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Preservice elementary teachers' pedagogical content knowledge related to area and perimeter: A teacher development experiment investigating anchored instruction with web-based microworlds

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Preservice Elementary Teachers' Pedagogical Content Knowledge Related to Area and Perimeter: A Teacher Development Experiment Investigating Anchored Instruction With Web-Based Microworlds

by

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DEDICATION

This work is dedicated to my family. To my loving wife Karen, without whose faithful and steadfast support this dissertation could not have been completed. Thank you for putting aspects of your life on hold during this journey. I love you. To my daughter Madison, this endeavor began before you were born. You have patiently waited for "the paper" to be done. I cannot tell you how glad I am to be able to finally tell you, "YES Madison, Daddy can play – whenever you want!"

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PRESERVICE ELEMENTARY TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE RELATED TO AREA AND PERIMETER: A TEACHER DEVELOPMENT EXPERIMENT INVESTIGATING ANCHORED INSTRUCTION WITH WEB-BASED MICROWORLDS

Matthew S. Kellogg

ABSTRACT

Practical concepts, such as area and perimeter, have an important part in today's school mathematics curricula. Research indicates that students and preservice teachers (PSTs) struggle with and harbor misconceptions regarding these topics. Researchers suggest that alternative instructional methods be investigated that enhance PSTs' conceptual understanding and encourage deeper student thinking. To address this need, this study examined and described what and how PSTs learn as they engage in anchored instruction involving web-based microworlds designed for exploring area and perimeter. It's focus was to examine the influences of a modified teacher development experiment (TDE) upon 12 elementary PSTs' content knowledge (CK) and knowledge of student thinking (KoST) regarding principles, relationships, and misconceptions involving area and perimeter as they develop simultaneously in a problem-solving environment.

The learning of meaningful mathematics is a personal and independent activity, as one struggles to create and reason through their own mathematical realities and misconceptions. This study describes PSTs' reasonings, misconceptions, and difficulties as they grappled with new knowledge or reconciled new knowledge with prior understandings. Quantitative and qualitative research methods, including case-subject analysis, were used. Instructional sessions similar to Steffe's (1983) teaching episodes comprised this study's intervention.

Results indicate that prior to intervention most of the PSTs possessed a procedural knowledge of area and perimeter and were bound by a dependency on formulas; their KoST pertaining area and perimeter was relatively underdeveloped. They seemed unaware of prevalent misconceptions students acquire while working with these concepts (specifically, units of measure and perceived relationships). The PSTs displayed an ineffective use of drawings to support their responses. Their preoccupation with finding what they judged as "the answer" to various problem-solving situations hindered their ability to properly diagnose and address student thinking and limited their meaningful interaction with the microworlds (MWs). A majority of PSTs felt the MWs were a valuable learning tool for themselves but not for their future students. The planned intervention played a role in the PSTs becoming more perceptive of the difficult mathematics involved with area and perimeter and better equipped to anticipate and address those difficulties with future students.

CHAPTER 1

INTRODUCTION

The notion that many students in elementary through high school struggle with understanding mathematical concepts has been sufficiently documented, as evidenced by performance on national and international assessments (Beaton et al., 1996; Kenny & Kouba, 1997; Rutledge, Kloosterman, & Kenney, 2009). A recent focus in mathematics education, however, has been on the difficulties that elementary in-service and preservice teachers have with the content they are expected to teach. Surveys of elementary preservice teachers report their feelings of apprehension and inadequacy about the mathematical content they will have to teach, as well as their inability to meet current expectations regarding the appropriate use of technology to aid and enhance that instruction (Abdal-Haqq, 1995; Ball, Lubienski, & Mewborn, 2001; Sanders & Morris, 2000; Swafford, Jones, & Thorton, 1997).

In response to these and other concerns regarding the state of mathematics education in America, several leading organizations - including the National Council of Teachers of Mathematics (NCTM), the Mathematics Association of America (MAA), the National Research Council (NRC), and state and national governmental agencies - have issued reports and documents echoing the challenges, laying the framework, and outlining standards to improve mathematics education and the preparation of mathematics teachers (International Society for Technology in Education [ISTE], 1993; NCTM, 1989, 1991, 2000; NRC, 2000; U. S. Department of Education [USDOE], 2000). A common thread within the recommendations of these organizations is the importance placed on teachers of mathematics conceptualizing their content knowledge and being able to incorporate multiple approaches with which to apply that knowledge when teaching. What follows describes a mixed-methods study conducted within an intact methods of teaching elementary mathematics course, taught by the researcher. The study focuses on preservice teachers as they experience innovative technology-based anchored instruction. The study emerges from a noticeable lack of research detailing instructional approaches for addressing the inadequate content knowledge of teachers, specifically on the topics of area and perimeter, as well as their limited perceptions of how and what students think regarding these concepts. This study suggests that such detail is needed if educators are to better understand how to intervene effectively in the mathematics training of teachers to facilitate their knowledge growth so as to influence ultimately student learning.

