ABSTRACT

Title of dissertation: EX-ANTE ASYMMETRIC INFORMATION IN CREDIT MARKETS AND MACROECONOMIC FLUCTUATIONS

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This dissertation analyzes macroeconomic effects of ex-ante information acquisition problems between lenders and borrowers in credit markets. It examines the ways in which the costs associated with the screening of privately informed heterogeneous loan applicants affect contractual arrangements, efficient allocation of credit and macroeconomic fluctuations. Screening costs can take on two different forms: i) the direct costs incurred by borrowers through collateral requirements and credit limits, ii) the resource costs incurred by lenders for spending time, effort and money on screening loan applicants, which are passed on to borrowers through higher loan rates. This dissertation is organized into two parts, analyzing macroeconomic implications of these two types of screening costs.

In the first chapter, entitled "Collateralization, Credit Allocation and Investment Dynamics", we focus on the case where screening can be achieved by collateral requirements and credit limits. We analyze how changes in collateral availability affect aggregate investment and output dynamics through the *misallocation* of bank

credit across heterogeneous investors. We develop a theoretical model of the credit market where banks cannot fully assess the credit risk of investors, and collateral capacity of investors becomes crucial in the design of financial contracts that facilitate efficient allocation of credit.

In the proposed framework, there are observable and unobservable components to a borrower/investor's credit risk. Banks are able to group borrowers into risk classes based on their observable characteristics such as their collateral capacity. However, within a quality group there are still higher and lower risk borrowers that cannot be observationally distinguished. For any group of borrowers with common observable quality, banks may design either pooling or separating contracts. Pooling contracts offer efficient loan amounts but entail cross subsidization of high risk borrowers, while separating contracts offer efficient loan rates but entail credit rationing of low risk borrowers. We show that a financial shock that reduces the collateral capacity of borrowers may switch the financial contracts designed for low quality borrowers from pooling to separating, which increases credit rationing within low quality group and reallocates credit in favor of high quality borrowers. Thus, our framework generates a flight to quality in bank lending, which is documented in the empirical literature to precede the recessions and predict future reductions in real GDP. This flight to quality occurs in our framework because banks tighten credit standards, or the terms of financial contracts, more for relatively risky or low quality borrowers when financial conditions worsen. The differential access to credit across these investors decreases aggregate investment efficiency and real economic activity.

In the second chapter, entitled "Costly Screening in Credit Markets, Net Worth Effects and Business Cycles", we focus on the case where screening can be achieved by incurring resource costs. We build a dynamic general equilibrium model featuring costly screening of loan applicants to examine how aggregate shocks are amplified and propagated through net worth effects compared to a standard model of ex-post monitoring costs, which has become the work-horse framework to analyze macro-financial linkages.

In the model, loan applicants tend to hide the prospects of their projects absent screening, and lenders have access to a costly screening technology to evaluate the creditworthiness of loan applicants, which is a way to economize on the agency costs induced by pooling all borrowers, but is an agency cost in and of itself, making loan terms, and in turn investment, dependent on borrower net worth. Borrower screening is inversely proportional to borrower net worth: as the borrower's stake in investment rises the need for screening falls. When borrower net worth is low, borrowers are subject to higher screening; and since screening costs are passed on to borrowers through higher loan rates, lower net worth increases the cost and decreases the availability of external funding. When there is an adverse aggregate productivity shock, borrower income or net worth falls and external finance becomes more expensive. As a result, both internal and external sources of finance shrink, decreasing aggregate investment and output, and further decreasing borrower net worth. In this way aggregate shocks are propagated through their effects on borrower net worth. Thus, we show that the costly screening framework can be an alternative to widely assumed monitoring costs in generating net worth effects that enhance the propagation of aggregate shocks. One advantage of the screening framework is that it yields wealth effects that induce persistent dynamics especially in bad times when borrower screening is more likely, which may create longer and deeper busts than booms. In addition to that, by yielding efficient risk pricing and quantity rationing endogenously as in actual bank lending practices, the screening framework constitutes an empirically plausible alternative to monitoring costs to motivate the agency costs in unsecured lending.

EX-ANTE ASYMMETRIC INFORMATION IN CREDIT MARKETS AND MACROECONOMIC FLUCTUATIONS

by

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Dedication

To Elfero... And to my Dad...

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List of Abbreviations

C&I	Commercial and Industrial
FRB	Federal Reserve Board
FRED	Federal Reserve Economic Data
IC	Incentive constraint
LHS	Left hand side
RC	Rate of collateralization/recovery
RHS	Right hand side

BGG	Bernanke, Gertler and Gilchrist (1999)
CF	Carlstrom and Fuerst (1997)
CRS	Constant returns to scale
NK	New Keynesian
RBC	Real Business Cycle

Chapter 1

Collateralization, Credit Allocation and Investment Dynamics

1.1 Introduction

Macroeconomic studies of credit market imperfections, following the seminal works of Bernanke and Gertler (1989), and Kiyotaki and Moore (1997), have primarily emphasized the role of borrower net worth or collateral capacity in shaping aggregate investment and output dynamics through the *volume* of credit obtained by a *representative* investor in the economy. The focus of this paper is to examine how changes in collateral capacity affect aggregate investment and output dynamics through the *allocation* of credit across *heterogeneous* investors.

It is now well known that credit market imperfections may enhance the propagation and amplification of exogenous productivity shocks.[1] Matsuyama (2007) argues that the allocation of credit might matter because some aggregate productivity changes may be caused by an endogenous shift in the allocation of credit across investors with different productivity levels. This paper builds a theoretical framework in which lower collateralization may lead to a recession through inefficient reallocation of credit towards high quality investors, due to tighter screening and credit rationing of low quality investors by banks. Thus, a financial shock that reduces the collateral capacity of investors, such as an increase in uncertainty over

[1] See Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) among many others.

investment returns, translates into an adverse productivity shock through a flight to quality in bank lending across investors.

We construct a model of financial contracting under adverse selection and limited collateral. Groups of investors with different observable quality, i.e., with different average productivity and average risk rating, finance investment projects through bank loans, pledging part of their projects. Borrower risk is private information within a quality group, which leads banks to design financial contracts that may either pool borrowers with common observable quality independent of unobservable risk, or separate relatively high and low risk borrowers. For a group of borrowers with common observable quality, a pooling contract offers the efficient (full information) loan amount, but the pooling loan rate entails cross subsidization of high risk borrowers by the low risk ones. On the contrary, separating contracts offer type specific efficient loan rates, but screening entails credit rationing of low risk borrowers, which disincentivizes high risk borrowers from pretending to be low risk. The main finding is that a fall in the collateral capacity of investors, among other model primitives, may switch the financial contracts designed for low quality investors from pooling to separating, which leads to an inefficient reallocation of credit in favor of high quality investors, because of tighter screening and credit rationing of low quality investors by banks. This flight to quality in bank lending due to tightening credit standards reduces aggregate investment efficiency and thereby real economic activity.

Both cross subsidization in a pooling regime and credit rationing in a separating regime hurt low risk borrowers and benefit high risk borrowers within a quality group. As a result, the equilibrium lending regime for borrowers with common observable quality is determined by whether pooling or separating contracts are more profitable for low risk borrowers. Consider the situation in which there is only one quality group of borrowers, and a low risk borrower in this group is indifferent between a pooling and a separating contract, which constitutes a regime switching point. When investment technology is decreasing returns to scale, i.e., project returns are concave in funds, a pooling regime yields higher physical investment than a separating regime at the regime switching point. Therefore, a pooling regime generates higher future capital, output and lending in the economy. The reason is that, given the amount of available funds, high risk borrowers in a separating regime over-invest while low risk borrowers under-invest in capital production compared to a pooling regime due to credit rationing. Thus, for a given level of resources devoted to the investment technology, a separating regime yields lower aggregate physical investment than a pooling regime, slowing economic growth.

Empirical work by Asea and Blomberg (1998) and Lown and Morgan (2006) document that bank lending standards for allocating business loans systematically change over the cycle between tightness and laxity, and significantly affect economic activity. In these studies, tighter standards are associated with greater use of non-price loan terms, such as collateral requirements and credit limits, and greater variance in the loan rates charged by the banks. The latter work, in particular, finds that tighter standards are usually followed by slower loan growth and often precede recessions. Switches between pooling and separating regimes in the model can be interpreted as switches between lax and tight lending standards. A pooling regime is a lax lending regime as it eliminates credit rationing and stimulates aggregate investment, which generates higher output and lending. A separating regime is a tight lending regime, which promotes efficient risk pricing at the expense of credit rationing, dampening aggregate investment, future output and lending.

Competitive banks in equilibrium will offer a menu of separating contracts to borrowers with common observable quality whenever cross subsidization under pooling becomes relatively more costly for low risk borrowers than credit rationing in terms of lost profits. The paper shows that the relative cost of cross subsidization increases in the relative riskiness and measure of high risk borrowers, and decreases in the degree of collateralization and the curvature of investment technology. A fall in the collateral capacity of investors increases expected bank losses when borrowers fail to repay, which increases loan rates as competitive banks need to compensate for these losses. For a given fall in collateralization, banks need to increase loan rates more for higher risk borrowers, as they fail and lose collateral more often. Thus, the loan rate under pooling increases more than the separating loan rate for a low risk borrower, increasing the degree of cross subsidization under a pooling regime. On the other hand, it is also true that the separating loan rate for high risk borrowers increases more than the separating loan rate for low risk borrowers, further incentivizing high risk borrowers more to pretend to be low risk. In this case, banks need to ration credit more for low risk borrowers in a separating regime in order to disincentivize high risk borrowers. Unless investment technology is “too concave”, lower collateralization makes a separating contract more attractive for a low risk borrower despite credit rationing, because a lower loan size does not hurt profits as much as a higher loan price when the marginal return on investment is not too high. Thus, a low risk borrower optimally trades off a lower loan amount for a lower loan rate because the cost of obtaining the efficient loan amount due to cross subsidization becomes too high. Thereby, lower collateralization increases the likelihood of a separating regime, and can either tighten an existing separating regime with a higher degree of credit rationing or switch a pooling regime into separating.

The paper shows that when there is a financial shock that reduces the collateral capacity of investors, banks switch to separating low quality investors first, which leads to a reallocation of credit towards high quality investors, in line with the stylized fact referred to as a flight to quality in bank lending. Accordingly, credit flows away from borrowers subject to higher agency costs, and flows in favor of higher quality borrowers in bad times or when borrower balance sheets deteriorate.[2] Asea and Blomberg (1998) document that the proportion of high quality bank loans (relatively safe loans) increases in periods with a high variance across loan rates charged by banks, suggesting that a flight to quality occurs when there is tighter separating across borrowers by banks. Moreover, both Lang and Nakamura (1995) and Asea and Blomberg (1998) document that increases in the proportion of safe bank loans are associated with future reductions in real economic activity.[3] The

[2]See Bernanke, Gertler and Gilchrist (1996) for a review of evidence on a flight to quality.

[3]Section 1.6.2 documents similar findings using data from the Survey of Terms of Business Lending, showing that decreases in the proportion of risky bank loans are highly predictive of future reductions in real GDP.

model in this paper features multiple groups of borrowers that differ in expected productivity and average risk levels (quality), which are publicly observable. We assume that higher quality borrowers can pledge project returns more easily as they are subject to lower agency costs. Within each quality group there are two types of borrowers, high and low risk, that cannot be observationally distinguished. When there is a financial shock that reduces the collateral capacity of all investors, banks switch to separating low quality borrowers first, as they are subject to higher agency costs. Therefore, whenever credit flows from (unobservably) low risk borrowers to high risk borrowers within the same quality group, due to a regime switch from pooling to separating for this group, the increased inefficiency affects the relative creditworthiness among different quality groups, causing credit to flow to observably higher quality groups with lower average risk levels. Thus, when financing conditions deteriorate, credit standards tighten more for low quality borrowers, which leads to a flight to quality. The inefficient reallocation of credit within and between quality groups decreases aggregate investment efficiency and in turn real economic activity.

Collateralization is defined here as the value of available collateral relative to the cost of the loan. It depends positively on the pledgeability of a project and negatively on the loan size, which in turn increases in the expected return on investment projects. Consider a financial shock that increases the uncertainty in investment returns, keeping the expected return on investment constant. This reduces the pledgeable investment returns, which means that the expected recovery rate of the loan in case of default falls. As a fall in the pledgeability of projects decreases collateralization, the uncertainty shock may lead to a flight to quality due to tighter screening and credit rationing of low quality investors by banks. Thus, although expected productivity and project technology of investors are unchanged, inefficient reallocation of credit due to lower collateralization decreases the growth path of the economy in the same manner as an adverse productivity shock. Conversely, consider an increase in the expected productivity of investors (an aggregate productivity shock), keeping the pledgeability of their projects unchanged. Given the allocation of credit across investors, higher expected productivity increases aggregate investment due to a rise in the project returns. However, the allocation of credit may change in an opposing way due to lower collateralization, increasing flight to quality and decreasing investment efficiency. In this case, the composition effect may dominate, if reallocation of credit decreases investment efficiency in a way that offsets the increase in investment productivity. Therefore, an increase in investment productivity may paradoxically create a bust through inefficient reallocation of credit, if collateralization is not kept proportionally high.

In terms of policy implications, this framework provides justification for loan guarantees or other government sponsored programs targeted towards disproportionately rationed small and young businesses and innovative firms, which disproportionately account for job creation and growth. When there is a negative disturbance in the economy that reduces collateral availability, such as a decrease in the value of available collateral or an increase in uncertainty about pledgeable investment returns, small business lending might be subsidized more to avoid excessive credit rationing of these firms that would weigh on job creation and growth.

Related Work This paper is mainly related to the literature on financial imperfections and credit regime switches. Modeling of a two sector overlapping generations framework with a financial friction in investment production is similar to Matsuyama (2007) and Martin (2008, 2009). Adverse selection in credit markets is modeled as in Stiglitz and Weiss (1981).

Matsuyama (2007) investigates credit composition effects in a model with homogeneous agents undertaking investment projects that differ in productivity, investment requirement and pledgeability. Borrowing constraints arise due to lack of contract enforceability. In Matsuyama (2007), credit flows only to the project with the highest return to lenders. When borrower net worth is low, the funded project may be the most pledgeable one rather than the most productive one, reducing investment specific productivity. This paper differs from Matsuyama (2007)in that borrowers differ in riskiness, which is private information, creating an adverse selection problem. This introduces contractual regime switches and a flight to quality due to lower collateralization. Moreover, projects are decreasing returns to scale, which endogenizes investment levels and enables credit to be allocated across many types of investors at the same time. This paper thus brings heterogeneity in risk and quality to the analysis of credit composition effects, complementing Matsuyama (2007).

Martin (2008) shows how changes in borrower wealth may lead to switches between pooling and separating regimes under adverse selection, generating endogenous output fluctuations. A decrease in collateralizable borrower wealth in bad times decreases screening possibilities and thereby increases pooling incentives, stimulating output when the regime switches from separating to pooling. This paper differs from Martin (2008) in the modeling of adverse selection and the collateral capacity of investors.[4] Thus, unlike in Martin (2008), lower collateralization in this paper increases the likelihood of a separating regime rather than a pooling regime, as cross subsidization under pooling becomes too large compared to credit rationing under screening. In this way, lower collateralization leads to a flight to quality, which lowers investment efficiency and economic activity.

This paper also has similarities to Azariadis and Smith (1998) and Reichlin and Siconolfi (2004), who study credit regime switches absent borrower net worth or collateral availability. Azariadis and Smith (1998) study a model in which optimism about interest rates increases loanable funds, which in turn eliminates the rationing regime, stimulating output and increasing interest rates in a self-fulfilling way. What eliminates the rationing regime and stimulates investment here is an increase in the collateral capacity of investors, which induces a more efficient use of available loanable funds, rather than an increase in loanable funds themselves. Similarly, Reichlin and Siconolfi (2004) study a model in which a rise in loanable funds in booms leads to a separating regime, which increases the proportion of riskier and wasteful investment undertaken, inducing busts. However, in their model the reversion in output is not due to inefficient allocation of credit itself, but due to the

[4] Martin (2008) models adverse selection as in De Meza and Webb (1987), in which expected borrower productivity is private information. Therefore, more productive investors are not only rationed compared to the efficient allocation, but also rationed compared to less productive investors in a separating regime, increasing the relative cost of screening.

assumption of high setup costs associated with riskier projects.

In the remainder, the chapter is organized as follows: Section 1.2 introduces the agents and the basic setup. Section 1.3 characterizes the loan contracts in a partial equilibrium setting, which is embedded in a general equilibrium framework in section 1.4. Section 1.5 analyzes how changes in model primitives, including collateralization, affect the likelihood of a separating regime versus a pooling regime, and presents a numerical illustration. Section 1.6 analyzes lending regime switches, which induce a reallocation of credit within and between groups of investors with common observable quality, altering aggregate investment dynamics. This section also reviews existing evidence and provides new evidence on flight to quality. Section 1.7 concludes.

1.2 Basic Setup

The economy consists of overlapping generations of two sets of measure one, two-period lived agents: households and entrepreneurs. Households are endowed with labor skills and their only role in this model is to provide funds for entrepreneurs, who undertake investment projects that produce capital. A continuum of competitive firms produce final goods using capital produced by entrepreneurs and labor supplied by households. Final goods are consumed or invested in capital production. Agents save or invest final goods in the first period and consume only in the second period. A continuum of competitive banks intermediate funds between households (savers) and entrepreneurs (investors).

Firms produce final goods using a constant returns to scale technology $Y_t = AF(K_t, L_t)$, where K_t is capital, L_t is labor, and A is total factor productivity. Let $y_t = Y_t/L_t = AF(K_t/L_t, 1) = Ag(k_t)$, where $k_t = K_t/L_t$ and $g(k_t)$ satisfies $g'(k_t) > 0 > g''(k_t)$ and Inada conditions. Factor markets are competitive, thus factor prices are $q_t = Ag'(k_t)$ for capital and $w_t = A[g(k_t) - g'(k_t)k_t]$ for labor. Capital is assumed to depreciate fully upon use for expositional purposes.

Households are endowed with a unit of time when young, which they use to supply labor inelastically. They save all their income w_t in banks and earn a gross return $(1+r_{t+1})$ when old, which yields consumption $(1+r_{t+1})w_t$ at $t+1$. Thus, aggregate supply of funds will be $S_t(A, k_t) = w_t(A, k_t)$.

Entrepreneurs undertake an investment project when young, transforming final goods into capital. A project may succeed or fail depending on entrepreneurial ability. Investing i_t units of final goods in a project yields $\bar{\gamma} f(i_t)$ units of capital in period $t+1$ on average, where $f(.)$ is an increasing and concave production technology, satisfying $f(0) = 0$ and Inada conditions. $\bar{\gamma}$ is expected investment productivity, which is observable and common among a group of investors.[5] Investors within a group may be high or low risk in terms of success: measure λ of investors are low risk (G: good risk) and measure $1 - \lambda$ of investors are high risk (B: bad risk). A high risk investor has a lower success probability $p^B < p^G$, but a higher productivity $\gamma^B > \gamma^G$

[5] Note that there may be several groups of investors with different expected productivities $\bar{\gamma}$. The setup is constructed for a representative group of investors with a given expected productivity, which is observable by banks.

conditional on success. A project yields γ^0 for all investors conditional on failure regardless of investor type. Thus, expected productivity $\bar{\gamma} = p^j\gamma^j + (1-p^j)\gamma^0$ is the same for $j \in \{G, B\}$, where $0 \leq \gamma^0 < \bar{\gamma} < \gamma^G < \gamma^B$.[6]

A $j \in \{G, B\}$ type entrepreneur investing i_t^j in a project obtains a stochastic rental income in period $t+1$ given by

$$q_{t+1}\gamma^j f(i_t^j) \quad \text{with probability } p^j$$

$$q_{t+1}\gamma^0 f(i_t^j) \quad \text{with probability } (1-p^j).$$

Since entrepreneurs have no internal funds, all investment is financed through bank loans. Given project outcomes, entrepreneurs honor bank loans and consume the rest of their rental income in period $t+1$.

Banks are risk neutral and competitive. They collect deposits from households at time t at a gross return $1 + r_{t+1}$. They enter into a loan contract with a young entrepreneur who claims to be a $j \in \{G, B\}$ type, investing in a project with observable technology $f(.)$ and failure productivity γ^0. Project success is observable and verifiable by banks ex-post. However, ex-ante borrower risk may be private information. A contract is a pair $(i_t^j, R_{t+1}^j(\gamma))$ specifying loan size i_t^j and state contingent repayment $R_{t+1}^j(\gamma)$ per unit of loan, where $\gamma \in \{\gamma^0, \gamma^j\}$. Optimal contracts are obtained by maximizing expected borrower profits subject to bank participation, based on observable and unobservable characteristics of loan applicants. Private information may lead banks to design pooling or separating contracts in order to mitigate adverse selection.

[6] The equality $\bar{\gamma} = p^j\gamma^j + (1-p^j)\gamma^0$ for $j \in \{G, B\}$ implies that $\frac{p^G}{p^B} = \frac{\gamma^B - \gamma^0}{\gamma^G - \gamma^0}$.

Timeline of events in a given period of the model is as follows:

1. Investment outcomes of old entrepreneurs are realized.
2. Firms hire labor from young households and rent capital from old entrepreneurs, produce output and make factor payments.
3. Young households deposit savings in banks.
4. Old entrepreneurs honor loans given investment outcomes and consume the rest of their rental income.
5. Old households earn returns on savings and consume those.
6. Young entrepreneurs borrow funds to undertake investment projects given their productivity and riskiness.

1.3 Loan Contracts

In this section, time subscripts are omitted for simplicity, and loan contracts designed for a given group of investors are characterized in a partial equilibrium setting, to be embedded in a general equilibrium framework in the next section.

Assumption 1.1. *$f(\iota) = \iota^\theta$ for $\theta \in (0,1)$ and $\frac{\gamma^0}{\bar{\gamma}} < \theta$.*

The functional form of $f(.)$ is chosen for simplicity and tractability. θ measures the elasticity of capital production with respect to the funds invested. Thus, a 1% increase in funds increases capital production by θ%. The precise role of Assumption 1.1 will be clear below. Given θ, Assumption 1.1 ensures γ^0 is not too high, so that a contract can be characterized as risky debt defined by a pair (i, R), where i is the loan amount and R is the loan rate. If projects fail, entrepreneurs default and banks confiscate the project outcome $\gamma^0 f(i)$, which serves as collateral.

The only source of uncertainty in the model is idiosyncratic investment productivity, which is diversifiable. Thus, banks can guarantee a risk free return r on household deposits. Given risk free rate r and expected rental rate q^e, contract (i^j, R^j) signed with a j-type borrower yields expected profits

$$\Pi^{e,j} = p^j[q^e \gamma^j f(i^j) - R^j i^j] \tag{1.1}$$

$$B^e = p^j R^j i^j + (1 - p^j) q^e \gamma^0 f(i^j) - (1 + r) i^j \tag{1.2}$$

for entrepreneurs and banks, respectively. Superscript e denotes expectation as of today for realizations tomorrow.

First consider the full information benchmark in which a borrower's risk is observable:

Proposition 1.1. *Under full information, contracts (i^j, R^j) for $j \in \{G, B\}$ maximize (1.1) subject to nonnegative (1.2). They satisfy*

$$f'(i^j) = \frac{1 + r}{q^e \bar{\gamma}} \tag{1.3}$$

$$p^j R^j i^j + (1 - p^j) q^e \gamma^0 f(i^j) = (1 + r) i^j \tag{1.4}$$

Proof. See Appendix A.1. □

Equation (1.3) implies that the full information investment level, denoted as i^*, is the same for both types. Equation (1.4) is the break even (zero profit) condition for the bank, which is induced by competition. Substituting (1.4) in (1.1) for R^j we obtain

$$\Pi^{e,j} = q^e \bar{\gamma} f(i^*) - (1+r)i^*$$

as expected profits under full information, which is also the same for both types.

Let $c^{e,j} = \frac{q^e \gamma^0}{(1+r)} \frac{f(i^j)}{i^j} \leq 1$ denote the expected recovery rate of the loan, which is the expected rate at which repayment in the event of failure (collateral) covers the cost of the loan for a bank. We will be refer to $c^{e,j}$ as collateralization and the recovery rate interchangeably. We can rewrite (1.4) as

$$R^j = [1 - (1-p^j)c^{e,j}]\frac{1+r}{p^j} \tag{1.5}$$

which defines a trade off between the loan rate and the recovery rate given by $\frac{\partial R^j}{\partial c^{e,j}} = -\frac{1-p^j}{p^j}(1+r)$. A marginal decrease in the recovery rate $c^{e,j}$ should be compensated by increasing the loan rate R^j by $\frac{1-p^j}{p^j}(1+r)$ for $j \in \{G, B\}$.[7] As high risk projects fail more often, the increase in the loan rate for a marginal decrease in the recovery rate is higher for a bad type borrower. If the expected recovery rate is $c^{e,j} = 1$, then $R^j = 1 + r$ for $j \in \{G, B\}$, which is the lowest loan rate chargeable, equal to the risk free rate. If the expected recovery rate is $c^{e,j} = 0$, then $R^j = \frac{1+r}{p^j}$ for $j \in \{G, B\}$, which is the highest loan rate chargeable. For any given loan amount, if the expected recovery rate is less than one, the loan rates satisfy $1 + r < R^G < R^B$.

[7]Note that under full information all $(R^j, c^{e,j} \in [0, \frac{q^e \gamma^0}{(1+r)} \frac{f(i^j)}{i^j}])$ pairs satisfying (1.5) constitute equilibria given i^j as in (1.3). A debt contract in which banks recover as much as they can in failure, i.e., $c^{e,j} = \frac{q^e \gamma^0}{(1+r)} \frac{f(i^j)}{i^j}$, is a special case of these equilibrium contracts.

Now suppose there is asymmetric information and banks are not able to distinguish high and low risk borrowers directly. As long as $R^G < R^B$ at the full information loan amount, it is always optimal for high risk borrowers to mimic being low risk. Therefore, as long as the expected recovery rate at i^* is less than one; or equivalently if $\frac{q^e \gamma^0}{(1+r)} \frac{f(i^*)}{i^*} < 1$, which is ensured by Assumption 1.1 above, there will be an agency problem. In this case banks either pool all loan applicants or design self selecting contracts to separate them, depending on which type of contract is more profitable to low risk borrowers.

