PLATFORM VARIABLE IDENTIFICATION USING SENSITIVITY ANALYSIS FOR PRODUCT PLATFORM DESIGN

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PLATFORM VARIABLE IDENTIFICATION USING SENSITIVITY ANALYSIS FOR PRODUCT PLATFORM DESIGN

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I dedicate this thesis to my family for their unconditional love and support

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SUMMARY

The recent trend of mass customization has redefined the way companies do business. Each individual customer is now their own market, requiring products specific to their wants and needs at mass production prices. This need for ever-increasing variety is a significant challenge for industry that many times leads to ballooning manufacturing costs and lower product performance. One approach that has received widespread attention and implementation is to develop families of products from standardized product platforms. While, many methods have been developed to address different challenges within product platform design, they are not without their limitations/tradeoffs and therefore leave much room for development and improvement.

The Product Platform Constructal Theory Method (PPCTM), developed by Dr. Gabriel Hernandez, is a novel approach for developing product platforms that enable customizable products. Rooted in the tenants of hierarchic systems theory and constructal theory, the PPCTM solves for the product platform as a problem of optimization of access in a geometric space. The result is a hierarchical organization of the modes for managing variety and the specification of their commonality across the product platform. Overall, the PPCTM offers an extremely comprehensive product platform design method, with the ability to accommodate multi-platform design, multiple design specifications, non-uniform demand modeling, and multi-objective decision-making. One limitation of this method is that the selection of platform variables and the modes for managing product variety must be pre-specified or determined ad hoc by the designer. This thesis

seeks to address this limitation through the integration of a sensitivity-based analysis method to determine the effect of platform variable variation on the family performance.

The result of this work is a Sensitivity-based PPCTM that facilitates the selection of common platform variables, such that modes for managing variety can be ranked and applied to the space element hierarchy. The proposed method is illustrated with three examples: the design of a line of customizable pressure vessels, universal electric motors, and finger pumps.

CHAPTER ONE

BACKGROUND AND MOTIVATION

1.1 Introduction and Motivation: Designing for Mass Customization

In the early 18th century, manufacturing mostly consisted of a network of skilled craftsman servicing the needs of their local community. Patrons were met individually and would solicit a product or service tailored specifically to their need. All that changed as machines began to revolutionize manufacturing and the once small weavers cottages were replaced by large mills; the Industrial Revolution had begun. A second industrial revolution came in the early 20th century when Henry Ford introduced the moving assembly line, ushering in the age of mass production. Nearly a century later a third revolution has begun, marrying the customization of craft production with the efficiency of mass production; welcome to Mass Customization.

1.1.1 What is Mass Customization

In today's highly competitive global market, consumers are once again king, forcing companies to fulfill their individual wants and needs or lose out to a company that will. "Customers can no longer be lumped together in a huge homogenous market, but are individuals whose individual wants and needs can be ascertained and fulfilled," says Joseph Pine II (Pine, 1993). Numerous studies from the automotive industry as well as surveys of manufacturing firms confirm this notion (Alford, et al., 2000, MacDuffie, et al., 1996, Womack, et al., 1990), showing a significant increase in the number of product variants offered. This trend has redefined the way companies do business, requiring manufacturers to provide products with increased variety, in a shorter period of time and at a lower cost. Companies have recognized that mass production alone is no longer sufficient to meet changing customer demands, and manufacturing is shifting to the paradigm of Mass Customization.

Stan Davis coined the term mass customization in 1987 to describe the process of creating greater competitive advantage through mass delivery of customized products (Davis, 1987). Pine declared mass customization "the new frontier in business competition," stating the most successful companies must have the ability to produce and distribute individually customized goods and services at mass production efficiencies (Pine, 1993). Evidently, Pine was correct, as today customization is available in every area of industry.

More than simply meeting customer demands, mass customization also presents a distinct economic benefit for the manufacturer. As greater variety is introduced into the market, production volume per part decreases shrinking the economic benefit of mass production (Ericsson and Erixon, 1999).

Figure 1-1, which shows the relative cost per product associated with mass production and mass customization as a function of production volume. Also included is the price customers are willing to pay. As production volume decreases, mass production quickly losses the economic advantage over mass customization. Better still, at low to medium production volume, comparison of the difference between the cost of production and the price consumers are willing pay shows a much larger profit margin using mass customization.

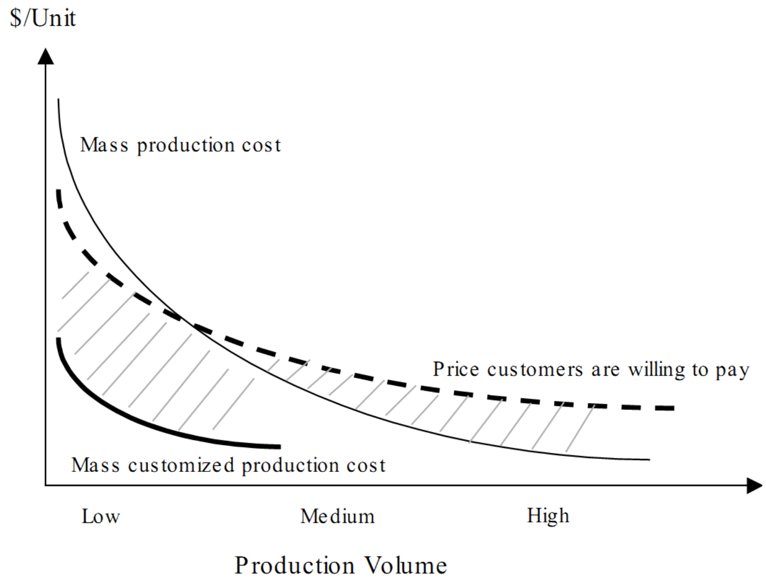

Figure 1-1: The Economic Implications of Mass Customization (Tseng and Jiao, 1998)

While the advantages of mass customization should now be obvious, the transition from mass production to mass customization is not without its challenges. Many of these challenges are rooted in the difference between mass production and mass customization shown in Table 1-1. Uncertain demand, small market niches, and shortened product development cycles all make it increasingly difficult to design products with variety.

Table 1-1: The Differences Between Mass Production and Mass Customization (Pine, 1993)

	Mass Production	*Mass Customization*
Focus	Efficiency through stability and control	Variety and customization through flexibility and quick responsiveness
Goal	Developing, producing, marketing, and delivering goods and services at prices low enough that nearly everyone can afford them	Developing, producing, marketing and delivering affordable goods and services with enough variety and customization that nearly everyone finds exactly what they want
Key Features	Stable Demand	Fragmented demand
	Large, homogeneous markets	Heterogeneous niches
	Low-cost, consistent quality, standardized goods, and services	Low-cost, high-quality, customized goods and services
	Long Product development cycles	Short product development cycles
	Long product life cycles	Short Product life cycles

Traditionally speaking, the easiest way to provide added variety is through the addition of more products. However, as Anderson identifies, adding inflexible products manufactured using inflexible techniques can lead to what he calls the cost of variety (Anderson, 1997). These costs include the cost of excessive parts, the cost of additional manufacturing processes and operations, the cost of insufficient product development, and the cost due to slow responsiveness to customer needs. The goal of mass customization, therefore, is to develop methods that minimize these costs, enabling affordable customization.

1.2 Product Platforms

One method, which has received significant attention in literature and widespread implementation in practice, is the development of product families. A *product family* is a stream of related products that share common features, functions and components, which can be adapted to meet a variety of market niches (Meyer and Lehnerd, 1997). This internal sharing within the family allows manufacturing enterprises to use standard equipment, processes and assembly lines, which help to reduce production cost and increase efficiency, addressing many of the challenges posed in the previous section.

While maximizing family commonality as a design strategy may seem simple enough, this task is far from trivial and is embodied in the selection of a product platform. Meyer and Lehnard (Meyer and Lehnerd, 1997), define a product platform as "a set of common components, modules, or parts from which a stream of derivative products can be efficiently developed and launched." An effective platform is the key to developing a successful family of products.

Many companies have been successful implementing a product family/product platform approach: Volkswagen shares a platform across multiple models and reduced development and production costs (Wilhelm, 1997), HP used modularization to delay differentiation so that more varieties can run on the same assembly line (Feitzinger and Lee, 1997), and Honda developed a stretchable automotive platform to better satisfy American and Japanese needs (Naughton, et al., 1997). In some instances platform development can lead to increased learning during product evolution and reduced testing and certification time in aircraft (Sabbagh, 1996) as well as aircraft engines (Rothwell and Gardiner, 1990). Sony built the iconic Walkman line on a platform based design producing 85% of its varieties by "minor rearrangements of existing features" (Sanderson, 1997). Even more noteworthy, in the 1970's, Black & Decker was able to realize an annual savings of over $1.8 million per year, by developing a family of universal electric motors through standardization of motor components (Meyer and Lehnerd, 1997).

These examples show the significant benefits platform based product development offers, and it is therefore extremely important to develop systematic approaches that can be used to design successful product platforms and families. Over the past decade, a flurry of research has occurred in the area of product family design, leading to the development of many methods, which will be reviewed in the next chapter. While these methods aim to address different challenges within product platform design, they are not without their limitations and tradeoffs and therefore leave much room for development and improvement.

The Product Platform Constructal Theory Method (PPCTM) is a novel approach for developing product platforms that enable customizable products (Hernandez, 2001). Hernandez proposes to solve the product platform design problem as a problem of optimization of access in a geometric space to address several limitations of existing methods. Williams later improved this method through the incorporation of product demand modeling and multi-objective optimization (Williams, 2003). Through its original inception, and subsequent augmentations, the PPCTM offers an extremely comprehensive product platform design method, with the ability to accommodate multi-platform design, multiple design specifications, non-uniform demand modeling, and multi-objective decision making. One limitation of this method is that the selection of platform variables and the modes for managing product variety must be pre-specified or determined ad hoc by the designer. This research seeks to address this limitation through the integration of a sensitivity based analysis method to determine the effect of platform variable variation on the family performance.

1.3 Research Question and Hypotheses

The primary goal of this research is the augmentation of the PPCTM to alleviate the limitation presented above. In order to achieve this goal, this research seeks to answer the following research question:

Primary Research Question
How can the Product Platform Constructal Theory Method be augmented to enable the selection of common platform variables?

To address this research question, a hypothesis is now posed, which will be investigated and validated.

Hypothesis 1
Incorporating sensitivity analysis into the PPCTM will yield the effects of varying design variables away from their optimums as is done during commonization, showing the designer which variables can support the most commonization.

The result of this work is an augmented PPCTM that eliminates the need for *a priori* platform specification, providing a systematic means of selecting platform variables.

1.4 Organization of this Thesis

Chapter 1 provides the background and motivation for this work, as well as the research question this thesis seeks to address. In Chapter 2, relevant literature is reviewed, including descriptions of important concepts, product family design methods, and the Product Platform Constructal Theory Method. Additionally, a gap analysis is conducted on existing literature. In Chapter 3, the augmented PPCTM is presented, which will infuse sensitivity analysis into the existing methods. In Chapter 4, the method is applied to two examples. In Chapter 5, the research questions and their associated hypotheses are reviewed and conclusions are drawn. Lastly, the limitations and suggestions for future work are outlined, and closing remarks are made.

CHAPTER TWO

LITERATURE REVIEW

With the background and motivation of this thesis discussed in the previous chapter, this chapter will survey relevant literature on the topic of product platform design as well as lay the fundamental groundwork for the topic of this thesis. This chapter also provides a detailed overview of the existing PPCTM along with a critical analysis describing a key limitation.

2.1 Product Platform Design

2.1.1 Overview

As stated in Chapter 1, the growing trend of mass customization has many companies using platform based product development strategies to create families of products that meet the increased need for product variety without ballooning manufacturing costs. Various definitions for what constitutes a product platform have been presented in literature. Robertson and Ulrich define a platform generally as a collection of assets that are shared by a set of products (Robertson and Ulrich, 1998), while others focus on industry and product specific applications (Sanderson and Uzumeri, 1995). Regardless of the definition, the main principle behind a platform strategy is the deliberate reuse and standardization of components and/or features to reduce the overall production cost and development time, while still offering a diverse range of products.

The benefits of platform-based strategies are widely displayed in industry. Successful product examples include airplanes, computers, power tools, and automobiles (Jiao, et al., 2007). Volkswagen has long been a leader in platform development within the automotive industry, having saved an estimated $1.5 billion per year in development and capital costs in the late 1990's (Wilhelm, 1997). Today, VW is continuing their platform strategy having recently announced the Modular Transverse Matrix (MQB in German) which they are proclaiming as "the beginning of a new era." The MQB strategy will standardize many vehicle component parameters across Volkswagen's several brands and classes, including a uniform mounting position of all engines whether conventional, electric or hybrid (see Figure 2-1). Using this strategy VW describes the potential to produce all vehicle models on the same assembly line and expects cost savings up to 20 percent and assembly time reductions of up to 30 percent (Volkswagen, 2012).

Figure 2-1: Volkswagen's new Modular Transverse Matrix drive systems (http://www.volkswagenag.com)

While the benefits of product platforms have been well documented, introducing excessive commonality has significant drawbacks. Increased commonality can lead to loss of performance as well as loss of distinctiveness for individual products within the family. For example, Volkswagen even with all of their platform success, had unexpected technical difficulties with the Audi TT, as the common A-platform was not well suited for such a high end vehicle (Weck, et al., 2003). Additionally, Ulrich and Eppinger point out that platform-based approaches can sometimes result in as high as 10 times the development cost of a single product (Ulrich and Eppinger, 2004). Therefore, the fundamental problem for designers is how to design effective platforms that reduce manufacturing and development costs while balancing the inherent tradeoff between commonality and performance.

To address this problem, considerable research effort has been invested over the last decade into product platform design by both industry and academia in order to understand how platforms should be systematically developed and what factors determine a platform's success. To that end, numerous product platform approaches exist in literature, which will now be reviewed.

Generally speaking, almost all product platform approaches fall into one of two categories: bottom-up or top-down (Simpson, et al., 2001). Bottom-up methods describe a redesign or reconfiguration of an existing product line to reduce internal variety and standardize components. For example, after developing 100+ lighting control products for individual customers, Lutron redesigned its product line such that all 100+ models could be manufactured using just 15-20 standard components (Pessina and Renner, 1998). Other examples of bottom up platform strategies include Black & Decker as described in the previous chapter (Lehnerd, 1987), John Deere (Shirley, 1990), and Volkswagen (Whitney, 1993). Several formal methods have been proposed in literature, such as Kalpakjian's group technology (Kalpakjian, 1997), Ericsson and Erixon's modular functional deployment (Ericsson and Erixon, 1999), and Siddique and Rosen's product family reasoning system (Siddique, 2000). Such methods benefit from the existing knowledge developed while designing the original product line, making it faster and easier for designers to modify the product family as well as more accurately estimate the costs of new parts.

Top-down platform design consists of an up front, a priori, decision to design a family of products based around a common platform. The major advantage of this type of approach is that by tackling product standardization from the very beginning, costly "bottom-up" redesign can be avoided and the process of adding more products later on may be made smoother (Wheelwright and Clark, 1992). Sanderson and Uzumeri cite how Sony managed the development of their Walkman products as one example of successful top-down development (Sanderson and Uzumeri, 1995). Another is Kodak's platform-based single use camera, which uses a standardized lens, viewfinder, and flash while changing packaging to attract different markets. This strategy enabled Kodak to develop products faster and more cheaply, so they could regain market share and overtake Fuji (Wheelwright and Clark, 1992).

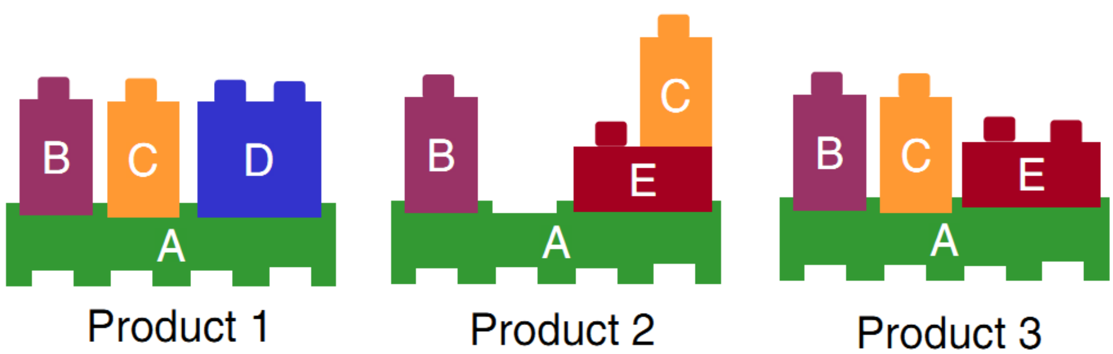

Figure 2-2: Example illustration of a module-based platform (Weck, et al., 2003)

Irrespective of top-down or bottom-up, there are two basic approaches for developing the actual product platform and subsequent derivatives. One is to derive product variants by adding, substituting or removing one or more functional *modules* to the platform. Such an approach of platform development is called *module-based* or *configurational* platform design (Simpson, et al., 2001). Figure 2-2 illustrates three products developed using a modular approach, each assembled with different combinations of components A, B, C, D, E. It is important to notice that even though all three products share components A, B, and C, only the base, A, constitutes a platform. As an industry example consider again the Sony Walkman family, which was built around key modules and platforms using flexible manufacturing to produce a wide variety of products at low costs, introducing over 250 models in the U.S. during the 1980's (Sanderson and Uzumeri, 1995).

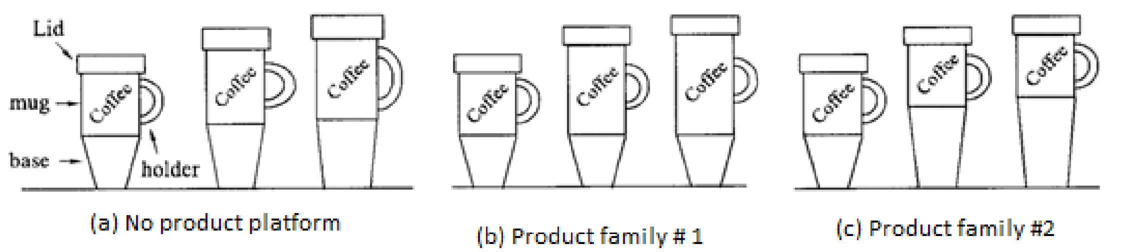

Figure 2-3: Example illustration of scale-based platforms (Khire, 2006)

The other approach is a *scale-based* approach referring to "the capability of a product platform to be scaled or stretched by varying one or more design parameters to satisfy different customer or market requirements" (Simpson, et al., 2001). Scale-based approaches generally require that all product variants be described by the same variables, and these variables will take on different instantiated values for the different variants. As an example, observe the product family consisting of three coffee mugs shown in Figure 2-3. Each mug consists of a lid, mug, base and handle. For scale-based design, the product platform is defined by those features, which are dimensionally standardized across the family. Figure 2-3(a) represents no product platform as all of the components are dimensionally unique. Figure 2-3(b & c), shows two different scale-based platforms: (b) having common lid, handle, and base dimensions, and (c) having common lid, mug and handle dimensions. Product variety customization is then offered through scaling of the mug or base length for families (b) and (c), respectively. Concerning product family terminology, the design variables that define the platform are referred to as platform design variables and design variables that are not shared are called non-platform design variables, or scaling variables. Additionally, recall that additional commonality generally conflicts with optimal performance. Consider if stability is an important performance characteristic, then there will likely be different tradeoffs between the different families shown in Figure 2-3. This brings to light a key issue in product family optimization; namely, the selection of platform and non-platform design variables.

Scale-based design has become increasingly common in many industries. The universal motor example presented is the first chapter has received considerable attention over the years as a quintessential platform success. Additionally, as shown in Figure 2-4, Boeing developed much of its 7X7 family using a fixed front and tail, then "stretched" the aircrafts to accommodate more passengers, carry more cargo, or increase flight range (Sabbagh, 1996).

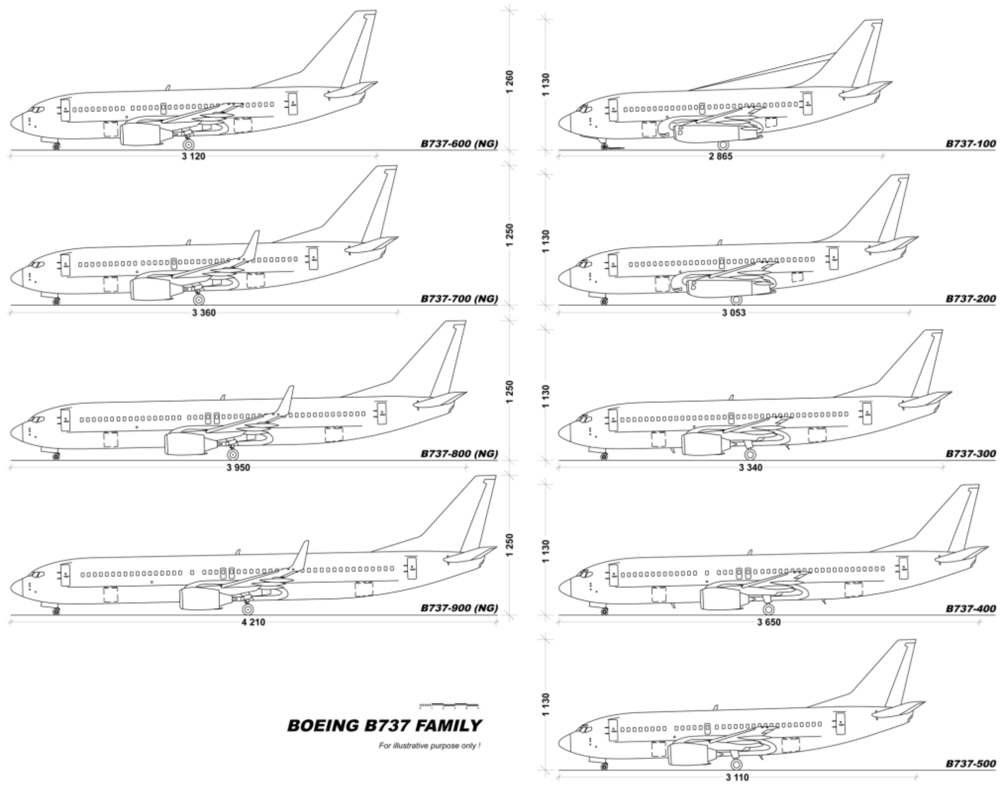

Figure 2-4: Boeing 737 Family Based on a Fixed Front and Tail (http://www.boeing.com/commercial)

With an overview of platform-based design presented, the review of product platform methods will be covered next. As a top down method is the focus of this thesis, the scope will be limited to only relevant top down methods.

2.1.2 Review of Existing Methods

Considerable work has been done on top-down product platform methods over the last decade and an extensive review of existing methods can be found here (Jiao, et al., 2007, Simpson, 2004). Generally speaking, top-down platform selection has three main components: identification and selection of the platform variables (platform configuration) and the extent of which those variables are shared, selection of optimal values for shared variables, and selection of optimal values for product variants (Dai and

Scott, 2007). Many of the examples in literature are concerned with only the last two tasks and begin with a preselected platform, as seen in Simpson and coauthors' Product Family Concept Exploration Method (Simpson, et al., 2001) and Messac and coauthors physical programming based method (Messac, et al., 2002). Due to the limitation of applying these methods to products with unknown platforms, several methods have been developed which begin to address the platform configuration problem. Messac et al. present the product platform penalty function to guide selection of common and scaling variables (Messac, et al., 2002). Nayak and coauthors use variation based modeling to minimize the deviation among design variables, while trying to meet the specified performance bound in the variation-based platform design method (Nayak, et al., 2002). These methods are generally categorized as two-stage approaches and can potentially lead to sub-optimality. More recent work by Khire et al. seeks to address this issue through the Selection Integrated Optimization (SIO) approach (Khire, 2008, 2006). This method uses a segregated mapping function which converts the combinatorial problem of platform selection into a continuous process. This work was further expanded to develop a complete framework for product platform planning (Chowdhury, 2011). In addition to the above methods, Simpson (Simpson, 2004) provides a review of other platform methods as summarized in Table 2-1.

Table 2-1: Literature review of Product Platform Design Methods adapted from (Simpson, 2004)

	Features of Product Family Design							
	Module-Based Family	Scale-Based Family	Specify Platform a priori?	Single-Objective	Multi-Objective	Models Manufacturing Cost	Models Market Demand	Considers Uncertainty?
(Allada and Jiang, 2002)	x		Y	x			x	Y
(Blackenfelt, 2000)	x		Y	x		x		Y
(Cetin and Saitou, 2004)	x		N		x			
(D'Souza and Simpson, 2003)		x	Y		x			
(Farrell and Simpson, 2003)		x	Y	x			x	
(Fellini, et al., 2004, Fellini, et al., 2005, Fellini, et al., 2006)	x	x	Y		x			
(Fujita, et al., 1998, Fujita, et al., 1999, Fujita, 2002)	x		Y	x		x	x	
(Gonzalez-Zugasti, et al., 2001)	x		Y	x	x	x	x	Y
(Hernandez, 2001, Kulkarni, 2005, Williams, 2003)		**x**	**Y**	**x**	**x**	**x**	**x**	**x**
(Kokkolaras, et al., 2002)	x		Y		x			
(Messac, et al., 2002, 2002)		x	Y		x			
(Nayak, et al., 2002)		x	N		x			
(Ortega, 1999)		x	N		x	x		
(Seepersad, et al., 2000, Seepersad, et al., 2002)		x	Y	x	x	x	x	Y
(Simpson and Mistree, 1999, Simpson, et al., 2001, Simpson, et al., 2001)		x	Y		x			
x shows that the method has the specified feature a blank indicates the feature is absent								

2.1.3 Limitations of Existing Methods

While considerable progress has been made through the development of the platform design methods reviewed above, Simpson points out that there are several major limitations associated with the majority of top-down product family design methods previously listed. Below these limitations will be listed along with recent works beginning to address them.

One main limitation is the extent to which platform variables are shared. For these existing methods, platform variables are either common to all products in the family or none. This can lead to a dramatic tradeoff between commonality and performance resulting in over designed lower end products, and reduced performance of higher end products. In order to reduce this loss in performance, designers should be able to specify different levels of commonality for different design parameters and components. Dai and Scott use sensitivity and cluster analysis to handle this problem (Dai and Scott, 2007), while other methods include pattern recognition and fuzzy logic (Freeman, 2011). A growing body of work is beginning to use genetic algorithms (GA) to handle varied levels of commonality (Khajavirad, et al., 2009, Simpson and D'Souza, 2004).

The second key limitation deals with the capability to specify multiple design specifications. Often times, existing platform approaches only consider offering variety for one design specification (e.g. motor torque, for an electric motor). This greatly inhibits designing truly customized products, as often times consumers require variety in multiple specifications, such as motor torque and motor power.

Lastly, of the methods surveyed, less than half deal with manufacturing costs or product demand. Most of these methods assume that maximizing product performance maximizes demand, maximizing commonality minimizes production costs, and that optimizing the tradeoff between the two leads to the most profitable product offering.

From the above review it is concluded that many of the existing product platform design methods suffer from one or more limitations. To address some of these limitations, Hernandez proposed the Product Platform Constructal Theory Method (PPCTM). Section 2.2 provides a complete discussion of this method.

2.2 The Product Platform Constructal Theory Method

As stated in Chapter 1, the Product Platform Constructal Theory Method (PPCTM) is a novel top-down approach for developing customizable products, which addresses the issues of multiple levels of commonality and multiple customizable specifications (Hernandez, 2001). Specifically, Hernandez shows that it is useful to abstract the platform optimization problem, such that the design of product platforms for customizable products can be represented and solved as a problem of access in a geometric space (Hernandez, et al., 2002). Williams goes on to augment this method to handle non-uniform demand modeling and multi-objective designs (Williams, 2003).

The focus of this thesis is to further extend the PPCTM to provide designers a systematic means of selecting platform variables. Before these additions are presented, details of the theoretical foundations, original conception and subsequent augmentations of the PPCTM must first be presented. Therefore, the author's objective in this section is to provide a comprehensive coverage of the PPCTM. Sections 2.2.1 and 2.2.2 detail the two theoretical underpinnings of this method, hierarchical systems theory and constructal theory. Section 2.2.3 introduces the original method proposed by Hernandez and goes on to detail the six steps of William's augmented method. This section is based on the works of Hernandez and Williams and is cited where appropriate. Lastly, section 2.2.4 provides a critical evaluation of the PPCTM and describes the existing limitation motivating this work.

2.2.1 Hierarchical Systems Theory

To develop a family of products that can be easily mass customized to meet a variety of needs, a design method must have an efficient organization strategy that can quickly respond to changes in requirements. Hernandez, therefore, looked to the study of complex systems found in nature, specifically the works of Herbert Simon, as inspiration for how to organize the varied modes of managing variety (Simon, 1996). From their study of complex systems, Simon and Ando make two fundamental observations regarding the natural organization of complex systems (Simon and Ando, 1961):

1. Complexity, both in natural and artificial systems, frequently takes the form of a hierarchy, whereby *a hierarchic system is defined as being composed of interrelated subsystems that have in turn their own subsystems, and so on, until some elementary level of components is reached*.
2. In general, interactions inside subsystems (in a hierarchically-organized system) are stronger and/or more frequent than those interactions which occur between subsystems.

