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Stochastic-optimization of equipment productivity in multi-seam formations

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STOCHASTIC-OPTIMIZATION OF EQUIPMENT PRODUCTIVITY IN MULTI-SEAM FORMATIONS

by

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ABSTRACT

Short and long range planning and execution for multi-seam coal formations (MSFs) are challenging with complex extraction mechanisms. Stripping equipment selection and scheduling are functions of the physical dynamics of the mine and the operational mechanisms of its components, thus its productivity is dependent on these parameters. Previous research studies did not incorporate quantitative relationships between equipment productivities and extraction dynamics in MSFs. The intrinsic variability of excavation and spoiling dynamics must also form part of existing models. This research formulates quantitative relationships of equipment productivities using Branch-and-Bound algorithms and Lagrange Parameterization approaches. The stochastic processes are resolved via Monte Carlo/Latin Hypercube simulation techniques within @RISK framework.

The model was presented with a bituminous coal mining case in the Appalachian field. The simulated results showed a 3.51% improvement in mining cost and 0.19% increment in net present value. A 76.95yd^3 drop in productivity per unit change in cycle time was recorded for sub-optimal equipment schedules. The geologic variability and equipment operational parameters restricted any possible change in the cost function. A 50.3% chance of the mining cost increasing above its current value was driven by the volume of material re-handled with 0.52 regression coefficient. The study advances the optimization process in mine planning and scheduling algorithms, to efficiently capture future uncertainties surrounding multivariate random functions. *The main novelty includes the application of stochastic-optimization procedures to improve equipment productivity in MSFs.*

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NOMENCLATURE

The following are symbols, variables, and abbreviations used in various sections of the thesis to achieve the set objectives.

Symbol	Description
Btu	British thermal unit (One Btu is equal to the amount of heat required to raise the temperature of one pound of liquid water by 1^0 Fahrenheit at its maximum density, which occurs at a temperature of 39.1^0F)
sr_T	Overall stripping ratio
v_{wi}	Volume of the i^{th} waste material
v_{csi}	Volume of the i^{th} coal seam
t_{csi}	Tons of the i^{th} coal seam i.e. $t_{csi} = v_{csi} \times tonnage\ factor$
rf	Dragline reach factor
d	Depth of overburden
sf	Swell factor
w	Cut width
h	Height of spoil pile from the base of coal seam
β	Highwall angle
θ	Spoil pile angle

n	Waste block
j	Resource
k	Waste extraction scheduling period
s	Waste mining strip
$x_n^{j,k}$	A continuous variable representing the ratio of block n excavated by resource j in period k
$d_n^{j,k}$	Discounted unit cost of mining all the material in block n as waste in period k by resource j
wt_n	Tonnage of waste in block n
$c_n^{j,k}$	Cost in present value terms of resource j mining a ton of block n as waste in period k
b_s^k	A binary integer decision variable equal to one if mining-strip s is scheduled to be excavated in period k, otherwise zero
γ^k	Cost per energy consumed in period k (\$/J)
$\varepsilon_n^{j,k}$	Energy consumed by resource j in mining a unit ton of block n in period k
$e_n^{j,k}$	Energy consumed in hauling a unit ton of block n excavated by resource j in period k
ϑ^k	Permissible total energy cost in period k (\$/J)

$ml^{j,k}$ Lower mining capacity limit of resource j in period k

$mu^{j,k}$ Upper mining capacity limit of resource j in period k

$di^{j,k}$ Internal dump available for resource j to spoil material excavated in period k

$de^{j,k}$ External dump available for resource j to spoil material excavated in period k

sg Specific gravity of the material

$ht_n^{j,k}$ Total hours for a resource j to excavate a unit ton of block n in period k

$a^{j,k}$ Mechanical available hours of resource j in block n per period k

$u^{j,k}$ Minimum utilization requirements of resource j in block n per period k

md Minimum drop cut width

τ Clearance radius of the equipment revolving frame

φ Clearance radius of the boom point sheave

l_e Length covered per unit excavation

o_n^k Total length of block n in period k

ex_n External space adjacent to block n

$md^{j,k}$ Required minimum drop cut of resource j in period k

hu^k Maximum haulage unit(s) capacity in period k

i_n Uniaxial compressive strength of block n

$f_n^{j,k}$ Force required by resource j to excavate a unit ton of block n in period k

$rs_n^{j,k}$ Resource j labor requirement per ton production of block n in period k

$la^{j,k}$ Available labor for resource j in period k

d_c^j The excavation depth at which the spoil pile will rise a height, h_c from the base of the coal seam given resource j

h_n Depth of the block n

t_n Tons of material in block n

l_n Length of block n

w_n Width of block n (cut width)

m Coal seam-cuts

e Coal product destination

t Coal extraction scheduling period

g Set of exposed coal seam-cuts

$y_m^{e,t}$ Continuous variable representing the portion of coal seam-cut m extracted and transported to destination e in period t.

$\varpi_m^{e,t}$ Discounted revenue obtained by selling the final product in coal seam-cut m to destination e in period t

v_m Tonnage of material in coal seam-cut m

q_m Thermal coal quantity in coal seam-cut m

r	Proportion of thermal coal quantity recovered at treatment plant
p^t	Price of thermal coal in present value terms per unit of product
se^t	Selling cost of thermal coal in present value terms per unit of product
$\varsigma_m^{e,t}$	Cost in present value terms per unit of coal seam-cut m for mining and processing to destination e
cu^t	Upper bounds of the mining capacity in period t
cl^t	Lower bounds of the mining capacity in period t
pu^t	Upper bounds of the processing capacity in period t
pl^t	Lower bounds of the processing capacity in period t
rp^t	Upper bounds of the transportation capacity in period t
st^t	Upper bounds of the stockpile capacity in period t
$\aleph^t$	Maximum coal market limit in period t
$\beth^t$	Minimum coal market limit capacity in period t
φ_m^t	Labor requirement per unit excavation of coal seam-cut n in period t
$\check{g}^t$	Available skilled labor in period t
∂^t	Pit-to-Plant maximum haulage unit(s) capacity in period t
hr_m^t	Hours to extract a unit quantity of coal seam-cut m in period t
av^t	Excavation equipment available hours in period t

uu^t	Utilization requirement of excavation equipment in period t
$b_{i,m}$	Quality parameter i per unit quantity of coal seam-cut m
$B_i^{e,t}$	Upper bound of quality parameter i at destination e in period t
$L_i^{e,t}$	Lower bound of quality parameter i at destination e in period t
λ	Lagrange multiplier

1. INTRODUCTION

The deployment of large mining equipment has resulted in low-cost, bulk production operations in surface mines. In strip coal mining operations, these economies of scale favor increasingly the use of draglines, shovels, dozers and other support equipment for overburden and coal extraction. However, the selection of a particular dragline model with fixed design geometry might be economically inept in varying geological and operating domains. A comprehensive introduction to the research study is presented in this section. The introduction include: (i) background of research problem, (ii) statement of the problem; (iii) objectives and scope of the study; (iv) research methodology; (v) scientific and industrial contributions; and (vi) structure of the thesis.

1.1 BACKGROUND OF RESEARCH PROBLEM

The world's largest estimated recoverable coal reserves are located in the United States of America with the coal mines producing more than a billion tons coal per annum (EIA, 2012). With the primary energy consumption in the United States estimated to increase by 1.1 percent per annum from 2004 to 2030, coal production has seen a steady growth. This growth is estimated to continue through 2035 (See figure 1.1) with an average growth rate of 1.0 percent from 2015 (EIA, 2012). The current coal consumption rates, as illustrated in Figure 1.2, make coal production vital to the micro and macro-economic growth of the United States. Coal production in the United States totaled 1.08 billion tons, about 0.9 percent increment from the 2009 total of 1.07 billion tons (EIA, 2012).

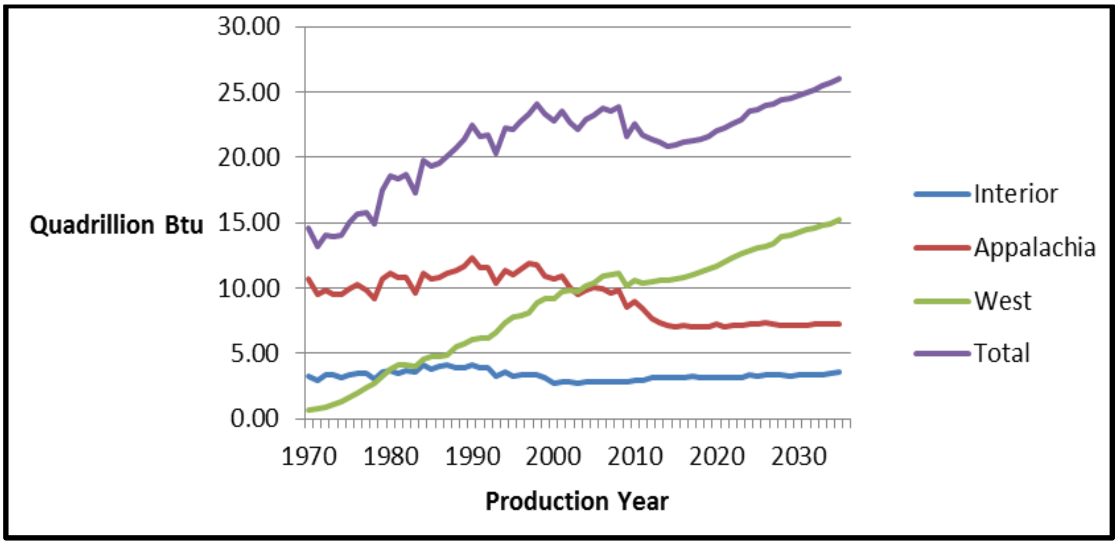


Figure 1.1 Coal Productions by Regions (quadrillion Btu) (EIA, 2012)

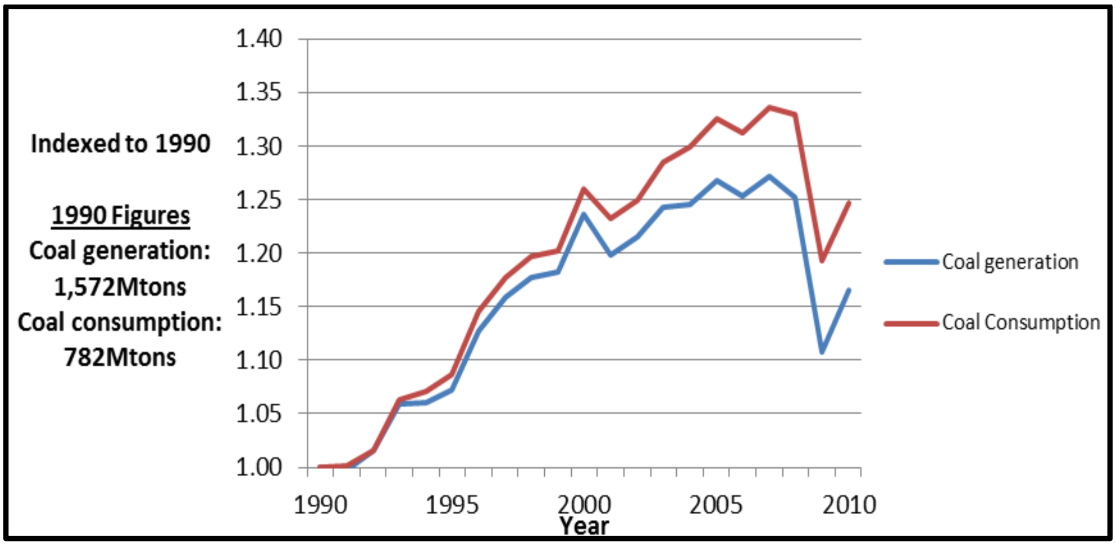


Figure 1.2 Coal Consumed and Generated in the Electric Power Sector (EIA, 2012)

Approximately, two-thirds of the coal is produced from surface mining operations. The increase in surface coal production can be attributed to the advent of larger trucks, shovels and more sophisticated draglines resulting in higher production efficiencies (Gershon, 1983). The mining method adopted by most United States surface coal mines is strip mining. In this method, draglines have extensively been engaged for overburden removal due to their economic advantages as compared to other extraction methods. In strip mining operations, the cost of waste material extraction is a significant portion of the overall mining cost. Similarly, in terms of equipment energy consumption, waste excavation and material spoiling are the most costly and energy-consuming sectors. The main factors that influence energy consumption in mining operations include: (i) equipment design and matching; (ii) explosives factor and degree of fragmentation; (iii) drilling patterns; (iv) working geometry and condition; and (v) loading/shift systems (Cooke and Randall, 1995). Figure 1.3 shows the distribution of excavation cost according to a survey by the mining association of Canada (2005). Based on Figure 1.3 and the energy-consumption parameters, it is implied that efficient stripping is a significant component in surface coal mine operation.

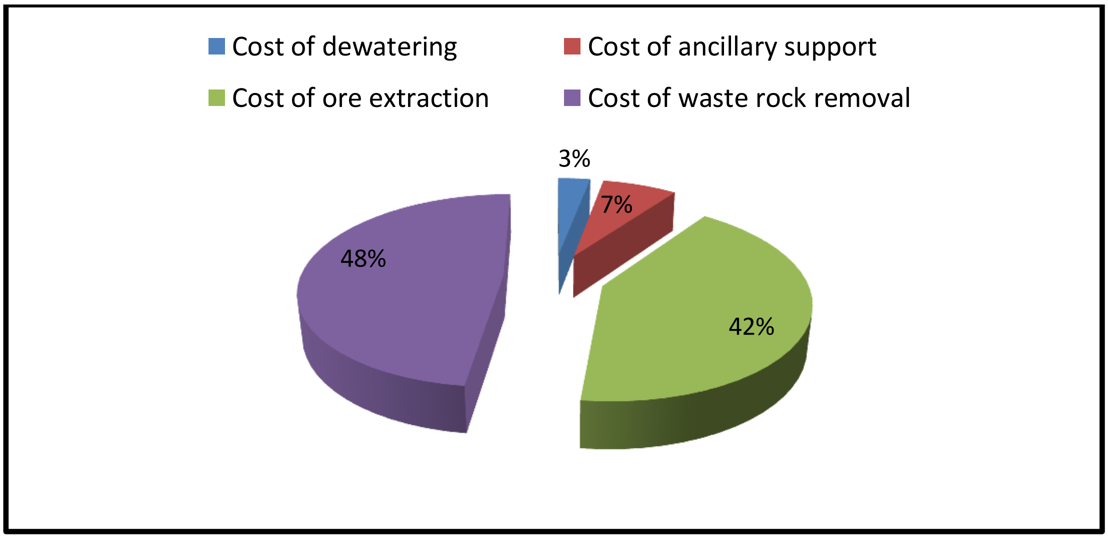


Figure 1.3 Strip Mining Cost Distributions (Mining Association of Canada, 2005)

The primary overburden excavation equipment in most strip mines is the dragline. With a typical machine weight of 8,350 tons and a capital investment of about

$200 million, draglines are massive and expensive equipment. The average bucket capacity is approximately 105 cubic yards and the equipment is continuously operated unless for preventive maintenance schedules. In 1999, among the 56 largest United States coal mines that use draglines, their total coal production was approximately 400 million tons (Gilewicz, 2000).

Due to the contribution of draglines to surface coal mining and the significant number of surface coal mines compared to underground coal mines, as illustrated by Figure 1.4, surface coal mining in the United States will continue to benefit from increased dragline productivity and optimal usage.

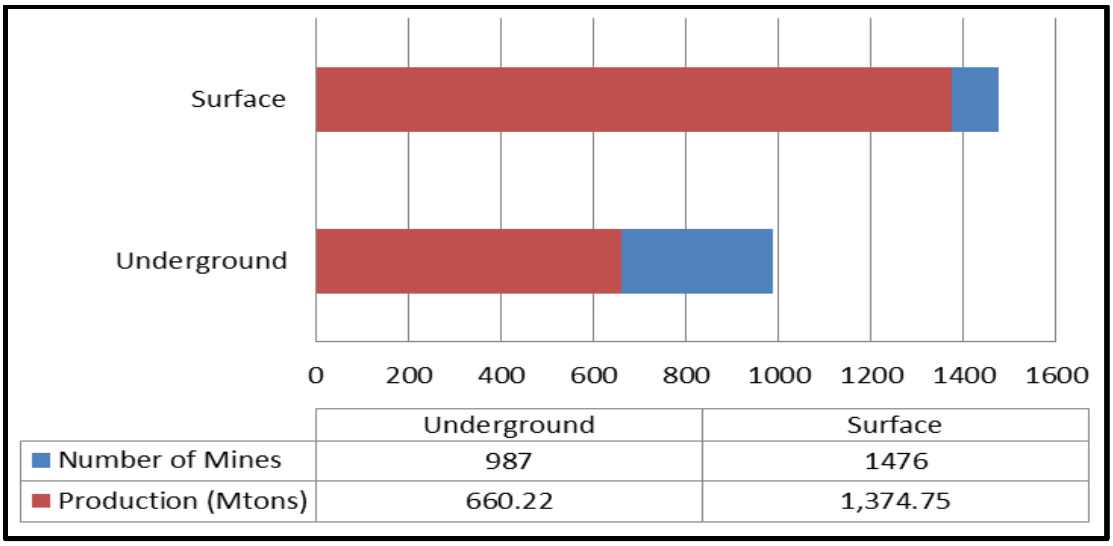


Figure 1.4 Distributions of U.S. Surface and Underground Coal Mines (EIA, 2012)

1.2 STATEMENT OF THE PROBLEM

Most of the surface coal deposits in central Appalachian region of southern WV, eastern KY, southwestern VA and western United States occur in multiple seams (Mark et al., 2007). In multi-seam formations (MSFs), optimal units and excavation scheduling of draglines are significant economic and sustainable mining requirement. Dragline applications in such environments tend to be challenging, and therefore need careful planning and scheduling. Since dragline operations are highly capital-intensive, investments in multiple units hugely impact the overall economic viability of projects. Although auxiliary equipment may be engaged to complement the dragline operations in these environments, sub-optimal dragline schedules, may result in huge economic losses. Sub-optimal dragline schedules are essential drawbacks in MSFs excavation, which must be resolved with applicable advanced research initiative.

The selection of a particular dragline model may not be economically sustainable due to varying geological and operating parameters as mining advances. With increasing overburden depth, material rehandling may occur as a result of shorter dragline reach. Material rehandling increases production cost significantly due to draglines spending time to handle material already excavated, and thus, has less time in exposing coal. Duration of dragline walking and operating pad preparations in the event of material rehandling may also reduce its productivity. There may be occurrences of differed revenues as some lower seams would not be timely recovered. Hence the key to gaining higher economic benefit in dragline operation is to select dragline unit(s) with optimal geometry to minimize material rehandling and ensure optimal recovery of lower coal seams. The selection and scheduling of dragline unit(s) is a comprehensive problem that should be resolved through detailed optimization models and appropriate research initiatives.

Ancillary operations in overburden removal, such as cast blasting and dozer-ripping to complement the extraction operations of draglines are also worth investigating. As the geologic conditions become more complex, the dragline reach and geometry require sufficient technical improvement to increase productivity. Since the dragline geometry is fixed, expected productivity is reduced due to varying overburden, coal seams and inter-burden depths. Even though there are several methods to increase the efficiency of stripping operations (improved design of dragline components), dragline productivity improvement by modification of the digging method is the most economic and efficient option (Demirel and Frimpong, 2009). Subsequently, optimal ancillary equipment selection and interaction with dragline digging dynamics could result in economic and efficient waste extraction sequence in MSFs.

Mine plans may also specify material schedules and sequencing that might require frequent movement of large equipment. In the occurrence of such sub-optimal excavation schedules and sequencing, equipment utilization is greatly reduced due to the frequency and the length of long deadheading periods, when the unit is unproductive. On the average, draglines take a step of approximately 6.56 feet within a period of 0.75 – 1 minute (Erdem and Düzgün, 2005).

Another challenge is the spoiling dynamics. The dynamics of material dumping is conditioned by the available spoiling area, operational safety, environmental constraints and production requirements. Major economic problems (loosing coal seams) may occur if the stripping and dumping dynamics are not optimally sequenced.

In MSF mining operations setups, optimal decisions regarding equipment selection and material schedules must be based on multivariable input constraints. These variable constraints are subject to future uncertainties, which might render an entire project uneconomic in the long term. Stochastic models are therefore required to completely define the underlying uncertainties associated with input parameters in these operations.

To develop the proposed optimal economic models, comprehensive stochastic-optimization formulations, provide a generic platform to simulate different scenarios.

Adequate knowledge of the challenging nature of MSFs provides understanding for improving the productivity of draglines while different scenario simulations offer a means to evaluate different operating conditions.

1.3 OBJECTIVES AND SCOPE OF THE STUDY

The primary objective is to maximize dragline and ancillary equipment productivity improvements and the associated economic benefits in MSF surface mining operations. The main components of the stochastic-optimization models include: (i) non-linear programming models for equipment allocation and material scheduling; (ii) stochastic-optimization models for equipment allocation and material scheduling; (iii) simulation of these models to produce a series of optimal solutions for different scenarios; and (iv) comprehensive risk analysis of the optimal solutions.

This work is limited to stochastic-optimization modeling of resource allocation and material excavation scheduling in MSFs using non-linear programming, Lagrange Parameterization, Monte Carlo and Latin Hypercube simulation techniques.

The developed models are verified and validated using a case study of a typical thermal coal producing mine with two fairly horizontal coal seams. The stochastic analyses are limited to the coefficients in the objective function models. Model experimentation is limited to different equipment capacities and variable overburden/inter-burden thicknesses. The direction of mining advancement and stripping is also assumed to be predefined. All analyses and discussions are limited to thermal coal seams and the United States economy.

1.4 RESEARCH METHODOLOGY

The research methodologies include in-depth analytical literature review, stochastic-optimization modeling, numerical modeling, computer simulation, risk profiling, and a comprehensive analysis of simulated results. The rationale for the research study is established from the detailed analytical literature review.

The generalized optimization models comprise the objective function and constraint modules. The objective function consists of profit maximization platforms. The constraint modules are divided into three sections: (i) physical, (ii) chemical and (iii) economic. Generalized Lagrange multiplier methods in association with the generalized reduced gradient algorithm are used to solve the optimization models.

Using the base case optimization models, stochastic models are developed, exploiting the intrinsic dynamics of the multivariate input parameters. Monte Carlo and Latin Hypercube simulation techniques in the SOLVER/@RISK software environments are used to simulate the stochastic models. The results obtained from the stochastic models are used to characterize the risk associated with the equipment productivity models.

1.5 SCIENTIFIC AND INDUSTRIAL CONTRIBUTIONS

This study contributes significantly to the existing body of knowledge and advances the frontiers in MSF excavation using stochastic-optimization modeling. The research work has formulated mathematical models of the excavation and spoiling dynamics, resource allocation, and material scheduling dynamics. Subsequently, the mathematical formulations are tailored towards improving the mechanics of dragline productivity in complex multi-seam coal formations. It also advances the body of knowledge governing the efficient material extraction for meeting downstream customer specifications of coal products.

Consequently, the optimal model also incorporates stochastic analyses to evaluate the risk associated with variable constraints. The optimal models result in significant benefits to multi-seam coal mining industry, such as minimizing rehandling, optimal coal extraction, desired coal quality attainment and optimal equipment sequencing and scheduling.

The study also advances the stochastic-optimization process in mine planning and scheduling algorithms, to efficiently capture future uncertainties surrounding multivariate random functions. The developed models and the resulting analysis form the basis for developing comprehensive economic models for MSF excavation. The research findings can also be extended to commercial mine planning and scheduling software for maximizing the efficiency and economic benefits of dragline operations.

1.6 STRUCTURE OF THE THESIS

Section 1 has provided the introduction of the research study. Section 2 contains the literature review. This section also identifies the various complexities in MSF excavation. Section 3 comprises the optimization models. The stochastic formulations are presented in Section 4. Section 5 comprises the computer models and the experimentation setup for each of the optimization models. Various matrices and flowcharts of each of the numerical solution models are also presented in this section. Section 6 focuses on the results analysis and discussion. Section 7 discusses the main conclusions of the research study and recommendations for future work. Finally, the reference section contains the bibliographic list of the comprehensive literature review.

2. LITERATURE REVIEW

The optimization algorithms investigated include: (i) linear programming, (ii) non-linear programming, (iii) mixed integer programming, and (iv) stochastic programming. The literature review also focus on application of these theories for optimizing material extraction in mining operations. This Section also contains the general description of multi-seam mine layouts, stripping methods and equipment allocation. Throughout the literature survey, the rationale and fundamental contributions of this MS research work has been completely established. All symbols, variables and abbreviations are explained in the nomenclature section.

2.1 EXTRACTION GEOMETRIES AND STRIP MINE LAYOUT

The geology of a mine layout is among the major determining factors in selecting appropriate stripping methods. The most common methods include: (i) simple side casting; (ii) chop cutting; (iii) extended benching; (iv) pull back mining; (v) terrace mining; and (vi) contour mining (Frimpong, 2011). Each of these methods can be modified for specific mining geometries.

2.1.1 Stripping Methods. The most common dragline method in singly-thin seam mining is the simple side cast. In this method, the dragline excavates from a bench immediately above the coal seam. Material is spoiled directly into the available space created by previous cuts. Material re-handling is often prevented by maintaining a cut width less than 40m (Satyanarayana, 2012). The advantages of simple side cast include: (i) simple system to adopt; (ii) swing angles can be kept to a minimum; and (iii) no re-handling if the digging depth is greater or equal to the overburden thickness. However, productivity is limited by the dragline's geometry and the overburden depth (Frimpong, 2011).

Chop cutting, is the stripping method usually applied in weak formations. This method is also suitable when the overburden depth is greater than the dragline digging depth. Unlike the simple side cast method, chop cutting method results in increased dragline swing angles ($\leq 180^0$). Increment in swing angles is estimated to result in 60% reduction in productivity (MA et al., 2006; Scott, 2010; and Frimpong, 2011).

Extended bench method involves the extension of the dragline bench towards the spoil pile. This method often results in material re-handling, and the challenge lies in minimizing the re-handled volume. The major advantage of the extended benching is the ability of the dragline to excavate thick overburden depths and spoil material beyond its digging limits (MA et al., 2006; Scott, 2010; and Frimpong, 2011).

An alternative to the extended benching method is the pull back method. This method allows the spoil to build up against coal seams and re-handle it later. However, the dragline swing angle increases to approximately 180^0. The increment in swing angle and the considerable spoil re-handling result in low productivity.

For deeper overburden depths, terrace mining is usually adopted. This method involves engagement of multiple draglines, hence, the economics is a limiting factor (MA et al., 2006; Scott, 2010; and Frimpong, 2011).

Contour mining is usually adopted to recover coal seams along hillsides with increasing overburden depths. The major demerit of contour mining is difficulty in dragline positioning which results in low productivity and unsafe working conditions (MA et al., 2006; Scott, 2010; and Frimpong, 2011).

2.1.2 Dumping Dynamics. Typically, overburden and inter-burden materials are dumped internally or externally. Although external dumping is not environmentally friendly, it is a necessary step in creating the initial access point for internal dumping

(Vasilyev et al., 1999; and Zaitseva et al., 2007). As shown in Figure 2.1, external dumps are created at locations away from the excavation domain. Internal dumps are created by in-pit dumping. However, internal dumping activity must be concurrent with the dynamics of mining advancement.

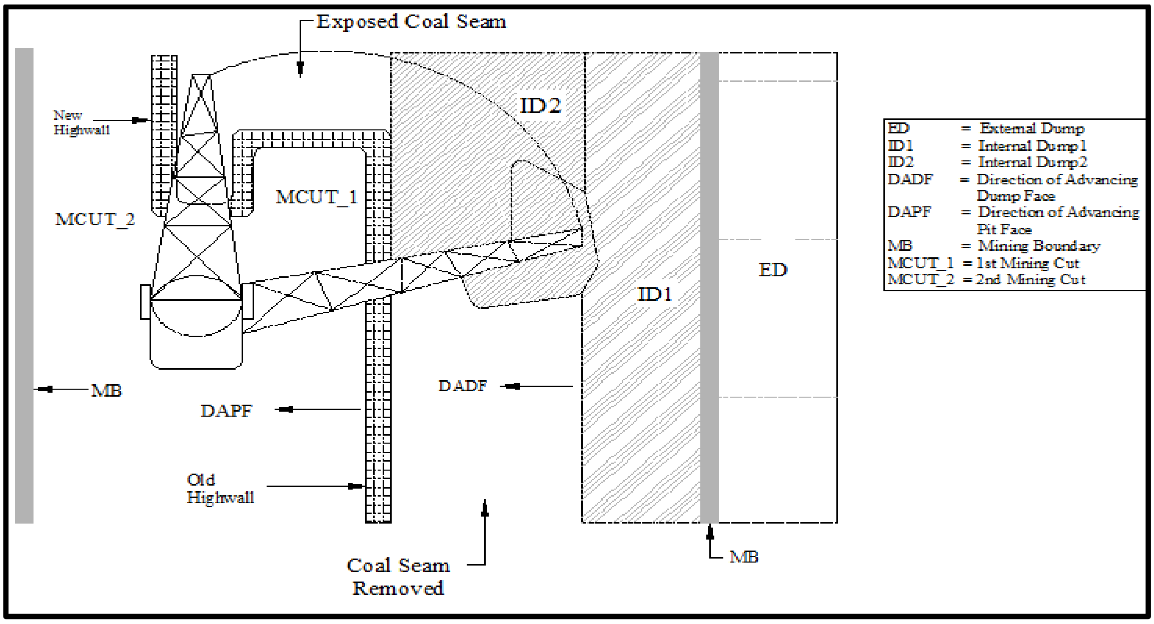


Figure 2.1 External and Internal Dumping Dynamics

Generally, increment in dumping capacity slacks at the beginning of new mining-cuts and also as a result of slower dumping advancement of preceding cuts. Internal dump capacity normally depends on the height of the dump layers, minimum permissible width of the operating floor of the dump, operational safety, dump slopes, economics and the excavation mechanisms. Dump failures possess environmental challenges and also affect coal recovery, mine safety and mining cost. Factors such as geometry and strength of the dump material, hydrogeological condition, load bearing capacity, and external load conditions affect dump stability (Kainthola et al., 2011). Draglines and other ancillary equipment operating from spoil piles, subject these dumps to external loads.

2.1.3 Multi-Seam Strip Mine Layout. A representative block model of MSF is obtained by precise modeling of the different structural characteristics. The material characteristics of these types of deposits include: topsoil, overburden, inter-burden and coal seams. A geologic block model of a typical MSF is illustrated in Figure 2.2. The complex structural geology of MSFs results in stripping ratio terms that must be accurately defined. This definition is often directed to meet specific needs of the excavation economics. The overall stripping ratio, sr_T , is given by equation (2.1).

$$sr_T = \begin{cases} \dfrac{\sum_{i=1}^{n} v_{wi}}{\sum_{i=1}^{m} v_{csi}} & \forall \left(\dfrac{bcm}{bcm}\right) \\ \dfrac{\sum_{i=1}^{n} v_{wi}}{\sum_{i=1}^{m} t_{csi}} & \forall \left(\dfrac{bcm}{ton}\right) \end{cases} \tag{2.1}$$

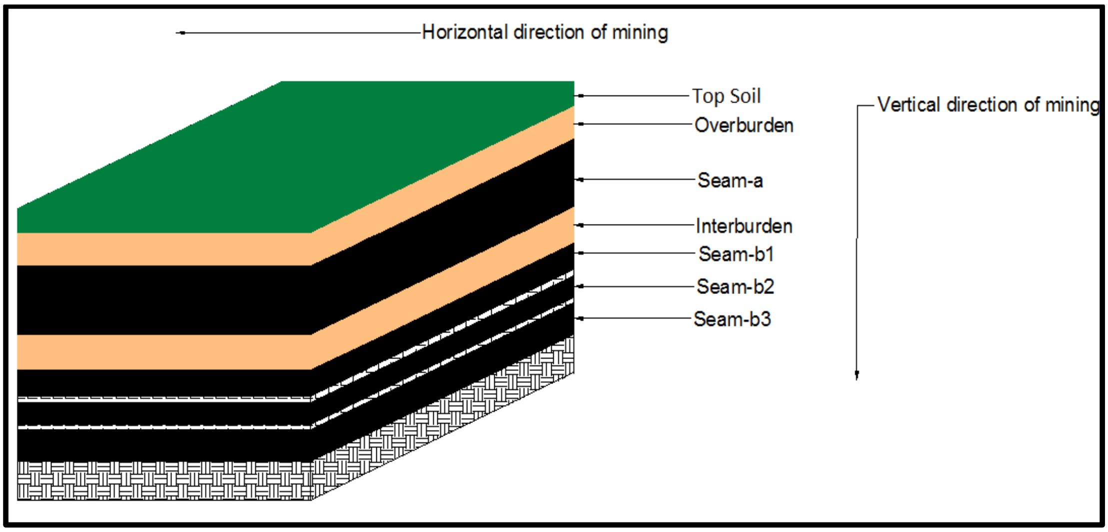


Figure 2.2 Block Model of an MSF

The determination of an economic stripping ratio (ESR) in MSFs comprises practices that minimize re-handling, optimize equipment interaction and maintain quality coal products. In this study, the ESR considers coal recovery and mining cost as dynamic variables determined by material characteristics; dragline reach and geometry; and the strip mine geometry.

