Supersymmetry Breaking: Models of Gauge Mediation with Gauge Messengers

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Supersymmetry Breaking: Models of Gauge Mediation with Gauge Messengers

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Abstract

With the start of the LHC, it becomes increasingly important to understand the experimental signatures that discriminate different extensions of the standard model.

Supersymmetry (SUSY), in particular the Minimal Supersymmetric Standard Model (MSSM), is one such extension that is specially attractive by its simplicity and elegance. However, if this symmetry is to be realized in nature, it must be spontaneously broken.

In this work we will try to understand the most general way in which SUSY breaking can happen in renormalizable field theories and the implications that this has on the minimal extension of the standard model (MSSM) mass spectrum.

The first two chapters are the introductory material: in chapter 1 we will introduce some of the key ideas necessary to understand supersymmetric field theories, and in chapter 2 we will briefly describe the the simplest supersymmetric version of the Standard Model.

In chapter 3 we will focus on understanding the role of R-Symmetry breaking in determining the soft terms gauge mediation of supersymmetry breaking (GMSB) can lead to. To do this we consider a model where both R-symmetry and SUSY are spontaneously broken. One starts with the model proposed by Intriligator, Seiberg and Shih (ISS) and adds a (dangerous) marginal operator, which we call a meson deformation. The inclusion of this operator leads to the spontaneous breaking of R-symmetry in the vacuum. One then gauges the $SU(5)_F$ of flavour and identifies it with the MSSM GUT gauge group, thus implementing GMSB. This[1] was the second explicit example where R-symmetry was spontaneously broken in the vacuum. As in the first[2], gaugino masses

turned out to be smaller than naively expected so that a mild splitting between scalar (squark and slepton) and gaugino masses exists.

After this, a general argument[3] showed that in fact gaugino masses are always significantly smaller than scalar masses if the universe is perturbatively stable. This arguments suggests that any viable vacuum should be (perturbatively) metastable, as had been previously noticed by Murayama and Nomura.

In chapter 4, we try to explore alternatives to this scenario by considering the possibility that the vacuum doesn't break supersymmetry by F-term vevs alone, but by a having *simultaneously* non-zero F and D-terms.

It turns out that this does not happen in models where the Kahler potential is canonical, and the superpotential is a cubic polynomial in the fields, but it can happen if either of these constraints is violated.

This leads us to consider a particular example, where we study a hidden sector model with $SU(3)$ gauge group, two flavours of quarks and one singlet. The superpotential is the most general consistent with the tree-level symmetries. The R-symmetry is anomalous, however, but one can still derive selection rules that constrain the form of the effective superpotential. The only extra term that is allowed is an instanton induced contribution. This term explicitly breaks the R-symmetry, but the resulting low energy superpotential is not generic and SUSY is still spontaneously broken.

While not a complete example of GMSB, this class of hidden sector models is interesting as it does not require metastability: the tension between the spontaneous breaking of an R-symmetry and the massless R-axion is bypassed by the naturally non-generic superpotential. These models usually have both F and D-term SUSY breaking, but these two vevs are not independent: in non-Abelian theories, the D-term vevs can only be induced by the F-term vevs of fields that are not gauge singlets.

The implementation of GMSB in scenarios where the F-terms are not gauge singlets is then considered in both its direct and semi-direct forms:

In chapter 5 we deal with direct gauge mediation with gauge messengers. In this version of gauge mediation, the spontaneously broken gauge group is identified with the MSSM GUT gauge group and generically leads to tachyonic squark or slepton masses. In the particular case where the GUT gauge group is $SU(5)$, we show that this problem can be solved if there are two independent sectors where SUSY is spontaneously broken or simply by using a solution of the doublet-triplet splitting problem where the vev responsible for the spontaneous breaking of the GUT symmetry is larger than the SUSY breaking scale. In both cases the effects gauge and non-gauge messengers have to combine if a viable spectrum is to be reached.[4]

We then finish out study in chapter 6 by considering the semi-direct version of gauge mediation with gauge messengers. As it is known, gaugino masses are screened from messenger interactions, at leading order in the SUSY breaking parameter F. Because of this, gaugino soft masses will be suppressed with respect to scalar soft masses. This leads to a scenario of mildly split SUSY, i.e. scalars are at least one or two orders of magnitude heavier than gauginos. This generically leads to some extra fine-tuning to get the EW breaking scale to occur at the correct scale.

Declaration

This dissertation is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university.

In particular, chapter 3 is based on the work:

"On the Diversity of Gauge Mediation: Footprints of Dynamical SUSY Breaking"
S. Abel, J. Jaeckel, V.V. Khoze and L. Matos
JHEP 03, 017 (2009), arXiv:0812.3119 [hep-ph]

Chapter 4 is based on:

"Some examples of F and D-term SUSY breaking models"
L. Matos
arXiv:0910.0451 [hep-ph]

And chapter 5 is based on:

"Gauge Mediation with Gauge Messengers in SU(5)"
L. Matos
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This thesis does not exceed the word limit for the respective Degree Committee.

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This thesis is dedicated both to her and to my girlfriend.

Contents

Chapter 1

Introduction

"Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which 'are' there."
— Richard Feynman, 1918-1988

1.1 Invitation:

Physics, over the centuries, has successfully allowed us to construct a language in which we can begin to understand the laws that govern nature. This has been an exciting journey as the new discoveries have forced us to drastically change the way we perceive the universe.

One of these important questions appeared in particle physics after the discovery of the W-bosons. We were asked to understand what unitarized the WW cross section: if nothing canceled the dangerous contributions from the longitudinal degrees of freedom, perturbative unitarity would be lost at the scale of $\sim$ 1TeV! Since unitarity is one of the cornerstones of quantum mechanics (as the total probability for *something* to happen should be 1), its violation would have grave consequences in our understanding of particle physics.

One proposed solution was the **Higgs Mechanism**[5][1]:

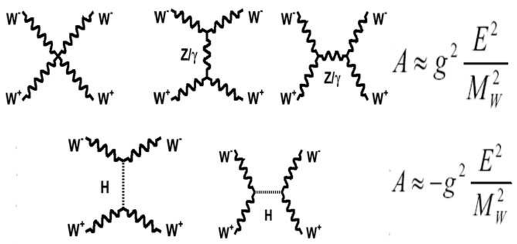


Figure 1.1: Cancellation of the terms that lead to the non-unitary behavior of the WW scattering amplitude.

This solution allowed us to understand these heavy vector bosons as gauge fields of a spontaneously broken gauge theory, which essentially made the dangerous contributions from the longitudinal degrees of freedom harmless! This idea allowed the construction of what has become the standard model (**SM**) of particle physics.

Even though the SM has been tested with a huge success, this was not the end of the story, since it does explain/include other puzzles: the existence of dark matter, neutrino masses or the origin of flavour, and it relies on the existence of a light scalar (the Higgs particle) of around 100GeV mass.

The reason why this last statement is a bit puzzling is that physicists believe the standard model for particle physics is actually an effective field theory valid only below some "cut-off" energy scale. However, being a scalar particle, the Higgs lacks a symmetry that protects its mass from receiving quadratic corrections from whatever physics exists at high energies:

$$m_H^2 = m_{H,0}^2 + \frac{k}{16\pi^2}\Lambda^2 \tag{1.1}$$

where Λ is the physical cut-off to the theory, k is some constant, and $m_{H,0}$ is the bare Higgs mass. There are some "natural" theoretical cut-offs that we can expect such as the Grand Unification (GUT) scale ($M_{GUT} \sim 10^{15}$GeV), or the scale at which gravity becomes strong, the Plank scale ($M_{Pl} \sim 10^{19}$GeV). The cancellation required between the bare mass and the quantum corrections to keep the Higgs mass close to the required $\sim$ 100GeV is some staggering $\sim$ 30 orders of magnitude!

[1]Historically, the proposal of the Higgs mechanism precedes the discovery of the W and Z particles, but it became more important after their discovery.

One natural way that such cancellations occur in nature is if there is some symmetry behind it.

Supersymmetry (SUSY) is one of the possible solutions to this need. One cool feature about it is that it is the only extension of the Poincaré group that allows for non-trivial scattering (the complete symmetry group is not a direct product of Poincaré and some internal symmetries). This is a highly non-trivial fact: it evades the Coleman-Mandula NO-GO theorem [6] by noting that one can extend the notion of group algebra to include $\boldsymbol{Z_2}$ **graded algebras** [7]:

$$[T^A, T^B] \equiv T^A * T^B - (-1)^{c_A+c_B} T^B * T^A \tag{1.2}$$

In practice, this means that, along with commutators of group generators, we also have anti-commutators. It turns out that some of these generators are naturally realized as spinorial objects, and when they act on a particle, they change its spin.

So, the way SUSY protects the Higgs mass is by putting it together (i.e in the same representation) with a fermionic superpartner (the Higgsino), and dictating that these two fields must have the same mass. As we know, fermion masses are protected by **chiral symmetry** from both quadratic *and* **linear corrections**, so that they only receive "harmless" logarithmic ones[2]:

$$m_H \sim m_{H,0} + \frac{k}{16\pi^2} Log(\frac{\Lambda}{m_Z}) \tag{1.3}$$

So SUSY essentially extends chiral symmetry from fermions to scalars, thus protecting them against the nasty quadratic and linear quantum corrections.

In fact all fields in the SM that get paired up with a superpartner whose spin differs by 1/2. This is partially bad: because we have not seen these fields in the colliders we've built so far, this means SUSY must be broken at some scale.

[2]Fermions (by dimensional analysis) cannot suffer quadratically divergent corrections to their mass. Suppose that there is a fermionic bare mass ($m_{f,0}$), in this case we could expect that $m_f \sim m_{f,0} + k_1\Lambda + k_2 m_{f,0} Log(\frac{\Lambda}{m_{f,0}})$. However, we know that all quantum corrections to the fermion bare mass must vanish in the limit that $m_{f,0} \rightarrow 0$. The logarithmic correction vanishes in this limit,but the cancellation of the linear one requires $k_1 = 0$!

The reason why this only partially bad is that below the SUSY breaking scale, all SM particles are much less sensitive to contributions to their mass coming from high energy physics than their superpartners (either due to gauge or chiral symmetry). So we have not seen these new SUSY particles because they are heavier than their SM friends.

This does give us lots of extra stuff to look for in the near future. If SUSY is to be responsible for the stabilization of the Higgs mass, at least some of these new particles should be visible in collisions at the LHC.

In fact SUSY is so powerful and interesting that it gives us a window to explore problems that are not (naively) related to collider physics as the existence of Dark Matter: because all superpartners differ in spin from the SM particles, it is possible to construct a matter parity where all SM fields have positive charge and superpartners have negative charge. This symmetry is usually called R-parity. One interesting consequence of it is that the lightest of the superpartners (LSP) must be stable, providing a natural mechanism by which the existence of Dark-Matter can be explained!

As we shall see along the way, SUSY can actually help in solving some other problems in particle physics, and even shed some light on the non-perturbative behavior of strongly coupled Yang-Mills theories.

So, hoping to have motivated that both from theoretical and experimental points of view it is interesting to study SUSY, let us look more carefully to the details of these supersymmetric worlds.

I dare not put all the books from where I learned the field theory and SUSY from: Peskin and Schroeder [8], Coleman's book [9], Weinberg's volume I and II [10, 11] are my main sources for field theory. For SUSY, I prefer Wess and Bagger [12], Martin's lectures [13], Bailin and Love's [14], Seiberg and Intrilligator's lecture notes [15, 16], Terning's book[17] and of course the unmissable Weinberg, volume III [18].

1.2 Supersymmetry

As we have said, Supersymmetry is the only non-trivial extension of the Poincaré group that allows the construction of interesting relativistic field theories. It can be most easily studied with the help of the algebra of the generators of the group. Being an extension of the Poincaré group, it has the same old momentum P_μ, and Lorentz generators $M_{\mu\nu}$, along with some new ones: Q^A_α, $\overline{Q}_{\dot{\alpha},A}$ (where A denotes the number of fermionic generators there are, $A = 1, ..., N$, and $\overline{Q}_{\dot{\alpha},A}$ is the Hermitian conjugate of Q^A_α).

These Q-generators transform as spinors under the Lorentz group and, unlike bosonic generators, they obey defining anti-commutation relations. We shall use the following conventions:

$$\begin{aligned} [A, B] &= AB - BA \\ \{A, B\} &= AB + BA \end{aligned} \tag{1.4}$$

The new relations that define the algebra of the group are:

$$\begin{aligned} \{Q^A_\alpha, \overline{Q}_{\dot{\beta}B}\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A_B \qquad \{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta} Z^{AB} \\ [P_\mu, Q^A_\alpha] &= 0 \qquad\qquad\quad [M_{\mu\nu}, Q^A_\alpha] = i(\sigma_{\mu\nu})^\beta_\alpha Q^A_\beta \end{aligned} \tag{1.5}$$

Where the σ's are the Sigma matrices[12], ϵ is the anti-symmetric symbol where $\epsilon_{12} = 1$, and $(\sigma^{\mu\nu})^\beta_\alpha = \frac{1}{4}\sigma^\mu_{\alpha\dot{\gamma}}\overline{\sigma}^{\nu\ \dot{\gamma}\beta} - (\mu \leftrightarrow \nu))$. The Z's are bosonic symmetry generators that commute with every other operator, they are known as the **central charge** and can generically be written as an anti-symmetric matrix.

In this work we will focus on the case where there is only one pair of fermionic generators Q_α, $\overline{Q}_{\dot{\alpha}}$. This case is known as $\boldsymbol{N = 1}$ **supersymmetry**. Because there is only one value the indices A,B in eq. 1.5 can take, the central charge Z vanishes in this kind of theories.

The third relation in (1.5) tells us that the momentum operator P_μ commutes with all the new generators of the algebra. This means that P^2 is a Casimir operator and so, useful to determine the irreducible representations (multiplets). Since all fields in a given multiplet must have the same P^2 eigenvalue, *all particles in the same multiplet must be mass degenerate.*

For particles of non-zero four-momentum, this algebra implies that the number of fermionic components equals the number of bosonic components (in a given multiplet).

The fermion number operator is defined as $(-1)^{N_F}$ and:

$$\begin{aligned} (-1)^{N_F}|B> &= |B> \\ (-1)^{N_F}|F> &= -|F> \end{aligned} \tag{1.6}$$

Where $|B>$ ($|F>$) is some bosonic (fermionic) state. Because of its definition, it must be that $\{(-1)^{N_F}, Q_\alpha\} = 0$.

Using this, and the cyclic properties of the trace operator:

$$\begin{aligned} Tr[(-1)^{N_F}\{Q_\alpha, \overline{Q}_{\dot\beta}\}] =& Tr[(-1)^{N_F} Q_\alpha \overline{Q}_{\dot\beta} + (-1)^{N_F} \overline{Q}_{\dot\beta} Q_\alpha] = \\ & Tr[\overline{Q}_{\dot\beta}(-1)^{N_F} Q_\alpha - \overline{Q}_{\dot\beta}(-1)^{N_F} Q_\alpha] = 0 \end{aligned}$$

On the other hand, using the explicit expression for the commutator we get:

$$0 = Tr[(-1)^{N_F}\{Q_\alpha, \overline{Q}_{\dot\beta}\}] = 2\sigma^\mu_{\alpha\dot\beta} Tr[(-1)^{N_F} P_\mu]$$

And for a multiplet with non-zero four momentum this implies:

$$Tr[(-1)^{N_F}] = N_B - N_F = 0 \qquad (1.7)$$

One could also wonder about what happens in a multiplet that has zero four momentum[3]. We shall come back to this point in a second.

Another useful property is that in global SUSY the energy of a state must be a positive semi-definite quantity:

$$\frac{1}{4} Tr[<\Phi|\{Q_\alpha, \overline{Q}_{\dot\beta}\}|\Phi>] = \frac{1}{4}||Q_\alpha|\Phi>||^2 + \frac{1}{4}||\overline{Q}_\alpha|\Phi>||^2 \geq 0$$

Using the expression for the anti-commutator we get (and $Tr[\sigma^\mu] = 2\delta^\mu_0$):

$$\frac{1}{4} Tr[<\Phi|\{Q_\alpha, \overline{Q}_{\dot\beta}\}|\Phi>] =<\Phi|P_0|\Phi>=<H> \geq 0 \qquad (1.8)$$

This result leads to a *very important check* on SUSY breaking: *unbroken global SUSY is equivalent to vanishing vacuum energy.*

$$Q_\alpha|0>=0 \iff <H>=0 \qquad (1.9)$$

[3]Note that zero four-momentum is a stronger condition than masslessness.

In fact this is so important that we shall repeat this fact in the form known as the **Witten index theorem**[19] (which we will review in more detail in a later section):

If $Tr((-1)^{N_F}) \neq 0$*, then SUSY is necessarily unbroken.*

Because $Tr((-1)^{N_F})$ can only receive contributions from states with zero four momentum, this quantity can only be non-zero if the theory contains states with vanishing energy, i.e. $< H > \equiv P^0 = 0$. This in turn means that the global vacuum of the theory has unbroken SUSY.

1.3 The building blocks:

To describe the irreducible representations of the Poincaré group we use functions in Minkowski space. Because we have extended this set of symmetries in a non-trivial way, the set of coordinates needed to describe "space-time" is increased (by two) [4]. This construction is known as **Superspace**[20], and formally consists of the coordinates $\{x^\mu, \theta, \overline{\theta}\}$, where the x^μ's are the usual Minkowski space-time coordinates and θ's are anticommuting (Grassmanian) variables[5].

Just like for the Poincaré group, irreducible representations are functions, subject to particular SUSY invariant constraints, defined in superspace. One useful way to impose these constraints is to use **superspace covariant derivatives**:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu \tag{1.10}$$

$$\overline{D}_{\dot\alpha} = \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \tag{1.11}$$

A nice way to write down functions in superspace is to expand them in the θ, $\overline{\theta}$ coordinates. Because of the anti-symmetry properties of these variables, it must be that a Taylor expansion must terminate (For $N = 1$ SUSY, the highest order term is $\theta^2\overline{\theta}^2 = \theta^\alpha\theta_\alpha\overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}$).

We will not go into the details of checking how to look for the irreducible representations, and limit ourselves to describe some of the *massless* representations of interest.

[4]One of the differences between SUSY and internal symmetries is that the space required to describe the irreducible representations is not a direct product between Minkowski and any other space.

[5]A detailed discussion of the properties of these objects and a formal definition of superspace can be found in [12, 21], and for a more hands on approach: [22]

1.3.1 Chiral fields:

A chiral superfield is one that obeys:

$$\overline{D}_{\dot{\alpha}}\Phi = 0 \tag{1.12}$$

Where Φ is a function in superspace, and $\overline{D}_{\dot{\alpha}}$ is the covariant derivative just defined.

In components, the solution is:

$$\Phi(y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{1.13}$$

Where $y = x + i\theta\sigma^m\overline{\theta}$. Even though there are two complex scalar fields, we will see that in most examples F(x) is not dynamical and can be integrated out, leaving us with a complex scalar and a Weyl spinor, and (on-shell) $N_F = N_B$.

Under a SUSY transformation, the components of a chiral field transform as:

$$\delta_\chi\phi = \sqrt{2}\chi.\psi \tag{1.14}$$

$$\delta_\psi\psi = \sqrt{2}\chi F + i\sqrt{2}\sigma^\mu.\overline{\chi}\partial_\mu\phi \tag{1.15}$$

$$\delta_\psi F = -i\sqrt{2}\partial_\mu\psi\sigma^\mu\overline{\chi} \tag{1.16}$$

Where χ is a spinor parameter, and $\overline{\chi}$ is its complex conjugate.

The important point is that the SUSY transformation of an F-term is a total derivative, so that $\int d^4xF$ is invariant under SUSY transformations.

Chiral fields have the cool property that they form a ring with respect to the usual addition and multiplication of functions (known as the **chiral ring**). This property will be extremely handy in section 1.4.

1.3.2 Vector field:

A vector field is a field that satisfies the condition:

$$V \equiv V^\dagger \tag{1.17}$$

In components this can be written as:

$$V(x,\theta,\overline{\theta}) = C(x) + i\theta\chi - i\overline{\theta}\overline{\chi} + \frac{i}{2}\theta^2(M(x)+iN(x)) - \frac{i}{2}\overline{\theta}^2(M(x)-iN(x)) - \theta\sigma^\mu\overline{\theta}v_\mu(x) + i\theta^2\overline{\theta}\overline{\lambda}(x + \frac{i}{2}\partial_m\chi(x)) - i\overline{\theta}^2\theta(\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}(x)) + \frac{1}{2}\theta^2\overline{\theta}^2(D(x) + \frac{1}{2}\Box C(x)) \tag{1.18}$$

Where C,D,M, N and v_μ are real fields. Clearly, if Λ is a chiral field, then $\Lambda+\Lambda^\dagger$ is a vector field. One of the uses for these fields is to describe gauge interactions.

Let us now look at the two following cases:

Abelian Case:

In an Abelian theory, under a local gauge transformation the vector field transforms as $v_\mu \to v_\mu + \partial_\mu k(x)$, where k(x) is a real function. We can then look at the effect of the following transformation:

$$V \to V + i(\Lambda - \Lambda^\dagger) \tag{1.19}$$

$$\Phi \to \exp(-i\Lambda)\Phi \tag{1.20}$$

Where now Λ is a chiral field and $i(\Lambda - \Lambda^\dagger)$ is taking the place of the real function k(x). If eq. 1.20 is correct, gauge invariance implies a much larger symmetry group in SUSY theories than in its non-SUSY cousins.

If $\Lambda = A + \sqrt{2}\theta\psi + \theta^2 F$, in terms of component fields we have:

$$\begin{aligned} C &\to C + i(A - A^*) \\ \chi &\to \chi + \sqrt{2}\psi \\ M + iN &\to M + iN + 2F \\ v_\mu &\to v_\mu + \partial_\mu(A + A^*) \\ \lambda &\to \lambda \\ D &\to D \end{aligned} \tag{1.21}$$

And it is comforting to see that the v_μ component indeed has the correct transformation law to be a $U(1)$ vector field.

A particular simple gauge, know as Wess-Zumino (WZ) gauge, is one where we use the freedom in eq. 1.20 to set C,χ, M and N to be zero[6]. It breaks SUSY but leaves the usual gauge invariance $v_\mu \to v_\mu + \partial_\mu k(x)$.[7]. Furthermore, the only fields that remain are v_μ, λ and D: A real vector field v_μ, a Majorana spinor λ_α, and a real scalar field D.

The generalization of the field strength is given by:

$$W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V \tag{1.22}$$

These superfields are chiral and gauge invariant:

$$\overline{D}_{\dot\beta} W_\alpha = 0 \qquad \overline{D}^2 D_\alpha (i(\Lambda - \Lambda^\dagger)) = 0 \tag{1.23}$$

In terms of components this gives:

$$W_\alpha = -i\lambda_\alpha(y) + (\delta_\alpha^\beta D(y) - \frac{i}{2}(\sigma^\mu \overline{\sigma}^{nu})_\alpha^\beta (\partial_\mu v_\nu - \partial_\nu v_\mu))\theta_\beta + \theta^2 \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \overline{\lambda}^{\dot\alpha}(y) \tag{1.24}$$

Where $y = x + i\theta\sigma\overline{\theta}$.

The non-Abelian Case

Generalizing the previous construction to non-Abelian gauge fields requires some more work, as now the generators of the gauge group have the following algebra:

$$[T^a, T^b] = i f^{abc} T^c \tag{1.25}$$

Where T^a are the generators of the group and f^{abc} are the structure constants of the group.

[6]We can only chose this gauge because this is a *massless* vector field. If it were a massive vector field *all the components* are physical (e.g. as in the Kahler potential).

[7] If we do not put any constraints on C, it is easy to see that the vacuum manifold in any SUSY theory is invariant under a gauge transformation where the gauge parameter is takes complex values, i.e. **the gauge group has been complexified**

The correct generalization of gauge transformations turns out to be;

$$exp(V') = exp(-i\Lambda^{\dagger})exp(V)exp(i\Lambda) \tag{1.26}$$

Where $\Lambda = 2g\Lambda^a T^a$, Λ is a chiral field and $V = 2gV^a T^a$.

And the correct generalization of the field strength is:

$$W_{\alpha} = \frac{1}{8g}\overline{D}^2 e^{-V}(D_{\alpha}e^V) \tag{1.27}$$

As before, in this gauge, the components of the field are: the real vector field v^a_{μ}, the Majorana spinor known as the gaugino λ^a_{μ} and the auxiliary real scalars D^a.

We can also study what happens to the components of a vector field under a SUSY transformation. We shall not write down all equations, the one that is of concern to us is the transformation of the D-term component:

$$\delta_{\chi}D = -\partial_{\mu}\overline{\psi}.\sigma^{\mu}.\chi + \overline{\chi}.\sigma^{\mu}\partial_{\mu}\overline{\psi} \tag{1.28}$$

Since the D-term component of a vector superfield transforms as a total derivative, $\int d^4xD$ is invariant under SUSY transformations.

1.4 SUSY Lagrangians:

We now come to the task of building SUSY Lagrangians. Obviously any SUSY action must be invariant under SUSY transformations.

We will now define two new quantities: $\mathcal{W}$ and K where $\mathcal{W}$ is a holomorphic function of chiral (super)fields[8] (i.e. is a chiral field) and K is a vector superfield. As we have noted, the F-term component and the D-term component of a chiral and vector

[8] When talking about chiral superfields, we will often omit the super prefix

superfields, when integrated over all space-time, give quantities that are invariant under SUSY transformations, so $\int d^4x\mathcal{W}|_{\theta^2}$ and $\int d^4xK|_{\theta^2\overline{\theta}^2}$ are SUSY invariant[9] .

K is the **Kahler potential** and it depends of the other fields in the model, $K(\phi, V, ...)$. The function $\mathcal{W}$ is split into two parts: $\mathcal{W} = W + \frac{\tau(\phi)}{16\pi i}W_\alpha W^\alpha$ where W is the **superpotential** and is a holomorphic function of chiral fields, $\frac{\tau(\phi)}{16\pi i}W_\alpha W^\alpha$ is the **kinetic term for the gauge fields**[10].

The function $\tau(\Phi)$ is a dimensionless holomorphic function of the chiral fields and its bare value is usually chosen to be $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$, where θ is the topological angle and not the superspace θ coordinate.

So

$$L = K|_{\theta^2\overline{\theta}^2} + (W + \frac{\tau}{16\pi i}W_\alpha W^\alpha)|_{\theta^2} + c.c. \tag{1.29}$$

By "c.c." we will generically mean the complex conjugate of the $X|_{\theta^2}$ term (for any X), i.e. only the chiral terms.

Or in a more action-looking form[11]:

$$S = \int d^4x(\int d^2\theta d^2\overline{\theta}K + \int d^2\theta(W + \frac{\tau}{16\pi i}W_\alpha W^\alpha) + c.c.) \tag{1.30}$$

And, for a **canonical Kahler potential**, one has:

$$K = \sum \chi_i^\dagger e^V \chi_i + \sum \Phi_i^\dagger \Phi_i \tag{1.31}$$

where χ_i is any chiral field that transforms non-trivially under the gauge transformations, Φ_i are gauge singlets and V is the vector field. [12]

Mathematically, the only constraint on the Kahler potential is that it must be a gauge invariant vector superfield, meaning that it will have an expansion similar to 1.18. Each of the components of K can be expressed as a function of the fields in the model.

[9] By $A|_{\theta^2}$ $(A|_{\theta^2\overline{\theta}^2})$ we mean the coefficient of the monomial proportional to θ^2 $(\theta^2\overline{\theta}^2)$ in the expansion of A.

[10] Note that by eq. 1.23, and by enforcing that τ is chiral, we have ensured that $\frac{\tau(\phi)}{16\pi i}W_\alpha W^\alpha$ is a chiral superfield, and the integral over space-time of its F-term is SUSY invariant.

[11] For a detailed description of Berezin integration see, for example [12], or assume that integration over a Grassman variable is equivalent to differentiation with respect to that same variable: i.e. $\int d^2\theta f(x,\theta) = \frac{d^2 f(x,\theta)}{d\theta^2}|_{\theta,\overline{\theta}=0} = f(x,\theta)|_{\theta^2}$

[12] We have absorbed the gauge coupling into the definition of the gauge generators.

Now, as we noted, under the transformation $K \to K + \Lambda + \Lambda^\dagger$, the space-time integral of the "D-term" component of K remains invariant. This transformation is known as a **Kahler transformation**. While closely related with gauge transformations, Kahler transformations are a new symmetry of the model.

Let us now look at the superpotential:

$$W = W(\Phi_i, \chi_i) \tag{1.32}$$

Is a gauge invariant function of Φ,χ but not their Hermitian conjugates.

In terms of components, one can expand this and get:

$$W(\Phi_i, \chi_i) = W(\phi_i, \chi_i) + \frac{\partial W}{\partial \Phi_i}(\Phi_i - \phi_i) + \frac{1}{2}\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}(\Phi_i - \phi_i)(\Phi_j - \phi_j) + (\Phi \leftrightarrow \chi) \tag{1.33}$$

Where $\Phi_i = \phi_i + \sqrt(2)\theta.\psi_i + \theta^2 F_{\phi_i}$.

$$W(\Phi_i, \chi_i)|_{\theta^2} = \frac{\partial W}{\partial \Phi_i} F_{\Phi_i} + \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}\Psi_{\phi_i}\Psi_{\phi_j} + (\Phi \leftrightarrow \chi) \tag{1.34}$$

In a U(1) theory there is another term that is allowed: $\chi V|_{\theta^2\overline{\theta}^2}$, where χ is some constant (in components, this is $\chi D(x)$. This term is known as the **Fayet-Iliopoulos term**[23]. Since in here the main focus will be about non-Abelian gauge theories, where this term is not allowed, we will ignore it.

From expanding eq. 1.31 it is easy to see that there is no kinetic term for the F-terms, so these fields are not dynamical. As it turns out the Lagrangian for them is so simple that they can easily be integrated out. Their equations of motion are:

$$F^\dagger_{\Phi_i} = -\frac{\partial W}{\partial \Phi_i} \tag{1.35}$$

1.5 SUSY breaking

In the case of a gauge symmetry, spontaneous symmetry breaking (SSB) happens if the vacuum is not invariant under the transformations:

$$< \delta\phi > \neq 0 \tag{1.36}$$

Where $\delta\phi$ is a gauge transformation of the field ϕ.

In the case of spontaneous breaking of SUSY the principle is the same, except that now the generators of the symmetry transformations are the Q's and $\overline{Q}$'s discussed before. Broken SUSY requires:

$$Q|0> \neq 0 \qquad or \qquad \overline{Q}|0> \neq 0$$

Which, as we know, implies that: $< H >> 0$

We are now interested in making this inequality more precise. To do this we will look at how SUSY breaking affects the transformation laws of the fields. In WZ gauge, a chiral and vector fields transform as:

$$\delta_\chi \phi = \sqrt{2}\chi\psi_\phi \tag{1.37}$$
$$\delta_\chi \psi = i\sqrt{2}\sigma^m\overline{\chi}D_m\phi + \sqrt{2}\chi F \tag{1.38}$$
$$\delta_\chi F = i\sqrt{2}\overline{\chi}\overline{\sigma}^m D_m\psi + 2igT^{(a)}\phi\overline{\chi}\overline{\lambda}^{(a)} \tag{1.39}$$
$$\delta_\chi v_\mu^{(a)} = -i\overline{\lambda}^{(a)}\overline{\sigma}^m\chi + i\overline{\chi}\overline{\sigma}^m\lambda^{(a)} \tag{1.40}$$
$$\delta_\chi \lambda^{(a)} = \sigma^{\mu\nu}\chi v_{\mu\nu}^{(a)} + i\chi D^{(a)} \tag{1.41}$$
$$\delta_\chi D^{(a)} = -\chi\sigma^m D_m\overline{\lambda}^{(a)} - D_m\lambda^{(a)}\sigma^m\overline{\chi} \tag{1.42}$$

Where D_m is the space covariant derivative ($D_m\phi = \partial_m\phi + igv_m\phi$ for a field transforming in the fundamental representation of the gauge group). Since the only vevs that do not break Lorentz invariance are those of ϕ, F and D, in a vacuum that respects these symmetries, one has:

$$\delta_\chi \psi = \sqrt{2}\chi F \tag{1.43}$$
$$\delta_\chi \lambda^{(a)} = i\chi D^{(a)} \tag{1.44}$$

This means that *SUSY is spontaneously broken if and only if F and/or D are non-zero in the vacuum.*

If perturbation theory can be applied, one can compute the vevs of these fields and check whether or not SUSY is spontaneously broken.

1.5.1 F-term SUSY breaking:

Even though it is very easy to spontaneously break a gauge symmetry, SUSY breaking is not a simple task. Assume that the superpotential is a generic function of n-variables and the Kahler potential is canonical. Then SUSY is unbroken if we can simultaneously solve all:

$$-\left(\frac{\partial W}{\partial \Phi_i}\right)^* = F_i(\phi_i) = 0 \tag{1.45}$$

Since these are n-complex equations in n-complex variables, they generically have solutions. In fact, it can be shown that if the Kahler potential is canonical, and in the absence of a F.I. term, one can always find a solution to the D-term equations along the space of solutions of the F-term equations. So *if SUSY is not broken by F-terms alone, it is not broken*[13]. This can change in cases where the solutions to the F-term equations lie at infinity (i.e. there are runaway directions to SUSY).

A generic(sufficient but not necessary) constraint due to **Nelson and Seiberg**[24], for models of F-term SUSY breaking is that they should have an R-symmetry that is spontaneously broken. An R-symmetry can be viewed as a rotational symmetry in the fermionic components of the superspace where:

$$\theta \rightarrow \theta' = e^{-i\alpha}\theta \qquad \overline{\theta} \rightarrow \overline{\theta}' = e^{i\alpha}\overline{\theta} \tag{1.46}$$

Since the superpotential $W|_{\theta^2}$ is invariant under this symmetry, it must be that the R-charge of W is 2.

By dimensional analysis, there must be at least one field X with non-zero R-charge and, because we are assuming that R-symmetry is spontaneously broken, a non-zero vev:

$$R(X) = r_x \neq 0 \qquad < X > \neq 0 \tag{1.47}$$

[13]We will look at this in more detail in the next section.

Let the remaining fields of which W depend on be called ϕ_i, and $R(\phi_i) = r_i$.

Then, by dimensional analysis, we can write the superpotential as:

$$W(\phi_i, X) = X^{2/r_x} g(y_i) \tag{1.48}$$

Where $y_i = \frac{\phi_i}{X^{r_i/r_x}}$ and g is some holomorphic function.

The F-term equations are then written as:

$$\frac{\partial W}{\partial X} = \frac{2}{r_x} X^{2/r_x - 1} g(y_i)$$
$$\frac{\partial W}{\partial y_i} = X^{2/r_x} \frac{\partial g(y_i)}{\partial y_i}$$

since $< X > \neq 0$, these are n-equations for n-1 variables, and may not have a solution.

As it turns out, it is extremely hard to build explicit examples where the R-symmetry is spontaneously broken at tree-level. The reason for this goes under the name of complexified R-symmetry transformation[14] (so it is an R-symmetry transformation where the parameter has been complexified). This transformation changes the norm of the F-terms and not just their phase[25].

$$\frac{\partial W}{\partial \phi_i} = W_i \rightarrow W_i' = e^{(2-r_i)(i\alpha - \beta)} W_i \tag{1.49}$$

By choosing the sign of β, and taking the limit $\beta \rightarrow \infty$, we can make some of the F-terms vanish asymptotically. So, as long as there are enough degrees of freedom to solve all the F-term equations for the fields with R-charge ≤ 2 or ≥ 2, one can find a complexified R-symmetry transformation such that the potential vanishes asymptotically.

As an example assume that we can solve all the F-term equations fields with R-charge ≤ 2. Then the remaining F-terms correspond to fields with R-charge $r_i > 2$. Then by choosing $\beta \rightarrow -\infty$:

$$W_i' = e^{(2-r_i)(i\alpha - \beta)} W_i \rightarrow 0 \tag{1.50}$$

So, if one wants **tree-level R-symmetry breaking**, it must be impossible to solve the F-term equations for the fields with R-charges ≤ 2 *and independently* the same must hold true for the fields with R-charge ≥ 2.

[14] Also known as the runaway problem.

Another approach is to use the **rank condition** to break SUSY. This is also an algebraic constraint on the kind of superpotential one can write down and assumes the existence of an R-symmetry. In this case the superpotential can be written as:

$$W = \sum \Phi_i f_i(\{y_j\}) \tag{1.51}$$

Where $R(\Phi_i) = 2$ and $R(y_j) = 0$, and there are n Φ-fields and q y-fields, with the constraint that $n > q$, i.e. there are more fields with R-charge 2 than fields with R-charge 0. In this case, there are n F-term equations for the Φ-fields but these depend on only $q < n$ variables, so generally they are not solvable and SUSY is spontaneously broken.

$$\frac{\partial W}{\partial \Phi_i} = f_i(y_j)$$

In these cases R-symmetry is often not broken. There are q F-term equations for the y-fields, and generically these can be solved, as one can simply set all the X_i's to zero:

$$\frac{\partial W}{\partial y_j} = X_i \frac{\partial f_i(y_j)}{\partial y_j} \tag{1.52}$$

More generically we can view these equations as an underconstrained system, since there are n-variables (the X's) and only q equations. Because of this, there will be (n-q) flat directions that can be expressed as linear combinations of X's (more generally, the existence of a flat direction can be shown in renormalizable models of F-term SUSY breaking). Determination of whether or not R-symmetry is broken requires the understanding of the mechanisms that stabilize these flat directions.[15]

1.5.2 D-term SUSY breaking:

This argument follows very closely the discussion of [12], with some more details.

In this section we will essentially ignore the presence of F.I. terms and show that if this happens (or simply consider a non-Abelian Gauge theory for which a F.I. term is not allowed), and it is possible to solve the F-term equations for finite values of the fields, one can always find a vacuum where the D-terms vanish.

[15]Since R-symmetry protects gaugino *Majorana* masses from being generated, this is important from the phenomenological point of view.

To show this, consider the explicit expression for the D-term in a global SUSY theory:

$$D^a = \phi_i^\dagger T^a_{(i)} \phi_i \tag{1.53}$$

Where one assumes a sum over the index i, and the generators are in the appropriate representation (labeled by i). Let us now assume that we have found a solution for finite values of the fields, for which $F_j(\{\phi_i\}) = 0$, and the fields have finite vevs.

This however does not uniquely fix one solution. One way to see this is to note that the F-terms have to transform under a representation of the gauge group (in particular, the same representation as the scalar part of the chiral superfield they belong to). Since they vanish in the vacuum, and are holomorphic, they are invariant not only under gauge transformation, but gauge transformations where the gauge parameter takes complex values. This means that complexified gauge transformations of the field vevs does not change the fact that the F-terms vanish in the vacuum:

$$\frac{\partial W}{\partial \phi_i} = 0 \to \frac{\partial W}{\partial \phi_i'} = (e^{i\alpha_a T^a})_{ji} \frac{\partial W}{\partial \phi_i} = 0$$

Where the α_a are complex numbers.

The D-term $D^a T^a$ represents a particular direction in the space spanned by the algebra of the gauge group, and we can perform a (gauge) rotation in this space to align the direction specified by D^a, to be along a vector that belongs to the Cartan subalgebra of the group, T^k. In this basis, there is only one non-vanishing component to the vector $D = D^a T^a$, namely $D^k T^k$ (no sum in k). This generator has eigenvalues $\mu_{k,i}$ or, dropping the k-index, μ_i.

Then the expression for the D^k becomes:

$$D^k = \phi_i^\dagger \mu_i \phi_i \tag{1.54}$$

There can now be two cases: either all the μ_i's have the same sign or they don't. Let us assume that they do, then we can perform a complexified gauge transformation along the direction given by T^k:

$$\phi_i \to e^{\mu_i \eta} \phi_i \tag{1.55}$$

Since T^k commutes with all other generators of the algebra, this will not generate any new D-term components. However the existing D^k transforms as:

$$D^k \rightarrow \phi_i \mu_i e^{2\mu_i \eta} \phi_i \tag{1.56}$$

So that if all $\mu_i > 0$, we can take $\eta < 0$, and as $\eta \rightarrow \infty$, $D^k \rightarrow 0$.

Now let us consider the case where not all the μ_i's have the same sign, then we note that D^k can be written as:

$$D^k = \frac{1}{2} \frac{\partial}{\partial \eta} \phi_i^\dagger e^{2\mu_i \eta} \phi_i \tag{1.57}$$

Now, the function $\phi_i^\dagger e^{2\mu_i \eta} \phi_i$ goes to ∞ when η goes to ∞ or $-\infty$, so that it must have a minimum somewhere between those two points. In that point, the gradient of the function vanishes, i.e. $D^k = 0$.

1.5.3 Combined F & D-term breaking:

The argument above shows that, quite generally, a necessary condition for SUSY breaking is the impossibility to solve the F-term equations for finite values of the fields. In other words: if we can solve $< F_i >= 0$ for all i and finite field vevs, then we can always find a solution where $D = 0$ and SUSY is unbroken.

A different question that can naturally arise is whether one can have a *combined* effect of F and D-term breaking in non-Abelian gauge theories. More precisely, we will be interested in finding out in what kind of models is SUSY spontaneously broken by F *and* D-term vevs. The answer to this question will be one of the main topics in chapter 4, so we will just summarize the results here.

As it turns out, roughly speaking, the only way this can happen is if there are dynamically generated terms in the superpotential and, in this case, the D-terms are not independent parameters.

Since there are few known models where this happens, and their field content is very constrained, the phenomenological applications of such models are very limited. This possibility is explored both in chapter 4 and in the last chapter.

1.6 Some results concerning O'Raifeartaigh models

We will now review some of the important results concerning the tree-level vacuum in O'Raifeartaigh (O'R) models and which will be of importance when studying gauge mediation.

By **O'R model** we mean a SUSY model with only chiral fields and that has a vacuum where SUSY is spontaneously broken (even though the original model constructed by O'Raifeartaigh spontaneously broke SUSY in the global vacuum, we will include the possibility that the "vacuum" is only metastable). Since it only has chiral fields, the model can only break SUSY through non-vanishing F-terms.

The first important result is the existence of tree-level flat directions[26].

The exact claim is that in O'R models where the superpotential is a cubic polynomial and the Kahler potential is canonical, (meta)stability of the vacuum implies the existence of a tree-level flat direction given by:

$$\phi_i = \phi_i^{(0)} + zF_i \tag{1.58}$$

where F_i is the goldstino direction in field space , and ϕ_i is the vev of the scalar superpartner of the goldstino. Of course this flat direction is lifted by quantum effects (since SUSY is broken). Note that no assumption here was made on there being an R-symmetry of the superpotential.

In some cases, when SUSY is broken because of the rank condition, it can happen that the only field that has non-vanishing R-charge and that can get a vev is the goldstino, so that the breaking of R-symmetry requires the understanding of the quantum corrections that lift this direction in field space. This brings us to the second result that was shown by Shih[27]:

In a model where the R-charges of the fields are restricted to 0 or 2, the vacuum is always stabilized at the origin of field space (i.e. R-symmetry is not spontaneously broken)[16]. So, to be able to spontaneously break R-symmetry, and generate Majorana gaugino masses, it is important that one of the fields in the model has an R-charge that is not 0 or 2.

[16]If the F-terms are not singlets under a gauged symmetry, then this result doesn't apply: R-symmetry *can* be spontaneously broken even if the R-charges are 0 or 2.

1.7 Non-perturbative SUSY breaking checks: the Witten Index

As we've seen, in SUSY theories, the number of fermionic and bosonic states must match. One exception to this rule is when we consider states of zero energy. So, answering the question of whether SUSY is broken dynamically or not, is a question of checking whether the energy of the vacuum is 0 or not. In particular if we can prove that there are states of zero energy, then SUSY must be unbroken.

There are several ways to quantify this, and they are essentially different versions of a quantity called the Witten index.

The **Witten index** is defined to be:

$$W_I = Tr((-1)^F) \tag{1.59}$$

And one can also define a **weighted Witten index**:

$$\mathcal{W}_I = Tr(C(-1)^F) \tag{1.60}$$

Where C is the charge conjugation operator, and the Tr is taken over all states in the theory.

We will look at the uses of the original version of the index. If it has a non-zero value for some value of the coupling constants, than it will take that value for all values of the coupling constants, as long as no new states of zero momenta appear or disappear (one way this might happen is when changing the values of the parameters of the model changes the asymptotic values of the potential).

The reason is that, for non-zero momenta, states with different statistics are paired up. As we smoothly change the parameters in the theory, some of the states can make the transition to having zero 4-momentum, or vice versa, but they *must do it in pairs*. For this reason, and unless non-perturbative dynamics brings about topological changes in the potential, the Witten index remains constant for *all* (non-zero) values of the parameters. It's for this reason that this provides a non-perturbative test on SUSY breaking. [17]

[17] Note that the opposite is not necessarily true: the vanishing of the Witten index may just mean that the vacuum is paired up with some massless fermionic state and SUSY is still unbroken.

The power of this index is not restricted to showing that SUSY is not broken by non-perturbative dynamics in a particular model, by doing a computation in the perturbative regime. Suppose we prove that in ”theory A” SUSY is not broken. Then SUSY is not broken in any theory whose low energy dynamics can be mapped to ”theory A”.

Consider the following example: Witten showed that the index is non-zero in SYM (with $SU(N_C)$ gauge group $W_I = N_C$), and so SUSY is not broken for any value of the gauge coupling.

Let us now take SQCD with quark masses: i.e. SYM and vector pairs of quarks such that every quark is massive. Because the Witten index does not depend of the values of the parameters, we can take the masses of the quarks to infinity. In this limit the quarks will be extremely heavy and we can integrate them out. Since the low energy dynamics of this theory is identical to SYM, we can conclude that SUSY is not spontaneously broken in SQCD with quark masses.

1.8 Regularization of SUSY theories:

Now that we understand how supersymmetric actions can be written down and what are the properties of the tree-level vacuum, we turn to the understanding of how quantum corrections can change these properties.

The first problem one encounters when attempting to study quantum corrections is that infinities seem to appear everywhere. This is because we need to do integrals that do not converge in an infinite 4-dimensional space. Even though SUSY puts extra constraints on the field theory, the same holds true for supersymmetric models.

To properly define the theory we have to introduce a regulator that renders the integrals finite and tractable. In a renormalizable theory, physical observables will not depend on this regulator, very much in the same way that in a gauge theory, a physical result cannot depend of the gauge we do our computation in.

So a regulator is a means to an end: computing physical observables. One property that it must have is to respect the symmetries of the theory that we want to keep (in some cases it is impossible to keep all the symmetries of the classical theory, and one says that *the symmetries broken by the regulator are anomalous*).

In non-SUSY theories a very popular way to regularize the integrals is **dimensional regularization**[28]. With this method, the integrals are made U.V. finite by changing the number of space-time dimensions from 4 to $4 - 2\epsilon$.

If one tries to apply this to SUSY theories, it introduces a mismatch between the gaugino and gauge boson degrees of freedom. This is bad, as the matching between fermionic and bosonic degrees of freedom is (part of) the reason why quadratic divergences are not present in SUSY theories (see also section 1.11), or in other words: it spoils the SUSY Ward identities. Because this mismatch is infinitesimal, it will not cause any new quadratic divergences, but it does change physical observables by a finite amount and the R.G. equations starting at two loops.

To regulate SUSY theories we need another method. One such possibility is **dimensional reduction (DR)** (see for example [29–31]). This is different from dimensional regularization because we keep space-time 4-dimensional, but compactify 2ϵ of these dimensions (for example into a "circle"). In this compactification the representation of the vector field breaks down to a vector in $(4 - 2\epsilon)$-dimensions plus the ϵ-scalars that transform in the adjoint representation of the gauge group[18]. To two loops this method regulates the integrals and respects the SUSY Ward identities. A proposed generalization of DR to all orders in perturbation theory was proposed in [32], and was named SDR. In this work, and because we will be mainly interested to two loop results, we will always use DR[19].

1.9 Renormalization

Renormalization essentially means to define what one means by mass and coupling constants in an interacting field theory. Take a free real scalar particle with mass m. It's very easy to write down a Lagrangian (density) that describes it:

$$L = \frac{1}{2}\phi\Box\phi - \frac{1}{2}m^2\phi^2 \tag{1.61}$$

[18] There are Kaluza-Klein modes, which have masses of the order of the 1/L, where L is the compactification scale. In DR these are ignored, so that 1/L acts as a cut-off scale

[19] In fact it will be $\overline{DR}'$ (or in some cases the scheme used to compute the NSVZ β-function), but details on what exactly this is are clarified after the next chapter

The mass is the parameter that we called m *and* is the position of the pole of the two point function (i.e. propagator):

$$< \phi(p)\phi(-p) >= \frac{i}{p^2 - m^2 + i\epsilon} \tag{1.62}$$

Now let this particle interact with other fields (or with itself). What does one mean by mass of a particle? We could mean a parameter in the Lagrangian that is the coefficient of a term $m^2\phi^2$, or the position of the pole of the two point function. In interacting field theories these two quantities no longer match. The same problem arises when trying to define a coupling constant with the help of parameters of the Lagrangian. This means that we need a set of equations (definitions) that specify exactly what we mean by mass and coupling constants of a field, these equations are the **renormalization conditions**.

Let us take a specific example to clarify this: consider the mass of a particle in an interacting quantum field theory. Generically the two point function takes the form:

$$< \phi(p)\phi(-p) >= \frac{iZ(\{parameters\}, p^2)}{p^2 - m(\{parameters\}, p^2)^2 + i\epsilon} + (regular) \tag{1.63}$$

Since the probability of a particle going from one point to another should be one, the first thing we need to do is to normalize the field:

$$\phi_r(p) = Z^{-1/2}\phi(p) \tag{1.64}$$

Where ϕ_r is the (re)normalized field. The mass of the field can then be specified to be the pole of the two point function *at some scale* μ. If we can measure this, we can simply specify:

$$m(\{parameters\}, p^2)^2|_{p^2=\mu^2} = m_r^2 \tag{1.65}$$

Where m_r is the measured mass at the energy scale μ, and is the **renormalized mass**.

A similar procedure can be adopted for all other coupling constants in the theory.

One can then write down the original (bare) Lagrangian in terms of renormalized quantities by expanding the original Lagrangian parameters around the renormalized ones:

$$L = \frac{1}{2}\phi\Box\phi - \frac{1}{2}m^2\phi\phi + interactions = \frac{1}{2}Z\phi_r\Box\phi_r - \frac{1}{2}Zm^2\phi_r^2 + interactions$$
$$= \frac{1}{2}\phi_r\Box\phi_r - \frac{1}{2}m_r^2\phi_r^2 + \frac{1}{2}\delta_Z\phi_r\Box\phi_r - \frac{1}{2}\delta_m\phi_r^2 + interactions \quad (1.66)$$

Where $\delta_Z = Z - 1$ and $\delta_m = m^2 Z - m_r^2$. This difference between renormalized and the bare parameters are called counter terms.

In this way, we have regained the definition of mass as the position of the pole of the two point function (propagator) and as a parameter in the Lagrangian. The difference with respect to the free field theory is that we now need to identify the energy scale at which we are specifying the mass, and there are “extra vertices” in the Lagrangian that we have called counter-terms.

Instead of doing this, one can choose to impose a set of rules (**scheme**) that (essentially) determines the coefficients of the counter-terms we should have so that the result of every loop calculation is finite. Examples of such schemes are MS, or $\overline{MS}$[20]. We can now explain what $\overline{\boldsymbol{DR}}'$ is(!): It is simply dimensional reduction with a subtraction scheme similar to $\overline{MS}$ with a slight twist for the ϵ-scalars mass parameter. For this mass, the subtraction scheme is chosen in such a way that physical observables (in fact the soft terms) do not depend on it[33][21].

A **renormalizable theory** is one where when we rewrite the bare Lagrangian in terms of the renormalized quantities we only get a finite number of counter-terms at every order of perturbation theory. An example of a SUSY renormalizable theory is one where the Kahler potential is canonical and the superpotential is a cubic polynomial in the fields.

1.9.1 The Renormalization Group equations

In the previous section we noted the need to renormalize the theory at some mass (energy) scale μ in order to define what we mean exactly by mass and coupling constants. When written in terms of these renormalized parameters the original Lagrangian gains counter-

[20]See for example [8] for a detailed explanation of what MS and $\overline{MS}$ are.

[21]More precisely, it is a scheme where the R.G. equations (next section) for the soft terms do not depend on this evanescent parameter. More details on this in section 1.12

terms that "absorb" these differences between bare (unrenormalized) and renormalized parameters.

Operationally, these counter-terms have to be re-computed at each order in perturbation theory so as to keep the renormalization equations valid.