Shulman (1986) outlines three categories of subject matter knowledge that a teacher of mathematics should possess; content knowledge (CK), pedagogical content knowledge (PCK), and curriculum knowledge. What a teacher knows and how they use that knowledge are critical elements to effective instruction. For this study, content knowledge was thought of as more than simply a collection of isolated facts and algorithms designed to produce correct answers; instead it also included a repertoire of interconnected and meaningful concepts and procedures (Ball, 1990). Although preservice teachers' content knowledge is often the intended focus of the mathematics courses they take, pedagogical content knowledge is left relatively underdeveloped (Brown & Borko, 1992) and therefore needs to be a primary focus of methods courses. A research method called the teacher development experiment (TDE) (Simon, 2000) provided a framework for studying the development of preservice teachers' content and pedagogical content knowledge (from both a psychological and social perspective) within a methods course. Domain-specific knowledge with respect to the pedagogical development of teachers of mathematics is currently lacking within the TDE research paradigm (Simon, 2000). This research study examined the specific concepts of area and perimeter and how preservice teachers' CK and PCK develop with respect to these concepts. Dewey (1964) espoused that content and methods were inseparable in teacher education. He wrote: "Scholastic knowledge is sometimes regarded as if it were something quite irrelevant to method. When this attitude is even unconsciously assumed, method becomes an external attachment to knowledge of subject matter" (p. 160). This study will attempt to follow Dewey's recommendation and study both CK and knowledge of student thinking (KoST). Increased KoST, a critical facet of pedagogical content knowledge (Brophy, 1991; Fennema & Franke, 1992; Shulman, 1986) and a focal point of this study, has been shown to change significantly how teachers interact with students, both mathematically and cognitively (Carpenter, Fennama, Franke, Levi, & Empson, 1999). Equally important is the role played by students within a mathematical learning environment. The NCTM *Curriculum and Evaluation Standards* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Principles and Standards for School Mathematics* (2000) all share a vision in which students are actively involved in learning meaningful mathematics. Before elementary students can learn the mathematics necessary for a successful future, classroom teachers need to be prepared to deliver that content effectively. For this vision to become a reality, teachers need many opportunities to attain, enhance, and explore their mathematical content knowledge in new and challenging ways (ISTE, 1999, 2008; NRC, 2001).

Integrating technology into the learning of mathematics has been shown to have positive effects on achievement, stimulate and enhance spatial visualization skills, and promote a more conceptual understanding of mathematics for students and teachers (Boers-van Oosterum, 1990; Dunham & Thomas, 1994; Groves, 1994; Rojano, 1996; Sheets, 1993). Research has shown that technology can be a valuable tool in promoting conceptual understanding of mathematics within preservice teachers (Keller & Hart, 2002; Wetherill, Midgett, & McCall, 2002) which lends support to a conceptual framework for appropriate uses of technology-supported mathematics activities (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000; Samatha, Peressini, & Meymaris, 2004). It would seem appropriate then that technology play a vital role in helping achieve the desired and necessary reform recommendations. As recently as 2000, the NCTM stated in its *Principles and Standards for School Mathematics*, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 11). However, in spite of such strong endorsements, as well as affirming research, many topics in mathematics which lend themselves to the visually stimulating qualities of technology are continually learned and taught through memorizing and algorithmic processes. In order to address the alleged deficiencies and bring about the recommendations for mathematics reform, new strategies for the delivery and learning of mathematical content need to be investigated. It would also seem reasonable and advantageous to expose preservice teachers to the same types of delivery methods that they are being challenged and encouraged to implement in their future classrooms.

Statement of the Problem

Teaching middle and high school mathematics for 12 years, combined with serving the last 10 years as a teacher educator, has revealed much to me regarding the mathematical understandings of both students and preservice teachers. An interesting, and somewhat troubling, realization has been that many of the preservice teachers I have worked with possess many of the same mathematical weaknesses and misconceptions (especially relating to measurement) as many classroom students discussed in the literature. To help combat such weaknesses, organizations such as the National Council of Teachers of Mathematics (NCTM, 1989, 2000) have advocated an increased emphasis on the teaching and learning of geometry at all levels; not just a traditional, procedural, and static view of geometry, but a dynamic, and visually stimulating discovery of the practical, problem-solving world of geometry (NCTM, 2000). Schmidt (2008) reported that measurement topics, such as area and perimeter, were part of the mathematics curriculum for all the top achieving countries, based on the TIMSS math assessment for seventh- and eighth-graders. These topics are part of a curriculum structure which appears to provide stability and a form of continuity across grades 1-8.

Geometry is a natural place for the development of visualization and spatial reasoning, which are valuable for many life skills (e.g., using maps, planning trip routes, approximating measurements, and designing landscapes). Geometric ideas are helpful in representing and solving many real-world situations. For example, when painting one's house, various area formulas must be applied correctly when deciding on how much paint to buy. The abilities to visualize, interpret, and properly represent measurement concepts are valuable skills for success in mathematics and in life (Clements & Battista, 1992).

Despite the practical value of and emphasis placed upon measurement topics such as area and perimeter, there is considerable research indicating that school students have an inadequate understanding of them (Beaton et al., 1996; Clements & Ellerton, 1996; Hart, 1987, 1993; Kenney & Kouba, 1997; Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988). Research also reveals that preservice and classroom teachers possess various degrees of misunderstandings regarding concepts surrounding area and perimeter (Menon, 1998; Reinke, 1997; Simon & Blume, 1994a; Tierney, Boyd & Davis, 1990; Woodward & Byrd, 1983). These studies also revealed that preservice teachers' understanding of student thinking regarding area and perimeter were severely lacking. This is especially troubling because students' dispositions towards mathematics are greatly influenced by their teacher's likes and dislikes, their expertise, and resulting comfort levels regarding the mathematics they teach (Ball & McDiarmid, 1989). What is also troubling is the lack of research exploring interventions designed to challenge and address area and perimeter shortcomings among preservice teachers. The opportunity for preservice teachers to reexamine and learn about familiar mathematics topics within new environments has the potential to turn the tide on the downward spiral described above.

Meeting the ongoing challenge of finding ways to effectively integrate content and methods within mathematics methods courses for elementary preservice teachers (PSTs) is also a priority of this research. Microworlds are a technology-based learning environment that facilitates exploring alternatives, testing hypotheses, and discovering facts regarding a specially designed context. An instructional strategy well suited to utilizing such an environment is anchored instruction. The major goal of anchored instruction is to develop useful and meaningful knowledge by designing learning and teaching activities around an “anchor” which is often a story, adventure, or situation that centers on solving problems that are of interest to the students (Cognition & Technology Group at Vanderbilt [CTGV], 1991). The latter provided the setting for this study. Anchored instruction may be a dynamic delivery method for geometric content and the use of such instructional approaches in the classroom have been strongly encouraged (NCTM, 2000). The impact of anchored instruction upon PSTs’ mathematical knowledge and their ability to apply that knowledge requires greater exploration. PSTs need many experiences with these new delivery methods to help them develop conceptual understandings of the content being delivered, to see and experience appropriate uses of technology in the teaching and learning of mathematics, and to help instill greater confidence for their future use (Chinnappan, 2000; Connors, 1997). However, there is scant research examining the different influences of anchored instruction upon PSTs’ mathematical content knowledge or their knowledge of student thinking.

