ABSTRACT<br>Title of dissertation: ESSAYS ON INFORMATION FRICTIONS, PRICES AND UNEMPLOYMENT<br>Camilo Morales-Jiménez<br>Doctor of Philosophy, 2016<br>Dissertation directed by: Professor S. Borağan Aruoba<br>Department of Economics<br>Professor L. Luminita Stevens<br>Department of Economics

I investigate the effects of information frictions in price setting decisions. I show that firms' output prices and wages are less sensitive to aggregate economic conditions when firms and workers cannot perfectly understand (or know) the aggregate state of the economy. Prices and wages respond with a lag to aggregate innovations because agents learn slowly about those changes, and this delayed adjustment in prices makes output and unemployment more sensitive to aggregate shocks.

In the first chapter of this dissertation, I show that workers' noisy information about the state of the economy help us to explain why real wages are sluggish. In the context of a search and matching model, wages do not immediately respond to a positive aggregate shock because workers do not (yet) have enough information to demand higher wages. This increases firms' incentives to post more vacancies, and it makes unemployment volatile and sensitive to aggregate shocks. This mechanism
is robust to two major criticisms of existing theories of sluggish wages and volatile unemployment: the flexibility of wages for new hires and the cyclicality of the opportunity cost of employment. Calibrated to U.S. data, the model explains $60 \%$ of the overall unemployment volatility. Consistent with empirical evidence, the response of unemployment to TFP shocks predicted by my model is large, humpshaped, and peaks one year after the TFP shock, while the response of the aggregate wage is weak and delayed, peaking after two years.

In the second chapter of this dissertation, I study the role of information frictions and inventories in firms' price setting decisions in the context of a monetary model. In this model, intermediate goods firms accumulate output inventories, observe aggregate variables with one period lag, and observe their nominal input prices and demand at all times. Firms face idiosyncratic shocks and cannot perfectly infer the state of nature. After a contractionary nominal shock, nominal input prices go down, and firms accumulate inventories because they perceive some positive probability that the nominal price decline is due to a good productivity shock. This prevents firms' prices from decreasing and makes current profits, households' income, and aggregate demand go down. According to my model simulations, a $1 \%$ decrease in the money growth rate causes output to decline $0.17 \%$ in the first quarter and $0.38 \%$ in the second followed by a slow recovery to the steady state. Contractionary nominal shocks also have significant effects on total investment, which remains $1 \%$ below the steady state for the first 6 quarters.

# ESSAYS ON INFORMATION FRICTIONS, PRICES AND UNEMPLOYMENT 

by

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## Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of <br> Doctor of Philosophy <br> 2016

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## Dedication

To my wife, parents and sister.

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# Chapter 1: The Cyclical Behavior of Unemployment and Wages under Information Frictions 

### 1.1 Introduction

Search and matching models are an appealing way to study fluctuations in the labor market, as they define unemployment in a manner that is consistent with statistical agencies' convention and describe in an attractive way the functioning of the labor market, how firms and workers are matched and how wages are negotiated. ${ }^{1}$ However, Shimer (2005) pointed out the low volatility of unemployment predicted by the standard search and matching model, hence giving rise to a large body of literature studying the amplifying effects of sluggish wages. This approach to the Shimer Puzzle has been criticized in recent years on the basis that, empirically, wages for new hires exhibit little rigidity while the opportunity cost of employment is pro-cyclical. ${ }^{2}$ In this chapter, I propose a new mechanism for sluggish wages based

[^0]on workers' noisy information about the state of the economy that is robust to the aforementioned critiques and that generates business cycle dynamics for unemployment and wages that are consistent with the empirical evidence. ${ }^{3}$

In my model, wages for new hires are flexible, but wages do not adjust immediately to the true state of the economy because agents learn slowly about aggregate shocks. This delayed adjustment in wages increases firms' incentives to expand employment, making unemployment volatile and sensitive to aggregate shocks. My model is able to explain $60 \%$ of overall unemployment volatility and generates wage semi-elasticities with respect to unemployment of around $-3 \%$, which is a conservative number in the literature.

The model presented in this chapter is in many respects similar to a standard RBC model with search and matching in the labor market. I introduce heterogeneous firms and assume that they differ in their permanent total factor productivity levels, which are public information. Hence, in equilibrium, the most productive firms are larger and pay higher wages. In order to distinguish between new hires coming from unemployment and job changers, I assume that workers search on the job for better-paid jobs. However, the most important distinction in this model versus the existing literature is that workers (households) face information frictions regarding aggregate conditions. In particular, the only source of aggregate uncer-
hires (job changers or new hires coming from unemployment) are more pro-cyclical than are wages for existing workers (e.g. Beaudry \& DiNardo, 1991; Bils 1985; Haefke, Sonntag \& van Rens 2013; Shin, 1994).
${ }^{3}$ Even though this chapter focuses on labor market fluctuations, sticky wages are potentially important for other macroeconomics questions. For example, Christiano, Eichenbaum, Evans (2005) and Smets and Wouters (2007) find that nominal wage stickiness is one of the most important frictions for understanding macroeconomic dynamics under nominal shocks.
tainty is aggregate total factor productivity (TFP), which is not directly observed by workers. Instead, workers form expectations based on a public and noisy signal that they receive each period. This implies that TFP shocks are only partially perceived by workers, who slowly learn about aggregate conditions as time goes by. This information friction affects households' and workers' decisions including consumption and saving. Firms and workers negotiate wages each period. Workers negotiate wages based on their beliefs about the aggregate state of the economy. Hence, after a positive productivity shock, wages remain relatively constant because workers do not immediately possess the proper information to demand higher wages, which generates sluggish wages within jobs. In other words, if productivity increases at time $t$, the wage demanded by workers at firm $j$ at time $t$ will not be very different from the wage that workers demanded at firm $j$ at time $t-1$.

The persistence in wages within jobs increases firms' incentives to hire workers in an expansion as they get to keep a larger fraction of the match surplus. However, in equilibrium, the high-paying/most-productive firms hire proportionally more new workers than the low-paying/less-productive firms in response to a positive productivity shock. This is because there is a significant increase in job-to-job flows as a consequence of the increase in employment, which reduces the average duration of a match for less productive firms and therefore the value of an additional worker. Given that firms have to pay a cost for recruiting new workers, low-wage less-productive firms end up paying this cost more frequently than more productive firms. ${ }^{4}$ In addition, an increase in aggregate TFP reduces the pool of unemployed

[^1]workers, which makes it more difficult for low-paying firms to find new workers but doesn't significantly affect high-wage firms, as they rely more on the pool of employed searchers to fill a vacancy.

In addition to this differential employment growth rate, I also find in my model that high-paying firms tend to exhibit more "flexible" wages in the sense that their wages increase more during expansions. This is a direct consequence of the differential employment growth rate. Notice that an increase in consumption and employment at firm $j$ increases the opportunity cost of employment at that firm because workers would prefer to enjoy more free time. ${ }^{5}$ In an expansion, highpaying firms have to offer higher wages in order to compensate their workers not only for the increase in consumption but also for the larger increase of employment. ${ }^{6}$ However, in an expansion, low-paying firms do not have to increase their wages as much as high-paying firms because, even though consumption increases, employment at low-paying firms is expanding at a lower rate. Hence, even though wages within jobs adjust slowly to the true state of the economy, the average wage for new hires exhibits a large positive response to productivity shocks on impact. This is because a new hire faces more and better-paying job opportunities in an expansion than in a recession. However, even after controlling for this composition effect, my model separations and vacancy duration across firms. In addition, they find that firms with higher employment growth have higher vacancy yields.
${ }^{5}$ Following Chodorow-Reich and Karabarbounis (2014), the flow opportunity cost of employment in my model is the sum of two components: (1) foregone unemployment benefits and (2) the foregone value of non-working activities in terms of consumption. Hence, the faster firm $j$ grows, the larger the opportunity cost of employment for its employees, as the foregone value of nonworking activities in terms of consumption increases.
${ }^{6}$ Notice that an increase in consumption makes the value of non-working activities rise in terms of consumption, given decreasing marginal utility of consumption.
generates wage semi-elasticities with respect to the unemployment rate for new hires and job changers of around $-3 \%$, which is similar to the estimate of Pissarides (2009) and larger than the estimates of Hagedorn and Manovskii (2013) and Gertler et al. (2014).

What does the empirical evidence tell us about the mechanism proposed in this chapter? Using employer-employee data for the U.S., Kahn and McEntarfer (2014) find that employment at high-wage firms is more sensitive to the business cycle. According to their estimates, the differential employment growth rate (high minus low-paying firms) is negatively correlated with the unemployment rate, and this difference is not driven by a more cyclically-sensitive product demand for high-paying firms or because high-wage firms suffer more from earnings rigidities. Hence, a decline in unemployment is associated with a larger increase in employment at highwage firms. In addition, they find that during a downturn, the distribution of new matches shifts towards low-paying firms, whose separation rate declines more than high-paying firms because of the reduction in job-to-job transitions. Therefore, even though net employment changes are more procyclical at high-paying firms, gross worker flows are more procyclical at low-paying firms. Using employer-employee data for the U.S., Haltiwanger, Hyatt and McEntarfer (2015) find that job-to-job flows do reallocate workers from lower-paying to higher-paying firms and that this reallocation is highly procyclical. They find that net employment growth for highwage firms is substantially greater in times of low unemployment compared with low-wage firms, which is driven by net poaching from low-wage to high wage firms. Similarly, Moscarini and Postel-Vinay (2012) find that employment growth is more
negatively correlated with the unemployment rate at large high-paying firms than at small low-paying firms. Moreover, they find that this fact holds mainly within, not across, sectors and states. In an earlier paper, Moscarini and Postel-Vinay (2008), using different data sources, conclude that "following a positive aggregate shock to labor demand, wages respond little on impact and start rising when firms run out of cheap unemployed hires and start competing to poach and to retain employed workers" (p, 2). Hence, wages increase for two reasons: first, workers are paid progressively more, and second workers move to higher-paying firms. ${ }^{7}$

Meanwhile, my assumption about information frictions finds empirical support in the work by Coibion and Gorodnichenko (2012). They compute forecast errors made by professional forecasters, consumers and firms, and document that forecast errors are not consistent with the predictions of a model with perfect information. Rather, they find that forecast errors follow a mean reverting process with a persistence between 0.8 and 0.9. According to their results, the behavior of forecast errors is more consistent with a model in which agents receive noisy signals about aggregate conditions, as I assume in this chapter. In addition, Carroll (2003) formulates and finds evidence in favor of a model in which consumers have a larger degree of information rigidity than other agents. Similarly, Roberts (1998) finds evidence of non-rational expectations in survey data, and Branch (2004) argues that surveys

[^2]reject the rational expectation hypothesis not because agents use an ad hoc expectation rule, but rather because agents optimally decide not to use a more complicated expectation (predictor) function.

I calibrate my model using U.S. data for the period 1964-2014. In order to address the cyclicality of wages for job stayers versus new hires, I use the Current Population Survey (CPS) microdata in order to compute the average wage for these two groups of workers controlling for composition effects (e.g. Solon, Barsky \& Parker, 1994; Haefke, Sonntag \& van Rens, 2013; Muller, 2012). Given that the driving force of this model is shocks to aggregate productivity, I follow the literature that investigates the effects of TFP innovations in order to estimate the fraction of business cycle moments that can be explained by aggregate temporary productivity shocks (e.g. Barnichon, 2010; Basu, Fernald \& Kimball, 2006; Blanchard \& Quah, 1989; Christiano, Eichenbaum \& Vigfusson, 2003, 2005; Gali, 1999). I find in the data that between 70 and $75 \%$ of overall business cycle volatility in labor market quantities such as unemployment, vacancies and the vacancy-unemployment ratio can be explained by temporary TFP innovations. In contrast, only $25 \%$ of the overall volatility in wages can be attributed to such transitory productivity shocks. For quantity variables, I find significant Impulse Response Functions (IRFs) to productivity shocks that exhibit a hump-shaped behavior, peaking one year after the TFP shock. The maximum responses indicate that, following a $1 \%$ increase in productivity, the total number of unemployed workers declines by $6 \%$, vacancies increase by $7 \%$ and the vacancy-unemployment ratio goes up by $15 \%$. I find that wages, adjusted for composition effects, are procyclical, but I do not find significant differ-
ences in the cyclicality of wages for different groups of workers. In contrast to labor market quantities, IRFs for wages are weak and delayed, peaking 2 years after the TFP shock. After a $1 \%$ increase in aggregate productivity, wage responses are very small in absolute value during the first 3 quarters (less than $0.2 \%$ ). Even though wages increase $1 \%$ above their trend 2 years after the shock, this effect is not statistically significant, indicating that wage responses to transitory TFP shocks are weak.

The model calibrated to the U.S. economy is able to explain between 60 and $70 \%$ of the volatility of unemployment, vacancies, and the vacancy-unemployment ratio and $90 \%$ of the volatility in output, consumption and investment that is due to TFP shocks. A graphical inspection reveals that the dynamics predicted by my model are very close to the dynamics estimated in the data. My model generates IRFs that are hump-shaped with peaks consistent with the empirical evidence that I present.

I also show that assuming sticky wages for continuing workers amplifies the unemployment response to productivity shocks, in contrast to previous literature for which the wage of job stayers is irrelevant for vacancy decisions. If a worker has to negotiate her wage for the following $n$ periods, she gives up using the new information she would otherwise be using in the future. Therefore, wages take longer to adjust to the true state of the economy, which increases the firm's incentives to post vacancies. Similarly, I show that assuming that firms face the same information frictions would reinforce my results. If firms observe their overall productivity at all times but cannot distinguish between idiosyncratic productivity shocks and aggregate shocks,
they will partially attribute aggregate shocks to idiosyncratic conditions. Hence, firms will underestimate the decline in the separation rate that is due to productivity shocks and will tend to post even more vacancies.

This work builds on the literature that addresses the Shimer puzzle (Shimer, 2005; Constain \& Reiter, 2008) by studying the amplifying effects of sluggish wages on job creation. ${ }^{8}$ This literature is large; some examples are: Blanchard and Gali (2010), Christiano, Eichenbaum and Trabandt (2014), Elsby (2009), Gertler and Trigari (2009), Hall (2005), Kennan (2009), Menzio (2005), and Venkateswaran (2013). This chapter differs in at least three aspects with respect to this literature. First, I propose a new mechanism for sticky wages based on workers that face information frictions regarding aggregate variables. This mechanism, in contrast to the previous literature, does not rely on any assumption about the persistence of aggregate shocks (Menzio, 2005) or the distribution of firms (Kennan, 2009). ${ }^{9}$ In contrast to Venkateswaran (2013), I show that assuming firms face information frictions does not generate sticky wages but can amplify the unemployment response to productivity shocks. ${ }^{10}$ As in Menzio (2005) and Kennan (2009), what drives sticky

[^3]wages in my model is the fact that workers are willing to work for wages that do not adjust to the true state of the economy. That is, it is not enough to explain why firms offer wages that are very persistent; workers need to be willing to accept them.

A second difference of this chapter with respect to the previous literature is that my model is able to generate significant unemployment volatility in spite of the procyclicality of the flow opportunity cost of employment (FOCE), which is the sum of the foregone unemployment benefits and the foregone value of nonworking activities valued in terms of consumption. According to Chodorow-Reich and Karabarbounis (2014), the FOCE is very procyclical, which weakens or breaks down the results of influential papers such as Hall and Milgrom (2008) and Hagedorn and Manovskii (2008). ${ }^{11}$ This point is also related to the argument of Brugemann and Moscarini (2010) that assuming rent rigidities (wages in excess of the value of unemployment) can account for at most $20 \%$ of the volatility in the job-finding rate. In this chapter, even though the FOCE is procyclical, I still find significant responses of labor market quantities to shocks. This is due to the timing of the model and the real part of the information friction. Given that households make consumption and saving decisions based on the same information friction, investment (capital accumulation) absorbs most of the shock in the initial periods, which prevents consumption and the FOCE from increasing. Hence, even though the FOCE eventually rises, it takes time because workers (not firms) have information frictions regarding

[^4]aggregate variables. To test this assumption, I show that my model predicts dynamics for investment that are consistent with the data and does a good job matching business cycle moments for consumption.

Finally, in contrast to previous literature, this chapter looks at the distributional implications of productivity shocks. I show how and why high-wage firms expand employment the most during an expansion and how this mechanism generates different wage dynamics across firms. In this chapter, even though the information friction is the same for all agents, wages at low-paying firms are less sensitive to the business cycle than wages at high-paying firms. This is a result that other models with sticky wages are unable to reproduce. In fact, in a standard New-Keynesian model, a higher cyclicality of wages at high-wage firms would indicate a lower degree of overall wage rigidity.

This chapter is also related to the literature about information frictions. This chapter is close in spirit to Lucas (1972), where agents' inability to distinguish between aggregate and idiosyncratic shocks generates money non-neutrality. Following Angeletos and La'O (2012) the information friction presented in this chapter has both a nominal and a real part. That is, noisy information about aggregate conditions affects not only price (wage) decisions, but also real allocations (saving, consumption). As explained above, the real part of the information friction plays an important role in explaining the dynamics of the model. Even though this information structure seems exogenous, paying limited attention to aggregate shocks is a standard result in the rational inattention literature that started with Sims (2003). For example, Mackowiak and Wiederhold (2009) present a model in which agents
optimally decide to receive a noisy signal about aggregate conditions, as I assume in this chapter, because acquiring information is costly. Similarly, Acharya (2014) and Reis (2006a, 2006b) show that agents optimally decide to update their information set sporadically when they face a cost of acquiring and processing information.

Finally, this chapter is related to the literature that studies the cyclicality of wages over the business cycle. On the one hand, many studies conclude that the degree of wage cyclicality is small, based in part on empirical evidence suggesting that nominal wages adjust, on average, every 4 quarters in the U.S. (e.g. Kahn, 1997; Barattieri, Basu \& Gottschalk, 2014). ${ }^{12,13}$ However, Pissarides (2009) argues that vacancy decisions depend only on the wage for new hires and points out that the wage elasticity with respect to unemployment for new hires is around $-3 \%$, in comparison with an elasticity of $-1 \%$ for job stayers. The Pissarides critique has been recently challenged by Gertler, Huckfeldt and Trigari (2014), who argue that the evidence presented by Pissarides is based only on job changers. Using PSID data, Gertler et al (2014) do not find that wages for new workers are more procyclical than wages for job stayers and find that the wage elasticity for job changers with respect to unemployment is $-1.7 \%$, which they argue is driven by changes in match quality. ${ }^{14}$ Whether or not wages for new hires are more procyclical than wages for

[^5]existing workers is still an open question and is beyond the scope of this chapter. ${ }^{15}$ Nevertheless, I use CPS microdata in order to construct the average wages for job stayers and new hires (adjusted for composition effects) and assess the predictions of my model. It is worth noting that in my model wages for new hires are flexible and I show that my model is able to reproduce a wage elasticity with respect to unemployment for new hires and job changers of around $-3 \%$, which is not a target in my calibration. ${ }^{16}$ Hence, this chapter points out that wage flexibility for new hires does not imply that wages adjust immediately to the true state of the economy.

The rest of this chapter is organized as follows: I present my model in Section 2 and explain its numerical solution in Section 3. Section 4 presents quantitative analysis. First, I look at the data for the U.S., estimate the fraction of the business cycle moments that can be explained by TFP shocks, and compute the business cycle dynamics of some relevant variables after an aggregate productivity shock. Then, I calibrate my model and compare the model's predictions with my empirical analysis. In Section 5, I discuss some alternative issues and extensions, and Section 6 concludes.

### 1.2 Theoretical Framework

The model presented in this section is, in many aspects, similar to a standard real business cycle model with search and matching in the labor market as in

[^6]Andolfatto (1996) and Merz (1995). I introduce job changers in this model following the theoretical framework of Moscarini and Postel-Vinay (2013) and Burdett and Mortensen (1998). The main difference of my model with respect to the relevant literature is that workers face information frictions about aggregate conditions. As in Lucas (1972), workers form expectations about current aggregate economic conditions based on noisy signals.

### 1.2.1 Model Overview

There are two types of agents in this economy, households and firms. There is a representative household in the economy made up of a continuum of workers that supplies capital and labor to firms and owns all firms in the economy. The household derives utility from consumption and leisure and discounts future utility at rate $\beta$. Capital is supplied in a perfectly competitive market at the capital rental rate $r$ and depreciates at rate $\delta_{k}$, while labor supply is subject to search frictions. I assume complete consumption insurance, which implies that workers seek to maximize income for the household. A worker can be employed or unemployed at each point in time. Unemployed workers receive unemployment compensation $b$ and are matched with a firm with probability $q$. Employed workers are separated from their job with exogenous probability $\delta_{h}$, in which case they must spend at least one period in unemployment before they can be matched with another firm. Employed workers can search on the job. An employed worker is matched with another firm with probability $\bar{i} \cdot q$, where $\bar{i}$ is the search intensity of employed workers relative to
unemployed workers and is fixed. However, employed workers only change jobs if they find a firm that offers an equal or better wage.

There is a continuum of firms indexed by $j$ with mass normalized to 1 . All firms produce a homogeneous good that is sold in a competitive market to the household and can be used for consumption or capital accumulation. A priori the only difference among firms is their (permanent) total factor productivity (TFP) level, which is denoted by $a_{j}$. Without loss of generality, I assume that $a_{j}$ is increasing in $j$. Hence, $a_{x} \geq a_{y}$ for all $x \geq y$. As in Moscarini and Postel-Vinay (2013), the most productive firms pay higher wages and are larger in equilibrium. ${ }^{17}$ Firms produce with capital $k_{j}$ and labor $h_{j}$, through a concave production function. Firms' output is denoted by $y_{j}=e^{a_{j}+a} k_{j}^{\alpha} h_{j}^{1-\alpha}$; where $a$ stands for aggregate TFP, which is common to all firms. At the beginning of each period, firms rent capital and open new vacancies, $v_{j}$. A vacancy is matched with a worker with probability $\tilde{q}$. If a vacancy is matched with an unemployed worker, the vacancy is filled with probability 1. However, if a vacancy is matched with an employed worker, the vacancy is filled only if the worker is coming from a less productive firm. As is standard, new workers (filled vacancies) become productive in the subsequent period. In order to avoid biasing my results in favor of high-wage firms, I assume a hiring cost of the

[^7]form $\frac{\kappa}{1+\chi}\left(\tilde{q}_{j} v_{j}\right)^{1+\chi}$, where $\chi>0$ and $\tilde{q}_{j}$ is the job filling rate for firm $j .{ }^{18}$
The total number of matches in the economy $m(v, s)$ is an increasing function in the total number of vacancies $\left(v=\int_{0}^{1} v_{j} d j\right)$ and the total number of job searchers $\left(s=u+\int_{0}^{1}\left(1-\delta_{h}\right) \bar{i} h_{j} d j\right.$, where $u=1-\int_{0}^{1} h_{j} d j$ is the number of unemployed workers). Following the literature, $m(v, s)$ is assumed to be homogeneous of degree 1. Hence, $q=m(\theta, 1)$ and $\tilde{q}=m\left(1, \theta^{-1}\right)$ where $\theta=v / s$ is labor market tightness.

Firms and workers negotiate wages, $w_{j}$, each period in order to split the expected match surplus according to a simple game: firms make a wage offer that can be accepted or rejected, in the latter case workers make a take-it-or-leave-it offer to firms with exogenous probability $\vartheta$. Hence, in steady state, $\vartheta$ is the fraction of the match surplus that goes to workers.

The only source of aggregate uncertainty is aggregate total factor productivity $a$, which follows an $\operatorname{AR}(1)$ process. However, $a$ is not directly observed by workers in this economy. Instead, every period there is a public and noisy signal $\hat{a}$ about the current level of aggregate TFP. This signal is observed by workers and firms, and this is common knowledge. Based on the expectations derived from this signal, workers make wage demands (in a sense that will be explained below) and the house-

[^8]hold makes consumption/savings decisions. Even though workers do not perfectly observe aggregate TFP, the idiosyncratic TFP level $a_{j}$ for each firm is public information. In the benchmark model, firms have perfect information about aggregate productivity. ${ }^{19}$

The timing of the model each period is as follows:

1. Aggregate TFP is realized.
2. The public signal is received and workers form expectations.
3. Wages are negotiated.
4. Firms rent capital and post vacancies.
5. Production takes place, and factors are paid.
6. The household makes a consumption decision based on the beliefs derived from the signal $\hat{a}$.
7. A fraction $\left(1-\delta_{h}\right) \bar{i} q$ of employed workers at firm $j$ is matched with another firm, a fraction $q$ of unemployed workers finds a new job, and a fraction $\delta_{h}$ of employed workers are endogenously separated from their jobs.
8. A fraction $\left(1-\delta_{h}\right) \bar{i} q F_{j}$ of employed workers leaves firm $j$ to join another firm, where $F_{j}$ is the probability for firm $j$ 's employees of being matched with a firm with higher $a_{j}$.
[^9]
### 1.2.2 Household

There is a representative household made up of a continuum of members with mass normalized to $1 .{ }^{20}$ The household is the owner of all firms in the economy, and it supplies capital and labor to firms. Capital is supplied in a perfectly competitive market at the rental rate $r$, while labor supply is subject to search frictions. I assume complete consumption insurance, which implies that workers seek to maximize income for the household. Consumption and savings decisions are made at the household level, but household members make their decisions based on the same information set $\mathcal{I}_{h}$. Throughout this chapter, $E_{\mathcal{I}_{h}}[x]$ is the expected value of $x$ conditional on the information set $\mathcal{I}_{h}$, and $E[x]$ is the expectation conditional on perfect information.

### 1.2.2.1 Consumption and Saving

Consumption and savings decision are made at the household level in order to maximize the utility function

$$
\begin{equation*}
\mathbb{U}(\omega, \Omega)=\frac{c^{1-\sigma}}{1-\sigma}-\Psi \int_{0}^{1} \frac{h_{j}^{1+\xi}}{1+\xi} d j+\beta E\left[\mathbb{U}\left(\omega^{\prime}, \Omega^{\prime}\right)\right] \tag{1.1}
\end{equation*}
$$

subject to the budget constraint (1.2) and a perceived law of motion for the

[^10]economy (1.3):
\[

$$
\begin{align*}
c+k^{\prime} & \leq\left(r+1-\delta_{k}\right) k+\int_{0}^{1} w_{j} h_{j} d j+\int_{0}^{1} \pi_{j} d j+b \cdot u-T  \tag{1.2}\\
\Omega^{\prime} & =\lambda^{h}(\Omega) \tag{1.3}
\end{align*}
$$
\]

where ' denotes next period's value. $\omega=\left\{k,\left\{h_{j}\right\}_{j=0}^{1}, \mathcal{I}_{h}\right\}$ is the vector of state variables for the representative household, and $\Omega$ is a vector that summarizes the aggregate state of the economy. $c$ is consumption, $k$ is capital, $w_{j}$ is the wage paid by firm $j$, and $\pi_{j}$ stands for firm $j$ 's profits. $u=\int_{0}^{1}\left(1-h_{j}\right) d j$ is the total number of unemployed workers, and $b$ is unemployment compensation, which is financed by lump sum taxes $(T=b \cdot u)$. The household and its members form expectations based on their information set $\mathcal{I}_{h}$ and on a perceived law of motion for the economy $\left(\lambda^{h}(\cdot)\right)$. Therefore, the problem for the household is given by:

$$
\begin{aligned}
\max _{c, k^{\prime}} & E_{\mathcal{I}_{h}}\{\mathbb{U}(\omega, \Omega)\} \\
& \text { s.t. }
\end{aligned}
$$

This leads to the first order condition for consumption:

$$
\begin{equation*}
c^{-\sigma}=\beta E_{\mathcal{I}_{h}}\left[\left(1-\delta+r^{\prime}\right) c^{\prime-\sigma}\right] \tag{1.4}
\end{equation*}
$$

It is worth noting that the consumption decision is also affected by information
frictions because the expectation in equation (1.4) is conditional on the information set $\mathcal{I}_{h}$. In other words, information frictions affect not only the wage bargaining process as described in section 1.2.5, but also real allocations. ${ }^{21}$ To the extent that aggregate shocks are partially perceived, the household will respond to productivity innovations by accumulating capital in an attempt to smooth consumption through time. As a result, the marginal disutility of labor (in terms of consumption) does not increase, which prevents wages from going up. This mechanism will be clear in section 1.2.5.

### 1.2.2.2 Workers

A worker can be employed or unemployed at each point in time. Unemployed workers receive unemployment compensation $b$ and are matched with a firm with probability $q$. Conditional on a match, a worker is matched with firm $j$ with probability $\left(\frac{v_{j}}{v}\right)$, where $v$ is the total number of vacancies in the economy and $v_{j}$ stands for firm $j$ 's vacancies. Hence, the value of unemployment $U(\omega, \Omega)$ is given by:

$$
\begin{equation*}
U(\omega, \Omega)=b+E\left\{Q\left((1-q) \cdot U\left(\omega^{\prime}, \Omega^{\prime}\right)+q \cdot \int_{0}^{1} W_{x}\left(\omega^{\prime}, \Omega^{\prime}\right) \frac{v_{x}}{v} d x\right)\right\} \tag{1.5}
\end{equation*}
$$

where $Q=\beta\left(\frac{c^{\prime}}{c}\right)^{-\sigma}$ is the stochastic discount factor between this and the next period and $W_{j}(\omega, \Omega)$ is the value of employment at firm $j$. Meanwhile, employed workers are separated from their job with exogenous probability $\delta_{h}$, in which case they have to spend at least one period in unemployment before they can be matched

[^11]with another firm. Following Moscarini and Postel-Vinay (2013), I assume that employed workers can search on the job. In particular, an employed worker at firm $j$ is matched with another firm with probability $\bar{i} \cdot q$. However, I assume that employed workers only change jobs if they find a firm that offers an equal or better wage. Throughout this chapter, I refer to jobs that pay higher wages as better jobs. ${ }^{22}$ Hence, the value of employment at firm $j$ is given by:
\[

$$
\begin{align*}
W_{j}(\omega, \Omega)= & w_{j}-\Psi \frac{h_{j}^{\xi}}{c^{-\sigma}} \\
& +E\left\{Q \left(\left(1-\delta_{h}\right)(1-\bar{i} q) W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)\right.\right. \\
& +\left(1-\delta_{h}\right) \bar{i} q \int_{0}^{1} \max \left\{W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right), W_{x}\left(\omega^{\prime}, \Omega^{\prime}\right)\right\} \frac{v_{x}}{v} d x \\
& \left.\left.+\delta_{h} U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right\} \tag{1.6}
\end{align*}
$$
\]

The first line in equation (1.6) is the net flow income of a worker employed at firm $j$. The second term $\left(\Psi \frac{h_{j}^{\xi}}{c^{-\sigma}}\right)$ is the value of non-working activities (or the marginal disutility of labor) in terms of consumption, which is derived from the household's utility function (1.1). The second line in equation (1.6) says that with probability $\left(1-\delta_{h}\right)(1-\bar{i} q)$ a worker is not exogenously separated from firm $j$ and is not matched with another firm. The third line captures that with probability $\left(1-\delta_{h}\right) \bar{i} q$ a worker is not exogenously separated from firm $j$, is matched with another firm, and picks the firm that gives her the higher continuation value. Finally, with probability $\delta_{h}$ a worker becomes unemployed.

Given that only weakly better jobs are accepted, $\max \left\{W_{j}(\omega, \Omega), W_{x}(\omega, \Omega)\right\}=$

[^12]$W_{x}(\omega, \Omega) \forall x \geq j$. Therefore, combining equations (1.5) and (1.6):
\[

$$
\begin{align*}
\left(W_{j}(\omega, \Omega)-U(\omega, \Omega)\right)= & w_{j}-z_{j} \\
& +E\left\{Q \left(\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right)\left(W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right.\right. \\
& +\left(1-\delta_{h}\right) \bar{i} q F_{j}\left(\tilde{W}_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right) \\
& \left.\left.-q\left(\bar{W}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right)\right\} \tag{1.7}
\end{align*}
$$
\]

Following Hall and Milgrom (2008), I define $z_{j}$ as the flow-opportunity cost of employment for firm $j . F_{j}$ is the probability of finding a weakly better job than $j$, $\tilde{W}_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)$ is the expected value of the new job for job changers leaving firm $j$, and $\bar{W}\left(\omega^{\prime}, \Omega^{\prime}\right)$ is the expected value of a new job for unemployed workers. These terms in turn satisfy:

$$
\begin{align*}
z_{j} & =b+\Psi \frac{h_{j}^{\xi}}{c^{-\sigma}}  \tag{1.8}\\
F_{j} & =\int_{j}^{1} \frac{v_{x}}{v} d x  \tag{1.9}\\
\tilde{W}_{j}\left(\omega^{\prime}, \Omega^{\prime}\right) & =\int_{j}^{1} W_{x}\left(\omega^{\prime}, \Omega^{\prime}\right)\left(\frac{v_{x}}{\int_{j}^{1} v_{y} d y}\right) d x  \tag{1.10}\\
\bar{W}\left(\omega^{\prime}, \Omega^{\prime}\right) & =\int_{0}^{1} W_{x}\left(\omega^{\prime}, \Omega^{\prime}\right) \frac{v_{x}}{v} d x \tag{1.11}
\end{align*}
$$

Notice that the net value of employment $\left(W_{j}(\omega, \Omega)-U(\omega, \Omega)\right)$ is a decreasing function in $z_{j}$ and therefore in consumption. An increase in consumption makes $z_{j}$ go up and reduces the net value of employment. As a consequence, wages must increase when consumption increases in order to compensate workers for the decline
in the value of employment.