One can now ask what is the physical meaning (if any) of this renormalization scale. Since the renormalization scale does not enter in the Lagrangian when written in terms of the unrenormalized fields, correlation functions (of unrenormalized fields) must not depend on them.

Let us take a simple example with one field and some dimensionless parameters collectively called "g" and the scale Λ from dimensional regularization (or a cut-off scale), then $G(x_1, x_2, ..., x_n, g, \Lambda) =< \phi(x_1)\phi(x_2)...\phi(x_n) >$ depends on g and Λ, but not on μ, the renormalization scale. We can write this correlation function in terms of renormalized fields:

$$< \phi(x_1)\phi(x_2)...\phi(x_n) >= Z^{-n/2} < \phi_r(x_1)\phi_r(x_2)...\phi_r(x_n) >= Z^{-n/2} G_r(x_1, x_2, ...x_n, g_r, \mu) \tag{1.67}$$

Where now both the renormalized fields and Z depend on the renormalized coupling constants g_r, and the renormalization scale. This dependence must be such that:

$$\frac{d}{d\mu} Z^{-n/2} G_r(x_1, x_2, ...x_n, g_r) = 0 \tag{1.68}$$

And these are the **Renormalization Group** equations[34][22]. By rearranging this a bit:

$$(\mu\frac{\partial}{\partial\mu} + \beta_r\frac{\partial}{\partial g_r} - \frac{n}{2}\gamma) G_r(x_1, x_2, .., x_n, g_r, \mu) = 0 \tag{1.69}$$

where we have used the definition for the $\boldsymbol{\beta}$ **function**:

$$\beta_r = \frac{\partial g_r}{\partial Log(\mu)} \tag{1.70}$$

[22]There are also the very similar looking Callan-Zymanzik equation. These are derived from the Ward-Identities of broken scale invariance and (in the deep Euclidean region.) have exactly the same form as these RG equations.

And **anomalous dimension**:

$$\gamma = \frac{\partial Log(Z)}{\partial Log(\mu)} \tag{1.71}$$

In a given theory (i.e. if we keep the I.R. physics constant), these equations constrain the way in which the correlation functions of the renormalized Lagrangian change as we change the renormalization scale. We shall get back to them after a small interlude.

1.9.2 The sliding scale

As we've seen in the previous section, there are flows associated with the specification of the renormalization scale and the change of the physical (renormalized) parameters of the theory. There is another way to do perturbation theory that is due to Wilson, and where this flow also arises.

Supposed that you want to study the dynamics of a system of heavy atoms in a weak external field. It will be very hard to solve the system exactly, so we may want to do an approximation based on what we know should happen: since the electrons are so much lighter than the nuclei, they respond to changes in the external field much faster than the heavy nuclei. So we can solve the equations of motion for the electrons for given nuclei positions. Calculate the field they generate (together with the external field), and then compute the equations of the motion for the nuclei in the mean field generated by the electrons and external field.

Wilson's idea about dealing with field theory is essentially the same: you separate the degrees of freedom of a system according to their momentum (or characteristic length and time of the dynamics). Then one solves the path integral for the higher energy degrees of freedom. This gives a different (more complicated) path integral in terms of only the low energy degrees of freedom. The last step is the solution of this path integral for the low energy degrees of freedom. Even though operationally one has to do approximations, this method is exact.

Let us make this a bit more precise: consider a QFT defined by some path integral with a momentum scale Λ (a momentum cut-off in *Euclidean space*), for a field ϕ with some boundary conditions, and parameters $\{g_i\}$. We can split this path integral to a sum over the several different momentum modes that the particle can have (consistent with the boundary conditions, and the cut-off).

$$Z = \int d\phi(x) e^{(-\int d^d x \mathcal{L}(\{g_i\},x))} = \Pi_{p<\Lambda}\Big(\int d\phi(p)\Big) e^{(-\int d^d x \mathcal{L}(\{g_i\},x))} \tag{1.72}$$

This gives the integral a shell-like structure where each layer corresponds to a different definite Euclidean momentum. We can now integrate out the shells of higher momentum, in terms of the lower energy degrees of freedom. This will give us a different path integral with a different cut-off but which will describe the low energy dynamics of the system. Integrating out a (sufficiently) thin shell in momentum space should not change the Lagrangian in the path integral by much:

$$Z = \Pi_{p<\Lambda}\Big(\int d\phi(p)\Big) e^{(-\int d^d x \mathcal{L}(\{g_i\},x))} = \Pi_{p<\Lambda-\epsilon}\Big(\int d\phi(p)\Big) e^{(-\int d^d x \mathcal{L}(\{g_i\},x)+\delta\mathcal{L}(\{g_i\},x))} \tag{1.73}$$

We can then compare it to the original Lagrangian we started with by reparameterizing the momentum cut-off (and space coordinates) of the low energy effective theory so that it matches with the cut-off of the original path integral[23]. If we for a moment forget that these two Lagrangians represent the same system, this gives us a map between two different path integrals that represent the same low energy dynamics:

$$\mathcal{L}(\{g_i\}, x) \to \mathcal{L}'(\{g_i'\}, x) \tag{1.74}$$

By performing a sequence of integrations of momentum shells and appropriate scalings we get a sequence of maps or a flow in the space of Lagrangians. If we now identify the cut-off scale of each of the Lagrangians with the renormalization scale of each of them, we get a flow in the space of renormalized parameters called the **Renormalization Group** (or RG) flow. This flow is governed by the differential equations we derived in the previous section. Let us now look more closely at them.

One of the quantities we defined was the beta-function for a coupling constant. As we have seen, to keep the low energy dynamics fixed, we must change the parameters of the Lagrangian as we change the renormalization scale. The beta-functions tells us exactly how this happens.

[23] These are not scale transformations. Coleman [9] has an interesting way of putting this: "Scale transformations (...) are very different animals [from the transformations of dimensional analysis that we are doing]".

Let us consider the particular Lagrangian:

$$L = L_0 + c_k O^k + ... \tag{1.75}$$

Where O^k is some operator of mass dimension k. The mass dimension of c_k is $4 - k$. If we look at the beta function for the coefficient c_k we can have three different kinds of behaviours:

$$\beta_{c_k}(\mu) <0 \tag{1.76}$$
$$\beta_{c_k}(\mu) =0 \tag{1.77}$$
$$\beta_{c_k}(\mu) >0 \tag{1.78}$$
$$\tag{1.79}$$

In the first case, the coupling constant grows in the IR (and goes to zero in the UV), so this is a **relevant operator**. The second case corresponds to a **marginal operator**, as the coefficient is the same at all mass scales. The third case corresponds to an operator that decreases in the IR and is called an **irrelevant operator**.

Let us look at this from another perspective. Suppose we've fixed our renormalization conditions and we instead ask how the correlation functions behave at lower energies. To do this we need to transform the correlation functions to momentum space and study their behavior at low momentum p[24]. In Fourier space the RG equations for the coefficient c_k can be written as:

$$(p\frac{\partial}{\partial p} - \beta_i \frac{\partial}{\partial g_i} + k - 4 - \gamma_k) c_k(\{g_i\}, p) = 0 \tag{1.80}$$

Where γ_k is the anomalous dimension of the operator O_k. Close to a **fixed point**, the beta functions vanish, and this equation can (approximately) be solved:

$$c_k(\{g_i\}, p) \approx c_k(\{g_i\}, M)(\frac{p}{M})^{k-4-\gamma_k} \tag{1.81}$$

Close to a **trivial fixed point** $\gamma_k = 0$ and we recover the familiar result that relevant operators have coefficients with *positive mass dimension*, marginal operators coefficients which are *dimensionless* and irrelevant operators have coefficients with *negative mass dimension*[25]

[24] Everything is done in *Euclidean* space, so it makes sense to talk about low momentum p.

[25] We may also have dangerous irrelevant,marginal or relevant operators! This prefix is added when a particular operator behaves differently in the UV and in the IR due to a change in anomalous

1.10 Renormalization in SUSY theories

This section is inspired in the ideas of Weinberg[18] regarding the non-renormalization theorems (first proved by Seiberg). Let us consider a SUSY theory now with the following Lagrangian density:

$$L = \phi_i^\dagger e^{-V} \phi_i|_{\theta^2\overline{\theta}^2} + (W(\phi_i) + \frac{1}{2g^2} W^\alpha W_\alpha)|_{\theta^2} + c.c. \tag{1.82}$$

Where W is the superpotential and W_α is the gauge field strength.

There are several non-dynamical fields in this Lagrangian: the auxiliary fields (F-component of chiral fields and D-components of vector fields). Their interactions are very simple, so they can be exactly integrated out.

However it is simpler to consider the theory without doing this. The reason is that the original Lagrangian has terms that have special properties (**holomorphicity**, **R-symmetry** and a **Peccei-Quinn symmetry**) that constrain the form of counter-terms that can appear in the renormalized theory. By integrating out these auxiliary fields these symmetries are no longer explicit and everything just becomes more complicated.

The **Peccei-Quinn symmetry** is the (perturbative) translational symmetry: $\frac{1}{2g^2} \rightarrow \frac{1}{2g^2} + i\chi$, and is the reason why we wrote down the coefficient of the vector field strength as $\frac{1}{2g^2}$. In the path integral, this $i\chi$ term is simply a complex phase (just like the topological θ angle) that couples to $Im(W^\alpha W_\alpha)$. Because $Im(W^\alpha W_\alpha)$ is a total derivative, it has no effect in perturbation theory, which makes the Peccei-Quinn symmetry a good (perturbative) symmetry.

We've already encountered R-symmetries. As we've seen, for an $N = 1$ SUSY theory, **R-symmetry** is a $U(1)$ symmetry where the superspace coordinates θ and $\overline{\theta}$ are charged: $R(\theta) = -R(\overline{\theta}) = 1$. If the superpotential has an R-symmetry it must then have $R(W) = 2$.

Here we shall consider the *renormalization of the Wilsonian action*, i.e. the flow of the operators as we integrate momentum shells (in a renormalizable theory where the momentum has been taken of to infinity)[26]. What this last bit means is that we only have to worry about renormalizable interactions.

dimension of the field: i.e. the sign of $k - 4 - \gamma_k$ changes as one goes from the UV to the IR. In other words: the β-function of a particular parameter changes sign along the R.G. flow.

[26]We will be mostly thinking about asymptotically free field theories.

We shall now show that perturbatively the superpotential is not renormalized at any order in perturbation theory and that the gauge coupling is renormalized only at one loop.

To do this we introduce two fictitious external fields, X and Y and consider the following auxiliary action:

$$L = \phi_i^\dagger e^{-V} \phi_i|_{\theta^2\overline{\theta}^2} + (YW(\phi_i) + \frac{1}{2} X\, W^\alpha\, W_\alpha))|_{\theta^2} + c.c. \tag{1.83}$$

The superpotential then has a (perturbative) R-symmetry where $R(Y) = 2$, $R(W_\alpha) = 1$ and $R(X) = R(\phi) = 0$, and is holomorphic (and has a Peccei-Quinn symmetry). Then, the Wilsonian effective action must respect these symmetries, even though the Lagrangian may have a prohibitively complicated form:

$$L_\mu = K_\mu(\phi, \phi^\dagger, V, X, X^\dagger, Y, Y^\dagger, ...)|_{\theta^2\overline{\theta}^2} + 2Re(B_\mu(\phi, W_\alpha, X, Y))|_{\theta^2} \tag{1.84}$$

Where we joined the renormalized effective superpotential and kinetic term for the gauge fields in the function we called B, and the subscript μ means that the effective action is considered at the scale μ. By dimensional analysis B can only be first order in Y or second order in W_α, as all other fields have R-charge 0:

$$B_\mu(\phi, W_\alpha, X, Y) = Y f_\mu(\phi, X) + h(X, \phi)\, W_\alpha\, W^\alpha \tag{1.85}$$

In principle f_μ and h_μ are general functions of X and ϕ, but the Peccei-Quinn symmetry forces them to take the form:

$$f_\mu(\phi, X) = f_\mu(\phi) \qquad\qquad h(\phi, X) = c_\mu X + l(\phi) \tag{1.86}$$

We now take the limit where x (the scalar component of X) goes to infinity and y (scalar component of Y) goes to 0. In this case there is a single term that can contribute to the operator $Y f_\mu(\phi)$ in the original Lagrangian: $2Re(Y f(\phi))$. (i.e. if we do a perturbative integration of one momentum shell for the high energy modes in this limit, there is only one vertex that can contribute to the said operator in the original theory, namely $2Re(Y f(\phi))$.) So:

$$f_\mu(\phi) = f(\phi) \tag{1.87}$$

If we set Y=0, the action has an extra symmetry where we give a charge of +1 to ϕ and 0 to all other fields. This dictates that every term in the action has equal number of ϕ and $\phi^\dagger$'s. This means that $l(\phi) = k$, a constant. And the action now takes the form:

$$L_\mu = K_\mu(\phi, \phi^\dagger, V, X, X^\dagger, Y, Y^\dagger, ...)|_{\theta^2\overline{\theta}^2} + 2Re(Y f(\phi) + (c_\mu X + k) W_\alpha W^\alpha)|_{\theta^2} \quad (1.88)$$

By inspecting the original Lagrangian (i.e. before we do the integration of the momentum shell), we see that the gauge propagators go as 1/x, while the gauge coupling goes as x, and every other propagator and vertex are x-independent. By taking the limit where y goes to zero, and counting powers of x in the the diagrams, one reaches the conclusion that there is no contribution (except for the tree-level interaction) to the term proportional to X and the constant term gets one-loop corrections only.

Non-perturbatively everything becomes more complicated, as both $U(1)$ symmetries are anomalous. In particular, the anomaly of the Peccei-Quinn symmetry is important if we are doing a calculation in an instanton background. Despite this, we can combine these two $U(1)$'s to form a non-anomalous symmetry that can be used to constrain the possible terms that can arise non-perturbatively (see Appendix A).

It turns out that for SQCD there are non-perturbative contributions: for $N_f = N_c - 1$ the main contribution is due to one instanton effects, and due to gaugino condensation for $N_f < N_c - 1$. The explicit form is:

$$W_{dyn} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{Det(\tilde{Q}Q)}\right)^{1/(N_c - N_f)} \quad (1.89)$$

Where Λ is the dynamically generated scale of the theory (more details in section 1.14).

After this long discussion we should stop to recap. the results: *in SUSY theories the Kahler potential gets renormalized at all orders in perturbation theory. The superpotential does not get renormalized in perturbation theory, and the form of the non-perturbative corrections that can appear are highly constrained by the non-anomalous symmetries of the model. The holomorphic gauge coupling only gets renormalized at one loop*[27].

[27] The holomorphic gauge coupling is the Wilsonian coupling, not the usual physical gauge coupling that appears in the 1PI action[35]. A more pedagogical discussion of the difference is done in [36], and we will also discuss this in a bit more detail in section 1.12.

1.11 An important sum rule of spontaneously broken SUSY

Unbroken SUSY requires a mass degeneracy between all component fields in the same superfield, so scalars have the same mass as fermions and gauge bosons as gauginos. If SUSY is spontaneously broken then this degeneracy will be broken as well, there is however a sum rule that is present. This takes a very simple form in the special case the Kahler potential is canonical and there are no D-term vevs: the weighted trace of the mass matrices vanishes. This is a critical property of supersymmetry, as it means that **quantum corrections do not introduce quadratic divergences to scalar masses**:

$\delta m^2 \supset$

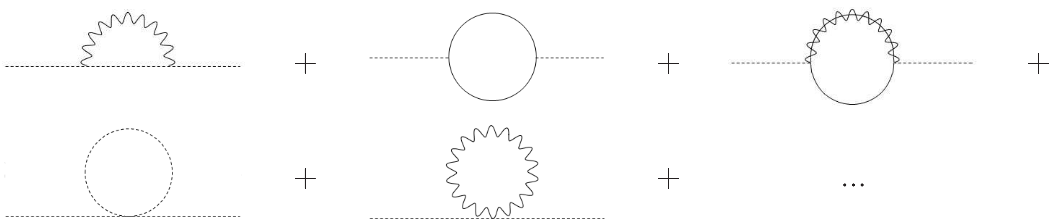


Figure 1.2: Cancellation of quadratic divergent corrections to scalar masses.

In generic field theories, the one loop corrected potential takes the form:

$$V_{tree+1-loop} = V(\phi) + k_0\Lambda^4 + k_2(\phi)\Lambda^2 + k_4(\phi)Log(\frac{\Lambda}{m(\phi)}) \quad (1.90)$$

Where the index of k represents the mass dimension of the coefficient and Λ is the cut-off of the theory.

These coefficients are constrained by SUSY: the term proportional to Λ^4 vanishes. Since the coefficient of this term (by dimensional analysis) can only depend on the spin/statistics of the particles running in the loop, it doesn't care whether SUSY is spontaneously broken or not. Because we know that when SUSY is unbroken the vacuum energy does not get corrected, this coefficient must vanish.

The second coefficient has mass dimension 2. The only way this can happen is if k_2 depends on the mass (m^2 in fact) and spin/statistics of the particles running in the loop. This coefficient must also vanish in the limit of unbroken SUSY, and it turns out that it is proportional to $Str(M^2)$.

This quantity is defined as:

$$Str(M^2) = \sum_j (-1)^j (2j+1) Tr(M_j^2) \tag{1.91}$$

Where j is the spin of the particle an M_j represents the mass matrix of all particles with spin j. Since this quantity vanishes even if SUSY is spontaneously broken, the 1-loop corrected potential only gets weak logarithmic corrections. Because masses depend on the second derivative of the potential around the minimum, in SUSY theories masses do not get corrections that depend quadratically on the cutoff scale.

If the D-terms were non-vanishing, the right hand side of the previous equation would change and the equation would be:

$$Str(M^2) = -2Tr[D] \tag{1.92}$$

Where $Tr[D] = D^a Tr[T^a]$. For non-Abelian gauge theories ($SU(N)$, $SO(N)$) the generators of the gauge group are automatically traceless. For a U(1) theory, the non-vanishing of the trace of the generator would imply the existence of a mixed $U(1)$-gravitational anomaly, so that D-term corrections to this sum rule are not of importance in the MSSM.

Another sort of corrections arises when the Kahler potential is not canonical. In this case [37], the supertrace depends on the curvature of the Kahler manifold (i.e. quantities derived from the third and higher order derivatives of the Kahler potential with respect to the chiral fields). This can happen due to the integration of some heavy fields (eg. vector multiplet of some spontaneously broken symmetry), so that if the theory is renormalizable and SUSY in the U.V. there will be no quadratic divergences.

An important detail is that there will be divergent logarithmic corrections to the scalar masses. These are cut at the scale at which we integrated the extra fields. Because of this, the coefficient of these terms can be reliably computed. In general there will also be finite threshold corrections that cannot be computed without knowledge of the U.V. theory.

If the high energy scale is "very high", these finite corrections will be subdominant[28]. If this is not the case, then a reliable computation of scalar masses requires the knowledge of the U.V. theory (up to the point where the $Str(M^2)$ vanishes). An interesting

[28] As the logarithms grow with the R.G. flow while the finite terms are invariant at one loop

example where this logarithmic term becomes important is in gauge mediation with gauge messengers.

1.12 Soft Breaking of Supersymmetry: Spurions

One way to look at soft SUSY breaking is by the spurion method[38]. A **spurion** is a fictitious (non-dynamical) superfield, which we couple to the fields in a particular model through renormalizable (and SUSY) interactions. We then allow this spurionic superfield to get vevs that are consistent with the unbroken symmetries of the model. In particular, the only vevs that are allowed by Lorentz invariance are x-independent vevs for the scalar, $\theta^2, \overline{\theta}^2$ and $\theta^2\overline{\theta}^2$ components of this spurionic field (scalar, F-term and D-term vevs). It was shown in [38] that no new types of divergences are introduced in this way. Because no quadratic divergences are introduced, any SUSY breaking term introduced in this way is called a **soft term**.

Applying this to a generic gauge theory with canonical Kahler potential and superpotential gives (at one loop):

$$\int d^4x d^4\theta Z_{\phi_i}(\{\phi_i, \phi_i^\dagger\}, V, \mu)\phi_i^\dagger e^V \phi_i + \int d^4x d^2\theta S(\{\phi_i\}, \mu) W_\alpha W^\alpha + W(\{\phi_i\}) \tag{1.93}$$

We can "promote" Z_{ϕ_i} and $S(\{\phi_i\}, \mu)$ to spurion fields, which generate the following soft terms:

$$A_{\phi_i} = Log(Z_{\phi_i})|_{\theta^2} \tag{1.94}$$

$$m^2_{\phi_i} = -Log(Z_{\phi_i})|_{\theta^2\overline{\theta}^2} \tag{1.95}$$

The A-terms (which are the coefficients of cubic scalar vertices), and the soft masses.

To compute gaugino masses we have to deal with a subtlety that we have avoided so far and is related with the fact that we are using dimensional reduction to regulate the theory.

In DR, the representation of the vector field in 4-dimensions decomposes to a d-dimensional vector and the ϵ-scalars. Both transform in the adjoint of the gauge group. Grisaru, Milewski and Zanon showed that there is an additional operator that one can add as it is gauge invariant and respects SUSY[39]:

$$\int d^4\theta O_{GMZ} = \epsilon \int d^2\theta Tr(W^\alpha W_\alpha) + h.c. \tag{1.96}$$

This harmless looking operator is called an evanescent term. The reason why it is not so harmless is that in perturbation theory this vertex appears in divergent loop diagrams. Because of this, this innocent looking operator can change the naive result one gets for physical observables [29].

Upon SUSY breaking, this operator gets renormalized, and:

$$\int d^4\theta T O_{GMZ} = \epsilon(\int d^2\theta (T|_0 + T|_{\theta^2}) Tr(W^\alpha W_\alpha) + h.c.) + T|_{\theta^2\overline{\theta}^2} A_\epsilon A^\epsilon \tag{1.97}$$

Where T is a renormalized spurion vector field. The last $T|_{\theta^2\overline{\theta}^2}$ term is the infamous ϵ-scalar mass mentioned in section 1.9. From the discussion there, it can be ignored.

At one loop, the computation of the gaugino mass yields a finite result, and this subtlety is not important, however it does make a difference at higher loops.

One then is lead to define the real (physical, and renormalized) gauge coupling[32]:

$$R \equiv S + S^\dagger + \epsilon T + \delta T^{(1)} \tag{1.98}$$

Where $\delta T^{(1)}$ is the coefficient of the counter-term for T proportional to $1/\epsilon$.

At leading order this gives:

$$R = S + S^\dagger + \frac{C(G)}{8\pi^2} Log(S + S^\dagger) - \sum_r \frac{C(r)}{8\pi^2} Log(Z_r) + O((S + S^\dagger)^{-1}) \tag{1.99}$$

Where this equation is consistent with the two loops R.G. equations for a softly broken SUSY theory.

So the **soft terms** are:

$$A_{\phi_i} = Log(Z_{\phi_i})|_{\theta^2} \tag{1.100}$$

$$m^2_{\phi_i} = -Log(Z_{\phi_i})|_{\theta^2\overline{\theta}^2} \tag{1.101}$$

$$m_\lambda = -Log(R(\{\phi_i\}, \mu))|_{\theta^2} \tag{1.102}$$

[29] This term also clarifies the difference between the holomorphic and real gauge couplings and why one is renormalized at one loop while the other is renormalized at all loop orders.

We could have also promoted the superpotential parameters to spurions. In this case we can have more terms:

- bilinear masses: $B\phi_i.\phi_j$ (where the $\phi_i.\phi_j$ is gauge invariant)
- linear terms: $C\Phi$ (where Φ is a singlet)

1.13 Analytic continuation into superspace

A method to compute the soft terms at leading order in the SUSY breaking parameters (here we will mainly focus on F-term SUSY breaking) [40, 41], called analytical continuation into superspace. This was later expanded in [32]. In here we will not go into the gory detail of checking every step but explain the main idea of the formalism.

Consider a model that has two sectors: the first (called hidden sector) consists of heavy superfields and a light superfield whose scalar and F-term components are non-zero in the vacuum: $< X >= M + \theta^2 F$. The second sector has some light fields which we call quarks (and will be the MSSM). These two sectors can only communicate through gauge interactions (so no tree-level superpotential couplings between fields in the two sectors are allowed).

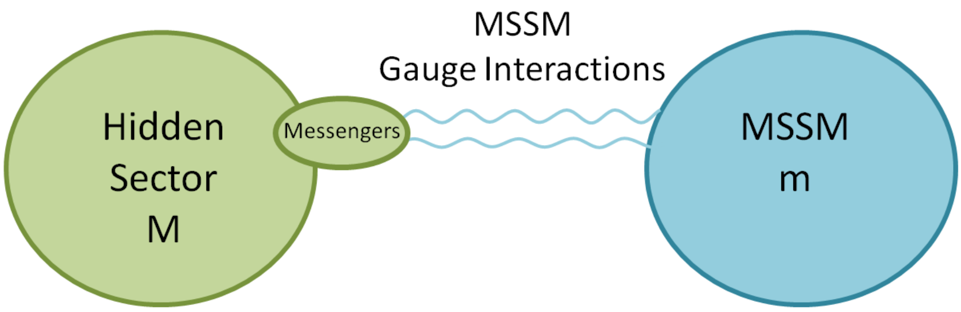


Figure 1.3: Schematic representation of a scenario where analytical continuation can be used

At low energy scales, we can integrate out the heavy fields as a function of the vevs of the light superfield. Since the masses of the particles we are integrating out must be much larger than the masses of the light particles, we need[30]:

$$M - F/M \gg m \tag{1.103}$$

[30]Note that this implies the no-tachyon condition $\frac{F}{M^2} < 1$.

This equation also guarantees that SUSY breaking effects in the low energy effective theory are small, and that using a SUSY formalism to describe it makes sense. Let us now consider *this theory, but with* $F = 0$.

Below the energy scale of these heavy particles, the action for the MSSM is very complicated, due to the renormalization of the Kahler potential and the real gauge coupling. Schematically:

$$K(Q, Q^\dagger, ..) = Z_{Q_i}(X, ...) Q_i^\dagger e^{2gV} Q_i + O((Q_i^\dagger Q_i)^2) \tag{1.104}$$

$$R = R(X, ...) \tag{1.105}$$

Where the quark fields are denoted by Q_i, $2gV = 2\sum_i g_i V_i$, a sum over the gauge groups under which the field Q_j is charged, $Z_{Q_j}(X, ...)$ is the wave-function renormalization for that field. The expression $O((Q_i^\dagger Q_i)^2)$ encodes all higher order gauge invariant operators that are suppressed by some power of M.

One now *assumes* that these equations are valid as superfield equations: when we turn F back on, the scalar vev X is replaced by its superfield vev in a way that is consistent with the non-anomalous symmetries of the theory. This procedure is known as **analytical continuation** to superspace[31].

So, upon analytical continuation, $Z_{Q_i}(X, ...)$ and $R(X, ...)$ will acquire θ^2 and $\theta^2\overline{\theta}^2$ components, thus acting as spurionic superfields.

As it turns out, at leading order in $\frac{F}{M^2}$, the relevant information necessary to compute the soft terms is encapsulated in the wave-function renormalization in the Kahler potential, the real gauge coupling, and the (unrenormalized) superpotential. This means that we can essentially ignore the $O((Q_i^\dagger Q_i)^2)$ terms in the Kahler potential!

Computationally, the wave-function renormalization and real gauge coupling can be determined by solving the R.G. equations for this theory, taking care to correctly match the quantities above and below the messenger mass threshold.

The soft terms (given by eq. 1.102) can then be computed by performing the analytical continuation of those quantities. Explicit examples will be given in sections 5, 6. We will not give too much emphasis on the soft bilinear parameter B (since the MSSM

[31] The nomenclature being borrowed from the context of complex analysis, where often functions (like the exponential, or the trigonometric functions) on the real axis can be analytically continued to the complex plane.

does not have singlets, no linear terms C are generated), but some comments will be made in chapter 2.3 regarding the μ, B_μ problem in gauge mediation.

The **validation of this analytical continuation** procedure can be done by performing a self-consistency check: see that the soft terms computed in this way obey the correct R.G. equations. For some cases, explicit evaluation of the relevant diagrams has confirmed the results of this method.

1.14 SYM+Matter

In this chapter we will study non-Abelian theories with vector matter, with particular emphasis to the unitary and orthogonal groups. This is based in the discussion of [15, 42].

1.14.1 Unitary Groups

Consider a theory with N_F flavors of quarks $(Q, \tilde{Q})$ transforming in the fundamental/anti-fundamental of a gauge symmetry $SU(N_C)$. We will be interested in understanding the vacuum of the theory. In the classical regime, if there is no superpotential, finding the vacuum amounts to finding the D-flat directions, i.e. the most general solutions to $D^a = 0$. In this case, the general solution is:

$$Q = \tilde{Q} = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & . & \\ & & & a_{N_F} \\ & & & \end{pmatrix} \tag{1.106}$$

For $N_F < N_C$, with a_i arbitrary. And

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & . & \\ & & & a_{N_c} \\ & & & \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & & \\ & \tilde{a}_2 & & \\ & & . & \\ & & & \tilde{a}_{N_C} \\ & & & \end{pmatrix} \tag{1.107}$$

With $|a_i|^2 - |\tilde{a}_j|^2$ =constant, for $N_F \geq N_C$.

These solutions have a gauge invariant form in terms of vevs of meson vevs $M_{ij} = Q^i.\tilde{Q}_j$, for the first case and both meson and baryon/anti-baryon vevs in the second case $M^i_j = Q^i.\tilde{Q}_j$, $B^{i_1 i_2 \ldots i_{N_C}} = Q^{i_1}Q^{i_2}...Q^{i_{N_C}}$, $\tilde{B}^{i_1 i_2 \ldots i_{N_C}} = \tilde{Q}^{i_1}\tilde{Q}^{i_2}...\tilde{Q}^{i_{N_C}}$, (and appropriate contractions of indices).

Generically, SUSY theories have the nice property that even if there is a superpotential, the vacuum manifold can be described in terms of gauge invariant operators [43]. There will be extra constraints coming from the requirement that the F-term equations have to be solved, but the light fields can always be described using gauge invariant operators.

To understand the dynamics of these fields one can use the Wilsonian action where all the heavy particles have been integrated out. If they turn out to be massless in the quantum theory they are called **moduli** fields, if they acquire a mass, they are **pseudo-moduli** fields.

This study would be very simple if we were not dealing with a gauge theory: if the low energy theory is weakly coupled, the non-renormalization theorems tell us that the superpotential is not renormalized. This would mean that the F-term equations are valid even quantum mechanically (as long as the Kahler potential is well behaved). In this case, the path integral involves a sum over backgrounds with different instanton numbers and this invalidates the symmetries we used to derive the non-renormalization theorems. A generalization of these can be made (see appendix A for a review).

Another important property is the behaviour of the gauge coupling with energy. This is dictated by the **NSVZ beta function**[35, 36]:

$$\beta_g = -\frac{g^3}{16\pi^2}\frac{3N_C - N_F + N_F\gamma(g^2)}{1 - N_C\frac{g^2}{8\pi^2}} \tag{1.108}$$

This is the exact solution for the gauge coupling beta function for a SQCD theory with N_F flavours and N_C colors, and γ is the anomalous dimension of the quarks. This equation tells us that for different numbers of flavours and colors, the theory will have different properties:

$N_F < N_C$

The theory is asymptotically free (and strongly coupled in the IR). There is a dynamically generated (ADS) superpotential:

$$W_{dyn} = (N_C - N_F)(\frac{\Lambda^{(3N_C - N_F)}}{Det(M)})^{1/(N_C - N_F)} \tag{1.109}$$

So the quantum theory in this regime doesn't have a vacuum unless one adds a mass deformation to the superpotential:

$$W_{tree} = Tr[m.M] \tag{1.110}$$

Where m if a $N_F \times N_F$ matrix with non-zero determinant.

In this case the potential has N_C minima:

$$< M >= (Det(m)\Lambda^{3N_C - N_F})^{1/N_C} m^{-1} \tag{1.111}$$

Which is in agreement with the Witten index (recall the argument of section 1.7).

$N_F = N_C$

As we've seen, there are no dynamically generated terms in the superpotential. But, it turns out, the quantum moduli space is deformed:

$$Det(M) - B\tilde{B} = 0 \rightarrow Det(M) - B\tilde{B} = \Lambda^{2N_c} \tag{1.112}$$

This can be seen by adding the superpotential: $W_{tree} = Tr[mM] + bB + \tilde{b}\tilde{B}$, where we give a large mass to one of the mesons. In this case the low energy theory must be the same as in the previous case. Then, by taking the limit $m, b, \tilde{b} \rightarrow 0$ one gets the result shown.

One easy way to implement this is to put it as a constraint in the superpotential with the help of a Lagrange multiplier.

This deformation removes all the singular points of the classical vacuum manifold (e.g the origin of field space)to create a smooth quantum moduli space. Chiral symmetry is everywhere broken. For regions close to the origin of field space the theory is strongly coupled and should be thought of as confining. The dynamical fields are the gauge invariant M, B and $\tilde{B}$.

Far away, the theory it is better understood as Higgsed. This is similar to QCD in the sense that chiral symmetry is broken due to dynamical effects.

$N_F = N_C + 1$:

For this case, the classical moduli space is exact, inspection of the anomaly equations for the global symmetries, indicates that the singularities of the Kahler potential are not smoothed out, but that the theory is described by massless composite mesons and baryons. Again there is a smooth transition from a Higgs phase to a confined phase (in this case without chiral symmetry breaking) according to how far away of the origin the vacuum is.

$N_C + 2 \leq N_F$:

If $N_C + 2 \leq N_F \leq 3N_C$ the theory is strongly coupled for field vevs close to the origin, and this strongly coupled region can be understood in terms of dual theories (which we will discuss later).

For $N_F > 3N_C$ the theory is not asymptotically free and can only make sense as an effective low energy QFT.

1.14.2 Orthogonal Groups

The discussion of orthogonal groups is slightly more subtle. Since we're assuming SUSY is not broken, the classical moduli space can be described using gauge invariant operators, as in the previous case. This means that for $N_F < N_C$ we need "Meson" degrees of freedom ($M \sim QQ$, with appropriate contractions), and for $N_F \geq N_C$ we need both "Meson"($M \sim QQ$ as before) and "Baryon" ($B \sim Q^{N_C}$, with appropriate contractions) degrees of freedom.

The discussion of whether or not there are dynamical contribution to the superpotential goes through in the same way (but with different group theory factors) as for Unitary groups. For $N_C = 3$ there is no dynamically generated superpotential for any N_F. For $N_C \geq 4$, and $N_F \leq N_c - 2$, there is a dynamically generated superpotential generated of the form:

$$W = \frac{1}{2}(N_C - N_f - 2)(\frac{16\Lambda^{3N_C - N_F - 6}}{Det(M)})^{1/(N_C - N_F - 2)} \tag{1.113}$$

For $N_F \geq N_C - 2$ no dynamically generated superpotential can be generated. If $N_F < 3(N_C - 2)$, the theory is asymptotically free, and becomes strongly coupled at low energies, the dynamics is more easily described with the help of a dual theory. Theories with more flavors only make sense as effective field theories.

One difference is that orthogonal groups are not centerless in even dimensions. This has the consequence that there are different (inequivalent) branches with different solutions. Take, for example, $N_F = N_C - 4$. On a general point in the moduli space, the vevs of the quarks break the $SO(N)$ gauge group to $SO(4)$. The center of this group consists of two elements: 1 and -1 (where 1 is the identity in the group)[32].

Now, in $SO(4)$, the gauge coupling essentially behaves as two independent quantities. Another way to see this is to note that $SO(4)$ is isomorphic to $SU(2) \times SU(2)$. And the dynamical contribution to the superpotential due to gaugino condensation breaks up in two. The contribution to the superpotential has the form:

$$W_{dyn} = \frac{1}{2}(\epsilon_L + \epsilon_R)(\frac{16\Lambda^{2(N_C-1)}}{Det(M)})^{1/2} \tag{1.114}$$

For this particular case, when $N_F = N_C - 4$, the ϵ_R, ϵ_L correspond to the two roots of unity. So that in one of the branches (where the two ϵ are equal), there is a dynamically generated superpotential, while if the two ϵ have opposite signs, there is none due to the "destructive interference" of the two terms.

1.15 Seiberg Duality:

As we saw, some of the theories of the previous sections become strongly coupled in the IR, so that a perturbative treatment of their dynamics is not possible. Fortunately in some cases one can find a "dual" description that is more tractable.

Let us explain what is meant by "dual". In these cases, we shall say that two theories are dual to each other if they both tend to fixed points in the IR, and the physics (scaling dimensions, correlation functions,...) of these (conformal) fixed points can be mapped

[32]Note that it is only in an even number of dimensions that both these elements belong to the group, as $Det(-1) = (-1)^N$

to each other (so strictly speaking these two theories are only dual to each other at these fixed points)[33].

In his work Seiberg argued that a SQCD theory with N_F flavors and N_C colors, with $N_F \geq N_C + 2$ is dual to another SQCD-like theory with N_F flavors and $N = N_F - N_C$ colors[44, 45]!

Let us explain this a little better: The first (**electric**) theory is SQCD with a global $SU(N_F)$ flavor symmetry and a $SU(N_C)$ gauge symmetry, and $N_C + 2 \leq N_F \leq \frac{3}{2}N_C$. It contains quarks/anti-quarks $(Q, \tilde{Q})$ that are fundamentals/anti-fundamentals under these two symmetry groups. This theory is asymptotically free and (is conjectured) flows to a fixed point in the IR. The gauge invariant operators of this theory are baryon/anti-baryon and meson operators: $B_E^{i_1 \ldots i_{N_c}} = Q^{i_1} \ldots Q^{i_{N_c}}$ (anti-baryons $\tilde{B}_E$ have $Q_i \to \tilde{Q}_i$) and $(M_E)^i_j = Q^i.\tilde{Q}_j$ [34]

The second (**magnetic**) theory consists of a SQCD theory with $SU(N_F)$ flavors symmetry and $N = N_F - N_C$ colors, with N_F and N_C in the same range as before. The field content consists of Mesons (M^i_j) that transform as the (adjoint $\oplus$ singlet) representations of the flavor group (and are singlets under the gauge group), and also quarks/anti-quarks $(q, \tilde{q})$ that are fundamentals/anti-fundamentals of the symmetry groups. The theory is IR free and goes to strong coupling in the UV. The superpotential is: $W = \lambda q.M.\tilde{q}$. The gauge invariant degrees of freedom are the mesons M^i_j and also baryons $B_M = q^{i_1} \ldots q^{i_{N_c}}$ (anti-baryons and $\tilde{B}_M$) and the magnetic mesons $(M_M)^i_j = q^i.\tilde{q}_j$.

What Seiberg showed is that there is a map between the behavior of the IR fixed points of the two different theories, so that the physics they describe is (in this sense) the same.

Before we look at the map, let us define some quantities:

$$\mu^{N_F} = (-1)^{N_f - N_c} \Lambda_E^{3N_C - N_F} \overline{\Lambda}_M^{3(N_F - N_C) - N_F} \tag{1.115}$$

$$C = \sqrt{-(-\mu)^{N_C - N_f} \Lambda^{3N_C - N_F}} \tag{1.116}$$

Where $\Lambda_E(\Lambda_M)$ is the dynamical scale of the electric (magnetic) theory.

[33] In special cases where we deform the the two theories in such a way that they flow from one conformal fixed point to another, the two theories can be dual to each other *along* the flow

[34] As we have seen before, these gauge invariant monomial operators are the building blocks that allow the description of the moduli space of the SUSY theory with any superpotential (that doesn't spontaneously break SUSY).

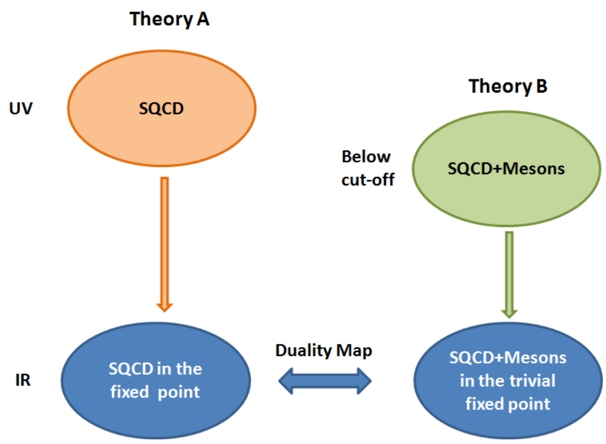


Figure 1.4: Seiberg Duality for $N_C + 2 \leq N_F \leq \frac{3}{2}N_C$

The map is then[35]:

$$B_E^{i_1 \ldots i_{N_C}} = C\epsilon^{i_1,\ldots i_{N_C} j_1 \ldots j_{\tilde{N}_C}} b_{j_1 \ldots j_{\tilde{N}_C}} \tag{1.117}$$

$$\tilde{B}_E^{i_1 \ldots i_{N_C}} = C\epsilon^{i_1,\ldots i_{N_C} j_1 \ldots j_{\tilde{N}_C}} \tilde{b}_{j_1 \ldots j_{\tilde{N}_C}} \tag{1.118}$$

$$(M_E)^i_j = \mu M^i_j \tag{1.119}$$

Because the fixed point of the second theory is a non-interacting fixed point (while the fixed point of the first theory happens for strong coupling), this is a strong-weak coupling duality.

Even though duality in these theories (at the fixed points) has not been proven explicitly, several very stringent tests have been made:

- The two theories must have the same global symmetries and the 't Hooft anomaly matching condition for these symmetries are satisfied;
- Matching of the moduli spaces of both theories;
- The I.R. of the dual of the dual theory matches the I.R. of the original theory;
- The duality is stable under deformations through F-components of chiral operators (i.e changes of the superpotential).

The last point is the most interesting for us: in chapter 3 we will explore a deformation of the duality map to build a phenomenologically viable model of gauge mediation.

[35]The dual of the magnetic mesons M_M is not present in the IR of the electric theory as it is massive, so we do not include them in the map.

Chapter 2

The softly broken MSSM:

"From wonder into wonder existence opens."
— Lao Tzu

2.1 The MSSM

The MSSM is the Minimal Supersymmetric Extension of the Standard Model. The fields are almost the same as those in the standard model, except that fermions and the Higgs are now in chiral multiplets and the vector bosons are now promoted to vector superfields. As it turns out, in order to give mass to both up and down type quarks, and to cancel $SU(2), U(1)$ anomalies, one has to add an extra Higgs chiral field. If we denote a field by how it transforms under the gauge groups $(SU(3), SU(2), U(1))$, the matter content of the MSSM is:

Quarks:

$$Q_i = (3, 2, \tfrac{1}{6}) \quad U_i^c = (\overline{3}, 1, -\tfrac{2}{3}) \quad D_i^c = (\overline{3}, 1, \tfrac{1}{3})$$

Leptons:

$$L_i = (1, 2, -\frac{1}{2}) \qquad\qquad E_i^c = (1, 1, 1)$$

Higgs:

$$H_u = (1, 2, \frac{1}{2}) \qquad H_d = (1, 2, -\frac{1}{2})$$

Where the superscript c means charge conjugate, and i stands for the family index.

The **vector superfields** are:

$$G = (8, 1, 1)$$
$$W = (1, 3, 1)$$
$$B = (1, 1, 1)$$

Where the 1's in the previous equation mean that the fields are singlets under that gauge group. The superpotential is taken to be:

$$W = \mu H_u H_d + y_u H_u Q U^c + y_d H_d Q D^c + y_l H_d L E^c \tag{2.1}$$

This is the minimal superpotential sufficient to produce a viable spectra for the quarks. It contains the familiar **Yukawa couplings** y_u, y_d and y_l, and the funny looking Higgs mass μ, called the **$\boldsymbol{\mu}$-term**[1].

Even though this superpotential is sufficient, it is not the most general superpotential allowed by gauge invariance. The following terms are also allowed:

$$\begin{aligned} W_{\Delta L=1} &= \lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D^c_k + \mu'^i L_i H_u \\ W_{\Delta B=1} &= \lambda^{ijk} U_i D^c_j D^c_k \end{aligned} \tag{2.2}$$

Where the first set of terms violate Lepton number and the second set violates baryon number. If these operators were allowed, they would lead to proton decaying extremely fast (way faster than the current experimental bounds). One way to solve this problem is by imposing an extra discrete Z_2 discrete symmetry, called **R-parity**, that forbids these operators:

$$R = (-1)^{2j+3B+L} \tag{2.3}$$

Where j stands for spin of the field, B is the baryon number and L is the lepton number.

[1]To make $H_u H_d$ gauge invariant, we have to contract it with an ϵ tensor

The MSSM has yet another problem for us: as we have seen, unbroken SUSY implies a degeneracy between scalar and fermion masses of particles in the same multiplet. In particular this would imply that for every fermion in the standard model, we would have a scalar with exactly the same mass. This is clearly in disagreement with observation!

This means that if SUSY is to be realized in nature, it must be spontaneously broken.

2.1.1 Electro-Weak symmetry breaking

One of the reasons why SUSY is interesting is that it is a symmetry that protects scalar masses against quadratic corrections and can thus give some explanation of why the Higgs mass is so light when compared with the GUT or the Planck scale. However, as we've just seen if SUSY is realized in nature, it must be broken.

As we want to avoid the nasty quadratic corrections to the Higgs mass, we want SUSY to be softly broken. As we have seen, a nice way to parametrize the ways this can happen is to use spurions, and the corresponding soft terms. The most general soft terms that can be generated for the MSSM are:

- **Gaugino Masses**: $M_i \lambda_i \lambda_i$, for all of the superpartners of the gauge bosons;
- **Trilinear Couplings**: $A_{ijk} \phi_i \phi_j \phi_k$, a gauge invariant cubic term in the squark, slepton and Higgs fields;
- **Scalar masses**: $m_\phi^2 \phi^\dagger \phi$ for all the slepton, squarks;
- $\boldsymbol{B_\mu}$ term is a mass term for the scalar components of the Higgs $B_\mu H_u H_d$;

To see wether or not Electro-Weak symmetry breaking occurs in our softly broken MSSM, we need to check the potential for the Higgs fields. This is given by:

$$V(H_u, H_d) = (|\mu|^2 + m_{H_u}^2)|H_u|^2 + (|\mu|^2 + m_{H_d}^2)|H_d|^2 + (B_\mu H_u H_d + c.c.) + \frac{1}{8}(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2 \tag{2.4}$$

We know that at low energies, the minimum of this potential should spontaneously break E.W. symmetry: $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. The only term in the previous equation that depends on the phases of the fields is the b-term (the m_Q^2 terms result from D-terms (loop integrals only generate D-terms...) so they must be real.) This means that we can rotate any phase away and CP is not spontaneously broken at tree-level in the Higgs sector.

Now there are two conditions that need to be imposed for the potential to have the right properties. In the absence of SUSY breaking the moduli space of any theory can be described using gauge invariant combinations of fields (D-flat directions). Since we have introduced SUSY breaking, any flat directions that existed are no longer flat, and we need to make sure that the superpotential is bounded from below. [2] This amounts to:

$$2B_\mu < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \tag{2.5}$$

Requiring that the origin in field space is not a minimum gives:

$$B_\mu^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) \tag{2.6}$$

One interesting detail that we can see from these equations is that if both $m_{H_u}^2$, $m_{H_d}^2$ are zero and $B_\mu = 0$, the equations can't be solved [3]. So that in this model spontaneous SUSY breaking is required to have E.W. symmetry breaking. The connection between these two constraints is even more evident if we require that the minimization of the potential is consistent with what we know from E.W symmetry breaking. One useful way to parametrize the way E.W. symmetry is broken is to use the ratio of the Higgs fields vevs and their overall size, or more precisely:

$$\begin{array}{c} tan(\beta) = \frac{<H_u>}{<H_d>} \\ m_z = \frac{1}{2}(g^2 + g'^2)(| < H_u > |^2 + | < H_d > |^2) \end{array} \tag{2.7}$$

At the minimum of the potential, this reads:

$$\begin{array}{c} sin(2\beta) = \frac{2B_\mu}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2} \\ m_Z^2 = \frac{|m_{H_d}^2 - m_{H_d}^2|}{\sqrt{1 - sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2 \end{array} \tag{2.8}$$

This equations are a bit strange: if no major cancellations happen, all $m_{H_d}^2$,$m_{H_d}^2$, B_μ and μ should be close to the E.W. scale:

$$m_{H_d}^2 \sim m_{H_u}^2 \sim B_\mu \sim \mu^2 \sim O(1 - 100)m_Z^2 \tag{2.9}$$

[2] In SUSY theories the potential is bounded from below, what should be read here is that there is that the messengers responsible for generating the soft terms decouple from the low energy theory, in such a way that the E.W. vacuum can be described using only the effective low energy theory with relevant or marginal operators

[3] In particular, if $m_{H_u}^2 = m_{H_d}^2$ the two equations can't be solved for any value of B_μ.

However, since μ is a SUSY preserving parameter, one is forgiven to think that its scale should be very high (e.g. GUT or Planck scale) and totally unrelated to the EW scale. This lack of understanding of the mechanism responsible for explaining the μ parameter is known as the **μ problem**[46].

One possibility is that the μ term is forbidden by some symmetry, like a $U(1)$ Peccei-Quinn symmetry and is only generated after SUSY breaking. This could explain why μ is of the same order of magnitude as all the other soft terms. In practice this can pose a problem to some of the mediation mechanisms which we shall review in the next section.

Since one can choose B_μ real, then there is no CP violation in the Higgs sector, and to a good approximation below the E.W. scale the mass eigenstates can be chosen to be eigenstates of the CP operator. Of particular interest are the two CP even Higgs fields, h and H, whose masses are given by:

$$m_{h,H} = \frac{1}{2}\Big(2\frac{B_\mu}{sin(2\beta)} + m_Z^2 \pm \sqrt{(2\frac{B_\mu}{sin(2\beta)} - m_Z^2)^2 + 8m_Z^2 B_\mu sin(2\beta)}\Big) \tag{2.10}$$

As this implies that at least one of them (by convention, h is taken to be the lightest state) has mass:

$$m_{h^0} < m_z|cos(2\beta)| \tag{2.11}$$

And we would expect a Higgs boson lighter than 91GeV, which is excluded by LEP data! Fortunately, this bound gets corrected by one loop effects:

$$\delta m_{h^0}^2 \propto \frac{y_t^2}{16\pi^2} m_t^2 Log(\frac{\overline{m}_{t1}\overline{m}_{t2}}{m_t}) \tag{2.12}$$

So that, a large top (m_t) and stop ($\overline{m}_{t1}, \overline{m}_{t2}$) masses give large corrections to the light Higgs mass which allow us to evade the experimental bounds. However this comes at the price that having the soft masses being large enough means that a larger tuning is required to keep m_Z naturally light in eq. 2.8.

In GUT models the μ problem takes the slightly different form known as the **doublet-triplet splitting problem**[47]: because all fields must form complete representations of the GUT group, there must exist a partner to the Higgs doublets. The lack of knowledge of the mechanism which explains why the rest of the Higgs GUT multiplet is absent from the low energy theory is the doublet-triplet splitting problem [4].

One reason why this is important is that: below the GUT scale, some of the vertices generated from the integration of the heavy fields allow for proton decay (dimension 6 vertices from integrating out the Higgsed vector bosons, dimension 5 operators from integrating out the heavy Higgs triplets and even lower dimension operators). While symmetries such as R-parity can be imposed that forbid the lower order operators (dimensions 3 and 4 operators), this is harder to do for dimension 5 and 6 operators. Compatibility with experimental data allows the determination very strong lower bounds on the mass of the rest of the Higgs GUT multiplet (triplets if they are $SU(5)$ fundamentals) and the energy at which GUT symmetry is broken.

Different solutions have been proposed, of which we give only some examples: references [46, 48, 49] deal with the μ-problem, we will mention some of the possible solutions for the doublet-triplet problem for theories with an $SU(5)$ gauge group in the chapter 5.

2.2 How and where is SUSY broken?

The next question that arises is then: how does this happen? Since this is still an open question, we will now address the much simpler question: how it does not happen.

SUSY cannot be spontaneously broken at tree-level within the SM model: the reason is that, as we've seen, at tree-level SUSY and vanishing of anomalies, requires that the supertrace of the mass matrix has to vanish for all particles in a given representation of the symmetry groups (i.e. same color representation, electric charge,...). In particular we know that the sum for the known down, strange and bottom quarks gives:

$$m_d^2 + m_s^2 + m_b^2 = 25GeV^2 \tag{2.13}$$

This implies that the sum of the mass of the scalar partners would have to total $50GeV^2$, which means that the mass of an individual squark cannot exceed $7GeV$! This is clearly ruled out by experiment.

