ABSTRACT

Title of dissertation: ESSAYS ON UNCERTAINTY, IMPERFECT INFORMATION, AND INVESTMENT DYNAMICS

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Understanding how imperfect information affects firms' investment decisions helps answer important questions in economics, such as how we may better measure economic uncertainty; how firms' forecasts would affect their decision-making when their beliefs are not backed by economic fundamentals; and what the impacts of firms' productivity uncertainty are in an environment of incomplete information. This dissertation provides a synthetic answer to these questions, both empirically and theoretically.

The first chapter provides empirical evidence to demonstrate that survey-based forecast dispersion identifies a distinctive type of second moment shocks other than the volatility shocks to productivity, i.e. uncertainty shocks. Such forecast disagreement disturbances can affect the distribution of firm-level beliefs regardless of whether or not belief changes are backed by changes in economic fundamentals. At the aggregate level, innovations that increase the dispersion of firms' forecasts lead to persistent declines in aggregate investment and output, and a slow recovery.

Conversely, the larger dispersion of future firm-specific productivity innovations, the standard way to measure economic uncertainty, generates the "drop-rebound-overshoot dynamics for aggregate investment and production. At the firm level, more productive firms increase investments given rises in future productivity dispersion, whereas investments drop when firms disagree more about the well-being of their future business conditions.

The second chapter presents a general equilibrium model of heterogeneous firms subject to the real productivity uncertainty shocks and informational disagreement shocks. As firms cannot perfectly disentangle aggregate from idiosyncratic productivity because of imperfect information, information quality drives the wedge of difference between the unobserved productivity fundamentals, and the firms' beliefs about how productive they are. Distribution of the firms' beliefs is no longer perfectly aligned with the distribution of firm-level productivity across firms. This model not only explains why, at the macro and micro level, disagreement shocks are different from uncertainty shocks, as documented in Chapter 1, but helps reconcile a key challenge faced by the standard framework to study economic uncertainty: a trade-off between sizable business cycle effects due to changes in uncertainty, and the right amount of pro-cyclicality of firm-level investment rate dispersion, as measured by its correlation with the output cycles.

ESSAYS ON UNCERTAINTY, IMPERFECT INFORMATION, AND INVESTMENT DYNAMICS

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Dedication

To Cathy, Shawn, Mom, and Dad

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1.1 Introduction

What is the impact of time-varying firms' uncertainty about future business conditions on their investment decisions and on the aggregate economy? This chapter shows that empirically, the dispersion of business-level forecasts, one of the standard measures of "uncertainty", differs from other canonical uncertainty proxies by their different aggregate and micro-level implications.[1] I provide novel aggregate and firm-level evidence to demonstrate that the measure of cross-sectional forecast disagreement helps identify shocks that are distinct from the uncertainty shocks, as in Bloom (2009), which denote the exogenous changes in the variance of firms' future productivity. Such newly identified shocks are labeled, *informational disagreement* shocks, as they affect the distribution of firm-level beliefs, even if the distribution of firms' productivity fundamentals is unchanged. Both shocks are considered *second moment* shocks, as they affect the variance of pure beliefs, and the variance of real

[1] Dispersion of private agents' point expectations about future inflation rate, GDP growth rates among others, i.e. the forecast disagreement is considered one of the standard measures of uncertainty about future price stability (Mankiw et al., 2004) and approximates the forecast error of the future aggregate economy (Bloom, 2014).

fundamentals respectively.

Distinguishing two different types of shocks helps better answer the question on whether or not the uncertainty shocks are a crucial driver of business cycle fluctuations. Within the framework of micro-level capital and labor adjustment frictions in the form of irreversible investment and non-convex adjustment costs (Bernanke, 1983; Pindyck, 1991; Bloom, 2009; Bloom et al., 2014), the "wait and see" channel of uncertainty shocks plays a pivotal role for uncertainty to have contractionary effects. In specific, increases in the variance of future demand or productivity can raise the option value of adopting a "wait and see" policy towards investment or hiring. As a significant number of firms pause their capital and labor adjustment actions, this leads to a large economic downturn. However, Bachmann and Bayer (2013) find that in a well calibrated model based on German data, the quantitative importance of such real-option effect can be very limited.[2] Empirically, Bachmann et al. (2013b) uncover that the magnitude of the impacts of identified uncertainty shocks largely depends on how we measure economic uncertainty in the data. Precisely, they found that for both Germany and the U.S., when uncertainty is measured by the dispersion of survey-based business forecasts, rises in the forecast dispersion will lead to a large recession and slow recovery. Otherwise, jumps in stock market volatility, another standard way of measuring uncertainty, would push the economy into a recession, followed by a quick rebound, and then the economy overshoots within a year (Bloom, 2009).

[2]Findings on the sign and the size of the predicted effect of uncertainty shocks can be very mixed. For example, very moderate effects have been found in a model absent the market frictions of price rigidity (Bundick and Basu, 2014), credit market friction (Gilchrist et al., 2014), or search friction (Leduc and Liu, 2015).

Therefore, I provide Vector Autoregression (VAR) evidence to demonstrate that the identified impacts of changes in the survey-based forecast disagreement, as in Bachmann et al. (2013b), do not correspond to the theoretical "wait and see" channel in a model of uncertainty shocks à la Bloom (2009). Instead, my joint identification exercise suggests that once isolated from disagreement shocks, the real option effect associated with the "wait and see" channel can be well identified: in response to uncertainty jumps, all major aggregate series experience "abrupt drops and quick rebounds". In addition, shocks that lead to more dispersed beliefs among firms about their own future profitabilities, even though there is no change to the *de facto* shape of distribution of productivity fundamentals, affect the economy through a separate channel. Larger forecast disagreement tends to trigger a bigger decline in investment, employment, and production, and is followed by a sluggish recovery. In the near term of the shocks, the real option effect due to changes in uncertainty about fundamentals may well explain 2 % of the variations in aggregate investment. However, the "wait and see" channel quickly decays within one year after the shocks of uncertainty. By contrast, as for the disagreement shocks, changes to the distribution of firm-level beliefs affect the aggregate investment dynamics all along, until the effect completely dies out after five years. Such evidence may well suggest that an important channel through which heterogeneous beliefs matter for the aggregate economy is mis-identified based on the current realm of theoretical models, which have the sole emphasis on the impacts of changes to variance of economic fundamentals.

Moreover, results show that the strong contractionary and slow recovery effects of disagreement shocks are robust, regardless of how we measure the cross-sectional belief difference. Following Bachmann et al. (2013b), I construct the business level forecast disagreement based on Philadelphia Fed's Business Outlook Survey (BOS) data. The surveyed manufacturing firms' beliefs about their future business conditions better reflect the actual decision makers' expectations. It thus helps explain why the distribution of firms' beliefs, whether or not they are backed by good or bad fundamentals, could strongly affect their business activities, such as investment. I also consider an alternative measure of belief dispersion using Philadelphia Fed's Survey of Professional Forecasters (SPF) data. Despite that SPF data captures expectations of institutional forecasters rather than actual firm producers, findings are that SPF forecast dispersion still helps identify information-based shocks that are orthogonal from concurrent changes in the variance of economic fundamentals.[3]

The "wait and see" channel, however, is very pronounced when we stick to the benchmark uncertainty measure following Bloom et al. (2014), the standard deviation of firm-level productivity shocks. This measure of uncertainty better aligns itself to the theoretical notion of "uncertainty", which approximates the forecast error about future productivity fundamentals. However, using an alternative measure of uncertainty, the news-based Economic Policy Uncertainty (EPU) index, as in Baker et al. (2015), I find that once the disagreement shocks are isolated, changes in the EPU proxy no longer have any significant aggregate impacts. This leads to the conclusion that pure information-based second moment disturbances may have

[3] A few papers have documented that based on U.S. and EU professional forecasters' data, the magnitude of disagreement in mean forecast about future economic aggregate variables is a poor proxy for agents' forecast uncertainty (D'Amico and Orphanides, 2008; Conflitti, 2010).

real impacts on business cycles by affecting the distribution of beliefs.

Using Compustat firm-level investment and operation data, this chapter further presents micro-level evidence to identify the separate channels through which uncertainty and disagreement would affect the firm-level investment. Firstly, I am able to disentangle the "wait and see" effect of uncertainty at the firm-level that is associated with short run drops, and the medium run "rebound and overshoot" of aggregate investment at the macro level. A model of uncertainty shocks in Bloom (2009) predicts that the *extensive margin* effect, i.e. more firms pause investment during more uncertain periods, dominates the *intensive margin* effect in the short run. In specific, the intensive margin operates through a convexity effect channel by which more dispersed firm-specific productivity means on average, firms' expected marginal product of capital increases. Higher expected productivity translates into larger aggregate investment. [4] As a result, the rebound and overshoot of aggregate investment follows a brief disruption of economic activities due to "wait and see" once the convexity effect dominates the "wait and see" effect in the medium-run. I present evidence showing that there is a linear negative association between firm-level investment and uncertainty measure, the trace for the "wait and see" effect. In addition, the firm-level investment rebound dynamics through a non-linear term of uncertainty, conditional on firm-level productivity growth rate can be also unraveled. Such micro evidence on rebound and overshoot contributes well to the literature that only focuses on the linear empirical relationship between uncertainty and firm-level

[4] Namely, this effect is also called Oi-Hartman-Abel effect, or simply, convexity effect (Oi, 1962; Hartman, 1972; Abel, 1983).

investment. [5] Specifically, I find that, conditional on the firm-specific Total Factor Productivity (TFP) growth rate, the more uncertain environment leads a firm to incur greater investment, a *productivity-enhancing* effect, in anticipation that they will become more productive next period since the variance of future productivity draws is larger.

Secondly, I provide micro-level evidence that is consistent with the very persistent contractionary effect of disagreement shocks at the macro level. Apart from the limited negative linear partial effect, conditional on a firm's productivity growth, greater disagreement significantly dampens the firm-level investment. This conditional investment elasticity has the opposite sign, different from that of the non-linear impact of productivity uncertainty. This finding is very important for the following reasons. First, the productivity-enhancing effect of uncertainty suggests that the cross-sectional *dispersion* of firm-level investment rate may shrink when productivity dispersion is subdued in good times, which is at odds with its procyclicality observed in the data (Bachmann and Bayer, 2014). Therefore, such evidence highlights a key shortfall faced by a model of uncertainty shocks, by which time-varying variance of productivity fundamentals cannot simultaneously generate a large recession and the pro-cyclicality of investment rate dispersion. Second, the productivity-dampening effect of forecast disagreement suggests that after a *second moment shocks*-triggered recession, the economy does not have to follow a counter-

[5] For example, Leahy and Whited (1995), and Gilchrist et al. (2014) finds the negatively correlated linear relationship between uncertainty and investment using U.S. firm-level panel data. However, using U.K. manufacturing firm level data, Bloom et al. (2007) finds that the linear effect is not statistically significant, but argues for a non-linear term of uncertainty, conditional on sales growth, as evidence for the "wait-and-see" channel. This interpretation may be erroneous because the non-linear term coefficient can well suggest an intensive margin effect of uncertainty.

factual quick rebound and overshoot. Consistent with findings in Bachmann and Bayer (2014), it is the otherwise large investment spikes of more productive firms that are persistently depressed, which leads to gradual declines in aggregate investment and slower recovery. Third, the different predictions about the cyclicality of investment rate dispersion can be well used to identify if the economy is shocked by informational second moment shocks, or the real second moment shocks about fundamentals.

This chapter unfolds as followed: Section 1.2 discusses the construction and the time-series properties of the benchmark and alternative measures of productivity uncertainty and forecast disagreement. Section 1.3 performs VAR analysis to provide aggregate evidence for the impacts of shocks to uncertainty and disagreement. Section 1.4 provides micro-based evidence using panel data to uncover relationships of uncertainty, disagreement and firm-level investment. Section 1.5 discusses the consistency of macro and micro evidence. It also highlights the importance of using disagreement to better explain the empirical findings that cannot be rationalized using models of uncertainty shocks only. Section 1.6 provides concluding remarks.

1.2 Measurements

Despite the limited consensus on what best measures economic uncertainty among private agents, a range of second moment measures based on time-series volatility and cross-sectional dispersion of key economic variables are considered the closet alternatives (Bloom, 2014). This selection of measures simply reflects the rationale that higher volatility of data series means greater difficulty for forecasting with good precision. Hence, commonly used uncertainty measures include: the volatility of a range of aggregate economic indicators including stock market price index, GDP growth rate, and Total Factor Productivity (TFP), along with the dispersion of idiosyncratic variables, such as firm-level TFP and in particular, that of forecasters' point expectations about future aggregate or idiosyncratic conditions. In this chapter, I comparatively study the dispersion of forecasts across firms and the cross-sectional dispersion-based measures of uncertainty only. In addition, I abstract from studying uncertainty measures based on stock market data, as they could partly capture the changes over time in the degree of financial frictions (Caldara et al., 2016).

Following Bachmann et al. (2013b), I construct the forecast disagreement measure (**DIS**) using Philadelphia Fed's Business Outlook Survey data. Consistent with Bloom et al. (2014), uncertainty is measured by the standard deviation of future log firm-level TFP innovations (**UNC**), estimated from Compustat Data. To avoid overuse of terminology, I call the former, "the measure of *disagreement*" and the latter, "the measure of economic *uncertainty*".[6] As robustness checks, the forecast dispersion index, based on the Survey of Professional Forecasters data (**SPF**), is considered an alternative measure of disagreement. The widely-used index of Economic Policy Uncertainty (**EPU**), which counts newspaper references of policy-related uncertainty keywords is also examined as another measure of uncertainty. I discuss

[6] Note that both Bachmann et al. (2013b) and Caldara et al. (2016) find that empirically forecast disagreement can have quite different macro implications as contrasted to other measures of uncertainty, despite they implicitly assume forecast disagreement is a measure of "uncertainty".

the data sources and construction method for **DIS** and **UNC** below and relegate the reader to Appendix A.1 for additional details on **SPF** and **EPU**.

1.2.1 Measure of Forecast Disagreement

I constructed the forecast disagreement index based on the Business Outlook Survey (BOS) monthly firm-level forecast data. Surveyed firms' forecasts better capture the difference in beliefs of actual decision makers, which can be directly used to examine the economic impacts of changes in firms' expectations. BOS surveys big manufacturing firms located in the Third Federal Reserve District, but the data is found to closely reflect the business outlook at the national level (Nakamura and Trebing, 2008).[7] I use data from January, 1970 to December, 2013 and then convert the series to quarterly frequency to explore its time series dynamics.

The BOS survey records the numbers of firms who report an increase, decrease or no change in their beliefs about the future business conditions. I focus on two questions in the survey probing their views about the "General Business Conditions" and their expected "New Orders" to be shipped in six months, relative to the survey date. The two survey questions are framed as follows:

- **General Business Conditions**: What is your evaluation of the level of general business activity six months from now versus [CURRENT MONTH]: *Decrease/No Change/Increase*

- **Company Business Indicators**: New Orders. Six months from now versus

[7]This third district covers the state of Delaware, the southern half of New Jersey, and the eastern two thirds of Pennsylvania. On average, about 100 to 125 firms responded to the survey each month, out of 250 who received the survey questionnaire (Trebing, 1998).

[CURRENT MONTH]: *Decrease/No Change/Increase*

Based on the fractions of responding firms for month t, with beliefs of increase and decrease in response to the surveyed question, as denoted by F_t^+ and F_t^- respectively, the disagreement index (**DIS**) can be defined below:

$$DIS_t = \sqrt{F_t^+ + F_t^- - (F_t^+ - F_t^-)^2}. \tag{1.1}$$

Figure 1.1 displays the two disagreement index series over time based on responses to both questions. It suggests that forecasts about general business conditions and about firm-specific new orders can be highly correlated, such that the disagreement series keeps very close track of each over time.[8] Without loss of generality, I use the disagreement index based on forecast data about the general business conditions as the benchmark disagreement measure labeled, **DIS**.

Equation 1.1 approximately measures the standard deviation of firm-level forecasts. It shows that increases in both fractions (larger F_t^+ and larger F_t^-) at the same time, i.e. more opposed views about future, thus disagreement, are adjusted for changes in the mean forecasts among firms. Mean forecast changes because firms become more optimistic (larger F_t^+ and smaller F_t^-) or more pessimistic (smaller F_t^+ and larger F_t^-). The closer this index is to 1 (when F_t^+ and F_t^- both get closer to 50 %), the greater is the magnitude of cross-sectional disagreement about their own future profitability. The complete optimism or pessimism is characterized by

[8] Trebing (1998) first finds that firms' responses to the question "general business condition" can be highly correlated with their responses to the question asking for more firm-specific conditions in the future such as shipments, and inventory among others.

$DIS_t = 0$ (when F_t^+ or F_t^- equals 1).

1.2.2 Measure of Economic Uncertainty

The benchmark uncertainty proxy is to capture the cross-sectional dispersion of future firm-specific log productivity innovations, a micro-level measure. Hence, greater variance in future productivity innovations leads to larger forecast errors of a firm's future business conditions. It can be seen from here that the dispersion of firm-specific *beliefs* about their future fundamentals, forecast disagreement, does not necessarily overlap with the dispersion of firms' actual *draws* of future fundamentals, i.e. real uncertainty.

Following Bloom (2014), idiosyncratic productivity is measured by firm-specific slow residual (or, firm-specific TFP). The log TFP innovations ($e_{i,t}$) are estimated based on the following first order auto-regressive equation about log productivity ($z_{i,t}$):

$$\hat{z}_{i,t+1} = \rho_z \hat{z}_{i,t} + \mu_i + \lambda_{t+1} + \sigma_{e,t} e_{i,t+1}. \tag{1.2}$$

where $\hat{z}_{i,t}$ denotes the estimated log TFP, as TFP is not directly observable. The specification controls for the firm fixed-effect (μ_i: time-invariant cross-firm difference in productivity) and the time fixed-effect (λ_t: cyclical changes in a firm's productivity over time, which are common to all firms). The log firm-level TFP data is directly from Imrohoroglu and Tüzel (2014), which adopts the Olley and Pakes (1996) method for estimation, exploiting a Compustat panel of firm-level annual data from 1963 to 2013. The estimation of panel data about firm-specific TFPs has controlled for the industry fixed-effects and the aggregate effect (yearly time fixed effects). In this case, period t corresponds to a year. For more details, see Imrohoroglu and Tüzel (2014).

The standard deviation, $\sigma_{e,t}$ of next year TFP shocks proxies for forecast uncertainty (**UNC**) regarding future firm-specific productivity to be realized. Uncertainty dated in year t about year $t+1$ is thus given by

$$UNC_t = \sigma_{e,t} \tag{1.3}$$

It shows that the more dispersed idiosyncratic TFP shocks in year t, the larger the forecast error for predicting firm i's productivity of next year, $t+1$. In order to compare and contrast the estimated annual uncertainty series with the disagreement series of higher frequency, the uncertainty series is interpolated for quarterly frequency. I will show that the results in this chapter do not depend on whether or not the interpolation is applied.

1.2.3 Exploratory Analysis

I firstly show the pairwise cross-correlations between disagreement measures (**DIS** and **SPF**), and leads and lags of uncertainty measures (**UNC** and **EPU**) using data of quarterly frequency. Following Bloom (2014), the SPF forecast dispersion series are constructed based on data from the year of 1990 going forward when Philadelphia Fed started managing the SPF survey. Table 1.1 summarizes the results of correlations. In general, proxies for disagreement are strongly and positively correlated with uncertainty measures. This is true regardless of whether the forecast disagreement among firms is about the “General Business Conditions” or about the “New Orders” (**DIS**), or disagreement is among professional forecasters (**SPF**) as they have different forecasts about future aggregate economic conditions such as the real GDP and the industrial production. Over the short-term horizon of three quarters of leads or lags, the positive co-movements between disagreement and uncertainty measures are significant. This may well suggest that more uncertain periods are associated with greater forecast disagreement among firms. It is thus important to understand if the belief difference measures the forecast uncertainty, and how disagreement and uncertainty would individually or jointly affect the economy, given their tight interactions.

More specifically, BOS-based disagreement measures have the largest correlations with the cross-sectional dispersion of future productivity innovations, **UNC** for periods when disagreement lags uncertainty ($h = 1, 2, 3$). This may imply that the past BOS disagreement indexes tend to indicate larger future productivity uncertainty. Also, we see **UNC** has its largest correlations with **SPF**-based measures when uncertainty lags or when it is contemporaneous with disagreement ($h = -1, 0$). Fluctuations in uncertainty could then be informative about rises in **SPF**-based forecast disagreement measures. In addition, I find that the **EPU** index tends to be much more correlated with the BOS disagreement indexes based on forecasts about general business conditions, relative to the index based on forecasts about new orders. This suggests that disagreement about general business conditions are correlated with fluctuations of policy uncertainty at the aggregate level. Now we move on to check the time evolution of proxies of uncertainty and disagreement.

The dashed line in Figure 1.2 indicates the time-varying BOS firm-level disagreement index regarding their forecasts about their future general business conditions. The solid line shows the interpolated quarterly time-variation in cross-sectional productivity uncertainty as estimated from Equation (1.2). This pair of series have been found to have strong positive correlations, based on the results from Table 1.1. As documented by Bachmann et al. (2013b) and Bloom (2014), forecast disagreement and uncertainty series are *counter-cyclical*: jumping before or during a recession and decaying right after a recession. Disagreement tends to quickly jump up and stays constant until further abrupt hikes reach its peak. The peaks quickly turn to huge busts after the recessions. However, it takes time for uncertainty to accumulate. For example, during the periods of 1985-1987 and 1995-1999 when uncertainty was climbing, belief dispersion was already quite stable at a high level. Similarly, when disagreement was undergoing rapid changes, underlying productivity uncertainty was sticky during 1982-1983, 1991-1993, and 2004-2006. Also, note that disagreement had more bounded variance in the second half of the sample compared to the first half of the time slice. In particular, we see BOS disagreement typically jumps before climbs of uncertainty, except that during the 2008-2009 recession, where very abrupt jumps in uncertainty and quick drops were followed by greater disagreement.

Once converted from monthly to quarterly data, Figure 1.3 shows that if we are using EPU to measure uncertainty as denoted by the solid line, hikes of uncertainty are also associated with recession periods. Also, we see more quick jumps and slumps, based on **EPU** of higher frequency, and the second half of the sample has larger variance in **EPU**. Similarly, we see **EPU** jumped before **DIS** picked up disagreement for the recession period 2008-2009. Also, note that **EPU** produces great spikes, for example, per the Black Monday in 1987, where we see little changes to the forecast disagreement.

Figures 1.4 and 1.5 compare the SPF forecast dispersion (dashed line) measure using real GDP forecasts to the two uncertainty measures **UNC** and **EPU**, respectively (solid line). These figures exhibit the SPF-based beliefs hiked tremendously during the recessions of 1990-1991 and 2008-2009, but less so for the brief period of 2000. Similarly, the SPF-based belief dispersion hiked following jumps in productivity uncertainty during the 2008-2009 Great Recession.

In summary, both forecast disagreement and uncertainty are counter-cyclical, and largely maintain synchronicity over time. However, it should be noted that they did suspend synchronizing now and then, and the chronological order of jumps may change for different episodes. Therefore, I proceed to examine the aggregate impacts of innovations to disagreement and uncertainty proxies, given their close but potentially different empirical relationships.

1.3 Aggregate Implications: Macro Evidence

To explore and quantify the dynamic aggregate effects of uncertainty and forecast disagreement over business cycles, I employ standard recursive ordering identification by estimating different VAR systems in order to identify and examine the impact of innovation changes to different measures.

Firstly, I isolate the exogenous changes that directly affect the dispersion of firms' *views* about the future, i.e. the *disagreement shocks* which affect distribution of firms' forecasts, by assuming that the exogenous changes to *real* productivity dispersion, the *uncertainty shocks*, do not shift the forecast dispersion within the same quarter. This benchmark identification shuts down the channel through which disturbances to productivity uncertainty would affect the forecast dispersion on impact (Scheme 1). Specifically, this scheme places the disagreement measure before the uncertainty proxy, as followed by other real macroeconomic variables, which is in line with the ordering adopted by Bloom (2009).

Secondly, I consider specifications that reverse the ordering between disagreement and uncertainty, so as to first disentangle uncertainty shocks. This ordering scheme assumes that the shocks that affect the dispersion of forecasts do not immediately drive the productivity dispersion this quarter (Scheme 2).

Then, I verify if the impulse responses of major macro variables to isolated disagreement shocks, using Scheme 1, are quantitatively similar to the impulse responses to the disturbances of disagreement, conditional on the restriction that forecast disagreement can be affected by uncertainty shocks on impact with Scheme 2. Similar comparisons can be performed in order to examine the identified impacts of the uncertainty shocks. Hence, we may conclude whether or not we can identify different types of shocks, and their different business cycle impacts if any. In particular, it will be interesting to see how the aggregate changes in the forecast dispersion, which are not originated from changes in the variability of real productivity fundamentals, could affect the economy in a different way as compared to the impacts of productivity uncertainty.

I first show the benchmark results, based on estimations of a range of trivariate VAR systems, which place various disagreement and uncertainty proxies prior to the U.S. aggregate investment series, as measured by the real gross private domestic investment. I select the aggregate investment series for the following reasons: (1) it is clearly a forward looking variable that is more closely related to forecasts and uncertainty about future; (2) as documented by Gilchrist et al. (2014), uncertainty shocks affect aggregate output primarily though the impacts on aggregate investment.

I present the estimated impulse responses of aggregate investment to one standard deviation jumps in the innovations to proxies for uncertainty and disagreement across various measurements. All variables are in log-levels with VARs estimated with four quarterly data lags. The sample period covers from 1970 Q1 to 2013 Q4, except for the system estimated using SPF data, which ranges from 1990 Q1 to 2013 Q4.

Figure 1.6 illustrates the impulse responses of aggregate investment, under the identification Scheme 1, where changes to uncertainty measures can respond to exogenous shocks that lead to more dispersed forecasts about the future. When uncertainty is measured by future TFP shocks innovations,**UNC**, aggregate investment drops on impact to the adverse uncertainty shocks until bottoming out at 1.5 to 2 % below the pre-shock level in about five quarters. Importantly, we find strong rebound and overshooting expansion, such that in five years, the mean prediction about aggregate investment is to reach 1-2 % above the pre-shock level. These "drop-rebound-overshoot" dynamics are consistent with the model-predicted real option effect of "wait and see" for uncertainty shocks. In addition, the identified effects of uncertainty shocks are robust regardless of how we measure forecast disagreement.

In response to shocks that trigger greater disagreement but are not due to fundamental changes in real productivity dispersion, aggregate investment experiences a persistent decline. Compared to the impacts of uncertainty shocks, the drops in investment reach the bottom with a greater decline of 3 % below the pre-shock level in about two years after the shock. Nonetheless, we see no sign of the "rebound-overshoot" path of investment, even five years following the disagreement shocks.

When uncertainty is measured as news-based reference frequency, **EPU**, we still see that aggregate investment can achieve a maximal drop of 3 % with limited recovery in response to increases in BOS disagreement. Conversely, the magnitude of the drops can be smaller and the estimated post-shock path to disagreement shocks is less precise when using **SPF** to proxy for disagreement. In addition, we get very imprecisely estimated impulse response paths of aggregate investment to jumps in **EPU** from two years after the shocks. Also, we only see weak rebound. Intuitively, the reason why the drop and rebound dynamics in case of uncertainty shocks is missing could be due to the fact that **EPU** captures more of the public *attention* towards uncertain public policies, which is no measure of the firms' productivity variance. For larger standard errors associated with impulse responses, it could be that as **EPU** counts for crisis and uncertainty related news, it already conditions itself on changes that directly affect private agents' beliefs. Therefore, the orthogonal changes in **EPU**, the supposedly identified uncertainty shocks would no longer have significant aggregate effects as disagreement shocks that affect dispersion of firms' beliefs are isolated first. In addition, since the SPF-based dispersion measure does not necessarily reflect the actual decision makers' forecast differences, this may also help explain the large variance of impulse responses of investment to both uncertainty and disagreement shocks when **SPF** enters the system.

Figure 1.7 shows the estimated impulse responses of aggregate investment, when Scheme 2 of identification is at work. The uncertainty shocks are first isolated when exogenous changes to disagreement do not, on impact, alter the distribution of real productivity fundamentals across firms. We found almost exact drops, quick rebound and overshoot of aggregate investment in response to the enlarged uncertainty shocks, in terms of the magnitude and timing of changes in aggregate investment dynamics. This is also true for the responses of investment to disagreement shocks, namely, persistent decline and very slow recovery if any. In addition, we found greater standard errors associated with estimations using **EPU** and **SPF**.

Table 1.2 summarizes the forecast error variance decomposition of aggregate investment at different forecast horizons, based on estimation of the trivariate VAR systems under identification of Scheme 1 ordering. For the near term of one quarter, innovations in **UNC** explain about 2 % of the variance of aggregate investment, while changes in the disagreement measure, **DIS** explain less than 1 % of variance in investment. However, in three years, innovations of disagreement proxies of both

DIS and **SPF** would explain around 30 % to 40 % of the variance, whereas changes in **UNC** explain at most 15 %. In about five years, more than half of the variance in investment, a significant magnitude of variations, can be explained by the dynamics of belief dispersion. Conversely, the fraction of the variance in aggregate investment, which can be explained by the dispersion of real productivity fundamentals, decays to roughly 12 % at most. In addition, when uncertainty is measured by **EPU** and the VAR estimation is coupled with SPF, we could still see a dominant fraction of aggregate investment dynamics as explained by the time-varying forecast disagreement.

I further show that with a larger VAR system, these empirical results are robust. In particular, consistent with the ordering considered in Bloom (2009), I estimate a 10-variable VAR system with Scheme 1 ordering: log(S&P500 stock market index) , log(disagreement measure), log(uncertainty measure), Federal Funds Rate , log(average hourly earnings in manufacturing) , log(consumer price index) , weekly average hours in manufacturing , log(non-farm payroll employment) , log(real gross private domestic investment) , and log(industrial production) . Such a benchmark identification proposes to isolate shocks to the belief dispersion, which are not due to changes in the variance of productivity fundamentals. Similarly, I consider the case when the disagreement and uncertainty measures are flipped in Scheme 2 ordering.

Rows (I) and (II) in Figure 1.8 display the impulse responses of aggregate investment, non-farm payroll employment, industrial production in logs to one standard deviation increase in innovations of **UNC** and **DIS**. Firstly, we still see the drop-rebound-overshoot pattern for the impulse responses of aggregate investment to increases in uncertainty shocks. The drop-rebound dynamics are also seen for the employment and industrial production, though limited overshoots are found. Note that the aggregate investment now bottoms at 1% below the pre-shock level around four quarters after the shocks, a smaller impact compared to the effect of uncertainty shocks identified for a trivariate system. On the contrary, in response to jumps in disagreement shocks, the aggregate investment, employment, and industrial production all undergo a persistent decline until reaching the bottom beyond two years. Despite the larger standard errors are associated with these impulse responses in the medium run to the disagreement shocks, we see no sign for rebounds, let alone overshoots. On expectation, it takes about five years for the investment, employment and production to fully return to the pre-shock levels subsequent to the disagreement shocks.

