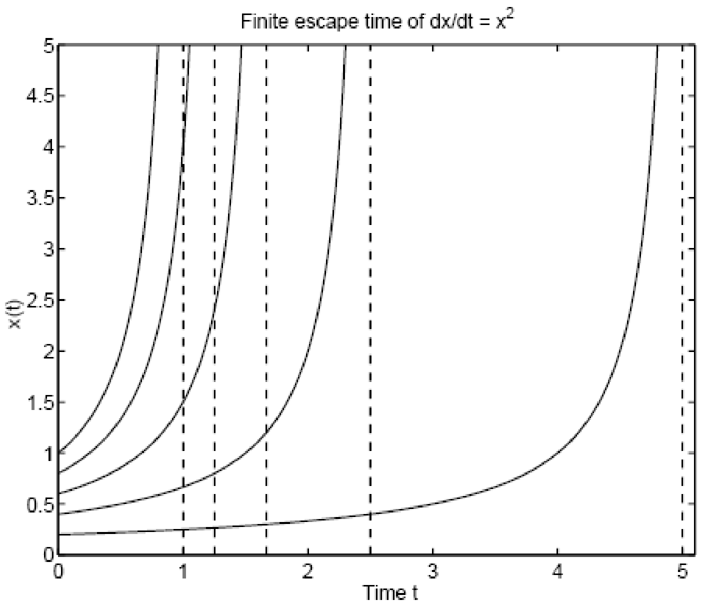


Figure 4.1: Finite escape time phenomenon.

Multiple isolated equilibria:

A linear system can have only one isolated equilibrium point, thus, it can have only one steady state operating point that attracts the state of the system irrespective of the initial state. A nonlinear system can have more than one isolated equilibrium point as shown in Figure 4.2. The state may converge to one of several steady state operating points, depending on the initial state of the system.

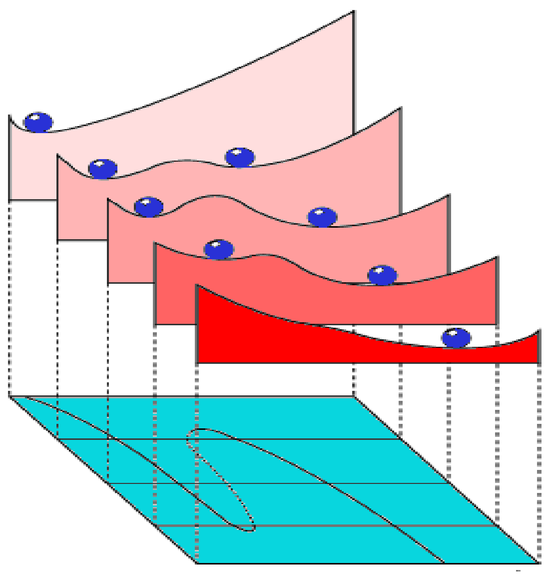


Figure 4.2: Multiple isolated equilibria.

Limit cycles:

For a linear time invariant system to oscillate, it must have a pair of eignvalues on the imaginary axis, which is a nonrobust condition that is almost impossible to maintain in the presence of perturbations. The amplitude of oscillation will be depend on the initial state. In real life, stable oscillation must be produced by nonlinear systems. There are nonlinear systems that can go into an oscillation of fixed amplitude and frequency, irrespective of initial state. This type of oscillation is known as a limit cycle as shown in Figure 4.3.

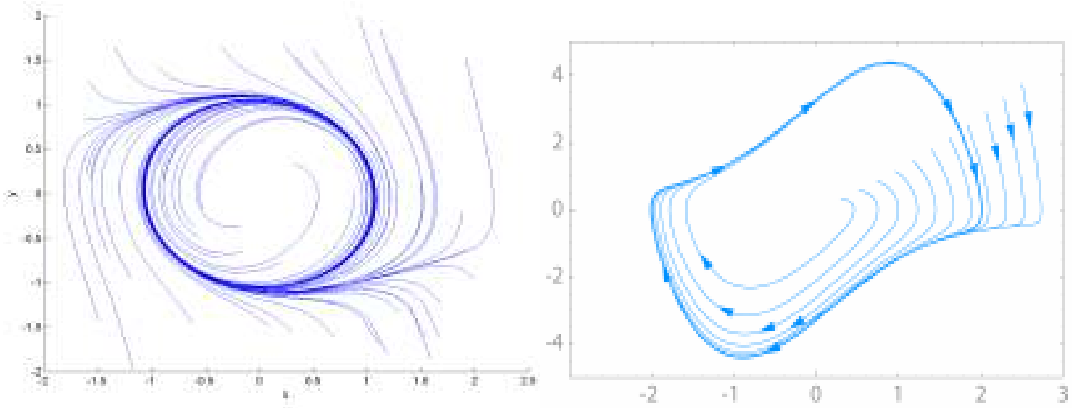


Figure 4.3: Two types of limit cycles.

These types of phenomena will be used in the most nonlinear subjects, so there are more than those types but will not be illustrated because of the concentration of the nonlinear subject in this report [26].

4.3 Common Nonlinearities:

In the following subsections, various nonlinearities which commonly occur in practice are presented.

Memoryless nonlinearities:

They are called memoryless, zero memory or static because the output of the nonlinearity at any instant of time is determined uniquely by its input at that instant; it does not depend on the history of the input, Figure 4.4 shows some of Memoryless common nonlinearities. Figure 4.5 shows characteristics of the hysteresis type which is a relay with hysteresis.

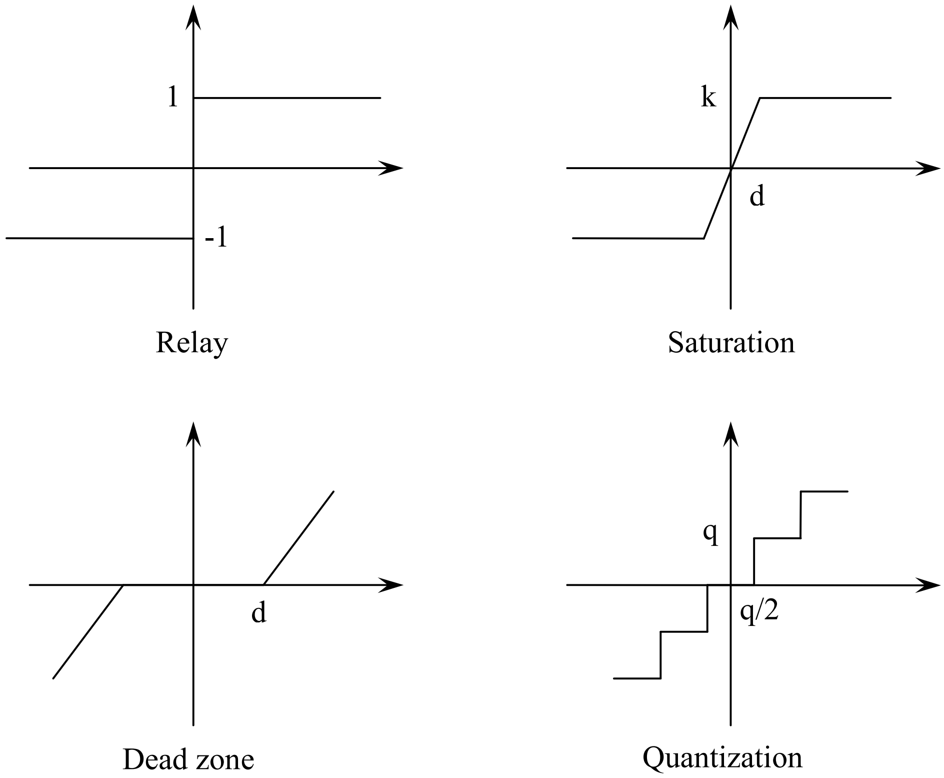


Figure 4.4 Memoryless common nonlinearities

Nonlinearity with memory

Quite frequently, we encounter nonlinear elements whose input-output characteristics have memory; that is, the output at any instant of time may depend on the whole history of the input [26].

4.4 Nonlinear Control Problem:

Generally, most of robotics systems are nonlinear systems. One common task in robotics system control is to demand the robot or parts of the body to follow a given reference trajectory [27]. Tracking control of system dynamics may change significantly. Hence, instead of trying to model the system, a more feasible solution is to schedule the gains at each operating point. Since human expert can describes the system in a natural language better than mathematical equations, fuzzy control is also commonly used in nonlinear control of robotics systems [12],[28].

