

ABSTRACT

Title of dissertation: ESSAYS ON SEARCH AND MATCHING
 IN THE LABOR MARKET

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I study the matching of heterogeneous workers and firms in the labor market. In particular, I examine how firm productivity shocks and the nature of decreasing returns affect *positive assortative matching*: outcomes where the best, most productive workers are employed by the most productive firms and the least productive workers are assigned to the least productive firms. Chapter one studies the relationship between worker-firm sorting and firm growth both empirically and theoretically. I use U.S. Census microdata to show that faster growing firms hire more productive workers, as measured by their lagged wages. This appears to be the first time that such a pattern has been documented. I interpret the pattern as evidence of positive assortative matching: faster growing, more productive firms tend to hire more productive workers. I develop a novel, analytically tractable search model and show that it can reproduce the observed patterns if there are strong complementarities in the production function. I also discuss the relationship between my empirical results and the previous literature on assortative matching. I argue that it is important to control for observable firm characteristics, which may proxy for the nature of the labor market facing the firm. Chapter two as-

sesses the theoretical properties of several models, some novel and some drawn from the previous literature. I focus on each model's sufficient conditions for positive assortative matching. The models in the existing literature have a variety of sufficient conditions for positive sorting, and it has not been clear why these conditions vary across models. To answer this question I develop generalizations of each model that preserve their sorting properties. By comparing the generalized models I am able to show that the differences stem from how firm size, or scale, is dealt with. This result suggests caution when developing and interpreting models of sorting, since seemingly innocuous assumptions can result in significantly different behavior.

ESSAYS ON SEARCH AND MATCHING
IN THE LABOR MARKET

by

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Preface

This dissertation deals with the matching of heterogeneous agents, particularly in the labor market. Much of my focus is on *positive assortative matching*: outcomes where the best, most productive workers are employed by the most productive firms and the least productive workers are assigned to the least productive firms. The question of positive assortative matching, or positive sorting, has interested economists at least since [Becker \(1973\)](#). Recently, the availability of linked employer-employee data has renewed interest in measuring and modeling labor market sorting. I study worker-firm sorting as it relates to decreasing returns at the firms level, firm employment dynamics, and search frictions. These models push the classical sorting models to incorporate more realistic features, while maintaining analytical tractability and providing intuition.

Chapter one studies the relationship between worker-firm sorting and firm growth both empirically and theoretically. Sorting in the labor market has been widely studied, but the relationship to firm growth has not received as much attention. I use U.S. Census microdata to show that faster growing firms hire more productive workers, as measured by their lagged wages. This appears to be the first time that such a pattern has been documented. I interpret the pattern as evidence of positive assortative matching: faster growing, more productive firms tend to hire more productive workers. I develop a novel, analytically tractable search model and show that it can reproduce the observed patterns if there are strong complementarities (i.e. log-supermodularity) in the production function. I also discuss the relationship between my empirical results and the previous literature on assortative matching. I argue that it is important to control for observable

firm characteristics, which may proxy for the nature of the labor market facing the firm.

Chapter two assesses the theoretical properties of several models, some novel and some drawn from the previous literature. I focus on each model's sufficient conditions for positive assortative matching (positive worker-firm sorting). The models in the existing literature have a variety of sufficient conditions for positive sorting. In particular, models in the (frictionless) tradition of [Becker \(1973\)](#) have supermodularity as a sufficient condition, while models based on the (frictionless) Dixit-Stiglitz framework have the much stronger property log-supermodularity as a sufficient condition. It has not been clear why these two widely used models differ in their predictions about sorting. To answer this question I develop generalizations of each model that preserve their sorting properties. By comparing the generalized models I am able to show that the differences stem from how firm size, or scale, is dealt with in each model. This result suggests caution when developing and interpreting models of sorting, since seemingly innocuous assumptions can generate different behavior.

I use directed search to extend my models to a frictional environment. I show that the exact same sufficient conditions, supermodularity and log-supermodularity, guarantee positive sorting under directed search. This result is particularly interesting because it contradicts the intuition that sorting should be more difficult to obtain in a setting with frictions. The intuition about sorting carries over unchanged from the frictionless models: what matters is how scale and decreasing returns are imposed, the search frictions are irrelevant for sorting. These results provide a useful benchmark in the rapidly expanding theoretical literature on

sorting under search frictions. In the past 15 years numerous models have been proposed, with almost as many implied sufficient conditions for positive sorting.¹ The relationship among these models, and with the frictionless counterparts, has not always been clear. My directed search models show how search frictions do not *necessarily* change the sorting properties of a model.²

Taken together the two chapters use a variety of tools to assess sorting in the labor market. Chapter one's model is based on random search, Nash bargaining, and convex vacancy costs. The frictional models in chapter two are based on directed search and concavity in employment or match production. This diversity reflects the fact that certain tools are better for certain jobs. Directed search often yields highly tractable results (see [Shimer \(2005\)](#), [Menzio and Shi \(2010\)](#) or [Schaal \(2012\)](#)) but is ill-suited to address the joint behavior of vacancy yields and the composition of hires discussed in chapter one.³ In addition, while concavity in employment or match production is an intuitive way to pin down firm scale, it makes firm dynamics complex since the value of a match depends both on future shocks and the firm's endogenous responses. Thus I follow [Coles and Mortensen \(2011\)](#), [Moscarini and Postel-Vinay \(2013\)](#) and others and rely on convex vacancy costs to study firm dynamics in chapter one. Convex vacancy also allow me to address the vacancy yield as documented by [Davis et al. \(2013\)](#).

¹See [Shimer and Smith \(2000\)](#), [Shi \(2001\)](#), [Teulings and Gautier \(2004\)](#), [Shimer \(2005\)](#), [Atakan \(2006\)](#), [Lentz \(2010\)](#), [Eeckhout and Kircher \(2010\)](#), [Eeckhout and Kircher \(2012\)](#), [Lise and Robin \(2013\)](#), [Lise et al. \(2013\)](#) and [Grossman et al. \(2013\)](#) among others.

²[Eeckhout and Kircher \(2012\)](#) make a similar point in the context of a model that nests some of my specifications. However, their focus is on the relationship between productivity and firm size, and they do not pursue the relationship between [Becker \(1973\)](#)'s partnership model and the Dixit-Stiglitz framework.

³[Kaas and Kircher \(2011\)](#) address the vacancy yield (see [Davis et al. \(2013\)](#)) in a directed search model with convex vacancy posting costs. However, it is not obvious how to extend such a model to heterogeneous workers while maintaining tractability. It seems likely that in such a model any sorting results would depend on the details of the worker and firm distributions, as well as the specification of the costs.

Alternatively, one might be concerned that the directed search models in chapter two take out too much of the search frictions. Letting firms commit to only hire one type of worker at a posted wage seems to be at odds with reality, where (some) employers surely meet a wide variety of workers and bargain over wages. But directed search is a useful abstraction and the main alternative, perfectly random search, is probably just as unrealistic.

The common thread running through the chapters is a concern with markets where (1) both firms and workers are heterogeneous, and (2) where firms may employ multiple workers. These two themes place the chapters in a relatively small section of the literature. A multitude of papers have modeled markets where firms are heterogeneous and employ multiple *homogeneous* workers. On the other hand, where workers and firms are differentiated by productivity it has been common to assume that each firm consists of a single job. Having diversity on both sides of the labor market in addition to a non-trivial firm scale decision is a relatively new but increasingly studied framework.

The labor literature has often abstracted from the question of firm scale, in order to make progress understanding sorting and related questions. However, it seems that more attention must be given to firm scale if the field is to make contact with the wider literatures on firm organization and macroeconomic dynamics. There is a great deal to be learned through such contact, since worker-firm sorting is clearly relevant for understanding industry/firm behavior, as well as exploring macroeconomic patterns. In addition, labor researchers cannot ignore the role of firm dynamics (i.e. job destruction, firm productivity shocks, reallocation) in the lives of workers, or the influence of macroeconomic shocks. Nonetheless, standard

macro, IO and trade models are clearly at odds with the single worker firm assumption that is at the heart of many labor models. The alternative assumption of linear production and convex vacancy costs (which I use in chapter one) is also difficult to reconcile with models of decreasing returns, although it does allow for a reasonable discussion of firm dynamics. It is hoped that these essays will help bridge the gap between pure models of worker-firm sorting and the voluminous literature on the firm, which has mostly abstracted from worker heterogeneity in the interest of modeling productivity, scale and dynamics.

Dedication

This thesis is dedicated to my parents, Dod and Dorothy Crane.

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Chapter 1: Firm Dynamics and Assortative Matching

1.1 Abstract

I study the relationship between firm growth and the characteristics of newly hired workers. Using Census microdata I obtain a novel empirical result: when a given firm grows faster it hires workers with higher past wages. These results suggest that productive, fast-growing firms tend to hire more productive workers, a form of positive assortative matching. This contrasts with prior research that has found negligible or negative sorting between workers and firms. I present evidence that this difference arises because previous studies have focused on cross-sectional comparisons across firms and industries, while my results condition on firm characteristics (e.g. size, industry, or firm fixed effects). Motivated by the empirical findings I develop a search model with heterogeneous workers and firms. The model is the first to study worker-firm sorting in an environment with worker heterogeneity, firm productivity shocks, multi-worker firms, and search frictions. Despite this richness the model is tractable, allowing me to characterize assortative matching, compositional dynamics and other properties analytically. I show that the model reproduces the positive firm growth-quality of hires correlation when worker and firm types are strong complements in production (i.e. the production function is strictly log-supermodular).

1.2 Introduction

Do fast growing firms hire different workers from slow growing or shrinking firms? How do firm recruitment strategies vary with employment growth? Do more productive workers match with more productive firms? I address these questions empirically and theoretically. Using linked Census microdata I document a new fact, that faster growing firms hire better workers. This suggests positive sorting of workers across firms, a result that has been elusive in the empirical literature. I extend the search and sorting model of [Shimer and Smith \(2000\)](#) to include multi-worker firms, productivity shocks, and firm growth. Once solved the extended model turns out to be highly tractable and yields analytical comparative statics. The model tightly links firm employment growth to productivity and the quality of hires, and I show that the empirical correlation can be reproduced if the production function exhibits strong complementarities between firm and worker types.

My focus is on how workers of differing productivity sort across firms in a non-random way. Textbook search models (e.g. [Pissarides \(2000\)](#) Chapter 1) assume homogenous workers and firms in order to focus on the number of workers employed, rather than the exact sorting of workers across jobs. Many authors have extended the bare bones model to capture heterogeneity in worker productivity or firm productivity, but less often both. This has been partially due to the difficulties of measuring sorting empirically and modeling it in a tractable way. However, significant dispersion in productivity across workers and firms has been well documented, and there is abundant anecdotal evidence of workers and firms sorting according to productivity. This evidence suggests that worker-firm sorting has

potentially significant consequences both empirically and in terms of welfare.

Recently, the increased availability of matched employer-employee data and the seminal contribution of Abowd, Kramarz and Margolis (1999) (AKM hereafter) have brought renewed interest to measuring and modeling sorting in the labor market. AKM jointly estimate worker and firm fixed effects from linked wage data. The estimated worker and firm effects may be interpreted as estimates of worker and firm productivity. Attention has centered on the question of *positive assortative matching*. Under positive assortative matching, or assortative matching for short, the best workers match with the best firms and the worst workers match with the worst firms. Negative assortative matching obtains when the worst workers match with the best firms, and the best workers with the worst firms. Perhaps surprisingly, AKM and the large subsequent empirical literature found mixed evidence of positive assortative matching. Authors have proposed a number of explanations, including match effects, limited mobility bias, and non-monotonicity of wages in firm productivity. All three may be serious issues, and it appears infeasible to simultaneously correct the AKM estimator along all these dimensions.

In this paper I use a different methodology and reach different conclusions. I find that, conditional on observable firm characteristics, faster growing firms tend to hire more productive workers. This suggests a kind of positive assortative matching, where fast growing and more productive firms match with more productive workers. My results contrast with much of the previous literature. The different results are due to a difference in methodologies. In my baseline specifications I condition on a variety of firm characteristics, including industry, size, and firm

fixed effects. The positive correlation changes dramatically when firm characteristics are not controlled for. Without firm characteristics the relation between firm growth and worker quality is highly nonmonotonic with a maximum around zero growth. This result echoes the findings in the previous literature, which have generally not conditioned on firm characteristics, and found mixed evidence of positive sorting.

I attribute these differences to segmentation in the labor market. When I condition on firm characteristics I implicitly compare firms to very similar firms, which are likely to inhabit similar labor markets.¹ Thus the baseline specifications, controlling for industry and other characteristics, pick up positive sorting within each segment of the labor market. The best doctors work for the best hospitals, the best pilots work for the best airlines, and so on. When I do not control for firm characteristics I am comparing nurses to pilots and hospitals to airlines. Given the significant differences in technology and human capital between sectors it is possible that sorting patterns between sectors differ from those within sectors. This argument suggests that while assortative matching may be the norm within any given labor market, the distribution of workers and firms across markets need not be assortative. Informally, intuition about assortative matching is based more on examples of specific industries or occupations, rather than economy-wide matching patterns. Thus it need not be surprising that I find evidence of positive sorting within narrow markets, but neither I nor the related literature find clear evidence

¹Controlling for industry, size and location clearly narrow the scope of comparison towards a single labor market. Including firm fixed effects makes the scope of comparison a single firm (using only time variation). This may seem to be a different kind of approach from controlling for industry, size, etc. However, we can interpret firm fixed effects as controlling for unobserved firm characteristics. It is highly doubtful that firm characteristics observable in the data are sufficient to precisely identify the labor market the firm faces. Under the assumption that firms do not jump between markets often, firm fixed effects are an effective means to proxy the firm's market in the absence of more detailed firm characteristics.

of positive sorting across the entire economy.²

A second important difference between my work and the previous literature is that I focus on firm employment growth. There is little, if any, previous research on how the quality of hires changes as a firm's employment growth varies. Firm growth is important to understand in its own right, but also serves as a plausible proxy for firm productivity. The measurement of firm productivity is critical for studying assortative matching. Recent papers have argued that the AKM fixed effects do not necessarily capture firm productivity. In my data I cannot measure productivity directly, so I propose firm growth as a proxy for firm productivity. This connection is supported by the extensive literature showing that more productive firms tend to grow relative to their less productive peers (See [Bartelsman and Doms \(2000\)](#), [Foster et al. \(2001\)](#), [Foster et al. \(2013\)](#) and the related literature). Firm growth also has the advantages of being trivial to compute and exhibiting significant time variation. Previous studies have abstracted from firm productivity dynamics, and thus do not use exploit the time dimension when studying sorting. The presence of time variation also allows me to use firm fixed effects as controls in some specifications.³

In addition, I use lagged worker wages to measure worker quality rather than AKM worker fixed effects. This is again motivated by questions regarding the

²A closely related issue is whether we can meaningfully rank workers and firms in disparate markets according to productivity. It is straightforward to construct at least a partial order of doctors or of pilots, but more difficult to compare productivity across occupations. When skill is multi-dimensional the assortative matching model is not appropriate, since it presumes a scalar worker (or firm) productivity variable. A generalized Roy model may be more appropriate when comparing different sectors.

³A potential concern with using employment growth to proxy productivity is that productivity growth may be caused by or concurrent with *downsizing* of the workforce. In such a case employment growth and productivity would be negatively correlated. While there is no doubt that there are examples of this phenomenon, it seems unlikely that productivity-enhancing downsizing is more common than newly-productive firms adding to their workforce. This is one area for future research.

consistency of the AKM fixed effects. Using lagged worker wages raises a number of potentially serious measurement issues, which I address in Section 1.3. Chief among these concerns is that on the job search generates a correlation between lagged and current wages independent of any worker productivity effects. I discuss this issue in detail in Section 1.3 and argue that this problem can be overcome by focusing on workers hired from nonemployment, as I do.

Motivated by the empirical findings, I develop a theory of sorting with search frictions and productivity shocks. My focus is on characterizing equilibrium matching patterns within a single market, and finding conditions under which (1) better workers match with better firms, (2) better workers match with faster growing firms. The model incorporates costly search, wage bargaining, ex-ante heterogeneity in worker and firm productivity, productivity shocks, and multi-worker firms. To my knowledge this is the first paper to characterize sorting in such an environment. Despite its richness the model is tractable, and I analytically derive restrictions on the production function which ensure that in equilibrium faster growing firms hire better workers, reproducing the empirical findings. The argument requires two steps. First, I show that strong production complementarities between worker and firm type (i.e. strict log-supermodularity of the production function) ensure positive assortative matching: better workers match with better firms. Second, I show that under positive assortative matching better firms are also faster-growing. This point, while intuitive, is not trivial to prove when workers are heterogeneous. Taken together these results imply a positive correlation between firm growth and the quality of hires as well as assortative matching by types.

An important feature of my model is that firms can employ multiple workers, whereas [Shimer and Smith \(2000\)](#) and others focus on single worker firms, or firms that cannot freely post additional vacancies. This difference turns out to matter a great deal. The reason is that when firms can post additional vacancies they have no opportunity cost of hiring a given worker. A central feature of the [Shimer and Smith \(2000\)](#) model is that both parties face a tradeoff between matching immediately or waiting for a better partner. If firms can employ multiple workers, they can hire a given worker today *and continue searching* (posting vacancies) in subsequent periods. This, along with linear production, eliminates the opportunity cost of hiring of for firms that are in contact with workers, greatly simplifying the analysis. This tractability allows me to explore other aspects of the model. I show analytically how quality of newly hired workers relates to the quality of incumbent workers at a firm, and how changing the frequency of productivity shocks changes matching patterns. These implications have not been well studied in the literature, and represent avenues for future research.

This paper also addresses the question of *recruiting intensity*. The recent work of [Davis et al. \(2013\)](#) has called into question some of the basic assumptions of the workhorse search model. They find that fast growing establishments tend to fill their job openings more quickly. This is at odds with the textbook search theory, which assumes the vacancy fill rate is exogenous to the firm and consequently employment growth is linear in vacancy postings at the firm level. The conclusion of [Davis et al. \(2013\)](#) is that firms are using other margins to attract workers, besides simply posting more vacancies. These margins may include advertising intensity, selectivity, or wage offers. The results in this paper show that selectivity

does indeed change with firm growth, but not in the direction one might expect: faster growing firms hire *better* workers on average. The theory reconciles the two empirical results through the wage margin. According to the theoretical model, a faster growing firm is willing to pay higher wages to all workers. Under positive assortative matching this allows the firm to attract high-type workers, while they continue to hire low type workers at the same time. Thus, faster growing firms fill vacancies faster by offering better wages and hiring more of the workers they meet.

1.2.1 Literature Review

The empirical literature on assortative matching in the labor market has developed quickly since [Abowd et al. \(1999\)](#). They used linked employer-employee data, similar to mine, to estimate worker and firm fixed effects. For example, if w_{ijt} is the log wage of worker i at firm j in period t and x_{it} is a vector of time-varying worker characteristics, an AKM regression has the form

$$w_{ijt} = x_{it}\beta + \theta_i + \psi_j + \varepsilon_{ijt}$$

where ε_{ijt} is an error term and θ_i and ψ_j are worker and firm fixed effects that can be estimated under some mild identification assumptions. The estimated worker fixed effect $\hat{\theta}_i$ measures how much more or less the worker is paid relative to his coworkers (holding x_{it} constant). The firm fixed effect $\hat{\psi}_j$ measures the average wage premium the firm pays, compared to other firms. If θ_i reflects worker productivity and ψ_j firm productivity then the empirical correlation $\text{Corr}[\hat{\theta}_i, \hat{\psi}_j]$ measures whether good workers match with good firms. If $\text{Corr}[\hat{\theta}_i, \hat{\psi}_j]$ is large

and positive, then productive workers tend to end up at productive firms. If is it close to zero then workers are not strongly sorted across firms. AKM found $\text{Corr}[\hat{\theta}_i, \hat{\psi}_j]$ to be small and *negative*. Subsequent work (see [Abowd et al. \(2002\)](#), [Susana Iranzo and Tosetti \(2008\)](#), [Andrews et al. \(2008\)](#), [Woodcock \(2008\)](#), [Lopes de Melo \(2013\)](#)) largely confirmed AKM’s finding in other data sets, finding negative or small positive correlations between worker and firm effects. These results have been especially puzzling given our tacit intuition that positive sorting is the norm, as least in some markets.

Many criticisms have been made of the AKM methodology as applied to sorting. Researchers have corrected the AKM methodology to address three major issues: [Andrews et al. \(2008\)](#) and [Andrews et al. \(2012\)](#) study limited mobility bias, [Woodcock \(2008\)](#) takes account of match effects, while [Lopes de Melo \(2013\)](#) addresses nonmonotonicity of wages in firm type.⁴ See also [Abowd et al. \(2012\)](#), who interpret the AKM fixed effects in the context of [Shimer \(2005\)](#)’s directed search model. Each of these papers uses a variant of the AKM methodology corrected for some possible biases, while abstracting from the other issues. It appears infeasible to correct the AKM estimator for all three issues simultaneously. Non-monotonicity of wages has received the most attention in the literature, perhaps reflecting a belief that it is a more serious problem than limited mobility bias and match effects. In an important contribution, [Lopes de Melo \(2013\)](#) shows that when the data is generated from a search model such as [Shimer and Smith \(2000\)](#), the AKM firm fixed effects are poor proxies for firm productivity. However, estimated worker effects appear to accurately reflect worker productivity. [Lopes de](#)

⁴[Eeckhout and Kircher \(2011\)](#), [Lise et al. \(2013\)](#), [Hagedorn et al. \(2012\)](#) and [Bagger and Lentz \(2012\)](#) also discuss nonmonotonicity of wages, but do not propose any correction to the AKM methodology.

Melo (2013) shows that in Brazilian data workers with high fixed effects tend to work in firms with other high fixed effect workers. In other words, workers are clustered with similar coworkers. This implies a kind of sorting, but without accurate firm productivity data we cannot tell how the workers are sorting. It might be that the best workers are clustered at the worst firms, or workers may be clustered according firm characteristics unrelated to firm productivity. To summarize, there are several potentially serious problems with using the AKM fixed effect methodology to study sorting. These issues have been addressed separately, but their interactions have not been explored. Correcting for the most serious issue, nonmonotonicity of wages, comes at the cost of only being able to study clustering of workers with similar coworkers, instead of studying sorting of workers across firms of different productivity.

Here I contribute to the literature by proposing an alternative way to identify sorting. I use firm growth to proxy firm productivity and lagged worker wages to proxy worker productivity. These measures have the advantages of being directly observable in the data. In my specifications I find evidence of positive sorting that is independent from the criticisms of the AKM fixed effect methodology. Of course, I face other measurement issues of my own, which I address in Section 1.3. Other researchers have departed from the AKM methodology and used other data to identify sorting. Bartolucci and Devicienti (2012) use firm balance sheet data to measure firm type directly. Lise et al. (2013) present an original strategy, using a method of simulated moments estimator to measure sorting from longitudinal worker data without links to firm data. They argue that assortative matching can be identified from the profile of worker wages after a spell of unemployment.

Intuitively, workers exiting unemployment tend to be mismatched, and the cross sectional distribution of wages will be compressed. As time passes job to job transitions should result in better wages for all workers. If sorting is important then then productive workers should experience more wage growth, as the degree of mismatch is reduced. Then assortative matching may be identified by the spreading out of wages across workers after an unemployment spell. Since they rely on worker side data only, they use the structure of their model to infer the sign and strength of assortative matching. To keep the model tractable they are forced to abstract from human capital accumulation, which may also produce a spreading out of wages. In contrast, I exploit the matched nature of my data to measure sorting in an arguably more direct fashion. My proxy for worker productivity is a lagged moving average of wages, which allows human capital to evolve over time. In addition, [Lise et al. \(2013\)](#) abstract from firm growth, which is part of my focus. In closely related work, [Bagger and Lentz \(2012\)](#) propose a structural search model and estimate it on linked Danish data. The model is based on [Lentz \(2010\)](#), where worker search intensity is endogenous. With complementarities in production, high productivity workers gain more from searching on the job, and consequently search more intensely and tend to match with better firms faster. Firm dynamics and productivity shocks are not modeled. Their estimation of the model requires a large number of auxiliary models and moments. My work complements theirs by focusing on directly observable moments in the data and presenting an alternative theoretical model. While [Bagger and Lentz \(2012\)](#) focus on search intensity and search while employed, I focus on transitions from unemployment and the role of firm dynamics.

