ABSTRACT

Title of Dissertation: ESSAYS ON CAPITAL FLOWS IN DEVELOPING COUNTRIES

Hidehiko Matsumoto, Doctor of Philosophy, 2018

Dissertation directed by: Professor Felipe Saffie
Department of Economics

In the first chapter, I develop a quantitative small-open-economy model to assess the optimal pace of foreign reserve accumulation by developing countries. The model features endogenous growth with foreign direct investment (FDI) entry and sudden stops of capital inflows to incorporate benefits of reserve accumulation. Reserve accumulation depreciates the real exchange rate and attracts FDI, which endogenously promotes productivity growth. When a sudden stop happens, the government uses accumulated reserve to prevent a severe economic downturn. The calibrated model shows that two factors are the key determinants of the optimal pace of reserve accumulation: the elasticity of the foreign borrowing spread with respect to debt, and the entry cost for FDI. The model suggests that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

The second chapter is a joint work with Felipe Saffie. In this chapter, we develop a small-open-economy model with endogenous firm and trade dynamics. Aggregate productivity of the economy increases through new firm entry and incumbent firms' innovation. Firms invest in two types of innovation: innovation to acquire new product lines, and innovation to start exporting their products. These innovation activities determine the extensive margins of imports and exports. The economy is also subject to sudden stops of capital inflows. The model can capture some of the empirical regularities of firm and trade dynamics during sudden stops: firms' innovation drops sharply, which causes a persistent decline in productivity and output; imports of goods decline substantially, while exports are almost unaffected; profits for exports increase due to a large real depreciation and lower production cost; the extensive margin of exports gradually expands after sudden stops. The model provides a tractable framework to study optimal capital policies in the context of endogenous firm and trade dynamics.

ESSAYS ON CAPITAL FLOWS IN DEVELOPING COUNTRIES

by

Hidehiko Matsumoto

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2018

Advisory Committee:
Felipe Saffie, Chair
Boragan Aruoba
Sebnem Kalemli-Ozcan
John Shea
Phillip L. Swagel

Dedication

To my wife.

Acknowledgements

First and foremost, I am deeply grateful to my main advisor Professor Felipe Saffie. This dissertation would not have been possible without his support. Since Professor Saffie came to the University of Maryland in 2014, he has been an exceptional advisor to me. He has been very patient and always encouraging me whenever I struggled in my research. He always made himself available for help and gave me much advice and guidance. This dissertation would have never been completed without his patient encouragement, advice, and support.

I also owe my gratitude to Professor Sebnem Kalemli-Ozcan for her support and guidance. She often spared her time for discussion with me despite her busy schedule. Her critical comments always guided me and my research toward a correct direction by showing the broad picture of the literature of international finance and where I should place my research in the literature.

My gratitude also goes to Professor Boragan Aruoba. He has been always kind to me since my first year in Maryland. He has been encouraging me throughout my Ph.D. life in Maryland, and I could not have managed to complete my dissertation without his help and support. I also learned many things about computational skills from his class and through discussions with him, which are the essential part of my dissertation.

I also thank Professor John Shea for his detailed comments and suggestions on my research and drafts. I thank Professor Phillip Swagel for serving as Dean's representative for my dissertation defense. I would also like to thank Professor Nuno Limao, Professor Ina Simonovska, and Professor Luminita Stevens for valuable suggestions and helpful comments.

I also thank my friends and classmates for many discussions, encouragements, and making my Ph.D. life happier. I also owe special gratitude to the staffs at the Department of Economics, especially to Vickie Fletcher, Terry Davis, Amanda Statland, and Mark Wilkerson for being supportive and making my life much easier. Finally, I owe my deepest thanks to my wife for her support, patience and love, and to our two children who were born during my Ph.D life.

Table of Contents

List of Tables

List of Figures

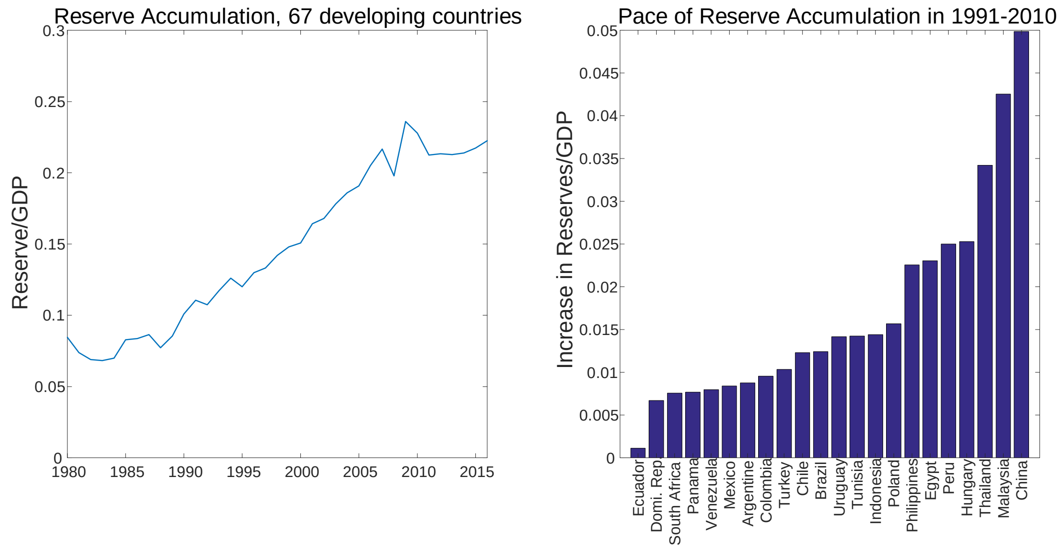
1 Reserve Accumulation, Foreign Direct Investment, and Economic Growth

1.1 Introduction

The active accumulation of foreign reserves by developing countries, especially those in East and Southeast Asia, is one of the most prominent developments in the international financial system in the past 25 years. The left panel in Figure 1.1 shows that the average reserve-to-GDP ratio across 67 developing countries has increased from less than 10% before 1990 to almost 25% by 2010. While many developing countries have built up reserve holdings in this period, there is a wide cross-country variation in how quickly these countries have accumulated reserves. The right panel in Figure 1.1 shows the average annual increase in reserve holdings as a percentage to GDP across developing countries in 1991-2010. It can be observed that Asian countries such as China, Malaysia, and Thailand have been accumulating reserves equivalent to 3.5-5% of GDP per year on average, while many Latin American countries are accumulating reserves less than 1% of GDP per year. Although the optimal reserve policy has been an active research area and a central policy question throughout the past decade, we still know little about the optimal pace of reserve accumulation, and why different countries accumulate reserves at different rates.

This chapter develops a quantitative small-open-economy model to study the optimal pace of reserve accumulation by developing countries. The main novelty of this chapter is twofold. First, the model incorporates the key benefits and costs of reserve accumulation

Figure 1.1: Reserve Accumulation by Developing Countries



into a quantitative framework. On the benefit side, the existing literature has identified two benefits of reserve accumulation: a growth-promoting effect and a precautionary effect. The growth-promoting effect goes through depreciating the real exchange rate and attracting foreign direct investment (FDI). The precautionary effect is that reserve holdings help to stabilize the economy in the face of volatile capital flows. Most existing theoretical papers studying optimal reserve policy incorporate only one of these effects. On the cost side, reserve accumulation crowds out domestic investment.[1] The second novel contribution is that this chapter addresses why different countries accumulate reserves at different paces, instead of trying to identify the unique optimal pace of reserve accumulation for a representative developing country.

The model is a small open economy with tradable and non-tradable sectors. The economy starts with scarce capital, accumulates capital by borrowing from abroad, and grows rapidly. The focus of the model is how reserve policy should be conducted during this transition

[1] Crowding-out of investment resulting from reserve accumulation is documented by Reinhart, Reinhart, and Tashiro (2016) at the macro level, and empirically shown by Cook and Yetman (2012) at the micro level.

period. To capture the two benefits of reserve accumulation mentioned above, I introduce two features into the model: endogenous growth with FDI entry and sudden stops of capital inflows. Endogenous growth is introduced to study the growth-promoting effect of reserve accumulation. The model framework is a version of the Schumpeterian growth model in which intermediate goods-producing firms endogenously innovate and increase aggregate productivity. I introduce FDI into the framework in order to capture the idea that reserve accumulation promotes growth in part by attracting FDI.[2] Sudden stops are introduced to capture the precautionary effect of reserve holdings. Sudden stops are modeled as an occasionally binding borrowing constraint on private debt and working capital financing.

The government reserve policy consists of two interventions. First, in normal times when the borrowing constraint is not binding, the government collects taxes at a fixed rate to accumulate reserves. Reserve accumulation causes real depreciation, which in turn shifts more labor to the tradable sector and reduces the real wage. This brings higher profits for intermediate firms, which induces more innovations and attracts FDI. Second, when the borrowing constraint binds, the government provides accumulated reserves to mitigate the shock to output, consumption, and investment. I call this intervention a bailout by the government. Since investment in innovation and FDI entry are forward-looking decisions, anticipation of future bailouts also induces investment in innovation and attracts FDI. Through these two interventions, the reserve policy achieves high and stable growth of the economy. The optimal pace of reserve accumulation is determined by the fixed tax rate that maximizes the expected utility of households.

[2] Dooley et al. (2007, 2014) argue that Asian countries' growth strategy is to repress the real wage by foreign exchange rate intervention and to attract FDI.

The reason why the reserve policy may improve welfare is because private agents do not internalize that their actions affect FDI entry decisions, and the reserve policy corrects this externality. The main benefit of receiving FDI is that foreign firms invest more in innovation than domestic firms and therefore contribute to productivity growth.[3] In order to attract FDI, the country can increase the growth rate by investing more in capital and shifting more labor to the tradable sector, which increases profits for foreign firms. Avoiding sharp drops in foreign firms' profits during sudden stops also helps to attract FDI. The reserve policy corrects the externality by bringing about more investment and a labor shift to the tradable sector in normal times, and by preventing a severe economic downturn during sudden stops.

On the other hand, reserve accumulation involves costs. As government collects tax revenue to accumulate reserves, private agents borrow more from abroad to compensate for the loss of resources. In the model, the interest rate on foreign borrowing is debt-elastic, and the larger debt-to-GDP ratio increases the interest rate spread. The debt-elastic spread causes two costs of reserve accumulation. First, it prevents a full offset by private agents, thus lowering consumption in the short run. The optimal reserve policy therefore balances the costs of short-run austerity with the benefits of higher long-run consumption. Second, the higher spread discourages investment in capital and innovation, a form of crowding out. This reduces the growth-promoting effect of reserve accumulation and worsens the trade-off between current austerity and future consumption.

In the quantitative analysis, I calibrate the model to a sample of eight developing countries. I target the productivity gain from FDI entry, the innovation rate of foreign firms

[3] Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012) show that foreign firms invest more in innovation than domestic firms in Indonesia and Spain respectively.

relative to domestic firms, and the value added share of foreign firms in the tradable sector, all from empirical papers. The frequency and duration of sudden stops are derived from sudden stop episodes for a sample of 33 countries over the period of 1980-2009. I solve the model globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint.

The first important result using this model is that the optimal pace of reserve accumulation and its welfare impact crucially depend on two characteristics of each country: the debt-elasticity of the foreign borrowing spread, and the FDI entry cost. In countries with a higher debt-elasticity of this spread, reserve accumulation causes severe crowding-out of investment, which reduces the growth-promoting effect. In countries with a larger FDI entry cost, reserve accumulation is not as effective in attracting FDI and the growth-promoting effect is therefore limited. In these cases, the optimal pace of reserve accumulation is slower, and the welfare gain is limited. The decomposition analysis shows that 72% of the growth-promoting effect of the reserve policy comes from real depreciation in normal times, and 28% comes from anticipation of future bailouts during sudden stops. The model also shows that reserve accumulation without bailouts cannot improve welfare, because without bailouts private agents do not increase foreign borrowing to compensate for the loss of resources as much, and thus reduce short-run consumption substantially.

Given these results, I evaluate each developing country's pace of reserve accumulation accounting for these two factors. I empirically estimate the debt-elasticity of the spread using panel regression for a sample of 22 developing countries. I find that a country's default history is significantly and positively associated with its elasticity of the spread with respect to debt. Accordingly I divide the 22 sample countries into 5 groups according to the number of past defaults, and estimate the elasticity for each group by interacting the debt-to-GDP ratio with dummy variables representing the default history. I also adjust the parameters for the FDI entry cost to match the FDI inflow-to-GDP ratio for each country. Using this model, I derive the optimal pace of reserve accumulation for each country, and compare it with the actual pace of reserve accumulation. The second important result is that many developing countries are roughly in line with the optimal pace suggested by the model, suggesting that these two factors can explain the observed cross-country variation in the pace of reserve accumulation. The correlation between the actual and optimal pace of reserve accumulation across 22 countries is 0.54. A few countries including China, however, seem to be accumulating reserves too quickly. This result may suggest that there is some other benefit of reserve accumulation in China and other countries that is not captured by the model in this chapter.

The remainder of the chapter is organized as follows. Section 1.2 reviews the related literature. Section 1.3 introduces the model. Section 1.4 discusses the mechanisms of how the reserve policy works in the model. Section 1.5 presents the calibration of the model and the quantitative analysis. Section 1.6 studies the key determinants of the optimal pace of reserve accumulation. Section 1.7 evaluates the actual pace of reserve accumulation by developing countries. Section 1.8 concludes.

1.2 Related Literature

Foreign reserve accumulation by developing countries has been an active research area in the last decade. One strand of literature focuses on the growth-promoting effect. As an empirical motivation, Aguiar and Amador (2011) find that there is a positive correlation between government net foreign asset growth and GDP growth across developing countries, which is in stark contrast to the prediction of neo-classical growth models. Gourinchas and Jeanne (2013) show that this correlation is driven mainly by reserve accumulation. Alfaro, Kalemli-Ozcan, and Volosovych (2014) find a similar correlation between reserve accumulation and growth, and further show that private capital inflows such as portfolio investment and FDI are in contrast positively correlated with productivity growth. Aizenman and Lee (2010) and Korinek and Servén (2016) develop models with a learning-by-doing externality in the tradable sector and study the optimal reserve policy. Attracting FDI is another proposed channel through which reserve accumulation promotes productivity growth. Dooley et al. (2007, 2014) argue that Asian countries' growth strategy is to repress the real wage by foreign exchange rate intervention in order to attract FDI from developed countries. Consistent with this view, Aizenman and Lee (2010) find that FDI inflows from Japan and Korea to China have increased along with China's reserve holdings since 2000.

Another strand of literature studies the precautionary benefits of reserve accumulation. Jeanne and Rancière (2011) model reserve accumulation as an insurance contract that pays off in a sudden stop, and quantify the optimal amount of reserve holdings. Bianchi, Hatchondo, and Martinez (2016) build a sovereign default model in which the government holds reserve assets to insure against future defaults and loss of access to international financial markets.

All of these theoretical papers focus on either the growth-promoting effect or the precautionary effect of reserve accumulation, but not both. The model in this chapter incorporates both effects into a unified framework and studies the interaction between the growth-promoting effect and the precautionary effect. In this sense, reserve policy in this chapter is similar to that in Benigno and Fornaro (2012). The key difference between my model and theirs is the growth process and how reserve policy promotes growth. Benigno and Fornaro (2012) assume that productivity in the tradable sector increases as more imported inputs are used for production, which is the externality in their model. Reserve policy promotes growth and improves welfare by inducing private agents to use more imported inputs. In my model, on the other hand, productivity in the tradable sector improves through endogenous domestic and FDI entry and incumbent firms' innovation. Entry and innovation are forward-looking decisions, and reserve policy induces more innovations and attracts FDI by increasing expected future profits for domestic and foreign firms. In particular, bailouts during sudden stops prevent sharp drops in firms' profits, and thus anticipation of future bailouts induces more innovations and attracts FDI, while in Benigno and Fornaro (2012) interventions during sudden stops help private agents to import more inputs and promote growth. In addition, reserve accumulation in my model crowds out domestic investment through a higher foreign borrowing spread, while in Benigno and Fornaro (2012) there is no crowding out by reserve accumulation.

There are a few papers that study cross-country differences in the amount or the pace of reserve accumulation. Obstfeld, Shambaugh, and Taylor (2010) consider the risk of double drain of capital and measure the optimal amount of reserve holdings relative to the size of the banking system. They show that even China does not appear to be an extreme outlier. Aguiar and Amador (2011) develop a neo-classical growth model with political frictions and show that differences in the degree of these frictions can explain cross-country differences in the speed of net public debt reduction. This chapter focuses instead on two other factors that can explain cross-country variation in the pace of reserve accumulation, the debt-elasticity of the spread on foreign borrowing and the FDI entry cost.

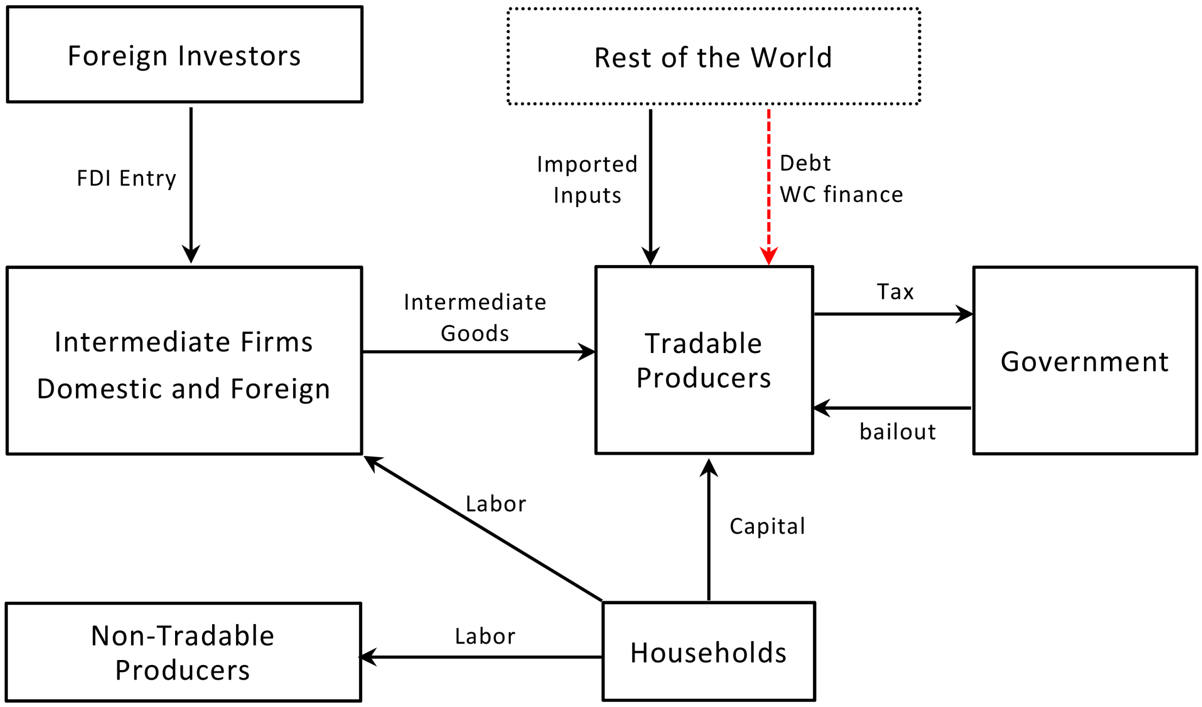
The model structure of this chapter rests on two strands of literature. First, the endogenous growth framework is based on a version of the Schumpeterian growth model developed by Ates and Saffie (2016). They incorporate the growth model developed by Klette and Kortum (2004) into a DSGE framework, and also introduce heterogeneous innovations. I extend Ates and Saffie (2016) by introducing FDI and innovation by foreign firms. Second, sudden stops are modeled as an occasionally binding constraint on foreign borrowing. The borrowing constraint in this model is similar to Bianchi (2016), and different from Mendoza (2010) in that the fraction of capital used as collateral is stochastic, and the collateral value of capital is set at book value rather than market value.

Lastly, this chapter shares with several recent papers the feature that crises have a permanent negative impact on productivity. Queralto (2015) builds a model based on the Comin and Gertler (2006) version of the product-variety expansion model and shows that the model can explain the permanent negative effect of the 1998 sudden stop on productivity in Korea. Gornemann (2015) also adopts the product-variety expansion model and develops a model that captures a very persistent negative effect of sovereign default on productivity. Ates and Saffie (2016) introduce heterogeneous innovations and financial selection into the Schumpeterian growth model and show that new firms created during sudden stops are fewer but better.

1.3 Model

The model framework is based on a standard infinite-horizon small open economy with tradable and non-tradable sectors. The overview of the model environment is as follows. The tradable goods producers use capital, a variety of differentiated intermediate goods, and imported inputs for production. Capital and differentiated intermediate goods are the two drivers of economic growth. The economy starts with scarce capital and grows quickly by accumulating capital. A variety of intermediate goods are produced by domestic and foreign firms. They are modeled as a version of the Schumpeterian growth model developed by Klette and Kortum (2004) and Ates and Saffie (2016), in which new entry and incumbent firms' innovations increase aggregate productivity endogenously. I extend their framework to incorporate FDI entry and foreign firms' innovations. The country is also subject to a sudden stop in the form of an occasionally binding borrowing constraint on foreign debt and working capital financing. In particular, the borrowing limit is set as a fraction of capital holdings of the country, and the fraction occasionally tightens with an exogenous probability. Unlike business cycle models such as Mendoza (2010), the possibility of sudden stops exists only in the transition periods of capital accumulation. In the transition where capital is scarce, there is a large need for foreign borrowing to accumulate capital, but the borrowing limit is tight. In this case, a negative shock to the borrowing limit causes a binding constraint and generates drops in output, consumption, investment, and FDI inflows. In the long run when capital has reached a steady state condition, the borrowing limit is large enough that the borrowing constraint never binds, and the economy follows a smooth balanced growth path. The focus of the analysis in this chapter is therefore how reserve policy should be conducted

Figure 1.2: Model Economy



during the transition period of capital accumulation.

Figure 1.2 presents a diagram for the model economy. There are six agents in the model. First, tradable goods producers produce goods by using capital, a variety of intermediate goods, and imported inputs. They also borrow from abroad using non-contingent one-period debt and within-period working capital financing. Because households in this model do not have direct access to the international financial market, tradable goods producers borrow from abroad to smooth consumption and accumulate capital on behalf of households.[4] Borrowing from abroad is subject to the stochastic borrowing limit as explained above. Second, intermediate goods producing firms produce a unit mass of differentiated intermediate goods using labor and sell output to tradable goods producers. There are domestic and foreign

[4]Even if households directly borrow from abroad, it would be an equivalent model. The assumption here is just to simplify the model by avoiding two agents facing the same borrowing constraint.

intermediate firms, and each firm produces one or more product line(s) of differentiated goods. There is endogenous entry and innovations by domestic firms and foreign firms, and through these activities aggregate productivity of intermediate goods increases over time. The detailed exposition of the firm dynamics and the growth process will be laid out below. Third, there are an infinite number of foreign firms who consider acquiring product lines from domestic firms and entering this country using FDI. Fourth, non-tradable goods producers produce goods using labor and sell output to households. Fifth, households consume tradable and non-tradable goods, accumulate and rent capital, and supply labor. They also invest in innovation to create new intermediate goods producing firms.

Given this model environment, the government engages in reserve policy to improve household's welfare. As explained above, reserve policy consists of two interventions. In normal times when the borrowing constraint is not binding, the government collects lump-sum taxes from tradable goods producers and accumulates reserves. When the borrowing constraint binds, the government provides accumulated reserves to tradable goods producers to help finance the working capital payment and prevent an economic downturn. I call this intervention a bailout by the government. I next turn to the characterization of each agent.

1.3.1 Tradable Goods Producer

Tradable goods are the numeraire of this model economy, and their price is normalized at one. The representative tradable goods producer uses capital K_t^D, unit-mass variety of intermediate goods $\{y_t(i)\}_{i=0}^1$, and imported inputs M_t to produce output Y_t^T following the

Cobb-Douglas production function:

$$Y_t^T = (K_t^D)^\alpha (I_t^M)^\theta (M_t)^{1-\alpha-\theta} \tag{1.1}$$

where I_t^M is the composite of intermediate goods:

$$I_t^M = \exp\left[\int_0^1 \ln y_t(i) di\right] \tag{1.2}$$

Before production materializes, a fixed fraction ϕ of the cost of intermediate goods and imported inputs needs to be paid. This payment is financed by within-period borrowing from abroad with no interest cost. In addition, the tradable goods producer borrows from abroad using one-period non-contingent debt B_t. Foreign borrowing is subject to an occasionally binding borrowing constraint. Specifically, the borrowing limit is given by $\kappa_t K_{t-1}$, where κ_t is a collateral shock and takes either of two values, κ_H or κ_L, following a two-state Markov process; and K_{t-1} is the capital stock of this country at the beginning of period t. κ_H is the value in normal times, and it is large enough that the borrowing constraint never binds. With exogenous probability P_{HL}, κ_t switches from κ_H to κ_L, which is small enough that the borrowing constraint may bind, depending on the state of the economy. In particular, when capital is scarce, the borrowing limit is low and at the same time there is a large incentive to borrow from abroad to accumulate capital. In this case, a collateral shock κ_L is likely to cause a binding borrowing constraint. As seen later, the binding borrowing constraint endogenously generates drops in output, consumption, investment in capital and innovation, and FDI inflows. When this happens, the government provides V_t units of reserves to help finance working capital payments and mitigate the negative impacts from a sudden stop. A negative collateral shock κ_L ends with probability P_{LH}, in which case κ_t switches back to the normal value κ_H.

Given these settings, the maximization problem of the representative tradable goods producer is as follows:

$$\max_{\{K_t^D, \{y_t(i)\}_{i=0}^1, M_t, B_t\}_{t=0}^\infty} E_o \sum\nolimits_{t=0}^{\infty} \beta^t \lambda_t \Pi_t^T$$

subject to the production function (1.1) and

$$\Pi_t^T = Y_t^T - r_t K_t^D - \int_0^1 p_t(i) y_t(i) di - P^M M_t - B_t + R_{t-1} B_{t-1} - T_t + V_t \tag{1.3}$$

$$-B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di + P^M M_t \right] - V_t \leq \kappa_t K_{t-1} \tag{1.4}$$

where λ_t is the marginal utility of tradable goods consumption by households, $p_t(i)$ is the price of intermediate goods i, P^M is the price of imported inputs, and R_{t-1} is the gross interest rate on foreign debt repaid at period t. T_t is a lump-sum tax that the government collects to accumulate reserves. Each period, the tradable goods producer chooses capital demand K_t^D, intermediate goods demand $\{y_t(i)\}_{i=0}^1$, imported inputs M_t, and foreign debt B_t to maximize the expected profit discounted by household's discount rate adjusted by the marginal utility λ_t. Let μ_t denote the Lagrange multiplier on the borrowing constraint (1.4). The first-order conditions with respect to the choice variables are as follows:

$$K_t^D : r_t = \alpha \frac{Y_t^T}{K_t^D} \tag{1.5}$$

$$y_t(i) : p_t(i)\left(1 + \phi\frac{\mu_t}{\lambda_t}\right) = \theta\frac{Y_t^T}{y_t(i)} \tag{1.6}$$

$$M_t : P^M\left(1 + \phi\frac{\mu_t}{\lambda_t}\right) = (1 - \alpha - \theta)\frac{Y_t^T}{M_t} \tag{1.7}$$

$$B_t : \lambda_t - \mu_t = \beta R_t E_t(\lambda_{t+1}) \tag{1.8}$$

$$\mu_t : \mu_t\left(B_t + \phi\left[\int_0^1 p_t(i)y_t(i)di + P^M M_t\right] - V_t - \kappa_t K_{t-1}\right) = 0,\ \mu_t \geq 0 \tag{1.9}$$

The first three equations are the demand functions for capital, intermediate goods, and imported inputs. When the borrowing constraint is slack, $\mu_t = 0$ and the demand functions for intermediate goods (1.6) and imported inputs (1.7) are the standard ones equating prices and marginal products. When the borrowing constraint binds, strictly positive μ_t appears as the external financing premium on working capital payments, which increases the effective cost of inputs. Equation (1.8) is the Euler equation with respect to foreign debt. Given that λ_t is the marginal utility of tradable goods consumption by households, it is the standard Euler equation except when the Lagrange multiplier on the borrowing constraint μ_t appears. This term captures the external financing premium on foreign debt when the borrowing constraint binds, which increases the effective real interest rate on foreign debt, as explained in Mendoza (2010). The last equation (1.9) is the complementary slackness condition for the borrowing constraint.

The gross interest rate on foreign borrowing R_t is endogenously determined. Following Schmitt-Grohé and Uribe (2003), R_t is a function of the aggregate debt-to-GDP ratio:

$$R_t = \overline{R} + \psi_b\left(\exp\left(\frac{B_t}{GDP_t} - \overline{b}\right) - 1\right) \tag{1.10}$$

where GDP is given by $Y_t^T - P^M M_t + P_t^N Y_t^N$ with $P_t^N Y_t^N$ being the non-tradable goods price times output. As shown in Schmitt-Grohé and Uribe (2003), this formulation guarantees that the debt-to-GDP ratio will converge to the given value $\bar{b}$ and R_t will be $\overline{R}$ in the long run, so that the balanced growth path is uniquely pinned down. In this model, however, this debt-elastic spread plays an important role along the transition path. As the government collects tax revenue, the tradable goods producer borrows more from abroad to compensate for loss of resources. This causes a higher interest spread through equation (1.10), which triggers two key consequences of reserve accumulation. First, because foreign borrowing becomes more costly, the tradable goods producer does not fully offset the collected tax by borrowing the same amount. This in turn leads to lower tradable goods consumption by households and causes real depreciation. This is the mechanism of how reserve accumulation causes real depreciation in the model. Real depreciation in turn shifts more labor to the tradable sector and brings higher profits for intermediate firms, which induces innovation and promotes growth. Second, higher interest rate implies a higher cost for investment, and thus crowds out investment both in capital and innovation. Section 4.2 discusses these key mechanisms and their implications for the optimal policy in more detail.

1.3.2 Intermediate Goods Producing Firms

There is a unit-mass variety of differentiated intermediate goods in the tradable sector, indexed by i. Following the versions of the Schumpeterian growth model developed by Klette and Kortum (2004) and Ates and Saffie (2016), a firm is defined as a collection of one or more product line(s) among these differentiated goods.[5] Each firm produces the

[5] The number of firms is endogenously determined by entry and innovation, as in Klette and Kortum (2004) and Ates and Saffie (2016). Although the model structure allows me to study the firm age and size

product line(s) it owns using labor, and innovates over other product lines. The production technology is given by:

$$y_t(i) = a_t(i)\ell_t(i) \tag{1.11}$$

where $a_t(i)$ is the labor productivity and $\ell_t(i)$ is labor input. Labor productivity $a_t(i)$ is heterogeneous across i, and improves over time by entry and innovations by domestic and foreign firms. The entry and innovation processes will be laid out in the next section. I first show that only the productivity leader produces goods for each product line, and explain how profit is determined by the size of the productivity lead over rival firms. As shown in equation (1.6), demand for each product line from the tradable goods producer is unit-elastic. This implies that the solution to the profit-maximization problem for a monopolist is to set the price infinitely high. In the Schumpeterian growth model, however, there are rival firms that can produce the same type of good with lower productivity, who could steal the market by setting a price slightly below the monopoly price. Therefore, through Bertrand competition, the profit-maximizing behavior by the productivity leader is to set the price equal to the marginal cost of the closest rival firm and monopolize the demand.

Next I explain how the productivity lead over the closest rival firm is determined. There are three different cases for a productivity increase. First, when domestic innovation happens on a product line, either by new entry or an incumbent's innovation, it improves productivity of the product line by a factor of $(1 + \sigma_t^D)$. This implies that the productivity leader of a domestic-owned product line has $(1 + \sigma_t^D)$ times higher productivity than that of the previous leader, which is the closest rival. Second, when a domestic-owned product line is

distribution, I focus on how reserve policy affects FDI and aggregate growth, and do not conduct firm-level analysis in this chapter.

acquired by foreign investors, which I call FDI entry, productivity increases by a factor of $(1+\sigma_t^F)/(1+\sigma_t^D)$ with $\sigma_t^F > \sigma_t^D$. Since the productivity leader of a domestic-owned product line has $(1+\sigma_t^D)$ times higher productivity than the closest rival, a productivity increase by a factor of $(1+\sigma_t^F)/(1+\sigma_t^D)$ upon FDI entry means that a product line acquired by foreign investors has $(1+\sigma_t^F)$ times higher productivity than the closest rival. Third, when a foreign incumbent firm innovates on a product line, productivity increases by a factor of $(1+\sigma_t^F)$. The productivity improvement process can be summarized as follows:

$$a_t(i) = \begin{cases} (1+\sigma_t^D)a_{t-1}(i) & \text{if domestic entry or innovation} \\ (1+\sigma_t^F)/(1+\sigma_t^D)a_{t-1}(i) & \text{if FDI entry} \\ (1+\sigma_t^F)a_{t-1}(i) & \text{if foreign innovation} \\ a_{t-1}(i) & \text{none of the above} \end{cases}$$

Under this process, the productivity lead over the closest rival is simply determined by whether the productivity leader is a domestic firm or a foreign firm. In the former case the productivity lead is by a factor of $(1+\sigma_t^D)$, and in the latter case it is by a factor of $(1+\sigma_t^F)$.

