

**NUMERICAL SIMULATION OF FREE SURFACE FLOW USING LAX DIFFUSIVE
EXPLICIT SCHEME**

*A Thesis Submitted In Partial Fulfillment of the
Requirements for the Degree Of*

**Master of Technology
in
Civil Engineering
(Water Resources Engineering)**

BY

KAMALINI DEVI

Roll .No- 212CE4488

Under the Guidance of
Prof. Kishanjit Kumar Khatua



**DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA-769008,**

May 2013



DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

DECLARATION

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the university or other institute of higher learning, except where due acknowledgement has been made in the text.

KAMALINI DEVI

**NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA-769008, ODISHA, INDIA**



CERTIFICATE

This is to certify that the thesis entitled, “**Numerical Simulation of Free Surface Using Lax Diffusive Explicit Scheme**” submitted by **Ms. Kamalini Devi** in partial fulfillment of the requirements for the award of Master of Technology Degree in **CIVIL ENGINEERING** with specialization in “**WATER RESOURCE ENGINEERING**” at the National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

Place:

Prof. K.K. Khatua
Dept. of Civil Engineering
National Institute of Technology
Rourkela-769008

ACKNOWLEDGEMENT

I express my sincere gratitude and sincere thanks to **Prof. K.K. Khatua** for his guidance, constant encouragement and support during the course of my Research work. I truly appreciate and value his esteemed guidance and encouragement from the beginning to the end of this works, his knowledge and accompany at the time of crisis remembered lifelong.

I sincerely thank to our Director **Prof. S. K. Sarangi**, and all the authorities of the institute for providing nice academic environment and other facilities in the NIT campus, I express my sincere thanks to Professor of water resource group, **Prof. K .C Patra, Prof. A.Kumar** and **Prof R. Jha** for their useful discussion, suggestions and continuous encouragement and motivation. Also I would like to thanks all Professors of Civil Engineering Department who are directly and indirectly helped us.

I am also thankful to all the staff members of Water Resource Engineering Laboratory for their assistance and co-operation during the course of experimental works. I also thank all my batch mates who have directly or indirectly helped me in my project work and shared the moments of joy and sorrow throughout the period of project work finally yet importantly, I would like to thank my Parents, who taught me the value of hard work by their own example.

At last but not the least, I thank to all those who are directly or indirectly associated in completion of this Research work

Date:.

Place:

Kamalini Devi
M. Tech (Civil)
Roll No -212CE4488
Water Resource Engineering

ABSTRACT

In an open channel or overland flow of shallow depth, flood wave propagation is the concept which requires governing equations for its solution. The computation of governing equations (both momentum equation and continuity equations) to be solved are generally called the Saint-Venant equations. These equations are highly nonlinear partial differential equations, the solutions of which are very much complex. Numerical approaches are generally employed to solve these equations and proper discretization with proper selection of grid size and time step provides the results more effectively and accurately. In the present research work the Saint – Venant equations are solved through the lax diffusive explicit finite difference scheme. In this the characteristic equations are simultaneously solved in both boundaries for dynamic wave, which leads to give very accurate result. Two types of downstream boundary conditions were considered together with the condition of discharge hydrograph at upstream end. The physical laws which govern two basic principles in the hydraulics of flow of water are principle of conservation of mass and principle of conservation of momentum. These two laws are of mathematical form generally expressed in partial differential equation form known as Saint-Venant equations. Conversion of these equations into ordinary partial differential equation forms and the simple discretization of this equation by explicit scheme using CFD tool are presented in this paper. During the time of flood, the flow in open channel is generally unsteady. Based on a simple explicit scheme using mat-lab software, the routing of flood in different section of study area in downstream locality is explored keeping the upstream flow hydrograph is as initial boundary condition. Upstream hydrograph at upstream boundary and the critical flow depth from critical flow condition at downstream boundary have been taken for the present analysis. Different discharge and stage hydrograph at downstream sections of the channel under sub critical flow conditions are explored and discussed. The study will be helpful for obtaining the nature of stage and flow hydrograph of a channel during flood. Mat-lab computing tool and suitable program has been performed successfully. The results of free surface flows of a channel using HEC-RAS software are also compared well. The present approach is found to be more effective than other existing numerical models which prove the adequacy of the present numerical model.

Key words: Finite difference method, explicit method, lax-diffusive scheme, HEC-RAS

TABLE OF CONTENTS

TITLE	PAGE NO.
CERTIFICATE	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iii
CONTENTS	iv
LIST OF FIGURES	vii
LIST OF TABLES	vii
NOTATIONS	viii

CHAPTER 1 INTRODUCTION

1.1. Overview	1
1.2. Equations Of Different Types Of Flow	1
1.2.1. Froude Number	2
1.2.2. Critical Flow	2
1.3. Hydrograph	3
1.4. Routing.....	4
1.4.1. Traditional Or Hydrological Method	6
1.4.1.1. Muskingum Method.....	7
1.4.2. Hydraulic Method	10
1.4.3. Derivation Of Saint Venant Equation	11
1.5. Frictional Slope	15
1.6. Application Of The Saint Venant Equation	15
1.7. Boundary Condition And Initial Condition	16
1.7.1. Flow Hydrograph	17
1.7.2. Stage Hydrograph	17

1.7.3. Stage And Flow Hydrograph	17
1.7.4. Rating Curve	17
1.7.5. Normal Depth From Manning’s Equation	18
1.8. Selection And Application Of Roughness Coefficient	18
1.9. Aim Of The Project.....	20
1.10. Organisation Of The Thesis	21

CHAPTER 2 LITERATURE REVIEWS

2.1. Overview	23
2.2. Previous Work On Flood Routing	23

CHAPTER 3 NUMERICAL METHODS AND ANALYSIS

3.1. Overview	30
3.2. Method Of Characteristics	30
3.2.1. Celerity.....	32
3.3. Finite Difference Method.....	35
3.3.1. Grid Generation	36
3.3.2 Explicit Method	36
3.3.3 Implicit Method	36
3.4. Stability	36
3.5. Explicit Finite-Difference Schemes	39
3.5.1. Conservation Form.....	40
3.5.2. Lax-Diffusive Scheme	40
3.5.2 Lax–Wendroff Method	42
3.6. Hec-Ras Model	42
3.7. Model Development And Methodology	43

CHAPTER 4 PROBLEM STATEMENT AND SOLUTION

4.1. Overview	46
4.2. Flood Routing In A River Section – A Case Study	46
4.3. Solution	47
4.3.1. Grid Generation	49
4.3.2. Boundary Condition.....	49
4.3.3. Downstream Boundary Condition	50
4.4. Numerical Solution	50
4.5. Results And Discussion	51
4.6. Result Obtained From Hec-Ras	55
4.7. Comparison Of The Results Between Lax Diffusive And Hec-Ras Model	61
CHAPTER 5 CONCLUSIONS AND SCOPE FOR FUTURE STUDY	
5.1. Conclusion	64
5.2. Scope Of Future Study.....	65
REFERENCES.....	66
PUBLICATION FROM THE WORK.....	66

LIST OF FIGURES

Title	Pages
Fig.1.1. Channel Routing	6
Fig.1.2. Muskingum wedge and Prism storage concept	8
Fig.1.3. Rating curve.....	18
Fig.3.1. The Upstream and Downstream Characteristic In x-t Plane	34
Fig.3.2. Grid in Finite Difference Scheme.....	37
Fig.4.1. Grid Map For Explicit Method.....	48

Fig.4.2. Known Flood Hydrograph at Upstream	49
Fig.4.3. Known Flood hydrograph at Upstream	51
Fig.4.4.Flood Hydrograph at 16km from Upstream	52
Fig.4.5. Flood Hydrograph at 28km from Upstream	52
Fig.4.6. Shifting of Flood Hydrograph	53
Fig.4.7. Flow Hydrographs Obtained From Lax-Diffusive Scheme	53
Fig.4.8. Depth Hydrograph Obtained From Lax-Diffusive Scheme	54
Fig.4.9. Depth Hydrograph at 16km from Upstream.....	54
Fig.4.10. Depth Hydrograph at 28km From Upstream.....	55
Fig.4.11. Shifting of Depth Hydrograph.....	55
Fig.4.12. Flow Hydrograph from HEC-RAS.....	56
Fig.4.13. Stage (m) Hydrograph from HEC-RAS	56
Fig.4.14. Comparison of flow Hydrograph at 16km from Upstream	62
Fig.4.15. Comparison of flowHydrograph at 28km from Upstream	62
Fig.4.16. Comparison of Stage Hydrograph at 16km from Upstream.....	63
Fig.4.17. Comparison of Stage Hydrograph at 28km from Upstream.....	63

LIST OF TABLES

TABLE 4.1. Computed Discharge (m^3/sec) Hydrograph Values at 16 Km Section from U/S end.....	57
TABLE 4.2. Computed Discharge (m^3/sec) Hydrograph Values at D/S End.....	58
TABLE 4.3. Computed Stage (m) Hydrograph Values at 16 Km Section from the U/S	59
TABLE 4.4. Computed Stages (m) Hydrograph Values at D/S End.....	60

NOTATIONS

g	Acceleration due to gravity
Q_b	Base flow
t_b	Base time
C	Celerity
c_n	Courant number
CFL	Courant-Friedrichs-Lewy
A	Cross sectional area of flow in a channel
S_o	Channel bottom slope
X	Distance along the channel
h_2	Depth at upstream
Z_1	Datum head at upstream
Z_2	Datum head at downstream
h_1	Depth at upstream
Q	Discharge of the channel
Y	Depth of flow of a channel
K_m	Expansion or contraction loss coefficient
k_m	Expansion or contraction loss coefficient
EGL	Energy grade line
Fr	Froude Number

S_f	Frictional slope
Δx	Grid size along the length
Δt	Grid size along the time axis
D_h	Hydraulic depth
R	Hydraulic radius
h_L	Head loss due to local cross sectional changes
h_l	Head loss i.e. minor loss due to cross sectional geometry change
I	Inflow to the channel
q_l	Lateral inflow per unit distance
q_l	Lateral flow per unit distance
h_f	Loss due to friction
h_a	Loss due to acceleration
n	Manning's roughness coefficient
V	Mean velocity of channel
M_c	Momentum fluxes per unit distance exchange between the channel
M_f	Momentum fluxes per unit distance exchange between the flood plain
O	Outflow from the channel
Q_p	Peak flow
t_p	Peak time
Y_1	Stage at upstream
Y_2	Stage at downstream
T	Time
T	Top width of the channel

S	Total storage of at observed reach
X	Weighing factor
B	Width of the channel or river
P	Wetted perimeter of the channel
V_1	Velocity at upstream
V_2	Velocity at down stream
α	Velocity distribution coefficient

CHAPTER 1

INTRODUCTION



INTRODUCTION

1.1 OVERVIEW

Flow is the movement of water from one place to another continuously in a current or stream. Two types of liquid flow are there in the study of hydraulics. First one is the flow within a conduit which has a free surface open to atmosphere, known as channel or river. The second one is pipe flow which is closed in all surfaces or flow within a closed conduit. In many ways the behaviours of these two types of flow are considered and used in similar way. Viscosity and gravity forces have played a major role in open channel and surface tension has a minor effect on flow in open channel. Pressure at free surface is assumed as constant in analysis purposes. In open channel, there are different types of flow including steady, unsteady, uniform, non-uniform, gradually varied, rapidly varied or combined of two or more than these. Steady flow is referred to as the flow in which water depth and velocity does not change with respect to time at a point. Unsteady flow is the flow in which flow parameters in which stage, velocity, and discharge change with time. In channel, flow may be steady but it may be either uniform or non-uniform flow depending on the flow depth and velocity change with space in successive cross-section of the channel. Uniform flow happens only when the cross-section is constant along the waterway. Uniform flow of channel can only occur in a prismatic channel in which cross section, roughness and slope in the flow direction are constant. But the flow in natural channel or river of variable cross section is considered as non-uniform flow. When the depth and velocity change gradually in the flow direction that the vertical acceleration can be neglected in a non-uniform flow is known as gradually varied flow; otherwise it is considered as the rapidly varied flow. Mainly in open channel flow, non-uniform flow occurs for its variable properties.

1.2 EQUATIONS OF DIFFERENT TYPES OF FLOW

Steady Flow

$$\frac{\partial v}{\partial t} = 0, \frac{\partial Q}{\partial t} = 0, \frac{\partial Y}{\partial t} = 0 \quad (1.1)$$

Unsteady flow

$$\frac{\partial v}{\partial t} \neq 0, \frac{\partial Q}{\partial t} \neq 0, \frac{\partial Y}{\partial t} \neq 0 \quad (1.2)$$



INTRODUCTION

Uniform flow

$$\frac{\partial v}{\partial x} = 0, \frac{\partial Q}{\partial x} = 0, \frac{\partial Y}{\partial x} = 0 \quad (1.3)$$

Non uniform flow

$$\frac{\partial v}{\partial x} \neq 0, \frac{\partial Q}{\partial x} \neq 0, \frac{\partial Y}{\partial x} \neq 0 \quad (1.4)$$

1.2.1 FROUDE NUMBER

The term Froude number f_r is a focal term used for knowing the type of flow in open channel. It is defined as the ratio of inertial force to gravitational forces. It is a dimensionless number which is represented by

$$f_r = \frac{v}{\sqrt{gd}} \quad (1.5)$$

$$V = \frac{Q}{A}$$

$$d = \frac{A}{T}$$

For f_r is less than 1, then it is called as subcritical flow. If it is greater than 1 it is characterized as supercritical flow. For Froude number is equal to 1 it referred as critical flow.

1.2.2 CRITICAL FLOW

The flow is critical when the specific energy is minimum. The variation of specific energy with respect to depth at a constant discharge shows a minimum depth which is named as critical depth. So the critical flow is the flow at minimum specific energy. And the critical depth is the depth of maximum discharge when the specific energy remains constant. During this critical flow, the velocity head is calculated and observed that it is half of the hydraulic depth. The general expression for critical flow is

$$Q^2/g = A^3/T \quad (1.6)$$



INTRODUCTION

So it obvious that sometimes flood happens due to the sudden high stage in channel. Often it comes so quickly that an alarm cannot be given before that. At that very moment most natural flows in streams and rivers change slowly with time. Other man made channels and canals have gates that permit a greater or lesser flow through their structures. Now a day to understand and predict the effects of flood in downstream region is a major challenge in hydrology and hydraulics.

In hydrology, the flood is a term generally means a sudden high stage in a river when the flow rate exceeds the maximum capacity of river body .The stage rises the level at which the river overflows its banks and obviously the inundation of the adjoining dry localities take place. The occurrence of flood primary affects on human lives, animals and the devastation of civil structures including residents along with other buildings, bridges, canals, siphons, transportation ways, sewerage systems, water treatment plants, power generation plants. Secondly it causes the most waterborne diseases and leads to shortages of food due to the destruction of crops which is to be harvested.

For prevention and control of flood a routing technique is applied to measure the flood magnitude in downstream areas. Two types of routing of flood are there. So routing is a process of determining the amplitude of a flood wave in every required downstream section resulting from the flood which moves from upstream to successive points along the waterway. It also estimates the magnitude of water flow, velocity and depth in required areas.

1.3 HYDROGRAPH

In unsteady flows, hydrograph is a term which is defined as a graphical representation of flow as a function of time at one concentration point during the flood. In a river network system, channel runoff is continuously transformed as it travels downstream reaches. A hydrograph is represented by a graph showing discharge at the concentration point versus time in river. The hydrograph is also referred as a time series of flow. Within each reaches, hydrograph characteristics change. The time series may be of a long period or only few selected rainfall events of few selected hours or days. . But during the flood, obviously it is the series of flow with a short interval of time due to the excess rain fall occurred frequently. Conventionally it is considered as two components



INTRODUCTION

(Direct runoff and base flow direct runoff and Base flow) of the hydrograph. Direct runoff is the effective rainfall that results directly from the series of rainfall after the losses subtracted from the gross rainfall. Base flow is the flow that is received from ground water storage unrelated to rain fall. Base flow separation involves dividing the hydrograph into a direct runoff component and a base flow component.

At the beginning, there is only base flow which is the ground water contribution to the river which is gradually depleting in an exponential form. After the storm subsides, the initial losses like interception and infiltration are met and then the surface flow begins. The hydrograph gradually rises and reaches its peak value after a time tp . Hydrograph characteristics like flow hydraulics, channel storage, subsurface contribution and lateral inflow. Additional attenuation comes in the presence of flood plain in the river due to overbank storage. Channel flow hydrograph then reshapes as the flow continuously travels to downstream.

From rain fall events if a storm water hydrograph has been generated for an upstream site, then we are commonly asked to predict what is the characteristic of the flood at some downstream observation point or points? Generally there are two things required for the downstream areas that how big will the flood peak be and how much time will the flow take to reach the flood peak? To answer these questions, we must route the flood from the upstream point of observation to the essential points at downstream.

1.4 ROUTING

Mathematically, flood Routing is the prediction of the magnitude, velocity and shape of a flood wave with respect to time at one or more points along a river channel. Due to channel irregularities, change of cross sectional geometry and roughness in bed and side, the flood wave configuration gets modified in required points. Routing is mainly applied to estimate how the proposed measures of magnitude (flow and stage) will affect the characteristics of flood waves in rivers in different localities. The other important purpose are the design of flood protection work, to forecast whether a flood possess a risk to health and safety, to provide information about how much water is present and the physical characteristic of the region so that adequate protection and economic solutions are taken immediately.



INTRODUCTION

Flood routing may be divided into two parts:

1. Flood routing through a reservoir.
2. Flood routing through a channel

Reservoir flood routing is done for accounting the storage availability in the reservoir where the storage and outflow are interdependent. This routing is applied when the inflow rate is higher than the outflow rate from the reservoir. Frequently the storage in the reservoir increases that the storage can be computed. Reservoir Routing is used to determine the peak-flow attenuation or reduction of peak that a hydrograph undergoes as it enters a reservoir. The continuity equation (conservation of mass) is solved for routing in the reservoir.

Flood routing through a channel is also same as the flood routing technique through a reservoir. In very long channels the entire flood wave travels a distance along the downstream resulting in at time interval and time of translation as well. The redistribution due to storage effects modifies the shape of the hydrograph, while the translation changes its position in time. In this routing, the translation and the attenuation are two terms used for describing the flood wave in downstream. Attenuation is a reduction of the peak flow due to dispersion of the flood wave over that interval of time. Flood translation or time lag is referred as the movement of a flood wave through a channel without attenuation caused by the time of travel of the flood wave between the two points. Keeping the peak discharge constant, the delay of the hydrograph occurs in observation points. This routing technique is used to analyze the effects of a channel on a hydrograph's peak flow and travel time. Inflow hydrograph and the channel characteristic data are needed for routing in a channel.