The main challenge in scheduling dragline operations in MSFs is the dragline reach geometry (refer to Figure 2.3). Assuming the toe of the spoil pile is allowed to rise up to a height, h from the base of the coal seam, as in Figure 2.4, the capacity of the spoil pile is increased and the reach factor (rf) is reduced by an amount equal to the increase in volume or the horizontal change (Equation 2.2)(Frimpong, 2011).

$$rf = d\left[\frac{1}{tan\beta} + \frac{sf}{tan\theta}\right] + \frac{w}{4} - \frac{h}{tan\theta} \tag{2.2}$$

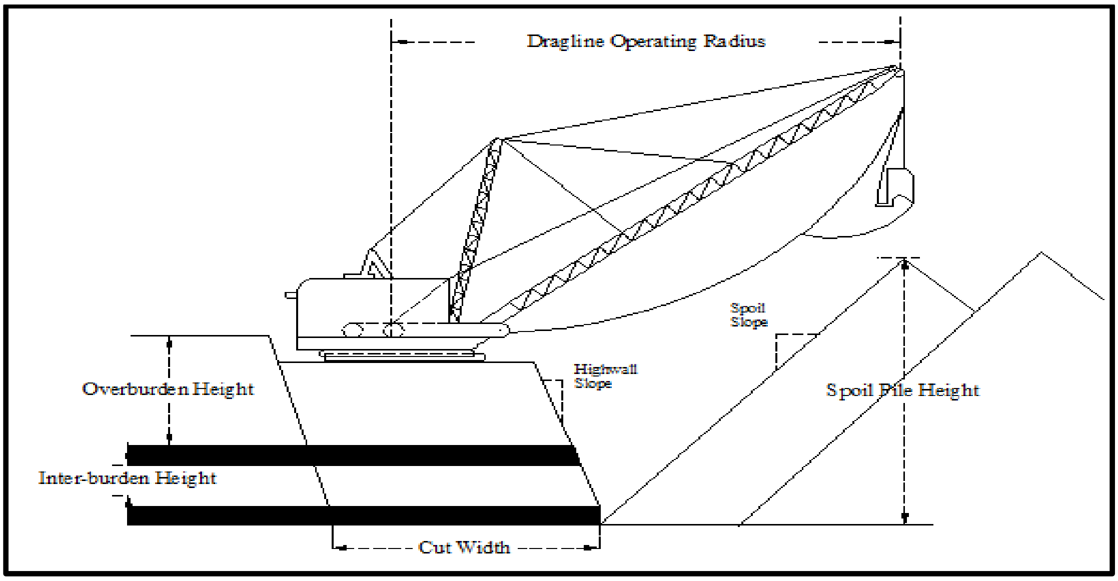


Figure 2.3 Dragline Reach Geometry

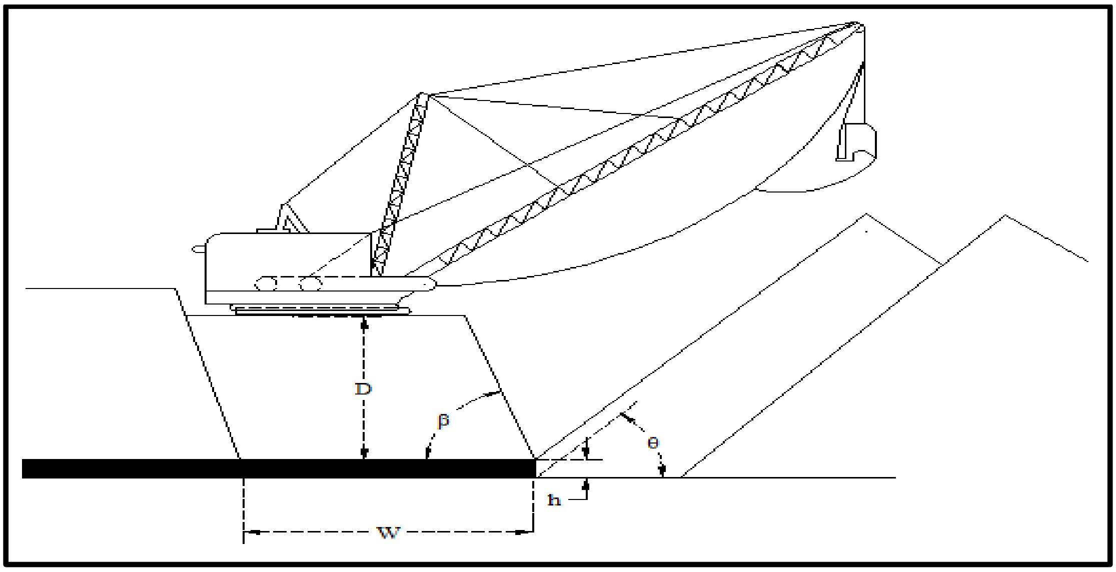


Figure 2.4 Dragline Material Re-handling Geometry

The overburden depth, d is the most sensitive parameter (Refer to Table 1.0 and Figure 1.0 in Appendix A). Hence, dragline operating dynamics in varying and large overburden must be optimally scheduled.

2.1.4 Resource Allocation in MSFs. Resource allocation problems in MSFs, like any other mining optimization problem, are governed by equipment selection and interaction, periodic productivity budgets, formation geometry, economics, and safety conditions. Material formation geometry presents structural uncertainties, which must be adequately defined by detail geological modeling. The geological models aid in determining accurate equipment-formation interactions and in effect, lead to proper equipment scheduling. In inclined MSFs, where material volume changes for every cut, equipment scheduling becomes more challenging, hence subjective decisions to this problem may be economically inefficient.

Draglines are the cheapest cost predominant equipment used in overburden stripping when the draglines' physical capabilities match the deposit's characteristics. This cost however increases significantly when the deposit's physical capabilities alter from the physical limitations of the dragline (Scott et al., 2010). An integrated system of ancillary operations is usually employed when the physical limitations of the dragline is exceeded.

Westcott et al. (2009) illustrate the following restrictions to achieving low-cost dragline excavation: (i) large deposits to ensure adequate strip length and sufficient reserves to justify the capital expenditure; (ii) gently dipping deposits, due to spoil instability on steep dips; and (iii) shallow deposits (draglines are restricted to 50 - 80 meters of overburden due to reach and dump height limitations).

Pre-stripping with alternative equipment is used to support dragline operations in thick overburden depths. Dozers, shovel and truck, and cast blasting are some of the alternatives. Shovel and trucks are flexible mining methods, suited for complex geological deposits, varying overburden depths and thickness, and smaller deposits (Westcott et al., 2009). This method also hauls material outside the digging domain to prevent spoil re-handling. Aiken and Gunnett (1990) stated that the shovel and truck system is used to excavate the upper and thinner overburdens within a deposit, while the dragline is engaged in deeper overburden.

Cast blasting moves approximately 25% to 50% of material without the engagement of other stripping equipment. Dozer push operations can follow cast blasting to achieve efficient excavation. This combination can excavate approximately 60% to 80% of overburden (MiningInfo, 2013). The cast blasting efficiency can be estimated using equation (2.3).

$$cast\ blast\ efficiency = 57.5\left(\frac{d}{w}\right) + 18 \tag{2.3}$$

2.2 CHALLENGES OF MSF EXTRACTION

Excavation in MSFs involves inter-relating variables, which influence economic decisions. The criteria for these economic decisions can be classified into three categories including: physical, economic and chemical (Falkie and Porter, 1973). The evaluation of these categories summarizes the extraction challenges.

2.2.1 Physical Variables. The physical challenges are divided into machine design, mine production and layout, external factors, and geological variables. The productivity of draglines is directly influenced by the mine layouts, pit geometries and the selected excavation method. Similarly, geologic characteristics define the stripping efficiency. As the geologic conditions become more complex, the dragline reach and geometry require sufficient technical improvement to increase productivity.

Spoiling dynamics is another major challenge in MSF extraction. During internal dumping, the mining dynamics and economics affect the dump capacity; the variability of which is achieved by changing the width of the operating floor (Zaitseva et al., 2007). Major economic problems (loosing coal seams) may occur in excavation of MSFs if the stripping and dumping dynamics are not optimally sequenced.

Material re-handling in strip mines could be controlled at the mine planning stage by selecting an appropriate excavation/spoiling dynamics which equipment selection and scheduling play an important role in. Wider pits are normally created in thick overburden formations to reduce re-handling and dragline walking time (Frimpong, 2011; Satyanarayana, 2012). In such situations, the main challenge is the increase in swing angles. Increasing swing angles result in reduced productivity, high maintenance cost and high clean-up time (Frimpong, 2011).

2.2.2 Economic Variables. Geologic uncertainties of MSFs and the stochastic nature of economic variables, result in difficulties during excavation schedules. A stable economic model in this domain thus demands the characterization of geologic uncertainties and a detail analysis of dependent economic variables.

The economic variables are classified as follows: (i) economics of mining lower seams; (ii) market demand and supply; (iii) economics of equipment selection and scheduling; (iv) cost of capital; (v) controllable operating costs; (vi) non-controllable operating costs; (vii) marketing and transportation costs; (viii) commodity price; (ix) production rate; and (x) capital expenditure (Falkie and Porter, 1973).

The major challenge is the determination of quantitative relationships that incorporate the physical parameters of the deposit. Similarly, the economic model must also define the stochastic nature of these parameters. Hence, from the economic standpoint, the limiting factor is the ESR as discussed in Section 2.1.3.

2.2.3 Chemical Variables. The quality of final coal products is controlled by chemical variables. These variables may include: ash, moisture, sulfur, volatile, calorific value (BTU), and fixed carbon content. In MSFs, these variables can occur randomly over the coal domain due to alteration zones and general morphological characteristics of the deposit.

Contrary to open pit mines, the large size and geometry of strip mine designs render selective mining unlikely. Thus, blending of different coal products to achieve desired quality is mostly conceivable at the processing stage.

However, initial mine design and material scheduling models should incorporate the knowledge of the quality trend of the formation. Due to geologic uncertainties of MSFs, incorporating chemical and physical variables into the overall economic model require a detailed model. Subjective approach obviously at this stage is very inefficient to meet these challenges.

2.3 PREVIOUS RESEARCH INITIATIVES AND OPTIMIZATION ALGORITHMS

2.3.1 Linear Programming (LP). An LP algorithm is defined by linear functions of unknown variables subject to a set of constraints, which are also linear equalities or linear inequalities in the unknown variables (Hillier and Hillier, 2010; Winston, 1994). LP problems were first shown to be solvable in polynomial time by Khachiyan (1979), and the concept has been adopted as a considerable field of optimization for several reasons (Fagoyinbo et al., 2011).

Erlandsson (1972) applied LP to investment analysis for an iron ore market. The objective was to obtain an optimal blend of ore product from different plants which met market specifications. This objective was achieved through: material balances, blending conditions, production capacities and market demands. Changes in total profit resulting from the different investment alternatives, together with investment and fixed costs, were used in a series of investment analyses. However, the analyses were based on single value inputs for stochastic input variables hence ignoring future uncertainties.

Falkie and Porter (1973) tackled the dynamics of economic decision-making in MSFs. The research led to the development of an economic decision-making model to aid operators and pit geologists in areas where selective mining was specially required or preferred. However, this work did not incorporate the detailed quantitative relationships for the various decision-making variables.

Bott and Badiozamani (1982) developed LP algorithms to optimize the mine planning, sequencing, and blending activities in MSFs. Due to the varying individual coal seam qualities, the authors identified coal blending as the optimal decision in meeting specified quality standards. The LP formulations included: pit geometry, material extraction, coal blending, and maximum resource utilization. However, transportation constraint and equipment selection were not captured. The economic impact of equipment selection and scheduling is vital in assessing feasibility of projects.

Olsson (1983) described the production operations in MSFs. The author identified major complexities in dragline operations as: careful planning, close dragline supervision and dragline movement. The material removal sequencing and equipment selection processes described by Olsson were based on heuristic algorithms and managerial inferences, which were complicated with numerous uncertainties. Stochastic simulation modeling is required to capture the uncertainties.

Gershon (1983), defined a more comprehensive LP model to solve some limitations of mine scheduling models. Gershon identified that most of the mine scheduling algorithms were not generic and hence their applications were limited. Also, existing optimization models compromised on valuable sectors in optimizing specific aspects of the mining operations. The modified algorithms thus incorporated market/production interactions, life of mine, and overall economic plans. In applying Gershon's generic LP models to MSFs however, specific equipment selection and sequencing must be thoroughly and independently addressed in the model formulations.

Tanaino et al. (1986) investigated open pit mining of a series of slightly inclined coal seams with temporal internal dumps. The authors developed mathematical algorithms to analyze the economic feasibility of mining a series of slightly inclined coal seams. This work had led to a reliable foundation for choosing mining operation technologies on the basis of current expenses and equipment costs. However, the excavation sequence must be carefully selected to minimize re-handling of material and losing of coal seams due to inaccessibility.

Tan and Ramani (1992) compared the mathematical feasibility of LP and dynamic programming (DP) models for open pit mine scheduling. The main objective of the mine scheduling problem was to find an ore production curve below the upper bound function of ore production and a stripping ratio curve. The major constraint established by the authors was that the optimal curve had to be within the feasible regions of ore and waste production that maximized the profit of the entire project. The optimal solutions obtained indicated that the LP models were more flexible than the DP models and could easily be formulated to define any mining geometry.

Ray et al. (1999) discussed the economic gains of cast blasting technique for overburden excavation, to using dragline or shovel/truck systems. The authors stated that cast blasting minimizes considerably the overburden handling cost. Hence an optimization model is necessary to investigate this statement and also determine optimal equipment schedules.

Awuah-Offei et al. (2003) emphasized the importance of predicting equipment needs well in advance of mining activities. Mining data collected were used to construct a simulation model of the haulage system in SIMAN simulation environment. The study was limited because the authors considered only the technical evaluations of the project. Also, the work did not include improving the productivities of the equipment within the mining domain.

Zaitseva et al. (2007) formulated LP models to analyze the effect of mining sequence on internal dump capacity. The pit development was modeled in spatial rectangular coordinates, and vertical planes were used to simplify the model. The dump development was modeled by the working flanks, functional relationship between the stable slope angles, and the geometry of the lower beds. However, quantitative relationships between equipment operating geometries and excavation dynamics were not established.

Zhou et al. (2007) studied the interaction between working bench advancement and stripping volume with variations in coal seam thickness. The model consisted of the annual mining advancement distance, annual stripping volume, maximum bulldozing depth, dragline bench height, and production reliability of the dragline mechanism. The authors concluded that dragline-bulldozer system offered greater flexibility in varying coal seam thicknesses than a pure dragline system. To expand this work, a generic optimization framework for optimal equipment selection is required.

Zhenming et al. (2011) used Visual Basic. NET and MATLAB to model a production scheduling optimization system for a coal mine. LP models were developed to optimize production plans, mineral processing and transportation schemes of the mine. The authors, however, eliminated the stochastic nature of the variables that defined the mine production schedule and economic plans.

Zhu et al. (2012) applied LP to optimize the mine plan of an iron ore mine. The LP algorithm was used to find optimal mine plan for a joint open pit and underground operation. The main objective was to increase the overall economic gain from the mining operation during the surface-underground transition period, by establishing mine plans with synthesized optimal results. The results showed optimal economic gains which was used to control the quality and quantity of the ore output. However, the uncertainties surrounding the input parameters were not defined which result in sub-optimal solutions with inherent risks of failure.

2.3.2 Non–Linear Programming (NLP). LP and NLP problems have always been treated separately; however, their methodologies have gradually become similar. One of the techniques commonly used to formulate constrained NLP problems is the Lagrange Multiplier Method (LMM). Lagrange multipliers are presented in a framework of differentiable functions, and are used to yield constrained stationary points. Their validity or usefulness often appears to be connected with the differentiation of the functions to be optimized. The usefulness of LMM for optimization is not limited to differentiable functions. Arbitrary real valued objective function can also be optimized over any set of variables using LMM (Pop, 2002; Evtushenko, 1977; and Little, 2008).

Everett III (1963) developed theorems to prove that the use of LMM constitutes a technique whose goal is maximization rather than location of stationary points of a function with constraints. He also illustrated that there were no continuity or differentiability restrictions on the functions to be maximized. The application of LMM could be extended to discrete, continuous, numerical or non-numerical functions.

Albach (1967) developed NLP optimization model for multi-stage production plans. Geological information of the orebody was the major source of uncertainty identified. Incorrect boring results and geostatistical analyses between boring points were some of the sources of uncertainties identified by the author. A chance-constrained programming problem was formulated to define the interaction between the production and investment plans. The model maximized a linear function subject to linear and non-linear constraints. However, this study did not consider the uncertainties with equipment scheduling.

Thomas et al. (1972) applied a non-linear automatic history matching technique for reservoir simulation models. The technique was based on Gauss-Newton least-square algorithm. The authors used this technique to automatically vary the reservoir parameters in other to obtain values representing the different field performance. The Gauss-Newton algorithm was used to match both linear and non-linear problems in a reasonable number of reservoir simulations. The results were compared to existing history matching techniques. The authors concluded that Gauss-Newton technique provided equivalent properties in fewer simulations ran. However, optimal solutions provided by Gauss-Newton's technique generally depends on the choice of initial values, hence the estimated uncertainty information is often inaccurate or insufficient (Chen et al., 2008).

Dagdelen and Johnson (1986) used Lagrange theory and parameterization concept of mining operations to optimize production schedules in an open pit mine. The authors identified that with the application of the new optimization algorithm, different scheduling conditions converged faster. However, the problem could be handled by any ultimate pit limit (UPL) algorithm such as Lerchs-Grossman (1965) graph theory based algorithm (Sattarvand and Niemann-Delius, 2008; Caccetta et al., 1998). The uncertainties surrounding the stochastic parameters were also not considered. Caccetta et al. (1998) extended this further by using subgradient optimization method, subsequent to applying Lagrange Multipliers to eliminate mining and milling constraints.

Gallagher et al. (1991) developed a new technique in locating an optimal model which was superior in performance to Monte Carlo techniques. The authors illustrated that, in providing a method for solving non-linear optimization problems, Monte Carlo techniques avoided the need for linearization. However, in practice, this technique is often prohibitive because of the large number of models that must be considered. A new class of methods, genetic algorithms (GA), has recently been devised in the field of artificial intelligence to curtail this problem. GAs', like the Monte Carlo methods, are completely non-linear, use random processes and require no derivative information.

Even though GA yields more efficient results, Monte Carlo integration is still regarded as an effective method for subsequent model appraisal. A major limitation of genetic algorithm is premature convergence and identification of fitness functions (Baluja and Caruana, 1995).

Zhao and Kim (1992) extended the work by Dagdelen and Johnson (1986) by demonstrating the absence of optimal production schedules using Lagrange parameterization. The authors illustrated this limitation under two conditions: (i) occurrence of duplicate optimal solutions for given scheduling periods, and (ii) stripping required beyond the minimum defined by the slope angles. Since the relaxed problem was solved by ULP algorithm, the convergence to optimal solutions under the two conditions was eliminated. The authors also showed that, the algorithm picks up all redundant optimal solutions simultaneously given a small Lagrange Multiplier and vice versa. The outcome of this study shows some limitations of the Lagrange parameterization technique.

Pendharkar and Rodger (2000), developed a generalized NLP model to eliminate excessive scheduling and inventory problems in coal industries. The authors considered production cost as a non-linear function of production volume. This technique provided better profit margins when compared to LP models. The model was limited to very simple cases, fails to address the stochastic nature of the variable input parameters, and also neglected optimal equipment selection.

2.3.3 Mixed Integer Programming (MILP). Optimization models normally may have both fractional and integer variables. Such a model is referred to as MILP when the objective function and the constraints are linear in nature (Hillier and Hillier, 2010; Winston, 1994). Though mixed integer non-linear programs (MINLP) also exist, these problems involve rigorous computational intelligence to reach optimality. As in the case of other operations research approaches, MILP have been used to solve mine production and resource scheduling problems. MILP models can be used to model diverse mining constraints such as selective mining, multiple equipment tasking, multiple ore processors, multiple material stockpiles, and blending strategies (Schouwenaars et al., 2001; Little et al., 2008; and Caccetta and Hill, 2003).

Muckstadt and Wilson (1968) applied MILP duality to schedule thermal generating systems. The authors presented a decomposable mixed-integer programming model for simultaneous economic consideration of unit commitment and short-term dispatch of thermal power generating equipment. The optimization model was developed to define the demand forecast as a discretize function that allowed a probabilistic estimate to be incorporated in the scheduling model. This attempt characterized the uncertainties surrounding the variable demand parameter of the system. The algorithm was extended to multiple periods and included time dependent constraints, which resulted in more practical optimal solutions. Even though duality is an important concept in constrained optimization, conventional duality theory leads to gaps for non-convex optimization problems (Chen, 2007).

Barbaro and Ramani (1983) used generalized multi-period MILP to model production schedules and processing facilities selection. The object function was to determine the best use of fixed resources in production schedules for multiple production units supplying different markets. The authors however, assumed linearity and certainty of all relationships. Further research is required to characterize the uncertainties with the multivariate random parameters.

Winkler (1996) showed the huge economic impact for using fixed cost components in complex mine planning and scheduling. The author used MILP as an extension to the LP model. The new MILP formulations were used to model successfully an underground hard rock coal mine. By these formulations, the variable and fixed cost components of the project were captured entirely. Thus, the economic impact was defined more efficiently. However, using MILP to model periodic fixed cost increases the computational complexity of the problem. The flexibility of the model to general optimization problems must also be ascertained.

Liu and Sherali (2000) applied MILP to solve coal shipping and blending problem for an electric utility company. Even though the solution procedure used heuristic approach in conjunction with branch-and-bound methods, it failed to characterize future uncertainties surrounding the various input parameters. The economic decision was made with single values representing stochastic parameters, a methodology which is inappropriate and inefficient.

Schouwenaars et al. (2001) proposed a new approach for planning optimal fuel paths of multiple vehicles using MILP. The new formulation incorporated directly, collision avoidance as mixed integer/linear constraints. The authors demonstrated that receding horizon strategies aimed at computing complete trajectories, can lead the system to unsafe conditions. The introduction of stochastic techniques to capture the uncertainties exhibited by the variable input parameters will make this new proposed algorithm more comprehensive and practically efficient.

Rahal et al. (2003) investigated the use of MILP for long-term scheduling in block caving mines. The effect of interferences in the mining cycle due to equipment breakdown and poor draw management were minimized. The main objective was to achieve production target while the deviation from the ideal draw profile was minimized. Results showed optimal production schedules and material drawing mechanisms however, the variability of the geologic parameters of the orebody and the mine operation parameters was not accounted for.

Caccetta and Hill (2003) applied branch and cut algorithm to open pit mine scheduling. An optimal production schedule over the life of a complex deposit for an open pit mine was determined. The results would be more comprehensive if the risk associated with variable parameters in a typical complex mining environment was characterized and implemented in the optimization algorithm.

Ramazan and Dimitrakopoulos (2003) developed an alternative MILP algorithm to tackle some of the limitations in traditional mixed integer programming (MIP) for scheduling excavation of multi-element deposits. Some of the limitations of the traditional MIP included: (i) infeasibility in generating optimal solutions with practical mining schedules; and (ii) inability to deal with in-situ variability of orebodies. Uncertainties surrounding the variability of orebodies were minimized by recourse parameters and probabilistic optimization. The incorporation of the dynamics of equipment selection and sequencing would make this algorithm more comprehensive for mine production scheduling.

Ramazan (2007) identified that mine production scheduling problem was typically an MIP type problem. However, the large number of integer variables required in formulating the problem made it impossible to solve. To overcome this limitation, the author proposed a new algorithm termed “Fundamental Tree Algorithm (FTA)”. The new algorithm was based on LP to aggregate material blocks and decreased the number of integer variables as well as the number of constraints required within the MIP formulation. After generating the fundamental trees for a given mineral deposit, an MIP model modified from traditionally known MIP formulations, was used to generate annual production schedules. Further research is thus necessary to improve on this proposed algorithm to make it more generic and applicable to practical situations.

Little et al. (2008) developed a new MIP model for production scheduling optimization in mining sublevel stope. The new algorithm was developed to reduce the excessive solution times (exponential increment) of MIP. The authors reviewed existing optimization models regarding production scheduling and proposed a classical MIP model that generated optimal results with less computational times. The model however excluded stope grade variability and identification of other stochastic processes.

Chicoisne et al. (2009) proposed a new decomposition method for the development of precedence constraint in large production scheduling. They extended the pioneering works of Dagdelen and Johnson (1986) by exploiting the structural properties of the optimal multipliers, and supplementing LP algorithms with heuristics to construct feasible solutions which approximate the bounds. However, the solutions obtain by the authors may not be ascertain since it is likely that the blocks scheduled to be mined in same time periods may be scattered throughout the geologic block model.

Askari-Nasab et al. (2010) applied MILP to large-scale pit production scheduling. The authors applied clustering algorithm to address the problem of dealing with numerous blocks by aggregating blocks into larger units. However, it is imperative for future research to develop and test different clustering techniques as well as the extension of the MILP framework to highlight stochastic characteristics of the input variables.

2.3.4 Stochastic-Optimization (SOP). Variables, such as commodity price, mining costs, geological trends of coal seams, and equipment periodic efficiency parameters are subject to uncertainties over the life of the mining project. Since the inception of SOP in the 1950s, it has continuously gained more popularity. SOP minimizes future unexpected occurrences by taking into account all possible future outcomes. It assumes that the optimal solution obtained is valid for all situations with evolving uncertainties. In the long run, this process achieves a better decision than considering only specific scenario(s) (Winston, 1994; Leite and Dimitrakopoulos, 2007; and Richmond, 2011).

Several authors have applied the concepts of stochastic simulation and SOP to engineering problems. In stochastic simulation, experiments are driven by randomly-generated input parameters defined by probability distributions. Output functions are then computed based on the selected inputs repetitively. The SOP technique couples the stochastic process with the optimizer such that optimal solutions are obtained for every multivariate random scenario. The variances of the expected values are minimized in these approaches.

Shih and Frey (1995) used a multi-objective chance-constrained optimization model to determine optimal coal blends. The coal quality parameters were treated as normally distributed random variables. The limitation of the traditional LP approach was mitigated by defining the intrinsic variability of coal characteristics. To extend this work further, coal blending can be considered during excavation, and the geologic variability (formation thickness, faults, coal seam inclinations, etc.) should be included.

Frimpong et al. (1998) applied geometrical, numerical and stochastic modeling techniques to model a mine production plan. The main objectives were to minimize total production cost and maximize operating profit for all active mining faces in the multi-bench operation. The study led to the development of a multivariate optimized pit shells simulator (MULSOP). Latin hypercube simulation technique was used to simulate the geometric models of the pit shells under different economic conditions. Stochastic and numerical modeling techniques were used to capture the uncertainties surrounding the geometric models. Even though stochastic processes capture uncertainties of random multivariate fields, as noted by the authors, the implementation of the model will require practical analyses of the field parameters.

Frimpong et al. (2002) expanded initial work in optimizing mine production plans to develop an SOP annealing optimizer. The new algorithm defined the stochastic processes governing ore reserves, commodity prices and overall production plans. An intelligent pit optimizer (IPO) was developed by the authors and then used to solve a pit optimization problem. The standard Gauss-Wiener process was used to model the uncertainties associated with the commodity prices. The results, as a form of validation, were compared to the results of 2D Lerchs-Grossman's algorithm applied to the same optimization problem. Neural networks were used for block pattern recognition; areas where the authors suggested further work. The future work will include training neural networks to recognize different geological structures such as faults that may affect optimal pit layouts.

Ta et al. (2005) applied SOP to allocate mine trucks based on truck load and cycle time uncertainties. The results showed improvement in the truck dispatch system by allocating trucks in complex environment. However, the authors omitted the variable nature of the orebody in scheduling shovel allocation for adequate blending constraints. A complete model is thus required to articulate completely the uncertainties surrounding equipment selection and scheduling.

Frimpong et al. (2007) used SOP to optimize a hedging scheme which mitigated risks while maximizing portfolio value. The authors used Weiner process to model the spot commodity price. The hedge position optimization program was however, found to be limited in the sense that the accuracy of the model was dependent on the accuracy of the spot and futures delivery price models. There was also no justification for selecting the Weiner process to model the spot commodity price against other price models.

Leite and Dimitrakopoulos (2007) used SOP and simulated annealing to model the open pit mine dynamics based on geologic uncertainty of a relatively low grade copper deposit. The stochastic results showed significant difference in the overall ore tonnage estimates and net present values compared to the conventional approach. The risk analysis results also showed relative low probability of deviation from the target for the stochastic models compared to conventional approach. However, the differences in the ore tonnages and the net present value can be attributed to differences in cut-off grades. This study showed the significance of stochastic process modeling in open pit mine modeling compared to conventional traditional approach.

Boland et al. (2008) developed techniques to generate multiple stochastic geological estimates that described more accurately the uncertain geology. The authors used multi- stage SOP to capture geological uncertainties. The study showed that nonanticipativity can be modeled with linear constraints involving variables already present in the model. The new stochastic models were logically more valuable in cases where the optimal mining schedules for each scenario, considered independently, showed significant variances. The stochastic model also yielded a higher net present value. However, further research is needed for more comprehensive stochastic models through the use of cutting planes.

Zheng (2010) applied stochastic integer programming to the natural gas industry. However, due to the difficulty in solving stochastic problems, the author proposed embedded Benders' decomposition solution algorithm (BDSA). BDSA applied successively to multi-stage stochastic LP. However, the solution to a multi-stage stochastic problem is limited by the size of the problem.

Richmond (2011) evaluated capital investment timing with stochastic modeling of time-dependent variables in open pit optimization. The author modeled the uncertainties surrounding commodity prices and operating cost using stochastic techniques. In the study, a simple ore scheduler was embedded in a floating cone-biased heuristics algorithm. The results showed that open pit optimization by processing different series of nested pit shells with constant economic and geologic parameters fail to capture future uncertainties. Reducing the computational time is a further ground to investigate in case of a complex multivariate random field.

Dimitrakopoulos (2011) developed and applied the concepts of stochastic simulation and SOP for modeling and integrating orebody uncertainty into mine design, production planning, and estimation of mining projects and procedures. The author identified that non-linear propagation of errors resulting from in-situ variability of orebody grades led to sub-optimal mine plans, bias production forecasts and inaccurate reserve estimates. Sequential simulation using high-order statistics was employed to simulate the deposit attributes because of the methods suitability to complex non-Gaussian spatial and complex non-linear geologic domains. The Results showed significant improvement in production, pit limits and overall net present value of the project. The methodology suggested a potential key contribution to the sustainable utilization of natural resources, supported by stochastic analysis of excavation schedules especially in MSFs.

Gupta et al. (2011) applied approximation procedures for SOP problems by means of a simple sampling base algorithm similar to the work of Agrawal et al (2008). The authors considered multi-stage versions of stochastic combinatorial optimization problems with recourse. The new algorithm sampled the probability distribution of the specific variables and constructs a partial solution as the resulting sample. However, the algorithms depended on the presence of cost–sharing functions with strictness properties and required a cross-monotonic cost-sharing scheme. It would therefore be worthwhile to develop approximation algorithms with fewer requirements on the cost-sharing functions and also minimize the linear loss in the various optimization stages. Immorlica et al. (2005) discussed further the limitations of cross-monotonic cost-sharing schemes.

Adeyefa and Luhandjula (2011) presented a review on multi-objective stochastic linear programming where they investigated previous and current algorithms governing optimization under uncertainties. The authors evaluated probability theories and multicriteria decision analyses in stochastic models. Complex state of combining randomness and multiplicity of objective functions were also investigated. The review also included the methodological approaches for solving multi-objective stochastic linear programs. The results of the investigation showed that SOP was more effective in finding realistic optimal solutions compared to the multi-objective approach. However, the applicability of multi-objective solution technique to linear problems is computationally more efficient. Thus, a hybrid approach which combines the computational merits of the two methods was suggested by the authors.

2.4 SUMMARY

Dragline productivity research initiatives have been carried out since the mid-1970s to optimize operating efficiency (Demirel, 2007). These have led to excavation and spoiling dynamic models developed for flat and inclined multi-seam coal strata. Economic models have also been developed over the years to identify multi-seam mining complexities and examine different excavation alternatives. Emphases in the current literature have been placed on subjective approach to investigating ancillary operations that complement dragline operations.

Despite these improvements, existing models lack SOP based algorithms for efficient decisions in multi-seam coal operations. These models fail to incorporate quantitative relationships of variables in equipment productivity improvement. The intrinsic variability of excavation and spoiling dynamics in MSFs are also not established in existing models. Coal blending activities are currently based on single value inputs, ignoring geologic uncertainties of the deposit.

The research advances SOP methods for improving dragline and ancillary operations in MSFs. The optimization models are formulated based on NLP and SOP techniques. The proposed methods incorporate the challenges and geometries of excavation in MSFs and present a pioneering effort in using SOP techniques to optimize equipment selection and scheduling within the domain of these challenges.

The research study contains original contributions to improving equipment productivity in complex operating conditions. The results obtained help in short-term mine planning and long-term risk characterization of future uncertainties.

3. OPTIMIZATION MODELING OF MATERIALS EXTRACTION

This section contains the model formulations divided into waste excavation and coal seam extraction. The optimization models are formulated based on NLP techniques. The nonlinearity in the optimization model is presented by the coal quality parameters. Refer to the nomenclature for the definitions of the symbols, variables and abbreviations.