Rows (III) and (IV) plot the impulse responses of the three aggregate variables in a system when uncertainty is measured by **EPU**. Consistent with what we found based on the tri-variate system, we no longer found the drop-rebound-overshoot dynamics of these series in response to increases in the **EPU**-based uncertainty measure. Moreover, these impulse response paths are estimated very imprecisely, and the empirical effects of the **EPU**-based uncertainty shocks seem very trivial. This finding contrasts the Bloom et al. (2014), in which rises in the **EPU** index exhibit significant impacts that a contraction and a slow recovery are followed after the shocks. In our VAR system, note that it is the increases in the disagreement shocks that drive major macroeconomic series to drop and to stay low without a quick rebound. Therefore, it's intrigue to understand why the documented strong effects of **EPU** are gone, and why the disagreement shocks initiate the contraction effects. Some explanations can be offered here: first,the **EPU** does not necessarily reflect the decision makers' forecast errors about future productivity, as it is not a direct measure of the future productivity variance. Rather, it is a measure of the intensity of public attention paid to crisis events. That's why the "wait and see" dynamics as predicted by a model of productivity uncertainty shocks may not directly apply to a case when **EPU** is changed. Second, given the strong correlations between **EPU** and **DIS** index highlighted in Table 1.1, it may suggest that in absent of the forecast disagreement series as in Bloom et al. (2014), changes in **EPU** depress the economy by picking up the channel of the disagreement shocks, through which the economy is affected.

Again, under the reversed ordering of disagreement and uncertainty measure, Figure 1.9 demonstrates that our findings are robust such that rises in uncertainty have a short-run recessionary effect, which is followed by a quick rebound of expansion. Differently, the effects of disagreement shocks can have more persistent dampening effects for a duration of up to two or three years following the shocks. Table 1.3 shows the forecast error variance decomposition of aggregate investment and industrial production at horizons up to five years, based on estimation of the larger VAR system with Scheme 1 ordering. It shows that at three-years up to five-years, changes in the disagreement measure over time can account for up to 15 % of the variance in aggregate investment and approximately 8% to 14 % of the variance in production. By contrast, less than 5 % of the variances in investment and production can be explained by variations in the uncertainty about productivity fundamentals.

When the forecast disagreement is measured by the SPF-based forecast dispersion index, the estimated impulse responses are attached with very large standard errors such that neither the impacts of uncertainty nor those of disagreement can be well-identified. Figures 1.10 and 1.11 show that regardless of ordering scheme, estimation of a large VAR system with **SPF** included introduces too much noise that the effects of uncertainty and disagreement shocks are no longer statistically distinguishable from zero. It could be well due to the fact that the SPF data does not capture the firms' own views. In addition, a smaller sample size from 1990 with only three marked recession periods is also responsible for not being able to deliver the intended effects.

In sum, based on the results from a range of VAR exercises, I may conclude that the survey-based disagreement index and dispersion of future firm-level productivity fundamentals identify two separate channels, through which the economy are very differently affected. However, one concern to this argument is that the policy, for example, the monetary authority's intervention, may endogenously react differently to movements in disagreement and uncertainty measures, which then leads to different aggregate impacts. Figure 1.12 shows the impulse responses of Federal Funds Rate to increases in uncertainty and disagreement shocks. It suggests that little evidence can be found to support the argument that the active management of monetary policy is the cause for the different aggregate effects. Hence, it is safe to conclude that there exist two types of *second moment* shocks. In addition, we propose that the disagreement shocks are related to the information diffusion among firms, thus "informational disagreement shocks", for the following reasons. First, these shocks, by construction, affect the firms' belief distribution, which are orthogonal to changes to the distribution of economic fundamentals that are relevant for firms' decision-making, i.e. "real uncertainty shocks". Second, we see that by including forecast disagreement series, the significant effects associated with the Economic Policy Uncertainty Index that counts the number of news coverage are muted. Last but not least, I find the "wait and see" effect of real uncertainty shocks robust as long as the empirical measure of uncertainty corresponds to the theoretical notion of future productivity variance. Now, we proceed to provide additional micro-based evidence that lends further credence to these arguments.

1.4 Firm-level Investment: Micro Evidence

In this section, I provide micro evidence to demonstrate how the firm-level business investment is differently associated with changes in disagreement and uncertainty measures. In addition, I examine the consistency from evidence on firm-level investments to macro evidence found in Section 1.3. Estimations of firm-level investment dynamics are based on Compustat annual data from 1970 to 2013. My identification strategy is to augment the baseline empirical firm-level investment equation following Bloom et al. (2007) and Gilchrist et al. (2014) by incorporating the aggregate measures of forecast disagreement and uncertainty:

$$\log[I/K]_{i,t} = \beta_{i,0} + \theta \log MPK_{i,t} + \beta_1 \log DIS_t + \beta_2 \log UNC_t + \epsilon_{i,t} \tag{1.4}$$

where $[I/K]_{i,t}$ denotes the firm-level investment rate, which is measured by a firm's investment-capital ratio in year t. $\beta_{i,0}$ denotes the firm-specific fixed effect such that the time-invariant differences across firms in investment rate are controlled for. $MPK_{i,t}$ refers to the empirical proxy for the marginal product of capital, a measure for future investment opportunities (Gilchrist et al., 2014). I consider a range of $MPK_{i,t}$ proxies including $[Y/K]_{i,t}$ (current sales-to-capital ratio), $[\pi/K]_{i,t}$ (current operating income-to-capital ratio) following Gilchrist and Himmelberg (1999), along with Tobin's Q measure, $Q_{i,t}$. In addition, the cash flow - capital ratio $[CF/K]_{i,t}$, an empirically found strong predictor of firm-level investment, is considered the fourth proxy.[9] In the benchmark estimation, marginal product of capital is measured by the sales-capital ratio. See details on defining these empirical measures in Appendix A.2 and the data summary in Appendix A.3.

In addition, I continue with the BOS measure of forecast disagreement regarding firm-level six-month future forecasts about the General Business Conditions (**DIS**), which is annualized from monthly data by averaging within a year, and the measure of uncertainty based on the dispersion of future firm-level TFP shocks (**UNC**). These measures have been found to have better identification of the effects associated with disagreement and uncertainty shocks at the aggregate level. Note that the regression equation (1.4) does not accommodate a time fixed effect per the presence of the time-varying second moment measures. Partial effect coefficients, β_1 and β_2, capture the signs and magnitudes of *linear* relationships between un-

[9]Little consensus has established on how we interpret why cash flow matters for firm's investment. Fazzari et al. (1988) argue that cash flow reflects the financial constraint of the firm. Gilchrist and Himmelberg (1999) find it is also a measure of expected marginal product of capital, or a measure related to future demand and profitability growth (Bond et al., 2004).

certainty or disagreement measures with the firm-level investment rate. Because the uncertainty proxy dated in year t measures the future productivity dispersion in year $t+1$, and disagreement dated in year t incorporates two-quarter ahead forecasts difference across firms, these linear coefficients exhibit the "en-ante" associations of firm-level investments with the dispersion of *to-be-realized* firms' productivities and the dispersion of *expectations*. In addition, I include lagged uncertainty and forecast disagreement dated in $t-1$ so as to explore the linear "ex-post" relationships about dispersion of realized productivities and "now-forecasts".

Moreover, a main task to establish the consistency between macro- and micro-based evidence is to identify the firm-level investment rebound dynamics associated with increases in productivity uncertainty, and the absence of a rebound in investment for greater forecast disagreement. By Bloom (2009), the key mechanism for rebound and overshoot of aggregate investment hinges on an intensive margin effect of uncertainty, i.e. the convexity effect, or Hartman-Abel effect (Hartman, 1972; Abel, 1983; Bloom, 2009). That is, firms see themselves more productive in expectation, as the variance of future idiosyncratic productivity increases. This implies that firms with larger productivity draws have larger investments when uncertainty goes up, because of the increased expected future productivity. Hence, to identify the "rebound and overshoot" dynamics, I further consider the *non-linear* associations of disagreement and uncertainty with the firm's investment rate, conditional on how productive a firm is. I use the estimated log firm-level TFP à la Imrohoroglu and Tüzel (2014), based on the Compustat firm sample to measure firm-specific productivity. Then, I merged the estimated TFP panel data with the firms' operation metrics. Thus, a futher augmented specification of the empirical equation about the firm-level investment rate is given by:

$$\begin{aligned}\log[I/K]_{i,t} =& \beta_{i,0} + \theta \log MPK_{i,t} + \beta_1 \log DIS_t + \beta_2 \log UNC_t \\ &+ \beta_3 TFP_{i,t} + \beta_4 \Delta TFP_{i,t} + \beta_5 [\Delta TFP_{i,t}]^2 \\ &+ \beta_6 \log DIS_t \times \Delta TFP_{i,t} + \beta_7 \log UNC_t \times \Delta TFP_{i,t} + \epsilon_{i,t} \end{aligned} \tag{1.5}$$

In particular, as shown in Equation (1.5), logged firm-level TFP ($TFP_{i,t}$) interacts with disagreement and uncertainty in terms of its growth rate $\Delta TFP_{i,t}$. By dropping the linear terms of disagreement and uncertainty, I also consider the case when the time fixed effect is controlled for to disentangle the non-linear relationships between firm-level investments and these interaction terms.

Table 1.4 summarizes the key estimation results. Columns 1 and 2 show that the associations of uncertainty or disagreement about the to-be-realized productivity fundamentals with firm-level investment rates are statistically positive, while the "ex-post" associations are statistically negative for the lagged linear terms. The positive associations are not surprising for the following reason: the estimated coefficients pick up the theoretically documented convexity or Hartman-Abel effects, as both proxies of uncertainty and disagreement could approximate for the firms' forecast errors about future economic fundamentals. Increased forecast errors lead to larger expected future productivity for all the firms, and thus larger investments on average. Important to note that such evidence is not directly comparable with the findings documented in other empirical works using micro-level data that find negative associations. Firstly, the empirical literature about uncertainty mainly relies on a particular type of proxies, which are mainly constructed from the firms' stock-market returns (Leahy and Whited, 1995; Bloom et al., 2007). However, as Caldara et al. (2016) shows, these uncertainty measures are highly correlated with the firms' financial distress measures, only through which, changes in uncertainty could have contraction effects. Secondly, those documented negative linear associations are "ex-post" regarding stock return realizations (Bloom et al., 2007). Therefore, based on our specification that includes both the contemporaneous and lagged disagreement and uncertainty terms, we still obtained the statistically significant negative partial effects for UNC_{t-1} and DIS_{t-1}. In the spirit of Bloom et al. (2007); Gilchrist et al. (2014), these negative relationships are considered evidence for the "wait and see" real-option effects. Firms pause without taking additional investments upon more dispersed productivity and more dispersed beliefs about "now".

Results in column 3 further suggests that controlling both uncertainty and disagreement measures, the positive "ex-ante" associations are robust, despite the reduced sizes of both convexity effects. Importantly, we see that the magnitudes of negative associations between firm-level investments and "now-cast" disagreement measure is no longer significant, and the magnitude is lessened by a tremendous degree. This suggests that changes in realized productivity dispersion are more likely linked to the "wait and see" motive in line with Bloom (2009) whereas the real option effect can be very trivial, if present at all, for ex-post disagreement changes.

Columns 4-6 show the estimation results for Equation (1.5). Firstly, we still see the following: (1) the convexity effects are pronounced for both uncertainty and forecast disagreement about to-be-realized productivity; (2) rises in the ex-post productivity dispersion are associated with drops in firm-level investment. Despite it still being significant at the 5 % level, the negative and linear real-option effect associated with forecast disagreement about the current period is very limited, when uncertainty and disagreement proxies are both controlled; (3) once controlled for both second moment proxies, all the partial effects related to uncertainty and disagreement are smaller, relative to the results presented in columns 4 and 5.

Particularly, we focus on the partial effects of the two interaction terms that involve firm-level TFP growths. According to columns 4-6, more productive firms, as measured by greater firm-level log TFP growths, are associated with larger firm-level investment rates when uncertainty is higher. This positive effect of the interaction term of uncertainty, conditional on productivity growth, confirms the hypothesis for the existence of rebound dynamics per higher uncertainty. Therefore, productivity uncertainty yields a *productivity-enhancing* effect because when the variance of future productivity increases, firms with larger productivity draw would have a greater expected productivity draw, which translates into a larger investment.

Conversely, column 5 finds a *productivity-dampening* effect for larger forecast disagreement. For firms that disagree more about their future business conditions by 1 %, more productive firms reduce their investments, while less productive firms increase their investments by 0.18 %. Therefore, we found both decreases and increases in investments, conditional on how productive firms are. It implies that, as the firm-level investment responds to changes in firm-specific productivity, large investment spikes that are otherwise taken by firms with greater productivity draws, are knocked down when firms disagree more about their future business conditions, even if the fundamental productivity distribution does not change. When controlling both second moment measures, column 6 finds statistically significant positive and negative signs, respectively, for the two interaction terms as well as significant effects. These results suggest that there are two separate channels through which forecast disagreement and future productivity uncertainty are related to firm-level investments. Column 7 illustrates that, with linear effects of disagreement and uncertainty captured by time fixed effects, non-linear impacts are robust with very little changes to the estimated coefficients, relative to the data in column 6.

Table 1.5 shows the results that validate our main findings are robust using different proxies for the marginal product of capital. Column 1 has the exact same numbers as column 6 in Table 1.4 to aid comparisons across the specifications. It shows that across the specifications of column 1-3, apart from the convexity effects and "wait and see" effects, the productivity-enhancing effect of uncertainty and the dampening effect of forecast disagreement are statistically significant. Interesting to note, we still find that the negative association between ex-post productivity uncertainty and firm-level investment is substantial, suggesting a strong "wait and see" channel. However, such real-option effect for forecast disagreement is very limited. In addition, the estimated elasticity for forecast disagreement, conditional on productivity growth, is consistently around -0.43, an economically significant number. For the case where $MPK_{i,t}$ is measured by Tobin's Q, the non-linear effects for both uncertainty and disagreement are no longer well estimated. This can be due to the fact that a significant number of year-firm observations of market value of total equity are missing.

I further provide additional evidence of robustness of these micro-level findings by controlling the lagged firm-level investment rate. As found by Gilchrist and Himmelberg (1999) and Eberly et al. (2012), lagged firm-level investment is a strong predictor of firm-level investment dynamics. Since the lagged investment is correlated with the unobserved firm-level individual effect, the OLS estimator is no longer consistent. Following Bloom et al. (2007), I adopt the Arellano-Bond GMM estimation procedure à al Arellano and Bover (1995); Blundell and Bond (1998). I consider the following all endogenous: the $TFP_{i,t}$, $MPK_{i,t}$, disagreement and uncertainty measures, along with their respective interactions with TFP growth rates. Three lags in log levels of these variables are included in the difference equation of the GMM system as instruments. In addition, the difference in log levels of these variables, up to three lags, are included in the level equation of the GMM system as instruments.

Table 1.6 summarizes the estimation results of dynamic specifications. It shows that the lagged investment is positively correlated with current firm-level investment rate, with elasticity estimated around 0.4-0.5. We observe little change to the signs and the magnitudes of the partial effects related to the linear relationships between uncertainty or disagreement and firm-level investment. However, the "wait and see" effect related to the firms' disagreement is found to be larger, which is at the similar degree of magnitude compared to the counterpart coefficients for uncertainty. Regarding the non-linear effects, the statistically significant coefficient for the interaction term of uncertainty and productivity growth rate is positive, around 0.2 when $MPK_{i,t}$ is measured by sales-capital ratio and profit-capital ratio, which is comparable with the magnitude of the estimate based on the static specifications. Using cash flows or Tobin's Q measures of $MPK_{i,t}$, such a partial effect of the interaction term is imprecisely estimated. By contrast, firms with greater productivity growth, conditional on a 1 % increasing disagreement environment, would cut their logged investment rates by 0.45 % unless $MPK_{i,t}$ is measured by Tobin's Q. This estimate of elasticity is of similar magnitude to that obtained from static estimations.

In summary, micro evidence exhibits that productivity uncertainty has the convexity effect and the "wait and see" effect, consistent with a model of uncertainty shocks. In addition, it shows a productivity-enhancing effect such that more productive firms would increase investment when productivity becomes more dispersed for the future. This serves the micro-foundation for a quick rebound and overshoot of investments at the aggregate level. On the contrary, the convexity effect can be stronger than its trivial "wait and see" effect associated with forecast disagreement. More importantly, it is the partial effect of the non-linear term, marked by a productivity-dampening effect that the forecast disagreement differs from productivity uncertainty by its distinctive effect on firm-level investment.

1.5 Discussion: Shock Identifications

At both the aggregate level and the firm level, we found that innovations to the firms' forecast disagreement can affect aggregate dynamics and the firm-level investment in a different way, as compared to the impacts of productivity uncertainty shocks. The distinct feature that distinguishes impulse responses of major macroeconomic aggregates to uncertainty shocks from those to disagreement shocks, is whether or not there is quick rebound and overshoot dynamics. At the firm level, whether the elasticity of the firm-level investment conditional on uncertainty productivity growth with respect to changes to second moment proxy is positive or negative, helps identify if disturbances originate from changes to the spread of real economic fundamentals, or from changes to the dispersion of heterogeneous beliefs.

A more general implication can be drawn to implement the identification of the different types of second moment shocks in the data. As larger productivity uncertainty pushes more firms into the inaction band, through the "wait and see" channel (Bloom, 2009), the dispersion of firm-level investment rates across firms is reduced. However, the non-linear productivity-enhancing effect associated with productivity uncertainty suggests that greater dispersion of future productivity shocks will translate into increased dispersion of investment rates. It is less clear how these two offsetting forces, due to jumps in productivity uncertainty, will drive the investment rate dispersion.[10] However, the presence of the similar but trivial "wait-and-see" channel, and the productivity-dampening effect associated with forecast disagreement could both reduce the dispersion of investment rates across firms.

Hence, this implication for identification is tested by estimating the forecasting equation shown below, regarding the firm-level investment rate dispersion. The

[10] Bachmann and Bayer (2014) is the first paper that highlights such ambiguity and theoretically examines the trade-off between real-option effect and productivity-enhancing effect, with respect to how the cyclicality of firm-level investment rates dispersion is determined.

specification in Caldara et al. (2016) is followed, which examines the near-term aggregate implications of various uncertainty measures.

$$\Delta \log \sigma_{t,I/K} = \beta_0 + \beta_1 UNC_{t-h} + \beta_2 DIS_{t-h} + \sum_{i=0}^{q} \theta_i \Delta \log \sigma_{t-h-i,I/K} + \epsilon_t \tag{1.6}$$

The dependent variable is the growth rate of standard deviation of firm-level investment-capital ratios $[I/K]_{i,t}$ across firms. The coefficients of β_1 and β_2 provide how changes to the dispersion of firm-level investment rates are related to the productivity dispersion and forecast disagreement $h > 0$ quarters in the past. Based on the OLS estimation, I estimate this equation to examine the effects of uncertainty and disagreement for different horizons: $h = 1, 2, 3, 4$. I consider $q = 1$ such that the growth rates of investment rate dispersion dated in quarters $t - h$ and $t - h - 1$ are controlled. The estimation results are summarized in Table 1.7.

As illustrated in Table 1.7, both uncertainty and disagreement increases are associated with slower growth of investment rate dispersion regardless of the length of forecast horizon. However, when forecast disagreement and productivity uncertainty are both controlled, only the increases in the past disagreement suggest the significant shrinkage in the growth rate of the investment rate dispersion in the future, whereas uncertainty does not help predict the dynamics of the investment rate dispersion. These results can be explained by the fact that productivity uncertainty could have shifted the dispersion of investment rates because of its high correlation with the disagreement series. Hence, we have verified the implication for identification that it is the changes in belief difference over time instead of changes to the productivity uncertainty, which contributes to the time variation of the investment rate dispersion.

In addition, we see that the macro- and micro-based evidence are consistent with each other. Aggregate rebound and overshoot of aggregate investment after jumps in uncertainty shocks could be consequences of the increased size of individual investment spikes taken by more productive firms. Qn the contrary, when belief changes are not backed by good or bad economic fundamentals, innovations that increase the dispersion of the firms' heterogeneous beliefs about future business conditions could increasingly dampen the size of investment spikes taken by more productive firms. As a result, in absent of the rebound dynamics, we could see a slower recovery of aggregate investment.

It shows that by isolating the time-varying disagreement shocks, we are better equipped to understand the mechanism that drives the dynamics of firm-level investment rate dispersion. As larger disagreement dampens investment spikes among more productive firms and induces investment jumps among less productive firms, the dispersion of the investment rate can be shrunk.

1.6 Conclusion

This chapter provides empirical evidence at both the aggregate level and the firm level to demonstrate that survey-based forecast dispersion identifies a different type of second moment shocks that affect firm-level belief dispersion, which are not backed by good or bad economic fundamentals. Such pure informational disagreement shocks differ from the canonical uncertainty shocks that directly affect the variance of real economic fundamentals, given their very different macro and micro implications.

Using firm level forecasts dispersion to measure disagreement, macro series such as aggregate investment, employment, and industrial production, all experience a persistent decline followed by a slow recovery in response to greater disagreement shocks. Conversely, when uncertainty is measured by the cross-sectional dispersion of future firm-specific productivity innovations that corresponds well to the theoretical concept of productivity uncertainty in the model of uncertainty shocks, the "wait and see" effect of drop-rebound-overshoot of macro aggregates is robust, following jumps in productivity uncertainty.

At the micro-level, conditional on being more productive, firm producers tend to invest more as a larger variance of future productivity increases the expected marginal product of capital. Such findings confirm the source at the firm-level, for a macroeconomic rebound. However, by creating informational confusion about future business conditions, innovations that trigger greater disagreement among firms dampen the size of investment spikes among more productive firms, which results in a more persistent economic downturn and a weak recovery.

Isolating disagreement shocks helps better explain two major facts that cannot be reconciled with a model of counter-cyclical uncertainty shocks only as found in Bachmann and Bayer (2014): (1) It is the dampened investment spikes during bad times that lead to drops in aggregate investment; (2) The dispersion of firm-level investments is pro-cyclical.

By isolating the informational disagreement shocks, this chapter finds that the dispersion of firms' heterogeneous beliefs is not a good measure of the concept of economic uncertainty, as defined to be the variance of future productivity in a model. In addition, given that the forecast disagreements have been found to be negatively correlated with changes in dispersion of firm-level investment rates. Understanding the dynamics about time-varying forecast disagreement helps answer why the firm-level investment rate dispersion is pro-cyclical in the data.

1.7 Tables

Table 1.1: Correlations Between Proxies of Disagreement and Uncertainty

Forecast	**DIS** (General)		**DIS** (Order)		**SPF** (Real GDP)		**SPF** (IP)	
Lag/Lead (h)	**UNC**	**EPU**	**UNC**	**EPU**	**UNC**	**EPU**	**UNC**	**EPU**
-3	0.230***	0.107	0.115	-0.02	0.270***	0.091	0.207**	0.154
-2	0.243***	0.138*	0.120	-0.01	0.350***	0.171*	0.248**	0.215**
-1	0.271***	0.223***	0.137*	0.090	0.401***	0.330***	0.273***	0.289***
0	0.307***	0.286***	0.163**	0.180**	0.417***	0.300***	0.271***	0.274***
1	0.339***	0.268***	0.187**	0.146*	0.393***	0.239**	0.243**	0.246**
2	0.372***	0.224***	0.213***	0.111	0.351***	0.265	0.190*	0.235**
3	0.400***	0.207***	0.242***	0.105	0.287***	0.197*	0.116	0.165

Notes: Numbers are the pairwise correlation coefficients between a time series of disagreement measure in quarter t and series of uncertainty dated at quarter $t + h$. h reflects leads and lags about uncertainty proxies **UNC** and **EPU**. "General" and "Order" respectively refer to the BOS disagreement index (**DIS**) using forecast data about General Business Condition and about New Order in six months relative to the survey date. Monthly disagreement index and monthly Economic Policy Uncertainty Index **EPU** are converted to quarterly using within-quarter averages. "GDP" and "IP" respectively refer to Survey of Professional Forecasters quarterly forecast dispersion series (**SPF**) regarding forecasts about Real GDP and about Industrial Production two quarters ahead. Quarterly data of **UNC** is obtained via interpolation of yearly data. Sample period: 1970Q1 - 2013Q4 except that series of **SPF** from 1990Q1-2013Q4 is used for computing its correlations with uncertainty proxies. * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

Table 1.2: Aggregate Investment: Forecast Variance Due to Innovations in Disagreement and Uncertainty (Trivariate VARs)

VAR System	Horizon:	One Quarter	One Year	Three Years	Five Years
(1)	**UNC**	2.22	11.32	14.39	11.56
	DIS	0.70	10.53	38.83	51.00
(2)	**EPU**	2.28	7.42	6.36	4.99
	DIS	0.10	10.68	42.19	55.82
(3)	**UNC**	0.54	4.80	7.52	15.38
	SPF	3.30	21.18	57.77	58.41
(4)	**EPU**	7.10	11.97	5.57	4.03
	SPF	4.61	16.55	27.67	30.11

Notes: Each cell number in a row denotes the fraction (in percent) of the total forecast error variance of log aggregate investment due to innovations in either uncertainty proxy (**UNC** or **EPU**) vs. in disagreement proxy (**DIS** or **SPF**) for a particular VAR system estimated. Column 1 refers to the four trivariate VAR systems estimated using different combinations of uncertainty and disagreement measures (Disagreement ordered before uncertainty proxy: Scheme 1 Ordering); see text for details on specification of these VARs. Sample period: 1970Q1 - 2013Q4 except those systems that involve series of **SPF**, which has data ranging from 1990Q1-2013Q4.

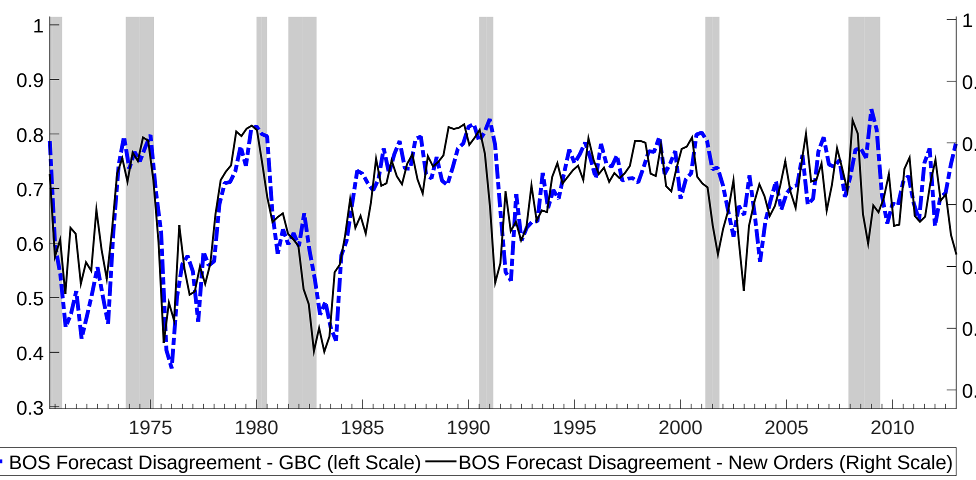
Table 1.3: Aggregate Investment and Industrial Production: Forecast Variance Due to Innovations in Disagreement and Uncertainty (Large VAR Systems)

VAR System	Horizon:	One Quarter	One Year	Three Years	Five Years
Aggregate Investment					
(1)	**UNC**	0.10	2.92	3.13	2.99
	DIS	2.58	8.21	13.45	13.11
(2)	**EPU**	1.39	0.90	0.57	1.00
	DIS	0.33	8.63	15.68	15.03
Industrial Production					
(1)	**UNC**	0.22	4.82	3.64	3.05
	DIS	3.90	7.22	7.56	7.76
(2)	**EPU**	0.51	0.26	0.24	0.33
	DIS	2.46	9.09	13.44	13.93

Notes: Each cell number in a row denotes the fraction (in percent) of the total forecast error variance of log aggregate investment due to innovations in either uncertainty proxy (**UNC** or **EPU**) vs. in disagreement proxy (**DIS** or **SPF**) for a particular VAR system estimated. Column 1 refers to the large VAR systems estimated using either pair of proxies **DIS** and **UNC**; system (1) or **DIS** and **EPU**; system (2) with disagreement measure ordered before uncertainty proxy: Scheme 1 Ordering; see text for details on specification of these VARs. Sample period: 1970Q1 - 2013Q4.

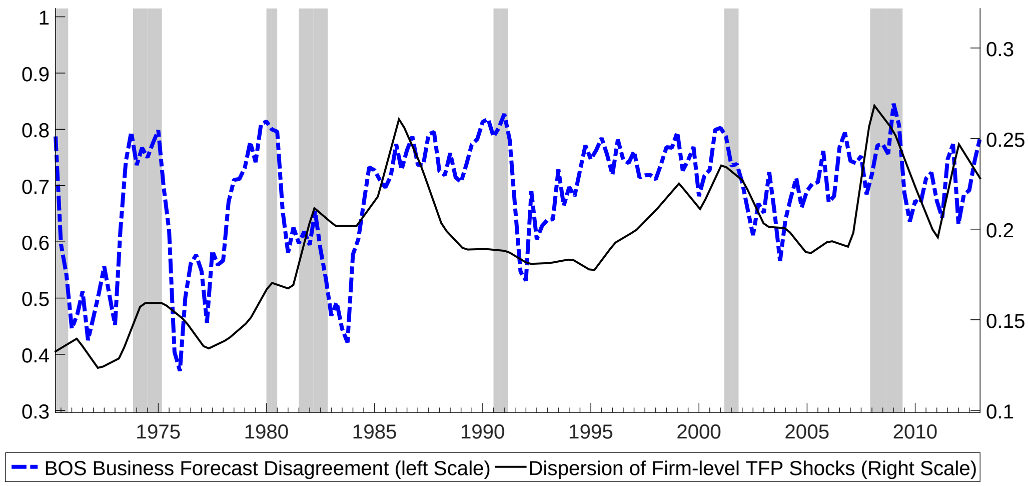
1.8 Figures

Figure 1.1: BOS Forecast Disagreement: Forecast of General Business Condition and Forecast of of New Orders



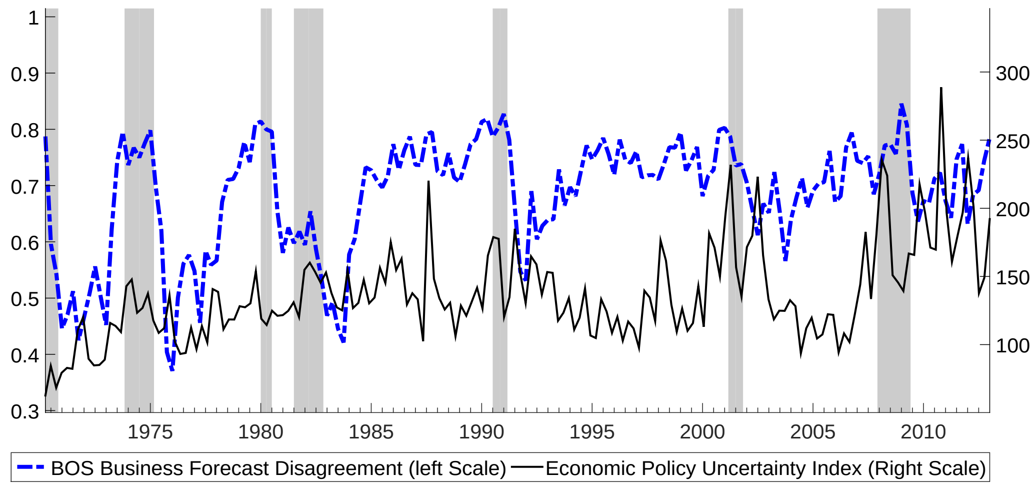
NOTES: Sample period: 1970:Q1 - 2013:Q4. The dashed line captures the magnitude of cross-sectional difference in six-month ahead forecast of "general business condition" and the solid line denotes the forecast difference in "new orders" forecasts among manufacturing firms based on Philadelphia Fed Business Outlook Survey. The disagreement indexes are constructed in line with Bachmann et al. (2013b). The shaded bars indicate the NBER-dated recession periods.

