ABSTRACT

Title of dissertation: ESSAYS ON SOVEREIGN DEBT CRISES

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This study analyzes two aspects of sovereign debt crises: first, the relationship between banking crises and sovereign defaults, second, how the debt dilution phenomenon affects sovereign default risk.

Episodes of sovereign default feature three key empirical regularities in connection with the banking systems of the countries where they occur: (i) sovereign defaults and banking crises tend to happen together, (ii) commercial banks have substantial holdings of government debt, and (iii) sovereign defaults result in major contractions in bank credit and production. The first essay provides a rationale for these phenomena by extending the traditional sovereign default framework to incorporate bankers that lend to both the government and the corporate sector. When these bankers are highly exposed to government debt a default triggers a banking crisis which leads to a corporate credit collapse and subsequently to an output decline. When calibrated to Argentina's 2001 default episode the model produces

default on equilibrium with a frequency in line with actual default frequencies, and when it happens credit experiences a sharp contraction which generates an output drop similar in magnitude to the one observed in the data. Moreover, the model also matches several moments of the cyclical dynamics of macroeconomic aggregates.

In the second essay we measure the effects of debt dilution on sovereign default risk and show how these effects can be mitigated with debt contracts promising borrowing-contingent payments. First, we calibrate a baseline model à la Eaton and Gersovitz (1981) to match features of the data. In this model, bonds' values can be diluted. Second, we present a model in which sovereign bonds contain a covenant promising that after each time the government borrows it pays to the holder of each bond issued in previous periods the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. This covenant eliminates debt dilution by making the value of each bond independent from future borrowing decisions. We quantify the effects of dilution by comparing the simulations of the model with and without borrowing-contingent payments. We find that dilution accounts for 84% of the default risk in the baseline economy. Similar default risk reductions can be obtained with borrowing-contingent payments that depend only on the bond market price. Using borrowing-contingent payments is welfare enhancing because it reduces the frequency of default episodes.

ESSAYS ON SOVEREIGN DEBT CRISES

by

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Dissertation submitted to the Faculty of the Graduate School of the
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of the requirements for the degree of
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Dedication

Para mi amor, Flopy.

Para mis otros *amores* Menchi, Doc, Berni, Pipo, Luli y Mamapata.

Acknowledgments

I owe my gratitude to all the people who have made this thesis possible and made my years at Maryland ones that I will always remember.

My advisors Enrique Mendoza, Carmen Reinhart and Pablo D'Erasmo provided great support and advice throughout these years. I initially started working with Carmen and soon Enrique and Pablo became involved in my dissertation. From them I learned how to do research in policy relevant issues with academic rigor and honesty. The three of them devoted many hours (on and off campus) in listening to me, discussing my progress and reading my drafts. Additionally, Carmen and Enrique provided financial support through their own research grants. The way I see economics has for ever been shaped by their example. I am immensely grateful to them. I wish I had better words to thank them, I just hope they know how much their support meant to me.

I thank Carlos Vegh for generously being part of my committee and also for insightful discussions about my research at earlier stages. Phillip Swagel gratefully accepted the invitation to be in my dissertation committee and I would like to thank him as well.

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Many institutions provided both financial and scientific support during the process of my dissertation. The Department of Economics at UMD provided financial support for five years. The Research Department at the Federal Reserve Bank of Richmond hosted me during the 2009 summer providing not only financial support but also an stimulating environment from which I benefited extensively. The America's center at the Federal Reserve Bank of Atlanta hosted me during the summer of 2010: the center gave me generous resources and Stephen Kay and Federico Mandelman were very welcoming and supportive of my research during my time in Atlanta. The Research Department at the International Monetary Fund was my residence during the summer of 2011: I benefited a lot from my discussions with Giovanni Favara and Rex Ghosh. Finally, the IMF Institute hosted me several times between 2010 and 2011 when I visited and worked on what is today the third chapter of my dissertation. To all of these people and institutions I am greatly thankful.

I want to also express my gratitude to my co-authors Juan Carlos Hatchondo and Leonardo Martinez. I learned a lot from them: they listened patiently at my research ideas and were crucial in shaping my understanding of sovereign default models. Both have proved to be great colleagues and honest friends. I look forward to many more years of research collaboration.

My interests in economics started when I was an undergrad student at Universidad Nacional de Tucuman, in Argentina. I had many very good professors there but Hugo Ferullo, Osvaldo Meloni, and specially Victor Elias deserve a special gratitude. They encouraged and helped me to go abroad and pursue graduate studies. Victor Elias was my mentor in Tucuman and he is still a source of consultation at every important decision in my career. My indebtness with him will never be paid off.

None of this could have been possible without the constant support of my friends at Maryland. I had the pleasure of becoming friends (and colleagues) with Abby Alpert, Juan Diego Bonilla, Pablo M. Federico, Teresa Fort, Carolina Gonzalez-Velosa, Daniel Hernaiz, Gabriel Lara, Antonio Lemus, Tim Moore, Rong Qian, Agustin Roitman, and M. Belen Sbrancia. Dani and Gabo (the "Chateau-boys") proved to be true friends and great room-mates, they were my family away from Tucuman (my hometown). Carito and Juandi were always there, through thick and thin and together with Abby and the Chateau-boys conformed the "Pop-Center family". I am very thankful to them for celebrating my successes as their owns and always be willing to waste some time talking about nothing, or going out for dinner, or having hours-long lunches. Pablo was also a great friend and room-mate. He spent many hours talking about life and economics with me and I look forward to many more. Antonio, Agustin and Rong became friends with me from the very beginning of my time at Maryland. With them I traveled, studied, talked about research and also relaxed and enjoyed life. Teresa, Tim and Belen were great friends, I am happy to have them in my life.

Of course I want thank my family. My wife Florencia was always there... in good times and bad. She was next to me from the early stages of my Ph. D. and always supported me with loving patience. She believed in me before I did. My parents (Carmen Rosa and Cesar) and my brothers (Bernardo and Augusto) loved me in the distance and encouraged me at every step in the process. I am sure I could never have finished without them in my life... thank you!

Above all I want to thank God.

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Chapter 1

Introduction

The ongoing crisis has generated a renewed attention to sovereign debt issues. In particular the interest seems to be two-folded: on the one hand, we want to understand the potential consequences of sovereign defaults. On the other hand, there is also a growing interest in how to design effective mechanisms to avoid (or dampen) these negative consequences.

This dissertation consists of two essays, each focusing on one of the above mentioned issues. The first essay studies the relationship between sovereign defaults, banking sector performance and economic activity. This can therefore be regarded as providing partial answers to the question ”what are the consequences of a sovereign default?”. The second essay takes on the topic of debt dilution, explains how the *dilution problem* generates higher financing costs for sovereigns and proposes a mechanism to ameliorate this problem. It can then be understood as a step forward in designing mechanisms to dampen the consequences of defaults.

The remainder of this introductory chapter is as follows: the first section overviews the analysis and results exposed in ”Sovereign Defaults and Banking Crises” (Chapter 2), and the second section summarizes the exposition in ”Debt Dilution and Sovereign Default Risk” (Chapter 3).

1.1 Sovereign Defaults and Banking Crises

Sovereign defaults and banking crises have been recurrent in emerging economies. Recent default episodes (e.g. Russia 1998, Argentina 2001) have shown that whenever the sovereign decides not to repay there is an adverse impact on domestic economies to a large extent through disruptions to the domestic financial systems. Why does this happen? Both in the Argentine and Russian experiences the banking sectors where highly exposed to government debt. In this way a government default directly decreased the value of the baking sector's assets. This forced banks to reduce credit to the domestic economy (a credit crunch) and in turn generated a decline in economic activity.

The current debt crisis in Europe also highlights the relationship between sovereign defaults, banking crises and economic activity. Most of the worries around Greece's possible default (or unfavorable restructuring) are related to the level of exposure that banking sectors in Greece and other European countries have to Greek debt. The concerns are not only on loosing what was invested in Greek bonds but mostly on how this shock to banks assets will undermine their lending ability and ultimately the economic activity as a whole.

This leads to the realization that sovereign default episodes can no longer be understood as events in which the defaulting country suffers mainly from international exclusion and trade punishments. The motivation above, the empirical evidence presented later on and the policy discussions (see IMF (2002b)) make it clear that we need to shift the attention to domestic financial sectors and how they channel the adverse effects of a default to the rest of the economy.

Grounded on three key regularities, namely that (i) defaults and banking crises tend to happen together, (ii) banks are highly exposed to government debt, and (iii) crises episodes are costly in terms of credit and output; we construct a theory that links defaults, banking sector performance and economic activity. This chapter provides a rationale for these phenomena by extending the traditional sovereign default framework to incorporate bankers that lend to both the government and the corporate sector. When these bankers are highly exposed to government debt a default triggers a banking crisis which leads to a corporate credit collapse and consequently to an output decline.

These dynamics that characterize a default and a banking crisis are obtained as the optimal response of a benevolent planner: faced with a level of spending that needs to be financed, and having only two instruments at hand (i.e. debt and taxes), the planner may find it optimal to default on its debt even at the expense of decreased output and consumption. The planner balances the costs and benefits of a default: the benefit is the lower taxation needed to finance the spending (lower than would otherwise have been necessary), the cost is the reduced credit availability and the consequently decreased output. Quantitative analysis of a version of the model calibrated to Argentina's 2001 default yields the following main findings:

1. First, we obtain default in equilibrium. One may argue that this is a trivial result given that we have a paper about defaults, but in this case it is not a trivial outcome. Our model economy is flexible enough to capture the case in

which all (or a fraction of) the debt is domestically held. In this scenario the typical reasoning in defaults models doesn't apply: here a default does not imply a transfer of resources from abroad. Here the ones that take the hit are the domestic bankers. In this way a default has distributional consequences in my model: when a default comes, bankers' assets lose value, but at the same time the funds that government need to raise as tax revenues decrease (due to having walked away on the debt) and therefore the tax rate charged over households' labor income is also reduced.

2. Second, the model exhibits a v-shaped behavior of output and credit, as observed in the data. It is important to highlight here that both the behavior of output and credit are endogenous results of the model (in contrast to widely assume endowment economies). Moreover it is the credit decline which generates the output decline, and this is the main (and most novel) mechanism of the paper. One can argue that government debt has a "liquidity" role in our framework: it increases the available funds in the economy, and therefore reduce the borrowing cost for firms. When defaults come, this role cannot be performed and then borrowing costs for firms increase. This is the way in which we obtain an endogenous cost of defaults: a default generates decreased lending ability of the bankers, which means a contraction in the supply of funds, which ends up with a higher borrowing cost for the corporate sector, and an output contraction.

3. Third, on a quantitative note, the model is able to generate an output decline

at default of roughly 9% (in deviation from trend), which is very close to the observed 10%. This is obtained at a default frequency 1.60% and at a Mean debt to output (16%) and mean exposure ratio (26%) matching the Argentine evidence. Overall the model does a very good job in accounting for business cycle regularities of Argentina and other emerging economies.

Yet another way of interpreting the contribution of this first essay is the following: the vast majority of the sovereign default literature has assumed exogenous cost-of-default structures. Most of the time a predetermined fraction of output is lost, and sometimes some exclusion from financial markets is assumed (reputational loss). If we are to take seriously these models and their predictions, then it is a desirable straightforward next step to put efforts to endogenize the costs of a default.

This is exactly what we do in the first essay, and we do it by extending the standard sovereign default framework to incorporate bankers that lend to both the government and the corporate sector.

1.2 Debt Dilution and Sovereign Default Risk

This essay is joint work with Juan Carlos Hatchondo and Leonardo Martinez, and constitutes the third chapter of the dissertation. This chapter measures the effects of debt dilution on sovereign default risk. The debt dilution problem appears because borrowers do not have the ability to commit to not dilute the market price of current debt issuances with future issuances. In the essay, the price of sovereign bond decreases (i.e., is diluted) when the government issues new bonds. If governments could commit to not dilute the price of current bond issuances with future issuances, this would allow them to sell bonds at a higher price. But it is well understood that governments have limited ability to make such commitment. Thus, dilution has received considerable attention in both academic and policy discussions. While previous studies suggest that debt dilution may be an important source of inefficiencies in debt markets, they do not quantify the effects of dilution.

First, the essay presents a calibrated sovereign default model (Eaton and Gersovitz (1981)) with dilution, which matches features of the data. Second, the essay modifies this baseline model by assuming that, when the sovereign issues debt, it pays to the holder of each existing bond the difference between the post-issuance bond price and the counterfactual bond price one would have observed without issuances. With this compensation, bond prices do not decline with debt issuances and thus, the debt dilution problem disappears. The essay quantifies the effects of dilution by comparing the simulations of the baseline and modified models. The essay shows that if the sovereign could eliminate dilution, the probability of default would decrease 84%.

The compensation used to measure dilution in our study may be difficult to implement in reality because it is determined using a counterfactual price. However, the essay shows that the gains from eliminating dilution could be obtained with simpler compensation schemes that could be implemented through debt contract covenants. For instance, most gains from eliminating dilution can be obtained by compensating bondholders with a predetermined share of debt issuance revenues. Furthermore, larger welfare gains can be obtained through lower default risk (at the expense of higher consumption volatility) by making the compensation to bond holders decreasing with respect to the post-issuance bond price. The analysis in this essay is also relevant for the study of other credit markets where the debt dilution problem is present.

Chapter 2

Sovereign Defaults and Banking Crises

2.1 Introduction

Sovereign defaults and Banking Crises have been recurrent in emerging economies. Recent default episodes (e.g. Russia 1998, Argentina 2001) have shown that whenever the sovereign decides not to repay there is an adverse impact on domestic economies to a large extent through disruptions to the domestic financial systems. Why does this happen? Both in the Argentine and Russian experiences (and also in others discussed below) the banking sectors where highly exposed to government debt. In this way a government default directly decreased the value of the baking sector's assets. This forced banks to reduce credit to the domestic economy (a *credit crunch*) and in turn generated a decline in economic activity.

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default in equilibrium, (2) v-shaped behavior of output and credit around crises episodes, (3) mean output decline in default episodes of approximately 8%, and (4) overall qualitative behavior of the model is in line with the business cycle regularities observed in Argentina and other emerging economies.

Quantitative models of sovereign default have received increased attention since the contributions of Aguiar and Gopinath (2006) and Arellano (2008). These papers extended the seminal framework of Eaton and Gersovitz (1981) to account for business cycle regularities in emerging economies. A large literature has emerged following this approach and several interesting aspects of the dynamics of sovereign debt have been studied recently.[1] The optimal default decision examined in most models in this literature comes from weighing the costs and benefits of default. The majority of the literature has assumed an exogenous cost-of-default structure.[2] This chapter proposes a channel to endogenize these costs via the role of government debt in the domestic credit market.

Mendoza and Yue (forthcoming) were the first to introduce endogenous costs of default. They assume that a sovereign default not only excludes the government from the international markets but also prevents the private sector firms from tapping foreign markets. In this way, a sovereign default forces the productive sector to use less efficient resources and hence generates an output cost. This chapter departs from Mendoza and Yue (forthcoming) in two relevant ways: first, it presents a model

[1]Yue (2010) studies debt defaults and renegotiations, D'Erasmo (2011) incorporates government reputation and endogenous default resolutions, Hatchondo and Martinez (2009) extend the model to incorporate long duration bonds, Hatchondo et al. (2011) study the effects of debt dilution, among others. Stahler (2011) presents a through account of the recent developments in this literature.

[2]See, for example, Chatterjee and Eyigungor (2011).

economy flexible enough to accommodate both external and domestic debt: when debt is at least partially domestically held default is a less attractive option (domestic residents' assets become worthless) and therefore makes *default-on-equilibrium* a more challenging outcome. Second, it acknowledges the high prevalence of government debt in banks' balance sheets and illustrates how a sovereign default diminishes credit availability in the economy.

Other researchers have recently and independently noticed the exposure of the domestic banking sector to government debt and have asked different questions about this phenomenon. Gennaioli et al. (2010) construct a stylized model of domestic and external sovereign debt where domestic debt weakens the balance sheet of banks. This potential damage suffered by the banking sector represents in itself a 'signaling' device that attracts more foreign lenders.[3] They understand the banks' exposure as a measure of financial institutions quality in the economy and derive a number of testable implications that they investigate empirically. Our analysis relates to Gennaioli et al. (2010) in that it also identifies the damage that financial institutions suffer when defaults come. We identify this reduced credit as the endogenous mechanism generating costs of defaults and also analyze the benefits side: how distortionary taxation changes when defaults happens. Additionally we develop a stochastic dynamic general equilibrium model that is able to account for a number of empirical regularities in emerging economies.

Bolton and Jeanne (2011) have an interesting paper on sovereign defaults and

[3]Basu (2009) develops a similar model that features domestic and foreign creditors and where domestic economic fragility allows the sovereign to borrow from international markets. Alessandro (2009) presents a related model in which the sovereign default increases the borrowing costs to domestic firms.

bank fragility in a model of contagion between financially integrated economies. With a stylized three-period model they show that financial integration without fiscal integration may result in an inefficient (from a global perspective) supply of government debt. This chapter relates to them in that both highlight the exposure of the banking sector to government debt and that sovereign defaults generate contractions in private credit. We depart from Bolton and Jeanne (2011) in that their focus is mostly on advance economies and contagion among them, whereas our chapter focuses on emerging economies. This chapter also differs on the optimal policy treatment of the default decision and the quantitative analysis of the business cycle behavior of models of sovereign defaults and banking.[4]

This chapter is also related to the optimal taxation literature. The closest paper to ours in this literature is Pouzo (2008). He builds on the work of Aiyagari et al. (2002) to analyze the optimal taxation problem of a planner in a closed economy with defaultable debt. This chapter differs from Pouzo (2008) in three crucial aspects: first, Pouzo (2008) relies on an exogenous cost of default whereas we propose an endogenous structure; second, Pouzo (2008) restricts the analysis to a closed economy setting (and therefore to domestic debt) whereas the environment to be studied here is flexible enough to accommodate both domestic and external debt; and third (on a more technical note), Pouzo (2008) solves for an equilibrium in which the government has commitment to a certain tax schedule but not to a repayment policy, whereas our analysis assumes no commitment on the side of the

[4]Yet another related work is the one by Livshits and Schoors (2009). They argue that defaults generate banking crises because of an inadequate prudential regulation which does not recognize the riskiness of the debt.

government.[5]

The contribution of this chapter is two-folded. First, it presents a theory of the transition mechanism of sovereign defaults to the banking sector and the rest of the economy. The existing literature on quantitative models of sovereign defaults has remained silent about the relationship of sovereign debt, default and the performance of the banking sector, this chapter fills this gap. Second, on a methodological note, this chapter presents a competing mechanism to endogenize the costs of sovereign defaults: a sovereign default generates a credit crunch, and this credit crunch generates output declines.