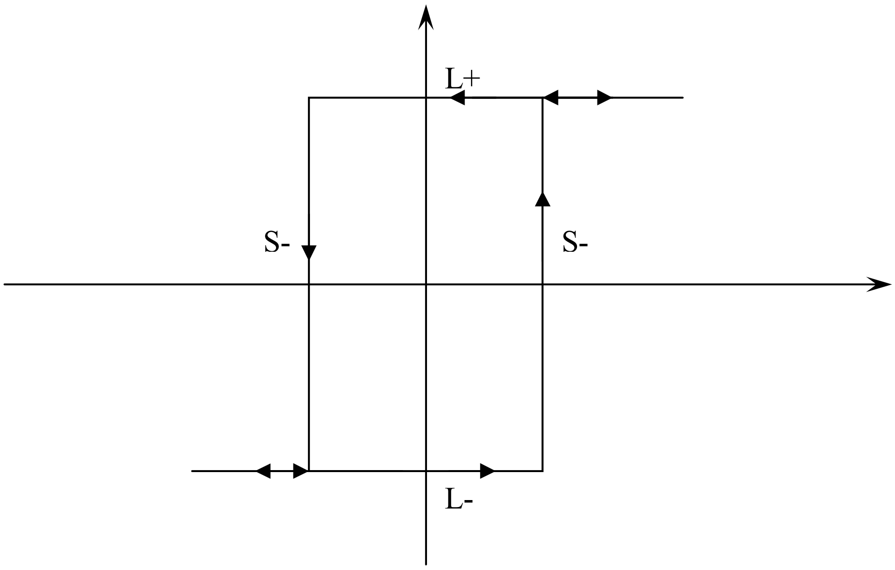


Figure 4.5 Relay with Hysteresis

A. Gain Scheduling Method

Nonlinear systems can be generally expressed by the following nonlinear autonomous system equation:

$$\dot{x} = f(x) + g(x)u \tag{4.3}$$

Where $x = [x_1, x_2, ..., x_n]^T \in R^{n\times 1}$ is the state vector, $u = [u_1, u_2, ..., u_m]^T \in R^{m\times 1}$ is the control input vector, $f(x)$ and $g(x) \in R^{n\times 1}$ are vector functions of states.

Assume $x^d(t) \in R^{n\times 1}$ is the given reference trajectory whose corresponding reference input is $u^d(t)$

$$\overset{*}{x}^d = f(x^d) + g(x^d)u^d \tag{4.4}$$

Taking Lyapunov linearization around the operating points (x^d, u^d), we have

$$\dot{x} = \dot{x}^d + A(x^d)(x - x^d) + B(x^d)(u - u^d) \tag{4.5}$$

Where

$$A(x^d) = \left.\frac{df}{dx}\right|_{x=x^d} \qquad B(x^d) = g(x^d) \tag{4.6}$$

Let $e = x - x^d, \dot{e} = \dot{x} - \dot{x}^d$ and (4.7)

System (4.5) is equivalent to

$$\dot{e} = A^d e + B^d u^e \tag{4.8}$$

where A^d and B^d are assumed to be transformed into diagonal CCF, which is valid for many robotics systems. Because the reference trajectory x^d is a function of time, the nonlinear system (4.3) can be linearized at frozen time τ so that the tracking problem of the nonlinear system is transformed into a stabilization problem of the linear system (4.8) in the error state space. The equilibrium points are shifted from the reference trajectory points $x^d(\tau)$ to the origin. However, the aforementioned conventional gain-scheduling method employs linearization between two consecutive operating points. If the system states vary significantly along the time axis, global stability will be a problem. An alternative solution is to utilize fuzzy rules containing expert knowledge to perform smoother interpolation of all the operating points in the control envelope [29].

B. Fuzzy Gain Scheduling

At some frozen times τ_i the corresponding control input can be approximated by (4.4), which is $x^d(\tau_i)$ or x^i shortly. In partitioning the state space, this x^i will be the centers of membership functions (MFs), LX^i [30]. The nonlinear system given by (4.3) can, therefore, be transformed into several local linearized systems

$$R^i : IF\ x^d\ is\ LX^i, THEN \quad \dot{e} = Ae + Bu^e \tag{4.9}$$

where A^i and B^i are system state matrices corresponding to x^i .

The control law to be designed is

$$R^i : IF\ x^d\ is\ LX^i\ ,\ THEN\quad u = u^d + u^e \qquad (4.10)$$

where u^d is the control input corresponding to the reference input x^d and u^e is the control input derived from error inputs.

The conventional approach of using the gain scheduling method is to design a linear controller for each local system in (4.9). The main advantage of this approach is that the powerful linear control theory may be applied. However, some simple nonlinear controllers like fuzzy PID controllers could be a better choice for handling the system nonlinearities. Then, the fuzzy PID controllers for local systems may be embedded in the global fuzzy gain scheduling rules to improve the integrity of the design [8], Moreover; the fuzzy Fed PID controller will give more optimal solution than any previous controllers as shown in the results in the following chapter.

CHAPTER 5

Hybrid Fuzzy Control Design

In this chapter, a fuzzy Fed PID controller is proposed to the enhanced control of the local linearized systems. By employing recursive feedback and appropriate tuning of conventional derivative gain, the fuzzy Fed PID controller guarantees sector conditions of the output [11]. Local stability analysis also explores the relationship between the conventional derivative gain and the fuzzy gain. Although the proposed controller is developed as a hybrid fuzzy Fed PID controller, the overall structure shows its potential to be a new form of standalone Fed FLC as depicted in Figure 5.1.

5.1 Hybrid Fuzzy Fed PID Control:

A fuzzy PID controller is proposed by discretizing the conventional PID controller and constructing from simple linear fuzzy rules in an incremental way. However in this chapter, a new type of fuzzy PID controller is proposed based on fuzzy Fed PID control structure using Mamdani versus the Takagi-Sugeno method [31].

The fuzzy Fed PID controller is constructed in an incremental way by employing both error signals and recursive feedback signals as inputs to Fed PID. The main idea is found in the integral side, where the integral side when it is fed by a deferential feedback gives us a null overshoot and a null steady state error, the enhancement is very significant using Fuzzy Fed PID controller. The most widely adopted conventional PID controller structure used in industrial applications is the following structure [32]:

$$u_{PID}(t) = K_P^C e_v(t) + K_I^C e_p(t) + K_D^C e_a(t) \qquad (5.1)$$

where K_P, K_I, and K_D are the conventional proportional, integral, and derivative gains, respectively, and $u_{PID}(t)$ is the controller output and $e_v(t)$ is the velocity error signal, $e_p(t)=\int e_v(t)$ is the proportional error signal and $e_a(t)=de_v(t)/dt$ is the acceleration error signal.

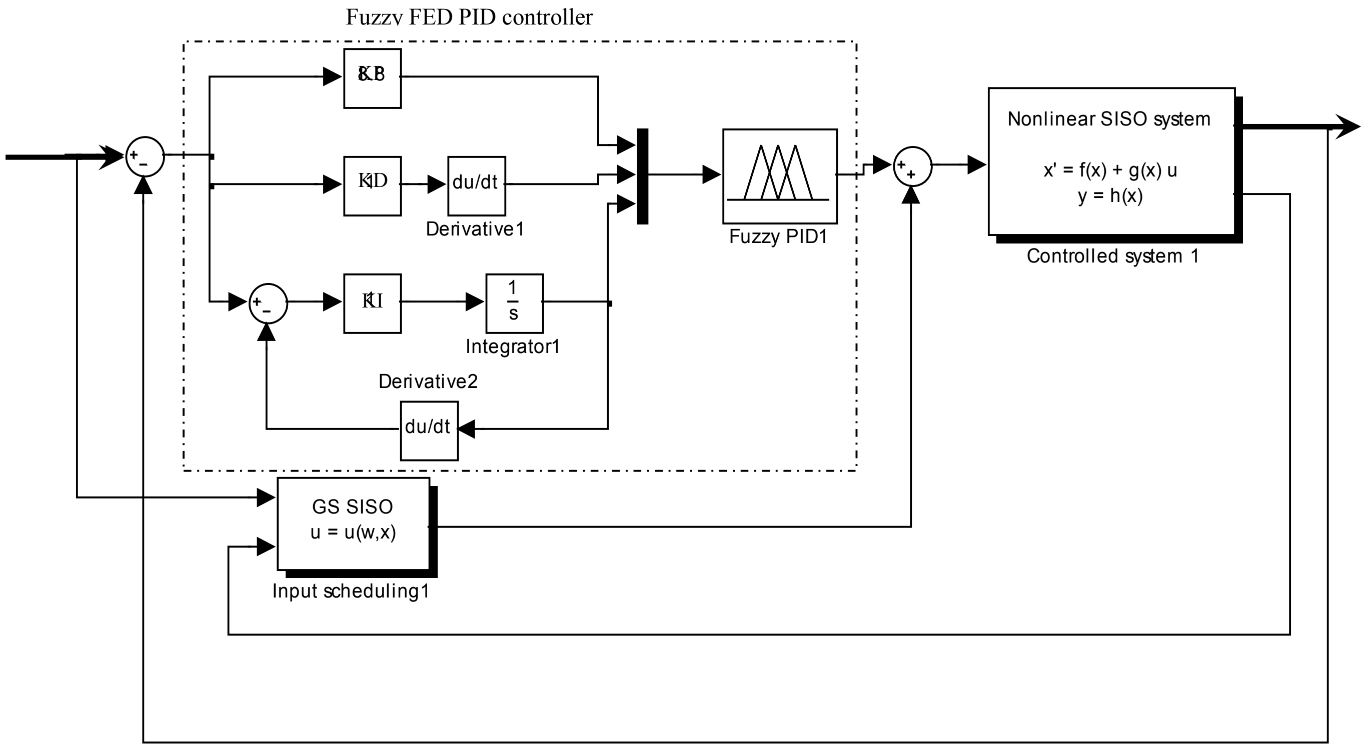


Figure 5.1: Overall Control Structure

The parameters in equation (5.1) form the PID controller and can be replaced by the following linear fuzzy rules:

$$R^j : IF\ e_p\ is\ LE_p^j\ AND\ e_v\ is\ LE_v^j\ , THEN\ u_{PID}\ is\ DU_{PID}^j \qquad (5.2)$$

Where LE_p^j *and* LE_v^j are the linguistic values of the error signals of the j[th] fuzzy rule and DU_{PID}^j is the desired function value of the output $u_{PID}(t)$

The first look to the Fed PID gives the following equation:

$$u_{PID}(t) = K_P^C e_v(t) + (0.5) K_I^C e_p(t) + K_D^C e_a(t) \qquad (5.3)$$

But the real output is differ when the Fed PID controller is used, where the Fed PID controller has overshoot and steady state error less than the conventional PID controller.