The generalized NLP algorithm is shown in equations (3.1) and (3.2).

$$\max\ (or \min)\, z = f(x_1, x_2, \dots, x_n) \tag{3.1}$$

$$s.t \quad \begin{array}{c} g_1(x_1, x_2, \dots, x_n)\ (\leq, =, or\ \geq) b_1 \\ g_2(x_1, x_2, \dots, x_n)\ (\leq, =, or\ \geq) b_2 \\ \vdots \\ g_m(x_1, x_2, \dots, x_n)\ (\leq, =, or\ \geq) b_m \end{array} \tag{3.2}$$

In this formulation, $f(x_1, x_2, \dots, x_n)$ is the NLP's objective function, and $g_1(x_1, x_2, \dots, x_n)\ (\leq, =, or\ \geq) b_1, \dots, g_m(x_1, x_2, \dots, x_n)\ (\leq, =, or\ \geq)\ b_m$ are the NLP's constraints (Winston, 1994).

3.1 GENERALIZED LAGRANGE MULTIPLIER TECHNIQUE

Assuming an arbitrary set of possible strategies S and real valued *payoff* function H, $H(x)$ is interpreted as the *payoff* which accumulates from employing the strategy $x \in S$. Thus, given the objective and the constraint functions in equations (3.3) and (3.4) respectively, there are n real valued functions $C^k[k \in (1, \dots, n)]$ defined on S, called the *Resource* functions. This implies that an amount $C^k(x)$ of the k^{th} *resource* is expended when strategy $x \in S$ is employed.

$$max_{x \in S} H(x) \tag{3.3}$$

$$C^k(x) \leq c^k, \forall k \in (1, \ldots . n) \tag{3.4}$$

3.1.1 LMM Theorem 1. For any choice of non-negative value λ^k, where $k \in (1, \ldots, n)$, if an unconstrained maximum of the Lagrangian function (equation 3.5) can be obtained, then the solution is a solution to that constrained maximization problem whose constraints are, in fact, the amount of each resource expended in achieving the unconstrained solution.

$$H(x) - \sum_{k=1}^{n} \lambda^k \, C^k(x) \tag{3.5}$$

Assuming x^* maximizes the function in equation (3.5), it implies that for all $x \in S$, equation (3.6) is valid.

$$H(x^*) \geq H(x) + \sum_{k=1}^{n} \lambda^k \, [C^k(x^*) - C^k(x)], \forall x \in S \tag{3.6}$$

This theorem is used to investigate the entire spectrum of constraints produced in the course of the solution to the optimization problem (Everett, 1963).

3.1.2 LMM Theorem 2 (Lambda Theorem). Given two optimum solutions produced by Lagrange multipliers for which only one *resource* expenditure differs, the ratio of change in the optimum *payoff*, to the change in that *resource* expenditure, is bounded between the two multipliers that correspond to the changed *resource*. Assuming

λ_1^k and λ_2^k are two sets of λ^{k}'s that produce solutions x_1^* and x_2^* respectively, the Lambda theorem is represented mathematical as follows:

$$\lambda_2^j \geq \left(\frac{H(x_1^*) - H(x_2^*)}{C^j(x_1^*) - C^j(x_2^*)} \right) \geq \lambda_1^j \tag{3.7}$$

Where

$$C^k(x_1^*) = C^k(x_2^*), \forall k \neq j \tag{3.8}$$

$$C^j(x_1^*) > C^j(x_2^*) \tag{3.9}$$

Based on this theorem, the effect of constraint relaxation can be examined. The starting set of multipliers that produce Lagrange solutions can also be identified (Everett, 1963).

3.1.3 LMM Theorem 3 (Epsilon Theorem). This theorem deals with the stability of the LMM. A solution that nearly maximizes the Lagrangian must be the solution that also nearly maximizes the *payoff* for the selected *resource* levels for stability. Hence, equation (3.10) is valid assuming $\bar{x}$ falls within ε of maximizing the Lagrangian.

$$H(\bar{x}) - \sum \lambda^k C^k(\bar{x}) > H(x) - \sum \lambda^k C^k(x) - \varepsilon \tag{3.10}$$

The theories discussed above are used to formulate the waste and coal seam extraction models. It should be noted that the LMM generates a mapping of the space lambda vector into the space of constraint vectors. This implies that there may be some inaccessible regions (caused by nonconcavity in the envelope of the set of achievable *payoff* points in the space of payoff versus constraint levels) consisting of vectors that are not generated by any lambda vectors. Thus, optimum payoff for constraints within such inaccessible regions; can be undiscovered by straight-forward application of the LMM (Everett, 1963).

The initial step in mine scheduling and optimization is the establishment of an accurate geologic block model of the deposit. Regular 3D models are used because of their efficiency with computerized optimization techniques. The block dimensions are determined by the physical characteristics of the coal seams and waste materials. Dip and azimuth, as well as the direction of maximum continuity of the coal seams, define its overall orientation. Numerical techniques such as geostatistics are used to assign the grade (quality) to each fixed-size block.

The objective function is to maximize the profit generated from mining and processing each block. The block economic values are determined by financial and metallurgical parameters. From the block model and pre-defined mining directions, excavation constraints are defined to identify sets of overlying and underlying blocks in each major cut.

The mining blocks are aggregated into larger units to reduce significantly the number of decision variables and constraint functions. The mining block aggregation is based on material type classification and relative spatial locations. It should be noted that, within a particular delineation zone, several strip-cuts can be defined based on the excavation and spoiling dynamics.

3.2 OPTIMIZATION MODELING OF WASTE EXTRACTION

The objective of the waste extraction NLP model is to minimize mining cost. This model includes resource (equipment) allocation for topsoil, overburden and inter-burden. The cost function incorporates the productivity of draglines and ancillary operations as well as the relative spatial locations of each mining strip.

Typical resource allocation dynamics in two seam mining comprises: draglines (operating from high and low walls), dozers, and shovel and truck systems scheduled for waste excavation. The dozer pushes the overburden to expose the first coal seam for extraction. The first inter-burden is fragmented by blasting and a dragline excavates the material to expose the second seam. A second dragline excavates the second inter-burden from the low-wall side of the pit (refer to Figure 3.1).

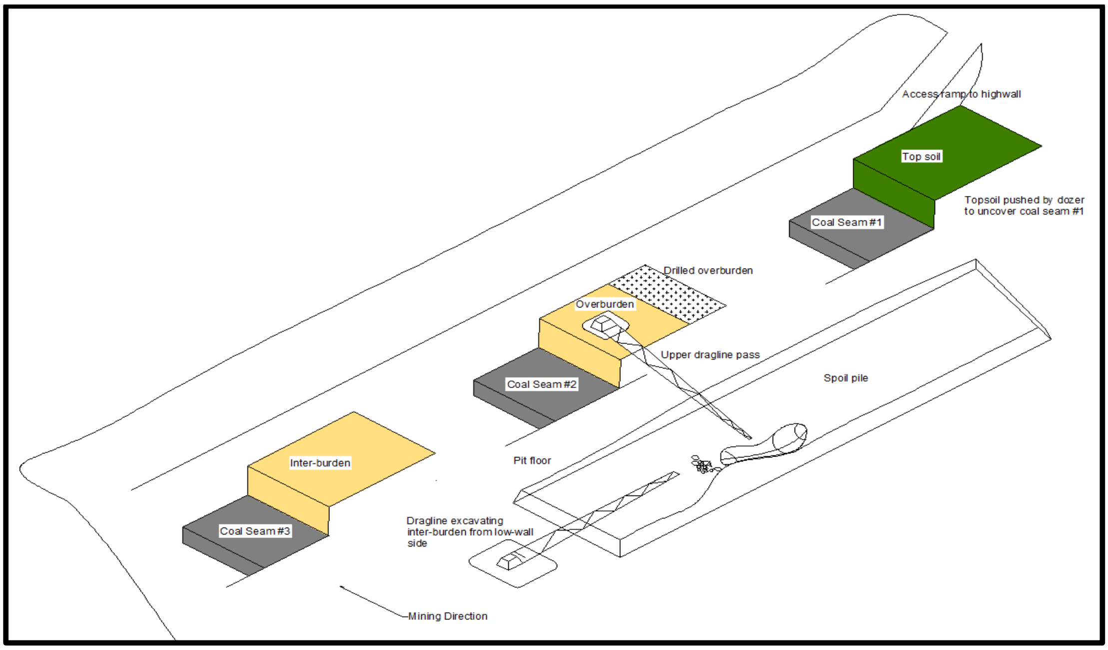


Figure 3.1 Resource Allocation Dynamics in Multi-Seam Formations

3.2.1 Objective Function. The indices in the objective function comprise mining block, $n \in \{1, \ldots, N\}$; resource, $j \in \{1, \ldots, J\}$; scheduling period, $k \in \{1, \ldots, K\}$; and mining-strip, $s \in \{1, \ldots, S\}$. The formulation as shown in equation (3.11) is the discounted cost of mining all the material in block n as waste in period k by resource j.

$$min\ Z(x_n^{j,k}) = min \sum_{s=1}^{S} \left\{ \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{n \in S} [\, x_n^{j,k} \times d_n^{j,k}] \right\} \tag{3.11}$$

$$d_n^{j,k} = wt_n \times c_n^{j,k}, \forall n \in \{1, \dots, N\}, j \in \{1, \dots, J\}, \qquad k \in \{1, \dots, K\} \tag{3.12}$$

3.2.2 Constraint Functions. The constraints include: (i) reserve; (ii) energy consumption; (iii) mining capacity; (iv) spoiling area availability and material rehandling; (v) equipment availability; (vi) equipment utilization; (vii) minimum mining width and resource interaction; (viii) haulage unit capacity and reach geometry; (ix) diggability; (x) labor; (xi) critical bench height; and (xii) non-negativity constraints.

3.2.2.1 Reserve. The reserve constraints are shown in equations (3.13) and (3.14). Equation (3.13) limits the portion of the block excavated by each resource in each period whiles equation (3.14) ensures the full excavation of all scheduled blocks.

$$x_n^{j,k} \leq 1, \forall n \in \{1, \dots, N\}, j \in \{1, \dots, J\}, k \in \{1, \dots, K\} \tag{3.13}$$

$$\sum_{j=1}^{k} \sum_{n \in s} x_n^{j,k} = 1, \forall j \in \{1, \dots, k\} \tag{3.14}$$

3.2.2.2 Energy consumption. This constraint considers the electrical energy and fuel consumptions by the resources during excavation and material haulage, and the chemical energy expended during cast blasting. Equation (3.15) regulates the energy consumption based on a minimum permissible total energy cost.

$$\sum_{j=1}^{J} \sum_{n \in s} \left(\gamma^k \times wt_n \times x_n^{j,k} [\varepsilon_n^{j,k} + e_n^{j,k}] \right) \leq \vartheta^k, \ \ \forall k \in (1, \dots, K) \tag{3.15}$$

3.2.2.3 Mining capacity. This constraint restricts the percentage of waste material excavated to the resource capacities. The minimum and maximum bounds are used to control the overall mining requirements of each resource. The capacities of the various resources are calculated based on their productivities per period and defined in ton-mile to highlight the waste removal sequencing. Equations (3.16) and (3.17) limit the mining capacities within lower and upper bounds respectively.

$$\sum_{n\in s}\left[wt_n \times x_n^{j,k}\right] \geq ml^{j,k}, \qquad \forall j \in \{1,\dots,J\}, \forall k\{1,\dots,K\} \tag{3.16}$$

$$\sum_{n\in s}\left[wt_n \times x_n^{j,k}\right] \leq mu^{j,k}, \qquad \forall j \in \{1,\dots,J\}, \forall k\{1,\dots,K\} \tag{3.17}$$

3.2.2.4 Spoiling area availability and material rehandling. Strip mining activity is such that internal dumping must be concurrent with block excavation (refer to Section 2). Due to this mechanism, dump area availability limits the material excavated. This constraint (equation 3.18) is however, not binding in shovel and truck systems (SHT) since material could be spoiled further away from the pit outline. The fix geometry of draglines results in fix dumping mechanisms, thus spoiling area availability is binding. Material rehandling may occur in dragline operations as the spoiling area decreases.

$$\sum_{n\in s}[wt_n \times x_n^{j,k} \times (1/_{sg})] \leq di^{j,k} + de^{j,k}, \forall k \in \{1,\dots,K\}, \forall j \in \{1,\dots,J\} \tag{3.18}$$

3.2.2.5 Equipment availability and utilization. These constraints are formulated as functions of the total hours a resource is allocated to a particular block; the strip mine geometry; and the overall capability of the equipment. The mechanical availability constraint is shown by equation (3.19) whiles equation (3.20) illustrates the equipment utilization constraint.

$$\sum_{n\in s}\left[wt_n \times x_n^{j,k} \times ht_n^{j,k}\right] \leq a^{j,k}, \forall j \in \{1, \dots, J\}, \forall k\{1, \dots, K\} \quad (3.19)$$

$$\sum_{n\in s}\left[wt_n \times x_n^{j,k} \times ht_n^{j,k}\right] \geq u^{j,k}, \forall j \in \{1, \dots, J\}, \forall k\{1, \dots, K\} \quad (3.20)$$

3.2.2.6 Minimum mining width and resource interaction. This constraint comprises the total length of the mining-strip, the external length of space outside the digging domain, and the minimum drop cut width (equation 3.21) for each resource (Hustrulid and Kuchta, 1998). The constraint function is illustrated by equation (3.22) for all blocks scheduled in all periods.

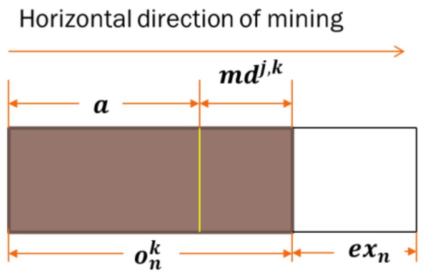


Figure 3.2 Resource Interaction Framework

$$md = \tau + \varphi \quad (3.21)$$

$$\sum_{n\in s}\left[wt_n \times x_n^{j,k} \times l_e\right] \leq o_n^k + ex_n - md^{j,k}, \forall j \in \{1, \dots, J\}, \forall k\{1, \dots, K\} \quad (3.22)$$

3.2.2.7 Haulage unit capacity and reach geometry. This constraint is *relaxed* for resources such as dozers and cast blasting technique (CBT). The constraint captures the truck capacities in the SHT system. The unit capacities are matched with the minimum and maximum production requirements of the shovel. Hence, given a maximum haulage capacity of resource j in period k as $hu^{j,k}$, the constraint is shown in equation (3.23).

$$\sum_{n \in s} [wt_n \times x_n^{j,k}] \leq hu^{j,k}, \forall k \in \{1, \dots, K\}, \forall j \in \{1, \dots, J\} \tag{3.23}$$

3.2.2.8 Diggability. The ease of excavation is determined by the material diggability and the break-out force exerted by the equipment. The constraint function defines equipment selection and scheduling based on the diggability index of the material. This ensures efficient equipment-formation interaction. The mathematical formulation is shown in equation (3.24).

$$\sum_{n \in s} [f_n^{j,k} \times wt_n \times x_n^{j,k}] \geq \sum_{n \in s} [i_n] \;, \forall k \in \{1, \dots, K\}, \;\; \forall j \in \{1, \dots, J\} \tag{3.24}$$

3.2.2.9 Labor. This constraint defines the labor requirement for each resource. The available skilled labor is matched to each resource labor requirement for all scheduled periods. The labor requirements are stated in personnel-hours and thus, it incorporates all subsidiary operational functions. Equation (3.25) shows the labor constraint.

$$\sum_{n \in s} (rs_n^{j,k} \times wt_n \times x_n^{j,k}) \; \leq la^{j,k} \;, \forall k \in \{1, \dots, K\}, \;\; \forall j \in \{1, \dots, J\} \tag{3.25}$$

3.2.2.10 Critical bench height. A critical digging depth constraint is defined beyond which dragline digging process is inefficient (the practical implications of the reach geometry parameters are explained in Section 2). The constraint function is *relaxed* for resources such as dozers, SHT system, and CBT. The mathematical formulations are shown as follows:

$$rf = d\left[\frac{1}{tan\beta} + \frac{sf}{tan\theta}\right] + \frac{w}{4} - \frac{h}{tan\theta} \tag{3.26}$$

$$d_c = \frac{tan\beta tan\theta}{tan\theta + sftan\beta}\left[rf - \left(\frac{w}{4}\right) + \left(\frac{h}{tan\theta}\right)\right] \tag{3.27}$$

$$h_n - \left[h_n \times \sum_{\forall k} x_n^{j,k}\right] \leq h_c^j, \qquad \forall j \in \{1, \dots, J\} \tag{3.28}$$

$$h_n = {}^{t_n}\!/_{[l_n \times w_n \times sg]} \tag{3.29}$$

$$\sum_{n \in s} \left[h_n \times x_n^{j,k}\right] \geq d_c^j \tag{3.30}$$

3.2.2.11 Non-negativity. To ensure that none of the decision variables assumes negative values, a non-negativity constraint (equation 3.31) is formulated.

$$x_n^{j,k} \geq 0 \tag{3.31}$$

3.2.3 Summary. The NLP optimization model is formulated for waste (topsoil, overburden and inter-burden) excavation in MSFs. The objective function is to minimize the cost of mining and hence improve dragline and ancillary equipment productivity. The constraint functions incorporate the excavation geometry, as well as the operating mechanisms of stripping equipment. Optimal solutions obtained are based on economic and technical considerations.

3.3 OPTIMIZATION MODELING OF COAL EXTRACTION

The objective function of the coal seam extraction model is to maximize the revenue generated from the mining activity. The model incorporates the variable geologic parameters of the coal seams, economic parameters, contractual specifications and extraction dynamics. MSFs are characterized by variable seam qualities in random fields. Since contractual agreements specify desired final coal qualities, blending is a necessary step to increase economic output where there is variability in coal quality variables. A summary of the entire optimization process is shown in Figure 3.2 where coal products with different qualities are transported to specific destinations.

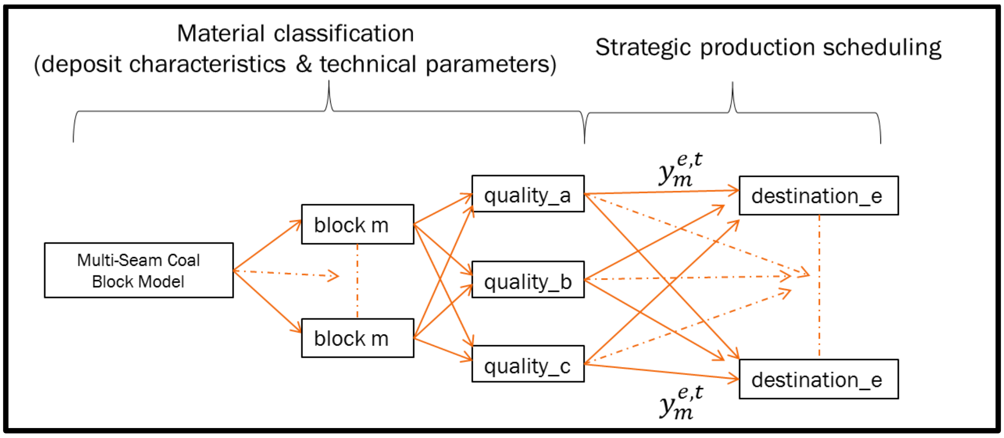


Figure 3.3 Summary of Coal Seam Extraction Process

3.3.1 Objective Function. The indices in the objective function include the seam-cut, $m \in \{1, \ldots, M\}$; product destination, $e \in \{1, \ldots, E\}$; scheduling period,

$t \in (1, \ldots, T\}$; and set of exposed seam-cuts, $g \in \{1, \ldots, G\}$. The objective function as shown in equation (3.32) is a function of the portion of seam-cut m extracted and transported to destination e in period t. The decision variable, $y_m^{e,t}$ is a continuous variable representing the portion of seam-cut m extracted and transported to destination e in period t.

$$\sum_{g=1}^{G} \left\{ max \sum_{e=1}^{E} \sum_{t=1}^{T} \left[\sum_{m \in g} \left(y_m^{e,t} \times \varpi_m^{e,t} \right) \right] \right\} \tag{3.32}$$

$$\varpi_m^{e,t} = [v_m \times q_m \times r \times (p^t - se^t)] - [v_m \times \varsigma_m^{e,t}], \forall m \in \{1, \ldots, M\}, t \in \{1, \ldots, T), e \in (1, \ldots, E\} \tag{3.33}$$

3.3.2 Constraint Functions. The constraints include: (i) reserve; (ii) mining capacity; (iii) processing capacity; (iv) transportation and stockpile capacities; (v) market condition and contractual agreement; (vi) labor; (vii) haulage capacity; (viii) extraction equipment availability and utilization; (ix) coal quality (calorific value, sulfur content, fixed carbon content, ash content, moisture content and volatile matter); and (x) non-negativity constraint.

3.3.2.1 Reserve. The reserve constraints are shown in equations (3.34) and (3.35). The portion of the seam-cut extracted is limited by equation (3.34) whiles equation (3.35) ensures the extraction of all seam-cuts in all periods.

$$y_m^{e,t} \leq 1, \forall m \in \{1, \ldots, M\}, e \in \{1, \ldots, E\}, t \in \{1, \ldots, T\} \tag{3.34}$$

$$\sum_{e=1}^{E} \sum_{m \in g} y_m^{e,t} = 1, \forall t \in \{1, \ldots, T\} \tag{3.35}$$

3.3.2.2 Mining capacity. Mining capacity is defined by production targets; based on which the extraction equipment is selected. The general geometric conditions of the excavation also define the mining capacity. Minimum and maximum limits are specified by equations (3.36) and (3.37) respectively.

$$\sum_{m \in g} [\, v_m \times \sum_{e=1}^{E} \left(y_m^{e,t}\right)] \geq cl^t \,, \forall t \in \{1, \dots, T\} \tag{3.36}$$

$$\sum_{m \in g} [\, v_m \times \sum_{e=1}^{E} \left(y_m^{e,t}\right)] \leq cu^t \,, \forall t \in \{1, \dots, T\} \tag{3.37}$$

3.3.2.3 Processing capacity. Coal processing capacity is represented by variable functions whose lower and upper bounds are dependent on the processing activity and the scheduling period. These bounds are functions of the mine life, stripping geometry, processing and stockpile facilities, economics, and mineable reserve. Equations (3.38) and (3.39) ensure that the total amount of the material mined in each scheduling period is within the upper and lower processing boundaries respectively.

$$\sum_{m \in g} [\, v_m \times \sum_{e=1}^{E} \left(y_m^{e,t}\right)] \leq pu^t \,, \forall t \in \{1, \dots, T\} \tag{3.38}$$

$$\sum_{m \in g} [\, v_m \times \sum_{e=1}^{E} \left(y_m^{e,t}\right)] \geq pl^t \,, \forall t \in \{1, \dots, T\} \tag{3.39}$$

3.3.2.4 Transportation and stockpile capacity. Equation (3.40) restricts coal production within the capacities of the transportation and the storage facilities (this also includes silo capacities). This constraint is a function of the economics of the project and market conditions.

$$\sum_{m \in g}\left[v_m \times \sum_{e=1}^{E}\left(y_m^{e,t}\right)\right] \leq rp^t + st^t, \forall t \in \{1, \dots, T\} \tag{3.40}$$

3.3.2.5 Market condition and contractual agreement. The quantity of material produced is bounded within regions that satisfy market conditions. The maximum and minimum boundaries are illustrated respectively by equations (3.41) and (3.42).

$$\sum_{m \in g}\left[v_m \times y_m^{e,t}\right] \leq \aleph^t, \forall t \in \{1, \dots, T\}, \forall e \in \{1, \dots, E\} \tag{3.41}$$

$$\sum_{m \in g}\left[v_m \times y_m^{e,t}\right] \geq \beth^t, \forall t \in \{1, \dots, T\}, \forall e \in \{1, \dots, E\} \tag{3.42}$$

3.3.2.6 Labor. This constraint limits the amount of coal extracted per period. The formulation incorporates excavation and treatment plant labor requirement, as well as all ancillary labor required in producing a unit ton of coal. The labor requirements are stated in personnel-hours and shown in equation (3.43).

$$\sum_{m \in g} \varphi_m^t \times \left[v_m \times \sum_{e=1}^{E}\left(y_m^{e,t}\right)\right] \leq \check{g}^t, \forall t \in \{1, \dots, T\} \tag{3.43}$$

3.3.2.7 Haulage capacity. For all coal seams, the capacities of haulage units must be adequate for all periods. Equation (3.44) ensures that the amount of coal scheduled to be extracted is limited to the maximum pit-to-plant and pit-to-stockpile(s) haulage capacities.

$$\sum_{m \in g} \left[v_m \times \sum_{e=1}^{E} \left(y_m^{e,t} \right) \right] \leq \partial^t, \forall t \in \{1, \dots, T\} \tag{3.44}$$

3.3.2.8 Equipment availability and utilization. The formulations include: the total hours to extract a unit quantity of seam-cut m in period t; the strip mining geometry; and the overall capability of production equipment. The treatment plant availability and utilization is neglected in this study. Equations (3.45) and (3.46) maintain respectively the availability and utilization of the equipment within predetermined boundaries.

$$\sum_{m \in g} \left[v_m \times \sum_{e=1}^{E} \left(y_m^{e,t} \right) \times hr_m^t \right] \leq av^t, \forall t \in \{1, \dots, T\} \tag{3.45}$$

$$\sum_{m \in g} \left[v_m \times \sum_{e=1}^{E} \left(y_m^{e,t} \right) \times hr_m^t \right] \geq uu^t, \forall t \in \{1, \dots, T\} \tag{3.46}$$

Market conditions require coal quality to be within specific acceptable boundaries. Quality distributions in coal seams are controlled by physical and chemical parameters, hence desired products can be achieved by selective mining, and at the processing stage. However, in strip mine operations, selective mining is limited by the overall mining geometry. Desired coal quality is mostly achievable at the treatment plant.

The general quality constraint formulations are shown in equations (3.47) and (3.48). The upper bound is given by equation (3.47) while equation (3.48) is the lower bound.

$$\frac{\sum_{m\in g} b_{i,m} \times \left[v_m \times q_m \times y_m^{e,t}\right]}{\sum_{m\in g}\left[v_m \times q_m \times y_m^{e,t}\right]} \leq B_i^{e,t}, \quad \forall t \in \{1, \dots, T\}, \forall e \in \{1, \dots, E\} \tag{3.47}$$

$$\frac{\sum_{m\in g} b_{i,m} \times \left[v_m \times q_m \times y_m^{e,t}\right]}{\sum_{m\in g}\left[v_m \times q_m \times y_m^{e,t}\right]} \geq L_i^{e,t}, \quad \forall t \in \{1, \dots, T\}, \forall e \in \{1, \dots, E\} \tag{3.48}$$

The coal quality is defined by energy, sulfur, fixed carbon, volatile matter, ash, and moisture contents. Thermal coal deposits are characterized by their heating value or Btu content. Within MSFs, the range of heating values is limited due to regionally scaled parameters. Btu content, however, needs to be within acceptable limits for efficient combustion purposes.

Similarly, the upper and lower limits of sulfur contents are restricted by market specifications, mining operations, and environmental constraints. The presence of sulfur in coal products may result in acid rain and contribute to other pollution-related health problems and as such, it is required to be within acceptable limits.

Another important quality parameter is fixed carbon content. Fixed carbon is the carbon found in the coal material which is left after volatile materials are burnt off. Fixed carbon content is also used as an estimate of the amount of coke produced from a coal sample (Miura et al., 2004).

Ash content is the non-combustible residue left after coal combustion. The geology of the formation, dilution and transportation operations contribute to ash content. The ash content in most commercial coal products is between 3% and 9% (NSP, 2001).

Occurrence of moisture in coal seams might be due to surface moisture, hydroscopic moisture, decomposition moisture and mineral moisture (Miura et al., 2004). Market conditions normally regulate the upper limits and mining conditions also specify lower limits to meet economic standards. The decision variables are therefore constrained within these limits to ensure an optimal blend.

Volatile matter content refers to the components of coal, except for moisture, which are liberated at high temperatures in the absence of air. This is usually a mixture of short and long chain hydrocarbons, aromatic hydrocarbons and some sulfur. The volatile matter of coal is determined under rigidly controlled standards (NSP, 2001).

3.3.2.9 Non-negativity. To ensure that none of the decision variables assume negative values, non-negativity constraint, equation (3.49) is formulated.

$$y_m^{e,t} \geq 0 \tag{3.49}$$

3.4 SUMMARY

The models are formulated based on the generalized NLP optimization techniques. The overall optimization model is divided into two sections: (i) waste extraction and (ii) coal seam extraction; for computational purposes. Geologic variables of MSFs, technical dynamics of stripping equipment and the downstream coal quality specifications are incorporated in the models. The solution parameters illustrate optimal resource allocation to reduce mining cost, and coal blending options that maximize revenue generated.

4. STOCHASTIC-OPTIMIZATION MODELING OF MATERIALS EXTRACTION

This section presents the stochastic-optimization (SOP) and simulation frameworks. All analyses and discussions are built on the mathematical basis of @RISK and frontline SOLVER, the software packages selected for the stochastic modeling. Refer to the nomenclature for the definitions of the symbols, variables and abbreviations.

4.1 STOCHASTIC-MODEL FORMULATION

The generalized multi-objective SOP problem is given by equations (4.1) and (4.2). In these formulations; $b^1(v), \ldots, b^k(v)$ are n-dimensional random vectors on a probability space (Adeyefa and Luhandjula, 2011). The stochastic process is limited to the coefficients in the objective function.

$$\min_{x \in C}(b^1(v)x, \ldots,\ b^k(v)x) \tag{4.1}$$

$$\mathrm{C} = \{x \in \mathbb{R}^n : Ax \leq b; x \geq 0\} \tag{4.2}$$

Equation (4.1) is transformed to its deterministic counterpart using the expectations and variances in the random multivariate fields (Equations (4.3) and (4.4)). It is assumed that the random data have probability distributions with finite expected values and variances.

$$\min_{x \in C}\{E(b^1(v))x, E(b^2(v))x, \ldots,\ E(b^k(v))x\} \tag{4.3}$$

$$\min_{x \in C}\{V(b^1(v))x, V(b^2(v))x, \ldots,\ V(b^k(v))x\} \tag{4.4}$$

$$\mathrm{C} = \{x \in \mathbb{R}^n : Ax \leq b; x \geq 0\} \tag{4.5}$$

In equations (4.3) and (4.4), E and V are the expected value and the variance respectively. Assuming x^* is an optimal solution for equation (4.3), ((4.4)) then x^* is the *expected value optimal solution (EVOS)* (*a variance optimal solution (VOS)*) for equation (4.1) subject to equation (4.2). The functions $E(b^1(v)), E(b^2(v)), \ldots, E(b^k(v))$ are the respective deterministic counterparts of $b^1(v), b^2(v), \ldots, b^k(v)$ obtained from stochastic simulation techniques (Adeyefa and Luhandjula, 2011). Thus, a solution to the optimization problem is generated for each simulation run of the random functions. The mean value of these solutions represents the EVOS. This approach describes the stochastic process of the random functions, and the optimal results incorporate the input function variabilities.

4.2 EXPECTATIONS AND VARIANCES IN RANDOM MULTIVARIATE FIELDS

If $x_i\ [i \in (1, n)]$ represents a set of random and continuous data; H_T is a function of many variables; and $f(x)$ is the probability density function of the set of x_i variables; then equations (4.7) and (4.8) represent respectively, the expected value and the variance of the function H_T (Narsing, 1997).

$$H_T = \emptyset[x_i, i \in (1, n)] \tag{4.6}$$

$$E(H_T) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \emptyset[x_i, (i \in 1, n)] * f(x_i) dx_i [i \in (1, n)] \tag{4.7}$$

$$\begin{gathered} Var(H_T) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} [\emptyset[x_i, i \in (1, n)] - E(H_T)]^2 \\ * f(x_i) dx_i [i \in (1, n)] \end{gathered} \tag{4.8}$$

Given that $x_1, x_2, \dots, x_n$ are drawn from $S_1, S_2, \dots, S_n$ sample spaces respectively, the expected value is illustrated as in equation (4.9).

$$E(H_T) = \iint \cdots \int_{S_1} \emptyset[x_1, x_2, \dots, x_n] * f(x_1)f(x_2) \dots f(x_n)\, dy_1\, dy_2 \dots dy_n$$

$$+ \iint \cdots \int_{S_2} \emptyset[x_1, x_2, \dots, x_n] * f(x_1)f(x_2) \dots f(x_n)\, dy_1\, dy_2 \dots dy_n$$

$$+ \iint \cdots \int_{S_n} \emptyset[x_1, x_2, \dots, x_n] * f(x_1)f(x_2) \dots f(x_n)\, dy_1\, dy_2 \dots dy_n \qquad (4.9)$$

From equation (4.9), the probability that the sample point $x_i\ [i \in (1, n)]$ is within $S_i\ [i \in (1, n)]$ is given by $P[x_n \in S_n | \mu, \sigma]$. The stochastic process is also used to determine the probabilities of the mining cost and the revenues exceeding specific figures (see equation (4.11)). The expected values and the various event occurrences are achieved by Monte Carlo and Latin Hypercube simulation models.

$$\sum_{i=1}^{n} P(x_n \in S_n | \mu, \sigma) = 1.0 \qquad (4.10)$$

$$P(x_i \geq x_{i\,min}) = 1 - F(x_{i\,min}) \qquad (4.11)$$

4.3 GENERATION OF RANDOM VARIATES

Inverse probability integral transformation (IPT) is used to generate random samples for the stochastic simulation. Other methods to generate random variates include: composition and accept-reject methods (Narsing, 1997). The IPT procedure involves formulating the quantile function of the distribution and then inverting the function (Devroye, 1986).