The theoretical literature on assortative matching has grown along with the applied literature. In his influential study, [Becker \(1973\)](#) examined assortative matching in the context of a frictionless marriage model with transferable utility. [Shimer and Smith \(2000\)](#), [Shi \(2001\)](#), [Shimer \(2005\)](#), [Atakan \(2006\)](#), and [Eeckhout and Kircher \(2010\)](#) reexamine Becker's results under various kinds of search frictions, while maintaining the assumption of one-to-one matching and no productivity dynamics. Usually stronger forms of complementarities are needed to ensure sorting when agents face search frictions. In my theoretical work I extend the widely used model of [Shimer and Smith \(2000\)](#) to incorporate multi-worker firms and firm productivity shocks. I find that introducing multi-worker firms considerably simplifies some of the technical complications in [Shimer and Smith \(2000\)](#), allowing me to characterize matching patterns and derive comparative statics in a straightforward fashion. [Lentz \(2010\)](#) and [Bagger and Lentz \(2012\)](#) discuss a similar tractability result in a different model, but do not address the firm dynamics that are my focus. [Lise and Robin \(2013\)](#) and [Eeckhout and Kircher \(2012\)](#) also allow for multi-worker firms, but again abstract from idiosyncratic firm dynamics. [Lise et al. \(2013\)](#) allow for match productivity shocks in a sorting model, but do not present analytical results and do not use data that (directly) measures firm dynamics. To summarize, I contribute to this fast-growing literature by modeling sorting with multi-worker firms that experience productivity shocks, and deriving analytical results.

A relatively new literature studies how firms fill job openings. [Davis et al. \(2013\)](#) document that faster-growing firms fill their vacancies more quickly. This suggests that growing firms are using other margins to attract workers, besides just posting

more job openings. [Kaas and Kircher \(2011\)](#) develop a theoretical directed search model, where growing firms post higher wages to attract homogeneous workers more quickly. [Belzil \(2000\)](#) documents that faster growing firms do indeed seem to pay identical workers higher wages.⁵ [Brown and Matsa \(2012\)](#) show that job postings of financially distressed firms attract fewer and less qualified applicants, suggesting that a shrinking, distressed firm will have to wait longer to fill its vacancies. In this paper I develop a related point, that fast growing firms hire better workers. By itself, this does not address vacancy fill rates. But my theory implies that fast growing firms pay better wages (similar to [Kaas and Kircher \(2011\)](#), albeit in a random search context) and are thus able to hire more of the workers they meet, increasing the speed with which vacancies fill.

Firm employment growth is the first difference of firm size, so my work on firm growth is related to the extensive literature on firm size and wages (see [Brown and Medoff \(1989\)](#) and [Abowd et al. \(1999\)](#), [Oi and Idson \(1999\)](#) and [Gordanier \(2012\)](#) among others). This literature has typically found that larger firms pay more and employ better workers, according to a variety of measures. This is consistent with my finding that faster growing firms (i.e. firms that are getting larger) hire better workers. However I find that the firm size and firm growth effects are distinct, since the growth effect survives the inclusion of firm fixed effects and controls for (beginning of period) firm size. In contrast, firm size effects are imprecisely estimated and ambiguously signed in this specification. This pattern suggests that part of the positive worker quality-firm size correlation found in the literature is due to unobserved time-invariant firm characteristics, rather than a firm's size

⁵Belzil uses Danish administrative data on matches and wages. The LEHD, which I use, only has quarterly earnings (and no hours data) so I cannot replicate Belzil's wage results.

at any particular point in time. This is consistent with models in the literature where firm size proxies productivity, opportunities for workers to specialize, or other unobserved characteristics.

Finally, there has been some attention devoted to the hiring policies of start-ups (see [Dahl and Klepper \(2007\)](#)) and high-growth firms (see [Coad et al. \(2011\)](#)). These papers study the correlation between indicators of a firm's long run prospects (long run growth, for example) and the characteristics of their hires. In other words they study primarily cross-sectional differences in hiring. In contrast, my focus is on within firm variation in growth and worker productivity. In addition, I emphasize the need for nonparametric methods to accurately identify the relationship between growth and quality of hires. Some of the most interesting findings in [Section 1.3.4](#) are highly nonlinear relations that would not be detected in a linear specification.

1.3 Empirics

My interest is in the relationship between firm growth and the productivity of new hires. I will show that when a given firm grows faster, it hires more productive workers. The productivity of the new hires is a characteristic of the workers themselves, not the firm they are hired into. To identify this phenomenon I need to measure firm growth and the productivity of newly hired workers. In addition, the data allow me to control for a number of observable and unobservable firm characteristics. Controlling for firm characteristics is critical to properly measuring the *ceteris paribus* impact of a change in firm growth on the quality of hires.

I take the quality of new hires to be the dependent variable, and regress it on firm

growth and a vector of controls. The empirical specification has the form

$$Q_{j,t} = m(g_{j,t}) + \beta \mathbf{Z}_{j,t} + \varepsilon_{j,t}$$

where $Q_{j,t}$ is a measure of the productivity of new hires at firm j in period t , $g_{j,t}$ is the employment growth rate of firm j in period t , $\mathbf{Z}_{j,t}$ is a vector of control variables, and $\varepsilon_{j,t}$ is the error term. The vector β are the coefficients on the controls and $m(\cdot)$ is an unknown function to be estimated. The function $m(\cdot)$ summarizes the relationship between firm growth and the quality of hires, so it is the focus of what follows.

The main research question is what happens to $Q_{j,t}$ when $g_{j,t}$ changes, holding all else constant. Thus it is important that $\mathbf{Z}_{j,t}$ include extensive controls for firm characteristics. The data I use includes industry, location, size and some other firm information. However, there is likely to be a large amount of unobserved firm heterogeneity imperfectly correlated with these characteristics. This cross-firm heterogeneity in growth rates and $Q_{j,t}$ will bias any estimate of $m(\cdot)$. Motivated by this concern I include firm fixed effects in my main specifications. These specifications use only within firm variation to identify $m(\cdot)$, and are thus robust to unobserved cross sectional heterogeneity. Inclusion of firm fixed effects brings the empirical specification closer to the thought experiment of the *ceteris paribus* impact of a change $g_{j,t}$ on $Q_{j,t}$.

It is straightforward to construct a firm employment growth variable, $g_{j,t}$, from the data. More complicated is the construction of $Q_{j,t}$, the productivity of new hires. In this context worker productivity are those characteristics that make the worker more productive regardless of job. Thus $Q_{j,t}$ does not include match-

specific productivity, or characteristics that are only valued by a narrow range of jobs. In the theoretical part of this paper I capture productivity by assuming that a higher “type” worker produces more on any given job than a low type worker. Thus, high type (high productivity) workers have an *absolute advantage* in all jobs. This simple setup ensures that the ranking of workers is unambiguous, and is widely used in the literature.

Intuition and the models discussed in Section 1.2.1 suggest that high productivity workers should earn more on average. If a worker produces more in all jobs, he ought to earn at least as much as a lower productivity worker. Then a worker’s average earnings are a noisy proxy for his productivity. My measure of the quality of hires is based on this idea. I build the quality of hires variable in two steps. First, I construct a productivity measure for each worker in each period. Since my data are quarterly this gives me a measure of a worker’s productivity in each quarter. This measure is essentially a backward looking moving average of earnings. Second, I find the set of workers hired by firm j in quarter t , and average their productivity measure. This gives me the average productivity of workers hired by the firm in that quarter.

Formally, let

$$q_{i,t} = \frac{1}{8} \sum_{k=1}^8 \text{earn}_{i,t-k} \quad (1.1)$$

where $q_{i,t}$ is the worker’s measured quality, and $\text{earn}_{i,t-k}$ is the worker’s total earnings (in all jobs) in quarter $t - k$. $q_{i,t}$ is thus a two year lagged moving average of earnings. I have experimented with longer and shorter moving averages and found no significant effect on my results. Longer horizons for the moving average reduce the noise from search frictions, unpaid leave, and firm-level shocks.

However, when there is human capital accumulation (and depreciation) the most recent earnings are the best indicators of current productivity. The eight quarter horizon was chosen to balance these two concerns.

Let $H_{j,t}$ be the set of workers *hired* by firm j in period t . Then my quality of hires variable can be written as

$$Q_{j,t} = \ln \left(\frac{1}{|H_{j,t}|} \times \sum_{i \in H_{j,t}} q_{i,t} \right).$$

This is simply the log average of $q_{i,t}$ over the set of hires. In practice I will use several variants of $Q_{j,t}$ to address concerns about measurement and selection. These variants are denoted by superscripts.

1.3.1 Nonparametric Estimation

Given $g_{j,t}$, $Q_{j,t}$ and $\mathbf{Z}_{j,t}$ I am prepared to estimate $m(\cdot)$. As will be made clear in the results section $m(\cdot)$ is highly nonlinear, so a linear regression is inappropriate. Here I follow [Davis et al. \(2013\)](#) and reformulate the problem as a dummy variable regression. Intuitively, the idea is to partition the domain of $m(\cdot)$ into narrow intervals (one percentage point wide in my application) and create indicator variables for each interval, or bin. An indicator variable is set to 1 if $g_{j,t}$ falls into the bin and zero otherwise. Then I regress $Q_{j,t}$ on the full set of indicator variables and $\mathbf{Z}_{j,t}$. The coefficient on a given indicator is then the sample average of $m(\cdot)$ within that bin conditional on $\mathbf{Z}_{j,t}$.⁶ Plotting the bin coefficients yields

⁶This formulation is similar to but distinct from more standard nonparametric methods, such as kernel regression. I am using comprehensive administrative data and include firm fixed effects in my regressions, which can result in millions of fixed effects. Under these circumstances estimating ordinary least squares requires special procedures. Estimating a kernel regression in the presence of this many fixed effects would not be feasible without additional specialized software. Ultimately, the dummy variable regressions are a convenient way to capture the shape

an estimate of $m(\cdot)$ on a discretized domain. where f_i are dummy coefficients to be estimated. All reported regressions are weighted by the number of hires. Unweighted and employment weighted results are similar.

1.3.2 Measurement and Selection Issues

Before turning to the data and results, there are several potential measurement problems that must be addressed. I show how alternative samples and dependent variables can be used to avoid some measurement problems. Where there are not good alternatives, I present evidence that the problems are not too severe in the data.

Recall that $Q_{j,t}$ is the log average of worker's earnings in all quarters, regardless of whether they were employed. In other words, it captures average earnings, not average earnings while employed. This choice is motivated by the fact that the extensive margin contains information about a worker's productivity. If a worker is unable to find a job, it tells us he may not be a high quality employee. In addition, in theoretical models average lifetime earnings are monotone increasing in worker productivity, but earnings while employed may not be. However, if shocks unrelated to worker productivity (e.g. health shocks or fertility) move the worker in and out of employment, then average earnings may not be an accurate proxy for productivity. To examine this possibility I use a measure of worker quality that excludes nonemployment. As an alternative to $q_{i,t}$ I define

$$q_{i,t}^{FQ} = \frac{\sum_{k=1}^8 \text{earn}_{i,t-k}^{FQ}}{\sum_{k=1}^8 \mathbf{1}\{\text{earn}_{i,t-k}^{FQ} > 0\}}$$

of the relationship.

where $earn_{i,t}^{FQ}$ is the earnings of the worker in jobs that lasted all of quarter t . Thus, periods of nonemployment and partial employment are excluded. The corresponding quality of hires measure is

$$Q_{j,t}^{FQ} = \ln \left(\frac{1}{|H_{j,t}^{FQ}|} \sum_{i \in H_{j,t}^{FQ}} q_{i,t}^{FQ} \right) \quad (1.2)$$

where $H_{j,t}^{FQ}$ is the subset of hires that had at least one quarter of full quarter employment in the previous two years. I report regressions using both all quarter earnings and full quarter earnings.⁷ While the magnitudes of the effects differ, the qualitative patterns are insensitive to the choice of variables.

Relatedly, note that $q_{i,t}$ will be a poor measure of productivity for new entrants to the full time labor force. Recent high school or college graduates will have zero or low lagged earnings for reasons unrelated to their desirability as workers. To mitigate this problem I only consider hires of workers aged 26 or older. The results are not sensitive to dropping this restriction, or replacing it with a requirement that the worker have positive earnings two or more years before being hired.

Next, there is a clear endogeneity issue when the worker's wages depend on the firm they work for. For example, if fast growing firms pay more, then wages of current employees will be positively correlated with firm growth, even if there are no differences in worker productivity. This concern motivates my use of a *lagged* moving average to measure the quality of new hires. $Q_{j,t}$ contains only wages from periods $t-1$ and earlier, before the workers were hired by firm j . Thus

⁷Both $Q_{j,t}^{FQ}$ and $Q_{j,t}$ involve taking a logarithm of earnings, so observations where the firm only hires zero-earnings workers are dropped. It might be wondered whether this creates a selection problem. Alternative specifications suggest that this is not a serious problem. In particular, the empirical results are largely unchanged when I use the raw average of worker earnings as the dependent variable. The same applies for the inverse-hyperbolic sine transformation.

fluctuations in firm j 's wages do not directly contaminate $Q_{j,t}$.

However, we might be concerned about the workers' previous employers. $q_{i,t}$ is a function both of a worker's inherent productivity as well as their luck in the labor market. As emphasized by the voluminous wage dispersion literature,⁸ workers of identical productivity may be paid different wages depending on which employers they find. Equally productive workers will have different $q_{i,t}$ because they had different previous employers, and have spent different durations in nonemployment. Thus we should think of $q_{i,t}$ as measuring the worker's true productivity plus some noise. Measurement error alone in $q_{i,t}$ will not likely cause serious bias, since $q_{i,t}$ goes into the dependent variable in my regressions.⁹ But with on the job search there is reason to think that a worker's lagged wage, including the "luck" component, influences his choice of whether to be hired. In particular, if a worker has a continuing job in period $t - 1$, his wage at that job can act as a reservation wage: workers will refuse any offer below what they currently earn. If firm wages are positively correlated with firm growth (as Belzil (2000) documents) then only a fast growing firm will be able to hire the worker away from the current employer. This means that the occurrence of a hire depends on the worker's $t - 1$ wage, *including any luck component of the wage*. Thus, faster growing firms will hire more workers from high wage firms, regardless of worker productivity. This creates a positive correlation between firm growth and $Q_{j,t}$, independent of any true differences in worker productivity. To address this issue

⁸See Mortensen (2003) and the literature reviewed therein. Burdett and Mortensen (1998) and Moscarini and Postel-Vinay (2013) each present models where homogeneous workers are paid differently.

⁹ $Q_{j,t}$ is a nonlinear transformation of $q_{i,t}$, so measurement error is not entirely innocuous. But note that the hire occurs at a large firm hiring multiple workers simultaneously. As more workers are hired, (uncorrelated) measurement error averages out of $\frac{1}{|H_{j,t}|} \times \sum_{i \in H_{j,t}} q_{i,t}$ before the log transformation.

I restrict attention to hires from nonemployment.¹⁰ Nonemployed workers do not have a current wage to act as a threat point, and so their choice of whether to be hired is a function of their productivity, not their (random) current employer. All the empirical results reported below calculate $Q_{j,t}$ and $Q_{j,t}^{FQ}$ on the subset of hires that came from nonemployment. Results including job-to-job hires (not reported) are qualitatively similar.

Variation in the reservation wage is another possible concern. It may be that workers with high lagged wages have higher reservation wages even after entering nonemployment. This could be because the worker falsely believes, based on their previous luck, that they can find another high wage job easily (see Hogan (2004)). Or it could simply be that some workers value leisure more, and so demand higher wages. These mechanisms will induce a positive correlation between lagged earnings while employed and firm growth, since only high growth, high wage firms can meet the reservation wages of these workers. But these mechanisms would also induce a weaker or negative correlation between lagged *all quarters* earnings and firm growth, since the more selective workers would spend more time unemployed. My empirical results run opposite to this prediction. I find that both all quarters earnings and full quarters earnings are positive functions of firm growth, and that all quarters earnings increase faster than full quarter earnings. Thus, while I cannot rule out variation in reservation wages, my results are inconsistent with reservation wages being the dominant force relating firm growth to measured

¹⁰The LEHD data I use only include data on employment status, not search activity. Thus I cannot distinguish between unemployed workers and those out of the labor force. All workers without jobs are taken together as nonemployed. Information on search activity would be useful for my purposes, but not critical. Recent work has shown that while self-reported search activity does predict future employment status, the link is noisy and unstable over time. Workers who exit the labor force are more likely to have low productivity, which my quality measure $q_{i,t}$ will capture.

worker quality.

1.3.3 Data and Sample

I use the linked employer-employee data from the Longitudinal Employer-Household Dynamics (LEHD) program at the U.S. Census Bureau. These files are derived from state level unemployment insurance records, which require firms to report quarterly wage and salary earnings of each covered employee to the state unemployment agencies. The main LEHD files consist of these earnings records, along with worker identifiers and firm identifiers. Limited worker and firm characteristics are also included. The worker identifiers are consistent across firms, time and states, so it is possible to identify the same worker at different points in time, or at different firms. The firm identifier, called a state employer identification number (SEIN) is similarly consistent across time, allowing me to follow an SEIN over time. I will use the SEIN as my firm concept in this paper.¹¹

Importantly, the LEHD files cover the vast majority of private non-farm jobs. Estimates put the coverage of such jobs at 96%. The main exclusions are federal workers, agricultural workers, and the self-employed. I draw firm (SEIN) data from six states: DE, IL, OH, IN, WI, VT. Importantly, when I construct worker earnings histories I draw on data from all states participating in the LEHD. This means that if a worker moves from New York and is hired by a firm in Chicago I include their New York earnings history in $q_{i,t}$. My sample period runs from 2000:Q1

¹¹The SEIN identifies the reporting entity for UI purposes. In single unit firms, the SEIN is synonymous with the firm. However, multi-unit firms may have several SEINs, and each SEIN may include several establishments (though the same establishment is never a part of two SEINs). Thus, a SEIN falls somewhere in between an establishment and a firm. My results are qualitatively unchanged if I restrict the sample to include only SEINs that did not report a change in the number of establishments. This sample restriction brings the data as close as possible to an establishment definition.

through 2008:Q2, the last quarter of data available in the research snapshot I use. The results are not sensitive to the inclusion of the Great Recession. I use as broad a sample as possible. In the reported results I only drop quarters where a given firm enters or exits, since there is good reason to think that selectivity and growth are determined by other factors in those quarters. The results are qualitatively unchanged when much more stringent restrictions are imposed. The construction of firm growth, hires from nonemployment and other variables requires some data processing. The main issue is accounting for the fact that the LEHD variables are reported quarterly, so they do not necessarily approximate high frequency, point-in-time data. For example, the raw LEHD earnings records for a quarter include all workers employed *for any duration* at a given firm in a given quarter. This does not necessarily reflect the firm’s actual employment at any point in time during the quarter. The solution is to define “beginning of quarter” and “end of quarter employment”.¹² In quarter t the beginning of quarter employment is the number of workers observed to work at the firm both in $t - 1$ and t . End of quarter employment is the number of workers employed both in t and $t + 1$. These measures more or less reflect firm employment on the first and last day of the quarter. Additional data processing details are in Appendix A.1. Table 1.1 presents employment weighted summary statistics on worker flows in my sample. As has been well documented, these flows are large: a firm typically hires and separates 17% of its workforce over the course of a quarter. The exact values are somewhat lower than those reported in Lazear and Spletzer (2013) (who use similar data), in part because I exclude entry and exit quarters. Average

¹²See http://lehd.ces.census.gov/doc/QWI_101.pdf for further documentation, and the use of these concepts in public-use data.

Variable	Mean	Std. Dev.
Hires Rate	0.17	0.25
Separation Rate	0.17	0.24
Employment Growth Rate	0.00	0.19
NE share of hires	0.53	0.22
EE share of hires	0.22	0.20

Table 1.1: Worker Flows

employment growth is indistinguishable from zero. This is partially a reflection of the states in the sample, and again partially due to my exclusion of entry and exit quarters. Importantly, the standard deviation of employment growth is 19 percentage points. To exclude outliers I will focus on the interval between -30% and 30% growth in my graphical results.

The last two rows of Table 1.1 report the share of hires that come from nonemployment (NE Hires) and from other jobs (EE Hires). NE hires account for 53% of total hires, while EE hires account for 22%.¹³ The remainder are hires that could not clearly be assigned to either of these categories. The shares of hires from nonemployment and employment are roughly in line with those reported by Fallick and Fleischman (2004) and Bjelland et al. (2011).

Table 1.2 reports summary statistics for the characteristics of new hires. The top two rows summarize the main dependent variables in the regressions: log average earnings ($Q_{j,t}$) and log average full quarter earnings ($Q_{j,t}^{FQ}$). Recall that these are logged averages of newly hired workers' lagged wages. They are calculated using only hires from nonemployment, to mitigate the selection concerns raised in Section 1.3.2. The sample average of $Q_{j,t}$ is 8.06, while the average of $Q_{j,t}^{FQ}$

¹³Some of these are workers who held a job for several periods, then were hired at a new firm but continued working at their old job. I exclude these workers for the same reason I exclude EE hires, to avoid the selection effect of lagged wages.

is 8.82. The difference between the two implies that, roughly speaking, average earnings while employed are 80% greater than average total earnings. Turning to the last two rows, this difference is clearly attributable to the fraction of time workers spend employed. “Fraction of Quarters Employed” is the average fraction of quarters a new hire had any employment. Like $Q_{j,t}$ and $Q_{j,t}^{FQ}$, this fraction is calculated on the 8 quarter lagged window. “Fraction of Quarters Fully Employed” is the fraction of quarters the worker held a job throughout the entire quarter. The means show that newly hired workers (from nonemployment) spent between 40% and 57% of the past 2 years at work. This explains the significant differences in the levels of $Q_{j,t}$ and $Q_{j,t}^{FQ}$, since only $Q_{j,t}$ includes time spent nonemployed.

Variable	Mean	Std. Dev.
$Q_{j,t}$	8.06	1.03
$Q_{j,t}^{FQ}$	8.82	0.70
Fraction of Quarters Employed	0.57	0.21
Fraction of Quarters Fully Employed	0.40	0.18

Table 1.2: Characteristics of New Hires

1.3.4 Results

In my baseline specifications I estimate $m(\cdot)$ using $Q_{j,t}$ and $Q_{j,t}^{FQ}$ as the dependent variables. In $\mathbf{Z}_{j,t}$ I include an extensive set of controls, including location (state), industry, quarter, interactions of all three, flexible controls for firm size, interacted industry and seasonal dummies, and firm fixed effects.¹⁴ After these controls are included, $m(\cdot)$ is identified by within firm variation that is not correlated with aggregate, industry or location effects. Figure 1.1 plots the estimates of $m(\cdot)$ using

¹⁴Firms change industries, so industry is not perfectly collinear with the firm fixed effects. The interactions of industry with other variables are decidedly not collinear with firm fixed effects.

$Q_{j,t}$ and $Q_{j,t}^{FQ}$ as the dependent variables. The horizontal axis is firm employment growth, restricted to the $[-30\%,30\%]$ interval. The vertical axis measures $Q_{j,t}$ (All Quarter Earnings) and $Q_{j,t}^{FQ}$ (Full Quarter Earnings). The estimates of $m(\cdot)$ have been normalized to average zero on the interval $[-30\%,0]$.¹⁵

For both specifications $m(\cdot)$ is relatively flat when growth is negative, but increases when growth is positive. Both relationships are overwhelmingly statistically significant. The t-statistic for the average slope on the $[-30\%,30\%]$ interval is 9.37 for $Q_{j,t}^{FQ}$ and 40.98 for $Q_{j,t}$. Thus the positive average relationships shown in Figure 1.1 are significant well beyond conventional thresholds.

In terms of magnitudes, the results for $Q_{j,t}$ imply that when a firm goes from zero growth to 30% growth they hire workers with roughly 9% higher lagged earnings. Using $Q_{j,t}^{FQ}$, the same change in employment growth leads to a 2% increase in lagged full quarter earnings. While both are statistically significant, the magnitude of the effect for $Q_{j,t}$ is larger. This is due to the inclusion of nonemployment. When a firm grows more quickly, they not only hire workers with higher wages in their old jobs (the $Q_{j,t}^{FQ}$ results), but they also hire workers who have experienced less nonemployment.

¹⁵For ease of presentation Figure 1.1 shows the three bin moving average of $m(\cdot)$, rather than the raw estimate. This has the effect of smoothing the relationship slightly, but this effect is quantitatively and qualitatively insignificant.