The remainder of the chapter is structured as follows. Section 2 presents stylized facts on sovereign defaults, banking crises and output and credit drops. Section 3 lays out the model and defines the equilibrium. Section 4 contains the numerical solution and the main results of the chapter. Section 5 has a conclusion and describes avenues for future research.

2.2 Stylized Facts

This section documents the stylized facts that motivate the theoretical/quantitative analysis presented in the rest of the chapter. It begins by describing the time clustering of default crises and banking crises. Then, it examines the exposure of the emerging economies' banking sectors to government debt. Finally, it presents the output and credit behavior around default episodes.

[5]This chapter is also related to the literature on optimal public policy without commitment. These papers solve for Markov-Perfect (and therefore, time-consistent) optimal policy. See Klein et al. (2008) and references therein.

	Default in period t
Banking Crises in:	
$t-2$ or $t-1$	14 (36%)
t	13 (33%)
$t+1$ or $t+2$	12 (31%)

Table 2.1: Timing of Defaults and Banking Crises.

Default and Banking crises tend to happen together. We follow Reinhart and Rogoff (2009) classification of banking crises to identify those crises that occurred in the temporal vicinity of a sovereign default. Of the 82 banking crises episodes documented in Reinhart (2010), 70 were coupled with default crises.[6] From those 70 episodes we only consider crises after 1970 (due to data limitations) and we identify those in which the sovereign default either preceded or coincided with the banking crisis: Table 2.1 shows that this occurred in 25 out of 39 post 1970 events.[7]

The relationship between Banking crises and Default episodes has been previously studied in the empirical literature. In particular, the question of whether a default causes a banking crisis or viceversa has been recently studied by Borensztein and Panizza (2008). They construct an index of banking crises that includes 149 countries for the period 1975-2000. In this sample they identify 111 banking

[6]They follow Demirgüç-Kunt and Detragiache (1998) in defining as a banking crisis any episode in which at least one of the following criteria is true: (1) The ratio of non-performing assets to total assets in the banking system exceeds 10 percent, (2) The cost of rescue operation was at least 2 percent of GDP, (3) Banking sector problems resulted in a large scale nationalization of banks, (4) Extensive bank runs took place or emergency measures (e.g. deposit freezes, prolonged bank holidays, generalized deposit guarantees) were enacted by the government. The mechanism highlighted in this chapter is closely related to (1).

[7]This timing of events, with the banking crisis occuring after or at the same time as the default event is consistent with the one we will assume in the model of the next Section, and hence the significance of documenting that this was the timing observed in about 2/3 of the actual twin default/banking crises.

Unconditional Prob. of a banking crisis	2.9
Prob. of banking crisis conditional on default	14.1
p-value on the test: $prob(bc/def) > prob(bc)$	0.0
Unconditional Prob. of a sovereign default	2.2
Prob. of default conditional on banking crisis	4.5
p-value on the test: $prob(def/bc) > prob(def)$	0.1

Table 2.2: Probabilities of Defaults and Banking Crises. From Borensztein and Panizza (2008)

crises (implying an unconditional probability of having a crisis equal to 2.9%) and 85 default episodes (unconditional default probability of 2.2%). Their results are summarized in the Table 2.2.

When conditioning on a sovereign default episode, the probability of a banking crisis increases by a factor of 5 and this conditional probability is statistically significant from the unconditional one (as denoted by a 0.0 p-value).

It can be argued that a banking crises can generate additional government spending (for example in the form of bailouts) that would make the sovereign more prone to a default. It is then imperative to examine the probability of having a default conditional on experiencing a banking crisis: this probability is only 2 percentage points higher than the unconditional probability and it is not statistically significant (at either the 1% and the 5% confidence levels) from the unconditional one.[8]

[8] A more recent empirical study on banking crises and sovereign defaults is the one by Balteanu et al. (2011). Using the date of sovereign debt crises provided by Standard & Poor's and the systemic banking crises identified in Laeven and Valencia (2008) they end up with a sample containing 121 sovereign defaults and 131 banking crises for 117 emerging and developing countries from 1975 to 2007. Among them they identify 36 "twin crises" (defaults and banking crises): in 17 of them a banking crisis preceded the sovereign default and in 19 the reverse was true.

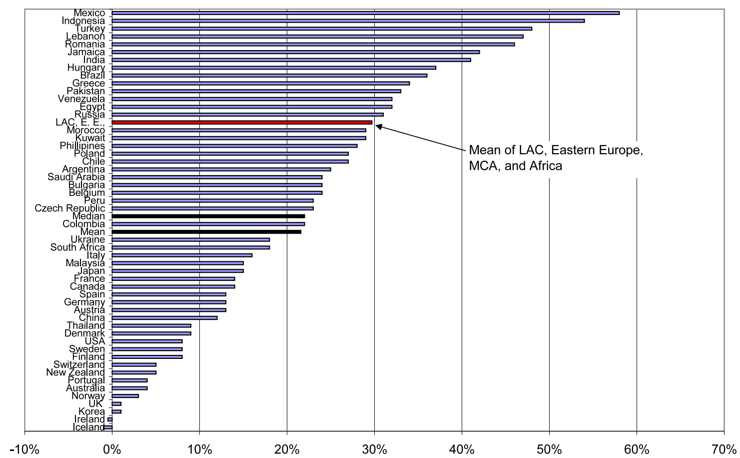

Figure 2.1: Banking Sector exposure to Gov't debt.

Even tough these results should be taken with a grain of salt, they suggest that a default may increase the probability of a banking crisis much more then the other way around. Overall, the evidence presented provides support for the assumed timing in the model.

The banking sector exposure to government debt. In this section we take a look at the degree of exposure of banks to government debt.[9] To do this we follow Kumhof and Tanner (2005) and define an *exposure ratio* in the following way:

$$\frac{\text{Financial Institutions' net credit to the gov't}}{\text{Financial Institutions' net total assets}}$$

As Figure 2.1 documents, this exposure ratio averages 30% for emerging economies. What is even more compelling is that for countries that actually defaulted (like Russia and Pakistan) this percentage was even higher.

[9]Between 2001 and 2002 in Argentina, considerations on the harm the banking system would take were in the front row of the discussions that eventually lead to the default decision. See Perry and Servén (2003) and Kumhof (2004).

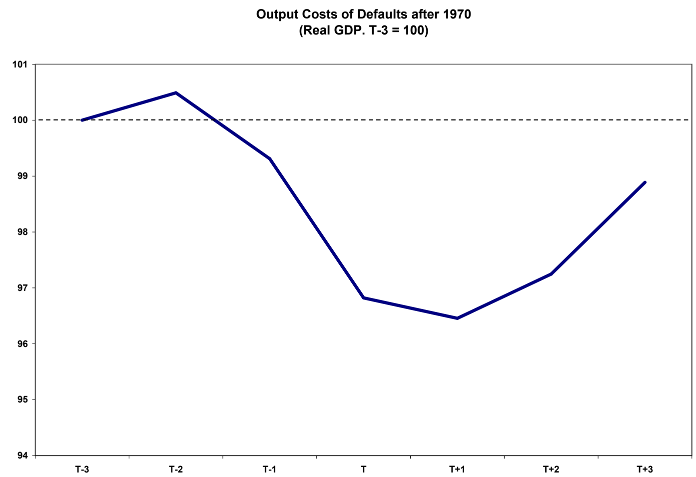

Figure 2.2: Costly Defaults and Banking Crises.

Crises episodes are episodes of decreased output and credit. It has been documented that output falls sharply in the event of a sovereign default (see for example Sturzenegger and Zettelmeyer (2005)). Figure 2.2 makes the same point in a visual way. We observe a v-shape behavior of GDP in the temporal vicinity of defaults.

Default and banking crises are also characterized by decreased credit to the private sector. To document this fact we use the Financial Structure Dataset constructed by Beck et al. (2009) to look at the behavior of *Private credit by deposit money banks and other financial institutions* around defaults and banking crises. Figure 2.3 plots this measure as a percentage of GDP and shows that when a default comes private credit shrinks and remains reduced for the subsequent periods.

To summarize the set of facts just reviewed: (1) Default and Banking crises

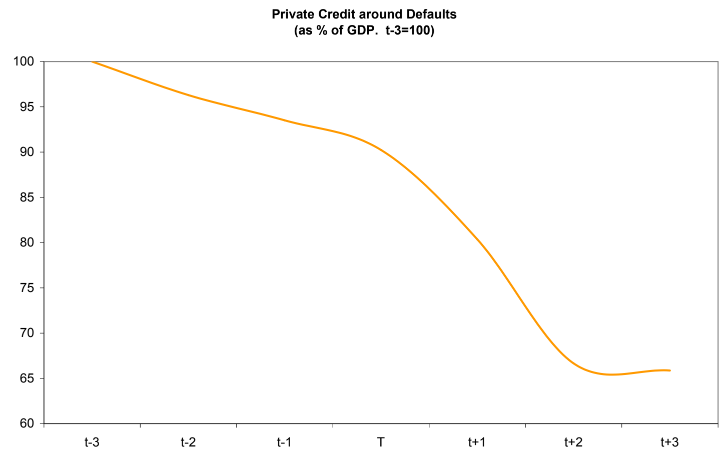

Figure 2.3: Private Credit.

tend to happen together (with 64% of banking crises happening together with or right after a default), (2) the banking sector is highly exposed to government debt (with emerging economies banking sector holding on average 30% of their assets in government bonds), and (3) crises episodes are episodes of decreased output and credit.

2.3 Environment

We analyze a closed economy under discrete time. There are four players in this economy: households, firms, bankers, and a (benevolent) government. In this framework the households do not have any intertemporal choice so they only make two decisions: how much to consume and how much to work (i.e., this is just a consumption/leisure problem from the households' point of view). The production in the economy is conducted by standard neoclassical firms that face only a working capital constraint: they need to pay a fraction of the wage bill up-front, hence their need for external financing.

The bankers lend to both firms and government from a pool of funds available to them each period. These bankers start the period with two assets: A and b. A represents an exogenous endowment that the bankers receive every period.[10] b represents the level of sovereign debt owned by the bank at the beginning of the period (this was optimally chosen in the previous period).

Finally, the government is a benevolent one (i.e., it tries to maximize the residents' utility). It faces a stream of spending that must be financed and it has three instruments to do so: labor income taxation, borrowing, and default. We do not assume any kind of commitment technology available to the government: this means that every period the government can default on its debt. This default decision is taken at the beginning of the period and influences the rest of the economic decisions. Therefore the following subsections examine how this economy works under both default and no-default, and ultimately how the sovereign optimally chooses its tax, debt, and default policies.

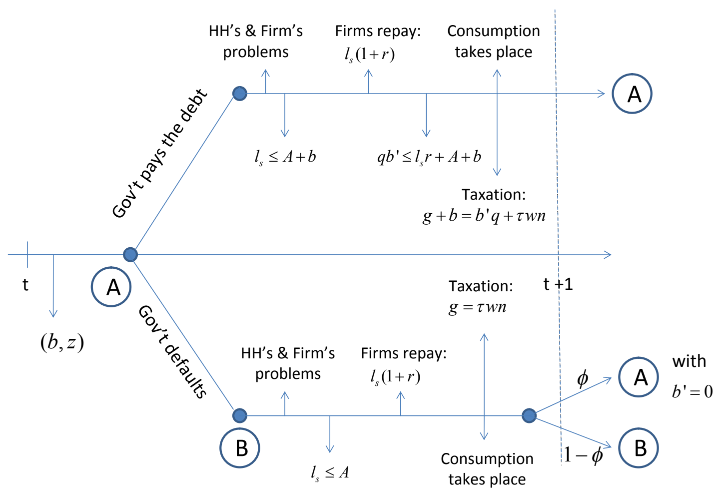

Figure 2.4: Timing of events.

2.3.1 Timing of events

The timing of events for a government that starts period t in good credit standing (i.e., not excluded from the credit market) is illustrated in Figure 2.4 and it proceeds as follows:

- Period t arrives:

  1. z_t (the TFP shock) is observed.

  2. Given that the government is not excluded from the credit market the pay-off relevant state variables are the level of debt and the level of the

[10]There are a number of ways to interpret this endowment A. One alternative is to model deposits dynamics, then A is composed of fresh deposits received by the bankers in period t. An alternative interpretation is that bankers are part of the household, and they are the only ones capable of conducting financial transactions, then it is optimal for the household (at the beginning of the period) to give this endowment A to the members of the household that will use it better: the bankers. Yet another interpretation is to think of the bankers as "international banks": A represents the flow of funds from the parent bank to its subsidiary.

TFP shock: (b_t, z_t). We are in node (A)

3. The government makes the default decision: $d_t \in \{0, 1\}$

   (a) if default is chosen ($d_t = 1$) we move to the lower branch of the tree and the following happens:

      i. government gets excluded and the credit market consists of only the (intra-period) loan market: firms borrow to meet the working capital constraint and bankers lend (l_s) up to the level of their endowment (A).

      ii. firms repay principal plus interests ($l_s(1+r)$) and all other markets (labor and goods) clear.

      iii. taxation (τ) and consumption take place.

      iv. at the end of period t a re-access *lottery* is played: with probability ϕ the government will re-access next period and get a fresh start ($b' = 0$) and with probability $1 - \phi$ the government will remain excluded and will start next period in node (B).

   (b) if repayment is chosen ($d_t = 0$) we move to the upper branch of the tree and the following happens:

      i. the credit market now consists of two markets: the market for working capital loans to firms and the market for government bonds. The bankers serve first the domestic market (l_s) up to the sum of their endowment and the repayed government debt they own ($A + b$).

ii. firms repay principal plus interests ($l_s(1+r)$) and the (intra-period) working-capital loan market closes.

iii. bankers now serve the sovereign bond's market. Each bond is traded at a price of q and bankers can lend (qb') only up to the total of their resources ($l_s r + A + b$).

iv. all other markets (labor and goods) clear, and taxation and consumption take place.

- Period t+1 arrives

2.3.2 Decision problems

In this section we describe the problems faced by each of the four economic agents in the economy. The variable d stands for the default decision and can take only two values: 0 (no default) or 1 (default).

2.3.2.1 Households' problem

As indicated above, the only decisions of the households are the labor supply and consumption levels. Therefore, the problem faced by the households can be expressed as:

$$\max_{\{c_t, n_t\}_0^\infty} \sum \beta^t U(c_t, n_t) \tag{2.1}$$

$$\text{s.t. } c_t + m = (1-\tau_t) w_t n_t + \Pi_t^F \tag{2.2}$$

where c_t stands for consumption, n_t denotes labor supply, w_t is the wage rate, τ_t is the labor-income tax rate, Π_t^F represents the firm's profits, and m is a constant spending aiming to capture the sum of investment and net exports.[11]

Plugging equation (2.2) into equation (2.1) the household's problem can be rewritten as:

$$\max_{\{n_t\}_0^\infty} \sum \beta^t U((1-\tau_t)w_t n_t + \Pi_t^F - m, n_t)$$

and the period-t FOC reads as:

$$U_c \ (1-\tau_t)w_t + U_n = 0 \tag{2.3}$$

Equation (2.3) can be rearranged into the familiar expression equating the marginal rate of substitution between leisure and consumption to the after-tax wage rate:

$$-\frac{U_n}{U_c} = (1-\tau_t)w_t \tag{2.4}$$

2.3.2.2 Firms' problem

The firms in this economy demand labor to produce the consumption good. They face a working capital constraint that requires them to pay up-front a certain fraction of the wages bill. In order to do so, firms will take intra-period loans from the bankers. Given these features, the firms' problem can be expressed as:

$$\max_{N_t, l_t^d} \Pi_t^F = z_t F(N_t) - w_t N_t + l_t^d - (1+r_t) l_t^d \tag{2.5}$$

$$\text{s.t. } \gamma w_t N_t \leq l_t^d \tag{2.6}$$

[11]The constant m is only added for calibration purposes so that the model can correctly match the consumption-to-output ratio.

where z is an aggregate technology shock, $F(N)$ is the production function, l_t^d is the demand for working capital loans, r_t is the rate charged for the loan, and γ is the fraction of the wages bill that must be paid up-front.

The working capital constraint is captured by equation (2.6). This equation will always hold with equality because firms do not need loans for anything else than paying $\gamma w_t N_t$, therefore any borrowing over and above $\gamma w_t N_t$ would be sub-optimal. Taking this into account and plugging the constraint into the objective function we obtain:

$$\max_{N_t} \Pi_t^F = z_t F(N_t) - (1 + \gamma r_t) w_t N_t$$

The period-t FOC is:

$$z_t F_N = (1 + \gamma r_t) w_t \tag{2.7}$$

Condition (2.7) equates the marginal product of labor to the marginal cost of hiring labor once the financial cost is factored in. Therefore, the optimality conditions from the firm's problem are equation (2.7) and equation (2.6) evaluated with equality.

2.3.2.3 Bankers' problem

Every period the bankers participate in two different credit markets: the loans market and the sovereign bonds markets. The working assumption is that they participate in these markets sequentially.[12]

The problem of bankers can be written in recursive form as:

$$W(b,z) = \max_{x,l_s,b'} v(x) + \delta E W(b',z') \tag{2.8}$$

$$s.t.\ x = A + (1-d)b + l_s r - (1-d)qb' \tag{2.9}$$

$$A + (1-d)b \geq l_s \tag{2.10}$$

where l_s stands for working capital loans supply, b' represents government bonds demand, A is the bankers' endowment, r is the interest rate on the working capital loans, and q is the price per sovereign bond.

$v(x)$ is the period return function of the banker, x is the end-of-period consumption of the banker, δ stands for the discount factor.

Equation (2.10) captures the implicit timing of the banking sector: the maximum the banker can lend to the firms is the sum of his endowment and the repayment of government debt.

