ABSTRACT

Title of dissertation: TWO ESSAYS IN MACROECONOMICS

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Castro and Coen-Pirani (2008) document that aggregate skilled hours and employment both became more volatile after the mid-1980s, in contrast to the simultaneous volatility decline of most aggregates, including overall hours and employment and unskilled hours and employment. In chapter 1, I propose that rising efficiency in matching skilled workers to vacancies accounts for this change. The rise of general-purpose information technology made the skills of well-educated workers more transferable across firms and industries, and this increased the suitability of unemployed skilled workers for a broader range of job vacancies. In turn this implies a larger increase in the flow of skilled labor into employment during economic booms. This causes skilled aggregates to be more volatile. I embed a simple search and matching mechanism in a typical dynamic general equilibrium model to demonstrate this idea.

The purpose of chapter 2 is to explore the contribution of capital-skill complementarity to short-run employment fluctuations. Given that such complementarity is a leading explanation for long-run changes in the skill premium, it is interesting

to check its short-run implications for employment volatility. The numerical results show that complementarity can make skilled employment more volatile than the unskilled, but it can not improve standard DSGE models' implications for overall labor market' volatility.

TWO ESSAYS IN MACROECONOMICS

by

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Dedication

To my parents for their consistent support and understanding

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Chapter 1

Matching Efficiency and Skilled Employment Volatility

1.1 Introduction

Conventional wisdom holds that skilled workers are less likely to be laid off during recessions and that skilled employment should be less volatile than its unskilled counterpart.[1] However, this was not the case in the U.S. beginning in the mid-1980s. Castro and Coen-Pirani (2008) present evidence that aggregate skilled hours and employment became at least as volatile as unskilled measures starting in the mid-1980s. One striking fact in their paper is that the standard deviation of the cyclical component of skilled aggregates increased in contrast to the simultaneous volatility decline of most macroeconomic aggregates, including total hours and employment. Thus, what happened to skilled workers? Why didn't skilled volatility decline?

I propose that rising efficiency in the labor matching process for skilled workers accounts for the change in relative volatility. Rising efficiency in the search process means that more unemployed workers get out of unemployment per unit of time for a given number of unemployed workers and vacancies. Specifically, in economic

[1] Following the labor literature's convention, I proxy the concept of skill with education. Specifically, a worker is counted as skilled if she has a college degree or higher. Otherwise, she is regarded as unskilled.

booms, increased matching efficiency leads to a larger number of vacancies filled, and thus employment expands more than before. The adoption and implementation of Information and Communication Technology (ICT) is my explanation for why search efficiency increased. One defining feature of ICT is that it lowers the specificity of human capital. When ICT is widely implemented in various industries, its standardization properties translate into a greater transferability of employee's skills across firms, industries and sectors of the economy.[2] Given that recent technology advance has been skill-biased, ICT affects matching efficiencies of the skilled and unskilled asymmetrically. ICT implies that skilled workers, when searching for jobs, face more suitable positions to apply for than before, which is not true for unskilled labor.

To make my idea clear, I embed a simple search and matching mechanism in an otherwise typical dynamic general equilibrium model. Search frictions allow vacancies and unemployment to co-exist. To capture the mechanism mentioned in the previous paragraph, I assume that the skilled and unskilled go through separate matching processes. To model matching efficiency in the labor market, I adopt a labor-vacancy matching function derived from the classic urn-ball framework. I add one feature to this standard scenario and enrich the matching function to embody the fact that the proportion of unemployed workers suitable for a particular vacancy

[2] Kambourov and Manovskii (2008) document the rising trends in occupation and industry mobility between 1968 and 1993. In addition to its impact on transferability of skills, the development of ICT makes it easier for skilled employees to search for jobs nationwide, such that geographic distance becomes a smaller factor when it comes to job application.

can change over time. The intuition of this feature is simple. Prior to the spread of ICT, human capital is firm-specific or job-specific. When firms post vacancies, only a small portion of the unemployed meet the position's requirements. This causes low matching rates. However, matching efficiency increases following wide implementation of ICT. ICT adoption causes the platform and working interface to become standardized, so that most of a skilled worker's existing knowledge continues to be effective in a new job. When a skilled worker is looking for job, her previous expertise satisfies the requirements of more vacancies than before, which dramatically increases the number of matches. Technically, for each type of worker, the matching function has a parameter governing the fraction of workers suitable for each associated vacancy. This parameter is related to matching efficiency and is assumed to have increased over time for skilled workers.

Castro and Coen-Pirani (2008) show that changes in capital-skill complementarity can account for most of the changes in relative volatility in the labor market. This paper identifies another mechanism that can explain the volatility increase in skilled employment. Apparently, these two hypotheses are competing. Both hypotheses emphasize the impact of technology advance on the cyclical behavior of employment. The Castro and Coen-Pirani (2008) explanation relies on the comparative advantage of skilled labor in adopting new technology. They argue that the end of technology diffusion reduces capital-skill complementarity. Once new equipment is widely used in the economy, skilled workers gradually lose their advantage in implementing new technology. Thus, complementarity falls and volatility of skilled employment rises. I argue, instead, that the general purpose feature of ICT expedites the standardization of production, making previously accumulated skills more transferable over jobs, firms and industries. The growing relevance of skill in the search process enlarges the possibility of matching workers with vacancies. Thus, economic booms witness a bigger swing in employment. In my opinion, the distinction between these hypotheses lies in the fact that Castro and Coen-Pirani (2008) and I focus on different aspects of technology. It is likely that these two mechanisms are complementary.

Broadly speaking, this paper is part of the literature on the relationship between technological change and the labor market. There is a growing literature on technology and skill premia. The difference between this paper and others is that I pay particular attention to the general-purpose nature of ICT. Bresnahan and Trajtenberg (1995) coined the term "General Purpose Technology" (GPT) to describe widely used technologies that transform both household life and the ways that firms conduct business. Examples are steam, electricity, internal combustion and ICT.[3] The existing skill premia literature centers on skill-biased technological change as an explanation for rising wage inequality due to changing wage differentials among different education levels.[4] Katz and Murphy (1992) and Acemoglu (1998,1999) propose that new equipment goods have improved the productivity of workers with

[3]Jovanovic and Rousseau (2005) present various stylized facts of GPT.

[4]There are exceptions. For example, Di Nardo et al (1996) and Lee (1999) hold that institutional changes, such as deunionization and the decline in the real minimum wage, play an important role in explaining changes in the wage premium. Wood (1995) claims that rising inequality is partly due to trade liberalization. For a comprehensive survey for this topic, see Acemoglu (2002) and Autor, Katz and Kearney (2005).

certain skills. Krusell, Ohanian, Rios-Rull and Violante (2000, henceforth KORV) show that capital-skill complementarity can explain the upward trend of the skill premium. Many economists argue that capital-embodied technology generates an increase in demand of skilled labor because skilled workers have a comparative advantage in adopting new equipment. When there is an acceleration in the speed of embodied technical change, the demand for skilled labor increases accordingly. Galor and Tsiddon (1997), Greenwood and Yorukoglu (1997) and Galor and Moav (2000) follow this argument. All these papers explore changes in relative prices of labor from the perspective of skill-biased technological progress. Instead, I turn attention to the quantity side of the labor market and analyze the implications of the general-purpose nature of ICT for employment volatility of different worker types. Aghion, Howitt and Violante (2002) have pointed out the importance of the general-purpose nature of ICT for the labor market, although they emphasize its implications for wage inequality rather than employment volatility.

This paper also draws on the labor search and matching literature. The search and matching model has become the workhorse in macroeconomic theories of the labor market. Mortensen and Pissarides (1994) provide an attractive mechanism generating equilibrium unemployment, which has been adopted to study various labor market issues. In this paper, I embed the search and matching process into an otherwise classical dynamic general equilibrium model.[5] I do not use the typical

[5]Shimer (2005) illustrates that the introduction of the search and matching function does not magnify employment volatility in the benchmark DSGE model. Shimer (2009) redefines this issue as the labor-wedge puzzle. My model also generates low volatilities for the labor market.

Cobb-Douglas matching function although it has properties consistent with many empirical results. The main reason is that the Cobb-Douglas matching function lacks micro-foundations and therefore its parameters do not have direct economic interpretations. I derive a simple matching function based on the urn-ball framework in which parameters have direct economic meanings.[6]

The paper is organized as follows. Section 2 presents empirical evidence on changes in employment volatility of skilled and unskilled workers in the U.S. Section 3 sets up the search and matching framework in a typical dynamic general equilibrium environment. Section 4 calibrates the parameters of the model and the analysis of numerical results is in section 5. Section 6 discusses alternative explanations for changes in the cyclical dynamics of the labor market, and section 7 summarizes the paper.

1.2 Empirical Observations

In this section, I re-produce the two main stylized labor market facts presented in Castro and Coen-Pirani (2008). First, the volatilities of skilled hours and employment increased beginning in the mid-1980s while those of the unskilled declined. If we take into account the simultaneous change in overall economic volatility, the pattern of relative employment volatility becomes quite remarkable. Secondly, the volatility of hours worked is primarily driven by that of employment. This result motivates my focus on the behavior of employment in my theoretical model.

[6]Other papers have also attemped to enrich the matching function by offering a micro-foundation. A comprehensive survey is Petrongolo and Pissarides (2001).

For the labor market measures, I rely on the CPS Merged Outgoing Rotation Group (MORG) dataset. I also use real GDP from NIPA. Following Castro and Coen-Pirani (2008), the time period of my sample is from 1979q1 to 2003q4. The MORG dataset is compiled by the NBER from extracts of the monthly CPS and contains data on weekly hours and earnings. Specifically, the original CPS outgoing rotation group data are recorded each month, and the Bureau of Labor Statistics extracts about 25,000 records of those outgoing households per month and assigns each household a weight such that aggregate statistics are representative of the U.S. population. One advantage of this dataset is that it contains comprehensive information, including weekly earnings. Earnings can be used to calculate wage weights, which are useful in controlling for composition effects when hours are aggregated across different demographic groups.

I follow standard sample-selection criteria to handle missing observations and coding errors and restrict attention to individuals in the labor force between 16 and 65 years of age that are not self-employed. I then convert monthly series into quarterly data. I have about 45,000 observations per quarter, of which on average about 11,000 hold at least a college degree. Variables constructed from the MORG data include total hours in efficiency units, employment in efficiency units and employment. These statistics are computed for all workers, and for skilled and unskilled labor respectively. To compute labor aggregates in efficiency units, real average wages for each demographic group are used as weights when aggregating across 240 subgroups. This method is analogous to the efficiency units approach suggested by Katz and Murphy (1992), KORV(2000), and Castro and Coen-Pirani (2008).

The main variables of interest are constructed as follows:[7]

Hours Worked in Efficiency Units: Using the efficiency units approach to control for the counter-cyclical dynamics of labor force quality, I construct a set of time-invariant weights for each demographic group. First, I divide the sample into 240 demographic groups based on age, sex, race and education. Second, I sum up weekly earnings and hours worked in each subgroup assuming that individuals within each subgroup are perfect substitutes. Next, a measure of the nominal wage rate is created by dividing total income by total hours within each group. As in Castro and Coen-Pirani (2008), I use the average real wages to weight hours worked across different groups.[8]

Employment: Aggregate employment in any given quarter is the total number of individuals, weighted with the CPS associated weights, who report being at work that quarter.[9] I compute this statistic for the entire labor force, and for skilled and unskilled labor. For the whole sample period, the average employment rate is 94%; the average skilled employment rate—defined as the ratio of the level of skilled employment to the entire labor force—is 23%; and the average unskilled rate, defined in a parallel way, is 71%.

[7]See Appendix A for details.

[8]I use the "CPI index for all urban consumers" from the BLS as the denominator when computing real wages.

[9]The MORG dataset contains several weights for different purposes. Following convention, I use "earning weight for all races" to get nationwide statistics for workers of different education levels. Notice that these CPS weights are not the same as the wage weights I construct to measure variables in efficiency units.

Employment in Efficiency Units: I use the same efficiency units approach discussed above for hours to construct employment efficiency unit aggregates.[10]

These time series from the MORG dataset display strong seasonality. Before any further data analysis, I need to deseasonalize them. The conventional way is to use the Census Bureau's seasonal adjustment program X12. Castro and Coen-Pirani (2008) claim that measurement errors also produce high frequency noise in these series. Following their practice, I apply a centered five-quarter moving average to these X12-adjusted series. I then log them and extract the cyclical component using a Hodrick-Prescott filter with a parameter of 1600. Volatility or variability is defined as the standard deviation of this cyclical component.

In Figure 1.1, I present the rolling standard deviations of GDP and hours in efficiency units of the skilled and unskilled, while Figure 1.2 demonstrates those for real GDP and skilled and unskilled employment.[11] For quarter t, I compute the standard deviation of the cyclical component of these variables using observations from period t to $t + 40$. Figures 1.1 and 1.2 display roughly the same patterns for hours and employment. Around the mid-1980s, the standard deviations of GDP and unskilled hours and employment decline sharply. By contrast, the volatility of skilled labor climbs over this period. A growing literature, reviewed by Stock and Watson (2003), has documented that the volatilities of most macro aggregates declined substantially starting in the mid-1980s. Clearly, skilled labor is one ex-

[10]In my calibration below, these weighted aggregates are used to compute the type-specific production weights for each type of worker.

[11]Please note that employment in Figure 1.2 is not in efficiency units.

ception. To display the differences between skilled and unskilled labor more clearly, Figure 1.3 shows the rolling standard deviations of skilled and unskilled employment relative to that of GDP. The ratio for the unskilled is roughly flat while the skilled ratio increases. This graph conveys a strong message that something fundamental in the labor market changed in the past three decades.

McConnell and Perez-Quiroz (2000) estimate the break date of aggregate volatility to be 1984q1. Accordingly, I divide the sample into two sub-periods: one from 1979q1 to 1983q4, and the other from 1984q1 to 2003q4.[12] For each sub-period, I compute the cyclical statistics of skilled and unskilled labor. Table 1.1 shows the same pattern as Figures 1.1 and 1.2. The standard deviations of skilled and unskilled labor change in opposite directions. In addition, Table 1.2 illustrates that a large proportion of fluctuations of hours results from employment regardless of skill level and period. To prove that the increased volatility of skilled aggregates is not an artifact of aggregation, Castro and Coen-Pirani (2008) rule out composition effects from sector, occupation and gender as explanations for the changing relative volatility of skilled and unskilled labor.

[12]One concern is that the first subperiod is so short that it cannot illustrate clearly the cyclical dynamics of the labor market. To correct this shortcoming of MORG dataset, Castro and Coen-Pirani(2008) also present evidence from the March CPS dataset, which is annual data from 1963 to 2002.

1.3 Model Set-up

Two types of economic agents co-exist in the model economy: one representative household, consisting of a continuum of infinitely-lived individuals who consume, search for jobs and supply labor in order to maximize discounted expected utility; and one representative firm, which uses labor and capital to maximize the expected discounted value of profit. There are three types of factor inputs: capital, skilled and unskilled labor. There are two technologies: one for matching unemployed workers seeking new jobs to vacancies posted by firms, the other for producing consumption goods using capital and labor inputs. Finally, the model economy contains one exogenous shock affecting total factor productivity. Time is discrete and is denoted by $t = 0, 1, 2, \cdots$.

1.3.1 Household

There is a representative household that includes a continuum of members with measure 1. Individuals are divided into two types: skilled or unskilled. These two types of workers go through different search and matching processes and enjoy different consumption paths.[13] The household acts as a social planner, maximizing the aggregate household utility by optimally allocating consumption goods among

[13] In the model economy, skilled and unskilled workers have different consumption because of the differences in their utility weights in the preference function. However, all workers of a given type enjoy the same amount of consumption regardless of employment status. This assumption seems realistic given that in the real economy consumption is different for workers of various education levels.

household members. The utility level depends on consumption and disutility from employment. The aggregate utility function is the weighted sum of the utilities of skilled and unskilled labor. The household optimally equalizes the marginal utility of consumption within household members of the same type, and also equalizes the weighted marginal utility of consumption across the two types of individuals. The aggregate utility for the household is:

$$\sum_{t=0}^{\infty} \beta^t E_0 \Big\{ \lambda_s \Big[\frac{l_s(c_{st})^{1-\sigma_s}}{1-\sigma_s} - \frac{(n_{st})^{1+\eta_s}}{1+\eta_s} \Big] + \lambda_u \Big[\frac{l_u(c_{ut})^{1-\sigma_u}}{1-\sigma_u} - \frac{(n_{ut})^{1+\eta_u}}{1+\eta_u} \Big] \Big\} \tag{1.1}$$

where the subscript "s" is related to skilled labor, while "u" represents unskilled workers. β is the subjective discount factor; $\lambda_i, i \in \{s, u\}$ is the subjective utility weight; l_i is the measure of each type of worker in the labor force, where $l_s + l_u = 1$;[14] c_{it} is consumption for type-i workers at period t; n_{it} is the measure of employment for each type of worker; and u_{it} is the corresponding unemployment measure at period t, where $l_i = n_{it} + u_{it}$. I assume that the household is able to insure its members against all idiosyncratic shocks, mimicking the complete markets allocation. Because utility is separable in consumption and leisure, all members of a given type of labor enjoy the same consumption regardless of their employment status.

The household chooses consumption for its members subject to:

$$0 = D_t + w_{st}n_{st} + w_{ut}n_{ut} + p_t q_t - p_t q_{t+1} - l_s c_{st} - l_u c_{ut}$$

where D_t is the dividend received by the household at period t; w_{it} is the state-contingent wage rate for type-i workers in period t, which is determined by Nash bargaining; p_t is the price per share at period t; q_t is the number of shares in

[14] I abstract from the labor-force participation decision in this paper.

the representative firm held by the representative household at the end of period $t-1$ and q_{t+1} is the number of shares carried over to period $t+1$. Due to the representative-agent assumption, the household is the only owner of the firm, which implies $q_t = q_{t+1}$ at the aggregate level. Accordingly, the above budget equation in equilibrium becomes:

$$0 = D_t + w_{st}n_{st} + w_{ut}n_{ut} - l_s c_{st} - l_u c_{ut} \tag{1.2}$$

The two types of workers go through separate matching processes. The laws of motion for labor flows are expressed as follows:

$$n_{st+1} = (1-\chi_s)n_{st} + M_s(u_{st}, v_{st}) \tag{1.3}$$

$$n_{ut+1} = (1-\chi_u)n_{ut} + M_u(u_{ut}, v_{ut}) \tag{1.4}$$

where $\chi_i, i \in \{s, u\}$ represents the type-specific separation rate; M_i is the matching function for each type of worker; and v_{it} is the type-specific vacancies posted by firms at period t. In the model economy, there is involuntary unemployment due to labor market frictions. Each period all unemployed workers search for new jobs, but the amount of job creation is also dependent on the number of vacancies posted.