Chorodow-Reich and Karabarbounis (2014) find empirically that the flow opportunity cost of employment $\left(z_{j}\right)$ is pro-cyclical and conclude that this procyclicality undermines the results of previous papers attempting to solve the unemployment volatility puzzle. A similar point is made by Brugemann and Moscarini (2010), who argue that rent rigidity, defined as the fraction of wages that do not depend on $z_{j}$, can account for at most $20 \%$ of the volatility in the job-finding rate. However, notice that in this chapter information frictions reduce the sensitivity of $z_{j}$ to productivity shocks. As explained above, to the extent that aggregate shocks are partially perceived, the household will respond to positive productivity innovations by accumulating capital in an attempt to smooth consumption through time, which prevents $z_{j}$ from increasing.

Finally, notice that the expectations in equations (1.5), (1.6) and (1.7) are not conditional on the household's information set $\mathcal{I}_{h}$. Instead, the expectations are conditional on perfect information. This is because equations (1.5) and (1.6) describe what a worker will actually receive in expectation and not what workers expect to receive. However, workers will have to form expectations about $W_{j}(\omega, \Omega)$ and $U(\omega, \Omega)$ in order to negotiate wages as described in section 1.2.5.

### 1.2.3 Firms

There is a continuum of firms indexed by $j$ with a mass normalized to 1 . Firms produce with capital and labor, and their output can be used for consumption or
for capital accumulation. At the beginning of each period, firms rent capital and open new vacancies, $v$. A vacancy is matched with a worker with probability $\tilde{q}$. As is standard in the literature, a filled vacancy becomes productive in the subsequent period. However, not all matches become productive. If a vacancy is matched with a worker that is currently employed at a better job, the match is dissolved. Hence, denoting $\tilde{q}^{u}$ as the probability of filling a vacancy with an unemployed worker and $\tilde{q}_{j}^{c}$ as the probability of filling a vacancy with a job changer, the job filling rate for firm $j\left(\tilde{q}_{j}\right)$ is given by:

$$
\begin{align*}
\tilde{q}_{j} & =\tilde{q}^{u}+\tilde{q}_{j}^{c}  \tag{1.12}\\
\tilde{q}^{u} & =\tilde{q} \cdot\left(\frac{u}{s}\right)  \tag{1.13}\\
\tilde{q}_{j}^{c} & =\tilde{q} \cdot\left(\int_{0}^{j} \frac{\left(1-\delta_{h}\right) \bar{i} h_{x}}{s} d x\right) \tag{1.14}
\end{align*}
$$

Notice that $\tilde{q}^{u}$ is the same for all firms. By contrast, the job filling rate varies across firms even though the probability of a match $(\tilde{q})$ is the same for all firms. $\tilde{q}_{j}^{c}$ and $\tilde{q}_{j}$ are higher for the most productive firms. As a consequence, lowproductivity firms rely more on the pool of unemployed workers. Hence, in an expansion, low-wage firms find it more difficult to fill a vacancy and to retain a worker than high-wage firms.

The problem for firm $j$ is given by:

$$
\begin{align*}
& \Pi_{j}\left(\omega_{f}, \Omega\right)=\max _{v_{j}, k_{j}} \pi_{j}+E\left[Q \Pi_{j}\left(\omega_{f}^{\prime},, \Omega^{\prime}\right)\right]  \tag{1.15}\\
& \text { s.t. } \\
& \pi_{j}=y_{j}-w_{j} h_{j}-r k_{j}-\frac{\kappa}{1+\chi}\left(\tilde{q}_{j} v_{j}\right)^{1+\chi}  \tag{1.16}\\
& y_{j}=e^{a_{j}+a} k_{j}^{\alpha} h_{j}^{1-\alpha}  \tag{1.17}\\
& h_{j}^{\prime}=\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right) h_{j}+\tilde{q}_{j} v_{j}  \tag{1.18}\\
& \Omega^{\prime}=\lambda^{f}(\Omega)  \tag{1.19}\\
& v_{j}, \quad k_{j} \geq 0 \tag{1.20}
\end{align*}
$$

where $a$ stands for aggregate TFP, which is common to all firms. $\omega_{f}=\left\{h_{j}\right\}$ is the vector of state variables for firm $j$, and equation (1.19) is the perceived law of motion for the economy. Denoting marginal labor productivity by $p_{j}=$ $(1-\alpha) e^{a_{j}+a} k_{j}^{\alpha} h_{j}^{-\alpha}$, the first order conditions with respect to $v_{j}$ and $k_{j}$ are given by:

$$
\begin{array}{ll}
v_{j}:-\kappa\left(\tilde{q}_{j} v_{j}\right)^{\chi}+E\left[Q \cdot J_{j}^{\prime}\left(\omega_{f}^{\prime}, \Omega^{\prime}\right)\right] & \leq 0 \\
k_{j}: p_{j}\left(\frac{h_{j}}{k_{j}}\right)\left(\frac{\alpha}{1-\alpha}\right)-r & =0 \tag{1.22}
\end{array}
$$

where $J_{j}\left(\omega_{j}, \Omega\right)$ is the firm's value of an additional worker, or the continuation
value of a filled vacancy:

$$
\begin{align*}
& J_{j}\left(\omega_{f}, \Omega\right)=\frac{\partial \Pi_{j}\left(\omega_{f}, \Omega\right)}{\partial h_{j}}  \tag{1.23}\\
& J_{j}\left(\omega_{f}, \Omega\right)=p_{j}-w_{j}+E\left[Q \cdot\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right) \cdot J_{j}\left(\omega_{f}^{\prime}, \Omega^{\prime}\right)\right] \tag{1.24}
\end{align*}
$$

Notice that even though the exogenous separation rate $\delta_{h}$ is the same for all firms, the total separation rate varies across firms. If we define $\delta_{h j}=1-\left(1-\delta_{h}\right)(1-$ $\left.\bar{i} q F_{j}\right)$ as firm $j$ 's total separation rate, we can see that low-wage (less-productive) firms have higher separation rates. Given that $F_{j}$ is lower for more productive firms, $\delta_{h j}$ is also lower for the most productive firms. Note that even though I am not assuming a cost per vacancy posted, labor market conditions affect the value a new vacancy through the firm specific separation rate $\delta_{h j}$. It will be shown that low-wage firms experience a larger increase in separations (quits) in expansions than high-wage firms. Hence, the value of a new worker increases less for less-productive firms in expansions.

### 1.2.4 Information Sets

I assume that workers (households) face information frictions in the sense that they do not perfectly know the current value of aggregate TFP $(a)$, which is the only source of aggregate uncertainty. I assume that there is a public signal ( $\hat{a}$ ), based on which workers form expectations. I assume that this public signal is also observed by firms, so that workers' beliefs are common knowledge. The public signal and
aggregate productivity are related as follows:

$$
\begin{equation*}
\hat{a}=a+n \tag{1.25}
\end{equation*}
$$

where $n$ is the noise of the signal. The aggregate TFP (a) and the noise $(n)$ are assumed to follow two independent $\mathrm{AR}(1)$ processes. I interpret the autocorrelation in this noise as waves of optimism or pessimism:

$$
\begin{array}{ll}
a^{\prime}=\rho_{a} \cdot a+e_{a}^{\prime} ; & e_{a} \sim N\left(0, \varsigma_{a}\right) \\
n^{\prime}=\rho_{n} \cdot n+e_{n}^{\prime} ; & e_{n} \sim N\left(0, \varsigma_{n}\right) \tag{1.27}
\end{array}
$$

In order to formally define the equilibrium of this economy and find the solution of this model, I have to assume that workers can perfectly observe the state of the economy with a lag of $\mathcal{T}$ periods where $\mathcal{T}$ is a large integer. Hence, the information set for the representative household is given by:

$$
\begin{equation*}
\mathcal{I}_{h}=\left\{\hat{a}^{\mathcal{T}}, \Omega_{-\mathcal{T}}\right\} \tag{1.28}
\end{equation*}
$$

where $\hat{a}^{\mathcal{T}}$ represents the last $\mathcal{T}$ realizations of $\hat{a}$, and $\Omega_{-\mathcal{T}}$ is the value of the vector $\Omega \mathcal{T}$ periods ago. This information set does not mean that the representative household does not perceive new productivity shocks at all. On the contrary, workers form expectations about current and future economic conditions based on Bayes' rule and this information set, in order to make their decisions. This assumption about
information implies that aggregate shocks are partially perceived by workers, who learn slowly about productivity innovations as time elapses while simultaneously continuing to receive positive or negative signals. Hence, if workers do not have enough information to conclude that the economy is in an expansionary path, they will not demand higher wages. Further, partial perception of aggregate shocks causes $c$ and $z_{j}$ to become more persistent, another avenue through which wage increases are muted somewhat.

Not surprisingly, empirical evidence suggests that agents do not form expectations based on perfect information. For example, Coibion and Gorodnichenko (2012) find that the expectations of firms, households, and central banks are more consistent with a model in which agents receive noisy signals about aggregate conditions, as is assumed in this chapter.

### 1.2.5 Wages

I assume that wages are completely flexible and are negotiated at the start of every period according to a simple game, through which firms and workers bargain over the match surplus $\left(S_{j}\right)$ :

$$
\begin{equation*}
S_{j}=J_{j}\left(\omega_{f}, \Omega\right)+W_{j}(\omega, \Omega)-U(\omega, \Omega) \tag{1.29}
\end{equation*}
$$

Notice that $w_{j}$ appears in functions $J_{j}\left(\omega_{f}, \Omega\right)$ and $W_{j}(\omega, \Omega)$ in accordance with equations (1.24) and (1.6). However, since $w_{j}$ is an endogenous variable, it is not
written as an argument for these functions. ${ }^{23}$ For expositional purposes, I will abuse notation slightly in this section and define functions $\vec{J}_{j}\left(w, \omega_{f}, \Omega\right)$ and $\vec{W}_{j}(w, \omega, \Omega)$ as:

$$
\begin{align*}
\vec{J}_{j}\left(w, \omega_{f}, \Omega\right)= & p_{j}-w+E\left[Q \cdot\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right) \cdot J_{j}\left(\omega_{f}^{\prime}, \Omega^{\prime}\right)\right]  \tag{1.30}\\
\vec{W}_{j}(w, \omega, \Omega)= & w-\Psi c^{\sigma} h_{j}^{\xi} \\
& +E\left\{Q \left(\left(1-\delta_{h}\right)(1-\bar{i} q F j) W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)\right.\right. \\
& \left.\left.+\left(1-\delta_{h}\right) \bar{i} q F_{j} \tilde{W}_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)+\delta_{h} U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right\} \tag{1.31}
\end{align*}
$$

Function $\vec{J}_{j}\left(w, \omega_{f}, \Omega\right)$ can be interpreted as the value of a filled vacancy for an arbitrary wage $w . \vec{W}_{j}(w, \omega, \Omega)$ is interpreted similarly. ${ }^{24}$ As a consequence, functions $\vec{J}_{j}\left(w, \omega_{f}, \Omega\right)$ and $J_{j}\left(w, \omega_{f}, \Omega\right)$ are related as follows:

$$
\begin{align*}
J\left(\omega_{f}, \Omega\right) & =\vec{J}_{j}\left(w_{j}, \omega_{f}, \Omega\right)  \tag{1.32}\\
W(\omega, \Omega) & =\vec{W}_{j}\left(w_{j}, \omega, \Omega\right) \tag{1.33}
\end{align*}
$$

where $w_{j}$ is the wage that holds in equilibrium.

[^13]
### 1.2.5.1 Wage negotiation

Wages in this economy are negotiated according to the following game:

1. The firm offers a wage $x$ to the worker.
2. The worker observes the firm's offer. Upon acceptance, the game ends with payoffs of $\vec{W}_{j}(x, \omega, \Omega)-U(\omega, \Omega)$ to the worker and $\vec{J}_{j}\left(x, \omega_{f}, \Omega\right)$ to the firm.
3. If the worker rejects the firm's offer, the match is destroyed with exogenous probability $1-\vartheta$ (with payoffs to both agents of 0 ); otherwise, the worker demands a wage $y$.
4. The firm observes this demand. Upon acceptance, the game ends with payoffs of $\vec{W}_{j}(y, \omega, \Omega)-U(\omega, \Omega)$ for worker and $\vec{J}_{j}\left(y, \omega_{f}, \Omega\right)$ for firm. If the firm rejects the worker's offer, the game ends with payoffs of zero for both agents.

The extensive-from representation of this game is given in Figure 1.1.

### 1.2.5.2 Equilibrium Wage and Discussion

Even though this model assumes information frictions, an important benchmark is the case in which all agents have perfect information. In this spirit, the following lemma establishes the equilibrium of this game under perfect information, which will be used to compare the results under information frictions.

Lemma 1. If all agents in the economy have complete and perfect information, the following strategy profiles constitute the unique sub-game perfect Nash equilibrium

Figure 1.1: Wage Determination Game
Firm offers a wage equal to x


Worker demands a wage equal to y
$\left(\vec{J}_{j}\left(y, \omega_{f}, \Omega\right), \quad \vec{W}_{j}(y, \omega, \Omega)-U(\omega, \Omega)\right)$


Note: This figure shows the extensive-form representation of the wage determination game. Firms and workers bargain over the match surplus $\left(S_{j}\right)$ by making wage offers/demands. Details are provided in the text.
of this game:

- For the worker:
- To accept only wage offers greater than or equal to $x^{*}$ where $\vec{W}_{j}\left(x^{*}, \omega, \Omega\right)$ $U(\omega, \Omega)=\vartheta \cdot S_{j}$
- To demand a wage equal to $y^{*}$ such that $\vec{W}_{j}\left(y^{*}, \omega, \Omega\right)-U(\omega, \Omega)=S_{j}$ and $\vec{J}_{j}\left(y^{*}, \omega_{f}, \Omega\right)=0$.
- For the firm:
- To offer $x^{*}$.
- To accept only wage demands that are less than or equal to $y^{*}$.

Proof. See Appendix A.2.1

Hence, under perfect information, the solution to this game coincides with the
solution to the Nash-Bargaining game when the worker's bargaining power is equal to $\vartheta$. Therefore, I will call $\vartheta$ the long-term bargaining power of workers.

Now, before characterizing the solution to this game with information frictions, the following lemmas tell us that, in equilibrium, firms cannot credibly communicate the true state of the economy to the workers.

Lemma 2. Suppose that agents are information-constrained as described in section 1.2.4. If there is an equilibrium in which firms' strategy is to reveal the aggregate state of the economy, the best strategy for firms is the same strategy described in Lemma 1.

Proof. See Appendix A.2.2

Lemma 3. If agents in the economy are information-constrained as described in section 1.2.4, then in equilibrium, firms do not follow a strategy in which they perfectly reveal the true state of the economy.

Proof. See Appendix A.2.3

Even though Lemmas 2 and 3 do not characterize the solution to this game, they make clear that a solution in which firms reveal the true state of the economy is not possible. The intuition is simple: firms have incentives to lie. Firms will always be tempted to tell workers that aggregate productivity is lower than it actually is, so wages can be lower. As a consequence, workers do not rely on firms' offer to form expectations about aggregate conditions. Before defining the solution for this game with information frictions, I make the following assumption:

Assumption 1. For all realizations of $a$ and $\hat{a}$,

$$
\begin{equation*}
\vec{J}_{j}\left(x^{* *}, \omega_{f}, \Omega\right) \geq 0 \tag{1.34}
\end{equation*}
$$

where $x^{* *}$ is such that:

$$
\begin{equation*}
E_{\mathcal{I}_{h}}\left[\vec{W}_{j}\left(x^{* *}, \omega, \Omega\right)-U(\omega, \Omega)\right]=\vartheta \cdot E_{\mathcal{I}_{h}}\left[S_{j}\right] \tag{1.35}
\end{equation*}
$$

That is, if both parties agree upon a wage $x^{* *}$ such that, according to the worker's information set, a fraction $\vartheta$ of the match surplus goes to the worker, the firm still gets a positive payoff for all realizations of the true productivity and the signal. I check that this assumption holds in my calibration. Next, the following lemma presents the solution to this game.

Lemma 4. If agents in the economy are information-constrained as described in section 1.2.4, the following strategy profiles constitute a Perfect Bayesian Nash equilibrium:

- For the worker:
- To accept only wage offers greater than or equal to $x^{* *}$ where:

$$
E_{\mathcal{I}_{h}}\left[\vec{W}_{j}\left(x^{* *}, \omega, \Omega\right)-U(\omega, \Omega)\right]=\vartheta \cdot E_{\mathcal{I}_{h}}\left[S_{j}\right]
$$

- To demand a wage equal to $y^{* *}$ such that:

$$
E_{\mathcal{I}_{h}}\left[\vec{W}_{j}\left(y^{* *}, \omega, \Omega\right)-U(\omega, \Omega)\right]=E_{\mathcal{I}_{h}}\left[S_{j}\right]
$$

- For the firm:
- To offer $x^{* *}$.
- To accept only wage demands that are less than or equal to $\tilde{y}^{* *}$ such that

$$
\vec{J}_{j}\left(\tilde{y}^{* *}, \omega_{f}, \Omega\right)=0
$$

## Proof. See Appendix A.2.4

Notice that in equilibrium, wages are a function of what workers would have demanded if given the chance, even though they do not get to make such a wage demand in equilibrium. This is because, if firms anticipate that workers will ask for a fraction $X$ of their perceived match surplus, they will offer a wage such that workers get $\vartheta \cdot X$ of the match surplus. Notice that this result is common in the literature. In the classical paper of Rubinstein (1982), there are no counter-offers in equilibrium because the first player to move makes an offer that takes into account what the other player would get in the second stage of the game. Similarly, Hall and Milgrom (2008) and Christiano et al (2005) assume that wages are negotiated according to an alternating wage offer game. In those papers, there are no counteroffers in equilibrium because firms compensate workers for what they would get if they had the chance to make a counter-offer. ${ }^{25}$ In this sense, this set-up introduces

[^14]information frictions in a tractable way, and the solution under perfect information of this game is the same as the Nash bargaining solution with workers' bargaining power equal to $\vartheta$.

Regarding the solution with information frictions, Lemma 4 is an important result for this chapter. Given that firms have incentives to lie about true productivity (Lemma 3), workers will only use their own information set to assess wage offers. Hence, wage demands will be based on information frictions. To the extent that aggregate TFP shocks are partially perceived, wage demands will be less sensitive to aggregate conditions because workers' expectations are smoother than aggregate shocks. Consequently, wages will be more sluggish under information frictions. Notice that assuming that firms face the same information friction would not affect the solution to this game, and therefore would not affect how sensitive wages are to productivity shocks. However, if firms observe their overall productivity $\left(a_{j}+a\right)$ at all times in addition to the signal $\hat{a}$ but cannot distinguish between aggregate and idiosyncratic TFP shocks, firms will partially attribute aggregate TFP innovations to idiosyncratic conditions. In that situation, firms will tend to post even more vacancies in expansions because, in addition to the effect of persistent wages, firms will underestimate the increase in separations and the decline in the job filling rate. This case is covered at the end of this chapter as an extension.

### 1.2.6 Equilibrium

We can now characterize the vector that describes the aggregate state of the economy as $\Omega=\left\{k,\left\{h_{j}\right\}_{j=0}^{1}, a^{\mathcal{T}}, \hat{a}^{\mathcal{T}}\right\}$. As before, $a^{\mathcal{T}}$ and $\hat{a}^{\mathcal{T}}$ refer to the last $\mathcal{T}$ realizations of $a$ and $\hat{a}$.

Definition 1. A recursive competitive equilibrium for this economy is a list of functions $\left\{\mathbb{U}(\omega, \Omega), W_{j}(\omega, \Omega), U(\omega, \Omega), \Pi_{j}\left(\omega_{f}, \Omega\right), J_{j}\left(\omega_{f}, \Omega\right)\right\}$ [Value Functions], $\left\{\left\{w_{j}(\Omega)\right\}_{j=0}^{1}, Q(\Omega), r(\Omega)\right\}$ [Prices], $\left\{\left\{h_{j}\left(\omega_{f}, \Omega\right), k_{j}\left(\omega_{f}, \Omega\right), v_{j}\left(\omega_{f}, \Omega\right), \pi_{j}\left(\omega_{f}, \Omega\right)\right.\right.$, $\left.\left.\tilde{W}_{j}(\omega, \Omega), z_{j}(\Omega)\right\}_{j=0}^{1}, \bar{W}(\omega, \Omega), c(\omega, \Omega), k(\omega, \Omega), y(\Omega), s(\Omega), \theta(\Omega)\right\}[$ Allocations], $\left\{\left\{\tilde{q}_{j}(\Omega), \tilde{q}_{j}^{c}(\Omega), F_{j}(\Omega)\right\}_{j=0}^{1}, q(\Omega), q^{u}(\Omega)\right\}\left[\right.$ Probabilities], and $\left\{\lambda, \lambda^{f}, \lambda^{c}\right\}[$ Law of motion] such that given a law of motion for $\{\hat{a}, a, n\}[E x o g e n o u s ~ v a r i a b l e s] ~$

- The representative household and workers optimize: Taking as given prices, probabilities and a perceived law of motion for the economy (1.3), $c(\omega, \Omega)$, $k^{\prime}(\omega, \Omega)$ satisfy optimality condition (1.4) and the household's budget constraint (1.2).
- Firms optimize: Taking as given prices, probabilities and a perceived law of motion for the economy (1.19), $v_{j}\left(\omega_{f}, \Omega\right), k_{j}\left(\omega_{f}, \Omega\right)$, and $h_{j}\left(\omega_{f}, \Omega\right)$ satisfy optimality conditions (1.21), (1.22) and the law of motion for $h_{j}$ (A.4).
- Wages and the stochastic discount factor: Wages are a solution to wage bargaining game 1.2.5.1 and the stochastic discount factor is consistent with $Q(\Omega)=\beta\left(\frac{c\left(\omega^{\prime}, \Omega^{\prime}\right)}{c(\omega, \Omega)}\right)^{-\sigma}$.
- Consistency of value functions: value functions $\mathbb{U}(\omega, \Omega), W_{j}(\omega, \Omega), U(\omega, \Omega)$, $\Pi_{j}\left(\omega_{f}, \Omega\right)$, and $J_{j}\left(\omega_{f}, \Omega\right)$ are consistent with equations (1.1), (1.6), (1.5), (1.15), and (1.24).
- Beliefs: at each point in time, workers' beliefs are determined by their information set $\mathcal{I}_{h}$, their perceived law of motion for the economy (1.3), and Bayes' rule.
- Law of motion: the household's and firms' decision rules imply a law of motion for the economy $(\lambda)$ that is consistent with the household's and firms' perceived law of motion: $\lambda^{f}=\lambda^{h}=\lambda$.
- Probabilities: probabilities $\tilde{q}_{j}(\Omega), \tilde{q}_{j}^{c}(\Omega), F_{j}(\Omega), q^{u}(\Omega)$, and $q(\Omega)$ are consistent with equation (1.12), (1.14), (1.9), (1.13) and $q(\Omega)=m(v(\Omega), s(\Omega)) / s(\Omega)$.
- Allocations: $\pi_{j}\left(\omega_{f}, \Omega\right), y_{j}\left(\omega_{f}, \Omega\right), z_{j}(\Omega), \tilde{W}_{j}(\omega, \Omega), \bar{W}(\omega, \Omega)$ and $\theta(\Omega)$ are consistent with equations (1.16), (1.17), (1.8), (1.10), (1.11), and $\theta(\Omega)=\left(\frac{v(\Omega)}{s(\Omega)}\right)$.
- Aggregation: $v, Y, s, u, k$, are consistent with:

$$
\begin{aligned}
& v(\Omega)=\int_{0}^{1} v_{j}\left(\omega_{f}, \Omega\right) d j \\
& y(\Omega)=\int_{0}^{1} y_{j}\left(\omega_{f}, \Omega\right) d j \\
& s(\Omega)=u(\Omega)+\int_{0}^{1} \bar{i} h_{j}\left(\omega_{f}, \Omega\right) d j \\
& u(\Omega)=\int_{0}^{1}\left(1-h_{j}\left(\omega_{f}, \Omega\right)\right) d j \\
& k(\Omega)=\int_{0}^{1} k_{j}\left(\omega_{f}, \Omega\right) d j
\end{aligned}
$$

- Exogenous variables: a, â, and $n$ evolve according to equations (1.25), (1.26) and (1.27).


### 1.3 Computation

In order to compute the solution to this model numerically, it is important to find and determine a law of motion for the economy, based on which the household forms expectations and makes decisions. This task may not be simple for a large vector $\Omega$, given a distribution of firms. Hence, I solve this model by combining the solution method for heterogeneous agent models proposed by Reiter (2009) and the Kalman Filter, which I used in a previous paper (Morales-Jiménez, 2014). In this section, I explain intuitively the logic behind this method.

First, the Reiter method solves heterogeneous agent models by taking a firstorder approximation of the model around the deterministic steady state of the economy. ${ }^{26}$ Assume that the following system of equations describes the equilibrium of the economy:

$$
\begin{equation*}
f\left(\Omega, \Omega^{\prime}, \Upsilon, \Upsilon^{\prime}, \mathbb{E}\right)=0 \tag{1.36}
\end{equation*}
$$

where $\Upsilon$ is the vector of endogenous variables of the economy and $\mathbb{E}$ is the vector of exogenous shocks. The Reiter method then finds the solution in three steps:

1. A finite representation of the economy is provided by discretizing the distri-

[^15]bution of agents.
2. The deterministic steady state of the economy is found by imposing $\mathbb{E}=0$ and finding the solution to:
\[

$$
\begin{equation*}
f^{*}=f\left(\Omega^{*}, \Omega^{*}, \Upsilon^{*}, \Upsilon^{*}, 0\right)=0 \tag{1.37}
\end{equation*}
$$

\]

3. The model is linearized numerically around the steady state, which yields the system of linear equations:

$$
\begin{equation*}
f_{1}^{*}\left(\Omega-\Omega^{*}\right)+f_{2}^{*}\left(\Omega^{\prime}-\Omega^{*}\right)+f_{3}^{*}\left(\Upsilon-\Upsilon^{*}\right)+f_{4}^{*}\left(\Upsilon^{\prime}-\Upsilon^{*}\right)+f_{5}^{*} \mathbb{E}=0 \tag{1.38}
\end{equation*}
$$

where $f_{i}^{*}$ is the partial derivative of (1.37) with respect to its $i$-th argument. This system is solved using a standard method such as Sims (2002) or Klein (2000).

Hence, the Reiter method induces a law of motion for the economy of the form:

$$
\begin{align*}
\Omega^{\prime} & =\mathbb{F} \Omega+\mathbb{E}  \tag{1.39}\\
\Upsilon & =\mathbb{G} \Omega \tag{1.40}
\end{align*}
$$

where $\mathbb{F}$ and $\mathbb{G}$ are matrices of coefficients. Therefore, the law of motion for the economy is described by: $\lambda=\{\mathbb{F}, \mathbb{G}\}$. The challenge for a model with information frictions comes from the fact that the law of motion $\lambda$ is derived from a perceived law of motion $\lambda^{h}$, which in equilibrium has to be equal to the actual law of motion
$\lambda$.
I exploit the linearity of the Reiter method and proceed as follows: ${ }^{27}$

1. Define a tolerance level.
2. Guess a linear law of motion for the economy $\lambda^{h\{1\}}=\left\{\mathbb{F}^{h\{1\}}, \mathbb{G}^{h\{1\}}\right\}$. A good initial guess may be the law of motion of the model under perfect information.
3. Let the household form expectations based on this guess and the Kalman filter.
4. Find the solution of the model using the Reiter method, which is given by a new law of motion $\lambda^{\{1\}}=\left\{\mathbb{F}^{\{1\}}, \mathbb{G}^{\{1\}}\right\}$.
5. If the maximum difference between $\lambda^{h\{1\}}$ and $\lambda^{\{1\}}$ is less than the predetermined tolerance level, stop and conclude that $\lambda^{h\{1\}}=\lambda$. Otherwise, update the household's perceived law of motion as follows:

$$
\begin{equation*}
\lambda^{h\{n+1\}}=d \cdot \lambda^{h\{n\}}+(1-d) \cdot \lambda^{\{n\}} ; \quad 0<d<1 \tag{1.41}
\end{equation*}
$$

where $d$ is a fraction that determines how smoothly the guess is updated.

## 6. Go back to step 3 .

[^16]
### 1.4 Quantitative Analysis

In this section, I assess the model's predictions in light of the empirical evidence for the United States for the period 1964 to 2014. Before taking a look at the data, it is important to highlight again two features of the model presented in this chapter. First, the main driving force in the model is productivity shocks. As a consequence, it would be incorrect to look only at the unconditional moments in the series, and I should try to identify the fraction of the business cycle that is driven by aggregate productivity shocks. ${ }^{28}$ Second, this is a business cycle model. Therefore, I should detrend all U.S. variables in order to make a correct comparison with my model. In order to do that, I follow the literature and filter all series (at quarterly frequency) using the Hodrick-Prescott filter with a smoothing parameter equal to $10^{5} .{ }^{29}$

### 1.4.1 U.S. Data

I present business cycle statistics for the quarterly time series (seasonally adjusted) of unemployment, vacancies, output, consumption, investment, aggregate TFP, and real wages (deflated by CPI) for job stayers, new hires, job changers and new hires from non-employment. ${ }^{30}$ All variables are HP-filtered in logs with a

[^17]smoothing parameter of $10^{5}$, which is a standard parameter in the literature.
Unemployment is the total number of unemployed people from the Current Population Survey (CPS). Vacancies are the composite help-wanted index computed by Barnichon (2010). Output is real output in the nonfarm business sector. Aggregate productivity is measured as the Solow residual as computed by Basu, Fernald, and Kimball (2006), which is available and updated at the website of the Federal Reserve Bank of San Francisco. Consumption consists of non-durable goods and services. Finally, investment is real gross private domestic investment. I include investment as a variable of interest because the impact of the information friction on investment plays an important role in my model.

Given the debate about the cyclicality of wages, I use the CPS microdata to construct the average wages for job stayers, new hires, job changers and new hires from non-employment adjusted for composition effects. In order to compute these wages, I follow Muller (2012) and Haefke et al. (2013) who also used the CPS microdata to construct similar series. Denoting $x_{i t}$ as a vector with individual level characteristics such as education, experience, sex, occupation and industry, the wage of individual $i$ at time $t\left(w_{i t}\right)$ is given by:

$$
\begin{equation*}
\log \left(w_{i t}\right)=x_{i t}^{\prime} \beta_{x}+\log \left(\hat{w}_{i t}\right) \tag{1.42}
\end{equation*}
$$

where $\hat{w}_{i t}$ is the part of wages that does not depend on individual characteristics, which may or may not depend on aggregate conditions. The average wage for
group $G\left(w_{t}^{G}\right)$ adjusted by composition effects is defined as:

$$
\begin{equation*}
\log \left(w_{t}^{G}\right)=\sum_{i \in G} \log \left(\hat{w}_{i t}\right) \omega_{i t} \tag{1.43}
\end{equation*}
$$

where $G=\{$ job stayers, job changers, new hires, new hires from non-employment $\}$, and $\omega_{i t}$ is the sample weight for individual $i$, which is provided by the Bureau of Labor Statistics (BLS). Since 1994, the CPS has asked individuals whether or not they still work at the same job as in the previous month, making it possible to identify job changers. However, it is not possible to identify job-to-job transitions prior that year. In order to have similar samples, all results regarding wages by group are restricted to the sample period 1994-2014. Appendix A. 4 provides more details about the CPS dataset, the methodology that I follow to construct $w_{t}^{G}$, and some auxiliary regressions and discussion. Given that the literature usually measures wages by the average hourly earnings of production and non-supervisory employees, which is available since 1964, the results for this series can be found in Appendix A. 4 as well.

