The Islamic University of Gaza
Deanery of Graduate Studies
Faculty of Engineering
Electrical Engineering Department

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عمادة الدراسات العليـا
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Parameterization for Time-Delay Systems Based on Passivity and LMI Approach

By

Mohammad H. Aburezeq

Supervisor

Dr. Hatem Elaydi

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بناءً على موافقة عمادة الدراسات العليا بالجامعة الإسلامية بغزة على تشكيل لجنة الحكم على أطروحة الباحث/ **محمد حسن عيد أبو رزق** لنيل درجة الماجستير في **كلية *الهندسة*** **قسم الهندسة الكهربائية–أنظمة التحكم** وموضوعها:

Parameterization for Time-Delay Systems Based on Passivity and LMI Approach

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د. حاتم علي العايدي **مشرفاً ورئيساً** ....

د. باسل محمود حمد **مناقشاً داخلياً** ....

د. إياد محمد أبو هدروس **مناقشاً خارجيًا** ....

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واللجنة إذ تمنحه هذه الدرجة فإنها توصيه بتقوى الله ولزوم طاعته وأن يسخر علمه في خدمة دينه ووطنه.

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أ.د. فؤاد علي العاجز

ABSTRACT

Control systems can be solved using optimization after being parameterized. Time-delays and uncertainty make it more difficult to obtain optimal solutions. In this work, it is proved that the stability properties of the time delay system can be easily and efficiency achieved using passivity properties in terms of Linear Matrix Inequality techniques (LMI) through effective and reliable optimization algorithms especially convex optimization tools. In this thesis we exploit an appropriate Lyapunov-Krasovskii function that contains both double and triple integral terms and to our knowledge no one have used triple integral term with combination of the passivity conditions; thus constitute the main contribution of this thesis. Thus, constitute moreover, Jensen’s inequality was utilized to deal with cross product terms that appeared when we derive the derivation of Lyapunov-Krasovskii function. Both delay-independent and delay-dependent cases are considered. New delay dependent stability bound for particular time delay systems is derived. This is clear through various numerical examples solved by convex optimization algorithm specifically by CVX toolbox under MATLAB package. Also we deal with the uncertainty that appeared in the control systems with delay. The above technique is used to construct passive robust controller renders the closed loop uncertain time delay system (UTDS) asymptotically stable; in addition, the stability analysis and synthesis of time varying systems with state and input delays is investigated using proposed method with " change of variables method" which make the solution of the particular problem easy and construct the controller directly by inverse transformation as well be seen in the sequel. The effectiveness of the proposed method is shown through several numerical examples. Based on the proposed method exploited in this thesis, at analysis phase, the time delay bound achieved by our approach is less conservative. In the synthesis phase concerns uncertain passive and uncertain H_∞ controller design less disturbance attenuation level of the time delay has been obtained using proposed method.

ملخص

" دراسة أنظمة التحكم الخطية ذات التأخير الزمني بناء على ما يعرف بخاصية الكمون في النظام عن طريق تقنية متباينة المصفوفات الخطية"

في هذه الأطروحة تم دراسة خاصية ما يعرف بالكمون في أنظمة التحكم الخطي ذات التأخير الزمني لما لهذه الخاصية من أهمية كبرى في حقل أنظمة التحكم الآلي ومن خلال هذه الدراسة تم ربط الخاصية تلك بمفهوم ما يسمى بدالة ليابونوف-كراسوفسكي حيث ومن خلال الدراسات والأبحاث في مجال التحكم الآلي تم التأكيد على عمق العلاقة بين مفهوم الكمون في الأنظمة و دالة ليبانوف وبالتالي يمكننا وباستخدام تقنيات فعالة وعبر برمجيات مخصصة لحل مثل هذه المسائل الحصول على نتائج لم تكن تحل بالطرق الأخرى الاعتيادية أو في أفضل الأحوال لم نكن لنحصل على نتائج مرضية وبسهولة كما سنلاحظ من خلال مناقشة هذه الأطروحة حيث تم تطبيق التقنية المذكورة على بعض المسائل التي نشرت في مجلات وأوراق علمية وبمقارنة تلك النتائج مع النتائج التي حصلنا عليها نجد أن نتائجنا أفضل من حيث أننا حصلنا على حد أعلى لقيمة التأخير الزمني التي ممكن أن يتعامل معه النظام دون أن يخرج نظام التحكم "تحت الدراسة" من حالة الاستقرار. علاوة على ذلك وعلى حد علمنا نستطيع أن نقول أننا نحن أول من استخدم التكامل الثلاثي مع خاصية الكمون حيث حصلنا على نتائج أفضل من تلك النتائج التي حصلنا عليها باستخدام التكامل الثنائي الذي استخدمناه نحن في هذا المبحث ناهيك عن النتائج الأخرى وكما سيتضح لاحقا من خلال حل بعض الأمثلة التي تزخر بها هذه الأطروحة ومن خلال المقارنات مع النتائج الموجودة حاليا وبالاضافة إلى ذلك لقد تم مناقشة متانة النظام ذات التأخير الزمني من خلال تصميم متحكم متين يأخذ بعين الإعتبار التشويش الذي يتعرض له النظام ويعمل هذا المتحكم على المحافظة على حالة الاستقرار للنظام مع موثوقية الأداء في مجال محدد بغض النظر عن التشويشات الخارجية.

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ABBREVIATIONS

AS	Asymptotically Stable
ARI	Algebraic Riccatti Inequality
AF	Analytic Function
BIBO	Bounded Input Bounded Output
LKF	Lyapunov-Krasovskii Functional
LTI	Linear Time Invariant
LMI	Linear Matrix Inequality
BRL	Bounded Real Lemma
PRL	Positive Real Lemma
PS	Passive System
SISO	Single Input Single Output
MIMO	Multi-Input-Multi-Output
SP	Strictly Passive
SFC	State Feedback Controller
OFC	Output feedback Controller
SF	Storage Function
SR	Supply Rate function
SS	State Space
TDS	Time Delay System
UTDS	Uncertain Time Delay System
TVS	Time Varying System
I/O	Input / Output

CHAPTER 1   INTRODUCTION

1.1. Background and Motivation

Time delay systems (TDS), in many references are known as dead time processes (DTP) and we encounter them in different branches in control fields, such as chemical engineering systems, lag transportations, product manufactories, robotics, telecommunications, biosystems, underwater vehicles and so on [1, 2, 3]. Time delay systems are difficult to deal with, because the presence of the delays may cause the system to be unstable or at least it degrades the performance of the control systems. From our knowledge during the courses studied in control theory we know that the delays in the systems produce a decrease in the system phase and also it impose a more restrictions and constraints on the system analysis and controller's design [2]. For these reasons and others the control issues of the time delay systems was one of the most important fields that attracted the attention of many engineers and researches. At the end, the engineers developed the first controller which takes the delays into account. This controller or compensator was the Smith predictor that was developed in 1957 but the Smith predictor has drawback that does not applicable for unstable systems. In spite of these efforts, several problems still remain open and every year many papers are written to deal with different aspect of time delay process control [3], and it motivates us to exploit different methods for studying the behavior of the time delay systems. In this work, we will deal with this topic from different point of view i.e. we will not follow the conventional ways based basically on the transfer function representation of the system, instead we will deal with state space representation of the system which is more suitable for modern optimization techniques such Linear Matrix Inequality (LMI) approach and passivity notion used in this theses. In literature, there are not much surveys for time delay systems based on passivity notions and LMI approach despite the importance for these concepts and the direct relation between passivity properties and stability criteria, and this is in turn motivates us to take and work under this topic. It is true that the notion of passivity and generalization of this notion (dissipativity) date back to early 1960. The first one who studied the concept of passivity was Popov [3] and he related this to the electrical networks which contain passive elements and does not generate energy. A key concept of dissipative and in turn the passive systems are that of storage functions and supply rate functions [2,3,4], and these concepts can be understood under certain conditions as a Lyapunov functions and in turn we can easily express these notions in terms of convex optimization approaches such as in our case Linear Matrix Inequality (LMI) method. The main idea behind studying the dissipativity and passivity properties of the system is that many important physical systems have certain input-output properties that are related to conservation, dissipation, and transport of energy [5], and this is in turn, lead us to so called energy based control theory that is strongly deals with Lyapunov function which is known and may be the first example for the LMI. The most commonly used representation for describing TDSs is functional differential equations [6], we will deal basically with such types of TDSs. In addition, we will discuss the problem of robustness of TDSs, since it is very important issue in control theory to guarantee the stability and performance criteria for the closed loop control systems despite of the unmodelling errors appeared in inaccurate mathematical model of the real plants and the disturbances affected the control system, or variation of the parameters of the model. These together impose more difficulties for designing the controller that renders the closed loop TDS stable, and such controllers are called robust controllers.

1.2. Research Problems

One of the most properties of passivity is that, the passive systems are minimum phase, and thus very easy to control via state and output feedback, even if they are highly nonlinear and/or coupled [5]. Another important class of passivity or strict passivity is a structural property which is not dependent on the numerical values of the parameters of the systems. Then passivity considerations may be used to establish stability even if there are large uncertainties or large variations in the system parameters [5]. In the light of these properties of passivity, in this research we will study the stability analysis and controller design and synthesis for continuous time delay systems with uncertainty based basically on the notion of passivity as a particular form of dissipativity and ensure stability and robustness. Two cases of time delay system's studying presented in this work; the first one was the independent delay case, in this type we excluded the delay from the studying and we take into account the delay matrix only, the second one dealt with the delay in the system (delay dependent case) and we take into account the effect of the delay on the performance of the system and using mathematical tools such as Jenson's inequality to get maximum upper bound of the delay that can the system tolerates it without destroy passivity and in turn the stability of the control system. Also we designed state feedback (SFC) for the first type of the time delay system described here. For the second type we constructed state feedback controller that satisfy H_∞ performance and state feedback controller that satisfy positive realness or passivity γ_p performance of the system. Let us summarize the stability and stabilization problems investigated in this thesis:

Given TDS which contains discrete and time-varying delays in the state or in the control or in both the state and the input control channels, obtain improved stability conditions with larger upper bound of delays that the system can be tolerate without affecting the stability criterion. As the case study we discuss the Construct state feedback controller and output feedback controller render the closed loop control system asymptotically stable, despite the size of delay. In addition for a given UTDS with discrete delay in the state and with perturbation in the control gain, construct robust controller renders the UTDS asymptotically robustly stable. Moreover, for TDS with varying delays in the input and state channels, design state feedback passive controller such that the closed loop control system is asymptotically stable. Finally, for a given TDS, construct state feedback controller such that the closed loop control system satisfy the H_∞ or passivity performances.

1.3. Research Objectives

In this subsection we will sum up the steps that will be followed to get accepted results based on the proposed approach exploited in this thesis. Firstly, as mentioned above that, the basic representation for time-delay system used in recent work is differential deference functional, so we construct such function, called a Lyapunov-Krasovskii function with quadratic and double integral terms which contain variable matrices to be found, hence guarantee the stability criterion by LMI optimization approach. Note that this method does not contain any tuning parameters (scalar or matrices) as in the case with the method in [7]. Then triple integral term used to reduce the conservatism of the TDS. Next, we deal with uncertainty in the controller itself and derive delay dependent stability and performance analysis for the robust control problem. After that, we used system transformation, in the sense to derive the upper bound of the delay; the system can be tolerated without destroying the passivity, hence, the stability and the performance criteria. In addition, the change of variables was used to make the computation effort easy and efficient. Finally, all aforementioned steps were casted in LMI optimization problem.

1.4. Literature Review

As mentioned above in the introduction section, there are two categories when deals with delays in the control systems, the first one is delay-independent criterion and the second is delay-dependent criterion, the later is less conservative, and the former is applicable when the delay in the system is small, and in turn these delays impose restrictions on the synthesis of controller and impose difficulties for studying; thus motivating the researchers and control engineers to investigate. In this subsection we briefly address the categories of time delay systems, approach used to derive effectiveness results and analysis and synthesis of time delay systems based on the proposed approach. The two categories are delay-independent and delay-dependent categories, the later is less conservative and in this thesis both input delay and state delay are considered, in addition the uncertainty in the system is discussed, and the approach exploited basically based on Lyapunov-Krasovskii functional contained both double and triple integrals and quadratic term with combination with passivity conditions, then the problem casted into optimization problem subject to LMI constraints. Now let us list some previous works related to ours:

1. In 1998, Lihua et. al. [8] studied the problems of robust passivity analysis and passification for a large class of uncertain systems with the uncertainty described by integral quadratic constraints. LMI solutions have been presented. Their results offered efficient solutions for several problems encountered in signal processing systems involving nonlinear elements. Their work was been done for system without time delay, but in my work I well give into consideration time delay in the control systems.

2. In 1999, Huang et. al. [9] presented an LMI approach to the strictly positive real (SPR) synthesis problem by finding an output feedback K such that the closed loop system is SPR. They also developed necessary and sufficient conditions for the plant state space matrices that guarantee the existing of a constant output feedback gain matrix K so that the closed loop system is SPR

and these conditions were casted as LMIs . They showed that the existence of K for the closed loop system to be SPR can be used to generate an adaptive control regulator that can stabilize any plant with arbitrary order and unknown parameters and regulate its output vector to zero. They worked with positive realness, since there is one to one relationship between passivity and positive realness. However, they only dealt with systems with no time delay.

3. In 2002, Fridman and Shaked, [10] proposed a delay-dependent solution for the problem of passive state feedback control of linear time invariant neutral and retarded type systems. The solutions provided sufficient conditions in the form of LMI.

4. In 2005, Peaucelle et. al. [11] presented non-conservative LMI conditions of robust strict G-passification. The main goal of the paper was to obtain necessary and sufficient conditions of robust passifiability and to develop techniques of robust passification for linear proper MIMO systems. A more general problem of G-passification of non-square systems was studied with conditions and design technique heavily relied on the methodology of (LMI) and using appropriate software. Again, this work was on systems with no time delays.

5. In 2005, Min Gang Hua et. al. [12] addressed dynamic output feedback passive control for neutral systems with delay in control input, and was concerned with the problem of passive control for a class of neutral systems with delay in control input. Then, they designed a dynamic output feedback passive controller which guaranteed the passivity of the systems, and derived passivity criterion in terms of LMIs. However they only addressed the specific kind of systems (neutral) and with delay only in control input.

6. In 2007, Zho Bao Yan et. al. [13] addressed the problem of passivity control for a kind of uncertain T-S fuzzy descriptor system. They gave a method to check the admissibility of the system. They proposed the controller that made the closed loop system admissible and strictly passive in terms of LMIs . There were no delays in the systems for this work.

7. In 2008, Magdi S. Mahmoud et. al. [14] established a new results for the problems of the dissipative analysis and state feedback synthesis of singular time delay systems in the states. The developed results encompassing all available results on H infinity approach, passivity and positive realness for singular time delay systems as special cases. Both delay dependent and delay independent cases were investigated and all sufficient stability conditions are cast as a linear matrix inequality. However, they used only the delay within the state of the systems..

8. In 2008, Nichil Chopra [15] studied the passivity of feedback interconnected of two passive systems when there were time varying delays in the communication. He transformed the two systems into scattering representation, transmitting the scattering variables, and using time varying gains in the communication path, passivity of the feedback interconnection can be guaranteed independent on the time varying delays. As shown he didn't use the LMI approach to solve the problem.

9. In 2010 Baozhu Du [16] his thesis devoted to study the stability and stabilization problems of dynamic systems with various types of time delays in the continuous time domain. In chapter 5 he studied H_∞ and passivity analysis via static and integral output feedback control for systems with input delay only. However, he did not take into account delays in both state and input channels.

1.5. Contributions

The main contributions of this thesis are the parameterization of time delay systems based on passivity and LMI approach. These contributions can be stated clearly as:

In the analysis phase, the delay bound of the time delay systems is improved compared with existing criteria using Lyapunov Krasovskii functional which contains double integral terms with unknown positive definite matrices completely defined with the software. Thus, there is no need to tune the parameters to get better results. In the sequel, we get improvement compared with other criteria that depend mainly on tuning parameters to achieve valuable results. Also Lyapunov Krasovskii functional is used which contains triple integral terms with passivity concepts, and this in turn will give us more improvement and less conservative results.

In the synthesis phase, positive real lemma (passivity) and H infinity methods with Lyapunov Krasovskii-functional mentioned above for the closed loop systems. Also, robustness stability and performance for time delay systems were discussed. Finally, stability analysis for systems with time delays and uncertain parameters in the system and in the controller were addressed.

1.6. Preliminary Work

1.6.1 Notations & Terminology

Let us list some useful definitions for the proceeding work to be more understood and clear

The matrix Q is said to be positive definite (positive semi definite) if the next inequalities hold $Q > 0$ ($Q \geq 0$) respectively. In the same fashion it said to be negative definite (negative semi definite) if the next hold $Q < 0$ ($Q \leq 0$). $Q = Q^T$ Symmetric matrix$Q$, Q^T transpose of matrix Q .

$\|.\|_\infty$ Infinity norm.

$\|.\|_2$ Euclidian norm or 2-norm.

$L_2[0, \infty)$ Refers to the space of square summable infinite vector sequences.

1.6.2 Definitions and Lemmas

In this subsection we will discuss some useful facts and lemmas that help us to derive appropriate mathematical expressions through that we will get the results indicate the effectiveness of the proposed method:

Definition 1 (Bounded Real Lemma) A continuous time linear TDS with disturbance $\omega \in L_2[0,\infty)$ and regulated output z is said to be satisfying the $\gamma_\infty or\ H_\infty$ performance if the following conditions hold:

1. The system is asymptotically stable for $\omega = 0$.

2. Under zero initial condition, for $\gamma_\infty > 0$ and $\tau \geq 0$, system $\sum$ satisfies $\int_0^\tau z^T(s)z(s)ds \leq \gamma_\infty^2 \int_0^\tau \omega^T(s)\omega(s)ds$.

Definition 2 (Positive Real lemma) A continuous time linear TDS with disturbance ω and regulated output z is said to be passive if there exists a scalar $\gamma_p \geq 0$, such that under zero initial conditions and for $\tau \geq 0$, $2\int_0^\tau \omega^T(s)z(s)ds \geq -\gamma_p \int_0^\tau \omega^T(s)\omega(s)$.

Lemma 1 (Schur Complement) [17]

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

Where $Q(x) = Q^T(x), R(x) = R^T(x), and\ S(x)$ depends affinely on x is equivalent to

$$R(x) > 0, Q(x) - S(x)R(x)^{-1}S^T(x) > 0,$$

Or

$$Q(x) > 0, R(x) - S(x)Q(x)^{-1}S^T(x) > 0.$$

Here we use the Schur complement to convert the nonlinear inequality into linear inequality.

Fact 1 for any real matrices $\Sigma_1, \Sigma_{2,}, \ and\ \Sigma_3$ with appropriate dimensions such that$0 < (\Sigma_3 = \Sigma_3^T)$, it follows that the next $\Sigma_1^T\Sigma_2 + \Sigma_2^T\Sigma_1 \leq \Sigma_1^T\Sigma_3\Sigma_1 + \Sigma_2^T\Sigma_3^{-1}\Sigma_2$ [7] holds.

Lemma 2 for any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, and a scalar $\gamma > 0$, vector function $x: [0,\gamma] \to R^n$ such that the integrations concerned are well defined, then

$$\gamma \int_0^\gamma x^T(s)Mx(s) \geq \left(\int_0^\gamma x(s)ds\right)^T M \left(\int_0^\gamma x(s)ds\right)$$

Lemma 3 [18] *for any scalar* $h > 0$ *and any constant matrix* $M = M^T > 0$ *the following inequality holds*

$$\frac{h^2}{2}\int_{t-h}^{t}\int_{s}^{t} x^T(u)Mx(u)duds \geq \left(\int_{t-h}^{t}\int_{s}^{t} x(u)duds\right)^T M \left(\int_{t-h}^{t}\int_{s}^{t} x(u)duds\right)$$

for proof see the Appendix.

Lemma 4 [19]: Let$\Upsilon, \Phi, \Psi, \Omega\ and\ F$ be real matrices of appropriate dimensions such that $\Omega > 0\ \ and\ F^TF \leq \mathrm{I}$ then we have the following

1. For a scalar $\varepsilon > 0$, $\Phi F\Psi + (\Phi F\Psi)^T \leq \varepsilon^{-1}\Phi\Phi^T + \varepsilon\Psi^T\Psi$.

2. For any scalar $\varepsilon > 0$ such that $\Omega - \varepsilon\Phi\Phi^T > 0$,

$$(\Upsilon + \Phi F\Psi)^T\Omega^{-1}(\Upsilon + \Phi F\Psi) \leq \Upsilon^{\mathrm{T}}(\Omega - \varepsilon\Phi\Phi^{\mathrm{T}})^{-1}\Upsilon + \varepsilon^{-1}\Psi^{\mathrm{T}}\Psi$$

Lemma 5[19]: For any matricesx, y $constant$ $\varepsilon > 0$, and time varying matrix $F(t)$ satisfying$F^T(t)F(t) \leq \mathrm{I}$, we have $x^TF(t)y + y^TF^T(t)x \leq \varepsilon x^Tx + \varepsilon^{-1}y^Ty$

1.7. Structure of the Thesis

The thesis is organized as follows:

Chapter 2 is dedicated to the study of the passivity analysis of TDSs independent of delays, passivity in the control theory and the relation between the passivity and the positive realness. Chapter 3 discuss the analysis and synthesis for SF controller independently on delay. Sufficient conditions are derived so the overall closed loop control system with time delay matrix renders passive, and hence asymptotically stable. Chapter 4 deals with systems that have the dependence of delays. Chapter 5 studies the construction of H_∞ and γ_∞ performance criteria and at the same time construct H_∞controller that meets the required performance criterion (disturbance attenuation bound). Chapter 6 concludes the work on this thesis.