1.3.2 Firm

The representative firm has access to a production function using one type of capital and two types of labor as inputs. I assume that the numbers of skilled and unskilled workers in the population are exogenously determined and there is no channel for an unskilled worker to upgrade her skill. The empirical labor literature, such as Katz and Autor (1999) and Autor, Katz and Krueger (1998), documents that skilled and unskilled workers are not perfect substitutes and that the elasticity of substitution between them is between 1 and 2, well short of infinity. Accordingly, the production function is:

$$y_t = e^{z_t} k_t^{\alpha} \left[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma} \right]^{\frac{1-\alpha}{\sigma}} \tag{1.5}$$

where z_t is the shock to general TFP; k is the capital stock; the elasticity of substitution between two types of workers is $\frac{1}{1-\sigma}$; and a_i is the type-specific weight in production. The firm is assumed to own capital, but has to employ labor from the household. To hire labor, the firm posts vacancies and goes through the matching process. The firm maximizes the discounted profit:[15]

$$\sum_{t=0}^{\infty} \beta^t E_0 \Lambda_{0,t} D_t \tag{1.6}$$

where $\Lambda_{0,t} = \dfrac{\partial u_t}{\partial c_{it}} \Big/ \dfrac{\partial u_0}{\partial c_{i0}}$

$$D_t = e^{z_t} k_t^{\alpha} \left[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma} \right]^{\frac{1-\alpha}{\sigma}} - w_{st} n_{st} - w_{ut} n_{ut} - f_s v_{st} - f_u v_{ut} + (1-\delta) k_t - k_{t+1}$$

$\Lambda_{0,t}$ is the stochastic discount factor of the household while D_t is the dividends. f_i is the unit cost of vacancy posting for type-i jobs, and δ is the depreciation rate of

[15] See Andolfatto (1996) for this type of set-up. In Appendix C, I derive an alternative set-up following Shimer (2009). Notice that the optimality conditions imply $\frac{\partial u_t}{\partial c_{st}} \Big/ \frac{\partial u_0}{\partial c_{s0}} = \frac{\partial u_t}{\partial c_{ut}} \Big/ \frac{\partial u_0}{\partial c_{u0}}$.

capital. The recursive expression for the above problem is :

$$J_t(z_t, k_t, n_{st}, n_{ut}) = \max_{\substack{\{v_{st}, v_{ut}, n_{st+1}, \\ n_{ut+1}, k_{t+1}\}}} \Big\{ e^{z_t} k_t^{\alpha} \left[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\right]^{\frac{1-\alpha}{\sigma}} - w_{st} n_{st} - w_{ut} n_{ut}$$
$$- f_s v_{st} - f_u v_{ut} + (1-\delta) k_t - k_{t+1} + \beta E_t \Lambda_{t,t+1} J(t+1|t) \Big\} \quad (1.7)$$

Each period, the firm optimally decides how many vacancies to post for a given type of labor and how much to invest in capital for future production. Because labor supply, like capital, is determined one period in advance, the firm faces the labor-matching constraints:

$$n_{st+1} = (1-\chi_s) n_{st} + M_s(u_{st}, v_{st})$$

$$n_{ut+1} = (1-\chi_u) n_{ut} + M_u(u_{ut}, v_{ut})$$

1.3.3 Matching Function

In this subsection, I derive a matching function based on an "urn-ball" framework. The aggregate matching function encapsulates search and matching frictions. The sources of frictions in the labor market are various, including co-ordination failures, mismatch and limited mobility. One reason why the Cobb-Douglas matching function is so popular is that its constant-returns-to-scale property is consistent with empirical outcomes.[16] However, this form of matching function looks like a black box, which limits the interpretation of both empirical and theoretical results. Moreover, this matching function is so abstract that certain features pertaining to the labor market cannot be added easily.

[16] See Petrongolo and Pissarides (2001).

Some existing research has attempted to establish micro-foundations for the matching function, in hopes of gaining a better understanding of the nature of matching frictions, and providing direct economic meanings for function parameters. The exponential-type matching function was first employed in a market context by Butters (1977), in which buyers and sellers contact each other in commodity markets.

In the typical static "urn-ball" scenario, unemployed workers each place a ball (job application) randomly in an urn (job-vacancy), where each urn belongs to one firm. Firms then pick one ball from their urn at random. In this model, matching frictions come from co-ordination failure between workers. For example, some urns may contain more than one ball while some others have nothing in them. The possibility of a given urn having at least one ball is $\big(1-(1-\frac{1}{v})^u\big)$, where u is is the number of balls and v is the number of urns. If the number of urns, v, goes to infinity, the limiting form of the above expression turns into $\big(1-e^{\frac{-u}{v}}\big)$. The expected number of matches is expressed as:

$$M = v\left(1 - e^{\frac{-u}{v}}\right) \tag{1.8}$$

In fact, Hall (1980), Pissarides (1979) and Peter (1984) all obtain similar forms for the matching process. The continuous-time version of the above equation is:

$$M = v\left(1 - e^{\frac{-u}{v}dt}\right) \tag{1.9}$$

which represents the expected number of urns (vacancies) receiving at least one application in a time period of length dt.[17]

[17] Mortensen and Pissarides (1999) enrich the continuous-time version by incorporating simulta-

Equation (1.8) satisfies intuitive properties of the matching function. For example, the number of matches is increasing in both arguments; $M(0, v) = M(u, 0) = 0$; and returns to scale are constant. Furthermore, the urn-ball framework can easily accomadate additional features, making the matching function more flexible and meaningful. Moreover, there are several papers, such as Montgomery (1991), Cao and Shi (2000), Julien et al (2000) and Burdett et al (2001), which model a trade-off facing agents between high match payoffs and the risk of not matching and generate an exponential equilibrium matching function in the limit.

To adapt the urn-ball scenario to my model economy, I assume that only a fraction α_i of unemployed workers of type i are qualified for a given vacancy.[18] As mentioned before, I assume that skilled and unskilled workers go through different search processes. Specifically, I assume that there are two labor markets, one for the skilled and the other for the unskilled. Each urn accepts only one type of application and is placed at the associated marketplace. Each type of worker goes to the corresponding labor market and tries her luck. Workers randomly put their balls in one urn without knowledge about their qualification for that particular urn. Firms can't identify ex ante which workers in the market are qualified when workers place balls, but firms know the fraction of unemployed workers in the market that are suitable, and this is constant over vacancies within a given market. Therefore, the

neous search on both sides of the labor market. The limiting form of their matching function is: $M = \lim_{dt \to 0} \left(v(1 - e^{\frac{-fu}{v} dt}) + u(1 - e^{\frac{-gv}{u} dt}) \right) \Big/ dt = fu + gv$, where f, g are parameters.

[18] For notation, I use α_i in this subsection and $i \in \{s, u\}$. In the subsequent subsections, α_s and α_u are used to represent the suitability of each type of worker to the associated vacancy, respectively.

associated probability of each urn having at least one suitable ball is $\big(1-(1-\frac{1}{v_i})^{\alpha_i u_i}\big)$. If we take the limit operation, then the expression becomes $\big(1-e^{\frac{-\alpha_i u_i}{v_i}}\big)$ and the corresponding matching function turns into:

$$M_i = v_i\left(1 - e^{\frac{-\alpha_i u_i}{v_i}}\right) \tag{1.10}$$

Based on this argument, it follows that the value of α_i is expected to fall between 0 and 1 in the discrete time formulation. In this paper, equation (1.10) is adopted as the matching function.

I can derive the same matching function in the continuous-time context. I assume that λ_i, the arrival rate of the Poisson process, is positively related to $\alpha_i u_i$, the number of unemployed workers of type i who are qualified for the job, and is negatively related to v_i, the number of vacancies of type i in this market. That is, $\lambda_i = \frac{\alpha_i u_i}{v_i}$. Therefore, the number of urns having at least one ball per unit of time is: $v_i\big(1-e^{-\lambda_i dt}\big)$ or $v_i\big(1-e^{\frac{-\alpha_i u_i}{v_i}dt}\big)$, where $dt = 1$.

1.3.4 Maximization Problem

For the household, the maximization problem can be expressed as:

$$V_t(z_t, q_t) = \max_{\{c_{st}, c_{ut}, q_{t+1}\}} \left\{ \lambda_s \left[\frac{l_s(c_{st})^{1-\sigma_s}}{1-\sigma_s} - \frac{(n_{st})^{1+\eta_s}}{1+\eta_s}\right] + \lambda_u \left[\frac{l_u(c_{ut})^{1-\sigma_u}}{1-\sigma_u} - \frac{(n_{ut})^{1+\eta_u}}{1+\eta_u}\right] + \beta E_t V(t+1|t) \right\} \tag{P.1}$$

s.t.

$$0 = D_t + w_{st}n_{st} + w_{ut}n_{ut} + p_t q_t - p_t q_{t+1} - l_s c_{st} - l_u c_{ut} \tag{1.11}$$

Following the conventional procedure, I get the optimality condition:

$$\lambda_s c_{st}^{-\sigma_s} = \lambda_u c_{ut}^{-\sigma_u} \tag{1.12}$$

Equation (1.12) shows that the household equalizes weighted marginal utilities of consumption across skilled and unskilled labor.

For the firm, the profit-maximizing problem is characterized as:

$$J_t(z_t, k_t, n_{st}, n_{ut}) = \max_{\substack{\{v_{st}, v_{ut}, n_{st+1}, \\ n_{ut+1}, k_{t+1}\}}} \Big\{ e^{z_t} k_t^{\alpha} \big[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\big]^{\frac{1-\alpha}{\sigma}} - w_{st} n_{st} - w_{ut} n_{ut}$$

$$- f_s v_{st} - f_u v_{ut} + (1-\delta) k_t - k_{t+1} + \beta E_t \Lambda_{t,t+1} J(t+1|t) \Big\} \quad \text{(P.2)}$$

$$\text{s.t.} \qquad n_{st+1} = (1-\chi_s) n_{st} + v_{st}\big(1 - e^{\frac{-\alpha_s u_{st}}{v_{st}}}\big) \qquad (1.13)$$

$$n_{ut+1} = (1-\chi_u) n_{ut} + v_{ut}\big(1 - e^{\frac{-\alpha_u u_{ut}}{v_{ut}}}\big) \qquad (1.14)$$

The optimality condition for capital is:

$$1 = \beta E_t \Lambda_{t,t+1} \big\{ \alpha e^{z_{t+1}} \big(k_{t+1}\big)^{\alpha-1} \big[(a_s n_{st+1})^{\sigma} + (a_u n_{ut+1})^{\sigma}\big]^{\frac{1-\alpha}{\sigma}} + 1 - \delta \big\}$$

In equilibrium, the cost of investing one unit of capital this period is equal to the expected value of the marginal product and the undepreciated portion of the capital next period.

These two agents interact with each other through two channels. One is through the labor market where the firm hires household members and pays wages for their work. The other is through the securities market. Household members hold the securities issued by the firm and receive dividends contingent on the state of the economy. In the model economy, wage rates are decided by axiomatic Nash Bargaining.

1.3.5 Nash Bargaining

At the start of each period, the household and firm bargain with one another over the skilled and unskilled wage rates after the productivity shock is realized. I assume that agents implement the axiomatic Nash bargaining solution from Nash (1953). I follow Shimer (2009) to set up the Nash bargaining problem as:[19]

$$\max_{\tilde{w}_i} \left\{ \tilde{V}_{it}^{\phi_i} \tilde{J}_{it}^{1-\phi_i} \right\} \tag{1.15}$$

where $i \in \{s, u\}$ indicates skilled or unskilled workers. Following Shimer (2009), $\tilde{V}_{it}$ represents the marginal utility to the household of having one more type-i worker employed at a wage $\tilde{w}_i$ in period t rather than unemployed, evaluated at the equilibrium levels of assets and employment.[20] Meanwhile, $\tilde{J}_{it}$ is the marginal profit to the firm of hiring one more type-i worker at a wage $\tilde{w}_i$ in period t, and ϕ_i is the bargaining weight of type-i workers vis-a-vis the firm.

The equilibrium wage rate $\tilde{w}_i$ solves the weighted geometric average of the gains from bargaining:

$$\tilde{w}_i = \arg\max \left\{ \tilde{V}_{it}^{\phi_i} \tilde{J}_{it}^{1-\phi_i} \right\}$$

The first-order condition is:

$$0 = \phi_i \tilde{J}_{it}(\cdot) \frac{\partial \tilde{V}_{it}}{\partial \tilde{w}_i} + (1 - \phi_i) \tilde{V}_{it}(\cdot) \frac{\partial \tilde{J}_{it}}{\partial \tilde{w}_i} \tag{1.16}$$

[19] $\tilde{w}_i$ is the type-specific arbitrary wage rate in the Nash bargaining context at period t. To simplify notation, I suppress the time-subscript 't'. In equilibrium, $\tilde{w}_i = w_{it}$.

[20] Some papers also incorporate rigid wages. If rigid wages are added to this bargaining framework, as in Hall (2005), wages will be the weighted sum of the current Nash-Bargaining solution and previous wages. This mechanism is designed to dampen the cyclical fluctuations of wages and to increase the volatility of employment.

After plugging in the expressions for $\tilde{J}_{it}(\cdot)$, $\tilde{V}_{it}(\cdot)$, $\frac{\partial \tilde{J}_{it}}{\partial \tilde{w}_i}$, $\frac{\partial \tilde{V}_{it}}{\partial \tilde{w}_i}$ and manipulating,[21] I find that the skilled and unskilled wages satisfy:

$$w_{st} = \phi_s(1-\alpha)e^{z_t}k_t^{\alpha}\Big[(a_s n_{st})^{\sigma}+(a_u n_{ut})^{\sigma}\Big]^{\frac{1-\alpha-\sigma}{\sigma}} a_s^{\sigma}(n_{st})^{\sigma-1}+(1-\phi_s)c_{st}^{\sigma_s}(n_{st})^{\eta_s} \quad (1.17)$$

$$w_{ut} = \phi_u(1-\alpha)e^{z_t}k_t^{\alpha}\Big[(a_s n_{st})^{\sigma}+(a_u n_{ut})^{\sigma}\Big]^{\frac{1-\alpha-\sigma}{\sigma}} a_u^{\sigma}(n_{ut})^{\sigma-1}+(1-\phi_u)c_{ut}^{\sigma_u}(n_{ut})^{\eta_u} \quad (1.18)$$

The wage formula is a linear combination of the marginal product of labor (MPL) and the marginal rate of substitution (MRS) between consumption and employment from the perspective of the household. The combination weights are the associated bargaining powers. Because MPL and MRS are both pro-cyclical, the wage rates are pro-cyclical and thus absorb most of the fluctuations of the exogenous shocks. As a result, the volatility of employment is quite low compared to output, as in most similar search models. Given that this paper does not focus on the solution to the Shimer (2005) puzzle, I abstract from this issue and concentrate on the relative volatilities of skilled and unskilled employment in the model.

1.3.6 Competitive Equilibrium

The competitive equilibrium is now formulated. The equilibrium results from the combination of the household's utility-maximizing and the firm's profit-optimizing problems. That is, the household solves problem (P.1) and the firm chooses optimal strategies for problem (P.2).

The aggregate state of the model economy is $SS_t = \{z_t, k_t, n_{st}, n_{ut}\}$ for period t. In equilibrium, the wages w_{st} and w_{ut} are functions of the state of the economy:

[21] Please check Appendix B for the detailed derivation.

$w_{st} = W_s(SS_t)$, $w_{ut} = W_u(SS_t)$. While skilled and unskilled employment and capital evolve according to $n_{st+1} = N_s(SS_t)$, $n_{ut+1} = N_u(SS_t)$, $k_{t+1} = K(SS_t)$.

A competitive equilibrium is a set of allocation rules, prices and aggregate laws of motion for the state variables that satisfy:

1. The household maximizes (P.1), taking as given the aggregate state SS_t and the pricing functions, W_s, and W_u. The household's solution is: $c_{st} = C_s(SS_t)$, $c_{ut} = C_u(SS_t)$;

2. The firm solves (P.2), given SS_t, W_s, and W_u. Its solution is: $n_{st+1} = N_s(SS_t)$, $n_{ut+1} = N_u(SS_t)$, $k_{t+1} = K(SS_t)$, $v_{st} = V_s(SS_t)$, $v_{ut} = V_u(SS_t)$;

3. The economy-wide resource constraint is satisfied:

$$e^{z_t} k_t^{\alpha} \left[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma} \right]^{\frac{1-\alpha}{\sigma}} + k_t(1-\delta) = k_{t+1} + l_s c_{st} + l_u c_{ut} + f_s v_{st} + f_u v_{ut}$$

1.4 Calibration

To proceed further, I calibrate the model economy in this section. First, some structural parameters are assigned values according to previous literature. Then, given these borrowed parameters, all other parameters are set so that the model matches certain stylized facts in the U.S. data. These unknown parameters govern the utility function, production sector, search and matching process, and exogenous technology process.

1.4.1 Parameters in the Utility Function: $\beta, \sigma_s, \sigma_u, \eta_s, \eta_u, \lambda_s, \lambda_u, l_s, l_u$

The subjective discount factor is set to match an annual rate of return of 5 percent. Given that the time period is set to one month, β is equal to 0.996.

The coefficient of relative risk-aversion, $\sigma_i, i \in \{s, u\}$, displays a wide range of estimates in existing literature. For example, using durable and non-durable goods, Rotemberg and Woodford (1997) estimate σ_i to be 0.16. Hall (1988) and Attanasio and Weber (1993) find σ_i to be around 3 using aggregate consumption data. Fuhrer (2000) estimates σ_i to be about 6 assuming habit formation in consumption. All these estimates regard individuals as identical while I have divided labor into two subgroups. I set $\sigma_s = \sigma_u = 1$, which is within the range of existing estimates. Thus, the consumption part of the utility function takes the log form.

η_s and η_u index the marginal disutilities for the representative household of skilled and unskilled employment, respectively. If I assume that each type of worker works a fixed number of hours per period, then η_s and η_u also govern the Frisch elasticities for the household of skilled and unskilled hours. Empirical estimates of this elasticity for the entire labor force have a wide range. MaCurdy (1981), Altonji (1986) and Browning, Deaton and Irish (1985) find elasticities between 0 and 0.5. Browning, Hansen and Heckman (1999) report estimates ranging from 0.5 to 1.6. Imai and Keane (2004) provide a high estimate of 3.8. In the benchmark version of my model, I set the elasticity to 0.5, implying $\eta_s = \eta_u = 2$, to make sure that the marginal disutilities of skilled and unskilled employment fall within a reasonable range. Again, I abstract from differences in this elasticity between the two types of workers. I check the robustness of numerical results to the values of η_s and η_u.

λ_s and λ_u are subjective weights of skilled and unskilled workers in the household utility. I set λ_s and λ_u to make sure that the steady state consumption ratio between skilled and unskilled labor is consistent with its empirical counterpart.[22] The Consumer Expenditure Survey (CES) dataset collects "total average annual expenditure" by education level since 1996. The average consumption ratio in the data is 1.67.

l_s and l_u are set to match the labor force shares of skilled and unskilled workers. Both of them are treated as exogenous values in my paper. For simulation II, these two values vary between two sub-periods.

1.4.2 Parameters for the Production Sector: $\alpha, \sigma, \delta, a_s, a_u, f_s, f_u$

α is the capital share of total income. I follow convention and set α to 0.35. σ governs the elasticity of substitution between two types of workers, for which the existing literature has a wide range of estimates. Autor, Katz and Krueger (1998) conclude that the elasticity of substitution is likely to fall into the interval between 1 and 2. Katz and Murphy (1992) estimate the elasticity as 1.4. In the benchmark version, I set $\sigma = 0.29$, corresponding to an elasticity $\frac{1}{1-\sigma} = 1.4$. In subsequent sections, I check the robustness of numerical results to other values of σ.

For the depreciation rate, I follow Shimer (2009) and use data from the Bureau of Economic Analysis. BEA's Fixed Asset Table reports the current cost net stock

[22]In the model economy, all workers of a given type optimallly consume the same amount. However, different type workers can consume different amounts at an optimum.

of fixed assets and consumer durables, which is used as a measure of the nominal capital stock. I get nominal GDP from NIPA. The annual average capital-output ratio in the US is 3.2, which implies that the ratio of capital to monthly output is 38.3. The depreciation rate is set to 0.0055 in order to be consistent with this ratio.[23] This way of fixing the depreciation rate allows me to abstract from issues such as the distinction between physical depreciation and economic depreciation.

a_s and a_u are type-specific production weights. The two types of workers are neither perfect substitutes nor equally productive in production. Hence, a_s and a_u are set to match the observed ratios of employment in efficiency units to employment for both types of workers. For the whole sample period of 1979-2003, the production weight for skilled workers is 12.58 while that for unskilled workers 7.28. In the following section, I normalize a_u to be 1 and set $a_s = 12.58/7.28$.

f_s and f_u are vacancy-posting costs, which are set to make sure that the steady-state values of n_s and n_u are equal to their empirical counterparts, as described further in section 1.5.