### 1.4.2 Business Cycle Statistics and TFP Shocks

Table 1.1 presents unconditional business cycle statistics. As has been previously documented in the literature, unemployment is one of the most volatile series. Unemployment is 10 times more volatile than TFP, and 8 times more volatile than output. Similarly, vacancies and the vacancy-unemployment ratio are also highly volatile relative to productivity and output.
Table 1.1: Business Cycle Statistics: U.S. Economy 1964:Q1- 2014:Q4

|  |  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{s}$ | $w^{u}$ | $w^{c}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Standard deviation Autocorrelation |  | $\begin{aligned} & 0.180 \\ & 0.960 \end{aligned}$ | $\begin{aligned} & 0.180 \\ & 0.949 \end{aligned}$ | $\begin{aligned} & 0.348 \\ & 0.957 \end{aligned}$ | $\begin{aligned} & 0.024 \\ & 0.941 \end{aligned}$ | $\begin{aligned} & 0.017 \\ & 0.957 \end{aligned}$ | $\begin{aligned} & 0.096 \\ & 0.915 \end{aligned}$ | $\begin{aligned} & 0.081 \\ & 0.211 \end{aligned}$ | $\begin{aligned} & 0.081 \\ & 0.194 \end{aligned}$ | $\begin{aligned} & 0.087 \\ & 0.174 \end{aligned}$ | $\begin{aligned} & 0.083 \\ & 0.234 \end{aligned}$ | $\begin{aligned} & 0.083 \\ & 0.222 \end{aligned}$ | $\begin{aligned} & 0.018 \\ & 0.907 \end{aligned}$ |
| Correlation Matrix | $u$ | 1 | -0.868 | -0.967 | -0.832 | -0.662 | -0.758 | 0.115 | 0.112 | 0.081 | 0.140 | 0.131 | -0.477 |
|  | $v$ |  | 1 | 0.967 | 0.781 | 0.575 | 0.784 | -0.132 | -0.134 | -0.098 | -0.161 | -0.151 | 0.531 |
|  | $v / u$ |  |  | 1 | 0.834 | 0.640 | 0.798 | -0.128 | -0.127 | -0.092 | -0.156 | -0.145 | 0.522 |
|  | $y$ |  |  |  | 1 | 0.894 | 0.821 | 0.168 | 0.172 | 0.181 | 0.148 | 0.152 | 0.802 |
|  | c |  |  |  |  | 1 | 0.629 | 0.310 | 0.302 | 0.309 | 0.299 | 0.301 | 0.734 |
|  | Inv |  |  |  |  |  | 1 | -0.021 | -0.015 | 0.005 | -0.048 | -0.040 | 0.689 |
|  | $w^{a}$ |  |  |  |  |  |  | 1 | 0.993 | 0.980 | 0.992 | 0.993 | 0.165 |
|  | $w^{s}$ |  |  |  |  |  |  |  | 1 | 0.973 | 0.984 | 0.985 | 0.171 |
|  | $w^{u}$ |  |  |  |  |  |  |  |  | 1 | 0.974 | 0.982 | 0.169 |
|  | $w^{c}$ |  |  |  |  |  |  |  |  |  | 1 | 0.999 | 0.147 |
|  | $w^{n}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0.151 |
|  | $a$ |  |  |  |  |  |  |  |  |  |  |  | 1 |

Notes: Statistics for the U.S. economy are based on: $u$ : Unemployment level. v: Help-wanted index (Barnichon, 2010). v/u: Vancancyunemployment ratio. $y$ : Real output in the nonfarm business sector. $c$ : Consumption of non-durable goods and services. Inv: Real private domestic investment. $w^{a}$ : Average wage in the economy. $w^{s}$ : Average wage for job stayers. $w^{u}$ : Average wage for new workers (workers who were unemployed in the previous period). $w^{c}$ : Average wage for job changers. $w^{n}$ : Average wage for new hires (new workers + job changers). $a$ : Solow residual (Basu, Fernald, \& Kimball, 2006). All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000 .
Table 1.2: Statistics for Business Cycle Driven by TFP: U.S. Economy 1964:Q1-2014:Q4

|  |  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{s}$ | $w^{u}$ | $w^{c}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Standard deviation <br> Autocorrelation |  | $\begin{aligned} & 0.137 \\ & 0.972 \end{aligned}$ | $\begin{aligned} & 0.127 \\ & 0.967 \end{aligned}$ | $\begin{aligned} & 0.254 \\ & 0.969 \end{aligned}$ | $\begin{aligned} & 0.023 \\ & 0.953 \end{aligned}$ | $\begin{aligned} & 0.012 \\ & 0.975 \end{aligned}$ | $\begin{aligned} & 0.083 \\ & 0.950 \end{aligned}$ | $\begin{aligned} & 0.020 \\ & 0.962 \end{aligned}$ | $\begin{aligned} & 0.020 \\ & 0.961 \end{aligned}$ | $\begin{aligned} & 0.023 \\ & 0.936 \end{aligned}$ | $\begin{aligned} & 0.019 \\ & 0.949 \end{aligned}$ | $\begin{aligned} & 0.020 \\ & 0.949 \end{aligned}$ | $\begin{aligned} & 0.017 \\ & 0.904 \end{aligned}$ |
| Correlation Matrix | $u$ | 1 | -0.986 | -0.996 | -0.960 | -0.920 | -0.937 | -0.857 | -0.866 | -0.850 | -0.811 | -0.818 | -0.856 |
|  | $v$ |  | 1 | 0.996 | 0.983 | 0.926 | 0.974 | 0.871 | 0.879 | 0.849 | 0.830 | 0.834 | 0.909 |
|  | $v / u$ |  |  | 1 | 0.972 | 0.916 | 0.960 | 0.888 | 0.897 | 0.873 | 0.842 | 0.848 | 0.888 |
|  | $y$ |  |  |  | 1 | 0.945 | 0.976 | 0.872 | 0.882 | 0.856 | 0.827 | 0.833 | 0.942 |
|  | $c$ |  |  |  |  | 1 | 0.861 | 0.911 | 0.911 | 0.897 | 0.897 | 0.900 | 0.790 |
|  | Inv |  |  |  |  |  | 1 | 0.759 | 0.774 | 0.749 | 0.693 | 0.703 | 0.974 |
|  | $w^{a}$ |  |  |  |  |  |  | 1 | 0.999 | 0.993 | 0.990 | 0.992 | 0.672 |
|  | $w^{s}$ |  |  |  |  |  |  |  | 1 | 0.990 | 0.985 | 0.987 | 0.695 |
|  | $w^{u}$ |  |  |  |  |  |  |  |  | 1 | 0.983 | 0.988 | 0.646 |
|  | $w^{c}$ |  |  |  |  |  |  |  |  |  | 1 | 1.000 | 0.590 |
|  | $w^{n}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0.598 |
|  | $a$ |  |  |  |  |  |  |  |  |  |  |  | 1 |

Notes: Statistics for the U.S. economy are based on: $u$ : Unemployment level. v: Help-wanted index (Barnichon, 2010). v/u: Vancancyunemployment ratio. $y$ : Real output in the nonfarm business sector. c: Consumption of non-durable goods and services. Inv: Real private domestic investment. $w^{a}$ : Average wage in the economy. $w^{s}$ : Average wage for job stayers. $w^{u}$ : Average wage for new workers (workers who were unemployed in the previous period). $w^{c}$ : Average wage for job changers. $w^{n}$ : Average wage for new hires (new workers + job changers). a: Solow residual (Basu, Fernald, \& Kimball, 2006). All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000 . These are the business cycle statistics that can be accounted by TFP shocks, as described in section 1.4.2.

Figure 1.2: Empirical Impulse Responses to a $1 \%$ Increase in TFP


Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags, where TFP is assumed to follow an exogenous $\operatorname{AR}(1)$ process. All variables are HP-filtered in logs using a smoothing parameter equal to $10^{5}$. All figures are expressed in percentage points. The $x$ axis represents quarters after the TFP shock. The shaded area represents $95 \%$ confidence intervals computed via bootstrap. The sample period is 1964Q1-2014Q4. The sample period for wages is 1994Q1-2014Q4.

Since only a fraction of these moments can be accounted for by productivity shocks, I follow the literature that investigates the effects of productivity innovations in order to estimate the properties of the business cycle that is driven by TFP shocks. ${ }^{31}$ Following Basu et al. (2006) and Gali (1999), I estimate bivariate nearVARs. In particular, for each variable $x$, I estimate the following system of equations:

$$
\begin{align*}
& a_{t}=\alpha_{a}+\rho_{a} \cdot a_{t-1}+e_{a}  \tag{1.44}\\
& x_{t}=\alpha_{x}+\sum_{i=1}^{3} \rho_{x}^{i} \cdot x_{t-i}+\sum_{i=0}^{3} \beta_{x}^{i} \cdot a_{t-i}+e_{x t} \tag{1.45}
\end{align*}
$$

In the first equation, I regress TFP $(a)$ on one lag of itself, which is an hypothesis that cannot be rejected. ${ }^{32}$ The second equation regresses each variable $x$ on the current $a$ and three lags of both itself and $a \cdot{ }^{33}$ Based on this estimation, I construct recursively the auxiliary variable $\tilde{x}$, which describes how variable $x$ evolves in response to TFP innovations:

$$
\begin{array}{ll}
\tilde{x}_{t}=x_{t} & t \leq 3 \\
\tilde{x}_{t}=\alpha_{x}+\sum_{i=1}^{3} \rho_{x}^{i} \cdot \tilde{x}_{t-i}+\sum_{i=0}^{3} \beta_{x}^{i} \cdot a_{t-i} & t>3 \tag{1.47}
\end{array}
$$

Table 1.2 presents business cycle statistics for these auxiliary variables. ${ }^{34}$ As expected, the standard deviations are lower and most of the correlations become

[^18]stronger. In particular, I estimate that $76 \%$ of overall unemployment volatility is due to productivity shocks. Similarly, around $70 \%$ of overall volatility in vacancies and the vacancy-unemployment ratio can be explained by TFP. However, productivity does not explain much of the observed volatility in wages. On average, productivity explains $25 \%$ of the standard deviation of wages for all groups.

It is important to note that Table 1.2 reports only the conditional correlations that are induced by TFP shocks. These conditional correlations represent the joint responses of endogenous variables to TFP, not the causal impact of one variable on the other. For example, Table 1.2 reports a strong, negative and significant conditional correlation between unemployment and wages. That is to say, an increase in wages is associated with a decrease in unemployment, which may sound counterintuitive given that firms' labor demand slopes down. However, this is exactly what the model predicts will happen in response to TFP shocks. As will be shown below, if productivity increases, wages increase because the marginal productivity of labor increases and because firms find it more difficult to find and retain new workers. Similarly, unemployment goes down in response to a higher productivity level because firms post more vacancies. Hence, TFP shocks induce a negative correlation between wages and unemployment.

To close this section, Figure 1.2 plots the Impulse Response Functions (IRFs) of the variables of interest to a $1 \%$ increase in aggregate TFP. Given that all of these variables are in logs and HP-filtered, the responses are percentage deviations around a trend and can be interpreted as elasticities. Some results from Figure 1.2 that will be used to assess my model predictions include: (1) Unemployment, vacancies,
and the vacancy-unemployment ratio are very sensitive to TFP shocks. In response to a $1 \%$ increase in TFP, unemployment declines $6 \%$ while vacancies rise by $7 \%$, which implies that the vacancy-unemployment ratio increases 15\%. (2) Responses are hump-shaped, which means that the largest response of these variables does not occur on impact. (3) Wages are positively correlated with TFP when they are adjusted for composition effects. However, wage responses are not statistically significant. (4) On average, wages peak 2 years (8 quarters) after a TFP shock, in contrast to 1 year (4 quarters) for unemployment and vacancies. (5) Wage responses to TFP shocks are very small in absolute value (less than $0.3 \%$ ) in the first three quarters.

### 1.4.3 Parameterization

I calibrate this model to quarterly frequency. I borrow the values for the intertemporal elasticity of substitution $(\sigma)$, the inverse of the Frisch elasticity of labor supply $(\xi)$, and the output elasticity of labor $(\alpha)$ from previous literature and set these parameters equal to $1,0.5$, and $0.33 .{ }^{35}$ Following the literature, I set $\vartheta$ equal to 0.5 , which implies equal bargaining power for workers and firms in steady state. The unemployment benefit $b$ is set to 0.041 following the evidence presented by Chodorow-Reich and Karabarbounis (2014). ${ }^{36}$ I set $\delta$ and $\beta$ so that the annual

[^19]depreciation rate is equal to $10 \%$ and the annual interest rate is equal to $5 \%$ in steady state.

Given firm heterogeneity in this model, it is important to have a matching function that guarantees that all matching probabilities are between 0 and 1 , which is not the case for the widely used Cobb-Douglas function. Hence, I follow den Han, Ramey and Watson (2000) and Hagedorn and Manovskii (2008) and assume the following function:

$$
\begin{equation*}
m(u, v)=\frac{u v}{\left(u^{l}+v^{l}\right)^{\frac{1}{l}}} \tag{1.48}
\end{equation*}
$$

I choose the parameter $l$ so that the job-finding probability $(q)$ is equal to 0.611 in steady state, which implies an average duration of unemployment equal to 15 weeks, consistent with evidence for the US economy. In steady state, the elasticity of matches with respect to vacancies $\left(\frac{\partial m(u, v)}{\partial v} \frac{v}{m(u, v)}\right)$ is equal to 0.454 , which is in the range reported by Petrongolo and Pissarides (2001). The exogenous separation rate $\delta_{h}$ is set such that the unemployment rate is equal to $5.5 \%$ in steady state.

I calibrate the distribution of the idiosyncratic TFP level $\left(a_{j}\right)$ such that: (1) marginal labor productivity $\left(p_{j}\right)$ is distributed according to a truncated normal between $[\underline{p}, \bar{p}]$ and (2) the mode of the distribution is 1 . Hence, everything is term of the mode (marginal) labor productivity across firms. ${ }^{37}$ The standard deviation of the normal distribution is calibrated to 0.2 , which is consistent with Long, Dziczek, Luria

[^20]and Wiarda (2008). ${ }^{38}$ Based on the evidence presented by Kahn and McEntarfer (2014), the extreme points of the distribution $(\underline{p}$ and $\bar{p})$ are calibrated such that the wage paid at the most productive firm is 5 times the wage paid at the least productive firm. I discretize the distribution for $a_{j}$ into 101 points. As to hiring costs, I calibrate the parameter $\chi$ to target the autocorrelation of aggregate vacancies, and $\kappa$ is set such that the total number of employed workers in steady state is equal to 0.945 .

I calibrate the disutility of labor parameter $\Psi$ such that the average of the ratio $\frac{z_{j}}{p_{j}}$ across firms is equal to 0.72 , which is consistent with the value found by Hall and Milgrom (2008). ${ }^{39}$ The value for $\bar{i}$ is calibrated such that the number of job changers per month in steady state is equal to $2.5 \%$ of the total population, which is consistent with the estimates of Fallick and Fleischman (2004). Finally, the persistence of aggregate TFP is calibrated to 0.95 and the standard deviation to 0.018 . Following Coibion and Gorodnichenko (2012), $\sigma_{n}$ and $\rho_{n}$ are calibrated such that the persistence of the forecasting error is equal to 0.8 and workers give a weight of $20 \%$ to new information. Tables 1.3 and 1.4 summarize the aforementioned calibration parameters and their sources when appropriate.

### 1.4.4 Model versus Data

Before turning to the dynamics, I first present Figure 1.3, which illustrates the role of heterogeneous firms in this model. Panel (a) plots the distribution of id-

[^21]iosyncratic TFP across firms $\left(f\left(a_{j}\right)\right)$ and the marginal labor productivity associated with each $a_{j}$, and panel (b) shows the wage rate $\left(w_{j}\right)$ and the probability of finding a better job conditional on a match for employed workers $\left(F_{j}\right)$. We can see that the most productive firms have higher marginal labor productivity and as a consequence pay higher wages. Panel (c) shows the average firm size as a function of the firm's labor productivity $p$ (solid black line), and the distributions of employment (dashed line). In particular, the dashed black line in panel (c) plots the fraction of workers that are currently employed in a firm with labor productivity equal to $p_{j}$. As in Moscarini and Postel-Vinay (2013) the most productive firms are larger in equilibrium, and therefore the distribution of employment is shifted to the right in comparison with the distribution for $p$.

Table 1.3: Parameters Externally Calibrated

| Parameter | Value | Description |
| :--- | :--- | :--- |
| $\sigma$ | 1 | Intertemporal elasticity of substitution |
| $\xi$ | 0.5 | Inverse of Frisch elasticity |
| $\alpha$ | 0.33 | Labor share in production function |
| $\rho_{a}$ | 0.95 | Persistence of productivity shocks |
| $\varsigma_{a}$ | 0.018 | Standard deviation of productivity shocks |

Notes: This table summarizes parameters that are externally calibrated. Details are reported in section 1.4.3.

Table 1.4: Parameters Internally Calibrated

| Parameter | Value | Description | Target |
| :---: | :---: | :---: | :---: |
| $a_{j}$ |  | Idiosyncratic TFP distribution | Marginal labor productivity distributed truncated normal with mean 1 , standard deviation 0.2 and truncated range $(0.5,2)$. |
| $b$ | 0.041 | Unemployment benefits | Fraction of $b$ over modal (marginal) labor productivity $=0.041$. |
| $\Psi$ | 0.8 | Disutility of labor parameter | Average $\frac{z_{j}}{p_{j}}$ equal to 0.72 . |
| $\varsigma_{n}$ | $5 \cdot \varsigma_{a}$ | Signaling parameter | Weight on new information $=20 \%$. |
| $\delta_{h}$ | 0.0356 | Exogenous separation rate | Unemployment rate $=5.5 \%$ |
| $\rho_{n}$ | 0.8 | Signaling parameter | Persistence of forecasting error $=0.85$. |
| $l$ | 4 | Matching function parameter | Unemployment duration $\approx 15$ weeks. |
| $\kappa$ | 0.8416 | Hiring cost function parameter | Total employment $=0.945$. |
| $\bar{i}$ | 0.6 | Relative search intensity of employed workers | Fraction of job changers $=2.5 \% /$ month |
| $\beta$ | 0.9879 | Discount factor | Annual interest rate $=5 \%$ |
| $\chi$ | 0.6 | Hiring cost function convexity | Persistence of vacancy index. |
| $\delta_{k}$ | 0.026 | Capital depreciation rate | Annual depreciation rate $=10 \%$. |

Notes: This table summarizes the parameters that were internally calibrated. Details are reported in section 1.4.3.

Panel (d) plots the separation rate $\left(\delta_{h j}\right)$ and the job filling rate $\left(\tilde{q}_{j}\right)$ associated with each level of marginal labor productivity. Since employed workers only accept jobs that pay a higher wage and unemployed workers always accept a job offer, the most productive firms have a higher job filling rate and a lower separation rate than less-productive firms. This also implies that low-paying firms rely more on the pool of unemployment while high-wage firms find most of their new hires from the pool of employment. Hence, it is not surprising that the labor productivity distribution of new workers (individuals that were unemployed in the previous period) is shifted to the left relative to the productivity distribution of all firms, while the distribution of job changers is shifted to the right (panel (e)) -new workers are more likely to find a job in a low-paying firm, in contrast to job changers, who are poached by the most productive firms.

In panel (f), we can also see that the distribution of overall employment is even more shifted to the right than the distribution of job changers. This is because the most productive firms have a low separation rate in equilibrium. In other words, a firm at the right tail of the productivity distribution has a higher job filling rate but also a lower separation rate than a firm at the middle of the distribution. Hence, a very productive firm doesn't have to post as many vacancies as a firm that is in the middle of the distribution.

Based on these distributions, Table 1.5 reports the average wage for different types of workers. The average wage for job stayers is higher than for any other group. This is because high-paying firms have the lowest separation rate, which gives a higher weight to employed workers at those firms. In contrast, the average

Figure 1.3: Firm and Employment Distribution in Steady State


Note: This figure plots the distributions of employment and productivity across firms in steady state along with the separation rate, job filling rate and wage associated with each firm.
wage for new workers (hired from unemployment) is the lowest among these groups of workers. As explained earlier, new workers are more likely to find a job at a low-paying firm.

Table 1.5: Average Wages in Steady State

| All workers | Stayers | New Hires | Changers | New Workers |
| :--- | :--- | :--- | :--- | :--- |
| 1.1419 | 1.1563 | 1.0072 | 1.0827 | 0.8776 |

Notes: This table reports the average wage for different groups of workers in steady state.

Next, Figures 1.4 and 1.5 plot the Impulse Response Functions of the aggregate variables of this model to a $1 \%$ increase in aggregate TFP (solid black lines). In
order to isolate the role of information frictions, I simultaneously plot the IRFs generated by a calibrated model in which agents have perfect information (dashed lines). In addition, Figure 1.6 plots the IRFs for the 10th, 25 th, 50 th, 75 th, and 90th percentiles of idiosyncratic productivity in the economy.

Based on these figures, we can see the role of information frictions in amplifying the unemployment response to productivity shocks. Since TFP shocks are partially perceived by workers, wages are less sensitive to aggregate productivity innovations (Figure 1.4). In particular, the assumed information friction has two reinforcing effects on wages. First, workers' expectations are highly sluggish. Hence, in a boom, workers do not demand a large increase in wages because they do not have enough information to conclude that the economy has entered an expansionary path. Second, given workers' beliefs, consumption does not change significantly on impact, so that a large fraction of the increase in aggregate output is absorbed by investment. This curbs the increase of the flow of opportunity cost of employment $\left(z_{j}\right)$, which makes wages even less responsive. Therefore, firms have more incentive to expand employment because wages adjust slowly to the true state of the economy.

However, firms' responses to this shock are not uniform. Actually, only the most productive firms experience an expansion in employment as a consequence of a positive aggregate TFP innovation. Since there is a large expansion in overall employment, there is a large flow of job changers that makes the separation rate increase for low-paying/less-productive firms. Hence, the value of a new hire is affected by two countervailing effects. On the one hand, the productivity increase, combined with sluggish real wages, tends to increase the value of an additional

Figure 1.4: Impulse Response Function to a 1\% Increase in Aggregate Productivity


Note: This figure plots model Impulse Response Functions (IRFs) to a $1 \%$ increase in aggregate TFP. Solid black lines are the IRFs for a model in which workers face information frictions, and dashed-lines are the IRFs generated by a model in which all agents have perfect information.
worker for firms in an expansion. On the other hand, an increase in the separation rate reduces the value of an additional worker because firms expect the match to not last as long. Hence, the value of an additional worker should increase more for highly-productive firms. Therefore, they expand employment the most. According to these results, the increase in the separation rate for low-paying firms is so large that they reduce their employment levels, as they are crowded out by the large expansion of highly productive firms. This implies that the differential employment growth rate between high and low paying firms is positive and procyclical, which is consistent with the empirical evidence (e.g. Kahn \& McEntarfer, 2014; Haltiwanger, et al., 2015).

Figure 1.5: Impulse Response Function to a 1\% Increase in Aggregate Productivity


Note: This figure plots model Impulse Response Functions (IRFs) to a $1 \%$ increase in aggregate TFP. Solid black lines are the IRFs of a model in which workers face information frictions, and dashed-lines are the IRFs generated by a model in which all agents have perfect information. $q, \tilde{q}$, and $\tilde{q}^{u}$ denote the job finding rate, the probability that a vacancy is matched with a worker, and the job filling rate (from unemployment), respectively.

This differential growth rate in employment implies a differential growth in the flow opportunity cost of employment $\left(z_{j}\right)$. Since high-paying firms are expanding employment the most, they also experience a larger increase in $z_{j}$, which makes their wages increase more than the wages for low-paying firms. The fact that wages increase more for the most productive firms does not imply that their workers have more or better information than workers employed at low-paying firms. Since workers can perfectly distinguish among firms and they know that high-productive firms are more sensitive to the business cycle, employees at the most productive firms demand a higher wage than employees at low-productive firms in response to an

Figure 1.6: Distributional Dynamics to a 1\% Increase in Aggregate Productivity












| -10 pctl |
| ---: |
| ---25 pctl |
| - |
| --70 pctl |
| --75 pctl |
| -90 pctl |

Note: This figure plots the Impulse Response Functions (IRFs) for a model with information frictions for different firms to a $1 \%$ increase in aggregate TFP. Solid gray lines are the IRFs for firms at the 10th percentile of idiosyncratic TFP. The dashedgray lines are the IRFs for firms at the 25th percentile. The solid x-marked black lines are the IRF for the median firm. The dashed black lines are the IRF for firms at the 75th percentile, and the solid black lines are the IRFs for firms at the 90th percentile. $z_{j}$ and $F_{j}$ denote the flow opportunity cost of employment for firm $j$, and the probability of finding a weakly better job than $j$, respectively.
increase in perceived productivity. Hence, the differential employment growth rate occurs despite the larger adjustment in wages for high-paying firms, which is also consistent with the empirical evidence. Kahn and McEntarfer (2014) do not find that the differential employment growth rate is driven by high-paying firms facing more sluggish wages. In fact, they show that high-paying firms reduce wages in recessions relative to low-paying firms.

These results imply different dynamics for the job-filling rate across firms $\left(\tilde{q}_{j}\right)$. In particular, since low-paying firms rely more on hiring from the pool of unemployment, they experience a large decline in $\tilde{q}_{j}$ because of the decline in unemployment.

By contrast, high-paying firms experience an initial decrease in the job-filling rate because of the large increase in the total number of vacancies. But as the pool of employment increases, the job filling rate for the most productive firms goes up because most of their new hires come from other firms.

Table 1.6 reports the business cycle statistics generated by a model in which workers face information frictions. In particular, I simulate the model for 100,000 periods and detrend all variables using the HP filter with a smoothing parameter of $10^{5}$. To facilitate comparison, Table A. 3 in Appendix A. 1 compares the business cycle moments generated by my model and those obtained from the data. Based on my simulations, we can see that a model in which workers face information frictions is able to explain $60 \%$ of the overall volatility of unemployment and around $70 \%$ of the overall volatility in other labor market quantities. Compared to my empirical estimates, my model is able to explain $90 \%$ of the unemployment volatility that is attributable to TFP shocks. Similarly, my model does a good job in terms of correlations, as the correlations predicted by the model are very close to those derived from the data. It is also worth noting that my empirical exercise helps us to reconcile some empirical inconsistencies of the search and matching model described in Hagedorn and Manovskii (2011). In particular, they argue that the contemporaneous correlation between the vacancy-unemployment ratio and productivity is significantly lower in the data than in the model and that the standard deviation of wages is higher than the wage elasticity with respect to productivity. According to my empirical analysis, conditioning on TFP shocks in the data increases the contemporaneous correlation between the vacancy-unemployment ratio and TFP from
0.52 to 0.89 and reduces the standard deviation of wages from 0.08 to 0.02 . My calibrated model generates a correlation between the vacancy unemployment ratio and TFP equal to 0.93 and a standard deviation of wages around 0.013. ${ }^{40}$

Figure 1.7 compares the IRFs estimated from the data with those generated by the model. With the exception of the IRFs for output and consumption, the model is able to explain very well the dynamics of these variables after a productivity shock. Even though the IRF for output predicted by the model does not lie in the confidence interval, the model is able to predict a hump-shaped response. It is worth noting that, by construction, the model must predict an impact response equal to $1 \%$. On the other hand, even though my model predicts too large a consumption response, it is worth noting that a calibrated model with perfect information shares this flaw. Also, notice that a smaller consumption response (as in the data) would reinforce my results on wages and unemployment, as the increase in the opportunity cost of employment would be smaller (less cyclical). On the other hand, the model with information frictions does a good job explaining the dynamics of both wages and investment. Recall that the real part of the information friction plays an important role in this model. Given that aggregate shocks are partially perceived by workers, most of the shock is absorbed by investment (capital accumulation). These IRFs tell us that the model does a good job of explaining the behavior of investment.

[^22]Table 1.6: Simulated Business Cycle

|  |  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{s}$ | $w^{u}$ | $w^{c}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Standard deviation Autocorrelation |  | $\begin{aligned} & 0.113 \\ & 0.960 \end{aligned}$ | $\begin{aligned} & 0.160 \\ & 0.925 \end{aligned}$ | $\begin{aligned} & 0.259 \\ & 0.960 \end{aligned}$ | $\begin{aligned} & 0.024 \\ & 0.934 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.989 \end{aligned}$ | $\begin{aligned} & 0.089 \\ & 0.926 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.981 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.981 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.979 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.981 \end{aligned}$ | $\begin{aligned} & 0.013 \\ & 0.980 \end{aligned}$ | $\begin{aligned} & 0.017 \\ & 0.871 \end{aligned}$ |
| Correlation Matrix | $u$ | 1 | $\begin{gathered} -0.799 \\ 1 \end{gathered}$ | $\begin{gathered} -0.928 \\ 0.965 \\ 1 \end{gathered}$ | $\begin{gathered} -0.885 \\ 0.903 \\ 0.943 \\ 1 \end{gathered}$ | -0.560 | -0.839 | -0.737 | -0.739 | -0.685 | -0.733 | -0.725 | -0.812 |
|  | $v$ |  |  |  |  | 0.420 | 0.983 | 0.619 | 0.619 | 0.616 | 0.630 | 0.626 | 0.932 |
|  | $v / u$ |  |  |  |  | 0.503 | 0.972 | 0.703 | 0.704 | 0.679 | 0.708 | 0.702 | 0.929 |
|  | $y$ |  |  |  |  | 0.695 | 0.927 | 0.849 | 0.849 | 0.824 | 0.849 | 0.844 | 0.955 |
|  | c |  |  |  |  | 1 | 0.400 | 0.966 | 0.966 | 0.964 | 0.964 | 0.964 | 0.970 |
|  | Inv |  |  |  |  |  | 1 | 0.613 | 0.613 | 0.592 | 0.618 | 0.611 | 0.472 |
|  | $w^{a}$ |  |  |  |  |  |  | 1 | 1.000 | 0.987 | 0.998 | 0.996 | 0.662 |
|  | $w^{s}$ |  |  |  |  |  |  |  | 1 | 0.986 | 0.998 | 0.996 | 0.663 |
|  | $w^{u}$ |  |  |  |  |  |  |  |  | 1 | 0.993 | 0.996 | 0.637 |
|  | $w^{c}$ |  |  |  |  |  |  |  |  |  | 1 | 0.999 | 0.663 |
|  | $w^{n}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0.657 |
|  | $a$ |  |  |  |  |  |  |  |  |  |  |  | 1 |

[^23]Figure 1.7: IRFs to $1 \%$ Increase in Aggregate Productivity. Data versus Model


Note: The solid black lines in this figure plot the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags, where TFP is taken to follow an exogenous $\operatorname{AR}(1)$ process. All variables are HP-filtered in logs using a smoothing parameter equal to $10^{5}$. All figures are expressed in percentage points. The $x$ axis represents quarters after the TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. The sample period is 1964Q1-2014Q4. The sample period for wages is 1994Q1-2014Q4. The dashed lines are the IRFs generated by a model with information frictions, and the dotted lines are the IRFs generated by a model in which all agents have perfect information.

Finally, the left panel of Figure 1.8 plots the IRF for average wages for each type of worker in my benchmark model. Notice that the average wages for new hires, job changers and new workers have a larger response on impact than the average wages for job stayers and all workers. However, these differences are driven primarily by heterogeneity across firms. To see this, note that average wages increase for two reasons: (1) because wages within firms increase and (2) because high-wage firms increase employment the most in an expansion. In order to see how important these two effects are, the right panel of Figure 1.8 plots the average wage for all groups of workers when wages are adjusted for this composition effect. In particular, I follow Horrace and Oaxaga (2001) and define the average wage for group $G$ adjusted for composition effects $\left(\tilde{w}^{G}\right)$ as the average wage for a fixed composition of workers across firms, where the composition of workers is given by the disribution of workers across firms in steady state.

Figure 1.8: Wage Responses to a 1\% Increase in Aggregate Productivity


Note: This figure plots the evolution of the average wage for different groups of workers in response to a $1 \%$ increase in aggregate productivity. The left panel plots the evolution of average wages not adjusted for composition effects. The right panel plots the evolution of average wages adjusted for composition effects.

By comparing the two panels of Figure 1.8, we can infer that the initial increase in the wages of new hires, job changers and new workers is due almost entirely to the large increase in employment at high-paying firms. However, when I control for the fact that high wage firms expand employment the most, wages of all groups have the same behavior -wage responses to aggregate shocks are gradual. Similarly, when controlling for this composition effect, there are not significant differences in wage responses for different groups of workers. This result is in line with previous empirical evidence. For example, using the National Longitudinal Survey of Youth (NLSY) and the Panel Study of Income Dynamics (PSID), Hagedorn and Manovskii (2013) find no significant differences in the cyclicality of wages for job changers and job stayers when they control for match quality. ${ }^{41}$

How does the wage flexibility in my model compare to the data? Pissarides (2009) finds that the wage semi-elasticity with respect to the unemployment rate for job changers is around $-3 \%$ in comparison for $-1 \%$ for job stayers. ${ }^{42}$ This evidence has been cited by Pissarides and others in favor of models with flexible wages and against models with sticky wages. In order to estimate this semi-elasticity in my model, I simulated the model for 100,000 periods, computed the average wages for all groups adjusted for composition effects and ran the following regression for each

[^24]group:
\[

$$
\begin{equation*}
\log \left(\tilde{w}_{t}^{G}\right)=\alpha_{0}+\beta_{u} \cdot u r_{t}+e_{t} \tag{1.49}
\end{equation*}
$$

\]

where $\tilde{w}_{t}^{G}$ is the average wage (adjusted for composition effects) for group $G$. ${ }^{43}$ $\alpha_{0}$ is a constant, $u r_{t}$ is the unemployment rate at time $t, e_{t}$ is an error term, and $\beta_{u}$ is the wage semi-elasticity with respect to the unemployment rate.