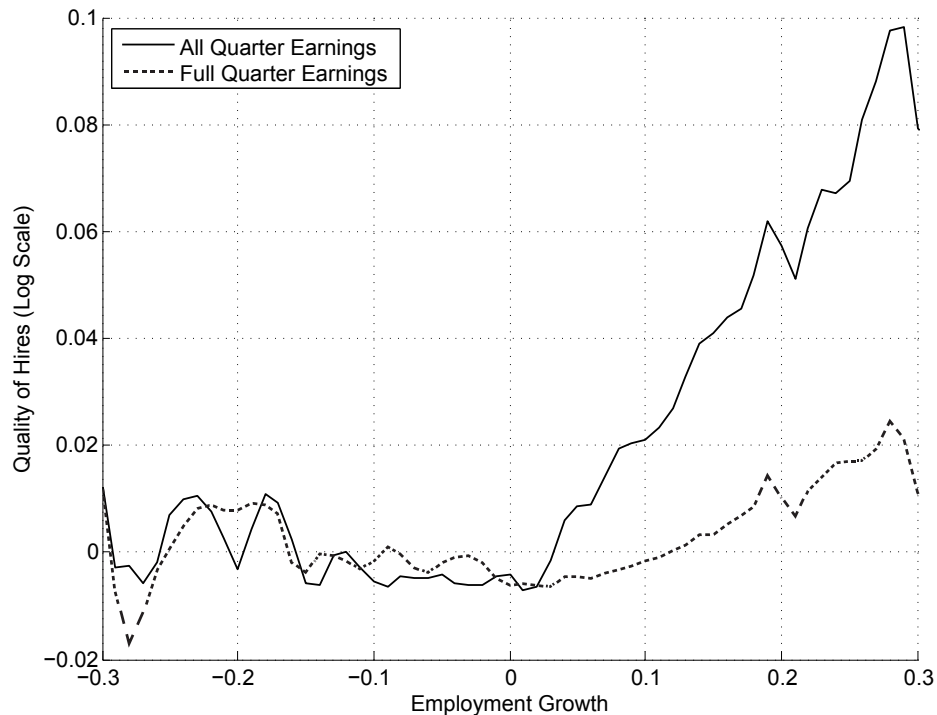


Figure 1.1: Baseline Specification

It might have been expected that increased firm growth would be associated with *lower* quality of hires, as growing firms exhaust their pool of potential employees and are forced to lower hiring standards. The results in Figure 1.1 show this is decidedly not the case. Instead, the results appear to support positive assortative matching. Firms which grow more quickly are presumably more productive, and are observed to be hiring more productive workers. In other words better workers sort themselves into more productive firms. As discussed earlier, the empirical literature has had difficulty identifying assortative matching. This paper contributes to that literature by showing evidence of assortative matching from a novel specification.

The results in Figure 1.1 and the associated t-tests establish that, *conditional on the control variables* ($\mathbf{Z}_{j,t}$), faster growing firms hire better workers. Now I turn to the importance of the control variables, in order to understand whether the positive

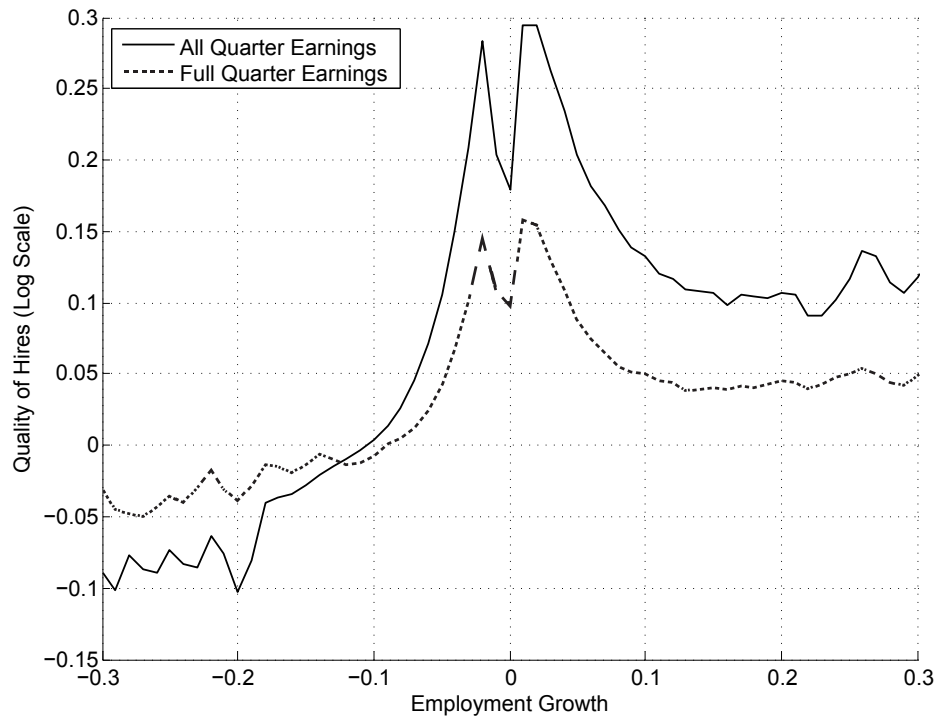


Figure 1.2: No Controls

growth-quality of hires correlation holds in the cross section. Figure 1.2 plots the estimated $m(\cdot)$'s when no control variables are included. It is apparent that the relationship is highly non-monotonic. These estimated $m(\cdot)$'s imply that low absolute values of growth are associated with better quality hires. Large positive or negative values of growth are associated with lower quality hires, with positive values having somewhat better average quality. A linear regression confirms that the average relationship on this interval is still positive and statistically significant, despite the non-monotonicity. Taken together, Figures 1.1 and 1.2 show that controlling for firm characteristics and other factors matters a great deal. Omitting controls would lead to the conclusion that the *absolute value* of growth is the most important factor influencing the quality of hires at a given firm. But this is not the case. The correct comparison in Figure 1.1 shows a monotone increasing relation between growth and quality of hires, once all else is held constant.

Figures 1.1 and 1.2 are extreme cases. Figure 1.2 has no controls, while Figure 1.1 includes controls for many firm characteristics, as well as seasonal, industry-level and aggregate fluctuations. There are many possible intermediate specifications, some of which I discuss in Appendix A.2. As more controls are added, the results look less like Figure 1.2 and more like the baseline specification. Controls for aggregate and industry level fluctuations are relatively unimportant. In contrast, firm fixed effects and controls for firm size and interacted industry-period-location dummies are critical in explaining the difference between Figures 1.1 and 1.2. These are all *firm characteristics*. The interpretation is simple: as more firm characteristics are included, the specification is closer to the ideal of holding all else constant.

1.3.5 Discussion

The empirical results demonstrate the importance of conditioning on firm characteristics. Without firm controls the relation between growth and quality of hires is ambiguous and non-monotonic. This echoes the results in the assortative matching literature, where AKM-style regressions find little correlation between worker type and firm type. AKM regressions typically do not condition on firm size, industry or other firm characteristics, so all inter-industry or cross-size variation is loaded into the estimated fixed effect. In addition, AKM-style regressions cannot include firm fixed effects as controls, since the firm fixed effect is the variable of interest.¹⁶ In this way the AKM models are similar to my specification without

¹⁶Mortensen et al. (2010) extend the AKM methodology to estimate worker fixed effects and firm-year fixed effects. With firm-year effects it would be possible to use time variation to study sorting while controlling for a time-invariant firm fixed effect. This is a potentially promising approach but it faces the same problems as the original AKM approach, as described in Section 1.2.1.

controls, where all inter-industry or cross-size variation is included. My specification without controls can be seen as an indirect replication of the typical AKM result: when no firm characteristics are conditioned on, there is little evidence of assortative matching.

The importance of firm characteristics may reflect segmentation in the labor market. When I condition on firm characteristics I implicitly compare firms only to very similar firms, which are likely to inhabit similar labor markets. This holds for firm fixed effects as well, under the assumption that firms do not change markets over time. Thus the baseline specifications, controlling for industry and other characteristics, pick up positive sorting within each segment of the labor market. The best doctors work for the best hospitals, the best pilots work for the best airlines, and so on. When I do not control for firm characteristics I am comparing doctors to pilots and hospitals to airlines. Firms in different markets may hire different workers and have different growth patterns for technological or other reasons. The comparison of Figures 1.1 and 1.2 suggests that markets with low firm volatility (growth near zero) are markets with higher quality workers. In this paper I do not seek to explain the differences between labor markets, or the entire joint distribution of firm growth and worker productivity. Rather I focus on the new fact that within markets (i.e. conditioning on firm characteristics), there does appear to be assortative matching. Based on my results, the AKM methodology may find different results when firm effects are estimated conditional on firm characteristics.

1.4 Theory

In this section I describe a general model that can account for the positive firm growth-quality of hires correlation described in Section 1.3. The model captures the intuition that if worker and firm productivity, are complementary in production then fast growing, highly productive firms should tend to hire more productive workers. Thus the model tightly links assortative matching to firm growth and the quality of hires. The model extends the influential work of [Shimer and Smith \(2000\)](#) to include multi-worker firms, idiosyncratic firm productivity shocks, and firm dynamics. Once solved, the extended model turns out to be highly tractable and yields analytical comparative statics. I derive sufficient complementarity conditions that ensure assortative matching and allow the model to replicate the observed growth-quality of hires correlation. Previously [Shimer and Smith \(2000\)](#) and [Lentz \(2010\)](#) have derived sufficient conditions for assortative matching, but their models abstracted from firm dynamics.

In the model workers and firms are characterized by a scalar type, interpreted as productivity. Search is random, and when workers and firms match the wage is determined by generalized Nash bargaining. Only unemployed workers search, though the qualitative results are unchanged if search while employed is allowed. I also allow firms to employ multiple workers by posting costly vacancies. Production is linear, so firm size is a function of the convex vacancy posting costs.

The most important feature of my model is that firms can employ multiple workers, whereas [Shimer and Smith \(2000\)](#) and others focus on single worker firms, or firms that cannot freely post additional vacancies. This difference turns out to matter

a great deal. The reason is that when firms can post additional vacancies (and production is linear), they have no opportunity cost of hiring a given worker. A central feature of the [Shimer and Smith \(2000\)](#) model is that both parties face a tradeoff between matching immediately or waiting for a better partner. By contrast, in my model, firms can hire a given worker today *and continue searching* (posting vacancies) in subsequent periods. This eliminates the opportunity cost of hiring for firms that are in contact with workers, greatly simplifying the analysis. A similar point is made by [Lentz \(2010\)](#) in a different context.

Section [1.4.1](#) introduces the model and derives the worker and firm decision rules. Section [1.4.2](#) briefly summarizes the steady-state equilibrium. In Section [1.4.3](#) I further characterize the worker's acceptance choices, and derive sufficient conditions for assortative matching. In Section [1.4.4](#) I turn to firm type and firm growth. In the body of the paper I skip some details of the firm's problem and sometimes focus on a model without productivity shocks, for the sake of exposition. Appendix [A.3](#) fully analyzes the firm's problem, and Appendix [A.4](#) analyzes sorting under productivity shocks. All results in the body of the paper carry through in the model with productivity shocks. Omitted proofs may be found in Appendix [A.5](#).

1.4.1 Environment

Workers and firms are risk neutral. Worker types are indexed by x . Firm types are indexed by y . Without loss of generality, I can assume that $x \in [0, 1]$ and $y \in [0, 1]$.¹⁷ Let $l(x)$ be the mass of workers of type x , and let $h(y)$ be the density

¹⁷See [Hagedorn et al. \(2012\)](#) for a discussion of normalizing agent types in this type of model.

of firm types. I normalize the mass of firms to unity, but allow the total mass of workers to vary. When a worker and firm are matched, their per-period output is $F(x, y) > 0$. I assume h, l and F are twice continuously differentiable. I also assume $F_1 > 0$ and $F_2 > 0$, so that higher types have *absolute advantage* over low types.

Each firm potentially employs a continuum of workers. Production of each worker-firm pair is independent of the firm's workforce. Let $N(x)$ be the mass of workers of type x working at a firm in a given period. Then total output at the firm is

$$Y = \int_0^1 N(x) F(x, y) dx$$

Within any match, the wage $w(x, y)$ is negotiated via Nash bargaining. Firms choose the level of vacancies to post each period. Each period some matches end due to exogenous separation shocks. Other matches end endogenously, when the worker and firm find it is more profitable to separate.

The main objects of interest are the *acceptance sets*. In equilibrium, each worker type x will be willing to match with a subset of firm types y . Let $A(x) \subset [0, 1]$ be the firm types that worker type x accepts offers from. The complement $A^c(x) = [0, 1] \setminus A(x)$ is the rejection set. Upon meeting, a worker and firm directly observe each other's types.

1.4.1.1 Productivity Shocks

Firm type, or productivity, is not constant over time. Each period there is a probability p that a given firm draws a new productivity value. The new value is drawn from the exogenous density $h(y)$. Note that when there is no firm exit, the

density of firm types is given by $h(y)$ as well. Conditional on drawing a new type, the new type is independent of the firm's old type (and everything else). A low value of p implies that productivity is more persistent. This process is a tractable way to model a persistent, mean-reverting process. It has been used by [Mortensen and Pissarides \(1994\)](#), [Lise et al. \(2013\)](#) and [Elsby and Michaels \(2013\)](#) among others.

1.4.1.2 Timing

Time is discrete. Within a period, the timing of events is as follows:

1. At the beginning of period t firm type y is realized.
2. Exogenous separations occur. Separated workers cannot search within the same period. They enter the matching market in $t + 1$.
3. Firms choose vacancy levels. Unemployed workers search, and meetings occur. Workers and firms who meet choose who they are willing to match with, anticipating the wage bargain and the future path of productivity.
4. Nash wage bargaining. After bargaining but before production either party can dissolve the match, resulting in endogenous separations.
5. Production occurs. Wages and unemployment benefits are paid. Consumption occurs.

1.4.1.3 Value Functions

In this section I briefly derive the firm and worker value functions. I describe the firm's problem in detail in [Section A.3](#). Each period the firm posts vacancies

v subject to a strictly convex, increasing, and differentiable cost function $c(v)$, where $c(0) = 0$ and $\lim_{v \rightarrow 0} c'(v) = 0$. Endogenous vacancy posting is important in what follows. Let $J(y)$ be the marginal net value of a vacancy to a type y firm, inclusive of posting costs. Since the firm is always free to adjust the number of vacancies, in equilibrium it must be that $J(y) = 0$. Otherwise the firm could profit by increasing or decreasing their postings. Let $J(x, y)$ be the firm's match surplus when paired with a type x worker, at the bargaining stage. It will turn out that $J(x, y)$ is the value both for newly formed matches as well as for continuing matches. If the match were to yield negative surplus for the firm ($J(x, y) < 0$), the firm could immediately dissolve the match and obtain a payoff of zero. Thus $J(x, y) \geq 0$ always. Assuming that $J(x, y) > 0$ the firm does not dissolve the match, and $J(x, y)$ can be written

$$\begin{aligned}
J(x, y) &= F(x, y) - w(x, y) + (1 - s)(1 - p)\beta J(x, y) \\
&\quad + (1 - s)p\beta \int_{A(x)} h(y') J(x, y') dy'
\end{aligned} \tag{1.3}$$

where $w(x, y)$ is the wage, and s is the exogenous separation probability. Linearity of production implies that the value of a match is independent of the firm's other workers. The firm value is composed of three terms. The first, $F(x, y) - w(x, y)$, is within-period production less wages. The next two terms represent continuation values. In the next period a separation shock arrives with probability s and a firm productivity shock arrives with probability p . If neither shock occurs (probability $(1 - s)(1 - p)$) the match remains unchanged, and the firm gets the discounted value $\beta J(x, y)$. Alternatively, if the productivity shock does arrive, the firm's

productivity changes to y' . If $y' \in A(x)$ the match continues to be acceptable to the worker and continues, with the firm receiving $J(x, y')$. The match is terminated if the separation shock arrives or the productivity shock makes the match unacceptable to the worker. In either case the firm is left with nothing.

Let $U(x)$ be the value of an unemployed worker, and let $W(x, y)$ be the value of that worker when matched with a type y firm. The value of unemployment satisfies

$$U(x) = \beta \int_{A(x)} f(y) W(x, y) dy + \beta \left(1 - \int_{A(x)} f(y) dy \right) U(x). \quad (1.4)$$

Here $f(y)$ is the arrival rate of offers from type y firms. Search is undirected so $f(y)$ is independent of worker type. If a worker receives an acceptable offer, he exits unemployment and receives continuation value $W(x, y)$. Otherwise the worker continues in unemployment. The worker receives a flow payoff of zero while unemployed. This assumption is made only for convenience.¹⁸

Turning to $W(x, y)$, assuming that $y \in A(x)$ so the match is acceptable and $W(x, y) > 0$, the value of employed worker is

$$\begin{aligned} W(x, y) &= w(x, y) + (1 - s)(1 - p)\beta W(x, y) \\ &\quad + (1 - s)p\beta \int_{A(x)} h(y') W(x, y') dy' \\ &\quad + (1 - s)p \left(1 - \int_{A(x)} h(y') dy' \right) \beta U(x) \\ &\quad + s\beta U(x). \end{aligned} \quad (1.5)$$

¹⁸If the worker does derive utility from unemployment $b > 0$, all matching and sorting results carry through unchanged for the net production function $\tilde{F}(x, y) = F(x, y) - b$.

This equation is similar to the firm value J , with the exception that the worker receives the value $U(x)$ when the match is dissolved. Within the period an employed worker receives the flow wage $w(x, y)$. If the match continues, the worker receives $W(x, y)$ or $W(x, y')$ depending on the productivity shock. Otherwise the worker goes into unemployment. If neither the separation nor the productivity shock arrive, the worker receives the continuation value $W(x, y)$. If the match is acceptable the worker remains at the firm, with value $W(x, y')$. If it is not acceptable the worker separates into unemployment. Finally, the exogenous separation shock moves the worker into unemployment with probability s .

1.4.1.4 Wage Bargaining

It will be convenient to define the worker surplus:

$$S(x, y) = W(x, y) - U(x).$$

This is the gain to the worker of being matched with a type y firm. Each period the worker and firm split the match surplus according to (generalized) Nash bargaining:

$$(1 - \eta) S(x, y) = \eta J(x, y) \tag{1.6}$$

where $\eta \in [0, 1]$ is the worker's bargaining power. In equilibrium matches form, or remain intact, if and only if the total surplus is positive

$$J(x, y) + S(x, y) > 0.$$

Some rearrangement of these equations yields expressions for the wage, the worker surplus and the firm surplus:

Fact 1. *Given a worker's acceptance set $A(x)$, the wage satisfies*

$$w(x, y) = \eta F(x, y) + (1 - \eta)\beta \int_{A(x)} f(y') S(x, y') dy' \quad (1.7)$$

the marginal value of a worker is

$$\begin{aligned} J(x, y) = & \theta(1 - \eta) \left(F(x, y) - \beta \int_{A(x)} f(y') S(x, y') dy' \right) \\ & + \theta(1 - s)p\beta \int_{A(x)} h(y') J(x, y') dy'. \end{aligned} \quad (1.8)$$

and the worker surplus is

$$\begin{aligned} S(x, y) = & \theta\eta \left(F(x, y) - \beta \int_{A(x)} f(y') S(x, y') dy' \right) \\ & + \theta(1 - s)p\beta \int_{A(x)} h(y') S(x, y') dy' \end{aligned} \quad (1.9)$$

where $\theta = \frac{1}{1 - (1 - s)(1 - p)\beta} > 0$.

Proof. See the Appendix. □

The expressions (1.7), (1.8) and (1.9) are fairly standard. In equation (1.7), we see that the bargained wage is a weighted average of current production and $\beta \int_{A(x)} f(y') S(x, y') dy'$, which is proportional to the worker's value of unemployment $U(x)$. This is the typical result in Nash bargaining models. Equations (1.8) and (1.9) show that the agents split production less the worker's threat point, $F(x, y) - \beta \int_{A(x)} f(y') S(x, y') dy'$, in accordance with the bargaining parameter η .

Finally, note that the future values of productivity do not appear in the wage expression, even though the wage bargain is forward looking. This is due to the choice of the productivity process.

1.4.1.5 Acceptance Sets

Equation (1.9) defines the worker surplus in terms of the worker acceptance set $A(x)$ as well as the surplus from other jobs, $S(x, y')$. My goal is to obtain an expression for the acceptance sets. To this end, I eliminate $S(x, y')$ from the right hand side of (1.9) by substitution:

Fact 2. *The worker surplus can be written as*

$$S(x, y, A) = \theta\eta \left[F(x, y) - \beta\theta \frac{\int_{A(x)} [\eta f(y') - (1-s)ph(y')] F(x, y') dy'}{1 + \theta\beta \int_{A(x)} [\eta f(y') - (1-s)ph(y')] dy'} \right] \quad (1.10)$$

which is a continuous function of (x, α, A) .

Proof. See the Appendix. □

Here I write S as a function of A to emphasize its dependence on the worker's acceptance decisions. At this stage I make no assumptions on A (such as optimality). Equation (1.10) has an intuitive interpretation. The worker's surplus is proportional to the term in square brackets. The first part, $F(x, y)$, is the productivity of the match. Obviously, surplus should be increasing in production, all else held constant. The second term captures both the probability of productivity shocks and worker's opportunity cost of matching with other firms. Consider what

happens when we eliminate productivity shocks by setting $p = 0$:

$$S(x, y, A) = \theta\eta \left[F(x, y) - \frac{\beta\theta\eta \int_{A(x)} f(y') F(x, y') dy'}{1 + \theta\beta\eta \int_{A(x)} f(y') dy'} \right].$$

The final term inside the brackets is a biased weighted average of net production in alternative jobs, where the weights are the meeting probabilities $f(y')$. Without productivity shocks, the ratio is the worker's opportunity cost: matching with a type y firm today precludes matching with another firm in $A(x)$ until the job ends exogenously. An increase in $F(x, y')$ increases the opportunity cost of matching with y , and naturally decreases $S(x, y, A)$.

Equation (1.10) allows me to begin characterizing the acceptance sets. First, note that current firm productivity only enters this expression through the initial $F(x, y)$ term. $F_2 > 0$ then implies $S_2 > 0$, so worker surplus is strictly increasing in firm type, regardless of worker type x or their choice of acceptance set. This is a strong result, summarized in the following proposition.

Proposition 1. *Under the assumed productivity process all workers rank firms the same: $S(x, y, A)$ is monotone increasing in y for all x and A . All workers prefer high y firms to low y firms.*

Increasing surplus implies that equilibrium acceptance sets will be intervals defined by a lower threshold:

$$A(x) = \{y | y > y(x)\} \tag{1.11}$$

where $y(x)$ is the threshold firm type for worker x . Knowing that the $A(x)$ are intervals, with all workers preferring high y to low x firms, simplifies analysis

considerably. Instead of considering acceptance sets which are arbitrary subsets of $[0, 1]$, I now focus on finding the thresholds $y(x)$. This result is a consequence of allowing firms to employ multiple workers and post multiple vacancies, along with linear production as in [Bagger and Lentz \(2012\)](#). When a vacancy matches with a worker, the firm can post another vacancy the next period at the same cost. Thus the firm faces no opportunity cost of matching. Match surplus is production less the opportunity costs of the worker and firm, so when firms have no opportunity cost (or more generally, no variation in opportunity cost) it is natural that workers prefer higher type firms. In the models of [Shimer and Smith \(2000\)](#), [Lopes de Melo \(2013\)](#), and [Hagedorn et al. \(2012\)](#) firms and workers are treated *symmetrically*, in that matching with a partner precludes any other productive activity, at least until the match exogenously dissolves. The result is that firms, like workers, vary in their opportunity cost of matching, and this two-sided selectivity leads to potentially complicated acceptance patterns.¹⁹

So far I have treated the acceptance set $A(x)$, or equivalently $y(x)$, as exogenous. Now I solve for the worker's optimal $y(x)$. Note that in equilibrium a worker will accept a job if and only if it yields positive surplus, given their acceptance set: $S(x, y, A) > 0$. Thus the optimal threshold $y(x)$ must satisfy

$$S(x, y(x), A(x)) = 0 \text{ if } y(x) > 0 \quad (1.12)$$

$$S(x, 0, [0, 1]) > 0 \text{ if } y(x) = 0 \quad (1.13)$$

The first case is of primary interest. If the threshold $y(x)$ is on the interior,

¹⁹My assumption is potentially more restrictive, but allows for tractable analysis. It is also motivated by the fact that in reality firms are large. The vast majority of workers are employed at multi-worker firm, not single worker firms.

then the surplus at the marginal firm must equal zero. Otherwise the worker would do strictly better by lowering his threshold and accepting more offers. It is possible that all jobs offer positive surplus. Then we are in the second case, where $A(x) = [0, 1]$, i.e. the worker accepts all jobs. Then it must be the case that the lowest ranked job still provides surplus. Substituting (1.10) into (1.12) and (1.13), imposing (1.11) and rearranging yields

$$F(x, y(x)) + \theta\beta(1-s)p \int_{y(x)}^1 h(y) (F(x, y) - F(x, y(x))) dy = \beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy \quad \text{if } y(x) > 0 \quad (1.14)$$

$$F(x, y(x)) + \theta\beta(1-s)p \int_{y(x)}^1 h(y) (F(x, y) - F(x, y(x))) dy \geq \beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy \quad \text{if } y(x) = 0 \quad (1.15)$$

Thus equations (1.14) and (1.15) are necessary conditions that $y(x)$ must satisfy. Intuitively, the left hand side of (1.14) is the value of matching with a firm of type $y(x)$ immediately, and the right hand side is the value of waiting for another acceptable offer. Matching with $y(x)$ yields utility proportional to $F(x, y(x))$ for some duration. Once matched, there is also the possibility of a productivity shock, captured by the second term on the left hand side. Roughly speaking, an increase in firm type to $y > y(x)$ increases the utility of the worker by $F(x, y) - F(x, y(x))$. On the right hand side, the benefit of waiting is the discounted expected difference between production at $y(x)$ and production at an alternative: $\beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy$.