CHAPTER 2 PASSIVITY ANALYSIS FOR TDS

2.1. Introduction to Passivity in Control Theory

Passive systems are the class of processes that dissipate certain type of physical or virtual energy, described by Lyapunov-like functions [4]. As mentioned in the previous chapter, the important concepts of passive systems are supply rate and storage function, see Figure (2.1).

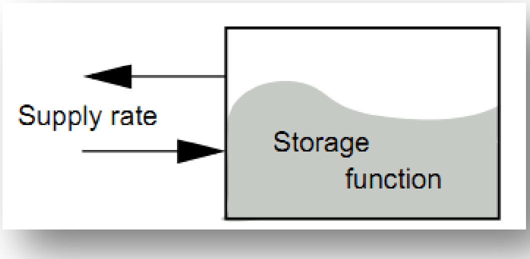


Figure (2.1) Illustration of supply rate and storage function

Passivity, originally a concept from electrical network theory, was first studied in control theory by Popov in the 1960's. The concept of passivity is related basically with the networks that consist of resistors, capacitors and inductors (RLC circuits) as shown in Figure (2.2).

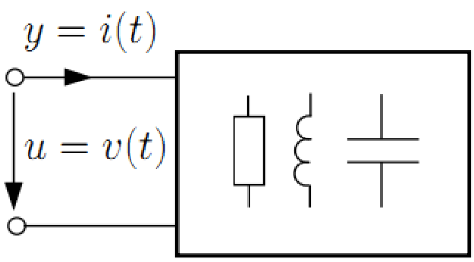


Figure (2.2) RLC circuit with power supply $p(t) = v(t)i(t)$

The differential equation of this circuit is:

$$L\frac{di}{dt}(t) + Ri(t) + Cx(t) = u(t) \tag{2.1}$$

where

$$x(t) = \int_0^t i(t')dt' \tag{2.2}$$

The energy stored in the system is

$$V(x,i) = \frac{1}{2}Li^2 + \frac{1}{2}Cx^2 \tag{2.3}$$

The time derivative of the energy when the system evolves is

$$\frac{d}{dt}V\big(x(t),i(t)\big) = L\frac{di}{dt}(t)i(t) + Cx(t)i(t) \tag{2.4}$$

Inserting the differential equation of the circuit we get

$$\frac{d}{dt}V\big(x(t),i(t)\big) = u(t)i(t) - Ri^2(t) \tag{2.5}$$

Integrating (2.5) from $t = 0\ to\ t = T$ gives

$$V\big(x(T),i(T)\big) = V[x(0),i(0)] + \int_0^T u(t)i(t)dt - \int_0^T Ri^2(t)dt \tag{2.6}$$

This means that, the energy at time $t = T$ is the initial energy plus the energy supplied to the system by the voltage u minus the energy dissipated by the resistor R. Note that if the input voltage u is zero, and if there is no resistance, then the energy $V(.)$ of the system is constant. Here $R \geq 0\ and\ V[x(0),\dot{x}(0)] > 0$, and it follows that the integral of the voltage u and the current i satisfies

$$\int_0^t u(s)i(s)ds \geq -V[x(0),i(0)] \tag{2.7}$$

The physical interpretation of this inequality is seen from the equivalent inequality

$$-\int_0^t u(s)i(s)ds \leq V[x(0),i(0)] \tag{2.8}$$

Which shows that the energy $-\int_0^t u(s)i(s)ds$ that can be extracted from the system is less than or equal to the initial energy stored in the system. The Laplace transform of the differential equation of the circuit is

$$(Ls^2 + Rs + C)X(s) = U(s)$$

This leads to the transfer function $\frac{X(s)}{U(s)} = \frac{1}{Ls^2+Rs+C}$. It is seen that the system has such transfer function is stable, and that, for $s = j\omega$, the phase of the function has absolute value less or equal to 90°, that is,

$$\left|\angle\frac{i}{u}(j\omega)\right| \leq 90^{\circ} \Rightarrow Re\left[\frac{i}{u}(j\omega)\right] \geq 0 \tag{2.9}$$

For all $\omega \in [-\infty, +\infty]$. As shown from (2.9), the system is stable and has positive real part on the $j\omega$ axis.

In the light of (2.9), and because (2.7) must holds for all inputs, one obtains the so called positive real lemma, and there is one to one relationship between them based on the Kalman-Yakubovich-Popov property [5, 6]. A system is said to be positive real if for all $t \geq t_0 \geq 0.\ u \in U$

$$\int_{t_0}^t y^T(t)u(t)dt \geq 0 \tag{2.10}$$

Whenever, $x(t_0) = 0$.

It is well known based on the positive real lemma stated in [7] that passivity conditions for LTI systems can be presented and solving using LMI approach under convex optimization technique. We will devote the next subsection for introducing LMI and Convex optimization technique.

One of the useful results for passive systems is that, parallel and feedback connections of passive systems are passive and that certain strict passivity properties are inherent see Figure (2.3).

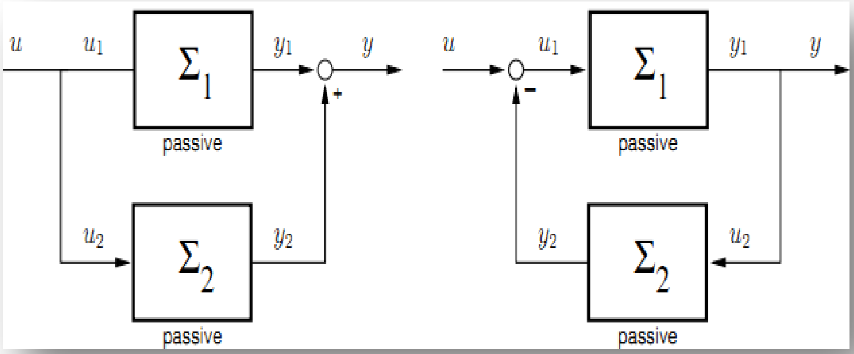


Figure (2.3) Parallel and feedback interconnection for passive systems

2.2. Convex Optimization and LMI Technique

Convex optimization problem is the one of the form

$$minimize\ f_0(x)$$
$$subject\ to\ f_i(x) \leq b_i, i = 1, \dots, m \tag{2.11}$$

Where, the functions $f_0, \dots, f_m : R^n \to R$ are convex, i.e. satisfy $f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y)$ for all $x, y \in R^n$ and all $\alpha, \beta \in R$ with $\alpha + \beta = 1, \alpha \geq 0, \beta \geq 0$

f_0 is the cost function to be optimized and in the control theory terminology, it corresponds to some performance characteristics of the control systems, such that minimization the overshoot of the closed loop system, or minimization the control energy required for the system, or so on. The constraints in (2.11) are in the form of LMI. The origin of LMI goes back as far as 1890, although they were not called this way at that time, when Lyapunov showed that, the stability of linear system $\dot{x} = Ax$ is equivalent to the existence of positive definite matrix P, which satisfies the matrix inequality $A^T P + PA < 0$. The term "Linear Matrix Inequality" was coined by Willems in 1970's to refer to this specific LMI, in connection with quadratic optimal control. As mentioned above LMI is a constraint in the form:

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i > 0 \tag{2.12}$$

Where

$x = (x_1, \dots, x_m)^T \in R^m$ is the vector of the m variables, $F_i = F_i^T > 0$ are given symmetric matrices. The inequality ">" means that the matrix $F(x)$ is positive definite, i.e., $u^T F(x) u > 0$ for all nonzero $u \in R^n$.

We can say that, if you cast a practical problem as a convex optimization problem, then you have solved the original problem [17, 21].

2.3. Passivity Properties of the Time Delay Systems

In this part of the thesis we will concentrate on the passivity conditions that the system will be met to guarantee the asymptotic stability of the linear time delay system. Let the system be described as:

$$\begin{aligned}
\dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau_1) + B_1 w(t) + B_2 u(t-\tau_2) \\
z(t) &= C_1 x(t) + D_1 u(t) + D_{11} w(t) \\
y(t) &= C_2 x(t) + D_2 w(t) \\
x(t) &= \varphi(t), t \geq 0.
\end{aligned} \tag{2.13}$$

Where

$x(t) \in R^{n_x}$ is the state; $u(t) \in R^{n_u}$ is the control input with $u(t) = 0$ for $t < 0$;

$y(t) \in R^{n_y}$ is the output measurment; w(t) $\in R^{n_p}$; exogenous input

$z(t) \in R^{n_z}$ is the controlled output;

φ(t) are continuous functions defined on (-∞, 0]. $A_0, A_1, B_1, B_2, C_1, C_2, D_1, D_{11}, D_2$ given exogenous constant matrices with appropriate dimensions, τ_1 *and* τ_2 are the state delay and the control delay respectively. The system (2.13) is approximate model for real system, namely for water quality system [14] and this is one of water quality studies on the River Nile. In a typical model, the state variables are the concentrations of pollutants P_A (represented a mixture of the low-levels in the bio-strata) and pollutant P_B (represented the mixture of the other levels in bio-strata). The control variables are signals proportional to the water speed and the amount of effluent discharged into the reach at pre-selected points. For more detail see [14] and references therein. So, we are considering this system as case study for analysis problem. Our task is to derive the passivity conditions for the system (2.13). Firstly, let us introduce the definition of passivity for time-delay control system (2.13):

Definition 2.1:
The time delay control system (2.13) is said to be passive if

$$\int_0^\infty w^T(t) z(t) dt > \beta, \forall w \in L_2(0,\infty), \tag{2.14}$$

where β some constant which depends on the initial condition of the system. In addition, the system is said to be strictly passive (SP) if it is passive and $D_{11} + D_{11}^T > 0$.

2.4. Stability Analysis of the Time Delay Systems

Begin to analysis stability of the system 2.13 in the sense of passivity notation we set u=0. Based on definition 2 in the previous chapter mainly PRL the next theorem can be exploited to derive passivity property of the system stated above:

Theorem 2.1 LTI TDS (2.13) is stable, if there exist positive definite matrices P and Q satisfying the linear matrix inequality (LMI)

$$\begin{bmatrix} A_0^T P + PA_0 + Q & PA_1 & PB_1 - C_1^T \\ * & -Q & 0 \\ * & * & -(D_{11} + D_{11}^T) \end{bmatrix} \le 0 \qquad (2.15)$$

Or equivalently, when $(D_{11} + D_{11}^T) \succ 0$, and there exist matrices $0 < P = P^T \in R^{n\times n}$ and $0 < Q = Q^T \in R^{n\times n}$ satisfying the algebraic Riccati inequality (ARI):

$$A_0^T P + PA_0 + Q + PA_1 Q^{-1} A_1^T P + (PB_1 - C_1^T)(D_{11} + D_{11}^T)^{-1}(B_1^T P - C_1) < 0 \qquad (2.16)$$

Then the system (2.13) is asymptotically stable and passive for all time delays in the state.

Proof: Define a Lyapunov functional V(x (t)) as follows:

$$V(x(t)) = x^T(t)Px(t) + \int_{t-\tau_1}^{t} x^T(s)Qx(s)ds \qquad (2.17)$$

Calculating the derivative of Lyapunov function V(x (t)) along the solution of (2.13), we get:

$$\begin{aligned} \dot V(x(t)) &= \dot x^T(t)Px(t) + x^T(t)P\dot x(t) \\ &\quad + x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1) \\ &= (x^T(t)A_0^T + x^T(t-\tau_1)A_1^T + w^T B_1^T)Px(t) \\ &\quad + x^T(t)P(A_0x(t) + A_1x(t-\tau_1) + B_1w(t)) \\ &\quad + x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1) \\ &= x^T(t)PA_0x(t) + x^T(t)PA_1x(t-\tau_1) + x^T(t)PB_1w(t) \\ &\quad + x^T(t)A_0^T Px(t) + x^T(t-\tau_1)A_1^T Px(t) + w^T(t)B_1^T Px(t) \\ &\quad + x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1) \\ &= x^T(t)\left(A_0^T P + PA_0 + Q\right)x(t) \\ &\quad + x^T(t-\tau_1)A_1^T Px(t) + x^T(t)PA_1x(t-\tau_1) \\ &\quad + w^T(t)B_1^T Px(t) + x^T(t)PB_1w(t) - x^T(t-\tau_1)Qx(t-\tau_1). \end{aligned} \qquad (2.18)$$

$$\dot{V}(x(t)) = \left[x(t) \quad x(t-\tau_1) \quad w(t)\right]^T \Sigma \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ w(t) \end{bmatrix}$$

when (2.19)

$$\Sigma = \begin{bmatrix} A_0^T P + PA_0 + Q & PA_1 & PB_1 \\ A_1^T P & -Q & 0 \\ B_1^T P & 0 & 0 \end{bmatrix}$$

So we can apply the following condition to demonstrate the passivity property for the control system (2.13):

$$\begin{aligned} \dot{V}(x(t)) - 2z^T(t)w(t) &= x^T(t)\left(A_0^T P + PA_0 + Q\right)x(t) \\ &\quad + x^T(t-\tau)A_1^T P\, x(t) + x^T(t)PA_1 x(t-\tau) + 2x^T(t)(PB_1 - C_1^T)w(t) \\ &\quad - x^T(t-\tau_1)Qx(t-\tau_1) - w^T(t)(D_{11} + D_{11}^T)w(t) \\ &= \Sigma^T(t)\prod\Sigma(t) \le 0 \end{aligned}$$

Where

$$\Sigma^T(t) = \left[x(t) \quad x(t-\tau_1) \quad w(t)\right]^T,$$

$$\prod = \begin{pmatrix} A_0^T P + PA_0 + Q & PA_1 & PB_1 - C_1^T \\ * & -Q & 0 \\ * & * & -(D_{11} + D_{11}^T) \end{pmatrix} \le 0 \qquad (2.20)$$

From Schur complement as shown in fact 1, and if (2.16) is satisfied then we conclude that (2.19) and (2.20) are hold. Hence,

$$\dot{V}(x(t)) \le 2z^T(t)w(t). \qquad (2.21)$$

Integrate (2.21) from t_0 to t_1, we have

$$\int_{t_0}^{t_1} z^T(t)w(t) > \frac{1}{2}\left[V(x(t)) - V(x(t_0))\right]. \qquad (2.22)$$

Since $V(x(t)) > 0$ for $x \ne 0$ and $V(x(t)) = 0$ for $x = 0$, it follows that as $t_0 = 0$ and $t_1 \to \infty$ that the system (2.13) is strictly passive and asymptotically stable, so the theorem is proved.
Let show a numerical example to illustrate Theorem 2.1:

Example 2.1:

Let the system matrices are as follow (River Nile system):

$$A_0 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.3 \\ -0.3 & -0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \quad D_{11} = [2].$$

Using the LMI solver and solving LMI (2.15) for the system we found:

P =

1.0218	0.7057
0.7057	3.0978

Q =

2.2562	0.6588
0.6588	1.1545

As shown $P = P^T > 0$ and $Q = Q^T > 0$; thus the system is asymptotically stable and strictly passive independent of the delay in the system. In our case the number of variables used to get these results is 26 while in [7&14] is 28 variables.

CHAPTER 3 CONTROLLER DESIGN VIA LMI TECHNIQUE

3.1. State Feedback Controller Design

Consider the system (2.13) with delays in the control input and in the state:
When we apply state feedback controller in the form

$$u(t) = Kx(t) \tag{3.1}$$

Where K is a constant gain matrix to be designed later, the closed loop system is as follow:

$$\begin{aligned} &\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau_1) + B_1 w(t) + B_2 K x(t-\tau_2) \\ &z(t) = (C_1 + D_1 K)x(t) + D_{11} w(t) \\ &y(t) = C_2 x(t) + D_2 w(t) \\ &x(t) = \varphi(t), t \geq 0. \end{aligned} \tag{3.2}$$

Theorem 3.1 Consider the system (3.2), if there exist positive definite matrices $Y = Y^T > 0, L = L^T > 0$ *and* $M = M^T > 0$, and matrix Z which satisfy the following LMI

$$\begin{pmatrix} YA_0^T + A_0 Y + L + M & A_1 Y & B_2 Z & B_1 - Y(C_1 + D_1 K)^T \\ * & -L & 0 & 0 \\ * & * & -M & 0 \\ * & * & * & -(D_{11} + D_{11}^T) \end{pmatrix} \leq 0 \tag{3.3}$$

Or if there exists $P = P^T > 0$ *and* $Q = Q^T > 0$ satisfying the algebraic inequality:

$$\begin{aligned} &A_0^T P + PA_0 + 2Q + PA_1 Q^{-1} A_1^T P \\ &+ PB_2 K Q^{-1} K B_2^T P \\ &+ (PB_1 - (C_1 + D_1 K)^T)(D_{11} + D_{11}^T)^{-1}(B_1^T P - (C_1 + D_1 K)) < 0 \end{aligned} \tag{3.4}$$

Then the system (3.2) is SP and asymptotically stable by the state feedback controller (3.3).

Proof Define a Lyapunov functional V(x(t)) as follows:

$$V(x(t)) = x^T(t) P x(t) + \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\tau_2}^{t} x^T(s) Q_2 x(s) ds \tag{3.5}$$

Calculating the derivative of Lyapunov function V(x(t)) along the solution of (3.2) , we get:

$$\dot{V}(x(t)) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)$$
$$+x^T(t)Q_1x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1)$$
$$+x^T(t)Q_2x(t) - x^T(t-\tau_2)Q_2x(t-\tau_2)$$
$$= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)$$
$$+ x^T(t)Q_1x(t) + x^T(t)Q_2x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1)$$
$$-x^T(t-\tau_2)Q_2x(t-\tau_2)$$

$$\begin{aligned}
= \; & x^T(t)\left(A_0^T P + PA_0 + Q_1 + Q_2\right)x(t) \\
& + x^T(t-\tau_1)A_1^T Px(t) + x^T(t)PA_1x(t-\tau_1) \\
& + x^T(t)PB_1w(t) + w^T(t)B_1Px(t) \\
& + x^T(t-\tau_2)K^T B_2^T Kx(t) + x^T(t)PB_2Kx(t-\tau_2) \\
& - x^T(t-\tau_1)Q_1x(t-\tau_1) - x^T(t-\tau_2)Q_2x(t-\tau_2).
\end{aligned} \tag{3.6}$$

Apply passivity condition as follows:

$$\begin{aligned}
\dot{V}(t) - 2z^T(t)w(t) = \; & x^T(t)\left(A_0^T P + PA_0 + 2Q\right)x(t) \\
& + x^T(t-\tau_1)A_1^T Px(t) + x^T(t)PA_1x(t-\tau_1) \\
& + x^T(t)PB_1w(t) + w^T(t)B_1Px(t) \\
& + x^T(t-\tau_2)K^T B_2^T Kx(t) + x^T(t)PB_2Kx(t-\tau_2) \\
& - x^T(t-\tau_1)Qx(t-\tau_1) - x^T(t-\tau_2)Qx(t-\tau_2). \\
& - x^T(t)C_1^T w(t) - w^T(t)C_1x(t) - x^T(t)K^T D_1^T w(t) \\
& - w^T(t)D_1Kx(t) - w^T(t)(D_{11} + D_{11}^T)w(t).
\end{aligned} \tag{3.7}$$

where

$$\Sigma^T(t) = \left[x(t) \quad x(t-\tau_1) \quad x(t-\tau_2) \quad w(t)\right]^T,$$

$$\Pi = \begin{pmatrix} A_0^T P + PA_0 + Q_1 + Q_2 & PA_1 & PB_2K & PB_1 - (C_1 + D_1K)^T \\ * & -Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -(D_{11} + D_{11}^T) \end{pmatrix} \le 0. \tag{3 . 8}$$

Or equivalently

$$\begin{aligned}
& A_0^T P + PA_0 + Q_1 + Q_2 + PA_1Q_1^{-1}A_1^T P \\
& + PB_2KQ_2^{-1}K^T B_2^T P + (PB_1 - (C_1 + D_1K)^T)(D_{11} + D_{11}^T)^{-1}(B_1^T P - (C_1 + D_1K)) < 0
\end{aligned} \tag{3.9}$$

Post and pre-multiplying the above inequality by P^{-1} we get the following inequality yields:

$$P^{-1}A_0^TPP^{-1} + P^{-1}PA_0P^{-1} + P^{-1}Q_1P^{-1} + P^{-1}Q_2P^{-1} + P^{-1}PA_1Q_1^{-1}A_1^TPP^{-1}$$

$$+P^{-1}PB_2KQ_2^{-1}K^TB_2^TPP^{-1} + P^{-1}(PB_1 - (C_1 + D_1K)^T)(D_{11} + D_{11}^T)^{-1}(B_1^TP - (C_1 + D_1K))P^{-1} < 0 \quad (3.10)$$

Let $P^{-1} = Y$ since $P > 0$ *so* $P^{-1} = Y > 0$, and rearrange the inequality (3.10), we get:

$$YA_0^T + A_0Y + YQ_1Y + YQ_2Y + A_1Q_1^{-1}A_1^T$$

$$+B_2KQ_2^{-1}K^TB_2^T + (B_1 - (YC_1^T + YK^TD_1^T))(D_{11} + D_{11}^T)^{-1}(B_1^T - (C_1Y + D_1KY)) < 0 \quad (3.11)$$

As shown the problem still non convex optimization since there is nonlinear (quadratic) terms YQ_1Y *and* YQ_2Y and products between the variables K *and* Y so if we define a new matrix $Z = KY$ and change of variables, since we can denote $YQ_1Y = L$ *and* $YQ_2Y = M$ and substitute into (3.10) we get:

$$YA_0^T + A_0Y + L + M + A_1Q_1^{-1}A_1^T$$

$$+B_2KQ_2^{-1}K^TB_2^T + (B_1 - (YC_1^T + Z^TD_1^T))(D_{11} + D_{11}^T)^{-1}(B_1^T - (C_1Y + D_1Z)) < 0 \quad (3.12)$$

Using Schur complement definition we can convert the nonlinear inequality (3.12) into linear matrix inequality as shown below:

$$\begin{pmatrix} YA_0^T + A_0Y + L + M & A_1Y & B_2Z & B_1 - Y(C_1 + D_1K)^T \\ * & -L & 0 & 0 \\ * & * & -M & 0 \\ * & * & * & -(D_{11} + D_{11}^T) \end{pmatrix} \leq 0 \quad (3.13)$$

From Schur complement as shown in fact 1, we notice that (3.12) and (3.13) are hold. Hence,

$$\dot{V}(x(t)) \leq 2z^T(t)u(t). \quad (3.14)$$

Integrate (3.14) from t_0 to t_1, we have

$$\int_{t_0}^{t_1} z^T(t)w(t) \geq \frac{1}{2}\left[V(x(t)) - V(x(t_0))\right]. \quad (3.15)$$

Since $V(x(t)) > 0$ for $x \neq 0$ and $V(x(t)) = 0$ for $x = 0$, it follows that as $t_0 = 0$ and $t_1 \to \infty$ that there is state feedback controller (3.2) render the system (3.1) strictly passive and asymptotically stable, so the theorem is proved.
From LMI (3.3) when the problem is solvable i.e. when the LMI (3.13) is feasible we can get the controller from the following equation:

$$K = ZY^{-1} \quad (3.16)$$

We can also get the same result by multiplying the LMI (3.13) by diag. $[P^{-1}, P^{-1}, P^{-1}, I]$ from both sides. Let us now see an example to show whether this method is workable or not.

