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Communication and Academic Vocabulary in Mathematics: A Content Analysis of Prompts Eliciting Written Responses in Two Elementary Mathematics Textbooks

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Communication and Academic Vocabulary in Mathematics: A Content Analysis of Prompts Eliciting Written Responses in Two Elementary Mathematics Textbooks

by

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Abstract

The purpose of this study was to investigate how writing in mathematics is treated in one 4^{th} grade National Science Foundation (NSF)-funded mathematics textbook titled *Everyday Mathematics* and one publisher-generated textbook titled *enVision MATH*. The developed framework provided categories to support each of the research questions. The results indicate that writing is supported in both traditional and NSF developed 4^{th} grade mathematics textbooks

Results also indicated the number of exercises and writing prompts was higher in the *enVision MATH* textbook. However, *Everyday Mathematics* had a higher percentage of exercises that were coded as writing prompts. The framework domains of *content strand* in *enVision MATH* and *Everyday Mathematics* are similar in percentages with the exception of prompts coded in the *other* category. *Everyday Mathematics* appeared to be the only textbook analyzed to support writing across different content areas. Furthermore, the content strand of *number sense* had the largest percentage of writing prompts coded between both textbook series. Other findings from this study suggest that the type of vocabulary coded within the writing prompts was similar in all categories between both textbook series analyzed. Additionally, vocabulary specific to the domain of mathematics and symbols appeared to have the largest percentage in this category for both textbook series.

The teacher and student editions were explored in *enVision MATH* and *Everyday Mathematics* to provide more depth to the research. An exploration of the teacher edition

indicated how writing was supported for instructional purposes. The teacher editions in both textbook series had the largest percentage of support in the form of one *sample* response. Within the *student edition* category, the layout varied in the *enVision MATH* and *Everyday Mathematics* textbook series. As a result, only the language of *Everyday MATH* could be analyzed for patterns in the *sections*, *sub-sections*, and *additional sub-sections* of where the prompts were located.

Although this investigation did not involve analyzing student responses to the writing prompts, the findings provide information regarding the expectations of the writer in order to construct a mathematical response. For example, the *domain specific vocabulary (*DSV) and *symbols* category was rated the highest in percentage for both textbooks indicating that students will need to have command of the language and symbols of mathematics in order to engage in meaning making written discourse.

Because most of the math prompts were specific to the *problem solving* category, it was determined after a linguistic analysis that the affordance of the prompt is much more complex than then binary categories of content and process Additionally, in order for students to respond to these content writing prompts, many process words known as *meta-language* (i.e., explanation, description, why question, how question) need to be comprehended in order for composition to begin.

In light of these findings, I recommend that special attention be given to the teacher and student editions regarding the implementation of writing in mathematics. The development of these materials has important implications regarding instruction and learning of mathematical concepts through writing, potentially impacting student performance on national and international assessments.

Chapter 1: Introduction

A Vignette

As an elementary-grades mathematics coach, I conducted "walk through" observations (Downey, English, Frase, Poston, & Steffy, 2004) of classrooms to gather evidence of best practices in mathematics instruction. In doing so, I collaborated with the literacy coach and noticed a discrepancy between the walk-through checklists for mathematics versus literacy. According to the county-produced literacy checklist, evaluators of teachers' literacy practices were asked to look for word walls (vocabulary and high frequency), conferring notes for writing, conferring notes for reading, leveled classroom libraries, book baggies with accountability forms, student writing samples on the bulletin board, leveled reading groups, and anchor charts. Conversely, the math checklist asked evaluators to find evidence of the district-adopted calendar kits and readily available manipulatives. Unlike the literacy checklist, the mathematics checklist did not include evidence of teacher use of these materials or any other instructional practice for mathematics. Where was the math word wall with content strand vocabulary? Where were the student math writings on bulletin boards (e.g., math stories, strategies for solving a problem, solution steps, explanations, and justifications)? Where was the math word of the day or the problem of the day posted? Where was the children's literature to support the mathematics topic? Where was the evidence of student conferencing notes regarding how students solved problems (i.e., documentation of strengths and weaknesses)? Where was evidence of the math groups? Where were the anchor charts for alternative and traditional strategy solutions? Where was the math?

As a math coach, my support for teachers centered on the content standards and small group instruction. This support was guided by the most pervasive resource in the mathematics classroom-- the textbook. My conversations with teachers primarily focused on how I could assist teachers in designing purposeful activities for small group instruction. From those conversations I developed activities for multiple grade levels throughout my school. Most of the activities centered on integrating mathematics writing through problem solving, journaling, and real world application of mathematics (i.e., newspapers). I also used technology, making sure each student had a spiral notebook to solve problems and write down the solution steps to the problems they answered on the computer. Interestingly, every activity I developed for small group instruction, for multiple grade levels, incorporated writing. After reflecting on my experiences of the "walk through" checklist and designing group activities that centered on writing, I began to understand that my coaching philosophy for teachers was centered on the process standard of communication, more specifically, that of writing.

A Case for Writing in Mathematics

The use of writing in mathematics teaching aligns with the recommendations of the National Council for Teachers of Mathematics (NCTM) process standards. The NCTM *Principles and Standards for School Mathematics* (PSSM) states that mathematics content standards are learned through five process standards: problem solving, reasoning and proof, communication, connections, and representation. Although the process of communication appeared to address my implementation of writing in mathematics, I also noticed how writing can be embedded within *each* of the process standards recommended by NCTM. Furthermore, after summarizing the research on elementary students' knowledge of number, the National Research Council (NRC, 2001) produced strands for mathematical proficiency. Resembling the NCTM's process strands, NRC (2001) proposed that in order to be proficient in mathematics, the recommendation of writing, throughout the interrelated strands of mathematical proficiency, should be evident. In addition, the *Curriculum Focal Points* (NCTM, 2006), which is similar to the PSSM, also has a theme of writing whereby the recommendations of reasoning, justification and communication are at the core of learning mathematics. More recent developments, such as the Common Core State Standards (CCSS) also have writing nested in the *Standards for Mathematical Practice* (CCSS, 2010).

Reform efforts in standards development acknowledge the impact of writing on cognition, a stance supported by the seminal research in early writing as problem solving by Bereiter and Scardamalia (1987). For example, the NCTM (2000) suggests that writing in mathematics can also help students "consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson" (p. 61). Similarly, Greenfield and Bruner (1969) observed that cultures with technologies such as written language and mathematical formalisms will "push cognitive growth better, earlier, and longer than others" (p. 654). Bruner (1986), maintained, "We teach a subject not to teach little living libraries on the subject, but rather to get a student to think mathematically for himself (sic)... to take part in the process of knowledge-getting. Knowledge is a process not a product" (p. 72).

The Influence of Standards Documents and Textbooks

Various organizations, such as the National Council of Teachers of Mathematics (NCTM), National Research Council (NRC), and members of the Council of Chief State School Officers (CCSSO) and the National Governor's Association, Center for Best Practices (NGA Center), have produced standards documents that highlight the use of writing in the mathematics classroom. For example the NCTM identified five process standards in the *Principles and Standards for School Mathematics* (NCTM, 2000). The NRC formulated the *Strands of Mathematical Proficiency* (NRC, 2001). Furthermore, members of the Council of Chief State School Officers (CCSSO) and the National Governors Association, Center for Best Practices (NGA Center) developed common standards for all states where communication is embedded throughout the content recommendation (CCSS, 2010).

The NCTM *Principles and Standards for School Mathematics* (2000) states that the content strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) should be taught through mathematical processes (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). Whether the processes are utilized in isolation or as a connected component, the process of writing can be demonstrated throughout these strands. For example, in order to problem solve one can write an explanation or description of the problem solving process by reasoning and proving one's mathematical thinking. Students can also write to describe the process of connecting the mathematics content in addition to providing an explanation of a particular mathematical representation.

The textbook publishing industry, as well as curriculum projects funded by the National Science Foundation (NSF), moved quickly to develop curriculum materials (i.e., textbooks) to align to standards recommendations from these various organizations. Publishers realize that in addition to the standards documents, the most common influence on content appears to be the textbook/curriculum program (Weis, Pasley, Smith, Banilower, & Heck, 2003). Thus, the mathematics textbook is typically researched as the dominant tool in classroom instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008).

Statement of the Problem

Although curriculum development projects, often funded by NSF, and textbook publishing companies profess to have an alignment to standards documents, only one study indicated an association between elementary textbook assessments and process standards alignment (Hunsader et al., 2006). Additionally, a search of the ERIC databases revealed that an analysis of the tasks that facilitate a written response in NSF funded textbooks and publisher-generated materials has not been conducted.

There is a lack of research on writing prompts in mathematics textbooks. Researchers note the affective and cognitive benefits of writing in mathematics (Alvermann, 2002; Burns, 2004; Countryman, 1992; Emig, 1977; McIntosh & Draper, 2001; Pugalee, 2004; Senk & Thompson, 2003; Shulman 1986; Urquhart, 2009; Urquhart & McIver, 2005). However, the language in the types of prompts has not been investigated.

The language of prompts and usage of prompts directly influence classroom opportunities for students to develop mathematical thinking. In order to construct a response in mathematics, the student must be able to comprehend the prompt while producing precise language to respond to the prompt. O’Connell et al. (2005) note that words are the building blocks for content understanding, emphasizing that in order to communicate, it is important for students to understand the words that express that content. The comprehension of mathematics encompasses not only vocabulary terms, but also the understanding of symbols (Thompson et al., 2008). These types of vocabulary have the potential to make the comprehension of mathematics a complex process.

The PSSM (NCTM, 2000) places an emphasis on vocabulary under the process strand of communication by recommending that students use mathematical vocabulary to express mathematical ideas in a precise manner. However, there are only two studies focused on the instructional implications of language and vocabulary in mathematics textbooks for middle grades learners (Haggarty & Pepin, 2002; Herbel-Eisenmann, 2007). The Haggarty and Pepin (2002) study examined and compared the layout of the mathematical textbooks used in France, Germany and England. Additionally, the study investigated the opportunities students had to perform mathematical processes through the use of the vocabulary and language in the directions. The Herbel-Eisenmann (2007) study examined one middle school National Science Foundation (NSF) funded textbook for the "voice" of that particular textbook. More importantly the researcher examined the linguistic choices (i.e., use of imperatives, pronouns, modal verbs and expressions) developed by the textbook authors in order to understand the role of the reader and how the relationship between the reader and the author is constructed.

Due to paucity of research in three areas - the alignment of elementary grades textbooks to the process standards, how writing prompts are situated in the elementary mathematics textbook, and the use of language within the prompts - an analysis of writing prompts in elementary mathematics textbooks is warranted. The following task is an example of a writing prompt used for analysis:

- How do you know 1/4 is greater than 1/5? Explain your thinking.
  (Urquhart, 2009)

I selected two elementary 4th grade textbooks with teacher editions: (1) the 2011 edition of *enVision MATH* published by Pearson Education, Inc. and (2) the third edition of books developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. Both of these textbooks were national versions and were therefore not modified to fit the needs of any one specific state.

Purpose of the Study

The purpose of this study is to examine writing prompts in mathematics textbooks. Specifically, I will explore the following questions:

1. How many writing prompts are included in one 4th grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?
2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?
3. What types of vocabulary are used in the writing prompts in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?
4. What types of prompts are provided in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?

Theoretical framework. I conducted this study through the lenses of three interwoven theoretical perspectives: cognitive, social, and rhetorical perspectives in writing. From a cognitive perspective, Vygotsky (1962) noted that writing makes a unique demand in that the writer must engage in "deliberate structuring of the web of meaning" (p. 100). In support of this perspective, many organizations [e.g., NCTM, NRC, Writing to Learn (WTL) activities - stemming from a 1983 movement Writing Across the Curriculum (WAC) and the National Writing Project (Nagin, 2003)] recognize writing as a tool for acquiring knowledge in the content areas. Vygotsky (1962) also noted how written language requires higher cognitive functions because a writer must also make a conscious attempt to portray meaning with the written symbol, wholly and intelligibly explaining it to a non-present reader.

From a social perspective, writing has the potential to facilitate communication. For example Englert, Mariage, and Dunsmore (2006) note the importance of Vygotsky and Bahktin's views of the social implication of writing by referencing the following statements:

"Higher psychological processes, such as writing and reading, have their origins in social processes that occur on an interpsychological plane, and that are mediated through language signs, symbols, actions and objects" (Vygotsky, 1978, p. 208). "Over time, these external semiotic mediators observed in their contextualized uses in activity settings become internalized and transformed to influence action" (Bakhtin, 1986, p. 208).

In addition, Dyson (1992, 1993) found that children in primary classrooms use writing as a vehicle for social interaction as they develop understanding about social purposes for writing (p. 29). In addition, justifying and explaining problem solutions have the potential to enrich oral conversations (Baxter, 2001).

Embedded within the cognitive and social perspectives in writing is what Bazerman (2008) calls *rhetorical specification*, whereby the focus of writing is in the following areas: the structure of language and the audience or purpose for the writing task. For example, Bazerman (2008) notes that research in rhetorical tradition found that the type of writing prompt has the potential to affect composing processes for audience or purpose (Matsuhashi, 982; Witte & Cherry, 1994). In addition, textual features are reported to be different depending on the prompt type affecting the purpose of the prompt (Reid, 1990). Regarding the rhetorical perspective in writing, thoughts and language are designed for the purpose of communication, not words in isolation (Bakhtin, 1986).

These three perspectives in writing theory provide a lens for understanding the cognitive, social, and rhetorical implications of investigating writing prompts in mathematics textbooks.

Summary of methods.

1. To determine the number of writing prompts, I conducted a simple count and tallied the writing prompts included in each textbook.
2. To determine how writing prompts varied across content strands, I the language of the prompt and aligned the prompt to the content strand.
3. To determine the types of vocabulary used in each prompt, I coded the words according to the extant work on typologies of academic vocabulary in the form of word lists (Baumann & Graves, 2010).
4. To determine the types of writing prompts included in each textbook, I classified the prompts based on the type of mathematical and linguistic processing required in order to respond to the prompt.

I developed an analytic framework using 11 dimensions with respective sub-categories based on (1) NCTM's *Principles and Standards for School Mathematics* content strands, (2) Baumann and Graves's (2010) classification scheme of academic vocabulary, and (3) research in mathematics writing prompt types (Burns, 2004; Dougherty, 1996; Urquhart, 2009; Whitin & Whitin, 2000) (see Appendix A). Using the framework as a way to record the data, I calculated the number of writing prompts per page, the number of exercises per page, page number, and the wording of the prompt. Then I further coded the prompt to determine the academic vocabulary used, and the total number of words and symbols (words coded and words not on list), mathematical content strand addressed (e.g., algebra, number sense, geometry, measurement). I also coded the type of prompt, features of the teacher edition that provided prompt support, and the student edition prompt location (see Appendix A).

Definition of Terms

The following section identifies important terms and definitions. The following terms are defined in this section: *academic vocabulary, domain specific vocabulary, general vocabulary, meta-language, symbols, prompt/writing task*, and *constructed response.*

Academic vocabulary. Baumann and Graves (2010) note that academic vocabulary is defined in two ways: 1) *domain specific* or the content used in disciplines like mathematics, and 2) *general academic* or the broad, all-purpose terms that appear across content areas but that may have different meanings depending on the context. In a classification typology, Baumann and Graves (2010) developed additional categories in classifying academic vocabulary to include *literary vocabulary* or the words that authors of literature use to describe characters, settings, and characters' problems and actions, *meta-language* or the terms used to describe the language of literacy and literacy instruction and words used to describe processes, and *symbols* or icons that are not conventional words.

Constructed response. A *Constructed Response* is an *open-ended* item in which students create or produce an answer or response in written form (McMillan, 2004). These types of items are different from *close-ended* items whereby the answer is selected from a number of alternatives or by filling in a blank. Multiple-choice, true/false, and matching are the common types of objective, or close ended assessment items. Conversely, *constructed response* items are items that require a written narrative for an answer (Banks, 2005). Constructed response items can range from a few sentences to a paragraph or essay. Many researchers believe these types of items are used as a vehicle for learning and as a tool for acquiring knowledge (Bruner, 1986; Bereiter & Scardamalia, 1987; Greenfield & Bruner, 1969; Nagin, 2003; Vygotsky (1962). These types of items are also included in many state and national assessments such as the National Assessment of Educational Progress (NAEP). The NAEP (2010) Glossary of Terms states a *constructed response* is a non-multiple-choice item that requires some type of written or oral response. Although constructed response items have similar definitions regarding the type of response required, analysis of responses was not the purpose of this study. Therefore the prompts that had the potential to evoke a written or constructed response were selected for analysis.

Domain specific vocabulary. Baumann and Graves (2010) define Domain Specific Academic Vocabulary as the content-specific terms and expressions found in content area textbooks and other technical writing (p. 6) in addition to the relatively low-frequency content-specific words and phrases that appear in content area textbooks and other technical materials (p. 9). Marzano and Pickering (2005) devised a *Building Academic Vocabulary Teacher's Manual* word list whereby 7,923 terms in 11 subject areas were extracted from national standards documents. These lists contain content specific words that are organized into four grade-level intervals where 86 of the terms are specific to the domain of mathematics. For purposes of this study, domain specific academic vocabulary has been modified to *domain specific vocabulary (DSV).*

General vocabulary. Baumann and Graves (2010) define General Academic Vocabulary as words that appear reasonably frequently within and across academic domains. The words may be polysemous with different definitions being relevant to different domains (p. 9). In addition, Coxhead (2000) developed an Academic word list based on terms that are most often found in academic texts. For purposes of this study, general academic vocabulary has been modified to *general vocabulary (GV).*

Meta-language. Based on the extant work on typologies of academic vocabulary, Baumann and Graves (2010) defined *meta-language* as terms used to describe the language of literacy and literacy instruction and words used to describe processes, structures, or concepts commonly included in content area texts (p.10). Marzano and Pickering (2005) *Building Academic Vocabulary Teacher's Manual* word list was also used for terms that are specific to *meta-language.* These word lists detail content specific vocabulary organized into four grade-level intervals. These terms are specific to describing processes in mathematics writing prompts in the written (textbook) curriculum that have the potential to facilitate writing.

Prompts/writing task. The term *prompt* is used interchangeably with *writing task* in this study. Research in the field of literacy and mathematics also uses the terms *prompt* and *writing task* interchangeably. For example, Murphy (2004) and Yancey (2004) analyzed writing prompts or writing tasks and used the terms interchangeably throughout their texts. In *Research on Composition*, Smagorinsky (2006), in a section titled "Writing Tasks," states that "writing is enhanced when tasks are motivating, interesting, appropriately challenging" (p. 34). Urquhart (2009) used writing tasks and prompts interchangeably by noting, "Whether writing their own word problems or preparing to write constructed responses, students need to be comfortable with certain words, know their definitions, and be able to use them in writing tasks" (p. 17). A constructed response is a type of task developed to elicit an answer in writing such as an essay, short answer or sentence completion (Hancock, 1994). Constructed response questions are similar to open-ended questions. Urquhart (2009) noted the three kinds of prompts (questions and statements) in learning of mathematics to be 1) content, 2) process, and 3) affective prompts.

Symbols. Baumann and Graves (2010) defined *symbols* as icons, emoticons, graphics, mathematical notations, electronic symbols, and so forth. Symbols are not conventional words.

Summary and Significance of Study

The content and process of mathematics learning and instruction are based upon reform recommendations stemming from national and international reports of mathematics achievement of students in the United States. Within these documents, writing is recommended to promote conceptual understanding of mathematics. These documents guide classroom instruction and curriculum. Furthermore, because textbooks are aligned with national standards and textbooks are typically the dominant tools for classroom instruction, this study examines how textbooks align to standards documents by investigating the treatment of writing in the written (textbook) curriculum. Although a number of researchers have conducted and reviewed studies regarding a curriculum analysis of mathematics textbooks, these previous researchers mainly focused on content standards with an emphasis on middle school textbooks (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008). This emphasis on middle grades curriculum left a gap in the literature for elementary grade level mathematics.

In addition, this study builds upon the importance of acquiring mathematics vocabulary for conceptual understanding (Beck, Mckeown, & Kucan, 2002; Fisher & Frey, 2008; Graves, 2006, 2009, 1986; Graves, Sales, & Ruda, 2008; Marzano & Pickering, 2005; Nagy, 1988; Nagy and Herman, 1987; Ruddell & Shearer, 2002; Stahl & Fairbanks, 1986). As Draper, Broomhead, Jensen and Siebert (2010) stated, “Students do not usually enter content area classrooms knowing how to read and write the specialized print and non print texts of the various disciplines” (p. 2). Additionally, Alvermann (2002) noted that writing raises the cognitive bar by having students problem solve and think critically, and that students should be encouraged to write in many different ways despite the teachers’ content area expertise.

In the elementary grades, the opportunity for students to communicate mathematically using terms and symbols would better prepare K-5 learners with the tools needed for secondary education. This study provides findings in the area of writing, vocabulary, and mathematics that inform the field of how to prepare students for academic success in the upper grades where content area literacy is a focus.

Chapter 2: Literature Review

Integrating literacy practices into mathematics is recommended by reform efforts supporting "depth not breadth" in teaching mathematical concepts. More specifically, the NCTM (2000) recommends using the process strand of communication (both written and oral) to support conceptual development. These recommendations guide the development of textbooks that serve as the most pervasive mathematics instructional resource in classrooms. Therefore, the purpose of this study was to examine writing prompts in two mathematics textbooks at one grade level.

Mathematics Standards

An *Agenda for Action* (1980) and *A Nation at Risk* (1983) are two reports that provided detailed information of the mediocrity happening in mathematics education in our country. These reports helped to advance the field of mathematics by advocating standards to align with reform recommendations of higher-level mathematical thought.

NCTM standards documents. Within the NCTM standards, higher-level mathematical thought processes, such as those connected with writing, are nested within the documents. The documents produced by the NCTM are the *Curriculum and Evaluation Standards for School Mathematics* (1989), *Principles and Standards for School Mathematics* (2000), and the *Curriculum Focal Points* (2006).

Curriculum and evaluation standards for school mathematics. The development of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) became a national model for mathematics instruction. The NCTM produced this important document as "statements of criteria for excellence in order to produce change"

(NCTM, 1989, p. 2). One theme common to the NCTM Standards and to the recent changes in mathematics education is that "the study of mathematics should emphasize reasoning so that students can believe that mathematics makes sense" (NCTM, 1989, p. 29).

Principles and standards for school mathematics. Another document that impacted the development of curriculum materials was the production of the *Principles and Standards for School Mathematics* (NCTM, 2000). This document updated the 1989 *Curriculum and Evaluation Standards* while building an emphasis on teaching the content strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) through mathematical processes (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). Because writing is embedded within each of the NCTM (2000) process strands, a brief overview of each strand, respectively, is noted below:

Problem Solving:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving (NCTM, 2000, p. 52).

Reasoning and Proof:

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs

- Select and use various types of reasoning and methods of proof (NCTM, 2000, p. 56).

Communication:

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely (NCTM, 2000, p. 60).

Connections:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics (NCTM, 2000, p. 64).

Representation:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena (NCTM Principles and Standards for School Mathematics, 2000, p. 67).

In analyzing the NCTM (2000) process strands, it is the strand of Communication, more specifically communicating in written form, which guides my study. Furthermore, it can be noted that the processes are all interwoven components, where the use of writing can be implemented naturally throughout *each* process strand.

Curriculum focal points. Following the *Principles and Standards for School Mathematics* (2000), the *Curriculum Focal Points* (NCTM, 2006) were developed. These Focal Points consist of the most important mathematical topics for each grade level. They comprise related ideas, concepts, skills, procedures and processes that form the foundation for understanding and using mathematics. By using the frameworks of other high performing countries, such as Japan and Singapore, the Curriculum Focal Points have been integral in the revision of many state math standards for Pre-K through grade 8 (NCTM, 2011). The Curriculum Focal Points note:

> Three curriculum focal points are identified and described for each grade level, pre-K–8, along with connections to guide integration of the focal points at that grade level and across grade levels, to form a comprehensive mathematics curriculum. To build students' strength in the use of mathematical processes, instruction in these content areas should incorporate—
>
> - the use of the mathematics to solve problems;
> - an application of logical reasoning to justify procedures and solutions; and
> - an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics (What are the NCTM Curriculum Focal Points, 2011).

"The purpose of identifying these grade-level curriculum focal points and connections is to enable students to learn the content in the context of a focused and cohesive curriculum that implements problem solving, reasoning, and critical thinking" (p. 10). The *Curriculum Focal Points* are similar to the *Principles and Standards for School Mathematics* whereby the focus of reasoning, justification and communication are at the core of learning mathematics. In examining the nature of the wording of the *Curriculum Focal Points*, writing is also nested within the recommendations.

National Research Council. In addition to the documents and standards developed by the NCTM there are also mathematical proficiency strands that arose from a synthesis of research in mathematics. These strands were formulated based on the 2001 NRC report, *Adding it Up, Helping Children Learn Mathematics.* Within this report mathematics proficiency was stated as a goal for all students. The NRC's Mathematics Learning Study Committee (Kilpatrick, Swafford, and Findell, 2001) clarified mathematical proficiency as five interrelated strands:

- *Conceptual understanding*, the integrated and functional grasp of mathematical ideas, which enables students to learn new ideas by connecting those ideas to what they already know.
- *Procedural fluency*, the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- *Strategic competence,* the ability to formulate and represent problems.
- *Adaptive reasoning*, the capacity for logical thought, explanation, and justification.

- *Productive disposition*, the belief that mathematics makes sense and is useful (NRC, 2001, p. 116).

Similar to the NCTM's process strands, in order to be proficient in mathematics, the support for writing is evident.

Common Core Standards. The release of the Common Core State Standards (CCSS) is an effort to promote democracy, equity, and economic competitiveness in the standards movement that began over 20 years ago during the publication of the NCTM *Curriculum and Evaluation Standards for School Mathematics*. In 2010 the NCTM, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) produced a joint public statement regarding the support of the implementation of CCSS by stating:

> By initiating the development of the CCSS, state leaders acknowledged that common K–grade 8 and high school standards culminating in college and career readiness would offer better support for national improvement in mathematics achievement than our current system of individual state standards. The CCSS provides the foundation for the development of more focused and coherent instructional materials and assessments that measure students' understanding of mathematical concepts and acquisition of fundamental reasoning habits, in addition to their fluency with skills. Most important, the CCSS will enable teachers and education leaders to focus on improving teaching and learning, which is critical to ensuring that all students have access to a high-quality mathematics program and the support that they need to be successful (National

Council of Teachers of Mathematics, Common Core Standards Joint Statement, 2010, para. 2).

In 2009, 48 states adopted the CCSS and established goals of implementing standards to include directives of the initiative (Common State Standards Initiative, 2010,"In the States," section, para.1). The CCSS developed a set of standards titled, *Standards for Mathematical Practice* integrating the components of the process standards of NCTM and the proficiency standards from the NRC. The Standards for Mathematical Practice lists recommendations in the form of standards similar to the NCTM and the NRC:

- Make sense of problems and persevere in solving them; mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- Reason abstractly and quantitatively; mathematically proficient students make sense of quantities and their relationships in problem situations including the use of mathematical symbols, quantitative reasoning, and the meaning of quantities.
- Construct viable arguments and critique the reasoning of others; mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- Model with mathematics; mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation.

- Use appropriate tools strategically; mathematically proficient students consider the available tools when solving a mathematical problem.
- Attend to precision; mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose.
- Look for and make use of structure; mathematically proficient students look closely to discern a pattern or structure.
- Look for and express regularity in repeated reasoning; mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts (Common Core State Standards Initiative, 2010, p.6).

In addition to the NCTM's process standards and the NRC's proficiency standards, the CCSS recommendations have the process of writing nested within each of the Standards for Mathematical Practice while specifically stating the importance of the acquisition of symbols for proficiency. Clearly the NCTM, NRC and CCSS recommendations have the potential to utilize the process of writing within the learning of mathematics.

In the area of curriculum, the Standards recommendations provide the framework for curriculum and instructional development. In support of standards and reform in curriculum materials, Pattison and Berkas (2000) note that the process of integrating standards into the curriculum emphasizes learning and growth for all as the natural and desired outcome of reform in the schools.

Summary. Reform recommendations for school mathematics resulted in the development of standards documents from the National Council for Teachers of

Mathematics, the National Research Council, and the members of the Council of Chief State School Officers and the National Governors Association. In analyzing these standards documents, a common thread among these resources is that in order for students to become mathematically proficient students must be able to *reason* mathematically. Consequently, mathematics instruction should focus on strategies that utilize the process of reasoning. If instruction focuses on the process of *reasoning* specifically, the mathematical standards from the various sources will be adhered to effortlessly. Although there is some reference to writing mathematically in the standards, using writing in the service of learning mathematics can be utilized as a strategic method for mathematical proficiency in most every standard developed.

Mathematics Textbooks

The mathematics textbook is an important tool in the mathematics classroom. The mathematics textbook is developed based on the standards and recommendations from various documents and reports regarding research in mathematics teaching and learning. Because the textbook is the dominant tool in the mathematics classroom (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008) with direct claims to an alignment with standards recommendations, an analysis their of open-ended, writing prompts is warranted.

In an effort to investigate the types of prompts in a mathematics textbook, it is important to understand two components of mathematics curriculum: (1) forces that impact major developments in the mathematics textbook; and (2) research in the area of mathematics textbook content analysis. A review of these two components is included in the following section.

The National Mathematics Advisory Panel. *The National Mathematics Advisory Panel* under the *U.S. Department of Education* (2008) produced a detailed report based on scientific research of instructional materials both nationally and internationally. The Panel included scientists, scholars, and professional members. Based on the research findings in instructional materials the Panel noted U.S. mathematics textbooks were excessive in length and often encompassed non-mathematical content compared to mathematics textbooks from other countries that ranked higher than the U.S. on international assessments. Based on these findings in instructional materials, the Panel made the following recommendations for textbook publishers:

> Publishers must ensure the mathematical accuracy of their materials. Those involved with developing mathematics textbooks and related instructional materials need to engage mathematicians, as well as mathematics educators, at all stages of writing, editing and revising. (p. 26).

Trends in International Mathematics and Science Study (TIMSS). The 2007 Trends in International Mathematics and Science Study (TIMSS) provided data regarding the mathematics and science achievement of U.S. 4th- and 8th-grade students compared to that of students in other countries. Findings from the 2007 TIMSS also provided an analysis of the results by listing reasons for mathematics underachievement in the U.S. Two of these reasons related directly to the textbook: (1) textbooks in the United States are not as challenging as are those in other nations and (2) United States curriculum is “a mile wide and an inch deep,” lacking a focus at each grade (p. 3).

Development of textbooks aligned to standards. In the mid to late 1990's, the National Science Foundation (NSF) provided funding to curriculum projects directed at developing materials known as "standards-based curricula," that is, to projects whose goal was to develop curriculum materials aligned with the vision outlined in the NCTM Standards. An attractive feature of the standards based curriculum materials (vs. publisher developed materials) are the professed alignment to the process standards of the new learning goals supported by the NCTM, (i.e., mathematical thinking, reasoning, problem solving, with an emphasis on connections, applications, and communications). The National Science Foundation provided major funding to establish projects for the development, piloting, and refinement of these *Standards*-based mathematics programs. As Tarr et al. (2008) explains:

> In response to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 2000) and in an effort to influence and strengthen the quality of U.S. mathematics textbooks, the National Science Foundation (NSF) has invested an estimated $93 million in K-12 mathematics curriculum development efforts (NRC 2000). Curriculum development teams…worked together to produce mathematics textbooks that embodied "standards-based" characteristics, including active engagement of students, a focus on problem solving, and attention to connections within mathematical strands as well as to real-life contexts (p. 248).

These projects brought together mathematics specialists (mathematics educators, mathematicians, and classroom teachers) who wrote and revised materials, the classroom teachers who tested the materials with their students for several years and provided feedback to the writers, and the commercial publishers who produced and distributed the completed curricula (Reys, Robinson, Sconiers, & Mark, 1999).

Development of the mathematics textbook. One of the major influences on content and instruction is textbook/curriculum programs (Weiss et al., 2003). As states adopted the standards that reflected the NCTM vision, the publishing industry moved quickly to make adaptations to their textbooks (Stein, Remillard, & Smith, 2007). More recently, the publishing industry has revised their textbooks to include the CCSS. For example Pearson Scott Foresman (2011) notes:

> Only Pearson offers complete and cohesive support to implement the new Common Core Standards and provide the easiest possible transition. We combine the resources and expertise of the world's leading assessment company with evolving and continually improving instructional materials, content experts and professional development to help you, your teachers, and your students succeed at every step along the way (Pearson, 2011, n.p).

In addition, *Everyday Mathematics* (2010), a National Science Foundation funded curriculum project textbook notes alignment to the CCSS by stating:

> We believe these new standards present us with a wonderful opportunity to continue to refine and improve *Everyday Mathematics*, as we have done over many years and three editions. By summer 2011, McGraw-Hill Education will publish the *Everyday Mathematics Common Core State Standards Edition* (©2012). This updated edition will include new and revised lessons at every grade level to ensure that *Everyday Mathematics* meets and exceeds CCSS. The *Everyday Mathematics CCSS Edition* will provide a comprehensive set of print

and digital components to help you meet your students' instructional needs (*Everyday Mathematics*, 2010, n.p.).

Although textbook companies are adhering to the recommendations currently, this was not always a focus. Traditionally, mathematics curricula of the 1970's and the 1980's and their relationship to student learning were not viewed as important aspects of scholarly investigation (Grouws, 1992). However, two factors assisted in changing this view. The first factor relates to the research in the area of instructional support regarding the role of the textbooks as a dominant tool in mathematics instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008). Secondly, national reports regarding student achievement garnered attention for the role and use of the textbook in the classroom.

Textbooks and teachers' use. The textbook is used in many facets in the mathematics classroom. The mathematics textbook is not only researched as the dominant tool used in mathematics instruction, but also has the value of providing professional development in mathematics content. The 2000 National Survey of Science and Mathematics Education investigated the use of the textbook in K-12 classrooms. The findings from the survey data indicated that commercially published materials were used in 87% of classrooms in grades K-4 and 97% of classrooms grades 5-8 (p. 81). According to the survey data, Weiss, Pasley, Smith, Banilower and Heck (2003) found that *Everyday Mathematics* published by McGraw-Hill/Merrill Company, and *enVision MATH* published by Addison Wesley Longman, Inc/Scott Foresman, had significant market share (over 50%) in both elementary and middle mathematics school curriculum. Additionally, they reported that 71% of lessons in the textbook were used for instructional strategies (p. 10). Further analysis also revealed that the determining influence regarding lesson content was state/curriculum standards, the textbook program, followed by the state/district accountability system. Additionally, in the reports from the Elementary section of the 2000 National Survey of Science, Status of Elementary School Mathematics Teaching, Malzahn (2002) noted that 78% of classes in grades K-5 completed textbook problems routinely. These findings suggest that along with the standards, the mathematics textbook has significant influence in the classroom, potentially affecting opportunities and thus student achievement levels.

Research on mathematics textbook and content analysis. There is a paucity of research in the area of elementary mathematics textbooks and investigation of process standards. The content strands encompass the majority of content analysis in mathematics textbooks. In addition, the majority of content analyses are conducted with middle and high school grades textbooks.

Selection of Textbooks. The NRC, in a 2004 report, stated, "the conduct of a content analysis requires identifying either a set of standards against which a curriculum is compared or an explicitly contrasting curriculum" (p. 74). Researchers who analyze mathematics textbooks and their effects on achievement generally use two criteria for selecting textbooks: selection of widely-used series and both NSF-funded and non NSF-funded curricula (Hodges et al., 2008; Johnson, Thompson, & Senk, 2010; Tarr, et al., 2008). In addition a study conducted by Tarr et al. (2008) regarding mathematics textbooks and their use in middle grades classrooms incorporated both NSF-funded textbooks and publisher developed mathematics textbooks with "significant market share" based on the 2000 Mathematics and Science Education Survey (Weiss, Banilower, & Smith, 2001).

Process strands and textbook analysis. Research on textbook analysis is limited. However, one study focused on two (grades 3-5) elementary textbook series (one NSF funded and one publisher-generated) regarding process standards. Although the analysis was conducted regarding textbook publishers' use of assessment, the authors' use of elementary grades textbooks and process standards alignment was pertinent to my study. Hunsader et al. (2006) developed a modified framework for the analysis of one mainstream curriculum compared to one NSF curriculum. The results suggested that neither of the publishers, whose assessments were analyzed in this study, integrated these processes into their assessments with any regularity. More importantly the researchers noted the importance of teacher decision when textbook assessments fail to reflect the process standards.

In examining Hunsader et al.'s (2006) framework, the "communication in written form" category and the "reasoning" (justify, explain one's thinking) category were coded separately. The author's determined that items that required students to "explain their thinking" or provide a "justification" required writing. It can therefore be concluded that problems which required students to reason required students to communicate in written form.

Content strands and textbook analysis. Funding by the NSF and the Carnegie Corporation of New York led to an evaluation of eight of the most widely used textbook series from major publishers, along with four sets of materials developed from the NSF. The American Association for the Advancement of Science (AAAS), *Project 2061* investigated the extent to which textbooks address six important mathematics concepts and whether or not the material is satisfactory for use in classrooms where literacy in mathematics is a goal for all students. The six middle school benchmarks (number concepts, number skills, geometry concepts, geometry skills, algebra graph concepts, and algebra equation concepts) were selected as the content criteria. The findings from the analysis of the curriculum were reported in a "Good News" and "Bad News" category. The findings appeared contradictory. For example, in the "Good News" category the findings suggest that the top two series contain both in-depth mathematics content and excellent instructional support. However, in the "Bad News" category the findings suggest that a majority of textbooks are particularly unsatisfactory in providing a purpose for learning mathematics, taking account of student ideas, and promoting student thinking. This study was fundamental in providing the middle grades with an awareness of the degree of coverage in content strands.