Table 1.7: Wage Semi-Elasticities With Respect To Unemployment Rate

| Job Stayers | New Hires | Job Changers | New Workers |
| :---: | :---: | :---: | :---: |
| -3.05 | -3.08 | -3.11 | -2.97 |

Notes: This table presents the wage semi-elasticities with respect to the unemployment rate generated by this model. Wages are adjusted for composition effects following the methodology of Horrace and Oaxaga (2001).

This semi-elasticity is reported in Table 1.7 for job stayers, new hires, job changers, and new workers. It is worth noting that these values are not a target in my calibration, but we can see that they are all around $-3 \%$. That is, this model is robust to the Pissarides critique. In my model, wages for new hires are flexible, and wage semi-elasticities with respect to the unemployment rate are around $-3 \%$. If these semi-elasticities were lower (in absolute terms) than $-3 \%$, as argued by Gertler et al. (2014), my model predictions would be reinforced as wages would be

[^25]less cyclical, which would further increase firms' incentives to expand employment. However, it is worth pointing out that my model is not able to endogenously generate a difference in the elasticity between job stayers and new hires. This is because all wages are negotiated period by period. In the next section, I extend this model by assuming that wages are negotiated at the moment in which new matches are formed and once a year after that. However, this extension does not significantly reduce the wage semi-elasticity for job stayers.

### 1.5 Robustness

This section addresses the robustness of my results to variations in some of the assumptions that underlie my analysis. In particular, I will consider: (1) allowing sticky wages for job stayers, (2) allowing firms to face information frictions, and (3) allowing a different HP smoothing parameter in the data.

I show that: (1) In contrast to previous literature, assuming that wages for job stayers are sticky amplifies the unemployment response to productivity shocks. When workers negotiate wages for the following $n$ periods, they give up using the flow of information that they would otherwise receive for the next $n$ periods, which makes wages even more sluggish. (2) Assuming that firms face information frictions reinforces my results, as firms underestimate the cost of recruiting new workers in expansions and expand employment even more. (3) Using a smoothing parameter equal to 1,600 makes wages less cyclical in the data and does not have a significant impact on other variables.

### 1.5.1 Sticky Wages for Job Stayers

In contrast to previous literature in which the wage of job stayers is irrelevant for vacancy decisions, assuming sticky wages for continuing workers amplifies the unemployment response to productivity shocks in my model. If a worker has to negotiate her wage for the following $n$ periods, she gives up using the new information she otherwise would be using in the future. To see this clearly, suppose that workers observe everything with a lag of 2 periods. In other words, if the economy is shocked at time $t$, workers do not know about this shock until period $t+2$. Hence, if a worker has to negotiate her wage at time $t$ for the following 4 periods and there is a positive productivity shock at time $t$, she will not demand a higher wage for the following 4 periods because she doesn't know about the productivity shock yet. At time $t+2$, workers will know about the productivity shock and would like to demand higher wages, but they cannot because their wages are fixed for at least another 2 periods. Given that firms have perfect information, they anticipate that they will keep a larger fraction of the match surplus for the following 4 periods, which will in turn create more incentive to post vacancies.

Figure 1.9 illustrates this point in the case of a positive productivity shock of $1 \%$. The left panel illustrates the evolution of the true productivity (solid black line) and the perceived productivity by workers at each point in time (dashed line), which is derived from the Kalman Filter. Hence, if wages for job stayers are flexible, continuing workers will negotiate wages each period based on their perceived productivity level. Hence, we can define the difference between the solid and dashed
lines (gray area) as the information rent that firms capture. This is because firms are producing according to a productivity that is equal to the solid line but are paying labor as if productivity was equal to the dashed line.

Figure 1.9: Flexible versus Sticky Wages for Job Stayers


Note: This figure illustrates the amplifying effects of sticky wages for job stayers. The left panel plots a situation in which job stayers negotiate their wages every period. The right panel plots the situation of an unemployed worker who finds a job 4 quarters after a TFP shock, when job stayers negotiate their wages every 4 periods and new hires negotiate their wages when they are matched with a firm.

Now, suppose that workers negotiate their wages every 4 periods and new hires negotiate their wages when they are matched with a firm. In this case, workers must form expectations about future economic conditions based on their beliefs about the current state of the economy. The right panel of Figure 1.9 illustrates the case of an unemployed worker who finds a job 4 periods after the productivity shock. The red dashed line is workers' perceived productivity at each point in time. For example, in period 4, when the unemployed worker finds a job, she thinks that the true productivity is equal to 0.3 . Given that she has to negotiate her wage for the
following 4 periods, she forecasts future economic conditions based on her beliefs. Hence, she negotiates wages as if productivity was evolving as in the black dashed line in the right panel of Figure 1.9. For example, as of period 4, the worker forecasts a productivity equal to 0.22 in period 8 and negotiates wages accordingly. When period 8 actually arrives, the worker has received new information and perceives that aggregate TFP is equal to 0.4 , but she cannot renegotiate her wage. Hence, the information rent for firms increases, giving firms more incentives to expand employment. Figure A. 1 in Appendix A. 1 plots the IRFs of this model when wages for new hires are flexible but are negotiated only once annually (every 4 quarters) thereafter. ${ }^{44}$ Even though the difference with respect to my baseline model is small, the difference is not insignificant. In particular, the model with sticky wages for job stayers captures very well the dynamics of unemployment for the first 10 quarters and has a larger vacancy response.

However, this model is not able to generate a significant difference between the wage semi-elasticity for job stayers and new hires. As shown in Table 1.8, even if wages are re-negotiated once a year, the wage semi-elasticity for job stayers generated by this model is about $-3 \%$ in comparison with $-1 \%$ reported by Pissarides (2009). This is because every period there is a fraction of job stayers re-negotiating their wages according to the economic conditions prevailing in that quarter that makes the overall average be cyclical.

[^26]Table 1.8: Wage Semi-Elasticities With Respect To Unemployment Rate. Model Sticky Wages for Job Stayers

| Job Stayers | New Hires | Job Changers | New Workers |
| :---: | :---: | :---: | :---: |
| -2.91 | -3.04 | -3.07 | -2.91 |

Notes: This table presents the wage semi-elasticities with respect to the unemployment rate generated by a model in with sticky wages for job-stayers. Wages are adjusted for composition effects following the methodology of Horrace and Oaxaga (2001).

### 1.5.2 Firms Face Information Frictions

Assuming that firms as well as workers face information frictions reinforces my results. Suppose assume that firms observe their overall productivity $\left(a_{j}+a\right)$ at all times but cannot decompose unexpected changes into aggregate and idiosyncratic shocks. Hence, if firms and workers form expectations about aggregate conditions based on the signal $\hat{a}$, firms will partially attribute aggregate shocks to idiosyncratic conditions. Therefore, in response to aggregate innovations, firms will underestimate the increase in quits and future wage changes that will result from a positive TFP shock. This increases firms' incentive to post more vacancies, as their perceived value of an additional worker is greater than is the actual value.

Figure A. 2 in Appendix A. 1 plots the impulse response functions generated by this model when I allow the information friction to affect firms as well as workers, holding other the model parameters at their benchmark values. As expected, introducing information frictions on the firms side reinforces my results, as the IRFs for labor market quantities are larger than in the benchmark model. Figure A. 3 in

Appendix A. 1 plots the IRFs when I re-calibrate the model parameters, and Table A. 2 in Appendix A. 1 presents the simulated business cycle moments. In this new set-up, the main results do not change relative to benchmark results. The unemployment response to productivity shocks is large and wage responses are delayed. However, the responses tend to peak earlier than predicted by my empirical estimates. In terms of business cycle moments, the model in which both firms and workers face information frictions does a good job in terms of standard deviations and correlations. However, the autocorrelation of the labor market quantities become smaller. This is because, on impact, firms overreact to aggregate shocks, and they compensate for this in later periods when they have amassed more information. For example, in response to a positive TFP shock, firms post a lot of vacancies on impact, but they reduce the number of vacancies (post less) as they learn about aggregate conditions and realize that the value of an additional worker is not as high as they had thought. This does not happen when firms have perfect information, as they perfectly predict the value of an additional worker and the convexity of the hiring cost function induces firms to smooth the number of vacancies they post.

### 1.5.3 HP Filter

For my benchmark empirical results, I detrend the data using the HP filter with a smoothing parameter equal to $10^{5}$. However, it is common in macroeconomics to use a smoothing parameter equal to 1,600 for quarterly data. Figure A. 4 in Appendix A. 1 plots the IRFs of my model along with those estimated using a
smoothing parameter equal to 1,600 . Responses for labor market quantities, output and investment are not substantially different from those reported in Figure 1.2. However, the wage responses are less cyclical in the sense that the maximum responses are smaller and less significant, indicating that wage responses to transitory TFP shocks are weak. Why do the wage responses change with the smoothing parameter? On the one hand, we expect wages and productivity to be correlated in the long run. If there is a permanent increase in TFP, we should expect higher wages in the economy. However, whether and how wages adjust to purely transitory shocks is not clear. Hence, the smaller the smoothing parameter we use, the smaller the fluctuations that can be explained by transitory shocks since larger fluctuations are attributed to a long run trend. However, it is worth noting that less cyclical wages in the data favor wage stickiness as a driving force of the business cycle. Therefore, my baseline smoothing parameter is conservative in the sense that it reduces the evidence for wage stickiness.

### 1.6 Conclusion

I propose a new mechanism for sluggish wages based on workers' noisy information about the state of the economy. In my model, workers receive noisy signals about the current state of the economy and learn slowly about aggregate conditions. Hence, wages do not immediately respond to a positive aggregate shock because workers do not (yet) have enough information to demand higher wages. This delayed adjustment in wages increases firms' incentives to post more vacan-
cies, making unemployment more volatile and sensitive to aggregate shocks. My calibrated model is able to explain $60 \%$ of the overall unemployment volatility and displays unemployment and wage dynamics consistent with the data. I find that the unemployment response to TFP shocks is large and hump-shaped, peaking after one year. In contrast, wage responses are delayed and weak, peaking instead after two years.

My model is robust to two major critiques of existing theories of sluggish wages and volatile unemployment: the flexibility of wages for new hires and the cyclicality of the opportunity cost of employment. On the one hand, my model assumes flexible wages for new hires and generates a wage semi-elasticity with respect to the unemployment rate for new hires equal to $-3 \%$, which is similar to the estimate of Pissarides (2009) and larger than the estimates of Hagedorn and Manovskii (2013) and Gertler et al (2014). On the other hand, my model predicts a very pro-cyclical opportunity cost of employment, as the value of non-working activities in terms of consumption increases in expansions.

Consistent with recent empirical evidence (e.g. Kahn \& McEntarfer, 2014; Haltiwanger et al., 2015), my model predicts that high-wage highly productive firms expand employment more than low-wage firms and also exhibit larger wage adjustments in expansions. This implies that the distribution of new hires shifts to the most productive and high paying firms in response to positive productivity shocks. This has important consequences for new hires, as they find more and better paying jobs in expansions.

In this chapter, I examine the data for the United States and estimate the
fraction of business cycle moments that can be attributed to productivity shocks. In order to allow for differences in the cyclicality of wages for job stayers and new hires, I use the Current Population Survey to construct average wages for these groups of workers controlling for composition effects. According to my results, between 70 and $75 \%$ of the overall volatility in labor market quantities such as unemployment and vacancies can be attributed to transitory TFP innovations. In contrast, only $25 \%$ of the overall volatility in wages can be explained by transitory productivity innovations. I find significant and hump-shaped Impulse Response Functions (IRFs) in the data to productivity shocks for unemployment, vacancies and the vacancyunemployment ratio. These responses peak 4 quarters after the shock, and imply that a $1 \%$ TFP shock reduces unemployment by $6 \%$, increases vacancies by $7 \%$ and increases the vacancy-unemployment ratio by $15 \%$. By contrast, the empirical IRFs for wages are weak and delayed. A $1 \%$ TFP shock increases wages by $1 \%$ after 8 quarters. My model is able to reproduce the dynamics that I estimate in the data and is able to explain $90 \%$ of the unemployment and vacancy volatility that is due to transitory productivity shocks.

In the robustness section, I show that assuming sticky wages for job stayers increases the unemployment response to productivity shocks. This result is in sharp contrast to existing studies, in which wage stickiness for incumbent workers is irrelevant for hiring decisions as long as wages for new hires are flexible. In my model, if a new hire has to negotiate her wage for the subsequent $n$ periods, she gives up using the new information that she otherwise would be using in the future, which will reduce the gap between the wage she actually demands and the wage she
should be demanding. Therefore, if wages for new hires do not initially adjust to an aggregate shock (because of the information friction), sticky wages for job stayers increase the time it will take for a worker's wage to adjust, which further increases firms' incentive to increase employment in expansions.

However, the wage semi-elasticity generated by this model for job stayers is significantly higher than reported by Pissarides (2009), which is also the case when job stayers negotiate their wages once a year. This is because every period there is a fraction of job stayers re-negotiating their wages according to the economic conditions prevailing in that quarter that makes the overall average be cyclical.

# Chapter 2: Information Frictions, Nominal Shocks, and the Role of Inventories in Price-Setting Decisions 

### 2.1 Introduction

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. Models with sticky information, rational inattention, or dispersed information display output and inflation dynamics that are consistent with the empirical evidence: inflation exhibits inertia, responses to monetary shocks are delayed and persistent, and anticipated disinflations do not result in booms (Ball, Mankiw \& Reis, 2005; Klenow \& Willis, 2007; Mankiew \& Reis 2002, 2010; Nimark, 2008; Woodford, 2002).

However, an implicit assumption in the existing literature is that pricing managers do not interact with production managers within firms. Pricing managers set firms' prices with limited or noisy information regarding not only aggregate variables but also their own input prices and demand, while production managers hire all the labor and capital that is necessary to produce the quantity demanded at given prices. As stated by Hellwig and Venkateswaran (2012), if this assumption were relaxed, nominal shocks would not have real effects on the economy because
the firm's input prices and demand contain all the information that is needed to infer the firm's best responses to nominal shocks in the standard framework used in existing literature. Hence, it remains unclear why nominal shocks would have real effects when prices are flexible and there is perfect communication within firms.

This chapter contributes to the literature by presenting a model with perfect communication within firms, flexible prices, output inventories, and real information frictions in which nominal shocks have real effects. ${ }^{1}$ This model is close in spirit to the islands model of Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). Intermediate producers observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time. Intermediate goods firms face idiosyncratic shocks, and as a consequence cannot perfectly infer the aggregate state of the economy. Intermediate producers set their output prices, determine production, and make inventories decisions based on their information set.

In this model, inventories are the link between information frictions, perfect communication within firms, and non-neutrality of nominal shocks. This is because inventories help smooth cost shocks and thus affect pricing and production decisions. The idea that inventories smooth cost shocks has been extensively explored in the literature (Bils \& Kahn, 2000; Khan \& Thomas, 2007; Ramsey \& West, 1999).

[^27]In almost every model with inventories, firms accumulate inventories in response to favorable marginal cost shocks. The resulting increase in production increases current marginal cost, smoothing marginal cost through time. In this model, I show that this cost smoothing also implies that firms' prices are smoothed through time under monopolistic competition.

The cost-smoothing role of inventories helps to explain the non-neutrality of nominal shocks for the following reason: given that firms only observe their nominal input prices and demand, they will accumulate inventories (by producing more) as long as they think that they are facing temporarily low real input prices. After a contractionary nominal shock, firms observe lower nominal input prices. They do not know the source of this change, but they know that it could be due to a either positive productivity innovation or a nominal shock. Since productivity shocks have a positive probability, firms will increase their stock of inventories. This will prevent firms' current prices from decreasing as much as they would in a model without inventories, which will distort relative prices, and make current profits and household income go down. As a consequence, aggregate demand falls.

I study a quantitative version of my model and find that a one-percent decrease in the money growth rate causes output to decline $0.17 \%$ in the first quarter and $0.38 \%$ in the second quarter, followed by a slow recovery to the steady state. I also find that contractionary nominal shocks have significant effects on total investment, which remains $1 \%$ below the steady state for the first 6 quarters. The investment response to an aggregate nominal perturbation is $-0.67 \%$ in the first quarter and reaches its trough response of $-2.26 \%$ in the second quarter.

I compare the model with information frictions to a model with perfect and complete information, and I find that information frictions make the model more consistent with the empirical evidence. In a model with complete and perfect information, inventory investment is pro-cyclical, and the standard deviation of inventory investment is small. In contrast, in the model with information frictions, inventory investment is counter-cyclical, and its standard deviation is closer to the data. Also, given the role of inventories, prices are more stable in absolute terms and relative to output in the model with information frictions.

This chapter also contributes to the literature by illustrating the general point that if firms make dynamic decisions (such as capital accumulation or inventory investment) and if their nominal input prices and demand do not perfectly reveal the state of nature, the economy exhibits money non-neutrality even under flexible prices and perfect communication within firms (Proposition 3). This non-neutrality occurs because firms need to forecast future aggregate conditions in order to make dynamic decisions. Hence, when current input prices and demand do not perfectly reveal aggregate conditions, firms make forecast errors because their inference about the state of the nature is wrong, and their real decisions deviate from the decision that would have been taken under perfect information. Thus, investment is key for money non-neutrality. Similarly, these results point out that firms input prices and demand contain noisy but important information about aggregate conditions, implying that how firms process information is key for understanding real responses to monetary shocks. The existing literature abstracts from this issue.

I solve the model by combining the Kalman-Filter with the solution method
for heterogenous agent models proposed by Reiter (2009). The idea behind my solution method is to guess a linear law of motion for the aggregate variables and to find the steady state of the economy using the Kalman Filter. Then, the economy is linearized around this steady state following the methodology of Reiter (2009), which generates a new law of motion for the economy. The law of motion is updated until a fixed point is reached.

This chapter is related to the literature on information frictions and aggregate supply. In this chapter, nominal shocks have real effects mainly because firms have imperfect information, not because prices are sticky. As argued by Ball, Mankiw and Reis (2005), models with information frictions may be able to improve upon the implausible inflation-output dynamics of the new Keynesian models. Mankiw and Reis (2002) assume that pricing managers update their information set every period with an exogenous probability and show that nominal disturbances can produce persistent real responses. Klenow and Willis (2007) assume that firms receive information regarding macro state variables every $A_{T}$ periods in a staggered fashion and find that greater values for $A_{T}$ lead to a delayed, hump shaped response of inflation and a stronger output response to nominal shocks. The assumption that agents receive information about macro state variables with a lag has microfundation in the papers of Reis (2006) and Acharya (2012). Reis (2006) shows that producers optimally do not process current news about aggregate variables when firms have to pay a cost of acquiring new information. Similarly, Acharya (2012) shows that firms optimally update their information about idiosyncratic shocks more often than their information about aggregate shocks when the cost of updating both types of
information is the same but the standard deviation of the idiosyncratic disturbances is greater. Unlike this chapter, these articles implicitly assume imperfect communication within firms. Namely, pricing managers do not observe the firm's input prices and demand at all times, but production managers do observe this information and use it to make optimal production decisions.

A key assumption of the model presented in this chapter is that firms face a signal extraction problem. Firms need to form expectations about aggregate conditions based on perfectly observed input prices and demand, which contain important but noisy information about the state of the nature. This assumption follows Lucas (1972), who assumes that producers face real idiosyncratic shocks and aggregate nominal shocks, and need to form beliefs about the idiosyncratic and aggregate part of their demand in order to make production decisions. Hence, nominal innovations have real effects on the economy because firms make forecast errors by misinterpreting price changes. A signal extraction problem also appears in Nimark (2008), who studies a model with sticky prices and information frictions. Nimark assumes that firms face Calvo-type nominal regidities and observe their idiosyncratic marginal cost, but do not have perfect information regarding the economy-wide average marginal cost, which is needed in order to set firms' prices optimally. Nimark shows that these assumptions help explain a gradual and persistent inflation response to nominal shocks. Similarly, recent literature on dispersed information assumes that producers face a signal extraction problem. For example, Woodford (2001) and Paciello and Wiederholt (2011) assume that pricing managers observe some aggregate variables such as productivity and markups with noise. In contrast
to these articles, this chapter assumes that the person making the pricing decision perfectly observes everything that happens inside the firm, including input prices, input quantities and quantity sold at given prices.

This chapter also builds on the work of Angeletos and La'O (2012), who make a clear distinction between real and nominal information frictions. According to their terminology, an information friction is considered real if it affects the firm's decision of a real variable. For example, Angeletos and La'O assume that firms make capital decisions based on the same limited or noisy information used to set prices. In this chapter, the information friction is real because it affects inventory decisions.

This work is also part of a recent literature studying monetary models with inventories. Jung and Yun (2013) show that the relationship between current inflation and the marginal cost of production weakens in a model with inventories and Calvo-type nominal rigidities. Krytsov and Midrigan (2013) point out that countercyclical markups produced by inventories, rather than nominal rigidities, can account for much of the real effects of monetary policy. Even though I do not study markups per se in this chapter, I also find that the relationship between prices and the marginal cost of production breaks down when firms can accumulate inventories. When a firm's cost increases drastically, the firm reduces production and sells a fraction of its inventory holdings. This reduction in the stock of inventories prevents the firm's price from rising as much as it would in a model without inventories. In contrast to previous work, inventories in my model are crucial to explaining why there are real responses to monetary shocks. This is not true in Jung and Yun
(2013) and Krytsov and Midrigan (2013), which both assume some type of nominal price rigidity, so that monetary policy is effective even without inventories.

Finally, this work is related to previous studies exploring the implications of the cost smoothing motive of inventory investment (e.g Bils \& Kahn, 2000; Eichenbaum, 1989; Khan \& Thomas, 2007a, 2007b). In contrast to the existing literature, my work studies the role of inventories in pricing decisions in a setting with monopolistically competitive firms. This will be relevant to understanding what makes prices more or less responsive to monetary shocks.

This chapter is divided into five sections. In section two, I present the model setup and discuss some properties of the decision rules. In section three, I solve the model when all agents have perfect information. In section four, I solve the model under a particular information friction. Section five concludes.

### 2.2 Model

The model is close in spirit to Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). There are three agents in this economy: a representative household, a representative final good producer, and a continuum of intermediate goods firms. Households supply labor and capital to the intermediate goods firms, and they purchase a final good that can be used for consumption and investment. The final good producer aggregates the intermediate goods of the economy through a constant returns to scale production function, sells its output in a competitive market to the household, and cannot accumulate
inventories. Intermediate goods producers sell their product in a monopolistically competitive market to the representative final good firm and can accumulate output inventories.

Households derive utility from consumption and leisure and discount future utility by $\beta$. Households supply labor and capital to the intermediate goods producers in perfectly competitive and sector specific markets, and they own all intermediate and final goods firms. Capital depreciates at rate $\delta_{K}$ and can be augmented by using the final good as investment: $K_{t}=\left(1-\delta_{K}\right) K_{t-1}+X_{t}$.

I assume a continuum of differentiated industries with measure one and indexed by $j$. Each industry is represented by an intermediate goods firm that produces with capital, $k$, and labor, $h$, through a concave production function. Each intermediate goods firm can accumulate output inventories, and its output is denoted by $y=\left(k^{\alpha} h^{1-\alpha}\right)^{\gamma}$, where $\gamma<1$. I provide an explicit motive for inventory accumulation by assuming that intermediate goods firms face idiosyncratic shocks to their demand and input prices. At the beginning of each period, an intermediate good firm is identified by its inventory holdings, $I$, its current demand, $d$, and its current input prices, $q$. An intermediate goods firm sets its output price and determines current production, which is devoted to sales and inventory investment.

I assume that intermediate goods firms always observe their nominal input prices and demand but do not observe current aggregate variables. Firms observe the nominal wage and rental rate of capital of their sector. As a consequence, firms know how much it costs to produce $y$ units and how many units of their product they can sell at price $p$ for any $y, p>0$.

Finally, I follow the literature and assume a cash-in-advance constraint for the nominal output: $P_{t} Y_{t}=M_{t}$, where $Y_{t}$ denotes total aggregate production. The productivity of the final good firm (aggregate total factor productivity), $A$, and money balances, $M$, follow $\mathrm{AR}(1)$ processes in logs, and these are the only sources of aggregate uncertainty in the model.

### 2.2.1 Household's Problem

The representative household owns all the economy's firms, and supplies labor and capital to the intermediate goods producers in competitive and sector-specific markets. Each period, the household allocates its total income between money holdings, consumption and investment, in order to maximize its expected discounted lifetime utility. The monetary authority is assumed to pay interest on money holdings and as a consequence there is not revenue from seigniorage. Hence, the household's problem reads:

$$
\begin{align*}
& U=\max _{\left\{C_{t}, h_{j t}, k_{j t}, K_{t+1}, M_{t+1}, X_{t}\right\}} \sum_{t=0}^{\infty} \beta^{t}\left(\frac{C_{t}^{1-\sigma}}{1-\sigma}-\Psi \frac{\left(\int_{0}^{1} \phi_{w, j t} h_{j t} d j\right)^{1+\eta}}{1+\eta}\right)  \tag{2.1}\\
& \text { s.t. } \\
& M_{t+1}+P_{t} C_{t}+P_{t} X_{t} \leq \int_{0}^{1} W_{j t} h_{j t} d j+\int_{0}^{1} R_{j t} k_{j t} d j+\Pi_{t}^{F}+i_{t} M_{t}  \tag{2.2}\\
& K_{t}=\int_{0}^{1} \phi_{r, j t} k_{j t} d j  \tag{2.3}\\
& K_{t+1}=\left(1-\delta_{K}\right) K_{t}+X_{t} \tag{2.4}
\end{align*}
$$

Where $C_{t}$ is consumption, $M_{t}$ is money balances, $X_{t}$ represents fixed capital
investment, $K_{t}$ is the stock of capital at the beginning of period $t, i_{t}$ is the nominal interest rate, and $\Pi^{F}$ stands for aggregate nominal dividends from the economy's firms. $h_{j t}$ is the labor supply to sector $j$, and $W_{j t}$ is the nominal wage in that sector. $\phi_{w, j t}$ is a sector-specific preference shock that is i.i.d. across sectors and independent of all other shocks. $\log \left(\phi_{w, j t}\right)$ is distributed normal with zero mean and variance $\sigma_{w}^{2} . R_{j t}$ is the nominal rental rate of capital in sector $j$ at time $t$, and $k_{j t}$ is the supply of capital to that sector at time $t$. I assume that at the beginning of each period, each unit of "general" capital can be"transformed" into $\frac{1}{\phi_{r, j t}}$ units of type- $j$ capital. ${ }^{2} \phi_{r, j t}$ is a sector-specific shock that is i.i.d. across sectors and independent of other shocks, and $\log \left(\phi_{r, j t}\right)$ is distributed normal with zero mean and variance $\sigma_{r}^{2}$.

From the first order conditions, the supplies of type- $j$ labor and capital are given by:

$$
\begin{align*}
\phi_{w, j t}\left(\int_{0}^{1} \phi_{w, j t} h_{j t} d j\right)^{\eta} & =\frac{W_{j t}}{P_{t}} C_{t}^{-\sigma}  \tag{2.5}\\
\frac{R_{j t}}{\phi_{r, j t}} & =\frac{R_{i t}}{\phi_{r, i t}} \tag{2.6}
\end{align*} \quad \forall i, j
$$

Hence in equilibrium:

$$
\begin{align*}
W_{j t} & =\phi_{w, j t} W_{t}  \tag{2.7}\\
R_{j t} & =\phi_{r, j t} R_{t} \tag{2.8}
\end{align*}
$$

[^28]Where $W_{t}$ and $R_{t}$ are the aggregate nominal wage and rental rate of capital. ${ }^{3}$
After substituting equations (2.7) and (2.8) in the household's problem, we have:

$$
\begin{align*}
& U=\max _{\left\{C_{t}, H_{t}, K_{t+1}, M_{t+1}, X_{t}\right\}} \sum_{t=0}^{\infty} \beta^{t}\left(\frac{C_{t}^{1-\sigma}}{1-\sigma}-\Psi \frac{H_{t}^{1+\eta}}{1+\eta}\right)  \tag{2.9}\\
& \text { s.t. } \\
& M_{t+1}+P_{t} C_{t}+P_{t} X_{t} \leq W_{t} H_{t}+R_{t} K_{t}+\Pi_{t}^{F}+i_{t} M_{t}  \tag{2.10}\\
& K_{t+1}=\left(1-\delta_{K}\right) K_{t}+X_{t} \tag{2.11}
\end{align*}
$$

Where $H_{t} \equiv \int_{0}^{1} \phi_{w, j t} h_{j t} d j$. Then, the optimality conditions are given by:

$$
\begin{align*}
C_{t}^{-\sigma} & =\beta \mathbb{E}\left[\frac{i_{t+1}}{P_{t+1} / P_{t}} C_{t+1}^{-\sigma}\right]  \tag{2.12}\\
C_{t}^{-\sigma} & =\beta \mathbb{E}\left[\left(\frac{R_{t+1}}{P_{t+1}}+\left(1-\delta_{K}\right)\right) C_{t+1}^{-\sigma}\right]  \tag{2.13}\\
\Psi H_{t}^{\eta} & =\frac{W_{t}}{P_{t}} C_{t}^{-\sigma} \tag{2.14}
\end{align*}
$$

### 2.2.2 Final Good Firm Problem

There is a representative final good firm that sells its product, $S_{t}$, to the household in a competitive market. This firm produces using the intermediate goods of the economy through a constant returns production function. Hence, the problem

[^29]for the final good firm reads:
\[

$$
\begin{align*}
& \pi_{t}^{f}=\max _{s_{j t}}\left\{P_{t} S_{t}-\int_{0}^{1} p_{j t} s_{j t} d j\right\}  \tag{2.15}\\
& \text { s.t. } \\
& S_{t}=A_{t}\left(\int_{0}^{1} \chi_{j t}^{\frac{1}{\epsilon}} s_{j t}^{\frac{\epsilon-1}{\epsilon}} d j\right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.16}
\end{align*}
$$
\]

Where $\pi_{t}^{f}$ stands for nominal profits, $A_{t}$ is aggregate total factor productivity, $s_{j t}$ is the amount of the intermediate good $j$ used in the production of the final good, and $\chi_{j t}$ is a good-specific technology shock that is i.i.d. across sectors and independent of all other shocks. $\log \left(\chi_{j}\right)$ is distributed normal with zero mean and variance $\sigma_{\chi}^{2}$. Therefore, by cost minimization, the demand for intermediate good $j$ is given by:

$$
\begin{equation*}
s_{j t}=\chi_{j t}\left[A_{t}^{\epsilon-1} S_{t}\left(\frac{P_{t}}{p_{j t}}\right)^{\epsilon}\right] \tag{2.17}
\end{equation*}
$$

Below I will assume that intermediate firm $j$ takes $d_{j t} \equiv \chi_{j t}\left[A_{t}^{\epsilon-1} S_{t} P_{t}^{\epsilon}\right]$ as given. Throughout this chapter, I define $d_{j t}$ as firm $j$ 's nominal demand in period $t$. Therefore, it is convenient to re-write $s_{j t}$ as follows:

$$
\begin{equation*}
s_{j t}=d_{j t} p_{j t}^{-\epsilon} \tag{2.18}
\end{equation*}
$$

I assume that intermediate goods firms always observe $d_{j t}$, which means that they know how many units of their output they can sell at different prices. In-
termediate goods firms know that their nominal demand depends on aggregate $\left(S_{t}, P_{t}, A_{t}\right)$ and idiosyncratic $\left(\chi_{j t}\right)$ variables, but they cannot infer these components separately by observing $d_{j t}$. In equilibrium the profits of the final good firm are zero, $S_{t} \equiv C_{t}+X_{t}$, and the price of the final good is given by:

$$
\begin{equation*}
P_{t}=\frac{1}{A_{t}}\left(\int_{0}^{1} \chi_{j t} p_{j t}^{1-\epsilon} d j\right)^{\frac{1}{1-\epsilon}} \tag{2.19}
\end{equation*}
$$