The formulations for the IPT are illustrated by equations (4.12) to (4.15).

$$\forall\ (a,b) \in [0,1]\ \times\ [0,1]: a \leq b \tag{4.12}$$

$$\{x: F(x) \geq b\} \subset \{x: F(x) \geq a\} \tag{4.13}$$

$$\Rightarrow inf\{x: F(x) \geq b\} \geq inf\{x: F(x) \geq a\} \tag{4.14}$$

$$\Leftrightarrow\ F^{-}(b) \geq F^{-}(a) \tag{4.15}$$

Subsequently, if the function F^{-} is non-decreasing, it implies that

$$U \leq F(x) \Rightarrow F^{-}(U) \leq F^{-}\big(F(x)\big) \tag{4.16}$$

Therefore, given the properties $F^{-}\big(F(x)\big) \leq x\ and\ F\big(F^{-}(U)\big) \geq U,$ and F non-decreasing by definition, equations (4.17) and (4.18) are obtained (Whiteley, 2008; Devroye, 1986).

$$F^{-}(U) \leq x \Rightarrow F\big(F^{-}(U)\big) \leq F(x) \tag{4.17}$$

$$F^{-}(U) \leq x \Rightarrow U \leq F(x) \tag{4.18}$$

The summary of the IPT algorithm procedure is as follows:

(i) Given a probability density function (PDF) which integrates into a cumulative density function (CDF)

(ii) Generate a uniformly distributed random number sequence $X \in \{0, \dots, 1\}$.

(iii) Compute the random variable with $F(y)$ given as $R = F^{-1}(X)$ (refer to Figures 4.1 and 4.2).

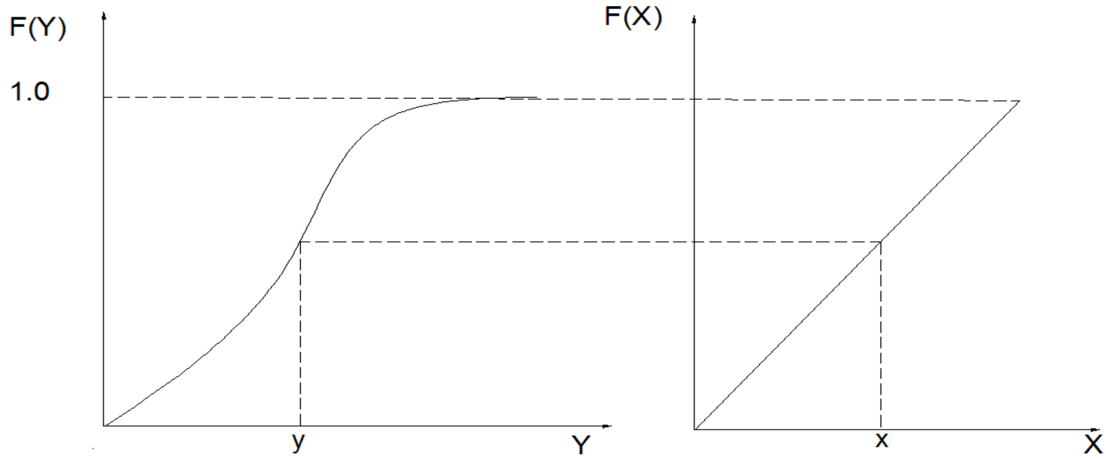


Figure 4.1 Cumulative Density Function

Figure 4.2 CDF of the Uniform PDF

If a random variable X follows a uniform distribution in the interval [0,1], the IPT algorithm states that the random variable $R = F^{-1}(X)$ has a continuous CDF, $F(y)$ (Narsing, 1997).

4.4 MONTE CARLO SIMULATION MODELING

This sampling technique relies on repeated random sampling to compute its results. The value of a complex function of multiple integrals is estimated by simulating a large number from the CDF of the random variables. However, the efficiency and convergence characteristics of a Monte Carlo simulation (MCS) is controlled by the selected random variate generation method, the variance reduction technique (VRT), and the number of samples drawn (Hammersley and Handscomb, 1964; Billinton and Li, 1994).

4.4.1 Variance Reduction Techniques. The VRT in @RISK is the crude and stratified sampling methods (Palisade, 2012). Other VRTs used to sample random variates include: importance, control variates, antithetic and orthogonal sampling (Hammersley and Handscomb, 1964; Billinton and Li, 1994).

Equation (4.19) shows the formulation for the crude Monte Carlo (CMC) sampling. In this equation, $Z(Y_i)$ is the sample function computed using the random sample Y_i generated from the PDF, $f(y)$. N is the number of samples ($i \in (1, \dots, N)$) and ℓ is the estimate through simulation. Equation (4.20) is the variance of the estimate (Kleijnen et al., 2010).

$$\hat{\ell}_N = \frac{1}{N}\sum_{i=1}^{N} z(Y_i) \tag{4.19}$$

$$Var(\hat{\ell}_N) = \frac{1}{N}\int (z(y) - \ell)^2 f(y)\, dy \tag{4.20}$$

Given the CMC estimation problem (equation (4.21)) and some finite random variable H obtained from h_i, such that the probabilities $p_i = P(H = h_i)$ are known, the stratified sampling estimator of ℓ is formulated as shown in equation (4.23). The variance of the unbiased estimator is given in equation (4.24).

$$\ell = E\big(Z(Y)\big) \tag{4.21}$$

$$\ell = E\left(E\big(Z(Y)\big)\right) = \sum_{i=1}^{m} p_i\, E(Z(Y)|H = h_i) \tag{4.22}$$

$$\hat{\ell}_N^* = \sum_{i=1}^{m} p_i \frac{1}{N_i} \sum_{j=1}^{N_i} Z(Y_{ij}) \tag{4.23}$$

$$Var\big(\hat{\ell}_N^*\big) = \sum_{i=1}^{m} \frac{p_i^2 \sigma_i^2}{N_i} \tag{4.24}$$

While the stratified sampling technique is more efficient than the CMC, both methods increase the precision of the estimates obtained from given number of iterations (Hammersley and Handscomb, 1964; Billinton and Li, 1994; and Kleijnen et al., 2010).

4.4.2 Convergence Rate of MCS Technique. The rate of convergence of MCS is dependent on the chosen number of simulations N to achieve the desired accuracy and the confidence interval on the accuracy (Lapeyre, 2007). The error in Monte Carlo estimate is inversely proportional to the number of samples.

Using the Central Limit Theorem, assuming $X_i\ (i \geq 1)$ is a sequence of independent identically distributed random variables such that $E(X_1^2) < +\infty,$ if the variance of X_1 is given by equation (4.25), then equation (4.27) shows the convergence for a Gaussian random variable with variance 1 and mean 0.

$$\sigma^2 = E(X_1^2) - E(X-1)^2 = E((X_1 - E(X_1))^2) \tag{4.25}$$

$$\left(\frac{\sqrt{n}}{\sigma}\epsilon_n\right) \tag{4.26}$$

$$\epsilon_n = E(X) - \frac{1}{n}(X_1 + \cdots + X_n) \tag{4.27}$$

The efficiency of the MCS depends on the variance of the estimate and the computer run-time. Even though large sample sizes reduce the error estimates, optimal numbers must be used to reduce run-times (Hammersley and Handscomb, 1964; Lapeyre, 2007).

4.5 LATIN HYPERCUBE SIMULATION MODELING

This technique is applied in uncertainty analysis to generate sample values from a multi-dimensional distribution. The process comprises the division of the probability ranges of PDFs for basic input random variables into N equivalent intervals (non-overlapping intervals of equal probabilities). The random selection of the intervals maintains the independence between the variables. Latin Hypercube simulation (LHS) utilizes the stratification of the theoretical PDFs of input random variables (Novak, et al., 1997). In the @RISK environment, the values from within the selected stratifications are chosen randomly. The number of stratifications of the CDF is equivalent to the number of iterations (Palisade, 2012).

Considering each sample (i, j), the sample values of X and Y (two independent, uniform distributed variables) are given respectively by equations (4.28) and (4.29).

$$X = F_X^{-1}((i - 1 + \varepsilon_X)/n), \forall \varepsilon_X \in \{0,1\}, X \in \{0,1\} \tag{4.28}$$

$$Y = F_Y^{-1}((j - 1 + \varepsilon_Y)/n), \forall \varepsilon_Y \in \{0,1\}, X \in \{0,1\} \tag{4.29}$$

In equations (4.28) and (4.29), F_X *and* F_Y are the CDFs of X and Y respectively, ε_X and ε_Y are random numbers, and n is the sample size (Cheng, 2000).

The simulation process involves the utilization of the centroids of the intervals shown in equations (4.28) and (4.29) through an IPT (refer to Section 4.3) of the PDFs.

Every interval of each variable is however, implemented only once and covers the multi-dimensional space of the random variates. This sampling technique is useful in the analysis of situations where low probability outcomes are represented in input PDFs, for high sampling efficiency.

4.6 RANDOM FUNCTIONS IN STOCHASTIC MODELING

The following are the PDFs used in this research study: lognormal, Gaussian, and uniform. The stochastic simulation process require the generation of random variates from the PDFs. BestFit (Palisade, 2012) is used to model the probability distributions of the input variables based on which the appropriate distributions are selected. This section illustrates the algorithmic derivation of the random variates from the probability distributions.

4.6.1 Lognormally Distributed Random Variates. Equations (4.30) and (4.31) illustrate the PDF and CDF respectively (Weisstein, 2003).

$$f(x) = \frac{1}{S\sqrt{2\pi x}} e^{-(\ln x - M)^2/(2S^2)} \tag{4.30}$$

$$F(x) = \frac{1}{2}\left[1 + erf\left(\frac{\ln x - M}{S\sqrt{2}}\right)\right] \tag{4.31}$$

The mean and the variance are given by equations (4.32) and (4.33) respectively.

$$\mu = e^{M+S^2/2} \tag{4.32}$$

$$\sigma^2 = e^{S^2+2M}\left(e^{S^2} - 1\right) \tag{4.33}$$

Using the Box-Muller algorithm, assuming s_1 and s_2 are two independent and standard uniform Variates, then a pair of statistically independent standard normal Variates is given by equations (4.34) and (4.35) (Narsing, 1997).

$$a_1 = \sqrt{(-2\ln s_1)}cos\ 2\pi\ s_2 \tag{4.34}$$

$$a_2 = \sqrt{(-2\ln s_1)}sin\ 2\pi\ s_2 \tag{4.35}$$

The joint probability is given by equation (4.36) where $V = s_2$ and $U = -\ln s_1$

$$f(a_1, a_2) = f(u, v) \cdot |J| = \frac{1}{2\pi} e^{\left[-\frac{1}{2}(a_1^2 + a_2^2)\right]}, \infty > a_2\ and\ a_1 > -\infty \tag{4.36}$$

Based on the formulations above, equations (4.37) and (4.38) represent the frameworks from which a pair of independent random variates may be generated.

$$x_1 = \mu + \sigma\sqrt{(-2\ln s_1)}cos\ 2\pi\ s_2 \tag{4.37}$$

$$x_2 = \mu + \sigma\sqrt{(-2\ln s_1)}sin\ 2\pi\ s_2 \tag{4.38}$$

4.6.2 Normally Distributed Random Variates. The PDF and the CDF are shown in equations (4.39) and (4.40) respectively (Weisstein, 2003).

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}, \quad x \in \mathbb{R}, \quad \sigma^2 \neq 0 \tag{4.39}$$

$$F(x; \mu, \sigma^2) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x - \mu}{\sigma\sqrt{2}}\right)\right] \tag{4.40}$$

Using the Box-Muller algorithm, and given s_1 and s_2 as two independent and standard uniform variates, then a pair of statistically independent standard normal variates is given by equations (4.41) and (4.42) (Narsing, 1997).

$$a_1 = \sqrt{(-2\log s_1)} cos\ 2\pi\ s_2 \tag{4.41}$$

$$a_2 = \sqrt{(-2\log s_1)} sin\ 2\pi\ s_2 \tag{4.42}$$

The joint probability is given by equation (4.43) where $V = s_2$ and $U = -\log s_1$

$$f(a_1, a_2) = f(u, v) \cdot |J| = \frac{1}{2\pi} e^{\left[-\frac{1}{2}(a_1^2 + a_2^2)\right]}, \infty > a_2\ and\ a_1 > -\infty \tag{4.43}$$

Equations (4.44) and (4.45) are derived from the formulations above, and represent the frameworks from which a pair of independent random variates may be generated.

$$x_1 = \mu + \sigma\sqrt{(-2\ln s_1)} cos\ 2\pi\ s_2 \tag{4.44}$$

$$x_2 = \mu + \sigma\sqrt{(-2\ln s_1)} sin\ 2\pi\ s_2 \tag{4.45}$$

4.6.3 Uniformly Distributed Random Variates. Given minimum and maximum values m and n, the PDF and CDF for this distribution are illustrated in equations (4.46) and (4.47) respectively (Park and Bera, 2009).

$$f(x) = \begin{cases} \dfrac{1}{n-m} & , \forall\ m \le x \le n \\ 0 & , \forall\ x < m\ or\ x > n \end{cases} \tag{4.46}$$

$$F(x) = \begin{cases} 0 & ,\forall\, x < m \\ \dfrac{(x-m)}{(n-m)} & ,\forall\, x \in [m,n] \\ 1 & ,\forall\, x \geq n \end{cases} \tag{4.47}$$

The random variates are generated as explained in Section 4.3. In the simulation process, the uniform distribution is used to generate other PDFs. Figures (5.3) and (5.4) show the PDF and the CDF plots of the uniform distribution.

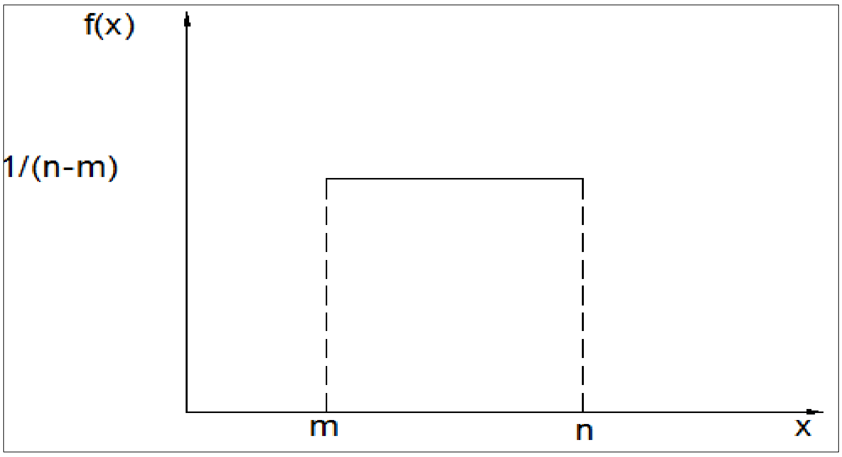


Figure 4.3 Uniform Probability Distribution Function

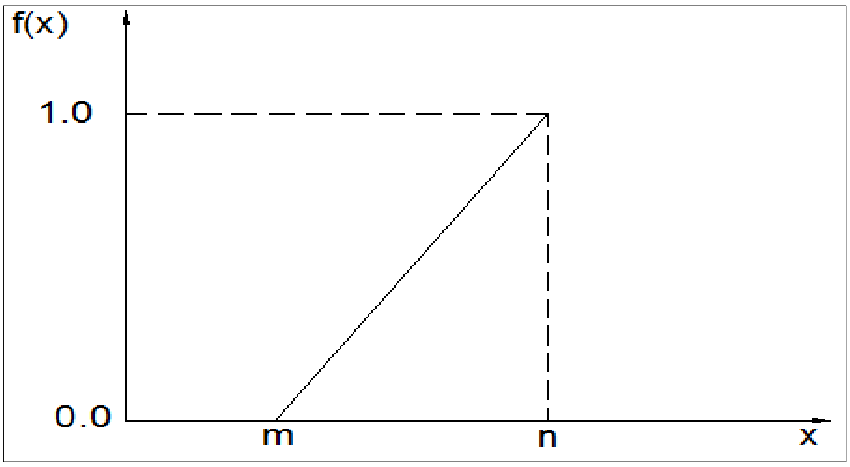


Figure 4.4 Uniform Cumulative Distribution Function

4.7 SUMMARY

The stochastic variables associated with excavation in MSFs are identified and modeled using PDFs. The variables identified include: mining cost, processing cost, commodity price, selling price, tonnage of material, plant recovery, and thermal coal quantity. The stochastic model is limited to the coefficient variables in the objective functions. Simulation of the variable parameters is limited to MCS and LHT while the random variates are generated by IPT functions. For the SOP, the expected value of the objective function is minimized. The mean value of the optimal solutions from each simulation run represents the expected value optimal solution. The results lead to real-time risk analysis for a comprehensive economic model.

5. COMPUTER MODELING AND EXPERIMENTAL ANALYSIS

The computer modeling framework and experimentation setups are presented in this section. The numerical solution algorithms and environment for solving the optimization models are also discussed. Two sections of the computer models are established and run as an independent functionality. The first section is based on non-linear programming (NLP). The solution algorithm for the first section is the Generalized Reduced Gradient (GRG). The second section is on stochastic-optimization (SOP) model. A bituminous coal mining case is used to validate the SOP model.

5.1 NUMERICAL MODELING

Numerical modeling techniques are used to create a set of equations and inequalities to solve the optimization models. The optimizer is initiated by evaluation of the Jacobian (JC) matrix of partial derivatives (PD) of the problem functions with respect to the decision variables. Finite difference method (FDM) is used to approximate the JC matrix as shown in equation (5.1).

$$\frac{f_i(x + \delta e_j) - f_i(x)}{\delta} \tag{5.1}$$

$$\delta = eps\left|1 + x_j\right| \tag{5.2}$$

In equations (6.1) and (6.2), the $[i, j]^{th}$ element of the JC matrix is approximated, where x is the JC parameter set; e_j is the j^{th} unit vector; and eps is a perturbation factor approximately equal to the square root of the machine precision (Fylstra, 1998).

Mining patterns and equipment schedules that satisfy the objective functions and the constraint models are investigated by the optimizer. Due to the magnitude of the models, dynamic Markowitz refactorization is used to improve stability. This approach, coupled with the sparse representation of the matrix, results in better memory usage (Fylstra, 1998).

5.1.1 NLP Solution Algorithm. The NLP model is formulated based on Lagrange multiplier method (LMM). This technique forces the constraint functions into the objective formulations as explained in section 3. The solution algorithm to the NLP model is the GRG algorithm implemented in SOLVER. GRG algorithm is guaranteed to find a local optimum only on problems with continuously differentiable functions, and also in the absence of numerical difficulties (Fylstra, 1998). The NLP models reach optimality when SOLVER finds a local optimum (where the Karush-Kuhn-Tucker (KKT) conditions are satisfied within the specified convergence tolerance).

Figure 5.1 shows the flowchart for the LMM technique. In this flowchart, constraint functions are identified and then multiplied by the Lagrange multipliers. The products are subtracted from the objective function as a means of forcing the constraint functions into the objective equation. The Lagrangian function is then maximized given the variables selected to obtain the optimal solution, and the procedure terminates. If the Lagrangian function is not maximized, new non-negative real numbers are generated by increasing the exponential value, k by unity. The new non-negative real values are then multiplied with the constraint functions and introduced into the objective function. The entire procedure is repeated until an optimal solution is obtained.

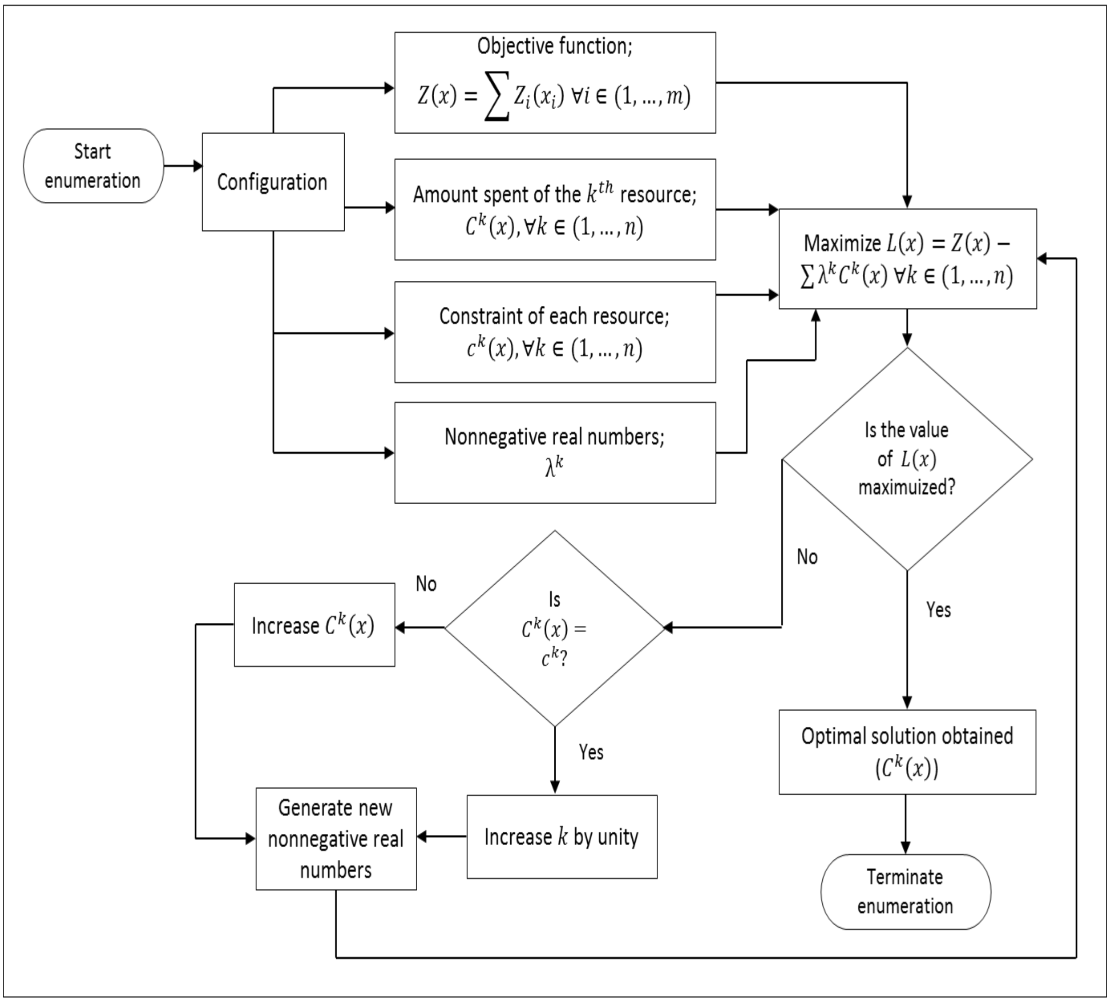


Figure 5.1 Generalized Lagrange Multiplier Method

5.1.2 Model Fitting. The stochastic parameters identified for each of the SOP models are used in the BestFit (Palisade, 2012) framework. This program attempts to model each of the random stochastic variables with probability density functions (PDFs). Maximum likelihood estimators and Levenberg-Marquardt methods are implemented in BestFit to define the PDFs for input data. The maximum likelihood estimation is illustrated as follows: assuming the probability of observing data vector x given the parameter θ is $f(x|\theta)$, if individual observations, x_is' are statistically independent of one another, then the PDF for the data $x = (x_1, \dots, x_m)$ is given by equation (5.3) (Myung, 2003). The maximum likelihood estimate of θ is that value of θ that maximizes the function in equation (5.4).

$$f(x = (x_1, x_2, \ldots, x_m)|\theta) = f_1(x_1|\theta) f_2(x_2|\theta) \cdots f_n(x_m|\theta) \tag{5.3}$$

$$l(\theta) = \sum_{i=1}^{m} \log(f(x_i\,|\theta)) \tag{5.4}$$

The Levenberg-Marquardt method is used to find the minimum of a function given in equation (5.5) which is a sum of squares of nonlinear functions. Assuming $J_i(x)$ is the JC of $f_i(x)$, λ_k are nonnegative scalars, and I is the identity matrix, then the Levenberg-Marquardt method searches in the direction given by the solution p to equation (5.6) (Weisstein, 2013).

$$F(x) = \frac{1}{2}\sum_{i=1}^{m} [f_i(x)]^2 \tag{5.5}$$

$$(J_k^T J_k + \lambda_k I)p_k = -J_k^T f_k \tag{5.6}$$

Three data types are allowed in BestFit: (i) density; (ii) cumulative; and (iii) sample. A summary of the PDF creation process for each data type are as follows (Palisade, 2012):

(i) Density data: the input data is sorted in ascending order, descriptive statistics are generated, data is normalized, and PDFs are created from the normalized data.

(ii) Cumulative data: the process is similar to the density data; however, cumulative distribution functions (CDFs) are created as the final step.

(iii) Sample data: the input data are sorted in ascending order, descriptive statistics are generated, data are converted to histograms, and PDFs are created from the histogram plots.

5.1.3 Stochastic Modeling. @RISK (Palisade, 2012) forms the platform of the stochastic modeling where quantitative risk analysis is performed. Numerical values are assigned to risk profiles using empirical data and qualitative assessments. Unlike deterministic analysis, stochastic risk analysis considers the interdependence of uncertain input parameters and also determines the impact of different inputs relative to the overall outcome.

Figure 5.2 illustrates the flowchart for the risk simulation. If LHT is selected, the PDF input data are stratified to obtain equal intervals on the probability curve. A sample is then drawn from each stratification. The MCS technique begins with the construction of a cumulative PDF and a cumulative normal PDF. The risk simulation model is divided into three sections: (i) sampling; (ii) standard recalculation; and (iii) output specifications. The sampling stage encompasses the construction of CDF and a cumulative uniform normal probability distribution function. Random variates are then generated using IPT.

The random variates are used in the simulation to produce series of optimal solutions for specific scenarios. The output file comprises frequency and cumulative plots of the objective function, tornado graphs for sensitivity analysis, target probability plots, and a general description of the stochastic process.

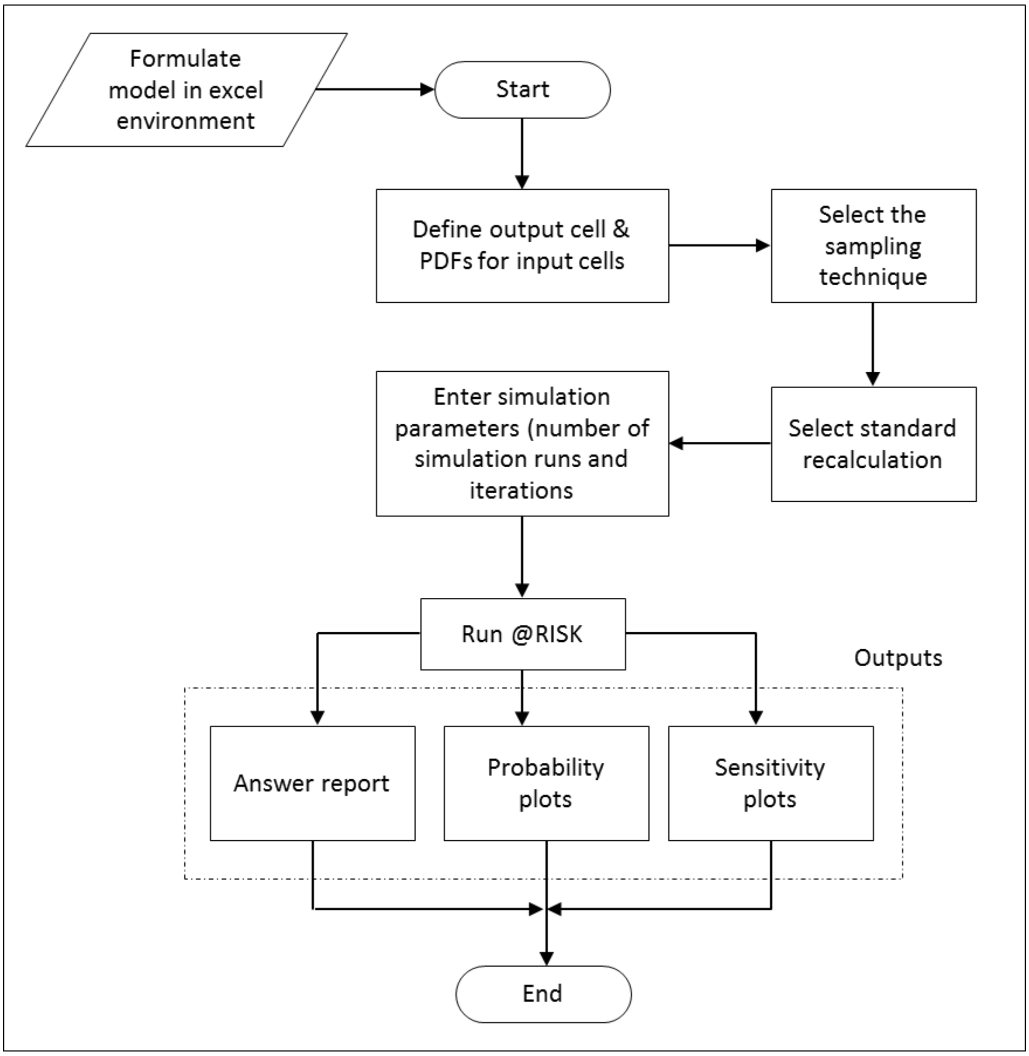


Figure 5.2 Stochastic Modeling and Simulation Flowchart

5.1.4 Optimal Number of Iteration. As a requirement, the optimal number of iterations for the stochastic optimization runs must be determined. This process determines the number of iterations required for a chosen level of precision in the results. Similarly, the optimal time, precision, tolerance and convergence values required by SOLVER for each of the models must be determined. Even though precision is based on the number of iterations of the simulation run, the relationship between iterations and precision depends on the relationship between the variables in the precision (Banks et al., 2000).

Experimental designs are created for this purpose, where the parameters are varied gradually to obtain distributions for the mean values and the variances of each run. A graph representing the mean and variance against the number of iterations is plotted to determine zones of parametric stability. Similarly, the underlying parameters (time, precision, tolerance and convergence) in SOLVER are varied gradually to determine stable zones. These parameters can also be obtained from the procedure outlined below (Banks et al., 2000):

(i) run simulation for a sample of H_0 iterations

(ii) calculate the sample variance σ^2 and the sample standard deviation σ from the simulation output

(iii) find the z-value of $[1-\left(\frac{\alpha}{2}\right)]$ percentile of the standard normal distribution , where $(1-\alpha)$ is the confidence level

(iv) Based on equations (5.7) and specified error ε, set the initial estimate of the number of iterations required as the smallest integer I_0 that satisfies equation (5.8). In equation (5.7), $\hat{x}$ and x are respectively the estimate of the mean and the actual mean.

$$P(|(\hat{x}-x)| \leq \varepsilon) \geq 1-\alpha \tag{5.7}$$

$$I_0 \geq \left(\frac{Z_{\frac{\alpha}{2}} * \sigma}{\varepsilon}\right)^2 \tag{5.8}$$

The results from these experimentations are case-specific and represent the optimal run parameters of the respective cases.

5.2 DATA GATHERING AND EXPERIMENTATION SETUP

5.2.1 Verification and Validation. The generated model has been verified by means of evaluating the solution outputs with the introduction of each constraint function. The verification process involves the identification and correction of errors in the mathematical formulations to achieve a reliable analysis of different experimentations. The objective functions are tested with varying input parameters and their outputs are checked with known results. The constraint functions are introduced systematically into the modeling framework to obtain their ranges of influence on the feasible solution sets.

During the verification process, it is realized that the coal qualities and the critical digging depth formulations introduces some degree of discontinuity in the model. Once the model is verified, relevant data from published literature, company websites and international energy outlook websites are used for the validation process.

The resources involved in the validation process include draglines (P&H 9010C); dozers (CAT D11); shovels (CAT 7495HD); trucks (CAT 793F); and cast blasting technique. The various experimentations include varying the operating cost parameters, coal seam properties, digging geometries, dumping dynamics, and different excavation scenarios.

No quantitative error check analyses is necessary for the validation process, however, the different optimal solutions obtained are evaluated with practical scenarios. For example, with similar constraints for all resources, the optimal solution depends on variations in the operating cost parameters. Also, infeasible solutions are obtained when resources are forced into digging domains which violate their corresponding critical digging depth parameters. Detailed results of the verification and the validation process are illustrated in Appendices A.

5.2.2. Extraction Strategies for Simulation. The strategies include: (i) conventional MSF mining; (ii) combined MSF-equipment-processes extraction method; and (iii) modified MSF-equipment-processes extraction method.

5.2.2.1 Conventional MSF mining. This simulation strategy comprises dozer pushing topsoil to expose upper coal seams and draglines excavating overburden and inter-burden material to expose lower coal seams. The evaluation also considers pre-stripping of portions of the topsoil with the shovel and truck system. This strategy is selected to mimic a typical subjective-decision in MSF extraction. The excavation and dumping dynamics in this method are subject to physical and economic conditions as mining progresses. Equipment productivities and efficiencies are directly affected by these parameters; hence the economic gains are subject to the sustainability of initial decisions.

Material volume re-handling, deadheading of draglines, losing lower seams and frequent movement of large excavation unit(s) have substantial economic impacts which could be overlooked by subjective analyses. Even though past economic trends and mining activities are applied in this method, future uncertainties in economic values and geologic depositions could render the final decisions sub-optimal. Detailed comparison of the simulated results to the optimal solutions is provided in the case study model.