Purpose of the Study

The purpose of this study was to examine levels of knowledge in the context of anchored instruction with geometry microworlds upon PSTs’ CK and KoST related to area and perimeter. In particular, it focused on their understandings, misconceptions, written and verbal explanations of that knowledge, and achievement on written area and perimeter tests – within the context of a mathematics methods course for PSTs. Previous research has shown that preservice elementary teachers have contextual and conceptual shortcomings regarding area and perimeter, and because the majority of this research has focused on revealing and measuring such misconceptions, little is known about the underlying causes of these misconceptions, how they may interfere with PSTs' ability to diagnose and address future students' difficulties, or what alternative instructional methods may help alleviate the area and perimeter misconceptions that PSTs have. In short, this study served three purposes: (a) further understand PSTs' cognitions of area and perimeter and how they change and develop through planned intervention, (b) examine the interplay between PSTs' CK and their KoST, and (c) develop and describe the use of anchored instruction, that integrates the use of web-based microworlds designed for exploring perimeter and area, as a potential learning environment for influencing PSTs' CK and KoST.

Conceptual Framework

There is considerable research indicating that students have an inadequate and procedural-based understanding of the concepts of area and perimeter (Beaton et al., 1996; Clements & Ellerton, 1996; Hart, 1987, 1993; Kenney & Kouba, 1997; Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988; Rutledge, Kloosterman, & Kenney, 2009). Research also reveals that preservice and classroom teachers possess varying degrees of misunderstandings regarding these same concepts (Menon, 1998; Reinke, 1997; Simon & Blume, 1994a; Tierney, Boyd & Davis, 1990). The methods coursework and teaching practicum provide preservice teachers with much needed theoretical and practical experiences; however, opportunities for preservice teachers to investigate carefully mathematical content that students find difficult, reflect upon why they find it difficult, and then plan appropriate intervention and follow-up appear to be lacking.

An emerging methodology for studying the development of teachers is the teacher development experiment (TDE) (Simon, 2000). This methodology builds on the central principle of the constructivist teaching experiment (Cobb & Steffe, 1983; Steffe & Thompson, 2000), that is, knowledgeable and skillful researchers can study teacher development by fostering development as part of a continuous cycle of analysis and intervention. Simon (2000) presents the TDE methodology as an adaptation and extension of two groundbreaking research approaches; the development of the constructivist teaching experiment (Cobb & Steffe, 1983; Steffe & Thompson, 2000) and, later, the whole-class teaching experiment (Cobb, Yackel, & Wood, 1993; Cobb, 2000). The constructivist teaching experiment is used to collect and coordinate individual and group data on children's concept development in particular areas of mathematics (Simon, 2000). The teaching experiment is primarily an exploratory tool directed towards understanding the progress students make while learning particular mathematical concepts over an extended time (Steffe & Thompson, 2000). The teaching experiment has been eloquently described by Steffe and Thompson as "a living methodology designed for the exploration and explanation of students' mathematical activity" (p. 274).

The TDE begins with an instructional issue that the teacher/researcher is striving to resolve (Simon & Tzur, 1999). In this study, the issue was that of finding mediums to effectively blend the presentation of content and methods. The contributions of the whole-class teaching experiment reside in attempting to understand mathematical learning as it occurs in the social context of the classroom (Cobb, 2000). It is common practice for the whole-class teaching experiment to expand the teaching experiment to include analysis of classroom social norms, socio-mathematical norms, and individual's mathematical beliefs and values (Cobb, 2000). This expansion of a teaching experiment to include these social aspects, however, may result in sacrificing some details of the individual's mathematical (and for this study, pedagogical) understandings and development (Simon & Blume, 1994b). The goals of this study could not allow for such potential sacrifices, and thus a conscious effort was made to minimize the methodological influences of the whole-class teaching experiment. Admittedly, the social interactions occurring within a classroom can play a role in learning, but they were not a focus of analysis in this study. Although the teaching experiment and whole-class teaching experiment focus primarily on mathematical development within classroom communities consisting of students and a teacher, the TDE is concerned with an additional academic community – the teacher educator and a group of teachers or preservice teachers. Simon (2000) posits that "the TDE can allow researchers to generate increasingly powerful schemes for thinking about the development of teachers' mathematical and pedagogical knowledge in the context of teacher education opportunities" (p. 338).

The focus of this TDE is an attempt to answer the question, "How do preservice teachers endeavor to develop their content knowledge (CK) and knowledge of student thinking (KoST), as related to area and perimeter, that is beyond what they already know?" The goal is to produce an account in which I describe how the preservice teacher goes about resolving conflicts in current knowledge and incorporating new knowledge (i.e., both of content and of student thinking about that content); thus, addressing the instructional issue presented earlier. The development of the TDE employed in this study is based on the interplay of four main constructs. First, and foremost, it is built around the major tenants of anchored instruction which, to summarize briefly, involves facilitating the learning of new knowledge anchored in a context of meaningful activities that are supported collaboratively (CTGV, 1990, 1991, 1992, 1993). Second, it is guided by Shulman's (1987) model for developing pedagogical reasoning. Third, Wales and Stager's (1977) program for problem solving, called *Guided Design*, provides a model for the social interaction between myself (the researcher) and the participants (preservice teachers), and among the participants themselves. Finally, this study's framework is supported by current thinking about the benefits technology, particularly web-based microworlds, suggest for student learning of mathematics. This notion is firmly supported and guided by Marzano's (1998) meta-analysis examining effective instructional techniques.