Example 3.1:

Consider unstable nominal system, i.e. let the matrix A_0 has at least one pole in the right half plane then apply theorem 3.1 to get the controller which stabilizes the system. Let the system represented as follows:

$$A_0 = \begin{bmatrix} 1 & -2 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad D_{11} = [1].$$

The eigenvalues of the system are 1.4742 and -2.3742. It is clear that the system is unstable because it has pole in the right half plane.
Using LMI (3.3) we can get controller with gains that stabilizes the unstable system
$K = [\text{-0.1191} \quad 0.0693]$
When simulating the system under initial conditions the system response goes to infinity as time goes to infinity, hence the open loop system is unstable, see Fig.(3.1)

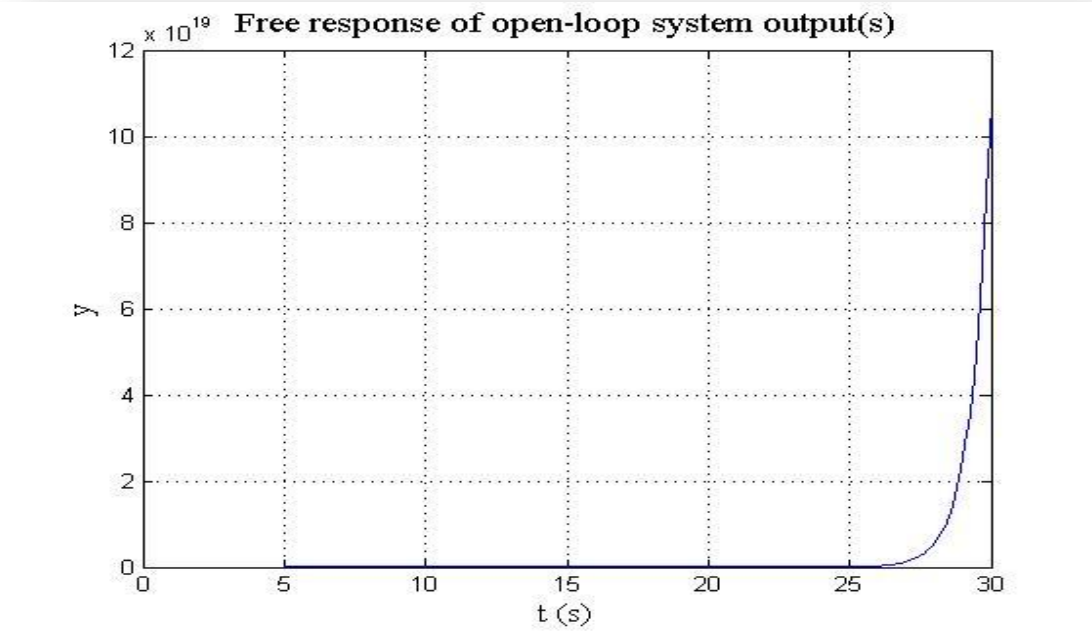


Figure (3.1) Open loop free response of the system in example (3.1)

It is clear from the Fig. (3.1), that is the open loop system is unstable.
Now applying obtained controller we get the free response of the closed loop control system. The obtained controller actually stabilizes the unstable plant considered in this example, and this is clear from the free response of the closed loop control system when the system is affected by initial conditions and by feedback controller obtained the system became stable and this is in turn clear from the Fig. (3.2)

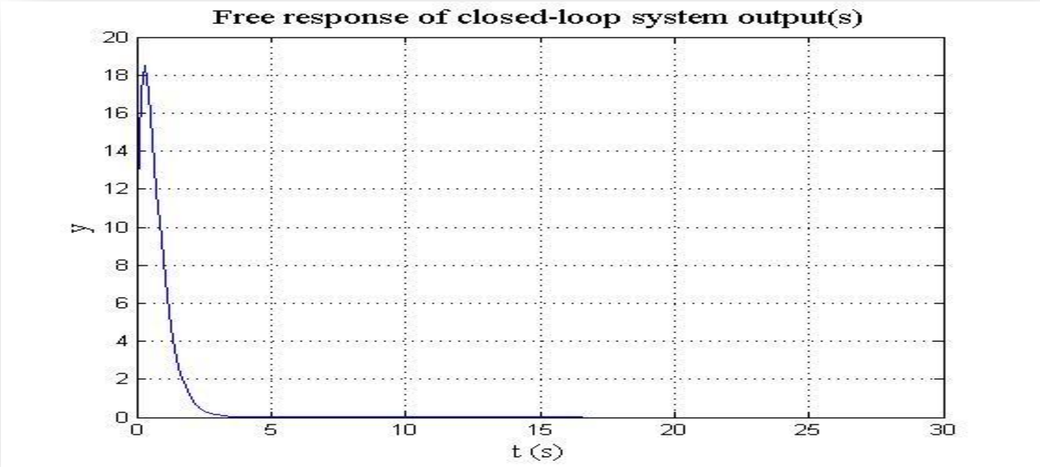


Figure (3.2) Closed loop free response of the system in example (3.1)

Similarly, for the states of the closed loop system, the obtained controller stabilized the system and this is assured by convergence the states to the equilibrium state (the origin) as the time goes to infinity.

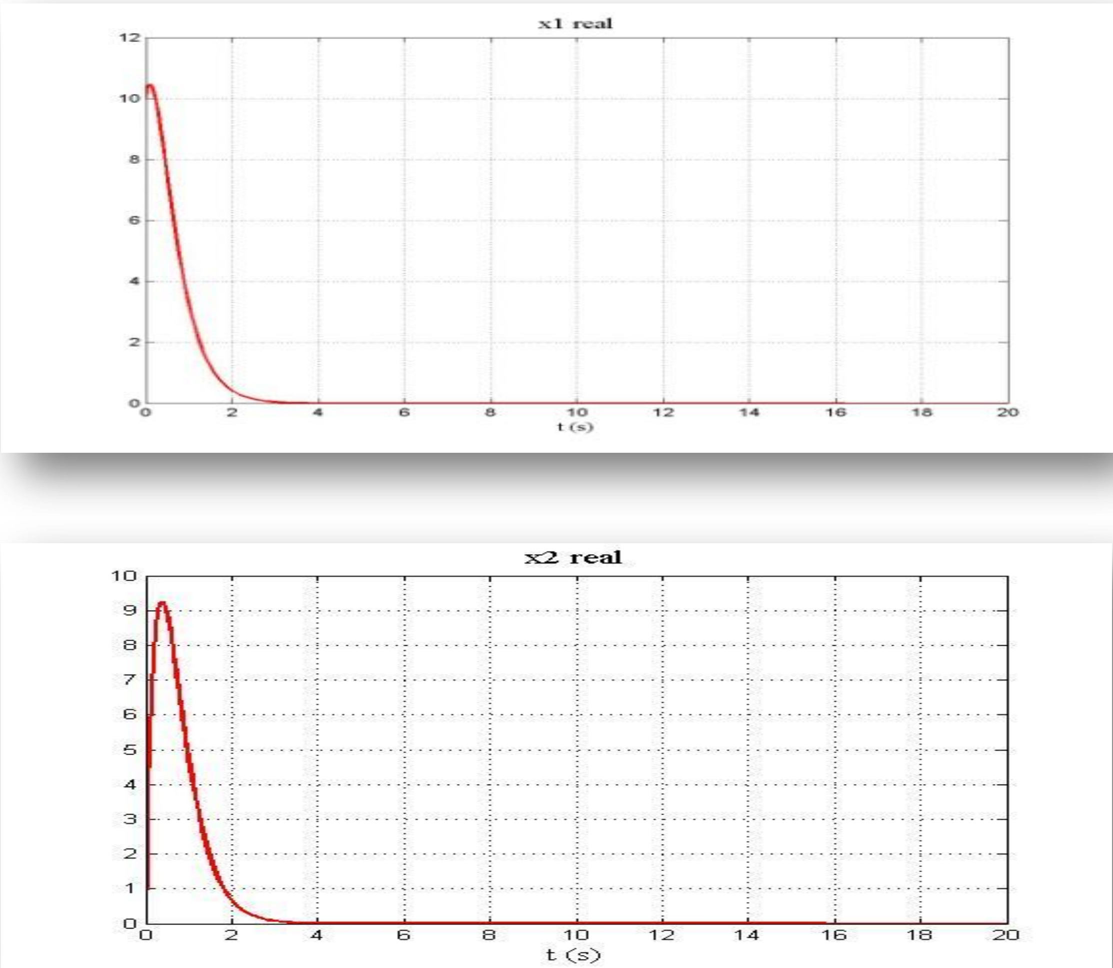


Figure (3.3) Closed loop state trajectories of the system in example (3.1)

When applying the step command to the closed loop system, the output will track the input and staying in the prescribed trajectory, this in turn confirms the fact that the closed loop system is stable (asymptotically stable) by the state feedback controller. See Fig. (3.4).

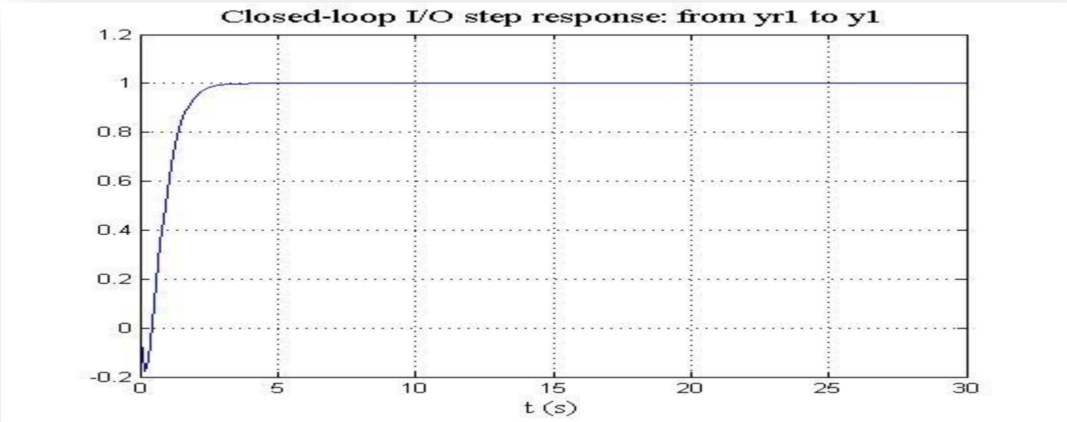


Figure (3.4) Closed loop step response of the system in example (3.1)

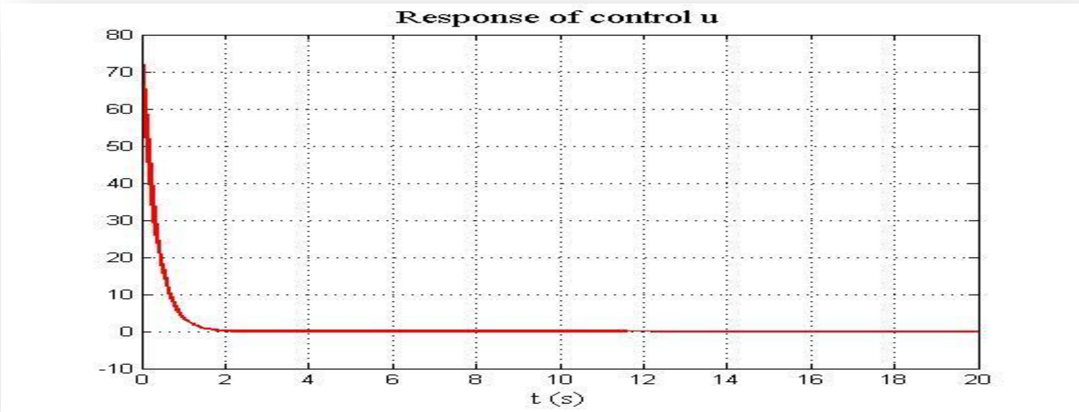


Figure (3.5) Control input of the system in example (3.1)

Fig. (3.5) shows the control input signal from the controller obtained.

3.2. Stabilization by Output Passive Controller Design

In many cases, it is difficult to measure all the states of the system and to construct the state feedback controller. In this case, we can design the output feedback controller since we can always get the measurements through sensors. In this section we construct dynamic output feedback controller for the next system:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + B_1 w(t) + B_2 u(t)$$
$$y(t) = C_2 x(t) + D_2 w(t) \qquad (3.17)$$

$x(t) \in R^{n_x}$ *is the state*; $u(t) \in R^{n_u}$ *is the control input*; $w(t) \in R^{n_u}$ *is exogenious inputs* $y(t) \in R^{n_y}$, is the output measurement.

Required to construct linear dynamical output controller in order k in the following form:

$$\dot{x}_r = A_r x_r + B_r y$$
$$u = C_r x_r + D_r y \qquad (3.18)$$

where $x_r \in R^k$, vector state of the controller.

A_r, B_r, C_r and D_r *, are gain matrices with appropriate dimensions.*

In the particular case $k = 0$ we have output static controller $u = D_r y$. The closed loop control system equation (3.17) and (3.18) when $k \neq 0$ has the following form:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau) + B_1 w(t) + B_2 (C_r x_r(t) + D_r \{C_2 x(t) + D_2 w(t)\}) \\ &= A_0 x(t) + A_1 x(t-\tau) + B_1 w(t) + B_2 C_r x_r(t) + B_2 D_r C_2 x(t) + B_2 D_r D_2 w(t) \\ &= (A_0 + B_2 D_r C_2) x(t) + B_2 C_r x_r(t) + (B_1 + B_2 D_r D_2) + A_1 x(t-\tau) \\ \dot{x}_r(t) &= A_r x_r(t) + B_r C_2 x(t) + B_r D_2 w(t) \end{aligned} \qquad (3.19)$$

Let us define the equations above as:

$$\dot{x}_{cl}(t) = A_{0cl} x_{cl} + A_{1cl} x_{cl} + B_{1cl} w$$
$$x_{cl} = Col(x(t), x_r(t)) \qquad (3.20)$$

$$Let\ \dot{x}_{cl}(t) = \dot{\bar{x}}(t),\ A_{0cl} = \bar{A}_0, A_{1cl} = \bar{A}_1, B_{1cl} = \bar{B}_1$$

We then can rewrite equation (3.20) in the compact form as shown below:

$$\dot{\bar{x}}(t) = \bar{A}_0 \bar{x}(t) + \bar{A}_1 \bar{x}(t-\tau) + \bar{B}_1 w(t)$$

$$\bar{Z}(t) = (C_1 + D_{12} D_r C_2) x(t) + D_{12} C_r x_r(t) + (D_{11} + D_{12} D_r D_2) w(t) \qquad (3.21)$$

$$Let\ \bar{C}_1 = \begin{pmatrix} C_1 + D_{12} D_r C_2 & D_{12} C_r \end{pmatrix}\ and\ \bar{D}_{11} = D_{11} + D_{12} D_r D_2$$

Then the final form of the closed loop control system with output feedback controller yields:

$$\dot{\bar{x}}(t) = \bar{A}_0 \bar{x}(t) + \bar{A}_1 \bar{x}(t-\tau) + \bar{B}_1 w(t)$$
$$\bar{Z}(t) = \bar{C}_1 \bar{x}(t) + \bar{D}_{11} w(t) \qquad (3.22)$$

So we can define the above matrices according to the equation (3.20) as follow:

$$\bar{A}_0 = \begin{pmatrix} A_0 + B_2 D_r C_2 & B_2 C_r \\ B_r C_2 & A_r \end{pmatrix}; \bar{A}_1 = \begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix}; \bar{B}_1 = \begin{pmatrix} B_1 + B_2 D_r D_2 \\ B_r D_2 \end{pmatrix} \qquad (3.23)$$

Theorem 3.2 For a given symmetric positive definite matrix Q if there exists positive definite symmetric matrix P and gain matrices A_r, B_r, C_r *and* D_r such that the following linear matrix inequality (LMI):

$$\begin{pmatrix} A_0^T P + PA_0 + C_2^T D_r^T B_2^T P + PB_2 D_r C_2 + Q & C_2^T B_r^T + PB_2 C_r & A_1^T P & 0 & PB_1 + PB_2 D_r D_2 - (C_1 + D_{12} D_r C_2)^T \\ C_r^T B_2^T P + B_r C_2 & A_r + A_r^T + Q & 0 & 0 & B_r D_2 - C_r^T D_1^T \\ * & * & * & -Q & 0 \\ * & * & * & 0 & -Q \\ * & * & * & * & -((D_{11} + D_{12} D_r D_2)^T + (D_{11} + D_{12} D_r D_2)) \end{pmatrix} \leq 0 \quad (3.24)$$

holds, then the state delay system (3.22) is asymptotically stable and passive using the output feedback passive controller (3.18).

proof: First let us define the $\bar{Q} = \begin{pmatrix} Q & 0 \\ 0 & Q \end{pmatrix} > 0$ and $\bar{P} = \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} > 0$,

As in the case of Theorem 3.1 concerned of static feedback controller we define a Lyapunov functional V(x(t)) as follows:

$$V(x(t)) = \bar{x}^T(t)\bar{P}\bar{x}(t) + \int_{t-\tau}^{t} \bar{x}^T(s)\bar{Q}\bar{x}(s)ds$$

Calculating the derivative of Lyapunov function V(x (t)) along the solution of (3.22), we get:

$$\dot{V}(\bar{x}(t)) = \dot{\bar{x}}^T(t)\bar{P}\bar{x}(t) + \bar{x}^T(t)\bar{P}\dot{\bar{x}}(t) + \bar{x}^T(t)\bar{Q}\bar{x}(t) - \bar{x}^T(t-\tau)\bar{Q}\bar{x}(t-\tau)$$

$$= \bar{x}^T(t)(\bar{A}_0^T\bar{P} + \bar{P}\bar{A}_0 + \bar{Q})\bar{x}(t) + \bar{x}^T(t-\tau)\bar{A}_1^T\bar{P}\bar{x}(t) + \bar{x}(t)\bar{P}\bar{A}_1\bar{x}(t-\tau)$$

$$+ \bar{w}^T(t)\bar{B}_1^T\bar{P}\bar{x}(t) + \bar{x}^T(t)\bar{B}_1\bar{P}\bar{w}(t) - \bar{x}^T(t-\tau)\bar{Q}\bar{x}(t-\tau)$$

To obtain the condition for passivity we apply the following equation:

$$\begin{aligned} \dot{V}(\bar{x}(t)) - 2Z^T(t)\bar{w}(t) = {} & \bar{x}^T(t)(\bar{A}_0^T\bar{P} + \bar{P}\bar{A}_0 + \bar{Q})\bar{x}(t) \\ & + \bar{x}(t)\bar{P}\bar{A}_1\bar{x}(t-\tau) + \bar{x}^T(t-\tau)\bar{A}_1^T\bar{P}\bar{x}(t) \\ & + \bar{x}^T(t)\bar{P}\bar{B}_1\bar{w}(t) + \bar{w}^T(t)\bar{B}_1^T\bar{P}\bar{x}(t) \\ & - \bar{x}^T(t-\tau)\bar{Q}\bar{x}(t-\tau) - \bar{x}^T(t)\bar{C}_1^T\bar{w}(t) \\ & - \bar{w}^T(t)\bar{C}_1\bar{x}(t) - \bar{w}^T(t)(\bar{D}_{11} + \bar{D}_{11}^T)\bar{w}(t) \end{aligned} \quad (3.25)$$

Collect the same terms together we get:

$$\begin{aligned} \dot{V}(\bar{x}(t)) - 2Z^T(t)\bar{w}(t) = {} & \bar{x}^T(t)(\bar{A}_0^T\bar{P} + \bar{P}\bar{A}_0 + \bar{Q})\bar{x}(t) \\ & + 2\bar{x}(t)\bar{P}\bar{A}_1\bar{x}(t-\tau) \\ & + 2\bar{x}^T(t)(\bar{P}\bar{B}_1 - \bar{C}_1^T)\bar{w}(t) \\ & - \bar{x}^T(t-\tau)\bar{Q}\bar{x}(t-\tau) \\ & - \bar{w}^T(t)(\bar{D}_{11} + \bar{D}_{11}^T)\bar{w}(t) \end{aligned} \quad (3.26)$$

$$= \Sigma^T(t)\Pi\Sigma(t),$$

$$\Sigma^T(t) = \begin{bmatrix} \bar{x}(t) & \bar{x}(t-\tau_1) & \bar{w}(t) \end{bmatrix}^T$$

$$\Pi = \begin{pmatrix} \bar{A}_0^T\bar{P}+\bar{P}\bar{A}_0+\bar{Q} & \bar{A}_1\bar{P} & \bar{P}\bar{B}_1-\bar{C}_1^T \\ * & -\bar{Q} & 0 \\ * & * & -(\bar{D}_{11}^T+\bar{D}_{11}) \end{pmatrix} \tag{3.27}$$

Now, if we simply substitute the corresponding values of the matrices $\bar{A}_0, \bar{A}_1, \bar{B}_1, \bar{C}_1, \bar{D}_{11}, \bar{P}, \bar{Q}$ into (3.27) we exactly get (3.24), after that and after some calculations we can derive the output passive controller that stabilizes the overall closed loop system and render the system passive and asymptotically stable. This completes the proof of Theorem 3.2.