1.4.3 Parameters for the Matching process: $\chi_s, \chi_u, \phi_s, \phi_u, \alpha_s, \alpha_u$

To estimate the separation rates for skilled and unskilled workers, I follow the method used in Shimer (2005) by inferring the job-finding rate first and then

[23] Alternatively, I can set the monthly depreciation rate to match the empirical counterpart and then derive the associated capital stock. Although the steady-state values of the capital stock obtained through these two approaches are different, they have quite negligible effects on the cyclical dynamics of the model.

computing the associated separation rate. The data are the basic monthly CPS from 1994 to 2006. I choose January 1994 as the starting period because the CPS survey switched to dependent interviewing at that time. Starting in 1994, respondents who have been laid off in successive months were not asked for the duration of their unemployment in the second month. This switch in survey methodology reduced measures of short-term unemployment.[24] In this paper, I first compute the separation rates without any modification to the data. I get $\chi_s = 0.0108$ and $\chi_u = 0.0201$. Next I scale separation rates up to match the separation rate of the entire labor force in Shimer (2005) who adjusts his measure for the 1994 switch in CPS survey methdology. The scaled-up estimates are $\chi_s = 0.0198$ and $\chi_u = 0.0369$. I set $\chi_s = 0.0108$ and $\chi_u = 0.0201$ for the benchmark model in simulation I. For robustness, I substitute the scaled-up separation rates and compute the associated numerical results. Simulated results are robust to this change.

Hall (2005) shows that separation rates can be estimated through the Job Openings and Labor Turnover Survey (JOLTS), or directly from flows in the CPS. Gottschalk and Moffitt (1998) directly compute separation rates from the Survey of Income and Program Participation (SIPP). All these estimates demonstrate that the separation rate is roughly constant over time and that recessions involve no significant increases in the rate of departure from jobs. In sum, it is reasonable to estimate the separation rate based on my abbreviated sample period and to keep the separation rate fixed over time in the model economy.

Existing literature adopts different values for the bargaining power, ϕ. Shimer

[24] See Abraham and Shimer (2001).

(2005) sets ϕ to 0.72 based on the Hosios's (1990) efficiency condition, while Hagedorn and Manovskii (2008) use bargaining power along with other structural parameters to match the volatility of labor market quantities and find ϕ to be 0.05. In this paper, I assume symmetric Nash-Bargaining power for both the skilled and unskilled, namely $\phi_s = \phi_u = 0.5$. In fact, the wage rates in my model are a bargaining-power weighted combination of MPL and MRS, both of which are pro-cyclical. As I show later, my results are robust to different values of ϕ_s and ϕ_u.

The parameters α_s and α_u govern matching efficiency for skilled and unskilled workers. These are key parameters in my model, and I discuss how I set them in section 1.5.

1.4.4 Parameters for the Exogenous Process: ρ_z, σ_z

Shimer (2009) estimates the productivity shock's properties by using the BLS annual data on multifactor productivity growth, first calculating the average annual growth rate, the unconditional standard deviation of annual productivity growth and the annual autocorrelation, then deriving the implied monthly parameters. I assume that the productivity shock has a deterministic trend and borrow $\rho_z = 0.98$ and $\sigma_z = 0.005$ from Shimer (2009).

Parameter values are listed in Table 1.3.

1.5 Numerical Results

In this section, I solve the dynamic model numerically. I take first-order linear approximations of optimal policy functions around steady-state values. To illustrate my main mechanism in the model economy, I carry out two experimental simulations. The first simulation aims to show the potential importance of matching efficiency for employment volatility. Specifically, I use prior research or data moments to pin down all parameter values except for α_s, α_u, f_s, and f_u, as described in the previous section. I regard α_s as a free parameter and allow its value to increase within a reasonable range to represent improvement in matching efficiency for skilled workers due to the spread of ICT. I assume α_u to be 0.35 while f_s and f_u are adjusted in response to changes in α_s to guarantee that steady-state values of n_s and n_u are consistent with the data. This experiment gives us a direct idea of the mechanism at work in the model. Next, I undertake a disciplined exercise of simulating the effects of changes in matching technology between two periods, 1979-1983 and 1984-2003. I pin down parameters for each period using moments not related to skilled employment volatility. I then show that the model can generate increased volatility of skilled employment in the second period.

To obtain the summary statistics of interest, I simulate the model 3000 times for each set of structural parameters. For each simulation, I set the number of periods to be 360, corresponding to thirty years. I convert simulated monthly series to quarterly data using 3-month averages. Finally, I calculate the standard deviation of these percentage deviations from the steady state for variables of interest.

1.5.1 Simulation I

In this paper, I propose that rising matching efficiency can account for the increased volatility of skilled employment. In the model, the key parameter to this mechanism is α_s, governing the fraction of skilled unemployed workers suitable for a given vacancy. My first simulation illustrates the positive relationship between matching efficiency and employment volatility. To isolate this relationship from others, I fix all parameters except for α_s, f_s and f_u. In particular, I set α_u to be 0.35 and assume it to be constant.[25] My procedure is as follows: I choose a value for α_s, pin down f_s and f_u to make sure that n_s and n_u are consistent with their empirical counterparts, simulate my model for this set of parameters and record the volatilities of variables of interest. I repeat this process for a range of values for α_s. Results are shown in Figures 1.4 and 1.5. As α_s increases, the volatility of skilled employment rises relative to that of output, while the relative volatility of unskilled employment remains flat. Other factors being controlled for, a more efficient matching process makes skilled employment more volatile. Intuitively, rising matching efficiency increases the number of matches in booms for a given number of unemployed workers and vacancies. Therefore, skilled unemployed workers have a higher job-finding rate, worker flows out of unemployment rise and employment fluctuates more than before. Meanwhile, Figure 1.5 shows the impact of changes in α_s on the skill premium, defined as the average ratio of skilled to unskilled wages implied by the model. The figure shows that increases in matching efficiency for

[25]The volatility pattern is robust to the value of α_u.

skilled workers generate a modest increase in the skill premium.

Several structural parameters calibrated above have a wide range of plausible estimates. I check the robustness of numerical results to these parameters. I re-do the exercise plotted in Figure 1.4 for values of the reverse labor supply elasticity, η_s and η_u, ranging from 0.25 to 3; for values of σ, where $\frac{1}{1-\sigma}$ is the substitution elasticity in production, ranging from 0.1 to 0.5; and for scaled-up separation rates, namely $\chi_s = 0.0198$ and $\chi_u = 0.369$. The volatility pattern shown in Figure 1.4 is robust to these tests. I also explore robustness to alternative values of ϕ_s and ϕ_u, which represent both the bargaining power and weights for wage determination. Specifically, ϕ_i is the weight on the marginal product of type-i labor in wage determination, while $1-\phi_i$ is the weight on the MRS between consumption and labor. In the model economy, the volatility of the MRS is less than that of the MPL. When the value of ϕ_i declines, wage fluctuations diminish accordingly. As a result, employment becomes more volatile. However, this does not affect the impact of α_s on the relative volatilities of skilled and unskilled employment.

1.5.2 Simulation II

Now that I have established that increased matching efficiency can potentially explain higher volatility for skilled workers, the next step is to assess whether this mechanism can potentially explain the observed rise in skilled volatility quantitatively. I calibrate the model to two subperiods, 1979-1983 and 1984-2003. Table 1.4 presents summary statistics on the labor market for these subperiods. Over time, the skilled share of the labor force increased at the expense of the unskilled share, and the wage ratio between skilled and unskilled workers rose. I use these changes to pin down parameters, and then examine the extent to which my calibrated model reproduces the observed volatility changes between periods.

Specifically, for each period I allow $\alpha_s, \alpha_u, f_s, f_u, \lambda_s$, and λ_u to vary while all other parameters are fixed at their values from simulation I. Note that α_s and α_u govern matching efficiency for skilled and unskilled workers while f_s and f_u are vacancy-posting costs. The first four parameters are set in each period to ensure that the model economy matches the observed long-run averages of the skilled labor share l_s, skilled employment share n_s, unskilled employment share n_u, and wage ratio $\frac{w_s}{w_u}$, whose values for two subperiods are listed in Table 1.4.[26] The last two parameters, λ_s and λ_u, are subjective utility weights for skilled and unskilled workers, which are set to guarantee that the consumption ratio is valid over changing periods.

Table 1.5 presents calibrated parameter values for each subperiod while Table 1.6 shows the simulated volatilities. Table 1.7 summarizes the volatility ratios of skilled and unskilled employment relative to output implied by the simulation II and compares them with the data. The last column of Table 1.7 shows that the model economy generates a 50% rise in relative volatility of skilled employment, compared to the 217% increase observed in the data.

Tables 1.5 and 1.6 show that the calibrated value of α_s increases between peri-

[26]Note that $l_u = 1 - l_s, u_s = l_s - n_s, u_u = l_u - n_u$. The values of l_s and l_u are directly determined by the associated measures of skilled and unskilled labor. Also, both are treated as exogenous parameters in the models.

ods, representing a rise in matching efficiency for skilled workers; and that consistent with the previous numerical results, employment volatility of the skilled goes up in the later period. However, Table 1.5 also shows that the calibrated α_u increases, while Table 1.6 displays a negligible change in unskilled volatility. This seems to be a challenge to my hypothesis: why does unskilled volatility not increase substantially when α_u goes up? My explanation is that there is a decline in the number of unemployed unskilled workers in the labor force between periods. Note that the number of matches of type-i workers is dependent on not only vacancies, v_i, and matching efficiency, α_i, but also the amount of unemployment, u_i. The increased α_i pushes the number of matches upward while the drop in u_i pulls this number down. On net, fewer unemployed unskilled workers get jobs in booms and unskilled employment expands less in the later period. One might worry that this argument concerns level changes while volatility depends on percentage deviations. Given the accompanying decline in the share of the unskilled in the labor force, it is not clear whether the decline in the level of unskilled unemployment is sufficient to prevent a rise in unskilled volatility driven by rising matching efficiency. To clarify this issue, I derive an approximate expression for employment volatility for type-i workers from the law of motion for employment, which is a function of α_i, the matching efficiency of type i workers and the steady-state values of unemployment $\bar{u}_i$, employment $\bar{n}_i$, and vacancies $\bar{v}_i$. For workers of type i, we have:[27]

$$var(n_i) = F_i\Big(\frac{\alpha_i \bar{u}_i}{\bar{v}_i}, \frac{\bar{u}_i}{\bar{n}_i}, \cdot\Big) \tag{1.19}$$

[27]See Appendix D for the detailed derivation.

where F_{i1} and F_{i2} are both positive, so that employment volatility is increasing in both $\frac{\alpha_i \bar{u}_i}{\bar{v}_i}$ and $\frac{\bar{u}_i}{\bar{n}_i}$. In contrast to unskilled workers, the steady-state number of unemployed skilled workers, $\bar{u}_s$, remains roughly constant between periods while matching efficiency of skilled workers increases. Therefore, the product of $\alpha_s \bar{u}_s$ increases substantially, leading to a volatility rise in skilled employment. In contrast, unskilled workers have a decline in $\bar{u}_u$, which offsets the effect of the rise of $\bar{\alpha}_u$. Taking into account all relevant factors, these competing parameter changes for the unskilled parameters roughly cancel each other out and the overall effect on unskilled employment volatility is negligible.

Table 1.6 also documents the volatility of job-finding rates for skilled labor, unskilled labor and the overall labor force in the two subperiods. The volatility of the job-finding rate for skilled labor has a higher percentage increase in the second period than that for unskilled labor.[28]

Next, I plot impulse responses to a favorable TFP shock to compare model dynamics between the two periods. Each quadrant of Figure 1.6 contains two sets of responses for a given variable, with the impulse responses for the second period marked with crosses ("x"). In both periods, when the economy has a positive shock to TFP, vacancies, job-finding rates and employment jump up for both types of workers, which leads to a drop in unemployment. The response of vacancies is

[28]Elsby, Michaels and Solon (2009) study outflow rates from unemployment since WWII using CPS data. They estimate job-finding rates using a variety of different methods. Their estimated rates do not show an upward or downward trend. They do not study trends in volatility in these rates, and they look at the overall labor force rather than skilled and unskilled workers separately.

smaller in the second period than in the first period following a good shock, due to the increased vacancy-posting cost. For skilled workers, the increase in matching efficiency dominates the effect of lower vacancies, so skilled workers have a much higher job finding rate in the later period, resulting in a larger percentage increase in employment. For unskilled workers, the increase in the job-finding rate is only slightly higher in the second period, because the lower vacancies almost completely offset the increase in matching efficiency, and consequently the initial response of unskilled employment is only slightly higher in the second period compared to the early period.

Under my first order approximation, impulse responses to negative TFP shocks are just the inverse of Figure 1.6, which implies skilled employment falls more than unskilled employment during downturns. Intuitively, this is because the model economy has a higher vacancy posting cost for skilled than for unskilled positions. In recessions, firms post fewer vacancies for skilled workers due to this higher cost, and the outflow from unemployment drops accordingly.

1.6 Discussion

In this section, I discuss several alternative explanations for changes in the cyclical dynamics of skilled and unskilled employment. These potential candidates include deunionization, international trade, changing age composition of the labor force, and tax cuts.

DiNardo et al (1996) and Lee (1999) provide evidence that the large decline in unionization in recent decades has made labor markets more competitive and increased wage dispersion.[29] Can deunionization explain rising instability of skilled employment relative to unskilled? A priori, we would expect that labor unions aim to stabilize wage rates and employment. Considering the strong bargaining power of labor unions, one might expect employment of union members to be more stable than that of non-union members. If so, deunionization would be expected to increase employment volatility, especially for unskilled workers who are likely to be unionized. Even if at the aggregate level labor unions make employment more sensitive to various shocks by increasing wage stickiness, it is still hard to argue that deunionization contributes a lot to the observed patterns of employment volatility, because of timing. The US deunionization started in the 1950's. We did not observe a decline in relative unskilled employment volatility until the 1980s.

Globalization has transformed international economic activities substantially in recent decades. Is it possible that globalization plays an important role in changes in employment volatility? Existing empirical and theoretical literature generates diverging results. Intuitively, material offshoring may make labor demand more elastic since firms can easily replace domestic workers and force unskilled workers to move to low-wage industries. Empirical studies by Slaughter (2001) and Senses (2007) are consistent with this intuition. This implies that workers in industries with foreign competition may have more risk of being displaced by material offshoring. However, other papers present conflicting results. Munch (2005), using a panel of Danish

[29] One side effect of unionization is to hinder employment growth, as discussed in Leonard (1992), and Bronars, Deere and Tracy (1994).

workers, finds that the impact of offshoring on the probability of displacement is modest while Geishecker (2008) documents a large effect for Germany. Meanwhile, Bergin, Feenstra and Hanson (2007) find that outsourcing makes employment more volatile overseas, but reduces employment fluctuations in the US. In my opinion, this channel may be helpful in explaining the volatility decline in the overall and unskilled employment, but is unlikely to explain the increasing relative volatility of skilled employment. Mankiw and Swagel (2006) show that service offshoring exerts a positive influence on US total employment. However, the level rise in US total employment should not have any direct implication for relative volatility of skilled and unskilled workers. If material or service outsouring is an important factor affecting employment volatility, it should have more influence in industries with high levels of imports and outports. However, Castro and Coen-Pirani (2008) check sector effects and find that the increase in relative skilled employment volatility affects all sectors of the economy.

Changes in the demographic composition of the workforce can lead to variation in business cycle volatility. Clark and Summers (1981), Rios-Rull (1996), Gomme et al (2005) and Jaimovich and Siu (2009) show that the cyclical volatilities of employment and hours are U-shaped as a function of age. Is it possible that demographic changes in the labor force account for changes in the cyclical dynamics of employment? The answer appears to be negative. Jaimovich and Siu (2009) claim that the associated demographic effects of the US baby boom and baby bust contribute to the Great Moderation. That is, changes in the age composition have dampened the volatility of labor in recent years rather than increasing it. Thus, the demographic change may be helpful in understanding the changing cyclical volatility of overall employment, but can not explain the rising volatility of skilled labor.

As for the significant tax cuts in 1981 and 1986, Castro and Coen-Pirani (2008) have already analyzed their potential effect on employment volatility. They assert that tax cuts may induce already employed workers to work more, but have a small effect on the decision of whether to work or not. Given that a large fraction of the volatility of hours worked comes from that of employment, tax cuts do not seem to an important factor. Tax cuts may have a larger impact on employment decisions for women than for men. Castro and Coen-Pirani (2008) narrow down the observations to white male workers and find the same volatility pattern.

1.7 Conclusion

I propose that rising matching efficiency helps explain the recent increase in employment volatility of skilled workers. Although there are various potential factors affecting the cyclical behavior of the labor market, such as institutions, fiscal policy and demographic composition, it is not clear that any of these factors can explain the rising volatility of skilled labor. While other factors may also contribute to changing labor market dynamics, I assert that technology advance in matching plays an important role.

Chapter 2

Capital-Skill Complementarity and Employment Volatility Over the Business Cycle

2.1 Introduction

The purpose of this paper is to explore the contribution of capital-skill complementarity to short-run employment fluctuations. In general, the quantities and prices of labor inputs are both key variables of interest in evaluating macroeconomic models. Given that capital-skill complementarity is a leading explanation for long-run changes in the skill premium, it is interesting to check its short-run implications for employment volatility. Does complementarity cause skilled employment to be more volatile than unskilled employment? Does allowing for complementarity improve upon existing DSGE models' implications for labor market volatility? Does investment-specific technological change matter when complementarity is present? The answer to the first question is yes; to the second, no; to the third, yes.

The concrete definition of complementarity used in this paper is borrowed from Krusell, Ohanian, Rios-Rull and Violante (2000, henceforth KORV (2000)). Their contribution is to use observable variables to capture unobservable skill-biased technology changes and to choose parameter values that allow the model to match the observed secular trend in the skill premium. In their framework, complementarity first means that the elasticity of substitution between unskilled labor and the equipment-skill composite is larger than that between skilled labor and equipment. Loosely speaking, equipment is complementary to skilled labor and is a substitute for unskilled labor.

Another interesting implication of complementarity in KORV is that the relative marginal product of skilled to unskilled employment is an increasing function of the equipment-skill ratio. Thus, changes in the equipment-skill ratio affect the relative volatility of skilled and unskilled employment. The second implication is also the main intuition of this paper. Without complmentarity, the standard DSGE model implies that the relative labor demand curve is only determined by the relative wage. With complementarity, the relative labor demand also depends on the equipment-skill ratio. If there is a positive shock and the equipment-skill ratio increases, then the relative demand for skilled labor increases. As a result, skilled employment has a higher percentage change than unskilled employment.[1] Therefore, complementarity can make skilled employment more volatile in DSGE models. This mechanism presumes that the equipment-skill ratio varies over the business cycle. In fact, Lindquist (2004) documents that this ratio is procyclical and lags output over the cycle in the data.

To check the short-run effect of complementarity for employment, I set up a benchmark model in which capital is divided into structures and equipment and production features equipment-skill complementarity. For comparison purposes, I also set up a traditional model and an extended model. The traditional model features

[1] Please check Figure 2.1.

a CES production function without complementarity while the extended one has an additional exogenous shock, namely a shock to investment-specific technology change. Comparing the traditional and benchmark models, we realize the contribution of complementarity. Moreover, Greenwood, Hercowitiz and Krusell (1997, henceforth GHK (1997)) and (2000) argue for the importance of investment-specific change at both long-run and short-run horizons. The extended model explores the combined effect of complementarity and investment shocks.

The paper is related to several lines of economic literature. First, this paper is connected with the literature on labor market volatility. Real business cycle theories have been criticized for failing to duplicate some significant facts regarding the labor market, such as the fluctuations of labor hours. Initially, RBC models relied on high degrees of labor supply elasticity to generate realistic labor market volatility. Indivisible labor, introduced by Rogerson (1984) and Hansen (1985), helps reconcile the low degree of the intertemporal elasticity implied by micro evidence with observed fluctuations in the labor market, as indivisible labor transforms an individual utility function with a low degree of intertemporal elasticity into a representative agent's utility with an infinite elasticity. In this paper, I examine if complementarity can improve on the ability of RBC models to match observed labor market volatility.