Specifically, this study examined the influence of anchored instruction that incorporates geometry microworlds on enhancing and deepening particular facets of preservice teachers' pedagogical content knowledge regarding area and perimeter – namely content knowledge and knowledge of student thinking. The assumption is that enhanced content knowledge, combined with appropriate intervention, will result in a more conceptually developed knowledge of student thinking. Although other pertinent dimensions of PCK exist, this study specifically examined two of them, content knowledge and knowledge of student thinking. Below, I describe each component of the framework that guided the development and execution of this study.

Anchored Instruction

Cognitive psychologists claim that knowledge is formed when small chunks of information are woven together within a contextual framework (Klock, 2000). Anchored instruction can scaffold an environment in which knowledge can be formed in that manner. Cobb, Yackel, and Wood (1992) state that there is a disconnect between how mathematics is learned and how it is eventually used in one's environment, and that a constructivist instructional approach can help address this dilemma. Although they were talking about students in the classroom, their statement is very relevant to the typical mathematical instruction received by elementary preservice teachers (Ball, 1988; Ball & Bass, 2000). Anchored instruction is grounded in and derived from constructivist theories of knowledge and is a specific application of situated cognition. It is a research-based paradigm for learning through technology-assisted problem solving developed by the Cognition & Technology Group at Vanderbilt (CTGV), under the leadership of John Bransford, who derived their insights from the work of Dewey (1933) and Hanson (1970). Anchored instruction is a "model that emphasizes the creation of an anchor of focus [typically, technology-based] around which instruction can take place" (Bauer, Ellefsen, & Hall, 1994, p. 131). Videodiscs have often been used to provide an environment to anchor instruction and problem solving to a meaningful context, as is the case with the Vanderbilt Group; however, research has shown that the appropriate choice of the anchor while implementing anchored instruction is more important than media attributes in the teaching of problem solving (Shyu, 1999).

This study involved actively engaging preservice teachers in thinking about and planning for how best to address students' misconceptions in mathematics (a realistic and relevant activity). To help facilitate this activity the context (or anchor) was situated within a learning environment whose instructional sequence explored documented student misconceptions regarding area and perimeter (the authentic content). Geometry microworlds, specifically designed for the mathematical content in this study, provided the dynamic environment to help participants focus on the relevant features of the problem-solving activities.

Format for Instructional Sequence

An instructional goal of developing the participants' content knowledge before addressing their knowledge of student thinking is supported from the literature. Bransford, Vye, Kinzer, and Risko (1990b) acknowledge the critical role that content knowledge plays in thinking and problem-solving. Shulman's (1987) model of developing pedagogical reasoning and action for effective teaching involves a cycle which begins with *Comprehension* and *Transformation*. Shulman proposes that understanding must occur before teaching can take place. Comprehension includes understanding critically a set of ideas to be taught, when possible, in more than one way. Once ideas are comprehended, they must be transformed in some manner before they can be taught and learned by students. An important aspect of this study is the planned development and transformation of content knowledge into knowledge of student thinking - a necessary pedagogical tool. Other research suggests that PCK needs to be built upon other forms of professional knowledge (e.g., content knowledge) (Rowan, Schilling, Ball, & Miller, 2001). In addition, features of Wales and Stager's (1977) "Guided Design" was implemented to provide a model through which I observed, discussed, and interviewed participating preservice teachers as they explored and wrestled with concepts individually and cooperatively with peers. The model includes: (a) introducing (verbally) an interesting problem and a general framework (which included a microworld) for solving the problem, (b) providing time for participants to generate individually and test their own strategies, (c) providing participants time to work with one or two other participants to develop a "group" consensus, and (d) sharing and comparing each group's solution to the strategies used and conclusions attained by an expert (the researcher and supporting research literature). The above processes are not meant to imply that transforming content knowledge into pedagogical content knowledge occurs within a set of fixed stages, phases, or steps. Instead, teacher education can only attempt to provide preservice teachers with the understanding, performance abilities, and a setting in which to develop the tools they will need to teach effectively.

Technology Integration

Other aspects of the intervention used in this study were supported by a meta-analysis of research on instruction performed by Marzano (1998). Based on the findings of over 100 research studies, Marzano identified instructional techniques that had a positive, significant impact on mathematical achievement. Specifically, four of those instructional techniques were shown to have an effect size greater than one and are especially pertinent to research involving instruction that incorporates the use of microworlds. The instructional techniques involve (a) having students represent new knowledge in image-based representations, (b) using computer-based manipulatives to explore new knowledge and practice applying it, (c) generating and testing hypotheses about new knowledge, and (d) modeling of new concepts to students in a direct fashion followed by them applying the concepts to different situations.

All four of these practices were utilized as part of the teaching experiment. Web-based microworlds provided the environment for these instructional techniques to be utilized. The dynamic learning environments afforded by today's technologies have been shown to stimulate and promote a conceptual understanding of mathematics within preservice teachers (Keller & Hart, 2002; Wetherill, Midgett, & McCall, 2002) which also lends support to a theoretical framework for appropriate uses of technology-supported mathematical activities (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000; Samatha, Peressini, & Meymaris, 2004). Microworlds provide such an environment. The epistemology underlying microworlds is derived from constructivism (Jonassen, 1991b); however, microworlds can also support goal-orientated environments in which learning occurs through discovery and exploration (Rieber, 1992). Rieber explains that one way to reach this compromise is by incorporating aspects of guided discovery into the learning activity which would naturally be constrained by the boundaries imposed by a particular microworld.

Microworlds, functioning as cognitive tools (i.e., open-ended learning environments), have been shown to assist in the learning of powerful and fundamentally different mathematics (Jonassen & Reeves, 1996; Pea, 1986), enhance student thinking (Lederman & Niess, 2000), support cognitive processes such as logical reasoning and hypothesis testing (Lajoie, 1993), provide specific feedback appropriate to guide in the learning of new material (Roblyer & Edwards, 2000), and encourage the exploration of mathematical ideas (Jensen & Williams, 1993). In summary, research provides a strong basis for the belief that anchored instruction that integrates web-based microworlds and provides opportunity for students to be immersed in a community of learners has the potential to enhance content knowledge and move it along the continuum of transformation into a useful knowledge of student thinking.