Figure 1.2: BOS Forecast Disagreement and Dispersion of Firm-level TFP Shocks



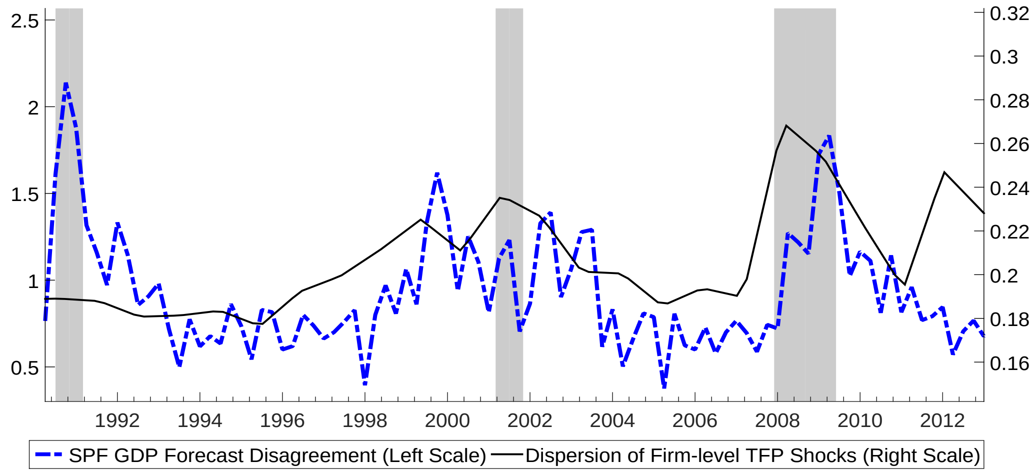
NOTES: Sample period: 1970:Q1 - 2013:Q4. The dashed line captures the magnitude of cross-sectional difference in six-month ahead forecast of "general business condition" among manufacturing firms based on Philadelphia Fed Business Outlook Survey data. The disagreement index is constructed in line with Bachmann et al. (2013b). The solid line depicts the estimate of dispersion of firm-level TFP innovations based on Compustat non-financial firms' data in line with Bloom et al. (2014).The shaded bars indicate the NBER-dated recession periods.

Figure 1.3: BOS Forecast Disagreement and Economic Policy Uncertainty



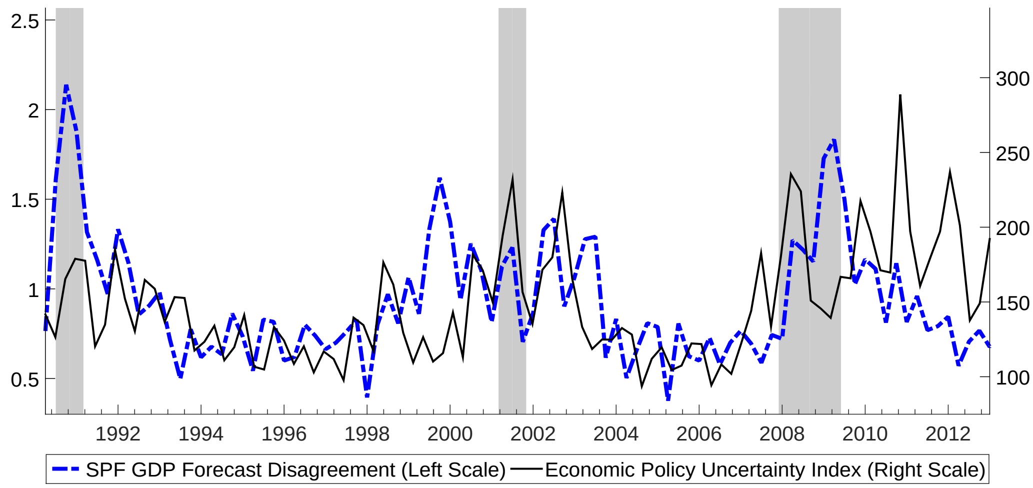
NOTES: Sample period: 1970:Q1 - 2013:Q4. The dashed line captures the magnitude of cross-sectional difference in six-month ahead forecast of "general business condition" among manufacturing firms based on Philadelphia Fed Business Outlook Survey data. The disagreement index is constructed in line with Bachmann et al. (2013b). The solid line depicts the media-based estimate of economic policy uncertainty based on Baker et al. (2015).The shaded bars indicate the NBER-dated recession periods.

Figure 1.4: SPF Forecast Disagreement and Dispersion of Firm-level TFP Shocks



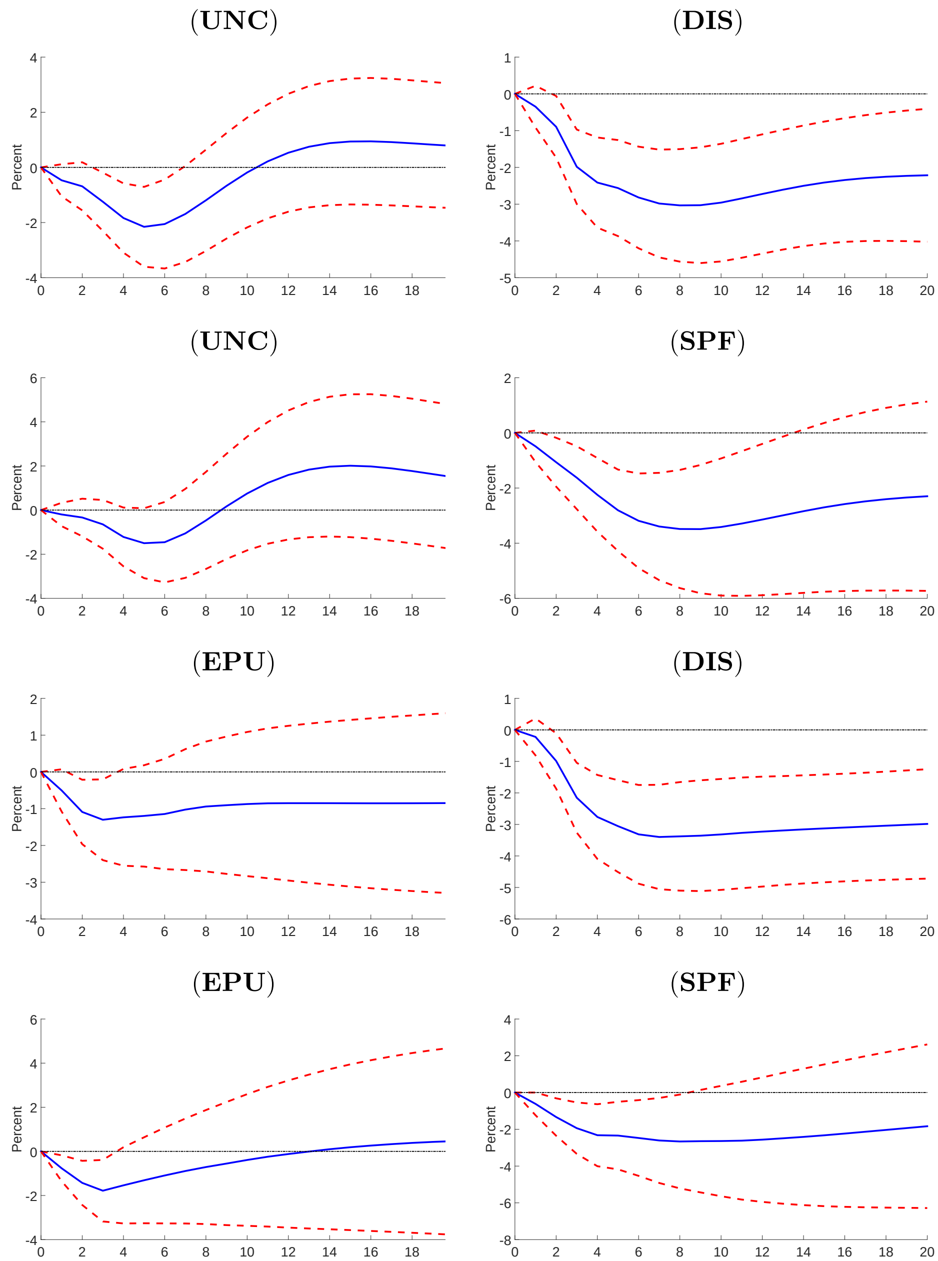
NOTES: Sample period: 1990:Q1 - 2013:Q4. The dashed line captures the magnitude of 75 percentile relative to 25 percentile difference in six-month ahead forecast of "Real GDP" among professional forecasters published by Philadelphia Fed Survey of Professional Forecasters (SPF) data. The solid line depicts the estimate of dispersion of firm-level TFP innovations based on Compustat non-financial firms' data in line with Bloom et al. (2014).The shaded bars indicate the NBER-dated recession periods.

Figure 1.5: SPF Forecast Disagreement and Economic Policy Uncertainty



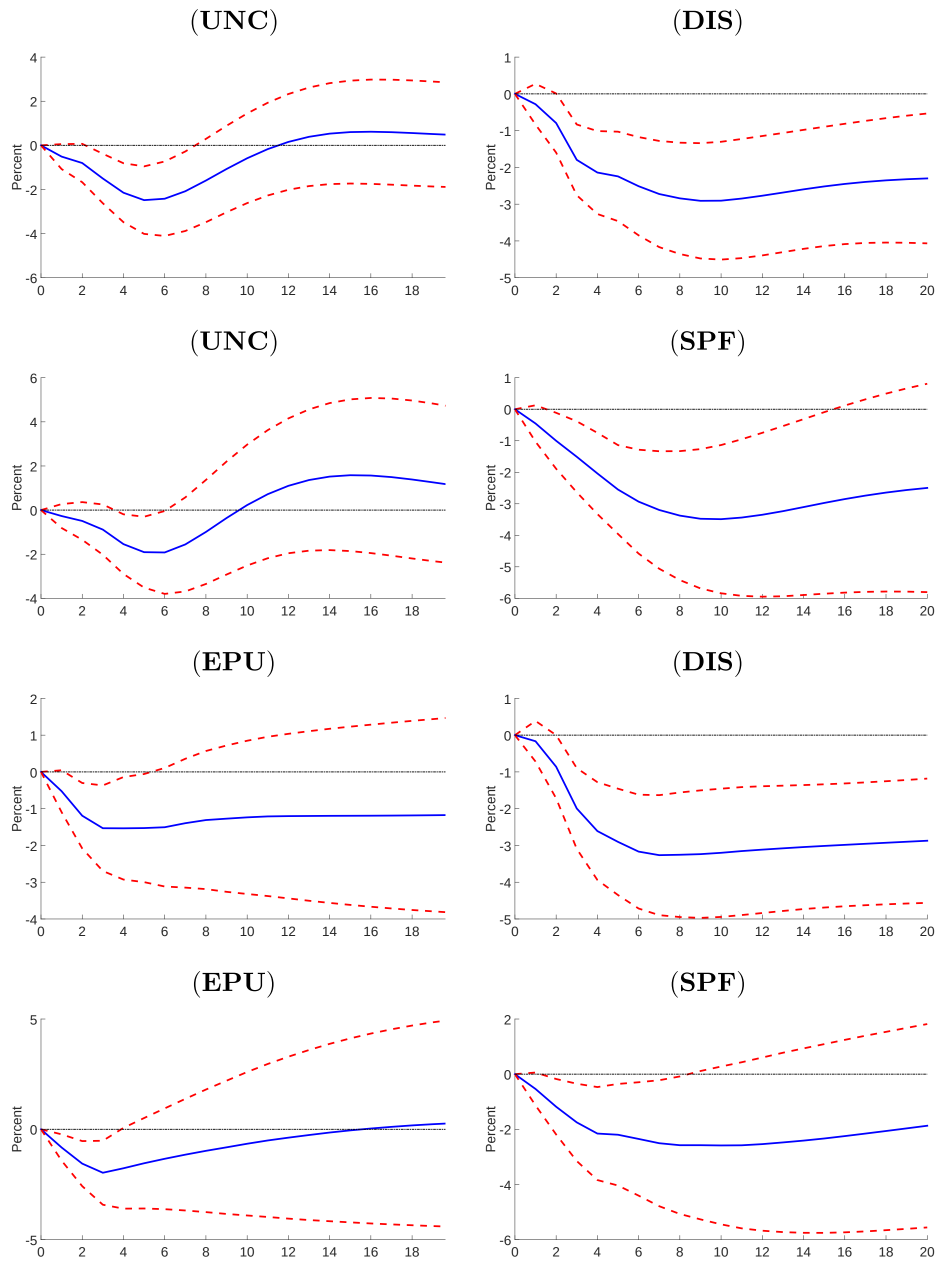
NOTES: Sample period: 1990:Q1 - 2013:Q4. The dashed line captures the magnitude of 75 percentile relative to 25 percentile difference in six-month ahead forecast of "Real GDP" among professional forecasters published by Philadelphia Fed Survey of Professional Forecasters (SPF) data. The solid line depicts the media-based estimate of economic policy uncertainty based on Baker et al. (2015).The shaded bars indicate the NBER-dated recession periods.

Figure 1.6: IRFs of Aggregate Investment: Uncertainty and Disagreement Shocks (Trivariate VAR - Ordering: Disagreement, Uncertainty and Investment)



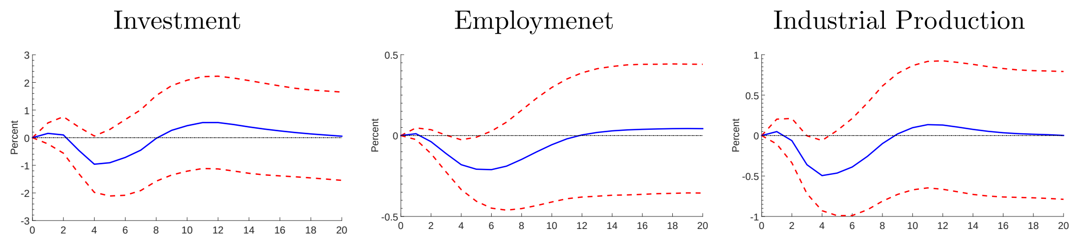
NOTES: This figure plots impulse responses of U.S. Real Gross Private Domestic Investment to 1 % increase in disagreement proxies (**DIS** and **SPF**) and uncertainty proxies (**UNC** and **EPU**) based on the estimation of a tri-variate VAR system (all in log levels) with 4 lags using quarterly data; see the text for details. Left column: responses to uncertainty shocks. Right column: responses to disagreement shocks. Each row shows the estimated responses to a particular pair of measures as indicated in the brackets. **DIS** is based on forecast data for "General Business Condition". **SPF** is based on forecast data for Real GDP. Sample period: 1970Q1 - 2013Q4 except for VAR systems involving SPF data: 1990Q1-2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations

Figure 1.7: IRFs of Aggregate Investment: Uncertainty and Disagreement Shocks (Trivariate VAR - Reversed Ordering: Uncertainty, Disagreement and Investment)

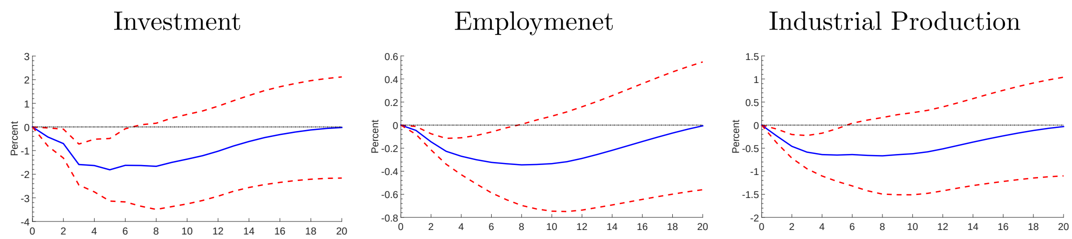


NOTES: This figure plots impulse responses of U.S. Real Gross Private Domestic Investment to 1 % increase in disagreement proxies (**DIS** and **SPF**) and uncertainty proxies (**UNC** and **EPU**) based on the estimation of a tri-variate VAR system (all in log levels) with 4 lags using quarterly data; see the text for details. Left column: responses to uncertainty shocks. Right column: responses to disagreement shocks. Each row shows the estimated responses to a particular pair of measures as indicated in the brackets. **DIS** is based on forecast data for "General Business Condition". **SPF** is based on forecast data for Real GDP. Sample period: 1970Q1 - 2013Q4 except for VAR systems involving SPF data: 1990Q1-2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations

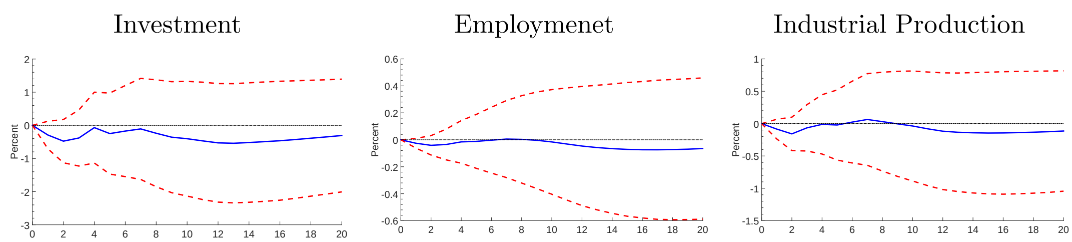
Figure 1.8: Aggregate Implications: Uncertainty and Disagreement Shocks
[Disagreement Index (**DIS**) Ordered Before Uncertainty)]



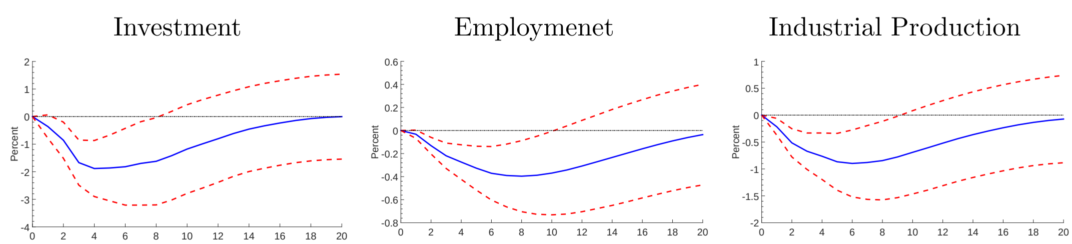
(I) Responses to an Uncertainty Shock (**DIS-UNC**)



(II) Responses to an Disagreement Shock (**DIS-UNC**)



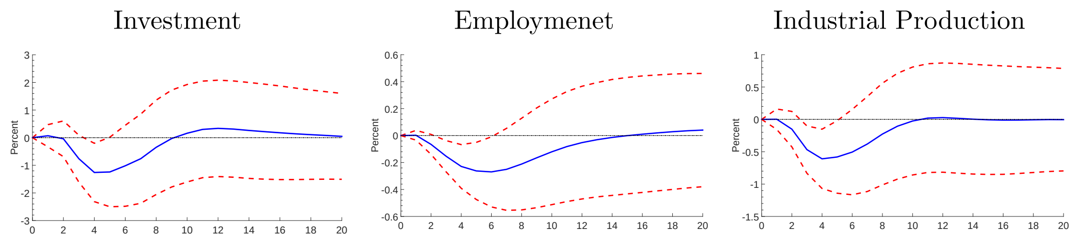
(III) Responses to an Uncertainty Shock (**DIS-EPU**)



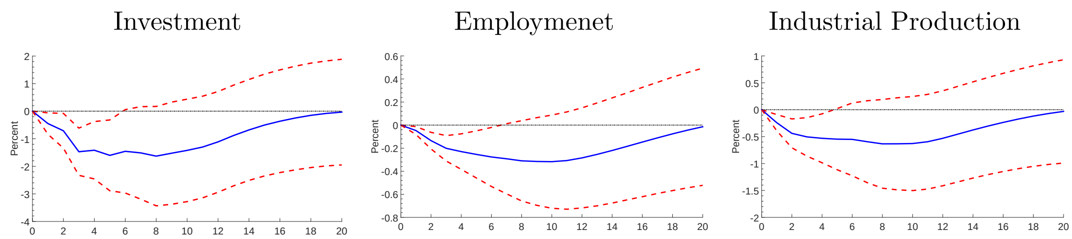
(IV) Responses to an Disagreement Shock (**DIS-EPU**)

NOTES: This figure plots impulse responses of U.S. real private domestic investment (first column), non-farm payroll employment (second column), and industrial production (third column) to 1 % increase uncertainty (**UNC** or **EPU**)and disagreement proxies (**DIS**), obtained from estimation of a ten-variable system of VAR with Scheme 1 Cholesky recursive ordering; see details in text. The frequency of data is quarterly and the VARs are estimated with 4 lags. The sample covers 1990Q1 to 2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations.

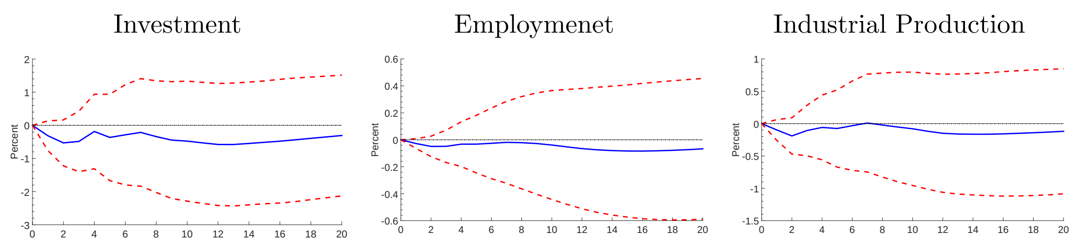
Figure 1.9: Aggregate Implications: Uncertainty and Disagreement Shocks
[Uncertainty Measure Ordered Before Disagreement Index (**DIS**)]



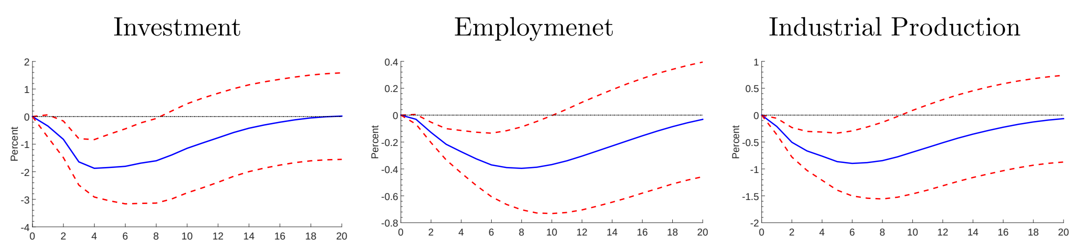
(I) Responses to an Uncertainty Shock (**UNC-DIS**)



(II) Responses to an Disagreement Shock (**UNC-DIS**)



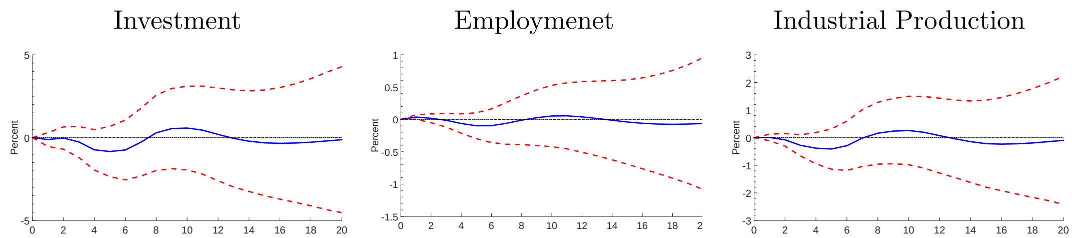
(III) Responses to an Uncertainty Shock (**EPU-DIS**)



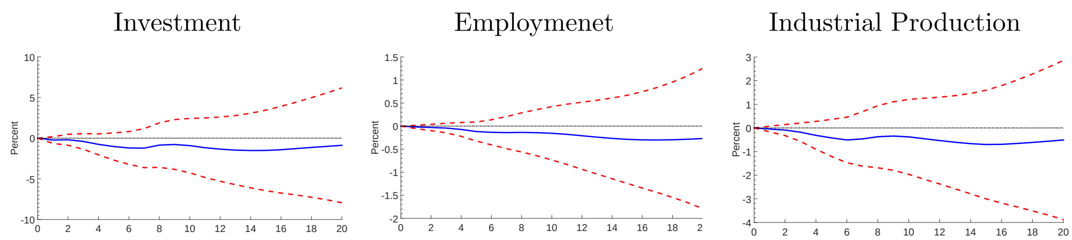
(IV) Responses to an Disagreement Shock (**EPU-DIS**)

NOTES: This figure plots impulse responses of U.S. real private domestic investment (first column), non-farm payroll employment (second column), and industrial production (third column) to 1 % increase uncertainty (**UNC** or **EPU**)and disagreement proxies (**DIS**), obtained from estimation of a ten-variable system of VAR with Scheme 2 Cholesky recursive ordering; see details in text. The frequency of data is quarterly and the VARs are estimated with 4 lags. The sample covers 1990Q1 to 2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations.

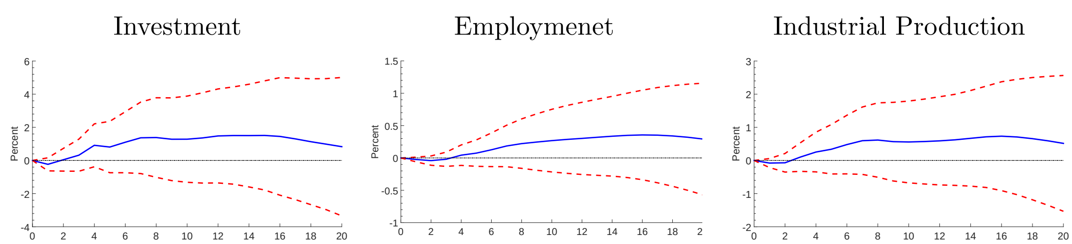
Figure 1.10: Aggregate Implications: Uncertainty and Disagreement Shocks
[Disagreement Index (**SPF**) Ordered Before Uncertainty)]



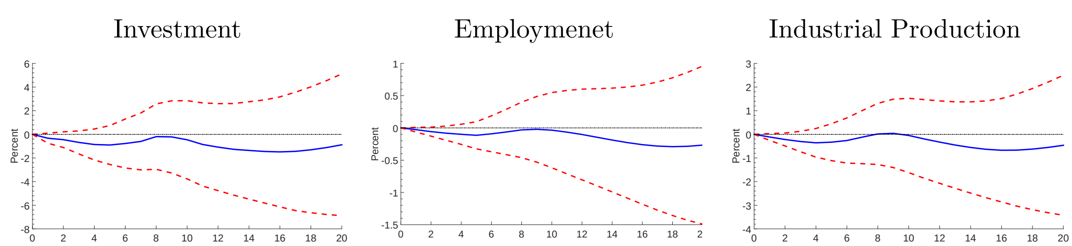
(I) Responses to an Uncertainty Shock (**SPF-UNC**)



(II) Responses to an Disagreement Shock (**SPF-UNC**)



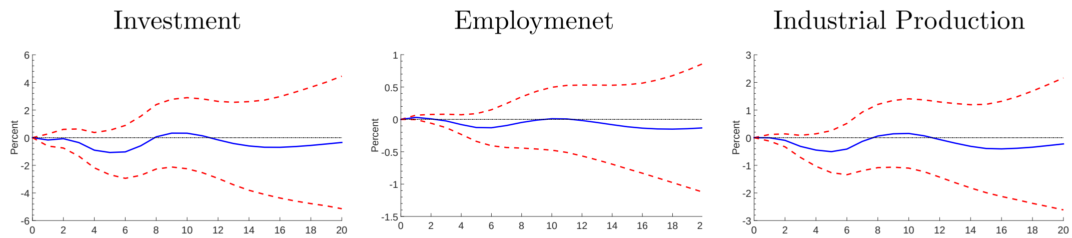
(III) Responses to an Uncertainty Shock (**SPF-EPU**)



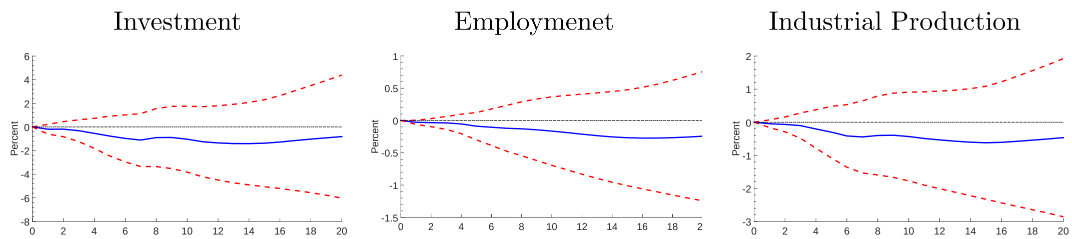
(IV) Responses to an Disagreement Shock (**SPF-EPU**)

NOTES: This figure plots impulse responses of U.S. real private domestic investment (first column), non-farm payroll employment (second column), and industrial production (third column) to 1 % increase uncertainty (**UNC** or **EPU**)and disagreement proxies (**SPF**), obtained from estimation of a ten-variable system of VAR with Scheme 1 Cholesky recursive ordering; see details in text. The frequency of data is quarterly and the VARs are estimated with 4 lags. The sample covers 1990Q1 to 2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations.

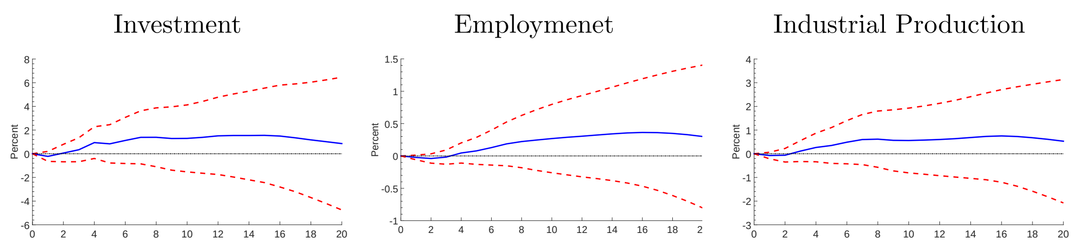
Figure 1.11: Aggregate Implications: Uncertainty and Disagreement Shocks
[Uncertainty Measure Ordered Before Disagreement Index (**SPF**)]



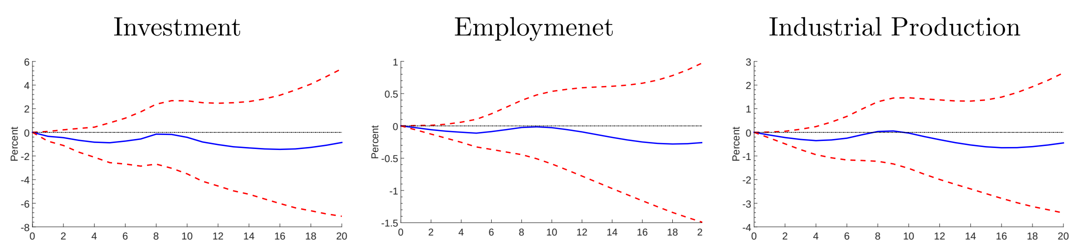
(I) Responses to an Uncertainty Shock (**UNC-SPF**)



(II) Responses to an Disagreement Shock (**UNC-SPF**)



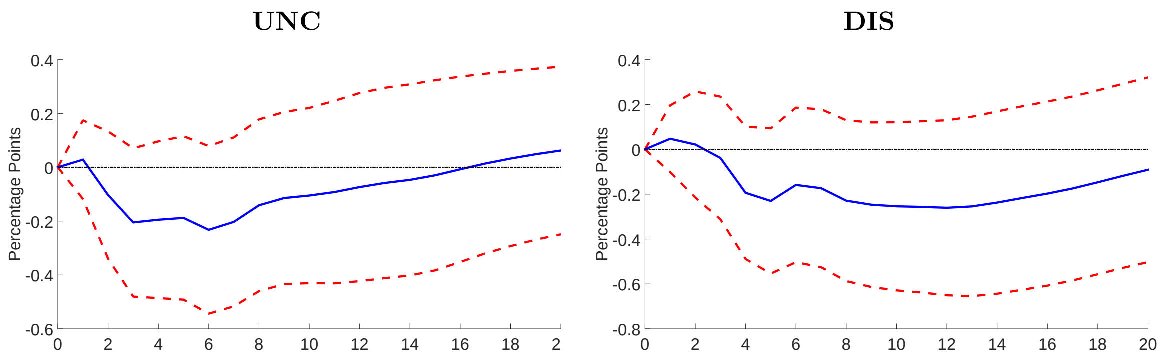
(III) Responses to an Uncertainty Shock (**EPU-SPF**)



(IV) Responses to an Disagreement Shock (**EPU-SPF**)

NOTES: This figure plots impulse responses of U.S. real private domestic investment (first column), non-farm payroll employment (second column), and industrial production (third column) to 1 % increase uncertainty (**UNC** or **EPU**)and disagreement proxies (**SPF**), obtained from estimation of a ten-variable system of VAR with Scheme 2 Cholesky recursive ordering; see details in text. The frequency of data is quarterly and the VARs are estimated with 4 lags. The sample covers 1990Q1 to 2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations.