Finally, I assume that total aggregate factor productivity, $A_{t}$, follows an $\operatorname{AR}(1)$ process in logs:

$$
\begin{align*}
\log \left(A_{t}\right) & =\rho_{A} \log \left(A_{t-1}\right)+\varepsilon_{A, t}  \tag{2.20}\\
\varepsilon_{A, t} & \sim N\left(0, \sigma_{A}^{2}\right) \tag{2.21}
\end{align*}
$$

### 2.2.3 Intermediate Goods Firms Problem

In each industry $j$, there is a single intermediate producer that supplies its product in a monopolistically competitive market to the final good firm. Each intermediate producer chooses employment, capital, the price of its product, and the stock of inventories for the next period. The cost of borrowing one unit of type- $j$ capital in period $t$ is given by the nominal rental rate $R_{j t}$, and the nominal wage of type- $j$ labor is given by $W_{j t}$. Hence the problem for the intermediate good firm in
sector $j$ is given by:

$$
\begin{align*}
& V\left(I_{0 j}, d_{0 j}, R_{0 j}, W_{0 j}\right)_{0}=\max _{\left\{p_{j t}, s_{j t}, y_{j t}, k_{j t}, h_{j t}, I_{j t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t} \pi_{j t}  \tag{2.22}\\
& \text { s.t. } \\
& \pi_{j t}=p_{j t} s_{j t}-R_{j t} k_{j t}-W_{j t} h_{j t}  \tag{2.23}\\
& s_{j t}=d_{j t} p_{j t}^{-\epsilon}  \tag{2.24}\\
& y_{j t}=s_{j t}+I_{j t+1}-I_{j t}  \tag{2.25}\\
& y_{j t}=\left(k_{j t}^{\alpha} h_{j t}^{1-\alpha}\right)^{\gamma}  \tag{2.26}\\
& I_{j t+1} \geq 0 \tag{2.27}
\end{align*}
$$

$\pi_{j t}$ is the current nominal profit, $p_{j t}$ is the price of good $j$, and $Q_{0, t}$ is the stochastic discount factor for the economy's firms: $Q_{0, t}=\beta \frac{u^{\prime}\left(C_{t}\right) / P_{t}}{u^{\prime}\left(C_{0}\right) / P_{0}}$. Equation (2.24) is the firm's demand, which was defined in equations (2.17) and (2.18). Now, by cost minimization, we can re-write this problem as follows:

$$
\begin{align*}
& V\left(I_{0 j}, d_{0 j}, q_{0 j}\right)_{0}=\max _{\left\{p_{j t}, s_{j t}, y_{j t}, I_{j t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t} \pi_{j t}  \tag{2.28}\\
& \text { s.t. } \\
& \pi_{j t}=p_{j t} s_{j t}-q_{j t} y_{j t}^{\frac{1}{\gamma}}  \tag{2.29}\\
& s_{j t}=d_{j t} p_{j t}^{-\epsilon}  \tag{2.30}\\
& y_{j t}=s_{j t}+I_{j t+1}-I_{j t}  \tag{2.31}\\
& I_{j t+1} \geq 0 \tag{2.32}
\end{align*}
$$

Where $q_{j t} \equiv\left(\frac{R_{j t}}{\alpha}\right)^{\alpha}\left(\frac{W_{j t}}{1-\alpha}\right)^{1-\alpha}$ is the nominal price of the firm's inputs. Notice that $q_{j t}$ can be decomposed as follows:

$$
\begin{align*}
q_{t j} & =\varphi_{j t} \bar{q}_{t}  \tag{2.33}\\
\varphi_{j t} & =\phi_{r, j t}^{\alpha} \phi_{w, j t}^{1-\alpha}  \tag{2.34}\\
\bar{q}_{t} & =\left(\frac{R_{t}}{\alpha}\right)^{\alpha}\left(\frac{W_{t}}{1-\alpha}\right)^{1-\alpha} \tag{2.35}
\end{align*}
$$

Where $\bar{q}_{t}$ is the "aggregate" nominal input price, and $\varphi_{j t}$ is an idiosyncratic shock that is i.i.d. across sectors and is distributed log-normal with zero mean and variance $\sigma_{\varphi}^{2} \equiv \alpha \sigma_{r}^{2}+(1-\alpha) \sigma_{w}^{2}$. The above problem is strictly concave, and the following first-order conditions pin down the firm's optimal decisions: ${ }^{4}$

$$
\begin{align*}
p_{j t} & =\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{q_{j t}}{\gamma}\right) y_{j t}^{\frac{1-\gamma}{\gamma}}  \tag{2.36}\\
\left(\frac{q_{j t}}{\gamma}\right) y_{j t}^{\frac{1-\gamma}{\gamma}} & \geq E\left[Q_{t, t+1}\left(\frac{q_{j t+1}}{\gamma}\right) y_{j t+1}^{\frac{1-\gamma}{\gamma}}\right] \tag{2.37}
\end{align*}
$$

Equation (2.36) states that the firm's price is equal to a markup times the marginal cost of production regardless of the production allocation. On the other hand, according to equation (2.37), inventories are used to smooth the marginal

[^30]Since $\epsilon>1$ and $\gamma \leq 1$, the first term in the firm's objective is strictly concave, and the second term is convex. Hence, this problem is strictly concave.
cost of production through time, and this equation holds with equality if $I_{j t+1}>0$. Suppose, for example, that a firm expects its marginal cost to go up in future periods due to an increase in the price of its inputs, $q_{j}$. In anticipation, the firm should increase its production and inventory stock in the current period, in order to sell those additional units when $q_{j}$ goes up. This would make the current and future marginal cost move in opposite directions, smoothing the firm's marginal cost. We have a similar story when a firm expects its demand, $d_{j}$, to increase. For the purposes of this work, the following lemmas will be useful.

Lemma 5. $p_{j t}$ is strictly decreasing in $I_{j t}$

Proof. See appendix B.1.1

In order to understand Lemma 5, suppose that the stock of inventories of a firm increases unexpectedly. Therefore, given that the firm will eventually sell those additional units, the firm's price will have to decrease at some point in order to induce consumers to buy more.

Lemma 6. Assuming that $\epsilon>1$ and that $\gamma \leq 1$, the optimal decision rules for $p_{j t}$ and $I_{j t+1}$ have the following properties:

- The current optimal price $\left(p_{j t}^{*}\right)$ is strictly increasing in the firm's current demand $\left(d_{j t}\right)$ and input prices $\left(q_{j t}\right)$.
- The current optimal price $\left(p_{j t}^{*}\right)$ is weakly increasing in the firm's future demand ( $\left.d_{j t+1}\right)$ and input prices $q_{j t+1}$.
- The optimal next period's stock of inventories $\left(I_{j t+1}^{*}\right)$ is weakly decreasing in the firm's current demand $\left(d_{j t}\right)$ and input prices $q_{j t}$. Moreover, if the initial stock of inventories is positive $\left(I_{j t}>0\right), I_{j t+1}^{*}$ is strictly decreasing in $d_{j t}$ and $q_{j t}$.
- The optimal next period's stock of inventories ( $I_{j t+1}^{*}$ ) is weakly increasing in the firm's future demand $\left(d_{j t+1}\right)$ and input prices $\left(q_{j t+1}\right)$.

Proof. See appendix B.1.2.

Intuitively, given that inventories are used to smooth cost shocks, a firm will sell inventories when its demand or input price increase. This will lower current marginal cost below what it would otherwise be in the absence of inventories. Similarly, if a firm expects its demand or input price to go up in the future, it will accumulate inventories by increasing its current production. This will make the current marginal cost, and thus the firm's current output price, increase relative to what it would otherwise be in the absence of inventories.

Lemma 7. At the firm level, inventories impose an upper bound on the expected increase in the firm's price. In particular,

$$
\begin{equation*}
1 \geq \mathbb{E}\left[Q_{t, t+1} \frac{p_{t+1}}{p_{t}}\right] \tag{2.38}
\end{equation*}
$$

Proof. See appendix B.1.3

This lemma implies that, with monopolistic competition, inventories smooth not only the marginal cost of production but also firms' prices. Intuitively, sup-
pose that a firm expects its price to go up in the following period and that $p_{t}<$ $E\left[Q_{t, t+1} p_{t+1}\right]$ so that $E\left[Q_{t, t+1} \frac{p_{t+1}}{p_{t}}\right]>1$. Notice that this firm could increase its profits by producing more today and selling those extra units in the next period. On the one hand, the increase in current production would make the current marginal cost go up, increasing $p_{t}$. On the other hand, according to lemma 5, the increase in the stock of next period's inventories will make $p_{t+1}$ decrease. As a consequence, the firm will accumulate inventories up to the point where $p_{t}=E\left[Q_{t, t+1} \cdot p_{t+1}\right]$. In that situation, the marginal benefit of selling one extra unit today $\left(p_{t}\right)$ will be equal to the marginal benefit of selling one extra unit in the next period $\left(E\left[Q_{t, t+1} \cdot p_{t+1}\right]\right)$.

### 2.2.4 Money And Nominal Shocks

I sidestep the micro-foundations of money and impose a cash-in-advance constraint on nominal output:

$$
\begin{align*}
P_{t} Y_{t} & =M e^{\mu_{t}}  \tag{2.39}\\
\mu_{t} & =\rho_{\mu} \mu_{t-1}+\varepsilon_{\mu, t}  \tag{2.40}\\
\varepsilon_{\mu, t} & \sim N\left(0, \sigma_{\mu}^{2}\right) \tag{2.41}
\end{align*}
$$

This assumption is standard in the literature. For example, Angeletos and La'O (2012) impose a similar restriction on total aggregate expenditure. Given these assumptions, inflation is zero in the deterministic steady state, in which $\varepsilon_{\mu, t}=$ $\varepsilon_{A, t}=0$.

### 2.3 Solving the Model With Perfect Information

In this section, I solve this model assuming perfect information. As I will show, nominal shocks do not have real effects on this economy. However, the optimal decision rules depicted in this subsection will help to explain why nominal shocks have real effects when a particular information friction is introduced. I start by defining the competitive equilibrium of this economy and establishing that this economy exhibits the classical dichotomy. Next, I report the impulse response functions to a productivity shock and compare them with those generated by two alternative models: ( $i$ ) one in which there is no heterogeneity across sectors and firms cannot accumulate inventories, and (ii) one model in which there is heterogeneity across firms but firms cannot accumulate inventories.

### 2.3.1 Competitive Equilibrium with Perfect and Complete Information

Definition: A competitive equilibrium with perfect and complete information in this economy is a sequence of prices $\left\{P_{t}, W_{t}, R_{t}, i_{t}, p_{j t}\right\}$, allocations $\left\{C_{t}, K_{t}, I_{t}\right.$, $\left.Y_{t}, X_{t}, H_{t}, y_{j t}, h_{j t}, k_{j t}\right\}$, a distribution of intermediate goods firms $\left\{\lambda(I, q, d)_{t}\right\}$, and exogenous variables $\left\{\mu_{t}, A_{t}\right\}$, such that given the initial conditions $K_{0}, \lambda(I, q, d)_{0}$ :

1. Households optimize taking prices, exogenous variables, the distribution of intermediate goods firms and initial conditions as given. The sequence $\left\{C_{t}, K_{t+1}\right.$, $\left.M_{t+1}, X_{t}, H_{t}\right\}$ satisfies equations (2.12), (2.13), (2.14), (2.10), and (2.11) along
with the transversality conditions:

$$
\begin{align*}
& \lim _{t \rightarrow \infty} \beta^{t} u^{\prime}\left(C_{t}\right) K_{t}=0 .  \tag{2.42}\\
& \lim _{t \rightarrow \infty} \beta^{t} u^{\prime}\left(C_{t}\right) M_{t}=0 . \tag{2.43}
\end{align*}
$$

2. The final good producer optimize taking prices, exogenous variables, the distribution of intermediate goods firms and initial conditions as given:

$$
\begin{align*}
P_{t} & =\frac{1}{A_{t}}\left(\int_{0}^{1} \chi_{j t} p_{j t}^{1-\epsilon} d j\right)^{\frac{1}{1-\epsilon}}  \tag{2.44}\\
C_{t}+X_{t} & =A_{t}\left(\int_{0}^{1} \chi_{j t}^{\frac{1}{\epsilon}} s_{j t}^{\frac{\epsilon-1}{\epsilon}} d j\right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.45}
\end{align*}
$$

3. Intermediate goods producers optimize taking $\left\{P_{t}, W_{t}, R_{t}, i_{t}, q_{j t},\left\{p_{z t}\right\}_{z \neq j}\right\}$, exogenous variables, the distribution of intermediate goods firms, and initial conditions as given. The sequence $\left\{y_{j t}, I_{j t+1}, p_{j t}\right\}$ satisfies equations (2.36), (2.37), (2.25), and (2.26) along with the transversality condition:

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \beta^{t} u^{\prime}\left(C_{t}\right) I_{t}=0 \tag{2.46}
\end{equation*}
$$

4. The distribution of intermediate goods firms evolves according to

$$
\begin{equation*}
\lambda\left(I^{\prime}, q^{\prime}, d^{\prime}\right)_{t+1}=\int \mathbf{1}_{\left\{I(I, q, d)=I^{\prime}\right\}} \cdot \operatorname{pr}\left(q^{\prime} \wedge d^{\prime} \mid q, d\right) \cdot d \lambda(I, q, d)_{t} \tag{2.47}
\end{equation*}
$$

Where $\mathbf{1}_{\left\{I(I, q, d)=I^{\prime}\right\}}$ is an indicator function that is equal to 1 if a firm with
initial stock of inventories $I$, input price $q$, and demand $d$, chooses a stock of inventories for the next period equal to $I^{\prime}$.
5. Markets Clear:

$$
\begin{align*}
H_{t} & =\int_{0}^{1} \phi_{w, j t} h_{j t} d j  \tag{2.48}\\
K_{t} & =\int_{0}^{1} \phi_{r, j t} k_{j t} d j  \tag{2.49}\\
Y_{t} & =C_{t}+X_{t}+I_{t+1}-I_{t} \tag{2.50}
\end{align*}
$$

6. The money growth rate and log total factor productivity follow $\operatorname{AR}(1)$ processes:

$$
\begin{gather*}
\mu_{t}=\rho_{\mu} \mu_{t-1}+\varepsilon_{\mu, t}  \tag{2.51}\\
\log \left(A_{t}\right)=\rho_{A} \log \left(A_{t-1}\right)+\varepsilon_{A, t} \tag{2.52}
\end{gather*}
$$

Proposition 1. The set of real allocations $\left\{C_{t}, K_{t}, I_{t}, Y_{t}, X_{t}, H_{t}, y_{j t}, h_{j t}, k_{j t}\right\}$ and distribution of intermediate goods firms $\left\{\lambda(I, q, d)_{t}\right\}$ that are consistent with the existence of a competitive equilibrium is independent of the path for money.

Proof. See appendix B.1. 4

Hence, this economy exhibits the classical dichotomy. As long as prices are flexible and all agents in this economy have perfect and complete information, real and nominal variables can be analyzed separately.

### 2.3.2 Numerical Analysis

I now examine impulse responses for a parameterized version of the model. The time period is one quarter. I draw on existing literature for the values of $\sigma, \eta$, $\delta$, and $\epsilon$. The intertemporal elasticity of substitution $(\sigma)$ is set to 2 . The inverse of the Frisch elasticity $(\eta)$ is equal to 0.4 . The rate of capital depreciation $\delta$ is fixed to 0.017, and the elasticity of substitution $(\epsilon)$ is set to 5 .
$\beta$ is selected so that the model has a real interest rate of $6.5 \%$ per year in steady state. The preference parameter $\Psi$ is calibrated to set the average hours worked in steady state to one-third of available time. The parameter associated with the capital share $(\alpha)$ is chosen so that the annual capital-output ratio in steady state is equal to 2.2, a value consistent with US data from 1960 to 2013.

In order to calibrate the persistence and standard deviation of the productivity shock $\left(\rho_{A}\right.$ and $\left.\sigma_{A}\right)$, I use the series for Total Factor Productivity from the Federal Reserve Bank of San Francisco for the period between 1960 and 2013. Detrending these series using the Hodrick-Prescott filter and estimating an $\operatorname{AR}(1)$ process to the detrended series yields a value of 0.8 for $\rho_{A}$ and 0.013 for $\sigma_{A}$.

I use the sweep-adjusted M1S series to calibrate the parameters associated with the money growth rate. Detrending the log series using the Hodrick-Prescott filter and estimating an $\operatorname{AR}(1)$ to this data yields a value of 0.9 for $\rho_{\mu}$ and 0.0084 for $\sigma_{\mu}$.

Finally, I assume that the standard deviations of the idiosyncratic shocks are equal so firms' demand and input prices are equally informative about aggregate
conditions. This standard deviation is calibrated so that the stock of inventories represents $13 \%$ of total GDP in the model with no information frictions. This is consistent with the inventories-output ratio for finished manufactured goods for the U.S. This implies a standard deviation of idiosyncratic shocks equal to $9 \%$.

### 2.3.3 Model Dynamics with Perfect Information

Given this set of parameters, I find the deterministic steady state and report it in table 2.1. ${ }^{5}$ Figure 2.1 displays the intermediate firms' decision rules for different levels of the nominal demand $d_{j}$ and input prices $q_{j}$, and the first panel of Figure 2.2 shows the ergodic distribution of inventories for this model. As stated in Lemmas 5 and 6 , the price decision rule is strictly decreasing in the initial stock of inventories. Also, notice that firms accumulate inventories when they face low input prices or demand because in those situations the marginal cost of production is low. Another feature of this figure is that the higher the initial stock of inventories, the smaller the impact of a cost or demand shock on the firm's price. For instance, when a firm's input price increases, the impact on the firm's price can be smoothed as long as the firm has a positive initial stock of inventories. According to the ergodic distribution, $45 \%$ of firms do not have inventories at a typical point in time, and $95 \%$ have an initial stock of inventories between zero and 0.5.

[^31]Figure 2.1: Intermediate Goods Firms' Decision Rules


Note: This Figure shows the intermediate goods firms decision rules in steady state in a model with perfect and complete information. $\mathrm{q}(\mathrm{j})$ - Nominal input price. $\mathrm{d}(\mathrm{j})$ - Nominal demand. p- firm's output price. y - firm's production. I- end of period inventories.

Table 2.1: Steady State Values. Model with Perfect and Complete Information

| Variable | Value | Description |
| :---: | :---: | :--- |
| $Y$ | 1.02 | Output |
| $C$ | 0.89 | Consumption |
| $I$ | 0.13 | Inventories |
| $K$ | 8.97 | Capital |
| $P$ | 0.69 | Price index |
| $W$ | 1.00 | Nominal Wage |
| $\frac{I}{Y}$ | 0.13 | Inventories-Output ratio |
| $\frac{K}{Y}$ | 8.80 | Capital-Output ratio |

Note: This table reports the steady state values for the model variables in the model with perfect and complete information.

To compute the impulse responses of this model, I take a first order approximation of the economy around the deterministic steady state, following the methodology proposed by Reiter (2009). ${ }^{6}$ This methodology allows a higher order representation of the cross-sectional distribution in the state vector and has the advantage that the solution is fully non-linear in the idiosyncratic (presumably large) shocks but linear in the aggregate (presumably small) shocks. ${ }^{7}$

Figure 2.3 plots the impulse response functions to a $1 \%$ increase in aggregate total factor productivity, $A$. The figure shows that inventories decline initially, then exhibits a hump shaped increase. These dynamics are the net results of several competing forces. First, the increase in productivity creates an incentive to accumulate more inventories for intermediate firms that are also facing a positive idiosyncratic productivity shock. In contrast, intermediate firms that are facing a negative idiosyncratic shock know that they will face a better shock with a high probability

[^32]Figure 2.2: Ergodic Distributions of Inventories


Note: This Figure plots the ergodic distribution of inventories in a model with perfect and complete information (left panel) and in a model with information frictions (right panel).
in the next period, and therefore they have an incentive to sell their stock of inventories in the current period. Second, firms expect total demand to keep increasing for another three quarters, which creates an incentive to accumulate inventories in the current period. Third, there is a big initial jump in total demand. Hence, firms have an incentive to use their stock of inventories in the current period in order to keep their prices relatively constant and take advantage of the increase in aggregate demand. As a result of these competing effects, most firms decide to sell a fraction of their inventories initially and wait until next period to accumulate inventories, making inventory investment countercyclical. However, inventory investment is procyclical in the data (e.g. Ramey \& West, 1999; Bils \& Kahn, 2000; Khan \& Thomas, 2007). As I will discuss in the next section, one important assumption that drives the response for inventories is that firms know what is happening in the economy. Once I modify this assumption, inventory investment will become procyclical.

Figure 2.3: Impulse Response Functions to a Productivity Shock. Model with Perfect and Complete Information


Note: This figure plots the impulse response functions to a $1 \%$ increase in total aggregate productivity in a model with perfect and complete information. All figures are deviations with respect to the deterministic steady state. The change in inventories is expressed as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. The intermediate good price is the average of input prices.

### 2.4 Solving the Model with Information Frictions

I now introduce a particular information friction in this economy. I assume that final goods firms observe aggregate variables with a lag of $\mathcal{T}$ periods but receive information about their input prices and demand in real time. For simplicity, I set $\mathcal{T}$ equal to 1 , which implies that firms do not observe the current level of the aggregate variables. As stated before, one contribution of this chapter is to provide a model with perfect communication within firms in which nominal shocks have real effects. The following proposition shows why this is important:

Proposition 2. Suppose that all agents in the economy except firms have perfect and complete information. Moreover, assume that intermediate goods producers cannot hold inventories, so their problem becomes:

$$
\begin{align*}
& V\left(q_{0}, d_{0}\right)_{0}=\max _{\left\{p_{t}, s_{t}, y_{t}\right\}} E \sum_{t=0}^{\infty} Q_{0, t}\left(p_{t} s_{t}-q_{t} y_{t}^{\frac{1}{\gamma}}\right)  \tag{2.53}\\
& \text { s.t. } \\
& s_{t}=d_{t} p_{t}^{-\epsilon}  \tag{2.54}\\
& y_{t}=s_{t} \tag{2.55}
\end{align*}
$$

If prices are flexible, and if there is perfect communication within firms such that pricing managers perfectly observe their input prices and demand, then nominal shocks do not have real effects on the economy regardless of the information friction on aggregate variables.

Proof. See appendix B.1.5.

Hellwig and Venkateswaran (2012) prove a result similar to Proposition 2 for a simpler model. ${ }^{8}$ If firms do not accumulate inventories or make other dynamic decisions such as investment, then as long as firms observe their current demand and input prices, information frictions are irrelevant. The intuition is simple: in such a model firms only need to know their current demand and input prices in order to infer their best response. A firm does not need to know the actual value of $C, X, P$ or even its own demand shock $\chi$, because $d$ and $q$ contain all the information that is relevant. This proposition implies, for example, that the models of Mankiw and Reis (2002), Paciello and Wiederhold (2011), and Klenow and Willis (2007) would not display real responses to monetary shocks if perfect communication within firms was allowed. However, Proposition 2 does not hold when intermediate goods firms can accumulate inventories or capital. This result is summarized in the following proposition.

Proposition 3. Suppose that all agents in the economy except intermediate firms have perfect and complete information. If intermediate goods firms can accumulate inventories or capital and their input prices and demand do not reveal the aggregate state of the economy, the economy exhibits money non-neutrality.

Proof. See appendix B.1.6.

These results are related to Angeletos and La'O (2012), who distinguish between nominal and real information frictions. Notice that one key difference be-

[^33]tween the problem faced by firms in Propositions 2 and 3 is the existence of real information frictions in the latter setting. Nominal shocks have real effects in the environment specified in Proposition 3 because investment decisions are based on noisy information about the state of nature, which makes the information friction real. In the environment of Angeletos and La'O (2012), however, nominal shocks would not have real effects if input prices and demand were perfectly observed. This is because firms could perfectly infer the aggregate state of the economy based on that information. ${ }^{9}$

Intuitively, when firms makes investment decision, future aggregate conditions play an important role in firms' problem. This is because the stock of inventories or the stock of capital affect future profits. Hence, when current input prices and demand do not perfectly reveal current aggregate conditions, firms make forecast mistakes because their inference about the state of the nature is wrong.

For instance, assume that firms accumulate inventories and that the aggregate input prices go down keeping everything else constant. If firms observe the aggregate state, they will react by adjusting output prices down, and real variables will be unchanged. But, if firms only observe aggregate variables with a lag, they will initially only observe their own input prices going down. Firms do not know the source of that movement. They only know that it could be because (i) the

[^34]aggregate economy has experienced a positive productivity shock, (ii) the aggregate economy has experienced a contractionary nominal shock, (iii) the firm has experienced a positive idiosyncratic shock, or (iv) a combination of these. Therefore, firms' responses will be a combination of the optimal responses for each case. Given that firms want to accumulate inventories when they are shocked by a positive idiosyncratic shock, they will respond to lower input prices by accumulating inventories, which has a positive effect on the firm's current price. How strong their responses are will depend on their expectations and the probability for each case. This points out why inventories help to explain the non-neutrality of money when perfect communication within firms is assumed.

In light of proposition 3, it is worth explaining why this chapter introduces money non-neutrality by allowing the firm to accumulate inventories and not capital as the previous proposition also suggests. As this chapter shows, inventories impose an endogenous upper bound on firms' expected price increases and make firms' prices more persistent, and these features may have important implications for the transition mechanism of monetary policy that have not been discussed in the previous literature. However, I do not mean to suggest that inventories are more relevant than capital accumulation for the monetary authority. That question could be addressed by future work. The main message of this chapter is that investment decisions are key for money non-neutrality under noisy information, flexible prices and perfect communications within firms. Similarly, in the spirit of Lucas (1972), this work aims to point out that firms' input prices and demand contain noisy but important information about aggregate conditions, and how firms process that in-
formation is also key for understanding real responses to monetary shocks. The relevant literature, including Angeletos and La'O (2012), abstracts from this signal extraction problem faced by firms.

### 2.4.1 Recursive Competitive Equilibrium

Given the information friction that was introduced above, it is convenient to define the competitive equilibrium in recursive form. Denote $\xi$ as the vector of aggregate state variables, which will be defined below. The household's recursive optimization problem is:

$$
\begin{align*}
U(K, M, \xi) & =\max _{M^{\prime}, K^{\prime}, C, H, X} \frac{C^{1-\sigma}}{1-\sigma}-\Psi \frac{H^{1+\eta}}{1+\eta}+\beta E\left[U\left(K, M, \xi^{\prime}\right)\right]  \tag{2.56}\\
\text { s.t. } & \\
M^{\prime}+P C+P X & \leq W H+R K+\Pi^{F}+i M  \tag{2.57}\\
K^{\prime} & =\left(1-\delta_{K}\right) K+X  \tag{2.58}\\
\xi^{\prime} & =\omega^{h}(\xi) \tag{2.59}
\end{align*}
$$

Where equation (2.59) is the household's perceived law of motion of $\xi$. The solution to this problem is given by decision rules $M^{\prime}(K, M, \xi), K^{\prime}(K, M, \xi), C(K, M, \xi)$, $H(K, M, \xi), X(K, M, \xi)$. Similarly, the intermediate goods firms' recursive opti-
mization problem is:

$$
\begin{align*}
& V\left(I, q, d, \xi_{-1}\right)=\max _{p, s, y, I^{\prime}} \pi+E_{\left\{q^{\prime}, d^{\prime}, \xi, Q \mid q, d, \xi_{-1}\right\}}\left[Q V\left(I^{\prime}, q^{\prime}, d^{\prime}, \xi\right)\right]  \tag{2.60}\\
& \quad \text { s.t. } \\
& \pi=p s-q y^{\frac{1}{\gamma}}  \tag{2.61}\\
& s=d p^{-\epsilon}  \tag{2.62}\\
& y=s+I^{\prime}-I  \tag{2.63}\\
& I^{\prime} \geq 0  \tag{2.64}\\
& \xi^{\prime}=\omega^{F}(\xi) \tag{2.65}
\end{align*}
$$

Where equation (2.65) is the firms' perceived law of motion of $\xi$. Since firms observe aggregate variables with a one period lag, the firm's problem depends on $\xi_{-1}$ and not on $\xi$ as in the household's problem. Hence, the solution in this case is given by decision rules $p\left(I, q, d, \xi_{-1}\right), s\left(I, q, d, \xi_{-1}\right), y\left(I, q, d, \xi_{-1}\right), I\left(I, q, d, \xi_{-1}\right)$.

Given the assumed information friction, the vector of aggregate state variables will be given by:

$$
\begin{equation*}
\xi=\left[\mu, A, \Lambda, K, \mu_{-1}, A_{-1}\right]^{\prime} \tag{2.66}
\end{equation*}
$$

Given that the only two sources of aggregate uncertainty are the productivity and nominal shocks, agents in this economy can perfectly infer the current distribution of firms $(\Lambda)$ and stock of capital $(K)$ by observing $\xi_{-1}$. This is why $\Lambda_{-1}$ and
$K_{-1}$ are not relevant for the law of motion of the economy.
The household's decision rule for capital accumulation along with the firms' decision rules for inventories induce a law of motion for the aggregate variables $\omega(\xi)$. In the recursive rational expectations equilibrium the actual and the perceived law of motions are equal. To economize on notation, I henceforth let $x(\cdot)$ denote the decision rule for $x$.

Definition: A recursive competitive equilibrium is defined by pricing functions $\{P(\xi), W(\xi), R(\xi), i(\xi), q(\xi)\}$, a law of motion for the aggregate variables $\omega(\xi)$, and a set of decision rules $\left\{C(\cdot), K^{\prime}(\cdot), M^{\prime}(\cdot), H(\cdot), X(\cdot), s(\cdot), y(\cdot), I(\cdot), p(\cdot)\right\}$ with associated value functions $\left\{U(K, M, \xi), V\left(I, q, d, \xi_{-1}\right)\right\}$ such that:

1. $K(\cdot), M(\cdot), C(\cdot), H(\cdot), X(\cdot)$ and $U(K, \xi)$ solve the household's recursive optimization problem, taking as given $P(\xi), W(\xi), R(\xi), i(\xi)$, and $\omega(\xi)$.
2. $p(\cdot), s(\cdot), y(\cdot), I(\cdot)$ and $V\left(I, q, d, \xi_{-1}\right)$ solve the intermediate goods firms' problem, taking as given $q(\xi), P(\xi), W(\xi), i(\xi)$, and $\omega(\xi)$.
3. The final good producer optimizes taking as given $P(\xi), W(\xi), R(\xi), i(\xi)$, and $\omega(\xi):$

$$
\begin{align*}
P(\xi) & =\frac{1}{A_{t}}\left(\int_{0}^{1} \chi_{j t} p(\cdot)^{1-\epsilon} d j\right)^{\frac{1}{1-\epsilon}}  \tag{2.67}\\
C(\cdot)+X(\cdot) & =A_{t}\left(\int_{0}^{1} \chi_{j t}^{\frac{1}{\epsilon}} s_{j t}^{\frac{\epsilon-1}{\epsilon}} d j\right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.68}
\end{align*}
$$

4. Markets clear:

$$
\begin{align*}
H(\cdot) & =\int \phi_{w, j} h_{j} d j  \tag{2.69}\\
K(\cdot) & =\int \phi_{r, j} k_{j} d j  \tag{2.70}\\
Y_{t} & =C(\cdot)+X(\cdot)+I(\cdot)-I \tag{2.71}
\end{align*}
$$

5. The perceived law of motion for the aggregate variables is consistent with the actual law of motion:

$$
\begin{equation*}
\omega(\xi)=\omega^{h}(\xi)=\omega^{F}(\xi) \tag{2.72}
\end{equation*}
$$

6. The distribution of firms evolves according to

$$
\begin{equation*}
\lambda\left(I^{\prime}, q^{\prime}, d^{\prime}, \xi^{\prime}\right)=\int \mathbf{1}_{\left\{I\left(I, q, d, \xi_{-1}\right)=I^{\prime}\right\}} \cdot \operatorname{pr}\left(q^{\prime} \wedge d^{\prime} \mid q, d\right) \cdot d \lambda(I, q, d, \xi) \tag{2.73}
\end{equation*}
$$

Where $\mathbf{1}_{\left\{I\left(I, q, d, \xi_{-1}\right)=I^{\prime}\right\}}$ is an indicator function that is equal to 1 if a firm with initial stock of inventories $I$, input price $q$, and demand $d$, chooses a stock of inventories for the next period equal to $I^{\prime}$.