Note that the output feedback from the integrator is taken from the output of the defuzzification process which gives the best results showing in the illustrative example.

The example will be illustrated to make sure of the proposed results, which gives the minimum overshoot and minimum steady state error. In the example, the proposed controller is used in Mamdani and Takagi-Sugeno fuzzy control with an inverted pendulum robot, that robot is used in the most of our applications because of nonlinearity problem and marginally stability. The dynamic equation of the inverted pendulum robot is given by

$$\ddot{\theta} = \frac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta - F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \tag{5.4}$$

Where θ is the angle between the pendulum and the vertical, the angular velocity is expressed by $\dot{\theta}$, the force which acts on the cart is F, the gravity acceleration g is 9.8m/sec^2 , mc and mp are the mass of cart and the mass of pole respectively, and l is the half length of the pendulum. The system equation is written as follow:

$$\dot{x} = f(x) + g(x)u \tag{5.5}$$

Where

$$f(x) = \begin{bmatrix} \dot{\theta} \\ \dfrac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta \cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 \\ \dfrac{-F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix}$$

The last two equations are used for simulation without a previous technique of linearization because of two methods are used, the first one is the gain scheduling method which divides the system into small areas to relent using of iterations, the second method is the fuzzy PID controller that uses the linguistic formulas and by default it makes a linearization of the nonlinear system. The addition of the two methods is called hybrid fuzzy PID controller [33],[34].

Let us discuss briefly the pendulum and give the numerical calculations and membership functions equations. As discussed, the angular position is θ, the angular velocity is $\dot{\theta}$, the external force F is applied to the cart. The gravity acceleration, g is 9.8 m/s^2 , the mass of the cart, m_c is 1.0kg, the mass of the pole, m_p is 0.1kg and the half length of the pole, l is 0.5m. Say that x=[θ $\dot{\theta}$]^T and u=F. Assume that the pole angle is required to follow a particular trajectory θ^d , now we can calculate the corresponding control input, u^d, at a frozen times. Then the system can then be linearized to $\dot{x} = \dot{x}^d + A^d(x - x^d) + B^d(u - u^d)$ where x^d=[θ^d $\dot{\theta}^d$]^T, A^d=df/dx$|_{x=x^d}$, and B^d=g(x^d).
The simulation program used to simulate hybrid fuzzy Fed PID control is the MALAB Simulink program. The membership functions are shown bellow and the error signals are the membership functions for the inputs where input 1 is the error signal, input 2 is error differentiation, and input 3 is the error integration. Also function1, 2, and 3 are:

$$\begin{aligned}
\mu_{pos}(\theta) &= \exp\left(-(K_u / T_u)(\theta - 1.5)^2\right) && (Function\ 1)\\
\mu_{zero}(\theta) &= \exp\left(-(K_u / T_u)\theta^2\right) && (Function\ 2)\\
\mu_{neg}(\theta) &= \exp\left(-(K_u / T_u)(\theta + 1.5)^2\right) && (Function\ 3)
\end{aligned} \tag{5.6}$$

Figure 5.2 shows the Nyquist diagram for the inverted pendulum which determine the gain of oscillation K_u and the frequency of oscillation T_u:
Frequency of oscillation is T_u= 3.5867e-7 Hz and the gain of oscillation is K_u= 27.

$$\begin{aligned}
\mu_{pos}(\theta) &= \exp\left(-(75278110.8)(\theta - 1.5)^2\right) && (Function\ 1)\\
\mu_{zero}(\theta) &= \exp\left(-(75278110.8)\theta^2\right) && (Function\ 2)\\
\mu_{neg}(\theta) &= \exp\left(-(75278110.8)(\theta + 1.5)^2\right) && (Function\ 3)
\end{aligned} \tag{5.7}$$

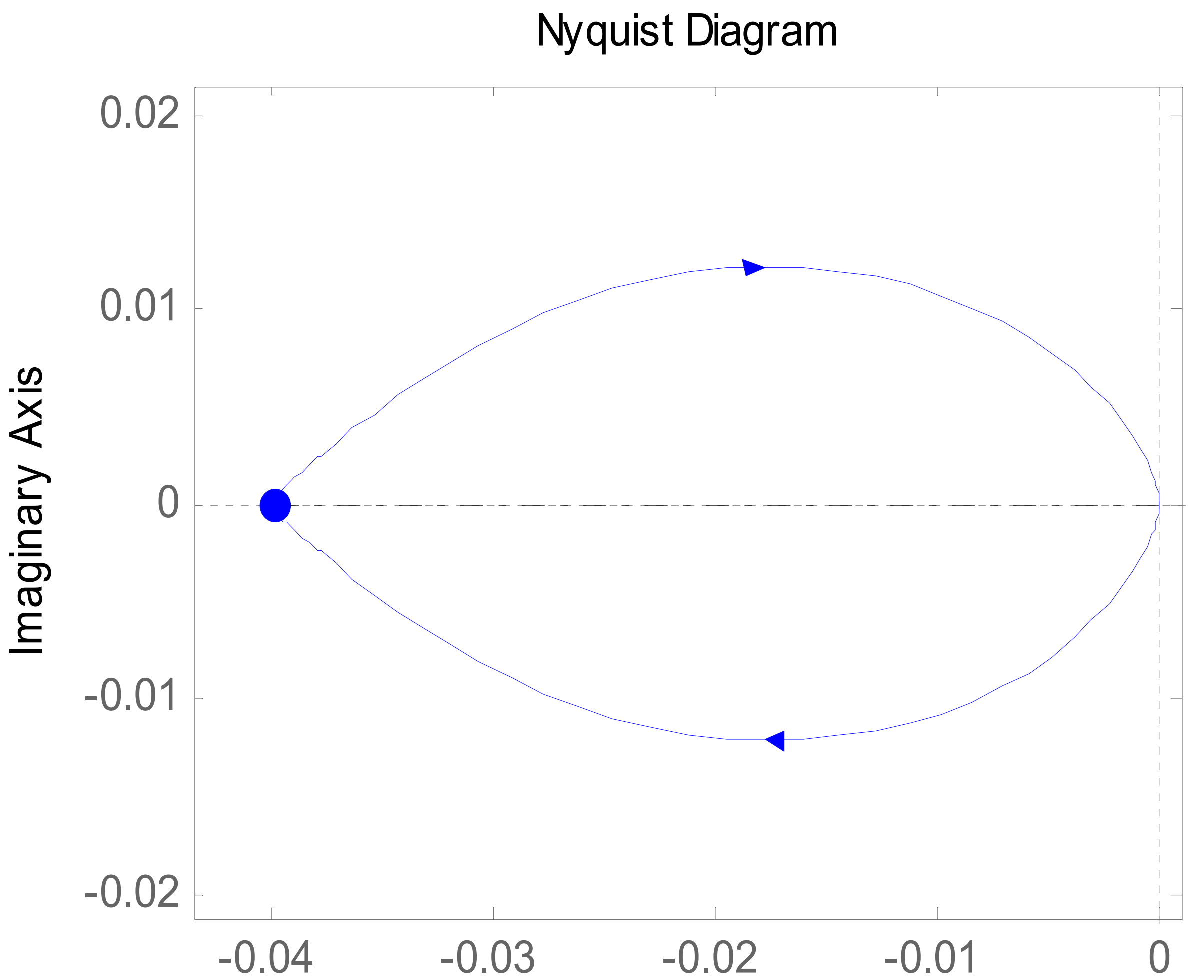


Figure 5.2: Nyquist diagram of the open loop inverted pendulum

5.2 Fuzzy PID Controller Rules:

Beside the point of the amount of masses and measurements of the pendulum, the most point to be focused is the Fed Fuzzy PID controller that makes lower overshoot and minimum steady state error. This technique always makes the best results shown in Figure 5.3, the fuzzy rules of the Fed PID controller using Takagi-Sugeno shown bellow is better than the results of Hybrid fuzzy Fed PID controller:

For the fuzzy proportional integrator differentiator:

