ABSTRACT

Title of dissertation: ESSAYS ON AUCTION THEORY

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This dissertation studies two features of high-value auctions that are not explicitly captured by the standard models in the auction theory literature. The first is that bidders in auctions for valuable assets sometimes have binding budget constraints. Standard models of auctions assume that bidders can submit any bid up to their valuation (or willingness to pay). An existing literature has developed models where bidders may face binding budget constraints and from these models has concluded that the presence of budget constraints has important implications for the relative performance of different auction formats, and as a consequence argues that the presence of budget constraints should be an important factor used in choosing an auction format.

Chapters 2 and 3 develop and study a model of budget constraints where the budget constraint is chosen explicitly in the model in response to a principal-agent problem between each bidder and a corresponding principal. In previous literature, the budget constraint is assumed to be given by some exogenous procedure, and hence is not affected by changes in the auction rules. The model presented here,

however, allows the choice of budget constraint to depend on the auction rules, and the main result of Chapter 2 shows that allowing for this effect leads to outcomes that closely resemble the classic results from the auction literature without budget constraints.

Chapter 3 investigates the theoretical predictions of Chapter 2 in an experiment involving undergraduate students at the University of Maryland. The experiment is designed to evaluate the decisions made by the subjects acting as the person responsible for deciding on a budget for the bidder. We perform treatments where the bidding behavior is simulated by computerized agents and ones where half the subjects in each session play the role of the bidder. Our results indicate that the subjects take the auction rules into account when deciding on their respective bidder's budget, and the direction of the response in the data agrees with the theoretical predictions.

Chapter 4 studies a separate feature of high-value auctions that is not captured by the standard auction models. That is, the bidders in the auction may have valuations for the auctioned item that depend on the the identities of the other winning bidders. If the auction determines the structure of the market the bidders will compete in after the auction, the bidders' values for the items will be affected by who participates in that market. The typical notion of efficiency in the auction literature corresponds to maximization of producer surplus in this model, but the auctioneer may also be concerned with total surplus in this environment. The main results show that these two notions of efficiency do not agree in this model, and that a sequential auction favors maximization of producer surplus, while a sealed-bid auction can favor maximization of total surplus. The key distinction between the two is that the sequential auction is assumed to reveal the identity of early winners to the later winners, while the sealed-bid auction reveals no information to the participants until the auction concludes.

ESSAYS ON AUCTION THEORY

by

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1. INTRODUCTION

An important reason for the success and appeal of auction theory is that the rules for bidding in auctions can faithfully be represented in a formal game theoretic model. This is in contrast to other applications of game theory to economic problems, where representing the environment in a formal economic model often involves non-trivial assumptions about the consequences of the players' actions. Yet, standard auction models do make assumptions about the environment in an auction that may be violated in practice. The importance of these deviations from the standard auction models is the subject of much of the auction literature, and this dissertation is a contribution to that literature.

The goal of this dissertation is to examine two important departures from the standard models. Chapters 2 and 3 are concerned with the possibility that bidders in an auction may not be able to submit a bid of any amount due to budget constraints. Standard auction models typically assume that bidders may submit any positive bid amount to the auctioneer, but it has been reported in the literature that bidders in auctions for valuable assets are often constrained [Cramton, 1995, Salant, 1997, Bulow et al., 2009].

Chapter 4 examines a model where bidders' valuations for the auctioned goods depend on private information available to the bidder and the identity of the other winners at the auction. This is a case where the valuations of the bidders exhibit allocative externalities and is likely a feature of auctions when the outcome affects the nature of competition between the bidders in some market.

Both budget constraints and allocative externalities are most likely present in auctions for valuable assets. The size of the transaction means that bidders are more likely to have their funding constrained either by their own firm or by external credit markets, and the fact that the firms are typically purchasing the assets for use in some market means that they are likely concerned with the ramifications the auction outcome might have for the competitive structure of these markets.

In the sections that follow, I give a brief overview of the motivation for each chapter and summarize the results.

1.1 Budget Constraints

In the existing literature on the effect of budget constraints on auction outcomes, budget constraints are cited as an important deviation from standard models because it is shown that incorporating budget constraints into otherwise standard models invalidates classic results, such as revenue equivalence [Myerson, 1981, Riley and Samuelson, 1981] between the first- and second-price auction formats [Che and Gale, 1998]. It is argued further that in the presence of budget constraints sellers should prefer atypical auction formats, because these formats would raise more money for the seller and improve the auction's efficiency [Che and Gale, 1996, Maskin, 2000, Pai and Vohra, 2008].

The existing models for budget constraints, however, generally assume that the distribution of budget constraints is determined exogenously, and is not affected by a change in the auction rules. Since many explanations for the existence of budget constraints in the literature suggest that they may be the result of an agency problem between the bidder and some third party, one could ask whether modeling the budget constraint as an explicit choice would change the results from the budget constraints literature. This is the question that is addressed by Chapter 2 of this dissertation.

The results in Chapter 2 show that by explicitly modeling the choice of a budget for each bidder prior to the auction, one can restore many of the results from the literature on auctions without budget constraints, such as revenue equivalence. The reasoning behind these results is simple. The players in the model who are responsible for setting the budgets (the principals) incorporate the consequences of a change in auction rules into their decision. Under the right conditions, this has the effect of completely canceling out any effect the change in auction rules might have on the outcome.

In Chapter 3, we provide experimental evidence for the primary conclusion drawn in Chapter 2, that the principals adjust their behavior to a change in the auction rules. In sessions at the Experimental Economics Lab at the University of Maryland involving undergraduate students, we conducted four separate treatments to test the basic predictions of the theory. In two treatments, subjects played the role of the principals, setting budgets for their computerized bidders in the first- and second-price auctions. In the other two treatments, half the subjects played the role of principals and half the subjects played the role of bidders in both the first- and second-price auctions.

The results of the experiment show that the principals react to the auction rules by setting significantly higher budgets in the second-price auction compared to the first-price auction. This is the most basic prediction of the theory developed in Chapter 2. We find that this result is supported by the data whether the role of the bidders is played by computerized subjects or human subjects. A more detailed prediction of the theory is that a bidder with a given value for the good should be constrained with the same *probability* in both auction formats. We show support for this prediction in the treatments with computerized bidders. In the treatments with human bidders, testing this hypothesis would require making strong assumptions about the intentions of the bidders when they are constrained.

1.2 Allocative Externalities

Chapter 4 of the dissertation studies a model where the bidders' payoffs are affected by the identity of the other winners. When the outcome of an auction determines the structure of some market, it is likely that the bidders' payoffs will be affected by the identities of the winners. For example, a bidder may value one unit of a good differently depending on whether or not a direct competitor wins the other unit.

In auctions with externalities such as these it is generally not possible for the seller to allocate the goods efficiently [Jehiel and Moldovanu, 2001], but Das Varma

[2002b] argues that if the seller can reveal the identities of the opposing bidders she can improve the performance of the auction, both in terms of efficiency and revenue. However, in this environment there are alternative ways to think about efficiency, because one may define efficiency in terms of the surplus in the resulting market. For example, one may also be concerned about the consumer surplus generated by the downstream market. Chapter 4 develops a model that distinguishes between two notions of efficiency and shows that the sequential auction (or an auction that reveals the identities of winning bidders as the auction progresses) favors one but not the other.

This chapter is also connected to the literature in Industrial Organization on the incentives for a monopolist to deter entry. It has been argued in this literature that sequentially auctioning resources instead of auctioning them all at once reduces a monopolist's incentives to deter entry and hence can lead to more competitive market structures [Krishna, 1993]; however, more recent work has qualified this result somewhat, showing that the results depend on assumptions about the relative size of the competitors [Gale and Stegeman, 2001]. Although Chapter 4 is more focused on efficiency than market structure, the results do support the idea that sequential auctions may lead to less competitive market structures.

2. ENDOGENOUS BUDGET CONSTRAINTS IN AUCTIONS

by Justin E. Burkett

2.1 Introduction

Bidders likely face budget constraints in many real-world auctions, especially in the sale of valuable assets such as wireless spectrum [Cramton, 1995, Salant, 1997, Bulow et al., 2009], and these constraints potentially have important strategic effects on the outcomes of auction models that cannot be captured in standard frameworks.

Current literature on the subject [most notably Che and Gale, 1998] argues that incorporating budget constraints into the standard independent private values model invalidates some well-known results like the revenue equivalence theorem [Riley and Samuelson, 1981, Myerson, 1981]. For example, in a model where bidders valuations are private and i.i.d. Che and Gale [1998] show that the first-price auction both raises more revenue and is more efficient than the second-price auction with budget constraints. Further work extends these results to show that the all-pay auction dominates the first-price auction in terms of revenue and efficiency [e.g., Che and Gale, 1996, Maskin, 2000, Pai and Vohra, 2008].

The earlier literature offers various explanations for the underlying cause of the budget constraints, including imperfect capital markets and agency problems [Che and Gale, 1998]. However, these papers derive their results from models where the budget constraint is treated as an exogenous random variable.[1] A potential advantage of this approach is that it allows one to be agnostic about the source of the budget constraints and focus on the strategic effects introduced by the constraints, but it ignores the possibility that the process generating the budget constraints may be affected by a change in auction rules.[2]

If one tries to describe explicitly an agency problem that generates budget constraints for the bidders, it seems that a description of the auction rules should be included as well. Otherwise, one would have to assume that the parties funding the auction take no interest in the auction design. Explicitly including a description of the auction rules in the agency problem would allow the budget to vary according to the rules, an effect that cannot adequately be captured in a model that treats the budget constraints as a primitive. The purpose of this paper is to explore how budget constraints might vary between different auction formats when the mechanism generating the budget constraint is made explicit.

I develop a model where the bidder's budget constraint is the endogenous result of an agency problem between the bidder and a principal responsible for funding the bidder's bid. Results from the model suggest that budget constraints

[1] The bidders' types are two-dimensional, including a valuation and a budget, distributed according to some commonly known prior distribution.

[2] Strictly speaking, the distribution of the budget constraints could be specified differently for each auction format, but without an explicit description of the mechanism generating the budget constraint it is not clear how to do this.

do not invalidate the standard results from the auction literature when they are treated as endogenous choices. For example, when I restrict attention to the case of independent signals between bidders, I find no difference in the expected revenue or efficiency between the first- and second-price auctions. Although a special case, the independence case corresponds to much of the existing literature on budget constraints.

In the more general case where bidders' information is allowed to be affiliated I characterize symmetric, equilibrium strategies for the bidders and principals. I also partially characterize the relative performance of the first- and second-price auctions, showing that the first-price auction must be more efficient. Under affiliation the principal prefers the first-price auction to the second-price auction and reacts by relaxing the budget constraint. The effect on revenue is less clear due to the complicated nature of the model and two counteracting effects. However, I am able to solve for the equilibrium in an example and show that the expected revenue in that case is higher in the second-price auction. This agrees with the revenue ranking from classic symmetric, affiliated values model [Milgrom and Weber, 1982].

Motivated by existing explanations for the existence of budget constraints, I model the budget constraint as the outcome of a principal-agent problem between the bidder and some principal responsible for funding the bidder's bid. I believe that this basic setup covers many possible explanations for the origin of budget constraints. For example, there is a large corporate finance literature suggesting that capital market imperfections are the results of agency problems [Shleifer and Vishny, 1997].

The details of the model are easily summarized by the following situation. An item is auctioned to one of N firms. Within each firm there is a manager interested in purchasing the asset, but the manager must get funding approval from the firm's board of directors.[3] An agency problem arises because the board of directors knows that the manager will tend to overpay for the asset relative to its true value to the firm because the manager has an empire-building motive or simply receives some private payoff from managing the asset.[4]

Both the manager and the board observe signals about the value of the asset to the firm upon which they base their choices. The board observes its signal first and decides on a budget for the auction, after which the manager observes an additional signal and decides on a bid to place at the auction that is consistent with the budget constraints.

I consider versions of the model with "hard" and "soft" budget constraints. In the hard budget constraint case, the budget is a fixed value which the bid may never exceed. In the soft budget constraint case the board is able to provide a price list to the manager for bids of different sizes.[5] The hard budget constraint case is more common in the existing literature, so I will primarily focus on this case here

[3] Throughout the paper I use the convention that the manager is male and the representative of the board is female.

[4] A modern reference for this description of managerial motives is Jensen [1986], but the idea can be traced back as far as Schumpeter [1934].

[5] In other words, the bidder is allowed to submit a bid of any size but must incur a cost of $c(b)$ to submit the bid, so that in the event that he wins the item he receives $v - c(b)$, where v is his valuation.

(Sections 2.4 and 2.5). It also may be the appropriate model if we impose a limited liability condition on the principal, rendering schemes that require the bidder to pay a fee to the principal infeasible. With soft budget constraints the principal is able to perfectly manipulate the bidder's objective and incentivize the bidder to behave exactly as the principal would given the same information (Section 2.6). One may interpret these two alternatives as representing two extreme descriptions of the form of a budget constraint. Cramton [1995] suggests that in practice budget constraints fall somewhere in between the two (i.e., they take the form of step functions).

2.2 Related Literature

Following early work on the effect of budget constraints on auction outcomes when valuations are known [Che and Gale, 1996], Che and Gale [1998] are the first to compare revenue between auction formats when both budgets and valuations are treated as private information. A further extension of this work is given in Che and Gale [2006] which develops techniques for comparing revenues between auction formats when types are multidimensional and independent. Several papers [Fang and Perreiras, 2002, 2003, Kotowski, 2010, Kotowski and Li, 2011] extend the model of Che and Gale [1998] to allow for valuations to be affiliated, but they retain the assumption that the bidders budgets are determined exogenously. Another direction of research has examined the effect of exogenous budget constraints on auctions of multiple goods [Brusco and Lopomo, 2008, Hafalir et al., 2012].

Several related papers make the financing decision endogenous in an auction.

Benoit and Krishna [2001] consider a multiple-object auction setup in complete information where bidders are allowed to choose their own budgets (at zero marginal cost). They show that at least one bidder chooses to restrict his budget in every equilibrium of their game. Zheng [2001] describes a model where bidders are able to supplement an existing cash position by borrowing at some fixed interest rate (common to all bidders) prior to the auction to finance their bids in a first-price auction. In addition, the bidders may choose after the auction to default on their bids. These two features are shown to have strong impacts on bidding behavior, such as low-budget bidders bidding more than high-budget bidders for some values of the interest rate.

Rhodes-Kropf and Viswanathan [2005] consider a model similar to the one of Zheng [2001] where bidders supplement a cash position by going to a competitive financing market before and/or after a first-price auction takes place.[6] They consider a variety of financing schemes and ask the question of which schemes lead to efficient outcomes in the auction. In a similar vein Zheng [2010] considers a situation where a social planner auctioning a good to cash constrained bidders has a choice of financing schemes to offer bidders and would like to choose the one that maximizes the efficiency of the outcome.

Another branch of literature considers the mechanism design problem, in the spirit of Myerson [1981], of auctioning an item (or items) to bidders with exogenous budget constraints. Various approaches to the difficult problems of a seller maximizing revenue or efficiency have been taken, each imposing different restric-

[6] Another variation of this idea is presented in Hyde and Vercammen [2002].

tions [Laffont and Robert, 1996, Che and Gale, 1999, 2000, Maskin, 2000, Malakhov and Vohra, 2008, Pai and Vohra, 2008].[7] There is a similar line of work in the computer science literature.[8]

Finally, this paper is similar to work in the industrial organization literature that examines the incentives of owner-manager pairs in oligopoly models [Fershtman and Judd, 1987].

2.3 Model

The model extends the Milgrom and Weber [1982] model of auctions with affiliated values to allow for a budgeting stage prior to a sealed-bid auction for a single good. To each of N bidders I add a principal responsible for setting their bidder's budget constraint prior to the auction (there are a total of $2N$ players in the game). Each bidder plays the role of the manager in the firm and each principal plays the role of the board of directors. Given their expected valuations of the asset, both are interested in maximizing the expected profit of the firm at the auction (expected valuation minus expected payment). This could be modeled by making both equity holders in the firm, so that they each receive a constant fraction of the firm's profits. The agency problem is the result of a systematic difference in how principals and bidders value the asset.

Temporarily ignoring the principals and the budget constraints, the relation

[7] For example, Laffont and Robert [1996] restrict all bidders to have the same budget constraint, while Maskin [2000] assumes that the budget constraint is common knowledge.

[8] See Kotowski [2010] for a list of papers in this area.

between the bidders from opposing firms is as it is in the Milgrom and Weber [1982] model. So I am assuming that the bidders are symmetric and have valuations for the good which may incorporate both private and common value components. In the notation of Milgrom and Weber [1982], bidder i values the object according to $u_i^B(T_i, \{T_j\}_{j \neq i})$, where u^B is symmetric in its last $N-1$ arguments,[9] increasing in T_i, nondecreasing in T_j $(j \neq i)$, nonnegative and continuous.[10] I use T_i to represent the signal of bidder i. Capital letters will be used for random variables, while bold typeface indicates a vector. Note that I am implicitly assuming here that the bidders' valuations only depend on bidder-specific information.

The bidders' signals are assumed to be affiliated and symmetrically distributed on $[\underline{t}, \bar{t}]^N$ according to some bounded, atomless density $f(\mathbf{t})$ with respect to the Lebesgue measure. Affiliation is equivalent to the density being log-supermodular almost everywhere.[11] The signals are symmetrically distributed if for any permutation $\pi(\mathbf{t})$ of $\mathbf{t}$, $f(\mathbf{t}) = f(\pi(\mathbf{t}))$.

The bidders (or agents) have no capital with which to make their bids, so they must rely entirely on the funding decision of their principal. In the first stage of the game, each of the principals privately observes some signal about the value of

[9] In other words, if $\pi(\mathbf{T}_{-i})$ is any permutation of the vector of opposing signals then I am assuming that $u_i^B(T_i, \mathbf{T}_{-i}) = u_i^B(T_i, \pi(\mathbf{T}_{-i}))$.

[10] Milgrom and Weber [1982] also allow for the utility function to depend on a vector of signals (labeled $S_1, S_2, \ldots$ in the paper) that are not specific to any bidder. I do not make use of these signals here, so they are omitted.

[11] Milgrom and Weber [1982] discuss the affiliation property in detail. The affiliation property is also known outside economics as multivariate total positivity of order 2 [Karlin and Rinott, 1980].

the asset to the company, and based on that signal sets a budget constraint for the bidder at auction. I first consider the case where the principal may only select a hard budget constraint (i.e., the principal allocates a fixed amount of funds which the bidder must not exceed). I briefly consider the case of a soft budget constraint at the end of the paper (Section 2.6).

Both the bidder and the principal are assumed to receive a zero payoff from losing or not participating in the auction. The principal may set a budget constraint that is low enough to prevent the bidder from winning.

Two conditions on the relationship between each principal and bidder motivate the principal's use of a budget constraint in the model. The first condition is that given the same information the bidder would be willing to bid more for the asset than the principal would. As mentioned in the introduction this could be the result of the bidder's empire-building motives or gaining some private payoff from controlling more assets irrespective of the payoff to the firm.

The second is that the bidder is better informed about the value of the asset to the firm than the principal. This could be the result of the bidder's specialized knowledge about the market that the firm operates in. The assumption motivates the principal to employ the bidder to decide on a bid rather than submitting a bid directly herself.

These conditions are formalized in the model by specifying that the relationship between the principal i's valuation, $u_i^P(T_i, \{T_j\}_{j \neq i})$, and bidder i's valuation is such that for all realizations $\mathbf{t}$ the principal's valuation is smaller. To simplify the solution to the model, I use a linear relationship between the two given by

$$u_i^B(\mathbf{t}) = u_i(\mathbf{t}) > \delta u_i(\mathbf{t}) = u_i^P(\mathbf{t}) \tag{2.1}$$

with $0 < \delta < 1$ representing how the principal discounts the bidder's assessment of the value of the good to the firm. Equivalently one could call $1/\delta$ the amount by which the bidder overstates the value of the asset due to his interest in empire-building.[12]

The principal and bidder are assumed to receive a constant fraction of the payoff to the firm, and thereby are interested in maximizing the firm's payoff directly. Specifically, let σ_P and σ_B be the principal's and bidder's shares of firm stock and suppose that firm i wins the item for a payment of p. Then bidder i receives $\sigma_B(u_i(\mathbf{t})-p) = \sigma_B(1-\delta)u_i(\mathbf{t})+\sigma_B(\delta u_i(\mathbf{t})-p)$ and principal i receives $\sigma_P(\delta u_i(\mathbf{t})-p)$, where I let $\delta u_i(\mathbf{t})$ be the value to the firm. Note that the term $\sigma_B(1-\delta)u_i(\mathbf{t})$ is the bidder's private gain from empire-building. Because the shares, σ_B and σ_P, are assumed to be constant and non-zero they cannot affect the decisions of the principals and bidders, so they are omitted from the rest of the paper.

Principal i receives a signal, S_i, in the first stage which is informative because it is affiliated with the bidder i's signal. Affiliation between S_i and T_i and the previous assumptions made on the distribution of $\mathbf{T}$ imply that all of the signals in

[12] All of the results in the paper go through if one assumes that $u_i^B(\mathbf{t}) > u_i^P(\mathbf{t})$ for all $\mathbf{t}$ and that the principal's valuation can be written as $u_i^P(\mathbf{t}) = d(u_i^B(\mathbf{t}))$ where $d(x) < x$ for all x and $d(x)$ is increasing and convex. These conditions preserve the assumptions made in Theorem 1. The results on soft budget constraints (Section 2.6) can also be extended to allow for such a relationship between the bidder's and principal's valuations.

the model are affiliated; however, allowing for arbitrary dependencies between the principals' signals turns out to be problematic for the specification of the equilibrium described here. So I make the further assumption that conditional on $T_i = t_i$, S_i is independent of the other signals. In other words, principal i's signal provides no additional information about other signals in the model once bidder i's signal is known. I believe this assumption complements the assumption that the bidders are better informed about the valuation of the good. The bidders in the model generally have better information about the environment than the principals, and this seems natural if bidder is a specialist. Formally, one may write the joint distribution of $(\mathbf{s}, \mathbf{t})$ as $f(\mathbf{s}, \mathbf{t}) = f_{\mathbf{T}}(\mathbf{t}) \prod_{i=1}^{N} f_{S_i \,|\, T_i}(s_i \,|\, t_i)$.

Figure 2.1 illustrates the information structure of the model with $N = 4$. The nodes in the graph represent the signals of all of players. Bidders' signals are labeled B_i, and principals' signals are labeled P_i. The edges represent the statistical dependence between the signals.

Finally, I assume that information is private to each of the principal-agent pairs throughout the first and the second stages. In other words, nothing is learned by firm i about the signal or budget of firm j after the conclusion of the first stage.

2.4 *Equilibrium*

I describe symmetric equilibrium strategies in this model for the first- and second-price auctions. As in Milgrom and Weber [1982] I consider the second-price auction first. The strategies consist of a function mapping principals' signals into

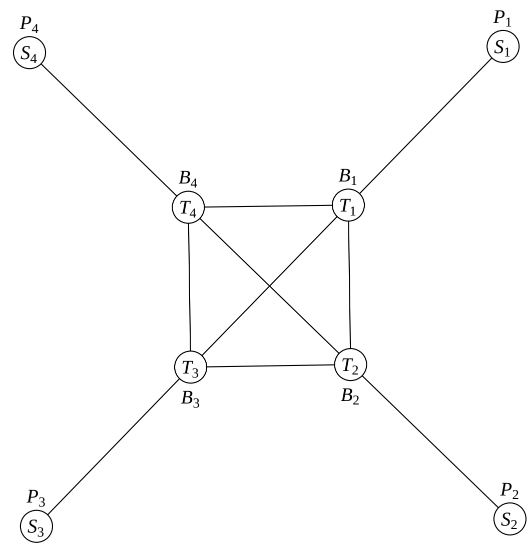


Fig. 2.1: Information Structure ($N = 4$)

budgets, $w(s)$, called the budget function, and a bid function for the bidders, $B(s, t)$, that depends on signals received by the bidder and the principal. I distinguish between the "constrained" bid function $B(s, t)$ that incorporates the budget into the bid and the "unconstrained" bid function $b(t)$ representing the bid that each bidder would make in the absence of a budget constraint.[13] In considering the problem facing the principal and bidder of a particular firm it is helpful to recognize that I can treat the strategies (bids and budgets) of the opposing firms as fixed throughout the first and the second stage. This is a consequence of the assumption that firms do not observe the actions of the other firms until the auction is finished.

[13] The unconstrained bid function is not a function of the principal's signal, because the principal's signal does not enter into the value of the asset. Also due to the conditional independence of the principal's signal, the bidder safely disregards this information.

Taking the perspective of bidder i, the basic idea behind the equilibria of the first- and second-price auction is to recognize that if the opposing principals use symmetric, increasing strategies and the opposing bids take a form that is increasing in both signals, the bids that bidder i faces are affiliated (see footnote 14). If one then appropriately modifies the definitions of the key objects in the Milgrom and Weber [1982] paper, the unconstrained choice of bidder i takes the same form as the equilibrium strategy in Milgrom and Weber [1982], and quasi-concavity of the objective implies that $B(s,t)$ takes the form $B(s,t) = \min\{b(t), w(s)\}$. Using this conclusion, I can then provide a result for the existence of a symmetric equilibrium for the principals.

2.4.1 Second-Price Auction

Consider the decision of bidder i given some arbitrary budget constraint. Suppose the equilibrium $b(t)$ is increasing and continuous. Then the selection of a budget constraint in bid space is equivalent to the selection of a "cutoff" type in type space. Define the function $\hat{t}(s)$ as $w(s) = b(\hat{t}(s))$, so that a selection $\hat{t}(s)$ is equivalent to the selection $w(s)$.

The description of equilibrium strategies is simplified if I define the random variable $\widetilde{T} = \min\{T, \hat{t}(S)\}$. In equilibrium $\widetilde{T}$ will represent the information that is revealed by an opposing bidder's bid. To describe the equilibrium I start by assuming that the opposing principals all use the increasing strategy $\hat{t}(s)$ and that the opposing bidders' strategies take the form

$$B(s,t) = \min\{b(t), b(\hat{t}(s))\} = b(\min\{t, \hat{t}(s)\}) = b(\tilde{t}) \tag{2.2}$$

where $b(t)$ is some increasing function. The second equality follows because $b(t)$ is increasing and min is nondecreasing. If the opposing bids take this form then the highest opposing bid is submitted by the opposing bidder with the highest realization of $\widetilde{T}$. In this sense, one can think of $\widetilde{T}$ as representing the type of an opposing bidder.