First consider a separating regime, in which contracts are designed to screen out borrower types:

Proposition 1.2. *Separating contracts (i^j, R^j) for $j \in \{G, B\}$ maximize (1.1) subject to nonnegative (1.2) and incentive constraints*

$$\Pi^{e,j}(i^l, R^l) \leq \Pi^{e,j}(i^j, R^j) \quad \textit{for } j, l \in \{G, B\}. \tag{1.6}$$

They satisfy (1.5) and

$$f'(i^B) = \frac{1+r}{q^e \bar{\gamma}} \tag{1.7}$$

$$\underbrace{[q^e \bar{\gamma} f(i^G) - (1+r) i^G]}_{\Pi^{e,G,s}} = \underbrace{[q^e \bar{\gamma} f(i^B) - (1+r) i^B]}_{\Pi^{e,B,s}} - (1 - \frac{p^B}{p^G})[1 - c^{e,G}](1+r) i^G \tag{1.8}$$

Proof. See Appendix A.1. □

Equation (1.7) implies that, given q^e and r, bad type investment is the same as in the full information case. It is the good types who must bear the cost of separation. Good type loan amount is characterized by Equation (1.8), which derives from the binding incentive constraint of high risk borrowers and break even conditions. Given the binding incentive constraint of bad type borrowers, the incentive constraint of good type borrowers reduces to $\frac{p^G}{p^B} \geq \frac{[1-c^{e,G}]}{[1-c^{e,B}]}\frac{i^G}{i^B}$, which is satisfied for all $p^G > p^B$ as long as $i^G \leq i^B$.

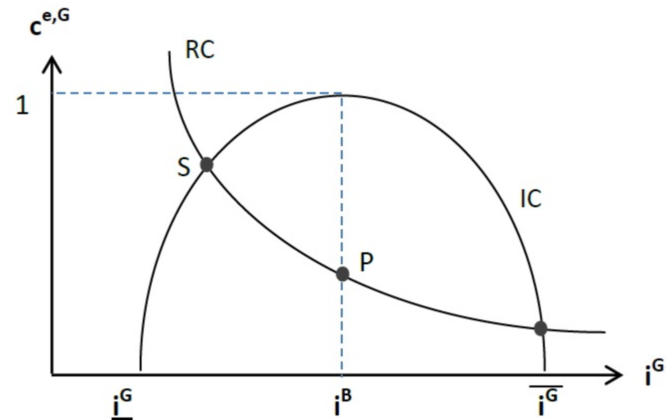

Figure 1.1: Equilibrium i^G in a separating contract

Corollary 1.1. *Under Assumption 1.1, $i^G < i^B$ in a separating contract.*

In Figure 1.1, RC depicts recovery rate $c^{e,G} = \frac{q^e\gamma^0}{1+r}\frac{f(i^G)}{i^G}$ as a decreasing function of i^G, and IC depicts incentive constraint (1.8) as an inverse U-shaped function $c^{e,G}$ of i^G, given q^e and r. RC is downward sloping because the marginal cost of i^G is constant for a bank but the marginal return in failure is decreasing in i^G. IC is inverse U-shaped because for a given expected recovery rate $c^{e,G}$, in order to make a good type contract unattractive to bad type borrowers, banks must either restrict or expand the good type loan amount compared to the bad type efficient loan amount. On the other hand, as the recovery rate increases, a good type contract becomes less attractive for a bad type borrower because the bad type loan rate falls more than the good type loan rate for a given increase in the recovery rate. Thus, for $c^{e,G} = 0$, banks would restrict or expand the good type loan amount heavily to achieve separation, as depicted in the figure by loan levels $\underline{i^G}$ and $\overline{i^G}$. For higher expected recovery rates, the distortion in loan amounts required for separation becomes milder. The loan amount is undistorted and $i^G = i^B$ only if the loan is expected to be recovered fully; i.e., $c^{e,G} = 1$ and $R^j = 1 + r$.[8]

Assumption 1.1 implies that the recovery rate expected from a good type borrower at the bad type efficient loan amount, which is depicted as point P in the figure, is less than one. In this case the two curves RC and IC intersect at two different points. Of these two $\{c^{e,G}, i^G\}$ pairs, point S, with a lower loan amount and a higher expected recovery rate, offers higher profits to the good type and is thus preferred by good type borrowers.[9] Therefore, good type credit is rationed compared to bad type credit under a separating regime. Note that high risk borrowers make more profits than low risk borrowers under a separating regime.

Now consider a pooling regime, in which a common contract is designed for all type of borrowers, without screening:

Proposition 1.3. *A pooling contract* $(i^j, R^j) = (i, R)$ *for* $j \in \{G, B\}$ *maximizes*

[8] The intuition is similar to the arguments in Martin (2008, 2009), where loan limits and collateral requirements are two different tools to achieve separation. In this setup recovery rates act as collateral, as there are no contract enforcement or moral hazard problems.

[9] Along the IC curve, bad types are indifferent between any two points. Between the two points of intersection, the one with a higher collateralization (lower loan rate) is more attractive for good type borrowers as they succeed and repay their debt more often.

expected profits of the borrower pool

$$\lambda\Pi^{e,G,p} + (1-\lambda)\Pi^{e,B,p} = q^e\bar{\gamma}f(i) - \bar{p}Ri - (1-\bar{p})c^e(1+r)i \tag{1.9}$$

subject to the break even condition of the bank lending to the pool

$$\bar{p}Ri + (1-\bar{p})c^e(1+r)i = (1+r)i \tag{1.10}$$

where $\bar{p} = \lambda p^G + (1-\lambda)p^B$ *and* $c^e = \frac{q^e\gamma^0}{1+r}\frac{f(i)}{i}$. *It satisfies*

$$f'(i) = \frac{1+r}{q^e\bar{\gamma}} \tag{1.11}$$

$$R = [1-(1-\bar{p})c^e]\frac{1+r}{\bar{p}} \tag{1.12}$$

Proof. See Appendix A.1. □

Equation (1.11) implies that, given q^e and r, investment in a pooling regime attains the full information level. However, given expected recovery rate c^e, the pooling loan rate is higher than the full information loan rate for a good type borrower and lower than the full information loan rate for a bad type borrower. Thus, pooling entails cross subsidization of bad type borrowers by good type borrowers, which can be seen from the pooling profits

$$\Pi^{e,G,p} = [q^e\bar{\gamma}f(i) - (1+r)i] - (\frac{p^G}{\bar{p}} - 1)[1-c^e](1+r)i \tag{1.13}$$

$$\Pi^{e,B,p} = [q^e\bar{\gamma}f(i) - (1+r)i] + (1 - \frac{p^B}{\bar{p}})[1-c^e](1+r)i \tag{1.14}$$

Equation (1.14) implies that, given q^e and r, high risk borrowers always prefer a pooling contract over a separating contract. Since both pooling and separating contracts yield the same level of investment for high risk borrowers, they prefer a pooling contract as it yields a lower loan rate because of cross subsidization. Note that high risk borrowers make more profits than low risk borrowers under the pooling regime as well.

Lemma 1.1. *The equilibrium contract will be the contract that yields higher profits to low risk borrowers.*

When a pooling contract yields higher profits than a separating contract for both types, the equilibrium contract in this economy will be a pooling contract. When a separating contract yields higher profits to good type borrowers and lower profits to bad type borrowers than a pooling contract, the equilibrium contract in this economy will be a separating contract. In the latter case, any deviation by a bank from offering separating to pooling contracts will only attract bad type borrowers and will yield bank losses, which rules out deviation.

Lemma 1.2. *Whenever* $[1-\frac{p^B}{p^G}][1-c^{e,G}](1+r)i^G < [\frac{p^G}{\bar{p}}-1][1-c^e](1+r)i$*, separating contracts are offered in equilibrium.*

Given q^e and r, a low risk borrower compares expected profits $\Pi^{e,G,s}$ under a separating contract, given by equation (1.8), and expected profits $\Pi^{e,G,p}$ under a pooling contract, given by equation (1.13). A low risk borrower prefers separating contracts whenever $\Pi^{e,G,s} > \Pi^{e,G,p}$. Lemma 1.2 states that a separating contract is preferred by good type borrowers whenever the cost of screening given by the left hand side is lower than the cost of cross subsidization given by the right hand side. Pooling contracts are offered in equilibrium whenever separating contracts are not preferred by good type borrowers.

Remark 1.1. *Separating contracts are always preferred by good type borrowers if* $[1-\frac{p^B}{p^G}] < [\frac{p^G}{\bar{p}}-1]$. *Since the pooling loan amount* i *is higher than the separating loan amount* i^G, *expected recovery rates satisfy* $[1-c^e] > [1-c^{e,G}]$. *Thus, for* $[1-\frac{p^B}{p^G}] < [\frac{p^G}{\bar{p}}-1]$, *which is the case when* λ *or* $\frac{p^B}{p^G}$ *are low enough, the inequality in Lemma 1 always holds.*

Remark 1.2. *We can rearrange (1.5) and (1.12) as* $[1-c^{e,G}] = \frac{p^G}{1-p^G}\frac{R^G-(1+r)}{1+r}$ *and* $[1-c^e] = \frac{\bar{p}}{1-\bar{p}}\frac{R-(1+r)}{1+r}$, *respectively. Thus, the inequality in Lemma 1.2 can be stated as* $[\frac{1-p^B}{1-p^G}-1][R^G-(1+r)]i^G < [1-\frac{1-p^G}{1-\bar{p}}][R-(1+r)]i$. *Accordingly, given the relative riskiness and measure of high risk borrowers, a lower risk premium payment makes a contract more attractive to a low risk borrower.*

Let us define $C^{sp} = \frac{[1-\frac{p^B}{p^G}]}{[\frac{p^G}{\bar{p}}-1]}\frac{[1-c^{e,G}]}{[1-c^e]}\frac{i^G}{i}$ as the relative cost of separating and pooling contracts. For $C^{sp} > 1$ the relative cost of a separating contract is high, so that pooling is preferred by low risk borrowers. As C^{sp} decreases, the likelihood of a separating regime increases and when $C^{sp} < 1$ separating contracts are preferred.

Lemma 1.3. *Given* q^e *and* r, *contractual terms satisfy*

i) $i = (\frac{q^e\bar{\gamma}\theta}{1+r})^{\frac{1}{1-\theta}}$ *and* $c^e = \frac{\gamma^0}{\bar{\gamma}\theta}$ *in a pooling regime,*

ii) $i^B = i$ *and* i^G *satisfies* $IC(\frac{i^G}{i^B}) = \frac{1}{\theta}(\frac{i^G}{i^B})^\theta - [1-(1-\frac{p^B}{p^G})(1-c^{e,G})]\frac{i^G}{i^B} - (\frac{1}{\theta}-1) = 0$ *in a separating regime, where* $c^{e,G} = \frac{\gamma^0}{\bar{\gamma}\theta}(\frac{i^G}{i^B})^{\theta-1}$.

Proof. See Appendix A.1. □

Lemma 1.3 shows that c^e, i^G/i and $c^{e,G}$ are independent of q^e and r, which implies that C^{sp} is independent of q^e and r as well. This means that general equilibrium prices have no effect on the equilibrium lending regime determined by C^{sp}, but do affect the contractual terms (i^j, R^j) for $j \in \{G, B\}$.

Proposition 1.4. *The equilibrium lending regime depends only on* $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$, *and is independent of* q^e *and* r.

Proof. See Appendix A.1. □

Proposition 1.4 implies that for a group of borrowers with common observable quality, the equilibrium lending regime, or lending standards, depends only on the relative measure and riskiness of high and low risk borrowers within this group, governed by λ and p^B/p^G, respectively; the collateral capacity of investors, governed by $\gamma^0/\bar{\gamma}$ given investment technology; and the curvature of investment technology, governed by θ. Neither changes in the risk free rate nor expected prices can affect lending standards in this model without aggregate uncertainty.

Proposition 1.5. *For each triple* $\{\frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$ *there exists a unique* $\lambda \in (0, 1)$ *such that a low risk borrower is indifferent between a pooling and a separating contract.*

Proof. See Appendix A.1. □

Proposition 1.5 states that there are quadruples $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$ that constitute a lending regime switching point for a group of borrowers. Before analyzing how changes in λ, $\frac{p^B}{p^G}$, $\frac{\gamma^0}{\bar{\gamma}}$ and θ may alter the equilibrium lending regime for a group of borrowers, we close the model in the next section and analyze what happens to aggregate investment at a regime switching point, assuming there exists only one group of borrowers in credit markets with expected productivity $\bar{\gamma}$ and average riskiness $\bar{p}$, which index this group's observable quality.

1.4 General Equilibrium

In any given period, the endogenous state is the capital stock k, while the exogenous states are $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$, which determine the lending regime in that period, together with $\bar{\gamma}$ and A. Let $x = \{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta, \bar{\gamma}, A\}$ and let $\tilde{i}^B(q^e, r, x)$ and $\tilde{i}^G(q^e, r, x)$ denote the loan amounts given by the equilibrium contract.

Aggregate demand for funds is given by $I = \lambda \tilde{i}^G + (1-\lambda)\tilde{i}^B$, which is equal to aggregate supply of funds $S = w = A[g(k) - g_k(k)k]$ in equilibrium, so that

$$w(A, k) = I(q^e, r, x). \tag{1.15}$$

Capital production is given by $k' = \lambda\bar{\gamma}f(\tilde{i}^G) + (1-\lambda)\bar{\gamma}f(\tilde{i}^B)$, which defines the capital accumulation frontier as

$$k' = k'(q^e, r, x). \tag{1.16}$$

As there is no aggregate uncertainty, the expected price of capital will be equal to the actual price of capital $q' = Ag_k(k')$ next period, so that

$$q^e = q'(A, k'). \tag{1.17}$$

Equations (1.15)-(1.17) characterize the solutions to $k'(k, x)$, $q^e(k, x)$ and $r(k, x)$, which pin down all other variables in the economy.[10]

[10] See Appendix A.2 for a full set of equilibrium equations.

Let the quadruple $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$ constitute a regime switching point and let superscripts $j \in \{p, s\}$ denote pooling and separating lending regimes respectively.

Proposition 1.6. *Given savings $S(w(A,k))$, at a regime switching point*

i) individual investment amounts satisfy $i^G < i = i^* < i^B$,

ii) the resulting capital stock satisfies $k'^s < k'^p$,

iii) the risk free rate satisfies $r^s < r^p$,

iv) future capital prices satisfy $q^{e,p} < q^{e,s}$.

Proof. See Appendix A.1. □

Proposition 1.6 states that savings in a given period will determine the aggregate resources devoted to capital production regardless of the lending regime. However, in a pooling regime borrowers obtain the full information loan amounts and invest efficiently, whereas in a separating regime high risk borrowers over-invest and low risk borrowers under-invest relative to the pooling regime. Thus, a pooling regime yields higher capital production (physical investment) for a given amount of resources devoted to investment due to concavity of the capital production function $f(.)$. This implies that a pooling regime results in higher risk free returns, higher future output and lending, and lower future capital prices than a separating regime.

Empirical evidence by Lown and Morgan (2006) suggests that changes in bank lending standards for allocating business loans have significant effects on economic activity. They show that tightening credit standards, i.e., greater use of non-price loan terms such as higher collateral requirements and credit limits, are usually followed by slower loan growth and often precede recessions in the US. We may interpret the switches between pooling and separating regimes in the model as switches between lax and tight lending standards induced by bank competition. A pooling regime is a lax lending regime as it eliminates credit rationing and stimulates aggregate investment, which entails higher output and lending. A separating regime is a tight lending regime, which promotes efficient risk pricing at the expense of credit rationing, which in turn dampens aggregate investment, future output and lending. Note that in a separating regime collateralization will be higher on average to facilitate efficient risk pricing.

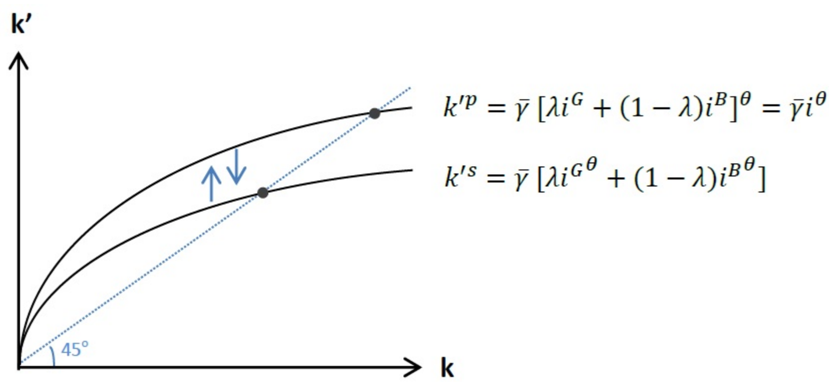

Figure 1.2: Aggregate investment at a regime switching point

Figure 1.2 depicts the capital accumulation frontier under the two regimes at a regime switching point. If the lending regime switches from pooling to separating, aggregate investment falls as a result of an inefficient reallocation of credit from low risk borrowers to high risk borrowers. In this case the economy's growth path shifts down as would be the case under a negative productivity shock. The degree of investment inefficiency in a separating regime is proportional to the degree of credit rationing i^B/i^G under a separating contract, which in turn depends on $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$. In the next section we analyze how changes in $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$ affect the equilibrium lending regime for a group of borrowers, by changing bank incentives to design pooling or separating contracts.

1.5 Determinants of Lending Regime

We have seen that the likelihood of a separating regime is inversely proportional to the relative cost of a separating contract for a low risk borrower, given by $C^{sp} = \frac{[1-\frac{p^B}{p^G}]}{[\frac{p^G}{\bar{p}}-1]} \frac{[1-c^{e,G}]}{[1-c^e]} \frac{i^G}{i}$, which depends only on $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$. In what follows, we are going to analyze how changes in $\{\lambda, \frac{p^B}{p^G}, \frac{\gamma^0}{\bar{\gamma}}, \theta\}$ affect the relative cost C^{sp} of a separating contract, and thereby affect the relative likelihood of a separating regime.

Proposition 1.7. *The effects of λ and $\frac{p^B}{p^G}$ on C^{sp} keeping other parameters constant:*

i) $\frac{\partial[1-\frac{p^B}{p^G}]/[\frac{p^G}{\bar{p}}-1]}{\partial\lambda} > 0 \;\rightarrow\; \frac{\partial C^{sp}}{\partial\lambda} > 0$

An increase in the measure of low risk borrowers λ decreases the degree of cross subsidization, and thereby increases the likelihood of a pooling regime.

ii) $\frac{\partial[1-\frac{p^B}{p^G}]/[\frac{p^G}{\bar{p}}-1]}{\partial p^B/p^G} > 0, \;\; \frac{\partial[1-c^{e,G}]/[1-c^e]}{\partial p^B/p^G} > 0, \;\; \frac{\partial i^G/i}{\partial p^B/p^G} > 0 \;\rightarrow\; \frac{\partial C^{sp}}{\partial p^B/p^G} > 0$

An increase in the relative success rate of high risk borrowers $\frac{p^B}{p^G}$ decreases the relative risk premium under a pooling contract and increases the relative loan size under a separating contract. Overall, an increase in $\frac{p^B}{p^G}$ decreases the relative cost of cross subsidization, and thereby increases the likelihood of a pooling regime.

Proof. See Appendix A.1. □

λ does not affect the quantity and the recovery rate of a loan under the two regimes, but as λ increases, cross subsidization becomes smaller since a larger fraction of low risk borrowers subsidize a smaller fraction of high risk borrowers. Thus, the relative cost of pooling falls and pooling becomes more likely. Note that as λ approaches 1, $\bar{p}$ approaches p^G and the cost of pooling vanishes.

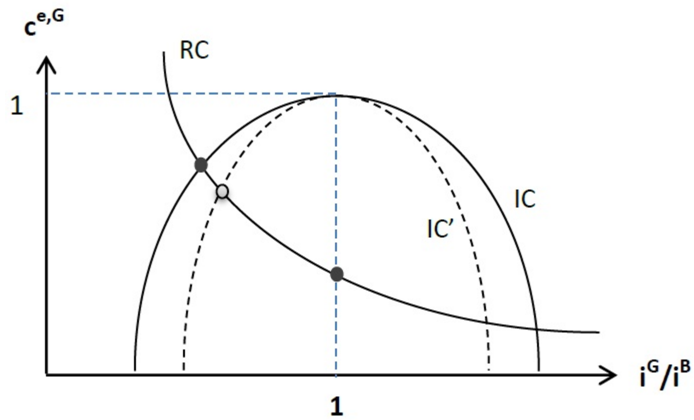

Figure 1.3: The effect of an increase in p^B/p^G

Figure 1.3 depicts the effect of an increase in $\frac{p^B}{p^G}$ on the RC and IC curves, plotted against i^G/i^B. An increase in $\frac{p^B}{p^G}$ makes the IC curve tighter, increasing i^G/i^B for any rate of collateralization. The reason is that, as borrower risks become closer, the separating loan rates become closer as well and good type contracts become relatively less attractive for bad type borrowers. Thus, the degree of rationing needed to disincentivize high risk borrowers in a separating contract declines. However, an increase in $\frac{p^B}{p^G}$ increases the loan rate for good types relative to bad types in a separating contract. Note that i^B (or i) stays constant, but i^G increases with an increase in $\frac{p^B}{p^G}$. Therefore, recovery rate $c^{e,G}$ falls while c^e does not change, increasing the relative loan rate for good types in a separating contract. Meanwhile, as high and low risk borrowers become more similar, the degree of cross subsidization in a pooling contract becomes smaller. Overall, an increase in $\frac{p^B}{p^G}$ makes a pooling contract more attractive for low risk borrowers, because the desired loan amounts become affordable under a pooling contract.

Remark 1.2 stated that a separating contract is more attractive to a low risk borrower, i.e., $C^{sp} < 1$, whenever $\frac{[R^G-(1+r)]}{[R-(1+r)]}\frac{i^G}{i} < [1 - \frac{1-p^G}{1-\bar{p}}]/[\frac{1-p^B}{1-p^G} - 1]$. Given the relative riskiness and measure of high and low risk borrowers, what makes a separating contract attractive to a low risk borrower despite credit rationing is the lower risk premium associated with it. A separating contract may result in higher profits than a pooling contract due to lower costs, although the former entails a lower amount of investment. Therefore, an increase in the relative interest payment under a pooling contract increases the likelihood of a separating regime, and the equilibrium lending regime becomes separating if the relative interest payment under a pooling regime exceeds a certain threshold. Below we show how lower collateralization $\frac{\gamma^0}{\bar{\gamma}}$ and higher investment elasticity θ (lower curvature in investment technology) increase the relative interest payment under a pooling regime given λ and $\frac{p^B}{p^G}$, increasing the likelihood of a separating regime despite higher credit rationing.

<u>A decrease in collateralization:</u> Consider a decrease in γ^0 given $\bar{\gamma}$. As depicted in Figure 1.4, a decrease in γ^0 shifts the RC curve down for a given $\bar{\gamma}$, decreasing both $c^{e,G}$ and c^e. This price effect increases the loan rates in both separating and pooling contracts. For a given decrease in collateralization, banks need to increase

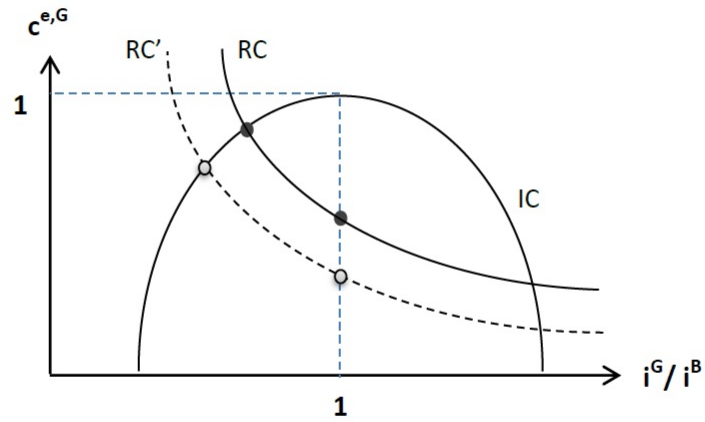

Figure 1.4: The effect of a decrease in $\gamma^0/\bar{\gamma}$

loan rates more for higher risk borrowers to break even, because they fail and lose collateral more often. Thus, the pooling loan rate increases more than the low risk loan rate in a separating regime, increasing the degree of cross subsidization under a pooling regime. Meanwhile, as γ^0 decreases, i^G decreases while i stays constant. The reason is that, as $c^{e,G}$ gets lower, the high risk loan rate increases more than the low risk loan rate in a separating regime, incentivizing high risk borrowers more to lie about their type. Therefore, the need for credit rationing to disincentivize high risk borrowers rises. Thus, a decrease in γ^0 not only raises the loan rate, but tightens the loan limit in a separating contract. As a result $[1 - c^e]i^B$ increases and the cost of pooling rises, but the effect on $[1 - c^{e,G}]i^G$ is not transparent.

Proposition 1.8. *For a low risk borrower, the cost of pooling and separating contracts both decrease in p^B/p^G and $\gamma^0/\bar{\gamma}$, and increase in θ.*

Proof. See Appendix A.1. □

Proposition 1.8 states that the cost of separating for good types also rises as a result of a decrease in γ^0 given $\bar{\gamma}$. Thus, the likelihood of a separating regime may increase or decrease as a result of a decrease in γ^0, depending on whether the cost of pooling or the cost of separating rises more. For relatively high θ, the cost of separating increases less than the cost of pooling with a decrease in γ^0, increasing the likelihood of a separating regime. For relatively low θ, the cost of separating increases more than the cost of pooling with a decrease in γ^0, increasing the likelihood of a pooling regime. The reason is that, when θ is high, the curvature of the investment production function decreases, making the marginal product less sensitive to changes in the quantity invested. Thus, when θ is high, a lower loan amount in a separating regime does not increase the marginal return on investment as much as a higher loan rate in a pooling regime increases the marginal cost of investment. Cross subsidization becomes so high in a pooling regime that low risk borrowers choose to signal their type by accepting a lower loan amount at a cheaper rate, decreasing their interest repayment. This is in line with the findings in Melnik and Plaut (1986), who analyze the structure of loan commitment contracts and document that "borrowers may trade off more favorable values of some loan variable for less favorable values of some other loan variable". In particular, borrowers may "purchase" themselves lower interest rates in exchange for lower credit commitments as is the case when collateral capacity of investors falls in our model with a sufficiently high θ. By decreasing the loan size, banks increase the rate of collateralization on the loan, which facilitates a lower loan rate as desired by low risk borrowers.

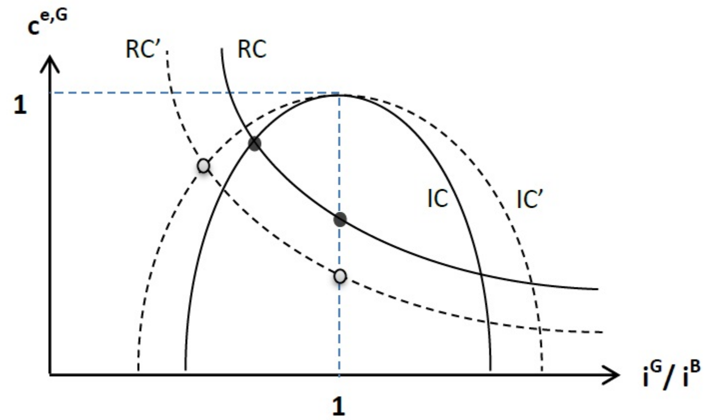

Figure 1.5: The effect of an increase in θ

A decrease in the curvature of investment technology: Figure 1.5 depicts the effect of an increase in θ. An increase in θ shifts the RC curve down, and it also flattens the IC curve. As a result the degree of rationing increases, since as the curvature of production technology falls, the disincentivizing effect of i^G relative to i^B becomes smaller. When θ is high, the IC curve becomes flatter and less sensitive to changes in i^G/i^B at any rate of recovery. Thus, shocks in RC must be absorbed more by a change in i^G/i^B and less by a change in $c^{e,G}$.