This argument does not exclude the possibility of there being a extremely heavy 4th generation of quarks that would contribute to the sum rule. However, it is highly

[4] In the simplest scenario (which motivated the name), the Higgs fields transform as $SU(5)$ fundamentals. Since we do not observe the triplets at low energy, they must be very heavy.

unlikely that, even in this case, SUSY is spontaneously broken in the MSSM[47]. The argument relies on two main observations:

The first is that the only possible non-vanishing D-terms (below the E.W. scale) are associated with the broken generators of the $U(1)_{hypercharge} \times SU(2)_{EW}$, let's call them $D_{U(1)}$ and $D_{SU(2)}$ (so this does not take into account the possibility of there being some local U(1) that is broken at some higher scale with non-vanishing D-term).

The second observation is to note that if one looks at some particular linear combination of the possible eigenvalues for squarks of positive and negative electric charge one can reach the conclusion that at least one of the squarks should be lighter than the u or d quark (respectively).

The squark masses can be written as:

$$\begin{aligned} M^2_{(u)} &= \begin{pmatrix} W_{(u),ij}W^{jk}_{(u)} - \frac{g'}{6}D_{U(1)} + \frac{g}{2}D_{SU(2)} & W_{(u),ikj}W^{j}_{(u)} \\ W^{ijk}_{(u)}W_{(u),k} & W^{ij}_{(u)}W_{(u),jk} + \frac{2g'}{3}D_{U(1)} \end{pmatrix} \\ M^2_{(d)} &= \begin{pmatrix} W_{(d),ij}W^{jk}_{(d)} - \frac{g'}{6}D_{U(1)} - \frac{g}{2}D_{SU(2)} & W_{(d),ikj}W^{j}_{(d)} \\ W^{ijk}_{(d)}W_{(d),k} & W^{ij}_{(d)}W_{(d),jk} - \frac{g'}{3}D_{U(1)} \end{pmatrix} \end{aligned} \tag{2.14}$$

Where the u and d index mean up or down type quarks and W_i, W_{ij}, W_{ijk} are the first, second and third derivatives of the superpotential, indices are raised and lowered by complex conjugation.

Now we can take the following expectation value:

$$\begin{aligned} \begin{pmatrix} 0 \\ v_u^* \end{pmatrix}^{\dagger} M^2_{(u)} \begin{pmatrix} 0 \\ v_u^* \end{pmatrix} &= m_u^2 + \tfrac{2}{3}g'D_{U(1)} \\ \begin{pmatrix} 0 \\ v_d^* \end{pmatrix}^{\dagger} M^2_{(d)} \begin{pmatrix} 0 \\ v_d^* \end{pmatrix} &= m_d^2 - \tfrac{1}{3}g'D_{U(1)} \end{aligned} \tag{2.15}$$

Since this is a weighted sum over all the squark masses, at least one of the up-type squarks must be lighter than $m_u^2 + \frac{2}{3}g'D_{U(1)}$ and one of the down type quarks must be lighter than $m_d^2 - \frac{1}{3}g'D_{U(1)}$. If we now further assume that after EW-symmetry breaking quarks do not mix with gauginos, this means that either one of the up-type quarks must be lighter than the u-quark, or one of the down-type squarks must be lighter than the d-quark, which is experimentally excluded.

Both these arguments may be interpreted to suggest that SUSY is not broken in the Standard Model at tree-level. This may be a good thing: if SUSY were broken at tree-level the scale of SUSY breaking would be related with some parameter in the superpotential (or F.I. term). One could then ask why this parameter is so much smaller than the GUT or Planck scales.

Let us then look for radiative ways to communicate SUSY breaking to the MSSM!

2.3 The Mediation Mechanism:

"You see, wire telegraph is a kind of a very, very long cat. You pull his tail in New York and his head is meowing in Los Angeles. (...) And radio operates exactly the same way: you send signals here, they receive them there. The only difference is that there is no cat."
— Albert Einstein, 1879-1955

If SUSY is realized in nature, it must be broken. There are then two questions we should ask: how does this happen, and how does the MSSM know about it! In this chapter we will argue that gauge interactions are a good way to communicate SUSY breaking from the hidden sector to the MSSM.

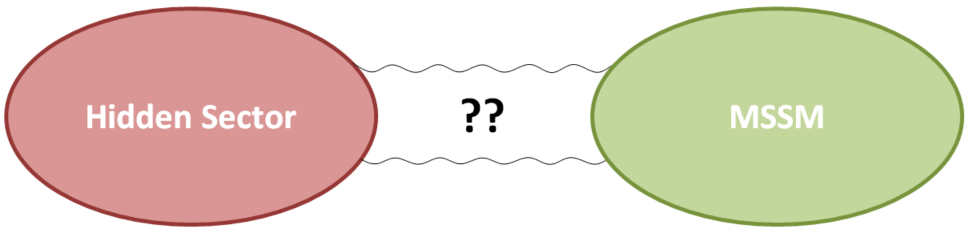


Figure 2.1: What is the interaction responsible for the mediation?

Gravity is one of the most natural ways of connecting the hidden sector to the MSSM: since many particles have a mass (or somehow know about broken scale invariance), gravity will *necessarily* play a role in determining the soft terms. However, because gravity becomes weakly coupled at low energies, how important its contribution is depends strongly on the SUSY breaking scale. The natural scenario to study this is supergravity (SUGRA)(see [12, 14, 18, 21] for some introductory lectures).

Because, at tree-level, there are no superpotential interactions between the MSSM and the hidden sector, all vertices that involve hidden sector and MSSM fields are suppressed by (a huge) M_{pl}. The soft masses will be approximately given by:

$$m_0 \sim \frac{F}{M_{Pl}} \tag{2.16}$$

So the SUSY breaking scale required to have soft terms of the order of the weak scale is quite high, $\sqrt{F} \approx 10^{10} - 10^{11}$GeV.

The problem with this scenario is that we do not really understand the dynamics at the Plank scale, so that generically we cannot compute the soft terms and see that a particular model is compatible with experiment. A very popular approximation known as **CMSSM** consists of the assumption that both gaugino ($m_{1/2}$) and squark masses (m_0) are universal and flavor blind, and the trilinear couplings are proportional to the Yukawa couplings ($A_{ijk} = A_0 y_{ijk}$) *at the GUT scale*. One can, in this case, perform the RG evolution of these parameters to low energy and perform fits to constrain the parameter space.

However, since gravity is not expected to respect global symmetries (such as flavor), it is not clear how good an assumption flavor blindness is in gravity mediation.

On the positive side, there is no μ problem, one popular solution is known as the Giudice-Masiero mechanism[48].

If the Kahler potential contains the terms:

$$\mathcal{K} \supset \frac{S^\dagger}{M_{pl}} H_u H_d + \frac{S^\dagger S}{M_{pl}^2} H_u H_d + cc \tag{2.17}$$

Where S is spurion field: $< S >= s + \theta^2 F_s$. Then these terms generate both a μ and a B_μ term to be of the correct size:

$$\mu \sim \frac{F^\dagger}{M_{Pl}} \tag{2.18}$$

$$B_\mu \sim \frac{F^\dagger F}{M_{Pl}^2} \tag{2.19}$$

So that we have that $\mu^2 \sim B_\mu \sim m_0^2$, without any big fine-tuning problems for EW symmetry breaking.

In this work, we will generically assume that the main contribution to the soft SUSY breaking terms of the MSSM comes from gauge interactions. This scenario is known as **Gauge Mediation** (see Fig. 1.3 for a scheme of how it looks like). Up to now gauge mediation has taken many forms. In the "simplest" incarnation one assumes the existence of three sectors: MSSM, messengers and hidden sector. The **hidden sector** is the sector where SUSY is broken (dynamically or at tree-level), the **messenger sector** is a set of fields that couples to the hidden sector and the MSSM gauge fields, and then there's the **MSSM**.

In this picture the soft terms are of order:

$$m_0 \sim k \frac{g^2}{16\pi^2} \frac{F}{M_{mess}} \tag{2.20}$$

Where M_{mess} is the messenger mass. Because $M_{mess} \ll M_{Pl}$, the SUSY breaking scale required to generate the right order of magnitude for the soft terms is much lower in gauge mediation than in gravity mediation: $\sqrt{F} \approx 10^5 - 10^6$GeV in this case. This means that, while present, gravity induced corrections make only a very small contribution to the SUSY soft breaking terms.

One of the immediate advantages of this approach is flavor independence: since the SUSY breaking scale is so low, the gauge interactions are flavor blind and no new large contributions to Flavor Changing Neutral Currents (FCNC) are generated.

Putting aside all our theoretical prejudices, the most important test of a theory is its experimental verification. The LHC will play a pivotal role in understanding what physics extends our current version of the Standard Model. However, in order to understand the data that will come out of this major experiment we need to know the physical observables at low energies ($\leq$ 1000GeV).

However, in SUSY theories, the natural scale for renormalization is just above the SUSY breaking scale. Even in models with a low energy SUSY breaking scale, such as gauge mediation, this is pretty high when compared with the center of mass energies at colliders. So, in order to compare theoretical results with experimental ones, we need to integrate the R.G. equations for a field theory where SUSY is (softly) broken.

There is a useful program to do this [50]. A recent study on how experimental results constrain the soft terms was done in [51]: since different mediation mechanisms have different SUSY breaking scales and result in different values for the soft terms (at low energies), this was also used to see which mediation mechanism is preferred.

Coming back to the simple scenario of GMSB, one feature that seems to permeate nearly all models of this type is **metastability**. The argument that justifies the *raison d'etre* of metastability is very simple: since MSSM gauginos have not been observed, they should be massive. Since in minimal models no other MSSM adjoints are present[5], the only way to make these fields massive is by giving them some Majorana masses.

On the other hand, in models where the superpotential is the most general consistent with the symmetries, SUSY breaking usually requires the existence of an R-symmetry[6]. But on one hand, Majorana gaugino masses are forbidden if there is an unbroken R-symmetry; on the other, the spontaneous breaking of a global R-symmetry leads to a massless Goldstone boson called R-axion, which is very problematic. So having a *general superpotential* **and** at the same time ensuring the *massiveness of the R-axion and gauginos*, implies a **metastable vacuum**.

There are different ways in which metastability can exist: the vacuum can be *perturbatively* metastable, in which case the tunneling from the metastable vacuum to a lower minimum of the theory can be described using only perturbation theory, or, if the true vacuum of the theory lies outside the regime of validity of the low-energy effective description one is using, it is *non-perturbatively* metastable.

In either case, the life-time of the vacuum must be sufficiently large (as we have not observed (so far!) the universe decaying to a SUSY ground state). This means that the amount by which the R-symmetry is broken must be very small:

$$W = W_R + \epsilon W_{R-breaking} \tag{2.21}$$

This can happen if, for example, $W_{R-breaking}$ is a *dangerously irrelevant operator* (eg. [54]), or an *irrelevant operator* as in the original ISS construction (i.e. $W_{R-breaking} \propto \frac{Det(M)^{1/N_F}}{\Lambda^{(N_F-3N)/N}}$, in the regime $N_F > 3N$). Since ϵ is a very small number, the SUSY vacuum, in these cases, is expected to be *parametrically*(like $1/\epsilon$) far away from the SUSY breaking vacuum, and is (parametrically) stable.

[5]This is not always the case, e.g. [52, 53]

[6]This symmetry usually needs to be spontaneously broken[24], but (e.g.if SUSY is broken by the rank condition) this is not a requirement.

It may also happen that the operator is ”dangerously” marginal as in a meson deformations of the ISS. One such case will be studied in chapter 3, where we give special emphasis to the role of R-symmetry in determining the MSSM soft terms.

More generically, SUSY breaking requires non-zero F and/or D-terms. In chapter 4 we will check in what conditions it is possible to have non-zero D-terms in the global vacuum of a model that does not have an F.I. term, and study some examples where this happens. In these cases it turns out that the F and the D-terms are not independent quantities, so that F-term SUSY breaking includes D-term SUSY breaking.

Broadly speaking gauge mediation models can then be separated into two main classes:

- ”normal” messenger models;
- gauge messenger models;

The first class of models has been the most widely studied in the literature: *all the messengers are chiral fields*. These may couple to the SUSY breaking spurions at tree level (**direct mediation**), or only at loop level (**semi-direct mediation**). Usually, calculable models of this type rely on ISS type constructions or retrofitting: any existing hidden sector gauge dynamics is either irrelevant (i.e. the gauge coupling becomes small at low energies), or becomes strong and has been integrated out. In both cases, the low energy dynamics can be described using some Wess-Zumino model, where SUSY is broken due to some O'R. type mechanism. These models can all be described using the framework of GGM [55].

The models in the second class haven't been so widely studied and required a generalization of the framework of GGM [56]: The main idea is that the MSSM gauge groups are the remnant of some larger semi-simple gauge group that is Higgsed at some higher energies. This Higgsing mechanism is not SUSY meaning that the Higgsed vector fields couple to the spurion through Kahler potential interactions and act as messengers. So, the main difference from the models of the previous type is that we now have *vector fields as messengers*. The study of this type of models will be the main topic of chapters 5 and 6.

It turns out that in models where the messengers are chiral fields, the vacuum should be perturbatively metastable[3]. Direct mediation models are usually simpler and give a less split spectrum than semi-direct mediation examples. As we shall see, this is because below the messenger mass scale gaugino masses are *screened* from quantum corrections coming from messenger interactions (in chapter 6 we will show that gaugino masses are not generated at three loops). This means that in semi-direct mediation gaugino masses are smaller than scalar masses, which may be problematic to get EW symmetry breaking.

In models where it is assumed that the vacuum is metastable, one needs to understand why or how it got there in the first place. In some of the known examples (e.g. the ISS), the reason why we seat in the metastable minimum is that thermal corrections in the early universe made this point in field space to be the *global minimum*, and it is only as the universe cooled down that the true SUSY minimum became visible.

Some of these models suffer from the $\boldsymbol{\mu/B_\mu}$ **problem**[49]. As we've seen, we may impose a symmetry that forbids the μ term at tree-level like the Peccei-Quinn symmetry where both Higgs carry the same (and non-zero) $U(1)$ charge. We also usually assume that at the SUSY breaking scale this symmetry is spontaneously broken. The problem then is that, generically, both μ *and* and B_μ are generated at the same order in perturbation theory. One way to see this is to look at the wave-function renormalization of the field responsible for SUSY breaking, call it X:

$$K \supset Z_X(H_u, H_d, ...)X^\dagger X \supset \frac{\partial^2 Z_X}{\partial H_u \partial H_d} H_u.H_d X^\dagger X \tag{2.22}$$

Below the SUSY breaking scale $X = x + \theta^2 F$, so that both a μ and a B_μ terms are generated:

$$\mu = \frac{\partial^2 Z_X}{\partial H_u \partial H_d} F^\dagger x \sim \frac{k}{(16\pi)^n} \frac{F^\dagger}{X^\dagger} \tag{2.23}$$

$$B_\mu = \frac{\partial^2 Z_X}{\partial H_u \partial H_d} F^\dagger F \sim \frac{k}{(16\pi)^n} \frac{F^\dagger F}{X^\dagger X} \tag{2.24}$$

Where $\frac{\partial^2 Z_X}{\partial H_u \partial H_d} \sim \frac{k}{(16\pi)^n}|X|^{-2}$, and where we've omitted the gauge indices. So its clear that both μ and B_μ are generated at the same order in perturbation theory, and B_μ is larger than μ^2, which generically causes problems for EW symmetry breaking.

As we have mentioned, in GUT theories one must also explain why there is a missing partner of the Higgs doublets at low energy. There is a variety of proposals for the solution of this problem: the sliding singlet[57, 58], the missing partner [59], also a dynamically generated potential [60]. As we shall see, the solution to this problem can have interesting implications in models of gauge mediation with gauge messengers.

In this work, for simplicity, we shall limit ourselves to cases where the GUT group is $SU(5)$. An extensive work in the context of gauge mediation with SO(10) unified theories was done in [61] where it is studied the possibility that the SO(10) unified MSSM is the Seiberg magnetic dual of another high energy field theory.

2.4 Summary of the next chapters:

So, before plunging into the main body of the thesis, we pause to summarize the structure of what follows: In the next chapter we will present the meson deformed ISS model. This is a deformation of the ISS model by a dangerously irrelevant operator. This helped us to understand the role of R-symmetry breaking and the influence of the light chiral (adjoint) messengers in determining the soft SUSY breaking terms.

Soon after this work, the decisive factor in determining the soft gaugino masses was uncovered in [3]: the vacuum should be perturbatively metastable. One of the assumptions of this argument is that SUSY is broken by F-terms alone, so chapter 4 is devoted to an attempt to evade this by having simultaneous F & D-term SUSY-breaking. Amongst other things we will see that (generically) this can only happen if there is a dynamically generated contribution to the superpotential and if the minimization of the tree-level potential implies the existence of F-terms that are not gauge singlets. As a consequence any model of combined F & D-term breaking with direct or semi-direct gauge mediation requires the understanding of gauge mediation with gauge messengers.

In chapters 5 and 6 we will review the calculation of the soft terms in models with gauge messengers. One particular problem is that, in most cases, squarks and/or sleptons turn out to be tachyonic. We will show how this can be solved either by having two hidden sectors or by extending the MSSM field content to solve the doublet-triplet splitting.

Chapter 3

Meson-Deformed ISS:

"God not only plays dice, He also sometimes throws the dice where they cannot be seen."
— Stephen Hawking, 1942-

3.1 Motivation

Clearly it is the interaction of the dynamical supersymmetry breaking (DSB) sector with the visible sector that plays a crucial role in BSM phenomenology. As we have seen, the Nelson-Seiberg theorem [24] tells us that in order to have a stable SUSY breaking vacuum, we need a spontaneously broken R-symmetry. As we were then reminded by Intriligator, Seiberg and Shih [62] (ISS), if we are willing to admit some amount of metastability, then the R-symmetry does not have to be exact but can be only approximate. Since we don't want the metastable vacuum to decay to the real one, this symmetry should be weakly violated. In any case, *R-symmetry* and *SUSY breaking* effects have to be communicated to the MSSM if a viable soft spectrum is to be achieved.

This gives us (at least) two questions: why should there be an approximate R-symmetry in the first place (or why should it be only violated by operators with small coefficients) and how does this approximate symmetry constrain the soft terms we get upon gauge mediation of the SUSY breaking effects to the MSSM. In this chapter we will explore these questions, with special emphasis on the second one.

In the context of ISS deformed models, an answer to the first question was given in[54, 63]: Murayama and Nomura noted that because the ISS model breaks supersymmetry in a magnetic Seiberg-dual formulation, the R-symmetry breaking couplings of the DSB sector can naturally be suppressed by powers of Λ_{ISS}/M_{Pl} (or some other high scale) where Λ_{ISS} is of order the Landau pole in the theory[1] due to the compositeness of some of the fields. Thus the magnetic theory can maintain an approximate R-symmetry even if the underlying electric theory has no R-symmetry and is generic. The phenomenology of this scenario is similar to standard gauge mediation although, because of the weakness of the coupling to the DSB sector, the scale of supersymmetry breaking has to be much higher than is normally assumed.

An alternative method of dealing with the R-symmetry question is to assume that it is broken spontaneously. Several examples of both one-loop and tree-level R-symmetry breaking were developed in Refs. [2, 27, 64–68] and very minimal models of *direct* mediation (i.e. where the "quarks" of the dynamical SUSY breaking sector play the role of messengers) [69–73] based on a "baryon"-deformation of the ISS model were developed in Refs. [2]. These followed earlier developments in Refs. [74–86].

A distinction between the phenomenology of the two kinds of model was drawn in Ref.[66] where it was noted that, whereas the explicit mediation models are rather similar to standard gauge mediation, the direct mediation models can differ significantly, with much heavier scalar superpartners than usual. Several questions remained however which we will address in this chapter. At first sight, one might suspect that this kind of spectrum indicates a residual approximate R-symmetry in the model, possibly because it is broken spontaneously at one-loop – indeed this would seem to be a mildly split version of the argument presented in Ref. [87].

On closer inspection however, the precise reason for the suppression of gaugino masses is a little more complicated, and as was shown later had to do with the fact that we were expanding the theory around its global minimum[3].

Moreover the ISS-like DSB sector itself may become phenomenologically important because, in direct mediation, it contains states charged under SM gauge groups that are light (typically of order 1 TeV).

In retrospective, the paper on which this chapter is based followed the story of F-term SUSY breaking to its logical conclusion: we catalogued the possible ways in which

[1]Strictly speaking it is the mass scale governing the identification of the composite meson $Q\tilde{Q}$ of the electric ISS theory with the elementary meson Φ of the magnetic theory, $Q\tilde{Q} = \Lambda_{ISS}\Phi$

supersymmetry and R-symmetry breaking ends up in the visible sector, using various exemplary models of different types of breaking and gauge mediation (direct or indirect).

Perhaps more interestingly we studied the role of the pseudo-moduli in gauge mediation, and found that they can have a relevant role to play in gauge mediation.

Generally the visible sector phenomenology ranges from a mildly split spectrum to a very heavy scalar (split-SUSY like) spectrum. In addition, in direct mediation the pseudo-Goldstone modes are expected to enter the visible spectrum, giving a rich source of new TeV mass particles associated with the SUSY breaking sector. This is similar to the effects of light pseudomoduli which have been found in [88] in the context of explicit R-symmetry breaking models.

We will also note that explicit mediation and spontaneously broken R-symmetry can be problematic in ISS-like models, due to the possibility that messengers become tachyonic. Thus the best prospect for direct gauge mediation (i.e. with explicit messengers) is explicit R-symmetry breaking of the form discussed in Ref. [63].

3.1.1 Overview

The starting point for the present chapter is, the model of Ref. [2], which introduced into the ISS superpotential a so-called "baryon deformation". This projected out some of the R-symmetry to satisfy the condition that some fields get R-charges different from 0 and 2 [27]. This baryon-deformed, or *ISSb* model, is a natural deformation of the ISS model which at tree-level has a runaway to *broken* supersymmetry. Upon adding the Coleman-Weinberg contributions to the potential, this direction is stabilized and R-symmetry is spontaneously broken. By gauging part of the flavour symmetry of the *ISSb* model, and identifying this with the parent $SU(5)$ of the Supersymmetric Standard Model (SSM), gauge mediation is implemented in a simple way.

In [89] it was shown that the Landau pole problem that usually plagues direct gauge mediation can be avoided: this is because the ISS model itself runs into a Landau pole above which a well-understood electric dual theory takes over. This results in a net reduction in the effective number of messenger flavours coupling to the SSM *above* the scale Λ_{ISS}, and this in turn prevents the Standard Model coupling running to strong coupling – a scenario dubbed "deflected gauge unification".

In the next sections we will generalize these observations to a much wider class of models. We will begin, in section 3.2, by introducing an alternative way to spontaneously break the R-symmetry of the ISS model: adding a meson term (with some singlet fields) to the superpotential. We call this the "meson-deformed" ISS model, or *ISSm* model. This bears some resemblance to the class of models considered previously in [67], although now the R-symmetry is broken radiatively rather than at tree-level, thus allowing it to be somewhat simpler.

We then, in section 3.3, go on to show how the supersymmetry breaking can subsequently be mediated, first in subsection 3.3.1 with an explicit mediation where we introduce an additional messenger sector, and then in subsection 3.3.2 with direct mediation.

In the first case the phenomenology is similar to the standard gauge mediation picture [90] – that is gauginos and scalars have similar masses governed by a single scale and related by functions of the gauge couplings and group theory indices. The absence of tachyonic messenger states requires additional *explicit* R-symmetry violating messenger mass terms, and gauge mediation is very similar to what was discussed by Murayama and Nomura [63].

In the second case, studied in subsection 3.3.2, we find that the directly mediated meson-deformed model *does* avoid tachyons without explicit R-symmetry breaking and gives phenomenology of a different sort, similar to that of the baryon-deformed model: the gaugino masses are suppressed.

We then turn to the task of understanding *why* gauginos are so light compared to the scalar spectrum, and give a particular argument. This has been generalized by Komargodski and Shih[3].

However, in this class of models there is a slight twist: since the stabilization of the potential requires the inclusion of the one-loop Coleman-Weinberg contributions, the F-terms no longer obey their tree-level relations and generate non-trivial contributions to gaugino masses at the leading order in F/M, and it is this effect which gives the main contribution to the gaugino masses.

In addition, in this regime, we shall find that the contribution from the adjoint pseudo-Goldstone modes, whose mass is lifted only at one-loop, can become important. In subsection 3.3.2 we consider this second point in more detail. We shall see that the pseudo-Goldstone modes can have a significant impact on the SSM mass spectrum. This is because their one-loop suppressed mass makes them behave like a mediating sector with a correspondingly lower messenger scale.

In order to give TeV scale SUSY breaking in the visible sector, the scale of hidden SUSY breaking is typically taken to be order $\frac{16\pi^2}{g^2}$ TeV or slightly higher. Thus the one-loop suppressed masses of the pseudo-Goldstone modes are typically around the scale of SUSY-breaking in the visible sector. This is a generic prediction: models of direct gauge mediation predict additional (with respect to the MSSM) scalar and fermion states in the visible sector, corresponding to pseudo-Goldstone modes, whose masses are close to the weak scale. We also note that the gaugino masses do not necessarily obey the usual relation where their mass ratios scale with the ratios of the coupling constants.

In section 3.4 we repeat the entire analysis for the baryon-deformed model, and find that the picture is similar. Finally, in section 3.5 we present the discussion of these results.

3.2 Meson-deformed ISS theory

As summarized in the Introduction, there are two simple types of deformation one might contemplate adding to the ISS model in order to make it spontaneously break R symmetry and generate Majorana gaugino masses in the visible sector. The first was presented in Refs. [2, 66] and corresponds to adding a baryonic operator to the original model. That possibility will be examined and extended in section 3.4. Here we will discuss an alternative possibility which is to add appropriate mesonic deformations to the original model.

We will work entirely in the low-energy magnetic (i.e. relevant to collider phenomenology) description of the ISS model [62]; it contains N_f flavours of quarks and anti-quarks, φ and $\tilde{\varphi}$ respectively, charged under an $SU(N)$ gauge group, as well as an $N_f \times N_f$ meson Φ_{ij} which is a singlet under this gauge group. This is an $SU(N)$ gauge theory with $N = N_f - N_c$ which is weakly coupled in the I.R.. The ISS superpotential is given by

$$W_{ISS} = h(\Phi_{ij}\varphi_i.\tilde{\varphi}_j - \mu_{ij}^2\Phi_{ji}) . \tag{3.1}$$

The coupling h is related to the different dynamical scales in the electric and magnetic theories (or equivalently the mapping between the two gauge couplings). The parameter

μ_{ij}^2 is derived from a Dirac mass term $m_Q Q\tilde{Q}$ for the quarks of the electric theory: $\mu^2 \sim \Lambda_{ISS} m_Q$ where the meson field $\Phi_{ij} = \frac{1}{\Lambda_{ISS}} Q_i \tilde{Q}_j$ and where Λ_{ISS} is the Landau pole of the theory. Equation 3.1 gives the tree-level superpotential of the magnetic ISS SQCD theory; there is also the non-perturbatively generated

$$W_{\rm dyn} = N \left(\frac{Det_{N_f} h\Phi}{\Lambda_{ISS}^{N_f - 3N}} \right)^{\frac{1}{N}} \tag{3.2}$$

which gives negligible[2] contributions to physics around the SUSY-breaking vacuum.

The flavor symmetry of the magnetic model is initially $SU(N_f)$. When we do *direct* mediation, see section 3.3.2, an $SU(5)_f$ subgroup of this symmetry is gauged and identified with the parent $SU(5)$ of the Standard Model, so that $N_f \geq N + 5$. On the other hand *indirect* mediation, considered in section 3.3.1, involves the introduction of *explicit* messengers and in that case N_f is a free parameter.

To visualise the the general set-up, let us first consider a simple example, which is appropriate for either case: we shall choose an $SU(2)$ gauge group for the magnetic dual theory and $N_f = 7$ flavours, with the flavour symmetry broken by μ_{ij} to $SU(2)_f \times SU(5)_f$. We will refer to this as the 2-5 model which was the also the prototype model[3] considered in Refs. [2, 66]. The matter field decomposition under the $SU(2)_f \times SU(5)_f$ flavour subgroup and the charge assignments under $SU(2)_{gauge} \times SU(2)_f \times SU(5)_f \times U(1)_B \times U(1)_R$ are given in Table 3.1. Note that we use an f-suffix to stand for "flavour" but

one should remember that in direct mediation $SU(5)_f$ contains the gauge group of the Standard Model.

In the case of the 2-5 model, by a gauge and flavour rotation, the matrix μ_{ij}^2 can be brought to a diagonal 2-5 form:

$$2 - 5 \text{ Model}: \qquad \mu_{ij}^2 = \begin{pmatrix} \mu_Y^2 \mathbf{I}_2 & 0 \\ 0 & \mu_X^2 \mathbf{I}_5 \end{pmatrix}, \quad \mu_Y^2 > \mu_X^2 \,. \tag{3.3}$$

[2]The only exception to this is the R-axion field. For this the explicit R-symmetry breaking contained in $W_{\rm dyn}$ gives a contribution to the mass [2] which importantly facilitates the evasion of astrophysical bounds [91–93]. For a recent discussion of the R-axion detection prospects at the LHC see [94].

[3]We will show momentarily that the meson-deformed ISS model actually requires a slightly more general flavour-breaking pattern which can be described by 1-1-5 and 2-2-3 models or their generalisations. For baryon-deformations all of these models, including the simplest 2-5 scenario will also work.

2-5 Model	$SU(2)_{\rm mg}$	$SU(2)_f$	$SU(5)_f$	$U(1)_B$	$U(1)_R$
$\Phi_{ij} \equiv \begin{pmatrix} Y & Z \\ \tilde{Z} & X \end{pmatrix}$	$\mathbf{1}$	$\begin{pmatrix} Adj + \mathbf{1} & \bar{\Box} \\ \Box & \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} & \Box \\ \bar{\Box} & Adj + \mathbf{1} \end{pmatrix}$	0	2
$\varphi \equiv \begin{pmatrix} \phi \\ \rho \end{pmatrix}$	$\Box$	$\begin{pmatrix} \bar{\Box} \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \bar{\Box} \end{pmatrix}$	$\frac{1}{2}$	R
$\tilde{\varphi} \equiv \begin{pmatrix} \tilde{\phi} \\ \tilde{\rho} \end{pmatrix}$	$\bar{\Box}$	$\begin{pmatrix} \Box \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \Box \end{pmatrix}$	$-\frac{1}{2}$	$-R$

Table 3.1: *The 2-5 Model. We show the ISS matter field decomposition under the gauge $SU(2)$, the flavour $SU(2)_f \times SU(5)_f$ symmetry, and their charges under the $U(1)_B$ and R-symmetry. Both of the $U(1)$ factors above are defined as tree-level symmetries of the magnetic ISS formulation in eq. 3.1. The (small) non-perturbative anomalous effects described by eq. 3.2 are not included. In the absence of baryon-deformations, the R-charges of magnetic quarks, $\pm R$, are arbitrary and can always be re-defined by considering instead a linear combination of $U(1)_B$ and $U(1)_R$ factors.*

Now consider adding the following deformation[4] involving the meson plus some additional singlet fields A, B, C:

$$W_{\rm meson-def} = h(m_1 A^2 + m_2 BC + \lambda AB\, {\rm tr}(\Phi))\,. \tag{3.4}$$

Here we chose to scale all the superpotential parameters with h. The meson deformation of the ISS model is characterised by the dimensionless coupling constant λ. In the electric-dual ISS formulation this deformation is $\sim \frac{1}{M_{Pl}} AB{\rm tr}(Q\tilde{Q})$ and thus

$$\lambda \sim \frac{\Lambda_{ISS}}{M_{Pl}} \ll 1 \tag{3.5}$$

The new singlet fields are constrained to have R-charges given in Table 3.2; these are different from 0 or 2, so spontaneous R-symmetry breaking is a possibility [27, 68].

The combined effect of $W_{ISS} + W_{\rm meson-def}$, gives a *generic* R-symmetry preserving superpotential which defines the low-energy magnetic formulation of our meson-deformed

[4] A similar deformation involving a meson operator and two singlet fields was previously considered in Ref. [64]. Their model, however, contained a runaway direction to a supersymmetric vacuum. For generic values of parameters, this makes the non-supersymmetric R-breaking vacuum of [64] short-lived and unstable to decay in the runaway direction. We will see below that our version of the meson-deformed model defined by eqs. 3.4, 3.1 with a 2-2-3 or 1-1-5 flavour patterns does not have a supersymmetric runaway, and the resulting susy-breaking vacuum is stabilised.

	$U(1)_R$
A	1
B	-1
C	3

Table 3.2: *R-charges of A, B, C singlet fields of the meson deformation in eq. 3.4.*

ISS theory. This is a self-consistent approach since, as pointed out in Ref. [66], R-symmetry breaking in the electric theory is controlled by a small parameter.[5] Terms quadratic in the meson Φ that could arise from lower dimensional irrelevant operators in the electric theory are forbidden by R-symmetry. Thus, our deformation is described by a *generic* superpotential and $W_{ISS} + W_{\text{meson-def}}$ gives its leading-order terms.

Being an exact symmetry of the tree-level magnetic superpotential, the R-symmetry of this model is actually spontaneously-broken, as we have already alluded to above. We shall consider this R-symmetry breaking before we discuss the SUSY breaking and its mediation.

First note that for any non-zero $\langle AB \rangle$ we can define an effective μ^2 term

$$\mu^2_{\text{eff}} = \mu^2 - \lambda \langle A \rangle \langle B \rangle. \tag{3.6}$$

Thus the magnetic quarks acquire vevs precisely as they do in the undeformed ISS but with μ^2 replaced by μ^2_{eff};

$$\langle \rho \rangle = \langle \tilde{\rho} \rangle = 0 \tag{3.7}$$

$$\langle \phi \tilde{\phi} \rangle = \mu^2_{\text{Y eff}}. \tag{3.8}$$

The vevs of $\text{tr}(\Phi)$ and C will simply set $\langle F_A \rangle = \langle F_B \rangle = 0$; that is

$$\langle \text{tr}(\Phi) \rangle = -\frac{2m_1 \langle A \rangle}{\lambda \langle B \rangle} \tag{3.9}$$

$$\langle C \rangle = -\frac{\lambda \langle A \rangle \langle \text{tr}(\Phi) \rangle}{m_2} = \frac{2m_1 \langle A \rangle^2}{m_2 \langle B \rangle}. \tag{3.10}$$

[5] In principle, it is known that the apparent R-symmetry of the magnetic formulation of the ISS SQCD is an approximate symmetry of the underlying electric theory: it is broken by the anomaly as per eq. 3.2. (At the same time, the anomaly-free combination of $U(1)_R$ and the axial symmetry $U(1)_A$ is broken explicitly by the mass terms of electric quarks m_Q.) However, the R-symmetry is broken in the electric theory in a controlled way [66] by small parameter, $m_Q/\Lambda_{ISS} = \mu^2/\Lambda^2_{ISS} \ll 1$. As such the R-symmetry is preserved to that order in the superpotential.

At this point the full tree-level potential is

$$V = \sum_{i=3}^{7} h^2 |(\mu^2_{\text{eff}})_{ii}|^2 + |F_C|^2 = 5h^2|\mu_X^2 - \lambda\langle AB\rangle|^2 + h^2 m_2^2 |B|^2 \tag{3.11}$$

so there is a runaway to unbroken SUSY in the direction $B \to 0$ and $A = \mu_X^2/\lambda B \to \infty$ along which the R-symmetry is broken.

Now, in order to end up with broken SUSY we would like to stabilize this type of runaway with Coleman-Weinberg terms in the one-loop potential. (Note that alternatively one could stabilize the model at tree-level using a more complicated potential and R-symmetry as discussed in Ref. [67].) We therefore need a runaway to *broken* SUSY since the Coleman-Weinberg contributions vanish where SUSY is unbroken. The classical runaway vacuum becomes non-supersymmetric if the components of the $\mu^2_{X\,ij}$ matrix on the right hand side of eq. 3.11 are no longer degenerate. This is easily achieved by breaking the flavour group into three rather than two factors.

For example, one can consider a 2-2-3 model. Here the original $SU(7)_f$ of the ISS $SU(2)_{\text{mg}}$ gauge theory is broken to $SU(2)_f \times SU(2)_f \times SU(3)_f$. This realisation can be thought of as the 2-5 model above where the $SU(5)_f$ flavour subgroup was further broken to $SU(2)_f \times SU(3)_f \times U(1)_{\text{traceless}}$ by splitting the eigenvalues of the μ^2_{ij} matrix. This does not cause problems for either explicit or direct mediation. Indeed in the case of direct gauge mediation the $SU(2)_L$ and $SU(3)_c$ components of μ^2_{ij} (or equivalently m_Q in the electric theory) renormalize differently below the GUT scale and so they are not expected to be the same [6].

Alternatively, one can consider an even simpler example of a 1-1-5 model with $N_f = 7$ and $N_c = 6$ so that the magnetic 'number of colours', $N = 1$, and the magnetic group is trivial. By splitting the eigenvalues of the μ^2_{ij} matrix we choose the flavour breaking to have the 1-1-5 pattern, $SU(7)_f \to U(1)_f \times U(1)_f \times SU(5)_f$. For the case of direct mediation the SM gauge group is $SU(5)_f$.

[6]Note that renormalization of μ^2 above the scale Λ_{ISS} would be understood as renormalization of m_Q in the electric theory.

N-P-X Model	$SU(N_P)_f$	$SU(N_X)_f$	$SU(N)_{\rm mg}$	$U(1)_B$	$U(1)_R$
$\Phi_{ij} \equiv \begin{pmatrix} Y & N & Z \\ \tilde{N} & P & M \\ \tilde{Z} & \tilde{M} & X \end{pmatrix}$	$\begin{pmatrix} 1 & \square & 1 \\ \bar{\square} & Adj+1 & \bar{\square} \\ 1 & \bar{\square} & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & \square \\ 1 & 1 & \square \\ \bar{\square} & \bar{\square} & Adj+1 \end{pmatrix}$	1	0	2
$\varphi \equiv \begin{pmatrix} \phi \\ \sigma \\ \rho \end{pmatrix}$	$\begin{pmatrix} 1 \\ \bar{\square} \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \\ \bar{\square} \end{pmatrix}$	$\square$	$\frac{1}{N}$	R
$\tilde{\varphi} \equiv \begin{pmatrix} \tilde{\phi} \\ \tilde{\sigma} \\ \tilde{\rho} \end{pmatrix}$	$\begin{pmatrix} 1 \\ \square \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \\ \square \end{pmatrix}$	$\bar{\square}$	$-\frac{1}{N}$	$-R$

Table 3.3: *The N-P-X Model. We indicate ISS matter field decomposition under the flavour subgroup $SU(N_P)_f \times SU(N_X)_f$. In direct mediation we would gauge $SU(N_P)_f \times SU(N_X)_f \times U(1)_{\rm traceless}$ or its subgroup, and identify it with the SM gauge group. We also show the gauge $SU(N)$ and the charges under the $U(1)_B$ and R-symmetry as in Table 3.1.*

To give a unified treatment of the 1-1-5 and the 2-2-3 models one can consider a general N-P-X model with $N + N_P + N_X = N_f$ and the μ_{ij}^2 matrix given by:

$$\mu_{ij}^2 = \begin{pmatrix} \mu_Y^2 \mathbf{I}_N & 0 & 0 \\ 0 & \mu_P^2 \mathbf{I}_{N_P} & 0 \\ 0 & 0 & \mu_X^2 \mathbf{I}_{N_X} \end{pmatrix} , \quad \mu_Y^2 > \mu_P^2 \, , \, \mu_X^2 \, , \quad \mu_P^2 \neq \mu_X^2 \, , \tag{3.12}$$

which corresponds to $SU(N_f) \to SU(N)_f \times SU(N_P)_f \times SU(N_X)_f$ as well as traceless $U(1)$ combinations which commute with the right hand side of eq. 3.12. For simplicity, the rank of top left Y-corner is identified with N, the number of magnetic colours, thus the original ISS rank condition which is responsible for the SUSY-breaking vacuum is arranged so that $F_\Phi = 0$ when $\Phi = Y$, see eq. 3.8, and $F_\Phi \neq 0$ when Φ is either P or X. The corresponding decomposition of ISS magnetic matter fields and their charges for this models are given in Table 3.3.

The minimization with respect to C and $\mathrm{tr}(\Phi)$ are as in eqs. 3.9-3.10 before, but minimization with respect to A, results in

$$\langle A \rangle = \frac{N_P \mu_P^2 + N_X \mu_X^2}{N_P + N_X} \frac{1}{\lambda \langle B \rangle} \, , \tag{3.13}$$

and consequently the potential

$$
\begin{aligned}
V &= \sum_{i=N+1}^{N_f} h^2 |(\mu^2_{\rm eff})_{ii}|^2 + |F_C|^2 \\
&= h^2 N_P \left(\mu_P^2 - \frac{N_P\mu_P^2 + N_X\mu_X^2}{N_P+N_X}\right)^2 + h^2 N_X \left(\mu_X^2 - \frac{N_P\mu_P^2 + N_X\mu_X^2}{N_P+N_X}\right)^2 + h^2 m_2^2 |B|^2 \\
&= h^2 \frac{N_P N_X}{N_P+N_X} (\mu_X^2 - \mu_P^2)^2 + h^2 m_2^2 |B|^2. \qquad (3.14)
\end{aligned}
$$

Again there is a runaway but now to broken supersymmetry as desired.

Note that in the case of explicit mediation the flavour symmetries in the ISS sector are divorced from the gauge symmetries of the Standard Model. In that case one can have a breaking of flavour symmetry that is more general than eq. 3.12, in terms of μ^2_{ii}. Defining the average μ_{ii} of the unbroken $SU(N_f - N)$ factor as

$$\overline{\mu^2} = \frac{1}{N_f - N} \sum_{i=N+1}^{N_f} \mu^2_{ii} \qquad (3.15)$$

we have

$$\langle A \rangle = \frac{\overline{\mu^2}}{\lambda \langle B \rangle} \qquad (3.16)$$

and then the generalisation of eq. 3.14 reads

$$V = h^2 \sum_{i=N+1}^{N_f} (\mu^2_{ii} - \overline{\mu^2})^2 + h^2 m_2^2 |B|^2 \qquad (3.17)$$

It is worth re-emphasizing that even in the limit $A, C \to \infty$ and $B \to 0$ the scalar potential V is non-zero, so we have a runaway to *broken* SUSY. Proceeding to one-loop, the Coleman-Weinberg contribution to the potential is therefore expected to lift and stabilize this direction at the same time as lifting the pseudo-Goldstone modes.

The Coleman-Weinberg effective potential [95] sums up one-loop quantum corrections into the following form:

$$V^{(1)}_{\rm eff} = \frac{1}{64\pi^2} {\rm Str}\, \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \equiv \frac{1}{64\pi^2} \left({\rm Tr}\, m^4_{\rm sc} \log \frac{m^2_{\rm sc}}{\Lambda^2} - 2 m^4_{\rm f} \log \frac{m^2_{\rm f}}{\Lambda^2} + 3 m^4_{\rm v} \log \frac{m^2_{\rm v}}{\Lambda^2} \right) \qquad (3.18)$$

where Λ is the UV cutoff[7], and the scalar, fermion and vector mass matrices are given by [96]:

$$m^2_{\rm sc} = \begin{pmatrix} W^{ab}W_{bc} + D^{\alpha a}D^{\alpha}{}_{c} + D^{\alpha a}_{\ \ c}D^{\alpha} & W^{abc}W_b + D^{\alpha a}D^{\alpha c} \\ W_{abc}W^b + D^{\alpha}{}_{a}D^{\alpha}{}_{c} & W_{ab}W^{bc} + D^{\alpha}{}_{a}D^{\alpha c} + D^{\alpha c}_{\ \ a}D^{\alpha} \end{pmatrix} \tag{3.19}$$

$$m^2_{\rm f} = \begin{pmatrix} W^{ab}W_{bc} + 2D^{\alpha a}D^{\alpha}{}_{c} & -\sqrt{2}W^{ab}D^{\beta}{}_{b} \\ -\sqrt{2}D^{\alpha b}W_{bc} & 2D^{\alpha c}D^{\beta}{}_{c} \end{pmatrix} \qquad m^2_{\rm v} = D^{\alpha}{}_{a}D^{\beta a} + D^{\alpha a}D^{\beta}{}_{a} \tag{3.20}$$

As usual, $W_c \equiv \partial W/\partial \Phi^c = F^{\dagger}_{\Phi^c}$ denotes a derivative of the superpotential with respect to the scalar component of the superfield Φ^c and the raised indices denote Hermitian conjugation, i.e. $W^{ab} = (W_{ab})^{\dagger}$. The D-terms are $D^{\alpha} = g z_a T^{\alpha a}_{\ \ b} z^b$ and they can be formally switched off by setting the gauge coupling $g = 0$, which we shall do for simplicity. All the above mass matrices will generally depend on field expectation values. The effective potential $V_{\rm eff} = V + V^{(1)}_{\rm eff}$ is the sum of the F-term (tree-level) potential and the Coleman-Weinberg contributions. To find the vacua of the theory we now have to minimize $V_{\rm eff}$.

Now we can check the lifting of the classical runaway direction by quantum effects in the Coleman-Weinberg potential. We have done this numerically using *Mathematica* and have also checked it with *Vscape* program of Ref. [97]. The non-supersymmetric vacuum is stabilised and in Table 3.4 we give values of the vevs for the 1-1-5 meson-deformed ISS model for a specific choice of external parameters. It is worth noting at this point that all the tree-level relations we have just derived get slightly shifted by the one-loop minimization. As we shall see, these one-loop effects often give the leading contribution to the mediation of SUSY-breaking and so it is important to keep track of them. This is shown in Table 3.4 where in the generic N-P-X model vevs develop along the direction

$$\begin{aligned} \langle\tilde{\phi}\rangle &= \xi\,\mathbf{I}_N \qquad \langle\phi\rangle = \kappa\,\mathbf{I}_N \\ \langle Y\rangle &= \eta\,\mathbf{I}_N \qquad \langle P\rangle = p\,\mathbf{I}_{N_P} \qquad \langle X\rangle = \chi\,\mathbf{I}_{N_X}, \end{aligned} \tag{3.21}$$

[7]Which is traded for a renormalization scale at which the couplings are defined.

VEV	$\kappa/\mu_X = \xi/\mu_X$	η/μ_X	p/μ_X	χ/μ_X	A/μ_X	B/μ_X	C/μ_X
Tree-level constrained	4.761	0.000	−0.128	−4.824	30.709	7.598	248.220
Unconstrained	4.761	0.003	−0.113	−4.828	30.797	7.562	248.960

Table 3.4: *The 1-1-5 Model: Stabilized vevs for a meson-deformed ISS theory with $N_f = 7$, $N_c = 6$, $h = 1$, $m_1/\mu_X = m_2/\mu_X = 0.03$, $\mu_Y/\mu_X = 5$, $\mu_P/\mu_X = 3$ and $\lambda = 0.01$. We show both the* constrained *vevs (i.e. the vevs obtained when the tree-level relations are enforced) and the true* unconstrained *vevs resulting from complete minimization.*

accompanied by the A, B, C vevs as before. These are the most general vevs consistent with the tree-level minimization.

3.3 Models of Mediation: from the meson-deformed ISS to the Standard Model

In the context of gauge mediation one can consider two distinct scenarios of how supersymmetry and R-symmetry breaking is transmitted to the visible Standard Model sector. The first class is ordinary gauge mediation (i.e. mediation with explicit messengers), and the second class involves the models of direct gauge mediation. In this section we discuss how these two possibilities can be realized for the SUSY breaking models we have outlined in the previous section

3.3.1 Gauge mediation with explicit messengers

We begin in this subsection with explicit mediation. In this scenario one imagines that there is a third sector – the messenger fields – that is responsible for generating the SUSY breaking operators required in the visible sector. The approach in this chapter is to try to have a preserved R-symmetry that is broken spontaneously. What we shall find is that we fall foul of the tachyonic messenger problem: ultimately we have to reintroduce explicit R-symmetry breaking messenger masses to avoid this and we are forced back to the explicit mediation scenario of Ref. [63].

To show this, let is first introduce an additional set of mediating fields f and $\tilde{f}$ transforming in the fundamental (and antifundamental respectively) of the Standard

Model gauge groups. For concreteness we can take f and $\tilde{f}$ to be (anti)-fundamentals of the underlying GUT gauge group, e.g. $SU(5)_{\rm GUT}$. In explicit mediation these messengers couple to the ISS sector via additional messenger coupling in the superpotential

$$W_{\rm mess} = \,{\rm Tr}(\tau\Phi)\, f \cdot \tilde{f} \tag{3.22}$$

where τ_{ij} is an arbitrary coupling which from the electric theory perspective should scale as Λ_{ISS}/M_{Pl} as in Ref. [63]. We remind the reader that there are no constraints on this coupling coming from the Standard Model, and that the ISS parameters, such as N, N_f are essentially unconstrained.

In order to see how the SUSY breaking enters the visible sector we need to exhibit the mass matrices for messenger fields explicitly. At tree-level the SUSY breaking enters into the scalar mass-squared matrices through the non-zero F_Φ-terms to which the messenger fields, f and $\tilde{f}$ couple. In general the matrices are given by (ignoring the D-terms)

$$\begin{aligned} m^2_{\rm sc} &= \begin{pmatrix} W^{ab}W_{bc} & W^{abc}W_b \\ W_{abc}W^b & W_{ab}W^{bc} \end{pmatrix}, \\ m_{\rm f} &= W_{ab}\,, \end{aligned} \tag{3.23}$$

with the W_{ac} being the SUSY preserving mass of the fermions, and the off-diagonal terms $W^{abc}W_b$ containing the SUSY breaking. In this case $W_{f\tilde{f}} = {\rm Tr}(\langle\tau\Phi\rangle)$ is the Dirac mass of the fermionic superpartners, ψ_f and $\psi_{\tilde{f}}$, and the SUSY breaking contribution appears first in the tree-level mass-squared of the scalars, $S = (f, \tilde{f}^*)$. We have:

$$m^2_{\rm sc} = \begin{pmatrix} |{\rm Tr}(\tau\Phi)|^2 & {\rm Tr}(\tau^\dagger F^\dagger_\Phi) \\ {\rm Tr}(\tau F_\Phi) & |{\rm Tr}(\tau\Phi)|^2 \end{pmatrix}. \tag{3.24}$$

Now, in order to avoid tachyonic messengers we must here impose the usual explicit mediation constraint that:

$$|{\rm Tr}(\tau\langle\Phi\rangle)|^2 > |{\rm Tr}(\tau\langle F_\Phi\rangle)| \tag{3.25}$$

which is effectively a *lower* bound on the amount of spontaneous R-symmetry breaking (since $\langle\Phi\rangle$ is charged under R-symmetry). In particular this generally prevents us arranging a split scenario with gauginos much lighter than squarks and sleptons, since this would be a signature of approximate R-symmetry. (The situation is drastically different in models of direct mediation as we shall see in the following sections.)

As we have said dimensional arguments give $\tau \sim \lambda \sim \Lambda_{ISS}/M_{Pl} \ll 1$ so the tachyonic inequality is delicate. If one assumes that $\Phi \sim \mu$ then it seems that the inequality is actually always violated when $\tau \ll 1$. But note that the same inequality can be equivalently written in terms of singlet vevs,

$$\tau\Phi \sim \tau m_1 \frac{A}{\lambda B} \sim \frac{\tau}{\lambda^2}\frac{m_1\mu^2}{B^2} \tag{3.26}$$

which shows that the situation is quite complicated and can only be analyzed numerically. For the values in Table 3.4 taking $\tau \sim \lambda$ violates the inequality which suggests that it may be problematic in general to avoid tachyonic messengers.

An explicit R-breaking mass term is a way to overcome this tachyon so that, as in Ref. [63], eq. 3.22 becomes

$$W_{\rm mess} = \operatorname{Tr}(\tau\Phi)\, f\cdot\tilde{f} + M_f\, f\cdot\tilde{f} \tag{3.27}$$

Hence explicit gauge mediation and spontaneous R-symmetry breaking are inconsistent when the DSB is based on the ISS model. Note that we could have also added a term $\frac{A^2 f\cdot\tilde{f}}{M_{Pl}}$; however since we have $\langle A\rangle \sim \mu_P \ll \Lambda$ the effective mass that this induces for the messengers is even smaller than $\langle \operatorname{Tr}(\tau\Phi)\rangle$.

From here on the calculation of the SUSY spectrum is rather standard with values for gaugino masses being generated being of the same order as those for scalar masses; and so one expects a similar phenomenology to normal explicit gauge mediation [90], with the diagram that induces the gaugino mass in the present explicit mediation case as shown in Fig. 3.1.