The distribution of $\widetilde{T}$ is a function of the signals of the principal and the bidder, along with the strategy used by the principals. Each $\widetilde{T}$ is a composition of nondecreasing functions of affiliated random variables. A straightforward application of Theorem 23 in Milgrom and Weber [1982] shows that $T_1, \widetilde{T}_2, \ldots, \widetilde{T}_N$ are affiliated.[14] Also, affiliation between T_i and S_i and a similar argument implies that $S_i, T_i, \widetilde{\mathbf{T}}_{-i}$ are affiliated. It follows that the first order statistic of $\widetilde{\mathbf{T}}_{-i}$ (denoted $\widetilde{T}_{(1)}$) and T_i are affiliated [Milgrom and Weber, 1982, Theorem 2].

Now define the function $v(x,y) = E[u(\mathbf{T}) \,|\, T_i = x, \widetilde{T}_{(1)} = y]$.[15] Given the assumptions on u and affiliation, $v(x,y)$ is increasing in x and nondecreasing in y.

[14] By Theorem 23, $Z = (S,T)$ being affiliated is equivalent to the following inequality holding for any nondecreasing g, increasing set A, and sublattice S,

$$\mathrm{E}[g(Z) \,|\, AS] \geq \mathrm{E}[g(Z) \,|\, S] \geq \mathrm{E}[g(Z) \,|\, \widehat{A}S]$$

Let $\widetilde{Z}_1 = T_1$ and $\widetilde{Z}_i = \widetilde{T}_i$ for $i \neq 1$. For some nondecreasing function h, $h(\widetilde{Z})$ is the composition of two nondecreasing functions of Z and hence $h(\widetilde{Z}) = g(Z)$ for some g. Therefore, the above inequality holds for any h and $\widetilde{Z}$ is affiliated.

[15] This function will play a role analogous to the $v(x,y)$ defined in Milgrom and Weber [1982].

If the opposing bidders adopt the strategy $b(\tilde{t})$, and bidder i makes a bid according to $b(t')$, then bidder i wins in the event that $t' > \widetilde{T}_{(1)}$. Assuming that the opposing bids take the form $b(\tilde{t}) = v(\tilde{t}, \tilde{t})$, bidder i's payoff from bidding according to $b(t')$ when his true signal is t is

$$\int_{\underline{t}}^{t'} (v(t, x) - v(x, x))g(x \,|\, t)\, dx \tag{2.3}$$

where $g(x \,|\, t)$ is used for the density of $\widetilde{T}_{(1)}$ conditional on t.[16] By direct analogy with Milgrom and Weber [1982], this function is maximized by choosing $t' = t$. For $t' < t$, it is increasing. Therefore, a choice of $\tilde{t} = \min\{t, \hat{t}\}$ satisfies the Karush-Kuhn-Tucker conditions for an optimum.

Lemma 1. *Assume the $N - 1$ other bidders use the strategy $b_{-i}(x) = v(x, x)$ and that the other principals use the same increasing budget function given by $\hat{t}_{-i}(s)$, then the unconstrained best response of bidder i is $b(t) = v(t, t)$. Given a budget $\hat{t}(s)$, the constrained best response is to bid* $\min\{b(t), b(\hat{t}(s))\} = b(\tilde{t})$.

The unconstrained best response defined above is the bidder's expected payoff conditional on his own information and the information contained in the event that his bid is just equal to the second highest bid. In that sense, this equilibrium is analogous to the one prescribed by Milgrom and Weber [1982].

The budget constraint manifests itself in the above strategies by forcing to the bidder to bid as if his signal were lower than it is. That is, there is a sense in which

[16] Due to the principals' signals being independent condition on the realizations of the bidders' signals $g(x \,|\, t, s) = g(x \,|\, t)$.

the bidder is not conditioning on the "correct" information when he makes his bid. However, this is a natural consequence of treating the budget constraints for the principal as a choice in type-space rather than bid-space.

Given the description of second stage play, the principal's problem is to select a budget constraint that has the effect of lowering the bid of her bidder on average. Left unconstrained the bidders will always select bids that are higher than the principals would like him to.[17]

Letting $f(t \mid s)$ be the density of the bidder's signal conditional on the principal's signal, the payoff to a principal receiving a signal of s and choosing a budget constraint consistent with $\hat{t}$ is given by

$$\begin{aligned} &\int_{\underline{t}}^{\hat{t}} \int_{\underline{t}}^{t} (\delta v(t,x) - v(x,x)) g(x \mid t) f(t \mid s) \, dx \, dt \\ &+ \int_{\hat{t}}^{\bar{t}} \int_{\underline{t}}^{\hat{t}} (\delta v(t,x) - v(x,x)) g(x \mid t) f(t \mid s) \, dx \, dt \end{aligned}$$

The first and second terms correspond to the payoff to the principal when the bidder is unconstrained and constrained. After canceling terms and rewriting, the first order condition for this problem can be written as follows

[17] Consider, for example, the case where the principals observe their bidders' signals directly and decide on bids directly. In this case, the symmetric equilibrium is for the principals to bid according to the function $b(t) = \delta v(x,x)$ where $v(x,x)$ is defined as in Milgrom and Weber [1982].

$$
\begin{aligned}
0 &= \int_{\hat{t}}^{\bar{t}} (\delta v(t, \hat{t}) - v(\hat{t}, \hat{t}))g(\hat{t} \,|\, t)f(t \,|\, s)\, dt \\
&= \mathrm{E}[\delta u(T_i, \mathbf{T}_{-i}) - u(\hat{t}, \mathbf{T}_{-i}) \,|\, T_i \geq \hat{t}, \widetilde{T}_{(1)} = \hat{t}, s] \\
&\quad \times P(T_i \geq \hat{t} \,|\, \widetilde{T}_{(1)} = \hat{t}, s) \qquad (2.4)
\end{aligned}
$$

where the second line replaces the function v with the utility function u. The equation is satisfied by setting $\hat{t}$ so that the expected payoff to the principal conditional on the principal's information, the budget constraint binding, and the highest opposing bidder having an effective type of $\hat{t}$ is zero. For some values of s, there may be no $\hat{t}$ in the support of T_i for which the right hand side of (2.4) is positive, in which case the principal optimally sets $\hat{t} = \underline{t}$ and prevents the bidder from participating.

To prove that an equilibrium exists I take advantage of work on the existence of monotone pure strategy equilibria in games of incomplete information [Athey, 2001, Reny, 2011]. Using results from these papers I am able to show that existence follows from a single crossing condition being satisfied. A sufficient condition for a single crossing condition to hold in this model is that the right-hand side of equation (2.4) be increasing in s. A sufficient condition for this to be true is that $\delta u(t_i, \mathbf{t}_{-i}) - u(t_i, \mathbf{t}_{-i})$ is nondecreasing in $(t_i, \mathbf{t}_{-i})$, since affiliation between S_i and $(T_i, \mathbf{T}_{-i})$ then implies that the expectation is increasing in s [Milgrom and Weber, 1982, Theorem 5]. The second term, $P(T_i \geq \hat{t} \,|\, \widetilde{T}_{(i)} = \hat{t}, s)$, is the expected value of a nondecreasing function of T_i, so it also must be nondecreasing in s.

Theorem 1. *If the right-hand side of equation (2.4) is increasing in s, a symmetric equilibrium in the second-price auction described above is given by $b(t) = v(t, t)$ and*

$w(s) = b(\hat{t}(s))$, where $\hat{t}(s)$ solves (2.4) when $\hat{t}(s) > \underline{t}$.

Proof. Given Lemma 1, the principals may be thought of as competing against each other in an odd sort of auction, where each is asked to name a value for $\hat{t}$ and their bid is calculated according to $\min\{v(t,t), v(\hat{t},\hat{t})\}$ so that there bid is random from their perspective. Note that conditional on t the bid is not increasing in $\hat{t}$ for all $\hat{t}$, but the expected bid is increasing for all $\hat{t} \in (\underline{t}, \bar{t})$. I show that this game satisfies the conditions in Athey [2001] including her single crossing condition, and hence possesses an equilibrium in increasing strategies.

The right-hand side of equation (2.4) being an increasing function of s implies that if π is the principal's objective we have $\pi_{\hat{t}s} > 0$. This in turn implies the single crossing condition.

I need to verify assumptions A1-A3 in Athey [2001] to apply Theorem 6 in that paper. A1 and A2 follow by assumptions made on the joint density and the utility function. A3 in this case requires that $\mathrm{E}[\delta u_i(\mathbf{T}) \,|\, s_i, W_i(\hat{t}_i', \hat{t}_{-i})]$ be increasing in s and nondecreasing in $\hat{t}'$, where $W_i(\hat{t}_i', \hat{t}_{-i})$ represents the event that the principal i wins with $\hat{t}_i'$ when the other principals set budget constraints according to $\hat{t}_{-i}$. Employing Theorem 5 from Milgrom and Weber [1982] again, this expression is increasing in s_i and nondecreasing in $\hat{t}_i'$.

Since the random variable $\widetilde{T}$ is not a primitive of the model, I also need to verify that in equilibrium its distribution satisfies the assumptions of the Athey [2001] paper. In particular, it should not have any mass points except at the lower bound of the support. But as long as $\hat{t}(s)$ is increasing this is true.

Therefore, Theorem 6 applies and an equilibrium in increasing strategies exists. Furthermore, a symmetric equilibrium, which is required for the arguments made above, also exists. This model satisfies the requirements of Theorem 4.5 in Reny [2011], which guarantees the existence of a symmetric equilibrium in increasing strategies when the bids are restricted to a finite action space. Following the arguments in Athey [2001] but considering only sequences of symmetric equilibria as the bid space becomes finer, it follows that a symmetric equilibrium exists when the bid space is a continuum.

□

2.4.2 First-Price Auction

The analysis of the first-price auction proceeds in the same way, beginning with the problem faced by the bidder. As in the second-price auction, I am able to make use of the equilibrium identified in the Milgrom and Weber [1982] paper for the first-price auction. With $v(x, y)$ and $g(x \mid t)$ defined as above, define the following unconstrained bid function

$$b(t) = \int_{\underline{t}}^{t} v(y, y)\, dL(y \mid t)$$

where

$$L(y \mid t) = \exp\left\{ -\int_{y}^{t} \frac{g(z \mid z)}{G(z \mid z)}\, dz \right\}$$

Now suppose that the opposing bidders bid according to $b(t)$ when they are unconstrained. Again, since $b(t)$ is an increasing, continuous function, the principal's choice of a budget constraint in bid space is equivalent to the choice of a cutoff type in type space. So if the opposing principals adopt the increasing strategy $\hat{t}(s)$, the opposing bids take the form

$$B(s,t) = \min\{b(t), b(\hat{t}(s))\} = b(\min\{t, \hat{t}(s)\}) = b(\tilde{t})$$

Again, define the random variable $\widetilde{T} = \min\{T, \hat{t}(S)\}$ with conditional density $g(x \mid t)$. I can now represent the payoff to bidder i of bidding $b(t')$ as

$$\int_{\underline{t}}^{t'} (v(t,x) - b(x)) g(x \mid t)\, dx \tag{2.5}$$

where I am again using the fact that a bid of $b(t')$ against opposing bids of $b(\tilde{t})$ wins in the event that $t' > \widetilde{T}_{(1)}$. The idea behind the following lemma is exactly the same as in the second-price auction. The new objects $v(x,y)$ and $g(x \mid t)$ have the same properties as their analogues in Milgrom and Weber [1982], so the proof that it is optimal for the unconstrained bidder to select $t' = t$ proceeds in the same way. The constrained bidder cannot select t, but as Milgrom and Weber [1982] show the objective is increasing for $t' < t$, so it must be optimal to select $t' = \hat{t}$. Finally, the argument for why the bid function must be continuous goes through in the same way.[18]

[18] The continuity argument in Milgrom and Weber [1982] depends on the symmetry of the signals. For differentiability, one may either rescale the signals to make the bid function differentiable [Milgrom and Weber, 1982] or notice that if the bid function is monotonic it must be differentiable almost everywhere. Combined with continuity these two properties imply that the differential

Lemma 2. *Let* $b(t) = \int_{\underline{t}}^{t} v(x,x)\, dL(x \,|\, t)$ *where* $L(x \,|\, t) = \exp\left\{-\int_x^t \frac{g(z \,|\, z)}{G(z \,|\, z)}\, dz\right\}$. *If the opposing bidders bid according to* $b_{-i}(\tilde{t})$ *with* $\hat{t}_{-i}(s)$ *increasing, then the unconstrained best response of bidder* i *is* $b(t)$. *Due to the quasi-concavity of bidder* i*'s objective, the choice* $\min\{b(t), b(\hat{t}(s))\} = b(\tilde{t})$ *is a constrained best response.*

The principal's payoff in the first-price auction can be written as the payoff in the second-price auction with the payment term replaced

$$
\begin{aligned}
&\int_{\underline{t}}^{\hat{t}} \int_{\underline{t}}^{t} (\delta v(t,x) - b(t)) g(x \,|\, t) f(t \,|\, s)\, dx\, dt \\
&+ \int_{\hat{t}}^{\bar{t}} \int_{\underline{t}}^{\hat{t}} (\delta v(t,x) - b(\hat{t})) g(x \,|\, t) f(t \,|\, s)\, dx\, dt
\end{aligned}
$$

The first order condition of the principal's problem in this case can be written as

$$
\begin{aligned}
0 &= \int_{\hat{t}}^{\bar{t}} \left\{ \left\{ \delta v(t,\hat{t}) - b(\hat{t}) \right\} g(\hat{t} \,|\, t) - b'(\hat{t}) G(\hat{t} \,|\, t) \right\} f(t \,|\, s)\, dt \\
&= \int_{\hat{t}}^{\bar{t}} \left\{ \delta v(t,\hat{t}) - b(\hat{t}) - b'(\hat{t}) \frac{G(\hat{t} \,|\, t)}{g(\hat{t} \,|\, t)} \right\} g(\hat{t} \,|\, t) f(t \,|\, s)\, dt \\
&= \mathrm{E}\left[\delta u(T_i, \mathbf{T}_{-i}) - b(\hat{t}) - b'(\hat{t}) \frac{G(\hat{t} \,|\, T_i)}{g(\hat{t} \,|\, T_i)} \;\Big|\; T_i \geq \hat{t}, \widetilde{T}_{(1)} = \hat{t}, s \right] \\
&\quad \times P(T_i \geq \hat{t} \,|\, \widetilde{T}_{(1)} = \hat{t}, s)
\end{aligned} \tag{2.6}
$$

An equilibrium in monotone pure strategies exists here as long as I can show that the same single crossing condition is satisfied. For the first-price auction the single crossing condition is satisfied immediately without the additional assumptions

equation defines the behavior of the bidder almost everywhere which is all that is required for a Bayesian Nash Equilibrium.

made in the second-price auction case. The integrand in the expectation in equation 2.6 is increasing in $\mathbf{T}$ because the utility function is increasing by assumption and the term $G(\hat{t} \,|\, t)/g(\hat{t} \,|\, t)$ is decreasing in t by affiliation.

Theorem 2. *A symmetric equilibrium in the first-price auction described above is given by* $b(t)$ *and* $w(s) = b(\hat{t}(s))$*, where* $b(t)$ *is defined in Lemma 2 and* $\hat{t}(s)$ *solves (2.6) when* $\hat{t}(s) > \underline{t}$*.*

Proof. The discussion preceding the proof shows that the Athey [2001] single crossing condition is satisfied here. The remainder of the proof proceeds exactly as in Theorem 1. □

2.5 Revenue and Efficiency

The primary focus of the existing literature on budget constraints in auctions has been on their effect on expected revenue and to a lesser extent on expected efficiency (or social surplus). For the first and second-price auctions, Che and Gale [1998] find that the first-price auction dominates the second-price auction both in terms of expected revenue and expected efficiency.

The strongest results comparing the revenue and efficiency in this model come from considering a special case of the model where an independence condition is satisfied between bidders. In this case we are able to show that the first- and second-price auctions perform equivalently in terms of both efficiency and revenue. Later in the section I return to the question of the relative performance of the first- and second-price auctions with affiliated values. In that case I am able to rank the auctions on efficiency and provide a partial result for revenue. There are two counteracting effects on revenue so it is difficult to determine the relative ranking in general. However, I am able to give an example where the second-price auction raises more revenue than the first-price auction.

2.5.1 Independent Signals

I make use of the affiliation property twice in the model described above. The first use of affiliation relates the signals of the bidders, while the second use of affiliation relates the signals of a particular principal to her corresponding bidder. In the independence case I make the stronger assumption that the bidders' signals are statistically independent. In other words I may write the joint distribution of random variables in the model as $f(\mathbf{s}, \mathbf{t}) = \prod_{i=1}^{N} f_{S_i,T_i}(s,t)$, where $f_{S_i,T_i}(s,t)$ is affiliated.

Since independent signals are trivially affiliated, the strategies described in Theorems 1 and 2 are also equilibria with independent signals. The important effect of independence is to simplify the principal's decision. The key step is to recognize that the distribution of the effective types of the opposing bidders, represented by $g(x \mid t)$ in the previous expressions, no longer depends on t when the independence condition is satisfied. Therefore, one may rewrite the principal's payoff in the first-price auction as

$$\begin{aligned}
&\int_{\underline{t}}^{\hat{t}} \int_{\underline{t}}^{t} (\delta v(t,x) - b(t)) g(x) f(t \mid s)\, dx\, dt \\
&\quad + \int_{\hat{t}}^{\bar{t}} \int_{\underline{t}}^{\hat{t}} (\delta v(t,x) - b(\hat{t})) g(x) f(t \mid s)\, dx\, dt \\
= \; &\int_{\underline{t}}^{\hat{t}} \int_{\underline{t}}^{t} (\delta v(t,x) - v(x,x)) g(x) f(t \mid s)\, dx\, dt \\
&\quad + \int_{\hat{t}}^{\bar{t}} \int_{\underline{t}}^{\hat{t}} (\delta v(t,x) - v(x,x)) g(x) f(t \mid s)\, dx\, dt
\end{aligned}$$

where I have used the fact that under independence $b(t) = \frac{1}{G(t)} \int_{\underline{t}}^{t} v(x,x) g(x)\, dx$. By inspection, the principal's payoff function is the same under both auctions when the independence condition holds implying that the principal must make the same choice for $\hat{t}$ in both auctions.

To complete the equivalence argument, observe that due to the way the equilibrium strategies are defined the resulting bids are equivalent to the bids that would be observed in an independent private values auction without budget constraints with signals distributed according to $\widetilde{T} = \min\{T, \hat{t}(S)\}$. Therefore, the revenue equivalence theorem applies.

Since the principals and the bidders disagree about the value of the asset to the firm, the efficiency of the auction is potentially ambiguous. The auction may be called efficient if it always allocates to the bidder with the highest valuation, but one may also define efficiency in terms of the valuation of the principals. In this model, however, there is no ambiguity because the bidder with the highest valuation is always paired with the principal with the highest valuation.

With independence between bidders, the two auctions perform the same in terms of efficiency. With the same budget constraint function in both auctions, the same realization of signals must yield the same winner, and the two auctions cannot differ in their allocations.

Theorem 3. *When signals are independent across bidders, the first- and second-price auctions with budget constraints are equivalent, both in terms of expected revenue and expected efficiency.*

The equivalence in terms of efficiency and revenue also implies that the two auctions are payoff equivalent for both the principal and the bidder. In fact, the bidder's bidding behavior makes the type of auction (first- or second-price) and the distribution of the opposing bidder's signals irrelevant to the principal. The principal's decision is completely determined by the valuation function and the joint distribution of her signal and the bidder's, $f(t \mid s)$. This is a direct consequence of treating the principal's decision in type-space though. The actual budget constraints (i.e., $b(\hat{t})$) do vary between the auction formats because the bid functions differ.

Note that we will not have full efficiency here because the budget constraint will occasionally bind, so it is possible that the bidder with the highest realization of T loses to another bidder because he is budget constrained.

To illustrate these results it is helpful to consider a linear example.

There are 2 firms. Each principal receives a signal, S, distributed uniformly on $[0, 1]$. Conditional on the principal's signal, s, the bidder's signal is uniformly distributed on $[s, s+1]$, so the relation between the two signals is $T = S + \varepsilon$

where $\varepsilon \sim U[0,1]$.[19] The bidder has a private value for the item given by his signal, t, and the principal values the item at δt.

For either auction, principal i chooses $\hat{t}$ to solve

$$\mathrm{E}[\delta T_i - \hat{t} \,|\, T_i \geq \hat{t}, s] = 0$$

which has the solution $\hat{t}(s) = \frac{\delta}{2-\delta}(s+1)$. So the bidder is budget constrained when $\varepsilon > \frac{\delta}{2-\delta} - \frac{2-2\delta}{2-\delta}s$. Note that as δ approaches 1 the principal relaxes the budget constraint eventually leaving the bidder unconstrained. When $\delta = 1$ the principal can trust the bidder to bid exactly as she would if she were to observe T.

In the second-price auction, a unconstrained bidder with signal t has a dominant strategy to bid his value, t. From the seller's perspective (or the perspective of an opponent) individual bids are therefore distributed according to the random variable W where $W = \min\{S + \varepsilon, \frac{\delta}{2-\delta}(S+1)\}$.

In the first-price auction, the unconstrained best response of a bidder with signal t is

$$b(t) = \mathrm{E}[W \,|\, W \leq t]$$

and to the seller bids are distributed according to $b(W)$.

[19] The joint density of T and S can be written as $f(t,s) = \mathbf{1}\{s \leq t \leq s+1\}\mathbf{1}\{0 \leq s \leq 1\}$ which is affiliated. The bidder's signals are clearly independent of one another.

In both auctions, the seller's expected revenue is the expected value of $W_{(2)}$ where $W_{(2)}$ is the second order statistic of W. Note that the auction from the seller's perspective is equivalent to a independent, private values auction where values are drawn according to the distribution of W.

Finally, either auction allocates the good inefficiently if, for example, $t_1 < t_2$ but $W_1 > W_2$ which occurs with positive probability.

2.5.2 Affiliated Signals

When signals between bidders are affiliated, the first- and second-price auction are no longer equivalent from the perspective of the principal. In fact, given a signal s if the principal were to choose the same budget constraint in both auctions, she would earn a higher payoff from the first-price auction when bidder's signals are affiliated. The reason for this is that the principal expects to pay less in the first-price auction when she sets the same budget constraint because affiliation between the bidders causes the average bid in the second-price auction to be higher [Milgrom and Weber, 1982].

The principal's preference for the first-price auction leads the principal to relax the budget constraint in the first-price auction relative to the second-price auction. Specifically, I show below that for a given s, the principal chooses a greater $\hat{t}$ in the first-price auction.

Theorem 4. *In first- and second-price auctions with the same distribution of signals, let $\hat{t}^F(s)$ and $\hat{t}^S(s)$ be equilibrium strategies of the principals in the first- and* *second-price auctions. It must be that* $\hat{t}^F(s) \geq \hat{t}^S(s)$ *for all* s.

Proof. Let $\gamma(x \mid y) = \frac{g(x \mid y)}{G(x \mid y)}$ and note that affiliation implies that $\gamma(x \mid y)$ is increasing in y [Milgrom and Weber, 1982, Lemma 1]. Using this and replacing $b'(\hat{t})$ the right hand side of (2.6) can be written as

$$\begin{aligned} & \int_{\hat{t}}^{\bar{t}} \left(\delta v(t,\hat{t}) - v(\hat{t},\hat{t})\right) g(\hat{t} \mid t) f(t \mid s)\, dt \\ + & \int_{\hat{t}}^{\bar{t}} \left(\gamma(\hat{t} \mid t) - \gamma(\hat{t} \mid \hat{t})\right) \left(v(\hat{t},\hat{t}) - b(\hat{t})\right) G(\hat{t} \mid t) f(t \mid s)\, dt \end{aligned} \tag{2.7}$$

The first term corresponds to the first order condition for the second-price auction, (2.4). The second term must be positive for $t > \hat{t}$ because γ is increasing in its second argument and $v(\hat{t},\hat{t}) > b(\hat{t})$. This is true for any symmetric, increasing strategy used by the opposing principals.

Next, suppose that there exists equilibria such that $\hat{t}^F(s) < \hat{t}^S(s)$ for some s where the inequality holds on a set of nonzero measure. I show that the first order condition in the first-price auction at such a $\hat{t}^F$ cannot hold. Let $\phi(\hat{t}(s), s, \hat{t}')$ be the first order condition for the principal in the second-price auction when she receives signal s, sets budget $\hat{t}$ and the opposing principals set budgets according to $\hat{t}'$. So we have $\phi(\hat{t}^S(s), s, \hat{t}^S) = 0$. For some $s' < s$, $\phi(\hat{t}^F(s), s', \hat{t}^F) = 0$ where $\hat{t}^F(s) = \hat{t}^S(s')$. Since the single crossing condition holds strictly, $\phi(\hat{t}^F(s), s, \hat{t}^F) > 0$. This and the argument in the previous paragraph imply that (2.7) must be positive for such an $\hat{t}^F$.

□

Relaxing the budget constraint must improve the efficiency of the auction because the allocation is more likely to depend on the bidders' signals and not the principals'. Since it is the bidders' signals that determine the value of the asset in the model, this must improve efficiency.

Corollary 1. *The first-price auction with budget constraints is more efficient than the second-price auction with budget constraints.*

Proof. I show that Theorem 4 implies for a given realization of signals if the first-price auction allocates inefficiently the second-price auction must too. The idea is similar to the one used in Theorem 1 of Che and Gale [1998]. Suppose that bidder 1 has the highest signal, t_1, but bidder 2 wins in the first-price auction so $\min\{t_1, \hat{t}^F(s_1)\} < \min\{t_2, \hat{t}^F(s_2)\}$. First, bidder 1's budget constraint must bind (otherwise bidder 1 would win). There are two cases, (i) $\hat{t}^F(s_1) < t_2 < \hat{t}^F(s_2)$ and (ii) $\hat{t}^F(s_1) < \hat{t}^F(s_2) < t_2$. In the second-price auction, each of the principals chooses a lower $\hat{t}$ (Theorem 4), so bidder 1's budget binds and by examining (i) and (ii) it is clear that bidder 2 must be the winner.

The first-price auction, however, might allocate efficiently when the second-price auction does not. This occurs if, for example, $t_2 < t_1 < \hat{t}^F(s_1) < \hat{t}^F(s_2)$ (so bidder 1 wins in the first-price auction) but $\hat{t}^S(s_1) < t_2 < t_1 < \hat{t}^S(s_2)$ (so bidder 2 wins in the second-price auction).