Figure 1.12: IRFs of Federal Funds Rate: Uncertainty and Disagreement Shocks
[Disagreement Index (**DIS**) Ordered Before Uncertainty)]



NOTES: This figure plots impulse responses of U.S. Federal Funds Rate to 1 % increase uncertainty (**UNC**) or disagreement proxy(**DIS**), obtained from estimation of a ten-variable system of VAR with Scheme 1 Cholesky recursive ordering; see details in text. The frequency of data is quarterly and the VARs are estimated with 4 lags. The sample covers 1990Q1 to 2013Q4. Area between red dashed lines defines 95 % confidence interval based on 1000 bootstrap simulations.

Table 1.4: Uncertainty, Disagreement, and Firm-level Investment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\log[Y/K]_{i,t}$	0.804***	0.799***	0.799***	0.732***	0.721***	0.724***	0.735***
	(0.014)	(0.014)	(0.015)	(0.015)	(0.014)	(0.015)	(0.015)
$\log[UNC]_t$	0.405***		0.252***	0.364***		0.243***	
	(0.020)		(0.021)	(0.020)		(0.021)	
$\log[UNC]_{t-1}$	-0.561***		-0.513***	-0.520***		-0.489***	
	(0.022)		(0.023)	(0.021)		(0.022)	
$\log[DIS]_t$		0.340***	0.320***		0.342***	0.319***	
		(0.020)	(0.021)		(0.019)	(0.020)	
$\log[DIS]_{t-1}$		-0.174***	-0.013		-0.187***	-0.039**	
		(0.019)	(0.020)		(0.019)	(0.019)	
$TFP_{i,t}$				0.699***	0.704***	0.699***	0.623***
				(0.023)	(0.022)	(0.023)	(0.022)
$\Delta TFP_{i,t}$				-0.252**	-0.508***	-0.133	-0.095
				(0.118)	(0.038)	(0.121)	(0.120)
$(\Delta TFP_{i,t})^2$				0.080***	0.073***	0.082***	0.066***
				(0.014)	(0.013)	(0.014)	(0.013)
$\log[UNC]_t \times \Delta TFP_{i,t}$				0.117		0.285***	0.293***
				(0.072)		(0.083)	(0.082)
$\log[DIS]_t \times \Delta TFP_{i,t}$					-0.189**	-0.403***	-0.405***
					(0.092)	(0.110)	(0.110)
No. Obs	88415	89432	77456	77456	89432	77456	77456
$R^2(Within)$	0.192	0.183	0.199	0.244	0.233	0.248	0.274
Time Fixed-Effect	N	N	N	N	N	N	Y

Notes: Sample covers annual data from 1970 - 2013. Dependent Variable: yearly firm-level investment-capital ratio in log $\log[I/K]_{i,t}$. Measure of Uncertainty: UNC_t, dispersion of next year $t+1$ log TFP shocks. Measure of Disagreement: DIS_t, annualized dispersion index of six-month ahead forecasts for the "General Business Conditions" among manufacturing firms; see text for details. Firm-level fixed effects are included for all specifications (not reported). Estimations are done through OLS. Bootstrapped S.E. in parentheses based on 100 repetitions and is clustered at the firm level. Significance levels: 10% *, 5% **, 1% ***

Table 1.5: Robustness Checks: Various Measures of Marginal Product of Capital

	(1)	(2)	(3)	(4)
$\log[UNC]_t$	0.243***	0.314***	0.312***	0.145***
	(0.021)	(0.022)	(0.022)	(0.038)
$\log[UNC]_{t-1}$	-0.489***	-0.619***	-0.680***	-0.382***
	(0.022)	(0.022)	(0.023)	(0.040)
$\log[DIS]_t$	0.319***	0.390***	0.451***	1.517***
	(0.020)	(0.022)	(0.022)	(0.089)
$\log[DIS]_{t-1}$	-0.039**	-0.060***	-0.080***	-0.063
	(0.019)	(0.020)	(0.021)	(0.082)
$\log[UNC]_t \times \Delta TFP_{i,t}$	0.285***	0.461***	0.251***	-0.107
	(0.083)	(0.092)	(0.090)	(0.224)
$\log[DIS]_t \times \Delta TFP_{i,t}$	-0.403***	-0.448***	-0.469***	-0.214
	(0.110)	(0.125)	(0.122)	(0.434)
$TFP_{i,t}$	0.699***	0.506***	0.874***	0.516***
	(0.023)	(0.028)	(0.027)	(0.037)
$\Delta TFP_{i,t}$	-0.133	0.115	-0.178	-0.457
	(0.121)	(0.135)	(0.131)	(0.288)
$(\Delta TFP_{i,t})^2$	0.082***	0.073***	0.060***	0.029
	(0.014)	(0.016)	(0.016)	(0.021)
$\log[Y/K]_{i,t}$	0.724***			
	(0.015)			
$\log[\pi/K]_{i,t}$		0.527***		
		(0.020)		
$\log[CF/K]_{i,t}$			1.380***	
			(0.254)	
$\log Q_{i,t-1}$				0.427***
				(0.016)
No. Obs	77456	77456	77456	20451
$R^2(Within)$	0.248	0.156	0.123	0.212

Notes: Sample covers annual data from 1970 - 2013. Dependent Variable: yearly firm-level investment-capital ratio in log $\log[I/K]_{i,t}$. Measure of Uncertainty: UNC_t, dispersion of next year $t+1$ log TFP shocks. Measure of Disagreement: DIS_t, annualized dispersion index of six-month ahead forecasts for the "General Business Conditions" among manufacturing firms; see text for details. Firm-level fixed effects are included for all specifications (not reported). Estimations are done through OLS. Bootstrapped S.E. in parentheses based on 100 repetitions and is clustered at the firm level. Significance levels: 10% *, 5% **, 1% ***

Table 1.6: Robustness Checks: GMM Estimation of Dynamic Specifications

Regressand: $\log[I/K]_{i,t}$	(1)	(2)	(3)	(4)
$\log I/K_{i,t-1}$	0.544***	0.422***	0.419***	0.459***
	(0.013)	(0.009)	(0.009)	(0.021)
$\log[UNC]_t$	0.203***	0.166***	0.113***	0.173***
	(0.020)	(0.020)	(0.021)	(0.040)
$\log[UNC]_{t-1}$	-0.279***	-0.405***	-0.381***	-0.326***
	(0.021)	(0.020)	(0.023)	(0.044)
$\log[DIS]_t$	0.273***	0.402***	0.433***	0.943***
	(0.022)	(0.020)	(0.020)	(0.098)
$\log[DIS]_{t-1}$	-0.179***	-0.280***	-0.260***	-0.940***
	(0.020)	(0.020)	(0.021)	(0.096)
$\log[UNC]_t \times \Delta TFP_{i,t}$	0.223**	0.175*	0.054	-0.375
	(0.098)	(0.101)	(0.103)	(0.251)
$\log[DIS]_t \times \Delta TFP_{i,t}$	-0.442***	-0.485***	-0.420***	0.283
	(0.135)	(0.141)	(0.145)	(0.540)
$TFP_{i,t}$	0.714***	0.338***	0.559***	0.461***
	(0.027)	(0.038)	(0.042)	(0.052)
$\Delta TFP_{i,t}$	-0.250*	0.064	-0.041	-0.564*
	(0.140)	(0.139)	(0.150)	(0.336)
$(\Delta TFP_{i,t})^2$	0.092***	0.061***	0.053***	0.011
	(0.014)	(0.015)	(0.017)	(0.029)
$\log[Y/K]_{i,t}$	1.073***			
	(0.059)			
$\log[Y/K]_{i,t-1}$	-0.717***			
	(0.059)			
$\log[\pi/K]_{i,t}$		0.375***		
		(0.055)		
$\log[\pi/K]_{i,t-1}$		0.230***		
		(0.046)		
$\log[CF/K]_{i,t}$			0.949	
			(0.586)	
$\log[CF/K]_{i,t-1}$			6.001***	
			(0.632)	
$\log Q_{i,t-1}$				0.425***
				(0.038)
$\log Q_{i,t-2}$				-0.186***
				(0.033)
Goodness of Fit (Corr)	0.451	0.422	0.348	0.499
Test Serial Correlation (P-value)	0.150	0.147	0.003	0.262

Notes: Sample covers annual data from 1970 - 2013. Firm-level fixed effects are included for all specifications (not reported). See details about measurements and instruments used for GMM estimation in text. Goodness of Fit is computed as the correlation coefficient of predicted log investment rate with the actual series. Bootstrapped S.E. in parentheses based on 100 repetitions and is clustered at the firm level. Significance levels: 10% *, 5% **, 1% ***

Table 1.7: Predictions: Changes in Dispersion of Firm-level Investment Rates

Horizon h	One-Quarter Forecast			Two-Quarter Forecast		
$\log[UNC]_{t-1}$	-0.013**		-0.008			
	(0.006)		(0.005)			
$\log[DIS]_{t-1}$		-0.016**	-0.012**			
		(0.006)	(0.005)			
$\log[UNC]_{t-2}$				-0.023**		-0.010
				(0.011)		(0.009)
$\log[DIS]_{t-2}$					-0.036***	-0.031***
					(0.011)	(0.010)
Adj. R^2	0.571	0.575	0.577	0.307	0.358	0.361
Horizon h	Three-Quarter Forecast			Four-Quarter Forecast		
$\log[UNC]_{t-3}$	-0.032**		-0.016			
	(0.014)		(0.011)			
$\log[DIS]_{t-3}$		-0.047***	-0.039***			
		(0.013)	(0.012)			
$\log[UNC]_{t-4}$				-0.032**		-0.015
				(0.015)		(0.012)
$\log[DIS]_{t-4}$					-0.048***	-0.042***
					(0.012)	(0.012)
Adj. R^2	0.209	0.282	0.295	0.083	0.174	0.183

Notes: Sample covers quarterly data from 1970Q4 - 2013Q4. Dependent variable $\Delta \log \sigma_{t,I/k_i}$ is the quarterly growth rate of firm-level investment rate dispersion in quarter t, which is measured by the quarter t and $t-1$'s log difference in cross-sectional standard deviation of quarterly firms' investment-capital ratios. Quarterly firm-level investment rates dispersion series, and quarterly productivity uncertainty series are interpolated from annual data. Specification includes a constant and two past lags of $\Delta \log \sigma_{t-h,I/k_i}$ and $\Delta \log \sigma_{t-h-1,I/k_i}$ (estimates not reported). Newey-west standard errors are reported in brackets. Significance levels: 10% *, 5% **, 1% ***

Chapter 2: Disagreement vs. Uncertainty: Investment Dynamics and Business Cycles

2.1 Introduction

In this chapter, I develop a general equilibrium theory that finds that changes in the magnitude of disagreement about aggregate productivity among firms can be an important driver of business cycles. The theory incorporates two intrinsically different forces: shocks to the dispersion of idiosyncratic productivity, and shocks to the dispersion of heterogeneous information. The former, known as swings in *uncertainty*, is argued to be a strong factor that triggered significant damages during the 08-09 recession (Bloom et al., 2014; Christiano et al., 2014). However, the business cycle impacts of time-varying information quality that shapes the distribution of firms' beliefs are largely under-explored. More dispersed information causes more firms to act on imprecise signals, which prevent them from being well informed of the economic status quo and from forming good forecasts of future profitability. Larger information dispersion can increase the heterogeneity of firms' beliefs about aggregate as well as firm-specific productivity, which results in greater *informational disagreement*, even if the distribution of productivity fundamentals is unchanged.

The model shows that larger informational disagreement makes more productive firms believe the unobserved good idiosyncratic productivity draws are not good enough to justify investment, which leads to capital mis-allocation and contraction. As it takes time for firms to update beliefs with good precision, post-recession recovery of aggregate investment and production can be sluggish.

Conventionally, uncertainty is measured by the mean-preserving spread of a distribution about real economic *fundamentals*, for example, cross-sectional dispersion of firm-specific productivity, and time-series volatility of aggregate productivity. Shocks that increase fundamental uncertainty raise firms' forecast error and force firms to "wait and see" such that aggregate investment and labor hours drop (Bloom, 2009).

This chapter is motivated by a key shortfall of models of real uncertainty shocks. If uncertainty shocks are critical triggers of economic downturns, the post-recession recovery should take place immediately after the adverse shocks, and the overshoots of aggregate investment lead to an ensuing economic expansion. Therefore, unless additional negative aggregate TFP shocks are imposed,[11] the model predictions are at odds with the empirics (Bloom et al., 2014). Therefore, building upon the evidence documented in Chapter that disturbances that affect the forecast disagreement could have very persistent contractionary effects, this chapter builds a model by incorporating firms' heterogeneous beliefs in order to better align a model of uncertainty shocks with the data. Precisely, heightened productivity uncertainty

[11] Economists have been long critical about what are the negative technology shocks that drive business cycles. For example, Summers (1986).

shocks can still generate abrupt disruptions of economic activities, while informational disagreement shocks, by driving the distribution of firms' beliefs, can slow down the recovery path.

In addition, the model economy addresses another key challenge using a standard framework to study economic uncertainty. Models with shocks to fundamental uncertainty cannot generate sizable business cycles while at the same time deliver the right amount of procyclicality of the firm-level investment rates dispersion as found in the data (Bachmann and Bayer, 2014), which is measured by its correlation with cycles of aggregate output. This is because after economic uncertainty is resolved, decayed "wait-and-see" effect pushes firms to restart investing and hiring, which expands the investment rate dispersion, whereas shrinkage of productivity dispersion as uncertainty subdues tightens the dispersion of firm-level investment rates. Given that forecast disagreement is negatively correlated with investment rate dispersion shown in Chapter , the model relies on information frictions to generate the procyclicality of investment rate dispersion. Importantly, unlike Bachmann and Bayer (2014), the model does not impose restrictions on the size of "wait-and-see" effect associated with uncertainty shocks.

Regarding the model ingredients, this chapter builds an imperfect information environment in which firms care about the difference between aggregate and idiosyncratic productivity. However, firms can only imperfectly disentangle the aggregate from idiosyncratic draws through noisy signals. How much they are "uncertain" about future profitability is driven by both factors: the dispersion of future idiosyncratic productivity, i.e. the *real uncertainty* about fundamentals, and the *imprecision* of the information contained in their signals. This chapter answers the following question: how can changes in the dispersion of pure noises, which drive cross-sectional disagreement about aggregate productivity, have real and sizable effects on business cycles? Importantly, without disregarding real uncertainty shocks, how can we disentangle effects of informational disagreement from those caused by real uncertainty?

The theoretical framework extends real business cycle general equilibrium models of heterogeneous firms with firm-level non-convex adjustment costs (Khan and Thomas, 2003, 2008) and those with uncertainty shocks (Bloom et al., 2014) to aid comparisons with existing work. It deviates from these benchmarks in the following ways: (1) subject to information frictions, firms cannot distinguish the aggregate from the idiosyncratic component despite their observations of the total TFP. (2) Idiosyncratic productivity is more persistent than the aggregate counterpart as suggested in the data (Cooper and Haltiwanger, 2006). Thus, firms extract separate beliefs about the levels of the two productivity components based on the observed total TFP and a noisy signal that indicates aggregate productivity. (3) Information precision of the signals is governed by an aggregate variable, the dispersion of firm-specific signals, or equivalently, the standard deviation of noises within the public signals. I show that modeling the imperfect signal either as public or private does not affect the results. It is the aggregate information precision that is time-varying and subject to exogenous disturbances. More dispersed information renders larger informational disagreement among firms. (4) Firm-level adjustment of capital incurs both convex and non-convex adjustment costs, though, for simplicity, labor hiring is free of adjustment frictions.

The model economy is hit by two types of exogenous disturbances. The first is, *second moment* shocks to non-fundamentals, i.e., dispersion of noisy signals,[12] along with those that affect fundamentals of real productivity: *uncertainty* shocks, which capture perturbations to the dispersion of idiosyncratic productivities across firms. The drawn distinction intends to emphasize that deterioration in information quality does not necessarily suggest the economy is undergoing larger variability in real productivity. The second type of exogenous disturbance is *first moment* shocks to the levels of aggregate productivity, idiosyncratic productivity, and the signal noise. All shocks are orthogonal.

The key mechanism arising from information frictions is that more productive firms underestimate their idiosyncratic productivity when they disagree more about future aggregate productivity. This is because imperfect information prevents firms from perfectly disentangling productivity components. For larger informational disagreement, firms increasingly mis-attribute more of the productivity variation due to the more persistent idiosyncratic productivity shocks to the less persistent aggregate counterpart and vice versa. As a result, the magnitude of *insufficient* firm-level investment response to idiosyncratic productivity increases. This leads to greater capital mis-allocation and a drop in aggregate investment. In addition, firms assign smaller weights to new productivity draws when they know their information is

[12] A large number of very recent theoretical works study the effect of confidence, sentiment, exuberance, and news on business cycles. This literature mainly focuses on the first moment time variation of aggregate noise shocks (Lorenzoni, 2009; Angeletos and La'O, 2011; Schmitt-Grohé and Uribe, 2012; Blanchard et al., 2013; Benhabib et al., 2015b). Few studies have explored the second moment shocks that change the variance of signal noise.

getting less precise. Therefore, as firms carry the erroneous perceptions over time, capital mis-allocation and contraction effects can be persistent.

The model predicts that pure informational second moment shocks can generate a *real* recession followed by a slow recovery even if fundamentals are not changed. Conversely, the real-option effect of “wait-and-see” brought about by jumps in fundamental uncertainty is only short-lived. In the medium run, when firms are pushed out of the inaction band, the pent-up investment triggers a quick rebound. Therefore, this chapter provides a theoretical explanation for why the impulse responses of aggregate investment to the changes in fundamental uncertainty and informational disagreement can differ as we found in Chapter . These results suggest that a sharp drop of aggregate investment as followed by a slow recovery can be a result of adverse shocks to both real uncertainty and informational disagreement without triggering negative aggregate TFP shocks. In addition, since informational disagreement shocks generates mis-perceptions about productivity, this model is able to deliver that more productive firms decrease their investments when they disagree more. Such model prediction is also consistent with firm-level evidence documented in Chapter .

In addition, the model finds that absent aggregate TFP shocks, greater informational disagreement shocks generates a recession while at the same time shrinks investment rate dispersion. When fewer firms form beliefs with good precision, firms disagree more about future aggregate productivity. Very productive firms increasingly believe that the *de facto* good idiosyncratic productivity draws are not that good, and less productive firms further embrace a more optimistic view. This generates greater underinvestment and overinvestment from both sides, which leads to an increasing shrinkage in dispersion of firm-level investment rates. It turns out time-varying forecast disagreement is crucial to give the right amount of procyclicality of investment rate dispersion.

A strong implication of the model is that we can rely on the dispersion of firm-level investment rates as the key data moment to identify whether firms become more uncertain because of more variability in real productivity, or because they are more misinformed. In addition, the model suggests that pure noise dispersion can drive important business cycles even if, on average, the economy does not have aggregate noise.

The rest of this chapter proceeds as follows: Section 2.2 summarizes the related literature. Section 2.3 illustrates a simple partial equilibrium model that compares and contrasts effects of fundamental uncertainty and informational disagreement. Section 2.4 gives the description of a full DSGE model. Section 2.5 discusses parameter values used to solve the full model. Section 2.6 presents the numerical results of the model. Section 2.7 concludes.

2.2 Related Literature

This chapter is related to several strands of the literature. Firstly, this chapter contributes to the stream of work that finds uncertainty and, more recently, stochastic volatility shocks can affect investment and hiring. Through two well-documented channels, i.e. the convexity and the real-option mechanism, changes in real uncertainty affect business cycles. Specifically, theories with convex capital adjustment cost or with convex marginal product of capital function in productivity predict that higher uncertainty increases investment given that expected marginal revenue of capital is higher (Oi, 1962; Hartman, 1972; Abel, 1983). When micro-level non-convexity is considered, the contractionary real-option effect dominates the expansionary convexity effect in the short run, though the convexity effect will kick in very quickly (Bloom, 2009; Bloom et al., 2014). By raising the expected marginal product of capital while forcing some firms to pause investment until more precise information arrives, this model finds that larger informational disagreement can affect business cycles similarly through these two channels despite sizes of effects are limited.

Secondly, the recent literature finds that the sign and the quantitative importance of real uncertainty shocks for business cycles are sensitive to the model structure and parameterization. Counterfactual expansionary effects or moderate negative impacts on output or investment are found if the model lacks additional market frictions such as price rigidity (Bundick and Basu, 2014), credit market friction (Gilchrist et al., 2014), or search friction (Leduc and Liu, 2015). Bachmann and Bayer (2013) find that, when calibrating with German data, the role of the real-option effect, as a key channel through which uncertainty can trigger a recession, is very limited. This chapter shows that the impacts of real uncertainty can survive in the presence of information frictions. However, to be able to generate the right amount of procyclicality of investment rate dispersion, it is crucial to differentiate shocks due to time-variation in information quality from real uncertainty shocks.

An emerging literature argues that various sorts of information frictions that affect precision of agents' learning endogenously determine uncertainty and drive business cycles: changes in estimated tail risk for forecasting (Orlik and Veldkamp, 2014), time-variation in the cost of information acquisition (Benhabib et al., 2015a), and agents' learning from the actions of others subject to information externalities (Fajgelbaum et al., 2015).[13] Instead of modeling how exactly information precision is shifted by endogenous actions, this chapter treats precision of signals firms receive as distinctive sources of exogenous perturbations. Importantly, the merit of this modeling approach is that results do not rely on the assumption that worsened information precision is due to adverse first moment shocks. Rather, informational disagreement shocks can be the primitive shocks that drive the cycles.

Evidence from Eisfeldt and Rampini (2006) suggests that the cost of capital reallocation across firms must be countercyclical, given that capital reallocation is procyclical and the benefit of reallocation as measured by firm-level productivity dispersion is countercyclical. This chapter rationalizes jumps in cross-sectional disagreement as the *information cost* that prevents more productive firms from accumulating capital when aggregate output is low. The closely related paper that generates similar capital mis-allocation based on information frictions is David et al. (2014). However, their paper models information frictions such that firms cannot perfectly learn their firm-specific demands via private information. Instead, this chapter builds information frictions on imperfect disentangling such that firms can-

[13]This stream of work addresses the concern in the literature on uncertainty shocks that the source of exogenous disturbances to uncertainty is unclear. Other endeavors to endogenize uncertainty shocks include Bachmann and Moscarini (2011) and Decker et al. (2014)

not distinguish aggregate from idiosyncratic productivity. I show that, as long as the additional information on which firms rely for disentangling purposes is imperfect, capital misallocation associated with mis-perceptions among firms is a natural consequence regardless of whether information is public or firm-specific.

The idea that imperfect disentangling of two different types of shocks triggers partial adjustment in decision variables can be dated back to Lucas (1972). Recent papers apply the noisy and dispersed private information to study optimal monetary policy (Woodford, 2001; Adam, 2007; Lorenzoni, 2010) and business cycles (Lorenzoni, 2009; Blanchard et al., 2013). Differently, within a neoclassical framework, this chapter studies the business cycle effects of second moment time variations in information quality rather than the impacts of the first moment shocks to aggregate noise.

2.3 A Simple Model

I present a three-period partial equilibrium simple model to illustrate the key mechanisms at work in the full model. Specifically, firms care about differentiating between aggregate and firm-specific productivity as they are separately driven by different shocks. However, firms cannot perfectly disentangle what fraction of their observed total productivity should be attributed to aggregate component and what fraction to idiosyncratic counterpart. Therefore, firms rely on additional noisy information to form separate beliefs. I show that changes in precision of information, which shifts the extent of cross-sectional disagreement, affect how precise firms' believes about their draws of productivity components are and how good the firms' expectations about future marginal product of capital are.

Firstly, results suggest that firms' investments in response to changes in either fundamental uncertainty or informational disagreement are affected by two offsetting forces: contractionary real-option effect and expansionary convexity effect. Secondly, I show that the capital misallocation due to rises in informational disagreement is the key mechanism that shrinks the dispersion of firm-level investment rates while drives down aggregate investment.

2.3.1 Environment

The economy is populated by a unit measure of firms and each firm is indexed by i. With the same initial capital stock $k_0 > 0$ across firms, firm i produces output for three periods. Firms' profit function is given by AK technology for the first two periods

$$y_{i,t} = A_{i,t} k_{i,t-1} \tag{2.7}$$

where total factor productivity $A_{i,t}$ has an aggregate component X_t and an idiosyncratic component $Z_{i,t}$ such that $A_{i,t} = X_t Z_{i,t}$. $k_{i,t-1}$ is the predetermined capital stock; the only factor input for $t = 1, 2$. Output in period 3 (last period) has decreasing returns to scale (DRTS) in capital such that

$$y_{i,3} = A_{i,3}^{1-\alpha} k_{i,2}^{\alpha} \tag{2.8}$$

$\alpha \in (0,1)$ captures the magnitude of DRTS. Assuming DRTS for last period production is simply for the purpose of deriving tractable analytical results. [14]

Productivity and Uncertainty. Firm i enters period 1 with observed TFP $A_{i,1}$, produces output $y_{i,1}$, makes investment decision $I_{i,1}$ but is uncertain about the to-be-realized $A_{i,2}$. Once $A_{i,2}$ is known to the firm at the beginning of period 2, uncertainty clears as firm knows perfectly that TFP will stay constant such that $A_{i,3} = A_{i,2}$. The firm then produces $y_{t,2}$ and decides on investment $I_{i,2}$ given the expected gain from producing in the last period. Uncertainty affects period 1's investment decision *only*.

I use lower cases to denote productivity factors in natural log, which are assumed to follow AR(1) processes.

$$x_1 = \sigma_{v,0} \cdot v \quad , \quad x_2 = \rho_x x_1 + \sigma_v \cdot v_2 \tag{2.9a}$$

$$z_{i,1} = \sigma_{e,0} \cdot e_i \quad , \quad z_{i,2} = \rho_z z_{i,1} + \sigma_e \cdot e_{i,2} \tag{2.9b}$$

Period 2 log productivity components are linked to their period 1 realizations through an auto-regressive system with persistence $\rho_j \in (0,1)$ for $j \in \{x, z\}$. Evidence suggests idiosyncratic productivity is more persistent $\rho_z > \rho_x$ (Davis and Haltiwanger, 1992; Cooper and Haltiwanger, 2006), as is assumed throughout the model section of this chapter.

In period 1, each log productivity component is written in products of the

[14] In the full quantitative model, this assumption is relaxed and the results derived here are not affected.

realized *first moment* shock innovations: v and e_i and the corresponding predetermined *second moments*: standard deviations $\sigma_{v,0}$ and $\sigma_{e,0}$ of innovations. These first-moment innovations are i.i.d. draws from $\mathbf{N}(0,1)$. We call v and e_i that affect period 1 TFP components respectively TFP shocks and firm-level TFP shocks, similarly for v_2 and $e_{i,2}$ with respect to period 2 TFP.

It exhibits that the second moments σ_v and σ_e scale the standard deviations of two future TFP shocks: v_2 and $e_{i,2}$. These second moments are treated as parameters here for simplicity. Stochastic shocks to σ_v and σ_e are known as *uncertainty* shocks à la Bloom (2009). Specifically, shocks to σ_v that affect the volatility of future aggregate productivity are called the *macro uncertainty* shocks, whereas shocks to σ_e that shape the dispersion of future idiosyncratic productivity across firms are known as the *micro uncertainty* shocks. It will be shown that qualitatively, the effects of σ_v and σ_e are the same. In addition, since these second moments are predetermined variables, it implies that firms observe them and know how volatile aggregate productivity and how dispersed firm-specific productivity will be next period.

Noisy Information and Disagreement. Since aggregate and idiosyncratic components are governed by different dynamics, firms care about individually the aggregate and idiosyncratic productivity realized in period 1. In order to pin down investment $I_{i,1}$ in period 1, firms form expectations about period 2 TFP $A_{i,2}$ conditional on their beliefs of realized productivity components.

However, I assume firms do not separately observe x_1 and $z_{i,1}$ though they observe the total sum of the log productivity factors $a_{i,1}$. In addition, firms have access to noisy public signals s about aggregate productivity with an i.i.d. noise shock $\xi \sim \mathbf{N}(0, 1)$.

$$a_{i,1} = x_1 + z_{i,1} \tag{2.10}$$

$$s \quad = x_1 + \sigma_\xi \cdot \xi \tag{2.11}$$

Firms are assumed to know how imprecise the signal is given the standard deviation σ_ξ of noise shocks. Knowing both $a_{i,1}$ and s facilitates the firm to extract separate beliefs about the two productivity components $\mathbb{E}(x_1|s, a_{i,1}) = x_{i,1|1}$ and $\mathbb{E}(z_1|s, a_{i,1}) = z_{i,1|1}$.

I will delegate later sections to show that as long as the signals are imperfect, regardless of whether it's public or firm-specific, model predictions are robust. Hence, we see firms disagree about future x_2 and $z_{i,2}$. It is that different forecasts about future productivity components $\rho_x x_{i,1|1}$ and $\rho_z z_{i,1|1}$ are given by the heterogeneity in current beliefs $x_{i,1|1}$ and $z_{i,1|1}$.

It is clear that if firms are perfectly informed of the two components as $\sigma_\xi \to 0$, firms would not disagree about current aggregate productivity since signal truly reveals the aggregate state $s = x_1$. Everyone expects the future aggregate TFP with identical forecast $\rho_x x_1$. In this case, dispersion of heterogeneous beliefs about future idiosyncratic productivity $z_{i,2}$ is simply governed by the real productivity dispersion as $\mathbb{E}(z_{i,2}) = \rho_z z_{i,1}$ as $z_{i,1}$ is observed.

Therefore, imprecision of information $\sigma_\xi \neq 0$ could shift the cross-sectional disagreement about aggregate and idiosyncratic productivity components for non-fundamental reasons. Magnitude of belief differences changes even if nothing changes to the distribution of aggregate and idiosyncratic productivity. [15] When modeled as exogenous shocks in the full model, changes in σ_ξ are called informational *disagreement shocks*. Calling it informational is because it generates disagreement per signal noisiness, which differs from fundamental *uncertainty* that captures TFP volatility and firm-level TFP spread σ_v and σ_e. Such distinction is to emphasize the idea that changes in measurement error in public signals may not necessarily imply or must be due to swings in fundamental uncertainty.

To characterize the simple model, Figure 2.13 plots the time line along which firms form beliefs and make investment decisions. The reason why a third period of production is needed is because some firms may take delayed investment in period 2 after staying inaction in period 1. To justify period 2 investment, there should be expected gain from producing in period 3.