However there is one feature of the present set-up that is rather interesting. The SUSY breaking effects in the visible sector, i.e. the gaugino and squark masses, are all proportional to the combination $W^{abc}W_b = Tr(\tau^\dagger F_\Phi^\dagger)$. But as we have seen in the previous section, the F-terms at the minimum (with VEV-less messengers, so that the SM gauge groups are not Higgsed) are given at tree-level by

$$F^\dagger_{\Phi_{ij}} = h\delta_{ij}(\mu_{ii}^2 - \overline{\mu^2})\,, \tag{3.28}$$

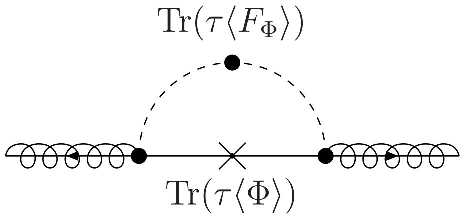


Figure 3.1: *One-loop contribution to the gaugino masses from messengers f, $\tilde{f}$. The dashed (solid) line is a bosonic (fermionic) messenger. The blob on the scalar line indicates an insertion of the F-term VEV into the propagator of the scalar messengers and the cross denotes an insertion of the R-symmetry breaking VEV into the propagator of the fermionic messengers.*

which clearly obeys

$$\mathrm{Tr}(F_\Phi^\dagger) = 0\,. \tag{3.29}$$

This can be seen to result from the minimization of the tree-level potential with respect to A for a given B VEV:

$$\frac{\partial V}{\partial A} = \lambda B \mathrm{Tr}(F_\Phi^\dagger) = 0\,. \tag{3.30}$$

Thus (at tree-level) the mediation of SUSY-breaking to the visible sector requires non-degenerate couplings τ_{ii}, and indeed we can write

$$\mathrm{Tr}(\tau F_\Phi) = h(\overline{\tau\mu^2} - \bar{\tau}\overline{\mu^2})\,. \tag{3.31}$$

That is, only if *both* τ and μ have non-degeneracy can there be unsuppressed SUSY breaking mediation, even though SUSY breaking *per se* requires non-degeneracy only in the latter.

However, as we have said, when the full minimization is performed, tree-level relations such as $\mathrm{Tr}(F_\Phi^\dagger) = 0$ are no longer expected to hold (for example, with the unconstrained values in the table we find $\mathrm{Tr}(F_\Phi^\dagger) = -0.034\mu_X^2$): typically one finds $\mathrm{Tr}(F_\Phi^\dagger) = \mu^2/(16\pi^2)$, since the effective F-term for mediation is one-loop suppressed. Thus when the τ are degenerate one can still get $m_\lambda \sim \frac{\mu^2}{16\pi^2 M_f}\frac{g^2}{(16\pi^2)} \sim 1$ TeV if $\mu^2/M_f \sim 10^7$ GeV.

3.3.2 Direct gauge mediation

Now, let us compute gaugino masses for the direct gauge mediation scenario from the meson-deformed ISS sector. We first consider the effects of those direct messengers which obtain R-symmetry breaking masses at tree-level and which couple directly to the largest F-terms. These transform in the fundamental representation of the SM gauge groups, and this constitutes a strictly one-loop and formally leading order effect. Then we will include additional, formally higher-loop, contributions from the pseudo-Goldstone modes transforming in both adjoint and (bi-)fundamental representations of the Standard Model gauge groups. It will turn out that the latter contributions can be of the same order.

Strict one-loop contributions to gaugino masses

To present a general discussion relevant for any deformation of the ISS model, by mesons, baryons or otherwise, we shall consider models of the form

$$W = h\Phi_{ij}\varphi_i.\tilde{\varphi}_j - h\mu_{ij}^2\Phi_{ji} + W_{\mathrm{meson-def}}(A_a, \Phi) + W_{\mathrm{baryon-def}}(A_a, \phi, \tilde{\phi}) \tag{3.32}$$

where A_a denote generic singlets. The superpotential depends on Φ linearly, this is dictated by the R-symmetry of the model and is a central feature of direct mediation in the ISS context.

To keep the presentation simple in what follows we shall concentrate here on the 1-1-5 model, so that the parent gauge symmetry of the SM (in this case $SU(5)_f$) is non-split. This discussion can also be straightforwardly generalised to the 2-2-3 and other N-P-X models by an appropriate reassembling of building blocks below.

The all important messenger/SUSY-breaking coupling in the superpotential is, in this class of models:

$$\frac{1}{h}W \supset \Phi_{ij}\varphi_i.\tilde{\varphi}_j \supset \rho X\tilde{\rho} + \phi Z\tilde{\rho} + \rho\tilde{Z}\tilde{\phi} + \phi Y\tilde{\phi}\,. \tag{3.33}$$

The field Φ is the pseudo-Goldstone mode, although note that F_ϕ and $F_{\tilde{\phi}}$ are non-zero as well as F_Φ – this will be important in what follows.

Gaugino masses are generated at one-loop order as indicated in Fig. 3.2. The fields propagating in the loop are fermion and scalar components of the direct mediation 'mes-

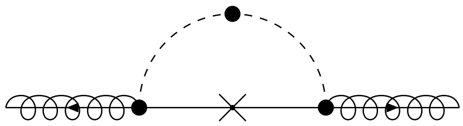


Figure 3.2: *One-loop contribution to the gaugino masses. The dashed (solid) line is a bosonic (fermionic) messenger. The blob on the scalar line indicates an insertion of $\langle F_\chi \rangle$ into the propagator of the scalar messengers and the cross denotes an insertion of the R-symmetry breaking VEV into the propagator of the fermionic messengers.*

sengers'. Since gaugino masses are forbidden by R-symmetry one crucial ingredient in their generation is the presence of non-vanishing R-symmetry breaking vevs. We are at this point interested in the contribution to the gaugino mass coming from those messenger fields transforming in the fundamental of $SU(5)$, which formally give the leading-order contribution. (We shall consider the contribution from the X fields separately in section 3.3.2.)

First we exhibit the mass matrices of messenger fields. As before, they are given by (ignoring the D-terms)

$$m_{\rm sc}^2 = \begin{pmatrix} W^{ab}W_{bc} & W^{abc}W_b \\ W_{abc}W^b & W_{ab}W^{bc} \end{pmatrix}, \qquad m_{\rm f} = W_{ac}\,. \tag{3.34}$$

The fundamental messengers are ρ, $\tilde{\rho}$ and Z, $\tilde{Z}$: we may define a messenger fermion multiplet,

$$\begin{aligned} \psi &= (\rho_i\,, Z_i)_{\rm ferm}\,, \\ \tilde{\psi} &= (\tilde{\rho}_i\,, \tilde{Z}_i)_{\rm ferm}\,, \end{aligned} \tag{3.35}$$

where $i = 1..5$. Then $\mathcal{L} \supset \psi\, m_{\rm f} \tilde{\psi}^T$ where the fermion messenger mass matrix is

$$m_{\rm f} = \mathbf{I}_5 \otimes \begin{pmatrix} \chi & \xi \\ \kappa & 0 \end{pmatrix}, \tag{3.36}$$

written in terms of the vevs χ, κ and ξ (c.f. 3.21):

$$\langle X \rangle = \chi I_5 \quad , \quad \langle \phi \rangle = \kappa \quad , \quad \langle \tilde{\phi} \rangle = \xi \, . \tag{3.37}$$

For the scalar mass-squared matrix, we can define equivalent multiplets

$$S = (\rho_i, Z_i, \tilde{\rho}_i^*, \tilde{Z}_i^*)_{\,\mathrm{sc}} \, . \tag{3.38}$$

To proceed one can diagonalize the mass matrices and compute the full one-loop contribution to the gaugino mass. That is we define the diagonalisations:

$$\hat{m}_{\mathrm{sc}}^2 = Q^\dagger m_{\mathrm{sc}}^2 Q \tag{3.39}$$
$$\hat{m}_{\mathrm{f}} = U^\dagger m_{\mathrm{f}} V \tag{3.40}$$

with eigenvectors

$$\begin{aligned} \hat{S} &= S.Q \\ \hat{\psi}_+ &= \psi.U \\ \hat{\psi}_- &= \tilde{\psi}.V^* \end{aligned} \tag{3.41}$$

Here, the m_{f} diagonalisation is in general a biunitary transformation.

In order to calculate the gaugino mass, we need the gauge interaction terms given by

$$\mathcal{L} \supset i\sqrt{2} g_A \lambda_A (\psi_1 T^A S_1^* + \psi_2 T^A S_2^* + \tilde{\psi}_1 T^{*A} S_3 + \tilde{\psi}_2 T^{*A} S_4) + H.C. \tag{3.42}$$
$$= i\sqrt{2} g_A \lambda_A (\hat{\psi}_{+i} \hat{S}_k^* (U_{i1}^\dagger Q_{1k} + U_{i2}^\dagger Q_{2k}) + \hat{\psi}_{-i} \hat{S}_k (Q_{k3}^\dagger V_{1i} + Q_{k4}^\dagger V_{2i})) + H.C. \tag{3.43}$$

Then the diagram in Figure 3.2 amounts to[8]

$$M_{\lambda_A}^{(\rho,Z)} = 4N g_A^2 \,\mathrm{tr}(T^A T^B) \sum_{ik} (U_{i1}^\dagger Q_{1k} + U_{i2}^\dagger Q_{2k})(Q_{k3}^\dagger V_{1i} + Q_{k4}^\dagger V_{2i}) \, I(\hat{m}_{\mathrm{f},i}, \hat{m}_{\mathrm{sc},k}) \tag{3.44}$$

where $I(\hat{m}_{\mathrm{f}}, \hat{m}_{\mathrm{sc}})$ is the appropriate one-loop integral with a fermion and a scalar. Here the "N" reinstates the possibility of an $SU(N)_{mg}$ gauge group. In the diagonal mass-

[8]More precisely, there are actually two diagrams of this type which are mirror images of each other.

basis

$$I(a,b)=\int \frac{d^4k}{(2\pi)^4}\frac{a}{k^2-a^2}\frac{1}{k^2-b^2}=\frac{-a(\eta+1)}{16\pi^2}+\frac{1}{16\pi^2}\frac{a}{a^2-b^2}\left[a^2\log\left(\frac{a^2}{\Lambda^2}\right)-b^2\log\left(\frac{a^2}{\Lambda^2}\right)\right] \tag{3.45}$$

and

$$\eta=\frac{2}{4-D}+\log(4\pi)-\gamma_E. \tag{3.46}$$

This integral is UV-divergent, but the divergences cancels in the sum over eigenstates as required.

Using 3.44 we can now evaluate gaugino masses in Figure 3.2 generated by fundamental messengers ρ, $\tilde{\rho}$ and Z. Numerical values for the gaugino mass for a few different values of parameters of the model are given in the Tables in section 3.3.3.

It is instructive to complement these numerical calculations by a simple analytic estimate, and in particular explain the smallness of these gaugino mass contributions. When the F-terms are small compared to μ^2 one can expand eq. 3.44-3.45. We define a matrix of 'weighted' F-terms as:

$$\mathcal{F}^{ab}=W^{abc}W_c\,, \tag{3.47}$$

and to the leading order in $\mathcal{F}$ obtain,

$$M_{\lambda_A}=\frac{g_A^2}{8\pi^2}\,N\,\mathrm{tr}(T^AT^B)\,\mathrm{Tr}(\mathcal{F}\cdot m_{\mathrm{f}}^{-1})+\mathcal{O}(\mathcal{F}^3)\,. \tag{3.48}$$

This is a well-known leading order in $\mathcal{F}$ approximation which is basis-independent. In the Appendix we give the derivation of eq. 3.48 in the general settings relevant to our model(s).

Clearly the matrix $\mathcal{F}$ is determined entirely by the contribution in eq. (3.33) to be

$$\mathcal{F}=W^{abc}W_c=h\begin{pmatrix}F_\chi & F_{\tilde{\phi}}\\ F_\phi & 0\end{pmatrix} \tag{3.49}$$

and since $m_{\rm f}^{-1} = \begin{pmatrix} 0 & \frac{1}{\kappa} \\ \frac{1}{\xi} & -\frac{\chi}{\xi\kappa} \end{pmatrix}$ we find

$$M_{\lambda_A}^{(\rho,Z)} = \frac{g_A^2}{8\pi^2} N\,\mathrm{tr}(T^A T^B) \left(\frac{F_{\tilde{\phi}}}{\xi} + \frac{F_\phi}{\kappa} \right) + \mathcal{O}(\mathcal{F}^3) \tag{3.50}$$

Now consider the minimization condition for the tree-level potential, $V = \sum_c |F^c|^2$ with respect to Y^*.

$$\frac{1}{2}\frac{\partial V}{\partial Y^*} = 0 = \sum_c W^{Yc} F_c = \kappa F_{\tilde{\phi}} + \xi F_\phi + W^{YA_a}_{\rm meson-def} F_{A_a} \tag{3.51}$$

(For the constrained 1-1-5 vevs shown in Table 3.4 this trivially sets $\eta = 0$.) This equation together with eq. 3.50 implies that the tree-level leading order gaugino mass is zero

$$M_{\lambda_A}^{(\rho,Z)} = 0 + \mathcal{O}(\mathcal{F}^3) \tag{3.52}$$

unless the additional singlet fields appearing in the meson deformation have non-zero F-terms as well. (This would require an additional source of SUSY breaking beyond the O'Raifeartaigh breaking of the ISS sector, and is therefore unattractive.) As we have stressed, these relations are perturbed when the potential is stabilized by one-loop effects (e.g. η is non-zero in the unconstrained model of Table 3.4): then the estimate in eq. 3.50 is still reasonably good, with the F-terms being derived from the one-loop equations.

This leading order suppression for the gaugino mass explains the relative smallness of our numerical results in Table 3.5 which shows the "reduced gaugino masses" $m_{1/2}$ defined by

$$M_{\lambda_A} = \frac{g_A^2}{16\pi^2} m_{1/2}\,. \tag{3.53}$$

In particular these values are much smaller than those derived for the scalars in Table 3.6 where we show the "reduced scalar masses" m_0 defined by

$$m^2_{\rm sferm} = \sum_A \frac{g_A^4}{(16\pi^2)^2} C_A S_A m_0^2\,, \tag{3.54}$$

where C_A and S_A are the standard Casimir/Dynkin indices as in Ref. [98]. We note that this suppression is also related to that in Ref. [99], which tells us that F_Φ does not contribute to the gaugino masses at leading order because of the structure of m_{f} (in particular the zero entry). Here we find that the argument extends to quite general models of direct mediation.

Additional contributions to gaugino masses

The effects considered above have so far generated rather small contributions to gaugino masses. Thus, we have to consider additional contributions, due to the adjoint X and P as well as the bifundamental M and $\tilde{M}$ messengers. These messengers are massless at tree-level and acquire masses only at loop-level. Thus their contributions to gaugino masses are formally a higher-loop effect. After a careful consideration we find that these indeed give a contribution to the gaugino masses which comparable to the strict one-loop effect described above. Scalar masses being unsuppressed at leading order are not significantly effected.

For 1-1-5 type models where the SM gauge group is $SU(5)_f$, the new contributions arise from the X_{ij} fields with $i, j = 1\ldots 5$. They contribute through the diagram shown in Figure 3.3. Note that the scalar vertex exists because the Coleman-Weinberg potential induces an R-symmetry violating mass term. The fermion mass-propagator is also absent at tree-level: since it is a Majorana term (and the X-fermions have R-charge 1) it also violates R-symmetry and by the non-renormalization theorem it vanishes in the absence of both R-symmetry and supersymmetry breaking. The naive expectation is therefore that this contribution will be three-loop suppressed. As we shall see, this is not the case, and in fact the contribution can be competitive with the previous contributions. This is because the X modes are pseudo-Goldstone modes: *all* their masses arise at one-loop, and the lightness of these modes corresponds to a suppression of the effective messenger scale of the adjoints whose mass is in fact similar to M_{SUSY}.

Let us estimate these effects in more detail. First the mass-insertions: the scalar mass-squareds come from the Coleman-Weinberg term

$$V_{\text{eff}}^{(1)} \supset \text{Str}(\frac{\mathcal{M}^4}{64\pi^2} \log \mathcal{M}^2). \tag{3.55}$$

In particular there are terms involving $\bar{W}_{\rho Z} W^{Z\rho} \bar{W}_{\rho\tilde{\rho}} W^{\tilde{\rho}\rho} = h^4 \xi^2 |\delta X_{ij}|^2$ where $X = \langle X \rangle + \delta X$. Since typically $\xi \gg \mu \gg \kappa$ one expects R-symmetry conserving mass-squared

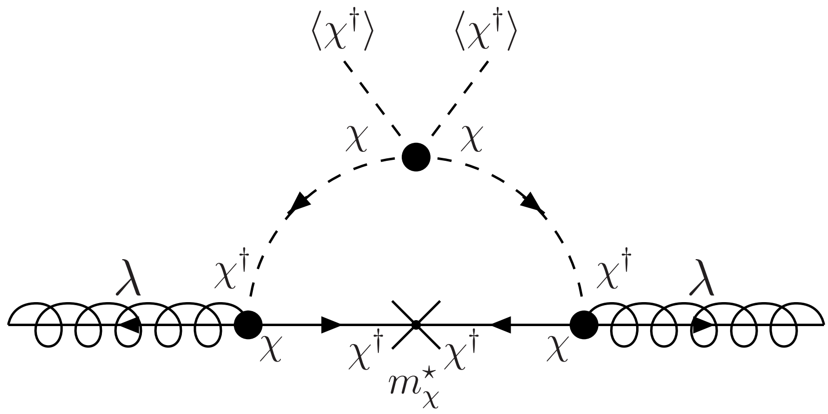


Figure 3.3: *One-loop contribution to the gaugino masses from X-messengers. The dashed (solid) line is a bosonic (fermionic) component of X. The blob on the scalar line indicates an insertion of $\langle F_\chi\rangle$ into the propagator of the scalar messengers and the cross denotes an insertion of the R-symmetry breaking VEV into the propagator of the fermionic messengers.*

for the adjoints of order

$$m^2_{XX^*} \sim \frac{h^4\xi^2}{64\pi^2} \tag{3.56}$$

at the minimum. R-symmetry violating masses are induced by terms such as $W_{\rho\tilde{\rho}}\bar{W}^{\tilde{\rho}\rho}W_{\rho\tilde{\rho}}\bar{W}^{\tilde{\rho}\rho} \supset h^4\langle X\delta X^\dagger\rangle^2 + h.c = h^4\chi^2(\delta X^*_{ij}\delta X^*_{ji}) + h.c.$ Hence we expect a neutral mass-squared matrix for $\mathbf{X} = (X^A, X^*_A)$ (where A is the adjoint index) of the form

$$\begin{aligned} m^2_{\mathbf{X}} &\sim \frac{\delta_{AB}}{64\pi^2}\begin{pmatrix} a & b \\ b^* & a \end{pmatrix}, \\ & a\sim\xi^2 \;\; ; \;\; b\sim\chi^2 \,. \end{aligned} \tag{3.57}$$

Assuming b is real, the diagonalisation of this matrix is $\hat{m}^2_{\mathbf{X}} = (\mathcal{Q}^X)^T m^2_{\mathbf{X}} \mathcal{Q}^X = \frac{h^4}{64\pi^2}\mathrm{diag}(a+b,\, a-b)$ where

$$\mathcal{Q}^X = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \tag{3.58}$$

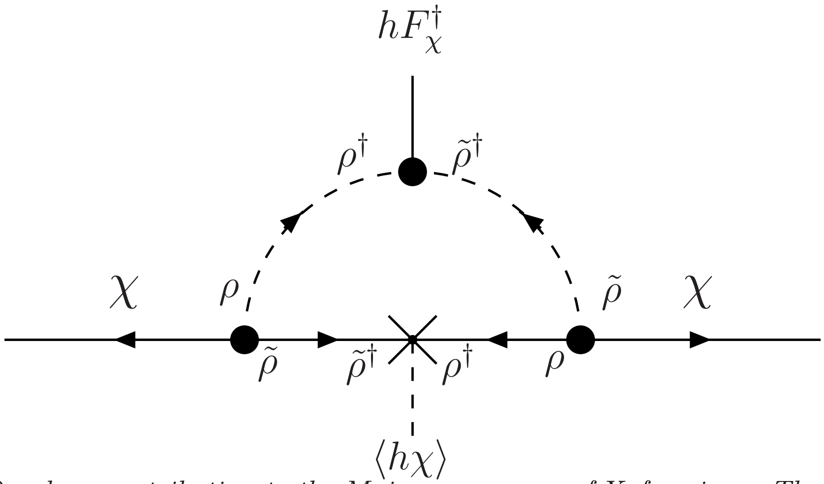


Figure 3.4: *One-loop contribution to the Majorana masses of X-fermions. The dashed (solid) line is a bosonic (fermionic) messenger. The blob on the scalar line indicates an insertion of $\langle F_\chi\rangle$ into the propagator of the scalar messengers and the cross denotes an insertion of the R-symmetry breaking VEV into the propagator of the fermionic messengers.*

We will call the two eigenvalues $\hat{m}^2_{X_\pm}$.

The R-breaking mass term for the adjoint fermion is generated from diagram shown in Figure 3.4. The topology is identical to the one-loop gaugino diagram with internal states ψ, $\tilde\psi$ and S with the mass matrices and diagonalisations as in eqs. 3.39 and 3.40, although of course the vertices are different: they come from the $W \supset h\rho X\tilde\rho$ coupling and are given by

$$V \supset h\,(X\psi_1)S_3^* + h(X\tilde\psi_1)S_1\ + h.c. \tag{3.59}$$

In terms of the previous mass eigenstates these become

$$V \supset hX\,(\hat\psi_{+i}\hat{S}_k^*(U_{i1}^\dagger Q_{3k}) + \hat\psi_{-i}\hat{S}_k(Q_{k1}^\dagger V_{1i}) + H.C. \tag{3.60}$$

where the diagonalisation matrices Q, U and V are exactly the same as in eqs. 3.39-3.40. Defining $X_{ij} = \sqrt{2}\,X_A T^A_{ij}$, and with standard Feynman parametrization we find that the diagram in Figure 3.4 generates

$$M_{\psi_X} = 4Nh^2 \mathrm{tr}(T^A T^B) \sum_{ik} (U^\dagger_{i1} Q_{3k})(Q^\dagger_{k1} V_{1i})\, I(\hat{m}_{\mathrm{f},i}, \hat{m}_{\mathrm{sc},k})\,, \tag{3.61}$$

where $I(\hat{m}_{\mathrm{f},i}, \hat{m}_{\mathrm{sc},k})$ is the same integral 3.45 as in 3.44.

Note that although the diagram in Figure 3.4 is similar to the fundamental contribution to the one-loop gaugino mass, there is less suppression. This is because the couplings of $\rho, \tilde{\rho}$ and Z, $\tilde{Z}$ to X are not degenerate as they are for the gaugino, indeed there is no equivalent of the $h\rho X\tilde{\rho}$ coupling for the $Z, \tilde{Z}$ fields at all; hence unitarity does not operate in the same way. Following the same steps as for the gaugino in the Appendix we obtain a non-vanishing leading order result in $\mathcal{F}$,

$$M_{\psi^A_X} \approx 4\, N\, h^2 \mathrm{tr}(T^A T^B) \sum_{ijk} \mathcal{A}_{jk} (U^\dagger_{i1} V_{1j})(U^\dagger_{k1} V_{1i})\, (\hat{m}_\mathrm{f})_i\, J(\hat{m}^2_{\mathrm{f}\,i},\, \hat{m}^2_{\mathrm{f}\,j},\, \hat{m}^2_{\mathrm{f}\,k}) \tag{3.62}$$

where the matrix $\mathcal{A}_{ij}$ was defined in eq. B.3 and the function J is given by

$$J(a,b,c) = \frac{1}{8\pi^2} \frac{a^2 b^2 \log\left(\frac{a}{b}\right) + a^2 c^2 \log\left(\frac{c}{a}\right) + b^2 c^2 \log\left(\frac{b}{c}\right)}{(a^2 - b^2)(a^2 - c^2)(b^2 - c^2)}. \tag{3.63}$$

A very rough simple estimate is

$$M_{\psi^A_X} \sim \frac{h^2 \chi}{32\pi^2} \frac{F_X}{\xi^2}. \tag{3.64}$$

This should be compared to the equivalent contribution to the gaugino mass in section 3.3.2 which did vanish at this order (see eqs. 3.48,3.52).

Having determined the masses of X messengers we can now make an estimate for their contribution to the gaugino mass. The general expression is

$$M^{(X)}_{\lambda_A} \;=\; g^2_A\, N_X\, \left(I(M_{\psi_X}, \hat{m}_{X_+}) - I(M_{\psi_X}, \hat{m}_{X_-}) \right)\,, \tag{3.65}$$

where N_X is the rank of the X lower-right corner in eq. 3.12, which in the case of 1-1-5 type models is $N_X = 5$, and $I(m_a, m_b)$ was defined in eq.3.45.

Equation 3.65 allows us to evaluate gaugino masses generated by adjoint X-messengers. Numerical values for the full mass expressions (without relying on estimates and expansions in $\mathcal{F}$) for the model given in Table 3.4 are presented in Table 3.5 in section 3.3.3.

In this table we give contributions from the ρ and Z messengers in the first column and from the X messengers in the second column. The third column gives the similar contribution from M messengers which we will comment on momentarily (see eq. 3.68). The last column is the total result. Other tables in the same subsection follow the same structure and give results for other models.

To understand the order of magnitude can also be understood with the help of the following analytical estimates. As we have seen the masses are of the order

$$\begin{aligned} M_{\psi_X^A} &\sim \frac{h^3\chi}{32\pi^2}\frac{\mu^2}{\xi^2} \\ \hat{m}^2_{X_\pm} &\sim \frac{h^4}{64\pi^2}(\xi^2 \pm \chi^2). \end{aligned}$$

Thus for $h \lesssim 1$ we expect $M^2_{\psi_X^A} \ll \hat{m}^2_{X_\pm}$ and we find

$$M^{(X)}_{\lambda_A} = \frac{g_A^2\, N_X}{32\pi^2}\, M_{\psi_X^A} \log\left(\frac{\hat{m}^2_{X_+}}{\hat{m}^2_{X_-}}\right) \sim \frac{g_A^2 h^3\, N_X}{2(16\pi^2)^2}\,\frac{\chi^3\mu^2}{\xi^4}, \tag{3.66}$$

where the last expression is valid for $\chi \lesssim \xi$. Note that, although in a "mass-insertion approximation" the leading order diagram is in principle three-loop, there is only a two-loop $1/(16\pi^2)^2$ suppression.

In addition to the contribution from the adjoint X fields we have a contribution from the M and $\tilde{M}$ fields. As can be seen from Table 3.3 these are bifundamentals under the $\mathrm{SU}(N_X)$ and $\mathrm{SU}(N_P)$ groups.

$$M_{\psi_M} = Nh^2\left[\sum_{ik}(U^{P\dagger}_{i1}Q^X_{3k})(Q^{X\dagger}_{k1}V^P_{1i})\, I(\hat{m}^P_{\mathrm{f},i}, \hat{m}^X_{\mathrm{sc},k}) + (X \leftrightarrow P)\right]. \tag{3.67}$$

Here, the labels P and X indicate the diagonalisation matrices for the $\mathrm{SU}(N_P)$ and $\mathrm{SU}(N_X)$ blocks, respectively (see Table 3.3). In particular, the X is the diagonalisation for the ρ and Z messengers whereas P corresponds to the σ and N.

The corresponding contribution to the gaugino mass is,

$$M^M_{\lambda^A} = 4\,\mathrm{tr}(T^AT^B)N_P\sum_{k=1}^{2}\mathcal{Q}^M_{1k}(\mathcal{Q}^M)^T_{k2}I(M_{\psi_M}, \hat{m}_{M,kk}), \tag{3.68}$$

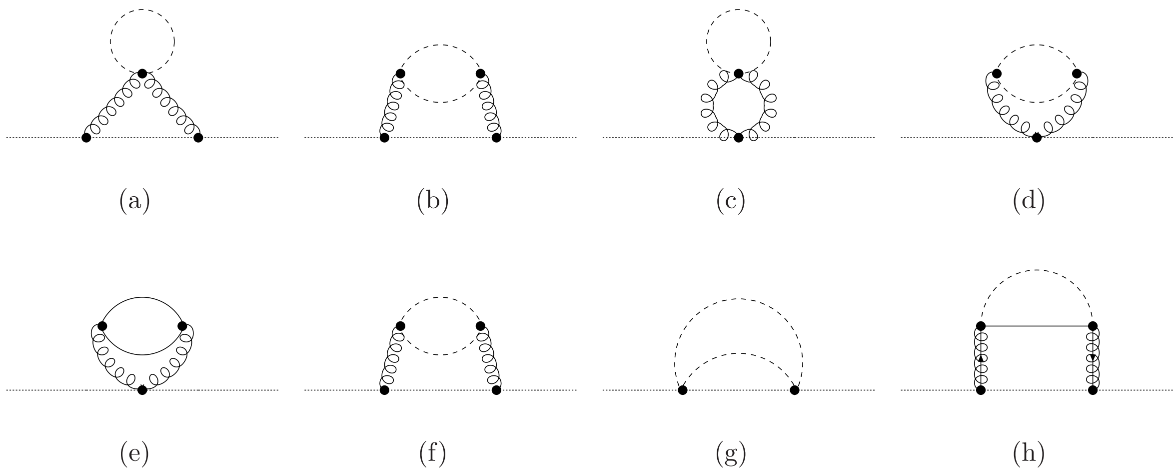


Figure 3.5: Two-loop diagrams contributing to the sfermion masses. The long dashed (solid) line is a bosonic (fermionic) messenger. Standard model sfermions are depicted by short dashed lines.

where $\mathcal{Q}^M$ is the M-analog of $\mathcal{Q}^X$ matrix given in eq. 3.58. As mentioned earlier these contributions are shown in the third column of Table 3.5 and similar ones in the subsection 3.3.3.

Scalar masses

Having determined the gaugino masses in the preceding subsections, we now outline the procedure for the generation of sfermion masses of the supersymmetric standard model. As in Ref. [66] we follow the calculation of Martin in Ref. [98] adapted to our direct mediation models.

Sfermion masses are generated by the two-loop diagrams shown in Fig. 3.5. In [98] the contribution of these diagrams to the sfermion masses was determined to be,

$$m_{\tilde{f}}^2 = \sum_{mess.} \sum_{a} g_a^4 C_a S_a(mess.)[\text{sum of graphs}], \tag{3.69}$$

where we sum over all gauge groups under which the sfermion is charged, g_a is the corresponding gauge coupling, $C_a = (N_a^2 - 1)/(2N_a)$ is the quadratic Casimir and $S_a(mess.)$ is the Dynkin index of the messenger fields (normalized to 1/2 for fundamentals).

As in the calculation of the gaugino mass we use the propagators in the diagonal form and insert the diagonalisation matrices directly at the vertices. For the diagrams 3.5(a) to 3.5(f) we have closed loops of purely bosonic or purely fermionic mass eigenstates of our messenger fields. It is straightforward to check that in this case the unitary matrices from the diagonalisation drop out. We then simply have to sum over all mass eigenstates the results for these diagrams computed in Ref. [98].

The next diagram 3.5(g) is slightly more involved. This diagram arises from the D-term interactions. D-terms distinguish between chiral and antichiral fields, in our case ρ, Z and $\tilde{\rho}, \tilde{Z}$, respectively. We have defined our scalar field S in 3.38 such that all component fields have equal charges. Accordingly, the ordinary gauge vertex is proportional to a unit matrix in the component space (cf. eq. 3.42). This vertex is then 'dressed' with our diagonalisation matrices when we switch to the $\hat{S}$ basis, 3.43. This is different for diagram 3.5(g). Here we have an additional minus-sign between chiral and antichiral fields. In field space this corresponds to a vertex that is proportional to a matrix $V_D = \mathrm{diag}(1, 1, -1, -1)$. We therefore obtain,

$$\text{Fig. 3.5(g)} = \sum_{i,m} (Q^T V_D Q)_{i,m} J(\hat{m}_{0,m}, \hat{m}_{0,i}) (Q^T V_D Q)_{m,i}, \tag{3.70}$$

where J is the appropriate two-loop integral for Fig. 3.5(g) which can be found in [98].

Finally, in 3.5(h) we have a mixed boson/fermion loop. The subdiagram containing the messengers is similar to the diagram for the gaugino mass. The only difference is the direction of the arrows on the gaugino lines. Indeed the one-loop sub-diagram corresponds to a contribution to the kinetic term rather than a mass term for the gauginos. (The mass term will of course contribute as well but will be suppressed by quark masses.) Using eq. 3.43 we find,

$$\text{Fig. 3.5(h)} = \sum_{ik} (|U^\dagger_{i1} Q_{1k} + U^\dagger_{i2} Q_{2k}|^2 + |Q^\dagger_{k3} V_{1i} + Q^\dagger_{k4} V_{2i}|^2) L(\hat{m}_{1/2,i}, \hat{m}^2_{0,k}) \tag{3.71}$$

where L is again the appropriate loop integral from [98].

Summing over all diagrams we find the sfermion masses which are typically significantly larger than the gaugino masses calculated earlier. Indeed, the scalar masses roughly follow the estimate

$$m^2_{\tilde{f}} \sim \frac{g^4}{(16\pi^2)^2} \mu^2. \tag{3.72}$$

This is precisely the leading order effect which in our direct mediation scenario is absent for the gaugino masses.

So far we have taken into account the ρ, Z (or similarly the σ, M) contributions which as we just explained give a non-vanishing leading order effect. In distinction to our earlier calculation of the gaugino masses we do not need to include the sub-dominant contributions from other messengers (which were massless at tree-level)[9].

3.3.3 Signatures in the directly mediated meson-deformed model

Here we present and summarize our result for gaugino and sfermion masses for a variety of our meson-deformed models. These results are most conveniently expressed in terms of the reduced gaugino ($m_{1/2}$)

$$M_{\lambda_A} = \frac{g_A^2}{16\pi^2} m_{1/2}\,, \tag{3.73}$$

and scalar masses (m_0^2)

$$m^2_{\rm sferm} = \sum_A \frac{g_A^4}{(16\pi^2)^2} C_A S_A m_0^2. \tag{3.74}$$

We similarly define reduced masses for the pseudo-Goldstone components of the direct messengers (appearing in Tables 3.7, 3.11, 3.17) by including a factor of $16\pi^2$,

$$m_{\rm reduced} = 16\pi^2 m_{\rm phys} \tag{3.75}$$

The first three Tables 3.5, 3.6 and 3.7 summarize our results for the mass spectrum at the high scale for meson-deformed 1-1-5 model specified in Table 3.4.

[9]Inclusion of such effects would be actually not completely straightforward because our mass-insertion technique breaks down when used in the two-loop diagrams for the scalars. The reason for this can be traced to the non-cancellation of the UV cutoff dependent terms. This problem would disappear if one performs a complete higher-loop calculation. In any case since the leading order result for scalars was non-vanishing we do not expect any significant changes from this.

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$	X	$\tilde{M}$ M	total
Tree-level constrained	8.22×10^{-5}	0.00	0.00	8.22×10^{-5}
Unconstrained (tree level masses)	5.34×10^{-3}	0.00	0.00	5.34×10^{-3}
Unconstrained (CW improved masses)	2.81×10^{-3}	4.49×10^{-3}	8.30×10^{-5}	7.38×10^{-3}

Table 3.5: *Contributions to the reduced gaugino mass $m_{1/2}$ for the meson-deformed 1-1-5 model of Table 3.4.*

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$
Tree-level constrained	0.48
Unconstrained (tree level masses)	0.48
Unconstrained (CW improved masses)	not consistent

Table 3.6: *Contributions to the reduced sfermion masses m_0 (only ρ, $\tilde{\rho}$, Z, $\tilde{Z}$ contribution) for the meson-deformed 1-1-5 model of Table 3.4. The third line in the table indicates that the use of the full CW corrected masses is inappropriate in this case (see text).*

Particle	Reduced Mass/μ_X
sfermions	0.48
gauginos	7.4×10^{-3}
χ_f	0.13
χ_s	1.33, 2.35
M_f, $\tilde{M}_f$	0.42
M_s, $\tilde{M}_s$	9.58, 9.73

Table 3.7: *Reduced masses for the various particles charged under the SM gauge group for the meson-deformed 1-1-5 model of Table 3.4, with $M_{\text{SUSY}}/\mu_X = 2.7$.*

The following four Tables 3.8, 3.9, 3.10 and 3.11 give results for the same 1-1-5 model but with a different choice of parameters. Comparing the last lines in Table 3.5 and Table 3.9 we see that the contribution from the X messengers can be of the same order but the relative sizes of the different contributions can vary quite significantly.

In total both models give rather similar predictions. with scalars being two orders of magnitude heavier than the gauginos. This is a "slightly" split-SUSY scenario which is expected in all of our direct mediation ISS-SSM models.

In addition, as can be seen from Tables 3.7, 3.11, some of the messengers which are charged under the Standard Model gauge group are relatively light with masses somewhere in between the scalars and the gauginos.

VEV	$\kappa/\mu_X = \xi/\mu_X$	η/μ_X	p/μ_X	χ/μ_X	A/μ_X	B/μ_X	C/μ_X
Tree-level constrained	4.76	0.00	-0.09	-2.50	17.14	13.61	215.92
Unconstrained	4.76	2.00×10^{-3}	-0.08	-2.51	17.20	13.56	217.38

Table 3.8: *Stabilized vevs for a meson model with* $N_f = 7$, $N_c = 6$, $h = 1$, $m_1/\mu_X = 0.05$, $m_2/\mu_X = 0.01$, $\mu_Y/\mu_X = 5$, $\mu_P/\mu_X = 3$ *and* $\lambda = 0.01$.

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$	X	$\tilde{M}$ M	total
Tree-level constrained	5.91×10^{-5}	0.00	0.00	5.91×10^{-5}
Unconstrained (tree level masses)	3.45×10^{-3}	0.00	0.00	3.45×10^{-3}
Unconstrained (CW improved masses)	1.78×10^{-3}	7.06×10^{-4}	1.34×10^{-5}	2.50×10^{-3}

Table 3.9: *Contributions to the reduced gaugino mass* $m_{1/2}$ *for the meson-deformed 1-1-5 model of Table 3.8.*

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$
Tree-level constrained	0.53
Unconstrained (tree level masses)	0.54
Unconstrained (CW improved masses)	not consistent

Table 3.10: *Contributions to the reduced sfermion masses* m_0 *(only* ρ, $\tilde{\rho}$, Z, $\tilde{Z}$ *contribution) for the meson-deformed 1-1-5 model of Table 3.8.*

The remaining six tables in this subsection give an example for a 2-2-3 model – a model with a non-trivial magnetic group. This model has very similar features with the only exception being that the reduced gaugino and sfermion masses differ for the different gauge groups. This shows that one can achieve a deviation from the simple scaling of the full physical masses with the gauge couplings eqs. 3.73 and 3.74 because $m_{1/2}$ and m_0 now actually depend on the index A specifying the gauge group.

Particle	Reduced Mass/μ_X
sfermion	0.54
gauginos	2.50×10^{-3}
χ_f	8.83×10^{-2}
χ_s	2.39, 2.71
M_f, $\tilde{M}_f$	0.24
M_s, $\tilde{M}_s$	10.20, 10.16

Table 3.11: *Reduced masses for the various particles charged under the SM gauge group for the meson-deformed 1-1-5 model of Table 3.8, with $M_{\rm SUSY}/\mu_X = 2.7$.*

VEV	$\kappa/\mu_X = \xi/\mu_X$	η/μ_X	p/μ_X	χ/μ_X	A/μ_X	B/μ_X	C/μ_X
Tree-level constrained	4.56	0.00	−1.40	−4.33	33.25	12.63	105.07
Unconstrained	4.56	2.00×10^{-3}	−1.34	−4.22	33.56	12.47	103.04

Table 3.12: *Stabilized vevs for a meson model with $N_f = 7$, $N_c = 5$, $h = 1$, $m_1/\mu_X = 0.03$, $m_2/\mu_X = 0.05$, $\mu_Y/\mu_X = 5$, $\mu_P/\mu_X = 3$ and $\lambda = 0.01$.*

Contribution (in units of μ_X)	σ, $\tilde{\sigma}$, N, $\tilde{N}$	P	$\tilde{M}$ M	total
Tree-level constrained	-4.50×10^{-3}	0.00	0.00	-4.50×10^{-3}
Unconstrained (tree level masses)	4.10×10^{-3}	0.00	0.00	-4.40×10^{-3}
Unconstrained (CW improved masses)	-4.52×10^{-2}	-4.17 $\times 10^{-4}$	-3.60×10^{-4}	-4.60×10^{-2}

Table 3.13: *Contributions to the reduced mass $m_{1/2}^{(2)}$ of the SU(2) gaugino for the meson-deformed 2-2-3 model of Table 3.12.*

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$	χ	$\tilde{M}$ M	total
Tree-level constrained	2.80×10^{-3}	0.00	0.00	2.80×10^{-3}
Unconstrained (tree level masses)	1.10×10^{-2}	0.00	0.00	1.10×10^{-2}
Unconstrained (CW improved masses)	1.05×10^{-2}	1.05×10^{-2}	-2.40×10^{-4}	2.10×10^{-2}

Table 3.14: *Contributions to the reduced gluino mass $m_{1/2}^{(3)}$ for the meson-deformed 2-2-3 model of Table 3.12.*

We have generated the soft SUSY breaking terms of the SSM at the high (messenger) scale. In order to determine the mass spectrum at the Electro-Weak scale the soft SUSY breaking parameters given in the tables should be renormalization group evolved. But we expect that the overall pattern remains the same.

Contribution (in units of μ_X)	σ, $\tilde{\sigma}$, N, $\tilde{N}$
Tree-level constrained	2.93
Unconstrained (tree level masses)	2.94
Unconstrained (CW improved masses)	not consistent

Table 3.15: *Contributions to the reduced masses $m_0^{(2)}$ of the SU(2) sfermions (only ρ, $\tilde{\rho}$, Z, $\tilde{Z}$ contribution) for the meson-deformed 2-2-3 model of Table 3.12.*

Contribution (in units of μ_X)	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$
Tree-level constrained	1.74
Unconstrained (tree level masses)	1.74
Unconstrained (CW improved masses)	not consistent

Table 3.16: *Contributions to the SU(3) sfermion masses $m_0^{(3)}$ (only σ, $\tilde{\sigma}$, N, $\tilde{N}$ contribution) for the meson-deformed 2-2-3 model of Table 3.12.*

Particle	Reduced Mass/μ_X
sfermions SU(2)	2.95
sfermions SU(3)	1.74
gauginos SU(2)	4.60×10^{-2}
gauginos SU(3)	2.10×10^{-2}
χ_f	0.41
χ_s	14.46, 15.06
P_f	0.62
P_s	5.40, 8.56
M_f, $\tilde{M}_f$	0.47
M_s, $\tilde{M}_s$	11.79, 11.56

Table 3.17: *Reduced masses for the various particles charged under the SM gauge group for the meson-deformed 2-2-3 model of Table 3.12 with $M_{\rm SUSY}/\mu_X = 2.96$.*

In summary, we see that all our direct models have the following features: 1) A heavy scalar spectrum; 2) The pseudo-Goldstone direct messengers are relatively light and the effective low energy theory is always extended away from the MSSM; 3) We can have deviations from the standard gaugino/sfermion mass pattern dictated by the Standard Model gauge couplings.

3.4 The baryon-deformed ISS theory and its mediation patterns

In this section we revisit models with the hidden sector given by baryon-deformed ISS theory introduced in [2, 66]. These models form extensions/deformations of the ISS which are complimentary to the meson deformations discussed above. We will extend the analysis to include the effects of the X and M messengers.

3.4.1 The baryon-deformed model

We start with an ISS model with $N_c = 5$ colours and $N_f = 7$ flavours, which has a magnetic dual description as an $SU(2)$ theory, also with $N_f = 7$ flavours and following [2, 66] we deform this theory by the addition of a baryonic operator. The resulting superpotential is given by

$$W = \Phi_{ij}\varphi_i.\tilde{\varphi}_j - \mu_{ij}^2\Phi_{ji} + m\varepsilon_{ab}\varepsilon_{rs}\varphi_r^a\varphi_s^b \tag{3.76}$$

where $i, j = 1...7$ are flavour indices, $r, s = 1, 2$ run over the first two flavours only, and a, b are $SU(2)$ indices. This is the superpotential of ISS with the exception of the last term which is a baryon of the magnetic $SU(2)$ gauge group. Note that the 1,2 flavour indices and the 3...7 indices have a different status and the full flavour symmetry $SU(7)_f$ is broken explicitly to $SU(2)_f \times SU(5)_f$. As before, the *direct* gauge mediation is implemented by gauging the $SU(5)_f$ factor and identifying it with the parent $SU(5)$ gauge group of the Standard Model. The matter field decomposition under the magnetic $SU(2)_{gauge} \times SU(5)_f \times SU(2)_f$ and their $U(1)_R$ charges are given in Table 3.1 with $R = 1$.

Using the notation established in the previous sections for the meson model the baryon-deformed model defined by eq. 3.76 is a 2-5 model. It is straightforward to consider alternatives such as a 1-5 model where the magnetic gauge group is empty and the baryon deformation is a linear operator,

$$W_{1-5} = \Phi_{ij}\varphi_i.\tilde{\varphi}_j - \mu_{ij}^2\Phi_{ji} + k\varphi_1, \tag{3.77}$$

or, for example, a 2-2-3 model as before. In all of those models Landau poles inherent in the direct mediation can be avoided by using the deflected unification mechanism of [89]. This works most effectively in the 1-5 model due to its minimal matter content.

The discussion of these models is virtually identical to that which we will now present for the 2-5 model.

At the Lagrangian level this baryon-deformed model respects R-symmetry. Thanks to the baryon deformation, the structure of R-charges allows for spontaneous R symmetry breaking and it was shown in [2] that this does indeed happen. We also stress that our baryon deformation is the leading order deformation of the ISS model that is allowed by R-symmetry of the full theory imposed at the Lagrangian level. As explained in [66] this is a self-consistent approach. For example, terms quadratic in the meson Φ that could arise from lower dimensional irrelevant operators in the electric theory are forbidden by R-symmetry. Thus, our deformation is described by a *generic* superpotential and 3.76 gives its leading-order terms.

Using the $SU(2)_f \times SU(5)_f$ symmetry, the matrix μ^2_{ij} can be brought to the form 3.3. The baryon operator can be identified with a corresponding operator in the electric theory. Indeed the mapping from baryons B_E in the electric theory to baryons B_M of the magnetic theory, is $B_M \Lambda_{ISS}^{-N} \leftrightarrow B_E \Lambda_{ISS}^{-N_c}$ (we neglect factors of order one). Thus one expects

$$m \sim M_{Pl} \left(\frac{\Lambda_{ISS}}{M_{Pl}} \right)^{N_f - 2N} = \frac{\Lambda^3_{ISS}}{M^2_{Pl}}, \tag{3.78}$$

where M_{Pl} represents the scale of new physics in the electric theory at which the irrelevant operator B_M is generated.

The F-term contribution to the potential at tree-level is

$$\begin{aligned} V &= \sum_{ar} |Y_{rs}\tilde{\phi}^a_s + Z_{r\hat{i}}\tilde{\rho}^a_{\hat{i}} + 2m\varepsilon_{ab}\varepsilon_{rs}\phi^b_s|^2 \qquad (3.79) \\ &+ \sum_{a\hat{i}} |\tilde{Z}_{\hat{i}r}\tilde{\phi}^a_r + X_{\hat{i}\hat{j}}\tilde{\rho}^a_{\hat{j}}|^2 + \sum_{as} |\phi^a_r Y_{rs} + \rho^a_{\hat{i}}\tilde{Z}_{\hat{i}s}|^2 + \sum_{a\hat{j}} |\phi^a_r Z_{r\hat{j}} + \rho^a_{\hat{i}} X_{\hat{i}\hat{j}}|^2 \\ &+ \sum_{rs} |(\phi_r . \tilde{\phi}_s - \mu^2_Y \delta_{rs})|^2 + \sum_{r\hat{i}} |\phi_r . \tilde{\rho}_{\hat{i}}|^2 + \sum_{r\hat{i}} |\rho_{\hat{i}} . \tilde{\phi}_s|^2 + \sum_{\hat{i}\hat{j}} |(\rho_{\hat{i}} . \tilde{\rho}_{\hat{j}} - \mu^2_X \delta_{\hat{i}\hat{j}})|^2 \end{aligned}$$

where a, b are $SU(2)_{mg}$ indices. The flavor indices r, s and $\hat{i}$, $\hat{j}$ correspond to the $SU(2)_f$ and $SU(5)_f$, respectively. It is straightforward to see that the rank condition works as in ISS; that is the minimum for a given value of X, Y, Z and $\tilde{Z}$ is along $\rho = \tilde{\rho} = 0$ and

$$\langle \phi \rangle = \frac{\mu^2_Y}{\xi} \mathbf{I}_2 \,, \qquad \langle \tilde{\phi} \rangle = \xi \, \mathbf{I}_2, \tag{3.80}$$

where ξ parametrizes a runaway direction that will be stabilized by the Coleman-Weinberg potential eq. 3.18. This then gives $Z = \tilde{Z} = 0$. In addition Y becomes diagonal and real (assuming m is real). Defining $\langle Y_{rs} \rangle = \eta \, \mathbf{I}_2$, the full potential is

$$V = 2\left|\eta\,\xi + 2m\frac{\mu_Y^2}{\xi}\right|^2 + 2\left|\eta\frac{\mu_Y^2}{\xi}\right|^2 + 5\mu_X^4. \tag{3.81}$$

Using R symmetry we can choose ξ to be real[10]. Minimizing in η we find

$$\eta = -2m\left(\frac{\xi^2}{\mu_Y^2} + \frac{\mu_Y^2}{\xi^2}\right)^{-1}. \tag{3.82}$$

Substituting $\eta(\xi)$ into eq. 3.81 we see that $\xi \to \infty$ is a runaway direction along which

$$V(\xi) = 8m^2\mu_Y^2\left(\frac{\xi^6}{\mu_Y^6} + \frac{\xi^2}{\mu_Y^2}\right)^{-1} + 5\mu_X^4. \tag{3.83}$$

Since in the limit $\xi \to \infty$, the scalar potential V is non-zero, we have a runaway to broken supersymmetry, hence the Coleman-Weinberg potential again lifts and stabilizes this direction, which is indeed the case [2]. As in eqs. 3.21 we parametrize the pseudo-Goldstone and runaway vevs by

$$\langle \tilde{\phi} \rangle = \xi \, \mathbf{I}_2 \qquad \langle \phi \rangle = \kappa \, \mathbf{I}_2 \tag{3.84}$$

$$\langle Y \rangle = \eta \, \mathbf{I}_2 \qquad \langle X \rangle = \chi \, \mathbf{I}_5. \tag{3.85}$$

Stabilized vevs for a 2-5 and a 1-5 model are shown in Tables 3.18 and 3.19, respectively. Constrained vevs in these tables arise from using the tree-level equations of motion eqs. 3.80 and 3.82. Again, the difference between constrained and unconstrained vevs is rather small but the general discussion of subsection 3.3.2 indicates that this difference has crucial effects on the generation of gaugino masses in direct mediation.

Explicit mediation has been studied in [66] and leads to the usual standard GMSB pattern (as also discussed for the meson-deformed model in subsection 3.3.1).

[10] The phase of ξ corresponds to the R-axion.

Vev	κ/μ_X	ξ/μ_X	η/μ_X	χ/μ_X
Tree-level constrained	1.10	8.18	−0.08	−0.35
Unconstrained	1.10	8.18	−0.08	−0.35

Table 3.18: *Stabilized vevs for a 2-5 baryon-deformed model with* $N_f = 7$, $N_c = 5$, $h = 1$, $m/\mu_X = 0.3$ *and* $\mu_Y/\mu_X = 3$.

Vev	κ/μ_X	ξ/μ_X	η/μ_X	χ/μ_X
Tree-level constrained	1.76	5.11	−0.05	−0.21
Unconstrained	1.76	5.11	−0.05	−0.20

Table 3.19: *Stabilized vevs for a 1-5 baryon-deformed model with* $N_f = 6$, $N_c = 5$, $h = 1$, $k/\mu_X^2 = 0.3$ *and* $\mu_Y/\mu_X = 3$.

3.4.2 Signatures in the directly mediated baryon-deformed model

The basic equations for calculating gaugino and scalar masses are the same as in subsection 3.3.2. Only the VEV configurations and the structure of the messenger mass matrices know about the difference in the deformation.

Our results for the soft SUSY breaking parameters at the messenger scale are presented below following the same structure as before. The first three tables correspond to the 2-5 model given in Table 3.18. The next three correspond to the 1-5 model specified in Table 3.19.