This problem can be rewritten as:

$$W(b,z) = \max_{l_s,b',\mu} v(A + (1-d)b + l_s r - (1-d)qb') + \delta E W(b',z') + \mu[A + (1-d)b - l_s]$$

Assuming differentiability of $W(b,z)$, the first-order conditions are:

$$l_s \ :\ v'(x)r - \mu = 0 \tag{2.11}$$

$$b' \ :\ -v'(x)q(1-d) + \delta E W_b(b',z') = 0 \tag{2.12}$$

$$\mu \ :\ A + (1-d)b - l_s \geq 0 \ \ \& \ \mu[A + (1-d)b - l_s] = 0 \tag{2.13}$$

[12] The assumption of sequential banking is no different from the day-market/ night-market or the decentralized-market/ centralized-market assumption commonly used in the money-search literature (see Lagos and Wright (2005)).

The envelope condition reads as:

$$W_b(b, z) = v'(x)(1 - d) + \mu(1 - d) \tag{2.14}$$

Then, rearranging equation (2.11) we get:

$$r = \frac{\mu}{v'(x)} \tag{2.15}$$

Combining equations (2.12), (2.14) updated one period, and (2.15), and focusing on the case of $d = 0$ (when government is not excluded and debt is actually traded) we get:

$$q = \delta E \left\{ \frac{v'(x')}{v'(x)}(1 - d')(1 + r') \right\} \tag{2.16}$$

To interpret this optimality condition, define the Stochastic Discount Factor (SDF) of the banker as:

$$\delta E \left\{ \frac{v'(x')}{v'(x)} \right\} \equiv \Lambda = SDF \tag{2.17}$$

and re-write the next period's payoff of the sovereign bond as:

$$\vartheta' = (1 - d')(1 + r') \tag{2.18}$$

This expression shows that in case of default ($d' = 1$) the lender not only losses his investment in sovereign bonds but also losses the future gains that those bonds would have created had them been repaid, which are summarized in r'.

Combining (2.16), (2.17), and (2.18) we obtain this standard asset-pricing equation:

$$q = \begin{cases} E\{\Lambda\vartheta'\} & \text{if } d = 0 \\ 0 & \text{if } d = 1 \end{cases} \tag{2.19}$$

This equation is the condition pinning down the price of debt subject to default risk in this model. In standard sovereign default models with risk neutral foreign lenders, Λ is replaced with the (inverse of the) world's risk free rate, which represents the lenders' opportunity cost of funds. In contrast, here domestic bankers arbitrage the price of today's debt against the expected marginal benefit of further lending.

2.3.2.4 Government Budget Constraint

The government needs to tax labor income to pay for both the exogenous spending and (in case it decides not to default) the debt obligations . Its budget constraint can be expressed as:

$$g + (1 - d_t)B_t = \tau_t w_t n_t + (1 - d_t)B_{t+1}q_t \tag{2.20}$$

where B_t stands for debt (with positive values meaning higher indebtness), g is the exogenous government spending, τ_t is the labor income tax-rate, w_t is the wage rate, and n_t stands for labor.

2.3.3 Competitive Equilibrium given Government Policies

Definition 1 *A Competitive Equilibrium given Government Policies is a sequence of allocations* $\{c_t, x_t, n_t, N_t, l_t^d, l_t^s, b_{t+1}\}_{t=0}^{\infty}$ *and prices* $\{r_t, w_t, \Pi_t^F\}_{t=0}^{\infty}$ *such that given sovereign bond prices* $\{q_t\}_{t=0}^{\infty}$*, government policies* $\{\tau_t, d_t, B_t\}_{t=0}^{\infty}$*, and shocks* $\{m, g, z_t\}_{t=0}^{\infty}$ *the following holds:*

1. $\{c_t, n_t\}_{t=0}^{\infty}$ *solve the households' problem in (2.1) - (2.2).*

2. $\{N_t, l_t^d\}_{t=0}^{\infty}$ *solve the firm's problem in (2.5) - (2.6).*

3. $\{x_t, l_t^s, b_{t+1}\}_{t=0}^{\infty}$ *solve the banker's problem in (2.8) - (2.10).*

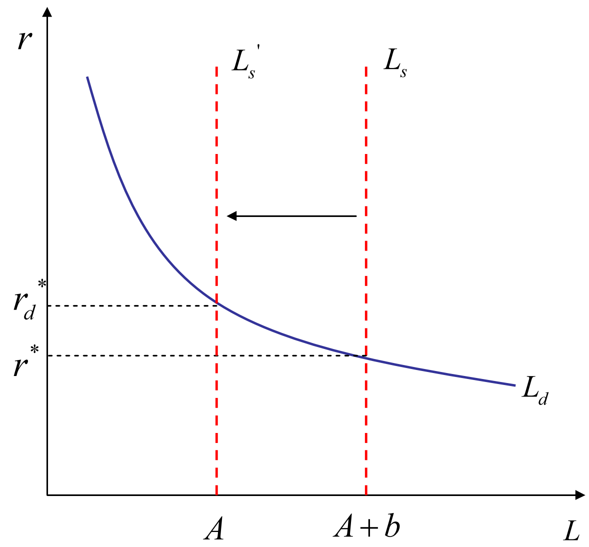

Figure 2.5: Loan Market in period t.

4. Markets clear:

$$n_t = N_t,\ b_t = B_t,\ l_t^d = l_t^s$$

and the Aggregate Resources Constraint holds:

$$c_t + x_t + g + m = zF(n_t) + A$$

2.3.3.1 Loan Market Characterization

The main mechanism of the model is to highlight how a sovereign default generates a credit crunch, which in turn shows up as an increase in the borrowing costs for the corporate sector (firms) and a subsequently economic slowdown.

This mechanism puts the financial sector in the spotlight and Figure 2.5 shows how the private credit market reacts to a sovereign default. Both the demand for loans and the supply for loans can be obtained from the partial equilibrium conditions coming from the firms' and bankers' optimization problems.

Given that the working capital loan is always risk free (because firms are assumed to never default on the loans) the bankers are going to supply inelastically the maximum amount they can. This inelastic supply curve is affected by a default: when the government defaults, bankers' holdings of government debt become non-performing and therefore they cannot be used in the private credit market. This is graphed as a shift to left of the L_s curve Figure 2.5. All this ends up in firms facing higher borrowing costs: $r_d^* > r^*$. The planner (whose problem is defined in the following subsection) takes into account how a default will disrupt this market.

2.3.4 Determination of Government Policies

We focus on Markov-Perfect equilibria where government policies are functions of pay-off relevant state variables: the level of public debt and the technological shock.

The benevolent planer wants to maximize the welfare of the residents. To do so it has three policy tools: taxation, debt and default; but it is subject to two constraints: (1) the allocations that emerge from the government policies should represent a competitive equilibrium, and (2) the government budget constraint must hold.

The optimization problem of the government can be recursively written as:

$$\mathcal{V}(b,z) = \max_{d\in\{0,1\}} (1-d)\mathcal{V}^{nd} + d\mathcal{V}^{d} \tag{2.21}$$

Given that there are two kind of residents (households and bankers) the overall objective function of the planner is a convex combination of the value functions of these two residents. Then:

$$\mathcal{V}^i(b,z) = \theta V^i(b,z) + (1-\theta)W^i(b,z)$$

where $i = d, nd$ and θ is the weight assigned to the households' *happiness* in the planner's objective function. The parameter θ gives the model certain flexibility. Letting θ be equal to one, implies that the planner will not take into account the welfare of bankers, putting the environment closer to the traditional Eaton and Gersovitz (1981) approach, where the lenders are foreigners and therefore no part of the planners objective function. Moving θ to zero implies that the planner will only care about bankers.

Following (2.8) $W^i(b, z)$ represents the banker's value function. The household's value function, on the other hand, is defined as: $V^i = U(c, n) + \beta E V^i(b', z')$.

Therefore, the value of no default is:

$$\mathcal{V}^{nd}(b, z) = \max_{n,c,b',x} \left\{ \theta V^{nd}(b, z) + (1-\theta) W^{nd}(b, z) \right\}$$

subject to:

$$\begin{aligned}
V^{nd}(b, z) &= U(c, n) + \beta E V^{nd}(b', z') \\
W^{nd}(b, z) &= v(x) + \delta E W^{nd}(b', z') \\
g + b &= \tau w n + b' q && \text{(gov't b.c.)} \\
c + x + g + m &= zF(n) + A && \text{(resources const.)}
\end{aligned}$$

$$\left.\begin{aligned}
x &= (A + b)(1 + r) - qb' \\
r &= \tfrac{znF_N}{b+A} - \tfrac{1}{\gamma} \\
-\tfrac{U_n}{U_c} &= (1 - \tau) w \\
w &= \tfrac{zF_N}{(1+\gamma r)} \\
q &= \delta E \left\{ \tfrac{v'(x')}{v'(x)} (1 - d')(1 + r') \right\}
\end{aligned}\right\} \quad \text{(comp. eq. conditions)}$$

Rewriting the constraints in terms of allocations only we obtain:

$$
\begin{aligned}
V^{nd}(b,z) &= U(c,n) + \beta E V^{nd}(b',z') \\
W^{nd}(b,z) &= v(x) + \delta E W^{nd}(b',z') \\
g + b &= \frac{znF_N}{(1+\gamma r^*)} + \frac{U_n(c,n)n}{U_c(c,n)} + b'q^* \\
c + x + g + m &= zF(K,n) + A \\
x &= (A+b)(1+r^*) - b'q^*
\end{aligned}
$$

where

$$
\begin{aligned}
r^* &= \frac{znF_N}{b+A} - \frac{1}{\gamma} \\
q^* &= \delta E \left\{ \frac{v'(x')}{v'(x)}(1-d')(1+r') \right\}
\end{aligned}
$$

V^{nd} and W^{nd} represent the values of the household and the banker under no-default, respectively.

In case the sovereign decides to default it gets excluded from the credit market in that period. There is a probability ϕ that the government will regain access to the financial market in which case its debt is forgiven (i.e. it gets a *fresh start*). Then, the value of default can be written as:

$$
\mathcal{V}^d(z) = \max_{n,c,x} \left\{ \theta V^d(z) + (1-\theta) W^d(z) \right\}
$$

subject to:

$$V^d(z) = U(c,n) + \beta E\left\{\phi V^{nd}(0,z') + (1-\phi)V^d(z')\right\}$$

$$W^d(z) = v(x) + \delta E\left\{\phi W^{nd}(0,z') + (1-\phi)W^d(z')\right\}$$

$$g = \tau w n \qquad \text{(gov't b.c.)}$$

$$c + x + g + m = zF(n) + A \qquad \text{(resources const.)}$$

$$\left.\begin{aligned} x &= A(1+r) \\ r &= \tfrac{znF_N}{A} - \tfrac{1}{\gamma} \\ -\tfrac{U_n}{U_c} &= (1-\tau)w \\ w &= \tfrac{zF_N}{(1+\gamma r)} \end{aligned}\right\} \qquad \text{(comp. eq. conditions)}$$

2.3.4.1 Recursive Competitive Equilibrium

Definition 2 *The Markov Perfect Equilibrium for this economy is (i) a borrowing rule $b'(b,z)$ and a default rule $d(b,z)$ with associated value functions $\{\mathcal{V}(b,z), \mathcal{V}^{nd}(b,z), \mathcal{V}^d(z)\}$, consumption(s) and labor plans $\{c(b,z), x(b,z), n(b,z)\}$ and taxation rule $\tau(b,z)$, (ii) an equilibrium pricing function for the sovereign bond $q(b',z)$, such that:*

1. *Given the price $q(b',z)$, the borrowing and default rules solve the sovereign's maximization problem in (2.21)*
2. *Given the price $q(b',z)$ and the borrowing and default rules; the consumption and labor plans $\{c(b,z), x(b,z), n(b,z)\}$ are consistent with competitive equilibrium.*
3. *Given the price $q(b',z)$ and the borrowing and default rules; the taxation rule $\tau(b,z)$ satisfies the government budget constraint.*
4. *The price equilibrium function satisfies equation (2.19)*

2.4 Quantitative Findings

2.4.1 Functional Forms and Stochastic Processes

The period utility function of the households is:

$$U(c,n) = \frac{\left(c - \frac{n^{\omega}}{\omega}\right)^{1-\sigma_c}}{1-\sigma_c}$$

The above preferences (called GHH after Greenwood et al. (1988)) have been traditionally used in the Small Open Economy - Real Business Cycle literature.[13] These preferences shut-off the wealth effect on labor supply and therefore help avoiding the potentially undesirable effect of having a counter-factual increase in output on default periods.[14]

The return function of the bankers is:

$$v(x) = \frac{x^{1-\sigma_b}}{1-\sigma_b}$$

The production function available to the firms is:

$$F(N) = N^{\alpha}$$

The only source of exogenous uncertainty in this economy is a productivity shock z_t. This shock follows an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t$$

where ε_t is an *i.i.d.* $N(0, \sigma_z^2)$.

[13] See, for example, Mendoza (1991).

[14] Using GHH preferences the marginal rate of substitution between consumption and labor does not depend on consumption, and therefore the labor supply is not affected by wealth effects. For a related analysis about how important are GHH preferences in generating output drops in the Sudden Stops literature, see Chakraborty (2009).

2.4.2 Calibration

The model is calibrated at an annual frequency using data for Argentina for the period 1980-2005. Table 2.3 contains the parameter values.

Curvature of labor disutility	ω	2.5	Frisch wage elasticity
Labor share in output	α	0.70	Standard value
Household risk aversion	σ_c	2	Standard value
Banker risk aversion	σ_b	0	Risk-Neutral bankers
Probability of redemption	ϕ	0.50	Mean exclusion spell
Government Spending	g	0.15	Gov't Spending/GDP = 20%
Investment + Net Exports	m	0.075	$(I + NX)$/GDP = 10%
Banker's discount factor	δ	0.96	Standard value RBC
Working capital requirement	γ	0.51	Wages bill/Private credit = 51%
Weight of hh. in planner's obj. function	θ	0.5	Equal weight
Household's discount factor	β	0.80	Mean Debt/ output $\cong$ 16%
Banker's endowment	A	0.2	Mean exposure ratio $\cong$ 35%
TFP autocorrelation coefficient	ρ	0.90	GDP autocorrelation
Std. dev. of innovations	σ_z	2.7%	GDP std. dev.

Table 2.3: Benchmark Calibration.

The parameters *above the line* are set to independently match moments from the data or are parameters that take common values in the literature. The labor share in output (α) and the risk aversion parameter for the households (σ_c) are set to 0.7 and 2 respectively, which are standard values in the quantitative macroeconomics literature. Banker's risk aversion σ_b is set to zero to feature risk-neutral bankers. The value of the exogenous spending level g is set up to 0.15 to match the ratio of General Government Expenditures to GDP for Argentina in the period 1985-2000 of 20% (from World Bank's World Development Indicators). Parameter m is meant to capture the level of investment (I) plus net exports (NX) and it is set to 0.075 so that the ratio (I+NX)/GDP generated by the model is equal to the observed ratio, 10%. The working capital requirement parameter (γ) is taken directly from the Argentine data. In the model γ is equal to the ratio of private credit to wage payments, the data shows that for Argentina this ratio was 51% (for the period 1993-2001).[15]

The discount factor for the bankers (i.e., the lenders) takes a usual value in RBC models with annual frequency, 0.96. It is important to realize that the exact value of δ is not crucial in itself but in how it compares with the household's discount factor (discussed below). The weight that households' utility receives on the planner's overall objective function (θ) is set in the benchmark calibration to 0.5 in order to give all the residents (bankers and households) the same weight. It is crucial to see that if θ takes the value of one, then the model features 'international banks' and hence facilitates the comparison with previous literature that focused on external debt and default: this is exercise is done as part of a sensitivity analysis, later in the manuscript.[16]

There are two more *above the line* parameters to discuss: the curvature of labor disutility (ω) and the probability of redemption (ϕ). The value of ω is typically chosen to match empirical evidence on the Frisch wage elasticity, $1/(\omega - 1)$. The estimates for this elasticity vary considerably: Greenwood et al. (1988) cite estimates

[15]In order to construct this ratio we took data for Private Credit from IMF's International Financial Statistics, and data for Total Wage-Earners Remuneration from INDEC(Argentina's Census and Statistics Office).

[16]Setting $\theta = 1$ features international banks but does not make our model collapse to the ones in Aguiar and Gopinath (2006) and Arellano (2008). The reason is very clear: in our model these international banks are not deep-pocket, but are only able to lend up to $A + (1 - d)b$.

from previous studies ranging from 0.3 to 2.2, González and Sala Lorda (2011) find estimates from -13.1 to 12.8 for Mercosur countries. Here we take $\omega = 2.5$ as the benchmark scenario (implying a Frisch wage elasticity of 0.67) and conduct some sensitivity analysis later.

The probability of redemption is governed by the parameter ϕ. The evidence collected by Gelos et al. (2004) is that emerging economies remain excluded on average 4 years after a default. This finding applies only to external defaults. It can be argued that governments have a additional mechanisms (regulatory measures, moral suasion, etc.) to place their debt in domestic markets making the domestic exclusion shorter than the external exclusion. Therefore the benchmark calibration will be $\phi = 0.5$, which given the annual calibration implies a mean exclusion of 2 years. Alternative values for ϕ are considered in the sensitivity analysis section.

The parameters *below the line* $\{\beta, A, \rho\,, \sigma_z\}$ are simultaneously determined in order to match a set of meaningful moments of the data. The moments matched by the model are all taken from Argentine data and they are the mean domestic debt to output ratio, the mean exposure of the banking sector to government debt, and the autocorrelation and standard deviation of GDP.

Given that the model features both a closed economy the correct debt-to-output ratio to match is the Domestic Debt to GDP. To do so I take the ratio of Total Debt to Output from Reinhart and Rogoff (2010) and extract only the Domestic Debt part of it by using the share of Domestic Debt to Total Debt form Reinhart and Rogoff (2011).

$$\underbrace{\frac{TD}{Y}}_{\text{from Reinhart and Rogoff (2010)}} \times \underbrace{\frac{DD}{TD}}_{\text{from Reinhart and Rogoff (2011)}} = \underbrace{\frac{DD}{Y}}_{\text{relevant debt ratio}}$$

This exercise gives a mean Domestic Debt to GDP ratio of 16.5% for the period 1980-2002. As it was documented in section 2 the banking sector of virtually every emerging economy is highly exposed to government debt. The mean *exposure ratio* (defined in section 2) for Argentina was approximately 25%.

