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Use of a Game-Based App as a Learning Tool for Students

with Mathematics Learning Disabilities to Increase Fraction Knowledge/Skill

by

Orhan Simsek

A dissertation submitted in partial fulfillment of the requirement for the degree of Doctor of Philosophy in Curriculum and Instruction with an emphasis in Special Education Department of Teaching Learning College of Education University of South Florida

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Keywords: App, Math, Fraction, and Mathematics Learning Disabilities

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DEDICATION

This dissertation is dedicated to my family

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ABSTRACT

The aim of this study was to investigate the effectiveness of a game-based app (*Motion Math: Fraction*) to help students with Mathematics Learning Disabilities (MLD) to gain fraction skills including comparison, estimation, and word problem solving in an after school program. The researcher used multiple baseline design by extending with follow-up phase to determine whether students retained the knowledge they learned while engaging with the app. Even though six students participated to the study, the researcher withdrew two of them and analyzed data came from four students. The result o the study showed that all of the students improved their fractions skills after engaging with *Motion Math: Fraction* and maintained the knowledge after no longer playing. The researcher presented recommendations for further studies, for implementation into classroom, and recommend for app developers to increase app efficiency for students who have different learning profiles, and needs variety learning materials while learning the content matters.

CHAPTER ONE: INTRODUCTION

A newly graduated registered nurse... administered one-half grain of morphine when, in fact oneeight grain was ordered, reasoning that since 4 plus 4 equals 8, ¹/₄ plus ¹/₄ equals 1/8 (instead of 1/2). Although the patient survived, the dose was enough to depress her respiration to a life threatening level. This was not an isolated incident... (Grillo, et al., 2001, p.168).

Mathematics knowledge and skills are needed for success in college and for many careers (Fuchs et al., 2013; Jordan, Hansen, Fuchs, Siegler, Gersten, & Micklo, 2013). However, studies indicated that approximately 10% of students in the United States have mathematical learning disabilities (MLD) caused by psychological processing deficits (Berch & Mozzacco, 2007; Geary, 2011). In addition to that, Geary (2011) stated, "the large scale studies in Great Britain indicated that about 23%," (roughly 10% have disability, and other 13% have difficulty in mathematics), "of adults are functionally innumerate, that is, they do not have the mathematical competencies needed for many routine day-today activities" (p. 3). The students face difficulties in functioning at the level of their typical peers in mathematics in school and the problem continue after school as well. After graduation, many of these students work in low paid jobs, and their life satisfaction is low compared to their typical peers (NMAP, 2008).

Even some specific mathematics skills, such as understanding and applying knowledge of fractions, are very challenging to learn for many students and adults (Hecht, Vagi, &Torgesen, 2007; Mazzocco & Devlin, 2008). Therefore, the National Mathematics Advisory Panel (NMAP, 2008) has stressed the importance of teaching fractions, and also Common Core State Standards

Initiative (CCSS, 2015) encourages schools to provide instruction in fractions skills early on, starting at third grade. Early fractions difficulties seen in elementary schools are a strong predictor for later achievement in mathematics (Watts, Duncan, Siegler, & Davis-Kean, 2014).

Considering these types of difficulties, NCLB (2002) and the Individuals with Disabilities Education Act of 2004 (IDEA, 2004) recommend the usage of Response to Intervention (RTI) for the purpose of identification and then deliver to evidence-based interventions. The RTI model also might reduce the disproportionate number of students of color being determined as needing special education services (Yell, Thomas, & Katsiyannis, 2012). The RTI model consists of three tiers, and each one requires a different level, and intensified explicit and systematic instruction for students that vary from whole classrooms to single individuals.

However, there are many problems for the proper delivery of scientific/evidence-based interventions through the RTI model. Teachers' limited knowledge of evidence-based interventions (Collier, 2010; Darling-Hammond, 2010; Ingersoll, 2002; Snow, Griffin, & Burns, 2005), and their pedagogical knowledge, lack of resources (Haager, Klingner, & Vaughn, 2007), and including money and time (Rosenfield, & Berninger, 2009), are some of the barriers. Many teachers implementing RTI in the field express concern about time. Therefore, strategies are needed that reduce barriers specifically generated by lack of time to deliver explicit instructions to students needing differentiated instructions.

Area of Concern

Fraction skills play a critical role in developing future mathematic concepts, such as algebra and in high functioning skills for a productive and successful life (Fuchs et al., 2013; Geary, 2011; National Mathematics Advisory Panel [NMAP], 2008; Siegler, Fazio, Bailey, &

Zhou, 2013). However, statistics shows that 50 percent of students in US have difficulty with basic level fraction skills (Misquitta, 2011). While considering students with a Mathematics Learning Disabilities, the level of the problem associated with fraction content is more serious for these students compared to their peers without the disability.

Theory and empirical based studies indicate the mathematical disability centers on the conceptual understanding of fractions. Berch and Mazzocco (2007) identified conceptual knowledge of fractions "as the awareness of what fraction symbols mean and the ability to represent fractions in multiple ways" (p. 122). Nevertheless, many students are not able to figure out the meaning of fraction symbols. Their previous knowledge and experience with whole number concepts lead students to read and compute fractions in the way of whole number concepts, such as whole numbers that do not decrease with multiplications, do not increase with division, and "the number with more digits is not necessarily larger, unlike with whole numbers" (Jordan et al., 2013 p. 46; NMAP, 2008; Ni & Zhou, 2005; Siegler, Fazio, Bailey, & Zhou, 2013). For instance, students may read $\frac{3}{4}$ as 3 and 4, and make computations based on what they read. Since students with MLD have weak working memory, they often calculate with their fingers (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Krasa, & Shunkwiler, 2009). In the case of fraction computation, the level of the problem increases because students with MLD are not able to use their fingers for computations involving fractions (Wu, 2008). Another challenge associated with developing fraction skills is "not use halving value" (dividing one into two equal pieces, 1/2) at upper levels. For instance, at the beginning of fraction instruction, generally fractions are taught as parts of a whole, but later on students see that some fraction might be bigger than a whole. However, before upper levels, students were taught only with considering halving values, dividing half, instead of providing some examples with odd

denominators (Pothier & Sawada, 1983; Pitkethly & Hunting, 1996). Therefore, Misquitta (2011) said that fractions should be taught based on the recommendation of The National Council of Teachers of Mathematics Standard (2006), which states "that fractional content incorporates understanding of fractions as part of the number line, understanding of the relationship of fractions to whole numbers, fraction equivalence..." because conceptual knowledge consists of those skills (p. 110). However, many teachers do not differentiate strategies, such as using number lines instead of only using pizza slices, in terms of students' needs (Maccini & Gagnon, 2006; Sigler, Thompson, & Schneider, 2011). This makes the problem more serious and also leads to widening the achievement gap between students with and without disabilities because of not grasping the logic behind the fraction.

Gap in Knowledge

The use of technology - specifically mobile devices, such as iPad -, is exponentially growing in the field of education due to its potential for increasing academic skills of students. Mobile devices enable students to learn whenever or wherever they want (Geist, 2011). Students do not need to be at school or any certain place at any certain time to be engaged in learning. By downloading various apps to iPads, teachers easily customize the devices in terms of the needs of their students to deliver instruction successfully. According to Walker (2011), there are around 560,000 apps that were created by almost 100,000 different publishers. People can reach these apps from a variety of places including the Apple Store. An interesting statistic also revealed by Walker (2011) is that each day, 775 new apps are developed and made available to download through the Store. The data show that "15 billion apps have been downloaded from the Apple Store in the past three years" (p.1), underscoring the growing importance of this technology.

Recent statistic showed that total download apps from 2008 to 2015 was 100 billion apps from only Apple Store at the end of the June (Statista, 2015).

The most popular categories in those apps are game based apps, which account for 74,379 apps (Walker, 2011). However, the number of educational based apps is less than many other categories. Teachers are also having difficulty identifying appropriate apps in terms of needs of students, even though Yerushalmy and Botzer (2011) stated, "we consider mobile learning to be an important aspect of future changes in the curriculum and in the nature of the classroom" (Yerushalmy, & Botzer, 2011, p.192; Walker, 2011). Therefore, more studies are needed to determine the effectiveness of educational apps to increase students' academic achievement. However, there is an apparent gap in the literature. Riconscente (2013) stated, "although hundreds of iPad apps on the market claim to improve learning, no published studies were found of controlled experiments that tested the effectiveness of an educational iPad app for increasing learning outcomes" (p. 187). While looking at specific content areas, such as fractions, there are just two studies conducted considering the effectiveness of *Motion Math: Fraction* app. The app was created at the Stanford School of Education in 2010 (Apple, 2015). It is described as an award winning fraction game. In this game, a star falls from the sky and players aim to carry it back to the sky. They can only do this task by placing fractions on the correct point on the number line. In this game, fractions may be seen in several forms: denominator/ numerator, percent, decimal, and pie chart.

Uncertainty That Causes Me Concerns

Considering the effectiveness of *Motion Math: Fraction* app, two different studies were conducted (Farmer, 2013; Riconscente, 2013). Riconscente's (2013) study consisted of students without disabilities while Farmer's (2013) study focused on low performing students selected by

administrators. However, Farmer did not discuss selection criteria and procedures for participants. Neither of these studies included students with MLD, nor investigated the app's effectiveness on fraction skill of this population. Therefore, it is not clear whether the app helped this group of students to learn fractions skills. Furthermore, in the study, the researcher integrated the *Motion Math: Fraction* app into classroom activities (Farmer, 2013). However, the researcher did not look at its usefulness for outside of school practice. Since teachers cite lack of time to provide differentiated instructions based on students' needs in classrooms, the impact of outside usage of the app should be examined.

Purpose of the Study

The aim of this study is to test the directional hypothesis; a-) participants (students with MLD) will increase their fraction skills by playing the *Motion Math: Fraction* app 20 minutes daily for two weeks, b-) participants will maintain the level of fraction skills they while playing the *Motion Math: Fraction* app 20 minutes daily for two weeks after no longer playing the app, c-) greater amounts of time interact with the app will result greater achievement gain for the students.

While conducting the study, the report for effective implementation of single subject design was adhered which provided by What Works Clearinghouse (WWC) (Kratochwill, Hitchcock, Horner, Levin, Odom, Rindskopf, & Shadish 2010). WWC "identify studies in education field and provide credible and reliable evidences" about effectiveness of interventions which used to improve certain skills of students (WWC, 2015). In that report several criteria were highlighted as requirements for evaluating a scientific based intervention, such as having at least four participants, and having at least five data points during the baseline of a study in which single subject design is used as

method. Therefore, having at least four participants for scientific credibility (Kratochwill et al., 2010) is needed in this study. Students with the needs in the area of fraction instruction as stated in their IEP were included in the study. The students are from different grade and age groups from a public charter school in the Southeast part of US.

Variables in this study are *Motion Math: Fraction* app as an independent variable and the students' fraction skills as a dependent variable.

Data were collected by employing a single subject experiment; specifically multiple baseline-AB type-design. The researcher use thirty-five items from the National Assessment of Educational Progress (NAEP; U.S. Department of Education, 2014), which have been released from 1990 to 2013 and other items from different studies to measure the dependent variable (Fuchs et al., 2013; Siegler et al., 2011). These items were categorized as easy, medium, or hard and distributed into 5 questions sheets while considering their difficulty levels. Each of these five question sheets includes 13 items. For each data point, researcher administered one of these sheets as paper and pencil tests. Professionals in the field of mathematics education were asked to evaluate the items regarding the relations of the items and the domain interest considering content validity (Johnson & Turner, 2003).

A single subject experiment were used as a method to collect data; specifically a multiple baseline design (AB design). For data analysis, first visual analysis were conducted to see differences between baseline phase and treatment phase considering level, trend, variability, and immediate effects of an intervention, overlapping data, and consistency of data patterns within and between phases (Fisher, Kelley, & Lomas, 2003; Hersen & Barlow, 1976; Kazdin, 1982; Kennedy, 2005; Morgan & Morgan, 2009; Parsonson & Baer, 1978). In addition to visual analysis, the researcher calculated effect size by using Percent Non-Overlapping Data (PND),

Percent Exceeding Median Data (PEM), and Percent of All Non-Overlapping Data (PAND) (Parker, Vannest, & Davis, 2011). A third step for data analysis was that Kenward-Roger Model was employed to estimate change in level and to estimate degree of freedom (Ferron, Bell, Hess, Rendina-Gobioff, Hibbard, 2009). The researcher interpreted results in terms of a *p*-value of .05. **Conceptual Framework for the Study**

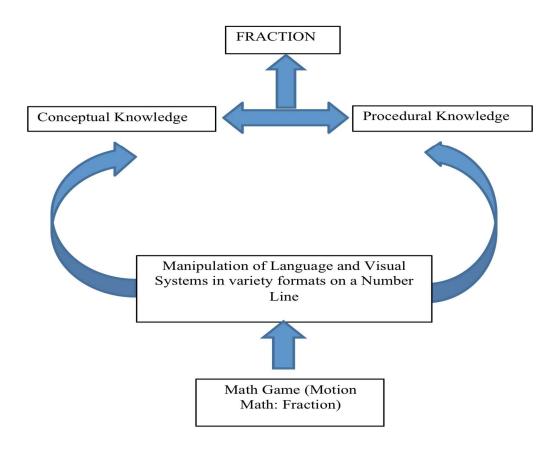


Figure 1. Conceptual Framework of the Study

Mathematical Learning disabilities (MLD) was identified "as a deficit in conceptual or procedural competencies that define the mathematical domain, and these, in theory, would be due to underlying deficits in the central executive or in the information representation or manipulation (i.e., working memory) systems of the language or visuospatial domains" (Geary, 2004, p. 9). The identification of learning disability is accepted as a conceptual frame and guidance of this research.

Geary (2004) described conceptual and procedural knowledge as types of mathematical knowledge. By employing visual and language systems, such as representing information on a number line, acquisition of that knowledge is promoted. However, gaining fraction knowledge is different from that of whole number knowledge (Siegler, Fazio, Bailey, & Zhou 2013). Sigler and his associates stated that, "learning fractions requires a reorganization of numerical knowledge, one that allows a deeper understanding of numbers than is ordinarily gained through experience with whole numbers" (p. 13), because of the unique features of a whole number, and fractions. Therefore, while representing information on a number line variety forms of language and visual systems should be provided to students who have deficits to manipulate the information by using variety tools, such as apps on iPdas.

Understanding fractions requires representing magnitudes, principles, and notations of rational numbers (Siegler, Thomson, & Schneider 2011). Indeed, this is known as conceptual knowledge of fractions. Misquitta (2011) stressed the relationship of conceptual knowledge and procedural knowledge considering the acquisition of these types of knowledge. Conceptual knowledge is described as understanding fractions symbols, operations symbols, relationship of numbers, and their rational quantities (Hecht, & Vagi, 2010), and this is hard to gain for students with MLD. Procedural knowledge is known as the process of computation (NMAP, 2008). Conceptual knowledge and procedural knowledge and procedural knowledge jointly reinforce each other. When conceptual knowledge increases, procedural knowledge also increases.

NMAP (2008) highlighted the employment of conceptual and procedural knowledge as essential elements to understand rational numbers. Several strategies are stressed which include

using the concrete to represent the abstract (CRA) and strategy instruction (Josep & Hunter, 2001; NMAP, 2008; Owen & Fuchs, 2002; Test & Ellis; 2005). Interestingly, even though studies conducted by researchers in the field of general education are stressing conceptual knowledge, procedural knowledge is more commonly used in the field of special education.

Unlike recommendations of NMAP (2008), and NCTM (2008), the reasoning of researchers in the field of focusing on procedural skills is that students with MLD have problem because of working memory deficit, which leads to difficulties in calculation and processing; therefore, many interventions adopted for the students focus on teaching calculation and process of calculation. A working memory deficit leads to difficulties in calculation and processing, which are related to procedural knowledge. However, only focusing on procedural knowledge and minimizing the importance of conceptual knowledge leads students to memorize the processes instead of understanding the meaning and relations.

Engaging with mathematic games has the potential to increase conceptual knowledge and number sense, which are interchangeably used (Berch, & Mazzocco, 2007). Siegler and Ramani (2009) investigated the effectiveness of board games to increase mathematical knowledge of preschool students by physically interacting with the number line integrated into the games. By playing the board games, students manipulate a token on the number line, and this helps them to develop a mental representation of the number line by providing concrete hints about magnitude of numbers. Result of the study showed significant improvement of the students' knowledge of comparisons, estimation, identification and counting of numbers. In light of this information, it is thought that game based mathematical apps on mobile devices, such as *Motion Math: Fraction*, might have the potential for manipulation of language and visual system in variety form on a number line to increase a form of mathematical knowledge of students with MLD.

Significance of the Study

This study adds to the evolving body of research that is designed to determine whether or not the *Motion Math: Fraction* app helps students with MLD to improve fraction skills. Besides the practical significance of the research, there is also theoretical significance of this research.

Limitations and Delimitations

Even though a single subject experiment has many advantages, the design also has weaknesses, such as generalizability. The six students participating in this study do not represent the entire population from which they are selected since these participants were not selected randomly from population and small number of participants (Cakiroglu, 2012). However, to overcome this problem, the researcher explicitly described the procedures used in conducting the study including sampling procedure, data collection, and data analysis. This detailed explanation allows other researchers to replicate the study.

For the purpose of delimitation, the researcher used several inclusion and exclusion criterions to draw boundaries of the study. Students in various grades were chosen from a public charter school in the southeast part of US, these students had MLD, and their needs were detailed in the Individualized Education Plan (IEP). The researcher also considered results of several tests (e.g., Woodcock Jonson III, Northwest Evaluation Association Standardized Assessment) that specifically focus on cognitive processing, and fraction computation to include or exclude students for the study.

Because of the comorbidity feature of learning disabilities in mathematics, the researcher included students who have Learning Disabilities, Attention Deficit-Hyperactivity Disorder (ADHD), and Autism Spectrum Disorder. On the other hand, the researcher excluded students

with having hearing or vision problems, and also excluded students who were in the category of Emotional Behavior Disorder (EBD).

The researcher conducted the study at a public charter school, which is defined as a fullday ESE school serving students who have learning related disability. Before choosing participants for the study, the researcher sent consent forms to all families who have children participating in the after school program. After receiving the families' responses to participate, the researcher chose students who best fits for the study based on their IEPs, FCAT scores, and other achievement test scores including Northwest Evaluation Association Standardized Assessment (NWEA). As a last step, the researcher asked students about their agreement to participate and have them to sign the consent form.