The notion of hierarchic systems can be described as a system in which internal subsystems are organized in ranks, where each subsystem represents a lower rank in the hierarchy. Consider for example, the structure of biological systems. With a cell as the building block, cells are organized into tissues, tissues into organs, and organs into systems all following a hierarchy. Even within the cell, there are sub-systems such as: the nucleus, cell membrane, and mitochondria. Artificial systems, such as a corporate structure, also typically follow such a system. Figure 2-5 shows an illustration of a hierarchic system compared with a non-hierarchic system, where each "A#" and "B#" represents a system or a subsystem of organization A and B. On the left, there are three distinct levels, where subsystem elements belong to only one higher level element. On the other hand, the right side of Figure 2-5 shows a non-hierarchic organization as B4 is not completely contained by either B2 or B3, such that higher or lower levels of organization cannot be distinguished.

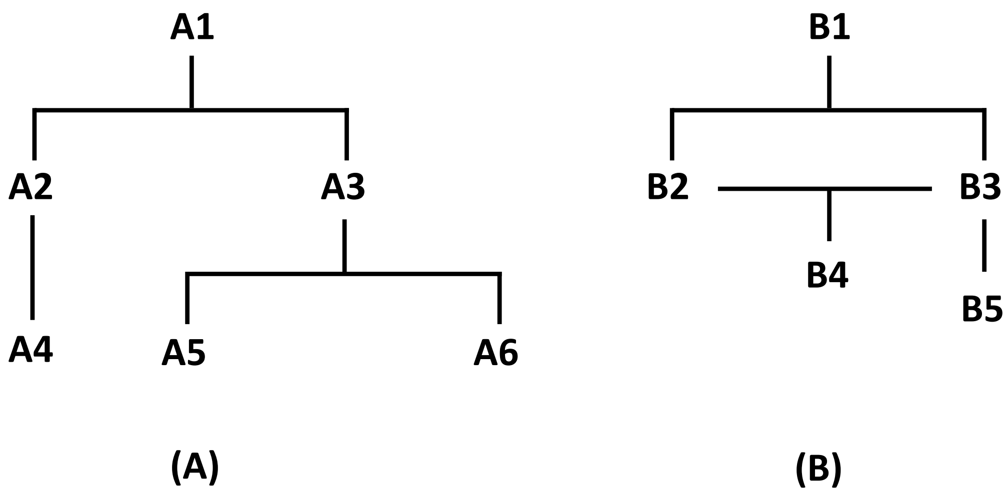

Figure 2-5: Hierarchic (A) vs. Non-Hierarchic (B) Organization of Systems (Williams, 2003)

Additionally, Simon and Ando investigated the concept of near-decomposability to develop several theorems (Courtois, 1985). Simply stated, they observed that favorable conditions arise when each stable subsystem operates nearly independently of the processes happening within the other subsystems. For example, in the hierarchy shown in Figure 2-5, near-decomposability means that the processes occurring in A4 should have very little influence on the processes happening in A5 and A6, as shown.

From these studies, Simon concluded that complex structures adapt and evolve more efficiently when they are organized hierarchically (Simon, 1996). For that reason, Hernandez chose hierarchic organization and near-decomposability as key foundations for the PPCTM, developing the following two fundamental posits (Hernandez, 2001):

Posit 1: Potential for rapid adaptation and/or response is higher in complex systems when they are organized hierarchically.

Posit 2: In hierarchically organized systems, the high-frequency (short run) responses tend to be associated with the lowest levels of the hierarchy and the low-frequency (long run) ones with the interactions of these subsystems, i.e., the higher levels of the hierarchic organization.

2.2.2 Constructal Theory

Constructal theory began as a result of studying optimal access in flow and traffic problems, seeking to maximize global performance through hierarchical organization. This optimum access problem must therefore determine the optimum arrangement of paths that link all points of a set space, S, with a common destination O.

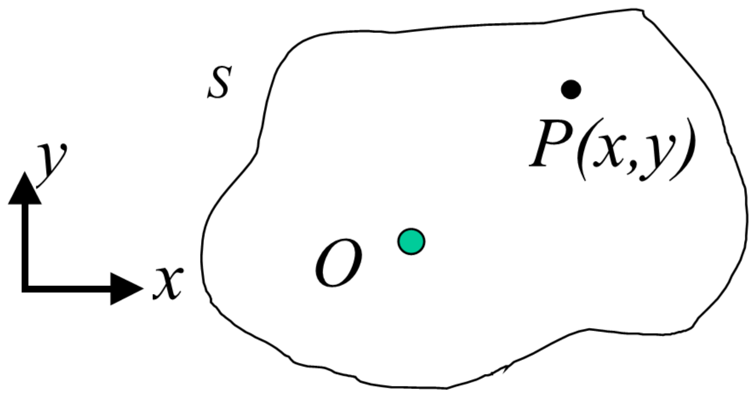

Figure 2-6: A Finite-Size Area with a Common Destination (Bejan, 1997)

To explain constructal theory, consider the street network example posed by (Bejan, 1996). Imagine the space S, shown in Figure 2-6, is a developing village and point *O* represents the marketplace where residents go to buy and sell goods. For residents that live at various points *P(x,y)* the question arises, *"What is the fastest way to travel from any point P to point O?*" The obvious answer is to create a straight path from every point to *O*. However, this is not feasible if there are multiple modes of transportation (i.e. walking, compared with driving an automobile) each with space requirements such that villagers can no longer live on that land. Therefore, patches of land must be allocated as residential areas, with a finite number of streets connecting these areas with the marketplace, as shown in Figure 2-7. This then raises the question, "*How does one determine the optimal street network*?"

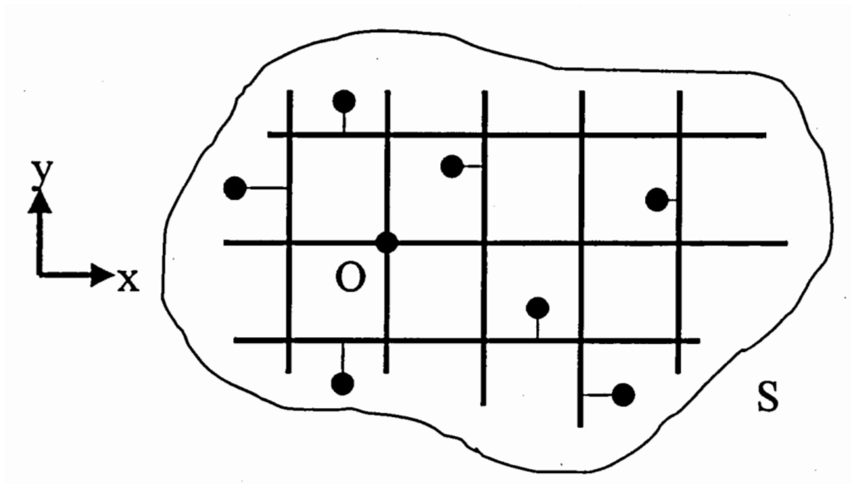

Figure 2-7: Example of a Street Network for a Finite-Size Area (Carone, 2003)

Bejan suggests that the best way to solve this problem is to optimize small areas of space, which can then be joined into larger areas of space that are optimized, and so on until the entire area is covered. Specifically, this example requires allocating a finite length of street to each finite patch of land, and then connecting these streets in such a ways as to optimize travel time as shown in Figure 2-8.

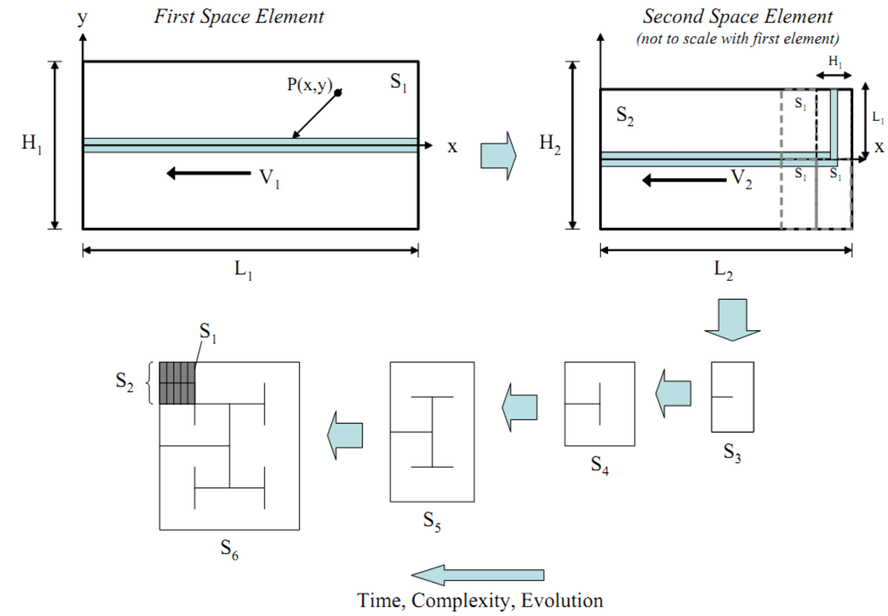

Figure 2-8: Assembly of Space Elements in a Constructal Manner (Chamberlain, 2007) modified from (Bejan, 2000)

Consider first the space S_1, the smallest space scale to be optimized. The minimum size for this area is constrained by the minimum size for living space for the residents of the land. For simplicity we will assume that this space is rectangular and the roadway must be a straight path; consider as a frame of reference this area would represent something similar to a neighborhood. There are then two modes of transportation within this space: walking from some point, P(x,y), at a speed V_0 and driving along the access street at a velocity of V_1 (where $V_1 > V_0$). The first step is then to determine the optimal form of this area, specifying dimensions H_1 and L_1, which optimizes the access of the population out of S_1. Once these smallest spaces are determined the problem remains to connect these areas together such that each subspace, S_1, can access the common destination O. To solve this problem, Bejan's Constructal Theory approach repeats the preceding geometric optimization connecting the smaller space elements using a faster access way, where $V_2 > V_1$, to form the space element S_2. The street network development proceeds in this manner, progressively building larger area elements until the total area S is covered.

A number of the assumptions and constraints made in this example are not realistic, such as the even distribution of the population and the straight roadways. However, this example illustrates the fundamental essence of the constructal method; that access problems can be solved through the optimization of the smallest space elements and the hierarchic assembly of these elements until the entire space is covered. For further explanation of this example and other applications of constructal theory readers are referred to (Bejan, 1996, 2000).

While Hernandez goes on to show that this sequential optimization process yields suboptimal results, he also suggests that this limitation of multi-stage optimization can be overcome through the implementation of more effective solution algorithms. Therefore, constructal theory provides a key foundation for the PPCTM and leads Hernandez to develop the following three posits (Hernandez, 2001):

Posit 3: System complexity results from a natural process of systems to provide paths of easier access.

Posit 4: Each path of access within the optimized system structure is unique and does not cross with other paths: the resulting structure is hierarchic.

Posit 5: The design of a hierarchic structure to provide easier access should proceed in a specific time direction: from the optimization of the basic elements at the smallest scale towards the optimal arrangement of these elements into higher-order assemblies, the process being one of repeated maximization of access (or minimization of losses) subject to constraints.

With the theoretical foundations established, the Product Platform Constructal Theory Method is presented next.

2.2.3 Steps in the Product Platform Constructal Theory Method

In the PPCTM, Hernandez was able to abstract Bejan's street network problem to product platform development, where access in a geometric space is analogous to managing product performance and commonality for a desired amount of variety (Hernandez, 2001). This product variety is defined as the space of customization, and comprises the set of all feasible combinations of values of product specifications that a manufacturing enterprise is willing to satisfy as well as the associated market demand.

Figure 2-9 shows a geometric representation of a space of customization. This space can be one dimensional, two dimensional, or multidimensional.

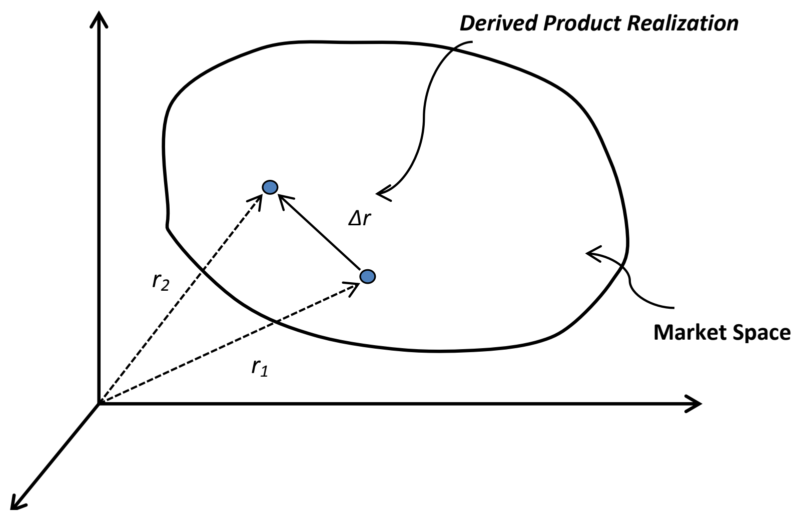

Figure 2-9: Product Platform Design as a Problem of Access in Geometric Space (Hernandez, et al., 2003)

Mathematically, let the space of customization be the set, M^N:

$$M^N \equiv \{r_1, r_2, \dots, r_N\} \qquad (2\text{-}1)$$

where N is the dimension of space given by the number of design specifications, $r_1, r_2, \dots, r_N$, required for the product family, such as desired pressure and volume for a pressure vessel or power and torque for a motor.

Using this formulation any product variant, i, within the space of customization can be represented by an N-dimensional vector, $\boldsymbol{r}$:

$$\boldsymbol{r}_i = r_{i1}\hat{e}_1 + r_{i2}\hat{e}_2 + \cdots + r_{iN}\hat{e}_N \tag{2-2}$$

where $\hat{e}_k$ is the unit vector for each product specification direction. New products, $\boldsymbol{r}_j$, can then be developed from an existing product, $\boldsymbol{r}_i$, through product customization represented by:

$$\Delta\boldsymbol{r}_{ji} = \sum_{k=1}^{N}(r_{jk} - r_{ik})\hat{e}_k = \sum_{k=1}^{N}\Delta r_{jik}\hat{e}_k \tag{2-3}$$

Thus, the new product $\boldsymbol{r}_j$ is given by:

$$\boldsymbol{r}_j = \boldsymbol{r}_i + \Delta\boldsymbol{r}_{ji} \tag{2-4}$$

Using this formulation, the goal of the designer is to develop a finite number of product platforms which can be customized to fulfill any product specification with the space of customization.

Consider, as an example, a family of pressure vessels where a customer could be concerned with volume and pressure. The space of customization for these vessels is two-dimensional, where each specification represents a dimension in the space, and each point within the space signifies a specific product that the manufacturer wants to fulfill. While the main objective for the manufacturer is reduction in cost, multiple objectives could also be included allowing the designer to maximize or minimize other goals .

With the design specifications and objectives determined, the designer must "access" all variants within the space through derivatives of baseline platforms. Accessing these variants is achieved through various modes of managing customization (Δr in Figure

2-9), which can be any generic approach in product design or its manufacturing process for achieving product customization (includes: modular design, platform design, dimensional scaling, etc.). The core crux of the PPCTM is determining the baseline platforms from which we can access all of the product variants within the space of customization. Building upon the foundations of constructal theory, this problem is formulated and solved as a hierarchical multi-stage optimization problem through Hernandez's Product Platform Constructal Theory Method.

While the PPCTM addressed several of the major limitations of other methods, Christopher Williams, in his Master's thesis, goes on to address several more limitations. Through the incorporation of the utility based compromise decision support problem, Williams is able to handle multiple objectives and competing goals as is seen in most complex systems. Additionally, he expands the method to incorporate non-uniform demand modeling, for both continuous and discrete models.

With these augmentations included, the six steps of the PPCTM are now presented in Figure 2-10.

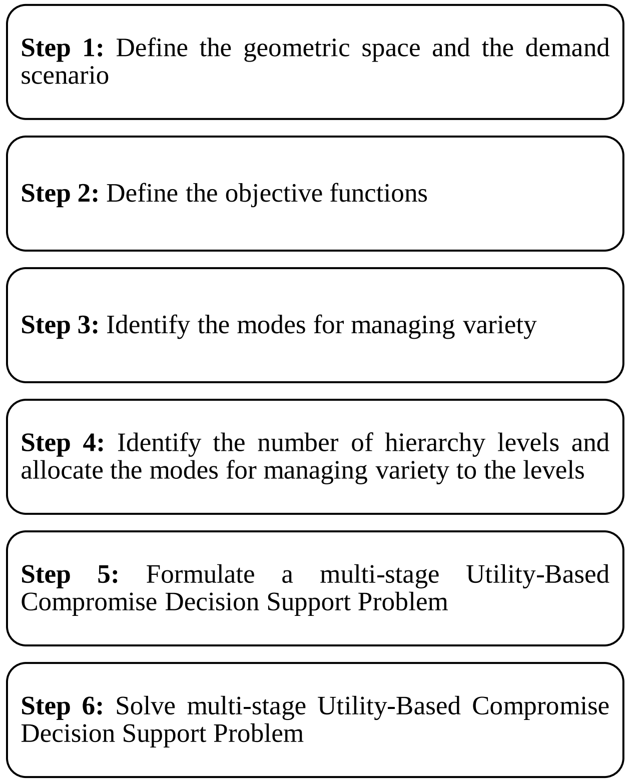

Figure 2-10: Flow Chart of the Product Platform Constructal Theory Method (Williams, 2003)

Step 1: Define the Market Space and Demand Scenario

In the first step of the PPCTM, the space of customization is defined by three components: specification of variety to be offered, determining the range of variety to be offered, and analysis of market demand. The number of specifications that will be offered determines the dimension of the space. Additionally, the ranges over which these parameters will vary must be specified and linked to associated demand models for the product.

Step 2: Define the Objective Functions

The second step of the PPCTM is the identification of the objective to be improved. Examples of common objectives for a family of products include, the minimization of cost, the maximization of profit, or maximization of performance parameters such as strength, efficiency or mass. Through the adoption of the utility-based compromise Decision Support Problem, the PPCTM also provides the designer the opportunity to define multiple objectives. An objective function can be formulated in one of two ways: either as a discretized analysis of the space using a summation equation:

$$O = \sum_{i=r_{1,min}}^{r_{1,max}} \sum_{j=r_{2,min}}^{r_{2,max}} \dots \sum_{k=r_{n,min}}^{r_{n,max}} Obj(r_{1,i}, r_{2,j}, \dots, r_{n,k}) \tag{2-5}$$

or as a continuous analysis using an integral:

$$O = \int_{i=r_{1,min}}^{r_{1,max}} \int_{j=r_{2,min}}^{r_{2,max}} \dots \int_{k=r_{n,min}}^{r_{n,max}} Obj(r_{1,i}, r_{2,j}, \dots, r_{n,k})\, dr_1 dr_2 \dots dr_n \tag{2-6}$$

In both formulas, r_{min} and r_{max} refer to the lower and upper bounds of each dimension of the market space, respectively.

Step 3: Identify the Modes for Managing Variety

Once the market space, product demand, and objection function have been defined, the designer must identify how to vary the product design in order to satisfy all of the required specifications of the market space. Examples of common modes for managing variety suggested by Williams include component commonality, dimensional commonality, standardization and modularity (Williams, 2003). Using these modes can be viewed as either increasing common features or components across sections of the market or ways of adjusting the product to realize greater variety.

Step 4: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety to the Levels

Having just identified the modes for managing product variety, in this step it is determined how they will be utilized. In the PPCTM the previously defined space of customization will be divided up into smaller subspaces through a series of stages. In this step, the number of stages must be defined and the appropriate modes for managing variety assigned to each stage. At the present state, the PPCTM has no clear way to select the number of stages or determine which modes should be assigned to which particular stage. However, this assignment is extremely critical as the results depend heavily upon how this is done. Hernandez suggests that modes capable of the smallest divisions should be used first and those which are more discrete should be used in later stages and states that more than one mode can be assigned to each stage. Additionally, constraints exist such that each lower level space element must be smaller than the higher elements, such that each space element of each stage can be combined together at the next stage, as shown in Figure 2-11.

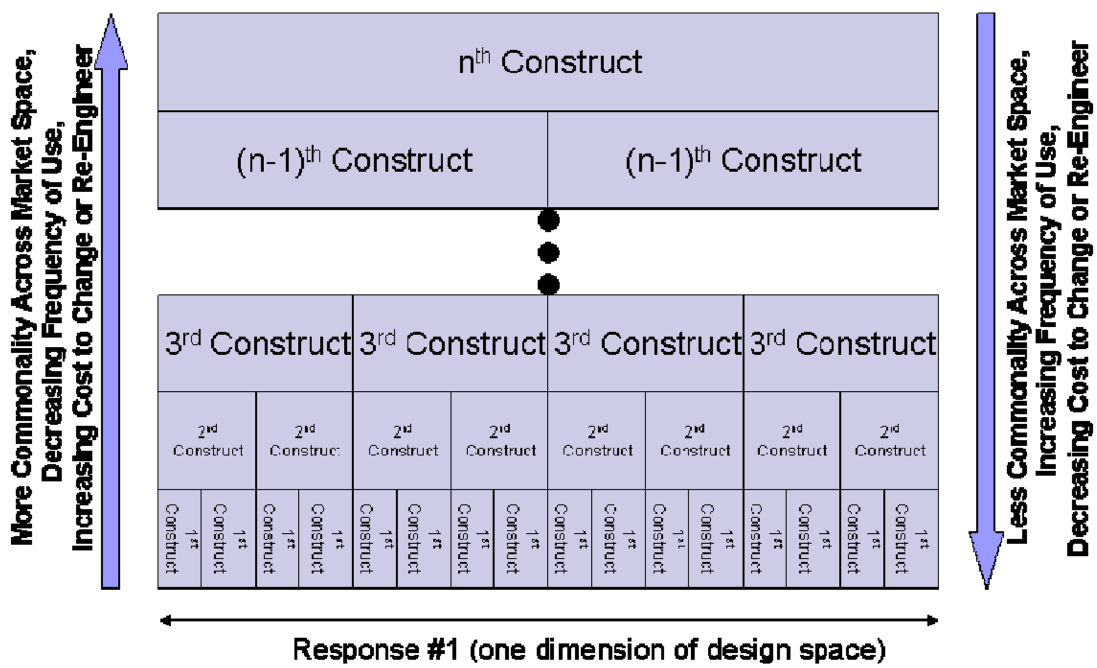

Figure 2-11: Hierarchical Ranking Space Elements in a Single Dimension of the Market Space (Chamberlain, 2007)

Step 5: Formulate a Multi-Stage Utility-Based Compromise Decision Support Problem

With the modes for managing variety and associated stages given, the design problem must now be formulated as a sequential utility based compromise Decision Support Problems. The fundamental decision in each stage is the determination of the size and shape of each subspace. Starting at the first stage with the smallest elements, the decision variables must be identified. These decision variables represent the range of commonality for each mode for managing variety, $\Delta r(i)$. For a problem with N parameters, the decision variables for any stage are:

$$\Delta r(i) = [\Delta r_1(i), \Delta r_2(i), \dots, \Delta r_N(i)] \tag{2-7}$$

In order for the formulation to maintain the tenants of hierarchical theory, there exists a constraint on the ranges of commonality such that each subsequent space element is larger than the previous:

$$\Delta r_j(i+1) \geq \Delta r_j(i) \tag{2-8}$$

The goal in each Utility-Based Compromise Decision Support Problem (u-cDSP) is the minimization of the deviation variable associated with the expected utility of the objectives. A typical u-cDSP formulation is shown in Figure 2-12.

For Each Stage i

Given: The N-dimensional market space $M^N = (r_1, r_2, \ldots, r_N)\}$
The decision variable of the previous stages $\Delta r(1),\ldots, \Delta r(i-1)$
The modes of managing product variety to be utilized at Stage i

Find: The value of decision variable $x(i) = [\Delta r_1(i), \Delta r_2(i), \Delta r_N(i)]$
The deviation variables, $d^-_{x,i}$ and $d^{\mp}_{x,i}$

Satisfy: Bounds: $\Delta r_{j,min}(i) \leq \Delta r_j(i) \leq \Delta r_{j,max}$
Constraints: $\Delta r_j(i) \geq \Delta r_j(i-1)$
$d^-_{x,i}, d^+_{x,i} \geq 0$
$d^-_{x,i} * d^+_{x,i} = 0$
Goals: $E\left[u(o_{x,i})\right] + d^-_{x,i} + d^+_{x,i} = 1$

Minimize: $Z_i = 1 - U_i = \sum_{x=1}^{y} k_x\,(d^-_{x,i} + d^+_{x,i})$; where $U_i = \sum_{x=1}^{y} k_{x,i} u(o_{x,i})$

Figure 2-12: Formulation of the Multi-Stage Utility-Based Compromise Decision Support Problem (Williams, 2003)

Step 6: Solve the Multi-Stage Utility-Based Compromise Decision Support Problem

The final step in the PPCTM is the solution of the multi-stage utility-based compromise Decision Support Problem. The key outcome of this solution is the determination of the values of the ranges for each mode for managing product variety, *Δr(i)*. In the original implementation of the PPCTM, Hernandez suggests the use of dynamic programming to solve this problem and overcome the suboptimal results associated with a purely constructal solution. This implementation requires the designer to first develop response surfaces that approximate the ranges of each mode, only then can the problem be solved by moving through each response surface to calculate the objective function. As this technique proves to be very tedious, in his later work Hernandez moved away from dynamic programming to exhaustive searches to solve the multi-stage problem, suggesting that any generic solution algorithm can be used to solve this step. Subsequent work has similarly used exhaustive searches, as well as genetic algorithms (Kulkarni, 2005, Williams, 2003). With the completion of this step, a baseline platform is presented with the optimal ranges for each mode for managing variety such that all products within the space of customization can be realized.

2.2.4 Critical Evaluation of the Product Platform Constructal Theory Method

The PPCTM, as developed by Hernandez, and extended by Williams, is a technique that enables a designer to develop platforms for customizable products while handling issues of multiple levels of commonality, multiple product specifications, and the inherent tradeoffs between platform extent and performance.

While this list covers most of the previously listed platform design method limitations, there is still one major limitation that needs to be address. As pointed out during the explanation of Step 3 in Section 2.2.3, when identifying the number of design stages and their associated modes for managing product variety, the designer is not guided by any formal process or quantitative data. None of the previous PPCTM examples (Carone, 2003, Hernandez, 2001, Kulkarni, 2005, Williams, 2003) has provided any insight into how this selection should be made, rather they specify the platform layout *a priori* based on designer experience.

In order to make this method more applicable to developing product platforms for new products, this limitation must be overcome. Consider the development of a new biomedical peristaltic pump, an example that will be revisited later in this thesis. This pump was initially developed for portable hemodialysis, and presents several performance advantages over existing pumps on the market. With these advantages, it would be beneficial to develop a family of pumps that can be utilized in a vast range of applications for pharmaceuticals to beverage processing. However, since the design space outside of the current application has not been well explored, a designer attempting to utilize the PPCTM would not be able to determine the number of stages or which of the only five design parameters to associate with them. It is therefore the focus of this thesis to extend the PPCTM to address this limitation.

2.3 Summary

In Section 2.1, the author presents a literature review of product family design theory, including definitions, methods and limitations so that the reader can firmly grasp the current state of the art and how existing methods aim to tackle the product family design problem. In Section 2.2, the Product Platform Constructal Theory Method is presented, discussed and critically analyzed. Theoretical foundations are laid in Section 2.2.1 and 2.2.2, with discussions of hierarchical systems theory and constructal theory. Section 2.2.3 discusses Hernandez's abstraction of these theories into the area of product family design, culminating in the introduction and explanation of the six steps of the PPCTM. In Section 2.2.4 a critical analysis of the PPCTM reveals one key limitation of the existing method in that it lacks a formal process for how to organize the modes for managing product variety and identify the number of stages. It is therefore the focus in this research to extend the PPCTM to alleviate this limitation.

CHAPTER THREE

MODIFIED PRODUCT PLATFORM CONSTRUCTAL THEORY METHOD WITH SENSITIVITY BASED PLATFORM IDENTIFICATION

In this chapter, the augmented Product Platform Constructal Theory Method will be presented. These augmentations aim to extend the existing approach so that future designers using the PPCTM will have a systematic means of selecting platform variables and determining their hierarchy. Specifically, the author wishes to answer the primary research question presented in Chapter 1:

Primary Research Question:
How can the Product Platform Constructal Theory Method be augmented to enable the selection of common platform variables?

Without a means of selecting platform variables, the PPCTM is severely limited when applied to new products and therefore makes this problem extremely important. To address this limitation and answer the primary research question, we present a sensitivity-based approach, integrated into the existing PPCTM to enable hierarchical ranking of design variables based on the impact of the variables on the product performance. In Section 3.1, a sensitivity index will be presented which serves as the backbone of the approach to be applied. Next, in Section 3.2, the five steps of the new sensitivity analysis will be presented and its infusion into the existing PPCTM will be presented. The new seven step sensitivity based method will be presented in Section 3.3, accompanied by a tutorial example of the method's application to designing a platform of customizable pressure vessels. Lastly, a summary of the chapter will be presented in Section 3.4.