The autocorrelation and standard deviation of Argentine GDP were computed at an annual frequency from the time series available from INDEC (the Argentine Census and Statistics Office) for the period 1980-2005. The autocorrelation of the cyclical component of GDP is 0.28 and the standard deviation is 4.11%.[17]

2.4.3 Results

This section examines the performance of the benchmark calibration of the model in accounting for some key statistical moments of business cycles and government debt. Table 3.2 reports the moments from the simulations of the model and compares them with the moments from Argentine data.

Overall the benchmark calibration of the model is able to account for several salient facts of the Argentine economy, namely a ratio of consumption volatility to output volatility grater than 1, countercyclical sovereign spreads, and a high and positive correlation between output and consumption.[18] All of these moments

[17] The series obtained from INDEC is real GDP, and it was first logged and then H-P filtered using a smoothing parameter of 6.25 (as suggested by Ravn and Uhlig (2002)).

[18] These facts also characterize many other emerging economies, as documented in Neumeyer and Perri (2005).

	Data	Model
$\sigma(c)/\sigma(y)$	1.17	1.82
$\sigma(n)/\sigma(y)$	0.57	0.94
$corr(R_s,\, y)$	-0.62	-0.81
$corr(c,\, y)$	0.97	0.99
$corr(N,\, y)$	0.52	0.99
$corr(R_s,\, N)$	-0.58	-0.77
Mean output drop	9.40%	8.23%
Mean credit drop	27.09%	8.17%
Default rate	2.56%	1.60%
Mean debt/ output	16.5%	16.84%
Gov't Spending/ output	20%	19.75%
Mean Exposure Ratio	36%	36.12%

Table 2.4: Simulated Moments and Data. R_s stands for bond's spread. The data for sovereign spreads is taken from J.P. Morgan's EMBI, which represents the difference in yields between an Argentine bond and a US bond of similar maturity. The spreads generated by the model are the difference between the the interest rate paid by the government and the one paid by the private sector.

were not targeted by the calibration process and are reproduced by the model. The benchmark calibration also captures nicely the negative correlation between employment and sovereign spreads, as well as procyclical employment.

The model does well at replicating the mean output drop. Data from INDEC indicates that in the recent Argentine default episode GDP fell 9.4 percentage points from trend. The benchmark calibration delivers an average decrease of 8.23 percent. The sovereign default triggers a credit crunch in the model and it in turn generates an output collapse. This collapse is due to a reduced access to the labor input, which is the only variable input in the economy. The fact that the economy cannot resort to a substitute input generates a sharp output decline. It is again important to remark that the mean output drop was not among the *targeted* moments in the calibration procedure.

The credit drop that drives the endogenous cost of default is a novel contribution of our analysis. The benchmark calibration of the model is able to produce a mean credit drop of 8.17% which accounts for 30% of the actual credit drop observed in the 2001-02 Argentine default.

The model does not do well in terms of matching the mean default rate, but this deserves further consideration. According to Reinhart and Rogoff (2009) Argentina has defaulted on its domestic debt 5 times since its independence in 1816, implying a default probability of 2.5%. It is typical in quantitative studies of sovereign default to calibrate the models to the observed frequency of default. Other models in the literature when calibrated to obtain the observed default frequency usually do poorly in some key dimensions, like the mean debt ratio.

2.4.3.1 Output dynamics around defaults

An important result of the chapter is to provide a framework able to deliver endogenous output declines in default periods. Figure 4 is constructed from the model simulations in the following way: first, we identify the simulation periods where default happens; second, we construct a time series of 8 years before and 3 years after each default; third, we average across default episodes to construct a series of the mean output behavior around defaults.

As it is clear from Figure 2.6 the model features an output decline (and a consequent consumption decline) in the default period. The model is also able to

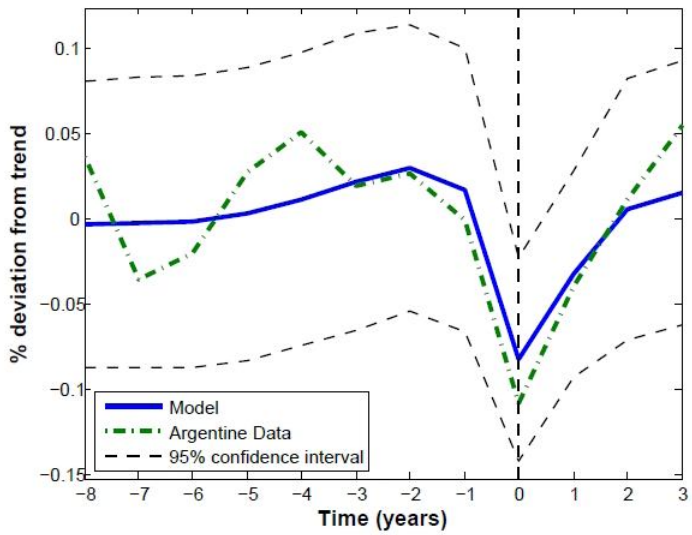

Figure 2.6: Output around defaults.

deliver a v-shape behavior of output around defaults: a strategic default is the *optimal crisis resolution* mechanism - - due to worsening economic conditions, the sovereign finds it optimal to default on its obligations (and assume the associated costs) instead of increasing the revenues required for repayment.

2.4.3.2 Endogenous cost of default: Credit contraction

Why does a default generate such a costly output decline? This chapter proposes a *credit crunch* explanation: given that bankers hold government debt as part of their assets, when a default comes a considerable fraction of these assets losses value, therefore their lending ability contracts and as a consequence credit to the

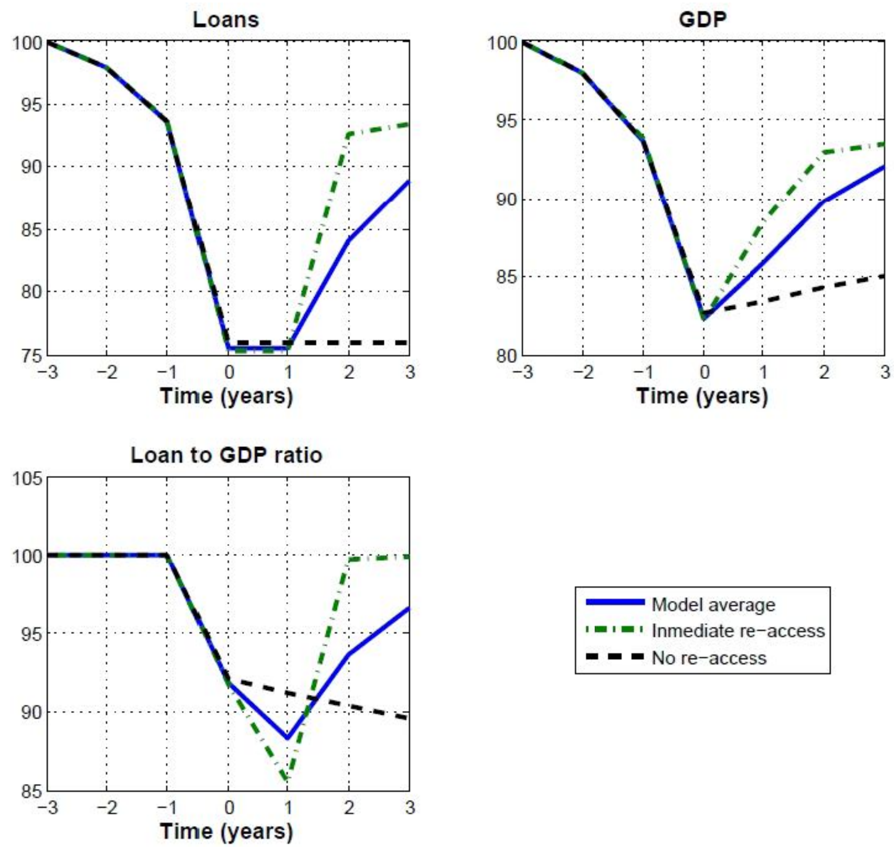

Figure 2.7: Credit crunch.

private sector diminishes. Given that the productive sector is in need of external financing, a credit crunch translates into an output decline.[19]

Figure 2.7 shows how a credit crunch looks in the model. The benchmark parametrization of the model is able to produce a collapse in the private sector credit (i.e., working capital loans to firms, in the model).

Figure 2.7 also shows how the exclusion from the credit market affects the credit availability and consequently output. In the top panels of this figure we can see the workings of a credit crunch: firms are in need of external financing, therefore when loanable funds shrink, output shrinks with them.

[19]Bolton and Jeanne (2011) identify this phenomenon as banks becoming *illiquid*.

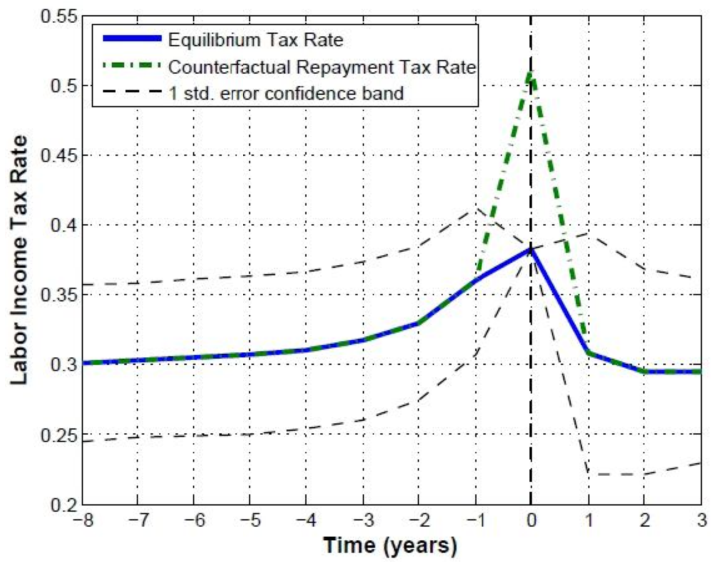

Figure 2.8: Labor tax rate around defaults.

We can also see the effect of exclusion from financial markets: if the government remains excluded then the private credit reduces (and remains low) and the output decline becomes more protracted. On the other hand, an immediate re-access to the credit market implies a rapid recovery in both credit and output.

Compared to the behavior observed in the data (i.e. Figure 2.3 in Section 2) the model is unable to generate any persistence. An interesting extension is to allow the bankers endowment A to be endogenously chosen: in this way bad states of the nature (where default will sometimes happen) give little incentive to build-up A. When a default hits, then there is even less incentives to accumulate A and this (possibly coupled with longer exclusion spells for the defaulting government) leads to lower private credit after a default.

2.4.3.3 Benefit of defaults: *reduced* taxation

As argued in the introduction, the optimal default decision comes from balancing costs and benefits of defaults. The costs of default were already discussed above: output declines because of a credit contraction. The benefits on the other hand come from *reduced* taxation. Figure 2.8 shows the behavior of the labor income tax rate around defaults: we plot the equilibrium tax rate and also the "counterfactual" tax rate that would have been necessary if instead of defaulting the government had repaid.

The reduced taxation is precisely the difference between the counterfactual repayment rate and the equilibrium tax rate: for the benchmark calibration this difference is of 13 percentage points on average. Appendix A shows that this rationale also goes through in a modified model where the sovereign has access only to lump-sum (i.e. non-distortionary) taxation.

2.4.3.4 Sovereign bonds market

In this section we analyze the behavior of the government bonds market. To this end, we show a set of charts that are often used in the default literature for this purpose. As discussed above, the model performs quite well with respect to the sovereign bond market dynamics: it produces default in bad times and (therefore) countercyclical spreads.

Figure 2.9 shows the equilibrium default region and the bond price function, respectively. With respect to top panel, the white area represents the repayment area: it decreases (until it vanishes) while the indebtness level increases and/or

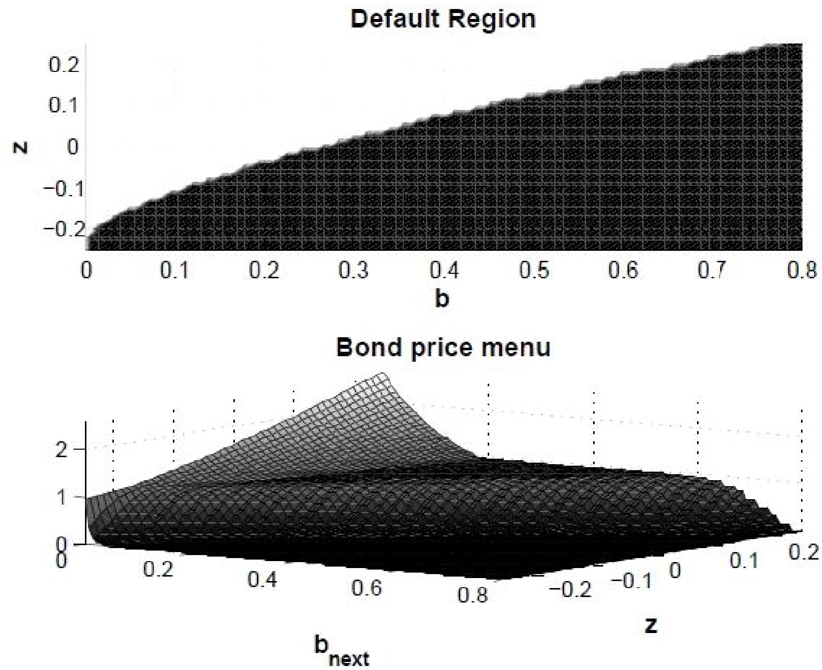

Figure 2.9: $d(b, z)$ and $q(b', z)$.

the level of the technology shock decreases. The bottom panel presents the price schedule that the government faces. As it was expected, the price the sovereign receives for each bond decreases with TFP and also with the total level of indebtness.

Now we turn to the behavior of spreads in the run up to a default. Figure 8 shows that the spreads generated by the model are able to mimic the behavior of the EMBI spreads in that they are relatively flat until the year previous to a default when they spike. On the other hand the mean spreads are higher than the observed ones away from default and lower than the data in period before.

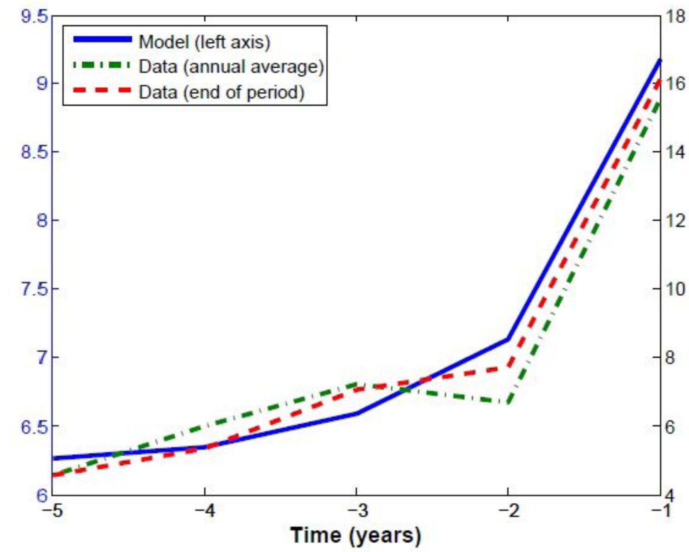

Figure 2.10: Spreads before a default.

2.4.4 Sensitivity Analysis

In this section we perform a sensitivity analysis to gain a better understanding of how a set of key moments in the baseline calibration react to changes in the underlaying parameters. Table 2.5 summarizes the findings of this exercise.

Let's first study how the model economy reacts to changes in the re-entry probability (ϕ). If the government could re-access immediately after a default ($\phi = 1$) then the overall costs of a default (exclusion form credit markets are among them) are reduced. Lower costs of default makes default a more attractive choice, so we see that for $\phi = 1$ default is more frequent. Consequently less debt can be sustained. If on the other hand we lower ϕ making re-access less frequent then the exclusion-cost-of-default is larger and the government is able to sustain higher debt ratios. The endogenous cost of default in this chapter works through a reduction in credit availability: the higher the debt the government defaults on, the higher the

	Moments (in %)					
	Default rate	$E\{b/y\}$	$E\{R_s\}$	Exposure	$y \downarrow$	Credit $\downarrow$
Data	2.5	16.50	7.43	36	9.40	27.09
Benchmark	1.60	16.84	6.17	36.12	8.23	8.17
re-entry (benchmark: $\phi = 0.5$)						
$\phi = 0.05$	0.73	24.91	4.31	42.93	5.72	6.88
$\phi = 0.1$	0.91	24.81	4.72	44.21	6.06	7.05
$\phi = 1$	1.63	16.53	5.99	36.95	10.02	10.92
w-k constraint (benchmark: $\gamma = 0.51$)						
$\gamma \to 0$	0	0	–	0	*n.a.*	*n.a.*
$\gamma = 0.1$	0	0	–	0	*n.a.*	*n.a.*
$\gamma = 0.3$	2.56	0.04	6.80	0.16	−0.81	0.01
$\gamma = 0.6$	0.75	38.47	4.84	47.10	12.38	14.12
$\gamma = 0.7$	0.27	63.46	4.40	56.13	18.81	19.37
Frisch wage elasticity (benchmark: $\omega = 2.5$)						
0.5 ($\omega = 3$)	3.51	9.52	7.57	24.32	6.84	4.46
1 ($\omega = 2$)	1.06	36.09	5.13	49.21	9.29	14.87
Banker's endowment (benchmark: $A = 0.2$)						
$A = 0.1$	0	109.91	0	75.31	*n.a.*	*n.a.*
$A = 0.3$	12.25	0.43	77.13	1.12	−0.35	0.10
$A = 0.5$	0	0	–	0	*n.a.*	*n.a.*
HH weight on $\mathcal{V}(b, z)$ (benchmark: $\theta = 0.5$)						
$\theta = 0$	0	39.96	0	22.22	*n.a.*	*n.a.*
$\theta = 1$	0.89	35.16	4.90	48.16	7.40	9.69

Table 2.5: Sensitivity Analysis.

cost, and this we see as we increase the value of ϕ.

Next, let's examine how the model behaves with different values of γ. This parameter governs the tightness of the working capital constraint, $\gamma \in (0, 1]$. A high (low) value of γ means that firms need to pay up front a higher (lower) ratio of their wage bill, therefore working capital loans are more (less) important for production. We can see that the model performs as expected: for lower values of γ (cases in which private credit is not that important for production) default is not very costly, consequently the government feels tempted to default too much. Creditors understand this and contract lending in the government bonds market. Therefore we see that for values of $\gamma \to 0$, the mean debt ratio is zero and then the **observed** default rate is also zero. On the other hand, high values of γ make default very costly. Consequently the observed debt ratios are higher and the observed default rates are low. In these rare (i.e. rarer than the benchmark) defaults, the costs in terms of output and credit drops considerable larger than in the benchmark calibration.