□

Interestingly, Che and Gale [1998] also find that the first-price auction is more efficient in their model. Recall that in their model the budget constraint is treated as an exogenous random variable for each bidder which does not depend on the type of auction. The result of this is that the incentive for bidders to shade their bids in the first-price auction also causes the budget constraints to bind less often relative to the second-price auction. In contrast, here the budget constraints are binding less often because the principals are choosing to relax the budget constraint in response to the first-price auction offering a higher payoff with affiliated signals. Also note that when bidders receive independent signals there is no efficiency advantage to the first-price auction in this model, and the Che and Gale [1998] model makes an analogous assumption about independence of bidders' signals.

The efficiency result is likely restricted to the symmetric environment I consider here, as the second-price auction tends to outperform the first-price auction on efficiency in standard models when the environment is asymmetric (see for example Proposition 2 in Maskin [1996]).

When considering expected revenue, there are two counteracting effects here and without an explicit solution for the equilibria it is difficult to determine which one dominates. The first effect identified by Milgrom and Weber [1982] is also known as the linkage principal and has been shown to lead the second-price auction to raise more revenue. This effect is certainly present in the model, but so is the effect of the principal relaxing the budget constraint in the first-price auction which favors the first-price auction in terms of revenue.

Example 2.5.2 shows that for at least some distributions the Milgrom and Weber [1982] ranking holds. Note that this is the opposite ranking of the one found in Che and Gale [1998]. I leave the question of whether or not this ranking always holds in the model for future research.

Let $N = 2$ and suppose that the joint distribution of $(t_1, t_2) \in [0, 1]^2$ is given by $f(t_1, t_2) = 3 \min\{t_1, t_2\}$.[20] Assume that S_i and T_i are perfectly correlated, and the bidders have private values with $u(\mathbf{t}) = t_i$ for bidder i.

In the second-price auction, the solution to the principal's problem is to set $\hat{t}(s) = \delta s$. The result is that the bidder is always budget constrained, bidding according to δs. Since the budget constraints always bind in this case, bidder 1 and principal 1 win when $s_1 > s_2$ or $t_1 > t_2$. The principal's expected payment is then $\delta \mathrm{E}[S_2 \mid S_2 < s_1] = \frac{2}{3}\delta s_1$.

The solution to the first-price auction is more involved. I start by assuming that the strategy of the opposing principal is linear (i.e., $\hat{t}(s) = s/\alpha$). From this I calculate the following

$$\begin{aligned} g(\hat{t} \mid t) &= \frac{2\alpha}{2 - t} \\ G(\hat{t} \mid t) &= \frac{2\alpha\hat{t} - t}{2 - t} \\ b(t) &= \frac{2\alpha}{4\alpha - 1} t \end{aligned}$$

Using the assumption that $\hat{t}(s) = s/\alpha$, (2.6) becomes

$$\left(\delta s - \frac{2s}{4\alpha - 1}\right) \frac{2\alpha}{2 - s} - \frac{2\alpha}{4\alpha - 1} \frac{s}{2 - s} = 0$$

which has the solution $\alpha = \frac{3+\delta}{4\delta}$. Again, the budget constraint always binds so principal 1 wins when $s_1 > s_2$. principal 1's expected payment can be written as

[20] This is the distribution from Example 6.2 in Krishna [2002] truncated to $[0, 1]^2$.

$$b(\hat{t}(s_1))P(S_2 < s_1 \mid s_1) = \frac{2}{3}\delta s_1 P(S_2 < s_1 \mid s_1) = \frac{2}{3}\delta s_1 \frac{s_1}{2 - s_1}$$

Since $\frac{s_1}{2-s_1} < 1$ for $s_1 \in (0,1)$, the principal's expected payment is lower in the first-price auction.

2.6 Soft Budget Constraints

Up to this point, the principal has been constrained to hard budget constraints. An alternative would be to allow the principal to setup a cost schedule for the bidder that assigns a charge to the bidder for each bid he might select in the auction, so that a bid of b might cost the bidder $c(b)$ to place. One way for the principal to implement this might be to allow the bidder to borrow any amount from her but to apply a financing charge to each bid, $c(b) - b$.

There turns out to be an natural choice for this cost schedule (in the first- and second-price auctions at least) that induces the bidder to bid exactly as the principal would were she to have the same information as the bidder. This implies that the game is equivalent to a one stage game where the principals observe both S and T (although having observed T the information contained in S is irrelevant) and decide on bids directly.

Consider the second-price auction, and suppose that the principals observe T and may decide on a bid directly. Define $w(x,y) = \mathrm{E}[u(\mathbf{T}) \mid T_i = x, T_{(1)} = y]$.[21] Then in the symmetric equilibrium each principal bids according to $b(t) =$

[21] $T_{(1)}$ is the first order statistic of the other bidders' signals.

$\delta w(t,t)$ [Milgrom and Weber, 1982]. Now consider the cost schedule given by $c(b) = b/\delta$, so that if the bidder wins the item his payoff is $u(\mathbf{t}) - b/\delta$ where b is the payment made. In this case the bidder's equilibrium strategy is to set $b/\delta = w(t,t)$, which is exactly the bid that principal would make given the same information as the bidder.

The cost schedule $c(b) = b/\delta$ has the same effect on the first price auction. This is easily seen by observing that the bidder's objective function can be written as

$$\begin{aligned} & \int_{\underline{t}}^{t'} (v(t,x) - b(x)/\delta)h(x \mid t)\, dx \\ = \; & \frac{1}{\delta} \int_{\underline{t}}^{t'} (\delta v(t,x) - b(x))h(x \mid t)\, dx \end{aligned}$$

where to avoid confusion I use $h(x \mid t)$ to represent the density of $T_{(1)}$ given t. So the bidder's objective is a monotonic transformation of what the principal's objective would be if the principal observed t and submitted a bid in the auction directly. If every principal imposes the same cost schedule, then the game is equivalent to the one where the principals all observe their bidders' signals and participate directly in the auction, because the objective functions coincide.

Theorem 5. *When the principal's are allowed to use arbitrary cost schedules to finance bids in the first- and second-price auctions, it is an equilibrium for the principals to choose the cost schedule $c(b) = b/\delta$. The result is that the bids submitted coincide with hypothetical bids submitted by principals participating in an auction with all of the information available to them.*

Because the bids submitted coincide with the hypothetical game involving only principals, the following corollary is immediate.

Corollary 2. *In the game with soft budget constraints, the revenue and efficiency of the first- and second-price auctions are identical to that of the hypothetical games where the principals observe T and submit bids directly.*

In other words, in this model introducing soft budget constraints does not change the revenue rankings of Milgrom and Weber [1982]. Also note that the auction is fully efficient when the budget constraints are soft, because the firm with the highest value of T must win the auction in the symmetric equilibrium.

2.7 Conclusion

Given the classic symmetric auction models (the standard independent private values model and the Milgrom and Weber [1982] model) an important question for auction theory has been how robust are these results to changes in the environment. Budget constraints are cited as an example of a feature of real-world auctions that would lead to failure of the revenue equivalence theorem [Krishna, 2002]. However, results from models that incorporate budget constraints largely treat them as exogenous. In the model presented here I show that incorporating budget constraints endogenously can reverse this conclusion and restore revenue equivalence between the first- and second-price auction.

Taken together, the results from the hard and soft budget constraint versions of the model suggest that incorporating endogenous budget constraints into auction models may not significantly change the qualitative results on revenue and efficiency from the classic (symmetric) auction models.

In the hard budget constraint case I find the following. When signals are independent between bidders, revenue and efficiency equivalence holds between the first- and second-price auctions. When signals are allowed to be affiliated as in Milgrom and Weber [1982], the auctions are no longer equivalent in terms of efficiency but the revenue ranking of Milgrom and Weber [1982] holds in at least some cases. Whether this ranking always holds or not is a question left for future research.

In the soft budget constraint case, there is no difference between the auctions with budget constraints and auctions without budget constraints where the principal bids directly. That is, the principal is able to perfectly control the behavior of the bidder.

The previous literature on the effect of budget constraints on auctions suggests that the first-price auction and even the all-pay auction should be preferred to the second-price auction by a revenue or efficiency maximizing seller. However, in auctions where budget constrained bidders are likely to exist (e.g., spectrum auctions) the formats tend to resemble second-price auctions. For example, the U.S. Federal Communications Commission uses a Simultaneous Multiple Round Ascending auction. My results favor the second-price auction in many respects and hence seem better aligned with observed practice.

3. AN EXPERIMENT ON AUCTIONS WITH ENDOGENOUS BUDGET CONSTRAINTS

by Lawrence M. Ausubel, Justin E. Burkett, and Emel Filiz-Ozbay

3.1 Introduction

Beginning with important articles Che and Gale [1996, 1998], and active literature has explored the implications of budget constraints in auctions. This literature models environments in which bidders have well-defined values for the items being auctioned, but bidders may also be subject to binding budget constraints. For example, a telecommunications entrant values a spectrum license at $800 million, but may only have access to a budget of $500 million. The existing literature identifies a number of interesting consequences. For example, a standard format such as the second-price auction may not yield efficiency in the sense of allocating items to the bidders who value them the most, as the highest-value bidder may have only the second-highest budget. More surprisingly, budget constraints may cause first-price auctions to outperform second-price auctions; with the bid-shading of the first-price auction, bidders are less likely to find their bids constrained by their budgets than in the second-price auction. This upsets revenue equivalence and results in the first-price auction generating higher revenues. Moreover, since bids are relatively more likely to reflect bidders' values than their limited budgets, the first-price auction may also yield greater efficiency than the second-price auction.

However, most of the conclusions of the existing literature are dependent on the modeling assumption that budget constraints are *exogenous*. Recent work by Burkett [2011] (see Chapter 2) demonstrates that the conclusions change qualitatively if budgets are allowed to be *endogenous*, determined as a solution to the principal-agent problem that gives rise to the budget constraint. A principal seeking to constrain the purchases of its agent ought to set a relatively more stringent budget when the agent participates in a first-, as opposed to a second-price auction, as identical budgets will leave the agent in the first-price auction unconstrained in more states of the world. Comparing two auction formats, while assuming that the principal sets the budget independently of the auction format, may make little more sense than comparing two auction formats by assuming that bidders use the same bid function in both auction format.

In this paper, we attempt to test the above reasoning experimentally. The "bidder" (the agent) can be thought of as a manager at a firm that is bidding on an asset. This manager has an empire-building motive for acquiring the asset, above and beyond profit maximization for the firm. The "principal" (the human subject of greatest interest) can be thought of as a senior executive or corporate board that is responsible for allocating the firm's investments and that seeks to constrain the empire building. In laboratory experiments, we test whether the budget constraints are exogenous with respect to the auction format – or whether the principal sets a lower budget for a first-price auction than for a second-price auction.

Our experimental results can be seen most easily in Figure 3.1, which displays box plots of the selected budgets for each decile of signals from $[0, 100]$ for both auction formats. The boxes indicate interquartile ranges range (IQR) and the whiskers extend to the furthest data point within $1.5 * \text{IQR}$. The gray (left) boxes give the budgets selected by the principal in first-price auctions and the black (right) boxes give the budgets selected in second-price auctions. It is evident to the naked eye that budgets are set significantly tighter in first-price than in second-price auctions for all signal deciles except $[0, 10]$. Exogeneity of the budget choice is also rejected by statistical tests.

Figure 3.1 displays clear results with a pair of human subjects in each experiment – one taking the role of the principal and one taking the role of the bidder. The results are even sharper when the human bidders are replaced by computerized bidders, as displayed later in Figure 3.5. Since the computerized bidders consistently follow predetermined rules, we are able to elicit more information about the principals behavior in these sessions. Using this additional information, we show that these data support the prediction that the principals constrain the same set of bidder types across auction format, a key implication of the theoretical model.

Burkett (2011) demonstrated theoretically, in the model used here, that the principal tightens the budget precisely so as to neutralize the change in auction format from second- to first-price. Consequently, the second-price auction with an endogenous budget constraint generates exactly the same theoretical expected revenues as the first-price auction with an endogenous budget constraint – a res-

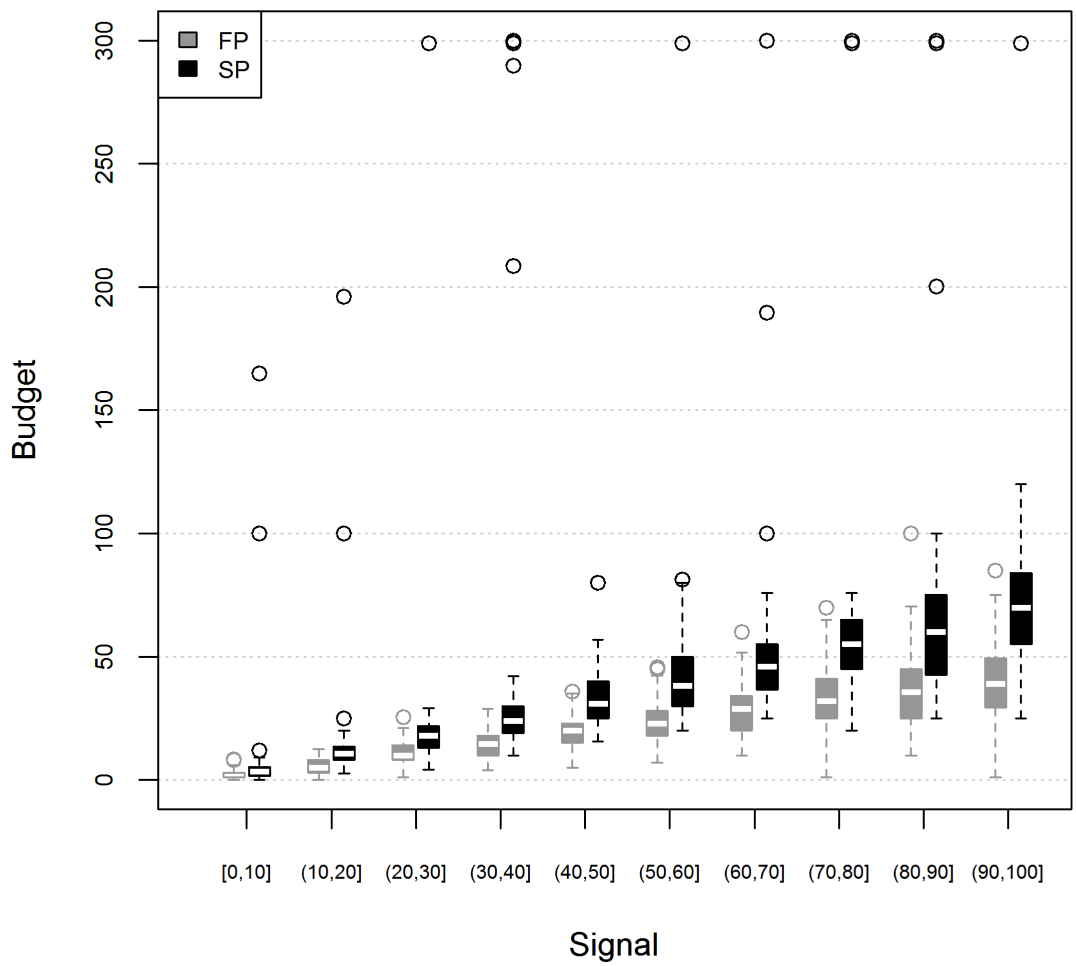


Fig. 3.1: Budgets in First- and Second-Price Auctions with Human Bidders

urrection of full revenue equivalence. While we test – and reject – this hypothesis in the paper, this experimental finding is unsurprising in light of the traditional experimental literature and is what we had expected to find. The experimental auctions literature without budget constraints has consistently found that bidders in the first-price auction bid higher than the risk-neutral Nash equilibrium, leading to higher expected revenues in the first-price auction.[1] Given this prior evidence, it would have been surprising if adding a pre-auction budgeting decision by a principal had eliminated the difference in expected revenues between the two auction formats.[2]

The rest of this paper is structured as follows. In Section 3.2, we specify the theoretical model and explore its properties. In Section 3.3, we describe the experimental design, and in Section 3.4, we give the experimental results. Section 3.5 concludes.

3.2 Model

The models tested in the experiment are standard first- and second-price sealed-bid, independent private values auction models with two bidders that have

[1] See Cox et al. [1982]; Cox et al. [1988], as the seminal papers; and Kagel [1995] for a detailed survey. Risk aversion [Cox et al., 1988], anticipation of regret [Filiz-Ozbay and Ozbay, 2007], joy of winning [see e.g. Goeree et al., 2002] , fear of loosing [Delgado et al., 2008], and level-k thinking [Crawford and Iriberri, 2007] are offered as possible explanations of the overbidding phenomenon.

[2] One interpretation of the results from Burkett [2011] is that the budgets in the model function like bids that are not always "active." If the subjects recognize this, one might expect similarities between the budgeting decisions in this experiment and bidding decisions in the existing literature.

been extended to include a budgeting stage prior to the auction. In the budgeting stage each bidder receives a budget from a principal. Both the principal and the bidder receive a payoff in the event that the bidder wins the item at the auction; however, the principal's payoff is always lower than the bidder's. This is due to an additional private payoff that the bidder receives from the item that does not accrue to the principal. It is this private payoff that motivates the principal to constrain the bidder with a budget.

Formally, the game occurs in two stages. In the first stage, each principal receives a signal about the value of the item and decides on a budget (bid cap) for the bidder based on this information. Neither the principal's signal nor the budget choices are observed by any other principal-bidder pair. Having observed their budgets, each bidder in the second stage observes their valuation for the good and decides on a bid for the auction which may not exceed the bid cap set by the principal. The winner of the auction is the principal and bidder team with the highest bid. We consider first-price and second-price payment rules.

3.2.1 Payoffs

The payoffs to both parties are determined only by the information received by the bidder. Specifically, we assume that if bidder $i \in \{1, 2\}$ observes a valuation of t_i, principal i has a valuation for the item given by δt_i, where $0 < \delta < 1$. If bidder i submits the winning bid in the auction and pays a price p, then bidder i receives a payoff proportional to $t_i - p$ and principal i receives a payoff proportional to $\delta t_i - p$.[3] That is, the bidder and the principal are both risk neutral and receive a payoff that is determined by the difference between their respective valuations and the price paid for the good.

3.2.2 Information

The signal received by principal i is denoted by s_i, assumed to be uniformly distributed on $[0, 100]$. The signals of principals i and j are independent. The principal does not observe her valuation for the good, but knows that her valuation for the good, δt_i, is uniformly distributed on $[0, s_i]$. In other words, s_i determines the upper limit of the principal's valuation. Based on the realization of s_i, the principal decides on a budget for the bidder, given by w_i. Having observed her budget, w_i, bidder i observes her valuation for the object, t_i, which given the assumption on the principal's valuation is uniformly distributed on $[0, s_i\delta]$. The timing of the game is depicted in Figure 3.2. The dashed edges indicate the dependence relations between the signals, while the solid edges indicate the actions taken by the participants (budgets were always referred to as caps in the experiment).

We chose these distributions for the experiment, because we wish to focus on the budgeting decision and hence would like the game to be as simple as possible from the principal's perspective. Note that as δ decreases (increases) the upper limit on

[3] The payoffs are proportional to those expressions to avoid double counting the total profits. For example, the bidder and the principal might be equity holders in a firm with shares σ_b and σ_p, respectively (where $\sigma_b + \sigma_p \leq 1$). The bidder is assumed to receive $\sigma_b\,(t_i - p))$ and the principal to receive $\sigma_p\,(\delta t_i - p))$. This formulation identifies the term $(1-\delta)\sigma_b t_i$ (the difference between the two payoffs) as the bidder's private payoff from obtaining the good.

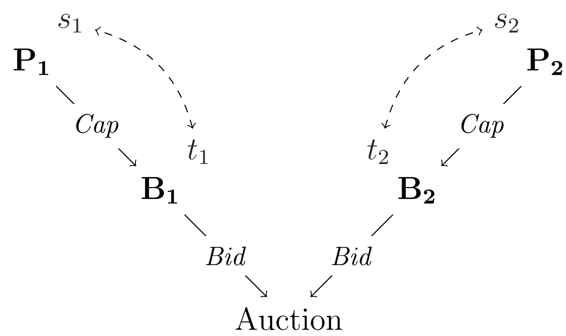


Fig. 3.2: Structure of the Game

the bidder's valuation increases (decreases) and the agency problem becomes more (less) severe.

3.2.3 Equilibrium

We consider a symmetric equilibrium of this model. The equilibrium will consist of the bidder's unconstrained choice of bid for a given valuation, $b(t)$, and the principal's choice of budget for each possible signal, $w(s)$. It can be shown that given these two choices the bid submitted to the auction is $\min\{b\left(t\right), w\left(s\right)\}$.

Assuming that the bidders employ bidding strategies that are strictly increasing and continuous functions of their valuations, each principal's choice of budget constraint is equivalent to choosing a cutoff type, $\hat{t}$, above which the bidder is constrained. In other words for a choice of budget constraint, $w(s)$, we can define a cutoff type as the t that satisfies $w\left(s\right) = b(\hat{t}\left(s\right))$ where $b(t)$ is the bidder's unconstrained choice of bid corresponding to the value t.

The first consequence of this representation is that the bid submitted at the auction is now $b(\min\{t, \hat{t}\left(s\right)\})$, so that the winning bidder is the bidder with the higher value of $\min\{t, \hat{t}(s)\}$. We refer to this quantity as the bidder's effective type. The bids submitted at the auction are then equivalent to the bids submitted in a standard independent private values auction where valuations are distributed according to $\min\{t, \hat{t}(s)\}$.

As is shown in [Burkett, 2011], the equilibrium choice of $\hat{t}(s)$ is the same in the first- and second-price auctions when bidders' signals are independent and is the solution to the following equation.

$$\mathrm{E}[\delta t \,|\, t \geq \hat{t}(s), s] = \hat{t}(s) \tag{3.1}$$

A detailed derivation of the equilibrium is in Appendix A. In our setup, the solution to Equation (3.1) is $\hat{t}(s) = \frac{s}{2-\delta}$. This in turn implies that the distribution of effective types is given by the following.

$$G(x) = P\left(\min\left\{t, \hat{t}(s)\right\} \leq x\right) = \left(2 - \delta - \delta \ln\left(\frac{2-\delta}{100} x\right)\right) \frac{x}{100}$$

In the second-price auction the bidder still has a weakly dominant strategy to bid her own value. That is, in the second-price auction $b_{SP}(t) = t$. In the first-price auction, the equilibrium bid functions are determined according to the expected value of the opponent's effective type given a winning bid.

$$\begin{aligned} b_{FP}(x) &= \mathrm{E}\left[\min\left\{t, \hat{t}(s)\right\} \,\middle|\, \min\left\{t, \hat{t}(s)\right\} \leq x\right] \\ &= \frac{4 - 3\delta - 2\delta \ln\left(\frac{2-\delta}{100} x\right)}{4 - 2\delta - 2\delta \ln\left(\frac{2-\delta}{100} x\right)} \frac{x}{2} \\ &\sim \frac{7}{16} x \end{aligned} \tag{3.2}$$

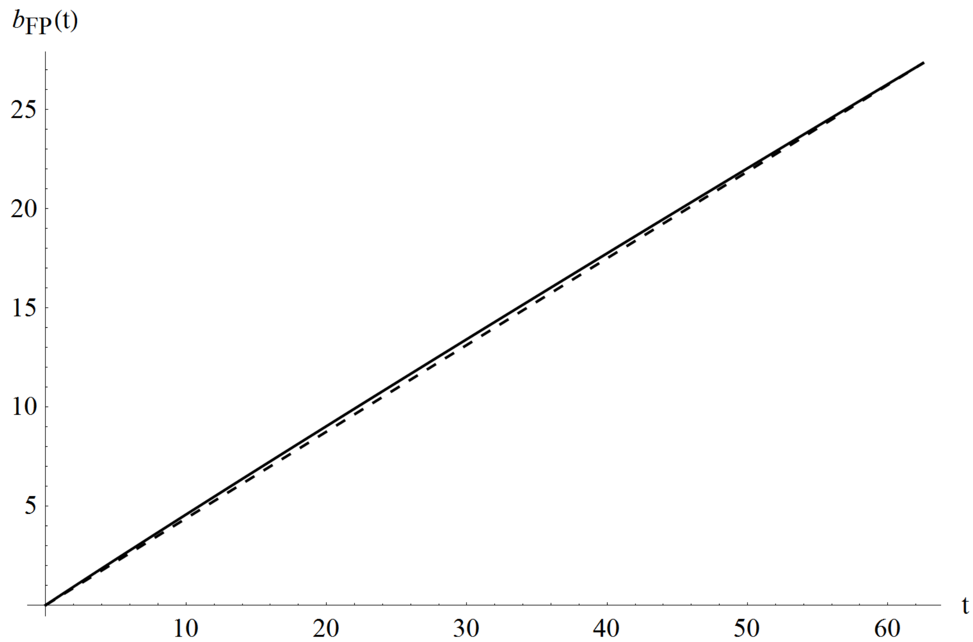


Fig. 3.3: Unconstrained First-Price Bid Function ($\delta = 2/5$)

Although this functional form is apparently complicated, Figure 3.3 shows that it is approximately linear for $\delta = 2/5$, the value used in the experiment. The dashed line is the straight line connecting the end points of the bid function (the slope is 7/16). In the figure the highest value of t shown is $t = 125/2$, which is the highest possible value of t when $\delta = 2/5$.

To summarize, in the second-price auction the equilibrium unconstrained bid function is given by $b_{SP}(t) = t$ and the budget function is given by $w_{SP}(s) = b_{SP}\left(\hat{t}_{SP}(s)\right) = \frac{s}{2-\delta}$. In the first-price auction, the unconstrained bid function is given by $b_{FP}(t)$ in Equation (3.2), and the budget function is given by $w_{FP}(s) = b_{FP}\left(\hat{t}_{FP}(s)\right) = b_{FP}\left(\frac{s}{2-\delta}\right)$.

The notable results from this analysis are that the first- and the second-price auction raise the same expected revenue and have the same expected efficiency.[4] This is a direct consequence of the bids being determined by the distribution of the effective types, $\min\{t, \hat{t}(s)\}$, which as noted above is unchanged between the first- and second-price auctions.