Research Questions

This study described and presented findings regarding an instructional approach that incorporates a form of anchored instruction (The Cognition and Technology Group at Vanderbilt [CTGV], 1992) in which area and perimeter microworlds assisted in providing a rich and dynamic learning environment for both an individual and cooperative approach to situated problem solving. The primary research question examined by this study was, *"In what ways do preservice elementary teachers' (PSTs') content knowledge and pedagogical content knowledge, related to area and perimeter, change as a result of experiencing anchored instruction integrated with web-based microworlds, designed for investigation of area and perimeter?"* In particular:

1. What is the PSTs' content knowledge regarding area and perimeter prior to involvement in the teaching episodes?
2. What is the PSTs' knowledge of student thinking regarding area and perimeter prior to involvement in the teaching episodes?
3. How does PSTs' content knowledge regarding area and perimeter change, if at all, during the course of this study?
4. How does the PSTs' knowledge of student thinking regarding area and perimeter change, if at all, during the course of this study?
5. In what ways, if at all, is the PSTs' knowledge of student thinking regarding area and perimeter related to their content knowledge of those same concepts?

Definitions

The following is a list of the terms that will be used throughout this study:

Pedagogical content knowledge: A kind of content knowledge that is useful for teaching. It includes "the ways of representing and formulating the subject that make it comprehensible to others; an understanding of what makes the learning of topics easy or difficult; the concepts and preconceptions that students of different ages and backgrounds bring with them" (Shulman, 1986, p. 9).

Content knowledge: A facet of PCK that refers to the amount and organization of facts and concepts, including an explanatory framework, about a subject in the mind of a teacher as well as why those facts and concepts are true (Shulman, 1986).

Knowledge of student thinking: A facet of PCK that involves organizing content knowledge in a way that would enable a teacher to understand children's thinking about content areas and appropriately address any shortcomings or misconceptions (Swafford, Jones, & Thorton, 1997).

Procedural knowledge: Many theories of learning and development indicate that procedural and conceptual knowledge lie on a continuum. For this study, they will be separated into the two ends of the continuum. Procedural knowledge will be defined as the ability to execute sequential actions in performing mathematical rules, algorithms, or procedures – typically it involves knowing HOW but not usually WHY.

Conceptual knowledge: A generalizable knowledge that goes beyond isolated facts, procedures, and the words themselves. Someone possessing conceptual understanding has knowledge that is organized, connected, and capable of being communicated in a meaningful way.

Inert knowledge: Knowledge that can usually be recalled when someone is specifically asked to do so but is not available to use spontaneously in a problem-solving situation.

Manipulative: a concrete or symbolic artifact that students interact with to facilitate a deeper understanding of an abstract concept.

Applet: A small, stand-alone version of a computer program or application designed to run on the Internet within a Web browser (i.e., Internet Explorer) and commonly used to add interactivity to websites.

Microworld: A Microworld is a term coined at the MIT Media Lab Learning and Common Sense Group. It means, literally, a tiny world inside which a student can explore alternatives, test hypotheses, and discover facts that are true about that world (i.e., relationships between mathematical concepts such as area and perimeter). (Retrieved July 26, 2006, from: http://www.umcs.maine.edu/~larry/microworlds/microworld.html)

Anchored instruction: "A model that emphasizes the creation of an anchor or focus [typically technology-based] around which instruction can take place" (Bauer, Ellefsen, & Hall, 1994, p. 131).

Situated cognition: The notion that cognition is not confined to the individual, but is connected to social activity and the environment that best reflects the way in which the knowledge will be used (Collins, 1991).

CHAPTER 2

REVIEW OF THE LITERATURE

The purpose of this study was to examine the changes, if any, in PSTs' content knowledge and knowledge of student thinking related to concepts and misconceptions regarding area and perimeter, written and verbal explanations of that knowledge, and achievement on written area and perimeter tests after experiencing anchored instruction with geometry microworlds. This chapter is organized into three main sections of research. The first section provides an overview of knowledge domains useful for teaching, while focusing on two specific domains (i.e., content knowledge and knowledge of student thinking). The second section examines student and teacher knowledge and understanding of area and perimeter. The third section contains a brief summary of the role of technology in preservice teacher education and its effect on learning, followed by a discussion about anchored instruction and microworlds.

Writing about PSTs also involves writing about students and teachers. To avoid confusion in this study, I use the term "preservice teacher (PST)" to mean someone studying mathematics as one of several subjects that will be taught (as with an elementary teacher) or only mathematics (typically future secondary teachers). Unless otherwise noted, the term "students" is reserved for students from Kindergarten to the end of secondary school. The term "teacher" will refer to someone who has graduated from college and teaches mathematics at the elementary, middle, or secondary level.

Knowledge Domains and the Craft of Teaching

There is little doubt that what a teacher knows impacts what is done in the classroom and ultimately what students learn (Fennema & Franke, 1992; Hill, Rowan, & Ball, 2005). It would seem reasonable then for those involved with teacher education to make every attempt to equip today's preservice teachers with the knowledge necessary to teach, as well as the ability to conceptualize and communicate that knowledge. However, there is very little consensus when it comes to defining what critical knowledge is needed to ensure that students learn mathematics. Many types of knowledge useful for teaching have been identified. For example there is general pedagogical knowledge, content knowledge (also referred to as subject matter knowledge), pedagogical content knowledge (which encompasses knowledge of student cognitions and knowledge of curriculum and school contexts), and knowledge of learners and their characteristics, beliefs, and attitudes (Manouchehri, 1997; Shulman, 1986). This study focused on two of these knowledge types: content knowledge and knowledge of student cognitions, which will be referred to as "knowledge of student thinking." Researchers such as Brophy (1991), Fennema and Franke (1992), and Shulman (1986) have identified these two components of teacher knowledge as critical in the teaching and learning process.