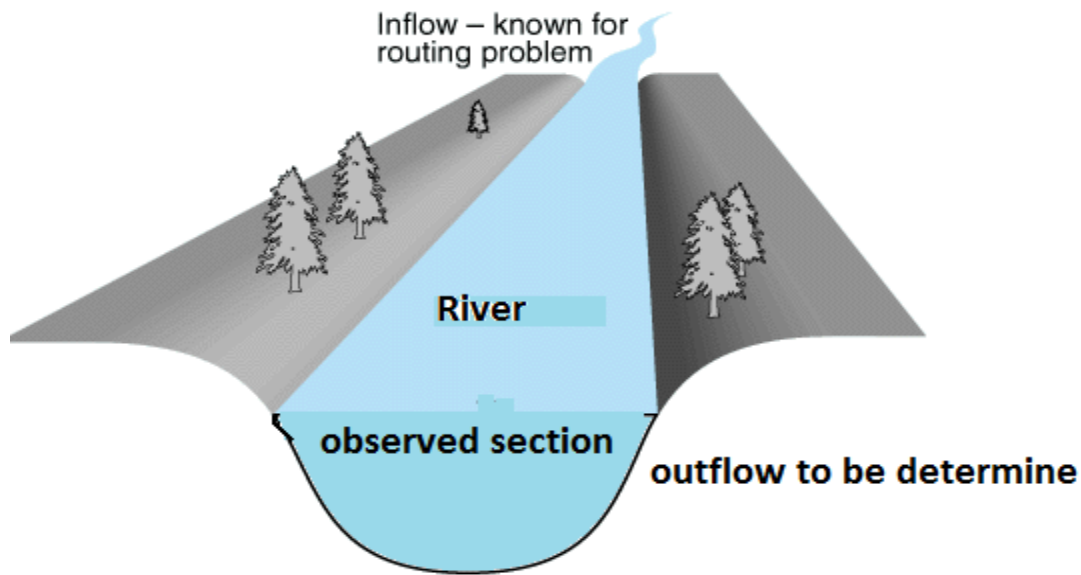


Fig.1.1 Channel Routing

But the accuracy of flood routing is an important subject for research because when the flow phenomenon in a channel is non-uniform and unsteady, it is very difficult to obtain a true picture of the flow hydrograph. Accurate information of the flood peak attenuation and the duration of the high water levels (translation) obtained by channel routing are most important in the application of flood forecasting, reservoir design and flood protection works. For channel routing, two methods are mainly used. A variety of routing methods are available in each group. They are

1. Traditional or hydrological method and
2. Hydraulic routing.

1.4.1 TRADITIONAL OR HYDROLOGICAL METHOD

Hydrologic routing methods employ essentially the equation of continuity, on the other hand hydraulic methods use continuity equation along with the equation of motion of unsteady flow hence better than hydrologic methods. Hydraulic routing is based on the solution of partial differential equations of unsteady overland flow. Saint-Venant equations are used for its solution.

Saint-Venant equations are consists of both continuity and momentum equations.



INTRODUCTION

$$I=O + \frac{\Delta S}{\Delta t} \quad (1.7)$$

Hydrologic method

Hydrological methods for channel routing need the mass balance of inflow, outflow and the volume of storage for the solution. These methods of routing require a storage-stage-discharge-relation to determine the outflow for each time. Hydrological methods involve numerical techniques that determine translation or attenuation to an inflow hydrograph.

There are three types of method in hydrological routing:

Level pool method (Modified Puls)

Muskingum method

Series of reservoir models

1.4.1.1 MUSKINGUM METHOD

The Muskingum method is a routing method of flood, used widely. A direct relationship between the reservoir storage and the out flow is taken into consideration for it's development. Two parts of storage within a reach are present in Muskingum method; they are prism storage and wedge storage. When the water is in steady motion, the storage is known as the prism storage. The one is the storage under actual flow surface.

The fig1 (a) shows the storage during the rising stages of the flood wave. At that another case, in wedge storage, it is either positive storage or negative storage. When it is positive, the storage is added to prism storage. It is mainly occurs in rising stage. In falling stage of water the wedge storage is subtracted from prism storage and refers as negative.

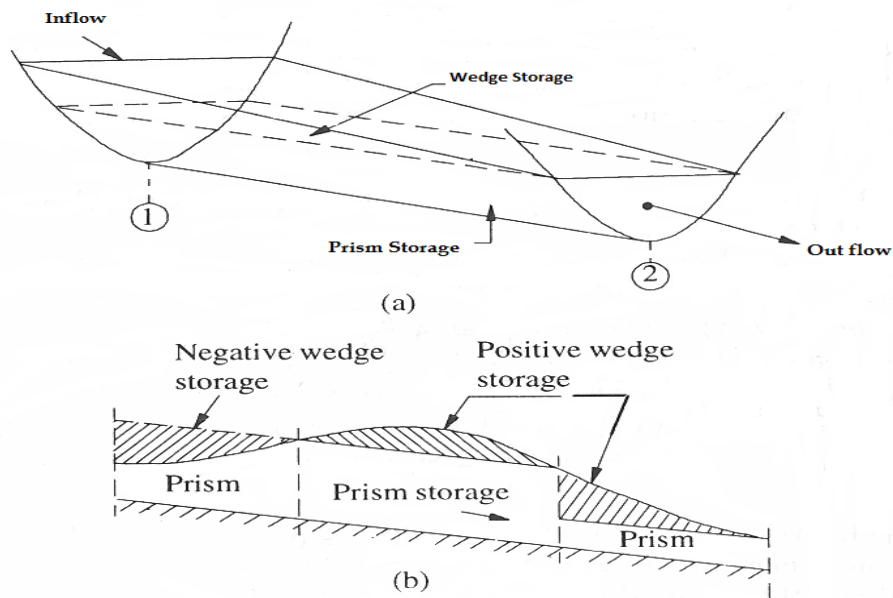
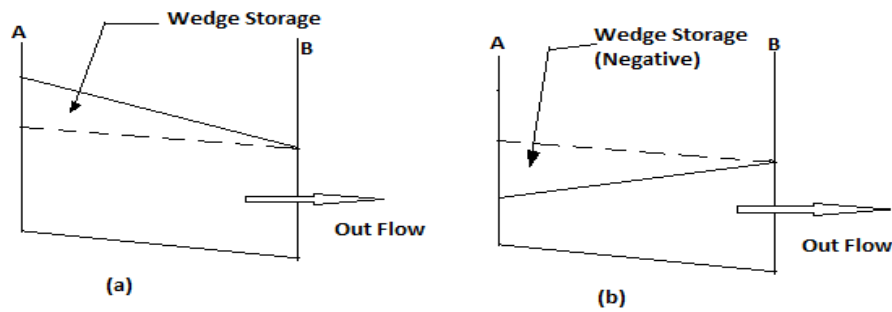


Fig.1.2 Muskingum wedge and Prism storage concept [16]

In the storage reservoir, the existing storage is the function of the outflow. But in case of drainage channel it is assumed that the storage is the function of both inflow and outflow. The water surface varies along the channel. It is not horizontal always. So the prism storage and wedge storage are considered for the knowing the capacity of the reservoir. Prism storage exists in prismatic channel having uniform flow through out. The volume of water between the actual surface of water and the top surface of prism storage is known as wedge storage. If there is a fixed depth at the end of the section, then the wedge storage is changing from positive to



INTRODUCTION

negative storage. There is an assumption that, the flow section during the flood is directly proportional to the discharge.

If the volume of the prism storage is KO , where the K is a proportional constant and the volume of wedge storage is $KX(I-O)$, then the total storage is the sum of that two component.

$$\text{The total storage} = K(XI + (I-X)O) \quad (1.8)$$

Where X is a weighting factor and it ranges between 0 to 0.5

This equation is widely used Muskingum storage equation for flood routing.

The weighting factor X depends on wedge storage. The wedge storage is zero in reservoir and it is taken 0.5 at case of full wedge. It is observed that the value is 0.2 at the case of natural River. K is known as storage time constant is defined as the travelling time of the flood wave along the channel. From equation (1.8) the storage value at the time of j and $j+1$ is written below

$$S_j = K(XI_j + (1 - X)O_j) \quad (1.9)$$

$$\text{And } S_{j+1} = K(XI_{j+1} + (1 - X)O_{j+1}) \quad (1.10)$$

Collecting similar terms and simplifying

$$O_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 O_j \quad (1.11)$$

The equation (1.11) is known as Muskingum equation of channel.

$$C_1 = \frac{0.5\Delta t - KX}{K(1-X) + 0.5\Delta t} \quad (1.12)$$

$$C_2 = \frac{0.5\Delta t + KX}{K(1-X) + 0.5\Delta t} \quad (1.13)$$

$$C_3 = \frac{K(1-X) - 0.5\Delta t}{K(1-X) + 0.5\Delta t} \quad (1.14)$$

$$C_1 + C_2 + C_3 = 1 \quad (1.15)$$

A assumption is there for choosing the time interval Δt that $K > \Delta t > 2KX$ If the value Δt is less than $2KX$. The negative coefficient values are avoided. The coefficient C_1 is generally negative.



INTRODUCTION

For simulation by this method initial and boundary condition is required. So the values of out flow, K and X of the upstream section is necessary.

First step for the procedure to routing is to know K and X by choosing a value Δt . The second method is calculate C1, C2, C3. The inflow, outflow, initial conditions are calculated for next time step. Then the repeat of the procedure is carried out.

1.4.2 HYDRAULIC METHOD

Hydraulic routing is based on the solution of partial differential equations of unsteady open-channel flow. So As Floods in overland flow is very complicated, flood wave routing and solving the governing equations of flood wave propagations are very much complex also. The cause of this complex analysis is due to unsteady non uniform flow, occurred when flood happens so quickly .The wave equations known as the dynamic wave equations are highly nonlinear which solution are difficult in hydrologic case study. The hydraulic models require the gathering of a lot of data related to river geometry and consume a lot of computer resources in order to solve the Saint-Venant equations numerically. Since it is very complex, by this procedure the solution can be obtained only by assuming a uniform flow. Therefore, the results obtained by this method are not accurate.

Now a days to understand, predict and solve the flood wave routing theory including the governing equations in required sections is an important issue in hydrology. Some flows including flood, tidal flow, flow at headrace and tailrace channel, flow below the bridge, flow through are the example of unsteady flow. In unsteady open channel flows and corresponding depth of water changes with respect time and space.

But for one-dimensional applications, the velocity and depth which are two parameters, changes with time and longitudinal distance. Flood wave propagation in overland and open channel flow may be described by the complete equations of motion for unsteady non-uniform flow. As the flow parameters are changing along the waterway it is assumed that flow is one-dimensional. so Saint Venant equation for gradually varied flow used for it. Momentum equations are *dynamic wave* equations. These are derived from principle of continuity and momentum principle which includes the bed and friction slope. To solve this analytically; it is found very much complex.



INTRODUCTION

But for approximate solution, some methods are widely used. For subcritical flow these methods are able to analyze numerically using the standard boundary and initial conditions. A number of existing software solutions are used to simulate the flood characteristics for one dimensional. Internal boundary conditions have two flow equations, related by special conditions and each is associated with two equations describing the water flow. Some of the methods are described later in this method which can be use different technique. The grids are first generated by knowing the value from initial and boundary condition. This condition includes initial elevation value and flow values. For steady and unsteady flow of open channel these condition are applicable. For unsteady flow, it is assumed that flow is steady for approximate result. So before doing unsteady flow analysis, the steady flow analysis is done for carry out values for initial condition for unsteady analysis. But a major problem comes in every analysis i.e. reflection at downstream end. The interpolations of boundary values are taken for known values of boundary.

The equations (Saint-Venant equations) used for hydraulic routing are also called as governing equation. They are

Continuity equation

$$\frac{\partial y}{\partial t} + D_h \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0 \quad (1.16)$$

Momentum equation

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} - S_b + S_f \right) = 0 \quad (1.17)$$

1.4.3 Derivation of Saint Venant equation

The 1-D Saint Venant equation was derived by Adhemar Jean Claude Barre de Saint-Venant, and is commonly used in open-channel flow including surface runoff. They consist of two partial differential equations. The first one satisfies the continuity equation and the second one obtained from Newton's second law that is momentum equation. They are the simplification of the two dimensional shallow water equations, which are also known as the two dimensional Saint Venant equations. Momentum equation is also applied as the energy equation in river hydraulics since



INTRODUCTION

the both equations (momentum equation and energy equation) are derived from Newton's second law of motion or Navier–Stokes equations.

Derivation

The law of continuity for gradually varied unsteady flow is

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (1.18)$$

It is considered that this flow is established in an infinitesimal space dx between two channel section. The discharge Q changes with distance with a rate $\frac{\partial Q}{\partial x}$ and depth changes with time at a rate $\frac{\partial y}{\partial t}$. Thus change of discharge through in space in time dt is $\left(\frac{\partial Q}{\partial x} dx dt\right)$.

Corresponding change in channel storage is space in time dt is $(T dx) \frac{\partial y}{\partial t} dt$ which is equal to $\frac{\partial A}{\partial t} dx dt$. Since $A = T \cdot y$; where A = the flow area; T = top width, y = depth of the flow.

Since water is incompressible, net change in discharge should be equal to the change in storage.

$$\frac{\partial Q}{\partial x} dx dt = -\frac{\partial A}{\partial t} dx dt \quad (1.19)$$

$$\text{Dividing by } dx dt \quad \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (1.20)$$

In a channel, top width t and bottom width b are continuous function of y . The change in flow area dA and for small change in flow depth dy may be approximated as $B dy$. So the Equation (1.20)

$$\text{May be written as } \frac{\partial Q}{\partial x} + \frac{B \partial y}{\partial t} = 0 \quad (1.21)$$

If there is volumetric inflow for outflow rate of q_1 per unit length of the reach dx .

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \pm q_1 \quad \text{or} \quad (1.22)$$

$$\frac{\partial Q}{\partial x} + \frac{B \partial y}{\partial t} = \pm q_1 \quad (1.23)$$



INTRODUCTION

We know $Q=AV$, substituting in first term $\frac{\partial Q}{\partial x}$, expanding and noting that $\frac{\partial A}{\partial t} = B \frac{\partial y}{\partial t}$

Hydraulic depth $D=A/B$, The equation (1.23) may be written as

$$\frac{\partial y}{\partial t} + D_h \frac{\partial v}{\partial x} + V \frac{\partial y}{\partial x} = 0 \quad (1.24)$$

This equation is known as Continuity Equation

Dynamic Equation:

This equation

$$z_1 + y_1 + \frac{v_1^2}{2g} = z_2 + y_2 + \frac{v_2^2}{2g} + h_l \quad (1.25)$$

Loss in unsteady flow is due to friction and acceleration i.e.

$$\text{Loss} = h_f + h_a \quad (1.26)$$

$$s_f = \frac{h_f}{\partial x} \quad (1.27)$$

$$\therefore h_f = s_f * \partial x \quad (1.28)$$

In unsteady flow, velocity varies with time, $\frac{\partial v}{\partial t}$ exists. This $\frac{\partial v}{\partial t}$ is the acceleration and due to creation of this acceleration, a force is to be produced causing loss of energy and this loss of energy is h_a in head form per unit weigh

From the Newton's second law of motion the force term (F)/unit time= $m * a$ /unit time

Where m =mass of the flow and a =acceleration due to gravity are both vector quantity

Since the acceleration is the time rate of change of velocity, then

Newton's second law of motion is written as

$$F = \frac{1}{g} \times \frac{dv}{dt} \quad (1.29)$$



INTRODUCTION

Work done by this force = force * distance

$$F = \frac{1}{g} \times \frac{dv}{dt} \times \partial x \quad (1.30)$$

This work done is the energy lost due to acceleration h_a

$$h_a = \frac{1}{g} \times \frac{dv}{dt} \times \partial x \quad (1.31)$$

Substituting h_f and h_a in equation 1.25

$$z_1 + y_1 + \frac{v_1^2}{2g} = z_2 + y_2 + \frac{v_2^2}{2g} + \frac{1}{g} \times \frac{dv}{dt} \times \partial x + S_f \times \partial x \quad (1.32)$$

$$[y_2 - y_1] + [z_2 - z_1] + \left[\frac{v_2^2}{2g} - \frac{v_1^2}{2g} \right] + \frac{1}{g} * \frac{dv}{dt} * \partial x + S_f * \partial x = 0 \quad (1.33)$$

$$\partial y + \partial z + \partial \left(\frac{v^2}{2g} \right) + \frac{1}{g} * \frac{dv}{dt} * \partial x + S_f * \partial x = 0 \quad (1.34)$$

$$\frac{\partial y}{\partial x} + \frac{\partial z}{\partial x} + \frac{\partial}{\partial x} \left(\frac{V^2}{2g} \right) + \frac{1}{g} \left(\frac{\partial v}{\partial t} \right) + S_f = 0 \quad (1.35)$$

$$g \frac{\partial y}{\partial x} + g \frac{\partial z}{\partial x} + \frac{2V\partial V}{2\partial x} + \frac{\partial v}{\partial t} + g * S_f = 0 \quad (1.36)$$

Since

$$\frac{\partial z}{\partial x} = -S_b \quad (1.37)$$

$$\text{So, } \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} - S_b + S_f \right) = 0 \quad (1.38)$$

This is Saint-Venant shallow water wave equation or dynamic equation of GVF. These describe the gradually varied flow of an incompressible in viscid fluid. As it formed by applying Newton's second law of motion a no of fundamental assumptions which are summarized as

- Flow is one-dimensional such that the velocity is constant over a cross section and water level is horizontal
- Vertical accelerations are negligible so that Hydrostatic pressure prevails that is the streamline curvature is small.



INTRODUCTION

- The channel is prismatic, i.e., the channel cross section and the channel bottom slope do not change with distance. The variations in the cross section or bottom slope may be taken into consideration by approximating the channel into several prismatic reaches.
- Friction or resistance effects and turbulence can be represented by using the steady state flow i.e. Manning's equation.
- Bottom slope or bed slope of the channel is small resulting in the cosine of the angle between the bed levels.