5.2.2.2 Combined MSF-equipment-processes extraction method. This simulation strategy examines the optimal engagement of different equipment units with unique mechanical geometries. The option captures the flexibility and adaptability of equipment units with specific operational functionalities to optimize material extraction.

This strategy aims at increasing equipment productivities, minimizing material re-handling, and increasing stripping efficiency, thus reducing mining cost. The simulation is based on the following equipment units and their excavation geometries: (i) draglines; (ii) dozers; (iii) shovel and truck; and (iv) cast blasting technique. The simulation set up as explained in the case study model investigates the engagement and interaction of these units and their tons-excavated; subject to varying geologic and operational conditions.

5.2.2.3 Modified MSF-equipment-processes extraction method. This strategy includes modification of the dragline mining method and the engagement of a secondary dragline on the spoil pile. The secondary dragline re-handles material to create sufficient internal dumping space and also prevents loosing of lower seams. Secondary draglines also prevent deadheading of primary units. Modification of stripping methods may be necessary in complex operating environment to increase equipment efficiency and overall improvement in economic output. However, the capital expenditure could diminish the economic gains in sub-optimal modifications.

5.3 CASE STUDY MODELING

A bituminous mining case has been carried out to verify and validate the SOP models. This includes the formulation of flowcharts on the excavation and spoiling dynamics, geologic block modeling, and the implementation of the optimization algorithms. All symbols, abbreviations and variables are defined in the nomenclature.

5.3.1 General Geology. Coal occurs in three areas in Virginia: the Richmond and Farmville basins, the Valley Coal fields, and the Southwest Virginia coal field. The coal-bearing strata in these fields are generally gently dipping and fairly horizontal. The material types include: (i) sandstone; (ii) silt-stone; (iii) shale; and (iv) occasional thin clastic and calcareous zones of marine origin. The general geology is the Pottsville formation along the east side of the Appalachian coal field. This formation consists of a gray conglomerate, fine to coarse grained sandstone, and is known to contain limestone, siltstone and shale.

The formation also contains anthracite and bituminous coal seams (USGS, 2011). The major formations within the Pottsville age are Pocahontas, Lee, Norton, and Wise (Henderson, 1979). The Wise formation is part of the upper Pottsville age while the Lee and Pocahontas are part of the lower Pottsville age. The Norton formation belongs to the middle Pottsville age and contains overlying Gladeville sandstone (Butts, 1914). This case study considers mining in the Wise formation.

5.3.2 The Wise Formation. This is the youngest coal-bearing formation in the Southwest Virginia field. This field is bituminous and altered to coke by igneous intrusions in some areas. The formation comprises a 2,070 foot-thick mass of shale and at least nineteen coal seams lying between the Gladeville and Harlan sandstone (Henderson, 1979). The Wise formation constitutes the surface rock in most of the quadrangle lying south of Pound River.

Glamorgan coal seam is the first seam and lies immediately above the Gladeville sandstone with thickness greater than 2 feet. Forty feet above the Glamorgan is the Blair coal seam, which is persistent in the southeastern part of the quadrangle and ranges between 2 and 5 feet in thickness. The Clintwood coal seam lies about 200 feet above the Gladeville sandstone and has thicknesses between 6 and 12 feet (Henderson, 1979).

Clintwood coal seam is overlain by 20 to 40-foot thick sandstone. About 250 feet above the Clintwood seam is the Bolling coal seam (divided into two by 20 to 40 feet of shale). The lower Bolling coal seam varies between 18 inches to 4 feet while the upper is

18 inches to 5 feet thick. The upper Bolling coal seam is overlain by 50 to 80 feet thick micaceous sandstone.

Approximately 360 feet above the upper Bolling coal seam are the Standiford seams divided into two seams by a 20-foot inter-burden. The thickness of the lower seam is about 2.5 feet while the upper seam is approximately 3 feet thick. The Standiford coal seam is overlain successively by the Taggart, Low Splint, Phillips, Pardee and the High Splint coal seam. The High Splint coal seam underlies a small portion of Black Mountain and about 400 feet above the Pardee seam (Henderson, 1979).

Mining in two of the major seams (thereinafter referred to as seam #1 and seam #2) with dimensions shown in Table 5.1 is considered. The basin has variable coal properties shown in Table 5.2 and the mining area dimension are given as 4 km by 2 km. Figure 5.3 shows the geologic block model of the formation.

Table 5.1. Multi-Seam Formation Extent and Parameters

Description	**Value**
Topsoil thickness (ft)	3 - 10
Overburden thickness (ft)	80 - 120
Inter-burden thickness (ft)	40 - 50
Seam #1 thickness (ft)	20
Seam #2 thickness (ft)	40
Coal partings thickness in seam #2 (ft)	4

Table 5.2. Coal Quality Parameters (Henderson, 1979)

Description	**Value**	
	Seam #1	Seam #2
Moisture (%)	2.1 - 3.7	1.5 - 4.3
Volatile matter (%)	34.5 - 37.5	32.8 - 38.4
Fixed carbon (%)	51.3 - 54.7	55.4 - 61.6
Ash (%)	5.6 - 10.5	1.7 - 4.6
Sulfur (%)	0.7 - 1.1	0.4 - 0.8
BTU	12,790 - 13,910	13,720 - 14,810

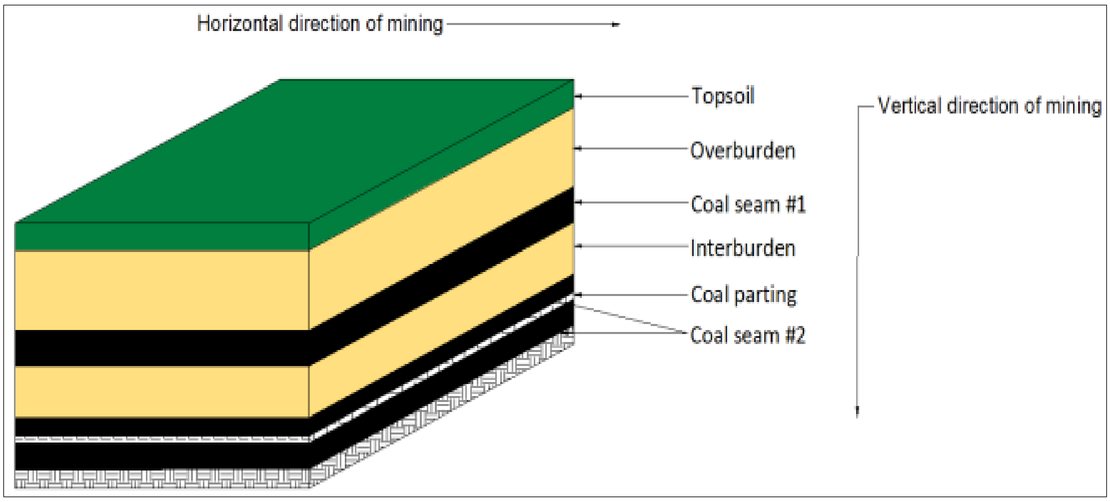


Figure 5.3 Block Model of the Formation

5.3.3 Flowcharts of the Extraction Process. The flowchart spans from preliminary geologic and technical operation analyses to life of mine plans. There are, however, separate categories of the mine planning process with unique details. Figure 5.4 illustrates the MSF operations. The flowchart begins with a detailed description of the geologic domain (number of coal seams, structural geology of the formation, etc.), production parameters and economic variables. Managerial and technical requirements and the extraction mechanisms are then used for production analyses, mine scheduling, material flow allocation, financial analyses and risk evaluation. Life of mine plans are developed based on these analyses and compared with external factors (environmental and safety compliance, investment returns, and managerial specifications). These processes are repeated to obtain optimal equipment selection plans, extraction sequences and sustainable economic strategies.

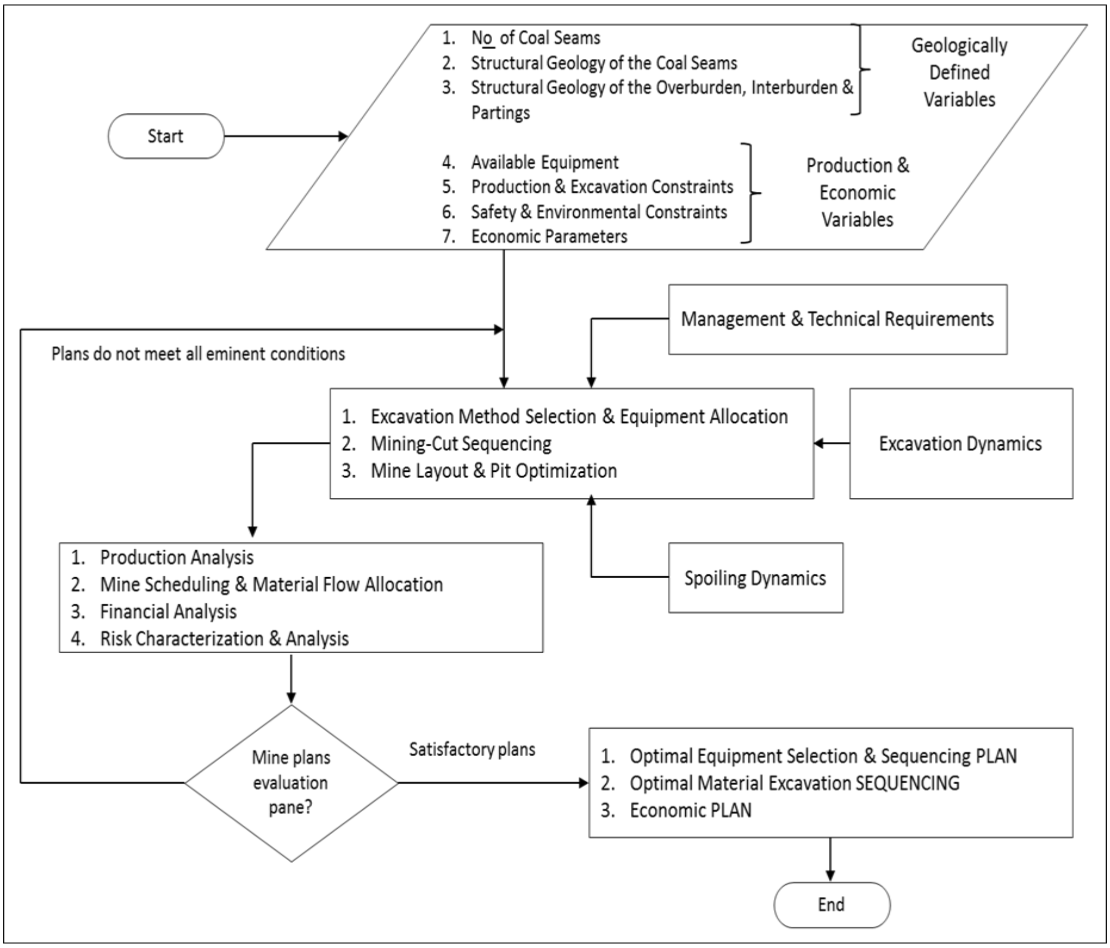


Figure 5.4 Flowchart of Multi-Seam Mining

5.3.4 Diggability Index and Equipment Options. Diggability index rating method and classification charts determine the ease of excavation based on which suitable equipment options are established. The index rating method is dependent on the following rock mass parameters: (i) uniaxial compressive strength (UCS); (ii) joint spacing; (iii) weathering; and (iv) bedding spacing. The ease of excavation of the coal seams, overburden and inter-burden as well as the rock parameters are illustrated in Table 1.0 in Appendix B. Based on these parameters and the general depositional characteristics of the formation, the following equipment and excavation options are investigated as possible extraction decisions: (i) dragline; (ii) ripper-dozer; (iii) cast blasting technique (CBT); and (iv) shovel and truck (SHT) system.

5.3.5 Capital (CAPEX) and Operating (OPEX) Cost Estimates. The CAPEX and OPEX estimates of each resource are shown as follows (x represents the maximum volume excavated per hour):

5.3.5.1 Dragline. P&H 9010C dragline model with bucket capacity of 75yd^3 is considered in this study. The CAPEX is estimated at $80 million per unit. The operating dimensions and model specifications are provided in Table 2.0 in Appendix B. The OPEX estimates are divided into equipment and labor operating cost. The equipment operating costs consist of 67% parts and 33% fuel and lubrication while the labor operating costs consist of 78% operator labor and 22% repair labor. Equation (5.9) gives the OPEX estimates ($/yd^3) (Bradley, 2002).

$$0.304 \times (swing\ angle)^{0.269}[1.984(x)^{-0.390} + 12.19(x)^{-0.888}] \tag{5.9}$$

5.3.5.2 Dozer. CAT D11 of blade capacity 57yd^3 is considered for the case study. The CAPEX is estimated at $2 million per unit. Table 3.0 in Appendix B illustrates the model specifications of the Dozer. The OPEX estimates are based on excavation and material relocation. The equipment operating costs average 47% parts and 53% fuel and lubrication. The labor operating costs average 86% operator labor and 14% repair labor. Equation (5.10) gives the OPEX estimates ($/yd^3) (Bradley, 2002).

$$5.81e^{-3}(dist.)^{0.904} \times 1.014e^{0.015(\%\,gradient)} \times 1.6[0.993(x)^{-0.430} + 14.01(x)^{-0.945}] \tag{5.10}$$

5.3.5.3 Cast blasting technique (CBT). CBT is efficient in minimizing labor and equipment costs, and offsets the cost of mechanical excavation of overburden. This method reduces the time required by dragline to swing and cast material by 25 to 30% (Ray et al., 1999). It was therefore estimated that the overall stripping cost is approximately 12.1% less the stripping cost of draglines. The CAPEX is also estimated at $2 million.

5.3.5.4 Shovel and truck system (SHT). CAT 7495HD cable shovel is selected for this analysis with 40-80yd^3 bucket capacity. The CAPEX is estimated at $25 million per unit. The model specifications are shown in Table 4.0 in Appendix B. A CAT 793F truck with nominal payload capacity of 249.45 tons is selected with an estimated CAPEX of $3 million per unit. The shovel and truck system is treated as one entity. Thus, the OPEX ($/yd^3) for this system is given by equation (5.11) (Bradley, 2002).

$$0.023(dist.)^{0.616} \times 0.877e^{0.046(\%\,gradient)} \times [0.407(x)^{-0.225} + 13.07(x)^{-0.936}] \tag{5.11}$$

5.3.6 Market Contractual Agreement. Bituminous coal is supplied to four destinations with unique quantity and quality specifications. Refer to Tables 5.0 and 6.0 in Appendix B for the minimum and maximum capacities as well as the coal quality specifications (based on operating paradigm of the power plants and environmental conditions) of each destination. Overseas export is neglected in this study, therefore, market conditions are specified with no external macro-economic influence. On-site stockpiles (STK) are defined in the optimization model to account for material mined and processed beyond the demand limits. Transportation cost and contractual agreement are factored into the comprehensive cost model for each destination. Table 7.0 (in Appendix B) shows the mining and transportation costs per destination.

5.3.7 Economic and Miscellaneous Parameters. A 10% discount rate for all prices and costs is selected based on commodity price and overall coal market analysis. \$41.01 per ton coal price is considered and a constant \$5.42 per ton selling price is assumed for all destinations. The maximum processing capacity is calculated as 880t/hr (minimum capacity is given as 32% of maximum capacity). The following are some assumptions: a period is a calendar year; 90% plant recovery of seam #2; 100% plant recovery of seam#1; coal extraction equipment capacity of 4808.48yd^3/hr; 100% thermal coal quantity; 0.2 hr/t of labor required in all periods; 90% extraction equipment availability; each strip is mined completely in a period; similar schedules exist for all periods; and the specific gravity of coal is 0.04t/ft^3. The stripping ratio is calculated as 4.89:1 (t:t). The economic analysis and parameters are limited within the United States economy. All other input data for the model are provided in Appendix B.

5.3.8 Waste Extraction Model. Considering the extraction sequence shown in Figure 5.5, the overburden and inter-burden are divided into four strips. Each strip is scheduled to be mined completely per period k. Due to the general geometry of strip mines, all the blocks within a particular mining strip are assumed to be aggregated into a single unit with dimensions equal to the strip dimensions. This allows for continuous extraction of a particular strip without frequent equipment movement. The following indices for the objective function are valid for all periods:

(i) mining blocks; $n \in \{1, \dots . N\}$

(ii) resource; $j \in \{1, \dots ,4\}$ where $j = 1$(dragline); $j = 2$(dozer); $j = 3$(shovel & truck); and $j = 4$(cast blasting)

(iii) scheduling period; $k \in \{1, \dots ,4\}$

(iv) mining strip $s \in \{1, \dots ,4\}$

The objective function is given in equation (5.12).

$$
\begin{aligned}
min \Bigg[& 0.28 * 10^6 \begin{pmatrix} c_1^{11} & c_1^{21} & \cdots & c_1^{j1} \end{pmatrix} \begin{pmatrix} x_1^{11} \\ x_1^{21} \\ \vdots \\ x_1^{41} \end{pmatrix} \\
& + 9.17 * 10^6 \begin{pmatrix} c_2^{12} & c_2^{22} & \cdots & c_2^{j2} \end{pmatrix} \begin{pmatrix} x_2^{12} \\ x_2^{22} \\ \vdots \\ x_2^{42} \end{pmatrix} \\
& + 9.17 * 10^6 \begin{pmatrix} c_3^{13} & c_3^{23} & \cdots & c_3^{j3} \end{pmatrix} \begin{pmatrix} x_3^{13} \\ x_3^{23} \\ \vdots \\ x_3^{43} \end{pmatrix} + \cdots \\
& + w_4 c_4^{4,4} x_4^{4,4} \Bigg]
\end{aligned}
\tag{5.12}
$$

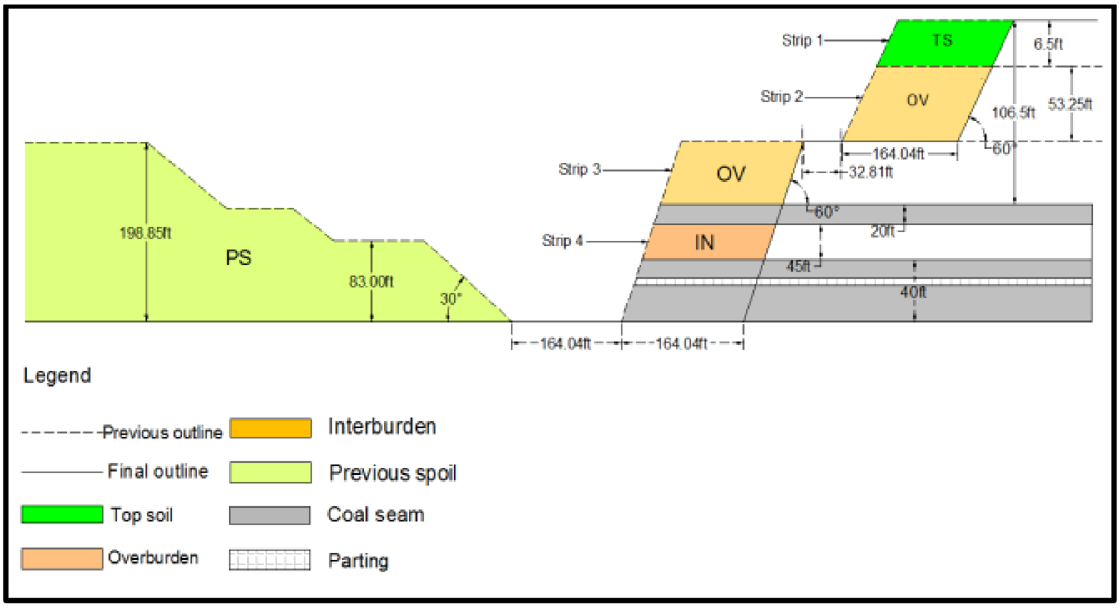


Figure 5.5 Waste Extraction Sequence

Equation (5.12) is subject to the following constraints: (i) reserve; (ii) energy consumption; (iii) mining capacity; (iv) spoiling area availability and material rehandling; (v) equipment availability; (vi) equipment utilization; (vii) minimum mining width and resource interaction; (viii) haulage unit capacity and reach geometry; (ix) diggability; (x) labor; (xi) critical bench height; and (xii) non-negativity constraints.

The reserve constraints are given in equations (5.13) and (5.14).

$$\begin{bmatrix} x_1^{11} & \leq & 1 \\ x_1^{21} & \leq & 1 \\ x_1^{31} & \leq & 1 \\ \vdots & \vdots & \vdots \\ x_4^{4,4} & \leq & 1 \end{bmatrix} \tag{5.13}$$

$$\begin{bmatrix} x_1^{11} & +x_1^{21} & +x_1^{31} & +\cdots+ & x_1^{4,1} & = & 1 \\ x_2^{12} & +x_2^{22} & +x_2^{32} & +\cdots+ & x_2^{4,2} & = & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ x_4^{1,4} & +x_4^{2,4} & +x_4^{3,4} & +\cdots+ & x_4^{4,4} & = & 1 \end{bmatrix} \tag{5.14}$$

Based on the given energy consumption rates (refer to Table 8.0 in Appendix B) and cost per energy of $0.05, equations (5.15) to (5.18) are derived. The total permissible energy cost for all periods are given respectively as $ 0.90 million, $ 9.50 million, $ 10.00 million, and $ 11.00 million.

$$Period\ 1:\quad 0.05 \times 0.28 * 10^6 (\varepsilon_1^{11} + e_1^{11} \quad \cdots + \cdots \quad \varepsilon_1^{41} + e_1^{41}) \begin{pmatrix} x_1^{11} \\ x_1^{21} \\ \vdots \\ x_1^{41} \end{pmatrix} \le 0.9 * 10^6 \tag{5.15}$$

$$Period\ 2:\quad 0.05 \times 9.17 * 10^6 (\varepsilon_2^{12} + e_2^{12} \quad \cdots + \cdots \quad \varepsilon_2^{42} + e_2^{42}) \begin{pmatrix} x_2^{12} \\ x_2^{22} \\ \vdots \\ x_2^{42} \end{pmatrix} \le 9.5 * 10^6 \tag{5.16}$$

$$Period\ 3:\quad 0.05 \times 9.17 * 10^6 (\varepsilon_3^{13} + e_3^{13} \quad \cdots + \cdots \quad \varepsilon_3^{43} + e_3^{43}) \begin{pmatrix} x_3^{13} \\ x_3^{23} \\ \vdots \\ x_3^{43} \end{pmatrix} \le 10 * 10^6 \tag{5.17}$$

$$Period\ 4:\quad 0.05 \times 7.5 * 10^6 (\varepsilon_4^{14} + e_4^{14} \quad \cdots + \cdots \quad \varepsilon_4^{44} + e_4^{44}) \begin{pmatrix} x_4^{14} \\ x_4^{24} \\ \vdots \\ x_4^{44} \end{pmatrix} \le 11 * 10^6 \tag{5.18}$$

The capacities of the various resources are calculated based on their productivities per period. Equations (5.19) to (5.21) represent the productivity formulations for draglines, dozers and shovels respectively (Frimpong, 2011; Assakkaf, 2003).

$$P_D\left(\frac{lcyd}{hr}\right)=\frac{k}{c_t}\left(\frac{n_{\underline{0}}\ of\ passes}{hr}\right)\times bc\left(\frac{lcyd}{pass}\right)\times a\times u\times bf \tag{5.19}$$

$$P_{DZ}\left(\frac{lcyd}{hr}\right)=\frac{60min\times Blade\ Load}{Push\ time+Return\ time+Manuever\ time} \tag{5.20}$$

$$P_s=\frac{k}{c_t}\left(\frac{n_{\underline{0}}\ of\ passes}{hr}\right)\times d_c\left(\frac{lcyd}{pass}\right)\times ff_s\times a\times u\times s_f\times p_f \tag{5.21}$$

$$k=\begin{cases}60 & \forall c_t\ in\ minutes\\ 3600 & \forall c_t\ in\ seconds\end{cases}$$

From equations (5.19) to (5.21), the productivities are calculated as: 4617 yd^3/hr, 190 yd^3/hr, 4808.48 yd^3/hr and 13,851yd^3/hr for the dragline, dozer, SHT, and CBT respectively. Equations (5.22) and (5.23) are the maximum and minimum capacities respectively. Short-term periodic variations in productivity are not captured in this study. Refer to Table 9.0 in Appendix B for the input parameters.

$$\begin{bmatrix} w_1\times x_1^{11} & \leq & 14.96*10^6\\ w_1\times x_1^{21} & \leq & 0.62*10^6\\ \vdots & \vdots & \vdots\\ w_4\times x_4^{4,4} & \leq & mu^{4,4}\end{bmatrix} \tag{5.22}$$

$$\begin{bmatrix} w_1\times x_1^{11} & \geq & ml^{1,1}\\ w_1\times x_1^{21} & \geq & ml^{2,1}\\ \vdots & \vdots & \vdots\\ w_4\times x_4^{4,4} & \geq & ml^{4,4}\end{bmatrix} \tag{5.23}$$

Mechanical availability plays a major role in the overall efficiency of excavation. The mechanical available hours per period k for all resources are given in Table 10.0 in Appendix B. The total hours required by each resource to excavate a unit ton of block n are also given in Table 11.0 in Appendix B. Mechanical availability constraint is relaxed for CBT. Equations (5.24) and (5.25) are the mechanical availability and utilization constraints. These constraints are extensions of the mining capacity constraints.

$$\begin{bmatrix} 4.45 * 10^{-4} w_1 \times x_1^{11} & \leq & 5400 \\ 6.77 * 10^{-3} w_1 \times x_1^{21} & \leq & 5400 \\ \vdots & \vdots & \vdots \\ h_4^{4,4} w_4 \times x_4^{4,4} & \leq & mu^{4,4} \end{bmatrix} \quad (5.24)$$

$$\begin{bmatrix} 4.45 * 10^{-4} w_1 \times x_1^{11} & \geq & mu^{1,1} \\ 6.77 * 10^{-3} w_1 \times x_1^{21} & \geq & mu^{2,1} \\ \vdots & \vdots & \vdots \\ h_4^{4,4} w_4 \times x_4^{4,4} & \geq & mu^{4,4} \end{bmatrix} \quad (5.25)$$

The next constraint is the minimum mining width/resource interaction constraint as explained in Section 3. Assuming the dragline and the SHT system covered 0.05ft per unit ton of topsoil excavated, 0.0014ft per unit ton of overburden and inter-burden excavated, external length of space outside the digging domain is $100ft$, and a minimum working space requirement calculated by equation (5.26), the minimum mining width constraint is shown in equation (5.27). Equation (5.26) is estimated using the parameters provided in Tables 2 and 4 in Appendix B.

$$Minimum\ width = \tau + \varphi \quad (5.26)$$

$$\begin{bmatrix} 0.05 w_1 \times x_1^{11} & \leq & 13123.4 + 100 - dm^{11} \\ 0.0014 w_1 \times x_1^{31} & \leq & 13123.4 + 100 - dm^{31} \\ \vdots & \vdots & \vdots \ + \ \vdots \ - \ \vdots \\ 0.0014 w_4 \times x_4^{14} & \leq & 13123.4 + 100 - dm^{14} \\ 0.0014 w_4 \times x_4^{34} & \leq & 13123.4 + 100 - dm^{34} \end{bmatrix} \quad (5.27)$$

The haulage unit capacity is matched with the maximum and minimum mining capacities of the shovel. Thus, given a constant maximum haulage capacity for each of the periods as 12.37*10^{6}tons, the constraint is shown in equation (5.28). The minimum haulage limit is not specified for this mining case.

$$\begin{bmatrix} w_1 x_1^{31} & \leq & 12.37 * 10^6 \\ w_2 x_2^{32} & \leq & 12.37 * 10^6 \\ w_3 x_3^{33} & \leq & 12.37 * 10^6 \\ w_4 x_4^{34} & \leq & 12.37 * 10^6 \end{bmatrix} \tag{5.28}$$

The labor requirements are defined in personnel-hours assuming all available labor is skilled in all operations. Given personnel-hours per ton excavated by the dragline, dozer, SHT, and CBT respectively as 0.05, 0.05, 0.1 and 0.05, equation (5.29) restricts the amount of material excavated by each resource. The available hours for each period are calculated as 0.5*10^6. Refer to Table 11.0 in Appendix B for the required hours per ton of material excavated.

$$\begin{bmatrix} 0.05 w_1 x_1^{11} + & 0.05 w_1 x_1^{21} + \cdots + & 0.05 w_1 x_1^{41} \leq 0.5 * 10^6 \\ 0.05 w_2 x_2^{12} + & 0.05 w_2 x_2^{22} + \cdots + & 0.05 w_2 x_2^{42} \leq 0.5 * 10^6 \\ \vdots \quad\quad \vdots & \vdots \quad\quad + \cdots + & \vdots \quad \vdots \quad \vdots \\ 0.05 w_4 x_4^{14} + & 0.05 w_4 x_4^{24} \ + \cdots + & rs_4^{4,4} w_4 x_4^{4,4} \leq 0.5 * 10^6 \end{bmatrix} \tag{5.29}$$

A critical digging depth for draglines is given as 12ft, hence, equation (5.30) is derived to ensure digging efficiency.

$$\begin{bmatrix} hd_1 \times x_1^{11} & \geq & 12 \\ hd_2 \times x_2^{12} & \geq & 12 \\ hd_3 \times x_3^{13} & \geq & 12 \\ hd_4 \times x_4^{14} & \geq & 12 \end{bmatrix} \tag{5.30}$$

The dump volumes available for all periods are stated in Table 12.0 in Appendix B. These volumes are assumed to be decreased by the volume of the previous cuts. Based on these parameters and the digging geometry, equation (5.31) is formulated.

$$\begin{bmatrix} w_1 x_1^{11} \times (1/0.02) & \leq & di^{11} + de^{11} \\ w_1 x_1^{21} \times (1/0.02) & \leq & di^{21} + de^{21} \\ \vdots & \vdots & \vdots \\ w_4 x_4^{4,4} \times (1/0.08) & \leq & di^{j,k} + de^{j,k} \end{bmatrix} \tag{5.31}$$

The non-negativity constraint is shown in equation (5.32).

$$\begin{bmatrix} x_1^{11} & \geq & 0 \\ x_1^{21} & \geq & 0 \\ x_1^{31} & \geq & 0 \\ \vdots & \vdots & \vdots \\ x_n^{j,k} & \geq & 0 \end{bmatrix} \tag{5.32}$$

5.3.9 Coal Seam Extraction Model. As shown in Figure 5.6, extraction of seam #1 starts in period t, assuming t corresponds to the completion period of block $OV3$. This assumption ensures that significant portions of the coal seam are exposed for blending. The assumption also allows for efficient equipment interaction (waste excavation equipment and coal seam extraction processes). Similarly, given comparable waste excavation and coal seam extraction rates, the extraction of seam #2 will commence in period $t + 5$, assuming $t + 5$ corresponds to the starting period of block $OV4$ (refer to Figure 5.7).

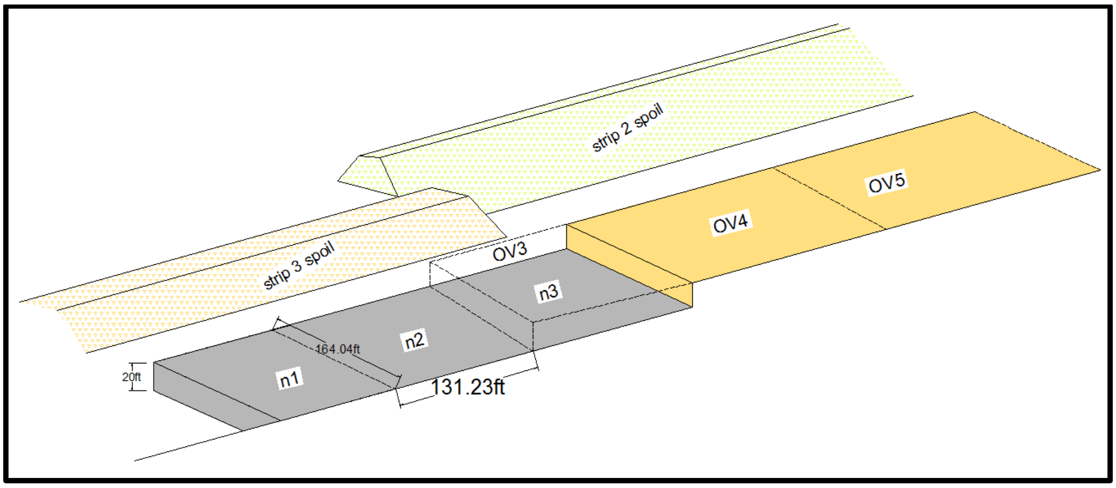


Figure 5.6 Excavation sequence of coal seam #1

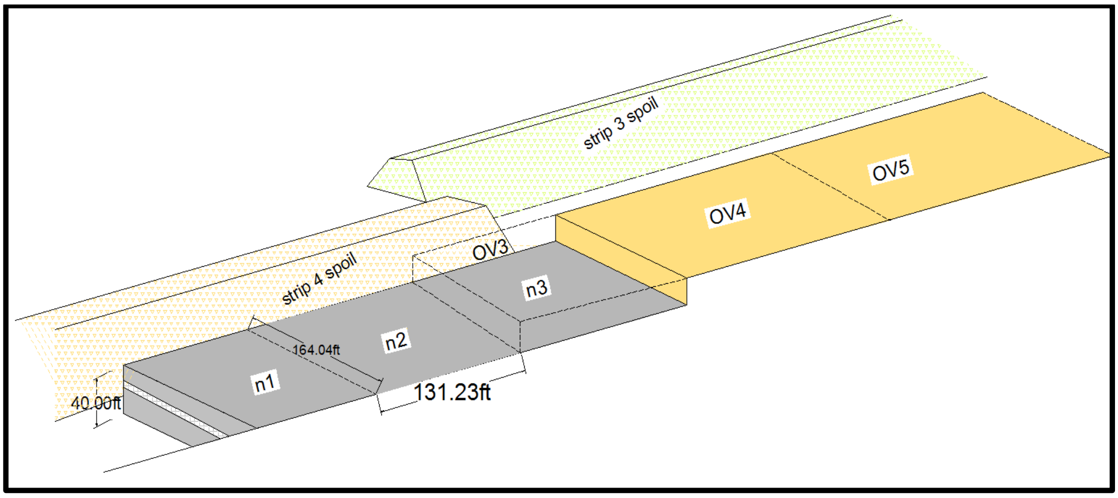


Figure 5.7 Excavation sequence of seam #2

The following are the indices for the objective function as defined in equation (5.33):

(i) Seam cut; $m \in \{1, \dots, M\}$

(ii) Product destination; $e \in \{1, \dots, 5\}$

(iii) Scheduling period; $t \in \{1, \dots, 2\}$; each seam is exposed fully per scheduling period

(iv) Set of exposed seam cuts ; $g \in \{1, \dots, 2\}$

$$\sum_{g=1}^{2}\left\{max\sum_{e=1}^{5}\sum_{t=1}^{2}\left[\sum_{m\in g}\left(y_m^{e,t} \times \varpi_m^{e,t}\right)\right]\right\} \tag{5.33}$$

Given the objective function in equation (5.33), the decision variables $y_m^{e,t}$ are constrained by the following: (i) reserve; (ii) mining capacity; (iii) processing capacity; (iv) transportation and stockpile capacities; (v) market condition and contractual agreement; (vi) labor; (vii) haulage capacity; (viii) extraction equipment availability and utilization; (ix) coal quality (calorific value, sulfur content, fixed carbon content, ash content, moisture content and volatile matter); and (x) non-negativity constraint.