I assume that the productivity step sizes σ_t^D and σ_t^F are increasing in capital scarcity, defined as k_{ss}/k_{t-1}, where $k_{ss} = K_{t-1}/A_t$ is the capital stock K_{t-1} normalized by the aggregate productivity A_t along the balanced growth path, and $k_{t-1} = K_{t-1}/A_t$ is the same variable in the transition, which trends upward over time. Specifically I assume the following functional forms:

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}}\right)^\rho \tag{1.12}$$

$$\sigma_t^F = \sigma^F \left(\frac{k_{ss}}{k_{t-1}}\right)^\rho \tag{1.13}$$

with $\rho > 0$ and $\sigma^F > \sigma^D$. In the transition in which capital is scarce, $k_{ss}/k_{t-1} > 1$ and the step sizes are large. As capital accumulates and k_{t-1} gets close to k_{ss}, k_{ss}/k_{t-1} declines toward 1 and the step sizes converge to σ^D and σ^F along the balanced growth path. The idea behind this assumption is that capital scarcity indicates the technological distance from the world frontier, and when there is a large distance, the economy can grow faster through the catching-up effect.

Now I show how profit for each intermediate firm is determined. As explained above, demand from the tradable goods producer (1.6) is unit-elastic, and thus the profit-maximizing behavior by the productivity leader is to set the price equal to the marginal cost of the closest rival. Given the productivity leader's productivity $a_t(i)$ for a product line i at period t, productivity of the closest rival is given by $a_t(i)/(1+\sigma_t^s)$, where $s = D, F$ indicates whether the productivity leader is a domestic firm or a foreign firm. The optimal price $p_t(i)$ and corresponding profit $\pi_t(i)$ are given as follows:

$$p_t(i) = \frac{W_t(1+\sigma_t^s)}{a_t(i)} \tag{1.14}$$

$$\pi_t(i) = \frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t} - W_t\ell_t(i) = \frac{\sigma_t^s}{1+\sigma_t^s}\frac{\theta Y_t^T}{1+\phi\mu_t/\lambda_t} \tag{1.15}$$

where W_t is the real wage. The middle term of equation (1.15) implies that from the individual firm's viewpoint, larger demand from the tradable goods producer and a cheaper real wage bring higher profits. Using the expression in (1.14), the profit can be written as the last term of equation (1.15). This expression shows that the profit for each product line depends only on the owner type $s = D, F$, and is independent of the product-line specific productivity $a_t(i)$. It follows from the middle term of (1.15) that the labor input also depends only on the owner type. Hereafter I use π_t^D and π_t^F to denote the profits for a domestic-owned and a foreign-owned product line respectively, and ℓ_t^D and ℓ_t^F to denote the corresponding labor input. From equation (1.15) labor input is given as follows:

$$\ell_t^s = \frac{\pi_t^s}{\sigma_t^s W_t} \tag{1.16}$$

There are two more things to note from equation (1.15). First, the assumption $\sigma^F > \sigma^D$ implies $\pi_t^F > \pi_t^D$, i.e. foreign-owned product lines yield higher profits than domestic-owned product lines. Second, profits are affected by the borrowing constraint on the tradable goods producer through the Lagrange multiplier μ_t. When the borrowing constraint binds, the tradable goods producer faces a higher effective cost of buying intermediate goods, which thus reduces their demand. Equation (1.15) shows that the smaller demand directly translates into a lower profit for the intermediate goods producing firms.

1.3.3 Innovation and Firm Dynamics

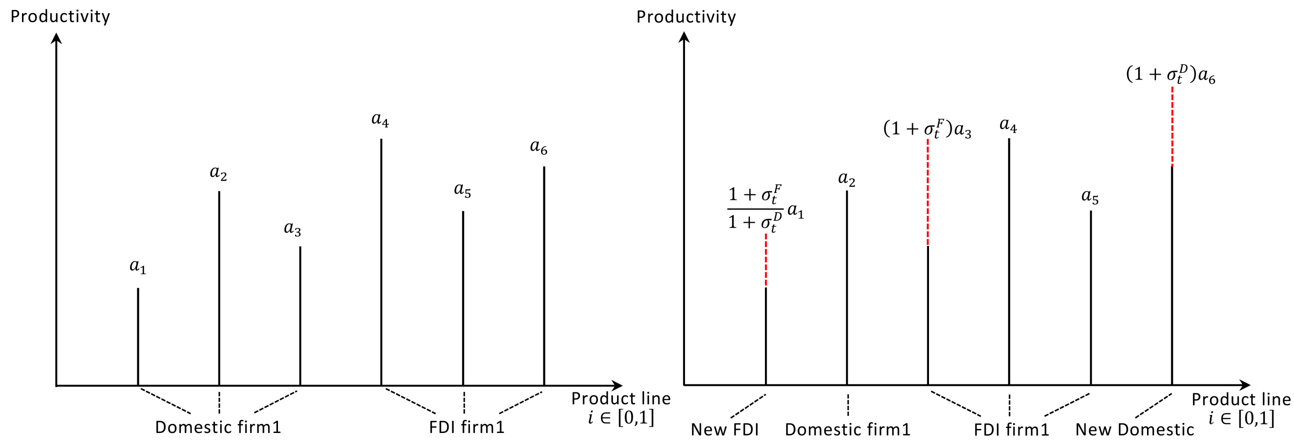
In the previous subsection I showed that only the productivity leader produces each product line and that its profit depends only on the owner type. I now turn to firm dynamics with a focus on how productivity leaders change through innovation. The firm dynamics in this economy are characterized by four different types of innovations: domestic entry, FDI entry, domestic incumbent innovation, and foreign incumbent innovation. Figure 1.3 illustrates an example of the evolution of firms from one period to the next. The left panel shows 6 product lines with productivity a_1 to a_6. The first three lines are owned and produced by domestic firm 1, and the other three lines by foreign firm 1. In the next period, depicted in the right panel, foreign investors acquire product line 1 from domestic firm 1 via FDI entry. FDI entry improves productivity of product line 1 by the factor of $(1+\sigma_t^F)/(1+\sigma_t^D)$. For product line 3, foreign incumbent firm 1 succeeds in innovation and improves its productivity by the factor of $(1+\sigma_t^F)$. Thus domestic firm 1 loses two product lines and shrinks its business. For product line 6, there is domestic entry and a new domestic firm obtains the product line from foreign firm 1. Through entry and innovation, firms compete with each other and endogenously enter, exit, expand and shrink, and increase the overall productivity of the country.

This framework captures several features of FDI entry and foreign firms documented by empirical studies: (1) Most FDI entry is through the acquisition of domestic firms rather than greenfield investment.[6] (2) Some domestic firms are forced to exit through competition with foreign firms.[7] In later sections I will also show: (3) Foreign firms innovate more often than domestic firms.[8] (4) In a crisis, foreign firms invest more in innovation than domestic firms and are more likely to survive.[9] I now move on to the characterization of entry and innovation. I start from innovation by foreign incumbent firms.

Innovation by Foreign Incumbent Firms Let θ_{t-1} denote the fraction of product lines owned by foreign firms at the beginning of period t. Consider a foreign firm that owns n product lines. As seen before, operating profit depends only on the owner type and is independent of the individual firms's productivity. Therefore the total operating profit of

[6] Navaretti and Venables (2004) shows that 90% of FDI is in the form of acquisitions.
[7] See Aitken and Harrison (1999).
[8] See Arnold and Javorcik (2009) and Guadalupe et al. (2012).
[9] See Alfaro et al. (2012).

Figure 1.3: Firm Dynamics



this firm is $n\pi_t^F$. I assume that a firm with n product lines has n opportunities to innovate. The idea behind this assumption is that an incumbent firm's innovation is based on and spins off from the existing technology in practice. Following Akcigit and Kerr (2015), the success probability of innovation i_t^F for each innovation opportunity is a concave function of tradable goods Z_t^F that a firm invests in innovation:

$$i_t^F = \eta^F \left(\frac{Z_t^F}{A_t} \right)^{1-\rho^F} \tag{1.17}$$

where $\eta^F > 0$ is the productivity coefficient and $0 < \rho^F < 1$ is the parameter that governs the concavity. As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model by the structure that the operating profit is independent of productivity. Undirected innovations by many firms imply that each product line faces the same replacement probability. Let d_t denote this probability, and define as $P(i, n, p)$ the probability of having

i successes in n trials for a binomial process with success probability p. Namely,

$$P(i,n,p) = \binom{n}{i} p^i (1-p)^{1-i}$$

The value of a foreign firm with n product lines can be written in a recursive form as follows:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{n} P(i,n,i_t^F) \left(\sum_{j=0}^{n} P(j,n,d_t) E_t \left(V_{t+1}^F(n+i-j) \right) \right) \right] \right\}$$

The first two terms are the operating profit minus the innovation investment cost. R^F is the world interest rate, and foreign investors who own foreign firms discount future profits by this rate. The bracketed term is the expected value of this firm next period. The first summation adds up the expected value over the $n+1$ cases for the number of successful innovations, from 0 to n. The second summation adds up over the $n+1$ cases for the number of replacements, again from 0 to n. Thus for a foreign firm with n product lines, there are $(n+1)^2$ different possible combinations of the number of successful innovations and replacements. Note, however, that the expected value of the firm in each case, $E_t\left(V_{t+1}^F(n+i-j)\right)$, depends only on the number of the product lines, $n+i-j$, and not on the specific combination of the number of innovations and replacements. For example, 3 innovations and 2 replacements will give the same expected value as 4 innovations and 3 replacements, namely $E_t(V_{t+1}^F(n+1))$.

Following Ates and Saffie (2016), I use a guess-and-verify method to show that the value of a foreign firm with n product lines is equal to n times the value of a foreign firm with a single product line:

$$V_t^F(n) = nV_t^F(1)$$

The formal proof is left to the Appendix. This linear relation enables us to aggregate the firm dynamics in a tractable way and study how the firm dynamics affect the endogenous growth of the entire economy, without keeping track of the firm size distribution. The value of a foreign firm with a single product line is given by:

$$V_t^F(1) = \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F}(1 + i_t^F - d_t) E_t(V_{t+1}^F(1)) \right\} \tag{1.18}$$

Taking into account equation (1.17), the first-order condition with respect to Z_t^F gives the optimal investment condition:

$$\eta^F \left(1 - \rho^F\right) \left(\frac{Z_t^F}{A_t}\right)^{-\rho^F} \frac{1}{A_t} \frac{1}{R^F} E_t(V_{t+1}^F(1)) = 1 \tag{1.19}$$

Since I assume $0 < \rho^F < 1$, investment Z_t^F and the probability of successful innovation i_t^F are increasing in the expected value of a product line next period.

FDI Entry FDI entry takes the form of an acquisition of a domestic-owned product line by foreign investors. There are infinitely many foreign investors who can consider acquiring a product line and entering this country via FDI. There are three types of cost for FDI entry. First, foreign investors need to pay a fixed fraction $0 < \lambda < 1$ of the discounted expected value of a product line $(1/R^F)E_t(V_{t+1}^F(1))$ to the domestic firm that owns the product line. λ can be interpreted as the negotiation power of the domestic owner firm against foreign investors.[10] Second, there is a fixed entry cost $A_t C^F$. This is in line with Helpman, Melitz,

[10] For the domestic owner firm to be willing to sell a product line to foreign investors, the incentive compatibility condition must be satisfied. This condition is given by $\lambda(1/R^F)E_t(V_{t+1}^F(1)) \geq E_t(\Lambda_{t,t+1} V_{t+1}^D(1))$ where the right-hand side is the expected value of a domestic-owned product line discounted by the households' stochastic discount factor. Because this condition is always satisfied under the calibrated parameter

and Yeaple (2004) in that FDI entry is characterized by a large fixed entry cost. Third, there is a congestion cost of entry, which is linearly increasing in the aggregate number of product lines acquired by FDI in each period. Since there is an infinite number of potential foreign investors, FDI entry continues until the congestion cost pushes down the net expected profit of entry to zero. Therefore FDI entry in each period, denoted by e_t^F, is determined by the following zero-profit condition:

$$A_t \chi^F \left(\frac{e_t^F}{1-\theta_{t-1}} \right) = (1-\lambda) \frac{1}{R^F} E_t(V_{t+1}^F(1)) - A_t C^F \tag{1.20}$$

where χ^F is a coefficient on the congestion cost. The denominator $1-\theta_{t-1}$ is the fraction of domestic-owned product lines at the beginning of period t. This is introduced to capture the idea that it is more costly to find a good product line to acquire as the number of domestic-owned product lines falls. Both the congestion cost and the fixed entry cost increase over time along with the aggregate productivity of the economy A_t, so that FDI entry e_t^F will be constant in the long run. Similarly to innovation by foreign incumbent firms, FDI entry e_t^F is an increasing function of the expected value of a product line.

Innovation by Domestic Incumbent Firms Characterization of domestic incumbent firms is similar to foreign incumbent firms, but different in one key aspect: there is the possibility that product lines are acquired by foreign investors via FDI. Consider a domestic firm with n product lines. This firm has n innovation opportunities as assumed for foreign firms. For each opportunity, the firm invests Z_t^D units of tradable goods, and the probability

values, I do not consider it explicitly.

of successful innovation i_t^D is given by the following equation:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t} \right)^{1-\rho^D} \tag{1.21}$$

with $0 < \rho^D < 1$. Let Q_t denote the price that foreign investors pay to the domestic owner firm to acquire a product line via FDI. From the last subsection this is given by:

$$Q_t = \lambda \frac{1}{R^F} E_t(V_{t+1}^F(1)) \tag{1.22}$$

Using Q_t, the replacement probability d_t, and the probability for a binomial process $P(i, n, p)$, the value of a domestic firm with n product lines can be recursively written as follows:

$$\begin{aligned} V_t^D(n) \quad = \quad & \max_{Z_t^D} \Big\{ n\pi_t^D - nZ_t^D \\ & + \left[\sum_{i=0}^{n} P(i, n, i_t^D) \left\{ \sum_{j=0}^{n} P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t \left[\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k) \right] \right) \right\} \right] \\ & + \left[\sum_{j=0}^{n} P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \Big\} \end{aligned}$$

Compared to the value of a foreign firm, the additional terms are the third summation in the second line and the third line. The third summation in the second line adds up the expected value over the $n - j + 1$ cases for the number of product lines acquired via FDI by foreign investors, from 0 to $n-j$, given that j product lines are replaced. $e_t^F/(1-\theta_{t-1})$ is the probability that each product line is acquired via FDI by foreign investors. Note that the expected value of the firm next period is discounted by the households' stochastic discount factor $\Lambda_{t,t+1}$. Note also that the number of product lines the firm owns next period is given by $n+i-j-k$. The third line adds up the acquisition price of FDI entry over the same $n-j+1$ cases given j replacements. Using the same guess-and-verify method, it can be shown that a linear relation holds for the value of a domestic firm:

$$V_t^D(n) = nV_t^D(1)$$

and the value of a domestic firm with a single product line is given by:

$$V_t^D(1) = \max_{Z_t^D}\left\{\pi_t^D - Z_t^D + \left[i_t^D + (1-d_t)\left(1-\frac{e_t^F}{1-\theta_{t-1}}\right)\right]E_t(\Lambda_{t,t+1}V_{t+1}^D(1)) + (1-d_t)\frac{e_t^F}{1-\theta_{t-1}}Q_t\right\} \tag{1.23}$$

The first-order condition with respect to Z_t^D gives the optimal condition for domestic innovation investment:

$$\eta^D\left(1-\rho^D\right)\left(\frac{Z_t^D}{A_t}\right)^{-\rho^D}\frac{1}{A_t}E_t(\Lambda_{t,t+1}V_{t+1}^D(1)) = 1 \tag{1.24}$$

Domestic Entry Finally, entry of new domestic firms comes from innovation by households and poaches a product line from incumbent firms. Households invest Z_t^E units of tradable goods to create new firms. The number of firms created from Z_t^E units of investment is given by:

$$e_t^D = \eta^E\left(\frac{Z_t^E}{A_t}\right)^{1-\rho^E} \tag{1.25}$$

The optimal investment Z_t^E satisfies that the marginal benefit of investment is equal to the marginal cost, therefore:

$$\eta^E\left(1-\rho^E\right)\left(\frac{Z_t^E}{A_t}\right)^{-\rho^E}\frac{1}{A_t}E_t(\Lambda_{t,t+1}V_{t+1}^D(1)) = 1 \tag{1.26}$$

1.3.4 Aggregation and Productivity Growth

I now characterize how firm dynamics translate into macroeconomic dynamics, specifically the transition of the share of product lines owned by foreign firms, and productivity growth.

First, replacement of a product line happens through three different channels: domestic incumbent innovations, foreign incumbent innovations, and domestic entry. Thus the replacement rate d_t is the sum of these three probabilities:

$$d_t = (1 - \theta_{t-1})i_t^D + \theta_{t-1}i_t^F + e_t^D \tag{1.27}$$

Note that the successful innovation probabilities by incumbents, i_t^D and i_t^F, are multiplied by the share of domestic-owned and foreign-owned product lines respectively. Next I derive the transition equation of θ_t, the share of product lines owned by foreign firms. θ_t increases for two reasons: foreign incumbent innovation over domestic-owned product lines, and FDI entry. θ_t decreases for two reasons: domestic incumbent innovation and domestic entry over foreign-owned product lines. The transition of θ_t is thus given by the following law of motion:

$$\begin{aligned}\theta_t &= \theta_{t-1} + \theta_{t-1}(1-\theta_{t-1})i_t^F + e_t^F - \theta_{t-1}(1-\theta_{t-1})i_t^D - \theta_{t-1}e_t^D \\ &= \theta_{t-1} + e_t^F - \theta_{t-1}e_t^D + (i_t^F - i_t^D)\theta_{t-1}(1-\theta_{t-1}) \end{aligned} \tag{1.28}$$

Next I derive the expressions for aggregate productivity and its growth. I first combine (1.15) and (1.16) to obtain the ratio between labor input by domestic-owned product lines

ℓ_t^D and foreign-owned product lines ℓ_t^F:

$$\frac{\ell_t^D}{\ell_t^F} = \frac{1+\sigma^F}{1+\sigma^D}$$

Combining this with total labor in the tradable sector $L_t^T = (1-\theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F$, I obtain the following expressions for labor input by domestic-owned and foreign-owned product lines:

$$\ell_t^D = \frac{1+\sigma^F}{(1-\theta_{t-1})\sigma^F + \theta_{t-1}\sigma^D} L_t^T$$

$$\ell_t^F = \frac{1+\sigma^D}{(1-\theta_{t-1})\sigma^F + \theta_{t-1}\sigma^D} L_t^T$$

Plugging these equations and the production function for intermediate goods (1.11) into (1.2), the composite of intermediate goods I_t^M can be written as follows:

$$I_t^M = A_t L_t^T \frac{(1+\sigma_D)^{\theta_{t-1}}(1+\sigma_F)^{1-\theta_{t-1}}}{\theta_{t-1}(1+\sigma_D) + (1-\theta_{t-1})(1+\sigma_F)} \tag{1.29}$$

where aggregate productivity A_t is given by:

$$A_t = \exp\left[\int_0^1 \ln a_t(i) di\right]$$

and L_t^T is total labor hired by intermediate goods producing firms. As is clear from the expression, aggregate productivity A_t grows as productivity of each product line $a_t(i)$ improves. Using the four different innovation rates and the innovation sizes, the growth rate of A_t is characterized as follows:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \left(\frac{1+\sigma_F}{1+\sigma_D}\right)^{e_t^F} (1+\sigma_D)^{e_t^D} (1+\sigma_D)^{(1-\theta_{t-1})i_t^D} (1+\sigma_F)^{\theta_{t-1} i_t^F} \tag{1.30}$$

The four terms in the right-hand side correspond respectively to FDI entry, domestic entry, domestic incumbent innovation, and foreign incumbent innovation. This completes the characterization of firm dynamics and its effect on aggregate productivity growth.

1.3.5 Non-Tradable Goods Producer

The non-tradable goods producer hires labor from households and produces non-tradable goods. The production function is given as follows:

$$Y_t^N = A_t (L_t^N)^{1-\alpha^N} \tag{1.31}$$

where $0 < 1 - \alpha^N < 1$ is the labor share in non-tradable goods production. I assume that total factor productivity in non-tradable goods production increases at the same rate as aggregate productivity in the tradable sector. This assumption comes from the empirical fact that productivity spillovers from multinational firms to domestic firms happen through worker mobility.[11] Since this spillover to non-tradable goods production is not internalized by those making innovation investment decisions, it works as an externality that may cause too little innovation. In the appendix, I study a version of the model with slow productivity spillovers to the non-tradable sector, and find that this alteration does not affect the optimal reserve policy or its welfare impact substantially. This spillover guarantees that production

[11] Dasgupta (2012) reviews the relevant empirical literature.

of tradable goods and non-tradable goods will grow at the same rate in the long run. Let P_t^N denote the non-tradable goods price. Since the law of one price holds for tradable goods between this country and the rest of the world, the non-tradable goods price P_t^N determines the real exchange rate of this country. Thus I call P_t^N the real exchange rate, and an increase in P_t^N is real appreciation. The first-order condition of the non-tradable goods producer is simply given by:

$$W_t = P_t^N A_t(1-\alpha^N)(L_t^N)^{-\alpha^N} \tag{1.32}$$

Since labor is mobile between the tradable and non-tradable sectors, the real wage W_t is common in both sectors. Using P_t^N and W_t the profit for non-tradable goods producer is given by:

$$\Pi_t^N = P_t^N Y_t^N - W_t L_t^N \tag{1.33}$$

which is paid to households.

1.3.6 Household

The representative household consumes tradable goods C_t^T and non-tradable goods C_t^N, supplies labor L_t elastically, accumulates and rents capital K_t to the tradable goods producer, and invests Z_t^E units of tradable goods in domestic entry. They receive the wage income $W_t L_t$, capital income $r_t K_{t-1}$, and profits from tradable goods producers Π_t^T, non-tradable goods producers Π_t^N, and domestic intermediate goods producing firms $(1-\theta_{t-1})(\pi_t^D - Z_t^D)$. They also receive FDI inflow $e_t^F Q_t^F$, which is revenue from the sales of domestic-owned product lines to foreign investors. The representative household's optimization problem is then given as follows:

$$\max_{\{C_t^T, C_t^N, L_t, K_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} [\ln C_t - \psi (L_t)^{\omega}]$$

$$C_t = \left[(\gamma)^{1/\varepsilon} (C_t^T)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{1/\varepsilon} (C_t^N)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.34}$$

subject to

$$C_t^T + P_t C_t^N + I_t + Z_t^E = W_t L_t + r_t K_{t-1} + \Pi_t^T + \Pi_t^N + (1-\theta_{t-1})(\pi_t^D - Z_t^D) + e_t^F Q_t^F \tag{1.35}$$

$$I_t = K_t - (1-\delta) K_{t-1} + \frac{\psi_k}{2} K_{t-1} \left(\frac{K_t}{K_{t-1}} - (1+\overline{g}) \right)^2 \tag{1.36}$$

where γ is a parameter to determine the weight of tradable goods in composite consumption C_t, ε is the constant elasticity of substitution between tradable and non-tradable consumption, and δ is the depreciation rate of capital. Capital accumulation is subject to a capital adjustment cost that slows down the transition process, and ψ_k is the parameter the governs the size of this cost. The functional form of the capital adjustment cost is taken from Neumeyer and Perri (2005) to be consistent with long-run growth of the economy.

Optimal investment in domestic entry Z_t^E is determined by equation (1.25). The first-order conditions for the rest of the choice variables can be summarized as follows:

$$\frac{C_t^T}{C_t^N} = \frac{\gamma}{1-\gamma} (P_t^N)^{\varepsilon} \tag{1.37}$$

$$\psi \omega (L_t)^{\omega-1} = \frac{W_t}{C_t} \left(\gamma \frac{C_t}{C_t^T} \right)^{1/\varepsilon} \tag{1.38}$$

$$\lambda_t \left[1 + \psi_k \left(\frac{K_t}{K_{t-1}} - (1+\bar{g})\right)\right] = \beta E_t \left[\lambda_{t+1} \left\{ r_{t+1} + 1 - \delta - \frac{\psi_k}{2} \left((1+\bar{g})^2 - \left(\frac{K_{t+1}}{K_t}\right)^2 \right) \right\} \right] \tag{1.39}$$

where λ_t is the marginal utility of tradable consumption given by:

$$\lambda_t = \frac{1}{C_t} \left(\gamma \frac{C_t}{C_t^T}\right)^{1/\varepsilon} \tag{1.40}$$

The stochastic discount factor $\Lambda_{t,t+1}$ is then given by $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. Equation (1.37) relates the optimal ratio of tradable and non-tradable goods consumption to the real exchange rate. Equation (1.38) gives the optimal labor supply L_t, and equation (1.39) is the Euler equation with respect to capital.

1.3.7 Government

The government in this model engages in reserve policy to improve household's welfare. The reserve policy consists of two types of interventions. First, when $\kappa_t = \kappa_H$ and the borrowing constraint is loose, the government collects T_t units of tradable goods through a lump-sum tax and accumulates reserves. In general T_t can be any function of the state of the economy, but in this chapter I consider a simple tax rule that the government collects a fraction τ of tradable goods output Y_t^T each period. Second, when $\kappa_t = \kappa_L$ and the borrowing constraint binds, the government provides accumulated reserves to the tradable goods producer to help finance working capital payments, which I call a bailout.

The government keeps accumulating τY_t^T units of reserves each period until it becomes suboptimal to do so. There are two reasons why accumulating reserves becomes suboptimal at some point in the transition. First, the benefit from attracting FDI becomes smaller as capital accumulates and the step size $\sigma_t^F(k_t)$ becomes smaller. Second, as capital accumulates, the collateral value becomes large enough at some point that the borrowing constraint never binds and there is no need for bailouts. For these reasons it is optimal for the government to stop accumulating reserves once capital is sufficiently accumulated.[12]

Bailouts in sudden stops are modeled as follows. The borrowing constraint being binding implies that the tradable goods producer cannot borrow as much as they would if the borrowing constraint was loose. Let $Y_t^{T,loose}$ denote tradable goods output when the constraint is loose. The shortage of foreign borrowing, denoted by S_t, can be written as follows:

$$S_t = \max\left\{-B_t + \phi\left(1-\alpha\right)Y_t^{T,loose} - \kappa_L K_{t-1}, 0\right\}$$

The first two terms are the borrowing amount when the constraint is loose, and $\kappa_L K_{t-1}$ is the borrowing limit, so that the gap is the shortage of foreign borrowing. A negative gap implies that the tight borrowing limit $\kappa_L K_{t-1}$ is still large enough to cover the necessary amount of borrowing, and in this case a max operator sets $S_t = 0$. When S_t is positive, the borrowing constraint binds without a bailout, and the government transfers reserves to the tradable goods producer to cover the shortage up to the amount of reserves at hand. The size of a bailout, denoted by V_t, is given as:

$$V_t = \min\left\{S_t, R^F F_{t-1}\right\}$$

[12] It is also optimal for the government not to rebate reserves after reserve accumulation stops, because rebating reserves would reduce the growth rate in the model. The redundant reserves are lost from the economy, but the welfare loss is limited (smaller than 0.02% of permanent consumption) because the value of goods received after 40 periods or more in the future is heavily discounted.

where $R^F F_{t-1}$ is the amount of reserves at the beginning of period t.

Note that the size of a bailout is dependent on the amount of private debt, and thus the anticipation of bailouts induces private agents to borrow more. I investigate if this is the optimal bailout policy by trying two different types of bailout policies. First, I study a bailout policy in which the government lends reserves to help finance working capital payments, and tradable producers repay these loans after production. I found that this type of intervention has a similar effect on productivity growth, but the welfare gain is limited compared to the benchmark policy described here. Second, in the appendix I also study a bailout policy in which there is a ceiling on the size of bailouts so that the bailout size is independent of private debt, and I found the benchmark bailout policy described here achieves higher welfare.

Given the tax rule in normal times and bailouts in sudden stops, the amount of reserves F_t follows the transition equation given as follows:

$$F_t = \begin{cases} R^F F_{t-1} + T_t & \text{when } \kappa_t = \kappa_H \\ R^F F_{t-1} - V_t & \text{when } \kappa_t = \kappa_L \end{cases} \tag{1.41}$$

1.3.8 Market Clearing Conditions

To close the model, this subsection lists the market clearing conditions. The capital market, labor market, and non-tradable goods market clearing conditions are given as follows:

$$K_{t-1} = K_t^D \tag{1.42}$$

$$L_t = L_t^T + L_t^N \tag{1.43}$$

$$Y_t^N = C_t^N \tag{1.44}$$

and labor in the tradable sector satisfies:

$$L_t^T = (1-\theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F \tag{1.45}$$

This completes the exposition of the model economy. The appendix formally defines the equilibrium of the model economy and the stationarized equilibrium conditions that I use to solve the model numerically.

1.4 Discussion of Reserve Policy

This section elaborates on the key mechanism of the model, namely how the reserve policy attracts FDI, promotes growth, and improves welfare. The section starts by describing the main externality in the model and explains how the reserve policy corrects it. The section then discusses the cost of the reserve policy, and explains the key trade-off that the reserve policy faces, i.e. lower consumption in the short run and higher consumption in the long run.

1.4.1 Benefit of Reserve Policy

The key externality in the model is that private agents do not internalize that their actions affect FDI entry decisions by foreign investors. The main benefit of receiving FDI is that foreign firms innovate more often than domestic firms, and thus contribute to higher productivity growth. To attract more FDI, households should invest more in capital and innovation to increase the growth rate of the economy, and make more labor available for foreign firms.

Higher growth and more labor will bring higher profits for foreign firms and attract more FDI entry.

The reserve policy is intended to correct this externality through two channels. The first one is the real depreciation channel. In normal times when the borrowing constraint is loose, the government collects taxes from private agents to accumulate reserves. As some resources are taken away by the government and profit for the tradable producer falls, households reduce tradable goods consumption. One thing to note here is that private agents have an incentive to offset reserve accumulation by borrowing more from abroad. There are two reasons why they have an incentive to borrow more from abroad. First, they have an incentive to compensate for the loss of resources. Second, anticipation of future bailouts induces them to borrow more. As shown in Jeanne (2012), if reserve accumulation is completely offset by private foreign borrowing, it would have no effect on the consumption path, and thus it would not cause real depreciation. The key factor that prevents full offset in this model is the debt-elastic foreign spread. As private agents borrow more from abroad, the interest spread rises through equation (1.10) and makes foreign borrowing more costly. Therefore offset is only partial, and households reduce tradable goods consumption.

Next I provide a partial equilibrium intuition for how this reduction in tradable goods consumption leads to real depreciation and a labor shift to the tradable sector. First, I combine equations (1.1), (1.7), (1.15), (1.29) and (1.32) to obtain the wage equality condition across sectors:

$$W_t = A_t F(K_{t-1}, \theta_{t-1}) \left(L_t^T\right)^{-\frac{\alpha}{\alpha+\theta}} = P_t^N A_t (1-\alpha^N)(L_t - L_t^T)^{-\alpha^N} \tag{1.46}$$

where $F(K_{t-1}, \theta_{t-1})$ is a function of the state variables. Since production is concave in labor in both the tradable sector and the non-tradable sector, the marginal product of labor is decreasing in labor in both sectors. Solving the second equation for P_t^N and plugging it into the optimality condition between tradable and non-tradable goods consumption (1.37), along with non-tradable goods production (1.31), we have:

$$\frac{C_t^T}{A_t(L_t - L_t^T)^{1-\alpha^N}} = \frac{\gamma}{1-\gamma}\left(\frac{A_t F(K_{t-1}, \theta_{t-1})\left(L_t^T\right)^{-\frac{\alpha}{\alpha+\theta}}}{A_t(1-\alpha^N)(L_t - L_t^T)^{-\alpha^N}}\right)^{\varepsilon}$$

Note that labor in the tradable sector L_t^T is the only endogenous variable in this equation except C_t^T and total labor supply L_t, given the state variables. Therefore, this equation indicates that the reduction in tradable goods consumption C_t^T caused by reserve accumulation requires a labor shift across sectors. Specifically, a labor shift to the tradable sector, i.e. higher L_t^T, decreases the right-hand side by decreasing the marginal product of labor in the tradable sector (numerator) and increasing the marginal product of labor in the non-tradable sector (denominator), and recovers the equality. This labor shift also reduces non-tradable goods production in the denominator of the left-hand side, but since the right-hand side is lower, the reduction in non-tradable goods production is smaller than the reduction in tradable goods consumption.[13] Note also that the fraction inside the parenthesis in the right-hand side is the real exchange rate P_t^N. Therefore, reserve accumulation causes real depreciation and a labor shift to the tradable sector. In addition, it is clear from equation (1.46) that this labor shift also reduces the real wage W_t. This mechanism is in line with the

[13] It can be easily seen that an opposite labor shift, i.e. lower L_t^T, is not consistent with a reduction in C_t^T. Lower L_t^T would increase the right-hand side, and at the same time decrease the left-hand side even more by increasing non-tradable goods production.

empirical findings by Rodrik (2008) that real depreciation promotes productivity growth by shifting more production resources to the tradable sector.