1.5 FRICTIONAL SLOPE

Frictional slope or the slope of the energy grade line (EGL) is defined as the line showing the total energy at any considered point in a river. This line represents the elevation of the total head (total energy) of flow at that section. The total energy is calculated from a reference known as the datum line. Mathematically it is the sum of the depth of flow, the piezometric height (or pressure head i.e. $p/\rho g$), and the velocity head (kinematic head i.e. $V^2/2g$). The energy grade line is the drop in slope for which the flow occurs in a channel. This slope of the line is known as the energy gradient. But in open channel pressure is zero so only water elevation and velocity head are taken into account.

If water elevation and velocity are h_1 and v_1 respectively in one section at upstream and the corresponding values for second section at downstream are h_2 and v_2 then by energy equation

$$h_1 + \alpha_1 \frac{v_1^2}{2g} = h_2 + \alpha_2 \frac{v_2^2}{2g} + h_l \quad (1.39)$$

The total head loss is calculated from

$$h_l = k_m \left[\alpha_1 \frac{v_1^2}{2g} - \alpha_2 \frac{v_2^2}{2g} \right] + h_1 - h_2 \quad (1.40)$$

1.6 APPLICATION OF THE SAINT VENANT EQUATION

- The 1-D Saint Venant Equation is commonly applied in dam break analysis, storm pulses in an open channel, as well as storm runoff in overland flow.



INTRODUCTION

- Simplifying the 1-Dimensional equation is used exclusively in models including HEC-RAS computer model, MIKE, and MIKE SHE because it is significantly easier to solve than the full shallow water equations.
- The propagation of flood waves through reservoirs and along rivers.
- The discharge released from hydro-power plants.
- The propagation of storm-induced floods in estuaries i.e. in flood routing problems.
- Saint Venant equation is commonly used to model open-channel flow and computation of surface runoff.

1.7 BOUNDARY CONDITION AND INITIAL CONDITION

The data required for unsteady flow analysis are boundary conditions for both external boundaries and the initial condition. Initial flow (base flow) and water depth at zero time level are applied as the initial condition at the beginning of the simulation. Boundary conditions are entered first because it is a given hydrograph, available from data.

A flow hydrograph can be used as the upstream boundary condition or downstream boundary condition. But it is commonly used as upstream boundary condition. For a reach of river, there

are n computational nodes, which bound $n-1$ finite difference cells. From these cells $2n-2$ equations can be developed. But another two equations are necessary for solution. These equations are provided by the boundary conditions given at upstream and downstream reach.

These equations are required at upstream boundary end and downstream boundary end for subcritical flow. For the supercritical flow, boundary conditions are applied in upstream end only.

The equation of flood hydrograph (the discharges verses time) is needed for upstream boundary at all reaches which are not connected to any other reaches. But for downstream, there are several boundary conditions available for the problems. There are different types of conditions can be specified for the both upstream and downstream boundaries.



INTRODUCTION

- 1) A flow hydrograph
- 2) A stage hydrograph
- 3) Stage and Flow hydrograph
- 4) A single valued rating curve
- 5) Normal depth from Manning's depth
- 6) Critical flow depth

1.7.1 Flow hydrograph

A flow hydrograph can be used as the boundary condition for upstream or downstream boundary. But it is commonly used as the upstream boundary.

1.7.2 Stage hydrograph

The hydrograph resulting from water depth with variation of time is used as either in upstream boundary or in downstream boundary.

1.7.3 Stage and Flow hydrograph

Both stage and Flow hydrograph of upstream section can be used combinedly for upstream boundary. Both stage and Flow hydrograph of downstream section can also be used combinedly for downstream boundary.

1.7.4 Rating curve

Rating curve is a graphical representation of discharge versus stage for a concentration point on a river. The flow is measured across the river channel with a flow meter. Then numerically the corresponding stages are calculated. This curve is a single valued relationship. This curve is only used as the boundary condition for downstream.



INTRODUCTION

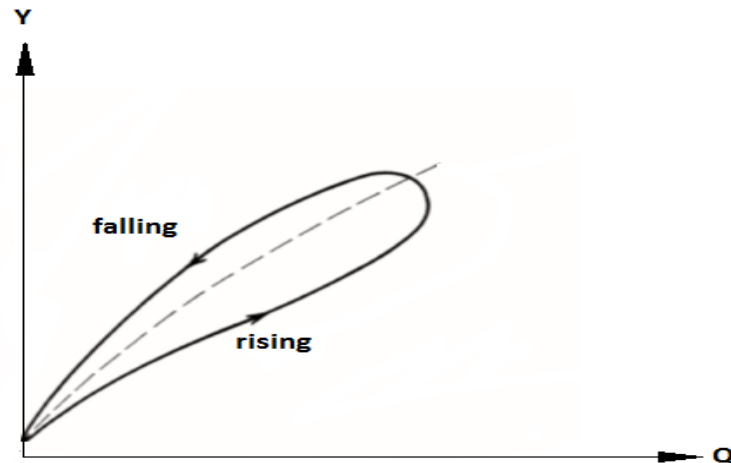


Fig.1.3 Rating curve

1.7.5 Normal depth from Manning's equation

The normal depth obtained from Manning's equation (for uniform flow) as downstream boundary condition for an open ended reach where the river is too long. From Manning's n the stage value can be estimated for each computed flow. The friction slope is required for the evaluating of normal depth. For friction slope (s_f) the Manning's coefficient or roughness coefficient (n) is required

$$S_f = \frac{n^2 v^2}{R^{4/3}} \quad (1.41)$$

1.8 SELECTION AND APPLICATION OF ROUGHNESS COEFFICIENT

The information is found from the U.S. geological Survey which are available here. For all hydraulic computations the roughness characteristic are required in open channel. They have computed the entire roughness coefficient by observing channel geometry and site condition for different channels. The selection of roughness coefficient are thus useful in estimating the roughness characteristics of similar channel. So it is very helpful to engineers to consider and apply a roughness where the geometry appearance and roughness characteristics of the channel are known.

Manning $n = 0.024-0.027$



INTRODUCTION

Description of channel:- Bed consists of smile covered cobbles and gravels deposits over smooth to rough rock, banks are composed of clay, short grass and exposed tree roots.

Manning $n=0.028-0.032$

Description of channel:-Bed consist of well-rounded boulders sand, gravel and boulders; $d_{50}=135-175\text{mm}$, $d_{84}=204-325\text{mm}$. Banks are composed of gravel and boulders ,and have tree and brush cover.

Manning $n=0.033-0.036$

Description of channel:-Bed consists of coarse gravel and cobbles with scattered boulders with some exposed bedrock. If the channel is bordered by railroad also it has n value 0.33 to 0.36. The Banks are made of gravel and rock and also have light vegetation .Bed is sand and gravel with several fallen trees in the reach. Banks are lined with overhanging trees and underbrush.

Manning $n=0.036-0.041$

Description of channel:-Bed is gravel and boulders well-rounded small boulders. $d_{50}=172-195\text{mm}$; $d_{84}=265-360\text{mm}$. The left bank is lined with overhanging bushes. The right bank is lined with trees and mildly sloped and has some boulders, brush, and weed cover.

Manning $n=0.036-0.041$

Description of channel:-Bed is gravel and boulders well-rounded small boulders. $d_{50}=172-195\text{mm}$; $d_{84}=265-360\text{mm}$. The left bank is lined with overhanging bushes. The right bank is lined with trees and mild sloped and has some boulders, brush, and weed cover.

Manning $n=0.041-0.043$

Description of channels:-Bed is composed of sand, gravel and small boulders with scattered large angular rocks $d_{50}=93\text{mm}-142\text{mm}$, $d_{84}=157\text{mm}-285\text{mm}$. Banks are composed of gravel and boulders, also having trees and brush along the tops. Banks are irregular and eroded, and have sparse cover of grass.



INTRODUCTION

Manning $n=0.044-0.050$

Description of channel:-Bed is sand and gravel with several outcrops in the reach and mostly coarse sand. Banks are steep and lined with overhanging trees and bushes angular boulders as much as 2ft in diameter and fairly steep and contain medium growths of underbrush and large trees.

Manning $n=0.051-0.060$

Description of channel:- If the bed consists mostly rock and very irregular size of 5ft in diameter of coarse sand and a few outcrops. Banks are lined with boulder, small trees, and bushes and are heavily lined with overhanging birch trees. Bed and bank consist of boulders $d_{50}=210\text{mm}, d_{84}=375\text{mm}$

6. Critical flow depth

Critical flow depth at the end section (downstream)of the channel is considered for the downstream boundary condition. The critical-flow depth is occurred when the channel or open channel flow ends at a steep bank of the channel. Because the critical-flow depth is the depth at which the flow occurs due to minimum energy.

Mathematically for critical flow condition

$$Q^2/g=A^3/B \tag{1.42}$$

$$Q(L, t) = \sqrt{\frac{gA^3(L,t)}{B}} \tag{1.43}$$

1.9 AIM OF THE PROJECT

It is concluded from the review of all literatures that, numerical works are very less in research area now for its complexity. It is a qualitative work in worldwide. It is a matter of concern that the program writing is very much hard. But the present research here is a simple explicit method which gives accurate result. The objectives of the present research work are:



INTRODUCTION

- Discretization of Saint-venant equation (both continuity and momentum) by finite difference method (Lax diffusive) by using CFD (computational fluid dynamics) tool.
- Approaching this explicit forward difference method to a natural river channel i.e. to demonstrate the potential of the methods used in this research, models are applied to simulate flood routing problems in a wild river basin Persian Gulf area.
- Applying MATLAB software for routing the flood and find out the flood hydrograph (both flow hydrograph and depth hydrograph) in different downstream study section.
- Comparison of this result obtained from present approach with HEC-RAS Computer model

1.10 ORGANISATION OF THE THESIS

This research paper is the combination of 6 main chapters. The first chapter contains the general introduction, overview of literatures are given in chapter 2, numerical methods and analysis are described in chapter 3, chapter 4 comprises problem statement and its solution with result, conclusion and future study are presented in chapter 5 and finally references are given in chapter 6.

A brief study about open channel flow with different types of flow is discussed in first chapter. It also includes hydrograph definition, types of flood routing technique, application of Saint-Venant equation in different field, boundary condition and initial condition and at last the selection of roughness coefficient.

Many eminent scientists and researcher's literature which are related to the present research are given in chapter 2. This chapter emphasizes the research related to numerical works and various flood routing techniques in different field.

Various numerical methods with their derivation are presented in chapter 3. It also describes the characteristic curve method which is most important procedure when solving in finite difference explicit scheme. Also detail methodology about the HEC-RAS model is given in this chapter.



INTRODUCTION

Application of the lax diffusive method in a practical field and its process are outlined in chapter 4. Generating grids, setting the initial and boundary condition in proper place are also included in this chapter. The results coming from this approach is compared with HEC-RAS numerical model. The figures and tables are presented here.

The discussion about the results and conclusion are finally presented in chapter 5. The recommendation for future study is listed out here.

References that have been made in different chapters are provided in last chapter i.e. chapter6.

CHAPTER 2

LITERATURE REVIEWS



2.1 OVERVIEW

Scientists and researchers are doing and approaching many different ways for solving numerical method in water resources engineering. The methods are very much complex in the case of unsteady flow. But among them some are easy to understand and predict the flood wave by simulation process. Scientists and researchers are approaching a better developed method day by day by modifying the previous one. In computational fluid dynamics, hydraulic flood routing problem is solved by Saint-Venant equation which includes 1D unsteady continuity equation and momentum equation. These equations may be described by the complete equation of motion for unsteady non uniform flow, known as dynamic wave equation which is proposed by Saint-Venant in 1871. But now routing the flood is a challenging area for research in water resource engineering. The flow characteristics expressed by the momentum equation terms, are dimensionless (Woolhiser & Liggett, 1967). In flood routing problems the Saint-Venant equation is solved by Preissmann four point implicit finite difference scheme in channel and flood plains (Rashid & Chaudhry, 1995). Method of characteristics is applied for solving the 1-D shallow water equation which is mostly used in explicit method (H. Eihanty, G.J.M Copeland 2003). The principal objective of this report is to present descriptive method to simulate the flood flow in downstream where the observation takes place.

2.2 PREVIOUS WORK ON FLOOD ROUTING

R. W. Carter and R. G. Godfrey (1960) worked on storage and flood routing. They considered a reach, where the stage storage method is associated with the mean gauge height. From predetermined inflow and outflow discharge the storage capacity is determined in this method. As the basic equation i.e. the law of continuity is used in reservoir routing process, the storage is considered and as well as determined as a function of outflow discharge only. Before selecting an appropriate technique, some assumptions are discussed to make enable to the user. The storage at every reaches of river channels is used extensively as an index of the duration and shape of flood waves at all points along a river. For computing and evaluating

stream flow records this storage index and the techniques of the flood routing are used. For long term gauging stations it is also used in hydrologic studies for computing natural flood



LITERATURE REVIEWS

hydrographs at all successive points on the river where major storage reservoirs have been constructed.

Zbigniew Kundzewicz (1983) determined Hydrodynamic Determination of Parameters of Linear Flood Routing Models. They have done different methods of hydrodynamic determination of parameters such as approximate hydraulic and hydrological flood routing models are analyzed and compared with respect to assumptions and the final equations for parameters. It is shown that the values of parameters obtained by different methods of physical interpretation are close to each other. Thus the problem of method for hydrodynamic determination of parameters is not critical. The physical interpretation of parameters of approximate flood routing models is cheaper than other methods of identification. Thus, in cases where accuracy requirements are not strictly attention to the simplifications discussed in the paper can be considered useful and practical. The hydrodynamic equations of open channel flow formulated by de Saint Venant in 1871 have been widely accepted as a faithful representation of the process. However this model puts severe demands on quantity and quality of data as well as on computational requirements. That is why the applicability of the complete nonlinear equations is restricted and numerous approximate flood routing models are being developed. Therefore it is very useful to establish the relationships between the parameters of linear flood routing models and the hydrodynamic parameters with full physical significance. One of the crucial steps in the process of modeling is the determination of model parameters. In the case of hydrodynamic models the parameters with full physical significance. One of the crucial steps in the process of modeling is the determination of model. In the case of hydrodynamic models the parameters have a physical sense and can be either measured or assessed in the field. The parameters of approximate conceptual or black box system models are usually identified from inflow and outflow data. There is a large number of techniques for identification of model parameters at one's disposal. However, due to the ill-posedness of the inverse problem and badly shaped topography of the criterion function of the optimization task, identification of the model parameters is not simple.

Muthiah Perumal (1994) worked on Hydrodynamic Derivation of a Variable Parameter Muskingum Method. They approached to present a variable parameter directly by deriving. The reservoir routing Muskingum method is used in this paper. prismatic cross section of varying



LITERATURE REVIEWS

shape from the Saint-venant equation for flood routing in channels and uniform flow obeying either Manning's or Chezy's law are observed well in this channel. This approach can also be used for the simultaneous computation of the stage hydrograph where there are corresponding inflows are given or the routed hydrograph is present. The first paper briefly describes the solution procedure for the discharge hydrograph and the second paper (Perunal, 1994) presents a verification of that methodology and used in a better way.

R.S.M Mizanur, M.Hanif Chaudhry(1995) carried on flood routing problem in main channel and flood plain. For simulation they modeled using Saint venant equation of continuity and momentum. They assumed the flow as one dimensional. The finite difference scheme is i.e. pressimann implicit method is used for the computation of flood hydrograph at 9 different sections along the channel. For boundary condition the upstream hydrograph values are found out by electrified butterfly valve in supply pipe. They observed the variation of depth and flow at 9 sections by using capacitance probes and a computerized data attainment system. The initial conditions are taken by conducting experiment .For verification of the result the two tests datas are presented in this paper.

Piter L.F. Betura & Claude Michel (1997) used the quadratic lag- and- route method for flood routing study. Experiment is done in a wide channel. Hydrologic flood routing method is used. Although the parameter of lag-and-rout method is not used in physical parameter of the channel but extensively it is used now a days. They proposed a new version of that method by assuming the storage characteristic of the reservoir is quadratic.

DambaravjaaOyunbaatar, GomboDavaa, DashzevegBatkhoo (2004) concluded with some results on application of flood routing models in the Kherlen River Basin. This research paper is based on application of some flood routing models in the Kherlen River. For reconstruction of missing hydrograph or for forecasting of the possible outflow rate at downward of a specified time interval, flood routing model are normally used. Linear regression model is implemented to use for forecasting by updating with the given equation with another new flow. Muskingum-Cunge model is used for more detailed analysis and measurement for estimate the different parameters. The origin of Kherlen River is started from southern slope of Khentei mountain at a approximate elevation with 1750 m and drains into Dalai nuur of china. The basin area of river in



LITERATURE REVIEWS

Mongolia territory is 116455 km² with length 10190 km. Generally, surface runoff in this river up to 56-76% in the form of rainfall in sunny period and snow melts in spring periods. The upper forest area is the main portion of runoff. Runoff of this river is gradually losses through the slope with sandy soil by infiltration and also by evaporation. The highest flood discharge of Kherlen River reaches 1320 m³/sec at Baganuur station in the year 1933,1954,1959,1967 etc.

P.Sreeja, Kapil Gupta (2008) experimented on a drainage channel by analyzing Saint-Venant equation. They worked on the transfer function formulation in the flood study of urban cities. They initiate the space constrains for the control of the flood flow so as to utilize the full capacity of the river water at affected areas. For optimization the existing structures like gate are kept for controlling the flood flow. They also studied different flow and their condition in laboratory model by attaching suitable gates in upstream and downstream. The result found by them is so accurate and the experiment result is matched with the natural river study result. These results using simplified method matches correctly with the experimental observed values.

Muthiah Permal, Bhabagrahi Sahoo, Tommaso Moramarco, Silvia Barbetta (2009) worked on hydrologic routing i.e. Muskingum method which is used for the simulation of flood hydrograph at different sections. They used multi linear Muskingum method for analysis. They analysed in a compound channel by using time consumed scheme. It is similar with the Muskingum method of linear flood routing. The comparison purpose they choose the river Tiber and one experiment in laboratory.

D. NageshKumar , Falguni Baliarsingh, K. Srinivasa Raju (2010) researched on Muskingum method but in a better way i.e. extended Muskingum method for routing purpose in Hirakud reservoir, Mahanadi. They routed the flood from upstream. To control the flood flow in reservoir release i.e. spillway is observed. Another uncontrollable inflow is observed which is coming from lower tributary. They used a linear programming for derivation of the coefficients which are used for inflow at upstream.