Haggarty and Pepin (2002) also conducted a study on middle grades mathematics textbooks. The researchers investigated the similarities and differences of middle grades mathematics textbooks in three countries in Europe (England, France and Germany). The aim of the research was to understand the range of ways in which the common content was presented in the textbooks. The research also investigated the ways teachers used the textbooks. In order to highlight the feature of teacher pedagogy, the concept of angles was examined in the three textbooks. Through a procedure of coding questions surrounding the concept of angles and teacher interviews, the findings suggest that different textbooks and teaching styles offer different opportunities to learn the content. Each of the textbooks had different levels of instruction for the concept of angle. One of the fundamental findings in this study was acknowledging the use of language or mathematics vocabulary in each of the materials. Although this study was conducted for middle grades textbooks on the content strand of geometry, the study indicates the importance of acknowledging mathematics language and vocabulary in the textbook.

In another exploration of the vocabulary of mathematics prompts, Herbel-Eisenmann (2007) investigated the voice in the mathematics textbook by identifying and categorizing words in one NSF funded student edition, *Thinking with Mathematical Models (TMM)*. By investigating the linguistic choices made by the textbook authors, the researcher categorized words based on four categories: imperatives, pronouns, modal verbs and expressions. Herbel-Eisenmann's investigation (2007) heightened awareness of the importance of language choice to achieving some of the goals of the Standards. This study also provided a window into investigating how the process standards were situated in mathematics textbooks. However, the focus of the study was on understanding the language to determine the voice of the mathematics textbook, not necessarily a focus on student learning or teacher development.

Summary. Research on textbooks has consisted primarily of middle and high school textbooks consisting of a review of content strands. In agreement, Johnson (2010) noted that studies of mathematics textbooks generally focus on a single content area, such as data analysis, probability, or reasoning and proof. The limited research in this area of process standard investigation needs to be addressed. In addition the paucity of research on content analyses of elementary grades textbooks is limited. An emphasis on the role of the textbook and research investigating vocabulary in the prompts of mathematics textbooks is warranted.

Mathematical Writing Prompts

There are many types of writing prompts that facilitate a constructed response with the type based on the purpose for writing in mathematics. In this section, I review (1) the types of writing prompts, (2) the formats of writing and (2) the role of language and vocabulary for communicating in mathematics.

Types of writing prompts. Within the field of mathematics, there are four types of mathematics writing prompts. These types of prompts are 1) content 2) process 3) affective and 4) narrative prompts (Baxter et al., 2001; Dougherty, 1996; Shield and Galbraith, 1998;Urquhart, 2009). A *content* prompt, according to Urquhart (2009), focuses on mathematical concepts and relationships. Student responses can be in the form of defining, comparing and contrasting, and explaining (Dougherty, 1996). A *process* type of prompt requires students to reflect on why they use various solution strategies or the steps they take to solve a problem (Dougherty, 1996) More specifically, *process* prompts require students to explicate their learning process (Urquhart, 2009). The third type of prompt consists of a task in which students write or journal about opinions and feelings (Baxter et al., 2001; Shield & Galbraith, 1998). The *narrative* prompt is a type of journal writing prompt. These types of prompts are commonly used for purposes of high stakes testing. Within this type of prompt, the constructed response can be in the form of a response that portrays math content in an imaginary or real world sense. Furthermore, mathematical *narrative* content and themes are embedded within children's literature (Burns, 2004; Whitin & Whitin, 2000).

Formats of mathematical writing. Depending on the type of writing prompt there are two types of writing formats in mathematics: math journals and journal writing.

Baxter, Woodward and Olson (2001) note that *math journals* are intended to reinforce mathematics concepts by describing or explaining mathematical ideas or reasoning. In *journal writing*, the student would write about opinions or feelings regarding the mathematics content (Shield & Galbraith, 1998).

Prompts for journals. In journal writing, the prompts consist of a task in which students write about opinions and feelings, that is, an *affective* prompt (Baxter et al., 2001; Shield & Galbraith, 1998). Another type of journal writing prompt is a *narrative* prompt. However, in math journals the writing prompt consists of a task that has expository purposes such as describing or explaining a mathematical process or content. Aspinwall and Aspinwall (2003) conducted a study with 23 fifth-grade students regarding writing prompts for journals. The writing prompts were scored in four categories: algorithms and computations, limited understanding, utilitarian value, and conceptual understanding. In analyzing the data the researchers noted that open-ended prompts provided teachers with a window into students' perceptions and knowledge. The researchers also noted that student responses to the open-ended prompts provided teachers with information that was essential for planning purposeful instruction. Although it was not revealed where or how the researchers obtained the prompts, the findings regarding the usefulness of mathematical writing for instructional purposes are useful for future studies.

Writing prompts for journaling tasks can also be developed by teachers. For example, Baxter et al. (2005) examined how writing revealed four low-achieving seventh-grade students' mathematical proficiency. The researchers' interest for the purpose of the study stemmed from reform recommendations on communicating in mathematics. The questions aimed at identifying what writing in mathematics revealed as students were encouraged to write about their mathematical ideas and reasoning through the use of teacher-developed writing prompts. The mathematical prompts consisted of an average of 30 prompts comprised of affective components, new concepts learned, or justification of an answer. Using data gathered from classroom observations, students' journals, and interviews with the teacher, the researchers were able to understand the role of written communication in mathematical proficiency. Based on conceptual and affective coding of the responses, the findings suggest that writing was a way for students to communicate their feelings to the teacher regardless of the prompt. In addition, the answers provided the teacher with valuable information regarding students' mathematical proficiency while planning mathematics lessons centered upon student understanding. Although the findings support the benefits of implementing writing in the mathematics classroom, it was not clear from the study how the teacher developed her prompts. Did the teacher use the textbook for writing prompt ideas or were the prompts derived on her own with no support? These questions need to be addressed if we are to understand the types of prompts that assist in facilitating mathematical proficiency.

In a study conducted by Dougherty (1996), first year heterogeneous eighth-grade algebra students were given prompts that focused on content, process, and affective components of mathematics. The prompts were given to students in the form of a nightly homework assignment. The goal was to have students reflect on the mathematics topics completed in class that particular day. Each type of prompt developed was to provide insight regarding the content students were learning or the process students had to undergo to solve a mathematics problem. Furthermore prompts were developed to provide information regarding feelings or attitudes of particular mathematics topics. An analysis of the prompt responses provided findings that these three particular types of mathematics prompts provided the students with a resource to assess their growth, and instructional benefits of detecting trends from within and across the mathematics classes regarding the progression of comprehension of particular topics, skills, concepts, and attitudes/beliefs using beginning of the year and end of the year assessments.

Collaborative journals. In a self-study, Fequa (1997) explored math journals with her kindergarten class. The teacher became interested in how to enhance her students' understanding of math concepts. While reflecting on her own classroom practice and student learning, the teacher decided to use a large book (*big book journal)* for a class math journal rather than using individual journals. Using a big book journal alleviated two of the teacher's concerns. First, the activity differed from the traditional individual writing assignment, and second, it focused on real problem solving in their classroom, rather than using arbitrary, "made up" story problems. The findings from using the big book journal were many. Students interacted as they discussed how to solve a problem and the teacher recorded the student responses. The journal also provided students with the opportunity to think about and use various symbols (including letters, words and mathematical drawings). The journal also allowed students to represent their thoughts in a meaningful way while being actively involved in reasoning, comparing and counting.

Powell (1997) also found journals to be a useful tool in the mathematics classroom. This classroom study actually analyzed responses in journals that related to the Greatest Common Factor (GCF) and the Least Common Multiple (LCM). The method to collect the data was done qualitatively by reviewing the responses noted in the journals of the students. The findings suggest that journaling captured the verbal representation of student thinking. Journaling provided the teacher a way to capture, examine, and respond to a student's mathematical thinking. In this study journaling also provided an opportunity for students to reflect on mathematical experiences, to examine their written reflections, and to reflect on their ideas critically. This type of reflective thinking enabled the student to become an active learner. Through the use of journaling in this case study, the researcher noted that the writing helped the students develop confidence in their understanding of mathematics and become more thoroughly engaged with mathematics.

Short response. Scheibelhut (1994) conducted a classroom project with first grade students and preservice teacher's implementation of writing in mathematics. Students were asked to solve various problems and respond to various affective questions regarding mathematics in short response formats. After reviewing the responses of the first-grade students' writing, the preservice teacher was convinced that incorporating writing into mathematics had many advantages. Through writing, the children were able to make sense out of mathematics and recognize its relationship to their everyday lives. The writing of the students also provided the pre-service teachers with insight into the attitudes and needs of the individual students and may have uncovered reasons for mathematics anxiety.

Writing and problem solving (k-12). Using writing to solve a mathematical problem can range from listing steps in the solution process to justifying why an answer is correct. Cognitively Guided Instruction (CGI) is a developmental program based on students' reasoning. Through this program, based on the premise of attending to student reasoning, understanding the reasoning, and teaching in a manner that reflects this knowledge, teachers can and will provide children with a mathematics education better than if they did not have this knowledge (Sowder, 2007). Therefore, student reasoning in verbal or written form provides a window into where the student's level of knowledge exists and serves as a guide for future instruction.

For example, Parker (2007) used the philosophy of CGI with a mathematics curriculum to assist 32 second-grade students to improve their ability to justify solutions to word problems in writing. Over a four week period, students were given mathematics story problems to solve where the explanation process of the solution was the focus. The gradual release of student's oral description into written responses was investigated. The method of analysis used to score the responses on the pre-and post tests was taken from the framework developed in the Wisconsin Knowledge and Concepts Examination (WKCE) criterion referenced test scoring rubric. The findings suggest that oral sharing of strategies aided the transition to written expression. In addition, the students with both low and high reading ability developed language for expressing thoughts mathematically.

Evans (1984) examined the use of writing to problem solve in short response format. The researchers were two fellow fifth grade teachers. One classroom was an experimental group while the other was a control group. CTBS scores were analyzed from both classes. The scores showed that the control group achieved higher scores due to a gifted population of about six students. The experimental group used writing with computation during math instruction. The control group used no writing during math instruction. Writing in the experimental group consisted of two methods: how to perform a computation and definitions. The findings suggest that the students with the lowest pretest scores in the experimental group made the most gains. It was further noted from the findings that writing gave the researchers one more tool to help less capable students grow. This classroom research study provided information on the benefits of writing for low achieving students; however, it lacked information on how much time was spent on writing in the experimental group as well as specifics on sample size.

Brown’s (1993) classroom study encompassed the use of writing in mathematics to motivate below grade level seventh-grade students. The study was conducted over several weeks during a unit on addition. The researcher provided the students with the opportunity to write authentic addition problems to exchange with their peers. The peers solved the problems using computation. The researcher evaluated the student samples on the basis of whether the response was actually an illustration of the desired operation. In conjunction with the English teacher, the sample writing problems were bound and put into a problem-solving notebook for other classes to use. Findings noted by this researcher revealed students understood more through this activity than they could verbally communicate. The researcher also noted that the students experienced a feeling of achievement and success in mathematics. Although this research study provided information regarding the importance of teacher judgment, more information was needed regarding the research methods used in the study.

As noted in the previous sections, writing in mathematics has many benefits. Additionally, writing can serve as a catalyst for discourse in oral form. In the section below information regarding how writing is used to facilitate discourse in oral form through math logs and free writing is discussed.

Writing and oral discourse. Steele (2001) explored how a teacher used "math logs" for 15 minutes at the beginning of each day in order to facilitate communication during a problem solving activity. These "math logs" served as a way for students to verbalize their responses by thinking about how they worked out solutions, organized their responses, evaluated their own approaches and clarified their thinking while drawing upon prior knowledge for conceptual development. Writing in mathematical logs was used in order for students to organize their thoughts in anticipating the teacher's questions and their possible answers. The teacher not only asked students questions, she was also an active listener. She was always open to change her initial plan based on students' predictions and ideas. Thus, this study demonstrated how the teacher successfully used probing questions to get the students thinking more algebraically.

Elbow and Sorcinelli (2006) also supported the notion of using writing in the form of free writing to facilitate student conversation. Free writing is a non-threatening written response used by the students to respond to a question. Additionally, Elbow & Sorcinelli (2006) stress the benefit of free writing by stating that "students will have more to say in discussion, and be less afraid to speak up, if you start with a few minutes of free writing. Two minutes of quick free writing after you ask a question will make all the difference in the world " (p. 3).

Writing and metacognition. Pugalee (1997) highlighted samples of mathematics responses from a second year algebra course. Problem solving tasks were administered to the students. The responses were then reviewed by the researcher for comprehension of concepts. The findings from the responses suggested that students were aware of metacognitive behaviors while solving problems and were able to communicate those aspects of the problem solving process. These writing responses also provided the teacher with examples of mathematical thinking to share with other students and also provided the teacher with information to make instructional decisions about the abilities of the students.

In a second study, Pugalee (2001) investigated whether students' writing about their mathematical problem solving processes showed evidence of a metacognitive framework. Twenty ninth-grade algebra students provided written descriptions of their problem-solving processes as they worked with six selected mathematics problems. Qualitative responses were classified in groups and subgroups based on similarity, orientation, organization, execution and verification. The findings suggest that a metacognitive framework was present in the writing of the subjects. Additionally, the findings supported the premise that students' writing can provide a source of information for teachers to assess how their students learn and think about mathematics.

Steele's (2005) study explored the use of writing to help students develop schemata for algebraic thinking within one month. Schema knowledge consists of identification, elaboration, planning and execution of knowledge. Eight seventh-grade pre-algebra students participated in a teaching experiment in which they solved algebraic problems related in mathematical structure. The students were given problems to solve individually, then to write about their thinking by reflecting. Students then met in small groups to discuss their problem solving approaches. Qualitative methods of data analysis were implemented to determine the effectiveness of writing to develop schema knowledge. Interviews and field notes were organized based on patterns and themes. The findings suggest that through explaining in writing the generalizable patterns in relationships between the quantities in the problems, they made their algebraic thinking explicit. This explicitness helped the students to develop schemata knowledge needed for solving similar algebraic problems.

Writing and assessment. Bolte (1997) examined the combined use of concept maps and interpretive essays as a method of assessment in three mathematics courses. The population studied consisted of 23 prospective elementary teachers enrolled in a mathematics content course, 63 students enrolled in a Calculus I course, and 17 prospective secondary mathematics teachers enrolled in a Survey of Geometries course. The students were asked to construct a concept map regarding a list of terms related to a familiar topic. After the concept map was completed, the students wrote an accompanying interpretive essay in which they clarified and developed the relationships expressed on the map. The essays were to give students the opportunity to reflect on the relationships illustrated on their concept map and refine their thoughts. Each concept map and interpretive essay was scored using an holistic scoring criteria. The concept map criteria's focus was on organization. The findings suggested that the combined use of these instruments provided substantial insight into the degree of connectedness of students' knowledge with respect to the given topics and enabled the instructor to assess the degree to which the mathematical material was being integrated into the learner's knowledge base. Additional information on how the scaffolding of instruction was included would be beneficial.

In addition, many high stakes assessments include items that involve the use of writing in the form of constructed responses to assess knowledge. For example, the National Assessment of Educational Progress (NAEP) assessments are conducted periodically in mathematics for grades 4, 8, and 12 (IES, 2010). The framework used for the NAEP assessments consists of five content areas (number/operations, measurement, geometry, data analysis/statistics/probability, and algebra). The questions are submitted in two formats: multiple choice and constructed response. Additionally, the constructed response format includes short and extended responses. The assessment of student mathematical knowledge through these items may require students to construct a few sentences, a paragraph, or full page response. Although the results from the constructed responses are combined with the multiple choice items, the importance of writing for assessment in national reporting is valued.

Summary. The findings regarding the types of writing in mathematics provide useful information as well as identify gaps. Many researchers focusing on communication in mathematics for teaching and learning (Burns, 2004; McIntosh & Draper, 2001; Pugalee 2004, 2005; Senk & Thompson, 2003; Shulman 1986) agree that teachers can learn about their students' thinking through the students' writing as well as the students' spoken words. Miller (1991) states "students who will not ask questions in class may express their confusion privately in writing” (p. x). The act of writing includes many benefits, such as learning mathematics content, providing a window into student thinking, affording teachers with information on planning, having students’ problem solve while focusing on their mathematical thinking process (metacognition), and opportunities to facilitate conversations through the use of the writing task. The research reported in this section provided useful information regarding the writing task parameters, but the authors did not specify the origins of the writing prompts or how they were derived. In addition, the importance of communicating in mathematics using the language of mathematics, more specifically vocabulary, was not mentioned across the findings. The lack of information about the prompts and the limited focus on vocabulary needs to be addressed.

Mathematics Language

In order to communicate mathematically, the language of mathematics is an important factor. In order for students to read the mathematical prompts and construct a response, mathematical vocabulary, meta-language, and symbols need to be addressed.

Vocabulary in mathematics. Communicating in writing has many advantages for learning mathematical concepts. However, many complex features of mathematics language make written communication in mathematics a challenging task. For example, when students read a prompt and construct a written response, mathematical vocabulary and signs/symbols require the student to transmediate, or interpret across one sign system to another (words to signs or diagrams). For this reason, special attention must be given to the unique characteristics of mathematics vocabulary and symbols that influence a student's ability to comprehend mathematics text (Thompson, Kersaint, Richards, Hunsader, & Rubenstein, 2008). Words and symbols need to be acquired and conceptually understood or "known" in order to communicate in mathematics.

For example, Nagy and Scott (2000) (as cited in Lehr, Osborn & Heibert, 2000) identified several dimensions that describe the complexity of what it means to know a word. First, word knowledge is *incremental*, which means that readers need to have many exposures to a word in different contexts before they "know" it. Second, word knowledge is *multidimensional*. Words have multiple meanings (e.g., *sage*: a wise person; an herb) and serve different functions in different sentences, texts, and conversations. Third, word knowledge is *interrelated* in that knowledge of one word (e.g., *urban*) connects to knowledge of other words (e.g., *suburban*, *urbanite*, *urbane*).

The following sections will explain the different types of mathematics vocabulary and the various mathematical signs/symbols important for mathematical writing prompts in the written (textbook) curriculum that facilitate a constructed response.

Domain specific vocabulary. According to Baumann & Graves (2010), academic vocabulary was found in content area textbooks and other technical writing and can be classified in two ways. The first definition is recognized as *domain specific academic vocabulary*, i.e., content specific words used in different domains such as geometry, biology, civics and geography. Brozo and Simpson (2007) define academic vocabulary as word knowledge that makes it possible for students to engage with, produce, and talk about texts that are valued in school. These words have been referred to as technical vocabulary (Fisher & Frey, 2008) or content specific vocabulary (Hiebert & Lubliner, 2008) or as Tier 3 words (Beck, Mckeowen, and Kucan, 2002). Graves and Bauman (2010) provide the following terms as examples of *domain specific vocabulary* according to their classification scheme*: apex, bisect, geometry, polyhedron, Pythagorean Theorem, scalene triangle*.[1] For purposes of this study, Domain Specific Academic Vocabulary has been modified to *domain specific vocabulary (DSV).*

General vocabulary. The second definition of academic vocabulary is defined as *general vocabulary*, i.e., the broad, all-purpose terms that appear across content areas but may vary in meaning because of the discipline itself. These types of words are

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[1] These terms were adopted from content area textbooks, informational trade books, internet sources and Marzano's & Pickering's (2005) *Building Academic Vocabulary* word list.

challenging to learn and use during communication efforts because, depending on the domain, the word will have different meanings. For example, Hiebert and Lubliner (2008) describe academic vocabulary as, "words whose meanings often change in different content areas, (e.g., form, process)" (pp. 111-112). Graves and Bauman (2010) provide the following terms as examples of *general academic vocabulary* according to their classification scheme: *analyze, assume, code, conduct, context, document, error, link, minor, period, project, range, register, role, and sum.*[2] For purposes of this study, General Academic Vocabulary has been modified to *general vocabulary (GV).*

Meta-language. Meta-language, according to Graves and Bauman (2010), can be defined as terms that are used to describe processes, structures, or concepts commonly included in content area texts. Graves and Bauman (2010) provide the following terms as examples of *meta-language* according to their classification scheme: *calculate, compare, estimate, explain, investigate, model, observe, and prove*. Although these words may be used in different content domains, the meaning of the term remains the same.[3]

An example of meta-language in mathematics writing can be found in prompts that facilitate a constructed response. For example, Urquhart (2009) developed a list for the most commonly used terms that facilitate a constructed response on state tests in mathematics: *analyze, describe, evaluate, narrate, reflect/question, summarize and*

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[2] These terms were adopted from Coxhead's (2000) word list. However, when terms do not fit into the classification system of general academic vocabulary or domain specific vocabulary because they describe a process, then the term will generally fit into the category of academic vocabulary called "meta-language."

[3] These terms were adopted from Marzano's & Pickering's (2005) *Building Academic Vocabulary* word list and Pilgreen's (2007) *Academic Terms for Book Parts*.

synthesize. The terms *describe, narrate, reflect/question, and synthesize* were not listed in the Coxhead (2000) word list. However, further analysis of the words placed these terms in the category of *meta-language* in Graves and Bauman's (2010) classification scheme. These words all describe processes in mathematics *and* have the *same* meaning across different domains – hence the definition of meta-language.

Using mathematical language to communicate is a complex process. In order to achieve this task, students need to be familiar with not only mathematics vocabulary including meta-language, but also signs and symbols. In understanding the nature of signs and symbols in mathematics communication, the field of semiotics is discussed.

Signs and symbols. Understanding how semiotics relates to the field of mathematics communication is important for instructional purposes. Historically, the definition of semiotics began with the philosopher Charles Sanders Peirce who discussed the meaning of "sign" as a part of a mediated language system consisting of three parts: the sign or signifier (conveys information); a signified (an object or idea that the sign is related throughout), and lastly, an interpretant (which is an interpreted further sign of the object) defining a three part system of meaning (Malcolm and Goguen, 1998). Discourse occurs when the sign receiver (listener or reader) understands the information that the sign producer (speaker or writer) intends to convey (Thompson, et.al, 2008). Similarly, Pirie (1998) lists symbolic language (using mathematics symbols) as one of the means to communicate in mathematics. In addition to acquiring meaning of vocabulary in a written mathematics prompt, the mathematics learner also has to acquire meaning of mathematical signs and symbols in order to achieve mathematical literacy. The complexity of learning and communicating math symbols and words is similar and should be treated with the same understanding as learning a foreign language. For example, Thompson et al. (2008) classify a math student as a *mathematics language learner*. The authors underscore the importance of providing many opportunities to learn and use the language of mathematics on a consistent basis in order for proficiency to occur in mathematical communication.

Rubenstein and Thompson (2001) specify that symbols are the hallmark of mathematics. They discuss the implications of teaching symbols within the area of communication, i.e., reading, writing, and speaking. Regarding oral communication, students must translate symbols into spoken language. In written communication, students must produce symbols, and in reading symbols, students must be able to understand the concept represented by the symbol. Hodges et al. (2008) note educators believe that using visual representations, such as symbols, drawings, and graphs, helps middle-school students reason about and understand mathematics. Moreover, these representations support students' learning and help them communicate their mathematical ideas (Hodges et al., 2008).

Baumann & Graves (2010) state that symbols can be presented as icons, emoticons, graphics, mathematical notations, and electronic symbols that are not conventional. Baumann and Graves (2010) also list examples of *symbols* according to their classification scheme: X^{-24}, $a^2 + b^2$, >, <, ☺, ;), $, %, #, and @ (p.10).

Vocabulary and signs/symbols are important components for communicating in mathematics. In order to communicate mathematically in written form, it is important to understand how mathematical vocabulary and signs/symbols are situated within a task that requires such a response. A modified version of the Baumann and Graves (2010) word classification system will be used as the framework for analyzing the vocabulary and symbols in the prompts of mathematics textbooks that have the potential to facilitate a written response.

Summary. Understanding the types of vocabulary and symbols needed for mathematical writing is important. Reading a mathematical writing prompt and facilitating a constructed response requires a learner to understand the language of mathematics. A review of mathematical language provides insight into the complex nature of vocabulary, meta-language, and symbols needed to communicate mathematically. Investigating the literature regarding content analysis of mathematics textbooks and the types of mathematical writing has guided me to formulate my research questions.

The following sections will provide a lens for the significance of writing in mathematics. First, a brief review of the literature regarding writing theory and the correlation to mathematics will be discussed followed by a review of the literature of writing to learn.

Writing Theories and Mathematics Correlation

Cognitive, social and rhetorical features are interwoven components in the complex process of writing. From a cognitive perspective, NCTM (2000) suggests that writing in mathematics can also help students "consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson" (p. 61). Similarly, Greenfield and Bruner (1969) observed that cultures with technologies such as written language and mathematical formalisms will "push cognitive growth better, earlier, and longer than others" (p. 654). Bruner (1986) maintained, "We teach a subject not to teach little living libraries on the subject, but rather to get a student to think mathematically for himself (sic)... to take part in the process of knowledge-getting. Knowledge is a process not a product" (p. 72).

From a socio-cultural perspective, mathematics tasks that facilitate written responses also have the potential to facilitate discourse in oral form. Baxter et al. (2001) suggest that written assignments that encourage students to justify and explain problem solutions have the potential to support and extend oral conversations. In support of this notion, Bereiter and Scardamalia (1987) note that empirical support from studies has shown that children write longer texts and texts of higher quality when they are provided with a “conversational partner” during writing (Daiute, 1986; Daiute & Dalton, 1993). O’Connell & O’Connor (2007) also mention the benefits of students writing to facilitate oral discourse and schema building:

> As students struggle to get their thoughts into words, they are challenged to process the ideas in order to restate them, elaborate on them, or conjecture about them. As they listen to their own and others’ thinking they often recognize their confusions, question their understandings, and fold others’ ideas into their own in order to modify and refine their knowledge (p. 1).

Supporting the importance for writing in mathematics, Connolly & Vilardi (1989) claim that writing develops thought processes useful in doing mathematics: abilities to define, classify, or summarize; methods of close, reactive reading; meta-cognition (an awareness of one’s own thinking and learning); and an awareness of attitudes and identification of mistakes and errors. Regarding the different ways writing can be used in the mathematics classroom, cognitive, social as well as rhetorical perspectives in terms of audience and purpose are nested within the constructed response. Cognitive, social and rhetorical theories of writing also define theoretical implications of writing in mathematics.

Writing To Learn

Writing is an important component across academic disciplines in education. The influence of writing as an instructional tool in the mathematics curriculum was highlighted during the 1980's as a part of the Writing Across the Curriculum (WAC) movement. Romberger (2000) defines WAC as a pedagogical movement that began as a response to a perceived deficiency in literacy among college students. WAC is premised on theories that maintain that writing is a valuable learning tool that can help students synthesize, analyze, and apply course content. Within this movement, writing to communicate--or what James Britton (1975) calls "transactional writing"--means writing to accomplish something, to inform, instruct, or persuade. Writing to learn, is different. We write to ourselves as well as talk with others to objectify our perceptions of reality; the primary function of this "expressive" language is not to communicate, but to order and represent experience to our own understanding. In this sense language provides us with a unique way of knowing and becomes a tool for discovering, for shaping meaning, and for reaching understanding (p. x).

Nagin (2003) notes that Writing to Learn (WTL) rejected the notion that writing serves primarily to translate what is known onto the page. Instead, advocates of WTL suggest teachers use writing to help students discover new knowledge to sort through previous understandings, draw connections, and uncover new ideas as they write. As part of the WAC program, WTL activities may also be used to encourage reflection on learning strategies and improve students' metacognitive skills (Brewster & Klump, 2004). Elbow and Sorcinelli (2006) acknowledge some of the cognitive factors by stating how low stakes writing (a type of freewriting that is used more informally and tends to be ungraded) has the potential to facilitate students' reflection, their discovery of new knowledge, their ability to draw connections, and develop metacognitive skills and uncover new ideas without having the fear of being graded.

Forsman (1985) provided a practical rationale for writing to learn. She stated "as teachers we can choose between (a) sentencing students to thoughtless mechanical operations and (b) facilitating their ability to think. If students' readiness for more involved thought processes is bypassed in favor of jamming more facts and figures into their heads, they will stagnate at the lower levels of thinking. But if students are encouraged to try a variety of thought processes in classes; they can, regardless of their ages, develop considerable mental power. Writing is one of the most effective ways to develop thinking" (p. 162).

Langer and Applebee (1987) present a project regarding the role that writing plays in content area learning in the secondary school curriculum. Within this project, writing was used by teachers as a way to help students review what they had learned by using logs or journals for writing. Within these journals, summarizing new material, note-taking, and study exercises were frequent practices for teachers to write about. However the most frequent use of writing was the review and summarizing of new learning in science classes. Another form of writing researched was impromptu writing. This type of writing asked students to write after specific events, i.e. after the presentation of a guest speaker, writing about the rules of a game, or after the reading of a book. Writing to learn was used as a tool for evaluation. Using this method, teachers used student writing as a means to assess what students have learned.

Similarly, Nuckles, Hubner, Dumer, and Renkl (2010) discuss the findings regarding two longitudinal studies that investigated journal writing while reporting an expertise reversal effect. In the experimental groups, students wrote regular journal entries over a term while receiving a combination of cognitive and metacognitive prompts. Initially, the control group received no prompts. The findings from the data (analyzed using a SOLO taxonomy ranging from six levels of knowledge), suggest that the experimental group applied more cognitive and metacognitive strategies in their journals and showed higher learning outcomes than the control group. The experimental group also showed increasingly higher performance ratings on the mid-year assessment than the control group. However, towards the end of the semester, the writers in the experimental group scored lower than the control group. The researchers describe this negative impact as the expertise reversal effect. In the study, this type of effect describes how the external guidance of prompts was beneficial initially during instruction, but later interfered with students' application of strategies. The implications from this type of effect can have a negative impact in cognitive and motivational factors in learning. The researchers believe that more research is needed regarding the extraneous factors of "overscripting or overprompting" and the effects on student learning.

Through the National Writing Project, Nagin (2003) notes that writing is a tool for thinking while emphasizing how the facilitation of such instruction can foster active learning and critical reflection. More specifically, "writing is a complex activity; more than just a skill or talent, it is a means of inquiry and expression for learning in all grades and disciplines” (p. 3). Writing in journals has the power to impact learning from a metacognitive stance by supporting the monitoring of comprehension and evaluation of learning outcomes (Nuckles et al., 2010).

Summary

A review of the research regarding mathematical standards developed in support of reform recommendations underscores the importance of utilizing mathematical process standards to acquire mathematics content. More specifically, the process of *reasoning* was found as a central component in attaining mathematics proficiency throughout the various standards documents. Through the process of writing to reason mathematically, it appeared the additional process standards would be adhered to logically. Furthermore, the standards documents also provide textbook publishing companies with a type of framework for the development of the content within the mathematics textbook. Because mathematics textbooks were found to be a dominant tool in the mathematics classroom, it would be reasonable to state that textbooks should have prompts that facilitate a constructed response whereby students can communicate by way of mathematical reasoning. Conversely, research regarding how mathematics textbooks adhere to mathematical process standards specifically is limited. Because the limited amount of research investigated middle grades textbooks primarily, a paucity of research was noted for elementary grades mathematics textbooks.

Furthermore, an examination of writing in mathematics revealed there are many benefits of mathematics writing. For example, writing can be used as a tool for learning, communicating, solidifying understanding, and as a method to inform instruction. However, the limited reporting in the research regarding the nature of the prompts (i.e., how the prompts were compiled and/or what resources were used for the prompts) and the mathematical language necessary for communication were not discussed in the findings of the literature reviewed.

In light of these findings, the research questions developed for this study were addressed using an analytic framework developed from the research literature (see Appendix A).

Chapter 3: Methods

In the mathematics classroom, writing is recommended to promote students' conceptual understanding of mathematics content (Alvermann, 2002; Bereiter & Scardamalia, 1987; Bruner, 1986; Burns, 2004; Countryman, 1992; Emig, 1977; Greenfield & Bruner, 1969; McIntosh & Draper, 2001; NCTM, 2000; Pugalee 2004; Senk & Thompson, 2003; Shulman 1986; Urquhart, 2009; Urquhart & McIver, 2005; Vygotsky 1962). In addition, writing can help students acquire vocabulary needed to communicate mathematically (Beck, Mckeown, & Kucan, 2002; Fisher & Frey, 2008; Graves, 2006, 2009, 1986; Graves, Sales & Ruda, 2008; Marzano & Pickering, 2005; Nagy, 1988; Nagy and Herman, 1987; Ruddell & Shearer, 2002; Stahl & Fairbanks, 1986). Although many benefits of writing are noted, the most common influence on mathematics content appears to be the textbook/curriculum program (Weis, Pasley, Smith, Banilower & Heck, 2003). Furthermore, the mathematics textbook is researched as the dominant tool in classroom instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008). Because writing is acknowledged to promote conceptual understanding *and* the textbook is regarded as the dominant tool for mathematics content, the purpose of this study was to examine the nature of writing prompts in mathematics textbooks. Specifically, I explored the following questions:

1. How many writing prompts are included in one 4th grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?

2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?

3. What types of vocabulary are used in the writing prompts in one 4th grade NSF-funded

mathematics textbook and one publisher-generated textbook?

4. What types of prompts are provided in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?

This chapter consists of seven sections. The first section describes the methods used for textbook sample selection. The second section explains the selection of writing prompts used for analysis. The third section illustrates how the analytic framework was developed through the use of a pilot study. The fourth section describes each of the framework dimensions. The fifth section reveals the parts of the textbooks used for analysis. The sixth section explains the check-coding system used for determining reliability of the framework dimensions. The final section discusses the sources of influence for determining reliability.

Textbook Sample Selection

The selection of textbooks occurred in two phases. In the first phase, I considered the grade level of the textbook to analyze. In the second phase, I considered the specific textbook.

Grade level selection. In selecting mathematics textbooks for the study, I considered the results of my literature review and my experience as a mathematics coach. The majority of published textbook analyses were conducted in middle and upper grade levels (Johnson, 2010). With the paucity of research on elementary grade level mathematics textbooks, I selected the elementary grades as my focus.

In order to select a specific grade within the elementary school, I considered curricular expectations and students' developmental levels. Whereas writing in the primary grades is often focused on letter formation, idea development, spelling, and page arrangement (Clay, 1977), in the intermediate grades, students are expected to write in many genres for many purposes (Boscolo, 2008). Many students have also developed the ability to explain their thoughts (Baxter, 2001). Therefore, I felt that the intermediate grades would be a context in which writing could be used within mathematics.

In addition to writing development, I also considered curricular expectations and testing constraints. For example, in many states fourth grade students are required to write in both expository and narrative forms on high stakes assessments (IES, 2010). Additionally, the National Assessment of Educational Progress (NAEP) assesses fourth graders in writing for national reporting and the Trends in International Mathematics and Science Study (TIMMS) reports internationally.

Based upon the developmental level of the students and the high-stakes accountability of writing in fourth grade, I selected the fourth-grade to conduct a mathematics textbook analysis.

Textbook selection. Johnson (2010) noted that the selection of textbooks for a content analysis is based upon two criteria: (1) researchers' selection of widely-used series and (2) researchers' selection of both NSF and non-NSF funded curricula (Hodges, Cady, & Collins, 2008; Reys & Reys, 2006; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). In addition, a third criterion regarding the importance of textbook alignment to the standards documents is a critical question that many states investigate when adopting textbooks (Reyes & Reyes, 2006). More specifically, the professed future alignment of the textbooks to the Common Core State Standards (CCSS) contributed to my selection of textbooks as well. Brief descriptions of these three criteria are explained below.

Widely-used textbooks with significant market share. Textbooks that are classified as widely-used have significant market share if a large percentage of states in the nation adopt the textbook series produced by the publisher (Jones, 2004; Tarr et al., 2008). According to the 2000 National Survey of Science and Mathematics Education (funded by NSF and conducted by Horizon Research Inc.) *Everyday Mathematics* published by McGraw-Hill/Merrill Company and *enVision MATH* published by Addison Wesley Longman, Inc. /Scott Foresman accounted for over 50% of the textbook usage in grades K-4 mathematics classes nationally (Weiss et al., 2003). Therefore, these textbooks have "significant market share" according to findings of the survey data.

NSF and non-NSF materials. Reform recommendations of higher-level mathematical thought were beginning to guide the development of mathematical standards and practices in the late 80's. One theme common to the NCTM Standards and to the recent changes in mathematics education is that "the study of mathematics should emphasize reasoning so that students can believe that mathematics makes sense" (NCTM, 1989, p. 29). According to Senk and Thompson (2003), "By 1991, the NSF had issued calls for proposals that would create comprehensive instructional materials for the elementary, middle and high schools consistent with the calls for change in the *Curriculum and Evaluation Standards* [NCTM, 1989]" (pp. 13-14). As a result of this project, *Everyday Mathematics* was developed as one of three comprehensive instructional programs at the elementary grades funded by the NSF. Textbooks that are not funded by NSF are generally considered to be publisher-generated. By selecting NSF and non-NSF materials, I captured two contrasting perspectives from which these materials are produced.