CHAPTER 4 DELAY DEPENDENT PASSIVE CONTROLLER ANALYSIS AND DESIGN

As we have seen from the previous discussion we notice that all our works concern the so called the delay-independent delay criterion, from the name of this criterion it is understood that in this method the size of delay does not take into account and we know that this criterion is more conservatism than the delay-dependent criterion, especially when there is small delays in the system. In the following section we will deal with delay-dependent stability criterion for the time delay passive system and derive sufficient conditions for stability in the term of linear matrix inequality (LMI) as will be clear in the sequel.

4.1. Delay-Dependent Stability Analysis

Let us again show the dynamical system (2.13) in its nominal form i.e. when only exogenous inputs will affect the plan.

All the matrices and the arguments are identical for the system (2.13). The following theorem gives us the first result on the delay dependent stability for the system (2.13).
Theorem 4.1: For a given positive scalar τ^*, the system (2.13) with time invariant delay is asymptotically stable and strictly passive if there exist $P = P^T > 0,$ $Q = Q^T > 0$ *and* $R = R^T > 0,$ such that the following like Riccati inequality holds:

$$\begin{aligned} & A_0^T P + PA_0 + 2Q + PA_1 Q^{-1} A_1^T P \\ & + (PB_1 - C_1^T)(D_{11} + D_{11}^T)^{-1}(B_1^T P - C_1) + \tau_1 \Theta + \tau_1^{-1} R < 0 \end{aligned} \tag{4.1}$$

Where

$$\begin{aligned} \Theta = & A_0^T RA_0 + A_0^T RA_1 + A_0^T RB_1 \\ & + A_1^T RA_0 + A_1^T RA_1 + A_1^T RB_1 \\ & + B_1^T RA_0 + B_1^T RA_1 + B_1^T RB_1 \end{aligned}$$

Or equivalently it is satisfying the following linear matrix inequality (LMI):

$$\begin{bmatrix} A_0^T P + PA_0 + Q - R & A_1 P + R & PB_1 - C_1^T & \tau_1^2 A_0^T R \\ * & -(Q+R) & 0 & \tau_1^2 A_1^T R \\ * & * & -(D_{11}^T + D_{11}) & \tau_1^2 B_1^T R \\ * & * & * & -\tau_1^2 R \end{bmatrix} < 0 \tag{4.2}$$

Then the system (2.13) will be strictly passive (SP) and asymptotically stable for all delays belonging for $0 \leq \tau^* \leq t$

Proof: let us define the following Lyapunov-Krasovskii functional for the system (2.13) as follows:

$$V(x(t)) = x^T(t)Px(t) + \int_{t-\tau_1}^{t} x^T(\theta)Qx(\theta)d\theta + \tau\int_{t-\tau}^{t}\int_{s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds \qquad (4.3)$$

The derivative along the trajectories of (2.13) leads to the following equality:

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &+ x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1) \\ &+ \tau_1^2\dot{x}^T(t)R\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \end{aligned} \qquad (4.4)$$

Using the Jensen's inequality (lemma 2) the last term can be bounded as follows:

$$\begin{aligned} &-\tau_1\int_{t-\tau_1}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \leq -\left(\int_{t-\tau_1}^{t}\dot{x}(\theta)d\theta\right)^T R\left(\int_{t-\tau_1}^{t}\dot{x}(\theta)d\theta\right) \\ &= \begin{bmatrix} x(t) \\ x(t-\tau_1)\end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau_1)\end{bmatrix} \\ &= -x^T(t)Rx(t) + 2x^T(t)R(t-\tau_1) - x^T(t-\tau_1)Rx(t-\tau_1). \end{aligned} \qquad (4.5)$$

Therefore we get the following derivative for (4.5):

$$\begin{aligned} \dot{V}(x(t)) &\leq x^T(t)A_0^T Px(t) + x^T(t-\tau_1)A_1^T Px(t) + W^T(t)B_1^T Px(t) \\ &+ x^T(t)PA_0x(t) + x^T(t)PA_1x(t-\tau_1) + x^T(t)PB_1W(t) \\ &+ x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1) \\ &+ \tau_1^2\Theta - x^T(t)Rx(t) + 2x^T Rx(t-\tau_1) - x^T(t-\tau_1)Rx(t-\tau_1) \\ &\leq x^T(t)\left(A_0^T P + PA_0 + Q - R\right)x(t) + x^T(t-\tau_1)A_1^T Px(t) \\ &+ x^T(t)PA_1x(t-\tau_1) + W^T(t)B_1^T Px(t) + x^T(t)PB_1W(t) \\ &+ \tau_1^2\Theta - x^T(t-\tau_1)Qx(t-\tau_1) + 2x^T Rx(t-\tau_1) - x^T(t-\tau_1)Rx(t-\tau_1) \end{aligned} \qquad (4.6)$$

Let us denote the term $\dot{x}^T(t)R\dot{x}(t)$ as Θ so after manipulation this term according to the system (2.13) yields:

$$\begin{aligned} \Theta &= A_0^T RA_0 + A_0^T RA_1 + A_0^T RB_1 \\ &+ A_1^T RA_0 + A_1^T RA_1 + A_1^T RB_1 \\ &+ B_1^T RA_0 + B_1^T RA_1 + B_1^T RB_1 \end{aligned}$$

Now let us applied the following equation for guaranteeing the passivity conditions for the system (2.13):

$$\begin{aligned}\dot{V}(t)-2z^T(t)w(t) &= x^T(t)\left(A_0^T P + PA_0 + Q - R\right)x(t) + x^T(t-\tau_1)A_1^T Px(t)\\ &+ x^T(t)PA_1x(t-\tau_1) + W^T(t)(B_1^T P - C)x(t) + x^T(t)(PB_1 - C_1^T)W(t)\\ &+ \tau_1^2\dot{x}^T(t)R\dot{x}(t) - x^T(t-\tau_1)Qx(t-\tau_1)\\ &- W^T(t)(D_{11}^T + D_{11})W(t) + 2x^T Rx(t-\tau_1) - x^T(t-\tau_1)Rx(t-\tau_1)\end{aligned} \quad (4.7)$$

Note that in the above equation we used the fact that

$2z^T(t)w(t) = z^T(t)w(t) + w^T(t)z(t)$

We can rewrite (4.7) in compact form as following:

$\dot{V}(t) - 2z^T(t)w(t) = \zeta^T \Sigma \zeta$

$$where \quad \zeta = \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ w(t) \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} A_0^T P + PA_0 + Q - R & PA_1 + R & PB_1 - C_1^T \\ * & -(Q+R) & 0 \\ * & * & -(D_{11}^T + D_{11}) \end{bmatrix} + \tau_1^2 \begin{bmatrix} A_0^T \\ A_1^T \\ B_1^T \end{bmatrix} R \begin{bmatrix} A_0 & A_1 & B_1 \end{bmatrix} \quad (4.8)$$

From Schur complement, it is easy follows that (4.1) and (4.2) are hold. Hence

$\dot{V}(x(t)) \le 2z^T(t)u(t).$

If $\Sigma \prec 0$, *then* $-\dot{V}(x(t)) + 2z(t)w(t) > 0$ and from which it follows that:

$$\int_{t_0}^{t_1} [z^T(t)\ w(t)] > \frac{1}{2}\left[V(x(t_1)) - V(x(t_0))\right]$$

Since $V(x(t)) > 0$ *for* $x \ne 0$ *and* $V(x(t)) = 0$ *for* $x = 0$, it follows that as $t_1 \to \infty$ the system (4.1) is strictly passive and asymptotically stable for all state delays that satisfy $0 \le \tau^* \le t$. This completes the proof.

Let us show the following example from the reference [16] to demonstrate the effectiveness of our method:

4.2. Numerical Example 4.1:

Consider the same system as in the example 2.1, and this system represents the water quality model for the Nile River as mentioned in the chapter 2, for convenience I mention the system here

$$A_0 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.3 \\ -0.3 & -0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \quad D_{11} = [2].$$

Using the LMI solver, especially CVX software, that works under Matlab package and solving LMI (4.3) for the system we found:

$$\begin{bmatrix} A_0^T P + P A_0 + Q - R & A_1 P + R & P B_1 - C_1^T & \tau_1^2 A_0^T R \\ * & -(Q+R) & 0 & \tau_1^2 A_1^T R \\ * & * & -(D_{11}^T + D_{11}) & \tau_1^2 B_1^T R \\ * & * & * & -\tau_1^2 R \end{bmatrix} < 0$$

P =
10.7206 4.9587
4.9587 7.7119
Q =
12.5853 4.1859
4.1859 2.5124
R =
1.3180 0.7526
0.7526 2.9588

As shown from the results we can see that we get $P = P^T > 0, Q = Q^T > 0$ and $R = R^T > 0$. Based on the theorem 4.1 we conclude that the system in example 4.1 which represents the water-quality model under consideration is asymptotically stable and strictly passive (SP) for any τ_1 satisfying $0 < \tau_1 \leq 1.1493$ and we notice that the upper bound delay using our approach is larger than in the work in reference [7], since the delay amount obtained was 0.4 seconds.
To verify the result let us now follow the conventional way to determine whether the system is stable or not, i.e. we can get the transfer function of the previous example then check state responses for to the system and show the behavior of the system, if the states when $t \to \infty$ go to the equilibrium i.e. to the origin then the system is asymptotically stable.

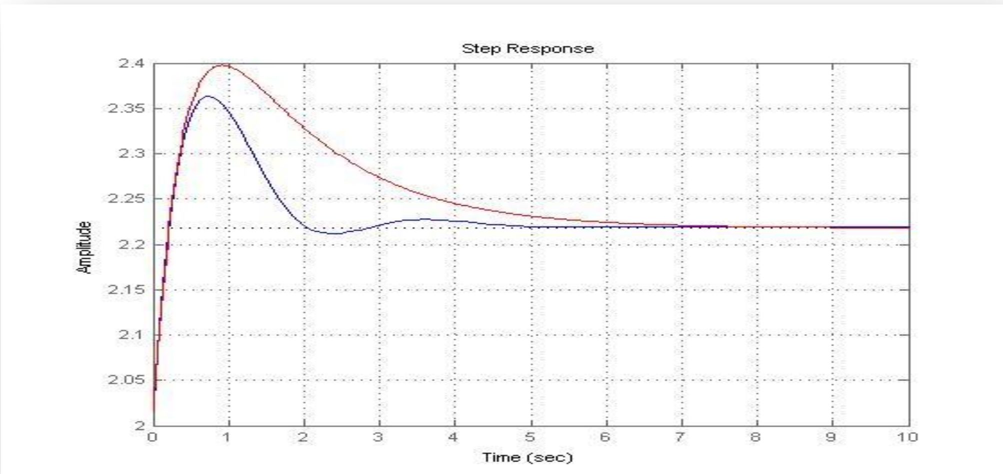
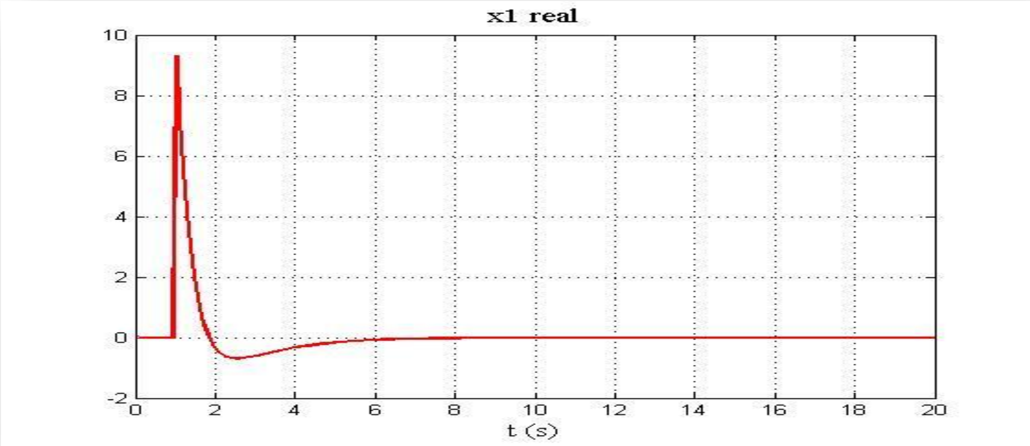
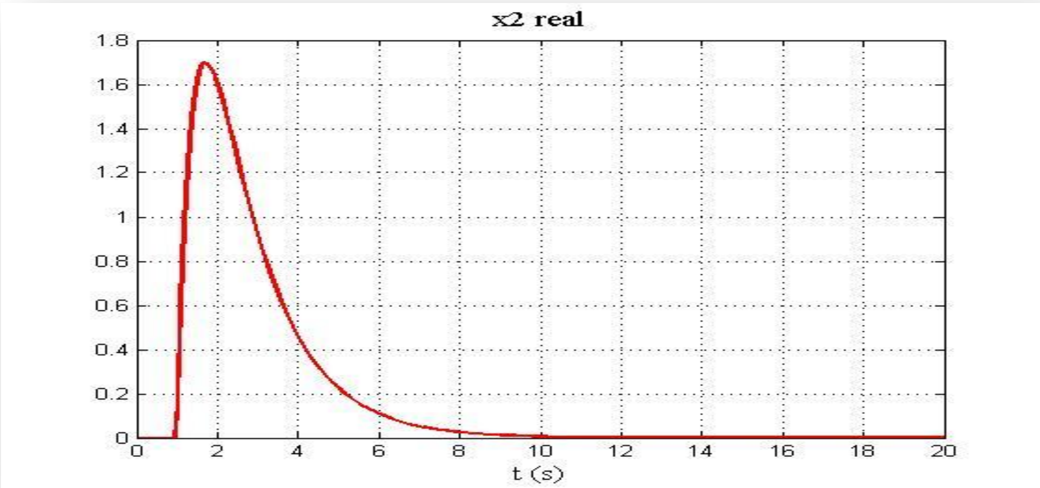


Figure 4.1 Step response for Example 4.1

In addition, the trajectories of the systems under initial conditions convergent to the equilibrium point (the origin) when the time goes to infinity. This is obvious from the response to the initial conditions for the two states of the control system. See Fig. (4.1) and (4.2).



(a)



(b)

Figure 4.2 (a), (b)Open loop state responses for Example 4.1

Figure (4.1) shows the step response of the time delay original system for example (4.1) (blue curve), and the approximated system by Pade approximation (red curve), also shown the state trajectories of the system. Fig. (4.2) shows the states converge to zero as time goes to infinity, so the system is asymptotically stable, and this is very clear from the step and state trajectories response of the system.

4.3. Lyapunov-Krasovskii Functional with Triple Integrals

In this section we will use the new Lyapunov-Krasovskii functional that include a triple integral term and we will get an improved feasible region of stability criterion, i.e. we expect to get larger upper bound of the delay for the time delay system under consideration.

Theorem 4.2: For a given positive scalar τ^*, the system (4.1) with time invariant delay is asymptotically stable and strictly passive if there exist $R = R^T > 0$, $S = S^T \geq 0,\ P = P^T > 0$ *and* $Q = Q^T > 0,$ such that the following LMI holds:

$$\begin{bmatrix} A_0^T P + PA_0 + Q - R & PA_1 + R & PB_1 - C_1^T & 0 & 0 & 0 & \tau^2 A_0^T R \\ * & -Q - R & 0 & 0 & 0 & 0 & \tau^2 A_1^T R \\ * & * & -(D_{11}^T + D_{11}) & 0 & 0 & 0 & \tau^2 B_1^T R \\ * & * & * & -\tau^2 S & \tau S & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & \left(\frac{\tau^2}{2}\right)^2 S & 0 \\ * & * & * & * & * & * & -\tau^2 R \end{bmatrix} < 0 \qquad (4.10)$$

Proof: consider the following Lyapunov-Krasovskii functional candidate containing a triple integral term:

$V = V_1 + V_2 + V_3 + V_4$

Where

$$V_1 = x^T(t)Px(t)$$

$$V_2 = \int_{t-\tau}^{t} x^T(\theta)Qx(\theta)d\theta$$

$$V_3 = \int_{t-\tau}^{t}\int_{s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds \qquad (4.11)$$

$$V_4 = \frac{\tau^2}{2}\int_{t-\tau}^{t}\int_{s}^{t}\int_{\theta}^{t} \dot{x}^T(v)S\dot{x}(v)dv d\theta ds,$$

Notice that the first three functional V_1, V_2 *and* V_3 are identical to the functional from Theorem 4.1, and in similar way we will derive the derivative as shown below:

$$\dot{V}_1 = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)$$

$$\dot{V}_2 = x^T(t)Qx(t) - x^T(t-\tau_1)Qx(t-\tau_1)$$

$$\dot{V}_3 = \tau_1^2 \dot{x}^T(t)R\dot{x}(t) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \qquad (4.12)$$

$$\dot{V}_4 = \left(\frac{\tau^2}{2}\right)^2 \dot{x}^T(t)S\dot{x}(t) - \left(\frac{\tau^2}{2}\right)\int_{t-\tau}^{t}\int_{s}^{t} \dot{x}^T(\theta)S\dot{x}(\theta)d\theta ds$$

$$= \left(\frac{\tau^2}{2}\right)^2 \ddot{x}^T(t)S\ddot{x}(t) - \left(\frac{\tau^2}{2}\right)\int_{t-\tau}^{t}\int_{s}^{t} \ddot{x}^T(\theta)S\ddot{x}(\theta)d\theta ds$$

By using lemma 3 the upper bound of double integral term of $\dot{V}_4$ can be calculated as shown below:

$$-\left(\frac{\tau^2}{2}\right)\int_{t-\tau}^{t}\int_{s}^{t}\ddot{x}^T(\theta)S\ddot{x}(\theta)d\theta ds \le -\left(\int_{t-\tau}^{t}\int_{s}^{t}\ddot{x}(\theta)d\theta ds\right)^T S\left(\int_{t-\tau}^{t}\int_{s}^{t}\ddot{x}(\theta)d\theta ds\right)$$
$$=\begin{bmatrix}\dot{x}(t)\\ \int_{t-\tau}^{t}\dot{x}(t)ds\end{bmatrix}^T\begin{bmatrix}-\tau^2 S & \tau S\\ * & -S\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\ \int_{t-\tau}^{t}\dot{x}(t)ds\end{bmatrix} \tag{4.13}$$

and we can write the previous quantity as :

$$-\tau^2\dot{x}^T(t)S\dot{x}(t)+2\tau S\dot{x}^T(t)\int_{t-\tau}^{t}\dot{x}(t)ds-\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)^T S\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right) \tag{4.14}$$