Safa Elbashir (2011) worked on Flood routing in natural channels using Muskingum methods. Accurate information of the flood peak attenuation and the duration the high water levels obtained by channel routing are of most important in flood forecasting operations and flood protection works (Subramanya, 2008.) This study implements two hydrological methods for channel routing, the basic Muskingum and the constant coefficient Muskingum-Cunge methods



LITERATURE REVIEWS

on the River Brosna, Co. Offaly in Ireland. Previous researches have reported the simplicity and applicability of these methods on most natural streams within certain limits. These limitations are encountered in the River Brosna where the available outflow data included a significant degree of error which makes it difficult to use for comparison and modeling purposes. Moreover, other factors influenced the implementation and the accuracy of these methods, in particular the backwater effects due to a weir located nearly four kilometers upstream the selected reach and the gradient of the channel which was very small (0.00047) to dampen the error in the routing procedure. This error is found to be greater when using a minimum time increment in the routing calculation. The results of this study showed that the hydrological methods failed to simulate the outflow hydrograph in the selected reach. Determining the models parameters was not possible by using the basic Muskingum method, whereas, the constant coefficient Muskingum-Cunge method calibrated some negative values for the attenuation, which contradicted the diffusivity of the flood wave and confirmed the significant effect of the weir located downstream the river. The conclusion is that an alternative method is needed to account for the factors that these methods neglect.

Mehdi Delphi (2011) applied Characteristics Method For Flood Routing (Case Study: Karun River). This is the case for flows in rivers or channels and of the ground run off. The transition of turbulent Navier-Stokes equations are tackled by using the Leibnitz formula. A series of approximation allowing to neglect one or several terms of the Saint-Venant equations is presented. In this study we used Characteristics method as a simplified form Kinematic wave equation for flood routing in the length 61 KM of Karun River. Moreover the process is carried out by MIKE 11 model for estimating the accuracy and agreement of the method together. The results showed that the Characteristics method is applied for this reach of river. In engineering, predict the flow as well as the depth at the time of flood is a challenging chapter now. The objective of analysis of flood routing is to know the flood hydrograph and stage hydrograph at successive points along the waterway. They determined the both hydrograph values by solving the both continuity and momentum equation of Saint Venant. For solving that partial derivative equation analytically, some terms are ignored. They removed the inertia terms by converting the hydraulic equation into diffusive equation. Although the flow is three dimensional but for analysis purposes, it is assumed as one-dimensional as it follows a certain path. This flood



LITERATURE REVIEWS

routing analysis is important mainly because of the urbanization near the river bank. Particularly for the unobstructed river channel the flow comes to the flood plain. So the storage characteristic has a major effect on that. This storage characteristic depends on channel cross section along with the flood plain. But for unavailability of the accurate data about the channel geometry, by some empirical relationship the wetted flow is determined and then cross sectional size is found. Then the routing process is carried on. The classic method to solve nonlinear equation of partial differences equations with two variables is known as Characteristics Method.

Junqiang Xia, Binlian Lin, Yanping Wang (2012) modeled a flood routing study manually in the lower yellow river. In this study the flood routing is done by finite volume method but in a improvement procedure which is known as spatial reconstructed method. It is a two dimensional study which is applied in flood routing problem of the river in the year of 2004 and 2006. For simulation of flow the manmade model is applied. Observation is taken between the model result and the natural river routing result. Relatively same result comes in both predictions. They also used different roughness coefficient values for different channel condition.

Doiphode Sanjay L, Oak Ravindra A (2012) worked on Dynamic flood routing and unsteady flow modeling: a case study of upper Krishna river. The movement of a flood wave in a river channel is a highly complicated process of unsteady and non uniform flow. Flood routing is a mathematical method (Model) for indicating the change in magnitude, shape and celerity of a flood wave which propagates through a river. The HEC-RAS is capable of performing one-dimensional water surface profile calculations for unsteady flow for a full network of channels, a branching system or a single river reach. Unsteady flood model in HEC-RAS was set up using available survey data of Krishna and Koyna River. The Krishna River reach of length about 233 Km from the downstream of Dhom Dam to Sangli city was considered for the flood routing studies. For Koyna River the reach from downstream of Koyna dam to Krishna –Koyna confluence at Karad was also incorporated. In the year 2005, maximum flood occurred in the Sangli. For upstream boundary of the model the available flood release hydrographs of the worst year 2005 from various reservoirs in upper reaches of Sangli was used. Contribution of discharge from the tributaries and the local catchments was also incorporated. Calibration of model was done using the gauge discharge data available at Karad and Sangli. The results of model run



LITERATURE REVIEWS

showed that the stage and discharge worked out from the model had a good agreement with observed stage and discharge. Hence, it was concluded that the model set up can be reliably used to get the flood flow profiles at Sangli. Studies were also conducted to estimate the changes in the hydrographs under the estimated worst scenarios. The analysis of available flood data was done to identify the flood sensitivity of Sangli, due to Koyna and Dhom dam flood water release. It was found that a flood situation at Sangli mainly depends on the water release from Koyna dam. Maximum limit for the flood release from Koyna dam could be 1690 cumecs, so that the flood level would reach the danger level of 540.77m at Irwin Bridge Sangli.

Val'Érie Dos Santos Martins, Mickael Rodrigues, Mamadou Diagne (2012) developed a multi-model approach to saint-venant equations with a stability study by LMIS. They worked with the study of the nonlinear Saint-Venant Partial Differential Equation (PDE). The proposed approach by them is based on the multi-model concept. This concept which takes into account is some Linear Time Invariant (LTI) models defined around a set of operating points. This method allows describing the dynamics of this nonlinear system in an infinite dimensional space over a wide operating range. A stability analysis of the nonlinear Saint-Venant PDE is proposed both by using Linear Matrix Inequalities (LMIs) and an Internal Model Boundary Control (IMBC) structure. The method is applied both in simulations and real experiments through a micro channel, thus the theoretical results developed in the paper. The multi-model structure is well adapted for nonlinear systems because it allows determining a set of linear models defined around some predefined operating points. Each local model (sub-model) is defined as a Linear Time Invariant (LTI) model dedicated to a specific operating point. The multi-model is based on weighting functions, which ensure the transition between the different local models. These functions represent the degree of validity of each local model. This degree is a function of the system inputs, outputs and time. The data used are those of the water channel of Valence. Comparisons between initial experimental results using a PI-controller (done some years ago) and simulations with the presented integral controller using the theoretically tuned gain are realized. New experiments are implemented, too, with these theoretical gains found by LMI ideas.

CHAPTER 3

NUMERICAL METHODS

AND ANALYSIS



3.1 OVERVIEW

In the study of open channel i.e. the unsteady flow which is described by two nonlinear hyperbolic partial differential equations depends on the variables i.e. flow velocity (v) and flow depth (y). These set of equation is the partial derivatives of time and distance. But for practical applications in field, It is required to know the value of the variables instead of the values of their derivatives. Excepts for some simplified case a closed form solution of these equations is not available. Therefore the governing equations are solved numerically for which different numerical methods can be possible for the complex solution. In this chapter various numerical models with its solutions are given which can be applied in practical field for simulation of flood.

Various numerical methods are given below. They are

- 1) Method of characteristics
- 2) Finite element method
- 3) Finite difference methods
- 4) Finite volume method
- 5) Spectral method

The characteristic method is used in different field. It is used the finite difference methods and finite element method in some ways. The numerical integration of nonlinear hyperbolic partial differential equations are solved by these three method. For the analysis of unsteady open channel flow present research is based on characteristic equation and finite difference method. So the detail study about the both two are given below:

3.2 METHOD OF CHARACTERISTICS

This Method is a graphical procedure for the solution for partial differential equations developed by Monge (1789) and named these procedures as the method of characteristics. It was used by Massau (1889) and Criya (1946) for analyzing surveys (both positive and negatives in open channels. It was used to investigate the propagation of flood waves in upstream and downstream



NUMERICAL METHODS AND ANALYSIS

in open channel and unsteady flow problems. Frequently all are using finite difference schemes as a method to solve shallow water equations for flow in open channels. The concept of characteristics curves is mostly applied in research field. It can be helpful in understanding the wave propagation at upstream and along the river. It is used for the explicit finite difference methods.

The Saint Venant equation for prismatic channels have been no lateral in flow or outflow given by

$$\frac{\partial y}{\partial t} + D_h \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} - S_b + S_f \right) = 0$$

By multiplying the continuity equation by an unknown multiplier, and adding it to momentum equation, the rearrange and the resulting equation obtain is

$$\frac{\partial v}{\partial t} + (v + \lambda D) \frac{\partial v}{\partial x} + \lambda \left[\frac{\partial y}{\partial t} + \left(v + \frac{g}{\lambda} \right) \frac{\partial y}{\partial x} \right] = g(S_0 - S_f) \quad (3.1)$$

Since $v = v(x, t)$ and $y = y(x, t)$,

The total derivatives of v and y are

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{dx}{dt} \quad (3.2)$$

$$\frac{Dy}{Dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} \frac{dx}{dt} \quad (3.3)$$

When comparing the equation (3.1) with equation (3.2) and equation (3.3) it shows that there are two total derivative term in equation 3.1

The first two terms of equation 3.1 represents the total derivatives of v and the last two terms inside the bracket represents the total derivatives of y

So $\frac{dx}{dt}$ and unknown multiplier λ can be defined as



NUMERICAL METHODS AND ANALYSIS

$$\frac{dx}{dt} = v + \lambda D = v + \frac{g}{\lambda} \text{ and} \quad (3.4)$$

$$\lambda = \pm \sqrt{\frac{g}{D}} \text{ or} \quad (3.5)$$

$$\lambda_{1,2} = \pm \sqrt{\frac{gB}{A}} = \pm \frac{g}{c} \quad (3.6)$$

as D = hydraulic depth= A/B and c =celerity

3.2.1 CELERITY

One-dimensional unsteady flow in an overland flow can be simulated with the governing equations however; certain conditions are there which require additional effect in the equations of motion. These conditions include the effects caused by celerity which is the main focused parameter in free-surface flows. For understanding the flood wave propagation in the channel in different type of flow (critical, subcritical and super critical), it is of great importance to know the celerity of a small wave. The term *celerity* is defined as the velocity of the progressing wave with respect to velocity of stationary water at steady state of the medium in which the wave is traveling. Mathematically the celerity (c)

$$C = \sqrt{gy} \quad (3.7)$$

From Froude no 3 types of flows are determined.

$$F_r = \frac{v}{\sqrt{gy}} = \frac{v}{c} \quad (3.8)$$

In subcritical flows, F_r is less than 1 .So it follows that $V < c$ in these flows. For critical flow F_r is equal to 1 so the term *celerity* of a small wave is equal to the flow velocity in the medium when the flow is critical .Similarly, it may be written that $V > c$ in the supercritical flows as F_r is greater than 1 for supercritical flow. So for the three type of flow different flow situations are possible for the wave propagation depending upon the relative magnitudes of the stationary water velocity V and celerity c , whether the flow is subcritical, critical, or supercritical. These three cases are



NUMERICAL METHODS AND ANALYSIS

- In subcritical flow, the wave travels in both directions. Wave carries a velocity $(V - c)$ in the direction of upstream and $(V + c)$ towards downstream direction since $V < c$.
- In critical flow, since $V = c$, the wave remains stationary at upper end of the medium and travels in the downstream direction at velocity $V + c$.
- In supercritical flow, since $V > c$, the wave carries at velocities $(V - c)$ and $(V + c)$ at the upstream and the downstream ends respectively. In other words, flow carries the wave towards downstream but does not carry in the upstream direction.

So from equation 3.6 we found that

$$\lambda_1 = \frac{g}{c} \text{ And } \lambda_2 = -\frac{g}{c} \quad (3.9)$$

So the equation 3.1 becomes

$$\frac{Dv}{Dt} + \frac{g}{c} \frac{Dy}{Dt} = g(S_0 - S_f) \quad (3.10)$$

$$\text{If } \frac{dx}{dt} = v + \frac{g}{\lambda} \text{ or } v + c \quad \text{for } \lambda_1 = \frac{g}{c} \text{ and} \quad (3.11)$$

$$\frac{Dv}{Dt} - \frac{g}{c} \frac{Dy}{Dt} = g(S_0 - S_f) \quad (3.12)$$

$$\text{If } \frac{dx}{dt} = v - \frac{g}{\lambda} \text{ or } v - c \quad \text{for } \lambda_2 = -\frac{g}{c} \quad (3.13)$$

When $v + c$ is plotted as a curve in $x-t$ plane it is represented towards downstream. This plot referred as positive characteristic, C^+ . In the same way when $v - c$ is plotted in $x-t$ plane it is along the upstream. This plot referred as negative characteristic, C^- .

Equation (3.10) and (3.12) are called the compatibility equations. Now the space variables have been eliminated, but they are in partial differential form so it is required to convert them into ordinary differential equations. Because the partial differential equations are valid for any values of x and t ; however, the transformed ordinary equations are valid only along the characteristics for unique solution. The final form of the equations to be presented for solution is obtained by transforming the partial form of Saint-Venant equation to ordinary derivatives so that derivatives taken in the proper directions, called characteristic direction.



NUMERICAL METHODS AND ANALYSIS

So the previous formula of the Saint venant equation i.e. equation 3.12 for gradually varied unsteady flow is

$$\frac{Dv}{Dt} - \frac{g}{c} \frac{Dy}{Dt} = g(S_0 - S_f) \quad (3.14)$$

By multiplying both side into area A the equation becomes

$$\frac{\partial Q}{\partial t} - \frac{\partial A}{\partial t} \frac{\partial x}{\partial t} = gA(S_0 - S_f) \quad (3.15)$$

$$\frac{dx}{dt} = v \pm c \quad \text{Or} \quad \frac{dx}{dt} = \frac{Q}{A} \pm c \quad (3.16)$$

So the equation 3.13 becomes

$$\frac{\partial Q}{\partial t} - \left(\frac{Q}{A} \pm c \right) \frac{\partial A}{\partial t} = gA(S_0 - S_f) \quad (3.17)$$

Considering the relation $Q=Q(t)$ or $A=A(t)$, this equation permit calculations of the other physical parameters at the upstream section of the channel. The upper and lower algebraic signs refer to positive and negative characteristics, respectively. At external points of upstream and downstream boundary, the negative and positive characteristics are considered for simulation process respectively.

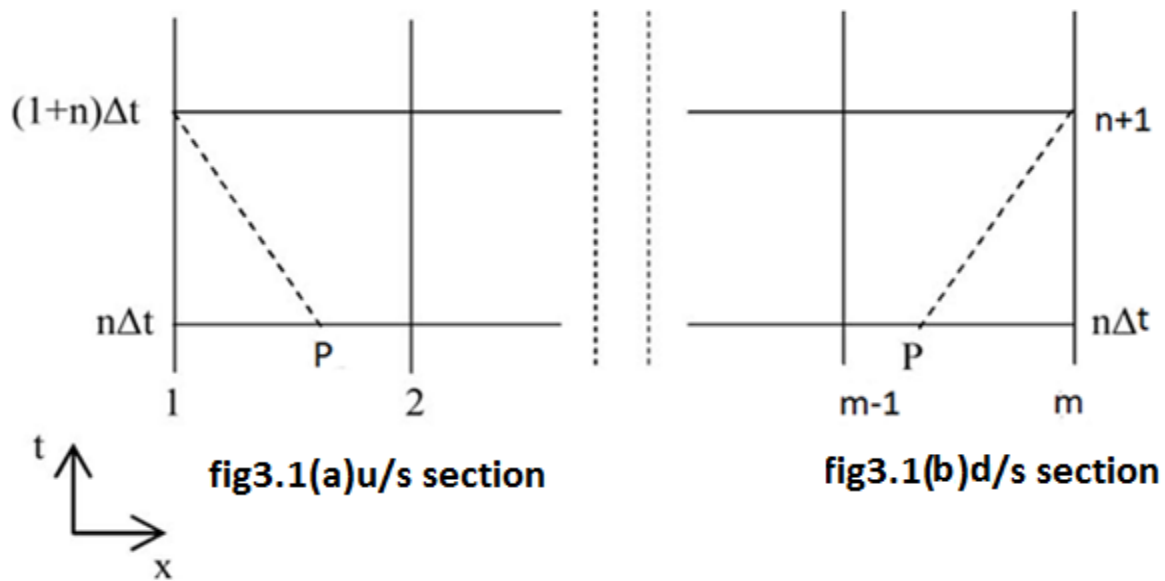


Fig.3.1 The Upstream and Downstream Characteristic In x-t Plane



NUMERICAL METHODS AND ANALYSIS

In this above fig the x axis represents the longitudinal direction for space variable and the ordinate represents the time level. It is already discussed that positive characteristic curve and negative characteristic curve are used in the external boundaries only. For internal grid points the finite difference scheme is applied. The negative characteristic will pass through the point p and $(1+n)\Delta t$ at upstream section shown in fig 3.1(a). Likewise the positive characteristic will pass through the point p and $(1+n)$ at downstream section shown in fig 3.1(b).

For finding out the unknown flow parameter at the upstream end, the solution is given by Equation (3.15)

$$\frac{Q_1^{n+1} - Q_P}{\Delta t} - \left(\frac{Q}{A} + c\right)_p \frac{(A_1^{n+1} - A_P)}{\Delta t} = gA(S_0 - S_f) \quad (3.18)$$

Similarly, at the downstream end denoting by P the inter section point between the positive characteristic passing through point $m=m+1$ and the time level line $t=n+1$ shown in fig.3(b) and considering the downstream boundary condition. Any downstream boundary condition from any conditions (Manning's equation, critical flow depth) can be applied.

The equation furnish

$$\frac{Q_N^{n+1} - Q_P}{\Delta t} - \left(\frac{Q}{A} - c\right)_p \frac{(A_{m-1}^{n+1} - A_P)}{\Delta t} = gA(S_0 - S_f) \quad (3.19)$$

Q_N^{n+1} Denotes the unknown discharge at the $n+1$ level and for y_N^{n+1} at the time level $n+1$ the downstream boundary condition should be applied.

3.3 FINITE DIFFERENCE METHOD

The method of characteristics is used for both Finite difference method and Finite element method. Finite difference method is the first technique for developing for approximating ordinary differential equation. It is based on performing Taylor series expansion and substituting the expressions into the differential equation. It also shows the problem through a series of values at a particular point. It is the simplest method for implement in any ordinary differential equation where the unknowns are found by replacing the derivative terms. As there is no unique solution in numerical method, if the data (initial data and boundary condition) are not known



NUMERICAL METHODS AND ANALYSIS

then it presents a ill posed solution. So that one will be very sure about the poor result or can know that the solution will fail.