3.3 Experimental Design

The experiments were run at the Experimental Economics Lab at the University of Maryland (EEL-UMD). All participants were undergraduate students at the University of Maryland. The main experiment involved five sessions of second-price sealed bid auctions (SP) and five sessions of first-price sealed bid auctions (FP). In our control treatments we had five sessions of FP and five sessions of SP where the bidders were computerized and principals were human subjects. In each session there were 16 subjects. No subject participated in more than one session. Therefore, we had 80 subjects per treatment with 320 subjects in total. The random draws were balanced in the sense that we used the same sequence of random number "seed" signals for each auction format, so the random value draws for SP matched the random draws for FP.[5] A new set of random draws was used for each session in

[4] In fact, one can make the stronger assertion that the two auction formats agree in their allocations for every possible realization of the signals. This is a consequence of the winner being the one with the highest value of $\min\{t, \hat{t}(s)\}$ in both cases.

[5] The random draws were balanced within the main and control treatments not in between. This is because in the main treatments, we had eight bidders and eight principals in a session and in the control treatments we had sixteen principals in the lab where the bidders were computerized players.

each format, etc. Participants were seated in isolated booths. Each session lasted less than two hours.[6] Bidder instructions are in Appendix A. To test the subjects' understanding of the instructions, they had to answer a sequence of multiple choice questions. The auctions did not begin until each subject answered all of the multiple choice questions correctly. The experiment is programmed in z-Tree [Fischbacher, 2007].

We start with explaining the design for the main treatments where both principals and bidders were subjects. Later we will describe the control treatments with computerized bidders.

In each session, each subject participated in 30 auctions. The first 5 auctions were practice ones and they were only paid for the last 25 rounds. At the beginning of a session, each subject was assigned a role randomly: principal or agent[7]. The role of a subject was kept fixed throughout the session. There were eight principals and eight bidders in the lab in each session. At each round a principal was randomly matched with an agent and formed a team of two subjects. Then two teams were randomly matched to participate in an auction. We made sure that not the same group of people played against each other in two consecutive rounds.

In each auction, one fictitious item was sold to two randomly matched teams. All decisions were anonymous. At the conclusion of each auction, the players learned

[6] In a typical session, the instructions were described for 20-30 minutes while the actual play lasted about an hour.

[7] In the experiment, we call principals as Participant A, and agents as Participant B to avoid any name driven bias.

the outcome of the auction. In particular, each subject learned her actual value, her and opponent team's actual bids, whether her team had received the object, the price paid by the winning team, and her own payoff[8]. The screen shots of the experiment were in the instructions (see Appendix A)

In the beginning of an auction, each principal received a private signal from the uniform distribution from [0,100], independently. They did not know their value for the auctioned item at this time but they knew that the value was distributed uniformly on [0,s] when the principal's signal is s. Then the principal was asked to submit a bid cap for her bidder.

After each principal submitted the cap, each bidder observed her value and the cap set by the principal. The value of a bidder was 2.5 times more than the value of the corresponding principal. Therefore, the value of a bidder was from the uniform distribution on $[0, 2.5s]$ when the corresponding principal's signal is s. Then the bidder is asked to enter her bid which was not allowed to exceed the cap.

After each bidder submitted a bid in behalf of her team, the team with the highest bid won the auction and paid its bid (the opponent team's bid) in the first-price treatment (in the second-price treatment).

In the control treatments, where we aimed to better understand the principals'

[8] They learned the opponent's payoff when the opponent lost but we did not tell them the opponent's payoff when the opponent wins because in that case the subjects could determine the actual value of the opponent and his bidding strategy to some extends. Since we used random matching in each round to generate single shot games, we aimed to minimize the learning about the strategy of the other subjects.

behavior, the bidders were computerized. Again we tested first- and second-price auctions. All the specifications such as the distribution of values and signals, number of bidders in an auction, and the auction rules were the same as the main treatments. In each session, there were 16 principals in the experimental laboratory. The computerized bidders were programmed to play according to the equilibrium unconstrained bid functions as described in the theory section (Section 3.2). We provided with three tools to the human principals in order to explain them the bidding strategy of computerized bidders: 1) The graph of bidding function of the computerized bidder; 2) A table summarizing the bids corresponding to some actual values; 3) An interactive tool in the software. The graph and the table were given as hard copies, and the interactive tool was a numbered line on each principal's computer screen. The signal received by the principal in a round was pointed as the max value for the object on the numbered line. The principal could slide a black square between zero and the max value. The computer reported the corresponding unconstrained bid of the principal's computerized bidder every time the principal dropped the black square at a possible actual value on the line. We told the subjects that this tool is provided to help them understand the bidding strategy of the computerized bidders unless they were constrained by the principals bid caps. An example of the computer screen of a principal with computerized bidder can be seen in the instructions provided in Appendix A.

The role of a subject in the control treatments was to decide on her bid cap after observing her signal for the round. Once each principal submitted her bid cap, the corresponding computerized bidder bids the minimum of the unconstrained bid corresponding to the actual value observed by the computerized bidder and the bid cap set by the principal.

All the amounts in experiment were in Experimental Currency Units (ECU). Subjects received $8 as initial endowment to cover any possible losses in the experiment. The principals were more subject to potential losses since they did not know their values at the time of decision making. No subject lost all of the initial endowment. The final earnings of a subject was the sum of her payoffs in 25 rounds in addition to the initial endowment. The payoffs in the experiment were converted to US dollars at the conversion rate of 20 ECU=$1 (for the principals) and 80 ECU=$1 (for the agents).[9] Our calculations based on equilibrium predicted four times higher payoffs for the principals than the agents in their variable payoffs. This was because of the difference between the valuations of principals and the agents for the same auctioned item. Hence we set different conversion rates to make the earnings of subjects playing different roles comparable. By interpreting the sigma in footnote 2 as the conversion rate, one may note that the theory is independent of the conversion rates.[10] Cash payments were made at the conclusion of the experiment in private. The average principal and agent payments were $23 and $25 (including $7 participation fee).

[9] We are confident that using different exchange rates does not alter our findings since our findings in the main treatments and in the control treatments (where the agents are computerized and therefore there is only principals' exchange rate) are qualitatively the same.

[10] An alternative method to balance the earnings of principals and bidders could be to provide them with different endowments. We did not use this method since it could make some subjects think that they were less favored by the experimenter.

3.4 Experimental Results

The analysis presented in this section is based on 500 auctions we conducted per auction format with human bidders and 1000 auctions we conducted per auction format with computerized bidders. We specify it when we disregard some data. While testing differences between treatments, we report Mann-Whitney-Wilcoxon statistics for the session averages assuming that session averages are independent.[11]

We start by studying the strategies of principals in treatments with and without computerized bidders. We compare the bid caps submitted in the first- and second-price auctions with the equilibrium budgets. Figure 3.4 (same as Figure 3.1) shows the principals' choices of budgets across all sessions in first- and second-price auctions when we have human bidders. The figure shows box plots of the budgets[12] for each of ten bins based on the signal observed by the principals. Figure 3.5 shows the same plot for the budgets submitted in treatments with computerized bidders.

The most basic prediction of the theory is that the principals choose lower budgets in the first-price auction. In particular for the parameters used in the experiment, the theory predicts the cap set by a principal with signal s as $w_{SP}(s) = \frac{s}{2-\delta} = \frac{5}{8}s = 0.625s$, in the second-price auction and $w_{FP}(s) \sim \frac{35}{128}s \sim 0.273s$, in the

[11] We also performed t-statistics by using each observation and the results were not qualitatively different in any of the comparisons.

[12] The box plots were created using standard techniques. The white lines represent the median; the box represents the interquartile range (IQR); the whiskers extend to the furthest data point within 1.5*IQR; and the open circles are individual data points outside 1.5*IQR. In Figure 3.4, 24 out of 28 of the outliers in the second-price auction represent decisions made by one subject.

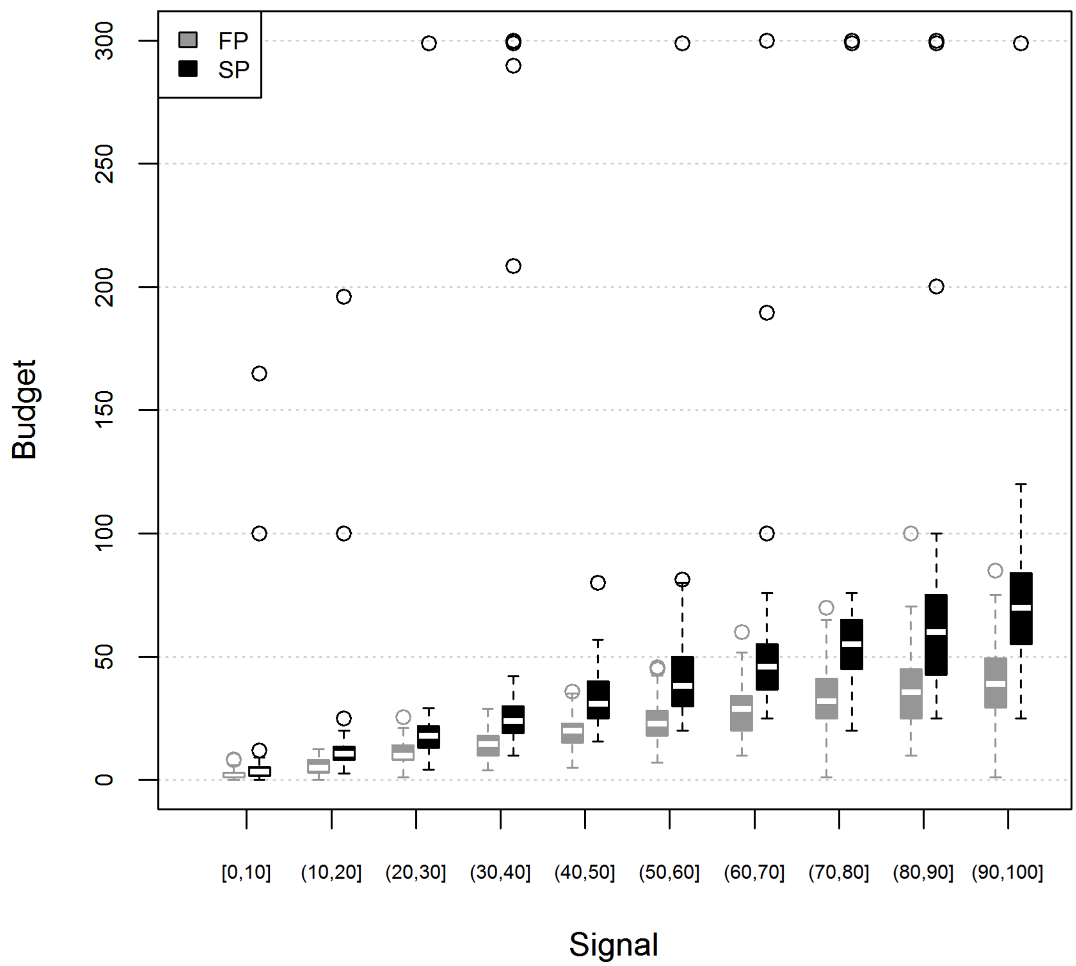


Fig. 3.4: Budgets in First- and Second-Price Auctions with Human Bidders

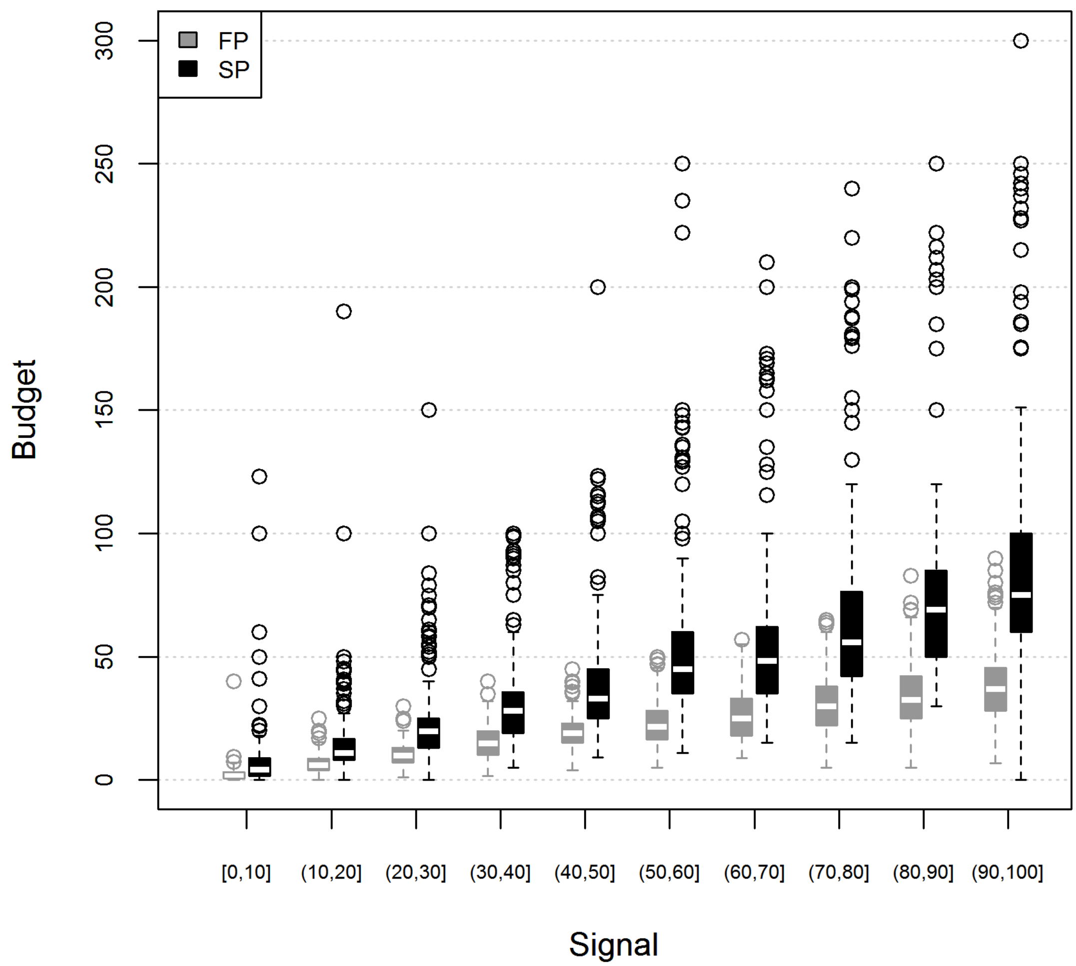


Fig. 3.5: Budgets in First- and Second-Price Auctions with Computerized Bidders

first-price auction.

Figures 3.4 and 3.5 clearly suggest that in the experiment, the principals set relatively lower budgets in first-price settings with and without human bidders, respectively. In order to further study the difference between the budgeting decisions of the principals in different auction formats statistically, we first run session-level fixed-effect and individual-level random-effect regressions of budgets set by the principals on the signals. The regression coefficients in Table 3.1 indicate that the principals in the first-price auctions set lower caps than the ones participating in second-price auctions for a given signal.[13] The estimated coefficients of signal are 0.743 in SP and 0.429 in FP with human bidders. They are 0.899 in SP and 0.397 in FP with computerized bidders.

There are several ways to characterize the behavior of the principals, but perhaps the most straight-forward is to examine the budget decided on as a fraction of their signal. This completely describes the behavior of the principals if they are choosing budgets that are linear functions of their signals (with zero intercept). Many of the principals' decisions in the data can be characterized by linear strategies, and note that equilibrium prescribes that the principals should be using linear strategies in the second-price auction and approximately linear strategies in the first-price auction. To quantify the fit of linear strategies, we calculated the R^2 values

[13] We also estimated these regressions controlling for the round number by including dummy variables indicating the first 10 rounds of the experiment in each treatment but these dummy variables were not significant at the 5% level and did not affect the estimates of interest when they were included, so they are excluded here.

	Human Bidders	Computerized Bidders	Equilibrium[14]
Constant	3.518 (3.774)	1.937 (1.578)	0
FP	-3.464 (3.786)	-1.099 (1.654)	0
Signal	0.743*** (0.032)	0.899*** (0.042)	0.625
FP× Signal	-0.314*** (0.033)	-0.502*** (0.046)	-0.352
N	2,000	4,000	

Standard errors in parentheses. * p<.05; ** p<.01; *** p<.001

Tab. 3.1: Regressions of Budget on Principal Signal

from regressions of the budget on the principal signals for each individual principal. For principals in the first-price auction, 75% of the principals had R^2 values above 0.79, 50% were above 0.87 and 25% were above 0.93. The corresponding numbers in the second-price auction were 0.87, 0.94 and 0.97. With computerized bidders in the first-price auction (second-price auction), 75% of the principals had R^2 values above 0.72 (0.79), 50% were above 0.86 (0.92), and 25% were above 0.93 (0.96).

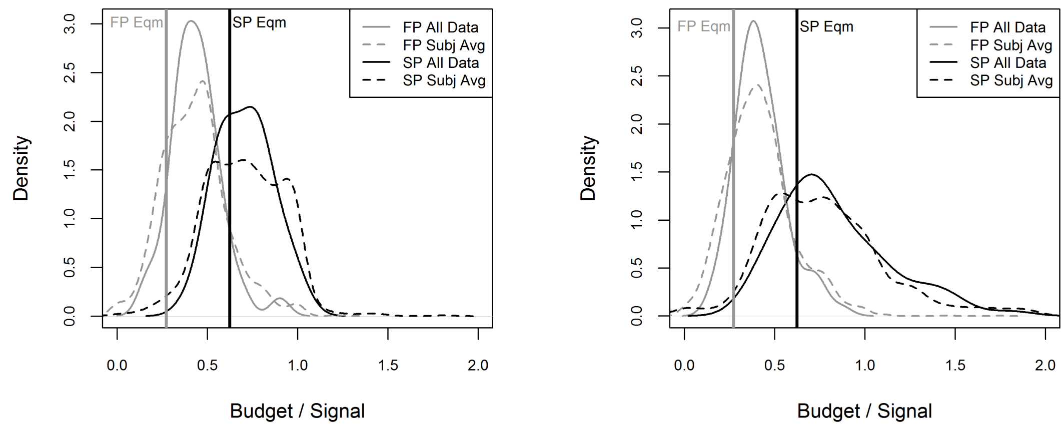
If we calculate a session average value of budget/signal for each principal and use a Mann-Whitney-Wilcoxon test to compare these fractions between the first- and the second-price auctions, we reject the hypothesis that the fractions in the first-price treatment are at least as high as those in the second-price treatment (p

= 0.004 when we have human bidders and p=0.004 when we have computerized bidders).[15] Figures 3.6(a) and 3.6(b) show the empirical density estimates of the average fractions for human and computerized bidders, respectively. In addition to showing the raw data (dashed line), we show the densities of the average budget fractions for each subject (solid line). In both figures, we have excluded data where the average fraction exceeds 2 to emphasize the range where most of the data is concentrated. In Figure 3.6(a), 0.1% of the raw first-price data (mean of 6.11) and 2.3% of the raw second-price data (mean of 46.22) is not shown. In Figure 3.6(b), 0.1% of the first-price data (mean of 4.79) and 9.7% of the second-price data (mean of 4.77) is not shown. The larger number of outliers in this last case is evident in Figure 3.5 above.[16] The vertical lines mark the equilibrium predictions (recall that this is approximate in the first-price case).

The theory predicts that the principals set the same cutoff value for the bidders in both auctions. This means that a principal who observed signal s will set the budget so that the set of types of bidders for whom the constraint binds are the same. More precisely, we calculate that cutoff type as $\frac{s}{2-\delta} \sim 0.625s$. This result of

[15] Note that we continue to reject this hypothesis if we exclude the first session of the second-price treatment. The subject who set the outlier budget levels in Figure 3.4 participated in that session.

[16] The larger fraction of outliers evident in the second-price treatments might be the result of the noisier feedback from the second-price design. The negative consequence of setting a high budget in either treatment is that one might have to (possibly) pay too high of a price for the item. In the first-price auction, the realization of this consequence requires that ones bidder also place a high bid, but in the second-price auction ones bidder must place a high bid and ones opponent must have a high budget and place a high bid.



(a) Human Bidders

(b) Computerized Bidders

Fig. 3.6: Empirical densities of (budget/signal) in all treatments

course depends on the bidders using equilibrium strategies for their bid functions. In the computerized bidder treatment, we are able to invert the bid functions used by the bidders to infer the principals' choice of cutoff type. To infer the cutoff type of the human bidders would involve making assumptions about the unconstrained behavior of the human bidders. Therefore, in the following analysis of the cutoff type, we only consider the computerized bidder treatment.

Given the inferred cutoff type choice in the computerized bidder treatment, we ran the following regression using $\hat{t}$ for the inferred cutoff type.

$$\hat{t}_i = \beta_0 + \ \beta_1 FP + \beta_2 Signal_i + \beta_3 (FP_i \times \ Signal_i) + D\gamma + \epsilon_{is}$$

The term $D\gamma$ represents session dummies to control for fixed effects at the session level. Including session-level fixed effects, prevents us from separately identifying β_0 and β_1, so these coefficients are not reported below. The s subscript in the error term indicates that there are subject-level effects included in the error term. Specifically we assume that

$$\epsilon_{is} = u_s + b_s Signal_i + v_i$$

where u_s is a subject-level random effect, b_s is a random coefficient also at the subject level, and v_i is a noise term. This specification allows for the subjects to have random, though uncorrelated, deviations in both the slope and the intercept terms. It also provides a way to explicitly model the heteroskedasticity in the data. This formulation also seems appropriate for this data as many of the subjects seem to be playing linear strategies. The results from the regression are below.[17]

Neither the likelihood ratio tests nor Wald tests on the significance of the null hypothesis $\beta_3 = 0$ reject at the 1% level for the experiments with computerized bidders. Therefore, the principals in FP and SP auctions do not constrain significantly different types of bidders when the bidders are computerized although they constrain a smaller set of bidder types than the equilibrium predicts.

[17] These regressions were also performed with controls for potential round effects. When a dummy for the first 10 rounds is included in the regressions, the coefficient on this dummy is positive and significant for the first-price treatment (but insignificant for the second-price treatment) suggesting that cutoff types were lower on average in the later rounds of the first-price treatment; however, including these controls affected neither the values of the β_2 or β_3 (in both cases the estimates changed by less than 0.001) nor the conclusions of the statistical tests reported so they omitted from the discussion.

	Coefficient (SE)
Signal	0.956*** (0.051)
FP $\times$ Signal	-0.001 (0.073)
χ^2 Tests of $\beta_3 = 0$	
Wald	0.00
LR	0.00
N	4,000

Tab. 3.2: Cutoff type regression

3.4.1 Revenue and Efficiency

The theory predicts that the principals choose to constrain the same set of types in both auction formats, and this leads to the revenue equivalence result. Revenue equivalence is, however, sensitive to the particular set of types the principals constrain. In our experiment, the principals constrain the fewer types than the theory predicts and in both formats the constrained types are the same. Moreover, as argued earlier the principals' behavior in the experiment is linear. The proposition below shows that the first-price is expected to raise higher revenue than the second-price if the principals' deviation from the equilibrium has these properties.

Proposition 1. *Suppose that the principals choose cutoff types according to linear strategies that constrain fewer types than equilibrium (i.e.* $\hat{t}(s) = \alpha s$ *with* $\alpha > 5/8$*).*

Then the first-price auction raises more revenue than the second-price auction with computerized bidders.

The results on seller revenue generated in four treatments as well as the equilibrium predictions based on the used draws are in Table 3.3. Aggregating average revenue to the session level, we performed Mann-Whitney-Wilcoxon tests that the session averages came from distributions with the same median. In line with Proposition 1, in the treatments with human bidders, the tests rejected this hypothesis between the first- and second-price auctions ($p = 0.008$). The hypothesis is rejected between the first-price auction and equilibrium ($p = 0.008$) as well. The test did not reject at the 5% significance level between the second-price auction and equilibrium ($p = 0.095$). In the treatments with computerized bidders, the results were roughly the same. The test rejects the hypothesis between the first- and second-price auctions ($p = 0.032$) and between the first-price auction and equilibrium ($p = 0.008$). The test did not reject at the 5% level between the second-price auction and equilibrium ($p = 0.056$).

Next we analyze the efficiency of allocation in the main treatments. Tables 3.4 and 3.5 summarize the rates of efficiency by using two different efficiency measures for human and computerized bidders cases, respectively. The first row is the percentages of auctions where the winning principal has the higher valuation. The second row is the average surplus that is realized. This measure is defined as the winning principal's value divided by the highest value of the two principals. The second measure tells how much of the efficient allocation surplus is realized by the

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Human Bidders	24.332 (0.658)	17.043 (0.173)	18.061 (0.402)	16.956 (0.363)
Computerized Bidders	23.15 (1.064)	16.653 (0.194)	18.81 (0.875)	16.648 (0.192)

Standard errors of session means are in the parenthesis

Tab. 3.3: Average Seller Revenue

auction outcome.

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Rate of efficient allocations	0.850 (0.021)	0.874 (0.010)	0.804 (0.023)	0.874 (0.010)
Realized Surplus	0.946 (0.008)	0.961 (0.003)	0.923 (0.012)	0.961 (0.003)

Standard errors of session means are in the parenthesis

Tab. 3.4: Efficiency in the treatments with human bidders

Using the Mann-Whitney-Wilcoxon (MWW) test and a significance level of 5%, the average rate of efficient allocations is not significantly different between first- and second-price (p = 0.205) or the first-price and equilibrium (p = 0.396), or the second-price and equilibrium (p = 0.057). Using MWW and a significance level of 5%, the average realized surplus is not significantly different between first- and second-price (p = 0.151), or the first-price and equilibrium (p = 0.222), but is significantly different between the second-price and equilibrium (p = 0.008).

The results on the efficiency of the allocations in the treatments with computerized bidders are presented in Table 3.5. There is no significant difference between first-price and second-price with respect to either measure and none of them are significantly different than the equilibrium prediction (all the p-values are greater than 0.346)

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Rate of efficient allocations	0.863 (0.017)	0.852 (0.007)	0.855 (0.009)	0.852 (0.007)
Realized Surplus	0.952 (0.008)	0.955 (0.002)	0.950 (0.003)	0.955 (0.002)

Standard errors of session means are in the parenthesis

Tab. 3.5: Efficiency in the control treatments

Tables 3.3, 3.4 and 3.5 indicate that having computerized bidders who play the equilibrium strategies or not do not alter the relative performance of the formats in terms of efficiency and revenue.