Evidently, the dominant contribution to the gaugino mass comes from unconstraining the vevs and putting in the full one-loop mass matrices. Overall this leads again to models with heavy scalars and, in distinction to our earlier paper [66] (where the constrained vevs were used), we do not need to fine tune the different μ^2 parameters to achieve a moderately split spectrum. It is remarkable that in all of the directly mediated ISS models gaugino masses are this sensitive to quantum corrections (due to the inevitable cancellation at tree-level).

Contribution	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$	χ	total
Tree-level constrained	4.17×10^{-5}	0.00	4.17×10^{-5}
Unconstrained (tree level masses)	1.74×10^{-3}	0.00	1.74×10^{-3}
Unconstrained (CW improved masses)	-1.57×10^{-3}	9.61×10^{-7}	-1.57×10^{-3}

Table 3.20: *Contributions to the reduced gaugino mass for the baryon-deformed 2-5 model of Table 3.18.*

Contribution	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$
Tree-level constrained	0.70
Unconstrained (tree level masses)	0.70
Unconstrained (CW improved masses)	not consistent

Table 3.21: *Contributions to the reduced sfermion masses (only ρ, $\tilde{\rho}$, Z, $\tilde{Z}$ contribution) for the baryon-deformed 2-5 model of Table 3.18.*

Particle	Mass/μ_P
sfermion	0.70
gauginos	1.57×10^{-3}
χ_f	1.92×10^{-2}
χ_s	2.92, 2.93

Table 3.22: *Reduced masses for the various particles charged under the SM gauge group for the baryon-deformed 2-5 model of Table 3.18.*

Contribution	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$	χ	total
Tree-level constrained	2.67×10^{-5}	0.00	2.67×10^{-5}
Unconstrained (tree level masses)	7.49×10^{-4}	0.00	7.49×10^{-4}
Unconstrained (CW improved masses)	-5.97×10^{-4}	3.60×10^{-7}	-5.96×10^{-4}

Table 3.23: *Contributions to the reduced gaugino mass for the baryon-deformed 1-5 model of Table 3.19.*

Contribution	ρ, $\tilde{\rho}$, Z, $\tilde{Z}$
Tree-level constrained	0.61
Unconstrained (tree level masses)	0.61
Unconstrained (CW improved masses)	not consistent

Table 3.24: *Contributions to the reduced sfermion masses (only ρ, $\tilde{\rho}$, Z, $\tilde{Z}$ contribution) for the baryon-deformed 1-5 model of Table 3.19.*

Particle	Mass/μ_P
sfermions	0.61
gauginos	5.96×10^{-4}
χ_f	1.10×10^{-2}
χ_s	2.921, 2.919

Table 3.25: *Reduced masses for the various particles charged under the SM gauge group for the baryon-deformed 1-5 model of Table 3.19.*

3.5 Conclusions

We have investigated different scenarios of gauge mediation which incorporate a dynamical SUSY breaking (DSB) sector coupled to a supersymmetric Standard Model. The DSB sector was realized in terms of two different types of deformations of the ISS model. These models generate all SUSY breaking parameters at the messenger scale in a calculable way from relatively simple supersymmetric Lagrangians. In all of the models investigated we find rather model independent signatures for the direct gauge mediation which include:

- Scalars are typically two orders of magnitude or more heavier than gauginos if we expand around the global minimum, but can otherwise be of the same order of magnitude as scalar masses.
- The low energy effective theory of the visible sector i.e. particles charged under the Standard Model gauge groups is necessarily extended by light pseudo-Goldstone messenger fields.

Because one requires to include quantum corrections to have a proper minimum of the potential, many of the models considered here do not strictly fall in the kind of models contemplated by Komargodski and Shihs argument[3]. Despite this, because the tree-level relations between the F-terms are weakly violated, gaugino masses are "abnormally" small. Contributions from non-standard (pseudo-moduli) messengers does not substantially change this conclusion.

In [100], a metastable vacuum was constructed in a model very similar to the meson deformed ISS that we presented here that leads to a viable soft mass spectrum.

More generally, in extra-ordinary gauge mediation[65], and then by using the framework of General Gauge Mediation (GGM) [55], it was shown that the scales for soft gaugino and scalar masses generically can only be described using two independent mass scales (in [55] the actual "size" of the parameter space for GGM was described and in [67] was discussed how this could be implemented in different models).

Finally we would like to briefly comment on how the usual little hierarchy problem of the supersymmetric Standard Model manifests itself. First of all, the non-observation of the Higgs at LEP requires that the mass of the lightest Higgs, $m_{h^0} > 115\,\mathrm{GeV}$. On the other hand, supersymmetric models predict an upper bound so that

$$(115\,\mathrm{GeV})^2 \,<\, m_{h^0}^2 \,<\, \cos^2(2\beta)m_Z^2 \,+\, \mathrm{rad.\,corr.}\,, \tag{3.86}$$

where the radiative corrections $\sim m_t^2 \log(m_{\tilde{t}}/m_t)$. To fulfill this one needs a rather large stop mass, which our models deliver. On the other hand, the conditions for Electro-Weak symmetry breaking require that at the E.W. scale

$$m_Z^2 = -2(m_{H_u}^2 + |\mu_{\mathrm{MSSM}}|^2) + \mathcal{O}(1/\tan^2(\beta)). \tag{3.87}$$

The scalar masses, including m_{H_u}, and their loop corrections are of the order of $m_{\tilde{t}}$ and are (as just argued) much bigger than the E.W. scale. This requires a fine-tuning of μ_{MSSM} of the order of 10^{-2}. In the direct mediation scenarios with a mildly split SUSY spectrum, $m_{\tilde{t}}$ is bigger than the minimal required value from eq. 3.86 resulting in a somewhat higher degree of fine-tuning of the order of $10^{-4} - 10^{-5}$. In this chapter we are treating μ_{MSSM} as a free parameter and do not attempt to solve this problem.

Chapter 4

Searching for F & D-term SUSY breaking:

"The possibility that we are living in a false vacuum has never been a cheering one to contemplate. Vacuum decay is the ultimate ecological catastrophe.[1]"
— Sidney Coleman, 1937-2007

We now understand that the reason why gaugino masses vanish (or are small) does not have to do with with how R-symmetry is broken, but to the fact that one is using the global minimum of the potential as a vacuum and SUSY is broken by F-term vevs alone.

This leads us to a picture where the vacuum is usually perturbatively metastable. Since the life-time of the vacuum has to be much larger than the age of the universe, this puts constraints on the amount of the metastability in the model, and consequently the size of the R-symmetry breaking operators. In the ISS deformed models, these turn out to be parametrically small, as they correspond to irrelevant operators in the U.V.. Their scale is suppressed by factors of $\frac{\Lambda_{ISS}}{M_{pl}}$, where Λ_{ISS} is the ISS dynamical scale.

In this chapter we investigate an alternative to this picture. Specifically we will investigate if it is possible to build calculable models with non-Abelian gauge symmetries that break SUSY by having both F *and* D-term vevs in the (global) vacuum. Models that

[1]This playful quote is from his paper "Gravitational effects on and of vacuum decay".

have this property, evade the argument of [3], and the universe can live in a perturbatively stable vacuum.

In some special cases, as first noticed in [69] and then [101], the superpotential can naturally be non-generic and SUSY can be broken even if there is no R-symmetry and the vacuum is stable. In these models there is no tension between the stability of the vacua, gaugino and axion masses. We will build a model with similar properties and where SUSY is broken by having non-zero F and D-term vevs.

The hope was that by coupling this type of hidden sector models with the MSSM a viable soft mass spectrum was achieved. However, since the allowed field content and superpotential couplings are highly constrained by symmetries, it is very hard to build a model where SUSY breaking is not communicated semi-directly.

Having non-zero D-terms clearly implies the spontaneous breaking of a gauge group. In this chapter we will investigate only cases where the hidden sector gauge group can have non-zero D-terms. The case where the MSSM GUT gauge group is Higgsed in a non-SUSY way is considered in the last chapters.

4.1 Constraints on models of combined F and D-term SUSY breaking:

In this section we will show that if the superpotential is renormalizable and the Kahler potential is canonical, the model can only break SUSY through F-terms (globally), i.e. when minimizing the function $V_D + V_F$ one always finds $V_D = 0$. To do this we will find a set of solutions that minimizes V_F and then show that it's always possible to chose one that solves $V_D = 0$. Since $V_D \geq 0$ this is the global minimum of the model.

Since we want to minimize V_F, we start by reviewing some known results about O'Raifeartaigh (O'R.) models[3, 26]:

Let us consider a renormalizable Wess-Zumino model with superpotential W (i.e. W is a degree 3 polynomial in the fields[2]), the Kahler potential is canonical. Then, $V_F = \sum_i |W_i|^2$, where W_i is the derivative of the superpotential with respect to the field ϕ^i and indices are raised and lowered by complex conjugation.

[2]If the theory is asymptotically free, the superpotential cannot have terms whose value for large field vevs grows faster than Φ^3

Since we are assuming that the minimum of the potential breaks SUSY, not all the W_i can be made to vanish simultaneously. At the minimum, the gradient of the potential must vanish, $W_{ij}W^j = 0$ (i.e. we are not considering cases where there are runaway directions, even though they might be interesting).

The tree-level boson mass is:

$$m_B^2 = \begin{pmatrix} \mathcal{M}_F^* \mathcal{M}_F & \mathcal{F}^* \\ \mathcal{F} & \mathcal{M}_F \mathcal{M}_F^* \end{pmatrix} \tag{4.1}$$

where $\mathcal{F}_{ij} = W_{ijk}W^k$, and $\mathcal{M}_F = W_{ij}$. In a consistent vacuum M_B^2 must be positive semi-definite.

We will now see that in the direction (W^i, W_i) this scalar mass matrix has a zero eigenvalue:

$$\begin{pmatrix} W_i \\ W^j \end{pmatrix}^\dagger \begin{pmatrix} \mathcal{M}_F^* \mathcal{M}_F & \mathcal{F}^* \\ \mathcal{F} & \mathcal{M}_F \mathcal{M}_F^* \end{pmatrix} \begin{pmatrix} W^i \\ W_j \end{pmatrix} = 2Re(W^i \mathcal{F}_{ij} W^j). \tag{4.2}$$

Since this mass matrix is positive semi-definite, this must vanish, otherwise it could be made negative by rotating the phase of W^i. An important point to make here is that in principle, one can have derivatives of D-terms and D-terms in the bosonic mass matrix as well. Since we will show that we can always choose a point where the D-terms vanish, it is self consistent to ignore D-term vevs. Derivatives of D-terms are unimportant as gauge invariance of the superpotential implies that $W_i D^{a,i} \equiv 0$. So $W^i \mathcal{F}_{ij} W^j = 0$. It's possible to show, by performing an expansion of the potential to 3rd order, and using this equation, that one actually has the stronger result: $\mathcal{F}_{ij} W^j = 0$.

If the superpotential is renormalizable, then the model has a classically flat direction parametrized by $\phi^i = (\phi^i)^{(0)} + zW^i$:

$$\delta W_i = W_{ij} z W^j + W_{ijk}(zW^j)(zW^k) = 0 \tag{4.3}$$

This expansion is exact since higher derivatives of W vanish. So, the F-terms are constant along this direction[3].

We will now consider the two possible situations:

[3]In a few moments we will show that V_D is constant along this direction as well

1. The F-terms do not break gauge symmetry;

2. The F-terms break gauge symmetry;

The first option is, for example, the case of the ISS model, the second case looks like the 3-2 or 4-1 models, except that in those cases the superpotentials are not 3rd order polynomials.

In the first case the non-vanishing F-terms are all gauge invariant. This means that if we perform a complex gauge transformation on the vevs of the fields, the potential remains invariant. In this case the symmetry group under which the potential V_F is invariant is enlarged from SU(N) to SL(N). This symmetry is enough to solve all the D-term equations.

One should note that this is not a true complex gauge transformation as only the vevs of the fields, and not the fields themselves, are being rescaled. Meaning, if we were performing a complexified gauge transformation, invariance of the potential would be guaranteed by a shift in the lowest component of the vector superfield. In the case we are considering, the SL(N) symmetry of the vacuum would still be there if there was no vector superfield (i.e. gauge symmetry).

We consider two cases to illustrate the point: one explicit example and a general argument.

Take an ISS model where we have gauged a U(1) baryon symmetry. The model has gauge group U(1) and the flavor group is SU(6) (for this number of flavors the magnetic gauge group of the ISS is empty, when $N_c = 5$). The field content is: quarks, ϕ, that transform under the fundamental of color and flavor, antiquarks, $\tilde{\phi}$, that transform as anti-fundamentals of color and flavor, and some mesons, Φ, that are color singlets and transform under the adjoint plus a singlet of flavor.

The superpotential is:

$$W = h(Tr[\tilde{\phi}\phi\Phi] - \mu^2 Tr[\Phi]) \tag{4.4}$$

SUSY is broken by the rank condition:

$$F_{\Phi_{ij}} = h(\tilde{\phi}_j \phi_i - \mu^2 \delta_{ij}) \tag{4.5}$$

Since the number of colors is less than the number of flavors, it's not possible to solve all these equations and SUSY is broken. The scalar potential is then minimized to be:

$$V_F = 5h^2\mu^4 \tag{4.6}$$

The moduli space up to global symmetries is given by:

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix}, \phi = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}, \tilde{\phi} = \begin{pmatrix} \tilde{\phi}_0 \\ 0 \end{pmatrix}, \text{with} \quad \tilde{\phi}_0\phi_0 = \mu^2 \tag{4.7}$$

The D-term potential in a random point of the pseudomoduli space is

$$V_D = \frac{g^2}{2}(|\phi_0|^2 - |\tilde{\phi}_0|^2)^2 \tag{4.8}$$

Under a complexified gauge transformation of the vevs,

$$\begin{aligned} \phi &\to e^{\alpha}\phi \\ \tilde{\phi} &\to \tilde{\phi}e^{-\alpha} \end{aligned} \tag{4.9}$$

And $V_D(\alpha) = (e^{2\alpha}|\phi_0|^2 - e^{-2\alpha}|\tilde{\phi}_0|^2)^2$. This vanishes when $\alpha = \frac{1}{4}Log[\frac{|\tilde{\phi}_0|^2|}{|\phi_0|^2}$. Note that on the F-term moduli space, neither ϕ or $\tilde{\phi}$ can vanish.

The general argument showing that in a SUSY theory pure D-term SUSY breaking is not possible[12] was reviewed in section 1.5.2, so we shall not repeat it here. This can be trivially extended to the case when there are non-zero F-terms that are gauge invariant[102].

We now look at models where the F-terms are not gauge invariant. In this case the vacuum manifold does not have the enhanced symmetry, and the previous argument does not hold. However, the vacuum manifold contains at least the direction parametrized by $\phi^i = (\phi^i)^{(0)} + zW^i$.

Since not all F-terms are gauge invariant, the heavy vector superfield will not be SUSY; for example there will be mixings between gauginos and chiral fermion mass matrices.

One can still investigate how different matrix elements are constrained by gauge invariance:

$$W_i(D^a)^{,i} \equiv W_i(T^a)^i_j\phi^j \equiv 0 \tag{4.10}$$

Because the superpotential must be gauge invariant, note that this is an identity and is not valid only at the minimum of V_F. Because of this we can get further identities from differentiating the previous expression:

$$\begin{aligned} W_{ij}(D^a)^{,i} + W_i(D^a)^{;i}_j \equiv 0 \\ W_{ijk}(D^a)^{,i} + W_{ij}(D^a)^{;i}_k + W_{ik}(D^a)^{;i}_j \equiv 0 \end{aligned} \tag{4.11}$$

At the minimum of the V_F, the previous expressions implies that:

$$W_i(T^a)^i_k W^k \equiv -W_{ik}W^k(D^a)^{,i} = 0 \tag{4.12}$$

We will now see that the D-terms are constant along the pseudo-moduli direction $\phi_i = \phi_i^{(0)} + zW_i$:

$$D^a(z) = D^a(0) + 2Re(z(\phi_i^{(0)})(T^a)^i_j W^j) + |z|^2 W_i(T^a)^i_j W_j = D^a(0) \tag{4.13}$$

The linear term in z vanishes because of(4.10), and the quadratic term in z vanishes by (4.12).

We will now show that even though the D-terms are constant along that particular pseudo-moduli direction, they can be made to vanish arbitrarily close to it. We will give first a specific example and discuss why this happens and then give a general argument.

Let us take a model considered in [80]: we have two chiral/antichiral fields that transform under a U(1), and a singlet. The superpotential is:

$$W = hS(\phi\tilde{\chi} - \mu^2) + m_1\phi\tilde{\phi} + m_2\chi\tilde{\chi} \tag{4.14}$$

Except for S, the untilded fields have charge +1 under the U(1) and the tilded fields have charge -1. The model has an R-symmetry with charges $R(S) = 2$, $R(\phi) = R(\tilde{\chi}) = 0$, $R(\tilde{\phi}) = R(\chi) = 2$, and the superpotential is general.

The F-term potential is given by

$$V_F = h^2|\phi\tilde{\chi} - \mu^2|^2 + m_1^2|\tilde{\chi}|^2 + m_2^2|\phi|^2 + |hS\tilde{\chi} + m_1\tilde{\phi}|^2 + |hS\phi + m_2\chi|^2 \tag{4.15}$$

The model breaks SUSY at tree-level, and the minimum is:

$$\begin{array}{rl} V_F = h^2\mu^4 & \text{if} \quad \mu^2 < \frac{m_1 m_2}{2h^2} \\ V_F = 2m_1m_2\mu^2 - \frac{m_1^2 m_2^2}{h^2} & \text{if} \quad \mu^2 > \frac{m_1 m_2}{2h^2} \end{array} \tag{4.16}$$

This as long as both m_1 and m_2 are positive. This model has pure F-term SUSY breaking: In the first case no charged field gets a vev. The $U(1)$ is unbroken and the D-terms vanish.

In the second case the $U(1)$ is broken. The vevs are:

$$\begin{array}{rl} \phi = & -\frac{\sqrt{m_2}\sqrt{m_1(h^2\mu^2 - m_1m_2)}}{hm_1} \\ \tilde{\phi} = & \frac{\sqrt{m_1(h^2\mu^2 - m_1m_2)}S}{m_1\sqrt{m_2}} \\ \chi = & \frac{\sqrt{m_1(h^2\mu^2 - m_1m_2)}S}{m_1\sqrt{m_2}} \\ \tilde{\chi} = & -\frac{\sqrt{m_1(h^2\mu^2 - m_1m_2)}}{h\sqrt{m_2}} \end{array} \tag{4.17}$$

for any value of S.

The flat direction is parametrized by:

$$\begin{array}{rl} S = & S^{(0)} - z\frac{m_1 m_2}{h} \\ \tilde{\phi} = & \tilde{\phi}^{(0)} - \frac{\sqrt{m_2}\sqrt{m_1(h^2\mu^2 - m_1m_2)}}{h}z \\ \chi = & \chi^{(0)} - \frac{\sqrt{m_2}\sqrt{m_1(h^2\mu^2 - m_1m_2)}}{h}z \end{array} \tag{4.18}$$

If we replace these vevs into V_D, we get that $V_D = \frac{1}{2}g^2\frac{(m_1^2 - m_2^2)^2(m_1m_2 - h^2\mu^2)^2}{h^4 m_1^2 m_2^2}$, which is generally non-zero unless $m_1 = m_2$. Note that this is a constant and independent of z, as we have seen it should be.

We will now deform the vevs by ϵ in the following way:

$$\begin{aligned} \phi &= -\frac{\sqrt{m_2}\sqrt{m_1(h^2\mu^2-m_1m_2)}}{hm_1} \\ \tilde{\phi} &= \frac{\sqrt{m_1(h^2\mu^2-m_1m_2)S}}{m_1\sqrt{m_2}} - \epsilon \\ \chi &= \frac{\sqrt{m_1(h^2\mu^2-m_1m_2)S}}{m_1\sqrt{m_2}} + \epsilon \\ \tilde{\chi} &= -\frac{\sqrt{m_1(h^2\mu^2-m_1m_2)}}{h\sqrt{m_2}} \\ S &= \frac{(m_1^2-m_2^2)\sqrt{m_1(h^2\mu^2-m_1m_2)}}{4m_1\sqrt{m_2}h^2\epsilon} \end{aligned} \tag{4.19}$$

In this vacuum

$$V_F = 2m_1m_2\mu^2 - \frac{m_1^2m_2^2}{h^2} + (m_1^2+m_2^2)\epsilon^2 \tag{4.20}$$

$$V_D = 0 \tag{4.21}$$

So, if $\epsilon \to 0$, we solve $V_D = 0$ and minimize V_F.

More generically will show that one can find a small deformation ϵ of the pseudo-moduli space, where the change in the D-terms is proportional to ϵz, while the change in the F-terms will only be proportional to ϵ. One can then take $\epsilon \to 0$ with $z\epsilon$ fixed, to solve $V_D = 0$, and V_F will approach it's minimum value. This means that if we ignore D-terms the model breaks SUSY at tree-level through F-terms, but as soon as we include D-terms they either vanish identically on the pseudo-moduli space or there is a runaway to broken SUSY.

The stabilization of this runaway direction has to be checked because in these models one has the extra contribution from gauge fields to the Coleman-Weinberg potential, and these are not positive definite. This fact can be seen, for example, in [103] and in the last example model we show. Some cases where some metastable vacua with both F and D-terms and no flat directions have been built [104].

We will consider that a gauge rotation of the fields has been made in such a way that there is only one non-vanishing D-term in a direction defined by an element of the Cartan subalgebra of the group, very much like when we dealt with the case that all the F-terms were gauge invariant.

We will chose that the deformation is a complexified gauge transformation in the direction defined by the same element of the Cartan subalgebra. This transformation will generically change both V_F and V_D.

Let T be the element of the Cartan subalgebra, then $T = diag(\{\mu_i\})$. Under this complexified gauge transformation The F-terms change as

$$\tilde{W}_i(z) = W_j(z)(e^{-\alpha_b T^b})_i^j = (W(0)^\dagger.(e^{-\alpha T})_i) = (W^\dagger.diag(\{e^{\alpha\mu_i}\}))_i \tag{4.22}$$

Where, again, we have used ϕ and dot product as a notation for the sum over all the fields and the generator is in the appropriate representation.

Then, the change in the potential is

$$V_F(\alpha) = W^\dagger.diag(\{e^{\alpha\mu_i}\}).W \approx V_F(0) + \alpha^2 W^\dagger.diag(\{\mu_i^2\}).W + O(\alpha^3) \tag{4.23}$$

This is because, as we've seen, the terms linear in the generator of the gauge group vanish. The previous example where we had a $U(1)$ gauge group agrees with this expression.

We now use the eq. 1.53 evaluated at $(\phi^i)' = \phi^i + zW^i$.

We get that:

$$\tilde{D}(z) = \phi^\dagger.diag(\{\mu_i\}).\phi \tag{4.24}$$
$$- 2Im(\alpha)(\phi^\dagger.diag(\{\mu_i^2\}).\phi + 2Re(z\phi^\dagger.diag(\{\mu_i^2\}).W) + |z|^2 W^\dagger.diag(\{\mu_i^2\}).W) \tag{4.25}$$

When z is very large this is solved by:

$$Im(\alpha) = \frac{\phi^\dagger.diag(\{\mu_i\}).\phi}{2|z|^2 W^\dagger.diag(\{\mu_i^2\}).W} \tag{4.26}$$

The solution α is small and the corrections in α^2 we ignored are negligible if we take $z \to \infty$. Note that $W_i\mu_i^2 W^i = \sum_i |\mu_i W^i|^2$ and should be greater than zero if SUSY is broken and the F-terms are not gauge invariant.

The conclusion is that in this class of models it's not possible to have a model where the global minimum has combined F and D-term SUSY breaking.

One should, of course, be careful to check if the description being used is correct and one doesn't go to a strong coupling regime or if one is using an effective theory, go outside its regime of validity.

4.1.1 Some consequences of this result

One interesting consequence of this result is in the context of tree-level gauge mediation. In this type of mediation, one requires the existence of F-terms that are not gauge invariant that lead to the existence of non-zero D-terms at tree-level through the minimization of the potential: $W^{\dagger}T^{a}W + g^{2}(M_{v})^{ab}D^{b} = 0$ (i.e. there may, in principle be vacua where $W^{\dagger}T^{a}W$ is non-zero, which leads to the existence of non-zero D-terms). However this result shows that this is not possible unless the vacuum is metastable, the Kahler potential is not canonical, or there is a dynamical contribution to the superpotential that is not a cubic polynomial in the fields. An interesting possibility is in GUT theories where the mechanism by which the doublet-triplet problem is solved through a dynamical mechanism (i.e. an instanton type contribution to the superpotential). This term is not polynomial in the fields and essentially affects the low energy physics of the model (i.e. it behaves in very much the same way as the ADS superpotential).

4.2 Adding non polynomial terms to the superpotential

In the last case, the discussion relied in the existence of the flat direction, so one question is: what happens if we do not have a flat direction. One such possibility is to consider the present model, but instead of a U(1) gauge symmetry, one considers a $SU(N_c)$ gauge symmetry with $N_c > 2$. In this case we have a SQCD theory with 2 flavours and a singlet and $N_c > N_F$. This theory has a dynamically generated term in the superpotential that comes from instanton contributions, and is not a polynomial of degree 3 (for simplicity we take $N_c = 3$).

The full superpotential is:

$$W = hS(\phi\tilde{\chi} - \mu^{2}) + m_{1}\phi\tilde{\phi} + m_{2}\chi\tilde{\chi} + \frac{\Lambda^{7}}{\phi.\tilde{\phi}\chi.\tilde{\chi} - \chi.\tilde{\phi}\phi.\tilde{\chi}} \tag{4.27}$$

At tree-level the superpotential is general and has an R-symmetry (this disallows shifts in the vev of S to absorb the linear term). However, for $N_f < N_C$ this symmetry is anomalous because of the dynamically generated ADS contribution, and the R-symmetry is broken to a discrete subgroup.

One may wonder if this model breaks SUSY globally. If this were to happen (SUSY is unbroken), we could integrate out the gauge degrees of freedom and use the low energy effective description with mesons:

$$W = hS(M_{12} - \mu^2) + m_1 M_{11} + m_2 M_{22} + \frac{\Lambda^7}{Det(M)} \tag{4.28}$$

The "F-term" equations are:

$$\begin{aligned} W_S &= h(M_{12} - \mu^2) \\ W_{M_{11}} &= m_1 - \tfrac{\Lambda^7 M_{22}}{Det(M)^2} \\ W_{M_{12}} &= hS + \tfrac{\Lambda^7 M_{21}}{Det(M)^2} \\ W_{M_{21}} &= \tfrac{\Lambda^7 M_{12}}{Det(M)^2} \\ W_{M_{22}} &= m_2 - \tfrac{\Lambda^7 M_{11}}{Det(M)^2} \end{aligned} \tag{4.29}$$

where W_ϕ is the derivative of the superpotential with respect to the field ϕ. These equations can be solved asymptotically with

$$\begin{aligned} S &= -\tfrac{\Lambda^7 M_{21}}{hDet(M)^2} \\ M_{11} &= \tfrac{m_2 \Delta^8}{\epsilon^2 \Lambda^7} \\ M_{12} &= \mu^2 \\ M_{21} &= \tfrac{\Delta^4 + \frac{m_1 m_2 \Delta^{16}}{\epsilon^3 \Lambda^{14}}}{\epsilon \mu^2} \\ M_{22} &= \tfrac{m_1 \Delta^8}{\epsilon^2 \Lambda^7} \end{aligned} \tag{4.30}$$

where Δ is some finite mass scale and $\epsilon \to 0$ is some dimensionless parameter. Note that in this limit all vevs but M_{12} go to infinity.

In the D-flat directions the Kahler potential is given by:

$$K = 2Tr(\sqrt{M^\dagger M}) \tag{4.31}$$

Which is valid as long as the theory is weakly coupled (i.e) mesonic vevs are larger than Λ.

The second derivative of the Kahler potential goes to 0 as $\epsilon \to 0$. One can, for simplicity take $m_1 = m_2 = m$, diagonalize M with unitary transformations and do the computation explicitly (using Mathematica for example) and see that in the runaway direction the $V_F = K^{-1}_{ab} W^a W^b$ goes to $2m^2\mu^2$.

This can be understood since as ϵ becomes very small, the scale at which the gauge symmetry is broken becomes large and the theory is weakly coupled. The reason why the potential doesn't slope to 0 for very large vevs of the meson field is easy to understand since in this regime the theory should be weakly coupled and the microscopic description should be good. In this regime, the instanton contribution is rather small compared with the tree-level superpotential, and there is an approximate R-symmetry that guarantees that SUSY is broken.

The reason why we can't just expand the Kahler potential around some scale to get a "canonical" Kahler potential for the meson fields (normalized by the square of the scale we are expanding around), is that there is no scale around which we can do this, i.e. there is no scale around which the minimization of V_F will justify the fact that we've neglected higher order terms in the expansion of the Kahler potential (V_F will be a function that slopes to 0 when the fields go to infinity). In other words, in a theory with a non-canonical Kahler potential SUSY is unbroken if the vev of the auxiliary components of all fields is much smaller than any other scale in the model (either 0 or vanishes asymptotically). For a non-canonical Kahler potential this means: $K^{-1,a,b} W_b \to 0$. We've shown that there is a limit in which $W_b \to 0$, but in this limit K^{-1} is divergent and the product does not go to zero in any direction in field space.

Witten Index

One may wonder if the statement that this model breaks SUSY is in disagreement with the Witten index argument, but note that taking m_1 and m_2 to infinity (the mass matrix has maximal rank) allows us to integrate out the quark fields ($m_{susy} \ll m_1, m_2$). The scalar field is then light and survives to low energy. Doing this integration one sees that at low energies the effective superpotential is $W = -hS\mu^2 + N_C \Lambda^3_{SYM}$. In this case the $Det(M) = m_1 m_2$, where M is the quark mass matrix (as in [105]), and Λ_{SYM} does not depend on the vev of S. There is no mass term for S in the superpotential and because of the linear term, the model breaks SUSY.

So, it is because of this clean separation of the dynamics into two independent sectors that one can have SUSY breaking. One should stress that it is only because of the tree-level R-symmetry, and in particular the rank condition. A discussion based on the

Konishi anomaly that SUSY breaking should hold non-perturbatively in models with a tree-level rank condition can be found in [69], and an example of another model where SUSY is broken at low energies without an exact R-symmetry in [101].

Numerical Calculations

Using Mathematica one can compute the eigenvalues of the matrix $M^\dagger M$ and do the square root. The sign is determined by the physical requirement that the potential should be bounded from bellow. It's straightforward to compute the Kahler metric and invert it to compute the V_F. One can then give values to the parameters and Mathematica provides different numerical algorithms that help finding the minimum of the potential.

As a test example, we use $m_1 = 1.2$,$m_2 = 0.45$,$h = 0.8$,$\Lambda = 0.05$ in units of μ, and $g = 0.75$. Using the mesonic degrees of freedom, we find that there is a minimum for:

$$\begin{aligned} S &= -23.36 \\ M_{11} &= 2.43 \\ M_{12} &= 0.16 \\ M_{21} &= 101.10 \\ M_{22} &= 6.49 \end{aligned} \tag{4.32}$$

with $V_F = 0.62\mu^4$. However, this minimizes V_F along the directions where $V_D = 0$, and may not be the right procedure. Instead of improving our understanding of the Kahler potential around the classical D-flat "pseudo-moduli space", as in [106, 107], we follow directly to the microscopic description.

We now minimize the potential using the microscopic description, and allowing for non-vanishing D-terms. The V_F potential is:

$$\begin{aligned} V_F = h^2|\phi.\tilde{\chi} - \mu^2|^2 + |m_1\phi_i - \frac{\Lambda^7(\chi.\tilde{\chi}\phi_i - \phi.\tilde{\chi}\chi_i)}{(\phi.\tilde{\phi}\chi.\tilde{\chi} - \chi.\tilde{\phi}\phi.\tilde{\chi})^2}|^2 + |hS\tilde{\chi}_i + m_1\tilde{\phi}_i - \frac{\Lambda^7(\chi.\tilde{\chi}\tilde{\phi}_i - \chi.\tilde{\phi}\tilde{\chi}_i)}{(\phi.\tilde{\phi}\chi.\tilde{\chi} - \chi.\tilde{\phi}\phi.\tilde{\chi})^2}|^2 \\ + |hS\phi_i + m_2\chi_i - \frac{\Lambda^7(\phi.\tilde{\phi}\chi_i - \chi.\tilde{\phi}\phi_i)}{(\phi.\tilde{\phi}\chi.\tilde{\chi} - \chi.\tilde{\phi}\phi.\tilde{\chi})^2}|^2 + |m_2\tilde{\chi}_i - \frac{\Lambda^7(\phi.\tilde{\phi}\tilde{\chi}_i - \phi.\tilde{\chi}\tilde{\phi}_i)}{(\phi.\tilde{\phi}\chi.\tilde{\chi} - \chi.\tilde{\phi}\phi.\tilde{\chi})^2}|^2 \end{aligned} \tag{4.33}$$

Where a sum over i is assumed. We chose some gauge fixing conditions: $\phi_2 = 0$, $\phi_3 = 0$, $\chi_3 = 0$ and minimized the potential with respect to the other fields, where the lower case index is a color index. Numerically we found that $\tilde{\phi}_3 = 0$,$\tilde{\chi}_3 = 0$.

The non-vanishing vevs are given (in units of μ) by:

$$\begin{aligned} S &= -1.51 \times 10^{-3} \\ \phi_1 &= 0.50 \\ \tilde{\phi}_1 &= 1.4 \times 10^{-3} \\ \tilde{\phi}_2 &= 1.36 \times 10^{-4} \\ \chi_1 &= 1.66 \times 10^{-3} \\ \chi_2 &= -2.28 \times 10^{-3} \\ \tilde{\chi}_1 &= 0.35 \\ \tilde{\chi}_2 &= -0.67 \end{aligned} \tag{4.34}$$

And $V_F + V_D = 0.58\mu^4$, which is lower than the previous result. This vacuum has $V_F = 0.54\mu^4$ and $V_D = 0.04\mu^4$, and so both D and F-terms are non-zero. This means that by moving away from the "D-flat" directions we were able to find a global minimum where SUSY is broken, and because the scale at which the gauge group is broken is much larger than Λ, the theory is weakly coupled.

We note that the scale at which gauge symmetry is broken is only one order of magnitude above the strong coupling regime, so quantum corrections are not negligible. One can change the parameters (for example, decreasing g will increase the vevs of the quarks), however, this does not change the fact that SUSY is broken. As a consequence of this fact, this model should be taken in the sense of an existence proof more than anything else.

To summarize what we have done: we have taken a model that had an R-symmetry and written the most general superpotential consistent with this. We noticed that for $N_f < N_c$ there are non-perturbative corrections to the potential that in practice make this symmetry anomalous. Despite having no continuous R-symmetry the model breaks SUSY because the superpotential is not generic and, for some range of the parameters, the global vacuum of the theory has a combined F and D-term SUSY breaking vacuum.

The way this work makes contact with the work of [108, 109] is that we can look at the gauge singlet as a Lagrange multiplier that deforms the "classical moduli space" of SQCD. On the other hand, the singlet also changes the behavior of the potential at infinity allowing the Witten index to change from the SQCD case. One can readily see that in this model it has to vanish (by looking at the case where the quark masses are much larger than the SUSY breaking scale).

4.2.1 The microscopic description:

As we've seen, there is no SUSY vacuum for finite values of the fields, in particular, if there is a vacuum it must exist for very large vevs so that a weakly coupled description should be valid. Because of this, it makes sense to do the minimization of the superpotential using the microscopic description. To a good approximation the Kahler potential should be canonical, and this makes the minimization easier. I will focus on the case with $N_F = N_C - 1$, but this argument can be used for larger number of colors as well. In particular I will present only the case $N_F = 2$ and $N_C = 3$ as it is the simplest. This case is particularly interesting as the gauge group can be completely Higgsed if the quark fields get vevs. We will show that the SUSY breaking scale cannot be made as small as one wants (i.e. it's unbroken), by assuming that this is possible, and then reaching a contradiction.

So, the superpotential in the following form:

$$W = \lambda \Phi(\phi.\tilde{\chi} - \mu^2) + m_1 \phi.\tilde{\phi} + m_2 \chi.\tilde{\chi} + \frac{\Lambda^7}{(\phi.\tilde{\phi}\chi.\tilde{\chi} - \phi.\tilde{\chi}\tilde{\phi}.\chi)} \tag{4.35}$$

Now, let $Det(M) = (\phi.\tilde{\phi}\chi.\tilde{\chi} - \phi.\tilde{\chi}\tilde{\phi}.\chi)$.

$$\begin{aligned} \frac{\partial W}{\partial \phi} &= \lambda \Phi \tilde{\chi} + m_1 \tilde{\phi} - \Lambda^7 \frac{\tilde{\phi}\chi.\tilde{\chi} - \tilde{\chi}\chi.\tilde{\phi}}{Det(M)^2} \\ \frac{\partial W}{\partial \tilde{\chi}} &= \lambda \Phi \phi + m_2 \chi - \Lambda^7 \frac{\chi\phi.\tilde{\phi} - \phi\chi.\tilde{\phi}}{Det(M)^2} \\ \frac{\partial W}{\partial \tilde{\phi}} &= m_1 \phi - \Lambda^7 \frac{\phi\chi.\tilde{\chi} - \chi\phi.\tilde{\chi}}{Det(M)^2} \\ \frac{\partial W}{\partial \chi} &= m_2 \tilde{\chi} - \Lambda^7 \frac{\tilde{\chi}\phi.\tilde{\phi} - \tilde{\phi}\phi.\tilde{\chi}}{Det(M)^2} \end{aligned} \tag{4.36}$$

To go any further, we need to choose an appropriate gauge. Since there is no flavor symmetry, we choose to do a QR decomposition, i.e. if Q is some $m \times n$ complex matrix (with $m > n$), it can be decomposed as $Q = UR$, where U is unitary and R is upper diagonal. This means that by a unitary transformation I can take my quark vevs to be an upper diagonal matrix (for the particular case at hand this means that there are only 3 independent Q-vevs and $\tilde{Q}$ is unconstrained.

Now we need to understand the D-flat directions. The easiest way to do this is to write down the generators of the Lie algebra of $SU(N)$ using the components of the generators of the Lie algebra of $GL(N)$:

$$T_j^{a,i} = T_s^{a,r}(A_r^s)_j^i \tag{4.37}$$

Where $T_j^{a,i}((A_r^s)_j^i)$ is the generator of the Lie Algebra of $SU(N)(GL(N))$, and $T_s^{a,r}$ is the component of $T_j^{a,i}$ along $(A_r^s)_j^i$. For fundamentals the generators of $(A_r^s)_j^i = \delta_j^s \delta_r^i$. Which means that $D^a = \phi^\dagger T^a \phi \to 0$ if and only if $D_r^s = \phi^\dagger (A_r^s)_j^i \phi \to \alpha \delta_r^s$. For $N_F < N_C$ the rank of the $Q.Q^\dagger$ (where the "." is a sum over flavors) is not sufficiently large and α has to be 0.

For the case where $N_F \leq N_C - 1$, the D-flatness conditions can be written as:

$$D_n^k = \phi^{\dagger, f_i, k}\phi_{f_i,n} - \tilde{\phi}^{f_i,k}\tilde{\phi}^\dagger_{f_i,n} \to 0 \tag{4.38}$$

Where f_i is a flavor index, and k,n are color indices. The most interesting constraints come from evaluating D_k^k, since for $k > N_f$ this implies that $\tilde{\phi}^{f_i,k} \to 0$.

So, at this point the vevs have the form:

$$Q = \begin{pmatrix} \phi_1 & \chi_1 \\ 0 & \chi_2 \\ 0 & 0 \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} \tilde{\phi}_1 & \tilde{\chi}_1 \\ \tilde{\phi}_2 & \tilde{\chi}_2 \\ \epsilon_{3,1} & \epsilon_{3,2} \end{pmatrix} \tag{4.39}$$

In this gauge, $Det(M) = \phi_1\chi_2(\tilde{\phi}_1\tilde{\chi}_2 - \tilde{\chi}_1\tilde{\phi}_2)$.

We note that this form does not solve all the D-flatness constraints, but as we shall see it will suffice to show that they are not compatible with the F-term constraints.

Let us now look at them:

$$\begin{aligned} \frac{\partial W}{\partial \phi_2}.\phi_1 &= m\tilde{\chi}.\phi \\ \frac{\partial W}{\partial \Phi} &= \lambda\phi.\tilde{\chi} - \mu^2 \end{aligned} \tag{4.40}$$

From these two equations, we see that if SUSY is to be unbroken (or broken at a scale arbitrarily lower than any other scale in the theory), it must be that $F^\dagger_{\phi_2}.\phi_1 \neq 0$, which in turn means that $\phi_1 \to \infty$. Also, because $F_\Phi = W_\Phi \to 0$, it must be that $\tilde{\chi}_1 \to 0$.

Now let us focus on the some of the other F-term constraints:

$$\begin{aligned} \frac{\partial W}{\partial \tilde{\phi}_1} &= (m - \Lambda^7 \frac{\chi_2 \tilde{\chi}_2}{Det(M)^2})\phi_1 \\ \frac{\partial W}{\partial \tilde{\phi}_2} &= -\Lambda^7 \frac{\chi_2 (F_\Phi + \mu^2)}{\lambda Det(M)^2} \end{aligned} \tag{4.41}$$

Since $\phi_{1,1} \to \infty$, and $F_\Phi \to 0$, the last two equations are only consistent if:

$$\begin{aligned} (m - \Lambda^7 \frac{\chi_2 \tilde{\chi}_2}{Det(M)^2}) &\to 0 \\ -\Lambda^7 \frac{\chi_2}{\lambda Det(M)^2} &\to 0 \end{aligned} \tag{4.42}$$

Or in other words, $\tilde{\chi}_2 \to \infty$. Now let us look at the D-flatness equations with this information:

$$D_2^2 = |\chi_2|^2 - |\tilde{\phi}_2|^2 - |\tilde{\chi}_2|^2 \to 0 \tag{4.43}$$

Since we've just shown that $\tilde{\chi}_2 \to \infty$, if this equation is to have a solution, it must be that $\chi_2 \to \infty$. This means that, at this point the vevs are of the form:

$$Q = \begin{pmatrix} \frac{1}{\delta_{1,1}} & \chi_1 \\ 0 & \frac{1}{\delta_{2,2}} \\ 0 & 0 \end{pmatrix} \quad \tilde{Q} = \begin{pmatrix} \tilde{\phi}_1 & \epsilon_{3,2} \\ \tilde{\phi}_2 & \frac{1}{\epsilon_{2,2}} \\ \epsilon_{3,1} & \epsilon_{3,2} \end{pmatrix} \tag{4.44}$$

We will now show that the constraints are not compatible with the solving the remaining F-term equations.

Take the following F-term:

$$\frac{\partial W}{\partial \chi_2} = m\tilde{\chi}_2 - \Lambda^7 \frac{\phi_1(\tilde{\phi}_1 \tilde{\chi}_1 - \tilde{\chi}_1 \tilde{\phi}_2)}{Det(M)^2} \tag{4.45}$$

If we multiply this equation by $\frac{\partial W}{\partial \tilde{\phi}_2}$, if SUSY is unbroken, the product $\frac{\partial W}{\partial \chi_2}\frac{\partial W}{\partial \tilde{\phi}_2} \to 0$. In particular this means that the F-term equations imply:

$$\begin{aligned} (m - \frac{F_{\phi_2}}{\phi_1}) m \frac{(\mu^2 + F_\Phi)}{\lambda} - \Lambda^{14} \frac{\mu^2 + F_\Phi}{\lambda Det(M)^3} &\to 0 \\ \frac{\mu^2 + F_\Phi}{\lambda Det(M)^2} \chi_2 &\to 0 \end{aligned} \tag{4.46}$$

Since $\chi_2 \to \infty$, the second equation only has a solution if $Det(M) \to \infty$, while the first equation is only soluble if $Det(M)$ is finite, which is a contradiction.

This shows that along all directions where the D-terms are much smaller than any other scale in the problem, the F-term equations cannot all be solved (in the sense that F-term vevs are very small). So that SUSY has to be broken in the region of parameter space where perturbation theory is valid and SUGRA corrections are irrelevant. As we've seen, for certain choices of parameters there are perturbatively stable minima.

4.3 Orthogonal groups:

For orthogonal groups the analysis goes through very much in the same way as for unitary groups. One can focus in the region of number of colors and flavors where there are dynamical contributions to the superpotential: $N_F < N_C - 2$. So, unlike what happens for the previous case, by giving vevs to the quarks one cannot fully Higgs the gauge group. Consequently there is always an unbroken $SO(N_C - N_F)$ gauge group.

For concreteness let $N_C = 6$, $N_F = 2$, with superpotential[4]:

$$W = m\phi_1.\phi_2 + \frac{1}{2}\lambda\Phi\phi_1.\phi_1 - \mu^2\Phi + \frac{\alpha}{(\phi_1.\phi_1\phi_2.\phi_2 - (\phi_1.\phi_2)^2)^{1/2}} \tag{4.47}$$

As we shall see, the F-term equations do not have solutions for finite values (including the origin) of the fields. This, however, does not mean that SUSY is broken, as there may be asymptotic solutions, or singularities in the inverse Kahler metric. The analysis of [42] shows that at the origin this function is smoothed out by quantum effects, which indicates that the renormalized Kahler potential should yield a smooth inverse Kahler metric for all points in field space.

This means that the only possible SUSY vacua must live in the regime where SUGRA corrections become important. In particular if we are able to find a stable vacuum, for finite values of the fields, it should be perturbatively stable.

I will now assume that quantum corrections to the Kahler metric can are small and analytical: (eg. $K_{quantum} = K_{classical} + \frac{\epsilon}{\Lambda}M^\dagger M$), where ϵ is small. Instead of using the macroscopic degrees of freedom and their classical Kahler potential to show that there are no SUSY vacua for finite values of the fields, I shall use the equivalent description in terms of microscopic degrees of freedom. Unlike what happened for unitary groups, we can't choose a gauge where the quark vevs are upper diagonal. For the present case

[4] I'm considering the branch where the dynamically generated superpotential *exists*, see section 1.14.2.

we can choose a gauge where:

$$Q = \begin{pmatrix} \phi_{1,1} & \phi_{2,1} \\ \phi_{1,2} & \phi_{2,2} \\ 0 & \phi_{2,3} \\ 0 & \phi_{2,4} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.48}$$

Where $\phi_{1,2}, \phi_{2,4}$ can be chosen to be real. In this gauge $Det(M) \equiv \phi_1.\phi_1\phi_2.\phi_2 - (\phi_1.\phi_2)^2 = (\phi_{1,1}\phi_{2,2} - \phi_{1,2}\phi_{2,1})^2 + (\phi_{1,1}^2 + \phi_{1,2}^2)(\phi_{2,3}^2 + \phi_{2,4}^4)$

We'll look with special attention to the following F-term equations:

$$\frac{\partial W}{\partial \Phi} = \frac{1}{2}\lambda\phi_1.\phi_1 - \mu^2 \tag{4.49}$$

$$\frac{\partial W}{\partial \phi_{2,1}} = m\phi_{1,1} + \frac{\alpha}{Det(M)^{3/2}}\phi_{1,2}(\phi_{1,1}\phi_{2,2} - \phi_{1,2}\phi_{2,1}) \tag{4.50}$$

$$\frac{\partial W}{\partial \phi_{2,2}} = m\phi_{1,2} - \frac{\alpha}{Det(M)^{3/2}}\phi_{1,1}(\phi_{1,1}\phi_{2,2} - \phi_{1,2}\phi_{2,1}) \tag{4.51}$$

$$\frac{\partial W}{\partial \phi_{2,3}} = -\frac{\alpha}{Det(M)^{3/2}}\phi_{2,3}(\phi_{1,1}^2 + \phi_{1,2}^2) \tag{4.52}$$

$$\frac{\partial W}{\partial \phi_{2,4}} = -\frac{\alpha}{Det(M)^{3/2}}\phi_{2,4}(\phi_{1,1}^2 + \phi_{1,2}^2) \tag{4.53}$$

$$\frac{\partial W}{\partial \phi_{2,5}} = 0 \tag{4.54}$$

$$\frac{\partial W}{\partial \phi_{2,6}} = 0 \tag{4.55}$$

$$\tag{4.56}$$

We can immediately see that both $\phi_{1,1}$ and $\phi_{1,2}$ cannot be finite if all F-term equations go to zero. Since if both these values were finite, $\frac{\partial W}{\partial \phi_2}.\phi_1 = m\phi_1.\phi_1 \to 0$, and this is clearly not compatible with $\frac{\partial W}{\partial \Phi} = \frac{1}{2}\lambda\phi_1.\phi_1 - \mu^2 \to 0$.

Since $F_{\phi_1} \to 0$, $F_{\phi_2}.F_{\phi_2} \to 0$ also. Evaluating this quantity in this particular gauge gives that:

$$F_{\phi_1}.F_{\phi_1} = \frac{2(F_\Phi + \mu^2)}{\lambda}(m^2 + \frac{\alpha^2}{Det(M)^3}) \tag{4.57}$$

This means that whatever the vevs are, the combination $Det(M) \equiv \phi_1.\phi_1\phi_2.\phi_2 - (\phi_1.\phi_2)^2$ is finite, if SUSY is to be unbroken. Equally, if $\frac{\partial W}{\partial \phi_{2,3}} \to 0$, then $\phi_{2,3} \to 0$ and $\phi_{2,4} \to 0$ if SUSY is unbroken.

So, if SUSY is unbroken, and because $\phi_1.\phi_1$ must be finite, $Det(M) = (\phi_{1,1}\phi_{2,2} - \phi_{1,2}\phi_{2,1})^2 + \delta_d$ is finite where $\delta_d = \frac{2(F_\Phi+\mu^2)}{\lambda}(\phi_{2,3}^2 + \phi_{2,4}^2)$.

In particular, $Det(M) = \pm\alpha(\frac{\lambda F_{\phi_2}.F_{\phi_2}}{2(F_\Phi+\mu^2)} - m^2)^{-1/2}$

In order not to carry all dependence on F_Φ, F_ϕ, let us just define, for simplicity, $\phi_{1,1}\phi_{2,2} - \phi_{1,2}\phi_{2,1} = \Delta_D$, where Δ_D is finite and can be computed from the expressions above.

Then:

$$\phi_{2,1} = \frac{\Delta_D + \phi_{1,1}\phi_{2,2}}{\phi_{1,2}} \tag{4.58}$$

Also, $\phi_{1,1} = \pm i(\phi_{1,2} + \delta_{1,2})$, where $\delta_{1,2} = \sqrt{\phi_{1,2}^2 + \frac{2(F_\Phi+\mu^2)}{\lambda}} - \phi_{1,2}$ is real as long as $\phi_{1,2}$ is sufficiently large (in which case $\delta \approx \frac{2(F_\Phi+\mu^2)}{\lambda\phi_{1,2}}$).

We now turn to the D-flatness condition. One of them reads:

$$2Im(\phi_{1,1}\phi_{1,2}^\dagger + \phi_{2,1}\phi_{2,2}^\dagger) = D \tag{4.59}$$

And if SUSY is unbroken, $D \to 0$.

Replacing all vevs and remembering that $\phi_{1,2}$ was chose to be real, one gets:

$$-\phi_{1,2}^2 + \Delta_{1,2} + Im(\phi_{2,2}\frac{\Delta_D^\dagger}{\phi_{1,2}}) - (1 + \frac{\delta_{1,2}}{\phi_{1,2}})|\phi_{2,2}|^2 = D \tag{4.60}$$

And $\delta_{1,2} \to 0$, $\Delta_{1,2}$ is finite. Since $\phi_{1,2} \to \infty$, there is no solution to this equation with $D \to 0$. (Since $|\frac{\phi_{2,2}\Delta_D}{\phi_{1,2}}| < |\phi_{2,2}|$, when Δ_D is finite and $\phi_{1,2} \to \infty$).

So there are no finite or asymptotic solutions to $V = V_F + V_D = 0$. While this does not mean that there is a stable vacuum, it does show that (neglecting gravity corrections) there is no SUSY vacuum. As of the conclusion of this work, I have not found a stable vacuum in this model.

4.4 Conclusions

So, we've shown that it is hard to build models that simultaneously break SUSY by non-zero F and D-term vevs. In fact, unless the Kahler potential is not the canonical one, or the superpotential is not a cubic polynomial in the fields, the global minimum can only break SUSY by having non-zero F-terms.

We've then solved this by noting that in SQCD, for $N_F < N_C$, there is a dynamical term that evades the perturbative non-renormalization theorems. Since the R-symmetry is anomalous, this non-perturbative term explicitly breaks the R-symmetry, but since the low energy effective superpotential is not generic, the model still breaks SUSY.