2.3.2 Productivity Beliefs, Disagreement, and Expectation

I examine how firms form separate productivity beliefs $x_{i,1|1}$ and $z_{i,1|1}$ and form expectations about future productivity in this section. Belief formations can be crucial as they determine firms' investment decisions. Greater σ_ξ, lower the quality of information contained in a received signal. I will show σ_ξ not only affects firm's forecasts but also shifts the cross-sectional disagreement about future aggregate

[15] Besides $\sigma_\xi \neq 0$, another key requirement is that firms should have different priors. That is, they differ in idiosyncratic productivity draws. However, this requirement is trivial in the sense that shutting down productivity heterogeneity means shutting down cross-sectional dimension.

productivity.

Productivity Beliefs. Assume TFP shocks v and e_i and noise shocks ξ are orthogonal. Note that variability of period 1 TFP shocks $\sigma_{v,0}$ and $\sigma_{e,0}$ are predetermined. Applying Bayes' Rule, we can characterize the firms' posterior beliefs about aggregate productivity x_1 and idiosyncratic productivity $z_{i,1}$. The magnitude of information precision affects these perceptions.

Lemma 1 *With imperfect information, upon observing s and $a_{i,1}$, firm i's posterior expectations of x_1 and z_i are given by*

$$\begin{bmatrix} x_{i,1|1} \\ z_{i,1|1} \end{bmatrix} = \kappa \begin{bmatrix} a_{i,1} \\ s \end{bmatrix} \tag{2.12}$$

$$\textit{where} \quad \kappa = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{b}/(\mathbf{a}+\mathbf{b}+\mathbf{c}) & \mathbf{c}/(\mathbf{a}+\mathbf{b}+\mathbf{c}) \\ 1-\mathbf{b}/(\mathbf{a}+\mathbf{b}+\mathbf{c}) & -\mathbf{c}/(\mathbf{a}+\mathbf{b}+\mathbf{c}) \end{bmatrix}$$

$$\mathbf{a} = 1/\sigma_{v,0}^2, \quad \mathbf{b} = 1/\sigma_{e,0}^2, \quad \mathbf{c} = 1/\sigma_\xi^2$$

Proof. See Appendix B.1 ■

Lemma 1 suggests that with noisy signals, firms' posterior beliefs about each productivity component are linear combinations of observables $a_{i,1}$ and s weighted by precision parameters (inverse of variances) $\mathbf{a}, \mathbf{b}, \mathbf{c}$. The weights can be well summarized in a Kalman gain matrix κ that elements sum up to 1. Changes in information quality (or, informational disagreement) $\mathbf{c}$ clearly shifts individual beliefs. The following proposition sees that precision of information can shift cross-sectional forecast disagreement.

Proposition 1 *Greater imprecision of signals raises cross-sectional dispersion of forecasts about future aggregate productivity.*

Proof. By Lemma 1, the standard deviation of cross-sectional forecasts of $A_{i,2}$ in period 1 after applying law of large numbers $\int_0^1 z_{i,1}\mathrm{d}i = 0$ is given by $\sigma_{\mathbb{E}(x_2)} = \rho_x \sigma_{x_{i,1|1}} = \frac{\rho_x\sqrt{\mathbf{b}}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}$. Hence

$$\frac{\partial \sigma_{\mathbb{E}(x_2)}}{\partial \sigma_\xi} > 0 \tag{2.13}$$

■ *Q.E.D.*

Then we compute firm i's expected period 2 TFP $A_{i,2}$ exploiting the auto-regressive system (2.9a) and (2.9b). The following lemma summarizes the key result.

Lemma 2 *For $\rho_z > \rho_x$,*

$$\mathbb{E}A_{i,2} = \exp[\mu_i + 0.5\mathbf{\Sigma}(\sigma_v, \sigma_e, \sigma_\xi)] \tag{2.14}$$

where

$$\mu_i = (\rho_x + \mathbf{M})x_1 + (\rho_z - \mathbf{N})z_{i,1} - \mathbf{P}\xi \tag{2.15}$$

is the mean forecast of future TFP in log. Terms $\mathbf{M}, \mathbf{N}, \mathbf{P}$ *and* $\mathbf{Q}$ *are positive and functions of disagreement* σ_ξ *in period 1 and predetermined uncertainty* $\sigma_{v,0}$ *and* $\sigma_{e,0}$*.* $\mathbf{M}'(\sigma_\xi) > 0$*,* $\mathbf{N}'(\sigma_\xi) > 0$ *, and* $\mathbf{Q}'(\sigma_\xi) > 0$*. Forecast variance* $\mathbf{\Sigma}$ *is a function of period 1 realized parameters of uncertainty and disagreement.*

$$\mathbf{M} = \mathbf{a} \cdot \mathbf{d},\ \mathbf{N} = \mathbf{b} \cdot \mathbf{d},\ \mathbf{P} = \sqrt{\mathbf{c}} \cdot \mathbf{d},\ \mathbf{Q} = (\rho_z - \rho_x)^2/(\mathbf{a} + \mathbf{b} + \mathbf{c})$$

$$\mathbf{d} = (\rho_z - \rho_x)/(\mathbf{a} + \mathbf{b} + \mathbf{c}) > 0,\ \mathbf{\Sigma} = \sigma_v^2 + \sigma_e^2 + \mathbf{Q}$$

Proof. See Appendix B.2 ■

$\mathbb{E}$ is firm i's expectation operator conditional on its information set at the beginning of period 1. Lemma 2 shows that the expected next period TFP depends on period 1 realizations of TFP shocks v, firm-level TFP shocks e_i, noise shocks ξ, three second moment parameters of uncertainty and disagreement σ_v, σ_e and σ_ξ and predetermined uncertainty $\sigma_{v,0}$ and $\sigma_{e,0}$. Larger TFP and firm-level TFP shocks increase firm's expected total TFP.

In addition, more persistent idiosyncratic shocks $\rho_z > \rho_x$, firms would mis-attribute part of the variation of productivity due to aggregate changes to the more persistent idiosyncratic shocks via a positive term $\mathbf{M}$. Vice versa, firms mis-attribute part of variation due to the idiosyncratic changes in productivity to the less persistent aggregate shocks through a negative term $-\mathbf{N}$. This mechanism of making perception errors is known as *mis-attribution of signals*.

Presence of noisy signals makes firms' expected next period total productivity negatively affected by rising noise shocks ξ. This is because firms know that they will mis-attribute signals and the errors of having noises within signals is captured by $\mathbf{P} > 0$. Apart from the real uncertainty terms σ_v^2 and σ_e^2, $\mathbf{Q}$ captures the additional forecast variance brought by informational disagreement in period 1.

If firms have perfect information (zero noise variation such that $s = x_1$) across firms as $\sigma_\xi \to 0$, the mis-attribution of signals effect is completely gone as $\mathbf{M}$, $\mathbf{N}$, $\mathbf{P}$ and $\mathbf{Q}$ all collapse to zero. This mis-attribution mechanism is pivotal as it differentially enhances and mutes the marginal impact of aggregate productivity shocks v and idiosyncratic shocks e_i on firms' expected total productivity relative to the perfect information benchmark. [16] In addition, by $\frac{\partial \mathbf{M}}{\partial \sigma_\xi} > 0$ and $\frac{\partial \mathbf{N}}{\partial \sigma_\xi} > 0$, we see that the larger disagreement among firms when firms act upon more imprecise signals to extract separate beliefs, the magnitudes of both enhancing and dampening effects will increase.

2.3.3 Informational Disagreement vs. Real Uncertainty

In this section, I show that imperfect information brings forth a distinctive effect of capital mis-allocation due to the mis-perceptions among firms about productivity realizations. Rises in disagreement reduce aggregate investment and shrinks the dispersion of firm-level investment rates. In addition, changes in both real uncertainty and informational disagreement can trigger the expansionary convexity

[16] Note that if $\rho_x = \rho_z$, there is no mis-attribution effect. With no difference in persistence between aggregate and idiosyncratic productivity shocks, firms do not need to differentiate them at all because firms' investment decisions in response to these level shocks will be no different. However, if $\rho_x > \rho_z$, though contradictory to the empirical evidence, mis-attribution mechanism is still there but the directions of enhancing and dampening effects will be reversed. The bottom line is that firms would always want to mis-attribute the changes due to less persistent shocks to more persistent changes. (See Appendix B.10 for details about other scenarios.)

effect and contractionary real-option effect. Mis-perception effect and convexity effect shift aggregate investment on the intensive margin, whereas real-option effect operates through the extensive margin.

2.3.3.1 Firm-level Investment

I study the effects of real uncertainty and informational disagreement (σ_v, σ_e and σ_ξ) on investment in period 1 when firms are subject to uncertainty and imperfect information.

Non-convex and Convex Capital Adjustment Costs. Use δ to denote the capital depreciation rate in period 1. Investment in period 1 is given by $I_{i,1} = k_{i,1} - (1-\delta)k_0$. For simplicity, I assume after period 2 and 3 productions, capital stock $k_{i,1}$ and $k_{i,2}$ will be fully depreciated. Therefore, period 2 investment is easily characterized as $I_{i,2} = k_{i,2} \geq 0$. Firms would only incur non-negative investment in period 2. By contrast, firms may invest ($I_{i,1} > 0$), disinvest ($I_{i,1} < 0$) and take no investment action ($I_{i,1} = 0$) in period 1.

I assume that if investment is non-zero $I_{i,t} \neq 0$ in period 1 or 2, firm has to pay a non-convex fixed cost c_k per unit of existing capital stock. In addition, following Lee and Shin (2000), I assume the fixed cost is avoidable in period 2 if a firm already paid the cost for non-zero investment in period 1. This assumption of cost avoidance is for the purpose of maintaining tractability of results only, the full model will be in line with standard fixed cost assumption as in Gilchrist et al. (2014). Investment in period 1 is also subject to a quadratic convex adjustment cost $\frac{1}{2}I_{i,1}^2$.

Firms' Problem. Firm i maximizes the sum of three expected dividends with no inter-temporal discounting.

$$\Pi = \max_{I_{i,1},I_{i,2}} A_{i,1}k_0 - I_{i,1} - \frac{1}{2}I_{i,1}^2 - c_k[\mathbb{I}_{c_k}k_0 + (1-\mathbb{I}_{c_k})k_{i,1}] + \mathbb{E}[A_{i,2}k_{i,1} - k_{i,2} + A_{i,3}^{1-\alpha}k_{i,2}^{\alpha}] \tag{2.16}$$

where

$$\mathbb{I}_{c_k} = \begin{cases} 1, & \text{if } I_{i,1} \neq 0 \\ 0, & \text{if } I_{i,2} \neq 0 \end{cases} \quad , \quad A_{i,3} = A_{i,2} = e^{x_2 + z_{i,2}} \quad , \quad I_{i,1} = k_{i,1} - (1-\delta)k_0$$

Solving the problem backwards, conditional on realized $A_{i,2}$, if firms take positive investment in period 2, then investment for period 3 production is given by

$$k_{i,2}^* = \alpha^{\frac{1}{1-\alpha}} A_{i,2} > 0 \tag{2.17}$$

Equation (2.17) says that higher realized period 2 productivity induces larger capital demand in order to reap higher profit from period 3 production.

Firms with non-zero investment in period 1. If a firm has paid a fixed cost for non-zero investment in period 1, total profit from period 2 and period 3 production conditional on realized $A_{i,2}$ is given by

$$\Pi_{i,2}(I_{i,1} \neq 0; A_{i,2}) = A_{i,2}(k_{i,1} + \psi)$$
$$\text{where} \qquad \psi = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$$

This shows if firms adjust capital in period 1, it is always optimal to invest positive $k^*_{i,2}$ to reap revenue of period 3 production rather than have zero investment with a smaller profit $A_{i,2}k_{i,1}$. The marginal gain from investing in period 2 as captured by ψ appears not dependent on investment actions in period 1. The optimal investment in period 1 is given by

$$I^*_{i,1} = \mathbb{E}A_{i,2} - 1 \tag{2.18}$$

Investment in period 1 positively responds to expected productivity in period 2, the forecast. It must be the case that $\mathbb{E}A_{i,2} > 1$ for positive investment ($I_{i,1} > 0$) and $\mathbb{E}A_{i,2} < 1$ for negative investment ($I_{i,1} < 0$). Then we can express the expected total profit for firms taking non-zero action (Adj) of investment or disinvestment in period 1 in the following

$$\Pi^{Adj} = A_{i,1}k_0 + \mathbb{E}A_{i,2}I^*_{i,1} + \mathbb{E}A_{i,2}(1-\delta)k_0 + \mathbb{E}A_{i,2}\psi - I^*_{i,1} - \frac{1}{2}I^{*2}_{i,1} - c_k k_0 \tag{2.19}$$

The first four terms in Equation (2.19) consist of the revenue to firms with non-zero action in period 1: period 1 output, expected gain from taking additional non-zero investment, expected gain from production using the net depreciation capital stock, and the expected return from taking positive investment in period 2. The other terms capture the cost expenditure including investment goods input, quadratic and fixed costs of capital adjustment.

Firms taking no investment action in period 1. Firms will take positive investment in period 2 for $\psi > 0$ even if they do not adjust capital stock in period 1. However, those firms that enter period 2 with existing capital stock $(1-\delta)k_0$ have to pay a fixed cost to have optimal investment as given by Equation (2.17). The period 2 and period 3 total profit conditional on taking no investment in period 1 is

$$\Pi_{i,2}(I_{i,1} = 0; A_{i,2}) = \max\{A_{i,2}((1-\delta)k_0 + \psi) - c_k(1-\delta)k_0, A_{i,2}(1-\delta)k_0\}$$

Expected total profit for these firms with no action (Non-Adj) is thus given by

$$\Pi^{Non-Adj} = A_{i,1}k_0 + \mathbb{E}A_{i,2}(1-\delta)k_0 + \mathbb{E}[\max\{A_{i,2}\psi - c_k(1-\delta)k_0, 0\}] \tag{2.20}$$

For firms taking no action in period 1, they also receive output from period 1 and the expected return from net depreciation capital. However, they also retain an *option value* from waiting in period 1 as captured by the third term in Equation (2.20). This option is said to be "in the money" when the realized period 2 productivity $A_{i,2}$ is greater than a fixed cost that a firm has to pay for such a delayed capital adjustment.

Option of Waiting. The option value is defined as the expected value of payoffs from all scenarios. The value cannot be negative because if the realized period 2 productivity is small enough, the firm can walk away from this option. Therefore, the higher the option value, the more likely a firm would wait a period without taking any investment or disinvestment actions. Exploiting the property of max function and truncated log-normality, we can re-express the option value as below:

$$V_{option} = \int_{\underline{A}}^{\infty} [\psi A_{i,2} - c_k(1-\delta)k_0] \mathrm{d}\hat{F}(A_{i,2}) = [1 - \hat{F}(\underline{A})][\psi\phi\mathbb{E}A_{i,2} - c_k(1-\delta)k_0] \geq 0 \tag{2.21}$$

where $\underline{A} = \frac{c_k(1-\delta)k_0}{\psi}$. $\phi = \frac{\Phi(\sqrt{\boldsymbol{\Sigma}} - \frac{\log \underline{A} - \mu_i}{\sqrt{\boldsymbol{\Sigma}}})}{\Phi(-\frac{\log \underline{A} - \mu_i}{\sqrt{\boldsymbol{\Sigma}}})} > 1$. Φ is the CDF of standard normal distribution and $\hat{F}(A_{i,2})$ is the posterior cumulative distribution about $A_{i,2}$ which follows log-normal distribution $\sim \ln \mathbf{N}(\mu_i, \boldsymbol{\Sigma})$ with mean and variance as in Equation (2.15).

Profit Maximization Then we can recast firm's problem using Equations (2.19) and (2.20)

$$\Pi = \max_{I_{i,1}} \{\Pi^{Adj}, \Pi^{Non-Adj}\} \tag{2.22}$$

We then define a Ψ function that captures the gain from taking non-zero investment action in period 1 relative to waiting.

$$\Psi = \Pi^{Adj} - \Pi^{Non-Adj} = \frac{(\mathbb{E}A_{i,2})^2}{2} - \zeta \cdot \mathbb{E}A_{i,2} + \gamma \tag{2.23}$$

where $\gamma = \frac{1}{2} - c_k k_0[1-(1-\delta)(1-\hat{F}(\underline{A})]$. I further assume $\zeta = 1-\psi+\psi\cdot\phi[1-\hat{F}(\underline{A})] > 0$ and $\gamma > 0$ to examine equilibrium of interest. We plot this *difference* function in Figure 2.14 in which it traces out a parabolic and symmetric function about the expected value of period 2 total productivity. To simplify notations, use E_i to denote firm i's forecast of $A_{i,2}$. Two roots E_i^I and E_i^D of function Ψ make firms are indifferent between taking non-zero investment and waiting in period 1 ($\Pi^{Adj} = \Pi^{Non-Adj}$). Note that the two trigger points are firm-specific because each firm has its own posterior belief of the distribution of next period total TFP. Therefore, we have the following lemma and we can characterize the optimal policy function for investment.

Lemma 3 (Investment/Disinvestment Thresholds) *For $\zeta > 0$ and $\gamma > 0$, there exist thresholds about expected next period TFP factor E^I and E^D such that $\Pi^{Adj} = \Pi^{Non-Adj}$ where $E^I = \zeta + \sqrt{\zeta^2 - 2\gamma} > 0$ and $E^D = \zeta - \sqrt{\zeta^2 - 2\gamma} > 0$, which are functions of aggregate variables $\sigma_v, \sigma_z,$ and σ_ξ and depend on posterior beliefs about distribution of future $A_{i,2}$. Loss from waiting increases in E_i when $E_i > \zeta$ and decreases in E_i when $E_i \in (0, \zeta)$.*

Proposition 2 *Firm i invests if $E_i > E_i^I$, disinvests if $E_i < E_i^D$ and takes no action if $E \in [E_i^D, E_i^I]$.*

Proposition 2 suggests that firm i would take positive (negative) investment in period 1 only if its expected value of next period total productivity is sufficiently higher (lower) than some threshold. This suggests that with mediocre future forecast, the option value of waiting outweighs all the other gains from taking investment or disinvestment.

2.3.3.2 Effects of Uncertainty vs. Disagreement

Through three propositions in the following, I compare and contrast between effects of real uncertainty and those of informational disagreement upon firm-level and aggregate investment.

Capital Mis-allocation Effect. This is the effect that forces firms' forecasts to deviate from the truth, which pushes quantity of investment and disinvestment away from optimal. The effect arises from the imperfect information, and is determined by the magnitude of informational disagreement. We examine the quasi-elasticity of investment with respect to changes in the three second moments. Define $\mathbf{i}_{i,1} = \log(1 + I_{i,1})$,

Lemma 4 *In an economy with imperfect disentangling, firm-level investment and disinvestment would over-react to TFP shocks and under-react to firm-level TFP shocks relative to a perfect information scenario conditional on firm's taking non-zero investment.*

Proof. Evaluate the partial derivatives of $\mathbf{i}_{i,1}$ with respect to the TFP x_1 and firm-level TFP $z_{i,1}$. We have $\phi_x > 0$ and $\phi_z > 0$ in the following:

$$\phi_x = \frac{\partial \mathbf{i}_{i,1}}{\partial x_1} = \rho_x + \mathbf{M} > \rho_x = \phi_x^* \tag{2.24a}$$

$$\phi_z = \frac{\partial \mathbf{i}_{i,1}}{\partial z_{i,1}} = \rho_z - \mathbf{N} < \rho_z = \phi_z^* \tag{2.24b}$$

The inequality conditions are given by the fact that $\mathbf{M} > 0$ and $\mathbf{N} > 0$. ϕ_x^* and ϕ_z^* denote the corresponding partial derivatives in case of perfect information when $\sigma_\xi \to 0$. Regarding disinvestment, the overreaction and underreaction still go through as $|\phi_x| > |\phi_x^*|$ and $|\phi_z| > |\phi_z^*|$. ■ *Q.E.D.*

Lemma 4 is a direct result of firms' mis-attributing signals due to information frictions. Firms are unable to perfectly identify productivity components and thus mis-attribute variation of productivity due to changes in less persistent aggregate TFP shocks v to changes in more persistent idiosyncratic shocks e_i and vice versa. Presence of imperfect information creates a gap between firm's belief and the realized but unobserved draw, which leads to the over-reaction and under-reaction of firm-level investment.

Importantly, this gap of misperception is governed by how much firms disagree with each other, or how much firms are mis-led by the signal noisiness. The degrees of these mis-perception are exactly the magnitudes of amplified and dampened investment. Given $\mathbf{M}$ and $\mathbf{N}$ both increase in σ_ξ, we have the following:

Proposition 3 *Larger disagreement increases (1) the amplification of firm-level investment to TFP shocks v and (2) further dampens its response to idiosyncratic shocks e_i, conditional on firm's taking non-zero action.*

Proof. Evaluate the cross-partial derivative of $\mathbf{i}_{i,1}$ with respect to both TFPs and disagreement:

$$\frac{\partial \mathbf{i}_{i,1}^2}{\partial x_1 \partial \sigma_\xi} = \frac{\partial \mathbf{M}}{\partial \sigma_\xi} > 0 \tag{2.25a}$$

$$\frac{\partial \mathbf{i}_{i,1}^2}{\partial z_{i,1} \partial \sigma_\xi} = -\frac{\partial \mathbf{N}}{\partial \sigma_\xi} < 0 \tag{2.25b}$$

■ *Q.E.D.*

According to Equations (2.25a) and (2.25b), amplified and dampened effects of firm-level investment both increase in informational disagreement. This is because rising disagreement enhances the firm-level extent of signal misattribution for all firms. A key proposition regarding the capital misallocation is stated below:

Proposition 4 (Capital Mis-allocation Due to Firms' Mis-perceptions) *When aggregate TFP shocks are at $v = 0$, more productive firms increasingly cut investment and less productive firms increasingly increase investment in response to greater disagreement among firms.*

Given imperfect disentangling, firms with good realized draws of firm-level TFP shocks would not believe the draws are that good to justify investment. Therefore, Proposition 4 says more productive firms are not investing enough while less productive firms are investing too much relative to the reality, which are unobserved to all firms. Capital is hence mis-allocated to less productive firms. Critically, when the magnitude of mis-perception rises due to more imprecise information, the extent of capital mis-allocation rises. We call this effect the capital mis-llocation effect.

Convexity Effect. The following proposition considers another effect which applies to both uncertainty and disagreement conditional on firms' taking non-zero actions. We consider the case when all first moment shocks: TFP shocks v, firm-level TFP shocks e_i, and the noise shocks ξ are at zero in order to isolate the impacts of second moments.

Proposition 5 *When first moment shocks are at zeros, conditional on taking non-zero investment action, firm-level investment increases in both uncertainty and disagreement.*

Proof. By Equations (2.14) and (2.18), with shocks $v = 0$, $e_i = 0$ and $\xi = 0$, we have

$$\phi_{\sigma_v} = \frac{\partial \mathbf{i}_{i,1}}{\partial \sigma_v} > 0 \quad , \quad \phi_{\sigma_e} = \frac{\partial \mathbf{i}_{i,1}}{\partial \sigma_e} > 0 \quad , \quad \phi_{\sigma_\xi} = \frac{\partial \mathbf{i}_{i,1}}{\partial \sigma_\xi} > 0 \tag{2.26}$$

Similarly, for disinvestment, higher second moments all reduce the quantity a firm takes for disinvestment. ■ *Q.E.D.*

By Equation (2.18), the amount of firm-level investment is positively responding to firm's expected marginal product of capital, i.e. the expected productivity in period 2. When the marginal production function $A_{i,2} = e^{x_2 + z_{i,2}}$ is convex in log productivity component, higher variance increases the expected value $\mathbb{E}A_{i,2}$ due to Jensen's Inequality. What's new in this noisy information environment is that informational disagreement brings about a second source of forecast variance $\mathbf{Q}$ that is added to the real uncertainty σ_v and σ_e. [17] As a result, investment or disinvestment would respond to disagreement similarly as if it responds to changes in real productivity uncertainty.

Real-Option Effect. We consider the second shared effect of uncertainty and disagreement with respect to their impacts on firm's hazard of capital adjustment.

Proposition 6 *For $\underline{A} \geq e^{\sqrt{\boldsymbol{\Sigma}}}$, a firm with all first moment shocks at zero in face of larger uncertainty and greater disagreement sees greater gain from waiting and taking no investment action.*

Proof. See proof in Appendix B.3 ■

This proposition says rises in either uncertainty or informational disagreement could enlarge a firm's inaction region such that for $j \in \{x, z, \xi\}$

$$\frac{\partial E_i^I}{\partial \sigma_j} > 0 \quad , \quad \frac{\partial E_i^D}{\partial \sigma_j} < 0 \tag{2.27}$$

Therefore, as the probability of taking positive (negative) investment when $E_i > E_i^I$ ($E_i < E_i^I$) both decreases, a firm becomes more likely to be pushed into the enlarged inaction region of waiting. The reason is that larger uncertainty or disagreement increases the option value of waiting.

The following section is then devoted to check the macro implications of all these effects on aggregate investment. I show that the misallocation effect arising from jumps of informational disagreement can generate real recession.

[17] In the perfect information case, the forecast variance is just given by $\sigma_v^2 + \sigma_e^2$.

2.3.3.3 Aggregate Investment

Here we examine the aggregate economy when aggregate TFP shocks $v = 0$ and aggregate noise shocks $\xi = 0$ whereas firms differ in firm-specific TFP shocks as if we are considering the ergodic distribution of firms in a dynamic setting at steady state. As firm i would incur positive investment $E_i - 1 > 0$ if its forecast $E_i > E_i^I$ or disinvestment $E_i - 1 < 0$ if $E_i < E_i^D$. I assume a single crossing property in the following to exclude the possibility for multiple equilibria in order to ensure that conditional upon taking non-zero action, further favorable or unfavorable firm-level productivity draws should not push the firm back to the inaction region.

Assumption 1 *At steady state with $v = 0$ and $\xi = 0$, conditional on taking positive (negative) investment, increase of the option value of waiting for a firm in response to larger (lower) firm-level productivity, is bounded such that $e^{\mu_i + \boldsymbol{\Sigma}}|e^{\mu_i + \boldsymbol{\Sigma}} - \zeta(e_i)| \geq |\zeta'(\mu_i)e^{(\mu_i + \boldsymbol{\Sigma})} - \gamma'(\mu_i)|$ where $\mu_i = (\rho_z - \mathbf{N})e_i$.*

The assumption above says no firm with a better forecast of the future relative to an investing firm would want to pause and the reverse is true for disinvesting firms: with even worse expected future productivity, firms will take disinvestment. This global continuity assumption helps us to use idiosyncratic productivity shocks e_i to describe the entire firm distribution and to examine an aggregate "inaction band".

Lemma 5 *For $v = 0$ and $\xi = 0$, firms would invest if firm-specific productivity draw e_i is greater than e^I and disinvest if $e_i < e^D$ where e^I uniquely solves $E_i(e^I) = E_i^I$ and e^D uniquely solves $E_i(e^D) = E_i^D$.*

Proof. See proof in Appendix B.4 ■

Lemma 6 *If* $\frac{\partial E_i^I}{\partial \sigma_j} > E_i'(\sigma_j)$ *and* $\frac{\partial E_i^D}{\partial \sigma_j} < E_i'(\sigma_j)$*, higher uncertainty or disagreement expands the aggregate inaction band such that* $\frac{\partial e^I}{\partial \sigma_j} > 0$ *and* $\frac{\partial e^D}{\partial \sigma_j} < 0$*.*

Proof. See proof in Appendix B.5 ■

Lemma 6 holds when the magnitude of firm-level real option effect dominates the convexity effect, the inaction band of the aggregate economy expands in face of higher uncertainty and disagreement. This implies that more firms would do nothing for higher uncertainty or disagreement because they see larger gains from waiting relative to the higher expected marginal return from taking additional investment.

Then we can characterize the aggregate investment **I** by integrating the capital increments across firms who are investing and the firms who are disinvesting.

$$\mathbf{I} = \int_{e^I}^{\infty} (E_i - 1)\mathrm{d}\Phi(e) - \int_{-\infty}^{e^D} (1 - E_i)\mathrm{d}\Phi(e) \tag{2.28}$$

Proposition 7 *In an economy with more investing firms relative to disinvesting firms* $1 - \Phi(e^I) > \Phi(e^D)$*, in response to larger disagreement, aggregate investment responds (1) negatively to the capital mis-allocation effect; (2) positively to convexity effect; (3) negatively to the real-option effect that reduces the number of firms who are investing.*

Proof. We take the partial derivatives of $\mathbf{I}$ with respect to disagreement

$$
\begin{aligned}
\frac{\partial \mathbf{I}}{\partial \sigma_\xi} = &\int_{e^I}^{\infty} E_i[-\mathbf{N}_{\sigma_\xi}\sigma_{e,0}e_i + \mathbf{\Sigma}'(\sigma_\xi)]\mathrm{d}\Phi(e) - (e^{(\rho_z-\mathbf{N})e^I+\mathbf{\Sigma}} - 1)\frac{\partial e^I}{\partial \sigma_\xi} \\
&+ (e^{(\rho_z-\mathbf{N})e^D+\mathbf{\Sigma}} - 1)\frac{\partial e^D}{\partial \sigma_\xi} + \int_{-\infty}^{e^D} E_i[-\mathbf{N}_{\sigma_\xi}\sigma_{e,0}e_i + \mathbf{\Sigma}'(\sigma_\xi)]\mathrm{d}\Phi(e)
\end{aligned}
$$

Rearrange this equation, we have

$$
\begin{aligned}
\frac{\partial \mathbf{I}}{\partial \sigma_\xi} = &\underbrace{\int_{e^I}^{\infty} E_i\mathbf{\Sigma}'(\sigma_\xi)\mathrm{d}\Phi(e) + \int_{-\infty}^{e^D} E_i\mathbf{\Sigma}'(\sigma_\xi)\mathrm{d}\Phi(e)}_{intensive\ margin\ :\ convexity\ effect>0} + \underbrace{(e^{(\rho_z-\mathbf{N})e^D+\mathbf{\Sigma}} - 1)\frac{\partial e^D}{\partial \sigma_\xi}}_{extensive\ margin\ :fewer\ disinvesting\ firms>0} \\
&\underbrace{-(e^{(\rho_z-\mathbf{N})e^I+\mathbf{\Sigma}} - 1)\frac{\partial e^I}{\partial \sigma_\xi}}_{extensive\ margin\ :fewer\ investing\ firms<0} \underbrace{-\int_{e^I}^{\infty} E_i\mathbf{N}_{\sigma_\xi}e_i\mathrm{d}\Phi(e) - \int_{-\infty}^{e^D} E_i\mathbf{N}_{\sigma_\xi}e_i\mathrm{d}\Phi(e)}_{intensive\ margin:\ capital\ mis-allocation\ effect\ <\ 0}
\end{aligned}
\tag{2.29}
$$

To see why the sign of the mis-allocation effect is negative, we consider the case when $1 - \Phi(e^I) > \Phi(e^D)$. If $e^D < e^I < 0$, we have

$$
\begin{aligned}
&-[\int_{e^I}^{\infty} E_i\mathbf{N}_{\sigma_\xi}e_i\mathrm{d}\Phi(e) + \int_{-\infty}^{e^D} E_i\mathbf{N}_{\sigma_\xi}e(i)\mathrm{d}\Phi(e)] \\
=&-\mathbf{N}_{\sigma_\xi}[\int_{0}^{\infty} e^{-2\mathbf{N}\sigma_{e,0}z_i}e_i\mathrm{d}\Phi(e) - \int_{e^D}^{e^I} E_ie_i\mathrm{d}\Phi(e)] < 0
\end{aligned}
$$

In case of $e^D < 0 < e^I$, we can again rearrange the integral to be

$$
\int_{e^I}^{\infty}(E_i - 1)\mathbf{N}_{\sigma_\xi}e_i\mathrm{d}\Phi(e) + \int_{-\infty}^{e^D}(E_i - 1)\mathbf{N}_{\sigma_\xi}e(i)\mathrm{d}\Phi(e) - \int_{e^D}^{e^I}\mathbf{N}_{\sigma_\xi}e(i)\mathrm{d}\Phi(e) > 0
$$

Because investment (disinvestment) is positive (negative), the first two terms in the bracket are positive. The last term (with the minus sign) will be positive due to the fact that there are more investing firms. ■ *Q.E.D.*

Absent the channel of capital misallocation, in response to higher productivity uncertainty, aggregate investment is subject to two offsetting forces: expansionary convexity effect and contractionary real-option effect. To be consistent with VAR-based impulse response evidence in Chapter , wait-and-see effect should dominate in the short run and convexity effect later kicks in to generate rebound of aggregate investment.