Standards alignment. Mathematics standards documents provide recommendations for the content students learn. Because the textbook is the dominant tool used in classrooms (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008), many textbook companies profess to adhere to these standards documents. According to Reys and Reys (2006), most publishers claim to be aligned with the NCTM's *Principles and Standards for School Mathematics*; however, careful examination of materials is recommended to determine if this claim is actually true. The two textbooks I chose for analysis claim to be aligned to the newly developed CCSS (2010).

Overview of selected textbooks. For these three reasons (significant market share, NSF and non NSF funded materials, and standards alignment), I chose the 4th grade textbook from two series (with teacher editions): the 2011 edition of *enVision MATH* published by Pearson Education, Inc. and the 2012 third edition of books developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. Both of these textbooks are national versions and are not modified to fit the needs of any one specific state mathematics standards requirements. The textbook, *enVision MATH,* was not funded by NSF and is therefore labeled publisher-generated (Dingman, 2010).

enVision MATH. As the non-NSF funded program, Pearson (2011) posted the following statement on its website regarding the *enVision MATH* math program (www.pearsonschool.com: Scott Foresman-Addison Wesley enVision MATH © 2011):

> Daily Problem-Based Interactive Math Learning followed by Visual Learning strategies deepen conceptual understanding by making meaningful connections for students and delivering strong, sequential visual/verbal connections through the Visual Learning Bridge in every lesson. Ongoing Diagnosis & Intervention and daily Data-Driven Differentiation ensure that *enVision MATH* gives every student the opportunity to succeed (*Pearson, enVision MATH,* para 1).

In addition, Resendez, Azin, Strobel (2009) report the findings of the program-effects over a two-year longitudinal study:

> Results showed significant growth over the two-year period in math knowledge and skills among *enVision MATH* students across all grade levels and assessments. *EnVision MATH* students showed significant improvement in math concepts and problem solving, math computation, and math vocabulary. Moreover, there is evidence of accelerated growth rates during the second year of usage of *enVision MATH* in the areas of math concepts and problem solving and math vocabulary skills. This suggests that the cumulative effects of *enVision MATH* are getting stronger over time (p. 2).

According to Resendez et al. (2009), *enVision MATH* also aligns to the NCTM Curriculum Focal Points (NCTM, 2006) with future alignment to CCSS (2010) on the horizon:

> Pearson is making unprecedented levels of investment in new models for education and supporting key elements of the reform agenda: Common Core standards, college and career readiness, teacher effectiveness, school improvement, and custom solutions for schools and colleges (Pearson Education, Inc., 2011).

The materials provided by Pearson Education were one fourth grade *enVision MATH* Student Edition textbook and Lessons 1-20 Teacher Editions. The materials were obtained via email correspondences and phone communication directly from a Pearson Elementary Representative in the State of Florida. These materials were then analyzed and coded accordingly.

***Everyday Mathematics*.** Below is the language used by the NSF-funded series, UCSMP *Everyday Mathematics* posted on their website (http://everydaymath.uchicago.edu/about/):

> *Everyday Mathematics* is distinguished by its focus on real-life problem solving, balance between whole-class and self-directed learning, emphasis on communication, facilitation of school-family cooperation, and appropriate use of technology (UCSMP, Everyday Mathematics, “n.d.”, para 2).

In addition, several research documents support the Everyday Mathematics program. For example, in the What Works Clearinghouse National Topic Report (2007) from the United States Department of Education, *Everyday Mathematics* was evaluated as the most promising among the elementary school mathematics programs reviewed between the years of 2006 and 2007. In addition, Carroll (1998) conducted an analysis of

Everyday Mathematics with TIMMS international data findings. Carroll (1998) reported as follows:

> Because of its research base, its international perspective, and its unique approach to curriculum development, UCSMP's *Everyday Mathematics* differs substantially from other programs and has anticipated many of the concerns raised by the TIMMS report. In contrast to more traditional programs, in *Everyday Mathematics* students investigate mathematical concepts in greater depth each year as the curriculum moves from the primary grades, the emphasis shifts from number and number sense to algebra, geometry and data, with the goal that approximately half of the students who complete the program will be ready for algebra by seventh grade (p.10).

In addition, *Everyday Mathematics* was developed upon standards recommendations and documents:

> During the 1980s, a consensus emerged about how best to teach mathematics to children. The NCTM *Standards* (1989) expressed that consensus. *Everyday Mathematics* is based largely on the same body of research that led to the *Standards* consensus. Wright Group provides reports on correlations between *Everyday Mathematics* and national standards, including NCTM, NAEP, and the Stanford Achievement Test (UCSMP, About Everyday Mathematics, para 4-10).

In addition, the program's statement of future alignment to CCSS (2010) is as follows:

> Each grade-level author reviewed the content standards and developed a plan to adjust lessons so that Everyday Mathematics aligned 100% to the CCSS. Those plans are complete and we are now implementing those adjustments to the

Everyday Mathematics program. Our author and editorial team are well on their way and we will have a program that aligns to the CCSS ready for implementation in the 2011-2012 school year (McGraw-Hill Education, Everyday Mathematics 2011, para 2).

I obtained the following materials from McGraw-Hill Education-- *Everyday Mathematics* 2012 Common Core third edition student math journals (two sets) with accompanying math master books (1) from the McGraw-Hill Education area representative for Pinellas County, Florida. In addition, a Teacher Lesson Guide, two-volume set was provided. These materials were analyzed and coded according to the revised framework.

Selection of Writing Prompts

Writing in mathematics is an effective method for students to learn mathematics content (Alvermann, 2002; Burns, 2004; Countryman, 1992; Emig, 1977; McIntosh & Draper, 2001; Pugalee 2004; Senk & Thompson, 2003; Shulman, 1986; Urquhart, 2009; Urquhart & McIver, 2005). In particular, writing in mathematics can help students develop problem-solving abilities (Evans, 1984; Parker, 2007; Sowder, 2007) and metacognitive skills (Brewster & Klump, 2004; Nuckles et al., 2010; Pugalee, 1997, 2001; Steele, 2005). Teachers can also use students' writing to identify strengths and gaps in students' content knowledge (Britton, 1975; Nagin, 2003; Romberger, 2000) as well as to understand students' affective positions and feelings about mathematics content (Baxter et al., 2007; Dougherty, 1996; Shield & Galbraith, 1998; Urquhart, 2009). Writing in mathematics is a valuable tool in many areas of mathematics instruction.

Textbooks include a variety of close ended exercises and open-ended tasks. Specifically, the term “prompt” is defined and used interchangeably as a “writing task” when the answer is in the form of an expanded written or constructed response (Murphy, 2004; Smagorinsky, 2006; Urquhart, 2009; and Yancey, 2004). Because of the cognitive and instructional benefits of writing a constructed response, I focused on the prompts that required expanded written, narrative, and evaluative responses in mathematics textbooks. The identification of prompts was conducted in three phases. In the first phase, I identified the exclusion and inclusion criteria for certain mathematics problems. In the second phase, I identified terms that had the potential to facilitate a written response. In the third phase, I discussed the reliability measures for prompt selection and framework dimensions.

Excluded items in mathematics textbook analysis. Given this study focused on mathematical writing prompts that facilitated a writing response, I excluded items that were defined as “close-ended” math problems. Cooney, Sanchez, Leatham, and Mewborn (2004) state that closed-ended questions do not allow students to reveal their thinking processes and generally call for an answer as a single digit, figure, or mathematical object. In other words, the answers are predetermined and specific. I decided to exclude the following problem types from the selection of prompts because the items met the criteria of a close-ended problem:

- Problem types that require computation with digits specifically.
- Problem types that require a one-word answer.
- Problem types that require numerical answers in standard or word form.
- Problem types with multiple-choice answer selections.

Exercises that require computation with digits do not require a student to construct a response other than digits. An example of an exercise that requires computation with digits specifically appears in Figure 1.

Find the sum of 37 and 28

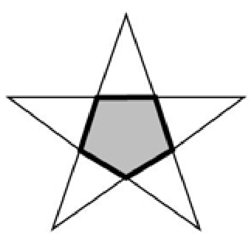
  37
+ 28

(Van deWalle, 2010).

Figure 1. Example of a computation specific problem type.

Also, I excluded exercises that led to a one-word answer. Exercises of this sort do not require the student to construct a response other than in a "one-word" form. An example of an exercise that requires a "one word" answer appears in Figure 2.

What is the shape of the figure inside the star?



The shape is a ________________________.

(4^{th} grade NAEP sample question, 2009)

Figure 2. Example of a "one-word" response problem type.

In addition, problem types that required numerical answers specifically in the form of digits written in standard or word form were excluded from the selection. Problems of this type do not require a student to construct a response other than in digit formation. An example of a problem type that requires an answer in numeric form, whether in standard or word form appears in Figure 3.

$\square - 8 = 21$

What number should be put in the box to make the number sentence above true?

Answer: ___________________________

(4th grade NAEP sample question, 2009)

Figure 3. Example of a "digit-specific" response problem type.

The final problem types excluded from the study were problems written in multiple-choice formats. These types of problems do not require a student to construct a response other than to identify the correct answer from a list of choices. An example of a problem type that is written in multiple choice format appears in Figure 4.

$2 + n$	5
$3 + n$	6
$4 + n$	7
$5 + n$	8

What number does *n* represent in the table?

A. 2
B. 3
C. 4
D. 5
(4th grade NAEP sample question, 2009)

Figure 4. Example of a "multiple-choice" response problem type.

Included items in mathematics textbook analysis. The criteria for the selection of prompts aligned closely to the characteristics of "open-ended" math problems. Cooney et al. (2004) describe "open-ended" math questions as those that require students to communicate their mathematical thinking, providing teachers with valuable information that can inform their teaching while eliciting multiple responses. In addition, the criterion for the selection of prompts also aligned closely to a "constructed response." A constructed response is a type of task developed to elicit an answer in writing, such as an essay, short answer or sentence completion (Hancock, 1994). Constructed response questions are similar to open-ended questions. For purposes of this study, writing prompts in the written curriculum that have the potential to facilitate an answer in one or more sentences were coded.

In order to determine the prompts that allowed students to communicate their mathematical thinking in the written curriculum, the language used within the prompt was analyzed. Based on empirical data, specific language functions that have the potential to facilitate a written response were used for my criteria selection in prompt identification. For example, Butler, Lord, Stevens, Malka, Borrego and Bailey (2004) compiled a list of terms from mathematics national standards documents and selected mathematics textbooks in which students produced or completed an oral or written task. Urquhart (2009) also produced a list of the "most-used" terms on constructed response items. A list of the terms included from each of these resources is provided in Appendix B. Prompts that include these terms have the potential to facilitate a written response. For example the term "explain" appears in both lists in Appendix B. A problem type that has the term “explain” has the potential to facilitate a written response. An example of a prompt that has the term “explain” appears in Figure 5.

Sample 1 How do you know 1/4 is greater than 1/5? Explain your thinking. Urquhart (2009)

Figure 5. A problem type with the term “explain” in the prompt.

In addition to the specific prompting items, the terms listed in Appendix B also have word *associations.* A word association is a term that is within the same family of words or meanings. An example of a word association can be described by a prompt that includes the word “write.” For example, the term “narrate” is used in Urquhart’s (2009) Word List. However an example of a prompt that includes the word “write” is not listed specifically. Urquhart (2009) notes that the word “write” is associated with the term “narrate.” Because the word “write” is not included in the Word Lists, the word “write” is *associated* with a particular term (narrate) and was identified as a prompt that has the potential to facilitate a written response. Depending on the context of the prompt, the associations between words on the list in Appendix B to words in the prompt were also identified when the word was not listed explicitly. An example of a prompt that included the word “write” and has an *association* with the term “narrate” appears in Figure 6.

Sample 2 Write a sequence of actions occurring over time by relating the story of evolution of the abacus through ancient, middle, and modern times. Urquhart (2009, p. 16)

Figure 6. Example problem type with a word *association* of “write to narrate”

In reviewing the *Everyday Mathematics* and *enVision MATH* textbooks for prompts that have the potential to facilitate a constructed response, I used the terms listed in Appendix B (Butler et al., 2004; Urquhart, 2009) and identified word associations when applicable to communicate my rationale for the selection of writing prompts in the written curriculum for analysis.

Developing the Analytic Framework: A Pilot Study

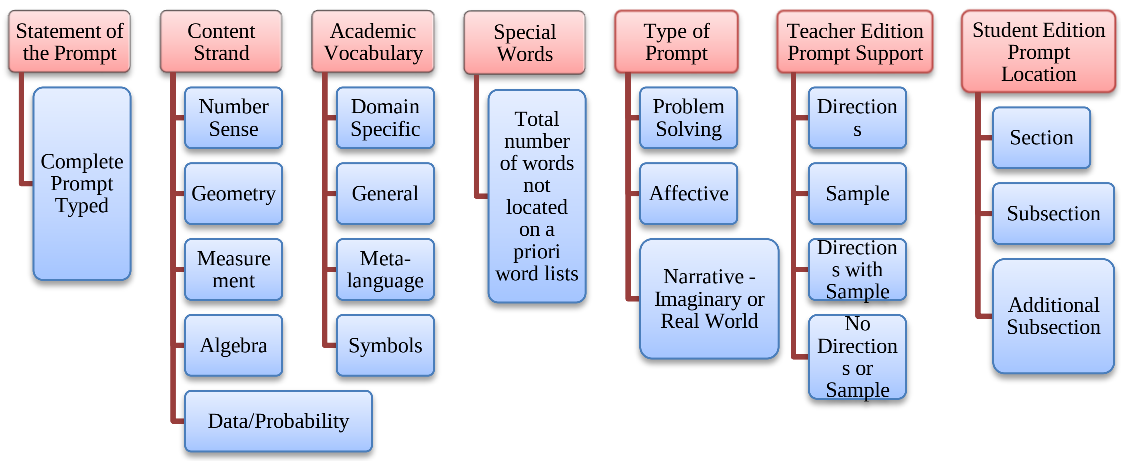


Figure 7. Analytic framework used in pilot study.

In order to develop a framework to guide the analysis of prompts, I conducted a pilot study to refine my methods. Using the first chapter from Harcourt Inc, *Harcourt Math Florida Edition* (2004), I analyzed five lessons (see Appendix C). Using each research question as a guide, I modified the framework (see Figure 10) in the following ways.

Question one. How many writing prompts are included in one 4th grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?

I isolated each exercise within Harcourt Chapter 1 that had a number or letter next to the exercise. If the exercise required a constructed answer in the form of a sentence or more, I coded the task as a writing prompt. I then typed the prompt verbatim and calculated the number of prompts located in the chapter. Eleven tasks were coded as writing prompts in the *Harcourt Math Florida Edition* in Chapter 1.

Framework revision from question one. Based on my analysis of these prompts, I added two dimensions to the framework: (1) *number of writing prompts/tasks per page* and (2) *number of exercises per page*.[4] Similar to Johnson et al. (2010) an "exercise" was defined as a problem or question that appears in an exercise set and is not solved or answered. In this study, the word "prompt/task" refers to an exercise that requires a constructed response. These two categories enabled me to calculate the proportion of writing prompts per page and to report my findings in the form of a percentage. For example, 11 writing prompts out of 186 mathematical exercises were coded for Chapter 1. The average number of writing prompts for Chapter 1 was 5% of the total exercises. I felt this type of information would be essential in reporting the relative emphasis placed on these tasks.

Question two. How do mathematical writing prompts vary across the content strands between one 4th grade NSF funded textbook and one publisher-generated textbook?

Using the established content strands (*number sense, geometry, measurement, algebra, data analysis)* identified by NCTM (2000), I categorized each prompt by strand. This identification process was conducted by analyzing the language within the prompt. For example, 11 writing prompts in Chapter 1 were coded under the category of *number*

[4] If a page had an exercise on it, it was counted as a "page." Only the pages that were counted had exercises.

sense based on the language that referred to number sense processes. The average number of prompts in the *number sense* category coded was 100% from Chapter 1.

Framework revisions from question two. In the revised framework the category of *other* was added to the categories. Although the pilot study did not have any prompts coded as *other*, the process of identifying the language helped to determine that a category of this nature should be developed in the event the language was not indicative of the language within each of the content strand categories.

Question three. What types of vocabulary are used in the writing prompts in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook? Within the original framework the academic vocabulary categories were as follows: *domain specific vocabulary (DSV)*, *general vocabulary (GV)*, *meta-language,* and *symbols*. Words that had the potential to be coded as academic vocabulary based on the definition of each of the vocabulary categories were scanned in an Excel document comprised of four vocabulary word lists. If the exact term was not found in the lists, then any possible derivatives of the word were located. If a derivative of the word was still not located, an association of the word was acknowledged in order to determine what type of academic vocabulary the term *could* potentially be coded. Word associations assisted in determining if the term *should* be in a specific word list. If an association was made to a particular term not found in the word lists, it was coded under *words not on list.*

Once the words were coded in the *academic vocabulary* domain, I counted the total number of the words in each of the following categories: *DSV, GV, Meta-language* and *Symbols.* I also counted the total number of words in the prompt in order to determine what percentage of words was academic vocabulary in the writing prompt. For example, the total words/symbols in the writing prompts that were coded as *academic vocabulary* for Chapter 1 was 72 out of 183 total words or 39%.

I then analyzed the total amount of words coded for each academic vocabulary category independently. The total count in each of the categories was then divided by the total number of words in order to determine which types of *academic vocabulary* were present. For example 37%, which was the majority of academic vocabulary coded from Chapter 1, was DSV. Furthermore, out of 72 total words identified as *academic vocabulary*, 7 of those words were not located on the a priori academic vocabulary lists. As a result, these words were placed in the *words not on list* category. Therefore, based on the definitions of the types of academic vocabulary, 10% of the words coded for Chapter 1 *should* be coded as academic vocabulary, but were not.

Framework revisions from question three. I made four revisions to the framework based on the analysis of the data from Question Three. The first revision was to change *special words* to *words not on list*. This domain name change appeared to be more representative of the status of the words. The second revision involved moving the dimension column next to academic vocabulary for ease of coding. The third revision was made for ease of check-coding regarding the co-rating of the framework and the word lists. For example, an Excel spreadsheet was developed to have all three word lists compiled into one spreadsheet instead of separate word lists. The lists were then color coded according to the academic vocabulary type. Furthermore, the Excel short-cut key of Ctrl-F was used to find the words in a quick simplistic manner versus going through each of the lists individually. The last revision included the change of the symbols list. The initial symbols list was vast in the amount of symbols listed whereby the majority of symbols were not indicative of elementary mathematics instruction. The new list contained symbols that were more common of elementary mathematics instruction.

Question four. What types of prompts are provided in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?

I coded the type of writing prompts according to the following categories: *problem solving, affective,* and *narrative* math content in an *imaginary* or *real world* sense. Based on the language within the prompt and the prompt affordance, the prompt was coded based on the categories. Simple descriptive statistics were used to determine a percentage of the *types* of writing prompts.

All of the prompts were coded in the category of *problem solving*. There were no prompts that afforded the response of a feeling or attitude. Additionally, there were no prompts that afforded the response of narrative. Therefore, 100% of the prompts coded in Chapter 1 were *problem solving* types of writing prompts.

Framework revisions from question four. In the revised framework, the category of *problem solving* was changed to *generic* prompt. I changed the category label based on the many mathematical connotations associated with the phrase *problem-solving*. Additionally, the category of *narrativizing* and *fictionalizing* mathematics content in an *imaginary* or *real world* sense was renamed for purposes of simplicity to *narrative* prompt.

Additional dimensions. The additional dimensions of teacher and student edition were not directly related to the research questions. However, the exploration of these dimensions within the framework provided more depth to the findings addressed in each of the research questions.

Teacher edition. The teacher edition provided information regarding how writing was supported in each of the textbooks. In the pilot framework the categories were *sample response* only, *support* only, a *sample with support*, or *no support or sample.* In reviewing the teacher edition for Chapter 1, the majority of writing prompts had only one *sample* student response as the form of support for the writing prompt. An example of prompt support from the teacher edition coded as *sample* only appears in Figure 8. The red colored sample response indicates the only support located in the teacher edition for this prompt:

- Which digit in the number 13,872 would be changed to form 19,872? How would the value of 13,872 change? A: The value would increase by 6,000.

Figure 8. Example prompt with support coded as *sample* only.

One problem had a brief description of instructional suggestions regarding the background knowledge needed for the prompt. In addition, this prompt also had a sample response of how the prompt should be answered. Therefore this prompt was coded as *support with sample.* An example of a prompt from the teacher edition coded as *support with sample* appears in Figure 9. The teacher edition provided both a *sample* response under the prompt along with *support* in the form of background knowledge for the topic of place value.

Prompt	Teacher Edition Support
Vocabulary Power What does the *place value* of a digit tell you? How does switching the positions of the digits in the number 52 affect that number's value? **Possible answer; A digit's place value tells you its value; the value decreases.**	**Vocabulary Power** The place value of a digit in a number determines the digit's value. For example, in the number 5,280, the digit 5 is in the thousands place, and so has a value of 5 x 1,000 = 5,000.

(*Harcourt, Inc.*, 2004, p.9)

Figure 9. Example of a prompt with support coded as *directions with sample*.

A code in the category of *directions* only would indicate that there was *no* sample response for the prompt. A code of *no directions or sample* indicates there was *no* support or sample located in the teacher edition for the prompt.

In the revised framework, the categories that had the term *directions* were changed to *support*. For example, the category of *directions* only was changed to *support* only, *sample with directions* was changed to *sample with support*, and *no directions or sample* was changed to *no support or sample.* The change from *directions* to *support* was made because the teacher edition did not provide explicit directions in teaching the prompt but rather support in various forms such as teaching the content within the prompt (see Figure 9).

Student edition. Determining where the writing prompts were located in the student edition had implications for the instruction of such prompts. For example, the majority of writing prompts in Chapter 1 were located within the Practice and Problem Solving sections of the student edition. Within the teacher edition, this section had instructional suggestions whereby students were encouraged to work on these particular problems for practice on their own. Therefore the majority of writing prompts in Chapter 1 were to be answered independently by the student. There were no modifications made to this framework dimension.

Summary of Pilot Study

The findings from the small-scale pilot study helped to refine the framework for analyzing writing prompts. Additionally, although this small-scale pilot utilized five lessons within the first chapter of the student edition, it assisted in my improvement of the framework reliability.

An analysis of the research questions across the framework dimensions provided for seven revisions to the framework. The first revision provided for additional dimensions to be added for purposes of calculating the average regarding the number of prompts. The second revision indicated that the category of *other* should be added to the content strands. The third revision changed the dimension of *special words* to *words not on list*. The fourth revision consisted of changing the symbols reference list to a more elementary mathematics *friendly* version. The fifth revision relocated the dimension of *words not on list* next to *academic vocabulary.* The sixth revision consisted of changing the name of *narrativizing and fictionalizing* math content in an *imaginary* or *real world* sense to *narrative* prompts in an *imaginary* or *real world* sense. The final revision consisted of changing *problem solving* to *generic* prompts.

The pilot study and the modification made to the framework, coupled with the research literature, provide an understanding of the framework presented.

Framework Dimensions

Modifications of the framework resulted in a framework with 10 dimensions: *number of writing prompts, number of exercises per page, statement of the prompt, content strand, academic vocabulary, words not on list, total number of words, type of prompt, teacher edition prompt support, student edition prompt location.* A table of the dimensions and code key are located in Appendix D. This framework of dimensions and code key was developed in the form of a matrix for the purposes of classification. (See Appendix A).

Furthermore, the framework dimensions were clustered according to themes in order to provide an understanding of the framework associations. For example, *number of* *writing prompts, number of exercises per page* and *student edition* were clustered as *page orientation.* The dimensions of *statement of the prompt, content strand, academic vocabulary, words not on list, total number of words, and type of prompt* were clustered as *prompt analysis*. The final dimension of *teacher edition prompt support* was identified as *prompt support.* In addition, the associations of the framework dimensions will assist in the organization of this section. (see Figure 10).

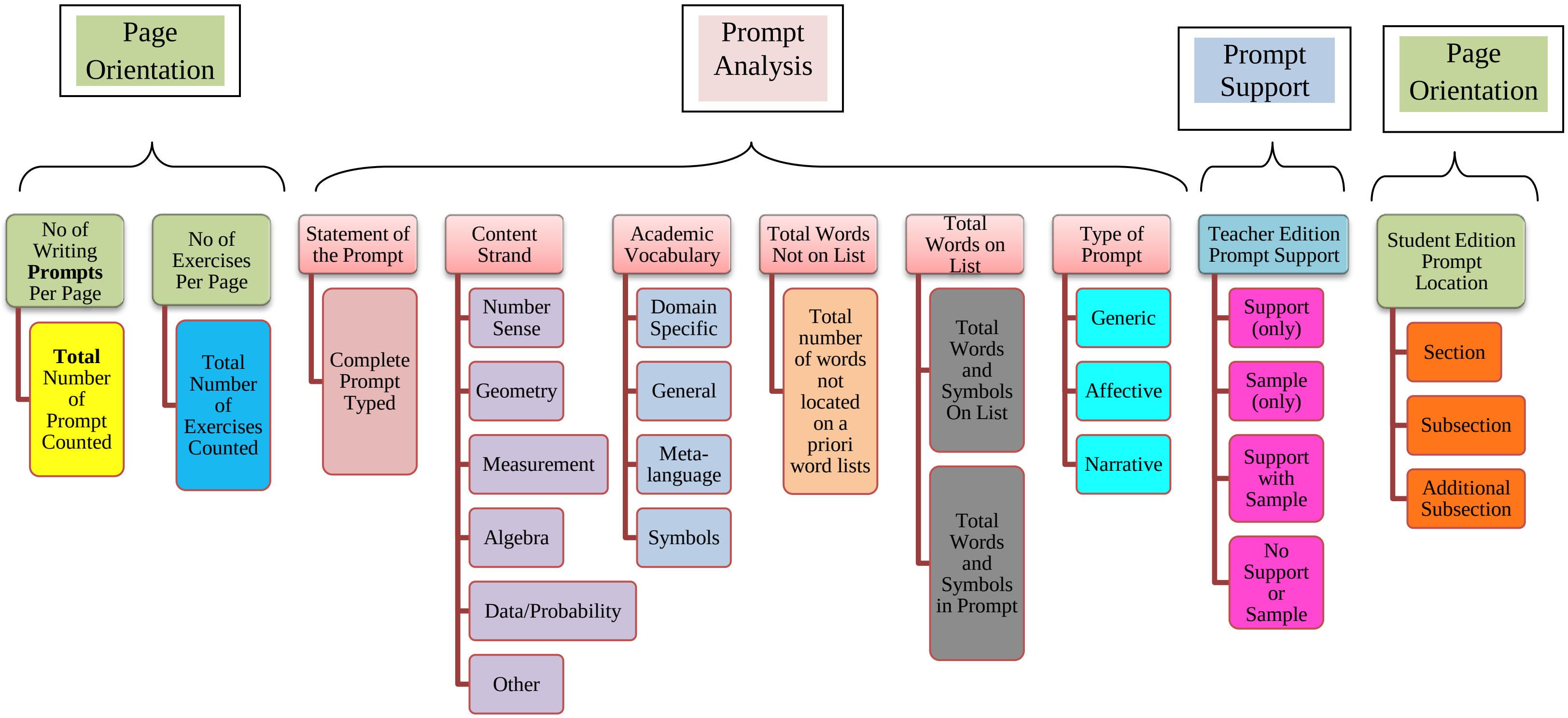


Figure 10. Clusters within framework dimensions

In the following section the dimensions within the cluster of *page orientation* will be described (see Figure 11).

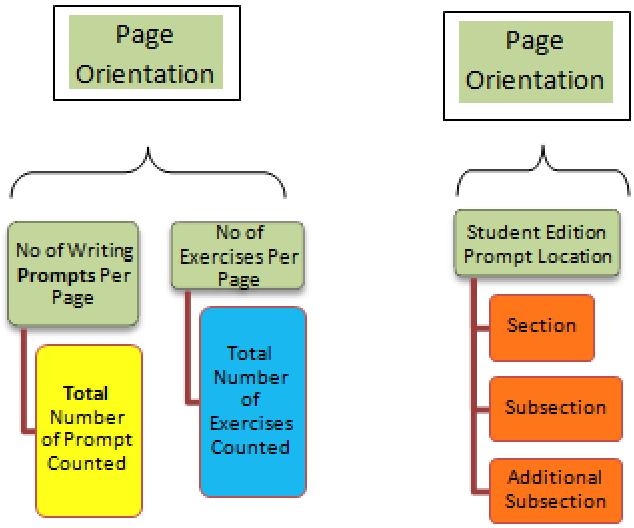


Figure 11. Framework dimensions within the cluster of *page orientation*

Number of writing prompts. Within the *number of writing prompts,* the number of writing prompts on each page was recorded. Simply put, if a page had two exercises coded as writing prompts, then the number indicated would be two. The number of writing prompts was then totaled and used to determine a percentage in the following section.

Number of prompts per page. Within the *number of prompts per page,* all of the exercises located on the pages of the writing prompts were counted. If a number or letter was used to identify an exercise in the student edition then it was counted. The total number of writing prompts was divided by the total number of exercises to determine the average number of writing prompts.

Student edition. Within the *student edition,* I noted the section, subsection and additional subsection titles of the prompt location in the student edition. The section location of writing prompts within the textbook provided information regarding *where* the prompts were located. I also determined the trends in prompt location or language patterns within the section titles of each textbook by conducting a simple count of the various patterns within the language of the titles.

The following dimensions within the cluster of *prompt analysis* will be described further in the next section (see Figure 12).

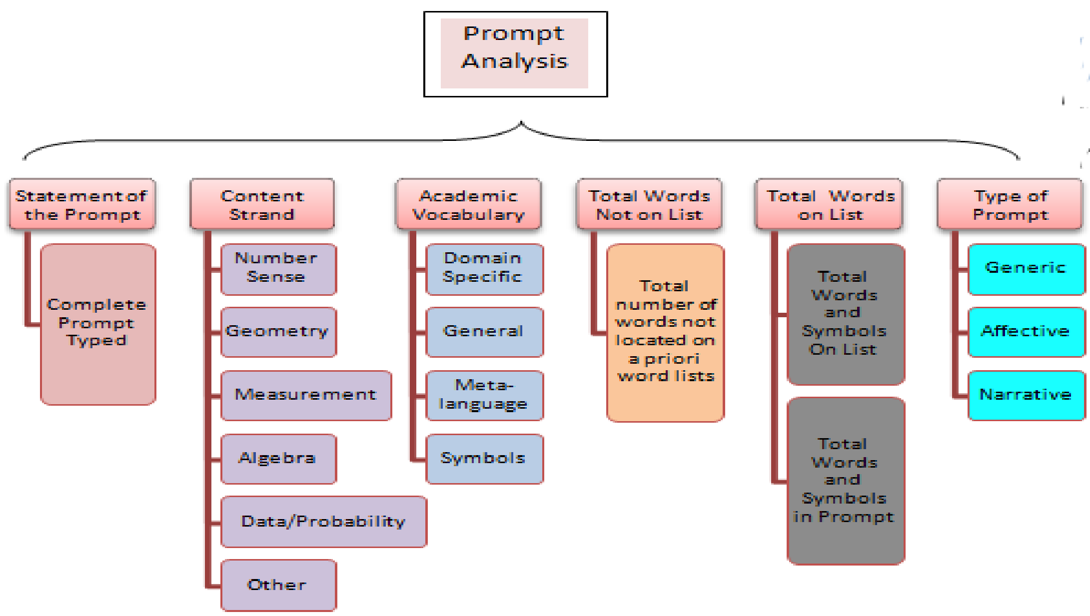


Figure 12. Framework dimensions within the cluster of *prompt analysis*.

Statement of the Prompt. Within the *statement of the prompt* domain, the exact wording from the prompt was recorded. By recording the words in the prompt I was able to analyze the language that led to coding with the *content strand, academic vocabulary* and *type of prompt* dimensions.

Content strand. Within the *content strand* domain, the language within the writing prompt was coded to determine its alignment with a particular content strand/s (see tables 1-5). For example the content within the writing prompt may have appeared in the following five content areas: *number/operations, geometry, algebra*, *measurement* and *data analysis/probability* (NCTM, 2000). The content area of each writing prompt was coded in the specific content category. Most elementary mathematics textbooks are divided into content sections, which make it generally uncomplicated regarding identification of the content strand. However an analysis of the language within the prompt and the title of the lesson allowed for the prompt to be coded in more than one content strand. If the prompt was categorized in multiple strands, the codes were reflected in the framework. The following section includes a description of the content as outlined from the NCTM *Principles and Standards* (2000) content strand expectations in Grades 3-5 and a sample of a writing prompt within each particular strand.

Number and operations. Number and operations is typically the largest strand for content expectations within the NCTM *Principles and Standards* (2000) at the elementary grades. Students are expected to understand numbers, operations, and number relationships while computing fluently and making reasonable estimates (NCTM, 2000). Table 1 presents the topics within the content strand of Number and Operations according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

Table 1

Topics within Number and Operations-Grades 3-5

Category	*Topic*
Number and Operations	Place value
	Base ten number system
	Whole numbers
	Negative numbers
	Decimals
	Fractions
	Percents
	Factors
	Multiplication of numbers
	Division of numbers
	Addition of numbers
	Subtraction of numbers
	Estimation of numbers

An example of a prompt that would be coded in the category of Number & Operations appears in Figure 13. This prompt would be coded in the Number & Operations category because of the fraction symbol.

> You see a sign in a shop window that reads "$\frac{1}{2}$ OFF SALE" What does this mean to you?
>
> Sullivan & Lilburn (2002)

Figure 13. Example prompt coded Number & Operations.

Algebra. The Algebra content strand consists of students' understanding, representing and analyzing mathematical situations, patterns, relations, functions, structures, and quantitative relationships using algebraic symbols and models (NCTM, 2000). Table 2 presents the topics within the content strand of Algebra according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

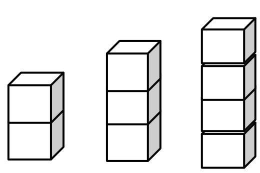
Table 2

Topics within Algebra-Grades 3-5

Category	*Topics*
Algebra	Patterns
	Functions
	Properties
	Variables
	Letter
	Symbol
	Rate of change

An example of a prompt that would be coded in the category of Algebra appears in Figure 14. This prompt would be coded in the Algebra strand because of the unknown pattern.

What is the surface area of each tower of cubes (include bottom)? As the towers get taller, how does the surface area change?



Principles and Standards, (NCTM, 2000)

Figure 14. Example prompt coded Algebra.

Geometry. Geometry consists primarily of analyzing properties and relationships of geometric figures and shapes. Table 3 presents the topics within the content strand of Geometry according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

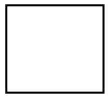
Table 3

Topics within Geometry-Grades 3-5

Category	*Topics*
Geometry	2 dimensional shape
	3 dimensional shape
	Triangles
	Pyramids
	Classes of Shapes
	Congruent
	Similar
	Coordinate system
	Horizontal lines
	Vertical lines
	Rotational symmetry
	Designs
	Geometric objects
	Geometric patterns
	Geometric paths
	Geometric models

An example of a prompt that would be coded in the category of Geometry appears in Figure 15. This prompt would be coded in the Geometry category because of the focus on the figure "square."

Write down everything you know and everything you can find out about this square.



Sullivan & Lilburn (2002)

Figure 15. Example prompt coded Geometry.

Measurement. The measurement content strand consists of understanding measurable attributes of objects, units and systems while applying appropriate techniques, tools and formulas to determine measurements (NCTM, 2000). Table 4 presents the topics within the content strand of Measurement according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

Table 4

Topics within Measurement-Grades 3-5

Category	*Topics*
Measurement	Length
	Area
	Width
	Height
	Size of an angle
	Measurement unit
	Standard unit
	Customary system
	Metric system
	Units of measurement
	Perimeter
	Volume
	Irregular shape

Table 4 (*continued*)

Category	*Topics*
Measurement	Weight
	Time
	Money
	Temperature
	Surface Area

An example of a prompt that would be coded in the category of Measurement appears in Figure 16. This prompt would be coded in the Measurement category because of the weight reference of "1 pound" in the prompt.

> What objects can you find in your home that have 1 pound marked on them? Ask someone at home to help you make a list.
>
> Sullivan & Lilburn (2002)

Figure 16. Example prompt coded Measurement.

Data Analysis/Probability. The Data Analysis/Probability content strand consists of collecting and analyzing data using appropriate statistics while developing and evaluating inferences and predictions from the data. The student must also apply basic concepts of probability (NCTM Principles and Standards, 2000). Table 5 presents the topics within the content strand of Data Analysis/Probability according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

Table 5

Topics within Data Analysis/Probability-Grades 3-5

Category	*Topics*
Data Analysis/Probability	Data
	Data set
	Categorical Data
	Numerical Data
	Observations
	Surveys
	Experiments
	Tables
	Graphs
	Line Plot
	Bar graph
	Line Graph
	Measures of center
	Median
	Degree of likelihood
	Likely
	Unlikely
	Equally likely
	Certain
	Impossible

An example of a prompt that would be coded in the category of Data Analysis/Probability appears in Figure 17. This prompt would be coded in the Data Analysis/Probability category because of the probability reference in the prompt.

> If two coins are tossed, what could happen?
>
> (Sullivan & Lilburn, 2002)

Figure 17. Example prompt coded Data Analysis/Probability.

Other. Based on the pilot study, the category of *other* was developed for prompts that could not be categorized within the five content strand categories. If the language within the writing prompts was not indicative of the language within the content strands then the prompt was coded under the category of *other*. An example of a prompt coded in the category of *other* appears in Figure 18. This prompt would be coded as *other* because the language within the prompts is not indicative of the language associated to the mathematics topics indicated in Tables 1-5.