To see the effect of this labor shift on FDI entry and innovation, I derive the expression for tradable output Y_t^T in terms of labor by combining (1.1), (1.7) and (1.29):

$$Y_t^T = A_t G(K_{t-1}, \theta_{t-1})(L_t^T)^{\frac{\alpha}{\alpha+\theta}}$$

where $G(K_{t-1}, \theta_{t-1})$ is another function of the state variables. Thus tradable output is an increasing function of labor in the tradable sector. Now recall the expression for firms' profit when the constraint is not binding from equation (1.15):

$$\pi_t^s = \theta Y_t^T - W_t \ell_t^s = \frac{\sigma_s}{1+\sigma_s} \theta Y_t^T$$

It follows that a labor shift to the tradable sector increases profits for intermediate firms. From the viewpoint of individual firms, they enjoy higher profits due to larger demand and a cheaper real wage.

The second channel through which the reserve policy attracts FDI and corrects the externality is the precautionary channel. When the borrowing constraint binds, bailouts help the tradable goods producer finance working capital payments, and prevent sharp drops in profits for intermediate firms. Since FDI entry and innovation investment decisions are forward-looking, anticipation of future bailouts increases the expected value of firms and induces more FDI entry and innovation today.

In summary, the reserve policy consists of reserve accumulation in normal times and bailouts in sudden stops. Anticipation of these policy interventions in the future increases the expected profits for intermediate firms today, and attracts more FDI entry and induces more innovation today. There is also a feedback loop that growth induces further growth. Harrison and Rodríguez-Clare (2010) review the empirical literature on FDI entry and summarize the extensive evidence that FDI is attracted to less risky and growing markets. The main mechanism here is consistent with this empirical fact, and in this model the reserve policy achieves high and stable growth. However, whether the reserve policy can improve welfare depends on its cost. The next section discusses this point.

1.4.2 Cost of Reserve Policy

There are two types of costs associated with the reserve policy in the model. First, as explained above, private agents do not fully offset reserve accumulation by foreign borrowing, so that consumption of tradable goods becomes lower in the short run. Consumption of non-tradable goods also becomes lower because reserve accumulation shifts labor to the tradable sector. As is clear from the discussion in the previous subsection, lower consumption and a labor shift to the tradable sector are the essential parts of the mechanism of how reserve policy works. In this sense they are the unavoidable cost of the reserve policy. At the cost of short-run lower consumption, the reserve policy promotes productivity growth and households enjoy higher consumption in the long run. Therefore, for the reserve policy to improve welfare, the long-run gain of higher consumption must exceed the short-run loss of lower consumption. This is the key trade-off that the reserve policy faces.

The second cost is a crowding-out effect of reserve accumulation. Reinhart, Reinhart, and Tashiro (2016) show that there is a strong negative correlation between the reserve-to-

GDP ratio and the investment-to-GDP ratio in Asian countries after 2000, suggesting that active reserve accumulation crowds out investment. Cook and Yetman (2012) use micro-level data to empirically show that reserve accumulation reduces bank lending in emerging Asian countries. In my model the crowding-out effect results from the debt-elastic spread on foreign borrowing. As the government accumulates reserves by collecting taxes, private agents borrow more from abroad to compensate for the loss of resources, at least partially. Higher borrowing then increases the interest rate on foreign debt through equation (1.10). To see how the higher interest rate on foreign debt crowds out investment in capital, recall that profits for the tradable producers are evaluated in terms of the marginal utility of tradable consumption by households λ_t. The Euler equation with respect to foreign debt therefore takes the standard form except the Lagrange multiplier on the borrowing constraint μ_t:

$$\lambda_t - \mu_t = \beta R_t E_t(\lambda_{t+1})$$

Comparing this equation with the Euler equation with respect to capital investment (1.39), it can be seen that a higher interest rate on foreign debt requires a higher capital return, and thus crowds out capital investment. A higher interest rate on foreign debt also crowds out investment in domestic firm entry and innovation. To see this, I arrange the Euler equation with respect to foreign debt to obtain the expression for the stochastic discount factor as follows:

$$E_t(\Lambda_{t,t+1}) = \frac{\beta E_t(\lambda_{t+1})}{\lambda_t} = \left(1 - \frac{\mu_t}{\lambda_t}\right)\frac{1}{R_t} \tag{1.47}$$

This equation indicates that a higher interest rate on foreign debt reduces the stochastic discount factor, implying that households value future tradable consumption less compared to tradable consumption today. Since investment in domestic firm entry and innovation are forward-looking, the lower stochastic discount factor leads to lower investment in domestic entry and innovation.

As seen from equation (1.10), the parameter ψ_b is crucial for the size of the crowding-out effect. If ψ_b is large, reserve accumulation is likely to cause severe crowding-out and slow down growth. In this case, the long-run gain from higher consumption is likely to be limited, and the welfare impact of reserve policy is smaller or can be even negative.

In summary, the reserve policy brings higher consumption in the long run at the cost of lower consumption in the short run. The optimal reserve policy is determined to hit the balance between the marginal gain and the marginal loss. The policy analysis section shows how the value of ψ_b and the FDI entry cost affect the optimal reserve policy and its welfare gain.

1.5 Quantitative Analysis

This section calibrates the model parameters, demonstrates the baseline simulation, and discusses the model features. I solve the model numerically by two steps: First I divide the equilibrium conditions by productivity level A_t to stationarize the equations. In the second step I solve the stationarized model globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint. The stationarized equilibrium conditions and the details of the solution procedure are left to the appendix.

1.5.1 Calibration

One period in the model is meant to be annual. There are 30 parameters to be determined in the model, except the debt-elasticity of spread ψ_b which is estimated from the data below. I use conventional values in the literature if available, and calibrate the other parameters to target the data for a sample of eight developing countries from 1990-2010.[14] Table 1.1 presents 19 externally-determined parameter values. Six parameters regarding preferences are set to conventional values in the literature. The discount factor $\beta = 0.96$ and gross return on the safe asset $R^F = 1.02$ are standard values for annual models. The weight on tradable goods in consumption $\gamma = 0.34$ is set following Mendoza (2005) and Durdu, Mendoza, and Terrones (2009). The elasticity of substitution between tradable and non-tradable goods in consumption, $\varepsilon = 0.6$, is in the middle of the range discussed in Mendoza (2005). The coefficient on labor disutility $\psi = 0.525$ is set so that labor supply in the long run is equal to 1. The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza (1991). Regarding the production parameters, capital's share in tradable production $\alpha = 0.3$ and the capital depreciation rate $\delta = 0.1$ are set to conventional values. The imported input price P^M is set to be 1, and labor's share in non-tradable production $1 - \alpha^N = 0.75$ is taken from Schmitt-Grohé and Uribe (2016). The share of intermediate goods θ is set so that the imported inputs-to-GDP ratio matches the data at 14%. The fraction of the input cost subject to the working capital requirement ϕ is determined by the method adopted in Mendoza (2010), in which the ratio of domestic credit to private firms relative to GDP is

[14]The countries are Chile, Colombia, Malaysia, Mexico, Poland, Thailand, Tunisia, and Vietnam. These counties are chosen based on the availability of data for the calibration, such as the manufacturing share of FDI inflows and JP Morgan EMBI Global spread data.

Table 1.1: Externally-Determined Parameters

Variable		Value	Source and Target
β	Discount factor	0.96	Standard
R^F	Return on reserve asset	1.02	Standard
γ	Tradable share in consumption	0.34	Mendoza (2005)
ε	CES between T and NT	0.6	Middle value in literature
ψ	Labor disutility	0.525	Unit labor supply
ω	Frisch elasticity $1/(\omega-1)$	1.455	Mendoza (1991)
α	Capital share	0.3	Standard
θ	Intermediate input share	0.56	Imported input/GDP 14%
P^M	Imported input price	1	Normalized value
$1-\alpha^N$	Labor share in non-tradable	0.75	Schmitt-Grohe Uribe (2016)
δ	Capital depreciation	0.1	Standard
ϕ	Share of input subject to WC	1.18	Private credit/GDP 47.0%
$\bar{b}$	Long-run debt/GDP	-0.36	Data
$\overline{R}$	Long-run interest rate	1.0635	Consistent with BGP growth
P_{HL}, P_{LH}	Probability matrix of κ_t	0.080, 0.851	Frequency and duration of SS
ρ^D, ρ^E, ρ^F	Concavity of innovation investment	0.5	Akcigit and Kerr (2015)

used as a proxy for working capital. This ratio is 47% on average for the sample countries, and this results in $\phi = 1.18$. This parameter is set to 1 in Neumeyer and Perri (2005) and 1.25 in Uribe and Yue (2006). The long-run debt-to-GDP ratio $\bar{b}$ is set to the average of the sample countries in recent years at 36%. The long-run interest rate on foreign borrowing $\overline{R}$ is set to be consistent with the long-run growth rate satisfying $\beta\overline{R} = 1 + \overline{g}$, where the long-run growth rate $\overline{g}$ is determined below. The only uncertainty in the model is stochastic borrowing limit coefficient κ_t. κ_t follows a two-state Markov process with a 2×2 transition matrix. I follow Jeanne and Rancière (2011) and derive the average frequency and duration of sudden stop episodes in the following way: a given country in a given year is in a sudden stop if the capital inflow-to-GDP ratio drops more than 5% from the previous year. Using the same sample of 33 countries as in Jeanne and Rancière (2011) over the period of 1980-2009, the unconditional probability of sudden stops is 8.6%, and each sudden stop episode continues for two years with probability 14.9%. Accordingly I set $P_{HL} = 0.080$ and $P_{LH} = 0.851$.

For the parameters related to innovation and FDI entry, the concavity parameters governing investment, ρ^D, ρ^E, ρ^F, are set to 0.5 following Akcigit and Kerr (2015) and their literature review. The remaining eight parameters, $\eta^D, \eta^E, \eta^F, \chi^F, \lambda, C^F, \sigma^D, \sigma^F$, are jointly determined to match eight moments in the data to those in the model in the long run. Each of the following moments are tightly related to the above eight parameters in the same order. (1) The ratio of R&D expenditure in the manufacturing sector to GDP is closely related to η^D, which governs the scale of domestic innovation. This ratio is in general small in developing countries and high in developed countries. I set the long-run ratio in the model to match the average of developed countries in the recent data, which is 2.4%. (2) The domestic entry rate is closely related to η^E. The data is taken from The World Development Indicators. The average of sample countries in 2007 is 8.56%. (3) The innovation rate of foreign firms relative to domestic firms identifies η^F, which governs foreign incumbents' innovation. Guadalupe, Kuzmina, and Thomas (2012) document that foreign firms in Spain conduct product innovations 1.387 times more often than domestic firms. I target this value. (4) The value added share of foreign firms in the manufacturing sector identifies χ^F, the congestion cost of FDI entry. This target is meant to pin down the economic presence of foreign firms in the tradable sector in the model. Ramstetter (2009) reports this value for Malaysia, Thailand, and Vietnam, and Ramondo (2009) reports this for Chile. The average of these four countries is 32.25%, which I set as a target. (5) The ratio of FDI inflows to the manufacturing sector relative to GDP helps to pin down the cost of acquisition for FDI. The average FDI inflow-to-GDP ratio of the sample countries over 1990-2009 is 3.8%. Data for FDI inflows by sector is available at the International Trade Centre's website. The data are available only for five of the eight sample countries. The average of the ratio of FDI inflows to the manufacturing sector relative to total FDI inflows for these countries in 2012 is 50%, thus I set 1.9% as a target. (6) As there is no data or reliable estimation for the fixed cost of FDI C^F, I follow Fillat and Garetto (2015) and set the fixed entry cost so that it is 72% of the operational profit of foreign-owned product lines. (7) The long-run growth rate helps to pin down σ^D, the productivity gain of domestic innovation. It is set to 2.1%, which is the average growth rate of developed countries in recent years. (8) The productivity gain from FDI entry identifies σ^F. Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012) estimate the productivity gain from FDI entry in Indonesia and Spain using the propensity score matching method to control for firm characteristics and mitigate the cherry-picking effect of FDI entry choice. These papers show that in the year of entry, firm productivity increases by 11%. Hence I set $(1 + \sigma^F)/(1 + \sigma^D) = 1.11$. When I evaluate each country's reserve policy below, I allow the FDI congestion cost χ^F and the fixed entry cost C^F to vary across countries to match variation in the relevant data moments within the sample.

Finally, the borrowing limit coefficient κ_L, the capital adjustment cost parameter ψ_k, and the exponent on the catch-up term ρ are determined to target the model behavior in the transition, because these parameters are irrelevant along the balanced growth path. κ_L and ψ_k are set to match the sudden stop dynamics of the model with the data shown in the next subsection. ρ governs the growth rate of the economy in the transition, and thus is set to match the average growth rate of the model economy in the first 30 periods with the average growth rate of the sample countries from 1980 to 2010. The average growth rate in the data

Table 1.2: Jointly-Determined Parameters

Variable		Value	Target	Model
η^D	Domestic innovation coeff.	0.24	Manu. R&D/GDP 2.4%	2.40%
η^E	Domestic entry coeff.	0.74	Domestic entry rate 8.56%	8.56%
η^F	Foreign innovation coeff.	0.21	Relative innovation rate 1.387	1.387
χ^F	Coeff. of FDI congestion cost	0.15	FDI value-added in manu. 32.25%	32.25%
λ	Share of FDI firm value paid	0.93	Manu. FDI inflow/GDP 1.9%	1.86%
C^F	Fixed entry cost	0.037	Fixed entry cost/profit 72%	72%
σ^D	Domestic innovation size	0.21	Long-run growth rate 2.1%	2.1%
σ^F	Foreign innovation size	0.34	11% productivity gain upon FDI entry	

Table 1.3: Parameters Related to Transitional Dynamics

Variable		Value	Target
κ_L	Borrowing limit coefficient	0.89	SS dynamics
ψ_k	Capital adjustment cost	15	Drop in investment in SS
ρ	Exponent on catch-up term	0.25	Avg. growth 3% in transition

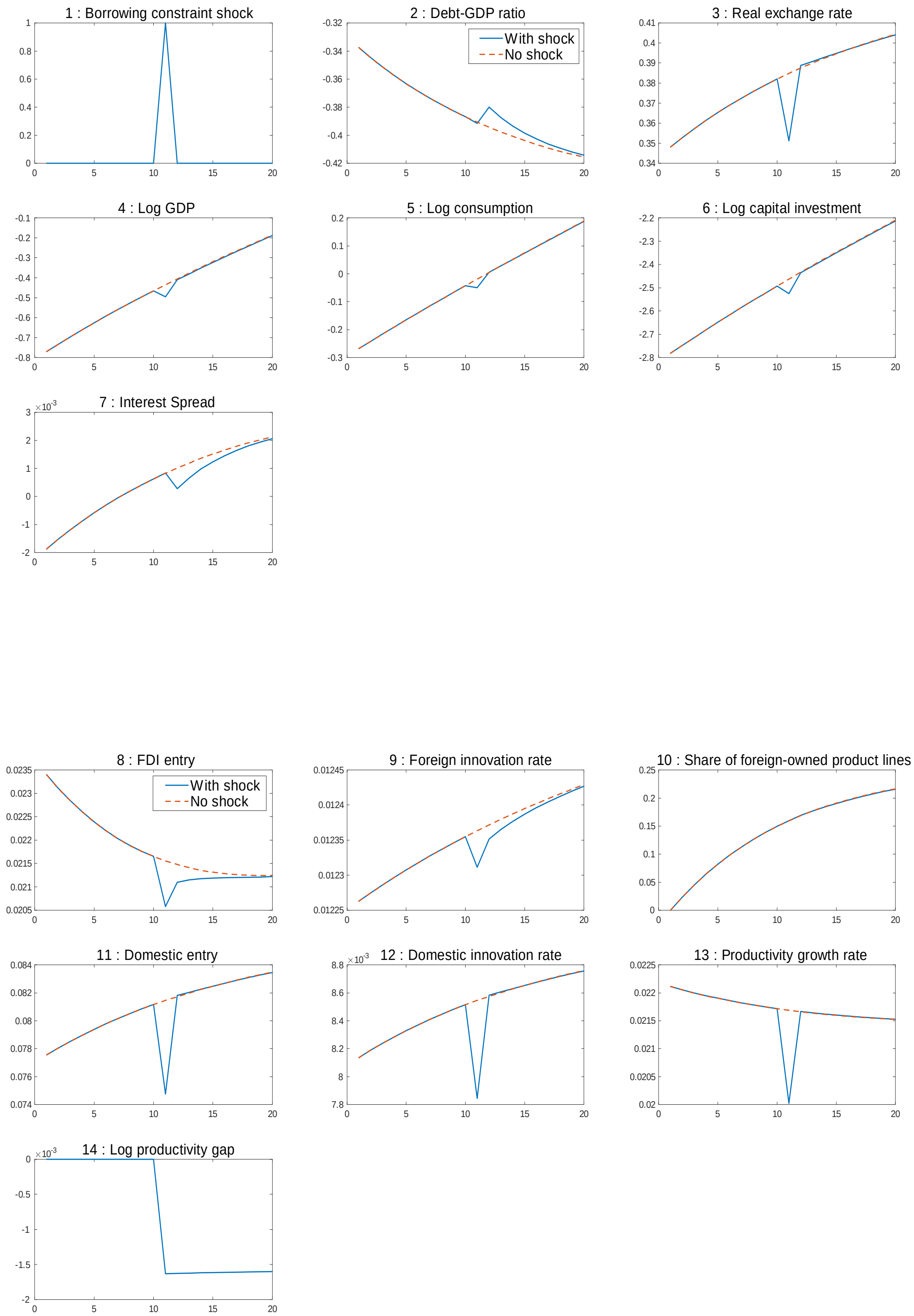
is about 3%. With $\rho = 0.25$, along with the initial capital holdings $k_{-1} = 0.5k_{ss}$, the average growth rate of the first 30 periods is 2.9%. In determining these parameter values, I do not target the pace of capital accumulation. According to the Penn World Table, the capital holdings of the sample countries in 1980 is 28% of that in 2010. In the model, the initial capital K_{-1} is about 30% of the capital holdings at period 30, which is close to the data.

1.5.2 Quantitative Performance of the Model

This section documents the quantitative performance of the model and demonstrates the role of the reserve policy. I start by showing a sample dynamic simulation path to give an idea of what the transition dynamics and sudden stops look like. Figure 1.4 presents a sample dynamic path without policy intervention. The debt-elasticity of spread is set at

$\psi_b = 0.0542$ as a benchmark; below, I estimate this parameter using a group of developing countries. Initial capital is 50% of its long-run level as measured by the productivity-adjusted value, initial debt is 33% of GDP to match with the data in 1980, and the initial share of foreign-owned product lines is 0. There is a one-time shock to the borrowing constraint at period 11. The solid lines are the paths with this shock, and the dashed lines are the smooth paths without shocks as a reference. Panel 2 shows the debt-to-GDP ratio, which increases gradually as capital accumulates and the borrowing limit expands. When a sudden stop happens the ratio jumps up, consistent with the stylized facts of sudden stops. The real exchange rate in Panel 3 drops sharply in a sudden stop. Panels 4-6 show that GDP, consumption, and investment all fall in a sudden stop, and investment drops the most, which is also in line with the empirical regularities of sudden stop events illustrated by Mendoza (2010). The interest spread in Panel 7 increases over time along with the debt-to-GDP ratio, and it declines in a sudden stop. This decline in the spread in the sudden stop may look odd, but the effective interest rate actually increases according to the external financing premium captured by the Lagrange multiplier on the borrowing constraint. Turning to innovation and FDI entry, Panels 8, 9, 11, and 12 show that FDI entry, foreign innovation, domestic entry, and domestic innovation all drop in a sudden stop. They also show that the size of the drop is much larger for domestic innovation compared to foreign innovation. This difference comes from that the fact that the stochastic discount factor by domestic households drops due to the binding borrowing constraint, as shown in equation (1.47), and thus domestic firms reduce their investment substantially, while foreign firms discount future profits by the fixed rate $1/R^F$. These different responses by domestic and foreign firms are consistent with the empirical facts documented by Alfaro and Chen (2012) that domestic firms reduce

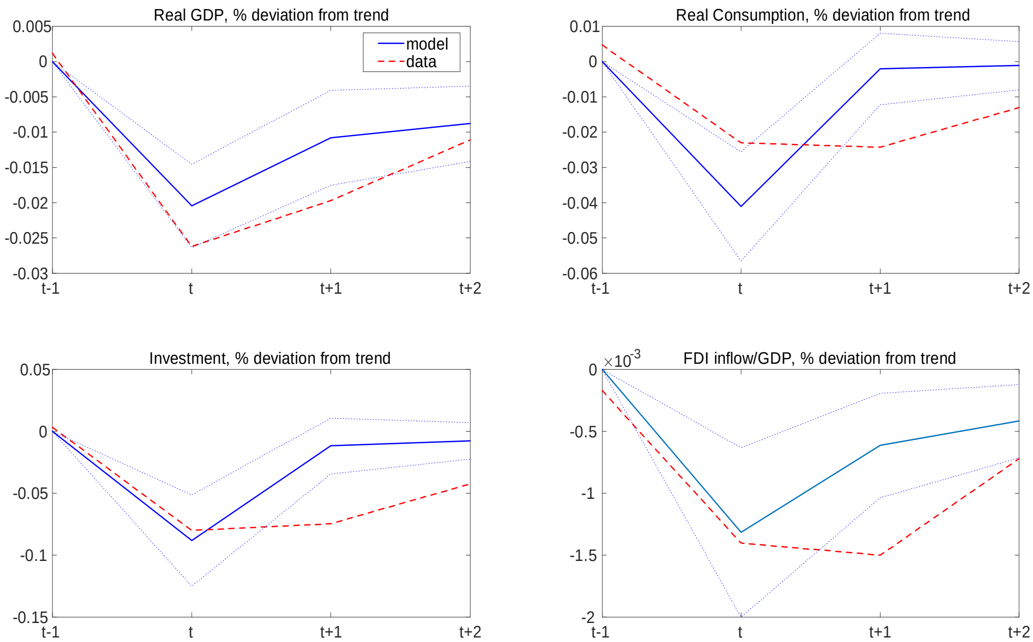
Figure 1.4: Model Simulation without Reserve Policy



investment to a larger extent than foreign-owned firms in crisis. The last panel shows that a sudden stop has a permanent level effect on productivity. Cerra and Saxena (2008) provide empirical evidence that the negative effect of a crisis on productivity is very persistent or almost permanent. This makes a clear contrast to previous business-cycle models of sudden stops such as Mendoza (2010). Permanent negative effects on productivity suggest that the cost of sudden stops may be underestimated in business-cycle models and point to the importance of the endogenous growth framework, especially when the focus is a normative analysis of policies trying to fight sudden stops.

To take a closer look at the quantitative performance of the model in capturing sudden stop episodes, I compare the average dynamics of sudden stop events in the model and in the data. I simulate the model with the same initial conditions as above 1,000 times, and compute deviations of the key variables from the smooth paths without sudden stops. Then I take the average of these deviations from the smooth paths in all sudden stop events. For the data, I derive deviations of the key variables from their smooth trends using the Hodrick-Prescott filter, using data for the same 33 developing countries in 1980-2009 that I used to determine the Markov transition matrix for sudden stops. The data for GDP, consumption, and investment are taken from the World Development Indicators, and the FDI inflow-to-GDP ratio is computed using the data from Broner, Didier, Erce, and Schmukler (2013). Then I take the average of these deviations for all the sudden stop events. Figure 1.5 shows the results for real GDP, real consumption, investment, and the FDI-to-GDP ratio for the model and the data. The dotted lines are one standard deviation bands for the model dynamics. It is clear that the model captures the quantitative dynamics of average sudden stop episodes in the data quite well.

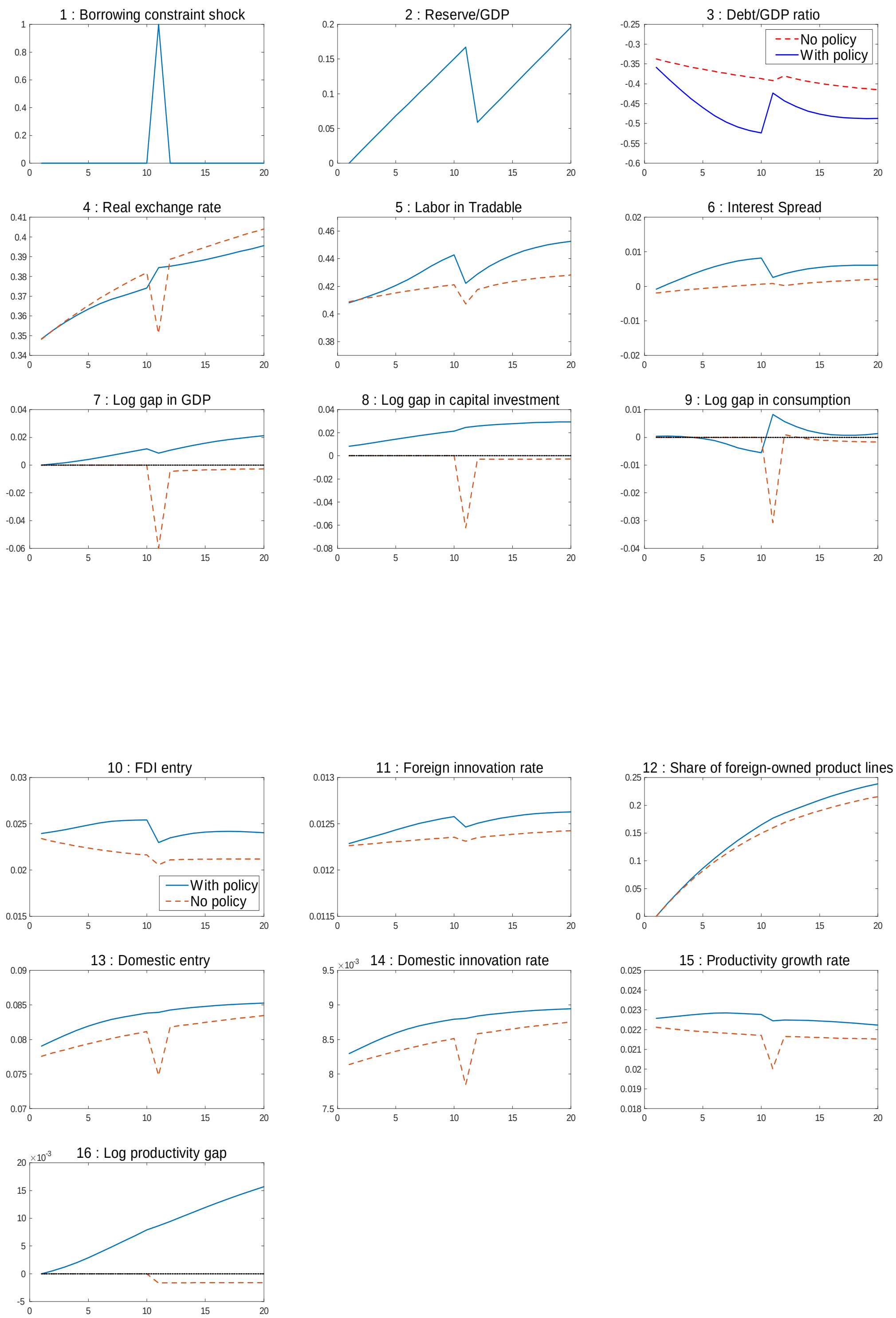
Figure 1.5: Sudden Stop Dynamics, Model and Data



Next I introduce reserve policy and demonstrate how it affects the transitional dynamics of the economy. I set a fixed tax rate τ as 3%, which is the optimal policy for this economy as shown in the next section. The government stops accumulating reserves when capital holdings adjusted by the productivity level reach 95% of the balanced growth path level as is true in the optimal policy.[15] Figure 1.6 presents the simulation results for the same one-time shock as above. The solid lines are the paths with the reserve policy and the dashed lines are the paths without policy. Panel 7,8,9, and 16 show the log gaps from the path without shocks, with and without policy. As the tax will be zero in the long run, both paths will converge to the same balanced growth path measured by productivity-adjusted values, but they are different in productivity levels. Panel 2 shows that reserves are accumulated in

[15] To be precise, the tax rate is declining linearly from 3% to 0 as k_{t-1}/k_{ss} increases from 85% to 95%. This enables me to avoid an abrupt change in the decision rules and makes numerical solution easier and more accurate.

Figure 1.6: Model Simulation with Reserve Policy



normal times, and provided to private agents when a sudden stop happens. Responding to reserve accumulation, private agents borrow more to compensate for the collected tax as shown in Panel 3. As the debt-to-GDP ratio increases, the foreign spread rises as shown in Panel 6, which makes the offset only partial and reduces tradable consumption. This in turn causes a real depreciation and a labor shift to the tradable sector as shown in panel 4 and 5. As shown in Panel 7 and 8, GDP and capital investment are always larger with reserve accumulation, but consumption is lower in the short run because the offset is partial. It may look puzzling that investment is larger with reserve accumulation, because the higher interest spread is likely to crowd out investment. The reason is that the increased labor in the tradable sector and the higher growth rate of productivity under reserve accumulation increase the marginal product of capital, and this positive effect dominates the negative effect from the higher spread. When a sudden stop happens, the government gives accumulated reserves to the private agents, which prevents sharp drops in GDP, investment, consumption, and real exchange rate. The latter 7 panels are related to firm dynamics and productivity growth. With reserve accumulation all types of entry and innovation are larger. These panels also show that intervention in a sudden stop prevents a drop in domestic entry and innovation, and achieves high and stable growth. The productivity level is higher by 1.7% after 20 periods, and the gap goes to almost 2.5% in the long run.

1.6 Optimal Reserve Policy

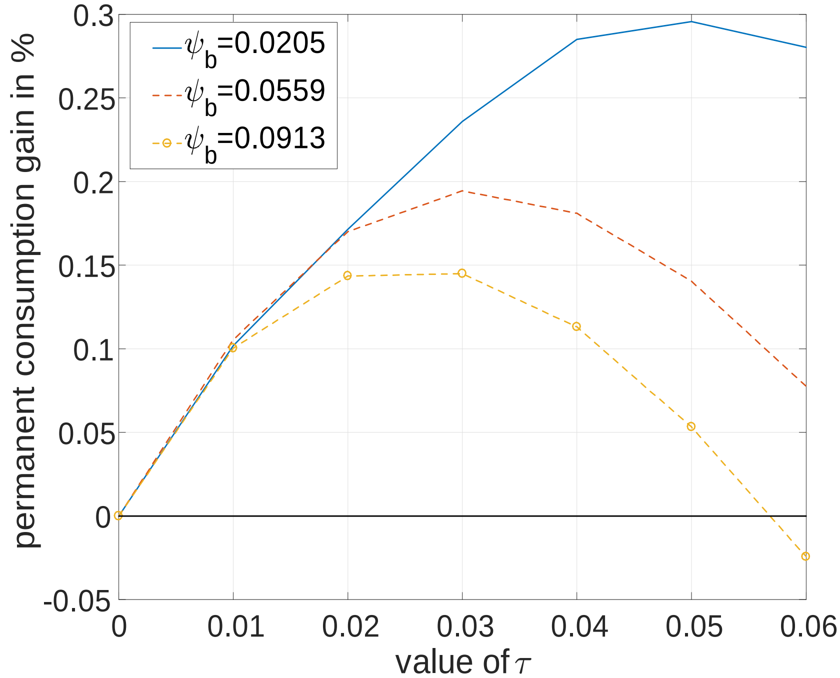
This section studies how the optimal reserve policy is determined, and shows the first main result of the chapter: the debt-elasticity of the interest rate spread and the FDI entry cost are key determinants of the optimal pace of reserve accumulation. As discussed in the model section, I consider a simple policy rule that the government collects a fixed fraction τ of tradable output every period to accumulate reserves, and eliminates the tax once the economy has accumulated enough capital. For the analyses below with different parameters and tax rates τ, I found that it is optimal to stop collecting taxes when capital holdings adjusted by the productivity level reach 95% of the balanced growth level. Thus this rule is fixed for all of the analyses hereafter.[16]

1.6.1 Role of Debt-Elasticity of Spread

In this subsection I show how the debt-elasticity of the spread ψ_b affects the optimal reserve policy. To have a quantitatively reasonable value for ψ_b, I estimate it using data for developing countries. The detailed estimation method is described in the next section, in which each country's reserve policy is evaluated quantitatively. The result is that the mean debt-elasticity of the spread over 22 developing countries is 0.0542, which implies that if the debt-to-GDP ratio increases by 10%, the spread increases by 54.2 basis points or 0.542%. I also estimate ψ_b for five subgroups of countries, according to the number of past defaults for each country. The estimated values for ψ_b are 0.0205, 0.0382, 0.0559, 0.0736, and 0.0915 from the fewest defaults to the most defaults. To clarify how different values of ψ_b affect the

[16] Again, to be precise, I assume the tax rate is declining linearly from τ to 0 as capital accumulates from 85% to 95%. See footnote 15.