An increase in θ has a similar effect on the terms of pooling and separating contracts to a decrease in $\gamma^0/\bar{\gamma}$. An increase in θ decreases collateralization c^e in pooling and $c^{e,G}$ in separating regimes, increasing the loan rates in both regimes, but increasing the pooling loan rate relatively more than the separating loan rate for low risk borrowers. Thus, both the degree of cross subsidization under pooling and the degree of credit rationing under separation rise with a higher θ. As a result, the likelihood of a separating regime may increase or decrease with an increase in θ, depending on whether the cost of pooling or the cost of separating rises more.

As before, for relatively high initial θ, the cost of separating increases less than the cost of pooling with a further increase in θ, increasing the likelihood of a separating regime. Below we numerically illustrate how changes in $\gamma^0/\bar{\gamma}$ and θ affect the likelihood of a separating regime for different initial values.

1.5.1 Numerical Illustration

This section presents a numerical illustration that shows how the relative magnitude of the costs associated with the two regimes for a low risk borrower changes with the investment elasticity $\theta \in (0,1)$ and the degree of collateralization $\frac{\gamma^0}{\bar{\gamma}\theta} \in (0,1)$, given $\frac{p^B}{p^G}$ and λ.[11] We choose values for $\frac{p^B}{p^G}$ and λ using data collected by the Federal Reserve Bank's Survey of Terms of Business Lending.[12]

Table 1.1 presents the time series averages of the following loan terms aggregated over all C&I loans made by sample commercial banks during the survey weeks: 1) weighted-average effective loan rate, 2) percent of loan value secured by collateral,

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[11]Note that $\frac{\gamma^0}{\bar{\gamma}\theta} = c^e$ reflects the degree of collateralization under a pooling regime (or under full information), while for a low risk borrower the degree of collateralization in a separating regime is $c^{e,G} = c^e(i^G/i)^{\theta-1} > c^e$.

[12]This survey collects data (FR 2028A) on commercial and industrial (C&I) loans made by 348 commercial banks and 50 U.S. branches and agencies of foreign banks to domestic customers during the first full business week of the mid-month of each quarter. Respondents provide information on price and non-price terms of individual loans made during the survey week, such as the interest rate, loan size, maturity, collateralization, and loan risk ratings. From the sample data, aggregate estimates of the terms of business loans extended during the survey week are constructed. The estimates provide measures of the cost of business credit and lending terms. The public release of the data covers the period 1997/2-2012/2.

risk group	loan rate (percent)	collateralization (percent)	average loans ($ thousands)	total loans ($ millions)	success rate (percent)	measure (thousands)
all C&I	4.83	39.35	592.38	88,008.41	97.15	148.57
minimal	3.91	30.73	1,284.85	5,886.57	98.75	4.58
low	4.22	25.79	1,083.52	16,656.10	98.42	15.37
moderate	4.90	42.39	551.21	30,729.49	96.89	55.75
acceptable	5.48	49.68	472.36	20,180.64	95.43	42.72

Source: The Federal Reserve Bank's Survey of Terms of Business Lending, 1997/2-2012/2.

Table 1.1: Terms of Business Lending

3) average loan size, 4) total value of loans. The table reports values both for all C&I loans and for subsets representing risk categories ranging from minimal risk to acceptable risk. The fifth column reports implied repayment (success) probabilities, calculated using equation (1.5), $R = [1-(1-p)c]\frac{1+r}{p}$, where R equals the average loan rate reported, c equals the average collateral rate reported, and risk free rate r is taken as 3 percent.[13] The relative measures of risk groups are not reported in the data. Thus, as a proxy, the last column reports the implied measures of each group, calculated as the time series averages of total value of loans divided by the average loan size.

Accordingly, the borrower pool in the data (all C&I loans) has a success probability of around 97 percent. Note that loan terms and the success probability of the moderate risk group roughly reflect those of the overall borrower pool. Thus, the minimal and low risk groups are treated as low risk borrowers, while the acceptable

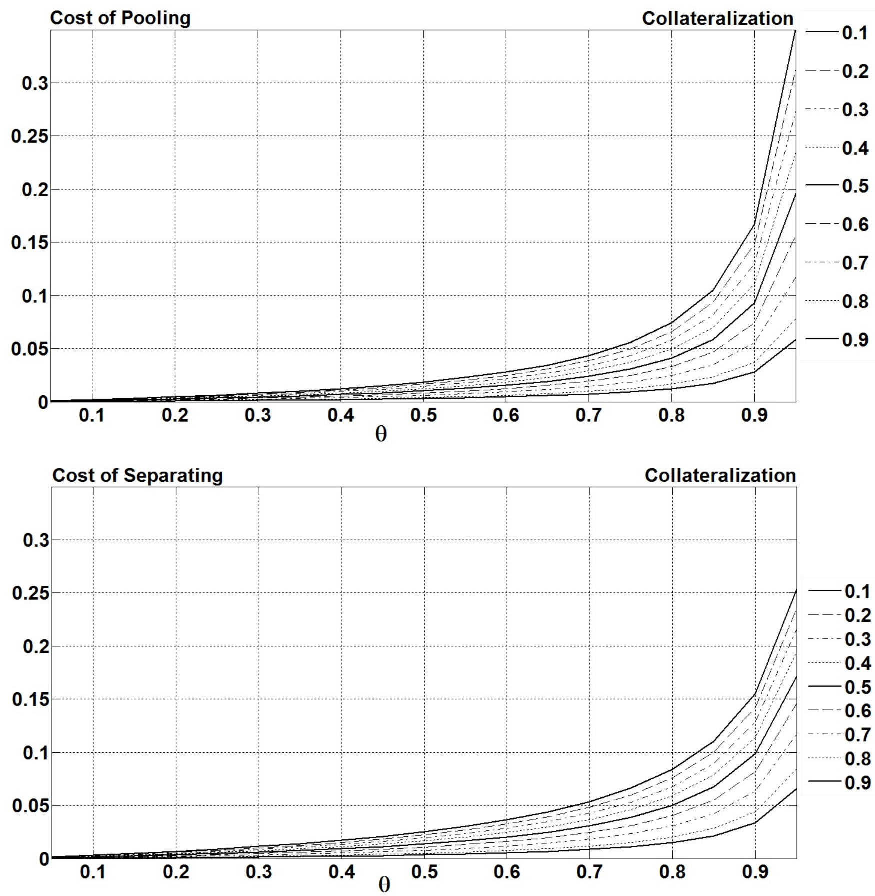
[13] The average 3-Month Treasury Bill rate (quarterly, secondary market rate) in the U.S. for the sample period is around 2.7 percent. *Source:* FRED (TB3MS).

risk group is treated as high risk borrowers when parameterizing the model.[14] Low risk borrowers have a success probability of roughly 99 percent and high risk borrowers have a success probability of roughly 95 percent. Thus, $\frac{p^B}{p^G}$ is set to $95/99 \approx 96$ percent. To match the success probability of the overall borrower pool, which is around 97 percent, we set the measure of low risk borrowers λ to roughly 0.5.[15]

Figure 1.6 plots how the costs of pooling and separating contracts for low risk borrowers change with the investment elasticity $\theta \in (0, 1)$ and collateralization $\frac{\gamma^0}{\bar{\gamma}\theta} \in (0, 1)$ given $\frac{p^B}{p^G}$ and λ. The vertical axes on the left specify the cost of a pooling or separating contract for a low risk borrower in terms of percentage losses from the full information profits. The costs of separating and pooling contracts are computed

[14]One problem is that the table reports observable characteristics of the loan applicants in the data, which are used to parameterize unobservable characteristics of the loan applicants in the numerical example. Unobservables cannot be measured. Thus, observables are taken as a proxy for unobservables to illustrate the model dynamics.

[15]Note that $\frac{p^B}{p^G}$ or λ do not affect the qualitative outcomes illustrated in the figures below, but may shift the levels on the Y axes. In the last column of Table 1.1, the measure of low risk borrowers is roughly half the measure of high risk borrowers. Note that the total measure of high and low risk borrowers roughly matches that of the moderate risk group, which is treated as the borrower pool in the numerical example. If we set the measure of low risk borrowers λ to roughly 1/3 instead of 1/2, pooling incentives would be lower. In the last column of Table 1.1, the measure of all C&I loans is significantly greater than the total measure of the four risk groups, because many of the loans (20 percent) are not rated by the respondents. Thus, we choose to set the relative measures so as to match the implied probability of success for the borrower pool.



Plotted for $\frac{p^B}{p^G} = \frac{0.95}{0.99} \approx 0.96$ and $\lambda = 0.5$.

Figure 1.6: The cost of asymmetric information

as

$$C^s = \frac{[1 - \frac{p^B}{p^G}][1 - c^{e,G}](1+r)i^G}{[q^e \bar{\gamma} f(i) - (1+r)i]} = \frac{\theta}{1-\theta}[1 - \frac{p^B}{p^G}][1 - c^{e,G}]\frac{i^G}{i} \tag{1.18}$$

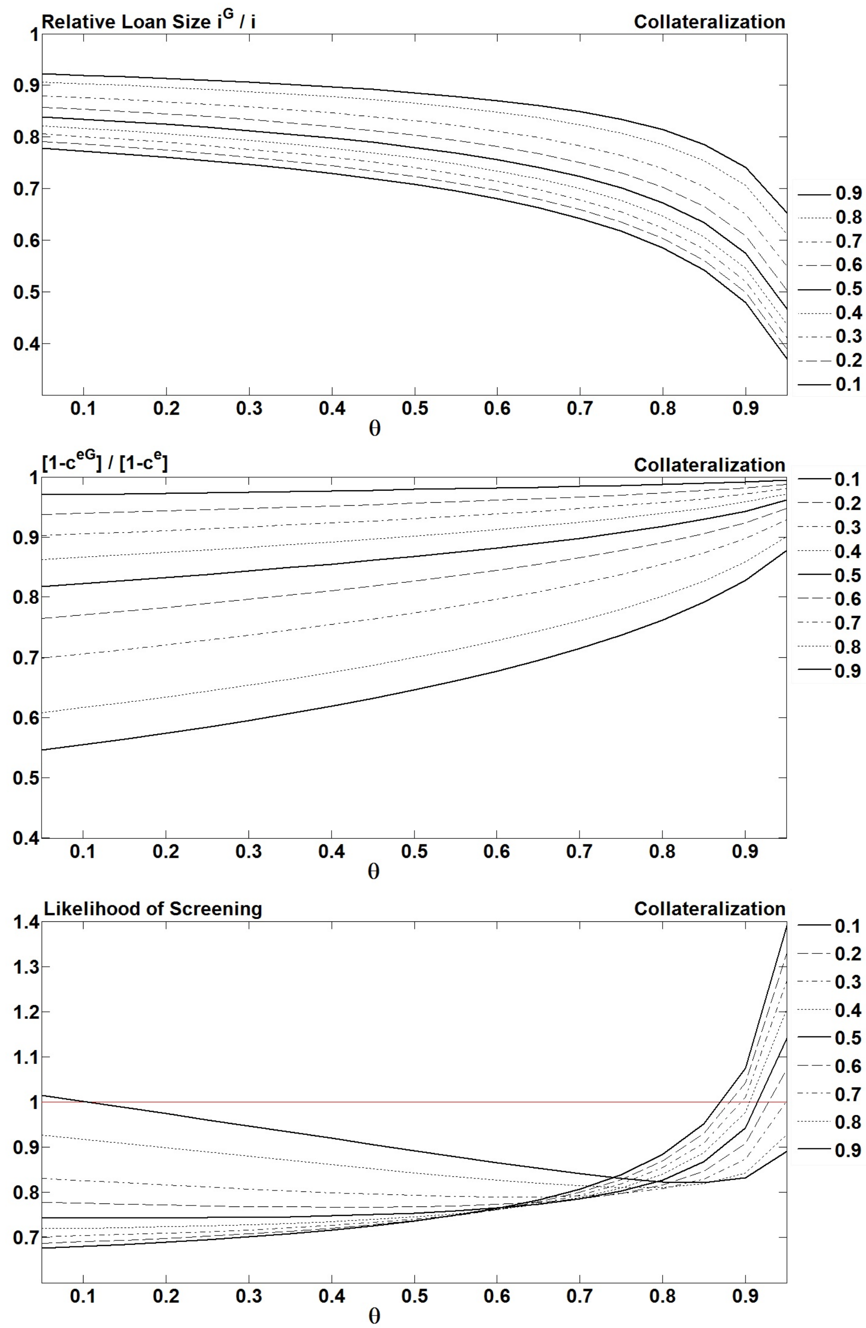
$$C^p = \frac{[\frac{p^G}{\bar{p}} - 1][1 - c^e](1+r)i}{[q^e \bar{\gamma} f(i) - (1+r)i]} = \frac{\theta}{1-\theta}[\frac{p^G}{\bar{p}} - 1][1 - c^e] \tag{1.19}$$

respectively, and plotted against investment elasticity θ for different rates of collateralization. The legends on the right specify the alternative rates of collateralization between zero and one, and the curves represent rising rates of collateralization from top to bottom.

Figure 1.6 confirms that both the cost of pooling and separating contracts increase in θ and decrease in $\frac{\gamma^0}{\bar{\gamma}\theta}$ for a low risk borrower, as stated in Proposition 1.8. Thus, informational frictions cause greater profit losses for low risk borrowers with higher investment elasticity θ and lower collateralization $\frac{\gamma^0}{\bar{\gamma}\theta}$. The reason is that when the investment elasticity θ is high, the loss in investment, due to either a higher loan rate from cross subsidization, or the credit rationing resulting from separation, is higher. Collateralization helps eliminate these losses, decreasing the loan rates under both regimes and decreasing the degree of rationing under a separating regime. For a collateralization rate of around 40 percent, which is the average rate for the borrower pool documented in Table 1.1, the agency costs could amount to a 10 percent loss in profits for a low risk borrower in this setup, if the investment elasticity is relatively high (θ close to 0.9).

Figure 1.7 plots how the relative likelihood of a separating regime C^{sp} changes with θ and $\frac{\gamma^0}{\bar{\gamma}\theta}$, along with its relative price $[1-c^{e,G}]/[1-c^e]$ and relative quantity $\frac{i^G}{i}$ components, given $\frac{p^B}{p^G}$ and λ. In the top panel, in line with Figure 1.4, we see that the relative loan size $\frac{i^G}{i}$ for low risk borrowers increases with collateralization for any level of investment elasticity θ. On the other hand, in line with Figure 1.5, $\frac{i^G}{i}$ decreases with θ for any collateralization level. For a collateralization rate of around 40 percent, the degree of rationing could exceed 30 percent for a low risk borrower in this example, if investment elasticity is relatively high (θ close to 0.9).

The second panel depicts the impact of θ and collateralization on $[1-c^{e,G}]/[1-c^e]$, which governs the loan rate for low risk types under separation relative to pooling. We see in this graph that the relative loan rate for low risk borrowers



Plotted for $\frac{p^B}{p^G} = \frac{0.95}{0.99} \approx 0.96$ and $\lambda = 0.5$.

Figure 1.7: The likelihood of a separating regime

decreases with collateralization for any level of θ. Collateralization decreases the loan rate for low risk borrowers in both regimes but this decrease is larger in a separating regime in percentage terms.[16] On the other hand, $[1 - c^{e,G}]/[1 - c^e]$ increases with θ at any level of collateralization. Therefore, a higher investment elasticity θ implies a higher loan rate for low risk borrowers in both regimes but this increase is more pronounced under separation in percentage terms.

In sum, relative price and quantity effects move in opposite directions with changes in collateralization and investment elasticity θ. The relative price of a loan under a separating contract decreases in collateralization but increases in θ, while the relative loan size i^G/i increases in collateralization but decreases in θ. The total effect on the likelihood of a separating regime depends on whether the relative price or quantity effect dominates. The third panel in Figure 1.7 depicts the overall effect of changes in collateralization and θ on the likelihood of a separating regime, given by the inverse of the relative cost of a separating contract $1/C^{sp} = C^p/C^s$. $C^{sp} = 1$ indicates the regime switching threshold and a lower C^{sp} indicates a rising likelihood for a separating regime. The graph confirms that for relatively low θ, an increase in collateralization increases the likelihood of separation as the relative price effect dominates, while for relatively high θ an increase in collateralization decreases the likelihood of separating as the relative quantity effect dominates. As we discussed above, the reason is that when investment elasticity θ is low, the curvature of the investment production function is high. Therefore, a higher collateralization affects

[16]In absolute terms, a marginal increase in collateralization decreases the pooling loan rate more than the separating loan rate for a low risk borrower, because the borrower pool fails more often than a low risk borrower. However, in percentage terms, since separating loan rate is already low, the decrease in the separating loan rate could exceed the decrease in the pooling loan rate.

the loan quantity less than the loan price as the former is more costly in terms of lost profits. On the other hand, the effect of an increase in θ on the likelihood of a separating regime depends on the initial level of θ. For θ close to one, an increase in θ increases the likelihood of a separating regime (the relative quantity effect dominates) at any level of collateralization. Otherwise, the effect of an increase in θ on the likelihood of a separating regime is ambiguous.

The curvature of investment technology θ, which is referred to as investment elasticity, is central to how collateralization affects screening incentives. The capital production function uses the final good as an input, which is a variable input such as raw (intermediate) materials. In this case θ could be associated with the variable input's share in production, provided that there is a second fixed factor in place that consumes the rest of the profits, although not specifically modeled. The fixed factor could be land or structures owned by entrepreneurs, who possess the fixed factor's share in profits, which is $1-\theta$.

Valentinyi and Herrendorf (2008) document that capital's share in investment good producing sectors is around 1/4, which is lower than the conventional 1/3 share of capital in final good producing sectors. Therefore, the intermediate input share in capital production, θ, would be around 3/4 in this case. On the other hand, in the standard RBC model final goods are transformed into capital one for one, although there might be quadratic adjustment costs.[17] As a result, in the remainder

[17]In this model, without any further assumptions, $\theta = 1$ would imply complete rationing of low risk borrowers from the market because banks would only offer the high risk borrower specific loan contract in equilibrium, which yields negative profits for low risk borrowers. In this case the

we will focus on the case in which curvature of investment technology is limited . Thereby, we postulate that θ is sufficiently high that lower collateralization increases screening incentives and that a marginal increase in θ increases screening incentives at any level of collateralization.

1.6 Credit Regime Switches

In this section we analyze how exogenous changes in collateral capacity affect the allocation of credit across groups of investors with different observable and unobservable risk levels, which in turn alters aggregate investment and shifts the growth path of the economy. We focus on the case in which the curvature of investment technology is limited, so that lower collateralization increases separating incentives.[18]

Consider groups of loan applicants with the same investment elasticity (technology) θ, who differ in expected productivity and average risk levels, which are publicly observable. Loan applicants with the same expected productivity $\bar{\gamma}$ and average risk level $\bar{p}$ represent firms with similar quality. Within each quality group there are two types of borrowers, high and low risk, that cannot be observationally distinguished as before. Below we analyze how a fall in the collateral capacity of investors may lead to an inefficient reallocation of credit, both across borrowers with different unobservable risk levels within the same quality group, and between

economy would attain investment efficiency and there would be no regime switching.

[18] The analysis in the alternative case with a relatively low θ is presented in Appendix A.3 for completeness.

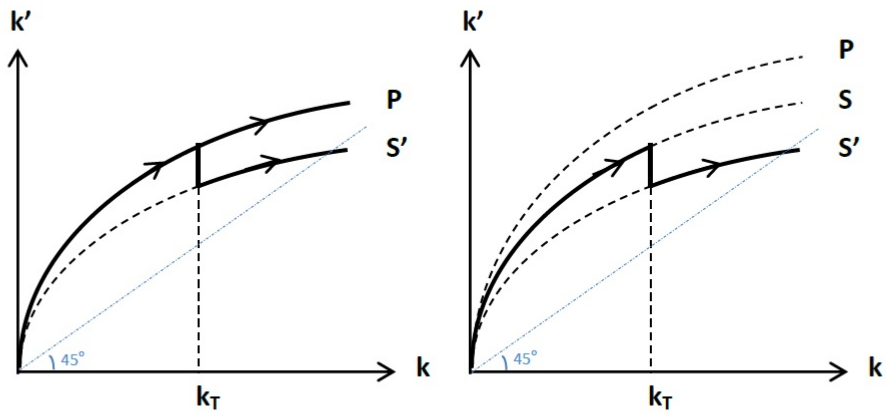
different quality groups.

1.6.1 Reallocation of Credit within a Quality Group

Suppose there are only two firms operating in the economy with a given capital production technology θ. They have the same expected productivity $\bar{\gamma}$ and collateralization given by $c^e = \frac{\gamma^0}{\bar{\gamma}\theta}$, but they differ in risk levels, which is private information. As discussed above, a fall in collateralization increases separating incentives if investment technology is not too concave, i.e., if θ is not too low. On the other hand, regardless of the value of θ, a decrease in collateralization increases the degree of rationing (i.e., decreases i^G/i) for a low risk borrower under a separating regime. Below we illustrate the effects of a fall in collateralization given investment technology, such as a fall in γ^0 given $\bar{\gamma}$, which corresponds to a financial shock increasing the uncertainty in investment returns while keeping the expected returns unchanged, or a rise in $\bar{\gamma}$ given γ^0. We show that a decline in collateralization may reduce aggregate investment through inefficient reallocation of credit across investors.

Figure 1.8 illustrates the possible effects of a decrease in γ^0 given $\bar{\gamma}$ on aggregate investment, under the presumption that θ is relatively high. That is, suppose at time T there is a fall in the pledgeable returns of projects, while overall productivity of investors is not changed. The graphs depict the growth path of the economy before and after time T under two scenarios, in which the initial regime is pooling (P) or separating (S).

Panel (a) shows that, if the lending regime is initially pooling, a decrease in



(a) Initially Pooling (b) Initially Separating

Figure 1.8: An increase in γ^0 given $\bar{\gamma}$, for relatively high θ

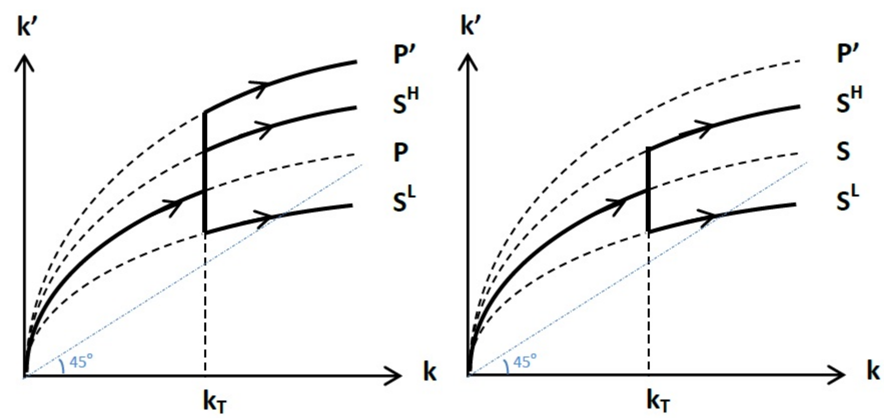
γ^0 would increase the likelihood of a separating regime, in which case banks may continue pooling investors at a higher degree of cross subsidization, or may switch to separating investors, reallocating credit from low risk to high risk investors within a given quality group. In the former case, a decrease in γ^0 would not affect the composition of credit in the economy, as the pooling loan amount $i = [\frac{q^e\bar{\gamma}\theta}{1+r}]^{\frac{1}{1-\theta}}$ obtained by high and low risk borrowers would not change. The decrease in γ^0 would only increase the pooling loan rate $R = [1-(1-\bar{p})c^e]\frac{1+r}{\bar{p}}$, which redistributes profits from low risk borrowers to high risk borrowers as given by equations (1.13)-(1.14), increasing the inequality in profit distribution. In the latter case, a switch to the separating regime would reallocate both profits and credit from low risk to high risk investors, decreasing aggregate investment efficiency by moving the growth path from P to S' in the economy.

Panel (b) shows that, if the lending regime is initially separating, a decrease in γ^0 would affect the composition of credit by increasing the degree of rationing

i^B/i^G under separating contracts. In this case, the growth path of the economy moves from S to S', increasing the inefficiency in aggregate investment by moving the economy further away from the efficient growth path P. Thus, given savings $S = w(A, k)$ in the economy, a decrease in γ^0 may reallocate credit from low risk borrowers to high risk borrowers of a given quality. As a result aggregate investment falls, in which case the growth path of the economy shifts down, decreasing future capital, output and lending in the economy. In this way, although the average efficiency of the productive technology is unchanged in the economy, i.e., $\bar{\gamma}$ and θ are constant, inefficient reallocation of credit due to lower collateralization may decrease investment specific productivity in the same way as an adverse productivity shock.

Now consider an increase in the overall productivity of investors $\bar{\gamma}$ at time T, keeping γ^0 unchanged. In this case there will be two opposing effects on aggregate investment. Keeping the composition of credit constant, an increase in $\bar{\gamma}$ increases aggregate investment as investors become more productive. However, an increase in $\bar{\gamma}$ also decreases collateralization $\frac{\gamma^0}{\bar{\gamma}\theta}$, which may change the composition of credit in an opposing way, under the presumption that θ is relatively high. Figure 1.9 illustrates the possible effects of an increase in $\bar{\gamma}$ given γ^0 on aggregate investment when θ is relatively high. In this case, a decrease in collateralization increases screening incentives and may change the composition of credit by reallocating credit from low risk borrowers to high risk borrowers with a given quality.

Panel (a) shows that, if the lending regime is initially pooling, an increase in $\bar{\gamma}$ could either keep the composition of credit unchanged and redistribute profits



(a) Initially Pooling (b) Initially Separating

Figure 1.9: An increase in $\bar{\gamma}$ given γ^0, for relatively high θ

from low risk borrowers to high risk borrowers, or switch the lending regime into separating and reallocate credit from low risk borrowers to high risk borrowers. In the former case, although lower collateralization decreases pooling incentives, the economy does not switch lending regimes. The growth path shifts from P to P', where investment is efficient at a higher productivity level, although the inequality between high and low risk borrower profits grows. In the latter case, lower collateralization switches the lending regime into separating, in which case the growth path may shift up or down depending on whether the opposing composition or productivity effects dominate. The growth path may shift down from P to S^L when the composition effect dominates, in which case reallocation of credit from low risk borrowers to high risk borrowers decreases investment efficiency in a way that offsets the increase in investment productivity. The growth path may shift up from P to S^H when the productivity effect dominates, in which case aggregate investment increases but not as far as the efficient level because of the composition effect.

Panel (b) shows that, if the lending regime is initially separating, an increase in $\bar{\gamma}$ would increase the degree of rationing i^B/i^G under separating contracts, reallocating credit further away from low risk borrowers to high risk borrowers. Once again the total effect on aggregate investment depends on the opposing composition and productivity effects. The growth path may shift down from S to S^L when the composition effect dominates, or up from S to S^H when the productivity effect dominates as discussed above. Thus, good prospects on aggregate investment productivity may paradoxically create a bust due to inefficient reallocation of credit across borrowers if collateralization is not kept proportionally high.