3.3.1 GRID GENERATION

Before generating grid for finite difference scheme it is a important factor to focus that there are two methods in finite difference scheme. They are explicit and implicit finite difference scheme.

3.3.2 EXPLICIT METHOD

Explicit method is an approach in computational fluid dynamics used in numerical analysis for obtaining solutions of time-dependent ordinary as well as partial differential equations. It is required in simulations of physical process in flood routing, operating problem. Explicit methods calculate the unknowns at a later time interval from the known values at a present time series. If the n level flow parameters are known then explicit method computes for $n+1$ time level.

3.3.3 IMPLICIT METHOD

Implicit method is an approach to find a solution by solving one equation or a no of equations simultaneously involving both the parameters of present time level and the next one. Implicit methods require an extra computation which is much harder to implement in the calculation process.

Grid generation for an explicit method applies small time steps to minimize the error in the result boundary. By considering numerical stability the regular grids are used in space and time. Impractically for such problems, explicit methods take much less computational time to achieve given accuracy but larger time steps used in an implicit method. Explicit or implicit method should be used depends upon the problem which solution is to found. To take suitable regular grid size, it is required to check the stability of the finite differences scheme.

3.4 STABILITY



NUMERICAL METHODS AND ANALYSIS

The stability of the explicit scheme is determined by the Courant-Fredrichs-Lewy (or Courant) condition. The Courant number (CFL), which is the ratio of the physical speed of the wave to the speed of the numerical signal. It should be less than unity so the grid size will be found from that condition. Updated calculations of the water stage is used to evaluate the celerity of the wave, c and for the water velocity u , the discharge value is used. Hence CFL condition is applied at each time step. This condition is implemented at each time step to evaluate the value of flow at the advanced time step. Numerically by this method the time step must be kept small enough so that information will be most accurate. Mathematically

$$\text{Courant no} = c_n = \frac{V \pm C}{\Delta x / \Delta t} \quad (3.20)$$

Where $0 < c_n \leq 1$

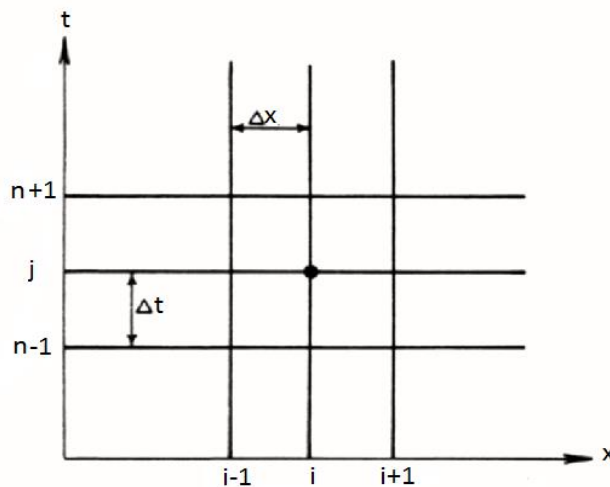


Fig.3.2 Grid in Finite Difference Scheme

After generating regular grids i.e. Δx and Δt the main factor for the approximate solution is to follow the principle of finite difference method. That is the derivatives in the partial differential equation are linearly solved by approximating the values for every grid points with combinations of function values. If u is the dependent variable and x and t independent variables (often space



NUMERICAL METHODS AND ANALYSIS

and time) then approximating the first-order derivatives for the space variables there are three methods for explicit scheme. They are

1. Forward difference Method

$$u_x \approx \frac{\partial u}{\partial x} \approx \frac{u_{i+1} - u_i}{\Delta x} \quad (3.21)$$

$$u_t \approx \frac{\partial u}{\partial t} \approx \frac{u^{n+1} - u^n}{\Delta t} \quad (3.22)$$

2. Backward difference Method

$$u_x \approx \frac{\partial u}{\partial x} \approx \frac{u_i - u_{i-1}}{\Delta x} \quad (3.23)$$

$$u_t \approx \frac{\partial u}{\partial t} \approx \frac{u^n - u^{n-1}}{\Delta t} \quad (3.24)$$

3. Central difference Method

$$u_x \approx \frac{\partial u}{\partial x} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta x} \quad (3.25)$$

$$u_t \approx \frac{\partial u}{\partial t} \approx \frac{u^{n+1} - u^{n-1}}{\Delta t} \quad (3.26)$$

Where i and n are referred as the variables for the present space and time level respectively. $i+1$, $n+1$ refer to the variables for the forward or next space and time level. $i-1$, $n-1$ refer to the variables for the past space and time level respectively.

Approximation of second-order derivatives has one method

Central difference Method

$$u_{xx} \approx \frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2} \quad (3.27)$$

$$u_{tt} \approx \frac{\partial^2 u}{\partial t^2} \approx \frac{u^{n+1} - 2u^n + u^{n-1}}{(\Delta t)^2} \quad (3.28)$$



Where i.e. refer to the variables for the present space and time level respectively. i+1, n+1 refer to the variables for the forward or next space and time level. i-1, n-1 refer to the variables for the past space and time level respectively.

Approximating values for mixed derivatives there is one method

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} \right) \quad (3.29)$$

$$\left(\frac{\partial^2 u}{\partial x \partial y} \right)_{i,j} = \frac{\left(\frac{\partial u}{\partial y} \right)_{i+1,j} - \left(\frac{\partial u}{\partial y} \right)_{i-1,j}}{2\Delta x} \quad (3.30)$$

$$\left(\frac{\partial u}{\partial y} \right)_{i+1,j} = \frac{u_{i+1,j+1} - u_{i+1,j-1}}{2\Delta y} \quad (3.31)$$

$$\left(\frac{\partial u}{\partial y} \right)_{i-1,j} = \frac{u_{i-1,j+1} - u_{i-1,j-1}}{2\Delta y} \quad (3.32)$$

So the approximating values for mixed derivatives

$$\left(\frac{\partial^2 u}{\partial x \partial y} \right)_{i,j} = \frac{u_{i+1,j+1} - u_{i+1,j-1} - u_{i-1,j+1} + u_{i-1,j-1}}{4\Delta x \Delta y} \quad (3.33)$$

Likewise explicit method, there are 3 methods in implicit method. They are

- Forward Difference Method,
- Backward Difference Method and
- Central Difference Method

The difference between these scheme is only in time level. In explicit method, the known variables are in present time level and the unknown variables are of forward time level. But for another method, the known variables are of forward time level and the parameters which are to be find out are of current time level.

3.5 EXPLICIT FINITE-DIFFERENCE SCHEMES

Here in this research by the method of Explicit Finite-Difference Schemes, the solution of governing equations of 1-D flow in open channel are solved. Several explicit finite-difference



schemes are there which have been proposed for the solution of Saint-Venant equations. Here Lax-diffusive hyperbolic partial differential equation is applied for solving the nonlinear problem for flood routing. Two explicit methods are given below.

3.5.1 CONSERVATION FORM

Conservation form of a derivative must form a telescoping series. In other words, when the terms are added over a grid, only the boundary terms should remain and the interior points should cancel out. For non-conservative form, the derivative is split apart to give accurate solution.

The conservation form of the governing equations (Saint-Venant equations) in the matrix form may be written as

$$U_t + F_x + S = 0 \tag{3.34}$$

Where U_t represents the derivative of variables with respect to time or a function of one independent variable t

F_x represents the derivative of variables with respect to space in one direction or a function of one independent variable

S represents the constant terms including slopes (bed slope and friction slope)

In which

$$U = \begin{pmatrix} A \\ VA \end{pmatrix}; F = \begin{pmatrix} VA \\ V^2A + \frac{1}{2}gAy \end{pmatrix}; S = \begin{pmatrix} 0 \\ -gA(S_0 - S_f) \end{pmatrix} \tag{3.35}$$

And $\frac{1}{2}Ay =$ moment of flow area about the free surface

3.5.2 LAX-DIFFUSIVE SCHEME

Lax scheme is a explicit method, which follows that it is unconditionally stable. It is first-order accurate in time and second-order accurate in space. In fact, all stable explicit differencing schemes for solving the governing equation i.e. Saint Venant equation are subject to the Courant-Friedrichs-Lewy (or CFL) condition for stability criterion. The CFL constraints determines the



NUMERICAL METHODS AND ANALYSIS

maximum allowable time-step for lax diffusive explicit method. This scheme is inherently unstable. Lax presented this scheme by slightly varying the unstable scheme. This scheme is one simple method to program which yields satisfactory results. In this scheme the partial derivatives and other variables are approximated so the general formulae for Lax-diffusive scheme are

By forward difference scheme

$$U_t = \frac{1}{\Delta t} (U_i^{n+1} - U^*) \quad (3.36)$$

Again putting the value of

$$U^* = \frac{1}{2} (U_{i+1}^n - U_{i-1}^n) \quad (3.37)$$

By central difference formula

$$f_x = \frac{1}{2\Delta x} (f_{i+1}^n - f_{i-1}^n) \quad (3.38)$$

The frictional slope

$$S_f = \frac{1}{2} (S_{f_{i+1}} - S_{f_{i-1}}) \quad (3.39)$$

So by putting $\frac{\partial U}{\partial t}$ and $\frac{\partial f}{\partial x}$ in original formula

We have the lax diffusive method

$$U_i^{n+1} = \frac{1}{2} (U_{i+1}^n + U_{i-1}^n) - \frac{1}{2} \frac{\Delta t}{\Delta x} (F_{i+1}^n + F_{i-1}^n) - S^* \Delta t \quad (3.40)$$

U_i^{n+1} represents unknown variable i.e. the area (A) which is found out from the solution of continuity equation of the next time level (n+1) and it refers to the discharge (VA) found out from momentum equation. So, the values of variables of interest i.e. depth and velocity are also determined from the values of A and VA which have been determined at the (n+1) time level. Then proceed to the next time step.



3.5.2 LAX–WENDROFF METHOD

Lax-Wendroff explicit scheme is also selected in order to solve the Saint-Venant Equations. The Lax–Wendroff is a method of computational fluid dynamics tool, named after Peter Lax and Burton Wendroff, for solving the hyperbolic partial differential equations numerically. In this method, the solution is based on explicit finite difference which is second-order accurate in both space and time. This method is like the lax diffusive method of explicit time integration where the unknown variables are determined or evaluated at the forward time by knowing the known values at current time level. So the general formulae for Lax- Wendroff scheme are

$$U_i^{n+1} = U_i^n - \frac{1}{2} \frac{\Delta t}{\Delta x} (F_{i+1}^n + F_{i-1}^n) + \frac{1}{4} \frac{(\Delta t)^2}{(\Delta x)^2} [(U_{i+1}^n + U_i^n)(F_{i+1}^n - F_i^n) - (U_i^n + U_{i-1}^n)(F_i^n - F_{i-1}^n)] \quad (3.41)$$

Similarly in lax diffusive method U_i^{n+1} represents the area (A) and discharge (VA) which is found out from the solution of continuity equation and momentum equation of the next time level (n+1). Then proceed to the next time step. The method is second-order, with stability requirement satisfies the Courant-Friedrichs-Lewy (or CFL) condition. The solution is obtained at a courant number of one. The result coming from this scheme is same as the lax- diffusive explicit method.

3.6 HEC-RAS MODEL

Hydrology engineering center refer river analysis system is known as HEC RAS model. It is developed by U.S. Army corps of engineer of hydrologic engineering center. This software allows to analysis of one dimensional steady flow, unsteady flow calculation and perform sediment transport along with the mobile bed computation for unsteady flow.

HEC-RAS Model is designed in such a way that it solves and performs calculations for both natural waterway and manmade canal in multitasking environment. It is mainly applied for 4



NUMERICAL METHODS AND ANALYSIS

river analysis application in steady flow, simulation for unsteady flow, movable sediment transport system and water quality. So there are various types of components intended for calculating the problems given by the user. Through a full network of system this software is designed to simulate the unsteady flow in a channel for subcritical flow, mixed flow regime like combination of subcritical flow, supercritical flow, and hydraulic jumps and draw down.

Obviously a modeler wants to formulate different plans at the time of its study. So every plan requires some information for its simulation. For calculation purpose the set of information about hydraulic properties like geometric data are entered by the user. Then simulation is done. The results are shown in graphical and tabular format.

The Saint-Venant equations (both continuity equation and momentum equations) for one dimensional steady and unsteady flow of river are given by:

$$\frac{\partial y}{\partial t} + D_h \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0 \quad (3.42)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = g(S_o - S_f) \quad (3.43)$$

Where V is the flow velocity; y is the flow depth; $D_h = A/B$ is hydraulic depth; A is flow area, B is top width of the channel; S_o is the channel bottom slope, S_f is the slope of energy grade line; x is the distance along the channel length; t is time and g is the acceleration due to gravity.

These equations can be solved by the River Analysis System (HEC-RAS) software which use the Preissman implicit scheme. The model and methodology of the schemes are described below

3.7 MODEL DEVELOPMENT AND METHODOLOGY

The most successful and accepted procedure for solving the one dimensional unsteady flow equation is the four point implicit scheme also known as the box scheme. Under the scheme, methods of solution are as follows

$$f_i = f_i^j \quad (3.44)$$

$$\Delta f_i^j = f_{i+1}^{j+1} - f_i^j \quad (3.45)$$

Or



$$f_i^{j+1} = f_i^j + \Delta f_i^j \quad (3.46)$$

Size of grid taken as (i, j) ; where i=space interval, j= time interval

General implicit finite difference scheme forms are

1. Time derivative

$$\frac{\partial f}{\partial t} \approx \frac{\Delta f}{\Delta t} = \frac{0.5(\Delta f_{i+1} + \Delta f_i)}{\Delta t} \quad (3.47)$$

2. Space derivative

$$\frac{\partial f}{\partial x} \approx \frac{\Delta f}{\Delta x} = \frac{(f_{i+1} - f_i) + \theta(\Delta f_{i+1} - \Delta f_i)}{\Delta x} \quad (3.48)$$

3. Function value

$$f = 0.5(f_{i+1} + f_i) + 0.5\theta(\Delta f_{i+1} + \Delta f_i) \quad (3.49)$$

f refers to both V and y in the partial derivatives and f stands for S_f and V as a coefficient .

A. Continuity Equation

$$\frac{\partial A}{\partial t} + \frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} - q_l = 0 \quad (3.50)$$

The above equation can be written for channel or flood plain:

$$\frac{\partial Q_c}{\partial x_c} + \frac{\partial A_c}{\partial t} = q_f \quad (3.51)$$

$$\frac{\partial Q_f}{\partial x_f} + \frac{\partial A_f}{\partial t} + \frac{\partial S}{\partial t} = q_c + q_l \quad (3.52)$$

Where the subscripts c and f refer to the channel and flood plain, respectively

q_l is the lateral inflow per unit length of flood plain and q_c and q_f are the exchange of water between the channel and flood plain

$$\frac{\partial Q_c}{\partial x_c} + \frac{\partial A_c}{\partial t} = q_f \quad (3.53)$$



$$\frac{\partial Q_f}{\partial x_f} + \frac{\partial A_f}{\partial t} + \frac{\partial S}{\partial t} = q_c + q_l \quad (3.54)$$

The exchange of mass is equal but not opposite in sign such that

$$\Delta x_c q_c = - \Delta x_f q_f \quad (3.55)$$

B. Momentum Equation

The equation states that the rate of change in momentum is equal to the external forces acting on the system. For single channel

$$\frac{\partial Q}{\partial t} + \frac{\partial(VQ)}{\partial x} + gA \left(\frac{\partial Z}{\partial x} + S_f \right) = q_c + q_l \quad (3.56)$$

The above equation can be written for the channel and for the flood plain:

$$\frac{\partial Q}{\partial t} + \frac{\partial(VQ)}{\partial x} + gA \left(\frac{\partial Z}{\partial x} + S_f \right) = q_c + q_l \quad (3.57)$$

The above equation can be written for the channel and floodplain:

$$\frac{\partial Q_c}{\partial t} + \frac{\partial(V_c Q_c)}{\partial x} + gA_c \left(\frac{\partial Z}{\partial x_c} + S_{fc} \right) = M_f \quad (3.58)$$

$$\frac{\partial Q_f}{\partial t} + \frac{\partial(V_f Q_f)}{\partial x} + gA_f \left(\frac{\partial Z}{\partial x_f} + S_{ff} \right) = M_c \quad (3.59)$$

Where M_c and M_f are the momentum fluxes per unit distance exchange between the channel and flood plain, respectively.

CHAPTER 4

PROBLEM STATEMENT

AND SOLUTION



PROBLEM STATEMENT AND SOLUTION

4.1. OVERVIEW

To demonstrate the potential of the methods used in this research, models are applied to simulate the flood routing problem in a wild river basin Persian Gulf area. For flood routing in a river section using numerical approaches need boundary condition for its solution. The upstream stage or the upstream discharge hydrograph data are necessary for its solution. So the discharge hydrograph is available here for its boundary condition. The solution procedure of the nonlinear hyperbolic Saint Venant equation are discussed in this chapter by generating suitable grids and setting proper initial and boundary condition. Then the solution is made by writing the program using explicit lax diffusive method by mat-lab and the results obtained from that are shown in different figures. The values are given in four columns for two different location. A computer model i.e. HEC-RAS is also used for comparison of the results. These values coming from HEC-RAS are also given in that tables. The comparison of both result coming from present approach and the model are shown in a single figure for one location.

4.2. FLOOD ROUTING IN A RIVER SECTION – A CASE STUDY

A hypothetical flood routing [2] discharge hydrograph of upstream boundary in a wide rectangular river has been considered here and is given by : A rectangular river section is assumed with width (B) = 120m, Average longitudinal Bed slope (S_o) considered is 0.00061, The value of Manning's roughness co-efficient n for the bed surface is 0.023. Let the Base flow(Q_b) in the river to be 100 m³/sec.

The U/S discharge hydrograph (Q_t) is generally sinusoidal in nature and is a function of time (t), base flow (Q_b) and peak flow(Q_p) is given by :-

$$Q(t) = \frac{Q_p}{2} \sin\left(\frac{\pi t}{t_p} - \frac{\pi}{2}\right) + \frac{Q_p}{2} + Q_b \text{ for } t < t_b \quad (4.1)$$

$$Q(t) = \frac{Q_p}{2} \cos\left(\pi \frac{t-t_p}{t_b-t_p}\right) + \frac{Q_p}{2} + Q_b \text{ for } t_p < t \leq t_b \quad (4.2)$$

$$Q(t) = Q_b \text{ for } t > t_p . \quad (4.3)$$

Where peak time (t_p), base time (t_b), and peak flow (Q_p) are assumed to be 5 hr, 15hr, and 200 m³/sec respectively. Friction slope S_f is calculated from Manning's equation



PROBLEM STATEMENT AND SOLUTION

$$S_f = \frac{n^2 V^2}{R^{4/3}}, \quad (4.4)$$

Where n =Manning's roughness coefficient, $R=A/P$ is the hydraulic radius; A is the flow area; P is the wetted perimeter.