Finally, we study the bidders' behavior in the experiment. One way to evaluate the behavior of the bidders is to do the following test. First, the theory predicts that each bidder's value pins down their choice of unconstrained bid. So we can calculate each bidder's predicted choice conditional on the budget given to them in the experiment as the minimum of the predicted unconstrained bid and the observed budget. We then compare the distribution of this value to the observed bids using a

Kolmogorov-Smirnov test and find that the test does not reject the null hypothesis that the distributions are the same at the 5% level (p-values are 0.969 for both auctions). This suggests that the results on revenue and efficiency are driven by the differences in budgeting behavior of the principals.

3.5 Conclusion

Prior to Vickrey [1961], it might well have been commonplace for observers to wonder, "Why would a seller ever want to use a second-price, rather than a first-price auction? Given any set of bids, the seller would always do better to charge the winner the highest bid." The obvious oversight is in not taking into account that bidding behavior is affected by a change in the rules.

It seems to us that the literature on budget constraints often makes a similar oversight. When budget constraints are present, they can be presumed to exist for a reason; quite likely, they arise from a principal-agent problem. When a principal imposes a budget on an agent, the principal should determine a budget appropriate for the auction format. Just like the bidding behavior, the principals actions to constrain the bidding behavior should also be affected by a change in the rules.

In this paper, we found clear experimental evidence that principals set demonstrably lower budgets for bidders when the format will be a first-price, rather than second-price auction. This holds true robustly, both with human bidders and with computerized bidders.

The endogeneity of budgets may be helpful in explaining one of the main predictive failures of the literature on auctions with budget constraints. Recall not only that the literature predicts the second-price auction to be outperformed by the first-price auction, but it predicts both formats to be outperformed by the all-pay auction. Nevertheless, while the all-pay auction is a good model for phenomena such as political lobbying, the all-pay format is hardly ever used for conventional sales of valuable assets. Perhaps sellers recognize that the principals would neutralize the effect of the all-pay auction by imposing even more stringent budget constraints on their bidders.

While the first-price auction generated higher revenues in our experiments, this came about for a different reason than in the literature with exogenous budget constraints. In the literature, the higher revenues resulted from budgets that were the same, for different auction formats. In our experiments with endogenous budget constraints, the higher revenues resulted from subjects setting higher budgets than in the equilibrium solutions, both with second-price and first-price formats. Our experimental design is not well suited for finding what motivated the setting of higher-than-equilibrium budgets; this is an interesting question for future research.

4. ALLOCATIVE EXTERNALITIES AND EFFICIENCY IN SEALED-BID AND SEQUENTIAL AUCTIONS

by Justin E. Burkett

4.1 Introduction

When auctions involve firms as participants, the outcome likely affect the structure of some downstream market, and the firms' expectations about this downstream interaction presumably affects their valuations of the good being auctioned. In some important cases the firms' valuations will be affected by the identities of the winners and losers in the auction. For example, the auction may determine whether or not a new entrant is allowed into a market and the incumbents would be adversely affected by the additional competition. In an auction without possible new entrants, a bidder may consider some bidders as stronger competitors than others. In a market with significant switching costs between incompatible products, competition is likely stronger between firms producing compatible products than it is between firms with incompatible products [Klemperer, 1995].

In this environment, revealing the identity of winners midway through the auction may have important implications for auction outcomes. This paper characterizes the implications for the auction's efficiency by comparing a sealed-bid format, where the winners are not revealed until the end of the auction to a sequential format, where winners' identities are revealed during the course of the auction. I also discuss a related issue, that defining efficiency in this environment is potentially problematic. For example, one allocation may maximize social surplus by choosing the bidders with the lowest cost of production, but not maximize the bidders' ex-post payoffs (producer surplus). The auction literature focuses on the second, but if we are considering an auction run by the government these two ideas need not coincide. For example, some allocations may lead to very competitive markets that benefit consumers but not the firms.

Auctions where the bidders care about the identities of the winners and losers, insofar as the identities enter into their valuation functions, fall into the category of auctions with externalities. Philippe Jehiel and Benny Moldovanu have written a number of papers on the difficulties that arise in auctions with externalities and provide a general overview of the literature in Jehiel and Moldovanu [2006]. In the presence of externalities several important complications arise that do not appear in standard auction models. For example, efficient allocation mechanisms do not exist except in special cases [Jehiel and Moldovanu, 2001].

One can distinguish between informational externalities, where bidders possess relevant information about the other bidders' valuations,[1] and allocative (or identity-dependent) externalities, where bidders care about the identities of the winners and losers.

[1] Milgrom and Weber [1982] is an early example of an auction with informational externalities.

Specifically, this paper considers the problem of selling two units of a homogeneous good to bidders who care about the identity of the winner of the other unit (i.e. there are allocative externalities). The participants have private information about a parameter determining their costs of production and value the good differently depending on the identity and cost parameter of the other winning bidder. The setup allows for multiple interpretations for efficiency, because it is not assumed that the producer surplus maximizing allocations always coincide with the ones that maximize social surplus. A seller who is therefore interested in allocating the goods to the bidders with the lowest production costs is not always interested in allocating to the bidders with the highest expected profit.

I focus on two standard auction formats, a sealed-bid, uniform price auction where the goods are allocated to the highest bidders at a price equal to the highest rejected bid, and sequential auctions where one unit of the good is sold in each round at a price equal to the second highest bidder in that round. The key difference between the two formats is that one reveals the identity of the winner of the first unit.[2] This fact leads to the result in this model that while the sealed-bid format in equilibrium selects the participant with the lowest cost, the sequential format is more likely to select the set of participants with higher joint profits. This suggests that the social surplus maximizing seller would prefer the sealed-bid format in this environment.

The impact of allocative externalities on auction design has been considered

[2] Of course, this is an assumption. One could imagine a sequential format where the winner of the first unit is not revealed to the remaining participants.

in several papers including Jehiel et al. [1999], Aseff and Chade [2004], Das Varma [2002a,b], and Das Varma and Lopomo [2010]. Jehiel et al. [1999] develop an optimal mechanism to sell one unit of a good when bidders are privately informed about their vector of payoffs when each of the other bidders is a winner. They show that the endogenous participation constraints along with the multidimensional private information have interesting implications for the optimal choice of reserve price. The optimal reserve price is never small, for example, it is either zero or much larger than zero. Aseff and Chade [2004] also consider revenue maximizing design for a two good auction.[3]

This paper is most closely related to a series of papers by Das Varma and coauthors [Das Varma, 2002a,b, Das Varma and Lopomo, 2010] that analyze an environment where a single good is auctioned in an environment where some of the bidders experience allocative externalities when the good is won by an opponent of a given type. They find that an ascending format can raise more revenue in this environment than a sealed-bid format [Das Varma, 2002a]; revealing the identities of the bidders can generally increase the efficiency, defined in terms of bidders' willingness to pay, and revenue performance of the auction [Das Varma, 2002b]; but that when considered as a model of entry, the sealed-bid format (i.e. a format which does not reveal bidders' identities) may be more efficient because the dynamic format in their model introduces a free riding effect among incumbents causing strategic non-participation [Das Varma and Lopomo, 2010].

[3] See also Figueroa and Skreta [2009] and Brocas [2007] for more examples of revenue maximizing mechanism design in this type of environment.

The theme throughout these papers is that revealing the identity of the bidders to other bidders, either directly or through a dynamic auction procedure can have important implications for the performance of standard mechanisms. This paper continues with this idea and extends the analysis to an environment where multiple goods are auctioned and the assumptions made on the payoffs and information structure are weaker. For instance, my model allows for payoffs to depend on the cost information of the opponents and does not impose strong distributional assumptions. Another significant difference is in the assumptions about how the externalities are transmitted. In this paper, the externalities affect the payoffs of other winners, but are not assumed to affect the losers of the auction in a non-uniform way.[4]

This paper is also related to the IO literature on monopolists' deterrence of entry [e.g. Gilbert and Newberry, 1982]. For example, Krishna [1993] argues that when resources are auctioned off sequentially rather than all at once there can be a tendency for a monopolists position to weaken over time, because the monopolist can strategically reduce the cost of acquiring those resources over time by waiting to purchase. In a related paper, Gale and Stegeman [2001] present a model where duopolists sequentially acquire goods and show that in their model the modal prediction is that the duopolists split the goods unevenly, and this result is driven in part by a strategic advantage to winning goods early in the sequence of auctions. Both of these results provide contrast to the results of Gilbert and Newberry [1982]

[4] In other words, if there is an impact on the losers of the auction, through post-auction interaction perhaps, this impact affects all of the losing bidders in the same way, irrespective of their identity.

that suggest that a monopolist tends to preempt entry when allowed to bid on the rights to a new innovation. Whereas these papers develop models of complete information, my paper is concerned with an incomplete information model. I also do not focus on market concentration but rather on the efficiency of the auction outcomes.

Other related papers in this literature include Rodriguez [2002], who considers a sequence of license auctions to firms participating in a post-auction market where all the bidders enter the same market and there is complete information. Katsenos [2008] considers a similar model to mine comparing sealed-bid and sequential auctions in an incomplete information model, but also assumes that all bidders participate in the same post-auction market. In other words, there is no difference in the identities of the bidders in his model, whereas that is the focus of the current paper.

The next section will describe the model, and the following section presents the results, first for the sealed-bid auction then for the sequential auction.

4.2 Model

There are $2N$ ($N \geq 3$) bidders competing in an auction for two homogeneous goods. Each bidder demands at most one of the goods and has a valuation that depends on the type of the other winner. Bidders may be one of two types, A and B, each of which is represented by N bidders in the auction. The types of bidder are assumed to be common knowledge among the bidders. Since I restrict attention the use of anonymous mechanisms for the seller, I do not take a stance on whether the seller can observe the type or not. It could be that the seller must use an anonymous mechanism because from her perspective the bidders are identical ex-ante.

I assume that winning the good allows each of the bidders to produce in a downstream market, and the expected profits from the market determine each firm's willingness to pay for a good. For example, the goods might represent licenses for wireless spectrum. The profit is determined as a function of a firm's costs, c_i, and the costs and type of the firm that wins the other unit.

Suppose that the type of the firms, A or B, represents the market that the firms will compete in, so if two firms of type A win goods at the auction they will compete against each other in market A, but if an A and a B win, they will be monopolists in the downstream markets A and B. Formally, suppose that each firm has a parameter c_i that determines its cost of production and that an A firm (firm i) receives a payoff $\pi_{A,A}(c_i, c_j)$ if it wins with firm j and firm j is an A firm, and a payoff of $\pi_{A,B}(c_i, c_k)$ if it wins with firm k and firm k is a B firm. I assume throughout that all the payoff functions, π, are continuous, strictly decreasing in their first argument, and strictly increasing in their second argument. I make the following assumptions on the payoff functions (assumed to hold for all c_i, c_j):

A1 $\pi_{A,A}(c_i, c_j) < \pi_{A,B}(c_i, c_j)$

A2 $\pi_{A,A}(c_i, c_j) = \pi_{B,B}(c_i, c_j)$

A3 $\pi_{A,B}(c_i, c_j) = \pi_{B,A}(c_i, c_j)$

A4 $\pi_{A,B}(c_i - d, c_j) - \pi_{A,B}(c_i, c_j) \geq \pi_{A,B}(c_i, c_j) - \pi_{A,B}(c_i, c_j - d)$

A4' $\pi_{A,A}(c_i - d, c_j) - \pi_{A,A}(c_i, c_j) \geq \pi_{A,A}(c_i, c_j) - \pi_{A,A}(c_i, c_j - d)$

Assumption 1 says that an A bidder would prefer to win against an B bidder over an A bidder with the same cost. One way to think about this assumption is that all A firms produce goods that are close substitutes to each other, while the goods produced by B firms are not close substitutes for each A's good. Keeping the wireless spectrum example, it could be that the market (A or B) represents the region of the country the firm operates in.

Assumptions 2 and 3 are symmetry assumptions, implying that there is nothing inherently different about being an A or B type. Assumption 3 also allows me to not distinguish between the allocations $\{A, B\}$ and $\{B, A\}$.

Assumption 4 and 4' say that reducing a firm's own cost by d increases the payoff more that increasing that firm's opponent's cost by d. Roughly, one's own costs are more important that one's rival's.

I normalize the payoffs of the bidders who lose the auction to 0. This is an important distinction between this model and the model of Das Varma [2002a], for example. In that paper the reservation value of the bidders depends on the type of the winning bidder. When losing bidders suffer negative externalities, they are willing to bid above their valuation to prevent their rivals from winning. In a model where all the bidders expect to interact in a downstream market it is likely that losers in the auction will care about the identity of the winning bidder.

This paper does not examine this effect, or in other words, assumes that the firms' outside opportunities are unaffected (or affected uniformly) by the outcome of the auction. This paper focuses on the interaction between an auction for multiple units of a good and bidders' expectations about the likely winners in the even that they win one of the units. Since Das Varma [2002a] is a model of a one unit auction it cannot capture this interaction.

Costs are assumed to be private information to each firm and distributed independently according to some commonly known distribution, $F(c)$ on a closed interval, $[0, 1]$. For convenience, I assume that the distribution function is strictly increasing on its domain (i.e. there are no gaps).

As an illustration of the above assumptions, the following example describes a simple version of a model of competition when consumers have switching costs, and is a slight modification of Example 0 in Klemperer [1995]. Switching costs are costs that consumers incur when switching from being a Firm A customer to a Firm B customer, and may be the result of compatibility differences between products or simply transaction costs [for an overview see Farrell and Klemperer, 2007].

Example 1. *Suppose that firms of type A manufacture products that are compatible with those of other type A firms but incompatible with products made by a type B firm. Assuming N consumers have a reservation price of R for one unit of a good. A fraction σ^A of consumers must pay a switching cost of s to buy from a B firm, while a fraction $\sigma^B = 1 - \sigma^A$ pay s to buy from A.*

If an A and a B type firm are in the market (with marginal costs c^A and c^B), $s \geq R - c^A > 0$, and $s \geq R - c^B > 0$, then in the unique non-cooperative equilibrium the firms price the goods as if they were monopolists in their respective markets

($p^A = p^B = R$) and earn profits $\pi^A(c^A, c^B) = \sigma^A N(R - c^A)$ and $\pi^B(c^B, c^A) = \sigma^B N(R - c^B)$.

If there are two As in the market (with marginal costs c_1^A and c_2^A), the firms compete for the share of customers in their segment (if $\min\{c_1^A, c_2^A\} \geq R - s$ so the B customers are not served). Under price competition the higher cost firm (firm 1) earns 0 and the lower cost firm (firm 2) earns $\pi^A(c_2^A, c_1^A) = \sigma^A N(c_1^A - c_2^A)$.

I first compare the outcomes of a sealed-bid (Vickrey) auction and sequential second-price auctions in this environment.

4.3 Results

4.3.1 Sealed-Bid Auction

Consider the problem of a seller in this environment who would like to maximize social surplus and allocate to the most efficient firms, the one's with the lowest cost. In the first case, consider the sealed-bid auction with Vickrey pricing rule, where the seller collects bids, awards the objects to the two highest bidders, and charges them the value of the third highest bid.

The Vickrey auction rules suggest that it is an equilibrium for the bidders to report their expected value for a good, and it is. However, with externalities the value of the good depends on the expected allocation, and in equilibrium the bidders' expected values have to be consistent with the allocation rule (they have to be correct about their expectations).

There is an symmetric equilibrium in this model, however, because from any bidder's perspective there are $N-1$ bidders of the same type and N bidders of the other type. By symmetric equilibrium I mean that the bids depend on costs symmetrically for each of the types of firms, or in other words, that bids are functions of costs and not types.

Proposition 2. *The unique symmetric equilibrium of the sealed-bid Vickrey auction is given by A firms bidding*

$$b_A(c) = \frac{N}{2N-1}\mathrm{E}[\pi_{A,B}(c, c_B^{(1)})] + \frac{N-1}{2N-1}\mathrm{E}[\pi_{A,A}(c, c_A^{(1)})] \tag{4.1}$$

The terms $c_A^{(1)}$ and $c_B^{(1)}$ refer to the lowest of the $N-1$ A firms and N B firms respectively.

B's use the analogous bidding function:

$$b_B(c) = \frac{N}{2N-1}\mathrm{E}[\pi_{A,B}(c, c_A^{(1)})] + \frac{N-1}{2N-1}\mathrm{E}[\pi_{B,B}(c, c_B^{(1)})] \tag{4.2}$$

Where the terms $c_A^{(1)}$ and $c_B^{(1)}$ now refer to the lowest of the N A firms and $N-1$ B firms respectively.

Proof. See Appendix □

In words, this means that the bidders' bid according to the weighted expected value of competing against an bidder of the same type and a bidder of the other type in the downstream market. Since these forms are symmetric between A and B firms, the firms submitting the highest bids must be the firms with the lowest costs irrespective of type.

Corollary 3. *In a symmetric equilibrium the sealed-bid Vickrey auction allocates to the two firms with the lowest cost, irrespective of their identities.*

The efficiency of this auction format depends on how we define efficiency in this environment. Although the auction allocates to the firms with the lowest cost, it does not always allocate to the firms with the largest ex-post valuation. This is because the firms also care about the identity of the other winner. Consider the case where $c_A^{(1)} < c_A^{(2)} < c_B^{(1)}$, but

$$\pi_{A,A}(c_A^{(1)}, c_A^{(2)}) + \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) < \pi_{A,B}(c_A^{(1)}, c_B^{(1)}) + \pi_{A,B}(c_B^{(1)}, c_A^{(1)}) \tag{4.3}$$

Continuity of the payoff functions and the assumptions on the cost distribution imply that this event has positive probability.

In other words, my assumptions allow for the lowest cost firms to win the object, and not have the highest ex-post valuations. If the auction allocates to an A and a B firm the allocation must also maximize the firms' ex-post valuations. Suppose that $c_A^{(1)} < c_B^{(1)} < c_A^{(2)}$, and consider the following inequalities

$$\begin{aligned} &\pi_{A,B}(c_A^{(1)}, c_B^{(1)}) + \pi_{A,B}(c_B^{(1)}, c_A^{(1)}) \\ &> \pi_{A,B}(c_A^{(1)}, c_A^{(2)}) + \pi_{A,B}(c_A^{(2)}, c_A^{(1)}) \\ &> \pi_{A,A}(c_A^{(1)}, c_A^{(2)}) + \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \end{aligned} \tag{4.4}$$

The first inequality follows from Assumption A4, and the second follows from A1. What A4 is ruling out essentially is that given the allocation $\{A, B\}$ we can increase the firms' joint payoff by replacing a low cost firm with a higher cost one.

4.3.2 Sequential Auction

I next consider a sequential auction where the identity and bid of the winner of the first auction is known to those bidding in the second auction. The two auctions are assumed to be standard second-price auctions. There are two reasons for making this choice. The first is tractability. Note that in the second stage the environment becomes asymmetric as the valuations no longer have the same distribution across types, and it is well known that closed form solutions to the asymmetric first-price auction are only known in special cases. The second is that the second-price auction is the Vickrey auction for a single good, and is therefore, the natural sequential auction to compare to the sealed-bid Vickrey auction.

Again, the symmetry of the environment in the first stage suggests looking for a symmetric equilibrium in the first stage, meaning that the first stage bids are determined by costs and not identities. The first step is to recognize that in the second auction we are in an environment with private values and no externalities.[5] It is an equilibrium in the second stage, therefore, for the bidders to bid their expected value if they win the good.

The derivation of first round bid strategies is based on the idea that the bidders would not be willing to pay more than their expected payment in the second round (conditional on winning), because for a given realization of opponents' costs if in equilibrium they win in the first round, they could always lower their bid and win in the second round. The full details are given in the appendix.

[5] Implicitly I am assuming here that the winning bid is revealed from the first stage of the auction and that the second round bidders can use this information to back out the winners cost.

To state the following proposition, I need to define the following five events. The events are defined from the perspective of an A type bidder, and I use $\mathbf{c}$ to represent the vector of realized costs for all bidders. The analogous events for a type B bidder are not shown.

$$
\begin{aligned}
E_1(c_A) = \Big\{ \mathbf{c} \,|\, c_A^{(1)} \leq c_A \leq \max(c_A^{(2)}, c_B^{(1)}) \cap \\
\pi_{A,A}(c_A, c_A^{(1)}) \geq \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \geq \pi_{A,B}(c_B^{(1)}, c_A^{(1)}) \Big\} \\
E_2(c_A) = \Big\{ \mathbf{c} \,|\, c_A^{(1)} \leq c_A \leq \max(c_A^{(2)}, c_B^{(1)}) \cap \\
\pi_{A,A}(c_A, c_A^{(1)}) \geq \pi_{A,B}(c_B^{(1)}, c_A^{(1)}) \geq \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \Big\} \\
E_3(c_A) = \Big\{ \mathbf{c} \,|\, c_A^{(1)} \leq c_A \leq \max(c_A^{(2)}, c_B^{(1)}) \cap \\
\pi_{A,B}(c_B^{(1)}, c_A^{(2)}) \geq \pi_{A,A}(c_A, c_A^{(1)}) \geq \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \Big\} \\
E_4(c_A) = \Big\{ \mathbf{c} \,|\, c_B^{(1)} \leq c_A \leq \max(c_B^{(2)}, c_A^{(1)}) \cap \\
\pi_{A,B}(c_A, c_B^{(1)}) \geq \pi_{A,B}(c_A^{(1)}, c_B^{(1)}) \geq \pi_{B,B}(c_B^{(2)}, c_B^{(1)}) \Big\} \\
E_5(c_A) = \Big\{ \mathbf{c} \,|\, c_B^{(1)} \leq c_A \leq \max(c_B^{(2)}, c_A^{(1)}) \cap \\
\pi_{A,B}(c_A, c_B^{(1)}) \geq \pi_{B,B}(c_B^{(2)}, c_B^{(1)}) \geq \pi_{A,B}(c_A^{(1)}, c_B^{(1)}) \Big\}
\end{aligned}
$$

In four of these events (1,2,4 and 5), the A bidder loses the first auction and wins the second auction. In the fifth (E_3), the A bidder loses the second auction despite having the second lowest cost because of the externality imposed by an A bidder winning the first auction. With these events defined, I can state the equilibrium bid functions.

Proposition 3. *With sequential second-price auctions, there is an equilibrium where* *an A bidder in the first round bids according to*

$$b_A^1(c) = \Pr[E_1(c)]\,\mathrm{E}[\pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \mid E_1(c)] + \Pr[E_2(c)]\,\mathrm{E}[\pi_{A,B}(c_B^{(1)}, c_A^{(1)}) \mid E_2(c)]$$
$$+ \Pr[E_3(c)]\,\mathrm{E}[\pi_{A,B}(c, c_A^{(1)}) \mid E_3(c)] + \Pr[E_4(c)]\,\mathrm{E}[\pi_{A,B}(c_A^{(1)}, c_B^{(1)}) \mid E_4(c)]$$
$$+ \Pr[E_5(c)]\,\mathrm{E}[\pi_{B,B}(c_B^{(2)}, c_B^{(1)}) \mid E_5(c)]$$

and an A bidder who does not win in the first round bids in the second round according to

$$b_A^2(c) = \begin{cases} \pi_{A,A}(c, c_A^{(1)}) & \text{if an } A \text{ wins the first good} \\ \pi_{A,B}(c, c_B^{(1)}) & \text{if a } B \text{ wins the first good} \end{cases}$$

B bidders' equilibrium bid functions are defined analogously, and importantly are symmetric in the first round.

Although complicated, the equilibrium bid functions have a simple interpretation. A bidder bids her expected second round payment in the first round and her (known) value in the second round.

Returning to the question of efficiency, it is interesting to observe that because the first round bid functions are symmetric between the types the first good must go to the bidder with the lowest cost. However, once the first good is assigned, the second auction need not allocate to the lowest cost firm.

Corollary 4. *In a symmetric equilibrium, the sequential second-price auctions allocate the first good to the lowest cost firm, but the second good may or may not go to the firm with the second lowest cost.*

Combining these observations with the ones from the sealed-bid auction, it is clear that a seller interested in allocating the goods to the lowest cost firms (perhaps because this would maximize total surplus in the market) should prefer the sealed-bid format over the sequential format in this environment. The reason is simple. The sealed-bid format does not allow the bidders to condition their bids on information about the realized identities of the other winner, while the sequential auction does.

As with the sealed-bid format, if the sequential format allocates one good each to an A and B bidder then the allocation must also maximize the ex-post valuations of the bidders. The reasoning is not exactly the same as in the sealed-bid case though. In the second round of the sequential auction the bidders know their ex-post valuations, so the second round must select the highest ex-post valuation from among the second round bidders. However, when the second round selects a bidder of the same type as the first round, the valuation of the first round bidder is affected (negatively) through the externality.

So if in the second round after the first good has been allocated to an A the second good is allocated to a B (so $c_A^{(1)} \leq c_B^{(1)} \leq c_A^{(2)}$), the first round winner must still have the highest valuation because from A1 it follows that

$$\pi_{A,B}(c_A^{(1)}, c_B^{(1)}) > \pi_{A,B}(c_B^{(1)}, c_A^{(1)}) > \pi_{A,A}(c_A^{(2)}, c_A^{(1)})$$

However, when the second good goes to an A (after an A wins the first round), the first round winner's payoff is reduced and this can lead to an allocation that does not maximize the ex-post payoffs of the bidders.

The next proposition will show that it must be that the sequential auction maximizes the ex-post valuations of the bidders more often than the sealed-bid auction. For this proposition, I introduce the following notion of efficiency, where in this application efficiency means maximization of the bidders' ex-post valuations.

Definition 1. *Let E_i be the union of events in which auction i allocates the goods inefficiently. Auction 1 is called more efficient than auction 2 if $E_1 \subseteq E_2$. It is strictly more efficient if $E_1 \subset E_2$.*

Using this definition of efficiency, I can now state

Proposition 4. *With efficiency understood to mean maximization of ex-post valuations, it must be that the sequential auction is more efficient than the sealed-bid auction.*

Proof. Let E_S (E_{SB}, respectively) be the events in which the sequential auction (sealed-bid auction) does not maximize the ex-post valuations of the bidders. I need to show that $E_S \subseteq E_{SB}$. As discussed above, both auctions maximize the ex-post valuations when the goods are allocated to bidders of different types. So I restrict attention to realizations that lead to allocations to the same type. Without loss of generality I consider cases where the allocation is to two A bidders. E_S is characterized by the set of inequalities

$$c_A^{(1)} \leq c_A^{(2)} \leq c_B^{(1)}$$

$$\pi_{A,A}(c_A^{(1)}, c_A^{(2)}) + \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) < \pi_{A,B}(c_A^{(1)}, c_B^{(1)}) + \pi_{A,B}(c_B^{(1)}, c_A^{(1)})$$

And E_{SB} is characterized by the inequalities

$$c_A^{(1)} \leq c_A^{(2)} \leq c_B^{(1)}$$

$$\pi_{A,A}(c_A^{(1)}, c_A^{(2)}) + \pi_{A,A}(c_A^{(2)}, c_A^{(1)}) < \pi_{A,B}(c_A^{(1)}, c_B^{(1)}) + \pi_{A,B}(c_B^{(1)}, c_A^{(1)})$$

$$\pi_{A,A}(c_A^{(2)}, c_A^{(1)}) \geq \pi_{A,B}(c_A^{(1)}, c_B^{(1)})$$

The only difference is the addition of the third inequality for E_{SB}. So it must be that $E_{SB} \subseteq E_S$. □

To summarize the results of this section, I have shown that in this model a seller interested in allocating the goods to the firms with the lowest cost prefers to run a sealed-bid auction, which always allocates to the bidders with the lowest cost realizations (this is not true for the sequential format). Next I have shown that the sequential format is always more likely to allocate to the bidders with the highest ex-post allocations. Therefore, these two notions of efficiency lead to opposite results in this model.