> Do you know anyone who has visited or lived in this country? If so, ask that person for an interview. Read about the country's customs and about interesting places to visit there. Use encyclopedias, travel books, the travel section of a newspaper, or library books. Try to get brochures from a travel agent. Then describe below some interesting things you have learned about this country.
>
> (*Everyday Mathematics* 4th Grade Student Journal, 2010)

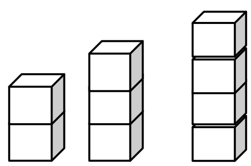
Figure 18. Example prompt coded Other.

An analysis of the language within the prompt assisted in determining which content strands had the majority of writing prompts. In addition, an analysis of the prompt language also provided information regarding the type of academic vocabulary identified within the writing prompt.

Academic vocabulary. Within the *academic vocabulary* domain, I recorded an analysis of the academic vocabulary within each of the writing prompts. I used a classification system based on empirical academic vocabulary categories or typologies (Baumann & Graves, 2010). Although various topologies have been developed for word structures and categories, Baumann and Graves (2010) used the most recent work on typologies of academic vocabulary (Fisher & Frey 2008; Harmon,Wood, & Hendrick, 2008; Hiebert & Lubliner, 2008) and developed a classification scheme. This classification scheme consists of five types of academic words and conceptual representations; (1) domain-specific vocabulary, (2) general vocabulary, (3) literary vocabulary, (4) meta-language, and (5) symbols. A modified version of the Baumann and Graves (2010) word classification scheme (see Appendix E) was used as a guide for developing this dimension. The modifications of this dimension included the elimination of the Literary Vocabulary, which is not relevant to my study.

Domain specific vocabulary. Baumann and Graves (2010) define Domain Specific Academic Vocabulary as the content-specific terms and expressions found in content area textbooks and other technical writing (p. 6). For purposes of this study this framework category has been renamed *domain specific vocabulary (DSV).* Within the DSV category, words in the prompt were coded based on the Baumann and Graves (2010) suggested source list: Building Academic Vocabulary Mathematics Word List (Marzano & Pickering, 2005) and adopted content area textbooks, informational trade books, and Internet sources. The Building Academic Vocabulary Mathematics Word List was drawn from national standards documents. For purposes of coding, I used the Building Academic Vocabulary Mathematics Word List (See Appendix F) as my primary source for word classification in the category of DSV. The other resources recommended were used if the terms were not found in the Building Academic Vocabulary Mathematics Word List. An example of a wording in a prompt that would be coded in the category of DSV is underlined and appears in Figure 19. The mathematical phrase "surface area" is coded according to the category of DSV. The words "surface" and "area" specifically are not analyzed in isolation. This phrase "surface area" and the word "cube" were found in the Building Academic Vocabulary Mathematics Word List.

What is the surface area of each tower of cubes (include bottom)? As the towers get taller, how does the surface area change?

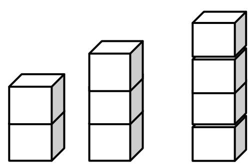


(NCTM *Principles and Standards*, 2000)

Figure 19. Example of words coded for *Domain Specific Vocabulary*.

General vocabulary. Based on the extant work on typologies of academic vocabulary, Baumann and Graves (2010) define General Academic Vocabulary as words that appear reasonably frequently within and across academic domains. The words may be polysemous, with different definitions being relevant to different domains. For purposes of this study this framework category has been renamed *general vocabulary (GV)*. Within the GV category, words in the prompt were coded based on the Baumann and Graves (2010) suggested source list: the Coxhead's (2000) Academic Word List. The Coxhead (2000) Academic Word List is the result of a corpus-based study of identifying 570 word families, about 3000 words altogether, of academic text coverage. For purposes of coding, I used the Coxhead (2000) Academic Word List (See Appendix G) as my primary source for word classification in the category of GV. If the word was polysemous, with different definitions being relevant to different domains, then the word was coded as GV. An example of a wording in a prompt that was coded in the category of GV is underlined and appears in Figure 20. The word area was found in the Coxhead (2000) Academic Word List. The word “change” is a polysemous word having two different meanings within different domains (i.e., “Change” for a dollar vs. how does the surface area “change?”).

What is the surface area of each tower of cubes (include bottom)? As the towers get taller, how does the surface area change?



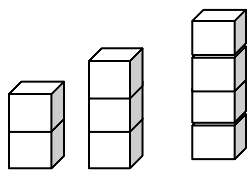
(NCTM *Principles and Standards*, 2000)

Figure 20. Example of words coded for *General Vocabulary*.

Meta-language. Baumann and Graves (2010) define *meta-language* as terms used to describe the language of literacy and literacy instruction as well as words used to describe processes, structures, or concepts commonly included in content-area texts. Within the *meta-language* category, words in the prompt were coded based on the Baumann and Graves (2010) suggested resource lists: Building Academic Vocabulary English Language Arts Word List (Marzano & Pickering, 2005) and Academic Terms for Book Parts (Pilgreen, 2005). The Building Academic Vocabulary English Language Arts Word List was drawn from national standards documents. The Academic Terms for Book Parts was drawn from English learners literacy center tutoring session (Grades 1-12) located at the University of La Verne. For purposes of coding, I used the Building

Academic Vocabulary English Language Arts Word List (See Appendix H) and Academic Terms for Book Parts (See Appendix I) as my primary sources for word classification in the category of *meta-language*. An example of a wording in a prompt that was coded in the category of *meta-language* is underlined and appears in Figure 21. The word "how" was found in the Building Academic Vocabulary Language Arts Word List (Marzano and Pickering, 2005).

What is the surface area of each tower of cubes (include bottom)? As the towers get taller, how does the surface area change?



(NCTM *Principles and Standards*, 2000)

Figure 21. Example of words coded for *Meta-language*.

Symbols. Baumann and Graves (2010) define *symbols* as icons, emoticons, graphics, mathematical notations, electronic symbols, and so forth that are not conventional words. Within the *symbol* category, words in the prompt were coded based on the Baumann and Graves (2010) suggested source list: Computer keyboard, online emoticons, Internet images, clipart, symbol-specific websites. For purposes of coding, the Fry and Kress (2006) *Reading Math Symbols Word List* in *The Reading Teacher's Book of Lists* was used as my primary source for symbol classification in the category of *symbols* (see Appendix J). An example of a symbol in a prompt that was coded in the category of *symbols* is underlined and appears in Figure 22. For example, *3*, ÷, $\frac{1}{2}$ have six symbols. Because the symbols word list has both the fraction as a whole, $\frac{1}{2}$, *and* the fraction bar (/), and two digits (1, 2) this particular symbol was analyzed as four symbols. Therefore the following symbols in the prompt were calculated as six symbols total (3,÷,$\frac{1}{2}$, 1, \, 2)

Write some different stories about $\underline{3 \div}\ \frac{1}{2}$?   (Sullivan and Lilburn, 2002)

Figure 22. Example code for *Symbols.*

Words not on list. Within the *words not on list* domain, I recorded words that were not identified in the academic vocabulary a priori word lists but *should* be according to the definitions of the academic vocabulary categories. Because the framework was developed from the most extant work on typologies of academic vocabulary by the Baumann and Graves (2010) word classification system, words that specifically met the criteria of the categories were analyzed and coded. However, if a word was not listed in the academic vocabulary word lists, it was coded in the dimension *words not on list.* This information was used to provide information regarding how many potential words were considered academic vocabulary in the writing prompts.

Total. Within the *total*, two categories were used for counting: *total words in writing prompt* and the *total number of academic vocabulary words*. These totals were used to determine the average percent of words that were considered academic vocabulary within the writing prompt.

Type of prompt. Within the *type of prompt* , the categories were modified from the pilot study to include the following: *affective*, *narrative* and *generic*. Prompts that were identified as *affective* elicited the response of a feeling or opinion (Baxter et al., 2007; Shield & Galbraith, 1998). Prompts identified as *narrative* elicited a type of storytelling aspect similar to the content and themes that are embedded within children's literature (Burns, 2004; Whitin & Whitin, 2000). Furthermore, prompts *not* coded as *affective* or *narrative* were coded as *generic*. These *generic* prompts were expository in nature in which the prompt affordance provided more of a problem-solving or explaining a process in mathematics (Baxter, Woodward, & Olson, 2005). For purposes of this research project, I used the category *generic* to code writing prompts that aligned with the expository definition.

Simple calculations in the following categories were used to determine which category had the largest percent of writing prompt types. In order to determine the type of writing prompt, the student edition was used as a resource to determine whether the prompt language afforded the response of a narrative, affective, or generic type of writing prompt. In addition, an investigation of the type of *generic* prompt was conducted by analyzing the language within the generic prompt stem to determine the nature of the generic prompts identified.

Within the next section, the *prompt support* will be described (see Figure 23).

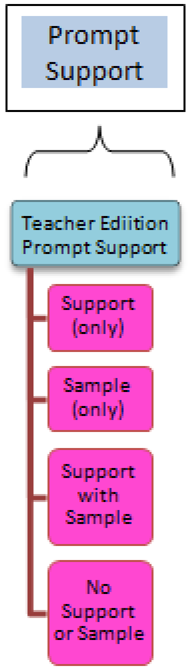


Figure 23. Framework dimension within *prompt support.*

Teacher edition. Within the *teacher edition*, pages were reviewed for instructional suggestions or recommendations related to the prompt. I reviewed the section in the teacher edition according to the location of the prompt in the student edition. There were four categories under the dimension of Teacher Edition with codes for each phrase: (1) support only; (2) sample provided only; (3) support with sample provided; and (4) no support or sample provided. These categories were coded according to the information provided in the teacher edition. This analysis revealed whether or not instructional support was provided for writing tasks in mathematics.

The 10 framework dimensions: *number of writing prompts, number of exercises per page, statement of the prompt, content strand, academic vocabulary, words not on list, total, type of prompt, teacher edition prompt support, student edition prompt location* described above were developed and refined to analyze the writing prompts identified in both textbooks.

Analysis

I reviewed 100% of the numbered or lettered exercises in the student edition in *Everyday Mathematics* and *enVisionMATH*. Because an analysis of this nature had not been previously conducted, elimination of certain sections of the textbooks may have altered the results. A common framework for a curriculum analysis investigates tasks in the activities or exercise sections of the textbook (Jones, 2004; Jones & Tarr, 2007). However, Johnson (2010) noted, in addition to the exercise and activities sections of textbooks, narrative portions of textbooks should also be examined for examples related to the content researched. Similar to Johnson (2010) in the proposed framework, I examined all sections of the student edition including the narrative portions for lettered or numbered exercises that provided the student with the opportunity to develop a constructed response. I then coded the writing prompts across the framework. In order to determine the reliability of the prompts coded within the two textbooks, the process of interrater reliability is described in the next section.

Reliability of Framework Dimensions

The reliability of my framework dimensions was the percentage of agreement that I had with another rater. In order to determine reliability of the coding of my dimensions, myself and two co-raters (doctoral student and faculty) coded the prompts. According to Miles and Huberman, (1994) this type of "check-coding" allowed my definitions to become sharper through discussion and possible modifications with the two co-raters. Weber (1990) noted, "to make valid inferences from the text, it is important that the classification procedure be reliable in the sense of being consistent: Different people should code the same text in the same way" (p.12). The closer the scores are between my co-raters and me, the higher the reliability. For example Miles and Huberman (1994) suggested that the percent of agreement should be close to 80%. For reliability purposes the next section will discuss the procedures used to monitor reliability, the description of the reliability in prompt selection, and the reliability of the coding of the dimensions of the framework.

Procedures used to monitor reliability. To ensure that the reliability of prompt selection and coding across the framework dimensions was reliable, I implemented a check-coding system with a doctoral-level mathematics student who had recently defended her dissertation proposal and was currently in "candidacy" and a recent Ph.D. graduate in Reading/Language Arts. Because the framework dimensions were developed from extant research, I selected these two co-raters for their expertise in order to strengthen my framework through conversations based upon the analysis of coded data. Furthermore, because I developed the codes for the framework, and coded 90% of the data, I wanted to determine how close I was to the final decision of my co-raters. Therefore I was the referent for purposes of coding.

The co-raters were familiar with my topic through conversations and the reading of my proposal. I corresponded with the co-raters approximately 12 times via email, telephone and face-to-face meetings (December of 2011 and January through March 2012). In addition, copious notes were taken during our conversations to provide information to strengthen the framework and codebook (see Appendix K). In order to ensure the co-raters had a common understanding, the first meeting consisted of a training session.

Reliability training. During my first session, I trained both co-raters using the pilot study as my guide. Additionally, a codebook was used as a reference for selection of the prompts and coding the prompts across the framework dimensions (see Appendix K). After the training session, I gave the co-raters the same textbook used in the pilot study and asked them to code the chapter using the framework. In order to determine the reliability of my prompt selection, the raters used the criteria of terms provided in Appendix B and in the coded book (see Appendix K). After the selection of prompts, the co-raters and I compared our coding and discussed any discrepancies. After the writing prompts were discussed, a blank framework in the form of an excel document was given to each rater. Next the co-raters rated the prompts along the dimensions of *content strand, academic vocabulary, type of prompt* and *teacher edition*. The co-raters used Appendices E-I for academic vocabulary with the codebook as a reference tool to code across the dimensions. Once the coding was complete the co-raters and I compared our coding across the dimensions and found consistency in our selections. After the training using the pilot study, we felt there was a common understanding of the analytical framework and the co-raters were ready to code on their own.

Lessons coded. I coded 100% of the textbook's sections and content areas that had a numbered or lettered exercise. The pages that consisted of a numbered or lettered exercise were titled *readable pages* for purposes of this study. Pages that were not coded did not have a numbered or lettered exercise on the page. The two co-raters reviewed 10% of the readable pages in order to assess agreement on the prompts to be included for analysis. I developed an itemization of the number of exercises within each chapter in order to provide ease of selection for the 10% of readable pages to be co-coded. Based upon discussion, the co-raters collectively selected the same two chapters from each textbook, totaling 10% of the readable pages within each textbook (see table 6). Because my two co-raters and I coded 10% of the readable pages, the tasks were then triple coded.

Table 6

Number of Readable Pages that were Triple Coded

Textbook	No. of Readable Pages Coded by Researcher	10% of Readable Pages for Co-rators	Lessons Selected by Co-rators Totaling 10%
enVision MATH	360	36	Lesson 13 Lesson 19
Everyday Mathematics	414	41	Lesson 10 Lesson 11

Reliability of prompt selection in enVision MATH. The reliability of the prompt selection was calculated based on the total number of prompts rather than percentages. Within the two lessons from the *enVision MATH* textbook I coded 32 tasks as writing prompts, Rater 1 coded 26 tasks as writing prompts, and Rater 2 coded 35 tasks as writing prompts. After analysis, there were a total of 37 prompts recognized. Of the 37 total prompts recognized, 22 were identified across all 3 coders resulting in a baseline agreement of 59%. Our discussion focused on the 15 remaining prompts that were not in full agreement. After review of the prompt, we came to a final agreement of 34 tasks that would be coded as writing prompts (see Table 7).

Table 7

Percentage of Agreement of Prompt Selection for enVision MATH textbook

Raters	No. of Prompts Identified Lesson 13	No. of Prompts Identified Lesson 19	Total No. of Prompts in Both Lessons
Researcher	23	9	32
Rater 1	19	7	26
Rater 2	27	8	35
Total	27	10	37
Baseline agreement	17	5	22
Final No. in Agreement	24	10	34

Final Decision. As noted in the table above, 37 unique prompts were identified across all three co-coders and 34 were included for analysis. After discussion, the co-coders and I collectively decided to eliminate three tasks as writing prompts because of the nature of the constructed response. For example, if the prompt could be answered in a one word response, the prompt was not included for final coding. In all three of the eliminated prompts, the prompt affordance was in the form of a one word answer. The following is a prompt that was eliminated based on the affordance of a one-word answer:

- Is it reasonable to say that the mass of Roger's backpack is twice as much as Marta's backpack?

Of the 32 prompts I coded individually, 100% of those prompts were included in the final count of 34 prompts agreed upon for analysis. The additional two prompts were identified by my co-coders. Therefore, the reliability of content strand selection for the *enVision MATH* was calculated using the final number of prompts as the referent [5].

Reliability of prompt selection in Everyday Mathematics. After reviewing the coding it was determined that all three coders had 100% of the coding consistent with one another. For example, of the 21 prompts coded in both lessons, 100% of those prompts were the same prompts among both co-coders and me. Therefore, there were no prompts identified by one only one rater and there was 100% baseline agreement.

There are several reasons that might explain why the agreement was higher in *Everyday Mathematics* than *enVision MATH*. First, this textbook was coded second and the previous coding may have made the prompt selection easier. Second, the layout of the *Everyday Mathematics* textbook has fewer tasks per page, sometimes having only one or two tasks per page to analyze. Third, because the total number of exercises in *Everyday Mathematics* is fewer than *enVision MATH,* there were fewer prompts affording a constructed response in the form of a sentence or more, thus making it easier to identify the prompts to be coded.

Reliability of coding across framework dimensions. An analytic framework was developed consisting of 10 dimensions: *number of writing prompts per page, number of exercises per page, statement of prompt, content strand, academic vocabulary, type of prompt, teacher edition prompt support, student edition prompt location, total number of academic vocabulary*, and *words not on list*. Four of the 10 dimensions did not require code-checking because the codes to be assigned to these dimensions were obvious:

[5] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

number of exercises per page, student edition prompt location and *academic vocabulary total.* Additionally, I decided not to check-code the *words not on list* dimension since this section was used as a category to place words that each of the raters *believed* to be academic vocabulary but could not locate in the a priori word lists. Therefore, the dimensions that were less obvious regarding coding assignment were *content strand, academic vocabulary, type of prompt* and *teacher edition prompt support.* These dimensions will be discussed in the following section. Furthermore, because the check-coding of the prompts and the dimensions was done at the same time, the check-coding in this section was based upon the prompts that were in the baseline of agreement for each of the lessons in both textbooks.

Reliability of coding of prompts in content strand. Based on NCTM's *Principles and Standards for School Mathematics* (2000), codes for *content strand* were *number sense, geometry, measurement, algebra, data analysis,* and *other*. The language of the prompt assisted in determining which of the content strands the prompt was coded. Additionally the titles of the lessons and the lesson section titles assisted in providing the appropriate codes. Furthermore the language within Tables 1-5 and the codebook also guided the process of coding appropriately.

Reliability of coding of prompts in content strand for enVision MATH. The reliability of content strand selection for enVision MATH was calculated using the final number of prompts agreed upon as the referent[6]. There were two differences in coding

[6] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

between the researcher and the co-raters. As the researcher, if the language within the prompt was located in more than one content strand, then I coded it accordingly into multiple content strands. Once I made the co-raters aware of the language in the prompt and how that language assisted my decision, they agreed regarding my codes. For example, the following prompt is an example of a prompt coded in the content strand of *number sense* and *measurement*:

- Blake jogged 1.7 miles one morning. His sister jogged $1\frac{3}{4}$ miles that same day. Who jogged farther? Explain your answer (*enVision MATH*, p. 283).

The prompt was coded in the content area of *measurement* because of the terms *miles* and *day*. In addition, the prompt was also coded in the content area of *number sense* because the symbols needed to answer the prompt were in fraction and decimal formation. Additionally, the term *farther* indicated the process of subtraction.

Before discussion, Rater 1 missed the coding in two areas for content strand. Based on discussion, Rater 1 agreed with the oversight and changed the coding decision. Before discussion, Rater 1 had approximately 90% agreement with my coding. Additionally, Rater 2 missed the same two codes in the two areas for content strand and changed the coding. Before discussion, Rater 2 had approximately 90% agreement with my coding. After our discussion, 100% of the prompts were coded in the appropriate content strand based on our final decision (see Table 8).

Table 8

Percentage of Agreement of Coding for Content Strand for enVision MATH

Before Discussion		
Co-coders	No. of Prompts	%
Researcher	22	100
Rater 1	20	91
Rater 2	20	91

Note. Percent is determined by number of prompts for a coder/final number of prompts (*n*=22).

***Reliability of coding of prompts in content strand for Everyday Mathematics*.** The reliability of content strand selection for the *Everyday Mathematics* was determined using the final number of prompts as the referent[7]. As the researcher, I had one code that was different from our final decision. Before discussion, I had 95% of the codes in agreement with Rater 1. After discussion, I agreed with Rater 1 who had 100% of the codes determined in our final decision. Before discussion, Rater 2 had 57% of the codes determined in our final decision. Based on discussion, it was determined that Rater 2 missed coding several prompts due to an oversight in the language of the prompt. After discussion, Rater 2 agreed with me and Rater 1 and changed the codes to reflect 100% agreement (see Table 9).

[7] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Table 9

Percentage of Agreement of Coding for Content Strand for Everyday Mathematics

Before Discussion		
Co-coders	No. of Prompts	%
Researcher	20	95
Rater 1	21	100
Rater 2	12	57

Note. Percent is determined by number of prompts per coder/final number of prompts (*n*=21).

Reliability of coding of academic vocabulary. Based on Baumann and Graves's (2010) classification scheme, the codes for *academic vocabulary* included *domain specific vocabulary*, *general vocabulary*, *meta-language* and *symbols.* These four categories were derived from the most recent work on typologies of academic vocabulary (Fisher & Frey, 2008; Harmon,Wood & Hendrick, 2008; Hiebert & Lubliner, 2008). The academic vocabulary within the writing prompts was coded based on location of specific terms in the academic vocabulary a priori word lists (Baumann & Graves, 2010; Coxhead, 2000; Fry & Kress, 2006; Marzano & Pickering 2005). Appendices F-J and the codebook (see Appendix K) also guided the process of coding appropriately. Additionally, word associations and derivatives were acknowledged during the coding process. If a word was not located in the a priori word lists, even though the word was classified by the definition, it was placed in the *words not on list* section. After the coding was completed in this section for each textbook, discussions regarding words omitted and words missed were reviewed with each of the co-raters.

Reliability of coding of academic vocabulary for enVision MATH. The reliability of academic vocabulary selection for enVision MATH was calculated using the final number of words agreed upon as the referent[8]. As the researcher, I missed one word due to an oversight that Rater 1 and Rater 2 had located in the word lists. Before discussion I had 99% of the codes in agreement with the final decision. Before discussion, Rater 1 had 89% of the codes determined in our final decision and Rater 2 had 90% of the codes in agreement with the final decision. After discussion Rater 1 and Rater 2 had changed the codes to reflect 100% agreement (see Table 10).

Table 10

Percent of Agreement of Coding for Academic Vocabulary for enVision MATH

Before Discussion		
Co-coders	No. of Academic Vocabulary	%
Researcher	136	99
Rater 1	122	89
Rater 2	124	90

Note. Percent is determined by number of words per coder/final number of words (n=137).

[8] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

During our discussion of the words missed, I simply had to show Rater 1 and Rater 2 where the words were located on the a priori word lists. For the omitted words, Rater 1 and Rater 2 had two word associations that were placed in the *words not on list* category after our discussion. For example, Rater 2 coded the term *translation* as *domain specific* because *slide transformation* was located in the *domain specific* list. In addition, Rater 2 also understood that a *slide transformation* is a type of *translation*. However, because the words are associated and not derivatives, the term was placed in the *words not on list* category.

Reliability of coding of academic vocabulary for Everyday Mathematics. The reliability of academic vocabulary selection for *Everyday Mathematics* was calculated using the final number of words agreed upon as the referent[9]. As the researcher, I missed 10 words word due to an oversight that Rater 1 and Rater 2 had located in the word lists. Before discussion I had 91% of the codes in agreement with the final decision. The words that I missed were commonly used so they resulted in the same word being missed across multiple writing prompts. Before discussion, Rater 1 had 84% of the codes determined in our final decision and Rater 2 had 82% of the codes in agreement with the final decision. Similar to our previous discussions based on words missed and words omitted, Rater 1 and Rater 2 had changed the codes to reflect 100% agreement (see Table 11).

[9] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Table 11

Percent of Agreement of Coding for Academic Vocabulary for Everyday Mathematics

Before Discussion		
Co-coders	No. of Academic Vocabulary	%
Researcher	91	90
Rater 1	85	84
Rater 2	83	82

Note. Percent is determined by no of words per coder/final number of words (*n*=101).

Reliability of coding of type of prompt. Based on the research in mathematics writing prompt types (Burns, 2004; Dougherty, 1996; Urquhart, 2009; Whitin & Whitin, 2000), the codes for *Type of Prompt* include *narrative*, *affective* and *generic* problem. The percentage of agreement in this domain was 100% among the researcher, Rater 1 and Rater 2 for both *enVision MATH* and *Everyday Mathematics* textbooks. The check-coding system indicated that the researcher and the co-raters coded 100% of the prompts the same. If a writing prompt did not require the student to answer in the form of an affective/attitude response, nor did it require the students to write a story, then the writing prompt was coded as *generic*. The high reliability in this domain may be a result from the high percentage (99%) of the writing prompts in both textbooks coded in the *generic* category.

Reliability of coding of teacher edition. The codes for teacher edition were based on the amount of support aligned to the writing prompt: *support*, *sample, support with sample,* and *no support or sample*. The reliability of this coding was based on identifying the type of support in the teacher edition for the writing prompts. Discussion for the reliability in this dimension was determined on the location of the support within each of the teacher editions.

Reliability of coding of teacher edition for enVision MATH. The coding for the teacher edition in this textbook was based on writing prompt support. The reliability of the coding of the teacher edition for *enVision MATH* was determined using the final number of prompts agreed upon as the referent[10]. As the researcher, I coded one of the prompts differently due to an oversight that Rater 1 and Rater 2 had located. After discussion, I agreed with 100% of the final decision of Rater 1 and Rater 2. Rater 1 and Rater 2 both had 100% of the codes in agreement with the final discussion. Overall there was a high percentage of agreement within the *enVision MATH* textbook from all three raters (see Table 12).

Table 12

Percent of Agreement of Coding for Teacher Edition for enVision MATH

Before Discussion		
Co-coders	No. of Academic Vocabulary	%
Researcher	21	95
Rater 1	22	100
Rater 2	22	100

Note. Percent is determined by number of words per coder/final number of prompts (*n*=22).

[10] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Reliability of coding of teacher edition for Everyday Mathematics. The coding of the teacher edition for this textbook was based on writing prompt support. The reliability of teacher edition for *Everyday Mathematics* was calculated using the final number of prompts agreed upon as the referent[11]. As the researcher, I had 100% of the codes in agreement with the final decision. Similarly, Rater 1 also had 100% of the codes in agreement with the final decision. Before discussion Rater 2 had 75% of prompts in agreement with the final decision. Based on discussion it was determined that an oversight occurred with Rater 2. After discussion, Rater 2 agreed with the researcher and Rater 1 to reflect 100% agreement of the final decision (see Table 13).

Table 13

Percent of Agreement of Coding for Teacher Edition for Everyday Mathematics

	Before Discussion	
Co-coders	No. of Academic Vocabulary	%
Researcher	21	100
Rater 1	21	100
Rater 2	16	76

Note. Percent is determined by no of words per coder/final number of prompts (n=21).

[11] I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Summary of Reliability of Framework Dimensions

The reliability of the coding within the framework led to co-coding in five of the dimensions for each textbook investigated: *statement of the prompt, content strand, academic vocabulary, type of prompt, and teacher edition prompt support*. Most discrepancies in coding were based on an oversight and were adjusted to reflect 100% of the final agreement. The training session integrating the codebook (see Appendix K) and collaborative discussions were important in achieving the reliability.

Sources of Influence

There are two sources of influence that have the potential to affect the reliability of my study. The first source of influence in the study is my bias interfering in training my co-raters. In order to reduce this training bias, I selected two raters instead of one to assist in coding the data within each of the dimensions. In an effort to obtain at least 80% agreement, discussions with additional modifications to the framework categories were addressed. My second source of influence was how the textbooks were chosen for the study. Within my literature review, it is noted that research between publisher-generated and NSF funded textbooks is common. Because *Everyday Mathematics* is the elementary level textbook for NSF funded textbooks and the only textbook to have a new third edition series 2012 titled "Common Core," this textbook was chosen. In choosing a publisher-generated textbook, *enVision MATH* was chosen because of the significant market share obtained by the publisher Pearson Scott Foresman in addition to alignment with the "Common Core."

The results and findings of my analysis from the two textbooks are discussed in Chapter 4.

Chapter 4: Results

The purpose of this study was to examine writing prompts in mathematics textbooks. Specifically, the study was designed to explore the following research questions:

1. How many writing prompts are included in one 4th grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?
2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?
3. What types of vocabulary are used in the writing prompts in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?
4. What types of prompts are provided in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?

Urquhart (2009) described a mathematical *writing prompt* as a task (i.e., questions and statements) that elicit particular responses. Urquhart categorized writing prompts as content focused, process focused, or affective. In addition, Smagorinsky (2006) noted that writing is enhanced when the *writing task* is interesting, motivating, and at the appropriate level of understanding.[12]

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[12] Research in the field of composition has suggested that the terms *writing prompt* and *writing task* can be used interchangeably (Murphy 2004; Yancey 2004; Smagorinsky, 2006; Urquhart, 2009); therefore, I have used both of these terms in the research.

In order to examine writing prompts in mathematics textbooks, I selected the 4th grade text from two widely-used textbooks and the corresponding teacher editions: *enVision MATH* published by Pearson Education, Inc. and the third edition of mathematics texts developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. Both series have significant market shares in the U.S.

In order to address each of the research questions, I developed a framework to analyze the prompts within the *enVision MATH* and *Everyday Mathematics* 4th grade textbooks within the series. Ten dimensions in the framework were developed based on the four research questions: *number of writing prompts, number of exercises per page, prompt, content strand, academic vocabulary, type of prompt, teacher edition, student edition, total,* and *words not on list*. A table of the dimensions and code key are located in Appendix D.

Numbered or Lettered Exercise

The unit of analysis for the data in the *student edition* was a numbered or lettered exercise and the number of words. Within the student edition the authors of the *enVision MATH* and *Everyday Mathematics* textbooks separated each chapter into a series of *lessons*. Therefore each lesson was explored for numbered or lettered exercises. A numbered or lettered exercise in the *student edition* was a problem type that required a student response. The response could be in the form of a closed-ended response, whereby the answer to the exercise was visible (i.e., multiple choice, true/false, or matching) or an open-ended/constructed response whereby the answer was not visible and required the student to construct an answer (Cooney et al., 2004). Exercises were coded as writing prompts only if the response to the exercise required a constructed response in the form of one or more sentences. Within the academic vocabulary domain, the unit of analysis was the number of words associated with the writing prompt. I analyzed 100% of each student edition and determined the numbered or lettered exercises that were identified as writing prompts. The *enVision MATH* textbook used black numbers with a period after the number to identify exercises. The *Everyday Mathematics* textbook used blue numbers and letters with a period after the numbers and letters to identify exercises. Additionally, the numbers of words in the prompt coded within the *Everyday Mathematics* and *enVision MATH* textbooks were also used as the unit of analysis when analyzing the academic vocabulary domain.

I analyzed a total of 34 lessons, 20 from *enVision MATH* and 14 from *Everyday Mathematics*. Table 14 provides a more detailed description of the lesson topics analyzed in each textbook. All numbered and lettered exercises from each textbook were counted. There were no numbered or lettered exercises unaccounted for. In total, the 34 lessons from both textbooks included 3,185 exercises with 2,481 in *enVision MATH* and 704 in *Everyday Mathematics*.

Table 14

Lesson Number and Title within the enVision MATH and Everyday Mathematics Textbooks

Lesson No.	*enVision Math*	*Everyday Mathematics*
1	Numeration	Naming and Constructing Geometric Figures
2	Adding and Subtracting Whole Numbers	Using Numbers and Organizing Data
3	Multiplication Meanings and Facts	Multiplication & Division: Number Sentences & Algebra
4	Division Meanings and Facts	Decimals and Their Uses
5	Multiplying by 1-Digit Numbers	Big Numbers, Estimation, and Computation
6	Patterns and Expressions	Division; Map Reference Frames; Measures of Angles
Projects		Algorithm Projects
7	Multiplying by 2-Digit Numbers	Fractions and Their Uses; Chance and Probability
8	Dividing by 1-Digit Divisors	Perimeter and Area
9	Lines, Angles, and Shapes	Fractions, Decimals, and Percents
10	Understanding Fractions	Reflections and Symmetry
11	Adding and Subtracting Fractions	3-D Shapes, Weight, Volume, and Capacity
12	Understanding Decimals	Rates
Projects		Algorithm Projects
13	Operations with Decimals	
14	Area and Perimeter	
15	Solids	
16	Measurement, Time, and Temperature	
17	Data and Graphs	
18	Equations	
19	Transformations, Congruence, and Symmetry	
20	Probability	

Writing Prompts

To determine how many of the total exercises were writing prompts, I isolated the student exercises that were identified with a number or a letter. If the exercise afforded the opportunity of a response using one or more sentences, it was coded as a prompt for written response. For example, the following prompt from the *enVision MATH* textbook was coded as a writing prompt:

- How does using commas to separate periods help you read large numbers?

From the 20 lessons analyzed in the *enVision MATH* textbook, 323 tasks were coded as writing prompts out of 2,481 exercises (13%). From the 14 lessons analyzed in *Everyday Mathematics*, 140 tasks were coded as writing prompts out of 704 exercises (20%). Table 15 shows a description of the tasks analyzed and coded as writing prompts within both textbooks.

Table 15

Exercises and Prompts within the enVision MATH and Everyday Mathematics Textbooks.

Textbook	Total No. of Exercises	Total No. of Writing Prompts	%
enVision MATH	2481	323	13
Everyday Mathematics	704	140	20

Although *enVision MATH* (N=323) included more writing prompts than *Everyday Mathematics* (N=140), *Everyday Mathematics* had a higher percentage of writing prompts (20%) than *enVision MATH* (13%).

Content Strand

To address the second research question, I examined how mathematical writing prompts varied across the content strands. In the combined data from both *enVision MATH* and *Everyday Mathematics* textbooks, a total of 62% of the writing prompts were coded in the *number sense* strand, 17% in the *geometry* strand, 18% in the *measurement* strand, 9% in *algebra*, 10% in *data analysis/probability,* and 6% were coded as *other*. Table 16 provides a more detailed description of the breakdown across content strands[13].

Table 16

Number and Percentage of Writing Prompts by Content Strand within the enVision MATH and Everyday Mathematics (EM) Textbooks

	enVision MATH (*N*=323)		*EM* (*N*=140)	
Content Strand	No.	% textbook total	No.	% textbook
Number Sense	213	66	75	53
Geometry	55	17	25	18
Measurement	48	14	38	27
Algebra	28	8	15	11
Data Analysis	34	10	15	11
Other	0	0	29	21

[13] The total number of writing prompts included in the analysis for content strand exceeds the previously stated totals (*enVision N*=378 and *Everyday Mathematics N*=197) therefore making the percentage above 100% for total because some prompts were coded in more than one content strand. However, the total number of prompts in each textbook remains the same for *enVison Math* (*N*=323) and *Everyday Mathematics* (*N*=140). This additional coding was based on the language within the prompt and/or the lesson section title in the textbook.

If a prompt had language that was used and identified within two content strands, the prompt was coded in both content strands. For example, the following prompt was coded in both the *number sense* and *measurement* categories:

- How many hundredths are in one-tenth? Explain using pennies and a dime.

The language of "hundreths" and "one-tenth" was coded as *number sense* (see Table 1). In addition, the language of "pennies" and "dimes" was coded as *measurement* (see Table 4). This prompt was located in the lesson section titled "Using Money to Understand Decimals." In total, 55 *enVision MATH* prompts were dually coded and 57 *Everyday Mathematics* prompts were dually coded.

Across the content strands both textbooks included approximately the same percentage of prompts in Geometry, Measurement, Algebra, and Data Analysis. The exceptions were: *number sense* and *other*. Both the *enVision MATH* textbook and *Everyday Mathematics* textbook had the largest percentages of prompts recorded in the *number sense* category. However, there were differences in the percentages recorded for each textbook that may be explained by the fact that 21% of *Everyday Mathematics* prompts were coded in the content strand of *other* and *enVision Math* had 0% coded in this category. Prompts coded in the section of *other* did not have any mathematical content language needed to identify a content strand category. Within the *Everyday Mathematics* textbook, these prompts were identified in lessons titled *My Country Notes.* These prompts dealt with particular questions associated with countries around the world.

Content strand and textbook. As indicated in Table 16, both of the textbooks had the highest percentage of writing prompts coded as *number sense* tasks. However, the category of *other* had the largest percent *difference* between the two series. Only the

Everyday Mathematics textbook had writing prompts coded as *other* such as the following:

- To which country would you most like to travel in your lifetime? Explain your answer (p. 325).

Because the language in this prompt does not lend itself to one of the five content strands in mathematics, I selected the code of *other*. Unlike the *Everyday Mathematics* textbook, 0% of the writing prompts in the *enVision MATH* textbook were coded as *other*; resulting in 100% of the prompts coded in at least one of the five content strands. As indicated by the language use in the aforementioned prompt, the *Everyday Mathematics* textbook integrated the content area of social studies into this particular mathematical writing prompt giving students the opportunity to integrate and connect mathematics in real world applications. Figure 24 illustrates the percentage of prompts in each textbook for each content strand[14].