1.If (input1 is -ve) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
2.If (input1 is -ve) and (input2 is -ve) and (input3 is zero) then (output1 is Function1)
3.If (input1 is -ve) and (input2 is -ve) and (input3 is +ve) then (output1 is Function1)
4.If (input1 is -ve) and (input2 is zero) and (input3 is -ve) then (output1 is Function1)
5.If (input1 is -ve) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
6.If (input1 is -ve) and (input2 is zero) and (input3 is +ve) then (output1 is Function2)
7.If (input1 is -ve) and (input2 is +ve) and (input3 is -ve) then (output1 is Function2)
8.If (input1 is -ve) and (input2 is +ve) and (input3 is zero) then (output1 is Function3)
9.If (input1 is -ve) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)
10.If (input1 is zero) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
11.If (input1 is zero) and (input2 is -ve) and (input3 is zero) then (output1 is Function2))
12.If (input1 is zero) and (input2 is -ve) and (input3 is +ve) then (output1 is Function2)
13.If (input1 is zero) and (input2 is zero) and (input3 is -ve) then (output1 is Function2)
14.If (input1 is zero) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
15.If (input1 is zero) and (input2 is zero) and (input3 is +ve) then (output1 is Function2)
16.If (input1 is zero) and (input2 is +ve) and (input3 is -ve) then (output1 is Function2)
17.If (input1 is zero) and (input2 is +ve) and (input3 is zero) then (output1 is Function2)
18.If (input1 is zero) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)
19.If (input1 is +ve) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
20.If (input1 is +ve) and (input2 is -ve) and (input3 is zero) then (output1 is Function2)
21.If (input1 is +ve) and (input2 is -ve) and (input3 is +ve) then (output1 is Function3)
22.If (input1 is +ve) and (input2 is zero) and (input3 is -ve) then (output1 is Function2)
23.If (input1 is +ve) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
24.If (input1 is +ve) and (input2 is zero) and (input3 is +ve) then (output1 is Function3)
25.If (input1 is +ve) and (input2 is +ve) and (input3 is -ve) then (output1 is Function3)
26.If (input1 is +ve) and (input2 is +ve) and (input3 is zero) then (output1 is Function3)
27.If (input1 is +ve) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)

Figure 5.3 illustrates the membership functions of the inputs and outputs of the desired controller, the blue color (left) for the membership function point to the negative input, the green (mid) one point to the zero membership and the red (right) point to the positive membership for each input

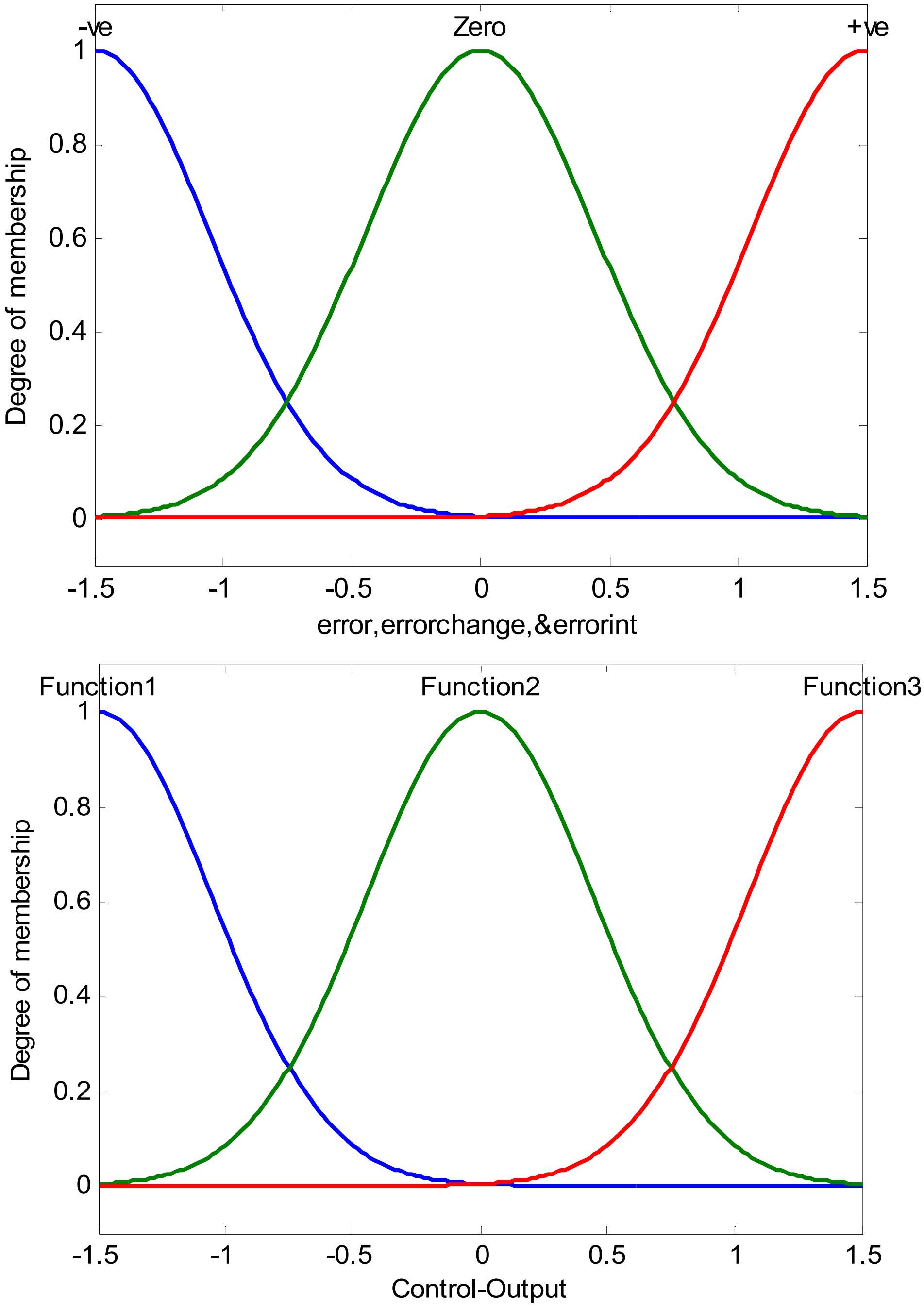


Figure 5.3: membership functions of the inputs to the controller and the output

Figure 5.4 illustrates the step response of hybrid fuzzy Fed PID controller versus conventional PID controller using Mamdani technique, the results are shown in Figure 5.4 clearly give the best steady state error and the best overshoot but give a delay:

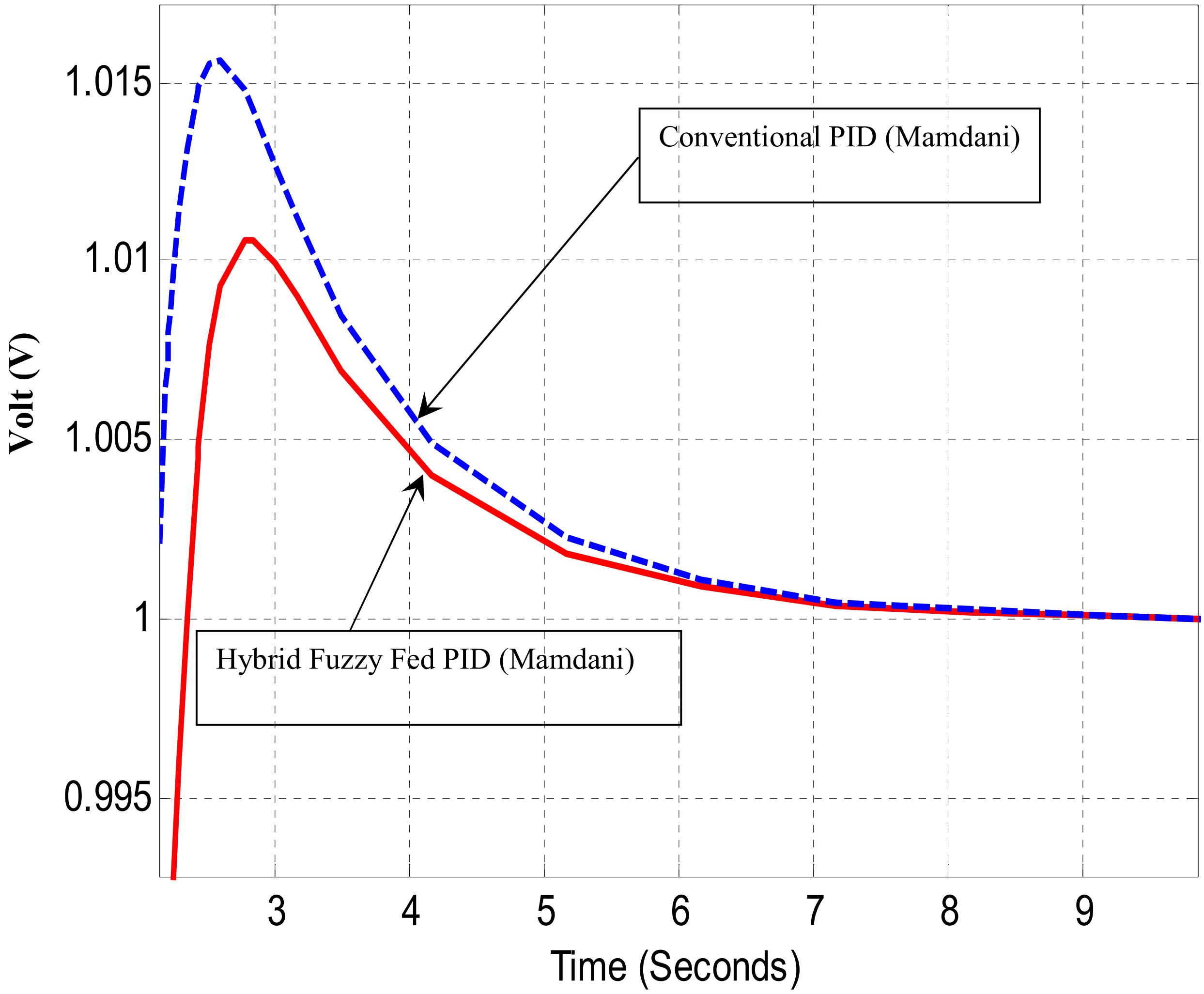


Figure 5.4: Stabilization control of the PID versus Fed PID (Mamdani) Long rule

As illustrated in the Figure 5.4, the overshoot of Hybrid Fuzzy Fed PID using the Mamdani method is less value than the overshoot of the conventional PID controller that satisfy the idea of using the fuzzy control is better than conventional PID in maximum overshoot and the steady state error.