Discussion: An important point to note here is that with only one group of borrowers with the same average productivity, a deterioration of collateral capacity of borrowers causes credit to flow from *unobservably* low risk borrowers to *unobservably* high risk borrowers. In the next section, we show how allowing for multiple groups of borrowers with different observable average productivity and average risk levels (quality) makes the model consistent with the stylized fact that credit flows in favor of higher-quality borrowers in bad times due to a *flight to quality*, a phenomenon documented by Lang and Nakamura (1995) and Bernanke et al. (1996) among many others. Accordingly, credit flows away from borrowers subject to higher agency costs, and flows in favor of higher quality borrowers during periods of recession or when borrower balance sheets deteriorate.

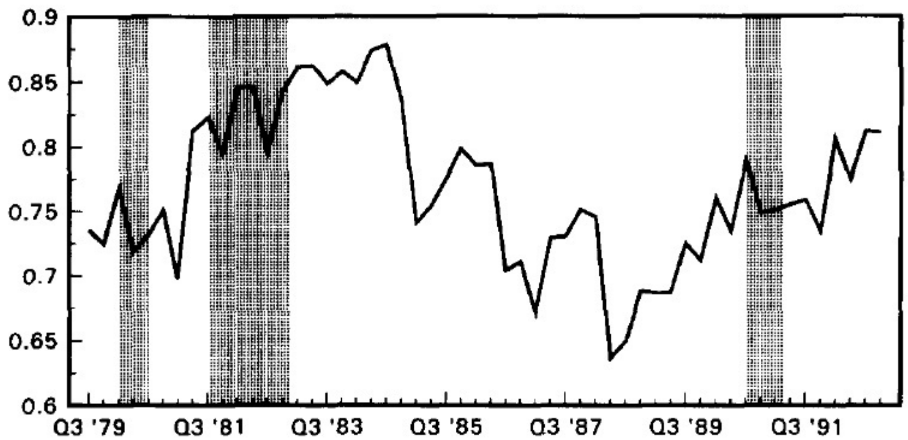
1.6.2 Reallocation of Credit between Quality Groups

In this section we allow for multiple groups of borrowers that differ in expected productivity and average risk levels (quality) to show how a financial shock that reduces the collateral capacity of investors may lead to a recession through inefficient reallocation of credit towards high quality investors (a flight to quality) due to tighter screening and separating across low quality investors by banks (tightening standards). First we review some empirical evidence on a flight to quality in bank lending, and then we show how a fall in the collateral capacity of investors can generate a flight to quality in the model.

<u>Flight to Quality in Bank Lending: Empirical Evidence</u> Using data from the Federal Reserve Board's Survey on Terms of Business Lending between the period 1977 to 1993, Lang and Nakamura (1995) and Asea and Blomberg (1998) analyze individual loan terms on new commercial and industrial loans made by a sample of commercial banks, including loan size, loan rate and bank risk rating of loans, to look for evidence for a flight to quality in bank lending. They categorize high quality loans as relatively safe loans, that are made at or below the prime rate plus 1 percent in each bank. Asea and Blomberg (1998) document that the proportion of safe loans made by banks increases in periods with a high variance across loan rates charged by banks, suggesting that a flight to quality occurs in periods with a tighter bank screening and separating across loan applicants. Moreover, both Asea and Blomberg (1998) and Lang and Nakamura (1995) document that increases in the proportion of safe bank loans are associated with future reductions in real economic

activity.

Figure 1.10: Flight to quality: Proportion of safe bank loans 1979-1992

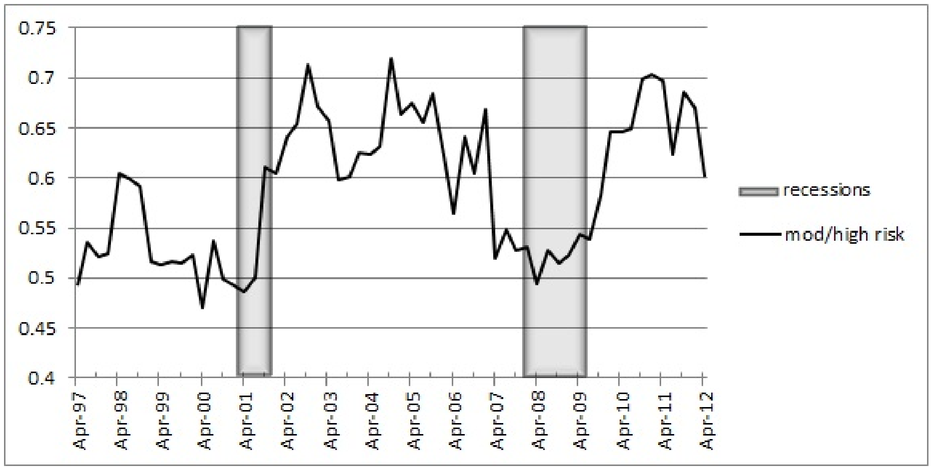


Source: Figure 1 in Lang and Nakamura (1995).

Figure 1.10 is borrowed from Lang and Nakamura (1995), showing the evolution of the proportion of safe bank loans over time. Shaded areas depict U.S. recessions. Evidently, a flight to quality in bank lending precedes all three recessions between 1979-1992. A bivariate vector auto-regression conducted by Lang and Nakamura confirms that increases in the proportion of safe loans (flights to quality) are highly predictive of future reductions in real GDP.

Here we conduct a similar exercise to Lang and Nakamura (1995) for more recent periods. The survey data for individual bank loans are not publicly available. However, as stated in section 1.5.1, the Federal Reserve reports aggregate estimates of the terms of business lending for all C&I loans and for four subsets representing risk categories reported by banks, namely minimal risk, low risk, moderate risk and acceptable (high) risk loans. We categorize low quality loans as relatively high risk loans, labeled as moderate and high risk groups by banks. As seen in Table 1.1

Figure 1.11: Proportion of risky bank loans 1997-2012



Source: FRB Survey of Terms of Business Lending, 1997/2-2012/2.

in section 1.5.1,these two groups are charged average loan rates above the average rate for all C&I loans. Figure 1.11 plots the evolution of the proportion of risky (low quality) bank loans over time. We see that the proportion of risky bank loans decreases prior to both the 2001 and 2008 recessions. Moreover, Granger causality tests confirm that decreases in the proportion of risky bank loans are highly predictive of future reductions in real GDP.[19] Although test results do not suggest a direct causality, they suggest that reallocation of credit across groups of borrowers is associated with fluctuations in real economic activity.

Flight to Quality in Bank Lending: Theoretical Framework In this section we allow for multiple groups of borrowers that differ in expected productivity and average risk levels (quality), which are publicly observable. Within each group there are two types of borrowers, high and low risk, that cannot be observationally distinguished

[19]See Appendix A.4 for test results.

as before. Therefore, whenever credit flows from (unobservably) low risk borrowers to high risk borrowers within the same quality group due to a decrease in the collateral capacity of investors, increased inefficiency affects the relative creditworthiness among different quality groups, making credit flow to observably higher quality groups, with lower average risk levels. Thus, credit flows away from borrowers more subject to agency costs, i.e., low quality borrowers who are more likely to be screened in this context, consistent with a flight to quality.

Suppose there are two groups of loan applicants, type-1 and type-2, operating in the economy. These types have a common investment production function parameter θ, but possibly different expected productivities satisfying $\bar{\gamma}_1 \geq \bar{\gamma}_2$. Assume that p^B/p^G and λ are the same in these two groups for simplicity, while $\bar{p}$ is different. Thus, lenders divide loan applicants in credit markets into groups of different observable quality, with the groups having similar risk dispersion but different average risk levels.

As $\bar{\gamma}_1 \geq \bar{\gamma}_2$, under full information type-1 borrowers would obtain a (weakly) higher loan amount as they are more productive on average. Given general equilibrium prices, type-1 and type-2 borrowers would obtain $i_1 = [\frac{q^e \bar{\gamma}_1 \theta}{1+r}]^{\frac{1}{1-\theta}}$ and $i_2 = [\frac{q^e \bar{\gamma}_2 \theta}{1+r}]^{\frac{1}{1-\theta}}$, respectively. Under asymmetric information about borrower risk, whether these borrowers obtain the desired loan amounts depends on the equilibrium credit regime, which in turn depends on collateralization, among other primitives. Remember that the investment elasticity θ is presumed to be relatively high, so that lower collateralization increases the likelihood of separation.

First consider the case in which $\bar{p}_1 > \bar{p}_2$, so that type-2 borrowers are riskier than type-1 borrowers on average. We assume observably riskier borrowers are subject to higher agency costs, which means that they can pledge a smaller part of their project returns. Therefore, γ_1^0 and γ_2^0 are such that $\frac{\gamma_1^0}{\bar{\gamma}_1} > \frac{\gamma_2^0}{\bar{\gamma}_2}$, so that high quality borrowers (who have high productivity and low risk on average) can pledge project returns more easily. In this case, type-2 borrowers are more likely to be screened by lenders, and they obtain a lower loan amount. Type-2 borrowers may represent small or family owned businesses in the economy with a lower collateral capacity, while type-1 borrowers may represent corporate firms with a higher collateral capacity.

When type-2 borrowers are more likely to be screened than type-1 borrowers, there are three possible lending regime scenarios: pool borrowers within each group (pp regime), pool type-1 borrowers and separate type-2 borrowers (ps regime), or separate borrowers within each group (ss regime). In this case there are two regime switching points: the first is where a low risk borrower of type-2 is indifferent between a pooling (pp) and a separating regime (ps), and the second is where a low risk borrower of type-1 is indifferent between a pooling (ps) and a separating regime (ss).

Suppose the economy is initially in good times, pooling borrowers within both high quality (type-1) and low quality (type-2) groups. In this case borrowers obtain the efficient, full information loan amounts given above. Consider a financial shock that reduces the collateral capacity of investors, i.e., suppose γ_j^0 for $j \in \{1, 2\}$ falls keeping $\bar{\gamma}_j$ constant so that investors can pledge a smaller part of their project returns. In this case lower collateralization affects the low quality group first as they are more likely to be screened by lenders. Thus, the economy switches to separating low quality borrowers first, in which case low risk borrowers in the low quality group are rationed compared to high risk ones. In this case, given general equilibrium prices, borrowers obtain $i_1 = [\frac{q^e\bar{\gamma}_1\theta}{1+r}]^{\frac{1}{1-\theta}}$, $i_2^B = [\frac{q^e\bar{\gamma}_2\theta}{1+r}]^{\frac{1}{1-\theta}}$ and $i_2^G(q^e, r) < i_2^B(q^e, r)$ in high and low quality groups, respectively.[20] As quality is observable, relative creditworthiness between type-1 and type-2 borrowers, determined by their relative productivity levels, should be recovered in equilibrium. That is, the amount of credit extended to high risk borrowers of type-1 relative to high risk borrowers of type-2 should reveal relative productivity of the two types in equilibrium. Thus, when collateral capacity of investors deteriorates, credit will not only flow away from low risk to high risk borrowers within a group of low quality borrowers, but will also flow away in favor of higher quality borrowers, with a lower average risk level.[21] This result can be formalized as follows:

Proposition 1.9. *Given $S(A, k) = I_1 + I_2$, if $\frac{\gamma_1^0}{\bar{\gamma}_1} > \frac{\gamma_2^0}{\bar{\gamma}_2}$, at a regime switching point*

i) investment levels satisfy $\frac{I_1^{pp}}{I_2^{pp}} < \frac{I_1^{ps}}{I_2^{ps}}$ and $\frac{I_1^{ss}}{I_2^{ss}} < \frac{I_1^{ps}}{I_2^{ps}}$

i) the capital stock satisfies $k'^{ss} < k'^{ps} < k'^{pp}$,

ii) the risk free rate satisfies $r^{ss} < r^{ps} < r^{pp}$,

iii) future capital prices satisfy $q^{e,pp} < q^{e,ps} < q^{e,ss}$.

[20] Note that in this case general equilibrium prices also change but we use the same prices q^e and r with a slight abuse of notation.

[21] Technically, with only two groups of borrowers there could be a flight *from* quality if the regime switches from ps to ss. However, we assume that there are always prime borrowers, who are pooled by banks and who attract more credit when regime switches occur. Thus, we assume in a model with only two groups of borrowers that high quality borrowers are always pooled.

Proof. See Appendix A.1. □

Proposition 1.9 states that, whenever the regime switches from pooling to separating type-j borrowers, for $j \in \{1, 2\}$, the economy moves to a less efficient growth path. When lenders switch from pooling to separating type-j borrowers, low risk type-j borrowers are rationed credit compared to high risk type-j borrowers. This inefficiency affects the relative creditworthiness across type-j and non type-j borrowers with observable quality, reallocating credit away from type-j borrowers to non type-j borrowers. Thus, when lenders switch from pooling to separating type-j borrowers, credit is reallocated away from low risk type-j borrowers to all other borrowers, decreasing the overall ratio $\frac{I_j}{I_{j'}}$ of credit obtained by type-j borrowers relative to non type-j borrowers, decreasing aggregate investment efficiency.

Now consider the alternative case for type-1 and type-2 borrowers, in which $\bar{\gamma}_1 \geq \bar{\gamma}_2$ but $\bar{p}_1 < \bar{p}_2$, so that borrowers who are more productive on average are riskier on average. In this case we assume γ_1^0 and γ_2^0 are such that $\frac{\gamma_1^0}{\bar{\gamma}_1} < \frac{\gamma_2^0}{\bar{\gamma}_2}$, so that highly productive but riskier borrowers are subject to higher agency costs and they can pledge project returns less easily. Thus, there is a tradeoff between productivity and pledgeability: high productivity firms with low collateralization are more likely to be screened than low productivity firms with high collateralization.For example Type-1 borrowers could represent high-tech firms with a new but advanced technology, which are highly productive but are subject to higher agency costs due to absence or uncertainty about the collateral value of their projects, while type-2 borrowers could represent low-tech firms with a well established but standard technology, which are less productive but are subject to lower agency costs.

When type-1 borrowers are more likely to be screened than type-2 borrowers, a decrease in the collateral capacity of investors affects the high productivity (type-1) group first as they are more likely to be screened by lenders. Thus, when the economy switches from pooling to separating type-1 borrowers, credit will flow away from high productivity borrowers to low productivity borrowers due to higher agency costs faced by the former. This is consistent with the stylized fact that high-tech firms with innovative technologies are more likely to be rationed credit due to absence or uncertainty of collateral value for their investment.[22] The allocative inefficiency in this case decreases aggregate investment even more than the previous case, because in this case highly productive high-tech firms are rationed credit, decreasing aggregate investment productivity more intensely.

A natural policy recommendation implied by this framework is promoting government sponsored credit programs in times of tight credit, such as increasing Small Business Administration guaranteed loans for disproportionately rationed small and young businesses and innovative firms, which disproportionately account for job creation and growth. When there is a negative disturbance in the economy that reduces collateral availability, such as a decrease in the value of available collateral or an increase in uncertainty about pledgeable investment returns, small business lending might be subsidized more to avoid excessive credit rationing, which otherwise would weigh on job creation and growth.

[22] See Himmelberg and Petersen (1994) and Luigi Guiso (1998) among others for evidence on the U.S. and Italian high-tech firms, respectively.

1.7 Concluding Remarks

This paper analyzes how credit is reallocated within and between groups of investors with different observable quality, i.e., different average productivity and average risk rating, when exogenous shocks affect borrowers' financing conditions. The main finding is that worsening financing conditions may lead to recessions through inefficient reallocation of credit towards high quality investors, due to tighter screening and credit rationing of low quality investors by banks.

When collateral capacity of investors deteriorates, borrowers with lower observable quality are affected first, as they are subject to higher agency costs measured by the pledgeability of their project returns. For low quality borrowers with observable average risk but unobservable individual risk levels, banks offer a menu of separating contracts: one with the efficient loan amount at a higher loan rate and another with a lower than efficient loan amount at a lower loan rate. In this case low risk borrowers of low quality self select the contract that offers cheap credit at the expense of a lower credit amount. This leads to a reallocation of credit towards higher risk borrowers within the low quality group, which affects the relative creditworthiness across groups of borrowers with different quality, and leads to a reallocation of credit towards high quality borrowers. This inefficiency in the aggregate allocation of credit decreases aggregate physical investment, which in turn decreases aggregate output and lending, and further decreases aggregate investment in the economy. Although the main objective of this paper is theoretical, the model captures the stylized evidence that a flight to quality in bank lending occurs when borrower balance sheets deteriorate and banks tighten credit standards *disproportionately*, leading to a decrease in economic activity.

One future extension is to incorporate aggregate uncertainty to the current framework to analyze how current and future prospects about aggregate neutral and investment specific productivity alter the allocation of credit through the financial contracts available to investors. In particular one could ask whether the allocation of credit may serve as an amplifier of aggregate productivity shocks. In principle, a productivity shock may change the allocation of credit in an amplifying or dampening way. Consider a positive and persistent shock to total factor productivity. A high productivity today increases expectations on future productivity and future prices, boosting the value of available collateral. Thus, anticipated returns for banks in the event of a borrower failure increase. However, a high productivity today also boosts lending and investment, which expands the set of states in which a borrower will fail. Thus, expected returns for banks may fall due to increased risk of default. As a result, the cost of lending, and thereby incentives to pool or separate borrowers, may rise or fall depending on the distribution of aggregate productivity, among other model primitives. Using the FRB's Senior Loan Officer Opinion Survey, Bassett et al. (2012) document that uncertainty about the economic outlook is the number one reason banks tighten credit standards. Adding first and second moment shocks to aggregate productivity, this framework can be used to examine how pessimism and uncertainty about future realizations of aggregate productivity affect bank incentives for setting lending standards, which in turn affect the allocation of credit across groups of borrowers and thereby economic activity.

Chapter 2

Costly Screening in Credit Markets, Net Worth Effects and Business Cycles

2.1 Introduction

Existing dynamic models with credit market frictions mostly assume ex-post asymmetric information between lenders and borrowers.[1] In particular, monitoring costs have been widely used to motivate agency costs associated with unsecured lending, which makes borrower net worth matter for finance and creates a channel through which aggregate shocks are amplified and propagated. This chapter studies a dynamic model featuring ex-ante asymmetric information in credit markets and costly screening of loan applicants. It examines the conditions under which loan applicants are screened in equilibrium, and how costly screening can create net worth effects that enhance the propagation of aggregate shocks compared to a standard dynamic model with ex-post monitoring costs.

The findings suggest that screening costs in credit markets can generate net

[1]Ex-post asymmetric information in credit markets refers to informational asymmetries between lenders and borrowers after the loan is disbursed. Examples include contract enforcement, moral hazard and bankruptcy verification problems. Informational asymmetries prevailing before the loan is disbursed refer to ex-ante asymmetric information, which are associated with the inability of lenders to evaluate borrower riskiness and quality accurately, such as adverse selection.

worth effects that are at least as strong as those generated by widely assumed monitoring costs both qualitatively and quantitatively. One advantage of the screening framework is that it yields wealth effects that induce persistent dynamics, especially in bad times when screening is more likely, which may create busts that are deeper and longer than booms. Moreover, the screening framework yields type specific loan terms endogenously, which makes screening costs an empirically plausible alternative to monitoring costs to motivate agency costs in unsecured lending.

Entrepreneurs with privately observed investment projects, which may be high or low quality in terms of profitability, apply for unsecured loans to obtain external finance as a supplement to their net worth. Moreover, investment projects yield a risky outcome. In principle, there are three potential types of information asymmetry in this setup. First, entrepreneurs are better informed about the quality of their projects ex-ante, and low quality project holders have an incentive to pretend to have a high quality project in order to borrow at a lower rate. Second, entrepreneurs are better informed about the outcome of their risky projects ex-post, and successful entrepreneurs have an incentive to pretend to go bankrupt in order not to pay back their debt. Third, entrepreneurs may deviate from investing the loan in their contracted project ex-post and simply use the loan for another purpose, to finance a riskier project, for instance, or just to consume.

Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) (hereafter CF) and Bernanke, Gertler and Gilchrist (1999) (hereafter BGG) are the pioneering works studying the second problem, in which borrowers tend to hide project outcomes absent monitoring. In the literature following CF and BGG, monitoring costs have been widely used in models with credit market frictions, studying a wide variety of issues regarding firm finance and real-financial interactions.[2] Meanwhile, Holmstrom and Tirole (1997) is a seminal work studying the third, moral hazard problem, in which borrowers tend to undertake riskier projects with high private benefits absent monitoring. In this chapter we instead study the first, adverse selection problem, in which borrowers tend to hide project qualities absent screening, which is also costly. Lenders choose to screen borrowers whenever screening costs are low enough, whenever project qualities are sufficiently distinct, which increases the uncertainty about project outcomes, and whenever borrowers with high quality projects are relatively scarce. Thus, screening is more likely during adverse economic conditions.

Bernanke and Gertler (1990) argue that monitoring costs are tractable and simple to implement, but empirically too small to initiate financial distress. Similarly, Wang and Williamson (1998) state that the costs that appear to be most important for real world financial intermediaries are not ex post verification costs but ex ante costs of information acquisition. For lenders, these costs are primarily associated with the screening of loan applicants. As an alternative to monitoring costs, Kiyotaki and Moore (1997) model agency costs as arising from enforcement problems, which lead to secured lending, as borrowers cannot be forced to repay

[2] See among many others, Cooley and Quadrini (2001), who show that financing constraints associated with costly monitoring can explain firm growth patterns. Christiano, Motto, and Rostagno (2010) analyze financial factors in economic fluctuations making use of a costly state verification framework. Covas and den Haan (2010) use the costly monitoring framework to match the cyclical behavior of debt and equity issuance.

unless the debt is secured. Thus, aggregate shocks affect the value of collateral and the ability to obtain external finance, which amplifies and propagates the initial shock. However, regardless of whether loans are secured or unsecured, both Kiyotaki and Moore and CF/BGG type models disregard the fact that there are diverse types of loan applicants ex-ante, who are screened and thus charged type specific loan terms by the lenders, and in some cases are credit rationed or simply denied a loan. This paper studies ex-ante borrower heterogeneity and costly screening in credit markets as opposed to ex-post verification and enforcement problems, and shows that a screening model is equally capable of creating net worth effects.

Following the modeling approach in CF, this paper embeds this particular ex-ante asymmetric information problem in an otherwise standard RBC model, ruling out ex-post monitoring, moral hazard and enforcement problems. The financial contract is based on De Meza and Webb (1987), who study the effects of private information about project types on aggregate investment and on the financial structure of borrowers, and on Wang and Williamson (1998), who study optimality of debt contracts under costly screening of borrower types. Our findings suggest that the ex ante asymmetric information problem creates net worth effects that enhance the propagation of aggregate shocks as much as ex-post verification problems. As lenders are especially likely to resort to costly screening during adverse economic conditions, net worth effects that induce persistent investment and output dynamics tend to occur especially in bad times, which makes economic busts deeper and longer than booms. In addition, in our model different productivity groups are charged different loan rates, and there is credit rationing in the sense that applicants with low quality investment projects do not obtain loans. Thus, screening costs may constitute an empirically plausible alternative to monitoring costs to motivate agency costs in unsecured lending, enabling models to generate risk pricing and credit rationing.[3]

Previous dynamic equilibrium studies with ex-ante information frictions, which are closely related to this paper include Kurlat (2010), House (2006) and Hobijn and Ravenna (2009). Kurlat (2010) focuses on adverse selection in financial markets and studies a dynamic economy featuring ex-ante asymmetric information about project quality. The main difference between this paper and Kurlat's is that the latter features a "lemons" problem as in Akerlof (1970), where borrowing constrained entrepreneurs sell past projects with privately observed quality to finance new ones. In this paper, entrepreneurs are allowed both to borrow to raise external funds and to use the outcomes of past projects to raise internal funds. This paper especially focuses on the effects of ex ante information acquisition problems on borrowing and the screening outcome that arises to separate borrowers as in actual credit markets. In Kurlat (2010), asymmetric information gives rise to an adverse selection problem that causes bad projects to drive good ones out of the market, whereas in our model asymmetric information causes good projects to draw in bad ones.[4]

House (2006) studies how ex ante asymmetric information about both riski-

[3]Note that relationship lending, which mitigates both ex-ante and ex-post asymmetric information between lenders and borrowers, is an important feature of lines of credit extended to small businesses, as shown in Berger and Udell (1995). This paper abstracts from relationship lending to focus on the wealth effects induced by ex-ante asymmetric information per se.

[4]This distinction is due to De Meza and Webb (1987).

ness and expected returns of investment projects distorts loan markets and affects the stability of an economy. Aggregate shocks affect the distribution of a loan pool, which in some cases destabilizes the economy, and in others makes the economy excessively stable (the opposite of a financial accelerator). Stabilizer equilibria may arise when financial frictions cause overinvestment in equilibrium (low types also invest) as in De Meza and Webb (1987). In this case, an increase in net worth can curtail overinvestment. This paper also uses the De Meza and Webb (1987) framework. However, intermediaries here have access to a costly screening technology to separate borrower types, overcoming the overinvestment problem but introducing a new friction in terms of a resource cost. Thus, a financial accelerator arises because of the latter friction, and separation (instead of a loan pool) allows the model to generate type specific loan terms and credit rationing as in actual credit markets.

Hobijn and Ravenna (2009) also feature an adverse selection problem in credit markets and costly screening. The authors incorporate an imperfect credit market into a standard New Keynesian (NK) model of monetary policy, in which consumers with different credit scores need to finance credit good purchases in advance. They obtain an endogenous risk profile of interest rates and analyze how endogenous securitization of loans amplifies the propagation mechanism in their NK model, which adds a new channel for monetary policy to influence the economy. The main difference of this work from theirs is that in the latter, financial frictions have no effect on the balance sheets of the agents, meaning that there is no net worth effect to cause propagation of aggregate shocks. Instead, propagation is achieved by nominal rigidities in the NK model, while adverse selection leads to endogenous securitization of loans, which amplifies this propagation. This paper focuses more on the mechanism by which an adverse selection problem can result in net worth effects on aggregate fluctuations.

The remainder of this chapter is organized as follows: section 2.2 presents the model, characterizes the equilibrium and discusses the solution methodology; section 2.3 presents the model economy's impulse responses to aggregate and investment specific productivity shocks and discusses/compares the implications of different agency cost models; and section 2.4 concludes.

2.2 Model

The economy consists of a continuum of infinitely lived consumers, firms and financial intermediaries. Consumers consist of η measure of entrepreneurs and $1-\eta$ measure of non-entrepreneurs. Firms produce non-storable final goods using capital rented and labor supplied by both types of consumers. Entrepreneurs transform final goods into storable investment goods using a risky investment technology.