Second, a growing literature has documented and discussed secular changes in the labor market. In order to explain the behavior of the skill premium, KORV (2000) use a partial equilibrium approach and asks whether a model with complementarity and the given capital stock series can reproduce the observed annual fluctuations of the skill premium. By relying on the same partial equilibrium method, Castro and Coen-Pirani (2008) try to explain the increasing volatility of skilled employment since the mid-1980's. As explained earlier, my paper adopts a dynamic general equilibrium model, instead of partial equilibrium, to examine the implications of complementarity for labor quantity dynamics by skill. Lindquist (2004) constructs a dynamic general equilibrium model mainly to explain why the skill premium is acyclical over the business cycle. When constructing the model, Lindquist (2004) assumes that skilled workers and unskilled workers constitute one unit of measure, abstracting from the issue of unemployment. I abstract from skill accumulation for the sake of simplicity while I allow for voluntary unemployment within the household in the model.

I construct models with fluctuations in employment instead of total hours worked. In the data, the fluctuation of total hours worked comes from both employment and average hours per worker. However, the fluctuation of employment is responsible for more than 80% of that of total hours. Thus, it is natural to model the cyclical dynamics of employment and allow for the presence of unemployment.

The paper is organized as follows. Section 2 documents empirical evidence for variables of interest and presents the cyclical properties of the US economy. Section 3 constructs the benchmark model with complementarity that contains two types of capital, equipment and structures. Section 4 calibrates the model. Section 5 sets up two control models, which help identify the exact impact of complementarity. Section 6 summarizes the paper.

2.2 Empirical Observations

In this section, I present the business cycle dynamics of employment, output, consumption, and investment in the US data. I log these variables and apply the widely-used Hodrick-Prescott filter to extract the cyclical components. Volatilities are calculated based on these cyclical components. Labor market variables are computed using the CPS Merged Outgoing Rotation Groups (MORG) dataset from 1984q1 through 2006q4. Output, consumption and investment are seasonally adjusted quarterly data from NIPA accounts over the same period. I document these statistics as a standard to judge how well the model with capital-skill complementarity performs. The volatility ratios of employment to GDP and skilled to unskilled employment are the main indicators I use to assess the models' performance.

2.2.1 MORG dataset

The MORG dataset is compiled by the NBER from extracts of the CPS Annual Earnings file and contains monthly data on weekly hours and earnings. The original CPS outgoing rotation group data are recorded each month. The Bureau of Labor Statistics extracts about 25,000 records of those outgoing households per month and assigns each household an earnings weight such that aggregate statistics are representative of the U.S. population. I follow the BLS's suggestion and use the earnings weights to aggregate individual observations. I aggregate the original monthly data into a quarterly series of 92 observations for the variables of interest.

I apply standard sample selection criteria to handle missing observations and coding errors.[2] After applying these criteria, I get about 45,000 individuals per quarter, of which on average about 10,600 hold at least a college degree. I restrict attention to individuals in the labor force between 16 and 65 years of age that are not self-employed.

The variables constructed from the MORG data include employment, total hours in efficiency units, total usual hours, average hours per worker and average wage rates. These statistics are calculated for the entire labor force, and for the skilled and unskilled workers respectively. The MORG variables used to construct these statistics include employment status, class status,[3] usual weekly earnings (including overtime, tips and commissions), usual weekly hours worked, and a set of demographic variables including age, sex, race and education. Weekly earnings are top-coded in the CPS. I follow Castro and Coen-Pirani (2008) to adjust these top-coded earnings. They suggest that top-coded earnings be multiplied by 1.3 to ensure that average earnings in the top decile remain constant from December 1988 to January 1989. In addition, I follow Castro and Coen-Pirani (2008) and compute real weekly earnings by dividing nominal weekly earnings by the Consumer Price Index (CPI).

In order to compute skilled and unskilled aggregates, I follow the efficiency-units method suggested by Katz and Murphy (1992), KORV (2000) and Castro and Coen-Pirani (2008). A skilled worker is defined as one with at least a four-year college degree and an unskilled worker is an individual with less than 16 years

[2]These criteria are conventional. Please check Wu (2010) for details.

[3]Class status refers to whether the worker's job is private, government, or self-employed.

of education. To compute aggregate hours in a way that controls for composition effects, I use the real average wages as weights when aggregating the hours worked by individuals in 240 demographic groups.

Now I briefly summarize some variables of interest as follows:

Employment: Aggregate employment in any given quarter is the total number of individuals, weighted with their CPS earnings weight, who report being at work that quarter. For the sake of comparison, I calculate this statistic for total employment, skilled employment and unskilled employment. For the whole sample period, the average total employment rate is 94%, the average skilled employment rate[4] is 23% and the average unskilled employment rate[5] is 71%.

Hours Worked in Efficiency Units: I use the efficiency-units approach to compute the total hours worked. These statistics are used to calculate the volatility ratio of employment to hours. To aggregate workers in different groups, I construct time-invariant weights for each demographic group. I use the average wages for each group as weights to aggregate total hours worked across different subgroups as in Castro and Coen-Pirani (2008).

Total Usual Hours Worked: This is defined as the simple aggregation of hours worked across 240 subgroups without weights. This statistic is computed for reference.

[4]The skilled employment rate is defined as the ratio of skilled employment to the entire labor force.

[5]The unskilled employment rate is defined as the ratio of unskilled employment to the entire labor force.

Average Hours Worked: This is defined as total hours in efficiency units divided by employment.

Average Wage Rates: This is defined as total earnings divided by employment.

These variables constructed from the MORG dataset display strong seasonal components. Thus, before any further data analysis, I have to deseasonlize these series. I use three methods to adjust the raw series: regressing raw series on quarter dummies, the Census Bureau's X12-ARIMA and the combination of X12-ARIMA and centered five quarters MA[6] suggested by Castro and Coen-Pirani (2008). Then I apply the Hodrick-Prescott filter with parameter 1600 to these seasonally adjusted series.

Tables 2.1, 2.2 and 2.3 shows the standard deviations of these variables from these three different methods. Table 2.1 is used to compare with my model economies.

2.2.2 NIPA data

I take other time series from NIPA accounts. These variables include output, consumption and investment. I apply the Hodrick-Prescott filter to real quarterly logs of GDP, consumption and investment and calculate standard deviations for their variabilities. These moments are used to evaluate the model's performance. Following GHK (1997), I also construct logs of real GDP, consumption and investment in

[6]Castro and Coen-Pirani (2008) suggest applying a centered 5 quarters moving average filter to the seasonally adjusted series to remove high frequency variations. Comparing these computed aggregates, the results using quarter dummies and X12-ARIMA are similar while the results from the combined method of X12 and MA5 reduce the labor market fluctuations significantly.

consumption units. That is, I divide the nominal macro series by the price deflator of nondurable consumption goods and services. Then I apply the Hodrick-Prescott filter to get the associated cyclical statistics. I use this procedure to guarantee that the statistics in the data are consistent with their counterparts in the model with investment shocks.

Table 2.4 summarizes the key moments computed from the MORG and NIPA data.

2.3 The Benchmark Model

This section outlines a dynamic stochastic general equilibrium model. The model economy contains two types of economic agents: one representative household, consisting of a continuum of infinitely-lived individuals, and one representative firm, which uses labor and capital to maximize the expected discounted value of profit. The production function uses four types of factor inputs: capital structures, capital equipment, skilled and unskilled labor. The production function features capital-skill complementarity. Finally, the model economy includes one exogenous shock affecting total factor productivity. Time is discrete and is denoted by $t = 0, 1, 2, \cdots$.

2.3.1 The Household

There is a representative household that includes a continuum of members with measure 1. Individuals are divided into two types: skilled or unskilled.[7] In this paper, I focus on the extensive margin of the labor market. One reason for this is that more than three quarters of total hours fluctuations stem from the extensive margin. In addition, policymakers and the public pay more attention to the employment rate than to total hours worked. By concentrating on employment, the model can be regarded as an attempt to enrich general equilibrium models with unemployment. RBC models characterize agents as continuously adjusting their hours, while the indivisible labor hypothesis of Hansen (1985) highlights the extensive margin of the labor market. A variant of standard RBC models is proposed by Cho and Cooley (1994) who allow representative agents to choose both margins. In my model, the household maximizes the aggregate utility by optimally supplying labor and allocating consumption goods among household members. The utility level depends on consumption and dis-utility from employment. The aggregate utility function is the weighted sum of the utilities of skilled and unskilled labor. The household optimally equalizes the marginal utility of consumption within household members of the same type, and also equalizes the weighted marginal utility of consumption

[7] In the model economy, skilled and unskilled workers have different consumption because of the differences in their utility weights in the preference function. However, the same type of workers enjoy the same amount of consumption regardless of employment status. This kind of assumption seems realistic given that in the real economy consumption is different for workers of various education levels.

across the two types of individuals. The aggregate utility for the household is:

$$\sum_{t=0}^{\infty}\beta^t E_0\Big\{\lambda_s\Big[\frac{l_s(c_{st})^{1-\sigma_s}}{1-\sigma_s}-\frac{\chi_s(n_{st})^{1+\eta_s}}{1+\eta_s}\Big]+\lambda_u\Big[\frac{l_u(c_{ut})^{1-\sigma_u}}{1-\sigma_u}-\frac{\chi_u(n_{ut})^{1+\eta_u}}{1+\eta_u}\Big]\Big\} \quad (2.1)$$

where the subscript "s" is related to skilled labor, while "u" represents unskilled workers. β is the subjective discount factor; $\lambda_i, i \in \{s, u\}$ is the subjective weight that the household places on the utility of the two types; $\chi_i, i \in \{s, u\}$ governs the disutility of labor supply; l_i is the measure of each type of worker in the labor force, where $l_s + l_u = 1$; c_{it} is consumption for type-i workers at period t; n_{it} is the measure of employment for each type of worker; and u_{it} is the corresponding unemployment measure at period t, where $l_i = n_{it} + u_{it}$. I assume that the household is able to insure its members against all idiosyncratic shocks, mimicking the complete markets allocation. Because utility is separable in consumption and leisure, all members of a given type of labor enjoy the same consumption regardless of their employment status.

In the model, the household makes decisions regarding not only the supply of two kinds of labor, but also the accumulation of two kinds of capital. Why distinguish capital equipment from capital structures? The intuition is that capital equipment is more connected to technological change and thus more complementary with skilled employment than capital structures. In addition, these two types of capital have quite different accumulation patterns in the data. GHK (1997) demonstrate the important role of investment-specific technological change. Supporting evidence includes the continuously falling price of equipment and faster means of communication and transportation. Based on these observations, GHK (1997) divide capital into two categories. Capital equipment is subject to increasing efficiency in its production, while capital structures keep a roughly constant production efficiency over time. In an extension below, I add investment shocks into the benchmark model.

At the beginning of each period, an exogenous neutral technology shock is realized. Based on the technological state and capital stock, the household decides how many skilled and unskilled workers to supply. Given that the measure of the labor force in the model economy is unity,[8] the employment rate is equal to the sum of the measures of skilled and unskilled employees. Moreover, the household owns capital structures and equipment and rents them out to the firm. At the end of the period, household members receive wage payments and capital returns from the firm. As the firm owners, household members get corporate profits. At this stage, the household decides how much to consume for instantaneous utility and how much to invest for next period's capital stock. That is, they allocate their resources between consumption, structures and equipment investment. The aggregate budget constraint from the household's perspective can be expressed as:

$$0 = \pi_t + w_{st} n_{st} + w_{ut} n_{ut} + r_{st} k_{st} + r_{et} k_{et} - l_s c_{st} - l_u c_{ut} - i_{st} - i_{et} \tag{2.2}$$

where π_t is the profit received by the household at period t; w_{it} is the state-contingent wage rate for type-i workers in period t; r_{st} and r_{et} are the return rates for capital structures and equipment at period t, respectively; k_{st} and k_{et} are the stocks for

[8] I abstract from fluctuations in the labor participation rate. According to the MORG dataset, the mean participation rate over the sample period is around 60%. This rate is determined not only by economic factors, but also by other non-economic concerns, such as health. Therefore, macroeconomic models usually step away from modeling labor force participation.

capital structures and equipment at period t; and i_{st} and i_{et} are investment in capital structures and equipment at period t, respectively.

For capital structures and equipment, the laws of motion take the conventional form:[9]

$$0 = k_{s,t+1} - (1 - \delta_s)k_{st} - i_{st} \tag{2.3}$$

$$0 = k_{e,t+1} - (1 - \delta_e)k_{et} - i_{et} \tag{2.4}$$

where δ_s and δ_e are depreciation rates for capital structures and equipment.

2.3.2 The Firm

The firm in the model uses a production technology with capital-skill complementarity. The firm rents capital structures and equipment and hires two types of workers from the household. At the end of the period, the firm pays wages and capital rents. The input prices are determined in perfectly competitive factor markets. I borrow the production function from KORV (2000):

$$y_t = e^{z_t} k_{st}^{\alpha_{ks}} \left\{ (a_u n_{ut})^{\theta_1} + \left[k_{et}^{\theta_2} + (a_s n_{st})^{\theta_2} \right]^{\frac{\theta_1}{\theta_2}} \right\}^{\frac{1-\alpha_{ks}}{\theta_1}} \tag{2.5}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{zt}, \qquad \epsilon_{zt} \sim N(0, \sigma_z^2) \tag{2.6}$$

where z_t is the shock to general TFP; and a_i is the type-specific weight in production. KORV (2000) uses the above function to capture the idea that capital equipment is more complementary to skilled employees than unskilled employees.

[9] There is no capital adjustment cost in the model. One reason is that calibrating the adjustment cost parameters is problematic in this type of model, as discussed in GHH (2000).

Technically, this requires that the substitution elasticity between unskilled employment and the skill-equipment composite, $\frac{1}{1-\theta_1}$, is larger than that between skilled employment and equipment, $\frac{1}{1-\theta_2}$. In turn, this requires $\theta_1 > \theta_2$. Implicitly, this production specification assumes that the substitution elasticity between unskilled workers and capital is identical to that between unskilled and skilled workers, which is supported by the evidence in Johnson (1997). Capital equipment and structures have asymmetric positions in production because they have different substitution possibilities with labor and are not perfect substitutes for each other.

Capital-skill complementarity is also reflected in the marginal product ratio of skilled to unskilled labor. Specifically, when $\theta_1 > \theta_2$, the relative marginal product of skilled workers is an increasing function of capital equipment. The more capital is accumulated, the more productive skilled workers are relative to unskilled on the margin. The marginal product ratio of skilled and unskilled labor is:

$$\frac{MPL_s}{MPL_u} = \big(\frac{a_s}{a_u}\big)^{\theta_1}\big(\frac{n_{ut}}{n_{st}}\big)^{1-\theta_1}\big[\big(\frac{k_{et}}{a_s n_{st}}\big)^{\theta_2} + 1\big]^{\frac{\theta_1-\theta_2}{\theta_2}} \tag{2.7}$$

where $\frac{a_s}{a_u}$ is referred to as the relative efficiency effect, $\frac{n_{ut}}{n_{st}}$ is the relative quantity effect, and $\frac{k_{et}}{a_s n_{st}}$ is the capital-skill complementarity effect.

The factor input markets are perfectly competitive. The rents of factor inputs are determined solely by their marginal contribution to production. Firms, as price takers, maximize the current profit:[10]

$$\pi_t = \max_{\substack{\{n_{st}, n_{ut},\\ k_{st}, k_{et}\}}} \big\{ y_t - w_{st} n_{st} - w_{ut} n_{ut} - r_{st} k_{st} - r_{et} k_{et} \big\} \tag{2.8}$$

[10] Note that perfect competition in factor markets and constant returns to scale jointly guarantee that the profit is zero in equilibrium.

$$\text{s.t.} \quad y_t = e^{z_t} k_{st}^{\alpha_{ks}} \left\{ (a_u n_{ut})^{\theta_1} + \left[k_{et}^{\theta_2} + (a_s n_{st})^{\theta_2} \right]^{\frac{\theta_1}{\theta_2}} \right\}^{\frac{1-\alpha_{ks}}{\theta_1}} \tag{2.9}$$

2.3.3 Competitive Equilibrium

The competitive equilibrium is now formulated. This equilibrium is the combined outcome of the household's utility-maximizing and the firm's profit-optimizing problems. The dynamic programming problem facing the representative household is:

$$V(z_t) = \max_{\substack{\{c_{st}, c_{ut}, \\ k_{s,t+1}, k_{e,t+1}\}}} \left\{ \lambda_s \left[\frac{l_s (c_{st})^{1-\sigma_s}}{1-\sigma_s} - \frac{\chi_s (n_{st})^{1+\eta_s}}{1+\eta_s} \right] + \lambda_u \left[\frac{l_u (c_{ut})^{1-\sigma_u}}{1-\sigma_u} - \frac{\chi_u (n_{ut})^{1+\eta_u}}{1+\eta_u} \right] + \beta E_t V(t+1|t) \right\} \tag{P.1}$$

$$\text{s.t.} \quad 0 = \pi_t + w_{st} n_{st} + w_{ut} n_{ut} + r_{st} k_{st} + r_{et} k_{et} - l_s c_{st} - l_u c_{ut} - i_{st} - i_{et} \tag{2.10}$$

$$0 = k_{s,t+1} - (1-\delta_s) k_{st} - i_{st} \tag{2.11}$$

$$0 = k_{e,t+1} - (1-\delta_e) k_{et} - i_{et} \tag{2.12}$$

The maximization problem of the firm is:

$$\pi_t = \max_{\substack{\{n_{st}, n_{ut}, \\ k_{st}, k_{et}\}}} \left\{ y_t - w_{st} n_{st} - w_{ut} n_{ut} - r_{st} k_{st} - r_{et} k_{et} \right\} \tag{P.2}$$

$$\text{s.t.} \quad y_t = e^{z_t} k_{st}^{\alpha_{ks}} \left\{ (a_u n_{ut})^{\theta_1} + \left[k_{et}^{\theta_2} + (a_s n_{st})^{\theta_2} \right]^{\frac{\theta_1}{\theta_2}} \right\}^{\frac{1-\alpha_{ks}}{\theta_1}} \tag{2.13}$$

The aggregate state of the model economy is $SS_t = \{k_{st}, k_{et}, z_t\}$ for period t. Assume that the equilibrium wages and rents are functions of the state of the economy: $w_{st} = W_s(SS_t)$, $w_{ut} = W_u(SS_t)$, $r_{st} = R_s(SS_t)$, $r_{et} = R_e(SS_t)$. Suppose that capital structures and equipment evolve according to $k_{s,t+1} = K_s(SS_t)$, $k_{e,t+1} = K_u(SS_t)$.

A competitive equilibrium is defined based on the above assumptions. Specifically, equilibrium is a set of allocation rules, prices and transfer functions and an aggregate law of motion for the state variables that satisfy:

1. The household maximizes (P.1), taking as given the aggregate state SS_t and the pricing and transfer functions, W_s, W_u, R_s and R_u. The household's solution is:

   $c_{st} = C_s(SS_t)$, $c_{ut} = C_u(SS_t)$, $n_{st} = N_s(SS_t)$, $n_{ut} = N_u(SS_t)$, $k_{st+1} = K_s(SS_t)$,

   $k_{et+1} = K_u(SS_t)$;

2. The firm solves (P.2), given SS_t, W_s, W_u, R_s and R_u. Its solution can be characterized as: $\tilde{n}_{st} = \tilde{N}_s(SS_t)$, $\tilde{n}_{ut} = \tilde{N}_u(SS_t)$, $\tilde{k}_{st} = \tilde{K}_s(SS_t)$, $\tilde{k}_{et} = \tilde{K}_e(SS_t)$;

3. The economy-wide resource constraint is satisfied:

   $$e^{z_t} k_{st}^{\alpha_{ks}} \left\{ (a_u n_{ut})^{\theta_1} + \left[k_{et}^{\theta_2} + (a_s n_{st})^{\theta_2} \right]^{\frac{\theta_1}{\theta_2}} \right\}^{\frac{1-\alpha_{ks}}{\theta_1}} + k_{st}(1-\delta_s) + k_{et}(1-\delta_e)$$

   $$= k_{s,t+1} + k_{e,t+1} + l_s c_{st} + l_u c_{ut};$$

4. All markets clear: $\tilde{n}_{st} = n_{st}, \quad \tilde{n}_{ut} = n_{ut}, \quad \tilde{k}_{st} = k_{st}, \quad \tilde{k}_{et} = k_{et}$.