4.3. SOLUTION

The accuracy of dynamic wave can be calculated by different approaches, assuming steady water profiles which are required for calculation in some conditions. The finite difference explicit scheme i.e. Lax-diffusive scheme is used for the interior grids and characteristic equations are used for both boundaries.

4.3.1. GRID GENERATION

It is divided the x - t plane into numbers of grids that the grid interval along the x -axis is Δx and the grid interval along the t -axis is Δt . Although it is not necessary but for easier calculation it is assumed that, the grid size is uniform along each axis. For the space j level a subscript $(i,j), (i+1,j), (i+2,j), (i+3,j), (i+4,j), (i+5,j)$, are used for different grids for space. For the time axis i level a subscript $(i,j), (i,j+1), (i,j+2), (i,j+3), (i,j+4), (i,j+5)$, are used for different grids for time. To refer to different variables at these grid points, the number of the spatial grid as a subscript and that of the time grid as a superscript are used. The known time level is denoted by superscript j and the unknown time level is denoted by $j + 1$.

The stability of numerical scheme is ensured by the courant condition. For a fixed spatial grid Δx , the value of Δt satisfying the courant condition is determined.

The grids are generated by the courant's law

$$\text{i.e. } c_n = \frac{V \pm C}{\Delta x / \Delta t} \quad (4.5)$$

and the $0 < c_n < 1$

Where V is the velocity of the flow;

Δx is grid size along the length



PROBLEM STATEMENT AND SOLUTION

Δt is grid size of the time

So after considering much values of Δx and Δt , the Δx and Δt are taken 1000 metre and 120 second respectively for accurate result.

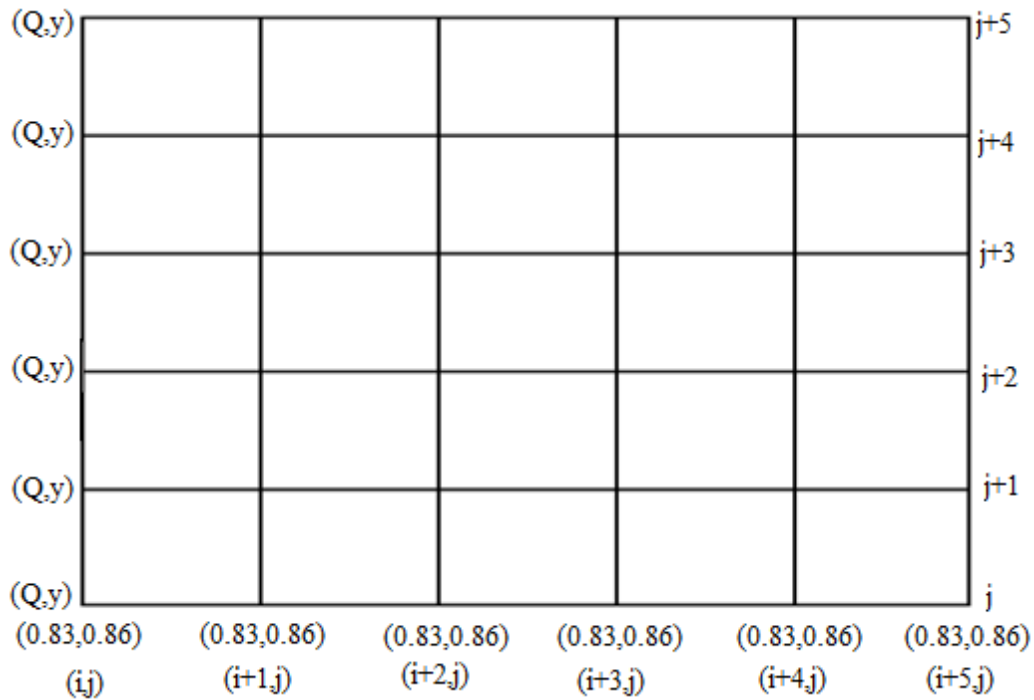


Fig.4.1 Grid Map For Explicit Method

The graph shows the space level and time level in x-axis and y-axis respectively. The flow along a longitudinal axis indexed by the abscissa x. The time series axis indexed by the ordinate y. The j level represents the values of flow variables of current time level. These values are set for the initial condition which is same at every node along the channel as lateral inflow. The values for initial condition is given in terms of $Q(x,0)$. The j+1, j+2, j+3 level show the corresponding to the next time level. The values of flow in a time series are given at the boundary condition for upstream at left boundary. The methods developed in this research assumed that the lateral inflow and outflow are $100 \text{ m}^3/\text{sec}$. The values for upstream boundary is given by a inflow hydrograph at a concentration point at a time series, given in terms of $Q(0,t)$.

The computation process for the solutions of the governing equations of motion is limited to evaluating them at a finite number of points along the channel. The two approaches are here



PROBLEM STATEMENT AND SOLUTION

for computing approximate solutions to these equations. First the values, at location of the points along the channel in advance time level at upstream section are fixed. Then the values are fixed at those grids, that adjusted as initial condition needed for the solution.

Then the broad range of the method of characteristics are included at upstream boundary as it is explicitly solves the solution. The characteristic equations are solved simultaneously by the equations given for the upstream and downstream boundary. Positive characteristic form is used at downstream boundary and negative characteristic form of the equations is used for upstream flow characteristics in explicit methods, tracing in whole or in part on the $x-t$ plane. In the method of characteristics, the locations and times at which flows and elevations are computed are irregular and vary as the flow is unsteady. This method is advantageous with great accuracy.

4.3.2. BOUNDARY CONDITION

The given U/S discharge hydrograph (Q_t) at a concentration point as a function of time is shown by Fig.4.2. It is shown that the peak flow is $300 \text{ m}^3/\text{sec}$ and the time of the peak flow is 5 hour. It continuously flows for 15 hour then emerges out. After 15 hour, the flow is equal to the base flow i.e. $100 \text{ m}^3/\text{sec}$. In a two-dimensional coordinate system the abscissa shows the time in hour and the flow values in series are indexed by the ordinate y.

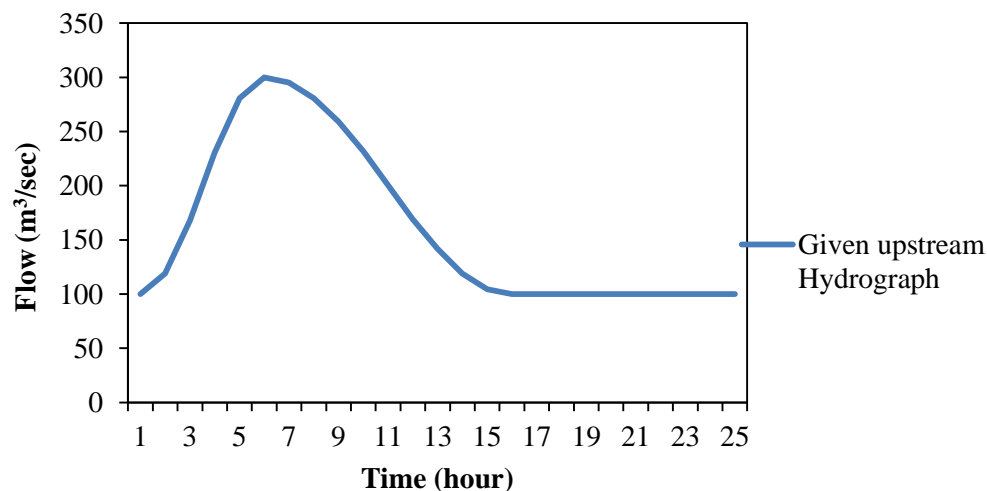


Fig.4.2 Known Flood Hydrograph at Upstream



PROBLEM STATEMENT AND SOLUTION

4.3.3 DOWNSTREAM BOUNDARY CONDITION

Here at end section when $x=L$, the critical flow depth is considered as the downstream end condition or downstream boundary condition, which occurs when the channel flow ends at a steep bank of the channel at 100km from upstream.

The critical flow condition is given by

$$\frac{Q^2}{g} = \frac{A^3}{T} \quad (4.6)$$

4.4. NUMERICAL SOLUTION

If we write conservation of mass and momentum equations for each grid point, we have $2n$ equations (n = number of reaches on the channel). These equations cannot be written for the downstream end as the next spatial grid values are unknown. However, we have $2(n+1)$ unknowns, i.e., two unknowns for each grid point. Thus, for the solution of Saint-Venant equation, two more equations are needed. These are provided by the boundary conditions. For initial conditions, values of stage(y) and flow(Q) at the beginning of the time step are to be specified at all the spatial nodes along the channel. The two boundary conditions required by the model for this explicit method are the inflow discharge hydrograph at the upstream boundary, and the critical flow depth at the downstream boundary. The lax diffusive method is used at internal grid points; whereas at external points of the boundary, the characteristic method including positive and negative characteristics is considered for downstream and upstream boundary condition respectively. The characteristic equations are

$$\frac{dx}{dt} = \frac{Q}{A} \pm c \quad (4.7)$$

$$\frac{dQ}{dt} - \left(\frac{Q}{A} \pm c\right) \frac{dA}{dt} = gA(S_0 - S_f) \quad (4.8)$$

$$C = \text{celerity} = \sqrt{gT} \quad (4.9)$$



PROBLEM STATEMENT AND SOLUTION

The positive characteristic equations are simultaneously solved by the downstream boundary condition and the negative characteristic equations are simultaneously solved by the upstream boundary condition given by the hydrograph to find out the unknown flow parameters at both boundaries. Once the values of A and VA have been determined at the $(j+1)$ time level. Then the next time step $(j+2)$ variables y and V are determined, the values of variables of interest of other time steps are proceeded to determine. So the lax diffusive method for each node is applied and the boundary conditions are applied at boundaries for the solution.

4.5. RESULTS AND DISCUSSION

Taking $\Delta t = 120$ sec and total duration of time = 20 hour, the time axis is divided by 600 grids. By approaching matlab software the governing equations is solved by lax-diffusive method. The known inflow hydrograph is obtained from this approach is shown by fig4.3. The peak flood is $300 \text{ m}^3/\text{sec}$.

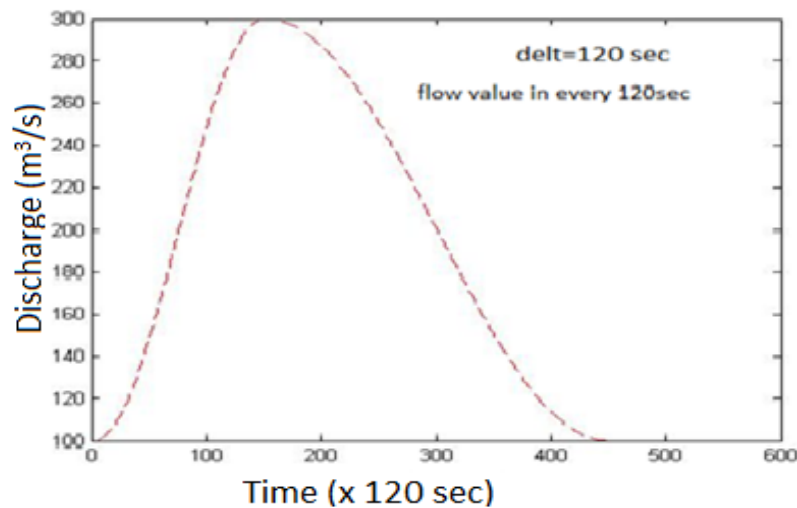


Fig.4.3 Known Flood hydrograph at Upstream

Two sections are considered for observation and comparison with HEC-RAS. Sections after 16km and 28km from upstream are considered. After writing the program by Mat-lab software the flow hydrograph is obtained at the section 16km from upstream is shown in fig.4.4.



PROBLEM STATEMENT AND SOLUTION

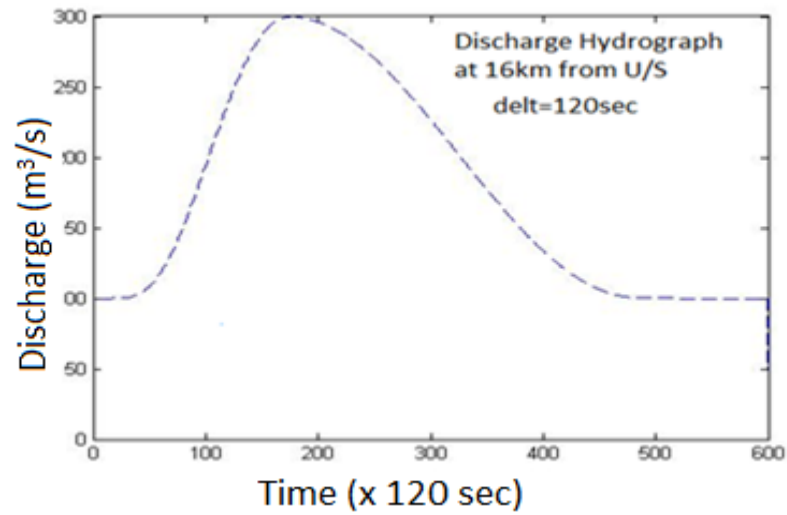


Fig.4.4.Flood Hydrograph at 16km from Upstream

The hydrograph is obtained at the section 28km from upstream is shown in fig.4.5.

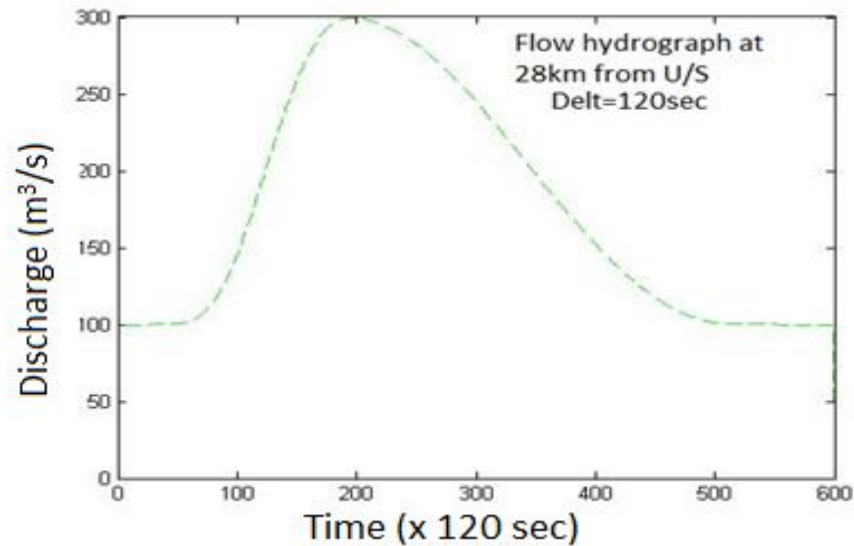


Fig.4.5 Flood Hydrograph at 28km from Upstream

Combining the three hydrographs of different sections, it can be clearly seen that shifting of hydrograph occurs along the downstream of the channel shown by Fig.4.6



PROBLEM STATEMENT AND SOLUTION

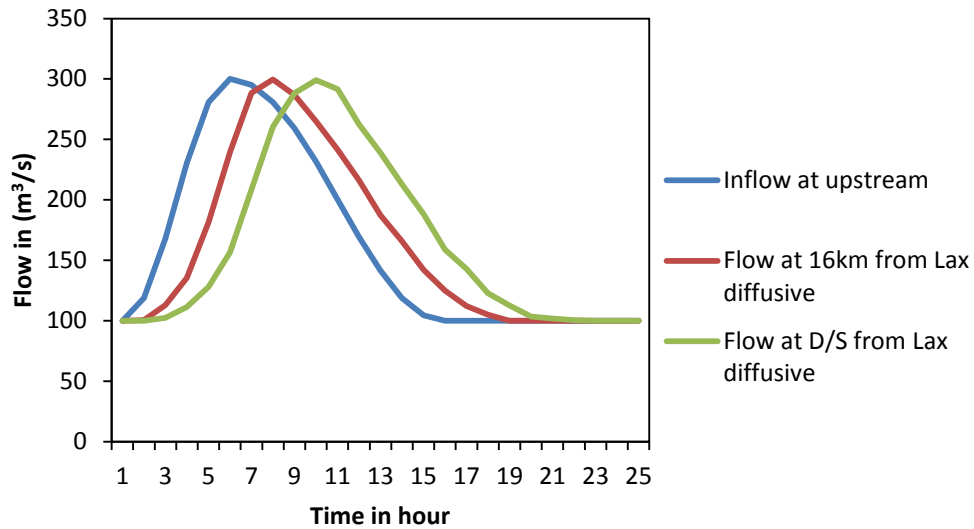


Fig.4.6 Shifting of Flood Hydrograph

From lax diffusive explicit scheme, flow hydrographs at different sections can be obtained which are shown in the fig.4.7. In this figure Δx and Δt are kept 1km and 120sec. 101 hydrographs are shown in this fig as the total length of the channel is considered as 100km.

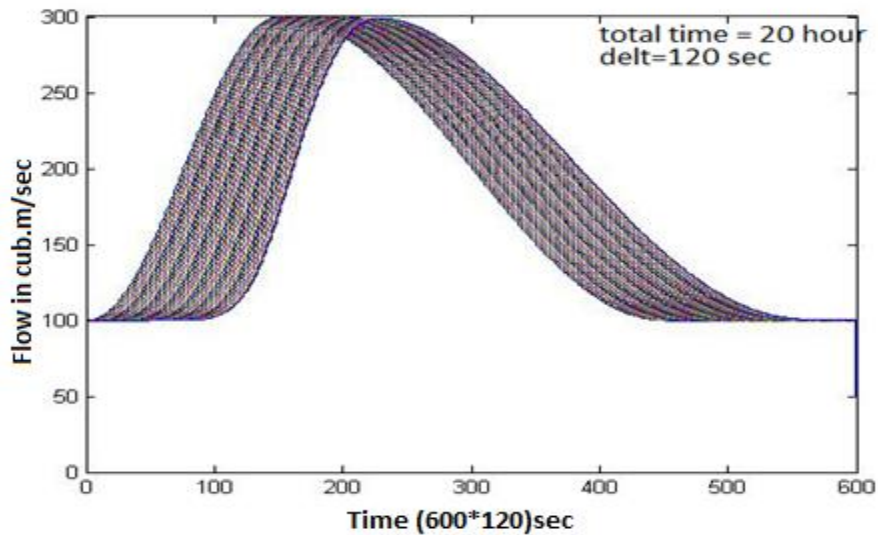


Fig.4.7. Flow Hydrographs Obtained From Lax-Diffusive Scheme



PROBLEM STATEMENT AND SOLUTION

Like the flow hydrographs, the stage hydrographs is also determined by simultaneous solution of negative characteristic equation with the upstream boundary condition at upstream boundary. So the stage hydrograph at upstream is obtained from lax diffusive method is shown in fig.4.8.