Investment technology differs among entrepreneurs because investment projects undertaken may be high or low quality in terms of their returns. Entrepreneurs who undertake their investment projects (labeled as investors) obtain external funds from financial intermediaries who in turn obtain funds from non-entrepreneurs (households) and possibly other entrepreneurs who do not undertake their investment projects (labeled as non-investors). As the quality of investment projects that entrepreneurs hold is private information, low quality project holders have an incentive to lie about the quality of their projects unless they are screened by the intermediaries. Thus, intermediaries may use a costly screening technology to separate borrowers with different quality of projects and make type specific loans to borrowers. If a project is successful, the borrower repays the loan; if not, the borrower goes bankrupt and restructures in the next period.

2.2.1 Timeline

Here is a timeline of events in a given period of the model:

1. Aggregate productivity is realized.

2. Firms hire labor and rent capital from both households and entrepreneurs, make factor payments and produce output given aggregate productivity.

3. Entrepreneurs learn the quality of their projects and become investors or noninvestors given idiosyncratic and aggregate states for investment.

4. Households and noninvestors make their consumption-saving decisions over their labor and capital income. In order to save, they purchase investment goods from banks in exchange for final goods.

5. Investors exchange their capital for final goods with banks to raise internal funds and they enter into a loan contract with banks to raise external funds.

6. Investors undertake their investment projects. If they are successful, they repay the loans; if not, they go bankrupt.

7. Investors who are still solvent make their consumption-saving decisions.

Thus, capital purchased by consumers from the banks comes from three distinct sources. First, before investment takes place, investors exchange their accumulated capital for final goods with banks to raise internal funds. Second, after investment takes place, successful entrepreneurs repay the loans of final goods taken out from banks in terms of newly created capital. Third, entrepreneurs who are still solvent make their consumption-saving decisions, exchanging a portion of their new capital for consumption goods with banks and saving the rest.

2.2.2 Households' Problem

Households are identical and decide how much to work, consume and save in each period t. They enter period t with 1 unit of time endowment and k_t units of accumulated capital. They supply labor l_t to firms for wage w_t and rent capital to firms for rental rate r_t. They consume c_t of their labor and capital income. Since the consumption good is nonstorable, in order to save for the future they purchase claims to investment goods from the banks at price q_t.[5] Thus, households choose c_t, l_t and k_{t+1} to maximize lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t) \tag{2.1}$$

subject to the period t budget constraint

$$c_t + q_t k_{t+1} = [r_t + q_t(1 - \delta)]k_t + w_t l_t. \tag{2.2}$$

[5]Put differently, households transform their unconsumed income into investment goods at a transformation rate of $1/q_t$. This rate is the risk free rate on household deposits in terms of investment goods.

β denotes the subjective discount factor of the representative household and δ denotes the rate of depreciation of capital.[6] Thus, the solution to the household's problem yields labor supply and savings supply (investment good demand) choices characterized by the first order conditions

$$\frac{u_l(t)}{u_c(t)} = w_t \tag{2.3}$$

$$q_t u_c(t) = \beta E_t u_c(t+1)[r_{t+1} + q_{t+1}(1-\delta)] \tag{2.4}$$

respectively.

2.2.3 Firms' Problem

Firms are identical and decide how much labor to hire and capital to rent from households and entrepreneurs in each period t in order to produce output Y_t of the final good, which is consumed or transformed into capital. They maximize profits

$$\Pi_t = Y_t - r_t K_t - w_t H_t - w_t^e H_t^e \tag{2.5}$$

where K_t is the aggregate capital stock of all agents, H_t is aggregate household labor and H_t^e is aggregate entrepreneurial labor. Output at time t is given by $Y_t = z_t F(K_t, H_t, H_t^e)$, where F is a constant returns to scale (CRS) production technology and z_t is aggregate productivity.

The solution to the firms' problem yields the first order conditions

$$r_t = z_t F_1(t),\ w_t = z_t F_2(t),\ w_t^e = z_t F_3(t) \tag{2.6}$$

[6]Households also own an equal equity share in each of the firms and the banks, profits of which are zero in equilibrium.

which characterize aggregate factor demands, payments of which exhaust the aggregate output completely.

2.2.4 Entrepreneurs' Problem

Entrepreneurs enter period t with 1 unit of time endowment and k_t^e units of accumulated capital; and they choose how much to invest, consume and save during period t. At the beginning of each period, entrepreneurs supply labor inelastically to firms for wage w_t^e and rent capital to firms for rental rate r_t. Then they privately face an investment project with a random quality, which consists of a risky technology for transforming final goods into investment goods. They invest i_t units of final goods in their projects, and after project outcomes are revealed they consume c_t^e and they save the rest of their resources to accumulate capital k_{t+1}^e. They maximize lifetime utility

$$E_0 \sum_{t=0}^{\infty} (\beta^e)^t c_t^e, \tag{2.7}$$

subject to period t budget constraint

$$c_t^e + q_t k_{t+1}^e = y_t^e, \tag{2.8}$$

where, y_t^e denotes ex-post entrepreneurial income in period t, the form of which will be specified below. Note that entrepreneurs are risk neutral and they discount the future at rate β^e.

Investment projects yield a risky return $x_t \in \{0, \bar{x}_t\}$ of capital goods for each unit of final goods invested, where $0 < \bar{x}_t$. However, investment projects differ in quality, which can be either high or low. Entrepreneurs can face a high quality project (type H) with probability α or a low quality project (type L) with probability $1-\alpha$. A high quality project has a higher success probability $p^H > p^L$ of obtaining a nonzero return. Thus, investing i_t units of final goods in a type $j \in \{H, L\}$ project yields an expected return of $p^j \bar{x}_t i_t$ capital goods in period t, which is higher for a high quality project.

Entrepreneurs may finance investment projects both internally and externally. In order to raise internal funds, entrepreneurs may use their labor and capital income, together with their undepreciated capital exchanged for final goods with the banks. These transactions yield a net worth of

$$n_t = w_t^e + k_t^e[r_t + q_t(1-\delta)] \tag{2.9}$$

in terms of final goods.

Given their observed project type $j \in \{H, L\}$, in order to raise external funds entrepreneurs may borrow $b_t^j \geq 0$ from the banks with gross interest rate $R_t^j(x_t)$ depending on the project outcome (and possibly other borrower characteristics) so that they can invest up to $i_t^j \leq n_t + b_t^j$ final goods in their projects.[7,8] Given $R_t^j(x_t)$

[7]Since the type of a borrower with project quality $j \in \{H, L\}$ is private information, entrepreneurs may have an incentive to lie about their types and try to borrow on more favorable terms, lowering banks' expected profits. Thus, we are anticipating here that banks may design contracts in which type $j \in \{H, L\}$ project holders pay $R_t^j(x_t)$ on their loans b_t^j. Note that $R_t^j(x_t) = R_t(x_t)$ and $b_t^j = b_t \; \forall j \in \{H, L\}$ is also an outcome, which pools the types.

[8]No moral hazard assumption implies $i_t^j \geq b_t^j$. Thus, borrowers use the loans only for investment purposes, i.e. they do not consume the loan.

and the loan amount b_t^j, they choose i_t^j to maximize expected profits from investment

$$\pi_t^{e,j} = q_t[p^j(\bar{x}_t i_t^j - R_t^j(\bar{x}_t)b_t^j) - (1-p^j)R_t^j(0)b_t^j] \tag{2.10}$$

which should exceed the opportunity cost $(i_t^j - b_t^j)$ of investing out of their net worth. Moreover, required repayment on loans should satisfy $0 \leq R_t^j(x_t)b_t^j \leq x_t i_t^j$ by limited liability assumption, which implies $R_t^j(0)b_t^j = 0 \;\forall j \in \{H, L\}$. Thus, when project is unsuccessful, banks cannot extract resources from the borrower, and the last term in brackets in the profit function is redundant.

Thus, ex-post income y_t^e of an entrepreneur with net worth n_t, a j quality project, a loan amount $b_t^j \geq 0$ and investment $i_t^j \geq 0$ will be given by

$$y_t^{e,j} = \mathbf{1}_{\{i_t>0\}}\{q_t[x_t i_t^j - R_t^j(x_t)b_t^j] + (n_t + b_t^j - i_t^j)\} + (1 - \mathbf{1}_{\{i_t>0\}})n_t \tag{2.11}$$

where the index function $\mathbf{1}_{\{.\}}$ takes the value 1 if $i_t > 0$ (investor) and 0 if $i_t \leq 0$ (non-investor). Thus, if an entrepreneur with a $j \in \{H, L\}$ quality project invests, he consumes and saves out of his profits from investment plus uninvested net worth. If he does not invest, he consumes and saves out of his initial net worth n_t. Hence, non-investors may also exchange final goods for investment goods with banks for saving purposes, as households do.

An entrepreneur who is still solvent after investment outcomes are realized will compare the trade off between the benefit of consuming today and the cost of foregoing future returns on internal funds. As entrepreneurs take as given the loan rate and amount before making their investment decisions, we must consider the bank's problem prior to solving the entrepreneurs' problem.

2.2.5 Banks' Problem

Banks are identical and should be thought of as capital mutual funds as in CF, through which capital acquired by households and non-investors in exchange for final goods becomes funding for investors to undertake projects.

In each period t, each bank enters into a loan contract with an entrepreneur who has net worth n_t, which is common knowledge, and who has project type $j \in \{H, L\}$, which is private information.[9] A loan contract can be structured either to pool the two types or to separate them by using a costly screening technology.

A separating contract features screening of borrower types, and type specific loan amounts and loan rates contingent on project outcomes. A separating loan contract offered by the bank to an entrepreneur with net worth n_t, who reports his type to be $j \in \{H, L\}$, consists of a nonnegative loan rate and a nonnegative loan amount pair $(R_t^j(x_t), b_t^j)$, which maximizes expected profits $\pi_t^{e,j}$ of the borrower given by (2.10) subject to the participation constraint of the bank

$$q_t[p^j R_t^j(\bar{x}_t) b_t^j + (1-p^j) R_t^j(0) b_t^j] \geq b_t^j + s_t^j \mu \frac{(b_t^j)^2}{n_t} \tag{2.12}$$

We should also include the feasibility constraint $0 \leq R_t^j(x_t) b_t^j \leq x_t i_t^j$ implied by limited liability. Note that an additional constraint is the participation constraint $\pi_t^{e,j} \geq i_t^j - b_t$ of the borrower, which may or may not be satisfied (trivially satisfied) in a separating equilibrium.[10,11]

[9] Note that contracting takes place after the aggregate state is realized and firm production occurs. Thus, n_t is taken as given during contracting and there is no aggregate uncertainty.

[10] A subset of loan applicants might not find borrowing optimal at the offered contractual terms, or banks may not supply loans for a subset of loan applicants.

[11] Note that costly screening reveals borrower types perfectly, so that an incentive constraint is

In equation (2.12), the left hand side gives the expected return to the bank from the loan in terms of final goods, which should exceed the cost of lending given by the right hand side. The first term on the right hand side is the opportunity cost of lending, which is simply the loan amount since the contracted loans are intraperiod. The second term is the total cost of screening a borrower's type, which depends on the parameter μ, and which is convex in the loan amount and inversely related to a borrower's net worth.[12] $s_t^j \in \{0, 1\}$ identifies if a type j borrower is screened, taking the value one if screening takes place and zero otherwise.[13]

A pooling contract offered by the bank does not involve screening, which implies $s_t^H = s_t^L = 0$. A pooling contract consists of a common loan rate $R_t(x_t) = R_t^H(x_t) = R_t^L(x_t)$ and a common loan amount $b_t = b_t^H = b_t^L$ maximizing the joint expected profits $\alpha\pi_t^{e,H} + (1-\alpha)\pi_t^{e,L}$ of the borrowers subject to the bank's participation constraint

$$\alpha\ [p^H R_t(\bar{x}_t) + (1 - p^H)R_t(0)]\ b_t + (1 - \alpha)\ [p^L R_t(\bar{x}_t) + (1 - p^L)R_t(0)]\ b_t \geq b_t \quad (2.13)$$

not necessary in this setup.

[12]Note that any functional form that is increasing in the leverage ratio b_t/n_t would work in this setup as screening costs. This specific cost function, quadratic in the loan amount and decreasing in borrower net worth, is chosen for analytical simplicity. Although just as ad hoc as the monitoring costs used in CF/BGG models that are linearly increasing in net worth, this functional form accounts for the fact that lenders become pickier as the loan amount to be disbursed rises, and that smaller/younger firms (borrowers) with lower wealth are subject to higher agency costs in credit markets.

[13]Note that the contract space is limited to pure strategies as in CF and BGG. Actually, it turns out in this setup that lenders would choose to screen borrowers with probability one even when randomization is allowed.

and the participation constraints $\pi_t^{e,j} \geq i_t^j - b_t$ for each type $j \in \{H, L\}$.

In equation (2.13), the left hand side gives the expected return to the bank from the loan, consisting of the probability of lending to an investor with a high quality project, which is α, times the expected repayment from a high type borrower $[p^H R_t(\bar{x}_t) + (1 - p^H) R_t(0)] b_t$, plus the probability of lending to an investor with a low quality project, which is $1 - \alpha$, times the expected repayment from a low type borrower, which is $[p^L R_t(\bar{x}_t) + (1 - p^L) R_t(0)] b_t$. This return should exceed the loan amount in the right hand side, which is simply the opportunity cost of the funds for the bank.

Thus, banks' problem is to find the equilibrium contract, either a separating or a pooling contract, and to offer the equilibrium loan rates and amounts to borrowers with net worth n_t and a type $j \in \{H, L\}$ project, which can be revealed or unrevealed by the contract. Below we demonstrate that with risk neutral competitive banks, the equilibrium contract is determined by high quality borrower preferences.

2.2.6 Solution to the Banks' and Entrepreneurs' Problems

Given q_t and the contractual terms (b_t^j, R_t^j), an entrepreneur with net worth n_t and a j type project has two independent decisions which are both linear: whether to borrow (to invest) and whether to invest his net worth. An entrepreneur will optimally borrow only if it is profitable, i.e. if $q_t p^j [\bar{x}_t - R_t^j] b_t^j \geq 0$. On the other hand, he will optimally invest his net worth only if $q_t p^j \bar{x}_t n_t \geq n_t$, or $q_t p^j \bar{x}_t \geq 1$.

For the banks, we will first characterize the equilibrium contract under symmetric information, and then demonstrate how asymmetric information changes the outcome.

Proposition 2.1. *Under symmetric information, the equilibrium contract* (b_t^j, R_t^j) *for* $j \in \{H, L\}$ *satisfies* $R_t^H = \frac{1}{q_t p^H} < \frac{1}{q_t p^L} = R_t^L$*, and only high type entrepreneurs borrow and undertake physical investment in the economy.*

Proof. Given q_t and n_t, a bank chooses b_t^j and R_t^j to maximize a borrower's expected return such that it makes nonnegative profits in expectation:

$$\max_{b_t^j,\ R_t^j} q_t p^j [\bar{x}_t - R_t^j] b_t^j \quad \text{such that} \quad q_t p^j R_t^j b_t^j - b_t^j \geq 0$$

Competition yields zero profits for the bank (break even) so that it charges $R_t^j = \frac{1}{q_t p^j}$ for any positive amount of loans.[14] Plugging this into borrower's expected return function and taking the derivative with respect to b_t^j yields the Kuhn-Tucker condition $q_t \leq \frac{1}{p^j \bar{x}_t}$ ($=$ if $b_t^j > 0$). Thus, in equilibrium we have $R_t^H = \frac{1}{q_t p^H} > \frac{1}{q_t p^L} = R_t^L$, and since $p^L < p^H$ we have $q_t = \frac{1}{p^H \bar{x}_t} < \frac{1}{p^L \bar{x}_t}$ from the first order condition, so that $b_t^H > b_t^L = 0$ and only high types borrow in equilibrium. Moreover, rearranging the first order condition as $q_t p^L \bar{x}_t < q_t p^H \bar{x}_t = 1$ we conclude that only high types will invest their net worth, in which case low types may lend part of their net worth to high types indirectly by saving in banks. The risk free rate $R_t^f = \frac{1}{q_t}$ is greater

[14] Technically, the break even constraint is $[R_t^j - \frac{1}{q_t p^j}] b_t^j = 0$. Thus, when $b_t^j = 0$, $R_t^j > 0$ is indeterminate. In this case banks simply deny a loan for type j, and do not offer a loan rate. We can equivalently say that banks would charge $R_t^j = \frac{1}{q_t p^j}$ for any positive loan amount but at this loan rate the loan applicant chooses not to borrow so that $b_t^j = 0$ in equilibrium. This is the case of price rationing.

than the expected return $p^L \bar{x}_t$ on investment for the low types, so that they prefer lending their net worth instead of investing it. $\square$

Corollary 2.1. *Under symmetric information capital's price would be* $q_t = \frac{1}{p^H \bar{x}_t}$ *and the aggregate economy behaves like an economy with transformation rate* $p^H \bar{x}_t$.

Since high type entrepreneurs are the only ones to undertake physical investment in the economy, the transformation rate in the economy reflects their technology. Note that the transformation rate is equal to the risk free return on deposits in terms of investment goods.

When type is private information, low types would have an incentive to lie about their types and borrow at $R_t^H < R_t^L$, which yields negative expected profits to the banks. Thus, banks design contracts either by pooling the two types or separating them using a costly screening technology to ensure nonnegative profits. If types are sufficiently distinct from each other and/or the screening cost is sufficiently low, equilibrium will entail separation as discussed below.

Proposition 2.2. *Under asymmetric information, a pooling contract* (b_t, R_t) *satisfies* $R_t = \frac{1}{q_t[\alpha p^H + (1-\alpha)p^L]}$. *Under equilibrium pooling, the price of capital is* $q_t = \frac{1}{\bar{x}[\alpha p^H + (1-\alpha)p^L]} \equiv q_t^p$, *in which case both types may borrow to invest but only high types would invest their net worth.*

Proof. In a pooling contract, given q_t and n_t, a bank chooses b_t and R_t to maximize borrowers' joint expected return such that it makes nonnegative profits in expectation

$$\max_{b_t,\ R_t} q_t[\alpha p^H + (1-\alpha)p^L][\bar{x}_t - R_t]b_t \quad \text{such that} \quad q_t[\alpha p^H + (1-\alpha)p^L]R_t b_t - b_t \geq 0$$

Competition yields zero profits for a bank so that it charges $R_t = \frac{1}{q_t[\alpha p^H + (1-\alpha)p^L]}$ on loans. Plugging this into borrowers' joint expected return function and taking the derivative with respect to b_t yields the Kuhn-Tucker condition $q_t \leq \frac{1}{[\alpha p^H + (1-\alpha)p^L]\bar{x}_t}$ (= if $b_t > 0$). Thus, stipulating that $b_t > 0$ in equilibrium we get $q_t = \frac{1}{[\alpha p^H + (1-\alpha)p^L]\bar{x}_t}$, in which case we have $R_t = \bar{x}_t$ and both types may borrow. Moreover, rearranging the first order condition as $q_t p^L \bar{x}_t < q_t[\alpha p^H + (1-\alpha)p^L]\bar{x}_t = 1 < q_t p^H \bar{x}_t$ we see that only high types will invest their net worth and low types will lend part of their net worth to high types by saving in banks. Note that the risk free rate is still greater than the expected return for low type investment. □

Corollary 2.2. *A pooling contract generates a pecuniary externality, since low types do not internalize the effect of their borrowing on prices.*

Borrowing of low types decreases the average transformation rate in the economy since low types are using funds that otherwise would be invested by the more efficient high types. Thus, the lower transformation rate implies lower capital accumulation, which yields lower income for the households and lower net worth for the entrepreneurs in the next period.

In a separating contract banks would like to screen only borrowers who claim to be high types, since only low types have an incentive to lie about their types. Thus, $s_t^H = 1$ and $s_t^L = 0$ in a separating equilibrium.

Proposition 2.3. *Under asymmetric information, a separating contract* (b_t^j, R_t^j) *for* $j \in \{H, L\}$ *satisfies* $R_t^L = \frac{1}{q_t p^L}$ *and* $R_t^H = \frac{1}{q_t p^H}[1 + \mu \frac{b_t^H}{n_t}]$. *Under a separating equilibrium, the price of capital is* $q_t = \frac{1}{p^H \bar{x}_t}[1 + 2\mu \frac{b_t^H}{n_t}] \equiv q_t^s$, *which yields a loan* *amount* $b_t^H = [\frac{q_t p^H \bar{x}_t - 1}{2\mu}]n_t$ *for high type borrowers. Since* $q_t > \frac{1}{p^H \bar{x}_t}$*, investing net worth is also optimal for high types.*

Proof. In a separating contract, given q_t and n_t, a bank chooses b_t^j and R_t^j to maximize borrower's expected return such that it makes nonnegative profits in expectation:

$$\max_{b_t^j,\ R_t^j} q_t p^j [\bar{x}_t - R_t^j] b_t^j \quad \text{such that} \quad q_t p^j R_t^j b_t^j - b_t^j - s_t^j \mu \frac{(b_t^j)^2}{n_t} \geq 0$$

Competition yields zero profits for a bank so that it charges $R_t^j = \frac{1}{q_t p^j}[1 + s_t^j \mu \frac{b_t^j}{n_t}]$ for any positive amount of loans. Plugging this into borrower's expected return function and taking the derivative with respect to b_t^j yields the Kuhn-Tucker condition $q_t \leq \frac{1}{p^j \bar{x}_t}[1 + 2\mu s_t^j \frac{b_t^j}{n_t}]$ (= if $b_t^j > 0$). Thus, given $s_t^H = 1$ and $s_t^L = 0$, in equilibrium we get $R_t^L = \frac{1}{q_t p^L}$ and $R_t^H = \frac{1}{q_t p^H}[1 + \mu \frac{b_t^H}{n_t}]$. Moreover, stipulating that high types should borrow in equilibrium, we get from the high types' first order condition that $q_t = \frac{1}{p^H \bar{x}_t}[1 + 2\mu \frac{b_t^H}{n_t}]$. Rearranging this, we calculate the optimal loan amount for high types as $b_t^H = [\frac{q_t p^H \bar{x}_t - 1}{2\mu}]n_t$ and high types would also invest their net worth since $q_t > \frac{1}{p^H \bar{x}_t}$. □

Lemma 2.1. *Given* q_t *and* n_t*, if parameters are such that* $\frac{p^H}{p^L} > [1 + 2\mu \frac{b_t^H}{n_t}]$*, low type agents do not borrow and invest in a separating equilibrium.*

Proof. The first order condition above implies that $\frac{1}{p^L \bar{x}_t} \geq \frac{1}{p^H \bar{x}_t}[1 + 2\mu \frac{b_t^H}{n_t}] = q_t^s$, with equality only if low types are also borrowing. Thus, in case $\frac{p^H}{p^L} > [1 + 2\mu \frac{b_t^H}{n_t}]$ we have $q_t p^L \bar{x}_t < 1$ (or $\bar{x}_t > \frac{1}{q_t p^L} = R_t^L$), so that low types would not borrow and would not invest their net worth. □

Assumption 2.1. *Parameters are such that* $\frac{p^H}{p^L} > [1 + 2\mu \frac{b_t^H}{n_t}]$.

With assumption 2.1, we stipulate that low type borrowers are denied a loan/do not borrow in a separating equilibrium, as in the full information benchmark. Thus, a pooling contract would be preferable to a low type entrepreneur, since in that case he may borrow a positive amount at a given rate. Note that, for any given positive $\frac{b_t^H}{n_t}$, μ and p^H, we can set p^L low enough to satisfy this condition.

Lemma 2.2. *The equilibrium contract will be the contract preferred by high type entrepreneurs.*

If a pooling contact is preferred by both high and low type entrepreneurs, the equilibrium contract will be a pooling contract. If a high type entrepreneur prefers a separating contract, and a low type entrepreneur prefers a pooling contract, the equilibrium contract will be a separating contract. Any deviation by a bank from offering separating to pooling contracts will only attract low type borrowers, invalidating pooling.

Lemma 2.3. *Given q_t and n_t, a high type borrower prefers a separating contract over a pooling contract whenever parameters satisfy $[\bar{x}_t - \frac{1}{\bar{p}q_t}]b_t < [\bar{x}_t - \frac{1}{p^H q_t}(1 + \mu\frac{b_t^H}{n_t})]b_t^H$, where $\bar{p} = [\alpha p^H + (1-\alpha)p^L]$.*

Proof. A high type entrepreneur prefers a separating contract over a pooling contract if it entails higher profits given q_t and n_t, i.e., $q_t p^H[\bar{x}_t - R_t]b_t < q_t p^H[\bar{x}_t - R_t^H]b_t^H$, where R_t and R_t^H are as in Propositions 2.2 and 2.3, respectively. □

Assumption 2.2. *Parameters are such that $[\bar{x}_t - \frac{1}{\bar{p}q_t}]b_t < [\bar{x}_t - \frac{1}{p^H q_t}(1 + \mu\frac{b_t^H}{n_t})]b_t^H$.*

With assumption 2.2, we guarantee the existence of a separating equilibrium given q_t and n_t. Note that, given positive b_t^H and b_t offered for some q_t and n_t, if we set p^L low enough that $\bar{p}$ approaches $\frac{1}{q_t \bar{x}_t}$ or set $[1+\mu\frac{b_t^H}{n_t}] < \frac{p^H}{\bar{p}}$ low enough, the condition in Assumption 2.2 is satisfied. Thus, the equilibrium will entail a separating contract only if types are sufficiently distinct from each other (p^H is high enough relative to p^L) and/or screening costs are sufficiently low (μ is sufficiently low), and/or there are sufficiently many low type borrowers (α is sufficiently low, so that $\bar{p}$ is sufficiently low).

Proposition 2.4. *Under Assumptions 2.1 and 2.2, the equilibrium contract will be a separating contract, in which investment is undertaken only by high type entrepreneurs.*

Proof. Follows from Lemmas 2.1 - 2.3. □

In the remainder, we will proceed maintaining Assumptions 2.1 and 2.2, by which equilibrium entails separation and only high type entrepreneurs will borrow and invest in equilibrium. Note also that when $q_t^s = \frac{1}{p^H \bar{x}_t}[1+2\mu\frac{b_t^H}{n_t}] < \frac{1}{[\alpha p^H + (1-\alpha)p^L]\bar{x}_t} = q_t^p$, a separating contract would be more *efficient* as it yields a higher transformation rate. Rearranging $q_t^s < q_t^p$ we have $[1+2\mu\frac{b_t^H}{n_t}] < \frac{p^H}{\bar{p}}$ as a condition for separation to be more efficient.[15]

[15] Under the parametrization used later, Assumptions 2.1, 2.2 and the efficiency condition will be satisfied. Note that whenever the efficiency condition for separation holds, so do Assumptions 2.1 and 2.2.