CHAPTER TWO: LITERATURE RIVEW

Mathematics Learning Disabilities

Specific Learning Disability (SLD) is one of the biggest categories under IDEIA 2004. Approximately, 10% of the students in the United States are in this category. In itself, SLD is separated into several subcategories: reading, writing, and mathematics disability. Even though the prevalence and the impacts of reading and mathematics disability are almost at the same level, many researchers have highlighted reading as a more crucial skill for an effective and productive life. However, awareness of mathematics disability is increasing, with several researchers describing the issue as "the birth of a new discipline" (Berch, & Mazzocco, 2007; Krasa, & Shunkwiler, 2009). Mathematic skills are as important as reading skills, and, in some cases, computation error can be life threatening. For instance, referring to the quote provided at the beginning of the first chapter, each pharmaceutical drug consists of an amount of ingredients. If a pharmacist puts more or less amount of some ingredient into a combination of a pharmaceutical drug, it may hurt patients, and might even cause death.

Although mathematic skills are important and useful for people, some of their cognitive deficits have negative impact on these skills. Several terms are used in defining their problems, such as dyscalculia, and mathematics difficulty (Berch, & Mazzocco, 2007). The occurrence of students with disabilities and difficulties might vary depending on the terms or criteria used. The difficulties represents a bigger group of students than disabilities because, in the case of difficulties, researchers use several cut off points; some use a criterion of being one grade level below from their peers, while others use below 35th percentile on a test (Eastburn, 2010; Krasa, & Shunkwiler, 2009). These different criteria differentiate from below average to low average

score of students' on tests (Gersten, Jordan, & Flojo, 2005). However, these students having difficulties in mathematics may not have mathematics disabilities. Since there are no clearly defined boundaries for the disabilities, identifying a large group of people with potential mathematic disabilities might prevent future academic failures. On the other hand, lack of clear criteria makes it difficult to comment about results of studies for generalizations (Berch, & Mazzocco, 2007).

Jordan et al., (2006) claim that mathematics difficulty can be due to environmental causes, instead of biological causes. In that case, when students receive instruction based on their needs, they may perform above average on standardized achievement tests. Since their scores are above average, they do not qualify for the category of difficulty due to their score.

One of the reasons for the use of a variety of terms is the definition of the disability. Still there is no consensus on models that have been used to identify or determine whether students are eligible for special education services. Even though in much of the research, discrepancy model has been stressed, the model has weaknesses; such as until students fail, it is hard to see any action against to problem of students to learn any content area (Berch, & Mazzocco, 2007).

MLD is defined as "a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to ... do mathematical calculations" (IDEA 2004). In the description of specific MLD, several important points were stressed, such as providing scientific based intervention in terms of the needs of students with the disability, the usage of discrepancy model because of its inefficacy to identify students, and exclusion of mental retardation and sensory impairment from the category of the disability (Simsek, 2013).

Prevalence of Mathematics Learning Disabilities

The prevalence of the disability might change depending on the criteria and math tasks considered in defining the disability. Berch and Mazzocco (2007) provide an example that shows how the numbers change by stating, "the cumulative incidence of dyscalculia in children up to 19 years was 5.9% (using Minnesota regression formula), 9.8% (using the discrepancy formula) and 13.8% (using the low-achievement formula)" (p. 54). Interestingly the numbers of children who were 7 years old were very small and the percentage varies only from 1.3% to 2.1% in the category of disability. However, the predicted percentage of the disability ranges from 5.9% and 13.8%. Pointing to the importance of conducting early screening tests to identify the disability before early adolescence. Furthermore, Mazzocco and Myers (2003) stated that the use of tests for the determination of early math ability showed that 63% of kindergarten students determined as having dyscalculia were still in the same category in third grade. This study is also important because it stresses the importance of assessing students' performance at multiple times. For that, Fuchs, Compton, Fuchs, Hollenbeck, Craddock, & Hamlett (2008) suggested dynamic assessment. On the other hand, delaying to identify students by waiting until students fail on standardized mathematics tests using the discrepancy model might cause academic failures.

Characteristics of Mathematics Learning Disabilities

Since researchers focus on a variety of math tasks, each of them claims a different task as a defining feature of the disabilities. This approach leads to other problems, such as generalization of the results of the studies included students with MLD. Some researchers stress the relationship math achievement and spatial skill, working memory, and phonological processing, however, others mention verbal skill and its contribution to the disability (Floyd, Evans, & McGrew, 2003; Krasa, & Shunkwiler, 2009). Fletcher (2005) found statistical differences between students with only mathematics disability (MD), students with only reading disability (RD), and comorbidity group of students who had both types of disability (MD/RD). In the research, it was claimed that students with comorbidity of math and reading disability showed difficulty related to language. Finding based on the Woodcock-Johnson Psycho-Education Test Battery- Revised included "statistically significant differing profiles in sustained attention, procedural learning, concept formation, phonological awareness, rapid naming, vocabulary, paired associative learning, and visual motor subtests, thus indicating that MD, RD, and MD/RD students learn differently" (Eastburn, 2010, p. 28); even students within the category of mathematics disability showed different characteristics (Berch, &Mazzocco, 2007; Krasa, & Shunkwiler, 2009).

Allsopp, Kyger, and Lovin (2007) emphasized the knowledge about learning characteristics of students with disabilities, from teachers' point of view, is critical to plan and successfully deliver instructions based on their needs. Otherwise, students do not understand even if teachers use quality instructions and variety of materials. The researchers classified common characteristics of students in eight different categories.

Characteristics	Description			
Learned helplessness	Students' repeated failure leads them to be reluctant			
	to try something different and they wait for someone			
	else to help them.			
Passive learning	These students do not actively participate in			
	classroom activities, and they have problems seeing			
	relationships between numbers. They do not employ			
	what they learned to a new problem situation.			
Memory difficulties	As these students have problem with short term and			
	working memory, retrieving information from long			
	terms memory, they do not make basic calculations			
	and have difficulty with multistep problems.			

Table 2.1. Common Characteristics of Students with Mathematics Difficulty

Table 2.1	. (Contin	ued)
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Characteristics	Description
Attention difficulties	Even though people are required to focus on content to learn, students with learning disability and having attention problem encounter a variety of stimuli that distract them. Those students have difficulty to pick relevant stimuli for mathematics instruction; therefore, they most likely miss critical points to solve problem, which requires multiple steps.
Cognitive/Metacognitive thinking deficits	Metacognitive skill is known as thinking about thinking. However, students with the disability have problems with this skill. They do not monitor what they are learning, specifically the planning, sequencing, and goal settings. Since students do not self-monitor, they cannot check their answers, and the answers are most likely wrong.
Processing deficit	As their central nervous system processes information differently, these students have problems with interpreting the things they see, hear, and feel. This leads them to miss the concept of what they learned. Furthermore, the processing of information is very slow when compared to their peers.
Low level of academic achievement	One of the common characteristics of these students is their low academic achievement, and this might be seen not only in mathematics but also in other areas, such as reading. Students with processing deficits need more time than their peer to be proficient in some certain concepts. However, in many cases, it does not work this way; thus, learning for them gets more difficult.

Characteristics	Description					
Math anxiety	There is strong correlation between mathematics					
	anxiety and poor mathematical knowledge (Ashcraft,					
	& Krause, 2007). Math anxiety has negative impact					
	on mathematics knowledge, course grades, and					
	students' performance on standardized tests.					
	Students' performance on standardized tests. Students' anxiety in early grades might make an					
	effect of snowball and leads students not like math.					
	Since math anxiety co-opt working memory					
	resources, working load is increased. "Which means					
	that anxiety-induced consumption of WM may					
	shrink this available capacity below the level needed					
	to successfully solve difficult math problems"					
	(Ramirez, Gunderson, Levine, & Beilock, 2013. p.					
	189).					

Table 2.1. (Continued)

As stated above, knowing the characteristics of students with MLD is critical while delivering instruction successfully based on the needs of the students. Considering the characteristics, teachers might develop various strategies. For instance, students with MLD develop math anxiety and this shrinks the capacity of working memory; therefore, the students need encoding and decoding strategies to gain mathematics skills. Sigler et al., (2013) stated working memory and inattentive behavior as reason of fraction problem, in that case teachers should use activities to increase students' on task behavior.

Bryant et al., (2000) created a form to identify common behaviors of students with MLD. At the beginning, the researchers asked hundreds of randomly selected teachers about common characteristics of the students, and read through studies in the field of special education. The researchers came up with 32 common behaviors of students with MLD. These 32 items were used to create a first version of the rating scale. At the next step, 75 experts were invited to examine the behaviors, and 36 of the experts accepted to participate. All of them had doctoral degree. By adding one more characteristic on the list of items, 2/3 of the experts agreed on 33 items showing common characteristics of students with MLD. Four common behaviors mostly agreed on were stated as problems in "word problem solving", "has difficulty with multi-step problem", "has difficulty with the language of math" and " fails to verify answers and settles for first answers" (p. 175).

The distance between numbers, such as one and two is larger than the distance between eight and nine in early ages and grades. However, this problem is persistent in later grades for students with MLD. Geary (2011) explained the problem as "due to a deficit or delay in the system for representing approximation magnitude" (p. 7). This skill, approximation, should be taught especially while teaching fraction concept. Teachers should teach approximation skill by using number line, and ask student where the numbers, such 4/5, on a number line.

Children at early ages and even at early grades use fingers to count; however, they need to develop different strategies for big numbers (Geary, 2011). Using fingers at later grades is common characteristics of students with MLD, and these students even make more errors while counting. For instance, while making addition (5+3=?), students with MLD start from 5, and count 6, 7, and find the answer as 5+3=7.

Students with MLD make more errors while solving multi-step problems since they misalign numerals while writing down partial answers, and carrying and borrowing numbers. However, Gear (2011) claimed that these problems were developmental and not persistent. Children with MLD and LA eventually will learn, but several years later.

Difficulties retrieving basic facts are another common characteristics of the students with MLD. Biggest reason for this problem is intrusion; retrieving irrelevant information from long-term memory to working memory to solve problems. In addition to intrusion, there might be several other mechanisms can cause problem in retrieving basic arithmetic tasks (Geary, 2011).

Fractions. Fractions are one of the most difficult content areas in mathematics for students with or without disabilities to comprehend (Charalambous & Pitta-Pantazi, 2007; Hecht & Vagi, 2010; Pitkethly & Hunting, 1996) and with which to be proficient (Misquitta, 2011). However, the number of studies focusing on students with MLD specifically in the area of fractions is few (Mazzocco & Devlin, 2008).

Even though fraction skills are essential for better functioning in many jobs, 50% of students from middle and high school have difficulty with basic level fraction skills (Fuchs et al., 2013; Misquitta, 2011). Geary (2004) estimates the percentage of students with MLD is around 10%, while almost 40% of students are specifically at risk for problems in comprehending fractions. Problems with fractions for students with MLD are more complex than others who are without the disability, or who are considered at-risk for mathematics difficulties but who are not identified (Groebecker, 1999). Mazzocco and Devlin (2008) investigated the performance of three groups of students: students with MLD, low performing students, and students without disabilities on naming skill of fractions, sequencing/ordering fractions based on magnitude, and determining equivalency of fractions. The result of the study showed that even though three groups of students had a degree of difficulty with fractions, students with MLD performed significantly lower than their peers who were at risk and their typical peers. Furthermore, a high correlation (rs > 0.80) was found between high schools students' mathematics achievement and their fraction knowledge. Fraction skills of fifth grade students are also known as an important predictor for further academic achievement in algebra (Mancini, & Ruhl, 2000; Siegler, Fazio, Bailey, & Zhou, 2013).

Students struggle with learning fractions for a variety of reasons. Students' knowledge of whole number operations appears to be one factor. Mack (1990) conducted a research to

determine the role of prior whole number knowledge to learn fractions. The researcher delivered fraction instruction individually to eight sixth grade students. She aimed to build informal fraction knowledge. At the beginning, students' informal knowledge was activated separately from symbols and procedural knowledge and this was meaningful for individuals. However, as instruction ensued students' lack of procedural knowledge inhibited their abilities to construct informal fraction knowledge on individuals' previous learning. Behr, Wachsmuth, Post, and Lesh (1984) investigated students' understanding of rational numbers, order and equivalency of rational numbers through clinical interviews as students compared using different types of fractions pairs (i.e. same numerator, and denominators) by using manipulate tools. Although many students were successful in grasping fraction knowledge, some of them had difficulty understanding the concept of fractions because their prior knowledge related to whole number concepts. As students continued to receive fraction instruction, the effects of prior whole number knowledge decreased.

Experience with whole numbers can sometimes interfere with students' abilities to develop conceptual understandings of fractions. Berch and Mazzocco (2007) identified conceptual knowledge of fractions "as the awareness of what fraction symbols mean and the ability to represent fractions in multiple ways" (p. 122). Previous knowledge and experience with whole number concepts can lead students to read and compute fractions in ways similar to what they have done with whole number concepts (Misquitta, 2011; NMAP, 2008; Ni & Zhou, 2005; Siegler, et all. 2011; Siegler et all. 2013). For instance, students may read ³/₄ as 3 *and* 4, and make computation based on what they read (i.e., "three and four is seven"). Therefore, researchers have recommended the use of instructional practices that facilitate conceptual understandings of fractions including use of variety examples in different situations to increase

students' understanding of fractions. Misquitta (2011) stressed, based on the recommendation of The National Council of Teachers of Mathematics Standards (NCTM, 2006), "that fractional content incorporates understanding of fractions as part of the number line, understanding of the relationship of fractions to whole numbers, fraction equivalence..." because conceptual knowledge consist of those skills (p. 110). Indeed, teaching fractions through a number line appears to have promise as an effective instructional practice (Siegler, et al., 2010). In several studies, researchers have suggested that use of a number line is critical to gain early number skills (Case, & Griffin, 1990; Case, & Okamoto, 1996). The number line helps students to encode and store fraction information by incorporating individuals' understanding based on magnitudes of the numbers, which is more easily retrieved from long terms memory (Siegler, et al., 2011). Unfortunately, teachers often fail to employ this practice including exposing students to a variety of fraction examples in different contexts due to several factors including lack of mathematics content knowledge and pedagogy of teachers, teachers' difficulties with class management skills, lack of resources, and too little instructional time made available to teachers (Brownell, Sindelar, Kiely, & Danielson, 2010; Collier, 2010; Maccini & Gagnon, 2006; Rosenfield, & Berninger, 2009).

In order to increase students' mathematical proficiency greater emphasis in the mathematics curriculum has been placed on how students are engaged in learning and doing mathematics (i.e., mathematical practice). NCTM (2000) has recommended triggering the skill of reasoning, and problem solving. Allsopp et al., (2007) categorized the standards of NCTM as processing big ideas as follows: problem solving, reasoning and proof, connections, communications, and representations. This view also emphasizes the importance of conceptual knowledge while working with fractions.

Conceptual knowledge and number sense are interchangeable, and engaging with mathematic games has the potential to increase this skill (Berch, & Mazzocco, 2007). Siegler and Ramani (2009) investigated the effectiveness of board games to increase mathematical knowledge of preschool students by physically interacting with the number line integrated into the games. By playing the board games, students manipulate a token on the number line, and this helps them to develop a mental representation of the number line by providing concrete hints about the magnitude of numbers. Results of the study showed significant improvement of the students' knowledge of comparisons, estimation, identification and counting of numbers.

Even though studies conducted to determine effective teaching strategies for fraction skills stressing the usage of number line concepts and teaching fractions as a number on the line, in classrooms teachers still use "parts of a whole" concept. This leads to inaccurate conceptualization of fractions when they have continual values (Riconscente, 2013). Since only relying on procedural knowledge, many students have problems understanding and processing the knowledge of fraction as numbers; therefore, they have difficulty placing fractions appropriately on a number line (National Mathematics Advisory Panel, 2008).

Fraction interventions for students with disability in math. Regarding the effective instructional practices for students with MLD to teach fraction skills, the researcher completed a literature review of studies in the field. In this review, studies focusing on fraction skills of students with learning disabilities and at- risk students were included. Inclusion criteria were: studies published in peer-reviewed journal between 1990- 2014; Studies those are empirical in nature considering the effectiveness of an intervention to improve fraction skills (i.e., identifying and representing fractions, comparing fractions considering their magnitude, adding, subtracting, multiplying and dividing fractions). ; Participant samples that included students identified with

learning disabilities, students considered by the authors to be at-risk for failure in mathematics, and students in grades K-8; The following data bases were included: ERIC EBSCO, Education Full text, PYCHOINFO, JSTOR. Key words utilized included *learning disability, struggling, difficulty, at-risk, fraction, elementary, middle school, mathematics, arithmetic,* and *number sense* entered in different combinations. Additionally, the references of published meta-analyses were examined for studies meeting the inclusion criteria.

A total of ten studies met inclusion criteria. See the table below.

Table 2.2. Studies for Students with Mathematics Difficulty in Fractions

Study	Participant	Grade	Design	Setting	Dependent Variable	Race	Results
Baker, Young, & Martin (1990)	6LD	5	Experimental	Sydney, Australia	Fraction, and spelling	NR	Results were in favor of one to one group over group instruction.
Bottge (1999)	2LD, 4OHI, 11 at-risk, 49 typical	8	Experimental and Quasi- experimental	Rural school district, Upper Midwest, U.S.	Computation and problem solving skill (addition and subtraction skills were also considered)	NR	Effective on transferring skill, but not on computation and word problems (ES =28)
Bottge, Heinrichs, Mehta, & Hunge (2002	7LD, 1ED, 34 typical	7	Quasi- experimental	Rural school district, Midwest, U.S.	Tests for computation and word problems (fraction addition and subtraction)	NR	Even though students without disability benefited from it, significant difference was not found for others. (ES = 25)

Study	Participant	Grade	Design	Setting	Dependent Variable	Race	Results
Butler, Miller, Crehan, Babbit, & Pierce (2003)	42LD, 8MLD,	6,7,8	Quasi- Experiment	Urban school district, south- western U.S. Resource room	Fraction equivalency	NR	ES = 0.26. Higher means for CRA group
Flores & Kaylor (2007)	30 at-risk in mathematics	7	Quasi- experiment	Rural school district, south- western U.S.	Percentage of correct answer to the questions regarding addition, subtraction and multiplication of fraction skill of the students	18 Hispanic, 6 White, 6African American	Significant findings were reported.
Fuchs et al. (2013)	259 at-risk	4	Experimental	U. S.	Fraction number line, assessing magnitude, and fraction computation. NAEP Total	51% African American, 26% White, 19% Hispanic, 4% other	ES (0.29 to 2.50)
Gersten, & Kelly (1992)	26 LD	Secondary	Pretest posttest, single subject design	Resource room setting	Fraction skill was assessed by using criterion referenced test consisted of 30 questions	NR	Students improved their score almost 51.5 percent from pre-test to post-test.
Jordan, Miller, & Mercer (1999)	5 LD, 1 ED, 6 OHI, 18 Gifted, 97 typical	4	Experimental	South- eastern U.S.	Fraction	52 White, 11 No- White	Treatment group received intervention via CSA performed better than control group

Table 2.2 (Continued)

Study	Participant	Grade	Design	Setting	Dependent Variable	Race	Results
Joseph & Hunter (2001)	3 LD	8	Single subject, Multiple baseline design	Urban school district, Ohio	Fraction	White	Participant improved their problem solving skills, and keep in maintenance phase.
Test & Ellis (2005)	3LD, 3ID	8	Single subject	Small- town, south- east. U.S.	Adding and subtraction fraction, steps to complete strategy	3 White, 3 African American	5 out of 6 improved their skills on fraction problem solving.