Equations (5.34) and (5.35) are the reserve constraints.

$$\begin{bmatrix} y_1^{1,1} & \leq & 1 \\ y_1^{2,1} & \leq & 1 \\ y_1^{3,1} & \leq & 1 \\ \vdots & \vdots & \vdots \\ y_m^{e,t} & \leq & 1 \end{bmatrix} \tag{5.34}$$

$$\begin{bmatrix} y_1^{1,1} & +y_1^{2,1} & +y_1^{3,1} & + \cdots & +y_1^{5,1} & = 1 \\ y_2^{1,1} & +y_2^{2,1} & +y_2^{3,1} & + \cdots & +y_2^{5,2} & = 1 \\ \vdots & \vdots & \vdots & & \vdots & \\ y_m^{e,t} & +y_m^{e,t} & +y_m^{e,t} & + \cdots & +y_m^{e,t} & = 1 \end{bmatrix} \tag{5.35}$$

The mining capacity constraints are shown in equations (5.36) and (5.37) where the upper limits are calculated as 31.16 x 10^6 tons for all periods. The lower limits are also calculated as 1.72 x10^6 tons and 3.44 x10^6 tons respectively for the first and second periods.

$$\begin{bmatrix} \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,1})] \geq 1.72 \times 10^6 \\ \\ \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,2})] \geq 3.44 \times 10^6 \end{bmatrix} \tag{5.36}$$

$$\begin{bmatrix} \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,1})] \leq 31.16 \times 10^6 \\ \\ \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,2})] \leq 31.16 \times 10^6 \end{bmatrix} \tag{5.37}$$

The next constraint controls the processing capabilities of the treatment plant. A minimum bound (equation (5.38)) is set to maintain a constant rate of operation while a maximum bound (equation (5.39)) controls the upper limit of material treated in all periods. The maximum and minimum limits are given respectively as $5.28 * 10^6$tons and $1.69 * 10^6$tons.

$$\begin{bmatrix} \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,1})] \geq 1.69 \times 10^6 \\ \\ \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,2})] \geq 1.69 \times 10^6 \end{bmatrix} \tag{5.38}$$

$$\begin{bmatrix} \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,1})] \leq 5.28 \times 10^6 \\ \\ \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,2})] \leq 5.28 \times 10^6 \end{bmatrix} \tag{5.39}$$

Coal products are transported by rails to thermal plants. The capacities of the transportation facilities and on-site capacities must be adequate to meet the total coal extracted and treated. Transportation capacities must also meet contractual agreements. Equation (5.40) is formulated given maximum rail capacity and on-site stockpile capacities $\forall t$ as 4.38 x 10^6tons.

$$\begin{bmatrix} \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,1})] \leq 4.38 \times 10^6 \\ \\ \sum_{m \in g} [v_m \times \sum_{e=1}^{5} (y_m^{e,2})] \leq 4.38 \times 10^6 \end{bmatrix} \tag{5.40}$$

The market coal quantity requirements are provided in Table 5.0 in Appendix B. Based on these parameters, equations (5.41) and (5.42) are derived illustrating the upper limits and lower limits respectively for all periods.

$$\begin{bmatrix} \sum_{m \in g} [v_m \times y_m^{1,1}] + \sum_{m \in g} [v_m \times y_m^{1,2}] \leq 1.26 \times 10^6 \\ \vdots \qquad + \qquad \vdots \qquad \vdots \qquad \vdots \\ \sum_{m \in g} [v_m \times y_m^{5,1}] + \sum_{m \in g} [v_m \times y_m^{5,2}] \leq 2.52 \times 10^6 \end{bmatrix} \tag{5.41}$$

$$\begin{bmatrix} \sum_{m \in g} [v_m \times y_m^{1,1}] + \sum_{m \in g} [v_m \times y_m^{1,2}] \geq 0.68 \times 10^6 \\ \vdots \qquad + \qquad \vdots \qquad \vdots \qquad \vdots \\ \sum_{m \in g} [v_m \times y_m^{5,1}] + \sum_{m \in g} [v_m \times y_m^{5,2}] \geq 0 \end{bmatrix} \tag{5.42}$$

Similar to the waste excavation model, equation (5.43) limits the quantity of coal produced to the labor available and required per unit production. It is assumed that all labor is equally skilled in all operations and the required hours per unit production is given as 0.2. The available labor hour is 0.7 x 10^6 per year and constant for all periods.

$$\begin{bmatrix} \sum_{m\in g} \times 0.2 \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,1} \right) \right] & \leq & 0.7 * 10^6 \\ \sum_{m\in g} \times 0.2 \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,2} \right) \right] & \leq & 0.7 * 10^6 \end{bmatrix} \tag{5.43}$$

Equations (5.44), (5.45) and (5.46) are the haulage, equipment availability and utilization constraints respectively. The maximum haulage capacity is calculated as 12.37 x 10^6tons, the extraction equipment available hours is given as 5400 hours, and the minimum utilization hours to meet production targets is calculated as 1500 hours for all periods. The hours required to produce a unit ton of coal is fairly equivalent to the SHT system in the waste extraction model (SHT system is used for the coal seam extraction).

$$\begin{bmatrix} \sum_{m\in g} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,1} \right) \right] & \leq & 12.37 * 10^6 \\ \sum_{m\in g} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,2} \right) \right] & \leq & 12.37 * 10^6 \end{bmatrix} \tag{5.44}$$

$$\begin{bmatrix} \sum_{m\in g} \times 9.26 \times 10^{-4} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,1} \right) \right] & \leq & 0.54 * 10^4 \\ \sum_{m\in g} \times 9.26 \times 10^{-4} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,2} \right) \right] & \leq & 0.54 * 10^4 \end{bmatrix} \tag{5.45}$$

$$\begin{bmatrix} \sum_{m\in g} \times 9.26 \times 10^{-4} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,1} \right) \right] & \geq & 0.15 * 10^4 \\ \sum_{m\in g} \times 9.26 \times 10^{-4} \left[v_m \times \sum_{e=1}^{5} \left(y_m^{e,2} \right) \right] & \geq & 0.15 * 10^4 \end{bmatrix} \tag{5.46}$$

The next constraints control the quality of coal products transported to the five destinations. Contractual agreement and power requirements demand coal products at specified qualities (refer to Table 6.0 in Appendix B). The variability of the coal properties in each cut are captured to determine the critical blends in meeting these specifications. Blending is however limited to the coal cuts exposed per given period t.

The fifth destination, on-site stockpile, has no other quality limitations aside the coal parameters. This ensures that all coal products which do not meet specifications of the four destinations are stockpiled.

The recovery of seam #2 might be lowered due to the presence of coal partings (refer to Figure 5.3). This occurrence is modeled by the recovery parameter, r in the objective function. The variable coal quality parameters are given in Table 5.2. The coal quality formulations are shown in Section 3.

Equation (5.47) is the non-negativity constraint to ensure all positive decision variables.

$$\begin{bmatrix} y_1^{11} & \geq & 0 \\ y_1^{21} & \geq & 0 \\ y_1^{31} & \geq & 0 \\ \vdots & \vdots & \vdots \\ y_m^{e,t} & \geq & 0 \end{bmatrix} \tag{5.47}$$

5.3.10 Risk Simulation Modeling and SOLVER Parameters. The objective is to determine the effect of multiple uncertainties on the stochastic input parameters (geologic variability, economic uncertainties and excavation/spoiling dynamics). The optimal simulation parameters are obtained from experimental designs, where parameters are varied gradually to obtain distributions for the mean values and variances of reach run. 1000 to 15,000 iterations in intervals of 1000 are run for each of the models (cost and revenue functions).

The results are graphed to determine zones of parametric stability. Figures (5.8) to (5.10) show the results for the mean values and the standard deviations against the number of iterations for each function. Parametric stabilities are observed after 8000 iterations for mining cost, 10000 iterations for revenue and 3000 iterations for net present value.

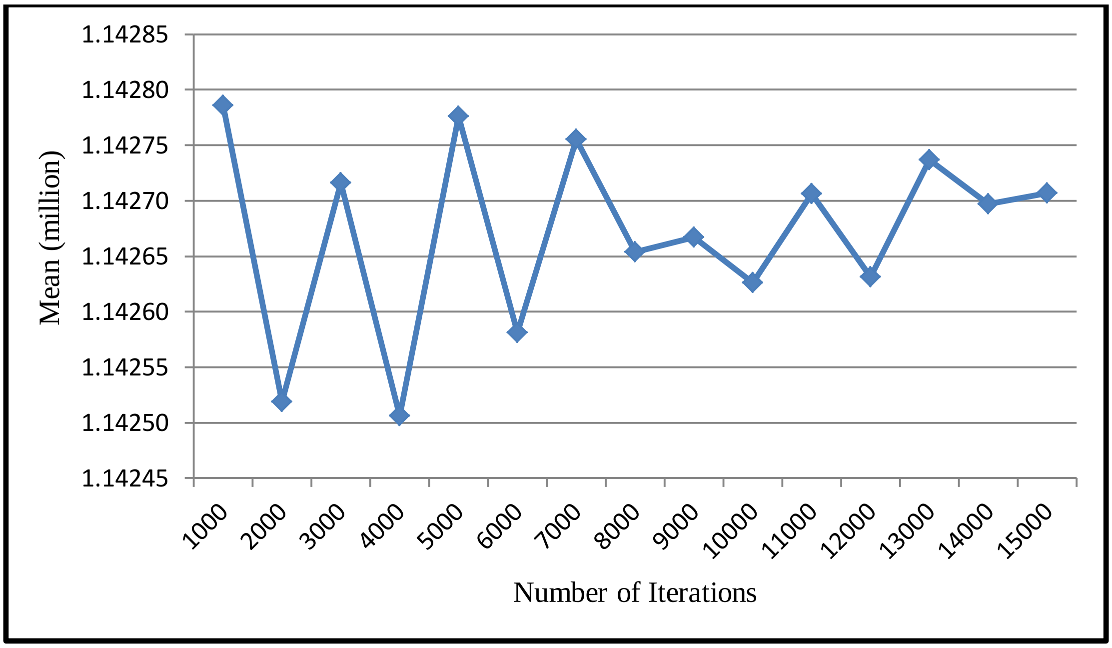


Figure 5.8 Mean Mining Cost vs. Number of Iterations

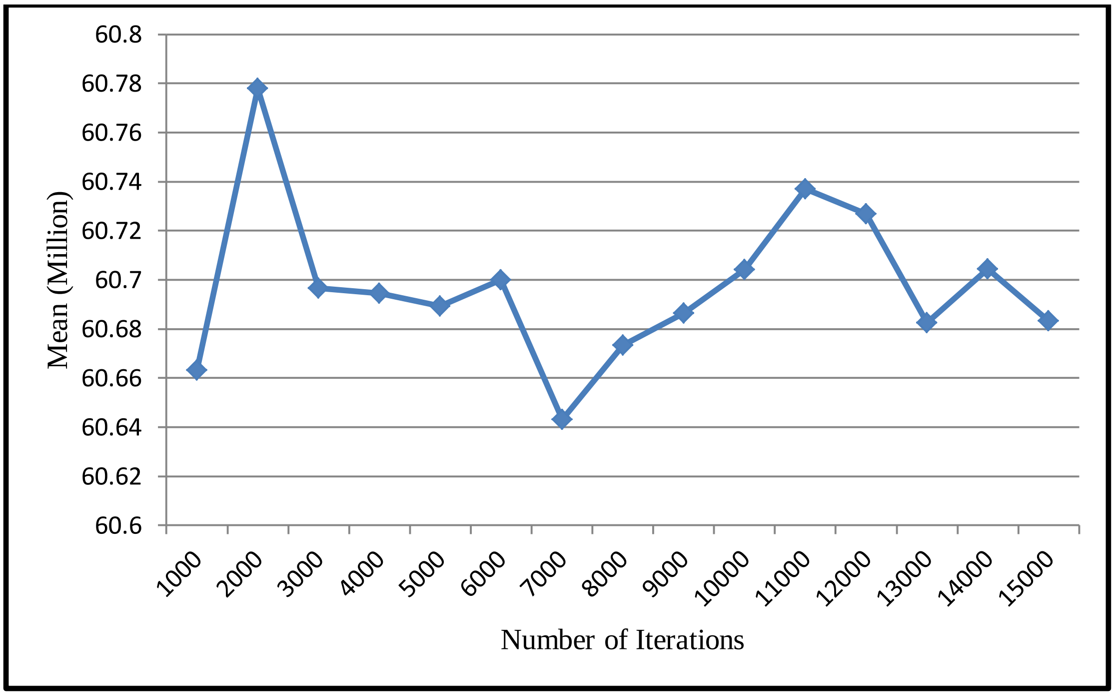


Figure 5.9 Mean Revenue vs. Number of Iterations

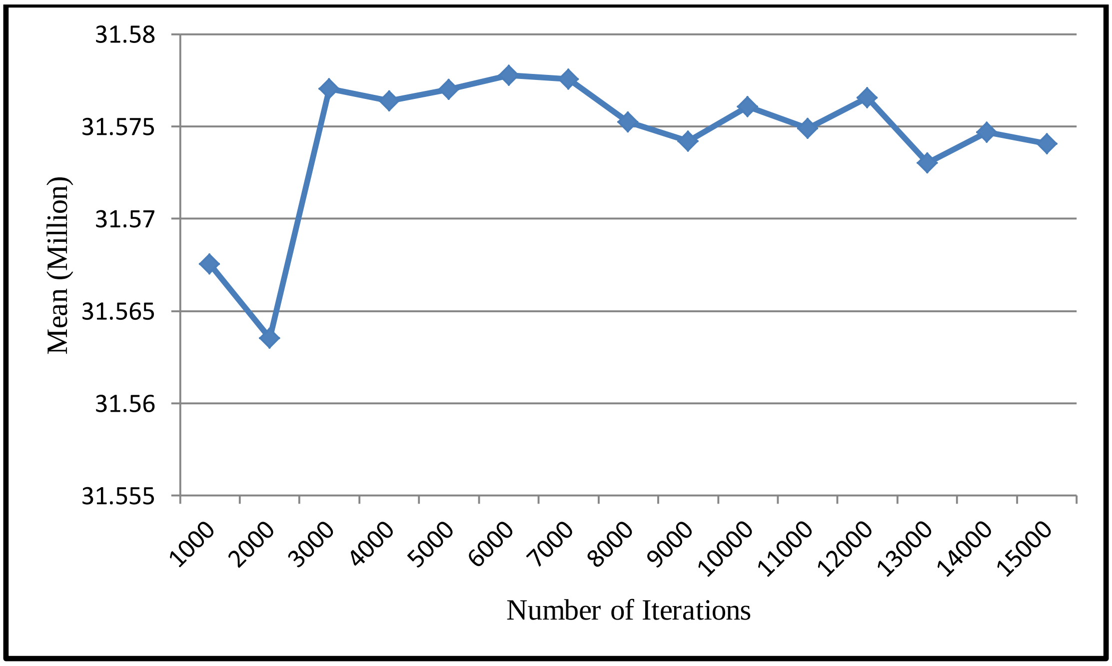


Figure 5.10 Mean NPV vs. Number of Iterations

BestFit (Palisade, 2012) is used to fit probability distributions to the available data. Normal PDFs are applied to mining, processing, and capital costs with ±20% (values determined from qualitative and quantitative data analysis). Lognormal PDF is applied to commodity price with the same truncations as discussed above. Uniform PDFs are used to model the processing recoveries, thermal coal quantities, selling price, and the total tons of material.

The required number of iterations, maximum time, precision, degree of tolerance and convergence parameters is defined in SOLVER. The primary and dual tolerance are maintained at 0.0000001; precision and convergence of 0.6; 150 number of iterations; and the maximum time is set to 15seconds.

5.4 SUMMARY

Numerical modeling frameworks, solution algorithms, and flowcharts are defined for the experimental analysis of the developed models. The generalized reduced gradient; the solution algorithms to the NLP models, are discussed. Procedures to determine optimal simulation parameters, model fitting algorithms, and optimal SOLVER parameters are also discussed. The application of the SOP model is presented with an MSF bituminous coal mining case located in Southern Virginia. The results produce optimal resource allocations, improve equipment productivity and ensure quality coal products. All input data are provided in the Appendices A and B.

6. DISCUSSION OF RESULTS

This section focuses on a detailed discussion of the results of the MSF mining case presented in Section 5. The discussion includes: waste extraction sequencing, equipment allocation and productivities, coal blending and transportation schedules, and economic evaluations of excavation alternatives. The details of the model outputs and definitions of symbols are provided, respectively, in Appendix C and the nomenclature.

6.1 ANALYSIS OF WASTE EXTRACTION MODEL

The optimized mining cost is $ 1.14 million for the 4 strips (see Figure 5.5) and approximately $149.34 million for the entire deposit. These figures show -3.51% change compared to the conventional traditional methods (dozer pushes topsoil and dragline excavates overburden/inter-burden). Figure 6.1 depicts the equipment allocations and the percentage of material excavated. The optimized results allocate 96.92% of the topsoil (strip-1) to the dozer in the first period. This allocation is influenced by the material properties and the dumping mechanisms.

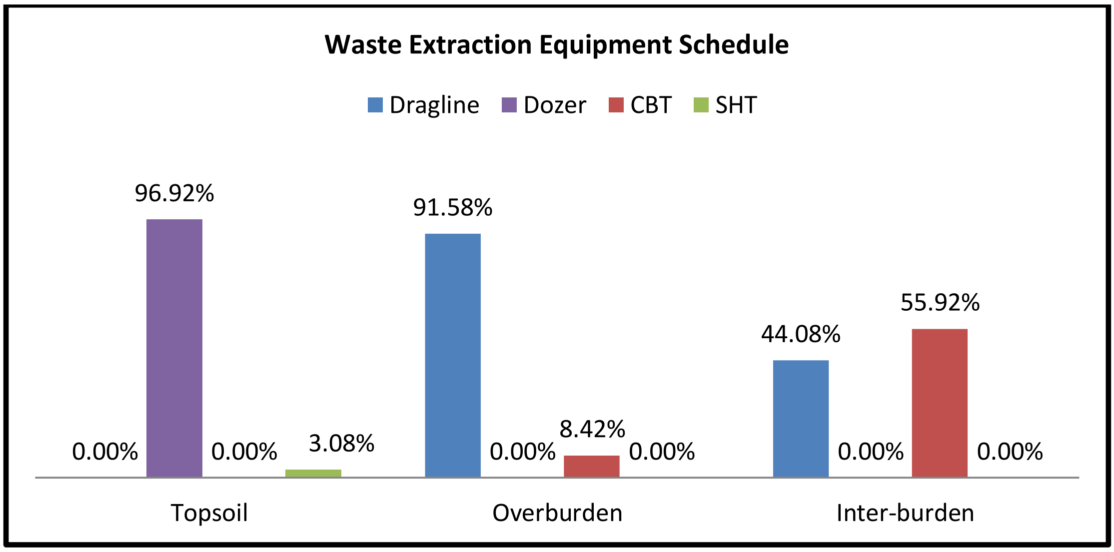


Figure 6.1 Waste Excavation Equipment Schedule

Due to the technical limitations on the dozer, 3.08% of the remaining topsoil material is allocated to the SHT system. The selection of SHT is influenced by the energy consumption specifications, the critical digging depth and OPEX figures. Experimental analysis of 100% dozer application in topsoil shows a +2.05% cost difference compared to the optimal results.

The dragline is allocated to 95.21% of overburden (strip-2) in period 2, 87.94% of overburden (strip-3) in period 3, and 44.08% of inter-burden (strip-4) in period 4. This schedule is influenced by the OPEX figures, specified energy costs, internal dumping concurrency with mining advancement, critical digging depth, material re-handling mechanisms, and the haulage requirement.

As mining progresses, the dragline spoiling distance reduces due to its fixed digging geometry, hence the percentage allocation reduces. Wider pits are normally created in thick overburdens to reduce material re-handling and dragline walking times (Frimpong, 2011; Satyanarayana, 2012). In such situations, the main challenge is the increment in swing angles. Increased swing angles result in reduced productivity, high maintenance cost and high clean-up times (MA et al., 2006; Scott, 2010; and Frimpong, 2011). Due to these factors, experimental analyses show +1.02% increments in mining cost for a 100% dragline allocation in the overburden. Engagement of a secondary dragline with an approximate cost of $100 million increases this cost difference. Subsequently, a 77yd^3 drop in productivity per unit change in cycle time is also recorded for the sub-optimal solution. The drop in productivity analyses exclude deadheading periods during material re-handling; an area which increases mining cost and reduces productivity. The mining cost distributions per equipment engagement in the overburden are shown in Figure 6.2.

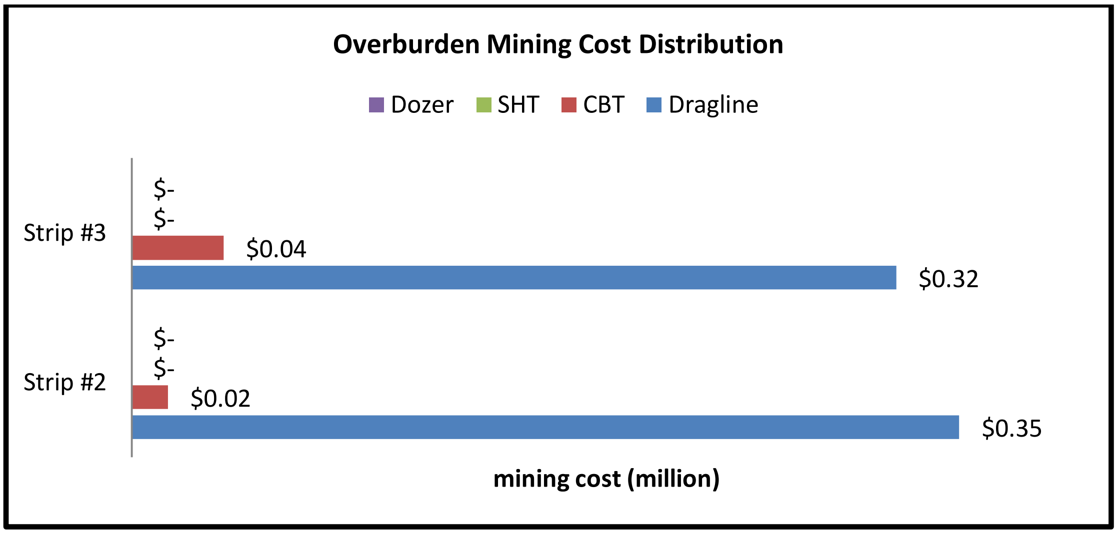


Figure 6.2 Mining Cost Distributions per Equipment Allocation

The CBT technique is allocated to 5% of overburden (strip-2) in period 2, 12% of overburden (strip-3) in period 3, and 56% of inter-burden (strip-4) in period 4 (as a result of the digging constraint on the dragline). Figure 6.3 shows the CBT allocations in the various mining strips with a moving average trend line.

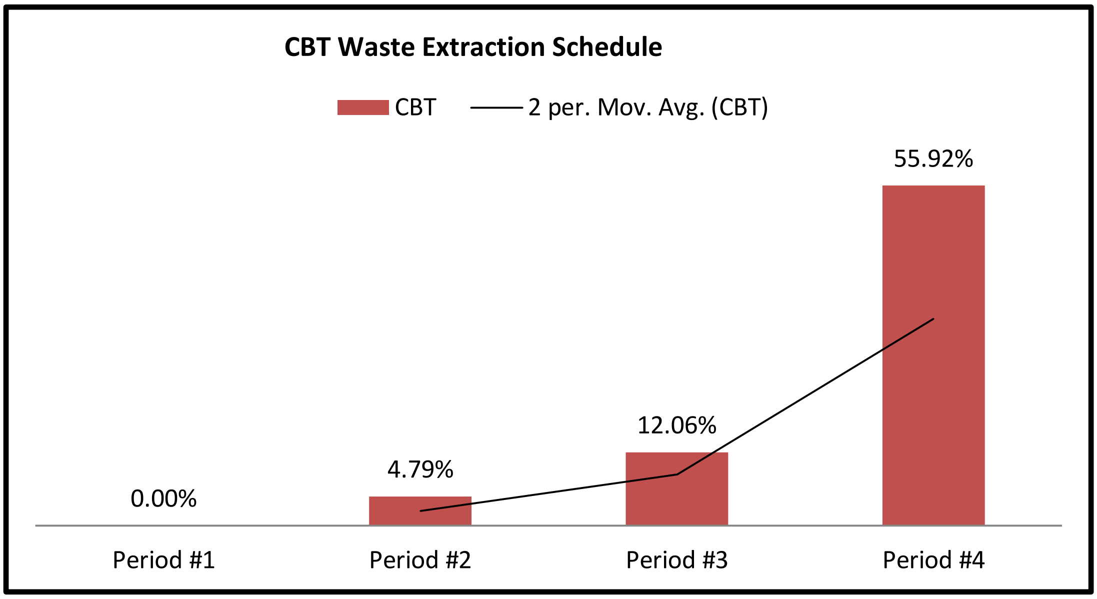


Figure 6.3 CBT Waste Extraction Schedule per Mining Strip

CBT reduces the time required by dragline to swing and cast material by 25 to 30% (Ray et al., 1999). With a calculated OPEX of $0.035/ton (13.77% less than the dragline OPEX), the allocation is restricted by energy specifications, internal dumping dynamics, haulage capacity constraints, and the geologic variability of the deposit. The trend line, as shown in Figure 6.3, illustrates an increment in percentage allocation as mining progresses. This is due to the material re-handling and spoiling geometry constraints on the dragline. The mining cost for 100% allocation of CBT in overburden and inter-burden is approximately 14% less than 100% dragline allocation in the same material. These figures exclude the possible high cost of material relocation, internal dumping technical difficulties, environmental considerations, and operational safety parameters.

Figure 6.4 shows the waste excavation cost distributions. In this schedule, the binding constraints include: dragline dump area availability, energy consumption rates, mining capacities of all resources, reserve constraints, and utilization rates. These parameters illustrate the geologic variability and equipment dynamic operational parameters in MSFs, and restrict any possible change in the cost function.

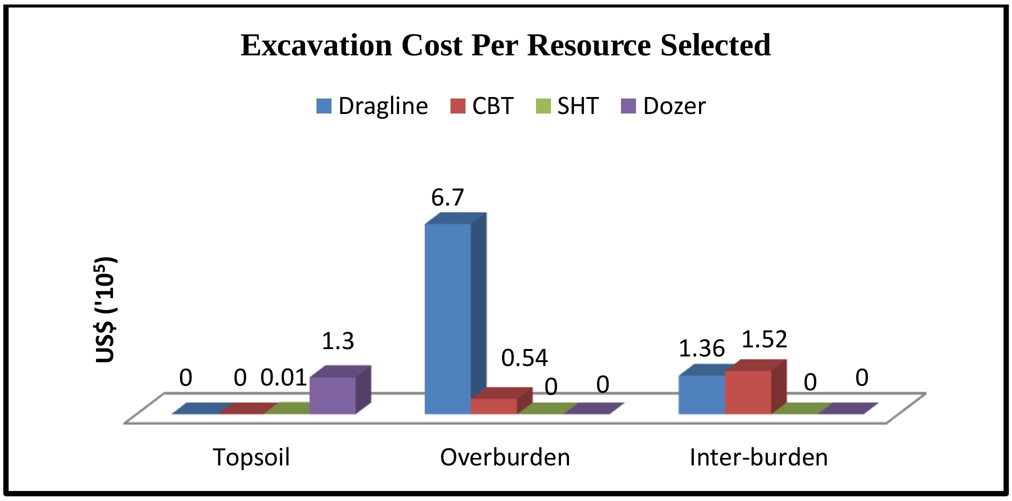


Figure 6.4 Waste Extraction Cost Distribution

6.2 ANALYSIS OF COAL SEAM EXTRACTION MODEL

The optimal revenue is $ 61.56 million for the first two coal-strips. The schedule includes: (i) 23.78% and 18.30% respectively of seam #1 and #2 to destination 1; (ii) 23.99% and 14.78% respectively of seam #1 and #2 to destination 2; (iii) 18.49% and 14.23% respectively of seam #1 and #2 to destination 3; and (iv) 33.74% and 16.87% respectively of seam #1 and #2 to destination 4.

Approximately, 36% of seam #2 is also stockpiled due to the maximum capacity limits on the destinations. Seam #1 is, however, fully mined and transported to all destinations (see Figures 6.5 and 6.6).

Stockpiling could be advantageous in situations where future increment in market prices is expected, and also in situations where good hedging conditions are defined.