As stated above, (1.14) and (1.15) are *necessary* conditions that an optimal $y(x)$ must satisfy. It turns out that they are sufficient conditions as well. This is because the left hand side of (1.14) is increasing in $y(x)$, while the right hand side is decreasing. This means that at most one point satisfies the system. The following proposition formalizes the argument.

Proposition 2. *The worker's optimal threshold $y(x)$ is the unique solution to equations (1.14) and (1.15).*

Proof. See the Appendix. □

Discussion

Consider the worker's acceptance choice in equation (1.14) when there are no productivity shocks ($p = 0$):

$$F(x, y(x)) = \beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy. \quad (1.16)$$

This condition is formally identical to the reservation wage expression for a model of sequential search with an exogenous wage offer distribution. These models were first studied by McCall (1970), Gronau (1971) and Mortensen (1970). The version I use can be found in Mortensen (1986) and Rogerson et al. (2005). In this model there is a single worker type and an exogenous wage offer distribution. Each period an unemployed worker searches for a job, drawing wage offers from the distribution $\bar{f}(w)$. Jobs end with probability s each period. Then the values

of unemployment U and employment $W(w)$ are

$$\begin{aligned} U &= \beta \int_{w^*}^{\bar{w}} W(w) \bar{f}(w) dw + \beta \left(1 - \int_{w^*}^{\bar{w}} \bar{f}(w) dw \right) U \\ W &= w + \beta(1-s)W + \beta sU \end{aligned}$$

where w^* is the worker's choice of reservation wage and \bar{w} is the upper support of the offer distribution. This leads to the following expression for the reservation wage:²⁰

$$w^* = \frac{\beta}{1-\beta} \int_{w^*}^{\bar{w}} \bar{f}(w) [w - w^*] dw. \quad (1.17)$$

Comparing this with equation (1.16) we see that the two decision rules have the same form, with the reservation wage w^* replacing the “reservation match quality” $F(x, y(x))$ in equation (1.17). It is interesting that the two decision rules should be so similar, given that wages are endogenous in my model. The similarity is a result of Nash bargaining and allowing firms to post multiple vacancies. Nash bargaining ensures that the wage is an additive function of a worker effect and a firm effect, as we can see in equation (1.7). Vacancy posting along with linear production implies that firms do not have an opportunity cost of hiring, so the firm effect in the wage equation is proportional to $F(x, y)$. If there were an opportunity cost, as in [Shimer and Smith \(2000\)](#), the firm effect would be $F(x, y)$ less the value of the firm's vacancy. Thus, my model can be interpreted as a general equilibrium extension to the [McCall \(1970\)](#) model, which retains the same simple decision rules as the partial equilibrium model. The advantage of the current model is that it allows for comparisons between different worker types, which will be taken up

²⁰See [Rogerson et al. \(2005\)](#) and [Mortensen \(1986\)](#) for explicit derivations and discussion.

in Section (1.4.3).

1.4.1.6 Vacancy Posting

The results in this section and the next are not strictly necessary for understanding the assortative matching results in Section 1.4.3. The interested reader may skip directly to that part of the paper.

The firm posts vacancies at a convex cost until the expected value of an additional vacancy is zero. Formally, this means

$$c'(v) = \int_0^1 q(x)J(x,y)dx.$$

This is derived explicitly from the firm's problem in Section A.3. The left hand side is the marginal cost of a vacancy, and right hand side is the expected marginal value of the vacancy. $J(x,y)$ is the value to the firm of meeting a type x worker, and $q(x)$ is the probability the vacancy attracts such a worker within the period. The right hand side simply integrates this value across worker type. Using the Nash bargaining solution this marginal condition can be written

$$c'(v) = \frac{(1-\eta)}{\eta} \int_0^1 q(x)S(x,y)dx$$

Recalling that $c(v)$ is strictly convex, I can invert $c'(v)$ and arrive at an explicit expression for v :

$$v(y) = c'^{-1} \left(\frac{(1-\eta)}{\eta} \int_0^1 q(x)S(x,y)dx \right). \quad (1.18)$$

Recall from Section 1.4.1.5 that $S(x, y)$ is increasing in y . This means that a higher type firm has a higher return to matching with a given worker. Then equation (1.18) implies that better firms post more vacancies: $v'(y) > 0$. This is a natural prediction, and turns out to be useful in Section A.4 for characterizing assortative matching under productivity shocks.

1.4.1.7 Meeting and Matching

To close the model, I formalize the relationship between vacancies, unemployment, and the meeting rates $f(y)$ and $q(x)$. I make the standard “balanced meeting” assumption for random search models. The sorting results in Section 1.4.3 do not depend on the details of the matching technology, as long as search is random. Let $u(x)$ be the mass of unemployed type x workers. Then total unemployment is $U = \int u(x)dx$, and I can define $\tilde{u}(x) = \frac{u(x)}{U}$ as the density of unemployed workers. Let $v(y)$ be the vacancy posting of a type y firm, and recall that $h(y)$ is the mass of type y firms in the economy. Then total vacancy posting is $V = \int v(y)h(y)dy$ and I can define $\tilde{v}(y) = \frac{v(y)h(y)}{V}$ as the share of vacancies that are posted by type y firms.

Following the literature, I assume that the total number of meetings between workers and firms in a period is given by $M(U, V)$, an increasing, concave, linearly homogenous meeting function. The meeting function takes total unemployment and total vacancies as inputs, and gives the total number of contacts between workers and firms.

I assume that each worker contacts at most one vacancy, and each vacancy contacts

at most one worker. This implies

$$M(U, V) \leq \min \{U, V\}.$$

Contacts are made in a random, fashion. Thus each worker has an equal probability of making a contact, which is given by $\frac{M(U, V)}{U}$. Likewise, each vacancy has a probability $\frac{M(U, V)}{V}$ of making a contact. Conditional on a worker making a contact, the vacancy is drawn at random from the population of vacancies. Thus, the arrival rate of type y vacancies for a searching worker is given by

$$f(y) = \frac{M(U, V)}{U} \tilde{v}(y). \quad (1.19)$$

Symmetrically for vacancies I have

$$q(x) = \frac{M(U, V)}{V} \tilde{u}(x) \quad (1.20)$$

where $q(x)$ is the probability a vacancy contacts a type x worker.

1.4.2 Equilibrium

In a steady state equilibrium the mass of unemployed workers of each type must remain constant over time. This requires that the outflows from unemployment equal the inflows:

$$u(x) \int_{y(x)}^1 f(y) dy = (l(x) - u(x)) \left(s + (1 - s)p \int_0^{y(x)} h(y) dx \right) \quad \forall x.$$

For a given x , the left hand side is the stock of unemployed times the rate at which they meet acceptable jobs. The right hand side is the mass of employed workers times the combined rate of endogenous and exogenous separations. This can be rearranged to yield

$$u(x) = \frac{\left(s + (1-s)p \int_0^{y(x)} h(y) dx \right) l(x)}{\left(s + (1-s)p \int_0^{y(x)} h(y) dx \right) + \int_{y(x)}^1 f(y) dy} \quad \forall x \quad (1.21)$$

which makes clear that $u(x)$ is a continuous function of $f(y)$ and $y(x)$.

In addition to balancing worker flows, a steady state general equilibrium requires that (1) worker acceptance choices are made optimally, (2) vacancy posting decisions are made optimally and (3) meeting rates are consistent with the meeting function and the stocks of unemployed and vacancies. The exogenous parameters are the scalars β, s, p, η and the functions $l(x), h(y)$ and $F(x, y)$. The endogenous objects are $y(x), v(y), f(y), q(x)$ and $u(x)$, which are jointly determined by equations (1.14), (1.15), (1.18), (1.19), (1.20), and (1.21). I reproduce the equilibrium conditions below, with intermediate notation eliminated.

First is the worker's acceptance set choice, for all x :

$$F(x, y(x)) + \theta\beta(1-s)p \int_{y(x)}^1 h(y) (F(x, y) - F(x, y(x))) dy = \beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy \quad \text{if } y(x) > 0 \quad (1.22)$$

$$\begin{aligned}
F(x, y(x)) + \theta\beta(1-s)p \int_{y(x)}^1 h(y) (F(x, y) - F(x, y(x))) dy \geq \\
\beta\theta\eta \int_{y(x)}^1 f(y) (F(x, y) - F(x, y(x))) dy \quad \text{if } y(x) = 0. \quad (1.23)
\end{aligned}$$

Equations (1.22) and (1.23) define $y(x)$ as a function of other model objects. Some additional algebra shows that $y(x)$ is a *continuous* function of $f(y)$, a fact which will be useful in showing existence of an equilibrium. Next is the firm's vacancy decision:

$$v(y) = c'^{-1} \left((1-\eta)\theta \int_0^1 q(x) \left[F(x, y) - \beta\theta \frac{\int_{y(x)}^1 [\eta f(y') - (1-s)ph(y')] F(x, y') dy'}{1 + \theta\beta \int_{y(x)}^1 [\eta f(y') - (1-s)ph(y')] dy'} \right] dx \right) \quad \forall y. \quad (1.24)$$

Note that equation (1.24) defines $v(y)$ as a continuous function of $q(x)$, $f(y)$ and $y(x)$. Finally, recall the meeting rates

$$f(y) = \frac{M \left(\int_0^1 u(x) dx, \int_0^1 v(y) h(y) dy \right)}{\int_0^1 u(x) dx} \frac{v(y) h(y)}{\int_0^1 v(y) h(y) dy} \quad \forall y \quad (1.25)$$

$$q(x) = \frac{M \left(\int_0^1 u(x) dx, \int_0^1 v(y) h(y) dy \right)}{\int_0^1 v(y) h(y) dy} \frac{u(x)}{\int_0^1 u(x) dx} \quad \forall x. \quad (1.26)$$

Equations (1.25) and (1.26) define $f(y)$ and $q(x)$ as continuous functions of $u(x)$ and $v(y)$.

Expressions (1.21), (1.22), (1.23), (1.24), (1.25) and (1.26) define a mapping of the endogenous objects $\{y(x), v(y), f(y), q(x), u(x)\}$ into new values $\{y'(x), v'(y), f'(y), q'(x), u'(x)\}$.

Under some regularity conditions, the continuity of the mapping ensures that a fixed point theorem can be applied, so an equilibrium exists. In matching models the sticking point in proving equilibrium existence is usually showing that the steady state flows equation defines a continuous mapping from acceptance sets to

the masses of searchers (see [Shimer and Smith \(2000\)](#), [Nöldeke and Tröger \(2009\)](#) and [Smith \(2011\)](#)). In my setup this is not an issue, because [\(1.21\)](#) and [\(1.24\)](#) explicitly solve for the masses of unemployed and vacancies as a function of other variables. In general the model may have multiple equilibria. Unfortunately, this is a common feature of models with two-sided heterogeneity and search.²¹

1.4.3 Assortative Matching

In this section I analyze the sorting of workers across firms. I focus on deriving sufficient conditions for positive and negative assortative matching. Intuitively, positive assortative matching means that better workers tend to match with better firms. In the context of my model, positive assortative matching is equivalent to $y'(x) \geq 0 \forall x$. That is, better workers (higher x) set their threshold firm type $y(x)$ higher. Similarly, negative assortative matching is equivalent to $y'(x) \leq 0 \forall x$. The case of no sorting is also of interest, and in my model this translates to $y'(x) = 0 \forall x$. The empirical pattern in [Section 1.3](#) relates firm growth to worker type. In this section I derive a condition, strict log-supermodularity, under which better firm types hire better worker types. In [Section 1.4.4](#) I show that under strict log-supermodularity better firms are also faster growing. Together, the firm type-worker type and firm type-growth relations imply that firm growth and quality of hires are positively correlated, reproducing the empirical results.

In what follows I use the worker's threshold condition to derive an expression for $y'(x)$. The sign of $y'(x)$, and thus the nature of sorting, depend on the sign of a simple expression that compares match production at the marginal firm to

²¹See [Sattinger \(1995\)](#), [Shimer and Smith \(2000\)](#) and [Lopes de Melo \(2013\)](#).

expected production at firms in the acceptance set. The expression can be unambiguously signed when production $F(x, y)$ is strictly log-supermodular or strictly log-submodular, as described below. These are sufficient conditions for positive and negative assortative matching, respectively. Interestingly, these sufficient conditions rely only on the production function, and not the distribution of agent types or any other parameter. In this sense they are similar to the sorting conditions derived by [Becker \(1973\)](#), [Shimer and Smith \(2000\)](#), [Lentz \(2010\)](#) and [Eeckhout and Kircher \(2010\)](#) in the context of other models. Given that the sorting conditions are independent of agent distributions and other parameters, it might be asked whether weaker sufficient conditions can be found. I show that any weaker sufficient conditions must depend on the distribution of types. Specifically, if the production function is not strictly log-supermodular or submodular one can find distributions of agents that lead to non-assortative matching.

For simplicity I focus on the case of no productivity shocks: $p = 0$. All of the intuition and results carry over to the case in which productivity shocks occur, but at the cost of additional notation and technicalities. I prove the main results for the general model in [Appendix A.4](#).

Consider the worker's threshold condition [\(1.14\)](#) when there are no productivity shocks:

$$F(x, y(x)) \left(1 + \beta\theta\eta \int_{y(x)}^1 f(y) dy \right) - \beta\theta\eta \int_{y(x)}^1 f(y) F(x, y) dy = 0 \quad (1.27)$$

This expression implicitly defines $y(x)$ in terms of x , $F(x, y)$, and the other parameter values. As a consequence [\(1.27\)](#) can also be used to derive an expression for $y'(x)$. While this expression is cumbersome, it shows that the *sign* of $y'(x)$,

and thus the nature of sorting, depend only on a simple condition, shown below.

Proposition 3. *When the worker's choice of $y(x)$ is given by (1.27), $y'(x) > 0$ if and only if*

$$\int_{y(x)}^1 w(y) \frac{F_1(x, y)}{F(x, y)} dy > \frac{F_1(x, y(x))}{F(x, y(x))} \quad (1.28)$$

and $y'(x) < 0$ if and only if

$$\int_{y(x)}^1 w(y) \frac{F_1(x, y)}{F(x, y)} dy < \frac{F_1(x, y(x))}{F(x, y(x))}, \quad (1.29)$$

where $w(y) = \frac{f(y)F(x, y)}{\int_{y(x)}^1 f(y)F(x, y) dy} > 0$, and $\int_{y(x)}^1 w(y) dy = 1$.

Proof of Proposition 3. Equation (1.27) implicitly defines $y(x)$, so differentiating it I can obtain an expression for $y'(x)$. Applying the implicit function theorem I have

$$y'(x) = - \frac{F_1(x, y(x)) \left(1 + \beta\theta\eta \int_{y(x)}^1 f(y) dy\right) - \beta\theta\eta \int_{y(x)}^1 f(y) F_1(x, y) dy}{F_2(x, y(x)) \left(1 + \beta\theta\eta \int_{y(x)}^1 f(y) dy\right)} \quad (1.30)$$

The denominator is always positive, so the sign of $y'(x)$ depends only on the numerator, implying $y'(x) > 0$ if and only if

$$\beta\theta\eta \int_{y(x)}^1 f(y) F_1(x, y) dy > F_1(x, y(x)) \left(1 + \beta\theta\eta \int_{y(x)}^1 f(y) dy\right).$$

Dividing this expression by (1.27) I arrive at

$$\frac{\int_{y(x)}^1 f(y) F_1(x, y) dy}{\int_{y(x)}^1 f(y) F(x, y) dy} > \frac{F_1(x, y(x))}{F(x, y(x))}$$

which can be rearranged to yield equation (1.28) as desired. A symmetric argu-

ment shows that equation (1.29) follows as well. \square

In equation (1.28) $\frac{F_1(x,y(x))}{F(x,y(x))}$ reflects the marginal change in output at the marginal firm type when we increase the worker type. The term $\int_{y(x)}^1 w(y) \frac{F_1(x,y)}{F(x,y)} dy$ reflects the expected change in productivity at firms in the acceptance set when we increase worker type. If output at the the marginal firm increases more than at the average accepted firm, the relative value of waiting for better offers is lower, and the worker lowers the acceptance threshold $y(x)$. Alternatively, if expected productivity at accepted firms increases more than productivity at the marginal firm, the worker prefers to wait for better offers and raises the acceptance threshold. In this case we have (locally) positive assortative matching, and $y'(x) > 0$.

Having shown that satisfying equation (1.28) is equivalent to assortative matching, the question is under what conditions does (1.28) hold? Clearly, a sufficient condition is that $\frac{F_1(x,y)}{F(x,y)}$ be increasing in y . Then any weighted average $\int_{y(x)}^1 w(y) \frac{F_1(x,y)}{F(x,y)} dy$ must exceed the value $\frac{F_1(x,y(x))}{F(x,y(x))}$. The economic interpretation of this condition is straightforward. When $\frac{F_1(x,y)}{F(x,y)}$ is increasing in y , it means that output is *more* sensitive to worker type at high y firms. Increasing worker type increases output the most at the highest y firms, encouraging the worker to raise the acceptance threshold and generating positive assortative matching.

The condition $\frac{\partial}{\partial y} \frac{F_1(x,y)}{F(x,y)} > 0$ is equivalent to strict log-supermodularity. In particular, a differentiable, positive, increasing function is strictly log-supermodular if and only if its logarithm is supermodular:

$$\frac{\partial^2}{\partial x \partial y} \ln F(x,y) > 0$$

which is equivalent to

$$\frac{\partial}{\partial y} \frac{F_1(x, y)}{F(x, y)} > 0$$

as asserted. Returning to equation (1.29), it is clear that $\frac{\partial}{\partial y} \frac{F_1(x, y)}{F(x, y)} < 0$ ensures $y'(x) < 0$, so matching is negative assortative. In turn $\frac{\partial}{\partial y} \frac{F_1(x, y)}{F(x, y)} < 0$ is equivalent to strict log-submodularity:

$$\frac{\partial^2}{\partial x \partial y} \ln F(x, y) < 0.$$

Finally, equations (1.28) and (1.29) imply there is no sorting, $y'(x) = 0$, if and only if $\frac{\partial}{\partial y} \frac{F_1(x, y)}{F(x, y)} = 0$. These results are summarized in the following Proposition.

Proposition 4. *Consider the economy with no productivity shocks. Matching is positive assortative, $y'(x) > 0$, if F is strictly log-supermodular. Matching is negative assortative, $y'(x) < 0$ if F is strictly log-submodular. There is no sorting if $\frac{\partial}{\partial y} \frac{F_1(x, y)}{F(x, y)} = 0$.*

Proposition 4 provides sufficient conditions for positive and negative assortative matching. Note that the conditions depend only on the production function F . They are independent of the distribution of agent types as well as the other parameters of the economy. A natural question is whether these conditions are the minimal ones that ensure assortative matching, or whether weaker conditions may suffice. The following Proposition shows that strict log-supermodularity is necessary for positive assortative matching, in the sense that if F is not globally strictly log-supermodular we can find a distribution of vacancies that ensures $y'(x) < 0$ somewhere.

Proposition 5. *If there exists y^* and x^* such that $\frac{\partial^2}{\partial x \partial y} \ln F(x^*, y^*) < 0$, then there exists a set of meeting rates $f(y)$ such that $y'(x^*) < 0$.*

Proof. See the Appendix. □

The strategy of proof is to adjust the meeting rates $f(y)$ to keep the worker's threshold constant at $y^* = y(x)$, but make sure sorting is (locally) negative. Keeping in mind that $\frac{\partial^2}{\partial x \partial y} \ln F(x^*, y^*) < 0$, there is some $y^{**} > y^*$ such that $\frac{F_1(x^*, y^*)}{F(x^*, y^*)} > \frac{F_1(x^*, y^{**})}{F(x^*, y^{**})}$. If $f(y^{**})$ is large enough, the worker will expect to meet type y^{**} firms much more often than any alternative. The increased weight on y^{**} means that the behavior of F around (x^*, y^{**}) dominates sorting decisions. Appealing to Proposition 3 then shows that $y'(x^*) < 0$, as desired.

Similar to other assortative matching results, my conditions on the production function are sufficient for *any* values of the other parameters and distributions, and are the weakest conditions on the production function that ensure assortativity for all other parameter values. If we consider a restricted set of type distributions, or other parameters, it may be possible to find alternative necessary and sufficient conditions. This is left for future work.

1.4.3.1 Discussion

This paper is one of many to study the theory of sorting in a search environment.²²

The two papers most similar to this one are [Shimer and Smith \(2000\)](#) and [Eeckhout and Kircher \(2010\)](#). While neither allow for productivity shocks, the models can still be compared. In the random search model of [Shimer and Smith \(2000\)](#), positive assortative matching requires supermodularity not only of $\ln F$ but also

²²See [Shimer and Smith \(2000\)](#), [Shi \(2001\)](#), [Shimer \(2005\)](#), [Atakan \(2006\)](#), [Eeckhout and Kircher \(2010\)](#), [Lentz \(2010\)](#) and [Eeckhout and Kircher \(2012\)](#).

of $\ln F_1$, $\ln F_2$ and $\ln F_{12}$.²³ Thus my log-supermodularity requirement is weaker than theirs. This can be attributed to my allowing multiple worker firms. In [Shimer and Smith \(2000\)](#) firms have an opportunity cost of matching, which is increasing in firm type. This tends to make high type firms less attractive to workers, since the higher firm opportunity cost comes out of the worker's wages. In my model the workers get a constant share of match output (plus a function of the worker opportunity cost). This makes high type firms more attractive, and encourages high type workers to wait for high type firms.

[Eeckhout and Kircher \(2010\)](#) work in a directed search framework. They derive *root-n supermodularity* as a sufficient condition for positive assortative matching. This condition requires that the n 'th root of the production function be positive, where n is the elasticity of substitution of the matching function. They show that root-n supermodularity is always weaker than log-supermodularity, and thus weaker than the requirements of [Shimer and Smith \(2000\)](#). This is due to the differences between directed and random search. See [Eeckhout and Kircher \(2010\)](#) for an illuminating discussion of these differences, and the relation of their result to that of [Shimer and Smith \(2000\)](#). My condition, while weaker than Shimer and Smith's, is still stronger than root-n supermodularity. This is because my model is one of random search, while [Eeckhout and Kircher \(2010\)](#) study directed search.

1.4.4 Firm Growth and the Quality of Hires

In Section [1.4.3](#) I showed that strict log-supermodularity ensures assortative matching between worker types and firm types: high type firms hire better workers. To

²³While [Shimer and Smith \(2000\)](#) do not discuss supermodularity of $\ln F$, [Eeckhout and Kircher \(2010\)](#) show that their conditions, along with monotonicity, imply log-supermodularity.

link these results to the empirical results in Section 1.3, I need a connection between firm type and firm *growth*. Intuitively, if high type firms tend to grow faster, then strictly log-supermodular match production will lead to a positive firm growth-quality of hires correlation, as seen in the data. On the other hand, if there is a weak or ambiguous correlation between firm type and firm growth then the assortative matching condition in Section 1.4.3 will be irrelevant for understanding the firm growth-quality of hires correlation.

It is natural to think that high type firms will tend to be faster growing: the results in Sections 1.4.1.6 and 1.4.3 show that under strict log-supermodularity better firms both post more vacancies and hire more workers per vacancy. In practice, the relationship between firm growth and firm quality is complicated. Higher type firms tend to post more vacancies and hire more workers. *Ceteris paribus*, this increases growth rates. However, a high type firm will also tend to be larger to begin with and have larger outflows of workers, both of which reduce growth rates. Sorting out these effects is complicated by the heterogeneity of the workforce, which causes firms to occasionally hire a substantial number of workers while simultaneously separating a large portion of their incumbent workers. This can happen in the model when there is a negative productivity shock.

To show that firm growth is positively correlated with firm type (and therefore worker type), it suffices to show that a stochastic dominance property holds. Let g be the (random) growth rate of a firm, and let $F_g(g|y)$ be the CDF of growth rates conditional on the firm being of type y . Assume for the moment that if $y' > y$, then $F_g(\cdot|y')$ dominates $F_g(\cdot|y)$ (in the sense of first order stochastic dominance). Such a dominance property implies that firm type and firm growth have a positive

correlation coefficient, which is the desired result. So the main problem is to prove dominance.

Proposition 6. *Assume that matching is positive assortative. Then $F_g(\cdot|y')$ dominates $F_g(\cdot|y)$ whenever $y' > y$.*

Proof. See the Appendix. □

The argument has three parts

- **Step 1:** Show that firm growth $g(y, m)$ is an exact function of current productivity y and a set of state variables m , where m includes the time elapsed since the last shock and the employment distribution at the time of the shock. In addition, $h(y|m) = h(y)$.
- **Step 2:** Show that $g_1(y, m)$ is positive.
- **Step 3:** Use the population distribution of m and y to show that $g_1(y, m) > 0$ is a sufficient condition for $F_g(\cdot|y')$ to dominate $F_g(\cdot|y)$ whenever $y' > y$.

The specification of the productivity process helps in Step 1. Step 2 follows in part from Proposition 7 (see below), which helps pin down the compositional dynamics of employment by worker type.