Table 2.2 (Continued)

Note: LD: Learning disability, ID: Intellectual Disability, ED: Emotional Disturbance, OHI: Other Health Impairment, NR: Not Reported.

Detailed analysis of the findings from the listed studies meeting criteria showed in the above chart generated common themes for interventions.

Concrete-representational-abstract instruction. Butler, Miller, Crehan, Babbitt, and Pierce (2003) and Jordan, Miller, and Mercer (1999) examined the effectiveness of a concreterepresentational-abstract (CRA) instruction (also referred to as the "graduated sequence model" and concrete-semi-concrete-abstract instruction) to increase the understandings of fractions of students who were struggling with understanding rational numbers. Students improved with respect to both procedural knowledge and conceptual understandings of fractions. Butler et al. (2003) compared the use of explicit CRA instruction to use of explicit representational to abstract (RA) only instruction for the purpose of teaching equivalency of fractions. The CRA group had the opportunity to manipulate concrete materials before transitioning to the representational and abstract levels. At Concrete phase of "C," students were introduced to solving word problems related to equivalency of fractions by using concrete manipulative, and then students transitioned to drawing pictures of fractional quantities at the "R" level, and then at the "A" level students solved word problems at the abstract level. The result of Butler et al.'s (2003) study showed the efficacy of CRA intervention (ES = 0.26) by considering the mean score of students in all subtest. However, researchers did not assign students randomly; therefore, results should be considered with caution. Jordan et al. (1999) examined the effectiveness of a concrete-semi-concrete-abstract (CSA) sequence of instruction compared to a control group receiving traditional instruction (based on the adopted textbook) without CSA. Students in the CSA group first manipulated objects to solve problems related to equivalency of fractions, then drew pictures to solve problems, and finally solved problems without the support materials or drawings. The treatment group (improved 29.3 of their mean score) outperformed the control group (improved 11.31 of their mean score).

Anchored/Contextualized instruction. Bottge (1999) and Bottge, Heinrichs, Mehta, and Hung (2002) examined the efficacy of contextualized math instruction (anchored instruction) to teach problem solving skills to students with MLD, at risk, and without disabilities by employing video based problems. In this approach, researchers provided real-life problems through videos (e.g., using fraction and measurement skills to constructing a cage for birds given information, such as width of the cage). Students worked collaboratively to solve problems by engaging in fraction and measurement skills. The focus of instruction was on the problem solving and their reasoning skills of students; therefore, fraction content was not directly taught as part of the intervention. While problem solving, students were required to convert given numbers into different formats, such as converting feet to inches. Students in remedial and pre-algebra classes increased their scores on the test regarding transferring skill, however, results for computation and word problem solving skills did not show significant differences.

Strategy instruction. Strategy instruction involves planning, attention, and self-regulatory, and using mnemonic. Joseph, & Hunter (2001) and Test, & Ellis (2005) conducted studies evaluating the effect of strategy instruction and fractions. Joseph and Hunter (2001) conducted a single subject multiple baseline design study to determine the effectiveness of a cue card strategy for adding and multiplying fractions. Initially, teachers demonstrated how cue cards were used to solve basic addition and subtraction problems with different type of denominators including common and uncommon. When students get proficiency to apply the strategy, they were needed to employ the strategy to problem solving questions. All students improved their skills even in the maintenance phase in which cue cards were removed. However, by using this strategy, students gained only procedural knowledge. Test and Ellis (2005) used a mnemonic called LAP to teach students needed special services for mathematics. At the "L" step, students look at the denominator to determine like or unlike denominators, and signs. The next "A" step is Ask questions (i.e. will the smallest denominator divide into the largest denominator and even number of times? p. 14), and then students "P"-ick a type of the fraction. After proficiently completing these steps of activities, students were taught how to add and subtract fractions. During the final step, students reduced the results found by calculation to lowest number. Results showed that five out of six students improved both skills; task completion and fraction problem solving: they reached mastery level. Their mean scores in intervention and maintenance phases were more than 80%. However, the sixth participant had 56.7% mean score in intervention and 55% mean score in maintenance phases.

Direct instruction. Direct instruction is generally characterized by one to one or teaching in separate classroom. Direct instruction has several components and these identified in following order;

(a) Organizing central concepts and strategies in ways that allow application across multiple contexts; (b) providing clear and systematic methods of teacher communication, decreasing the likelihood of student misunderstanding or confusion; (c) the use of formats involving structured verbal exchanges between students and teachers, allowing for increased student engagement, ongoing progress monitoring, and repeated verbal practice; (d) strategically integrating skills to ensure efficient learning and understanding; and (e) arranging Instructional concepts into tracks in which learning develops across the length of the program while providing ongoing review and generalization (Flores, & Kaylor, 2007, pp. 85-86).

Three studies evaluated the use of direct instruction to teach fraction skills (Baker, Young and Martin, 1990; Flores & Kaylor, 2007; Gersten and Kelly, 1992). Flores and Kaylor (2007) investigated the effectiveness of direct instruction program on students' fraction performance. Thirty students, their age range was from 12 to 14, in academically at risk category participated in this study. The majority of the participants were from minority groups living in a rural school district from the south eastern part of the U.S. Researchers employed pre and post test which included "performance assessment, open-ended questions, and multiple choice items" and analyzed data by using *t*-tests. The results showed significant improvement after intervention. Even though a majority of students performed below 50 percentile on pre-test, twenty-six participants increased to above 75 percentile on post-test. Furthermore, the intervention improved students' on-task behavior. However, several questions remained or were unanswered such as questioning the effectiveness of utilizing the intervention in traditional general education classroom that might include students with learning disabilities. Baker, Young and Martin (1990) also conducted a study to investigate the effects of direct instruction on fraction and spelling skills. Unlike the study of Flores and Kaylor (2007), they had small number of participants (n=6) and it took place in remedial setting. They compared the effectiveness of two type instructions: small group versus one to one. Even though all factors were same including sequence of instruction and materials, students in the direct instruction group spent more time achieve

mastery level in both programs: fraction and spelling programs. Results showed significant improvement in fraction and spelling skills of all students. All students reached the mastery level in fraction program, and 4 out of 6 students reached master level in spelling program. "Student 5" in one to one instruction and "Student 6" in direct instruction scored 72%. On task behavior for both groups of students were noted as a high but not different from each other's. Gersten and Kelly (1992) used another form of direct instruction by employing coaching with videodisc instruction. Four special education teachers delivered fraction content and they were observed in term of several criterions; such as providing informational feedback, inappropriate feedback (i.e., only saying you are wrong), and whether using praise. After completing observations, researchers interviewed with each teacher (n=4). In this session, they responded 17 semistructured questions. Three of the questions were related coaching method, and teachers stated their views about most and least beneficial parts of the process. Other questions included teachers' thoughts on videodisc and fraction curriculum. Results of the study showed that students in these teachers' classroom increased their scores on criterion-referenced test from pretest to post-test by 51.5 percent. Furthermore, researchers highlighted the importance of conceptual understanding of procedures while calculating.

Explicit instruction. The type of instruction incorporates detailed explanation, modeling of problem solving, guided practice, and providing feedback. Fuchs et al. (2013) conducted the research to determine the effects of Fraction Challenge intervention developed by Fuchs and Schumacher (2010) to increase the understanding of fraction concepts of students who are in the at-risk category for math. In terms of true experimental research design, the authors compared intervention and control groups. Main differences between control and intervention groups were that comparison group received instruction relaying on procedures and part whole relation, even

though intervention group received instruction that demanded less computation. Furthermore, in the instruction of comparison group, number line had less importance. The researchers stated " the ES favoring intervention over control children was 0.92 *SD*s, and the achievement gap for control students remained large (1.09 at pretest; 0.96 at post), while the gap for intervention students decreased substantially (from 1.07 to 0.08)" (p. 696).

Summary of findings. Fractions are known as one of the most difficult content area in mathematics to understand and to be proficient. Researchers conducted several studies considering various interventions in terms of the needs of students with MLD. Common interventions included graduate sequences (CRA), anchored instruction, strategy instruction, direct instruction, and explicit instruction. The results of these studies showed that students with disabilities and at-risk benefited from the interventions, with the exception of anchored instruction, to varying degrees. Studies utilizing anchored instruction had different results. For instance, Bottge (1999) found a small positive improvement on students' academic skills, but Bottge et al. (2002) reported a negative effect size. Interestingly, in a majority of the studies reviewed the authors did not employ number line concept to increase conceptual knowledge of fractions for students with MLD and at risk (Baker, Young, & Martin, 1990; Bottge, 1999; Bottge et al., 2002; Butler et al., 2003; Flores, Kaylor, 2007; Jordan et al., 1999; Test, & Ellis, 2005). Since the study of Butler et al., (2003) took place at the end of the semester, researchers had no chance to look at maintenance effects of the intervention. Furthermore, they included students with variety disabilities categories (i.e. EBD, MMR, ADHD); therefore, it leads us to be cautious about generalizability of the studies.

Technology

Mathematical skill is critical to be a competitive citizen for economic success and quality of life (Seo, & Bryant, 2009). However, individuals with MLD struggle to be a competitive citizen. Even though they need more time and special services in terms of their needs, the current trend in education is inclusion (Misqutta, 2011), and students do not always benefit in inclusive classrooms from instruction (Seo & Bryant, 2009). Students with MLD need more time to process when teachers introduce new concepts and they need differentiated practices compared to their peers who are not struggling in mathematics. Nevertheless, teachers state lack of time although they want to provide instruction based on the students' needs. When teachers lack knowledge, problems increase (Darling-Hammond, 2010). Because of problems including quality teachers, lack of time, and resources, researchers, such as Ross and Bruce (2009), support use of technology which "could provide the sequencing and scaffolding that teachers might have difficulty providing" (p. 713). Technology also provides real learning opportunities for people to learn mathematics (Allsopp, Kyger, & Lovin, 2007; National Council of Teachers of Mathematics, 2008).

Mobile Learning

Franklin (2011) defined mobile learning (M-Learning) as "learning that happens anywhere, anytime" on any devices (p.261). With M-Learning, people can reach the content faster and efficiently. M-Learning does not require people to be any specific location for the learning process; it brings the content to people where they are.

Students participate in learning activities, such as drill and practices (most of the applications for mobile devices have been created for these activities) in education field out of classroom by using the important accessibility and portability features of mobile devices (Cakir,

2011). M-Learning also supports group work, increases the opportunity of communication and cooperative learning by improving students' motivation to engage with learning activities in classrooms.

Mobile devices such as phones, smartphones, mp3, mp4 players, iPods, netbooks, laptops, tablets, iPads, and e-readers have become very popular for different users all over the worlds (El-Hussein & Cronje, 2010; Franklin, 2011; Kalinic, Arsovki, Stefanovic, Arsovski, & Rankovic, 2011). The younger population is known as digital natives since these devices are commonly used among them, specifically the devices especially common among students at universities (Cheon, Lee, Crooks, & Song, 2012; Kalinic et al., 2011; Park, Nam, & Cha, 2012). Therefore, this common usage of mobile devices changed learning pattern and activities, and the idea of learning by using these devices became a trend in many fields (Jeng, Wu, Huang, Tan, & Yang, 2010).

Applications on mobile devices help all learners from different ages, levels, and even abilities. For instance, note taking, agenda, and typing applications; Dragon Dictation, are accessible for all learners to increase their productivity. Furthermore, many other apps support students learning in content areas. For instance, mobile devices increase students' academic achievement including mathematics (Cumming, Draper Rodrigues, 2013; Farmer, 2013), increase on task behavior of primary grade students having Emotional Behavior Disorders (EBD) during independent academic activities (Flower, 2014), support in development of communication skills for second language learners (Demski, 2011), and offer modeling for students with Autism Spectrum Disorder (Burton, Anderson, Prater, & Dyches, 2013; Hammond, Whatley, Ayres, & Gast, 2010). Mobile learning provides opportunities for learners to build their own knowledge in different contexts, and help learners construct their own understanding.

There are around 560,000 apps that were created by almost 100,000 different publishers, and each day, 775 new education apps were developed and made available to download through the Store (Walker, 2011). People access those apps from variety sources, including Apple Store. The data show that "15 billion apps have been downloaded from the Apple Store in the past three years" (Walker, 2011. p.1), underscoring the growing importance of this technology. Recent statistic showed that total download apps from 2008 to 2015 is 100 billion apps from only Apple Store at the end of June (Statista, 2015).

Apple sold approximately 300,000 iPads in the first day, April 3, 2010, which was released, and at the end of the first year 14.8 million units of iPads were sold (Harvey, 2010; Walling, 2014), and the number of sold iPads continues to skyrocket. A variety features lead people to buy the device. One important feature from the point of view of a researcher in the field of special education is to provide opportunities for students who are struggling to access content in a variety of ways (Misur, 2012). However, the integration of those devices into education settings is not easy because of a variety of reasons, such as cost, and distractibility features (Brown, Ley, Evett, & Standen, 2011). At the beginning, people tend to resist new technology due to lack of understanding.

Game-based apps. Balci (2015) identified educational games as "a game created for the purpose of teaching a subject in the form of software that runs on a computer such as desktop, laptop, handheld, or game console" (p.1). Game-based software (apps) on mobile devices is popular since they increase students' engagement regarding their motivations (Franklin, 2011; Hill, 2011). Many of these game based apps were developed for different purposes, but the main goal was to increase engagement of students and increase the time students were exposed to content matter. However, the number of studies examined the effectiveness of applications on

mobile devices to deliver elementary mathematics instruction to improve academic achievements is few even though many studies indicated positive correlation between engagement and academic success in mathematics (NCTM, 2008). In the long run, by increasing their work performance on academic tasks, it is possible to decrease the achievement gap between students with and without disabilities by using the apps on mobile devices in education settings (Rosen, & Beck-Hill, 2012). By downloading game-based educational apps, mobile devices can be easily customized to support individuals' special learning needs. Since these apps provide fun activities, students on task behavior was increase, and it helped students to learn difficult content such as fraction (Brown et al., 2011).

Due to a variety of reasons, the market for iPad and use of them in education settings has skyrocketed (Hill, 2011; Price, 2011). iPads are user friendly, less than to textbooks in weight, can be easily updated versus text which become obsolete, and can connect to the internet faster than many other devices. Regarding apps on iPads, they offer fun activity for educational contents besides delivering instruction (Carr, 2012). Teachers meaningfully introduce mathematics instruction to students by using game based apps on mobile devices and this probably increases outcomes. For this assumption, apps have been created to deliver instructions for any content matters should be tested.

Murray and Olcese (2011) investigated apps on iPad regarding whether students and teachers do things with or without it in regular education settings. The researchers reported that a small number of apps on iPad support students and teachers for meaningful learning and teaching methods. Nevertheless, many of these apps were created not taking into consideration any modern learning theories. Therefore, choosing appropriate apps designed to meet pedagogical needs of students is critical.

Using real-world examples by interactive games was suggested since it is natural that students in elementary schools like to play academic games in mathematics (Griffin, 2007); therefore, game-based learning gain popularity among teachers in regards to teaching mathematics instruction. Use of mathematics games promise benefits for students due to games increase engagements and motivations of students (Carr, 2012). Taking into consideration the feature of games, and the students with MLD, these games, such as *Motion Math: Fraction*, might help them to overcome math anxiety by increasing their motivation for trying to solve problem again and again when they are not successful. Because, while playing a game, "losing is not losing", and "hard is not bad and easy is not good" (Turkay, Hoffman, Kinzer, Chantes, & Vicari, 2014, p. 9). Since students have this notion, they never lose their motivation to play. Playing interactive games increase the excitement and interest of students about learning mathematics (Griffin, 2007). Besides that, gaming in mathematics provides multiple opportunities for students, such as providing corrective feedback (Allsopp et al., 2007). If the apps provide corrective feedback, students may learn from their errors, and this is the most important form of learning.

Granted that mathematics knowledge consists of two type of knowledge; conceptual and procedural. However, in the field of special education, procedural knowledge, getting the correct answer, is highlighted rather wondering how students reach the answer, conceptual understanding (Allsopp, et al., 2007). Therefore, game-based apps facilitate problem solving skills of students with MLD and conceptual understanding of the targeted content in the app (Carr, 2012).

Even though thousands of apps are in the market, interestingly, the number of educational based apps is not as extensive as many other categories (Walker, 2011). Besides that, teachers

have difficulty identifying the appropriate apps for specific students' needs, although Yerushalmy and Botzer (2011) stated "we consider mobile learning to be an important aspect of future changes in the curriculum and in the nature of classroom" (p.192). Therefore, more studies are needed to determine the effectiveness of educational apps to increase students' academic achievement, and to inform teachers about the use of apps to deliver specific contents. There is an apparent gap in the literature. Riconscente (2013) stated, "although hundreds of iPad apps on the market claim to improve learning, no published studies were found of controlled experiments that tested the effectiveness of an educational iPad app for increasing learning outcomes" (p. 187). While looking at specific content areas, such as fractions, few studies was conducted considering the effectiveness of game based educational apps.

Bearing in mind teachers' claim about lack of time to prepare materials for students who needs differentiated instruction, apps can be critical for teachers and students. Increasing the amount of exposure to mathematics instructions using game-base apps might escalate the likelihood of students' benefits. For instance, when students used *Motion Math: Fraction* (one of the apps to teach the concepts of fraction to students who are from grade 3-5) out of the classroom, it may increase their exposure to mathematics skills, specifically fraction skills.

Another important point of this study is that teachers should be aware of the opportunities provided by technologies. For instance, since one of the reasons of learning problem in fractions was stated as in attentive behavior (Brown et al., 2011; Siegler, 2011), game-based apps that increase students' engagement improve the possibility of students' success in the content area of fraction. Furthermore, these game based apps might be use as virtual manipulatives (Carr, 2012; Riconscente, 2013). Virtual manipulatives have advantages considering the weight of concrete manipulative generally used in CRA strategy, it is hard to organize them, and when one piece of

a combination is lost, you cannot use the set anymore. However, virtual manipulatives have many advantages, such as easy to organize, never lose, and there is no weight problem while carrying. Besides these advantages, Mendiburo, and Hasselbring (2014) virtual manipulatives were effective as much as concrete manipulatives during instruction delivery.