4.4 Conclusion

The results show that a seller interested in allocating to firms with the lowest cost would prefer to run a sealed-bid Vickrey auction over a sequential auction in this environment. The typical definition of efficiency prescribes that a seller maximize the ex-post valuations of the bidders, and in this environment a seller prefers a sequential auction in this case. This second result agrees with the spirit of the Das Varma [2002b], who shows that a seller auctioning a single good in an environment with allocative externalities can raise revenue and expected efficiency by revealing the identities of bidders.

The implication of this paper is that revealing information about bidders through a sequential auction format harms the ability of the auction to allocate to the bidders with the lowest cost. It is interesting to note that one criticism of dynamic auction formats is that they may allow for bidders to implement collusive strategies [Klemperer, 2004]. This paper provides another rationale that is closely related, when there are allocative externalities there need not be any “active” collusion for the sequential format to favor the bidders.

APPENDIX

A. CHAPTER THREE SUPPORTING MATERIALS

A.1 Derivation of Equilibrium

Let t represent the valuation of the bidder, δt represent the valuation of the principal, and s represent the signal received by the principal. The distributions of these random variables in the experiment are

$$s \sim U\left[0, 100\right]$$

$$t \sim U\left[0, \frac{s}{\delta}\right]$$

$$\delta t \sim U\left[0, s\right]$$

The relevant density functions are

$$f\left(s\right) = \frac{1}{100} \qquad 0 \leq s \leq 100$$

$$f\left(t \mid s\right) = \frac{\delta}{s} \qquad 0 \leq t \leq \frac{s}{\delta}$$

A.1.1 Second Price Auction

Let $b_2(t)$ be the bidder's unconstrained choice of bid in the second price auction. It is a weakly dominant strategy in this environment for the bidder to set $b_2\left(t\right) = t$.

Let $w_2(t)$ be the budget set by the principal. Since the bidder's unconstrained choice is the identity functions, the budget set by the principal is equivalently represented in terms of t as $\hat{t}_2(s) \equiv w_2(s)$. Also let the density of the opposing bids be given by $g(x)$. The principal's objective is now

$$\int_0^{\widehat{t_2}} \int_0^t (\delta t - x)\, g(x)\, f(t \mid s)\, dx\, dt + \int_{\widehat{t_2}}^{\frac{s}{\delta}} \int_0^t (\delta t - x)\, g(x)\, f(t \mid s)\, dx\, dt \quad \text{(A.1)}$$

The first term represents the payoff in the event that the budget constraint does not bind, and the second is the payoff when the constraint does bind. The first order condition is

$$\begin{aligned} \int_{\hat{t}_2}^{s/\delta} \left(\delta t - \hat{t}_2\right)\ g\left(\hat{t}_2\right)\ f(t \mid s)\ dt &= 0 \\ \int_{\hat{t}_2}^{s/\delta} \left(\delta t - \hat{t}_2\right) \frac{\delta}{s} dt &= 0 \\ \frac{s}{s - \delta \hat{t}_2} \int_{\hat{t}_2}^{s/\delta} \left(\delta t - \hat{t}_2\right) \frac{\delta}{s} dt &= 0 \\ \frac{s + \delta\ \hat{t}_2}{2} - \hat{t}_2 &= 0 \\ \hat{t}_2 &= \frac{s}{2 - \delta} \end{aligned}$$

For the second line, note that $g\left(\hat{t}_2\right)$ is a constant. In the third line, we are dividing by $P\left(t > \hat{t}_2 \middle|\ s\right)$ to make the left side a conditional expectation. The final line is the principal's equilibrium choice of $\hat{t}_2$, which in the second price auction is also the equilibrium choice of budget constraint. The constrained bid submitted by the bidder can be written as

$$B(s, t) = \min\left\{t, \frac{s}{2 - \delta}\right\}$$

Where it depends on both the bidder's and the principal's information.

A.1.2 First Price Auction

We give a brief treatment of the equilibrium derivation in the first price auction and refer the reader to Burkett (2011) for a more complete treatment. The first observation to make is that if the bidders choose unconstrained bids according to some monotonic and continuous bid function $b_1(t)$, then for a choice of budget $w_1(s)$ we can define a cutoff type $\hat{t}_1(s)$ by $b_1\left(\hat{t}_1(s)\right) \equiv w_1(s)$. The eventual bid submitted can thus be represented by $b_1\left(\min\left\{t, \hat{t}_1(s)\right\}\right)$. If we define $\tilde{t}_1 \equiv \min?\{t, \hat{t}_1(s)\}$, then in a symmetric monotone equilibrium the bidder with the higher value of $\tilde{t}_1$ will be the winner. Letting $G(x)$ and $g(x)$ represent the distribution and density functions of the equilibrium $\tilde{t}_1$for each bidder, the unconstrained choice of bid takes the usual form

$$b_1(t) = \frac{1}{G(t)} \int_0^t x\ g(x)\, dx$$

with $G(x)$ playing the role of the usual type distribution. Now writing the principal's objective in terms of the choice of $\hat{t}_1$ we get

$$\int_0^{\hat{t}_2} G(t)\left(\delta t - b_1(t)\right) f(t \mid s) dt + \int_{\hat{t}_2}^{s/\delta} G\left(\hat{t}_1\right)\left(\delta t - b_1\left(\hat{t}_1\right)\right)\ f(t \mid s)\ dt$$

And plugging in for $b_1(t)$ this becomes

$$\int_0^{\hat{t}_2} \left(G(t)\delta t - \int_0^t x\ g(x)\, dx\right) f(t \mid\ s) dt + \int_{\hat{t}_2}^{s/\delta} \left(G(\hat{t}_1)\delta t - \int_0^{\hat{t}_1} x\ g(x)\, dx\right)\ f(t \mid\ s)\ dt$$

For a given $G(x)$, this is the same objective as the one in the second price auction, so the solution for $\hat{t}_1$ is the same

$$\hat{t}_1 = \frac{s}{2-\delta}$$

In order to determine $b_1(t)$, we need to find $G(x)$, the distribution of $\min\left\{t, \frac{s}{2-\delta}\right\}$.

We get

$$G(x) = \left\{2 - \delta - \delta \ln\left(\frac{2-\delta}{100}x\right)\right\}\frac{x}{100}$$

This makes the unconstrained bid function

$$b_1(x) = \left(\frac{4 - 3\delta - 2\delta \ln\left(\frac{2-\delta}{100}x\right)}{4 - 2\delta - 2\delta \ln\left(\frac{2-\delta}{100}x\right)}\right)\frac{x}{2}$$

A.2 *Proof of Proposition 1*

Suppose the principals choose a cutoff type strategy according to $\hat{t}_\alpha(s) = \alpha s$, where in equilibrium $\alpha = \alpha^* = 5/8$ in both auction formats, and let the distribution of $\widetilde{t_\alpha} \equiv \min\{t, \alpha s\}$ be given by $G_\alpha(x)$ with $G_\alpha^{(i)}(x)$ being the distribution of the i^{th} order statistic. Finally, denote the equilibrium unconstrained bid function derived above by $b_{\alpha^*}(x) = \frac{1}{G_{\alpha^*}(x)}\int_0^x y\, dG_{\alpha^*}(y)$. Then the experimental expected revenue in the first price auction ($E\left[R_\alpha^{FP}\right]$) and the second price auction ($E[R_\alpha^{SP}]$) with principals following a strategy $\hat{t}_\alpha(s)$ can be written as

$$E\left[R_\alpha^{FP}\right] = \int_0^{100\alpha} b_{\alpha^*}(x)\; dG_a^{(1)}(x) \qquad E\left[R_\alpha^{SP}\right] = \int_0^{100\alpha} x\, dG_a^{(2)}(x)$$

As the theory shows, with $\alpha = \alpha^*$ the two expressions are equal. Also, using a standard revenue equivalence argument we have

$$E\left[R_\alpha^{SP}\right] = \int_0^{100\alpha} b_\alpha(x)\; dG_a^{(1)}(x)$$

where $b_\alpha(x)$ is defined analogously to $b_{\alpha^*}(x)$. So the first price auction raises more revenue when $\alpha > \alpha^*$ if and only if

$$\int_0^{100\alpha} b_{\alpha^*}(x)\, dG_a^{(1)}(x) > \int_0^{100\alpha} b_{\alpha}(x)\, dG_a^{(1)}(x)$$
$$\Longleftrightarrow \int_0^{100\alpha} \left(b_{\alpha^*}(x) - b_{\alpha}(x)\right)\, dG_a^{(1)}(x) > 0$$

In fact, one can show that for $\alpha > \alpha^*$ and all $x > 0$, $b_{\alpha^*}(x) > b_{\alpha}(x)$, so that the above expression holds. Note that $b_{\alpha^*}(x) > b_{\alpha}(x)$ if $\frac{g_\alpha(x)}{G_\alpha(x)} < \frac{g_{\alpha^*}(x)}{G_{\alpha^*}(x)}$ [Maskin and Riley, 2000] and it is straightforward to verify that $\frac{g_\alpha(x)}{G_\alpha(x)}$ decreases in α when $\alpha^* < \alpha < 5/2$. For $\frac{g_\alpha(x)}{G_\alpha(x)}$, we obtain after simplifying

$$\frac{g_\alpha(x)}{G_\alpha(x)} = \frac{1}{x} - \frac{10\alpha}{5x - 2\alpha x \ln\left(\frac{x}{100\alpha}\right)}$$

When $\alpha \geq 5/2$, the principal leaves the bidder unconstrained with probability 1 and increases in α no longer affect $\frac{g_\alpha(x)}{G_\alpha(x)}$, so the revenue raised is the same as that when $\alpha = 5/2$.

A.3 Participant Instructions

INSTRUCTIONS FOR FIRST PRICE SEALED-BID AUCTIONS

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US$.

- In this experiment, you will participate in a sequence of auctions. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.
- At the beginning of the session, you will be assigned to one of the two types: **A** and **B.**
- Your type is fixed throughout the experiment.
- A single good will be auctioned off in each period.

Matching in Each Period

In each period, each type A subject will be randomly matched with a type B subject and form a team of two. Then each team will be randomly matched with another team of two subjects and then two teams will participate in an auction. You will never know whom you are matched. You will not be matched with the same group of subjects in any two consecutive periods.

- Each team consists of a type A and a type B subjects.
- Each team participates in an auction to obtain an auctioned good.
- Two teams participate in an auction.

Values

For Type A subjects:

At the beginning of each period, each type A subject *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. The

MAX VALUEs of the two type A subjects in two teams that are participating in the same auction are independently determined and most likely different.

Type A subject does not know her exact VALUE for the good at the time of decision making. All she knows is that her VALUE is a number contained in the interval [0, MAX VALUE]. Therefore, her VALUE is at minimum zero and cannot exceed her MAX VALUE. Again, any number between 0 and her MAX VALUE is equally likely. For example, let's say a type A subject receives a MAX VALUE of 45.32. Then her VALUE is uniformly distributed on interval [0, 45.32] and it can be any number less than or equal to 45.32. Let's say her VALUE is 21.00. This means that if her team obtains the good at the end of the period, she will receive 21.00 ECU from us.

For Type B Subjects:

Each Type B subject knows the true value of her Type A teammate. Each Type B subject's value for obtaining the good is 2.5 times her Type A teammate's value.

Type B's Value = 2.5 x Type A's Value

Auction

Each auction occurs in two stages. In the first stage only the Type A subjects will be active. In the second stage, only the Type B subjects are active. Specifically,

Stage 1:

- Each Type A only observes her MAX VALUE. Her true value is something less than this MAX VALUE.
- Each Type A subject decides on a CAP which is the maximum amount she allows her type B teammate to bid.

Stage 2:

- Each Type B observes the exact value of the good for herself.
- Each Type B also observes the CAP decided on by her Type A teammate.
- Each Type B subject decides on how much she wants to bid in behalf of her team in the auction. Type B subjects are not allowed to bid above the CAP. The bid decided on is simply labeled BID.

- After both teams participating in the auction submit their BIDs, the one who has **the highest BID obtains the good** and **pays <u>her BID</u>**. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

- The following table summarizes the progression of stages

	Team 1		**Team 2**	
	Type A	**Type B**	**Type A**	**Type B**
Stage 1	Sees: MAX VALUE Chooses: CAP		Sees: MAX VALUE Chooses: CAP	
Stage 2		Sees: CAP,VALUE Chooses: BID (≤ CAP)		Sees: CAP,VALUE Chooses: BID (≤ CAP)

Earnings in a Period

When your team obtains the good at the end of a period (if your BID is the highest), then you will receive your VALUE for the good and will pay the team's **BID**. If your team does not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

Earnings = Your VALUE – Your BID (If you obtained the auctioned good);

Earnings = 0 (If you did not obtain the auctioned good).

Recall that a Type B subject's value is 2.5 times more than her Type A teammate. Moreover, at the time of decision making, each Type B subject knows her value. However, Type A subjects only know their maximum possible value but not their actual value.

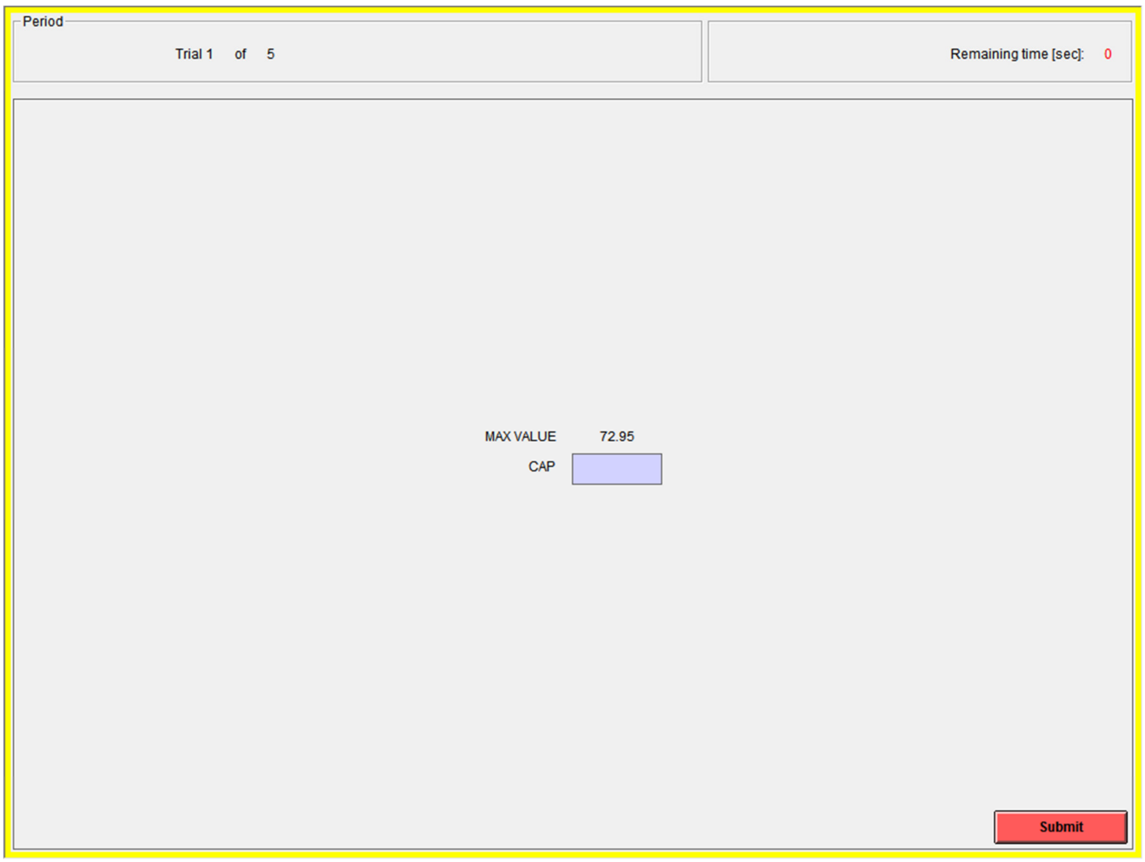
Sequence of Auctions:

When the current period is over, the next period will start. Each period, you will be randomly matched with a new teammate and participate in a new auction with a different opponent team. Your VALUE of the good in each period is independent of your VALUE in the previous periods.

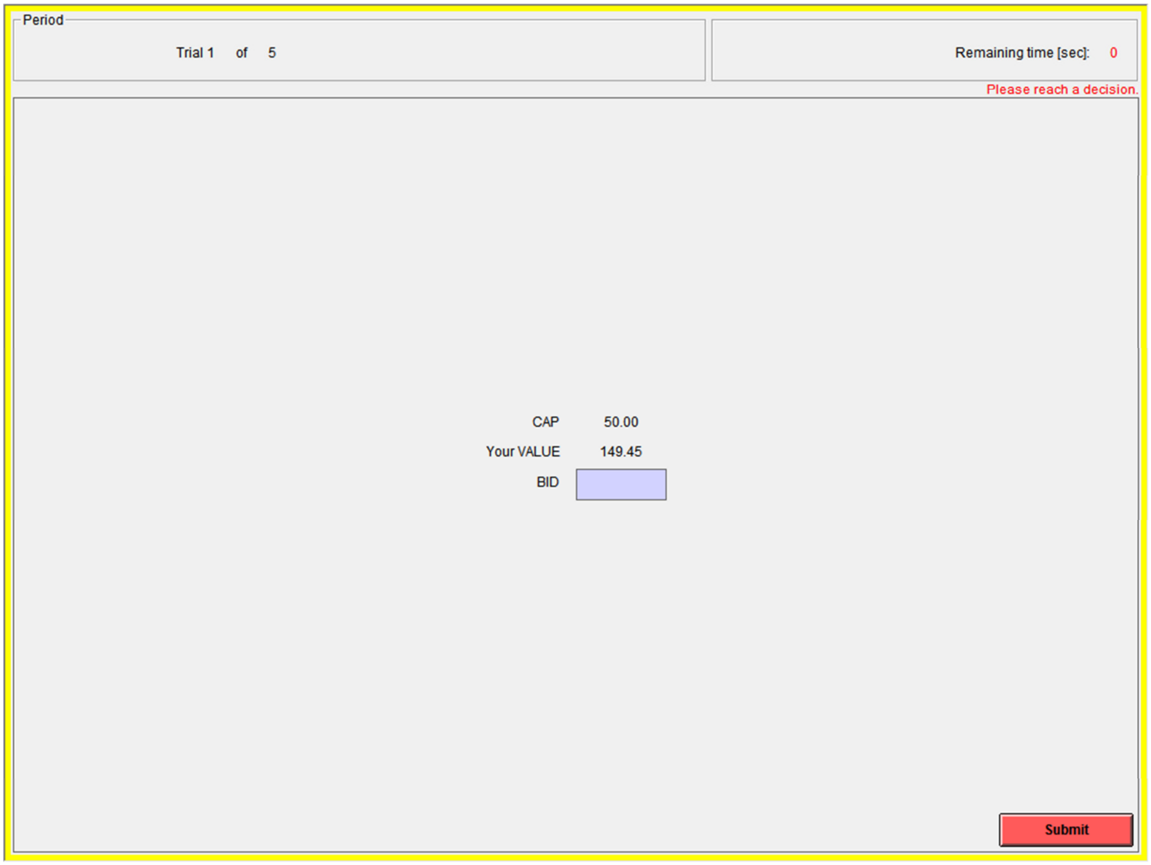
Screens

Below, there is an example of how Type A's and Type B's screens may look.

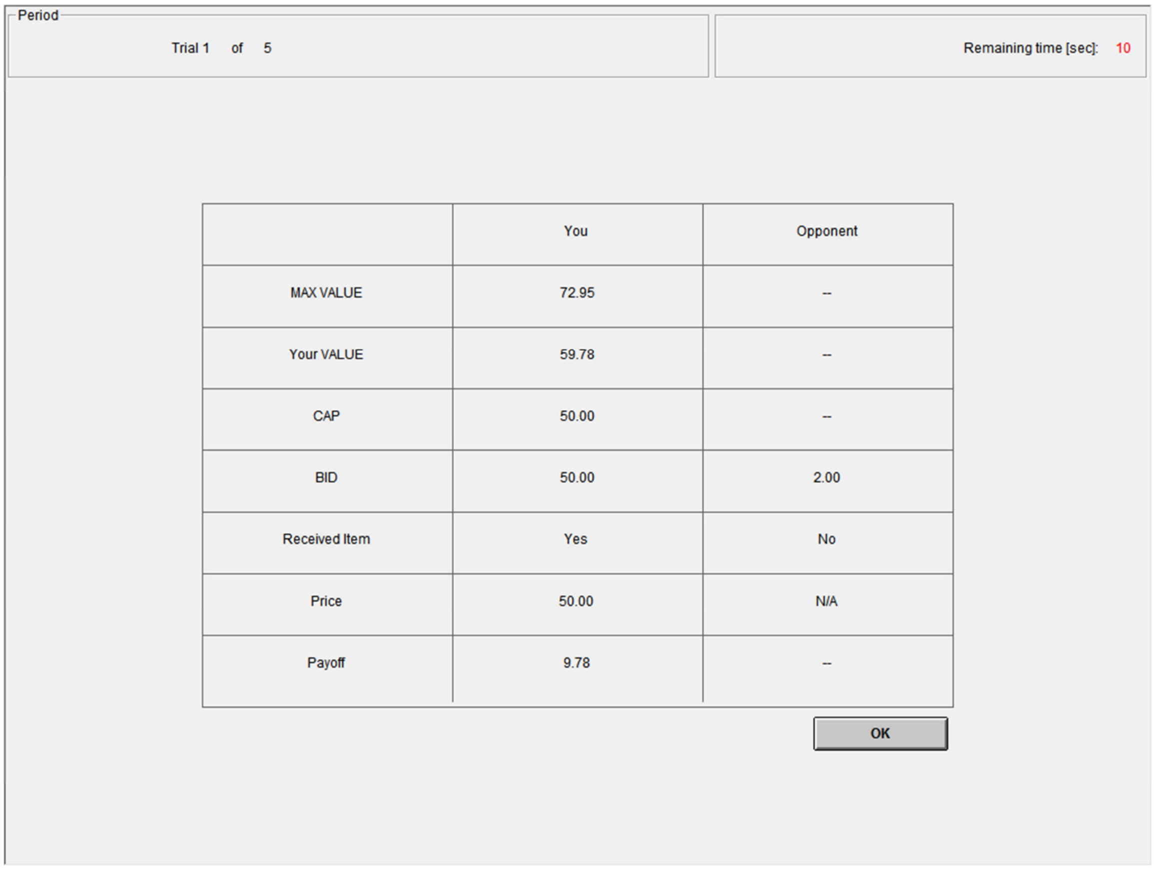
If you are a Type A subject, you see your MAX VALUE. Remember that this is the highest amount your actual value can be. You DON'T know your actual value before the auction is over. You need to enter a CAP for your Type B teammate in the text box on your screen and click on SUBMIT.



If you are a Type B subject, you see your value for the good and the CAP that your Type A teammate decided on. After observing your value, you will enter your BID in the text box on your screen and click on SUBMIT. Your bid has to be *less than or equal to* your CAP.



The screen below shows the information that a Type A person will see at the conclusion of a period. It displays Type A's MAX VALUE, VALUE and CAP. Then it shows the BID that the Type B teammate decided on. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Note that much of the information about your opponent is hidden.



Period

Trial 1 of 5

Remaining time [sec]: 10

	You	Opponent
MAX VALUE	72.95	--
Your VALUE	59.78	--
CAP	50.00	--
BID	50.00	2.00
Received Item	Yes	No
Price	50.00	N/A
Payoff	9.78	--

OK

The next screen shows the information that a Type B person will see at the conclusion of a period. It displays Type B's VALUE, CAP and BID. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Again, much of the information about your opponent is hidden.

Period

Trial 1 of 5

Remaining time [sec]: 0

	You	Opponent
Your VALUE	149.45	--
CAP	50.00	--
BID	50.00	2.00
Received Item	Yes	No
Price	50.00	N/A
Payoff	99.45	--

OK

Example

The tables below indicate all the MAX VALUEs, VALUEs and BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment Type A subjects will observe only their own MAX VALUEs and Type B subjects will observe their own VALUES. No subject will know the values received by the opponent team.

1.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**5.00**	20.50
Type B's VALUE	141.00	20.20
Type B's BID	**5.00**	9.03
Received Item	**No**	**Yes**
Price	N/A	9.03
Type A's Payoff	**0**	-0.95
Type B's Payoff	**0**	11.17

2.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**36.00**	20.50
Type B's VALUE	**141.00**	20.20
Type B's BID	**36.00**	9.03
Received Item	**Yes**	**No**
Price	36.00	N/A
Type A's Payoff	**20.40**	0
Type B's Payoff	**105.00**	0

3.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**70.00**	20.50
Type B's VALUE	**141.00**	20.20
Type B's BID	**59.44**	9.03
Received Item	**Yes**	**No**
Price	59.44	N/A
Type A's Payoff	**-2.96**	0
Type B's Payoff	**81.56**	0

In all of the examples above, Type A of Team 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 20.50. Team 2's BID is 9.03.

Type A of Team 1 observes a MAX VALUE of 84.62. Her true value (which she does not know at the time of decision making), is 56.40. Therefore, the value of the Type B subject of Team 1 is 141 (2.5x56.40=141).

Each of the three tables corresponds to different choices of CAP and BID for Team 1. In the first table, Type A chose a cap of 5.00 and Type B chose a BID of 5.00.