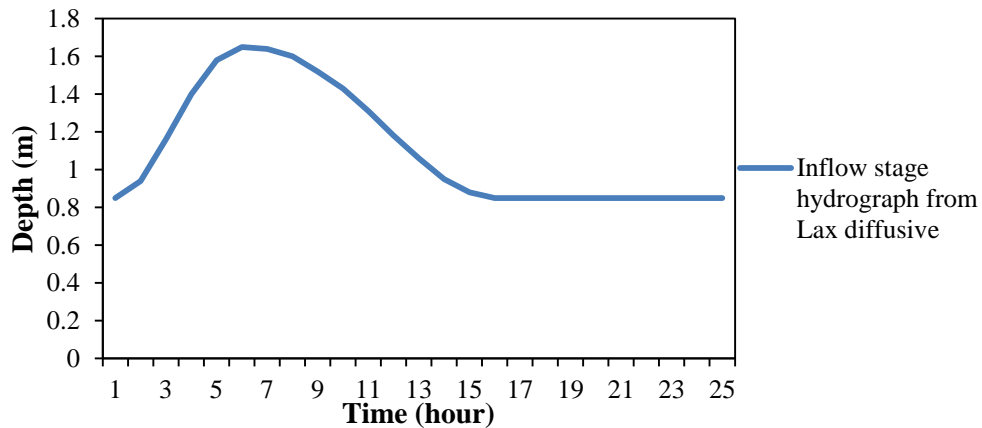


Fig.4.8. Depth Hydrograph Obtained From Lax-Diffusive Scheme

By applying the diffusion process at the intermediate sections the stage hydrograph at 16km from upstream is obtained which is shown in fig.4.9.

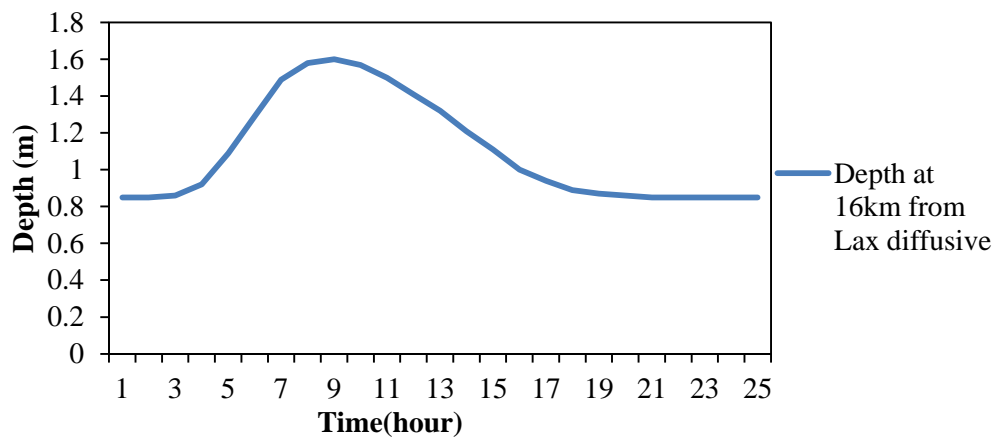


Fig.4.9 Depth Hydrograph at 16km from Upstream

By applying the same diffusion process of lax diffusive method at the intermediate sections, the stage hydrograph at 28km from upstream is obtained which is shown in fig.4.10.



PROBLEM STATEMENT AND SOLUTION

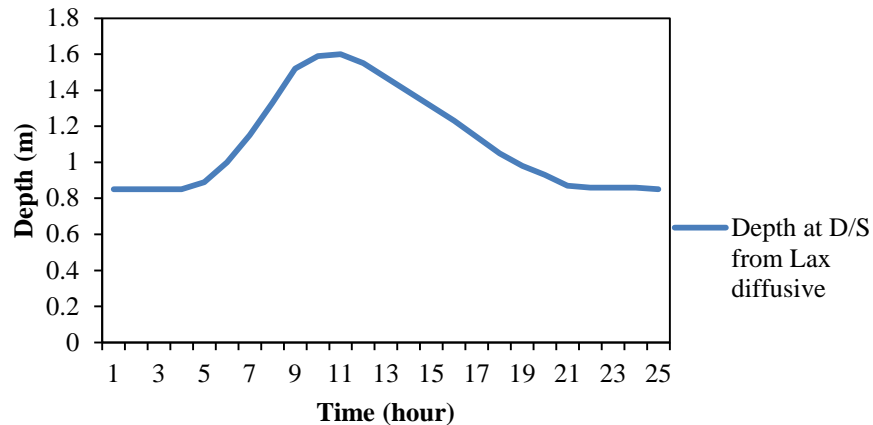


Fig.4.10 Depth Hydrograph at 28km From Upstream

Combining the three stages in one figure, it can be clearly observed that by channel routing, the hydrographs of different sections are shifting along the downstream of the channel shown by Fig.4.11

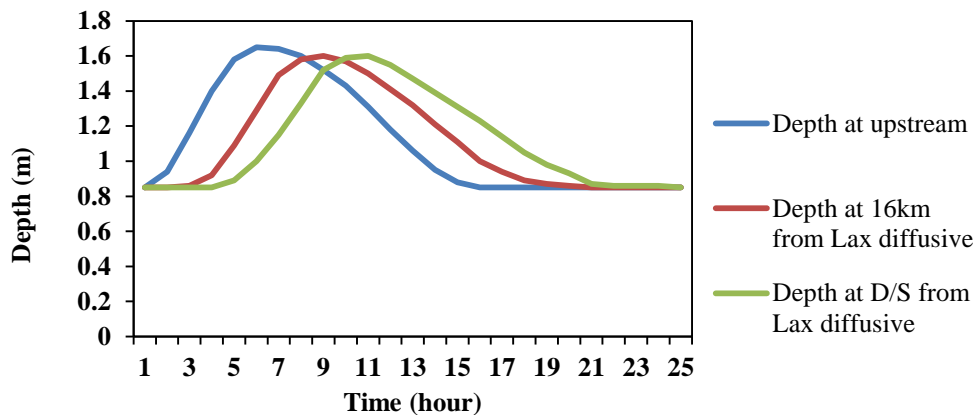


Fig.4.11 Shifting of Depth Hydrograph

4.6. RESULT OBTAINED FROM HEC-RAS

In HEC-RAS computer model, the same inflow hydrograph is put for routing purpose. This model is used for comparison of unsteady flow routing data, computed from explicit method. To compute the flow and water elevation in different section of interest, some basic data are needed.



PROBLEM STATEMENT AND SOLUTION

To perform this computation in a wide rectangular channel, geometric cross sections data for every section are required.

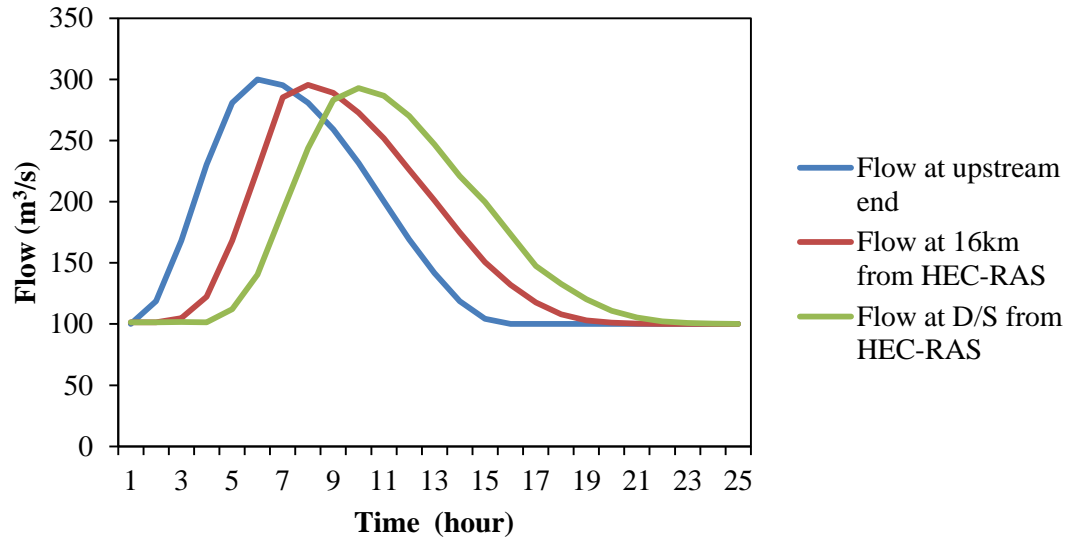


Fig.4.12 Flow (m³/s) Hydrograph from HEC-RAS

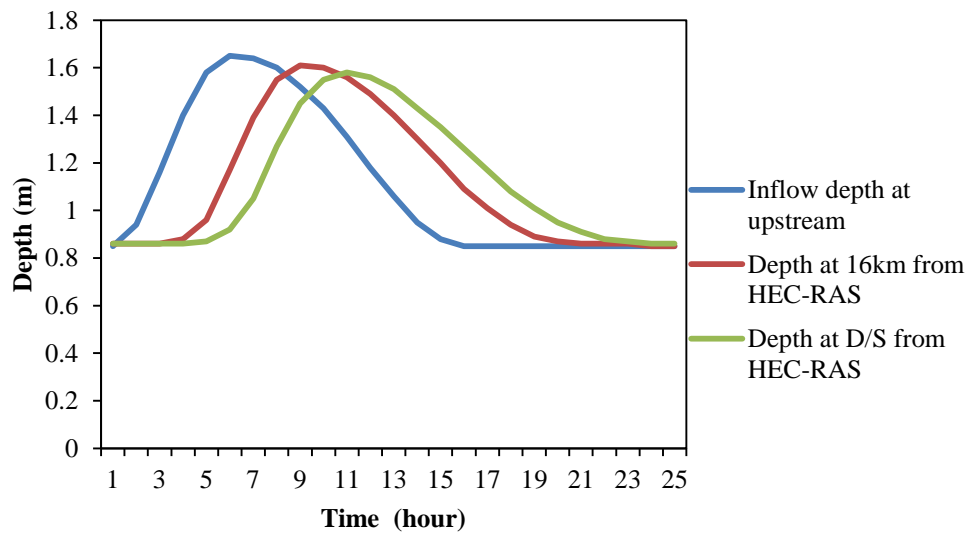


Fig.4.13 Stage (m) Hydrograph from HEC-RAS

They are computed from bed slope of the channel, reach length and cross sectional geometry of upstream. The steady state flow data is required for putting the initial condition and to know the



PROBLEM STATEMENT AND SOLUTION

steady state water elevation. At each station the cross sectional geometry data and the initial depth as well as the flow data are put to smoothen the calculation. Each cross sectional data set is identified by a river reach and river station label. Maximum 500 data points can be used to describe each cross section. The unsteady flow data set is also necessary at upstream section. This set is a set of the complete values of inflow hydrograph. By putting all these values for 28km reach length, the hydrographs of different cross section are found out. The initial flow and initial depth are put as $100 \text{ m}^3/\text{sec}$ and 0.86 m respectively. 600 values of flow at a time series of a flow hydrograph are given at upstream as boundary condition. Then friction slope value as 0.00061 is given at downstream for calculation of normal depth. Two observation sections i.e. 16km and 28km from upstream are selected for comparison purpose. The flow hydrograph and depth hydrograph are obtained for that two sections. The Δt and Δx are assumed as 120sec and 1000 meters respectively. The flow hydrographs at 16km and 28km including inflow hydrograph are shown in fig.4.12.

Like the flow hydrographs the stage hydrographs at 16km and 28km including inflow depth hydrograph are shown in fig.4.13.

Table 1 and table 2 contain the values of computed discharge hydrograph at two sections 16km and 28km from upstream of the river. The 1st column values represent the time of 25 hour whereas the second column shows the values of inflow values in m^3/sec at upstream.

The both values at two observed locations obtained from lax diffusive and HEC-RAS computer model are presented in m^3/sec in 3rd and 4th column respectively.

TABLE 4.1 Computed Discharge (m^3/sec) Hydrograph Values at 16 Km Section from U/S end

Time (hour)	Flow at U/S end (m^3/sec)	Flow at 16 km from U/S end (m^3/sec)	
		Lax diffusive method	HEC-RAS
1	100.00	100.00	101.39
2	118.80	100.55	101.49
3	168.00	112.98	104.86
4	230.40	135.34	122.39



PROBLEM STATEMENT AND SOLUTION

5	280.80	181.39	168.00
6	300.00	239.48	226.15
7	295.20	288.49	285.36
8	280.80	299.64	295.43
9	259.20	286.93	289.03
10	231.60	264.93	272.65
11	200.40	241.69	251.73
12	169.20	216.12	226.30
13	141.60	187.34	201.13
14	118.80	165.78	174.96
15	104.40	142.12	150.60
16	100.00	124.92	132.07
17	100.00	112.18	117.65
18	100.00	105.04	107.93
19	100.00	100.01	103.06
20	100.00	100.00	101.06
21	100.00	100.00	100.34
22	100.00	100.00	100.11
23	100.00	100.00	100.03
24	100.00	100.00	100.01
25	100.00	100.00	100.00

TABLE 4.2 Computed Discharge (m^3/sec) Hydrograph Values at D/S End

Time (hour)	Flow at U/S end (m^3/sec)	Flow at D/S end (m^3/sec)	
		Lax diffusive method	HEC-RAS
1	100.00	100.00	101.33
2	118.80	100.00	101.45
3	168.00	102.36	101.57
4	230.40	111.38	101.47
5	280.80	127.94	112.20
6	300.00	156.48	140.40
7	295.20	208.49	192.43
8	280.80	260.36	243.67



PROBLEM STATEMENT AND SOLUTION

9	259.20	288.16	283.10
10	231.60	299.00	292.92
11	200.40	291.74	286.60
12	169.20	262.59	270.25
13	141.60	239.03	246.79
14	118.80	212.96	220.94
15	104.40	188.05	199.77
16	100.00	158.84	173.28
17	100.00	142.78	147.48
18	100.00	122.78	132.80
19	100.00	112.64	120.19
20	100.00	103.30	110.92
21	100.00	101.71	105.26
22	100.00	100.40	102.31
23	100.00	100.00	100.93
24	100.00	100.00	100.35
25	100.00	100.00	100.00

Table 4.3 and table 4.4 contain the values of computed stage hydrograph at two sections 16km and 28km from upstream of the river. The 1st column values represent the time of 25 hour whereas the second column shows the values of inflow depth values in m³/sec at upstream. The both values at two observed locations obtained from lax diffusive and HEC-RAS computer model are presented in metre in 3rd and 4th column respectively.

TABLE 4.3. Computed Stage (m) Hydrograph Values at 16 Km Section from the U/S

Time (hour)	Depth at U/S end (m)	Depth at U/S end (m)	
		lax diffusive method	HECRAS
1	0.85	0.85	0.86
2	0.94	0.85	0.86
3	1.16	0.86	0.86
4	1.40	0.92	0.88
5	1.58	1.09	0.96
6	1.65	1.29	1.17
7	1.64	1.49	1.39
8	1.60	1.58	1.55



PROBLEM STATEMENT AND SOLUTION

9	1.52	1.60	1.61
10	1.43	1.57	1.60
11	1.31	1.50	1.56
12	1.18	1.41	1.49
13	1.06	1.32	1.40
14	0.95	1.21	1.30
15	0.88	1.11	1.20
16	0.85	1.00	1.09
17	0.85	0.94	1.01
18	0.85	0.89	0.94
19	0.85	0.87	0.89
20	0.85	0.86	0.87
21	0.85	0.85	0.86
22	0.85	0.85	0.86
23	0.85	0.85	0.86
24	0.85	0.85	0.85
25	0.85	0.85	0.85

TABLE4. 4 Computed Stages (m) Hydrograph Values at D/S End

Time (hour)	Depth at U/S end (m)	Depth at D/S end (m)	
		lax diffusive method	HECRAS
1	0.85	0.85	0.86
2	0.94	0.85	0.86
3	1.16	0.85	0.86
4	1.40	0.89	0.87
5	1.58	1.00	0.92
6	1.65	1.15	1.05
7	1.64	1.33	1.27
8	1.60	1.52	1.45
9	1.52	1.59	1.55
10	1.43	1.60	1.58
11	1.31	1.55	1.56
12	1.18	1.47	1.51
13	1.06	1.39	1.43



PROBLEM STATEMENT AND SOLUTION

14	0.95	1.31	1.35
15	0.88	1.23	1.26
16	0.85	1.14	1.17
17	0.85	1.05	1.08
18	0.85	0.98	1.01
19	0.85	0.93	0.95
20	0.85	0.87	0.91
21	0.85	0.86	0.88
22	0.85	0.86	0.87
23	0.85	0.86	0.86
24	0.85	0.85	0.86
25	0.85	0.85	0.86

4.7. COMPARISON OF THE RESULTS BETWEEN LAX DIFFUSIVE AND HEC-RAS MODEL

In this research the flow and stage values at two observed station, obtained from the numerical analysis of lax diffusive method are compared with the HEC-RAS computer model. The Fig.4.14 and Fig.4.15 represents the comparison of flow values at 16km and 28km from upstream of channel respectively.

Like the flow comparison the stage values at two observed station, obtained from the numerical analysis of lax diffusive method are compared with the HEC-RAS computer model. The Fig.4.16 and Fig.4.17 represents the comparison of flow values at 16km and 28km from upstream of channel respectively.