Figure 1.7: Welfare Impact with Different ψ_b

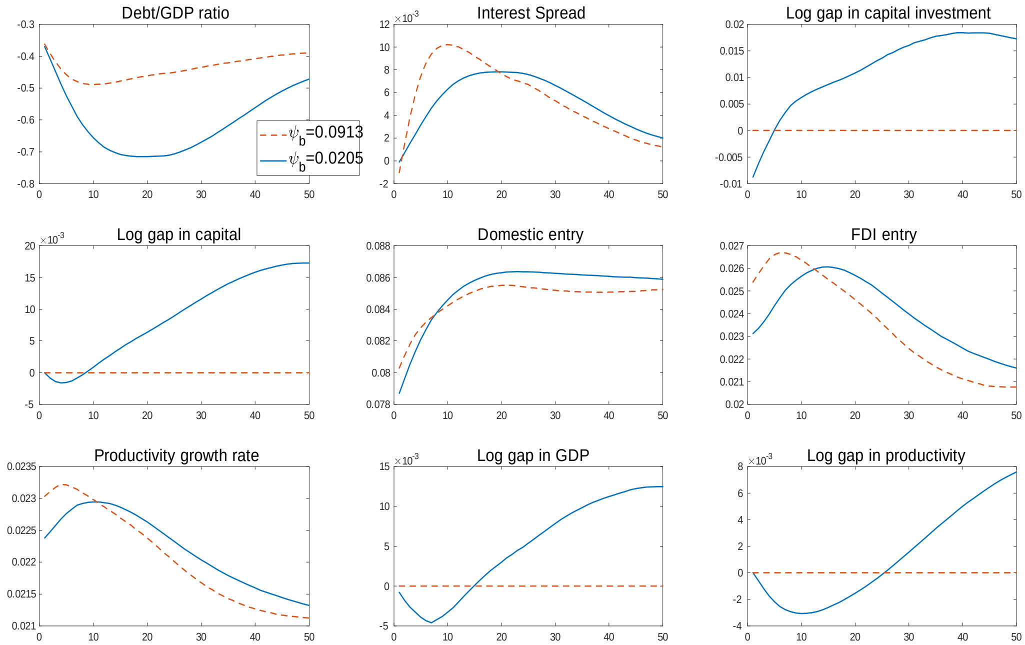


optimal reserve policy, I use 0.0205, 0.0559, and 0.0913 for the following analysis.

The optimal reserve policy is derived in the following way: Given each value for ψ_b, I first numerically solve the model without reserve policy, simulate the model with stochastic shocks for 300 periods 100,000 times, and compute the expected utility. Next I solve the model with reserve policy $\tau = 0.01$ to 0.06 and compute the expected utility in the same way. The welfare gain/loss for each reserve policy is evaluated in terms of the permanent consumption gain/loss in percentage terms, as is common in the literature. The result is summarized in Figure 1.7. It is clear that the welfare gain from the reserve policy is larger with a smaller debt-elasticity of spread ψ_b. It can also be observed that the optimal pace of reserve accumulation τ is higher with smaller ψ_b.

To understand the role of ψ_b, Figure 1.8 plots the dynamics of key variables from simulations with $\psi_b = 0.0205$ and $\psi_b = 0.0913$. Log gaps refer to logged variables for $\psi_b = 0.0205$

Figure 1.8: Role of ψ_b



minus the logged variables for $\psi_b = 0.0913$. The tax rate τ is set to 0.04 for both simulations, and shocks are shut down to see the difference in a clean setting. The first panel shows that the debt-to-GDP ratio is always smaller with a more elastic spread, implying that private agents offset reserve accumulation less with a more elastic spread. The smaller offset makes reserve accumulation more effective at suppressing tradable consumption, so that for the first several periods the economy grows faster with a more elastic spread. However, as the debt-to-GDP ratio becomes larger, the interest spread becomes higher with a more elastic spread, as shown in the second panel. This higher spread discourages capital investment and slows down capital accumulation as shown in the third and fourth panels. Lower capital in the tradable sector leads to smaller profits for intermediate goods producing firms, which thus discourages domestic and foreign entry, and slows down productivity growth. As a result, GDP and the productivity level will eventually be lower with a more elastic spread.

This mechanism has an important implication for the optimal pace of reserve accumulation. As discussed in the previous section, the key trade-off that the reserve policy faces is lower short-run consumption against higher long-run consumption. The debt-elasticity of the foreign spread determines the extent of crowding-out due to reserve accumulation, and thus is the key determinant of the long-run gain from the reserve policy. If the spread is more elastic to private debt, reserve accumulation causes more severe crowding-out and reduces the growth-promoting effect of reserve accumulation. In this case, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

1.6.2 Role of the FDI Entry Cost

Another key determinant of the optimal reserve policy in the model is the FDI entry cost. There is a vast literature on the determinants of FDI inflows, and many factors have been identified as significant determinants, such as the host country's institutions, relative labor endowments, and so on.[17] FDI entry costs in the model can be interpreted as these implicit factors that govern the size of FDI inflows to the country.[18]

To show why the FDI entry cost is important for the reserve policy, I change the FDI entry cost parameters from the baseline calibration to create an economy with a larger FDI entry cost. In particular, I target the ratio of manufacturing FDI inflows to GDP to 1.08% rather than the benchmark 1.9%. I use this lower value in the next section to evaluate the reserve policy in some countries. This target requires me to increase the coefficient

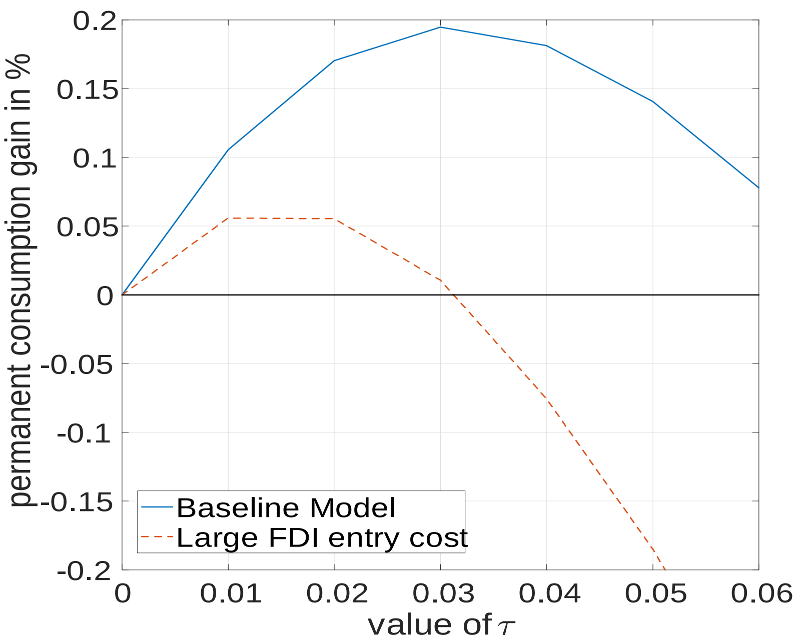
[17] Blonigen (2005) and Blonigen and Piger (2011) review the literature on the determinants of FDI.

[18] Alternatively, in the appendix I estimate the FDI entry cost across countries using the index for the cost to start up new businesses.

Table 1.4: Parameters with large FDI entry cost

	Variable	Value	Target	Model
χ^F	Coeff. of FDI congestion cost	0.32	Manu. FDI inflow/GDP 1.08%	1.08%
C^F	Fixed entry cost	0.040	Fixed entry cost/profit 72%	72%
σ^D	Domestic innovation size	0.24	Long-run growth rate 2.1%	2.1%
σ^F	Foreign innovation size	0.38	11% productivity gain upon FDI entry	

Figure 1.9: Welfare Impact with Different FDI Entry Cost



on the FDI entry congestion cost χ^F. I also adjust the fixed entry cost C^F to keep the fixed entry cost-to-profit ratio at 72%, and adjust the innovation step sizes σ^D and σ^F to have the same long-run growth rate as in the baseline model, keeping the relative size $(1+\sigma^F)/(1+\sigma^D) = 1.11$ unchanged. The other parameters are left unchanged. New parameter values are summarized in Table 1.4.

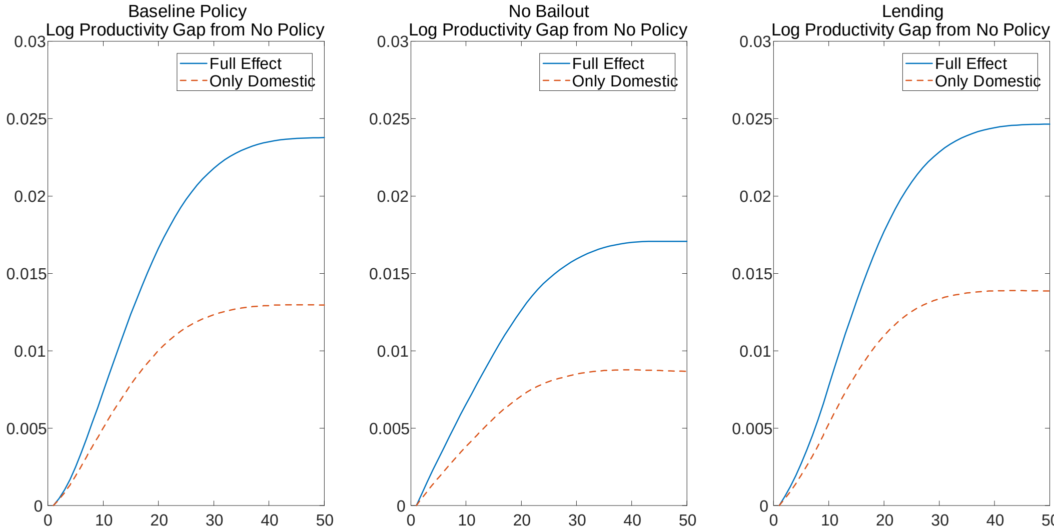
Given these new parameter values, I compute the welfare impact of reserve policy for different rates of accumulation τ. Figure 1.9 presents the results, with the results from the baseline model for comparison. Both models are solved assuming $\psi_b = 0.0542$. It is clear that the welfare gain is substantially smaller for the case with a larger FDI entry cost. The figure also shows that the optimal pace of reserve accumulation is slower, and as the pace becomes faster the welfare impact quickly turns negative. This result suggests that attracting FDI is an important channel through which the reserve policy improves welfare. If some factors of the country impede FDI inflow and the reserve policy is not effective in attracting FDI, the optimal pace of reserve accumulation is slower, and the welfare impact is likely to be limited.

1.6.3 Decomposition of Policy Effect

So far I study the effect of the reserve policy as a whole, but the reserve policy consists of two types of interventions, reserve accumulation in normal times and bailouts in sudden stops. This subsection conducts a decomposition analysis of the policy effect. Specifically, to highlight the effect of bailouts on growth and welfare, I compare two different types of bailout schemes to the baseline policy. The first alternative scheme is that the government accumulates reserves but never uses them for bailouts. I call this a "no-bailout" scheme. The second scheme is that the government provides accumulated reserves to private agents to help finance working capital payments, but private agents need to repay these reserves to the government after production. I call this a "lending" scheme. The model parameters are the same as the baseline model, and the debt-elasticity of the spread is set at its benchmark value $\psi_b = 0.0542$. Borrowing constraint shocks are shut down to highlight the effect of bailout policies through anticipation without actual bailouts.

Figure 1.10 shows the simulation results for the three different policy schemes: the baseline scheme, the no-bailout scheme, and the lending scheme. The solid curves show the productivity gaps relative to the case without policy. The dashed curves are created by

Figure 1.10: Effect of Different Bailout Schemes on Growth

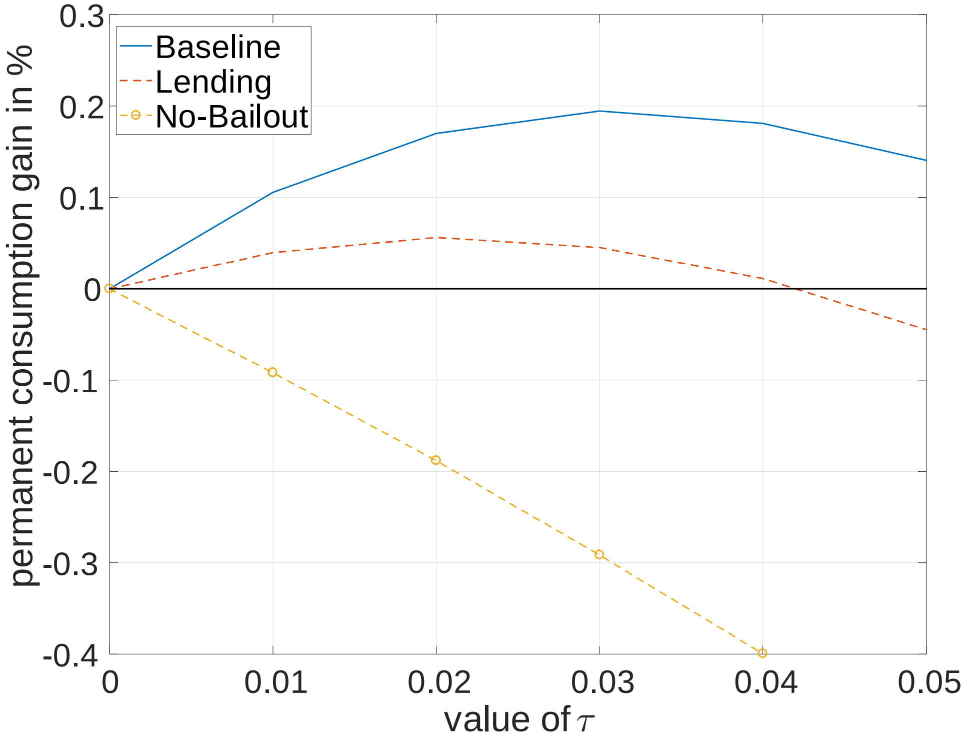


fixing the FDI entry rate and foreign innovation rate to their values in the case without policy. Therefore they show the policy effects on growth through only the domestic entry and innovation channels, and the gaps between the solid curves and the dashed curves show the policy effects on growth through the FDI entry and foreign innovation channels.

There are three important observations from this figure. First, note that the no-bailout scheme promotes growth only through the channel of reserve accumulation causing real depreciation. Therefore the gap between the impact of the baseline policy and the no-bailout scheme is the effect of the policy on growth through the anticipation of bailouts. Comparing the baseline policy and the no-bailout scheme tells us that 72% of the effect of the reserve policy on growth comes from the real depreciation channel, and 28% comes from the bailout anticipation channel. Second, the effect of the lending scheme on growth is almost the same as the effect of the baseline policy. This reveals that the bailout anticipation channel is actually due to the anticipation that bailouts will help working capital finance, and not the anticipation of transfers. Third, in all three schemes the effect on promoting growth comes from domestic and foreign factors roughly equally. In each case, 55%, 51%, and 56% of the growth-promoting effect comes from domestic entry and innovation, and the rest comes from FDI entry and foreign innovation.

Next I study the effect of these three policy schemes on welfare. I compute the welfare gain/loss from the three policy schemes using the same method as above, namely simulating the model with each policy 100,000 times with stochastic shocks, and measuring the welfare gain/loss in terms of the permanent consumption gain. Figure 1.11 shows the welfare gain/loss for each policy scheme for different rates of reserve accumulation. The first observation is that reserve accumulation alone without any type of bailout cannot improve welfare. There are two reasons for this result. First, the growth-promoting effect is limited without bailouts, and thus the long-run gain from higher consumption is smaller. Second, since there are no bailouts, private agents do not increase foreign borrowing to compensate for reserve accumulation as much. This reduces short-run consumption even more, and increases the short-run cost of reserve accumulation. The second observation is that the lending scheme substantially improves the welfare impact from the reserve policy compared to the no-bailout scheme. The welfare plot indicates that about 70% of the welfare improvement over the no-bailout scheme comes from lending, i.e. helping working capital financing. The rest of the welfare improvement comes from rebating reserves in bailouts. In the lending scheme, private agents actually do not receive any rebate of accumulated reserves, and all reserves are lost from the economy. In contrast, in the baseline policy some of the reserves are rebated to private agents depending on the number of sudden stops, and thus the loss of resources is smaller. Since the productivity gain is almost the same between the baseline policy and the lending scheme, the welfare gap between the baseline policy and the lending scheme comes

Figure 1.11: Welfare Impact of Different Bailout Schemes



solely from rebating reserves.

1.7 Evaluation of Reserve Policy

This section conducts the second main analysis of the chapter: evaluation of the actual reserve policies of developing countries. The last section shows that the optimal reserve policy and its welfare impact are crucially dependent on the debt-elasticity of the spread and the FDI entry cost. In the first subsection I estimate the debt-elasticity of the spread for developing countries from the data. Then I proceed to evaluate whether observed reserve accumulation policies of developing countries are roughly optimal, and whether the model can quantitatively explain observed variation in the pace of reserve accumulation across countries.

1.7.1 Estimation of Debt-Elasticity of Spread

There is a large amount of literature on the determinants of the interest rate spread in developing countries.[19] I first follow Dell'Erba, Hausmann, and Panizza (2013) and conduct a parsimonious panel regression to estimate the relationship between the spread and the debt-to-GDP ratio. The regression equation is given as follows:

$$S_{i,t} = \beta_0 + \beta_1 \text{debtGDP}_{i,t} + \alpha_i + \tau_t + \varepsilon_{i,t}$$

where $S_{i,t}$ is the interest rate spread on external borrowing in percentage points, α_i is a country-specific fixed effect, τ_t is a time-specific fixed effect, and $\varepsilon_{i,t}$ is an error term. The data for the spread is taken from JP Morgan's EMBI Global, as is common in the literature. Since the available time period of this data is different across countries, this is an unbalanced panel regression with 22 countries with maximum time period 1994-2015. The debt-to-GDP ratio is computed using the data from Lane and Milesi-Ferretti (2017). The data is annual and the total number of observations is 379. The result is that β_1 is estimated to be 0.0542 with a standard error of 0.0073 and a t-value of 7.45. This means that as the debt-to-GDP ratio increases by 10%, the spread increases by 54.2 basis points or 0.542%. This is similar to the results of other papers that include more controls, such as 0.0447 in Dell'Erba, Hausmann, and Panizza (2013) and 0.0567 in Kennedy and Palerm (2014).

Next I differentiate countries into several groups with different debt-elasticities of the spread. I found that the number of past defaults is significantly associated with a high

[19] One of the main interests in the literature is whether the developing countries' spread is determined by the global factors or the countries' fundamentals. See for example Kennedy and Palerm (2014) and their literature review.

Table 1.5: Estimation of Debt-Elasticity of Spread

Explanatory Variables	Coefficient (S.E.)	t-value
β_1: Debt-GDP ratio	0.0205* (0.0107)	1.92
β_2: Debt-GDP ratio × Default	0.0177*** (0.0042)	4.23

elasticity of the spread, which is consistent with the findings of Reinhart and Rogoff (2009). According to the data in Reinhart (2010), the number of defaults before the sample period for the sample 22 countries varies from 0 to 9. Accordingly, I divide the sample countries into five groups with the number of defaults 0 or 1, 2 or 3, 4 or 5, 6 or 7, and 8 or 9, and assign variables from 0 to 4 for each group. Then I estimate the following regression:

$$S_{i,t} = \beta_0 + \beta_1 \text{debtGDP}_{i,t} + \beta_2(\text{debtGDP}_{i,t} \times \text{Default}_i) + \alpha_i + \tau_t + \varepsilon_{i,t}$$

The result is presented in Table 1.5. This result implies that the debt-elasticity of the spread for countries with 0 or 1 default is 0.0205, and the elasticity increases by 0.0177 as the number of defaults increases by two: 0.0382 for 2 or 3 defaults, 0.0559 for 4 or 5 defaults, 0.0736 for 6 or 7 defaults, and 0.0913 for 8 or 9 defaults.

1.7.2 Evaluation of Each Country's Reserve Policy

Given the estimated debt-elasticity of the spread, this subsection evaluates each country's reserve policy. I proceed with the following steps: (1) I adjust the FDI entry cost parameters and innovation step sizes to match the FDI inflow–to-GDP ratio in the model to the data.[20][21] (2) Given the new parameters and given the estimated ψ_b, I solve the model and find the optimal reserve accumulation pace τ that maximizes household's expected utility. (3) I compute the average pace of reserve accumulation for each country from the data. In particular, I divide increases in reserve holdings by GDP, both expressed in terms of current US dollars, for every year from 1991-2010, and take the average across years. The data is from the World Development Indicators. (4) I solve the model assuming that each country's reserve accumulation pace τ corresponds to the data, and compute the expected utility.[22] The results are presented in Figure 1.12 and Tables 6.

Figure 1.12 lines up the sample countries in the order of the observed pace of reserve accumulation from the slowest to the fastest, along with the optimal pace suggested by the model. Overall, most developing countries are roughly in line with the optimal pace suggested by the model. The correlation between the actual and the optimal pace of reserve accumulation across these 22 countries is 0.54. It shows that Panama and Chile may have more room for welfare gains by accumulating reserves more quickly. On the other hand, Turkey, Indonesia, and China seem to be accumulating reserves too fast.

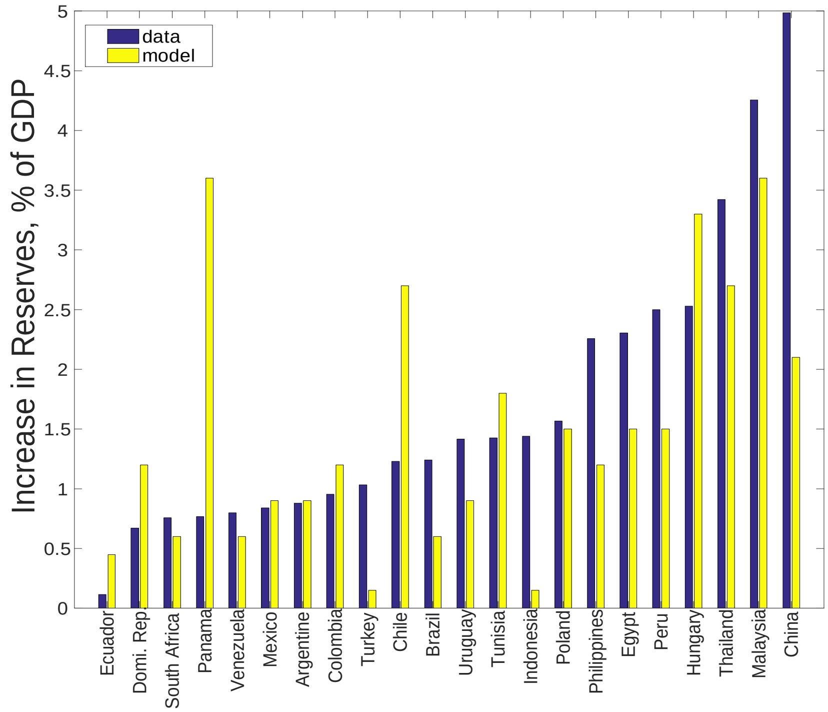
Table 1.6 presents more detailed results including the welfare gain/loss by the actual and optimal pace of reserve accumulation. It can be observed that the welfare gain from the actual policy is close to the optimal level for many countries. This is because the welfare gain is not very sensitive to the pace of reserve accumulation around the optimal pace, and thus a small deviation from the optimal pace does not reduce the welfare gain. Therefore,

[20] As I did for the baseline model, I assume that 50% of FDI inflow goes into the manufacturing sector. Accordingly the target is 50% of the FDI–to-GDP ratio for each country.

[21] It may be too much to let a single FDI entry cost parameter χ^F explain all the cross-country differences in the FDI inflow-to-GDP ratio. In the appendix, I alternatively estimate the FDI entry cost using the Starting a Business Index from the World Bank's Doing Business Surveys.

[22] In the model a 1% tax on tradable output corresponds to 0.6% of GDP on average in the first 30 periods.

Figure 1.12: Reserve Accumulation Pace, Data and Model



many countries are in fact accumulating reserves at the optimal pace or very close to the optimal in terms of welfare. Looking at each country in detail, most Latin American countries have high elasticity of the spread because of their default history. This reduces the optimal pace of reserve accumulation, but most Latin American countries are actually in line with the optimal pace. Only three countries, China, Indonesia, and Turkey, incur a welfare loss due to an accumulation pace that is substantially faster than the optimal pace.This overall result suggests that the debt-elasticity of the foreign borrowing spread and the FDI entry cost can explain why different countries accumulate reserves at different rates. In countries with high debt-elasticity of the spread and/or the large FDI entry cost, the optimal pace of reserve accumulation is slow. If these countries accumulate reserves at a pace similar to Asian countries such as Malaysia or Thailand, it may cause a large welfare loss through

Table 1.6: Evaluation of Reserve Accumulation Pace

Country	Accum. Pace (%)		Welfare (%)		Elasticity of Spread	FDI Inflow / GDP (%)
	Actual	Optimal	Actual	Optimal		
Argentine	0.9	0.9	0.05	0.05	0.0736	2.16
Brazil	1.2	0.6	0.01	0.04	0.0913	2.12
Chile	1.2	2.7	0.20	0.21	0.0913	5.57
China	5.0	2.1	-0.13	0.18	0.0382	3.24
Colombia	1.0	1.2	0.08	0.08	0.0736	2.60
Dominican Rep.	0.7	1.2	0.09	0.12	0.0736	3.11
Ecuador	0.1	0.5	0.01	0.02	0.0913	1.67
Egypt	2.3	1.5	0.10	0.13	0.0382	2.62
Hungary	2.5	3.3	0.35	0.37	0.0382	5.05
Indonesia	1.4	0.2	-0.11	0.01	0.0382	0.76
Malaysia	4.3	3.6	0.39	0.39	0.0205	4.69
Mexico	0.8	0.9	0.06	0.06	0.0913	2.48
Panama	0.8	3.6	0.16	0.43	0.0382	6.05
Peru	2.5	1.5	0.07	0.13	0.0913	3.47
Philippines	2.3	1.2	0.01	0.06	0.0205	1.51
Poland	1.6	1.5	0.09	0.13	0.0382	2.71
South Africa	0.8	0.6	0.02	0.02	0.0382	1.22
Thailand	3.4	2.7	0.18	0.21	0.0205	2.96
Tunisia	1.4	1.8	0.16	0.16	0.0559	3.32
Turkey	1.0	0.2	-0.05	0.01	0.0559	1.04
Uruguay	1.4	0.9	0.03	0.06	0.0913	2.44
Venezuela	0.8	0.6	0.04	0.04	0.0913	2.06

severe crowding-out of investment and/or little gain from FDI. Such over-accumulation could occur in principle for countries with serial default history and/or small FDI inflows, typically Latin American countries. However, these countries are not actually accumulating reserves quickly, and thus are not subject to welfare losses from over-accumulation in practice.

1.8 Conclusion

In the past decade, foreign reserve accumulation by developing countries has been both an active research area and a central area of policy debate. However, our understanding regarding the optimal reserve policy and its benefits and costs is still limited. We also know little about the reason for the wide variation in the amount and the pace of reserve accumulation across developing countries.

This chapter contributes to our understanding on these issues by developing a quantitative framework to assess the optimal reserve policy, incorporating the key benefits and costs of reserve accumulation. On the benefit side, I combine elements from two strands of literature, endogenous growth and sudden stops, to incorporate the growth-promoting effect and the precautionary effect of reserve accumulation. I also introduce FDI into the endogenous growth framework, which constitutes an important channel through which reserve accumulation promotes growth. On the cost side, I introduce crowding out of investment resulting from reserve accumulation.

Using the model, I identify two factors that are important determinants of the optimal pace of reserve accumulation: the debt-elasticity of the interest rate spread, and the FDI entry cost. In countries with a high debt-elasticity of the spread, active reserve accumulation severely crowds out investment, which reduces the growth-promoting effect. In countries with large FDI entry costs, reserve policies are not effective in attracting FDI, and the growth-promoting effect is limited. In these cases, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

I show that both real depreciation by reserve accumulation in normal times and bailouts during sudden stops are important in terms of both the productivity gain and the welfare gain. Quantitative analysis shows that 72% of the growth-promoting effect comes from real depreciation by reserve accumulation in normal times, and 28% is from the anticipation of future bailouts in sudden stops. I also show that real depreciation alone cannot improve welfare without bailouts, and 70% of the welfare gain from bailouts comes from helping to finance working capital payment.

Accounting for differences in the debt-elasticity of the spread and the FDI entry cost across countries, most developing countries are roughly in line with the optimal pace of reserve accumulation suggested by the model. This result implies that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

In addition, the model developed in this chapter provides a useful framework to study broader research areas. One possible extension is to introduce a pecuniary externality into the borrowing constraint, as in Mendoza (2010) and Bianchi and Mendoza (2013), and to study optimal macroprudential policies. The key difference from their models is that the model in this chapter has endogenous growth, thus enabling the study of the interaction between macroprudential policies and growth. It is important to incorporate growth into the framework because countries subject to a sudden stop of capital inflows are developing countries, which grow faster than the rest of the world during tranquil times. The model here also incorporates innovations by heterogeneous firms in the endogenous growth framework, as originally developed in Ates and Saffie (2016). Thus the model enables the study of the effects of policy on various types of firms.

2 Firm and Trade Dynamics in Sudden Stops

2.1 Introduction

Business cycles in emerging economies are characterized by volatile international capital flows. In particular, sudden stops of capital inflows often cause severe economic downturns such as declines in production, consumption, and asset prices. Theoretical literature has made important progresses in capturing these events by embedding financial frictions into dynamic stochastic general equilibrium (DSGE) models, represented by the seminal work by Mendoza (2010). This literature has studied optimal policies such as capital controls to prevent sudden stops (see Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Benigno, Chen, Otrok, Rebucci, and Young (2016) for example). However, empirical studies have revealed that the effects of sudden stops are not limited to aggregate production and consumption. First, real exchange rate depreciation during sudden stops causes contrasting dynamics in imports and exports, mainly through the extensive margins (Alessandria, Kaboski, and Midrigan (2010), Alessandria, Pratap, and Yue (2015)). Second, the growth rate of the economy declines during sudden stops, and the output level hardly recovers its original trend, pointing to persistent losses in productivity (Cerra and Saxena (2008), Blanchard, Cerutti, and Summers (2015)). Third, sudden stops affect firm entry and innovation at the micro level, which causes a persistent effect on economic growth (Ates and Saffie (2016)). These elements are important in designing optimal policies to deal with volatile capital flows, but most existing papers miss these elements.

This chapter develops a framework that can incorporate all of these elements. The key novelty of our model is twofold. First, economic growth and the extensive margins of imports and exports are determined by endogenous firm dynamics. Second, sudden stops affect non-exporting and exporting firms differently. In particular, non-exporting firms reduce profits during sudden stops, while exporting firms increase profits by taking advantage of the lower relative marginal cost compared to foreign rival firms. The model is a small open economy with tradable and non-tradable sectors. The final tradable sector produces goods using a variety of differentiated intermediate goods. These intermediate goods are produced by many domestic and foreign firms, and through new firm entry and incumbent firms' innovation, productivity of the intermediate sector increases endogenously. Whether each type of good is imported or exported is determined by firms' endogenous activities. Firms own and produce several types of intermediate goods. For each product line, firms invest in innovation to start exporting to the foreign market. When they succeed in innovation, they start exporting their products to the foreign market. Conversely, if foreign firms succeed in innovation, which happens with an exogenous probability, this country starts importing goods from those foreign firms. Sudden stops are modeled following Bianchi and Mendoza (2018). There is a fixed amount of productive asset in the economy, which is used as a collateral for foreign borrowing. When stochastic shocks to aggregate productivity and the interest rate switch from a good state to a bad state, the borrowing constraint binds. The binding constraint then reduces the asset price and tightens the borrowing constraint further, which amplifies the effect of a bad shock on the economy. During sudden stop events, non-exporting firms face lower domestic demand and reduce profits. In contrast, exporting firms increase profits because sudden stops reduce the real wage and the relative marginal cost compared to foreign rival firms. Then firms invest more to export, and the extensive margin of exports expands gradually.