3.1 Foundations for the Sensitivity Analysis

As previously mentioned, effective platform variable selection is at the core of any successful product family. At its present state, the PPTCM cannot be used to determine this platform configuration, as selection of platform variables and the modes for managing product variety are not guided by a systematic method. In all of the previously used examples, such as the universal electric motor (Hernandez, et al., 2002), pressure vessel (Hernandez, et al., 2003), cantilever beam (Williams, et al., 2007), organization of the modes for managing variety and the corresponding platform variables is predetermined, with little explanation as to how these selections were made. This is very limiting especially when looking forward to new products where designer experience may be lower.

To solve similar configuration problems, both Fellini et al. (Fellini, et al., 2004) and Dai and Scott (Dai and Scott, 2007) have implemented sensitivity analysis for product platform specification. The former uses the acquired sensitivity information to impose a penalty function based on performance deviations from a specified value. This method is limited by requiring analytical solution of the objective function, and strict constraints on the span of the design variants. The latter develops a sensitivity index, which does not require these constraints due to the numerical nature of their solution. Due to this benefit, the author wishes to extend this sensitivity index to the PPCTM forming the basis for the sensitivity approach to follow.

To determine platform sensitivity to changes in the design variables, a sensitivity index is introduced, which is an adapted form of the finite differencing method. The first step in this method is to instantiate baseline individual product variants at distributed locations over the space of customization. Prior to this the designer must have already defined the product objective function and elicited customer preferences if multiple objectives are to be used. Using the defined objective function a decision support problem is formulated and solved to find the individual variants and their associated design variables which minimize (or maximize) the objective function. Z^* is defined to be the value of the minimum objective of the individual variant, and $\boldsymbol{x}^* = x_1^*, x_2^*, \ldots, x_n^*$ are the corresponding design variable values for each variant. We must note, when designing for a continuous space of customization, the space must be discretized at a sufficiently small increment to ensure the selected baseline variants provide an adequate representation of the design space. A generic problem formulation for instantiating the individual variants is shown below in Figure 3-1.

Given:	The N-dimensional space of customization $M^N = (r_1, r_2, \ldots, r_N)\}$ The level of discretization for the space of customization An appropriate mathematical model User preferences for objectives(if needed)	
Find:	The values of the design variables, $\mathbf{x_j}$	
Satisfy:	Bounds:	$x_{j,min} \leq x_j \leq x_{j,max}$
	Constraints:	Defined by designer (e.g. failure criteria, design limits)
Minimize:		Objective function for each individual variant: Z_i

Figure 3-1: Problem Formulation for Developing Baseline Variants

Given as inputs for this formulation are the N-dimensional space of customization, the step size of discretization, the model of the system and user preferences if multiple objectives are to be used. Each dimension of the space of customization, $r_1, r_2, \ldots, r_N$, represents a design specification over which the designer wishes to offer variety. For example, in the pressure vessel example to come, the manufacturer wishes to offer variety in terms of volume of the vessel and pressure that can be contained. These two specifications form a two dimensional space, as shown in Figure 3-2, and the black square is the space of customization showing the extent of variety to be offered. The blue points represent the individual baseline variants that will be solved for dictated by step size of discretiztion within the space. For each of these variants, the goal is to find values of the design variables, $\mathbf{x_i}$, and the associated objective function (e.g. radius, length, thickness and cost of the vessel), such that: the design specifications are met, the objective function is minimized, and any design constraints are satisfied.

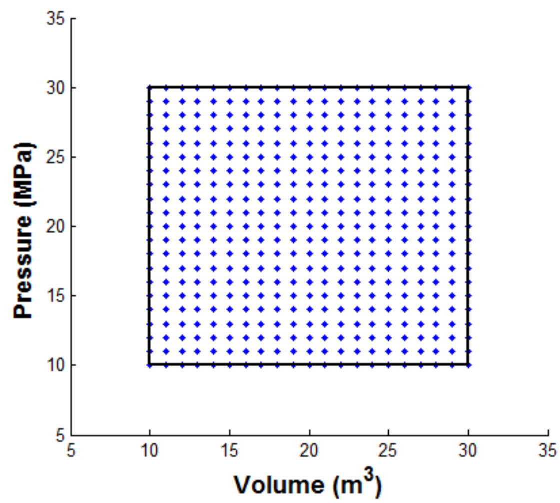

Figure 3-2: Example Discretized Space of Customization

Next, all of the output design variables must be normalized with respect to the maximum and minimum values from the individual variants. The sensitivity with respect to each design variable is then calculated as shown in Equation 3-1 (Dai and Scott, 2007). $SI_{x_i}^{e}$ is the sensitivity of product *e* to changes in variable x_i:

$$SI_{x_i}^e = \frac{|Z^* - Z_+^*| + |Z^* - Z_-^*|}{2\Delta x_i} \tag{3-1}$$

where Δx_i is the step size for design variable, Z^* is the minimum of the objective function for that variant, Z_+^* is the objective function at one step above, and Z_-^* is the objective function at one step below.

The step size, Δx_i, for each design variable is determined by the designer on a case by case basis using knowledge of the design variables and changes that would be physically feasible. For example, if a certain design variable comes in fixed incremental sizes, then at a minimum, the step size for that variable should be set at that incremental size. Anything smaller would produce much less realistic sensitivity data. If there are not such constraints on the design variables, another guideline for determining the step size would be to take a percentage of the difference between the maximum value and the minimum value of the design variable observed over the space. These are the maximum and minimum values actually used for the variants, not the upper and lower bound of the variable itself.

Each individual sensitivities represent the local sensitivity for each product variant. It is important to explain that the sensitivity index is not trying to calculate the gradient of the objective function at the minimum value. Rather, the index is meant to characterize how much of an impact varying this design variable would have on the performance of that variant so that this sensitivity can be compared to determine which variables are suited for higher or lower levels of commonization. To illustrate this point, consider Figure 3-3 comparing objective function plots for two different design variables x_1 and x_2 of a single product. The variables x_1^* and x_2^* represent the baseline values for each variable with Z^* representing the minimized objective function value. Varying the design variable values by a normalized value, Δx, yields a reduction in performance as shown by the increase in the objective function. Comparing the two plots shows that for variable x_2, there is a much greater performance loss, ΔZ, than there is for variable x_1 if the design variables were to vary away from their minimums as a result of commonization. This would be reflected by a greater sensitivity index value for x_2 than for x_1.

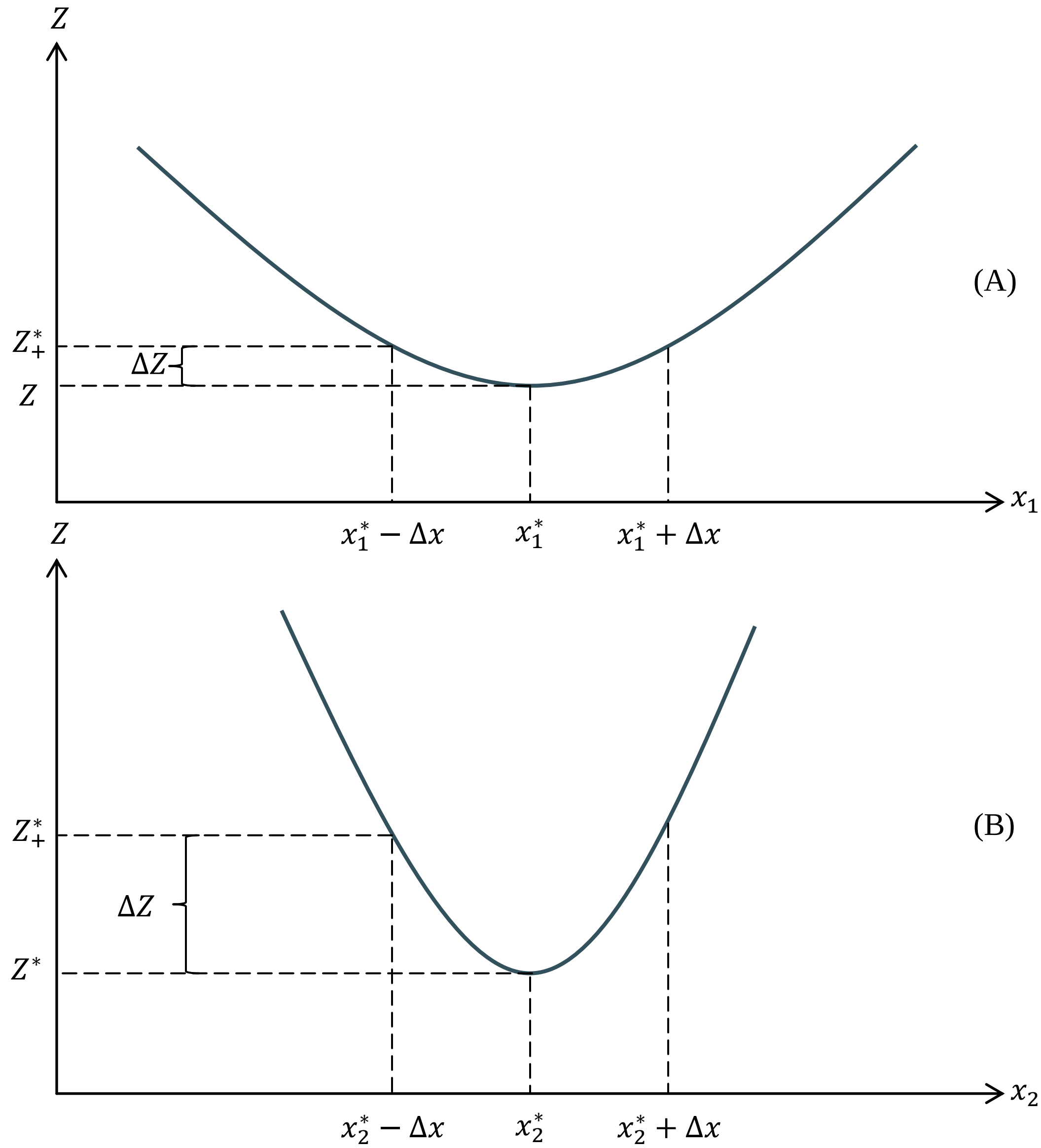

Figure 3-3: Comparison of change in objective function as two platform variables are varied

Once the sensitivities for all of the variants have been determined, to aggregate this sensitivity information over the entire space of customization, a weighted sum average is taken of all local sensitivities yielding a global sensitivity value which expresses how sensitive the overall performance is to changes in that design variable. The designer must determine weight values heuristically if there are preferences that are not incorporated into the objective function. Generally speaking, the author recommends using the specified demand scenario to determine these weighting values. For uniform demand scenarios the weight values would be equal to unity, whereas for variable demand weighting values proportional to demand at the specific location in the space are suggested. Therefore, the global sensitivity is given by Equation 3-2:

$$G_{x_i} = \frac{\sum_{i=1}^{n} D^e * SI_{x_i}^e}{\sum_{i=1}^{n} D^e} \tag{3-2}$$

where $SI_{x_i}^e$ is the sensitivity of product e to changes in variable x_i and D^e is the demand for product e given by the demand profile at that location within the space of customization.

It is important to note that multiple objectives should be incorporated into a single model objective function and used to evaluate the sensitivity. Therefore, there is no need to calculate sensitivities with respect to separate objectives as they are already accounted for. These global sensitivities should then be ranked from greatest sensitivity to least. This rank can then be used for determining the number of platform subspace levels and the corresponding variables. The core concept is that within the PPCTM construct, variables with the greatest sensitivity should embody the smallest space elements, as small changes away from the optimum value have a large impact on performance and would need the highest number of unique designs. In the larger space elements, variables with lower sensitivity should be used, as making these variables common over multiple variants, and a larger range of area, will have little effect on performance. Determination of the number of levels is currently determined heuristically based on the resulting sensitivities. Generally speaking, modes for managing variety with similar sensitivities can be grouped together within a level and solved simultaneously. At the greatest extreme there could be a space level for each individual design variable given significant variations in sensitivities.

3.2 Incorporating Sensitivity Analysis into the PPCTM

In the previous section, a sensitivity index was introduced and serves as the backbone of the sensitivity-based approach presented here. Building from this foundation, a five step sensitivity analysis is now introduced, as shown in Figure 3-4. This analysis is infused between Step 3 and Step 4 of the existing method and will enable designers to appropriately allocate modes for managing variety to the design stages in Step 4.

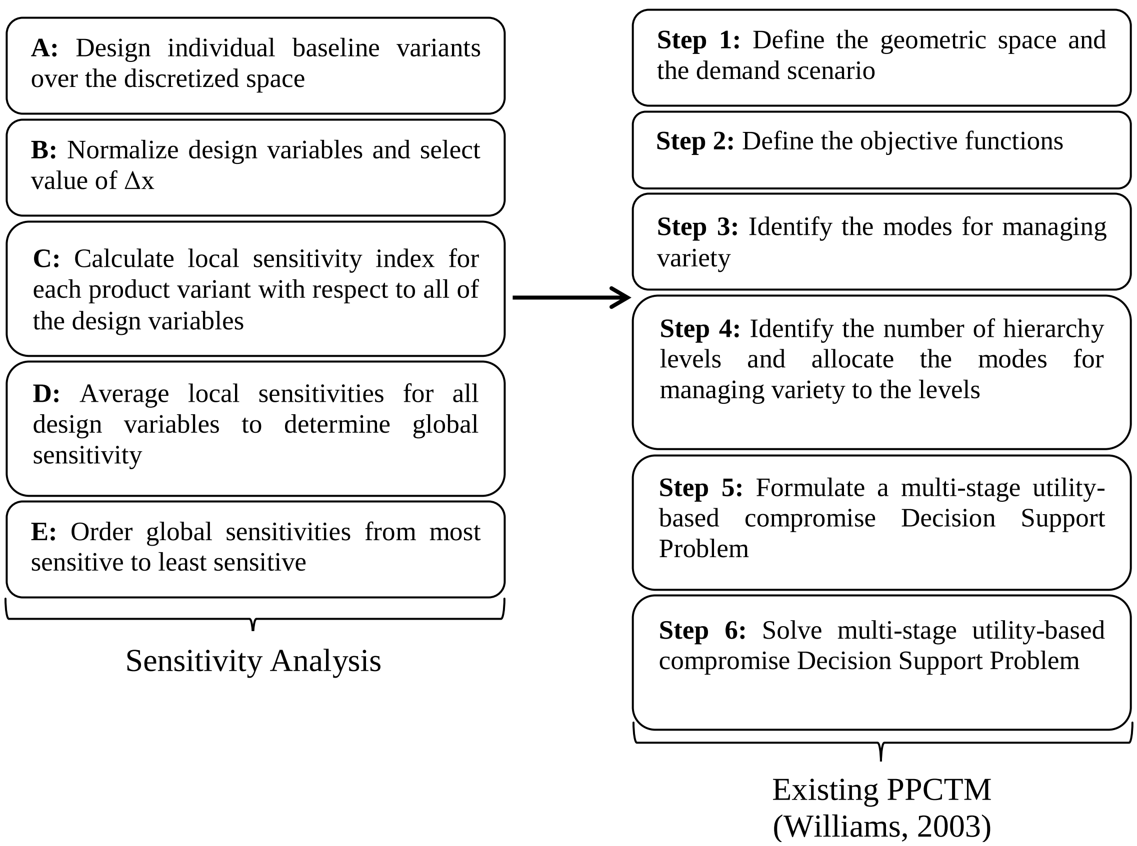

Figure 3-4: Implementing Sensitivity Analysis into the PPCTM

From the first three steps in the existing method, the sensitivity analysis takes as inputs the geometric space, the demand scenario, objective function, and the identified modes for managing variety. The sensitivity analysis steps are then implemented and the output is a list of ranked modes for managing variety that will then be used in the current step 4 to determine the number of levels and allocate the modes for managing variety commonalized within that space.

3.3 Modifying the Problem Formulation and Solution Method

In addition to the inclusion of sensitivity analysis found in Step 4, modifications to the solution method used by Williams and Hernandez have also been implemented in this work. In previous works, the decision formulations have been separated into separate stages for each space element. Williams (2003) noted that a sequential solution of these stages will not work as the space element decisions are highly coupled as well as optimal solutions at one stage may not provide the global optimal solution over the entire space. He therefore suggests a recursive solution manner, but still maintains the multi-stage formulation, which increases the computational expense of the solution. This work suggests a natural extension to a combination of the multiple stages, such that a single stage is formulated where all of the space element dimensions are solved for simultaneously. For this reason, the author has omitted any reference to stages, and discusses solely the hierarchy of space elements. Additionally, it is the author's opinion that strict enforcement of utility based compromise decision support problem formulation is too restrictive, since in many cases a more general formulation is sufficient. This is especially true of single objective problems, where the added tools of the u-cDSP present little benefit for the additional complexity. As such this work uses a more generic formulation for the problem of determining the size of the space elements and the values of their associated platform variables. This formulation follows the same structure as the u-cDSP and is therefore readily upgraded if necessary. A generic problem formulation for determining the size of the space elements is shown below in Figure 3-5.

Given:	The N-dimensional market space $M^N = (r_1, r_2, \ldots, r_N)\}$ and demand scenario The modes for managing variety and the hierarchy of the elements they're associated with	
Find:	The values of decision variables $\Delta r_i = [\Delta r_1, \Delta r_2, \ldots, \Delta r_N]$ The values of the design variables within each space element	
Satisfy:	Bounds:	$\Delta r_{i,min} \leq \Delta r_i \leq \Delta r_{i,max}$ $x_{j,min} \leq x_j \leq x_{j,max}$
	Constraints:	$\Delta r_i \geq \Delta r_{i-1}$ Failure criteria, design limits, etc. set by designer
Minimize:	The value of the objective function, $Z_{avg} = \frac{1}{D_{tot}}\left[\left(\sum_{r_{1,min}}^{r_{1,max}} \ldots \sum_{r_{n,min}}^{r_{n,max}} D_i(r_1, r_2, \ldots, r_N) Z_i(r_1, r_2, \ldots, r_N)\right) + Cpenalty\right.$	

Figure 3-5: Generic Formulation for Determining the Extent of the Space Elements

In the above formulation, Δr_i represents the decision variables to be solved for, which define the size of each space element in the hierarchy and thus set the levels of commonality for design variables. The goal is to determine the size of the decision variables, which minimizes the objective function. The objective function is formulated as a discrete analysis over the space of customization, where the space of customization is divided into a number of nodes, which serve to approximate the variants across the space of customization. The general form of the objective function is shown in Equation 3-3.

$$Z_{avg} = \frac{1}{D_{tot}}\left[\left(\sum_{r_{1,min}}^{r_{1,max}} \ldots \sum_{r_{n,min}}^{r_{n,max}} D_i(r_1, r_2, \ldots, r_N) Z_i(r_1, r_2, \ldots, r_N)\right) + C_{penalty}\right] \quad (3\text{-}3)$$

The objective function formulation has two parts: a summation of objective values for the individual variants and a commonality penalty function. The summation is taken with respect to each dimension of the space $(r_1, r_2, \dots, r_N)$, where the *max* and *min* subscripts refer to the upper and lower bounds for the associated dimension. The value to be summed is the product of the individual demand (D_i) and objective function value (Z_i) at each node within the space. The commonality penalty function serves to incorporate the goal for commonality across the product family by penalizing the platform for added variety in values of the design variables. The commonality penalty function can be defined as an added cost for equipment or ordering, as will be shown in the first example, or a goal to minimize the deviation within each design variable. Other commonality measures can also be used, so long as the main principle is that maximizing commonality, minimizes the penalty. Lastly, to determine the average value across the space of customization, the summation and commonality penalty are divided by the total demand. The total demand is calculated as a sum of the demands for each node in the discretized space.

The final step is to solve the decision support problem, Figure 3-5, yielding the size of the different space elements $(\Delta r_1, \Delta r_2, \dots, \Delta r_N)$ and the associated design variable values made common within each space. The general solution method (adopted from Williams, 2003) involves iterating through the different size combinations of space elements, commonizing the design parameters across each element, evaluating the objective function for products within each space, then calculating/comparing the overall objective function for each iteration. The following steps are used:

- The particular geometric market space, demand scenario, assigned modes for managing variety and objective function are taken as inputs.
- An initial starting value for the size of the space elements is given and used to define the bounds on each sub-space.
- The values of the design variables to be commonized for each space are determined such that the individual objective function of the products within that space is minimized and the constraints for all variants are met (Decision 0).
- The average objective function of the entire market space is evaluated including the individual objective function values, the commonality penalties, and the demand. This is the main output of this algorithm.
- These steps are then repeated using a new iteration of space element sizes, and the output is compared with the previous iteration's output.
- This process continues until the designer is satisfied that the best solution has been found (i.e. a stopping criteria has been met).

It is observed within these steps that there are actually two levels of decision problems to be solved. In the primary level, the decision variables defining the size of the space elements must be determined; however, inherent to this decision is the selection of the actual value of the variable to be made common across each sub-space, referred to as Decision 0 in the above steps. The problem formulation for the primary decision is presented in Figure 3-5, whereas Decision 0 is formulated similarly to the baseline variants, Figure 3-1. The primary difference for Decision 0 is that the givens are the bounds on the subspace rather than the entire market and the goal is to find a common value of each design variable such that the average objective value is minimized within the subspace. As there are two formulations, two separate solution routines must be implemented.

The general characteristics of the Decision 0 problem, assuming a scalable design, are a large set of continuous design variables, a well defined individual objective function that is stated explicitly in terms of these design variables, along with linear and nonlinear constraints. Many optimization methods can be used for Decision 0, however because this decision will need to be solved many times for each space element, and often includes many design variables, the primary selection criteria is efficiency. Gradient based methods are therefore best suited for solving Decision 0, and this work therefore uses such a method facilitated through Matlab's *fmincon* function, specifically using the interior point algorithm.

The characteristics of the primary decision also include continuous decision variables, representing the size of the space elements, however the number of decision variables is typically much lower than the number of design variables. Additionally, the size of a decision variable corresponding to a higher space element level is bounded by the size of the lower level (e.g. $\Delta r_1 \leq \Delta r_2 \leq \Delta r_3$), further reducing the size of the problem. Next, the objective function is not explicitly defined in terms of the decision variables, which requires that any gradient based approach must solve the gradients numerically thus reducing efficiency. Therefore, the benefits of a gradient based approach for the primary decision are negligible, and this work therefore elects to use an exhaustive search method thereby trading marginal added cost for extra confidence in the final solution.

3.4 The Augmented Product Platform Constructal Theory Method with Sensitivity Analysis

As stated at the beginning of this chapter, the core thrust in this thesis is to provide the designer with a means of selecting platform variables so that they can be ranked and used within the framework of the PPCTM. In the previous two sections, a method for sensitivity analysis was introduced to achieve this goal and alleviate the limitations of the PPCTM. The sensitivity analysis is now incorporated into the PPCTM through the addition of a new step, "Step 4: Implement Sensitivity Analysis and Rank Modes for Managing Variety." With this inclusion, the new seven-step PPCTM is presented in Figure 3-6.

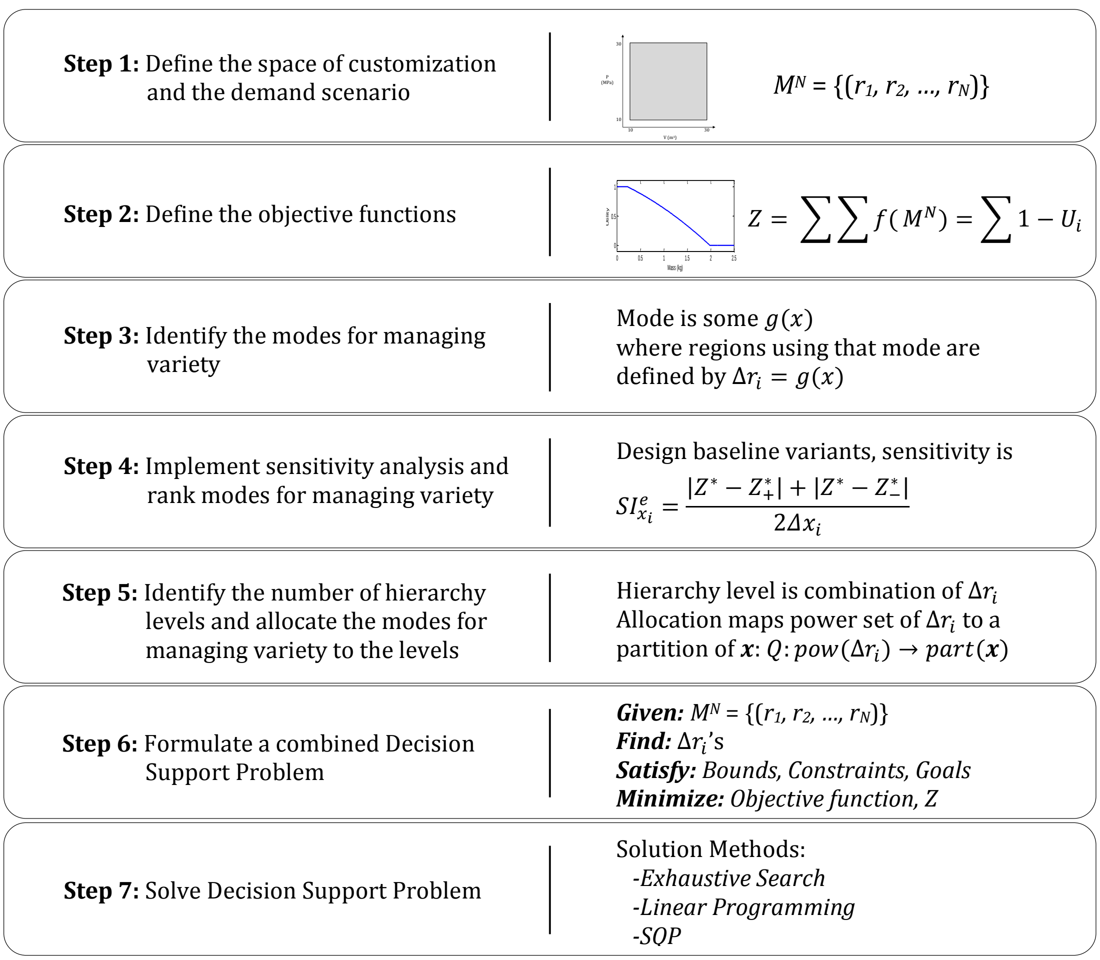

Figure 3-6: The seven step augmented PPCTM

To assist in the explanation of the augmented PPCTM, each step of the method is illustrated using a tutorial example of developing a product platform for a line of customizable pressure vessels. The pressure vessel example has been presented in several previous works on the PPCTM (Hernandez, 2001, Hernandez, et al., 2003, Williams, 2003) making it well suited as an initial validation of this work.

3.4.1 Pressure Vessel Problem Description

A manufacturer of pressure vessels, looking to gain a competitive edge, wishes to provide customized vessels to meet their customer's needs. The manufacture knows that added variety can lead to large increases in cost and therefore wishes to develop a family of platforms that can efficiently offer customizable pressure vessels while keeping costs down.

For this example, the conceptual design of the pressure vessel has been predetermined. Each vessel consists of a cylindrical container capped at both ends by hemispherical heads as shown in Figure 3-7. The center body shell is manufactured from two sheets of rolled plate which are welded together to form a cylinder. Each head is forged from a single sheet and then welded to the center body. All of the welds used are single-welded butt joints with a backing strip.

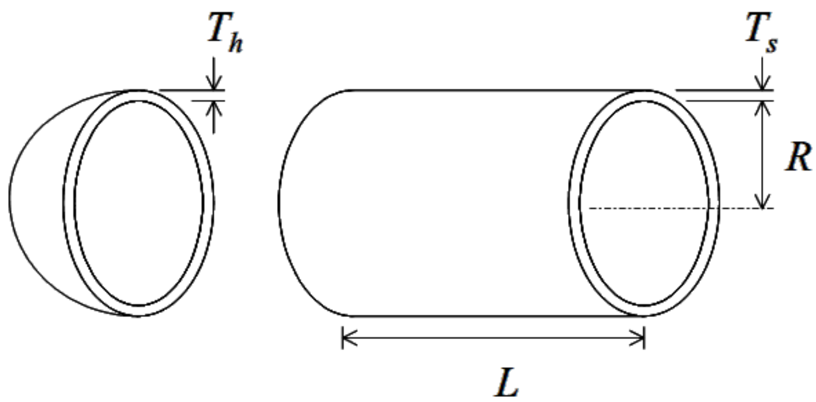

Figure 3-7: Pressure Vessel Schematic

The manufacturer wishes to offer customization with a given range of pressures between 10 to 30 MPa, and volumes between 10 and 30 m^3. In order to characterize any desired pressure vessel within this range, the manufacturer must determine the following design variables: length (L), radius (R), and the head and shell plate thickness (T_h and T_s).

3.4.2 Pressure Vessel Model

In order to implement the Augmented PPCTM, there must be a fully described model for the system being studied. In this section, the pressure vessel model is presented.

As previously stated, each pressure vessel consists of a cylindrical shell capped at both ends by hemispherical heads. The shell and heads are produced from carbon steel sheets (ASME SA 203 grade B) and then rolled or forged into the final shape. Sheets of this material are available in thicknesses ranging between 2 – 76.2mm and lengths up to 7m. Available equipment limits the maximum radius to 1.5m.

To achieve the desired specification for volume, various combinations of shell length and radius can be used. The volume of a vessel is given by:

$$V = \pi R^2 L + \frac{4}{3}\pi R^3 \quad (3\text{-}4)$$

Variation in the allowable pressure can be achieved by varying the shell and head thickness. For a given pressure, the design of the vessel must satisfy the following constraints on minimum thickness of the shell and head (Bednar, 1986):

$$T_s \geq \left(\frac{P}{\sigma_y - 0.6P}\right) R \quad (3\text{-}5)$$

$$T_h \geq \left(\frac{P}{2\sigma_y - 0.2P}\right) R \quad (3\text{-}6)$$

where σ_y is the yield strength of the material (1077 MPa), and P is the desired pressure.