[14] The total number of writing prompts included in the analysis for content strand exceeds the previously stated totals (*enVision N*=378 and *Everyday Mathematics N*=197) therefore making the percentage above 100% for total because some prompts were coded in more than one content strand. However, the total number of prompts in each textbook remains the same for *enVison Math* (*N*=323) and *Everyday Mathematics* (*N*=140). This additional coding was based on the language within the prompt and/or the lesson section title in the textbook.

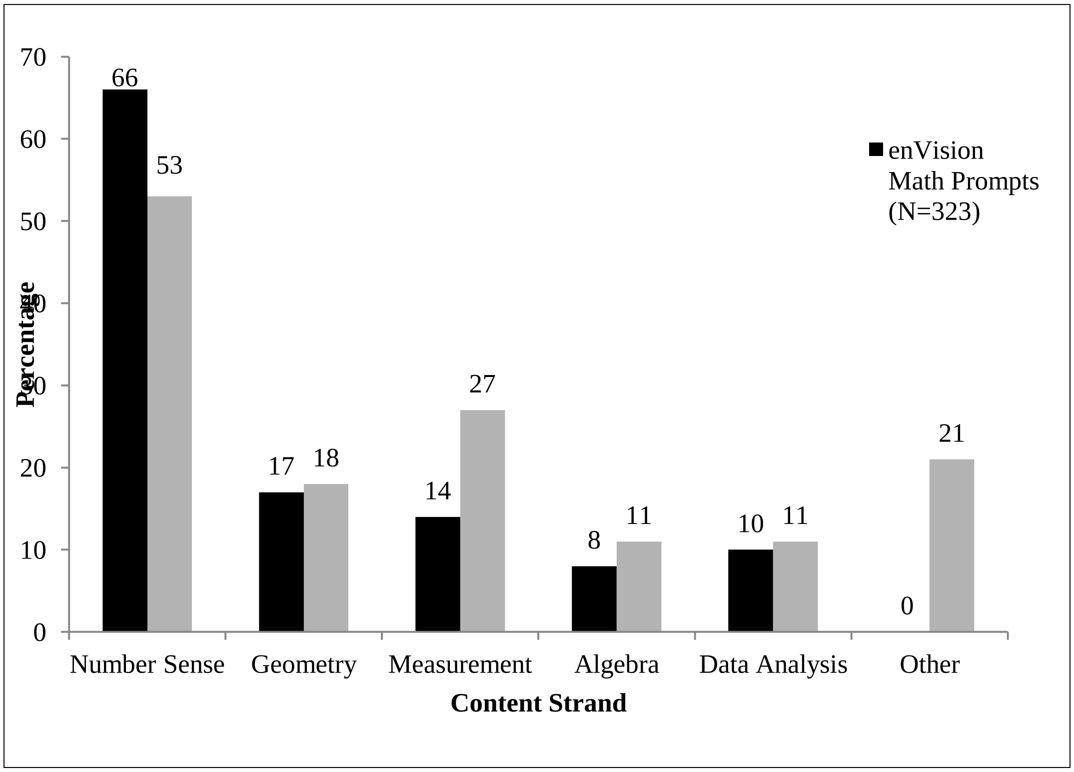

Figure 24. Percentage of prompts within each content strand of *enVision MATH* and *Everyday Mathematics* (EM) textbooks.

Academic Vocabulary

The third research question related to the type of vocabulary coded within the prompts. There were four different codes in the framework category for Academic Vocabulary (see Appendix A). First a word in the prompt was coded as *domain specific vocabulary* if the term was explicit to the domain of mathematics (see Appendix F). The following prompt used bolded font to indicate the vocabulary identified and coded as *domain specific vocabulary*:

- Describe how you would order the continent's **area** using place value.

Second, words coded as *general vocabulary* were generally polysemous in nature or had more than one meaning depending on the content area (see Appendix G). The following prompt uses bolded font to indicate the vocabulary identified and coded as *general vocabulary*:

- If you buy an **item** that costs $8.32, why would you pay with one $10 bill, 3 dimes, and 2 pennies?

Third, words coded as *meta-language* usually described a process (see Appendix H and I). The following prompt uses bolded font to indicate the vocabulary identified and coded as *meta-language:*

- **Why** do you only need to look at the number of dollars to know that $5.12 is greater than $4.82?

Fourth, the final code of *symbols* categorized all the signs and symbols conducive to understanding the mathematics writing prompt (see Appendix J). The following prompt uses bolded font to indicate the vocabulary identified and coded as *symbols*:

- Describe how to order **7,463, 74,633**, and **74,366** from least to greatest.

Table 17

Vocabulary Items and Symbols in the enVision MATH and Everyday Mathematics Textbooks.

Textbook	No. of Total Words & Symbols	No. of Words & Symbols Coded AV	No. of Prompts	Average No. of AV Per Prompt
enVision MATH	5748	2157	323	6.67 (7)
EM	3211	843	140	6.02 (6)

Overall, the largest percentage of Academic Vocabulary was in the *symbols* category for *enVision MATH* and *Everyday Mathematics*. The *symbols* category accounted for 35% of the Academic Vocabulary between the two textbooks, with the second highest average of 33% coded as *domain specific vocabulary* and 27% as *meta-language. General vocabulary* had the lowest average of 5% between the two textbooks. Table 18 provides detailed information regarding these percentages.

Table 18

Type of Academic Vocabulary within the Writing Prompts in the enVision MATH and Everyday Mathematics (EM) Textbooks.

	enVision MATH		*EM*		*enVision Math & EM*
Type of Academic Vocabulary	*n*	%	*n*	%	Total %
Domain Specific Vocabulary	730	34	259	31	33
General Vocabulary	117	5	42	5	5
Meta-language	540	25	261	31	27
Symbols	770	36	281	33	35
Total	2157	100	843	100	100

As indicated in Figure 25, the greatest percentage of vocabulary items was in the *symbols* category of the Academic Vocabulary category. Rubenstein and Thompson (2001) specify that, in order to read and write in mathematics, students must produce symbols and be able to understand the concept represented by the symbols. For actual words, the academic vocabulary strand with the largest percentage between *enVision MATH* and *Everyday Mathematics* was the *domain specific vocabulary* category. The words in this category were specific to the domain of mathematics and would generally be located in mathematics standards and in a mathematics textbook glossary (Baumann & Graves, 2010; Marzano & Pickering, 2005). The academic vocabulary category of *meta-*

language had the largest percentage difference (6%) between the two textbooks. The percentage of *general vocabulary* was not only the same for both textbooks but also the lowest percentage in each textbook.

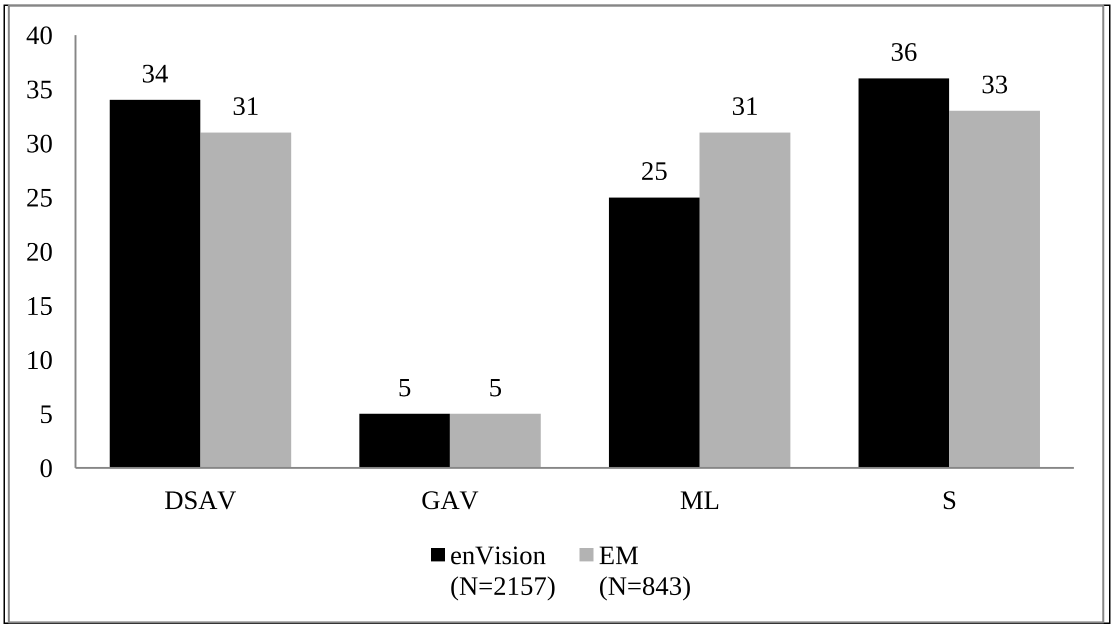

Figure 25. Percentage of academic vocabulary within the writing prompts in the *enVision MATH* and *Everyday Mathematics* (EM) textbook.
Note. DSV = Domain Specific Vocabulary; GV = General Vocabulary; ML = Meta-language; S = Symbols

Included in the percentages for *Academic Vocabulary* were derivatives. For example if the word *explain* was located in the prompt, the word was coded as meta-language since *explanation* is the derivative found in the meta-language word list. A total of 440 words were identified as derivatives of the word lists.

Academic vocabulary and words per prompt. In total, 2,157 out of the 5,748 total words within the 323 prompts located in the *enVision MATH* textbook were coded as academic vocabulary. Therefore, an average of 6.67 academic vocabulary words per prompt was determined. In addition, 5,748 total words were counted within the 323 coded prompts. Therefore, an average of 18 words per prompt was indicated. Because an average of 7 words per prompt were coded as academic vocabulary out of the 18 average words per prompt, approximately 37% of the words within the prompt were coded as academic vocabulary for *enVision MATH* (see Table 19).

Similarly, 843 words out of the 3,211 total words within the 140 prompts located in the *Everyday Mathematics* textbook were coded as academic vocabulary. Therefore an average of 6.02 academic vocabulary words per prompt was determined (see Table 17). In addition, 3,211 total words were counted within the 140 coded prompts. Therefore, an average of 23 words per prompt was indicated. Because an average of 6 words per prompt were coded as academic vocabulary out of the 23 average words per prompt, approximately 27% of the words within the prompt were coded as academic vocabulary for *Everyday Mathematics* (see Table 19).

Table 19

Percent of Academic Vocabulary per Prompt within the enVision MATH and Everyday Mathematics Textbooks.

	Total No. of Words & Symbols	Total No. of AV Words	No. of Prompts Words per Prompt	Average No. of Prompts	Average No. of per Prompt	% of AV
EV	5748	2157	323	7	18	37
EM	3211	843	140	6	23 `	27

Note. EV = *enVision Math*; EM = *Everyday Mathematics*; AV=Academic Vocabulary

Words Not On List

The category *words not on list* related to all of the words in the prompt that were identified as academic vocabulary according to the definitions of *DSV*, *GV*, *meta-*

language, and *symbols* but were not located on the a priori academic vocabulary word lists. Once identified as *academic vocabulary*, the words were then scanned in the academic vocabulary word lists (see Appendix F-J) for purposes of categorizing. If the word or the derivative of the word was not located in one of the vocabulary word lists, it was placed in the *words not on list* category. Overall, within the *enVision MATH* and *Everyday Mathematics* textbooks1,679 words were placed in the *words not on list* category. Although many of the words were duplicates, they were labeled in the *words not on list* category as DSV, GV, or ML by definition of the academic vocabulary categories (see Appendix A). For example, *pennies* and *dimes* were located on more than one occasion and coded as DSV by association to the term *money* in the DSV word list. The number of each of the words that *could* potentially be in the a priori academic vocabulary word lists can be found in Table 20.

Table 20

Words Not on List Within the Writing Prompts in the enVision MATH and Everyday Mathematics (EM) Textbooks.

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Academic Vocabulary Category	*n*
Domain specific vocabulary	591
General vocabulary	296
Meta-language	792
Total	1679

Type of Prompt

The final research question related to the type of prompt located within each textbook. The language used within the prompt had the potential to determine the type of prompt: affective or expository. Affective prompts (Baxter et al., 2007; Shield & Galbraith, 1998) are prompts that intend to elicit opinions or feelings. Because *enVision MATH* did not have any prompts coded as *affective,* the following prompt from *Everyday Mathematics* is used as an example of an *affective* prompt. The language used within the prompt required a constructed response of an opinion or feeling:

- What are some things you have enjoyed on the World Tour? (p. 325).

Expository responses are responses that do not involve feelings or opinions but more problem-solving or explaining a process in mathematics (Baxter, Woodward & Olson, 2005). I used the category, *generic,* to code writing prompts that aligned with the expository definition. The two prompts below were coded as *generic*:

1) Explain why the value of 5 in 5,264 is 5,000 (*enVision MATH*, p. 4).

2) Feng said the name of this angle is ∠SRT. Is he right? Explain. (*Everyday Mathematics,* p. 6).

Because the study included only Grade 4 mathematics textbooks, primarily for the focus of high-stakes writing for national and international reporting, I decided to include another type of writing prompt in the framework (see Appendix A). The additional prompts are commonly used for purposes of high-stakes testing. I labeled this type of prompt as a *narrative* prompt. For these narrative prompts, the constructed response could be in the form of a response that displayed math content in an imaginary or real world sense. Furthermore, *narrative* content and themes are embedded within children's literature (Burns, 2004; Whitin & Whitin, 2000). The following math prompt was noted by Burns (2004) to facilitate a story construction. This type of prompt was coded as a *narrative* type in the framework:

- Write a story entitled, "If I Were One Centimeter High" (p. 105).

Overall, *enVision MATH* and *Everyday Mathematics* had the largest percentage of writing prompts coded in the cognitive category of *generic*. *Generic* prompts accounted for an average of 93% of the prompts across the two textbooks. An average of 4% of the prompts in the *Everyday Mathematics* textbook were coded within the category of *affective*. *Affective* prompts were only located in the *Everyday Mathematics* textbook. Within the *narrative* category, *enVision MATH* textbook had only one prompt (<1%) coded in this category but *Everyday Mathematics* had approximately 18% of prompts in this category. Table 21 provides detailed information regarding these percentages.

Table 21

Type of Prompt in the enVision MATH and Everyday Mathematics (EM) Textbook.

	enVision MATH		*EM*	
Type of Prompt	*n*	%	*n*	%
G	322	99	110	78
A	0	0	5	4
N (r)	1	<1	25	17
Total	323	99	140	99

Note. G = Generic; A = Affective; N Narrative, r = Real World.

Type of prompt and textbook. The greatest percentage regarding the type of prompt was within the *generic* category. I coded almost 100% of the prompts from *enVision MATH* as *generic*. Because of this high percentage, I conducted an additional analysis. Within the field of mathematics, there are three types of mathematics writing prompts. These types of prompts are (a) content (b) process and (c) affective prompts (Dougherty, 1996; Urquhart, 2009). Because the majority of prompts were *not* categorized as *affective*, further analysis of whether the prompts were *content* or *process* types of mathematics prompts was conducted. A *content* prompt according to Urquhart (2009) is one that attends to mathematical concepts and relationships. Student responses can be in the form of defining, comparing and contrasting, and explaining (Dougherty, 1996). The following prompt was defined by Urquhart (2009) as a *content* prompt:

- How do you know $\frac{1}{4}$ is greater than $\frac{1}{5}$? Explain your thinking. (p.7).

A *process* type of prompt invoked student responses regarding the selection of the various strategies or the steps used to solve a process problem (Dougherty, 1996). More specifically, *process* prompts ask the students to explain their learning process in solving a problem (Urquhart, 2009). Doughtery and Simmons (2006) identify the following prompt as a *process* type prompt:

- I can justify my solution to a volume problem by…(p. 34).

Generic Prompt

The high percentage of domain specific vocabulary and symbols coded within the prompts in the *generic* category (see Table 21) indicate knowledge of the *content* of mathematics required in order to construct a response. Additionally, mathematical processes such as problem solving, reasoning and proof, connections, communication, and representations (NCTM, 2000) also need to be generated in order to construct a response to a mathematical writing prompt. Therefore the ambiguity of the binary category of *content and process* prompts led to a deeper investigation of the language features within the prompt. More specifically a linguistic analysis of the mathematical prompt was conducted in order to determine how these stems effect potential constructed responses or affordances of the prompts.

A linguistic analysis of the prompt stem led to the development of a taxonomy of the language used most often in the stems of the 98% of the *generic* prompts (see Appendix M). Fang and Schleppegrell (2010) noted that authors of textbooks base their prompts on a mood system in the form of making statements: (declarative mood), asking questions (interrogative mood), and issuing commands (imperative mood). Based on this interpretation, the mathematical prompt stems were divided into two sections of *questions* and *commands.* Then I identified the *type of question* and the *type of command*. The *type of question* was divided into four types: (1) how questions, (2) why questions, (3) what questions, (4) when questions. The *type of statement* category was divided into three types based on the stem language: (1) describe, (2) explain, (3) construct. Within each of the types are the different variations of the questions and commands used within the prompts.

The findings indicated that 203 prompts were categorized as *questions* and 254 prompts were categorized as *commands* (see Table 21). The total within these two categories was greater than the total number of prompts (*N*=430) due to the fact that 27 of the mathematical prompts had a stem (question or statement) in the beginning of the prompt and a stem (question or command) at the end of the prompt. The following prompt is an example of a mathematical prompt having two stems (in bold type font) in the form of a *question* and a *command*:

- Gina pays for an item that costs $6.23 with a $10 bill. **What is** the least number of coins and bills she could get as change? **Explain.**

These findings of a dual stem indicate the complexity students may encounter when having to answer both a question *while* providing an explanation to a command.

The analysis of the *type of question* indicates there were 13 variations of *how* questions, 11 variations of *why* questions, 9 variations of *what* questions and 2 variations of *when* questions. In the *type of command* category, findings indicate there were 3 variations of *describe* commands, 7 variations of *explain* commands, 7 variations of *construct* commands using *write*, *make* and *give* as stem words (see Appendix M).

A further analysis of the *types of question* category indicate the variations of *how* were the most common form of question stem. The second most common form of question stem were the variations of *why*. Even though the percentages were lower in the categories of *what* and *when*, students were also encouraged to construct responses to these forms of questions (see Table 21). In the *types of command* category, the most common command required the student to *explain* a response. The second most common command required the student to respond by the use of a construction to the command words of *write*, *give* and *make* (see Table 22).

Table 22

Number of Mathematical Prompt Stems of Generic Category

Question Stems	*n*
How	111
Why	64
What	26
When	2
Total	203

Table 22 (*continued*)

Command Stems	
Explain	174
Describe	30
Write	48
Give	1
Make	1
Total	254

The results of the analysis of prompt stems indicated a multitude of question and command stem variations for students to decipher in order to construct a response. As the students construct a response to mathematical prompts, they must also consider processes such as problem solving, reasoning and proof, communication, connections and representations flexibly while utilizing mathematical vocabulary and symbols. Strategically, problem solving strategies such as pattern recognition, working backwards, guess and test, experimentation/simulation, reduction/expansion, organized listing/exhaustive listing, logical deduction, and divide and conquer (Krulik & Rudnick, 1995) should also be implemented during the construction process of the prompt. Furthermore, mathematical process and problem solving strategies should also incorporate the structures of writing during composition. Fang and Schleppegrell (2010) note literacy structures of listing, description, explanation, sequence, compare/contrast, cause/effect, and problem/solution are encouraged in writing and reading within the content areas. The projected constructed response of the generic prompt should utilize mathematical process standards while integrating mathematical strategies and literacy structures. For example, in order for a student to construct a response to a problem, many of the problem solving processes can be used simultaneously (such as reasoning and proving) while making connections and representations. Additionally, problem solving strategies such as *pattern recognition* and *logical deduction* can also be utilized while implementing the literacy structures of *descriptions and sequences*. This interwoven, recursive process of the complex nature of integrating writing in the mathematics content area can be found in the form of a model in Appendix M.

Affective Prompt

Only *Everyday Mathematics* had prompts coded within the *affective* category. These types of prompts require students to construct an answer that is associated with an attitude or feeling about mathematics. According to Dougherty (1996), these types of prompts provide a more holistic view of how students view mathematics. The following prompt was coded as *affective* from the *Everyday Mathematics* textbook:

- What are some things you have enjoyed on the World Tour?

The prompts coded as *affective* were located in a section titled *World Tour.* This section infused the content area of social studies within the *Everyday Mathematics* student textbook. Although words specific to the domain of mathematics were not located in these prompts, the prompts were coded as *affective* because they included language indicating a feeling or attitude. Additionally, these prompts were located in the student edition of the *Everyday Mathematics* textbook.

Narrative Prompt

Everyday Mathematics also had the majority of prompts coded *narrative.* These prompts were coded in a lesson section entitled, “My Country Notes,” and were related to touring a country. More specifically, the prompts asked questions such as, “what types of clothes should one pack when visiting a favorite capital?” or “why a particular country was chosen to visit?” Therefore, all of the prompts coded in this section were further classified as *real world* and not *imaginary*. In addition, only one prompt (<1%) was located in this category of the framework within the *enVision MATH* textbook. Figure 26 provides more information regarding the percentages calculated within this category of the framework.

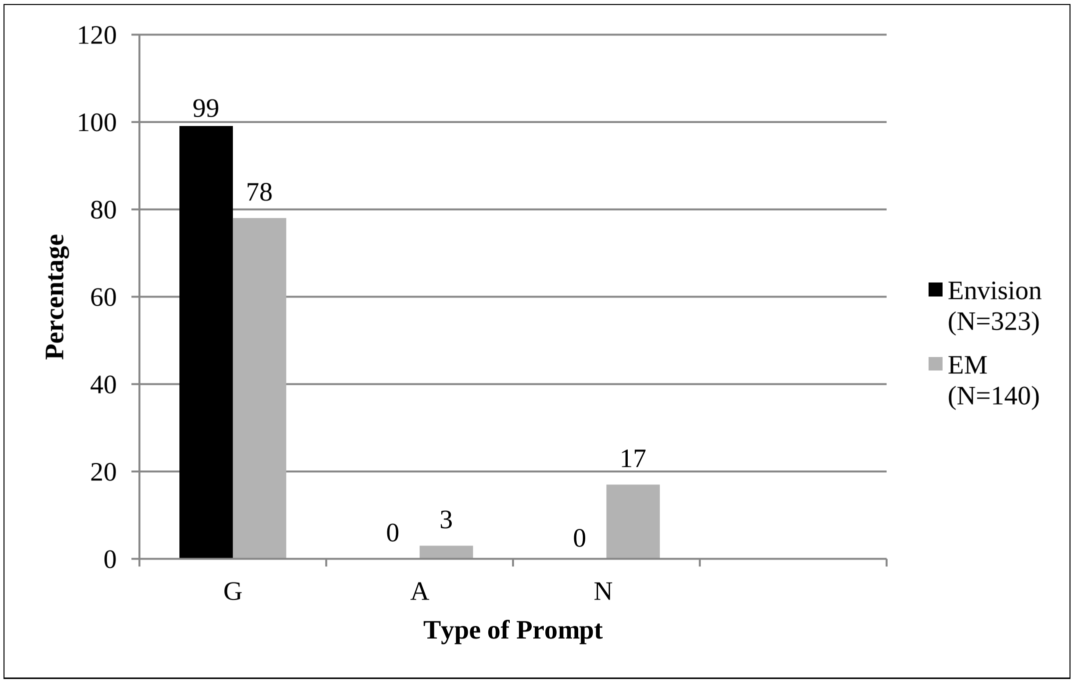


Figure 26. Percentage of the types of prompts in the *enVision MATH* and *Everyday Mathematics* (EM) textbook.
Note. G = Generic; A = Affective; N = Narrative

Other Framework Categories

Although the framework was designed specifically to align to the research questions (see Appendix A) by examining the nature of writing in two mathematics textbooks, the additional categories of *teacher edition* and *student edition* assisted in providing another layer of analysis regarding the prompts. Exploration of the teacher edition enhanced the research questions by providing information on how the writing prompts were supported from an instructional standpoint. In addition, an examination of the writing prompt location in the student edition also had instructional implications. Information regarding the sections and subsections and additional subsections of where the prompts were located in the student edition provided information of how *enVision MATH* and *Everyday Mathematics* situate writing in mathematics.

Teacher edition. The category of *Teacher Edition* was comprised of four sections. The first section labeled *support* provided the teacher with support only. This support was in the form of a phrase such as a sentence/s or a paragraph of instructional guidelines or building content knowledge. This section did not provide a sample or example student response.

The second category of *sampl*e indicated the teacher edition provided support in the form of a sample or example student response. The teacher edition did *not* provide support regarding the writing prompt. Rather, the teacher's edition *only* included a *sample* or example for purposes of instruction. The teacher had to rely on her own experience in teaching writing in mathematics. Although student responses can take various forms, only one sample answer was given as a guide for instruction. A novice teacher or one who has low content knowledge in mathematics may find a one-sample response challenging from an instructional standpoint. The third category of *support with sample* included both support *and* a sample. The last category *no support or sample* indicated no support or sample was provided in the teacher edition as support for the writing prompt.

As indicated in Table 23, the greatest percentage of instructional support for the writing prompts was coded as a *sample* category. This finding indicated that the teacher edition provided only a *sample* student response as the sole form of instructional support.

The teacher editions from *enVision MATH* and *Everyday Mathematics* provided instructional support in the form of a sample response for 427 of the writing prompts coded. Overall, 14 writing prompts had *no sample or support* in the teacher editions. The section of *support* accounted for 22 (16%) of the writing prompts coded in *Everyday Mathematics*.

Table 23

Type of Support in the enVision MATH and Everyday Mathematics (EM) Textbook.

	enVision MATH		*EM*	
Type of Prompt	*n*	%	*n*	%
Support (only)	0	0	22	16
Sample (only)	148	46	68	49
Support with Sample	170	53	41	29
No Support or Sample	5	2	9	6
Total	323	101	140	100

Teacher edition and textbook. The largest percentage coded in the domain of *teacher edition* can be found in the *sample* category of the framework. Over 75% of writing prompts identified within the *enVision MATH* and *Everyday Mathematics* textbooks had support for the writing prompts in the form of a sample response or example answer. Close to 100% of the prompts within the *enVision MATH* textbook were identified in the *sample* and *support with sample* category. Furthermore, the greatest difference between the two textbooks was in the *support* section. Both *enVision MATH* and *Everyday Mathematics* have support for over 90% of the writing prompts (see Figure 27).

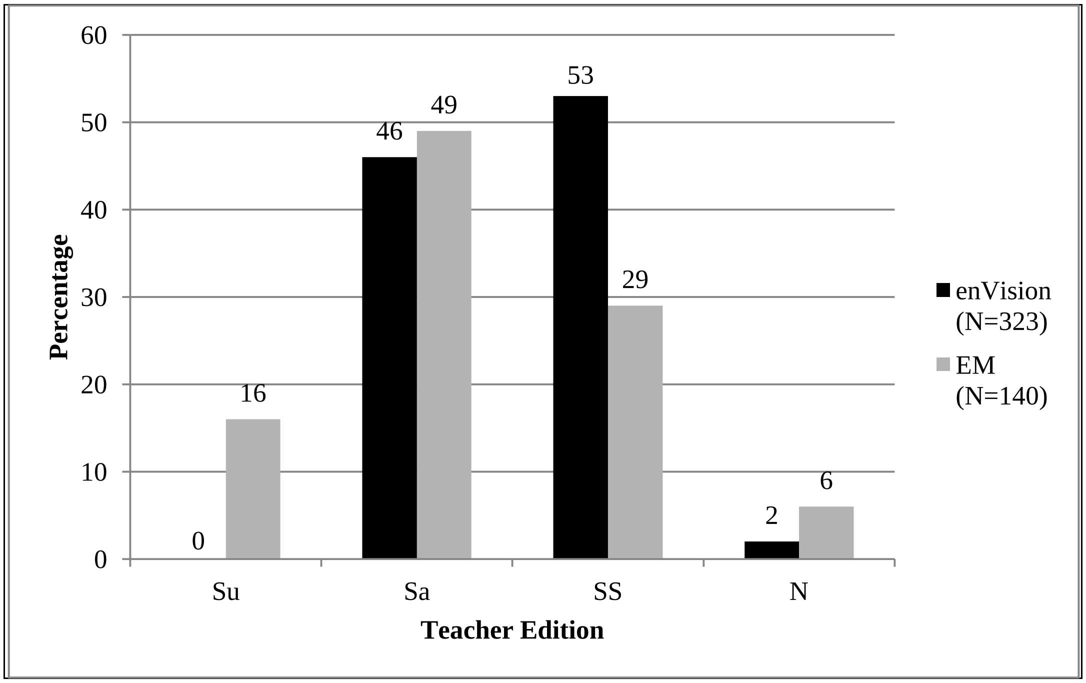


Figure 27. Percentage of types of support for the prompts within the Teacher Edition in the *enVision MATH* and *Everyday Mathematics* (EM) textbooks.
Note. Su= Support (only); Sa=Sample (only) ; SS=Support with Sample; N=No Support or Sample

Student edition. The domain section of *student edition* in the framework contained three sections titled: *section, sub-section,* and *additional sub-section.* The layout of the student editions of both textbooks varied greatly. Although the lesson numbers were close in range (*N*=20 and *N*=13) the number of section titles within these lessons differed to a great extent.

Student Edition and textbook. Upon analysis of the three categories within the dimension of *Student Edition*, the *enVision MATH* textbook had more coding in each of the categories than *Everyday Mathematics.* Because there were limited sub-sections or additional sub-sections located within the *Everyday Mathematics* textbook, the language was too complex and varied to analyze for patterns. Because each topic section had a different title, the language analyzed within the title provided no pattern for analysis; most every topic section title had a different heading using different language in *sections, subsections,* and *additional subsections*. (see Appendix N). Additionally, the language within the section titles of the *Everyday Mathematics* student textbook contained words specific to mathematics. Therefore a simple calculation of the amount of DSV was conducted within the sections of each lesson. Approximately 101 words were calculated to be DSV in *Everyday Mathematics* section titles of the student edition and 11 words in the section titles of the *enVision MATH* textbook.

Conversely, only the *enVision MATH* textbook provided data in this domain across all three categories for patterns in language in the section titles. Since there are titles in the sections, sub-sections and additional sub-sections, the analysis of the language within the titles of these categories revealed patterning. This patterning found in the language of the section titles allowed for a visual representation in the form of a graph to be developed. Figure 28 provides an example of *section*, *sub-section* and *additional sub-section* titles of the prompt location within the student edition of *enVision MATH*.

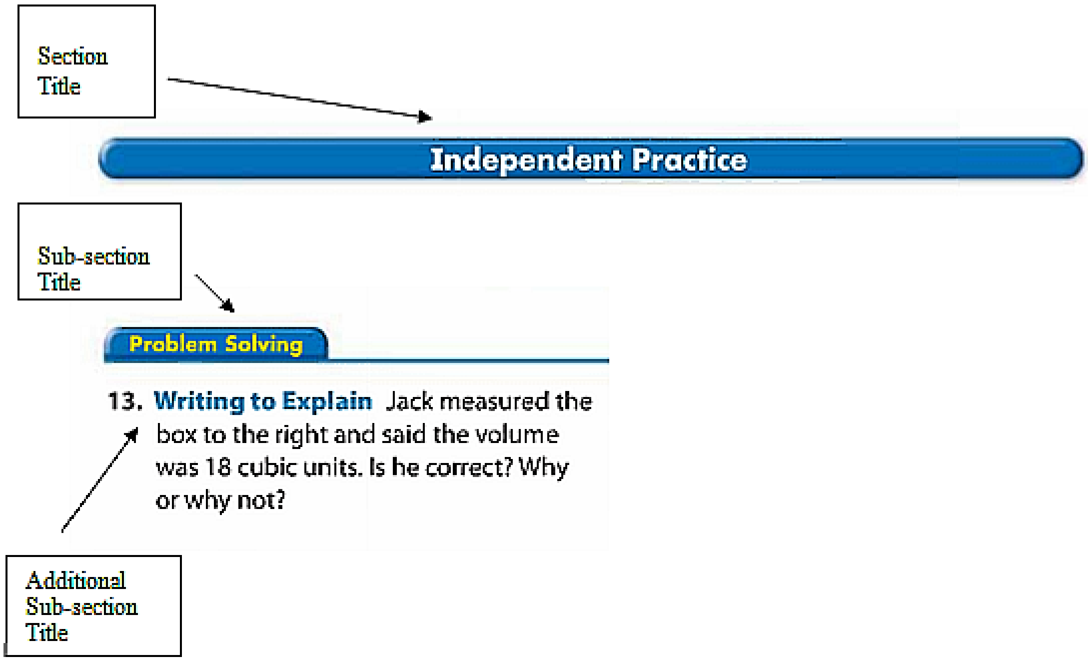


Figure 28. Example of “section titles” for a writing prompt within a student edition page.

As indicated in Figure 29, the largest percentage of writing prompts was located in the sections of *guided practice* and *independent practice.* The lowest percentages are in the *algebra*, *enrichment*, and *practice* sections.

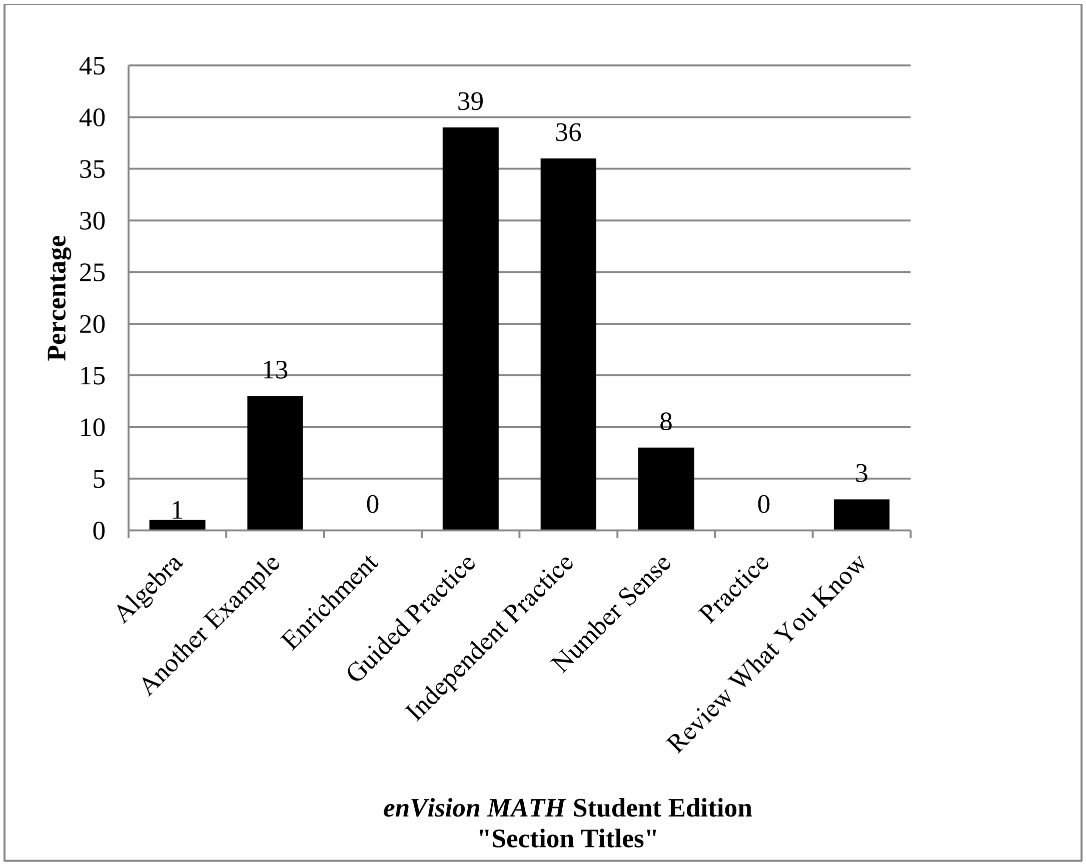


Figure 29. Percentage of prompts within the Student Edition "Section Titles" for the *enVision MATH* textbook.

The language within the titles of the *sections*, *sub-sections* and *additional sub-sections* illustrated different words were used more often than others. For example, the word *understand* was located 117 times in the sub-section or additional sub-section title of where the writing prompt was located in the student edition. The second highest percentage was the language *problem* or *problem solving.* The lowest number of writing prompt section titles had the word *reasoning* within the title of the section. A more detailed description of the percentages of the language within the section titles of the writing prompt location can be found in Table 24.

Table 24

Number of Category Language within the Student Edition for enVision MATH

	enVision MATH
Category Language	*n*
Writing	94
Understand	117
Explain	89
Reasoning	53
Problem/Problem Solving	100
Total *N* of Words	453

Cross Analysis

As revealed in the previous sections within this chapter, the analysis of prompts within the content strands revealed trends within the framework dimensions. As a result, I determined an additional analysis across the dimensions was necessary to provide a context for the findings of the individual strands. Therefore, using a matrix, I cross analyzed the results from my analysis of *content strand* categories (i.e., number sense, geometry, measurement, algebra, data analysis and other*)* with (1) the categories of *academic vocabulary* (i.e., domain specific vocabulary, general vocabulary, meta-language, and symbols), (2) *type of prompt* (generic, affective, and narrative) and (3) *teacher edition information* (i.e., support, sample, support with sample, and no support or sample). In order to determine if any patterns were revealed, simple calculations, using the data from each of the categories were used during the cross analysis. The findings from the matrix analysis are discussed in the following section.