In the quantitative analysis, we solve the model using a version of the policy-function iteration method to deal with an occasionally binding borrowing constraint. The model is calibrated to the Brazilian economy as a benchmark. We simulate the model for many periods and take the average dynamics of the economy around sudden stop episodes. We find that the model captures the aggregate dynamics of sudden stops quantitatively well. Sudden stops are likely to happen when a good exogenous shock of high aggregate productivity and a low interest rate is followed by a bad shock of low productivity and a high interest rate. This is consistent with the sudden stop dynamics in Mendoza (2010). When the borrowing constraint binds, the asset price drops by almost 5%, and the debt-to-GDP ratio shrinks by 1%. Sudden stops also cause a drop in GDP of 1% and consumption of 2% from their trends.

More importantly, firm and trade dynamics in the model are consistent with some of the empirical evidence reported in the literature. First, new entry and incumbent firms' innovation substantially decline during sudden stops. This slows down aggregate productivity growth and in turn has a persistent negative effect on GDP, consistent with the empirical findings in Cerra and Saxena (2008) and Blanchard, Cerutti, and Summers (2015). Second, while imports of intermediate goods decline during sudden stops, exports of intermediate goods are not affected, which is roughly in line with empirical facts documented in Alessandria, Kaboski, and Midrigan (2010) and Alessandria, Pratap, and Yue (2015). Moreover, profits from exporting rise by 9% relative to the trend and stay higher for several years after sudden stops. This is because a drop in the real wage reduces the relative marginal cost of production compared to foreign rival firms. Firms then invest more in innovation to export, and the extensive margin of exports expands gradually, which is also consistent with the empirical fact shown in Alessandria, Pratap, and Yue (2015). In short, our model can capture several empirical regularities about sudden stops that most existing papers miss.

One additional feature of the model is that the model structure allows us to have the firm-level moments at least at the balanced growth path, such as the firm size distribution in terms of output, profit, and factor inputs, for the entire firms and also for exporters and non-exporters separately. We plan to use these firm-level moments for calibration in future research. We discuss this point below in the model section and in the appendix.

The remainder of the chapter is organized as follows. Section 2.2 reviews the related literature. Section 2.3 introduces the model. Section 2.4 presents calibration of the model and the quantitative analysis. Section 2.5 concludes.

2.2 Related Literature

The key contribution of our work is that we bridge two strands of existing literature to provide a tractable framework to study firm and trade dynamics during sudden stops. The first strand is the literature on the relationship between short-run shocks and long-run growth. Comin and Gertler (2006) develop a model in which short-run shocks to the economy causes medium-term business cycles using a product-variety expansion type of endogenous growth framework. Following Comin and Gertler (2006), Queralto (2015) introduces a financial intermediary into a similar growth framework to explain a persistent loss in total factor productivity after the sudden stop in Korea. Gornemann (2015) also adopts a similar framework to capture a persistent output loss after sovereign crises. Ates and Saffie (2016) introduce a financial selection of new firm entry into a version of the Schumpeterian growth model developed by Klette and Kortum (2004), and show that sudden stops have a persistent effect on growth through the composition of new firm entry. The firm dynamics and growth part of our model is based on Ates and Saffie (2016). The key difference from this literature is that we model sudden stops as an occasionally binding borrowing constraint with an endogenous collateral value, so that the model provides a framework to study optimal capital policies in the context of endogenous firm dynamics. We also incorporate heterogeneous innovations and trade dynamics into the model. Clementi and Palazzo (2016) and Akcigit and Kerr (2015) point to the importance of heterogeneous firm and innovation dynamics in the short run and in the long run. Alessandria, Kaboski, and Midrigan (2010) and Alessandria, Pratap, and Yue (2015) show that crises have contrasting effects on imports and exports through sharp devaluations.

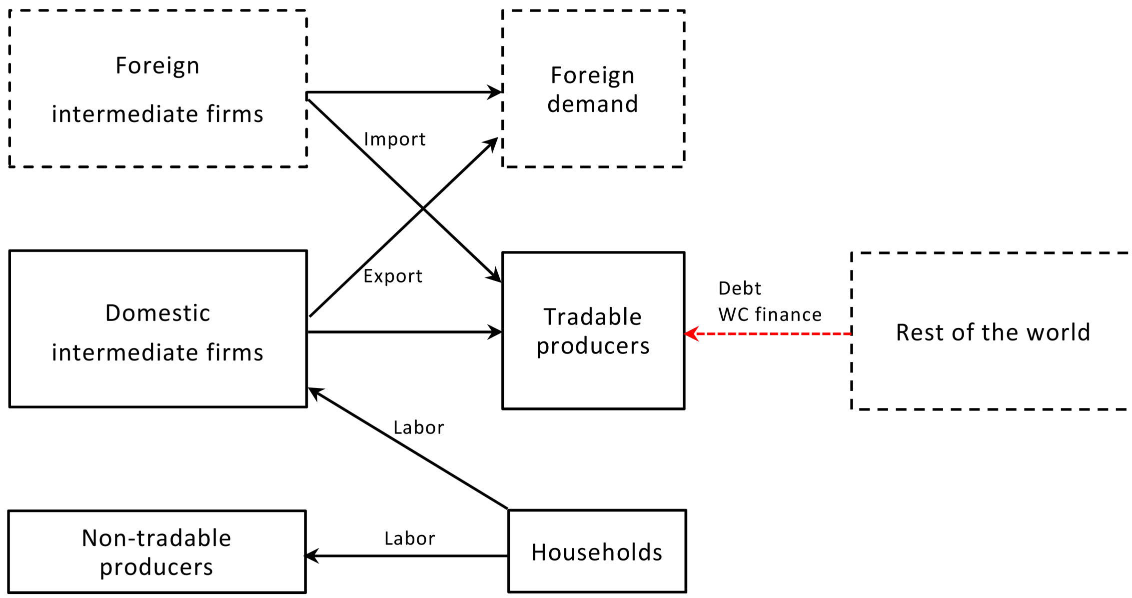
The second strand is the literature that studies optimal policies to manage capital flows, following the seminal work by Mendoza (2010). Bianchi (2011) studies optimal policies to deal with over-borrowing by private agents in an endowment economy. Benigno, Chen, Otrok, Rebucci, and Young (2016) use the same model as in Bianchi (2011) and study several alternative policies to manage over-borrowing. Jeanne and Korinek (2013) set up a stylized three-period model and study the optimal combination of ex-ante and ex-post policies to deal with over-borrowing. Benigno, Chen, Otrok, Rebucci, and Young (2013) also study ex-ante and ex-post policies in a dynamic model with endogenous production of tradable and non-tradable goods. The key contribution of our work to this literature is that we incorporate endogenous firm dynamics and growth, so that the model enables us to study the interaction between capital policies and growth. Ma (2017) sets up a model with both endogenous growth and an occasionally binding borrowing constraint with an externality, and studies the optimal ex-ante and ex-post policies. Our model differs from his work in that our model has heterogeneous firm dynamics and also trade dynamics, both of which are relevant for designing optimal policies.

2.3 Model

The model framework is based on an infinite-horizon small open economy with tradable and non-tradable sectors. The key features of the model are endogenous firm dynamics and the extensive margins of imports and exports. The final tradable goods producers use a variety of differentiated intermediate goods for production. These intermediate goods are produced by domestic and foreign intermediate firms. The intermediate sector is modeled as a version of the Schumpeterian growth model developed by Ates and Saffie (2016), which is a discrete time version of Klette and Kortum (2004) that incorporates aggregate risk. We extend their framework in two dimensions. First, domestic firms invest in two types of innovations: innovations to acquire more product lines, and innovations to start exporting. Second, there are endogenous extensive margins of imports and exports. When firms succeed in innovation to start exporting, they start exporting their products to the foreign market. Conversely, if foreign firms succeed in innovation, this country imports goods from those foreign firms. These extensive margins of imports and exports endogenously shift over time through firm dynamics.

The overview of the model environment is presented in Figure 2.1. The final tradable goods producers use a variety of differentiated intermediate goods produced by domestic and foreign firms. They also borrow from abroad using a non-contingent one-period bond and

Figure 2.1: Model Economy



within-period working capital financing. They are subject to shocks to aggregate productivity and the interest rate on the foreign bond. Because households in this model do not have direct access to the international financial market, tradable goods producers borrow from abroad to smooth consumption on behalf of households. Domestic and foreign intermediate firms produce intermediate goods using labor and productive assets, both internationally immobile. There are endogenous extensive margins of imports and exports as explained above. Non-tradable firms produce goods using labor and sell output to households. Households consume tradable and non-tradable goods, and supply labor to the domestic intermediate firms. Next we turn to the characterization of each agent.

2.3.1 Final Tradable Goods Producer

The final tradable good is the numeraire of this model economy, and its price is normalized at one. The representative tradable goods producer uses unit-mass variety of intermediate goods $\{y_t(i)\}_{i=0}^1$ to produce output Y_t^T:

$$Y_t^T = \exp(\varepsilon_t^A) \exp\left[\int_0^1 \ln y_t(i) di\right] \tag{2.1}$$

where ε_t^A is a stochastic productivity shock. Before production materializes, a fixed fraction ϕ of the cost of intermediate goods needs to be paid. This working capital payment is financed by within-period borrowing from abroad with no interest cost. In addition, the tradable goods producer borrows from abroad using a one-period non-contingent bond. These foreign borrowings are subject to the borrowing constraint as follows:

$$-B_t + \phi\left[\int_0^1 p_t(i) y_t(i) di\right] \leq \kappa Q_t L_{t-1} \tag{2.2}$$

where κ is a fixed parameter on the collateral value, B_t is bond holdings in period t, L_{t-1} is the amount of internationally immobile productive asset (land) holdings by final tradable producers, and Q_t is the asset price. Productive assets are used for production by intermediate firms, and final tradable producers earn rental rates R_t^L from intermediate firms. These assets are in fixed unit supply, and the market clearing condition is $L_t = 1$. Final tradable producers are owned by households, thus they discount future profits using households' discount rate.[23] Given these settings, the maximization problem by the representative final

[23] The assumption that final tradable producers instead of households borrow from abroad and own assets is just to make algebra simpler. Even if households borrow from abroand and own assets, it would be an

tradable producer is given as follows:

$$\max_{\{\{y_t(i)\}_{i=0}^{1}, L_t, B_t\}_{t=0}^{\infty}} E_o \sum_{t=0}^{\infty} \beta^t \lambda_t \Pi_t^T$$

subject to the production function (2.1) and

$$\Pi_t^T = Y_t^T - \int_0^1 p_t(i) y_t(i) di - B_t + \exp(\varepsilon_{t-1}^R) R B_{t-1} - Q_t L_t + (Q_t + R_t^L) L_{t-1} \tag{2.3}$$

$$-B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di \right] \leq \kappa Q_t L_{t-1}$$

where λ_t is the marginal utility of tradable goods consumption by households, $p_t(i)$ is the price of intermediate good i, and $\exp(\varepsilon_t^R)R$ is a stochastic gross interest rate on the foreign bond. Each period, the final tradable producer chooses intermediate goods demand $\{y_t(i)\}_{i=0}^{1}$, productive asset holdings L_t, and foreign bond holdings B_t to maximize the expected profit discounted by household's discount rate adjusted by the marginal utility λ_t. Let μ_t denote the Lagrange multiplier on the borrowing constraint (2.2). The first-order conditions with respect to the choice variables are as follows:

$$y_t(i) : p_t(i) \left(1 + \phi \frac{\mu_t}{\lambda_t} \right) = \frac{Y_t^T}{y_t(i)} \tag{2.4}$$

$$L_t : \lambda_t Q_t = \beta E_t \left[\lambda_{t+1} (Q_{t+1} + R_{t+1}^L) + \mu_{t+1} \kappa Q_{t+1} \right] \tag{2.5}$$

$$B_t : \lambda_t - \mu_t = \beta R \exp(\varepsilon_t^R) E_t(\lambda_{t+1}) \tag{2.6}$$

equivalent model with some additional algebra.

$$\mu_t : \mu_t \left(-B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di \right] - \kappa Q_t L_{t-1} \right) = 0, \; \mu_t \geq 0 \tag{2.7}$$

The first equation is the demand function for intermediate goods. When the borrowing constraint is slack, $\mu_t = 0$ and the demand function for intermediate goods (2.4) is the standard one equating price and marginal product. When the borrowing constraint binds, strictly positive μ_t appears as the external financing premium on working capital payments, which increases the effective cost of inputs. Equation (2.5) is the Euler equation with respect to productive asset holdings. As explained in Bianchi and Mendoza (2018), the asset price Q_t reflects the discounted future marginal product of assets R^L_{t+s} discounted by the asset return. Equation (2.6) is the Euler equation with respect to foreign bond holdings. Given that λ_t is the marginal utility of tradable goods consumption by households, it is the standard Euler equation except when the Lagrange multiplier on the borrowing constraint μ_t appears. This term captures the external financing premium on foreign debt when the borrowing constraint binds, which increases the effective real interest rate on foreign debt, as explained in Mendoza (2010). The last equation (2.7) is the complementary slackness condition for the borrowing constraint.

2.3.2 Intermediate Firms

There is a unit-mass variety of differentiated intermediate goods in the tradable sector, indexed by i. These intermediate goods are produced by domestic and foreign firms. Each firm produces one or more product line(s) across these intermediate goods. This section explains static production and profits in the intermediate sector. The next section explains firm dynamics.

Intermediate firms use productive assets and labor for production. Because these production resources are internationally immobile, domestic firms use domestic assets and labor, and foreign firms use foreign assets and labor. Firms produce intermediate goods using the following production function:

$$y_t = a_t \left(\ell_t\right)^{\alpha} \left(h_t\right)^{1-\alpha} \tag{2.8}$$

where a_t is productivity, ℓ_t is asset input, and h_t is labor input. Productivity is heterogeneous across firms. There are potentially many firms, including domestic and foreign firms, that could produce each type of intermediate good. To see which firm actually produces each type of good, let us focus on the demand for intermediate goods by the final tradable producer given by (2.4). Because this is a unit-elastic demand function, the profit-maximizing behavior would be to set the price infinitely high if a seller was monopolistic.[24] In this model, however, there are rival firms which can produce the same type of good, and they would set a slightly lower price and steal the demand. Therefore, through Bertrand competition, the firm with the lowest marginal cost sets the price equal to the marginal cost for the second-best rival firm, and monopolizes demand. As a result, only the firm with the lowest marginal cost produces each type of good.

The marginal cost of producing intermediate goods for each firm depends on three elements. The first elements are the production factor prices, namely the asset rental rate R_t^L and labor wage W_t. These costs are symmetric across all domestic firms, but foreign firms face foreign factor prices, R_t^{L*} and W_t^*. The second element is whether goods are exported or sold domestically. We assume international trade is subject to a symmetric iceberg cost,

[24] To see this, note that unit-elastic demand implies that sales, price times quantity, are independent of price. By setting the price infinitely high, quantity would be zero. In this case, sellers could earn the same sales with no production cost, which maximizes the profit.

and firms need to ship $1 + \xi$ units of goods to export 1 unit of goods. The third element is the productivity of each firm. Different firms have different productivity across different product lines.

As explained in the next section on firm dynamics, firms need to make innovation to export their products. Before making innovation to export, firms can sell their products only domestically. Once a firm innovates to export, previous exporters lose their access to the foreign market because their products become obsolete. This means that there is at most one firm that can export their products. This assumption divides product lines into three types: domestic lines, exporting lines, and importing lines. For domestic lines, domestic firms produce intermediate goods, and there is no trade. The second-best rival firms are also domestic firms. For exporting lines, domestic firms sell their products not only domestically but also export to the foreign market. In this case, the second-best rival firms are domestic firms in the domestic market, and foreign firms in the foreign market. For importing lines, the final tradable producers of this economy import goods from foreign firms. The second-best rival firms are domestic firms. Hereafter we use D, X, M to denote these three types of product lines. Next we elaborate on firms' production, price setting, and profits for each of three types of lines.

(D) Domestic lines

For domestic lines, domestic firms produce intermediate goods to the domestic final tradable producer. The second-best rival firms are also domestic firms. Because the factor prices are symmetric across all domestic firms, the only difference between the producing firm and the rival firms are the productivity. As explained in the next section, the producing firms for domestic lines have $1+\sigma^D$ productivity lead over the rival firms. Let $a_t(i)$ denote the productivity level of the producing firm for product line i. Then the marginal cost for the domestic rival firm is given as:

$$\widetilde{MC}_t^D(i) = \frac{1}{a_t(i)/(1+\sigma^D)}\overline{\alpha}\left(R_t^L\right)^{\alpha}\left(W_t\right)^{1-\alpha} = p_t^D(i) \tag{2.9}$$

where $\overline{\alpha} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. Recall that the optimal price is equal to the marginal cost for the second-best rival firms. Thus, this is the optimal price that the producing firms set, which we denote by $p_t(i)$. The profit for the producing firm is then given by:

$$\pi_t^D(i) = p_t(i)y_t(i) - R_t^L \ell_t(i) - W_t h_t(i)$$

Combining with the demand function from the final tradable producers (2.4), the profit can be written as follows:

$$\pi_t^D = Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}\frac{\sigma^D}{1+\sigma^D} \tag{2.10}$$

There are three things to note. First, the profit does not depend on the productivity level specific to the product line $a_t(i)$. In the appendix, we show that asset and labor inputs for each product line also do not depend on the productivity. This property enables us to study the aggregate dynamics of the economy without keeping track of heterogeneous productivity levels across product lines. Second, the Lagrange multiplier on the borrowing constraint μ_t affects the profit. In particular, when the borrowing constraint binds, μ_t is strictly positive and the profit declines. This is because when the borrowing constraint binds, the final tradable producer reduces demand for intermediate goods. Lower demand then translates into lower profits for intermediate firms. Third, factor prices do not affect profits. The reason is as follows. When domestic factor prices become cheaper, production cost for the producing firm goes down. But at the same time, the optimal price goes down because the marginal cost for the rival firms also goes down. These cancel out each other and thus the profit is not affected by a change in domestic factor prices. This makes a clear contrast to the case of exports discussed below.

(X) Exporting lines

For exporting lines, domestic firms sell their products not only domestically but also in the foreign market. Price setting and profit in the domestic market are essentially the same as domestic lines, except that the producing firms on exporting lines have larger productivity lead over the domestic rival firms, which is denoted by $1+\sigma^X$. The optimal price and profit in the domestic market are accordingly given as follows:

$$\widetilde{MC}_t^X(i) = \frac{1}{a_t(i)/(1+\sigma^X)}\overline{\alpha}\left(R_t^L\right)^{\alpha}\left(W_t\right)^{1-\alpha} = p_t^X(i) \tag{2.11}$$

$$\pi_t^X = Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}\frac{\sigma^X}{1+\sigma^X} \tag{2.12}$$

Because the productivity lead is larger, a profit is also larger.

In the foreign market, foreign final tradable producers demand intermediate goods. To specify foreign demand, we assume the following production function by foreign countries:

$$Y_t^* = \exp\left[\int_0^1 \ln y_t^*(i)di\right]$$

Foreign production is not subject to any shocks, and Y_t^* grows at a constant rate. Then foreign demand for each type of good is given by:

$$p_t^*(i) = \frac{Y_t^*}{y_t^*(i)} \tag{2.13}$$

This is again a unit-elastic demand function, thus the same logic holds and the firm with the lowest marginal cost sets the price equal to the marginal cost for the second-best rival firms. In the foreign market, the second-best rival firms are foreign firms. Foreign firms use foreign assets and labor, therefore their marginal cost is given as follows:

$$\widetilde{MC}_t^*(i) = \frac{1}{a_t(i)/(1+\sigma^X)} \overline{\alpha} \left(R_t^{L*}\right)^{\alpha} \left(W_t^*\right)^{1-\alpha} = p_t^*(i) \tag{2.14}$$

This is the price that domestic exporting firms set in the foreign market. Domestic exporters are subject to an iceberg cost, and need to ship $1+\xi$ units of goods to export 1 unit of goods. Then, combining with the demand function (2.13), the profit from exporting can be written as follows:

$$\pi_t^* = Y_t^* \left(1 - \frac{1+\xi}{1+\sigma^X} \frac{\left(R_t^L\right)^{\alpha} \left(W_t\right)^{1-\alpha}}{\left(R_t^{L*}\right)^{\alpha} \left(W_t^*\right)^{1-\alpha}} \right) \tag{2.15}$$

The key difference from a domestic profit is that a profit from exporting depends on factor prices. In particular, when domestic factor prices R_t^L and W_t decline, the profit becomes higher. This is because a decline in domestic factor prices makes domestic production cheaper, while the price is still determined by the foreign factor prices. Therefore in this case the profit goes up.

(M) Importing lines

For importing lines, the final tradable producer imports intermediate goods from foreign firms. Recall that demand by the final tradable producer is unit-elastic given by (2.4). This implies that the payment to foreign firms is independent of the price and always given as follows:

$$p_t(i)y_t(i) = \frac{Y_t^T}{1 + \phi\mu_t/\lambda_t} \tag{2.16}$$

But we still need to know the price $p_t(i)$, because the price determines the amount of imports $y_t(i)$, which affects production of final tradable goods Y_t^T. As discussed above, the price is equal to the marginal cost for the second-best rival firm, which is a domestic firm in this case. As shown in the next section, we assume that the productivity lead by foreign firms is the same as domestic exporting firms, $1 + \sigma^X$. Then the optimal price is exactly the same as the case for exporting lines, which is given by (2.11). Note that in equation (2.16), $Y_t^T / (1 + \phi\mu_t/\lambda_t)$ is given and $p_t(i)$ is the same as case for exporting lines. This means that output $y_t(i)$ is also the same as exporting lines.[25]

This completes the exposition of static production and profits for intermediate firms. Table 2.1 summarizes three product lines. Next we turn to the firm dynamics and explain how productivity increases over time along with imports and exports of intermediate goods.

2.3.3 Innovation and Firm Dynamics

Each firm is a collection of product lines for which the firm has the highest productivity. Firm dynamics are characterized by new firm entry, innovation by incumbent firms, and

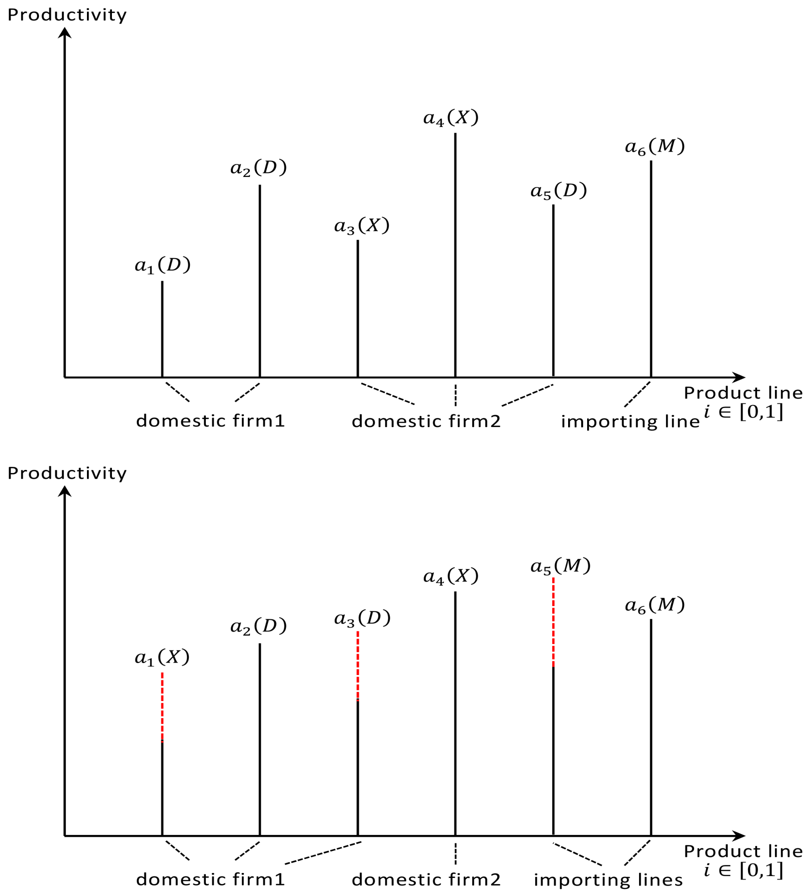
[25] The profit can be different between the domestic exporting firms and foreign firms, because factor prices are different and foreign firms are subject to an iceberg cost. But profits for foreign firms are out of our interest.

Table 2.1: Three Types of Product Lines

type (symbol)	who produces	second-best rival	profit
Domestic (D)	domestic firm	domestic firm	$Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^D}{1+\sigma^D}$
Exporting (X)	domestic firm	domestic firm (domestic) foreign firm (export)	$Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^X}{1+\sigma^X}$ $Y_t^* \left(1 - \frac{1+\xi}{1+\sigma^X} \frac{\left(R_t^L\right)^\alpha (W_t)^{1-\alpha}}{\left(R_t^{L*}\right)^\alpha (W_t^*)^{1-\alpha}}\right)$
Importing (M)	foreign firm	domestic firm	not relevant

innovation by foreign firms. Figure 2.2 illustrates an example of the evolution of firms from one period to the next. In the top figure, domestic firm 1 produces two product lines, both of which are domestic lines. Domestic firm 2 produces three product lines, of which the first two are exporting and the last line is domestic. There is also one foreign product line. In the model, domestic firms invest in two types of innovation: innovation to acquire new domestic product lines, and innovation to make their own domestic product line to be an exporting line. The bottom figure shows one possible case in the next period. In this example, domestic firm 1 succeeds in an exporting innovation on product line 1, and now this product line is exporting with $1 + \sigma^X$ productivity lead over other firms. For product line 3, domestic firm 1 succeeds in a domestic innovation and acquires this line. A domestic innovation improves productivity of the line by a factor of $1 + \sigma^D$. Foreign innovation happens on any product line with an exogenous probability i^F. In this figure, foreign innovation happens on product line 5, and domestic firm 2 loses this product line. Foreign innovation improves productivity by a factor of $1+\sigma^X$. There is also new domestic firm entry, not depicted in this figure. New domestic firm entry can happen on any product line, and a new firm starts with a single domestic product line with $1 + \sigma^D$ productivity lead.

Figure 2.2: Firm Dynamics



Through entry and innovations, aggregate productivity of the intermediate sector increases over time. In addition, these firm dynamics endogenously determine which product lines are imported and exported. Firm dynamics change the status of each product line over time and determine the extensive margins of imports and exports. Next we explain the innovation decisions by firms.

Innovation by Domestic Incumbent Firms Let us consider a domestic firm that produces n^D domestic product lines and n^X exporting product lines. We model innovations by firms in the following way.

(1) Domestic innovation

A firm with n^D domestic lines and n^X exporting lines has $n^D + n^X$ domestic innovation opportunities in total. For each innovation opportunity, a firm invests Z_t^D units of final tradable goods to make a domestic innovation to acquire new domestic product lines. The idea behind this assumption is that a domestic innovation is a spin-off from the existing technologies. Success in each innovation gives one new domestic product line. This implies that a firm with n^D domestic lines and n^X exporting lines can acquire $n^D + n^X$ new domestic lines at most. The success probability of a domestic innovation is given as follows:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t} \right)^{1/\rho} \tag{2.17}$$

where A_t is the average productivity of intermediate firms including foreign firms. This functional form for innovation is consistent with Akcigit and Kerr (2015), and shown to be empirically supported. A firm with n^D domestic lines and n^X exporting lines invest $(n^D + n^X)Z_t^D$ units of final tradable goods in total.

(2) Exporting innovation

For each domestic product line, a firm invests Z_t^X units of final tradable goods to make an exporting innovation. The success probability of an exporting innovation is given by the following function:

$$i_t^X = \eta^X \left(\frac{Z_t^X}{A_t} \right)^{1/\rho} \tag{2.18}$$

When a firm succeeds in an exporting innovation on their own domestic line, this line becomes an exporting line and the firm can export this type of good to the foreign market.

As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model by the structure that the operating profit is independent of the productivity level. Undirected innovations by many firms imply that each product line faces the same replacement probability. Let d_t denote the replacement probability, and define as $P(i,n,p)$ the probability of having i successes in n trials for a binomial process with success probability p. Namely,

$$P(i,n,p) = \binom{n}{i} p^i (1-p)^{1-i}$$

The value of a domestic firm with n^D domestic lines and n^X exporting lines can be written in a recursive form as follows:

$$\begin{aligned} V_t(n^D,n^X) = \max_{Z_t^D,Z_t^X} \{ & n^D\pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D+n^X)Z_t^D - n^D Z_t^X \\ & + \sum_{i=0}^{n^D+n^X} P(i,n^D+n^X,i_t^D) \sum_{j=0}^{n^D} P(j,n^D,d_t) \sum_{k=0}^{n^D-j} P(k,n^D-j,(1-d_t)i_t^X) \sum_{m=0}^{n^X} P(m,n^X,d_t) \\ & E_t\left[\Lambda_{t,t+1}V_{t+1}(n^D+i-j-k,n^X+k-m)\right]\} \end{aligned}$$

The first line is the operating profits minus the innovation investment cost. The second and third line add up the expected value of a firm across all the possible combinations of innovations and replacement on n^D domestic lines and n^X exporting lines in the next period. The first summation adds up across all the possibilities for domestic innovations from 0 success to $n^D + n^X$ successes. The second summation adds up over the number of domestic lines being replaced from 0 to n^D. The third summation adds up over the number of successful exporting innovations. We assume that exporting innovations materialize only if replacement does not happen on that line. Thus the effective success probability is given by $(1-d_t)i_t^X$. The last summation adds up over the number of exporting lines being replaced from 0 to n^X.

Following Ates and Saffie (2016), we use a guess-and-verify method to show that the value of a firm with n^D domestic lines and n^X exporting lines is equal to the sum of n^D times the value of a single domestic line and n^X times the value of a single exporting line:

$$V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$$

The formal proof is left to the Appendix. This linear relation enables us to aggregate the firm dynamics in a tractable way and study how the firm dynamics affect endogenous growth and the extensive margins of imports and exports, without keeping track of the firm size distribution. The value of a single domestic line is given by:

$$\begin{aligned} V_t(1,0) \quad = \quad & \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\ & \left. + \left(i_t^D + (1-d_t)(1-i_t^X)\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\} \end{aligned} \tag{2.19}$$

and the value of a single exporting product line is:

$$V_t(0,1) = \max_{Z_t^D} \left\{ \pi_t^X + \pi_t^* - Z_t^D + i_t^D E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\} \tag{2.20}$$

Taking into account equation (2.17), the first-order condition with respect to Z_t^D gives the optimal condition for domestic innovation:

$$\eta^D \frac{1}{\rho} \left(\frac{Z_t^D}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1 \tag{2.21}$$

and the first-order condition regarding exporting innovation is:

$$(1-d_t)\eta^X \frac{1}{\rho} \left(\frac{Z_t^X}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} \left(E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] - E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] \right) = 1 \tag{2.22}$$

Assuming $\rho > 1$, the probability of successful innovation is increasing and concave in investment. Note also that investment is forward-looking in the sense that as the expected value of a product line increases, firms invest more in innovation.

Domestic Entry Entry of new domestic firms comes from innovation by households. New domestic firms poach a product line from incumbent firms, and start with a single domestic line. Households invest Z_t^E units of final tradable goods to create new firms. The number of firms created from Z_t^E units of investment is given by:

$$e_t = \eta^E \left(\frac{Z_t^E}{A_t} \right)^{1/\rho} \tag{2.23}$$

The optimal investment Z_t^E equates that the marginal benefit of investment to the marginal cost, so that the following holds:

$$\eta^E \left(1 - \rho^E\right) \left(\frac{Z_t^E}{A_t} \right)^{-\rho^E} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1 \tag{2.24}$$

Productivity Growth and Extensive Margins of Trade We now characterize how firm dynamics translate into macroeconomic dynamics, specifically productivity growth and the extensive margins of trade. We denote the share of domestic lines by θ_t^D, and the share of exporting lines by θ_t^X. Then the share of importing product lines is given by $1-\theta_t^D-\theta_t^X$. First, replacement of a product line happens for three reasons: domestic entry, domestic innovations, and foreign innovations. Thus the replacement rate d_t is the sum of these three probabilities:

$$d_t = (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D + e_t + i^F \tag{2.25}$$

Note that the successful innovation probability by incumbent firms i_t^D is multiplied by the share of domestically-owned product lines, a sum of domestic lines and exporting lines. Next we derive the transition equation of the share of domestic lines θ_t^D. θ_t^D increases by new domestic firm entry and domestic innovations by incumbent firms. θ_t^D decreases by exporting innovations and foreign innovations. The transition of θ_t^D is thus given by the following law of motion:

$$\theta_t^D = \theta_{t-1}^D + (1-\theta_{t-1}^D)(e_t + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D) - \theta_{t-1}^D((1-d_t) i_t^X + i^F) \tag{2.26}$$

The transition equation of the share of exporting lines θ_t^X is derived in a similar way. θ_t^X increases by exporting innovations, and decreases by any of domestic entry, domestic innovations, and foreign innovations. The transition of θ_t^X is thus characterized by the following equation:

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D (1-d_t) i_t^X - \theta_{t-1}^X (e_t + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D + i^F) \tag{2.27}$$

The share of foreign lines are then given by $1-\theta_t^D-\theta_t^X$. As discussed above, this country imports foreign intermediate goods. Therefore, endogenous variations in the share of foreign product lines correspond to the extensive margin of imports. And endogenous variations in the shares of exporting lines determine the extensive margin of exports.