PROBLEM STATEMENT AND SOLUTION

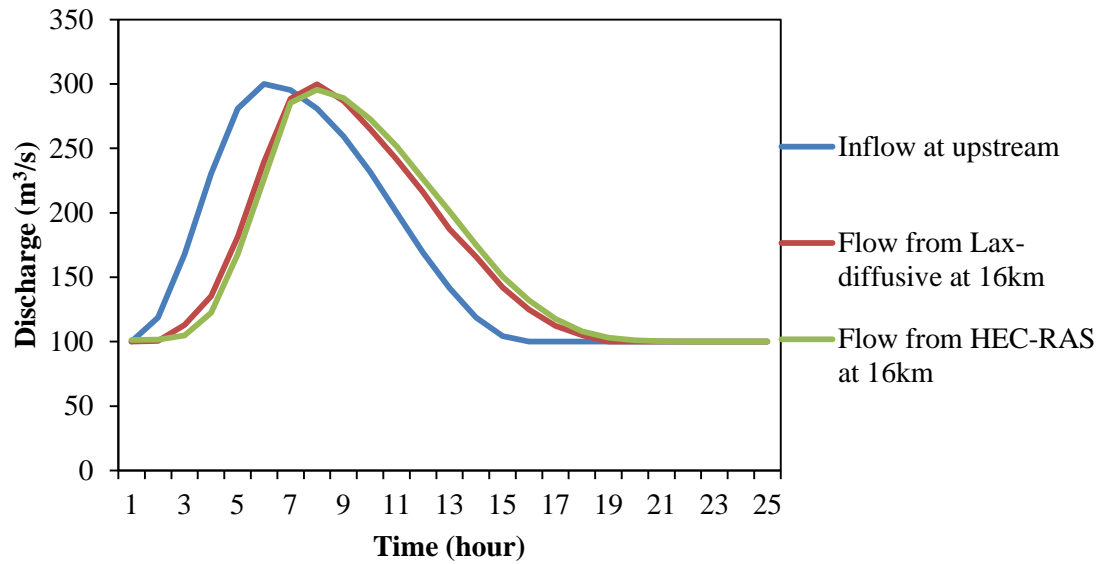


Fig.4.14 Comparison of flow hydrograph at 16km from upstream

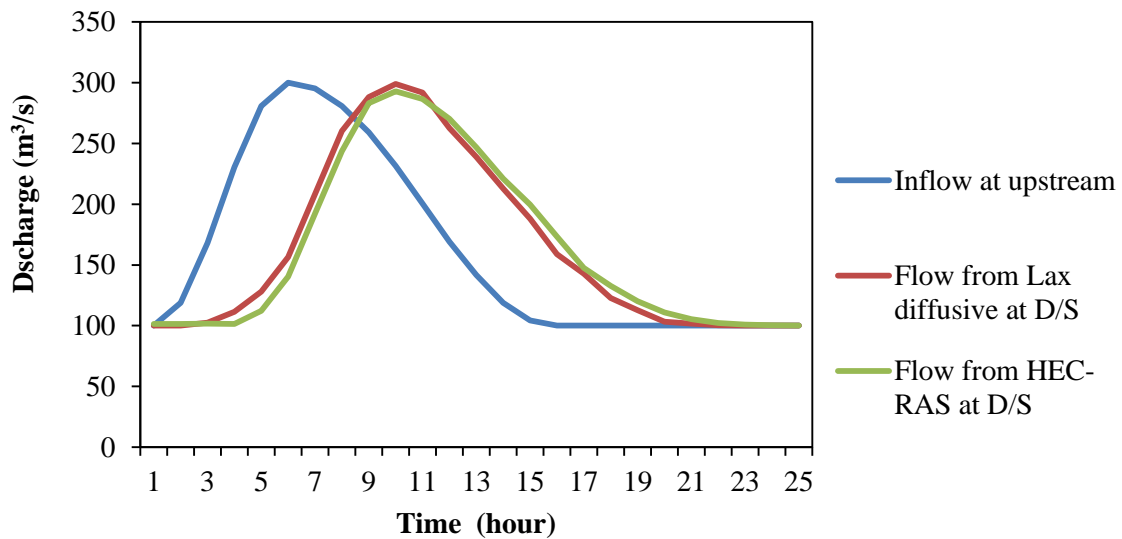


Fig.4.15 Comparison of flow hydrograph at 28km from upstream



PROBLEM STATEMENT AND SOLUTION

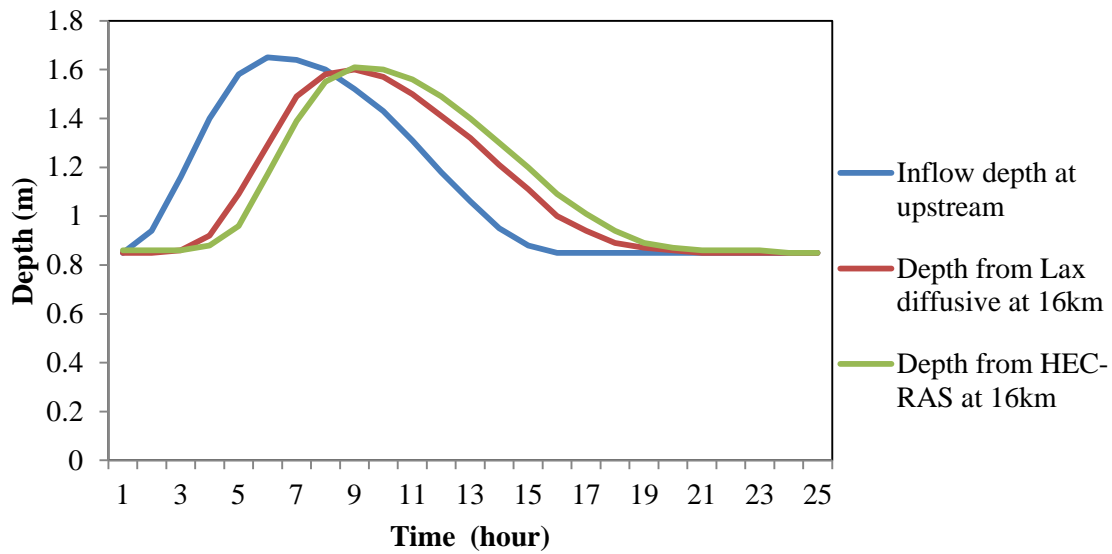


Fig.4.16. Comparison of Stage Hydrograph at 16km from Upstream

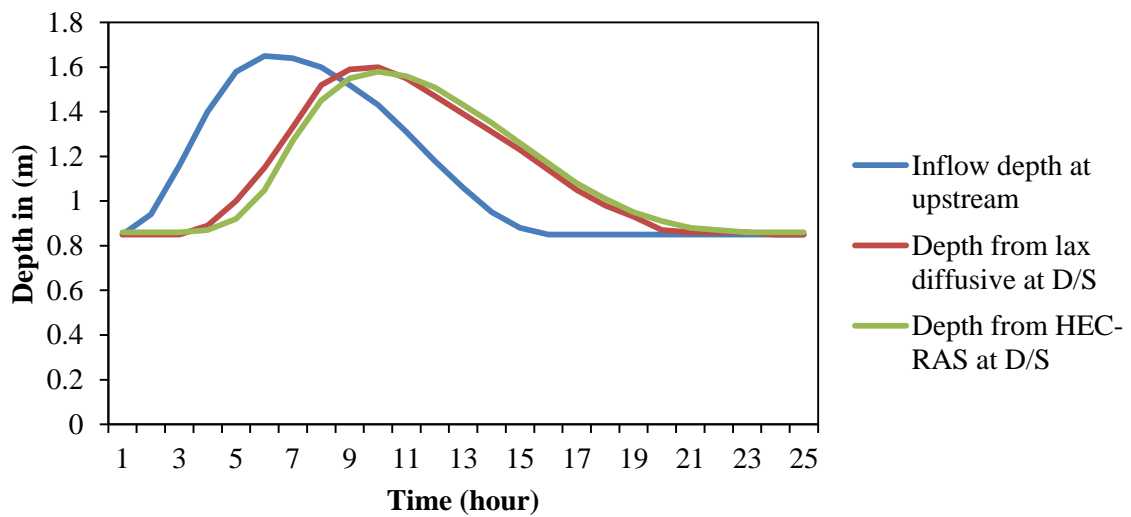


Fig.4.17. Comparison of Stage Hydrograph at 28km from Upstream

CHAPTER 5
CONCLUSIONS AND
SCOPE OF FUTURE STUDY



CONCLUSIONS AND SCOPE OF FUTURE STUDY

5.1 CONCLUSIONS

In this research the solution for unsteady flow in a wide rectangular river through explicit numerical scheme and HEC-RAS computer model are presented.

In the present research, Flood translation or time lag is occurred as the movement of a flood wave occurs from upstream to downstream through a channel. This routing technique is known as channel routing used to analyze the effects of a channel on a hydrograph's peak flow and travel time.

So from routing of the channel, the peak flow and time lag of hydrograph are obtained from present approach of lax diffusive explicit method. The hydrograph at observed sections are occurred due to delay of the inflow hydrograph to come towards downstream. So the inflow hydrograph, when changed at observed sections has peak discharge constant. Because the hydrograph without attenuation is caused due to the time of travel of the flood wave between the two points. Keeping the peak discharge constant, the delay of the hydrograph occurs in observation points at 16km and 28km.

From the result of explicit method, it is seen that the magnitude of wave attenuation is less than $1\text{m}^3/\text{sec}$ which is equal 0.33% of inflow at 16 km from upstream and is equal to $5\text{m}^3/\text{sec}$ which is 1.6% of inflow in HEC-RAS computer model.

The magnitude of wave attenuation is less than $1\text{m}^3/\text{sec}$ which is equal to 0.33% of inflow in explicit method and $8\text{m}^3/\text{sec}$ which is equal to 2.6% of flow in HEC-RAS computer model at 28km from upstream end.

The peak flow depth determined by lax-explicit method is 1.6 meters at 16km from upstream and 1.61 meters in HEC-RAS computer model.

The peak flow depth determined by lax-explicit method is 1.6 meters at 28km from upstream and 1.58 meters in HEC-RAS computer model.

The arrival time of peak flow in explicit method at 16 km after upstream is two hour after the peak of inflow occurs. The HEC-RAS computer model gives same result at that section. At



CONCLUSIONS AND SCOPE OF FUTURE STUDY

28km from upstream, the arrival time of peak flow in explicit method is four hour after the peak of inflow occurs and the HEC-RAS model also gives the same result.

From the present research work, it is found that the present explicit numerical method is giving a similar result with that obtained from HEC-RAS model.

As there is a base flow at initial state before flood occurs, there should not be any losses due to infiltration to soil. And in explicit method it can be seen that there is no reduction of peak flow. So it seems more accurate from the HEC-RAS model.

The present method is easier and user friendly, which can be successfully applied for flood flow routing, modeling, in other river routing studies. It can be also applied in Flood predictions, Evaluation of flood control measures, Judgment of effects in urbanization, Flood warning, Storm water detention, pond storage, Flood mitigation.

5.2 SCOPE OF FUTURE STUDY

This project is an approach towards a better understanding for flood routing problems. This study can be accomplished with few additional features in future. Some of the future scopes are as:

- By using implicit scheme, the routing of flood can be solved numerically for approximate solution and different explicit techniques may be applied successfully in practical field.
- More and more studies can be done for river with flood plain of Natural River.
- Different software's can be used to analyze the numerical results and for comparison purpose also.

REFERENCES



REFERENCES

REFERENCES

- 1] Akbari G. , Firozi B., 2010 *Implicit And Explicit Numerical Solution of Saint-Venant Equations For Stimulating Flood Wave In Natural Rivers*,5th National Congress On Civil Engineering ,Ferdowsi University Of Mashhad, Mashhad,Iran.
- 2] Akbari G. And Firoozi B. *Characteristics of Recent Floods in Persian Gulf Catchment*, BIOINFO Civil Engineering Volume 1, Issue 1, 2011, Pp-07-14
- 3] Amanda Jane Crosseley ,B.Sc ,M.Sc, , October 1999, *Accurate and Efficient Numerical Solutions For The Saint Venant Equations For Open Channel Flow* University of Nottingham.
- 4] Appendix 4-C, *Open Channel Theory*, MDOT Drainage Manual.
- 5] ASCE Task Committee ,*Unsteady Flow Modeling Of Irrigation Canals*, *Journal of Irrigation and Drainage Engineering* ASCE 119(4):615-630, 1993
- 6] Bautista E, Clemmensa J, and Strelkoff T, Member of ASCE, 2003,*General Characteristics of Solution To The Open Channel Flow*, Feed Forward CONTROL PROBLEM.DOI-10-1061/ASCE 0733-9437(2003)129:2.(129).
- 7] CEL 251, Hydrology, *Surface Flow: Flood and Flood Routing*.
- 8] Chapter-8:*Flood Routing Area Method*: U.S.Goel. Survey Techniques Water-Resource Inv.Book-3,Chapter-A 2
- 9] D. Nagesh Kumar ,Falguni baliarsingh, K. Srinivasaraju ,12 August 2010,*Extended Muskingum Method For Flood Routing*.
- 10] Dambaravjaa Oyunbaatar, Gombo Davaa, Dashzeveg Batkhuu, *Some Results of Application of Flood Routing Models in The Kherlen River Basin*.
- 11] Doiphode Sanjay L, OakRavindra A. July-Sept 2012, *Dynamic Flood Routing and Unsteady Flow Modeling: A Case Study of Upper Krishna Rive* .
- 12] Elhanafy H. And Copeland G.J.M, *Modified Method of Characteristics For The Shallow Water Equation*, Civil Engg. Dept. Strathelyde University.U.K.
- 13] HECRAS User's manual
- 14] Houk.I.E, 1918, *Calculation of Flow In Open Channels*: Miami Conservancy Dist.Tech. Repts,Pt4,283p.



REFERENCES

- 15] Hulsing, Harry, 1966, *Measurement of Peak Discharge at Dams by Indirect Methods*, U.S. Geol Survey Techniques Water-Resource Inv. Book-3, Chapter-A 5.
- 16] Junqiang Xia, Binliang Lin, Yanping Wang, 10 July 2012, *Modeling of Man-Made Flood Routing In The Lower River, China*.
- 17] K. Subramanyam, *Flow in Open channel Book*, TMH publication 2009
- 18] Kalita H.M And Sharama A.K., 2012 *Efficiency And Performance of Finite Difference Schemes in Solution of Saint-Venant Equation*, International Journal of Civil And Structural Engineering, Vol-2, No.3.
- 19] Kranjcevic L., Crnkovic B. And Zic N.C., 2006, *Improved Implicit Numerical Scheme For One Dimensional Open Channel Flow Equation*, 5th International Congress Society Of Mechanics, September, 21-23, Croatia.
- 20] Mehdi Delphi, 29 September, 2011, *Application of Characteristics Method For Flood Routing (Case Study: Karun River)*.
- 21] Moghaddam M.A. and Firoozi B., 2011, *Development of Dynamic Flood Wave Routing In Natural Rivers Through Implicit Numerical Method*, American Journal of Scientific Research, ISBN 1450-223X Issue 14.
- 22] Muthiah Perumal, Bhagirathi Sahoo, Tommaso Moramarco, Silvia Barbetta, 2009 *Multilinear Muskingum Method for Stage-Hydrograph Routing in Compound Channels*.
- 23] Muthiah Perumal, 1994 of *Hydrodynamic Derivation of a Variable Parameter Muskingum Method*.
- 24] P. Sreeja, Kapil Gupta, December 2008 of *Transfer Function Formulation of Saint-Venant's Equations For Modeling Drainage Channel Flow: An Experimental Evaluation*.
- 25] Peter L.F Bentura, Claude Michel, April 1997 of *Flood Routing in a Wide Channel With a Quadratic Lag-and-Route Method*.
- 26] R. W. Carter And R. G. Godfrey, 1960 of *Storage and Flood Routing*
- 27] R.S.M. Mizanur Rashid, M. Hanif Chaudhry, 1995 of *Flood Routing In Channels with Flood Plains*.
- 28] Safa Elbashir, 2011 of *Flood Routing in Natural Channels Using Muskingum Methods*, Dublin Institute of Technology.



REFERENCES

- 29] Simmons, D.B. And Richardson E.V,1962, *The Effect of Bed Roughness on Depth – Discharge Relation in Alluvial Channels*. U.S Goel Survey Water Supply Paper 1498-E,26p.
- 30] Tommaso Moramarco,Claudia Pandolfo, and Vijay P.Singh .F. ASCE of *Accuracy of Kinematic Wave And Diffusion Wave Approximations For Flood Routing I:Steady Analysis*.
- 31] Tommaso Moramarco,Ying Fan,And Rafael L. Bras of *Analytical Solution For Channel Routing With Uniform lateral Inflow*, Fellow, ASCE
- 32] Val Érie Dos Santos Martins, Mickael Rodrigues, Mamadou Diagne of *A Multi–Model Approach to Saint–Venant Equations: A Stability Study By LMIS*.
- 33] Yong G.L,(2010),*Two Dimensional Depth Averaged Flow Modeling With An Unstructured Hybrid Mesh*, Journal of Hydraulic Engineering, American Society of Civil Engineering ,136(10,Pp 12-23.
- 34] Zbigniew Kundzewicz, August 1983, *Hydrodynamic Determination of Parameters of Linear Flood Routing Models*.



PUBLICATIONS FROM THE WORK

PUBLICATIONS FROM THE WORK

A: PUBLISHED

- Kamalini Devi, Bhabani Shankar Das , Kishanjit K. Khatua “*Effect Of Roughness Coefficient On Solution Of Saint-Venant Equations In River Management*” Proceedings 3rd International Conference on Sustainable Innovative Techniques Architecture, Civil And Environmental Engineering (SITACEE-2014) in Organized by Krishi Sanskriti, 2014
- Bhabani Shankar Das ,Kamalini Devi, Kishanjit K. Khatua “*Regulation of unsteady flow in open channel by using inverse explicit method and comparison with HECRAS*” Proceedings 3rd International Conference on Sustainable Innovative Techniques Architecture, Civil And Environmental Engineering (SITACEE-2014) In Organized by Krishi Sanskriti, 2014

B: ACCEPTED FOR PUBLICATION

Kamalini Devi, Bhabani S Das and Kishanjit K Khatua, “*Solution of Saint-Venant equation in open channel using different roughness*” Proceedings of 1st International Conference on Innovative Advancements in Engineering and Technology (IAET–2014) , March, 2014, Jaipur, Rajasthan, India.

C: COMMUNICATED FOR PUBLICATION

Kamalini Devi, Bhabani S Das and Kishanjit K Khatua, “*Numerical simulation of free surface flow using Lax-diffusive explicit scheme*” 19th IAHR-APD 2014 congress, 21-24 September 2014, Water Resources University, Hanoi, Vietnam.