To be consistent with the evidence, in case of greater informational disagreement, combined effects of capital mis-allocation and real-option should dominate convexity effect in the short run. For a slow recovery after the adverse informational shocks, the capital misallocation effect should be sizable and persistent enough to offset the effect when more firms are pushed out of inaction region and restart investing. I delegate the full model to quantitatively assess their joint impacts on aggregate investment. In addition, I delegate Appendix B.8 to show that building upon firm-specific signals, these effects on aggregate investment still hold.

2.3.3.4 Dispersion of Investment Rates

The simple model predicts that if uncertainty jumps, then more firms pause investing and hiring. Therefore, dispersion of investment rates could shrink. However, uncertainty jumps suggest that productivity shocks will be more dispersed next period. Hence, conditional on more firms restarting to invest and hire when they get out of the inaction band, the dispersion of investment rates should be larger. The trade-off of real-option effect and convexity effect drives the correlation between uncertainty shocks and investment rate dispersion across firms.

Conversely, larger disagreement not only pushes firms into inaction region to wait for clearer information, but generates capital mis-allocation, which reinforces the shrinkage in dispersion of investment rates. On the intensive margin, such capital-misallocation effect could partly offset the convexity effect that may expand investment rate dispersion. Therefore, quantitative model will gives a full assessment of interplay of these effects in terms of driving the movement of investment rate dispersion.

2.4 The Full Model

A dynamic stochastic general equilibrium framework is built to quantitatively evaluate the impacts of uncertainty and disagreement on firm-level and aggregate investment. I consider firm-specific signals instead of public signals about the aggregate productivity in the full model. In addition, results in the simple model section suggest that macro and micro uncertainty affect firm-level and aggregate investment via same channels: convexity effect and real-option effect, at least qualitatively. The full model is thus devoted to compare and contrast effects associated with disagreement shocks and those due to micro-level uncertainty shocks that shift cross-sectional productivity dispersion.

Precisely, the model economy is hit by exogenous shocks to dispersion of information or signals (informational disagreement shocks) and shocks to dispersion of idiosyncratic productivity (real uncertainty shocks). Heterogeneity on the production side is driven by persistent differences in firm-specific productivity and different draws of signals about aggregate productivity each period. Firms make investment decisions subject to a fixed cost and a quadratic capital adjustment cost.

The model differs from the neoclassical model with non-convex adjustment costs (Khan and Thomas, 2008) in that firms infer the unobserved productivity components with imperfect signals. The precision of signals is determined by the dispersion of firm-specific signals. This setup thus augments the framework with conventional uncertainty shocks (Bloom, 2009) by having non-fundamental shocks to dispersion of signal noises.

2.4.1 Firms

2.4.1.1 Technology

There are a large number of production units in the model economy. [18] In period t, firm i produces output $y_{i,t}$ using predetermined capital stock $k_{i,t-1}$ and labor $n_{i,t}$ via a Cobb-Douglas Decreasing Returns to Scale (DRS) production technology:

$$y_{i,t} = e^{x_t + z_{i,t}} k_{i,t-1}^{\alpha_k} n_{i,t}^{\alpha_n}. \tag{2.30}$$

[18] I do not differentiate the terms ”firm”, ”establishment” or ”plant”

The log stochastic productivity has a common component x_t (aggregate productivity) and a firm-specific component $z_{i,t}$ (idiosyncratic productivity). α_k and α_n respectively refers to share of capital and labor in production. $\alpha_k + \alpha_n \in (0,1)$ captures the degree of decreasing returns to scale for all firms. Firm i has an infinite horizon.

2.4.1.2 Imperfect Information: Recursive Signal Extraction

At the beginning of period t, firm i does not separately observe the realized components of productivity. Rather, it solves a signal extraction problem at the beginning of each period upon observing the productivity sum $a_{i,t}$ and the signal $s_{i,t}$. These exogenous processes are defined in the following.

Exogenous Processes. The aggregate and idiosyncratic productivity components x_t and $z_{i,t}$ of firm i's total productivity, $a_{i,t} = x_t + z_{i,t}$, are assumed to follow stationary AR(1) processes:

$$x_t = \rho_x x_{t-1} + \sigma_v v_t \tag{2.31a}$$

$$z_{i,t} = \rho_z z_{i,t-1} + \sigma_{e,t-1} e_{i,t}. \tag{2.31b}$$

$\rho_j \in (0,1)$ with $j \in \{x, z\}$ are persistence parameters with $\rho_z > \rho_x$. Innovations $v_t \sim \mathbf{N}(0,1)$ and $e_{i,t} \sim \mathbf{N}(0,1)$ are identically and independently distributed over time and across firms. These are *first moment* shocks that affect productivity levels. $\sigma_{e,t}$ are time-varying standard deviations that scale the dispersion of next period $t+1$ idiosyncratic productivity shocks, which are realized at the beginning of period t. The dynamics of productivity uncertainty is given by:

$$\log(\sigma_{e,t}) = (1-\rho_{\sigma_e})\log(\bar{\sigma}_e) + \rho_{\sigma_e}\log(\sigma_{e,t-1}) + \eta_{\sigma_e}\epsilon_{\sigma_e,t}. \tag{2.32}$$

The standard normal innovations ϵ_{σ_e} known as *uncertainty shocks* affect the dispersion of firms' cross-sectional idiosyncratic productivity. $\bar{\sigma}_e$ is the unconditional mean of dispersion of productivity shocks $e_{i,t}$.

The signal that contains information regarding the aggregate productivity is contaminated by the idiosyncratic noise shocks $\xi_{i,t}$, which are an i.i.d. draw from $\mathbf{N}(0,1)$ over time and across firms such that

$$s_{i,t} = x_t + \sigma_{\xi,t}\xi_{i,t}. \tag{2.33}$$

The common parameter $\sigma_{\xi,t}$ captures the spread of heterogeneous information quality or "noisiness" across firms at the beginning of period t, a non-fundamental shiftier of cross-sectional disagreement. I similarly assume that $\sigma_{\xi,t}$ in log follows a stationary AR(1) process with unconditional mean $\bar{\sigma}_\xi$,

$$\log(\sigma_{\xi,t}) = (1-\rho_{\sigma_\xi})\log(\bar{\sigma}_\xi) + \rho_{\sigma_\xi}\log(\sigma_{\xi,t-1}) + \eta_{\sigma_\xi}\epsilon_{\sigma_\xi,t} \tag{2.34}$$

The innovation term $\epsilon_{\sigma_\xi,t} \sim \mathbf{N}(0,1)$ denotes shocks to the dispersion of firm-specific noises. Similarly to modeling imperfect information as public signal, larger $\sigma_{\xi,t}$ makes more firms acting on imprecise firm-specific signals, which also measures the aggregate information precision. Thus, I call $\epsilon_{\sigma_\xi,t}$ shocks to information precision that affect disagreement, the informational *disagreement shocks*. $\rho_{\sigma_\xi} \in (0,1)$ is the persistence parameter and η_{σ_ξ} is the standard deviation of the innovation to disagreement.

Swings in this noise spread can be interpreted as firms exogenously hold more or less dispersed beliefs about the latent unknown, which arises from the sources other than the true dynamics of economic fundamentals. [19]. I assume uncertainty shocks and disagreement shocks $\{\epsilon_{\sigma_e}, \epsilon_{\sigma_\xi}\}$ are mutually orthogonal for the sake of identifying their individual contributions to investment dynamics. Note that with this assumption, firms disagree due to dispersed information does not necessarily suggest the real economy is undergoing changes in uncertainty.

Information Set. The only information learned by the firm in order to infer the productivity components is the received noisy signal about aggregate productivity and the observable productivity sum. I assume firms do not learn from the other firms' information sets and thus do not act upon other firms capital and labor decisions. [20] In addition, I assume that firms would never know what the history of true realizations of aggregate productivity shocks is.

[19]The driver of distributional changes in firms' beliefs about fundamentals may be due to a sunspot variable that "conveys no information about technology, preference or endowments and does not directly enter the equilibrium conditions" (Woodford, 1990) It is possible that changes in belief dispersion are in fact endogenous and driven by optimal information updating by firms. While this is an interesting idea, it is beyond the scope of this chapter.

[20]This model thus abstracts from the complication that firms need to care about what other firms think about what others think, i.e. higher order beliefs. Implicitly, firms are assumed not to communicate with each other and do not coordinate to reach a consensus or maintain a given noise dispersion.

As firm i enters period t, it carries a few state variables that characterize the imperfect information environment: (1) period $t-1$ posterior beliefs about the then productivity components $x_{i,t-1|t-1}$ and $z_{i,t-1|t-1}$; (2) posterior variance and co-variance matrix, or forecast variance about x_{t-1} and $z_{i,t-1}$ in period $t-1$, $\hat{\Sigma}_{t-1|t-1}$. This variance matrix is an aggregate variable as all firms are subject to the same second moment dynamics that affect the precision of forecasting; (3) $t-1$ uncertainty realizations that govern how dispersed the period t's idiosyncratic productivity shocks $\sigma_{e,t-1}$.

As period t unfolds, firm i knows (1) how disagreed they are among themselves about aggregate productivity x_t due to imprecise information $\sigma_{\xi,t}$; (2) how uncertain the $z_{i,t+1}$ productivity will be: $\sigma_{e,t}$; (3) observes the realized total productivity sum $a_{i,t}$, and (4) receives a signal $s_{i,t}$.

Signal Extraction. Firm i's profit maximization problem consists of solving a signal extraction problem and making optimal labor and investment decisions. Bayesian firms use a recursive Kalman Filtering way to optimally update prior beliefs to form posterior estimates of productivity components. Acting upon the posterior beliefs about current productivity $x_{i,t|t}$ and $z_{i,t|t}$, firms compute future expectation of marginal product of capital in order to pin down the investment decision given they know the shock persistence.

How firms form new beliefs are derived in Appendix B.6. Important to note that the uncertainty realizations $\sigma_{e,t-1}$ about current period productivity shocks $e_{i,t}$, along with the disagreement $\sigma_{\xi,t}$ affect the *precision of forming beliefs* about current period t. However, *precision of future expectation* about period $t+1$ productivity will depend on both current informational imprecision $\sigma_{\xi,t}$ and the newly realized uncertainty $\sigma_{e,t}$.

2.4.1.3 Capital Adjustment: Non-convex and Convex Costs

I assume firms have to pay a fixed cost $c_k > 0$ per unit of their existing capital stock $k_{i,t-1}$ as long as it decides to invest or disinvest ($I_{i,t} \neq 0$) each period. In addition, capital adjustment incurs a quadratic adjustment cost given by

$$\frac{\theta}{2}[\frac{I_{i,t}}{k_{i,t-1}}]^2 k_{i,t-1} \tag{2.35}$$

where $\theta > 0$ indexes the level of the cost.

Empirical evidence lends support to modeling adjustment costs (Cooper and Haltiwanger, 2006). In addition, non-convex fixed cost is necessary to generate a region of inaction in which firms do not take investment and disinvestment actions in equilibrium (Bloom, 2009). Firm-level non-convex adjustment cost is critical for uncertainty and disagreement to affect aggregate investment through the real-option effect channel. Quadratic adjustment cost is to attenuate the excessive responses of investments to productivity shocks.

2.4.1.4 Profit Maximization

Each firm can be denoted by its predetermined stock of capital $k_{i,t-1}$ and its previous period posterior estimates of productivity components $m_{i,t-1|t-1} =$

$\{x_{i,t-1|t-1}, z_{i,t-1|t-1}\}$. Then we can fully describe the distribution of firms over the Borel algebra $\mathbf{S}$ for the space $S = \mathrm{R}^+ \times \mathrm{R}^2$ on which the probability measure μ_{t-1} is defined. μ_{t-1} denotes the firm distribution in the end of period $t-1$ (beginning of period t) and is varying over time. The capital stock $k_{i,t}$ at any point of time is non-negative.

The aggregate state of the economy at the beginning of period t is described by $\Omega_t = \{x_t, \sigma_{e,t}, \sigma_{\xi,t}, \sigma_{e,t-1}, \mu_{t-1}\}$ as disagreement, and uncertainty are time-varying. The reason why $\sigma_{e,t-1}$ enters the aggregate state vector is because it matters for firms to form current period posterior beliefs about productivity components. Law of large numbers average out idiosyncratic productivity shocks and noise shocks and they do no enter as aggregate state variables. I assume a mapping Γ of Ω_t as some aggregate laws of motion, which moves the firm distribution over time such that $\mu_t = \Gamma(\Omega_t)$.

Labor Demand. Given existing capital stock $k_{i,t-1}$, aggregate law of motion $\Gamma(\Omega_t)$, and wage w_t, along with a stochastic discount factor $\beta Q_{t+1|t}$ in numeraire of consumption goods, firm i maximizes expected profit among options of being inaction and taking non-zero investment subject to the adjustment costs.

Firm i's labor demand can be separately determined apart from the dynamic programming problem by solving a static optimization problem each period:

$$\max_{n_{i,t}} e^{x_t + z_{i,t}} k_{i,t-1}^{\alpha_k} n_{i,t}^{\alpha_n} - w_t n_{i,t}$$

to have the optimal labor demand $n_{i,t} = [\frac{\alpha_n e^{x_t + z_{i,t}} k_{i,t-1}^{\alpha_k}}{w_t}]^{\frac{1}{1-\alpha_n}}$. Net the wage bill payment, firm i's operating profit is $(1-\alpha_n)y_{i,t}$ where

$$y_{i,t} = [\frac{\alpha_n}{w_t}]^{\frac{\alpha_n}{1-\alpha_n}} \exp[\frac{x_t + z_{i,t}}{1-\alpha_n}] k_{i,t-1}^{\frac{\alpha_k}{1-\alpha_n}} \tag{2.36}$$

Profit Maximization. Then firm's dynamic optimization problem is defined as:

$$\mathbf{V}(k_{i,t-1}, m_{i,t-1|t-1}; \Omega_t) = \max_{k_{i,t}} \{\mathbf{V}^{Adj}, \mathbf{V}^{Non-Adj}\} \tag{2.37}$$

where

$$\begin{aligned}\mathbf{V}^{Adj}(k_{i,t-1}, m_{i,t-1|t-1}; \Omega_t) =& \max_{k_{i,t}} (1-\alpha_n)y_{i,t} - I_{i,t} - c_k k_{i,t-1} - \frac{\theta}{2}[\frac{I_{i,t}}{k_{i,t-1}}]^2 k_{i,t-1} \\ &+ \beta \mathbb{E} Q_{t+1|t} \mathbf{V}(k_{i,t}, m_{i,t|t}; \Omega_{t+1}) \end{aligned} \tag{2.38}$$

$$\begin{aligned}\mathbf{V}^{Non-Adj}(k_{i,t-1}, m_{i,t-1|t-1}; \Omega_t) =& \max_{k_{i,t}} (1-\alpha_n)y_{i,t} \\ &+ \beta \mathbb{E} Q_{t+1|t} \mathbf{V}((1-\delta)k_{i,t-1}, m_{i,t|t}; \Omega_{t+1}) \end{aligned} \tag{2.39}$$

y_t by Equation (2.36) and $I_{i,t} = k_{i,t} - (1-\delta)k_{i,t-1}$.

2.4.2 Households

I assume there is a representative household who has quasi-linear utility in labor hours and owns all the firms. It solves the following lifetime utility maximization problem:

$$\mathbf{W}(\Omega_t) = \max_{\{c_t, n_t^h\}_{t=0}^{\infty}} \log(c_t) + \psi(1 - n_t^h) + \beta \mathbb{E}_t \mathbf{W}(\Omega_{t+1}) \tag{2.40}$$

subject to

$$c_t = w_t n_t^h + \int_S \Pi_{i,t} \mu(\mathrm{d}[k_{i,t-1}, x_{i,t-1|t-1}, z_{i,t-1|t-1}]). \tag{2.41}$$

ψ is the marginal disutility of labor. Take wage w_t as given, household chooses consumption c_t and total labor hours n_t^h, which are to be allocated among firms. The household does not save but receives the profits of all the firms each period [21]. Optimization yields the following first order conditions:

$$w_t = \psi c_t \tag{2.42}$$

$$\Lambda_t = \frac{1}{c_t} \tag{2.43}$$

Λ_t is the Lagrangian multiplier associated with the budget constraint and has the interpretation of the marginal utility of consumption. In general equilibrium, this term enters firm i's stochastic discount factor.

[21] Household can save through buying shares of firms. Abstract from saving, the results about identifying responses of aggregate investment dynamics to different types of second moment shocks are not changed.

2.4.3 Recursive Competitive Equilibrium

Use prime and subscript -1 to respectively denote future and predetermined variables, a recursive competitive equilibrium is defined as collection of functions $\{\mathbf{V}, \mathbf{W}, N, K, \lambda, \Lambda, C, w, N^h, \Gamma, \Xi\}$ such that

1. Given predetermined capital stock k_{-1}, observables a, and signal s, wage w, and stochastic discount factor βQ, the firm's value function $\mathbf{V}$, the policy function of optimal capital stock demand K, and labor demand policy N satisfy the firm's recursive problem (2.37).

2. Given wage w, the welfare function $\mathbf{W}$ satisfies household's utility maximization problem (2.40). The marginal value of consumption Λ and the policy function for consumption C satisfy (2.43) and (2.42).

3. The labor market clears with wage w. The labor supply N^h and demands satisfy

$$N^h = \int_S N(k_{-1}, m_{-1|-1}, \mu_{-1}; x, z, \xi, \sigma_e, \sigma_\xi)\mu(\mathrm{d}[k_{-1}, m_{-1|-1}])$$

4. The goods market clears:

$$\begin{aligned} C = \int_S \{ [\frac{\alpha_n}{w}]^{\frac{\alpha_n}{1-\alpha_n}} \exp[\frac{a}{1-\alpha_n}] k_{-1}^{\frac{\alpha_k}{1-\alpha_n}} - [K - (1-\delta)k_{-1}] \\ - \mathbb{I}_{c_k} c_k k_{-1} - \frac{\theta}{2} \frac{[K-(1-\delta)k_{-1}]^2}{k_{-1}} \} \mu(\mathrm{d}[k_{-1}, m_{-1|-1}]) \end{aligned}$$

where $K = K(k_{-1}, m_{-1|-1}, \mu_{-1}; x, z, \xi, \sigma_e, \sigma_\xi)$ and $\mathbb{I}_P$ and $\mathbb{I}_{c_k}$ some indicator functions such that

$$\mathbb{I}_{c_k} = \begin{cases} 1 & : K \neq (1-\delta)k_{-1} \\ 0 & : K = (1-\delta)k_{-1} \end{cases}$$

5. Stochastic discount factor is given by:

$$\Xi = \frac{C}{C'} = \frac{w}{w'}$$

6. The aggregate law of motion defines the dynamics of the probability measure μ_t of firms over space S: $\mu = \Gamma(x, \sigma_e, \sigma_\xi, \sigma_{e,-1}, \mu_{-1})$

7. The state variables of posterior beliefs m satisfy the recursive time-varying Kalman Filter conditions given by Equations (B.47a), (B.47b) and (B.47c). In addition, the productivity level and second moment stochastic processes are given by Equations (2.31a)-(2.34).

2.4.4 Approximate Aggregation

Firm i enters period t carrying the key state variable, $\mu(k_{i,t-1}, m_{i,t-1|t-1})$, end of period $t-1$ joint distribution of firm-level capital stock and beliefs about aggregate productivity and firm-level productivity across firms. Firm's investment decision in period t and newly formed posterior beliefs, which move firms in the distribution over time, depend on a range of aggregate state variables and μ_{t-1}, i.e. defined in Ω_t

vector. Hence the aggregate law of motion $\mu_t = \Gamma(\Omega_t)$ cannot be written in closed form.

Following literature on the general equilibrium of heterogeneous agents, I assume firms are bounded rational such that they only use a finite number of distributional moments to infer evolution of joint distribution over time, and the market-clearing prices (Krusell et al., 1998). Specifically, I use the cross-sectional means of capital stocks $\bar{k}_{t-1}$ and mean of posterior beliefs about the log aggregate productivity $\bar{x}_{t-1|t-1}$ to describe μ_{t-1}, which turns out to be sufficient to describe the joint distribution. [22] Market-clearing wage w_t taken as given for firms' investment decisions is also assumed to be function of these two means.

Firms thus take the following log-linear perceived laws of motion to infer the distribution dynamics and equilibrium wage:

$$\begin{bmatrix} \log(\bar{k}_t) \\ \bar{x}_{t|t} \\ \log(w_t) \end{bmatrix} = \Gamma_0 + \Gamma_1 \begin{bmatrix} \log(\bar{k}_{t-1}) \\ \bar{x}_{t-1|t-1} \end{bmatrix} + \Gamma_2 \begin{bmatrix} \log(\sigma_{\xi,t}) \\ \log(\sigma_{e,t}) \\ \log(\sigma_{e,t-1}) \end{bmatrix} \tag{2.44}$$

Where Γ_0, Γ_1 and Γ_2 are conformable vectors or matrices of coefficients. It should be noted that the *de facto* aggregate productivity x_t is not included as aggregate state variable because firms never truly act upon it per the information frictions. In addition, a number of second moment state variables are to augment the laws of motion in order to capture the aggregate impacts of uncertainty and disagreement shocks.

[22] By law of large numbers, mean of posterior beliefs about idiosyncratic productivity should be intrinsically zero, which does not affect the aggregate dynamics.

The lagged uncertainty enter the laws of motion simply because it directly affects the newly formed posterior beliefs about current period productivity components.

2.4.5 Sketch of Model Solution

The equilibrium definition requires that with the acceptable error tolerance, these approximate laws of motions should "rationalize" the actual equilibrium dynamics. Precisely, the actual market-clearing wage series, aggregate capital stock and mean forecast of TFP series move closely enough as if their actual dynamics are following these laws. Therefore, these equations will be determined in equilibrium.

Taking conjectured policy function for a firm's optimal investment as solved under a given parameter conjecture of aggregate laws of motion, I simulate the economy for a fixed number periods after burning 1000 initial periods of data. Simulation is done following Young (2010), which moves firm density across grid nodes over time. Then a non-linear solver is used to clear the labor market to obtain a time series of equilibrium real wage. Then I re-estimate the equation systems (2.44) using OLS based on the simulation data and update the $\Gamma_j, j \in \{0, 1, 2\}$ coefficients. The equilibrium is solved by looping over policy function and updating laws of motion coefficients until full convergence is reached.

To save computational complexity, the exogenous processes: uncertainty, and disagreement are discretized into two-state Markov chain processes using Tauchen method. Aggregate, idiosyncratic productivity and noise shocks are discretized into three states. The exact solution algorithm is detailed in Appendix B.11.

2.5 Parameter Values

The full set of parameter values that feeds into the full quantitative model is pinned down through combinations of estimations and calibrations.

2.5.1 Measurement and Estimation

First, to measure the productivity uncertainty process defined in the model, I estimate Equation (1.2) based on estimated TFP panel using annual firm-level data from Compustat sample in Imrohoroglu and Tüzel (2014). The dispersion of next year productivity shocks denotes the level of current year uncertainty. For quarterly frequency, annual uncertainty series is linearly interpolated with a year. Second, the disagreement index constructed from Philadelphia Fed Business Outlook Survey data is used to approximate for cross-sectional disagreement; see details in Chapter . Empirical evidence has shown that magnitude of cross-sectional disagreement is not measuring the right concept of productivity uncertainty as defined in our model.

Following Leduc and Liu (2015), parameters on the persistence and innovation S.D.s for fundamental uncertainty and informational disagreement processes are obtained by estimating a large VAR system with both uncertainty and disagreement measures included along with major macro aggregates. Specifically, estimations are based on the VAR system with Cholesky ordering restriction that augments the ordering structure as specified in Bloom (2009). I have shown estimated impulse responses of aggregate investment, output and employment to shocks to disagreement and uncertainty in Chapter .

Here, to reiterate, I examined the ordering that puts uncertainty measure first and also the case when disagreement is put prior to uncertainty. These ordering assumptions are to isolate the non-fundamental sources of second moment shocks that shift cross-sectional disagreement but do not immediately affect the dispersion of real idiosyncratic productivity. Or, jumps in productivity uncertainty shocks do not quickly translate into a disagreement spike within a quarter. It shows ordering between uncertainty and disagreement does not affect the estimation results and the estimates are robust across VAR specifications. I plot the IRFs of uncertainty and disagreement to their own innovations in Figure 2.15.

Figure 2.15 shows that after reaching the peak of jumps in one year, it takes four years for uncertainty to fully decay. On average, uncertainty falls about 25 % of its peak within a year. Then by the AR(1) formulation, the quarterly uncertainty shock persistence is implied by $\rho_{\sigma_e} = (1-0.25)^{1/4} = 0.93$. By contrast, within a year, the disagreement index drops about 85.7 % off its peak, the quarterly persistence is thus given by $\rho_{\sigma_\xi} = (1-0.857)^{1/4} = 0.615$. On impact, the percent jump directly translates into the standard deviation numbers that capture changes in innovations to uncertainty or disagreement. Hence, the estimation shows that uncertainty shocks are more persistent than that of informational shocks to disagreement. However, shocks to uncertainty has one tenth of its innovation size relative to that of disagreement shocks.

Parameter values for the aggregate productivity and idiosyncratic productivity processes are directly borrowed from Cooper and Haltiwanger (2006), which estimates the persistence and standard deviation using constructed plant capital series based on data on retirements and investment constructed from the Longitudinal Research Database (LRD). Following Edmond and Veldkamp (2009), I convert their annual persistence numbers to quarterly counterparts using $\rho_{quarter} = \rho_{annual}^{1/4}$. Then I convert S.D. of their aggregate productivity innovations by $\sigma_{quarter} = \sigma_{annual}/(1 + \rho_{quarter} + \rho_{quarter}^2 + \rho_{quarter}^3)$. Hence, $\rho_x = 0.93 < \rho_z = 0.97$, $\sigma_v = 0.014$ and the unconditional dispersion of log idiosyncratic productivity $\bar{\sigma}_e = 0.15$. I note that the sample mean of cross-sectional annual S.D. of log firm-level TFPs based on Compustat data is 0.42, which falls in the range of 0.3 as in Gilchrist et al. (2014) and 0.64 as in Cooper and Haltiwanger (2006). Therefore, the magnitudes of firm-level productivity dispersion $\bar{\sigma}_e$ are close enough regardless of whether it's based on Compustat or LRD sample, despite I set it to be the number that is more consistent with Cooper and Haltiwanger (2006)'s other estimates on productivity levels.

For the sample average of business outlook disagreement index is around 0.68, given that this index does not perfectly correspond to the informational noisiness or dispersion measure defined in the full model, I thus simply set the unconditional noise dispersion $\bar{\sigma}_\xi = \sqrt{0.68^2 - 0.15^2} = 0.66$, assuming that the dispersion of real productivity and the dispersion of noises are additively shaping the business outlook disagreement index. In Table 2.8, I summarize the parameter values as estimated based on the VAR system.

2.5.2 Calibration

Some parameter values are standard as in the literature. One period corresponds to a quarter. I take numbers on the capital and labor shares in production technology from Khan and Thomas (2008) such that $\alpha_k = 0.256$ and $\alpha_n = 0.64$ so that capital cost share is about one third. The subjective discount factor $\beta = 0.99$ implies an annualized real interest rate of 4 % at the steady state.

A stationary distribution of firms is computed by solving the model with all aggregate shocks turned off. I discretize idiosyncratic productivity and the firm-specific noise into three states. Wage is pinned down by having the wage taken by the firms close enough to the market clearing wage. Equilibrium is solved based on the exact firm mass distribution over the grids. The steady state firm distribution in capital stock helps calibrate parameters on capital depreciation, adjustment costs, and marginal disutility of labor $(\delta, \theta, c_k, \psi)$. They are jointly pinned down by matching the model-implied moments to the data moments of the cross-sectional distribution of plant-level annual investment rates as in Cooper and Haltiwanger (2006).[23] Table 2.9 lists the moment targets and reports the calibrated parameters. The process of calibrating the model is supposed to align the model so as to generate the right fraction of firms who incur large investment spikes as defined to have annual investment rate greater than 20%, the *lumpy investment*, and the fraction of firms with disinvestment, i.e. annualized investment rate smaller than -1%.

Table 2.9 shows that the calibrated adjustment costs, both convex and non-

[23] The moment targets are estimates of annual rates. I convert my model-predicted quarterly investment rates to get annual equivalents according to $\frac{k_{i,t}}{k_{i,t-1}} - (1-\delta)^4$.

convex, are small in size, which are in line with estimates based on Simulated Method of Moment estimation of a partial equilibrium model as in Cooper and Haltiwanger (2006). The calibrated quarterly capital depreciation rate is 2.37 %, which is close enough to 2.5 % as usually assumed in the literature, for example, Gilchrist et al. (2014) and Bloom (2014). The marginal dis-utility of labor falls in the range of $2 \sim 4$ as commonly documented in the literature, e.g. Hansen (1985).

2.6 Quantitative Results

2.6.1 Steady State

The full quantitative model generates a number of key moments of firm-level investment rate distribution at steady state, which well match the data targets. Table 2.10 summarizes the statistics of the distribution. It should be noted that a few trade-offs for matching the data moments exist. First, despite the band of firms' inaction region is not targeted in the calibration process, presence of fixed cost for capital adjustment still delivers 2.4 % of firms with no investment or disinvestment actions taken at steady state.[24] Larger fixed cost parameter c_k, the greater the range of inaction band is. However, as the fixed cost is symmetric for both investment and disinvestment efforts, higher c_k will reduce the average investment rates across firms. Second, to generate sufficient mass of investment spikes, the convex adjustment cost parameter θ cannot be overly large. Also, it cannot be too small as the model will generate excessively large investment drops among disinvesting firms.

[24] However, this number is smaller than the 8 % found in the U.S. data (Cooper and Haltiwanger, 2006).

Despite all these, it is important that the model is able to generate the right fraction of investing firms at steady state, which is about seven to eight times the mass of disinvesting firms. This well matches the U.S. micro-level data. Moreover, as shown in the simple model, this relative mass guarantees that in response to rises in informational disagreement, drops in investments among more productive firms outweighs the increased investments from those less productive firms, which leads to drops in aggregate investment.

2.6.2 Macro Implications: Aggregate Investment Dynamics

In order to examine the effects of informational disagreement shocks and real uncertainty shocks on aggregate investment, I compute the Impulse Response Functions (IRFs) of aggregate investment rate to one standard deviation (S.D.) increases in shocks to productivity uncertainty, and to the dispersion of signal noises. Impulse responses averages out across 10000 simulations. For each simulation, exogenous increases in shocks are imposed in period 101 after first 100 quarters burned. Firm-level investments are then aggregated and divided by the aggregated capital stock.