2.2.7 Aggregation

Given that we have a separating equilibrium, in which investment is only undertaken by high type entrepreneurs, the amount of final goods devoted to capital formation (investment) by a high type entrepreneur will be determined by the sum of his loan $b_t^H = [\frac{q_t p^H \bar{x}_t - 1}{2\mu}]n_t$ from the bank, as derived in Proposition 2.3, and his net worth n_t. Thus, $i_t^H = b_t^H + n_t$ yields

$$i_t^H(q_t, n_t) = [\frac{q_t p^H \bar{x}_t - 1}{2\mu} + 1]n_t \tag{2.14}$$

which can be aggregated into total investment I_t^H by the help of linearity

$$I_t^H(q_t, n_t^H) = \eta\alpha i_t^H(q_t, n_t^H) \tag{2.15}$$

where n_t^H denotes the mean net worth holdings of high type borrowers and $\eta\alpha$ is the measure of high type borrowers in the economy. Thus, aggregate investment I_t^H depends only on mean net worth holdings n_t^H of high type borrowers, and capital's price q_t. Expected capital output at the aggregate level will be given by

$$I_t^{H^S}(q_t, n_t^H) = p^H \bar{x}_t I_t^H(q_t, n_t^H). \tag{2.16}$$

which is the capital supply curve for the economy, that is upward sloping in q_t.

Note that $R_t^H = \frac{1}{q_t p^H}[1 + \mu \frac{b_t^H}{n_t}] = \frac{1}{2}[\frac{1}{q_t p^H} + \bar{x}_t]$ is the same for all high type borrowers since $\frac{b_t^H}{n_t} = \frac{q_t p^H \bar{x}_t - 1}{2\mu}$ implies that all high type borrowers will have the same $\frac{b_t^H}{n_t}$ in equilibrium. Thus, all high type borrowers will invest a common multiple $\bar{d}_t = \frac{i_t^H}{n_t}$ of their net worth, which we call the leverage ratio.

As discussed above, after investment outcomes are realized, entrepreneurs with positive ex-post income y_t^e given by equation (2.11) compare the trade off between the benefit of consuming today and the cost of foregoing the future return on internal funds.[16] The internal solution to this trade off is characterized by

$$q_t = \beta^e E_t\ [r_{t+1} + q_{t+1}(1-\delta)][ER_{t+1}] \tag{2.17}$$

where ER_{t+1} stands for expected return to internal funds for investment, which is calculated as

$$ER_{t+1} = \frac{\alpha[q_{t+1}p^H(\bar{x}_{t+1}i^H_{t+1} - R^H_{t+1}b^H_{t+1})] + (1-\alpha)n_{t+1}}{n_{t+1}}. \tag{2.18}$$

This equation tells us that at time $t+1$ an entrepreneur with net worth n_{t+1} will have a high quality project with probability α, and in that case he will pour his net worth into his project, which has an expected return of $[q_{t+1}p^H(\bar{x}_{t+1}i_{t+1} - R^H_{t+1}b^H_{t+1})]$. With probability $1-\alpha$ he will have a low quality project and will keep his net worth n_{t+1}. Note that ER_{t+1} is greater than one, which incentivizes entrepreneurs to postpone consumption and accumulate net worth. This may lead to complete self financing. We assume entrepreneurs to be more impatient than households so that entrepreneurial consumption will be high enough to ensure an ongoing need for external financing ($i^H_t > n_t$), since complete self financing would render the agency costs irrelevant.[17]

Plugging the common leverage factor $\bar{d}_{t+1} = \frac{i^H_{t+1}}{n_{t+1}}$ into equation (2.18), we have

$$ER_{t+1} = \alpha\ [q_{t+1}p^H(\bar{x}_{t+1}\bar{d}_{t+1} - R^H_{t+1}(\bar{d}_{t+1} - 1))] + (1-\alpha) \tag{2.19}$$

[16] See Appendix B.1 for an explicit statement and solution to this problem.

[17] Note that from the perspective of entrepreneurs, equation (2.17) need not hold with equality as there may be corner solutions due to risk neutrality. However, we stipulate an internal solution in equilibrium and compute the aggregate prices that support it.

which implies that equation (2.17) is common to all solvent entrepreneurs since ER_{t+1} is independent of individual net worth.

Finally, aggregating entrepreneurial budget constraints given by equations (2.8) - (2.11) across all high and low types, we obtain

$$K_{t+1}^{H} = \eta\alpha p^{H} n_t^{H}[\bar{x}_t\bar{d}_t - R_t^{H}(\bar{d}_t - 1)] - \eta\alpha\frac{c_t^{H}}{q_t} \tag{2.20}$$

$$K_{t+1}^{L} = \eta(1-\alpha)\frac{n_t^{L} - c_t^{L}}{q_t} \tag{2.21}$$

where K_t^j denotes aggregate entrepreneurial capital stock for type $j = H, L$ and n_t^j denotes average entrepreneurial net worth. Similarly, c_t^j now denotes average entrepreneurial consumption with a slight abuse of notation. These aggregation results suggest that one needs to keep track of only mean net worth holdings of each type of entrepreneur to see how net worth affects the aggregate economy.

2.2.8 Equilibrium

Assuming a linear investment technology makes it simple to aggregate over entrepreneurs with considerable net worth heterogeneity since only mean net worth holdings of each type of entrepreneur affect the aggregate economy. Thus, we consider a high type entrepreneur who holds the average level of net worth among all high type entrepreneurs, and a low type entrepreneur who holds the average level of net worth among all low type entrepreneurs, together with the representative household, firm and bank in this economy. Letting small letters other than prices indicate population averages and capital letters indicate aggregate amounts, a competitive equilibrium is defined as below:

Competitive Equilibrium A competitive equilibrium for this economy consists of a set of quantities Y_t, K_{t+1}, K^H_{t+1}, K^L_{t+1}, H_t, H^e_t, c_t, k_{t+1}, l_t, c^H_t, c^L_t, k^H_{t+1}, k^L_{t+1}, n^H_t, n^L_t, i^H_t, i^L_t, b^H_t, b^L_t, s^H_t, s^L_t, a set of prices r_t, w_t, w^e_t, R^H_t, R^L_t, q_t and a set of exogenous processes z_{t+1}, $\bar{x}_{t+1}$ that are stationary functions of a set of aggregate states $(K_t, K^H_t, K^L_t, z_t, \bar{x}_t)$, such that

1. Given prices r_t, w_t and q_t, the representative household optimally chooses c_t, l_t and k_{t+1} to solve the households' problem;

2. Given prices r_t, w^e_t, q_t and R^j_t for a type $j \in \{H, L\}$ entrepreneur and the productivity state $\bar{x}_t$, the j type entrepreneur holding the mean net worth n^j_t chooses c^j_t, k^j_{t+1} and i^j_t (given b^j_t) optimally to solve the entrepreneurs' problem;

3. Given prices r_t, w_t and w^e_t, and the aggregate state z_t, the representative firm chooses K_{t+1}, H_t and H^e_t to solve the firms' problem;

4. Given capital's price q_t, productivity state $\bar{x}_t$ and the mean net worth holdings n^j_t of a type $j \in \{H, L\}$ borrower, the representative bank chooses R^j_t, b^j_t and s^j_t to solve the banks' problem;

5. Consumption goods, investment goods and labor markets clear:

$$Y_t = z_t F(K_t, H_t, H_t^e)$$

$$= (1-\eta)c_t + \eta[\alpha(c_t^H + i_t^H) + (1-\alpha)(c_t^L + i_t^L)] + \eta\mu[\alpha s_t^H \frac{(b_t^H)^2}{n_t^H} + (1-\alpha)s_t^L \frac{(b_t^L)^2}{n_t^L}]$$

$$[K_{t+1} - (1-\delta)K_t] = \eta\alpha p^H \bar{x}_t i_t^H + \eta(1-\alpha)p^L \bar{x}_t i_t^L$$

$$H_t = (1-\eta)l_t$$

$$H_t^e = \eta;$$

6. Perceptions of the aggregates are correct:

$$K_t^H = \eta\alpha k_t^H$$

$$K_t^L = \eta(1-\alpha)k_t^L$$

$$K_t - K_t^H - K_t^L = (1-\eta)k_t.$$

Thus, given that $s_t^H = 1$ and $s_t^L = b_t^L = i_t^L = 0$ in the equilibrium contract, the competitive equilibrium for this economy is characterized by twelve equations in twelve unknowns K_{t+1}, K_{t+1}^H, K_{t+1}^L, H_t (or $(1-\eta)l_t$), c_t, c_t^H, c_t^L, n_t^H, n_t^L, i_t^H, R_t^H, q_t given the exogenous processes for z_t and $\bar{x}_t$. These equations are:

Household equilibrium conditions

$$\frac{u_l(t)}{u_c(t)} = z_t F_H(t) \tag{2.22}$$

$$q_t u_c(t) = \beta E_t u_c(t+1)[z_{t+1}F_K(t+1) + q_{t+1}(1-\delta)] \tag{2.23}$$

$$(1-\eta)c_t + q_t[K_{t+1} - K_{t+1}^H - K_{t+1}^L] = [z_t F_K(t) + q_t(1-\delta)][K_t - K_t^H - K_t^L] + z_t F_H(t)H_t \tag{2.24}$$

Market clearing conditions

$$K_{t+1} = (1-\delta)K_t + \eta\alpha p^H \bar{x}_t i_t^H \tag{2.25}$$

$$(1-\eta)c_t + \eta\alpha(c_t^H + i_t^H + \mu\frac{(i_t^H - n_t^H)^2}{n_t^H}) + \eta(1-\alpha)c_t^L = z_t F(t) \tag{2.26}$$

Terms of the equilibrium contract

$$R_t^H = \frac{1}{q_t p^H}[1 + \mu(\frac{i_t^H}{n_t^H} - 1)] \tag{2.27}$$

$$i_t^H = [\frac{q_t p^H \bar{x}_t - 1}{2\mu} + 1]n_t^H \tag{2.28}$$

Entrepreneurial equilibrium conditions

$$n_t^H = z_t F_\eta(t) + \frac{K_t^H}{\eta\alpha}[z_t F_K(t) + q_t(1-\delta)] \tag{2.29}$$

$$n_t^L = z_t F_\eta(t) + \frac{K_t^L}{\eta(1-\alpha)}[z_t F_K(t) + q_t(1-\delta)] \tag{2.30}$$

$$K_{t+1}^H = \eta\alpha n_t^H p^H[\bar{x}_t \frac{i_t^H}{n_t^H} - R_t^H(\frac{i_t^H}{n_t^H} - 1)] - \eta\alpha\frac{c_t^H}{q_t} \tag{2.31}$$

$$K_{t+1}^L = \eta(1-\alpha)\frac{n_t^L - c_t^L}{q_t} \tag{2.32}$$

$$q_t = \beta^e E_t\ [z_{t+1}F_K(t+1) + q_{t+1}(1-\delta)]\{\alpha\ q_{t+1}p^H[\bar{x}_{t+1}\frac{i_{t+1}^H}{n_{t+1}^H} - R_{t+1}^H(\frac{i_{t+1}^H}{n_{t+1}^H} - 1)] + (1-\alpha)\} \tag{2.33}$$

2.2.9 Solution Methodology

Given the deterministic steady state implied by the equations characterizing the equilibrium, model parameters are set to match various empirical regularities and microeconomic findings for the U.S. economy. Equilibrium equations are log-linearized by transforming the variables to represent percentage deviations from steady state. Decision rules are computed by the method of undetermined coefficients using the Schmitt-Grohe and Uribe (2004) algorithm.

Parametrization A period in the model is a quarter. Household preferences are given by $u(c, 1-l) = log(c) + \nu(1-l)$, where $\nu = 2.8$ is chosen so that $l = 0.3$ and work hours account for 30 percent of household time endowment in steady state. We set $\beta = 0.99$, implying a 4 percent annual real rate of interest.

Consumption good production technology is Cobb-Douglas with a capital share of 36 percent, a household labor share of 63.99 percent, and an entrepreneurial labor share of 0.01 percent. Entrepreneurial labor is needed to ensure a positive net worth in each period, which is essential in the financial contract. The labor share for entrepreneurs is set to an arbitrarily small number, so that the model with $\mu = 0$ collapses to a standard RBC model. The capital depreciation rate is set to $\delta = 0.02$.

We set the probability of success for a high type entrepreneur to be $p^H = 99.75$ percent, implying an annual bankruptcy rate of 1 percent. This value is consistent with the quarterly failure rate for the U.S. businesses reported in Fisher (1999) using the Dun-Bradstreet dataset over the period 1984-1994, which is 0.974 percent. The measure of entrepreneurs, η, has no effect on the steady state output ratios in the model economy. Thus, we set $\eta = 0.5$, which is a simple normalization. The measure of high type entrepreneurs, α, turns out to affect only the high and low type shares of the steady state output ratios of entrepreneurial aggregates, given a constant total entrepreneurial share as determined by other model parameters. Thus, we also set $\alpha = 0.5$ as a simple normalization.[18]

[18]Note that η and α can be set to match the corresponding actual measures for entrepreneurs

We set screening parameter μ to 0.5 percent to match a leverage ratio (ratio of debt over equity, b/n) of around 1. This is the value implied by 1987-2009 quarterly data for the U.S. non-financial business sector as calculated in Chugh (2010). This value of leverage implies a 0.03 percent share of total screening costs in output. Finally we define $\beta^e = \gamma\beta$, where $\gamma = 99.25$ percent is set to match an annual risk premium of 3 percent, given by $Rq - 1$ in the model economy. Thus, entrepreneurs discount the future more heavily than do households, which ensures external financing ($i > n$) in equilibrium. Complete self financing is not an interesting case since it eliminates agency costs as discussed above.

For the investment specific productivity process $\bar{x}_t$, we assume that transformed investment specific productivity follows an AR(1) process

$$\hat{x}_t = \phi\hat{x}_{t-1} + e_t \tag{2.34}$$

where $e_t \sim N(0, s^2)$, $\phi = 0.95$ and $s = 0.01$, and where $\hat{x}_t = log(\frac{\bar{x}_t}{\bar{x}})$ and long run average $\bar{x}$ is set to be equal to $1/p^H$, so that the model with $\mu = 0$ collapses to a standard RBC model with a one-to-one transformation rate, i.e., capital's price is unity.

Transformed aggregate productivity is assumed to follow an AR(1) process

$$\hat{z}_t = \rho\hat{z}_{t-1} + \epsilon_t \tag{2.35}$$

where $\epsilon_t \sim N(0, \sigma^2)$, $\rho = 0.95$ and $\sigma = 0.01$, and where $\hat{z}_t = log(\frac{z_t}{z})$ and the long run average is $z = 1$.

in the U.S. economy as well, but this is irrelevant to the aggregate outcomes of the model.

Steady State Given the parametrization above, the average high type entrepreneur's steady state consumption share $\frac{c^H}{n^H}$ is around 5 percent, and the average low type entrepreneur's steady state consumption share $\frac{c^L}{n^L}$ is around 7 percent. The internal rate of return in the steady state is $\frac{1}{\gamma} - 1 = 0.75$ percent and the steady state price of capital is $q = 1.01$.[19] Household consumption is about 76 percent of output and investment is about 23.7 percent of output in the steady state.

2.3 Results

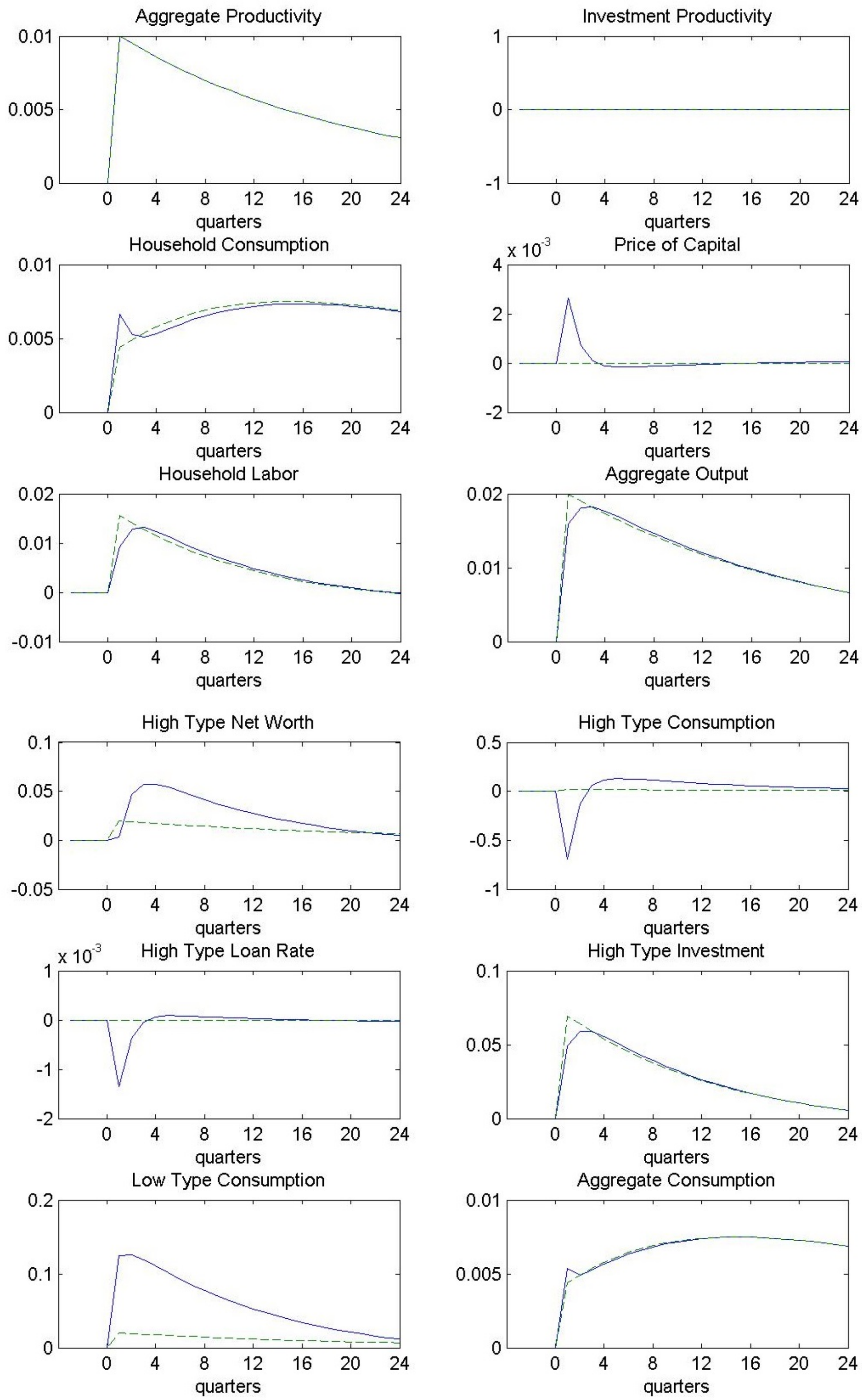
In this section we present and discuss the impulse responses of the model economy to aggregate and investment specific productivity shocks respectively, and make comparisons to the CF and BGG models.

Figure 2.1 presents the behavior of households, entrepreneurs and the aggregate economy in response to a $\sigma = 1$ percent unexpected increase in aggregate productivity from its steady state level.[20] We will discuss the impact effect first and then discuss the dynamics.

When aggregate productivity rises, increases in the marginal product of labor and the marginal product of capital pull up factor prices. As a result, households

[19]Given that in the steady state $q = 1.01$, $\frac{b^H}{n^H} = 1$ and $\bar{x} = \frac{1}{p^H}$, and assuming $p^H = 99.75\%$ and $\alpha = 0.5$, setting $p^L \leq 97.75\%$ so that $\bar{p} \leq 98.75\%$ satisfies Assumptions 2.1 and 2.2 and the efficiency condition.

[20]Household labor approximates aggregate labor since entrepreneurial labor share is negligible and fixed. High type and low type entrepreneurial behavior should be interpreted as the behavior of the entrepreneurs with the average level of net worth among all high type and low type project holders respectively.



Costly Screening Model (–), Standard RBC Model (- -)

Figure 2.1: Model Economy's Response to an Aggregate Productivity Shock

become wealthier and would like to increase both consumption and savings. So consumption increases on impact and investment demand also increases, which leads to increased household lending. Increased investment demand pulls up capital's price. Moreover the increase in wages encourages more labor supply as there is no income effect on labor supply. As aggregate capital is predetermined, output follows the behavior of labor and increases on impact. Note that household consumption rises more on impact compared to a standard RBC model, as the impact increase in capital's price encourages more consumption.

On the entrepreneurs side, increased factor prices increase entrepreneurial net worth, but since capital is predetermined, the rise in net worth on impact is limited. Increased lending by households increases the leverage ratio of the average high type entrepreneur as net worth rises modestly, while the loan amount rises more than net worth does. Entrepreneurial investment also increases on impact as both net worth and loans increase. Meanwhile, the increase in the expected future capital price decreases current consumption and increases saving for high types as the return on internal funds rises. As for the low type entrepreneurs, the wealth effect (increase in net worth) makes them increase both consumption and saving on impact as was the case for households.

We see that the movements in aggregate labor, output and investment in response to a positive aggregate productivity shock display propagation of this shock over time. This results from the delayed peak response of high type entrepreneurial net worth to increased productivity, which increases capital's price and the return on internal funds on impact. As saving of high types increases sharply on impact to exploit the increased internal return, next period's net worth increases sharply. But after the initial period, capital's price starts declining and net worth starts increasing less. After the price of capital goes back to its steady state level, net worth starts declining and eventually tracks the movement of productivity as in a standard RBC model.[21]

Investment mainly follows the movement of high type net worth. After its initial increase, household consumption displays a reverse hump, mimicking the movement in capital's price and reflecting the hump-shaped behavior in investment. An increase in investment supply must be accompanied by an increase in savings supply (or investment demand) by households and a resulting decrease in household consumption. As consumption decreases after the initial impact, labor supply increases more, pulling consumption up again in the next period. Then labor supply starts falling gradually, displaying a hump-shaped behavior overall, which is followed by output. Five quarters after the initial impact, net worth effects seem to disappear and the economy starts tracking the persistent productivity shock towards the steady state as in a standard RBC model.

High type consumption rises after its initial decline due to the spike in entrepreneurial savings. As the internal return to investment gradually falls, entrepreneurial savings decline and consumption picks up. Low type consumption also shows a hump-like shape, as low types de-accumulate capital and thereby decrease their net worth, saving less for future investment, lending more to high types and

[21]See appendix B.2 to see the effect of an aggregate productivity shock on capital's price q_t as well as the amount of investment in a supply/demand diagram.

consuming the proceeds. Aggregate consumption is a weighted average of household and entrepreneurial consumption, overall showing a hump-shaped behavior as well.

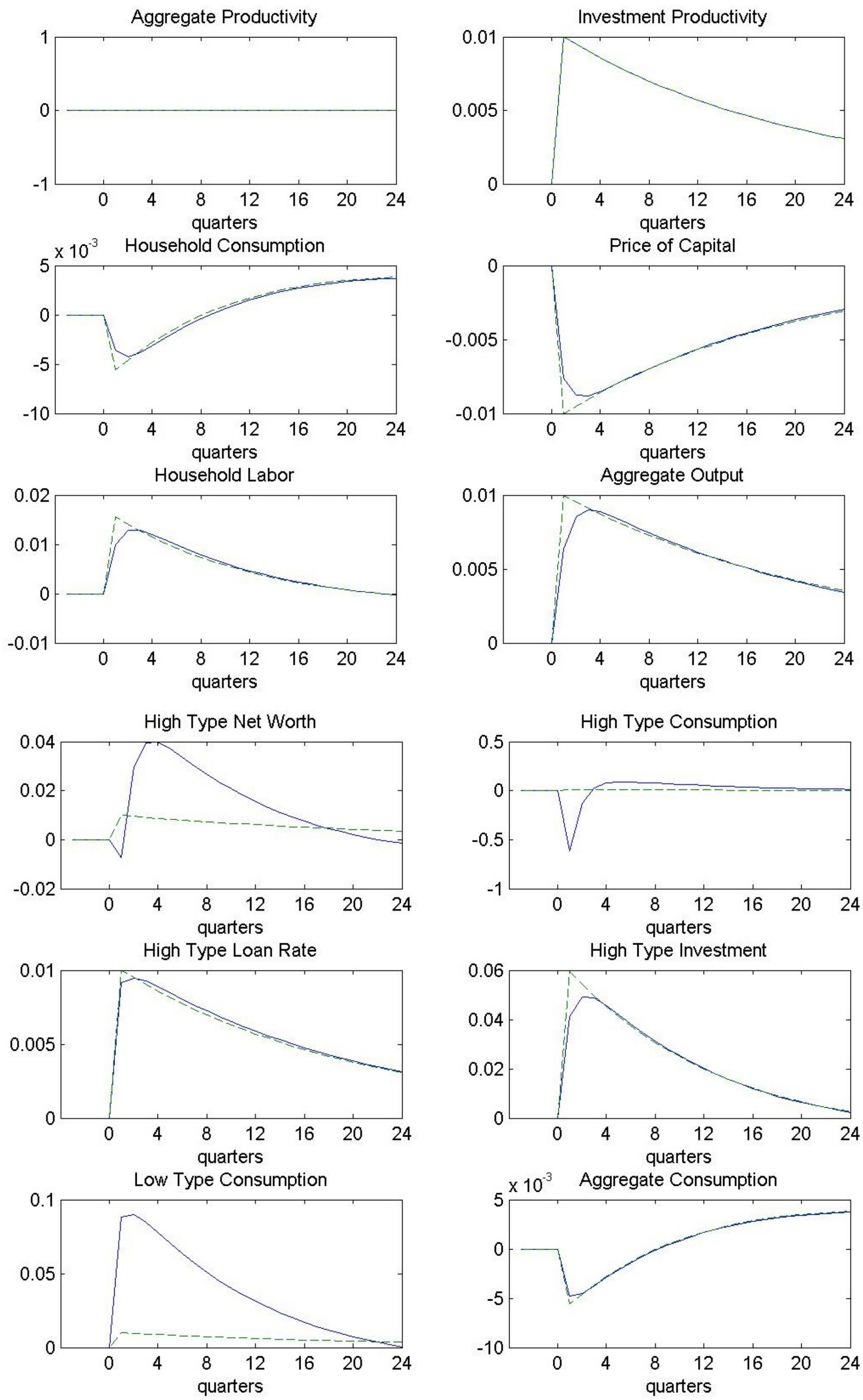
Impulse responses to an aggregate productivity shock in the model show both qualitative and quantitative parallels with the CF model.[22] As a matter of fact, both models also share a drawback: the risk premium, i.e., the difference between the risky loan rate and the riskless return, is procyclical in both models whereas the data shows the opposite. The risk premium in the model is given by $R^H q - 1$. Since the contract is intraperiod, the riskless return for the supplied funds is 1, whereas the risky loan rate is R^H in terms of investment goods, the price of which is q. A positive productivity shock raises the price of capital q and lowers the loan rate R^H on impact, as can be seen from the impulse responses. The increase in q offsets the decrease in R^H, and $R^H q$ increases slightly on impact (less than 0.1 percent increase from its steady state level). Thus, the risk premium $R^H q - 1$ increases.[23]

Figure 2.2 presents the behavior of households, entrepreneurs and the aggregate economy in response to a 1 percent unexpected increase in investment specific productivity from its steady state level.

When investment productivity rises, the transformation rate in the economy rises, facilitating more capital production and pulling down the price of capital. A fall in the price of capital increases the relative price of consumption, which makes households consume less and save more on impact. Household labor increases

[22]See Appendix B.3 for the impulse responses to a productivity shock in the CF model.