The advantage of such models is clear, it allows us to have massive R-axions in the global vacuum of the model. Also, because the superpotential is not a cubic polynomial in the fields, (and at the minimum SUSY is not broken by F-terms alone) Komargodski and Shih's argument[3] would not apply to the messenger sector.

It is not completely clear *why* the superpotential should have the R-symmetry at tree-level in the first place. The main point is that it is *consistent* to impose such a symmetry, even if it turns out to be anomalous. The reason for this is simple: while in generic field theories, renormalizability of the (effective) field theory implies that all operators that are consistent with the symmetries should be included in the Lagrangian, in SUSY theories the (non-perturbative) non-renormalization theorems ensure that the superpotential can only receive dynamical contributions. In other words, if a term is not present in the superpotential at the cut-off scale, it won't be generated by the R.G equations.

In practice, the inclusion of messengers is the main problem of this sort of models as the rank condition and the field content necessary to have these dynamical terms in the superpotential put severe constraints on the possible field content of the model.

The inclusion of the messenger sector via semi-direct mediation will be studied in the last chapter of this thesis. Next we will see what happens when we have F-terms breaking the MSSM GUT group.

Chapter 5

Gauge Messengers in direct gauge mediation:

"And remember, no matter where you go, there you are."
— Confucius, 551- 479 BC

5.1 Introduction:

A lot of work has been done in in recent years trying to understand the most general way to describe the sort of mass spectra one can expect to find at the LHC if SUSY is realized in nature and is communicated to the MSSM through gauge interactions ([1, 41, 55, 56, 66, 67, 80, 103, 106, 110–115]). A framework known as general gauge mediation [55] (GGM) was constructed and, under very general assumptions, it describes (in a model independent way) the possible set of soft terms one can expect with only a small number of parameters. Sum rules for the squark/slepton masses were also derived.

From this work other features that were necessary for a particular model to be viable have emerged. One example is the requirement that the universe should be in a perturbative metastable vacuum [3, 24, 27]. This conclusion arose from the fact that if this is not assumed, gaugino masses vanish at leading order in $\frac{F}{M}$. This would generally give a small hierarchy between gaugino masses and the soft scalar masses, which leads to some tuning to get the correct Electro-Weak (E.W.) symmetry breaking scale.[1]

In all these models, it was implicitly assumed that the field whose F-component was not 0 was a gauge singlet. In the previous chapter we considered a more general case: we allowed F-terms of fields that were not gauge singlets, and clarified the relation between this and having a (global) vacuum where SUSY is broken by non-zero F *and* D-term vevs.

Having understood better how to build such models, and their vacuum properties, we now turn our attention to communicating the SUSY breaking effects to the MSSM. Since it is highly constraining, we will not require the models to have non-vanishing D-terms in the vacuum.

In this chapter we will study models with F-terms that do not transform trivially under the *MSSM GUT gauge group*. This is still different from the models described by the GGM framework, as it allows for a new type of messengers: gauge messengers. These are Higgsed vector fields that couple to SUSY breaking vevs through Kahler interactions and thus act as messengers.

In [110], it was noted that the predictions of GGM could be generalized by allowing the presence of this new type of messenger field, and an extension of this framework was constructed in [56]. One particular difference with the old perturbative picture of gauge mediation is that gaugino masses are generated at leading order in $\frac{F}{M}$ even if the vacuum is not metastable.

This scenario is then richer than the one described by GGM: it allows for a more general class of soft terms and the sum rules of GGM are changed [110]. The main motivation of this work is then to explore the role of gauge messengers and see whether these models lead to qualitatively different conclusions.

One problem that arose when trying to build models of this type is that some of them have tachyonic squark and/or slepton masses [41, 56, 110]. This is because the leading order (one loop) contribution to the soft scalar masses is always tachyonic. In [56] was shown that this contribution can be suppressed with respect to the two loop corrections.

[1] A more general argument that any vacuum of a model with low energy SUSY breaking model should be metastable can be made by saying that any spontaneously broken R-symmetry should be only approximate in order not to have a massless R-axion. This is a much weaker assumption: it only requires that SUSY is restored somewhere in field space (e.g. non-perturbatively), while the previous argument requires that this vacuum must be accessible within the regime of validity of perturbation theory.

But even these are often negative, so that squarks/sleptons remain tachyonic even at two loops.

In this chapter we will address this problem, and propose possible solutions.

Its structure is as follows: in section 2 we give a description of the basic model we will be considering. The messengers will be adjoints of $SU(5)$. We will show that in the vacuum, and because we will couple these adjoints to some F-terms, the $SU(5)$ is naturally broken to the MSSM gauge group $SU(3) \times SU(2) \times U(1)$. This is different from many models where the GUT breaking respects SUSY and the choice of symmetry breaking pattern is chosen.

In section 3 is a review of the results of [56, 110].

In section 4 we explore the problem of tachyonic squarks and sleptons and show that in a large class of models the sign of the soft masses depends only on the field content of the messenger sector, and not on the parameters: F/M turns out to be universal for both gauge and non-gauge messengers.

In section 5 we show how the constraints used to derive the previous result can be evaded with two examples, and we present the conclusions in section 6.

5.2 A toy model:

The model for the messenger sector consists of two fields that are adjoints under the $SU(5)$ GUT and a singlet. The superpotential is given by:

$$W = -\mu^2\Phi + \lambda\Phi Tr(Y_0Y_0) + mTr(Y_2Y_0) + \overline{\lambda}Tr(Y_2Y_0Y_0) \tag{5.1}$$

Where Φ is the singlet and Y_0 and Y_2 are adjoints of $SU(5)$. This superpotential has an R-symmetry such that $R(Y_0) = 0$ and $R(Y_2) = R(\Phi) = 2$, and is general. The Y-fields can be written using the generators of the Lie Algebra of $SU(5)$: $Y_i = Y_i^{(a)}T^{(a)}$, where $T^{(a)}$ are the generators (mode details in the appendix C). Note that linear terms in the Y-fields vanish as the generators of $SU(5)$ are traceless.

Let us start by analyzing this model in the limit where $\lambda = 0$:

$$W = -\mu^2\Phi + mTr(Y_2Y_0) + \overline{\lambda}Tr(Y_2Y_0Y_0) \tag{5.2}$$

In this case there are two independent sectors: one composed by the singlet and it's superpotential (singlet sector), and the second consisting of the adjoint and its superpotential (adjoint sector). SUSY is broken in the singlet sector as $F_\Phi = \mu^2$, and Φ is the Goldstino. The adjoint sector can have several SUSY solutions:

- $Y_2 = 0$, $Y_1 = 0$;
- $Y_2 = 0$, $Y_0 = \frac{m}{3\lambda}\, diag(\{1, 1, 1, 1, -4\})$;
- $Y_2 = 0$, $Y_0 = \frac{m}{\lambda}\, diag(\{2, 2, 2, -3, -3\})$;

Where we use the notation $diag(\{x_1, x_2, ..., x_n\})$ to denote a diagonal matrix with elements $x_1, ..., x_n$. Since SUSY is not broken in the adjoint sector the degeneracy between these vacua is not lifted and (ignoring SUGRA corrections) all should be considered on equal footing.

Let us consider now the beta function associated with the GUT gauge group: the extra adjoints give a very large negative contribution above their mass threshold:

$$\begin{aligned} b' =& b_{MSSM} - S_{messengers} = 3 \times N_c - S_{matter} - S_{messengers} = \\ & 3 \times 5 - 3 \times \frac{3}{2} - 3 \times \frac{1}{2} - 1 - 2 \times 5 = -2 \end{aligned} \tag{5.3}$$

Where we take as matter content: the MSSM [116, 117] (a fundamental $\overline{5}$, an anti-symmetric 10 and two Higgs, 5 and $\overline{5}$) and two adjoints for the messenger sector.

Above the GUT scale the gauge coupling will not be asymptotically free. This means that we are implicitly assuming that above the GUT scale the MSSM is actually the dual low energy description of some theory valid at energies well above the GUT scale (other examples where the MSSM is considered to be the dual of another theory are considered in [89, 118]). In this context, the singlets of the low energy theory could be thought of as composites of some other fields.

Now let us turn on the λ parameter: If λ is small enough the solutions will change by a small amount and in the minimum the symmetry breaking pattern should be one of the exhibited by the previous solutions (a more detailed discussion of the general minimization of the potential is presented in the appendix A).

At leading order in λ one gets:

V	*Solutions*
μ^4	$Y_2 = 0$ $Y_0 = 0$ $\Phi = y$
$\mu^4 - \frac{40}{9}\lambda\frac{m^2\mu^2}{\overline{\lambda}^2}$	$Y_2 = y\ diag(1,1,1,1,-4)$ $Y_0 = \frac{m}{3\overline{\lambda}}(1+2\lambda(\frac{\mu}{m})^2)diag(\{1,1,1,1,-4\})$ $\Phi = y\frac{3\overline{\lambda}}{2\lambda} + 3y\frac{\overline{\lambda}\mu^2}{m^2} - y\lambda(\frac{20}{3\overline{\lambda}} + \frac{18\overline{\lambda}\mu^4}{m^4})$
$\mu^4 - 60\lambda\frac{m^2\mu^2}{\overline{\lambda}^2}$	$Y_2 = y\ diag(\{1,1,1,-3/2,-3/2\})$ $Y_0 = \frac{2m}{\overline{\lambda}}(1+2\lambda(\frac{\mu}{m})^2)diag(\{1,1,1,-3/2,-3/2\})$ $\Phi = y\frac{\overline{\lambda}}{4\lambda} + y\frac{\overline{\lambda}\mu^2}{2m^2} - y\lambda(\frac{15}{\overline{\lambda}} + \frac{3\overline{\lambda}\mu^4}{m^4})$

Table 5.1: Structure of the different vevs in the toy model: for $\lambda > 0$ the global minimum breaks $SU(5)$ to the SM gauge groups.

Where, as expected, there is a flat direction associated with the fields with R-charge 2, parametrized here by y. An important point to make is that there are no new complex phases associated with these vevs, so there are no new sources of CP violation.

We noted that when $\lambda = 0$ there were three possible solutions and that they all should be considered on equal footing. By coupling the adjoint sector to the singlet sector (SUSY breaking sector), this degeneracy is broken. By choosing λ to be positive we see that the preferred vacuum is the one required in the MSSM and none of the others.

The non-vanishing F-terms are:

$$\begin{aligned} F_\Phi &= \frac{\partial W}{\partial \Phi} = -\mu^2 + 30m^2\frac{\lambda}{\overline{\lambda}^2} \\ F_{Y_2^{(23)}} &= \frac{\partial W}{\partial Y_2^{(23)}} = -5\sqrt{3}\mu^2\frac{\lambda}{\overline{\lambda}^2} \\ F_{Y_2^{(24)}} &= \frac{\partial W}{\partial Y_2^{(24)}} = -3\sqrt{5}\mu^2\frac{\lambda}{\overline{\lambda}^2} \end{aligned} \quad (5.4)$$

Where:

$$T^{23} = diag(1/(2\sqrt{6}), 1/(2\sqrt{6}), 1/(2\sqrt{6}), -3/(2\sqrt{6}), 0)$$
$$T^{24} = diag(1/(2\sqrt{10}), 1/(2\sqrt{10}), 1/(2\sqrt{10}), 1/(2\sqrt{10}), -2/\sqrt{10})$$

So, some of the F-terms are not invariant under the GUT gauge group. As we will see this is a necessary and sufficient condition for the existence of gauge messengers.

Gauge messengers:

Now let us consider the gauge messengers. The form of a fermionic mass matrix squared for a generic superpotential and field content is given by:

$$m_f^2 = \begin{pmatrix} W_{ij}.W^{jk} + 2D_i^a D^{a,k} & \sqrt{2}D_i^{a,j} W_j \\ \sqrt{2}D_{,l}^{b,k} W^l & D_l^b D^{a,l} + D_l^a D^{b,l} \end{pmatrix} \tag{5.5}$$

Where $W_i(W_{ij})$ is the first (second) derivative of the superpotential with respect to the fields ϕ_i (and ϕ_j), D^a is the D-term: $D^a = g\sum_i \phi_i^\dagger T^a \phi_i$ and $D^a_{,j}$ is its derivative with respect to ϕ_j. Indices are raised and lowered by complex conjugation.[2]

In the usual models of gauge mediation the only non-vanishing F-term is associated with a gauge invariant direction. This means that the off-diagonal terms $\sqrt{2}D_i^{a,j}W_j$ in the fermionic mass matrix (squared) vanish, so that the only fields that feel the effects of SUSY breaking are the scalars.

This doesn't have to happen: in our case, when λ is not zero, there are some F-terms that are not gauge invariant under the full GUT group. This means that the Higgsing of the vector superfields is not SUSY: there will be some mixing between the Higgsed gauginos and the fermionic components of messengers. The mass spectrum of the components of these Higgsed vector superfields will not be SUSY, and because of this they will act as (gauge-)messengers.

For this particular case, it's the bifundamentals of the $SU(3)\times SU(2)$ that are Higgsed and act as gauge messengers.

[2]Gauge invariance of the superpotential in the form of: $D^{a,j}_{,k} W_j + D^{a,j} W_{jk} \equiv 0$ was used to simplify the mass matrix.

The mass matrix for these fields at leading order in λ is:

$$\left(\chi^\dagger, \tau^\dagger, \psi^\dagger\right) (M_f^{bif.})^2 \begin{pmatrix} \chi \\ \tau \\ \psi \end{pmatrix} \tag{5.6}$$

$$(M_f^{bif.})^2 = \begin{pmatrix} \frac{25(m^2+4\lambda\mu^2)g^2}{\overline{\lambda}^2} & \frac{25(m^2+2\lambda\mu^2)g^2}{2m\overline{\lambda}}y & 0 \\ \frac{25(m^2+2\lambda\mu^2)g^2}{2m\overline{\lambda}}y & \frac{25g^2}{4}y^2 & \frac{10i\lambda\mu^2 g}{\overline{\lambda}} \\ 0 & -\frac{10i\lambda\mu^2 g}{\overline{\lambda}} & \frac{25(4m^2+16\lambda\mu^2+\overline{\lambda}^2y^2)g^2}{4\overline{\lambda}}y^2 \end{pmatrix} \tag{5.7}$$

So we can see that the F-terms couple to the gauge fields at tree-level and the model has gauge messengers. The fermionic masses are approximately given by:

$$m^2_{g.m.,\pm} = 25\frac{g^2}{\overline{\lambda}^2}m^2 + \frac{25}{4}y^2g^2 + 100\lambda\frac{g^2}{\overline{\lambda}^2}\mu^2 \pm 10g\lambda\frac{y}{\sqrt{4m^2+(\overline{\lambda}y)^2}}\mu^2 \tag{5.8}$$

$$m_l^2 = \frac{(8m^2\overline{\lambda}y + (\overline{\lambda}y)^3)^2\lambda^2\mu^4}{m^4(4m^2+(\overline{\lambda}y)^2)} \tag{5.9}$$

And $m_{g.m.,\pm}$ are the masses of the gauge messengers[3].

We note that even though this is F-term breaking the mass splittings come from (tree-level) Kahler potential interactions, not from the superpotential. This is the main difference from the usual models of gauge mediation.

5.3 The soft terms:

The soft terms for a model of gauge mediation with gauge messengers has been recently computed in [56, 110, 113]. We shall present a short review of these calculations[56] using only wave-function renormalization techniques [32, 41].

The main differences from the usual scenarios of gauge mediation are:

- gaugino masses are generated at leading order in $\frac{F}{M}$ expansion (even without metastability);
- soft scalar masses are generated at one loop;

[3] To compute the eigenvalue m_l we computed the fermionic mass matrix to order λ^3 and then extracted the eigenvalues, which we then expanded to order λ^2.

- trilinear couplings are generated at one loop even at the messenger scale;

Gaugino Masses:

Diagrammatically we have that the contributions to gaugino masses are given in Figure 5.1. In most models of gauge mediation the second diagram actually doesn't give any contribution.

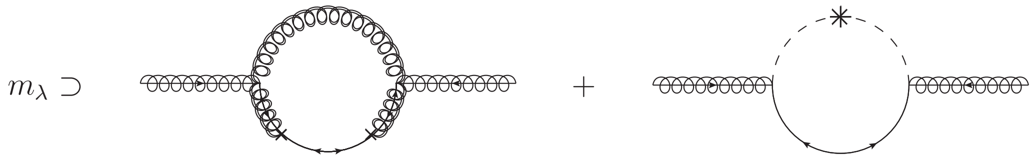


Figure 5.1: Where the external legs correspond to the un-Higgsed MSSM gauginos, and the doubled wavy-solid (wavy) lines are Higgsed gauginos (gauge bosons), solid (dashed) lines are messenger fermions (scalars). A cross is a mass insertion and a double cross is an F-term insertion. These diagrams give the leading order in F/M gaugino mass contribution.

We will now calculate this contribution: Let us assume that we've fixed some useful gauge (eg. unitary gauge) to perform the calculations and that we call our goldstino field X, so that in the vacuum $< X >= x + \theta^2 F$. Unlike in the usual scenario we will allow x to not be a gauge singlet. So F dictates the scale of SUSY breaking while x is one of the vevs responsible for the breaking of the GUT gauge group to the MSSM gauge groups.

For now let us set F to 0 and work in the SUSY limit.

For the unbroken gauge multiplet, at a given scale μ, the Lagrangian interaction is determined by the X-dependent gauge function $S(X, \mu)$, (where the reason why S can only depend on X is that it must be a holomorphic function of X):

$$L \supset \int d^2\theta^2 S(x,\mu) W^{a\alpha} W^a_\alpha + h.c. \tag{5.10}$$

If we now turn on F a little bit ($F/M^2 << 1$), and since the dependence of S on $< X >$ is holomorphic, the only way that S can change (at one loop) is by the change $< X > \to X$, where X is now a spurion field. This replacement is called analytical continuation into superspace, since the continuation of $< X >$ to superspace induces a continuation of both gauge coupling and wave-function renormalization to superspace as well (from their dependence on X). The validity of the procedure relies on the fact that this continuation gives the correct R.G. equations for the soft terms.

And S is given by:

$$S(x,\mu) = \frac{\alpha^{-1}(x,\mu)}{16\pi} - \frac{i\Theta}{32\pi^2} \quad (5.11)$$

Where Θ is the topological vacuum angle.

So, to compute gaugino masses at leading order in F we need only to solve the R.G. equations for the gauge coupling in the SUSY limit and the continue them to superspace. We note that even though S is holomorphic in the goldstino field, $\alpha^{-1} = 16\pi Re(S)$ is not. The one loop R.G. equation is

$$\frac{d}{dt}\alpha^{-1} = \frac{b}{2\pi} \quad (5.12)$$

where $t = Log(\mu)$ and $b = 3N_c - N_f$ for an $SU(N)$ theory. Let us call b' the β-function coefficient in the U.V. (i.e. above the GUT scale), and b_i the β-function coefficient of the i-th gauge group below the GUT scale (so $SU(2)$, $SU(3)$ or $U(1)$). So that the expressions for the holomorphic gauge coupling are given by:

$$S(\mu) = S(\Lambda_{U.V.}) + \frac{b'}{32\pi}Log(\frac{\mu}{\Lambda_{U.V.}})S_a(\mu) = S(\Lambda_{U.V.}) + \frac{b'}{32\pi^2}Log(\frac{\Lambda_x}{\Lambda_{U.V.}}) + \frac{b_a}{32\pi^2}Log(\frac{\mu}{\Lambda_x}) \quad (5.13)$$

Where $\Lambda_{U.V.}$ is some U.V. cutoff. And the first expression is valid above the messenger mass threshold and the second expression is valid below.

The gaugino mass (at this order)[4]is given by

$$m_\lambda = g^2(\mu)S|_{\theta^2} \quad (5.14)$$

So, below the scale $x =< X >$ the gaugino mass is given by the θ^2 component of the gauge function S which is:

$$m_{\lambda_a} = \frac{\alpha_a(\mu)}{4\pi}(b_a - b')\frac{F}{x} \quad (5.15)$$

[4]the gaugino mass, being an observable, depends on the physical gauge coupling, not the holomorphic one. However at one loop there is no difference

This generalizes for multiple mass thresholds (as long as $F/M^2 << 1$). However, it is well known that if the hidden sector superpotential is a cubic polynomial in the fields and one is sitting at the global minimum, the contribution from normal messengers vanishes to leading order in F/M. which means that the only possible non-vanishing contribution is from gauge messengers, and this is given by:

$$m_{\lambda_a} = -\frac{\alpha_a(\mu)}{2\pi}(N_c' - N_{c,a})\frac{F}{x} \tag{5.16}$$

Where $N_c', N_{c,a}$ are the number of colors in the GUT, MSSM "a-th" gauge group, and $-2(N_c' - N_{c,a})$ is the contribution from the Higgsed vector superfields ($(N_c' - N_{c,a})$ from the eaten would be Goldstone Bosons and -3$(N_c' - N_{c,a})$ from the vector field). Explicit computations for the toy model at hand have been done and and it has been checked that this contribution is non-zero.

Scalar Masses:

Scalar masses can be generated at one loop. This is because the gauge messengers couple to non-zero F-terms already at tree-level. The diagrams are presented in Figure 5.2. The

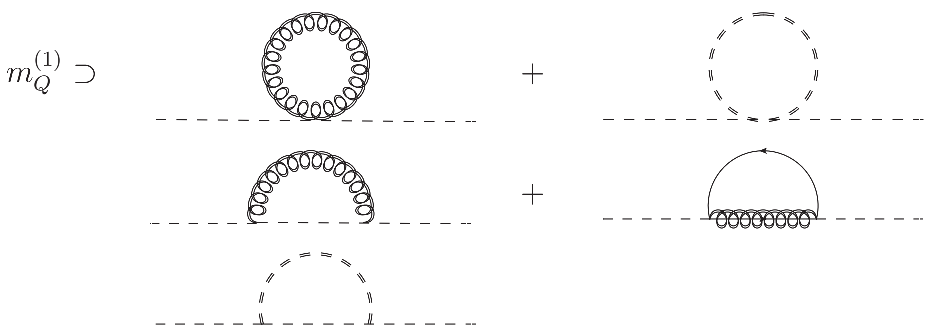


Figure 5.2: Where the external legs correspond to the MSSM squarks, and the doubled wavy-solid (wavy) lines are Higgsed gauginos (gauge bosons), double dashed lines are the scalar messengers. The leading F/M contribution is given by the diagram with the Higgsed gaugino (with 4 mass insertions).

crucial observation is that in the SUSY limit the mass of the Higgsed vector superfields is given by $(M_v^2)^{AB} = \Phi^\dagger\{T^A, T^B\}\Phi$. One can take the simplifying assumption that all the masses are the same and that we've chosen a basis where they are diagonal, i.e. $(M_v^2)^{AB} = (\Phi, \Phi)\delta^{AB}$, where the inner product is defined as the (degenerate) eigenvalue of the matrix $\Phi^\dagger\{T^A, T^B\}\Phi$.

The one loop R.G. equation for the Quark superfield is given by:

$$\frac{d}{dt}Log(Z_Q) = \frac{C}{\pi}\alpha \tag{5.17}$$

where C is the Casimir of the Quark superfield representation under GUT gauge group ($C = \frac{N^2-1}{2N}$ for an $SU(N)$ fundamental).

So that below the messenger threshold the wave-function renormalization function is given by:

$$Z_Q(M_v, \mu) = Z_Q(\Lambda_{U.V})\left(\frac{\alpha(\Lambda_{U.V.})}{\alpha(M_v)}\right)^{\frac{2C'}{b'}}\left(\frac{\alpha_a(M_v)}{\alpha_a(\mu)}\right)^{\frac{2C_a}{b_a}} \tag{5.18}$$

and b' is the beta-function coefficient of the GUT gauge coupling and C,b are the corresponding constants for the MSSM gauge couplings.

The gauge coupling below the messenger threshold is given by:

$$\alpha^{-1}(\mu) = \alpha^{-1}(\Lambda_{U.V.}) + \frac{b'}{4\pi}Log(\frac{(X,X)}{\Lambda^2_{U.V.}}) + \frac{b}{4\pi}Log(\frac{\mu^2}{(X,X)}) \tag{5.19}$$

We now need to continue these expressions into superspace and extract the SUSY breaking soft terms. The first step is to canonically normalize the fields: upon analytically continuation Z picks up θ^2,$\overline{\theta}^2$ and $\theta^2\overline{\theta}^2$ terms with the constraint that it must be a real function. So the θ^2 and the $\overline{\theta}^2$ components are the complex conjugates of each other:

$$Z = z + Z|_{\theta^2}\theta^2 + (Z|_{\theta^2})^\dagger\overline{\theta^2} + Z|_{\theta^2\overline{\theta}^2}\theta^2\overline{\theta}^2 \tag{5.20}$$

Where the z is the "scalar" component of Z, $Z|_{\theta^2}$ is the θ^2 component of Z and $Z|_{\theta^2\overline{\theta}^2}$ is the $\theta\overline{\theta}^2$ component of Z.

Then after the redefinition of canonically normalized fields:

$$Q' = z^{1/2}(1 + \frac{Z|_{\theta^2}}{z}\theta^2)Q \tag{5.21}$$

Where Q is the normal Quark superfield and Q' is the canonically normalized quark superfield.

If we define the terms in the potential as:

$$V = \sum_i m_{Q_i}^2 Q_i^\dagger Q_i + A_{Q_i} Qi \frac{\partial W}{\partial Qi} + c.c. + (\frac{\partial W}{\partial Qi})^* \frac{\partial W}{\partial Qi} \tag{5.22}$$

And integrate out the auxiliary components of the quark fields using the previous expressions, if follows that:

$$A_{Q_i} = Log(Z_{Q_i})|_{\theta^2} \tag{5.23}$$

$$m_{Q_i}^2 = -Log(Z_Q)|_{\theta^2\overline{\theta}^2} \tag{5.24}$$

Because the correct mass threshold for gauge messengers is of the form (x, x) and not $X^\dagger X$, the expansion of the gauge coupling into superspace is given by:

$$\alpha^{-1}((X,X)) = \alpha^{-1}((x,x)) + \frac{b'}{4\pi}(\theta^2 \frac{(x,F)}{(x,x)} + \overline{\theta}^2 \frac{(F,x)}{(x,x)} + \theta^2\overline{\theta}^2 \frac{(F,F)(x,x) - (x,F)(F,x)}{(x,x)^2}) \tag{5.25}$$

$$\alpha_a^{-1}(\mu_s) = \alpha_a^{-1}(\mu) + \frac{b' - b_a}{4\pi}(\theta^2 \frac{(x,F)}{(x,x)} + \overline{\theta}^2 \frac{(F,x)}{(x,x)} + \theta^2\overline{\theta}^2 \frac{(F,F)(x,x) - (x,F)(F,x)}{(x,x)^2}) \tag{5.26}$$

Where $\alpha_a^{-1}(\mu_s)$ is the gauge coupling below the gauge messenger mass threshold and $\alpha^{-1}((X,X))$ is the gauge coupling at the messenger mass threshold (after analytical continuation).

If we now replace these expressions for the gauge couplings in the expression for the squark masses, we get:

$$\begin{aligned} m_Q^2 =& \frac{g^2(\mu)}{8\pi^2}((C - \chi C') + c\frac{b'}{b}(\chi - 1))\frac{(F.F)(x,x) - (x,F)(F,x)}{(x,x)^2} + \\ &+ 2\left(\frac{g^2(\mu)}{16\pi^2}\right)^2 ((bC + b'(C' - 2C)) + (\chi^2 - 1)\frac{b'}{b}(bC' - Cb'))\frac{(x,F)(F,x)}{(x,x)^2} \end{aligned} \tag{5.27}$$

Where $\chi = \frac{\alpha(M)}{\alpha(\mu)}$. Where we note that the one loop contribution is often tachyonic since $C \leq C'$ and $(F.F)(x,x) - (x,F)(F,x) \geq 0$.

The case with two messenger thresholds is a simple generalization of this, and at leading order (i.e. ignoring the running of the gauge coupling) the result is a direct sum of the result we got for gauge messengers and the usual result for normal messengers. So the contributions for the soft terms coming from different messengers add up[5].

[5] See appendix C for more details

5.3.1 A-terms:

The relevant diagram to compute is:

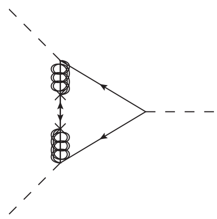


Figure 5.3: Main diagram contributing to the trilinear couplings at leading order in F/M.

As we saw in the previous section, the A-terms are given by:

$$A_Q = Log(Z_Q)|_{\theta^2} \tag{5.28}$$

Being that this expression is readily extracted from considering the expression for the wave-function renormalization: 5.18, and the expression for the gauge coupling 5.19 analytically continued to superspace.

The result is given by:

$$A(\mu) = \frac{\alpha(\mu)}{2\pi}(C' - C)\frac{(x, F)}{(x, x)} + \frac{\alpha(\mu)}{2\pi}(C' - C\frac{b'}{b})(\chi - 1)\frac{(x, F)}{(x, x)} \tag{5.29}$$

Where $\chi = \frac{\alpha(M)}{\alpha(\mu)}$.

so that the main contribution is

$$A(\mu) \approx \frac{\alpha(\mu)}{2\pi}(C' - C)\frac{(x, F)}{(x, x)} \tag{5.30}$$

5.3.2 Suppression of the one loop contribution to squark masses:

An important result of [56] was to show that when the scalar partner of the goldstino (sgoldstino) gets a large vev, these corrections are suppressed.

It is a well known result that in O'R. models where the superpotential is a cubic polynomial in the fields, the vev of the scalar partner of the goldstino parametrizes a flat direction of the potential[3, 26]:

$$x \to x' = x + zF \Rightarrow \partial_i W(x) \to \partial_i W(x') = \partial_i W(x) \tag{5.31}$$

The one loop contribution is proportional to the coefficient $\frac{(F,F)(x,x)-(x,F)(F,x)}{(x,x)^2}$, which along the flat direction scales as:

$$\begin{aligned} &\frac{(F,F)(x+zF,x+zF)-(x+zF,F)(F,x+zF)}{(x+zF,x+zF)^2} = \\ &\frac{(F,F)(x,x)-(x,F)(F,x)}{(x+zF,x+zF)^2} \to \frac{(F,F)(x,x)-(x,F)(F,x)}{|z|^4(F,F)^2} \end{aligned} \tag{5.32}$$

Where we took the limit $zF \gg x$. Comparing with the suppression one gets for the usual two loop contribution:

$$\frac{(F,x+zF)(x+zF,F)}{(x+zF,x+zF)^2} \to \frac{1}{|z|^2} \tag{5.33}$$

So that the ratio between the one and the two loop contribution is:

$$\frac{(m_Q^2)^{(1)}}{(m_Q^2)^{(2)}} \approx \frac{4\pi}{\alpha(\mu)} \frac{C-C'}{(bC+b'(C'-2C))} \frac{(F,F)(x,x)-(x,F)(F,x)}{(F,F)^2} \frac{1}{|z|^2} \tag{5.34}$$

so that if we stabilize z sufficiently far away from the origin the two loop corrections can dominate over the one-loop ones.

$$z \geq \sqrt{\frac{4\pi}{\alpha(\mu)} \frac{C-C'}{(bC+b'(C'-2C))} \frac{(F,F)(x,x)-(x,F)(F,x)}{(F,F)^2}} \tag{5.35}$$

We note that since $(F,F)(x,x)-(x,F)(F,x)$ can be small when compared with (F,F) due to some alignment, one can have a sufficient suppression of the one loop correction without necessarily requiring a very large value of z.

If instead of z we had used $y = \frac{z}{|F|}$ to parametrize the flat direction, the limit is approximately given by:

$$y > \frac{4\pi}{g^2} M_v \tag{5.36}$$

Where M_v is the mass of the Higgsed gauginos evaluated at the origin of the pseudo-moduli space $y = 0$ (If there is some alignment between F and x this lower bound can be violated).

5.4 Model Building Constraints:

In this section we will show that in a large class of models, for large sgoldstino vevs, one can show that for every messenger in a model:

$$\frac{F}{M} \approx \frac{1}{z} \tag{5.37}$$

Where the pseudomoduli direction is given by: $X_i = x_i^{(0)} + zF_i$[6]. What this means is that in a large class of models, F/M is the same for all (gauge and non-gauge) messengers. In other words: even if the mass thresholds of the messengers are different and the F-terms they couple to are different, F/M will be the same for all messengers. What this implies is that the sign of the squark/slepton masses depends only on the field content of the theory, not on it's parameters [7].

Let us be more precise about what kind of models we are considering:

- The Kahler potential is canonical;
- SUSY should be broken at tree-level (no runaway directions);
- The vev of the sgoldstino parametrizes the only flat direction and it should be everywhere stable (i.e. no metastability);
- The messengers couple linearly to the goldstino;
- The fermionic mass matrices for (non-gauge) messengers factorize into 2×2 matrices;
- In order to suppress the one loop tachyonic contribution from gauge messenger to scalar masses the sgoldstino is the largest vev in the model;

For example: the first four constraints are easily satisfied in renormalizable theories that break SUSY by virtue of the rank condition (and we sit at the global minima). The fifth condition essentially tells us that the messenger sector should not contain three fields that can mix in complicated ways.

An important result [3, 27] is that in the global minima one has:

$$\frac{\partial}{\partial X} Det(W_{ij}) = 0 \tag{5.38}$$

[6] Since the Kahler potential is canonical, in the vacuum, $F_i = W_i$, so both quantities can be used

[7] The overall scale, i.e. the value at which z is stabilized, will of course depend on the parameters of the model. Also, if the mass splittings are too large R.G. effects should be taken into account.

Where X is the vev of the scalar partner of the goldstino, and W_{ij} is the second derivative of the superpotential with respect to the fields ϕ_i, ϕ_j. This is nothing but the argument that gaugino masses vanish at leading order in F/M unless the vacuum is metastable.

We also note that if the superpotential for the messengers can be written in the form, and there is an R-symmetry:

$$W = fX + (M^{ij} + XN^{ij})\phi^i\phi^j \tag{5.39}$$

then[27]:

$$Det(M + XN) = Det(M) \tag{5.40}$$

wether or not there is metastability, so that $\frac{\partial}{\partial X} Det(W_{ij}) = 0$, and the following argument still applies.

By manipulating eq. 5.38, and choosing a basis where the fermionic mass matrix is diagonal, one can rewrite it as:

$$Tr(\frac{W_{ijk}F^k}{m_i}) = 0 \tag{5.41}$$

And $W_{ij} = m_i\delta_{ij}$.

When the fermionic mass matrix factorizes to a 2×2 matrix, this means that:

$$\frac{W_{11k}F^k}{m_1} = -\frac{W_{22k}F^k}{m_2} \tag{5.42}$$

So, the contribution to the soft scalar masses is exactly the same for both fermionic mass eigenstates (at leading order in F/M).

We now write down the general dependence of the mass matrix for the messengers on the vevs of the model:

$$W_{ij} = m_{ij} + W_{ijk}X^k + W_{ijl}\phi^l + O(\phi^2) \tag{5.43}$$

Where m_{ij} is some mass matrix and W_{ijk} is the third derivative of the superpotential, and we have separated the dependence on the goldstino field X from the other fields (ϕ's). The higher order terms are absent if the superpotential is a cubic polynomial in the fields. Since messengers couple to the goldstino, the term $W_{ijk}X^k$ cannot be identically 0.

For large vevs of X[8] we have:

$$Tr(W_{ij}) \approx W_{iik}X^k \tag{5.44}$$

This means that:

$$\begin{aligned} Tr(W_{ij}) &= W_{iik}X^k \\ Det(W_{ij}) &= constant \end{aligned} \tag{5.45}$$

For the mass matrices we are considering, this implies that one of the mass eigenstates is very light and the other is very heavy:

$$\begin{aligned} m_H &\approx W_{iik}X^k \\ m_L &\approx \tfrac{constant}{W_{iik}X^k} \end{aligned} \tag{5.46}$$

So, for the heavy field, the contribution to the soft masses is given by:

$$\frac{W_{iik}F^k}{m_i} \approx \frac{W_{iik}F^k}{W_{iik}X^k} = \frac{W_{iik}F^k}{W_{iik}(x^{k,(0)} + zF^k)} \approx \frac{W_{iik}F^k}{zW_{iik}F^k} = \frac{1}{z} \tag{5.47}$$

And eq. 5.42 tells us that this contribution is the same for both mass eigenstates.

Now, for the gauge messengers, we've just shown that the contribution to soft masses is proportional to the square of $\frac{(F,X)_i}{(X,X)_i}$, where the inner product $(A,B)_i$ is defined as the i-th eigenvalue of $A^\dagger\{T^a,T^b\}B$.

For large vevs of X this is given by:

$$\frac{(F,X)}{(X,X)} = \frac{(F,x+zF)}{(x+zF,z+zF)} \approx \frac{z(F,F)}{z^2(F,F)} = \frac{1}{z} \tag{5.48}$$

As we wanted to show.

5.4.1 Constraining models with gauge messengers:

We've seen that in a variety of models the ratios F/M for the different fields approach a universal value for large values of the sgoldstino vev. This means that the sign of the squark and slepton masses generated is a function only of group theory factors (i.e.

[8] Note that eq. 5.36 gives a bound on how big the other (non-gauge invariant) vevs can be in order to have an appropriate suppression of the one loop contributions.

the representations of the messenger fields) and does not depend on the superpotential parameters. In particular, apart from R.G. effects, the ratios of squarks and sleptons depend only on the representations of the messenger fields.

It is interesting to note that in these models, for large sgoldstino vevs, we recover the condition for non-tachyonic scalar soft masses that was derived in [41]:

$$b'(2 - \frac{C'}{C_i}) < b_i \tag{5.49}$$

Where b' (b_i) are coefficients of the beta functions for the gauge couplings above (below) the GUT scale and C' (C_i) are the Casimirs of the representation under which the field transforms above (below) the GUT scale.

The main difference though is that equality of F/M for all messengers is not assumed.

This constraint can be easily evaded if we assume metastability. There are different reasons why this scenario is probably not preferred. Metastability usually requires the existence of an approximate R-symmetry [24, 54, 63, 83, 103]. Unlike in normal models of gauge mediation (e.g. ISS[103]) to suppress the tachyonic contribution to squarks we do not want the vacuum to be close to the origin of field space. It is hard to see how a polynomial superpotential could have an approximate R-symmetry far away from the origin.

If the gauge coupling (above the GUT scale) is asymptotically free, the mechanism of [57] becomes available to stabilize the vacuum far away from the origin. However, this doesn't always happen. Also, the one loop contributions to the effective potential don't necessarily lead to the stabilization of flat directions[56, 80, 102, 112].

Another thing to consider is wether such local minima should be preferred with respect to global ones. In [76, 79] it was shown that generically thermal corrections in the early universe make vacua close to the origin of field space preferred, this is because close to the origin of field space there are usually more light fields, so thermal corrections are smaller. Metastable vacua far away from the origin would not be favoured.

It is also possible that the number of messengers is sufficiently small so that eq, (5.49) is verified. However, two adjoints is already too many...

One could think of building a model with one adjoint field (plus messengers in other representations), but this is very hard. Assume:

- There is only one adjoint field Y (plus fields in other representations);
- In order to break $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$, Y is the only non-gauge invariant field whose F-term and scalar component can get vevs (singlets can have scalar and F-term vevs);
- Above the GUT scale, the model is SUSY;
- The sgolstino vev parametrizes a flat direction;

The second constraint is actually not very constraining as fundamental, symmetric and anti-symmetric vevs don't break $SU(5)$ to the MSSM gauge groups. More complicated representations are likely to contribute to the beta function so that eq. 5.49 is violated.

The third condition means that no field that couples to any SUSY breaking vev can be heavier than the GUT scale[9]. In other words: the only SUSY breaking parameters allowed at the GUT scale are F-term vevs of dynamical fields, there are no spurions.

In order to have gauge messengers, there must exist a non-gauge invariant F-term. This means that in the vacuum:

$$Y = y + \theta^2 F_y \tag{5.50}$$

And these vevs break $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$.

Since there is only one adjoint,it must be that:

$$F_y \propto f(y) \tag{5.51}$$

Since y is the only non-gauge invariant vev. If we assume the superpotential is a polynomial function of the fields, we have that $f(0) = 0$.

However in models where SUSY is broken due to the rank condition, or where SUSY is spontaneously broken and the superpotential is a cubic polynomial, the vev of the goldstino parametrizes a flat direction. Since F_y is non-zero, Y is part of the goldstino (generally there may be more non-zero F-terms, so that the goldstino is a particular linear combination of these fields). In any case, the scalar component of Y, y, parametrizes a flat direction. So F_y cannot depend on y and must vanish.

[9]We are ignoring the possibility of non-zero D-terms.

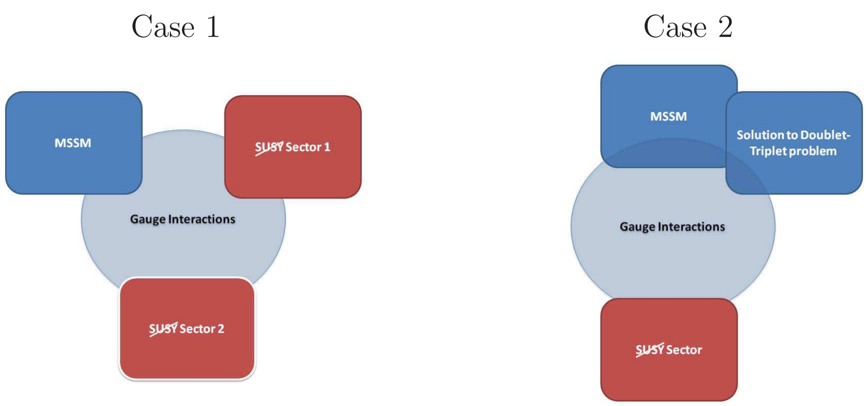
So, with these constraints it's not possible to build a model of gauge mediation where gauge messengers exist.

Another possible way around this problem which we shall not explore here is to embed the $SU(5)$ into a product of several groups [70, 119].

5.5 Possible solutions:

A simple possible solution to this problem is to have a second independent flat direction that does not couple to (all) normal messengers. This violates one of the constraints and allows us to address the problem of tachyonic scalar masses.

We will now consider two ways in which this can happen: Where it's assumed that



the solution to the doublet-triplet problem implies the existence of an adjoint field whose vev breaks $SU(5)$ to the MSSM gauge groups, or some other field whose vev contributes to the mass of the Higgsed vector superfields.

5.5.1 Case 1:

In this case we create the second flat direction by adding a second sector where SUSY is spontaneously broken and that does not contain gauge messengers. There are then two goldstino fields. By changing the ratio of the vevs of the two sgoldstino fields we can enhance the contributions from normal messengers to the soft terms, and get non-tachyonic squarks and sleptons.

As an example consider that we couple the model we presented in section 1, and add a second sector with 2 chiral messengers in fundamental/anti-fundamental pairs $(Q_1, Q_2, \tilde{Q}_1, \tilde{Q}_2)$. The superpotential is:

$$W = -\mu^2\Phi + \lambda\Phi Tr(Y_0Y_0) + mTr(Y_2.Y0) + \overline{\lambda}Tr(Y_2Y_0Y_0) + \lambda_2\overline{\Phi}Q_1.\tilde{Q}_1 - \overline{\mu}^2\overline{\Phi} + m1Q_2\tilde{Q}_1 + m_2Q_1\tilde{Q}_2 \tag{5.52}$$

Where $\Phi, \overline{\Phi}, Q_2, \tilde{Q}_2$ have R-charges equal to 2, and $Y_0, Q_1, \tilde{Q}_1$ have R-charge 0. Since there are no couplings between the fields of the two sector, we can study them independently.

To simplify the discussion even further, we shall take $m_1 = m_2 = \overline{m}$. For $m_2^2 > \lambda_2\mu^2$, the quarks do not get vevs, and the minimum of the potential is given by:

$$\begin{aligned} Q_1 = 0;\ Q_2 = 0; \\ \tilde{Q}_1 = 0;\ \tilde{Q}_2 = 0; \end{aligned} \tag{5.53}$$

And Φ is undetermined at tree-level.

Furthermore, since in this sector there are no gauge messengers [27], quantum corrections stabilize the vev of the goldstino at the origin of field space. It is not relevant that R-symmetry is not spontaneously broken in this sector, what is important is that the sgolstino vev of the sector with gauge messengers is non-zero (as this breaks the R-symmetry). At the origin of field space, both quarks get masses equal to $\overline{m}$, and couple to an F-term equal to $\lambda\mu^2$, so that

$$\frac{F}{M}_{n.s.} = \frac{\lambda\mu^2}{\overline{m}} \tag{5.54}$$

Where this contribution only affects soft scalar masses, and n.s. stands for "normal messenger sector".

We have already studied the other sector, and for large sgoldstino vevs F/M is approximately given by:

$$\frac{F}{M}_{g.s.} = \frac{4\lambda\mu^2}{\overline{\lambda}y} \tag{5.55}$$

Where g.s stands for gauge messenger sector and y parametrizes the vev along the sgoldstino direction. These contributions affect both soft gaugino and scalar masses. In particular the contribution to the scalar masses is negative.

We will now specify the region in parameter space that we will be considering. In order to keep the gauge couplings in the perturbative regime, all the particles should be reasonably heavy. Also, to keep the scale of the soft terms much lower than the GUT scale to avoid a large tuning for E.W. symmetry breaking, $\sqrt{F} \ll M_{GUT}$. There are several ways to do this. The way we will do it is by choosing the parameters around the region where the F-term equations become degenerate and SUSY stops being broken, i.e. if the superpotential was:

$$W = X_1 f_1(\phi) + X_2 f_2(\phi) \tag{5.56}$$

We would choose the parameters in such a way that the two $f_i(\{\phi_j\})$'s vanish at the same point in field space. A possible reason for this to happen could be an approximate symmetry of the superpotential of the high energy theory (e.g. it is only violated by some non-perturbative term).

In this work we will assume that the flat direction can be stabilized far away from the origin. In [80] it was shown that in models with gauge messengers, even if the R-charges of the fields are 2 and 0 the sgolstino vev can be stabilized away from the origin. So, Coleman-Weinberg corrections are a possible mechanism to achieve this.

The scalar two loop contributions are given by:

$$m_q^2 \approx \frac{\alpha(\mu)^2}{8\pi^2}(NC_i(\frac{F}{M}_{n.s.})^2 + ((C' - 2C_i)b'' + b_i' C)(\frac{F}{M}_{g.s.})^2) \tag{5.57}$$

where C', C_i are the quadratic Casimirs for the particular MSSM quark (for a fundamental of $SU(5)$, $C' = \frac{12}{5}$, and $C_3 = \frac{4}{3}, C_2 = \frac{3}{4}$), for this model $b' = -4, b_3 = 1$, and $b_2 = -3$. (gaugino and trilinear couplings are both non-zero at leading order in F/M and given by the respective expressions), and we can choose the parameters in such a way that the vev of the field y is such that all the squarks/sleptons are non-tachyonic.

We will now give an example point: In units of μ. We assume that the goldstino

flat direction in the gauge messenger is stabilized for $y = 23, 85$, and for the non-gauge messenger sector is stabilized at the origin. Computing the mass of the Higgsed vector fields, allows us to match μ in units of M_{GUT}: $\mu = 0.022 M_{GUT}$.

μ	λ	λ_2	λ_3	$\frac{\mu_2}{\mu}$	$\frac{m}{\mu}$	$\frac{\overline{m}}{\mu}$	g_{GUT}
1	0.5	0.5	0.1	1.1×10^{-4}	0.1291	1	1

Table 5.2: Sample parameter point in this two sector model.

The masses of the other fermionic fields are (in units of M_{GUT}): The "effective" F/M

Adjoints $SU(3)$	$(0.5, 0.0002)$
Adjoints $SU(2)$	$(0.5, 0.0002)$
Fundamentals	0.017
Bifundamentals	$(1, 1, 1.92 \times 10^{-9})$

one loop contribution is $\frac{F}{M}^{(1)} = \sqrt{\frac{(F.F)(x,x)-(x,F)(F,x)}{(x,x)^2}} = 4 \times 10^{-13} M_{GUT}$.

The two loop F/M contribution is:

$$\begin{aligned}(F/M)_{g.s.} &= 9,00 \times 10^{-13} M_{GUT} \\ (F/M)_{n.s.} &= 1,03 \times 10^{-11} M_{GUT}\end{aligned} \tag{5.58}$$

Where in the sector with the gauge messengers, all F/M's are approximately the equal to the value $F/M_{g.m.}$, and for the sector with the quarks F/M is given by F/M_{fund}.

We can now compute the squark and slepton masses, which we summarize in the next table: Where these soft masses are computed at the messengers scale (i.e. close to

Field	Q	U	D^c	L	E^c	H_u
$\frac{m_Q}{M_{GUT}}$	3.53×10^{-13}	2.96×10^{-13}	2.29×10^{-13}	2.60×10^{-13}	3.16×10^{-13}	2.61×10^{-13}

Table 5.3: Squark and sfermion soft scalar masses.

the GUT scale) and we took all gauge coupling to be equal, and equal to 1.

A-terms are proportional to:

$$A \approx \frac{g^2}{8\pi^2}(C - C')\frac{(F,x)}{(x,x)} \approx 3 \times 10^{-14} M_{GUT} \tag{5.59}$$

While the gaugino masses are around[10]:

$$m_\lambda \approx -\frac{g^2}{8\pi^2}(5 - N_i)\frac{(F, x)}{(x, x)} \approx -4 \times 10^{-14} M_{GUT} \tag{5.60}$$

If M_{GUT} is take to be 10^{16}, then squarks and sleptons have masses around 3TeV and gauginos have masses around 400GeV, at the messenger scale.

More generally, and in the worst case, one may expect an approximate upper bound on the ratio:

$$\frac{m_\lambda}{m_Q} \lesssim \frac{1}{4\pi} \tag{5.61}$$

So that even if the sgolstino of the sector with gauge messengers is not stabilized very far away from the origin, the splitting between gauginos and squarks/sleptons is around one order of magnitude.

In this model one does not expect unification to be automatic. At one loop, and with the parameters we used, it is very simple to calculate the gauge couplings as a function of the energy scale:

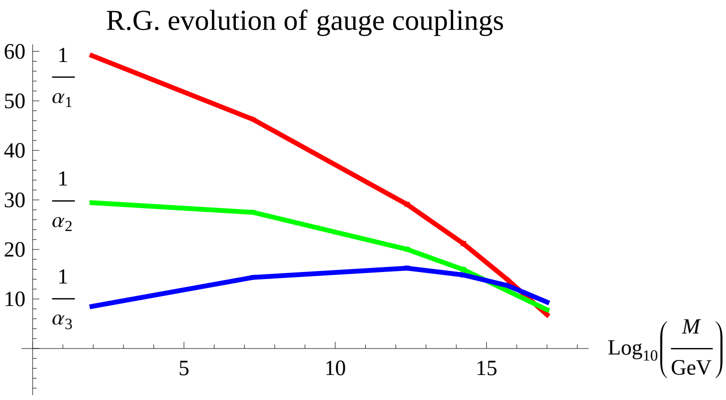


Figure 5.4: One Loop R.G. equations for the Gauge Couplings.

As we can see, unification is possible without a large fine-tuning (but is not automatic).

[10]There is no problem with a negative gaugino mass, as the sign can simply be absorbed by a phase redefinition.

5.5.2 Case 2:

In this case there is a flat direction to which the gauge messengers couple, but the normal messengers do not. Unlike in the previous example this flat direction is not associated with SUSY breaking. So the normal messengers have masses of order $m = X$ (where X is the sgoldstino vev), and gauge messengers have masses of order Φ, where Φ is the vev along the second flat direction. In this case the suppression of the one loop tachyonic contribution of the gauge messengers to the soft squark/slepton masses is lost.

The expressions of the soft terms is approximately:

$$\begin{aligned} m_Q^2 &\approx -(\tfrac{g^2}{16\pi^2})N_{g.m.}(\tfrac{F}{\Phi})^2 + (\tfrac{g^2}{16\pi^2})^2 N_{n.m.}(\tfrac{F}{X})^2 \\ m_\lambda &\approx \tfrac{g^2}{16\pi^2}\overline{N}_{g.m.}\tfrac{(F,X)}{(\Phi,\Phi)} \end{aligned} \tag{5.62}$$

Where the group theory factors associated with the number of messengers are encoded in $N_{g.m.}$, $N_{n.m.}$ and $\overline{N}_{g.m.}$. Where X is the sgoldstino vev and Φ is the vev along the flat direction. One then needs $\frac{\Phi}{X}$ to be large enough so that $m_Q^2 > 0$.