### 2.4.2 Computation with Information Frictions

I solve this problem for small deviations around the steady state by following the methodology of Reiter (2009). This methodology has the feature that the law of motion for the aggregate variables is linear. Denoting $\mathbf{Y}$ as the vector of jump
variables, this economy can be described by the following two equations:

$$
\begin{align*}
& \widehat{\xi^{\prime}}=F \widehat{\xi}+\mathbf{V}  \tag{2.74}\\
& \widehat{\mathbf{Y}}=G \widehat{\xi} \tag{2.75}
\end{align*}
$$

Where $\widehat{x}$ denotes the deviation in levels of $x$ around the steady state, $F$ and $G$ are coefficient matrices, and $\mathbf{V} \equiv\left[\varepsilon_{\mu}, \varepsilon_{A}, \mathbf{O}_{\mathbf{1 \times ( \mathbf { 2 } \times \mathbf { n } \times \mathbf { n z } + \mathbf { 4 } )}}\right]^{\prime}$ is the vector of i.i.d. shocks. $n i$ is the number of grid points for the stock of inventories and $n z$ is the number of grid points for the idiosyncratic shocks.

To find the equilibrium of this economy, I start with a guess for matrices $F$ and $G$. Given this guess, the household's and firms' decision rules induce a law of motion and two new matrices $F^{(n e w)}$ and $G^{(n e w)}$. In equilibrium, these matrices have to be equal. If they are not, I update these matrices until a fixed point is reached.

One should note that the intermediate goods firms face a signal extraction problem. They observe their current input price $(q)$ and demand $(d)$ but do not have information about the current aggregate variables. These firms need to form expectations about the evolution of their input prices and demand in order to make their pricing and inventory decisions. To see this notice that:

$$
\begin{align*}
& d=\chi D  \tag{2.76}\\
& q=\varphi \bar{q} \tag{2.77}
\end{align*}
$$

Where $D \equiv A^{\epsilon-1}(C+X) P^{\epsilon}$ is the aggregate nominal demand, and $\bar{q} \equiv$
$\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}$ is the aggregate nominal input price. Since the law of motion for the aggregate variables is linear, I use the Kalman Filter to compute the expectations of the intermediate goods firms. Taking logs in equations (2.76) and (2.77) we get:

$$
\begin{align*}
& \log (d)=\log \left(D^{s s}\right)+D^{s s} \widehat{D}+\log (\chi)  \tag{2.78}\\
& \log (q)=\log \left(\bar{q}^{s s}\right)+\bar{q}^{s s} \widehat{\bar{q}}+\log (\varphi) \tag{2.79}
\end{align*}
$$

Where $x^{s s}$ denotes the value of $x$ in steady state. Notice that firms observe $\log (d)$ and $\log (q)$, but they do not observe $\widehat{D}, \widehat{\bar{q}}, \chi, \varphi$. Therefore, this signal extraction problem can be expressed as:

$$
\begin{align*}
{\left[\begin{array}{c}
\log (d) \\
\log (q)
\end{array}\right] } & =\left[\begin{array}{c}
\log \left(D^{s s}\right) \\
\log \left(\bar{q}^{s s}\right)
\end{array}\right]+\left[\begin{array}{c}
G_{D} \\
G_{\bar{q}}
\end{array}\right] \widehat{\xi}+\left[\begin{array}{l}
\chi \\
\varphi
\end{array}\right]  \tag{2.80}\\
\widehat{\xi^{\prime}} & =F \widehat{\xi}+\mathbf{V} \tag{2.81}
\end{align*}
$$

Where $G_{D}$ and $G_{\bar{q}}$ are the rows of matrix $G$ associated with the jump variables $D$ and $\bar{q}$. Hence, this system can be solved using the Kalman Filter.

### 2.4.3 Impulse responses with Information Frictions

Assuming the same parameter values as for the perfect information model, I report the steady state for this economy in Table 2.2 and the ergodic distribution of inventories in the second panel of Figure 2.2. The only significant difference between
the steady state with perfect information and the steady state with information frictions is that the stock of inventories now represents $15 \%$ of total output. Given that aggregate uncertainty is greater with information frictions and given that the final goods firms value function $\left(V\left(I, q, d, \xi_{-1}\right)\right)$ is strictly concave, intermediate firms have more incentive to invest in inventories, which provide insurance against negative shocks from the point of view of the firm. ${ }^{10}$

Table 2.2: Steady State Values. Model with Information Frictions

| Variable | Value | Description |
| :---: | :---: | :--- |
| $Y$ | 1.02 | Output |
| $C$ | 0.89 | Consumption |
| $I$ | 0.15 | Inventories |
| $K$ | 8.97 | Capital |
| $P$ | 0.69 | Price index |
| $W$ | 1.00 | Nominal Wage |
| $\frac{I}{Y}$ | 0.15 | Inventories-Output ratio |
| $\frac{K}{Y}$ | 8.80 | Capital-Output ratio |

Note: This table reports the steady state values for the endogenous model variables in the model in which final goods firms do not have information about current aggregate variables.

### 2.4.3.1 Productivity shock

Figure 2.4 plots the impulse response functions to a $1 \%$ increase in productivity, and Figure 2.5 compares these impulse responses with those generated by the model with perfect information. One of the most striking results is that inventories increase after the productivity shock in the model with information frictions. To explain this result, suppose for simplicity that the idiosyncratic $\operatorname{cost} \varphi$ has a uni-

[^35]form distribution. ${ }^{11}$ This implies that the nominal price of frims' inputs $q$ is also distributed uniform between $\left[q^{l}, q^{u}\right]$ with mean $\bar{q}$ as shown in Figure 2.6. Firms located between $\left[q^{l}, \bar{q}\right]$ have more incentive to accumulate inventories than those located between $\left[\bar{q}, q^{u}\right]$. After a positive aggregate productivity shock, the average input price $\bar{q}$ decreases to $\bar{q}-\phi$, where $\phi>0$. Figure 2.6 also shows how the distribution shifts. Given that firms do not know that the economy has experienced a positive productivity innovation, all the firms have an incentive to accumulate more inventories. Firms located between $\left[\bar{q}, q^{u}-\phi\right]$ (part A in Figure 2.6) are in the right tail of the new distribution, but they are not sure that the distribution has changed. As a consequence, those firms do not sell as many inventories as they would under full information. In the model with perfect information, those same firms know that the economy has been shocked, they know that their input price is relatively high, and they know about the big jump in total demand. Therefore, these firms sell a high volume of inventories when the economy experiences a positive productivity innovation in a model with perfect information. Similarly, firms facing an input price between $\left[q^{l}, \bar{q}\right]$ (part B of Figure 2.6) attach some probability under imperfect information that they are facing low real input prices with respect to the whole distribution. Therefore, they accumulate more inventories than they would absent information frictions. Finally, firms between $\left[b^{l}-\phi, b^{l}\right]$ (part C in Figure 2.6) know that the input price distribution has changed, since their input price has probability zero under the old distribution. Hence, those firms accumulate inventories not only

[^36]because they know that their input price is relatively low, but also because they have better expectations about the evolution of the economy, and they know that aggregate demand will keep increasing for another couple of periods.

The aggregate price index falls in the model with information frictions as the economy is able to produce more goods at a lower price. However, in comparison with the model with perfect information, the magnitude of the price decline is smaller. This is because the firms in the right tail of the idiosyncratic input price distribution do not sell as many inventories. Hence, these firms set a higher price. Since firms accumulate more inventories under imperfect information, current profits decline. This explain why the increase in the aggregate demand and output is smaller under imperfect information, since household's income is expanding at a slower rate.

Figure 2.4: Impulse Response Functions to a Productivity Shock. Model with Information Frictions


Note: This Figure plots the impulse response functions to a $1 \%$ increase in the total aggregate productivity in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 2.5: Impulse Response Functions to a Productivity Shock. Model with Perfect and Complete Information Vs Model with Information Frictions


Note: This Figure compares the impulse response functions to a $1 \%$ increase in total aggregate productivity for two different models. Complete and Perfect Information- Model with heterogeneous firms and output inventories in which agents have perfect and complete information. Information Frictions- Model with heterogeneous firms and output inventories in which firms observe aggregate variables with one period lag.

Figure 2.6: Distribution For The Input Price $q$


Note: This figure illustrates how the distribution of final goods firms changes after a productivity shock. Assuming that the idiosyncratic cost $(\varphi)$ distributes uniform, the distribution of the input price is also uniform between $\left[q^{l}, q^{u}\right]$ (Before shock). After a productivity shock, the distribution shifts to the left (After shock). Firm in part A and B, do not have enough evidence to conclude that the economy was shocked, and they think that they have more incentives to accumulate inventories. Only firms in part C conclude that the distribution changed.

### 2.4.3.2 Nominal Shock

Figure 2.7 plots the impulse response functions of this economy to a $1 \%$ decrease in the money growth rate. After the shock, intermediate goods firms observe a decrease in their nominal input price and nominal demand. They do not know the source of these changes. They only know that they could be facing a positive productivity shock (aggregate or idiosyncratic), a contractionary nominal shock, or a combination of both. Given that there is some probability that they are facing a positive productivity shock, firms accumulate inventories in the first period. As explained above, this response is amplified by the fact that firms located in the right tail of the input price distribution do not sell their stocks of inventories as much as they would under perfect information.

The large increase in inventories reduces current profits $\left(\Pi^{F}\right)$, and as conse-
quence household income. Since households want to smooth their consumption, they consume a part of their capital and work more. In the second quarter, when firms see that the economy was shocked by a lower money growth, they realize that they made a mistake by accumulating inventories. So they reduce their production and sell a large fraction of their inventories.

The dynamics of total investment (capital plus inventory) follow the output dynamics. However, the magnitude of fluctuations is larger for investment than for output. Notice that output decreases $0.18 \%$ in the first quarter while investment goes down by $0.67 \%$. The output and investment troughs are in quarter two, when output decreases $0.38 \%$ and investment falls $2.26 \%$.

### 2.4.3.3 Business Cycle Moments

Following Cooley and Hansen (1995), Tables 2.3 and 2.4 show variables' standard deviations, cross-correlations with output, and correlations with the money growth rate from simulating the model with perfect information (Table 2.3), and the model with information frictions (Table 2.4). For each table, the economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended using the Hodrick-Prescott filter. To assess these models, I compare these tables with the numbers reported in Table 2.5, which presents business cycle statistics for the U.S. economy.

It is not surprising that the standard deviations increase in the model with information frictions, since this model adds more uncertainty to the intermediate
firms' problem, and generates real responses to nominal shocks. Also, total investment and change in inventories become the most volatile variables in the model with information frictions, which is consistent with the empirical evidence. Similarly, prices become more stable in the model with imperfect information. The standard deviations of the price level and inflation are smaller, and they are even smaller in relative terms when compared to output. This is because firms carry more inventories on average to smooth shocks. The correlations with output in the model with information frictions are also closer to the data. In particular, inventory investment is pro-cyclical in the model with information frictions, and total investment is strongly correlated with output.

Finally, Figure 2.8 shows the optimal price series (left panel) for a firm facing a particular series of demand and input price shocks (right panel). The black line in the left panel shows the optimal price series for a firm that cannot accumulate inventories; the red line shows the price set by a final goods firm that can accumulate inventories and that has perfect information; and the blue line shows the price set by a firm that can accumulate inventories but that faces the information friction. Table 2.6 presents some statistics for this simulation.
Table 2.3: Business Cycles Statistics: Model with Perfect Information

| Variable | $\mathrm{SD}(\%)$ | Relative | Cross Correlation of Output with |  |  |  |  |  |  |  |  | Corr with $\mu$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | X | $\mathrm{x}(+1)$ | $\mathrm{x}(+2)$ | $\mathrm{x}(+3)$ | $\mathrm{x}(+4)$ |  |
| Output | 1.232 | 1.000 | 0.009 | 0.156 | 0.381 | 0.665 | 1.000 | 0.665 | 0.381 | 0.156 | 0.009 | -0.729 |
| Consumption | 0.209 | 0.170 | 0.420 | 0.521 | 0.632 | 0.722 | 0.772 | 0.361 | 0.045 | -0.179 | -0.313 | -0.129 |
| Capital | 0.412 | 0.334 | 0.607 | 0.644 | 0.636 | 0.553 | 0.368 | 0.022 | -0.224 | -0.382 | -0.467 | 0.367 |
| Hours | 0.670 | 0.544 | -0.175 | -0.028 | 0.214 | 0.544 | 0.959 | 0.705 | 0.475 | 0.282 | 0.149 | -0.892 |
| Price level | 2.060 | 1.670 | -0.006 | -0.088 | -0.229 | -0.416 | -0.622 | -0.443 | -0.284 | -0.166 | -0.076 | 0.495 |
| Inflation | 1.630 | 1.320 | 0.104 | 0.179 | 0.238 | 0.261 | -0.228 | -0.201 | -0.150 | -0.115 | -0.090 | 0.344 |
| Investment | 9.116 | 7.402 | -0.058 | 0.089 | 0.322 | 0.624 | 0.990 | 0.686 | 0.423 | 0.208 | 0.066 | -0.795 |
| Change Inv | 0.124 | 0.101 | 0.150 | 0.318 | 0.502 | 0.691 | -0.078 | -0.115 | -0.128 | -0.101 | -0.102 | 0.298 |
| Real Int rate | 0.042 | 0.034 | -0.197 | -0.055 | 0.186 | 0.515 | 0.948 | 0.706 | 0.484 | 0.297 | 0.167 | -0.909 |
| Money growth | 0.177 | 0.144 | 0.439 | 0.317 | 0.084 | -0.266 | -0.729 | -0.650 | -0.547 | -0.439 | -0.353 | 1.000 |
| Note: This table presents the standard deviations, cross-correlation with output, and correlations with the money growth rate after simulation model with perfect information. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial serin logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output. Fixed In capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate. |  |  |  |  |  |  |  |  |  |  |  |  |

Table 2.4: Business Cycles Statistics: Model with Information Frictions

| Variable | SD(\%) | Relative | Cross Correlation of Output with |  |  |  |  |  |  |  |  | Corr with $\mu$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | x | $\mathrm{x}(+1)$ | $\mathrm{x}(+2)$ | $\mathrm{x}(+3)$ | $\mathrm{x}(+4)$ |  |
| Output | 0.979 | 1.000 | -0.007 | 0.124 | 0.341 | 0.562 | 1.000 | 0.562 | 0.341 | 0.124 | -0.007 | 0.287 |
| Consumption | 0.221 | 0.225 | 0.274 | 0.378 | 0.473 | 0.558 | 0.639 | 0.385 | 0.087 | -0.098 | -0.211 | 0.544 |
| Capital | 0.920 | 0.939 | 0.211 | 0.244 | 0.206 | 0.160 | -0.064 | 0.086 | -0.098 | -0.129 | -0.156 | 0.545 |
| Hours | 0.974 | 0.995 | -0.105 | -0.047 | 0.106 | 0.263 | 0.597 | 0.231 | 0.230 | 0.125 | 0.068 | -0.309 |
| Price level | 1.460 | 1.488 | 0.017 | 0.022 | -0.030 | -0.058 | -0.171 | 0.078 | -0.027 | -0.010 | -0.011 | 0.553 |
| Inflation | 1.37 | 1.404 | -0.005 | 0.055 | 0.031 | 0.119 | -0.265 | 0.112 | -0.019 | 0.001 | -0.009 | 0.236 |
| Investment | 7.150 | 7.308 | -0.069 | 0.055 | 0.280 | 0.510 | 0.983 | 0.549 | 0.368 | 0.163 | 0.041 | 0.203 |
| Change Inv | 7.430 | 7.59 | 0.020 | -0.033 | -0.015 | -0.156 | 0.262 | -0.112 | 0.016 | -0.004 | 0.010 | -0.189 |
| Real Int rate | 0.061 | 0.062 | -0.133 | -0.047 | 0.111 | 0.355 | 0.544 | 0.308 | 0.252 | 0.145 | 0.075 | -0.262 |
| Money growth | 1.586 | 1.620 | -0.047 | 0.011 | 0.094 | 0.179 | 0.287 | 0.452 | 0.411 | 0.182 | 0.066 | 1.000 |

Note: This table presents the standard deviations, cross-correlation with output, and correlations with the money growth rate after simulating the model with information frictions. In this model, final goods firms observe aggregate variables with one period lag. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output. Fixed Inv- Fixed capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate.

Table 2.5: Business Cycles Statistics. US Economy 1967-2012

| Variable | $\mathrm{SD}(\%)$ | Relative | Cross Correlation of Output with |  |  |  |  |  |  |  |  |  |  | Corr |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | x | $\mathrm{x}(+1)$ | $\mathrm{x}(+2)$ | $\mathrm{x}(+3)$ | $\mathrm{x}(+4)$ | $\mathrm{with} \mu$ |  |  |
| Output | 1.55 | 1.00 | 0.29 | 0.49 | 0.70 | 0.88 | 1.00 | 0.88 | 0.70 | 0.49 | 0.29 | -0.13 |  |  |
| Consumption | 0.86 | 0.56 | 0.34 | 0.53 | 0.69 | 0.82 | 0.84 | 0.77 | 0.65 | 0.50 | 0.31 | -0.19 |  |  |
| Hours | 1.60 | 1.03 | 0.08 | 0.28 | 0.49 | 0.70 | 0.87 | 0.90 | 0.85 | 0.73 | 0.56 | -0.34 |  |  |
| Price level | 1.31 | 0.85 | -0.65 | -0.67 | -0.65 | -0.56 | -0.42 | -0.27 | -0.12 | 0.04 | 0.20 | -0.30 |  |  |
| Inflation | 1.47 | 0.95 | -0.41 | -0.31 | -0.17 | 0.01 | 0.22 | 0.38 | 0.50 | 0.57 | 0.59 | -0.39 |  |  |
| Investment | 6.97 | 4.50 | 0.32 | 0.48 | 0.65 | 0.80 | 0.91 | 0.80 | 0.60 | 0.37 | 0.13 | -0.14 |  |  |
| Change Inv | 17.3 | 11.16 | 0.07 | 0.22 | 0.39 | 0.53 | 0.67 | 0.51 | 0.27 | 0.04 | -0.15 | -0.30 |  |  |
| Real Int rate | 1.31 | 0.84 | -0.31 | -0.16 | 0.01 | 0.24 | 0.42 | 0.50 | 0.53 | 0.50 | 0.47 | -0.52 |  |  |
| Money growth | 2.61 | 1.68 | 0.11 | 0.06 | 0.02 | -0.04 | -0.13 | -0.21 | -0.26 | -0.26 | -0.21 | 1.00 |  |  |

Note: This table presents the standard deviations, cross-correlation with output, and correlations with the money growth rate for the US economy. Series were logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output.

Figure 2.7: Impulse Response Functions to a Nominal Shock. Model with Information Frictions


Note: This Figure plots the impulse response functions to a $1 \%$ increase in the nominal interest rate in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 2.8: Simulated Price for Three Different Models


Note: This Figure plots the simulated output price charged by a particular final goods firms in three different models. The right panel plot the simulated input price and demand. The left panel shows the price charged by the final goods firm. No inventories- The final goods firm cannot accumulate inventories. No info frictions- firm can accumulate inventories and has perfect and complete information. Info frictions- firm can accumulate inventories but observes aggregate variables with one period lag.

Table 2.6: Price Statistics for a Simulated Final Goods Firm

| Model | Mean | St. Dev | First Autocorrelation | Correlation with |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | q(j) | d(j) |
| No Inventories | 0.878 | 0.079 | 0.031 | 0.998 | -0.067 |
| No Information Frictions | 0.851 | 0.062 | 0.548 | 0.673 | -0.049 |
| Information Frictions | 0.845 | 0.061 | 0.551 | 0.626 | 0.069 |

Note: This table present the mean, standard deviation, first autocorrelation, and the correlation with the input price and demand for a final goods firm. No inventoriesprice charged by a final goods firm in a model in which firms cannot accumulate inventories and have perfect information. No Information Frictions- price charged by a final goods firm in a model in which firms can accumulate inventories and have perfect and complete information. Information Frictions- price charged by a final goods firm in a model in which firms observe aggregate variables with a lag and can accumulate inventories.

Notice that the correlation between firms' output prices and input prices is very strong (0.998) when firms cannot accumulate inventories. In contrast, when firms can accumulate inventories, this correlation decreases by almost $40 \%$. Hence, inventories break the strong relationship between current input prices and current output prices. Also, introducing inventories adds persistence to prices. The first autocorrelation of the output price increases from almost zero to 0.55 . As discussed above, inventories are used to smooth the marginal cost of production, which also implies price smoothing in the context of monopolistic competition.

### 2.4.4 Persistence of Idiosyncratic Shocks

In this chapter, I have assumed that idiosyncratic shocks are completely transitory. This is an important assumption that helps to explain real responses to monetary shocks. In order to see this, notice that equation (2.37), which governs the inventory decisions, would always hold with strict inequality in steady state if
idiosyncratic shocks were permanent, and therefore the optimal inventory decision would be zero $\left(I_{j t+1}^{*}=0\right)$. This is because, the role of inventories is to smooth production decision and therefore prices. If idiosyncratic shocks were permanent, firms will respond by adjusting their prices and keeping their inventories level constant in response to a idiosyncratic shock. To illustrate this, Figure 2.9 plots the impulse response functions of the aggregate price index and the aggregate output to a $1 \%$ decrease in the money growth rate when the persistence of idiosyncratic shocks are equal to: (1) zero (solid black line/baseline), (2) 0.3 (dashed line), and (3) 0.5 (dotted line). As we can see, the closer idiosyncratic shocks are to be permanent, the larger the price response and the smaller the output response.

Figure 2.9: Price and Output Responses. Different Persistence of Idiosyncratic Shocks


### 2.5 Conclusions

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. However, an assumption in the existing literature is that pricing managers do not interact with production managers
within firms. If this assumption is relaxed, nominal shocks would not have real effects on the economy in existing models. Hence, it is not clear why nominal shocks have real effects when prices are flexible and there is perfect communication within firms (input prices and demand are perfectly observed by pricing managers).

In this chapter, I present a model with information frictions, output inventories, and perfect communication within firms in which nominal shocks have real effects on the economy. In this model, intermediate goods firms observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time. In this model, inventories helps to explain the non-neutrality of nominal shocks for the following reason: given that firms only observe their nominal input prices and demand, they will accumulate inventories (by producing more) as long as they think that they are facing low real input prices. After a contractionary nominal shock, firms observe lower nominal input prices. They do not know what the source of this change is, but they know that it could be due to a positive productivity innovation or due to a nominal shock. Since positive shocks have a positive probability, firms will increase their stock of inventories. This will prevent firms' prices from decreasing, which will distort relative prices, and will make current profits and households' income go down. As a consequence, aggregate demand and real output fall.

According to my model simulations, a negative nominal shock reduces output by $0.17 \%$ in the first quarter and by $0.38 \%$ in the second quarter, followed by a slow recovery to the steady state. Contractionary nominal shocks have also significant effects on investment, which remains $1 \%$ below the steady state for the first 6 quarters.

Investment responds to an aggregate nominal perturbation by $-0.67 \%$ in the initial quarter and reaches its trough in the second quarter when it falls by $2.26 \%$. I also find that information frictions make the model more consistent with the empirical evidence on inventory behavior. In the model with information frictions, inventory investment is counter-cyclical; and its standard deviation is closer to the data.

I show that this model does not generate real effects of nominal shocks when there is perfect communication within firms if firms do not accumulate inventories or capital, even when firms have imperfect information about aggregate shocks. In contrast, I show that if firms make investment decisions (capital accumulation or inventory decisions) and if their nominal input prices and demand do not perfectly reveal the aggregate state of nature, the economy exhibits money non-neutrality even under flexible prices and perfect communication within firms (Proposition 3). In those situations, firms need to forecast future aggregate conditions in order to make optimal current decisions. Hence, when current input prices and demand do not perfectly reveal aggregate conditions, firms make forecast errors because their inference about the state of the nature is wrong, and their real decisions deviate from the decision that would have been taken under perfect information.

This chapter introduces money non-neutrality by allowing firm to accumulate inventories and not capital as the Proposition 3 also suggests. However, this does imply that inventories are more relevant than capital accumulation for the monetary authority. The relative importance of inventories versus capital accumulation is left for future work. The main point of this chapter is that investment decisions are key for money non-neutrality under flexible prices and perfect communication within
firms. Similarly, in the spirit of Lucas (1972), this work points out that firms input prices and demand contain noisy but important information about aggregate conditions, and how firms process that information is key for understanding real responses to monetary shocks. The relevant literature, including Angeletos and La'O (2012), abstracts from this signal extraction problem.

Appendix A: Appendix for Chapter 1
A. 1 Other Figures and Tables
Figure A.1: IRFs to 1\% Increase in Aggregate Productivity. Data versus Model with Sticky Wages for Job Stayers

Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags (solid black lines), where TFP is taken to follow an exogenous AR(1) process. All variables are HP-filtered in logs using a smoothing parameter equal to $10^{5}$. All figures are expressed in percentage points. The $x$ axis represents quarters after TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. Sample period is 1964Q1-2014Q4. The sample period for wages is 1994Q1-2014Q4. The dashed lines are the IRFs generated by a calibrated model with information frictions, in which wages for job stayers are negotiated every 4 quarters. The dotted lines are the IRFs generated by a calibrated model, in which all agents have perfect information and in which wages for job stayers are negotiated every 4 quarters.










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Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags (solid black lines), where TFP is taken to follow an exogenous $\operatorname{AR}(1)$ process. All variables are HP-filtered in logs using a smoothing parameter equal to $10^{5}$. All figures are expressed in percentage points. The $x$ axis represents quarters after TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. Sample period is 1964Q1-2014Q4. The sample period for wages is 1994Q1-2014Q4. The dashed lines are the IRFs generated by a calibrated model with information frictions affecting only workers, and the dotted lines are the IRFs generated by a non-calibrated model in which all agents face information frictions (firms and workers). For details see section 1.5.2.
Figure A.3: IRFs to 1\% Increase in Aggregate Productivity. Data versus Model with Information Frictions Affecting Firms and Workers
















Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags (solid black lines), where TFP is taken to follow an exogenous $\mathrm{AR}(1)$ process. All variables are HP-filtered in logs using a smoothing parameter equal to $10^{5}$. All figures are percentage points. The $x$ axis represents quarters after TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. The sample period is $1964 \mathrm{Q} 1-2014 \mathrm{Q} 4$. The sample period for wages is $1994 \mathrm{Q} 1-2014 \mathrm{Q} 4$. The dashed lines are the IRFs generated by a calibrated model, in which only workers face information frictions (benchmark). The dotted lines are the IRFs generated by a re-calibrated model with information frictions in which both firms and workers face information frictions.
Figure A.4: IRFs to $1 \%$ Increase in Aggregate Productivity. Data versus Model (Data filtered with smoothing parameters equal to 1,600 )












Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags (solid black lines), where TFP is taken to follow an exogenous $\mathrm{AR}(1)$ process. All variables are HP-filtered in logs using a smoothing parameter equal to 1,600 . All figures are expressed in percentage points. The $x$ axis represents quarters after TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. The sample period is 1964Q1-2014Q4. The sample period for wages is 1994Q1-2014Q4. The dashed lines are the IRFs generated by a calibrated model with information frictions, and the dotted lines are the IRFs generated by a calibrated model in which all agents have perfect information.

Table A.1: Statistics for Business Cycle Driven by TFP: U.S. Economy 1964:Q1-2014:Q4

| Standard deviation |  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{s}$ | $w^{u}$ | $w^{c}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.14 | 0.13 | 0.25 | 0.02 | 0.01 | 0.08 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
|  |  | (0.03) | (0.02) | (0.05) | (0.00) | (0.00) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (-) |
| Autocorrelation |  | 0.97 | 0.97 | 0.97 | 0.95 | 0.98 | 0.95 | 0.96 | 0.96 | 0.94 | 0.95 | 0.95 | 0.90 |
|  |  | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.32) | (0.32) | (0.30) | (0.32) | (0.31) | (-) |
| $u$ |  | 1 | -0.99 | -0.99 | -0.96 | -0.92 | -0.94 | -0.86 | -0.87 | -0.85 | -0.81 | -0.82 | -0.86 |
|  |  |  | (0.04) | (0.02) | (0.04) | (0.09) | (0.05) | (0.47) | (0.45) | (0.41) | (0.49) | (0.47) | (0.05) |
| Correlation Matrix | $v$ |  | 1 | 0.99 | 0.98 | 0.93 | 0.97 | 0.87 | 0.88 | 0.85 | 0.83 | 0.83 | 0.91 |
|  |  |  |  | (0.015) | (0.04) | (0.08) | (0.05) | (0.47) | (0.45) | (0.42) | (0.49) | (0.48) | (0.05) |
|  | $v / u$ |  |  | 1 | 0.97 | 0.92 | 0.96 | 0.89 | 0.90 | 0.87 | 0.84 | 0.85 | 0.89 |
|  |  |  |  |  | (0.04) | (0.08) | (0.05) | (0.47) | (0.45) | (0.42) | (0.49) | (0.48) | (0.05) |
|  | $y$ |  |  |  | 1 | 0.95 | 0.98 | 0.87 | 0.88 | 0.86 | 0.83 | 0.83 | 0.94 |
|  |  |  |  |  |  | (0.04) | (0.09) | (0.44) | (0.43) | (0.37) | (0.46) | (0.45) | (0.06) |
|  | c |  |  |  |  | 1 |  |  | 0.91 | 0.90 | 0.90 | 0.90 | 0.79 |
|  |  |  |  |  |  |  | $(0.03$ | (0.48) | $(0.47)$ | (0.42) | (0.51) | (0.49) | (0.03) |
|  | Inv |  |  |  |  |  | 1 | 0.76 | 0.77 | 0.75 | 0.69 | 0.70 | 0.97 |
|  |  |  |  |  |  |  |  | (0.49) | (0.47) | (0.43) | (0.51) | (0.50) | (0.02) |
|  | $w^{a}$ |  |  |  |  |  |  | 1 |  |  |  |  | 0.67 |
|  |  |  |  |  |  |  |  |  | (0.02) | (0.07) | (0.03) | $(0.03)$ | (0.50) |
|  | $w^{s}$ |  |  |  |  |  |  |  | 1 | 0.99 | 0.99 | 0.99 | 0.70 |
|  |  |  |  |  |  |  |  |  |  | (0.07) | (0.06) | (0.05) | (0.48) |
|  | $w^{u}$ |  |  |  |  |  |  |  |  | 1 | 0.98 | 0.99 | 0.65 |
|  |  |  |  |  |  |  |  |  |  |  | (0.09) | (0.07) | (0.45) |
|  | $w^{c}$ |  |  |  |  |  |  |  |  |  | 1 | 1.00 | 0.59 |
|  |  |  |  |  |  |  |  |  |  |  |  | (0.01) | (0.52) |
|  | $w^{n}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0.60 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | (0.51) |
|  | $a$ |  |  |  |  |  |  |  |  |  |  |  | 1 |

Notes: This table reports the business cycle moments driven by TFP shocks and their standard deviation (in parentheses) computed via bootstrap. For more details see Table 1.2.

Table A.2: Simulated Business Cycle. Calibrated Model in which Both Firms and Workers Face Information Frictions.

|  |  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{s}$ | $w^{u}$ | $w^{c}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Standard deviation |  | 0.125 | 0.165 | 0.273 | 0.025 | 0.013 | 0.082 | 0.018 | 0.018 | 0.016 | 0.018 | 0.017 | 0.018 |
| Autocorrelation |  | 0.900 | 0.803 | 0.912 | 0.935 | 0.986 | 0.937 | 0.975 | 0.975 | 0.974 | 0.974 | 0.974 | 0.871 |
| Correlation Matrix | $u$ | 1 | -0.757 | -0.918 | -0.850 | -0.465 | -0.928 | -0.736 | -0.736 | -0.703 | -0.735 | -0.726 | -0.824 |
|  | $v$ |  | 1 | 0.954 | 0.791 | 0.254 | 0.890 | 0.538 | 0.538 | 0.523 | 0.538 | 0.533 | 0.895 |
|  | $v / u$ |  |  | 1 | 0.870 | 0.368 | 0.966 | 0.664 | 0.664 | 0.640 | 0.664 | 0.657 | 0.921 |
|  | $y$ |  |  |  | 1 | 0.769 | 0.949 | 0.940 | 0.940 | 0.931 | 0.940 | 0.938 | 0.974 |
|  | c |  |  |  |  | 1 | 0.543 | 0.929 | 0.928 | 0.945 | 0.929 | 0.934 | 0.634 |
|  | Inv |  |  |  |  |  | 1 | 0.810 | 0.810 | 0.786 | 0.809 | 0.802 | 0.965 |
|  | $w^{a}$ |  |  |  |  |  |  | 1 | 1.000 | 0.998 | 1.000 | 1.000 | 0.847 |
|  | $w^{s}$ |  |  |  |  |  |  |  | 1 | 0.997 | 1.000 | 0.999 | 0.847 |
|  | $w^{u}$ |  |  |  |  |  |  |  |  | 1 | 0.998 | 0.999 | 0.836 |
|  | $w^{c}$ |  |  |  |  |  |  |  |  |  | 1 | 1.000 | 0.847 |
|  | $w^{n}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0.844 |
|  | $a$ |  |  |  |  |  |  |  |  |  |  |  | 1 |

Notes: Statistics for the simulated economy when both firms and workers face information frictions: u: Unemployment level. $v$ : Vacancies $v / u$ : Vancancy-unemployment ratio. $y$ : Output. c: Consumption. Inv: Investment. $w^{a}$ : Average wage in the economy. $w^{s}$ : Average wage for job stayers. $w^{u}$ : Average wage for new workers (workers who were unemployed in the previous period). $w^{c}$ : Average wage for job changers. $w^{n}$ : Average wage for new hires (new workers + job changers). a: Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000 .