Cross Analysis within *enVision MATH.*

Content Strand and Academic Vocabulary. Within the content strand of *number sense*, the matrix analysis revealed that symbols were the most frequent form of academic vocabulary used in *number sense* prompts. Approximately 43% of the academic vocabulary coded in *number sense* was comprised of *symbols*. Within the *geometry* content strand, the largest percentage of academic vocabulary was *domain specific vocabulary*. Approximately 54% of the academic vocabulary in geometry was classified as *domain specific vocabulary.* An analysis of the content strand of *measurement* was similar to *number sense* in that the largest percentage of academic vocabulary was coded as *symbols*. Within the *algebra* content strand, 33% of the academic vocabulary was coded as *symbols* and 35% was coded as *domain-specific vocabulary*. Within the content strand of *data/probability* the largest percentage (35%) was coded as *domain specific vocabulary* (see Table 25).

Content Strand and Type of Prompt. Findings in the content strand of *number sense* indicated 99% of prompts were categorized as *generic* prompts. Less than 1% of prompts in number sense were located in the *narrative* category. Furthermore, results indicated that 100% of the prompts in *geometry, measurement, algebra,* and *data/probability* were coded as *generic* prompts. There were no prompts coded as *affective* within the *enVision MATH* 4th grade textbook (see Table 25).

Content Strand and Teacher Edition Prompt Support. The cross analysis of content strand with teacher edition revealed the most common form of support for *number sense* prompts was both *sample* and *support with a sample*. Approximately 49% of the support was in the form of a *sample* and 48% was in the form of *support with a* *sample*. The largest percent of teacher-edition prompt support for *geometry*, *measurement*, *algebra* and *data/probability* was coded as *support with a sample* (see Table 25).

Table 25

Cross Analysis Percentage of Content Strand to the Framework Dimensions within the enVision MATH Textbook.

enVision Math	Academic Vocabulary				Type of Prompt			Teacher Edition			
Content Strand	DSV	GV	ML	S	G	A	N	Su	Sa	SS	N
Number Sense		(*n*=1421) [a]				(*n*=215)			(*n*=215)		
%	26	6	25	43	99	0	<1	0	49	48	2
Total No.	(364)	(80)	(360)	(617)	(214)	(0)	(1)	(0)	(106)	(104)	(5)
Geometry		(*n*=359)				(*n*= 55)			(*n*= 54)		
%	54	6	24	15	100	0	0	0	31	68	0
Total No.	(193)	(24)	(87)	(55)	(55)	(0)	(0)	(0)	(17)	(37)	(0)
Measurement		(*n*=286)				(*n*=47)			(*n*=46)		
%	28	5	27	39	100	0	0	0	36	63	0
Total No.	(82)	(15)	(77)	(112)	(47)	(0)	(0)	(0)	(17)	(29)	(0)
Algebra		(*n*=190)				(*n*=28)			(*n*= 28)		
%	33	4	27	35	100	0	0	0	43	53	3
Total No.	(63)	(8)	(52)	(67)	(28)	(0)	(0)	(0)	(12)	(15)	(1)

Table 25 (*continued*)

enVision Math	Academic Vocabulary				Type of Prompt			Teacher Edition			
Content Strand	DSV	GV	ML	S	G	A	N	Su	Sa	SS	N
Data/Probability	(*n*=194)				(*n*=35)			(*n*=35)			
%	35	10	31	24	100	0	0	0	42	54	3
Total No.	(68)	(20)	(60)	(46)	(35)	(0)	(0)	(0)	(15)	(19)	(1)

Note. In the Academic Vocabulary category, DSV = Domain Specific Vocabulary; GV = General Vocabulary; ML = Meta-language; S = Symbols. In the Type of Prompt category, G = Generic; A = Affective; N=Narrative. In the Teacher Edition category Su= Support only; Sa=Sample only; SS= Support with Sample; N=No Support or Sample.

[a] The total number within each domain included in this analysis may exceed the previously stated totals because some prompts were coded in more than one content strand. This additional coding was based on the language within the prompt and/or the lesson or section title in the textbook. If a prompt had language that was used and identified within two content strands, the prompt was coded in both content strands. However, the total number of prompts in each textbook remains the same for *enVison Math* (*N*=323) and *Everyday Mathematics* (*N*=140) and the total number of academic vocabulary words remains the same (*enVision Math* (*N*=2,157) and *Everyday Mathematics* (*N*=843).

Cross Analysis within *Everyday Mathematics.*

Content Strand and Academic Vocabulary. Within the content strand of *number sense,* the matrix analysis revealed that *symbols* were the most frequent form of academic vocabulary coded in the *number sense* prompts. Approximately 39% of the academic vocabulary coded in *number sense* was comprised of *symbols*. Within the *geometry* content strand, the largest percentage of academic vocabulary was *domain specific vocabulary*. Approximately 43% of the academic vocabulary in geometry was coded as *domain specific*. An analysis of the content strand of *measurement* was similar to *number sense* in that the largest percentage of academic vocabulary was coded as symbols. Approximately 45% of the academic vocabulary in *measurement* was coded as *symbols.* The *algebra* content strand was similar to *number sense* in that the largest percentage of academic vocabulary was coded as *domain specific*. Approximately 45% of the words coded in the *algebra* strand were coded as *domain specific*. Within the *data analysis/probability* content strand, 39% were coded as domain specific and 36% were coded as *meta-language*. Therefore the *data analysis/probability* were only separated by a 3% difference. The final category of *other* indicates that 72% of the prompts were coded as *meta-language* (see Table 26).

Content Strand and Type of Prompt. Findings in the content strand of *number sense* indicated 97% of prompts are categorized as *generic* prompts. Furthermore, results indicated that close to 100% of the prompts in *geometry*, *measurement*, *algebra,* and *data/probability* were coded as *generic* prompts. In the category of *other,* 75% of the prompts were coded as *narrative* and 17% were coded as *affective* (see Table 26).

Content Strand and Teacher Edition Prompt Support. The cross analysis of content strand with teacher edition revealed the most common form of support for *number sense, algebra* and *data analysis/probability* prompts. Approximately 63% of the support was in the form of a *sample* in the *number sense* category, 64% in *algebra,* and 53% in *data analysis/probability*. The largest percentage of teacher edition support for the content strand of *geometry* was in the form of *support with sample*. Within the *measurement* content strand 44% was coded as *support with sample* and 41% were coded as *sample.* The largest percentage of teacher edition prompt support for the category of *other* was coded in the *support* category (see Table 26)

Table 26

Cross Analysis Percentage of Content Strand to the Framework Dimensions within the Everyday Mathematics Textbook.

Everyday Mathematics	Academic Vocabulary				Type of Prompt			Teacher Edition			
Content Strand	DSV	GV	ML	S	G	A	N	Su	Sa	SS	N
Number Sense		(*n*=609)[a]				(*n*=75)			(*n*=75)		
%	23	23	14	39	97	0	3	1	63	28	8
Total No.	(142)	(142)	(85)	(240)	(73)	(0)	(2)	(1)	(47)	(21)	(6)
Geometry		(*n*=110)				(*n*= 18)			(*n*= 23)		
%	43	2	37	17	94	0	6	0	48	52	0
Total No.	(48)	(2)	(41)	(19)	(17)	(0)	(1)	(0)	(11)	(12)	(0)
Measurement		(*n*=272)				(*n*=32)			(*n*=32)		
%	30	5	19	45	97	0	3	6	41	44	9
Total No.	(81)	(14)	(53)	(124)	(31)	(0)	(1)	(2)	(13)	(14)	(3)
Algebra		(*n*=77)				(*n*=14)			(*n*= 14)		
%	45	5	31	18	100	0	0	0	64	35	0
Total No.	(35)	(4)	(24)	(14)	(14)	(0)	(0)	(0)	(9)	(5)	(0)

Table 26 (*continued*)

Everyday Mathematics	Academic Vocabulary				Type of Prompt			Teacher Edition			
Content Strand	DSV	GV	ML	S	G	A	N	Su	Sa	SS	N
Data/Probability		(*n*=77)				(*n*=15)			(*n*=15)		
%	39	6	36	18	93	0	7	7	53	27	13
Total No.	(30)	(5)	(28)	(14)	(14)	(0)	(1)	(1)	(8)	(4)	(2)
Other		(*n*=149)				(*n*=29)			(*n*=29)		
%	15	11	72	1	7	17	75	69	7	17	7
Total No.	(23)	(16)	(108)	(2)	(2)	(5)	(22)	(20)	(2)	(5)	(2)

Note. In the Academic Vocabulary category, DSV = Domain Specific Vocabulary; GV = General Vocabulary; ML = Meta-language; S = Symbols. In the Type of Prompt category, G = Generic; A = Affective; N=Narrative. In the Teacher Edition category, Su= Support only; Sa=Sample only; SS= Support with Sample; N=No Support or Sample.

[a.] The total number within each domain included in this analysis may exceed the previously stated totals because some prompts were coded in more than one content strand. This additional coding was based on the language within the prompt and/or the lesson or section title in the textbook. If a prompt had language that was used and identified within two content strands, the prompt was coded in both content strands. However, the total number of prompts in each textbook remains the same for *enVison Math* (*N*=323) and *Everyday Mathematics* (*N*=140) and the total number of academic vocabulary words remains the same (*enVision Math* (*N*=2,157) and *Everyday Mathematics* (*N*=843).

Chapter 5: Discussion

International assessment results regarding U.S. students in mathematics are discouraging. For example, the 2007 Trends in International Mathematics Study (TIMSS) reported that only 10% of U.S. fourth graders and 6% of U.S. eighth-graders performed at or above the advanced international benchmark level in mathematics (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008). In an attempt to address the low performance of U.S. students, recommendations within standards documents were developed upon the premise of teaching for "depth not breath" (ASCD, 1997).

Various organizations have supported these recommendations through the development of standards-based documents such as the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (NTCM, 2000), the National Research Council's mathematics proficiency strands (NRC, 2001), and the Common Core State Standards' *Standards for Mathematical Practice* (CCSS, 2010). From the review of these standards and the literature, it is clear that the process of writing is important in mathematics instruction. NCTM (2000) notes: "Writing in mathematics can help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson" (p. 61).

A review of relevant literature also revealed that many researchers focus on communication in mathematics for teaching and learning (Burns, 2004; McIntosh &

Draper, 2001; Pugalee 2004, 2005; Senk & Thompson, 2003; Shulman 1986). More specifically, writing is reported to have many benefits, such as providing a window into student thinking (Baxter et al., 2005; Bolte,1997; Herbel-Eisenmann, 2007), while providing teachers with information regarding planning for instructional purposes (Aspinwall & Aspinwall, 2003; Baxter, et al., 2005). Moreover, writing is a vehicle to support students' problem solving processes (Alvermann, 2002; Bereiter & Scardamalia, 1987; Evans, 1984; Parker, 2007; Sowder, 2007) because it supports metacognition (Brewster & Klump, 2004; Fequa, 1997; Powell, 1997; Pugalee, 1997, 2001; Scheibelhut, 1994). Futhermore, Writing to Learn (WTL) is based upon the premise of writing for learning (Brewster & Klump, 2004; Elbow & Sorcinelli, 2006; Forsman, 1985; Langer & Applebee, 1987; Nuckles, et al., 2010; Nagin 2003; Vygotsky, 1962). Writing also provides an avenue to facilitate conversation (Bakhtin, 1986; Baxter, 2001; Dyson, 1992, 1993; Englert, Mariage, & Dunsmore, 2006; Vygotsky, 1978).

The importance of writing in mathematics, the pervasiveness of the textbook as the dominant teaching tool (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al 2008), and the limited research regarding how writing prompts are supported in mathematics textbooks provided the rationale for this inquiry. Therefore, the purpose of this study was to examine writing prompts in two widely used mathematics textbooks: The fourth grade versions of *enVision MATH* published by Pearson Education, Inc. and the third edition of mathematics texts developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. I selected two textbooks with different educational philosophies in order to understand how writing was incorporated in NSF-funded and publisher-generated textbook curricula.

I developed an analytic framework using 10 dimensions with respective sub-categories based on (1) NCTM's *Principles and Standards for School Mathematics* content strands, (2) Baumann and Graves's (2010) classification scheme of academic vocabulary, and (3) research in mathematics writing prompt types (Burns, 2004; Dougherty, 1996; Urquhart, 2009; Whitin & Whitin, 2000) (see Appendix A). Using the framework as a way to record the data, I calculated the number of writing prompts per page, the number of tasks per page, page number, and the wording of the prompt. Then I further coded the prompt to determine the academic vocabulary and the total number of words and symbols (coded and words not on list). I also coded the type of prompt, features of the teacher edition that provided prompt support, and student edition prompt location (see Appendix A).

In addition, I developed the framework to answer each of the following research questions:

1. How many writing prompts are included in one 4th grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?
2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?
3. What types of vocabulary are used in the writing prompts in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?
4. What types of prompts are provided in one 4th grade NSF-funded mathematics textbook and one publisher-generated textbook?

Based on my analysis of these two textbooks, there are six major findings related to my research questions and these are explicitly discussed in the following sections.

1. The Questionable Focus on Number Sense

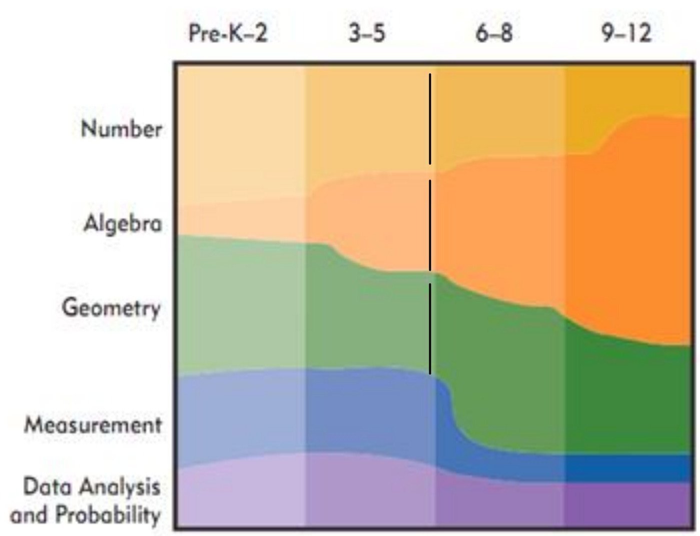
The NCTM *Principles and Standards for School Mathematics* (NCTM, 2000) indicate the following discrete content strands: *number sense, geometry, measurement, algebra, data analysis.* To categorize writing prompts by content strand, I used the language in the lesson title and within the prompt as well as the topic language listed in NCTM's (2000) content strands (see Tables 1-5). Furthermore if the language within the prompt was not connected to a particular content strand, the code of *other* was used.

In both textbooks, most of the writing prompts were coded in the *number sense* category. This finding indicates that the majority of prompts are related to the following content: *place value*, *base ten number system*, *whole* and *negative numbers, decimals, fractions, percents*, *factors, multiplication, division, addition, subtraction* and *estimation of numbers*. For example, *enVision MATH* had approximately 56% of prompts located within *number sense* while *Everyday Mathematics* had approximately 38% of prompts in this category. On average, approximately 50% of the prompts in both textbooks were in the strand of *number sense*. Given the evidence that mathematical thinking and problem solving are crucial in mathematics development (Cobb, 1986b; Cobb, Yackel, & Wood, 1989; Confrey, 1987; Thompson, 1985; von Glasersfeld, 1983), it seems contradictory that the preponderance of prompts focused on *number sense* rather than other mathematical content. The answer to this question, I believe, is two-fold: standards documents and state assessments.

Standards documents and state assessments. The high percentage of prompts coded in the *number sense* strand aligns to the National Assessment of Educational Progress (NAEP, 2005) framework. The NAEP framework, which was developed to assess students' mathematical thinking at the national level, includes a majority of *Number and Operations* tasks for 4th graders (National Assessment Governing Board, 2008). Additionally high-stakes state assessments also have a majority of *number sense* tasks on their assessments. For example, Florida, Texas, and California collectively represent about 25% of the total national market in textbook adoption (Tyson, 1997). Interestingly, Florida and Texas state assessments also have the majority of tasks in the *number sense* category, according to the Florida Comprehensive Assessment Test (FCAT), Test Design Summary (2009), Texas Assessment of Knowledge and Skills (TAKS), Blueprint for Grades 3-8 Mathematics (2010).

One of the factors textbook publishing companies use to develop content within the textbooks relates to standard documents (Reys & Reys, 2006). Additionally, standards documents drive the content on state assessments. NAEP and two of the three states with the largest market share in textbook adoption have the largest percent of assessment items in the *number sense* strand.

However, an evaluation of the PSSM (NCTM, 2000) regarding the focus of the various content strands per grade level indicates a balanced approached for the content strand of *number*, *algebra* and *geometry* at the end of the grade level band 3-5 (see Figure 30). Because the content strands for the grade level band 3-5 appear to have an equal focus, shouldn't the strands of *number, algebra,* and *geometry* have similar percentages of writing prompts instead of the majority of prompts located in the strand of number sense?



(PSSM, Executive Summary, 2000, p. 4)

Figure 30. Emphasis of the content standards across the grade bands.

Number sense as constrained skill. If the reason for the emphasis on number sense is related to standards and textbooks, then the reason is not a mathematical one given the need for students to develop mathematical thinking in geometry and algebra (Battista, 2007; Moses & Cobb, 2001a; Paul, 2003;). For example, according to Clements and Sarama (2007) early childhood and primary grades *number and operations* is arguably the most important area in mathematics learning and one of the best developed areas in mathematics research (p. 466). However these claims are only relevant to children in *early childhood* and *primary grades.* Although number sense in the middle and high school grades encompasses important content such as whole numbers, fractions, decimals, percents, proportions, and integers and number theory (NCTM, 2000), students in the intermediate grades are also encouraged to develop mathematical skills and strategies in other content areas such as algebra. This focus on other content strands is in preparation for future success in mathematics. For example, *algebra* appears to have significant importance and has been identified as the "Gate-Keeper" for future success beyond the early grades school mathematics curriculum (Stinson, 2004). Additionally, Moses and Cobb (2001a) noted that the content associated with Algebra possesses gate-keeping power for college mathematics.

> In support of this finding (as cited in Stinson 2004, p. 11) Algebra is the "gateway" to advanced mathematics and science in high school, yet most students do not take it in middle school (U.S. Department of Education, 1997, p. 5-6). Furthermore, students who enrolled in algebra as eighth-graders were more likely to reach advanced mathematics courses (e.g., algebra 3, trigonometry, calculus). Additionally students who enrolled in algebra as eighth graders and completed an advanced math course during high school were more likely to apply to a four year college than those eighth-grade students who did not enroll in algebra as eighth-graders but who also completed an advanced math course during high school (U.S. Department of Education, 1999, p. 1-2).

The continued emphasis on number sense through the intermediate grades appears to be analogous to the inappropriate practice of focusing on lower-level skills in the field of literacy. Scott Paris described the following, "In general, letter knowledge, phonics, and concepts of print are highly constrained, phonemic awareness and oral reading fluency are less constrained, and vocabulary and comprehension are least constrained" (2005, p. 187). These skills are "constrained" in that "skills such as alphabet knowledge are most related to decoding in early childhood, whereas unconstrained skills such as vocabulary are related to a wide range of academic skills throughout life" (p. 188).

Although phonics is an integral part of emergent reading, the continued instruction of phonics can potentially hinder the analysis of reading comprehension skills (Dennis, 2012, Dennis & Parker, 2010; Paris, 2005). Could this analogy to constrained skills in literacy align to the heavy focus of number-sense instruction in the intermediate grades and potentially constrain mathematical skills such as measurement, algebra, and geometry in preparation for middle school and beyond? Shouldn't intermediate students communicate by way of reasoning, problem solving, and justifying thinking while also utilizing the process skills of connecting and representations? As a potential solution and as an attempt to provide more of a balance in the types of writing tasks across content strands, teachers could modify the writing tasks (when applicable) by changing the language in the prompt to utilize vocabulary and processes within the other content areas. The modification of textbook writing tasks to facilitate more of a balance in other mathematics content areas will require training in the use of the teacher edition, mathematics vocabulary, and writing strategies and processes.

Implications for teachers. The suggestion to modify writing prompts has implications for teacher training programs for both inservice and preservice teachers.

The topic of number sense is promoted with the NCTM (2000) *Principles and Standards for School Mathematics* in grades K-12. Therefore, the importance of number sense concepts is acknowledged throughout the upper grades. However, the large portion of writing tasks in the content strand of number sense is a concern regarding the importance of other mathematical content areas such as algebra and the gate-keeping components of mathematics (Stinson 2004). The large portion of writing prompts in this area may be seen as a type of constraint for mathematical thinking in other content areas. An attempt to address this concern is the modification of writing prompts in mathematics textbooks to include domain specific vocabulary associated with other mathematics content areas such as geometry, algebra, etc. This modification of prompts could provide more of a balance to facilitate writing within other mathematical content areas. However, the revising of prompts would require the implementation of educational training programs. The implications for teacher educators and professional development is to assist preservice and inservice teachers in identifying where the writing prompts are located in the curriculum and then to modify or develop further prompts for instruction in the different content strands. Regardless of the textbook scope and sequence, teachers can locate writing prompts in the lesson and modify the language and vocabulary to meet the expectations of upcoming content if there are no writing prompts within the lesson *or* if the number of writing prompts are minimal. This information has the potential to provide insight to the field of mathematics by investigating how this type of knowledge could assist preservice and inservice teachers in identifying prompts that are suitable for their instructional goals.

Content strand summary. The need for students to encounter writing prompts across content areas is an important consideration for textbook publishing companies, teacher education programs and professional development. First, writing provides students with an opportunity to solidify their thinking by reflecting on their work and clarifying their thoughts while utilizing vocabulary and the language needed to communicate effectively (NCTM, 2000; O'Connell & O'Connor, 2007; Rubenstein & Thompson, 2002; Thompson & Chappell, 2007). For example in the prompt below, students could explain a process such as reasoning while utilizing the vocabulary needed to construct a response:

- Why do you think a square can also be called a rectangle, but a rectangle cannot be called a square?

An answer to this prompt could provide teachers with evidence of students' mathematical understanding because their writing offers teachers a window into their thinking (Sowder, 2007). In addition, the teacher could have information regarding the use of metacognitive processes (Pugalee, 2001) during the construction of an answer to the prompt. Writing provides a window into the acquisition of the vocabulary and language needed to develop a written response.

2. The Importance of Concept Development Through Mathematical Vocabulary

In order to communicate effectively in mathematics, language is important as students use specified content vocabulary. To understand the type of vocabulary needed to construct a response to a mathematical prompt, the language within the identified writing prompt was investigated. Based on this investigation, the domain of *academic vocabulary* was developed to encompass four categories (based upon a modified classification scheme developed by Baumann and Graves, 2010) derived from the most recent work on typologies of academic vocabulary (Fisher & Frey 2008; Harmon, Wood, & Hendrick, 2008; Hiebert & Lubliner, 2008). Four of the five categories were adapted from the Baumann and Graves (2010) classification scheme: *domain specific vocabulary* (DSV) included words specific to mathematics only; *general vocabulary (GV*) indicated words that appeared reasonably frequently within and across academic domains. The words could be polysemous, with different definitions being relevant to different domains. *Meta-language* was the term used to describe words associated with processes, structures, or concepts commonly included in content area texts. *Symbols* was the term for mathematical notation. The fifth category of Literary Vocabulary was not relevant to my study and therefore was not used in the classification scheme.

An additional analysis across the dimensions of the framework was conducted to provide a context for the findings of the individual content strands within the framework. The use of a matrix assisted in the cross analysis of the *content strand* categories (i.e., *number sense, geometry, measurement, algebra, data analysis* and *other)* with (1) the categories of *academic vocabulary*,(i.e., domain specific vocabulary, general vocabulary, meta-language, and symbols), (2) *type of prompt* (generic, affective, and narrative) and (3) *teacher edition information* (i.e., support, sample, support with sample, and no support or sample). Descriptive statistics, using the data from each of the categories, revealed some interesting patterns. The framework and cross analysis of the dimensions indicated important findings associated with conceptual development and academic vocabulary.

For example, the highest percentages of academic vocabulary within the *enVision MATH* and *Everyday Mathematics* textbooks were coded as *symbols* and *Domain specific vocabulary (DSV).* In other words, across all math prompts, mathematical symbols (e.g. +, -, %) and Domain specific vocabulary (e.g. rhombus, meter, prism) appeared most frequently. The cross analysis also supported this finding of symbols and domain specific vocabulary having the largest percentage of vocabulary within each of the mathematics content strands. Because the majority of mathematics writing prompts for the elementary grade levels were coded within the concept of *number sense*, it is important to note the types of vocabulary most often encountered within these prompts. The high percentage of academic vocabulary containing symbols in the writing prompts aligned to the notion of *symbols* being the hallmark of mathematics (Thompson & Rubenstein, 2001). As such, the complexity of writing a response to a prompt with symbols could require students to read the symbol, interpret the symbol, and then use the symbol in the prose if needed. As Tall (1993) found, mathematics symbols can evoke a process or a concept. For example the following statements are samples of mathematics problems whereby symbols were used and interpreted in two ways:

- 3+2 is either the process of addition of 2 and 3 or the concept of sum.
- $\frac{3}{4}$ can mean (amongst other interpretations) the process of division of 3 by 4 or the concept of fraction $\frac{3}{4}$.
- +2 denotes the process of shifting 2 units to the right and also the concept of a signed number of +2 (p. 2).

The possibility of two or more processes or concepts within the prompt increases the difficulty level of reading and interpreting prompts as well as the process of interpreting and using symbols in mathematics. In a separate issue, the high percentage of domain specific vocabulary in geometry, algebra, and data analysis/probability prompts could also alter the requirements on students by involving not only symbols but words that are specific to the domain of mathematics. These words are content specific (Hiebert & Lubliner, 2008; Jetton & Alexander, 2004) and generally not used outside of mathematics. Additionally these terms are also noted as technical terms (Fisher & Frey, 2008; Harmon et al., 2008) with low frequency of use (Beck, et al., 2002, 2008). Approximately one-third of the total numbers of words analyzed within the prompts of both textbooks were coded as highly technical complex vocabulary such as symbols and domain words. In addition to symbols, domain words such as scale, outlier, divide, triangle, mode, and median, were located in the prompts.

Instructional implications. The instructional implications regarding the use of vocabulary acquisition in mathematics are paramount. Teacher education courses and professional development in mathematics education should consider the integration of vocabulary strategy instruction (Murray, 2004; Thompson & Chappell, 2007; Thompson & Rubenstein, 2000, 2007, Rubenstein 2007) and literacy (Allen 2007; Beck, Frey & Fisher, 2008; 2009; McKeown & Kucan, 2002, 2008; Marzano, 2004). Given the vocabulary knowledge required for students to answer writing prompts, textbook publishing companies should consider including some of the best practices in vocabulary instruction in their Teacher Editions. For example, publishers could implement a "professional development" segment within the Teacher Edition or possibly as a supplemental guide for strategy instruction within this area focusing on the area of symbols. This type of support would assist instruction regarding students' ability to transmediate, or interpret, one sign system to another (words to signs/diagrams or signs/diagrams to words). This type of guide would include literacy strategies in vocabulary instruction coupled with word lists.

3. Word lists as Instructional Resources

The academic vocabulary within the writing prompts was identified using a priori word lists (Baumann & Graves, 2010; Coxhead, 2000; Fry & Kress, 2006; Marzano & Pickering 2005). For example, Domain specific vocabulary (DSV) was identified using the Marzano and Pickering (2005) *Building Academic Vocabulary Teacher's Manual* word list whereby 7,923 terms in 11 subject areas were extracted from national standards documents. These lists contain content-specific words organized into four grade-level intervals where 86 of the terms were specific to the domain of mathematics. General vocabulary (GV) was located using the Coxhead (2000) *Academic Word List* based on terms that were most often found in academic texts. Additionally, the terms under the category of meta-language were based on Marzano and Pickering's (2005) *Building Academic Vocabulary Teacher's Manual*. These word lists detailed content-specific vocabulary organized into four grade-level intervals. Additionally, these terms were specific to mathematics writing prompts that have the potential to facilitate writing.

The symbols were identified using Baumann and Graves (2010) definitions of non-conventional words such as icons, emoticons, graphics, mathematical notations, electronic symbols, and so forth. Furthermore, the *Reading Math Symbols Word List* developed by Fry and Kress (2006) was also used to determine the classification of a *symbol.*

Each of the lists mentioned above was then transferred into an Excel document for ease of locating academic vocabulary. Words that had the potential to be considered academic vocabulary based on the definition of the different types of vocabulary were scanned in the Excel word list document to determine the appropriate coding. If the term was not located in any of the lists, then the possible derivative or association of the term was considered. However, if the word, the association, or derivative was not located in the word lists, but the word had the potential to be considered academic vocabulary, it was placed in the *words not on list* dimension. Examples are provided below.

Derivatives. During the co-rating session of this study, the co-raters missed a few words because the co-raters were not familiar with word derivatives and associations for certain academic vocabulary. For example, the term *multiplication* is in the DSV list. However this term has derivatives of *multiply, multiplied, multiplier, multiple,* etc. If the term multiply was encountered, it should be coded as DSV because it is a derivative of multiplication. However, my co-raters missed these terms. Due to my familiarity with the lists, I was able to help my co-raters identify some of the derivatives of terms they missed.

Associated Terms. Additionally, words that were not only derivatives of academic vocabulary but *associated* with academic vocabulary were not included in these lists. As a result, many terms that should have been coded were labeled as *words not on list*. For example, the term *day* is found in the DSV list. However, the actual days of the week, *Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday*, are not located in the *DSV* category. Therefore because of the word structure, these word associations were coded into the *words not on list* word list.

The words included in the *words not on list* dimension *should* be in the a priori word lists but were not. For example, the terms *gallon*, *dollar, milliliter*, and *trapezoid* are vocabulary that *should* be included in the DSV list but were not. Furthermore, the word lists including process words in the *meta-language* category should also be updated. This category had the majority of words indicated in the *words not on list* category. The words *answer* and *know* are not in the meta-language word list but were located on multiple counts in the writing prompts. For example, the word *answer* was located 71 times and the word *know* was located 50 times within the writing prompts. These words are vocabulary associated with a process that students need to know in order to construct a response. Therefore, these words *should* be included in the *meta-language* word list.

This word list provides information regarding the type of words that need to be included in newly revised academic vocabulary lists.

Improving the specificity of word lists. The academic vocabulary word lists should be updated and revised to include different derivatives and word associations of vocabulary needed in order to communicate mathematically. These derivatives and associations of words have the potential to create abstract meanings. For example, Jetton and Shanahan (2012) used the terms *nominalization* to describe how mathematical operations such as *add* or *divide* are turned into *addition* and *division* but have completely different meanings. Veel (1999) noted that it is possible for a student to be able to divide but not know the concept of division. The transition from knowing *how* to add or divide versus the *conceptual understanding* of addition or division are processes that may need to be deciphered when constructing a response to a mathematical prompt. These content and process words are vocabulary that teachers need to know for instruction and students need to acquire for communication purposes.

Word lists provide an opportunity for teachers to understand the depth and breadth of the vocabulary, and subsequently, the concepts of all the different derivatives and associations necessary for thinking mathematically. In addition, word lists can be used during the composition process as a student aid. Similar to the popular literacy Dolch Word list, which complied words that need to easily be recognized in order to achieve reading fluency (Dolch, 1936), a mathematics word list based on achieving mathematical literacy per content strand is encouraged due to the ambiguity of the mathematical language used in the prompts.

4. Ambiguity of Prompts

I used the categories of *affective*, *narrative*, and *generic* to code the types of prompts textbook publishers utilized in two mathematics textbooks. An *affective* prompt is one that has language that elicits an opinion, feeling or attitude towards math (Baxter et al., 2007, Shield & Galbraith, 1998). A *narrative* writing prompt requests the writer to construct an answer that displays math content in imaginary or real world sense. Narrative math content is encouraged in the field of mathematics as an instructional tool and supported through the use of children's literature (Burns, 2004; Rubenstein & Thompson, 2002; Shiro, 1997, Thompson, 1997; Whitin & Whitin, 2000). The final category of *generic* prompt is inclusive of all of the prompts that were *not* coded as *affective* or *narrative*.

Generic prompts. The *generic* prompt category accounted for 93% of total prompts within both textbooks. According to the research in mathematics writing, these generic prompts were classified as either *content* or *process* prompts (Dougherty, 1996; Urquhart, 2009). For example, I coded the following *enVision MATH* prompt as *generic* as it required the students to utilize both *processes* and *content* in order to construct a response:

- Can a circle and a square ever be congruent? Why or why not? (p. 454).

Similarly, the following prompt from *Everyday Mathematics* also requires the student to use both *content* and *process* skills:

- Feng said the name of the angle is ∠SRT. Is he right? Explain (p. 8).

For both of the constructed responses, the content of geometry is required. In addition, the process skill of explaining the answer and justifying the response is required. Therefore, the use of both content and process skills was required for all of the generic prompts.

Questions or commands. Given that content or process prompts were not mutually exclusive categories, I conducted a linguistic analysis of the prompt stems. A further analysis of the prompt stems indicated that prompts fell in the category of questions or commands (Fang & Shleppegrell, 2010). Within the stems analyzed in the *generic* category, there were multiple variations of questions and statements providing yet another dimension of complexity in constructing an answer. For example, in the "*How Question*" section, 13 types of question stems using the word *how* were recorded: *how can you, how would you, how could you, how could, how would, how does, how did, how can, how many, how are*, *how is,* and *how*.

For purposes of instruction, teachers need to keep in mind that students will need to process the command and/or question while devising a response that uses language structures of listing, description, explanation, sequence, compare/contrast, cause/effect, and problem/solution (Fang & Shleppegrell, 2010). Furthermore the student will need to incorporate problem solving processes such as pattern recognition, working backwards, guess and test, experimentation/simulation, reduction/expansion, organized listing/exhaustive listing, logical deduction and divide and conquer (Krulik & Rudnick, 1995) while integrating mathematical processes of problem solving, reasoning and proof, communication, connections and representation. Clearly, what seems like a simple prompt can mask a series of complicated mathematical processes that are made more complicated through the prompt's linguistic structure.

Rhetorical structures as affordances in mathematics questions. An investigation of the *prompt affordance* provides an understanding of the interwoven recursive process of integrating literacy structures with mathematical strategies and processes in order to construct an answer to a mathematical prompt (see Appendix N). The implications for teacher education and professional development strongly encourage the use of best practices in the process of writing while incorporating problem solving strategies in mathematics. Although this claim may be easily stated, the difficulty of teaching writing in this context provides a challenge. For example, Hill and Resnick (1995) state:

> Most writing instructors today realize that the most difficult part of any real writing task is analyzing a complex rhetorical situation and deciding what combination of writing strategies would stand the best chance of accomplishing the writer's purposes within that situation (p. 146).

Because of the rhetorical affordance regarding the various process and strategies to be utilized by the student, the written response to a writing prompt *could* be completed in various forms. The implication of various responses could potentially affect instruction. Bazerman (2008) calls for *rhetorical specification* whereby the focus of writing is delimited by the structure of language and the audience or purpose for the writing task. For example, the prompt below and the possible answers illustrate the various responses based on the language used by the individual:

- Why is $\frac{1}{4}$ less than $\frac{1}{2}$?

1) I know $\frac{1}{4}$ is less than $\frac{1}{2}$ because when comparing fractions that have a 1 in the numerator, you can look at the denominator. The larger the number in the denominator, the smaller the fraction.

2) $\frac{1}{4}$ is less than $\frac{1}{2}$ because if I had a whole cookie and cut the cookie into fourths and took one piece, it would be smaller than if I had a whole cookie and cut it into two pieces and took 1 piece.

3) I know $\frac{1}{4}$ is smaller than $\frac{1}{2}$ because if you use a number line and divide the number line into fourths, $\frac{1}{2}$ is equal to $\frac{2}{4}$ and $\frac{2}{4}$ is greater than $\frac{1}{4}$.

These three answer constructions are completely different. For example, the first answer deals with the concept of numerators and denominators regarding size, the second answer portrays the concept of whole, and the third answer involves equivalent fractions. Although all three are correct, what if the teacher has a different response in mind? Should the student have to guess what that particular answer *could* be? The student's guessing work is especially complicated with the prompt stem, "how would you…?" This potential mismatch regarding the rhetorical analysis of what the teacher and student potentially have in mind as a response to a prompt provides important instructional implications for mathematics teacher educator coursework, inservice professional development and textbook publishing companies.

Because these mathematics prompts may have some overlapping meanings regarding the affordance of the prompt and the process the writers should undertake in order to answer the prompt, the topic of strategy instruction should be addressed. For example, the Self Regulated Strategy Development (SRSD) model (Graham, 2006) has been noted as an effective approach for students to develop their mathematics writing based on the following areas: Develop Background Knowledge, Describe It, Model It, Support It, and Independent Use. This model affords students an opportunity to learn writing strategies used by highly skilled writers. Strategies such as planning, drafting and revising are maintained through the use of self regulating components (i.e., goal setting, self assessment, self instruction, self reinforcement, and imagery) as students progress through a series of six stages. Because mathematics prompts afford opportunities for the use of strategies and structures in both mathematics and reading, this type of model seems useful.

Instructional implications. The coursework for preservice teachers and professional development for inservice teachers should encompass instruction that is geared toward building content knowledge and pedagogical content knowledge in mathematics. More specifically, teachers will need content knowledge of the mathematics concepts and the pedagogical knowledge of how students learn mathematics. For example, Sowder (2007) explains Grossman's (1990) important components for preservice teachers and professional development programs emphasizing mathematics content knowledge and pedagogical content knowledge below:

1) an overarching knowledge and belief about the purposes for teaching (mathematics);

2) knowledge of students' understandings, conceptions, and potential misunderstandings (in mathematics);

3) knowledge of (mathematics) curriculum and curricular materials, and

4) knowledge of the instructional strategies and representations for teaching particular topics (in mathematics) (p. 164).