Now combine all the derivatives mentioned above we get the following:

$$\begin{aligned}\dot{V}(x(t)) \le\ & x^T(t)A_0^T Px(t)+x^T(t-\tau)A_1^T Px(t)+W^T(t)B_1^T Px(t)\\ & x^T(t)PA_0x(t)+x^T(t)PA_1x(t-\tau)+x^T(t)PB_1W(t)\\ & +x^T(t)Qx(t)-x^T(t-\tau)Qx(t-\tau)-x^T(t)Rx(t)\\ & +\ \tau^2\Theta+2x^T Rx(t-\tau_1)-x^T(t-\tau_1)Rx(t-\tau_1)+\left(\frac{\tau^2}{2}\right)^2\ddot{x}^T(t)S\ddot{x}(t)\\ & +2\tau S\dot{x}^T(t)\int_{t-\tau}^{t}\dot{x}(t)ds-\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)^T S\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)\end{aligned} \tag{4.15}$$

$$\begin{aligned}\le\ & x^T(t)\left(A_0^T P+PA_0+Q-R\right)x(t)+x^T(t-\tau)A_1^T Px(t)\\ & +x^T(t)PA_1x(t-\tau)+W^T(t)B_1^T Px(t)+x^T(t)PB_1W(t)\\ & +\ \tau^2\Theta-x^T(t-\tau_1)Rx(t-\tau_1)+2x^T Rx(t-\tau_1)-x^T(t-\tau)Qx(t-\tau)\\ & +2\tau S\dot{x}^T(t)\int_{t-\tau}^{t}\dot{x}(t)ds-\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)^T S\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)+\left(\frac{\tau^2}{2}\right)^2\ddot{x}^T(t)S\ddot{x}(t)\end{aligned} \tag{4.16}$$

Where

$$\begin{aligned}\Theta = & A_0^T RA_0+A_0^T RA_1+A_0^T RB_1\\ & +A_1^T RA_0\ +A_1^T RA_1+A_1^T RB_1\\ & +\ B_1^T RA_0+B_1^T RA_1+B_1^T RB_1\end{aligned}$$

Now for passivity analysis and to show that the system is asymptotically stable and strictly passive (SP) we will go to apply the passivity condition in the similar way as in the Theorem 4.1

$$\dot{V}(t) - 2z^T(t)w(t) = x^T(t)\left(A_0^T P + PA_0 + Q - R\right)x(t)$$
$$+ x^T(t-\tau)A_1^T Px(t) + \tau^2 \dot{x}^T(t)R\dot{x}(t)$$
$$+ x^T(t)PA_1 x(t-\tau) + W^T(t)(B_1^T P - C)x(t)$$
$$+ x^T(t)(PB_1 - C_1^T)W(t) + \tau^2 \dot{x}^T(t)S\dot{x}(t)$$
$$- x^T(t-\tau)Qx(t-\tau) - x^T(t-\tau)Rx(t-\tau)$$
$$- W^T(t)(D_{11}^T + D_{11})W(t) + 2\tau \dot{x}^T(t)S\int_{t-\tau}^{t}\dot{x}(t)ds$$
$$- \left(\int_{t-\tau}^{t}\dot{x}(t)ds\right)^T S\left(\int_{t-\tau}^{t}\dot{x}(t)ds\right) + \left(\frac{\tau^2}{2}\right)^2 \ddot{x}^T(t)S\ddot{x}(t) \tag{4.17}$$
$$+ x^T(t)Rx(t-\tau) + x(t-\tau)Rx(t)$$

Let us now write Eq.(4.17) in the compact form as follows:

$$\xi^T \Pi \xi \quad \text{when} \quad \xi^T = \begin{bmatrix} x(t) \\ x(t-\tau) \\ w(t) \\ \dot{x}(t) \\ \int_{t-\tau}^{t}\dot{x}(s)ds \\ \ddot{x}(s) \end{bmatrix} \tag{4.18}$$

$$\Pi = \begin{bmatrix} A_0^T + PA_0 + Q + R & PA_1 + R & PB_1 - C_1^T & 0 & 0 & 0 \\ * & -(Q+R) & 0 & 0 & 0 & 0 \\ * & * & -(D_{11} + D_{11})^T & 0 & 0 & 0 \\ * & * & * & -\tau^2 S & \tau S & 0 \\ * & * & * & * & -S & 0 \\ * & * & * & * & * & \left(\frac{\tau^2}{2}\right)^2 S \end{bmatrix}$$
$$+ \tau^2 \begin{bmatrix} A_0 \\ A_1 \\ B_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T R\begin{bmatrix} A_0 & A_1 & B_1 & 0 & 0 & 0 \end{bmatrix} \tag{4.19}$$

as in the proof of Theorem 4.1 and applying Schur complement we conclude that the LMI (4.10) holds, so the theorem is proved.

4.4. Numerical Example (4.2):

Consider the same system as in the example 1 and this system represents the water quality model for the Nile River, for convenience I will rewrite the system here

$$A_0 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.3 \\ -0.3 & -0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \quad D_{11} = [2].$$

Using the CVX toolbox, and solving LMI (4.10) for the system we found:

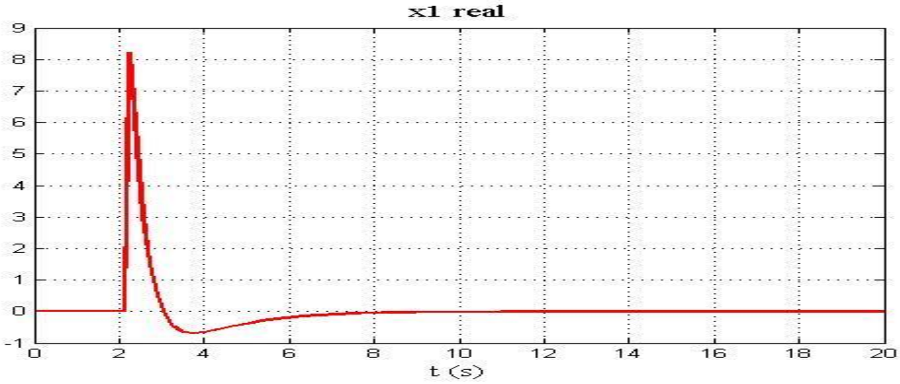
$$\begin{bmatrix} A_0^T P + PA_0 + Q - R & PA_1 + R & PB_1 - C_1^T & 0 & 0 & 0 & \tau^2 A_0^T R \\ * & -Q-R & 0 & 0 & 0 & 0 & \tau^2 A_1^T R \\ * & * & -(D_{11}^T + D_{11}) & 0 & 0 & 0 & \tau^2 B_1^T R \\ * & * & * & -\tau^2 S & \tau S & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & \left(\frac{\tau^2}{2}\right)^2 S & 0 \\ * & * & * & * & * & * & -\tau^2 R \end{bmatrix}$$

P =
13.5166 7.8249
7.8249 9.9649
Q =
13.1563 7.3203
7.3203 5.4082
R =
1.1927 0.3883
0.3883 1.7824
S =
1.0e-013 *
0.8590 -0.0094
-0.0094 0.8652

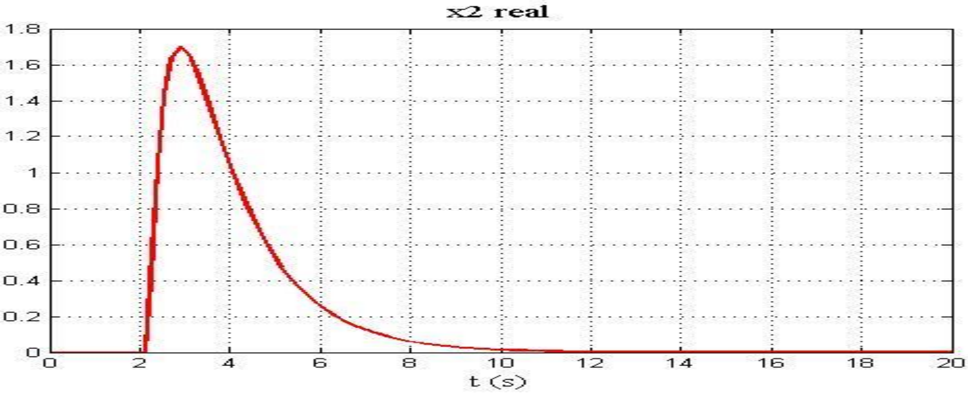
Since all matrices are positive definite this means that the water equality model under consideration is asymptotically stable and strictly passive for any τ satisfying

$$0 \leq 2.1898 \leq t$$

And this is confirmed when we simulate the system under the influence of the initial conditions. The state of the system convergent to equilibrium as the time goes to infinity as shown in Fig. (4.3), we conclude that the system is strictly passive, hence, asymptotically stable and tolerates delay up to 2.1898 seconds.



(a)



(b)

Figure 4.3 (a)and (b)state trajectories for Example 4.2

It is obvious from the above figures that the delay is affected the state trajectories of the system, but the system still passive and hence asymptotically stable. For comparison see the next table.

Table 1 UPPER BOUND OF TIME DELAY

LKF with tuning scalar parameters	0.3621
LKF without tuning parameters, with Jenson's inequality method	1.41925~1.4193
LKF with triple integral term	2.1898

We conclude that both theorems gave us the different upper bounds of delay for this system and in the both cases we get improvements over the existing results as shown in the comparison between the results obtained.

CHAPTER 5 CONTINIOUS TIME UTDS ANALYSIS AND SYNTHESIS

5.1. Overview of H_∞ Control Theory

Robustness is very important in control system design because real engineering systems are affected by external disturbances and measurement noises and there are always differences between the real plant and the mathematical models used for design. So, a control engineer is required to design a controller that will stabilize the plant, if it is not stable originally, and satisfy certain performance levels in the presence of disturbance signals, noise interference, unmodelled plant dynamics and plant-parameter variations. These design objectives are best realized via the state feedback mechanism [1]. As already mentioned above that there is close relation between passivity and robustness and this relation established via Kalman-Yakubovich-Popov lemma and this in turn motivates us to discuss the robustness issue in the perspective of passivity. In this section we will discuss H_∞ approach which addresses the robustness issue of the control systems, and this approach called H_∞ optimal control theory. In the H_∞ control design framework, the H_∞ robustness in this thesis will be taken the same as the performance objectives, that is, to minimize the H_∞ norm of the closed loop control system to guarantee the desired performance specifications. This will be clear in the subsequent sections in this chapter. Figure (5.1) shows the standard H_∞ configuration.

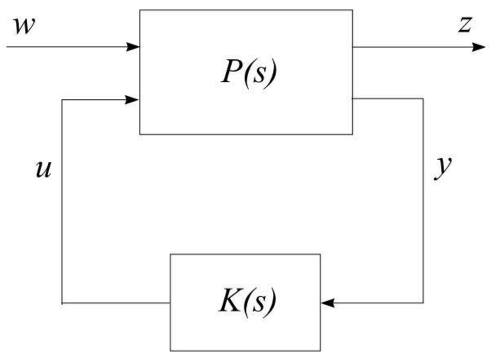


Figure (5.1) the standard H_∞ configuration

Where w denotes the external inputs of the plant, z denotes the output signals to be minimized/penalized that includes both the performance and robustness measurements, y is the measurements available to the controller K and u is the vector of control signals. $P(s)$ is called the generalized plant or interconnected system. The objective is to find the stabilizing controller K to minimize the output z, in the sense of energy, over all w with energy less than or equal to 1. Thus, it is equivalent to minimizing the H_∞ norm of the transfer function from w to z.

5.2. H infinity Controller Design for Independent Delay UTDS

Let the UTDS be described as follows:

$$\dot{x}(t) = (A +\Delta A)x(t) + (A_d +\Delta A_d)x(t-\tau_1)$$
$$+(B_1 + \Delta B_1)u(t) + B_w w(t)$$

$$z(t) = Cx(t) + D_w w(t) \tag{5.1}$$

Where A is the nominal state matrix, ΔA is the perturbation in the state matrix, A_d is the delay state matrix, ΔA_d is perturbation in the delay state matrix, B_1 is the control input matrix, ΔB_1 is the perturbation in the control input matrix, and B_w exogenous input matrix. All matrices are in the appropriate dimensions. In this subsection we will design H_∞ robust controller that render the closed loop control system asymptotically robustly stable despite of the uncertainty affected the system, in the same time it is acceptable to achieve γ_∞ performance criteria, i.e. the controller should minimize the infinity norm of the closed loop system that corresponds to disturbance attenuation level. The new quantities in the system (5.1) are $\Delta A, \Delta A_d, and\ \Delta B_u,$ are time invariant parameter uncertainties, and assumed to be in the following form:

$$[\Delta A \quad \Delta A_d \quad \Delta B_1\ ] = HF[N_1 \quad N_2 \quad N_3 \quad] \tag{5.2}$$

H, $N_1, N_2, and\ N_3,$ are constant matrices and $F \in R^{p\times k}$ is the uncertain matrix satisfying

$$F^T F \leq \mathrm{I} \tag{5.3}$$

The controller has the following form:

$$u(t) = (K + \Delta K)x(t) \tag{5.4}$$

Where $K \in R^{m\times n}$ is the controller gain to be designed, and ΔK is the controller gain perturbation with the norm bounded additive form:

$$\Delta K = \Delta_1 = H_1 F_1 E_1 \tag{5.5}$$

Where $H_1\ and\ E_1$ are known matrices, and F_1 is unknown matrix satisfying

$$F_1{}^T F_1 \leq \mathrm{I} \tag{5.6}$$

The closed loop descriptor uncertain time delay system under the state feedback controller (5.4) is seems as follows:

$$(\Sigma_\Delta)\text{: E}\ \dot{x}(t) = A_c x(t) + A_{d_c} x(t-\tau) + Bw(t)$$

$$z(t) = Cx(t) + Dw(t) \tag{5.7}$$

$$x(t) = \phi(t), t \in [-\tau, 0], \tau > 0$$

Where $A_c = A_K + \Delta A_K + (B_1 + \Delta B_1)\Delta K,\ A_{d_c} = A_d + \Delta A_d, A_K = A + B_1 K,$
$$and\ \Delta A_K = \Delta A + \Delta B_1 K = MF\widetilde{N_1}, \widetilde{N_1} = N_1 + N_3 K$$

Theorem 5.1: Consider the UTDS (Σ_Δ) and controller perturbation Δ_1 in (5.5) and (7.6), then if there exists symmetric positive definite matrices $X^T = X > 0$, $Q^T = Q > 0$ and matrix Y with appropriate dimensions and scalars $\varepsilon_{1,}\varepsilon_2\ and\ \varepsilon_3$ such that the next inequalities hold

$$X^T E^T = EX \geq 0 \tag{5.8}$$

$$\begin{bmatrix} \Pi_{11} & A_d & B + C'D & \Pi_{14} & X'E_1' & \varepsilon_2 B_1 H_1 & 0 \\ * & -Q & 0 & N_2' & 0 & 0 & 0 \\ * & * & -\gamma^2{}_\infty I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{1,}I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & \varepsilon_2 H_1' N_3' \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} \tag{5.9}$$

where $\Pi_{11} = AX + XA' + (\varepsilon_1 + \varepsilon_3)M * M' + B_1 Y + Y' B_1 + C'C$

$$\Pi_{14} = X' N_1' + Y' N_3'$$

then H_∞ control problem is solvable and the closed loop system is robustly stable by the H_∞ controller $u(t) = Kx(t), K = YX^{-1}$ with disturbance attenuation criterion γ_∞ .

Proof: Construct a Lyapunov-Krasovskii functional with matrices $P > 0, Q > 0$
Define LKF candidate as:

$$V\big(x(t)\big) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds \tag{5.10}$$

Differentiating $V\big(x(t)\big)$ along the solution of (7.5) gives:

$$\dot V\big(x(t)\big) = 2x^T(t)PA_c x(t) + 2x^T(t)PA_{dc}x(t-\tau) + 2x^T(t)PB_w w(t)] \\ +x^T(t)Qx(t) - x^T(t-\tau)Q(x-\tau) \tag{5.11}$$

Using lemma (4), it follows that:

$$2x^T(t)PA_{dc}x(t-\tau) - x^T(t-\tau)Q(x-\tau)$$

$$\leq x^T(t)P[A_d Q^{-1} {A_d}^T + A_d Q^{-1} N_2^T$$
$$\times (\varepsilon_1 I - N_2 Q^{-1} N_2^T)^{-1} N_2 Q^{-1} {A_d}^T + \varepsilon_1^{-1} MM^T]Px(t)$$
$$+x^T(t) A_d^T P x^T(t-\tau)$$

$$2x^T(t)PA_c x(t) \leq x^T(t)\{[PA + A^T P + PB_1 K + K^T B_1^T P]$$
$$[(\varepsilon_1 + \varepsilon_3)PMM^T P + \varepsilon_1^{-1} N_1^T N_1 + \varepsilon_3^{-1} K^T N_3^T N_3 K]$$
$$+P^T B_1 H_1 (\varepsilon_2^{-1} I - \varepsilon_3^{-1} H_1^T N_3^T N_3 H_1)^{-1} H_1^T B_1^T P\}x(t)$$

We have $\Delta A_K^T P + P^T \Delta A_K = P^T(B_1 + \Delta B_1)\Delta K + \Delta K^T((B_1 + \Delta B_1)^T P$

$$\leq \varepsilon_2 P^T(B_1 + \Delta B_1)H_1 H_1^T (B_1 + \Delta B_1)^T P + \varepsilon_2^{-1} E_1^T E_1$$

and $\varepsilon_2(B_1 + \Delta B_1)H_1 H_1^T(B_1 + \Delta B_1)^T \leq B_1 H_1(\varepsilon_2^{-1} I - \varepsilon_3^{-1} H_1^T N_3^T N_3 H_1)^{-1} H_1^T B_1^T +$
$\varepsilon_3^{-1} M M^T$

for any scalars $\varepsilon_i > 0, i = 1,2,3$ so that $\varepsilon_1 I - N_2 Q^{-1} N_2^T > 0$,
$(\varepsilon_2^{-1} I - \varepsilon_3^{-1} H_1^T N_3^T N_3 H_1)^{-1} > 0$

satisfying above inequalities, it follows that $\dot{V}(x(t)) \leq \eta^T(t)\Pi\eta(\mathrm{t})$

where $\eta^T = [x^T(t) \quad x^T(t-\tau) \;\; w^T(t)]$,

$$\Pi = \begin{bmatrix} \Pi_{11} & A_d^T P & PB \\ * & -Q & 0 \\ * & * & \Pi_{33} \end{bmatrix} \tag{5.12}$$

Where, $\Pi_{11} = (5.12)\ and\ \Pi_{33} = (5.13)$. Next the robust H_∞ performance of the closed loop control system (5.7) will be considered under the feedback controller that render the system asymptotically stable and achieved disturbance attenuation under the reducing the infinity norm of the closed system. Let introduce the following:

$$\Im = \int_0^\infty [z(t)^T z(t) - \gamma_\infty^2 w(t)^T w(t)]dt$$

Assuming (5.3) with zero initial conditions we obtain, the closed loop control system (5.7) satisfies H_∞ performance $\gamma_\infty > 0$, that is:

$$\Im = \int_0^\infty [z(t)^T z(t) - \gamma_\infty^2 w(t)^T w(t) + \dot{V}(x(t))]dt \tag{5.13}$$

consider (5.13) and (5.12), rearrange and put it in the form (5.12), after that multiply the LMI by $diag[P^{-1}, I, I] and\ put\ X = P^{-1}, Y = KP^{-1}$ and apply Schur complement, we get (5.9). Refer to inequality (5.9), and put $\gamma^2 = \bar{\gamma}$, and since other variable matrices depend affine on the parameters of the problem so we have the following optimization problem

$$Mminimize\ \bar{\gamma}$$

Subject to

$$\begin{bmatrix} \Pi_{11} & A_d & B + C'D & \Pi_{14} & X'E_1' & \varepsilon_2 B_1 H_1 & 0 \\ * & -Q & 0 & N_2' & 0 & 0 & 0 \\ * & * & -\bar{\gamma}_\infty I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{1,}I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & \varepsilon_2 H_1' N_3' \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \tag{5.14}$$

$$E^T X = XE \geq 0$$
$$\varepsilon_i > 0, i = 1,2,3\ , X > 0, \bar{\gamma}_\infty > 0$$

5.3. Examples

Example (5.1):

Consider the same system as in [20] and the same parameters used there:

$A = \begin{bmatrix} 0.1 & 1 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.5 & 0.2 & 0.1 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0.5 & -0.1 & 0 \\ 0 & 0.1 & -0.2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 0.1 & 0 & -0.1 \\ 0.2 & 0.5 & 0.1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 0.1 \end{bmatrix}$, $M = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}$, $N_1 = [0.1 \quad 0 \quad 0.1]$, $N_3 =$

$[0 \quad 0.1]$, $H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ and $E_1 = [0.1 \quad 0 \quad 0.3]$

By solving optimization problem (5.14) using CVX package, we get the solution as follows:

$$X = \begin{bmatrix} 49.0626 & -19.7549 & 0 \\ -19.7549 & 9.1214 & 0 \\ 0 & 0 & 1.0178 \end{bmatrix}$$
$$Y = \begin{bmatrix} -50.6690 & -237.5030 & 22.1911 \\ -74.9826 & -241.6567 & -3.9298 \end{bmatrix}$$
$$Q = \begin{bmatrix} 120.6000 & 0 & 0 \\ 0 & 32.3000 & 0 \\ 0 & 0 & 85.7035 \end{bmatrix}$$

$$\bar{\gamma}_\infty = 0.0094, \varepsilon_1 = 2.6147, \varepsilon_2 = 2.5913\ and\ \varepsilon_3 = 2.6147$$

The state feedback H_∞ controller of the system (5.7) for example (5.1) is given by

$$K = \begin{bmatrix} -90.0095 & -220.9796 & 21.8032 \\ -95.3156 & -232.9267 & -3.8611 \end{bmatrix}$$

with disturbance attenuation $\gamma_\infty = \sqrt{\bar{\gamma}_\infty} = \quad 0.0971$. On the other hand in [20] H_∞ optimization problem does not exploited, hence the robustness issue does not considered there, only the state feedback controller addressed and no any information about how much the system robustly stable.