The manufacture's objective in developing this product platform is to minimize the average cost per vessel over the entire market. As a result, a means of modeling the cost of the vessels is needed. The analysis of cost presented has been carried over from that used by (Hernandez, 2001) and (Williams, 2003) in their previous applications of the PPCTM to this problem. The total cost of manufacturing pressure vessels is comprised of four components: material cost, welding cost, ordering cost, and equipment cost (Note: labor costs and plant utilities costs are assumed to be included with welding and equipment costs).

The material cost is determined by the amount of material that must be purchased to build each vessel. This cost is comprised of two parts: the cost of the material used in each vessel and the cost of the material wasted when cutting the raw steel plates to the required dimensions. The material cost ($C_{material}$) is given by:

$$C_{material} = 2\pi\rho(C_s R T_s L + C_h R^2 T_h) + C_{waste} \tag{3-7}$$

where ρ is the density of the material (7800 kg/m^3), C_s is the cost per kilogram of processed shell steel ($0.80 per kg), and C_h is the cost per kilogram of forged steel for the head ($2 per kg).

The cost of the wasted material (C_{waste}) is given by:

$$C_{waste} = 2\pi\rho C_p T_s R(L_o + L) \tag{3-8}$$

where L_o is the length of the raw steel plate, and C_p is the cost per kilogram of the raw steel plate.

The welding cost (C_{weld}) is composed of the cost of the longitudinal welds across the shell and the cost of the circumferential welds around the head. The longitudinal welding cost ($C_{longweld}$) is given as:

$$C_{longweld} = V_l \rho C_w \tag{3-9}$$

where the volume of the welding material, V_l is given by,

$$V_l = 2\pi\left(\frac{T_s}{cos30°}\right)^2\left(\frac{60}{360}\right)L = \frac{4}{9}\pi {T_s}^2 L \tag{3-10}$$

and C_w is the cost of hand welding the material ($15 per kg hand welded).

The circumferential welding cost ($C_{circweld}$) is given as:

$$C_{circweld} = V_s \rho C_w \tag{3-11}$$

where the volume of the welding material, V_s is given by,

$$V_s = 4\pi^2 \left(\frac{T_s}{cos30°}\right)^2 \left(\frac{60}{360}\right) R = \frac{8}{9}\pi^2 {T_s}^2 R \tag{3-12}$$

The total welding cost becomes

$$C_{weld} = 2\pi\rho \left(\frac{2}{9} C_w {T_s}^2 L + \frac{4}{9} C_w \pi {T_s}^2 R\right) \tag{3-13}$$

After combining the above equations, the total cost for a single pressure vessel, not including equipment and ordering cost is therefore,

$$C(L, R, T_s, T_h) = 2\pi\rho \left(C_s R T_s L + C_h R^2 T_h + \frac{2}{9} C_w {T_s}^2 L + \frac{4}{9} C_w \pi {T_s}^2 R + C_p T_s R (L_o + L)\right) \tag{3-14}$$

In addition to material costs there is also a cost associated with ordering the raw material, C_{order}. A fee of $250 is assessed each time an order for raw material is place as to cover shipping, handling and stocking the inventory. This cost is based on the number of different sized sheets of raw material that must be ordered (based on different values of raw length and different thicknesses required); it is not related to the quantity of sheets ordered and is therefore not a function of demand. The cost for ordering is given by:

$$C_{order} = \sum_{i=1}^{m} 250 \tag{3-15}$$

where m is the number of distinct sheets of steel required (i.e. the number of distinct values of L_o, T_s and T_h.

The cost of purchasing manufacturing equipment (C_{equip}), namely the forging presses and associated dies to make the heads and bend the shells, is evaluated with:

$$C_{equip} = \sum_{p=1}^{N_p} (50000 R_p) \tag{3-16}$$

where N_p is the number of pressed needed, and R_p is the radius of the die used for each press.

This concludes the pressure vessel model. The cost equation presented in Equation 3-14 is the total cost of producing one vessel and must be evaluated for each individual variant. The ordering cost and equipment cost (Equations 3-15 and 3-16, respectively) serve as commonality penalty functions and are calculated for the entire family of vessels.

3.4.3 Step 1: Define the space of customization and the demand scenario

As described in Section 2.2.3, the space of customization is defined by three components: specification of variety to be offered, determining the range of variety to be offered, and analysis of market demand. In this application, the manufacturer wishes to offer customization with respect to the volume and the pressure of the vessel. Therefore, there are two independent design specifications resulting in a two-dimensional space of customization ($N=2$) where the first dimension, r_1, is associated with volume and the second, r_2, is associated with pressure. The range in each direction was established in the problem description and will be from 10 to 30 m^3 and 10 to 30 MPa for volume and pressure, respectively. The resulting space of customization is illustrated in Figure 3-8.

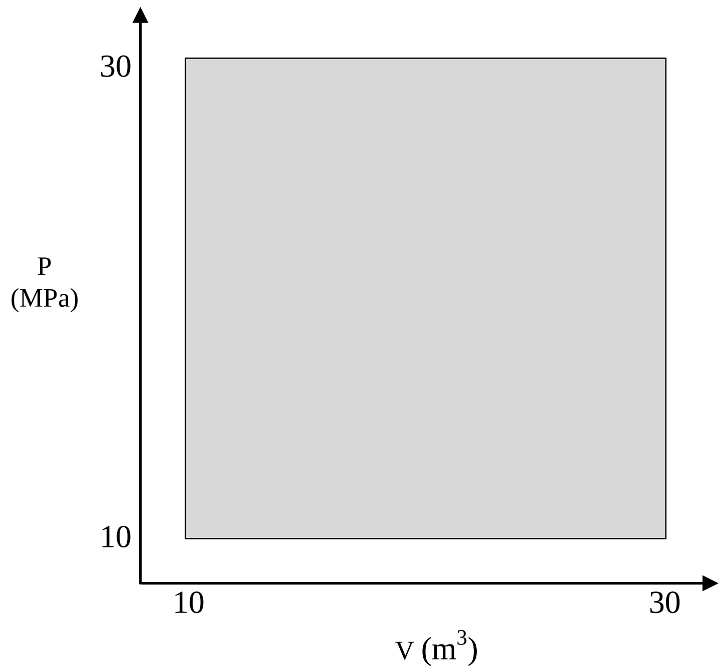

Figure 3-8: Pressure Vessel Space of Customization

The third component is to define the market demand scenario. For this example, a discrete pyramid is used as shown in Figure 3-9, where the highest demand is for the vessels in the center of the space of customization and then steps down moving out toward the edges.

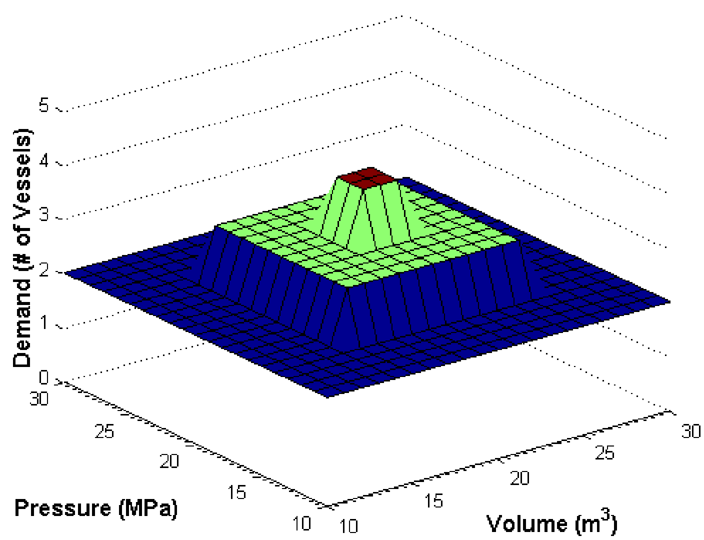

Figure 3-9: Pressure Vessel Demand Scenario

3.4.4 Step 2: Define the Objective Functions

From the problem description, the manufacturer's objective is to minimize the cost to produce customized pressure vessels over a desired range. In the cost equations described in Section 3.4.2 there are two levels of cost. There is the cost to produce individual vessels described by Equation 3-14 and then there is the cost of ordering and equipment, Equations 3-15 and 3-16. Therefore, average cost is simply the summation over the market space of the individual cost of each specific variant multiplied by the demand, plus the ordering and equipment costs, divided by the total demand. The final formulation of the objective function for the entire space is shown in Equation 3-17.

$$C_{average} = \frac{1}{D_{total}}\left[\sum_{P=10}^{30}\sum_{V=10}^{30}\left(D_{V,P}C_{V,P}\right) + C_{equip} + C_{order}\right] \qquad (3\text{-}17)$$

where $D_{V,P}$ and $C_{V,P}$ are the demand and cost for a specific product variant at location (V,P), and D_{total} is the total demand of products.

It's important to point out that the equipment cost and the ordering cost serve the role of commonality objectives over the entire space. Each cost is related to the number of different variations in design variables and presents a tradeoff between the cost of manufacturing and reduction in material due to individually designed vessels. Since these costs are associated with the commonality over the entire space they must be excluded for the baseline designs during sensitivity analysis, which looks at the sensitivity at discrete instances within the space of customization. As such, the objective function used for the sensitivity analysis will be the minimization of cost to produce a single vessel shown in Equation 3-18.

$$C(L, R, T_s, T_h) = 2\pi\rho\left(C_s R T_s L + C_h R^2 T_h + \frac{2}{9} C_w {T_s}^2 L + \frac{4}{9} C_w \pi {T_s}^2 R + C_p T_s R(L_o + L)\right) \quad (3\text{-}18)$$

3.4.5 Step 3: Identify the Modes for Managing Variety

A mode for managing variety is any approach for achieving customization within the space of customization. For this example, customization must be offered for both volume and pressure. To that end, the designer has identified 3 modes for managing variety; one in the pressure direction and two in the volume direction. Generally speaking the modes of managing variety are dimensional scaling of the design variables or commonization of a design variable.

Dimensional Customization of Shell Length from Stock Plate (Volume)

One mode of providing customization in the volume of a vessel is to vary the shell length of the vessel. Changes in volume are achieved by cutting the shell length from a piece of stock material while the radius remains constant, as shown in Figure 3-10. To calculate the necessary length for a specified volume, *V*, the manufacturer can use:

$$L = \frac{V}{\pi R^2} - \frac{4}{3} R \quad (3\text{-}19)$$

The main decision for this mode is to determine what size stock lengths to carry. Having stock lengths closer to the lengths needed reduces waste costs, however, increases in the amount of different stock lengths also increases the ordering cost.

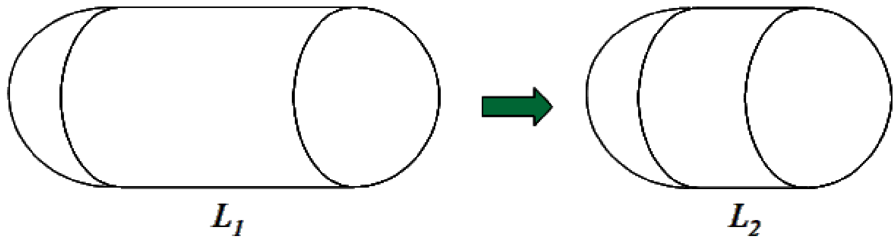

Figure 3-10: Dimensional Customization of the Shell Length
(Hernandez, 2001)

<u>*Commonization of the Vessel Radius (Volume)*</u>

In this mode, changes in volume are made by modifying the radius of the vessel. Modifying the radius presents a cost savings in material as you can reduce the amount of waste material cut from the stock length mentioned above. However, since each different radius used requires the purchase of press and die, the increase in equipment cost limits the amount of variation in radius making for a great example of the tradeoff between commonality and customization.

<u>*Commonization of the Shell and Head thickness (Pressure)*</u>

In this mode, modifications in the thickness of the shell and head enable the designer to change the range of pressure that can be accommodated. While it is not realistic to have infinite variation of thicknesses to meet desired pressures, the general principle is based on the fact that if a vessel with given dimensions of L and R, and plate thickness T_h and T_s, can satisfy the pressure constraint for a pressure P_1, then these thicknesses also satisfy the constraints for any pressure $P \leq P_1$. Therefore, reductions in ordering costs can be made by using shell and head thicknesses which accommodate a larger range of pressures. The tradeoff remains that larger thicknesses increase material cost and offset the reduction in ordering cost.

3.4.6 Step 4: Implement Sensitivity Analysis and Rank Modes for Managing Variety

In Step 4 the proposed sensitivity analysis as described in Section 3.1 and 3.2 will be demonstrated. Recall that the purpose of the sensitivity analysis is to identify which variables are best suited to handle increased commonality and thus provide the designer with a means of determining how to use the modes for managing variety in the constructal hierarchy.

The first step of the sensitivity analysis is to discretize the space of customization and solve for baseline designs at each point. For this example, the space will be discretized by increments of 2 m^3 in the volume direction and 2 MPa in the pressure direction. At this level of discretization there will be 121 baseline variants. For each baseline variant we will solve for the values of the four design variables which minimize the cost of the individual vessel, while satisfying the desired volume and pressure. Additionally, each vessel must obey the bounds on the design variables and stay below the failure criteria of the material. The problem formulation for the baseline vessels is shown in Figure 3-11.

Given:	The 2-dimensional market space $M^2 = \{(V, P)\}$		
	Discretization in 2 m^3 and 2 MPa steps (121 vessel variants)		
	Complete and certain mathematical models (Section 3.4.2)		
Find:	Design Variables, **x**:		
	x = R, L, T_s, T_h		
Satisfy:	Bounds:	$0.5 \leq R \leq 1.5$ m	
		$3 \leq L \leq 7$ m	
		$1 \leq T_s \leq 75$ mm	
		$1 \leq T_h \leq 75$ mm	
	Constraints:		Failure Prevention
			$T_s \geq \left(\frac{P}{\sigma_y - 0.6P}\right)R$
			$T_h \geq \left(\frac{P}{2\sigma_y - 0.2P}\right)R$
Minimize:	The cost of each vessel variant, $C_{V,P}$ (Equation 3-18)		

Figure 3-11: Problem formulation for Baseline Pressure Vessels

The above formulation was solved for each variant using the Matlab constrained optimization function, *fmincon*, specifically the interior point algorithm. A sample of the baseline variants, ranging from 18 – 24 m^3 and 18 – 24 MPa, is presented in Table 3-1. The full table can be found in the Appendix. Additionally, Figure 3-12 provides a visualization of the cost function over the discretized design space. As would be expected there is a continuous increase in cost as the vessels get larger and contain a higher pressure.

Table 3-1: Section Design Variables for Baseline Vessels

V (m^3)	P (Mpa)	R (m)	Ts (mm)	Th (mm)	Lo (m)	Cost ($)
18	18	0.840	14.181	7.031	7.000	4159.04
	20	0.840	15.775	7.814	7.000	4676.27
	22	0.840	17.372	8.597	7.000	5204.79
	24	0.840	18.972	9.380	7.000	5744.66
20	18	0.882	14.897	7.386	7.000	4625.94
	20	0.882	16.571	8.208	7.000	5201.98
	22	0.882	18.248	9.031	7.000	5790.72
	24	0.882	19.930	9.854	7.000	6392.24
22	18	0.922	15.573	7.721	7.000	5093.45
	20	0.922	17.323	8.581	7.000	5728.46
	22	0.922	19.077	9.441	7.000	6377.60
	24	0.922	20.834	10.301	7.000	7040.99
24	18	0.961	16.216	8.040	7.000	5561.52
	20	0.961	18.038	8.935	7.000	6255.65
	22	0.961	19.864	9.830	7.000	6965.39
	24	0.961	21.694	10.726	7.000	7690.82

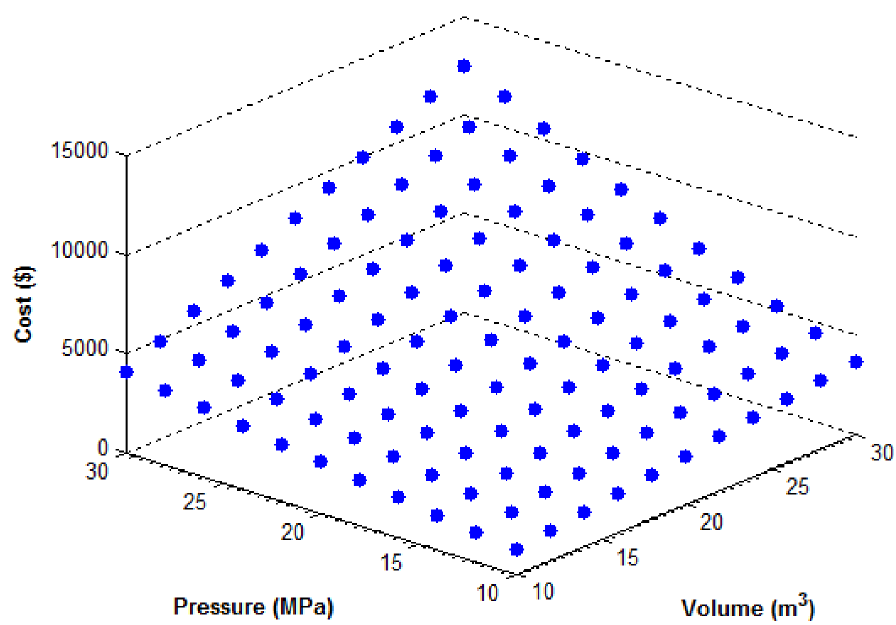

Figure 3-12: Cost of the Baseline Vessels

Inspection of the baseline vessels presented in Table 3-1 reveals that for each of the specified volumes the L variable is set to the upper bound of 7 meters. At first glance this might appear to be an error, however, a quick analysis of the design variables shows that these are indeed correct. For any specified volume there is a frontier of combinations of radius and length that will satisfy it. Figure 3-13 shows an example of this frontier for an example vessel having V = 16 m^3 and P = 16 MPa. Each point represents a combination that satisfies the volume specification and has an associated minimum thickness to satisfy the pressure specification. Seen in the right-hand plot, as radius decreases so does cost, making it desirable to drive the radius down until eventually the length reaches the upper bound. A similar result is seen over the entire space of customization resulting in the length going to the upper bound in all cases.

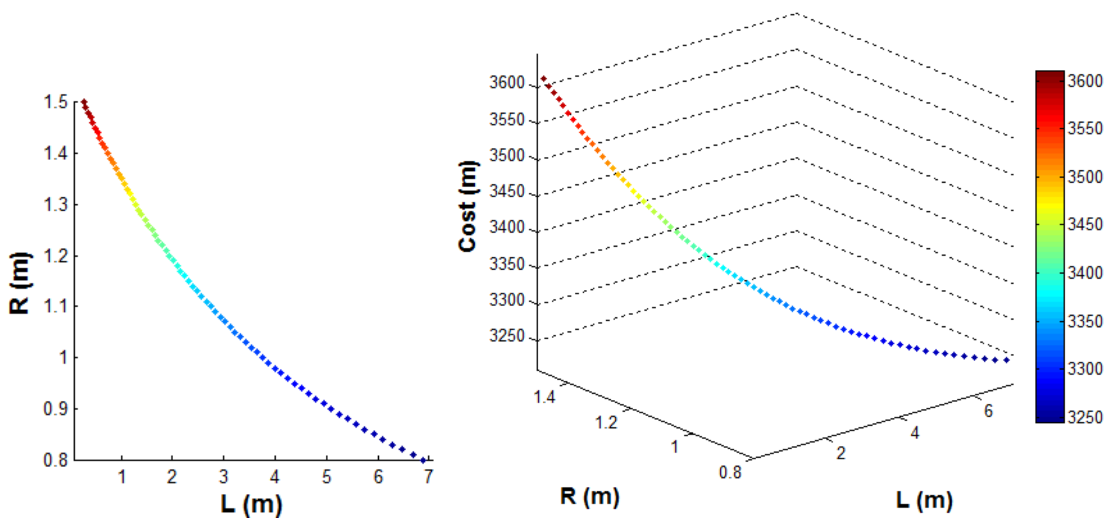

Figure 3-13: Design Variable Exploration for V = 16 m^3, P = 16 MPa

Once the baseline variants have been instantiated over the space of customization, the next step is to select the sensitivity step size. As stated in Section 3.1, the step size (Δx) is chosen at the discretion of the designer. In this example, the step size is chosen to be 5% of the range of each variable for the baseline designs. For example, the radius varies from a minimum value of 0.637 m to a maximum of 1.065, for a range of 0.428 m. Therefore the step size for radius will be 5% of the range, or 0.0214 m. The same process is followed for the other variables with the length being an obvious exception. For the length the author has chosen to use a step size of 5% of the common value. The step size used for each variable is shown in Table 3-2.

Table 3-2: Sensitivity Analysis Step Size

	Design Variables			
	ΔR (m)	ΔL (m)	ΔT_s (mm)	ΔT_h (mm)
Step Size	0.0214	0.35	1.2	0.6

With the step size determined, the local sensitivity is calculated for each product variant with respect to all four design variables. A sample of the local sensitivities from the center of the space of customization, ranging from 18 – 24 m^3 and 18 – 24 MPa, is presented in Table 3-3. The full table can be found in the Appendix.

Table 3-3: Sample of Local Sensitivities

		Design Variables			
V (m^3)	P (Mpa)	R	L	T_s	T_h
18	18	365.47	18.12	289.84	69.16
	20	467.20	23.16	296.23	69.16
	22	581.46	28.83	302.63	69.15
	24	708.33	35.12	309.05	69.15
20	18	395.58	20.79	305.71	76.00
	20	505.69	26.58	312.54	75.93
	22	629.36	33.07	319.38	75.85
	24	766.69	40.29	326.22	75.77
22	18	430.61	23.52	320.80	83.01
	20	550.47	30.07	328.08	82.92
	22	685.10	37.43	335.36	82.82
	24	834.58	45.59	342.65	82.71
24	18	487.85	26.32	335.21	90.91
	20	623.64	33.64	343.02	91.02
	22	776.16	41.87	350.86	91.14
	24	945.51	51.01	358.72	91.27

The global sensitivity of each variable is calculated using Equation 3-2, such that each variant is weighted by the associated demand for the point in the space of customization. The global sensitivities for the four variables are presented in Table 3-4.

Table 3-4: Pressure Vessel Global Sensitivities

	Design Variables			
	R	L	T_s	T_h
Global Sensitivity	578.25	30.31	308.90	76.08

The final part of the sensitivity analysis is to order the global sensitivities from most sensitive to least sensitive. As there are only four variables, the order of sensitivities is readily apparent. Vessel radius shows the most sensitivity to changes away from the optimum and should therefore have the least commonality. Shell and head thickness are in the middle, although shell thickness is considerably more sensitive to changes. Vessel length showed the least sensitivity which is expected based on the exploration shown in Figure 3-13.

3.4.7 Step 5: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety

In this step, the sensitivity data is used to define when and how each of the modes for managing variety will be used. Without this data the designer would be required to make design decisions based on trial and error or run an exhaustive search of all possible levels of commonality.

<u>*First Space Element*</u>

The first space element, S_1, represent the lowest level of the hierarchy and smallest area divisions within the space of customization. The designer therefore wishes to assign those modes of managing variety with the highest sensitivity to these elements as those modes are not well suited for variation away from the optimal value for a particular variant and should therefore have least amount of commonality over the space. Between the two modes of managing variety in the volume direction, changes to the radius of the vessel have the highest sensitivity and will therefore be selected as Mode V_1. In the pressure direction, the sole mode for managing variety, commonization of the head and shell thickness, will be selected as Mode P_1. The two thicknesses could be treated separately and allowed to have different levels of commonality, however, in this example the author will follow previous works which treated them as one mode.

The size of the first space elements is set by the values of ΔV_1 and ΔP_1. They represent the extent to which these variables are made common. An example implementation of the first space elements is shown in Figure 3-14.

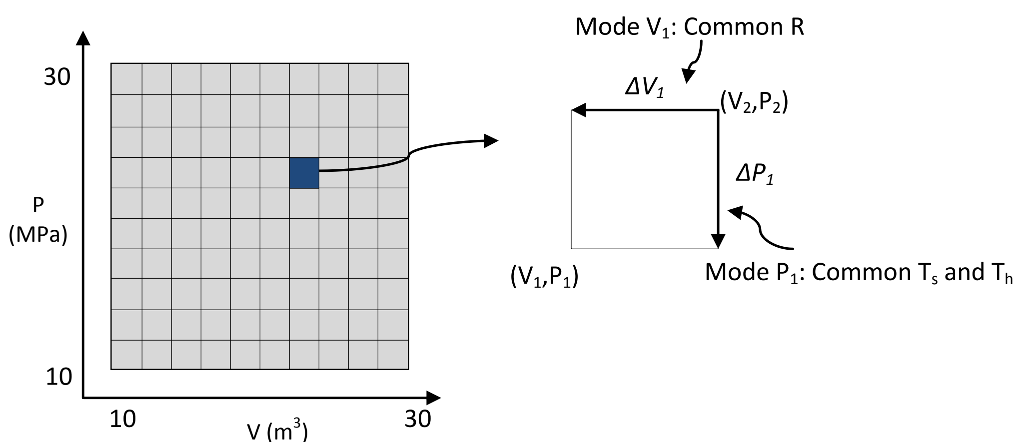

Figure 3-14: The First Space Element of the Space of Customization

Second Space Element

The second space element, S_2, is composed of a number of first space elements, S_1, in the volume direction. For second space element, the designer should select the mode for managing variety with the next highest sensitivity. In this example, the next mode for managing variety in the volume direction is dimensional customization of shell length from stock plate, which is also the final mode. This mode is selected as Mode V_2. The size of the second space element will be determined by ΔV_2 in the volume direction and is fixed by ΔP_1 in the pressure direction as this is the only mode associated with pressure. An example implementation of the second space elements is shown in Figure 3-15.

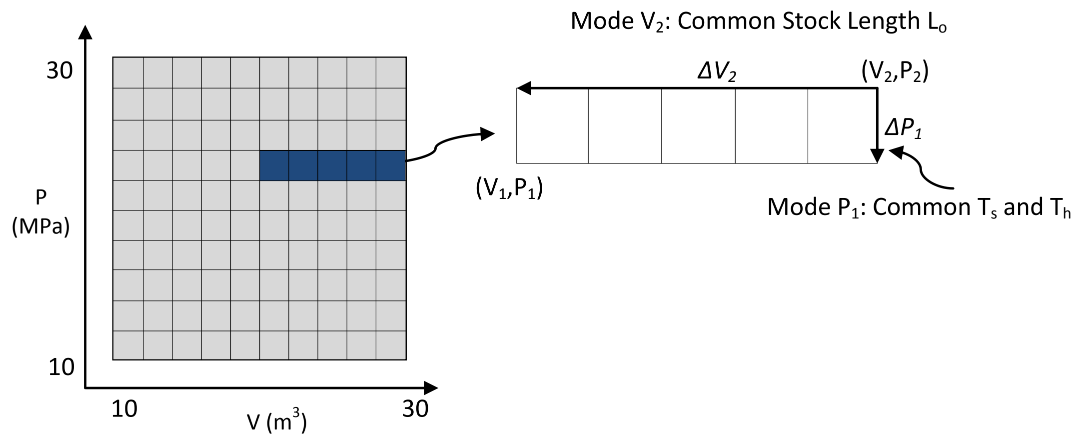

Figure 3-15: The Second Space Element of the Space of Customization

3.4.8 Step 6: Formulate a Combined Decision Support Problem

In this example, the sole objective is the minimization of the average cost to produce pressure vessels over the space of customization. As such, a multiple objective formulation such as the u-cDSP provides no added benefit over a simpler single objective formulation, which will be used in this example. The formulation for the problem of designing a platform for customizable pressure vessels is presented in Figure 3-16. Given as inputs are the market space and the hierarchy of modes of managing variety recently defined. The desired outputs are the size of the space elements, as given by the decision variables ΔV_1, ΔP_1, and ΔV_2, as well as the specific common values of the design variables for each space element. Bounds are provided for the design variables as well as on the decision variables. Lastly, in addition to minimum allowable thicknesses, constraints are implemented, per hierarchical and constructal theory, such that each subsequent higher space element must be greater than or equal in size to a lower space element.