Furthermore, inadequate knowledge of important mathematical ideas can lead to "missed opportunities for fostering meaningful connections between key concepts and representations" (Borko & Putnam, 1995, p. 44). More specifically, if teachers are going to use writing as a springboard for conversation, the importance of content knowledge and pedagogical content knowledge should be addressed in teacher education and professional development programs.

Additionally, training in the implementation of oral discourse strategies should be encouraged for the positive impact regarding the importance of engaging in conversation to solidify learning and facilitate writing. For example, Chapin, O'Connor and Anderson (2003) recommend discourse practices in order to facilitate conversation that supports the development of students' reasoning and students' abilities to express their thoughts clearly:

1) implementing talk moves that engage students in discourse;

2) facilitating the art of questioning;

3) using student thinking to propel discussions;

4) setting up a supportive environment; and

5) orchestrating the discourse.

In addition to these practices, Stein, Engle, Smith, and Hughes (2008) propose the Five Practices Model whereby the teachers' role is to:

1) anticipate student responses to challenging mathematical tasks;

2) monitor student's work on and engagement with the tasks;

3) select particular students to present their mathematical work;

4) sequence the student responses that will be displayed in specific order; and

5) connect different students' responses and connect the responses to key mathematical ideas.

Facilitation of writing prompts for purposes of discussion provides an opportunity for teachers and students to learn important mathematics content while enhancing the benefits of social interaction for learning.

Many mathematics educators and researchers view mathematics instruction as a social interaction process. For example, Steele (2009) notes the findings from Cobb, Yackel and Wood (1991) that support children's opportunities to talk about their mathematical understanding. Students construct a more powerful way of thinking about mathematics through social interactions with a more knowledgeable person (p. 211). This knowledgeable person has the potential to be the teacher. In order for teachers to facilitate this type of environment where various responses are accepted for the same prompt, a thorough knowledge of the content should be acquired. This acquisition of knowledge in the form of professional development can also be conducted through the use of the Teacher Edition. For example, although textbooks are acknowledged as the dominant tool in the mathematics classroom for what is taught, they also have the value of providing professional development within their content (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008).

5. Teachers' Editions

I specifically analyzed the teacher editions of the two textbooks to provide insight as to the type of written support teachers receive regarding prompt instruction. I examined each textbook for *support, sample, support with sample,* and *no support or sample* to gather data to the corresponding prompt coded in the student edition. If the prompt was coded in *support* then some form of directional support was provided to the teacher without a sample response. The category of *sample* identified prompts that only had support in the teacher edition in the form of a student sample response. The category of *support with sample* categorized prompts that had support in the teacher edition in the form of support with a sample student response. The final category of *no support or sample* signified that the teacher edition provided no support for the prompt.

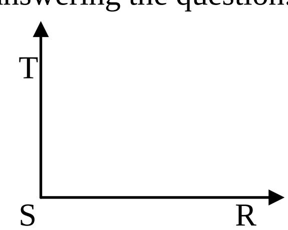
Support and sample responses. Two of the most salient findings regarding the *teacher edition* are related to the *support* and *sample* categories. Both *enVision MATH* and *Everyday Mathematics* had the majority of prompts coded in the *sample* and *support with sample* categories. In other words, a majority of prompts in the teacher edition provided the teacher with a sample and the teacher editions in both textbooks provided only *one* sample student response. This structure is problematic given the fact that a majority of the prompts are written in a way for students to construct a variety of responses based on the multiple interpretations of the prompt.

Additionally, further analysis of the *support* category provided information that the teacher editions are also limited regarding support for the prompt. For example, the support was not in the form of directions because the type of support did not provide teachers explicit information regarding how to teach writing through the prompts nor did they provide information to the teachers of the various forms of sample responses. Although the *no support or sample* category had the lowest number of prompts recorded for teacher support, this finding provides information that some of the prompts had *no* support at all. Furthermore, coding in this area implies that the teacher is left to his/her own discretion regarding instruction on the prompt. The novice teacher or one with low content knowledge in mathematics may find writing prompts coded in the area *no support or sample* a challenge to teach. However, after further examination, the ambiguity of the prompt affordance leaves the mathematics educator at a potential standstill regarding instruction. Although the teacher edition provided one sample response as the most common form of support the dilemma of how we treat these prompts in mathematics education remains a question.

This data is unsettling. The limited support for writing instruction in the teacher edition provides a key implication for textbook publishing companies. In an effort to address the ambiguity of prompts, textbook publishing companies could change the language within the prompts to be more specific. For example the second bullet in the following prompts are examples of prompts that have been modified from the original version to provide clarity:

- Can a circle and a square ever be congruent? Why or why not? (p. 454).
- List the differences between a circle and a square.

The second of these prompts is more specific in requesting the process of developing a list as a strategy for answering the question.



- Feng said the name of the angle is ∠SRT. Is he right? Explain (p. 8).
- List the different ways of naming the angle above? Explain your reasoning.

The second of these prompts has more specific language of using a list and an explanation in the form of reasoning to name the angle in all of the correct formats (i.e., $\angle$TSR and $\angle$RST).

Instructional support

Instructional support through the modification of the prompts has implications for professional development and teacher education programs. Changing the language of the prompt also has the potential to differentiate instruction in mathematics. Furthermore, attention to the amount of academic terms within the prompt has the potential to affect the cognitive level within the prompt. Therefore, the specificity of the language within the prompts could impact the layout of the teacher edition so that it encompasses professional development components. For example using the model developed for prompt strategy instruction (see Appendix N) publishers could select language in the prompt by using one or more of the following: 1) reading structures, 2) mathematical problem solving strategies and/or 3) process skills during the development of the prompt. The following are examples of four prompts using language that is more specific in order to eliminate some of the ambiguity of responses.

1) $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}$ - What pattern do you notice in the following set of fractions? Write the answer in a sequence.
2) Explain how $\frac{1}{2}$ is greater than $\frac{1}{4}$ by comparing and contrasting.
3) Name a fraction that is greater than $\frac{3}{4}$. Justify your answer by using guess and test.

4) Pick any three fractions in the box above and order from least to greatest. Next, pick one of the strategies listed in the strategy box to explain how you know your answer is correct.

These four prompts were developed in an instructional type of hierarchy. For example, the first problem relates to the patterning of fractions, the second relates to comparing and ordering fractions which is a little more complex than noticing a pattern. The third problem now asks the student to select a fraction larger than the one indicated. The request of justifying an answer using a guess and test will indicate that the student *should* select a few fractions to determine the correct solution, and the fourth problem allows the student to use fractions of choice and a strategy of choice. Furthermore, a student should not progress to the next problem in the sequence if there is an indication the problem cannot be solved. This type of formative assessment would provide a window into student thinking allowing for the teacher to assign tasks that are more complex based on the language or remediation before the next task in the textbooks can be attempted.

This type of hierarchy is based on Norman Webb's (2002) three levels of cognitive complexity in mathematics tasks. For example, Level 1 mathematics items include the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. Level 2 mathematics items require students to make some decisions as to how to approach the problem or activity and Level 3 mathematics items require reasoning, planning, using evidence, and a higher level of thinking. The writing tasks mentioned are similar to these complexity levels whereby instruction would benefit by progressing through the levels in a type of hierarchy. This progression would inform instruction similar to a “triage” manner regarding intervention and enrichment.

Teaching as triage. The teacher edition *could* provide support for the teacher using the metaphor of *triage*. For example, if the student can answer the first problem then he or she is ready to construct a response to the succeeding problems. Furthermore, the teacher edition can guide the teacher with prescriptions for intervention as needed. As one will notice, the last problem (4) allows the student to select from a menu of options in both content and process. This type of student selection indicates the importance of self selected topics during writing instruction (Bereiter & Scardamalia, 1986).

Curriculum and professional development. Designers of curriculum are encouraged to not only adhere to reform recommendations but to also provide professional development for the instruction of writing prompts. This type of support is needed regarding the complexity of writing in mathematics and the imperative focus of standardized constructed items in the near future regarding national assessments. As stated in the PARCC Item Development correspondence:

> Designers of curricula assessments and professional development must all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. Separating the practices from the content is not helpful and is not what the standards require. The practices to do not exist in isolation; the vehicle for engaging in the practices is mathematical content (p. 45).

As a result, instructional programs for integrating writing in mathematics should be developed with the elements of literacy structures, mathematical strategies and mathematical processes. Instruction regarding how to reflexively move from each element is encouraged as writing is a complex process. In support of a new paradigm for writing instruction in mathematics, Moje (2008) notes:

> We need to consider the larger contexts in which strategies are drawn up and the practices that various strategies support. It may be most productive to build Disciplinary literacy instructional programs rather than merely encourage content teachers to employ literacy teaching practices and strategies (p. 96).

Additional research in these areas should be encouraged in order to fully implement writing in mathematics with success.

Types of curriculum: intended versus implemented. The *intended curriculum* is represented by goals and directives set forth in standards documents and policy, as well as their appearance in the teacher edition. The *implemented* curriculum is what actually is taught in the classroom (Schmidt et al., 2000; Valverde et al., 2002). Valverde et al. (2000) note:

> The inclusion of a learning goal in the intended curriculum does not guarantee that it will be covered. Including an intention as a goal does not guarantee that the opportunity to attain that goal will actually be provided in the classroom but does greatly increase the probability that it will (p. 8).

Within this study, other influences could have a potential impact on what is implemented by the teacher and encountered by the student. However, these influences were not analyzed. Tarr, et al. (2008) note teacher knowledge and beliefs have the potential to impact the implemented curriculum. Although textbooks are acknowledged as the dominant tool in the mathematics classroom for what is taught, they have the value of providing professional development within its content (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008).

6. Student Edition

The total number of exercises within the *enVision MATH* textbook (*n*=2481) was more than the number of exercises in the *Everyday Mathematics* textbook (*n*=704). Although more writing prompts were coded within the *enVision MATH* textbook (*n*=323) than the *Everyday Mathematics* (*n*=140) the percentage of writing prompts was higher for *Everyday Mathematics* (20%) than *enVision MATH* (13%).

The analysis also illustrated how the potential opportunity for learning was impacted by the number of exercises within the two textbooks. For example, one can speculate that because *Everyday Mathematics* has fewer exercises the chances are increased that the writing prompts will be addressed during instruction of the lesson. Winfield (1987) notes that opportunity to learn may be measured by "time spent in reviewing, practicing, or applying a particular concept or by the amount and depth of content covered with particular groups of students" (p. 439). Fewer exercises for review, practice, and application *may* increase the chance of writing tasks being selected for depth of content. Conversely, conventional wisdom regarding the benefits of “choice” might be appealing; however, a large number of exercises *may* decrease the opportunity for the students to encounter the writing prompt as its selection is due to teacher decision. Although writing prompt and exercise selection were not measured in this study, future research should investigate the impact of choice. Is it the case that more choices do not equate to quality of instruction. If teachers have fewer exercises to select from are the chances of encountering each of those tasks increased?

In addition, the student edition could parallel the notion of "triage" as mentioned above. The student edition can include tasks that are colored or structured in such a way as to indicate their importance and their difficulty. With such a structure, students can self-select the tasks based on their instructional needs.

Limitations

The study has several limitations. The first set of limitations relates to the generalizability of findings. The sample was small; therefore the findings may not apply to other textbooks series or to other materials within the series studied. Although the textbooks I selected were widely used, market share data does not provide information regarding the actual percentage of students using the textbooks in the United States. In addition, I selected textbooks that were published by two different textbook publishing companies having different educational philosophies. However, the sample consisted of only two textbooks.

The second set of limitations relates to the reliability and validity of the findings in the analytic framework developed. Although inter-rater reliability was calculated, threats to reliability in the training and execution of the coding of the prompts may exist. This is especially relevant with the dimension of academic vocabulary and the word derivatives and associations. Coder fatigue may also be present because 10% of both student editions in the *enVision MATH* and *Everyday Mathematics* textbooks yielded a large number of prompts to be analyzed across the framework dimensions.

To ensure reliability of the framework, one doctoral student and one Ph.D. literacy researcher coded 10% of the lessons. Two types of reliability were calculated regarding the framework (These percentages of agreement are reported in Chapter 3.)

The first measure consisted of the percentage of agreement in choosing the same tasks as writing prompts. The second measure consisted of the percentage of agreement in choosing the same codes across framework dimensions.

The validity of the framework refers to how accurate the framework measures important features of writing prompts. A thorough review of extant literature regarding writing in mathematics coupled with reform recommendations provided direction regarding the development of the dimensions and categories across the framework. Although there were many forms of prompt affordances, only the prompts that provided a potential construction of more than a one-word answer were used for analysis in my framework.

Recommendations for future research

Aligned with reform efforts in mathematics instruction, new assessment tools based on two assessment consortia will require students to construct responses to literacy rich mathematical prompts as part of a national assessment in the near future. More specifically Shaughnessy (2011) noted:

> The Partnership for the Assessment of Readiness for College and Career (PARC) and Smarter-Balanced Assessment Consortium (SBAC) have obtained federal grants to development assessment tools, both formative and summative, to assess students' proficiency with the content and practices specified in the Common Core State Standards for mathematics (CCSSM) by the start of 2014 (NCTM Summing It Up, para.1).

Currently, states must decide which assessment consortia to adopt. Regardless of the states' selection, both consortium will have students constructing a response to a mathematical prompt as a measure of ability. Within this vein, mathematical literacy to including instruction in mathematical writing will be recommended. Results from my study coupled with the high stakes demand of writing in mathematics provide valuable information regarding five projected areas for future research.

The first area for future research would be to identify the different varieties of cognitive demands of writing prompts based on the language and vocabulary used in the prompts. Identifying if prompts are low level or high level in complexity according to Norman Webb's Depth of Knowledge levels (Webb, 2002) ratings would inform the field of mathematics regarding the differentiation of writing tasks for instruction. Based on this information, writing task language could have the potential to be modified in order to increase the level of complexity or lower the level of complexity.

The second area for future research would be to include within an analytical type of framework coding for the graphics combined with the writing prompts. Identification of whether or not a graphic was used in the teacher edition could provide useful information regarding transference of information as another issue of complexity in composing a written construction.

The third area for future research would be to analyze student responses to mathematical writing prompts. Identification of the language within the prompts correlating to the language within the constructed response could have major instructional implications in the area of vocabulary.

The fourth area for research aligns to the social aspect of writing. Observations of teacher and student oral discourse surrounding the constructed responses could be a valuable contribution to the field of mathematics. For example, the types of responses from students' explanation of answers, and teacher questioning could provide the field of mathematics with information regarding the conversation "moves" that facilitate writing in mathematics.

Along the lines of teacher questioning, the final area for future research would be in the area of teacher instruction. Data regarding how teachers use the prompts and what teachers are really assigning in writing prompts would be worth knowing. For example, using mathematical writing prompts at the beginning, middle or end of a lesson would also inform teachers regarding the most appropriate application of mathematical writing prompts based on the goals of the lesson or teacher. The final area for future research would be how teachers can use analytic rubrics more effectively in the classroom for written responses in mathematics.

Conclusions

The majority of extant literature related to writing in mathematics has given limited attention to the treatment of writing in mathematics textbooks especially in the elementary grades. This study explored writing prompts in two different textbooks: a publisher generated textbook and an NSF-funded textbook. I developed an analytic framework to analyze the language of writing prompts. This study was not developed to determine which textbooks were best at supporting writing in mathematics. Rather the study was an attempt to provide an understanding of how writing in mathematics is promoted through the use of tasks that require a student to construct a response.

Writing in mathematics helps students solidify understanding through the use of the process strand of *communication*. As noted in the *Principles and Standards for School Mathematics*, (NCTM 2000):

> As students are asked to communicate about mathematics they are studying—to justify their reasoning to a classmate or to formulate a question about something that is puzzling—they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about mathematics. (p. 63).

Similarly, the *Curriculum Focal Points* (NCTM, 2006) also support the use of writing in mathematics through the implementation of reasoning, justification and communicating. Additionally, the NRC developed interrelated strands for mathematical proficiency integrating the use of writing. Further recommendations through the CCSS also support the use of writing within the *Standards for Mathematical Practice* (CCSS, 2010).

This study was developed to inform the field of mathematics how textbooks support these reform recommendations of writing in mathematics through an investigation of writing prompts. Additionally, textbooks are known to have an influence on classroom instruction since they are used often as instructional tools (Ball & Cohen, 1996). An investigation of the prompt affordances through an analysis of the vocabulary and language used in the mathematical prompt stems provided salient discussion regarding the complexity of instruction and composition in this area and the implications for instruction and textbook publishing companies.

Although prompts relating to *number sense* were recorded as the largest strand category in both textbooks, the other strands *should* be acknowledged in writing. Students need to become familiar with the vocabulary used when constructing responses to prompts in other mathematics content areas.

Research regarding best practices in vocabulary instruction relating to literacy should help inform the field of mathematics regarding the importance of integrating such strategies. Additionally, the a priori word lists should be updated and revised to include the different derivatives and word associations of vocabulary needed in order to communicate mathematically. These derivatives and associations of words have the potential to create abstract meanings.

The lack of support found in the teacher edition for these types of prompts is a clear indication that the area of teacher support for writing in mathematics needs to be reconsidered in the teacher editions. The first reason for this implication is that the complexity of the language of the mathematical prompts stems, coupled with the vocabulary, indicates these prompts are ambiguous in nature. The ambiguity of these prompts allows for various processes to be used therefore providing many opportunities for variety of responses.

Differences in the textbooks were also discussed. In light of the finding that the *enVision MATH* had more writing prompts coded, there were more overall exercises for students to encounter. The large amount of exercises in this textbook could affect what teachers choose to assign and instruct. If teachers are unfamiliar with the content and find the support lacking in the teacher edition regarding prompt directions, the writing prompts *may* be skipped. The omission of tasks, due to teacher selection, could affect students' potential opportunity to learn.

Because the mathematics textbook is researched as the dominant tool in classroom instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008), it was encouraging to find that textbook developers are adhering to reform recommendations of writing in mathematics. Although the textbooks explored are different in their philosophies, there were a few recommendations for both textbooks in order to improve student textbooks and teacher editions. These recommendations welcome the collaboration of literacy and mathematics researchers and experts in order to develop the instructional tools needed for successful implementation of writing in mathematics. Discussions centered upon the following five ideas would be constructive regarding the development of textbooks and instructional materials: 1) vocabulary used in the prompts and the types of vocabulary needed to facilitate potential response, 2) the multiple strategies and processes that could potentially be used by students in order to construct a response, 3) teacher development resources coupled with the teacher edition regarding the variety of prospective answers, 4) teacher development resources regarding prompt instruction using a triage approach, 5) development of a balanced number of writing prompts in all content areas. This collaborative union would benefit the fields of both literacy and mathematics.

Before we can begin to implement the process of writing in the mathematics classroom, a love for the discovery of mathematical knowledge through the mere act of communication should be embraced in all facets within the teaching and learning of mathematics.

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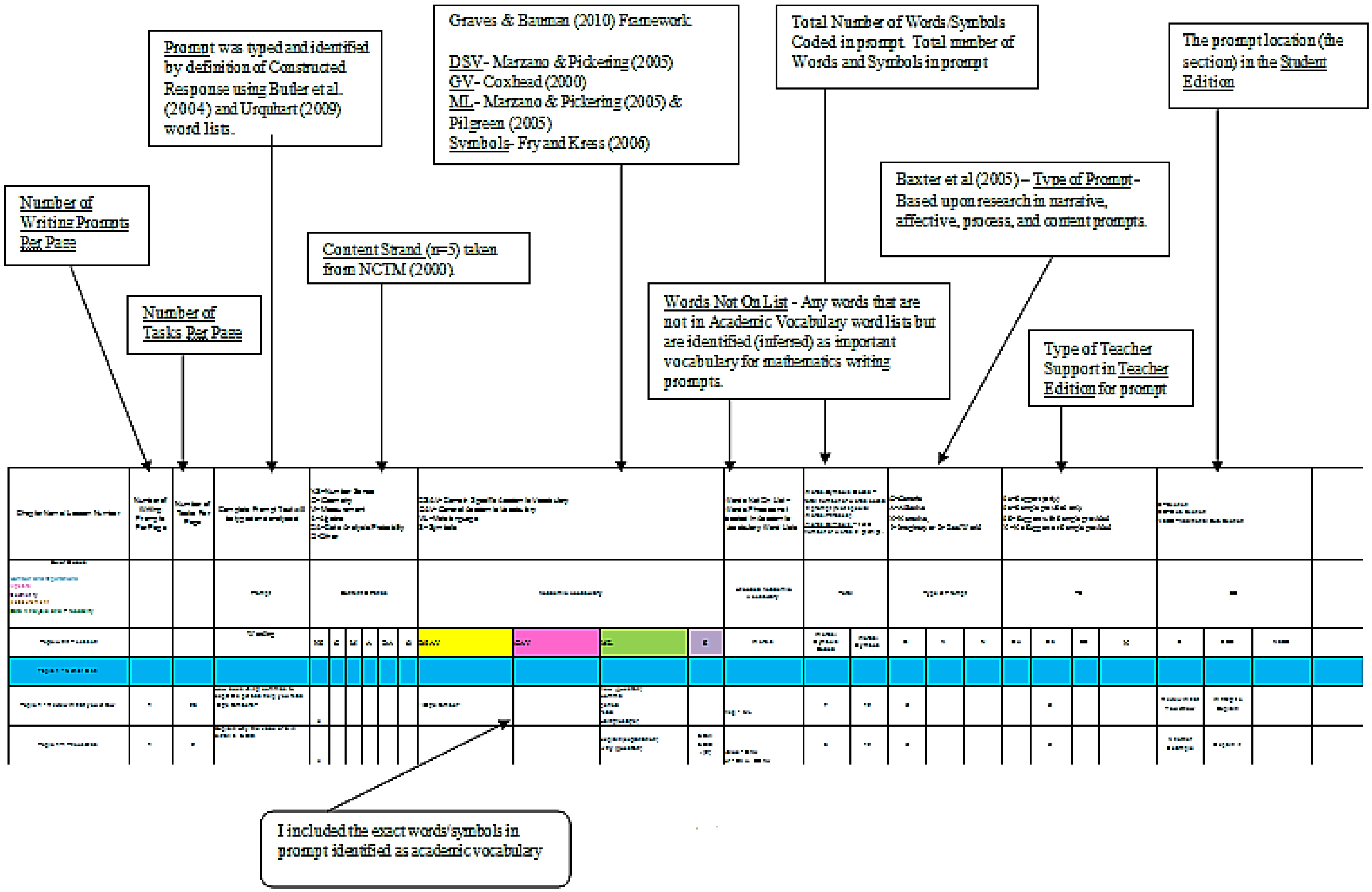
Appendices

Appendix A: Curriculum Analysis Framework



Appendix B: List of Terms for Identification of Prompts

Resource	Term (noun)	Term (Verb)
Butler et al. (2004)	Analysis	Analyze
	Classification	Classify
	Definition	Define
	Explanation	Explain
	Generalization	Generalize
	Hypothesis	Hypothesize
	Identification	Identify
	Justification	Justify
	Organization	Organize
	Prediction	Predict
	Synthesis	Synthesize
Urquhart (2009)	Description	Describe
	Narration	Narrate
	Reflection	Reflect
	Question	Question
	Summarization	Summarize

Appendix C: Pilot Framework (revisions in bold)

Question 1	Question 1	Question 1	Question 2					Question 3							Question 4			Additional Information				Additional Information		
Number of writing prompts per page	**Number of tasks per page**	Complete prompt text will be typed and analyzed.	NS=Number Sense G=Geometry M=Measurement A=Algebra DA=Data Analysis/Probability **O - Other**					DSAV=Domain Specific Academic Vocabulary GAV=General Academic Vocabulary ML=Metalanguage S=Symbols				**Special Words= words not found in Academic Vocabulary Word Lists/ Words Not On List**	Words/Symbols Coded = total number of words coded in prompt. Words/Symbols = total number of words in prompt.		PS=Problem Solving, A=Affective FN=Fictionalizing & Narratizing Math Content I=Imaginery or R=Real **World - Eliminate this and change to**			D=Directions provided only S=Sample provided only DS=Directions & Sample provided N=No directions or				S=Section SS=Sub Section AdSS=Additional Sub Section		
# of writing prompts	**# of tasks**	Prompt	Content Strand					Academic Vocabulary				Special Words	Total		Type of Prompt			TE				SE		
		Wording	NS	G	M	A	DA	DSAV	GAV	ML	S	Word/s	Words/ Symbols Coded	Words/ Symbols	PS	A	FN = I or R **N**	D	S	DS	N	S	SS	AdSS
1	**2**	Which digit in the number 13,872 would be changed to form 19,872? How would the value of 13,872 change?	x1					digit number value	change	form how	13,872 19,872		8	19	x				x			Learn		
2	**29**	Explain how to find the value of the digit 7 in the number 76,308.	x1					value digit number		explain how	7 76,308	find - ML	8	14	x				x			Check		
		If you add a ten thousands digit that is 2 times the ones digit to the number 2,794, what is the new number? Explain?	x1					add "ten thousands" digit ones		explain	2 2,794		8	22	x				x			Practice and Problem Solving	Write About It	
1	**21**	Explain how its period helps you identify the place-value of the digit 9 in 952,700. In 1969, the Apollo 11 astronauts traveled 952,700 miles.	x1					digit miles	period	explain how	9 952,700 1969 11		10	24	x				x			Check		
2	**21**	Ms. Diaz wrote the number 46,152,780. The answer is 6,000,000. What is the question?	x1					number		question	46,152,780 6,000,000	**wrote - GAV** **answer - GAV**	6	14	x				x			Practice and Problem Solving	What's The Question	
		Saturn takes about 10,760 days to orbit the sun. Is it correct to read this number as ten-million, seven-hundred sixty? Explain.	x1					number "ten million seven hundred sixty"		explain correct read	10,760		6	19	x				x			Practice and Problem Solving	Fast Fact and Science	
2	**10**	24,613,351 is one-million more than 14,613,351. Describe his error.	x1					"one million" "more than"	error	describe	24,613,351 14,613,351		6	8	x				x			Practice and Problem Solving	What's The Error	
	10	What does the place-value of a digit tell you? How does switching the positions of the digits in the number 52 affect that number's value	x1					digit numbers value	affect	tell how	52	**place value - DSAV** **switch - ML** **position - ML**	10	25	x					x		Practice and Problem Solving	Vocabulary Power	
1	**6**	In Example B, why is 4,000 not a reasonable number?	x1					number		why	4,000	**reasonable - ML**	4	10	x				x			Learn		
2	**8**	Explain whether the number of students in your class is a good benchmark for the number of students in your school.	x1					number benchmark		explain			3	21	x				x			Check		
		Explain when you use a benchmark number.	x1					benchmark number		explain			3	7	x				x			Practice and Problem Solving	Write About It	
Total	**Total**		**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Total**	**Tota**	**Total**	**Total**			
11	**186**		**11**	**0**	**0**	**0**	**0**	**27**	**4**	**17**	**17**	**7**	**72**	**183**	**11**	**0**	**0**	**0**	**10**	**1**	**0**			

Appendix D: Curriculum Analysis Framework Dimension Descriptions

Dimension	Categories	Abbreviations
Prompt	Wording of Prompt	Exact wording in prompt
Content Strand	Number & Operations	N
	Algebra	A
	Geometry	G
	Measurement	M
	Data Analysis & Probability	DA
	Other	O
Academic Vocabulary	Domain specific vocabulary	DSV
	General vocabulary	GV
	Meta-language	ML
	Symbols	S
Type of Prompt	Generic	G
	Affective	A
	Narrative	N
Teacher Edition (TE)	Support (only)	Su
	Sample (only)	Sa
	Support with Sample	SS
	No Support or Sample provided	N
Student Edition (SE)	Section	S
	Sub Section	SS
	Additional Sub Section	AdSS

Appendix E: Vocabulary Classification Scheme

Domain-specific academic vocabulary: The relatively low-frequency, content specific words and phrases that appear in content area textbooks and other technical writing materials.	Math: apex, bisect, geometry, polyhedron, Pythagorean theorem, scalene triangle Science: anticyclone, barometric pressure, dew point, isobar, meteorology, virga Social Studies: atoll, buttle, escarpment, geography, tectonic plate, terminal moraine	• Content-specific vocabulary (Hiebert & Lubliner, 2008) • Technical vocabulary (Fisher & Frey, 2008) • "Language" of academic domains (Jetton & Alexander, 2004) • Academically technical terms (Harmon, Wood & Hedrick, 2008) [15]Tier 3 words - (Beck, McKeown, & Kucan, 2002, 2008)	• Building Academic Vocabulary: Teacher's Manual (Marzano & Pickering, 2005) [all but the "English Language Arts" Word Lists • Adopted content area textbooks • Informational trade books • Internet sources
General academic vocabulary: Words that appear reasonably frequently within and across academic domains. The words may be polysemous, with different definitions being relevant to different domains.	Analyze, assume, code, conduct, context, document, error, link, minor, period, project, range, register, role, sum (all selected from Coxhead's 2000, list)	• General academic vocabulary (Hiebert & Lubliner, 2008) • Academic words (Coxhead, 2000) • General academic vocabulary (Townsend 2009) • Specialized vocabulary (Fisher & Frey, 2008) • [16]Tier 2 words. (Beck, McKeown, & Kucan, 2002, 2008)	• Coxhead's (2000) Academic Word List [www.victoria.ac.nz.lals/resources /wordlist/default.aspx]

[15] Tier 3 words are low frequency words that occur in specific domains.

[16] Tier 2 words have a high frequency of use with multiple meanings across different domains.

Meta-language: Terms used to describe the language of literacy and literacy instruction and words used to describe processes, structures, or concepts commonly included in content area texts.	Language of Literacy and instruction: epic, genre, glossary idiom, infer, interrogative, main idea, outline, sonnet, summarize, table of contents. Processes in Content Area Texts: calculate, compare, estimate, explain, investigate, model, observe, prove	• Academic language (Pilgreen 2007) • School-task vocabulary (Hiebert & Lubliner, 2008)	• Building Academic Vocabulary (Marzano & Pickering, 2005) [just the “English Language Arts” Word Lists] • “Academic terms for Books Parts” (Pilgreen, 2007, pp. 243-244) Pending…
Symbols: Icons, emoticons, graphics, mathematical notations, electronic symbols, and so forth that are not conventional words.	X-24, >, $A^2+B^2=C^2$, %, 0, ™, (o,o), $,	• Symbolic representations (Harmon, Wood, & Hedrick, 2008)	• Computer keyboard, online emoticons, Internet images, clipart, symbol-specific websites.

Note. From “What is Academic Vocabulary” by J.F. Baumann and Michael F. Graves, 2010, *Journal of Adolescent Literacy*, p. 9-10. Copyright 2010 by the International Reading Association.

Appendix F: Domain Specific Academic Vocabulary

2-dimensional shape
2-dimensional shape combination
2-dimensional shape decomposition
2-dimensional shape slide
2-dimensional shape turn
2-dimensional space
3-deminsional shape
3-deminsional shape cross section
3-dimensional shape
3-dimensional shape combination
above
absolute error
absolute function
absolute value
acceleration
acute angle
add radical expressions
addend
addition
addition algorithm
addition counting procedures
addition of fractions
algebraic expression
algebraic expression expansion
algebraic function
algebraic representation
algebraic step function
alternate interior angle
angle
angle bisector
angle measurement tool
angle of depression
angle unit
approximate lines
arc
area
area
area model
area of irregular shapes
area under curve
array
associative property
asymptote of function
axis of symmetry
bar graph
base 10
base 60
base e
basic number combinations
behind
below
benchmarking
between calendar
biased sample
binary system
bivariate data
bivariate data transformation
bivariate distribution
blue print
box & whisker plot
capacity
cardinal number
cartesian coordinates
categorical data
centimeter
central angle
central limit theorem
certainty (probability)
certainty of conclusions
chance
chord circle without center
circle
circle formula
circular function
circumference
circumference formula
classes of functions
classes of triangles
clock
cluster
coin
combination
combining like terns
common denominator
common fractions
commutative property
complementary angle
complementary event
complex number
complex problem
composite number
compound event
compound interest
conditional probability
confidence interval
congruence
conjecture
conjugate complex number
conservation of area
constant
constant difference
constant rate of change
constant ratio
continuity
continuous probability distribution
control group
convert large number to small number
convert small number to large number
coordinate geometry
coordinate plane
coordinate system
corner
correlation
corresponding angles
corresponding sides
cosine
counter example
counting procedure
critical paths method
cube
cube number
cube root

cubic unit
curve fitting
curve fitting median method
cylinder
data
data cluster
data collection method
data display error
data extreme
data gap
data set
day
decibel
decimal
decimal addition
decimal division
decimal estimation
decimal multiplication
decimal subtraction
decreasing pattern
deductive argument
deductive prediction
defining properties of shape/figures
density
dependent events
derivation
diagram
difference
different size units
dilation
dilation of object in a plan
direct function
direct measure
direction
discrete probability
discrete probability distribution
dispersion
distance
distance formula
distributive property
divide radical expressions
dividend
divisibility
division
domain of function
elapsed time
empirical verification
english system of measurement
enlarging transformation
equal ratios
equation
equation systems
equilateral triangle
equivalent forms
equivalent forms of equations
equivalent forms of inequalities
equivalent fractions
equivalent representation
estimate answer
estimation
estimation of fractions
estimation of height
estimation of length
estimation of width
even numbers
event likelihood
expanded notation
expected value
experiment
experimental design
experimental probability
exponent
exponent
exponential function
exponential notation
extreme value
faces of a shape
factorial
factorial notation
factors
fair chance
fibonacci sequence
finite graph
flip transformation
foot (measurement)
force
formal mathematical induction
formula for missing values
fraction
fraction addition
fraction division
fraction inversion
fraction multiplication
fraction subtraction
fractions of different size
frequency
frequency distribution
front-end digits
front-end estimation
function
function composition
function notation
geometric function
geometric pattern
geometric pattern extension
global/local behavior
gram
graph
graphic representation of function
graphic solution
greater than
greatest common factor
grid
grouping
growing pattern
growth rate
guess and check
height
histogram
horizontal axis
hour
identity property
imaginary number
improbability
improper fraction
in from

inch
increasing pattern
independent events
independent trials
indirect measure
inductive reasoning
inequality
inequality solutions
inflection
input/output table
inside
integer
intercept
interest
intersecting lines
intersection of shapes
invalid argument
inverse function
investigation
irrational number
irregular polygon
irrelevant information in a problem
isometric
isosceles triangle
iterative sequence
large sample
law of large numbers
law of probability
least common multiple
left
length
less than
limit
limited sample
line equation
line graph
line segment
line segment congruence
line segment similarity
line symmetry
line through point not on a line
linear arithmetic sequence
linear equation
linear geometric sequence
linear log function
linear pattern
linear units
lists
location
logarithmic function
logic and
logic if/then
logic none
logic not
logic or
logic some
logical all
mass
mathematical expression
mathematical theories
matrix
matrix addition
matrix division
matrix equation
matrix inversion
matrix multiplication
matrix subtraction
maximum
mean
measure of height
measurement
measures of central tendency
measures of length
measures of width
measuring cup
median
meter
method selection
metric system
midpoint
minimum
minimum/maximum of function
minute
mixed numbers
mode
model
money
monitor progress of a problem
monomial
monte carlo simulation
multiple
multiple problem solving strategies
multiple strategies for proofs
multiplication
multiplication algorithm
multiply radical expressions
mutually exclusive events
natural log
natural number
nature of deduction
near
negative exponent
negative number
networks
nominal data
nondecimal numeration system
nonlinear equation
nonlinear function
nonroutine vs. routine problems
normal curve
number
number line
number of faces
number pairs
number property
number sentence
number subsystems
number systems
number theory
number triplet
numeral
numeric pattern
obtuse angle
odd numbers
odds

open sentence
order of operations
ordered pairs
ordinal number
orientation
outcome
outliers
outside
overestimation
parallel box plot
parallel lines
parallelogram
parallelogram formula
parameter
parameter estimate
parametric equation
part of whole
path
pattern
pattern addition
pattern division
pattern extension
pattern multiplication
pattern recognition
pattern subtraction
percent
percents above 100
percents below 1
perimeter
perimeter formula
periodic function
permutation
perpendicular bisector
perpendicular lines
perspective
phase shift
pi
pictorial representation
pie chart
place holder
planar cross section
plane
plane figure
point of tangency
polar coordinates
polygon
polynomial
polynomial addition
polynomial division
polynomial function
polynomial multiplication
polynomial solution by bisection
polynomial solution by sign change
polynomial solution successive approximation
polynomial subtraction
population
positive number
postulate
pound
powers
precision of estimation
precision of measurement
prediction
prime factor
prime factorization
prime number
prism
probability
probability distribution
problem formulation
problem space
problem types
process of elimination
product
projection
proof
proof paragraph
proportion
proportional gain
protractor
pyramid
pythagorean theorem
quadratic equation
quadrilateral
quartile deviation
quotient
radical expression
radical function
radius
random number
random sample
random sampling technique
random variable
range
range of estimations
range of function
rate
rate of change
rational function
rational number
real numbers
real-world function
reciprocal
rectangle
rectangle formula
rectangle prism
rectangular coordinates
recurrence equation
recurrence relationship
recursive equation
recursive sequence
reduced form
reference set
reflection in plan
reflection in space
reflection transformation
regression coefficient
regression line
relative distanced
relative error
relative frequency
relative magnitude
relative magnitude of fractions
relative size
relatively prime
relevant information in a problem
reliability
remainder
repeating pattern

representativeness of
sample
restate a problem
reversing order of
operations
rhombus
richter scale
right
right angle
right triangle geometry
roman numeral
root
roots & real numbers
roots to determine cost
roots to determine profit
roots to determine revenue
rotation
rotation in plane
rotation symmetry
rounding
ruler
same size units
sample
sample selection
techniques
sample space
sample statistic
sampling distribution
sampling error
scalar
scale
scale drawing
scale map
scale transformation
scatter plot
scientific notation
second (time)
sequence
series
series circuit
set
shape combination
shape division
shape pattern
shape similarity
shape symmetry
shape transformation
shrinking pattern
shrinking transformation
sigma notation
significant digits
similar figures
similar proportions
similarity
similarity vs. congruence
simplification
sine
sinusoidal function
size
slide transformation
slope
slope intercept formula
smallest set of rules
solid figure
solution algorithm
solution probabilities
sound attern
speed
sphere
spreadsheet
spurious correlation
square
square number
square root
square units
standard deviation
standard measure of
weight
standard measures of time
standard vs. non standard
units
statistic
statistical experiment
statistical regression
stem & leaf plot
step function
straight edge & compass
strategy efficiency
strategy generation
technique
studies
subset
substitution for unknowns
subtract radical
expressions
subtraction
subtraction algorithm
successive approximations
sum
summary statistic
supplementary angle
surface area
surface area cone
surface area cylinder
surface area sphere
survey
symbolic representation
synthetic geometry
systems of inequalities
table
table representation of
functions
table representation of
probability
tallies
tangent
temperature
temperature estimation
temperature measurement
term
tessellation
tetrahedron
theorem
theorem direct proof
theorem indirect proof
theoretical probability
thermometer
time interval
time zone
transversal
trapezoid formula
treatment group
tree diagram model
trial & error
triangle

triangle formula
triangle sides
trigonometric ratio
trigonometric relation
truncation
truth table proof
two way tables
u.s. customary system
under
underestimation
unit analysis
unit conversation
unit differences
unit size
univariate data
univariate distribution
unknown
unlike denominators
upper/lower bounds
valid argument
validity
variability
variable
variable change
variance
vector
vector addition
vector division
vector multiplication
vector subtraction
velocity
venn diagram
verbal representation of a problem
verification
vertex
vertex edge graph
vertical axis
volume
volume formula
volume measurement
volume of cylinder
volume of irregular shapes
volume of prism
volume of pyramid
volume of rectangular solids
week
whole number
width
work backward
written representation
year
zero