2.4 Calibration

Before solving the competitive equilibrium numerically, it is necessary to assign values to structural parameters. In this section, I follow conventional procedures.

First, some structural parameters are assigned values according to widely accepted RBC literature. Then given these borrowed parameters, all other parameters are set in such a way that the model matches certain stylized facts in the U.S. data. These unknown parameters govern the utility function, production function and the exogenous processes.

2.4.1 Parameters in the Utility Function: $\beta, \sigma_s, \sigma_u, \eta_s, \eta_u, \lambda_s, \lambda_u, \chi_s, \chi_u,$ l_s, l_u

The subjective discount factor, β, is the inverse of the time-discount rate, which in equilibrium equals the steady state return on capital. In standard RBC models, it is conventional to assume that the steady state annual rate of return on capital is about 5 percent. Thus, the corresponding quarterly subjective discount factor, β, is equal to 0.987.

The coefficient of relative risk-aversion, $\sigma_i, i \in \{s, u\}$, governs the curvature of utility with respect to consumption. Estimates of σ display a certain degree of divergence in the empirical literature. For example, to make their model replicate the impulse responses implied by a VAR estimated using U.S. data, Rotemberg and Woodford (1997) set σ to 0.16. Hall (1988) and Attanasio and Weber (1993) estimate σ to be around 3, using aggregate consumption data. Fuhrer (2000) finds σ to be about 6, assuming habit formation in consumption. All these estimates regard individuals as identical while my model has divided labor into two subgroups. To follow the RBC convention, I set $\sigma_s = \sigma_u = 1$ so that the consumption part of the utility function takes the log form.

η_s and η_u are the marginal disutilities for skilled and unskilled labor. If I assume that each type of worker works a fixed number of hours per period, then η_s and η_u also govern the Frisch elasticities for the household of skilled and unskilled hours. Traditional RBC models take the log-log utility function or $\frac{\left(c^{\alpha}(1-h)^{1-\alpha}\right)^{1-\eta}-1}{1-\eta}$, use total hours worked as the measure of labor input, and assume that the steady state value of labor supply is one third of non-sleeping time. This implies that the Frisch elasticity is 2.[11] However, empirical estimates of the Frisch elasticity are quite different. Studies such as MaCurdy (1981), Altonji (1986) and Browning, Deaton and Irish (1985) find elasticities between 0 and 0.5. More recently, Browning, Hansen and Heckman (1999) report estimates ranging from 0.5 to 1.6. Imai and Keane (2004) provide a relatively high estimate of 3.8. In the benchmark version of my model, I set the elasticity to 0.5, implying $\eta_s = \eta_u = 2$, to make sure that the marginal disutilities of skilled and unskilled employment fall within a reasonable range. Again, I abstract from differences in this elasticity between the two types of workers.

λ_s and λ_u are subjective weights of skilled and unskilled workers in the household utility. I set λ_s and λ_u to make sure that the steady state consumption ratio between skilled and unskilled labor is consistent with its empirical counterpart.[12]

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[11]The Frisch elasticity is defined as $\left.\frac{\mathrm{d}log(h)}{\mathrm{d}log(w)}\right|_c$, where h is labor hours, w is wage rate, and c is consumption. For log-log utility form or $\frac{\left(c^{\alpha}(1-h)^{1-\alpha}\right)^{1-\eta}-1}{1-\eta}$, the Frisch elasticity is $\frac{1-h}{h}$. If we assume that $h = 0.33$, then the elasticity is 2.

[12]In the model economy, the social planner guarantees only that workers of the same type have the same amount of consumption.

The Consumer Expenditure Survey (CES) dataset collects "total average annual expenditure" by education level since 1996. I find the mean value of the consumption ratio to be 1.67.

χ_s and χ_u are free parameters which are used to guarantee that the steady state levels of skilled and unskilled employment are consistent with the long-run means implied by MORG data. Finally, l_s and l_u are the shares of skilled and unskilled workers in the labor force, which are estimated directly from MORG statistics.

2.4.2 Parameters for the Production Sector: $\alpha_{ks}, \theta_1, \theta_2, a_s, a_u, \delta_s, \delta_e$

α_{ks} is capital structures' share of output. GHK (2000) set α_{ks} to 0.13 while assuming that the overall capital share of output is 0.35. I follow their practice and choose the same value.

θ_1 is directly related to the substitution elasticity between unskilled employment and the capital-skill composite. The empirical literature offers a wide range of estimates. Based on different specifications of functional form, estimation technique and data, estimates of the substitution elasticity range from 0.5 to 3, which implies θ_1 between -1 and 0.667. A good summary is provided by Hamermesh (1993). I take the value from KORV (2000), $\theta_1 = 0.401$.

θ_2 is associated with the substitution elasticity between skilled employment and capital. Empirical work finds that this parameter is generally less than 1.2. To keep my production function as standard as possible, I borrow its value from KORV (2000). That is, $\theta_2 = -0.491$.

a_s and a_u are type-specific production weights. The two types of workers are neither perfect substitutes nor equally productive in production. Hence, a_s and a_u are set to match the observed ratios of employment in efficiency units to employment for both types of workers. For the whole sample period, the production weight for skilled workers is 12.58 while that for unskilled workers 7.28. Here, I normalize a_u to be 1 and set a_s to 12.58/7.28.

δ_s and δ_e are the depreciation rates for capital structures and equipment. GHK (2000) set the annual depreciation rates of capital structures and equipment to 0.056 and 0.124. The corresponding quarterly depreciation rates are 0.014 and 0.031, respectively. Therefore, I set $\delta_s = 0.014$ and $\delta_e = 0.031$.

2.4.3 Parameters for the Exogenous Process: ρ_z, σ_z

I follow the standard RBC literature to assume that the neutral technology shock is an AR(1) process. Specifically, $z_t = \rho_z z_{t-1} + \epsilon_{zt}$, $\epsilon_{zt} \sim N(0, \sigma_z^2)$. The associated parameter values are borrowed from Cooley and Hansen (1995).

2.4.4 Parameters for Control Models: $\alpha_k, \theta_3, \delta, \rho_q, \sigma_q$

Given that the purpose of this paper is to explore the short-run implications of capital-skill complementarity, it is necessary to set up and examine control models for comparison purposes. This paper has two control models, namely the traditional and extended models.[13] In the traditional model, I allow two types of labor inputs, namely skilled and unskilled workers, as in my benchmark model. However, there

[13] Table 2.5 lists the features of these three models.

is no complementarity and only one type of capital in production. According to Autor, Katz and Krueger (1998), the elasticity of substitution between skilled and unskilled workers falls into the interval between 1 and 2. Katz and Murphy (1992) estimate the elasticity as 1.4. In the traditional model, I use a CES function to combine skilled and unskilled labor inputs and set the elasticity of substitution to 1.4. Specifically, the production function is;

$$y_t = e^{z_t} k_t^{\alpha_k} \left[(a_u n_u)^{\theta_3} + (a_s n_s)^{\theta_3} \right]^{\frac{1-\alpha_k}{\theta_3}} \tag{2.14}$$

The output share of capital, α_k, is set to 0.35, and the monthly depreciation rate, δ is set to 0.025, according to classic RBC literature.

The extended version of the model includes shocks to investment-specific technology. One reason to do this is that GHK (1997) document that investment shocks have significant impacts both on long-run growth and on the business cycle. Fisher (2006) and Justinian et al (2009) confirm the importance of investment shocks in high-frequency cycles. The other reason is that the investment shock is highly associated with capital equipment, which also plays a key role distinct from structures in my model with capital-skill complementarity. The extended model is identical to the benchmark model except for the law of motion of capital equipment:

$$0 = k_{et+1} - (1 - \delta_e) k_{et} - e^{q_t} i_t \tag{2.15}$$

$$\text{where} \qquad q_t = \rho_q q_{t-1} + \epsilon_{qt}, \quad \epsilon_{qt} \sim N(0, \sigma_q^2) \tag{2.16}$$

q_t is also assumed to be independent of the neutral technology shocks. I borrow the associated parameter values from GHH (2000) and convert them into monthly frequency.

Table 2.6 summarizes parameter values for the benchmark and extended models while Table 2.7 presents calibration results for the traditional model.

2.5 Numerical Results

The cyclical behavior of the benchmark model is evaluated and discussed in this section. For comparison purposes, I also set up a traditional version of the DSGE model and an extended model with investment shocks. The main conclusions are listed below. First, capital-skill complementarity helps the model explain the observed volatility ratio of skilled to unskilled employment; Second, this complementarity does not improve upon the DSGE model's performance on overall employment volatility; Third, the investment shock is dominated by the neutral shock in such a way that their joint effect on the model economy is not significantly different from the case of having only the neutral shock.

I solve my DSGE models numerically. Given that the purpose of this paper is not to compare the welfare properties of different model economies, I apply the first-order perturbation method. To obtain the summary statistics of interest from the model, I simulate the model repeatedly to get 300 samples of artificial time series. For each sample, I keep the same number of periods as the corresponding time length in the data. After the raw series are generated, I transform them back to levels, extract the cyclical components, and calculate the corresponding standard deviations.

2.5.1 Volatility Ratio of Skilled and Unskilled Employment

As Tables 2.8 and 2.9 show, the benchmark model generates skilled employment that is more volatile than unskilled employment, while the traditional model implies that both have the same volatility.[14] Capital-skill complementarity contributes much to this result. Intuitively speaking, complementarity puts the two types of labor in asymmetric positions in the production function. Remember that the relative marginal product of skilled employment is identical to the relative demand of skilled labor, and complementarity adds the capital-skill ratio, $\frac{k_{et}}{a_s n_{st}}$, into the demand function. The impact of capital equipment on the relative demand, via the capital-skill-ratio mechanism, makes it possible for skilled employment to fluctuate more than unskilled employment. When there is a positive technology shock, equipment investment drives up the stock of capital equipment, which shifts upward the relative demand curve for skilled labor as Figure 2.1 shows. Keeping other factors constant, it is clear that skilled employment has a larger percentage change than unskilled labor under complementarity. Lindquist (2004) documents that the capital-skill ratio has a standard deviation 1.3 times that of output, is weakly procyclical and lags output over the cycle.[15] On the contrary, the traditional model can not produce this result due to its assumptions on the production function. The relative marginal product in the traditional model is a function only of the two

[14]The first row of Table 2.9 shows that complementarity causes the volatility ratio between skilled and unskilled employment to be larger than 1 while the traditional model makes the ratio to be 1.

[15]Please check Tables 1 and 2 in Lindquist (2004).

types of labor inputs. In turn, this implies that the employment ratio of skilled and unskilled is constant. It follows that both types have the same fluctuation in the traditional model.

In the benchmark model, the relative volatility is not sensitive to the degree of complementarity. This is contrary to what Castro and Coen-Pirani (2008) advocate. They argue that changes in the degree of complementarity in the US in recent years explain a large part of the recent volatility change in skilled hours. They estimate θ_2 to increase from -2.2 to -0.4 at the 1979-2003 period. I simulate the benchmark model for both values and record the results in Table 2.10. Castro and Coen-Pirani (2008) use a partial equilibrium framework to study relative labor demand by skill. By making the relative marginal product consistent with the long-run behavior of the skill premium, they pin down the long run change in the substitution elasticity. They then use the implied relative demand curve to back out a series of skilled hours, and explore this series' cyclical properties. However, in the DSGE framework, the volatility ratio is closely related to the cyclical movement of $\frac{k_{et}}{a_s n_{st}}$. When the capital-skill ratio increases in response to a positive shock, the relative demand for skilled labor increases, and the volatility of skilled employment becomes larger than that of unskilled employment. This is also why the benchmark model has a hard time generating a volatility ratio less than one. To produce a volatility ratio below one in the model with complementarity, the capital-skill ratio must be countercyclical. One way to accomplish this is to set $\eta_s = \eta_u = 0$, which makes the labor supply elasticity of both types infinite, dramatically increasing employment variability. In this case, then the economy has a positive shock, skilled employment immediately jumps up a lot, which dominates the movement of the capital-skill ratio and pulls down the relative demand curve. As a result, the benchmark model has skilled employment less volatile than unskilled employment. However, please note that the correlation between the capital-skill ratio and output becomes -0.391, which is contradictory to empirical evidence.

The impulse responses are illustrative in understanding the mechanics of the model economy. Figure 2.2 presents the responses of output, consumption, employment and investment to a neutral technology shock of one standard deviation. With a favorable neutral shock, all variables jump up. Following a positive response of equipment investment, the resulting increase of the equipment stock drives up the relative demand for skilled employment. One interesting fact is that the percentage increase of equipment investment is higher than that of structures investment. The main reason is their asymmetric positions in the production function. If k_s and k_e have symmetric positions in the production function, such as $y = e^z k_s^{\tilde{\alpha}} k_e^{\tilde{\alpha}} F(n_s, n_u)$, and have the same depreciation rates, these two types of capitals have the same investment pattern. Another determinant is the depreciation rate. Keeping other factors constant, investment in a given type of capital is lower when it has a high depreciation rate than when it has a low depreciation rate.

2.5.2 Volatility Ratio of Employment to Output

Capital-skill complementarity does not improve the ability of the DSGE model to explain the observed volatility ratio of employment to output compared with the traditional RBC model, as displayed by the second row of Table 2.9. One clever way to overcome this weakness is proposed by Rogerson (1984) and Hansen (1985). Although complementarity has a direct impact on the volatility ratio of skilled to unskilled labor, its influence on the volatility ratio of employment to output is ambiguous. Consider Figure 2.1. When a positive shock hits the economy, the increased equipment stock shifts up the relative demand curve, resulting in a higher ratio of skilled to unskilled employment. Given that skilled workers are more efficient and require higher wages, firms who hire more skilled workers choose to employ fewer unskilled workers. Thus, unskilled workers have a smaller percentage increase, which decreases the volatility of unskilled labor and consequently pulls down the volatility for total employment relative to output.

2.5.3 Investment-Specific Shocks

Investment-specific shocks have been identified by GHH (1997) and others as an important source of economic fluctuations. I examine the impact of investment shocks in the extended model and find that they can increase the volatility of skilled employment by 36% when co-existing with neutral technology shocks. Tables 2.11 and 2.12 record these results. Intuitively, when a positive investment shock appears, a given amount of output can be transformed to a larger amount of capital equipment. Firms have incentives to extend production and invest more in capital equipment. Thus, firms want to hire more workers. Since complementarity implies that equipment is favorably biased towards skill, it follows that skilled employment experiences a larger percentage change. Table 2.12 shows that when the economy has only investment shocks, the volatility ratio of skilled to unskilled employment is 7.5. Meanwhile, this model generates only about 15% of the observed output volatility. Adding investment shocks to neutral technology shocks causes the volatility ratio between skilled and unskilled labor to jump by 36%. However, overall output and employment volatility in the extended model is only slightly higher than in the benchmark model.

2.6 Conclusion

This paper examines the short-run implications of capital-skill complementarity for the labor market's volatility. Within the DSGE framework, complementarity can make skilled employment more volatile than unskilled employment. However, complementarity does not increase the volatility of total employment. In addition, investment specific shocks increase the relative volatility of skilled employment but not overall volatility.

Table 1.1: Volatilities of Employment, Hours and GDP

	GDP	Employment	**Skilled** Employment	**Unskilled** Employment	Hours	**Skilled** Hours	**Unskilled** Hours
1979-2003	0.0132	0.0081	0.0068	0.0092	0.0091	0.0085	0.0108
1979-1983	0.0218	0.0126	0.0051 (0.234)	0.0153 (0.702)	0.0143	0.0072	0.0183
1984-2003	0.0097	0.0065	0.0072 (0.742)	0.0067 (0.691)	0.0071	0.0088	0.0077

Notes:

1. Data on employment and hours are from MORG of CPS, and GDP is from NIPA.
2. All data are first converted to quarterly series，then are logged and detrended using the H-P filter with a smoothing parameter of 1600.
3. I use the Census Bureau's X12 program to remove seasonality from each series. Given that the CPS data contains high-frequency measurement errors, I also apply a centered 5-quarter moving average to the seasonally adjusted series from MORG.
4. Variables related to hours refer to aggregate hours in efficiency units.
5. Values in parentheses are the volatility ratios of the associated variables relative to GDP.

Table 1.2: Volatility Comparison between Employment and Hours

	Period of Time	Employment	Hours	Ratio
All Workers	1979-2003	0.0081	0.0091	0.89
	1979-1983	0.0126	0.0143	0.88
	1984-2003	0.0065	0.0071	0.92
Skilled Workers	1979-2003	0.0068	0.0085	0.80
	1979-1983	0.0051	0.0072	0.71
	1984-2003	0.0072	0.0088	0.82
Unskilled Workers	1979-2003	0.0092	0.0108	0.85
	1979-1983	0.0153	0.0183	0.84
	1984-2003	0.0067	0.0077	0.87

Notes:

1. Data on employment and hours are from MORG of CPS, and GDP is from NIPA.
2. All data are first converted to quarterly series, then are logged and detrended using the H-P filter with a smoothing parameter of 1600.
3. I use the Census Bureau's X12 program to remove seasonality from each series. Given that the CPS data contain high-frequency measurement errors, I also apply a centered 5-quarter moving average to the seasonally adjusted series from MORG.
4. Variables related to hours refer to the total hours in efficiency units.
5. The column labeled "Ratio" refers to the volatility ratio of "Employment" to "Hours".

Table 1.3: Calibration Results for Simulation I

Subjective discount factor	β	0.996
CRRA: skilled	σ_s	1
CRRA: unskilled	σ_u	1
Coefficient governing employment disutility: skilled	η_s	2
Coefficient governing employment disutility: unskilled	η_u	2
Utility weight: skilled	λ_s	0.626
Utility weight: unskilled	λ_u	0.374
Labor force share: skilled	l_s	0.2295
Labor force share: unskilled	l_u	0.7702
Capital share of output	α	0.35
Coefficient governing elasticity of substitution	σ	0.29
Depreciation rate of capital	δ	0.0055
Production weight: skilled	a_s	12.58/7.28
Production weight: unskilled	a_u	1
Vacancy-posting cost: skilled	f_s	--
Vacancy-posting cost: unskilled	f_u	--
Separation rate: skilled	χ_s	0.0108
Separation rate: unskilled	χ_u	0.0201
Bargaining power: skilled	ϕ_s	0.5
Bargaining power: unskilled	ϕ_u	0.5
Matching efficiency: skilled	α_s	--
Matching efficiency: unskilled	α_u	0.35
Autocorrelation of productivity shocks	ρ_z	0.98
Standard deviation of productivity shocks	σ_z	0.005

Notes: α_s varies between 0.25 and 0.65; f_s and f_u are set to guarantee that the share of each type of labor in total employment is consistent with the data.