[23]It is also clear by Equation (2.27) that the risk premium $R^H q$ is increasing in the leverage ratio b/n, which is procyclical.



Costly Screening Model (–), Standard RBC Model (- -)

Figure 2.2: Model Economy's Response to an Investment Productivity Shock

on impact to be able to supply more funds and sustain future consumption. As aggregate capital is predetermined, output follows the behavior of labor supply and increases on impact.

As the price of capital falls due to higher investment productivity, entrepreneurial net worth falls on impact because of the decreased value of accumulated capital. However, increased household savings lead to higher loan amounts, which increases high type investment on impact despite a lower net worth. After investment takes place, high type entrepreneurs decrease consumption and increase savings on impact, in order to exploit higher future returns from investment. Note that increased investment and savings pull net worth up in subsequent periods for high type entrepreneurs.[24] As for the low type entrepreneurs, they lend more to high types and therefore increase consumption in response to increased investment productivity. Aggregate consumption decreases on impact, as households and high type entrepreneurs decrease consumption.

Note that as net worth falls but borrowing rises, the leverage ratio rises again, pulling up the risk premium as before. The investment productivity shock decreases the price of capital q but raises the loan rate R^H on impact as can be seen from the impulse responses. The decrease in q is offset by the increase in R^H, and $R^H q$ increases slightly on impact (less than 0.1 percent increase from its steady state level). Thus, the risk premium $R^H q - 1$ increases.

As before, investment follows the path of investors' net worth and household

[24]See appendix B.2 to see the effect of an investment productivity shock on capital's price q_t as well as the amount of investment in a supply/demand diagram.

consumption follows the path of capital's price in the periods following the shock. Five quarters after the initial impact, net worth effects seem to disappear and the economy starts tracking the persistent productivity shock towards the steady state as in a standard RBC model.

2.3.1 Discussion

The main reason for the counterfactual behavior of risk premium in these models is that net worth cannot fully respond to shocks immediately, since loan contracts are intraperiod and there is no aggregate uncertainty during contracting. As discussed above, a positive productivity shock raises factor prices and capital's price, which lead to a modest rise in the value of net worth as the capital stock is predetermined. But loan amounts rise more than proportionally to net worth as a result of increased demand for investment goods. Thus, total investment rises with the increased loans, but the internal financing rate falls. A larger share of the total investment is now financed externally, so the leverage ratio increases. Thus, bankruptcy risk in the CF model increases, which drives the risk premium up. In the screening model, bankruptcy risk is constant but higher leverage leads to greater screening and therefore a higher risk premium.

BGG also assumes monitoring costs, but contracts in BGG are interperiod and the aggregate state of the economy affects contracting. A positive productivity shock increases net worth more than the loan amount, so leverage and bankruptcy risk falls, implying a countercyclical risk premium. In CF and screening models, the delayed response of net worth after a positive productivity shock leads to a counterfactual procyclical movement in the risk premium, but it is this delay that gives rise to propagation (hump-shaped behavior) in aggregate variables. In BGG, propagation is achieved by price stickiness and exogenous delays in investment, and agency costs mainly amplify the shocks. In CF there is no amplification, but agency costs do propagate the shocks over time.

The challenge is to set up a model of unsecured lending in which levels and movements in the leverage ratio, risk premium, loan rates and loan amounts are in line with the data, and in which the model matches the dynamics of the key macroeconomic variables and features both amplification and propagation of aggregate shocks as in the data. In addition, the underlying assumptions for credit markets, such as the motivation for agency costs, should be empirically plausible so that the model predictions are reliable. Ex-ante borrower heterogeneity and screening of loan applicants, which results in type specific price and quantity setting for loans, are realistic and therefore a promising candidate for motivating agency costs in unsecured lending.

In this simple model with credit market imperfections, we can see that one advantage of the screening framework over the monitoring framework is that wealth effects could induce more persistent dynamics during adverse economic conditions, when borrower screening is more likely. Lenders choose to screen borrowers only when i) screening technology is cheap, ii) borrower qualities are distinct enough so that screening out good borrowers yields efficiency gains, iii) high quality borrowers are scarce and screening them out eliminates the cross subsidization of low quality borrowers, which all depend on model parameters. The model is solved locally with parameters ensuring the screening outcome, which makes loan terms and in turn investment dependent on borrower net worth, resulting in persistent wealth effects. However, when there is less uncertainty about the quality of borrowers, and when low quality borrowers are scarce as could happen under favorable economic conditions, lenders would choose to pool borrowers instead of screening them out, eliminating net worth effects. In this case, the economy would behave the same as a standard RBC model as shown in Figures 2.1 and 2.2; however, the economy's steady state investment, output and consumption levels would be lower than the first best since pooling leads to a decrease in the aggregate investment productivity (economy's transformation rate). Thus, net worth effects that induce persistent investment and output dynamics tend to occur especially in bad times, which makes economic busts deeper and longer than booms.

2.4 Conclusion

The aim of this paper is to analyze the effects of a particular agency problem – ex-ante asymmetric information in credit markets and costly screening of loan applicants – on the dynamics of key macroeconomic and financial measures. We examine how aggregate shocks are amplified and propagated through net worth effects in a model of ex-ante screening costs compared to a standard model of ex-post monitoring costs, in particular the model by Carlstrom and Fuerst (1997). The principal contribution is to show that screening costs in credit markets can generate net worth effects that are qualitative and quantitatively similar to those generated by the more commonly assumed monitoring costs. Moreover, the screening model generates type specific loan terms endogenously and enables researchers to study many issues prevailing in actual bank lending practices, such as risk pricing and credit rationing.

In terms of future research, this model can be extended to endogenize the measure of low and high type projects to reflect the "tightness" of good projects associated with the aggregate state of the economy. As good projects start dominating the economy, say because of a large positive productivity shock, we expect pooling to be optimal since the total cost of screening would be high; however, when bad projects become more common, we expect separation to be optimal. Thus, one would get an asymmetric loan term setting outcome in credit markets; in good times lenders are less selective in making loans, while in bad times they are pickier, as one would expect to find in the data. This may explain how high risk borrowers become overindebted in good times, which has severe consequences when the economy busts. Another extension could be including collateral constraints in this adverse selection environment, as in Kiyotaki and Moore (1997), and letting collateral values be the tools to separate the types instead of assuming a costly screening technology. Thus, high types self select by choosing high collateral and low interest rates, whereas low types select low collateral and high interest rates. Thus, fluctuations in collateral values through asset price movements can both amplify and propagate the effect of productivity shocks as in Kiyotaki and Moore (1997), in addition to generating different loan terms offered to different risk groups.

Appendix A

A.1 Proofs

<u>*Proof of Proposition 1.1:*</u> Competition yields zero profits (1.4) for the bank. Substituting (1.4) in (1.1) and maximizing we have

$$\max_{i^j} q^e\bar{\gamma}f(i^j) - (1+r)i^j \quad \rightarrow \quad f'(i^j) = \frac{1+r}{q^e\bar{\gamma}} \qquad \square$$

<u>*Proof of Proposition 1.2:*</u> Incentive compatibility constraints are given by

$$\begin{aligned} IC^G: \quad & q^e\bar{\gamma}f(i^G) - [p^G R^G i^G + (1-p^G)c^{e,G}(1+r)i^G] \geq \\ & q^e\bar{\gamma}f(i^B) - [p^G R^B i^B + (1-p^G)c^{e,B}(1+r)i^B] \\ IC^B: \quad & q^e\bar{\gamma}f(i^B) - [p^B R^B i^B + (1-p^B)c^{e,B}(1+r)i^B] \geq \\ & q^e\bar{\gamma}f(i^G) - [p^B R^G i^G + (1-p^B)c^{e,G}(1+r)i^G] \end{aligned}$$

which, plugging in the break even conditions, become

$$\begin{aligned} IC^G: \quad & q^e\bar{\gamma}f(i^G) - (1+r)i^G \geq q^e\bar{\gamma}f(i^B) - [\frac{p^G}{p^B} - (\frac{p^G}{p^B} - 1)c^{e,B}](1+r)i^B \\ \Rightarrow \quad & q^e\bar{\gamma}f(i^G) - (1+r)i^G \geq q^e\bar{\gamma}f(i^B) - (1+r)i^B - [\frac{p^G}{p^B} - 1][1 - c^{e,B}](1+r)i^B \\ IC^B: \quad & q^e\bar{\gamma}f(i^B) - (1+r)i^B \geq q^e\bar{\gamma}f(i^G) - [\frac{p^B}{p^G} + (1 - \frac{p^B}{p^G})c^{e,G}](1+r)i^G \\ \Rightarrow \quad & \underbrace{q^e\bar{\gamma}f(i^B) - (1+r)i^B}_{\Pi^{e,B,s}} \geq \underbrace{q^e\bar{\gamma}f(i^G) - (1+r)i^G}_{\Pi^{e,G,s}} + [1 - \frac{p^B}{p^G}][1 - c^{e,G}](1+r)i^G \end{aligned}$$

IC^B must bind and IC^G must be satisfied. Plugging binding IC^B into IC^G we get $[\frac{p^G}{p^B} - 1][1 - c^{e,B}]i^B \geq [1 - \frac{p^B}{p^G}][1 - c^{e,G}]i^G \quad \Rightarrow \quad \frac{p^G}{p^B} \geq \frac{[1-c^{e,G}]}{[1-c^{e,B}]}\frac{i^G}{i^B}$, which is satisfied as long as $i^G \leq i^B$. In this case, since a good type has no incentive mimicking a bad type, the bad type contract is not distorted. However, binding IC^B characterizes the good type contract together with (1.4). Note that given i^B, $\Pi^{e,G,s}$ decreases in i^G for any given $c^{e,G} < 1$, implying $i^G < i^B$. □

Proof of Proposition 1.3: Expected profits $\lambda \Pi^{e,G,p} + (1-\lambda)\Pi^{e,B,p}$ of the borrower pool is

$$q^e \bar{\gamma} f(i) - \lambda[p^G Ri + (1-p^G)c^e(1+r)i] - (1-\lambda)[p^B Ri + (1-p^B)c^e(1+r)i]$$

$$= q^e \bar{\gamma} f(i) - [\bar{p}Ri + (1-\bar{p})c^e(1+r)i] = q^e \bar{\gamma} f(i) - (1+r)i$$

where the last equality follows from substituting in the break even condition (1.10) for the pool. Maximizing for i yields $f'(i) = \frac{1+r}{q^e \bar{\gamma}}$. □

Proof of Lemma 1.3: See the below proof. □

Proof of Propositions 1.4 - 1.5: Proposition 1.4 follows from Lemmas 1.1 - 1.3. Lemma 1.3 makes use of $f(\iota) = \iota^\theta$ and $\theta f(i^B) = f'(i^B)i^B = \frac{1+r}{q^e \bar{\gamma}} i^B$. Dividing both sides of (1.8) by $(1+r)i^B$ we get

$$[q^e \bar{\gamma} \frac{f(i^G)}{f(i^B)} \frac{f(i^B)}{(1+r)i^B} - \frac{i^G}{i^B}] = [q^e \bar{\gamma} \frac{f(i^B)}{(1+r)i^B} - 1] - (1 - \frac{p^B}{p^G})[1 - c^{e,G}]\frac{i^G}{i^B}$$

$$\rightarrow \quad IC(\frac{i^G}{i^B}) = \frac{1}{\theta}(\frac{i^G}{i^B})^\theta - [1 - (1 - \frac{p^B}{p^G})(1 - c^{e,G})]\frac{i^G}{i^B} - (\frac{1}{\theta} - 1) = 0$$

where $c^{e,G} = \frac{q^e\gamma^0 f(i^G)}{(1+r)i^G} = q^e\gamma^0 \frac{f(i^G)}{f(i^B)}\frac{f(i^B)}{(1+r)i^B}\frac{i^B}{i^G} = \frac{\gamma^0}{\bar{\gamma}\theta}(\frac{i^G}{i^B})^{\theta-1}$, which defines $\frac{i^G}{i^B}$ as a function of θ, $\frac{p^B}{p^G}$ and $\frac{\gamma^0}{\bar{\gamma}}$. Thus, $C^{sp} = \frac{[1-\frac{p^B}{p^G}]}{[\frac{p^G}{\bar{p}}-1]}\frac{[1-c^{e,G}]}{[1-c^e]}\frac{i^G}{i^B}$ only depends on λ, $\frac{p^B}{p^G}$ $\frac{\gamma^0}{\bar{\gamma}}$ and θ, proving Proposition 1.4. Solving $C^{sp} = 1$ for $\bar{p}$ we get

$$\bar{p} = p^G \frac{[1-c^e]i^B}{[1-\frac{p^B}{p^G}][1-c^{e,G}]i^G + [1-c^e]i^B} = p^B + \lambda(p^G - p^B)$$

$$\rightarrow \quad \lambda = \frac{\frac{[1-c^e]}{[1-\frac{p^B}{p^G}][1-c^{e,G}]\frac{i^G}{i^B}+[1-c^e]} - \frac{p^B}{p^G}}{1-\frac{p^B}{p^G}} \leq 1$$

as the unique λ that constitutes a regime switching point given $\frac{p^B}{p^G}$, $\frac{\gamma^0}{\bar{\gamma}}$ and θ. $\square$

Proof of Proposition 1.6: Regardless of the lending regime, total savings is equal to total investment in projects. Thus, $S(A,k) = I^s = I^p$. We compute

$$\frac{I^s}{I^p} = \frac{\lambda i^G + (1-\lambda)i^B}{i} = [\lambda\frac{i^G}{i^B} + (1-\lambda)]\frac{i^B}{i} = 1$$

$$\frac{k'^s}{k'^p} = \frac{\bar{\gamma}[\lambda(i^G)^\theta + (1-\lambda)(i^B)^\theta]}{\bar{\gamma}i^\theta} = [\lambda(\frac{i^G}{i^B})^\theta + (1-\lambda)](\frac{i^B}{i})^\theta < 1$$

$$\frac{q^{e,s}}{q^{e,p}} = \frac{\alpha(k'^s)^{\alpha-1}}{\alpha(k'^p)^{\alpha-1}} = (\frac{k'^s}{k'^p})^{\alpha-1} > 1$$

The second inequality follows from $(\frac{i^B}{i})^\theta = [\lambda\frac{i^G}{i^B} + (1-\lambda)]^{-\theta}$ and the concavity of $f(\iota) = \iota^\theta$, which implies $[\lambda\frac{i^G}{i^B} + (1-\lambda)]^\theta > [\lambda(\frac{i^G}{i^B})^\theta + (1-\lambda)]$.

Since $i^B = (\frac{q^{e,s}\bar{\gamma}\theta}{1+r^s})^{\frac{1}{1-\theta}}$ and $i = (\frac{q^{e,p}\bar{\gamma}\theta}{1+r^p})^{\frac{1}{1-\theta}}$ we compute $\frac{i^B}{i} = [\frac{q^{e,s}/(1+r^s)}{q^{e,p}/(1+r^p)}]^{\frac{1}{1-\theta}}$, which yields

$$\frac{q^{e,s}}{q^{e,p}} = (\frac{k'^s}{k'^p})^{\alpha-1} = [\lambda(\frac{i^G}{i^B})^\theta + (1-\lambda)]^{\alpha-1}[\frac{q^{e,s}/q^{e,p}}{(1+r^s)/(1+r^p)}]^{\frac{\theta}{1-\theta}(\alpha-1)}$$

$$\rightarrow \quad \frac{1+r^s}{1+r^p} = [\lambda(\frac{i^G}{i^B})^\theta + (1-\lambda)]^{\frac{1-\theta}{\theta}}(\frac{q^{e,s}}{q^{e,p}})^{\frac{1-\theta\alpha}{\theta(1-\alpha)}}$$

$$\rightarrow \quad \frac{1+r^s}{1+r^p} = [\lambda(\frac{i^G}{i^B})^\theta + (1-\lambda)]^{\alpha-1}[\lambda\frac{i^G}{i^B} + (1-\lambda)]^{1-\theta\alpha}$$

Now if we show that $[\lambda(\frac{i^G}{i^B})^\theta+(1-\lambda)] > [\lambda\frac{i^G}{i^B}+(1-\lambda)]^{\frac{1-\theta\alpha}{1-\alpha}}$, the last equation implies $r^s_t < r^p_t$. Since $\alpha \in (0,1)$ and $\frac{1-\theta\alpha}{1-\alpha}$ is strictly increasing in α (at rate $1-\theta$), RHS attains its maximum at $\alpha = 0$, in which case RHS<LHS. Thus, RHS<LHS for all $\alpha \in (0,1)$. Finally, since $\frac{i^B}{i} = [\frac{q^{e,s}/q^{e,p}}{(1+r^s)/(1+r^p)}]^{\frac{1}{1-\theta}} > 1$, we must have $i^G < i < i^B$ for $I^s = I^p$ to hold. $\square$

<u>*Proof of Propositions 1.7 - 1.8:*</u> First note that $[\frac{p^G}{\bar{p}}-1] = \frac{(1-\lambda)[1-\frac{p^B}{p^G}]}{1-(1-\lambda)[1-\frac{p^B}{p^G}]}$ and $\frac{[1-\frac{p^B}{p^G}]}{[\frac{p^G}{\bar{p}}-1]} = \frac{1}{(1-\lambda)}-[1-\frac{p^B}{p^G}]$. Thus, λ and $\frac{p^B}{p^G}$ decrease $[1-\frac{p^B}{p^G}]/[\frac{p^G}{\bar{p}}-1]$. Now IC can be rearranged as

$$IC(\frac{i^G}{i^B}) = \frac{1}{\theta}(\frac{i^G}{i^B})^\theta - [1-(1-\frac{p^B}{p^G})(1-c^{e,G})]\frac{i^G}{i^B} - (\frac{1}{\theta}-1) = 0$$

$$c^{e,G} = \frac{\gamma^0}{\bar{\gamma}\theta}(\frac{i^G}{i^B})^{\theta-1}$$

$$\rightarrow \quad IC(\frac{i^G}{i^B}) = [1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})]\frac{1}{\theta}(\frac{i^G}{i^B})^\theta - \frac{i^G}{i^B} - \frac{p^G}{p^B}(\frac{1}{\theta}-1) = 0 \qquad \text{(A.1)}$$

The derivative of i^G/i^B with respect to p^B/p^G, $\gamma^0/\bar{\gamma}$ and θ are as follows:

$$\frac{\partial i^G/i^B}{\partial p^B/p^G} = -\frac{IC_{p^B/p^G}}{IC_{i^G/i^B}} = -\frac{-(1-c^{e,G})i^G/i^B}{[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})](i^G/i^B)^{\theta-1}-1} > 0$$

$$\frac{\partial i^G/i^B}{\partial \gamma^0/\bar{\gamma}} = -\frac{IC_{\gamma^0/\bar{\gamma}}}{IC_{i^G/i^B}} = -\frac{-(p^G/p^B-1)(i^G/i^B)^\theta/\theta}{[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})](i^G/i^B)^{\theta-1}-1} > 0$$

$$\frac{\partial i^G/i^B}{\partial \theta} = -\frac{IC_\theta}{IC_{i^G/i^B}} = -\frac{-[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})]\theta^{-2}(i^G/i^B)^\theta(1-ln(i^G/i^B)^\theta)+\frac{p^G}{p^B}\theta^{-2}}{[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})](i^G/i^B)^{\theta-1}-1} < 0$$

Note that $[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})] > 1$ and $(i^G/i^B)^{\theta-1} > 1$, implying $IC_{i^G/i^B} > 0$. We also have $IC_\theta > 0$ since $x - xln(x) < 1$ for $x = (i^G/i^B)^\theta < 1$ and $[1+(\frac{p^G}{p^B}-1)(1-\frac{\gamma^0}{\bar{\gamma}})] < \frac{p^G}{p^B}$. The derivative of $\frac{[1-c^{e,G}]}{[1-c^e]}$ with respect to p^B/p^G is

$$\frac{\partial \frac{[1-c^{e,G}]}{[1-c^e]}}{\partial p^B/p^G} = \frac{1}{[1-c^e]}\frac{\partial[1-c^{e,G}]}{\partial i^G/i^B}\frac{\partial i^G/i^B}{\partial p^B/p^G} > 0$$

Thus, the relative cost of a separating contract increases in p^B/p^G:

$$\frac{\partial C^{sp}}{\partial p^B/p^G} = \underbrace{\frac{\partial[1-\frac{p^B}{p^G}]/[\frac{p^G}{\bar{p}}-1]}{\partial p^B/p^G}}_{(+)}\ \underbrace{\frac{\partial[1-c^{e,G}]/[1-c^e]}{\partial p^B/p^G}}_{(+)}\ \underbrace{\frac{\partial i^G/i^B}{\partial p^B/p^G}}_{(+)} > 0$$

For Proposition 1.8, using (1.15) and (A.1) we may write

$$C^s = \frac{\theta}{1-\theta}[\frac{i^G}{i^B} - \frac{1}{\theta}(\frac{i^G}{i^B})^\theta] + 1$$

Given θ, we compute $\frac{\partial C^s}{\partial i^G/i^B} = \frac{\theta}{1-\theta}[1-(i^G/i^B)^{\theta-1}] < 0$, implying that the cost of a separating regime falls in both p^B/p^G and $\gamma^0/\bar{\gamma}$. Given $C^p = \frac{\theta}{1-\theta}[\frac{p^G}{\bar{p}}-1][1-\frac{\gamma^0}{\bar{\gamma}\theta}]$ by (1.16), we compute $\partial C^p/\partial(\gamma^0/\bar{\gamma}) < 0$ and $\partial C^p/\partial\theta > 0$. We need to show $\partial C^s/\partial\theta > 0$. Using the differential of $C^s(i^G/i^B, \theta)$ we compute

$$\frac{\partial C^s}{\partial \theta} = \underbrace{\frac{\partial C^s}{\partial i^G/i^B}|_\theta}_{(-)} \underbrace{\frac{\partial i^G/i^B}{\partial \theta}}_{(-)} + \underbrace{\frac{\partial C^s}{\partial \theta}|_{i^G/i^B}}_{(+)} > 0$$

The positive sign of the last term follows from $\frac{\partial (i^G/i^B)^\theta}{\partial \theta} < 0$ holding $i^G/i^B < 1$ constant. □

<u>*Proof of Proposition 1.9:*</u> Let the first group of loan applicants has measure η and productivity $\bar{\gamma}_1$, and the second group of loan applicants has measure $1-\eta$ and productivity $\bar{\gamma}_2$. Let the second group be subject to higher agency costs (low quality group), so that lower collateralization switches the regime from pooling to separating for the second group first. Consider the case in which low risk borrowers within the second group are indifferent between pooling and separating contracts. That is, the economy is at a regime switching point where lending regime switches from pooling borrowers within both groups (pp regime), to pooling borrowers within the first group and separating borrowers within the second group (ps regime).

In this case, regardless of the lending regime, total savings is equal to total investment in projects. Thus, $S(A,k) = I^{ps} = I^{pp}$. We compute

$$I^{pp} = \eta i_1^{pp} + (1-\eta) i_2^{pp} = [\eta + (1-\eta)\frac{i_2^{pp}}{i_1^{pp}}] i_1^{pp}$$
$$I^{ps} = \eta i_1^{ps} + (1-\eta)\lambda i_2^G + (1-\eta)(1-\lambda) i_2^B = [\eta + (1-\eta)[\lambda \frac{i_2^G}{i_2^B} + (1-\lambda)]\frac{i_2^B}{i_1^{ps}}] i_1^{ps}$$

Given $q^e(pp)$ and $r(pp)$, we compute $\frac{i_2^{pp}}{i_1^{pp}} = \frac{\bar{\gamma}_2}{\bar{\gamma}_1}^{\frac{1}{1-\theta}}$ by Lemma 1.3. Similarly, given $q^e(ps)$ and $r(ps)$, we compute $\frac{i_2^B}{i_1^{ps}} = \frac{\bar{\gamma}_2}{\bar{\gamma}_1}^{\frac{1}{1-\theta}}$. Thus, we obtain

$$\frac{I^{pp}}{I^{ps}} = \frac{[\eta + (1-\eta)\ \frac{\bar{\gamma}_2}{\bar{\gamma}_1}^{\frac{1}{1-\theta}}]\ i_1^{pp}}{[\eta + (1-\eta)\ \underbrace{[\lambda\frac{i_2^G}{i_2^B} + (1-\lambda)]}_{<1}\ \frac{\bar{\gamma}_2}{\bar{\gamma}_1}^{\frac{1}{1-\theta}}]\ i_1^{ps}} = 1$$

$$\longrightarrow \quad \frac{i_1^{pp}}{i_1^{ps}} < 1 \quad \longrightarrow \quad \frac{\eta i_1^{pp}}{S(A,k) - \eta i_1^{pp}} \frac{S(A,k) - \eta i_1^{ps}}{\eta i_1^{ps}} = \frac{I_1^{pp}/I_2^{pp}}{I_1^{ps}/I_2^{ps}} < 1$$

which proves that the proportion of high quality loans (type 1) increases when the economy switches from pooling to separating low quality borrowers (type 2). For parts ii-iv we follow the same steps in the proof of Proposition 1.6. □

A.2 Equilibrium Conditions

Let $j \in \{p, s\}$ denote pooling or separating regimes. Given k_t, aggregate household savings and consumption satisfy

$$S_t = w_t, \quad C_{t+1}^{H,j} = (1 + r_t^j) w_t$$

Factor prices satisfy

$$w_t = A(1-\alpha)(k_t)^{\alpha}, \quad q_{t+1}^j = A\alpha(k_{t+1}^j)^{\alpha-1}$$

Terms of the equilibrium contract satisfy

$$\begin{aligned}
i_t &= \left(\frac{q_{t+1}^{e,p}\bar{\gamma}\theta}{1+r_t^p}\right)^{\frac{1}{1-\theta}}, \quad c_{t+1}^e = \frac{\gamma^0}{\bar{\gamma}\theta}, \quad R_t = [1-(1-\bar{p})c_{t+1}^e]\frac{1+r_t^p}{\bar{p}} \\
i_t^B &= \left(\frac{q_{t+1}^{e,s}\bar{\gamma}\theta}{1+r_t^s}\right)^{\frac{1}{1-\theta}}, \quad c_{t+1}^{e,B} = \frac{\gamma^0}{\bar{\gamma}\theta}, \quad R_t^B = [1-(1-p^B)c_{t+1}^e]\frac{1+r_t^s}{p^B} \\
&\frac{1}{\theta}\left(\frac{i_t^G}{i_t^B}\right)^{\theta} - [1-(1-\frac{p^B}{p^G})(1-c_{t+1}^{e,G})]\frac{i_t^G}{i_t^B} - (\frac{1}{\theta}-1) = 0, \\
c_{t+1}^{e,G} &= \frac{\gamma^0}{\bar{\gamma}\theta}\left(\frac{i_t^G}{i_t^B}\right)^{\theta-1}, \quad R_t^G = [1-(1-p^G)c_{t+1}^{e,G}]\frac{1+r_t^s}{p^G}
\end{aligned}$$

Aggregate entrepreneurial investment, capital production and consumption satisfy

$$I_t^p = i_t$$

$$K_{t+1}^p = \bar{\gamma} f(i_t)$$

$$C_{t+1}^{E,p} = q_{t+1}^p \bar{\gamma} f(i_t) - (1+r_t^p) i_t$$

$$I_t^s = \lambda i_t^G + (1-\lambda) i_t^B$$

$$K_{t+1}^s = \bar{\gamma}[\lambda f(i_t^G) + (1-\lambda) f(i_t^B)\,]$$

$$C_{t+1}^{E,s} = \lambda[q_{t+1}^s \bar{\gamma} f(i_t^G) - (1+r_t^s) i_t^G] + (1-\lambda)[q_{t+1}^s \bar{\gamma} f(i_t^B) - (1+r_t^s) i_t^B]$$

Goods and capital markets clear

$$I_{t+1}^{j} + C_{t+1}^{H,j} + C_{t+1}^{E,j} = A(k_{t+1}^{j})^{\alpha}$$

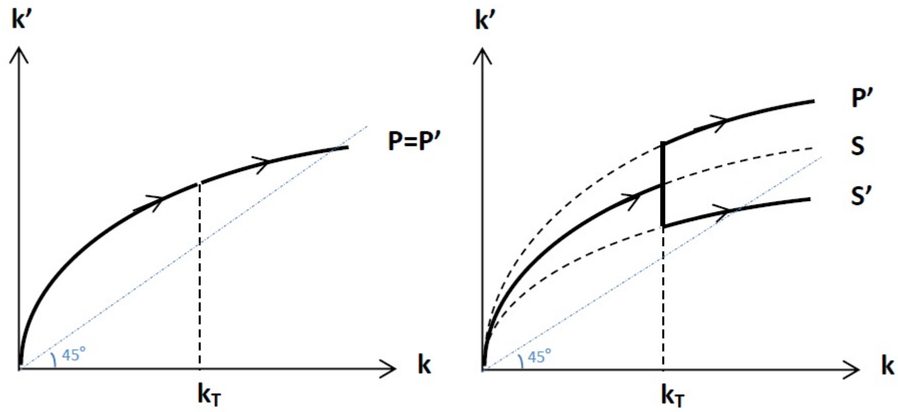
$$S_t = I_t^j$$

Expectations satisfy

$$q_{t+1}^{e,j} = q_{t+1}^{j} = \alpha(k_{t+1}^{j})^{\alpha-1}.$$

A.3 Regime switching when θ is low

In this section we analyze how exogenous changes in the collateral capacity of investors affect the allocation of credit across investors with different unobservable risk levels, under the assumption that the curvature of investment technology is high, i.e., θ is relatively low. In this case, lower collateralization *increases* pooling incentives unlike in Section 1.6; however, increases the degree of rationing for a low risk borrower under a separating regime as before.