Research has well documented that many novice teachers, especially elementary, struggle to varying degrees with the content they must teach including: multiplication and place value (Ball, 1988; Ma, 1999; Steinberg, Haymore, & Marks, 1985), division (Ball, 1990; Post, Harel, Behr, & Lesh, 1991; Simon, 1993), fractions (Khoury & Zazkis, 1994;

Lehrer & Franke, 1992; Leinhardt & Smith, 1985); functions and graphing (Even, 1993; Wilson, 1994; Stein, Baxter, & Leinhardt, 1990), geometry and measurement (Baturo & Nason, 1996; Heaton, 1992; Simon & Blume, 1994a), and proof (Ball & Wilson, 1990; Ma, 1999; Martin & Harel, 1989). Each of these areas represents subject matter that needs increased attention as part of teacher education. Rather than focusing on results related to teachers' lack of specific content knowledge, this portion of the literature review examines the difficulties teachers experience when they teach without a conceptual content knowledge, the cognitive issues that surround these difficulties, and approaches used to address these difficulties.

Content Knowledge and Pedagogical Content Knowledge

Before the literature is reviewed, it is important to delineate clearly the knowledge domains that will be discussed. Content knowledge (a facet of pedagogical content knowledge) consists of the amount and organization of facts, concepts, and principles, including an explanatory framework, about a subject in the mind of a teacher as well as why those facts and concepts are true (Shulman, 1986). Different subject matter areas all have content structures that must not only be learned by teachers but also be made clear, represented well, and categorized in useful ways. Teachers need to be able to explain why certain truths are accepted, and even how those truths relate to subject matter outside the domain being discussed. Content knowledge valuable for teaching should ideally scan the scope of Bloom's taxonomy when interacting within the classroom environment (Ball, 2003). Clearly, a teacher's content knowledge will be an integral part of their teaching, and a lack of it will very likely affect the quality of instruction (Grossman, Wilson, & Shulman, 1989) and ultimately student learning (Fennema & Franke, 1992). There is considerable research pertaining to various aspects of teachers' content knowledge or subject matter knowledge, but this review will primarily focus on efforts to enhance pre- and inservice teachers' content knowledge, and will appear later in the review.

The term *pedagogical content knowledge* was originally used by Lee Shulman to describe what he called at the time a "missing paradigm" in the research on teaching. Shulman acknowledged that content knowledge, which is "the amount and organization of knowledge in the mind of the teacher" (p. 9), is inseparable from PCK; however, PCK goes beyond a mere knowledge of subject matter (mathematics for example) to a dimension of content knowledge that is usable for teaching and learning. Pedagogical content knowledge (which includes knowledge of student thinking) facilitates the effective teaching of subject matter. It involves the most useful forms of representations of ideas, analogies, illustrations, examples, and explanations (Shulman, 1986). PCK can be defined as an understanding of how to represent specific topics in ways appropriate to the diverse abilities and interests of the learners (Grouws & Schultz, 1996). It has been described as the seamless interweaving of subject matter and pedagogy useful for teaching and learning (Ball & Bass, 2000).

Characterizing PCK

What makes a teacher an expert? Expertise in mathematics instruction develops over many years and takes on many different forms. Two critical areas that must be under ongoing construction, while on the road to becoming an expert, are knowledge about content and knowledge about students' thinking (Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992). Both these categories of knowledge are specific dimensions of pedagogical content knowledge (Shulman, 1987). When discussing mathematical content knowledge, researchers often use the terms procedural and conceptual to denote a distinction between two forms of content knowledge (Eisenhart et al., 1993).

As commonly used, procedural knowledge refers to mastery of symbolic representations, computational skills, and knowledge of procedures for identifying and solving various mathematical components, algorithms, and definitions. For example, a student with procedural knowledge of divisions of fractions will know the steps for writing down the problem, performing the division algorithm (first, invert the divisor, and then multiply the two fractions). Teaching a procedural knowledge for the division of fractions is exemplified by presenting a step-by-step procedure for producing an answer, often accompanied by strategies for remembering the steps of the algorithm. For example, “Yours is not to question why, just invert and multiply.” Such statements when presented in the context of “learning” about fractions are troubling on many levels. Any teacher who uses such instructional strategies, although they may not be classified a novice based on years of experience, would certainly possess a novice’s knowledge of mathematical content and pedagogy.

Novice PCK

Preservice elementary teachers (including student teachers) are obviously considered novices. As mentioned earlier, there have been many studies documenting the ways in which novice teachers struggle with the mathematical content they must teach. However, there is far less research examining novice teachers’ PCK and how that knowledge influences their thinking about student thinking and subsequently their instructional decisions. Borko et al. (1992) studied eight senior, preservice elementary teachers who had selected mathematics as a concentration and were intending on teaching middle school. They reported extensively about one specific preservice teacher called Ms. Daniels. Even though Ms. Daniels had the strongest mathematics background of any participant, that knowledge did not apparently serve her well when forced to make instructional decisions in front of students. Teaching situations revealed a limited repertoire of instructional representations. She was unable to generate meaningful examples in response to students' questions. During interviews it was revealed that Ms. Daniels put a greater importance on learning activities and accumulating "ideas that will work" than on the conceptual information presented in her methods courses. For whatever reason she apparently had not acquired the words, mental pictures, or the conceptual knowledge needed to produce an adequate explanation during whole-class instruction. Mapolelo (1999) had similar results while studying the PCK of three prospective middle school teachers who had been identified as "outstanding in mathematics" (p. 715). Their strong mathematics background did not apparently transfer directly into a classroom-ready pedagogical content knowledge. When given opportunity to teach, all of the student teachers in the study resorted exclusively to a lecture method that was procedural and explanation orientated. In most cases their explanations, although accurate, focused on procedures and did not encourage the students to connect mathematical concepts. The student teachers expressed confidence regarding the mathematical content they would be teaching; however, their content knowledge did not appear sufficiently supported by PCK to facilitate flexible, responsive teaching. They had difficulties responding to student questions and seemed ill-equipped to design meaningful activities that would enhance conceptual understanding.