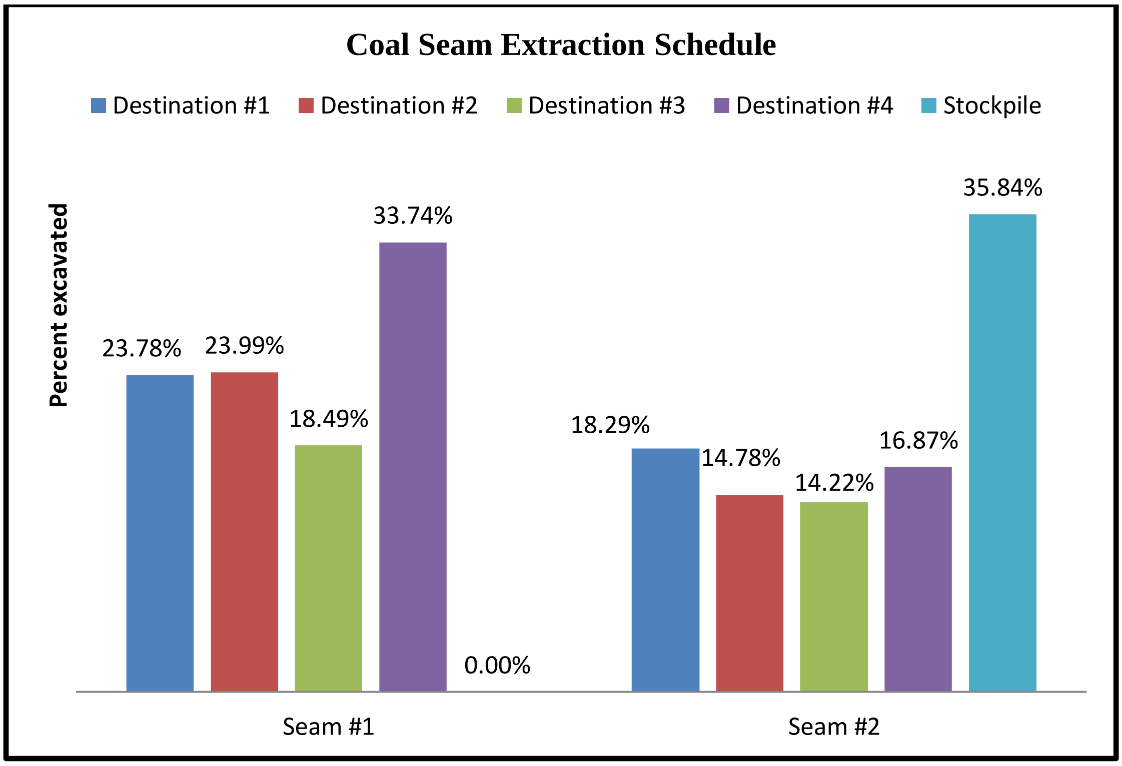


Figure 6.5 Seam #1 and #2 Extraction Schedule

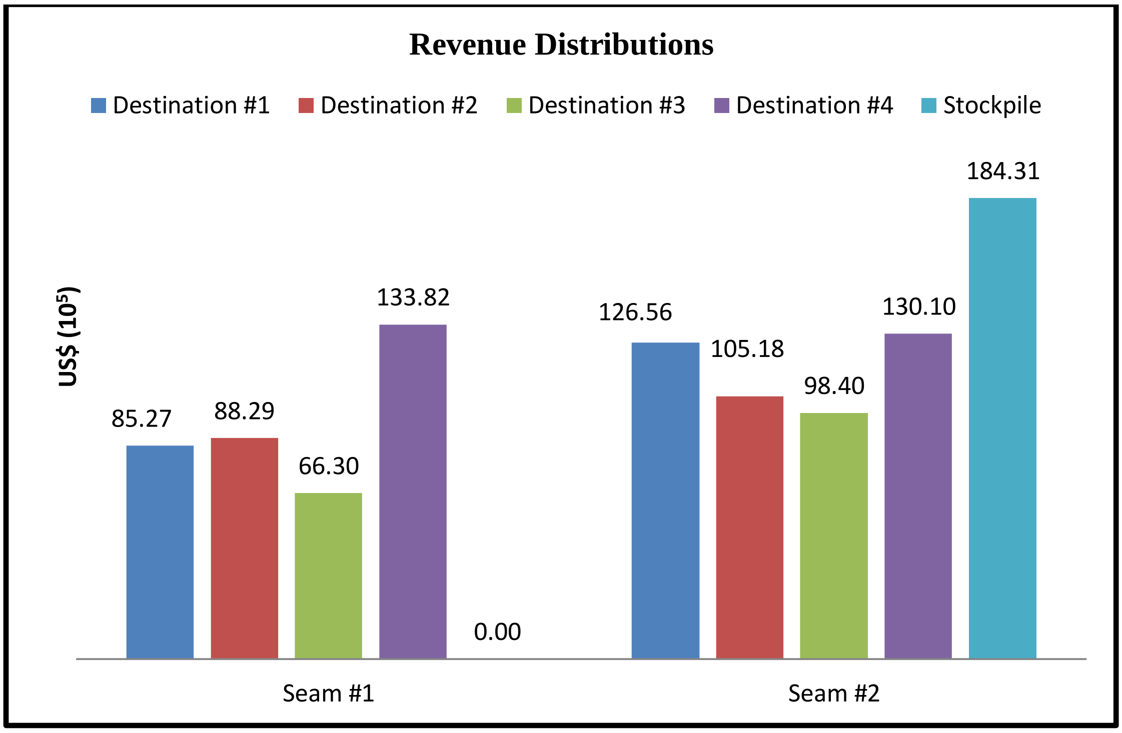


Figure 6.6 Revenue Distributions per Destinations

The coal quality specifications also determine the coal extraction schedule. Destination #3, for example, demands lower Btu levels, hence larger portions (18.49%) of seam #1 (12,790 Btu – 13,910 Btu) are transported compared to seam #2 (13,720 Btu – 14,810 Btu). Destinations #1 and #2 have similar quality specifications; however, the huge difference (29.52%) in allocation is due to the lower limit capacity differences.

The coal seam extraction and transportation costs also limit the model to find blending options with least cost expenditures. As shown in Figure 6.7, if all quantity, quality and capacity constraints are satisfied, the difference in the amount transported depends on the cost parameters (explained by the allocations in destinations #2 and #3). Contractual agreements, however, may specify strict specifications which render some other parameters non-binding (refer to destinations #1 and STK).

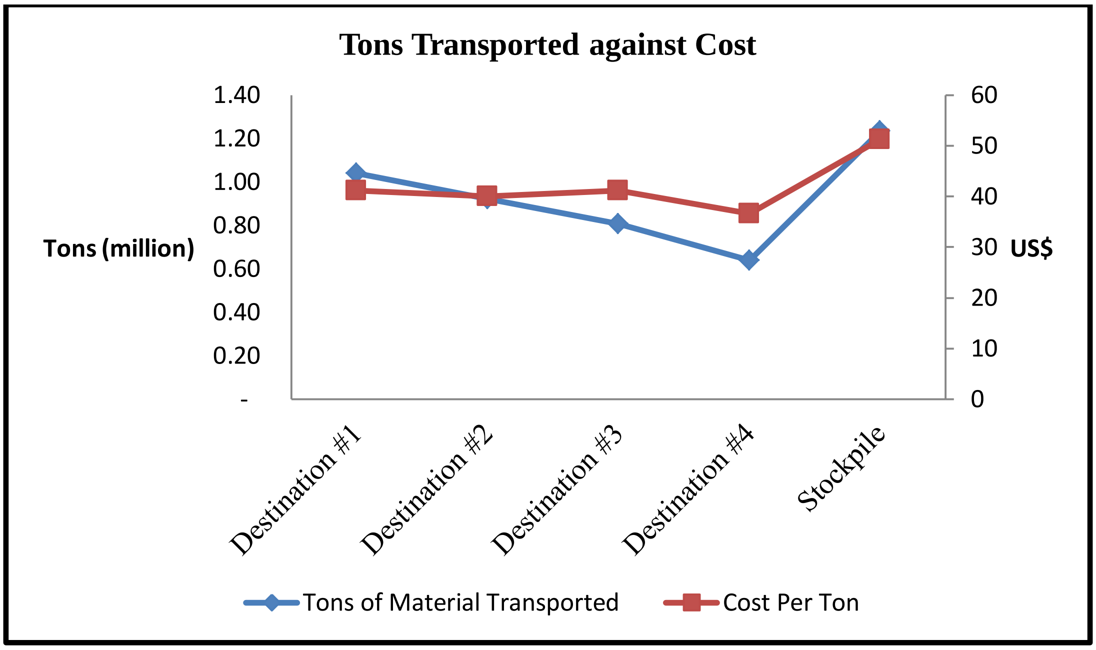


Figure 6.7 Influences of Cost Parameters on Coal Transportation Cost

The binding constraints in the coal extraction model include: (i) reserve; (ii) minimum and maximum coal quality and quantity of STK (iii) market demands for destinations #1, #3, and #4. Thus, the sensitive model parameters include the amount of reserve available and market contractual agreements. This implies that accurate numerical modeling and analyses of the formation geology, and the defined economic limiting factors are vital for an efficient blending scheme.

Even though coal seams are fairly homogenous within specific domains, highly disseminated (in terms of quality parameters) depositions could result in difficult blending decisions. This difficulty emanates from matching the general extraction sequencing with selective mining. The model is applicable in such situations by defining the set of coal blocks exposed per given periods. Similarly, different stripping scenarios could be evaluated to produce optimal decisions. These tools are efficient in analyzing different management and technical decisions for comprehensive and economically sustainable models.

6.3 STOCHASTIC SIMULATION AND OPTIMIZATION RESULTS

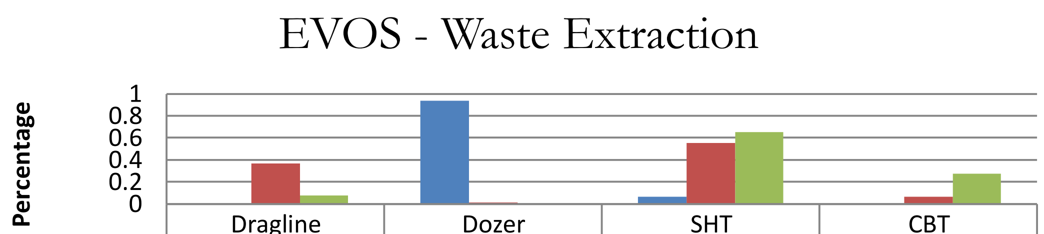
Monte Carlo and Latin Hypercube techniques are used to simulate the stochastic models. This is done in @RISK (Palisade, 2012) with 10,000 iterations in a single simulation run. The optimal simulation parameters are obtained from experimental designs as illustrated in Section 5.

BestFit is used to fit probability distributions to the available data using maximum-likelihood estimators. For density and cumulative data, BestFit uses the method of least squares to minimize the distance between the input curve points and the theoretical function (refer to Section 5). The fit statistics and the graphical results are shown in Figures 1 to 8 in Appendix C.

6.3.1 The Stochastic Model Input Data. Normal probability distribution functions (PDFs) are applied to mining, processing, and capital costs with ±20% truncations (values determined from qualitative and quantitative data analysis). Tables 1.0 to 3.0 in Appendix C contain the stochastic input parameters for mining, processing, and capital cost.

Lognormal PDF is applied to commodity price with the same truncations as stated above. Uniform PDFs are used to model the processing recoveries, thermal coal quantities, total tons of material, and selling price. Table 4.0 in Appendix C contains the stochastic input parameters for commodity price, processing recoveries, thermal coal quantities and material reserves. The various defined PDFs are the inputs to the stochastic simulation model.

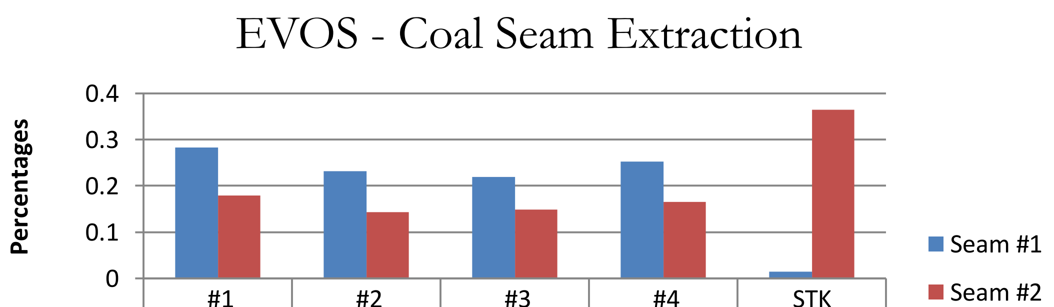
6.3.2 Stochastic-Optimization Results. The stochastic-optimization (SOP) is based on the process discussed in section 4.1 in section 4. Tables 5.0 to 24.0, and 26.0 to 44.0 in Appendix C show respectively the waste and coal extraction SOP results for each simulation run. As depicted in Appendix C, each simulation run generates a solution to the optimization problem posed by the particular coefficients generated in that run. Figures 9.0 to 17.0 in Appendix C show the SOP results of each resource. The SOP results summary for coal seam extraction are shown in Figures 18.0 to 27.0 in Appendix C. A mean of the solutions is calculated to obtain an expected value optimal solution (EVOS). Figures 6.8 and 6.9 illustrate the EVOS for the waste and coal seam extraction models respectively.



	Dragline	Dozer	SHT	CBT
Topsoil	0	0.937650993	0.062349007	0
Overburden	0.369475355	0.012979134	0.550331053	0.067214457
Inter-burden	0.073790696	0	0.654355549	0.271853755

Resources

Figure 6.8 Equivalent Value Optimal Solution – Waste Extraction



	#1	#2	#3	#4	STK
Seam #1	0.282461968	0.231468946	0.219006707	0.252154329	0.01490805
Seam #2	0.179444591	0.143271753	0.148051296	0.165536771	0.363695589

Destinations

Figure 6.9 Equivalent Value Optimal Solution – Coal Seam Extraction

The SOP results compared to the optimal results in sections 6.1 show greater variations in the overburden and inter-burden resource allocations (see Figure 6.10). This phenomenon and the related binding parameters are explained by the stochastic simulations results (refer to Section 6.3.3). The coal seam extraction variations however, remain fairly constant due to fewer variations in the coefficient parameters.

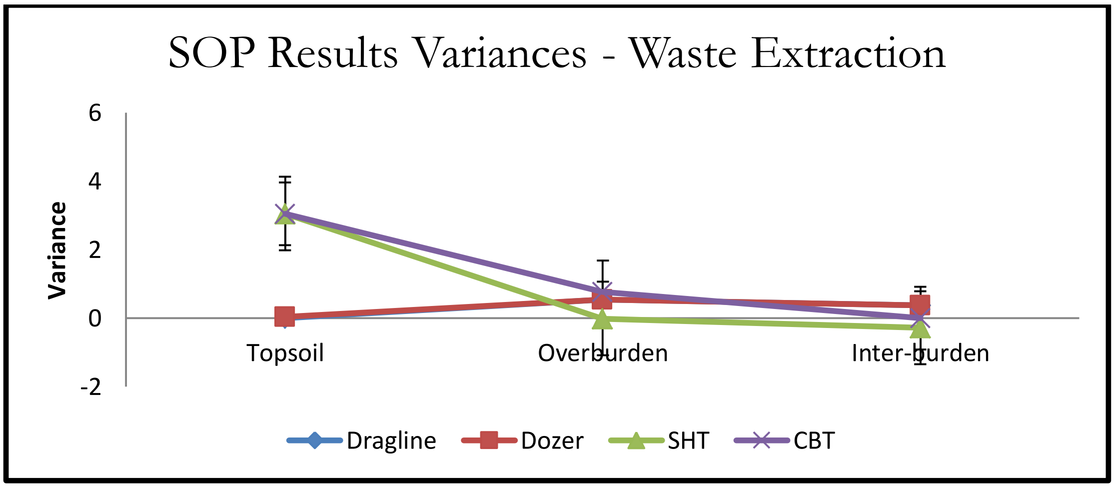


Figure 6.10 SOP Results Comparison – Waste Extraction

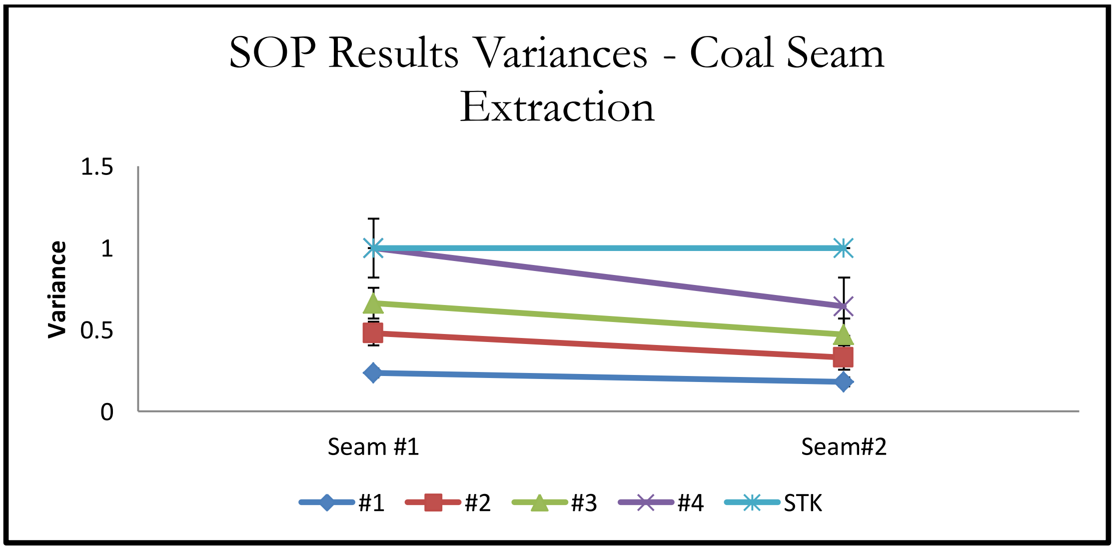


Figure 6.11 SOP Results Comparison – Coal Seam Extraction

6.3.3 Stochastic Simulation Results. The results, as shown in Figure 6.12, indicate a 50.3% chance of the waste mining cost increasing above its current value. The mean mining cost is $/t 0.04; minimum and maximum values are $/t 0.03 and $/t 0.06 respectively. These variations are driven by the tons of material allocated to the dragline. The failure-probability zone increases further with material re-handling. Minimal variations are, however, identified in the mining cost due to fairly close truncations and the approximation of tons-excavated by uniform PDFs. The 5th and the 95th percentile values are $/t 0.04 and $/t 0.05 respectively.

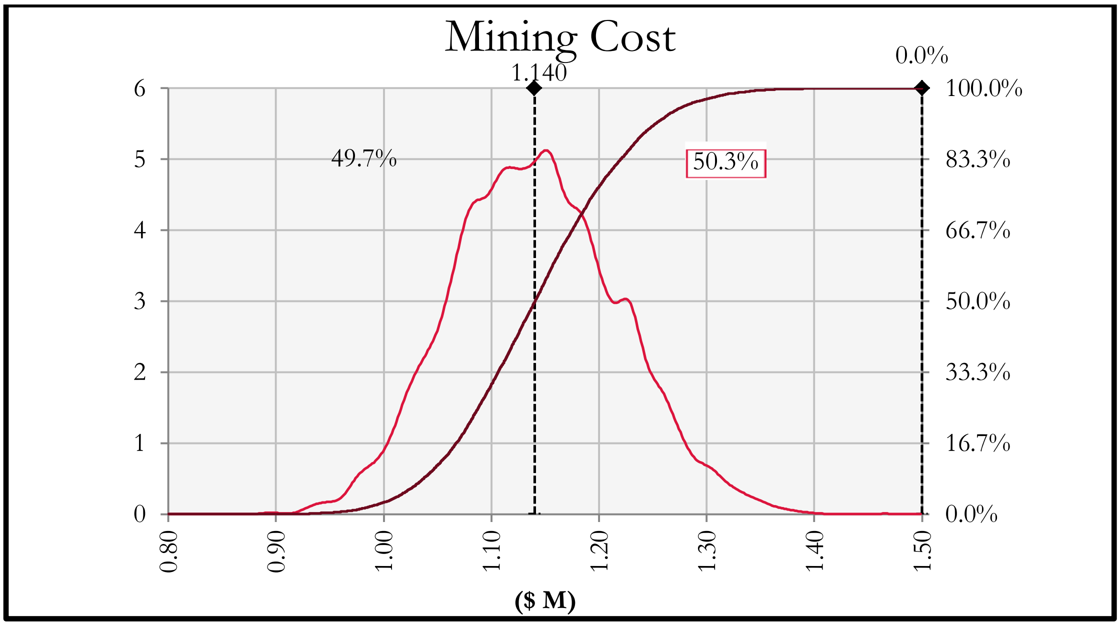


Figure 6.12 PDF Overlay with Cumulative Probability Curve (Mining Cost)

Figure 6.13 shows the tornado graph illustrating the impact of the various input parameters on the mining cost. In this figure, the overall mining cost varies between $/t 0.0410 and $/t 0.0455; $/t 0.0421 and $/t 0.0444; and $/t 0.0425 to $/t 0.0444 due to the tons excavated by draglines in period #2; CBT in period #4; and dozer in period #1 respectively. The tons excavated by the dragline in period #2 have the highest regression coefficient ($\hat{\beta}$) of 0.52.

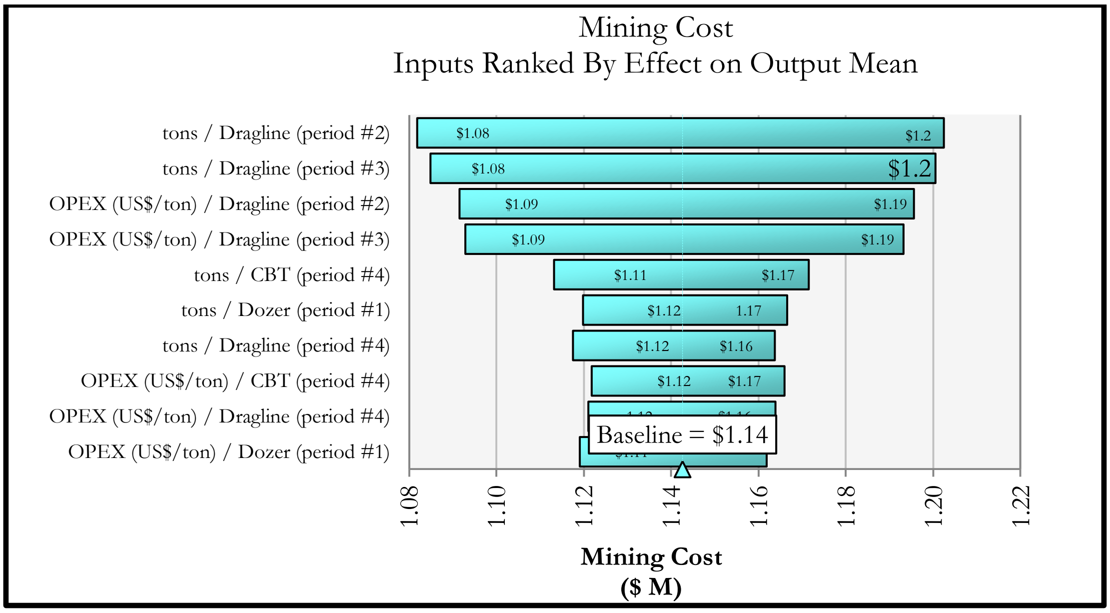


Figure 6.13 Tornado-Change in Output Mean Graph

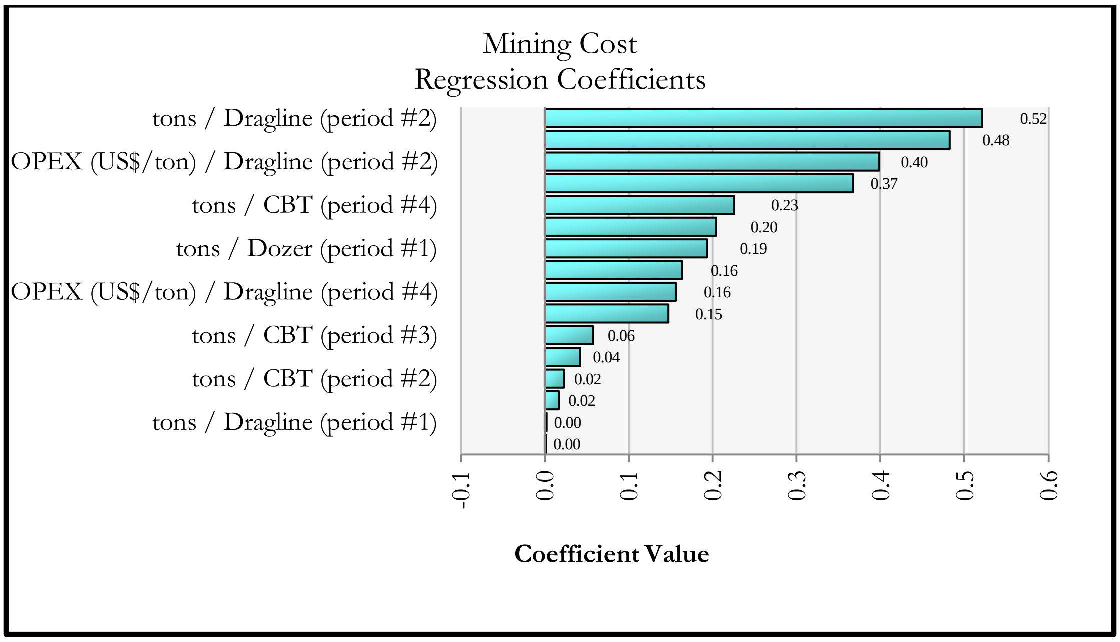


Figure 6.14 Tornado-Regression Coefficients (Mining Cost)

Even though the CBT is allocated to about 55.92% of inter-burden in period #4, its $\hat{\beta}$ is 33.3% more compared to the dragline's in the same strip. These closely matched figures are due to the similar OPEX parameters used in the model. The dozer, with about 97% allocation in period #1, has a $\hat{\beta}$ of 0.19 on the output mean. Figure 6.14 shows the tornado-regression coefficient.

From these analyses, a 31.3% reduction in dragline OPEX will result in 25% overall decrease in mining cost. This reduction can be achieved by optimal allocations and good engineering practices. Sensitivity plots also aid in understanding the operational dynamics. This analysis is done in TopRank (Palisade, 2012) and the results, as illustrated in Figure 6.15, show input percentiles varied between 0% and 100% in steps of 10%.

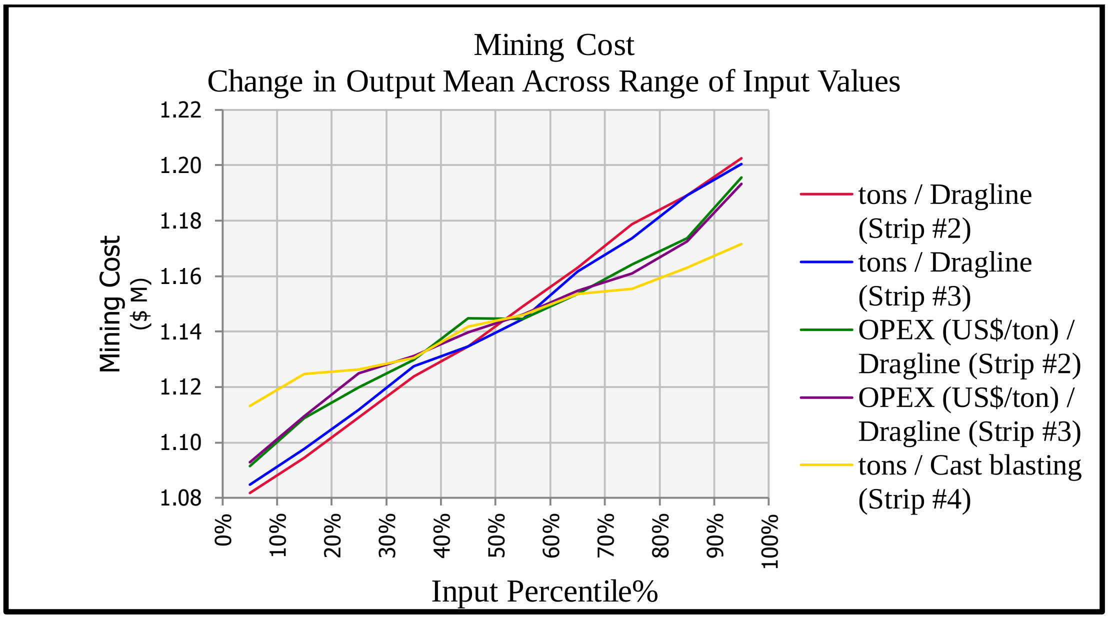


Figure 6.15 Sensitivity Analyses (Mining Cost)

The input variations are used to observe different disparities in the output mean. The effect of the CBT allocation has a gradual impact on the cost function, however, significant variations are observed between the 35 and 95 percentile ranges. These changes are due to the digging geometry constraint placed on the dragline as mining progresses.

The dragline's input effect is steady for the range of variations and similar gradual trends are observed for all other scheduled resources. The overall output mean variations are attributed to the operating cost figures, equipment availability, digging geometry constraints and the resource operating mechanisms. The available dump capacity constrained by the digging mechanism also impacts the equipment selection and the cost function.

The risk modeling includes Chi-Squared (Chi-sq), Anderson-Darling (A-D) and Kolmogorov-Smirnov (K-S) statistical tests applied in @RISK platform. Equal probability bin arrangement is chosen for the Chi-Squared binning. Parametric Bootstrap is run with 1000 number of re-samples at 95% parameter confidence level. Computational time increases with the number of re-samples selected, and thus, a sufficient number to reach desired results is critical.

The Weibull distribution is the appropriate fit for the mining cost risk profile with the following results for the statistical tests: (i) Chi-sq: 83.71; (ii) K-S: 0.0089; and (iii) A-D: 1.8453. A measure of the uncertainty in the random variable is also obtained through Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

The PDF and CDF for Weibull distribution are shown in equations (6.1) and (6.2) where x is a random variable, m is a shape parameter, and b is a scale parameter (Weisstein, 2003). The mining cost risk-profile is shown in Figure 6.16.

$$f(x) = \begin{cases} \frac{m}{b}\left(\frac{x}{b}\right)^{m-1} e^{-(x/b)^m} & , \forall m \in \mathbb{R}, b \in \mathbb{R}, x \geq 0 \\ 0 & , \forall m \in \mathbb{R}, b \in \mathbb{R}, x < 0 \end{cases} \tag{6.1}$$

$$F(x) = 1 - e^{-(x/b)^m} \tag{6.2}$$

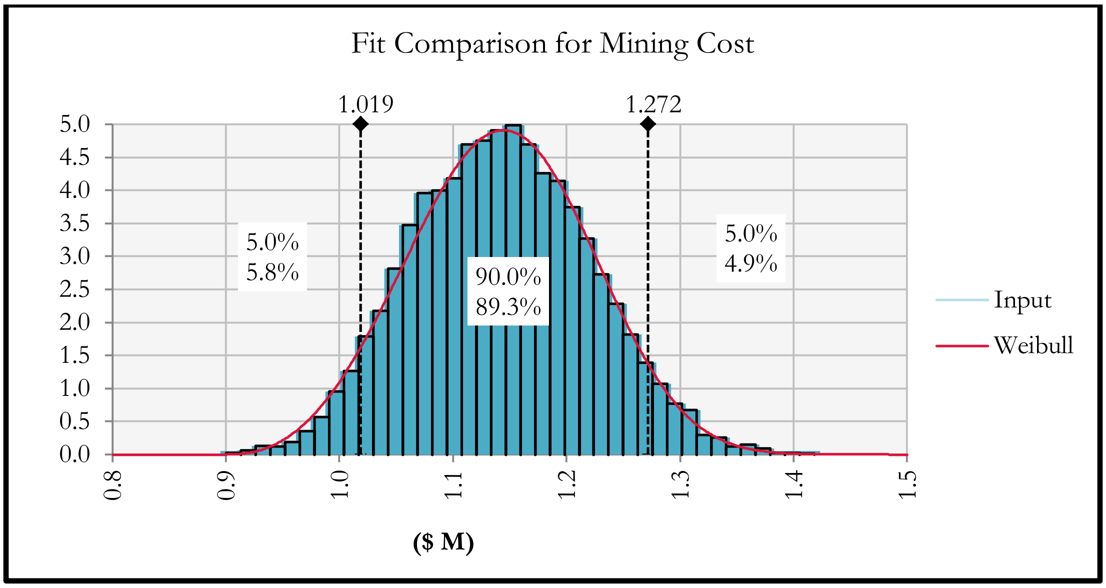


Figure 6.16 Mining Cost Risk Profile (Weibull Distribution)

Given the base model and the risk modeling, the triangular PDF and the BetaGeneral distribution best describe the dragline and dozer, and the CBT operating cost risk-profiles respectively (see results in Figures 28 to 30 in Appendix C). These results provide a robust platform for a comprehensive economic model.

The revenue risk analyses indicate a 53.7% probability of the revenue falling below the current estimates. As shown in Figure 6.17, the mean revenue is \$/t 11.91, minimum is \$/t 0.24 and the maximum is \$/t 22.15. The high probability of failure is attributed to the amount of coal product stockpiled in the second period, the price of coal and the thermal quantities present in the coal product. The 5th and 95th percentile values are \$/t 0.20 and \$/t 0.25 respectively. The vast differences are due to the different contractual market agreements and variable coal properties.

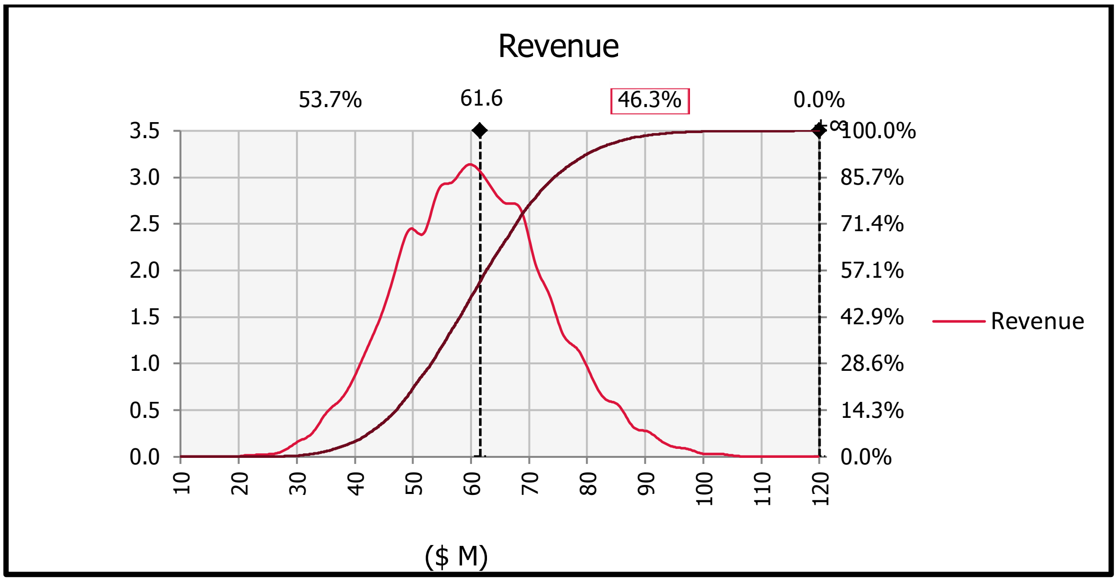


Figure 6.17 PDF Overlay with Cumulative Probability Curve (Revenue)

Tornado graphs are used to assess the inputs effect on the output mean. These plots are used to establish the low-base-high output future expectations. Figure 6.18 shows the tornado plot of changes in output values to a +1 standard deviation change in all input parameters.

The following analyses are achieved with the plot: (i) determination of the amount of variability in the output model and the sources of residual risk; (ii) identification of the parameters with the most variability that contributes in the outputs; (iii) investigation of variable distributions to check model quality; and (iv) preliminary identification of risk mitigation measures.