Next we derive the expressions for aggregate production and productivity growth. Aggregate production of final tradable goods can be written as follows:

$$Y_t^T = \exp\left[\int_0^1 \ln y_t(i)di\right] = A_t\left[\left(\ell_t^D\right)^\alpha\left(h_t^D\right)^{1-\alpha}\right]^{\theta_{t-1}^D}\left[\left(\ell_t^X\right)^\alpha\left(h_t^X\right)^{1-\alpha}\right]^{\theta_{t-1}^X}\left[\frac{1}{1+\xi}\left(\ell_t^M\right)^\alpha\left(h_t^M\right)^{1-\alpha}\right]^{1-\theta_{t-1}^D-\theta_{t-1}^X}$$

where $\ell_t^D, \ell_t^X, \ell_t^M$ are assets used by each product line, and h_t^D, h_t^X, h_t^M are labor hired by each product line. Note that ℓ_t^M and h_t^M are factors in foreign countries. Expressions for these variables are left to the appendix. A_t is the average productivity of intermediate firms, given by:

$$A_t = \exp\left[\int_0^1 \ln a_t(i)di\right]$$

A_t grows as productivity of each product line $a_t(i)$ improves through domestic firm entry, incumbent innovations, and foreign innovations.[26] The growth rate of A_t is thus characterized by the following equation:

$$\frac{A_{t+1}}{A_t} = 1+g_t = (1+\sigma^D)^{e_t+(\theta_{t-1}^D+\theta_{t-1}^X)i_t^D}(1+\sigma^X)^{\theta_{t-1}^D(1-d_t)i_t^X}(1+\sigma^X)^{i^F} \tag{2.28}$$

The three terms in the right-hand side correspond respectively to the sum of new firm entry and domestic innovations, exporting innovations, and foreign innovations. This completes

[26]Note that A_t is not necessarily the productivity level of this economy, because A_t includes prouctivity of foreign firms. But the long-run growth rate of this economy is determined by growth in A_t.

the characterization of firm dynamics and its effect on aggregate productivity growth and the extensive margins of imports and exports.

Firm-Level Distribution We model the firm and trade dynamics so that it is tractable enough to solve numerically with an occasionally binding borrowing constraint. But at the same time, the model structure allows us to have the firm-level moments such as the size distribution and the share of exporting firms, at least at the balanced growth path. This section sketches some of the model features.

In our model, each firm is characterized by the number of domestic and exporting lines it owns, (n^D, n^X). Each domestic and exporting line has the same output, profit, and factor inputs respectively. Therefore, it is easy to compute output, profit, and factor inputs for firms with different sizes. In addition, we can derive the whole distribution of the firm size in terms of the number of product lines. The detailed process is left to the appendix, but basically we iterate the laws of motion for firm size until the firm size distribution converges. Using the firm size distribution, we can compute the firm-level moments such as the mean and standard deviations of output, profit, and factor inputs. We can also derive these statistics for exporters and non-exporters separately, and compute the relative mean size between exporters and non-exporters. We plan to use these firm-level moments for calibration in future research.

2.3.4 Non-Tradable Goods Producer

The non-tradable firm hires labor from households and produces non-tradable goods. The production function is given as follows:

$$Y_t^N = A_t (H_t^N)^{1-\alpha^N} \tag{2.29}$$

where $0 < 1 - \alpha^N < 1$ is the labor share in non-tradable goods production. We assume that total factor productivity in non-tradable goods production increases at the same rate as aggregate productivity in the tradable sector. This spillover guarantees that production of tradable goods and non-tradable goods will grow at the same rate in the long run. Let P_t^N denote the non-tradable goods price. Since the law of one price holds for tradable goods between this country and the rest of the world, the non-tradable goods price P_t^N determines the real exchange rate of this country. Thus we call P_t^N the real exchange rate, and an increase in P_t^N corresponds to real appreciation. The first-order condition of the non-tradable goods producer is simply given by:

$$W_t = P_t^N A_t (1 - \alpha^N)(H_t^N)^{-\alpha^N} \tag{2.30}$$

Because labor is mobile between the tradable and non-tradable sectors, the real wage W_t is common in both sectors. Using P_t^N and W_t the profit for the non-tradable firm is given by:

$$\Pi_t^N = P_t^N Y_t^N - W_t H_t^N \tag{2.31}$$

which is paid to households.

2.3.5 Household

The representative household consumes tradable goods C_t^T and non-tradable goods C_t^N, supplies labor H_t elastically, and invests Z_t^E units of final tradable goods in domestic entry. They receive the wage income W_tL_t and profits from tradable goods producers Π_t^T, non-tradable goods producers Π_t^N, and domestic intermediate firms $\theta_{t-1}^D\left(\pi_t^D - Z_t^D - Z_t^X\right) + \theta_{t-1}^X\left(\pi_t^X + \pi_t^* - Z_t^D\right)$. The representative household's optimization problem is then given as follows:

$$\max_{\{C_t^T, C_t^N, H_t, Z_t^E\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left[\ln\left(C_t - A_t\frac{(H_t)^{\omega}}{\omega}\right)\right]$$

$$C_t = \left[(\gamma)^{1/\varepsilon}(C_t^T)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{1/\varepsilon}(C_t^N)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.32}$$

subject to

$$C_t^T + P_t^N C_t^N + Z_t^E = W_tH_t + \Pi_t^T + \Pi_t^N + \theta_{t-1}^D\left(\pi_t^D - Z_t^D - Z_t^X\right) + \theta_{t-1}^X\left(\pi_t^X + \pi_t^* - Z_t^D\right) \tag{2.33}$$

where γ is a parameter to determine the weight of tradable goods in composite consumption C_t, and ε is the constant elasticity of substitution between tradable and non-tradable consumption. Optimal investment in domestic entry Z_t^E is determined by equation (2.24). The first-order conditions for the rest of the choice variables can be summarized as follows:

$$\frac{C_t^T}{C_t^N} = \frac{\gamma}{1-\gamma}(P_t^N)^{\varepsilon} \tag{2.34}$$

$$A_t(H_t)^{\omega-1} = W_t \left(\gamma \frac{C_t}{C_t^T}\right)^{1/\varepsilon} \tag{2.35}$$

The marginal utility of tradable consumption used to discount the future profits of the final tradable sector is:

$$\lambda_t = \frac{1}{C_t - (H_t)^{\omega}/\omega} \left(\gamma \frac{C_t}{C_t^T}\right)^{1/\varepsilon} \tag{2.36}$$

Then the stochastic discount factor $\Lambda_{t,t+1}$ is given by $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. Equation (2.34) relates the optimal ratio of tradable and non-tradable consumption to the real exchange rate. Equation (2.35) gives the optimal labor supply H_t.

2.3.6 Market Clearing Conditions

To close the model, this subsection lists the market clearing conditions. The asset market, labor market, and non-tradable goods market clearing conditions are as follows:

$$1 = \theta_{t-1}^D \ell_t^D + \theta_{t-1}^X (\ell_t^X + \ell_t^*) \tag{2.37}$$

$$H_t = \theta_{t-1}^D h_t^D + \theta_{t-1}^X (h_t^X + h_t^*) + H_t^N \tag{2.38}$$

$$Y_t^N = C_t^N \tag{2.39}$$

Finally, the trade balance and the current account of this economy are given as follows:

$$\begin{aligned} TB_t \;=\; & \underbrace{Y_t^T - C_t^T - Z_t^E - \theta_{t-1}^D \left(Z_t^D + Z_t^X\right) - \theta_{t-1}^X Z_t^D}_{\text{final tradable output - absorption}} \\ & + \underbrace{\left(\theta_{t-1}^D + \theta_{t-1}^X\right) Y_t^*}_{\text{export of intermediate goods}} - \underbrace{\left(1 - \theta_{t-1}^D - \theta_{t-1}^X\right) \frac{Y_t^T}{1 + \phi\mu_t/\lambda_t}}_{\text{import of intermediate goods}} \end{aligned}$$

$$CA_t = TB_t + \left(\exp(\varepsilon^R_{t-1})R - 1\right) B_{t-1} = B_t - B_{t-1}$$

This completes the exposition of the model economy. The appendix formally defines the equilibrium of the model economy and the stationarized equilibrium conditions that we use to solve the model numerically.

2.4 Quantitative Analysis

This section calibrates the model parameters, demonstrates the baseline simulation, and discusses the model features. We solve the model numerically by two steps: First we divide the equilibrium conditions by the productivity level A_t to stationarize the equations. In the second step we solve the stationarized model globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint. The stationarized equilibrium conditions and the details of the solution procedure are left to the appendix.

2.4.1 Calibration

One period in the model is meant to be annual. There are 17 parameters to be determined in the model. We take conventional values from the literature if available, and calibrate the other parameters to target the Brazilian economy as a benchmark. Table 2.2 presents 12 externally-determined parameter values. The discount factor $\beta = 0.96$ and interest rate on foreign bonds $R = 1.04$ are standard values for annual models. The weight on tradable goods in consumption $\gamma = 0.31$ is set following Bianchi (2011). The elasticity of substitution between tradable and non-tradable goods in consumption, $\varepsilon = 0.6$, is in the middle of the

Table 2.2: Externally-Determined Parameters

Variable		Value	Source
β	Discount factor	0.96	Standard
R	Foreign bond interest rate	1.04	Standard
γ	Tradable share in consumption	0.31	Bianchi (2011)
ε	CES between T and NT	0.6	Middle value in literature
ω	Frisch elasticity $1/(\omega-1)$	1.455	Mendoza (1991)
α	Asset share in tradable	0.3	Standard
$1-\alpha^N$	Labor share in non-tradable	0.75	Schmitt-Grohe and Uribe (2016)
ξ	iceberg cost	0.21	Anderson and van Wincoop (2004)
ϕ	Fraction of input subject to WK	0.4	Middle value in literature
κ	Coefficient on borrowing constraint	0.14	Mendoza (2010)
ρ	Concavity of innovation investment	1.5	Middle value in literature
i^F	Foreign innovation rate	0.01	Domestic entry rate

range discussed in Mendoza (2005). The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza (1991). Regarding the production parameters, asset's share in tradable production $\alpha = 0.3$ is a standard value, and labor's share in non-tradable production $1-\alpha^N = 0.75$ is taken from Schmitt-Grohé and Uribe (2016). The iceberg cost of exports $\xi = 0.21$ follows the estimation by Anderson and van Wincoop (2004). The fraction of the input cost subject to the working capital requirement ϕ varies widely depending on how it is estimated. We set its value to 0.4, which is in the middle of the range in the literature such as Mendoza (2010) and Ates and Saffie (2016). The coefficient on the borrowing constraint κ is set to 0.14, which is close to 0.2 assumed in Mendoza (2010). The concavity parameter governing investment ρ is set to 1.5, which is the middle value in the literature such as Comin and Gertler (2006), Akcigit and Kerr (2015), and their literature review. The foreign innovation rate is set to 0.01, which is somewhat arbitrarily set to a low value so that the economy grows mainly through domestic innovation.Five parameters related to firm

Table 2.3: Jointly-Determined Parameters

Variable		Value	Target	Model
η^E	Domestic entry coeff.	1.30	GDP growth rate 2.5%	2.5%
η^D	Domestic innovation coeff.	0.71	Import/GDP ratio 12.8%	12.8%
η^X	Exporting innovation coeff.	0.68	Export/GDP ratio 12.8%	12.8%
σ^D	Domestic innovation size	0.11	R&D expenditure/GDP 1.0%	3.2%
σ^X	Exporting innovation size	0.47	Labor share in tradable 21.7%	27.8%

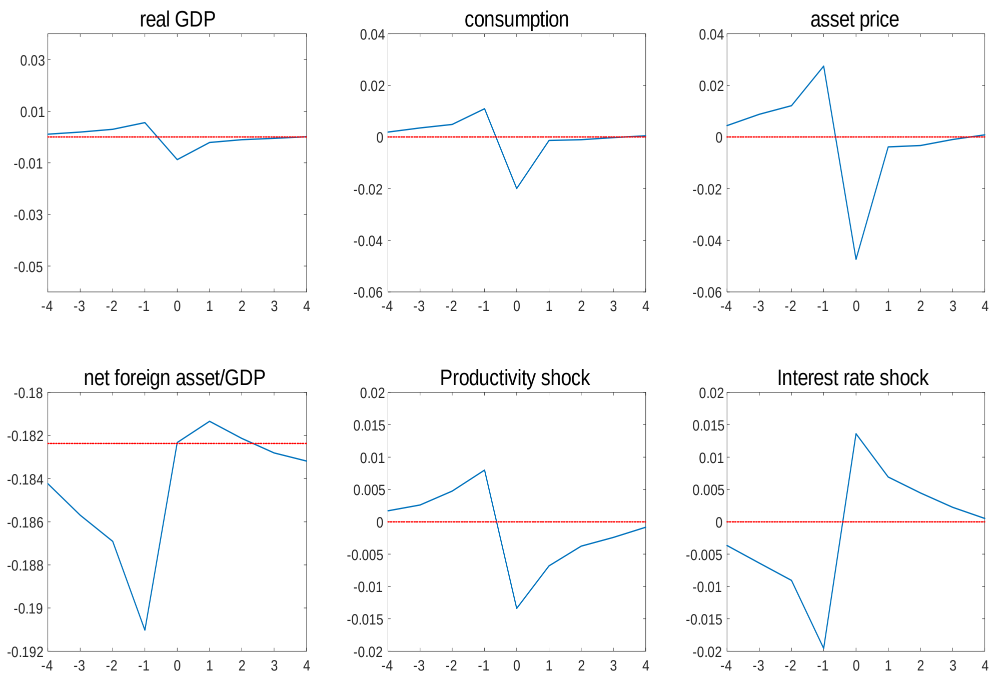
dynamics and growth, $\eta^E, \eta^D, \eta^X, \sigma^D, \sigma^X$, are jointly determined to match five moments at the balanced growth path of the model with Brazilian data in 2001-2011. The five targeted moments are the GDP growth rate, the import-to-GDP ratio, the export-to-GDP ratio, the R&D expenditure-to-GDP ratio, and the labor share in the tradable sector. Parameter values and targeted moments are listed in Table 2.3.The aggregate shocks to the economy affect the productivity of the final tradable sector ε_t^A and the interest rate on the foreign bond ε_t^R. We take the stochastic process for these shocks from Mendoza (2010), in which ε_t^A and ε_t^R follow a joint discrete Markov process with two realizations for each variable. In particular, ε_t^A takes ± 0.0134 and ε_t^R takes ± 0.0196 with the same autocorrelation 0.59 and the negative correlation -0.67 between ε_t^A and ε_t^R. We set Y_t^* so that the total revenue for exporting product lines is 2.1 times larger than that for domestic product lines at the balanced growth path. Alfaro, Chari, and Kanczuk (2017) report that the revenue for exporting firms is 2.1 times larger than that for non-exporting firms. As discussed above, each firm in our model can have several product lines, and each product line is actually not corresponding to each firm. We plan to use the firm-level moments for calibration in the next draft. Finally, foreign production cost R_t^{L*} and W_t^* are set to the domestic values R_t^L and W_t at the balanced growth path, taking into account that this country and foreign countries have a close productivity level.

2.4.2 Sudden Stop Dynamics

This section presents sudden stop dynamics in the model. We pick all sudden stop events from the 9,000-period simulation described above. Following Bianchi and Mendoza (2018), sudden stops are identified as events in which the current account adjusted for its trend is at least two standard deviations above its mean. With this definition of sudden stops, the unconditional probability of sudden stops in the model is 7.9%, which is in line with empirical estimations in Eichengreen, Gupta, and Mody (2006) and Jeanne and Rancière (2011). Figure 2.3 plots the average dynamics of the key macro variables before and after sudden stops in the simulation. The three panels in the top row show log deviations of real GDP, consumption, and the asset price from their linear trends.[27] On average, GDP drops by 1%, consumption drops by 2%, and the asset price drops by 5% when a sudden stop happens. The sharp drop in the asset price indicates that the borrowing limit substantially tightens during sudden stops, and an amplification effect sets in motion as in Mendoza (2010) and Bianchi and Mendoza (2018). The net foreign asset-to-GDP ratio in the left panel of the bottom row has a sharp spike, which indicates a sudden reversal of capital inflows. The last two panels show the average development of productivity and interest rate shocks that cause sudden stops. Before sudden stops happen, productivity is high and the interest rate is low, implying that the country is enjoying favorable shocks. During this period, the country increases foreign borrowing and the bond holdings-to-GDP ratio is below its mean. When these favorable shocks reverse to bad shocks of low productivity and a high interest rate, the

[27] Linear trend is created by taking log of 20-period series around each sudden stop (10 periods before and after sudden stop respectively) and taking a linear trend of this log series.

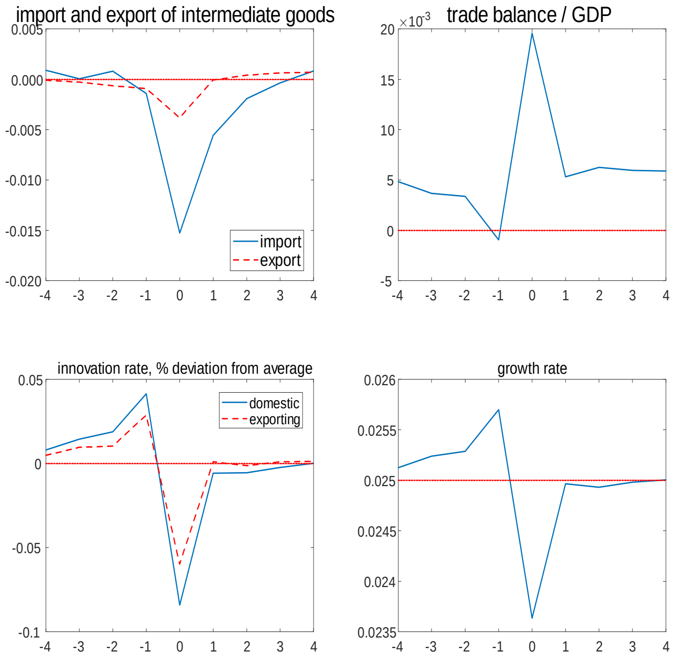
Figure 2.3: Sudden Stop Dynamics, Macro Variables



asset price declines and forces the borrowing constraint to bind. Then households are forced to cut consumption, which reduces the asset price further, and the amplification mechanism sets in motion. These developments of exogenous shocks and the subsequent endogenous dynamics are all consistent with Mendoza (2010).

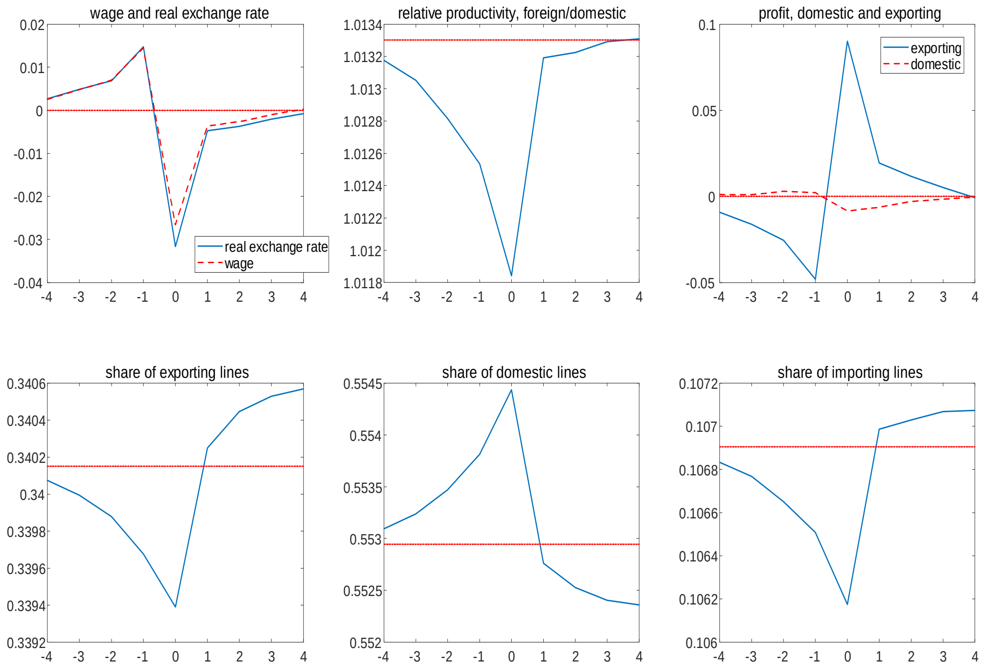
Next we examine the trade and growth dynamics, which are the novel features of our model. Figure 2.4 plots the average dynamics of the key variables around sudden stops in the model simulation. The first panel presents the average dynamics of imports and exports of intermediate goods. We observe that imports decline much more than exports and stay below the trend persistently, while exports are almost unaffected. A decline in imports occurs because the final tradable producer in this country is constrained by the borrowing limit and cannot buy as many intermediate goods as it wants, which reduces demand for intermediate

Figure 2.4: Sudden Stop Dynamics, Trade and Growth



goods. In contrast, foreign demand is not affected by sudden stops in this country, so that exports are unaffected. This clear contrast in import and export dynamics during crises is at least qualitatively consistent with the empirical facts documented in Alessandria, Kaboski, and Midrigan (2010) and Alessandria, Pratap, and Yue (2015). The second panel shows that the trade balance-to-GDP ratio improves in sudden stops, which is also in line with the empirical fact. The bottom two panels show firms' innovation and growth. The left panel shows that domestic innovation rate drops by 8% and exporting innovation rate drops by 6% in sudden stops. As a result, productivity growth rate drops sharply, which has a persistent effect on the economy. This persistent negative effect on the economy is consistent with empirical findings by Cerra and Saxena (2008) and Blanchard, Cerutti, and Summers (2015).

Figure 2.5: Sudden Stop Dynamics, Trade Margins



Next we show how the extensive margins of trade react in sudden stops. Figure 2.5 plots the key variables regarding the extensive margins of trade. The first panel shows that both the real exchange rate and the domestic wage drop by 3% in sudden stops. A sharp real depreciation is observed during sudden stops in the data as reported in Mendoza (2005). The real depreciation and cheaper wage result in a decline in the relative marginal cost of production for domestic firms compared to foreign firms. The second panel shows the relative productivity of the foreign economy compared to the domestic economy. Since productivity growth rate of the domestic economy drops in sudden stops, relative foreign productivity jumps up during sudden stops, which implies larger foreign demand for exports. Because of the decline in the relative marginal cost and larger foreign demand, profits from export jump up by 9% above the trend as shown in the third panel. In contrast, a profit in the domestic market does not react much. This large gap between the domestic and exporting profits in turn affects firms' investment decision. As shown in the third panel of Figure 2.4, both of the domestic and exporting innovation rates drop in sudden stops, but the exporting innovation rate drops less than the domestic innovation rate. This is because the profit from export is expected to be higher than its trend after sudden stops, thus firms invest more in exporting innovation compared to domestic innovation. These endogenous responses by firms determine how the extensive margins of trades develop when a sudden stop happens. The left panel in the bottom row of Figure 2.5 shows that the share of exporting lines gradually increases after sudden stops, which is an expansion of the extensive margin of exports. This occurs because profits from export are persistently higher after sudden stops, and firms invest more in exporting innovation. This gradual expansion of the extensive margin of exports is at least qualitatively in line with the empirical fact reported in Alessandria, Pratap, and Yue (2015). The middle panel shows that the share of domestic product lines declines in sudden stops. This occurs because the domestic innovation rate drops. The last panel shows that the extensive margin of imports jumps up after sudden stops. Whether the extensive margin of imports expands or shrinks in this model depends on the relative size of the changes in the share of domestic lines and exporting lines. In this simulation a decline in the share of domestic lines is larger than an increase in the share of exporting lines, thus the share of importing lines increases. This expansion of the extensive margin of imports in crisis seems not consistent with the empirical fact, and we plan to improve it.

2.5 Conclusion

This chapter develops a quantitative small-open-economy model with endogenous firm and trade dynamics. Firms engage in two types of innovations, innovations to acquire new product lines, and innovations to start exporting their products. These innovation activities determine the extensive margins of imports and exports endogenously. The economy is also subject to sudden stops of capital inflows in the form of an occasionally binding borrowing constraint. We show that the model can capture the aggregate dynamics of output, consumption, asset prices, and foreign debt quantitatively well. More importantly, firm and trade dynamics of the model during sudden stops are in line with some of the empirical regularities: firms' innovation rate drops sharply, which causes a persistent decline in productivity and output; imports of goods decline substantially, while exports are almost unaffected; profits for exports increase due to a large real depreciation and lower production costs; the extensive margin of exports expand gradually after sudden stops. The model provides a tractable framework to study optimal capital control policies in the context of endogenous firm and trade dynamics. Because capital policies affect firm and trade dynamics and growth, it is important to take these elements into account in designing optimal policies. We are planning to pursue this direction for future research.

Appendix to Chapter 1

Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the model economy and the stationarized equilibrium.

Equilibrium

Definition: *The equilibrium of the model economy is defined by the initial states* A_0, $R_{-1}B_{-1}$, K_{-1}, θ_{-1}, F_{-1}, κ_{-1}, *the stochastic process* $\{\kappa_t\}_{t=0}^{\infty}$, *the government policy rules* $\{T_t, V_t\}_{t=0}^{\infty}$ *and the following:*

1. *Tradable goods producer: Given prices* $\{r_t, W_t, R_t\}_{t=0}^{\infty}$ *and the government policy rules* $\{T_t, V_t\}_{t=0}^{\infty}$, $\{K_t^D, M_t, B_t, I_t^M, Y_t^T, \Pi_t^T, \mu_t\}_{t=0}^{\infty}$ *satisfy (1.1), (1.3), (1.5), (1.7), (1.8). (1.9), (1.29).*

2. *Foreign intermediate goods producing firms: Given prices* $\{W_t\}_{t=0}^{\infty}$ *and tradable goods output* $\{Y_t^T\}_{t=0}^{\infty}$, $\{\ell_t^F, Z_t^F, \pi_t^F, i_t^F, V_t^F, \sigma_t^F\}_{t=0}^{\infty}$ *satisfy (1.13), (1.15), (1.16), (1.17), (1.18), (1.19).*

3. *Domestic intermediate goods producing firms: Given prices* $\{W_t\}_{t=0}^{\infty}$ *and tradable goods output* $\{Y_t^T\}_{t=0}^{\infty}$, $\{\ell_t^D, Z_t^D, \pi_t^D, i_t^D, V_t^D, \sigma_t^D\}_{t=0}^{\infty}$ *satisfy (1.12), (1.15), (1.16), (1.21), (1.23), (1.24).*

4. *Foreign investors:* $\{e_t^F, Q_t^F\}_{t=0}^{\infty}$ *satisfy (1.20) (1.22).*

5. *Non-tradable goods producer: Given prices* $\{W_t, P_t^N\}_{t=0}^{\infty}$, $\{Y_t^N, L_t^N, \Pi_t^N\}_{t=0}^{\infty}$ *satisfy (1.31), (1.32), (1.33).*

6. *Households: Given prices* $\{r_t, W_t, P_t^N\}_{t=0}^{\infty}$, $\{C_t, C_t^T, C_t^N, L_t, K_t, Z_t^E, I_t, e_t^E, \lambda_t\}_{t=0}^{\infty}$ *satisfy (1.25), (1.26), (1.34), (1.35), (1.36), (1.37), (1.38), (1.39), (1.40).*

7. *Foreign reserve:* $\{F_t\}_{t=0}^{\infty}$ *follows the transition equation given by (1.41).*

8. *Aggregate variables* $\{A_t, \theta_t, d_t\}_{t=0}^{\infty}$ *satisfy (1.27), (1.28), (1.30).*

9. *Prices* $\{r_t, W_t, P_t^N, R_t\}_{t=0}^{\infty}$ *and labor in tradable sector* $\{L_t^T\}_{t=0}^{\infty}$ *satisfy (1.10), (1.42), (1.43), (1.44), (1.45).*

Stationarized Equilibrium

To stationarize the model, I divide the equilibrium conditions by aggregate productivity A_t. I denote stationarized variables by the lower-case letters, and use g_t to denote the productivity growth rate A_{t+1}/A_t. I also make some arrangements and reduce the number of equations. The following is the complete list of equations to characterize the stationarized equilibrium of the model:

Tradable goods producers

$$y_t^T = \left(\frac{k_{t-1}}{1+g_{t-1}}\right)^{\alpha} (i_t^M)^{\theta}(m_t)^{1-\alpha-\theta}$$

$$i_t^M = L_t^T \frac{(1+\sigma_D)^{\theta_{t-1}}(1+\sigma_F)^{1-\theta_{t-1}}}{\theta_{t-1}(1+\sigma_D)+(1-\theta_{t-1})(1+\sigma_F)}$$

$$w_t = \frac{\theta_{t-1}(1+\sigma_D)+(1-\theta_{t-1})(1+\sigma_F)}{(1+\sigma_D)(1+\sigma_F)} \frac{\theta y_t^T}{L_t^T} \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$r_t^k = \alpha \frac{y_t^T}{k_{t-1}/(1+g_{t-1})}$$

$$(1-\alpha-\theta)\frac{y_t^T}{m_t} = P^M \left(1 + \phi \frac{\mu_t}{\lambda_t}\right)$$

$$1 - \frac{\mu_t}{\lambda_t} = \beta R_t E_t \left(\frac{\lambda_{t+1}}{\lambda_t(1+g_t)}\right)$$

$$R_t = \overline{R} + \psi_b \left(\exp\left(\frac{b_t}{gdp_t} - \overline{b}\right) - 1\right)$$

$$\mu_t \left[-b_t + \frac{\phi(1-\alpha)y_t^T}{1+\phi\mu_t/\lambda_t} - \kappa_t\right] = 0$$

Foreign intermediate goods producing firms

$$\sigma_t^F = \sigma^F \left(\frac{k_{ss}}{k_{t-1}}\right)^\rho$$

$$\pi_t^F = \frac{\sigma_t^F}{1+\sigma_t^F} \theta y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$v_t^F = \pi_t^F - z_t^F + \left[i_t^F + (1-d_t)\right](1+g_t)\frac{1}{R^F} E_t(v_{t+1}^F)$$

$$i_t^F = \eta^F (z_t^F)^{1-\rho^F}$$

$$\eta^F (1-\rho^F)(z_t^F)^{-\rho^F}(1+g_t)\frac{1}{R^F} E_t(v_{t+1}^D) = 1$$

Domestic intermediate goods producing firms

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}}\right)^\rho$$

$$\pi_t^D = \frac{\sigma_t^D}{1+\sigma_t^D} \theta y_t^T \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$v_t^D = \pi_t^D - z_t^D + \left[i_t^D + (1-d_t)\left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right)\right](1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) + (1-d_t)\frac{e_t^F}{1-\theta_{t-1}}q_t^F$$

$$i_t^D = \eta^D (z_t^D)^{1-\rho^D}$$

$$\eta^D(1-\rho^D)(z_t^D)^{-\rho^D}(1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) = 1$$

FDI entry

$$\frac{e_t^F}{1-\theta_{t-1}} = \chi^F\left[(1-\lambda)(1+g_t)\frac{1}{R^F}E_t(v_{t+1}^F) - c^F\right]$$

$$q_t^F = \lambda(1+g_t)\frac{1}{R^F}E_t(v_{t+1}^F)$$

Aggregate variables

$$d_t = e_t^D + (1-\theta_{t-1})i_t^D + \theta_{t-1}i_t^F$$

$$\theta_t = \theta_{t-1} + e_t^F - \theta_{t-1}e_t^D + (i_t^F - i_t^D)\theta_{t-1}(1-\theta_{t-1})$$

$$1+g_t = \left(\frac{1+\sigma_t^F}{1+\sigma_t^D}\right)^{e_t^F}(1+\sigma_t^D)^{e_t^D}(1+\sigma_t^F)^{(1-\theta_{t-1})i_t^D}(1+\sigma_t^F)^{\theta_{t-1}i_t^F}$$

Non-tradable goods producer

$$y_t^N = (L_t - L_t^T)^{1-\alpha^N}$$

$$w_t = P_t^N(1-\alpha^N)(L_t - L_t^T)^{-\alpha^N}$$

Households

$$c_t^T + b_t + k_t + z_t^E = y_t^T - P^M m_t - \theta_{t-1}\pi_t^F - (1-\theta_{t-1})z_t^D + (1-\delta)\frac{k_{t-1}}{1+g_{t-1}} + \frac{R_{t-1}b_{t-1}}{1+g_{t-1}} + e_t^F q_t^F - \tau_t - \psi_t^K$$

$$\frac{c_t^T}{y_t^N} = \frac{\gamma}{1-\gamma}(P_t^N)^{\varepsilon}$$

$$\psi\omega(L_t)^{\omega-1} = \frac{w_t}{c_t}\left(\gamma\frac{c_t}{c_t^T}\right)^{\frac{1}{\varepsilon}}$$

$$1+\psi_k\left(\frac{k_t(1+g_{t-1})}{k_{t-1}}-(1+\overline{g})\right) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t(1+g_t)}\left\{r_{t+1}+1-\delta-\frac{\psi_k}{2}\left((1+\overline{g})^2-\left(\frac{k_{t+1}(1+g_t)}{k_t}\right)^2\right)\right\}\right]$$

$$\lambda_t = \frac{1}{c_t}\left(\gamma\frac{c_t}{c_t^T}\right)^{1/\varepsilon}$$

Domestic firm entry

$$e_t^D = \eta^E(z_t^E)^{1-\rho^E}$$

$$\eta^E(1-\rho^E)(z_t^E)^{-\rho^E}(1+g_t)E_t(\Lambda_{t,t+1}v_{t+1}^D) = 1$$

Foreign reserve transition

$$f_t = R^F\frac{f_{t-1}}{1+g_{t-1}} + \tau_t - v_t$$

The stationarized equilibrium is characterized by 33 variables $\{y_t^T$, k_t, g_t, i_t^M, m_t, L_t^T, θ_t, w_t, μ_t, λ_t, r_t^k, σ_t^F, π_t^F, v_t^F, z_t^F, i_t^F, σ_t^D, π_t^D, v_t^D, z_t^D, i_t^D, e_t^F, q_t^F, d_t, R_t, y_t^N, L_t, P_t^N, c_t^T, b_t, z_t^E, e_t^D, $f_t\}_{t=0}^{\infty}$ and the above 33 equations, given the initial state $R_{-1}b_{-1}/(1+g_{-1})$, $k_{-1}/(1+g_{-1})$, θ_{-1}, $f_{-1}/(1+g_{-1})$, κ_{-1}, the government policy $\{\tau_t, v_t\}_{t=0}^{\infty}$, and the stochastic process $\{\kappa_t\}_{t=0}^{\infty}$.