Example (5.2):

Consider the same system as in [21]

$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$, $A_d = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$, $M = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $N_1 = [1 \quad 0.5]$, $N_2 = [1 \quad 0.4]$, $N_3 = 0.2$, $C = \begin{bmatrix} 0.2 & 1 \\ 1.5 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

By solving optimization problem (5.17) using CVX package, we get the solution as follows:

$$X = \begin{bmatrix} 16.4807 & 8.7405 \\ 8.7405 & 6.6289 \end{bmatrix}$$

$$Q = \begin{bmatrix} 201.9 & 0 \\ 0 & 585.2 \end{bmatrix}$$

$$Y = [-128.0290 \quad -84.0550]$$

$$\bar{\gamma}_\infty = 0.0404, \varepsilon_1 = 0.9517, \varepsilon_2 = 0.6833 \; and \; \varepsilon_3 = 2.2436$$

The state feedback H_∞ controller of the system (5.7) with the system described in [21] is given by:

$$K = [-3.4704 \quad -8.1042]$$

with disturbance attenuation $\gamma_\infty = \sqrt{\bar{\gamma}_\infty} = 0.2011$

We conclude that we have better result since the controller we have designed gives good disturbance attenuation level despite of the all uncertainties in the system and in the controller itself. On the other hand in [21] the disturbance attenuation level was $\gamma_\infty = 2.5$. It is understood that by the controller obtained using our method attenuates disturbance effectively because we used the passivity conditions in our approach, but in the approach exploited in [21] such conditions do not used.

5.4. Positive Realness (Passive) Controller Design for Independent Delay UTDS

Follow the same procedure in the previous subsection but replace (5.13) by the following:

$$\mathfrak{I} = \int_0^\infty [2z(t)^T w(t) - \gamma_p w(t)^T w(t) + \dot{V}(x(t))] dt < 0 \tag{5.15}$$

Little manipulation we get the following optimization problem:

$$Mminimize\ \bar{\gamma}$$

Subject to

$$\begin{bmatrix} \Pi_{11} & A_d & B - X'C' & \Pi_{14} & X'E_1' & \varepsilon_2 B_1 H_1 & 0 \\ * & -Q & 0 & N_2' & 0 & 0 & 0 \\ * & * & -\gamma_p - (D + D') & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{1,}I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & \varepsilon_2 H_1' N_3' \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \qquad (5.16)$$

$$E^T X = XE \geq 0$$

$$\varepsilon_i > 0, i = 1,2,3\ , X > 0, \gamma_p > 0$$

$$\Pi_{11}\ as\ in\ 6.1 - C'C$$

Example (5.3):

Consider the same system as in [20]

$A = \begin{bmatrix} 0.1 & 1 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.5 & 0.2 & 0.1 \end{bmatrix}$, $A_d = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0.5 & -0.1 & 0 \\ 0 & 0.1 & -0.2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 0.1 & 0 & -0.1 \\ 0.2 & 0.5 & 0.1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 0.1 \end{bmatrix}$, $M = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}$, $N_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \end{bmatrix}$, $N_3 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$, $H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 0.1 & 0 & 0.3 \end{bmatrix}$

By solving optimization problem (5.16) using Matlab LMI Control Toolbox, we get the solution as follows:

$$X = \begin{bmatrix} 1.9893 & -0.4901 & 0 \\ -0.4901 & 1.8709 & 0 \\ 0 & 0 & 1.9105 \end{bmatrix}$$

$$Q = \begin{bmatrix} 2.3134 & 0 & 0 \\ 0 & 2.2634 & 0 \\ 0 & 0 & 2.2857 \end{bmatrix}$$

$$Y = \begin{bmatrix} -0.0817 & -3.9424 & -0.5374 \\ -1.2793 & -2.1531 & -1.9055 \end{bmatrix}$$

The state feedback passive controller of the system (5.7) with the system described in [20] is given by

$$K = \begin{bmatrix} -0.5989 & -2.2642 & -0.2813 \\ -0.9905 & -1.4104 & -0.9974 \end{bmatrix}$$

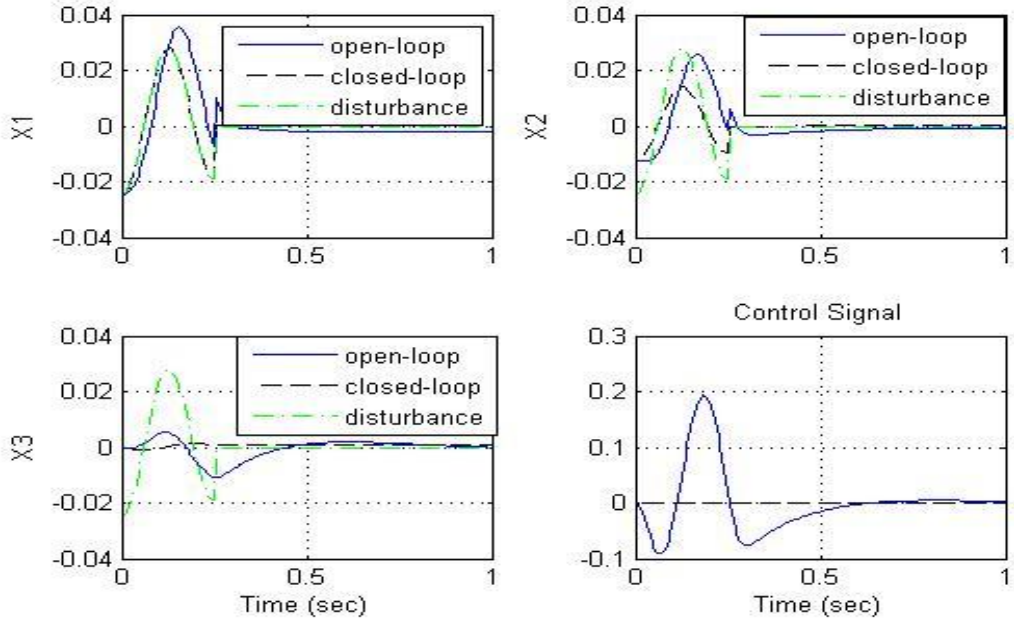


Figure (5.2) Robust performance and stability Example(5.3)

From the plot we can observe that the response of the closed control system with controller (black dashed) obtained is good for disturbance rejection (green), and in this example we achieved the robust stability and robust performance.

Example (5.4):

Again consider the system [21]:

$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$, $A_d = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$, $M = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $N_1 = [1 \quad 0.5]$, $N_2 = [1 \quad 0.4]$, $N_3 = 0.2$, $C = \begin{bmatrix} 0.2 & 1 \\ 1.5 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

By solving optimization problem (5.19) for the system in [21] we get the following results:

$$X = \begin{bmatrix} 1.2829 & -0.9874 \\ -0.9874 & 4.4460 \end{bmatrix}$$

$$Q = \begin{bmatrix} 393.42 & 0 \\ 0 & 156.95 \end{bmatrix}$$

$$Y = [-27.7295 \quad -29.9643]$$

$$\varepsilon_1 = 0.9517, \varepsilon_2 = 0.6833 \; and \; \varepsilon_3 = 2.2436$$

And the state feedback passive controller is given by:

$$K = [-32.3267 \quad -13.9191]$$

with disturbance attenuation level $\gamma_p = 0.4629$

As shown from result the state feedback controller based on passivity criterion gives us better result comparing with H infinity controller designed in [23], since the disturbance attenuation in [23] was $\gamma_\infty = 2.5$.

5.5. Delay Dependent State Feedback Passive Controller Design

Let us now study the delay dependent stability analysis for the UTDS (Σ_Δ), since the delay independent category achieved in the previous subsection may be conservative when the delays are small, in this subsection we will discuss the delay dependent category (technique) of robust state feedback controller design for UTDS (Σ_Δ). We go to design uncertain controller which renders the closed loop control system described in (5.7) asymptotically stable with prescribed disturbance attenuation level for any given delay satisfying $0 < \tau \le \bar{\tau}$. For simplicity let us make a little modification in the system(5.1), and assume that there is no uncertain parameter in the input matrix B_1 and the matrix E is an identity matrix with appropriate dimension.

Theorem 5.2: the system (Σ_Δ) is robustly stable for any time delay satisfying $0 < \tau \le \bar{\tau}$, if there exist matrices such that the following LMI holds:

$$\begin{bmatrix} L & \bar{d}L_1 & L_2 & L_3 \\ * & -J_1 & 0 & 0 \\ * & * & -J_2 & 0 \\ * & * & * & -J_3 \end{bmatrix} < 0 \tag{5.20}$$

Where$L = \quad (A + A_d)X + X(A + A_d)^T + B_1Y + Y^TB_1^T + \sum_{i=1}^{4} \varepsilon_i HH^T$
$+ \quad \varepsilon_2 B_1 H_1 H_1^T B_1^T + A_d(X_1 + X_2)A_d^T,$

$L_1 = [XA + Y^TB_1^T \quad XA_d^T \quad XN_1^T + B_1H_1^T \quad XN_d^T]$

$L_2 = [XN_1^T \quad XN_2^T \quad XE_1^T \quad A_d(X_1 + X_2)N_2^T]$

$L_3 = X^TC, J_1 = diag(X_1 - \varepsilon_5 HH^T, X_2 - \varepsilon_6 HH^T, \mathrm{I}\varepsilon_5, \mathrm{I}\varepsilon_6$

$J_2 = diag(\varepsilon_1, \varepsilon_3, \varepsilon_2, \varepsilon_4 I - N_d(X_1 + X_2)N_d^T) and\, J_3 = (D_w + D_w^T)$

Proof: Using Leibniz-Newton formula we can write

$$x(t-\tau) = x(t) - \int_{-\tau}^{0} \dot{x}\,(t+\theta)d\theta$$
$$= x(t) - \int_{-\tau}^{0} [\,A_c(t+\theta) + A_{d_c}x(t+\theta-\tau) + Bw(t+\theta)d\theta \tag{5.21}$$

Substituting (5.21) into (5.7) yields

$$\dot{x}(t) = A_cx(t) + A_{d_c}\{x(t) + \int_{-\tau}^{0} [\,A_c(t+\theta)$$

$$+A_{d_c}x(t+\theta-\tau)+Bw(t+\theta)]d\theta\}+Bw(t) \quad (5.22)$$

$$\dot{x}(t)=[A_c+A_{d_c}]x(t)-A_{d_c}\left\{\int_{-\tau}^{0}A_cx(t+\theta)d\theta+\int_{-\tau}^{0}A_{d_c}x(t+\theta-\tau)d\theta+\int_{-\tau}^{0}Bw(t+\theta)d\theta\right\}+Bw(t) \quad (5.23)$$

Here we get $A_c=(A+B_1K+\Delta A+B_1\Delta K)$

Define LKF as follows:

$$V(x)=x^T(t)Px(t)+V_1(x) \quad (5.24)$$

Where $V_1(x)=\int_{-d}^{0}\left(\int_{t+\theta}^{t}x^T(\tau)Q_1x(\tau)d\tau+\int_{t+\theta-d}^{t}x^T(\tau)Q_2x(\tau)\,d\tau\right)d\theta$

The derivative of $V(x)$ in (5.24) along the trajectories states i.e. the solution of the system (Σ_Δ) with $w=0,$ with respect to t is given by:

$$\dot{V}(x)=x^T(t)[A_c^TP+A_{dc}^TP+PA_c+PA_{dc}]x(t)$$

$$-2x^T(t)PA_{dc}\int_{-t}^{0}A_cx(t+\theta)d\theta-2x^T(t)PA_{dc}\int_{-t}^{0}A_{dc}x(t+\theta-\tau)d\theta+\dot{V}_1(x)$$

$$=2x^T(t)PA_cx(t)+2x^T(t)PA_{dc}x(t)-2x^T(t)PA_{dc}\left\{\int_{-t}^{0}A_cx(t+\theta)d\theta+\int_{-t}^{0}A_{dc}x(t+\theta-\tau)d\theta\right\}+x^T(t)(dQ_1+dQ_2)x(t)+s(x(t),t) \quad (5.25)$$

Where,

$$s(x(t),t)=-\int_{-d}^{0}x^T(t+\theta)Q_1x(t+\theta)d\theta-\int_{-d}^{0}x^T(t+\theta-d)Q_2x(t+\theta-d)\,d\theta \quad (5.26)$$

Using lemma 4 gives

■ $2x^T(t)PA_cx(t)\le x^T(t)\langle PA+A^TP+PB_1K+K^TB_1^TP+\alpha_1PHH^TP+\alpha_1^{-1}N_1^TN_1+\alpha_2PB_1H_1H_1^TB_1^TP+\alpha_2^{-1}E_1^TE_1\rangle x(t)$

■ $-2x^T(t)PA_{dc}\int_{-d}^{0}A_cx(t+\theta)d\theta\le dx^T(t)PA_{dc}P_1{A_{dc}}^TPx(t)$

$$+\int_{-d}^{0}x^T(t+\theta)A_c^TP_1^{-1}A_cx(t+\theta)d\theta$$

$$\le dx^T(t)PA_{dc}P_1{A_{dc}}^TPx(t)+\int_{-d}^{0}x^T(t+\theta)W_1x(t+\theta)d\theta$$

■ $-2x^T(t)PA_{dc}\int_{-d}^{0}A_{dc}x(t+\theta-\tau)d\theta\le dx^T(t)PA_{dc}P_2{A_{dc}}^TPx(t)$

$$+\int_{-d}^{0}x^T(t+\theta)A_{dc}^TP_2^{-1}A_{dc}x(t+\theta)d\theta$$

$$\le dx^T(t)PA_{dc}P_2{A_{dc}}^TPx(t)+\int_{-d}^{0}x^T(t+\theta)W_2x(t+\theta)d\theta$$

■ $PA_{dc}(P_1+P_2)A_{dc}^TP\le PA_d(P_1+P_2)A_d^TP+W_3$

Where,

$$W_1=(A+B_1K)^T(P_1-\alpha_5HH^T)^{-1}(A+B_1K)+\alpha_5^{-1}(N_1+H_1K)^T(N_1+H_1K)$$

$$W_2=A_d^T(P_2-\alpha_6HH^T)^{-1}A_d+\alpha_6^{-1}N_d^TN_d$$

$W_3 = P\{A_d(P_1+P_2){N_d}^T[\alpha_4 I - N_d(P_1+P_2)N_d^T]^{-1}N_d(P_1+P_2)A_d^T + \alpha_4 HH^T\}P$

For any scalars $\alpha_i > 0, i = 1,2,\dots,6, P_1 > 0$ and $P_2 > 0,$ $such$ $that P_1 - \alpha_5 HH^T > 0,$ $P_2 - \alpha_6 HH^T > 0, \alpha_4 I - N_d(P_1+P_2)N_d^T > 0$ we get

$$\dot{V}(x) \le x(t)^T P[\Phi(P, P_1, P_2, K, \bar{\alpha}_1, \bar{\alpha}_2, d)]Px(t)$$

Where

$\Phi(P, P_1, P_2, K, \bar{\alpha}_1, \bar{\alpha}_2, d) = (A + A_d)P^{-1} + P^{-1}(A+A_d)^T + B_1 K P^{-1} + P^{-1}K^T B_1^T + P^{-1}(\alpha_1^{-1}N_1^T N_1 + \alpha_3^{-1}N_2^T N_2 + \alpha_2^{-1}E_1^T E_1)P^{-1} + (\alpha_1 + \alpha_3)HH^T + \alpha_2 B_1 H_1 H_1^T B_1^T + dP^{-1}[\sum_{i=1}^{3} W_i + PA_d(P_1+P_2)A_d^T P]P^{-1}$

$$\bar{\alpha}_1 = [\alpha_1,,\dots,\alpha_4]^T \ and \ \bar{\alpha}_2 = [\alpha_5, \alpha_6]^T$$

Now, let $P^{-1} = X, P_1 = \bar{d}^{-1}X_1, P_2 = \bar{d}^{-1}X_2, Y = KX \Rightarrow K = YX^{-1}, \bar{\alpha}_1 = \bar{\varepsilon}_1, \bar{\alpha}_2 = \bar{d}^{-1}\bar{\varepsilon}_2$, where $\bar{\varepsilon}_1 = [\varepsilon_1, \dots, \varepsilon_4]^T$ and $\bar{\varepsilon}_2 = [\varepsilon_5, \varepsilon_6]^T$. Then from the inequality (6.20) it is follows that

$\varepsilon_4 I - N_d(P_1+P_2)N_d^T > 0, P_1 - \varepsilon_5 HH^T > 0, P_2 - \varepsilon_6 HH^T > 0$

and

$$\dot{V}(x) \le x(t)^T X^{-1}\left[\Phi\left(X^{-1}, \bar{d}^{-1}X_1, \bar{d}^{-1}X_2, YX^{-1}, \bar{\varepsilon}_1, \bar{d}^{-1}\bar{\varepsilon}_2, \bar{d}\right)\right]X^{-1}x(t) < 0. \qquad (5.27)$$

If this inequality holds then there is a scalar $c > 0$ such that $\dot{V}(x) \le -c\|x(t)\|^2$ that guarantees the stability of the closed loop system. Let us now define the next inequality to guarantee the asymptotic stability analysis for the UTDS based on the passivity conditions.

$$\dot{V}_1(x) = \dot{V}(x) - 2z^T(t)w(t) \qquad (5.28)$$

Where $\dot{V}(x)$ as in (5.27) and $-2z^T(t)w(t) \le -[z^T(t)w + w^T(t)z]$, substitute these into (5.28) we get

$$\dot{V}_1(x) = \dot{V}(x) - 2z^T(t)w(t) = \mathcal{M}^T\mathcal{B}\mathcal{M} < 0 \qquad (5.29)$$

Where $\mathcal{M} = [x^T(t) \quad w^T(t)]$, and

$$\mathcal{B} = \begin{bmatrix} \Phi\left(X^{-1}, \bar{d}^{-1}X_1, \bar{d}^{-1}X_2, YX^{-1}, \bar{\varepsilon}_1, \bar{d}^{-1}\bar{\varepsilon}_2, \bar{d}\right) & XC^T \\ * & -(D_w + D_w^T) \end{bmatrix} < 0 \qquad (5.30)$$

Rearrange and use some algebra then apply Schur complement we get (5.20).

This concludes the proof.

Example (5.5):

Consider the uncertain time delay system in [20], the system described as:

(Σ_Δ): E $\dot{x}(t) = A_c x(t) + A_{d_c} x(t-\tau) + Bw(t)$

$z(t) = Cx + Dw(t)$

With parameters as:

$$A = \begin{bmatrix} 0.1 & 1 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.5 & 0.2 & 0.1 \end{bmatrix}, A_d = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0.5 & -0.1 & 0 \\ 0 & 0.1 & -0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$$

$C = \begin{bmatrix} 0.1 & 0 & -0.1 \\ 0.2 & 0.5 & 0.1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 0.1 \end{bmatrix}, M = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}, N_1 = [0.1 \quad 0 \quad 0.1], N_3 =$

$[0 \quad 0.1], H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ and $E_1 = [0.1 \quad 0 \quad 0.3], N_2 = [.2 \quad 0 \quad -.1]$

Using CVX control toolbox we get the solution of LMI (5.20) as:

$$X = \begin{bmatrix} 0.1176 & -0.1009 & -0.1354 \\ -0.1009 & 0.5554 & -0.0095 \\ -0.1354 & -0.0095 & 0.1896 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 2.5639 & 4.6061 & -0.0847 \\ 4.6061 & 29.3359 & 7.6159 \\ -0.0847 & 7.6159 & 3.1463 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0.2230 & -0.4501 & -0.3915 \\ -0.4501 & 2.3219 & 0.4175 \\ -0.3915 & 0.4175 & 0.9687 \end{bmatrix}$$

$$Y = \begin{bmatrix} -0.2721 & -1.0888 & -0.1086 \\ -0.2739 & -1.6853 & -0.3159 \end{bmatrix}$$

$$\varepsilon_1 = 0.0454, \varepsilon_2 = 2.7632, \varepsilon_3 = 0.2603,$$
$$\varepsilon_4 = 3.7918e-006, \varepsilon_5 = 6.4960, \varepsilon_6 = 3.6612$$

From theorem 5.3.1 we conclude that this system has delay dependent solution. Furthermore, a state feedback passive controller can be obtained:

$$u(t) = 10^6 \begin{bmatrix} -1.4354 & -0.2785 & -1.0392 \\ -2.1183 & -0.4110 & -1.5335 \end{bmatrix}$$

which will stabilize the system for all admissible uncertainties and any $d \leq t \leq 3.9431$. We conclude based on the obtained result using proposed approach in this thesis that the system renders passive despite of the uncertainties affected the system and in addition, the system can tolerate the delay less than 3.9431, thus constitutes the one of main contribution in this thesis, since in the reference [20] only the delay-independent criterion considered and there is no information about the delay the system can be handled without affecting the stability analysis and performance of the closed loop control system.