Game-based apps (interventions) on mobile devices for students with MLD. Regarding the effective apps for students with MLD to learn fraction skills, the researcher conducted an analysis. In this analysis, studies focusing on game based apps to teach fraction skills to students from diverse groups were included. Inclusion criteria were: studies published in journals between 2010- 2015 since iPad was launched in 2010 (Falloon, 2013); and these studies are empirical in nature considering the effectiveness of an intervention to improve fraction skills (i.e., identifying and representing fractions, comparing fractions considering their magnitude, adding, subtracting, multiplying and dividing fractions). The following databases were included: ERIC EBSCO, Education Full text, PYCHOINFO, and JSTOR. Key words utilized included *disability, struggling, difficulty, at-risk, fraction, elementary, middle school, mathematics, arithmetic, number sense, mobile devices, iPad, hand-held devices, smart phones, and apps entered in different combinations. Additionally, the references of published meta-analyses were examined for studies meeting the inclusion criteria.*

A total of seven studies met inclusion criteria. See table below.

Study	Participants	Grade	Design	Setting	Dependent Variable	Race	Result
Brown, Ley, Evett, & Standen, (2011)	16 ID	2 to 5	Experimental; treatment and control group	Nottingham, UK	Fraction, decimal, and percentage	NR	Students improved their scores, but there is no significant difference between groups.
Bryant, Ok, Kang, Kim, Lang, Bryant, and Pfannestiel (2015)	6LD	4	An alternating treatments design (Single Case)	Texas	Multiplication facts described as prerequisite for rational numbers including fractions	4 Hispanic, 2 mixed race	The results of study showed there is no difference or minimal difference, and no intervention was better than the others.
Carr, (2012)	104	5	Quasi- experimental	Virginia	5 th grade math contents, including fraction	NR	Both groups improved their score, and result of the study showed that there is no significant difference and no evidence to reject the null hypothesis.

Table 2.3. Studies of Game Based Apps for Students with disabilities

Study	Participants	Grade	Design	Setting	Dependent Variable	Race	Result
Farmer, 2013	44	6	Quasi- Experimental	Gainesville, Georgia	Fraction	NR	Experiential group performed better than control group, but not significant difference was found
Nordness, Haverkost, & Volberding, 2011	3, 2LD, 1 EBD	2	Single- subject design	Midwest, Nebraska	Subtraction determined as important for further academic skills; fraction	NR	Jacob improved his score from 33% to 90%, Sarah improved her score from 16% to 71%, and John improved his score from 11% to 75% on the test
Kiger, Herro, & Prunty, 2012	87, 14% disabilities	3	Experimental and control group	Midwestern	Multiplication test	92 % white	Intervention group performed better than control group
Riconscente (2013)	122	5	Experimental; repeated measures crossover design	Southern California	Fraction	Latino, Caucasian	Significant improvement seen (p=.01)

Table 2.3. Continued

Note: LD: Learning disability, ID: Intellectual Disability, ED: Emotional Disturbance, NR: Not Reported

Since the researcher could not find common themes among the articles shown in the above chart, the researcher provided detailed information about individual articles.

Brown, Ley, Evett, and Standen (2011) investigated the effects of game based learning on mathematical skills, specifically fraction skills, of students with intellectual disability (ID). In an experimental study, they compared treatment and control groups consisting of 16 students with

ID to evaluate the effectiveness of the intervention. By employing math pair design, students were randomly assigned in two groups. Eight students played the intervention game that teaches fractions, and the others played the control game. Cheese Factory, game, allows the users to work at their own pace, and also adapt difficulty levels in terms of the students' abilities. Students' performances, before and after intervention, were recorded regarding the changes in understanding fraction concepts. Since there was high variability within the group of students, researchers also conducted qualitative analysis. Results of this study showed favor of the intervention group, while, the control group did not make notable improvement except for one student in the group. However, researchers underlined the distractive aspects of the game.

Bryant, Ok, Kang, Kim, Lang, Bryant, and Pfannestiel (2015) compared three type of instructions which were app-based instruction (AI), teacher-directed instruction (TDI), and combination of instructional approaches (CI) which was a combination of AI and TDI to teach multiplication facts described as prerequisite for rational numbers including fractions to six students identified as having learning disabilities. Math Drills and Math Evolve, iPad applications were used in this study. Math Drills provided an opportunity to drill and practice activities and students monitored their progress. Math Drills had two modes: in one, review mode, students were able to review the content, such as cues about blocks, number lines, and in practice mode which allowed students to change the types of questions, colors, etc. The Math Evolve app allowed students to change operator, and the level of difficulty. Participants used the apps during consecutive three-weeks period Monday - Friday. The results of study showed there is no difference or minimal difference between the instructional approaches, and no intervention was better than the others.

By employing a quasi-experimental study, Carr (2012) aimed to determine the impact of iPad devices on mathematic achievement of students who were in the fifth grade. Students participated in several activities in this study, such as "playing game-based learning applications, reviewing presentations, accessing online video tutorials, or using interactive manipulative" (p.270). Other participants in the control group did not use an iPad. Utilizing the district's benchmarks for fifth grade mathematics, students were taught math content including fractions. Difference of pre and posttest mean scores were 6.67% for the comparison group. The difference for the experimental group was 6.74%. Both groups improved their score, but the result of the study showed that there was no significant difference and no evidence to reject the null hypothesis. Providing, and supporting instruction with the iPad did not show more benefits than instruction delivered without the iPad for fifth grade students to increase their academic achievement of mathematics.

Kiger, Herro, and Prunty (2012) determined the effects of Mobile Learning Intervention to teach multiplication skills to third grade students in a Mid-Western elementary school. They included four classes, two of them were Mobile Learning Intervention classes and the other two classes were comparison, consisting of 87 students in total. Around 14% of the students had disabilities, 20% were economically disadvantaged, and 90% were Caucasian. Students were matched by their gender, race, economic status, disability, and performance. In Mobile Learning Intervention classes, students used iPod Touch devices in order to exercise multiplication. Each day, one or two math apps were introduced to these students, and they practiced for 10 minutes. In total, there were ten apps utilized; "Multiplication Genius Lite, Mad Math Lite, Pop Math, Flash To Pass, Math Drills Lite, Math Tappers: Multiples, Multiplication Flashcards To Go, Brain Thaw, Math Magic, and FlowMath" (p. 68). Some of these apps, for instance, Math Drills

Lite, were chosen to construct the background of further academic skills, such as fractions and algebra. There was not specific order to choose any apps for daily practice. On the other hand, in comparison classrooms, student did multiplication practices by using "business as usual" which is a kind of technique that incorporates flash cards, and fact triangles. Students in experimental group were also allowed to use websites to practice at home and sometimes play games in the lab; they increased the time students' exposure the content matter. Participants in both groups had almost the same technological environment. The result of the research showed that students who received the intervention made more correct answers than the comparison group. However, there were many variables that contribute to the successful implementation of mobile learning interventions, such as pedagogy of teachers, the attitudes of administrators, school facilities, and time spent to practice, and none of these variables were controlled.

Nordness, Haverkost, and Volberding (2011) examined the use of flashcard applications on iPods to increase one of the basic skills, two- digit subtraction, which is essential for higherlevel math skills such as fractions. Multiple baseline design was employed in this study. All participants were identified as needing special education services. The reason for choosing these students was that they performed significantly lower than their peers without disabilities in the subtraction portion of the district and curriculum based test. Researchers measured "correctly answered subtraction problems on the Nebraska Abilities Math Test" as the dependent variable for this study (p. 17). Math Magic, a software application, was the independent variable. Researchers programmed the app to solve two digit problems in ten minutes. Students completed the exercises three times a week. The results showed that students improved their scores on N-ABLES by using the Math Magic app while practicing two digit numbers from 0-20 for a ten minutes time frame three times a week. Their first participant Jacob, improved his score from

33% to 90%, Sarah improved her score from 16% to 71%, and John improved his score from 11% to 75% on the test.

Riconscente (2013) examined the effectiveness of Motion Math: Fraction to improve the fraction skills of students without disabilities, as well as their attitudes towards mathematics. The researcher conducted the study in a school setting to control extraneous variables, such as length of playing time and frequency. From low income mostly Latino and Caucasian families, 122 fifth grade students participated in this study, but due to incomplete data, the researcher dropped 20 students during analyses. Adapted items were used to measure the dependent variable. The researcher utilized "repeated measures crossover design", and a group of students was randomly assigned and received intervention for the first week. In the second week, the control group received the intervention. Students were tested before intervention, at midpoint and after intervention. There was no difference in the pretest between control and intervention groups (p =.415). The result of an independent *t*-test showed significant differences at mid-test in favor of group one that received the intervention first (p = 01). The result at the end of the intervention was that, both groups' performance was close to each other, and there was no significant difference (p = .559). Gaining a positive attitude towards playing the game was connected with the time they played; when a group was in the control condition, students' attitudes did not change, but after playing the game, significant changes in positive way were observed for both groups.

Farmer (2013) tested the hypothesis that "Math achievement will be significantly higher for students exposed to iPad "Motion Math" (MM) instruction compared to students who receive traditional math instruction" (p. 21). The researcher compared two groups in terms of quasiexperimental research design. The control group received instruction in traditional instruction

and practiced by using worksheets. Even though the intervention group was taught in the same way, they practiced by playing the game on iPads. The result of the study showed that "the experimental groups' average increase was 3.61 whereas the control groups' average increase was only 1.11" (p. 27), but the improvement was not significantly different from the control group.

Summary of findings. The researcher included all studies conducted regarding the effectiveness of game-based apps to teach fraction skills and others skills, such as multiplications and divisions, which were stated as prerequisite for fraction skills above section. Three of the studies (Brown et al., 2011; Bryant, et al., 2015; Nordness, et al., 2011) were specifically devoted to students with disabilities. In two studies (Brown et al., 2011; Nordness et al., 2011), students improved their academic skills significantly and they developed positive attitudes towards mathematics after engaging with the game based apps. However, participation selection in these studies were problematic, selection procedures were not clearly explained; therefore, people should be cautious about the results of these studies.

In the study of Kiger et al., (2012), 14 percent of the participants had disabilities, but there was no information about the type of disability and how they chose this participants. Other three studies (Carr, 2012; Farmer, 2013; Riconscente, 2013) did not included students with disabilities; therefore the impact of the application on students with disabilities was not evident.

Interestingly game based apps sometimes distracted students in the classroom (Brown, et al., 2011). While designing an instruction via game based apps, this aspect should be kept in mind. Another weakness of the studies reviewed was that controlling extraneous variables was not considered (Carr, 2012; Kiger, et al., 2012). Carr's study showed that more than one variable may effect fractions skills; therefore, it was hard to determine whether there was an effect of

playing the app on fraction skills of the students. Similarly, Kiger and associates (2012) allowed students to continue what they were doing in the classroom when they were at home, but there was no information on how much time each student or group spent on apps. This further proved that the available literature must have considered all aspects affecting game-based learning as all the environments have only been partially controlled. Therefore, conclusions about use of app are inconclusive by analyzing currently available.

CHAPTER THREE: METHOD

"We know that some methods of inquiry are better than others in just the same way in which we know that some methods of surgery, farming, road-making, navigating, or what-not are better than others. It does not follow in any of these cases that the "better" methods are ideally perfect...we ascertain how and why certain means and agencies have provided warrantably assertable conclusions, while others have not and cannot do so" (Phillips & Burbules, 2000, p. 4).

The purpose of this research is to test the following directional hypotheses;

- Participants (students with MLD) will increase their fraction skills by playing the Motion Math: Fraction app 20 minutes daily for two weeks.
- Participants will maintain the level of fraction skills they while playing the Motion Math: Fraction app 20 minutes daily for two weeks after no longer playing the app,
- 3. Participants will achieve greater gains in fraction skills with greater amounts of time interacting with the *Motion Math: Fraction* app.

Before providing deep details about the research hypotheses, the philosophical stance underlying the research, post-positivism, is described. Then, information about the design of the research, variables (dependent, independent variable), participants, data collection procedure, and data analysis process is discussed.

Philosophical Stance of the Study

In this research, the researcher has several hypotheses that need to be tested, and the postpositivism approach is appropriate for the research design (Phillips, & Burbules, 2000). A postpositivist lens suggests that absolute truth cannot be obtained but that truth can be approximated, where findings are probably true, and observations are imperfect. Since findings are an approximate truth, though not absolutely secure, hypotheses based on current evidence can be put forth and made available for public scrutiny.

It is assumed that there is bias in research, but this bias may be minimized using rigorous methods that include standardization of research procedures and treatment fidelity checks. Furthermore, a rigorous and detailed explanation is critical for later replication of research studies that can lead to generalizability, particularly single case design (Kratochwill, Hitchcock, Horner, Levin, Odom, Rindskopf, & Shadish, 2010).

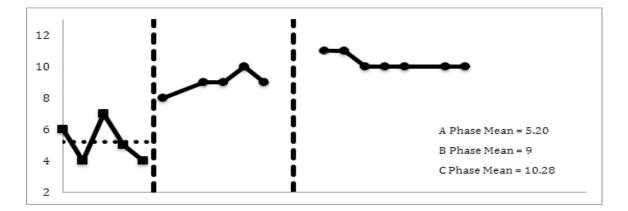
Toll (2012) states that "quantitative methods are logically consistent with post-positivist epistemology, and moreover when appropriate the ability to formulate empirical hypotheses with statistically tuned predictions allows for a more faithful application of the principle of falsification" (p. 1). However, as Phillips and Burbules (2000) contend "accepting this pursuit of knowledge does not necessitate a commitment to a claim of 'absolute truth' or its attainability" (p. 3). There is an independent reality that exists and that it can be known, although our knowledge of this reality is imperfect.

Observation is central in the design of this study and these observations help evaluate the hypothesis. However, because of inherent error in observation, multiple sources of data must be collected in order to increase the validity of the findings, and single case design allows for doing this. Theories and personal orientations guide observations; therefore, having "pure objective"

observation should not be expected. To improve objectivity based on theory of post-positivism in the process of data collection, an outsider (a doctoral student) scored the daily participant response sheets/probes.

Design

Cakiroglu (2012) identified single case design as "a scientific research methodology that is used to investigate a functional relationship between a dependent and an independent variable" (p. 21). Because the main purpose of this study is to determine whether the *Motion Math: Fraction* app is effective to teach fraction skills to students having difficulty with fraction skills, a quantitative, single case design is appropriate for this purpose (Kratochwill et al., 2010; Horner, & Spaulding, 2010). Considering the nature of the disability category and students' needs in fractions skills, single case design is a commonly used methodology to study the effects of interventions on academic and behavioral outcomes of individuals with disabilities (Kratochwill et al., 2010). The specific type of single case design, which was employed in this study, is a multiple baseline AB type design with a maintenance (follow-up) phase. Ferron and Scot (2005) identified multiple baseline design as an extension of simple case design. In this design, before introducing any intervention, researchers are required to measure interested behaviors or skills. And then, after obtaining a certain amount of stable data in baseline, researchers employ the intervention and repeatedly measure the interested behavior.



Graph 3.1. Multiple Baseline Design: AB type extended with Follow-up

The graphic shows the logic of the multiple baseline AB type design. According to the graphic, while the first participant receives an intervention and notable changes in his/her behavior are evident, the second participant is in baseline, not exposed to the intervention, and there is no important change in his/her behavior. In this way, researchers control the effects of history and maturation, allowing for greater confidence that any changes in the behavior of interest are due to the intervention (Ferron & Scot, 2005). This increases the internal validity of a study. Furthermore, the design provides more that three phase repetitions (i.e., instances of experimental effect) which reduces the threat to internal validity (Horner, et al., 2005). Having more than three phase repetitions within single case design is an important criterion for meeting the standards of a scientific study determined by What Works Clearinghouse (Kratochwill, et al., 2010).

Even though the single case research design is known to have problems with generalizability (Ferron & Scot, 2005), this limitation can be overcome with replication by other researchers. To facilitate replication, researchers need to provide explicit information about their design and procedures to allow other researchers to replicate the study.

Participants. The study took place at a public charter school in the South East of United States. To recruit participants, initially, the researcher asked the teacher working in an after

school math classroom for her referral to determine appropriate students for the study, and then the researcher requested consent from the families with middle school children in the after school program at the school. The number of students in the program varied from day to day, typically the number varied from 10 to 15 students. The families of six children provided signed consent for their child to participate in the study. Students in this study were from different grades levels: Ezeli, Jamie, and Alan were from 6th grade, Monica and Katie were from 7th grade, and Cambiasso was from 8th grade. After getting the consent form from families, and assent form from the students, data collection began on October 5, 2015. Two of the six original participants did not complete the study. After four sessions, Jamie said he did not want to play anymore. He kept coming to the class, but only sat at a corner in the room and did not play after that day. Therefore, Jamie was removed from the study. The other participant who did not complete the study, Katie, said she was no longer able to come for after school math program since she was required to attend another program that took place at the same time. Katie was also removed from the study. Final data analysis was conducted for the four remaining students in this study. Three of them were male, and one was female. Two of the participants were African-American, and the other two were Hispanic.

Monica was, 13 years old, in the 7th grade. She has been receiving special education services under the category of the Specific Learning Disabilities. Her last Northwest Evaluation Association standardized assessments results in mathematics showed that she earned an overall score of 189 in math (the mean for 6th graders at this time of the year was 223). Her score was in the 2nd percentile of same grade peers (when she was in 6th grade). She scored in the Low range for Operations and Algebraic Thinking, Geometry, The Real and Complex Number Systems, and Statistics and Probability. Based on Adaptive Diagnostic Assessment of Mathematics K-7, her overall grade level performance was at mid-third grade level. Her last test results showed that she improved her skills, which includes patterns within the operations, problem solving, ordered pairs, and integrating graphs from the level of 3.50 to 3.75, however, she was still well behind her peers 13 years of age, in the 7th grade.

Cambiasso was, 14 years old, in the 8th grade. He has been receiving services under the category of Autism Spectrum Disorder and Language Impairments. Although Cambiasso is reported to have received direct and specialized instruction based on his needs, he has not been able to consistently demonstrate understanding of any of his IEP mathematics objectives, which include "solve one-step problems involving unit rates associated with rations of fraction" and "find percentages in real-world contexts." According to his classroom teacher, Cambiasso has been working hard and makes honest attempts to successfully complete related assignments and tasks, but he has great difficulty even in basic mathematics concepts. He is well behind grade level mathematics expectations and requires high levels of remediation. Mathematics tasks that require more than one step are a particular area of difficulty, specifically word problems.

Ezeli was, 13 years old, in the 6th grade. He has receiving services under the category of Other Health Impairment including ADHD. Even though there is no information in his cumulative file about his performance on any standard tests, it records to indicate the following 4th grade level mathematics goals: recalling basic multiplication facts, solving multi-digit addition, subtraction and multiplication problems, which are critical areas for success with fractions. Although his participation and his focus in the math classroom have improved, he still needs prompting while following daily classroom activities.

Alan was, 13 years old, in the 6th grade. He has receiving services under the category of Specific Learning Disabilities, including support in reading, writing, math and social skills/work

habits. He also does see a therapist and is on medication for ADHD. He struggles with attention when learning and doing mathematics, which often hinders him developing understanding of new math concepts. He often makes mistakes with mathematics that he has previously mastered. He shows inconsistency in computation problems when working independently. He enjoys math, but will often rush through his work when he thinks he understands. However, even when mistakes are pointed out to him by his teacher, he refuses to make the changes. Then, he becomes upset and gets frustrated.