Appendix G: General Academic Vocabulary Word List

abandon
abstract
academy
access
accommodate
accompany
accumulate
accurate
achieve
acknowledge
acquire
adapt
adequate
adjacent
adjust
administrate
adult
advocate
affect
aggregate
aid
albeit
allocate
alter
alternative
ambiguous
amend
analogy
analyze
annual
anticipate
apparent
append
appreciate
approach
appropriate
approximate
arbitrary
area
aspect
assemble
assess
assign
assist
assume
assure
attach
attain
attitude
attribute
author
authority
automate
available
aware
behalf
benefit
bias
bond
brief
bulk
capable
capacity
category
cease
challenge
channel
chapter
chart
chemical
circumstance
cite
civil
clarify
classic
clause
code
coherent
coincide
collapse
colleague
commence
comment
commission
commit
commodity
communicate
community
compatible
compensate
compile
complement
complex
component
compound
comprehensive
comprise
compute
conceive
concentrate
concept
conclude
concurrent
conduct
confer
confine
confirm
conflict
conform
consent
consequent
considerable
consist
constant
constitute
constrain
construct
consult
consume
contact
contemporary
context
contract
contradict
contrary
contrast
contribute
controversy
convent
converse
convert
convince

cooperate
coordinate
core
corporate
correspond
couple
create
credit
criteria
crucial
culture
currency
cycle
data
debate
decade
decline
deduce
define
definite
demonstrate
denote
deny
depress
derive
design
despite
detect
deviate
device
devote
differentiate
dimension
diminish
discrete
discriminate
displace
display
dispose
distinct
distort
distribute
diverse
document
domain
domestic
dominate
draft
drama
duration
dynamic
economy
edit
element
eliminate
emerge
emphasis
empirical
enable
encounter
energy
enforce
enhance
enormous
ensure
entity
environment
equate
equip
equivalent
erode
error
establish
estate
estimate
ethic
ethnic
evaluate
eventual
evident
evolve
exceed
exclude
exhibit
expand
expert
explicit
exploit
export
expose
external
extract
facilitate
factor
feature
federal
fee
file
final
finance
finite
flexible
fluctuate
focus
format
formula
forthcoming
found
foundation
framework
function
fund
fundamental
furthermore
gender
generate
generation
globe
goal
grade
grant
guarantee
guideline
hence
hierarchy
highlight
hypothesis
identical
identity
ideology
ignorance
illustrate
image
immigrate
impact
implement
implicate
implicit

imply
impose
incentive
incidence
incline
income
incorporate
index
indicate
individual
induce
inevitable
infer
infrastructure
inherent
inhibit
initial
initiate
injure
innovate
input
insert
insight
inspect
instance
institute
instruct
integral
integrate
integrity
intelligence
intense
interact
intermediate
internal
interpret
interval
intervene
intrinsic
invest
investigate
invoke
involve
isolate
issue
item
job
journal
justify
label
labor
layer
lecture
legal
legislate
levy
liberal
license
likewise
link
locate
logic
maintain
major
manipulate
manual
margin
mature
maximize
mechanism
media
mediate
medical
medium
mental
method
migrate
military
minimal
minimize
minimum
ministry
minor
mode
modify
monitor
motive
mutual
negate
network
neutral
nevertheless
nonetheless
norm
normal
notion
notwithstanding
nuclear
objective
obtain
obvious
occupy
occur
odd
offset
ongoing
option
orient
outcome
output
overall
overlap
overseas
panel
paradigm
paragraph
parallel
parameter
participate
partner
passive
perceive
percent
period
persist
perspective
phase
phenomenon
philosophy
physical
plus
policy
portion
pose
positive
potential
practitioner
precede

precise
predict
predominant
preliminary
presume
previous
primary
prime
principal
principle
prior
priority
proceed
process
professional
prohibit
project
promote
proportion
prospect
protocol
psychology
publication
publish
purchase
pursue
qualitative
quote
radical
random
range
ratio
rational
react
recover
refine
regime
region
register
regulate
reinforce
reject
relax
release
relevant
reluctance

rely
remove
require
research
reside
resolve
resource
respond
restore
restrain
restrict
retain
reveal
revenue
reverse
revise
revolution
rigid
role
route
scenario
schedule
scheme
scope
section
sector
secure
seek
select
sequence
series
sex
shift
significant
similar
site
so-called
sole
somewhat
source
specific
specify
sphere
stable
statistic
status

stimulate
straightforward
strategy
stress
structure
style
submit
subordinate
subsequent
subsidy
substitute
successor
sufficient
sum
summary
supplement
survey
survive
suspend
sustain
symbol
tape
target
task
team
technical
technique
technology
temporary
tense
terminate
text
theme
theory
thereby
thesis
topic
trace
tradition
transfer
transform
transit
transmit
transport
trend
trigger

ultimate
undergo
underlie
undertake
uniform
unify
unique
utilize
valid
vary
vehicle
version
via
violate
virtual
visible
vision
visual
volume
voluntary
welfare
whereas
whereby
widespread

Appendix H: Meta-language – English Language Arts Word List

acronym
action segment
action verb
action word
active listener
actor
adjective
adjective clause
adjective phrase
adverb
adverb clause
adverb phrase
advertisement
advertising code
advertising copy
aesthetic purpose
aesthetic quality
affix
allegory
alliteration
allusion
almanac
alphabet
ambience
ambiguity
american literature
american psychological association
analogy
ancient literature
anecdotal scripting
anecdote
anglo-saxon affix
anglo-saxon root
animation
annotated bibliography
antonym
apology
apostrophe
appeal to authority
argumentation
articulation
artifact
asking permission
assonance
atlas
attack ad hominem
audience
audiotape
author
author's bias
author's purpose
autobiographical narrative
autobiography
auxiliary verb
back cover
background knowledge
ballad
bandwagon
beginning consonant
belief system
bias
bible
bibliography
biographical narrative
biographical sketch
biography
blend
blurring of genres
body language
body of the text
bolding
book
brainstorm
british literature
broadcast
broadcast advertising
business letter
bylaw
camera angle
cartoon
catalog
cause and effect
cd-rom
celebrity endorsement
censorship
central idea
chapter
chapter title
character
character development
character trait
characterization
chart
checklist
children's literature
children's program
chronological order
chronology
cinematographer
circumlocution
citation
clarification
clarity of purpose
climax
clincher sentence
close-up
closing
closing sentence
clue
cognate
coherence
cohesion
collective noun
colon
comma
command
commercial
commercialization
common feature

common noun
comparative adjective
compare & contrast
compile
complete sentence
complex sentence
composition
composition structure
compound adjective
compound noun
compound personal pronoun
compound sentence
compound verb
compound word
compound-complex sentence
comprehension
computer generated image
concept
conceptual map
concluding statement
conclusion
conjunction
conjunctive adverb
connotative meaning
consonance
consonant blend
consonant substitution
construct meaning
consumer document
content-area vocabulary
context
context clue
contract
contraction
contrast
contrasting expressions
controlling idea
convention
conversation
coordinating conjunction
copyright law
correlative conjunction
counter argument
couplet
cover
credibility
credit
criteria
critical standard
criticism
cross-reference
cue
cultural agency
cultural expression
cultural influence
cultural nuance
cultural theme
current affairs
cursive
custom
cutline
dash
date
debate
declarative sentence
decode
deconstruct
definition
delivery
demonstrative pronoun
denotative meaning
derivation
description
descriptive language
detail
diagram
dialect
dialogue
diary
dictation
dictionary
dictionary
digressive time
direct address
direct quote
directionality
directions
director
discussion
discussion leader
divided quotation
document
documentary
double negative
draft
drama
drama-documentary
dramatic dialogue
dramatic mood change
drawing
edit
editorial
elaboration
electronic media
e-mail
emotional appeal
emphasis
encyclopedia
ending
ending consonant
enunciation
epic
episode
essay
ethics
etiquette
etymology
everyday language
exaggerated claim
example
excerpt
exclamation mark
exclamatory sentence
explanation
explicit/implicit
exposition
expression
expressive writing
extend invitation
extended quotation
external/internal conflict
extraneous information
eye contact
fable
facial expression
facilitator

fact vs. opinion
fairy tale
false causality
familiar idiom
familiar interaction
fantasy
faulty mode of persuasion
fcc regulation
feature article
feature story
feedback
fiction
fictional narrative
field study
figurative language
figure of speech
film director
film review
filter (in photography)
first name
first person
flashback
folktale
follow/give directions
follow-up sentence
footnote
foreign word
foreshadowing
form
formal language
formal speech
format
friendly audience
friendly letter
front cover
fully developed character
future perfect verb tense
gender
generalization
genre
gesture
glittering generality
glossary
grammar
grammatical form
graphic artist
graphic organizer
graphics
greek affix
greek root
greeting
group discussion
guest speaker
guide words
heading
headline
hierarchic structure
high frequency word
historical fiction
historical theme
homeric greek literature
homonym
homophone
host
hostess
hostile audience
how question
humor
hyperbole
hyphen
idiom
illustration
imagery
imperative sentence
incongruity
inconsistency
indefinite adjective
indefinite pronoun
indentation
independent clause
index
inference
inflection
informal language
information source
interior monologue
interjection
internal conflict
internet
interpretation
interrogative pronoun
interrogative sentence
interview
intonation
introduction
investigate
invitation
irony
irregular plural noun
irregular verb
italics
jargon
job application
job interview
journal
juxtaposition
key word
keyboarding
knowledge base
language
language convention
last name
latin affix
latin root
layout
learning log
leave-taking
lecture
legend
letter
letter of request
letter-sound relationship
limited point of view
line (in a play)
linking verb
list
listening comprehension
listening skill
literal phrase
literary criticism
literary device
literature
literature review
log
logic
logical argument
logical fallacy
logo

logographic system
long vowel
lowercase
lyric poem
magazine
main character
main idea
manner of speech
map
margin
marketing
mass media
meaning clue
mechanics (language)
media generated image
media type
mediaeval literature
medium
memorandum
memory aid
mental image
message
metaphor
meter
methodology
microfiche
minor character
miscue
modern language
association
modern literature
modifier
modulation
mood
motive
movie
multimeaning word
multimedia presentation
multiple drafts
multiple sources
musical
mystery
myth
mythology
narration
narrator
native culture
native speaker
negative
negotiate
neoclassic literature
news
news broadcaster
news bulletin
newspaper
newspaper section
non verbal cue
nonfiction
norm
notes
noun
noun clause
noun phrase
novel
nuance
number word
numerical adjective
object
object pronoun
objective view
ode
omniscient point of view
onomatopoeia
opening monologue
opinion
oral presentation
oral report
oral tradition
order of events
organization
outline
overgeneralization
overstatement
overview
pacing
packaging
page format
pamphlet
parable
paragraph
parallel episodes
parallel structure
paraphrase
parody
parts of a book
passage
past perfect verb tense
past tense
pastoral
peer review
peer-response group
pen pal
performance review
period
periodical
persona
personal letter
personal narrative
personal pronoun
personal space
personification
perspective
persuasion
philosophical assumption
phone directory
phonetic analysis
photographer
phrase
phrase grouping
physical description
physical gesture
picture book
picture dictionary
pitch
plagiarism
plot
plot development
poem
poetic element
point of view
poise
policy statement
polite form
political cartoonist
political speech
posing a question
positive adjective
possessive noun

possessive pronoun
posture
predicate adjective
predictable book
preface
prefix
preposition
prepositional phrase
present perfect verb tense
present tense
presentation
preview
prewriting
primary source
print
prior knowledge
private audience
problem-solution
producer
production cost
programming
progressive verb form
projection
pronominal adjective
pronoun
pronunciation
proofread
prop
propaganda
proper adjective
proper noun
proposition of fact speech
proposition of policy speech
proposition of problem speech
proposition of value speech
proverb
public audience
public opinion trend
publication date
publish
pull-down menu
punctuation
purpose
question
question mark
questionnaire
quiz show
quotation
quotation marks
radio program
rating
r-controlled
reaction shot
readability
readers guide to periodical literature
reading strategy
reading vocabulary
recitation
recurring theme
red herring
redraft
reference source
reflexive pronoun
regular plural noun
regular verb
relative pronoun
relevant detail
repeats
rephrasing
report
representation
request
reread
research paper
resolution
resource material
respond to literature
restatement
resume
retell
revise
rhetorical device
rhetorical question
rhyme
rhyming dictionary
rhythm
role playing
romantic period literature
root word
rules of conversation
sales technique
salutation
sarcasm
satire
saying
scan
science fiction
script writer
second person
secondary source
self-correction
semicolon
sensory image
sentence
sentence combining
sentence structure
sequential order
set design
setting
shades of meaning
short story
short vowel
sight word
sign speech
signature
simile
simple sentence
singular noun
sitcom
skim
skit
slang
slanted materials
small talk
soap opera
social interests
sociocultural context
software
soliloquy
somber lighting
sound effect
sound system
source
special effect

specialized language
speech action
speech pattern
speed reading
speed writing
spelling
spelling pattern
spoken text
standard english
status indicator
stay on topic
stereotype
story element
story map
story structure
stream of consciousness
stress
structural analysis
style sheet format
stylistic feature
sub vocalize
subject
subject pronoun
subjective view
subject-verb agreement
subliminal message
subordinate character
subordinating connection
subplot
suffix
summarize
summary
summary sentence
superlative adjective
supernatural tale
supporting detail
suspense
syllabic system
syllabication
syllable
symbol
symbolism
synonym
syntax
synthesize
table
table of contents
tabloid newspaper
take turns
talk show
tall tale
target audience
target language
technical directions
technical language
telephone information service
television program
tempo
temporal change
tense
tension (in a story)
text
text boundary
text feature
text structure
textbook
textual clue
thank you letter
theater
theme music
thesaurus
thesis
thesis statement
third person
time lapse
time line
title
title page
tone
topic sentence
transition
translate
transparency
trickster tale
truth in advertising
typeface
typing
understatement
universal theme
uppercase
usage
verb
verb phrase
verbal cue
vernacular dialect
videotape
viewer perception
viewpoint
villain
visual aid
visual text
vocabulary
voice
voice inflection
voice level
volume
vowel combination
vowel sound
warranty
web site
when question
where question
why question
word borrowing
word choice
word family
word origin
word play
word processing
word reference
word search
written directions
written exchange

Appendix I: Meta-language Academic terms for Book Parts Word List

author index
bibliography
boldface type
caption
chapter
chart
column
conclusion
diagram
excerpt
figure
font size
font/print
glossary
graph (line/bar)
graph (pie)
handbook
illustration/picture
indentation
index
introduction
italicized type
map
page
paragraph
passage
preface
quotation
section
selection
subtitle/subheading
table
table of contents
title heading
title page
transition

Appendix J: Symbols

Primary Symbols

see	say
+	and or plus
×	times
=	is equal to or equals
<	is less than
¢	cent or cents
½	one-half
¾	three-quarters
%	percent
-	take away or minus
÷	is divided by
≠	is not equal to
>	is more than or is greater than
$	dollar or dollars
¼	one-quarter
⅓	one-third
#	number or pound

Intermediate Symbols

see	say
+	plus or positive
×	is multiplied by
=	is equal to or equals
<	is less than
* and ·	is multiplied by
?	a missing number
≅	is approximately equal to
≤	less than or equal to

(	open parenthesis
[	open bracket
@	at
:	is to
∴	therefore
r	set of real numbers
∪	union with or union
⊂	contained in or a subset of
∈	element of
⇔	equivalent
-	minus or negative
÷	is divided by
≠	is not equal to
>	is greater than
/	is divided by
∠	angle
⊥	is perpendicular to
≥	is greater than or equal to
	closed parenthesis
]	closed bracket
ø	null set, empty set or zero
::	as
≈	is approximately
n	set of natural numbers
∩	intersects or intersection
⊄	not a subset of
∉	is not an element of
∥	is parallel to

numeral
symbols

zero	0
one	1
two	2
three	3
four	4
five	5
six	6
seven	7
eight	8
nine	9
ten	10
eleven	11
twelve	12
thirteen	13
fourteen	14
fifteen	15
sixteen	16
seventeen	17
eighteen	18
nineteen	19
twenty	20
thirty	30
fourty	40
fifty	50
eighty	80
ninety	90
one hundred	100

Appendix K: Codebook

Below you will find each category listed in the Framework with specific directions for Co-Rating each category.

Chapter Name/Lesson Number

This section is not co-rated.

Number of Writing Prompts Per Page

1) Determine the number of prompts selected per page for coding and record in the section indicated. This category is aligned to Prompt Selection.

Number of Exercises Per Page

1) An exercise or prompt that is located on the page. In order for a exercise or prompt to be counted in this section the textbook author would have denoted a number next to the exercise or prompt. Only numbered or lettered exercises or prompts will be counted.

Textbook/Page Number

This section is not co-rated

Prompt Selection

1) Select only exercises on the page that are numbered.
2) Select only tasks on the page that have words in the prompt. Exercises that involve computation with digits specifically will NOT be selected.
3) Determine if prompt has the potential to facilitate a constructed response by identifying the language or terms within the prompt found in Appendix B.
4) Answer the prompt to determine the type of constructed response.
5) If the answer to the prompt has the potential to facilitate a one word response *or* has a multiple choice selection, the prompt will NOT be selected.
6) Tasks that require the student to write "rules" or "lists" are NOT selected.
7) If the prompt has the potential to facilitate a sentence or more, the prompt will be selected for coding.

Content Strand

1) Color Codes for Envision Topics are based upon the NCTM strands.

2) Identify the color of the Topic where the prompt was identified and select that strand based on the color assigned by the text book.
3) Determine if the language used in the Topic/Unit Title provides information on additional strand selection.

4) Read the prompt to determine if the language in the prompt provides for an additional strand to be selected. See Table 1-5 for a list of NCTM topics to assist in strand identification.
5) Academic Vocabulary (Items 3-8 repeat for each section)

Domain specific vocabulary (DSV)

1) Identify words that are specific to DSV by using the word lists in Table 1-5, mathematics textbook glossary of Harcourt Pilot study, and prior knowledge of mathematics terms to assist in identification of domain specific vocabulary.
2) Review the terms in Appendix F to assist in the identification of DSV.
3) Conduct a word search using the Ctrl Find Key in the Excel Spreadsheet of Academic Vocabulary Word Lists. Words are color coded according to the categories in the Academic Vocabulary section of the Framework.
4) Continue with the Ctrl Find key until you have exhausted the search and returned back to initial position.
5) Record findings in the appropriate Academic Vocabulary sections in Framework.
6) If a word is found in two or more Academic Vocabulary sections the word is coded appropriately in each section and <u>underlined</u>.
7) Identified words may be derivatives of the Academic Vocabulary found in the Word Lists. The derivative is noted next to the word coded in parenthesis.
8) If a word is not found in the Academic Vocabulary word list the rater may code the word in the Special Words section of the Framework with the appropriate classification of the Academic Vocabulary next to the word.

General vocabulary (GV)

1) Identify words that are specific to GV by recognizing words in the prompt that appear reasonably frequently within and across academic domains. The words may be polysemous, with different definitions being relevant to different domains.
2) Review the terms in Appendix G to assist in the identification of GV.
3) Conduct a word search using the Ctrl Find Key in the Excel Spreadsheet of Academic Vocabulary Word Lists. Words are color coded according to the categories in the Academic Vocabulary section of the Framework.
4) Continue with the Ctrl Find key until you have exhausted the search and returned back to initial position.
5) Record findings in the appropriate Academic Vocabulary sections in the Framework.
6) If a word is found in two or more Academic Vocabulary sections the word is coded appropriately in each section and <u>underlined</u>.
7) Identified words may be derivatives of the Academic Vocabulary found in the Word Lists. The derivative is noted next to the word coded in parentheses.

8) If a word is not found in the Academic Vocabulary word list the rater may code the word in the Special Words section of the Framework with the appropriate classification of the Academic Vocabulary next to the word.

Meta Language (ML)

Identify words that are specific to ML by recognizing words in the prompt that are used to describe the language of literacy and literacy instruction and words used to describe processes, structures, or concepts commonly included in content area texts.

1) Review the terms in Appendix H-I to assist in the identification of ML.
2) Conduct a word search using the Ctrl Find Key in the Excel Spreadsheet of Academic Vocabulary Word Lists. Words are color coded according to the categories in the Academic Vocabulary section of the Framework.
3) Continue with the Ctrl Find key until you have exhausted the search and returned back to the initial position.
4) Record findings in the appropriate Academic Vocabulary sections in the Framework.
5) If a word is found in two or more Academic Vocabulary sections the word is coded appropriately in each section and underlined.
6) Identified words may be derivatives of the Academic Vocabulary found in the Word Lists. The derivative is noted next to the word coded in parentheses.
7) If a word is not found in the Academic Vocabulary word list the rater may code the word in the Special Words section of the Framework with the appropriate classification of the Academic Vocabulary next to the word.

Symbols

1) Words in the prompt are NOT mathematics symbols.
2) Punctuation marks in the prompt are NOT mathematics symbols (i.e., commas including seriations (lists), hyphens used between words, periods, and question marks).
3) All numerals that represent numbers will be coded as symbols.
4) Any symbol that is NOT a word or part of the punctuation in the prompt will be coded as a symbol.
5) If a symbol is combined with another symbol the symbol will be coded as *one.* The parts that make the symbol, if those parts are in the symbols list, will also be counted independently.

Words Not On List

This section is not co-rated.

Total

Words and Symbols Coded

1) Count the number of words and symbols coded in each of the Academic Vocabulary sections.
2) Numerals are coded as symbols and counted as one number.
3) Commas, periods, colons, dollar signs, fraction symbols, within numbers are counted as one symbol *and* as individual symbols. For example (2,000,567 is counted as 3, one time for the whole number and two times for each comma).
4) Phrases are counted as individual words.
5) Underlined words are counted one time.
6) Special Words category is NOT counted.

Words and Symbols

1) Count the total number of all words and all symbols in the prompt.
2) Commas, periods, colons, dollar signs, fraction symbols, within numbers are counted as one symbol *and* as individual symbols. For example (2,000,567 is counted as 3, one time for the whole number and two times for each comma).
3) Phrases are counted as individual words

Type of Prompt

1) Because all the prompts coded are generic, the prompt will only be coded in this section if it is NOT coded in the other categories.
2) Affective prompts are coded in this section if the prompt involves the reader to write an opinion, feeling, or belief regarding the topic.
3) Narrative is coded in this section if the prompt provides the writer with information to write about math content in a fictional or narrative sense using real world or imaginary indicators.

Teacher Edition

Find the section of the Teacher Edition for the prompt coded. Read the section carefully to indicate the following codes listed below.

Support provided only (Su)

1) A prompt is coded in this section if the Teacher Edition only has teaching support for the prompt coded. Support includes any indicator of instructional notes for the prompt. Any information given to the teacher for the prompt other than a student sample is coded in this section.

Sample provided only (Sa)

1) A prompt is coded in this section if the Teacher Edition only has a sample of how the prompt should be answered for the prompt coded. No other directions or guidance is given for the prompt.

Support with Sample provided (SS)

1) A prompt is coded in this section if the Teacher Edition has both teaching support *and* a sample of how the prompt should be answered for the prompt coded.

No Support or Sample provided (N)

1) A prompt is coded in this section if the Teacher Edition has NO teaching support or sample answer provided.

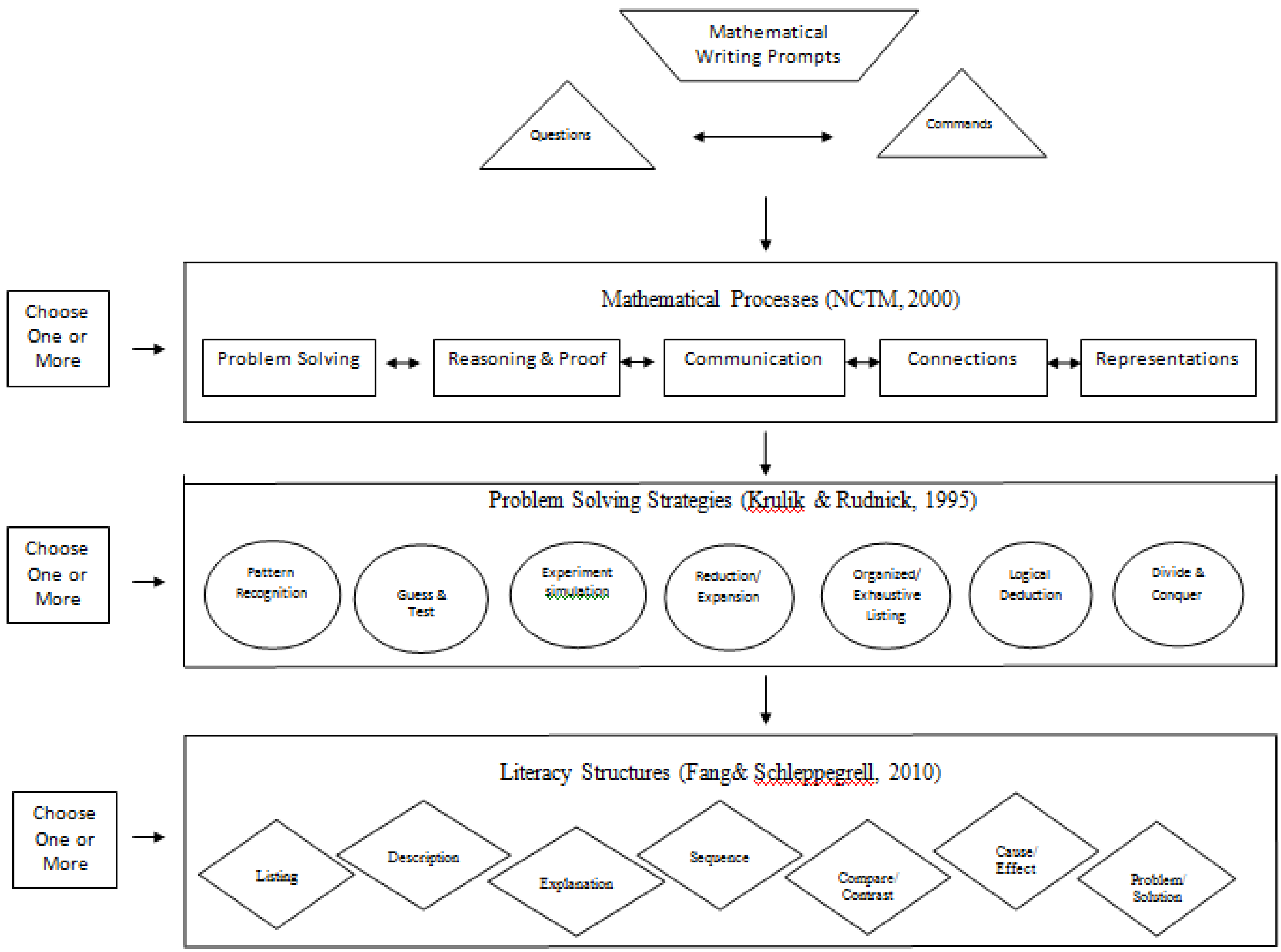
Student Edition (SE)

This section is not co-rated.

Appendix L: Linguistic Analysis of Mathematical Prompt Stems

Questions				Command		
Type of Question				Type of Command		
How Questions	Why Questions	What Questions	When Questions	Describe	Explain	Construct
How Can You	Why Would You	What Would Happen	When Will	Describe How You	Explain How You	Write A Problem
How Do You	Why Do You	What Do You	When Are	Describe How To	Explain Why You	Write A Word Problem
How Would You	Why Can You	What Was		Describe	Explain Your Answer	Write a Number Story
How Could You	Why Can't You	What Can You			Explain How To	Write a Question
How Could	Why Was	What Makes It			Explain Why	Write
How Would	Why or Why Not	What Does			Explain How	Give
How Does	Why Are	What Do			Explain	Make
How Did	Why Is	What Is				
How Can	Why Does	What				
How Many	Why Do					
How Are	Why					
How Is						
How						
13 Types	11 Types	9 Types	2 Types	3 Types	7 Types	7 Types

Appendix M: Model of "Affordances" within Mathematical Writing



Appendix N: Student Edition Section (S), Sub-sections (SS), and Additional Sub-section (AdSS) in *Everyday Mathematics* and *enVision MATH*

Everyday Mathematics				enVision MATH			
S	SS	AdSS	N	S	SS	AdSS	N
Angles			1	Algebra			1
An Algorithm for Multiplying a Fraction by a Whole Number			1	Algebra Connections	Write a Problem		2
A Bicycle Trip			2	Another Example	Explain It		32
A Floor Plan of My Classroom			1	Another Example	Explain It	Reasonableness	8
Algorithm Project 1			12	Enrichment	Practice	Number Sense	1
A Polygon Alphabet	Try This		2	Guided Practice			3
Areas of Triangles			1	Guided Practice	Do You Understand		67
Color Coded Population Maps			1	Guided Practice	Do You Understand	Number Sense	1

Comparing Decimals			1	Guided Practice	Do You Understand	Reasoning	3
Comparing Fractions			1	Guided Practice	Do You Understand	Writing to Explain	44
Circle Graphs			1	Guided Practice	Do You Understand	Write a Problem	1
Cube-Stacking Problems			1	Guided Practice	Do You Know How		1
Cellular Telephone Use			1	Guided Practice	Write A Problem		4
Converting Units of Measure			1	Guided Practice	Problem Solving	Writing to Explain	2
Decimal Addition and Subtraction			1	Independent Practice			13
Designing a Bookcase			1	Independent Practice	Algebra		1
Discount Number Stories			3	Independent Practice	Error Search		1
Do These Numbers Make Sense			5	Independent Practice	Problem Solving	Geometry	1
Evaluating Large Numbers	Facts About the Capital of the Country		2	Independent Practice	Problem Solving		29

Expected Spinner Results	Facts About the Capital of the Country		1	Independent Practice	Problem Solving	Number Sense	13
Estimating Weights in Grams and Kilograms	My Impressions About the Country		1	Independent Practice	Problem Solving	Reasoning	18
Finding Lines of Reflection	Facts About the Capital of the Country		1	Independent Practice	Problem Solving	Writing to Explain	30
Frieze Patterns	My Impressions About the Country		1	Independent Practice	Problem Solving	Error Search	3
Factor Pairs of Prime Numbers			2	Independent Practice	Writing to Explain		8
Fraction Review			1	Number Sense	Estimation and Reasoning		24
Finding Unknown Angle Measures			1	Practice			1
Growing Patterns			7	Review What You Know	Fraction Concepts		1
Head Sizes			3	Review What You Know	Writing to Explain		10

Insect Data			1				
Investigating Liters and Milliliters	Math Message: Eating Fractions		1	Total Prompts			323
Interpreting Remainders			3				
Internet Users			1				
Looking Back on the World Tour			4				
Largest Cities by Population			1				
Measuring Angles			1				
Math Boxes			2				
Measuring Capacity			1				
My Country Notes			11				
My Country Notes			6				
My Country Notes			1				
Measuring Land Invertebrates			1				
Multiplying Ones			1				

by Tens								
Making a 1-Ounce Weight			2					
Making a 1-Ounce Weight								
Modeling a Rectangular Prism			2					
Modeling a Rectangular Prism			1					
Multiplying Tens by Tens			1					
Ordering Fraction			1					
Open Sentences			1					
Parallelograms			4					
Probability			2					
Playing Card Probabilities			1					
Planning a Driving Trip			1					
Patterns in Multiplication			3					

Facts							
Product Testing			3				
Rates			2				
Review: Fractions, Decimals, and Percents			1				
Fraction and Mixed-Number Addition and Subtraction			1				
Rate Tables	Facts About the Capital of the Country		1				
Solving Number Stories	Impressions About the Country		1				
Taking Apart Putting Together	Facts About the Capital of the Country		1				
Using Coins to Add Fractions	Impressions About the Country		1				

Unit Prices	Facts About the Capital of the Country		3				
U.S. Traditional Addition 3	Impressions About the Country		2				
U.S. Traditional Addition: Decimals 3	Impressions About the Country	U.S. Traditional	2				
U.S. Traditional Multiplication 3	Algorithm Project 1	Algorithm Project	4				
U.S. Traditional Subtraction 3	Algorithm Project 3		2				
U.S. Traditional Subtraction: Decimals 3	Algorithm Project 4		2				
Using Your Student Reference Book	Algorithm Project 5		1				
What Do Americans Eat	Algorithm Project6		1				
What is the One?	Algorithm Project 7		1				
Total Prompts			140				

Permissions and IRB Application

Permissions

May 18, 2011

Good afternoon Christine,

Thank you for your email!

In response to your request below, ASCD is pleased to grant you permission to include passages from Building Academic Vocabulary, by Robert J. Marzano & Debra J. Pickering, in your forthcoming dissertation. Please include a proper reference or citations with the excerpts. If you wish to publish your work for commercial purposes, you are required to contact us again to secure additional rights to do so.

Should you have any questions, please do not hesitate to contact me and I will be happy to respond to any query.
Thank you for your interest in ASCD publications and good luck with your dissertation!

Best regards,

Matt

Matthew Mayer
ASCD
Rights & Permissions Project Coordinator
www.ascd.org

Dear Christine,
You have full permission to use the word list--and I would very much enjoy seeing a copy of your findings!
Sincerely,
Jan Pilgreen

Dr. Janice Pilgreen
Professor of Literacy Education
Reading Program Chair/Literacy ¢er Director
University of La Verne
1950 Third Street, La Verne, CA 91750
909.593.3511, X4624

Dear Christine Thank you for your message. I'd be interested in the results of your study. Please consider this email to be permission.

Best wishes

Averil Coxhead

IRB

Hello Ms. Joseph: If you are not interacting or collecting personal identifiable data from human subjects you would not need IRB review.If you are reviewing textbooks – then you would not submit an IRB application.

Regards, Various B. Menzel, CCRPResearch Compliance Administrator - Social & Behavioral StudiesUniversity of South FloridaPhone: (813) 974-6433Fax: (813) 974-7091E-mail: vmenzel@research.usf.edu Mailing: USF / Division of Research Integrity & Compliance12901 Bruce B. Downs Blvd., MDC Box 35Tampa, FL 33612 Deliveries: 3702 Spectrum Blvd., Suite 155 USF Research Park Please visit our website: http://www.research.usf.edu/cs/irb.htm for the most recent IRB information.

RE:Pro#4572

Reply |eirb to me, Various
show details Jun 27

It actually sounds like this may not qualify as Human Subjects research and may not need to be submitted to the IRB. I have copied Various Menzel, who is one of the individuals that reviews IRB studies. She should be able to provide both of us with a little more insight. Various, can you tell if Ms. Joseph needs to pursue IRB approval any further. She did receive the NHSR determination page when completing her application. Thank you, Amber McPhersonARC Help DeskDivision of Research Integrity & Complianceeirb@research.usf.edu (813) 974-2880ARC Login (eIRB &eCOI): https://arc.research.usf.edu/Prod
ARC@research.usf.edu;eIRB@research.usf.edu;eCOI@research.usf.edu