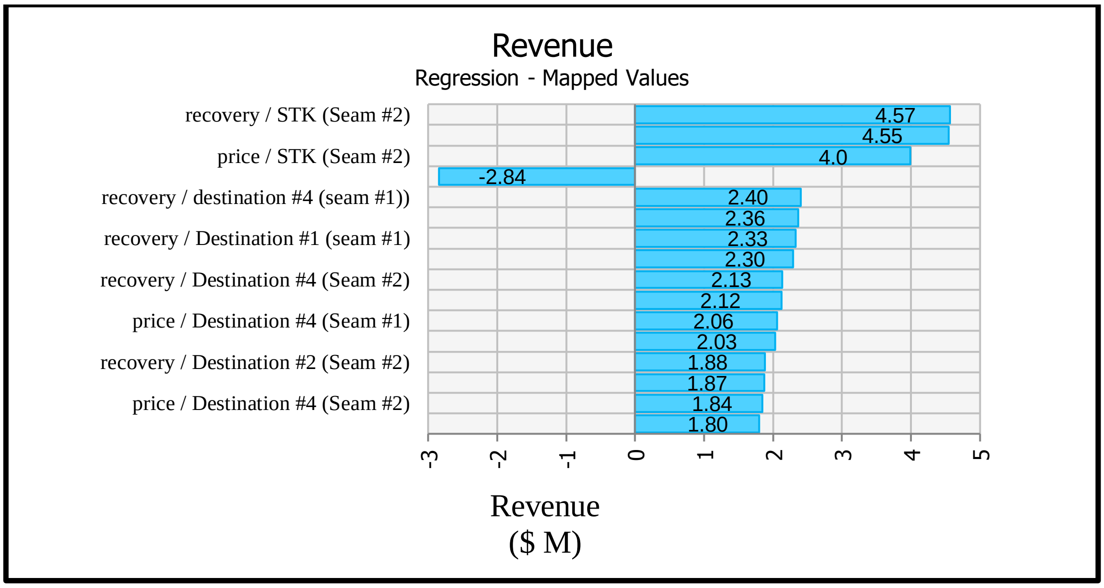


Figure 6.18 Tornado – Regression Mapped Values (Revenue)

From Figures 6.18 and 6.19, the plant recovery of the stockpiled material has the highest impact on the revenue with a regression-mapped value of $/t 0.88. The least impact is from the material transported to destination #3. These variations are due to capacity limits at the various destinations, future commodity prices and the cost parameters.

The variability with future coal prices increases the risk of stockpiling as shown by the tornado plots. Approximately 35.84% of seam #2 is stockpiled in period 2 at a cost of $/t 26.08 (-0.22 regression coefficient).

With the current price estimated at $41.01 per ton product, a variable change in this parameter will affect the viability of stockpiling. A 32.25% and 33.31% reductions respectively in recovery and thermal coal quantities at the stockpile result in 25% reduction in the overall revenue.

The least amount of coal (14.23%) is transported to destination #3 in period 2 at a cost of $/t 20.35, hence the less impact on the overall revenue.

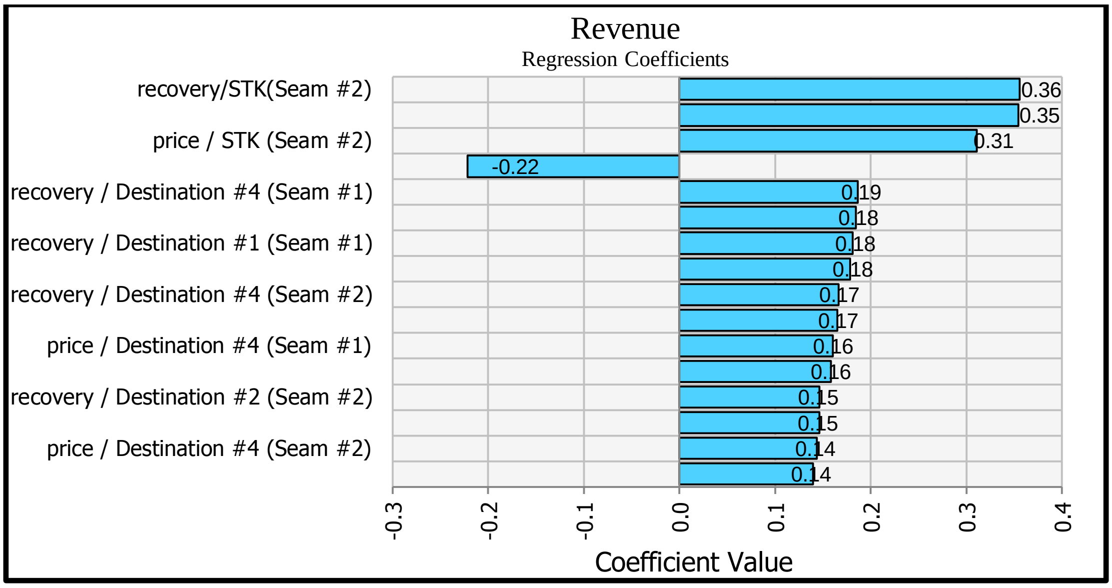


Figure 6.19 Tornado - Tornado-Regression Coefficients (Revenue)

The geologic variability is characterized in the tornado plots by the thermal quantities and the recovery. Despite the general homogeneity of coal seams, adequate numerical analysis of the formation geology is required to minimize risk. This activity is vital in MSF extraction due to possible alteration zones and complicated extraction sequences. The above discussions can be correlated with the waste extraction sequencing. Sub-optimal extraction sequences in MSFs result in loosing coal seams, and complicating coal seam extraction mechanisms, thus reducing recovery (as discussed in section 2).

A deferred revenue due to loosing of coal seams is similar to the stockpiling discussion above. Similarly, uncertainties surrounding future coal prices result in major setbacks in economic model. The maximum regression coefficient and the regression-mapped values for the coal price are 0.31 and $ 4.0 million respectively (refer to Figures 6.18 and 6.19).

Sensitivity analyses are also run to identity the parametric effects on output variables. This is done in TopRank (Palisade, 2012) and the results are shown in Figure

6.20. In this Figure, input percentiles are varied from 0% to 100% in steps of 10% to observe different variations in the output mean.

The cost to mine and transport coal to the stockpile has a negative gradient, thus having an inverse effect on the revenue function. The recovery of the coal product to destination #1 has similar sensitivity trends as the stockpiling parameters.

From this analyses, the major concerns are material stockpiling, geologic conditions and the commodity price. The contractual agreements for most of the destinations are met, hence their variability and influence is not observed.

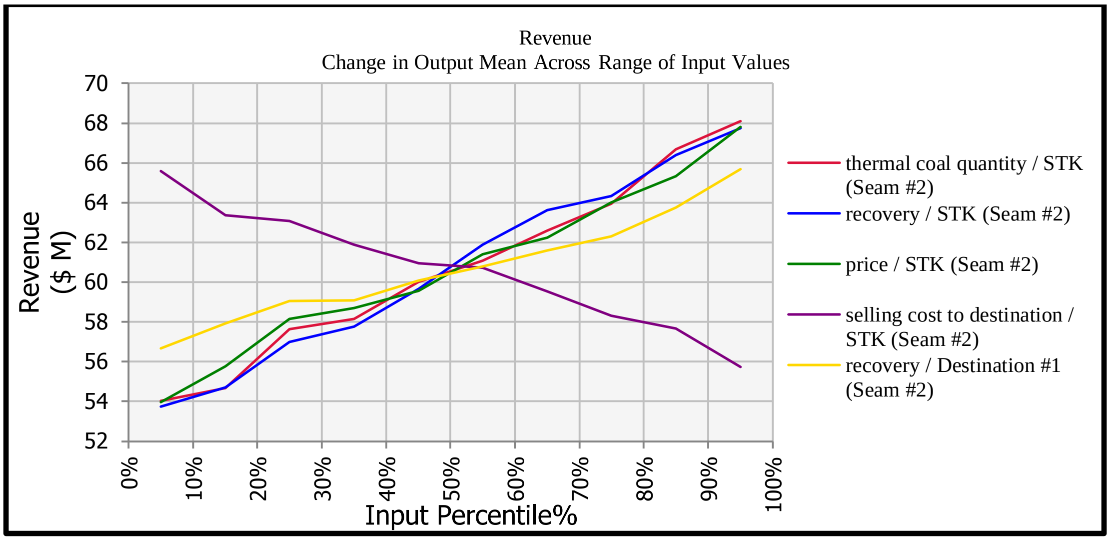


Figure 6.20 Sensitivity Analyses (Revenue)

The revenue risk-profile is shown in Figure 6.21 with similar setup parameters as the waste extraction model. The normal distribution is the appropriate fit for the revenue output with the following statistical test results: (i) Chi-sq: 129.99; (ii) K-S: 0.0201; and (iii) A-D: 5.6974. The PDF and CDF for normal distribution are shown in equations

(6.3) and (6.4) where x is the random variable, μ is the mean and σ^2 is the variance (Weisstein, 2003).

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}, \quad x \in \mathbb{R}, \quad \sigma^2 \neq 0 \tag{6.3}$$

$$F(x; \mu, \sigma^2) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \tag{6.4}$$

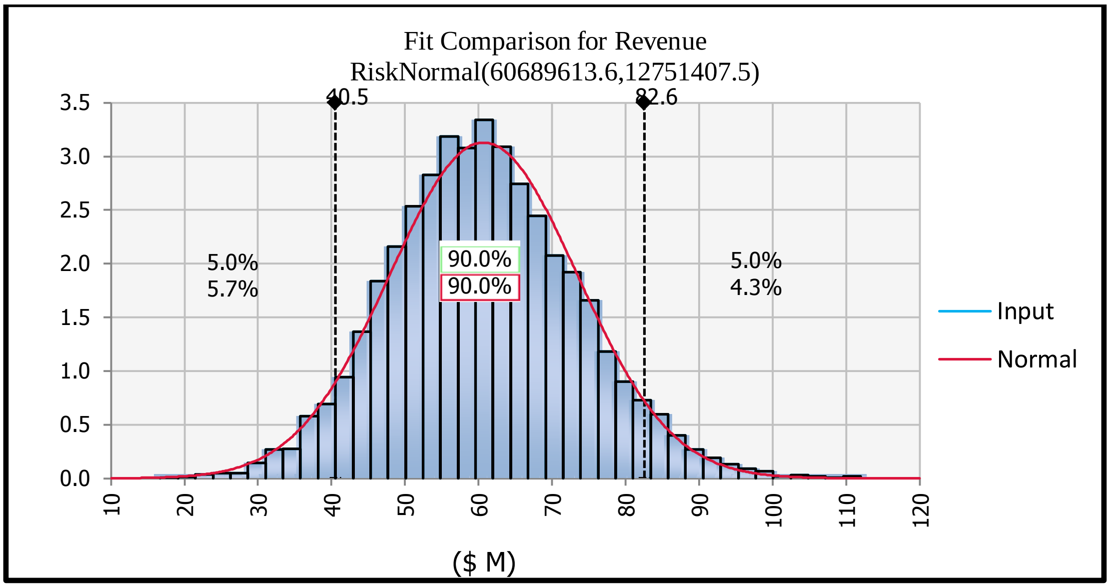


Figure 6.21 Revenue Risk Profile (Normal Distribution)

The net present value (NPV) is calculated for the first 20years to highlight the mining cost expenditure and the revenue functions. At a 10% discount rate, the NPV is $ 31.69 million. As shown in Figure 6.21, the probability of the NPV falling below its current estimate is 47.8%. The cost and the revenue functions are modeled with the Weibull distribution and the Normal distribution functions respectively. The distributions are obtained from the risk profiles of each parameter functions.

The results show a mean NPV of $31.19 million, minimum value of $16.87 million and a maximum value of $48.80 million. The 5th and the 95th percentile values are $25.06 million and $38.83 million respectively. Figure 6.22 shows the regression-mapped value plot of changes in output values to a +1 standard deviation change in all input parameters. The high risk in year 4 is attributed to the amount of material stockpiled, market prices and the waste extraction schedule.

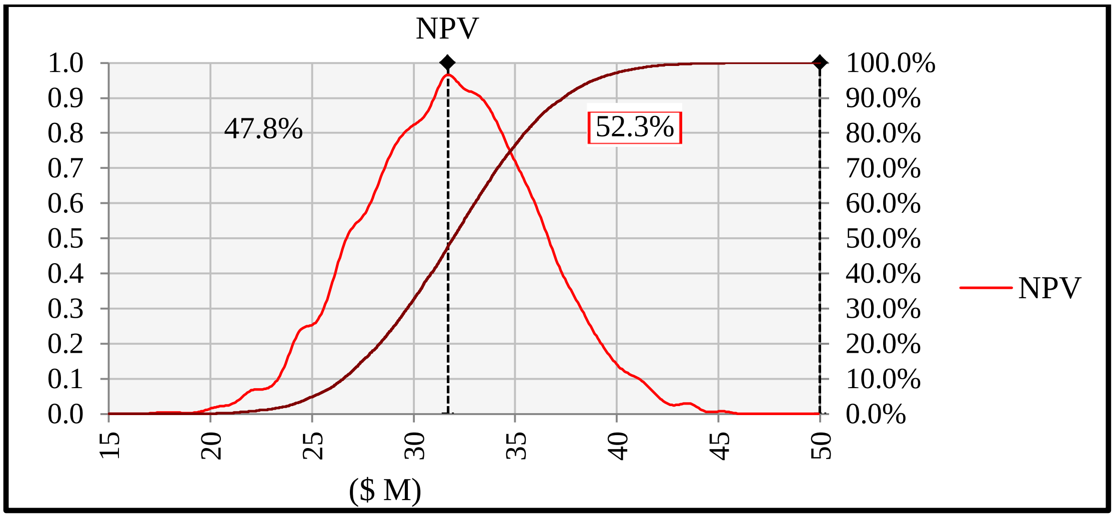


Figure 6.22 PDF Overlay with Cumulative Probability Curve of Revenue

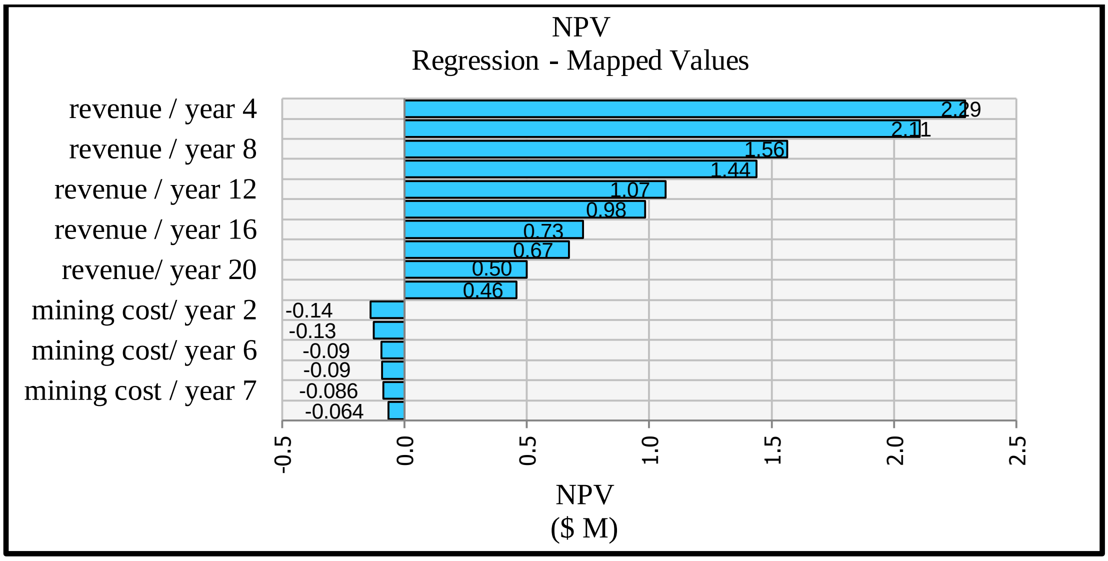


Figure 6.23 Tornado – Regression Mapped Values (NPV)

Sensitivity plots are also generated to analyze the effect of percentage changes in input parameters on the mean NPV (refer to Figure 6.23). The capital investment in the first year of the project is the most sensitive with a negative gradient.

All the revenue parameters are observed to have similar trends due to a fairly constant extraction rates for all periods. The change in the NPV mean across the range of input values is also observed to be gradual with no sharp gradients.

The NPV risk-profile is shown in Figure 6.24 with similar setup parameters to the waste extraction and revenue models. The normal distribution is the appropriate fit for the NPV function with the following results for the statistical tests: (i) Chi-sq: 75.04; (ii) K-S: 0.004; and (iii) A-D: 0.2552. The PDF and CDF for Normal distribution are shown in equations (6.3) and (6.4) respectively.

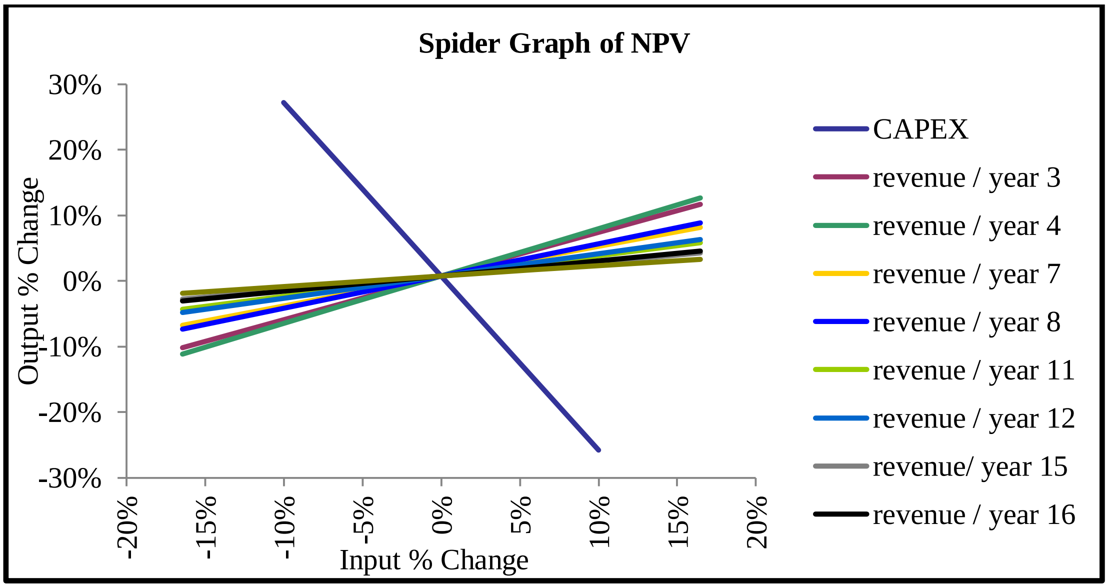


Figure 6.24 Sensitivity Analyses of Input Parameters on Mean NPV

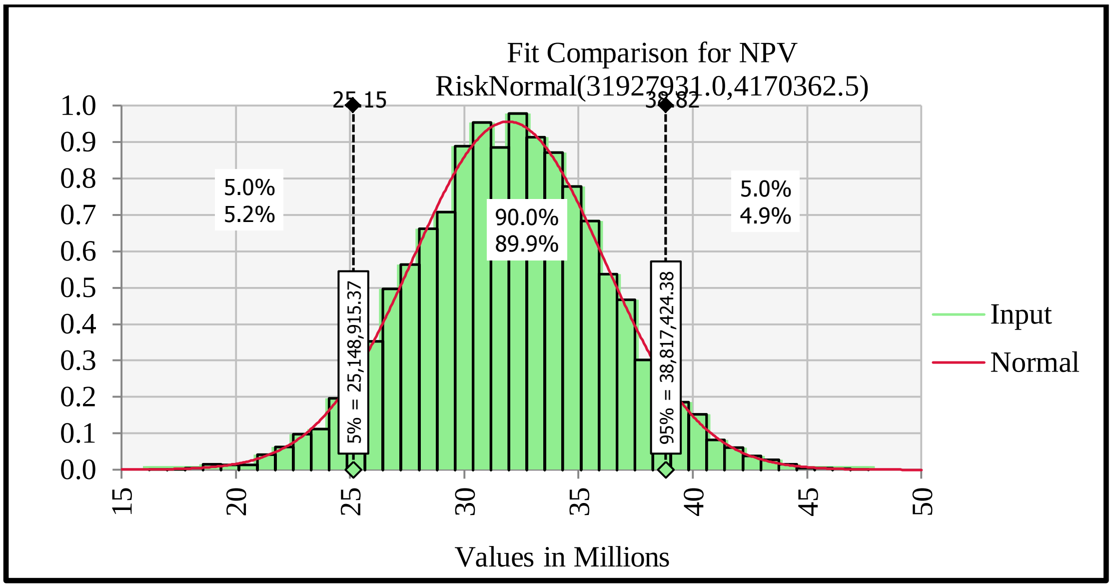


Figure 6.25 NPV Risk Profile (Normal Distribution)

6.4 SUMMARY

The SOP models improve the overall NPV and mining cost, respectively, by +0.19% and -3.51% compared to the conventional traditional schedule (dozer pushes topsoil and dragline excavates overburden/inter-burden). The EVOS for waste extraction equipment schedules include: (i) topsoil (93.77% by dozer and 6.23% by SHT); (ii) overburden (36.95% by dragline, 1.30% by dozer, 55.03% by SHT, and 6.72% by CBT); and (iii) inter-burden (7.38% by dragline, 65.44% by SHT, and 27.19% by CBT). The EVOS for coal seam extraction schedule include: (i) seam #1 (28.25% to destination #1, 23.15% to destination #2, 21.90% to destination #3, 25.22% to destination #4, and 1.49% stockpiled); and (ii) seam #2 (17.94% to destination #1, 14.33% to destination #2, 14.81% to destination #3, 16.55% to destination #4, and 36.37% stockpiled).

The stochastic simulation results indicate a 50.3% chance of the mining cost increasing above its current figure ($/t 0.04), 53.7% chance of the revenue falling below its current estimate ($/t 11.91), and a 52.3% probability of the NPV increasing above its current value ($ 31.69 million). The tons of material excavated by the dragline in strip #2 have the highest regression coefficient $(\hat{\beta})$ of 0.52 in the waste extraction model. The recovery of the material stockpiled has the highest impact on the revenue with a regression-mapped value of $ 4.57 million. The revenue in year 4 has the highest impact on the NPV with a regression-mapped value of $ 2.29 million (The high risk in year 4 is attributed to the amount of material stockpiled, market prices and the waste extraction schedule). The mining cost risk-profile is modeled with the Weibull distribution whiles the revenue and NPV risk-profiles are modeled with the Normal distribution.

7. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This section summarizes the research study presented throughout this thesis and provides conclusions and appropriate recommendations for future research directions in multi-seam formation (MSF) extraction.

7.1 SUMMARY

The upsurge in surface coal production can be attributed to the advent of larger trucks, shovels and draglines. This growth has resulted in higher production efficiencies (Demirel and Frimpong, 2009). Increasing the productivities and efficiencies of these equipment units is a key to improving economic values. The general excavation geometries and the complexities of extraction in MSFs render this activity a sensitive portion of the mine planning process. Sub-optimal resource allocation and coal seam extraction schedules could result in revenue losses, high production costs resulting in operating profit losses.

Dragline research initiatives have resulted in the development of excavation and spoiling dynamic models (Tanaino, 1986; Vasilyev et al., 1999; Zaitseva et al., 2007). Economic models have also been developed to identify mining complexities and examine different excavation alternatives (Falkie and Porter, 1973; Ray et al., 1999). Zhou et al. (2007) and Ray et al. (1999) studied dragline-bulldozer and cast blasting (CBT) techniques, respectively. The authors concluded that working bench advancement and the economic gains of ancillary operations require optimal schedules.

The use of mathematical programming to solve mine planning and scheduling problems has been shown to be very robust. Over the years, researchers have relied on algorithms such as linear programming (LP), non-linear programming (NLP), mixed integer programming (MIP) and goal programming (GP) to optimize long term production plans in mines. Stochastic-optimization (SOP) models have also been applied to minimize future unexpected occurrences by taking into account all possible outcomes. Despite these improvements, quantitative dragline and ancillary equipment production must be incorporated in optimization models. The intrinsic variability of excavation and spoiling dynamics in MSFs must also form part of existing models.

To develop the proposed economic models, comprehensive SOP formulations provided a generic platform to simulate different scenarios. Adequate knowledge of the challenges of MSFs provided understanding into improving equipment productivity while different scenario simulations offered a means to evaluate different operating conditions.

NLP mathematical models were developed for waste and coal seam extraction schedules in MSF. The general Lagrange Multiplier method (LMM) was used to develop the NLP mathematical models. The main objective was to maximize the net present value (NPV), thus minimize mining cost and maximize revenue from coal seam extraction. The mathematical expressions provided quantitative relationships between stripping equipment productivities, excavation and spoiling mechanisms, coal blending options and economic outputs.

The models were solved using the Generalized Reduced Gradient (GRG) Algorithm. The optimizer was initiated by evaluation of the Jacobian (JC) matrix of partial derivatives (PD) of the problem functions with respect to the decision variables. Finite difference method (FDM) was used to approximate the JC matrix. Due to the magnitude of the models, dynamic Markowitz refactorization was used to improve stability. This approach, coupled with the sparse representation of the matrix, results in better memory usage (Fylstra, 1998). The numerical solution algorithms were implemented in SOLVER (Frontline, 2012).

Stochastic variables associated with MSFs were identified and modeled using probability distribution functions (PDFs). The respective PDFs were obtained from detailed data analysis and model fitting. The SOP process minimized the expected value of the objective functions where expected value optimal solutions (EVOS) were obtained. Stochastic simulation was carried out using Monte Carlo and Latin Hypercube techniques. The simulation parameters were acquired from experimental designs, where parameters were varied gradually to obtain distributions for the mean values of each run. The results led to real-time risk analysis for the economic model.

A bituminous MSF mining case was used to validate the SOP models. The waste extraction dynamics, as well as coal seam extraction and transportation schemes were defined by mathematical functions. A comprehensive risk model was also developed for the optimal solutions based on which technical decisions were established. The optimal results include: (i) stripping equipment allocation to improve productivity; (ii) coal seam extraction options; and (iii) the characterization of future uncertainties.

7.2 CONCLUSIONS

This study comprises the use of an analytical literature review, mathematical modeling, stochastic process definitions, numerical modeling, computer simulation and risk characterization to achieve its objectives. Mathematical techniques were used to model excavation and spoiling dynamics in MSF extraction. These techniques provided detailed quantitative information on stripping equipment productivities and excavation complexities. The formulations resulted in equality and inequalities equations, which were solved using numerical solution techniques. Characterization of future uncertainties associated with the optimal solutions was achieved thorough SOP and risk analysis.

The primary objective was to maximize dragline and ancillary equipment productivity improvements and the associated economic benefits in MSF mining operations. The main components of the stochastic-optimization models included:

1. NLP models for equipment allocation and material removal scheduling in a typical multi-seam formation.
2. SOP models for equipment allocation and material removal scheduling in a multi-seam environment.
3. Simulation of these models to produce a series of optimal solutions for different scenarios.
4. Comprehensive risk analysis of the optimal solutions for the multi-seam operation.

Given the MSF case study, the following conclusions have been drawn from detailed simulation experimentation and result analyses of the waste extraction model:

1. The overall mining cost was $149.34 million for the entire deposit; a 3.51% decrease compared to the conventional traditional methods (dozer pushes topsoil and dragline excavates overburden/inter-burden).
2. The dozer was scheduled to excavate 93.77% of topsoil in period 1; influenced by the material properties and specified material relocation mechanisms (SOP results).

3. A 6.23% of topsoil was allocated to stripping by the shovel and truck (SHT) system due to critical digging depth limitation specified for the dragline (SOP results).

4. The overburden stripping schedule included: 36.93% by dragline, 1.30% by dozer, 55.03% by shovel and truck (SHT), and 6.72% by cast blasting technique (CBT) (SOP results).

5. The inter-burden extraction schedule included: 7.38% by dragline, 65.44% by SHT, and 27.19% by CBT (SOP results).

6. A 100% dozer application in topsoil showed a +2.05% cost difference compared to the optimal results.

7. The dragline spoiling distance reduced as mining progressed, hence the percentage allocation reduced.

8. A 100% dragline application in overburden resulted in +1.02% cost difference compared to the optimal results.

9. A 77yd^3 drop in productivity per unit change in cycle time was recorded for sub-optimal dragline schedules.

10. A 13.77% decrease in mining cost was recorded for 100% allocation of CBT in overburden and inter-burden, compared to the dragline.

11. The geologic variability and equipment dynamic operational parameters restricted any possible change in the cost function.

From the detailed simulation experimentation and result analyses of the coal seam extraction models, the following conclusions have been drawn:

1. Optimal revenue was $ 61.56 million for the first two coal-strips.

2. Seam #1 extraction schedule included: 28.25% to destination #1; 23.15% to destination #2; 21.90% to destination #3; 25.22% to destination #4, and 1.49% stockpiled.

3. Seam #2 extraction schedule included: 17.94% to destination #1; 14.33% to destination #2; 14.81% to destination #3; 16.55% to destination #4; and 36.37% stockpiled.

4. The sensitive model parameters included the amount of reserve available and market contractual agreements.

5. Stockpiling could be advantageous in situations where future increment in market prices is expected.

6. Accurate numerical modeling and analyses of the formation geology, and the defined economic limiting factors were vital for an efficient blending scheme.

7. Highly disseminated (in terms of quality parameters) depositions could result in complicated blending decisions.

From the stochastic simulation and risk analysis, the following conclusions have been drawn:

1. A 50.3% chance of the waste mining cost increasing above its current value.

2. Mining cost variations were driven by the tons of material allocated to the dragline (regression coefficient of 0.52) and the failure-probability zone increased further with material re-handling.

3. A 31.3% reduction in dragline OPEX resulted in 25% overall decrease in mining cost. This reduction can be achieved by optimal allocations.

4. The overall output mean variations were attributed to the operating cost figures, equipment availability, digging geometry constraints and the resource operating mechanisms.

5. The Weibull distribution was the appropriate fit for the mining cost risk-profile with the following statistical test results: (i) Chi-sq: 83.71; (ii) K-S: 0.0089; and (iii) A-D: 1.8453.

6. The triangular PDF and the BetaGeneral distribution was the appropriate fit for the dragline and dozer, and the CBT operating cost risk-profiles respectively.

7. The revenue risk analyses indicated a 53.7% probability of the revenue falling below the current estimates.

8. The high probability of failure from the revenue model was attributed to the amount of coal product stockpiled, the price of coal and the thermal quantities present in the coal product.

9. A 32.25% and 33.31% reductions respectively in recovery and thermal coal quantities at the stockpile resulted in 25% reduction in the overall revenue.

10. Uncertainties surrounding future coal prices resulted in major setbacks in economic modeling. The maximum regression coefficient and regression mapped values for the coal price were 0.31 and $ 4.0 million respectively,

11. The normal distribution was the appropriate fit for the revenue output with the following statistical test results: (i) Chi-sq: 129.99; (ii) K-S: 0.0201; and (iii) A-D: 5.6974.

12. At a 10% discount rate, the NPV was $ 31.69 million with 47.8% probability of failure.

13. The normal distribution was the appropriate fit for the NPV function with the following statistical test results: (i) Chi-sq: 75.04; (ii) K-S: 0.004; and (iii) A-D: 0.2552.

The SOP models improved the overall NPV and mining cost, respectively, by +0.19% and -3.51% compared to the conventional traditional schedule (dozer pushes topsoil and dragline excavates overburden/inter-burden). The geology of the formation, digging geometries of stripping equipment, material re-handling, coal seam quality variations, and contractual agreements were the sensitive parameters associated with the economic models. The concept involved is generally applicable and should not be limited to this case only.

7.3 RECOMMENDATIONS

Despite the significant contributions of the research study to equipment productivity improvement in MSFs, several areas require improvement through future research investigations. The following areas are suggested:

1. Some functions in the optimization model exhibit discontinuity; an area which should be investigated in future research for a more robust SOP model.

2. The research finding in (1) should be used in conjunction with the thesis results to provide resource allocation economic models to complex operating environments.

3. Due to the limited data available, there were no absolute justifications for the selected input PDFs during the stochastic simulation. Thus, the BestFit results cannot be generalized for the input parameters.

4. During the quantitative equipment productivity formulations, material extraction schedules were pre-defined. Future research initiatives should incorporate equipment productivity formulations with material extraction sequencing.

5. Although a diggability index was established for all equipment investigated, the dynamics of equipment-formation interaction should be investigated prior to allocation.

6. The research findings showed the in-depth application of stochastic processes in mine planning procedure. These concepts should be incorporated in commercial software packages for comprehensive decision models.

7. Other optimization algorithms, which curtail some limitations of the NLP algorithms, are worth investigating.

APPENDIX A

REACH FACTOR SENSITIVITY ANALYSIS, MODEL VERIFICATION & PLOTS ON CD-ROM

1. INTRODUCTION

Included with this thesis is a CD-ROM, which contains the reach factor sensitivity analysis and model verification. All documents have been prepared as Microsoft Word 2010 document files (Windows 2010).

APPENDIX B

CASE STUDY INPUT PARAMETERS & PLOTS ON CD-ROM

1. INTRODUCTION

Included with this thesis is a CD-ROM, which contains the case study input parameters. All documents have been prepared as Microsoft Word 2010 document files (Windows 2010).

APPENDIX C

STOCHASTIC-OPTIMIZATION MODELING & PLOTS ON CD-ROM

1. INTRODUCTION

Included with this thesis is a CD-ROM, which contains the stochastic-optimization modeling results. All documents have been prepared as Microsoft Word 2010 document files (Windows 2010).

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VITA

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