An alternative approach would be to work with the distribution of firm types conditional on growth. This might appear more natural, since the empirical results are stated in terms of worker types conditional on firm growth. However, it turns out there is not necessarily any stochastic dominance between the type distributions conditional on growth. This necessitates working with growth conditional on firm type to prove dominance and therefore a positive correlation between firm growth and the quality of hires.

1.4.4.1 Discussion

I have shown that the model implies a positive correlation between firm growth and quality of hires if production is strictly log-supermodular. The same condition implies positive assortative matching, so the pattern documented in Section 1.3 can be taken as indirect evidence for assortative matching between workers and firms. This paper is the first to characterize matching analytically in an environment with firm growth and productivity shocks. One difficulty is that in such a rich environment the connection between firm growth and productivity can become unclear. Proposition 6 proves that a positive connection between firm growth and productivity does indeed hold within the model.

As noted earlier, Lopes de Melo (2013), Eeckhout and Kircher (2011) and others have shown that AKM fixed effects are inconsistent when the data generating process is a model along the lines of Shimer and Smith (2000). Their critique does not apply to my model. Allowing firms to post multiple vacancies ensures there is no opportunity cost to hiring, so the wage is monotone increasing in firm type. Then fixed effects estimation should recover the firm types accurately. If my model is correct there is a puzzle: why do I find assortative matching in Section 1.3, when AKM and others find none? As I have discussed, one possibility is that assortative matching obtains only within narrow labor markets, while sorting across the whole economy is more ambiguous. Another possibility is that my results are based on firm *growth* while the fixed effects results assume a constant firm effect over the sample period. If productivity shocks are not too persistent, so that time-averaged productivity is close to the unconditional expectation of productivity for most firms, then we would expect fixed effects regressions to

reveal relatively little sorting. The reason is that while there is sorting between worker and firm types, the firm fixed effect is an average of firm productivity over the sample. This suggests an alternative identification strategy, which would jointly estimate worker effects and firm-year effects as in [Mortensen et al. \(2010\)](#). Allowing the firm effect to vary over time would get rid of the averaging problem.

1.4.5 Additional Results

Besides addressing the empirical correlation between firm growth and the quality of hires, the model can be used to derive additional results. Here I discuss three broad sets of results.

1.4.5.1 Vacancy Filling Rates

The model can be used to study the relationship between firm growth and how quickly vacancies fill. Recall from [Proposition 1](#) that each worker prefers high type firms to low type firms. This implies that vacancies at high type firms fill more quickly, since a larger share of workers are willing to match with high type firms. This, along with [Proposition 6](#), means that within the model faster growing firms also fill their vacancies more quickly.

This fact relates to the recent work of [Davis et al. \(2013\)](#). They find empirically that fast growing establishments tend to fill their job openings more quickly. This is at odds with the textbook search theory, which assumes the vacancy filling rate is exogenous to the firm and consequently employment growth is linear in the vacancy rate at the firm level. The conclusion of [Davis et al. \(2013\)](#) is that fast growing firms are using other margins to attract workers, besides simply posting

more vacancies. My model and the empirical results point to a particular channel. High growth firms are more productive, and are able to hire selective high type workers. The increased wages associated with a positive productivity shock allow the firm to attract workers that would otherwise reject an offer from the firm. In this way the firm hires more of the workers it meets, reducing vacancy durations.

1.4.5.2 Compositional Dynamics

Conditional on positive assortative matching, we can say something about the dynamics of worker composition at the firm.

Proposition 7. *Assume that matching is positive assortative: $y'(x) > 0$. At a given firm, the growth rate of employment of type x workers*

$$g_t(x) = \frac{N_t(x) - N_{t-1}(x)}{0.5(N_t(x) + N_{t-1}(x))} \quad (1.31)$$

is weakly increasing in x , as long as $N_t(x) > 0$.

Proof. See the Appendix. □

This result can be understood as follows. Fix a worker type x . When y falls below $y(x)$, a firm cannot hire any type x workers, and all incumbent type x workers leave. If y later rises above $y(x)$ the firm begins hiring type x workers again, and the growth rate is high because the initial stock is low. On the other hand, if y has been greater than $y(x)$ for a long time there will be a larger stock of type x workers at the firm. This implies the growth rate of type x workers will be lower. Better workers have higher thresholds $y(x)$, so the firm is more likely to have dipped below the threshold more recently.

These dynamics lead to another interesting result:

Corollary 1. *The quality of new hires at a firm is always (weakly) better than the quality of incumbents, in the sense of first order stochastic dominance.*

Proof. Follows immediately from Proposition 7. □

Since better workers have faster growth rates at a firm, it follows that the quality of new hires dominates the quality of incumbents. This holds regardless of whether the firm is growing or shrinking, since a negative shock leads the best workers to leave. So although the quality of new hires may be lower than that of incumbents one period ago, the quality is higher than those who remain. Consideration of separations leads to another result.

Proposition 8. *The quality of separating workers is always (weakly) better than the quality of incumbents, in the sense of first order stochastic dominance.*

Proof of Proposition 8. Separations are a mix of exogenous separations, at rate s , and endogenous separations in response to productivity shocks. Exogenous separation rates are constant across worker types, so if there is not a negative productivity shock the distribution of separations matches the distribution of incumbents. If there is a negative productivity shock then the best incumbents separate, and the distribution of separators will dominate the distribution of incumbents. □

Given that both hires and separators are better than incumbents, we might ask whether we can order hires and separators relative to each other. This turns out not to be possible. If there is a good productivity shock, hires will be better than

separators, since all separations will be exogenous. If there is a negative shock the opposite will hold.

1.4.5.3 Dynamics and Selectivity

Section 1.4.3 showed that the sufficient conditions for assortative matching are independent of the existence or frequency of productivity shocks. In this section I study worker selectivity and its relation to productivity shocks, under the assumption of positive assortative matching. I show that reducing the persistence of shocks makes workers less selective. Formally, the level of $y(x)$ is decreasing in p .

Proposition 9. *Let $f(y)$ be fixed. Under positive assortative matching, a reduction in p reduces $y(x)$ for all x .*

This is a partial equilibrium result: it depends on $f(y)$ being held constant. In reality, different values of p would induce different unemployment rates and vacancy postings, changing $f(y)$.²⁴ Nonetheless, the result provides some strong intuition for how productivity shocks shape matching decisions and sorting. The effect of more frequent shocks is to make firms more similar to each other, since the expected duration of current productivity is shorter. Since firms are effectively more similar to each other, each worker is willing to accept more firm offers and $y(x)$ declines. As p increases, all workers move toward accepting all offers²⁵.

²⁴There is a special case in which this result holds exactly in general equilibrium. In the special case of a quadratic meeting function $M = UV$, the values $f(y)$ depend only on the number of vacancies posted, not on the number of unemployed. If vacancy costs are highly convex, say $c(v) = \lim_{z \rightarrow \infty} v^z$ then vacancies per firm are constant, and $f(y)$ will be invariant to changes in p .

²⁵In this discrete time model, workers may still not accept all offers when $p = 1$. This is because a very low value of y , even for one period, may be enough to deter matching. However, the same model can be set in continuous time with no change in sorting results. In continuous time, the arrival rate of shocks can be driven to infinity, and workers accept all jobs in the limit.

1.5 Conclusion

This paper contributes to the theory and empirics of worker-firm sorting. On the empirical side, I document a previously unknown relationship between firm growth and the quality of hires: when a given firm grows faster it hires more productive workers. This relationship survives corrections for selection bias and the endogeneity of wages. Under the assumption that faster growing firms are more productive this pattern implies positive assortative matching. This evidence of assortative matching stands in contrast to the mixed results in the literature following [Abowd et al. \(1999\)](#). An important caveat is that when I do not condition on firm characteristics, I do not find evidence of assortative matching. This mirrors the results in the AKM literature, which do not typically control for firm characteristics. I argue that firm characteristics proxy for the labor market a firm faces. While assortative matching may be the norm within a given industry or occupation, it need not characterize cross-market variation in firm productivity, worker productivity, and wages. Thus my baseline results suggest positive sorting within labor market segments, while my alternative specification (and perhaps the AKM regression) captures the more ambiguous relationships that hold between markets.

Motivated by the empirical results, I develop a tractable model of worker-firm sorting within a single market. The model's tractability allows me to analytically characterize sorting, as well as firm compositional dynamics and the effect of productivity shocks on sorting. The model's central equilibrium condition, the worker's first order condition, corresponds exactly to a model of sequential search

with an exogenous wage distribution. I show that the model reproduces the positive correlation between firm growth and the quality of hires when worker and firm types are strong complements in production (i.e., the production function satisfies strict log-supermodularity). This result extends the results of [Becker \(1973\)](#) on complementarity and assortative matching to an environment with productivity shocks and firm growth.

There are many possibilities for future research. One obvious direction is to examine business cycle behavior. Since the Great Recession, there has been renewed interest in the composition of the unemployed and how hiring recovers after a downturn. [Mueller \(2012\)](#) shows that composition of the unemployed becomes *better* in recessions. [Davis et al. \(2013\)](#) and [Davis et al. \(2012\)](#) study the role of employer recruiting intensity in the Great Recession. These studies point to a role for worker and firm heterogeneity in understanding aggregate dynamics.

A.1 Data

In constructing worker and job flows I follow generally follow the definitions used in official LEHD data products.²⁶ The first task is defining firm employment. As noted in the main text, the total number of workers with earnings at firm j in quarter t does not necessarily reflect the firm size at any point in time. A firm with high turnover will have many workers over the course of a quarter, even though it only had a few employees at each point in time. To get around this issue I use beginning of quarter employment and end of quarter employment. Beginning of quarter employment in quarter t is the number of workers who reported positive earnings at firm in both $t - 1$ and t . Intuitively, this means that the worker was at the firm on the first day of quarter t . It is possible that the worker separated during $t - 1$ and was rehired sometime later in quarter t , but this is not a serious concern overall. Similarly, end of quarter employment is the number of workers with positive earnings at the firm in both quarters t and $t + 1$. These workers are likely to have been employed on the last day of quarter. Thus beginning of quarter employment and end of quarter employment serve as two “point in time” estimates of firm size. Letting $bop_{j,t}$ and $eop_{j,t}$ denote the two point in time measures, I define firm employment growth using the conventional [Davis et al. \(1998\)](#) formula

$$g_{j,t} = \frac{eop_{j,t} - bop_{j,t}}{0.5(eop_{j,t} + bop_{j,t})}.$$

²⁶See http://lehd.ces.census.gov/doc/QWI_101.pdf and the related documentation for more information on public-use LEHD data.

A.1.1 Full Quarter Earnings

Recall my definition of $q_{i,t}^{FQ}$:

$$q_{i,t}^{FQ} = \frac{\sum_{k=1}^8 \text{earn}_{i,t-k}^{FQ}}{\sum_{k=1}^8 \mathbf{1}\{\text{earn}_{i,t-k}^{FQ} > 0\}}.$$

The intention here is to measure the worker's average earnings per quarter *while employed*. The difficulty is that when a worker is hired or separated within a quarter I do not observe how much of the quarter they were employed for. The solution is to look for cases where I can reasonably confident that the worker was employed the entire quarter. In particular, if a worker is observed at firm j in period $t-1$, t and $t+1$ it is likely they were employed at j continuously throughout period t . I follow this logic in calculating $\text{earn}_{i,t}^{FQ}$:

$$\text{earn}_{i,t}^{FQ} = \text{earn}_{i,t} \times \mathbf{1}\{\text{earn}_{i,t-1} > 0\} \times \mathbf{1}\{\text{earn}_{i,t} > 0\} \times \mathbf{1}\{\text{earn}_{i,t+1} > 0\}.$$

A.1.2 Hires

I emphasize the importance of focusing on hires from nonemployment. Here again I must take account of the quarterly nature of the data. If a worker is observed to have worked at several firms in a quarter it is difficult to determine which jobs were concurrent, which jobs ended before others began, and whether there were any nonemployment spells in the quarter.

A hire occurs when a worker has positive earnings at firm j in quarter t , but zero earnings at j in quarter $t-1$. This implies that at some point in quarter t the worker was hired at j . If the worker has positive earnings *only* at j in quarter

$t - 1$ we may assume that they were nonemployed prior to being hired.²⁷ Thus, to measure hires from nonemployment (NE) I use only workers who were hired in t and had exactly one job in t .

Measuring job to job hires (EE) also requires some assumptions. I consider a hire to be an EE hire if the worker (1) was hired in t , (2) had another job that continued from $t - 1$ into t , and (3) no jobs from $t - 1$ continued into $t + 1$. These restrictions allow for job overlap or periods of nonemployment during t , but do not allow the worker's old jobs (from $t - 1$) to continue beyond the transition quarter. This measure doubtless includes some transitions that included nonemployment between jobs. It also excludes job to job flows involving very short duration jobs. These concerns are not too serious for me, since I only use the EE measures as a comparison for the NE measure. I have also tried several variations on these definitions and found no significant differences in the results.

A.2 Additional Empirical Results

Here I present results from some intermediate specifications which include some of the controls from the baseline specification. First, Figure 3 plots the estimated $m(\cdot)$'s when I only include size controls. These estimates fall somewhere in between Figure 1.1 and Figure 1.2. Apparently controlling for size brings us closer to the "within firm" specification, but does not eliminate the non-monotonicity seen in Figure 1.2.

²⁷The LEHD data are based on employer payroll records, which are organized (within the employer) by pay period. Since pay periods may vary across employers it is in principal possible that a worker could make a job-to-job transition near the first day of a quarter, but only be recorded at a single employer in each quarter. This problem is attenuated by the short duration of pay periods relative to a calendar quarter.

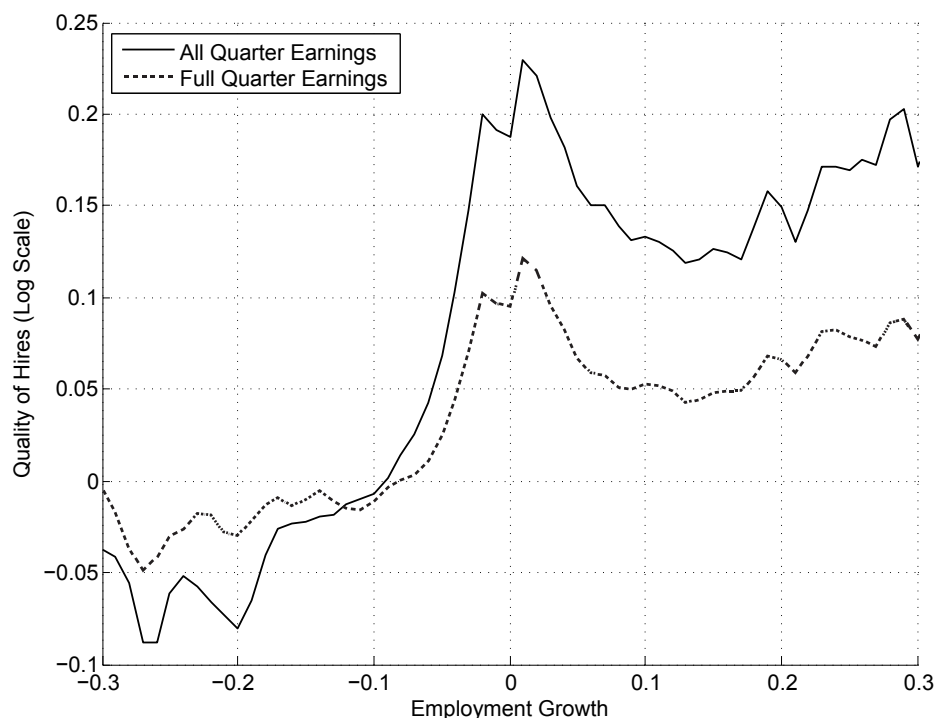


Figure 3: Size Controls Only

Next, Figure 4 adds firm fixed effects industry-season controls to the size controls. The results are much closer to the baseline specification, especially for all quarter earnings ($Q_{j,t}$). Both relations are on average positive and statistically significant. Qualitatively, the combination of firm fixed effects and size controls is enough to bring the results close to the the much richer baseline specification. A natural question is whether the firm fixed effects are truly necessary. Firm fixed effects are a convenient way to absorb all time invariant heterogeneity, but they might just proxy for data moments that are observable. In Figure 5 I experiment with a specification that is similar to the baseline specification, except that the firm fixed effects are replaced with a set of variables reflecting the firm's average worker and job flows. In particular, I include the mean and standard deviation of employment growth, the hires rate, and the separations rate over the observed lifetime of the firm. These six variables vary across firms but not over time, and capture the

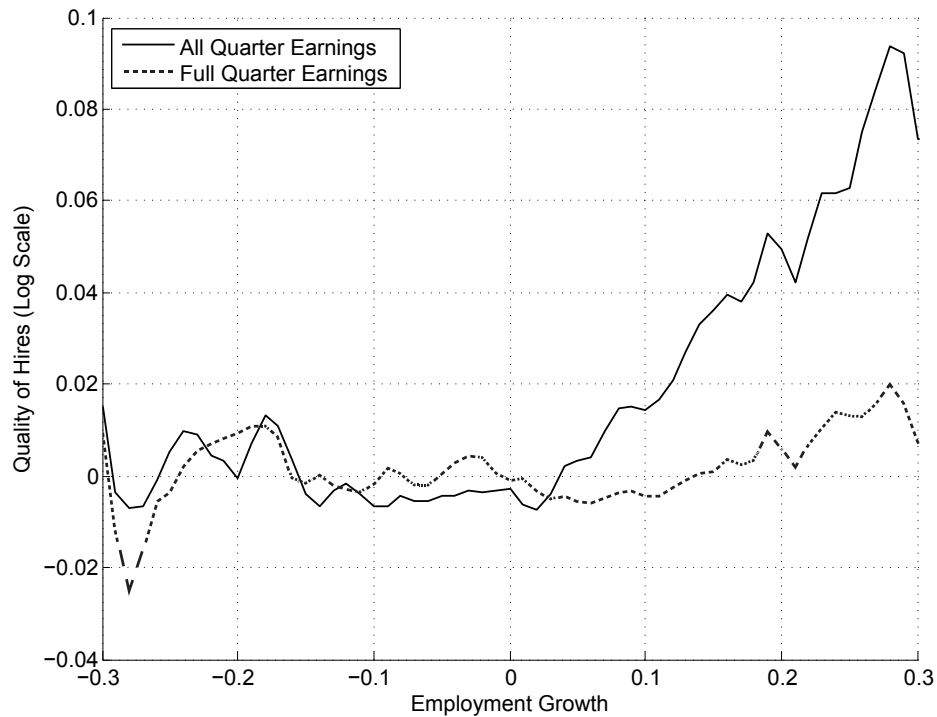


Figure 4: Size Controls and Firm Fixed Effects

firm's average level and volatility of worker and job flows. Thus, they capture some characteristics that would have been picked up by a firm dummy. In Figure 5 we see that this modified regression does produce a nearly monotone relationship between the variables, though the shape of the relation is quiet different from Figure 1.1. This suggests that while the worker/job flow variables are useful controls, they do not substitute for the firm fixed effects.

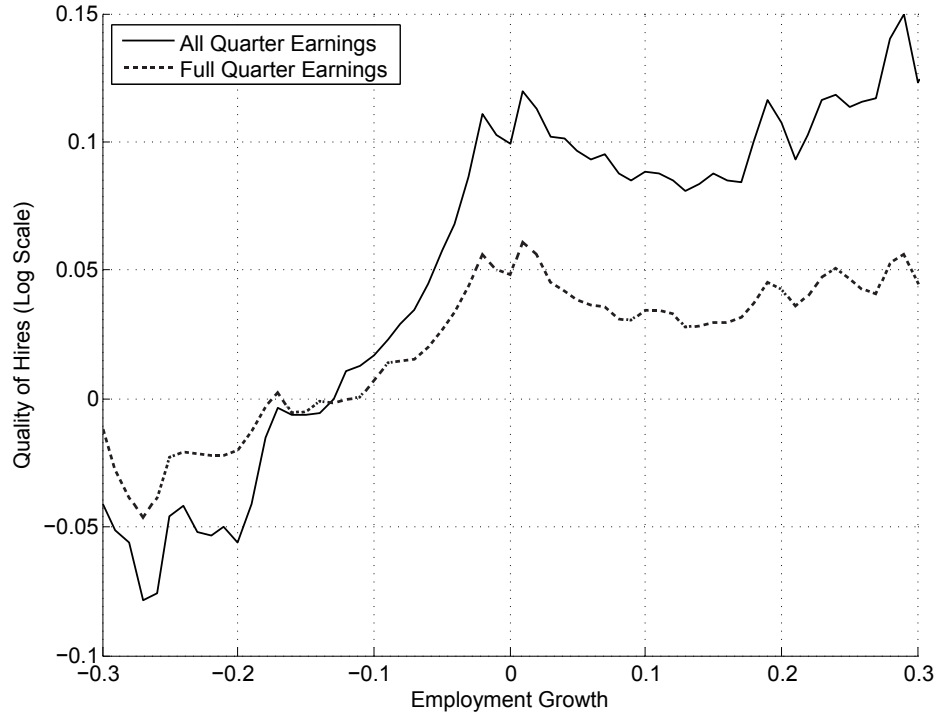


Figure 5: All Controls Except Firm Fixed Effects (Including Average Flows)

A.3 The Firm's Problem

The firm's profit maximization problem is

$$\begin{aligned}
 \pi(N_{-1}(x), y) = & \max_{N(x), v} \left\{ \int_0^1 N(x) [F(x, y) - w(x, y)] dx - c(v) \right. & (32) \\
 & + \beta(1-p)\pi((1-s)N(x), y) \\
 & \left. + \beta p \int_0^1 h(y') \pi((1-s)N(x), y') dy' \right\}
 \end{aligned}$$

subject to

$$0 \leq N(x) \leq N_{-1}(x) + v \cdot q(x) \quad \forall x \quad (33)$$

$$y \in A(x) \quad \text{if} \quad N(x) > 0 \quad \forall x. \quad (34)$$

Turning to (33) first, $N_{-1}(x) : [0, 1] \rightarrow \mathbb{R}_+$ is the inherited stock of workers from the previous period, $q(x) : [0, 1] \rightarrow \mathbb{R}_+$ is the arrival rate of workers of each type, v is the firm's vacancy postings (a choice variable) and $N(x)$ is the workforce in the current period (another choice variable). The constraint simply states that current employment of type x workers must be no greater than surviving employment from the past, $N_{-1}(x)$, plus new recruitment $v \cdot q(x)$. Note that the firm posts a single set of vacancies for all worker types.

The second constraint (34) imposes that the firm must be in the acceptance set of any worker they employ. If the worker did not find the match acceptable, they would depart into unemployment before production began.

In the profit maximization problem (32) the firm takes last period's employment and their current type as state variables. The firm chooses current employment and vacancy posting, taking the bargained wages $w(x, y)$ as given. Within the period, the firm enjoys profits net of wages $F(x, y) - w(x, y)$ from each worker, and pays the convex vacancy posting cost $c(v)$. If a productivity shock does not arrive the firm has discounted continuation value $\beta \pi((1-s)N(x), y)$, where s is the exogenous separation rate. If a shock does arrive the firm gets the expected value $\beta \int_0^1 h(y') \pi((1-s)N(x), y') dy'$.

The Lagrangian for the firm's problem is

$$\begin{aligned}
\pi(N_{-1}(x), y) &= \int_0^1 N(x) [F(x, y) - w(x, y)] dx - c(v) \\
&+ \beta(1-p)\pi((1-s)N(x), y) \\
&+ \beta p \int_0^1 h(y') \pi((1-s)N(x), y') dy' \\
&+ \int_0^1 \mu(x) (N_{-1}(x) + v \cdot q(x) - N(x)) dx.
\end{aligned}$$

Assume that $y \in A(x)$. Then the match yields positive surplus, and the hiring constraint will bind. In this case the FOC with respect to $N(x)$ is

$$\begin{aligned}
F(x, y) - w(x, y) + \beta(1-p)(1-s) \frac{d\pi}{dN_*} \\
+ \beta p(1-s) \int_0^1 h(y') \frac{d\pi}{dN_*} dy' - \mu(x) = 0.
\end{aligned}$$

Next, note that by the envelope condition

$$\frac{\partial \pi(N_{-1}(x), y)}{\partial N_{-1}(x)} = \mu_x \tag{35}$$

so when $y \in A(x)$ I can use the FOC to write

$$\begin{aligned}
\frac{\partial \pi(N_{-1}(x), y)}{\partial N_{-1}(x)} &= F(x, y) - w(x, y) \\
&+ (1-s)(1-p)\beta \frac{\partial \pi(N_*(x), y)}{\partial N_*(x)} \\
&+ (1-s)p\beta \int_0^1 h(y') \frac{\partial \pi(N_*(x), y')}{\partial N_*(x)} dy'.
\end{aligned} \tag{36}$$

On the other hand, if $y \notin A(x)$ then worker separates from the firm before pro-

duction occurs. This implies

$$\frac{\partial \pi(N_{-1}(x), y)}{\partial N_{-1}(x)} = 0 \text{ if } y \notin A(x) \quad (37)$$

Taking equations (36) and (37) together for $y \in A(x)$ I have

$$\begin{aligned} \frac{\partial \pi(N_{-1}(x), y)}{\partial N_{-1}(x)} &= F(x, y) - w(x, y) \\ &\quad + (1-s)(1-p)\beta \frac{\partial \pi(N_*(x), y)}{\partial N_*(x)} \\ &\quad + (1-s)p\beta \int_{A(x)} h(y') \frac{\partial \pi(N_*(x), y')}{\partial N_*(x)} dy' \end{aligned}$$

Which is identical to equation (1.3) when we set $\frac{\partial \pi(N_{-1}(x), y)}{\partial N_{-1}(x)} = J(x, y)$. This shows how we can formally derive equation (1.3) from the firm's problem. Taking the FOC with respect to v I have

$$c'(v) - \int_0^1 q(x)\mu(x)dx = 0.$$

The left hand side of this equation is the value of a vacancy. Optimality dictates that this be set to zero. Using equation 35 I can write

$$c'(v) = \int_0^1 q(x)J(x, y)dx.$$

A.4 Assortative Matching with Productivity Shocks

Consider the worker's threshold condition (1.14):

$$F(x, y(x)) - \beta\eta\theta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)][F(x, y) - F(x, y(x))] dy = \quad (38)$$

The following generalizes Proposition 3 to the case with productivity shocks.