Figure 5.5 illustrates the step response of hybrid fuzzy Fed PID controller (Takagi-Sugeno) versus conventional PID controller:

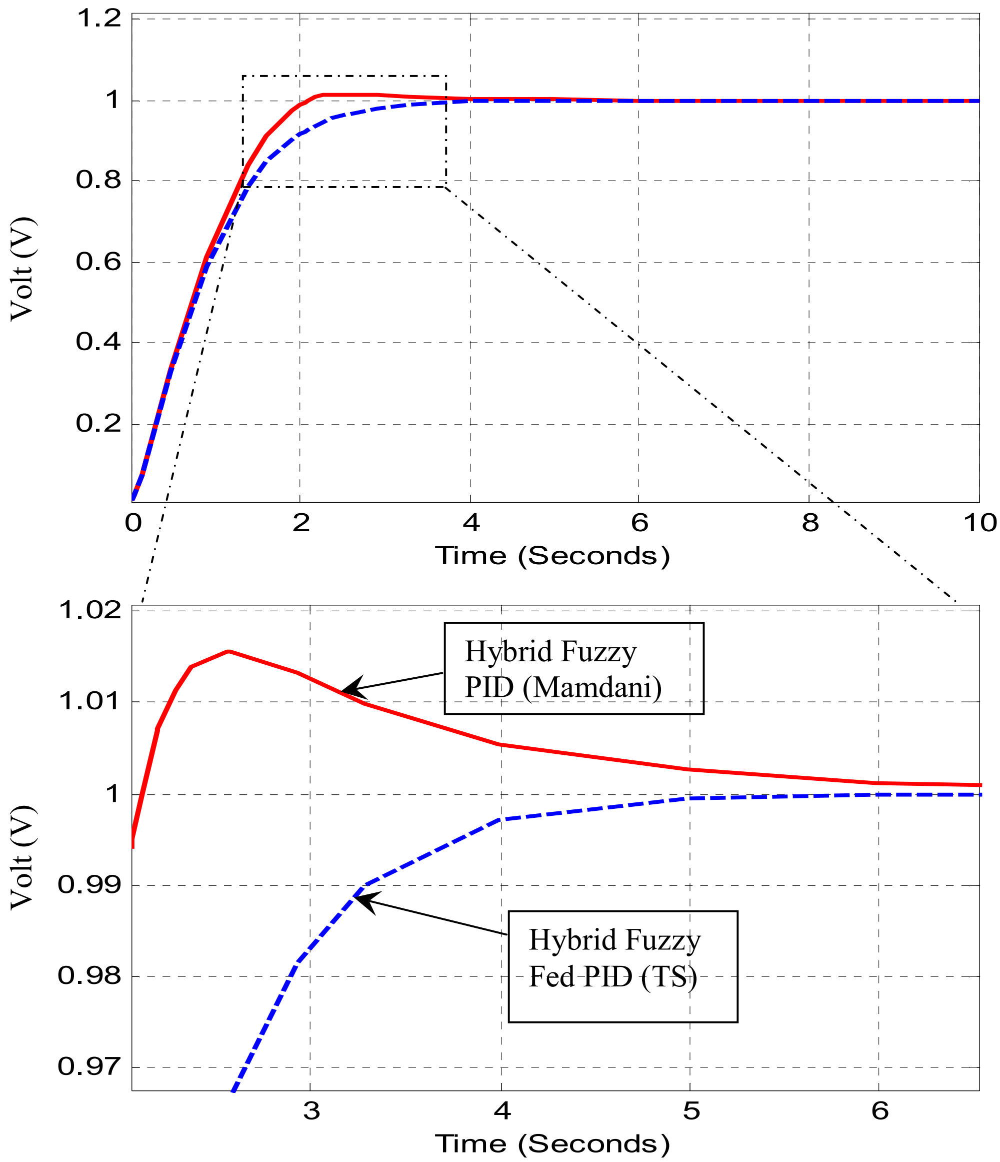


Figure 5.5: Stabilization control of the PID versus Fed PID (Takagi-Sugeno)

The figure illustrates the Mamdani versus Fed Sugeno Hybrid Fuzzy PID controller where the Fed Takagi-Sugeno achieves the zero overshoot but the Mamdani makes some overshoot, in addition the Fed Takagi-Sugeno has steady state error less value than Mamdani method.

Figure 5.6 illustrates the step response of hybrid fuzzy Fed PID controller (Takagi-Sugeno) versus hybrid fuzzy Fed PID controller (Mamdani):

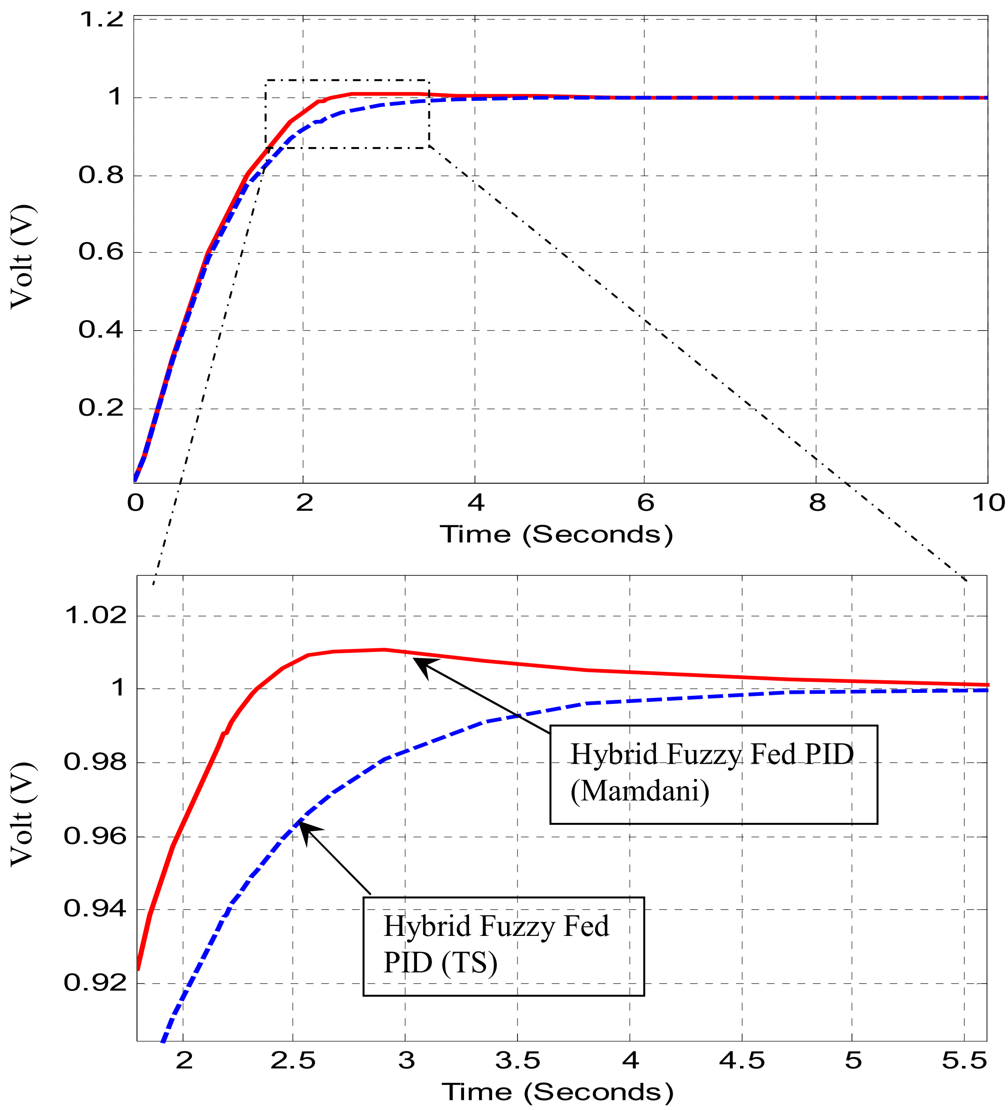


Figure 5.6: Stabilization control of the Fed PID (Mamdani) versus Fed PID (Takagi-Sugeno)

The Fed Takagi-Sugeno achieves the zero overshoot but the Fed Mamdani makes some overshoot, in addition the Fed Takagi-Sugeno has steady state error less value than Mamdani method. Anyway, when the Fed theorem is used the minimum steady state error and the minimum overshoot will be achieved.

5.3 Minimizing the number of rules of the Fuzzy PID Controller:

In this section the series reduction method will be used for redesigning inputs and the number of rules. Theoretically, the number of rules that cover all possible input variations for a five term fuzzy controller is ($n_1 \times n_2 \times n_3 \times n_4 \times n_5$). Where ($n_1 \times n_2 \times n_3 \times n_4 \times n_5$) are the number of membership functions or linguistic labels of the five input variables. In a particular case, if $n_1=n_2=n_3=n_4=n_5=5$, then the number of rules will be 3125 as shown in Figure 5.7. In practical applications, the implementation of such a large rule base will take a lot of reasoning time besides a large amount of process memory [35].