When it comes to the Frisch wage elasticity ($1/(\omega - 1)$) the behavior of the model is as expected. A higher wage elasticity of labor supply implies that when a default comes and firms have to decrease their labor demand (due to higher costs of financing) the decline in equilibrium labor is greater than with a more inelastic labor supply. Therefore (as expected) for lower values of ω (higher elasticity) the model features higher output drop at default and can sustain lager debt ratios, and for higher values of ω the opposite is true.

Banker's endowment A is a crucial parameter for the workings of the credit crunch. Basically a higher level of A makes government debt less important for the private credit market. The planner understands this and it is more tempted to default when A is larger. Lenders foresee this behavior and refrain from lending to the government. Therefore we observe a non-monotonic relation here: for low enough levels of A (e.g. $A = 0.1$) default is too painful for the firms and therefore the government decides to never default and with this it is able to sustain large debt ratios. Conversely, for large enough levels of A (e.g. $A = 0.3$) defaults are less painful for the private credit market and therefore the government is more tempted to default (we observe larger default rates) and it is able to sustain lower debt ratios. For even larger levels of A, everyone understands that debt is not very important for the credit markets and that governments will be tempted to default on it every time they have a chance, therefore nobody lends.

Finally, we examine the role of θ, the weight that households receive in the planner's objective function. If $\theta = 1$ it means the planner only cares about households or (what is the same) bankers are not residents but instead they are international banks. If this is the case we can compare the model against other models of external sovereign debt and default. In that dimension the model does a very good job: it obtains a reasonable default probability (0.90%) and achieves a debt to output ratio close to the data (external debt to output in Argentina was around 36%). On the other extreme if $\theta = 0$, then the "planner" only cares about the bankers and given the bankers are the only lenders, default would never be optimal. Therefore the planner can sustain higher debt ratios (higher than in the benchmark calibration).

2.4.5 Discussion of the assumptions

The model just described made a series of simplifying assumptions in order to isolate the effect that a sovereign default has on the banking sector and the productive sector of the economy. In this subsection we discuss them, ways to relax them and potential implications.

Constant g: in order to make the optimal fiscal policy as simple as possible we had assumed a constant level of government expenditure, g. While this is a useful first approximation relaxing this assumption could improve the model's quantitative performance. How to do it? The first thing to do is to make g valuable: a common approach is to include g in the household's utility function. Then, g becomes an extra fiscal policy instrument: the planner understands that higher g implies either higher τ or higher indebtness, but also takes into account the agents' preference for g. Then, when a default comes and consumption declines the planner finds it optimal to decrease g as well in order to satisfy the intratemporal optimality condition relating private and public consumption. Therefore the model would be able to account for the observed procyclicality of government spending.

Financial autarky: we presented a model of a closed economy where government debt is held by domestic bankers. While this is an attractive environment to study the implications of a sovereign default on the domestic financial sector it comes at the expense of having an unrealistic closed capital account. If we let the government borrow form abroad as well as from the domestic bankers then the model becomes one of domestic and external sovereign debt, a missing piece in the quantitative literature on sovereign defaults. Nonetheless, this extension would bring other forces into play. The possibility of selective defaults can in principle mitigate the negative effect that a sovereign default has on the domestic economy: by defaulting on foreign lenders the government could in principle afford to serve the domestic lenders. Allowing the private sector to borrow form abroad as well will decrease the relevance of the domestic credit market for domestic production. However, as long as a fraction of the domestic firms needs to borrow from domestic sources the mechanism proposed in the model will still play an important role.

Constant A: assuming bankers receive a constant endowment every period allows us to fix ideas and focus on the asset side of the bankers' balance sheet and how it responds to a sovereign default. Relaxing this assumption is clearly a desirable step towards bringing the model closer to the data. Allowing (for example) for deposits from the households will enrich the environment and make it able to capture another salient feature of emerging economies crises: bank runs. Anticipating the possibility of a sovereign default and fearing banks will not be able to fully repay the deposits, households will run on the banks and therefore put more pressure on the availability of loanable funds in the economy.

2.5 Conclusions

The prevalence of defaults and banking crises is a defining feature of emerging economies. Three facts are noteworthy about these episodes: 1- Default and Banking crises tend to happen together (with 64% of banking crises happening together with or right after a default), 2- the banking sector is highly exposed to government debt (with emerging economies' banking sector holding on average 30% of their assets in government bonds), and 3- crises episodes are episodes of decreased output and credit.

In this chapter we have provided a rationale for this phenomena. Bankers who are exposed to government debt suffer from a sovereign default that reduces the value of their assets (i.e. a banking crisis). This forces these bankers to decrease the credit to the productive private sector. This *credit crunch* translates in both lower and more costly financing for the productive sector, which generates an endogenous output decline.

This chapter presents a theory of the transition mechanism of sovereign defaults to the banking sector and the rest of the economy. A methodological contribution of this chapter is to present a competing mechanism to endogenize the costs of sovereign defaults: a sovereign default generates a credit crunch, and this credit crunch generates output declines.

The model shows an overall qualitative fit of the data (v-shape behavior of output around defaults, credit contractions at default). On a quantitative note, several dimensions are accurately accounted for: countercyclicality of bond spreads, mean default rate, mean output drop.

Some avenues for future research / possible extensions are:

- Incorporating an endogenous banker's net worth accumulation: this extension can in principle generate some persistence in the model and help to capture the time profile of domestic credit better.

- Bank leverage and regulatory policy: banks behavior is intentionally crude in the model (at this point simplifications are necessary to compute a rather complicated Markov Perfect optimal policy problem) but one could think of the potential implications that regulatory policy could have for bank leverage. If government debt (by regulation) is considered risk-free then banks have additional incentives to hold government debt. On the other hand a default not only decreases banks assets but also diminishes their leverage. Then the government faces a meaningful trade-off when considering both its debt position and the default policy.

- Endogenous government spending: making public spending valuable and letting the government optimally chose its level will increase the complexity of the planning problem but could also improve the model's ability to account for the observed procyclicality of government expenditures.

- Political economy of sovereign defaults. A default in this model implies redistribution from bankers (lenders) to households (taxpayers). It would be an interesting extension to study the politico-economic equilibrium that would arise if agents were allowed to vote on the default policies.

Chapter 3

Debt Dilution and Sovereign Default Risk [1]

3.1 Introduction

This paper presents a measure of the effects of the debt dilution induced by governments' borrowing decisions and shows how these effects can be mitigated with debt contracts that specify borrowing-contingent payments. If governments could commit to not dilute the value of current bond issuances with future borrowing, this would allow them to sell bonds at a higher price. Sovereign debt dilution may become a problem when governments do not have the ability to make such commitment.

Participants in various credit markets have made efforts to mitigate the dilution problem, which is suggestive of the relevance assigned to this issue. Corporate debt contracts often include covenants intended to limit debt dilution (Asquith

[1]Co-authored with Juan Carlos Hatchondo (Richmond Fed.) and Leonardo Martinez (IMF).
For comments and suggestions, we thank Fernando Alvarez, Luis Brandao-Marques, Fernando Broner, Satyajit Chatterjee, Pablo D'Erasmo, Bora Durdu, Enrique Mendoza, Dirk Niepelt, Jorge Roldos, Diego Saravia, and seminar participants at the U.S. Board of Governors of the Federal Reserve System, the University of Virginia, the Federal Reserve Bank of Richmond, the Federal Reserve Bank of Atlanta, the Universidad Nacional de Tucumán, the 2009 Wegmans conference, the Pontificia Universidad Católica de Chile, the Central Bank of Chile, the 2010 Royal Economic Society Meeting, the 2010 Eastern Economic Association Meeting, the 2010 SED conference, the 2010 Berlin Conference on Sovereign Debt and Default, the 2010 and 2011 AEA meetings, the ECB, and the 2011 NBER International Finance and Macroeconomics Workshop. Remaining mistakes are our own. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management, the Federal Reserve Bank of Richmond, or the Federal Reserve System.

et al. (2005), Smith and Warner (1979) and Rodgers (1964) discuss corporate debt covenants). In some cases, debt claims differ in their seniority—if existing debt is senior to new issuances, this may mitigate the dilution problem. A seniority structure is common in corporate debt and collateralized loans to households. In contrast, sovereign bonds typically do not present differences in legal seniority but include a pari passu clause and a negative pledge clause that prohibits future issuances of collateralized debt.[2] These clauses are intended to avoid making new debt senior to previously issued debt, but do not make existing debt senior to debt that will be issued in the future.

The weaker protection against sovereign debt dilution may be due in part to the weak enforcement of sovereign debt claims.[3] Overall, it seems clear that existing sovereign debt contracts do not eliminate the risk of debt dilution.

The possibility of sovereign debt dilution has also received considerable attention in both academic and policy discussions. Several studies describe the benefits of eliminating debt dilution. For instance, Bizer and DeMarzo (1992) show how dilution may lead to equilibria with higher debt levels and higher interest rates implied by higher default probabilities. It has also been argued that dilution may

[2]Sturzenegger and Zettelmeyer (2006) explain how in Ecuador's 2000 sovereign debt restructuring, the government exchanged defaulted bonds for new bonds that included a clause specifying that if a default occurred within 10 years following the restructuring agreement, the government would extend new bonds to the holders of the restructured debt. Sturzenegger and Zettelmeyer (2006) argue that the "effect of this was to offer a (limited) protection of bond holders against the dilution of their claims by new debt holders in the event of default." However, the inclusion of such debt covenants is much more an exception than a rule. It has also been argued that loans from institutions such as the International Monetary Fund or the World Bank receive de facto seniority over loans from private agents (see, for example, Saravia (2010)).

[3]This weak enforcement has lead to several proposals to induce more orderly sovereign debt restructurings (see, for example, Bolton and Skeel Jr (2005), Borensztein et al. (2004), G-10 (2002), IMF (2003), Krueger and Hagan (2005), and Paulus (2002)).

lead to excessive issuance of short-term debt (Kletzer (1984)), or of debt that is hard to restructure after a default (Bolton and Jeanne (2005)), which in turn could increase the likelihood and/or severity of sovereign debt crises. Bolton and Skeel Jr (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as observed in corporate bankruptcy procedures. Borensztein et al. (2004) suggest changes in national and international laws that may facilitate the introduction of debt contracts that provide some protection against debt dilution.[4] While these studies suggest that debt dilution may be an important source of inefficiencies in debt markets, they do not quantify the effects of dilution.

We do so using a default framework à la Eaton and Gersovitz (1981).[5] Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government's objective is to maximize the expected utility of private agents. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. Second, it decides how much to borrow. The government can borrow by issuing non-contingent long-duration bonds, as in Hatchondo and Martinez (forthcoming). The cost of defaulting is represented by an endowment loss that is incurred in the default period.

The most common modeling approach for the study of debt dilution is to focus

[4]Detragiache (1994), Eaton and Fernandez (1995), Niepelt (2008), Sachs and Cohen (1982), Tirole (2002), and UN (2004) also discuss inefficiencies raised by debt dilution.

[5]This framework has been used in many recent studies. See, for instance, Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009), Arellano (2008), Boz (2009), Cuadra et al. (forthcoming), D'Erasmo (2011), Durdu et al. (2010), Lizarazo (2005, 2006), and Yue (2010). These models share blueprints with the models used in studies of household bankruptcy—see, for example, Athreya et al. (2007), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2008), and Sánchez (2008).

on the effect of seniority clauses.[6] However, it is well known that seniority does not fully eliminate debt dilution if new borrowing increases the default probability (see, for example, Bizer and DeMarzo (1992)). Therefore, in general, one cannot measure accurately the effects of dilution by comparing equilibria with and without seniority. Furthermore, seniority clauses may not be useful to eliminate sovereign debt dilution in reality. Weak enforcement of sovereign debt claims may constitute an obstacle to implementing a meaningful seniority structure.

A second approach for the study of debt dilution is to compare equilibria obtained with long-duration and with one-period bonds.[7] However, one-period bonds do not only eliminate dilution but also increase rollover risk. We show that, in general, one cannot measure accurately the effects of dilution by comparing equilibria with one-period and with long-duration bonds: Lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk. Furthermore, while eliminating dilution with borrowing-contingent payments increases welfare, replacing long-duration bonds by one-period bonds in the presence of rollover risk decreases welfare.

We propose a new approach for the study of the effects of debt dilution. We

[6]For instance, Bi (2006) analyzes a model with one-quarter and two-quarter bonds and studies the effects of making earlier issuances senior to new issuances.

[7]Intertemporal debt dilution only appears with long-duration bonds. With one-period bonds, when the government decides its current issuance level, the outstanding debt level is zero (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). Thus, the government cannot dilute the value of debt issued in previous periods. Chatterjee and Eyigungor (2011) and Hatchondo and Martinez (forthcoming) show that in a sovereign default framework, equilibrium default risk is significantly higher with long-duration bonds than with one-period bonds.

modify the baseline model by assuming that sovereign bonds include a covenant specifying that after each time the government borrows it compensates existing bondholders by paying the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. With this borrowing-contingent payment, bonds' values become independent from future borrowing and thus, there is no dilution caused by borrowing decisions. We measure the effects of dilution by comparing simulations of the baseline model (with dilution) with the ones of the modified model (without dilution). We impose discipline to our quantitative exercise by calibrating the baseline model to match data from an economy facing default risk (Argentina before its 2001 default).

We find that, if the sovereign eliminates debt dilution, the number of defaults per 100 years decreases from 5.6 (with dilution) to 0.9 (without dilution). That is, dilution accounts for 84% of the default risk in the simulations of the baseline model. Reducing the default frequency is beneficial for welfare because defaulting is ex-ante inefficient. Thus, our exercise is indicative of the quantitative importance of dilution and supports the view that dilution should be a central issue in discussions of sovereign debt management and the international financial architecture (e.g., Borensztein et al. (2004)).

Eliminating dilution allows the government to choose a low default risk. With dilution, default risk is high even if the government chooses very low debt levels that almost certainly would not trigger a default in the following period. This is the case because future governments would increase the debt level. As long as a government cannot control the choices of future governments, it cannot choose low default risk.

Promising borrowing-contingent payments allow the government to moderate the borrowing levels that future governments will choose.

The government's ability to choose low default risk is reflected in its borrowing opportunities (i.e., the set of combinations of levels of debt and interest rates the government can choose from). Thus, eliminating dilution shifts the set of government's borrowing opportunities. The equilibrium combinations of debt and interest rate levels in the simulations without dilution are not part of the government's choice set with dilution. For no-dilution equilibrium debt levels, the equilibrium interest rate would be about 400 basis points higher in the economy with dilution. Borrowing-contingent payments weaken the government's incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

The borrowing-contingent payments that eliminate dilution may be difficult to implement in practice. This is because determining these payments requires knowledge of the price at which bonds would have traded in the absence of current-period borrowing. While that price can be easily computed in our simulations, it may be difficult to determine in practice.

However, we show that most gains from eliminating dilution can be obtained with two simple borrowing-contingent payments that depend only on the sovereign bond market price. In one, the sovereign promises to pay a predetermined share of current borrowing revenues to the holder of each bond issued in previous periods. In the other, borrowing-contingent payments are a decreasing function of the bond market price. The benchmark default frequency is reduced 69% with the first scheme and 86% with the second scheme.

It should be emphasized that our findings are not based on the assumption that the government cannot default on borrowing-contingent payments in the same way it can default on other payments. Sovereign debt contracts often contain an acceleration clause and a cross-default clause (for example, see IMF (2002a)). The first clause allows creditors to call the debt they hold in case the government defaults on a payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that in practice, when the government chooses to default on a payment it chooses to default on all its debt. The implementation of borrowing-contingent payments only requires that defaulting on borrowing-contingent payments would trigger acceleration and cross-default clauses and, therefore, a default on all government debt.

The borrowing-contingent payments studied in this paper resemble covenants commonly used in corporate debt contracts to transfer resources from debtors to creditors when credit quality deteriorates (for instance, because of an increase in indebtedness). For example, Asquith et al. (2005) document such "interest-increasing performance pricing" and find lower interest rates for contracts with this pricing.

Borrowing-contingent payments also resemble taxes used in previous studies for eliminating overborrowing by private debtors (see Bianchi (2011) and the references therein). In these studies, borrowing by one agent increases other agents' cost of borrowing and the probability of a crisis. Taxing private borrowing reduces the frequency of crises. In this paper, the borrowing by future governments increases the current government's cost of borrowing and the default probability. Borrowing-contingent payments "tax" borrowing by future governments and thus reduce default risk.

The rest of the article proceeds as follows. Section 3.2 introduces the model. Section 3 discusses the calibration. Section 3.4 presents the results. Section 3.5 concludes and discusses possible extensions of our analysis.

3.2 The model

We first discuss the baseline model with debt dilution and later introduce borrowing-contingent payments that allows us to quantify the role of debt dilution.

3.2.1 The baseline environment

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, where

$$\log(y_t) = (1-\rho)\,\mu + \rho\log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right],$$

where E denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u(c) = \frac{c^{(1-\gamma)} - 1}{1-\gamma}.$$

As in Hatchondo and Martinez (forthcoming), we assume that a bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ. In particular, a bond issued in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$.

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.

As in previous studies of sovereign default, the cost of defaulting is not a function of the size of the default. Thus, as in Arellano and Ramanarayanan (2008), Chatterjee and Eyigungor (2011), and Hatchondo and Martinez (forthcoming), when the government defaults, it does so on all current and future debt obligations. This is consistent with the behavior of defaulting governments in reality. As mentioned in the introduction, sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current.[8] Following previous studies, we also assume that the recovery rate for debt in default is zero.

Lenders are risk neutral and assign the value e^{-r} to payoffs received in the next period. Bonds are priced in a competitive market inhabited by a large number of

[8]The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued (see IMF (2002a)). For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed.

identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

When the government defaults, it faces an income loss of $\phi(y)$ in the default period. In Section 3.4.3 we show that our findings are robust to assuming that the government is excluded from capital markets after a default episode.[9] Following Chatterjee and Eyigungor (2011), we assume a quadratic loss function $\phi(y) = d_0 y + d_1 y^2$.