Given:	The 2-dimensional market space $M^2 = (V, P)$ Mode V_1: Commonalize R Mode V_2: Commonalize L_o Mode P_1: Commonalize T_s and T_h		
Find:	The values of decision variables $\Delta V_1, \Delta P_1, \Delta V_2$ The values of the design variables within each space element **x** = R, L, T_s, T_h		
Satisfy:	Relationships:	$\Delta V_1 = f(R)$ $\Delta V_2 = f(L_o)$ $\Delta P_1 = f(T_s, T_h)$	
	Bounds:	$0 \leq \Delta V_1 \leq 20$ $0 \leq \Delta P_1 \leq 20$ $0 \leq \Delta V_2 \leq 20$	$0.5 \leq R \leq 1.5$ m $3 \leq L \leq 7$ m $1 \leq T_s \leq 75$ mm $1 \leq T_h \leq 75$ mm
	Constraints:	$\Delta V_1 \leq \Delta V_2 \leq 20$ Failure Criteria for any Individual Variant: $\sigma_y \geq \frac{RP}{T_s} + 0.6P$ (from Eqn. 3-4) $\sigma_y \geq \frac{RP}{2T_s} + 0.1P$ (from Eqn. 3-5)	
Minimize:	The average cost over the space of customization (Eqn. 3-18): $C_{average} = \frac{1}{D_{total}}\left[\sum_{P=10}^{30}\sum_{V=10}^{30}\left(D_{V,P}C_{V,P}\right) + C_{equip} + C_{order}\right]$		

Figure 3-16: Formulation of the Decision Support Problem for the Pressure Vessel

3.4.9 Step 7: Solve the Decision Support Problem

The final step of the PPCTM is the solution of the Decision Support Problem presented in the previous step. As discussed in (Williams, 2003), there are two methods for analyzing the space of customization: analytical evaluation, or through numerical discretization of the space. An analytical solution was first used by (Hernandez, 2001) during the original implementation of the PPCTM where the objective function was integrated across the space with respect to the design specifications (volume and pressure). While this approach represents the most rigorous and exact technique, it is limited by the need for the objective functions as well as the demand scenarios to be solely functions of the design specifications. This requirement adds considerable complexity to the derivation of the objective functions, and ultimately excludes integration of objective or demand functions which cannot be solved analytically. To circumvent these limitations, Williams proposes a discrete analysis whereby the space is approximated by multiple discrete points across the space. Such an approach is advocated in this work, as it represents a natural extension of the discretization already implemented during sensitivity analysis, and discrete analysis is more easily implemented using common software packages. For a more in depth description of the two approaches please refer to (Williams, 2003).

Following Williams' discretized approach, the designer must select an appropriate solution technique to solve the previously presented problem formulation. Any appropriate solution algorithm can be used so long as its primary goal is the determination of the extent of defined space elements and the minimization of the objective function. As the pressure vessel problem contains relatively few decision variables, an exhaustive search of the different combinations of ΔV_1, ΔP_1, and ΔV_2 is implemented in this work. For a more complex problem more efficient algorithms can replace exhaustive search, however, in this work the benefit of fully exploring the design space outweighs the computational expense. In this implementation, market space discretization was set to 0.1 m^3 and 0.1 MPa in the volume and pressure directions, respectively. Additionally, a minimum value for the decision variables ($\Delta V_1, \Delta P_1, \Delta V_2$) was set to 1 MPa and 1$m^3$, as any point within a 1x1 element could be easily accessed through length customization and would not require a separate platform.

The results of the application of the Sensitivity-Based PPCTM are presented in Table 3-5. The specific values of the design variables are also determined using the PPCTM. A segment of the results are presented in Table 3-6 and the full table is presented in the appendix.

Table 3-5: Pressure Vessel Decision Variable Results

ΔV_1 (m^3)	**ΔP_1 (Mpa)**	**ΔV_2 (m^3)**	**Average Cost ($)**
5	1	20	5880.90

Table 3-6: Segment of Roadmap for Product Specifications and Associated Design Variables

V (m^3)	**P (Mpa)**	**R (m)**	**T_s (mm)**	**T_h (mm)**	**L_o (m)**
10-15	10-11	0.771	7.93	3.94	7
	11-12		8.65	4.30	
	12-13		9.38	4.66	
	13-14		10.10	5.02	
	14-15		10.83	5.38	
	15-16		11.56	5.74	
	16-17		12.29	6.10	
	17-18		13.02	6.46	
	18-19		13.75	6.81	
	19-20		14.48	7.17	
	20-21		15.22	7.53	
	21-22		15.95	7.89	
	22-23		16.68	8.25	
	23-24		17.42	8.61	
	24-25		18.15	8.97	
	25-26		18.89	9.33	
	26-27		19.63	9.69	
	27-28		20.37	10.05	
	28-29		21.11	10.41	
	29-30		21.85	10.77	
15-20	10-11	0.882	9.07	4.51	7
	11-12		9.90	4.92	
	12-13		10.73	5.33	
	13-14		11.56	5.74	
	14-15		12.39	6.15	
	15-16		13.23	6.56	
	16-17		14.06	6.98	
	17-18		14.90	7.39	
	18-19		15.73	7.80	
	19-20		16.57	8.21	
	20-21		17.41	8.62	
	21-22		18.25	9.03	
	22-23		19.09	9.44	
	23-24		19.93	9.85	
	24-25		20.77	10.27	
	25-26		21.61	10.68	
	26-27		22.46	11.09	
	27-28		23.30	11.50	
	28-29		24.15	11.91	
	29-30		25.00	12.32	

To aid the reader with interpreting the results and showing their utility, consider the following tutorial. The results inform the manufacturer that the best configuration of the modes of variety, such that average cost per vessel is minimized, is to commonalize the radius for every 5 m^3 of volume, commonalize the head and shell thickness for every 1 MPa of pressure, and to commonalize the stock plate length over the entire space of customization. Next, the common design variables for each of these spaces serve as a roadmap as to what dimensions are needed for any desired vessel. Consider a customer requesting a pressure vessel having a volume of 17 m^3 able to hold a pressure up to 25 MPa. Following the Table 3-6, the manufacture knows that the baseline design will have a radius of 0.882 m, a shell and head thickness of 22.77 and 10.27 mm, respectively, and will need to be cut from a stock length of 7 m. The length of the specific vessel requested, calculated from Equation 3-19, is:

$$L = \frac{V}{\pi R^2} - \frac{4}{3}R = 5.784 \text{ m}$$

3.4.10 Discussion of Results

Before closing this chapter and the tutorial example it is important to analyze the results and establish the validity of the method. The first step in establishing the validity of the method is to build confidence that the results produced are actually correct and the method has provided the platform with the lowest cost. To do this a small snapshot of the exhaustive search is shown in Table 3-7. From these results we can see that the reported results do indeed provide the lowest cost and the method has passed the first test of validation.

Table 3-7: History of Exhaustive Search

ΔV_1 (m^3)	ΔP_1 (Mpa)	ΔV_2 (m^3)	Average Cost ($)
2	1	20	6049.54
4	1	20	5882.99
5	1	20	5880.90
6	1	20	5918.62
10	1	20	6028.54
2	2	20	6149.38
4	2	20	6010.67
5	2	20	6015.01

The second way in which the validity is checked is through the comparison of the Sensitivity-Based PPTCM results with those of Williams' Augmented PPCTM. This is important in order to show the usefulness of the improved method. The goal of this work is to provide the designer with a better means of selecting modes for managing variety and should therefore provide equivalent or better results than those found using the previous method. For comparison, Williams' results are presented in Table 3-8.

Table 3-8: Williams' PPCTM Results

ΔV_1 (m^3)	ΔP_1 (Mpa)	ΔP_2 (m^3)	Average Cost ($)
2	1	20	6053.56

Comparing the results, we see that the Sensitivity-Based PPCTM platform has a lower average cost than Williams' PPCTM, but in similar ranges. We notice two differences between Williams' results and those presented in Table 3-5, namely a smaller ΔV_1dimension for the first space element and the second space element being allocated to the pressure dimension, ΔP_2, instead of ΔV_2. These differences are a direct result of how the modes for managing variety were selected. From the sensitivity analysis we observed that the pressure vessel length had the lowest sensitivity and was best suited for the higher commonality of the second space element. Without the sensitivity data, Williams chose to apply the length variable to the first space elements which lead to many different stock lengths needed, whereas the sensitivity-based results use one stock length. This increased the ordering cost associated with the stock lengths. Additionally, for the second space element Williams applied commonality of the radius to the pressure dimension whereas this work applied it to the volume dimension. This resulted in a greater number of distinct radii being used and a greater equipment cost. These two factors combined resulted in different space dimensions and higher average cost than the sensitivity-based results. Therefore, even for a platform with relatively few variables the Sensitivity-Based PPCTM helps the designer to make better design decisions, which produce improved results.

While the slightly lower cost using the sensitivity-based method is important, a more interesting/surprising observation is the similarity of the two methods results in terms of commonality, especially given the very different starting formulations and the expectations based on the sensitivity analysis. It was expected that the pressure vessel radius, R, which has the highest sensitivity, should have the least commonality compared with the other variables such as shell thickness, T_h, which has a much lower sensitivity. However, in the final results radius has greater commonality than shell thickness similar to what was found by Williams' using a different formulation. At first glance this may appear to be a major error with the sensitivity-based method, however a closer inspection reveals the source of the discrepancy lies with the objective function used. In the sensitivity calculation, this work only incorporated the individual cost function and excluded the equipment and ordering cost with the intent to separate individual performance from the overall commonality penalties. This exclusion ignores the relative weight of each commonality penalty, such as the very high equipment cost, which as it turns out has a much greater impact than the individual performance lost due to commonality. Had the objective function used included the commonality penalties, the author expects the sensitivity values would be different and suggest the levels of commonality found in Williams' and the present works' solution. Regardless, it is clear that future work should include the commonality penalties into the sensitivity objective function, especially if they have different magnitudes.

A second observation from the comparison with Williams' results is also noted. Williams assigned the pressure vessel radius as a mode for managing variety with respect to pressure whereas this work assigned the radius to be a mode for managing variety with respect to volume. Both possibilities are valid as the radius is coupled with respect to both the pressure and volume calculations and given the results for this simple example it turns out that both selections produce reasonable results. However, consider a situation of higher complexity where there are many design specifications and the design variables are highly coupled between them. How does one decide how to organize coupled design variables? This is a significant obstacle for the application of this method (and other methods) to higher complexity problems and requires further investigation in future work.

3.5 Summary

In this chapter, a new Sensitivity-Based PPCTM is presented along with a tutorial example of its application to the design of a family of customizable pressure vessels. The main objective of the sensitivity based approach is to provide the designer with a systematic method for assigning modes of managing variety to space elements based on how well suited a mode is for commonality. Section 3.1 discusses the underlying principles of the sensitivity analysis and presents the sensitivity index to be used. Section 3.2 details the specific steps of the sensitivity analysis and how they are infused into the PPCTM, followed by discussion of a more general problem formulation in Section 3.3. These sections serve as the theoretical backbone for answering the primary research question showing the specific augmentations to the PPCTM. Lastly, the new Sensitivity-Based PPCTM is presented in Section 3.4 accompanied by a tutorial example detailing each step. The pressure vessel example shows how sensitivity analysis can be implemented to effectively determine which design variables are best suited for commonality and how they should be ordered in the space hierarchy. Therefore, the proposed method and tutorial example present the authors answer to the Primary Research Question, and initial validation of Hypothesis 1. The pressure vessel example represents a good first problem for validation as it covers multiple extensions of the PPCTM, including multi-dimensional space and non-uniform demand. However, one limit of this problem is the small number of design variables making the importance of the sensitivity data less obvious. In the next chapter, the augmented PPCTM will be applied to two more examples, helping to further validate Hypothesis 1 and demonstrate the method's utility.

CHAPTER FOUR

CUSTOMIZED PRODUCT PLATFORM DESIGN: CASE STUDIES

A method for assessing extent of platform commonality and designing families of products for customization, along with an initial validation, was presented in Chapter 3. The objective in this chapter is to continue that validation through the presentation of two additional case study examples. Specifically, we design product platforms for families of customizable universal electric motors and customizable finger pumps. Like the pressure vessel, the universal electric motor has been used as a benchmark example in many product family publications and will serve well as a second validation of the proposed method and enable easy comparison with other published works.

The second example involves applying the method to a newly developed pumping technology. The novel pumping technology could be applied to a number of different industries and is well suited to be expanded into a product family. The true goal of any design method is to expand out of research and be applied to real product development. This example will therefore serve as "sanity check" for the applicability of the proposed method to the top down design of a new customizable product family.

4.1 Design of a Platform for Universal Electric Motors

4.1.1 Universal Electric Motor Description

Universal electric motors got their name for their ability to function on both alternating current (AC) and direct current (DC). In addition to this flexibility, universal motors can produce more torque per amount of current that any other type of single phase motor (Chapman). Owing to their high performance characteristics and flexibility, universal electric motors are used in a wide range of applications. Many household appliances such as electric drills, saws, blenders, and vacuums are all driven by a universal electric motor (Veinott and Martin).

A universal electric motor is composed of two main components: an armature (also called a rotor) and a field (also called a stator), see Figure 4-1. The armature consists of a solid metal shaft and slats around which wire is wrapped as many as a thousand times. The armature rotates within the field, which consists of a hollow metal cylinder also with slats wrapped longitudinally a few hundred times. The armature and field are wired in series so that both run on the same amount of current. As the current passes through the windings around the field, a large magnetic field is generated within the hollow cylinder. This field exerts a force on the armature which is also carrying current. Due to the geometry of the windings, current on one side of the armature is always flowing in the opposite direction of current the other side, generating a net torque causing the armature to spin within the field. This concludes the description of the universal electric motor, let us now consider the design problem statement.

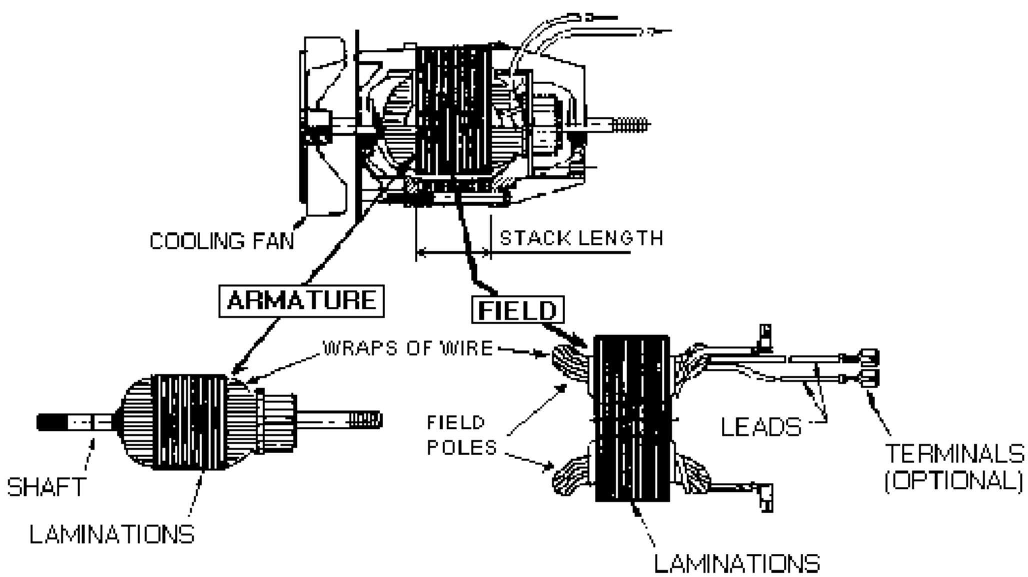

Figure 4-1: Schematic of a Universal Electric Motor (Simpson, 1998)

4.1.2 Universal Electric Motor Problem Statement

Imagine you are the project manager for the household appliances division of a major global conglomerate. Your company is looking to launch a new line of products that utilize universal electric motors. In the past, your company has spent a great deal of time and money designing and producing individual motors for each new product. This time, however, you would like to develop the family of motors around standard platforms that can be efficiently customized to meet new product demand.

More specifically, the problem is to develop a product platform for a family of universal electric motors that satisfies a range of torque requirements between 0.05 and 0.5 Nm. The other motor characteristics to be considered include motor mass M, efficiency η, power P, and magnetizing intensity H. From the performance standpoint the designer is particularly concerned with mass and efficiency, which are chosen to be minimized and maximized respectively. Additionally there are a number of other design constraints imposed, given below:

1. Power (P): The desired power for each motor in the family is 300 W.
2. Efficiency (η): The target for average efficiency for all of the motors is 70%, but it is never allowed to fall below 15%.
3. Mass (M): The target for average mass for all of the motors is 0.5 kg, with a maximum allowable mass of 2 kg.
4. Magnetizing intensity (H): All motor designs should maintain magnetizing intensities below 5000 Amp·turns/m to ensure that the magnetizing flux within the motor does not exceed the physical flux capacity of the steel.

4.1.3 Universal Electric Motor Model

The mathematical model of the universal electric motor used in this work is derived directly from the work of Simpson and coauthors (Simpson, et al., 2001). This section is meant only as a summary, therefore, for further explanation on the derivation of certain formulas, the reader is directed to (Simpson, et al., 2001), or the handbooks upon which this model was originally based (Chapman, 1991, Cogdell, 1996). This model takes in eight design variables as input (N_c, N_s, A_{wa}, A_{wf}, r_o, t, L, I) and returns as output the power (P), torque (T), mass (M), and efficiency (η) of the motor. The eight design variables and their bounds are described below.

1. Number of wire turns on the armature, N_c
2. Number of wire turns on each field pole, N_s
3. Cross-sectional area of armature wire, A_{wa}
4. Cross-sectional area of field wire, A_{wf}
5. Radius of the stator, r_o
6. Thickness of the stator, t

7. Stack length of the motor, L

8. Current drawn by the motor, I

The bounds on the motor variables are shown in Table 4-1.

Table 4-1: Bounds on the Motor Design Variables

Variable	Units	Min	Max
N_c	Turns	100	1500
N_s	Turns	1	500
A_{wa}	mm^2	0.01	1.0
A_{wf}	mm^2	0.01	1.0
r_o	cm	1.0	10.0
t	cm	0.5	10.0
L	cm	0.1	10.0
I	Amp	0.1	6.0

Mass of Electric Motor:

The mass of the universal electric motor is the sum of the masses of the armature, the field, and the windings on both the armature and field. In the case of this example, a greatly simplified motor model is used where the armature is modeled as a solid steel cylinder and the stator is modeled as a hollow steel cylinder. Thus the overall formula for the mass of the motor is:

$$Mass = M_{stator} + M_{armature} + M_{windings} \tag{4-1}$$

where

$$M_{stator} = \pi \cdot L \cdot \rho_{steel}({r_o}^2 - (r_o - t)^2) \tag{4-2}$$

$$M_{armature} = \pi \cdot L \cdot \rho_{steel}\left(r_o - t - l_{gap}\right)^2) \tag{4-3}$$

$$M_{windings} = \rho_{copper}\left[N_c \cdot A_{wa}(2L + 4\left(r_o - t - l_{gap}\right) + 2 \cdot N_s \cdot A_{wf}(2L + 4(r_o - t))\right] \tag{4-4}$$

Power Calculations:

The basic governing equation for the power output of a motor is given by the input power minus the power losses.

$$P = P_{in} + P_{losses} \tag{4-5}$$

where the input power is given by the product of the voltage and the current:

$$P_{in} = V_t \cdot I \quad (4\text{-}6)$$

There are a variety of reasons for losses within a motor including the heating of the wires, the interface between the brushes and the armature, the friction in the motor's bearings, as well as hysteresis and eddy currents in the motor core. Assuming that the motor is designed well and used properly, a number of these losses prove to be negligible and can be ignored, including thermal and frictional losses. As such, a simplified expression for power losses is presented based solely on the copper heating and the brush interfaces.

$$P_{losses} = P_{copper} + P_{brush} \quad (4\text{-}7)$$

where

$$P_{copper} = I^2(R_a + R_s) \quad (4\text{-}8)$$

and

$$P_{brush} = \alpha \cdot I \quad (4\text{-}9)$$

where α is typically given a value of 2 Volts.

In Equation 4-8, R_a and R_s are the resistances in the wire winding of the armature and the stator and can be further specified as functions of the design variables. These resistances can be calculated using the general equation for resistance in a wire given by:

$$Resistance = \frac{(Resistivity)(Length)}{Cross - Sectional\ Area} \quad (4\text{-}10)$$

From the above equation, where resistivity (ρ) is a property of the wire and the wire is assumed to have roughly rectangular cross sections, it can be shown that the resistances of the wire on the armature and the stator are:

$$R_a = \frac{\rho \cdot N_c \cdot \left(2L + 4\left(r_o - t - l_{gap}\right)\right)}{A_{cross-section,armature\ wire}} \quad (4\text{-}11)$$

and

$$R_s = \frac{2\rho \cdot N_s \cdot \left(2L + 4(r_o - t)\right)}{A_{cross-section, field\ wire}} \tag{4-12}$$

Efficiency Calculations:

Motor efficiency can be calculated directly from the equations for power given in Equations 4-5 and 4-6. The basic equation for efficiency is given by:

$$\eta = \frac{P}{P_{in}} = \frac{(P_{in} - P_{losses})}{P_{in}} \tag{4-13}$$

Torque Calculations:

The final equation to derive is an equation for torque of the motor. The torque output of the motor is given by:

$$T = K \cdot \phi \cdot I \tag{4-14}$$

Where K is a motor constant, ϕ is the magnetic flux in the motor and I is the current. Assuming the armature has simplex winding and the number of poles in the motor two, the motor constant can be reduced to:

$$K = \frac{N_c}{\pi} \tag{4-15}$$

Deriving the equation for magnetic flux is considerable more complicated. At the basic level, the equation for flux through a magnetic circuit is found by dividing the magnetomotive force ($\boldsymbol{J}$) by the total reluctance in the motor ($\boldsymbol{R}$), the bold font is meant to distinguish between reluctance and resistance calculations:

$$\boldsymbol{\phi} = \frac{\boldsymbol{J}}{\boldsymbol{R}} \tag{4-16}$$

where the magnetomotive force is simply the number of turns around one pole of the stator:

$$\boldsymbol{J} = N_s \cdot I \qquad (4\text{-}17)$$

The basic formula for reluctance is given by:

$$\boldsymbol{R} = \frac{Length}{(Permeability)(Area_{cross-sectional})} \qquad (4\text{-}18)$$

Permeability, μ, is expressed as relative permeability of the material multiplied by the permeability of free space, μ_o. For the whole motor, the total reluctance is the sum of the reluctances of the stator, rotor and two air gaps:

$$\boldsymbol{R} = \boldsymbol{R}_s + \boldsymbol{R}_r + 2\boldsymbol{R}_{air} \qquad (4\text{-}19)$$

where,

$$\boldsymbol{R}_s = \frac{(\pi(2r_o + t)/2)}{2 \cdot \mu_{steel} \cdot \mu_o \cdot t \cdot L} \qquad (4\text{-}20)$$

$$\boldsymbol{R}_r = \frac{1}{\mu_{steel} \cdot \mu_o \cdot L} \qquad (4\text{-}21)$$

$$\boldsymbol{R}_{air} = \frac{1}{\mu_{air} \cdot \mu_o \cdot L} \qquad (4\text{-}22)$$

and the permeability of steel (μ_{steel}) is calculated as a function of magnetizing intensity given by three sections of a curve using the following expressions:

$$\begin{array}{ll} \mu_{steel} = -0.2279 \cdot H^2 + 52.41 \cdot H & H \leq 220 \\ \quad + 3115.8 & 220 \leq H \leq 1000 \\ \mu_{steel} = 11633.5 - 1486.33 \cdot \ln(H) & H > 1000 \\ \mu_{steel} = 1000 & \end{array} \qquad (4\text{-}23)$$

The magnetizing intensity is given by:

$$H = \frac{N_c \cdot I}{l_c + l_r + 2 \cdot l_{gap}} \qquad (4\text{-}24)$$

where l_c is the mean magnetic path length of the stator/field, which is taken to be half of the stator's inner circumference, l_r is the diameter of the armature, and l_{gap} is the length of the air gap.

4.1.4 Implementing the Augmented Product Platform Constructal Theory Method

4.1.4.1 Step 1: Define the Space of Customization and Demand Scenario

In this example, variety will be offered for two specifications, namely, the power P (in Watts) and the torque T (in N·m). The first specification, power, is held constant at 300 Watts. Therefore, the space of customization is one dimensional, with torque ranging from 0.05 to 0.5 N·m, as shown in Figure 4-2. For simplicity, demand will be uniform across the space.

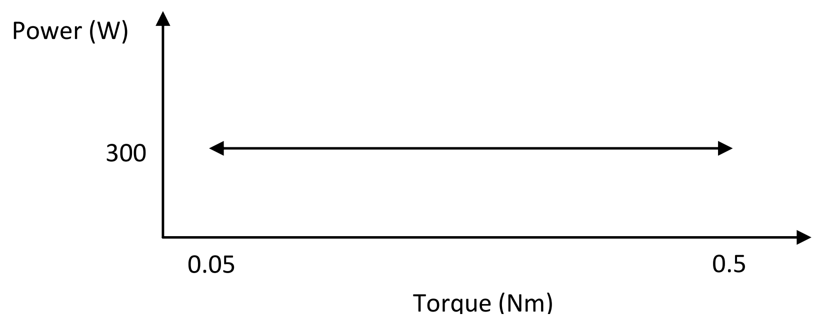

Figure 4-2: Electric Motor Space of Customization

4.1.4.2 Step 2: Define the Objective Functions

The goal in developing this product platform is to maximize overall performance within the product family, while managing the platform commonality. This performance is characterized by two objectives: minimization of average mass and maximization of average efficiency. General equations for mass and efficiency are given in Equations 4-1 and 4-13. Following these equations, average mass and efficiency is given by the summation of the product variants across the entire market space divided by the total number of variants, shown in Equations 4-25 and 4-26.

$$\bar{m} = \frac{1}{n}\sum_{i=1}^{n} M_i \tag{4-25}$$

$$\bar{\eta} = \frac{1}{n}\sum_{i=1}^{n} \eta_i \tag{4-26}$$

where *n* is the number of variants.

The next step when defining the objective functions is to combine the multiple objectives listed into a single aggregated objective function. This process is accomplished through the use of utility theory, where designer preferences can be quantified and then combined into a multi-attribute utility function. The steps for implementing utility theory are shown below, as described in (Seepersad, 2001).

The first step is to assess the utility functions for each objective. These functions are determined by first declaring the absolute design extremes, of what value is ideal (which will have a utility of 1) and what value is unacceptable (which will have a utility of 0). The values between these extremes are then determined through a series of hypothetical situations used to assess the designer's preferences. The utility values for the universal motor example are shown in Table 4-2.

Table 4-2: Universal Electric Motor Utility Function Assessment

Utility Value	Design Situation	Mass	Efficiency
1	The decision-maker's ideal attribute level – beyond which the decision-maker is indifferent to further improvements in the attribute.	0.25	0.95
0.75	The decision-maker is indifferent between obtaining a design alternative with a 'desirable' attribute value for certain and a design alternative with a 50-50 chance of yielding either a tolerable or an ideal attribute level.	0.75	0.65
0.50	The decision-maker is indifferent between obtaining a design alternative with a 'tolerable' attribute value for certain and a design alternative with a 50-50 chance of yielding either an unacceptable attribute value or an ideal attribute value.	1.25	0.4
0.25	The decision-maker is indifferent between obtaining a design alternative with an 'undesirable' attribute value for certain and a design alternative with a 50-50 chance of yielding either a tolerable or an unacceptable attribute value.	1.75	0.25
0	The decision-maker's unacceptable attribute level – beyond which he/she is unwilling to accept an alternative.	2	0.15

These points are then fitted with polynomial curves to establish the independent utility equations for motor mass and efficiency, shown in Equations 4-27 and 4-28.

$$u_{mass} = -0.0872m^2 - 0.3509m + 1.0825 \tag{4-27}$$

$$u_{\eta} = -1.0567\eta^2 + 2.3625\eta - 0.3022 \tag{4-28}$$

With the utility values assessed, the resulting utility functions are plotted below in Figure 4-3. Note the upper and lower saturations as indicated by the preferences shown in Table 4-2. Therefore, any mass below 0.25 kg or efficiency above 95% are considered ideal and given an equal utility of 1. Similarly, any mass above 2 kg or efficiency below 15% are considered unacceptable and given a utility of 0.

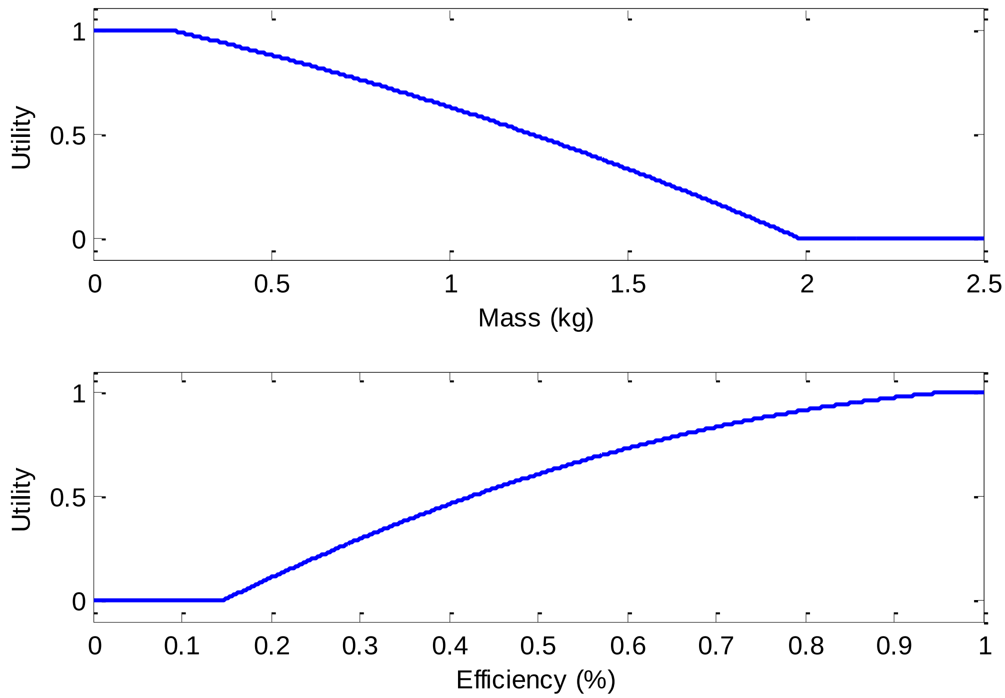

Figure 4-3: Utility Curves for Mass and Efficiency

The next step is to combine these individual utility functions into a multi-attribute utility function. This is accomplished through a weighted sum of the two utility functions shown in Equation 4-29:

$$U = k_{mass}u_{mass} + k_{\eta}u_{\eta} \tag{4-29}$$

where k_{mass} and k_{η} are scaling constants for mass and efficiency. For the case of this design problem $k_{mass} = 0.5$ and $k_{\eta} = 0.5$, as the design gives equal preference to both objectives.