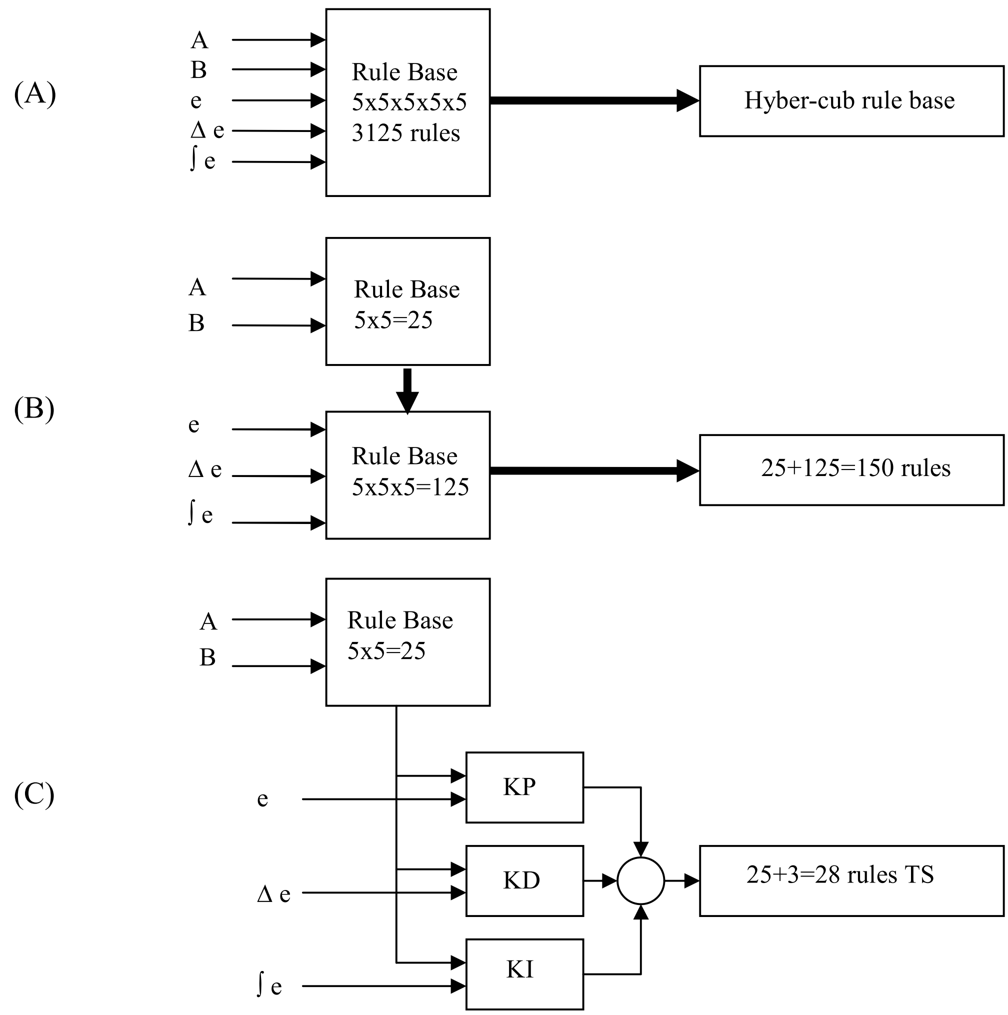


Figure 5.7: Some strategies in reducing Rule bases. (a) Original hyper-cube rule base. (b) Two rule bases in series. (c) Reduction based on Takagi-Sugeno

The 27 rules used in the inverted pendulum example can be reduced using the series method shown in the Figure 5.8

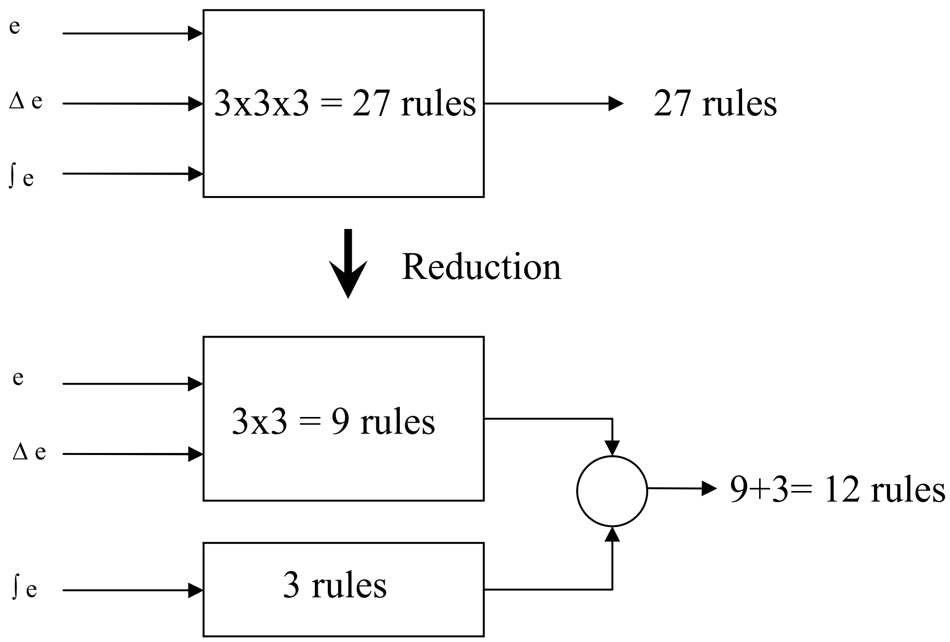


Figure 5.8: Reduction of the inverted pendulum Fed PID controller rules using series method

The fuzzy PID controller is divided into two main rules the PD rules and the Integrator rules, I divided the integrator rules to make a feedback from the output of the integrator with a deferential feedback to the input of the integrator, this technique always makes the best results referred to optimization in the direction of reducing the number of rules, the fuzzy rules of the Fed PID controller shown below:

For the fuzzy Fed integrator:
IF (e_p is –ve) THEN (u_{PID} is –ve)
IF (e_p is zero) THEN (u_{PID} is zero)
IF (e_p is +ve) THEN (u_{PID} is +ve)

For the fuzzy proportional differentiator:

IF (e_v is –ve) AND (e_a is –ve) THEN (u_{PID} is –ve)

IF (e_v is –ve) AND (e_a is zero) THEN (u_{PID} is –ve)

IF (e_v is –ve) AND (e_a is +ve) THEN (u_{PID} is zero)

IF (e_v is zero) AND (e_a is –ve) THEN (u_{PID} is –ve)

IF (e_v is zero) AND (e_a is zero) THEN (u_{PID} is zero)

IF (e_v is zero) AND (e_a is +ve) THEN (u_{PID} is +ve)

IF (e_v is +ve) AND (e_a is –ve) THEN (u_{PID} is zero)

IF (e_v is +ve) AND (e_a is zero) THEN (u_{PID} is +ve)

IF (e_v is +ve) AND (e_a is +ve) THEN (u_{PID} is +ve)

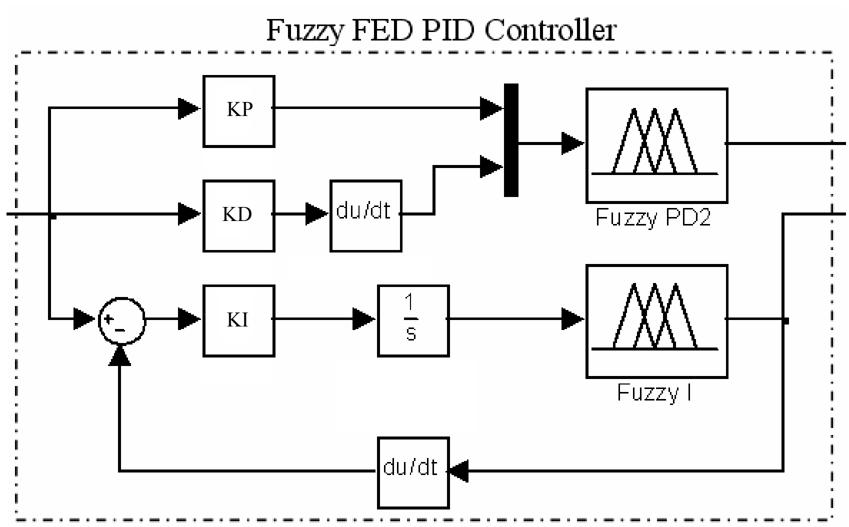


Figure 5.9: Fuzzy Fed PID controller with only 12 rule

The results of the reduced rules are shown in Figure 5.10 that illustrates a good overshoot and a good steady state error which nearly equals to that used in 27 rules but don't forget that the optimization in rules numbers qualifies big processes to be reserved. The 27 rule TS Fed PID achieves null overshoot and very small steady state error, and the 12 rule TS Fed PID has some overshoot and small steady state error. But the 27 rule wastes time, processing and money where that is not achieved in 12 rule TS Fed PID. A numerical values can be exactly illustrated in Table 5.1.