Table A.3: Business Cycle Moments (Data versus Model)

|  | $u$ | $v$ | $v / u$ | $y$ | c | Inv | $w^{a}$ | $w^{n}$ | $a$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Standard Deviation |  |  |  |  |  |  |  |  |
| Data | 0.18 | 0.18 | 0.35 | 0.02 | 0.02 | 0.10 | 0.08 | 0.08 | 0.02 |
| Filtered Data | 0.14 | 0.13 | 0.25 | 0.02 | 0.01 | 0.08 | 0.02 | 0.02 | 0.02 |
| Model | 0.11 | 0.16 | 0.26 | 0.02 | 0.01 | 0.09 | 0.01 | 0.01 | 0.02 |
|  | Auto-correlation |  |  |  |  |  |  |  |  |
| Data | 0.96 | 0.95 | 0.96 | 0.94 | 0.96 | 0.92 | 0.21 | 0.22 | 0.91 |
| Filtered Data | 0.97 | 0.97 | 0.97 | 0.95 | 0.98 | 0.95 | 0.96 | 0.95 | 0.90 |
| Model | 0.96 | 0.92 | 0.96 | 0.93 | 0.99 | 0.93 | 0.98 | 0.98 | 0.87 |
|  | Correlation with output |  |  |  |  |  |  |  |  |
| Data | -0.88 | 0.90 | 0.94 | 1.00 | 0.89 | 0.82 | 0.17 | 0.15 | 0.80 |
| Filtered Data | -0.96 | 0.98 | 0.97 | 1.00 | 0.95 | 0.98 | 0.87 | 0.83 | 0.94 |
| Model | -0.88 | 0.90 | 0.94 | 1.00 | 0.70 | 0.93 | 0.85 | 0.84 | 0.95 |
|  | Correlation with TFP |  |  |  |  |  |  |  |  |
| Data | -0.48 | 0.53 | 0.52 | 0.80 | 0.73 | 0.69 | 0.17 | 0.15 | 1.00 |
| Filtered Data | -0.86 | 0.91 | 0.89 | 0.94 | 0.79 | 0.97 | 0.67 | 0.60 | 1.00 |
| Model | -0.81 | 0.93 | 0.93 | 0.95 | 0.97 | 0.47 | 0.66 | 0.66 | 1.00 |

Notes: This table reports business cycle statistics for the U.S. economy and a simulated economy. Statistics reported in the Data, Filter data, and Model rows were previously presented in Tables 1.1, 1.2, and 1.6. $u$ : Unemployment level. $v$ : Vacancies $v / u$ : Vancancy-unemployment ratio. $y$ : Output. $c$ : Consumption. Inv: Investment. $w^{a}$ : Average wage in the economy. $w^{n}$ : Average wage for new hires (new workers + job changers). $a$ : Aggregate TFP.

## A. 2 Proofs

## A.2.1 Proof of Lemma 1

If all agents in the economy have complete and perfect information, the following strategy profiles constitute the unique sub-game perfect Nash equilibrium of this game:

- For the worker:
- To accept only wage offers greater than or equal to $x^{*}$ where $\vec{W}_{j}\left(x^{*}, \omega, \Omega\right)$ $U(\omega, \Omega)=\vartheta \cdot S_{j}$
- To demand a wage equal to $y^{*}$ such that $\vec{W}_{j}\left(y^{*}, \omega, \Omega\right)-U(\omega, \Omega)=S_{j}$ and $\vec{J}_{j}\left(y^{*}, \omega_{f}, \Omega\right)=0$.
- For the firm:
- To offer $x^{*}$.
- To accept only wage demands that are less than or equal to $y^{*}$.

Proof. I begin at the third stage of the game (i.e. when the worker makes an offer). At this stage, the firm will accept any wage demand $y$ as long as $\vec{J}_{j}\left(y, h_{j}, \Omega\right) \geq 0$. Hence, the worker will demand a wage $y^{*}$ such that $\vec{J}_{j}\left(y^{*}, h_{j}, \Omega\right)=0$ and she keeps all the match surplus. Thus, at the second stage (i.e. when the worker has to accept or reject the firm's offer), the worker knows that if she rejects this offer, her expected payoff at the third stage will be $\vartheta \cdot S_{j}$. Therefore, she will only accept
wage offers that are greater than or equal to $x^{*}$ where $\vec{W}_{j}\left(x^{*}, \omega, \Omega\right)-U=\vartheta \cdot S_{j}$. Finally, at the first stage of the game (i.e. when the firm makes an initial offer), the firm anticipates a payoff of zero if it makes an offer less than $x^{*}$ and a payoff of $\vec{J}_{j}\left(x, h_{j}, \Omega\right)$ if $x \geq x^{*}$. Hence, the firm offers exactly $x^{*}$ to the worker and she accepts it.

## A.2.2 Proof of Lemma 2

Suppose that agents are information-constrained as described in section 1.2.4. If there is an equilibrium in which firms' strategy is to reveal the aggregate state of the economy, the best strategy for firms is the same strategy described in Lemma 1.

Proof. Since we are considering the equilibrium of the game, if firms are following a revealing strategy, workers know it and behave rationally. As a consequence, workers can perfectly infer the current state of the economy based on the firm's wage offer.

Hence, a worker knows that she will receive, in expectation, $\vartheta \cdot S_{j}$ if she rejects a firm's offer. Therefore, the optimal strategy for workers is:

- Infer the current level of the aggregate productivity based on firm's offer $x$ :

$$
a=x^{-1}(a)
$$

- To accept only wage offers greater than or equal to $x^{*}$ where:

$$
\begin{gathered}
\vec{W}_{j}\left(x^{*}, \omega, \Omega\right)-U=\vartheta \cdot S_{j} \\
\vec{J}_{j}\left(x^{*}, \omega, \Omega\right)=0
\end{gathered}
$$

- To demand a wage equal to $y^{*}$ if she has the chance such that:

$$
\vec{W}_{j}\left(y^{*}, \omega, \Omega\right)-U=S_{j}
$$

Now, given the workers' strategy, the firm anticipates a payoff of zero if it makes an offer less than $x^{*}$ and a payoff of $\vec{J}_{j}\left(x, h_{j}, \Omega\right)-U$ if $x \geq x^{*}$. Given that $\vec{J}_{j}\left(x, h_{j}, \Omega\right)$ is strictly decreasing in $x$, the optimal strategy for firms, assuming that they follow a revealing strategy is:

- To offer $x^{*}$.
- To accept only wage demands that are less than or equal to $y^{*}$.

As a consequence, if there exists an equilibrium in which firms reveal the true state of the economy, in equilibrium firms offer exactly $x^{*}$ and workers will accept it. In other words, workers rationally believe that if a firm extents a wage offer $x$, it has to be the case that $x=x^{*}$.

## A.2.3 Proof of Lemma 3

If agents in the economy are information-constrained as described in section 1.2.4, then in equilibrium, firms do not follow a strategy in which they perfectly reveal the true state of the economy.

Proof. Suppose not. By Lemma A.2.2, if there is an equilibrium in which firms reveal the true state of the economy, firms always offer $x=x^{*}$ and workers accept all wage offers $(x)$ because they rationally believe that $x$ is always equal to $x^{*}$. However,
in order for these strategies to be an equilibrium, firms cannot have incentives to deviate.

Suppose that firms deviate to an strategy in which they offer $\tilde{x}=0.5 x^{*}$. Workers will accept this offer because they believe $\tilde{x}=x^{*}$, and firms will be better off because $J_{j}(\tilde{x})>J_{j}\left(x^{*}\right)$. Therefore, there is not an equilibrium in which firms reveal the true state of the economy.

## A.2.4 Proof of Lemma 4

If agents in the economy are information-constrained as described in section 1.2.4, the following strategy profiles constitute a Perfect Bayesian Nash equilibrium:

- For the worker:
- To accept only wage offers greater than or equal to $x^{* *}$ where:

$$
E_{\mathcal{I}_{h}}\left[\vec{W}_{j}\left(x^{* *}, \omega, \Omega\right)-U(\omega, \Omega)\right]=\vartheta \cdot E_{\mathcal{I}_{h}}\left[S_{j}\right]
$$

- To demand a wage equal to $y^{* *}$ such that:

$$
E_{\mathcal{I}_{h}}\left[\vec{W}_{j}\left(y^{* *}, \omega, \Omega\right)-U(\omega, \Omega)\right]=E_{\mathcal{I}_{h}}\left[S_{j}\right]
$$

- For the firm:
- To offer $x^{* *}$.
- To accept only wage demands that are less than or equal to $\tilde{y}^{* *}$ such that

$$
\vec{J}_{j}\left(\tilde{y}^{* *}, \omega_{f}, \Omega\right)=0
$$

Proof. I begin at the third stage of the game (i.e. when the worker gets to make an offer). At this stage, the firm will accept any wage demand $y$ as long as its expected value is greater than or equal to zero. Given the firm's strategy, the firm's offer does not reveal its information. Therefore, the worker will demand a wage $y^{* *}$ such that, given her information set, firm's value is zero. Thus, at the second stage (i.e. when the worker has to accept or reject the firm's offer), the worker knows that if she rejects this offer, her expected payoff at the third stage will be $\vartheta \cdot E_{\mathcal{I}_{h}}\left[S_{j}\right]$. Therefore, she will only accept wage offers that are greater than or equal to $x^{* *}$. Finally, at the first stage of the game (i.e. when the firm makes an offer), the firm anticipates a payoff of zero if it makes an offer less than $x^{* *}$ and a payoff of $\vec{J}_{j}\left(x, h_{j}, \Omega\right) \geq 0$ if $x \geq x^{* *}$. Hence, the firm offers exactly $x^{* *}$ to the worker and she accepts it.

## A. 3 Detailed Household's Problem

This appendix presents the household's problem in recursive form and the complete derivation of the employment and unemployment functions. The household's utility function is given by:

$$
\begin{equation*}
\mathbb{U}(\omega, \Omega)=\frac{c^{1-\sigma}}{1-\sigma}-\Psi \int_{0}^{1} \frac{h_{j}^{1+\xi}}{1+\xi} d j+\beta E\left[\mathbb{U}\left(\omega^{\prime}, \Omega^{\prime}\right)\right] \tag{A.1}
\end{equation*}
$$

Hence, the household's problem is:

$$
\begin{equation*}
\max _{c, k^{\prime},\left\{h_{j}^{\prime}\right\}_{j=0}^{1}} E_{\mathcal{I}_{h}}\{\mathbb{U}(\omega, \Omega)\} \tag{A.2}
\end{equation*}
$$

subject to the budget constraint, the law of motion of labor, and the perceived law of motion of the economy:

$$
\begin{align*}
c+k^{\prime} & =\left(r+1-\delta_{k}\right) k+\int_{0}^{1} w_{j} h_{j} d j+\int_{0}^{1} \pi_{j} d j+b \cdot u-T  \tag{A.3}\\
h_{j}^{\prime} & =\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right) h_{j}+q\left(\frac{v_{j}}{v}\right) u+\int_{0}^{j} \bar{i} q\left(\frac{v_{j}}{v}\right)\left(1-\delta_{h}\right) h_{x} d x  \tag{A.4}\\
u & =\int_{0}^{1}\left(1-h_{j}\right) d j  \tag{A.5}\\
\Omega^{\prime} & =\lambda^{h}(\Omega) \tag{A.6}
\end{align*}
$$

where $E_{\mathcal{I}_{h}}[\cdot]$ is the expectation conditional on the household information set $\mathcal{I}_{h} . \omega=\left\{k,\left\{h_{j}\right\}, \mathcal{I}_{h}\right\}$ is the vector of state variables for household, and $\Omega$ is a vector that summarizes the aggregate state of the economy. Letting $\phi_{c}$ and $\phi_{j}$ denote the

Lagrange multipliers for equations (A.3) and (A.4), the first order conditions are given by:

$$
\begin{array}{lll}
c: & E_{\mathcal{I}_{h}}\left\{c^{-\sigma}-\phi_{c}\right\} & =0 \\
k^{\prime}: & E_{\mathcal{I}_{h}}\left\{-\phi_{c}+\beta \phi_{c}^{\prime}\left(r^{\prime}+1-\delta_{k}\right)\right\} & =0 \\
h_{j}^{\prime}: & E_{\mathcal{I}_{h}}\left\{-\phi_{j}-E\left\{\beta \Psi h_{j}^{\prime \xi}+\beta \phi_{c}^{\prime}\left(w_{j}^{\prime}-b\right)\right.\right. & \\
& +\left(1-\delta_{h}\right)\left(1-\bar{i} q^{\prime} F_{j}^{\prime}\right) \beta \phi_{j}^{\prime}-q^{\prime} \int_{0}^{1} \beta \phi_{x}^{\prime}\left(\frac{v_{x}^{\prime}}{v^{\prime}}\right) d x & \\
& \left.\left.+\left(1-\delta_{h}\right) \bar{i} q^{\prime} \int_{j}^{1} \beta \phi_{x}^{\prime}\left(\frac{v_{x}^{\prime}}{v}\right) d x\right\}\right\} & =0 \tag{A.9}
\end{array}
$$

Hence, combining (A.7) and (A.9) and lagging one period:

$$
\begin{align*}
E_{\mathcal{I}_{h}}\left\{\left(W_{j}(\omega, \Omega)-U(\omega, \Omega)\right)\right\}= & E_{\mathcal{I}_{h}}\left\{w_{j}-z_{j}\right.  \tag{A.10}\\
& +E\left\{Q \left(\left(1-\delta_{h}\right)\left(1-\bar{i} q F_{j}\right)\left(W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right.\right. \\
& +\left(1-\delta_{h}\right) \bar{i} q F_{j}\left(\tilde{W}_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right) \\
& \left.\left.\left.-q\left(\bar{W}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)\right)\right\}\right\} \tag{A.11}
\end{align*}
$$

where:

$$
\begin{equation*}
\left(W_{j}\left(\omega^{\prime}, \Omega^{\prime}\right)-U\left(\omega^{\prime}, \Omega^{\prime}\right)\right)=\frac{\phi_{j}}{\beta \phi_{c}^{\prime}} \tag{A.12}
\end{equation*}
$$

Also from the first order conditions, we can verify that the optimality condi-
tions for $c$ is given by:

$$
\begin{equation*}
c^{-\sigma}=\beta E_{\mathcal{I}_{h}}\left[\left(1-\delta+r^{\prime}\right) c^{\prime-\sigma}\right] \tag{A.13}
\end{equation*}
$$

## A. 4 Wages

In this appendix, I present more details about my empirical exercise and some additional experiments using other wage series.

## A.4.1 Wage series

I use the Current Population Survey (CPS) to construct wage series adjusted for composition effects. The CPS is the main labor force survey for the U.S., and it is the primary source of labor force statistics such as the national unemployment rate. The CPS consists of a rotating panel where households and their members are surveyed for four consecutive months, not surveyed for the following eight months, and interviewed again for another four consecutive months. The CPS includes individual information such as employment status, sex, education, race, state, etc. However, individual earnings and hours worked are collected only in the fourth and eight interviews. In addition, since 1994, individuals have been asked if they still work in the same job reported in the previous month, making it possible to identify job changers. Following Muller (2012) and Haefke, Sonntag and van Rens (2013), my empirical model is based on the following equation:

$$
\begin{equation*}
\log \left(w_{i t}\right)=X_{i t} \beta+\log \left(\tilde{w}_{i t}\right) \tag{A.14}
\end{equation*}
$$

where $w_{i t}$ is the hourly wage rate for individual $i$ at time $t, X_{i t}$ is a vector of individual characteristics, and $\tilde{w}_{i t}$ is the component of the wage rate for individual $i$
at time $t$ that is orthogonal to individual $i$ 's characteristics. The hourly wage rate is constructed by dividing weekly earnings by weekly hours. Following Schmitt (2003), top-coded weekly earnings are imputed assuming a log-normal cross-sectional distribution for earnings. Following Haefke et al. (2013) I drop hourly wage rates below the 0.25 th and above the 99.75 th percentiles each month. In order to take into account changes over time in the regression coefficients, I estimate equation (A.14) period by period controlling for: education, a fourth order polynomial in experience, gender, race, marital status, state, 10 occupation dummies, and 14 industry dummies. ${ }^{1}$ Then, I use the residuals from this Mincer regression to construct the average wage for each group as:

$$
\begin{equation*}
\log \left(w_{t}^{G}\right)=\sum_{i \in G} \log \left(\tilde{w}_{i t}\right) \omega_{i t} \tag{A.15}
\end{equation*}
$$

where $G=\{$ All, Job stayers, New hires, Job Changers, New hires from unemployment (new workers) $\}$, and $\omega_{i t}$ is individual $i$ 's weight. ${ }^{2}$ Due to sample design, it is not possible to match individuals in the fourth quarter of 1995. Hence, with the exception of the average wage for all workers, wage series have a missing value in this period. In order to fill these missing observations, for continuity, I impute these series using the average wage for all workers. However, my results are robust to limiting my sample period to 1996-2014.

[^37]
## A.4.2 Alternative wage series (Haefke et al, 2013)

Haefke et al. (2013) constructed two wage series for production and nonsupervisory employees adjusted for composition effects, which they have kindly made available online. In this subsection, I show that these series tell a similar story: wage responses to TFP shocks are delayed and weak. In particular, Figure A. 5 plots the impulse response functions of these wages to a $1 \%$ increase in aggregate TFP. The sample period is 1984Q1-2006Q1, which is the same one used in Haefke et al. (2013). In order to fill in the missing values in 1985 and 1995, I impute these series using the real aggregate wage.

Figure A.5: IRFs to 1\% Increase in Aggregate Productivity. Average Wage for Production and Nonsupervisory Employees


Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with two lags (solid black lines), where TFP is taken to follow an exogenous $\mathrm{AR}(1)$ process. All variables are HP-filtered in logs with smoothing parameter equal to $10^{5}$. All figures are expressed in percentage points. The $x$ axis represents quarters after the TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap. The sample period is 1984Q1-2006Q1. The dashed lines are the IRFs generated by a calibrated model, in which only workers face information frictions (benchmark). The dotted lines are the IRFs generated by a calibrated model in which all agents have perfect information.

## A.4.3 Results for average wage of production and non-supervisory

 employeesIn order to illustrate the role of composition effects, Figure A. 6 plots the IRFs to a $1 \%$ increase in TFP of the average wage for production and non-supervisory employees.

Figure A.6: IRFs to 1\% Increase in Aggregate Productivity. Average Wage for Production and Nonsupervisory Employees





Note: This figure plots the impulse responses to a $1 \%$ increase in TFP from bivariate near-VARs with three lags, where TFP is taken to follow an exogenous AR(1) process. All variables are HP-filtered in logs. For the two figures at the left a smoothing parameter equal to $10^{5}$ was used. For the tow figures at the right a smoothing parameter equal to 1,600 was used. The sample period for the top row is 1964Q12014Q4. The sample period the bottom row is 1994Q1-2014Q4. All figures are expressed in percentage points. The $x$ axis represents quarters after the TFP shock. The shaded area represents the $95 \%$ confidence intervals computed via bootstrap.

This figure shows that this series is acyclical. Even ignoring the wide confidence intervals, the point estimates of these IRFs are very small in absolute terms.

## Appendix B: Appendix for Chapter 2

## B. 1 Proofs

## B.1.1 Lemma 5

$p_{j t}$ is strictly decreasing in $I_{j t}$

Proof. First, notice that $V(I, q, d)_{t}^{i}$ is a strictly increasing and concave function in $I$. Notice that the firm's problem could be written as follows:

$$
\begin{align*}
& V\left(I_{0}, d_{0}, q_{0}\right)_{0}=\max _{\left\{y_{j t}, I_{j t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t}\left[d_{j t}\left(y_{j t}+I_{j t}-I_{j t+1}\right)^{\frac{\epsilon-1}{\epsilon}}-q_{j t} y_{j t}^{\frac{1}{\gamma}}\right]  \tag{B.1}\\
& \text { s.t. } \\
& I_{j t+1} \geq 0 \tag{B.2}
\end{align*}
$$

Since $\epsilon>1$ and $\gamma \leq 1$, the first term in (B.1) is strictly concave, and the second term is convex. Hence, this problem is strictly concave. Then, using the envelope theorem, we get that $V(I, q, d)_{0}$ is strictly increasing in $I$. Thus, $V(I, q, d)$ is strictly
increasing and concave in $I$. Then, using the envelope theorem:

$$
\begin{align*}
\frac{\partial V(I, q, d)_{t}}{\partial I_{t}} & =\left(\frac{\epsilon-1}{\epsilon}\right) p_{j t}>0  \tag{B.3}\\
\frac{\partial^{2} V(I, q, d)_{t}}{\partial I_{t}^{2}} & =\left(\frac{\epsilon-1}{\epsilon}\right) \frac{\partial p_{j t}}{\partial I_{j t}}<0 \tag{B.4}
\end{align*}
$$

## B.1.2 Lemma 6

Assuming that $\epsilon>1$ and that $\gamma \leq 1$, the optimal decision rules for $p_{j t}$ and $I_{j t+1}$ have the following properties:

- The current optimal price $\left(p_{j t}^{*}\right)$ is strictly increasing in the firm's current demand $\left(d_{j t}\right)$ and input prices $\left(q_{j t}\right)$.

Proof. By the envelope theorem and the symmetry of the second derivatives, we get that:

$$
\begin{align*}
& \frac{\partial p}{\partial d} \propto \frac{\partial^{2} V}{\partial d \partial I}=\frac{\partial^{2} V}{\partial I \partial d}=p^{-\epsilon} \frac{(1-\epsilon)}{\epsilon} \frac{\partial p}{\partial I}>0  \tag{B.5}\\
& \frac{\partial p}{\partial q} \propto \frac{\partial^{2} V}{\partial q \partial I}=\frac{\partial^{2} V}{\partial I \partial q}=-\left(\frac{1}{1-\gamma}\right)\left(\frac{y}{p}\right) \frac{\partial p}{\partial I}>0 \tag{B.6}
\end{align*}
$$

- The current optimal price $\left(p_{j t}^{*}\right)$ is weakly increasing in the firm's future demand $\left(d_{j t+1}\right)$ and input prices $q_{j t+1}$.

Proof. Using these results and the optimality condition for inventories (2.37), we obtain:

$$
\begin{equation*}
p_{j t} \geq E\left[Q_{t, t+1} p_{j t+1}\right] \tag{B.7}
\end{equation*}
$$

Hence, for $X=\left\{d_{j t+1}, q_{j t+1}\right\}$ :

$$
\begin{array}{ll}
\frac{\partial p_{j t}}{\partial X}=E\left[Q_{t, t+1} \frac{\partial p_{j t+1}}{\partial X}\right]>0 & \text { if } I_{j t+1}^{*}>0 \\
\frac{\partial p_{j t}}{\partial X}=0 & \text { if } I_{j t+1}^{*}=0 \tag{B.9}
\end{array}
$$

- The optimal next period's stock of inventories $\left(I_{j t+1}^{*}\right)$ is weakly decreasing in the firm's current demand $\left(d_{j t}\right)$ and input prices $q_{j t}$. Moreover, if the initial stock of inventories is positive $\left(I_{j t}>0\right), I_{j t+1}^{*}$ is strictly decreasing in $d_{j t}$ and $q_{j t}$.

Proof. For $X=\left\{d_{j t}, q_{j t}\right\}$ and using the optimality condition for inventories (2.37), we obtain:

$$
\begin{align*}
\frac{\partial p_{j t}}{\partial X} & =E\left[Q_{t, t+1} \frac{\partial p_{j t+1}}{\partial X}\right]>0 & & \text { if } I_{j t+1}^{*}>0  \tag{B.10}\\
\frac{\partial p_{j t+1}}{\partial X} & =0 & & \text { if } I_{j t+1}^{*}=0 \tag{B.11}
\end{align*}
$$

Hence

$$
\begin{array}{ll}
\frac{\partial p_{j t+1}}{\partial X} \propto-\frac{I_{j t+1}}{X}<0 & \text { if } I_{j t+1}^{*}>0 \\
\frac{\partial p_{j t+1}}{\partial X} \propto-\frac{I_{j t+1}}{X}=0 & \text { if } I_{j t+1}^{*}=0 \tag{B.13}
\end{array}
$$

- The optimal next period's stock of inventories $\left(I_{j t+1}^{*}\right)$ is weakly increasing in the firm's future demand $\left(d_{j t+1}\right)$ and input prices $\left(q_{j t+1}\right)$.

Proof. From the second part of this lemma and for $X=\left\{d_{j t+1}, q_{j t+1}\right\}$

$$
\begin{array}{ll}
\frac{\partial p_{j t}}{\partial X} \propto \frac{I_{j t+1}}{X}>0 & \text { if } I_{j t+1}^{*}>0 \\
\frac{\partial p_{j t}}{\partial X} \propto \frac{I_{j t+1}}{X}=0 & \text { if } I_{j t+1}^{*}=0 \tag{B.15}
\end{array}
$$

## B.1.3 Lemma 7

At the firm level, inventories impose an upper bound for the increase in the firm's price. In particular,

$$
\begin{equation*}
1 \geq \mathbb{E}\left[Q_{t, t+1} \frac{p_{t+1}}{p_{t}}\right] \tag{B.16}
\end{equation*}
$$

Proof. This comes directly from multiplying both sides of equation (2.37) by $\epsilon /(\epsilon-$ 1)

## B.1.4 Proposition 1

The set of real allocations $\left\{C_{t}, K_{t}, I_{t}, Y_{t}, X_{t}, H_{t}, y_{j t}, h_{j t}, k_{j t}\right\}$ and distribution of final goods firms $\left\{\lambda(I, q, d)_{t}\right\}$ that are consistent with the existence of a competitive equilibrium is independent of the path for money.

Proof. Notice that we can re-write the set of equations that describe the competitive equilibrium in a form that does not involve the nominal interest rate. To see this, we need to define the real rental rate of capital $r_{t}=R_{t} / P_{t}$, the real wage rate $w_{t}=W_{t} / P_{t}$, and relative prices $\tilde{p}_{j t}=p_{j t} / P_{t}$ and $\tilde{q}_{j t}=q_{j t} / P_{t}$. Also, the stochastic discount factor becomes: $\tilde{Q}_{0, t}=\beta u^{\prime}\left(C_{t}\right) / u^{\prime}\left(C_{0}\right)$. By defining and replacing these variables in the set of equations that describe the competitive equilibrium, we get a system of equations that are independent of the nominal interest rate.

## B.1.5 Proposition 2

Suppose that all agents in the economy except firms have perfect and complete information. Moreover, assume that intermediate goods producers cannot hold inventories, so their problem becomes:

$$
\begin{align*}
& V\left(q_{0}, d_{0}\right)_{0}^{i}=\max _{\left\{p_{t}, s_{t}, y_{t}\right\}} E \sum_{t=0}^{\infty} Q_{0, t}\left(p_{t} s_{t}-q_{t} y_{t}^{\frac{1}{\gamma}}\right)  \tag{B.17}\\
& \text { s.t. } \\
& s_{t}=d_{t} p_{t}^{-\epsilon}  \tag{B.18}\\
& y_{t}=s_{t} \tag{B.19}
\end{align*}
$$

If prices are flexible, and if there is perfect communication within firms such that pricing managers perfectly observe their input prices and demand, then nominal shocks do not have real effects on the economy regardless of the information friction on aggregate variables.

Proof. In Proposition 1, I showed that the set of equations that describe the competitive equilibrium under perfect information could be written in a form that does not involve the nominal variables. Since the only equations that change under information frictions are those involving intermediate goods firms, I only need to show that those equations can be written in a form that does not involve nominal variables. First, notice that the intermediate goods firms problem can be re-stated as:

$$
\begin{equation*}
V\left(q_{0}, d_{0}\right)_{0}=\max _{\left\{p_{t}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t}\left(p_{t}^{1-\epsilon} d_{t}-q_{t}\left(d_{t} p_{t}^{-\epsilon}\right)^{\frac{1}{\gamma}}\right) \tag{B.20}
\end{equation*}
$$

Hence, from the first order condition, we find that:

$$
\begin{equation*}
p_{t}^{*}=\left[\left(\frac{\epsilon}{\epsilon-1}\right) \frac{q_{t}}{\gamma} d_{t}^{\frac{1-\gamma}{\gamma}}\right]^{\frac{\gamma}{\gamma+\epsilon(1-\gamma)}} \tag{B.21}
\end{equation*}
$$

And, using the definition of $d_{t}$, we have:

$$
\begin{equation*}
p_{t}^{*}=P_{t} \cdot\left[\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{q_{t}}{P_{t}}\right) \frac{\left(\chi_{t} A_{t}^{\epsilon-1}\left(C_{t}+X_{t}\right)\right)^{\frac{1-\gamma}{\gamma}}}{\gamma}\right]^{\frac{\gamma}{\gamma+\epsilon(1-\gamma)}} \tag{B.22}
\end{equation*}
$$

Therefore, the firm's relative price, $\left(p_{t}^{*} / P_{t}\right)$, is independent of the nominal variables, and therefore so is the set of allocations that are consistent with the
existence of a competitive equilibrium.

## B.1.6 Proposition 3

Suppose that all agents in the economy except firms have perfect and complete information. If intermediate goods firms can accumulate inventories or capital and their input prices and demand do not reveal the aggregate state of the economy, the economy exhibits money non-neutrality.

Proof. If firms accumulate inventories their problem becomes:

$$
\begin{align*}
& V\left(q_{0}, d_{0}\right)_{0}=\max _{\left\{p_{t}, s_{t}, y_{t}, I_{t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t}\left(p_{t} s_{t}-q_{t} y_{t}^{\frac{1}{\gamma}}\right)  \tag{B.23}\\
& \text { s.t. } \\
& s_{t}=d_{t} p_{t}^{-\epsilon}  \tag{B.24}\\
& y_{t}=s_{t}+I_{t+1}-I_{t}  \tag{B.25}\\
& I_{t+1} \geq 0 \tag{B.26}
\end{align*}
$$

And the optimality conditions are given by:

$$
\begin{align*}
& p_{t}^{*}=\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{q_{t}}{\gamma}\right)\left[d_{t} p_{t}^{*-\epsilon}+I_{t+1}^{*}-I_{t}\right]^{\frac{1-\gamma}{\gamma}}  \tag{B.27}\\
& p_{t}^{*} \geq E_{t}\left[Q_{t, t+1} p_{t+1}^{*}\right] \tag{B.28}
\end{align*}
$$

Notice that the optimal current price depends not only on the firm's current demand and input prices but also on current inventory investment, which according
to (B.28) depends on firm's expectations. Similarly, if a intermediate goods firm can accumulate capital, its problem becomes:

$$
\begin{align*}
V\left(q_{0}, d_{0}, k_{0}\right)_{0} & =\max _{\left\{p_{t}, s_{t}, y_{t}, x_{t}, k_{t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t}\left(p_{t} s_{t}-w_{t} y_{t}^{\frac{1}{1(-\alpha) \gamma}} k_{t}^{\frac{-\alpha}{(1-\alpha)}}-p_{x t} x_{t}\right)  \tag{B.29}\\
& \text { s.t. } \\
s_{t} & =d_{t} p_{t}^{-\epsilon}  \tag{B.30}\\
y_{t} & =s_{t}  \tag{B.31}\\
k_{t+1} & =\left(1-\delta_{K}\right) k_{t}+x_{t} \tag{B.32}
\end{align*}
$$

Where $p_{x t}$ is the price of investment. The optimality conditions are given by:

$$
\begin{align*}
p_{t}^{*} & =\left[\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{w_{t}}{\gamma(1-\alpha)}\right) d_{t}^{\frac{1}{\gamma(1-\alpha)}-1} k_{t}^{\frac{-\alpha}{(1-\alpha)}}\right]^{\frac{\gamma(1-\alpha)}{(1-\epsilon) \gamma(1-\alpha)+\epsilon}}  \tag{B.33}\\
p_{x t} & =E_{t}\left\{Q_{t, t+1}\left[\left(\frac{\alpha}{1-\alpha}\right) w_{t+1}\left(d_{t+1} p_{t+1}^{*-\epsilon}\right)^{\frac{1}{\gamma(1-\alpha)}} k_{t+1}^{* \frac{-\alpha}{1-\alpha}}+p_{x t+1}\left(1-\delta_{K}\right)\right]\right\} \tag{B.34}
\end{align*}
$$

Notice that in both cases investment decisions depend on firms' expectations. Hence, if firms' expectations under informational frictions are not equal to those under perfect and complete information, firms' decision rules would not be equal.