Figure 2.16 plots the impulse responses of quarterly aggregate investment rate given one S.D. jumps of two different second moment shocks, individually or jointly, imposed in quarter 2. The black line displays the impulse responses to increases in productivity uncertainty shocks only. Aggregate investment rate drops immediately in response to uncertainty jumps. This is due to the fact that more firms would pause, wait, and see until uncertainty is cleared when they see greater gain form waiting for one quarter. Drops in aggregate investment rate are almost 1 percentage point are economically large as the steady state aggregate investment rate is around 2.5 %. Afterwards, the convexity effect brought by uncertainty jumps quickly dominates the died wait-and-see channel. More firms are getting outside the inaction region as productivity draws are realized and productivity draws are more dispersed across firms. Therefore, ex-post, conditional on taking actions, on average, firms found themselves more productive and expect themselves being more productive. Restarting investments generates quick rebound of aggregate investment and aggregate investment even overshoots. Overall, the found "drop-rebound-overshoot" dynamics is consistent with the predictions of a model about uncertainty shocks (Bloom, 2009). Moreover, we see these effects associated with uncertainty shocks are robustly sizable even in the presence of imperfect information environment.

Focusing on the blue dashed line, given more dispersed information that triggers informational disagreement, we see a very trivial decline of aggregate investment in quarter 2. This can be a result of the interplay of three forces ongoing. First, the extensive margin real-option effect could be limited for not many firms pause adjusting capital in response to such informational second moment shocks. Second, conditional on the capital adjustment margin, the convexity effect is still present and firms have greater expected marginal product of capital as forecast error increases with informational noisiness. Third, on impact, firms start making perception mistakes due to the information frictions that firms cannot perfectly disentangle aggregate from idiosyncratic productivity. Firm-level investments are being cut among more productive firms as they would not believe their idiosyncratic productivity draws are good enough. Similarly, less productive firms are investing more. As a result, cuts in investments among a much larger mass of investing firms hammers aggregate investment. Hence, it shows that all these forces are offsetting each other right at the moment of shocks, which means very moderate changes to aggregate investment.

However, going forward, the mis-perception effect starts dominating convexity effect. We see a very gradual decline and much slower recovery compared to the impulse response to uncertainty shocks. Note that the wait-and-see effect is only short-lived and limited in case of informational disagreement shocks. The plot shows that it takes about three to four years for aggregate investment rate to return to the steady state. As firms' are dynamically extracting beliefs over time using Kalman Filtering, their beliefs are weighted averages of prior beliefs and new measurements of total factor productivity $a_{i,t}$ and productivity signal $s_{i,t}$. Thus persistence of imperfect information builds in some *belief inertia*. The gradually formed false beliefs means that more productive firms would slowly accumulates cuts in investments until the right capital targets are reached. The reason why the recovery is slow is because larger dispersion of noises also force firms to increasingly under-weigh new measurements of total factor productivity as the Kalman gain from learning is smaller when information gets nosier. Therefore, more productive firms' pessimistic beliefs won't go away quickly. Such gradual decline in aggregate investment won't stop until firms start weighing more on measurements when informational nosiness eventually decays. Firms finally realize that there was no fundamental shocks occurred at the very beginning. Hence, such mis-perception effect is critical to deliver a real recession even if shocks are not backed by economic fundamental changes. Moreover, such effect helps generate a gradual decline and slower recovery.

The red dotted line plots the impulse response of aggregate investment rate to one S.D. increases in shocks to both real and informational second moments imposed in quarter 2. In reality, it is possible that both shocks can be present, this exercise helps understand the relative roles for real uncertainty shocks and informational disagreement shocks in terms of shifting the aggregate investment dynamics. It shows that the "wait-and-see" effect associated with jumps in real uncertainty dominates in the short run, which generates a sharp decline of aggregate investment in quarter 3. However, conditional on firms taking investment and disinvestment actions, the mis-perception effect associated with informational disagreement shocks can be strong so that the recovery path is largely delayed. In addition, there is no overshoot of aggregate investment.

Important implications can be drawn from here. We still need second moment shocks to economic fundamentals, by affecting the spread of real productivities, to generate very abrupt disruptions of economic activities through channels of "wait-and-see". However, some informational disturbances are also indispensable, by injecting inertia and wrong firms' beliefs, in order to slow down the recovery path. Therefore, impulse response analysis shows how firms' imperfect learning can be critical to propagate the contraction effects of jumps in real uncertainty.

2.6.3 Micro Implications: Firm-level Investments

I then proceed to check if the three separate effects, i.e. convexity effect, wait-and-see effect, and the mis-perception effect, which are important to drive the aggregate investment dynamics, can be well identified at the micro-level and are differently associated with real uncertainty and informational disagreement shocks. The model is simulated for 400 quarters and for a panel of 2000 firms continuously given the optimal policy function for firm-level investment. I estimate the following equation based on the simulated data:

$$\begin{aligned}\log(1+[I/K]_{i,t}) =& \beta_{i,0} + \beta_1 \log(\sigma_{e,t}) + \beta_2 \log(\sigma_{\xi,t}) \\ & + \beta_3 \log(\sigma_{e,t}) \times \Delta z_{i,t} + \beta_4 \log(\sigma_{\xi,t}) \times \Delta z_{i,t} + \beta_5 \Delta z_{i,t} + \epsilon_{i,t} \end{aligned} \quad (2.45)$$

Here, t corresponds to a quarter. To account for the negative investment rates, the dependent variable is log transformed. Note that $\log(1+[I/K]_{i,t}) \approx [I/K]_{i,t}$ when investment-capital ratio is small. Therefore, the estimated coefficients can be roughly interpreted as semi-elasticity such that changes in investment rate are associated with the percentage changes in controlled variables. β_1 measures the partial linear effect of real uncertainty shocks on firm-level investment. Note that $\sigma_{e,t}$ is realized this period but governs the dispersion of to-be-realized idiosyncratic productivity shocks for quarter $t+1$. β_2 captures the linear effects of informational disagreement shocks on firm-level investment. $\sigma_{\xi,t}$ measures how dispersed, or noisy a firm's signal is at quarter t. The non-linear effects β_3 and β_4 are considered as we aim to gauge how the investments taken by more productive firms are affected in different ways by real and informational second moment shocks. β_5 captures the coefficient that is associated with the impacts of growth rate of firm-level TFP on firm-level investment rate. Table 2.11 summarizes the OLS estimations results.

Columns 1-2 display the point estimates for β_1 and β_2 when uncertainty and disagreement enters the specification separately. We see that the linear term coefficients are both negative and are statistically significant. This result suggest that wait-and-see channel dominates the convexity effect for both second moment shocks on impact. columns 3 and 4 show that by including lagged linear terms of uncertainty and disagreement, we can better decompose the linear effect into the convexity and wait-and-see channels. Uncertainty about future productivity variance triggers higher investments, whereas realized productivity variance pushes more firms into inaction by knocking down investments. Conversely, the negative effect is picked up by the contemporaneous $\sigma_{\xi,t}$, while the expansion effect is captured by the lagged disagreement term. This difference may be due to the fact that signal nosiness is modeled to be about the unobserved and *realized* productivity realizations. This helps explain why the similar linear effects associated with terms of uncertainty and disagreement are not concurrently dated. Apart from these linear effects, we still find the productivity-enhancing and dampening effects associated differently with uncertainty and disagreement shocks. Note that the magnitude of dampening effect due to jumps in disagreement is smaller than the enhancing effect associated with larger uncertainty shocks. However, this comparison only captures the difference of shock effects on impact. As we seen from Section 2.6.2, the accumulated effects of disagreement shocks over time can be economically significant. When both linear and non-linear terms of productivity uncertainty and informational disagreement enter the specification for estimation, column 5 still finds that more productive firms cut investments when they are more disagreed because of noisier information while they increase investments when they see larger productivity variance in the future. Therefore, these productivity-enhancing and dampening-effects are consistent with the empirical findings in Chapter . The negative effects associated with the interaction term of disagreement are direct results of the mis-perception effect of disagreement shocks due to imperfect information. Also, the productivity-enhancing effect associated with real uncertainty shocks confirms the micro-foundation for a quick rebound of aggregate investment.

Hence, we see that it is the combined convexity effect and wait-and-see effect associated with productivity uncertainty, and the interplay of all three effects including the mis-perception effect related to informational disagreement that determines the firm-level investment decision. These model-implied effects are well-identified at the micro-level, which helps understand the model-predicted aggregate investment dynamics. Moreover, both macro and micro implications from the quantitative model match the counterpart empirical findings documented in Chapter .

2.6.4 Cyclicality of Investment Rate Dispersion: Role of Informational Disagreement Shocks

Apart from identifying the different macro and micro-level effects associated with real productivity uncertainty and informational disagreement shocks, I explore the quantitative role for disagreement shocks to drive the dynamics of investment rate dispersion across firms. As found in Chapter , real productivity uncertainty have offsetting forces, i.e. convexity effect and wait-and-see effect, that complicates the movement of investment rate dispersion over time. According to Bachmann and Bayer (2014), in order to generate pro-cyclical investment rate dispersion as found in the data, they engineered a model of uncertainty shocks such that the wait-and-see channel is present but bounded, and impose that uncertainty shocks are negatively correlated with aggregate TFP shocks. Their model can deliver the right magnitude of pro-cyclicality only at the cost that uncertainty shocks are no longer significant business cycle drivers.

I delve into the question that without imposing additional restrictions on the property of real uncertainty shocks, whether the presence of information frictions helps deliver the right amount of pro-cyclicality of firm-level investment rate dispersion. Firstly, I compute correlation coefficient between yearly firm-level investment rate dispersion, and the annual U.S. real gross domestic product. Both series are HP detrended. Using annual data for correlation computations is because only yearly firm-level investment rates are readily available based on the Compustat sample from Chapter . Also, correlation coefficients based on yearly data are directly comparable to the data moment in Bachmann and Bayer (2014). It shows that the data moments in the first panel of Table 2.12, the choice of HP-filtering smoothing parameter matters. However, despite the variations, we see that investment rate dispersion is strongly pro-cyclical.

Then I simulate the model of quarterly frequency for 40 years (160 quarters) and average the investment rates and total output within a year to get annual data. Firstly, using a smoothing parameter of 100, the correlation between investment rate dispersion and output is around 0.73 when both second moment shocks are shut off but the TFP shocks at the aggregate level. This model-implied correlation clearly overshoots the 0.45 or 0.6 data moment. Such overshoot is also found in Bachmann and Bayer (2014) based on model simulations with only aggregate TFP shocks. The reason for this overshoot is that increases in aggregate TFP shocks select excessive amount of huge investment spikes in good times. Therefore, the correlation is overly large. Secondly, I further simulate the model with both aggregate TFP shocks and real productivity uncertainty shocks turned on. We see that the correlation number does not get closer to the data moment if not gets even worse. The reason why we have very trivial changes here is that despite wait-and-see channel that shrinks investment rate dispersion is short-lived, it partly offsets the convexity effect that productivity dispersion would drive more investment spikes. Therefore, unless we impose that aggregate TFP shocks and uncertainty shocks are negatively correlated, and the wait-and-see channel is bounded as in Bachmann and Bayer (2014), we won't be able to nail down the overshot number.

Finally, I simulate the model with all shocks turned on. It turns out that the simulation of full model gives a number of cyclicality that is closer to the data moments. To understand why a model with all these orthogonal shocks can jointly deliver the right amount of pro-cyclicality of investment rate dispersion, I estimate regression models of logged firm-level investment rate dispersion on three factors: aggregate productivity x_t, uncertainty measure $\sigma_{z,t}$ in log, along with informational disagreement $\sigma_{\xi,t}$ in log. As shown in Table 2.13, across specifications, we found that both expansion force and shrinkage force on the investment rate dispersion associated with real uncertainty are present with the former dominating the latter. Differently, the effects of rising informational disagreement are unambiguously reducing the investment rate dispersion because more productive firms are cutting investments and less productive firms are investing more. In addition, we see that the TFP shocks are the very significant driver of the dispersion of investment rate dynamics for it selects the right amount of investment spikes.

Here, we see the additional merit of considering stochastic informational second moment shocks, i.e. they helps better align a model of second moment shocks with the data *without* muting the business cycle effects of real uncertainty shocks. Moreover, the model setup does not have to impose shock correlations, or restrict the parameters associated with firm-level non-convex adjustment costs.

2.7 Conclusion

In this chapter, I study a general equilibrium model of heterogeneous firms that face shocks to aggregate productivity and more persistent firm-specific productivity in an environment with imperfect information. Firms care about each productivity component but can only imperfectly disentangle them from the total TFP through imperfect signals, regardless of whether the signals are public or private. I disentangle two distinctive sources of business cycles: uncertainty shocks as in Bloom (2009) and shocks to the precision of signals through which firms learn about aggregate productivity.

Information imprecision driven by pure information dispersion can shift the cross-sectional dispersion in firms' beliefs about aggregate and idiosyncratic productivity over time. I refer to the exogenous informational shocks that affect disagreement among firms as "informational disagreement shocks". When the level of pure disagreement rises, firms become increasingly confused. Investment decisions continue under-reacting to idiosyncratic productivity draws. When more productive firms perceive that good idiosyncratic productivity draws are not good enough to justify investments, this leads to underinvestment. Aggravated capital mis-allocation can have real and sizable impacts on business cycles by driving down aggregate investment, which is simply due to the presence of non-fundamental sources of informational disturbances, even if there are no adverse TFP shocks nor shocks that affect dispersion of real firm-level productivity.

This chapter makes three main contributions. (1) It generates the right amount of pro-cyclicality regarding the dispersion of investment rates, a key empirical regularity that, according to Bachmann and Bayer (2014), models of uncertainty shocks cannot explain well. (2) Through an aggregate investment channel, the model explains why we see macro aggregates undergo a quick rebound after a rise in volatility-based measures of uncertainty but a gradual decline and slow recovery in response to larger forecast disagreement in the data. (3) The model also suggests that we can use dispersion in firm-level investment rates as a key identifier to determine when the economy is hit by real uncertainty shocks and when the economy is simply driven by informational disagreement shocks.

2.8 Tables

Table 2.8: Parameter Values: Fundamental Uncertainty and Informational Disagreement

Parameter	Value	Interpretation
ρ_{σ_e}	0.930	Persistence of Micro Uncertainty
ρ_{σ_ξ}	0.615	Persistence of Informational Disagreement
$\bar{\sigma}_z$	0.150	Unconditional Mean of Uncertainty
$\bar{\sigma}_\xi$	0.660	Unconditional Mean of Informational Disagreement
η_{σ_e}	0.007	Standard Deviation of Uncertainty Shocks
η_{σ_ξ}	0.070	Standard Deviation of Informational Disagreement Shocks

Notes: Parameters are based on 10 variable Structural VAR estimations. See details in the text.

Table 2.9: Calibrated Parameters and Data Targets

Symbol	Value	Parameter	Target	Number
θ	0.0656	Quadratic Cost	Frac. investment spikes	18.6 %
c_k	0.0270	Fixed Cost	Frac. of disinvestment	10.4 %
ψ	3.3606	Marginal Dis. of Labor	Labor Hours	0.33
δ	0.0237	Capital Depreciation Rate	Investment Rate	12.2 %

Table 2.10: Calibrated Model: Steady State Moments

Moments	Data Source	Data	Model
Labor Hours	Standard	0.33	0.322
Fraction of disinvesting firms	LRD	10.4 %	12.5 %
Fraction of investment spikes	LRD	18.6 %	15 %
Average Investment Rate	LRD	12.2 %	10.2 %
Fraction of investing firms	LRD	81.6 %	85.1 %

Notes: The moments of the steady state cross-sectional firm-level investment rates are computed based on a continuous simulation of firm's grid-based investment policy function after 200 quarters are burned for a panel of 10,000 firms.

Table 2.11: Simulation and Estimation: Effects on Firm-level Investment Rates

	(1)	(2)	(3)	(4)	(5)
$\log(\sigma_{e,t})$	-0.0003***		0.0054***		0.0054***
	(0.0000)		(0.0001)		(0.0001)
$\log(\sigma_{\xi,t})$		-0.0002***		-0.0007***	-0.0004***
		(0.0000)		(0.0001)	(0.0001)
$\log(\sigma_{e,t-1})$			-0.0062***		-0.0062***
			(0.0001)		(0.0001)
$\log(\sigma_{\xi,t-1})$				0.0004***	0.0003***
				(0.0001)	(0.0001)
$\log(\sigma_{e,t}) \times \Delta z_{i,t}$			0.0134**		0.0135**
			(0.0067)		(0.0067)
$\log(\sigma_{\xi,t}) \times \Delta z_{i,t}$				-0.0028*	-0.0028*
				(0.0020)	(0.0020)
$\Delta z_{i,t}$			0.0520*	-0.0134	0.0416*
			(0.0280)	(0.0129)	(0.0309)
N	800000	800000	798000	798000	798000

Notes: Based on a simulated panel of 2000 firms for 400 periods. Dependent variable: $\log(1+I_{i,t}/k_{i,t-1})$ to account for negative investments. Measure of Uncertainty: $\sigma_{e,t}$, dispersion of quarter $t+1$ idiosyncratic productivity shocks $e_{i,t}$. Measure of Informational Disagreement: $\sigma_{\xi,t}$, dispersion of quarter t firm-specific noise shocks $\xi_{i,t}$. $z_{i,t}$: idiosyncratic productivity. Firm-level fixed effects are included for all specifications (not reported). Estimations are done through OLS. Robust standard errors reported in parentheses are clustered at the firm level. Significance levels: 10% *, 5% **, 1% ***

Table 2.12: Cyclicality of Investment Rate Dispersion

Correlations with HP-filtered Aggregate Output	
Data Moments	
Data[1] (Compustat; H-P $\lambda = 6.25$)	0.70***
Data[2] (Compustat; H-P $\lambda = 100$)	0.45***
Data[3] (Bachmann and Bayer, 2014)	0.60***
Models	
Only Aggregate TFP shocks	0.73***
Aggregate TFP shocks + Uncertainty Shocks	0.75***
Baseline Model	0.68***

Notes: Data moments [1]: correlations with the cyclical component of annual U.S. Real Gross Domestic after HP-detrend with filtering parameter of 6.25; Data moments [2]: correlations with the cyclical component of annual U.S. Real Gross Domestic after HP-detrend with filtering parameter of 100; These two data moments are computed based on yearly data from 1970 to 2013. Data moments [3]: correlations with the cyclical component of annual real gross value added of the nonfinancial private business sector from NIPA data after HP-detrend with filtering parameter of 100 (Bachmann and Bayer, 2014). Model with only Aggregate TFP shocks: both types of second moment shocks are off. Model-implied correlation coefficient is computed based on simulated series of investment rate dispersion for 40 years (160 quarters) averaging across 100 simulation samples. Real output and investment rate dispersion series are annualized and then HP detrended using filtering parameter of 100. Product Baseline Model: aggregate TFP shocks, aggregate uncertainty shocks, and informational disagreement shocks are all present. Aggregate TFP shocks + Uncertainty Shocks: informational disagreement shocks are shut off. Significance levels: 10% *, 5% **, 1% ***

Table 2.13: Simulation and Estimation: Dispersion of Firm-level Investment Rates

	(1)	(2)	(3)	(4)
$\log(\sigma_{z,t})$	1.0355***		0.9573***	0.8769***
	(0.0191)		(0.0191)	(0.0190)
$\log(\sigma_{z,t-1})$	-0.7345***		-0.6735***	-0.6566***
	(0.0193)		(0.0194)	(0.0193)
$\log(\sigma_{\xi,t})$		-0.0666***	-0.0604***	-0.0056
		(0.0103)	(0.0103)	(0.0101)
$\log(\sigma_{\xi,t-1})$		-0.3813***	-0.3792***	-0.4259***
		(0.0104)	(0.0104)	(0.0102)
x_t				25.2664***
				(0.2845)
Adj R^2	0.0046	0.0169	0.0209	0.0306

Notes: Based on a simulated panel of 2000 firms for 400 quarters. Dependent variable: logged standard deviation of investment-capital ratios across firms. Measure of Uncertainty: $\sigma_{z,t}$, dispersion of quarter $t+1$ idiosyncratic productivity shocks $e_{i,t}$. Measure of Informational Disagreement: $\sigma_{\xi,t}$, dispersion of quarter t firm-specific noise shocks $\xi_{i,t}$. Measure of aggregate productivity in log x_t. Robust standard errors to heteroskedasticity are reported in parentheses. Significance levels: 10% *, 5% **, 1% ***

2.9 Figures

Figure 2.13: Timeline Firm's Decision

$t=1$	$t=2$	$t=3$
(Period of Uncertainty and Disagreement) observes TFP $A_{i,1}$ produces $y_{i,1}$ receives signal s observes σ_v, σ_e and σ_ξ form beliefs $x_{i,1\|1}$ and $z_{i,1\|1}$ makes investment $I_{i,1}$	$A_{t,2}$ reveals produces $y_{i,2}$ makes investment $I_{i,2}$ knows $A_{i,3} = A_{i,2}$	produces $y_{i,3}$

Figure 2.14: s-S Policy

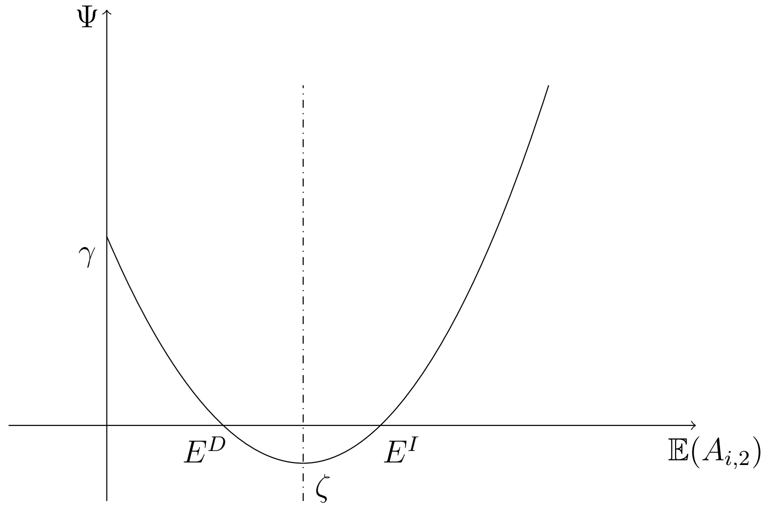


Figure 2.15: VAR-based IRFs: Bloom (2009) with Disagreement, Uncertainty and Investment

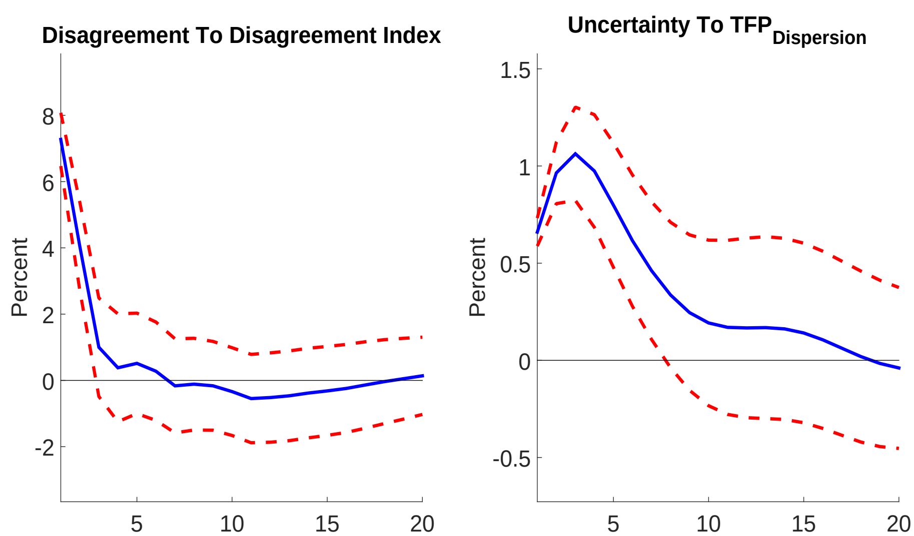
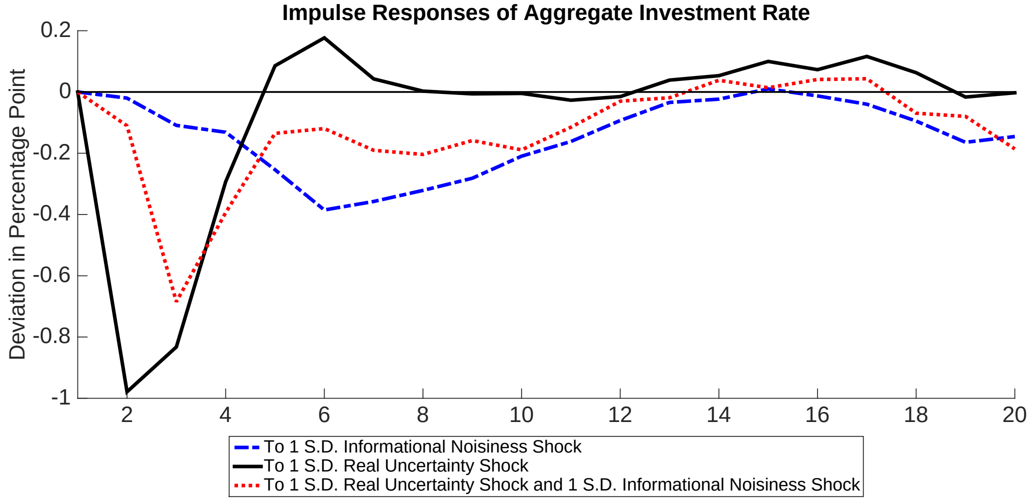


Figure 2.16: Model-based IRFs: Shocks to Uncertainty and Disagreement in Period 2



Concluding Remarks

A firm's uncertainty about its future productivity depends on both the underlying variance of its productivity, and the accuracy of the firm's information about this productivity. Literature that examine the nature and the business cycle impacts of uncertainty, mainly study the changes in the former, i.e. the volatility of economic fundamentals. This dissertation is devoted to empirically and theoretically assessing how information frictions interact with volatility changes of real productivity, in order to understand how the firms' beliefs about future profitability are determined, and how firm-level and aggregate investment are affected by changes in the firms' beliefs.

The first chapter finds that firms' forecast dispersion identifies a different type of second moment shocks other than shocks that affect the variance of real productivity. By affecting the distribution of firm-level beliefs, even if nothing changes to the economic fundamentals, greater heterogeneity in the firms' forecasts triggers persistent declines in aggregate investment, employment, and output as followed by a sluggish recovery. At the firm-level, increasing belief heterogeneity is found to render more productive firms to decrease investments even more, which leads to drops in aggregate investment. Conversely, more dispersed firm-specific future productivity shocks, conventional measure of economic uncertainty, generates the "drop-rebound-overshoot" dynamics for macro aggregates. At the micro-level, more productive firms increase investments when they see productivity variance becomes larger.

To explain these identified impacts of disagreement and productivity uncertainty in the data, Chapter 1.8 presents a general equilibrium model of heterogeneous firms, when the firms' forecasts about their future productivities do not necessarily overlap with the underlying productivity draws because of the imperfect information. When firms rely on noisy signals to disentangle the unobserved aggregate and idiosyncratic productivity, they disagree about the future productivities for pure informational reasons. The model finds that informational disagreement makes firms underestimate *de facto* idiosyncratic productivity draws. Therefore, more productive firms carry increasingly pessimistic forecasts and cut firm-level investments. At the aggregate level, non-fundamental second moment shocks can lead to a real recession by bringing down aggregate investment. Hence, imperfect information helps explain why firm-level and aggregate investment responds quite differently to changes in cross-sectional dispersion of beliefs, as opposed to the variance changes of real productivity. Theses results suggest that a sharp drop of aggregate investment as followed by a slow recovery can be a result of adverse shocks to both real uncertainty and informational disagreement without triggering negative aggregate technological shocks. Precisely, heightened productivity uncertainty shocks can still generate abrupt disruptions of economic activities, while informational disagreement shocks, by driving the distribution of firms' beliefs, can slow down the recovery path.

Nonetheless, uncertainty and disagreement variations are treated to be exogenous in this dissertation. In the future, endogenizing the process of information acquisition to justify how these second moments change over time can be an interesting extension. Empirically, understanding which sources of information noise, such as policy signals and private information, contributes the most to the time-varying disagreement changes among firms, and which types of firms will be more likely to be affected by belief-driven disturbances can be very useful. These research endeavors will greatly help examine the role of belief-driven shocks in other contexts on labor market dynamics, corporate borrowing, and taxation policy, among other domains.

Appendices

Addendum to Chapter 1

A.1 Alternative Measures of Disagreement and Uncertainty

Index of Economic Policy Uncertainty (**EPU**): based on the frequency of newspaper references to policy-related economic uncertainty, the index has been found to spike near tight presidential elections, Gulf Wars I and II, the 9/11 attacks, the failure of Lehman Brothers, the 2011 debt-ceiling dispute and other major battles over fiscal policy. See details in Baker et al. (2015).

Dispersion of Forecasts Measures Based on Philadelphia Fed's Survey of Professional Forecasters data (**SPF**): the survey is conducted quarterly among professional forecasters regarding their forecasts about major macroeconomic variables of the U.S. for the quarter of survey, and up to one year or two years ahead. The survey started in 1968 and the Federal Reserve Bank of Philadelphia took over the collection from the National Bureau of Economic Research, and maintained the survey since 1990. It has been claimed that measurement issues can be severe fore the data before 1990 and I use the data from 1990Q1 up to 2013Q4 in line with Bloom (2014).

Cross-sectional forecast dispersion measures the degree of disagreement among the expectations of different forecasters. I use the forecast dispersion index published by Philadelphia Fed's regarding forecasts about the U.S. real GDP and the industrial production. The exact measure of dispersion is taken as the difference between the 75th percentile and the 25th percentile (the interquartile range) of the point forecasts surveyed. To aid the comparisons with the BOS survey-based forecast disagreement which is constructed based on six month ahead forecast data, I consider two quarters ahead SPF forecast dispersion measure as benchmark SPF measure. As robustness checks, I found no evidence that one-year ahead SPF forecast dispersion measure would significantly alter the benchmark VAR estimation and the empirical impulse responses.

A.2 Measures of Operation Ratios For the Empirical Investment Equations

Firm-level data of annual frequency is used in the firm-level investment equation estimations. I stick to the Compustat fiscal year definitions so that a firm's operation is considered in year $t-1$ data entry if this firm has its end of the fiscal year from January through May of calendar year t otherwise in year t. The definitions of empirical measures are listed below:

1. $[I/K]_{i,t}$: investment-capital ratio, Capital Expenditures in year t divided by Property, Plant and Equipment - Total (Net) in year $t-1$.

2. $[Y/K]_{i,t}$: current sales-to-capital ratio, Sales/Turnover (Net) in year t divided by Property, Plant and Equipment - Total (Net) in year $t-1$.

3. $[\pi/K]_{i,t}$: current operating income-to-capital ratio, Operating Income Before Depreciation in year t divided by Property, Plant and Equipment - Total (Net) in year $t-1$.

4. $Q_{i,t}$: average Q measure, (market value of common equity in year t + book value of total liabilities in year t) then divided by book value of total assets in year t.

5. $[CF/K]_{i,t}$: cash flow-capital ratio, Income Before Extraordinary Items - Adjusted for Common Stock Equivalents in year t divided by Property, Plant and Equipment - Total (Net) in year $t-1$.

A.3 Compustat Data Summary

Compustat annual data from 1970 to 2013 is used for estimating the empirical relationship between firm-level investment and uncertainty/disagreement measures. I restrict the sample to reflect non-financial firms and also exclude regulated utility firms. In specific, I remove firm-year observations with Standard Industry Classification (SIC) codes between 6000 and 6999 (inclusive), and exclude range between 4900 to 4999 (inclusive). In addition, firm-year observations are dropped if a firm did not continuously operate in the investment margin at least for three years. I further make sure that firm-year observations included in the sample have the following properties:

- total asset is positive

- number of employees is greater than one.