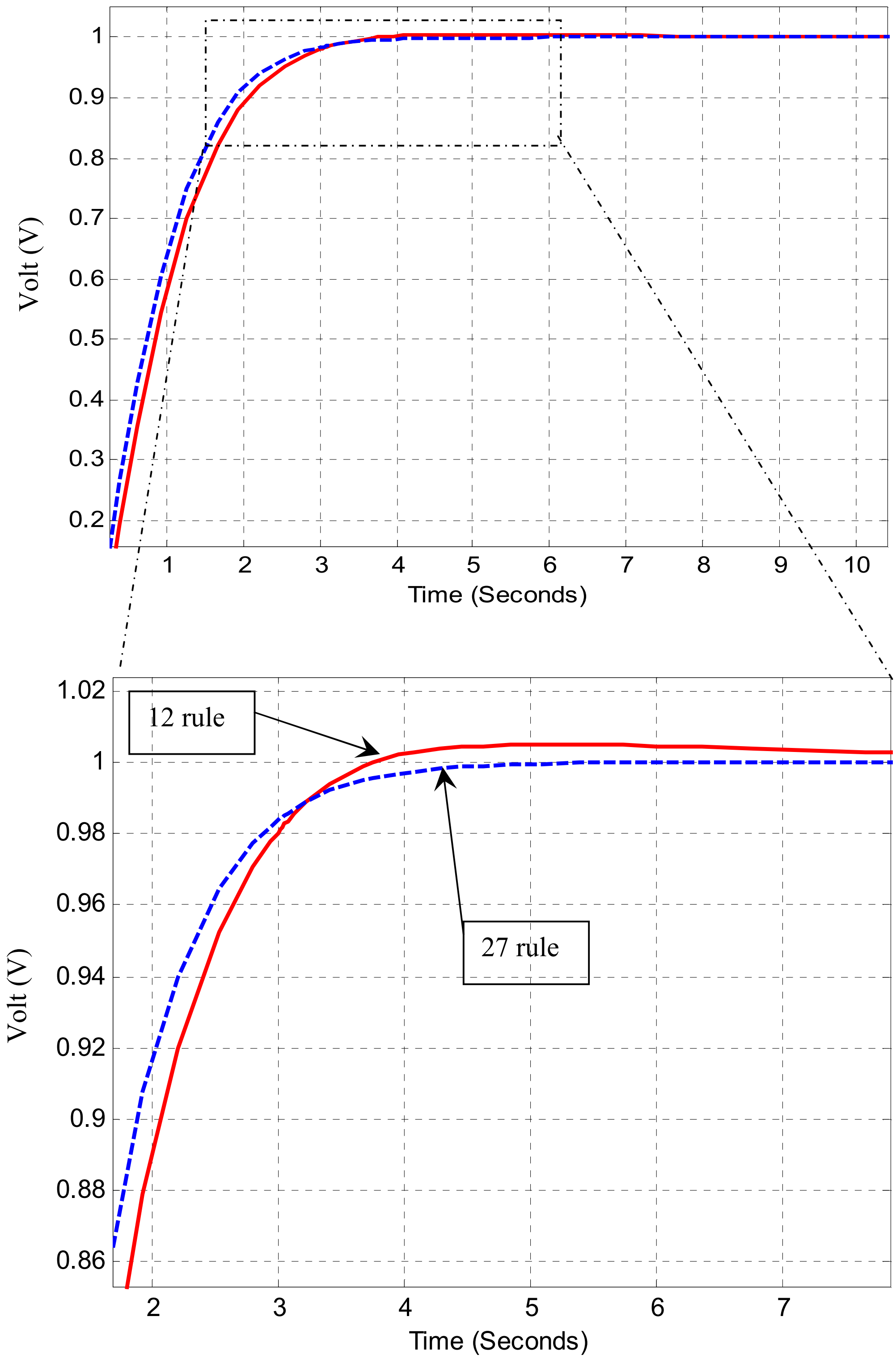


Figure 5.10: Fuzzy (TS) Fed PID controller with only 12 rule step response versus 27 rules step response.

Table 5.1 illustrates the cooked results of the proposed approach and compares it with three previous studies in two salient issues, steady state error and overshoot. The first study is the Hybrid Fuzzy PI plus Conventional D Control of Linear and Nonlinear Systems [8]. The second study is the study of hybrid fuzzy control of robotics systems which illustrates the output control [12]. While the other study view a fuzzy control system via linear matrix inequalities [36].

Table 5.1: SSE and maximum OS of various control approaches.

Desired Control Approach	***OS%***	***SSE***
Mamdani Conventional PID Control	1.60	3.10E-3
Mamdani Hybrid Fuzzy Fed PID Control	0.90	1.20E-3
Mamdani Hybrid Fuzzy PID Control	1.10	1.50E-3
Takagi-Sugeno Hybrid Fuzzy Fed PID Control	0.00	0.17E-3
Mamdani Hybrid Fuzzy Fed PID Control	0.90	1.20E-3
Takagi-Sugeno Hybrid Fuzzy Fed PID Control	0.00	0.17E-3
Hybrid Fuzzy PI+D Control of Nonlinear Systems [8]	2.61	0.00E-3
Hybrid Fuzzy PID Control of Robotics Systems [12]	0.47	4.00E-3
Fuzzy Control Systems via LMIs [36]	0.93	0.00E-3
27 rules Takagi-Sugeno Hybrid Fuzzy Fed PID Control	0.00	0.17E-3
12 rules Takagi-Sugeno Hybrid Fuzzy Fed PID Control	0. 34	1.80E-3

Figure 5.11 shows the step response of the previous work for the fourth part of Table 5.1 where figure (a) illustrates the Hybrid Fuzzy PI+D Control of Nonlinear Systems [8], figure (b) illustrates Hybrid Fuzzy PID Control of Robotics Systems [12] and figure (c) illustrates Fuzzy Control Systems via LMIs step response.

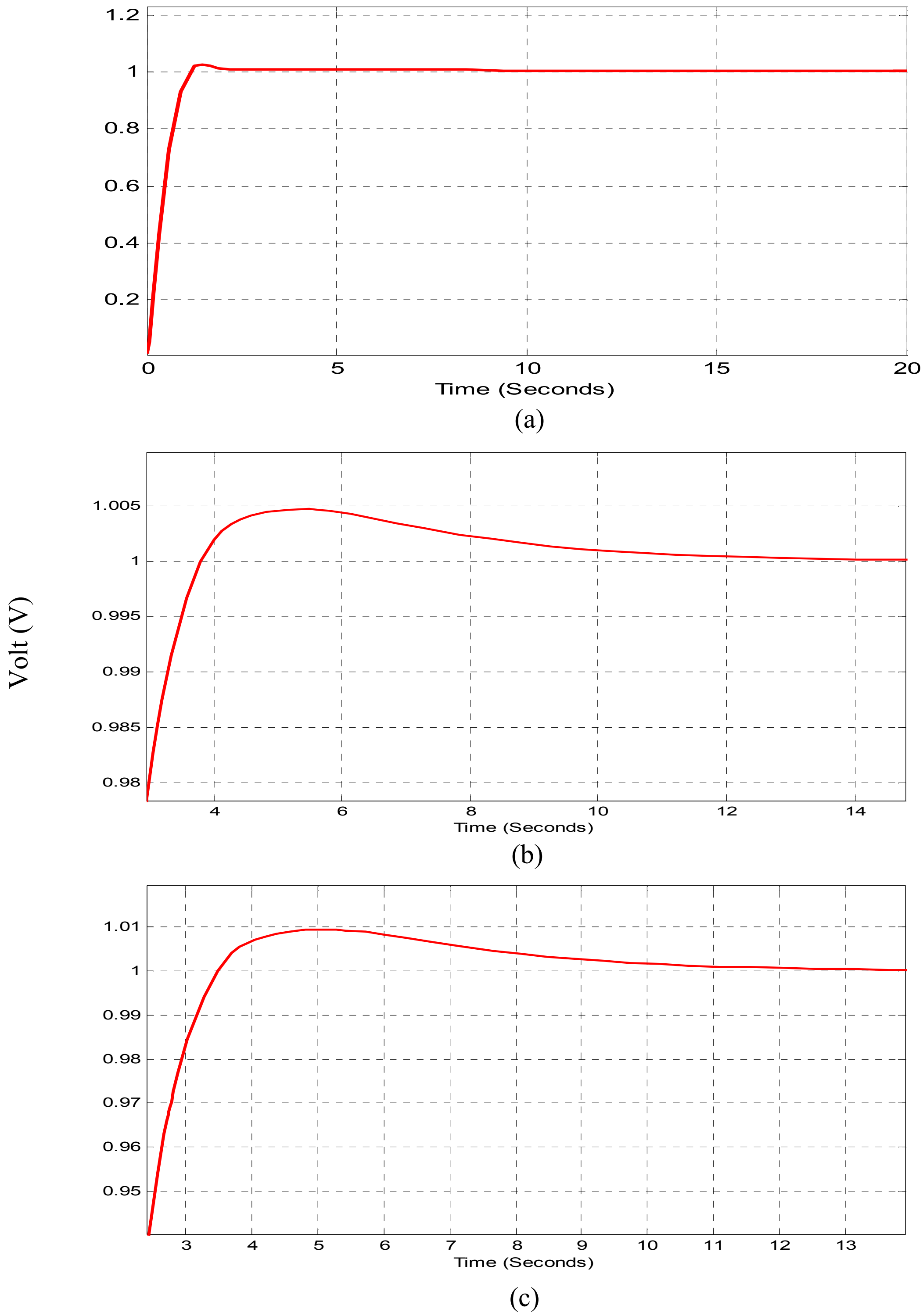


Figure 5.11: A step response for some previous work. (a) Hybrid Fuzzy PI+D Control of Nonlinear Systems. (b) Hybrid Fuzzy PID Control of Robotics Systems. (c) Fuzzy Control Systems via LMIs

The salient disadvantage in Fed PID control is the delay time. Delay time is the half time of the unit step response where the half time cross the curve. Delay time is proportional with rising time. Table 5.2 illustrates the results of the proposed approach and compares it with three previous studies in rising time.