Using the same notation as in Hamilton (1994) and under these assumption, we can summarize firms' expectations by the following signal extraction problem. Denoting $\xi$ as the vector of aggregate state variables of the economy and $y$ as the vector of contemporaneous variables that a firm perfectly observes (input prices and
demand), we get that:

$$
\begin{align*}
\xi_{t+1} & =\phi\left(\xi_{t}\right)+v_{t+1}  \tag{B.35}\\
& y_{t} \tag{B.36}
\end{align*}=a\left(x_{t}\right)+h\left(\xi_{t}\right)+w_{t} . ~ \$
$$

Where $\phi, a$, and $h$ are non-linear functions, $x_{t}$ is a vector of observed and exogenous variables, and $v_{t}$ and $w_{t}$ are vector of unobserved i.i.d. shocks. $v \sim$ $N(0, Q)$, and $w_{t} \sim N(0, R)$. Hence, this system can be linearized as follows:

$$
\begin{align*}
y_{t} & =a\left(x_{t}\right)+h_{t}+H_{t}\left(\xi_{t}-\hat{\xi}_{t \mid t-1}\right)+w_{t}  \tag{B.37}\\
\xi_{t+1} & =\phi_{t}+\Phi_{t}\left(\xi_{t}-\hat{\xi}_{t \mid t}\right)+v_{t+1} \tag{B.38}
\end{align*}
$$

Where $\hat{\xi}_{t \mid j}$ is the expected value of $\xi_{t}$ given information until period $j$. Hence, the contemporaneous inference about the aggregate state of the economy $\tilde{\xi}_{t \mid t}$ is given by:

$$
\begin{equation*}
\tilde{\xi}_{t \mid t}=\hat{\xi}_{t \mid t-1}+P_{t \mid t-1} H_{t}\left(H_{t}^{\prime} P_{t \mid t-1} H_{t}+R\right)^{-1} H_{t}^{\prime}\left[y_{t}-a\left(x_{t}\right)-h\left(\hat{\xi}_{t \mid t-1}\right)\right] \tag{B.39}
\end{equation*}
$$

Notice that under perfect and complete information $\tilde{\xi}_{t \mid t}=\hat{\xi}_{t \mid t}$ for all $t$. Therefore, if $\tilde{\xi}_{t \mid t} \neq \hat{\xi}_{t \mid t}$ firms cannot perfectly infer the aggregate state of the nature and, as a consequence, firms' decision rules will deviate from those under perfect and complete information. This occurs when $r+n<r+k+z$ where $r$ is the number of state variables, $n$ the number of perfectly observed variables by a firm, $k$ is the
number of non-zero elements in the main diagonal of $Q$, and $z$ is the number of non-zero elements in the main diagonal of $R$. In that case, the number of equations $(r+n)$ is greater than the number of unknown variables $(r+k+z)$. In other word, the number of variables observed by firms has to be lower than the total number of aggregate and idiosyncratic shocks that producers face. One should note that $z=0$ does not guarantee that $\tilde{\xi}_{t \mid t}=\hat{\xi}_{t \mid t}$, it only implies that firms can perfectly infer the value of $h\left(\xi_{t}\right)$

## B. 2 Computation of the Model With Perfect and Complete Informa-

 tionI approximate the model by assuming that the idiosyncratic shocks, $\varphi$ and $\chi$, and the inventories holdings, $I$, can only take values on the grids $\Gamma^{\varphi}=\left\{\varphi^{1} \ldots \varphi^{n b}\right\}$, $\Gamma^{\chi}=\left\{\chi^{1} \ldots \chi^{n \chi}\right\}$, and $\Gamma^{I}=\left\{0, I^{2} \ldots I^{n i}\right\}$. I find the transition probability matrices $\Pi^{\varphi}$ and $\Pi^{\chi}$ for $\varphi$ and $\chi$ using the Tauchen's method. Defining the variable $z \in \Gamma^{z}=\left\{z^{1} \ldots z^{n z \equiv n b \times n \chi}\right\}$ such that: $z=z^{r}$ if $\varphi=\varphi^{\operatorname{ceil(}(r / n \chi)}$ and $\chi=\chi^{\bmod (r, n \chi)}, \mathrm{I}$ specify the time varying distribution matrix $\boldsymbol{\Lambda}_{t}$ of size $(n i \times n z)$ such that the row $l$, column $r$ element represents the fraction of firms in state $\left(I^{l}, z^{r}\right) .{ }^{1}$ Following Costain and Nakov (2011), given the decision rule $I(I, z)=\arg \max _{\left\{I^{\prime} \in R^{+}\right\}} V\left(I^{\prime}, I, z\right)^{i}$, inventories holdings are kept on the grid $\Gamma^{I}$ by rounding $I(I, z)$ up or down stochastically without changing the mean. Specifically, for each $w \in\{1,2, \ldots n z\}$, define matrix

[^38]$\mathbf{R}^{\mathbf{w}}$ of size $(n i \times n i)$ as:
\[

\mathbf{R}^{\mathbf{w}}= $$
\begin{cases}\frac{I^{l^{t}(r, w)}-I_{t}^{* r, w}}{I^{t}(r, w)-I^{l_{t}(r, w)-1}} & \text { in column r, row } l_{t}(r, w)-1  \tag{B.40}\\ \frac{I^{* r}, w}{}-I^{\left.l^{(r} r, w\right)-1} \\ I^{t}(r, w)-I^{l_{t}(r, w)-1} & \text { in column r, row } l_{t}(r, w)\end{cases}
$$
\]

Where

$$
\begin{align*}
I_{t}^{* r, w} & =\arg \max _{\left\{I^{\prime} \in R^{+}\right\}} V\left(I^{\prime}, I=I^{r}, z=z^{w}\right)^{i}  \tag{B.41}\\
I^{l_{t}(r, w)} & =\min \left\{I \in \Gamma^{I}: I \geq I_{t}^{* r, w}\right\} \tag{B.42}
\end{align*}
$$

Hence, the evolution of $\boldsymbol{\Lambda}_{\boldsymbol{t}}$ can be computed as:

$$
\begin{align*}
\operatorname{vec}\left(\boldsymbol{\Lambda}_{\mathbf{t}+\mathbf{1}}\right) & =\left(\Pi^{z \prime} \otimes \mathbf{I}_{\mathbf{n i}}\right) \times \mathbf{R} \times \operatorname{vec}\left(\boldsymbol{\Lambda}_{\mathbf{t}}\right)  \tag{B.43}\\
\mathbf{R} & =\left[\begin{array}{cccc}
\mathbf{R}^{\mathbf{1}} & \mathbf{0}_{\mathbf{n i}} & \cdots & \mathbf{0}_{\mathbf{n i}} \\
\mathbf{0}_{\mathbf{n i}} & \mathbf{R}^{2} & \cdots & \mathbf{0}_{\mathbf{n i}} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0}_{\mathbf{n i}} & \mathbf{0}_{\mathbf{n i}} & \cdots & \mathbf{R}^{\mathbf{n z}}
\end{array}\right] \tag{B.44}
\end{align*}
$$

Where $\mathbf{I}_{\mathbf{n i}}$ is the identity matrix of size $n i .^{2}$ Similarly, the row $l$, column $r$ element of the pricing, inventory and profit functions $\left(p(I, z), I(I, z)\right.$, and $\left.\pi(I, z)^{i}\right)$
${ }^{2}$ define $\tilde{\boldsymbol{\Lambda}}_{t}$ such that:

$$
\begin{align*}
\operatorname{vec}\left(\tilde{\boldsymbol{\Lambda}}_{t}\right) & =\mathbf{R} \times \operatorname{vec}\left(\boldsymbol{\Lambda}_{t}\right)  \tag{B.45}\\
\tilde{\boldsymbol{\Lambda}}_{t}^{(w)} & =\mathbf{R}^{\mathbf{w}} \times \boldsymbol{\Lambda}_{t}^{(w)} \tag{B.46}
\end{align*}
$$

Where $\mathbf{X}_{t}^{w}$ is the column $w$ of matrix $\mathbf{X}_{t}$, and $\mathbf{0}_{\mathbf{x}}$ is the zeros matrix of size $n z$. Hence, the row $k$, column $w$ element of matrix $\tilde{\boldsymbol{\Lambda}}_{t}$ represents the fraction of firms in state $z=z^{w}$ that, regardless of their initial inventories holdings, have an stock of inventories equal to $I^{k}$ at the end of period t .
are given by:

$$
\begin{align*}
p\left(I^{l}, z^{r}\right)_{t} \times I\left(I^{l}, z^{r}\right)_{t} & =E\left[Q \times p\left(I\left(I^{l}, z^{r}\right), z\right)_{t+1} \times \Pi^{z}\left(:, z^{r}\right)^{\prime}\right] I\left(I^{l}, z^{r}\right)_{t}  \tag{B.51}\\
p\left(I^{l}, z^{r}\right)_{t} & =\left(\frac{\epsilon}{\epsilon-1}\right) \frac{q\left(z^{r}\right)_{t}}{\gamma}\left(d\left(z^{r}\right)_{t} p\left(I^{l}, z^{r}\right)_{t}^{1-\epsilon}+I\left(I^{l}, z^{r}\right)_{t}-I^{l}\right)  \tag{B.52}\\
\pi\left(I^{l}, z^{r}\right)_{t}^{i} & =d\left(z^{r}\right)_{t} p\left(I^{l}, z^{r}\right)_{t}^{1-\epsilon}-q\left(z^{r}\right)_{t}\left(d\left(z^{r}\right) p\left(I^{l}, z^{r}\right)_{t}^{1-\epsilon}+I\left(I^{l}, z^{r}\right)_{t}-I^{l}\right) \tag{B.53}
\end{align*}
$$

Where $d\left(z^{r}\right)_{t}$ and $q\left(z^{r}\right)_{t}$ are the values of $d$ and $q$ consistent with $z=z^{r}$. It is worth pointing out that the expectation in equation (B.51) is over the aggregate shocks of the economy. The expectation over the evolution of $z$ is written explicitly by multiplying by $\Pi^{z}$. Hence, the vector of aggregate variables is given by:

$$
\begin{equation*}
\vec{X}_{t} \equiv\left\{\operatorname{vec}\left(\boldsymbol{\Lambda}_{\mathbf{t}}\right), K_{t}, \Pi_{t}, P_{t}, D_{t}, \bar{q}_{t}, Q_{t}, Y_{t}, C_{t} H_{t}, r_{t}, w_{t}, \operatorname{vec}\left(I(I, z)_{t}\right)\right\} \tag{B.54}
\end{equation*}
$$

Vector $\vec{X}_{t}$ along with the vector of shocks $\vec{Z}_{t}=\left(\log \left(A_{t}\right), \mu_{t}\right)$ consist of $2(n i \times n z)+11$ endogenous variables that are determined by the following system Therefore, $\boldsymbol{\Lambda}_{t+1}$ can also be written as:

$$
\begin{align*}
\boldsymbol{\Lambda}_{t+1} & =\tilde{\boldsymbol{\Lambda}}_{t} \times \Pi^{z}  \tag{B.47}\\
\operatorname{vec}\left(\boldsymbol{\Lambda}_{t+1}\right) & =\operatorname{vec}\left(\tilde{\boldsymbol{\Lambda}}_{t} \times \Pi^{z}\right)  \tag{B.48}\\
\operatorname{vec}\left(\boldsymbol{\Lambda}_{t+1}\right) & =\left(\Pi^{z \prime} \otimes \mathbf{I}_{\mathbf{n i}}\right) \times \operatorname{vec}\left(\tilde{\boldsymbol{\Lambda}}_{t}\right)  \tag{B.49}\\
\operatorname{vec}\left(\boldsymbol{\Lambda}_{t+1}\right) & =\left(\Pi^{z \prime} \otimes \mathbf{I}_{\mathbf{n i}}\right) \times \mathbf{R} \times \operatorname{vec}\left(\mathbf{\Lambda}_{t}\right) \tag{B.50}
\end{align*}
$$

Where $\mathbf{I}_{\mathbf{n i}}$ is the identity matrix of size $n i$.
of equations

$$
\begin{align*}
C_{t}^{-\sigma} & =\beta \mathbb{E}\left[\left(r_{t+1}+\left(1-\delta_{K}\right)\right) C_{t+1}^{-\sigma}\right]  \tag{B.55}\\
\Psi H_{t}^{\eta} & =w_{t} C_{t}^{-\sigma}  \tag{B.56}\\
\frac{H_{t}}{K_{t}} & =\left(\frac{r_{t}}{w_{t}}\right)\left(\frac{1-\alpha}{\alpha}\right)  \tag{B.57}\\
C_{t}+K_{t} & =w_{t} H_{t}+r_{t} K_{t}+\Pi_{t} / P_{t}+\left(1-\delta_{K}\right) K_{t}  \tag{B.58}\\
P_{t}^{1-\epsilon} & =\mathbf{e}_{\mathbf{n i}}^{\prime}\left[\chi(z) p(I, z)_{t}^{1-\epsilon} \cdot * \mathbf{\Lambda}_{\mathbf{t}}\right] \mathbf{e}_{\mathbf{n z}}  \tag{B.59}\\
\left(\frac{Y_{t}}{A_{t}}\right)^{\frac{\epsilon-1}{\epsilon}} & =\mathbf{e}_{\mathbf{n i}}^{\prime}\left[\chi(z)^{\frac{1}{\epsilon}} \cdot * y(I, z)_{t}^{\frac{\epsilon-1}{\epsilon}} \cdot * \mathbf{\Lambda}_{\mathbf{t}}\right] \mathbf{e}_{\mathbf{n z}}  \tag{B.60}\\
\Pi_{t} & =\mathbf{e}_{\mathbf{n i}}^{\prime}\left[\pi(I, z)_{t}^{i} \cdot * \mathbf{\Lambda}_{\mathbf{t}}\right] \mathbf{e}_{\mathbf{n z}}  \tag{B.61}\\
p\left(I^{l}, z^{r}\right)_{t} \times I\left(I^{l}, z^{r}\right)_{t} & =E\left[Q \times p\left(I\left(I^{l}, z^{r}\right), z\right)_{t+1} \times \Pi^{z}\left(:, z^{r}\right)^{\prime}\right] I\left(I^{l}, z^{r}\right)_{t} \quad \forall l, z  \tag{B.62}\\
v e c\left(\mathbf{\Lambda}_{\mathbf{t}+\mathbf{1}}\right) & =\left(\Pi^{z /} \otimes \mathbf{I}_{\mathbf{n i}}\right) \times \mathbf{R} \times v e c\left(\mathbf{\Lambda}_{\mathbf{t}}\right)  \tag{B.63}\\
D_{t} & =A_{t}^{\epsilon-1}\left(C_{t}+K_{t+1}-\left(1-\delta_{K}\right) K_{t}\right) P_{t}^{\epsilon}  \tag{B.64}\\
Q_{t} & =\beta \frac{P_{t}}{P_{t+1}}\left(\frac{C_{t}}{C_{t+1}}\right)^{\sigma}  \tag{B.65}\\
\bar{q}_{t} & =P_{t}\left(\frac{r_{t}}{\alpha}\right)^{\alpha}\left(\frac{w_{t}}{(1-\alpha)}\right)^{1-\alpha}  \tag{B.66}\\
\log \left(P_{t}\right)+\log \left(Y_{t}\right) & =\mu_{t} \tag{B.67}
\end{align*}
$$

Notice that given a inventory decision rule, the price decision rule and current profit are given by equations (B.52) and (B.53). Following the notation of Costain and Nakov (2011), this set of equations form a first-order system of the form:

$$
\begin{equation*}
\mathbb{E}_{t} \mathcal{F}\left(\vec{X}_{t+1}, \vec{X}_{t}, \vec{Z}_{t+1}, \vec{Z}_{t}\right)=0 \tag{B.68}
\end{equation*}
$$

This system can linearized by computing numerically the jacobian matrices at the deterministic steady state, in order to express this system as a first-order linear expectational difference equation system:

$$
\begin{equation*}
\mathbb{E}_{t} \mathcal{A} \Delta \vec{X}_{t+1}+\mathcal{B} \Delta \vec{X}_{t}+\mathbb{E}_{t} \mathcal{C} \vec{Z}_{t+1}+\mathcal{D} \vec{Z}_{t}=0 \tag{B.69}
\end{equation*}
$$

Where $\mathcal{A} \equiv D_{\vec{X}_{t+1}} \mathcal{F}^{*}, \mathcal{B} \equiv D_{\vec{X}_{t}} \mathcal{F}^{*}, \mathcal{C} \equiv D_{\mathbf{E}_{t+1}} \mathcal{F}^{*}, \mathcal{D} \equiv D_{\mathbf{E}_{t}} \mathcal{F}^{*}$. Then this system of equations can be solve using the QZ decomposition describe in Klein(2000).

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[^0]:    ${ }^{1}$ Rogerson and Shimer (2011) assess in more detail how models with search frictions have shaped our understanding of aggregate labor market outcomes.
    ${ }^{2}$ For example, Rudanko (2009) shows in a model with long-term contracts that wage rigidity does not increase unemployment volatility as long as wages for new hires are flexible. Mortensen and Nagypal (2007) argue that the literature has overemphasized the need for sticky wages to increase unemployment volatility in the standard model of Mortensen and Pissarides (1994) and highlight three other features that could help explain the Shimer puzzle: (1) a low elasticity of the matching function with respect to vacancies, (2) a low value for the flow opportunity cost of employment, and (3) strong feedback from the job-finding rate to wages. Similarly, Pissarides (2009) critiques the assumption of sticky wages based on empirical evidence that wages for new

[^1]:    ${ }^{4}$ For example, Davis, Faberman, and Haltiwanger (2013) find a large heterogeneity in hires,

[^2]:    ${ }^{7}$ Similarly, there is a large literature that points out the existence of sectoral wage differences for the U.S. and differences in the cyclical behavior of employment across sectors. Some examples are: Abraham and Katz (1986), Davis and Haltiwanger (1991), Haisken-DeNew and Schmidt (1995), Horrace and Oaxaga (2001), Juhn, Muphy, and Pierce (1993), Krueger and Summer (1988), and Rielly and Zanchi (2003). One interpretation of these facts is that the sectors more subject to cyclical demand pay higher wages in order to compensate workers for higher unemployment risk (e.g. Barlevy, 2001, Okun, 1973, McLaughlin \& Bils, 2001).

[^3]:    ${ }^{8}$ There are alternative sources of fluctuations that increase unemployment volatility that are not studied in this chapter. For example, den Haan, Ramy, and Watson (2000) show that endogenous job destruction increases the response of unemployment to productivity shocks, and Carlsson and Westermark (2015) point out that sticky wages for job stayers may increase the strength of this channel. Similarly, recent literature has pointed out that sticky wages for job stayers may increase the unemployment volatility if firms face financial frictions (Schoefer, 2015) or if labor effort is variable (Bils, Chang \& Kim, 2014), even though sticky wages for continuing workers do not directly affect vacancy decisions in their models.
    ${ }^{9}$ Menzio (2005) and Kennan (2009) derive endogenous sticky wages based on firms that have private information about their labor productivity. In Menzio (2005), aggregate shocks cannot be very persistent. Otherwise, workers would demand higher wages. In Kennan's model, the standard deviation of idiosyncratic productivity cannot be large.
    ${ }^{10}$ Venkateswaran (2013) assumes firms that face information frictions regarding aggregate variables. In his model, after a positive productivity shock, firms do not offer higher wages because they partially attribute aggregate shocks to idiosyncratic conditions, which makes firms post more vacancies.

[^4]:    ${ }^{11}$ The Chodorow-Reich and Karabarbounis (2014) critique extends to all papers that assume a fixed and therefore acyclical FOCE, including Menzio (2005) and Kennan (2009).

[^5]:    ${ }^{12}$ For example, Christiano et al (2013) argue that a "successful model must have the property that wages are relatively insensitive to the aggregate state of the economy" (p, 3). Similary, Abraham and Haltiwanger (1995) find that the relation between aggregate wages and output does not always seem to be contemporaneous. They conclude that it is not possible to say whether real aggregate wages are procyclical or not and that in general the cyclicality is small.
    ${ }^{13}$ In contrast to other countries, there is no seasonal pattern in wage adjustments in the U.S. Le Bihan, Montornes and Heckel (2012), Lunnemann and Wintr (2009), and Sigurdsson and Sigurdardottir (2011) present evidence of nominal wage adjustment for France, Luxembourg and Iceland that exhibits seasonal patterns.
    ${ }^{14}$ They do not find that wages for job changers are more procyclical than wages for job stayers when they include match fixed effects.

[^6]:    ${ }^{15}$ For example, Hines, Hoynes, and Krueger (2001) argue that much of the cyclicality of wages estimated by Solon, et al (1994) comes from weighting the data by hours worked.
    ${ }^{16}$ Based on their empirical results, Gertler et al (2014) build a model in which the wage elasticity of job changers is driven by changes in match quality. Menzio and Shi (2011) also present a model in which job to job transitions are driven by random match quality.

[^7]:    ${ }^{17}$ While there is evidence in favor of a positive relationship between firm size and wages (e.g. Brown \& Medoff, 1989; Moscarini \& Postel-Vinay, 2008), there is also evidence indicating that firm age is important as well for understanding differences in cyclical behavior across firms (e.g. Haltiwanger, Jarmin \& Miranda 2013; Fort, Haltiwanger, Jarmin, \& Miranda 2013). In particular, Haltiwanger et al. (2015) point out the importance of classifying firms by wage instead of size. This chapter abstracts from firm entry and exit. Hence, even though in this chapter larger firms are more productive and pay higher wages, it is possible to think about the firm's size in the long run. However, I expect my results to be robust to firm entry and exit since firm size does not affect my mechanism.

[^8]:    ${ }^{18}$ Assuming a vacancy posting cost instead would disproportionally affect low-wage firms, as they have to post even more vacancies in expansions as a consequence of a larger decline in their job filling rate. However, in the context of this model, assuming a hiring cost function does not imply that vacancy decisions do not depend on labor market conditions. On the contrary, job-tojob transitions induce changes in the separation rate within firms that significantly influence the value of a new vacancy. Pissarides (2009) argues that hiring costs are a plausible assumption and discusses how assuming hiring rather than vacancy costs may change the results in the standard model. However, I show that my calibrated model with perfect information does not do a good job matching the unemployment and wage dynamics observed in the data. On the other hand, Gertler and Trigari (2009) and Gertler et al. (2014) assume a quadratic cost of adjusting employment in order to ensure a determinate equilibrium. I prefer a hiring cost over a cost of adjusting employment as a hiring cost does not bias my results in favor of high-wage firms. However, my results are not sensitive to assuming a cost of adjusting employment.

[^9]:    ${ }^{19}$ In section 1.5.2, I show that my results are reinforced when firms also face information frictions.

[^10]:    ${ }^{20}$ For expositional purposes, I derive in this section the value of employment and unemployment based on the model assumptions. For a detailed derivation of these value functions as in Merz (1995) and Andolfatto (1996), see appendix A.3.

[^11]:    ${ }^{21}$ Following the terminology of Angeletos and La'O (2012), the information friction is real since it affects both prices and real allocations.

[^12]:    ${ }^{22}$ As explained in section 1.2 .3 , firms with higher idiosyncratic productivity pay higher wages.

[^13]:    ${ }^{23}$ In contrast, the match surplus is independent of $w_{j}$.
    ${ }^{24}$ Notice that I do not index $w$ in equations (1.30) and (1.31) by firms $j$ in order to distinguish between an arbitrary wage $w$ and the equilibrium wage $w_{j}$. On the other hand, notice that the match surplus does not depend on $w$ :

    $$
    \vec{J}_{j}\left(w, \omega_{f}, \Omega\right)+\vec{W}_{j}(w, \omega, \Omega)-U(\omega, \Omega) \equiv J_{j}\left(\omega_{f}, \Omega\right)+W_{j}(\omega, \Omega)-U(\omega, \Omega)=S_{j}
    $$

[^14]:    ${ }^{25}$ Similarly, Matejka and McKay (2012) derive a model in which goods' prices are determined by consumers' beliefs when they face information frictions and firms have perfect information.

[^15]:    ${ }^{26}$ For a detailed application of the Reiter method, see Costain and Nakov (2011).

[^16]:    ${ }^{27}$ The linearity of the model makes the model tractable as I can compute expectations based on a linear filter. Otherwise, I would need to use non-linear filters (such as the particle filter), which would substantially increase the complexity of the problem for a large vector $\Omega$.

[^17]:    ${ }^{28}$ For example, Hall and Milgrom (2008) argue that a significant fraction of unemployment volatility is uncorrelated with productivity, and they estimate that $68 \%$ of unemployment volatility is driven by productivity shocks. In their paper, productivity is measured by output per hour. In this chapter, I measure productivity as the Solow residual computed by Basu, Fernald, and Kimball (2006). In my model, labor productivity is an endogenous variable, in contrast to TFP, which is the main driving force in the model.
    ${ }^{29}$ In the last section of this chapter, I discuss how my results change if I use a smoothing parameter equal to 1,600 . In general my results are not very sensitive to this parameter.
    ${ }^{30}$ New hires can be decomposed into two groups: new hires coming from unemployment and new hires coming from other jobs (job changers).

[^18]:    ${ }^{31}$ Examples in this literature are Barnichon (2010), Basu et al. (2006), Blanchard and Quah (1989), Christiano, Eichenbaum, and Vigfusson (2003, 2005), Gali (1999) and Shea (1998).
    ${ }^{32}$ Adding further lags does not improve explanatory power.
    ${ }^{33}$ This number of lags satisfies both the Akaike and Schwarz criteria.
    ${ }^{34}$ Table A. 1 in appendix A. 1 presents the standard deviation for these moments.

[^19]:    ${ }^{35}$ Peterman (2013) reviews the Frisch elasticities used in macro models (between 2 and 4) and estimates an elasticity for macro studies between 2.9 and 3.1 , which implies a value of 0.33 for $\xi$. In order to have a similar value to the standard literature, I set $\xi=0.5$, but a lower value would make the results of this chapter stronger, as $z_{j}$ would become less cyclical.
    ${ }^{36}$ Chodorow-Reich and Karabarbounis (2014) estimate that unemployment benefits are $21.5 \%$ of the marginal labor productivity. However, when adjusted by eligibility, claims and take up costs, $b$ declines to 0.041 . Given that I will have a distribution of labor productivity, I take a conservative approach, and I set $b$ to $0.041 \%$ of the model marginal labor productivity, which is equal to 1 .

[^20]:    ${ }^{37}$ Given that the distribution of employment is not uniform across firms in equilibrium, the median productivity across firms is not equal to the median productivity across workers.

[^21]:    ${ }^{38}$ They report that the standard deviation of $\log$ productivity across firms was 0.657 in 1997, while the median $\log$ productivity was 3.47 . Hence, as a fraction of the median, the standard deviation is approximately 0.2 .
    ${ }^{39}$ There is an extensive debate surrounding the value of the flow opportunity cost of employment $(z)$ in the literature, with parameterizations ranging from 0.4 (e.g. Shimer, 2005) to 0.955 (e.g. Hagedorn and Manovskii, 2008). A value around 0.72 is less controversial than these extremes.

[^22]:    ${ }^{40}$ Hagedorn and Manovskii (2011) also discuss an additional shortcoming: the correlation between vacancies and productivity is maximized when vacancies are led one or two quarters. My model is consistent with this fact. However, this is because I assume a strictly convex hiring cost function. Gertler and Trigari (2009) are also able to generate this pattern by assuming a quadratic adjustment cost in employment.

[^23]:    Notes: Statistics for the simulated economy under information frictions: $u$ : Unemployment level. v: Vacancies $v / u$ : Vancancyunemployment ratio. $y$ : Output. c: Consumption. Inv: Investment. $w^{a}$ : Average wage in the economy. w $w^{s}$ : Average wage for job stayers. $w^{u}$ : Average wage for new workers (workers who were unemployed in the previous period). $w^{c}$ : Average wage for job changers. $w^{n}$ : Average wage for new hires (new workers + job changers). $a$ : Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000.

[^24]:    ${ }^{41}$ Gertler et al. (2014) find the same result for a different sample period using the Survey of Income and Program Participation (SIPP) dataset. Below, I discuss the consequences of assuming sticky wages for job stayers.
    ${ }^{42}$ These numbers imply that an increase of one percentage point in unemployment (for example, from 5 to $6 \%$ ) makes wages for job changers and job stayers decrease by $3 \%$ and $1 \%$, respectively.

[^25]:    ${ }^{43}$ As before, I control for composition effects following the methodology of Horrace and Oaxaga (2001). The advantage of this methodology, in contrast to running a regression with firm dummies, is that the results are independent of the excluded variable. Haefke et al. (2013) also discuss the advantages of this methodology when constructing the average wage for new workers (production and non-supervisory employees).

[^26]:    ${ }^{44}$ This implies that, in each period, a fraction of continuing workers and all new hires will be negotiating their wages for the following 4 quarters.

[^27]:    ${ }^{1}$ Following the terminology of Angeletos and La'O (2012), if firms make certain production decisions based on noisy information (or limited attention), the information friction is considered real. In standard information friction models, firms set their nominal prices based on noisy or limited information, but real variables adjust freely to the true state of the nature, as if they were made under perfect information (Angeletos \& La'0, 2012, p 2). In the model of this chapter, production and inventory decisions are taken based on noisy information about aggregate variables, which makes the information friction real.

[^28]:    ${ }^{2}$ For instance, one can think of computers as being the capital good. Every sector needs computers in order to produce, but each sector needs some specific programs that must be installed or updated before they can be used in the production process.

[^29]:    ${ }^{3}$ In other words, the nominal wage and rental rate of capital in a sector with no idiosyncratic $\operatorname{shocks}\left(\phi_{w, j t}=\phi_{r, j t}=1\right)$

[^30]:    ${ }^{4}$ Notice that the firm's problem can be written as follows:

    $$
    \begin{aligned}
    & V\left(I_{0 j}, d_{0 j}, q_{0 j}\right)_{0}=\max _{\left\{y_{j t}, I_{j t+1}\right\}} E_{0} \sum_{t=0}^{\infty} Q_{0, t}\left(d_{j t}\left(y_{j t}+I_{j t}-I_{j t+1}\right)^{\frac{\epsilon-1}{\epsilon}}-q_{j t} y_{j t}^{\frac{1}{y}}\right) \\
    & \quad \text { s.t. } \\
    & I_{j t+1} \geq 0
    \end{aligned}
    $$

[^31]:    ${ }^{5}$ In the deterministic steady state $\sigma_{A}=\sigma_{\mu}=0$

[^32]:    ${ }^{6}$ Appendix B. 2 discusses in detail how I solved the model.
    ${ }^{7}$ For the purposes of this work, this will be particularly useful when computing the firm's expectations. Given the linearity of the solution in aggregate variables, firms can use a linear filter, such as the Kalman Filter, in order to compute their expectations.

[^33]:    ${ }^{8}$ This result is formalized in their Proposition 1.

[^34]:    ${ }^{9}$ Input prices and demand do not reveal the aggregate state of the economy as long as the number of variables observed by firms is lower than the number of aggregate and idiosyncratic shocks in the economy (see proof of proposition 3 in appendix B.1.6). In Angeletos and La'O (2012), firms would observe five variables: their productivity, their demand, the wage rate of their sector, tax rates, and the aggregate price index (price of investment); and firms will face the same number of shocks: productivity shocks, consumption preference shocks, labor preferences shocks, tax shocks, and nominal shocks.

[^35]:    ${ }^{10}$ Using language from consumer theory, firms have a precautionary motive for holding inventories.

[^36]:    ${ }^{11}$ I solve the model assuming that this schock is log-normal, but assuming a uniform distribution is helpful for discussing the intuition behind the results.

[^37]:    ${ }^{1}$ For occupation and industry, I use variables OCC1950 and IND1950 provided by IPUMS-CPS. Experience is defined as age minus years of education minus 6 .
    ${ }^{2}$ Following the literature, individual $i$ 's weight is the product of the individual weight reported by the BLS and hours worked.

[^38]:    ${ }^{1}$ Being more precise, $\Gamma^{z}=\left\{\left(\varphi^{1}, \chi^{1}\right),\left(\varphi^{1}, \chi^{2}\right) \ldots\left(\varphi^{2}, \chi^{1}\right),\left(\varphi^{2}, \chi^{2}\right) \ldots\left(\varphi^{n b}, \chi, 1\right) \ldots\left(\varphi^{n b}, \chi^{n \chi}\right\}\right.$, and its transition probability matrix is given by $\Pi^{z}=\Pi^{\varphi} \otimes \Pi^{\chi}$