The government cannot commit to future default and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables. As discussed by Krusell and Smith (2003), there may be multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.

[9] Hatchondo et al. (2007) solve a baseline model of sovereign default with and without the exclusion punishment and show that eliminating this punishment only affects significantly the debt level generated by the model.

3.2.2 Recursive formulation of the baseline framework

Let b denote the number of outstanding coupon claims at the beginning of the current period, and b' denote the number of outstanding coupon claims at the beginning of the next period. A negative value of b implies that the government was a net issuer of bonds in the past. Let d denote the current-period default decision. We assume that $d = 1$ if the government defaulted in the current period and $d = 0$ if it did not. The number of bonds issued by the government is given by $-[b' - (1-d)(1-\delta)b]$. Let V denote the government's value function at the beginning of a period, that is, before the default decision is made. Let $\tilde{V}$ denote its value function after the default decision has been made. Let F denote the conditional cumulative distribution function of next-period endowment y'. For any bond price function q the function V satisfies the following functional equation:

$$V(b,y) = \max_{d \epsilon \{0,1\}} \{d\tilde{V}(1,b,y) + (1-d)\tilde{V}(0,b,y)\}, \tag{3.1}$$

where

$$\tilde{V}(d,b,y) = \max_{b' \leq 0} \left\{ u(c) + \beta \int V(b',y') F(dy' \mid y) \right\}, \tag{3.2}$$

and

$$c = y - d\phi(y) + (1-d)b - q(b',y)\left[b' - (1-d)(1-\delta)b\right]. \tag{3.3}$$

The bond price is given by the following functional equation:

$$\begin{aligned} q(b',y) &= \int e^{-r}\left[1 - h(b',y')\right] F(dy' \mid y) \\ &\quad + (1-\delta)\int e^{-r}\left[1 - h(b',y')\right] \; q(g(h(b',y'),b',y'),y') F(dy' \mid y) \end{aligned} \tag{3.4}$$

where h and g denote the future default and borrowing rules that lenders expect the government to follow.If the government defaults (does not default), $h = 1$ ($h = 0$). The function g determines the number of coupons that will mature next period. The first term in the right-hand side of equation (3.4) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (3.4) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold in the next period.

Equations (3.1)-(3.4) illustrate that the government finds its optimal current default and borrowing decisions taking as given its future default and borrowing decision rules h and g. In equilibrium, the optimal default and borrowing rules that solve problems (3.1) and (3.2) must be equal to h and g for all possible values of the state variables.

Definition 3 *A Markov Perfect Equilibrium is characterized by*

1. a set of value functions $\tilde{V}$ and V,
2. a default rule h and a borrowing rule g,
3. a bond price function q,

such that:

(a) given h and g, V and $\tilde{V}$ satisfy equations (3.1) and (3.2) when the government can trade bonds at q;

(b) given h and g, the bond price function q is given by equation (3.4); and

(c) the default rule h and borrowing rule g solve the dynamic programming problem defined by equations (3.1) and (3.2) when the government can trade bonds at q.

3.2.3 A framework without debt dilution

In this section, we propose a modification to the model presented in Section 3.2.1 that will allow us to study an economy without debt dilution and, in turn, to measure the effects of debt dilution. We eliminate debt dilution—caused by borrowing decisions—by introducing a borrowing-contingent debt covenant. The covenant specifies that if the sovereign borrows, it has to pay to the holder of each previously-issued bond the difference between the counterfactual bond price one would have observed absent new borrowing and the observed bond market price.[10] This covenant eliminates debt dilution by making the value of each bond independent from future borrowing decisions.

We assume that a default on borrowing-contingent payments triggers acceleration and cross-default clauses that make all the government's debt obligations become current: if the government selectively defaults on borrowing-contingent payments, it has to cancel all current and future debt obligations, discounting fu-

[10]To be precise, these payment obligations do not only depend on the borrowing decision (how many bonds are issued in the current period). They also depend on the current income realization and the debt level, given that the counterfactual bond price one would have observed absent new borrowing requires knowledge of the income and debt level, and of the bond price function. While this price is easy to compute in our simulations, it would be difficult to determine in reality. In Section 4.4 we show that most benefits from eliminating dilution can be obtained with borrowing-contingent payments that do not depend on this counterfactual price.

ture debt obligations at the risk-free rate. Consequently, selectively defaulting on borrowing-contingent payments cannot be better than buying back all government debt. Therefore, the next subsection presents the recursive formulation of the framework with borrowing-contingent payments without giving the government the option to selectively default on borrowing-contingent payments (but giving the government the option to buy back its debt).

3.2.4 Recursive formulation of the framework without debt dilution

As before, let q denote the price function of sovereign bonds. Let $\tilde{b} \equiv (1-\delta)b < 0$ denote the interim number of next-period coupon obligations. As in Section 3.2.1, when the government wants to buy back its bonds, it does so at the secondary-market price. If the government issues $\tilde{b} - b' > 0$ bonds, borrowing-contingent payments are given by $-\tilde{b}[q(\tilde{b}, y) - q(b', y)]$. Suppose the bond price is higher when the debt level is lower because the default probability is increasing with respect to the debt level (as is always the case for the parameterizations we study). The equilibrium bond price is then given by

$$
\begin{aligned}
q(b', y) \quad = \quad & \int e^{-r} \left[1 - h\left(b', y'\right)\right] F\left(dy' \mid y\right) \\
& + (1-\delta) \int e^{-r} \left[1 - h\left(b', y'\right)\right] \max \left\{0, q(\tilde{b}', y') - q(g(h(b', y'), b', y'), y')\right\} F\left(dy' \mid y\right) \\
& + (1-\delta) \int e^{-r} \left[1 - h\left(b', y'\right)\right] q(g(h(b', y'), b', y'), y') F\left(dy' \mid y\right). \qquad (3.5)
\end{aligned}
$$

The first term of the right-hand side of equation (3.5) represents the expected value of the next-period coupon payment. The second term represents the expected value of borrowing-contingent payments. The third term represents the expected value of a bond at the end of next period—after the lender received the coupon and borrowing-contingent payments. Note that, because of the borrowing-contingent payments, the future value of a lender's investment may be affected by the income shock, a debt buyback, and a default, but not by new borrowing. Thus, there is no debt dilution in this framework.

The government's budget constraint reads

$$c = y - d\phi(y) + (1-d)b + q(b', y)(\tilde{b} - b') + \tilde{b}\max\{0, q(\tilde{b}, y) - q(b', y)\}. \tag{3.6}$$

The last term of the right-hand side of equation (3.6) represents the government's borrowing-contingent payment. After replacing equations (3.3) and (3.4) by equations (3.5) and (3.6) in the dynamic programming problem described in Section 3.2.2, we obtain the problem without debt dilution.

3.3 Calibration

Table 3.1 presents the calibration. We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion of 2, which is within the range of accepted values in studies of business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. The parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001, as in Hatchondo et al. (forthcoming). The parameterization of the output process is similar to the parameterization used in other studies that consider a longer sample

Borrower's risk aversion	γ	2
Interest rate	r	1%
Output autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log output	μ	$(-1/2)\sigma_\epsilon^2$
Duration	δ	0.0341
Discount factor	β	0.969
Default cost	d_0	-0.69
Default cost	d_1	1.01
Risk premium	α	4

Table 3.1: Parameter values.

period (see, for instance, Aguiar and Gopinath (2006)).

With $\delta = 3.41\%$, bonds have an average duration of 4.19 years in the simulations of the baseline model.[11] Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This duration is not significantly different from what is observed in other emerging economies. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of 4.77 years, with a standard deviation of 1.52.

We calibrate the discount factor and the output cost (two parameter values) to target three moments: a mean spread—i.e., the difference between the sovereign bond yield and the risk-free interest rate—of 7.4, a standard deviation of the spread of 2.5, and a mean debt level of 28% of the mean quarterly output in the pre-default

[11] We use the Macaulay definition of duration, which with the coupon structure in this paper is given by

$$D = \frac{1 + r^*}{\delta + r^*},$$

where r^* denotes the constant per-period yield delivered by the bond.

samples of our simulations (the exact definition of these samples is presented in Section 4.1).[12] The targets for the spread distribution are taken from the spread behavior in Argentina before its 2001 default (see Table 3.2). Regarding the debt level, for the period we study, Chatterjee and Eyigungor (2011) target a mean level of unsecured sovereign debt of 70% of quarterly output. Since our model is a model of external debt and Sturzenegger and Zettelmeyer (2006) estimate that 60% of the debt Argentina defaulted on was held by residents, we choose to target a mean debt level that is roughly 40% of the value targeted by Chatterjee and Eyigungor (2011). In Section 3.4.3 we show that our findings are robust to targeting a higher debt level.

3.4 Results

As in Hatchondo et al. (2006), we solve the models numerically using value function iteration and interpolation.[13] First, we show that debt dilution accounts for most of the default risk in the benchmark economy. Second, we present the welfare gains from eliminating dilution. Third, we discuss the robustness of our measurement of the effects of debt dilution. Fourth, we show that most gains from eliminating dilution can be obtained with simpler borrowing-contingent payment schemes that may be easier to implement. Fifth, we compare long-duration debt

[12] The discount factor value we obtain is relatively low but higher than the ones assumed in previous studies (for instance, Aguiar and Gopinath (2006) assume $\beta = 0.8$). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Sapriza (2008)).

[13] We use linear interpolation for endowment levels and spline interpolation for asset positions. The algorithm finds two value functions, $\tilde{V}(1, \cdot, \cdot)$ and $\tilde{V}(0, \cdot, \cdot)$. Convergence in the equilibrium price function q is also assured.

with borrowing-contingent payments to one-period debt. Finally, we compare the allocation without dilution with the allocation that the government could attain if it could trade a full range of one-period state contingent bonds.

3.4.1 Dilution and default risk

This subsection measures the effects of debt dilution. In order to do so, it presents simulation results from the models with and without debt dilution. Table 3.2 reports moments in the data and in our simulations.[14] As in previous studies, we report results computed for pre-default simulation samples. The exception is the default frequency, which we compute using all simulation periods. We simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. We focus on samples of 32 periods because we compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001.[15] In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths in which the last default was declared at least two periods before the beginning of each sample.

The moments reported in Table 3.2 are chosen so as to illustrate the ability of the model to replicate distinctive business cycle properties of emerging economies.

[14] The data for output and consumption were obtained from the Argentine Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that.

[15] The qualitative features of this data are also observed in other sample periods and in other emerging markets (see, for example, Aguiar and Gopinath (2007), Alvarez-Parra et al. (2011), Boz et al. (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that in the data we consider, the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income.

	Data	Dilution	No dilution
Defaults per 100 years		5.59	0.91
Mean debt market value		0.21	0.20
Mean debt face value	0.28	0.30	0.21
$E(R_s)$	7.44	7.21	0.95
$\sigma\left(R_s\right)$	2.51	2.54	0.68
$\sigma(\tilde{y})$	3.17	2.97	3.28
$\sigma(\tilde{c})/\sigma(\tilde{y})$	0.94	1.03	1.10
$\rho\left(\tilde{c},\tilde{y}\right)$	0.97	1.00	0.99
$\rho\left(R_s,\tilde{y}\right)$	-0.65	-0.81	-0.63

Table 3.2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. Other columns report the mean of the value of each moment in 500 simulation samples. Each sample consists of 32 periods before a default episode.

These economies feature a high, volatile, and countercyclical interest rate, and high consumption volatility. To compute the quarterly interest rate spread we first calculate the yield that makes the present value of future payments promised in a bond—which for the no-dilution case includes borrowing-contingent payments—equal to the bond price. Then, we calculate the quarterly spread as the difference between this yield and the risk-free rate. The annualized spread (R_s) is four times the quarterly spread.[16]

In Table 3.2, the logarithm of income and consumption are denoted by $\tilde{y}$ and $\tilde{c}$, respectively. The standard deviation of x is denoted by $\sigma(x)$ and is reported in percentage terms. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of $1,600$. Table 3.2 also reports the mean debt market value (computed as the mean b divided by $\delta+r^*$, where r^* is the mean equilibrium interest rate) and the mean debt face value (computed as the mean b divided by $\delta + r$).

Table 3.2 shows that the baseline model with dilution matches the data reasonably well. As in the data, in the simulations of the baseline model consumption and income are highly correlated, and the consumption volatility is higher than the income volatility. The model also approximates reasonably well the moments used as targets (the mean debt level, and the mean and standard deviation of the spread). Estimating the default probability in the data is difficult. Using a sample of 68 countries between 1970 and 2010, Cruces and Trebesch (2011) find a frequency

---

[16] In the economy without dilution, let

$$\mathcal{C}(b, y) = max\left\{q(b(1-\delta)(1-h(b, y)), y) - q(g(h(b, y), b, y), y), 0\right\}$$

denote the per-bond borrowing-contingent payment when the initial state is given by the vector (b, y). Let

$$q^{DF}(b', y; i) = e^{-i} \int \left[1 + \mathcal{C}(b', y') + (1-\delta) q^{DF}(g(h(b', y'), b', y'), y'; i)\right] F(dy' \mid y)$$

denote the price of a default-free bond that pays the coupon and $\mathcal{C}$ every period, where i denotes the constant rate at which future payments are discounted. In state (b, y), the yield of a defaultable bond is defined as the rate i^* that satisfies

$$q^{DF}(g(h(b, y), b, y), y, i^*) = q(g(h(b, y), b, y), y).$$

The annualized interest rate spread is therefore defined as $R_s = 4(i^* - r)$.

of 6.6 defaults every 100 years. Arellano (2008) targets a frequency of 3 defaults per 100 years because that is the number of defaults observed in Argentina during the last 100 years. The default frequency in our benchmark simulation is between those numbers. In Section 3.4.3, we show that, when the risk aversion of lenders is calibrated to generate a yearly default frequency of three defaults per 100 years, the effects of debt dilution are similar to the ones reported using our baseline calibration.

What are the quantitative effects of debt dilution? Table 3.2 shows that debt dilution accounts for 84% of the default risk in the simulations of the baseline model. The number of defaults per 100 years decreases from 5.59 in the baseline to 0.91 in the model without debt dilution. Debt dilution also accounts for 87% of the spread paid by the sovereign. The standard deviation of the spread decreases from 2.54 with debt dilution to 0.68 without debt dilution. The mean face value of outstanding bonds declines 32%. Most of this decline is explained by the lower interest rate in the simulations of the model without debt dilution: The mean market value of outstanding bonds decreases only by 9%.

In order to shed light on how eliminating debt dilution affects the government's optimal decisions it is illustrative to consider its first-order conditions. We use $f_j(x_1, ..., x_n)$ to denote the first-order derivative of the function f with respect to the argument x_j. The first-order condition in the benchmark economy (with dilution) is given by

$$u_1(c)q^{Dil}(b', y) = \beta \int V_1^{Dil}(b', y') \, F(dy' \mid y) - u_1(c)q_1^{Dil}(b', y)[b' - (1-d)\tilde{b}], \quad (3.7)$$

where the superindex Dil denotes variables in the economy with dilution. The left-hand side of equation (3.7) represents the marginal benefit of borrowing. By issuing one extra bond today, the government can increase current consumption by $q^{Dil}(b', y)$ units. The right-hand side of equation (3.7) represents the marginal cost of borrowing. The first term in the right-hand side represents the future cost of borrowing: by borrowing more, the government decreases expected future consumption. The second term in the right-hand side represents the current cost of borrowing: by borrowing more, the government decreases the issuance price of every bond it issues in the current period, which in turn decreases current consumption.

Assuming that the zero-profit bond price is decreasing in the debt level (as we find it is the case for the parameterizations we study) and the government chooses to borrow ($b' < \tilde{b}$), the first-order condition in the economy without dilution is given by

$$u_1(c)q^{\text{No dil}}(b', y) = \beta \int V_1^{\text{No dil}}(b', y')\, F(dy' \mid y) - u_1(c)q_1^{\text{No dil}}(b', y)b', \qquad (3.8)$$

where the superindex No dil is used to denote variables in the economy with no dilution.

The comparison of equations (3.7) and (3.8) shows how our modification to the baseline model affects the tradeoffs faced by the government when it issues debt. In equation (3.7), the current cost of borrowing depends on the new issuances: The government only internalizes as a cost the decline in the value of bonds issued in the current period. It does not internalize as a cost the decrease in the value of the debt issued in previous periods. In contrast, equation (3.8) shows that with our modification to the baseline model, the current cost of borrowing depends on the entire debt stock at the end of the period. That is, the government chooses its issuance level internalizing the dilution in the value of debt issued in previous periods.

Equation (3.8) illustrates the tradeoffs faced by the government for a given bond price. But the change in the government's tradeoffs also affect the bond price schedule the government faces when issuing debt. We illustrate that in Figure 3.1. The figure presents the spread demanded by lenders as a function of the face value of next-period debt. The figure also presents the combination of spread levels and next-period debt chosen by the government when its initial debt level is the average level in the simulations of each case considered in the graph.

Figure 3.1 shows that a shift in the government's choice set plays an important role in accounting for the reduction in spreads implied by the elimination of debt dilution: Even for the same debt levels, spread levels are higher in the benchmark than in the no-dilution model. For the equilibrium debt levels without dilution, equilibrium spread levels would be about 400 basis points higher in the economy with dilution. In the model without dilution, borrowing-contingent payments weaken the governments incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This in turn allows the government to pay a lower interest rate.