Numerical Solution

In this section I sketch the numerical solution method and present the accuracy of the solution.

Solution Method

The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. I set the equally-spaced grid points for the endogenous state variables, foreign debt $R_{t-1}b_{t-1}/(1+g_{t-1})$, capital $k_{t-1}/(1+g_{t-1})$, share of product lines owned by foreign firms θ_{t-1}, and foreign reserve holdings $f_{t-1}/(1+g_{t-1})$. I set 31 grid points for debt, capital, and reserves. I set 5 grid points for the share of foreign product lines, since the decision rules are close to linear over this state variable. There are also 2 states for the borrowing limit κ_t.

2. For each grid point, I set the initial guess for 5 variables: b_t, z_t^D, z_t^F, L_t^T, and the right-hand side of the Euler equation with respect to capital (RHSEE).

3. For each grid point, I do the following:

    (a) I leave the 5 variables I have made guess for as unknown variables, and express all the other endogenous variables in terms of the state variables and 5 unknowns. In this process I first assume that the borrowing constraint is not binding and proceed. Later I check if the constraint is satisfied. If it is not satisfied, I recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the 5 variables.

    (b) Using multi-dimensional linear interpolation over the next-period state variables and the guess for the 5 variables (b_t, z_t^D, z_t^F, L_t^T, RHSEE), I compute all the endogenous variables next period. I then calculate all the forward-looking expectation terms, such as the right-hand side of the Euler equations and the value functions.

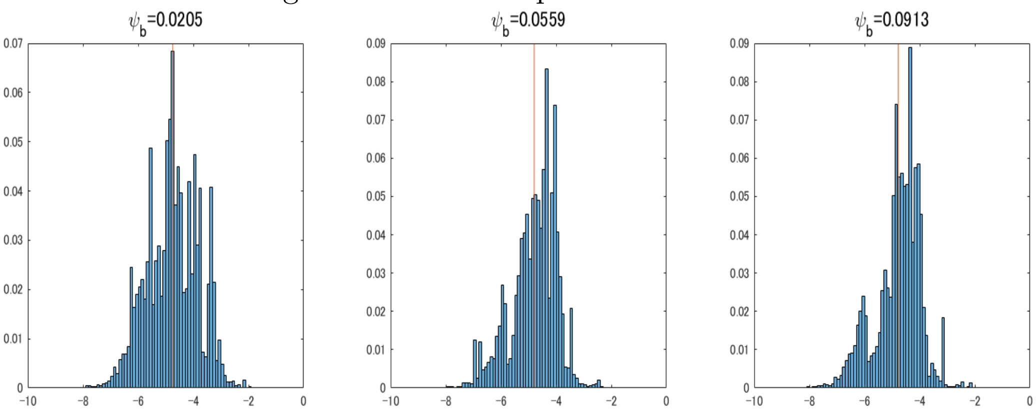
(c) All the equilibrium conditions are now the functions of the initial 5 unknowns. There are 4 equations I did not use in step (a), and the explicit expression for RHSEE, thus 5 equations in total. I solve for the 5 unknowns using non-linear solver.

4. I check the gap between the guess and the newly-obtained values for the 5 variables. If they are close enough, I stop. If not, I update the guess by the newly-obtained values, and go back to step 3. Repeat this process until the gap becomes sufficiently small.

Accuracy of Solution

Next I present the accuracy of the numerical solution obtained by the above method. Following Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), I compute the Euler equation error of the solution. I use the Euler equation with respect to foreign borrowing, because it is subject to the occasionally binding borrowing constraint, and thus likely to cause a larger error. For each value of ψ_b and τ, I simulate the model for 50 periods with the initial states used in the main analysis and stochastic shocks to the borrowing constraint. The reason why I stop simulation at period 50 is because the economy after period 50 follows a smooth path with no borrowing constraint binding, and thus errors are very small. I repeat this simulation 10,000 times. For each period t in each simulation i, I compute the Euler

Figure A.1: Euler Equation Error



equation error defined as follows:

$$error_{t,i} = \log_{10}\left[1 - \frac{c_{t,i}^{T,EE}}{c_{t,i}^{T}}\right]$$

where $c_{t,i}^{T}$ is tradable consumption computed directly from the decision rules, and $c_{t,i}^{T,EE}$ is computed by using the Euler equation with respect to foreign borrowing. Figure A.1 plots the distribution of the Euler equation errors obtained by this method. As a reference, I plot the distributions for the models with three different ψ_b that are used to study the effect of the debt-elasticity of the spread on the optimal policy, with the corresponding optimal τ. For each case, the average error is smaller than -4 and the maximum error is smaller than -2, which are reasonably small compared to the literature. For other models with different values of ψ_b and τ, the distributions of errors are similar.

Alternative Policy and Model

This section studies a bailout policy with an upper bound for the bailout size, and also the model with slow productivity spillover to the non-tradable sector as robustness checks.

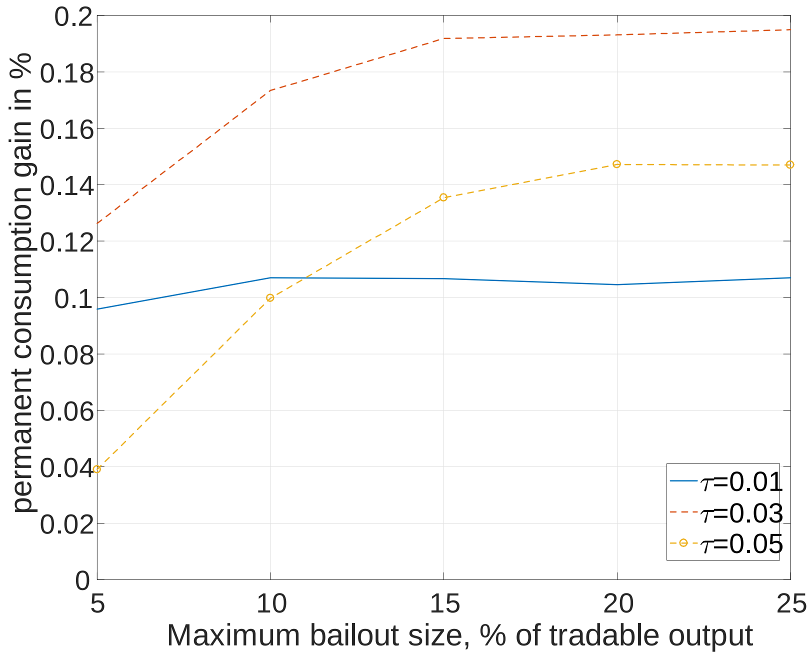
Bailout Policy with Upper Bound

For the bailout policy studied in the main text, the size of a bailout is dependent on the amount of private debt, thus there is a moral-hazard borrowing by private agents. This potentially makes reserve accumulation less effective in depreciating the real exchange rate and promoting growth. This section considers a bailout policy in which the size of a bailout is independent of the amount of private debt. Specifically, I introduce an upper bound for the size of a bailout in terms of a fixed fraction χ of tradable output Y_t^T. The size of a bailout $\widetilde{V}_t$ is then given as follows:

$$\widetilde{V}_t = \min\{V_t, \chi Y_t^T\}$$

where V_t is the size of a bailout for the baseline policy discussed in the main text. This bailout policy implies that as the private debt becomes larger, the size of a bailout hits the upper bound χY_t^T and is independent of the amount of the private debt. To compare with the baseline policy, $\widetilde{V}_t$ units of reserves are given to private agents upon bailouts. I try different values for χ with different tax rate τ to see how the optimal τ is affected by the upper bound χ. Figure A.2 presents the result. It shows that the higher upper bound monotonically gives higher welfare. It also shows that as χ becomes larger, the welfare impact is not affected by χ and becomes flat. This is because the size of a bailout is not likely to be larger than the upper bound, thus the upper bound never binds. This is essentially the baseline bailout policy in which there is no upper bound for the size of a bailout. This analysis therefore shows that the baseline bailout policy with full rebating is better than the bailout policy

Figure A.2: Bailout Policy with Upper Bound



with an upper bound.

Slow Productivity Spillover to Non-Tradable Sector

This subsection considers the model with slow productivity spillover from the tradable sector to the non-tradable sector. This might potentially have an important impact on the result, because slower productivity growth in the non-tradable sector compared to the tradable sector would cause real appreciation through the Balassa-Samuelson effect and might mitigate the policy effectiveness in promoting growth and attracting FDI. One possible way to introduce slow spillover is to model the non-tradable productivity A_t^N as follows:

$$A_t^N = (A_{t-1}^N)^{\iota}(A_t)^{1-\iota}$$

where ι is the parameter that governs the speed of productivity spillover. In terms of the stationarized model, the relative productivity $a_t^N = A_t^N/A_t$ is given by the following equation:

$$a_t^N = \left(\frac{A_{t-1}^N}{A_t}\right)^\iota = \left(\frac{a_{t-1}^N}{1+g_{t-1}}\right)^\iota$$

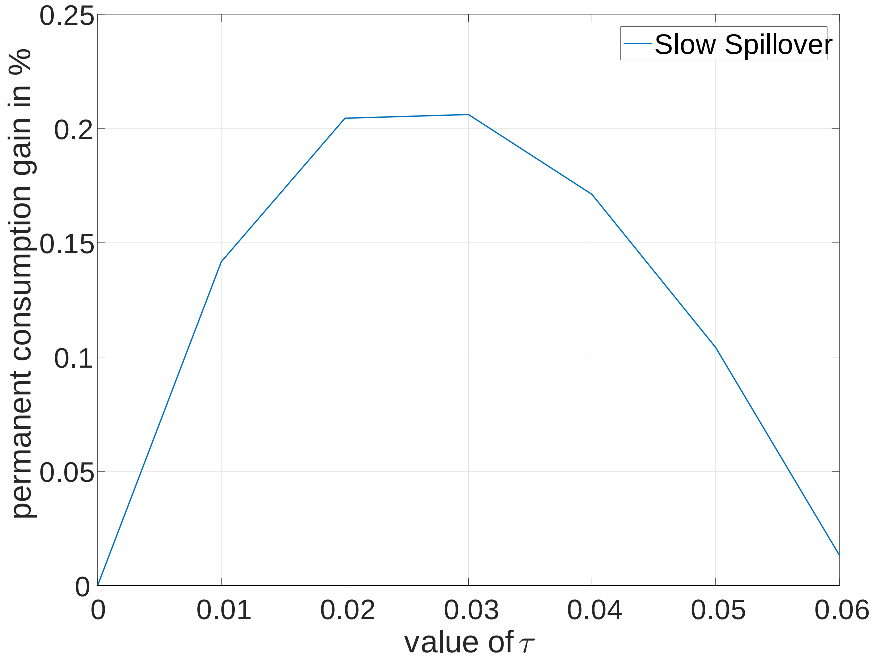
If $a_{-1}^N = 1$ and $\iota = 0.9$ for example, the relative productivity declines over time from 1 to 0.83 at the balanced growth path.

This specification, however, would require another state variable in the model and make the numerical solution substantially difficult, given that there are 5 state variables in the model. Therefore I consider a reduced form spillover function as follows:

$$a_t^N = \left(\frac{k_{-1}}{k_{t-1}}\right)^{\iota'}$$

With this functional form, the relative productivity declines over time from 1 to some value determined by the parameter ι'. I set the value for ι' to match the long-run relative productivity equal to 0.83, consistent with the case with $\iota = 0.9$ for the original specification. This gives $\iota' = 0.27$, and I adjust the other parameter values so that the long-run growth rate $\overline{g}$ is the same as the baseline model. I then try different values for τ and find the value that maximizes the expected utility of households. The result is presented in Figure A.3: The optimal pace of reserve accumulation is still $\tau = 0.03$, and the size of welfare gain is very close to the baseline model. Therefore I conclude that slow productivity spillover to the non-tradable sector does not change the main result of the chapter.

Figure A.3: Slow Spillover to Non-Tradable Sector



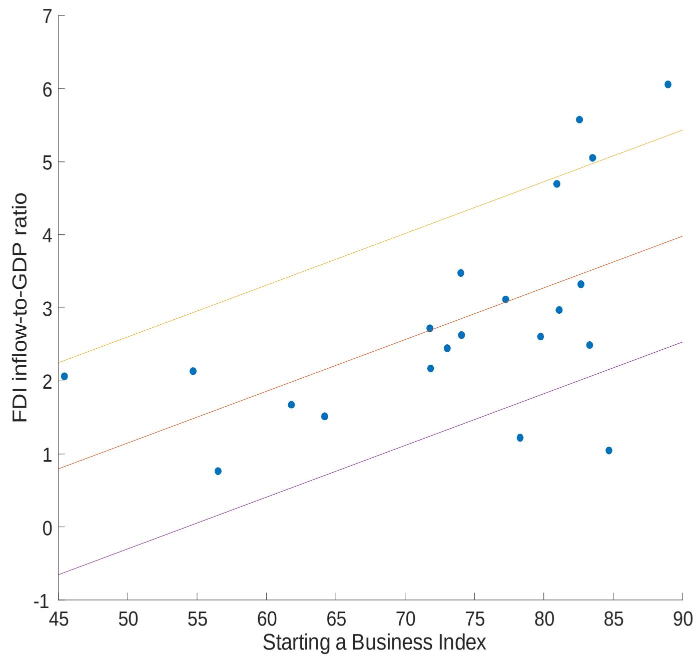
Policy Evaluation with Estimated FDI Entry Cost

This section presents an alternative analysis of reserve policy evaluation. In the main text, I adjust the FDI entry cost parameters to target the FDI inflow-to-GDP ratio for each country and evaluate the reserve policy. In this section, I estimate the FDI entry cost for each country using Starting a Business Index from the World Bank's Doing Business Surveys, and evaluate each country's pace of reserve accumulation.

Estimation of FDI Entry Cost

Starting a Business Index measures the effective cost of starting a new business in each country by taking into account the minimum capital requirement, number of procedures, and time and cost to start up a new business. The Index is focused on 100% domestically-

Figure A.4: Correlation between FDI inflow and Starting a Business Index



owned firms, but I use this Index as a proxy for cost to start a new business by foreign investors. To validate that this Index can be used as a proxy for FDI entry cost, I first show the correlation between the Index and the FDI inflow-to-GDP ratio across developing countries. Since the Index is not available for China, I remove China from the sample in the main text and use a sample of 21 developing countries. The Index takes a value between 0 and 100, and a higher value implies smaller cost to start a new business. The regression result is presented in Figure A.4, with a band for one standard deviation. It is clear that there is a positive correlation between the Index and FDI inflows. The slope is 0.071 with a standard deviation 0.024 and a t-value 2.90. This result validates that Starting a Business Index can be used as a proxy for FDI entry cost.

Given this result, I estimate the FDI entry cost parameter in the model using Starting a Business Index. To do this, I assume that the congestion cost coefficient for FDI entry is a function of the Index:

$$1/\chi_i^F = \beta_0 + \beta_1(\text{Index}_i)^{\beta_2}$$

where χ_i^F is the congestion cost coefficient for country i, and Index_i is the average of Starting a Business Index for country i in 2004-2017. The reason for taking the inverse of χ_i^F is because higher Index implies smaller cost. I then choose β_0, β_1, β_2 to minimize the sum of squared gaps in the FDI inflow-to-GDP ratios between the model and the data across countries. Namely,

$$\min_{\beta_0,\beta_1,\beta_2} \sum_{i=1}^{21} \left[\left(\frac{\text{FDI}}{\text{GDP}}\right)_i^{\text{data}} - \left(\frac{\text{FDI}}{\text{GDP}}\right)_i^{\text{model}} \right]^2$$

In doing this, I adjust the fixed entry cost C^F to keep the fixed entry cost-to-profit ratio at 72%, and the step sizes σ^D and σ^F to have the same long-run growth rate as in the baseline model for each country. As a result, I obtain $\beta_0 = 2.36$, $\beta_1 = 33.1$, and $\beta_2 = 10.9$. Figure A.5 plots the FDI inflow-to-GDP ratios using the estimated FDI entry cost χ_i^F along with the ratios in the data. The model captures the variation in the FDI inflow-to-GDP ratios across countries relatively well, although there are some large gaps for countries with high Index. The regression lines for the data and the model perfectly coincide.

Evaluation of Each Country's Reserve Policy

Now I evaluate each country's reserve policy using the estimated FDI entry cost. I follow the same steps as in the main text, namely, I derive the optimal pace of reserve accumulation for each country, and compare the welfare gain/loss between the actual pace and the optimal pace. The results are presented in Figure A.6 and Table A.7. It seems from Figure A.6 that there are more gaps between the actual pace and the optimal pace, compared to the analysis

Figure A.5: FDI Inflows based on Estimated FDI Entry Cost

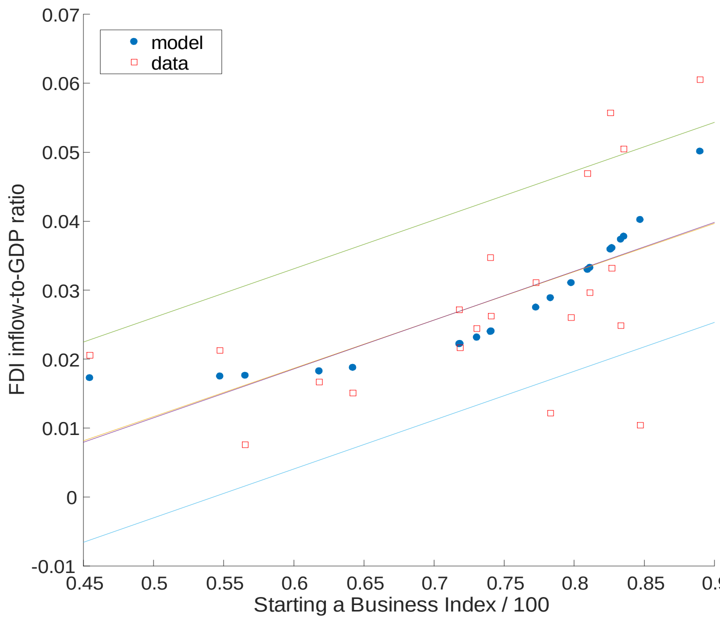
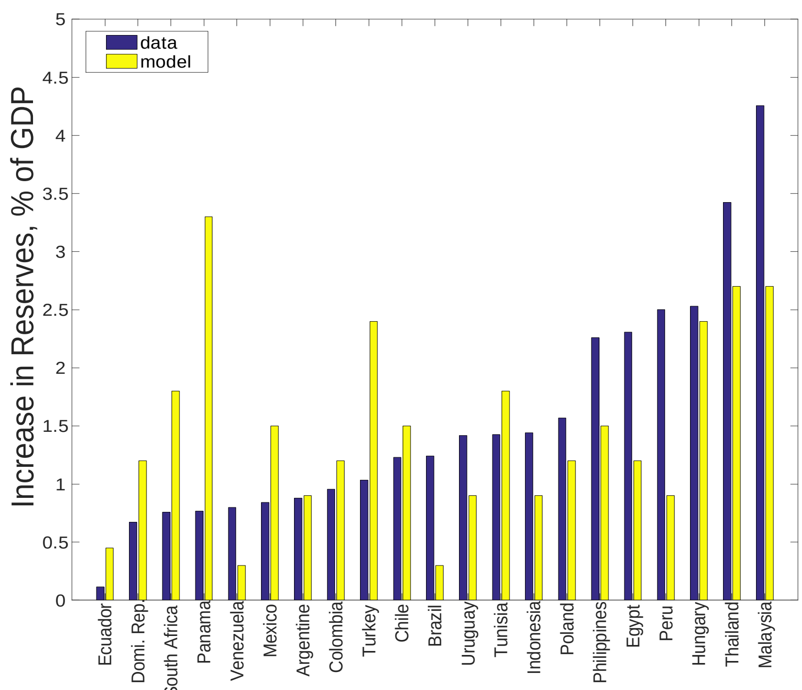


Figure A.6: Reserve Accumulation Pace using Estimated FDI Entry Cost



in the main text. It suggests that countries such as Ecuador, South Africa, Panama, and Turkey, may have room for welfare improvement by accumulating reserves more quickly. But Table A.1 shows that welfare gains for most countries are again close to the optimal level, suggesting that most countries are still roughly in line with the optimal pace. The largest discrepancy from the result in the main text is Turkey, which has a -0.05% welfare loss due to over-accumulation in the main analysis, but now has a significant positive welfare gain and still some room for welfare improvement by accumulating more quickly. This is because Turkey has a small FDI inflow-to-GDP ratio at 1.04% in the data, while Starting a Business Index is high at 84.7, and thus the estimated FDI inflow-to-GDP ratio is 4.02%, which is four times as large as the actual ratio.

Table A.4: Evaluation of Reserve Accumulation Pace

Country	Accum. Pace (%)		Welfare (%)		Elasticity of Spread	Estimated FDI/GDP
	Actual	Optimal	Actual	Optimal		
Argentine	0.9	0.9	0.05	0.05	0.0736	2.22
Brazil	1.2	0.3	-0.03	0.02	0.0913	1.75
Chile	1.2	1.5	0.14	0.14	0.0913	3.59
Colombia	1.0	1.2	0.11	0.12	0.0736	3.10
Dominican Rep.	0.7	1.2	0.08	0.09	0.0736	2.75
Ecuador	0.1	0.5	0.01	0.02	0.0913	1.82
Egypt	2.3	1.2	0.06	0.10	0.0382	2.40
Hungary	2.5	2.4	0.24	0.24	0.0382	3.78
Indonesia	1.4	0.9	0.03	0.05	0.0382	1.76
Malaysia	4.3	2.7	0.17	0.25	0.0205	3.30
Mexico	0.8	1.5	0.13	0.15	0.0913	3.73
Panama	0.8	3.3	0.15	0.37	0.0382	5.01
Peru	2.5	0.9	-0.13	0.05	0.0913	2.40
Philippines	2.3	1.5	0.06	0.09	0.0205	1.88
Poland	1.6	1.2	0.08	0.09	0.0382	2.22
South Africa	0.8	1.8	0.10	0.15	0.0382	2.89
Thailand	3.4	2.7	0.23	0.25	0.0205	3.32
Tunisia	1.4	1.8	0.18	0.19	0.0559	3.61
Turkey	1.0	2.4	0.17	0.22	0.0559	4.02
Uruguay	1.4	0.9	0.02	0.05	0.0913	2.31
Venezuela	0.8	0.3	0.00	0.02	0.0913	1.73

Proof of Linear Relations in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate producing firms.

Foreign Firms

I guess the linear relation $V_t^F(n) = nV_t^F(1)$. I first work on the value of a foreign firm with a single product line:

$$V_t^F(1) = \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{1} P(i,1,i_t^F) \left(\sum_{j=0}^{1} P(j,1,d_t) E_t \left(V_{t+1}^F (1+i-j) \right) \right) \right] \right\}$$

There are 4 cases next period, depending on whether innovation is successful or not, and replacement happens or not. Writing out all 4 cases and noting $V_{t+1}^F(0) = 0$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \Big\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[P(0,1,i_t^F) P(0,1,d_t) E_t \left(V_{t+1}^F(1) \right) \right] \\ &\qquad + P(1,1,i_t^F) P(1,1,d_t) E_t \left(V_{t+1}^F(1) \right) \\ &\qquad + P(1,1,i_t^F) P(0,1,d_t) E_t \left(V_{t+1}^F(2) \right) \} \\ &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1-i_t^F)(1-d_t) E_t \left(V_{t+1}^F(1) \right) \right] + i_t^F d_t E_t \left(V_{t+1}^F(1) \right) + i_t^F (1-d_t) E_t \left(V_{t+1}^F(2) \right) \right\} \end{aligned}$$

Using the linear relation $V_{t+1}^F(2) = 2V_{t+1}^F(1)$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1-i_t^F)(1-d_t) E_t \left(V_{t+1}^F(1) \right) \right] + i_t^F d_t E_t \left(V_{t+1}^F(1) \right) + 2 i_t^F (1-d_t) E_t \left(V_{t+1}^F(1) \right) \right\} \\ & \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[(1 + i_t^F - d_t) E_t \left(V_{t+1}^F(1) \right) \right] \right\} \end{aligned}$$

which is equation (1.18) in the main text. Next I work on the value of a foreign firm with n product lines:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{n} P(i,n,i_t^F) \left(\sum_{j=0}^{n} P(j,n,d_t) E_t \left(V_{t+1}^F(n+i-j) \right) \right) \right] \right\}$$

Using the linear relation $V_{t+1}^F(n+i-j) = (n+i-j)V_{t+1}^F(1)$,

$$\begin{aligned} V_t^F(n) &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^{n} P(i,n,i_t^F) \left(\sum_{j=0}^{n} P(j,n,d_t)(n+i-j) E_t \left(V_{t+1}^F(1) \right) \right) \right] \right\} \\ &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} E_t \left(V_{t+1}^F(1) \right) \left[\sum_{i=0}^{n} P(i,n,i_t^F) \sum_{j=0}^{n} P(j,n,d_t)(n+i-j) \right] \right\} \end{aligned}$$

Inside of bracket can be written as follows:

$$\sum_{i=0}^{n} P(i,n,i_t^F) \sum_{j=0}^{n} P(j,n,d_t)(n+i-j) = n + \sum_{i=0}^{n} P(i,n,i_t^F)i - \sum_{j=0}^{n} P(j,n,d_t)j = n + ni_t^F - nd_t$$

Note that the last two terms are just the expected number of successes for each binomial process. Thus $V_t^F(n)$ can be written as follows:

$$\begin{aligned} V_t^F(n) &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} n(1 + i_t^F - d_t) E_t \left(V_{t+1}^F(1) \right) \right\} \\ &= n \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} (1 + i_t^F - d_t) E_t \left(V_{t+1}^F(1) \right) \right\} \\ &= nV_t^F(1) \end{aligned}$$

This verifies that my initial guess $V_t^F(n) = nV_t^F(1)$ is correct.

Domestic Firms

I guess the linear relation $V_t^D(n) = nV_t^D(1)$. Again I first work on the value of a domestic firm with a single product line, this time taking into account acquisition by foreign investors:

$$\begin{aligned}
V_t^D(1) &= \max_{Z_t^D} \left\{ \pi_t^D - Z_t^D \right. \\
&+ \left[\sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1+i-j-k)\right] \right) \right\} \right] \\
&+ \left. \left[\sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \right\}
\end{aligned}$$

There are now 6 cases next period: Whether innovation is successful or not, and whether the product line is replaced, acquired, or survives. Writing out the second line,

$$\begin{aligned}
& \sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1+i-j-k)\right] \right) \right\} \\
= \; & P(0,1,i_t^D) P(0,1,d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1} V_{t+1}^D(1)\right) \\
& + P(1,1,i_t^D) P(0,1,d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1} V_{t+1}^D(2)\right) \\
& + P(1,1,i_t^D) P(0,1,d_t) P\left(1, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left(\Lambda_{t,t+1} V_{t+1}^D(1)\right) \\
& + P(1,1,i_t^D) P(1,1,d_t) E_t\left(\Lambda_{t,t+1} V_{t+1}^D(2)\right)
\end{aligned}$$

Using the linear relation $V^D_{t+1}(2) = 2V^D_{t+1}(1)$,

$$\begin{aligned}
&= \left[P(0,1,i^D_t)P(0,1,d_t)P\left(0,1,\frac{e^F_t}{1-\theta_{t-1}}\right)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right)\right] \\
&\quad+\left[P(1,1,i^D_t)P(0,1,d_t)P\left(0,1,\frac{e^F_t}{1-\theta_{t-1}}\right)2E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right)\right] \\
&\quad+\left[P(1,1,i^D_t)P(0,1,d_t)P\left(1,1,\frac{e^F_t}{1-\theta_{t-1}}\right)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right)\right] \\
&\quad+\left[P(1,1,i^D_t)P(1,1,d_t)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right)\right] \\
&= (1-i^D_t)(1-d_t)\left(1-\frac{e^F_t}{1-\theta_{t-1}}\right)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right) \\
&\quad+2i^D_t(1-d_t)\left(1-\frac{e^F_t}{1-\theta_{t-1}}\right)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right) \\
&\quad+i^D_t(1-d_t)\frac{e^F_t}{1-\theta_{t-1}}E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right) \\
&\quad+i^D_t d_t E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right) \\
&= i^D_t+(1-d_t)\left(1-\frac{e^F_t}{1-\theta_{t-1}}\right)E_t\left(\Lambda_{t,t+1}V^D_{t+1}(1)\right)
\end{aligned}$$

Next, writing out the third line,

$$\sum_{j=0}^{1}P(j,1,d_t)\left(\sum_{k=0}^{1-j}P\left(k,1-j,\frac{e^F_t}{1-\theta_{t-1}}\right)kQ_t\right)=(1-d_t)\frac{e^F_t}{1-\theta_{t-1}}Q_t$$

Therefore $V^D_t(1)$ can be written as follows:

$$V^D_t(1)=\max_{Z^D_t}\left\{\pi^D_t-Z^D_t+\left[i^D_t+(1-d_t)\left(1-\frac{e^F_t}{1-\theta_{t-1}}\right)\right]E_t(\Lambda_{t,t+1}V^D_{t+1}(1))+(1-d_t)\frac{e^F_t}{1-\theta_{t-1}}Q_t\right\}$$

which is equation (1.23) in the main text. Next I work on the value of a domestic firm with n product lines:

$$\begin{aligned} V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &+ \left[\sum_{i=0}^{n} P(i,n,i_t^D) \left\{ \sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k)\right] \right) \right\} \right] \\ &+ \left. \left[\sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \right\} \end{aligned}$$

Using the linear relation $V_{t+1}^D(n+i-j-k) = (n+i-j-k)V_{t+1}^D(1)$,

$$\begin{aligned} V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &+ \left[\sum_{i=0}^{n} P(i,n,i_t^D) \left\{ \sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \right) \right\} \right] \\ &+ \left. \left[\sum_{j=0}^{n} P(j,n,d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \right\} \\ &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &+ E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \left[\sum_{i=0}^{n} P(i,n,i_t^D) \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \right] \\ &+ \left. Q_t \left[\sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \right] \right\} \end{aligned}$$

The bracketed term in the second line is:

$$\begin{aligned} & \sum_{i=0}^{n} P(i,n,i_t^D) \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \\ =\ & n + ni_t^D - nd_t - \sum_{j=0}^{n} P(j,n,d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \\ =\ & n + ni_t^D - nd_t - \sum_{j=0}^{n} P(j,n,d_t) \left[(n-j) \frac{e_t^F}{1-\theta_{t-1}} \right] \\ =\ & n + ni_t^D - nd_t - n\frac{e_t^F}{1-\theta_{t-1}} + nd_t \frac{e_t^F}{1-\theta_{t-1}} \\ =\ & n \left[i_t^D + (1-d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}} \right) \right] \end{aligned}$$

The bracketed term in the last line is:

$$\sum_{j=0}^{n} P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k = n \left[(1-d_t)\frac{e_t^F}{1-\theta_{t-1}}\right]$$

Therefore $V_t^D(n)$ can be written as follows:

$$\begin{aligned} V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &\quad + n\left[i_t^D + (1-d_t)\left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right)\right] E_t\left[\Lambda_{t,t+1} V_{t+1}^D(1)\right] \\ &\quad \left. + n\left[(1-d_t)\frac{e_t^F}{1-\theta_{t-1}}\right] Q_t \right\} \\ &= nV_t^D(1) \end{aligned}$$

This verifies that my initial guess $V_t^D(n) = nV_t^D(1)$ is correct.