Table 5.2: Rising time various control approaches.

Desired Control Approach	***Rising Time (sec)***
Mamdani Conventional PID Control	1.876
Mamdani Hybrid Fuzzy Fed PID Control	2.135
Mamdani Hybrid Fuzzy PID Control	1.645
Takagi-Sugeno Hybrid Fuzzy Fed PID Control	1.952
Mamdani Hybrid Fuzzy Fed PID Control	1.783
Takagi-Sugeno Hybrid Fuzzy Fed PID Control	1.952
Hybrid Fuzzy PI+D Control of Nonlinear Systems [8]	1.034
Hybrid Fuzzy PID Control of Robotics Systems [12]	0.893
Fuzzy Control Systems via LMIs [36]	1.356
27 rules Takagi-Sugeno Hybrid Fuzzy Fed PID Control	1.952
12 rules Takagi-Sugeno Hybrid Fuzzy Fed PID Control	2.078

Simply, you can compare among various values of rising time. The rising time increase in each control when the improvement in overshoot and steady state error. In other words, when the Fed PID control is used, the improvement in overshoot and steady state error is achieved, then, the delay time will be increase.

CHAPTER 6

Conclusion

In this thesis, a new approach of control design of a hybrid fuzzy Fed PID controller was proposed using the Takagi-Sugeno method. Instead of analyzing the fuzzy controller by numerical calculations, the proposed design method focused on constructing the fuzzy rule base. The proposed controller demonstrated excellent control performance for nonlinear robot which depended on the hybridizing of the gain scheduling method and Fed PID Takagi-Sugeno controller which gave the best control specifications towards the conventional PID, fuzzy PID and hybrid fuzzy PID. The proposed problem was considered one of the hottest and useful topics in the area of fuzzy control field related with robotics systems. The research began with a brief description of PID control and a quick eye on the Fed PID control. Then the research goes to a detailed illustration to the fuzzy control and its implementation beside the small points which be lemmas today. The implementation of the hybrid fuzzy Fed PID control design was defined and designed in the fifth chapter and give the best results among the other old results without using the Fed PID. The nonlinear chapter was adopted without details of nonlinear systems, but only it described the system and the method which used in the design. Finally, made of reduction of the fuzzy rules, the conclusion and the future work.

Future research can be done in the area of Hybrid Fuzzy Fed PID controller design by extending the results obtained here to the case of output feedback controllers. The results can be extended to the problem of non-fragile implementation of such controllers. Our approach was able to provide resiliency with respect to the controller gains. However, in practice one might need to have resilience with respect to variations in the electronic components that the controller is made of. Also the effect of truncation of the parameters can be an important research direction. In the area of T-S fuzzy systems, we need to look for stability results that take into account the properties of fuzzy implications and membership functions to reduce the conservatism in our stability results. In other words, we did not utilize the membership functions in proving our stability results. However, if the membership functions are available, we should look for stability results that use the information of the membership functions. Another direction for future research is the approximation accuracy of T-S fuzzy systems. At present this is a very active area and several researchers have reported some relative success. Also the concentration on the robust control and the uncertainty field can be developed and applied for the Fed PID control [37]. The implementation of the new idea on the FPGA can be considered one of hot topics to be applied in the field of applied engineering.

Bibliography:

[1] E H Mamdani, “Application of Fuzzy Algorithms for the Control of a Dynamic Plant” *IEE* vol. 121, pp. 1585-1588, 1974.

[2] L. A. Zadeh, “Fuzzy logic = computing with words,” *IEEE Transactions on Fuzzy Systems*, vol. 4, pp. 103–111, May 1996.

[3] G. Chen, T. Pham, *Introduction to fuzzy sets, fuzzy logic, and fuzzy control systems*. Boca Raton: CRC PRESS, 2001.

[4] Z. Y. Zhao, M. Tomizuka, and S. Isaka, “Fuzzy gain scheduling of PID controllers,” *IEEE Transactions on Systems, Man and Cybernetics*, vol. 23, pp. 1392–1398, Oct. 1993.

[5] L.X. Wang, *A course in Fuzzy systems and controls*. Englewood Cliffs: Prentice-Hall, 1997.

[6] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*. Englewood Cliffs: Prentice-Hall, 1994.

[7] W. Li, “Design of a hybrid fuzzy logic proportional plus conventional integral-derivative controller,” *IEEE Transactions on Fuzzy Systems*, vol. 6, pp. 449–463, Nov. 1998.

[8] M.J. Er and Y.L. Sun, “Hybrid fuzzy proportional-integral plus conventional derivative control of linear and nonlinear systems,” *IEEE Transactions on Industrial Electronics*, vol. 48, pp. 1109–1117, Dec. 2001.

[9] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, pp. 338-353, 1965.

[10] E. H. Mamdani and S. Assilian, “An experiment with in linguistic synthesis with a fuzzy logic controller,” *International journal of Man-Machine studies*, vol. 7, pp. 1-13, 1975.

[11] Ch. Clifton, A. Homaifar and M. Bikdash, “Design of generalized sugeno controllers by approximating hybrid fuzzy PID controller,” *IEEE Transactions on Fuzzy Systems*, vol. 3, pp. 1906-1911, 1996.

[12] Y. Lei Sun, M. Joo Er, “Hybrid Fuzzy Control of Robotics Systems,” *IEEE Transactions on Fuzzy Systems*, vol. 12, pp. 755-765, Dec 2004.

[13] B. KUO, *Automatic Control Systems*, Englewood Cliffs: Prentice Hall, 1995.

[14] A. Tewari, *Modern Control Design with MATLAB and Simulink*, Chichester: Wiley, 2002.

[15] N. Nise, *Control Systems Engineering*, Hoboken: Wiley, 2004.

[16] B. KUO, *Digital Control Systems*, New York: Oxford, 1992.

[17] G. Franklin, J. Powell, A. Naeini, *Feedback Control of Dynamic Systems*, Englewood Cliffs: Prentice Hall, 2002.

[18] CH. Philips, H.Nagle, *Digital Control System Analysis and Design*, Englewood Cliffs: Prentice Hall, 1995.

[19] L. Jain, N. Martin, *Fusion of Neural Networks, Fuzzy Sets, and Genetic Algorithms Industrial Applications*, Boca Raton: CRC PRESS, 1999.

[20] H.Nguyen, N. Prasad, *Fuzzy Modeling and Control*, Boca Raton: CRC PRESS, 1999.

[21] M. Jamshidi, N. Vadiee, and T. Ross, *Fuzzy Logic and Control Software and Hardware Applications*, Englewood Cliffs: Prentice Hall, Inc. 1993.

[22] W. Pedrycz, *Fuzzy Control and Fuzzy Systems*, Third Avenue, New York: Wiley, 1993.

[23] B. Hamed, "Comparison of fuzzy logic and classical controller designs for nonlinear systems", PhD dissertation, New Mexico State University, Las Cruces, 1999.

[24] P. Moratori, A. Cruz, C. Lima, "Comparing Robustness and Performance of Hybrid and Non-Hybrid Fuzzy Controllers aimed to guide a simulated robot," *Proceedings of IEEE International Conference on Fuzzy Systems*, pp. 1126-1132, July 2006.

[25] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetic*, vol. 15, pp. 116-132, June 1985.

[26] H. K. Khalil, *Nonlinear Systems*. Englewood Cliffs: Prentice Hall, 2002.

[27] H. B. Kazemian, "The SOF-PID Controller for the Control of a MIMO Robot Arm," *IEEE Transactions on Fuzzy Systems*, vol. 10, pp. 523-532, Aug 2002.

[28] K. Kiimbla, M. Jamshidi, "Hybrid Fuzzy Control Schemes for Robotic Systems," *Proceedings of IEEE International Symposium on Intelligent Control*, pp. 171-176, August 1995.

[29] O. Castillo, P. Melin, "A General Method for Automated Simulation of Non-Linear Dynamical Systems using a New Fuzzy-Fractal-Genetic Approach," *IEEE Trans Evolutionary Computation,* vol. 3, pp. 2333-2340, 1999.

[30] K. J. Astrom, B. Wittenmark, *Adaptive Control*. New York: Addison Wesley, 1989.

[31] M. Valdés, J. Botía, A. Gómez-Skarmeta, "Some Applications of Hybrid Fuzzy Modeling," *Proceedings of IEEE International Conference on Fuzzy Systems*, pp. 607-612, 2005.

[32] K. K. Tan, Q. G.Wang, and C. C. Hang, *Advances in PID Control*. New York: Springer-Verlag, 1999.

[33] D. Basmadjian, *Mathematical Modeling of Physical Systems*, New York: Oxford, 2003.

[34] M. Spong, S. Hutchinson, M. Vidyasagar, *Robot Modeling and Control*, Hoboken: Wiley, 2006.

[35] R. Sehab, "Fuzzy PID Supervision for Nonlinear System: Design and Implementation," *Proceedings of IEEE North America Information processing society*, pp. 36-41, June 2007.

[36] K. Tanaka, T. Ikeda, H. Wang, "Fuzzy Control System Design via LMIs," *IEEE American Control Conference*, vol. 5, pp. 2873-2877, June 1997.

[37] J. Burl, *Linear Optimal Control H2 & H∞ Methods*, Harlow: Addison Wesley, 1999.