Independent variable (Motion Math: Fraction). Recently, researchers have been investigating the effects of apps on mobile devices to improve students' academic skills as well as behavior skills (Ciampa, & Gallagher, 2013). Specifically, there is growing interest among researchers who are interested in learning how mobile devices can address learning challenges of students by increasing physical interactions with games on mobile devices. For instance, the theory behind the development of Motion Math: Fraction was that "cognitive processes are deeply rooted in the body's interactions with the world" (Riconscente, 2013, p. 189), and that knowledge is gained though bodily relations with the app. One of the biggest advantages of Motion Math: Fraction is that the app can increase students' motivation that can maintain their attention helping them to process information more easily and meaningfully (Riconscente, 2013). When students fail to find the correct answer, the app motivates them by providing students with cues to help them answer correctly reducing the likelihood that students will get frustrated and anxious. This feature is very important for students with MLD because when they face any challenges in any academic content or give wrong answer to directed question, they often quit trying, engaging in learned helplessness and developing math anxiety (Allsopp, Kyger, & Lovin, 2007).

Motion Math: Fraction was developed at the Stanford School of Education in order to improve students' understanding of fractions, decimals, and percentages by using the number line in a game context. The game-based *Motion Math: Fraction* app is available for iPad, iPod, and iPhone. The app is described as an award winning fraction game. In this game, a star falls from the sky (depicted in figure 3.1.) and the goal for players is to carry it back to the sky. They can only do this by placing the fraction on the correct point on a number line. When students do not place a star at the correct point, the app provides several scaffolded clues to help the student determine the correct placement of fraction on the number line. The first clue includes arrows showing which side (left or right) star should be placed. If the student is still not able to the place star to the correct point, the next clue that is provided includes showing hash lines that divide the number line in equal parts. Similar fractions are also used as hints to helps students to compare fractions. The final clue actually provides rational numbers around the point students were expected to place star on the number line (Shown in figure 3.2.).



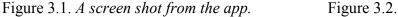


Figure 3.2. *Explicit clue*.

Motion Math: Fraction offers different levels of difficulty to its audience: beginner, medium, and expert. The app also provides additional challenges within each level. Besides the changing of difficulty, images used in each level are differentiated.

The constant feedback while physically interacting with the game is an important feature of *Motion Math: Fraction*. It provides reinforcement, such as verbal reinforcement; "PERFECT" which encourages students to play more.

In several studies, the use of the number line was stressed to increase conceptual knowledge of fraction (NMAP, 2008). While developing the app, the use of number lines was considered a central feature of the app. The app manipulates language and visual systems (i.e.,

number lines) in variety formats to facilitate conceptual understanding of fractions. In doing so, the app helps students to manipulate and process the information.

Siegler et al., (2013) stated, "fraction knowledge is associated with working memory, attention, and IQ" (p.16). Since *Motion Math: Fraction* requires students' bodily engagement where the user tilts the iPads to move the ball right or left. Such bodily movement has potential to positively affect students' attention to the game and working with fractions.

Furthermore, the app aligns with the Common Core State Standards (CCSS) since it address the following knowledge/skills: (a) master estimation of fraction, percent, decimal, and pie chart; (b) locate the many representations of fractions on a given number line; and (c) build automaticity in comparing fractions and therefore can support core instruction (CCSS, 2015). The app can also support core instruction by providing students with practice opportunities during the school day and after. *Motion Math: Fraction* appropriate for students from grades 3 to 5 considering addressed skills and grades levels (Motion Math, 2015). When we think about teachers' statements about time concerns in inclusive educational settings, the importance of this type of app might be understood because it provides students with opportunities to practice in and out of school thereby making it possible for students to engage in more response opportunities, increasing their opportunities to develop proficiency and maintaining their proficiency.

The Apple iTunes Preview page includes descriptive information about the "*Motion Math: Fraction*" app, which includes its category (education), when it was updated (Jan 14, 2014), version (1.4), and size (23.0 MB). Customer rating is a four out of five star based on the review on Apple Store.

In this research, "*Motion Math: Fraction*" app is the independent variable and it was systematically manipulated during the intervention period. To meet the standard determined by What Works Clearinghouse (WWC), I used a multiple baseline design (extended AB type by providing follow-up phase), which provide more than three different phase repetitions. Furthermore, each individual has different amounts of data points differentiate in each phases, and even from person to person to demonstrate an effect in each phase.

In various format, 65 items were used while collecting data (Items are in Appendix-1). The researchers took the items from different resources, such as released items by National Assessment of Educational Progress, and two prominent articles in the field (Fuchs et al. 2013; Siegler et al., 2011).

Analysis of the app's quality. An evaluation rubric for iPod/iPad Apps was created by Walker (2010), and revised by Schrock (2011) (See Appendix-3). The rubric can be used to evaluate apps according to several categories including curriculum connection, feedback, authenticity, differentiation, user friendliness, student motivation, and data reporting. This rubric was utilized to evaluate the quality of the *Motion Math: Fraction* app using three external reviewers. Three doctoral students each evaluated the quality of the app using the rubric. Two of the reviewers were male, and the other was female. One of them is in the instructional technology doctoral program and working in a National Science Foundation project to develop different types of games, and the two others are completing their cognates in instructional technology.

For the first domain on the rubric, curriculum connection, to determine the quality of *Motion Math: Fraction*, 2 out of 3 external reviewers stated that fraction skills are strongly reinforced in the app, and one stated that the targeted skill (fractions) is reinforced.

For the second domain (feedback), all of the doctoral students said that the app includes specific feedback and believed the feedback could be of help to students to improve their performances. Two important features of effective instructional games for students with disabilities is that they focus on the concept/skill in need of development by students and that they provide immediate and constant feedback is critical for students with disabilities (Allsopp, Kyger, & Lovin, 2007; Hattie, & Timperley, 2007).

For the third domain (authenticity), 2 external reviewers rated that the app as presenting fraction skills in an authentic format, but the third external reviewer rated the app as providing practice opportunities for fraction skills in a contrived game.

For the fourth domain (differentiation), only one external reviewer believed the app offers full flexibility. He also stated that when he looked at the sequence of questions, the sequence was changed based on the students' performances. For instance, if students had problems placing 1/3 on the number line, more questions are presented related to the same fraction until students reach mastery for that type of question. Therefore, the app is designed to differentiate questions based on individual responses. Developers of the app highlight the feature of it (Adauto, &Klein, 2010). However, the two other external reviewers said that it offers limited flexibility with respect to difficulty level (e.g., less difficulty, difficult, and more difficult). They said the app should have provided the opportunity to move back and forth within levels and change the speed limit in terms of the students' pace.

For the fifth domain (user friendliness), two reviewers said that the app can be used independently without any help from a teacher, adult, or peer, and that students would be able to easily navigate the app. However, one reviewer believed that students might need a teacher's help to learn how to use the app.

For the sixth domain (student motivation), all external reviewers believed that students would be motivated to use the app when prompted by a teacher.

For the seventh domain (reporting), all external reviewers rated the app as having reporting capabilities, providing electronic data to teachers and students related to performance.

The overall mean rating by external reviewers was 3.4 on a 4-point scale.

Social validity. The researcher employed the modified version of Instructional Materials Motivation Survey (IMMS) to measure the social validity of the *Motions Math: Fraction* app to evaluate the students' motivation and their thought about the intervention (Keller, 2009; See the modified version in Appendix-7). The reason of modification was that some of the statements were not measure what the researcher needed to determine students' motivation on the instructional material used in the study; therefore he modified majority of the statements and deleted some of them as well. This modification was mostly on wording since the researchers used game-based app instead of paper pencil type of instructional materials. Keller (2009) organized the survey into four categories including, attention of students, relevance of the material to students' interests, confidence level of students, and students' satisfaction with the material. Table 3.1 shows these categories and the question numbers within these categories.

Attention	Relevance	Confidence	Satisfaction
2	6	1	5
8	9	3 Reverse	14
11	10	4	21
12 Reverse	16	7 Reverse	27
15 Reverse	18	13	32
17	23	19 Reverse	36
20	26 Reverse	25	
22 Reverse	30	34 Reverse	
24	33	35	
28			
29 Reverse			
31 Reverse			

 Table 3. 1. IMMS Scoring Guide

Reverse: "The marked items as reverse (Table 3.1) are meant in a negative way"

In addition, a Likert Scale social validity checklist created by the researcher was employed. This measure consists of nine statements, one of the items is used in a negative manner, to see students' thoughts about *Motion Math: Fraction* (See it in Appendix-4). Generally items are about students thought for the features of the app and whether these features helped them to learn the intended contend area. For instance, "the images in the game helped me to learn fraction". The researcher used both tools after the intervention session.

Performance measurement tool. In this study, 65 fraction items in various forms were used during the data collection process (Sample Questions are in Appendix-1). Questions were received from different resources: 35 items that have been released between 1990 and 2013 from the National Assessment of Educational Progress (NAEP; U.S. Department of Education, 2014), 15 from the article of Fuchs et al. (2013) and fifteen from Siegler et al., (2011). Since fractions are seen in different forms, such as decimal, and pie chart, the researcher wanted to have a variety of questions representing different fractions concepts. Questions consist of multiple choices, comparison, and completion items. Questions were scored 0 (incorrect) and 1 (correct). With the chosen questions, the researcher created a question pool. And then, he equally

distributed the questions to five question sheets consisting of 13 questions in terms of their difficulty level: hard, medium, and easy. For the next step, the researcher sent the questions to two mathematics teachers: one was working at a charter school teaching elementary and middle grade mathematics, and other was PhD candidate at a University in field of math education. These teachers were chosen because of their expertise in teaching and research area. They checked the quality, clarity, and structure of questions. They gave the following suggestions; changing the order of options based on their property values, giving more space between questions, and working on wording. Next step was to meet a faculty at the measurement department considering reliability of the questions. He said that using questions taking from NAEP, and articles (Fuchs et al., 2013; Siegler et al., 2011) increase reliability, and checking clarity is another way for increasing reliability. But he recommended adding directions to question sheets. After reviewing several directions forms for mathematics questions, the researcher added directions to the question sheets.

Content validity. As a means for determining content validity (Johnson & Turner, 2003) of the question items, the researcher asked the mathematics teachers, who check the clarity, and quality of the questions described in previous section, to review the test items to determine whether these represent the targeted content, clarity of the items, appropriateness for participants, and whether the items align with the content in the app, and with the CCSS (Common Core State Standards). After having the completed content validity forms (See apeendix-2) from the mathematics teacher, and the PhD candidate (he got the degree in mathematics education), percentage of agreement was calculated. There are 65 questions and 6 criteria in the rubric including appropriateness for grade levels, clarity, alignment with CCSS, and alignment with the

app. The researcher found 377-agreement pointed out of 390 possible points resulting in 96% agreement on the criteria.

Data collection procedures. Data was collected during the fall semester in 2015 at a public charter school in South East of US. At that school, students received supplementary core courses in after school program. In after school program, students complete their assignments and receive extra instructional help in terms of specific content areas. Data collection procedures took place when students were in the after school program in classroom at the school on Monday through Friday for an 8-week period, October 5th through the first week of December with a 1-week break interruption.

For the fidelity of the intervention, the researcher used a 9-item fidelity checklist in order to determine treatment efficacy. These items included providing an iPad, launching the app, choosing the level of difficulty for students, observing students whether on task, ensuring students engaged with the app a certain amount of time (20 minutes), and administering the progress monitoring assessment after students finished playing the game (See Appendix-5). Already trained doctoral students observed the sessions and inter-observer agreement was 90%. Besides the researcher, one of the doctoral students scored student responses on the assessments to increase inter-rater reliability. No differences were found.

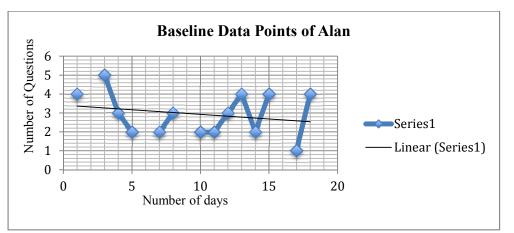
Even though different researchers suggest different numbers of data points for the baseline period to achieve stability, having at least five data points for each student is required for single case research design standards in order to calculate the stability of the baseline data points (Kratochwill et al., 2010; Neuman, McCormick, &International Reading Association, 1995). Considering that, in the baseline phase, when there were at least five data points for each individual, the researcher calculated stability for the baseline phase. In terms of the criteria stated

by Neuman et al. (1995), 85% (80-90%) of data points in any phase should be within a 15% range of the mean of all data points in that phase.

For instance, the mean of Alan's data points at that time was 2.92.

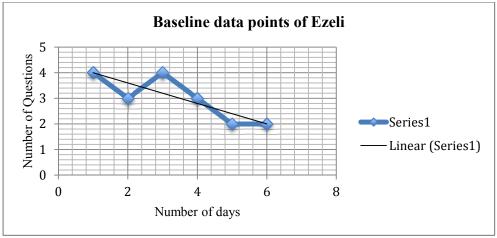
.15*2.92=0.438

Therefore, it is expected that 85% of data should be within the range of 2.482- 3.358. His data points in the baseline phase were 4, 5, 3, 2, 2, 3, 2, 2, 3, 4, 2, 4, 1, and 4. Nine out of 14 data points (roughly 64%) in the phase within a 15% range of the mean of all data points in that phase. However, the graph depicted in the below table shows that trend is downward, and variability is small.

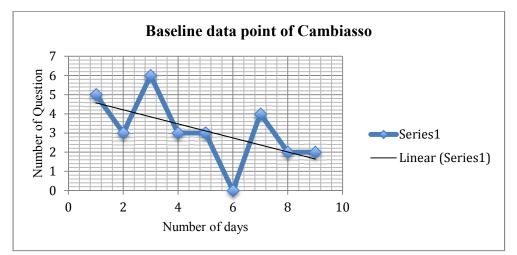


Graph 3.2. Baseline data points of Alan

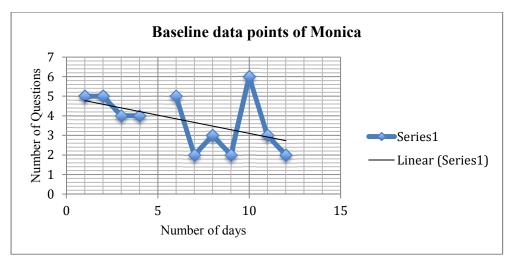
In baseline, data points for other participants also were not stable, but again trend lines for all of them were downward. See the graphs provided for each individual.



Graph 3. 3. Baseline data points of Ezeli



Graph 3.4. Baseline data points of Cambiasso



Graph 3.5. Baseline data points of Monica

According to Kratochvil, et al., (2010), "if the effect of the intervention is expected to be larger and demonstrates a data pattern that far exceeds the baseline variance, a shorter baseline with some instability may be sufficient to move forward with intervention implementation" (p. 19). Therefore, the researcher moved to the intervention phase even though unstable data set was seen for all participants and trends were all negative in direction.

Based on these baseline data for all participants, the researcher randomly selected order in which participants would receive the intervention. For this purpose, he wrote each of the participants' names on a different piece of paper, and then randomly selected one for the first intervention session. For example, Ezeli was the first participant selected. When Ezeli had at least three data points in the intervention phase and when there was notable change in the performance (Ferron, & Scot, 2005), the researcher selected another participant for the intervention period using the same random selection process as used with the first participant. This process continued until all students received the intervention. When students finished 10 sessions playing with the app in the intervention phase, they did not play the app for one week before maintenance assessment began.

Analysis. Analysis of the data consisted of visual analysis, calculation of effect sizes utilizing Percent Non-overlapping Data (PND), Percent of All Non-overlapping Data (PAND), and Percent Exceeding Median Data (PEM, and multilevel modeling. For the purpose of data analysis, in addition to the researcher, three graduate students, who took the course single-case experiments, completed visual analysis of the graphs, which were developed using Microsoft Excel program (an example provided under the title of design). These graduate students used six features to determine the effect of the intervention. These features included: level, trend, variability, immediate effect of the intervention, overlapping data points, and consistency of data patterns within and between phases (Fisher, Kelley, & Lomas, 2003; Hersen & Barlow, 1976; Kazdin, 1982; Kennedy, 2005; Morgan & Morgan, 2009; Parsonson & Baer, 1978). Krotochwil et al. (2010) identified level as "the mean score for the data within a phase," trend as "the slope of the best-fitting straight line for the data within a phase", and variability as "the range of standard deviation of data about the best fitting straight line" (p. 18). While considering immediacy of the effect, the graduate students examined whether there was recognizable change between the levels of the last four data points in the baseline data series and the level of the three data points of the intervention data series. Immediate effect was the statement of the influence of the independent variable on outcome variable.

After completing the visual analysis, visual analysts determined whether there were at least three indications of an effect at different points in time. Three indications of an effect is the accepted standard for determining whether an intervention (i.e., *Motion Math: Fraction* app) results in an experimental effect on the dependent variable (fraction knowledge/skill) (Krotochwill, et al.).

Besides the visual analysis, the researcher used Percent Non-overlapping Data (PND), which is commonly used by researchers to calculate the effect size of studies in which single subject design is used (Gast, 2010). Percent of All Non-overlapping Data (PAND), and Percent Exceeding Median Data (PEM) to determine the effect size. For this purpose, the researcher looked at the data points to learn whether baseline data points and intervention data points overlaps, and made calculation.

After visual analysis and calculation of PND, PAND, and PEM for effect size, data were analyzed by using a multilevel model for multiple-baseline (hierarchical liner model). To estimate the average change in level across phases, and estimate degree of freedom, Kenward-Roger method was utilized. Ferron, Bell, Rendina-Gobioff, and Hibbard (2009) stated that modification that was employed is suitable for the design of the study, and the observed level of variance in the baseline and treatment phase.

Below, Table 3.2 shows the relationship between research hypotheses, data collection methods, and analysis tools.

Table 3.2. Study Flow Chart

Research	Data Collection	Analysis	What I expected to learn
Hypotheses			
1. Playing 20	a) Single-Case	1.a. Visual Analysis	1.a.a. There any trend, slope, immediate change from baseline
minutes everyday for	Experiment	1.b. PND, PEM, PAND	to intervention
two weeks period		1.c. Statistical Models;	1.a.b. Whether or not there is/are overlapping data.
with Motion Math:		Kenward-Roger method.	1.a.c. Whether there is statistical significance, to learn
Fraction increases			confidence interval, and degree of freedom.
fraction skills of			
students with			
Mathematics			
Learning Disability.			
2. After playing	a) Single-Case	2.a. Visual Analysis;	2.a.a. Whether there is change in level between intervention
Motion Math:	Experiment	2. b. Statistical Models:	and follow-up phases.
Fraction during		hierarchical linear model	2.b.b. Whether there is stable or upward trend in follow-up
treatment, the		(Modifying Kenward-Roger)	phases.
participants will			
maintain the			
knowledge they			
gained after no			
longer playing the			
app.			
3. Greater amounts	a) Single-Case	3.a. Visual Analysis	3.a.a. Whether there is trend in intervention phase
of time interact with	Experiment	3.b. Statistical Models;	3.b.b. Whether statistical models providing information about
the app will result		hierarchical linear model	time is important for students' performance.
greater achievement			
gain for the students.			