Proposition 10. *When the worker's choice of $y(x)$ is given by (38), $y'(x) > 0$ if and only if*

$$\int_{y(x)}^1 w^*(y) \frac{F_1(x, y)}{F(x, y)} dy > \frac{F_1(x, y(x))}{F(x, y(x))} \quad (39)$$

and $y'(x) < 0$ if and only if

$$\int_{y(x)}^1 w^*(y) \frac{F_1(x, y)}{F(x, y)} dy < \frac{F_1(x, y(x))}{F(x, y(x))}, \quad (40)$$

where $w^*(y) = \frac{[\eta f(y) - (1-s)ph(y)]F(x, y)}{\int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)]F(x, y) dy}$ and $\int_{y(x)}^1 w^*(y) dy = 1$.

Proof. Equation (38) implicitly defines $y(x)$, so differentiating it I can obtain an expression for $y'(x)$. Applying the implicit function theorem I have

$$y'(x) = -\frac{F_1(x, y(x)) - \beta\eta\theta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] [F_1(x, y) - F_1(x, y(x))] dy}{F_2(x, y(x)) \left(1 + \beta\eta\theta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] dy\right)}.$$

It is easy to show that the denominator is positive by substituting in for θ . So the sign of $y'(x)$ depends only on the numerator, implying $y'(x) > 0$ if and only if

$$\beta\theta\eta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] F_1(x, y) dy > F_1(x, y(x)) \left(1 + \beta\theta\eta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] dy\right)$$

Dividing this expression by (38) I arrive at

$$\frac{\int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] F_1(x, y) dy}{\int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] F(x, y) dy} > \frac{F_1(x, y(x))}{F(x, y(x))}$$

Setting $w^*(y) = \frac{[\eta f(y) - (1-s)ph(y)]F(x,y)}{\int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)]F(x,y)dy}$ I have equation (39)

$$\int_{y(x)}^1 w^*(y) \frac{F_1(x,y)}{F(x,y)} dy > \frac{F_1(x,y(x))}{F(x,y(x))}$$

as desired. A symmetric argument yields equation (40). \square

Note that the weights $w^*(y)$ may be negative if $f(y)$ is small and $h(y)$ is large. The sorting results summarized in Proposition 4 relied on the weights $w(y)$ being *positive*. To generalize Proposition 4 I need to show a single-crossing property holds for $w^*(y)$. This requires two intermediate results.

Result 1 First, I need to show

$$\int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] F(x,y) dy > 0. \quad (41)$$

That is, although $[\eta f(y) - (1-s)ph(y)] F(x,y)$ may take negative values, the integral is always positive at an optimal threshold $y(x)$. To see this, rearrange (38) to get

$$F(x,y(x)) \left(1 + \beta\eta\theta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] dy \right) = \beta\eta\theta \int_{y(x)}^1 [\eta f(y) - (1-s)ph(y)] F(x,y) dy.$$

I have already argued in the proof of Proposition 10 that the term in parentheses is greater than zero, which then implies that the right hand side is positive, so equation (41) holds.

Result 2 Second, I need to show that there exists \bar{y} such that

$$[\eta f(y) - (1-s)ph(y)] < 0 \iff y < \bar{y}. \quad (42)$$

That is, $[\eta f(y) - (1-s)ph(y)]$ has a single-crossing property, and is strictly positive above some threshold. To see this recall that $h(y)$ is not only the density of productivity shocks but is also the density of firm types (this follows from the nature of the productivity process). Then I can write $f(y) = \frac{M(U,V)}{U} \frac{h(y)v(y)}{V}$, where $v(y)$ is the number of vacancies posted by a type y firm and V is the aggregate number of vacancies. Using this I factor out $h(y)$ from the expression:

$$[\eta f(y) - (1-s)ph(y)] = h(y) \left[\eta \frac{M(U,V)}{U} \frac{v(y)}{V} - (1-s)p \right]$$

Next, recall from Section **1.4.1.6** that vacancy posting $v(y)$ is increasing in y . This shows that $\left[\eta \frac{M(U,V)}{U} \frac{v(y)}{V} - (1-s)p \right]$ will turn positive once and only once, proving (42). Intuitively, the difference between $f(y)$ and $h(y)$ is the number of vacancies posted per firm. More productive firms post more vacancies, so it is understandable that above a threshold the difference $\eta f(y) - (1-s)ph(y)$ will be positive.

Finally, taking equations (41) and (42) together, it is clear that the weights $w^*(y)$ also satisfy the same single crossing property:

$$w^*(y) < 0 \iff y < \bar{y}. \quad (43)$$

Given that $w^*(y)$ satisfied this condition, I can show a useful result.

Lemma 1. Let $w^*(y)$ satisfy equation (43) and $\int_a^b w^*(y) = 1$. If $g(y)$ is increasing and positive then

$$\int_a^b w^*(y)g(y) > g(a)$$

and if $g(y)$ is decreasing and positive then

$$\int_a^b w^*(y)g(y) < g(a)$$

Proof. Say that $g(y)$ is increasing and positive. Then I want to show

$$\int_a^b w^*(y) (g(y) - g(a)) > 0.$$

Note that $g^*(y) = g(y) - g(a)$ is increasing and non-negative. Then the desired condition can be written as:

$$\int_{\bar{y}}^b w^*(y)g^*(y) > \int_a^{\bar{y}} (-w^*(y))g^*(y).$$

where $-w^*(y)$ is a positive weight for $y < \bar{y}$. Recall that $\int_a^b w^*(y)dy = 1$, so

$$\int_{\bar{y}}^b w^*(y) > \int_a^{\bar{y}} (-w^*(y))$$

$$\int_{\bar{y}}^b w^*(y)g^*(\bar{y})dy > \int_a^{\bar{y}} (-w^*(y))g^*(\bar{y})dy$$

Since $g^*(y)$ is increasing I have $\int_{\bar{y}}^b w^*(y)g^*(y) > \int_{\bar{y}}^b w^*(y)g^*(\bar{y})dy$ and $\int_a^{\bar{y}} (-w^*(y))g^*(\bar{y})dy > \int_a^{\bar{y}} (-w^*(y))g^*(y)$. Putting these all together I have

$$\int_{\bar{y}}^b w^*(y)g^*(y) > \int_a^{\bar{y}} (-w^*(y))g^*(y)$$

as desired. □

Substituting $g(y) = \frac{F_1(x,y)}{F(x,y)}$ in Lemma 1, it is clear that $\frac{\partial}{\partial y} \frac{F_1(x,y)}{F(x,y)} > 0$ is sufficient for positive assortative matching in the generalized model. The conditions for negative sorting and no sorting follow from analogous arguments. I summarize the generalized results below.

Proposition 11. *Consider the general model with productivity shocks. Matching is positive assortative, $y'(x) > 0$, if F is strictly log-supermodular. Matching is negative assortative, $y'(x) < 0$ if F is strictly log-submodular. There is no sorting if $\frac{\partial}{\partial y} \frac{F_1(x,y)}{F(x,y)} = 0$.*

A.5 Omitted Proofs

Proof of Fact Fact 1: First I derive an expression for $S(x, y)$. Rearranging (1.4) I can write

$$U(x) = \beta \int_{A(x)} f(y) S(x, y) dy + \beta U(x)$$

and rearranging equation (1.5) I have

$$\begin{aligned} W(x, y) &= w(x, y) + \beta W(x, y) - [s + (1-s)p] \beta S(x, y) \\ &\quad + (1-s)p\beta \int_{A(x)} h(y') S(x, y') dy' \end{aligned}$$

Putting these together I have

$$\begin{aligned}
S(x, y) &= w(x, y) + (1-s)(1-p)\beta S(x, y) \\
&\quad + (1-s)p\beta \int_{A(x)} h(y') S(x, y') dy' \\
&\quad - \beta \int_{A(x)} f(y) S(x, y) dy.
\end{aligned} \tag{44}$$

Next, note that the bargaining equation (1.6) implies $J(x, y) = \frac{1-\eta}{\eta} S(x, y)$. Using this in equation (1.3) yields

$$\begin{aligned}
J(x, y) &= F(x, y) - w(x, y) + (1-s)(1-p)\frac{1-\eta}{\eta}\beta S(x, y) \\
&\quad + (1-s)p\frac{1-\eta}{\eta}\beta \int_{A(x)} h(y') S(x, y') dy'.
\end{aligned} \tag{45}$$

Combining (44), (45), the Nash bargaining equation and rearranging I have

$$w(x, y) = \eta F(x, y) + (1-\eta)\beta \int_{A(x)} f(y) S(x, y) dy \tag{46}$$

Using this in the worker surplus expression I have

$$\begin{aligned}
S(x, y) &= \theta\eta \left(F(x, y) - \beta \int_{A(x)} f(y) S(x, y) dy \right) \\
&\quad + \theta(1-s)p\beta \int_{A(x)} h(y') S(x, y') dy'
\end{aligned} \tag{47}$$

where $\theta = \frac{1}{1-(1-s)(1-p)\beta}$. Similarly, the equation 45 yields

$$\begin{aligned}
J(x, y) &= \theta(1-\eta) F(x, y) - \theta\eta\beta \int_{A(x)} f(y) J(x, y) dy \\
&\quad + \theta(1-s)p\beta \int_{A(x)} h(y') J(x, y') dy'.
\end{aligned}$$

□

Proof. Start with equation (1.9):

$$S(x, y) = \theta\eta \left(F(x, y) - \beta \int_{A(x)} f(y) S(x, y) dy \right) + \theta(1-s)p\beta \int_{A(x)} h(y') S(x, y') dy'$$

Rearranging this yields

$$S(x, y) = \theta\eta F(x, y) + \theta\beta \int_{A(x)} [(1-s)ph(y') - \eta f(y')] S(x, y') dy' \quad (48)$$

Multiplying by $[(1-s)ph(y) - \eta f(y)]$ and integrating I have

$$\begin{aligned} \int_{A(x)} [(1-s)ph(y) - \eta f(y)] S(x, y) dy &= \\ \theta\eta \int_{A(x)} [(1-s)ph(y) - \eta f(y)] F(x, y) dy & \\ \left\{ \int_{A(x)} [(1-s)ph(y) - \eta f(y)] dy \right\} \theta\beta \int_{A(x)} [(1-s)ph(y') - \eta f(y')] S(x, y') dy' & \end{aligned}$$

Which implies

$$\int_{A(x)} [(1-s)ph(y) - \eta f(y)] S(x, y) dy = \frac{\theta\eta \int_{A(x)} [(1-s)ph(y) - \eta f(y)] F(x, y) dy}{1 - \theta\beta \int_{A(x)} [(1-s)ph(y) - \eta f(y)] dy}$$

And substituting this into (48)

$$S(x, y) = \theta\eta F(x, y) - \theta\eta \frac{\theta\beta \int_{A(x)} [\eta f(y) - (1-s)ph(y)] F(x, y) dy}{1 + \theta\beta \int_{A(x)} [\eta f(y) - (1-s)ph(y)] dy}$$

which is the desired result.

Continuity: S fails to be continuous only when $1 - \theta\beta \int_{A(x)} [(1-s)ph(y) - \eta f(y)] dy =$

0. For this to hold it must be that

$$1 - \beta\theta(1-s)p \left[\int_A h(y) dy \right] \leq 0.$$

After some rearranging this becomes

$$\frac{1 - \beta(1-s) + \beta(1-s)p \left(1 - \left[\int_A h(y) dy \right] \right)}{1 - \beta(1-s)(1-p)} \leq 0.$$

Note that $\left[\int_A h(y) dy \right] \leq 1$, so this is clearly a contradiction. So I have shown that

$1 - \beta\theta \left[\int_A ((1-s)ph(y) - \eta f(y)) dy \right] > 0$, and thus S is continuous. \square

Proof. Step 1 shows that there are never two points satisfying (1.14). Steps 2 and 3 show that (1.14) has a solution if and only if $y(x)$ is not a solution.

Step 1: There is at most one solution to (1.14). The derivative of the right hand side is

$$\begin{aligned} \frac{\partial}{\partial y^*} \beta\theta\eta \int_{y^*}^1 f(y) (F(x,y) - F(x,y^*)) dy &= \\ -\beta\theta\eta F_2(x,y^*) \int_{y^*}^1 f(y) dy &< 0 \end{aligned}$$

And the derivative of the left hand side is

$$\begin{aligned} \frac{\partial}{\partial y^*} \left[F(x,y^*) - b + \theta\beta(1-s)p \int_{y^*}^1 h(y) (F(x,y) - F(x,y^*)) dy \right] &= \\ F_2(x,y^*) \left[1 - \theta\beta(1-s)p \int_{y^*}^1 h(y) dy \right] & \end{aligned}$$

To see that $\left[1 - \theta\beta(1-s)p \int_{y^*}^1 h(y)dy\right]$ is positive, substitute in for θ to get

$$\left[1 - \theta\beta(1-s)p \int_{y^*}^1 h(y)dy\right] = \frac{1 - \beta(1-s) \int_{y^*}^1 h(y)dy}{1 - \beta(1-s)(1-p)} > 0.$$

Thus the left hand side of (1.14) is strictly decreasing and the right hand side is strictly increasing, so there is at most one solution.

Step 2: If there is no solution to (1.14) then (1.15) must hold. First, let

$$\begin{aligned} z(y^*) &= (F(x, y^*) - b) + \theta\beta(1-s)p \int_{y^*}^1 h(y)(F(x, y) - F(x, y^*)) dy \\ r(y^*) &= \beta\theta\eta \int_{y^*}^1 f(y)(F(x, y) - F(x, y^*)) dy \end{aligned}$$

From Step 1 we know that

$$\begin{aligned} z'(y^*) &> 0 \\ r'(y^*) &< 0 \end{aligned}$$

Note that

$$\begin{aligned} z(1) &> 0 \\ r(1) &= 0 \end{aligned}$$

Then, since z and r are continuous, the only way to have no solution to (1.14) is to have

$$z(x) > r(x) \quad \forall x.$$

This implies $z(0) > r(0)$, which is equivalent to (1.15) so I am done.

Step 3: If (1.15) holds then (1.14) has no solution. Reversing the logic from Step 2, if $z(0) > r(0)$ then $z(x) > r(x) \forall x$, so (1.14) cannot hold. \square

Proof of Proposition 5. The goal is to show that if F is not strictly log-supermodular then matching is not necessarily positive assortative. I start by assuming that

$$\frac{\partial^2}{\partial x \partial y} \ln F(x^*, y^*) < 0 \quad (49)$$

for some x^* and y^* , and then show that I can pick $f(y)$ to make $y'(x^*) < 0$. First, note that under these conditions we can find $y^{**} > y^*$ such that

$$\frac{F_1(x^*, y^*)}{F(x^*, y^*)} > \frac{F_1(x^*, y^{**})}{F(x^*, y^{**})}. \quad (50)$$

This will be used in the final step of the proof. Now I argue we can set $f(y)$ such that $y(x^*) = y^*$, and then show we can adjust $f(\cdot)$ to put more weight on y^{**} . It is trivial to show that there exist many meeting rate functions $f(\cdot)$ such that $y(x^*) = y^*$, so I simply assuming that we have $f(\cdot)$ such that the optimally condition (1.27) holds:

$$F(x, y^*) \left(1 + \beta\theta\eta \int_{y^*}^1 f(y) dy \right) - \beta\theta\eta \int_{y^*}^1 f(y) F(x, y) dy = 0. \quad (51)$$

Next, define

$$\bar{f} = \int_{y(x)}^1 f(y) dy \quad (52)$$

And define \bar{y} as the solution to

$$F(x, \bar{y}) = \frac{\int_{y^*}^1 f(y) F(x, y) dy}{\int_{y^*}^1 f(y) dy} \quad (53)$$

Note that $\bar{y} > y^*$. \bar{y} is the firm type that yields the expected match production within the acceptance set, while \bar{f} is the probability the gets an acceptable offer.

Then I can write

$$F(x, y^*) (1 + \beta\theta\eta\bar{f}) - \beta\theta\eta\bar{f}F(x, \bar{y}) = 0. \quad (54)$$

This can be interpreted as the indifference condition of a counterfactual problem. Say there is only one firm type \bar{y} , and the worker gets offers with probability \bar{f} . Obviously the worker accepts the offers and the decision problem is trivial. But, we can also examine what would happen if the worker were offered a one time chance to match with an alternative firm type y . Then this condition says that y^* is the lowest acceptable alternative firm type. Equivalently, we could imagine there is a measure zero population of firms with $y \neq \bar{y}$, and that the worker has to define their acceptance set on these alternatives as well.

Rearranging I have

$$\bar{f} = \frac{F(x, y^*)}{\beta\theta\eta(F(x, \bar{y}) - F(x, y^*))}.$$

Now define a new function

$$\bar{f}(y) = \frac{F(x, y^*)}{\beta\theta\eta(F(x, y) - F(x, y^*))} \quad (55)$$

That is, $\bar{f}(y)$ is the *total* arrival rate necessary to ensure the worker's reservation

type is y^* when all offers (except a set of measure zero) are of type y . $\bar{f}(y)$ is well defined for all $y > y^*$, and in particular $\bar{f}(y^{**})$ exists. In words, we can find an offer arrival rate that ensures the worker's reservation productivity is y^* when almost all firms are type y^{**} .

Equation (55) presumes a point mass of firms at type y . The results in Proposition 3 relied on continuous, differentiable densities. This is not a serious gap, since we can approximate the point mass as the limit of a sequence of smooth densities. The critical point here is that F is twice continuously differentiable. This means that $\frac{\partial^2}{\partial x \partial y} \ln F(x, z) < 0$ implies $\frac{\partial^2}{\partial x \partial y} \ln F(x, z') < 0$ for some neighborhood U around z . Then we can replace the point mass at z with a density that accumulates on U .

Recall from Proposition 3 that $y'(x^*) > 0$ if and only if

$$\int_{y^*}^1 w(y) \frac{F_1(x^*, y)}{F(x^*, y)} dy > \frac{F_1(x^*, y^*)}{F(x^*, y^*)} \quad (56)$$

where

$$w(y) = \frac{f(y)F(x, y)}{\int_{y^*}^1 f(y)F(x, y) dy} > 0. \quad (57)$$

Clearly, if $f(y^{**}) \gg f(y)$ then $w(y^{**}) \gg w(y)$ as well. Under these circumstances

$$\int_{y^*}^1 w(y) \frac{F_1(x^*, y)}{F(x^*, y)} dy \approx \frac{F_1(x^*, y^{**})}{F(x^*, y^{**})} < \frac{F_1(x^*, y^*)}{F(x^*, y^*)} \quad (58)$$

The inequality shows that $y'(x^*) < 0$, so we are done. \square

Proof of Proposition 6. The proof has three steps.

Step 1: Growth is an exact function of current y and a set of state variables m that satisfy $h(y|m) = h(y)$. Consider a firm in period t . The firm's

growth rate is defined as

$$g_t = 2 \frac{E_t - E_{t-1}}{E_t + E_{t-1}}$$

Where E_t is total employment, measured in the production stage. E_t satisfies the law of motion $E_t = H_t + (1 - s_t)E_{t-1}$, where H_t is total hires and s_t is the separation rate. We observe $s_t > s$ only when there is a negative productivity shock, leading to endogenous separations. Recall also that total hires $H_t = H(y_t)$ is a function only of current productivity y_t : both the number of vacancies posted and the acceptance decisions of workers only depend on y_t . Assuming $y_t = y_{t-1} = y_{t-2}$ and working recursively, we can write

$$E_t = H(y_t)(1 + (1 - s)) + (1 - s)^2 E_{t-2}$$

and assuming that $y = y_t$ has been constant since period $t - j$ we have

$$E_t = H(y) \sum_{i=0}^{j-1} (1 - s)^i + (1 - s)^j E_{t-j}. \quad (59)$$

Thus E_t is an exact function of (1) current productivity y , (2) the duration of the current productivity regime j , and (3) employment at the start of the regime E_{t-j} . Note that E_{t-j} is not independent of y : if y is very low, we expect E_{t-j} to be low as well, since productivity changed to y at the beginning of period $t - j$ or earlier. This means any high type worker will have separated before E_{t-j} is measured.

In what follows, I assume that a productivity shock arrived at the start of period $t - j$, and there has been no subsequent shock. Let $N : [0, 1] \rightarrow \mathbb{R}_+$ be the firm's

employment distribution in period $t - j - 1$. That is, the firm employed $N(x)$ workers of type x in the production stage of period $t - j - 1$. If the shock does not cause any separations then

$$\begin{aligned} E_{t-j} &= H(y) + (1-s)E_{t-j-1} \\ &= H(y) + (1-s) \int_0^1 N(x)dx. \end{aligned}$$

$\int_0^1 N(x)dx$ gives total employment at the end of period $t - j - 1$. At the start of $t - j$, productivity changes to y and exogenous separations occur. So hiring is given by $H(y)$ and separation by $\int_0^1 N(x)dx$. In the general case, where endogenous separations may occur, the law of motion is

$$E_{t-j} = H(y) + (1-s) \int_0^{x(y)} N(x)dx. \quad (60)$$

Here $x(y)$ is the highest worker type that is willing to work for type y firm. The integral makes clear that any high type workers, $x > x(y)$, separate from the firm at the start of $t - j$. Putting (59) and (60) together I have

$$E_t = H(y) \sum_{i=0}^{j-1} (1-s)^i + (1-s)^j \int_0^{x(y)} N(x)dx.$$

Thus, E_t is an exact function of y, j and $N(x)$. Note y is independent of $N(x)$ and j : since $N(x)$ is only a function of *lagged* productivity values, it contains no information about y . In particular, the conditional density of y , $h(y|j, N(x))$ is equal to the unconditional density $h(y)$. Letting $m = (j, N(x))$, I am done.

Step 2: $g_1(y, m)$ is positive. Define

$$E_{t-j}^* = \int_0^{x(y)} N(x) dx. \quad (61)$$

where $x(y)$ and $N(x)$ are defined in Step 1. E_{t-j}^* is the mass of workers that remain at the firm after the productivity shock, but before exogenous separations occur. Using this notation I can write period t employment as be employment

$$E_t = H \sum_{i=0}^{j-1} (1-s)^i + (1-s)^j E_{t-j}^*$$

where H is the level of hiring for a type y firm. Differencing I have

$$E_t - E_{t-1} = (1-s)^{j-1} (H - sE_{t-j}^*)$$

so the growth rate is

$$g = 2r^{j-1} \frac{H - sE_{t-j}^*}{H \left[\sum_{i=0}^{j-2} r^i + \sum_{i=0}^{j-1} r^i \right] + E_{t-j}^* [r^j + r^{j-1}]}$$

where $r = (1-s)$. Substituting in equation (61) and $H = \int_0^{x(y)} H(x) dx$ I have

$$g = 2r^{j-1} \frac{\int_0^{x(y)} H(x) dx - s \int_0^{x(y)} N(x) dx}{\int_0^{x(y)} H(x) dx \left[\sum_{i=0}^{j-2} r^i + \sum_{i=0}^{j-1} r^i \right] + \int_0^{x(y)} N(x) dx [r^j + r^{j-1}]}$$

Differentiating this expression with respect to y , I find that $g' > 0$ if and only if

$$x'(y) \frac{H(x(y)) - sN(x(y))}{H(x(y)) \left[\sum_{i=0}^{j-2} r^i + \sum_{i=0}^{j-1} r^i \right] + N(x(y)) [r^j + r^{j-1}]} >$$

$$x'(y) \frac{\int_0^{x(y)} H(x) dx - s \int_0^{x(y)} N(x) dx}{\int_0^{x(y)} H(x) dx \left[\sum_{i=0}^{j-2} r^i + \sum_{i=0}^{j-1} r^i \right] + \int_0^{x(y)} N(x) dx [r^j + r^{j-1}]}$$

Under the assumption of positive assortative matching, $x'(y)$ is positive and can be canceled on both sides. Note that the remaining term on the left hand side is the growth rate of type $x(y)$ workers at the firm, while the right hand side is the growth rate of total employment at the firm. Appealing to Proposition 7, we see that the left hand side must be larger, so we are done.