(a) Initially Pooling (b) Initially Separating

Figure A.1: An increase in γ^0 given $\bar{\gamma}$, for relatively low θ

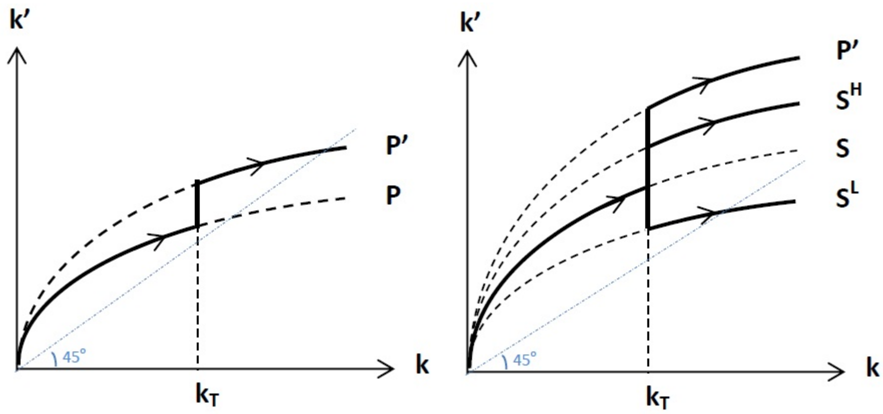
Figure A.1 illustrates the possible effects of a decrease in γ^0 given $\bar{\gamma}$ at time T on aggregate investment, when θ is relatively low. Panel (a) shows that, if the lending regime is initially pooling, a decrease in γ^0 will not change the composition of credit since the regime continues to be pooling. In this case, lower collateralization increases the pooling loan rate, reallocating profits from low risk borrowers to high risk borrowers.

Panel (b) shows that, if the lending regime is initially separating, a decrease in γ^0 may change the composition of credit in opposing ways, either by increasing the degree of rationing i^B/i^G, or switching the regime into pooling. In the first case, the growth path moves from S to S' as the credit is reallocated from low risk borrowers to high risk borrowers. In the second case, rationing becomes so high that the regime switches to pooling with a greater degree of cross subsidization, moving the economy from S to P'. Although lower collateralization paradoxically creates a boom, a decrease in collateralization increases the inequality in profit distribution.

One thing to note here is that, although higher collateralization increases screening incentives when θ is low, and a rise in collateralization may switch the lending regime from pooling to separating, reallocating credit from low risk borrowers to high risk borrowers, the inefficiency arising in aggregate investment could be minimal in this case, as higher collateralization brings pooling and separating loan terms closer. As collateralization approaches one, the separating regime coincides with the pooling regime.

Figure A.2 illustrates the possible effects of an increase in $\bar{\gamma}$ given γ^0 at time T on aggregate investment, when θ is relatively low. In this case, a decrease in collateralization increases pooling incentives and may change the composition of credit by reallocating credit from low risk to high risk borrowers, or vice versa.

If the lending regime is initially pooling, an increase in $\bar{\gamma}$ would keep the composition of credit unchanged, although lower collateralization would redistribute profits from low risk borrowers to high risk borrowers. Panel (a) shows that, in this case the growth path shifts from P to P', where investment is still efficient at a higher productivity level, but the inequality between the profits of high and low risk



(a) Initially Pooling (b) Initially Separating

Figure A.2: An increase in $\bar{\gamma}$ given γ^0, for relatively low θ

borrowers is higher.

Panel (b) shows that, if the lending regime is initially separating, an increase in $\bar{\gamma}$ would either increase the degree of rationing i^B/i^G under a separating regime or switch the lending regime into pooling and reallocate credit from high risk borrowers to low risk borrowers. In the latter case, the growth path shifts from S to P', where investment becomes more efficient at a higher productivity level, although the inequality between the profits of high and low risk borrowers becomes higher. In the former case, the separating regime carries on, but the growth path may shift up or down depending on whether the opposing composition or productivity effects dominate. The growth path shifts down from S to S^L when the composition effect dominates, while it shifts up from S to S^H when the productivity effect dominates. Therefore, when expected investment specific productivity is high, if collateralization is not kept equally high, inequality between the profits of high and low risk borrowers may rise and aggregate investment may fall as before.

A.4 Granger causality test results

Do lagged values of the proportion of risky loans made by banks help predict current fluctuations in real GDP? To test this, we use quarterly real GDP data (GDPC96) and quarterly data on the total value of loans for all C&I loans made by all commercial banks (EVANQ) in the sample of FRB's Survey on Terms of Business Lending, together with the total value of C&I loans made to moderate (EVAMNQ) and acceptable (EVAONQ) risk groups categorized by sample banks for the period 1997/2 to 2012/2. All data series are obtained from FRED.

We define RISKY as the proportion of moderate and high (acceptable) risk C&I loans, and we use the log of real GDP series, to conduct the following test

$$log(RGDP)_t = c + \sum_{i=1}^{L} \alpha_i \; log(RGDP)_{t-i} + \sum_{i=1}^{L} \beta_i \; RISKY_{t-i} + u_t$$

$$H_0 : \beta_1 = \beta_2 = ... = \beta_L = 0$$

The table below reports the p-values for rejecting the null hypothesis for lag lengths between 1 to 8 (quarters). Accordingly, decreases in the lagged values of the proportion of risky loans are highly predictive of current reductions in real GDP.

L	1	2	3	4	5	6	7	8
p-value	0	0.0077	0.0182	0.0414	0.0306	0.0239	0.0399	0.0295

Appendix B

B.1 Entrepreneurial Problem

An entrepreneur starts period t with accumulated capital k_t^e. After the firm production, he collects a net worth of $n_t = w_t^e + k_t^e[r_t + q_t(1-\delta)]$. Then he draws a high (low) type investment project with probability α $(1-\alpha)$, the success probability of which is p^H (p^L). If he is a high type, he is allowed to borrow b_t^H at rate R_t^H. If he is a low type, he does not borrow or invest. But he may lend part of his net worth. After investment outcomes are realized, all entrepreneurs maximize lifetime utility given by equation (2.7) subject to budget constraint given by equations (2.8) and (2.11).

The value function of an entrepreneur with a $j \in \{H, L\}$ type project is given by

$$V(k_t^e, j) = \max_{k_{t+1}^e,\ c_t} c_t + \beta^e[\alpha p^H V(k_{t+1}^e, H) + (1-\alpha)V(k_{t+1}^e, L)] \tag{B.1}$$

where

$$c_t + q_t k_{t+1}^e = \begin{cases} q_t[\bar{x}_t \frac{i_t^H}{n_t} - R_t^H \frac{b_t^H}{n_t}]n_t & \text{if high type} \\ \\ n_t & \text{if low type} \end{cases} \tag{B.2}$$

Note that, here i_t^H/n_t, R_t^H and b_t^H/n_t are all functions of parameters and q_t. Thus, they are constants from the perspective of an entrepreneur. Solving for c_t in (B.2) and plugging it in (B.1), the first order condition with respect to k_{t+1}^e (which is the same for both type of entrepreneurs) and the Envelope conditions are calculated as

$$q_t = \beta^e[\alpha p^H V'(k^e_{t+1}, H) + (1-\alpha)V'(k^e_{t+1}, L)] \tag{B.3}$$

$$V'(k^e_t, H) = [r_t + q_t(1-\delta)]q_t[\bar{x}_t \frac{i^H_t}{n_t} - R^H_t \frac{b^H_t}{n_t}] \tag{B.4}$$

$$V'(k^e_t, L) = [r_t + q_t(1-\delta)] \tag{B.5}$$

Updating (B.4) and (B.5) by one period and plugging them into (B.3) we get

$$q_t = \beta^e E_t \ [r_{t+1} + q_{t+1}(1-\delta)][\alpha q_{t+1} p^H (\bar{x}_{t+1} \frac{i^H_{t+1}}{n_{t+1}} - R^H_{t+1} \frac{b^H_{t+1}}{n_{t+1}}) + (1-\alpha)] \tag{B.6}$$

which is equivalent to equations (2.17)-(2.19) in the text.

B.2 Supply and Demand Curves for Investment

In this section we graph the investment supply and demand curves to illustrate how different shocks affect the capital's price q_t as well as the amount of investment.

Investment (capital) supply in the economy is given by equation (2.16), which can be re-written as

$$\begin{aligned} I_t^S(q_t, n_t^H) &= p^H \bar{x}_t I_t^H(q_t, n_t^H) \\ &= p^H \bar{x}_t \eta\alpha[\frac{q_t p^H \bar{x}_t - 1}{2\mu} + 1] n_t^H \end{aligned} \tag{B.7}$$

We can see that the supply of capital I_t^S is upward sloping in q_t and it shifts up in n_t^H, or equivalently z_t, and $\bar{x}_t$.[1] On the other hand, the capital demand curve of households is given by the Euler equation

$$q_t u_c(t) = \beta E_t u_c(t+1)[r_{t+1} + q_{t+1}(1-\delta)] \tag{B.8}$$

which is downward sloping in q_t, and similar to the supply curve, shifts up in z_t. As the price q_t of capital rises, the relative price of current consumption falls and the household optimally increases current consumption and decreases savings, which curbs the household demand for capital. On the other hand, an increase in aggregate productivity z_t yields a positive income effect on household consumption and saving,

[1]Note that high type entrepreneurs do not acquire capital through financial intermediaries. Thus, the capital supplied to non-investors is effectively $I_t^S - [K_t^H + 1 - (1-\delta)K_t^H]$, where the latter term is high type entrepreneurs' net capital acquisitions. Equivalently, we can deduce the capital supplied to households as $I_t^S - [K_{t+1}^H + K_{t+1}^L - (1-\delta)(K_t^H + K_t^L)]$. In the supply/demand analysis, we will neglect the capital acquisitions by entrepreneurs for simplicity, and treat I_t^S as the approximate value of the capital supplied to households.

shifting out the capital demand curve of households.

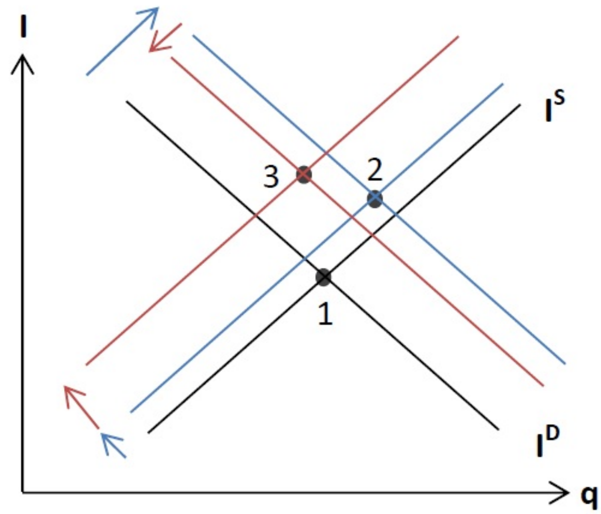


Figure B.1: Capital Supply and Demand after an Aggregate Productivity Shock

Figure B.1 illustrates the movements in the capital (investment) demand and supply curves, together with the capital's price and the level of investment in response to a positive aggregate productivity shock. A positive aggregate productivity shock raises wages and rental rates, which lead to a rise in household income, raising desired savings and shifting the investment demand curve out. In the meantime, entrepreneurial income and net worth also rise modestly, shifting the investment supply curve up a little, as capital stock is predetermined and entrepreneurial wages are small (see the blue curves). Thus, on impact the economy moves from point 1 to point 2 as shown in the graph, increasing both the level of investment and the price of capital. An increase in the expected future price of capital increases the return on internal funds, which raises high type entrepreneurial savings on impact, increasing the entrepreneurial net worth further in the next period, shifting the investment supply curve up even more. On the demand side, the investment demand curve follows the movement in aggregate productivity, shifting back down slowly together with the persistent productivity shock (see the red curves). Thus, in the period(s) following the shock the economy moves from point 2 to point 3 as shown in the graph, consistent with the movements in investment and the price of capital as shown in Figure 2.1. Accordingly, investment keeps increasing for a couple of periods after its initial increase on impact, showing a hump-shaped behavior. Both the level of investment and the price of capital return back to their steady state levels as the supply and demand curves keep shifting down together with the falling aggregate productivity.

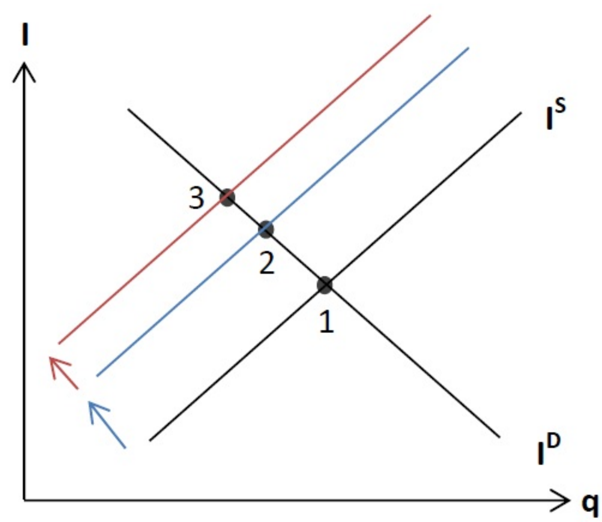


Figure B.2: Capital Supply and Demand after an Investment Productivity Shock

Figure B.2 illustrates the movements in the capital (investment) demand and supply curves, together with the capital's price and the level of investment in response to a positive investment productivity shock. A positive investment productivity shock raises the current return on investment, shifting up the investment supply curve, which moves the economy from point 1 to point 2 as shown in the graph. Thus, given the demand for investment, the price of capital falls and the level of investment rises. In the meantime, a positive investment productivity shock also raises the expected future return on investment, which increases entrepreneurial savings and raises entrepreneurial net worth in the next period(s). The rise in entrepreneurial net worth further shifts the investment supply curve up, increasing the level of investment and decreasing the price of capital even more, moving the economy from point 2 to point 3, in line with the movements in aggregate investment and capital's price as shown in Figure 2.2. Both the level of investment and the price of capital return back to their steady state levels as the supply curve shifts back down together with the decrease in investment productivity.

B.3 Impulse Responses of the CF Model

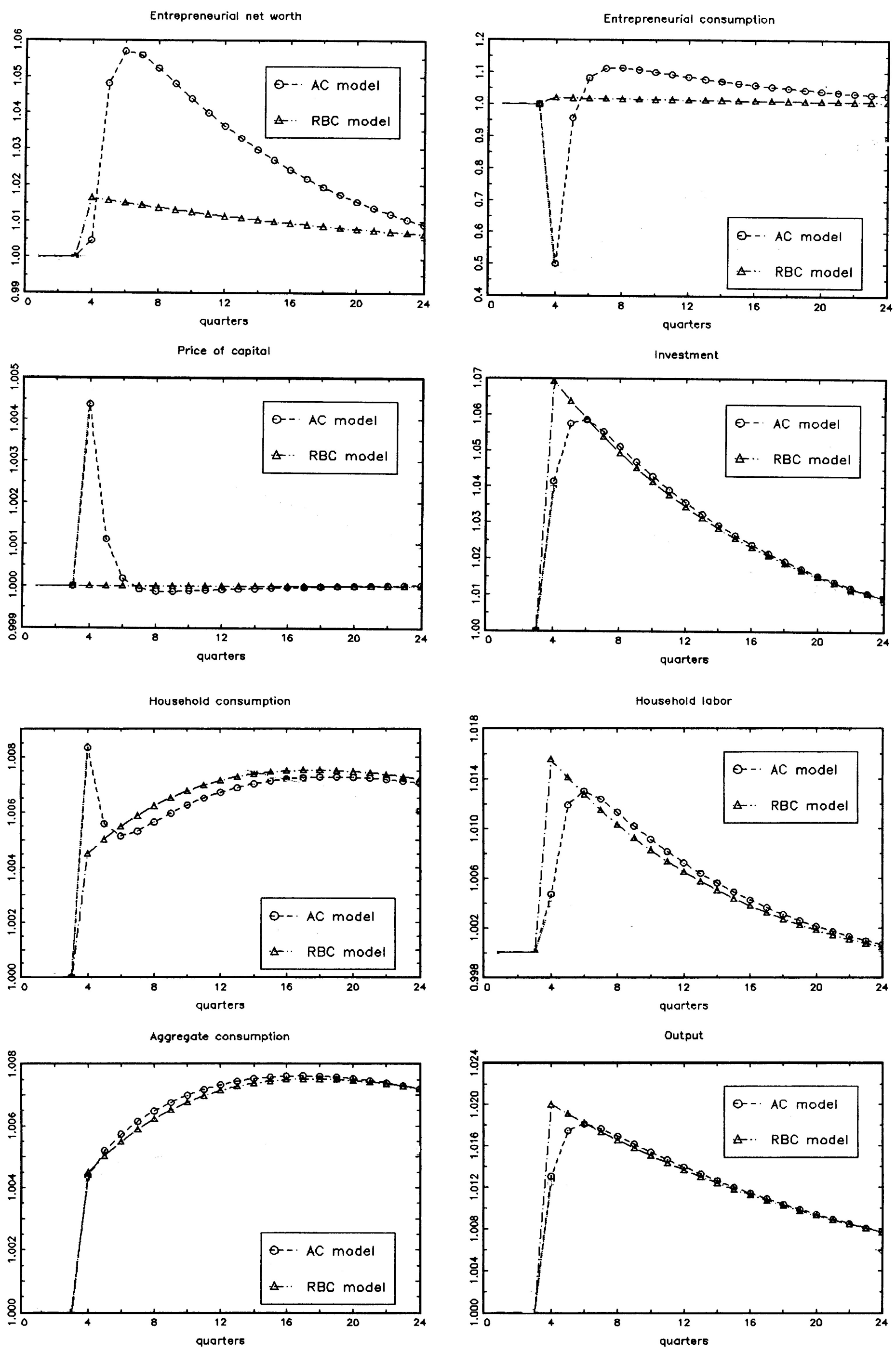


Figure B.3: The Response to a Productivity Shock in the CF Model

Figure B.3 is borrowed from CF, showing the impulse responses of the costly monitoring model and a standard RBC model to a 1% increase in aggregate productivity from its steady state level.[2] We see that the impulse responses of the costly screening model shown in Figure 2.1 matches those of the costly monitoring model shown in Figure B.3 both qualitatively and quantitatively.[3]

[2]Note that in the original Figure 2 of CF each panel contains the dynamics of three different models: 1) A standard RBC model in which agency costs are set to zero, 2) An RBC model with agency costs, and 3) An RBC model with agency costs, in which net worth is held constant, which is isomorphic to a standard cost-of-adjustment model. The third model's impulse responses are not shown in Figure B.3 to simplify comparison to Figure 2.1.

[3]Entrepreneurial net worth and consumption in Figure B.3 should be compared to high type net worth and consumption in Figure 2.1.

Bibliography

[1] Akerlof, G. (1970). *The Market for Lemons: Quality Uncertainty and the Market Mechanism*. Quarterly Journal of Economics, 85, pp. 488-500.

[2] Asea, P. and Blomberg, B. (1998). *Lending Cycles*. Journal of Econometrics, 83(1-2), pp. 89-128.

[3] Azariadis, C. and Smith, B. (1998). *Financial Intermediation and Regime Switching in Business Cycles*. American Economic Review, 88(3), pp. 516-536.

[4] Bassett, W., Chosak, M. B., Driscoll, J. and Zagrajsek, E. (2012). *Changes in Bank Lending Standards and the Macroeconomy*. Mimeo. Division of Monetary Affairs, Federal Reserve Board.

[5] Bernanke, B. and Gertler, M. (1989). *Agency Costs, Net Worth and Business Fluctuations*. American Economic Review, 79, pp. 14-31.

[6] Bernanke, B. and Gertler, M. (1990). *Financial Fragility and Economic Performance*. The Quarterly journal of Economics, 105(1), pp. 87-114.

[7] Bernanke, B., Gertler, M. and Gilchrist, S. (1996). *The Financial Accelerator and the Flight to Quality*. Review of Economics and Statistics, 78(1), pp. 1-15.

[8] Bernanke, B., Gertler, M. and Gilchrist, S. (1999). *The Financial Accelerator in a Quantitative Business Cycle Framework*. Handbook of Macroeconomics, 1(21), pp. 1341-1393.

[9] Berger, A. and Udell, G. (1995). *Relationship Lending and Lines of Credit in Small Firm Finance*. Journal of Business, 68(3), pp. 351-381.

[10] Carlstrom, C. and Fuerst, T. (1997). *Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis*. American Economic Review, 87(5), pp. 893-910.

[11] Christiano, L., Motto, R. and Rostagno, M. (2010). *Financial Factors in Economic Fluctuations.* European Central Bank, Working Paper Series, No 1192, May 2010.

[12] Chugh, S. (2010). *Firm Risk and Leverage-based Business Cycles*. University of Maryland. Mimeo.

[13] Cooley, T. F. and Quadrini, V. (2001). *Financial Markets and Firm Dynamics.* American Economic Review, 91(5), pp. 1286-1310.

[14] Covas, F. and Den Haan, W. J. (2010). *The Role of Debt and Equity Finance over the Business Cycle.* American Economic Review, forthcoming.

[15] De Meza, D. and Webb, D. (1987). *Too Much Investment: A Problem of Asymmetric Information.* Quarterly Journal of Economics, 102(2), pp. 281-292.

[16] Fisher, J. (1999). *Credit Market Imperfections and the Heterogeneous Response of Firms to Monetary Shocks.* Money, Credit and Banking, 31(2), pp. 187-211

[17] Guiso, L. (1997). *High-Tech Firms and Credit Rationing.* Journal of Economic Behavior and Organization, 35, pp. 39-59.

[18] Himmelberg, C. and Petersen, B. (1994). *R&D and Internal Finance: A Panel Study of Small Firms in High-Tech Industries.* The Review of Economics and Statistics, 76(1), pp. 38-51.

[19] Hobijn, B. and Ravenna F. (2009). *Loan Securitization and the Monetary Transmission Mechanism.* Federal Reserve Bank of San Francisco. Mimeo.

[20] Holmstrom, B. and Tirole, J. (1997). *Financial Intermediation, Loanable Funds, and the Real Sector.* Quarterly Journal of Economics, 112(3), pp. 663-691

[21] House, C. (2006). *Adverse Selection and the Financial Accelerator.* Journal of Monetary Economics, 53, pp. 1117-1134.

[22] Kiyotaki, N. and Moore, J. (1997). *Credit Cycles.* Journal of Political Economy, 105, pp. 211-248.

[23] Kurlat, P. (2010). *Lemons, Market Shutdowns and Learning.* MIT, mimeo.

[24] Lang, W. and Nakamura, L. (1995). *Flight to Quality in Banking and Economic Activity.* Journal of Monetary Economics, 36, pp. 145-164.

[25] Lown, C. and Morgan, D. (2006). *The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey.* Journal of Money, Credit, and Banking, 38(6), pp. 1575-1597.

[26] Martin, A. (2008). *Endogenous Credit Cycles.* Mimeo, Universitat Pompeu Fabra.

[27] Martin, A. (2009). *A Model of Collateral, Investment, and Adverse Selection*. Journal of Economic Theory, 144(4), pp. 1572-1588.

[28] Matsuyama, K. (2007). *Credit Traps and Credit Cycles*. American Economic Review, 97, pp. 503-516.

[29] Melnik, A. and Plaut, S. (1996). *Loan Commitment Contracts, Terms of Lending, and Credit Allocation*. The Journal of Finance, 41(2), pp. 425-435.

[30] Reichlin, P. and Siconolfi, P. (2004). *Optimal Debt Contracts and Moral Hazard Along the Business Cycle*. Economic Theory, 24, pp. 75-109.

[31] Schmitt-Grohe, S. and Uribe, M. (2004). *Solving Dynamic General Equilibrium Models Using a Second-order Approximation to the Policy Function*. Journal of Economic Dynamics and Control, 28, pp. 755-775.

[32] Stiglitz, J. and Weiss, A. (1981). *Credit Rationing in Markets with Imperfect Information*. American Economic Review, 71, pp. 393-410.

[33] Valentinyi, A. and Herrendorf, B. (2008). *Measuring Factor Income Shares at the Sectoral Level*. Review of Economic Dynamics, 11, pp. 820-835.

[34] Wang, C. and Williamson S. (1998). *Debt Contracts and Financial Intermediation with Costly Screening*. Canadian Journal of Economics, 31(3), pp. 573-596.