CHAPTER FOUR: FINDINGS

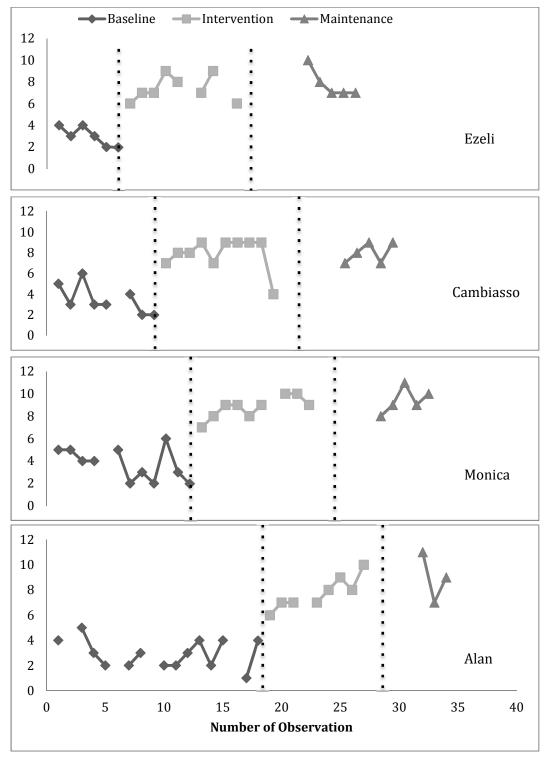
Visual Analysis

In addition to the researcher, three doctoral students who took a doctoral level single case research course served as peer reviewers and examined the graphs for each participant and completed visual analysis in terms of six features stated by Kratocwill et al. (2010). All three doctoral students observed a change in level for all four participants. When considering trends, all of the reviewers stated there is an upward trend in intervention phase for two participants: Monica and Alan. For other participants, there was no consensus on whether there was a trend. However, two of the reviewers highlighted a data point in intervention phase for Cambiasso, which changed the way of the trend line for the participant in the phase in negative manner. For the maintenance phase, the reviewers noted that two of the participants: Cambiasso and Monica, have an upward trend and the other two: Ezeli and Alan, have a downward trend. However, reviewers also stated that more data points were needed in order to reach an absolute conclusion about trends in the maintenance phase.

With respect to variability in the data, the reviewers observed that overall there not a high level of variability within phases for each participant. Only one data point for Cambiasso during the intervention phase was observed as an outlier.

All of the reviewers noted the immediate effect at the intervention for each participant. With respect to overlapping data, two peer reviewers observed only one data point in the intervention phase that overlaps with a data in baseline phase and this was for Cambiasso. Finally, all peer reviewers observed consistent data patterns across participants in baseline and

intervention phases. However in the maintenance phase, two participants have upward trends, while two other participants have downward trends.



Graph 4.1 Time series data for each participant in each phase.

Effect Size Calculation

Percent Non-Overlapping Data (PND)

To calculate PND, the highest data points in baseline (phase A) were identified and then data points in the treatment phase (B phase) that exceeded the highest data point at baseline were counted. Numbers of non-overlapping data in phase B were then divided by the total points in phase B to arrive at a percentage. After calculating PND values for each participant, an overall effect size was calculated by dividing the sum of PND values of each individual by the number of participants.

$$PND = \frac{n_{BnonoverlapA}}{n_B}. 100$$

For Ezeli, PND = $\frac{8}{8}. 100 = 100$
For Cambiasso, PND = $\frac{9}{10}. 100 = 90$
For Monica, PND = $\frac{9}{9}. 100 = 100$
For Alan, PND = $\frac{8}{8}. 100 = 100$
Effect Size = $\frac{100 + 100 + 90 + 100}{4} = 97.5$

The PND Scale below was used to determine the effectiveness of interventions (Campell & Herzinger, 2010; Scruggs, Mastropieri, & Castro, 1987).

90%+ = Highly Effective 70%-90% = Moderate Effective 50%-70% = Minimally Effective >50% = Ineffective The mean effect size (97.5) for participants is above 90 percent meaning the intervention can be considered to be highly effective based on PND.

Percent Exceeding Median Data (PEM)

Considering an increase, the median of the A phase was identified and then data points in B phase that exceed it were determined. As a second step, the number of data points in B phase that exceeded the median in A phase were divided by the number of data points in B phase, and then multiplied by 100 to find the percentage.

$$\text{PEM} = \frac{n_{B>md}}{n_B}. \ 100$$

For Ezeli, median of A phase is 3.

$$PEM = \frac{8}{8} \cdot 100 = 100$$

For Cambiasso, median of A phase is 3.

$$PEM = \frac{10}{10}. \ 100 = 100$$

For Monica, median of A phase is 4.

$$PEM = \frac{9}{9}$$
. 100= 100

For Alan, median of A phase is 3.

$$PEM = \frac{8}{8} \cdot 100 = 100$$

Effect Size= $\frac{100+100+100+100}{4} = 100$

The mean effect size (100) for participants is above 90 percent meaning the intervention can be considered to be highly effective based on PEM.

Percent of All Non-overlapping Data (PAND)

To calculate PAND, as a first step, the number of overlapping data was subtracted from total number of data points (n) in baseline and treatment phases, and then this number $(n_{Bremaning})$ was divided by the total number points (n) in baseline and treatment phases.

$$PAND = \frac{n_{Bremaning}}{n} \cdot 100$$

For Ezeli, PAND = $\frac{14}{14} \cdot 100 = 100$
For Cambiasso, PAND = $\frac{17}{18} \cdot 100 = 94.44$
For Monica, PAND = $\frac{20}{20} \cdot 100 = 100$
For Alan, PAND = $\frac{22}{22} \cdot 100 = 100$
Effect Size = $\frac{100 + 94.44 + 100 + 100}{4} = 98.61$

The mean effect size (98.61) for participants is above 90 percent meaning the intervention can be considered to be highly effective based on PAND.

SAS Analysis (Multilevel Modeling)

The researcher completed inferential analysis by using SAS. As a first step in this analysis the researcher employed the Kenward-Roger model. The purpose of this analysis was to estimate the average change in level across all phases and estimate the degree of freedom.

Solution for Fixed Effects										
Effect	EstimateStandardDFt Value $Pr > t $ AlphaLowerUppError \cdot									
Intercept	3.2626	0.3032	5.82	10.76	<.0001	0.05	2.5149	4.0102		
treat	4.6839	0.3126	27.9	14.98	<.0001	0.05	4.0435	5.3244		
follow	5.2364	0.3837	25	13.65	<.0001	0.05	4.4461	6.0267		

Table 4. 1 shows the results of SAS analysis.

Note: There are 13 questions and each of them worth 1 point.

The estimated average baseline level for all participants is 3.2626. This value increased by 4.6839 at the treatment phase. Therefore, students increased their average score from 3.2626 to 7.9465 (3.2626+4.6839) from baseline to treatment phases. P value for treatment effect is <. 0001. Participants continued to improve their score into the maintenance phase. The table shows that participants increased their mean score by 5.2364 from baseline, representing a mean score increase of 0.5525 (5.2364-4.6839) from the treatment to maintenance phase. In total, the average of maintenance phase is 8.499 and *p* value for maintenance phase is <. 0001. Kenward-Roger Model also provided information to estimate the degree of freedom with greater accuracy that showed a 95% confidence level that the actual treatment effect is between 4.0435 and 5.3244. Based on these data, use of a game-based app, *Motion Math: Fraction* is highly effective on the students' fraction skills.

The researcher also looked at the time effect in treatment and maintenance (follow-up) phases. Table 4.2 shows the findings of this analysis.

	Solution for Fixed Effects										
Effect	Estimat e	Standard Error	DF	t Value	Pr > t	Alpha	Lower	Upper			
Intercept	3.2972	0.2861	6.07	11.52	<.0001	0.05	2.5989	3.9954			
treat	3.4554	0.9023	29.5	3.83	0.0006	0.05	1.6113	5.2995			
follow	1.3457	2.8167	30.6	0.48	0.6362	0.05	-4.4019	7.0932			
treat*time	0.07316	0.04981	29	1.47	0.1527	0.05	-0.02871	0.1750			
follow*time	0.1367	0.09846	31.1	1.39	0.1749	0.05	-0.06408	0.3375			

Table 4.2 shows time effect in treatment and follow-up phases.

Time effect in treatment phase was 0. 07316 (p=0.1227) and follow-up phase was 0.1367 (p= 0.1749). Based on the data, time in both phases is not significantly effective to change the students' performance in either way.

Summary of Findings

Hypothesis 1. Participants (students with MLD) will increase their fraction skills by playing the *Motion Math: Fraction* app 20 minutes daily for two weeks.

Based on the findings from the visual analyses, PND, PEM, and PAND effect size calculations, and statistical analysis of the data set, it is apparent that all participants improved their fraction skills by using the game-based app, *Motion Math: Fraction*. The findings of visual analysis showed that there are more than three indications for the effectiveness of the interventions. There are changes in level between phases and these changes are clear for each participant. With respect to visual analysis, all three reviewers and the researcher confirmed an immediate effect across participants. All of the reviewers observed that there is a slight upward trend line for participants in the intervention phases, but two of the reviewers also highlighted a data point, which is the last data point in the intervention phase for Cambiasso, which might change the direction of trend line

for the participants in the phase since it is far away from the regression line. In maintenance phase, reviewers stated Cambiasso and Monica have upward trends and the other two participants, Ezeli and Alan, have downward trends. Therefore, reviewers noted that caution should be taken as the amount of data points in maintenance (followup) phase may not be enough to definitive conclude about whether there is a trend in maintenance phase for all participants and in what direction the trend might be.

With respect to variability, there is no high variability within the phases for all participants except for an outlying data point for Cambiasso in the intervention phase. The outlying data point in the intervention phase also overlaps the data points in the baseline phase for Cambiasso. There is no other overlapping data for all participants.

Overall, the visual analysis of the times series data show that, although there is some missing data for all participants, there is an immediate change in level, and upward trend in intervention phase for Ezeli, Monica and Alan, and downward trend in intervention phase for Cambiasso with only trivial overlapping data present for one participant, Cambiasso, and no high variability within the phases for all of them.

The researcher calculated effect size by using PND, PEM, and PAND. PND was 97.5, PEM was 100, and PAND was 98.61. Campell and Herzinger (2010), and Scruggs, Mastropieri, and Castro (1987) provided a scale to reach a conclusion about the results. They categorized the scale as follows:

90%+= Highly Effective 70%-90%= Moderate Effective 50%-70%= Minimally Effective

>50%= Ineffective

Based on this scale there is a functional relationship between dependent and independent variable and the intervention can be considered to be highly effective for these middle school youth with MLD to learn fraction skills.

By using the statistical analysis, the researcher found that the estimated average baseline level for all participants is 3.2626, and this value in intervention phase increased by 4.6839 from the baseline. This means that participants increased their mean score to 7.9465 in treatment phase. P value for treatment effect is <. 0001. Therefore, the directional hypothesis is accepted since there are significant differences.

Hypothesis 2. Participants will maintain the level of fraction skills they while playing the *Motion Math: Fraction* app 20 minutes daily for two weeks after no longer playing the app.

Based on the time series graph, reviewers and the researchers found that that there is both upward and downward trends among participants. Even though two of the participants: Cambiasso and Monica, had an upward trend, the other two participants, Ezeli and Alan, had downward trends in the maintenance phase. Therefore, the visual analysis reviewers suggested that caution should be used when reaching conclusion about the maintenance phase time series trends by conducting visual analysis. However, statistical analysis confirmed that participants retained their knowledge from treatment to maintenance (p <.0001). In fact, participants continued to improve their score even in maintenance phase by 5.2363 from baseline and by 0.5525 from treatment.

Hypothesis 3. Greater amounts of time interacting with the app will result in greater achievement gains for the students.

Considering the visual analysis, none of the reviewers were clear about whether there is an overall increasing or decreeing trend for the treatment phase among participants as an upward trend was observed for two of the participants and a downward trend was observed the other two participants. Therefore, it was difficult to reach a definitive conclusion about time effect through visual analysis of the data. The researcher also analyzed time effect in each phase by using the multilevel modeling. These data suggest that time did not have a significant impact on the performance of students in intervention (*p* value 0.1527), and in maintenance (*p* value0.1749) phases. Every day, students' performance increases by 0.07316 at intervention phase, and by 0.1367 at maintenance phase.

Finding from Social Validity Tools and Summary of This Findings

Considering goals, procedures and outcomes (Wolf, 1978), which are the dimensions of social validity, the researcher used two social validity surveys; one created by the researcher, and the other (Instructional Materials Motivation Survey) modified by the researcher. The Instructional Materials Motivation Survey (IMMS) included 29 items. Based on five levels of agreement (not true= 1, slightly true= 2, moderate true= 3, mostly true= 4, and very true= 5), the researcher measured students' motivation on the instructional material (*Motion Math: Fraction*) according to four categories: attention, relevance, confidence, and satisfaction. Tables 4.4 through 4.7 show individual participants and their responses to items:

Attention	Re	Relevance		nfidence	Satis	faction	
Item/Response	e Item	Item/Response		Item/Response		lesponse	
2/4		6/3		1/3	4	5/4	
8/4		9/-		3/4	1	4/4	
11/5		10/4		4/3	2	1/5	
12/3		16/3		7/-	27/4		
15/-		18/-		13/5		2/4	
17/4		23/-		19/4		36/3	
20/3		26/4		25/4			
22/3		30/4		34/3			
24/-		33/5	35/-				
28/5							
29/3							
31/4							
Total 38/10=	3.8 Total	23/6=3.83	Total	26/7=3.71	Total	24/6=4	

 Table 4.4.
 Monica's IMMS Result

- means that the items were not used.

Table 4.5. Cambiasso's IMMS Result

Atte	ention	Relevance		Cor	Confidence		action
Item/R	lesponse	Item/Response		Item/	Item/Response		esponse
2	2/1		6/1		1/4	5/3	
8	3/5		9/-		3/1	14	/3
1	1/4		10/2		4/2	21	./4
1	2/3		16/2		7/-	27/4	
1	15/- 18/-		13/1		32/5		
1	17/4 23/-		23/-	19/5		36/5	
2	20/1		26/4		25/3		
2	2/4	30/4			34/2		
2	4/-		33/3	35/-			
28/1							
29/3				-			
31/4							
Total	30/10=3	Total	16/6=2.66	Total	18/7=2.51	Total	24/6=4

- means that the items were not used.

Atter	ntion	Relevance Confidence		dence	Satisf	action	
Item/Re	esponse	Item/Response		Item/Response		Item/Response	
2/	/5	6/	/5	1,	/5	5,	/5
8/	/5	9	/-	3/	/4	14	/5
11	/5	10)/5	4/	/5	21	/5
12	2/1	16	5/5	7.	/-	27/5	
15	5/-	18/-		13/5		32/5	
17	//5	23/-		19/1		36/5	
20	/5	26/5		25/3			
22	2/5	30	/5	34/5			
24	l/-	33	/5	35/-			
28/5							
29/5							
31	31/5						
Total	46/10=4.6	Total	30/6=5	Total	28/7=4	Total	30/6=5

 Table 4.6. Ezeli's IMMS Result

- means that the items were not used.

Table 4. 7. Alan's IMMS Result

Att	ention	Relevance		Confi	Confidence		action
Item/	Response	Item/Response		Item/Response		Item/Response	
	2/5	6/	/5	1,	/5	5.	/4
	8/5	9	/-	3/	/5	14	/5
	11/5	10)/5	4/	/3	21	/5
	12/2	16	5/5	7.	/-	27/5	
	15/- 1		3/-	13/5		32/5	
	17/5	23/-		19/5		36/5	
,	20/5	26/1		25	5/5		
,	22/5	30/5		34/5			
	24/-	33	5/5	35/-			
,	28/5						
	29/5						
	31/5						
Total	46/10=4.6	Total	30/6=5	Total	28/7=4	Total	30/6=5

- means that the items were not used.

This analysis showed that individual students agreement on the items change from category to category. First realized finding is that average of Cambiasso' score in the category of confidence (2.51) and relevance (2.66) is very low and he thought that it is hard to see any connection what he needs to do and the covered instructional activities, and the app did not increased his confidence level as much as he expected while engaging fraction problems. However, Ezeli (confidence= 4, relevance= 5) and Alan (confidence=4, relevance= 5) stated that the content covered in the app is related to what they needs to do, and they thought that learning fraction skills by engaging with the app is easy.

Overall, individual student agreement as follow: the average scores for Ezeli was 4.62, for Cambiasso 3.03, for Monica 3.82, and for Alan 4.65. Even though students' scores varied from student to student, all reported they were satisfied with the app. All of the participants stated, *"The content matter provided by the app was easy to understand than I would like for it to be"*. The app has a variety of features that are eye catching and helped them to learn fractions. However, students expressed their concern about item 10, which was *"the practices brought by the app are so abstract that it was hard to keep my attention on it"*. And one of the participants, Cambiasso, stated that he got bored after playing for a while. This might be a reason for his last data point in the intervention phase. However, he had upward trend in even maintenance phase after removing the intervention.

While analyzing the researcher-created social validity survey, the researcher did realize the parallel findings to IMMS, such as when Cambiasso said he got bored; while all of the participants expressed that they learned the content matter. Even though he has upward trend in the intervention phase without considering the last data point: his last data point in that phase

changed the trend line from upward to downward since the data point is far away from the regression line and far below the average of data in the phase.

CHAPTER FIVE: DISCUSSION/IMPLICATIONS

Practical Implications

Providing evidence-based interventions to students with MLD in terms of their needs is vital for their learning. In this study, students with MLD significantly improved their fraction skills in an after school program by engaging with the Motion Math: Fraction app for a 20 minute period with little or no teacher support. The achievement gap between students with MLD and students without disabilities has steadily increased over time. Apps such as this may be an effective way to provide struggling learners with opportunities to engage in responding to mathematics tasks without requiring much teacher guidance. The fact that the app was effective in improving fraction skills for the four participants in this study makes the prospect of the impact apps could have in the mathematics classroom intriguing, particularly for students with MLD who require many response opportunities with feedback in order to improve; but who may not receive these opportunities. Instead of spending more time providing one-to-one direct instruction, teachers may be able to use their instructional time more effectively and reach more students by teaching students how to use a particular app and take more of a support role and scaffold their level of support (more or less) based on individual student needs as students engage with the app. Perhaps this would provide students with more time to engage with content matter in class and at home, on the bus or other places increasing their opportunities to respond with feedback thereby increasing levels of proficiency and knowledge at their pace.