Step 3: $g_1(y, m) > 0$ implies $F_g(\cdot|y')$ dominates $F_g(\cdot|y)$ whenever $y' > y$.

Using the results in Step 1, I can write the distribution of growth conditional firm type as

$$F_g(g|y) = \frac{\int_{m,y} \mathbf{1}\{g(y, m) < g\} \times \mathbf{1}\{y^* = y\} dF(m, y^*)}{\int_{m,y} \mathbf{1}\{y^* = y\} dF(m, y^*)}$$

Where $F(m, y)$ is the joint CDF of m and y and $F_g(g|y)$ is the CDF of growth rates conditional on y . Using the fact that $h(y|m) = h(y)$ this simplifies to

$$F_g(g|y) = \int_m \mathbf{1}\{g(y, m) < g\} dF(m) \quad (62)$$

From equation (62) it is clear that increasing y reduces $F_g(g|y)$ if $g_1(y, m) > 0$.

This means that $F_g(\cdot|y')$ dominates $F_g(g|y)$ whenever $y' > y$, so I am done. \square

Proof of Proposition 7. Define $j(x)$ by

$$\begin{aligned} y_{t-j(x)} &\leq y(x) \\ y_{t-i} &> y(x) \quad \forall i > j(x). \end{aligned}$$

That is, $t - j(x)$ is the most recent period when the firm's productivity was below the worker's acceptance threshold. This implies that $N_{t-j(x)}(x) = 0$, so I can write employment in t as the accumulation of surviving hires since $t - j(x)$:

$$N_t(x) = \sum_{i=0}^{i=j(x)-1} (1-s)^i H_{t-i}(x). \quad (63)$$

Plugging this expression into the growth rate formula I have

$$g_t(x) = 2 \frac{H_t(x) - s \sum_{i=1}^{i=j(x)-1} (1-s)^{i-1} H_{t-i}(x)}{H_t(x) + (2-s) \sum_{i=0}^{i=j(x)-1} (1-s)^{i-1} H_{t-i}(x)} \quad (64)$$

Clearly, a ceteris paribus increase in $j(x)$ reduces $g_t(x)$. This comes about through two channels. A higher $j(x)$ means there has been more time for the firm to accumulate workers, so the denominator of (1.31) is larger. Second, the larger stock of type x workers implies higher level of separations, reducing the numerator. The final step is to note that $H_t(x) = q(x)v(y_t)$: hires of type x workers equals the arrival rate $q(x)$ times the number of vacancies posted, $v(y_t)$. Substituting in this expression, the $q(x)$ cancel leaving

$$g_t(x) = 2 \frac{v(y_t) - s \sum_{i=1}^{i=j(x)-1} (1-s)^{i-1} v(y_{t-i})}{v(y_t) + (2-s) \sum_{i=0}^{i=j(x)-1} (1-s)^{i-1} v(y_{t-i})}. \quad (65)$$

This makes clear that changing x changes $g_t(x)$ only through $j(x)$, and not through

any differences in hiring subsequent to $t - j(x)$. Under positive assortative matching, increasing x lowers $j(x)$, so $g_t(x)$ increases in x . \square

Chapter 2: The Theory of Sorting in Labor Markets

2.1 Abstract

Becker's classic result shows that supermodularity of the production function is necessary for positive assortative matching in a partnership environment. On the other hand it is well known that log-supermodularity of the production function, a stronger condition, is necessary for positive sorting in environments with multi-worker firms and monopolistic competition. Both the partnership model and the monopolistic competition model are meant to study similar phenomena, so it is puzzling that the comparative static predictions are at odds with each other. I show that reason for the divergence is not in the difference between one-to-one and many-to-one matching, but rather in the difference between decreasing returns in employment and decreasing returns in production. I show that the monopolistic competition framework implies a price effect which makes high type firms less attractive for high type workers. As a result, strong complementarities are needed to ensure positive sorting. The price effect is absent from the partnership model, so weaker complementarities are sufficient to generate positive sorting. I show that these results and the related intuition carry over to models with search frictions, providing a pair of tractable benchmark models. These results highlight the connection between frictionless matching models and those that incorporate

search.

2.2 Introduction

In this paper I examine how workers are sorted across firms in a variety of environments. A large literature has developed which studied how heterogeneous workers match with heterogeneous firms. The focus of the literature is on *positive assortative matching*: outcomes where the best, most productive workers are employed by the most productive firms and the least productive workers are assigned to the least productive firms. There are two broad classes of models: one based on [Becker \(1973\)](#)'s one-to-one matching model, and one that resembles the [Dixit and Stiglitz \(1977\)](#) framework in that firms have a choice of scale and face decreasing returns. Becker's classic result shows that supermodularity of the production function (i.e. complementarity) is necessary for positive assortative matching in a frictionless partnership environment. Well-known papers in trade and industrial organization show that log-supermodularity of the production function is necessary for positive sorting in Dixit-Stiglitz-like models.

As discussed below, log-supermodularity is a substantially stronger condition than supermodularity, so it is puzzling why these two environments lead to different sorting conditions. I show that the answer is not in the difference between one-to-one and many-to-one matching, but rather in how decreasing returns to scale are imposed. I show that the concavity in *production* that Dixit-Stiglitz-like models impose makes it more difficult to obtain positive sorting relative to models that impose concavity in the *number of workers*. The latter class of models includes Becker's partnership model as a special case, in addition to a wide class of models

where firms face costs that depend on the number of workers, rather than the worker's productivity.

The result can be understood as follows. The social planner faces a tradeoff when deciding where to assign a given worker. Assigning the worker to a higher type firm results in an increase in match production, because higher type firms are always more productive. However, there is always a tradeoff (in order to have an internal solution). In Becker-like models, the cost is in terms of the size of the workforce, and is additive and independent of worker type. In Dixit-Stiglitz-like models the cost is in terms of match production, and so depends on the worker type. In particular, the cost is proportional to the worker productivity, so higher productivity workers face a larger (absolute) cost of matching with high type firms. As a result, simple supermodularity is not sufficient for positive sorting in Dixit-Stiglitz-like models. Stronger complementarity, in the form of log-supermodularity, is needed to induce high type workers to accept high type firms. The results show how simple differences in modeling assumptions can lead to dramatic difference in equilibrium behavior.

In the second section I extend my analysis to a directed search environment with multi-worker firms. I show that all my main results and intuition generalize. In particular I show that the supermodularity and log-supermodularity sorting conditions extend *unchanged* from the frictionless models to the directed search model. This result contrasts with the results in [Shimer and Smith \(2000\)](#) and [Eeckhout and Kircher \(2010\)](#), where positive assortative matching requires stricter conditions when there are search costs. My results are more in line with those of [Atakan \(2006\)](#) and [Lentz \(2010\)](#), whose frictional generalizations of the partnership model

require supermodularity to obtain positive sorting. However, this paper is apparently the only one besides [Eeckhout and Kircher \(2012\)](#) to provide an analysis of frictional sorting when firms face concave production functions. My Dixit-Stiglitz-like model can be regarded as a special case of theirs, which allows me to characterize precisely the necessary and sufficient conditions for sorting. My Becker-like model is apparently new to the literature.

The close correspondence between the frictionless and directed search models is particularly useful, since the growing literature on frictional sorting has led to a number of sufficient conditions for sorting (see [Shimer and Smith \(2000\)](#), [Shi \(2001\)](#), [Teulings and Gautier \(2004\)](#), [Shimer \(2005\)](#), [Atakan \(2006\)](#), [Lentz \(2010\)](#), [Eeckhout and Kircher \(2010\)](#), [Eeckhout and Kircher \(2012\)](#), [Lise and Robin \(2013\)](#), [Lise et al. \(2013\)](#) and [Grossman et al. \(2013\)](#) among others). Some of the differences in sorting are due to how search frictions are modeled, while others are due to the way firm scale is introduced or other features of the environment. My results highlight how the treatment of firm scale alone can change sorting, regardless of search frictions.

The paper proceeds as follows. In [Sections 2.2.1 and 2.2.2](#) I introduce some notation and concepts that will be used throughout the paper. Then I turn to frictionless models of matching. [Section 2.3.1](#) introduces the generalized partnership model, a model of multi-worker firms that nests Becker's partnership model. I show that the generalized partnership model shares the sorting behavior of the original Becker partnership model, and so is a useful tool for comparison. [Section 2.3.2](#) introduces the concave model, a generalization of the Dixit-Stiglitz framework that inherits its sorting properties. Then [Section 2.3.3](#) compares the two

models and shows which features are the source of their sorting behavior. Section 2.4 introduces search frictions, and largely follows the format of Section 2.3. Section 2.5 concludes.

2.2.1 Common Features

All the models under consideration share some key components. First, workers and firms are heterogeneous with respect to their productivity. Let $x \in [0, 1]$ denote a worker's type and $y \in [0, 1]$ denote a firm's type. Let $f(x, y) > 0$ be the *match production function*, or simply the production function. $f(x, y)$ is the efficiency units of labor generated when a type x worker matches with a type y firm. In certain cases this will be identical to physical output, but in general output will be an increasing, possibly nonlinear function of labor. Unless otherwise stated, I assume that all firms are producing a homogenous good whose price is normalized to unity. I assume that higher type agents are more productive:

$$f_1(x, y) > 0 \quad \forall x, y$$

$$f_2(x, y) > 0 \quad \forall x, y,$$

that is, I assume absolute advantage. The match production function will be the central object of interest in what follows.

A common component is lack of free entry. I assume that there is an exogenous mass $b(y)$ of type y firms, and an exogenous mass $l(x)$ of type x workers. Thus both the number and distribution of agents are fixed. However, firms choose the size of their workforce endogenously, so the distribution of *jobs* is determined in

equilibrium.

2.2.2 Supermodularity and Log-Supermodularity

The results in this paper involve properties of the match production function $f(x, y)$. Here I summarize some of these properties and their relationships.

Definition 1. A function $f(x, y)$ is *supermodular* if and only if

$$f(x', y') + f(x, y) \geq f(x, y') + f(x', y) \quad (2.1)$$

for all $y' > y, x' > x$. A function is *submodular* if and only if

$$f(x', y') + f(x, y) \leq f(x, y') + f(x', y)$$

Supermodularity is often called complementarity. If $f(x, y)$ is a production function, then equation (2.1) says that total output is larger when high types are paired with high types and low with low. When functions are differentiable, there is a simpler characterization:

Corollary 2. *If $f(x, y)$ is twice differentiable then $f(x, y)$ is supermodular if and only if*

$$f_{12}(x, y) \geq 0$$

for all y, x . It is submodular if and only if

$$f_{12}(x, y) \leq 0$$

for all y, x .

The cross-partial condition $f_{12}(x, y) \geq 0$ captures the idea of increasing differences: the marginal effect of increasing x is increasing in y . Now I turn to log-supermodularity, which has a natural definition:

Definition 2. A positive function $f(x, y)$ is *log-supermodular* if and only if $\ln f(x, y)$ is supermodular. It is *log-submodular* if and only if $\ln f(x, y)$ is submodular.

The following result can be obtained by combining Corollary 2 and Definition 2:

Corollary 3. *If $f(x, y)$ is twice differentiable then $f(x, y)$ is log-supermodular if and only if*

$$f_{12}(x, y) - \frac{f_1(x, y)f_2(x, y)}{f(x, y)} \geq 0 \quad (2.2)$$

for all y, x . It is log-submodular if and only if

$$f_{12}(x, y) - \frac{f_1(x, y)f_2(x, y)}{f(x, y)} \leq 0 \quad (2.3)$$

for all y, x .

Assuming that $f(x, y)$ is increasing in both arguments, Corollary 3 shows that log-supermodularity is a stronger condition than supermodularity. That is, since $\frac{f_1(x, y)f_2(x, y)}{f(x, y)}$ is positive some functions may have $f_{12}(x, y) \geq 0$ everywhere (supermodularity) but fail to have $f_{12}(x, y) - \frac{f_1(x, y)f_2(x, y)}{f(x, y)} \geq 0$ everywhere (log-supermodularity).

It is useful to consider some examples. The function $f(x, y) = x + y$ is both supermodular and submodular: the cross partial equals zero. Since it is on the borderline of being supermodular, it is not surprising that it fails log-supermodularity ($f_{12}(x, y) - \frac{f_1(x, y)f_2(x, y)}{f(x, y)} = -\frac{1}{x+y} < 0$). The function $f(x, y) = xy$ is supermodu-

lar, and is both log-supermodular and log-submodular. The function $\exp(xy)$ is strictly log-supermodular, and so of course is supermodular as well.

2.3 Frictionless Models

In this section I lay out two frictionless models of matching and sorting. The first is the generalized partnership model, which nests the one-to-one partnership model as a special case. The second model is the concave model, which nests monopolistic competition (e.g. Dixit-Stiglitz) models as special cases. I derive the sufficient conditions for assortative matching in a comparable fashion for both models. I show that the generalized partnership model replicates the sorting behavior of the partnership model (i.e. the sorting condition is identical in both models), and to explain why the sorting conditions differ between the partnership and the concave model.

2.3.1 Generalized Partnership Model

In this section I describe the generalized partnership (GP) model. The model is based on the classical assignment model, sometimes called the partnership model, where a finite number agents are matched one-to-one. The GP model is a generalization of the partnership model where agents on one side of the market (firms) can simultaneously match with several agents from the other side (workers). I show that the standard partnership model is a limiting case of the GP model. The key result is that sorting in the GP model is governed by the same property (supermodularity) as in the partnership model. In this sense the GP model generalizes [Becker \(1973\)](#)'s results to a setting with multi-worker firms.

2.3.1.1 Firm's Problem

Firms choose the size and composition of their workforce, while workers simply seek the highest available wage. The labor market is frictionless, so a firm can hire any amount of type x labor at the market wage $w(x)$. Production at each firm is linear, but there is a cost which is convex in the size of the workforce. A type y firm solves the problem

$$\max_{n(\cdot, y) \in \mathcal{N}} \int_0^1 n(x, y) [f(x, y) - w(x)] dx - c \left(\int_0^1 n(x, y) dx \right)$$

where $n(\cdot, y)$ is a function $[0, 1] \rightarrow \mathbb{R}_+$, and \mathcal{N} is the space of such functions. The firm's employment of type x workers is given by $n(x, y)$. $c(\cdot)$ is the strictly convex employment cost function. The employment cost function constrains firm size in equilibrium. Intuitively, firms will hire workers until their output (net of wages) equals the marginal cost of hiring another worker.

2.3.1.2 Social Planner's Solution

Since the welfare theorems apply, I focus on the social planner's solution. In this environment the social planner seeks to maximize total output net of employment costs, taking the number of workers and firms as fixed. Wages are a one-for-one transfer between agents, and so do not enter the problem.

Let $n(x, y) > 0$ be the mass of type x workers assigned to a type y firm. The social planner seeks an assignment to solve

$$\max_{n(\cdot, \cdot) \in \mathcal{N}^*} \int_0^1 b(y) \left[\int_0^1 n(x, y) f(x, y) dx - c \left(\int_0^1 n(x, y) dx \right) \right] \quad (2.4)$$

subject to

$$\int_0^1 b(y)n(x,y)dy \leq l(x). \quad (2.5)$$

where $n(\cdot, \cdot)$ is a function $[0,1]^2 \rightarrow \mathbb{R}_+^2$ and \mathcal{N}^* is the space of such functions. In other words, the social planner chooses the value of $n(x,y)$ for all x and y . Let $n(y) = \int_0^1 n(x,y)dx$ be the size of the workforce assigned to each type y firm. Then the net marginal product of a type x worker at a type y firm is

$$f(x,y) - c'(n(y)). \quad (2.6)$$

Assume the social planner's solution has $n^*(x,y) > 0$. Then it must be the case that

$$f(x,y) - c'(n(y)) \geq f(x,y') - c'(n(y')) \quad \forall y' \quad (2.7)$$

since otherwise output could be increased by reassigning some of the type x workers to another firm. The inequality (2.7) is critical in what follows. It implies that y is a local maximizer of $f(x,y) - c'(n(y))$, so if $f(\cdot, \cdot), c(\cdot)$ and $n(\cdot)$ are sufficiently differentiable, the first and second order conditions yield

$$f_2(x,y) - c''(n(y))n'(y) = 0 \quad (2.8)$$

$$\frac{d}{dy} [f_2(x,y) - c''(n(y))n'(y)] \leq 0. \quad (2.9)$$

respectively. Assuming that the second order condition holds strictly, I can apply the implicit function theorem to (2.8) obtain

$$\frac{dy}{dx} = - \frac{f_{12}(x,y)}{\frac{d}{dy} [f_2(x,y) - c''(n(y))n'(y)]}$$

where the denominator is strictly negative. This says that better firms are assigned to better workers if and only if $f_{12}(x, y)$ is positive. That is, matching is positive assortative if $f(x, y)$ is supermodular (see Corollary 2). In this example I made several assumptions about differentiability and smoothness. These assumptions are not necessary for the result. The following Proposition summarizes the result for the general case.

Proposition 12. *Consider the social planner's problem in the GP model (Equations (2.4) and (2.5)). If $f(x, y)$ is supermodular, then there is positive assortative matching. If $f(x, y)$ is submodular, then there is negative assortative matching.*

Proof. Note that if $f(x, y)$ is supermodular then $f(x, y) - c'(n(y))$ is supermodular as well for all functions $n(y)$. I have already argued that satisfying (2.7) is necessary for the social optimal assignment. By Topkis' theorem the (possibly set-valued) solution to the maximization problem

$$\operatorname{argmax}_{y \in [0,1]} [f(x, y) - c'(n(y))]$$

is nondecreasing in x for all possible $n(y)$, including the socially optimal $n(y)$. Higher type workers must be assigned to (weakly) higher type firms, so matching is positive assortative. The proof for submodularity and negative assortative matching is symmetric. \square

The proof shows that supermodularity is sufficient for assortative matching. It turns that there is a necessity result as well. In particular, let $y(x)$ be the social planner's optimal assignment of workers to firms. Then if matching is positive assortative (i.e. $y(x)$ is increasing) then $f(y, x)$ must be supermodular along $y(x)$.

That is, supermodularity of f along the equilibrium assignment is a necessary condition for positive sorting. Supermodularity of f globally, as in the Proposition, is sufficient for positive sorting. Similar arguments hold for all the Propositions that follow, so I will only focus on proving sufficiency to save space.

2.3.1.3 Related Literature

The GP model nests the classical assignment model (e.g. [Koopmans and Beckmann \(1957\)](#), [Shapley and Shubik \(1971\)](#), [Becker \(1973\)](#)). In the classical assignment model there are a fixed number of workers and fixed number of firms, each of which can employ a *single* worker. It has been known since [Becker \(1973\)](#) that supermodularity ensures positive assortative matching in the assignment problem. In GP model firms can employ multiple workers at a convex cost. Let $c(n) = \frac{1}{z}n^z$, and let $z \rightarrow \infty$. Then $c'(n) \rightarrow 0$ for $n \leq 1$ and $c(n) \rightarrow \infty$ for $n > 1$. In the limit each firm will employ a unit mass of workers, since the marginal cost jumps from zero to infinity at unity. This is isomorphic to [Koopmans and Beckmann \(1957\)](#)'s discussion of the problem.¹ For reference, the following Proposition summarizes the supermodularity condition for the limiting case of one-to-one matching, due to [Becker \(1973\)](#).

Proposition 13. *Consider the social planner's problem in the model of one-to-one matching. If $f(x, y)$ is supermodular, then there is positive assortative matching. If $f(x, y)$ is submodular, then there is negative assortative matching.*

¹The Becker model involves single workers matching with single firms. In the limit of the GP model each firm matches with a unit mass of workers. This difference is not significant, as [Koopmans and Beckmann \(1957\)](#) discuss. The main distinction between the GP model and the partnership model is that the firm's marginal product $f(x, y) - c'(n(y))$ is endogenous in the GP model.

2.3.1.4 Interpretation of the Model

The GP model, firm size is constrained by convex employment costs. If $c(n)$ were not convex, nothing would prevent a single firm from hiring the entire labor force. Given the assumption of absolute advantage, as $c(n)$ approaches linearity the highest type firm would grow at the expense all others.

The most familiar models of firms typically assume that the profit function is concave, either because of a fixed factor of production or because the firm faces downward sloping demand for its good. The GP model can be interpreted in this fashion as well.

The combination of linear production and convex employment costs can be interpreted in several ways. The core idea is that it becomes increasing costly for the firm to maintain a larger workforce. This could come from increased overhead costs (more management, and more layers of management, as the workforce expands). It is also worth noting that when workers are homogeneous the GP model is isomorphic to a standard model with a CES profit function. In particular, let profit be given by

$$\pi(n) = yn - yc(n)$$

where y is the firm's productivity parameter, n is employment, and $c(n)$ is the convex cost function. Here I assume that firm type multiplies costs as well as output. This assumption could be incorporated into the GP model without any significant changes. If I set

$$c(N) = N - N^\alpha$$

then the resulting profit function is yN^α , just as in the monopolistic competition

model. This shows that the GP model is not far removed from workhorse models of multi-worker firms.

2.3.2 Concave Model

In this section describe the concave (C) model of sorting. The firm's output is given by

$$G\left(\int_0^1 n(x,y)f(x,y)dx\right)$$

where $G(\cdot)$ is a positive, increasing and strictly concave function. Here $\int_0^1 n(x,y)f(x,y)dx$ are the efficiency units of labor hired by the firm. $G(\cdot)$ can capture the effect of a fixed factor of production, such as capital or managerial talent, that generates decreasing returns to scale. It can also capture the effect of imperfect competition. The critical difference between the GP model and the C model is that in the GP model curvature is a function of the quantity of *workers* (the function $c(n)$), while in the C model curvature is a function of *efficiency units* (i.e. productivity-weighted employment) or match production.

As before, in a frictionless competitive equilibrium firms take the market wage schedule as given, so the firm solves the problem

$$\max_{n(\cdot,y) \in \mathcal{N}} G\left(\int_0^1 n(x,y)f(x,y)dx\right) - \int_0^1 n(x,y)w(x)dx.$$

That is, the firm chooses the quantity and quality of workers to maximize output net of wages.

2.3.2.1 Social Planner's Solution

Exactly as in section (2.3.1), the welfare theorems apply so I focus on the social planner's problem. The planner maximizes the sum of output across all firms:

$$\max_{n(\cdot, \cdot) \in \mathcal{N}^*} \int_0^1 b(y) G \left(\int_0^1 n(x, y) f(x, y) dx \right) dy \quad (2.10)$$

subject to

$$\int_0^1 b(y) n(x, y) dy \leq l(x). \quad (2.11)$$

Letting $e(y) = \int_0^1 n(x, y) f(x, y) dx$, the marginal product of a type x worker at a type y firm is

$$G'(e(y)) f(x, y). \quad (2.12)$$

Assume the social planner's solution has $n^*(x, y) > 0$. Then it must be the case that

$$G'(e(y)) f(x, y) \geq G'(e(y')) f(x, y') \quad \forall y' \quad (2.13)$$

since otherwise output could be increased by reassigning some of the type x workers to another firm. (2.13) implies that y is a local maximizer of $G'(e(y')) f(x, y')$, so if the functions are sufficiently differentiable the first and second order conditions yield

$$G''(e(y)) e'(y) f(x, y) + G'(e(y)) f_2(x, y) = 0 \quad (2.14)$$

$$\frac{d}{dy} \left\{ G''(e(y)) e'(y) f(x, y) + G'(e(y)) f_2(x, y) \right\} \leq 0. \quad (2.15)$$

Assuming that the second order condition holds strictly, I can apply the implicit function theorem to (2.14) obtain

$$\frac{dy}{dx} = -\frac{G''(e(y))e'(y)f_1(x,y) + G'(e(y))f_{12}(x,y)}{\frac{d}{dy}\{G''(e(y))e'(y)f(x,y) + G'(e(y))f_2(x,y)\}}.$$

Substituting in equation (2.14) I have

$$\frac{dy}{dx} = -G'(e(y))\frac{f_{12}(x,y) - \frac{f_1(x,y)f_2(x,y)}{f(x,y)}}{\frac{d}{dy}\{G''(e(y))e'(y)f(x,y) + G'(e(y))f_2(x,y)\}},$$

where the denominator is strictly negative and $G'(e(y)) > 0$. Thus, $\frac{dy}{dx} > 0$ if and only if the numerator is positive. Recalling Corollary 3, this means that matching is positive assortative if and only if $f(x,y)$ is log-supermodular. As in the previous section, this example has relied on some differentiability assumptions. The following Proposition gives the general result.