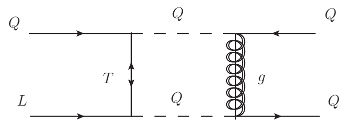
If $\frac{\Phi}{X}$ is large enough so that $m_Q^2 > 0$, a significant cancellation between the one loop contribution from gauge messengers and the two loop contribution from normal messengers is required to get $m_Q \sim m_\lambda$.

Example:

As a particular example, we now explore the possibility that this solution is actually connected to the solution of the doublet-triplet problem. We will now briefly review this problem[116, 117].

In the context of $SU(5)$ GUTs the doublet-triplet problem can be understood in the following way: take the two MSSM Higgs fields to be a fundamental/antifundamental pair of $SU(5)$. Below the energy at which the GUT symmetry is spontaneously broken these representations split to two $(3,1)$ and two $(1,2)$ under $SU(3)$ and $SU(2)$ respectively. The $(1,2)$'s are the MSSM Higgs fields. Since the triplets are absent in the low energy theory, they must be massive. Below these Higgs triplets mass scale, they can be integrated out. This generically generates dimension 5 operators[11] that allow for proton decay[120]:

[11] Strictly speaking the dimension 5 operators that we are considering are the effective vertices one gets from integrating out the Higgs triplet and Higgsed gaugino (i.e. Consider that the Higgs triplet and



Where T represents the Higgs triplets, Q and L are quarks/leptons (for solid lines) and squarks/sleptons (for dashed lines), and we suppressed the family and gauge indices. This diagram is suppressed by the Higgsino triplet mass and the SUSY breaking scale. Since there are very stringent bound on this decay, it means that the Higgs triplets should actually be very heavy.

This problem can be addressed if one assumes the existence of a sliding singlet that couples to the Higgs [58]:

$$W = \lambda_x(H_u.\tilde{Y}_0.H_d + \tilde{\Phi}H_u.H_d) + m_2 Tr[\tilde{Y}_0.\tilde{Y}_0] + \overline{\lambda}_x Tr[\tilde{Y}_0.\tilde{Y}_0.\tilde{Y}_0] \tag{5.63}$$

This superpotential has many different minima, all of which are SUSY. If all the Higgs vevs are 0, we have:

$$\begin{aligned} &H_u = 0 \quad H_d = 0 \qquad\qquad \tilde{Y}_0 = 0 \\ &H_u = 0 \quad H_d = 0 \quad \tilde{Y}_0 = \tfrac{2m_2}{3\overline{\lambda}_x} diag(\{2,2,2,-3,-3\}) \end{aligned} \tag{5.64}$$

Since we know that in any realistic model, below the E.W. scale, the Higgs doublets spontaneously break the $SU(2) \times U(1)$ symmetry by acquiring vevs, we can look for SUSY solutions that allow for this mechanism to happen. If only the Higgs triplets are stabilized at the origin, there is only one solution that is:

$$\tilde{Y}_0 = \tfrac{2m_2}{3\overline{\lambda}_x} diag(\{2,2,2,-3,-3\}) \quad \tilde{\Phi} = \tfrac{2m_2}{\overline{\lambda}_x} \tag{5.65}$$

Along this direction the Higgs triplets are very heavy while the doublets remain massless (at tree-level):

$$M = \frac{2m_2}{3\overline{\lambda}_x} diag(\{5,5,5,0,0\}) \tag{5.66}$$

Where M is the fermionic mass matrix for the Higgs fields. We will now add to this extended version of the MSSM the model we presented in the second section, and assume

Higgsed gaugino propagators are evaluated at zero momenta and contracted to a point). For clarity we present the full diagram.

that the solution which keeps the Higgs doublets light and the triplets heavy is the correct minimum of the potential.[12]

The adjoints of $SU(5)$ will decompose to $(8,0)+(0,3)+(3,2)+(\overline{3},2)$+singlet. There is no mixing between the adjoints of the two sectors and they all get masses close to the GUT scale. The bifundamentals of the two sectors do mix: one chiral pair is eaten by the gauginos and becomes heavy, the two other pairs are "light" and get masses of the order of $\frac{F}{M}$ due to a see-saw structure of the mass matrix.

If we take the parameters to be: The masses of the fermionic fields are (in units of

μ	λ	λ_2	$\frac{m}{\mu}$	λ_x	$\overline{\lambda}_x$	$\frac{m_2}{\mu}$	g_{GUT}
1	0.1	0.5	0.2887	0.0167	0.0025	0.05	1

Table 5.4: Sample parameter point for the model with doublet-triplet slitting.

M_{GUT}[13]): So that if the GUT scale is $10^{16}GeV$, the two "light" bifundamentals would be

Adjoints $SU(3)$	$(0.028, 0.011, 0.006)$
Adjoints $SU(2)$	$(0.028, 0.011, 0.006)$
Higgs triplets	0.017
Bifundamentals	$(1, 1, 1.63 \times 10^{-9}, 6.03 \times 10^{-10})$

around $10^7 GeV$. We note that both the Higgs doublets and triplets have SUSY spectra, i.e. they do not couple to any F-terms at tree-level, as the vev of the F-term of $\tilde{\Phi}$ is 0. The Higgs sector is different from the quark and leptonic sectors since they can know about SUSY breaking indirectly through loops with $\tilde{\Phi}$ bifundamentals[14].

Ignoring this effect which should be small if λ_x and $\overline{\lambda}_x$ are small, we can give an order of magnitude estimate for the contribution coming from gauge mediation. One needs to take into account the one loop contribution from gauge messengers and the two loop contributions from both gauge and normal messengers. The one loop F/M effect (given in eq. 5.27) is $(F/M)^{(1)} = 3,91 \times 10^{-10}$.

[12] In practice this may require some tuning.
[13] M_{GUT} is taken to be the mass of the Higgsed vector bosons
[14] So, even though the MSSM fields only know about SUSY breaking effects radiatively, strictly speaking this scenario is not pure gauge mediation.

The two loop F/M effects are given by:

$$\frac{F}{M}_{g.m.} = 1.12 \times 10^{-12} M_{GUT}$$
$$\frac{F}{M}_{n.m.} = 4,02 \times 10^{-9} M_{GUT} \qquad (5.67)$$

So that the order of magnitude for the soft terms (at the messengers scale):

$$m_Q \sim 10^{-11} M_{GUT}$$
$$m_L \sim 10^{-11} M_{GUT} \qquad (5.68)$$
$$m_\lambda \sim 4 \times 10^{-14} M_{GUT}$$

Where there are nearly three orders of magnitude between gaugino and scalar masses. However, the values we present are computed at the messenger scale and R.G. effects are important and should be taken into account, and R.G. effects may be sufficient to solve this problem.

Even though this model allows us to use a solution to the doublet-triplet problem to get non-tachyonic scalar masses, and is very simple, it should be improved in order to become more realistic. A mechanism that makes the bifundamentals heavier would help reducing the tuning required to get unification: Where this plot is the R.G. evolution

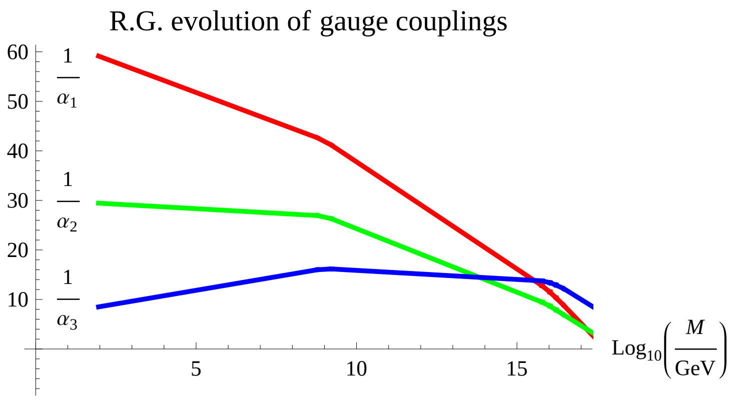


Figure 5.5: One loop R.G. equations for the Gauge Couplings.

of the gauge couplings for the sample point we just presented.

5.6 Conclusions

In this chapter we presented a model of F-term SUSY breaking with two $SU(5)$ adjoint chiral messenger fields and a singlet. Coupling these adjoint fields to the SUSY breaking sector broke the degeneracy between vacua with different symmetry breaking patterns. This gave us a natural mechanism that could explain why the $SU(5)$ GUT group is broken to the MSSM gauge group. In the particular model we presented this happened when one of the Yukawa couplings was < 1 and positive.

In it's simplest form the model was not viable as quarks and sleptons were tachyonic. We showed that in a large class of models that have gauge messengers, this is associated with the need to stabilize the sgoldstino vev far away from the origin, and is independent of the values of the parameters.

To solve this problem we proposed two scenarios:

SUSY is broken independently in two sectors, and gauge messengers exist in only one of them. There are two sgoldstinos that acquire different vevs. By choosing the ratio between these vevs it is possible to enhance the contributions from normal messengers and make both squark and sleptons non-tachyonic. We showed a concrete example where this scenario is realized, and that indeed squarks and sleptons can be non-tachyonic. Gaugino masses (and trilinear couplings) are generated at leading order in F/M at one loop (because of gauge messengers), and up to R.G. effects are lighter than scalars (up to one order of magnitude).

In the second scenario there are also two sectors,but SUSY is only broken in one of them. The SUSY breaking sector should have both gauge and normal messengers, while the sector where SUSY is not broken should have a field whose vev breaks the GUT symmetry. If this vev is larger than the sgoldstino vev, the contribution from normal messengers can be enhanced and squarks and sleptons can be non-tachyonic.

One natural realization of this scenario is the sliding singlet solution to the doublet-triplet problem (or other solutions to the doublet-triplet problem) together with the first model we presented, as the hidden sector. We showed that with this extension of the MSSM, there exists a region of parameter space and field vevs for which both squarks and sleptons are non-tachyonic. This is not a complete model, however, and some of its problems were identified.

We also did not solve the problems with the Witten hierarchy idea[121], but instead argued that it should be the SUSY breaking scale that is much lower than the GUT scale. This could be because of some approximate symmetry in the high energy theory: if unbroken, this symmetry would make the "natural" choice of parameters in the low energy model to be such that, despite the rank condition, SUSY is not broken. In other words: one does not need small parameters to have unbroken SUSY, but there should be some relations between the couplings of the low energy theory if the SUSY breaking scale is to be smaller than the GUT scale. It is this relation between the couplings of the superpotential that could be enforced by some (approximate) symmetry in the high energy theory.

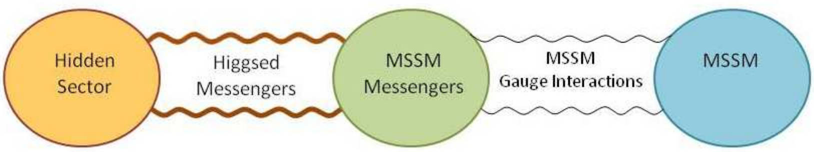
Chapter 6

Semi-Direct Gauge Mediation

"Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry."
— Richard Feynman, 1918-1988

In the previous section we studied the possibility of building phenomenologically viable models with a perturbatively stable vacuum and where there are simultaneously gauge and non-gauge messengers (it turns out that in most cases they come together).

In this section we investigate **semi-direct gauge mediation**. In this scenario the messenger fields couple to the SUSY breaking sector only through (hidden sector) gauge interactions. Since the superpotential for these messengers usually consists of a mass term, adding fields in this way does not introduce (at least perturbatively) new SUSY vacua. This makes semi-direct gauge mediation an attractive/easy mechanism to make viable models out of the early and rigid constructions where SUSY is dynamically broken (like the 3-2, the 4-1 and the model of chapter 4).



So, in this type of scenario one has three independent sectors:

- The hidden sector where SUSY is spontaneously broken

- The messenger sector that mediates the SUSY breaking
- The MSSM

The messengers are charged under both the hidden sector and the MSSM gauge groups. Because messengers only know about SUSY breaking radiatively, the MSSM soft terms usually come with one more loop factor than in direct gauge mediation. I will be considering mostly cases where the hidden sector has gauge messengers. Since this is a generalization of the result where only chiral messengers exist, the result for that particular case can be found straight-forwardly.

We will compute the soft terms using analytical continuation into superspace, so it will implicitly be assumed that SUSY is softly broken at low energies and $F \ll M$.

A useful formula is:

$$f(\mu^2 + \mu^2|_{\theta^2}\theta^2 + c.c. + \Delta\theta^2\overline{\theta}^2) = f(\mu^2) + \frac{\partial f(\mu^2)}{\partial \mu^2}\mu^2|_{\theta^2}\theta^2 + c.c. + (\frac{\partial f(\mu^2)}{\partial \mu^2}\Delta + \frac{\partial^2 f(\mu^2)}{\partial^2 \mu^2}\left|\mu^2|_{\theta^2}\right|^2)\theta^2\overline{\theta}^2 \tag{6.1}$$

This Taylor expansion is exact by virtue of the anti-commutative properties of Grassman variables.

What it tells us is that when we analytically continue a function into superspace, the extra components depend on the derivatives of the function and not on the function itself. So main ingredient to compute the leading F/M contribution to the soft terms is the knowledge of the beta-function coefficients and anomalous dimensions.

It is widely known that gaugino masses are screened from messenger interactions. What this means in practice is that (apart from effects from messenger Yukawa couplings) the leading order in the F/M expansion two loop contribution to the gaugino mass vanishes.

This can pose some fine-tuning problems when dealing with E.W. symmetry breaking: in order to have evaded detection, gaugino masses must be sufficiently heavy, but because of this splitting between gaugino and scalar masses, the large quantum corrections will tend to make the Higgs too heavy.

In order to understand how much of a problem this can be, we need to understand exactly what are the dominant effects that lead to the generation of gaugino masses in semi-direct gauge mediation (with or without gauge messengers). Since we will only use R.G. equations with terms up to two loops, we can only compute reliably gaugino masses up to two loops and squark scalar masses up to three loops. If some assumptions are made[1],we can compute gaugino masses up to 3-loops.

We will see that at leading order in F/M, gaugino screening is felt all the way up to the three-loop level, so the dominant term usually comes from the two loop NLO contribution in the F/M expansion. In order to have a viable spectrum one then needs to have $F/M \lesssim 1$, but in turn this leads to the existence of light fields charged under the MSSM gauge groups.

6.1 Gaugino Masses:

Since the messenger masses are SUSY at tree-level, the one loop contribution to the MSSM gaugino masses vanishes, and we need to evaluate multi-loop effects. This means that we will need to use the real gauge coupling instead of the holomorphic one. As we have seen, the main difference between these two is that the real gauge coupling knows about the GMZ evanescent term (i.e. is sensitive to the axial anomaly we mentioned in the first section) and it is renormalized at all loop orders.

In the $\overline{DR}'$ scheme, the relation between the holomorphic and real gauge coupling is[32]:

$$R(\mu) = 2Re(S(\mu)) + \frac{T_G}{8\pi^2} Log(Re(S(\mu))) - \sum_r \frac{T_r}{8\pi^2} Log(Z_r(\mu)) + O(Re(S(\mu))^{-1}) \quad (6.2)$$

Where S is the holomorphic gauge coupling, T_G is the Casimir for the adjoint rep. of the gauge group. Z_r, T_r are the wave-function renormalization, the Dynkin index for the field r, and $8\pi R = 1/\alpha$ (where $\alpha = \frac{g^2}{4\pi}$, and g is what is commonly called the gauge coupling)[2].

We will now review the gaugino screening argument[40]:

[1]We will assume that the $\overline{SDR}'$ scheme is consistent with the NSVZ beta function for the real gauge coupling in the limit where SUSY is not spontaneously broken.

[2]To avoid confusion with the holomorphic gauge coupling we will use the letter T instead of the more standard S for the Dynkin index.

At two loops (below the messenger scale), the real gauge coupling is given by:

$$\begin{aligned} R(\mu) =& R(\mu_0) + \frac{b'}{16\pi^2} Log(\frac{m^2_{mess}}{\mu_0^2}) + \frac{b}{16\pi^2} Log(\frac{\mu^2}{m^2_{mess}}) + \frac{T_G}{8\pi^2} Log(\frac{Re(S(\mu))}{Re(S(\mu_0))}) \\ & - \sum_m \frac{T_m}{8\pi^2} Log(\frac{Z_m(\mu_{mess})}{Z_m(\mu_0)}) - \sum_q \frac{T_q}{8\pi^2} Log(\frac{Z_Q(\mu)}{Z_Q(\mu_0)}) \end{aligned} \tag{6.3}$$

Where Z_Q is the wave-function renormalization for the quark fields, Z_m is the wave-function renormalization for the messengers. Beyond the tree-level approximation, the messenger mass threshold gets shifted due to interactions:

$$\mu_{mess,0} \rightarrow \mu_{mess,0}/Z_m \tag{6.4}$$

Where $\mu_{mess,0}$ is the bare messenger mass. Using this renormalized mass for the messengers gives:

$$\begin{aligned} R(\mu) =& R(\mu_0) + \frac{b'}{16\pi^2} Log(\frac{m^2_{mess,0}}{\mu_0^2}) + \frac{b}{16\pi^2} Log(\frac{\mu^2}{m^2_{mess,0}}) + \frac{T_G}{8\pi^2} Log(\frac{Re(S(\mu))}{Re(S(\mu_0))}) \\ & + \sum_m \frac{T_m}{8\pi^2} Log(Z_m(\mu_0)) - \sum_q \frac{T_q}{8\pi^2} Log(\frac{Z_Q(\mu)}{Z_Q(\mu_0)}) + O((S(\mu) + S(\mu)^\dagger)^{-1}) \end{aligned} \tag{6.5}$$

So no gaugino masses are generated to two loops.

If we assume that we are using a scheme that is compatible with the NSVZ exact beta-function, then we can (see appendix D) reliably compute gaugino masses up to three-loops. In the limit where the messenger Yukawa couplings are small (or non-existent), no gaugino masses are generated (at leading order in F/M):

$$\frac{dR}{dLog(\mu_{hm})} = 0 \tag{6.6}$$

Which gives:

$$m_\lambda \approx O(F^3/M^5) \tag{6.7}$$

If the assumption doesn't hold, one needs to know the MSSM gauge coupling beta-function to three-loops to reliably compute gaugino masses.

There were two effects that were not computed: the possible role of Yukawa messenger couplings, and $\frac{F^3}{M^5}$ two loop contributions to gaugino masses. In the messenger sector, Yukawa couplings can split the masses of the messenger fields. This will give (at low energies) a contribution to gaugino masses of the type: $O(g^6 Log(\frac{m_{Hmess}}{m_{Lmess}}))$, where m_{Hmess} (m_{Lmess}) is the mass of the heavy (light) messenger field.

The second effect comes from considering higher order terms in the F/M expansion. These cannot be computed with the methods we have used as these contributions come from irrelevant operators that are generated from the integration of the messengers. If $F \sim M^2$, these contributions are expected to be the dominant ones, and of order $O(g^4 \frac{F^3}{M^5})$.

6.2 Squark Masses:

Squark masses can be computed from the wave-function renormalization of the quark fields. In the particular case we are looking at (the MSSM messengers are normal messengers) squark masses appear at 3 loops if the hidden sector contains gauge messengers, and 4 loops otherwise.

This can be understood since if the hidden sector has gauge messengers, the MSSM messengers get soft masses at one loop. It then takes two loops to communicate these effects to the MSSM, making 3-loops. If the hidden sector does not have gauge messengers, it takes two loops to communicate the SUSY breaking effects first to the MSSM messengers, and two more loops to communicate them to the MSSM.

As before, the way to proceed with the computation is to analytically continue the Higgsed messenger mass threshold to superspace. The soft terms will depend on derivatives of the squark wave-function renormalization that can be re-expressed in terms of the beta function coefficients and anomalous dimensions of the fields (see Appendix D for a more detailed treatment).

The squark mass is given by:

$$m_Q^2 = -Log(Z_Q)|_{\theta^2\bar{\theta}^2} \tag{6.8}$$

Referring to eq. 6.1, we see that we will need both the first and second derivatives of $Log(Z_Q)$ below the MSSM messenger scale.

The wave-function renormalization for the Quark superfield is given by:

$$Log(Z_Q(\mu)) = -\int_{\mu}^{\mu_{mess}} \frac{d\mu'}{\mu'}\gamma_Q^{bm}(\mu') - \int_{\mu_{mess}}^{\mu_{hm}} \frac{d\mu'}{\mu'}\gamma_Q^{am}(\mu') - \int_{\mu_{hm}}^{\mu_0} \frac{d\mu'}{\mu'}\gamma_Q^{hm}(\mu') \tag{6.9}$$

Where we have transformed the differential R.G. equations to an integral form with boundary condition $Log(Z_Q(\mu_0)) = 0$. The $\gamma_Q^{bm}, \gamma^{am}, \gamma_Q^{hm}$'s are the (unknown) exact expressions for the anomalous dimensions of the quark fields below the messenger scale, above the messenger scale and above the Higgsed messengers.

The squark mass is given by:

$$m_Q^2 = \left(\frac{1}{2}\frac{\partial Log(Z_Q(\mu))}{\partial Log(\mu_{hm})}\frac{(F,F)(X,X)-(F,X)(X,F)}{(X,X)^2} + \frac{1}{4}\frac{\partial^2 Log(Z_Q(\mu))}{\partial^2 Log(\mu_{hm})}\frac{(X,F)(F,X)}{(X,X)^2}\right) \tag{6.10}$$

Where μ_{hm} is the mass for the Higgsed hidden sector messengers.

For large sgoldstino vevs the first term is highly suppressed, and in this regime, the previous expression becomes:

$$m_Q^2 \approx \frac{1}{4}\frac{\partial^2 Log(Z_Q(\mu))}{\partial^2 Log(\mu_{hm})}\frac{(X,F)(F,X)}{(X,X)^2} \tag{6.11}$$

This second derivative is given by:

$$\begin{aligned}\frac{d^2 Log(Z_Q)}{d^2 Log(\mu_{hm})} =& \frac{d^2 Log(\mu_{mess})}{d^2 Log(\mu_{hm})}\delta\gamma_Q^{mess}(\mu_{mess}) + \frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\frac{d\delta\gamma_Q^{mess}(\mu_{mess})}{dLog(\mu_{hm})} + \frac{d\delta\gamma_Q^{hs}(\mu_{hm}^2)}{dLog(\mu_{hm})}\\ &+ \frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\frac{d\delta\gamma_Q^{mess}(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{mess}} - \frac{d\gamma_Q^{am}(\mu^2)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}}\\ &- \int_{\mu}^{\mu_{mess}} \frac{d\mu'}{\mu}\frac{d^2\gamma_Q^{bm}(\mu^2)}{d^2 Log(\mu_{hm})} - \int_{\mu_{mess}}^{\mu_{hm}} \frac{d\mu'}{\mu}\frac{d^2\gamma_Q^{am}(\mu^2)}{d^2 Log(\mu_{hm})}\end{aligned} \tag{6.12}$$

Where we used:

$$\begin{aligned}\delta\gamma_Q^{hs}(\mu^2) &= \gamma_Q^{hm}(\mu^2) - \gamma_Q^{am}(\mu^2)\\ \delta\gamma_Q^{mess}(\mu^2) &= \gamma_Q^{am}(\mu^2) - \gamma_Q^{bm}(\mu^2)\end{aligned} \tag{6.13}$$

Now, because we have assumed the existence of Higgsed hidden sector messengers, the leading term is:

$$\frac{d^2Log(Z_Q)}{d^2Log(\mu_{hm})} \approx -\frac{d\gamma_Q^{am}(\mu^2)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}} \tag{6.14}$$

If there were no such messengers squark masses would be generated at four loops.

Let us look at this point in a bit more detail:

$$\frac{d^2Log(Z_Q(\mu))}{d^2Log(\mu_{hm})} \approx -\frac{d\gamma_Q^{am}(\mu^2)}{dg(\mu)}\frac{dg(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}} \approx -\frac{32}{(16\pi^2)^3}g^4_{\text{MSSM}}(\mu_{hm})g^2_{hs}(\mu_{hm})N_{mess}\delta C_{m,hs}C_Q \tag{6.15}$$

If there were no Higgsed messengers, the Casimir of representation of the messenger fields under the hidden sector gauge group would not change and this term would vanish.

So, if there are gauge messengers, the squark masses are given by:

$$m_Q^2 \approx C_Q\frac{8}{(16\pi^2)^3}g^4_{\text{MSSM}}(\mu_{hm})g^2_{hs}(\mu_{hm})N_{mess}\delta C_{m,hs}\frac{(X,F)(F,X)}{(X,X)^2} \tag{6.16}$$

So this is a three-loop mass contribution.

6.3 Summary of Results

Semi-direct gauge mediation is an attractive way to add messengers to rigid models of dynamical SUSY breaking as this procedure doesn't usually lead to any new SUSY vacua. However, since the messenger mass threshold is SUSY, gaugino masses are generated at two loops and at next to leading order in the expansion on F/M. This happens because, at leading order in F/M, there is an "accidental" screening of the effects of messenger interactions.

In some scenarios, as the 3-2 or 4-1 models, where SUSY is dynamically broken in a calculable way, there are Higgsed messengers. In these cases, squark masses squared are generated at three-loops. This contribution can (at least partially) be suppressed with respect to the four loop one, if the sgoldstino vev is very large and the MSSM and hidden sector gauge groups are asymptotically free.

So the scenario is one where gaugino masses are suppressed with respect to scalar masses, the difference being larger in models with gauge messengers (as in these, scalar masses are generated at three-loops instead of four).

Because of this, it doesn't seem likely that gauge messengers will play a role if nature decides to communicate SUSY breaking effects to the MSSM with a form of perturbative semi-direct gauge mediation: in these scenarios the splittings between gauginos and scalars will be too high. This means that in order to keep the Higgs mass close to the E.W. scale requires more tuning.

In order to evade these problems, models of semi-direct gauge mediation *without gauge messengers* and with $F/M \lesssim 1$ seem favoured. If this is to happen, some of the messengers may be light enough so as to be detected experimentally.

Appendix A

SQCD

"In the book of life, the answers aren't in the back."
— Charlie Brown

A.1 Non-perturbative renormalization theorems:

This section is inspired by Weinberg's discussion on the non-perturbative corrections to the superpotential.

Let us look at the Kahler potential first, in the case where $N_F < N_C$. If the rank of the meson field M_{ij} is maximal, the gauge group is completely Higgsed. In this case, we can integrate out the heavy fields, and get a low energy effective theory that consists of the light meson fields. The classical Kahler potential is:

$$K = 2\sqrt{M^\dagger M} \tag{A.1}$$

If we are close to a point in field space such that one of the eigenvalues of M vanishes, then the Kahler metric derived from this potential becomes singular. This is to be expected: close to this point there is an enhanced symmetry where some of the Higgsed fields become massless and must be included in the low energy description.

As we've seen, to all orders in perturbation theory the superpotential is not renormalized, the holomorphic gauge coupling is renormalized at one loop and the Kahler potential is generally a complicated function.

We can now generalize these arguments to include non-perturbative effects. We will constrain this discussion to points in field space where the gauge group is completely broken at low energies, and the group is asymptotically free. In this case, the gauge coupling can be taken to be small and the leading non-perturbative effects are given by instantons. Let us consider the (bare) Lagrangian density:

$$L = [\Phi^\dagger e^{-V}\Phi]_D + 2Re[W(\Phi)]_F - Re[i\frac{\tau}{8\pi}\sum W_a W^a]_F \quad \text{(A.2)}$$

And consider the auxiliary model:

$$L = [\Phi^\dagger e^{-V}\Phi]_D + 2Re[YW(\Phi)]_F - Re[i\frac{T}{8\pi}\sum W_a W^a]_F \quad \text{(A.3)}$$

This becomes the same as the original Lagrangian density when $Y = 1$ and $T = \tau$. In perturbation theory, this system has several symmetries that are sufficient to allow us to construct the full effective superpotential. Non-perturbatively, the path integral includes a sum over different non-trivial backgrounds (instantons with different winding numbers). Even though this invalidates the use of both the R-symmetry and the Peccei-Quinn symmetry. We can, however, combine these two $U(1)$'s and define a new, non-anomalous, "$U(1)_X$":

$$\begin{aligned}
\theta &\rightarrow e^{i\phi}\theta \\
\Phi &\rightarrow \Phi \\
V &\rightarrow V \\
Y &\rightarrow e^{2i\phi}Y \\
T &\rightarrow T + (C-S)\phi/\pi
\end{aligned} \quad \text{(A.4)}$$

Where C is the quadratic Casimir (N_c for $SU(N)$) and S is the total Dynkin index (given by $Tr(T^A T^B) = S\delta^{AB}$, where Tr is a sum over all chiral fields, as well as color indices).

One can now consider the effective Wilsonian action at some scale μ.

$$L_\mu = [A_\mu(\Phi, \Phi^\dagger, V, T, T^\dagger, Y, Y^\dagger, ...)]_D + 2Re[-i\frac{T}{8\pi}\sum W_a W^a + B_\mu(\Phi, W^a, T, Y)] \quad \text{(A.5)}$$

The reason why the term linear in T was separated so that the shift $T \rightarrow T + (C - S)\phi/\pi$ cancels the $U(1)_R$ anomaly. Because of the $U(1)_X$ symmetry and the holomorphic properties of B, we can deduce that the only way B can depend on T is through positive powers of $e^{2i\pi T}$ [1]. Another argument for this comes from considering that when the gauge coupling goes to 0, instanton effects must be suppressed, and this can only happen (since in the vacuum T is replaced by τ) if $|e^{2i\nu\pi T}| = |e^{2i\nu\pi\tau}| = e^{-2\nu\pi/g^2} \rightarrow 0$, i.e. $\nu > 0$. So, only instantons with positive winding number can contribute to the action.

Under this non-anomalous symmetry, the term $e^{2i\nu\pi T}$ has a definite R-charge, whose sign depends on wether $C > S$,$C = S$ or $C < S$ (for SQCD this means $N_C > N_F$, $N_C = N_F$, $N_C < N_F$). Let us restrict ourselves to SQCD (otherwise just replace C with N_C and N_F with S).

In the first case, $N_C > N_F$, the term under the anomalous R-symmetry, $e^{2i\nu\pi T}$ has a positive R-charge: $2(N_C - N_F)\nu$. The anomalous R-symmetry together with Lorentz invariance dictate that B has the form:

$$B_\mu = YW_\mu(\Phi) + W^bW^a l_{\mu,ab}(\Phi) + e^{\frac{2i\pi T}{N_C - N_F}} v_\mu(\Phi) \tag{A.6}$$

The function $l_{\mu,ab}(\Phi)$ is constrained in the same way as before. For $Y = 0$, there is a $U(1)_V$, where all Φ have charge 1, because of this all Y-independent functions must depend on equal numbers of Φ and $\Phi^\dagger$. This means that as before the holomorphic gauge coupling only gets renormalized at one loop.

So far, we've shown that when we set $Y = 1$ and $T = \tau$, the renormalized Lagrangian density is given by:

$$L_\mu = [A_\mu(\Phi, \Phi^\dagger, V, T, T^\dagger, Y, Y^\dagger, ...)]_D + 2Re[-i\frac{\tau_\mu}{8\pi}W_aW^a + W_\mu(\Phi) + e^{\frac{2i\pi T}{N_C - N_F}} v_\mu(\Phi)] \tag{A.7}$$

Where τ_μ is the one loop running gauge coupling, and all the non-perturbative complications have been hidden in the function $v_\mu(\Phi)$.

To determine this function we note that it does not depend on Y, so we can set it to 0. In this case, there is a $U(1)_d$ for each of the Φ fields, along with the flavour and color symmetries. Then, it can be shown that the only invariant is the determinant of the Meson operator $D \equiv Det(M_{ij})$. The set of $U(1)_d$ transformations is anomalous but, for each of them, one can form a non-anomalous symmetry by assigning a transformation

[1] The dependence of B in T must be such that shifts in T correspond to $U(1)_R$ transformations in B and instanton contributions to the path integral are given by $e^{2i\nu\pi T}$ ($e^{2i\nu\pi T^*}$) for positive (negative) winding number ν

law to T:.

$$\begin{aligned} \Phi &\rightarrow e^{i\alpha}\Phi \\ T &\rightarrow T + n(d)S_d\alpha/\pi \end{aligned} \tag{A.8}$$

This means that v_μ must be an homogeneous function of degree $-2n(d)S_d/(N_C - N_F)$ fore every representation of the fields. (where S_d is the Dynkin of any single field transforming under a particular representation of the gauge group, and n(d) the number of fields that transform in that particular representation, so $S_d = 1/2$ and $n(d) = N_f$, for SQCD). This means that in general for SQCD, the dynamically generated superpotential will be a homogeneous function of the $D^{-1/(N_C - N_F)}$!

Since the superpotential must have mass dimension 3, the coefficient that multiplies this factor must be proportional to $m^{(3C-S)/(C-S)}$. Since the theory is renormalizable, the Wilsonian action cannot depend on the cut-off or any other scale coming from a method used to regulate the theory. This is sufficient to determine the whole function up to an overall constant:

$$exp(\frac{2i\pi T}{C-S})v_\mu(\Phi) = C_{N_C,N_F}(\frac{\Lambda^{(3N_C-N_F)}}{Det(M)})^{1/(N_C-N_F)} \tag{A.9}$$

This constant has been determined using instantons (for $N_F = N_C - 1$) and was found to be $C_{N_C,N_F} = N_C - N_F$.

The other case that needs to be analyzed is when $N_C = N_F$. In this case, the term $e^{2i\nu\pi T}$ does not have an R-charge. So, even though a linear term in B is forbidden by R-invariance, it can still make an appearance in the Wilsonian action. For this case, instead of one Y-field, we introduce a field Y_r for every term of the form Φ^r in the superpotential (e.g. a mass term $m\Phi.\tilde{\Phi} \rightarrow y_2\Phi.\tilde{\Phi}$. The Wilsonian superpotential is then:

$$B_\mu = W_\mu(y_r, \Phi, e^{2i\nu\pi T}) + \sum W^b W^a l_{\mu,ab}(\Phi, e^{2i\nu\pi T}) \tag{A.10}$$

Where W_μ is linear in the y_r's. To determine the function $l_{\mu,ab}(\Phi, e^{2i\nu\pi T})$ we note that there is a non-anomalous symmetry under which

$$\begin{aligned} \Phi &\to e^{i\alpha}\Phi \\ T &\to T + N_C\alpha/\pi \\ Y_r &\to e^{-ir\alpha}Y_r \end{aligned} \tag{A.11}$$

Because of this symmetry, if we expand W_μ in powers of y_r and $e^{2i\pi T}$, a term proportional to $y_r^{n_r}(e^{2i\pi T})^a$, must be proportional to some Φ_Φ^N, where $N_\Phi = \sum rn_r - 2N_C a$. Since due to the R-symmetry $W_\mu(y_r, \Phi, e^{2i\nu\pi T})$ must be linear in the y_rs, a term proportional to $y_r(e^{2i\pi T})^a$ has:

$$N_\Phi = r - 2N_C a \tag{A.12}$$

While very generic, this is not too usefull. There are two main cases of interest: $r = 0$ and $a = 1$.

If $r = 0$, then we are not considering a superpotential coupling, but the renormalization of the gauge kinetic term, and:

$$N_\Phi = -2N_C a \tag{A.13}$$

So that no term with a positive number of Φ's is allowed (recall that only instantons with positive winding number ($a > 0$) contribute to the action). The terms that do not depend on Φ correspond to the one loop renormalized gauge coupling.

The second case of interest is $a = 1$, in this case it is easy to see that the term:

$$W_{dyn} = \lambda_0(aB\tilde{B} + bDet(M) + ce^{2i\pi T}) \tag{A.14}$$

is allowed to be generated. Where $B(\tilde{B})$ is a baryon (anti-baryon) operator, and M is a meson operator (as defined in the main section). This term corresponds to a one instanton correction (and $e^{2i\pi T} \sim \Lambda^{2N_C}$).

To go any further, we need to use a deformation of the superpotential by a mass term: i.e. add mM_{ii}. This breaks flavour symmetry from $N_F \to N_F - 1$. By taking the mass to infinity, we can integrate out one flavour and relate the original theory to a case that we already know: SQCD with $N_F < N_C$. It turns out that for consistency of the solutions in both theories, there *must be* a dynamically generated term in the superpotential, proportional to equation A.14.

This term is special however: because the theory is asymptotically free, no term in the superpotential should grow faster than ϕ^3 for large values of the fields. So this term should be interpreted as a quantum constraint on the moduli space of the theory with λ_0 being a Lagrange multiplier and not a true contribution to the superpotential.

So, at the end of the day the results are equivalent to saying that: the superpotential is not renormalized, and the moduli space is deformed according to:

$$Det(M) - B\tilde{B} = 0 \rightarrow Det(M) - B\tilde{B} = \Lambda^{2N_c} \tag{A.15}$$

where Λ is the dynamical scale of the theory.

For $N_C < N_F$ it can be shown (we refer to [18] or the lectures of Seiberg [15]) that the superpotential is not renormalized.

Appendix B

Meson-Deformed ISS

B.1 Leading order contribution to the gaugino mass

To develop a perturbative approximation of Eqs. (3.44)-(3.45) we note that when the F-terms are small compared to μ^2, we may first go to the "fermion-diagonal basis", by making a rotation on the scalars given by

$$Q_0 = \begin{pmatrix} U & \mathbf{0} \\ \mathbf{0} & V \end{pmatrix} \tag{B.1}$$

where the U and V matrices are the fermion-diagonalisation matrices defined in (3.40). In this basis the scalar mass-squareds are

$$\tilde{m}^2_{\rm sc} = Q_0^\dagger m^2_{\rm sc} Q_0 \approx \begin{pmatrix} \hat{m}^2_{\rm f} & \mathcal{A} \\ \mathcal{A}^\dagger & \hat{m}^2_{\rm f} \end{pmatrix} \tag{B.2}$$

where

$$\mathcal{A}_{ij} = U^\dagger_{ia} W^{abc} W_c V_{bj} = (U^\dagger \mathcal{F} V)_{ij} \tag{B.3}$$

in terms of the F-term matrix $\mathcal{F}^{ab} \equiv W^{abc} W_c$. Evaluating the diagram for the gaugino mass in this basis (cf. Eqs. (3.44)-(3.45)) and suppressing the overall factor $2g_A^2 {\rm tr}(T^A T^B)$, yields,

$$\int \frac{d^4k}{(2\pi)^4} \sum_{k,l=1}^{4} \sum_{i,j=1}^{2} (U_{i1}^\dagger Q_{0,1k} + U_{i2}^\dagger Q_{0,2k}) \left(\frac{1}{k^2 - \tilde{m}_{\mathrm{sc}}^2} \right)_{kl} \left(\frac{\hat{m}_f}{k^2 - \hat{m}_{\mathrm{f}}^2} \right)_{ij} (Q_{0,l3}^\dagger V_{1j} + Q_{0,l4}^\dagger V_{2j})$$
$$= \int \frac{d^4k}{(2\pi)^4} \sum_{i,j,k,l=1}^{2} (U_{i1}^\dagger U_{1k} + U_{i2}^\dagger U_{2k}) \left(\frac{1}{k^2 - \tilde{m}_{\mathrm{sc}}^2} \right)_{k,(l+2)} \left(\frac{\hat{m}_f}{k^2 - \hat{m}_{\mathrm{f}}^2} \right)_{ij} (V_{l1}^\dagger V_{1j} + V_{l2}^\dagger V_{2j})$$
$$= \int \frac{d^4k}{(2\pi)^4} \sum_{i,j,k,l=1}^{2} \delta_{ik} \left(\frac{1}{k^2 - \tilde{m}_{\mathrm{sc}}^2} \right)_{k,(l+2)} \left(\frac{\hat{m}_f}{k^2 - \hat{m}_{\mathrm{f}}^2} \right)_{ij} \delta_{jl}, \tag{B.4}$$

where, in the last step, we have made use of the unitarity of the U and V matrices.

The fermion propagator is already diagonal, but the boson propagator has off diagonal terms $\sim \mathcal{A}$. Expanding in powers of $\mathcal{A}$ we have,

$$\left(\frac{1}{k^2 - \tilde{m}_{\mathrm{sc}}^2} \right)_{k,(l+2)} = \left(\frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} \mathcal{A} \frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} + \frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} \mathcal{A} \frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} \mathcal{A}^\dagger \frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} \mathcal{A} \frac{1}{k^2 - \hat{m}_{\mathrm{f}}^2} + \cdots \right)_{kl} \tag{B.5}$$

Using that $\hat{m}_{\mathrm{f}}$ is a diagonal matrix we find to lowest order in $\mathcal{A}$,

$$M_{\lambda^A} = 2g_A^2 \,\mathrm{tr}(T^A T^B) \mathrm{Tr}(\mathcal{A} I^{(1)}(\hat{m}_{\mathrm{f}})) \tag{B.6}$$

where

$$I_{ij}^{(1)} = \mathrm{diag}(I(\hat{m}_{ii})) \tag{B.7}$$

and

$$I^{(1)}(m) = \int \frac{d^4k}{(2\pi)^4} \frac{m}{(k^2 - m^2)^3} = \frac{1}{32\pi^2} \frac{1}{m}. \tag{B.8}$$

Using the explicit form of $I^{(1)}$ we have the leading order contribution to the gaugino masses:

$$M_{\lambda^A} = \frac{g_A^2}{16\pi^2} \,\mathrm{tr}(T^A T^B) \mathrm{Tr}(\mathcal{A} \hat{m}_{\mathrm{f}}^{-1}) = \frac{g_A^2}{16\pi^2} \,\mathrm{tr}(T^A T^B) \mathrm{Tr}(\mathcal{F} m_{\mathrm{f}}^{-1}) \,. \tag{B.9}$$

This reproduces Eq. (3.48).

Appendix C

Gauge Messengers in Direct Gauge Mediation

C.1 Minimization of the potential

The model is:

$$W = -\mu^2\Phi + \lambda\Phi Tr(Y_0Y_0) + mTr(Y_2Y_0) + \overline{\lambda}Tr(Y_2Y_0Y_0) \tag{C.1}$$

We shall describe the possible vevs that the Y fields can have using the generators of $SU(5)$. The generators of the Cartan subalgebra can be written as:

$$\begin{aligned}
T^{21} &= diag(1/2, -1/2, 0, 0, 0) \\
T^{22} &= diag(1/(2\sqrt{3}), 1/(2\sqrt{3}), -1/(\sqrt{3}), 0, 0) \\
T^{23} &= diag(1/(2\sqrt{6}), 1/(2\sqrt{6}), 1/(2\sqrt{6}), -3/(2\sqrt{6}), 0) \\
T^{24} &= diag(1/(2\sqrt{10}), 1/(2\sqrt{10})1/(2\sqrt{10}), 1/(2\sqrt{10}), -2/\sqrt{10})
\end{aligned} \tag{C.2}$$

The other generators can simply be written with the help of the $SU(2)$ generators.

Then the Y-fields can be written as

$$Y_j = \sqrt{2}y_j^kT^k \tag{C.3}$$

Where we take the $\sqrt{2}$ factor to so as to canonically normalize fields:

$$K \supset Tr(Y_j^\dagger Y_j) = 2(y_j^k)^\dagger y_j^l Tr(T^kT^l) = (y_j^k)^\dagger y_j^l \tag{C.4}$$

We can now use the gauge degrees of freedom to align the Y_0 vevs along the directions spanned by the Cartan subalgebra (i.e. along the diagonal). We shall use letters from the middle of the alphabet (usually k) to mean that the field corresponds to a direction of the Cartan subalgebra of $SU(5)$, and a letter from the end of the alphabet to mean that the field is not along a direction spanned by the Cartan subalgebra (usually r).

We now analyze the F-term equations, we shall start by looking at the F-terms that are not along the Cartan subalgebra directions. By virtue of the gauge choice, the F-term equations for the y_2^r directions vanish (not for the Cartan subalgebra directions), and the F-term equations for the y_0^r fields also take a very simple form:

$$W_{y_2^r} = 0 \tag{C.5}$$

$$W_{y_0^r} = f_r(\{y_0^k\})y_2^r \tag{C.6}$$

Where $f_1(y_0^{21}, y_0^{22}, y_0^{23}, y_0^{24}) = m + \frac{1}{15}\overline{\lambda}(5\sqrt{6}y_0^{22} + 5\sqrt{3}y_0^{23} + 3\sqrt{5}y_0^{24})$, and different f's have different expressions. So, the solution to these equations is given by $y_2^r = 0$. The reason why we can do this for these fields is that they do not appear in any other F-term equations, so V_F will have a quadratic term in these y_2^r with a positive semi-definite coefficient given by $|f_r|^2$.

So, by choosing the Y_0 to be diagonal,one gets that due to the F-term equations Y_2 is also diagonal.

We will now subdivide the problem of minimizing $V_F = \sum W_i W^i$ into three cases: The symmetry breaking pattern is $SU(5) \to SU(4) \times U(1)^2$, $SU(5) \to SU(3) \times SU(2) \times U(1)$ and generically when:

$$\begin{aligned} \lambda \ll& \;\; 1 \\ \frac{\lambda}{\overline{\lambda}^2} <& \;\; 1 \end{aligned} \tag{C.7}$$

We start by analyzing the case: $SU(5) \to SU(4) \times U(1)^2$. This particular case is equivalent to choosing the vevs to lie along the direction spanned by T^{24}, so there are only three complex variables in the problem: y_0^{24} and y_2^{24} and Φ. We use R-symmetry to choose Φ to be real. so we can write $y_0^{24} = y_{0r} + i\ y_{0i}$, $y_2^{24} = y_{2r} + i\ y_{2i}$ and Φ is a real variable now.

Then the V_F potential is given by:

$$\begin{aligned} V_F = \; & \mu^4 + m^2(y_{0r}^2 + y_{0i}^2 + y_{2r}^2 + y_{2i}^2) - \tfrac{3\overline{\lambda}}{\sqrt{5}} m \; y_{0r}(y_{0r}^2 + y_{0i}^2 + 2(y_{2r}^2 + y_{2i}^2)) + \\ & + \overline{\lambda}^2 \tfrac{9}{20}(y_{0r}^2 + y_{0i}^2)(y_{0r}^2 + y_{0i}^2 + 4(y_{2r}^2 + y_{2i}^2)) + \\ & + \lambda(4m\Phi(y_{2r} \; y_{0r} + y1i \; y_{2i}) - 2\mu^2(y_{0r}^2 - y_{0i}^2) - \tfrac{12}{\sqrt{5}}\overline{\lambda}\Phi y_{2r}(y_{0r}^2 + y_{2i}^2)) \\ & + \lambda^2(y_{0r}^2 + y_{0i}^2 + 4\Phi^2)(y_{0r}^2 + y_{0i}^2) \end{aligned} \tag{C.8}$$

One then has to find the extremes of the potential by solving the system of equations corresponding to setting the gradient of the potential to 0 and check wether the solutions one find are local minima or maxima. One gets the following set of solutions:

$$solution_1 = \{y_{0r} = 0, y_{0i} = 0, y_{2r} = 0, y_{2i} = 0\} \tag{C.9}$$

$$\begin{aligned} solution_{2,3} = \{ & y_{0r} = \frac{\sqrt{5}(9m\overline{\lambda} \pm \sqrt{m^2(9\overline{\lambda}^2 - 160\lambda^2) + 16\lambda\mu^2(20\lambda^2 + 9\overline{\lambda})})}{40\lambda^2 + 18\overline{\lambda}^2}, y_{0i} = 0, y_{2i} = 0, \\ & \Phi = y_{2r}(\frac{-3m^2\overline{\lambda}m^2 + 24\lambda\overline{\lambda}\mu^2 \pm m\sqrt{m^2(9\overline{\lambda}^2 - 160\lambda^2) + 16\lambda\mu^2(20\lambda^2 + 9\overline{\lambda})}}{8\sqrt{5}\lambda(2\lambda\mu^2 - m^2)})\} \end{aligned} \tag{C.10}$$

$$\begin{aligned} solution_{4,5} = \{ & y_{0r} = \frac{3\sqrt{5}m\overline{\lambda}(m^2 + 2\lambda\mu^2}{80\lambda^3\mu^2 + 9\overline{\lambda}^2(m^2 + 4\lambda\mu^2)}, \\ & y_{0i} = \pm\sqrt{5}\sqrt{-\frac{(m^2 + 2\lambda\mu^2)(9\overline{\lambda}^2 m^4 + 90\lambda\overline{\lambda}m^2\mu^2 + 32\lambda^2(20\lambda^2 + 9\overline{\lambda})\mu^4)}{(80\lambda^3\mu^2 + 9\overline{\lambda}^2(m^2 + 4\lambda\mu^2))^2}}, \\ & y_{2i} = \frac{5my_{0i}}{5my_{0r} - 3\sqrt{5\overline{\lambda}}(y_{0i}^2 + y_{0r}^2)} y_{2r}, \\ & \Phi = -\frac{5m^2 - 6\sqrt{5\overline{\lambda}}my_{0r} + 9\overline{\lambda}^2(y_{0r}^2 + y_{0i}^2)}{2\lambda(5my_{0r} - 3\sqrt{5\overline{\lambda}}(y_{0r}^2 + y_{0i}^2))} y_{2r}\} \end{aligned} \tag{C.11}$$

Where we note that solutions 4 and 5 do not exist for small λ (and if $\lambda < 0$ these solutions are even more complicated).

The value of V_F for solutions 1,2 and 3 is:

$$V_F^{(1)} = \mu^4 \tag{C.12}$$

$$V_F^{(2,3)} = \frac{1}{8(20\lambda^2+\overline{\lambda}^2)^3}(5m^4(81\overline{\lambda}^4+3600\lambda^2\overline{\lambda}^2-3200\lambda^4)+200m^2\lambda(320\lambda^4-36\lambda^2\overline{\lambda}^2-81\overline{\lambda}^4)\mu^2+$$
$$+72(20\lambda^2\overline{\lambda}+9\overline{\lambda}^3)^2\mu^4 \pm 15m\overline{\lambda}((9\overline{\lambda}^2-160\lambda^2)m^2+16\lambda(20\lambda^2+9\overline{\lambda}^2)\mu^2)^{3/2}) \tag{C.13}$$

Where we note that solutions 2 and 3 are not equivalent and have different values for the potential at the minimum. One can now expand these solutions to linear order in λ and get the results mentioned in section 1.

The analysis of the case $SU(5) \to SU(3) \times SU(2) \times U(1)$ is very similar. We will just outline the main differences. The vevs are now chosen to be along a combination of the directions given by T^{23} and T^{24}: $y_0^{23} = \sqrt{5/3}y_0^{24}$, $y_2^{23} = \sqrt{5/3}y_2^{24}$. Again there are only 3 complex valued variables, and we can use R-symmetry to make one of them be real: $y_0^{24} = y_{0r} + i\ y_{0i}$, $y_2^{24} = y_{2r} + i\ y_{2i}$ and Φ is real.

The extremes of the potential are:

$$solution_1 = \{y_{0r} = 0, y_{0i} = 0, y_{2r} = 0, y_{2i} = 0\} \tag{C.14}$$

$$solution_{2,3} = \{y_{0r} = \frac{3\sqrt{5}}{8(30\lambda^2+\overline{\lambda}^2)}(3m\overline{\lambda} \pm \sqrt{m^2(\overline{\lambda}^2-240\lambda^2)+16\lambda\mu^2(\overline{\lambda}+30\lambda^2)}), y_{0i} = 0,$$
$$y_{2i} = 0, \Phi = -\frac{45m^2-24\sqrt{5}\lambda my1r+16\overline{\lambda}(y_{0r}^2+y_{0i}^2)}{6\lambda(15my_{0r}-4\sqrt{5}\lambda(y_{0r}^2+y_{0i}^2)}y_{2r}\} \tag{C.15}$$

$$solution_{4,5} = \{y_{0r} = \frac{3\sqrt{5}m\overline{\lambda}(m^2+2\lambda\mu^2)}{4(\overline{\lambda}^2m^2+4\lambda\mu^2(30\lambda^2+\overline{\lambda}^2))},$$
$$y_{0i} = \pm\frac{3\sqrt{5}}{4}\sqrt{-\frac{(m^2+2\lambda\mu^2)(\overline{\lambda}^4m^4+10\lambda\overline{\lambda}^2m^2\mu^2+32\lambda^2\mu^4(30\lambda^2+\overline{\lambda}^2))}{(m^2\overline{\lambda}^2+4\lambda\mu^2(30\lambda^2+\overline{\lambda}^2))^2}},$$
$$y_{2i} = \frac{15my_{0r}}{15my_{0r}-4\sqrt{5}\lambda(y_{0r}^2+y_{0i}^2)}y_{2r},$$
$$\Phi = -\frac{45m^2-24\sqrt{5}\lambda my1r+16\overline{\lambda}(y_{0r}^2+y_{0i}^2)}{6\lambda(15my_{0r}-4\sqrt{5}\lambda(y_{0r}^2+y_{0i}^2)}y_{2r}\} \tag{C.16}$$

As in the previous case these two last solutions only exist when λ is not small. The potential at the other three solutions is given by:

$$V_F^{(1)} = \mu^4 \tag{C.17}$$

$$V_F^{(2,3)} = \frac{1}{16(20\lambda^2+\overline{\lambda}^2)^3}(15m^4(\overline{\lambda}^4+600\lambda^2\overline{\lambda}^2-7200\lambda^4)+600m^2\lambda(720\lambda^4-6\lambda^2\overline{\lambda}^2-\overline{\lambda}^4)\mu^2+ \\ 16(30\lambda^2\overline{\lambda}+\overline{\lambda}^3)^2\mu^4 \pm 15m\overline{\lambda}((\overline{\lambda}^2-240\lambda^2)m^2+16\lambda(30\lambda^2+\overline{\lambda}^2)\mu^2)^{3/2}) \tag{C.18}$$

And if we expand these values we get the results quoted in section 1.