It does not appear that increased mathematics training (i.e., content knowledge) alone will develop or enhance pedagogical content knowledge. Meredith (1993) found that even preservice elementary teachers specializing in mathematics were often "baffled by learners' difficulties" (p. 332). A strong mathematical content knowledge does not seem to translate into understanding how students think about and learn mathematics or predicting common difficulties. Mapolelo (1993) reported that some middle grades student teachers, even though possessing extensive mathematics background, also lacked the ability to anticipate misconceptions that students might have regarding learning the concepts at hand. It seems apparent that research is needed to explore avenues to better equip preservice teachers with knowledge regarding the common misconceptions children have about elementary mathematics and how best to address them.

Expert-novice PCK Differences

Borko et al. (1992) reported that novice teachers are very concerned about their limited pedagogical content knowledge and the impact such a shortcoming may have on teaching and learning. Research also indicates that the PCK acquired by novice teachers is primarily procedural in content and application (Ball & Wilson, 1990; Fuller, 1996). Teachers possessing conceptual understanding of mathematics interact with both content and students in fundamentally different ways. Conceptual understanding involves knowledge of the underlying structure of mathematics, how various concepts connect, and a realization of the various relationships between ideas that facilitate meaningful explanations of mathematical procedures (Eisenhart et al., 1993). In the case of division of fractions, conceptual knowledge would include discussing the nature of fractions in general as well as specifics regarding the fractions to be divided. The meaning of division would be investigated – often exemplified by using concrete and semi-concrete models (i.e., Cuisenaire rods, Hershey bars, paper folding, or drawings). The expert teacher exhibits a greater propensity towards incorporating such learning tools into their instruction.

Fuller's (1996) qualitative research suggested that experienced teachers seem to possess a greater conceptual understanding of certain mathematical topics than their preservice counterparts. An example of such knowledge was the fact that the classroom teachers were much more likely to suggest using manipulative materials to help students understand mathematical concepts as opposed to the procedural-laden responses of preservice teachers. One shortcoming however to Fuller's (1996) study is the vagueness with which some of the findings are reported. It appears a lack of substantive follow-up (possibly interviews) to the instrument used, the Survey on Teaching Mathematics (Rich, Lubinski, & Otto, 1994), lent itself to this vagueness. For example, one of the expert teachers participating in the study indicated they would "draw pictures or use manipulatives to demonstrate" (p. 25) in response to a survey question involving a student who had a mathematical misconception. Although the teacher's response does seem to indicate a tendency toward conceptual-based instructional strategies, the reader is left to wonder exactly what pictures or manipulatives would have been used and why.

Other researchers have reported the conceptual approaches of expert teachers. Mitchell and Williams (1993) observed expert teachers, more than twice as often as their novice counterparts, incorporating technology to promote a focus on understanding content and process. Expert teachers not only present content differently than novices, but their more developed PCK enables them to more thoroughly synthesize mathematical material for the purpose of review. Livingston and Borko (1990) investigated how secondary mathematics student teachers prepared for and conducted review lessons as compared with their expert cooperative teachers. Review lessons provide a unique opportunity for a teacher to blend content knowledge and knowledge of student thinking in a setting that often includes improvisation. The main difference between the novice and the expert appears to be one of focus. Livingston and Borko (1990) reported that the expert teacher's focus is the student while the novice tends to focus on the content and task at hand. The expert teacher has more extensively developed schemata for PCK that includes more inclusive planning, a greater repertoire of explanations, representations, and knowledge of common errors and misconceptions. Novice teachers on the other hand seem to have a limited PCK about students – how they learn the subject matter, the common errors they make, as well as an awareness of the misconceptions they harbor. Although some instructional settings (e.g., reviewing for an exam) can produce clear distinctions between the expert and novice teacher, certain content areas appear to be troublesome to both.

Fractions seem to elicit procedural approaches to teaching and learning by both novice and experienced teachers (Fuller, 1996). In such cases performance and getting right answers takes priority over understanding. Instructional strategies involving certain mathematical topics (e.g., knowledge of fractions) also reveal varying levels of conceptual understanding among the expert teachers (Leinhardt & Smith, 1985). Perhaps teachers need to revisit difficult concepts and reflect upon their teaching practices in the hopes of transforming procedural approaches to conceptual. Procedures are a necessary part of mathematics; however, conceptual teaching would present a web of connected ideas encompassing fractions with the intent to help students understand how and why mathematical procedures produce right answers. Brown and Borko (1992) argue that without a conceptual understanding of mathematical ideas, teaching mathematics from a conceptual perspective is inconceivable.

To be considered complete, a mathematics education should include aspects of both procedural and conceptual knowledge. There is no serious conflict in their development or implementation (Ma, 1999). Thus, if the goal is to teach for mathematical understanding, then the teacher must incorporate instruction that facilitates the development of mathematical procedures within a framework of conceptual understanding (Wearne & Herbert, 1988). The expert teacher understands that procedures in mathematics should always be accompanied by conceptual representations (Hiebert & Carpenter, 1992). The importance of equipping pre- and inservice teachers with PCK useful for teaching cannot be overstated. Grossman (1991) articulates the importance of this domain of knowledge for the teaching and learning of mathematics:

> If teachers are to guide students in their journey into unfamiliar territories, they will need to know the terrain well. Both knowledge of the content and knowledge of the best way to teach that content to students, help teachers construct meaningful representations, representations that reflect both the nature of the subject matter and the realities of students' prior knowledge and skills. (p. 203)

Reforming Pedagogical Content Knowledge

The knowledge needed to teach is uniquely different in both content and purpose from the knowledge possessed by non-teaching peers. To Shulman (1987):

> The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content

> knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by students. (p. 15)