Table 1.4: Summary Statistics of Labor Market for 1979-1983, 1984-2003 and 1979-2003

	1979-1983	1984-2003	1979-2003
Skilled share of labor force	0.1840	0.2389	0.2295
Ratio of skilled employment to labor force	0.1780	0.2325	0.2231
Ratio of skilled unemployment to labor force	0.0060	0.0064	0.0063
Unskilled share of labor force	0.8157	0.7608	0.7702
Ratio of unskilled employment to labor force	0.7434	0.7040	0.7158
Ratio of unskilled unemployment to labor force	0.0723	0.0507	0.0544
Wage ratio of skilled to unskilled labor	1.66	1.87	1.82

Table 1.5: Calibration Results for Simulation II

		First Period (1979-1983)	Second Period (1984-2003)
Subjective discount factor	β	0.996	0.996
CRRA: skilled	σ_s	1	1
CRRA: unskilled	σ_u	1	1
Coefficient governing employment disutility: skilled	η_s	2	2
Coefficient governing employment disutility: unskilled	η_u	2	2
Utility weight: skilled	λ_s	0.808	0.873
Utility weight: unskilled	λ_u	0.191	0.127
Labor force share: skilled	l_s	0.1840	0.2389
Labor force share: unskilled	l_u	0.8157	0.7608
Capital share of output	α	0.35	0.35
Coefficient governing elasticity of substitution	σ	0.29	0.29
Depreciation rate of capital	δ	0.0055	0.0055
Production weight: skilled	a_s	12.58/7.28	12.58/7.28
Production weight: unskilled	a_u	1	1
Vacancy-posting cost: skilled	f_s	6.25	17.17
Vacancy-posting cost: unskilled	f_u	2.23	4.08
Separation rate: skilled	χ_s	0.0198	0.0198
Separation rate: unskilled	χ_u	0.0369	0.0369
Bargaining power: skilled	ϕ_s	0.5	0.5
Bargaining power: unskilled	ϕ_u	0.5	0.5
Matching efficiency: skilled	α_s	0.72	0.94
Matching efficiency: unskilled	α_u	0.48	0.61
Autocorrelation of productivity shocks	ρ_z	0.98	0.98
Standard deviation of productivity shocks	σ_z	0.005	0.005

Notes: Five parameters are re-calibrated at second stage, corresponding to the period of 1984-2003.

Table 1.6: Volatility Results of Simulation II

	First Period (1979-1983)	Second Period (1984-2003)
Output	0.030928	0.030954
Consumption	0.021462	0.021581
Skilled Employment	0.000061 (0.0020)	0.000093 (0.0031)
Unskilled Employment	0.000186 (0.0061)	0.000191 (0.0062)
Vacancies for the **skilled**	0.007598	0.005885
Vacancies for the **unskilled**	0.007795	0.007065
Skilled Unemployment	0.001797	0.003068
Unskilled Unemployment	0.001921	0.002357
Job-finding rate for the **skilled**	0.001865	0.003086
Job-finding rate for the **unskilled**	0.002150	0.002566
Overall Job-finding rate	0.002120	0.002626

Note: Values in parentheses are the volatility ratios of variables of interest relative to output.

Table 1.7: Volatility Comparison between the Data and Simulation II

Relative to output Volatility	First Period (1979-1983)	Second Period (1984-2003)	Percentage Change
Skilled Employment	Model: 0.0020 Data: 0.234	Model: 0.0031 Data: 0.742	Model: 50% Data: 217%
Unskilled Employment	Model: 0.0061 Data: 0.702	Model: 0.0062 Data: 0.691	Model: 2% Data: -2%

Notes: Relative volatilities refer to the volatility ratios between variables of interest to output or GDP; the last column labeled “Percentage Change” is the change of the relative volatilities between these two periods.

Figure 1.1: Rolling standard deviation (40 quarters ahead) of real GDP, Unskilled and Skilled Hours

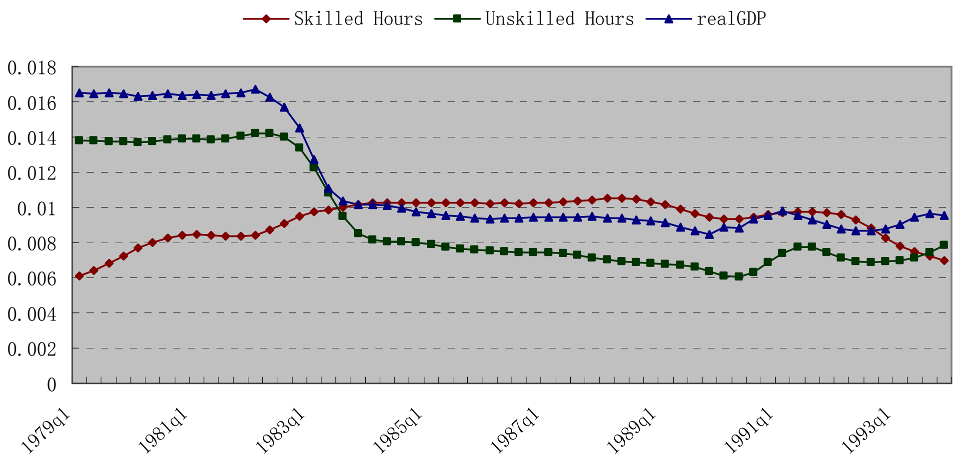


Figure 1.2: Rolling standard deviation (40 quarters ahead) of real GDP, Unskilled and Skilled Employment

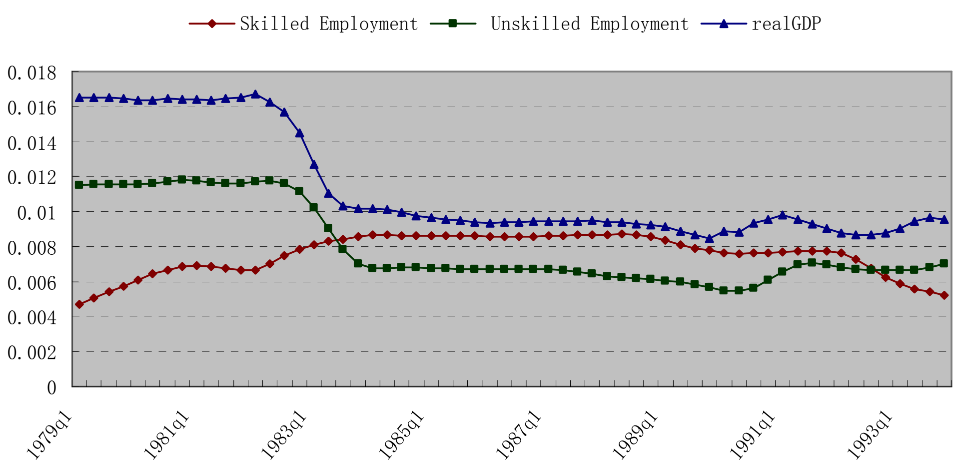


Figure 1.3: Ratio of rolling standard deviations (40 quarters ahead) of Unskilled and Skilled Employment to real GDP

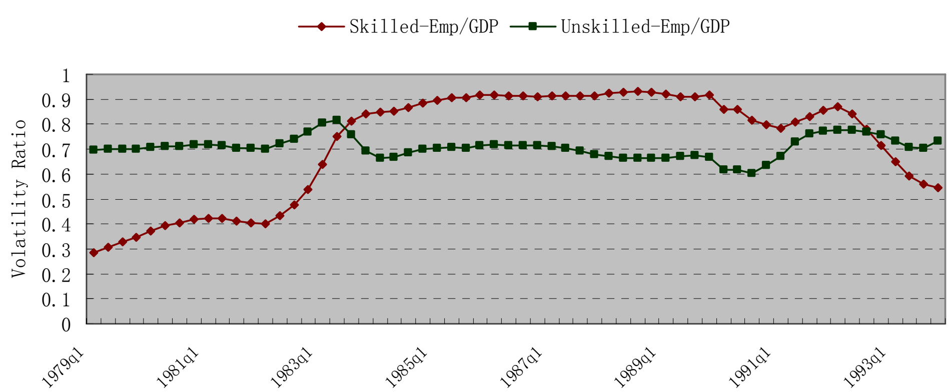


Figure 1.4: Volatility Ratios from Simulation I

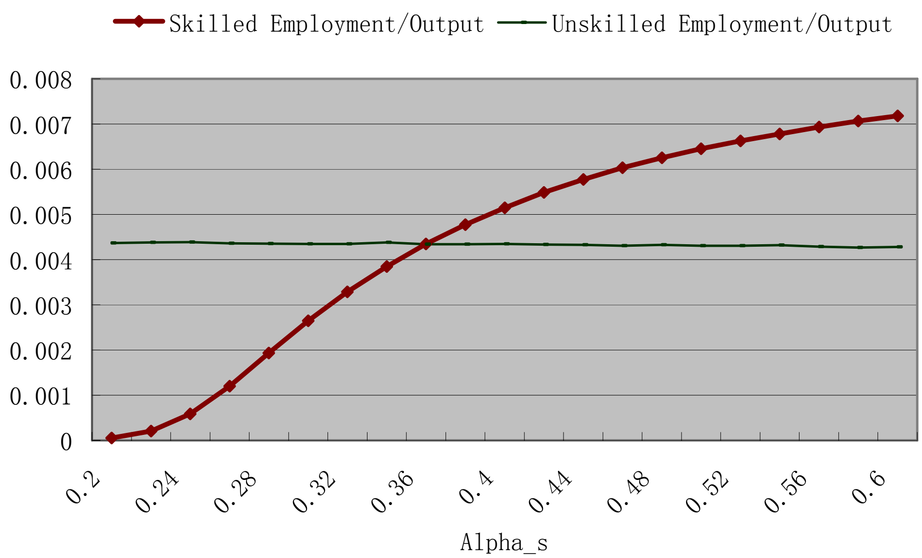


Figure 1.5: Skill Premium from Simulation I

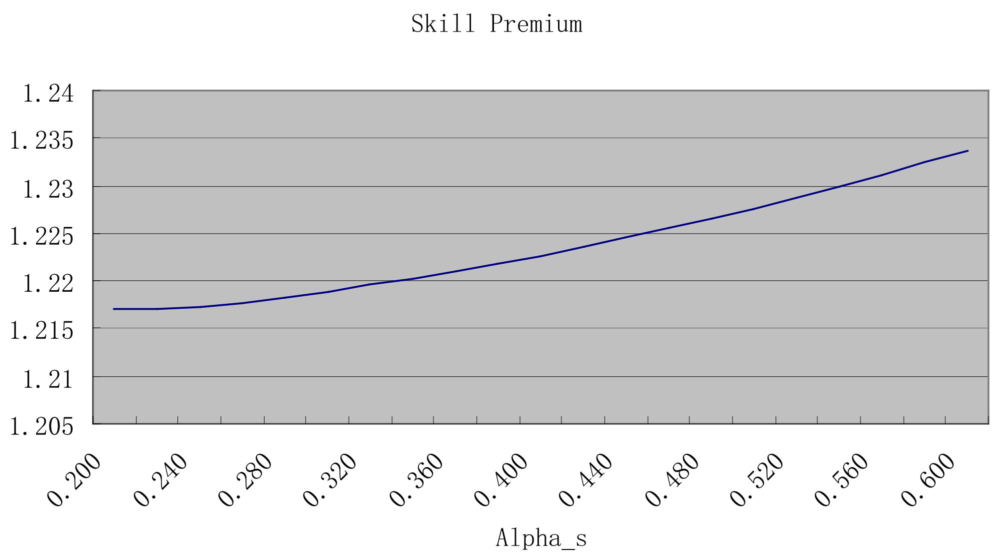


Figure 1.6: Impulse Responses from Simulation II

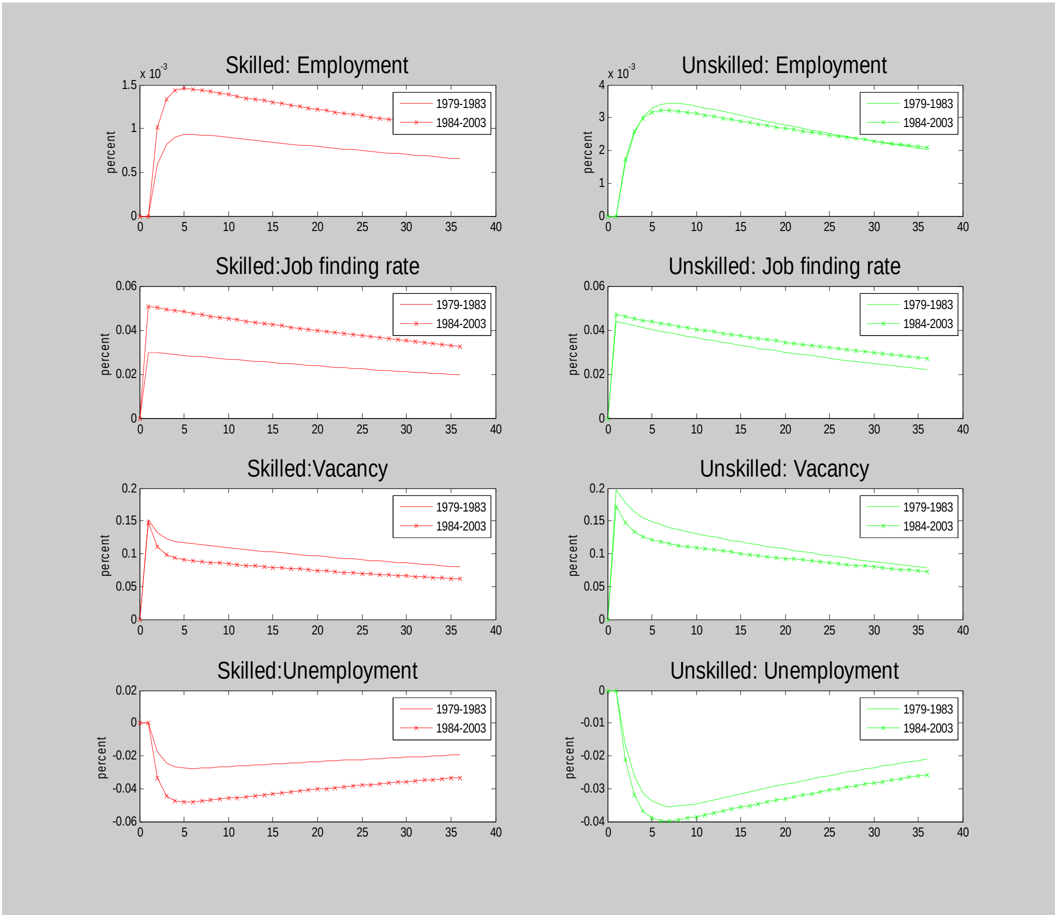


Table 2.1: Volatility of Employment, Hours and Wages: Seasonal Dummies

	Employment	Total Hours in Efficiency units	Total Usual Hours	Weekly Hours per Worker	Wage Rates per worker
All Workers	0.0077	0.0085	0.0099	0.0038	0.0087
Skilled labor	0.0108	0.0122	0.0119	0.0047	0.0128
Unskilled labor	0.0092	0.0105	0.0111	0.0034	0.0075

Note: The values in this table are computed using HP-filtered data that has been seasonally adjusted by regressing raw data on four quarterly dummies.

Table 2.2: Volatility of Employment, Hours and Wages: X12-ARIMA

	Employment	Total Hours in Efficiency units	Total Usual Hours	Weekly Hours per Worker	Wage Rates per worker
All Workers	0.0075	0.0085	0.0095	0.0035	0.0084
Skilled labor	0.0102	0.0114	0.0112	0.0043	0.0121
Unskilled labor	0.0085	0.0097	0.0103	0.0032	0.0072

Note: The values in this table are computed using HP-filtered data that has been seasonally adjusted by using the X-12 ARIMA method.

Table 2.3: Volatility of Employment, Hours and Wages: Castro and Coen-Pirani (2008)

	Employment	Total Hours in Efficiency units	Total Usual Hours	Weekly Hours per Worker	Wage Rates per worker
All Workers	0.0066	0.0071	0.0083	0.0016	0.0066
Skilled labor	0.0067	0.0082	0.0078	0.0021	0.0092
Unskilled labor	0.0070	0.0081	0.0089	0.0021	0.0053

Note: The values in this table are computed using HP-filtered data that has been seasonally adjusted by following Castro and Coen-Pirani.

Table 2.4: Volatility and Cross-Correlation: Data

	Volatility	Cross-correlation	Volatility (2)	Cross-correlation (2)
GDP	0.0093	1	0.0097	1
Consumption	0.0074	0.8222	0.0080	0.8458
Investment	0.0426	0.8358	0.0419	0.7325
Structures Investment	0.0522	0.5686	0.0574	0.4274
Equipment Investment	0.0432	0.8708	0.0412	0.8090
Aggregate Employment	0.0077	0.7022	0.0077	0.6306
Skilled Employment	0.0108	0.4538	0.0108	0.4091
Unskilled Employment	0.0092	0.5948	0.0092	0.5256
Skilled Wage	0.0128	0.2831	0.0128	0.3313
Unskilled Wage	0.0075	0.2187	0.0075	0.3565

Notes: 1. The second and third columns show the volatilities and cross-correlation of real variables divided by the GDP deflator;

2. The fourth and fifth columns present the volatilities and cross-correlation of real variables divided by the price deflator of nondurable consumption and services;

3. Variables related to employment are de-seasonalized through regression on quarterly dummies;

4. Investment is defined as Private Gross Domestic Investment minus Residential investment, Equipment investment is investment on equipment and software while Structure investment is investment on nonresidential structures;

5. The data covers the period of 1984q1 to 2006q4.

Table 2.5: Comparison of Model Set-up

	Capital-Skill Complementarity	Two Types of Labor	Two Types of Capital	Neutral Tech Shocks	Investment Shocks
Benchmark Version	✓	✓	✓	✓	✗
Traditional Version	✗	✓	✗	✓	✗
Extended Version	✓	✓	✓	✓	✓

Table 2.6 : Calibration Results for the Benchmark and Extended Models

Subjective discount factor	β	0.987
Utility weight: skilled	λ_s	0.625
Utility weight: unskilled	λ_u	0.375
CRRA: skilled	σ_s	1
CRRA: unskilled	σ_u	1
Coefficient governing employment disutility: skilled	η_s	2
Coefficient governing employment disutility: unskilled	η_u	2
Disutility weight of employment: skilled	χ_s	10.2
Disutility weight of employment: unskilled	χ_u	2.02
Labor share: skilled	l_s	0.233
Labor share: unskilled	l_u	0.767
Capital structure share of output	α_{ks}	0.13
Coefficient governing substitution elasticity of unskilled employment and capital equipment	θ_1	0.401
Coefficient governing substitution elasticity of skilled employment and capital equipment	θ_2	-0.495
Production weight: skilled	a_s	12.58/7.28
Production weight: unskilled	a_u	1
Depreciation rate of capital structures	δ_s	0.014
Depreciation rate of capital equipment	δ_e	0.031
Autocorrelation of neutral technology shocks	ρ_z	0.94
Standard deviation of neutral technology shocks	σ_z	0.007
Autocorrelation of investment shocks	ρ_q	0.89
Standard deviation of investment shocks	σ_q	0.0273

Note: Neutral Technology Shocks are used in the benchmark and extended models while investment shocks are only used in the extended model.