5.6. SFPC for TDS with TVD in the State and Control Channels

The time delay systems that contain the state delay and derivative of the state delay is very important type, since such systems arising basically in the field of power systems, since these time delay systems consider the natural models of fluctuations in the voltage and current in problems arising in transmission lines[22]. In this chapter we shall study the state feedback passive controller for time varying time delay system contained delays in the state and input channels and we will use the change of variables technique already used in the previous chapters to demonstrate the effectiveness of this method, since as will would be shown that this method easy to deal with, in addition it gives us better results comparison with the results given in[22]. So, consider the same system in[22]. I will rewrite it here for convenience

$$\begin{aligned}
\dot{x}(t) &= A_0x(t) + A_1x\big(t-\tau_1(t)\big) + A_2\dot{x}\big(t-\tau_2(t)\big) + B_1w(t) + B_2u(t-\tau_3(t)) \\
z(t) &= C_1x(t) + D_1u(t) + D_{11}w(t) \\
y(t) &= C_2x(t) + D_2w(t) \\
x(t) &= \phi(t), t \geq 0.
\end{aligned} \tag{5.31}$$

Since $\tau_i(t), i = 1,2,3$ are arbitrary differentiable functions satisfying:

$$\begin{Bmatrix} 0 \leq \tau_1(t) < \infty, 0 \leq \tau_2(t) < \infty, 0 \leq \tau_3(t) < \infty \\ \dot{\tau}_1(t) \leq \sigma_1 < 1, \dot{\tau}_2(t) \leq \sigma_2 < 1, \dot{\tau}_3(t) \leq \sigma_3 < 1 \end{Bmatrix} \tag{5.32}$$

Other notations are the same as in (2.13) in this thesis. So, we will go to construct the state feedback passive controller $u = Kx(t)$ which renders the closed loop time delay time varying control system passive, hence asymptotically stable. The closed loop control system will be shown as:

$$\begin{aligned}
\dot{x}(t) &= A_0x(t) + A_1x\big(t-\tau_1(t)\big) + A_2\dot{x}\big(t-\tau_2(t)\big) + B_1w(t) + B_2Kx(t-\tau_3(t)) \\
z(t) &= (C_1 + D_1K)x(t) + D_{11}w(t) \\
y(t) &= C_2x(t) + D_2w(t) \\
x(t) &= \phi(t), t \geq 0.
\end{aligned} \tag{5.33}$$

Theorem 5.3: Consider the closed system (5.33), for given $Q = Q^T > 0$, if there exist $Y = Y^T > 0$, $M = M^T > 0$ and matrix L,which satisfy the following LMI:

$$\begin{bmatrix}
\mathcal{F} & A_1Y & A_2Y & B_2L & \mathfrak{H} & YA_0^T \\
* & -(1-\sigma_1)M & 0 & 0 & 0 & YA_1^T \\
* & * & -(1-\sigma_2)M & 0 & 0 & YA_2^T \\
* & * & * & -(1-\sigma_3)M & 0 & L^TB_2^T \\
* & * & * & * & -(D_{11}+D_{11}^T) & B_1^T \\
* & * & * & * & * & -Q^{-1}
\end{bmatrix} < 0 \tag{5.34}$$

Where $\mathcal{F} = YA_0^T + A_0Y + 2M, \mathfrak{H} = B_1 - YC_1^T - L^TD_1^T$

then the system (5.32) is passive and asymptotically stable with the state feedback passive controller u = Kx(t).

Proof: Define a LKF V(x(t)) as follows:

$$V\big(x(t)\big) = x^T(t)Px(t) + \int_{t-\tau_1(t)}^{t} x^T(s)Qx(s)ds + \int_{t-\tau_2(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds + \int_{t-\tau_3(t)}^{t} x^T(s)Qx(s)ds \tag{5.35}$$

Calculating the derivative of (5.35) along the trajectories of (5.33) it follows that:

$$\begin{aligned}
\dot{V}(x_t) = {} & \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
& + x^T(t)Qx(t) - \big(1-\dot{\tau}_1(t)\big)x^T\big(t-\tau_1(t)\big)Qx\big(t-\tau_1(t)\big) \\
& + \dot{x}^T(t)Q\dot{x}(t) - (1-\dot{\tau}_2(t))\dot{x}^T(t)\big(t-\tau_2(t)\big)Q\big(t-\tau_2(t)\big) \\
& + x^T(t)Qx(t) - (1-\dot{\tau}_3(t))x^T\big(t-\tau_3(t)\big)Qx\big(t-\tau_3(t)\big) \\
\le {} & \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + 2x^T(t)Qx(t) \\
& -(1-\sigma_1)x^T\big(t-\tau_1(t)\big)Qx\big(t-\tau_1(t)\big) \\
& + \dot{x}^T(t)Q\dot{x}(t) - (1-\sigma_2)\dot{x}^T(t)\big(t-\tau_2(t)\big)Q\big(t-\tau_2(t)\big) \\
& -(1-\sigma_3)x^T\big(t-\tau_3(t)\big)Qx\big(t-\tau_3(t)\big) \\
= {} & x^T(t)[A_0^TP + PA_0 + 2Q]x(t) + 2x^T(t)PA_1x\big(t-\tau_1(t)\big) \\
& + 2x^T(t)PA_2\dot{x}\big(t-\tau_2(t)\big) + 2x^T(t)PB_2Kx\big(t-\tau_3(t)\big) \\
& + 2x^T(t)PB_1w(t) - (1-\sigma_1)x^T\big(t-\tau_1(t)\big)Qx\big(t-\tau_1(t)\big) \\
& + \dot{x}^T(t)Q\dot{x}(t) - (1-\sigma_2)\dot{x}^T(t)\big(t-\tau_2(t)\big)Q\big(t-\tau_2(t)\big) \\
& -(1-\sigma_3)x^T\big(t-\tau_3(t)\big)Qx\big(t-\tau_3(t)\big).
\end{aligned}$$

Now apply this equation:

$$\begin{aligned}
\dot{V}(x_t) - 2z^T(t)w(t) = {} & x^T(t)[A_0^TP + PA_0 + 2Q]x(t) + 2x^T(t)PA_1x\big(t-\tau_1(t)\big) \\
& + 2x^T(t)PA_2\dot{x}\big(t-\tau_2(t)\big) + 2x^T(t)PB_2Kx\big(t-\tau_3(t)\big) \\
& + 2x^T(t)(PB_1 - C_1^T + K^TD_1^T)w(t) - w^T(t)(D_{11} + D_{11}^T)w(t) \\
& + \dot{x}^T(t)Q\dot{x}(t)) - (1-\sigma_1)x^T\big(t-\tau_1(t)\big)Qx\big(t-\tau_1(t)\big) \\
& -(1-\sigma_2)\dot{x}^T(t)\big(t-\tau_2(t)\big)Q\big(t-\tau_2(t)\big) \\
& -(1-\sigma_3)x^T\big(t-\tau_3(t)\big)Qx\big(t-\tau_3(t)\big). \\
= {} & \eta^T(t)\Omega\eta(t)
\end{aligned} \tag{5.36}$$

Where

$$\eta(t) = \big[x(t) \quad x(t-\tau_1(t)) \quad \dot{x}\big(t-\tau_2(t)\big) \quad x\big(t-\tau_3(t)\big) \quad w(t)\big]^T,$$

Applying Schur complement as we already did in the previous subsequent chapters we get the following LMI:

$$\Omega = \begin{bmatrix} \Omega_{11} & PA_1 & PA_2 & PB_2K & PB_1 - (C_1 + D_1K)^T & A_0^T \\ * & -(1-\sigma_1)Q & 0 & 0 & 0 & A_1^T \\ * & * & -(1-\sigma_2)Q & 0 & 0 & A_2^T \\ * & * & * & -(1-\sigma_3)Q & 0 & K^TB_2^T \\ * & * & * & * & -(D_{11} + D_{11}^T) & B_1^T \\ * & * & * & * & * & -Q^{-1} \end{bmatrix} \tag{5.37}$$

Where $\Omega_{11} = A_0^TP + PA_0 + 2Q$

Pre- and post- multiplying Ω by $diag[P^{-1},P^{-1},P^{-1},P^{-1},I,I]$, we get

$$\begin{bmatrix} \mathcal{M}_{11} & A_1P^{-1} & A_2P^{-1} & B_2KP^{-1} & \mathcal{M}_{15} & P^{-1}A_0^T \\ * & -(1-\sigma_1)P^{-1}QP^{-1} & 0 & 0 & 0 & P^{-1}A_1^T \\ * & * & -(1-\sigma_2)P^{-1}QP^{-1} & 0 & 0 & P^{-1}A_2^T \\ * & * & * & -(1-\sigma_3)P^{-1}QP^{-1} & 0 & P^{-1}K^TB_2^T \\ * & * & * & * & -(D_{11}+D_{11}^T) & B_1^T \\ * & * & * & * & * & -Q^{-1} \end{bmatrix} \quad (5.38)$$

Where$\mathcal{M}_{11} = \mathrm{P^{-1}A_0^T + A_0P^{-1} + 2P^{-1}QP^{-1}}$, $\mathcal{M}_{15} = B_1 - P^{-1}((C_1 + D_1K)^T$

As shown from the above inequality it is not LMI since it contains nonlinear unknown terms, hence we will go to use as mentioned in the beginning of this section the change of variables method to make the previous inequality LMI. So, let $\mathrm{Y} = \mathrm{P^{-1}, P^{-1}QP^{-1}, L = KP^{-1} = KY}$ so the state feedback passive controller can be derived from $\mathrm{K = LP^{-1} = LY}$, put these quantities in the above inequality we get the next LMI:

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{11} & \mathrm{A_1Y} & \mathrm{A_2Y} & \mathrm{B_2L} & \mathcal{L}_{15} & \mathrm{YA_0^T} \\ * & -(1-\sigma_1)\mathrm{M} & 0 & 0 & 0 & \mathrm{YA_1^T} \\ * & * & -(1-\sigma_2)\mathrm{M} & 0 & 0 & \mathrm{YA_2^T} \\ * & * & * & -(1-\sigma_3)\mathrm{M} & 0 & \mathrm{L^TB_2^T} \\ * & * & * & * & -(\mathrm{D_{11}}+\mathrm{D_{11}^T}) & \mathrm{B_1^T} \\ * & * & * & * & * & -\mathrm{Q^{-1}} \end{bmatrix} \quad (5.39)$$

Where$\mathcal{L}_{11} = \mathrm{YA_0^T + A_0Y + 2M}$, and $\mathcal{L}_{15} = \mathrm{B_1 - YC_1^T - L^TD_1^T}$, put them in (5.39), we get (5.34). So the theorem is proved.

Example (5.6):

Let us see the same example as in [22]. The matrices describe the system are as follow:

$$\mathrm{A_0} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \mathrm{A_1} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \mathrm{A_2} = \begin{bmatrix} 0 & 0 \\ 0.3 & 0.2 \end{bmatrix}, \mathrm{B_1} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

$$\mathrm{B_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathrm{C_1} = [1 \quad 1], \mathrm{C_2} = [1 \quad 1], \mathrm{D_1} = \mathrm{D_2} = \mathrm{D_{11}} = [1],$$

$$\tau_1(\mathrm{t}) = 2.0 + 0.3\sin(\mathrm{t}), \tau_2(\mathrm{t}) = 3.5 + 0.4\cos(\mathrm{t}), \tau_3(\mathrm{t}) = 4.0 + 0.2\sin(\mathrm{t})$$

Hence we have$\sigma_1 = 0.3, \sigma_2 = 0.4$ and $\sigma_3 = 0.2$, select$\mathrm{Q} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$. So using Matlab toolbox we solve the LMI (5.34) and obtain that:

$$\mathrm{K} = [\,-0.3271 \quad -0.3119], \ \mathrm{P} = \begin{bmatrix} 0.3887 & 0.3235 \\ 0.3235 & 0.4568 \end{bmatrix}$$

The results obtained here almost similar to the results obtained in the [22].

5.7. PR and Passivity Analysis for TDS with Varying Delay

As mentioned in definition 2 and under this definition we can define the next statement: The closed loop system (5.39) is passive with $\gamma_p > 0$ if the following inequality holds:

$$\dot{V}\big(x(t)\big) - 2z^T(t)w(t) - \gamma_p w^T(t)w(t) \le \zeta^T \Pi \zeta$$

Where $\Pi =$

$$\begin{bmatrix} \mathcal{F} & A_1Y & A_2Y & B_2L & \mathfrak{H} & YA_0^T \\ * & -(1-\sigma_1)M & 0 & 0 & 0 & YA_1^T \\ * & * & -(1-\sigma_2)M & 0 & 0 & YA_2^T \\ * & * & * & -(1-\sigma_3)M & 0 & L^TB_2^T \\ * & * & * & * & \rho & B_1^T \\ * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \qquad (5.40)$$

$$\rho = -\gamma_p - (D0' + D0)$$

and all the others as the previous section.

$\dot{V}\big(x(t)\big) - 2z^T(t)w(t) - \gamma_p w^T(t)w(t) =$

$$\begin{aligned} & x^T(t)[A_0^TP + PA_0 + 2Q]x(t) + 2x^T(t)PA_1x\big(t-\tau_1(t)\big) \\ & + 2x^T(t)PA_2\dot{x}\big(t-\tau_2(t)\big) + 2x^T(t)PB_2Kx\big(t-\tau_3(t)\big) \\ & + 2x^T(t)(PB_1 - C_1^T + K^TD_1^T)w(t) - w^T(t)\big(\gamma_p + D_{11} + D_{11}^T\big)w(t) \\ & + \dot{x}^T(t)Q\dot{x}(t)) - (1-\sigma_1)x^T\big(t-\tau_1(t)\big)Qx\big(t-\tau_1(t)\big) \\ & \quad -(1-\sigma_2)\dot{x}^T(t)\big(t-\tau_2(t)\big)Q\big(t-\tau_2(t)\big) \\ & \quad -(1-\sigma_3)x^T\big(t-\tau_3(t)\big)Qx\big(t-\tau_3(t)\big). \end{aligned}$$

Rearrange and put the result in the dense form, we get (5.40). If there are

$Y = Y^T > 0, M = M^T \ge 0,$ *and matrix* L , Such that (5.40) holds, then the system (5.33) guaranteed to be asymptotically stable and strictly passive with disturbance attenuation levelγ_p. So using the MATLAB software, LMI toolbox or CVX program we can derive the passive controller $K = LY^{-1}$ for time varying delay system that renders the overall system asymptotically stable and strictly positive real (SPR), hence strictly passive (SP).

Example (5.7):

For the same system described in the[23],

$A_0 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}$,$A_2 = \begin{bmatrix} 0 & 0 \\ 0.3 & 0.2 \end{bmatrix}$,$B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_1 = C_2 = [1 \quad 1]$,$D_1 = D_2 = D_{11} = 1$, $\tau_1(t) = 2.0 + 0.3\sin(t)$, $\tau_2 = 3.5 + 0.4\cos(t)$, $\tau_3(t) = 4 + 0.2\sin(t)$. Hence we have $\sigma_1 = 0.3, \sigma_2 = 0.4$ *and* $\sigma_3 = 0.2$
using CVX tool we get the next result:

$$Y = \begin{bmatrix} 4.9512 & -3.4959 \\ -3.4959 & 4.0930 \end{bmatrix} > 0, M = \begin{bmatrix} 1.4670 & -0.8398 \\ -0.8398 & 0.6879 \end{bmatrix} > 0$$

it means that the time delay time varying control system (5.39) rendered strictly passive (SPR) by virtue of the passive controller, $K = [-0.2216 \quad -0.1400]$ when $\gamma_p = 0.5739$. In [23] the optimization problem not addressed, only the state feedback passive controller was obtained, and there was no information about the robustness of the closed loop control system.

CHAPTER 6 CONCLUSION

It is very important issue in control theory to deal with delays occurred in different channels of the control system. Since delays may force the system to be unstable, it is essential to take them into account when discussing the stability analysis and synthesis of the control systems. In this thesis the problem of stability analysis of linear uncertain time delay systems has been addressed based on the notion of passivity conditions. These conditions have been expressed using a linear matrix inequality approach. LMI approach is very efficient tool to solve such problems, because it can be solved numerically using reliable and available software packages, such as LMI toolbox and CVX toolbox under Matlab software. The problem which we dealt with and solved in this thesis was to find the largest bound for the time delay to ensure the global asymptotic stability of the time delay systems; in addition, using proposed LMI approach, the robust stability analysis and synthesis was addressed, so that the closed loop time delay control system is asymptotically robustly stable despite the uncertainty. To solve these problems, the Lyapunov-Krasovskii functional that contains triple integral term was exploited, to improve the feasible region of stability criterion. It was solvable and we get better results compared with some existing ones, namely we have got seven times greater bound of time delay than in the works published by Magdi Mahmud, since the bound of delay we obtained was 2.9s; on other hand Magdi Mahmud's method was 0.4seconds. The proposed solution obtained better results concerned the amount of delay which the system can tolerate in the presence of uncertain elements or behavior in the system parameters and the controller itself. Finally, future work can be extended to construct an efficient algorithm to auto tuning the parameters (scalars or matrices) to get optimal results; in addition, the proposed method exploited in this thesis can easily be extended for nonlinear systems, containing distributed delays and time delay systems with contain the interval matrices.

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Appendix A

***proof* of lemma 3: From lemma 2, the following inequality holds**

$$(t-s)\int_s^t x^T(u)Mx(u)du \ge \left(\int_s^t x(u)du\right)^T M\left(\int_s^t x(u)du\right)$$

Where $t-h \le s \le t$.

By using fact1inequality (42) is equivalent to the following

$$\begin{bmatrix} \int_s^t x^T(u)Mx(u)du & \int_s^t x^T(u)du \\ \int_s^t x(u)du & (t-s)M^{-1} \end{bmatrix} \ge 0.$$

Integrate (43) from $t-h$ *to* t ***yields***

$$\begin{bmatrix} \int_{t-h}^{t} \beta(t,s)ds & \int_{t-h}^{t}\int_s^t x^T(u)duds \\ \int_{t-h}^{t}\int_s^t x(u)duds & \int_{t-h}^{t}(t-s)M^{-1}ds \end{bmatrix} \ge 0.$$

Where $\beta(t,s)=\int_s^t x^T(u)Mx(u)du$.