Proposition 14. *Consider the social planner's problem in the C model (Equations (2.10) and (2.11)). If $f(x,y)$ is log-supermodular, then there is positive assortative matching. If $f(x,y)$ is log-submodular, then there is negative assortative matching.*

Proof. I have already argued that satisfying (2.13) is necessary for the social optimal assignment. Taking logs, the inequality becomes

$$\ln G'(n(y)) + \ln f(x,y) \geq \ln G'(e(y')) + \ln f(x,y') \quad \forall y'$$

so the associated maximization problem is

$$\max_{y \in [0,1]} [\ln G'(e(y)) + \ln f(x,y)].$$

If $\ln f(x, y)$ is supermodular, then by Topkis' theorem the (possibly set-valued) solution to this problem is nondecreasing in x for all possible $e(y)$, including the socially optimal $e(y)$. Higher type workers must be assigned to (weakly) higher type firms, so matching is positive assortative. The proof for log-submodularity and negative assortative matching is symmetric. \square

2.3.2.2 Related Literature

The C model assumes that the price of output is constant across firms and that there are decreasing returns to efficiency units. It turns out that the C model is formally equivalent to a much broader class of models, including models in which production is linear and prices are determined outside the production model. In particular, let $\int_0^1 n(x, y) f(x, y) dx$ be the output function of the firm and $p(y)$ be the predetermined price of the firm's output. Then equations (2.12) and (2.13) and Proposition 14 carry over with $p(y)$ replacing $G'(e(y))$. This generalization is possible because the marginal product (equation (2.12)) depends only on the variation of $G'(e(y))$ (or $p(y)$). It does not matter if the variation arises from curvature in the production function or from variation in prices. See Costinot (2009) for an example of this approach.

A case of special interest is the workhorse CES/Dixit-Stiglitz demand model. One can interpret $\int_0^1 n(x, y) f(x, y) dx$ as physical output and $G(\cdot)$ as the *revenue* generated by selling the output in a monopolistically competitive market. If the representative consumer's preferences are CES with an elasticity of substitution

$\sigma > 1$, it can be shown that

$$G' \left(\int_0^1 n(x, y) f(x, y) dx \right) = A \left(\int_0^1 n(x, y) f(x, y) dx \right)^{\frac{-1}{\sigma}}$$

where A is a constant. Revenues are concave because the price declines as production increases. See [Teulings \(1995\)](#), [Teulings \(2005\)](#) and [Costinot and Vogel \(2010\)](#) for examples of this setup.

2.3.3 Comparison

I now return to the initial question - why do the sufficient conditions for assortative matching differ between the partnership model ([Proposition 13](#)) and the C model ([Proposition 14](#))? [Proposition 12](#) establishes that supermodularity is the relevant condition both in the partnership and GP models. This means that the distinction between one-to-one matching and multi-worker firms is not the source of the difference.

Here I will consider the model equilibria under the assumptions of differentiability and continuity. This is only to aid intuition and save on notation. Recall the optimality conditions from the GP model:

$$f_2(x, y) - k'(y) = 0 \tag{2.16}$$

$$\frac{d}{dy} [f_2(x, y) - k'(y)] \leq 0 \tag{2.17}$$

and of the C model:

$$p'(y)f(x, y) + p(y)f_2(x, y) = 0 \quad (2.18)$$

$$\frac{d}{dy} \{p'(y)f(x, y) + p(y)f_2(x, y)\} \leq 0 \quad (2.19)$$

where I have set $k(y) = c'(n(y))$ and $p(y) = G'(e(y))$ to simplify notation. $k(y)$ is the marginal cost of adding a worker to a type y firm in the GP model, and $p(y)$ is the marginal product of an efficiency unit of labor at firm y in the C model (alternatively, $p(y)$ can be interpreted as a price, as in Section 2.3.2.2). Both $k(y)$ and $p(y)$ implicitly depend on the allocation of workers.

Starting with equation (2.16), in the GP model worker x is assigned to the firm y such that the marginal gain from moving up the firm type distribution ($f_2(x, y)$) equals the marginal increase in employment costs ($k'(y)$). Note that $k'(y)$ is independent of the worker type x being considered. This will be important later.

Turning to the C model, note first that equation (2.18) implies that $p(y)$ is decreasing in y : in equilibrium better firms have lower marginal product per efficiency unit. The reason is that a better firm always produces more efficiency units given the same workforce, so a lower equilibrium marginal product balances this effect in equilibrium. Put differently, better firms are farther out on the concave function $G(\cdot)$, so $G'(\cdot)$ (and thus $p(y)$) must be lower.

Equation (2.18) says that the worker is assigned to a firm such that the marginal gain from moving up the firm distribution, $p(y)f_2(x, y)$, is balanced by the decrease in marginal product per efficiency unit $p'(y)f(x, y)$ (since $p'(y) < 0$). Following the interpretation in Section 2.3.2.2 I will call $p'(y)f(x, y)$ the *price effect*, since

it is the impact on revenue of moving to a higher firm's price without changing match output.² I will call $p(y)f_2(x,y)$ the *production effect*, since it is the effect of changing the match output holding prices constant. Critically, the price effect includes $f(x,y)$, so assigning a worker to a higher firm type has a cost that is *proportional to the worker's match production* $f(x,y)$. This is intuitive, since the change in price affects all of a worker's output, not just the marginal units. Workers that produce more (i.e. high x) lose more in absolute terms from the price effect. Thus the price effect in (2.18) is increasing in worker type. This is in contrast to the GP model, where the cost component $k'(y)$ is independent of firm type.

Now assume that worker x happens to be assigned to firm y in each model. To get at assortative matching, consider whether a slightly higher type worker is assigned to a higher or lower firm type. In equation (2.16) increasing x has the effect

$$\frac{d}{dx} [f_2(x,y) - k'(y)] = f_{12}(x,y),$$

that is, $f_2(x,y)$ changes but the cost function $k'(y)$ is unaffected. If $f_{12}(x,y) > 0$ then the marginal benefit of increasing firm type is increasing in y , so $f_{12}(x,y) > 0$ is sufficient for (local) positive assortative matching. In the C model, the reasoning is more complex. An increase in x changes both the price and production effects in equation (2.18):

$$\frac{d}{dx} [p'(y)f(x,y) + p(y)f_2(x,y)] = p'(y)f_1(x,y) + p(y)f_{12}(x,y)$$

²The interpretation as a price is not important for the results. One can think of $p'(y)f(x,y)$ as a "marginal product per efficiency unit effect", but for clarity and intuition I treat it as a price here.

For the production effect, the direction of change is governed by $f_{12}(x, y)$, as before. The change in the price effect is $p'(y)f_1(x, y) < 0$: the change output multiplied by $p'(y)$.

Note that equation (2.16) is increasing in x if and only if $f_{12}(x, y) > 0$, while supermodularity is a necessary but *insufficient* condition for equation (2.18) to be increasing in x . The lack of sufficiency is due to the price effect. For (2.18) to be increasing in x the complementarities in production must be sufficient to overcome the negative effect of prices. In the C model increasing worker type increases match output, which increases the absolute losses due to lower output prices. In the GP model the cost term (the convex employment cost) is uniform across worker types, so high type workers do not bear any extra cost relative to low type workers.

To summarize, the price effect in the C model makes it more costly for high type workers to match with high type firms. High type workers produce more regardless of where they work, so the lower prices at high type firms are more costly. Strong complementarities are needed to overcome the price effect, so simple supermodularity is not sufficient for positive sorting. In the GP the costs are in terms of employment, and so are independent of worker type. Changing worker type affects the first order condition only through the change in production. In particular, there is nothing beyond the match production function that makes high type firms relatively less attractive to high type workers. Thus the weaker supermodularity condition is sufficient for positive sorting.

2.3.3.1 Discussion

The C model and Becker's partnership model are both intended to study how heterogeneous agents match in a simple, tractable environment. In particular, both have been used to study how workers are optimally assigned to firms. The models turn out to have different implications for sorting. The partnership model requires that production be supermodular to have positive sorting, while the C model requires log-supermodularity for positive sorting. This difference is puzzling given that both models are trying to capture essentially the same phenomenon. It is worth investigating the source of this difference, in order to shed light on the implicit assumptions in each model and the relationship between the models.

The partnership and C models differ in how firms are modeled. It might be thought that the divergence in sorting conditions results from the partnership model assuming one-to-one matching while the C model allows for multi-worker firms. I have shown that this is not the case. The essential features and conclusions of the partnership model carry over into models where multi-worker firms are of fixed size, and models where firms endogenously choose their size subject to a smooth employment cost function. Thus the divergence in sorting conditions is not an issue with single versus multi-worker firms, or with fixed employment versus endogenous employment. Rather, the divergence results from how firm scale is imposed: costs that depend on the number of workers make positive sorting relatively easy to achieve, while costs that depend on efficiency units/match production make positive sorting harder to achieve.

Given that the C model and the GP model differ, which is more reasonable? The answer to this question depends on the determinants of equilibrium firm size. If

the driving force in the firm's problem is limited demand for a firm's product (e.g. Dixit-Stiglitz monopolistic competition), then the C model is appropriate. On the other hand if the firm's output price is relatively fixed but the firm has a fixed factor of production (e.g. capital or managerial talent) then either model may be applicable. For example, consider the case where each firm is identified with a manager, whose talent y is variable. Better managers extract more when managing any worker (i.e. $f_2(x, y)$). Managing more workers is more difficult for any given manager, so net output has decreasing returns. If an increase in scale reduces the output of all matches by the same *absolute* amount then the GP model is correct: the output at all matches drops by $c'(n(y))/n$. Alternatively, an increase in scale may result in a *proportional* drop in output at all matches. Then the C model would be applicable. A topic for future research is disentangling which of these alternatives is empirically most relevant.

2.4 Directed Search

Much of the recent work on sorting in labor and trade has focused on frictional environments. Two strands have developed in the literature. [Shimer and Smith \(2000\)](#), [Shi \(2001\)](#), [Shimer \(2005\)](#), [Atakan \(2006\)](#) [Lentz \(2010\)](#) and [Eeckhout and Kircher \(2010\)](#), among others, have extended [Becker \(1973\)](#)'s partnership model to incorporate search frictions. [Teulings and Gautier \(2004\)](#), [Eeckhout and Kircher \(2012\)](#), [Grossman et al. \(2013\)](#) and others have worked with frictional models closely related to the C model.

In this section I extend my results to an environment with search frictions. I develop frictional versions of the GP and C models, and show that they share the

same sorting conditions as the frictionless models. The reasoning is the same as in the frictionless cases: the price effect in the C model makes it more costly for high type workers to match with high type firms.

These results generalize the frictionless models described earlier, and can serve as a useful benchmark in comparing models of frictional sorting. The sufficient conditions for sorting under search frictions can be complicated, and depend subtly on the way frictions are introduced. My focus is on two particularly tractable frameworks that highlight the similarity between classical assignment problems and frictional ones.

I focus on models of *directed search*. Firms post vacancies committing to hire a certain type of worker at a certain wage if contact is made. Workers observe the available contracts, and direct their search towards one of them. Thus the economy is divided into submarkets defined by a worker type and a wage offer. Search frictions are present in each submarket. The number of matches formed depends on the number of vacancies and unemployed workers in the submarket.

I will assume throughout that firms post vacancies at a linear cost k . The assumption of linear vacancy costs is critical in deriving the solution, and sets this paper apart from many recent papers which rely on (strictly) convex vacancy costs to pin down firm sizes.³ In this section, as above, firm size will be constrained by either convex employment costs or by concavity in the production function.

³See Moscarini and Postel-Vinay (2013), Coles and Mortensen (2011), Lise et al. (2013) and Crane (2014).

2.4.1 Matching

Each submarket is defined by (w, x) , the wage offered in the submarket and the worker type recruited there. Let $u(w, x)$ be the mass of workers that decide to search market (w, x) , and let $v(w, x)$ be the mass of vacancies posted in the market.

Then define

$$\theta(w, x) = \frac{v(w, x)}{u(w, x)}$$

as the market tightness in submarket (w, x) . In equilibrium $\theta(w, x)$ is taken as given by each firm and worker.

Matching within submarket (w, x) is governed by a standard matching function $m(u, v)$, assumed to be positive, increasing, concave and linearly homogeneous.

Let $q(w, x)$ be the *worker finding rate*:

$$\begin{aligned} q(w, x) &= \frac{m(u(w, x), v(w, x))}{v(w, x)} = m(\theta^{-1}(w, x), 1) \\ q(\theta(w, x)) &= q(w, x). \end{aligned}$$

$q(w, x)$ is the probability that a given vacancy meets a worker. Its counterpart is $p(w, x)$, the *job finding rate*:

$$\begin{aligned} p(w, x) &= \frac{m(u(w, x), v(w, x))}{u(w, x)} = m(1, \theta(w, x)) \\ p(\theta(w, x)) &= p(w, x). \end{aligned}$$

2.4.2 Frictional Generalized Partnership Model

The GP model of Section (2.3.1) can be extended to incorporate search costs. In the competitive equilibrium of the frictional GP model the firm seeks to maximize output less recruitment costs (wages and vacancies) and employment costs. Without loss of generality, I can assume that each firm recruits a *single* type of worker in equilibrium. The firm solves

$$\max_{x,v,w,n} n(f(x,y) - w) - kv - c(n)$$

subject to

$$n = vq(\theta(w,x)).$$

$c(n)$ is the familiar convex employment cost function. Taking the submarket tightness $\theta(w,x)$ as given, the firm is free to choose the type of worker recruited x , the wage offer w , and the number of vacancies to post v . These choices jointly determine the number of hires n .

2.4.2.1 Social Planner's Solution

It is well known that the welfare theorems apply in these kinds of directed search markets, so I will use the social planner's solution to pin down sorting patterns.⁴

The social planner maximizes net output, subject to the search frictions in the economy. Wages are simply a transfer from the social planner's perspective, so

⁴See [Eeckhout and Kircher \(2010\)](#) and [Eeckhout and Kircher \(2012\)](#). Efficiency does not obtain when firms are unable to post sufficiently detailed contracts, as in [Peters \(2010\)](#).

it is not useful to have submarkets indexed by w . Instead, I will consider submarkets indexed by (x, y) .⁵ The social planner chooses both the quantity and distribution of vacancies across submarkets as well as the distribution of workers across submarkets.

In section 2.3 the only choice variables were the assignments of workers, so it was sufficient to consider the marginal production of workers at a given firm. In this section the planner chooses both the assignment of workers (to submarkets) *and* the distribution of vacancies. Thus it will not suffice to work with the marginal conditions of workers independently of the vacancy distribution. To address this complication I will consider the problem in two stages. First, given a distribution $u(x, y)$ of workers across submarkets I find the optimal vacancy function $v(x, y)$. This gives me a (conditionally) optimal submarket tightness function $\theta(x, y)$. This optimal tightness turns out to depend only on the marginal product $f(x, y) - c'(n(y))$. Then I consider how to distribute workers given the optimal tightness $\theta(x, y)$. I show that when $f(x, y)$ is supermodular, the marginal social value of allocating a worker to a market is not necessarily supermodular, but is an increasing transformation of a supermodular function. As a result, the social planner's assignment is nondecreasing if (and only if) $f(x, y)$ is supermodular.

⁵In principle the social planner could index markets by a wage or some other characteristic as well as (x, y) , but due to concavity of the matching function such submarkets cannot improve on the outcome of (x, y) markets.

First Step The goal is to derive optimal tightness $\theta(x, y)$ given the distribution of workers. In general the social planner solves

$$\max_{u(\cdot, \cdot), v(\cdot, \cdot), n(\cdot)} \int_y b(y) \left\{ \int_x \left[m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) f(x, y) - v(x, y) k \right] dx - c(n(y)) \right\} dy \quad (2.20)$$

subject to

$$n(y) = \int_x m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) dx \quad (2.21)$$

$$l(x) = \int_y u(x, y) dy. \quad (2.22)$$

where $u(\cdot, \cdot)$ and $v(\cdot, \cdot)$ are functions $[0, 1]^2 \rightarrow \mathbb{R}_+^2$ and $n(\cdot)$ is a function $[0, 1] \rightarrow \mathbb{R}_+$.

Note that the term $m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) = v(x, y) \frac{m(u(x, y), v(x, y) b(y))}{v(x, y) b(y)}$, the employment of type x workers at a single type y firm. Substituting (2.21) into (2.20) and taking the first order condition with respect to $v(x, y)$ I have

$$m_2 \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) \left[f(x, y) - c' \left(\int_x m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) dx \right) \right] = k,$$

or in simpler terms

$$m_2(1, \theta(x, y)) [f(x, y) - c'(n(y))] = k \quad (2.23)$$

where $\theta(x, y) = \frac{v(x, y) b(y)}{u(x, y)}$. This condition determines submarket tightness as a function of x , y , and $n(y)$.

Note that $m_2(1, \theta)$ is strictly decreasing in θ , so from (2.23) I can write

$$\theta(x, y) = A(f(x, y) - c'(n(y))) \quad (2.24)$$

where $A(\cdot)$ is an increasing function. This says that market tightness should be higher where the marginal product of the workers is higher.

Second Step Now consider how to distribute workers. Returning to (2.20) the first order condition with respect to $u(x, y)$ is

$$m_1\left(\frac{u(x, y)}{b(y)}, v(x, y)\right) \left[f(x, y) - c' \left(\int_x m\left(\frac{u(x, y)}{b(y)}, v(x, y)\right) dx \right) \right] \leq 0 \quad (2.25)$$

with equality where $u(x, y) > 0$. Simplifying the notation, this means y maximizes $m_1(1, \theta(x, y)) [f(x, y) - c'(n(y))]$. Now since $m_1(1, \theta)$ is increasing in θ I can use (2.24) to write

$$m_1(1, \theta(x, y)) [f(x, y) - c'(n(y))] = \bar{A}(f(x, y) - c'(n(y)))$$

where $\bar{A}(\cdot)$ is increasing and positive. Thus maximizing $m_1(1, \theta(x, y)) [f(x, y) - c'(n(y))]$ turns out to be equivalent to maximizing $f(x, y) - c'(n(y))$, just as in equation (2.7). From this point the logic follows from Section (2.3.1), meaning that supermodularity is sufficient for positive sorting. The following proposition summarizes the result.

Proposition 15. *Consider the social planner's problem in the frictional GP model (Equations (2.20), (2.21) and (2.22)). If $f(x, y)$ is supermodular, then there is positive assortative matching. If $f(x, y)$ is submodular, then there is negative assortative matching.*

2.4.2.2 Discussion

My results differ from those of [Eeckhout and Kircher \(2010\)](#), who study directed search in a partnership model. They derive a sufficient condition for positive sorting (called root - supermodularity) that is stronger than supermodularity. As they discuss, time consuming search makes it harder to obtain positive sorting, since productive agents have a strong incentive to avoid unemployment (and unfilled vacancies), and so may be willing to accept a lower quality match to ensure that production occurs. This argument depends on search being time consuming. In their model, this is reflected in the fact that firms have a single job to post (and can only post one vacancy for it), and if they fail to meet a worker they receive nothing.

In the frictional GP model firms may post as many vacancies as they wish. Due to linear vacancy costs, the cost of recruiting workers is linear in the number of workers. As a result, the time cost of search in [Eeckhout and Kircher \(2010\)](#) is translated to a linear *monetary* cost in my model. When choosing which workers to recruit, the firm does not face the possibility of failing to recruit and remaining idle. Rather, the firm faces an additive cost of recruiting. This additive cost is constant across firm types, in contrast to the cost of failing to match, which is increasing in firm type. As a result, the relatively weak supermodularity condition ensures assortative matching in my model. This result parallels that of [Atakan \(2006\)](#), who focused on random search with Nash bargaining. He showed that when meetings can be obtained at a linear cost (equivalently, when there is no discounting of the future) then supermodularity is the necessary condition for positive sorting.

It is important to note that the frictional GP model is not a limiting case of the [Eeckhout and Kircher \(2010\)](#) model. It is possible to make employment costs sufficiently convex so that firms only attempt to recruit a unit mass of workers. But with linear vacancy costs, firms still absorb search costs as a monetary loss rather than a risk of not matching. To arrive at the [Eeckhout and Kircher \(2010\)](#) model, it would be necessary to have vacancy costs be *convex*. On the other hand, the frictional GP model can replicate Becker's model or the (frictionless) GP model as limiting cases. All that is needed is to make vacancy costs sufficiently low. Then firms post enough vacancies that all workers are hired, and essentially the only cost is wages.

2.4.3 Frictional Concave Model

The C model can be extended to the directed search environment by combining the arguments in Sections [2.3.2](#) and [2.4.2](#). In the frictional C model the firm faces decreasing returns to efficiency units and linear vacancy costs. Skipping to the social planner's problem and substituting in for $n(y)$, I have

$$\max_{u(\cdot,\cdot),v(\cdot,\cdot)} \int_y b(y) \left\{ G \left(\int_x m \left(\frac{u(x,y)}{b(y)}, v(x,y) \right) f(x,y) dx \right) - k \int_x v(x,y) dx \right\} dy \quad (2.26)$$

subject to

$$l(x) = \int_y u(x,y) dy. \quad (2.27)$$

So the first order condition with respect to $v(x, y)$ can be written

$$G' \left(\int_x m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) f(x, y) dx \right) m_2 \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) f(x, y) = k,$$

or in simpler terms

$$G'(e(y)) m_2(1, \theta(x, y)) f(x, y) = k \quad (2.28)$$

where $e(y) = \int_x m \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) f(x, y) dx$ and $\theta(x, y) = \frac{v(x, y)b(y)}{u(x, y)}$. This condition determines submarket tightness as a function of x , y , and $e(y)$. Note that $m_2(1, \theta)$ is strictly decreasing in θ , so from (2.28) I can write

$$\theta(x, y) = A(G'(e(y)) f(x, y)) \quad (2.29)$$

where $A(\cdot)$ is an increasing function.

Now consider how to distribute workers. Returning to (2.26) the first order condition with respect to $u(x, y)$ is

$$m_1 \left(\frac{u(x, y)}{b(y)}, v(x, y) \right) G'(e(y)) f(x, y) \leq 0 \quad (2.30)$$

with equality where $u(x, y) > 0$. Simplifying the notation, this means y maximizes $m_1(1, \theta(x, y)) G'(e(y)) f(x, y)$. Now since $m_1(1, \theta)$ is increasing in θ I can use (2.29) to write

$$m_1(1, \theta(x, y)) G'(e(y)) f(x, y) = \bar{A}(G'(e(y)) f(x, y))$$

where $\bar{A}(\cdot)$ is increasing and positive. Thus maximizing $m_1(1, \theta(x, y)) G'(e(y)) f(x, y)$ turns out to be equivalent to maximizing $G'(e(y)) f(x, y)$, just as in equation

(2.13). From this point the logic follows from section 2.3.2, meaning that log-supermodularity is sufficient for positive sorting. The following proposition summarizes the result.

Proposition 16. *Consider the social planner's problem in the frictional GP model (Equations (2.26) and (2.27)). If $f(x,y)$ is log-supermodular, then there is positive assortative matching. If $f(x,y)$ is log-submodular, then there is negative assortative matching.*

2.4.3.1 Discussion

Propositions 15 and 16 extend Propositions 12 and 14 to a search environment. The intuition extends as well: in the frictional C model high type workers suffer more from the negative price effect, and so may be tempted to match with lower type firms. Stronger complementarities (i.e. log-supermodularity) are necessary to overcome the price effect and induce positive assortative matching.

2.5 Conclusion

I have examined the conditions that lead to worker-firm sorting in a variety of models. Two distinct classes of models have been used to study this phenomenon: Becker's partnership and the Dixit-Stiglitz framework (and variants thereof). These models imply different sorting conditions. Perhaps surprising, the fundamental difference between the models is not that one is discrete and one-to-one while the other has an endogenous scale choice. Rather, I show that the concavity in *production* that Dixit-Stiglitz-like models impose makes it more difficult to obtain positive sorting relative to models that impose concavity in the

number of workers. The latter class of models includes Becker's partnership model as a special case, in addition to a wide class of models where firms face costs that depend on the number of workers, rather than the worker's productivity. Concavity in production makes it more costly for high type workers to match with high type firms, discouraging positive sorting.

In the process of demonstrating these results I derived the GP model, a convenient generalization of the partnership model. The GP model also coincides with the C model when workers are homogeneous. The GP model is also somewhat more tractable than the C model, and so may be the basis of future work where the C model has been used.

I also show that my results carry over unchanged to a model with directed search. The close correspondence between the frictionless and directed search models is particularly useful, since the growing literature on frictional sorting has led to a number of sufficient conditions for sorting. Some of the differences in sorting are due to how search frictions are modeled, others are due to the way firm scale is introduced or other features of the environment. My results highlight how the treatment of firm scale alone can change sorting, regardless of search frictions.

An important topic for future work is disentangling which margins are empirically most important for determining firm size. If limited demand is the driving factor, then the C model should be the focus of research. If fixed factors, like capital or managerial talent, are most important then either model might be best. Firm dynamics are another topic for future research. Initial results suggest that a dynamic, stochastic version of the GP model is not difficult to characterize.

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