Appendix to Chapter 2

Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the model economy and the stationarized equilibrium.

Factor Allocation

Before defining the equilibrium, we derive the expressions for asset and labor allocations. First we show that the total cost for production $R_t^L \ell_t(i) + W_t h_t(i)$ is equal to production $y_t(i)$ times the marginal cost. The latter can be written as:

$$y_t(i) \times MC_t(i) = a_t(i)(\ell_t(i))^{\alpha}(h_t(i))^{1-\alpha} \times \frac{1}{a_t(i)}\overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha} = \overline{\alpha}(R_t^L \ell_t(i))^{\alpha}(W_t h_t(i))^{1-\alpha}$$

Using the cost minimization condition $R_t^L \ell_t(i)/W_t h_t(i) = \alpha/(1-\alpha)$,

$$y_t(i) \times MC_t(i) = \frac{1}{\alpha} R_t^L \ell_t(i) = \frac{1}{1-\alpha} W_t h_t(i) = R_t^L \ell_t(i) + W_t h_t(i) \qquad \text{(B.1)}$$

Thus production times the marginal cost is equal to the total cost.

Next, profit for a product line can be written as follows:

$$\pi_t(i) = p_t(i) y_t(i) - (R_t^L \ell_t(i) + W_t h_t(i)) = (p_t(i) - MC_t(i)) y_t(i)$$

Recall that the optimal price is equal to the marginal cost for the second-best rival. Here we consider only the case in which the second-best rival is a domestic firm, but the case in which the rival is a foreign firm is similar. The rival's marginal cost is $(1+\sigma^s)$ times the marginal cost for the leader, where $s = D, X$ depending on the type of the product line. Therefore,

$$\pi_t(i) = \sigma^s MC_t(i) y_t(i)$$

Using (B.1),

$$\pi_t(i) = \sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i)$$

In the main text, we derived another expression for a profit in equation (2.10). Thus we have:

$$\sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i) = Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^s}{1+\sigma^s} \tag{B.2}$$

This equation shows that the asset and labor input $\ell_t(i)$ and $h_t(i)$ are independent of productivity $a_t(i)$, and depends only on the type of product lines, $s = D, X$. Combining this equation with $s = D$ and $s = X$, we obtain the relative factor input between domestic lines and exporting lines:

$$\frac{\ell_t^D}{\ell_t^X} = \frac{h_t^D}{h_t^X} = \frac{1+\sigma^X}{1+\sigma^D}$$

Equilibrium

Definition: *The equilibrium of the model economy is defined by the initial states* A_{-1}, B_{-1}, θ_{-1}^D, θ_{-1}^X, ε_{-1}^A, ε_{-1}^A, *the stochastic process* $\{\varepsilon_t^A, \varepsilon_t^R\}_{t=0}^\infty$, *and the following:*

1. *Tradable goods producer: Given prices* $\{\{p_t(i)\}_{i=0}^1, Q_t, R_t^L\}_{t=0}^\infty$, $\{\{y_t(i)\}_{t=0}^1, B_t, L_t, Y_t^T, \mu_t\}_{t=0}^\infty$ *satisfy (2.1), (2.3), (2.6), (2.5), (2.7).*

2. *Domestic intermediate goods producing firms: Given prices* $\{W_t, R_t^L\}_{t=0}^{\infty}$ *and tradable goods output* $\{Y_t^T, Y_t^*\}_{t=0}^{\infty}$, $\{\ell_t^s, h_t^s, Z_t^D, Z_t^X, \pi_t^s, i_t^s, V_t(1,0), V_t(0,1)\}_{t=0}^{\infty}$ *satisfy (2.9), (2.11), (2.17), (2.18), (2.19), (2.20), (2.21), (2.22), (B.2).*

3. *Non-tradable goods producer: Given prices* $\{W_t, P_t^N\}_{t=0}^{\infty}$, $\{Y_t^N, L_t^N, \Pi_t^N\}_{t=0}^{\infty}$ *satisfy (2.29), (2.30), (2.31).*

4. *Households: Given prices* $\{W_t, P_t^N\}_{t=0}^{\infty}$, $\{C_t, C_t^T, C_t^N, H_t, Z_t^E, e_t, \lambda_t\}_{t=0}^{\infty}$ *satisfy (2.23), (2.24), (2.32), (2.33), (2.34), (2.35), (2.36).*

5. *Aggregate variables* $\{A_t, \theta_t^D, \theta_t^X, d_t\}_{t=0}^{\infty}$ *satisfy (2.25), (2.26), (2.27), (2.28).*

6. *Prices* $\{Q_t, R_t^L, W_t, P_t^N\}_{t=0}^{\infty}$ *satisfy (2.37), (2.38), (2.39).*

Stationarized Equilibrium

To stationarize the model, I divide the equilibrium conditions by aggregate productivity A_t. I denote stationarized variables by the lower-case letters, and use g_t to denote the productivity growth rate A_{t+1}/A_t. I also make some arrangements and reduce the number of equations. The following is the complete list of equations to characterize the stationarized equilibrium of the model:

Tradable goods producers

$$y_t^T = \exp(\varepsilon_t^A) A_t \frac{(L_t^D)^{\alpha}(H_t^D)^{1-\alpha}}{\theta_{t-1}^D} \left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D}$$

$$\lambda_t - \mu_t = \beta \exp(\varepsilon_t^R) E_t(\lambda_{t+1})$$

$$\lambda_t q_t = \beta E_t \left[\lambda_{t+1}(q_{t+1} + r^L_{t+1}) + \mu_{t+1}\kappa q_{t+1}\right]$$

$$\mu_t \left[-b_t + \phi y^T_t \frac{1}{1+\phi\mu_t/\lambda_t} - \kappa q_t\right] = 0$$

Intermediate goods producing firms

$$R^L_t = \frac{1}{1+\sigma^D}\alpha y^T_t \frac{\theta^D_{t-1}}{L^D_t}\frac{1}{1+\phi\mu_t/\lambda_t}$$

$$W_t = \frac{1}{1+\sigma^D}(1-\alpha)y^T_t \frac{\theta^D_{t-1}}{H^D_t}\frac{1}{1+\phi\mu_t/\lambda_t}$$

$$L^X_t = L^D_t \frac{\theta^X_{t-1}}{\theta^D_{t-1}}\frac{1+\sigma^D}{1+\sigma^X}$$

$$H^X_t = H^D_t \frac{\theta^X_{t-1}}{\theta^D_{t-1}}\frac{1+\sigma^D}{1+\sigma^X}$$

$$\pi^D_t = \frac{\sigma^D_t}{1+\sigma^D_t}y^T_t \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$\pi^X_t = \frac{\sigma^X_t}{1+\sigma^X_t}y^T_t \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$\pi^*_t = y^*_t - \frac{1+\xi}{1+\sigma^X}\frac{1}{\omega^*}(r^L_t)^\alpha (w_t)^{1-\alpha}$$

$$1 = \frac{1+\sigma^X}{1+\xi}\overline{\alpha}\omega^* \frac{1}{\theta^X_{t-1}}(L^*_t)^\alpha (H^*_t)^{1-\alpha}$$

$$v_t(1,0) = \pi^D_t - z^D_t - z^X_t + \left[i^D_t + (1-d_t)(1-i^X_t)\right] E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) + (1-d_t)i^X_t E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$$

$$v_t(0,1) = \pi^X_t + \pi^*_t - z^D_t + i^D_t E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) + (1-d_t)E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$$

$$i^D_t = \eta^D (z^D_t)^{1/\rho}$$

$$\eta^D \frac{1}{\rho}(z^D_t)^{1/\rho - 1} E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) = 1$$

$$i_t^X = \eta^X (z_t^X)^{1/\rho}$$

$$(1-d_t)\eta^X \frac{1}{\rho}(z_t^X)^{1/\rho-1}\left(E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) - E_t(\Lambda_{t,t+1}v_{t+1}(1,0))\right) = 1$$

Aggregate variables

$$d_t = (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D + e_t + i^F$$

$$\theta_t^D = \theta_{t-1}^D + (1-\theta_{t-1}^D)(e_t + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D) - \theta_{t-1}^D((1-d_t)i_t^X + i^F)$$

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D(1-d_t)i_t^X - \theta_{t-1}^X(e_t + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D + i^F)$$

$$1 + g_t = (1+\sigma^D)^{e_t + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D}(1+\sigma^X)^{\theta_{t-1}^D(1-d_t)i_t^X}(1+\sigma^X)^{i^F}$$

$$a_t^* = \frac{1+\overline{g}}{1+g_t}a_{t-1}^*$$

Non-tradable goods producer

$$y_t^N = (H_t^N)^{1-\alpha^N}$$

$$w_t = P_t^N(1-\alpha^N)(H_t^N)^{-\alpha^N}$$

Households

$$c_t^T + z_t^E = y_t^T - \theta_{t-1}^D\left(Z_t^D + Z_t^X\right) - \theta_{t-1}^X Z_t^D + \left(\theta_{t-1}^D + \theta_{t-1}^X\right)Y_t^* - \left(1 - \theta_{t-1}^D - \theta_{t-1}^X\right)\frac{Y_t^T}{1+\phi\mu_t/\lambda_t} - b_t + \exp(\varepsilon_{t-1}^R)R\frac{b_{t-1}}{1+g_{t-1}}$$

$$\frac{c_t^T}{y_t^N} = \frac{\gamma}{1-\gamma}(P_t^N)^\varepsilon$$

$$(H_t)^{\omega-1} = w_t\left(\gamma\frac{c_t}{c_t^T}\right)^{\frac{1}{\varepsilon}}$$

$$\lambda_t = \frac{1}{c_t - (H_t)^\omega/\omega}\left(\gamma\frac{c_t}{c_t^T}\right)^{1/\varepsilon}$$

$$e_t^D = \eta^E (z_t^E)^{1/\rho}$$

$$\eta^E \frac{1}{\rho}(z_t^E)^{1/\rho-1}E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) = 1$$

Market clearing

$$H_t = H_t^D + H_t^X + H_t^* + H_t^N$$

$$1 = L_t^D + L_t^X + L_t^*$$

Numerical Solution

In this section we sketch the numerical solution method. The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. We set the equally-spaced grid points for the endogenous state variables, foreign debt $\exp(\varepsilon_t^R)Rb_{t-1}/(1+g_{t-1})$, share of domestic product lines θ_{t-1}^D, share of exporting product lines θ_{t-1}^X, relative productivity of foreign countries over the domestic country $a_t^* = A_t^*/A_t$. There are also 2 states for stochastic shocks ε_t^A and ε_t^R respectively.

2. For each grid point, we set the initial guess for five variables: c_t^T, $E_t(\Lambda_{t,t+1}v_{t+1}(1,0))$, $E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$, L_t^D, q_t.

3. For each grid point, we do the following:

   (a) We leave the five variables we have made guess for as unknown variables, and express all the other endogenous variables in terms of the state variables and five unknowns. In this process we first assume that the borrowing constraint is not binding and proceed. Later we check if the constraint is satisfied. If it is not satisfied, we recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the five variables.

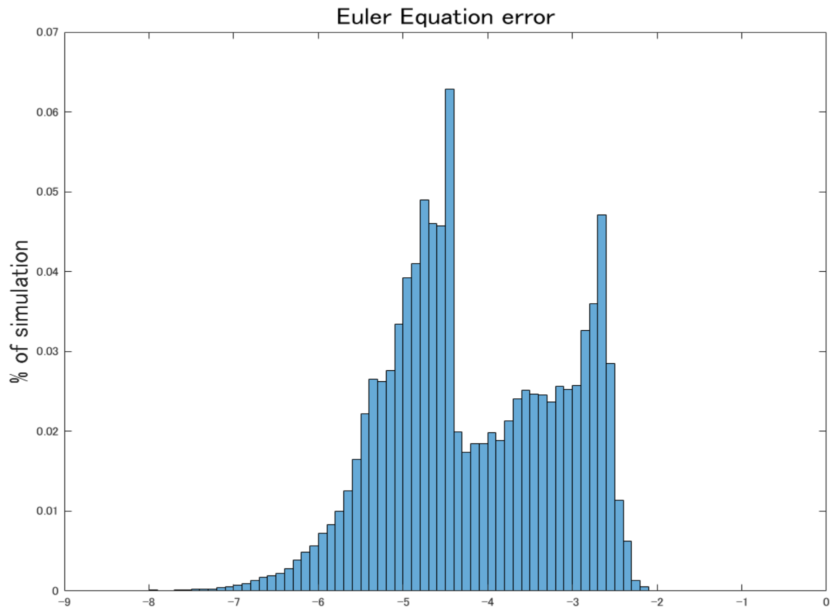
(b) Using multi-dimensional linear interpolation over the next-period state variables and the guess for the five variables (c_t^T, $E_t(\Lambda_{t,t+1}v_{t+1}(1,0))$, $E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$, L_t^D, q_t), we compute all the endogenous variables next period. we then calculate all the forward-looking expectation terms, such as the right-hand side of the Euler equations and the value functions.

(c) All the equilibrium conditions are now the functions of the initial five unknowns. There are five equations we did not use in step (a), thus five equations in total. We solve for the five unknowns using non-linear solver.

4. We check the gap between the guess and the newly-obtained values for the five variables. If they are close enough, we stop. If not, we update the guess by the newly-obtained values, and go back to step 3. Repeat this process until the gap becomes sufficiently small.

We check the accuracy of the numerical solution in the same way as in Chapter 1, namely using the Euler equation error. Because the model in this chapter is a business cycle model rather than a transition model as in Chapter 1, we simulate the model for 100,000 periods with stochastic shocks and compute the Euler equation error for each period. Figure B.1

Figure B.1: Euler Equation Error



plots the distribution of the Euler equation errors obtained by this method. The average error is smaller than -4 and the maximum error is smaller than -2, which are reasonably small compared to the literature.

Proof of Linear Relations in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate producing firms. We guess the linear relation $V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$ and prove it. We first work on the value of a firm with a single domestic product line:

$$\begin{aligned} V_t(1,0) \quad = \quad & \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\ & \left. + \left[\sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, i_t^X\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1+i-j-k,k)\right] \right) \right\} \right] \right\} \end{aligned}$$

Using the linear relation, the summations in the second line can be written as follows:

$$\begin{aligned}
& \sum_{i=0}^{1} P(i,1,i_t^D)\left\{\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1}V_{t+1}(1+i-j-k,k)\right]\right)\right\} \\
= & \sum_{i=0}^{1} P(i,1,i_t^D)\left\{\sum_{j=0}^{1} P(j,1,d_t)\left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1}[(1+i-j-k)V_{t+1}(1,0)+kV_{t+1}(0,1)]\right]\right)\right\} \\
= & E_t\left[\Lambda_{t,t+1}V_{t+1}(1,0)\right]\sum_{i=0}^{1} P(i,1,i_t^D)\sum_{j=0}^{1} P(j,1,d_t)\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(1+i-j-k) \\
& +E_t\left[\Lambda_{t,t+1}V_{t+1}(0,1)\right]\sum_{i=0}^{1} P(i,1,i_t^D)\sum_{j=0}^{1} P(j,1,d_t)\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(k) \\
= & (i_t^D+(1-d_t)(1-i_t^X))E_t\left[\Lambda_{t,t+1}V_{t+1}(1,0)\right]+(1-d_t)i_t^X E_t\left[\Lambda_{t,t+1}V_{t+1}(0,1)\right]
\end{aligned}$$

Therefore we have:

$$\begin{aligned}
V_t(1,0) = & \max_{Z_t^D,Z_t^X}\left\{\pi_t^D - Z_t^D - Z_t^X\right. \\
& \left.+\left(i_t^D+(1-d_t)(1-i_t^X)\right)E_t\left[\Lambda_{t,t+1}V_{t+1}(1,0)\right]+(1-d_t)i_t^X E_t\left[\Lambda_{t,t+1}V_{t+1}(0,1)\right]\right\}
\end{aligned}$$

Similarly, we can show that the value of a firm with a single exporting line is given as follows:

$$V_t(0,1)=\max_{Z_t^D}\left\{\pi_t^X+\pi_t^*-Z_t^D+i_t^D E_t\left[\Lambda_{t,t+1}V_{t+1}(1,0)\right]+(1-d_t)E_t\left[\Lambda_{t,t+1}V_{t+1}(0,1)\right]\right\}$$

which is equation (2.19) and (2.20) in the main text. Next I work on the value of a firm with general n^D domestic lines and n^X exporting lines:

$$\begin{aligned}
V_t(n^D,n^X) = & \max_{Z_t^D,Z_t^X}\{n^D\pi_t^D+n^X(\pi_t^X+\pi_t^*)-(n^D+n^X)Z_t^D-n^X Z_t^X \\
& +\sum_{i=0}^{n^D+n^X} P(i,n^D+n^X,i_t^D)\sum_{j=0}^{n^D} P(j,n^D,d_t)\sum_{k=0}^{n^D-j} P(k,n^D-j,i_t^X)\sum_{m=0}^{n^X} P(m,n^X,d_t) \\
& E_t\left[\Lambda_{t,t+1}V_{t+1}(n^D+i-j-k,n^X+k-m)\right]\}
\end{aligned}$$

Using the linear relation in the value function,

$$
\begin{aligned}
V_t(n^D, n^X) = &\max_{Z_t^D, Z_t^X} \{n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X)Z_t^D - n^D Z_t^X \\
&+ E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, d_t)(n^D + i - j - k) \\
&+ E_t[\Lambda_{t,t+1}V_{t+1}(0,1)] \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, d_t)(n^X + k - m)
\end{aligned}
$$

The second line can be written as follows:

$$E_t[\Lambda_{t,t+1}V_{t+1}(1,0)](n^D + (n^D + n^X)i_t^D - n^D d_t - n^D(1 - d_t)i_t^X$$

The third line can be written as follows:

$$E_t[\Lambda_{t,t+1}V_{t+1}(0,1)](n^X + n^D(1 - d_t)i_t^X - n^X d_t$$

Therefore $V_t(n^L, n^H)$ can be written as follows::

$$
\begin{aligned}
V_t(n^D, n^X) &= \max_{Z_t^D, Z_t^X} \{n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X)Z_t^D - n^D Z_t^X \\
&\quad + E_t[\Lambda_{t,t+1}V_{t+1}(1,0)](n^D + (n^D + n^X)i_t^D - n^D d_t - n^D(1 - d_t)i_t^X \\
&\quad + E_t[\Lambda_{t,t+1}V_{t+1}(0,1)](n^X + n^D(1 - d_t)i_t^X - n^X d_t)\big\} \\
&= \max_{Z_t^D, Z_t^X} \{n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X)Z_t^D - n^D Z_t^X \\
&\quad + n^D \left\{(i_t^D + (1 - d_t)(1 - i_t^X))E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] + (1 - d_t)i_t^X E_t[\Lambda_{t,t+1}V_{t+1}(0,1)]\right\} \\
&\quad + n^X \left\{i_t^D E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] + (1 - d_t)\right\} E_t[\Lambda_{t,t+1}V_{t+1}(0,1)]\big\} \\
&= n^D V_t(1,0) + n^X V_t(0,1)
\end{aligned}
$$

This verifies that the initial guess $V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$ is correct.

Firm Size Distribution

This section shows the law of motion for the share of each firm size and how to derive the firm size distribution. Each firm is characterized by the number of domestic and exporting lines it owns, (n^D, n^X). The law of motion for the firm size (n^D, n^X) is the formula that gives us the measure (number) of firms that own (n^D, n^X) given the firm size distribution in the previous period. Let $\delta_t(n^D, n^X)$ denote the measure of firms that own n^D domestic lines and n^X exporting lines at period t. Because the total measure of intermediate goods is one and each firm owns at least one product line, the measure of firms is between 0 and 1, i.e. $\delta_t(n^D, n^X) \in [0, 1]\ \forall t, n^D, n^X$. In order for a firm to become a firm with (n^D, n^X) in the next period, there are some conditions to be satisfied. For example, a firm with (i, j) at period $t-1$ can own at most $2i + j$ domestic lines, because this is the case in which all domestic innovations $(i+j)$ are successful, all exporting innovations fail, and no replacement on domestic lines happens. So, if a firm owns (i, j) that satisfies $2i+j < n^D$, this firm cannot become a firm with (n^D, n^X) in the next period.

Let (i, j) denote the number of domestic and exporting lines that a firm owns at period $t-1$. Let (k, m) denote the number of successes in domestic innovation and exporting innovation respectively. Then consider a case in which this firm becomes a firm with (n^D, n^X). This implies that the number of replacement on domestic and exporting lines, denoted by (r^D, r^X), are given by $r^D = i + k - m - n^D$ and $r^X = j + m - n^X$. The table below lists up all the notations:

symbol	description
n^D	domestic lines next period
n^X	exporting lines next period
i	domestic lines this period
j	exporting lines this period
k	successes in domestic innovation
m	successes in exporting innovation
r^D	replacements on domestic lines
r^X	replacements on exporting lines

These variables need to satisfy the following conditions:

- Number of successful innovations is limited by the number of existing lines:

(1) $k \leq i + j$

(2) $m \leq i$

- The sum of exporting innovation and replacements on domestic lines is limited by the number of existing domestic lines. The number of replacements on exporting lines is limited by the number of existing exporting lines:

(3) $r^D + m \leq i$

(4) $r^X \leq j$

- Minimum necessary number of successful innovations to achieve (n^D, n^X):

(5) $k \geq n^D - i$

(6) $m \geq n^X - j$

- Given i, minimum necessary number of exporting lines j to achieve (n^D, n^X):

(7) $j \geq n^D - 2j$

(8) $j \geq n^X - i$

- Non-negativity constraints:

(9) $i \geq 0$

(10) $j \geq 0$

(11) $k \geq 0$

(12) $m \geq 0$

(13) $r^D \geq 0$

(14) $r^X \geq 0$

As in the main text, let $P(i, n, p)$ denote the binomial probability for i successes in n trials with the success probability p. Incorporating all the conditions above and some additional conditions to keep the consistency across different conditions, the law of motion for firm size (n^D, n^X) is given as follows:

$$\begin{aligned}\delta_t(n^D, n^X) &= \sum_{i=0}^{\infty} \sum_{j=\max\left\{\begin{array}{c}0,\, n^D - 2i,\\ n^X - i,\, I_+\left(\frac{n^D + n^X}{2} - i\right)\end{array}\right\}}^{\infty} \delta_{t-1}(i, j) \sum_{k=\max\left\{\begin{array}{c}0,\, n^D - i,\\ n^D + n^X - i - j\end{array}\right\}}^{\min\left\{n^D, i+j\right\}} \sum_{m=\max\{0, n^X - j\}}^{\min\left\{n^X, i, i+k-n^D\right\}} \\ &\quad P(k, i+j, i_t^D) P(m, n^D + m - k, i_t^X) P(i + k - n^D - m, i, d_t) P(j + m - n^X, j, d_t)\end{aligned}$$

where $I_+(x)$ is the smallest integer that is equal to or greater than x. The special case is $(n^D, n^X) = (1, 0)$, because there is new firm entry. In this case e_t is added to the right-hand side.

To derive the firm size distribution at the balanced growth path, we use the values at the balanced growth path for new entry, innovation and replacement rates e_t, i_t^D, i_t^X, d_t, and iterate this law of motion for large enough (n^D, n^X) until the distribution converges for every firm size. For the parameter values used in the main text, the firm distribution is not very dispersed and more than 90% of firms own only one product line either domestic or foreign. But still there is heterogeneity in the sense that 57% of firms are a single domestic line and 35% are a single exporting line. In future research we plan to calibrate the parameters to match the key firm-level moments using this formula.

References

Aguiar, M., and M. Amador (2011): "Growth in the Shadow of Expropriation," *Quarterly Journal of Economics*, 126(2), 651–697.

Aizenman, J., and J. Lee (2010): "Real Exchange Rate, Mercantilism and the Learning By Doing Externality," *Pacific Economic Review*, 15(3), 324–335.

Akcigit, U., and W. R. Kerr (2015): "Growth through Heterogeneous Innovations," *NBER Working Paper*, 16443.

Alessandria, G., J. P. Kaboski, and V. Midrigan (2010): "Inventories, Lumpy Trade, and Large Devaluations," *American Economic Review*, 100(5), 2304–2339.

Alessandria, G., S. Pratap, and V. Yue (2015): "Export Growth and Aggregate Dynamics in Large Devaluations," *mimeo*.

Alfaro, L., A. Chari, and F. Kanczuk (2017): "The real effects of capital controls: Firm-level evidence from a policy experiment," *Journal of International Economics*, 108, 191–210.

Alfaro, L., and M. X. Chen (2012): "Surviving the Global Financial Crisis: Foreign Direct Investment and Establishment Performance," *American Economic Journal: Economic Policy*, 4(3).

Alfaro, L., S. Kalemli-Ozcan, and V. Volosovych (2014): "Sovereigns, Upstream Capital Flows, and Global Imbalances," *Journal of the European Economic Association*, 12(5).

Anderson, J. E., and E. van Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, 42(3), 691–751.

Arnold, J. M., and B. S. Javorcik (2009): "Gifted kids or pushy parents? Foreign direct investment and plant productivity in Indonesia," *Journal of International Economics*, 79(1), 42–53.

Aruoba, S. B., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2006): "Comparing solution methods for dynamic equilibrium economies," *Journal of Economic Dynamics and Control*, 30(12), 2477–2508.

Ates, S. T., and F. Saffie (2016): "Fewer but Better : Sudden Stops , Firm Entry , and Financial Selection," *mimeo*.

Barba-Navaretti, G., and A. Venables (2004): *Multinational Firms in the World Economy*. Princeton University Press.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2013): "Financial crises and macro-prudential policies," *Journal of International Economics*, 89(2), 453–470.

——— (2016): "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective," *Journal of Monetary Economics*, 84, 147–165.

Benigno, G., and L. Fornaro (2012): "Reserve Accumulation, Growth and Financial Crises," *CEP Discussion Paper*, (1161).

Bianchi, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(December), 3400–3426.

——— (2016): "Efficient Bailouts?," *American Economic Review*, 106(12), 3607–3659.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2016): "International Reserves and Rollover Risk," *NBER Working Paper*, 18628.

Bianchi, J., and E. G. Mendoza (2013): "Optimal Time-Consistent Macroprudential Policy," *NBER Working Paper*, (19704).

——— (2018): "Optimal Time-Consistent Macroprudential Policy," *Journal of Political Economy*, 126(2).

Blanchard, O., E. Cerutti, and L. H. Summers (2015): "Inflation and Activity - Two Explorations and their Monetary Policy Implications," *NBER Working Paper*, (21726).

Blonigen, B. A. (2005): "A Review of the Empirical Literature on FDI Determinants," *Atlantic Economic Journal*, 33, 383–403.

Blonigen, B. A., and J. Piger (2011): "Determinants of Foreign Direct Investment," *NBER Working Paper*.

Broner, F., T. Didier, A. Erce, and S. L. Schmukler (2013): "Gross capital flows : Dynamics and crises," *Journal of Monetary Economics*, 60, 113–133.

Cerra, V., and S. C. Saxena (2008): "Growth Dynamics : The Myth of Economic Recovery," *American Economic Review*, 98(1), 439–457.

Clementi, G. L., and B. Palazzo (2016): "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, 8(3), 1–41.

Comin, D., and M. Gertler (2006): "Medium-Term Business Cycles," *American Economic Review*, 96(3).

Cook, D., and J. Yetman (2012): "Expanding central bank balance sheets in emerging Asia : a compendium of risks and some evidence," *BIS Papers*, (66), 1–46.

Dasgupta, K. (2012): "Learning and knowledge diffusion in a global economy," *Journal of International Economics*, 87(2), 323–336.

Dell'Erba, S., R. Hausmann, and U. Panizza (2013): "Debt Levels, Debt Composition, and Sovereign Spreads in Emerging and Advanced Economices," *CID Working Paper*, (263).

Dooley, M. P., D. Folkerts-Landau, and P. Garber (2007): "Direct investment, rising real wages and the absorption of excess labor in the periphery," *G7 Current Account Imbalances: Sustainability and Adjustment*, (May), 103 – 132.

——— (2014): "The Revived Bretton Woods System's First Decade," *NBER Working Paper*, 20454.

Durdu, C. B., E. G. Mendoza, and M. E. Terrones (2009): "Precautionary demand for foreign assets in Sudden Stop economies: An assessment of the New Mercantilism," *Journal of Development Economics*, 89(2), 194–209.

Eichengreen, B., P. Gupta, and A. Mody (2006): "Sudden Stops and IMF-Supported Programs," *NBER Working Paper*, (12235).

Fillat, J. L., and S. Garetto (2015): "Risk, Returns, and Multinational Production," *Quarterly Journal of Economics*, 130(4), 2027–2073.

Gornemann, N. (2015): "Sovereign Default , Private Investment , and Economic Growth," *mimeo*, pp. 1–59.

Gourinchas, P.-O., and O. Jeanne (2013): "Capital Flows to Developing Countries: The Allocation Puzzle," *Review of Economic Studies*, 80(4), 1484–1515.

Guadalupe, M., O. Kuzmina, and C. Thomas (2012): "Innovation and Foreign Ownership," *American Economic Review*, 102(7), 3594–3627.

Harrison, A., and A. Rodríguez-Clare (2010): "Trade, foreign investment, and industrial policy for developing countries," *Handbook of Development Economics*, 5(C), 4039–4214.

Helpman, E., M. J. Melitz, and S. R. Yeaple (2004): "Export versus FDI with heterogeneous firms," *American Economic Review*, 94(1), 300–316.

Jeanne, O. (2012): "Capital Account Policies and the Real Exchange Rate," *NBER International Seminar on Macroeconomics 2012*.

Jeanne, O., and A. Korinek (2013): "Macroprudential Regulation versus Mopping Up after the Crash," *NBER Working Paper*.

Jeanne, O., and R. Rancière (2011): "the Optimal Level of International Reserves for Emerging Market Countries : a New Formula and Some Applications," *Economic Journal*, 121, 905–930.

Kennedy, M., and A. Palerm (2014): "Emerging market bond spreads: The role of global and domestic factors from 2002 to 2011," *Journal of International Money and Finance*, 43, 70–87.

Klette, T. J., and S. Kortum (2004): "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 112(5), 986–1018.

Korinek, A., and L. Servén (2016): "Undervaluation through foreign reserve accumulation: Static losses, dynamic gains,," *Journal of International Money and Finance*, 64, 104–136.

Lane, P. R., and G. M. Milesi-Ferretti (2017): "International Financial Integration in the Aftermath of hte Global Financial Crisis," *IMF Working Paper*, 17(115).

Ma, C. (2017): "Financial stability, Growth and Macroprudential Policy," .

Mendoza, E. G. (1991): "Real business cycles in a small open economy," *American Economic Review*, 81(4), 797–818.

——— (2005): "Real Exchange Rate Volatility and the Price of Nontradable Goods in Economies Prone to Sudden Stops," *Economia*, 6(1), 103–148.

——— (2010): "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, 100(5), 1941–1966.

Neumeyer, P. A., and F. Perri (2005): "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, 52(2), 345–380.

Obstfeld, M., J. C. Shambaugh, and A. M. Taylor (2010): "Financial Stability, the Trilemma, and International Reserves," *American Economic Journal: Macroeconomics*, 2(April), 57–94.

Queralto, A. (2015): "A Model of Slow Recoveries from Financial Crises," *mimeo*.

Ramondo, N. (2009): "Foreign plants and industry productivity: Evidence from Chile," *Scandinavian Journal of Economics*, 111(4), 789–809.

Ramstetter, E. D. (2009): "Firm- and Plant-level Analysis of Multinationals in Southeast Asia: the Perils of Pooling Industries and Balancing Panels," *ICSEAD Working Paper Series*, 22.

Reinhart, C. M. (2010): "This Time is Different Chartbook : Country Histories on Debt, Default, and Financial Crises," *NBER Working Paper*, (15815).

Reinhart, C. M., V. R. Reinhart, and T. Tashiro (2016): "Does reserve accumulation crowd out investment?," *Journal of International Money and Finance*, 63, 89–111.

Reinhart, C. M., and K. Rogoff (2009): *This Time Is Different : Eight Centuries of Financial Folly*. Princeton University Press.

Rodrik, D. (2008): "The Real Exchange Rate and Economic Growth," *Brookings Papers on Economic Activity*, (2), 365–412.

Schmitt-Grohé, S., and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61(1), 163–185.

——— (2016): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124(5), 1466–1514.

Uribe, M., and V. Z. Yue (2006): "Country spreads and emerging countries: Who drives whom?," *Journal of International Economics*, 69(1), 6–36.