One of the common features of students with the disabilities is being passive learner. With flipped learning model, teachers support students' active participation to classroom

activities (Zhonggen, & Guifang, 2016). In this model, after receiving instruction in the classroom, teachers might ask students to engage with app for a certain time of period, such as 20 minutes, at home or wherever they want except classroom. If they have problem and cannot figure out problem situations, they may take notes about the problems, and discuss when they come to classroom. In this way, teachers might increase the students' engagement and motivations.

Thousands of apps for mobile devices are in the market, and their developers claim benefits for their audiences (Douglas, Wojcik, &Thompson, 2012), even though there are no systematic studies evaluating their effectiveness (Riconscente, 2013).

Importantly, users of educational apps for students with disabilities need to understand the goals and functions of apps that can help people with and without disabilities and how to teach students to use them more effectively. Therefore, students and teachers need to learn how individual apps differ because each app is developed for specific purposes with a variety of features that may or may not support the needs of students with disabilities. The extent to which an app is appropriate for learning content is also an important consideration. In their research to determine the functions of apps, Douglas, Wojcik, and Thompson (2012) found that only 46 out of 508 apps were appropriate to use for mathematical purposes.

Research Implications

Each day thousands of apps come to stores. Some of them are meant for educational purposes, some of them are not. Since usage of apps in classrooms or out of classrooms for educational purposes does not have a long history, people in the field are cautious to assert that apps do or do not work to improve educational outcomes for students. This research might

trigger more studies about apps and their usage for academic purposes considering students with MLD.

The researcher did not intend to investigate whether participants were able to generalize the knowledge they gained by engaging with the app to fraction related tasks that were not represented in the app. However, based on a review of their performance and the types of fraction tasks to which they were successful responding, an interesting pattern emerged. The researcher reviewed student responses to the different types of questions in the daily assessment probes (i.e., comparison of fractions questions and problem based fraction questions). Even though the comparison types of questions were directly related to the content of the app, the problem-based questions were not included in the app. The word problems in that assessment probes required students to transfer their knowledge of fractions to solve fraction related word problems. While looking through the question sheets, the researcher realized that students' number of correct responses to questions that were directly related to the app increased and the students correctly answered almost all of them. However, the same was not true for word problem-based questions. A review of their IEP revealed that the four participants all have language related difficulties that could have affected their success with the problem-based questions. With respect to future researcher, researchers might consider how disability related characteristics, such as language difficulties, impact the effect of a mathematics related intervention.

Two participants, Cambiasso, and Monica, improved their fraction skills even during the maintenance phase even though they did not receive instruction on fractions in their mathematics classroom. Given the positive results combined with the lack of instruction on fractions outside of the participants' use of the app, it is possible that students developed practical math strategies

while dealing with the questions. This could possibly explain why some students increased their scores from intervention phase to maintenance phase. However, further studies should be conducted to investigate this issue to determine the effectiveness of the app and how they might aid in students' development of math problem solving strategies.

Even though the app provides drill and practice opportunities for students by using cues, prompts, and feedbacks, it has several weaknesses and one of them is not having an instruction delivery module. Students are assumed to have basic fraction knowledge. However, even though students receive fraction instruction in classrooms, they might still need to review basic concepts before starting to play. Considering mastery level (e.g., solving 80% of the questions correctly), none of the students reached that level. Therefore, it is suggested that this app could be improved by adding at least a review module. Research could be conducted that evaluates whether student performance improves with this enhancement to the app.

Theoretical Implications

The framework in figure 5.1 was developed based on the definition of mathematical disability that given by Geary (2004). Geary identified mathematical disability "as a deficit in conceptual or procedural competencies that define the mathematical domain, and these, in theory, would be due to underlying deficits in the central executive or in the information representation or manipulation (i.e., working memory) systems of the language or visuo-spatial domains" (Geary, 2004, p. 9). While designing this study, the researcher aimed to manipulate language and visual systems during information processing since it was highlighted in the definition for students with MLD. For this goal, the researcher specifically chose an app that engaged students in coding information in variety of visual formats including through images and numbers on number lines.

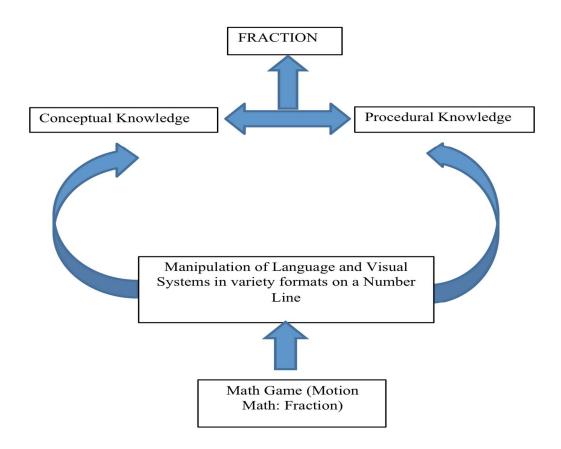


Figure 5.1 represents the idea behind the study.

Siegler and Ramani (2008) investigated the effectiveness of board games to increase mathematical knowledge of preschool students by physically interacting with the number line integrated into the games. By playing the board games, students manipulated a token on the number line, and this helped them to develop a mental representation of the number line by providing concrete hints about magnitude of numbers. Results of the study showed significant improvement of the students' knowledge of comparisons, estimation, identification and counting of numbers. Results of this study also support the notion that a number line also helped students to better understand magnitude of rational numbers (i.e., fractions) but doing so by engaging with a game-based app.

Suggestions to App developers

Each individual has a different learning profile, and they learn differently. Even students within the category of MLD learn differently. Some prefer visuals (**+@@@=5), and others learn easily by working with abstract math symbols (2+3=5). Additionally, considering the comorbidity issue (having more than one type of disability, such as math and reading disabilities), developers should explain what they mean when they provide problem situations for students or should provide some examples that module how to solve the problems. For instance, students are required to compare fractions in one problem situation, but the problem situation is addressed in written words. In this study, some students did not read the directions or did not understand the directions even after reading. After a while, they asked what they were required to do for that problem situation. Directions should be clear and simple so that students can easily understand, especially students with learning related disabilities.

If app developers use figures or images, they should consider cultural relevance, and students' experience with the figures. For instance, pizza is mostly consumed food in the US, and many app developers deliver fraction instruction by pizza slices. However, in many countries most of students could not eat pizza, and they do not have experience with. Therefore, while developing apps, the aim to teach content matters but so do factors such as these.

Even though the app provides a record about students' performance, it is limited. Teachers might need to learn how many times students received feedback, the nature of clues provided, and the type of question (e.g., "place ½ on the number line"). With such information, teachers could have information that they can use to provide enhanced support to students in targeted ways. Providing detailed information about the type of feedback should be another concern of app developers.

Limitations

In this study, the researcher used a single case research design, specifically multiple baseline AB design, and there were four participants. Because of having a small number of participants, generalizability is one of the limitations of single case research design. However, the researcher provided detailed information regarding sampling procedures, data collection procedures, and analysis of data such that other researchers might be able to replicate the study. Replications of single case studies are a viable method for establishing generalizability of results.

A second limitation is that this research did not occur in the classroom, which might have led to differential effects because of contextual differences. Additionally, utilization of qualitative methods might have helped further understand particular characteristics of the quantitative data. For instance, Cambiasso had a small upward trend during the intervention phase until the last data point in the phase, but he performed very below the average and the trend line turned downward. Interviewing Cambiasso about why this might have happened may have provided insight into this quantitative data pattern. Without such qualitative data, the researcher cannot explain the reason why.

Future Research Direction

As stated, the generalizability of single case research design is a limitation of this study. Therefore, the study should be replicated with larger numbers of participants and through using multiple methods; including observation, and interviewing with students in addition to single subject experiments. Another limitation of single case design is it is quasi-experimental in nature and as such consideration of the life at the place in which data was collected is not paramount. Students' low or high performance might be because of different reasons that are not captured by single case type data. For instance, even though Cambiasso had an upward trend and his average

performance was around 8 of 13 questions correct during the intervention phase, the last day, he only correctly solved 4 out of 13 problems. Considering the social validity tools, Cambiasso stated that he got bored after playing so many days, but there is no information about his reasons of getting bored. We need to know about possible reasons, such as is he hungry or tired? By using multiple methods, researchers might be able to obtain more conclusive results about the effectiveness of the app considering the above issues.

Family participation is critical for their children's education, and students spend almost 2/3 of their time at home or at other place with their families. One of the advantages of using game-based apps on mobile devices, is that they allow students to play whenever and wherever they want. While playing the game, they may have fun, and also they have opportunities to learn the targeted content without having pressure on them in a safe environment, such as at their home. Feeling safe might decrease math anxiety, a common issue for students with MLD. Therefore, in further studies, conducting such studies at home could be considered. In this case, family members could work as co-researchers (e.g., taking notes about how many minutes children play, when they play, etc.).

Interestingly students improved their scores during the maintenance phase even though students did not receive fraction instruction in their classroom. However, the researcher does not have knowledge about what they did at home. Therefore, in further studies, researchers might consider collaborating with families to follow students when they are at home. There might be several reasons to the students' improvement during maintenance. One possibility is that participants developed strategies to solve the problems as they engaged with the app during intervention, becoming more proficient with the strategy day by day and they were able to utilize these "self-learned" strategies in maintenance.

As stated before students' performances on word problems did not improve as much as their performance on fraction comparison questions that related directly to the type of questions posed in the app. Reasons for this should be investigated. In addition investigating student error patterns across the time series phases and how students' error patterns changed after engaging with the app could be another area of research focus.

Conclusion

As a result, use of the *Motion Math: Fraction* app resulted in increased fraction knowledge of the students. Although generalization of these findings is very difficult because of sample size and some missing data, the researcher can make causative inference based on these findings. Before making generalizations, this study needs replication by other researchers.

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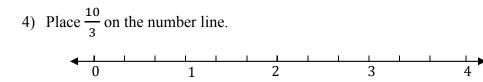
APPENDICES

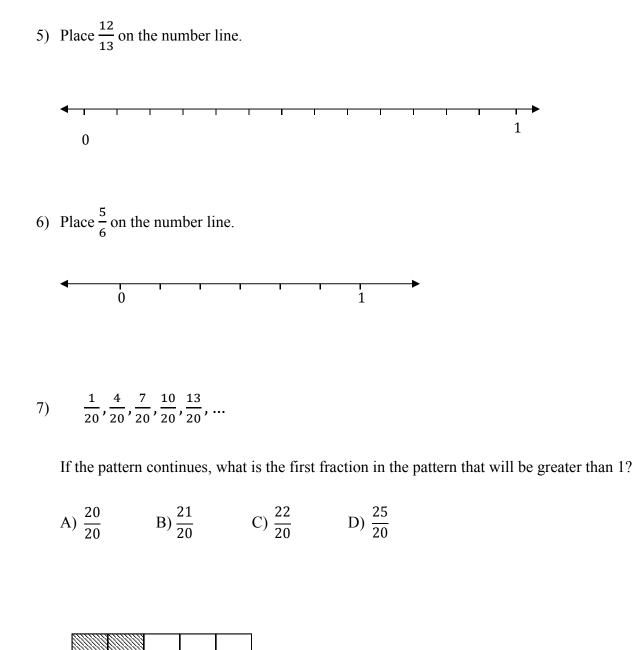
Appendix-1: Questions

Question Sheet-1

For first three questions, you are required to compare given fractions in terms of their property values by using sign (<, >, =). In questions 4, 5, and 6, you need to determine correct place for given fraction on the number line. Multiple-choice questions below are followed by four suggested answers. Select the one that is best in each case. Respond fully to the open-ended question. Show your work and clearly explain your answer. You will be graded on the correctness of your answer. Each question is 1 point.

- 1) Put the appropriate sign between numbers (=, <, >).
 - $\frac{1}{2}$ $\frac{4}{8}$
- 2) Put the appropriate sign between numbers (=, <, >).
 - $\frac{1}{8}$ $\frac{3}{4}$
- 3) Put the appropriate sign between numbers (=, <, >).
 - $\frac{7}{8}$ $\frac{7}{12}$

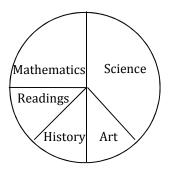




8)

What fraction of the figure is shaded?

Answer



Note: Figure NOT drawn to scale

9) The pie chart above shows the portion of time, Pat spent on homework in each subject last week. If Pat spent almost 2 hours on mathematics, how many hours would Pat spend on homework altogether?

A) 4 B) 8 C) 12 D) 16

10) Jose ate
$$\frac{1}{2}$$
 of a pizza

Ella ate $\frac{1}{2}$ of another pizza

Jose said that he ate more pizza than Ella, but Ella said they both ate the same amount. Explain why and show what Jose could be right.

11) Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?A) 0.91%B) 1.10%

- C) 52%D) 91%E) 95%
- 12) Rima and Eric have earned a total of 135 tokens to buy items at the school store. The ratio of the number of tokens that Rima has the number of tokens that Eric has is 8 to 7. How many tokens does Rima have?

A) 8 B) 15 C) 56 D) 72

13) In the past year and a half, Alfred's dog gained an average of $\frac{1}{4}$ pound each month.

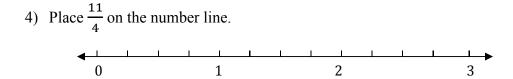
Today, Alfred's dog weighs 75.5 pounds. How much did the dog weigh a year and a half ago?

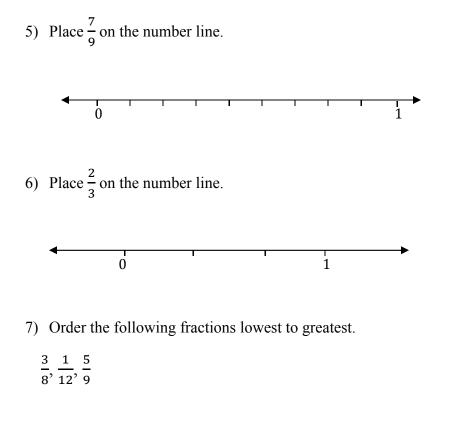
- A) 57.5 pounds
- B) 71.0 pounds
- C) 71.5 pounds
- D) 74.0 pounds
- E) 79.5 pounds

Question Sheet-2

For first three questions, you are required to compare given fractions in terms of their property values by using sign (<, >, =). In questions 4, 5, and 6, you need to determine correct place for given fraction on the number line. Multiple-choice questions below are followed by four suggested answers. Select the one that is best in each case. Respond fully to the open-ended question. Show your work and clearly explain your answer. You will be graded on the correctness of your answer. Each question is 1 point.

- 1) Put the appropriate sign between numbers (=, <, >).
 - $\frac{3}{6}$ $\frac{1}{2}$
- 2) Put the appropriate sign between numbers (=, <, >).
 - $\frac{1}{2}$ $\frac{3}{4}$
- 3) Put the appropriate sign between numbers (=, <, >).
 - $\frac{1}{20} \qquad \frac{7}{8}$

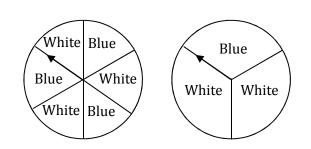






What fraction of the group of umbrellas is furled?

A)
$$\frac{1}{3}$$
 B) $\frac{3}{7}$ C) $\frac{4}{7}$ D) $\frac{5}{7}$



Lori has a choice of two spinners. She wants the one that gives her a greater probability of landing on blue.

Which spinner should she choose?

9)



Explain why the spinner you chose gives Lori the greater probability of landing on blue.

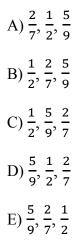
 $10) \qquad \qquad \boxed{\frac{4}{8}} \qquad \qquad \boxed{\frac{25}{50}} \qquad \qquad \boxed{\frac{5}{10}}$

These three fractions are equivalent. Give two more fractions that are equivalent to.



- 11) A recipe requires $1\frac{1}{3}$ cups of sugar. Which of the following ways describes how the measuring cups shown can be used to measure $1\frac{1}{3}$ cups of sugar a;
- A) Use the ¹/₂ cup three times.
 B) Use the ¹/₄ cup three times.
 C) Use the ¹/₂ cup twice and the ¹/₃ cup once.
 D) Use the ¹/₂ cup twice and the ¹/₂ cup once.
 E) Use the ¹/₄ cup once, the ¹/₃ cup once, and the ¹/₂ cup once.

12) In which of the following are the three fractions arranged from least to greatest?



13) The ratio of boys to girls to adults at a school party was 6:5:2. There were 78 people at the party. How many of them were adults?

A) 6

B) 12

- C) 18
- D) 30

E) 3

Question Sheet-3

For first three questions, you are required to compare given fractions in terms of their property values by using a sign (<, >, =). In questions 4, 5, and 6, you need to determine correct place for given fraction on the number line. Multiple-choice questions below are followed by four suggested answers. Select the one that is best in each case. Respond fully to the open-ended question. Show your work and clearly explain your answer. You will be graded on the correctness of your answer. Each question is 1 point.

1) Put the appropriate sign between numbers (=, <, >).

9	5
10	10

2) Put the appropriate sign between numbers (=, <, >).

 $\frac{4}{6}$ $\frac{3}{7}$

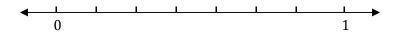
3) Put the appropriate sign between numbers (=, <, >).

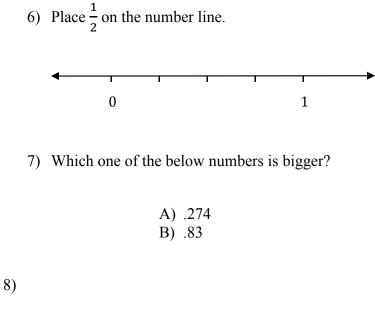
 $\frac{1}{2}$ $\frac{5}{10}$

4) Place $\frac{8}{3}$ on the number line.

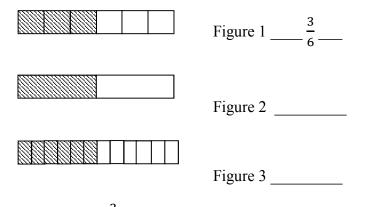


5) Place $\frac{4}{7}$ on the number line.





The shaded part of each strip below shows a fraction.



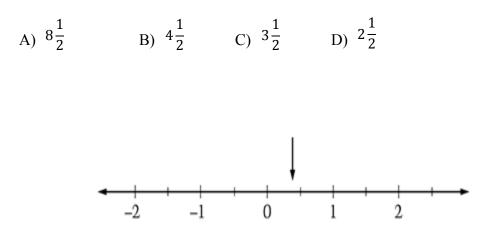
If the figure 1 shows $\frac{3}{6}$, find the corresponding fractions of Figure 2 and Figure 3, and compare these three fractions.