Lastly, the deviation function is formulated to minimize the deviation from the target utility (i.e. 1). This approach is carried over from utility theory, but is mathematically equivalent to maximizing the multi-attribute utility function. The resulting deviation function to be minimized is shown in Equation 4-30.

$$Z = 1 - U \qquad (4\text{-}30)$$

Equation 4-30 will be used for development of the individual baseline motor variants. In order to maximize performance over the entire space of customization it must be expanded into a summation across the discretized space. Before presenting the final summation, a commonality penalty function is introduced. In the pressure vessel example, the tradeoff between commonality and performance was incorporated to the cost model through an ordering cost and an equipment cost. Each of these factors served to penalize a platform that had little commonality. For the current example, the motor model has no such penalties, and would maximize performance through minimizing commonality. This work therefore incorporates a bulk commonality penalty function shown in Equation 4-31:

$$C_{penalty} = \frac{\sigma_{A_{wf}}}{\mu_{A_{wf}}} + \frac{\sigma_{N_s}}{\mu_{N_s}} + \frac{\sigma_t}{\mu_t} + \frac{\sigma_{A_{wa}}}{\mu_{A_{wa}}} + \frac{\sigma_{N_c}}{\mu_{N_c}} + \frac{\sigma_{r_o}}{\mu_{r_o}} \qquad (4\text{-}31)$$

where σ and μ are the standard deviation and the mean of the design variables across the space of customization. Other works have used a similar penalty aiming to incorporate the tradeoff between performance and commonality (Messac, et al., 2002, Khire and Messac 2008).

The final objective to be minimized over the space of customization is the summation of the individual deviation functions, Z_i, divided by the number of variants, n, plus the commonality penalty function, $C_{penalty}$, as shown in Equation 4-32.

$$Z_{avg} = \frac{1}{n}\sum_{i=1}^{n} Z_i + C_{penalty} \qquad (4\text{-}32)$$

4.1.4.3 *Step 3: Identify the Modes for Managing Variety*

As shown in the tutorial example, the modes for managing variety are the designer's means of building the elements of the space of customization that serve as platforms for delivering customized variants. The universal electric motor modes for managing product variety are:

1. Commonization of the number of wire turns on the armature, N_c
2. Commonization of the number of wire turns on each field pole, N_s
3. Commonization of the cross-sectional area of armature wire, A_{wa}
4. Commonization of the cross-sectional area of field wire, A_{wf}
5. Commonization of the radius of the stator, r_o
6. Commonization of the thickness of the stator, t

Keeping with Simpson's original implementation, current and length (*I* and *L*) are allowed to vary such that the desired torque and power requirements are met. They are therefore not considered modes for managing variety, but must still be determined for each product variant.

4.1.4.4 *Step 4: Implement Sensitivity Analysis and Rank the Modes for Managing Variety*

Following the proposed augmented PPCTM, sensitivity analysis will now be conducted for the 6 modes for managing variety. The first step is to solve for baseline design variants across the space of customization. For this example, the space will be discretized into 0.05 Nm increments. This discretization is chosen at the designer's discretion, but must be sufficiently small to capture sensitivity changes across the entire range of customization. The value of 0.05 Nm was chosen based on previous universal motor examples where 10 variants were offered based on 0.05 Nm torque increases.

For each baseline variant the 8 design variables which minimize the objective function must be determined, while satisfying the desired torque and power requirements. Additionally, each motor must obey the bounds on the design variables as well as the constraints on magnetizing intensity and feasibility. The problem formulation for the baseline product motors is shown in Figure 4-4.

Given:	The 1-dimensional space of customization M^1 Discretization in 0.05 Nm steps Baseline Variants, T = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50 Nm Universal Motor Equations (see Section 4.1.3)		
Find:	Design Variables, **x**: **x** = N_c, N_s, A_{wa}, A_{wf}, r_o, t, L, I		
Satisfy:	Bounds:	$100 \le N_c \le 1500$ turns	$1.0 \le r_o \le 10.0$ cm
		$1 \le N_s \le 500$ turns	$0.1 \le t \le 10$ mm
		$0.01 \le A_{wa} \le 1.0$ mm^2	$0.1 \le L \le 10$ cm
		$0.01 \le A_{wf} \le 1.0$ mm^2	$0.1 \le I \le 6.0$ Amp
	Constraints:	Magnetizing Intensity:	$H \le 5000$ Amp•turns/m (Eqn. 4-24)
		Feasible Geometry:	$t < r_o$
		Power:	$P = 300$ Watts (Eqn. 4-5)
		Efficiency:	$\eta \geq 0.15$ (Eqn. 4-13)
		Mass:	$M \le 2.0$ kg (Eqn. 4-1)
Minimize:	$Z = 1 - U$ where U is given by Equation 4-29		

Figure 4-4: Problem formulation for designing baseline universal motors

Similar to the pressure vessel example, this formulation was solved using the Matlab optimization function, *fmincon*. The ten individually optimized motor solutions along with their masses and efficiencies are shown in Table 4-3.

Table 4-3: Individual Baseline Motor Variants

	Design Variables								Responses	
Motor Torque	N_c (turns)	N_s (turns)	A_{wa} (mm^2)	A_{wf} (mm^2)	r_o (cm)	t (mm)	L (cm)	*I* (Amps)	η (%)	M (kg)
0.05	686	70	0.246	0.246	1.71	4.27	1.83	3.05	85.4	0.280
0.10	856	77	0.261	0.261	1.99	5.31	2.08	3.25	80.2	0.424
0.15	969	80	0.271	0.271	2.18	6.00	2.23	3.42	76.2	0.538
0.20	1052	81	0.279	0.279	2.31	6.53	2.35	3.58	72.8	0.633
0.25	1118	81	0.285	0.285	2.42	6.96	2.43	3.74	69.8	0.716
0.30	1171	81	0.290	0.290	2.51	7.32	2.50	3.89	67.0	0.788
0.35	1215	80	0.295	0.295	2.59	7.62	2.57	4.05	64.4	0.854
0.40	1252	79	0.300	0.300	2.66	7.89	2.62	4.21	62.0	0.912
0.45	1282	78	0.304	0.304	2.72	8.12	2.66	4.37	59.7	0.965
0.50	1307	76	0.308	0.308	2.77	8.33	2.70	4.54	57.4	1.013

With the baseline variants determined, sensitivity analysis will now be conducted for each of the 6 modes of managing variety. Before calculating the local sensitivities the designer must specify sensitivity step size. In this example, the step size is chosen to be 10% of the range of each variable for the baseline designs. The step size used for each variable is shown in Table 4-4.

Table 4-4: Electric Motor Sensitivity Step Size

	Design Variables					
	ΔN_c (turns)	ΔN_s (turns)	ΔA_{wa} (mm^2)	ΔA_{wf} (mm^2)	Δr_o (cm)	Δt (cm)
Step Size	62	1	0.006	0.006	0.106	0.405

Next, the local sensitivities are calculated for each baseline variants with respect to each of the 6 modes for managing variety using Equation 3-1. The global sensitivity for each mode is then calculated as the average value of the local sensitivities using Equation 3-2. The local sensitivities over the discretized space are shown in Table 4-5. The last row shows the averaged global sensitivity for each mode.

Table 4-5: Electric Motor Local and Global Sensitivities

	Design Variables					
Motor Torque	N_c (turns)	N_s (turns)	A_{wa} (mm^2)	A_{wf} (mm^2)	r_o (cm)	t (cm)
0.05	0.1967	0.0119	0.0215	0.0036	0.1960	0.0381
0.10	0.2064	0.0146	0.0336	0.0045	0.2213	0.0419
0.15	0.2163	0.0167	0.0440	0.0051	0.2393	0.0448
0.20	0.2257	0.0186	0.0535	0.0055	0.2536	0.0471
0.25	0.2347	0.0203	0.0625	0.0058	0.2655	0.0492
0.30	0.2433	0.0218	0.0710	0.0060	0.2755	0.0509
0.35	0.2514	0.0232	0.0795	0.0061	0.2842	0.0525
0.40	0.2592	0.0246	0.0870	0.0062	0.2916	0.0539
0.45	0.2667	0.0258	0.0944	0.0062	0.2980	0.0551
0.50	0.2738	0.0270	0.1016	0.0062	0.3036	0.0562
Global	0.2374	0.0205	0.0648	0.0055	0.2629	0.0490

The last step of the sensitivity analysis is to rank order the global sensitivities from most sensitive to least sensitive as shown in Table 4-6.

Table 4-6: Rank of Global Sensitivities

r_o	0.2629	1
N_c	0.2374	1
A_{wa}	0.0648	2
t	0.0490	2
N_s	0.0205	2
A_{wf}	0.0055	3

4.1.4.5 Step 5: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety

Using the ranked global sensitivities, the number of hierarchy levels and their associated modes for managing variety can now be determined. This work suggests that modes for managing variety with similar sensitivities be grouped together into the same hierarchy level. For this example, the designer notes three magnitudes of sensitivity values in Table 4-6 and therefore elects for three levels of space elements (T_1, T_2, and T_3). T_1 spaces will have common r_o and N_c, T_2 spaces will have common A_{wa}, t, and N_s, and T_3 space will have common A_{wf}. Each level of space elements are described below.

First Space Element

As described during the pressure vessel example, the first space element is the lowest level in the hierarchy and has least amount of commonality. The modes with the highest sensitivity are therefore chosen for this level in the hierarchy. As such, commonization of the radius, r_o, and commonization of the number of turns of wire on the armature, N_c, are selected as Mode T_1. The size of the first space elements is set by the value of ΔT_1 and will serve as the decision variable to set the amount of commonality for the first level. An example implementation of the first space elements is shown in Figure 4-5.

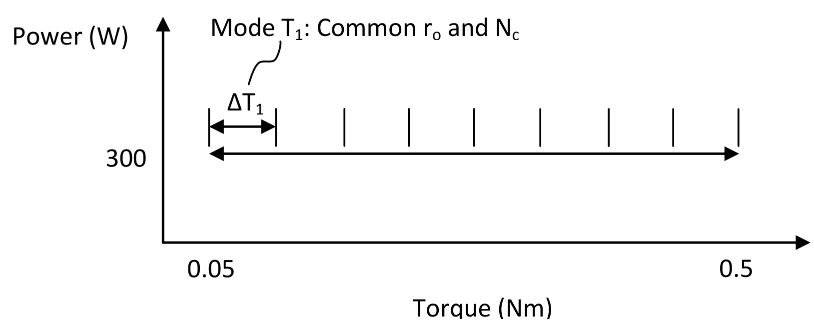

Figure 4-5: First Space Element of the Motor Space of Customization

Second Space Element

For the second space elements, commonization of the area of the wire on the armature, A_{wa}, thickness of the stator, t, and number of turns of wire on each pole of the field, N_s, are selected as Mode T_2. Each of the three has a middle level of sensitivity, all of similar magnitude. The decision variable to determine the size of the second space elements is ΔT_2. An example implementation of the second space elements is shown Figure 4-6.

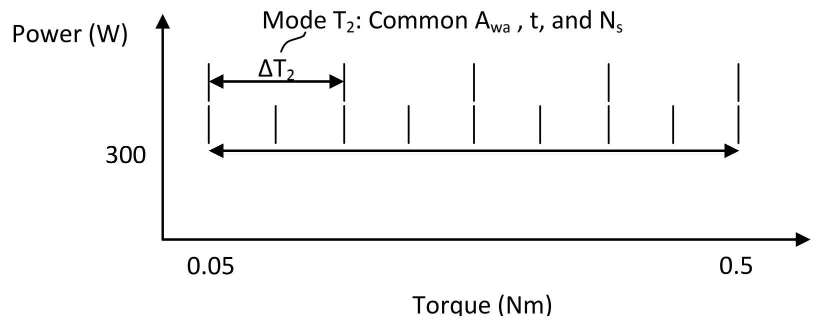

Figure 4-6: Second Space Element of the Motor Space of Customization

Third Space Element

The final space element in the hierarchy will commonalize the cross sectional area of the wire on the field, A_{wf}, which is selected as Mode T_3. The area of the wire on the field has the lowest sensitivity and is therefore best suited for the greater commonality of a higher space element. ΔT_3 is the last decision variable, which will set the size of the third level of space elements. An illustration of the final space element is shown Figure 4-7.

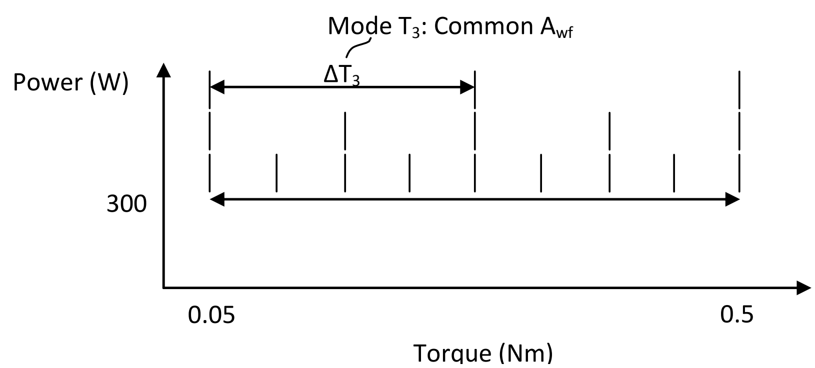

Figure 4-7: Third Space Element of the Motor Space of Customization

4.1.4.6 Step 6: Formulate a Combined Decision Support Problem

As stated in the problem description, the two objectives for this example are to maximize the average motor efficiency and to minimize the average motor mass over the space of customization. To handle these two competing objectives, this example utilizes the utility-based decision support problem (u-DSP) which is a simple extension of the formulation presented in Section 3.3. The u-DSP embodies a theoretical construct, which enables mathematical modeling of the designer's preference for multiple objectives such that they can be combined into a single expected utility function. Use of the aggregated utility function makes for easy comparison among design decisions, and does not require normalization of objective function values like a simple weighted sum. Development of the aggregated utility function, which combines the designer's preferences for both efficiency and mass, is discussed in Section 4.1.4.2. The formulation for the problem of designing a platform for customizable electric motors is presented in Figure 4-8. The main goal of this formulation is to find the size of the space elements, represented by the decision variables ΔT_1, ΔT_2, and ΔT_3, which maximize efficiency and minimize mass. Additionally, we wish to determine the specific instantiated value for each design variable which will be held common over each space element. Given as inputs are the motor space of customization and the hierarchy of modes for managing variety as presented in the previous section. Relationships are listed to show how each space element is related to the specified design variables. Bounds and constraints presented are similar to those used during development of the baseline motors. The only change is the addition of bounds on the space elements, which are determined by the space of customization, and the constraint that each subsequent higher space element must be greater than or equal in size to the next lower element. Lastly, the deviation function to be minimized has been adapted from the individual formulation to reflect an average over the whole space.

Given:	The 1-dimensional space of customization M^1: T = [0.05,0.5] Mode T_1: Commonization of r_o andN_c Mode T_2: Commonization of A_{wa}, t, and N_s Mode T_3: Commonization of A_{wf}		
Find:	Value of the decision variables ΔT_1, ΔT_2, ΔT_3 The values of the common design variables within each space element **x** = N_c, N_s, A_{wa}, A_{wf}, r_o, *t*, *L*, *I*		
Satisfy:	Relationships:	$\Delta T_1 = f(r_o, N_c)$ $\Delta T_2 = f(A_{wa}, t, N_s)$ $\Delta T_3 = f(A_{wf})$	
	Bounds:	$0.05 \leq \Delta T_1 \leq 0.5$ $0.05 \leq \Delta T_2 \leq 0.5$ $0.05 \leq \Delta T_3 \leq 0.5$	$100 \leq N_c \leq 1500$ turns $1 \leq N_s \leq 500$ turns $0.01 \leq A_{wa} \leq 1.0$ mm^2 $0.01 \leq A_{wf} \leq 1.0$ mm^2 $1.0 \leq r_o \leq 10.0$ cm $0.1 \leq t \leq 10$ mm $0.1 \leq L \leq 10$ cm $0.1 \leq I \leq 6.0$ Amp
	Constraints:	$\Delta T_1 \leq \Delta T_2 \leq \Delta T_3 \leq 0.5$	
		Magnetizing Intensity:	$H \leq 5000$ Amp•turns/m (Eqn. 4-24)
		Feasible Geometry:	$t < r_o$
		Power:	P = 300 Watts (Eqn. 4-5)
		Efficiency:	$\eta \geq 0.15$ (Eqn. 4-13)
		Mass:	$M \leq 2.0$ kg (Eqn. 4-1)

Minimize:

$$Z_{avg} = \frac{1}{n}\sum_{i=1}^{n} Z_i + C_{penalty}$$

where *i* = 1,2, … , *n* based on the level discretization from T= 0.05 to 0.5

Z_i is given by Equation 4-30 and $C_{penalty}$ is given by Equation 4-31

Figure 4-8: Formulation of the Electric Motor Decision Support Problem

4.1.4.7 Step 7: Solve the Decision Support Problem

As is discussed in Section 3.4.9 for the pressure vessel example, solution of the decision support problem is conducted over a discretized space rather than the continuous integration approach proposed by Hernandez. In this example, the space of customization is discretized in steps of 0.01 Nm. To solve the formulation over the discretized space a combination of exhaustive search and Matlab's constrained optimization function, fmincon, was used. Having only three decision variables, an exhaustive search was used to iterate through the different combinations of ΔT1, ΔT2, and ΔT3. The specific design variables for each space is determined using the constrained optimization algorithm as was done for the baseline motors. The resulting decision variables found using the Sensitivity-Based PPCTM are presented in Table 4-7, along with the average mass and efficiency over the space of customization. The specific values of the design variables within each space are shown in Table 4-8.

Table 4-7: Electric Motor Decision Variable Results

ΔT_1 (Nm)	ΔT_2 (Nm)	ΔT_3 (Nm)	Avg. Mass (kg)	Avg. Eff (%)	U_{avg}
0.15	0.15	0.45	0.664	64.3	0.793

Table 4-8: Roadmap for Product Specifications and Associated Design Variables

	Design Variables					
Motor Torque	A_{wf} (mm^2)	N_s (turns)	t (mm)	A_{wa} (mm^2)	N_c (turns)	r_o (cm)
0.05-0.10						
0.10-0.15		71	5.32	0.230	926	2.12
0.15-0.20						
0.20-0.25						
0.25-0.30	0.310	71	6.66	0.253	1123	2.48
0.30-0.35						
0.35-0.40						
0.40-0.45		66	7.51	0.269	1197	2.70
0.45-0.50						

4.1.5 Discussion and Comparison of Results

The results shown inTable 4-8 present a customizable platform enabling an electric motor manufacturer to provide virtually continuous variation for torques between 0.05 and 0.50 Nm while providing a high level of performance and commonality (i.e. at most three values for any single process variable). This is significant as no other prominent product platform method has been used to provide customization or such high variety for the electric motor example. In this section, the results will be discussed with the aim of validating the augmented method and establishing its usefulness. This will be accomplished through answering two questions:

I. *Does the Sensitivity-Based PPCTM produce better result than the previous version using ad hoc variable selection?*
II. *Does the Sensitivity-Based PPCTM produce better results than other product platform methods?*

The goal of adding sensitivity analysis to the PPCTM was to enable the designer to make better choices determining which modes for managing variety to assign to the different hierarchy levels leading to better platform performance. To show that the sensitivity-based method does improve performance, we must compare with results using the old PPCTM which assigned modes based on designer experience. As no previous PPCTM research has used the electric motor example, these results must be developed here. To do this the same problem formulation as Figure 4-8 has been run, however, in this case the designer has selected commonization of A_{wf} to be used in the Mode T_1, and commonization of r_o to be used in Mode T_3 (simply switching A_{wf} and r_o). Therefore, the comparison platform has the following mode allocation, which yielded the results in Table 4-9:

- Mode T_1: Commonization of A_{wf} andN_c
- Mode T_2: Commonization of A_{wa}, t, and N_s
- Mode T_3: Commonization of r_o

Table 4-9: Comparison Results using previous PPCTM

ΔT_1 (Nm)	ΔT_2 (Nm)	ΔT_3 (Nm)	Avg. Mass (kg)	Avg. Eff (%)	U_{avg}
0.15	0.15	0.45	0.693	61.7	0.780

Comparing the above results with the Sensitivity-Based method shows reduced performance resulting from a single mix up when assigning modes for managing variety. While the effect is relatively small, it certainly shows the importance of proper selection. The author therefore concludes that the Sensitivity-Based PPCTM does aid the designer in making better platform selections and improving results, which further validates the method.

To build confidence in the usefulness of this method it is also important to benchmark against other product family methods. The sensitivity based results are therefore compared with two product platforms developed using the PPCEM (Simpson et al., 2001). The first platform uses all six variables (N_c, N_s, A_{wa}, A_{wf}, r_o, t) as common platform variables, allowing only I and L to vary. The second has less commonality, using t and r_o as platform variables and allowing the rest to vary. It is important to note that the PPCEM platforms were developed for a family of 10 specific torque values (T = [0.05,0.10,0.125, 0.15,0.20,0.25,0.30,0.35,0.40,0.50]. The Sensitivity-Based platform was therefore used to instantiate products to meet these 10 torque values as shown in Table 4-10.

Table 4-10: Sensitivity-Based PPCTM customized motors

	Design Variables								Responses	
Motor Torque	N_c (turns)	N_s (turns)	A_{wa} (mm^2)	A_{wf} (mm^2)	r_o (cm)	t (mm)	L (cm)	*I* (Amps)	η (%)	M (kg)
0.05	926	71	0.230	0.310	2.12	5.32	1.83	3.05	81.1	0.292
0.10	926	71	0.230	0.310	2.12	5.32	2.08	3.25	76.6	0.410
0.125	926	71	0.230	0.310	2.12	5.32	2.23	3.42	74.5	0.459
0.15	926	71	0.230	0.310	2.12	5.32	2.35	3.58	72.5	0.503
0.20	1123	71	0.253	0.310	2.48	6.66	2.43	3.74	67.7	0.618
0.25	1123	71	0.253	0.310	2.48	6.66	2.50	3.89	64.7	0.675
0.30	1123	71	0.253	0.310	2.48	6.66	2.57	4.05	61.7	0.719
0.35	1197	66	0.269	0.310	2.70	7.51	2.62	4.21	58.2	0.833
0.40	1197	66	0.269	0.310	2.70	7.51	2.66	4.37	55.6	0.858
0.50	1197	66	0.269	0.310	2.70	7.51	2.70	4.54	50.8	0.884

Table 4-11 and Table 4-12 show the comparison results between the proposed method and two PPCEM families. The PPCTM family has a significantly higher average efficiency and lower average mass than the first PPCEM family. Much of this performance gain is a result of lower commonality in the proposed method. However, the proposed method also produces slightly better results compared with the second family, while also having significantly greater commonality measured by the number of different instantiated values of the design variables (35 compared with 60).

These results show that the proposed Sensitivity-Based PPCTM is on par with existing product family methods and can successfully instantiate a desired family within the space of customization. The proposed method has better performance than the benchmark method, even at a higher level of commonality. Thus, the usefulness of the Sensitivity-Based method is established and the method is validated.

Table 4-11: Comparison of PPCTM with PPCEM Family 1

	PPCTM		PPCEM I and L Vary		Percent difference	
Motor Torque	η	M (kg)	η	M (kg)	η	M (kg)
0.05	81.1	0.292	76.8	0.380	5.60	-23.21
0.10	76.6	0.410	72.2	0.520	6.09	-21.15
0.125	74.5	0.459	70.0	0.576	6.41	-20.31
0.15	72.5	0.502	67.9	0.625	6.75	-19.63
0.20	67.7	0.618	63.9	0.703	5.87	-12.11
0.25	64.6	0.675	60.2	0.759	7.24	-11.08
0.30	61.7	0.719	56.8	0.797	8.56	-9.77
0.35	58.2	0.833	53.6	0.820	8.62	1.55
0.40	55.6	0.858	50.5	0.830	10.18	3.42
0.50	50.8	0.884	44.8	0.820	13.37	7.77
				Average change:	7.87	-10.45

Table 4-12: Comparison of PPCTM with PPCEM Family 2

	PPCTM		PPCEM A_{wa}, A_{wf}, N_s, N_c, L, I Vary		Percent difference	
Motor Torque	η	M (kg)	η	M (kg)	η	M (kg)
0.05	81.1	0.292	74.7	0.397	8.57	-26.50
0.10	76.6	0.410	72.1	0.456	6.24	-10.09
0.125	74.5	0.459	71.1	0.477	4.77	-3.77
0.15	72.5	0.502	70.1	0.499	3.40	0.66
0.20	67.7	0.618	67.5	0.568	0.22	8.79
0.25	64.6	0.675	64.6	0.646	0.00	4.47
0.30	61.7	0.719	62.2	0.712	-0.87	1.00
0.35	58.2	0.833	59.9	0.774	-2.80	7.58
0.40	55.6	0.858	57.7	0.833	-3.57	3.05
0.5	50.8	0.884	53.8	0.941	-5.59	-6.09
				Average change:	1.04	-2.09

4.2 Design of a Platform for Customizable Finger Pumps

4.2.1 Finger Pump Description

The motivation for this example problem revolves around the development of a lightweight and efficient pumping device. Currently, peristaltic pumps, otherwise known as roller pumps, are widely used to pump a variety of different fluids. Some common applications include drug delivery, pumping of caustic chemicals, dialysis, and cardiac bypass. They are a type of positive displacement pump which uses a roller to push fluid through a flexible tube. One of the major benefits of this type of pump is that the fluid always remains in the tube and therefore never comes in contact with the pumping mechanism. This is a significant advantage when it is necessary to pump a sterile fluid, a very aggressive chemical, or any time you wish to ensure no cross contamination of your fluid. For that reason roller pumps have become very popular in biomedical applications as well as pumping chemicals in lab environments. Due to the nature of how the fluid is pumped, it is necessary for extremely stiff tubing to be used. This greatly reduces the pump's efficiency as well as increases the size and weight of the pump due to the large motor needed. With this increased size and power consumption it is very difficult for these pumps to be utilized in portable and size constrained application, and it would therefore be extremely beneficial to develop a small, light and efficient alternative to the roller pump.

It light of these limitations an alternative to peristaltic roller pumps has been developed to enable portable hemodialysis (Kang, 2010). This pump technology utilizes a series of fingers to push the fluid through a tube to achieve the desired flow rates. The finger pump, which still classifies as a positive displacement pump, maintains all of the benefits of roller pumps (i.e., no contamination of the fluid) with the added benefits of higher efficiency and reduction in size compared to similar flow rate pumps, as well as a reduction in clotting when pumping biological fluids. In addition to hemodialysis this technology, and its benefits, could also be utilized in many other applications where roller pumps are currently used, however each applications will require a different flow rate to be achieved requiring additional design work. A CAD model of the pump design and physical prototype are shown below in Figure 4-9.

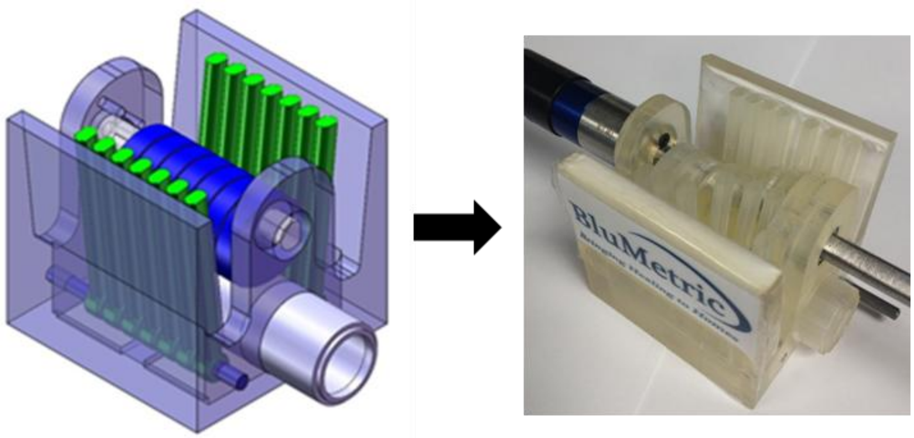

Figure 4-9: Pump Design and Physical Prototype (Courtesy Jane Kang)

4.2.2 Finger Pump Problem Statement

After the initial development of this pumping technology for portable hemodialysis it has become necessary to expand this technology to meet a greater range of market needs and expand the product portfolio. However, to individually design and manufacture a pump for each specific application would be far too costly and time consuming, thus limiting the feasibility of expansion and business success. We therefore propose the utilization of a top down design approach to develop a family of pumps, which can cover the full range of flow rates necessary. By developing a standardized product platform, we will be able to scale the current pump model to meet any need, thus reducing design time/cost, while allowing for customization.