In the first example the BIDs of two teams are 5.00 and 9.03. Since the highest bid (9.03) is submitted by Team 2, Team 2 obtains the good and pays its BID (9.03). In this period, the subjects in Team 1 earn zero, Type A of Team 2 earns 8.08 – 9.03 = -0.95 ECU, and Type B of Team 2 earns 20.20-9.03=11.17. Note that Type A of Team 2 loses money in this period because her Type B teammate is allowed to submit a bid that is higher than the Type A's true value.

In the second example, everything is the same except Type A of Team 1 choses a higher CAP (36.00) and Type B chooses a higher BID (36.00). Now Team 1 receives the item. The price is equal to the BID (36.00), so Type A of Team 1's payoff is 56.40 – 36.00 = 20.40 ECU and Type B of Team 1's payoff is 141.00-36.00=105.00 ECU.

In the third example, Type A of Team 1's CAP is now 70.00, and Type B's BID is now 59.44. The BIDs are 59.03 and 9.03. Team 1 receives the item for a price of 59.03, Type A's payoff is 56.40 – 59.03 = -2.96 ECU, and Type B's payoff is 141.00-59.03=81.97 ECU.

Total Payoffs

At the beginning of today's session both Type A and Type B subjects will receive an endowment of $8. The endowment is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings —your earnings from 25 auctions plus your initial endowment. The conversion rate is $1 = 20 ECU for Type A and $1 = 80 ECU for Type B. In addition to this sum, you will be paid a $7 participation fee.

Are there any questions?

Practice Questions: Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose a Type A subject observes a MAX VALUE of 76.00ECU. What is her possible VALUE?

a) any value from 0 to 100.00

b) any value from 0 to 76.00

c) any value from 0 to 76.00 but not 76.00

d) any value from 0 to 100.00 but not 76.00

2. Suppose a Type A subject entered 21.00 as the CAP. What are the possible BIDs that the Type B subject in her team can select?

a) Any BID is possible.

b) Any BID that is between 0 and Type B's VALUE is possible.

c) Any BID that is between 0 and Type A's VALUE is possible.

d) Any BID that is between 0 and the CAP is possible.

3. Fill the table below

	Team 1	Team 2
MAX VALUE (observed by Type A)	**43.00**	**37.40**
Type A's VALUE	**4.00**	**10**
Type A's CAP	**38.00**	**21.00**
Type B's VALUE		
Type B's BID	**6.00**	**8.00**
Received Item		
Price		
Type A's Payoff		
Type B's Payoff		

INSTRUCTIONS FOR SECOND PRICE SEALED BID AUCTIONS

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US$.

- In this experiment, you will participate in a sequence of auctions. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.
- At the beginning of the session, you will be assigned to one of the two types: **A** and **B.**
- Your type is fixed throughout the experiment.
- A single good will be auctioned off in each period.

Matching in Each Period

In each period, each type A subject will be randomly matched with a type B subject and form a team of two. Then each team will be randomly matched with another team of two subjects and then two teams will participate in an auction. You will never know whom you are matched. You will not be matched with the same group of subjects in any two consecutive periods.

- Each team consists of a type A and a type B subjects.
- Each team participates in an auction to obtain an auctioned good.
- Two teams participate in an auction.

Values

For Type A subjects:

At the beginning of each period, each type A subject *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. The

MAX VALUEs of the two type A subjects in two teams that are participating in the same auction are independently determined and most likely different.

Type A subject does not know her exact VALUE for the good at the time of decision making. All she knows is that her VALUE is a number contained in the interval [0, MAX VALUE]. Therefore, her VALUE is at minimum zero and cannot exceed her MAX VALUE. Again, any number between 0 and her MAX VALUE is equally likely. For example, let's say a type A subject receives a MAX VALUE of 45.32. Then her VALUE is uniformly distributed on interval [0, 45.32] and it can be any number less than or equal to 45.32. Let's say her VALUE is 21.00. This means that if her team obtains the good at the end of the period, she will receive 21.00 ECU from us.

For Type B Subjects:

Each Type B subject knows the true value of her Type A teammate. Each Type B subject's value for obtaining the good is 2.5 times her Type A teammate's value.

Type B's Value = 2.5 x Type A's Value

Auction

Each auction occurs in two stages. In the first stage only the Type A subjects will be active. In the second stage, only the Type B subjects are active. Specifically,

Stage 1:

- Each Type A only observes her MAX VALUE. Her true value is something less than this MAX VALUE.
- Each Type A subject decides on a CAP which is the maximum amount she allows her type B teammate to bid.

Stage 2:

- Each Type B observes the exact value of the good for herself.
- Each Type B also observes the CAP decided on by her Type A teammate.
- Each Type B subject decides on how much she wants to bid in behalf of her team in the auction. Type B subjects are not allowed to bid above the CAP. The bid decided on is simply labeled BID.

- After both teams participating in the auction submit their BIDs, the one who has **the highest BID obtains the good** and **pays <u>her opponent's BID</u>**. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

- The following table summarizes the progression of stages

	Team 1		Team 2	
	Type A	**Type B**	**Type A**	**Type B**
Stage 1	Sees: MAX VALUE Chooses: CAP		Sees: MAX VALUE Chooses: CAP	
Stage 2		Sees: CAP,VALUE Chooses: BID (≤ CAP)		Sees: CAP,VALUE Chooses: BID (≤ CAP)

Earnings in a Period

When your team obtains the good at the end of a period (if your BID is the highest), then you will receive your VALUE for the good and will pay the **opposing team's BID**. If your team does not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

Earnings = Your VALUE – Your Opponent's BID (If you obtained the auctioned good);
Earnings = 0 (If you did not obtain the auctioned good).

Recall that a Type B subject's value is 2.5 times more than her Type A teammate. Moreover, at the time of decision making, each Type B subject knows her value. However, Type A subjects only know their maximum possible value but not their actual value.

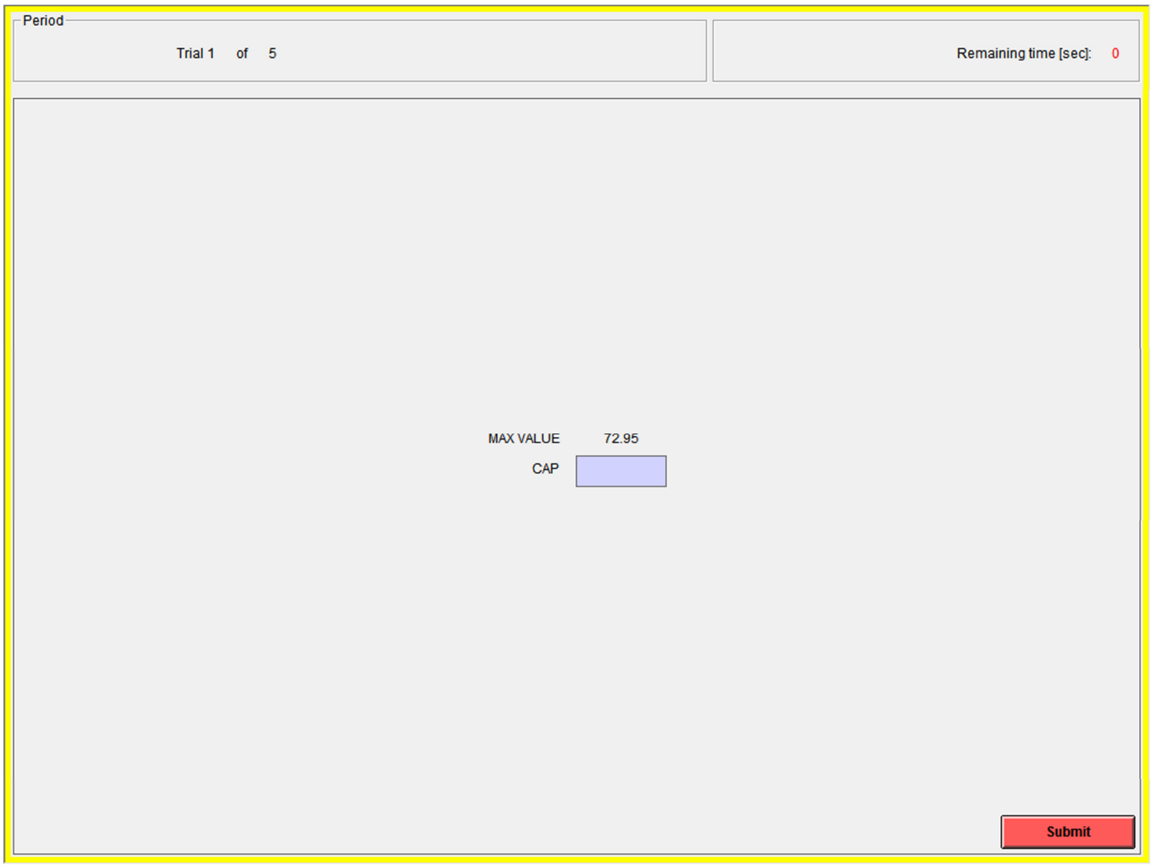
Sequence of Auctions:

When the current period is over, the next period will start. Each period, you will be randomly matched with a new teammate and participate in a new auction with a different opponent team. Your VALUE of the good in each period is independent of your VALUE in the previous periods.

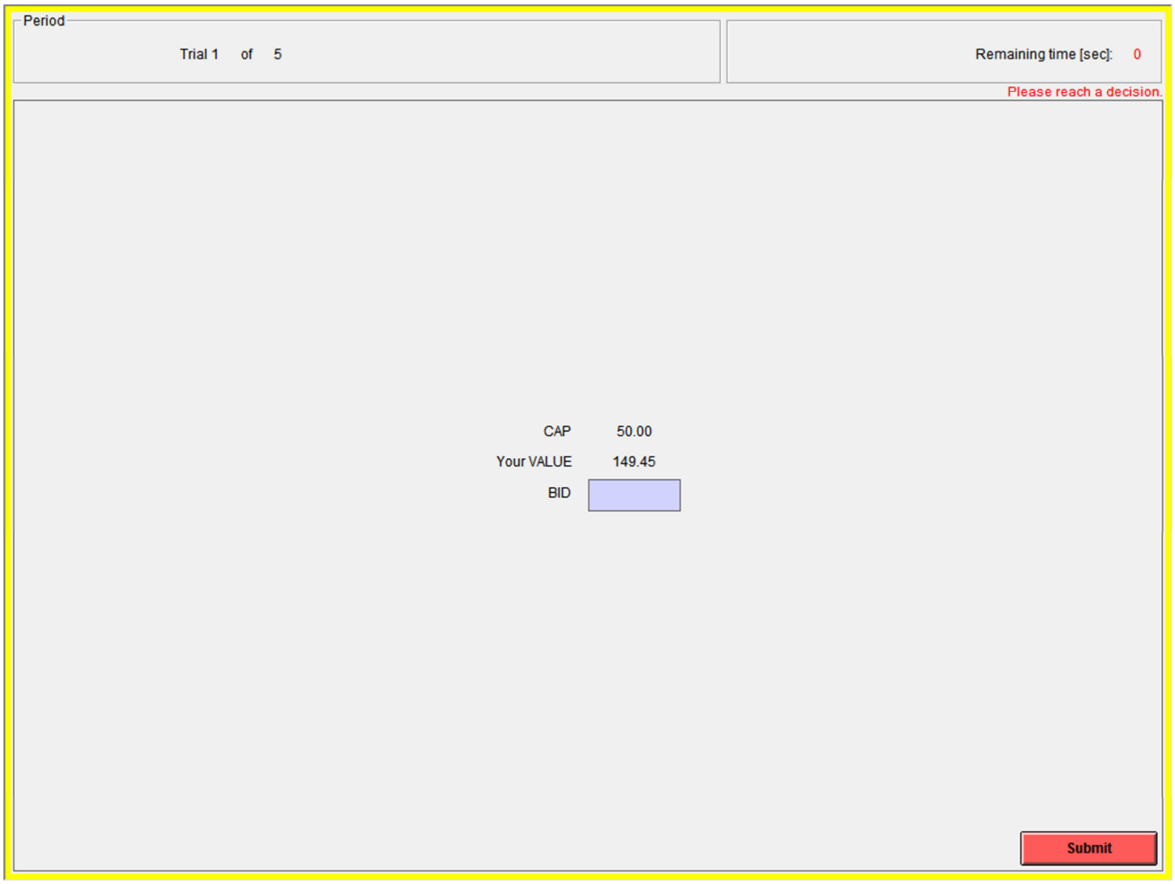
Screens

Below, there is an example of how Type A's and Type B's screens may look.

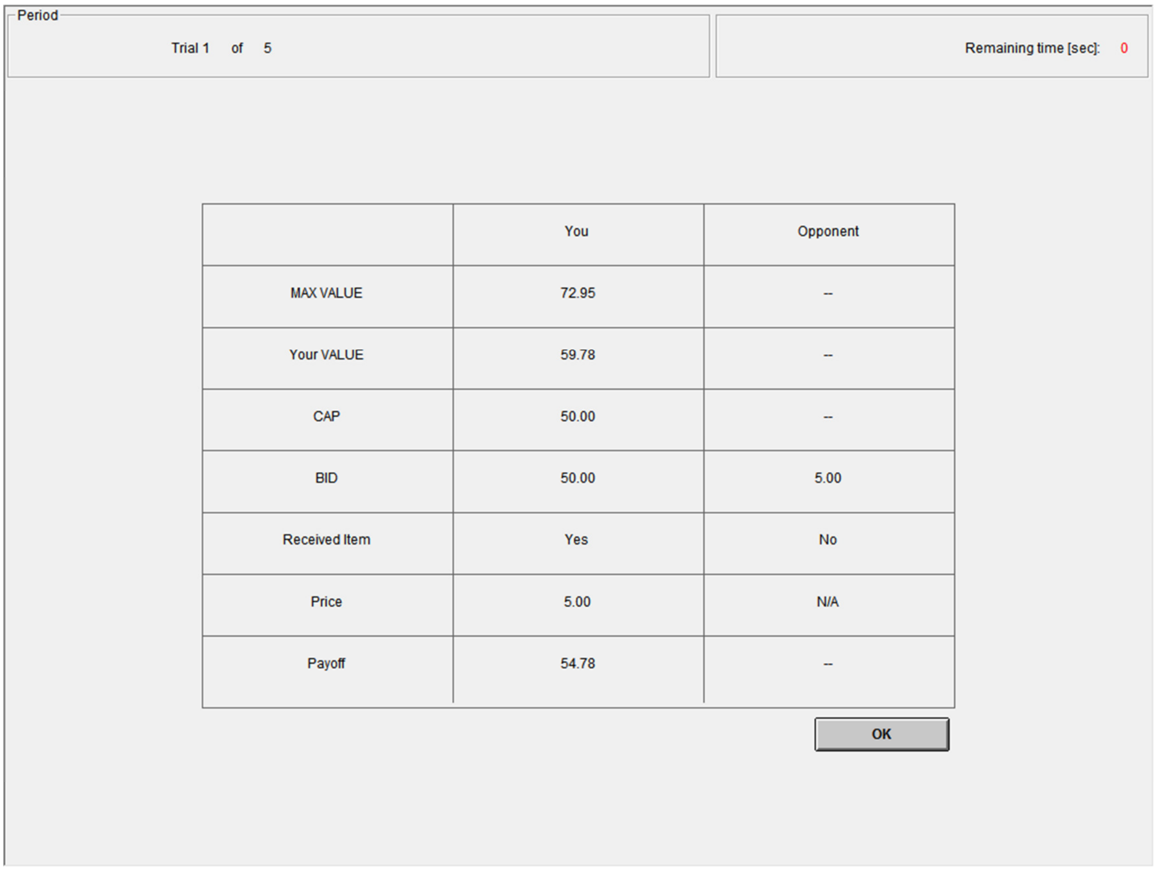
If you are a Type A subject, you see your MAX VALUE. Remember that this is the highest amount your actual value can be. You DON'T know your actual value before the auction is over. You need to enter a CAP for your Type B teammate in the text box on your screen and click on SUBMIT.



If you are a Type B subject, you see your value for the good and the CAP that your Type A teammate decided on. After observing your value, you will enter your BID in the text box on your screen and click on SUBMIT. Your bid has to be *less than or equal to* your CAP.



The screen below shows the information that a Type A person will see at the conclusion of a period. It displays Type A's MAX VALUE, VALUE and CAP. Then it shows the BID that the Type B teammate decided on. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Note that much of the information about your opponent is hidden.



Period

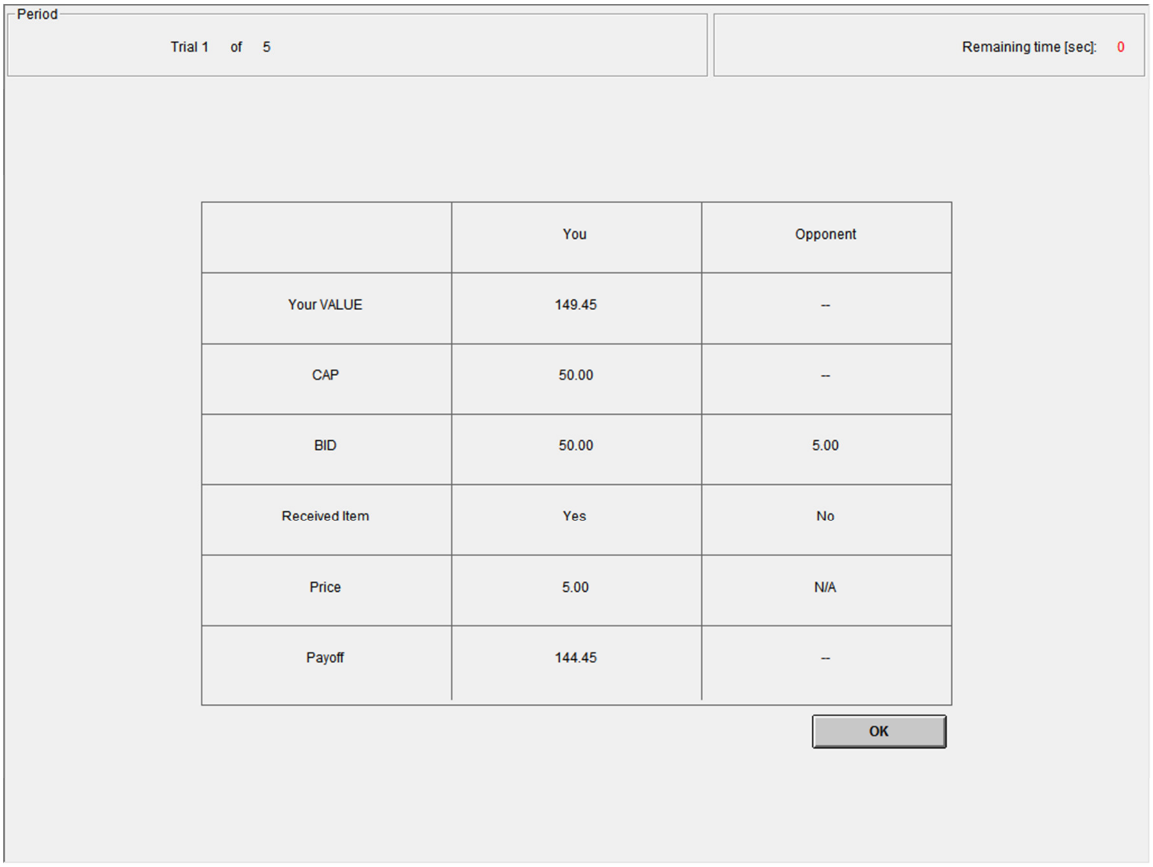
Trial 1 of 5

Remaining time [sec]: 0

	You	Opponent
MAX VALUE	72.95	--
Your VALUE	59.78	--
CAP	50.00	--
BID	50.00	5.00
Received Item	Yes	No
Price	5.00	N/A
Payoff	54.78	--

OK

The next screen shows the information that a Type B person will see at the conclusion of a period. It displays Type B’s VALUE, CAP and BID. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Again, much of the information about your opponent is hidden.


Period

Trial 1 of 5

Remaining time [sec]: 0

	You	Opponent
Your VALUE	149.45	--
CAP	50.00	--
BID	50.00	5.00
Received Item	Yes	No
Price	5.00	N/A
Payoff	144.45	--

OK

Example

The tables below indicate all the MAX VALUEs, VALUEs and BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment Type A subjects will observe only their own MAX VALUEs and Type B subjects will observe their own VALUES. No subject will know the values received by the opponent team.

1.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**12.00**	20.50
Type B's VALUE	**141.00**	20.20
Type B's BID	**12.00**	12.03
Received Item	**No**	**Yes**
Price	N/A	12.00
Type A's Payoff	**0**	-3.92
Type B's Payoff	**0**	8.20

2.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**36.00**	20.50
Type B's VALUE	**141.00**	20.20
Type B's BID	**36.00**	12.03
Received Item	**Yes**	**No**
Price	12.03	N/A
Type A's Payoff	**44.37**	0
Type B's Payoff	**128.97**	0

3.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	**84.62**	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	**70.00**	20.50
Type B's VALUE	**141.00**	20.20
Type B's BID	**59.44**	12.03
Received Item	**Yes**	**No**
Price	12.03	N/A
Type A's Payoff	**44.37**	0
Type B's Payoff	**128.97**	0

In all of the examples above, Type A of Team 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 20.50. Team 2's BID is 12.03.

Type A of Team 1 observes a MAX VALUE of 84.62. Her true value (which she does not know at the time of decision making), is 56.40. Therefore, the value of the Type B subject of Team 1 is 141 (2.5x56.40=141).

Each of the three tables corresponds to different choices of CAP and BID for Team 1. In the first table, Type A chose a cap of 12.00 and Type B chose a BID of 12.00.

In the first example the BIDs of two teams are 12.00 and 12.03. Since the highest bid (12.03) is submitted by Team 2, Team 2 obtains the good and pays its Opponent's BID (12.00). In this period, the subjects in Team 1 earn zero, Type A of Team 2 earns 8.08 – 12.00 = -3.92 ECU, and Type B of Team 2 earns 20.20-12.00=8.20. Note that Type A of Team 2 loses money in this period because her Type B teammate is allowed to submit a bid that is higher than the Type A's true value.

In the second example, everything is the same except Type A of Team 1 choses a higher CAP (36.00) and Type B chooses a higher BID (36.00). Now Team 1 receives the item. The price is equal to the Opponent's (Team 2's) BID (12.03), so Type A of Team 1's payoff is 56.40 – 12.03 = 44.37 ECU and Type B of Team 1's payoff is 141.00-12.03 = 128.97 ECU.

In the third example, Type A of Team 1's CAP is now 70.00, and Type B's BID is now 59.44. The BIDs are 59.03 and 12.03. Team 1 receives the item for a price of 12.03, Type A's payoff is 56.40 – 12.03 = 44.37 ECU, and Type B's payoff is 141.00-12.03 = 128.97 ECU.

Total Payoffs

At the beginning of today's session both Type A and Type B subjects will receive an endowment of $8. The endowment is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings —your earnings from 25 auctions plus your initial endowment. The conversion rates are $1 = 20 ECU for Type A and $1 = 80 ECU for Type B. In addition to this sum, you will be paid a $7 participation fee.

Are there any questions?

Practice Questions: Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose a Type A subject observes a MAX VALUE of 76.00ECU. What is her possible VALUE?

a) any value from 0 to 100.00

b) any value from 0 to 76.00

c) any value from 0 to 76.00 but not 76.00

d) any value from 0 to 100.00 but not 76.00

2. Suppose a Type A subject entered 21.00 as the CAP. What are the possible BIDs that the Type B subject in her team can select?

a) Any BID is possible.

b) Any BID that is between 0 and Type B's VALUE is possible.

c) Any BID that is between 0 and Type A's VALUE is possible.

d) Any BID that is between 0 and the CAP is possible.

3. Fill the table below

	Team 1	Team 2
MAX VALUE (observed by Type A)	**43.00**	**37.40**
Type A's VALUE	**4.00**	**10**
Type A's CAP	**38.00**	**21.00**
Type B's VALUE		
Type B's BID	**6.00**	**8.00**
Received Item		
Price		
Type A's Payoff		
Type B's Payoff		

INSTRUCTIONS FOR FIRST-PRICE SEALED BID AUCTIONS (computerized bidders)

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US$.

- In this experiment, we will run a sequence of auctions in which you will act as the buyer of a fictitious good. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.
- A single good will be auctioned off in each period.
- In each period, a computerized bidder will bid on behalf of you in the auction. We will tell you the bidding rule of the computerized bidder later in these instructions
- If you do not limit your computerized bidder, it will place an UNCONSTRAINED BID which may be higher than the amount you would like it to bid.
- Your task will be to determine a CAP, which is the maximum amount that you allow your computerized bidder to bid.
- The computerized bidder's ACTUAL BID is the lesser of its UNCONSTRAINED BID and the CAP.

Matching in Each Period

In each period, you will be randomly matched with another person in this room. You and that person will participate in the auction. You will never know who the other person is in your auction. You will not be matched with the same person in any two consecutive periods.

Values

At the beginning of each period, each person participating in the auction *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. Your MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. Your MAX VALUE and the MAX VALUE of the other person who participates in the same auction with you are independently determined.

Your VALUE for the good is the amount of ECU the experimenter will give you if you receive the item at the end of the period. You do not know your exact VALUE for the good at the time of decision making. All you know is that your VALUE is a number contained in the interval [0, MAX VALUE]. Therefore, your VALUE is at minimum zero and cannot exceed your MAX VALUE. Again, any number between 0 and your MAX VALUE is equally likely. For example, let's say you receive a MAX VALUE of 45.32. Then your VALUE is uniformly distributed on interval [0, 45.32] and it can be any number less than or equal to 45.32. Let's say your VALUE is 21.00. This means that if you get the good at the end of the period, you will receive 21.00 ECU from us.

To reiterate, MAX VALUE is never higher than 100 and a VALUE is never higher than the corresponding MAX VALUE. Each person in an auction receives independent MAX VALUEs and independent VALUEs. Hence, your VALUE and MAX VALUE are most likely different from your opponent's VALUE and MAX VALUE.