- Property, Plant and Equipment - Total (Net), the capital stock measure is positive

After removing outliers with statistical filters, I have the following data summary results: It shows that this unbalanced panel has 104799 firm-year observations in

Table A.14: Summary Statistics of Sample Firm Characteristics

Variable	Obs	Mean	Std. Dev.	Min	Max	Median
$[I/K]_{i,t}$	104799	0.23	0.17	0	1	0.19
$[Y/K]_{i,t}$	104799	5.13	3.44	0.03	15	4.48
$[\pi/K]_{i,t}$	104799	0.55	0.47	-0.5	2.5	0.44
$Q_{i,t}$	31091	1.42	1.3	0.03	58.04	1.08
$[CF/K]_{i,t}$	104799	0.14	0.5	-12.29	12.08	0.13
$TFP_{i,t}$	104799	-0.33	0.38	-5.53	3.36	-0.32

total. On average, each year, we have about 4333 firms in the sample. Due to the missing market value data, Tobin's Q measure has very few valid observations. Note that the $TFP_{i,t}$ is already in log and the data is directly from Imrohoroglu and Tüzel (2014) which is merged with firm profiles by fiscal year and Global Company Key (GVKEY). Following Gilchrist et al. (2014), I restrict the investment-capital ratio to be bounded between 0 and 1.

Addendum to Chapter 2

B.1 Proof of Lemma 1

Proof. Define vector X_i as firm i's information set known at the onset of period 1:

$$X_i = \begin{bmatrix} s \\ a_i \end{bmatrix} = \begin{bmatrix} x_1 + \sigma_\xi \xi \\ x_1 + z_{i,1} \end{bmatrix}.$$

$x_{i,1|1}$ and $z_{i,1|1}$ are linear projections of $x_1, z_{i,1}$ on X_i such that

$$x_{i,1|1} = \mu_x + \Sigma_{xX}\Sigma_{XX}^{-1}(X - \mu_X)$$

$$z_{i,1|1} = \mu_z + \Sigma_{zX}\Sigma_{XX}^{-1}(X - \mu_X)$$

where μ_x, μ_z, μ_X are prior means. Given the zero mean and orthogonality properties, variance co-variance matrix Σ_{XX} is defined as below. The firm index and period 1 index in the expectation operator are suppressed:

$$\Sigma_{XX} = \mathbb{E}(XX') = \begin{bmatrix} \sigma_{v,0}^2 + \sigma_\xi^2 & \sigma_{v,0}^2 \\ \sigma_{v,0}^2 & \sigma_{v,0}^2 + \sigma_{e,0}^2 \end{bmatrix}$$

By the matrix inverse property,

$$\Sigma_{XX}^{-1} = \frac{1}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2} \cdot \begin{bmatrix} \sigma_{v,0}^2 + \sigma_{e,0}^2 & -\sigma_{v,0}^2 \\ -\sigma_{v,0}^2 & \sigma_{v,0}^2 + \sigma_\xi^2 \end{bmatrix}$$

Similarly, $\Sigma_{xX} = [\sigma_{v,0}^2, \sigma_{v,0}^2]$ and $\Sigma_{zX} = [0, \sigma_{e,0}^2]$. Therefore, it yields

$$x_{i,1|1} = \frac{\sigma_{v,0}^2\sigma_\xi^2 \cdot a_i + \sigma_{v,0}^2\sigma_{e,0}^2 \cdot s}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2}$$
$$z_{i,1|1} = \frac{\sigma_{e,0}^2(\sigma_{v,0}^2 + \sigma_\xi^2) \cdot a_i - \sigma_{v,0}^2\sigma_{e,0}^2 \cdot s}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2}$$

Redefine $\mathbf{a} = 1/\sigma_{v,0}^2$, $\mathbf{b} = 1/\sigma_{e,0}^2$ and $\mathbf{c} = 1/\sigma_\xi^2$, we get the formulations of $x_{i,1|1}$ and $z_{i,1|1}$ in Lemma 1 ■ *Q.E.D.*

B.2 Proof of Lemma 2

Proof. By Lemma 1, $x_{i,1|1}$ and $z_{i,1|1}$ are linear combinations of standard normal variables and thus follow normal distribution. Hence, expectation of the exponential of these variables are log-normal. We have

$$\mathbb{E}e^{a_{i,2}} = \exp\left[\mathbb{E}(a_{i,2}) + 0.5\mathbf{\Sigma}\right] = \exp\left[\rho_x x_{i,1|1} + \rho_z z_{i,1|1} + 0.5\mathbf{\Sigma}\right]$$

where $\mathbf{\Sigma}$ is the forecast variance co-variance term of $A_{i,2}$ conditional on receiving the noisy signal, which measures the precision of expectation. Hence we have

$$\begin{aligned}\mathbf{\Sigma} &= \mathbb{E}(x_2 - \rho_x x_{i,1|1})^2 + \mathbb{E}(z_{i,2} - \rho_z z_{i,1|1})^2 + 2\mathbb{E}(x_2 - \rho_x x_{i,1|1})(z_{i,2} - \rho_z z_{i,1|1}) \\ &= \mathbb{E}(\rho_x x_1 + \sigma_v v_2 - \rho_x x_{i,1|1})^2 + \mathbb{E}(\rho_z z_{i,1} + \sigma_e e_{i,2} - \rho_z z_{i,1|1})^2 \\ &+ 2\mathbb{E}(\rho_x x_1 + \sigma_v v_2 - \rho_x x_{i,1|1})(\rho_z z_{i,1} + \sigma_e e_{i,2} - \rho_z z_{i,1|1})\end{aligned}$$

By Lemma 1, substituting out $x_{i,1|1}$ and $z_{i,1|1}$, we then have

$$\mathbf{\Sigma} = \sigma_v^2 + \sigma_e^2 + \frac{(\rho_z - \rho_x)^2}{\mathbf{a} + \mathbf{b} + \mathbf{c}}$$

Therefore, it yields

$$\mathbb{E}e^{a_{i,2}} = \exp\left[(\rho_x + \mathbf{M})x + (\rho_z - \mathbf{N})z_{i,1} - \mathbf{P}\xi + 0.5(\sigma_v^2 + \sigma_e^2 + \mathbf{Q})\right]$$

Where $\mathbf{Q} = \frac{(\rho_z - \rho_x)^2}{\mathbf{a}+\mathbf{b}+\mathbf{c}}$ and

$$\mathbf{M} = \frac{(\rho_z - \rho_x)\mathbf{a}}{\mathbf{a} + \mathbf{b} + \mathbf{c}} > 0, \ \mathbf{N} = \frac{(\rho_z - \rho_x)\mathbf{b}}{\mathbf{a} + \mathbf{b} + \mathbf{c}} > 0, \ \mathbf{P} = \frac{(\rho_z - \rho_x)\sqrt{\mathbf{c}}}{\mathbf{a} + \mathbf{b} + \mathbf{c}} > 0$$

It shows that $\mathbf{M}'(\sigma_\xi) > 0$, $\mathbf{N}'(\sigma_\xi) > 0$ and thus $\mathbf{Q}'(\sigma_\xi) > 0$. We see for perfect information case $\sigma_\xi \to 0$ when x_1 and $z_{i,1}$ are separately observed, $\mathbf{M}, \mathbf{N}, \mathbf{P}$ and $\mathbf{Q}$ terms go to zero. The expectation term is standard given by

$$\mathbb{E}e^{a_{i,2}} = \exp\left[\rho_x x_1 + \rho_z z_{i,1} + 0.5(\sigma_v^2 + \sigma_e^2)\right]$$

■ *Q.E.D.*

B.3 Proof of Propositions 6

Proof. • E_i^I and E_i^D are two roots of the gain from taking non-zero investment Ψ function. Within these two bounds, firms would not take any investment action for $\Pi^{Non-Adj} > \Pi^{Adj}$. By the implicit function theorem:

$$\frac{\mathrm{d}E_i}{\mathrm{d}\sigma_j} = -\frac{\partial\Psi/\partial\sigma_j}{\partial\Psi/\partial E_i} = \frac{[c_k(1-\delta)k_0 - \psi\cdot\phi E_i]\frac{\mathrm{d}\hat{F}(\underline{A})}{\mathrm{d}\sigma_j} + \psi E_i[1-\hat{F}(\underline{A})]\frac{\mathrm{d}\phi}{\mathrm{d}\sigma_j}}{E_i - \zeta}$$

It shows that for range $E_i^I \in (\zeta, \infty)$, denominator is positive whereas range $E_i^D \in (0, \zeta)$ leads to negative denominator where $\zeta = 1 - \psi(1-\phi)$.

By Equaton (2.21), the numerator of the equation above is exactly given by the partial derivative of the option value with respect to the second moment parameter:

$$\begin{aligned}\frac{\partial V_{option}}{\partial\sigma_j} &= \int_{\underline{A}}^{\infty} integral[\frac{\partial}{\partial\sigma_j}\frac{1}{\sqrt{2\pi}A_{i,2}}\mathbf{\Sigma}^{-0.5}e^{-\frac{(\log A_{i,2}-\mu_i)^2}{2\mathbf{\Sigma}}}]\mathrm{d}A_{i,2} \\ &= \int_{\underline{A}}^{\infty}\frac{integral}{2\sqrt{2\pi}A_{i,2}}\mathbf{\Sigma}^{-1.5}e^{-\frac{(\log A_{i,2}-\mu_i)^2}{2\mathbf{\Sigma}}}[\frac{(\log A_{i,2}-\mu_i)^2}{\mathbf{\Sigma}} - 1]\mathbf{\Sigma}'(\sigma_j)\mathrm{d}A_{i,2}\end{aligned}$$

where $integral = \psi A_{i,2} - c_k(1-\delta)k_0 > 0$, $\mathbf{\Sigma}'(\sigma_j) > 0$ and $A_{i,2} \geq \underline{A} > 0$. By the assumption of $\underline{A} \geq e^{\sqrt{\mathbf{\Sigma}}} > 1$, when all first moment shocks are at zeros: $v = 0$, $e_i = 0$, and $\xi_i = 0$ that makes $\mu_i = 0$, we have the following

$$\frac{(\log A_{i,2})^2}{\mathbf{\Sigma}} \geq 1$$

for all $A_{i,2} \geq \underline{A}$. Thus the numerator is positive.

The results above are largely due to the fact that a mean-preserving spread increase of a convex function (max function) such, as uncertainty or disagreement, increases the expected value of future TFP thus the option value of waiting. Therefore larger fundamental uncertainty and non-fundamental disagreement enlarges firm's inaction band by having $E_i^{I'}(\sigma_j) > 0$ and $E_i^{D'}(\sigma_j) < 0$.

• Therefore, firms see greater gain from waiting and pausing actions in case of larger uncertainty or in more disagreed environment. This is true regardless whether or not this is for macro or micro uncertainty.

■ *Q.E.D.*

B.4 Proof of Lemma 5

Proof. We consider the partial derivative of gain from taking non-zero action (Ψ) with respect to idiosyncratic TFP shocks e_i:

$$\begin{aligned}\frac{\partial \Psi}{\partial e_i} &= \frac{\partial}{\partial e_i}[\frac{e^{2(\mu_i+\mathbf{\Sigma})}}{2} - \zeta(e_i)e^{(\mu_i+\mathbf{\Sigma})} + \gamma(e_i)] \\ &= \mu_i'(e_i)[e^{2(\mu_i+\mathbf{\Sigma})} - \zeta(e_i)e^{(\mu_i+\mathbf{\Sigma})} - \zeta'(\mu_i)e^{(\mu_i+\mathbf{\Sigma})} + \gamma'(\mu_i)]\end{aligned}$$

$\mu_i'(e_i) = (\rho_z - \mathbf{N})\sigma_{e,0} = \frac{\rho_z\mathbf{a}+\rho_z\mathbf{c}+\rho_x\mathbf{b}}{\mathbf{a}+\mathbf{b}+\mathbf{c}} > 0$. For $e^{\mu_i+\mathbf{\Sigma}} > E_i^I > \zeta(e_i)$, by Assumption (1), We have $\frac{\partial\Psi}{\partial e_i} > 0$ while $\frac{\partial\Psi}{\partial e_i} < 0$ for $e^{\mu_i+\mathbf{\Sigma}} < E_D^I < \zeta(e_i)$. For e^I solves $E_i^I = e^{\mu_i+\Sigma}$ such that $\Pi^{Adj} = \Pi^{Non-Adj}$, firms would invest if $e_i > e^I$. Similarly, firms would disinvest if $e_i < e^D$ where e^I and e^D are common to all firms for a given ξ_i. ■

Q.E.D.

B.5 Proof of Lemma 6

Proof. By implicit function theorem, we have

$$\frac{\partial e^I}{\partial \sigma_j} = -\frac{\partial(E_i(e_i) - E_i^I)/\partial \sigma_j}{\partial(E_i(e_i) - E_i^I)/\partial e_i}$$
$$\frac{\partial e^D}{\partial \sigma_j} = -\frac{\partial(E_i(e_i) - E_i^D)/\partial \sigma_j}{\partial(E_i(e_i) - E_i^D)/\partial e_i}$$

By Assumption (1),

$$e^{\mu_i + \mathbf{\Sigma}} - \frac{|\zeta'(\mu_i)e^{(\mu_i+\mathbf{\Sigma})} - \gamma'(\mu_i)|}{|e^{\mu_i+\mathbf{\Sigma}} - \zeta(e_i)|} \geq 0$$

Hence, $\partial(E_i(e_i) - E_i^D)/\partial e_i > 0$ and $\partial(E_i(e_i) - E_i^I)/\partial e_i > 0$. Therefore, for $\frac{\partial E_i^I}{\partial \sigma_j} > E_i'(\sigma_j)$ and $\frac{\partial E_i^D}{\partial \sigma_j} < E_i'(\sigma_j)$,

$$\frac{\partial e^I}{\partial \sigma_j} > 0 \quad , \quad \frac{\partial e^D}{\partial \sigma_j} < 0$$

The inaction band expands for a given level of ξ_i. ■ *Q.E.D.*

B.6 Time-varying Recursive Kalman Filtering

Firm i's inference problem has a state space representation:

$$m_{i,t} = Fm_{i,t-1} + \zeta_{i,t} \qquad \text{(State Equation)}$$

$$n_{i,t} = Hm_{i,t} + u_{i,t} \qquad \text{(Measurements Equation)}$$

where

$$m_{i,t} = \begin{bmatrix} x_t \\ z_{i,t} \end{bmatrix}, \ F = \begin{bmatrix} \rho_x & 0 \\ 0 & \rho_z \end{bmatrix}, \ \zeta_{i,t} = \begin{bmatrix} \sigma_v v_t \\ \sigma_{e,t-1} e_{i,t} \end{bmatrix}, \ \zeta_{i,t} \sim \mathbf{N}(\mathbf{0}, \chi_t)$$

$$n_{i,t} = \begin{bmatrix} a_t \\ s_{i,t} \end{bmatrix}, \ H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad u_{i,t} = \begin{bmatrix} 0 \\ \sigma_{\xi,t} \xi_{i,t} \end{bmatrix}, \ u_{i,t} \sim \mathbf{N}(\mathbf{0}, R_t)$$

$$\chi_t = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_{e,t}^2 \end{bmatrix}, \ R_t = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{\xi,t}^2 \end{bmatrix}$$

Firm i uses a recursive linear projection algorithm each period to form new posterior estimates object $\hat{m}_{i,t|t} = \mathbb{E}(m_t|a_t, s_{i,t})$. The posterior expectations are functions of period $t-1$ posterior estimates $\hat{m}_{i,t-1|t-1}$, period $t-1$ posterior variance covariance matrix (matrix of imprecision) $\hat{\Sigma}_{t-1|t-1} = \mathbb{E}[(m_{i,t-1} - \hat{m}_{i,t-1|t-1})(m_{i,t-1} - \hat{m}_{i,t-1|t-1})']$, newly observed magnitude of disagreement $\sigma_{\xi,t}$ and predetermined uncertainty $\sigma_{e,t-1}$, as well as the observable object $n_{i,t}$. The projection rules are stated below:

$$
\begin{aligned}
\hat{m}_{i,t|t} &= (\iota - \kappa_t H)F\hat{m}_{i,t-1|t-1} + \kappa_t n_{i,t} && \text{(B.47a)}\\
\text{where} \qquad \kappa_t &= (F\hat{\Sigma}_{t-1|t-1}F' + \chi_t)H'[H(F\hat{\Sigma}_{t-1|t-1}F' + \chi_t)H' + R_t]^{-1} && \text{(B.47b)}\\
\hat{\Sigma}_{t|t} &= (I - \kappa_t H)(F\hat{\Sigma}_{t-1|t-1}F' + \chi_t) && \text{(B.47c)}
\end{aligned}
$$

ι is a 2 by 2 identity matrix and a prime denotes the matrix transpose. Equations (B.47b) and (B.47c) show that all firms attach the same weights to previous period posterior estimates and the new observables. Specifically, κ_t known as the Kalman gain is the optimal weights for $n_{i,t}$. κ_t resembles the weights we see in Lemma 1. Equation (B.47c) is the discrete time Riccati equation that updates posterior variance covariance matrix and the thus Kalman gain each period.

By Equation (B.47b), rising disagreement for higher noise dispersion passes larger R_t term into a higher discount of the Kalman gain whereas larger uncertainty terms increases the κ term. This suggests that noise dispersion makes firm underweigh the new observations of productivity draws rather than overweigh as uncertainty shocks do. It also implies noise dispersion shocks can create some expectation inertia such that firms still weigh more on previous period estimates.

Specifically, to derive Equations (B.47a)(B.47b)(B.47c), suppress the firm i index, the updating is through a linear projection rule as derived below following

Hamilton (1994):

$$\hat{m}_{t|t} = \hat{m}_{t|t-1} + \kappa_t \tilde{\upsilon}_t$$

$$\hat{\Sigma}_{t|t} = (I - \kappa_t H)\hat{\Sigma}_{t|t-1}$$

where

$$\begin{aligned}
\hat{m}_{t|t-1} &= F\hat{m}_{t-1|t-1} \\
\hat{\Sigma}_{t|t-1} &= F\hat{\Sigma}_{t-1|t-1}F' + \chi_t \\
\hat{\Sigma}_{t|t} &= (I - \kappa_t H)(F\hat{\Sigma}_{t-1|t-1}F' + \chi_t) \\
\tilde{\upsilon}_t &= n_t - H\hat{m}_{t|t-1} \\
\kappa_t &= \hat{\Sigma}_{t|t-1}H'(H\hat{\Sigma}_{t|t-1}H' + R_t)^{-1}
\end{aligned}$$

B.7 Goodness of Fit Checks

Following Krusell et al. (1998), the accuracy of the approximated aggregate laws of motion is measured based on the two goodness of fit statistics, i.e. adjusted R^2 and the standard error of residuals from regressions per the system (2.44). Aggregate time series are simulated given the optimal investment policy functions. The table below reports the statistics.

The aggregate laws of motion are estimated and converged to a tolerance range of smaller than 0.1 and the policy function is solved with convergence tolerance of 0.1. For more refined tolerance, the convergence may not be achievable. Also, for

Table B.15: Approximate Aggregate Laws of Motion

State Var	$\log(\bar{k}_t)$	$\log(w_t)$	$\bar{x}_{t\|t}$
constant	0.51904	-0.67227	0.00411
$log(\bar{k}_{t-1})$	0.88565	0.95341	-0.00047
$log(\bar{x}_{t-1\|t-1})$	0.33	1.7086	0.98236
$\sigma_{\xi,t}$	-0.02425	0.3058	0.00019
$\sigma_{e,t}$	-0.0976	2.1719	0.00067
$\sigma_{e,t-1}$	0.02589	-0.27906	-0.00019
$Adj.R^2$	0.9974	0.85	0.9704
s.e.	0.0002	0.051	0.0007

Notes: Regressions are done using OLS in line with Equations (2.44).

more more refined grids, a very slow speed of convergence will result. The aggregate capital stock equation is solved with very high precision. In order to improve the accuracy of other equations, I tried to augment with interactions and other higher order moments. However, adding more moments creates explosiveness in the sense that the path-searching may drift away from converging to an equilibrium. Therefore, I maintained with a smaller set of state variables. Estimated coefficients for the persistence of aggregate capital stock has comparable magnitudes and signs as in Khan and Thomas (2008).

B.8 Robustness Checks: Public Signals vs. Private Signals

I check if or not when signals are defined as firm-specific signals, the key results would be altered. The answer is no. Following Lemma 1, by derivation of Appendix

B.1, we can redefine information vector X_i at the onset of period 1 by labeling signal as firm-specific s_i attached with a firm-specific noise.

$$X_i = \begin{bmatrix} s_i \\ a_i \end{bmatrix} = \begin{bmatrix} x_1 + \sigma_\xi \xi_i \\ x_1 + z_{i,1} \end{bmatrix}.$$

Note that the aggregate parameter that governs the dispersion of information still measures the aggregate information precision σ_ξ: more dispersed information, more fraction of firms are acting upon less precise information with noises of larger magnitude $|\xi_i|$. As a result, firms still follow the same rule of extracting beliefs about current period aggregate and idiosyncratic productivity as given by Lemma 1.

The only difference resulting from firm-specific noises is that by derivation of Appendix B.2, firm i's expectation of future productivity will now depend on firm-specific noise ξ_i.

$$\mathbb{E}e^{a_{i,2}} = \exp\left[(\rho_x + \mathbf{M})x + (\rho_z - \mathbf{N})z_{i,1} - \mathbf{P}\xi_i + 0.5(\sigma_v^2 + \sigma_e^2 + \mathbf{Q})\right]$$

Therefore, firm's over-reaction of investment to aggregate TFP shocks and under-reaction of investment to idiosyncratic TFP shocks are still there. Change in non-fundamental disagreement parameter σ_ξ would change the extent of capital misallocation. On average, when first moment shocks are at zero, both jumps in uncertainty and disagreement can increase firm's option value of waiting and also increase expected value of marginal product of capital by increasing forecast variance term $\mathbf{\Sigma} = \sigma_v^2 + \sigma_e^2 + \mathbf{Q}$.

For aggregate implication, the following shows that we can still define the aggregate cutoff points in terms of real firm-specific productivity conditional on a distribution of firm-specific noises. The impact of disagreement would not be changed at all.

$$\begin{aligned}\frac{\partial \mathbf{I}}{\partial \sigma_\xi} = & \underbrace{\int_{-\infty}^{\infty}\int_{e^I}^{\infty} E_i \mathbf{\Sigma}'(\sigma_\xi) \mathrm{d}\Phi(e)\mathrm{d}\Phi(\xi) + \int_{-\infty}^{\infty}\int_{-\infty}^{e^D} E_i \mathbf{\Sigma}'(\sigma_\xi) \mathrm{d}\Phi(e)\mathrm{d}\Phi(\xi)}_{intensive\ margin\ :\ convexity\ effect > 0} \\ & + \underbrace{\int_{-\infty}^{\infty} (e^{(\rho_z - \mathbf{N})e^D - \mathbf{P}\xi_i + \mathbf{\Sigma}} - 1)\frac{\partial e^D}{\partial \sigma_j}\mathrm{d}\Phi(\xi)}_{extensive\ margin\ : fewer\ disinvesting\ firms > 0} - \underbrace{\int_{-\infty}^{\infty} (e^{(\rho_z - \mathbf{N})e^I - \mathbf{P}\xi_i + \mathbf{\Sigma}} - 1)\frac{\partial e^I}{\partial \sigma_j}\mathrm{d}\Phi(\xi)}_{extensive\ margin\ : fewer\ investing\ firms < 0} \\ & \underbrace{- \int_{-\infty}^{\infty}\int_{e^I}^{\infty} E_i \mathbf{N}_{\sigma_\xi} e_i \mathrm{d}\Phi(e)\mathrm{d}\Phi(\xi) - \int_{-\infty}^{\infty}\int_{-\infty}^{e^D} E_i \mathbf{N}_{\sigma_\xi} e(i) \mathrm{d}\Phi(e)\mathrm{d}\Phi(\xi)}_{intensive\ margin\ :\ capital\ mis-allocation < 0}\end{aligned}$$

The reason why macro implications are not affected is that firm-specific noises on average will equal to zero, and the noises are i.i.d. over time. Therefore, aggregation of firm-level investment decisions from intensive margin washes out the effect of firm-specific noise when log expectation is linear in firm-specific noises. Therefore, convexity effect and misallocation effect go through for aggregate. Regarding the real-option effect, firm-specific noise does not affect the cutoff points of investing mass and dis-investing mass conditional on distribution of firm-specific noises. It is that uncertainty and disagreement who will affect forecast variance alter the relative mass since firms see greater gain from waiting on average.

In sum, modeling signals as private or public do not affect the main results of this paper. The full model solution that builds on firm-specific information further confirms this robustness.

B.9 Robustness Checks: Noisy Signal About the Idiosyncratic Productivity

I examine the case when firm i receives a firm-specific signal about its idiosyncratic productivity rather than the aggregate plus a firm-specific noise. The information set at the onset of period 1 is reformulated below:

$$X_i = \begin{bmatrix} s_i \\ a_i \end{bmatrix} = \begin{bmatrix} z_{i,1} + \sigma_\xi \xi_i \\ x_1 + z_{i,1} \end{bmatrix}.$$

By the matrix inverse property,

$$\Sigma_{XX}^{-1} = \frac{1}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2} \cdot \begin{bmatrix} \sigma_{v,0}^2 + \sigma_{e,0}^2 & -\sigma_{e,0}^2 \\ -\sigma_{e,0}^2 & \sigma_{z,0}^2 + \sigma_\xi^2 \end{bmatrix}$$

Since $\Sigma_{x_1 X} = [0, \sigma_{v,0}^2]$ and $\Sigma_{zX} = [\sigma_{e,0}^2, \sigma_{e,0}^2]$, it yields

$$\begin{aligned} x_{i,1|1} &= \frac{\sigma_{v,0}^2(\sigma_{e,0}^2 + \sigma_{\xi,0}^2) \cdot a_i - \sigma_{v,0}^2\sigma_{e,0}^2 \cdot s_i}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2} \\ z_{i,1|1} &= \frac{\sigma_{e,0}^2\sigma_{\xi,0}^2 \cdot a_i + \sigma_{v,0}^2\sigma_{e,0}^2 \cdot s_i}{\sigma_{v,0}^2\sigma_{e,0}^2 + \sigma_{e,0}^2\sigma_\xi^2 + \sigma_{v,0}^2\sigma_\xi^2} \end{aligned}$$

Reevaluate Equation (2.14), using notations of inverse of variances, we have

$$\mathbb{E}(e^{x_2+z_{i,2}}) = [\rho_x + \frac{(\rho_z - \rho_x)\mathbf{a}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}]x_1 + [\rho_z - \frac{(\rho_z - \rho_x)\mathbf{b}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}]z_{i,1} + \frac{(\rho_z - \rho_x)\sqrt{\mathbf{c}}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}\xi_i + 0.5(\sigma_v^2 + \sigma_e^2 + \mathbf{Q})$$

We see that firms' investment would exactly under-react to idiosyncratic productivity and over-react to aggregate productivity in line with what we had in Appendix B.1.

In summary, as long as the idiosyncratic productivity component is more persistent than the aggregate component, amplification and dampening effects are still present. The reason is that firms still use imperfect information to disentangle the two components even if the information is about the idiosyncratic productivity.

B.10 Robustness Checks: Permanent and Transitory Component + Firm-specific Noisy Signal

Use notations x and z to denote two different but unobserved aggregate productivity components that enter production function. Absent the idiosyncratic shocks, a firm observes total productivity sum $a = x + z$ but relies on noisy signal $s_i = x + \sigma_\xi \xi_i$ to disentangle them. Without loss of generality, the signal can be defined about z as well. We can think of one is permanent and one is transitory component. Then the firm solves a signal extraction problem in line with Appendix B.1. Firm i has separate beliefs as below

$$x_{i,1|1} = \frac{\sigma_{v,0}^2 \sigma_\xi^2 \cdot a_i + \sigma_{v,0}^2 \sigma_{e,0}^2 \cdot s_i}{\sigma_{v,0}^2 \sigma_{e,0}^2 + \sigma_{e,0}^2 \sigma_\xi^2 + \sigma_{v,0}^2 \sigma_\xi^2}$$

$$z_{i,1|1} = \frac{\sigma_{e,0}^2 (\sigma_{v,0}^2 + \sigma_\xi^2) \cdot a_i - \sigma_{v,0}^2 \sigma_{e,0}^2 \cdot s_i}{\sigma_{v,0}^2 \sigma_{e,0}^2 + \sigma_{e,0}^2 \sigma_\xi^2 + \sigma_{v,0}^2 \sigma_\xi^2}$$

Using notations of inverse of variances, we have

$$\mathbb{E}(e^{x_2+z_{i,2}}) = [\rho_x + \frac{(\rho_z - \rho_x)\mathbf{a}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}]x + [\rho_z - \frac{(\rho_z - \rho_x)\mathbf{b}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}]z - \frac{(\rho_z - \rho_x)\sqrt{\mathbf{c}}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}\xi_i + 0.5(\sigma_v^2 + \sigma_e^2 + \mathbf{Q})$$

Clearly, in line with what we had in Appendix B.1. Specifically, to take an extreme example, if x is the permanent component with $\rho_x = 1$ whereas z is transitory with $\rho_z = 0$. We have the following such that

$$\mathbb{E}(e^{x_2+z_{i,2}}) = (1 - \frac{\mathbf{a}}{\mathbf{a}+\mathbf{b}+\mathbf{c}})x + \frac{\mathbf{b}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}z + \frac{\sqrt{\mathbf{c}}}{\mathbf{a}+\mathbf{b}+\mathbf{c}}\xi_i + 0.5(\sigma_v^2 + \sigma_e^2 + \mathbf{Q})$$

In summary, depending on the relative persistence of aggregate components, firm's investment would always *over-react* to the one with *smaller persistence* and *under-react* to the one with *larger persistence*.

B.11 Iterative Steps for Equilibrium Solution

The following are the general steps used to solve for the recursive competitive equilibrium. I borrow ingredients from the Approximate Aggregation procedure in Krusell et al. (1998).

a Assume that the aggregate capital stock $\bar{k}_{t-1}$ and the mean level of TFP forecasts $\bar{x}_{t-1|t-1}$ at the beginning of period t are sufficient to summarize the beginning-of-period distribution of firms, μ_{t-1}, we come up with Equation system (2.44). Taking parameter conjectures $\Gamma_j, j \in \{0, 1, 2\}$, solve for the individual firm's policy functions using Value Function Iteration that accounts for micro-level nonconvexity.

b Using the capital stock decision rule, simulate actual firm distribution following Young (2010) for T periods and compute the aggregate capital stock and posterior mean belief about TFP time series by taking the cross-sectional average. Each period in this model corresponds to a quarter. I set $T = 5000$ periods with the first 1000 periods burned. Using the same innovations that generate the panel, the goods market clearing condition generates a time series of the equilibrium real wage. Do OLS regressions on the burned-in sample using actual series and obtain new estimates of parameters $\Gamma'_j, j \in \{0, 1, 2\}$.

c Evaluate if $\max(|\Gamma'_j - \Gamma_j|) < \epsilon$, a tolerance range. If true, stop. Otherwise, update conjectures via $\Gamma_j = \lambda\Gamma_j + (1 - \lambda)\Gamma_j$, where λ is the convergence speed control on how much weights are assigned to the previous parameter conjectures.

d With updated parameters Γ_j, solve the individual policy functions again until the vector of Γ_j converges. Each converged parameter vector is associated with a particular set of exogenous innovations on which aggregation is obtained. Repeat the convergence for T_{mc} times and obtain the average $\frac{1}{T_{mc}} \sum_{t=1}^{T_{mc}} \Gamma_j$.

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