We now turn to the general case where no preferred symmetry breaking pattern is chosen. The first thing is to rescale all the Y-fields by $\frac{m}{\overline{\lambda}}$, as this simplifies the potential (if $\lambda = 0$ this makes the function to minimize independent of any parameter in the model).

Then we not that the fields y_2^k, due to their R-charge, can only appear in the $F_{y_0^k}$ terms. These are n-equations in n-variables (the y_2^k's) and can be solved. This obviously does not hurt the global minimization of the potential. The solutions are quite long and not particularly deep, so we shall not present them here. The bottom line is that one has only to consider the F-terms of the fields with R-charge equal to 2 in the minimization of the potential.

The potential can then be written as:

$$V_F = \frac{m^4}{\overline{\lambda}^2}v(y_0^{21},y_0^{22},y_0^{23},y_0^{24}) - 2\lambda\frac{\mu^2m^2}{\overline{\lambda}^2}Re((y_0^{21})^2+(y_0^{22})^2+(y_0^{23})^2+(y_0^{24})^2)+\mu^4+O((\frac{\lambda}{\overline{\lambda}^2})^2) \tag{C.19}$$

Where $v(x,y,z,w)$ is a complicated positive semi-definite function whose minimum is 0. The reason this must be so is that when $\lambda = 0$ the messengers decouple from the SUSY breaking sector, so their F-terms vanish in the vacuum. We will now use perturbation theory to minimize this potential where we will take $\frac{\lambda}{\overline{\lambda}^2} < 1$ as the small parameter. The minima of the function v can be found exactly since this amount to solving $F_{y_2^k} = 0$ when $\lambda = 0$, i.e.

$$y_0^{21}(15+5\sqrt{6}y_0^{22}+5\sqrt{3}y_0^{23}+3\sqrt{5}y_0^{24}) = 0 \tag{C.20}$$

$$5\sqrt{6}(y_0^{21})^2+y_0^{22}(30-5\sqrt{6}y_0^{22}+10\sqrt{3}y_0^{23}+6\sqrt{5}y_0^{24}) = 0 \tag{C.21}$$

$$5\sqrt{3}(y_0^{21})^2+5\sqrt{3}(y_0^{22})^2+2y_0^{23}(15-5\sqrt{3}y_0^{23}+3\sqrt{5}y_0^{24}) = 0 \tag{C.22}$$

$$\sqrt{5}(y_0^{21})^2+\sqrt{5}(y_0^{22})^2++\sqrt{5}(y_0^{23})^2+y_0^{24}(10-3\sqrt{5}y_0^{24}) = 0 \tag{C.23}$$

This system of equations has $2^4 = 16$ solutions out of which only three are independent:

$$Y_0 = diag(\{0, 0, 0, 0, 0\}) \tag{C.24}$$

$$Y_0 = \frac{m}{\bar{\lambda}} diag(\{2, 2, 2, -3, -3\}) \tag{C.25}$$

$$Y_0 = \frac{m}{3\bar{\lambda}} diag(\{1, 1, 1, 1, -4\}) \tag{C.26}$$

The rest being permutations of these solutions(there are $1 + \binom{5}{2} + 5 = 16$ of these).

One can then use perturbation theory and expand around the solutions we found to linear order in $\frac{\lambda}{\bar{\lambda}^2}$ to get approximate solutions to the potential. Since these approximations will respect one of the symmetry breaking patterns we have already studied we shall not repeat this operation here.

C.2 The messenger mass matrices

Let us now assume that $\lambda > 0$, and focus on the case when the GUT group is broken down to the MSSM gauge group. Due to the Higgsing of the GUT group the adjoints of SU(5) decompose under the unbroken $SU(3) \times SU(2) \times U(1)$ as $(8, 1) \times (1, 3) \times (3, 2) \times (\overline{3}, 2) \times (1, 1)$, i.e. one adjoint of $SU(3)$, one adjoint of $SU(2)$, a vector-like pair of bifundamentals and a singlet. The field content of the model then becomes:

GUT	$SU(3) \times SU(2) \times U(1)$	$Representation$
V	$\begin{pmatrix} V_{SU(3)} & \psi \\ \tilde{\psi} & V_{SU(2)} \end{pmatrix} + \rho 1$	$\begin{pmatrix} (8,1) & (3,2) \\ (\overline{3},2) & (0,3) \end{pmatrix} + singlet$
Y_2	$\begin{pmatrix} Y2_{adj3} & \tau \\ \tilde{\tau} & Y2_{adj2} \end{pmatrix} + \rho 2$	$\begin{pmatrix} (8,1) & (3,2) \\ (\overline{3},2) & (0,3) \end{pmatrix} + singlet$
Y_0	$\begin{pmatrix} Y0_{adj3} & \chi \\ \tilde{\chi} & Y0_{adj2} \end{pmatrix} + \rho 3$	$\begin{pmatrix} (8,1) & (3,2) \\ (\overline{3},2) & (0,3) \end{pmatrix} + singlet$
Φ	Φ	$singlet$

Let us now get the fermionic and bosonic masses for the particles in the model. We shall focus in the region of parameter space where $\lambda < 1, \overline{\lambda}y > m$.

If we focus first in the fermionic mass matrix for the adjoint fields of the unbroken $SU(3)$, (since we saw that all vevs are real values, we shall drop complex conjugation symbols to simplify the formulas) we get that the fermionic mass matrix for these fields is given by:

$$((Y2^a_{adj3})^\dagger, (Y0^a_{adj3})^\dagger)(M_f^{2,(adj)}) \begin{pmatrix} Y2^a_{adj3} \\ Y0^a_{adj3} \end{pmatrix} \tag{C.27}$$

And $Y2_{adj3} = \sum Y2^a_{adj3}\lambda^a$, where λ^a is a basis of the $SU(3)$ algebra. Where $M_f^{2,(adj)}$ is, at leading order in λ given by:

$$M_f^{2,(adj)} = \begin{pmatrix} 25m^2 + \frac{20}{9}\overline{\lambda}^2 y2r^3 + \frac{16}{9}\lambda\mu^2(\frac{\overline{\lambda}^2 y2r^2}{m^2} + 45) & \frac{10\sqrt{5}}{3}\overline{\lambda}y2rm + \frac{20\sqrt{5}}{3}\lambda\mu^2\frac{\overline{\lambda}y2r}{m} \\ \frac{10\sqrt{5}}{3}\overline{\lambda}y2rm + \frac{20\sqrt{5}}{3}\lambda\mu^2\frac{\overline{\lambda}y2r}{m} & 25m^2 + 80\lambda\mu^2 \end{pmatrix} \tag{C.28}$$

To linear order in λ and second order in $\frac{1}{\overline{\lambda}y}$ we get (assuming $\overline{\lambda} > 0$):

$$m_H^2 = 50m^2 + 160\lambda\mu^2 + \frac{4}{9}(5m^2 + 4\lambda\mu^2)\frac{\overline{\lambda}^2 y^2}{m^2} - \frac{225m^2}{4}\frac{5m^2 + 28\lambda\mu^2}{\overline{\lambda}^2 y^2} \tag{C.29}$$

$$m_l^2 = \frac{225}{4}m^2\frac{5m^2 + 28\lambda\mu^2}{\overline{\lambda}^2 y^2} \tag{C.30}$$

The scalar mass matrix can be written as:

$$((Y2^a_{adj3})^\dagger,\ (Y0^a_{adj3})^\dagger,\ (Y2^a_{adj3})^T,\ (Y0^a_{adj3})^T)(M_b^{2,(adj)}) \begin{pmatrix} Y2^a_{adj3} \\ Y0^a_{adj3} \\ (Y2^a_{adj3})^* \\ (Y0^a_{adj3})^* \end{pmatrix} \tag{C.31}$$

Or replacing the vevs:

$$M_b^{2,(adj)} = \begin{pmatrix} 25m^2+80\lambda\mu^2+\frac{4}{9}\frac{\overline{\lambda}^2 y^2(5m^2+4\lambda\mu^2)}{m^2} & \frac{10\sqrt{5}}{3}\frac{\overline{\lambda}y(m^2+2\lambda\mu^2)}{m} & -10\lambda\mu^2 & 0 \\ \frac{10\sqrt{5}}{3}\frac{\overline{\lambda}y(m^2+2\lambda\mu^2)}{m} & 25m^2+80\lambda\mu^2 & 0 & 0 \\ -10\lambda\mu^2 & 0 & 25m^2+80\lambda\mu^2+\frac{4}{9}\frac{\overline{\lambda}^2 y^2(5m^2+4\lambda\mu^2)}{m^2} & \frac{10\sqrt{5}}{3}\frac{\overline{\lambda}y(m^2+2\lambda\mu^2)}{m} \\ 0 & 0 & \frac{10\sqrt{5}}{3}\frac{\overline{\lambda}y(m^2+2\lambda\mu^2)}{m} & 25m^2+80\lambda\mu^2 \end{pmatrix} \tag{C.32}$$

And the eigenvalues are given by:

$$m^2_{H,\pm} = 50m^2+160\lambda\mu^2+\frac{4}{9}(5m^2+4\lambda\mu^2)\frac{\overline{\lambda}^2 y^2}{m^2} - \frac{225m^2}{4}\frac{5m^2+28\lambda\mu^2}{\overline{\lambda}^2 y^2} \pm \frac{5\lambda\mu^2(4\overline{\lambda}^2 y^2-45m^2}{2y^2\overline{\lambda}^2}) \tag{C.33}$$

$$m^2_{l,\pm} = \frac{225m^2}{4}\frac{5m^2+28\lambda\mu^2}{\overline{\lambda}^2 y^2} \pm \frac{225}{2}\frac{\lambda m^2\mu^2}{\overline{\lambda}^2 y^2} \tag{C.34}$$

For the adjoints of $SU(2)$ the mass matrix is very similar, the fermion masses are:

$$m^2_H = 50m^2+240\lambda\mu^2+\frac{4}{9}(5m^2-4\lambda\mu^2)\frac{\overline{\lambda}^2 y^2}{m^2} - \frac{225m^2}{4}\frac{5m^2+52\lambda\mu^2}{\overline{\lambda}^2 y^2} \tag{C.35}$$

$$m^2_l = \frac{225m^2}{4}\frac{5m^2+52\lambda\mu^2}{\overline{\lambda}^2 y^2} \tag{C.36}$$

And the scalar masses are:

$$m^2_H = 50m^2+240\lambda\mu^2+\frac{4}{9}(5m^2-4\lambda\mu^2)\frac{\overline{\lambda}^2 y^2}{m^2} - \frac{225m^2}{4}\frac{5m^2+52\lambda\mu^2}{\overline{\lambda}^2 y^2} \pm \frac{5\lambda\mu^2(4\overline{\lambda}^2 y^2-45m^2)}{\overline{\lambda}^2 y^2} \tag{C.37}$$

$$m^2_l = \frac{225m^2}{4}\frac{5m^2+52\lambda\mu^2}{\overline{\lambda}^2 y^2} \pm \frac{225}{2}\frac{\lambda\mu^2 m^2}{\overline{\lambda}^2 y^2} \tag{C.38}$$

So that for large values of the pseudomodulos vev one of the fields gets heavy while the other gets light. This is a simple consequence of R-symmetry together with the fact that these adjoints do not enter in the Higgs mechanism (i.e. they only get masses through the superpotential). Since the superpotential is R-symmetric, one can generically show that the $det(M)$ does not depend of the vev of the scalar partner of the goldstino y (i.e our flat direction). Since $Tr(M)$ does depend on y, it has to be that for large values of y one of the eigenvalues has to go with $m^{1-r}y^r$ while the other goes as $(m^{1+r}y^{-r}$, for some value of r. This gives us a sort of see-saw mechanism where as y increases one of the field becomes lighter and the other becomes heavier.

The complete fermionic mass matrices (we take all vevs to be real):

Adjoint of $SU(3)$

$$(Y2^a_{adj3}, Y0^a_{adj3}) M_f^{(adj)} \begin{pmatrix} Y2^a_{adj3} \\ Y0^a_{adj3} \end{pmatrix} \tag{C.39}$$

Where:

$$M_f^{(adj)} = \begin{pmatrix} \frac{1}{45}(45m^2 + 48\sqrt{5}m\overline{\lambda}y_0^{24} + 180\lambda^2\Phi^2 + 96\sqrt{5}\lambda\overline{\lambda}\Phi y_2^{24} + 64\overline{\lambda}^2((y_0^{(24)})^2 + (y_2^{(24)})^2) & \frac{2}{225}(15m + 8\sqrt{5}\overline{\lambda}y_0^{(24)})(15\lambda\Phi + 4\sqrt{5}\overline{\lambda}y_2^{(24)}) \\ \frac{2}{225}(15m + 8\sqrt{5}\overline{\lambda}y_0^{(24)})(15\lambda\Phi + 4\sqrt{5}\overline{\lambda}y_2^{(24)}) & \frac{1}{225}(15m + 8\sqrt{5}\overline{\lambda}y_0^{(24)})^2 \end{pmatrix} \tag{C.40}$$

(The adjoints $V_{SU(3)}$ of the unbroken $SU(3)$ are obviously massless)

The eigenvalues can be computed and are:

$$\begin{aligned} mass_{1,2} = \; & \tfrac{1}{45}((8\overline{\lambda}y_0^{(24)} + 3\sqrt{5}m)^2 + (4\sqrt{2\lambda}y_2^{(24)} + 3\sqrt{10}\lambda\Phi)^2 \\ & \pm 2Abs(4\overline{\lambda}y_2^{(24)} + 3\sqrt{5}\lambda\Phi)\sqrt{(8\overline{\lambda}y_0^{(24)} + 3\sqrt{5}m)^2 + (4\overline{\lambda}y_0[24] + 3\sqrt{5}\lambda\Phi)^2} \end{aligned} \tag{C.41}$$

Adjoint of $SU(2)$

$$(Y2^a_{adj3}, Y0^a_{adj3}) M_f^{(adj2)} \begin{pmatrix} Y2^a_{adj3} \\ Y0^a_{adj3} \end{pmatrix} \tag{C.42}$$

Where:

$$M_f^{(adj2)} = \begin{pmatrix} m^2 - \frac{8\overline{\lambda}y_0^{(24)}m}{\sqrt{5}} + 4\lambda^2\Phi^2 + \frac{16}{5}\overline{\lambda}(\overline{\lambda}((y_0^{(24)})^2 + (y_2^{(24)})^2) - \sqrt{5}\lambda\Phi y_2^{(24)}) & -\frac{2}{225}(5m - 4\sqrt{5}\overline{\lambda}y_0^{(24)})(-5\lambda\Phi + 2\sqrt{5}\overline{\lambda}y_2^{(24)}) \\ -\frac{2}{225}(5m - 4\sqrt{5}\overline{\lambda}y_0^{(24)})(-5\lambda\Phi + 2\sqrt{5}\overline{\lambda}y_2^{(24)}) & \frac{1}{25}(5m - 4\sqrt{5}\overline{\lambda}y_0^{(24)})^2 \end{pmatrix} \tag{C.43}$$

(The adjoints $V_{SU(2)}$ of the unbroken $SU(2)$ are obviously massless)

The eigenvalues can be computed and are:

$$\begin{aligned} mass_{1,2} = \; & \tfrac{1}{5}((4\overline{\lambda}y_0^{(24)} - \sqrt{5}m)^2 + (2\sqrt{2\lambda}y_2^{(24)} - \sqrt{10}\lambda\Phi)^2 \\ & \pm Abs(2\sqrt{(2)}\overline{\lambda}y_2^{(24)} - \sqrt{10}\lambda\Phi)\sqrt{2(4\overline{\lambda}y_0^{(24)} - \sqrt{5}m)^2 + (2\sqrt{(2)}\overline{\lambda} - \sqrt{10}\lambda\Phi)^2)} \end{aligned} \tag{C.44}$$

if y_2^{24} is sufficiently large, the splittings in the scalar mass matrices are small, so that F/M can be computed as the mass splitting over the mass (i.e. this is exact up to $(F/M)^2$ corrections). By doing this one gets that:

$$\frac{F}{M} = \frac{3\sqrt{5}\lambda\mu^2}{\overline{\lambda}y_2^{24}} \tag{C.45}$$

for both gauge and non-gauge messengers (for gauge messengers one has to do this for the fermionic mass matrix, but the discussion goes through almost word by word).

C.3 Soft terms in models with two mass thresholds

Let us now compute the leading order contribution to squark masses assuming that there are two mass thresholds (and one SUSY breaking scale). One of the thresholds concerns normal (non-gauge messengers) and the other is with respect to normal gauge messengers. Here we will assume that gauge messengers are heavier than non-gauge messengers.

The computation of the running gauge function is done by solving the beta-function R.G. equations, one gets three solutions: one above the GUT scale, one between the GUT scale (where the gauge messengers are integrated out) and the non-gauge messengers, and one below the scale of the non-gauge messengers. These are respectively:

$$\alpha^{-1}(\mu) = \alpha^{-1}(\Lambda_{U.V.}) + \frac{b''}{4\pi}Log(\frac{\mu}{\Lambda_{U.V.}}) \tag{C.46}$$

$$\alpha^{-1}(\mu) = \alpha^{-1}(\Lambda_{U.V.}) + \frac{b''}{4\pi}Log(\frac{(X,X)}{\Lambda_{U.V.}}) + \frac{b'}{4\pi}Log(\frac{\mu^2}{(X,X)}) \tag{C.47}$$

$$\alpha^{-1}(\mu) = \alpha^{-1}(\Lambda_{U.V.}) + \frac{b''}{4\pi}Log(\frac{\mu}{\Lambda_{U.V.}}) + \frac{b'}{4\pi}Log(\frac{X_2^\dagger X_2}{(X,X)}) + \frac{b}{4\pi}Log(\frac{\mu^2}{X_2^\dagger X2}) \tag{C.48}$$

Where b'',b',b are the gauge function beta-function coefficients in the three different regimes. And the mass of the gauge messengers is $\sqrt{(X,X)}$, and the mass of the normal messengers is X_2.

We shall introduce now the following notation for the real gauge coupling and the squark wave-function renormalization functions and spurion-like fields: if they have a subscript s they are to be understood as the analytically continued functions into superspace, while if they do not have an s subscript, they are ordinary (scalar part) functions.

Upon continuation to superspace one gets:

$$\begin{gathered}(X_s, X_s) = (x, x) + \theta^2(x, F) + \overline{\theta}^2(F, x) + \overline{\theta}^2\theta^2(F, F) \\ X_{2s} = x_2 + \theta^2 F_2\end{gathered} \tag{C.49}$$

And x_2 is the mass scale of the non-gauge messengers and F_2 is the component of the goldstino that they couple to. For the gauge messengers X is the goldstino and F is the vev of the F-term associated.

We can now see how the expectation values of the auxiliary components of the real gauge coupling get vevs upon this analytic continuation into superspace. Replacing the definitions of (X_s, X_s) and X_{2s} "spurions" into the equations of the real gauge couplings we get:

$$\alpha_s^{-1}(M) = \alpha^{-1}(M) + \frac{b''}{4\pi} Log(1 + \theta^2 \frac{(x, F)}{(x, x)} + \overline{\theta}\frac{(F, x)}{x, x} + \theta^2\overline{\theta}^2 \frac{(F, F)}{(x, x)}) \tag{C.50}$$

$$\begin{aligned}\alpha_s^{-1}(x_2) =& \alpha^{-1}(x_2) + \frac{b'' - b'}{4\pi} Log(1 + \theta^2 \frac{(x, F)}{(x, x)} + \overline{\theta}\frac{(F, x)}{(x, x)} + \theta^2\overline{\theta}^2 \frac{(F, F)}{(x, x)}) \\ &+ \frac{b'}{4\pi} Log(1 + \theta^2 \frac{F_2}{x_2^\dagger} + \overline{\theta}\frac{F^\dagger}{X} + \theta^2\overline{\theta}^2 \frac{|F|^2}{|x_2|^2})\end{aligned} \tag{C.51}$$

$$\begin{aligned}\alpha_s^{-1}(\mu) =& \alpha^{-1}(\mu) + \frac{b'' - b'}{4\pi} Log(1 + \theta^2 \frac{(x, F)}{(x, x)} + \overline{\theta}\frac{(F, x)}{(x, x)} + \theta^2\overline{\theta}^2 \frac{(F, F)}{(x, x)}) \\ &+ \frac{b' - b}{4\pi} Log(1 + \theta^2 \frac{F_2}{x_2^\dagger} + \overline{\theta}\frac{F^\dagger}{X} + \theta^2\overline{\theta}^2 \frac{|F|^2}{|x_2|^2}) + \frac{b}{4\pi} Log(1 + \theta^2 \frac{F2}{x2^\dagger} + \overline{\theta}\frac{F^\dagger}{X} + \theta^2\overline{\theta}^2 \frac{|F|^2}{|x_2|^2})\end{aligned} \tag{C.52}$$

One can now solve the R.G. equation for the wave function-renormalization Z_Q, and get that:

$$-Log(Z(\mu)) = 2\frac{c''}{b''} Log(\frac{\alpha(m_v)}{\alpha(\Lambda_{U.V.})}) + 2\frac{c'}{b'} Log(\frac{\alpha(x_2)}{\alpha(m_v)}) + 2\frac{c'}{b} Log(\frac{\alpha(\mu)}{\alpha(x_2)}) \tag{C.53}$$

Replacing the expression for the running gauge couplings in this expression and recalling that in a general model

$$m_Q^2 = -Log(Z(\mu))|_{\theta^2\overline{\theta}^2} \tag{C.54}$$

We get that:

$$
\begin{aligned}
m_Q^2 = \;& \tfrac{\alpha(\mu)}{2\pi}\big(c' - c'' - 2c'\tfrac{b''}{b'} - \chi 1(c'\tfrac{b''}{b'} + c'') + \chi 2(b - b')(1 - \tfrac{b''}{b'})c'\big)\tfrac{(F,F)(x,x)-(F,x)(x,F)}{(x,x)^2} \\
& + \tfrac{\alpha(\mu)^2}{8\pi^2}\big((b - b')c'\tfrac{|F_2|^2}{|x_2|^2} + (b'c' + b''(c'' - 2c') + 2c'b'\tfrac{b'}{b})\tfrac{(x,F)(F,x)}{(x,x)^2} + 2\chi 1 b''(c'\tfrac{b''}{b'} + c'')\tfrac{(F,x)(x,F)}{(x,x)^2} \\
& - 2\chi 2\tfrac{(1-\frac{b'}{b})c'(-F2^\dagger(x,x)b' + x2(F,x)(b'-b''))(F2(x,x)b' + x2^\dagger(x,F)(b''-b'))}{|x2|^2(x,x)^2}\big) + O(\chi 1^2, \chi 2^2)
\end{aligned}
\tag{C.55}
$$

Where $\chi 1 = \frac{\alpha(M)}{\alpha(\mu)} - 1$,$\chi 2 = \frac{\alpha(x2)}{\alpha(\mu)} - 1$. So that we see that if we set $\chi 1$ and $\chi 2$ to zero, we get a sum of the naive expectations for the masses.

Appendix D

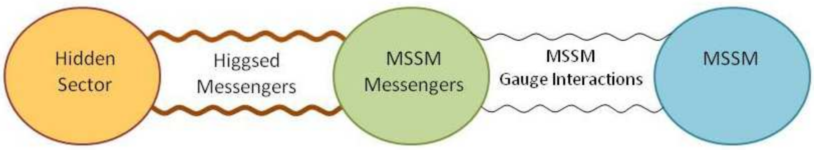
Semi-direct gauge mediation

In this appendix the calculation for the soft scalar and gaugino masses is done in detail. We will allways assume that both the hidden sector and the MSSM gauge couplings are small so that they serve as a good parameter in a perturbative expansion. So, when we present the leading order contribution to some quantity, it is shown the lowest order term in an expansion in both the hidden sector and MSSM gauge couplings.

To perform the computations we will symbolically solve the R.G. equations for the gauge couplings and wave-function renormalization of the quark fields (or in other words, express the differential R.G. equations in an integral form) and then express the soft terms with the help of the anomalous dimensions and beta-function coefficients.

D.1 Usefull quantities and notation:

Let us draw the picture for semi-direct gauge mediation again:



To simplify, we shall assume that the hidden sector can be described by a non-zero F-term and one messenger mass scale: the *Higgsed messengers* mass scale μ_{hm}.

Then we have the messenger sector which we will denominate as *normal messengers*, these have a much lower mass scale than the Higgsed messengers μ_{mess}.

Then we have the MSSM, with the quarks and leptons.

There are three well defined energy intervals: from the U.V. to the μ_{hm}, from μ_{hm} to μ_{mess} and finally below μ_{hm} to the I.R.. Because the field content of the theory in each of these energy intervals is different, the R.G. equations for the gauge couplings and the wave-function renormalization of the fields will be different in each of them.

We will denote the beta-function coefficients as β_g^{hm} *above the Higgsed messenger scale*, β_g^{am} if we're considering an energy range that is *between the Higgsed messenger and the normal messenger scales*, and β_g^{bm} is we are in an energy range that is *below the normal messenger scale*.

The same superscripts are used for the anomalous dimensions of a field "i": $\gamma_i^{hm}, \gamma_i^{am}$ and γ_i^{bm} above the Higgsed messenger scale, between this scale and the normal messenger scale, and below the normal messenger scale.

We will also use the notation $\delta\beta_g^{hm}$ to denote the change of the beta-function coefficient *at the Higgsed messenger mass scale* due to the integration of these fields, $\delta\beta_g^{mess}$ is the change of the beta-function coefficient *at the normal messenger mass scale*. Identical notation is used for the change in the anomalous dimensions of fields.

Group factors with a prime symbol are computed above the Higgsed messenger mass threshold and non-primed quantities are computed below it. For example, S' will be the total Dynkin index (sum over all fields) above the Higgsed messenger mass scale and S will be the same quantity below the Higgsed messenger scale. Generically $C_{r,i}1 = \sum_a t^a_{r,i} t^a_{r,i}$, $S_{r,i}\delta^a_b = tr(t^a_{r,i} t^b_{r,i})$ and $C_{g,i}\delta^a_b = tr(t^a_{adj,i} t^b_{adj,i})$ where $t^a_{r,i}$ is the a'th generator of the i'th gauge group for the representation r.

In order to compute gaugino masses, we need to understand the renormalization of the (normal) messengers mass and the renormalization of the real MSSM gauge couplings. To compute scalar masses we need to understand the wave-function renormalization of the quark and lepton superfields.

We will the same symbol the the Higgsed messenger mass before and after analytical continuation. Hopefully, it will be clear from the context which one we will be referring to:

$$\mu_{hm} \equiv \mu_{hm} + \mu_{hm}|_{\theta^2}\theta^2 + (\mu_{hm}|_{\theta^2})^* \overline{\theta}^2 + \Delta\theta^2\overline{\theta}^2 = (X,X) + (X,F)\theta^2 + (F,X)\overline{\theta}^2 + (F,F)\theta^2\overline{\theta}^2 \tag{D.1}$$

D.1.1 Renormalization of the (normal) messenger mass

A very useful quantity both for the calculation of squark and gaugino masses is the dependence of the renormalization of the messenger masses with the Higgsing scale of the hidden sector. It turns out that we will need both the first and the second derivatives of this quantity.

So, we need to evaluate:

$$\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})} = -\frac{dLog(Z_{mess}(\mu_{mess}))}{dLog(\mu_{hm})} \tag{D.2}$$

The messenger wave-function renormalization is given by:

$$Log(Z_{mess}(\mu)) = -\int_{\mu}^{\mu_{hm}} \frac{d\mu'}{\mu'}\gamma_m^{am}(\mu'^2) - \int_{\mu_{hm}}^{\mu_0} \frac{d\mu'}{\mu'}\gamma_m^{hm}(\mu'^2) + Log(Z_{mess}(\mu_0)) \tag{D.3}$$

Note the minus sign to give the correct R.G. equations.

So that:

$$\begin{aligned}\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})} =& \frac{1}{1+\gamma_m^{am}(\mu_{mess})}\Bigg(\gamma_m^{am}(\mu_{hm}) - \gamma_m^{hm}(\mu_{hm}) \\ &+ \int_{\mu_{mess}}^{\mu_{hm}} \frac{d\mu'}{\mu'}\frac{d}{dLog(\mu_{hm})}\gamma_m^{am}(\mu'^2)\Bigg)\end{aligned} \tag{D.4}$$

Because the anomalous dimension is a one loop quantity (meaning that in perturbation theory, the dominant contribution is given by one loop diagrams), $\frac{d}{dLog(\mu_{hm})}\gamma_m^{bhm}(\mu'^2)$ is two loops and represents the R.G. evolution of the "finite" term $\delta\gamma_m^{hm}(\mu_{hm})$ down to the messenger scale. At leading order, the equation reads:

$$\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})} \approx -\delta\gamma_m^{hm}(\mu_{hm}) \approx -\delta C_m \frac{\alpha_{hs}(\mu_{mess})}{\pi} \tag{D.5}$$

Since the effects of the renormalization of the mass of the Higgsed fields is very easy to take into account, from now on we will show only (for simplicity) the derivatives of the functions with respect to the renormalized mass. To find the final result one needs to insert $\frac{dLog(\mu_{hm})}{dLog(\mu_{hm,0})} = 1 - \gamma_{hm}$ in the appropriate places.

The second derivative of $Log(\mu_{mess})$ is given by:

$$\frac{d^2Log(\mu_{mess})}{d^2Log(\mu_{hm})} = \Bigg(-\Big(\frac{1}{(1+\gamma_m^{am}(\mu_{mess}))^2}\frac{d\gamma_m^{am}(\mu_{mess})}{dLog(\mu_{hm})}\Big(-\delta\gamma_m^{hm}(\mu_{hm}) + \int_{\mu_{hm}}^{\mu_0}\frac{d\mu'}{\mu'}\frac{d}{dLog(\mu_{hm})}\gamma_m^{am}(\mu'^2)\Big)$$
$$+ \frac{1}{(1+\gamma_m^{am}(\mu_{mess}))}\big(\frac{d\gamma_m^{am}(\mu_{hm})}{dLog(\mu_{hm})} - \frac{d\gamma_m^{hm}(\mu_{hm})}{dLog(\mu_{hm})} - \frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\frac{d\gamma_m^{hm}(\mu_{hm})}{dLog(\mu_{hm})} + \frac{d\gamma_m^{am}(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}}$$
$$+ \int_{\mu_{hm}}^{\mu_0}\frac{d\mu'}{\mu'}\frac{d^2}{d^2Log(\mu_{hm})}\gamma_m^{bhm}(\mu'^2))\Big)\Bigg) \tag{D.6}$$

And the leading order contribution is:

$$\frac{d^2Log(\mu_{mess})}{d^2Log(\mu_{hm})} \approx \big(\frac{d\gamma_m^{am}(\mu_{hm})}{dLog(\mu_{hm})} - \frac{d\gamma_m^{hm}(\mu_{hm})}{dLog(\mu_{hm})} + \frac{d\gamma_m^{am}(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}}\big) \tag{D.7}$$

In terms of the one loop beta function coefficients:

$$\frac{d^2Log(\mu_{mess})}{d^2Log(\mu_{hm})} \approx (C'_{m,hs}b'_{hs} + C_{m,hs}b^{hs} - 2C_{m,hs}b'^{hs})\frac{\alpha_{hs}^2(\mu_{mess})}{2\pi^2} \tag{D.8}$$

Where $b^{hs} = 3C^{hs} - S^{hs}$, and $b'^{hs} = 3C'^{hs} - S'^{hs}$, are the one loop coefficients of the hidden sector gauge coupling below and above the Higgsing scale (and recall $C_{m,hs}1 = \sum_a t^a_{m,hs}t^a_{m,hs}$, $S_{m,hs}\delta^a_b = tr(t^a_{m,hs}t^b_{m,hs})$ and $C_{hs} = C_{g,hs}\delta^a_b = tr(t^a_{adj,hs}t^b_{adj,hs})$).

More simply, the first derivative of the renormalized messenger mass is essentially the messenger A-terms and the second derivative the messenger soft masses.

D.1.2 Renormalization of the MSSM gauge coupling

The real gauge coupling at a scale μ below the messenger scale is given by:

$$g_{\text{MSSM}}(\mu) = -\int_{\mu}^{\mu_{mess}}\frac{d\mu'}{\mu'}\beta_g^{bm}(\mu) - \int_{\mu_{mess}}^{\mu_{hm}}\frac{d\mu'}{\mu'}\beta_g^{am}(\mu')$$
$$-\int_{\mu_{hm}}^{\mu_0}\frac{d\mu'}{\mu'}\beta_g^{hm}(\mu') + g_{\text{MSSM}}(\mu_0) \tag{D.9}$$

Where $\beta^{bm}, \beta^{am}, \beta^{hm}$ is the beta function coefficient below the messenger scale, above the messenger scale and above the hidden sector Higgs scale. For simplicity, we shall assume that all messengers transform under the same representation of the gauge group.

The derivative of the gauge coupling with respect to the Higgsed particles mass is:

$$\frac{dg_{\mathrm{MSSM}}(\mu)}{dLog(\mu_{hm})} = \frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\delta\beta_g^{mess}(\mu_{mess}) + \delta\beta_g^{hm}(\mu_{hm}) - \int_{\mu}^{\mu_{mess}} \frac{d\mu'}{\mu'}\frac{d\beta_g^{bm}(\mu)}{dLog(\mu_{hm})} - \int_{\mu_{mess}}^{\mu_{hm}} \frac{d\mu'}{\mu'}\frac{d\beta_g^{am}(\mu)}{dLog(\mu_{hm})} \tag{D.10}$$

The result for the derivative of the MSSM gauge coupling above the messenger scale can be easily derived as well.

We can look up the coefficient of these beta-functions up to two loops in [122]. If we assume that the three-loop β-function coefficients are the same as in the NSVZ β-function, we can write, neglecting terms of order $O(g^9)$:

$$\begin{aligned}\beta_{g_a}(\mu) =& \frac{1}{16\pi^2} g_a^3(\mu)(\sum_r S_{r,a} - 3C_{g,a}) + \frac{1}{(16\pi^2)^2} g_a^5(\mu)(-6C_{g,a}^2 + 2C_{g,a}\sum_r S_{r,a} + 4\sum_r S_{r,a}C_{r,a}) + \\ & \frac{4}{(16\pi^2)^2} g_a^3(\mu)\sum_{r,b,b\neq a} g_j^2(\mu) S_{r,a}C_{r,b} + \frac{12}{(16\pi^2)^3} C_{g,a}^3 g_a^7(\mu) + 4\frac{g_a(\mu)}{(16\pi^2)^3}\sum_{i,b}(S_{i,a}C_{i,b}(3C_{g,b} - \sum_j S_{j,b}) g_b^4(\mu)) \\ & - 8\frac{g_a(\mu)}{(16\pi^2)^3}\sum_{i,b,c}(S_{i,a}C_{i,b}C_{i,c} g_b^2(\mu) g_c^2(\mu)) + \frac{4}{(16\pi^2)^3} C_{g,a}^2 \sum_i S_{i,a} g_a^7 + \frac{8}{(16\pi^2)^3} C_{g,a}\sum_{i,b} S_{i,b}C_{i,b} g_b^2 g_a^5\end{aligned} \tag{D.11}$$

Where, like before, $C_{r,i}1 = \sum_a t^a_{r,i} t^a_{r,i}$, $S_{r,a}\delta^{i,j} = tr(t^i_{r,a} t^j_{r,a})$ and $C_{g,a}\delta^{i,j} = tr(t^i_{adj,a} t^j_{adj,a})$.

So that in semi-direct models, the renormalization of the MSSM gauge couplings depends on the hidden sector beta function through two loop contributions. I'm ignoring contributions from Yukawa couplings.

This means that:

$$\begin{aligned}\delta\beta^{mess}_{g_{\mathrm{MSSM}}}(\mu_{mess}) =& \frac{g^3_{\mathrm{MSSM}}(\mu_{mess})}{16\pi^2} N_m (1 + \frac{1}{(16\pi^2)} g^2_{\mathrm{MSSM}}(\mu_{mess})(2C_{g,\mathrm{MSSM}} + 4C_{m,\mathrm{MSSM}}) \\ & + \frac{4}{(16\pi^2)} g^2_{hs}(\mu_{mess}) C_{m,hs}) + O(g^7)\end{aligned} \tag{D.12}$$

Where $N_m = \sum_{i=mess} S_{i,\mathrm{MSSM}}$, and:

$$\begin{aligned}\delta\beta^{hm}_{g_{\mathrm{MSSM}}}(\mu_{hm}) =& \frac{4}{(16\pi^2)^2} g^3_{\mathrm{MSSM}}(\mu_{hm}) g^2_{hs}(\mu_{hm}) N_m \delta C_{m,hs} \\ & + \frac{8}{(16\pi^2)^3} g^5_{\mathrm{MSSM}}(\mu_{hm}) g^2_{hs}(\mu_{hm}) N_m (C_{g,\mathrm{MSSM}} - 2C_{m,\mathrm{MSSM}})\delta C_{m,hs} \\ & + \frac{4}{(16\pi^2)^3} g^3_{\mathrm{MSSM}}(\mu_{hm}) g^4_{hs}(\mu_{hm}) N_m \delta(C'_{m,hs}(3C'_{g,hs} - 2C'_{m,hs} + \sum_j S'_{j,hs})) + O(g^9)\end{aligned} \tag{D.13}$$

Where $\delta(C'_{i,hs}(3C'_{g,hs}-2C'_{i,hs}+\sum_j S'_{j,hs}) = C'_{i,hs}(3C'_{g,hs}-2C'_{i,hs}+\sum_j S'_{j,hs})-C_{i,hs}(3C_{g,hs}-2C_{i,hs}+\sum_j S_{j,hs})$.

For the hidden sector, the expression can be simplified:

$$\delta\beta^{hm}_{g_{hs}}(\mu_{hm}) = \frac{1}{(16\pi^2)^2}g^3_{hs}(\mu_{hm})(\sum_i(S'_i - S_i) - 3(C'_{g,hs} - C_{g,hs})) + O(g^5) \tag{D.14}$$

Expressions for the integrals can be computed approximately using perturbation theory. Neglecting terms of order $O(g^8)$:

$$\begin{aligned}\int_{\mu_{mess}}^{\mu_{hm}}\frac{d\mu'}{\mu'}\frac{d\beta^{am}_{g_{\rm MSSM}}(\mu')}{dLog(\mu_{hm})} &= \int_{\mu_{mess}}^{\mu_{hm}}\frac{d\mu'}{\mu'}(\frac{\partial\beta^{am}_{g_{\rm MSSM}}(\mu')}{\partial g_{\rm MSSM}(\mu')}\frac{dg_{\rm MSSM}(\mu')}{dLog(\mu_{hm})} + \frac{\partial\beta^{am}_{g_{\rm MSSM}}(\mu')}{\partial g_{hs}(\mu')}\frac{dg_{hs}(\mu')}{dLog(\mu_{hm})})\\ &\approx 3\delta\beta^{hm}_{g_{\rm MSSM}}(\mu_{hm})\frac{g^2_{\rm MSSM}(\mu_{hm})}{16\pi^2}(\sum_i S_{i,\rm MSSM} - C_{g,\rm MSSM})Log(\frac{\mu_{hm}}{\mu_{mess}})+\\ &\frac{8}{(16\pi^2)^2}\delta\beta^{hm}_{g_{hs}}(\mu_{hm})g^3_{\rm MSSM}(\mu_{hm})g_{hs}(\mu_{hm})\sum_i S_{i,\rm MSSM}C_{i,hs}Log(\frac{\mu_{hm}}{\mu_{mess}})\end{aligned} \tag{D.15}$$

Similarly, and keeping only terms to $O(g^5)$:

$$g_{\rm MSSM}(\mu_{mess})^3 = g_{\rm MSSM}(\mu_{hm})^3 - \frac{3}{(16\pi^2)}(\sum_i S_{i,\rm MSSM} - 3C_{g,\rm MSSM})g_{\rm MSSM}(\mu_{hm})^5 Log(\frac{\mu_{hm}}{\mu_{mess}}) \tag{D.16}$$

Replacing equations (D.4,D.13,D.15,D.16 and D.14) into equation (D.10), it all simplifies:

$$\frac{dg_{\rm MSSM}(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{mess}} \approx O(g^9) \tag{D.17}$$

This is essentially a different way to see how the gauge coupling is insensitive to the Higgsed messenger mass threshold, or in other words, it is nothing but the gaugino screening argument. Interestingly, there is no 3-loop contribution to gaugino masses, which is not *a priori* implicit by the gaugino screening argument.

D.1.3 Wavefunction renormalization of quarks/leptons:

At two loops, and ignoring Yukawa couplings, the anomalous dimension of a (general chiral) field "r" is given by:

$$\gamma_r = \sum_i C_{r,i}\frac{g_i^2}{4\pi^2} + \sum_i C_{r,i}(3S_{g,i} - 2C_{r,i} - \sum_s S_{s,i})\frac{g_i^4}{4(2\pi)^4} - 2\sum_{i,j\neq i} C_{r,i}C_{r,j}\frac{g_i^2 g_j^2}{4(2\pi)^4} \tag{D.18}$$

If the field is charged under different gauge groups. So, for the messenger fields, this is the expression to use, while the quarks are only charged under the MSSM gauge group.

The change of the anomalous dimension at the hidden sector Higgs threshold is then given by:

$$\begin{aligned}\delta\gamma_m^{hs}(\mu_{hm}) =& \delta C_{m,hs}\frac{g_{hs}^2}{4\pi^2} + C'_{m,hs}(3S'_{G,hs} - 2C'_{m,hs} - \sum_s S_{s,hs})\frac{g_{hs}^4}{4(2\pi)^4} - C_{m,hs}(3S_{G,hs} - 2C_{m,hs} - \sum_s S_{s,hs})\frac{g_{hs}^4}{4(2\pi)^4} \\ &- 2\delta C_{m,hs}C_{m,\mathrm{MSSM}}\frac{g_{hs}^2 g_{\mathrm{MSSM}}^2}{4(2\pi)^4}\end{aligned} \tag{D.19}$$

And is a one loop quantity, whereas the change of the anomalous dimension of the quark superfield at this mass scale is a three loop quantity:

$$\delta\gamma_Q^{hs}(\mu_{hm}) = O(g^6) \tag{D.20}$$

At the messenger scale, the change of the squark anomalous dimension is given by:

$$\delta\gamma_Q^{mess}(\mu_{mess}) = -C_{Q,\mathrm{MSSM}}N_{mess}\frac{g_{\mathrm{MSSM}}^4}{4(2\pi)^4} \tag{D.21}$$

We will also need:

$$\gamma_Q^a(\mu) \approx C_{Q,\mathrm{MSSM}}\frac{g_{\mathrm{MSSM}}(\mu)^2}{4\pi^2} \tag{D.22}$$

$$\frac{d\gamma_Q^a(\mu)}{dLog(\mu_{hm})} \approx C_{Q,\mathrm{MSSM}}\frac{g_{\mathrm{MSSM}}(\mu)}{2\pi^2}\frac{dg_{\mathrm{MSSM}}(\mu)}{dLog(\mu_{hm})} \approx C_{Q,\mathrm{MSSM}}\frac{g_{\mathrm{MSSM}}(\mu)}{2\pi^2}\delta\beta_{g_{\mathrm{MSSM}}}^{hm}(\mu_{hm}) \tag{D.23}$$

And it will is important to note that the MSSM gauge coupling here is above the normal messenger mass threshold.

D.2 Gaugino masses:

Gaugino masses are given by:

$$m_\lambda = g^2_{\mathrm{MSSM}}(\mu) R|_{\theta^2} = -\frac{1}{2g_{\mathrm{MSSM}}(\mu)} \frac{dg_{\mathrm{MSSM}}(\mu)}{dLog(\mu_{hm})} Log(\mu_{hm})|_{\theta^2} \tag{D.24}$$

Replacing the result of eq. D.17 this gives:

$$m_\lambda \sim O(g^8 F/M) + O(g^4 F^3/M^5) + O(g^4 Y^2) \tag{D.25}$$

So, if Yukawas are small, gaugino masses are only generated at two loops at NLO in the SUSY breaking parameters, a contribution that we cannot compute with this technique.

D.3 Squark masses:

Squark soft masses are given by:

$$m^2_Q = -Log(Z_Q(\mu))|_{\theta^2\bar{\theta}^2} \tag{D.26}$$

To get the wave-function renormalization we proceed as before: symbolically solve the coupled R.G. equations for the wave-function renormalization, gauge couplings,... (we will again ignore the effects of Yukawa couplings) and then perform the analytical continuation into superspace.

The wave-function renormalization is given by:

$$\begin{aligned} Log(Z_Q(\mu)) = &- \int_{\mu}^{\mu_{mess}} \frac{d\mu'}{\mu} \gamma_Q^{bm}(\mu^2) - \int_{\mu_{mess}}^{\mu_{hm}} \frac{d\mu'}{\mu} \gamma_Q^{am}(\mu^2) \\ &- \int_{\mu_{hm}}^{\mu_0} \frac{d\mu'}{\mu} \gamma_Q^{hm}(\mu^2) + Log(Z_Q(\mu_0)) \end{aligned} \tag{D.27}$$

Where, like before, $\gamma_Q^{hm}, \gamma_Q^{am}, \gamma_Q^{bm}$ are the anomalous dimensions of the quark superfield above the Higgs messenger, between this scale and the normal messenger scale, and below the normal messenger scale.

Since the fields are canonically normalized at scale μ_0, $Log(Z_Q(\mu_0)) = 0$.

To get the $\theta^2\overline{\theta}^2$ component we again use eq. (6.1). It tells us that:

$$f(\mu^2+\mu^2|_{\theta^2}\theta^2+c.c.+\Delta\theta^2\overline{\theta}^2)|_{\theta^2,\overline{\theta}^2}=(\frac{\partial f(\mu^2)}{\partial\mu^2}\Delta+\frac{\partial^2 f(\mu^2)}{\partial^2\mu^2}\left|\mu^2|_{\theta^2}\right|^2) \tag{D.28}$$

Applied to our particular case this is:

$$Log(Z_Q(\mu))|_{\theta^2,\overline{\theta}^2}=(\frac{\partial Log(Z_Q(\mu))}{\partial\mu^2}\Delta+\frac{\partial^2 Log(Z_Q(\mu))}{\partial^2\mu^2}\left|\mu^2|_{\theta^2}\right|^2) \tag{D.29}$$

So we need to compute the first and second derivatives of the wave-function renormalization. The first derivative is given by:

$$\begin{aligned}\frac{dLog(Z_Q)}{dLog(\mu_{hm})}=&\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\delta\gamma_Q^{mess}(\mu_{mess})+\delta\gamma_Q^{hs}(\mu_{hm}^2)-\\&\int_{\mu}^{\mu_{mess}}\frac{d\mu'}{\mu}\frac{d\gamma_Q^{bm}(\mu^2)}{dLog(\mu_{hm})}-\int_{\mu_{mess}}^{\mu_{hm}}\frac{d\mu'}{\mu}\frac{d\gamma_Q^{am}(\mu^2)}{dLog(\mu_{hm})}\end{aligned} \tag{D.30}$$

Where $\delta\gamma_Q^{mess}(\mu_{mess})$ is the change of the quark superfield anomalous dimension due to the integration of the messenger fields, and $\delta\gamma_Q^{hs}(\mu_{hm}^2)$ is the change in the anomalous dimension due to the integration of the hidden sector Higgsed fields.

We also need the second derivative of the wave-function renormalization:

$$\begin{aligned}\frac{d^2Log(Z_Q)}{d^2Log(\mu_{hm})}=&\frac{d^2Log(\mu_{mess})}{d^2Log(\mu_{hm})}\delta\gamma_Q^{mess}(\mu_{mess})+\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\frac{d\delta\gamma_Q^{mess}(\mu_{mess})}{dLog(\mu_{hm})}+\frac{d\delta\gamma_Q^{hs}(\mu_{hm}^2)}{dLog(\mu_{hm})}\\&+\frac{dLog(\mu_{mess})}{dLog(\mu_{hm})}\frac{d\delta\gamma_Q^{mess}(\mu)}{dLog(\mu_{hm})}|_{\mu=\mu_{mess}}-\frac{d\gamma_Q^{am}(\mu^2)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}}\\&-\int_{\mu}^{\mu_{mess}}\frac{d\mu'}{\mu}\frac{d^2\gamma_Q^{bm}(\mu^2)}{d^2Log(\mu_{hm})}-\int_{\mu_{mess}}^{\mu_{hm}}\frac{d\mu'}{\mu}\frac{d^2\gamma_Q^{am}(\mu^2)}{d^2Log(\mu_{hm})}\end{aligned} \tag{D.31}$$

We can see at what order the terms in this expression can be generated. The only one that can be generated at three loops is $-\frac{d\gamma_Q^{am}(\mu^2)}{dLog(\mu_{hm})}|_{\mu=\mu_{hm}}$, and all the others are generated at four loops.

This means that:

$$\frac{d^2Log(Z_Q)}{d^2Log(\mu_{hm})}\approx -C_Q\frac{32}{(16\pi^2)^3}g^4_{\rm MSSM}(\mu_{hm})g^2_{hs}(\mu_{hm})N_{mess}\delta C_{r,hs} \tag{D.32}$$

Now we just need to add all the terms:

$$Log(Z_Q(\mu))|_{\theta^2,\bar{\theta}^2} = (\frac{1}{2}\frac{\partial Log(Z_Q(\mu))}{\partial Log(\mu_{hm})}\frac{\Delta}{\mu_{hm}^2}) + \frac{1}{2\mu_{hm}^2}\frac{\partial}{\partial Log(\mu_{hm})}(\frac{1}{2\mu_{hm}^2}\frac{\partial Log(Z_Q(\mu))}{\partial Log(\mu_{hm})}\left|\mu^2|_{\theta^2}\right|^2) \tag{D.33}$$

Or simply:

$$Log(Z_Q(\mu))|_{\theta^2,\bar{\theta}^2} = (\frac{1}{2}\frac{\partial Log(Z_Q(\mu))}{\partial Log(\mu_{hm})}\frac{(F,F)(X,X)-(F,X)(X,F)}{(X,X)^2} + \frac{1}{4}\frac{\partial^2 Log(Z_Q(\mu))}{\partial^2 Log(\mu_{hm})}\frac{(X,F)(F,X)}{(X,X)^2} \tag{D.34}$$

We recognize this structure as the general structure of squark masses in models with gauge messengers. In this particular case, the term proportional to $\frac{(F,F)(X,X)-(F,X)(X,F)}{(X,X)^2}$ can be seen to be generated at three-loops, while the term proportional to $\frac{(X,F)(F,X)}{(X,X)^2}$ is a four loop quantity.

As we know, for large sgoldstino vevs, $\frac{(F,F)(X,X)-(F,X)(X,F)}{(X,X)^2} \sim X^{-4}$, while $\frac{(X,F)(F,X)}{(X,X)^2} \sim X^{-2}$.

In this limit:

$$Log(Z_Q(\mu))|_{\theta^2,\bar{\theta}^2} \approx \frac{1}{4}\frac{\partial^2 Log(Z_Q(\mu))}{\partial^2 Log(\mu_{hm})}\frac{(X,F)(F,X)}{(X,X)^2} \tag{D.35}$$

Where the second derivative is to be replaced by eq. D.31.

So, the dominant term in models with Higgsed messengers is given by:

$$m_Q^2 \approx C_Q\frac{8}{(16\pi^2)^3}g^4_{\rm MSSM}(\mu_{hm})g^2_{hs}(\mu_{hm})N_{mess}\delta C_{r,hs}\frac{(X,F)(F,X)}{(X,X)^2} \tag{D.36}$$

In other models, squark masses can only be generated at four loops. The four loop contribution can only be computed with the knowledge of the three-loop coefficients for the anomalous dimension of a chiral field (we shall not do this calculation here).

Colophon

This thesis was made in $\LaTeX\,2_\varepsilon$ using an addapted version of the "hepthesis" class [123].

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