Specifically, the problem is to develop a customizable product platform for a family of finger pumps that satisfies a range of flow rate requirements between 100 and 600 ml/min. From a performance standpoint, the goal is to minimize pump volume and maximize pump efficiency, while maximizing commonality within the family. Demand for these pumps is assumed uniform across the market space. In the next section the pump model will be introduced to describe the working principles behind this technology.

4.2.3 Finger Pump Model

The mathematical model of the finger pump used in this work is developed directly from the Master's Thesis of Jane Kang (Kang, 2010). It should be noted that this model is a condensed version of Ms. Kang's model and relies on several empirical relations and constants determined during the pumps development. While the model presented here covers all aspects relevant to the present work, readers are directed to (Kang, 2010) for further information and explanation of the finger pump technology. This model takes in five design variables as input (T_w, T_H, , F_w, N_f, V) and returns as output the achieved flow rate (FR), efficiency (η), and volume (*Vol*) of the finger pump. The five design variables and their bounds are described below.

1. Tube width or squeeze distance, T_w
2. Tube height, T_H
3. Finger width, F_w

4. Number of fingers on a side, N_f

5. Voltage, V

The bounds on the pump variables are shown in Table 4-13.

Table 4-13: Bounds on the Pump Design Variables

Variable	Units	Min	Max
T_w	cm	0.5	2.5
T_H	cm	0.5	3.0
F_w	cm	0.3	1.0
N_f	No.	5	12
V	Volts	2	12

To aid the reader in understanding how these variables relate to the physical pump, Figure 4-10 diagrams the variable dimensions on the pump components along with the overall dimensions of the pump body.

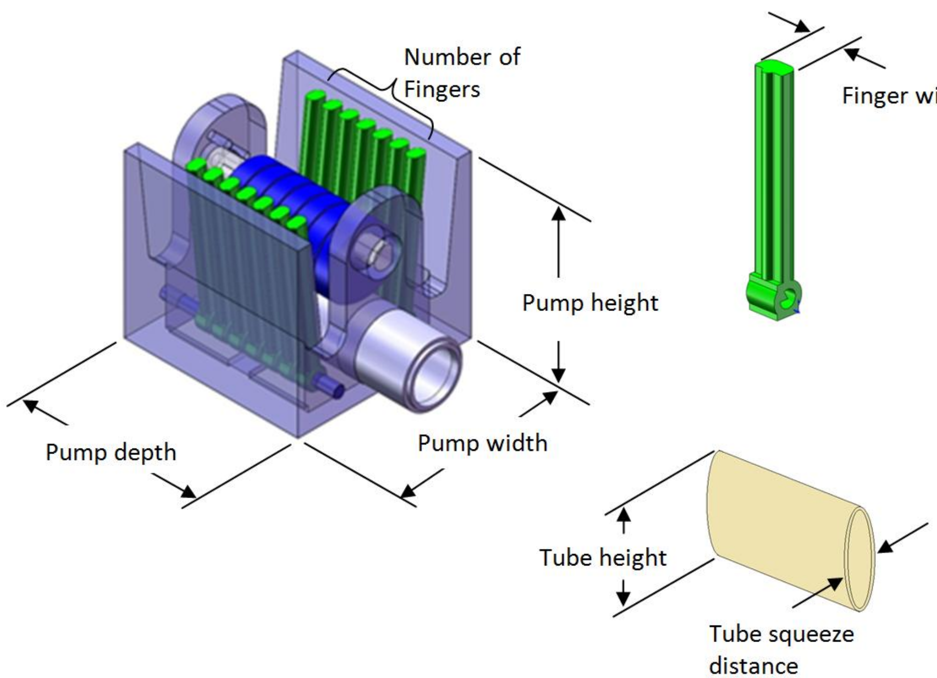

Figure 4-10: Finger Pump Design Variables

Flow rate Calculations:

Flow in the finger pump technology is generated by a motor driven cam, which sequentially presses the fingers onto the tube. This compresses the fluid filled tubing in order to push the fluid forward. Therefore, the volume of fluid displaced by a finger stroke (ml) and the rate of the strokes (/min) determine the flow rate produced by the pump, as shown in Equation 4-33.

$$Flow\ Rate\ = Volume\ per\ Stroke\ \cdot Rate\ of\ Strokes \qquad (4\text{-}33)$$

The volume per stroke is the volume of fluid in the section of tubing beneath the finger about to displace it and is therefore the product of the cross section of the tube and the finger width.

$$Volume\ per\ Stroke\ = Effective\ Cross\ Section\ \cdot Finger\ Width \qquad (4\text{-}34)$$

When the tube is inserted into the pump, it takes the shape of a long oval, as shown in Figure 4-10. While the oval cross section can be calculated as the product of the tube width and tube height with a constant, namely π/4, testing of the model using this value lead to over estimates of the pump flow rate as compared with experimental. This is due to a number of effects, such as head change, and back flow. To compensate for these losses the cross section has been adapted to an effective cross section using an “oval constant”. The oval constant is used as a lumped constant to account for pump losses and is determined using experimental data. The effective cross section is calculated using Equation 4-35.

$$Effective\ Cross\ Section = Oval\ Constant\ \cdot Tube\ Width\ \cdot Tube\ Height \qquad (4\text{-}35)$$

where for the current pump setup the oval constant was determined to be, OC = 0.589.

Assuming a 1:1 gearing from the motor to the cam shaft, each motor revolution results in one stroke for each finger in the row. The rate of stroke is therefore simply the product of the motor speed and the number of fingers in the row.

$$Rate\ of\ Strokes\ = No.of\ Fingers\ \cdot Motor\ Speed \qquad (4\text{-}36)$$

where the motor speed as a function of input voltage is approximated by the following linear fit of experimental data.

$$Motor\ Speed\ = 9.083\ \cdot Voltage + 6.514 \qquad (4\text{-}37)$$

Efficiency Calculations:

The first performance characteristic to be considered is pump efficiency. Efficiency is calculated by dividing the fluid power by the brake power as shown in Equation 4-38.

$$\eta = \frac{Fluid\ Power}{Brake\ Power} \qquad (4\text{-}38)$$

Fluid power refers to the theoretical power required to transport the fluid at a specified flow rate and pressure. In this example, pressure is set to blood pressure, 100mmHg, as the expected end use will be for biological applications. The equation for fluid power is given by:

$$Fluid\ Power\ = Flow\ Rate\ \cdot Pressure \qquad (4\text{-}39)$$

Brake refers to the power required to operate the pump. It is calculated by multiplying the required voltage and current as shown in Equation 4-40.

$$Brake\ Power\ = Voltage\ \cdot Current \qquad (4\text{-}40)$$

Volume Calculations:

The second performance characteristic to consider is the size of the pump. The volume of the pump is calculated by multiplying the three characteristic lengths of the pump as shown in Equation 4-41.

$$Volume = Dept \cdot Width \cdot Height \tag{4-41}$$

The pump depth is defined as the length of the pump in the flow direction and is a function of the number of fingers, the finger width and the size of the frame.

$$Depth \ = No.of\ Fingers \ \cdot Finger\ Width + \alpha \tag{4-42}$$

The width of the pump is defined as the same direction as the squeeze direction and can be calculated using Equation 4-43. The terms are doubled to reflect symmetry along the center plane of the pump.

$$Width \ = 2 \ \cdot [Tube\ Width + \beta] \tag{4-43}$$

The height of the pump is defined in the direction if the tube height and can be calculated by adding the tube height with the size of the frame.

$$Height \ = Tube\ Height + \gamma \tag{4-44}$$

α, β, and γ are constants used to incorporate the additional length of the pump frame and space for the cams. For the current implementation these values have been set to $\alpha = 1$, $\beta = 1$, and $\gamma = 2$ cm. Note that these constants can be adjusted to accommodate design changes such as added frame stiffness.

4.2.4 Implementing the Augmented Product Platform Constructal Theory Method

4.2.4.1 Step 1: Define the Space of Customization and Demand Scenario

For the finger pump example, variety will be offered for only one design specification namely pump flow rate. This represents a one dimensional space of customization, as shown in Figure 4-11, where flow rate customization is offered from 100 ml/min to 600 ml/min. Demand is selected to be a uniform distribution across the space.

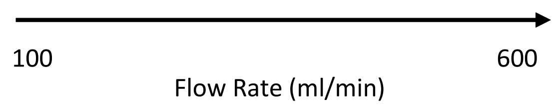

Figure 4-11: Finger Pump Space of Customization

4.2.4.2 Step 2: Define the Objective Functions

As mentioned in the problem description, the goal in developing this product platform is to maximize overall performance of the family of pumps. This performance is characterized by two objectives: maximization of average efficiency and minimization of average volume. The equations for efficiency and volume are given in Equations 4-38 and4-41. Using these equations, average efficiency and volume is given by the summation shown in Equations 4-45 and 4-46.

$$\bar{\eta} = \frac{1}{n}\sum_{i=1}^{n} \eta_i \tag{4-45}$$

$$\overline{Vol} = \frac{1}{n}\sum_{i=1}^{n} Vol_i \tag{4-46}$$

where n is the number of variants.

Following the procedure used in the previous example the objectives are now combined into a single aggregated multi-attribute utility function. First, the designer’s preferences are assessed to determine the utility values for each objective as shown in Table 4-14.

Table 4-14: Finger Pump Utility Function Assessment

Utility Value	Design Situation	Volume	Efficiency
1	The decision-maker's ideal attribute level – beyond which the decision-maker is indifferent to further improvements in the attribute.	50	0.2
0.75	The decision-maker is indifferent between obtaining a design alternative with a 'desirable' attribute value for certain and a design alternative with a 50-50 chance of yielding either a tolerable or an ideal attribute level.	125	0.15
0.50	The decision-maker is indifferent between obtaining a design alternative with a 'tolerable' attribute value for certain and a design alternative with a 50-50 chance of yielding either an unacceptable attribute value or an ideal attribute value.	175	0.1
0.25	The decision-maker is indifferent between obtaining a design alternative with an 'undesirable' attribute value for certain and a design alternative with a 50-50 chance of yielding either a tolerable or an unacceptable attribute value.	205	0.05
0	The decision-maker's unacceptable attribute level – beyond which he/she is unwilling to accept.	250	0.02

These points are then fitted with polynomial curves to establish the independent utility equations for pump efficiency and volume, shown in Equations 4-47 and 4-47.

$$u_{\eta} = -6.107\eta^2 + 6.776\eta - 0.1105 \qquad (4\text{-}47)$$

$$u_{Vol} = -1.419 * 10^{-5} Vol^2 - 8.781 * 10^{-4} Vol + 1.084 \qquad (4\text{-}48)$$

The resulting utility functions are plotted below in Figure 4-12.

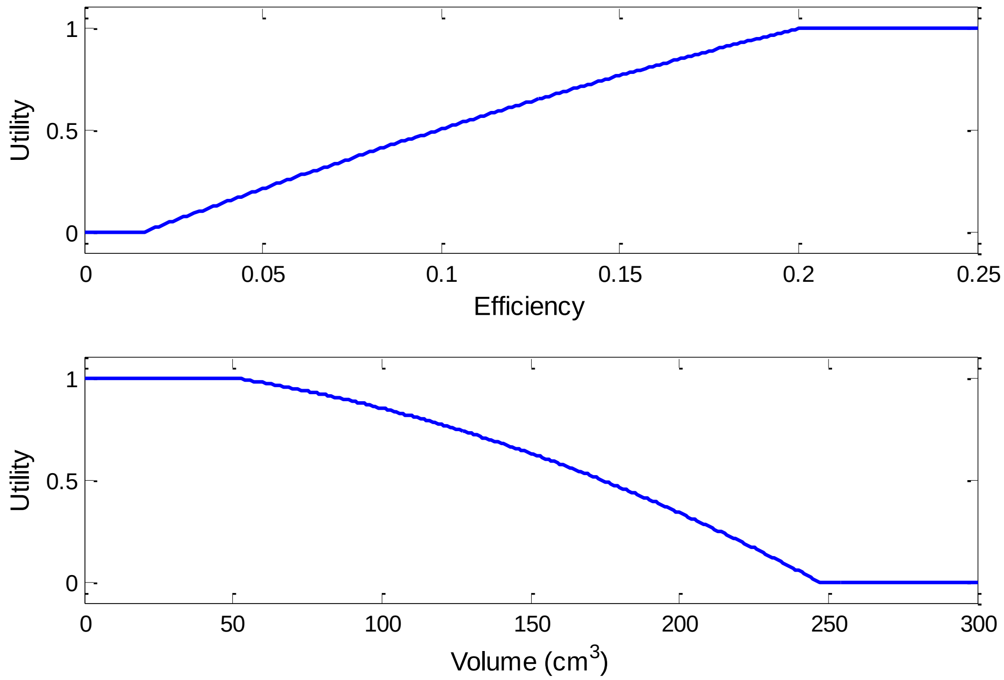

Figure 4-12: Utility Curves for Efficiency and Volume

These individual utility functions are now combined into a multi-attribute utility function. This is accomplished through a weighted sum of the two utility functions shown in Equation 4-49:

$$U = k_\eta u_\eta + k_{Vol} u_{Vol} \qquad (4\text{-}49)$$

where k_η and k_{Vol} and are scaling constants for efficiency and volume. For this design problem $k_\eta = 0.5$ and $k_{Vol} = 0.5$ as the designer gives equal preference to both objectives.

Lastly, the deviation function is formulated to minimize the deviation from the target utility (i.e. 1), which is equivalent to maximizing overall performance. The resulting deviation function to be minimized is shown in Equation 4-50.

$$Z = 1 - U \qquad (4\text{-}50)$$

Equation 4-50 is used to develop the individual baseline pumps. In order to maximize performance over the entire space of customization it must be expanded into a summation across the discretized space. Additionally, as there is no inherent tradeoff between commonality and performance, a commonality penalty function is included similar to the motor example. The bulk commonality penalty function is shown in Equation 4-51:

$$C_{penalty} = \frac{\sigma_{T_w}}{\mu_{T_w}} + \frac{\sigma_{T_H}}{\mu_{T_H}} + \frac{\sigma_{F_w}}{\mu_{F_w}} + \frac{\sigma_{N_f}}{\mu_{N_f}} \tag{4-51}$$

where σ and μ are the standard deviation and the mean of the design variables across the space of customization.

The final objective to be minimized over the space of customization is the summation of the individual deviation functions, Z_i, divided by the number of variants, n, plus the commonality penalty function, $C_{penalty}$, as shown in Equation 4-52.

$$Z_{avg} = \frac{1}{n}\sum_{i=1}^{n} Z_i + C_{penalty} \tag{4-52}$$

4.2.4.3 Step 3: Identify the Modes for Managing Variety

Customization of the finger pumps is achieved through the following modes for managing variety:

1. Commonization of the tube width, T_w
2. Commonization of the tube height, T_H
3. Commonization of the finger width, F_w
4. Commonization of the number of fingers, N_f

Note that voltage is not considered as a mode for managing variety, and is allowed to vary such that the desired flow rate is met. The necessary voltage is still important, however, as the power necessary affects efficiency.

4.2.4.4 *Step 4: Implement Sensitivity Analysis and Rank the Modes for Managing Variety*

With the four modes for managing variety defined, sensitivity analysis will now be conducted. First, baseline design variants are solved for across a discretized space of customization. For this example, the space has been discretized into 50 ml/min increments as smaller increments can be readily achieved through voltage adjustments to change the motor speed.

For each baseline design variant the five design variables which achieve the desired flow rate and minimize the objective function (Section 4.2.4.2) are determined. The problem formulation for the baseline pump variants is shown in Figure 4-13.

Given:	The 1-dimensional space of customization M^1 Discretization in 50 ml/min steps Baseline Variants, *FR* = 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600 ml/min Finger Pump Equations (see Section 4.2.3)		
Find:	Design Variables, **x**: **x** = T_w, T_H, , F_w, N_f, V		
Satisfy:	Bounds:	$0.5 \leq T_w \leq 2.5$ cm $0.5 \leq T_H \leq 3.0$ cm $0.3 \leq F_w \leq 1.0$ cm	$5 \leq N_f \leq 12$ $2 \leq V \leq 12$ Volts
Minimize:	$Z = 1 - U$ where U is given by Equation 4-49		

Figure 4-13: Problem formulation for designing baseline finger pumps

This formulation was solved using the Matlab optimization function, fmincon. The eleven individually optimized pumps along with their performance (efficiency and volume) are shown in Table 4-15.

Table 4-15: Individual Baseline Pump Variants

	Design Variables				Responses	
Flow Rate	T_w (cm)	T_H (cm)	F_w (cm)	N_f (No.)	η (%)	Volume (cm^3)
100	2.102	2.235	0.353	5	17.9	116.1
150	2.178	2.346	0.377	5	18.1	125.9
200	2.234	2.438	0.397	5	18.2	134.1
250	2.277	2.516	0.415	5	18.3	141.3
300	2.275	2.496	0.455	5	18.6	148.5
350	2.443	2.733	0.427	5	19.7	156.3
400	2.454	2.794	0.440	5	19.7	161.6
450	2.447	2.723	0.477	5	19.8	166.9
500	2.491	2.867	0.468	5	19.9	171.7
550	2.473	2.906	0.483	5	19.7	175.6
600	2.495	2.978	0.488	5	19.9	180.2

Next, sensitivity analysis is conducted for each of the four modes of managing variety. The step size for calculating the local sensitivities is chosen to be 25% of the range of each variable for the baseline designs. The step size used for each variable is shown in Table 4-16.

Table 4-16: Finger Pump Step Size

	Design Variables			
	Δ T_w (cm)	Δ T_H (cm)	Δ F_w (cm)	Δ N_f (fingers)
Step Size	0.100	0.1875	0.0375	1

Following the procedure defined in Chapter 3, the local sensitivities are calculated for each baseline variant with respect to each of the four modes of managing variety using Equation 3-1. The global sensitivity is then calculated for each mode using Equation 3-2.

The local sensitivities over the space of customization along with the combined global sensitivity are shown in Table 4-17.

Table 4-17: Finger Pump Local and Global Sensitivities

	Design Variables			
Flow Rate	T_w (cm)	T_H (cm)	F_w (cm)	N_f (fingers)
100	0.0142	0.0637	0.0210	0.3363
150	0.0615	0.1271	0.0312	0.3488
200	0.0264	0.0743	0.0195	0.4347
250	0.0211	0.0732	0.0188	0.3635
300	0.0149	0.1674	0.0293	0.4143
350	0.0576	0.1879	0.5480	0.5215
400	0.0315	0.0962	0.3776	0.5025
450	0.0583	0.0679	0.4745	0.4401
500	0.0328	0.1417	0.4248	0.5640
550	0.0490	0.1962	0.5916	0.5789
600	0.0822	0.1849	0.6605	0.5317
Global	0.0409	0.1255	0.2634	0.4578

The last step of the sensitivity analysis is to rank order the global sensitivities from most sensitive to least sensitive as shown in Table 4-18:

Table 4-18: Rank of Global Sensitivities

N_f	0.4578
F_w	0.2634
T_H	0.1255
T_w	0.0409

4.2.4.5 Step 5: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety

Having calculated the global sensitivities for each mode for managing variety, they are now used to determine the number of hierarchy levels and their associated modes. The guiding principles for this step are: 1) modes with the highest sensitivities are allocated to the lowest levels while modes with lower sensitivities are used on higher levels, and 2) modes with similar sensitivities can be grouped together in the same level. For this example, the design sees no clear groupings of the sensitivity values and therefore elects to assign each mode for managing variety to its own hierarchy level. Thus, there will be four levels of space elements (FR_1, FR_2, FR_3, FR_4). Each of these levels will now be allocated to one of the four modes for managing variety. Generally speaking the mode with the highest sensitivity, commonization of the number of fingers, would be assigned to the lowest level of the hierarchy. However, a closer investigation into this sensitivity value and the baseline pumps reveals this is not the best option. Table 4-15 shows that the baseline optimal designs all share a common number of fingers, the minimum value of five fingers. Therefore, while the sensitivity value is correct, indicating that changing the number away from the optimum will have the greatest impact on performance, we can already see that this value will be driven to the minimum value and made common across the entire space of customization. This mode for managing variety is therefore assigned to the highest level FR_4, where commonality is the highest. This choice is made mainly to reduce the computational expense. If for instance the designer followed the sensitivity information and chose to assign them to the lowest space element, the variable would still be found common in each separate space element it would just be more expensive to iterate through all of the smaller sized elements without any benefit.

The remainder of the modes for managing variety are assigned in order of highest sensitivity to lowest:

First Space Element (smallest space elements and lowest commonality over the space):

- Commonization of the finger width, F_w
- The decision variable for this level is ΔFR_1

Second Space Element (middle-range space elements):

- Commonization of the tube height, T_H
- The decision variable for this level is ΔFR_2

Third Space Element (middle-range space elements):

- Commonization of the tube width, T_w
- The decision variable for this level is ΔFR_3

Fourth Space Element (largest space elements and greatest commonality over the space)

- Commonization of the number of fingers, N_f
- The decision variable for this level is ΔFR_4

An example implementation of the hierarchic organization of these space elements is shown in Figure 4-14. In the next section the mathematical formulation to determine their size and specific parameter values will be developed.

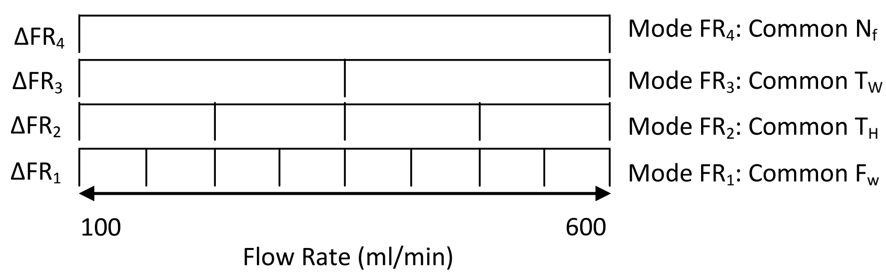

Figure 4-14: Organization of hierarchy for Finger Pumps

4.2.4.6 Step 6: Formulate a Combined Decision Support Problem

There are two goals when designing any new product platform – maximizing commonality and minimizing loss of performance for any product offerings. To determine the best combination of these two goals a utility-based decision support problem (u-DSP) formulation is used. The corresponding u-DSP formulation for the finger pump example is listed in Figure 4-15. In summary, the goal of this formulation is to find the size of the space elements, represented by the decision variables ΔFR_1, ΔFR_2, ΔFR_3, ΔFR_4, which minimize the aggregated objective function as discussed in Section

4.2.4.2. Given as inputs are the pump space of customization and the hierarchy of modes for managing variety presented in the previous section. Relationships are listed to show which design variables correspond to which space elements. The bounds and constraints given show the limits on the size of the space elements as well as the fact that each subsequent higher space element must be greater or equal in size to the lower elements.

Given: The 1-dimensional space of customization M^1: FR = [100,600]
Mode FR_1: Commonization of F_w
Mode FR_2: Commonization of T_H
Mode FR_3: Commonization of T_w
Mode FR_4: Commonization of N_f

Find: Value of the decision variables ΔFR_1, ΔFR_2, ΔFR_3, ΔFR_4
The values of the common design variables within each space element
x = T_w, T_H, , F_w, N_f, V

Satisfy: Relationships:
$\Delta FR_1 = f(F_w)$
$\Delta FR_2 = f(T_H)$
$\Delta FR_3 = f(T_w)$
$\Delta FR_4 = f(N_f)$

Bounds:

$50 \leq \Delta FR_1 \leq 500$	$0.5 \leq T_w \leq 2.5$ cm
$50 \leq \Delta FR_2 \leq 500$	$0.5 \leq T_H \leq 3.0$ cm
$50 \leq \Delta FR_3 \leq 500$	$0.3 \leq F_w \leq 1.0$ cm
$50 \leq \Delta FR_4 \leq 500$	$5 \leq N_f \leq 12$
	$2 \leq V \leq 12$ Volts

Constraints: $\Delta FR_1 \leq \Delta FR_2 \leq \Delta FR_3 \leq \Delta FR_4 \leq 500$

Minimize:

$$Z_{avg} = \frac{1}{n}\sum_{i=1}^{n} Z_i + C_{penalty}$$

where i = 1,2, … , n based on the level discretization from FR= 100 to 600
Z_i is given by Equation 4-50 and $C_{penalty}$ is given by Equation 4-51

Figure 4-15: Formulation of the Pump Decision Support Problem

4.2.4.7 Step 7: Solve the Decision Support Problem

As with the previous examples, solution of the decision support problem is conducted over a discretized space using a combination of exhaustive search and Matlab's constrained optimization function, fmincon. In this example, the space of customization is discretized in steps of 1 ml/min. The resulting decision variables found using the Sensitivity-Based PPCTM are presented in Table 4-19, along with the average volume and efficiency over the space of customization. The specific values of the design variables within each space are shown in Table 4-20.

Table 4-19: Pump Decision Variable Results

ΔFR_1 (ml/min)	ΔFR_2 (ml/min)	ΔFR_3 (ml/min)	ΔFR_4 (ml/min)	Avg. Volume (cm^3)	Avg. Eff (%)	U_{avg}
100	500	500	500	153.9	18.9	0.782

Table 4-20: Pump Roadmap and Design Variables

	Design Variables			
Pump Flow Rate (ml/min)	N_f (fingers)	T_w (cm)	T_H (cm)	F_w (cm)
100-150	5	2.34	2.65	0.31
150-200				
200-250				0.38
250-300				
300-350				0.45
350-400				
400-450				0.49
450-500				
500-550				0.54
550-600				

4.2.5 Discussion and Validation of Results

In the previous two examples, well studied benchmark products were used in order to compare with previous methods and validate claims that Sensitivity-based PPCTM can help improve decision making and produce better results. The goal in this example is to establish the true usefulness of the proposed method for top down customizable product platform design by applying it to a new product.

Beginning to establish this usefulness begs the question: *Did the application of the Sensitivity-based PPCTM successfully produce a platform that could be readily customized?* The answer is undoubtedly yes. The results shown in Table 4-20 represents a customizable platform enabling the manufacturer to provide continuous variation for flow rates between 100 and 600 ml/min. Individual variants were instantiated across this range in 1 ml/min increments in order to calculate the average efficiency and volume, and all 501 variants were able to achieve the desired flow rates and meet the design constraints. Additionally, this platform has very high commonality, sharing three of the four modes for managing variety across the entire space of customization. The fourth, finger width, has only five different values which is very low compared with the hundreds of flow rate achievable.

Having just discussed the high commonality, the reader is bound to wonder: at what cost? There is always a tradeoff between commonality and performance and if one sacrifices too much performance the product may no longer be competitive in the market. In light of this observation: *Can the application of the Sensitivity-based PPCTM mitigate the performance lose due to commonality?*

To answer this question, platform derived variants are compared with the individual baseline solutions found in Step 4 (Section 4.2.4.4). The individual pumps were developed for 11 specific flow rates (FR = [100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600]). Therefore, the pump platform must be used to instantiate products to meet these flow rates. For any values that fall on the border of two space elements (e.g. 200, 300, etc.), the higher space element value was used. The specific products and their performance are shown in Table 4-21.

Table 4-21: Sensitivity-based PPCTM Customized Pumps

	Design Variables					Responses	
Flow Rate	N_f (No.)	T_w (cm)	T_H (cm)	F_w (cm)	Voltage (V)	η (%)	Volume (cm^3)
100	5	2.34	2.65	0.31	2.44	19.1	125.9
150	⇩	⇩	⇩	0.31	3.32	18.0	125.9
200	⇩	⇩	⇩	0.38	3.56	19.8	142.5
250	⇩	⇩	⇩	0.38	4.29	18.1	142.5
300	⇩	⇩	⇩	0.45	4.34	19.8	154.8
350	⇩	⇩	⇩	0.45	4.96	19.0	154.8
400	⇩	⇩	⇩	0.49	5.17	20.0	163.1
450	⇩	⇩	⇩	0.49	5.73	18.3	163.1
500	⇩	⇩	⇩	0.54	5.77	19.9	173.4
550	⇩	⇩	⇩	0.54	6.29	18.6	173.4
600	⇩	⇩	⇩	0.54	6.80	17.3	173.4

Comparison of the baseline pumps performance and the proposed method is shown in Table 4-22. The efficiency and volume of each pump are presented along with the percentage difference of each platform pump from the baselines. For efficiency, a positive change denotes an improvement from the baselines to the PPCTM; for volume, a negative change denotes an improvement. Note as a quick sanity check, that no pump shows an improvement for both efficiency and volume as this would indicate that the baseline pump was not truly the best design. Most variants have comparable performances to their baseline counterparts, with some pumps improving slightly in one parameter and dropping in the other, or having minor reductions in both. Furthermore, averaging across the PPCTM family showed a negligible performance drop compared with the baseline pumps. As tallied at the bottom of Table 4-22, the PPCTM pumps lost only 0.8% in efficiency and are 1.2% larger in volume. It therefore seems safe to conclude that for the pump example the Sensitivity-based PPCTM was overwhelmingly successful at mitigating the performance loss due to commonality.