Table 2.7 : Calibration Results for the Traditional Model

Subjective discount factor	β	0.987
Utility weight: skilled	λ_s	0.625
Utility weight: unskilled	λ_u	0.375
CRRA: skilled	σ_s	1
CRRA: unskilled	σ_u	1
Coefficient governing employment disutility: skilled	η_s	2
Coefficient governing employment disutility: unskilled	η_u	2
Disutility weight of employment: skilled	χ_s	23.1
Disutility weight of employment: unskilled	χ_u	1.47
Labor share: skilled	l_s	0.233
Labor share: unskilled	l_u	0.767
Capital share of output	α_k	0.35
Coefficient governing substitution elasticity of skilled and unskilled employment	θ_3	0.29
Production weight: skilled	a_s	12.58/7.28
Production weight: unskilled	a_u	1
Depreciation rate of capital	δ	0.025
Autocorrelation of neutral technology shocks	ρ_z	0.94
Standard deviation of neutral technology shocks	σ_z	0.007

Table 2.8: Benchmark and Traditional Models -- Volatility and Correlation

	U.S. Economy		Model			
	1984q1 to 2006q4		Benchmark Model		Traditional Model	
	Volatility	Correlation	Volatility	Correlation	Volatility	Correlation
GDP	0.0093	1	0.0109	1	0.0103	1
Consumption	0.0074	0.8222	0.0046	0.9475	0.0034	0.8989
Investment	0.0426	0.8358	0.0950	0.9745	0.0349	0.9891
Structures Investment	0.0522	0.5686	0.1014	0.9669	--	--
Equipment Investment	0.0432	0.8708	0.1520	0.3115	--	--
Employment	0.0077	0.7022	0.0023	0.9758	0.0025	0.9792
Skilled Employment	0.0108	0.4538	0.0025	0.9690	0.0025	0.9792
Unskilled Employment	0.0092	0.5948	0.0022	0.9717	0.0025	0.9792
Skilled Wage	0.0128	0.2831	0.0092	0.9933	0.0079	0.9980
Unskilled Wage	0.0075	0.2187	0.0086	0.9983	0.0079	0.9980

Table 2.9: Benchmark and Traditional Models -- Volatility Ratio

	U.S. Economy	Model	
	1984q1 to 2006q4	Benchmark Model	Traditional Model
$\frac{\text{Skilled Employment}}{\text{Unskilled Employment}}$	1.17	1.13	1
$\frac{\text{Employment}}{\text{GDP}}$	0.83	0.215	0.243

Table 2.10: Benchmark Model -- Volatility Ratio

	U.S. Economy	The Benchmark Model		
		$\theta_2 = -0.495$	$\theta_2 = -0.4$	$\theta_2 = -2.2$
$\frac{\text{Skilled Employment}}{\text{Unskilled Employment}}$	1.17	1.136	1.137	1.091
$\frac{\text{Employment}}{\text{GDP}}$	0.83	0.215	0.210	0.207

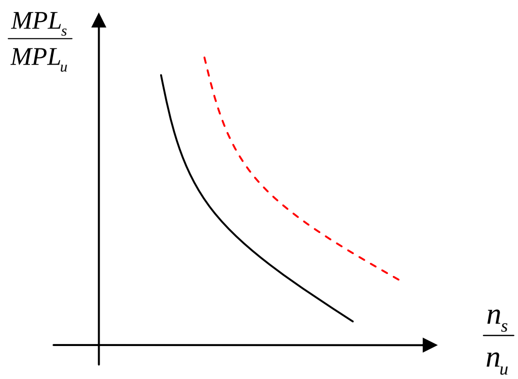
Table 2.11: Benchmark and Extended Models -- Volatility and Correlation

	U.S. Economy		Model			
	1984q1 to 2006q4		Benchmark Model		Extended Model	
	Volatility	Correlation	Volatility	Correlation	Volatility	Correlation
GDP	0.0093	1	0.0109	1	0.0110	1
Consumption	0.0074	0.8222	0.0046	0.9475	0.0047	0.9431
Investment	0.0426	0.8358	0.0950	0.9745	0.0977	0.9737
Structures Investment	0.0522	0.5686	0.1014	0.9669	0.1119	0.9278
Equipment Investment	0.0432	0.8708	0.1520	0.3115	0.3409	0.0932
Employment	0.0077	0.7022	0.0023	0.9758	0.0023	0.9751
Skilled Employment	0.0108	0.4538	0.0025	0.9690	0.0029	0.8977
Unskilled Employment	0.0092	0.5948	0.0022	0.9717	0.0022	0.9617
Skilled Wage	0.0128	0.2831	0.0092	0.9933	0.0099	0.9712
Unskilled Wage	0.0075	0.2187	0.0086	0.9983	0.0087	0.9980

Table 2.12: Extended Model: Different Shocks

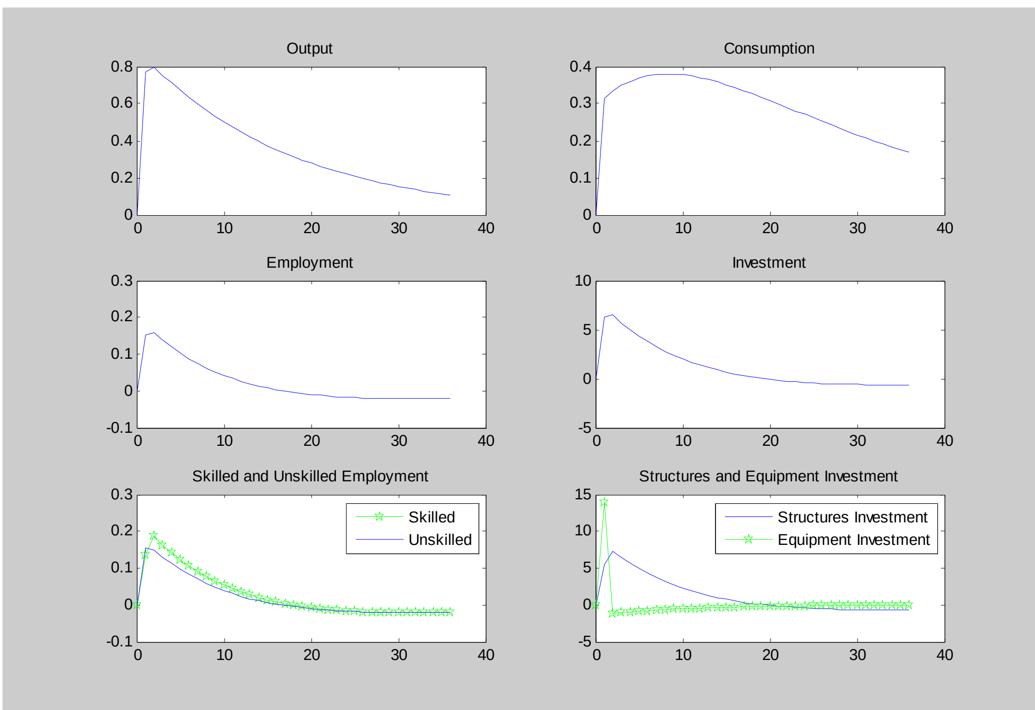
	U.S. Economy		The Extended Model					
	1984q1 to 2006q4		Two shocks		Neutral Shock only		Investment Shock only	
	Volatility	Correlation	Volatility	Correlation	Volatility	Correlation	Volatility	Correlation
GDP	0.0097	1	0.0110	1	0.0109	1	0.0015	1
Consumption	0.0080	0.8458	0.0047	0.9431	0.0046	0.9452	0.0005	0.7096
Investment	0.0419	0.7325	0.0977	0.9737	0.0968	0.9724	0.0166	0.9688
Structures Investment	0.0574	0.4274	0.1119	0.9278	0.1037	0.9653	0.0420	0.7843
Equipment Investment	0.0412	0.8090	0.3409	0.0932	0.1499	0.3046	0.2703	-0.5402
Employment	0.0077	0.6306	0.0023	0.9751	0.0023	0.9744	0.0005	0.9759
Skilled Employment	0.0108	0.4091	**0.0029**	0.8977	**0.0025**	0.9685	**0.0015**	0.9973
Unskilled Employment	0.0092	0.5256	0.0022	0.9617	0.0022	0.9703	0.0002	0.8227
Skilled Wage	0.0128	0.3313	0.0099	0.9712	0.0094	0.9934	0.0033	0.9972
Unskilled Wage	0.0075	0.3565	0.0087	0.9980	0.0088	0.9982	0.0007	0.9894

Figure 2.1: Relative Demand for Skilled Labor



Note: The solid line is the initial relative demand curve of skilled labor. When there is a positive technology shock, the increase of capital stock pushes up the demand curve upward. The red dashed line is the new one.

Figure 2.2: Extended Model: Impulse Responses to a Neutral Technology Shock

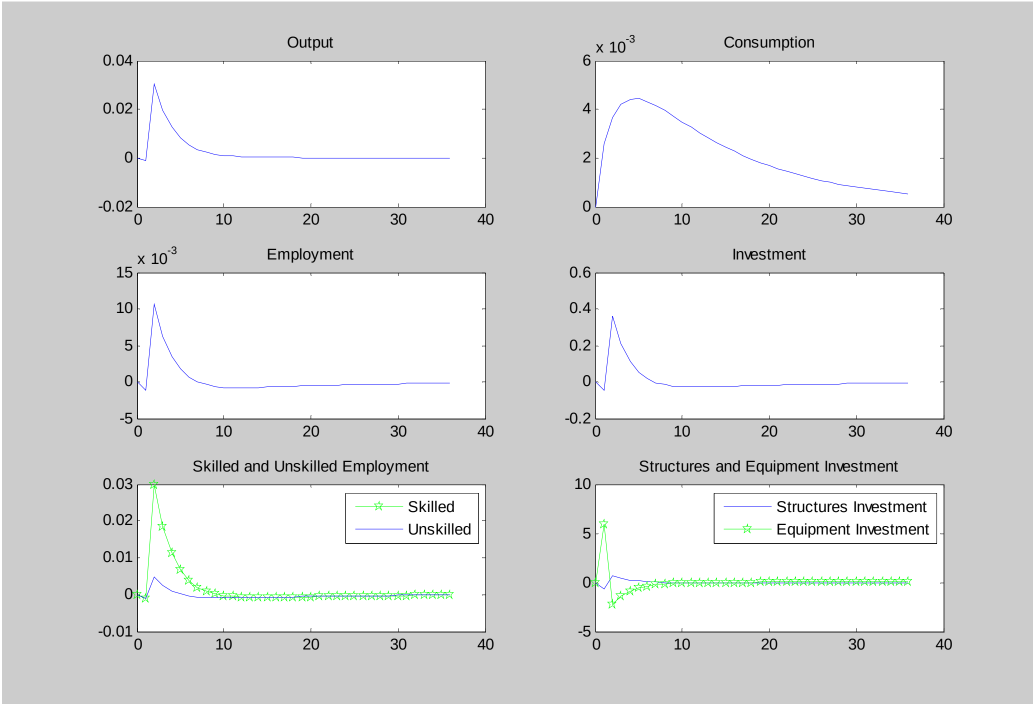


Notes: 1 One period is a quarter;

2 The unit of the vertical axis is percentage point. All variables are expressed as the percentage deviation from their steady state value;

3 The initial shock is equal to one standard deviation of the stochastic process.

Figure 2.3: Extended Model: Impulse Responses to an Investment Shock



Notes: 1 One period is a quarter;

2 The unit of the vertical axis is percentage point. All variables are expressed as the percentage deviation from their steady state value;

3 The initial shock is equal to one standard deviation of the stochastic process.

Appendix A

MORG Data

This appendix presents a detailed explanation on how I construct labor market variables used in this paper. I follow the method of Castro and Coen-Pirani (2008).

A.1 Sample Selection

I restrict attention to individuals who are between 16 and 65 years old, in the labor force and not self-employed. For individuals who reported being employed, I eliminate them if they have either zero or missing earnings, or have either zero or missing values on both "usual hours worked" and "actual hours worked last week". I use the weekly usual hours as hours worked for each employee. From 1994 on, if individuals who have zero or missing usual hours have actual hours of last week, I use actual hours to represent usual hours. I make this adjustment because individuals reporting "usual hours vary" are recorded as having missing usual hours since 1994.

A.2 Demographic Subgroups

I divide the sample into 240 demographic groups based on sex, race, age and education level. First, each individual is either male or female. Second, I restrict attention to three race categories, namely whites, blacks and others. Thirdly, I create ten 5-year age groups for people from 16 to 65 years old. Finally, I construct four education groups: no high school diploma, or equivalently less than 12 years of completed schooling; high school graduate, or equivalently 12 years of completed schooling; between high school and college graduate, or equivalently between 13 and 15 years of completed schooling; college graduate and beyond, or equivalently 16 years of completed schooling or above.

Individuals are classified by their highest grade of school completed. Between 1979 and 1991, individuals report their highest grade of school attended rather than completed. If one person reported 12 years of education but his/her 12th year of schooling is still ongoing, I classify this individual as having no high school diploma. From 1992 on, the BLS education classification switched from years of schooling to educational categories, which was a consistent transformation and did not cause a break in the series of education level. I assign education group with college degrees or above to the skilled category and all the other three education groups to the unskilled category.

A.3 Aggregation across Demographic Groups

In order to aggregate hours worked across different subgroups, time-invariant weights for each subgroup are needed. Following Katz and Murphy (1992), KORV (2000) and Castro and Coen-Pirani (2008), I use the average wage rate of each subgroup over the whole sample as the weights.

A.3.1 Aggregate Weights

First, I aggregate monthly data into quarterly series. After sample selection, each subgroup contains individuals from 1979q1 to 2003q4. Assuming that individuals within each group are perfect substitutes, I calculate the average wage rate for each group by dividing the total income over the total hours worked:

$$w_g = \frac{\sum_{i \in g} \mu_i * inc_i}{\sum_{i \in g} \mu_i * h_i}$$

where g is the group index of each demographic group; inc_i is the individual i's real weekly income, which is computed by dividing reported nominal earnings over the corresponding CPI; h_i and μ_i are, respectively, the individual hours worked and "earnings weight for all races".

A.3.2 Hours in Efficiency Units

Based on the wage weights, hours in efficiency units are computed as follows. For each quarter, I calculate the total usual hours worked for each demographic group. Then, I use wage weights to aggregate these 240 values and get the hours in efficiency units for each quarter.

$$H_t = \sum_{g \in G} w_g \left(\sum_{i \in g} h_{it} \mu_{it} \right)$$

Appendix B

Derivation of Wage Rates Resulting from Nash-Bargaining

First, by definition, we get:

$$V^{NB}(z_t, n_{st}, n_{ut}) = \max_{\substack{\{\varepsilon_s,\varepsilon_u, \\ c_{st},c_{ut}\}}} \Bigg\{ \lambda_s \Big[\frac{l_s(c_{st})^{1-\sigma_s}}{1-\sigma_s} - \frac{(n_{st}+\varepsilon_s)^{1+\eta_s}}{1+\eta_s} \Big] + \lambda_u \Big[\frac{l_u(c_{ut})^{1-\sigma_u}}{1-\sigma_u} - \frac{(n_{ut}+\varepsilon_u)^{1+\eta_u}}{1+\eta_u} \Big] \tag{P.3}$$

$$+ \beta E_t V(t+1|t) \Bigg\}$$

s.t.

$$0 = D_t + w_{st} n_{st} + \tilde{w}_s \varepsilon_s + w_{ut} n_{ut} + \tilde{w}_u \varepsilon_u - l_s c_{st} - l_u c_{ut}$$

$$0 = (1-\chi_s)(n_{st}+\varepsilon_s) + v_{st}\left(1 - e^{\frac{-\alpha_s(l_s - n_{st} - \varepsilon_s)}{v_{st}}}\right) - n_{st+1}$$

$$0 = (1-\chi_u)(n_{ut}+\varepsilon_u) + v_{ut}\left(1 - e^{\frac{-\alpha_u(l_u - n_{ut} - \varepsilon_u)}{v_{ut}}}\right) - n_{ut+1}$$

Based on the above definition, I derive the following expressions:

$$\tilde{V}_{st} = \frac{\partial V^{NB}(\cdot)}{\partial \varepsilon_s}\Big|_{\varepsilon_s=0} \tag{B.1}$$

$$\tilde{V}_{ut} = \frac{\partial V^{NB}(\cdot)}{\partial \varepsilon_u}\Big|_{\varepsilon_u=0} \tag{B.2}$$

Next, we get:

$$J^{NB}(z_t, k_t, n_{st}, n_{ut}) = \max_{\substack{\{\varepsilon_s,\varepsilon_u,v_{st},v_{ut}, \\ n_{st+1},n_{nt+1},k_{t+1}\}}} \Bigg\{ e^{z_t} k_t^{\alpha} \big[(a_s n_{st} + a_s \varepsilon_s)^{\sigma} + (a_u n_{ut} + a_u \varepsilon_u)^{\sigma}\big]^{\frac{1-\alpha}{\sigma}} - w_{st} n_{st} - \tilde{w}_s \varepsilon_s$$

$$- w_{ut} n_{ut} - \tilde{w}_u \varepsilon_u - f_s v_{st} - f_u v_{ut} + (1-\delta)k_t - k_{t+1} + \beta E_t \Lambda_{t,t+1} J(t+1|t) \Bigg\} \tag{P.4}$$

s.t.

$$0 = (1-\chi_s)(n_{st}+\varepsilon_s) + v_{st}\left(1 - e^{\frac{-\alpha_s(l_s - n_{st} - \varepsilon_s)}{v_{st}}}\right) - n_{st+1}$$
$$0 = (1-\chi_u)(n_{ut}+\varepsilon_u) + v_{ut}\left(1 - e^{\frac{-\alpha_u(l_u - n_{ut} - \varepsilon_u)}{v_{ut}}}\right) - n_{ut+1}$$

Then,

$$\tilde{J}_{st} = \frac{\partial J^{NB}(\cdot)}{\partial \varepsilon_s}\Big|_{\varepsilon_s=0} \tag{B.3}$$
$$\tilde{J}_{ut} = \frac{\partial J^{NB}(\cdot)}{\partial \varepsilon_u}\Big|_{\varepsilon_u=0} \tag{B.4}$$

The equilibrium wage rate w_i solves the weighted geometric average of the gains from bargaining:

$$\tilde{w}_i = \arg\max_{\tilde{w}_i} \left\{ \tilde{V}_{it}^{\phi_i} \tilde{J}_{it}^{1-\phi_i} \right\}$$

The first-order condition is: $\quad 0 = \phi_i \tilde{J}_{it}(\cdot)\frac{\partial \tilde{V}_{it}}{\partial \tilde{w}_i} + (1-\phi_i)\tilde{V}_{it}(\cdot)\frac{\partial \tilde{J}_{it}}{\partial \tilde{w}_i}$

From (P.3), I get the following:

$$\tilde{V}_{st}(w_s) = \lambda_s c_{st}^{-\sigma_s}(\tilde{w}_s - w_{st}) + V_{n_{st}}$$
$$V_{n_{st}} = -\lambda_s (n_{st})^{\eta_s} + \lambda_s c_{st}^{-\sigma_s} w_{st} + \beta(1 - \chi_s - \alpha_s e^{\frac{-\alpha_s(l_s - n_{st})}{v_{st}}})E_t V_{n_{st+1}}$$

and $\quad \tilde{V}_{ut}(w_u) = \lambda_u c_{ut}^{-\sigma_u}(\tilde{w}_u - w_{ut}) + V_{n_{ut}}$

$$V_{n_{ut}} = -\lambda_u (n_{ut})^{\eta_u} + \lambda_u c_{ut}^{-\sigma_u} w_{ut} + \beta(1 - \chi_u - \alpha_u e^{\frac{-\alpha_u(n_u - n_{ut})}{v_{ut}}})E_t V_{n_{ut+1}}$$

From (P.4), I get:

$$\tilde{J}_{st}(w_s) = w_{st} - w_s + J_{n_{st}}$$
$$J_{n_{st}} = (1-\alpha)e^{z_t}k_t^{\alpha}\left[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\right]^{\frac{1-\alpha-\sigma}{\sigma}} a_s^{\sigma}(n_{st})^{\sigma-1} - w_{st}$$
$$+ \beta(1 - \chi_s - \alpha_s e^{\frac{-\alpha_s(l_s - n_{st})}{v_{st}}})E_t \Lambda_{t,t+1} J_{n_{st+1}}$$

$$
\begin{aligned}
\tilde{J}_{ut}(w_u) =& w_{ut} - \tilde{w}_u + J_{n_{ut}} \\
J_{n_{ut}} =& (1-\alpha)e^{z_t}k_t^{\alpha}\big[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\big]^{\frac{1-\alpha-\sigma}{\sigma}} a_u^{\sigma}(n_{ut})^{\sigma-1} - w_{ut} \\
&+ \beta(1-\chi_u - \alpha_u e^{\frac{-\alpha_u(l_u-n_{ut})}{v_{ut}}})E_t\Lambda_{t,t+1}J_{n_{ut+1}}
\end{aligned}
$$

Now, I solve the Nash-bargaining problem for the skilled wage rate.

$$
\because \quad 0 = \phi_i \tilde{J}_{it}(\cdot)\frac{\partial \tilde{V}_{it}}{\partial \tilde{w}_i} + (1-\phi_i)\tilde{V}_{it}(\cdot)\frac{\partial \tilde{J}_{it}}{\partial \tilde{w}_i}
$$

$$
\therefore \quad 0 = \phi_s\lambda_s J_{n_{st}} - (1-\phi_s)c_{st}^{\sigma_s}V_{n_{st}}
$$

When plugging $V_{n_{st}}$, I get:

$$
\therefore \quad \phi_s\lambda_s J_{n_{st}} = (1-\phi_s)c_{st}^{\sigma_s}\Big\{-\lambda_s(n_{st})^{\eta_s} + \lambda_s c_{st}^{-\sigma_s}w_{st} + \beta\big(1-\chi_s-\alpha_s e^{\frac{-\alpha_s(l_s-n_{st})}{v_{st}}}\big)E_t V_{n_{st+1}}\Big\}
$$

Plugging $V_{n_{st+1}} = \frac{\phi_s\lambda_s J_{n_{st+1}}}{(1-\phi_s)c_{st+1}^{\sigma_s}}$ into the above equation and taking some manipulation, I get:

$$
w_{st} = \phi_s(1-\alpha)e^{z_t}k_t^{\alpha}\Bigg[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\Bigg]^{\frac{1-\alpha-\sigma}{\sigma}} a_s^{\sigma}(n_{st})^{\sigma-1} + (1-\phi_s)c_{st}^{\sigma_s}(n_{st})^{\eta_s}
$$

For the unskilled wage rate, I solve it in a parallel way and get the result:

$$
w_{ut} = \phi_u(1-\alpha)e^{z_t}k_t^{\alpha}\Bigg[(a_s n_{st})^{\sigma} + (a_u n_{ut})^{\sigma}\Bigg]^{\frac{1-\alpha-\sigma}{\sigma}} a_u^{\sigma}(n_{ut})^{\sigma-1} + (1-\phi_u)c_{ut}^{\sigma_u}(n_{ut})^{\eta_u}
$$

Appendix C

Equivalent Model Set-Up

In my paper, I adopt a complete-market structure following Andolfatto (1996). Here, I show another way to set up the model, which follows Shimer (2009). Time is discrete, which is denoted by $t = 0, 1, 2, \cdots$. The state of the economy at time t is denoted by $s^t = \{s_0, s_1, s_2, \cdots s_t\}$, which represents the history of the economy. $\Pi(s^t)$ represents the time-0 belief about the probability of observing a given history s^t at time t.