9)

Which decimal represents the shaded part of the figure?

A) 0.5 B) 0.28 C) 0.2 D) 0.02

10) Ted went to the beach at 10:30 a.m. and came back to the home at 2:00 p.m. How many hours is elapsed during this outdoor activity?

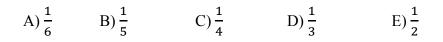


11) On the number line above, the arrow is pointing to a number that is closest to which of the following?

- A) 0.20
- B) 0.37
- C) 0.62
- D) 0.75
- E) 1.62



12) In the figure above, what fraction of rectangle *ABCD* is shaded?



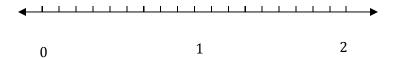
13) Which of the following ratios is equivalent to the ratio of 6 to 4?

A) 12 to 18
B) 12 to 8
C) 8 to 6
D) 4 to 6
C) 2 to 3

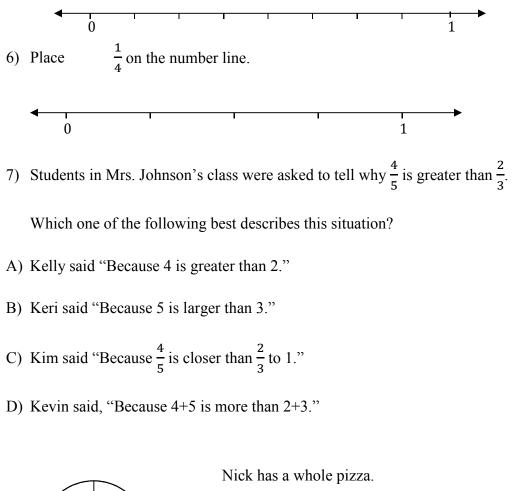
Question Sheet-4

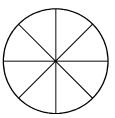
For first three questions, you are required to compare given fractions in terms of their property values by using sign (<, >, =). In questions 4, 5, and 6, you need to determine correct place for given fraction on the number line. Multiple-choice questions below are followed by four suggested answers. Select the one that is best in each case. Respond fully to the open-ended question. Show your work and clearly explain your answer. You will be graded on the correctness of your answer. Each question is 1 point.

- 1) Put the appropriate sign between numbers (=, <, >).
 - $\frac{1}{12} \qquad \frac{1}{5}$
- 2) Put the appropriate sign between numbers (=, <, >).
 - $\frac{8}{12} \qquad \frac{1}{2}$
- 3) Put the appropriate sign between numbers (=, <, >).
 - $\frac{3}{6}$ $\frac{3}{8}$
- 4) Place $\frac{13}{9}$ on the number line.



5) Place $\frac{3}{8}$ on the number line.





Nick says he will eat $\frac{1}{2}$ of the pizza.

He says he will give $\frac{3}{8}$ of the pizza to Sam and $\frac{3}{8}$ of the pizza to Joe.

Can Nick do what he says?

8)

Yes

()No

Explain or show why or why not.

9) Kim, Les, Mario, and Nina each had a string 10 feet long

Kim cut he	er into fifths			
Les cut his	into fourths			
Mario cut l	nis into sixths			
Nina cut he	er into thirds			
After the c	uts-off were ma	ade, who has the	longest piece of	f string?
Kim	B) Les	C) Mario	D) Nina	

10) There are 22 students in a class.

If there are 12 girls in the class, what is the ratio of the number of boys to the number of girls in this class?

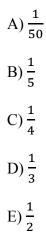
A) 10 to 12 or 10:12
B) 10 to 22 or 10:22
C) 12 to 10 or 12:10
D) 22 to 12 or 22:12

11) On the number line above, what number would be located at point *P*?

Answer

- 12) Jim has $\frac{3}{4}$ of a yard of string that he wishes to divide into pieces, each $\frac{1}{8}$ of a yard long. How many pieces will he have?
 - A) 3
 - B) 4
 - C) 6
 - D) 8

13) Of the following, which is closest in value to 0.52?



Question Sheet-5

For first three questions, you are required to compare given fractions in terms of their property values by using sign (<, >, =). In questions 4, 5, and 6, you need to determine correct place for given fraction on the number line. Multiple-choice questions below are followed by four suggested answers. Select the one that is best in each case. Respond fully to the open-ended question. Show your work and clearly explain your answer. You will be graded on the correctness of your answer. Each question is 1 point.

1) Put the appropriate sign between numbers (=, <, >).

4	4
12	6

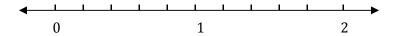
2) Put the appropriate sign between numbers (=, <, >).

6	7
7	10

3) Put the appropriate sign between numbers (=, <, >).

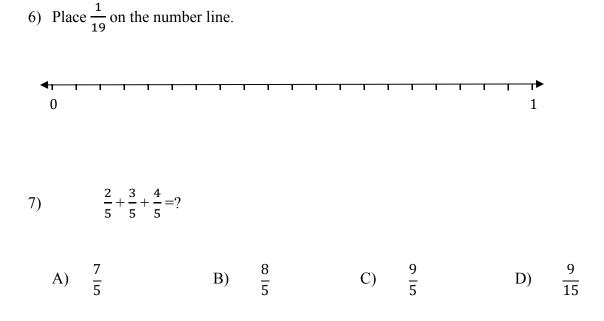


4) Place $\frac{7}{5}$ on the number line.

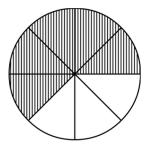


5) Place $\frac{1}{7}$ on the number line.





8) The figure below shows that a part of a pizza has been eaten.



Which one of the following fractions represents the part is left?

A)
$$\frac{3}{8}$$
 B) $\frac{3}{5}$ C) $\frac{5}{8}$ D) $\frac{5}{3}$

9)

In the diagram, what is the relationship between the number of Δs and the number of

s?

A) For every 1 \triangle , there are 2 \cancel{C} s

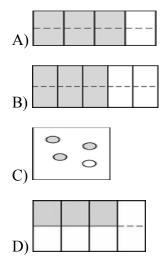
 $\overset{\frown}{a}$

- B) For every 1 \triangle , there are 10 \swarrow s
- C) For every 1 \triangle , there is 1 $\stackrel{\wedge}{\searrow}$
- D) For every 5 \triangle s, there is 1 \cancel{k}

10) If $1\frac{1}{3}$ cups of flour are needed for a batch of cookies, how many cups of flour will be needed for 3 batches?

A)
$$2\frac{1}{3}$$
 B) 4 C) 3 D) $4\frac{1}{3}$

11) Which picture shows that $\frac{3}{4}$ is the same as $\frac{6}{8}$?



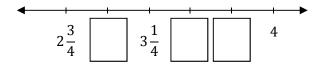
12) There were 90 employees in a company last year. This year the number of employees increased by 10 percent. How many employees are in the company this year?

A) 9

- B) 81
- C) 91
- D) 99
- E) 100

13) Jorge left some numbers off the number line below. Fill in the number that should go in

A, B, and C.



Appendix-2: (Content Valid	lity Check List
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Questions	Appropriate for 6 th grade	Appropriate for 7 th grade	Appropriate for 8 th grade	Clarity	Aligning with CCSS	Aligning with the App
1.1						
1.2						
1.3						
1.4						
1.5						
1.6						
1.7						
1.8						
1.9						
1.10						
1.11						
1.12						
1.13						
2.1						
2.2						
2.3						
2.4						
2.5						
2.6						

0.7	1			
2.7				
2.8				
2.9				
2.10				
2.11				
2.12				
2.13				
3.1				
3.2				
3.3				
3.4				
3.5				
3.6				
3.7				
3.8				
3.9				
3.10				
3.11				
3.12				
3.13				
4.1				
4.2				
4.3				
4.4				
	1	1	1	l

4.5				
4.6				
4. 7				
4.8				
4.9				
4. 10				
4.11				
4.12				
4.13				
5.1				
5.2				
5.3				
5.4				
5.5				
5.6				
5.7				
5.8				
5.9				
5.10				
5.11				
5.12				
5.13				
L	1		1	

Domain	4	3	2	1
Curriculum Connection	Skill(s) reinforced are strongly connected to the targeted skill or concept	Skill(s) reinforced are related to the targeted skill or concept	Skill(s) reinforced are prerequisite or foundation skills for the targeted skill or concept	Skill(s) reinforced in the app are not clearly connected to the targeted skill or concept
Feedback	Feedback is specific and results in improved student performance	Feedback is specific and results in improved student performance (may include tutorial aids)	Feedback is limited to the correctness of student responses and may allow students to try again	Feedback is limited to the correctness of student responses
Authenticity	Targeted skills are practiced in an authentic format/ problem-based learning environment	Some aspects of the app are presented in an authentic learning environment	Skills are practiced in a contrived game/simulation format	Skills are practiced in a rote or isolated fashion (e.g. flashcards)
Differentiation	App offers complete flexibility to alter settings to \Box meet student needs	App offers more than one degree of flexibility to adjust settings to meet student needs	App offers limited flexibility to adjust settings to meet student needs (e.g., few levels such as easy, medium, hard)	App offers no flexibility to adjust settings to meet student needs (settings cannot be altered)
User Friendliness	Students can launch and navigate within the app independently	Students need to have the teacher review how□to use the app	Students need to have the teacher review how to use the app on more than one occasion	Students need constant teacher supervision in order to use the app

Appendix-3: Instructional Quality Check for Apps

Student motivation	Students are highly motivated to use the app and select it as their first choice from a selection of related choices of apps	Students use the app as directed by the teacher	Students view the app as "more schoolwork" and may be off-task when directed by the teacher to use the app	Students avoid the use of the app or complain when use of the app is assigned
Reporting	Data is available electronically to the student and teacher as a part of the app	Data is available electronically to student on a summary page and may be screenshot to share with teacher	Data is available electronically to the student,□but is not presented□on a single summary page	The app does not contain a summary page

*Created by Harry Walker- John Hopkins University, 2010

ID	Statements	Strongly Agree (5)	Agree (4)	Neutral (3)	Disagree (2)	Strongly Disagree (1)
1	I learned fraction					
2	The images in the game helped me to learn fraction					
3	Number line help me to determine where I should place the numbers					
4	I feel better while solving fraction problems.					
5	I like to play the game on iPad.					
6	It (the app on iPad) is easy to use					
7	I feel confortable					
8	I am satisfied with Motion Math HD					
9	I am bored with it					
Comm	nent:	1	1	1	1	1

Appendix-4: Social Validity / Likert-type Scale for Students

Appendix-5: Intervention Fidelity Checklist

I

1	Teacher open app on iPad
2	Student begin from the already determined level
3	Students on task (record of time also, many times he/she prove prompts)
4	Students play 20 minutes
5	After 20 min. of playing on the app, teacher provides appropriate question sheet
6	Teachers read directions
7	Ask whether students have questions about directions
8	Students work on questions
9	When students finish, teachers collect question sheet
Comment:	

Appendix-6: Visual Analysis of the Data Set

In this paper, you are required to make visual analysis of provided data set in terms of several criterion, which are follows; level, trend, variability, immediate effects of an intervention, overlapping data, and consistency of data patterns within and between phases. Krotochwil et al. (2010) identify level as "the mean score for the data within a phase," trend as "the slope of the best-fitting straight line for the data within a phase", and variability as "the range of standard deviation of data about the best fitting straight line" (p. 18). While considering immediacy of the effect, the graduate students will look whether or not there is recognizable change between the levels of the last four data points in the baseline data series and the level of the three data points of the intervention data series. Immediate effect is the statement of the influence of the independent variable on outcome variable.

Level:

Trend:

Variability:

Immediate effects of the intervention:

Overlapping data:

Consistency of data patterns within and between phases:

	In Original Form		Not True	Slightly True	Moderate True	Mostly True	Very True
1	1	When I first looked at the app, I had the impression that learning fraction would be easy for me.					
2	2	There was something interesting at the beginning of this study that got my attention.					
3	3 Rev.	The app on iPad was difficult to understand than I would like for it to be.					
4	4	After receiving the introductory information provided by the researcher, I felt confident that I know what I was supposed to learn from this lesson.					
5	5	Completing the exercises by the app gave me a satisfying feeling of accomplishment.					
6	6	It is clear to me how the content of this material is related to things I already know.					
7	8	The app has features are eye- catching.					

Appendix-7: Instructional Materials Motivation Survey

8	10	Completing the levels within modules successfully was important to me.			
9	11	The quality of the images, and writing helped to hold my attention.			
10	12 Rev.	The practices brought by the app are so abstract that it was hard to keep my attention on it.			
11	13	As I worked on this lesson, I was confident that I could learn the content.			
12	14	I enjoyed engaging with the app so much that I would like to know more about this topic.			
13	16	The content of this material is relevant to my interests.			
14	17	The way the information is arranged on the iPad helped keep my attention.			
15	19 Rev.	The exercises provided by the app were too difficult.			
16	20	The app has things that stimulated my curiosity.			
17	21	I really enjoyed playing with the app.			
18	22 Rev.	The amount of repetition in the app caused me to get bored sometimes.			
19	25	After engaging with the app, I was confident that I would be able to pass a test on it.			
20	26 Rev.	The content provided was not			

		relevant to my needs because I already knew most of it.			
21	27	The feedback and reinforcement helped me feel rewarded for my effort.			
22	28	Illustration, images, etc., helped keep my attention on the lesson.			
23	29 Rev.	The style of writing is boring			
24	30	I could relate the content of the app to things I have seen, done, or thought about in my own life.			
25	31 Rev.	There are so many words on each scene that it is irritating.			
26	32	It felt good to successfully complete the levels within modules.			
27	33	The content of this lesson will be useful to me.			
28	34 Rev.	I could not really understand quite a bit of the material, the app.			
29	36	It was a pleasure to work on such a an app.			

Appendix-8: Permission from the App Developer

From:	Jacob Klein jacob@motionmathgames.com	
Subject:	Re: Your question to Motion Math	
Date:	April 24, 2014 at 4:52 PM	
To:	orhansimsek@mail.usf.edu	



Hi Orhan,

If you're willing to do the translation (there's not much text!), we could add Turkish in our next update. The voices however are more costly to do, so when the character says "Perfect!", etc, I'm not sure we have the budget to record those. If you have access to good recording equipment, of course, you could do that as well.

Thank you for your interest in our game, and be sure to read the published research on its efficacy.

Best,





Jacob Klein - CEO, Motion Math - jacob@motionmathgames.com - 917.568.0176

On Wed, Apr 23, 2014 at 9:36 PM, orhansimsek@mail.usf.edu <orhansimsek@mail.usf.edu> wrote:

Name	
Orhan	Simsek
Email	
orhans	simsek@mail.usf.edu
•	Yes, sign me up for the occasional Motion Math newsletter.
l'm a	
Other	
Message	
increa though resear	loctoral student at University of South Florida, and I am interested in mobile devices and apps to use mathematical skills of students with learning disabilities in math. I found Motion Math HD and I ht to use it in my dissertation. I am an international student from Turkey, and I want to conduct the rch in Turkey. However, the app not offering Turkish language. I just wonder about whether it is ble to add Turkish language.
I look	forward to hearing you.

Appendix-9: Permission for Picture Use

From: Jacob Klein jacob@motionmathgames.com Subject: Re: Your question to Motion Math Date: May 31, 2016 at 2:38 PM To: Orhan orhansimsek@mail.usf.edu

Congrats Orhan. Yes, you have my permission to use product images in your paper. I look forward to reading it.

Best, Jacob

Jacob Klein - CEO, Motion Math - 917.568.0176

On Wed, May 25, 2016 at 1:16 PM, Orhan <<u>orhansimsek@mail.usl.edu</u>> wrote: Hi Jacob;

I have finished the project and writing a paper for dissertation (PhD). I need to use two screen shoot images you provided on the web page, and I need your permission to use.

By the way, app is wonderful and students liked it.

I look forward to hearing from you

Orhan Simsek

Appendix-10: IRB Approval Letter



RESEARCH INTEGRITY AND COMPLIANCE Institutional Review Boards, FWA No. 00001669 12901 Bruce B. Downs Blvd., MDC035 • Tampa, FL 33612-4799 (813) 974-5638 • FAX(813)974-7091

July 31, 2015

Orhan Simsek College of Education Tampa, FL 33617

RE: **Expedited Approval for Initial Review** IRB#: Pro00022415 Title: Motion Math: Fraction: A Game-Based App for Students with Mathematics Learning Disability (MLD)

Study Approval Period: 7/30/2015 to 7/30/2016

Dear Mr. Simsek:

On 7/30/2015, the Institutional Review Board (IRB) reviewed and **APPROVED** the above application and all documents contained within, including those outlined below.

Approved Item(s): Protocol Document(s): Study Protocol-1

Study involves children and falls under 45 CFR 46.404: Research not involving more than minimal risk.

Consent/Assent Document(s)*: Child Assent.pdf Parental Permission Consent form.pdf *Please use only the official IRB stamped informed consent/assent document(s) found under the "Attachments" tab. Please note, these consent/assent document(s) are only valid during the approval period indicated at the top of the form(s).

It was the determination of the IRB that your study qualified for expedited review which includes activities that (1) present no more than minimal risk to human subjects, and (2) involve only procedures listed in one or more of the categories outlined below. The IRB may review research through the expedited review procedure authorized by 45CFR46.110 and 21 CFR 56.110. The research proposed in this study is categorized under the following expedited review category:

(5) Research involving materials (data, documents, records, or specimens) that have been collected, or will be collected solely for nonresearch purposes (such as medical treatment or diagnosis).

(7) Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

[Study involves children and falls under 45 CFR 46.404: Research not involving more than minimal risk.]

As the principal investigator of this study, it is your responsibility to conduct this study in accordance with IRB policies and procedures and as approved by the IRB. Any changes to the approved research must be submitted to the IRB for review and approval via an amendment. Additionally, all unanticipated problems must be reported to the USF IRB within five (5) calendar days.

We appreciate your dedication to the ethical conduct of human subject research at the University of South Florida and your continued commitment to human research protections. If you have any questions regarding this matter, please call 813-974-5638.

Sincerely,

HAL PAR

Kristen Salomon, Ph.D., Vice Chairperson USF Institutional Review Board

ABOUT THE AUTHOR

Orhan Simsek is the author of this work. He born in Turkey and earned his bachelor degree in Turkey. After receiving scholarship from National Ministry of Education, he came to USA to continue his education in graduate level. He got masters degree from University of Texas at San Antonio, and Ph.D. from University of South Florida in the program of special education. His research interest, which are follows; game-based apps on mobile devices to improve academic skills of students with disabilities and at-risk, mobile learning and integration into regular classroom setting, and evidence based interventions for students with Mathematics Learning Disabilities.

He lives with his wife, Fatma, and his two children; Ahmet Taha, and Mert Enes in Tampa, USA.