Figure 3.1 also helps us understand why consumption volatility is higher in the economy without dilution. As illustrated in Figure 3.1, when income is low, issuance

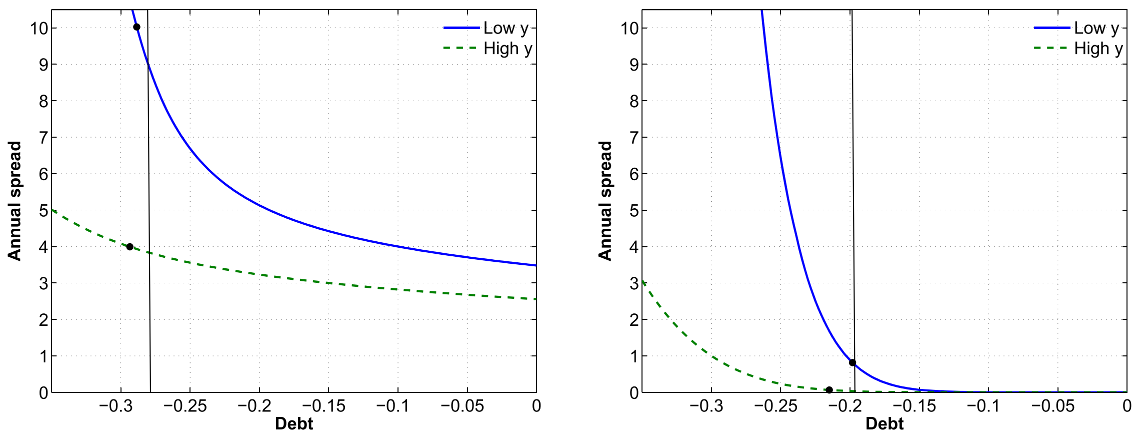

Figure 3.1: Menu of combinations of spreads and next-period debt levels $(\frac{b'}{\delta+r})$ from which the government can choose. The left panel corresponds to the baseline case. The right panel corresponds to the case without debt dilution. In each of these two cases, solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for that case. Vertical lines mark the government's debt level before its issuance decision. The low (high) value of y corresponds to an endowment realization that is one standard deviation below (above) the unconditional mean.

levels tend to be lower in the economy without dilution than in the benchmark with dilution (in the figure, issuance levels are represented by the horizontal distance between the dark dots and the vertical solid line).[17] Thus, the government is more effective in mitigating the effects of low income realizations on consumption in the benchmark with dilution.

[17]Notice that, for the same debt level, the spread curves in Figure 3.1 are steeper when income is lower. This implies that in the economy without dilution, for the same issuance level, borrowing-contingent payments would be larger when income is lower.

3.4.2 Welfare gains from eliminating dilution

We next show that it is welfare enhancing to implement the borrowing-contingent payments that eliminate debt dilution. Eliminating dilution reduces the frequency of defaults, and with that it reduces the deadweight losses caused by defaults. We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between continuing living in the benchmark economy with dilution and moving to an economy without dilution. The welfare gain of moving from the benchmark economy to the economy without dilution is given by

$$\left(\frac{V^{\text{No dil}}(b,y)}{V^{\text{Dil}}(b,y)}\right)^{\left(\frac{1}{1-\gamma}\right)} - 1.$$

Figure 3.2 presents welfare gains from implementing the borrowing-contingent payments that eliminate dilution. The figure considers two initial debt levels: zero, and the mean debt level in the simulations of the economy with dilution. The figure shows that for both cases there are positive welfare gains from eliminating dilution.

In order to eliminate dilution in an economy with positive debt levels, the government promises borrowing-contingent payments to holders of existing debt. This is costly for the government and explains why in Figure 3.2 welfare gains from eliminating dilution are lower when there is initial debt (except for lower income levels for which the government chooses to default).[18]

[18] Welfare gains are larger in a default period (when the government writes off all debt liabilities) than when the government enters the period without debt. After a default, the government wants to smooth out the income cost of defaulting. Thus, in a default period, the government has stronger incentives to borrow than when it enters the period without debt. This explains why the improvement in borrowing terms implied by the elimination of dilution is more valuable in default periods than when the government enters the period without debt.

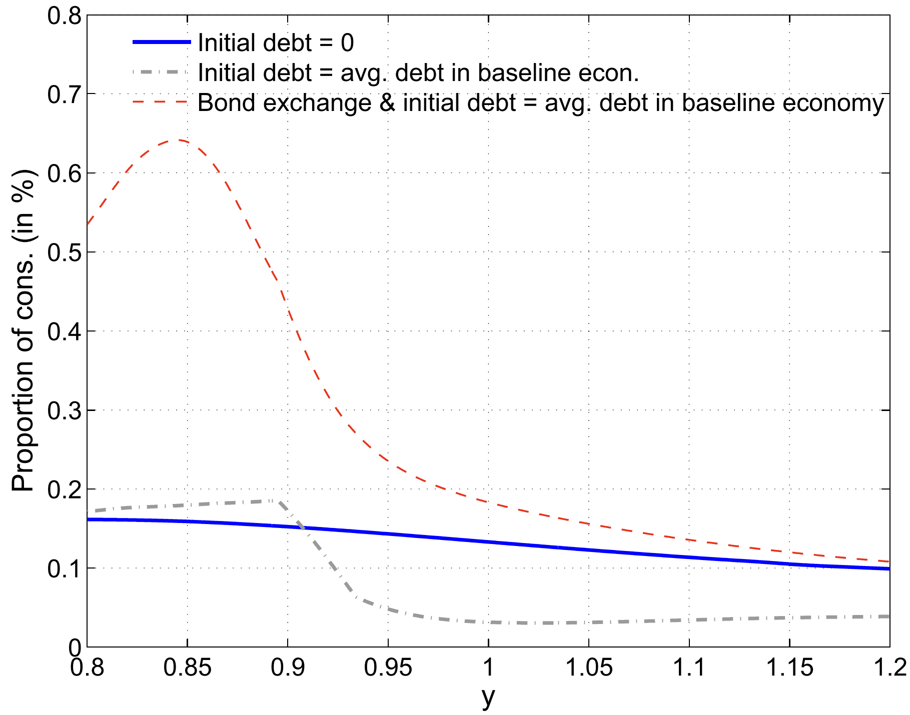

Figure 3.2: Consumption compensation (in percentage terms) that makes domestic agents indifferent between living in an economy with or without dilution. The figure was constructed assuming that the initial debt level is equal to zero or to the mean debt level in the simulations of the economy with dilution. A positive number means that agents prefer the economy without dilution.

Figure 3.2 also presents welfare gains from the case in which the government captures existing bondholders' capital gains from the introduction of borrowing-contingent payments that eliminate debt dilution. We assume the government captures these gains through a debt exchange: The government makes a take-it-or-leave-it debt buyback offer with the promise that these borrowing-contingent payments will be implemented only if this offer is accepted. Thus, the government offers bondholders to buy back previously issued bonds at the price that would have been observed if borrowing-contingent payments were never implemented. That price is lower than the no-dilution price at which the government would be able to issue debt after implementing borrowing-contingent payments. By assuming that the government makes a take-it-or-leave-it offer, we focus on the extreme case in which it reaps all capital gains.[19] The case in which borrowing-contingent payments are introduced without a debt exchange constitutes the other extreme case in which bondholders enjoy all these gains.

It should be mentioned that one may want to take our measure of the welfare gain with a grain of salt. In particular, one could argue that our measure is too low. Since there is no production in our setup, we cannot capture productivity gains from reducing the level and volatility of interest rates.[20] Several studies find evidence of a significant effect of interest rates on productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (2008), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Furthermore, the welfare gain is increasing in the level of debt, and we chose to calibrate our model to a low debt level to resemble the level of sovereign defaultable debt held by foreigners. In the next subsection we present a calibration with a higher debt level and show that eliminating dilution results in a larger welfare gain.

3.4.3 Robustness

In this subsection, we show that our finding of a strong effect of debt dilution on sovereign default risk is robust to changes in the benchmark economy we study. We focus on three changes to our benchmark. First, we modify the cost of defaulting

[19] It is also assumed that there are no output costs triggered by that debt exchange.

[20] The development of a sovereign default framework that accommodates effects of interest rates on factors allocation is the subject of ongoing research (see, for example, Mendoza and Yue (forthcoming) and Sosa-Padilla (2011)).

to allow for a higher debt stock. Second, we assume that lenders are risk averse. Third, we introduce sudden-stop shocks into the model (while we still assume risk-averse lenders). For all cases, we show that eliminating dilution leads to a significant reduction of the default frequency and increases welfare.

In order to present a case with higher debt levels, we assume a larger cost of defaulting. A defaulting economy is banned from international capital markets for a stochastic number of periods and it loses a fraction of its output in every period it remains excluded. The probability of reentry to capital markets is constant over time. This is the same cost of defaulting considered in most previous studies of sovereign default (e.g., Aguiar and Gopinath (2006) and Arellano (2008)). We assume that the probability of reentry equals 0.282, which is the value used by Arellano (2008). The only parameters that change compared to the benchmark parameterization are the ones that determine the output loss after a default. We set $d_0 = -0.7043$ and $d_1 = 0.9236$. This calibration implies a debt level of 78 percent of quarterly output, a value slightly higher than the one targeted by Chatterjee and Eyigungor (2011).

We introduce risk premium as Arellano and Ramanarayanan (2008). We assume that the price of sovereign bonds satisfies a no arbitrage condition with stochastic discount factor $M(y', y) = e^{-r-\alpha\varepsilon'-0.5\alpha^2\sigma_\epsilon^2}$. Several studies document that the risk premium is an important component of sovereign spreads and that a significant fraction of the spread volatility in the data is accounted for by the volatility in the risk premium (see, for example, Borri and Verdelhan (2009), Broner et al. (2007), Longstaff et al. (2011), and González-Rozada and Levy Yeyati (2008)).

The discount factor $M(y', y)$ is a special case of the discrete-time version of the Vasicek one-factor model of the term structure (see Vasicek (1977) and Backus et al. (1998)). With our formulation, the risk premium is determined by the income shock in the borrowing economy. The advantage of our formulation is that it avoids introducing additional state variables to the model. However, it may be more natural to assume that the lenders' valuation of future payments is not perfectly correlated with the sovereign's income.

Our third robustness exercise shows that our measurement of the quantitative effect of dilution on default risk is robust to assuming that there is a shock to the cost of borrowing that is not perfectly correlated with the sovereign's income. To this end, we introduce sudden-stop shocks. These shocks have received considerable attention in the international macroeconomics literature (see Durdu et al. (2009) and the references therein) but have been mostly ignored in the quantitative sovereign default literature (Chatterjee and Eyigungor (2011) is a notable exception). We add a shock s such that when $s = 1$ ($s = 0$) the government can (cannot) issue debt—the government can always buy back previously issued debt. We denote by π_1 (π_0) the probability of $s_{t+1} = s_t$ conditional on $s_t = 1$ ($s_t = 0$). Thus, the value functions with sudden stops are given by

$$V\left(b, y, s\right) = \max_{d\epsilon\{0,1\}} \{d\tilde{V}\left(1, b, y, s\right) + (1-d)\tilde{V}\left(0, b, y, s\right)\},$$

$$\tilde{V}(d, b, y, 1) = \max_{b' \leq 0} \left\{ u\left(c\right) + \beta \int \left[\pi_1 V(b', y', 1) + (1-\pi_1) V(b', y', 0)\right] F\left(dy' \mid y\right) \right\},$$

and

$$\tilde{V}(d,b,y,0)=\max_{b(1-\delta)\leq b'\leq 0}\left\{u\left(c\right)+\beta\int\left[\pi_0 V(b',y',0)+(1-\pi_0)V(b',y',1)\right]F\left(dy'\mid y\right)\right\}.$$

We calibrate π_1 and π_0 assuming that there is a 5.5% unconditional probability of observing $s=0$ and sudden-stop periods last for one year on average (using annual data for a sample of developing countries from 1980 to 2003, Eichengreen et al. (2008) estimate a 5.5% probability of observing a sudden stop).

Table 3.3 shows that there is a strong effect of debt dilution on sovereign default risk in the three economies studied in this subsection. The effects of dilution on other variables are also very similar across the different exercises. The exception is the debt level. When the government is excluded from capital markets after a default, the market value of debt is higher in the economy without dilution. The better borrowing terms that the government receives in the economy without dilution increases the value of having access to debt markets and, therefore, it makes defaulting more costly in that case. Eliminating dilution is also beneficial for welfare in all the cases studies.

3.4.4 Alternative borrowing-contingent payments

Implementing the borrowing-contingent payments that eliminate dilution would require knowledge of the fundamentals that determine bond prices (income and debt in the model) and the mapping from fundamentals onto bond prices. In this section, we study the effects of two simpler debt covenants.

We first study the case in which borrowing-contingent payments are a prede-

	Higher debt		Risk-averse lenders		Sudden Stops	
	Dilution	No dilution	Dilution	No dilution	Dilution	No dilution
Defaults per 100 years	4.52	1.02	3.10	0.42	5.07	0.86
Mean debt market value	0.55	0.56	0.20	0.18	0.23	0.19
Mean debt face value	0.78	0.59	0.28	0.19	0.32	0.20
$E(R_s)$	7.12	1.11	7.04	0.99	7.64	0.52
$\sigma\left(R_s\right)$	2.72	0.75	2.19	0.71	3.05	0.65
$\sigma(y)$	2.98	3.22	2.03	3.36	3.07	3.25
$\sigma(c)/\sigma(y)$	1.08	1.26	1.04	1.08	1.03	1.11
$\rho\left(c, y\right)$	0.99	0.98	1.00	0.99	0.99	0.99
$\rho\left(R_s, y\right)$	-0.79	-0.65	-0.82	-0.63	-0.80	-0.62
Welfare gain (% of cons.)		0.33		0.13		0.12

Table 3.3: Robustness exercises. The welfare gain corresponds to the average gain for the case of zero debt.

termined fixed share of borrowing revenues per unit of outstanding debt. We search for the optimal share of borrowing revenues the government should promise to existing creditors. We find that this share is such that on average the government pays to holders of debt issued in previous periods 36% of its issuance revenues.[21]

Table 3.4 presents simulation results for the economy with the fixed-share borrowing-contingent payments. The table shows these payments allow the government to achieve 83% of the decline in the default frequency and 69% of the ex-ante welfare gain it achieves with the borrowing-contingent payments that eliminate dilution.

The main difference between fixed-share borrowing-contingent payments and

[21] It should be mentioned that promising higher borrowing-contingent payments is not necessarily costly for the government because it allows the government to sell bonds at a higher price.

	Dilution	No dilution	Fixed share	Linear
Defaults per 100 years	5.59	0.91	1.71	0.81
Mean debt market value	0.21	0.20	0.17	0.19
Mean debt face value	0.30	0.21	0.20	0.20
$E(R_s)$	7.21	0.95	2.01	0.79
$\sigma\,(R_s)$	2.54	0.68	1.31	0.66
$\sigma(y)$	2.97	3.27	3.05	3.28
$\sigma(c)/\sigma(y)$	1.03	1.10	0.96	1.10
$\rho\,(c, y)$	1.00	0.99	0.99	0.99
$\rho\,(R_s, y)$	-0.81	-0.63	-0.64	-0.61
Welfare gain (% of cons.)		0.13	0.09	0.13

Table 3.4: Simulation results for different borrowing-contingent payments. The welfare gain corresponds to the average gain for the case of zero debt.

the borrowing-contingent payments that eliminate dilution is that the former are an increasing function of the post-issuance bond price $q(b', y)$ while the latter are a decreasing function of $q(b', y)$. Next, we study the effects of introducing borrowing-contingent payments that are a decreasing function of the post-issuance bond price (but do not depend on the bond price that would have been observed in the absence of borrowing). In particular, we assume that the covenant specifies that if the government issues debt in the current period, it has to pay $A - q(b', y)$ per bond issued in previous periods. We search for the optimal value of A and find that this value is 1.75% higher than the price of a risk-free bond without borrowing-contingent payments. Table 3.4 shows that this optimal borrowing-contingent function would allow the government to achieve the same welfare gain as with the borrowing-contingent payments that eliminate dilution, with a slightly lower default frequency. In summary, our findings indicate that simple borrowing-contingent payments could also reduce default risk significantly and be welfare enhancing.

3.4.5 Borrowing-contingent payments vs. one-period bonds

In this subsection, we show that in general one cannot measure the effects of debt dilution by comparing equilibria with long-duration and one-period bonds. With one-period bonds, when the government issues debt, the outstanding debt level is zero (either because the government honored its debt at the beginning of the period or because it defaulted on it) and, thus, the government cannot dilute the value of debt issued in previous periods. But with one-period bonds the government has to pay back its entire debt stock in every period, which increases its exposure to rollover risk.

Table 3.5 presents simulation results for two models without dilution: Our model with the borrowing-contingent payments that eliminate dilution, and a one-period bond model (i.e., a model with $\delta = 1$). The table presents results for both the baseline and the sudden-stop versions of the model. Table 3.5 shows that simulation results obtained with one-period bonds differ from those obtained with borrowing-contingent payments. Because one-period bonds may increase the government's exposure to rollover risk, differences in results are wider for the sudden-stop model, for which rollover risk is more significant. With sudden stops, lower default probabilities with one-period bonds are not only the result of the elimination of debt dilution but also the result of the lower debt levels chosen by the borrower to mitigate rollover risk.

	Baseline		Sudden stops	
	Borrowing-cont.	One-period	Borrowing-cont.	One-period
Defaults per 100 years	0.91	0.34	0.86	0.03
Mean debt (market value)	0.20	0.19	0.19	0.08
Mean debt (face value)	0.21	0.20	0.20	0.08
$E(R_s)$	0.95	0.44	0.52	0.15
$\sigma\,(R_s)$	0.68	0.45	0.65	0.27
$\sigma(y)$	3.28	3.10	3.25	3.22
$\sigma(c)/\sigma(y)$	1.10	1.23	1.11	1.28
$\rho\,(c, y)$	0.99	0.97	0.99	0.80
$\rho\,(R_s, y)$	-0.63	-0.73	-0.62	-0.41
Welfare gain (% of cons.)	0.13	0.13	0.12	-0.10

Table 3.5: Business cycle statistics without debt dilution. The welfare gain corresponds to the average gain for the case of zero debt.

In terms of welfare, when the government is subject to sudden-stop shocks and rollover risk is significant, replacing long-duration bonds by one-period bonds results in welfare losses. The ex-ante welfare loss from issuing one-period bonds instead of long-duration bonds is equivalent to a permanent consumption decline of 0.1%. In contrast, the ex-ante welfare gain from introducing borrowing-contingent payments is equivalent to a permanent consumption increase of 0.12%.

3.4.6 Borrowing-contingent payments vs. one-period state-contingent claims

Equation (3.6) makes clear that the borrowing-contingent payments needed to eliminate dilution are state-contingent. In this subsection, we explore how those borrowing-contingent payments perform against the best possible case with state-contingent one-period claims. It would be best for the government to transfer resources across periods using contracts with payoffs that are conditional on the past history of state realizations. For tractability reasons, in this subsection we consider a market structure in which the government can issue one-period Arrow-Debreu securities that pay off conditional on the next-period domestic income realization. The government is subject to the same limited liability constraint that is present in the benchmark economy.[22]

We assume that the government chooses how much it promises to pay next period for each realization of next-period income y' (payments can be negative). Let $b'(y')$ denote such government's promise. The government is subject to the same cost of defaulting that is present in the benchmark economy. Without loss of generality, we assume that the government only promises payments $b'(y')$ for which it would not choose to default. Lenders would not pay for a government's promise $b'(y')$ that the government would default on.