For the household:

$$V\big(z(s^t), a(s^t)\big) = \max_{\{c_s(s^t), c_u(s^t)\}} \Bigg\{ \lambda_s \Big[\frac{l_s (c_s(s^t))^{1-\sigma_s}}{1-\sigma_s} - \frac{(n_s(s^t))^{1+\eta_s}}{1+\eta_s} \Big] + \lambda_u \Big[\frac{l_u (c_u(s^t))^{1-\sigma_u}}{1-\sigma_u} - \frac{(n_u(s^t))^{1+\eta_u}}{1+\eta_u} \Big] + \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V(s^{t+1}|s^t) \Bigg\}$$

$$\text{s.t.} \quad 0 = a(s^t) + w_s(s^t) n_s(s^t) + w_u(s^t) n_u(s^t) - l_s c_s(s^t) - l_u c_u(s^t) - \sum_{s^{t+1}|s^t} q_t(s^{t+1}) a(s^{t+1})$$

where $a(s^t)$ is the household's assets; and $q_t(s^{t+1})$ is the time-t price of a unit of consumption over history s^{t+1} paid in units of history-s^t consumption. The optimality conditions are:

$$\lambda_s (c_s(s^t))^{-\sigma_s} = \lambda_u (c_u(s^t))^{-\sigma_u}$$

$$q_t(s^{t+1}) = \beta \frac{\Pi(s^{t+1}) (c_s(s^{t+1}))^{-\sigma_s}}{\Pi(s^t) (c_s(s^t))^{-\sigma_s}} = \beta \frac{\Pi(s^{t+1}) (c_u(s^{t+1}))^{-\sigma_u}}{\Pi(s^t) (c_u(s^t))^{-\sigma_u}}$$

For the firm:

$$J\big(z(s^t),k(s^t),n_s(s^t),n_u(s^t)\big)=\max_{\substack{\{v_s(s^t),v_u(s^t),n_s(s^{t+1}),\\ n_u(s^{t+1}),k(s^{t+1})\}}}\Big\{e^{z(s^t)}(k(s^t))^{\alpha}\big[(a_s n_s(s^t))^{\sigma}+(a_u n_u(s^t))^{\sigma}\big]^{\frac{1-\alpha}{\sigma}}$$

$$-w_s(s^t)n_s(s^t)-w_u(s^t)n_u(s^t)-f_s v_s(s^t)-f_u v_u(s^t)+(1-\delta)k(s^t)-k(s^{t+1})+\sum_{s^{t+1}|s^t}q_t(s^{t+1})J(s^{t+1}|s^t)\Big\}$$

s.t.

$$n_s(s^{t+1})=(1-\chi_s)n_s(s^t)+v_s(s^t)\big(1-e^{\frac{-\alpha_s u_s(s^t)}{v_s(s^t)}}\big)$$

$$n_u(s^{t+1})=(1-\chi_u)n_u(s^t)+v_u(s^t)\big(1-e^{\frac{-\alpha_u u_u(s^t)}{v_u(s^t)}}\big)$$

Appendix D

Expression for Employment Volatility

In this section, I log-linearize the law of motion for employment to derive an expression for employment volatility. To simplify the notation, I suppress the type-specific subscript s or u.

Denote the steady state value of variable x by $\bar{x}$, and denote the percentage deviation from its steady-state value by $\hat{x}$. That is, $\hat{x} = \frac{x-\bar{x}}{\bar{x}}$

For worker flow, the law of motion is: $n_{t+1} = (1-\chi)n_t + v_t\big(1 - e^{\frac{-\alpha u_t}{v_t}}\big)$. Log-linearizing on the above equation, we get:

$$\hat{n}_{t+1} = (1-\chi)\hat{n}_t + \chi\Big(1 - \frac{\frac{\alpha\bar{u}}{\bar{v}}}{e^{\frac{\alpha\bar{u}}{\bar{v}}}}\Big)\hat{v}_t + \frac{\chi\frac{\alpha\bar{u}}{\bar{v}}}{e^{\frac{\alpha\bar{u}}{\bar{v}}}-1}\hat{u}_t$$

Since $l = n_t + u_t$, we have: $0 = \bar{n}\hat{n}_t + \bar{u}\hat{u}_t$, or $\hat{u}_t = \frac{\bar{n}\hat{n}_t}{\bar{u}}$. Replacing $\hat{u}_t$ with $\frac{\bar{n}\hat{n}_t}{\bar{u}}$ in the above equation, we find:

$$\hat{n}_{t+1} = \Phi_{nn}\hat{n}_t + \Phi_{nv}\hat{v}_t \tag{D.1}$$

where $\Phi_{nn}\big(\frac{\alpha\bar{u}}{\bar{v}}, \frac{\bar{u}}{\bar{n}}\big) = 1 - \chi - \frac{\chi\frac{\alpha\bar{u}}{\bar{v}}}{e^{\frac{\alpha\bar{u}}{\bar{v}}}-1}\frac{\bar{n}}{\bar{u}}, \quad \Phi_{nn} > 0, \quad \frac{\partial\Phi_{nn}}{\partial\frac{\alpha\bar{u}}{\bar{v}}} > 0, \quad \text{and} \ \frac{\partial\Phi_{nn}}{\partial\frac{\bar{u}}{\bar{n}}} > 0$

$$\Phi_{nv}\big(\frac{\alpha\bar{u}}{\bar{v}}\big) = 1 - \frac{\frac{\alpha\bar{u}}{\bar{v}}}{e^{\frac{\alpha\bar{u}}{\bar{v}}}-1}, \quad \Phi_{nv} > 0, \quad \text{and} \ \frac{\partial\Phi_{nv}}{\partial\frac{\alpha\bar{u}}{\bar{v}}} > 0$$

Finally, we have $var(n) = \Phi_{nn}^2 var(n) + \Phi_{nv}^2 var(v) + 2\Phi_{nn}\Phi_{nv}cov(n,v)$, which implies:

$$var(n) = var(v)\frac{\Phi_{nv}^2}{1-\Phi_{nn}^2} + 2cov(n,v)\frac{\Phi_{nn}\Phi_{nv}}{1-\Phi_{nn}^2} \tag{D.2}$$

Bibliography

[1] Abraham, Katharine G. and Robert Shimer. (2001) "Changes in Unemployment Duration and Labor-Force Attachment." NBER Working Paper W8513.

[2] Acemoglu, Daron. (1998) "Why Do New Technologies Complement Skills? Directed Technical Change And Wage Inequality." Quarterly Journal of Economics 113 (4):1055-1089.

[3] Acemoglu, Daron. (1999) "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence." American Economic Review, 89 (5):1259-1278.

[4] Acemoglu, Daron. (2002) "Technical Change, Inequality, and the Labor Market." Journal of Economic Literature, 40 (1):7-72.

[5] Aghion, Philippe, Peter Howitt and Giovanni L. Violante. (2002) " General Purpose Technology and Wage Inequality." Journal of Economic Growth 7 (4):315-45.

[6] Altonji, Joseph G. (1986) "Intertemporal Substitution in Labor Supply: Evidence from Micro Data." Journal of Political Economy 94 (3):176-215.

[7] Andolfatto, David. (1996) "Business Cycles and Labor-Market Search." American Economic Review 86 (1):112-132.

[8] Attanasio, Orazio P. and Guglielmo Weber. (1993) "Consumption Growth, the Interest Rate and Aggregation." Review of Economic Studies 60 (3):631-649.

[9] Autor, David H., Lawrence F. Katz and Alan B. Krueger. (1998) "Computing Inequality: Have Computers Changed The Labor Market?" Quarterly Journal of Economics 113 (4):1169-1213.

[10] Bergin, P., R.C. Feenstra and G.H. Hanson. (2007) "Outsourcing and Volatility," NBER Working Paper 13144.

[11] Bresnahan, T.F. and M. Trajtenberg. (1995) "General Purpose Technologies:'Engines of Growth'?" Journal of Econometrics 65 (1):83-108.

[12] Bronars, Stephen, Donald R. Deere and Joseph S. Tracy. (1994) "The Effects of Unions on Firm Behavior: An Empirical Analysis Using Firm-Level Data." Industrial Relations: A Journal of Economy and Society 33 (4):426-451.

[13] Browning, Martin, Angus Deaton and Margaret Irish. (1985) “A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle.” Econometrica 53 (3):503-544.

[14] Browning, Martin, Lars Peter Hansen and James J. Heckman. (1999) “Micro Data and General Equilibrium Models.” Handbook of Macroeconomics, by J. B. Taylor and M. Woodford, vol.1 chap.8. Elsevier, 543-633.

[15] Burdett, Kenneth, Shouyong Shi and Randall Wright. (2001) “Pricing and Matching with Frictions.” Journal of Political Economy 109 (5):1060-1085.

[16] Butters, Gerard R. (1977) “Equilibrium Distribution of Sales and Advertising Prices.” Review of Economic Studies 44 (3):465-491.

[17] Cao, Melanie and Shouyong Shi. (2000) “Coordination, Matching, and Wages.” Canadian Journal of Economics 33 (4):1009-1033.

[18] Castro, Rui and Daniele Coen-Pirani. (2008) “Why Have Aggregate Skilled Hours Become So Cyclical since the Mid-1980s?” International Economic Review 49 (1):135-185.

[19] Cho, J-O. and T.F. Cooley. (1994) “ Employment And Hours Over The Business Cycle.” Journal of Economic Dynamics and Control 18 (2): 411-432.

[20] Clark, Kim B. and Lawrence H. Summers. (1981) “Demographic Differences in Cyclical Employment Variation.” NBER Working Paper W0514.

[21] Cooley, Thomas F. and Gary D. Hansen. (1995) “Money and the Business Cycle.” Frontiers of Business Cycle Research, Princeton: Princeton University Press, 175-216

[22] DiNardo, John, Nicole Fortin and Thomas Lemieux. (1996) “Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach.” Econometrica 64 (5):1001-1044.

[23] Elsby, Michael W.L., Ryan Michaels and Gary Solon. (2009) “The Ins and Outs of Cyclical Unemployment.” American Economic Journal: Macroeconomics 1 (1):84-110.

[24] Fisher, Jonas D. M. (2006) “The Dynamic Effects of Neutral and Investment-Specific Technology Shocks.” Journal of Political Economy 114 (3):413-451.

[25] Fuhrer, Jeffrey C. (2000) “Habit Formation in Consumption and its Implications for Monetary-Policy Models.” American Economic Review 90 (3):367-390.

[26] Galor, Oded and Daniel Tsiddon. (1997) “ Technological Progress, Mobility, and Economic Growth.” American Economic Review. 87 (3):363-382.

[27] Galor, Oded and Omer Moav. (2000) “Ability-Biased Technological Transition, Wage Inequality, and Economic Growth.” Quarterly Journal of Economics 115 (2):469-497.

[28] Geishecker, Ingo. (2008) “The Impact of International Outsourcing on Individual Employment Security: a Micro-Level Analysis.” Labour Economics 15 (3):291-314.

[29] Gomme, Paul, Richard Rogerson, Peter Rupert and Randall Wright. (2005) “The Business Cycle and the Life Cycle.” NBER Macroeconomics Annual 2004 19:415-592.

[30] Gottschalk, Peter and Robert Moffitt. (1998) “Job Instability and Insecurity for Males and Females in the 1980s and 1990s.” Boston College Working Paper 408.

[31] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. (1997) “Long-run implications of investment-specific technology change.” AER 87 (3):342-362.

[32] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. (2000) “The role of investment-specific technological change in the business cycle.” European Economic Review 44 (1):91-115.

[33] Greenwood, Jeremy and M. Yorukoglu. (1997) “1974.” Carnegie-Rochester Conference Series on Public Policy 46 (1):49-95.

[34] Hagedorn, Marcus and Iourii Manovskii. (2008) “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited.” American Economic Review 98 (4):1692-1706.

[35] Hall, Robert E. (1980) “A Theory of the Natural Unemployment Rate and the Duration of Employment.” NBER Working Paper W0251.

[36] Hall, Robert E. (1988) “Intertemporal Substitution in Consumption.” Journal of Political Economy 96 (2):339-357.

[37] Hall, Robert E. (1988) "Intertemporal Substitution in Consumption." Journal of Political Economy 96 (2):339-357.

[38] Hall, Robert E. (2005) "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years." NBER Macroeconomics Annual 101-137.

[39] Hall, Robert E. (2005) "Employment Fluctuations with Equilibrium Wage Stickiness." American Economic Review 95 (1):50-65.

[40] Hamermesh, D.S. (1993) "Labor Demand." Princeton, N.J: Princeton Univ. Press

[41] Hansen, Gary D. (1985) "Indivisible labor and the business cycle." Journal of Monetary Economics 16 (3):309-327.

[42] Hosios, Arthur J. (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment." Review of Economic Studies 57 (2):279-298.

[43] Imai, Susumn and Michael P. Keane. (2004) "Intertemporal Labor Supply and Human Capital Accumulation." International Economic Review 45 (2):601-641.

[44] Jaimovich, Nir and Henry E. Siu. (2009) "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility." American Economic Review 99 (3):804-826.

[45] Johnson, G.E. (1997) "Changes in Earings Inequality: The Role of Demand Shift." Journal of Economic Perspective 11 (2):41-55.

[46] Jovanovic, Boyan and Peter L. Rousseau. (2005) "General Purpose Technologies." NBER Working Paper 11093.

[47] Julien, Benoit, John Kennes and Ian King. (2000) "Bidding for Labor." Review of Economic Dynamics 3 (4):619-649.

[48] Justiniano, Alejandro, Giorgio E. Primiceri and Andrea Tambalotti. (2009) "Investment Shocks and Business Cycles." NBER Working Papers 15570.

[49] Kambourov, G. and I. Manovskii.(2008) "Rising Occupational and Industry Mobility in the United States: 1968-1997." International Economic Review 49 (1):41-79.

[50] Katz, L and K. Murphy. (1992) "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." Quarterly Journal of Economics 107 (1):35-78.

[51] Katz, Lawrence F. and David Autor. (1999) "Changes in the Wage Structure and Earnings Inequality." Handbook of Labor Economics vol3 PartA Chap12:1463-1555, Elsevier.

[52] Krusell, Per, Lee E. Ohanian, Jose-Victor Rios-Rull, and Giovanni L. Violante. (2000) "Capital-Skill Complementarity and Inequality: a Macroeconomic Analysis." Econometrica 68 (5):1029-1054.

[53] Lee, David. (1999) "Wage Inequality in the U.S. during the 1980s: Rising Dispersion or Falling Minimum Wage?" Quarterly Journal of Economics 114 (3):977-1023.

[54] Leonard, Jonathan S. (1992) "Unions and Emploment Growth." Industrial Relations: A Journal of Economy and Society 31 (1):80-94.

[55] Lindquist, Matthew J. (2004) "Capital-Skill Complementarity and Inequality Over the Business Cycle." Review of Economic Dynamics 7 (3):519-540.

[56] MaCurdy, Thomas. (1981) "An Empirical Model of Labor Supply in a Life Cycle Setting." Journal of Political Economy 89 (5):1059-1085.

[57] Mankiw,Gregory and Phillip Swagel. (2006) "The Politics and Economics of Offshore Outsourcing." Journal of Monetary Economics 53 (5):1027-1056.

[58] McConnell, M.M. and G. Perez-Quiros. (2000) "Output Fluctuations in the United States: What has changed since the early 1980's?" American Economic Review 90 (5):1464-1476.

[59] Montgomery, James D. (1991) "Equilibrium Wage Dispersion and Interindustry Wage Differentials." Quarterly Journal of Economics 106 (1):163-179.

[60] Mortensen, D.T. and C.A. Pissarides. (1994) "Job Creation and Job Destruction in the Theory of Unemployment." Review of Economic Studies 61 (3):397-415.

[61] Mortensen, Dale T. and Christopher A. Pissarides. (1999) "New Developments in Models of Search in the Labor Market." Handbook of Labor Economics vol3 partB:2567-2627.

[62] Munch, J.R. (2005) "International Outsourcing and Individual Job Separations." Working Paper, University of Copenhagen.

[63] Nash, John. (1953) "Two-Person Cooperative Games." Econometrica 21 (1):128-140.

[64] Peters, Michael. (1984) "Bertrand Equilibrium with Capacity Constraints and Restricted Mobility." Econometrica 52 (5):1117-1127.

[65] Petrongolo, Barbara and C.A. Pissarides. (2001) "Looking into the Black Box: A Survey of the Matching Function." Journal of Economic Literature 39 (2):390-431.

[66] Pissarides, Christopher. (1979) "Job Matchings with State Employment Agencies and Random Search." Economic journal 89 (356):818-833.

[67] Rios-Rull, Jose-Victor. (1996) "Life-Cycle Economies and Aggregate Fluctuations." Review of Economic Studies 63 (3):465-489.

[68] Rogerson, Richard. (1988) "Indivisible labor, lotteries, and equilibrium." Journal of Monetary Economics 21 (1):3-16.

[69] Rotemberg, Julio J. and Michael Woodford. (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." NBER Working Paper W0233.

[70] Senses, M.Z. (2007) "The Effects of Outsourcing on the Elasticity of Labor Demand," Mimeo, Johns Hopkins University.

[71] Shimer, Robert. (2005) "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." American Economic Review 95 (1):25-49.

[72] Shimer, Robert. (2009) "Labor Markets and Business Cycles." Book Manuscript.

[73] Slaughter, Matthew. (2001) "International Trade and Labor-Demand Elasticities." Journal of International Economics 54 (1):27-56.

[74] Stock, J.H. and M.W. Watson. (2003) "Has the Business Cycle Changed and Why?" NBER Macroeconomics Annual 2002, vol17:159-218.

[75] Wood, Adrian. (1995) "How Trade Hurt Unskilled Workers." Journal of Economic Perspectives 9 (3):57-80.