Auction

- Two persons participate in each auction. Each person is represented by a computerized bidder.
- You observe your MAX VALUE for the current period. **Your computerized bidder observes your true VALUE for the good.**
- After observing your MAX VALUE for the period, you need to decide the maximum amount that you will allow the computerized bidder to bid on your behalf. We call this amount your "CAP".
- You have been given two sheets explaining the bidding rule of your computerized bidder for each possible VALUE (unless you restrict it by a CAP). These two sheets provide you with the same information in two formats: one is a table, and one is a graph. The bids on the sheets are referred to as the computerized bidder's UNCONSTRAINED BID. Please take a look at these sheets and confirm that when your VALUE is, for example, 22.00 ECU, the UNCONSTRAINED BID will be 44.00.
- The computerized bidder's ACTUAL BID is the lesser of its UNCONSTRAINED BID and the CAP:

ACTUAL BID = minimum {UNCONSTRAINED BID , CAP}

- After both your and the other player's ACTUAL BIDs are submitted, the one who has **the highest ACTUAL BID obtains the good** and **pays <u>her ACTUAL BID</u>**. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

Earnings in a Period

If you obtain the good at the end of the period (if your ACTUAL BID is the highest), then you will receive your VALUE for the good and you will pay your **ACTUAL BID**. If you did not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

Earnings = Your VALUE – Your ACTUAL BID (If you obtained the auctioned good);

Earnings = 0 (If you did not obtain the auctioned good).

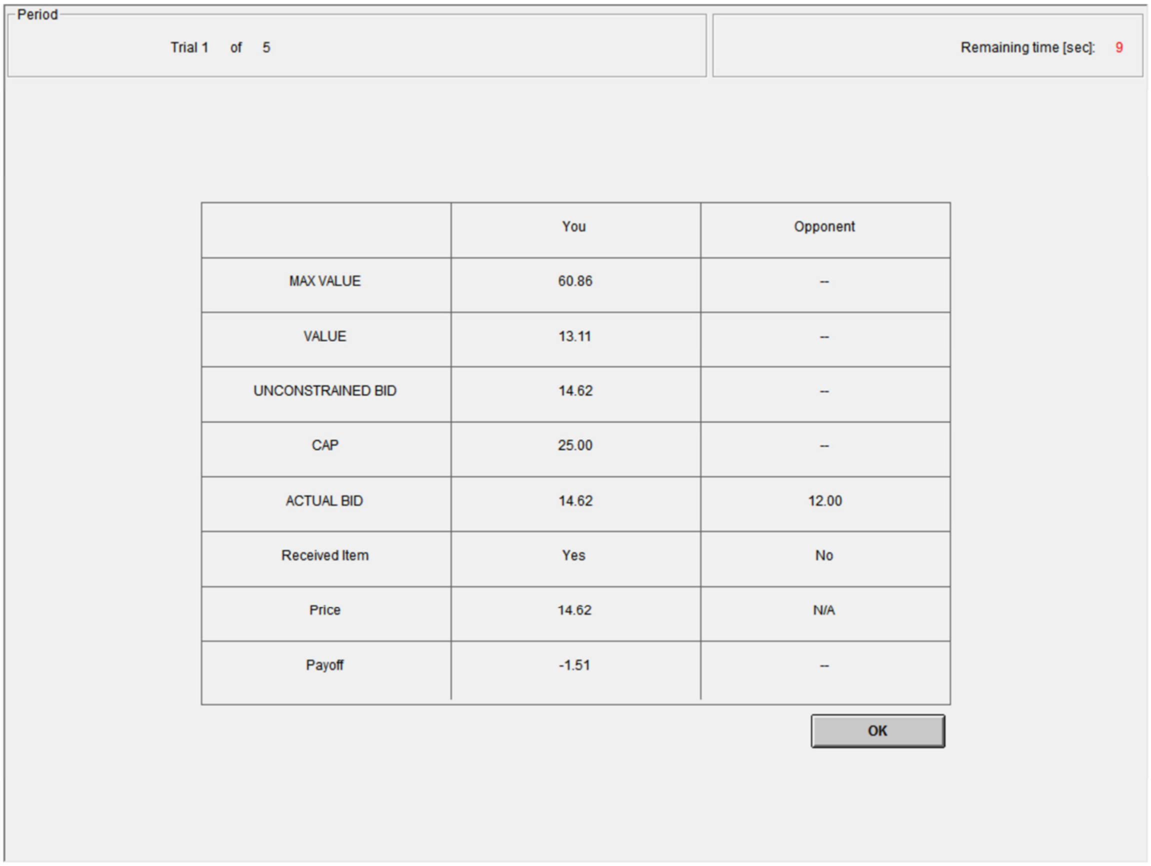
When the current period is over, the next period will start. Each period, you will be randomly matched with a new player and receive a new MAX VALUE. Therefore, your VALUE of the good in each period is independent of your VALUE in the previous periods.

Screens

Below, there is an example of how your screen may look. On top of your screen there is an interactive tool. The tool shows your MAX VALUE for the current period. Remember that your VALUE can be any number between zero and your MAX VALUE, but you do not know what it is. However, your computerized bidder knows your VALUE and bases its UNCONSTRAINED BID on your VALUE. By sliding the little black square between zero and your MAX VALUE, you may see the UNCONSTRAINED BID of your computerized bidder for the corresponding VALUE. This tool provides you with the exact same information as you may learn from the UNCONSTRAINED BID table or graph that we have provided to you. Please use whichever tool that you prefer in order to understand how the computerized bidder bids unless it is restricted by a CAP.

You need to enter your bidder's CAP for this period in the text box on your screen and click on SUBMIT. Remember that your ACTUAL BID will be what your computerized bidder's UNCONSTRAINED BID is **unless** you restrict it by a CAP, in which case it will be the lesser of the two numbers.

The screen below is an example of the results screen after the conclusion of one auction. It displays your true VALUE, the UNCONSTRAINED BID that corresponds to that value, and the ACTUAL BID. It will also give you the ACTUAL BID of your opponent and calculate the winner, the price of the item and your payoff for the round. Note that much of the information about your opponent is hidden.



	You	Opponent
MAX VALUE	60.86	--
VALUE	13.11	--
UNCONSTRAINED BID	14.62	--
CAP	25.00	--
ACTUAL BID	14.62	12.00
Received Item	Yes	No
Price	14.62	N/A
Payoff	-1.51	--

Example

The tables below indicate all the MAX VALUEs, VALUEs and UNCONSTRAINED BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment you will only observe your own MAX VALUE and will *only* know that your VALUE is not higher than your MAX VALUE. You will *not* know the MAX VALUE or VALUE of the other player.

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**59.44**	9.03
CAP	**5.00**	20.50
ACTUAL BID	**5.00**	**9.03**
Received Item	**No**	**Yes**
Price	N/A	9.03
Payoff	**0**	-0.95

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**59.44**	9.03
CAP	**36.00**	20.50
ACTUAL BID	**36.00**	**9.03**
Received Item	**Yes**	**No**
Price	36.00	N/A
Payoff	**20.40**	0

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**59.44**	9.03
CAP	**70.00**	20.50
ACTUAL BID	**59.44**	**9.03**
Received Item	**Yes**	**No**
Price	59.44	N/A
Payoff	**-2.96**	0

In all of the examples above, Player 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 20.50. Player 2's VALUE is 8.08 and therefore, her computerized bidder's UNCONSTRAINED BID is 9.03. However, since 20.50 > 9.03, the ACTUAL BID of Player 2 is her UNCONSTRAINED BID.

Player 1 observes a MAX VALUE of 84.62. Each of the three tables corresponds to a different choice of CAP that Player 1 might have chosen. Player 1's private VALUE is 56.40 (she does not observe this at the time of deciding on the CAP but her computerized bidder knows the VALUE). In the first table, Player 1 chose a cap of 5.00. If you check the provided bidding sheet, you will see that Player 1's computerized bidder's UNCONSTRAINED BID is 59.44. Since 5.00 < 59.44, the ACTUAL BID is 5.00.

In the first example the ACTUAL BIDs are 5.00 and 9.03. Since the highest bid (9.03) is submitted by Player 2, Player 2 obtains the good and pays her ACTUAL BID (9.03). In this period, Player 1 earns zero and Player 2 earns 8.08 – 9.03 = -0.95 ECU. Note that Player 2 loses money in this period because the bidder is unconstrained and allowed to submit a bid that is higher than the true value.

In the second example, everything is the same except Player 1 chose a higher CAP (36.00). Now the ACTUAL BIDs are 36.00 and 9.03 and Player 1 receives the item. The price is equal to her ACTUAL BID (36.00), so Player 1's payoff is 56.40 – 36.00 = 20.40 ECU.

In the third example, Player 1's CAP is now 70.00. The ACTUAL BIDs are now 59.03 and 9.03. Player 1 receives the item for a price of 59.03, so her payoff is 56.40 – 59.03 = -2.96 ECU.

Total Payoffs

At the beginning of today's session you will receive an endowment of 160 ECU which is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings —your earnings from 25 auctions plus your initial endowment. The conversion rate from ECU to dollars is $1 = 20 ECU. In addition to this sum, you will be paid a $7 participation fee.

Are there any questions?

Practice Questions

Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose your MAX VALUE is 76.00ECU. What is your possible VALUE?

a) any value from 0 to 100.00

b) any value from 0 to 76.00

c) any value from 0 to 76.00 but not 76.00

d) any value from 0 to 100.00 but not 76.00

2. Suppose your MAX VALUE is 76.00 and you entered 21.00 as your CAP. Your computerized bidder observed your private VALUE of 69.00. What will your ACTUAL BID be?

a) 69.00

b) 21.00

c) 54.00

d) 76.00

3. Suppose your MAX VALUE is 43.00 and you entered 38.00 as your CAP. Your computerized bidder observed your VALUE of 4.00. What will your ACTUAL BID be?

a) 4.00

b) 4.57

c) 38.00

d) 43.00

4. Suppose Player 1's ACTUAL BID is 26.15 and Player 2's ACTUAL BID is 63.00. Who will obtain the good and what price the winner will pay?

a) Player 1 wins and pays 26.15

b) Player 1 wins and pays 63.00

c) Player 2 wins and pays 63.00

d) Player 2 wins and pays 26.15

5. Suppose Player 1's ACTUAL BID is 31.00 and Player 2's ACTUAL BID is 24.00. Player 1's VALUE for the good was 38, and Player 2's VALUE was 46. What will be the earnings of each player from this period.

a) Player 1 earns 7, Player 2 earns zero.

b) Player 1 earns 7, Player 2 earns 22.

c) Player 1 earns 14, Player 2 earns zero.

d) Player 1 earns zero, Player 2 earns 22.

INSTRUCTIONS FOR SECOND-PRICE SEALED BID AUCTIONS (computerized bidders)

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US$.

- In this experiment, we will run a sequence of auctions in which you will act as the buyer of a fictitious good. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.
- A single good will be auctioned off in each period.
- In each period, a computerized bidder will bid on behalf of you in the auction. We will tell you the bidding rule of the computerized bidder later in these instructions
- If you do not limit your computerized bidder, it will place an UNCONSTRAINED BID which may be higher than the amount you would like it to bid.
- Your task will be to determine a CAP, which is the maximum amount that you allow your computerized bidder to bid.
- The computerized bidder's ACTUAL BID is the lesser of its UNCONSTRAINED BID and the CAP.

Matching in Each Period

In each period, you will be randomly matched with another person in this room. You and that person will participate in the auction. You will never know who the other person is in your auction. You will not be matched with the same person in any two consecutive periods.

Values

At the beginning of each period, each person participating in the auction *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. Your MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. Your MAX VALUE and the MAX VALUE of the other person who participates in the same auction with you are independently determined.

Your VALUE for the good is the amount of ECU the experimenter will give you if you receive the item at the end of the period. You do not know your exact VALUE for the good at the time of decision making. All you know is that your VALUE is a number contained in the interval [0, MAX VALUE]. Therefore, your VALUE is at minimum zero and cannot exceed your MAX VALUE. Again, any number between 0 and your MAX VALUE is equally likely. For example, let's say you receive a MAX VALUE of 45.32. Then your VALUE is uniformly distributed on interval [0, 45.32] and it can be any number less than or equal to 45.32. Let's say your VALUE is 21.00. This means that if you get the good at the end of the period, you will receive 21.00 ECU from us.

To reiterate, MAX VALUE is never higher than 100 and a VALUE is never higher than the corresponding MAX VALUE. Each person in an auction receives independent MAX VALUEs and independent VALUEs. Hence, your VALUE and MAX VALUE are most likely different from your opponent's VALUE and MAX VALUE.

Auction

- Two persons participate in each auction. Each person is represented by a computerized bidder.
- You observe your MAX VALUE for the current period. **Your computerized bidder observes your true VALUE for the good.**
- After observing your MAX VALUE for the period, you need to decide the maximum amount that you will allow the computerized bidder to bid on your behalf. We call this amount your "CAP".
- You have been given two sheets explaining the bidding rule of your computerized bidder for each possible VALUE (unless you restrict it by a CAP). These two sheets provide you with the same information in two formats: one is a table, and one is a graph. The bids on the sheets are referred to as the computerized bidder's UNCONSTRAINED BID. Please take a look at these sheets and confirm that when your VALUE is, for example, 22.00 ECU, the UNCONSTRAINED BID will be 55.00.
- The computerized bidder's ACTUAL BID is the lesser of its UNCONSTRAINED BID and the CAP:

ACTUAL BID = minimum {UNCONSTRAINED BID , CAP}

- After both your and the other player's ACTUAL BIDs are submitted, the one who has **the highest ACTUAL BID obtains the good** and **pays her opponent's ACTUAL BID**. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

Earnings in a Period

If you obtain the good at the end of the period (if your ACTUAL BID is the highest), then you will receive your VALUE for the good and you will pay your **opponent's ACTUAL BID**. If you did not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

Earnings = Your VALUE – Opponent's ACTUAL BID (If you obtained the auctioned good);

Earnings = 0 (If you did not obtain the auctioned good).

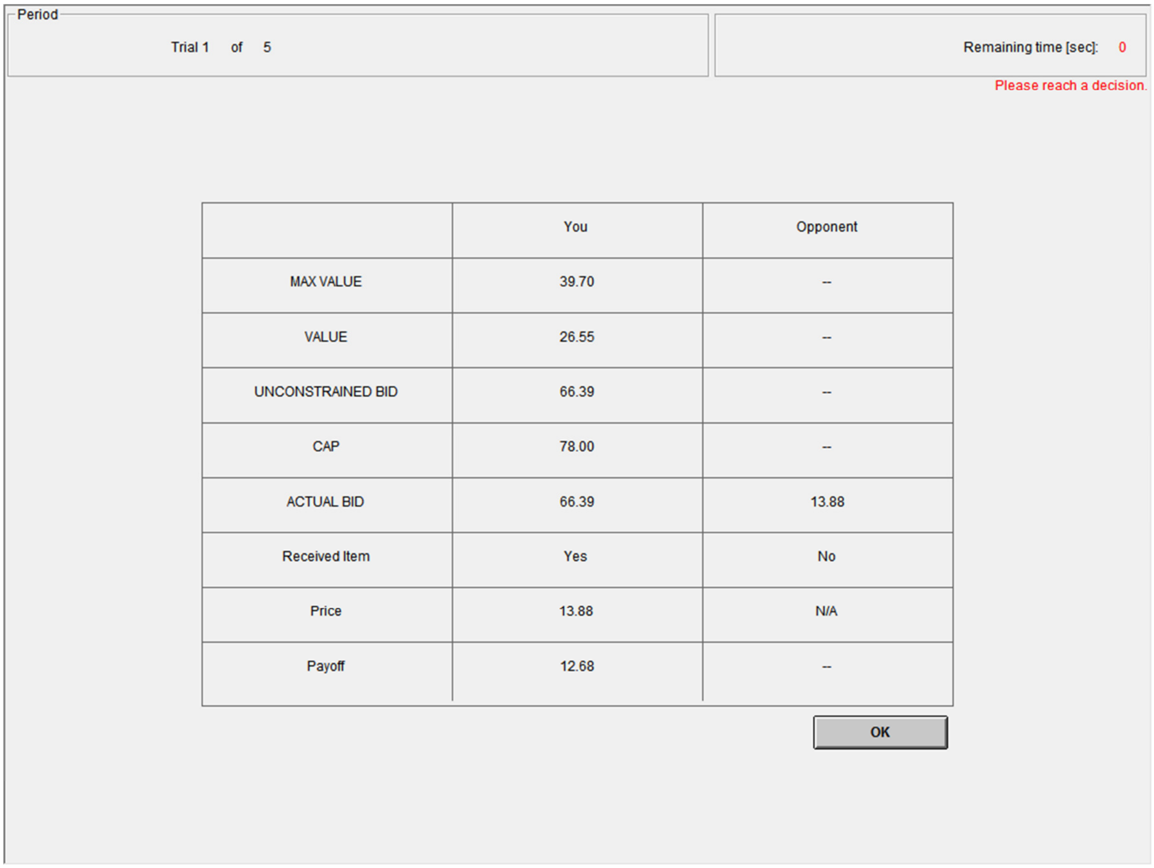
When the current period is over, the next period will start. Each period, you will be randomly matched with a new player and receive a new MAX VALUE. Therefore, your VALUE of the good in each period is independent of your VALUE in the previous periods.

Screens

Below, there is an example of how your screen may look. On top of your screen there is an interactive tool. The tool shows your MAX VALUE for the current period. Remember that your VALUE can be any number between zero and your MAX VALUE, but you do not know what it is. However, your computerized bidder knows your VALUE and bases its UNCONSTRAINED BID on your VALUE. By sliding the little black square between zero and your MAX VALUE, you may see the UNCONSTRAINED BID of your computerized bidder for the corresponding VALUE. This tool provides you with the exact same information as you may learn from the UNCONSTRAINED BID table or graph that we have provided to you. Please use whichever tool that you prefer in order to understand how the computerized bidder bids unless it is restricted by a CAP.

You need to enter your bidder's CAP for this period in the text box on your screen and click on SUBMIT. Remember that your ACTUAL BID will be what your computerized bidder's UNCONSTRAINED BID is **unless** you restrict it by a CAP, in which case it will be the lesser of the two numbers.

The screen below is an example of the results screen after the conclusion of one auction. It displays your true VALUE, the UNCONSTRAINED BID that corresponds to that value, and the ACTUAL BID. It will also give you the ACTUAL BID of your opponent and calculate the winner, the price of the item and your payoff for the round. Note that much of the information about your opponent is hidden.



	You	Opponent
MAX VALUE	39.70	--
VALUE	26.55	--
UNCONSTRAINED BID	66.39	--
CAP	78.00	--
ACTUAL BID	66.39	13.88
Received Item	Yes	No
Price	13.88	N/A
Payoff	12.68	--

Example

The tables below indicate all the MAX VALUEs, VALUEs and UNCONSTRAINED BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment you will only observe your own MAX VALUE and will *only* know that your VALUE is not higher than your MAX VALUE. You will *not* know the MAX VALUE or VALUE of the other player.

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**141.00**	20.20
CAP	**10.00**	24.50
ACTUAL BID	**10.00**	**20.20**
Received Item	**No**	**Yes**
Price	N/A	10.00
Payoff	**0**	-1.92

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**141.00**	20.20
CAP	**48.10**	24.50
ACTUAL BID	**48.10**	**20.20**
Received Item	**Yes**	**No**
Price	20.20	N/A
Payoff	**36.20**	0

	Player 1 (YOU)	Player 2
MAX VALUE	**84.62**	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	**141.00**	20.20
CAP	**90.00**	24.50
ACTUAL BID	**90.00**	**20.20**
Received Item	**Yes**	**No**
Price	20.20	N/A
Payoff	**36.20**	0

In all of the examples above, Player 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 24.50. Player 2's VALUE is 8.08 and therefore, her computerized bidder's UNCONSTRAINED BID is 20.20. However, since 24.50 > 20.20, the ACTUAL BID of Player 2 is her UNCONSTRAINED BID.

Player 1 observes a MAX VALUE of 84.62. Each of the three tables corresponds to a different choice of CAP that Player 1 might have chosen. Player 1's private VALUE is 56.40 (she does not observe this at the time of deciding on the CAP but her computerized bidder knows the VALUE). In the first table, Player 1 chose a cap of 10.00. If you check the provided bidding sheet, you will see that Player 1's computerized bidder's UNCONSTRAINED BID is 141.00. Since 10.00 < 141.00, the ACTUAL BID is 10.00.

In the first example the ACTUAL BIDs are 10.00 and 20.20. Since the highest bid (20.20) is submitted by Player 2, Player 2 obtains the good and pays her **opponent's** ACTUAL BID (10.00). In this period, Player 1 earns zero and Player 2 earns 8.08 – 10.00 = -1.92 ECU. Note that Player 2 loses money in this period because the bidder is unconstrained and allowed to submit a bid that is higher than the true value.

In the second example, everything is the same except Player 1 chose a higher CAP (48.10). Now the ACTUAL BIDs are 48.10 and 20.20 and Player 1 receives the item. The price is equal to her **opponent's** ACTUAL BID (20.20), so Player 1's payoff is 56.40 – 20.20 = 36.20 ECU.

In the third example, Player 1's CAP is now 90.00. The ACTUAL BIDs are now 90.00 and 20.20. Player 1 receives the item for a price of 20.20, so her payoff is 56.40 – 20.20 = 36.20 ECU.

Total Payoffs

At the beginning of today's session you will receive an endowment of 160 ECU which is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings —your earnings from 25 auctions plus your initial endowment. The conversion rate from ECU to dollars is $1 = 20 ECU. In addition to this sum, you will be paid a $7 participation fee.

Are there any questions?

Practice Questions

Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose your MAX VALUE is 76.00ECU. What is your possible VALUE?

a) any value from 0 to 100.00

b) any value from 0 to 76.00

c) any value from 0 to 76.00 but not 76.00

d) any value from 0 to 100.00 but not 76.00

2. Suppose your MAX VALUE is 76.00 and you entered 21.00 as your CAP. Your computerized bidder observed your private VALUE of 69.00. What will your ACTUAL BID be?

a) 69.00

b) 21.00

c) 54.00

d) 76.00

3. Suppose your MAX VALUE is 43.00 and you entered 38.00 as your CAP. Your computerized bidder observed your VALUE of 4.00. What will your ACTUAL BID be?

a) 4.00

b) 10.00

c) 38.00

d) 43.00

4. Suppose Player 1's ACTUAL BID is 26.15 and Player 2's ACTUAL BID is 63.00. Who will obtain the good and what price the winner will pay?

a) Player 1 wins and pays 26.15

b) Player 1 wins and pays 63.00

c) Player 2 wins and pays 63.00

d) Player 2 wins and pays 26.15

5. Suppose Player 1's ACTUAL BID is 31.00 and Player 2's ACTUAL BID is 24.00. Player 1's VALUE for the good was 38, and Player 2's VALUE was 46. What will be the earnings of each player from this period.

a) Player 1 earns 7, Player 2 earns zero.

b) Player 1 earns 7, Player 2 earns 22.

c) Player 1 earns 14, Player 2 earns zero.

d) Player 1 earns zero, Player 2 earns 22.

B. CHAPTER FOUR SUPPORTING MATERIALS

This appendix contains proofs of two of the propositions in Chapter 4.

Proof of Proposition 1: Suppose that the firms adopt the strategy described in the statement of the proposition. First note that from assumptions A2 and A3 and because the cost parameters are identically distributed, $b_A(c) = b_B(c)$. So if the bidders follow this strategy the two highest bids will correspond to the two lowest draws from $F(c)$. Now consider the problem of an A bidder who submits a bid z when the other bidders follow the prescribed strategies (the problem is symmetric for a B bidder). Since the lowest of the opposing cost draws determine the bidder's value, we can restrict attention to events where $c_A^{(1)} < c_B^{(1)}$ and $c_A^{(1)} > c_B^{(1)}$. The first event occurs with probability $\frac{N-1}{2N-1}$ (because there are $N-1$ out of $2N-1$ permutations of the bidders where an opposing A receives the lowest cost and each permutation is equally likely), while the second occurs with probability $\frac{N}{2N-1}$. This implies that an A bidder's expected valuation conditional on winning is given by $b_A(c)$. The important point is that this expected valuation conditional on winning is completely determined by the behavior of the opposing bidders and the bidders cost (not his report). The proposition then follows from the standard Vickrey logic.

There cannot be another symmetric equilibrium. Suppose there were and consider the problem of a given A bidder. If the other bidders use a symmetric strategy then by the argument above the bidder's expected value conditional on winning is still $b_A(c)$, so in any proposed symmetric equilibrium the bidder can improve his payoff by bidding $b_A(c)$.

Proof of Proposition 2: Optimality of the second round strategies is obvious, because in the second round the auction corresponds to a second price auction for a single good with independent private values.

In the definitions of the events $E_1(c_A), E_2(c_A), E_3(c_A), E_4(c_A), E_5(c_A)$, I show all the possible orderings of profit functions conditional on c_A being the second lowest cost signal. Now consider the problem of an A bidder in the first round (the argument for a B bidder is analogous) facing opposing bidders using a symmetric increasing strategy. The highest of the opponent's bid then corresponds to the lowest of the opponents' cost signals. So in $E_1(c_A)$, for example, an opposing A has the highest opposing bid in the first round.

The bidding strategy described in the statement of the proposition is then a bidder's expected payment conditional on losing in the first round (in equilibrium) and winning in the second round. Note that this is not exactly correct as in E_3 the bidder also loses in the second round, but losing gives the same payoff as winning and paying one's full value.

The key observation in the proof is that if the opponents all follow a symmetric strategy in the first round, then bid their (known) value in the second round, an A bidder's expected payment conditional on advancing to the second round and winning in that round is not affected by his bid in the first round (because the ordering of the signals does not change). This expected second round payment then determines his willingness to pay in the first round.

Suppose that the bidder bids above his expected payment and wins in the first round when the realizations were such that he would have won in equilibrium in the second round. In the event that $z > b_A(c_A^{(1)}) > b_A(c_A)$ or $z > b_B(c_B^{(1)}) > b_A(c_A)$, he has to pay an amount above $b_A(c_A)$ in the first round when he could have increased his expected payoff by waiting until the second round (expecting to pay $b_A(c_A)$). Lowering his bid in the first round would have no impact on his payoff in this case.

If the realizations were such that the bidder wins in the first round, then raising his bid has no effect on his expected payoff. If he lowers his bid and loses to some bid $b_A(c_A^{(1)})$ (the argument is the same if he lost to $b_B(c_B^{(2)})$), then he can infer that his expected payment in the second payment in the second round is (by symmetry of the bidders' expectations) $b_A(c_A^{(1)})$, so that he could have received the same expected payment by following equilibrium and winning in the first round.

If the realized signals were such that the bidder does not win in either round, the bidder would expect a negative payoff by bidding above $b_A(c_A)$ in the first round and winning because the price would necessarily be greater than $b_A(c_A)$ and from this the bidder could infer that the expected price in the second round is at least as high as the bidder's actual value in expectation. Obviously, lowering the bid has no effect on the payoff in this case.

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