Let $\tilde{W}(d, b, y)$ denote the value function of a government that has chosen the default decision d after starting the period with debt b and income y. For any b_0 and b_1, $\tilde{W}(1, b_0, y) = \tilde{W}(1, b_1, y)$. Thus, since $\tilde{W}(0, b, y)$ is decreasing in b, for any income level y, there exists a debt level $\underline{b}(y')$ such that the government defaults if and only if its debt level is higher than $-\underline{b}(y')$. For any y, $\underline{b}(y)$ satisfies $\tilde{W}(0, \underline{b}(y), y) =$

[22]Issuing long-term debt allows the government to bring resources forward from future periods, not only from the subsequent period. That is not an option in the economy with one-period Arrow-Debreu securities. Clearly, this limitation is not an issue in the absence of the limited liability constraint. But it can dampen the welfare gain from issuing Arrow-Debreu securities in the presence of such constraint.

$\tilde{W}(1, \underline{b}(y), y)$, where

$$\begin{aligned}\tilde{W}(d,b,y) &= \max_{b'(y')}\left\{u(c)+\beta\int W(b',y')F(dy' \mid y)\right\}\\ & \quad s.t. \quad c = y - d\phi(y) + (1-d)b - e^{-r}\int b'(y')F(dy' \mid y)\\ & \qquad b'(y') \geq \underline{b}(y') \text{ for all } y',\end{aligned}$$

and

$$W(b,y) = \max_{d\epsilon\{0,1\}}\{d\tilde{W}(1,b,y) + (1-d)\tilde{W}(0,b,y)\}.$$

Table 3.6 summarizes simulation results in the benchmark, and in the economies with the borrowing-contingent payments that eliminate dilution, and with state-contingent claims. Since there are no defaults with state-contingent claims, we cannot use the same criterion for selecting samples that we used for economies with defaults. In order to facilitate the comparison of simulations across economies, for the economy with state-contingent claims we use the same 500 samples of 32 periods used to compute the simulations in the benchmark economy (income shocks are the same, though the initial debt levels may differ).

Table 3.6 shows that the average ex-ante welfare gain from moving to an economy with state-contingent claims amounts to a permanent increase in consumption of 0.53%. There are three sources of welfare gains from using state-contingent claims. First, issuing state-contingent claims allows the government to avoid defaults. Second, when borrowing, the government is able to obtain more resources with state-contingent claims (as reflected in the higher market value of its debt). Third, the consumption process is more disentangled from the income process in the economy with state-contingent claims (as reflected in the lower consumption

	Benchmark	No dilution	State-contingent claims
Defaults per 100 years	5.59	0.91	0
Mean debt market value	0.21	0.20	0.30
Mean debt face value	0.30	0.21	0.30
$\sigma(y)$	2.97	3.28	2.97
$\sigma(c)/\sigma(y)$	1.03	1.10	0.67
$\rho\,(c, y)$	1.00	0.99	0.87
Welfare gain (% of cons.)		0.13	0.53

Table 3.6: Business cycle statistics in the benchmark economy and in the economies with the borrowing-contingent payments that eliminate dilution and with one-period state-contingent bonds. The welfare gain corresponds to the average gain for the case of zero debt.

volatility).

Welfare gains from moving to an economy with borrowing-contingent payments are 25% of those from introducing state-contingent claims. In contrast with state-contingent claims, borrowing-contingent payments do not eliminate defaults completely, reduce slightly the market value of the government's debt, and increase consumption volatility.

Our stylized framework is likely to overstate the advantages of introducing state-contingent claims over the introduction of borrowing-contingent payments. First, identifying the state is much more difficult in the real world than in our stylized model. In our model income shocks are the only source of uncertainty. However, Tomz and Wright (2007) argue that other determinants of the sovereigns' willingness to repay besides aggregate income play an important role in accounting for sovereign defaults. Identifying the shocks that affect default risk, measuring these shocks, and writing debt contracts with payments contingent on these shocks may be difficult. Second, writing debt claims contingent on income may suffer from verifiability and moral hazard issues that are not present in our stylized model. A government could manipulate the GDP calculation and final GDP data are available with a significant lag. It has also been argued that debt claims contingent on GDP may introduce moral hazard problems by weakening the government's incentives to implement growth-promoting policies, which improves the likelihood of repayment.[23]

3.5 Conclusions

We solved a baseline sovereign default framework à la Eaton and Gersovitz (1981) assuming that sovereign bonds contain a debt covenant promising that after each time the government borrows, it pays to the holder of each bond issued in previous periods the difference between the bond market price that would have been observed absent current-period borrowing and the observed market price. This covenant eliminates debt dilution—caused by borrowing decisions—by making the value of each bond independent from future borrowing decisions. We measured the effects of debt dilution by comparing the simulations of this model with the ones of the baseline model without borrowing-contingent payments. We found that even without commitment to future repayment policies and without optimally designed contingent claims, if the sovereign eliminates debt dilution, the default probability decreases 84%. We also showed that most gains from eliminating dilution can be obtained with borrowing-contingent payments that depend only on the bond market

[23] See, for instance, Krugman (1988). These issues could be addressed by indexing debt contracts to variables that the government cannot control such as commodity prices or trading partners' growth rates (see for instance Caballero (2002)).

price.

Our findings indicate that governments could benefit from committing to lower future borrowing levels, which governments could achieve through fiscal rules. Eliminating debt dilution should be an important motivation for the implementation of fiscal rules that could reduce significantly the risk of debt crises and the mean and volatility of interest rates (see Hatchondo et al. (2012)). Fiscal crises are occurring in countries with fiscal rules in part because of the weak enforcement of these rules. The borrowing-contingent covenants studied in this paper could enhance the enforcement of fiscal rules by providing incentives for lower future borrowing levels. Implementing these covenants would necessitate the same strict accounting norms necessary for the successful implementation of fiscal rules.

As in Chatterjee and Eyigungor (2011) and Hatchondo and Martinez (forthcoming), we assume that the government cannot choose the duration of its debt. Relaxing this assumption could enhance our understanding of the effects of debt dilution. It has been argued that the dilution problem may lead to excessive issuances of short-term debt (e.g., Kletzer (1984)). However, allowing the government to choose the duration of its debt would increase the computation cost significantly.[24]

[24]If one allows the government to choose a different duration of sovereign bonds each period, one would have to keep track of how many bonds the government has issued of each possible duration to determine government's liabilities (Arellano and Ramanarayanan (2008) study a model in which the government can choose to issue bonds with two possible durations). The computation cost of including additional state variables may be significant (see Hatchondo et al. (2006)).

Appendix A
Lump Sum Taxes

In this Appendix we present and solve the model studied in "Sovereign Defaults and Banking Crises" (i.e. Chapter 2) with the following modification: the government has now access to lump-sum taxation. The aim of this appendix is to highlight that even thought taxation is not distortionary anymore there is still a meaningful trade-off when taking the default decision.

A.1 Cost vs. Benefits of a Default

As we did in the previous chapters (and as it is usual in the literature of sovereign defaults) we are going to approach the default decision as an strategic one. Therefore, the sovereign (i.e. the decision maker) will balance the costs and benefits of a default and decide to default whenever the benefits exceed the costs.

The costs of a sovereign default were already discussed and analyzed in Chapter 2: a sovereign default generates a credit crunch which in turn creates an output decline. The benefits of a default have been described as "reduced taxation" in Chapter 2: by walking away on its debt the sovereign can afford to reduce the tax rate that it was charging to households.

The modification in this appendix is that the government now has access only to a lump-sum tax. With distortive taxation a tax-break after the default has a dual effect for the households: first, by changing the after-tax wage it alters their labor supply decision, second, given that the total revenues collected decrease there is an "income" effect that benefits households - they simply pay less taxes, and they like that. In the scenario of lump-sum taxation the first effect disappears but the second effect remains: a tax-break is still valued by the households (hence it remains being a "benefit" of defaulting) because of the income effect just mentioned.

The remainder of this appendix is as follows: section A.2 presents how the model equations should be modified in order to accommodate for lump-sum taxation, section A.3 shows the quantitative findings under this modified environment and makes the case that the main results and intuitions carry through even in absence of distortionary taxation.

A.2 Modified Model

In this section we present the needed alterations to the model in Chapter 2 in order to study the case of lump-sum taxation. The very first change to the model is in the household's budget constraint. Now it should look as:

$$c_t + m = w_t n_t - T_t + \Pi_t^F \tag{A.1}$$

where T_t stands for the lump-sum taxation that is levied in period t. After this change we immediately need to change the intra-temporal Euler equation that results from the maximization of the household. Now this optimality condition will read as:

$$-\frac{U_n}{U_c} = w_t \tag{A.2}$$

where the modification consist of the absence of $(1 - \tau_t)$ in the right-hand side. This is exactly what we mean by taxation being not distortionary: it does not generate a wedge between the marginal rate of substitution between consumption and leisure (left-hand side of equation A.2) and the wage rate (right-hand side of equation A.2).

The last modification to the model equations is in the government budget constraint. This change is very straight forward and points out that now taxation is lump-sum:

$$g + (1 - d_t)B_t = T_t + (1 - d_t)B_{t+1}q_t \tag{A.3}$$

A.3 Quantitative Findings

In this section we will present the quantitative performance of the modified model. The parametrization/calibration is the same as in Chapter 2. [1]

With only the exception of $corr(R_s, N)$ all other moments behave qualitatively in line with the data (and also in line with the benchmark specification of the model).

[1]The aim here is to see how the model performs under lump-sum taxation and not to match moments from the data. If we would attempt the latter, clearly we would need to re-calibrate the model given that we have changed its structure.

	Data	Benchmark	Lump-Sum
$\sigma(c)/\sigma(y)$	1.17	1.82	2.86
$\sigma(n)/\sigma(y)$	0.57	0.94	0.57
$corr(R_s, y)$	-0.62	-0.81	-0.29
$corr(c, y)$	0.97	0.99	0.88
$corr(N, y)$	0.52	0.99	0.90
$corr(R_s, N)$	-0.58	-0.77	0.02
Mean output drop	9.40%	8.23%	8.92%
Mean credit drop	27.09%	8.17%	19.34%
Default rate	2.56%	1.60%	0.74%
Mean debt/ output	16.5%	16.84%	42.88%
Gov't Spending/ output	20%	19.75%	17.20%
Mean Exposure Ratio	36%	36.12%	58.75%

Table A.1: Simulated Moments: Data vs. Benchmark model vs. Lump-sum model. R_s stands for bond's spread. The data for sovereign spreads is taken from J.P. Morgan's EMBI, which represents the difference in yields between an Argentine bond and a US bond of similar maturity. The spreads generated by the model are the difference between the the interest rate paid by the government and the one paid by the private sector.

A.3.1 Output and credit dynamics around defaults

In this subsection we document the dynamics of output and credit around defaults under the lump-sum taxation framework. The basic message of this subsection is: nothing changes. The behavior of credit and output in the new specification resemble those of the benchmark specification. In other words, the main mechanism of the paper (a default-induced credit crunch that generates an output decline) goes through regardless of the taxation instruments available to the government.

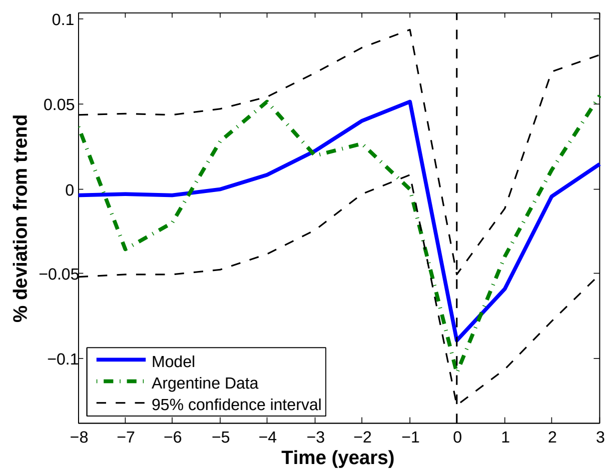

Figure A.1: Lump-Sum model: Output around defaults.

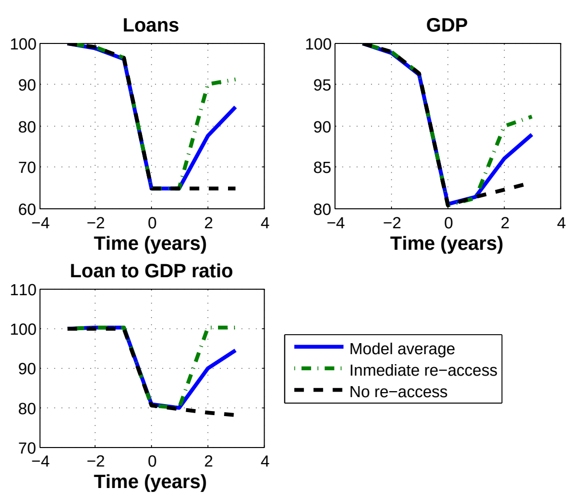

Figure A.2: Lump-Sum model: Credit around defaults.

A.3.2 Sovereign Debt Market

In this subsection we report how the sovereign bonds' market behaves under the lump-sum taxation specification. Figures A.3 and A.4 show the default region, the bond-price schedule, and the equilibrium spreads. Again the message is the same: regardless of the quantitative differences between the lump-sum taxation and the distortionary taxation versions, the qualitative behavior is the same.

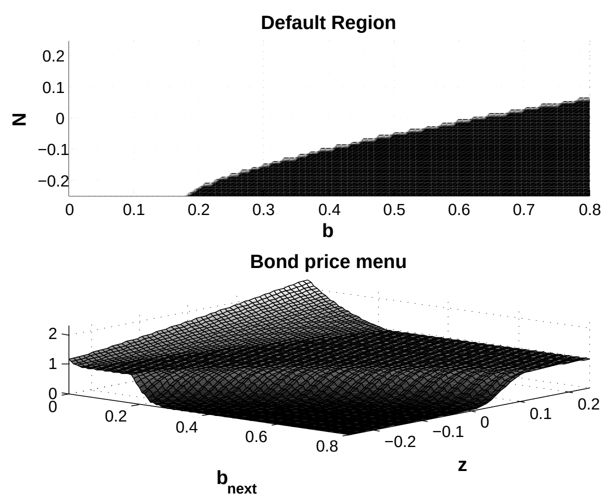

Figure A.3: Lump-Sum model: $d(b, z)$ and $q(b', z)$.

A.3.3 Taxation around defaults

Finally we analyze how the taxes move around a default in the lump-sum version of the model. Contrary to the previous sub-sections we are going to plot together the labor tax rate (from the distortionary-taxation version of the model)

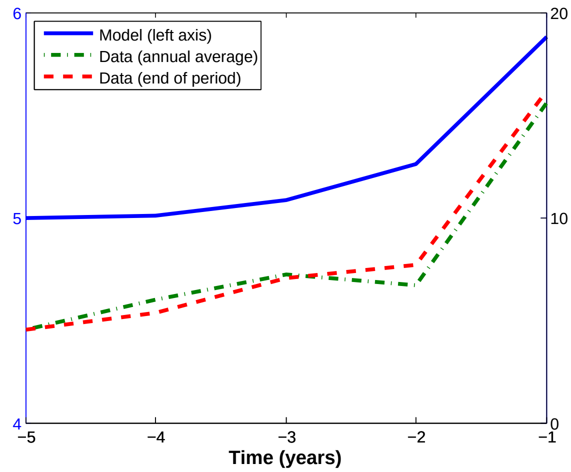

Figure A.4: Lump-Sum model: Spreads before a default.

and the lump-sum tax in the same graph to more easily see how things change when we move to a lump-sum taxation environment. Figure A.5 shows this exercise.

The take away from Figure A.5 is very straight-forward: in both cases the taxes increase in the run-up to the crisis, and then at the moment of default the actual tax levied is significantly lower than what should have been charged in the (counter-factual repayment case). Therefore the aforementioned intuition that a default allows the sovereign to afford a tax break is still valid in the lump-sum taxation environment.

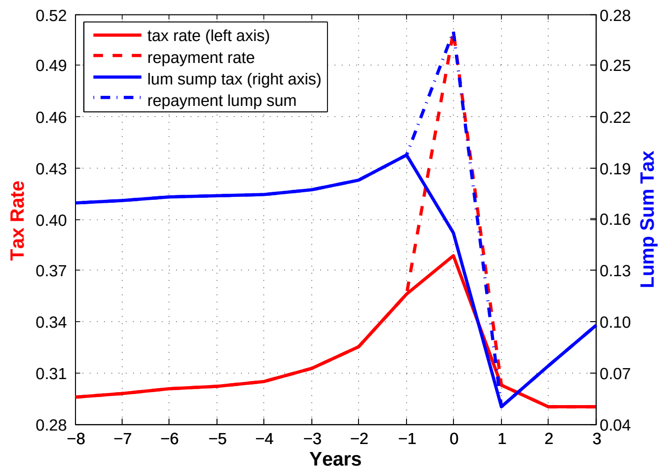

Figure A.5: Lump-sum taxes vs. Distortionary taxes.

Appendix B

Data Sources for Chapter 2

This appendix is aimed to provide further details about the sources of data used in Chapter 2 ”Sovereign Defaults and Banking Crises”.

B.1 Timing of Defaults and Banking Crises

The stylized fact reported in the body of the chapter (i.e. that sovereign defaults often coincide or precede banking crises) was constructed using the data provided by Reinhart (2010) which is publicly available at:

http://www.nber.org/papers/w15815

B.2 Banking Sector Exposure to Government Debt

As expressed in the chapter, we follow Kumhof and Tanner (2005) in documenting this fact by looking at the exposure ratio.

$$\frac{\text{Financial Institutions' net credit to the gov't}}{\text{Financial Institutions' net total assets}}$$

The data was taken from the IMF's International Financial Statistics (IFS).

B.3 Output and Credit

To construct the output and credit dynamics around defaults we used IMF's IFS series of Gross Domestic Product (GDP) and Private Credit.

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