APPENDIX B

MATLAB code programs

****

Chapter 3
The code for example 3.1

****

```
%This is for Example 1 determines the passivity
% and asymptotic stability for the system
warning off
clear all
close all
clc

A0=[-3 -2 ;1 0];
A1=[0 .3;-.3 -.2 ];
B=[.5;.4]; C0=[2 0]; D0=[2];
setlmis([]);
P=lmivar(1,[2 1]);
Q=lmivar(1,[2 1]);
lmiterm([1 1 1 P],A0',1);
lmiterm([1 1 1 P],1,A0);
lmiterm([1 1 1 Q],1,1);
lmiterm([1 1 2 P],1,A1);
lmiterm([1 1 3 P],1,B);
lmiterm([1 1 3 0],-C0');
 lmiterm([1 2 2 Q],-1,1);
lmiterm([1 3 3 0],-(D0+D0'));
lmiterm([-2 1 1 Q],1,1);
lmiterm([-3 1 1 P],1,1);
ff=getlmis;
[tt,PP]=feasp(ff);
P=dec2mat(ff,PP,P);
Q=dec2mat(ff,PP,Q);
P
Q
x=[P*A0+A0'*P+Q P*A1 (C0'-P*B);A1'*P -Q  zeros(2,1);(C0-B'*P)
zeros(1,2) -(D0+D0')]
eig(x)
if (P>0&Q>0)
   ('the system is strictly passive and  asymptotically stable')
else
 ('the system is unstable')
end


*********************************************************************
Chapter 2
*********************************************************************
%This is for Example 3.1 determines
%State feedback control design
warning off
clear all
close all
clc
A0=[1 0;-1 -2];A1=[0 0;.2 .1];
B1=[0;.1];B2=[0;1];C0=[1 1];D0=1;
% A0=[0 0;0 1];A1=[-1 -1;0 .7];B2=[0;1];B1=[0;0];C0=[ 0 0];D0=[0];
```

```
setlmis([]);
Y=lmivar(1,[2 1]);
L=lmivar(1,[2 1]);
M=lmivar(1,[2 1]);
Z=lmivar(2,[1 2]);
lmiterm([1 1 1 Y],1,A0','s');
lmiterm([1 1 1 L],1,1);
lmiterm([1 1 1 M],1,1);
lmiterm([1 1 2 Y],A1,1);
lmiterm([1 1 3 Z],B2,1);
lmiterm([1 1 4 0],B1);
lmiterm([1 1 4 Y],-1,C0');
lmiterm([1 1 4 -Z],-1,D0');
lmiterm([1 2 2 L],-1,1);
lmiterm([1 3 3 M],-1,1);
lmiterm([1 4 4 0],-(D0'+D0));
lmiterm([-2 1 1 Y],1,1);
lmiterm([-3 1 1 L],1,1);
lmiterm([-4 1 1 M],1,1);
ff=getlmis;
[tt,PP]=feasp(ff);
Y=dec2mat(ff,PP,Y);
 Z=dec2mat(ff,PP,Z);
  L=dec2mat(ff,PP,L);
  M=dec2mat(ff,PP,M);
  Y
  K=Z*Y^-1
```

****

Chapter 4

Example 4.1

****

```
%This program gives us exact strict feasible
%solution, since in CVX context there is no direct
% inequality such that P>0, so in this program i replaced
%P>=0 by P>=eye(n)
%solution of the problem , and in the same way i replaced the
constraints
%[]<=0 by []<=-[eye(n),........]. See the program for more details
%Done by Mohammed H.E.Aburezeq
%
clear all
close all
clc
A=[-3 -2;1 0];A1=[0 .3;-.3 -.2];B=[.5;.4];C=[2 0];D=[2];

cvx_begin sdp
variable P(2,2) symmetric
variable Q(2,2) symmetric
variable R(2,2) symmetric
tau=.2;
tau=1.41925;
% tau=2.1899
P>=eye(2)
Q>=eye(2)
R>=eye(2)
[A'*P+P*A+Q-R,   A1*P+R,        P*B-C',        tau*tau*A'*R;...
 P*A1'+R,        -(Q+R),        zeros(2,1),    tau*tau*A1'*R;...
 B'*P-C,        zeros(1,2),     -(D+D'),       tau*tau*B'*R;...
```

```
 tau*tau*R*A,   tau*tau*R*A1,   tau*tau*R*B,   -tau*tau*R]       <=-
[eye(2),zeros(2),zeros(2,1),zeros(2);...
 zeros(2),eye(2),zeros(2,1),zeros(2);...
 zeros(1,2),zeros(1,2),eye(1,1),zeros(1,2);...
 zeros(2),zeros(2),zeros(2,1),eye(2,2)]
cvx_end
P
Q
R
tau
```

Chapter 4

Example 4.1

Open loop state responses

```
clc
clear all
close all
A =[-6.0493  -11.5232   -7.1262;1.0000         0         0; 0
1.0000         0];
B=[1;0;0];C=[0.4822 -0.0578 -2.2823];D=[0];
TT=ss(A,B,C,D)

t=[0:0.01:4];
U=[zeros(size(t))];
X0=[0.4;.2];
 CharPoly=poly(A)
 Eigs0=eig(A)
 damp(A);
 X0=[04;.2;0.3];
  [Y0,t,Xo]=lsim(TT,U,t,X0);
      Xo(101,:);
      figure;
      subplot(211), plot(t,Xo(:,1)); grid;
      set(gca,'FontSize',18);
      ylabel('{\itx}_1 ')
      subplot(212),plot(t,Xo(:,2));grid;axis([0 4 -2 1]);
      set(gca,'FontSize',18);
      xlabel('\ittime(sec)');
ylabel('{\itx}_2 ');

*****************************************************
%Chapter5
%Section 5.1
%Numerical Example1
%H infinity controller for uncertain time delay system
*****************************************************************
clear all
close all
clc
cvx_begin sdp
E=[1 0 0;0 1 0;0 0 0];A=[0.1 1 0.1;0.1 0.3 0.1;0.5 0.2 0.1];Ad=[0.1 0
0.2;0.5 -0.1 0;0 0.1 -0.2];
B1=[0.1 0 ;0  1 ;-1 1 ];B=[0.1 0.2  ;0 0.1 ;0.1 0  ];C=[0.1 1 -
0.1;0.2 0.5 0.1];D=[1 0.1  ;.5 1 ];
M=[.1 ;.1 ;.2  ];N1=[.1 0 .1];N2=[.2 0 -0.1];N3=[0 .1 ];H1=[.1 ;.1
];E1=[0.1 0 0.3];
epsilon1=2.6147;epsilon2=2.5913;epsilon3=2.6147;
variable X(3,3) symmetric
variable Q(3,3) symmetric
```

```
variable Y(2,3)
variable gamma
minimize (gamma)
X>=eye(3)
Q>=eye(3)
 E'*X>=0
 X'*E'>=0
% gamma*eye(2)>=0
[A*X+X*A'+(epsilon1+epsilon3)*M*M'+B1*Y+Y'*B1'+C'*C,  Ad,
B+C'*D,      X'*N1'+Y'*N3',    X'*E1',        epsilon2*B1*H1,
zeros(3,1);...
    Ad',                                                  -Q,
zeros(3,2),    N2',             zeros(3,1),    zeros(3,1),
zeros(3,1);...
    B'+D'*C,                                      zeros(2,3),   -
gamma*eye(2,2),      zeros(2,1),      zeros(2,1),      zeros(2,1),
zeros(2,1);...
    N1*X+N3*Y,                                        N2,
zeros(1,2),-eye(1,1)*epsilon1,   zeros(1,1),     zeros(1,1),
zeros(1,1);...
    E1*X,                                            zeros(1,3),
zeros(1,2),    zeros(1,1),       -epsilon2*eye(1,1),  zeros(1,1),
zeros(1,1);...
    epsilon2*H1'*B1',                              zeros(1,3),
zeros(1,2),    zeros(1,1),      zeros(1,1),          -
epsilon2*eye(1,1), epsilon2*H1'*N3';...
    zeros(1,3),                                   zeros(1,3),
zeros(1,2),   zeros(1,1),      zeros(1,1),          epsilon2*N3*H1,
-epsilon3*eye(1,1)]<=0
cvx_end
X
Q
Y
K=Y*X^-1
gamma
gamma1=sqrt(gamma)
```

```
*******************************************************************
%Chapter5
%Section 5.2 (Second program for Numerical example1 witten in LMI
toolbox)
%H infinity controller for uncertain time delay system
********************************************************************
clear all
close all
clc
E=[1 0 0;0 1 0;0 0 0];A=[0.1 1 0.1;0.1 0.3 0.1;0.5 0.2 0.1];Ad=[0.1 0
0.2;0.5 -0.1 0;0 0.1 -0.2];
B1=[0.1 0 ;0  1 ;-1 1 ];B=[0.1 0.2  ;0 0.1 ;0.1 0  ];C=[0.1 1 -
0.1;0.2 0.5 0.1];D=[1 0.1  ;.5 1 ];
M=[.1 ;.1 ;.2  ];N1=[.1 0 .1];N2=[.2 0 -0.1];N3=[0 .1 ];H1=[.1 ;.1
];E1=[0.1 0 0.3];
epsilon1=0.1;epsilon2=2.5913;epsilon3=2.6147;alpha=0.0823
  setlmis([]);
X=lmivar(1,[3 1]);
 Y=lmivar(2,[2 3]);
Q=lmivar(1,[3 1]);
lmiterm([1 1 1 X],A,1,'s');
 lmiterm([1 1 1 Y],B1,1,'s');
  lmiterm([1 1 1 0],C'*C);
lmiterm([1 1 1 0],(epsilon1)*M*M');
```

```
lmiterm([1 1 2 0],Ad);
lmiterm([1 1 3 0],B);
lmiterm([1 1 3 0],C'*D);
lmiterm([1 1 4 -X],1,N1');
lmiterm([1 1 4 -Y],1,N3');
lmiterm([1 1 5 -X],1,E1');
lmiterm([1 1 6 0],epsilon2*B1*H1);
lmiterm([1 2 2 0],-Q);
lmiterm([1 2 4 0],N2');
lmiterm([1 3 3 0],-alpha*eye(2));
lmiterm([1 4 4 0],-epsilon1*eye(1));
lmiterm([1 5 5 0],-epsilon2*eye(1));
lmiterm([1 6 6 0],-epsilon2*eye(1));
lmiterm([1 6 7 0],epsilon2*H1'*N3');
lmiterm([1 7 7 0],-epsilon2*eye(1,1));
  lmiterm([-2 1 1 X],1,1);
  lmiterm([-3 1 1 Q],1,1);
ff=getlmis;
[tt,PP]=feasp(ff);
X=dec2mat(ff,PP,X);
Y=dec2mat(ff,PP,Y);
Q=dec2mat(ff,PP,Q);
X
Y
Q
gamma=(alpha)
K=Y*X^-1
ndec = decnbr(ff)
*****************************************************************
%Chapter5
%Section 5.1
%Numerical Example5.2
%H infinity controller for uncertain time delay system
*******************************************************************
clear all
close all
clc
cvx_begin sdp
%  E=[1 0 ;0 0];
A=[-2 1;0 -3];Ad=[-1 0;0 1];
B1=[1;1];B=[0.5;0];C=[0.2 1;1.5 1];D=[1;1];
M=[.5;.5  ];N1=[1 .5];N2=[1 .4];N3=[.2 ];H1=[.1  .1];E1=[0.1 1 ];
epsilon1=2.6147;epsilon2=2.5913;epsilon3=2.6147;
variable X(2,2) symmetric
variable Q(2,2) diagonal
variable Y(1,2)
variable alpha
minimize (alpha)
% alpha=2.5
X>=eye(2)
Q>=eye(2)
 [A*X+X*A'+(epsilon1+epsilon3)*M*M'+B1*Y+Y'*B1',  Ad,          B+C'*D,
X'*N1'+Y'*N3',        X'*E1',              epsilon2*B1*H1,
zeros(2,1),          C';...
    Ad',                                                 -Q,
zeros(2,1),          N2',                 zeros(2,1),
zeros(2,2),            zeros(2,1),              zeros(2,2);...
    B'+D'*C,                                         zeros(1,2),    -
alpha*eye(1,1),       zeros(1,1),          zeros(1,1),
zeros(1,2),            zeros(1,1),              zeros(1,2);...
```

```
    N1*X+N3*Y,                                  N2,
zeros(1,1),        -eye(1,1)*epsilon1,   zeros(1,1),
zeros(1,2),            zeros(1,1),             zeros(1,2);...
    E1*X,                                     zeros(1,2),
zeros(1,1),         zeros(1,1),         -epsilon2*eye(1,1),
zeros(1,2),            zeros(1,1),              zeros(1,2);...
    epsilon2*H1'*B1',                         zeros(2,2),
zeros(2,1),          zeros(2,1),          zeros(2,1),          -
epsilon2*eye(2,2),    epsilon2*H1'*N3',         zeros(2,2);...
    zeros(1,2),                               zeros(1,2),
zeros(1,1),          zeros(1,1),          zeros(1,1),
epsilon2*N3*H1,      -epsilon3*eye(1,1),     zeros(1,2);...
     C,                                        zeros(2,2),
zeros(2,1),         zeros(2,1),          zeros(2,1),
zeros(2,2),         zeros(2,1),      -eye(2)]<=0
cvx_end
X
Q
Y
K=Y*X^-1
alpha
gamma=sqrt(alpha)
*******************************************************************
%Chapter5
%Section 5.2 numerical example 5.2
%Positive realness passive  controller for uncertain time delay
system
******************************************************************
clear all
close all
clc
cvx_begin sdp
E=[1 0 0;0 1 0;0 0 0];A=[0.1 1 0.1;0.1 0.3 0.1;0.5 0.2 0.1];Ad=[0.1 0
0.2;0.5 -0.1 0;0 0.1 -0.2];
B1=[0.1 0 ;0  1 ;-1 1 ];B=[0.1 0.2  ;0 0.1 ;0.1 0  ];C=[0.1 1 -
0.1;0.2 0.5 0.1];D=[1 0.1  ;.5 1 ];
M=[.1 ;.1 ;.2  ];N1=[.1 0 .1];N2=[.2 0 -0.1];N3=[0 .1 ];H1=[.1 ;.1
];E1=[0.1 0 0.3];
epsilon1=2.6147;epsilon2=2.5913;epsilon3=2.6147;
variable X(3,3) symmetric
variable Q(3,3) diagonal
variable Y(2,3)
variable gamma
minimize (gamma)
X>=eye(3)
Q>=eye(3)
    E'*X'>=0
   X'*E'>=0
   gamma>=0
[A*X+X*A'+(epsilon1+epsilon3)*M*M'+B1*Y+Y'*B1',  Ad,      B-X'*C',
X'*N1'+Y'*N3',    X'*E1',        epsilon2*B1*H1,
zeros(3,1);...
    Ad',                                                -Q,
zeros(3,2),    N2',              zeros(3,1),    zeros(3,1),
zeros(3,1);...
    B'-C*X,                                   zeros(2,3),  -gamma-
(D+D'),       zeros(2,1),       zeros(2,1),     zeros(2,1),
zeros(2,1);...
    N1*X+N3*Y,                                  N2,
zeros(1,2),-eye(1,1)*epsilon1,   zeros(1,1),     zeros(1,1),
zeros(1,1);...
```

```
    E1*X,                                       zeros(1,3),
zeros(1,2),    zeros(1,1),       -epsilon2*eye(1,1),  zeros(1,1),
zeros(1,1);...
    epsilon2*H1'*B1',                         zeros(1,3),
zeros(1,2),    zeros(1,1),      zeros(1,1),         -
epsilon2*eye(1,1), epsilon2*H1'*N3';...
    zeros(1,3),                               zeros(1,3),
zeros(1,2),   zeros(1,1),      zeros(1,1),         epsilon2*N3*H1,
-epsilon3*eye(1,1)]<=0
cvx_end
X
Q
Y
K=Y*X^-1
Gamma
*********************************************************************
      %Chapter5
%Section 5.2
%Numerical example2
%Positive realness passive  controller for uncertain time delay
system
*********************************************************************
clear all
close all
clc
cvx_begin sdp
E=[1 0 0;0 1 0;0 0 0];A=[0.1 1 0.1;0.1 0.3 0.1;0.5 0.2 0.1];Ad=[0.1 0
0.2;0.5 -0.1 0;0 0.1 -0.2];
B1=[0.1 0 ;0  1 ;-1 1 ];B=[0.1 0.2  ;0 0.1 ;0.1 0  ];C=[0.1 1 -
0.1;0.2 0.5 0.1];D=[1 0.1  ;.5 1 ];
M=[.1 ;.1 ;.2  ];N1=[.1 0 .1];N2=[.2 0 -0.1];N3=[0 .1 ];H1=[.1 ;.1
];E1=[0.1 0 0.3];
% epsilon1=2.6147;epsilon2=2.5913;epsilon3=2.6147;
epsilon1=0.9517;epsilon2=0.6833;epsilon3=2.2436;
variable X(3,3) symmetric
variable Q(3,3) diagonal
variable Y(2,3)
variable gamma
minimize (gamma)
X>=eye(3)
Q>=eye(3)
     E'*X'>=0
     X'*E'>=0
      gamma*eye(2)>=0
[A*X+X*A'+(epsilon1+epsilon3)*M*M'+B1*Y+Y'*B1',  Ad,      B-X'*C',
X'*N1'+Y'*N3',    X'*E1',        epsilon2*B1*H1,
zeros(3,1);...
    Ad',                                              -Q,
zeros(3,2),    N2',             zeros(3,1),    zeros(3,1),
zeros(3,1);...
    B'-C*X,                                 zeros(2,3),  -gamma-
(D+D'),      zeros(2,1),      zeros(2,1),     zeros(2,1),
zeros(2,1);...
    N1*X+N3*Y,                                       N2,
zeros(1,2),-eye(1,1)*epsilon1,   zeros(1,1),     zeros(1,1),
zeros(1,1);...
    E1*X,                                       zeros(1,3),
zeros(1,2),    zeros(1,1),       -epsilon2*eye(1,1),  zeros(1,1),
zeros(1,1);...
```

```
    epsilon2*H1'*B1',                          zeros(1,3),
zeros(1,2),     zeros(1,1),      zeros(1,1),        -
epsilon2*eye(1,1), epsilon2*H1'*N3';...
    zeros(1,3),                               zeros(1,3),
zeros(1,2),    zeros(1,1),      zeros(1,1),        epsilon2*N3*H1,
-epsilon3*eye(1,1)]<=0
cvx_end
X
Q
Y
K=Y*X^-1
gamma
```

****

Chapter 5

Example 5.3

****

```
%Section 7
%Passive controller design
clear all
close all
clc
cvx_begin sdp
% % A=[-3 -2;1 0];A1=[0 .3;-.3 -.2];B1=[.5;.4];C=[2
0];D=[2];B2=[1;1];
A=[.19 0;0 1.19];A1=[-.8 -1; 0 -.7];B1=[0;1];B2=[1;1];C=[0 1];D=[0];
 variable Y(2,2) symmetric
 variable L(2,2) symmetric
 variable M(2,2) symmetric
%    variable R(2,2) symmetric
  variable Z(1,2)
  variable gamma
  minimize (gamma)
tau=2
Y>=eye(2)
L>=eye(2)
M>=eye(2)
%  R>=eye(2)
%  [L,M;M' Y]>=0
gamma*eye(1)>=1
[A*Y+Y*A'+B1*Z+Z'*B1'+L-M,    M+A1*Y+B2*Z,                B1-
Y*C'+Z'*D',            tau*tau*Y*A'+ tau*tau*Z'*B1';...
 M'+Y*A1'+Z'*B2',              -L-M,                   zeros(2,1),
tau*tau*Y*A1'+ tau*tau*Z'*B2';...
 B1'-C*Y+D*Z,              zeros(1,2),            -(gamma+D+D'),
tau^2*B1';...

 tau*tau*A*Y+tau^2*B1*Z,        tau*tau*A1*Y+tau^2*B2*Z,
tau*tau*B1,               -tau*tau*Y]<=-
[eye(2),zeros(2,2),zeros(2,1),zeros(2,2);...
    zeros(2,2),eye(2,2),zeros(2,1),zeros(2,2);...
    zeros(1,2),zeros(1,2),eye(1),zeros(1,2);...
    zeros(2,2),zeros(2,2),zeros(2,1),eye(2,2)]
cvx_end
Y
Z
L
M
%  R
tau
K=Z*Y^-1
```

```
gamma
***************************************************************************
Chapter 5
%Example 5.4
%State feedback passive controller for the TDS
%with time  varying delays in the state and input channels
***************************************************************************
clear all
close all
clc
A0=[0 1;-1 -2];A1=[0 0;.2 .1];A2=[0 0;0.3
0.2];B1=[0;0.1];B2=[0;1];C1=[1 1];C2=[1
1];D1=[1];D2=[1];D11=[1];Q=[0.2 0;0 0.2];
sigma1=0.3;sigma2=0.4;sigma3=0.2;
setlmis([]);
Y=lmivar(1,[2 1]);
W=lmivar(1,[2 1]);
L=lmivar(2,[1 2]);
 lmiterm([1 1 1 Y],1,A0','s');
 lmiterm([1 1 1 W],2*eye(2),1);
 lmiterm([1 1 2 Y],A1,1);
 lmiterm([1 1 3 Y],A2,1);
 lmiterm([1 1 4 L],B2,1);
 lmiterm([1 1 5 0],B1);
 lmiterm([1 1 5 Y],-1,C1');
 lmiterm([1 1 5 -L],-1,D1');
 lmiterm([1 1 1 Y],1,A0');
 lmiterm([1 2 2 W],-(1-sigma1)*eye(2),1);
  lmiterm([1 2 6 Y],1,A1');
   lmiterm([1 3 3 W],-(1-sigma2)*eye(2),1);
   lmiterm([1 3 6 Y],1,A2');
  lmiterm([1 4 4 W],-(1-sigma3)*eye(2),1);
  lmiterm([1 4 6 -L],1,B2');
  lmiterm([1 5 5 0],-(D11+D11'));
  lmiterm([1 5 6 0],B1');
  lmiterm([1 6 6 0],-Q^-1);
  lmiterm([-2 1 1 Y],1,1);
  lmiterm([-3 1 1 W],1,1);
  ff=getlmis;
[tt,PP]=feasp(ff);
Y=dec2mat(ff,PP,Y);
W=dec2mat(ff,PP,W);
    L=dec2mat(ff,PP,L);
Y
W
L
K=L*Y^-1
***************************************************************************
```

Chapter 5
CVX package under MATLAB software
Example 5.6

```
%Passive controller design for time delay system with varying delays
%in state and input channels
clear all
close all
clc
cvx_begin sdp
% % A=[-3 -2;1 0];A1=[0 .3;-.3 -.2];B1=[.5;.4];C=[2
0];D=[2];B2=[1;1];
```

```
% A=[.19 0;0 1.19];A1=[-.8 -1; 0 -.7];B1=[0;1];B2=[1;1];C=[0
1];D=[0];
A0=[0 1;-1 -2];A1=[0 0;.2 .1];A2=[0 0;0.3
0.2];B1=[0;0.1];B2=[0;1];C=[1 1];C2=[1
1];D1=[1];D2=[1];D11=[1];Q=[0.2 0;0 0.2];
sigma1=0.3;sigma2=0.4;sigma3=0.2;
  variable Y(2,2) symmetric
  variable M(2,2) symmetric
  variable L(1,2)
  variable gamma
  minimize (gamma)
Y>=eye(2)
M>=0
gamma*eye(1)>=0
[A0*Y+Y*A0'+2*eye(2)*M, A1*Y, A2*Y,B2*L,B1-Y*C'-L'*D1', Y*A0';...
 Y*A1',-(eye(2)-sigma1*eye(2))*M, zeros(2,2),zeros(2,2), zeros(2,1),
Y*A1';...
 Y*A2',zeros(2,2),-(eye(2)-sigma2*eye(2))*M,zeros(2,2),zeros(2,1),
Y*A2';...
 L'*B2', zeros(2,2),zeros(2,2),-(eye(2)-sigma3*eye(2))*M, zeros(2,1),
L'*B2';...
 B1'-C*Y-D1*L, zeros(1,2), zeros(1,2),zeros(1,2),-gamma*eye(1,1)-
(D11+D11'), B1';...
 A0*Y, A1*Y,  A2*Y,           B2*L,        B1,              -Q^-1]<=0
cvx_end
Y
L
M
K=L*Y^-1
gamma
```