Table 4-22: Comparison of Baseline Results with Sensitivity-based PPCTM

	Individual Baseline Pumps		Sensitivity-based PPCTM Pumps		Percent difference	
Flow Rate (ml/min)	η	Volume (cm^3)	η	Volume (cm^3)	η	Volume (cm^3)
100	17.9	116.1	19.1	125.9	6.7	8.4
150	18.1	125.9	18.0	125.9	-0.6	0.0
200	18.2	134.1	19.8	142.5	8.8	6.3
250	18.3	141.3	18.1	142.5	-1.1	0.8
300	18.6	148.5	19.8	154.8	6.5	4.2
350	19.7	156.3	19.0	154.8	-3.6	-1.0
400	19.7	161.6	20.0	163.1	1.3	0.9
450	19.8	166.9	18.3	163.1	-7.8	-2.3
500	19.9	171.7	19.9	173.4	0.0	1.0
550	19.7	175.6	18.6	173.4	-5.6	-1.3
600	19.9	180.2	17.3	173.4	-13.1	-3.8
				Average change:	-0.8	1.2

4.3 Summary

In this chapter, two case studies are presented to demonstrate how the Sensitivity-based PPCTM can be applied to the design of a customizable product platform. Through these case studies, the primary research question has been investigated and the research hypothesis of this thesis is validated. The universal electric motor example, presented in Section 4.1, is a benchmark example used widely throughout literature. The proposed sensitivity-based method was used to successfully develop a fully customizable platform, which fulfills a wide range of torque requirements. This sensitivity-based platform was compared with a platform developed using the former PPCTM and shown to enable better variable selection leading to improved results. Additionally, the developed platform was used to instantiate a family of motors, which produced superior results when compared with other methods in literature, both in terms of performance and commonality.

The finger pump example, presented in Section 4.2, is a newly designed pumping technology looking to expand into the biotech/pharmaceutical market. Niche industries such as these require customized products with high performance and minimal size. It is therefore very important that the manufacturer has a platform to quickly meet customer demands with minimal added manufacturing cost. As such, this design problem is well suited for the application of the proposed Sensitivity-based PPCTM. From the results shown in Section 4.2.5, it is evident that the pump platform is a success. The sensitivity-based platform provides continuous adjustment of the flow rate, while maintaining high commonality. Furthermore, comparison with individually optimized pumps showed negligible loss in performance.

These examples, along with the pressure vessel example in Chapter 3, show how the proposed method can be implemented to determine which design variables are best suited for commonality and how they should be ordered in the space element hierarchy. It is therefore concluded that the proposed method answers the Primary Research Question and the author’s hypothesis is validated.

CHAPTER FIVE

CLOSURE

The primary goal of this thesis is to present augmentations to the Product Platform Constructal Theory Method (PPCTM). Specifically, to provide the designer the ability to assess and organize the modes for managing variety through the infusion of sensitivity analysis. Chapter 2 discusses the existing state of the art along with a detailed overview of the PPCTM. Chapter 3 presents the foundations of this work, the augmented sensitivity based method, and a tutorial example of its use through the design of a line of customizable pressure vessels. Chapter 4 shows the utility of the augmented method through application to two example problems, namely, a family of universal electric motors and a family of finger pumps.

In this chapter, the development and presentation of the augmented PPCTM is brought to a close. In Section 5.1, we return to the research questions posed in Chapter 1 and review the answers that have been offered. The resulting contributions made are discussed in Section 5.2. In Section 5.3, limitations of the research are presented, and avenues of future work are described. Final remarks are given in Section 5.5, closing this chapter and the thesis.

5.1 Answering the Research Questions

In Chapter 1, the concept of mass customization and use of product platforms to achieve this customization are introduced. Appropriately designing a product platform presents a number of challenging research problems, requiring continuous work to develop and expand product platform design methods. The PPCTM is one such method which has led to a steady stream of research and progress in product platform development for customization. One limitation of this method is that the selection of platform variables and the modes for managing product variety must be pre-specified or determined ad hoc by the designer. This limitation motivated the following research question and hypothesis:

Primary Research Question
How can the Product Platform Constructal Theory Method be augmented to enable the selection of common platform variables?

Hypothesis 1
Incorporating sensitivity analysis into the PPCTM will yield the effects of varying design variables away from their optimums as is done during commonization, showing the designer which variables can support the most commonization.

The result of this work is an augmented PPCTM that eliminates the need for *a priori* platform specification, providing a systematic means of selecting platform variables. In particular, an additional step was added to the existing method in which a sensitivity analysis is carried out to determine which design variables are suited for the highest levels of commonality and which variables should have lower levels of commonality. The backbone of the sensitivity analysis is the sensitivity index discussed in Section 3.1. For successful implementation five sub-steps are presented in Section 3.2 which once infused come to form the new seven step Sensitivity-Based PPCTM shown in Section 3.4.

Validation of the method is established through its application to three example problems: a pressure vessel, an electric motor, and a novel pumping technology. The pressure vessel examples serves three purposes. First, it acts as a tutorial example for how to implement the sensitivity-based method. Second, the example shows the applicability of the sensitivity analysis to a two dimensional space and a non-uniform demand scenario. Third, comparison with the previous method's results showed reduced cost due to use of the sensitivity information even when there are only a few variables to chose from. The electric motor example represented a benchmark example, widely used in literature, which included a greater number of design variables and constraints. This example established usefulness for products with multiple design objectives as well as presented a means of handling the tradeoff between performance and commonality through the use of a commonality penalty function. Results showed that using the sensitivity information produced better results than an ad hoc selection of platform variables for customization, and when compared with individual variants found using other product platform methods. Lastly, the pump example validates the method through application to a real-world, newly developed technology. This example serves as a "sanity check" for the true merit of this method and for top-down methods as a whole. Most top-down methods are presented using well studied benchmark examples. While this is necessary for validation and comparison, the purpose of any method is to be widely applicable and prove effective where top-down is really needed; in new products. This example shows that applicability through the use of a recently developed product that was designed for a niche market, then expanding this product into a customizable platform that can serve many more business sectors.

Based on the results shown in this thesis and the above discussion it is therefore asserted that the primary research question has been answered and the hypothesis validated.

The Product Platform Constructal Theory Method has be augmented in order to enable the selection of common platform variables. This has been accomplished by incorporating sensitivity analysis into the PPCTM which yielded the effects of varying design variables away from their optimums as is done during commonization, enabling the designer to select the variables which can support the most commonization.

5.2 Contributions

The PPCTM was developed over a decade ago and has undergone several iterations. Each iteration has incrementally improved the method through added tools and techniques to more efficiently and effectively design for product customization and to be applicable to a greater number of domains. The goal of this work was to continue this improvement in a way that both independently contributes to the method and further enables the iterations that have come before it.

The primary contribution of this work is the infusion of a sensitivity-based analysis method into the Product Platform Constructal Theory Method. The use of this method provides the designer with the ability to determine which design variables are best suited for commonality and to effectively allocate the modes for managing variety to the space level hierarchy. As a result, this method removes largely the need for ad hoc design decisions that lead to reduced performance. It has been shown that the Sensitivity-Based PPCTM produces improved results when compared with previous versions of the method and with other product platform methods. As a secondary contribution, two example problems never used with the PPCTM are presented which can serve as additional benchmarks for future PPCTM development.

5.3 Future Work

An exciting aspect of research is that in the pursuit of answering a single question, one often finds that there are many more to explore. This work is no different and there have been several stones left unturned. In this section, we will describe three areas for future work that the author believes to be most important.

Connecting Sensitivity to Space Element Size

In this work, sensitivity analysis is used to assess and rank the different modes for managing variety based on their suitability for commonization. This ranking enables the designer to allocate the different modes to various levels of space elements. After this point the size of the space elements is determined using the decision support problem with the constraint that all space elements at the same hierarchy level must have the same size. It is this author's belief that the constraint for size uniformity may negatively impact performance by averaging out over localized regions that can handle greater commonality and other regions which require less commonality. It would therefore be advantageous to selectively determine space element sizes based on the local performance impact. The information needed to achieve this is already being determined during the sensitivity analysis, where local performance sensitivities are determined for all modes of managing variety. To utilize this information on a localized scale would require additional investigation to connect ranges of sensitivity values to the resulting size of the space elements. This would also require an improved problem formulation and solution method for handling the added complexity of non-uniform space elements.

Incorporating Modular Functionality

For the three examples in this thesis and all previous applications of the PPCTM only scalable product components are considered. This ignores modular changes in functionality, e.g. substituting a new motor for increased speed in the pump example, which are common in product design, especial when providing customization. A promising line of work would be to extend the existing PPCTM to incorporate modularity of function structures, as well as mixed scalable/modular components. Such an extension would include many challenging avenues of work, such as how to solve the problem when using mixed continuous and discrete parameters. The existing method has used only continuous parameters. In order to handle discrete variables both the sensitivity analysis and the overall solution method would require adaptation. Another major research avenue would concern how to map the different function structures to the space of customization. The addition or subtraction of each function structure would correspond to completely different segments of the space of customization. It may be necessary to map each functional module to its own individual market space. The union of these market spaces would then form the overall space.

Expanding the Complexity of the Problem

Across the board in product platform design research there is a need for methods to be expanded to higher complexity, higher dimensionality applications. The existing PPCTM has developed a strong framework to do so, but has yet to be proven with more than three design specifications and a handful of design variables. Many challenging research problems will emerge as the number of specifications and design variables increase. One particularly challenging question arose in the seemingly simple pressure vessel example, with respect to the coupling of design variables with multiple product specifications and how to organize them. It is a common practice in axiomatic design to try to decouple these systems and/or ignore the weak coupling. Therefore, research to determine the functional dependency of the coupled design variables and appropriate decoupling strategy could be investigated. Another approach would be to leave the design variables and specifications coupled and switch from the analytical models presented to the integration of numerical analysis methods such as FEA, CFD, etc. A second research avenue to investigate within higher complexity systems is the calculation and aggregation of sensitivity information. As the number of variables grows so does the computational cost, which could present challenges using the existing finite difference based approach. Therefore, future work should investigate how to increase the computational efficiency of the sensitivity calculation and how to aggregate the sensitivity information with respect to greater dimensionality.

5.4 Closing Remarks

Product family design and design related research is aimed at providing engineers with tools and methods to design more effectively and efficiently products and processes. The PPCTM is one such method that stands as an icon in design for customization and with each subsequent iteration it grows into an even more comprehensive platform. While I am sure research into PPCTM will continue to expand the method, this work represents a meaningful step forward in providing the designer valuable knowledge for improved decision making. It is important to note that while the PPCTM is meant to handle many varied design scenarios, it is not meant to replace the designer. The method is meant to build upon the work of engineers and designers developing new products, simulation models, and production techniques. Without the close supervision of a knowledgeable designer the results of any method would most certainly fall short. Therefore, it is our goal that engineers embrace this method as a means of improving their product design and reducing the number of design iterations.

Personally, I am excited by the many applications of the PPCTM and the potential for increased customization of future products. I hope that this work helps to further validate the original method and build future avenues of research. Lastly I hope that this work will inspire others to pursue research in design, as I was inspired by those before me.

APPENDIX A

PRESSURE VESSEL SPECIFICATIONS

A.1 Pressure Vessel Individual Baselines

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)	Cost ($)
10	10	0.637	5.95	2.96	7.000	1220.38
10	12	0.637	7.14	3.55	7.000	1481.33
10	14	0.637	8.34	4.14	7.000	1748.00
10	16	0.637	9.55	4.74	7.000	2020.42
10	18	0.637	10.75	5.33	7.000	2298.62
10	20	0.637	11.96	5.92	7.000	2582.64
10	22	0.637	13.17	6.52	7.000	2872.52
10	24	0.637	14.38	7.11	7.000	3168.30
10	26	0.637	15.60	7.71	7.000	3470.01
10	28	0.637	16.82	8.30	7.000	3777.70
10	30	0.637	18.04	8.89	7.000	4091.39
12	10	0.694	6.48	3.23	7.000	1465.39
12	12	0.694	7.79	3.87	7.000	1779.13
12	14	0.694	9.10	4.52	7.000	2099.87
12	16	0.694	10.41	5.16	7.000	2427.64
12	18	0.694	11.72	5.81	7.000	2762.49
12	20	0.694	13.04	6.46	7.000	3104.48
12	22	0.694	14.36	7.11	7.000	3453.63
12	24	0.694	15.68	7.75	7.000	3810.01
12	26	0.694	17.01	8.40	7.000	4173.65
12	28	0.694	18.33	9.05	7.000	4544.60
12	30	0.694	19.67	9.70	7.000	4922.92
14	10	0.747	6.97	3.47	7.000	1710.60
14	12	0.747	8.37	4.16	7.000	2077.25
14	14	0.747	9.78	4.86	7.000	2452.21
14	16	0.747	11.19	5.55	7.000	2835.52
14	18	0.747	12.60	6.25	7.000	3227.25
14	20	0.747	14.02	6.95	7.000	3627.43
14	22	0.747	15.44	7.64	7.000	4036.14
14	24	0.747	16.86	8.34	7.000	4453.42
14	26	0.747	18.29	9.03	7.000	4879.32
14	28	0.747	19.72	9.73	7.000	5313.91
14	30	0.747	21.15	10.43	7.000	5757.24
16	10	0.795	7.42	3.69	7.000	1955.98
16	12	0.795	8.92	4.43	7.000	2375.66
16	14	0.795	10.41	5.17	7.000	2804.98
16	16	0.795	11.92	5.91	7.000	3244.00
16	18	0.795	13.42	6.65	7.000	3692.79
16	20	0.795	14.93	7.40	7.000	4151.40
16	22	0.795	16.44	8.14	7.000	4619.90

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)	Cost ($)
16	24	0.795	17.95	8.88	7.000	5098.35
16	26	0.795	19.47	9.62	7.000	5586.82
16	28	0.795	20.99	10.36	7.000	6085.37
16	30	0.795	22.52	11.10	7.000	6594.07
18	10	0.840	7.84	3.90	7.000	2201.52
18	12	0.840	9.42	4.69	7.000	2674.33
18	14	0.840	11.01	5.47	7.000	3158.14
18	16	0.840	12.59	6.25	7.000	3653.02
18	18	0.840	14.18	7.03	7.000	4159.04
18	20	0.840	15.78	7.81	7.000	4676.27
18	22	0.840	17.37	8.60	7.000	5204.79
18	24	0.840	18.97	9.38	7.000	5744.66
18	26	0.840	20.58	10.16	7.000	6295.97
18	28	0.840	22.19	10.95	7.000	6858.78
18	30	0.840	23.80	11.73	7.000	7433.17
20	10	0.882	8.24	4.10	7.000	2447.21
20	12	0.882	9.90	4.92	7.000	2973.24
20	14	0.882	11.56	5.74	7.000	3511.66
20	16	0.882	13.23	6.56	7.000	4062.53
20	18	0.882	14.90	7.39	7.000	4625.94
20	20	0.882	16.57	8.21	7.000	5201.98
20	22	0.882	18.25	9.03	7.000	5790.72
20	24	0.882	19.93	9.85	7.000	6392.24
20	26	0.882	21.62	10.68	7.000	7006.63
20	28	0.882	23.30	11.50	7.000	7633.97
20	30	0.882	25.00	12.32	7.000	8274.35
22	10	0.922	8.61	4.29	7.000	2693.04
22	12	0.922	10.35	5.15	7.000	3272.38
22	14	0.922	12.09	6.00	7.000	3865.50
22	16	0.922	13.83	6.86	7.000	4472.50
22	18	0.922	15.57	7.72	7.000	5093.45
22	20	0.922	17.32	8.58	7.000	5728.46
22	22	0.922	19.08	9.44	7.000	6377.60
22	24	0.922	20.83	10.30	7.000	7040.99
22	26	0.922	22.60	11.16	7.000	7718.69
22	28	0.922	24.36	12.02	7.000	8410.82
22	30	0.922	26.13	12.88	7.000	9117.45
24	10	0.961	8.97	4.46	7.000	2938.99
24	12	0.961	10.77	5.36	7.000	3571.72
24	14	0.961	12.58	6.25	7.000	4219.65
24	16	0.961	14.40	7.15	7.000	4882.89
24	18	0.961	16.22	8.04	7.000	5561.52
24	20	0.961	18.04	8.94	7.000	6255.65
24	22	0.961	19.86	9.83	7.000	6965.39
24	24	0.961	21.69	10.73	7.000	7690.82
24	26	0.961	23.53	11.62	7.000	8432.06
24	28	0.961	25.37	12.52	7.000	9189.20
24	30	0.961	27.21	13.42	7.000	9962.35
26	10	0.997	9.31	4.63	7.000	3185.06
26	12	0.997	11.18	5.56	7.000	3871.26
26	14	0.997	13.06	6.49	7.000	4574.09

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)	Cost ($)
26	16	0.997	14.94	7.42	7.000	5293.67
26	18	0.997	16.83	8.34	7.000	6030.12
26	20	0.997	18.72	9.27	7.000	6783.52
26	22	0.997	20.62	10.20	7.000	7554.01
26	24	0.997	22.51	11.13	7.000	8341.68
26	26	0.997	24.42	12.06	7.000	9146.64
26	28	0.997	26.33	12.99	7.000	9969.02
26	30	0.997	28.24	13.92	7.000	10808.92
28	10	1.032	9.63	4.79	7.000	3431.24
28	12	1.032	11.57	5.75	7.000	4170.97
28	14	1.032	13.52	6.71	7.000	4928.80
28	16	1.032	15.46	7.67	7.000	5704.84
28	18	1.032	17.42	8.64	7.000	6499.21
28	20	1.032	19.37	9.60	7.000	7312.02
28	22	1.032	21.33	10.56	7.000	8143.41
28	24	1.032	23.30	11.52	7.000	8993.49
28	26	1.032	25.27	12.48	7.000	9862.38
28	28	1.032	27.24	13.44	7.000	10750.20
28	30	1.032	29.22	14.41	7.000	11657.07
30	10	1.065	9.94	4.95	7.000	3677.53
30	12	1.065	11.95	5.94	7.000	4470.86
30	14	1.065	13.95	6.93	7.000	5283.76
30	16	1.065	15.96	7.92	7.000	6116.35
30	18	1.065	17.98	8.91	7.000	6968.76
30	20	1.065	20.00	9.91	7.000	7841.12
30	22	1.065	22.02	10.90	7.000	8733.56
30	24	1.065	24.05	11.89	7.000	9646.21
30	26	1.065	26.09	12.89	7.000	10579.19
30	28	1.065	28.13	13.88	7.000	11532.65
30	30	1.065	30.17	14.87	7.000	12506.71

A.2 Pressure Vessel Sensitivities

		Design Variables			
V (m^3)	P (Mpa)	L	R	T_s	T_h
10	10	1.868	48.049	198.241	39.787
10	12	3.060	78.713	202.548	39.795
10	14	4.530	116.508	206.866	39.806
10	16	6.278	161.481	211.195	39.819
10	18	8.308	213.681	215.535	39.835
10	20	10.620	273.157	219.886	39.852
10	22	13.217	339.960	224.247	39.871
10	24	16.101	414.138	228.619	39.893
10	26	19.274	495.742	233.003	39.917
10	28	22.737	584.823	237.397	39.943
10	30	26.493	681.430	241.802	39.971
12	10	2.385	55.844	216.896	47.189
12	12	3.907	91.483	221.734	47.176
12	14	5.783	135.409	226.582	47.160
12	16	8.015	187.678	231.440	47.141
12	18	10.606	248.347	236.307	47.119
12	20	13.559	317.473	241.183	47.093
12	22	16.874	395.113	246.069	47.065
12	24	20.556	481.325	250.964	47.034
12	26	24.607	576.169	255.869	46.999
12	28	29.029	679.701	260.783	46.962
12	30	33.824	791.982	265.707	46.921
14	10	2.927	66.501	233.664	54.741
14	12	4.796	108.942	239.039	54.770
14	14	7.098	161.251	244.430	54.806
14	16	9.838	223.495	249.837	54.849
14	18	13.018	295.742	255.259	54.898
14	20	16.642	378.059	260.696	54.955
14	22	20.712	470.516	266.149	55.019
14	24	25.231	573.181	271.619	55.089
14	26	30.203	686.124	277.103	55.167
14	28	35.630	809.415	282.604	55.252
14	30	41.516	943.123	288.121	55.344
16	10	3.492	74.027	249.573	61.960
16	12	5.720	121.271	255.433	61.966
16	14	8.466	179.500	261.307	61.974
16	16	11.734	248.788	267.194	61.984
16	18	15.528	329.211	273.096	61.995
16	20	19.850	420.845	279.012	62.008
16	22	24.704	523.766	284.942	62.022
16	24	30.094	638.050	290.886	62.037
16	26	36.024	763.775	296.844	62.055
16	28	42.497	901.019	302.816	62.074
16	30	49.517	1049.859	308.803	62.094
18	10	4.075	82.181	264.412	69.160
18	12	6.675	134.629	270.747	69.160
18	14	9.880	199.271	277.096	69.159
18	16	13.694	276.192	283.459	69.158
18	18	18.121	365.473	289.836	69.157

V (m^3)	P (Mpa)	L	R	T_s	T_h
18	20	23.165	467.200	296.227	69.156
18	22	28.830	581.457	302.633	69.154
18	24	35.121	708.329	309.054	69.153
18	26	42.041	847.902	315.488	69.151
18	28	49.595	1000.262	321.938	69.149
18	30	57.788	1165.497	328.401	69.148
20	10	4.675	88.952	278.525	76.194
20	12	7.658	145.720	285.305	76.158
20	14	11.335	215.689	292.097	76.114
20	16	15.710	298.946	298.900	76.062
20	18	20.789	395.584	305.715	76.001
20	20	26.576	505.692	312.540	75.931
20	22	33.075	629.362	319.377	75.853
20	24	40.292	766.687	326.224	75.767
20	26	48.231	917.760	333.083	75.671
20	28	56.898	1082.673	339.954	75.567
20	30	66.297	1261.522	346.835	75.454
22	10	5.290	96.829	291.829	83.252
22	12	8.665	158.625	299.057	83.207
22	14	12.826	234.789	306.295	83.151
22	16	17.777	325.420	313.544	83.085
22	18	23.523	430.615	320.804	83.008
22	20	30.071	550.473	328.076	82.920
22	22	37.425	685.095	335.358	82.821
22	24	45.591	834.581	342.652	82.711
22	26	54.575	999.032	349.957	82.591
22	28	64.381	1178.549	357.273	82.459
22	30	75.016	1373.235	364.600	82.316
24	10	5.918	109.699	304.192	90.609
24	12	9.695	179.709	311.910	90.664
24	14	14.349	265.997	319.652	90.733
24	16	19.888	368.673	327.418	90.814
24	18	26.317	487.850	335.208	90.909
24	20	33.643	623.640	343.023	91.017
24	22	41.870	776.155	350.861	91.138
24	24	51.006	945.510	358.723	91.272
24	26	61.057	1131.819	366.610	91.420
24	28	72.028	1335.197	374.522	91.582
24	30	83.927	1555.760	382.457	91.757
26	10	6.558	116.343	316.444	97.466
26	12	10.744	190.593	324.576	97.488
26	14	15.902	282.107	332.729	97.516
26	16	22.041	391.003	340.903	97.549
26	18	29.165	517.398	349.099	97.588
26	20	37.283	661.412	357.315	97.632
26	22	46.401	823.165	365.552	97.681
26	24	56.526	1002.777	373.811	97.736
26	26	67.664	1200.369	382.091	97.796
26	28	79.823	1416.065	390.392	97.862
26	30	93.009	1649.987	398.715	97.933
28	10	7.210	125.908	328.043	104.477

V (m^3)	P (Mpa)	L	R	T_s	T_h
28	12	11.811	206.262	336.613	104.532
28	14	17.482	305.300	345.209	104.599
28	16	24.230	423.148	353.830	104.680
28	18	32.063	559.935	362.477	104.773
28	20	40.987	715.789	371.151	104.879
28	22	51.011	890.839	379.850	104.998
28	24	62.141	1085.218	388.575	105.131
28	26	74.386	1299.055	397.326	105.277
28	28	87.752	1532.484	406.104	105.436
28	30	102.248	1785.638	414.908	105.608
30	10	7.871	131.715	339.426	111.184
30	12	12.894	215.765	348.385	111.190
30	14	19.085	319.370	357.365	111.197
30	16	26.453	442.653	366.366	111.206
30	18	35.004	585.747	375.388	111.217
30	20	44.747	748.789	384.431	111.229
30	22	55.691	931.911	393.495	111.242
30	24	67.843	1135.254	402.580	111.257
30	26	81.211	1358.952	411.686	111.274
30	28	95.804	1603.146	420.814	111.292
30	30	111.630	1867.974	429.963	111.311

A.3 Pressure Vessel Design Roadmap

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)
10-13	10-11	0.721	7.41	3.69	7
	11-12		8.09	4.02	
	12-13		8.77	4.36	
	13-14		9.45	4.69	
	14-15		10.13	5.03	
	15-16		10.81	5.36	
	16-17		11.49	5.70	
	17-18		12.17	6.03	
	18-19		12.85	6.37	
	19-20		13.54	6.71	
	20-21		14.22	7.04	
	21-22		14.91	7.38	
	22-23		15.60	7.71	
	23-24		16.28	8.05	
	24-25		16.97	8.39	
	25-26		17.66	8.72	
	26-27		18.35	9.06	
	27-28		19.04	9.40	
	28-29		19.73	9.73	
	29-30		20.42	10.07	
13-16	10-11	0.795	8.17	4.06	7
	11-12		8.92	4.43	
	12-13		9.67	4.80	
	13-14		10.41	5.17	
	14-15		11.16	5.54	
	15-16		11.92	5.91	
	16-17		12.67	6.28	
	17-18		13.42	6.65	
	18-19		14.17	7.02	
	19-20		14.93	7.39	
	20-21		15.68	7.77	
	21-22		16.44	8.14	
	22-23		17.20	8.51	
	23-24		17.95	8.88	
	24-25		18.71	9.25	
	25-26		19.47	9.62	
	26-27		20.23	9.99	
	27-28		20.99	10.36	
	28-29		21.76	10.73	
	29-30		22.52	11.10	

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)
16-19	10-11	0.862	8.85	4.40	7
	11-12		9.66	4.80	
	12-13		10.47	5.21	
	13-14		11.29	5.61	
	14-15		12.10	6.01	
	15-16		12.91	6.41	
	16-17		13.73	6.81	
	17-18		14.54	7.21	
	18-19		15.36	7.61	
	19-20		16.18	8.01	
	20-21		17.00	8.42	
	21-22		17.82	8.82	
	22-23		18.64	9.22	
	23-24		19.46	9.62	
	24-25		20.28	10.02	
	25-26		21.10	10.42	
	26-27		21.93	10.83	
	27-28		22.75	11.23	
	28-29		23.58	11.63	
	29-30		24.41	12.03	
19-22	10-11	0.922	9.48	4.72	7
	11-12		10.35	5.14	
	12-13		11.22	5.57	
	13-14		12.09	6.00	
	14-15		12.96	6.43	
	15-16		13.83	6.86	
	16-17		14.70	7.29	
	17-18		15.57	7.72	
	18-19		16.45	8.15	
	19-20		17.32	8.58	
	20-21		18.20	9.01	
	21-22		19.08	9.44	
	22-23		19.95	9.87	
	23-24		20.83	10.30	
	24-25		21.71	10.73	
	25-26		22.60	11.16	
	26-27		23.48	11.59	
	27-28		24.36	12.02	
	28-29		25.25	12.45	
	29-30		26.13	12.88	

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)
22-25	10-11	0.979	10.06	5.00	7
	11-12		10.98	5.46	
	12-13		11.90	5.91	
	13-14		12.82	6.37	
	14-15		13.75	6.83	
	15-16		14.67	7.28	
	16-17		15.60	7.74	
	17-18		16.53	8.19	
	18-19		17.45	8.65	
	19-20		18.38	9.11	
	20-21		19.31	9.56	
	21-22		20.24	10.02	
	22-23		21.18	10.47	
	23-24		22.11	10.93	
	24-25		23.04	11.39	
	25-26		23.98	11.84	
	26-27		24.91	12.30	
	27-28		25.85	12.76	
	28-29		26.79	13.21	
	29-30		27.73	13.67	
25-28	10-11	1.032	10.60	5.27	7
	11-12		11.57	5.75	
	12-13		12.54	6.23	
	13-14		13.51	6.71	
	14-15		14.49	7.19	
	15-16		15.46	7.67	
	16-17		16.44	8.15	
	17-18		17.42	8.63	
	18-19		18.39	9.12	
	19-20		19.37	9.60	
	20-21		20.35	10.08	
	21-22		21.33	10.56	
	22-23		22.32	11.04	
	23-24		23.30	11.52	
	24-25		24.28	12.00	
	25-26		25.27	12.48	
	26-27		26.26	12.96	
	27-28		27.24	13.44	
	28-29		28.23	13.93	
	29-30		29.22	14.41	

V (m^3)	P (Mpa)	R (m)	T_s (mm)	T_h (mm)	L_o (m)
28-30	10-11	1.065	10.94	5.44	7
	11-12		11.95	5.94	
	12-13		12.95	6.44	
	13-14		13.95	6.93	
	14-15		14.96	7.43	
	15-16		15.96	7.92	
	16-17		16.97	8.42	
	17-18		17.98	8.91	
	18-19		18.99	9.41	
	19-20		20.00	9.91	
	20-21		21.01	10.40	
	21-22		22.02	10.90	
	22-23		23.04	11.40	
	23-24		24.05	11.89	
	24-25		25.07	12.39	
	25-26		26.09	12.89	
	26-27		27.11	13.38	
	27-28		28.13	13.88	
	28-29		29.15	14.38	
	29-30		30.17	14.87	

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