It would be hard to question the importance of developing expert teachers who possess a powerful and flexible pedagogical content knowledge; however, there are many opinions regarding what activities can develop such knowledge. Feinman-Nesmer and Buchmann (1986) argue that novice teachers do not acquire pedagogical content knowledge until they are faced with the challenges of actual classroom teaching. In lieu of personal experiences, which are not always possible or expedient, there are several recommendations. Ball and Bass (2000) encourage using opportunities to learn content that either simulate or are situated in the contexts in which subject matter is used. For example, some teacher educators use children's work as a site to analyze and interpret students' knowledge as well as an opportunity for pre- and inservice teachers to revisit the content themselves (Barnett, 1998; Schifter, 1998). Other researchers and teacher educators promote the use of video clips depicting exceptional classroom lessons or cases of classroom episodes as a means of fostering the development of PCK (Kellogg & Kersaint, 2004; Lampert & Ball, 1998). Reflecting upon previously learned content knowledge and the context in which it was learned has been suggested as a valuable platform from which to attempt the transformation of PCK (Meredith, 1993). There seems to be a building consensus that developing PCK should occur simultaneously with the development of CK (Good & Grouws, 1987; Stacey et al., 2001), and that without adequate CK, the acquiring of PCK is severely hampered (Hutchison, 1997). Zeichner and Tabachnick (1981) state that unless teacher education seeks to also reform the content knowledge of their preservice teachers along with their pedagogy, the lasting effects of methods classes will be weak. Brown and Borko (1992) would seem to agree when they argue that:

> Unless novice teachers experience good mathematics as students, see it modeled by teachers they respect, and are situated in a culture of teaching that accepts and practices good teaching, it will be difficult for them to implement and maintain good teaching in their classrooms (p. 227).

Innovative Interventions for Pre- and Inservice Teachers

Developing Meaningful Content Knowledge

As stated earlier, there is no shortage of research documenting that preservice teachers, especially elementary, struggle with the mathematical content they must teach. Sadly, many preservice teachers are not willing to take personal responsibility for their mathematical shortcomings. Sanders and Morris (2000) reported that the majority of the preservice elementary teachers in their study offered excuses ranging from technical terminology to non-coverage at their school for their knowledge deficits regarding the elementary mathematics they must teach. Some preservice teachers were embarrassed by poor test results and felt inadequate to tackle their lack of content knowledge. Fortunately, other evidence suggests that improvements in areas of content deficiency can be made. Preservice teachers' content knowledge has previously been thought to be developed adequately in university mathematics courses (Brown & Borko, 1992), but researchers are now recommending that it should be addressed in the methods courses from a different perspective (Manouchehri, 1997). Ball (1990) contends that mathematics methods courses can change not only the pedagogy of preservice teachers but also their mathematical knowledge if the course is constructed with that as a goal.

Mathematics methods courses have been the setting for several studies aimed at reforming preservice teachers' content knowledge into conceptual understanding well-suited for the classroom. Constructivist approaches to learning are often preferred by teacher educators. They can be useful in encouraging preservice teachers to investigate and more importantly challenge their prior learning and then promote the reconstruction of incorrect or weak mathematical ideas (Cobb, 1987). Stoddart, Connell, Stofflett, and Peck (1993), following constructivist principles, developed a five-week conceptual change content unit on rational numbers to investigate ways of improving elementary preservice teachers' mathematical understanding. Qualitative methods (i.e., interviews) were used to evaluate change in content understanding as a consequence of the conceptual change instruction. Although the findings indicated a substantial improvement in the content knowledge of the preservice teachers (n = 18) who received conceptual change instruction, a few limitations should be reported. The study offered no description of the posttest (i.e., Were the items the same or parallel?), and no interview samples (or vignettes) were provided. Lastly, the study reported that the participant's responses were "analyzed to evaluate change in content understandings as a consequence of the conceptual-change instruction" (p. 233); however, the method of analysis was not described nor were samples of participants' responses presented or discussed. Although Stoddart et al.'s findings were promising, the short duration of the study (5 weeks) and small sample size suggests a need for further work with larger samples investigating the influences of longer intervention integrating mathematical content into methods courses.

Quinn's (1997) research extended aspects of Stoddart et al.'s (1993) by integrating the study of mathematical content throughout a semester-long methods course. Quinn did not use a conceptual-change model but did design his elementary mathematics course around constructivist-based recommendations. An open classroom atmosphere was established where student questions were encouraged and valued, and learning activities were designed for participants to engage in hands-on, cooperative work. The course stressed the instilling in children a conceptual understanding of mathematics. A test devised to measure conceptual and intuitive understanding of mathematics was used for both the pre- and posttests. A correlated-groups *t* test comparing the preservice elementary teachers' pretest and posttest scores was statistically significant, $t(26) = 4.1$, $p < .001$, indicating the meaningful knowledge of mathematical content of the participants increased significantly during the course, albeit with a small sample size. An interesting side note from Quinn's study was the fact that, of the many content areas addressed in the course, geometry was one of the most troubling for the preservice elementary teachers – even after the semester-long intervention. Quinn would seem to suggest that changes in mathematical content courses for preservice teachers would only enhance their conceptual understanding of the mathematics they must teach. McGowen and Davis (2002) partially addressed Quinn's concerns by conducting a case study of one of the forty-six participants enrolled in a specially designed mathematics content course for preservice elementary teachers. A preservice teacher named Holly was selected for study because of her unique combination of very poor computational skills and outstanding higher-order thinking skills. Analysis of Holly's three separate takings, spread out over the course of a semester, of a 30 question paper-and-pencil competency exam of basic arithmetic computation (her scores were 20%, 50%, and 87%), along with interview data, revealed noticeable growth of her mathematical understanding. McGowen and Davis argue that preservice elementary teachers are in need of a mathematics foundation to build upon before they will be able to think about how to use their mathematical knowledge in the classroom. In other words, a strong foundation in content knowledge is essential to constructing pedagogical content knowledge truly useful for classroom instruction.

Constructing Pedagogical Content Knowledge

Relatively speaking, research examining the development of pedagogical content knowledge is still in its infancy. Although the line separating CK from PCK is blurry, with PCK containing elements of subject matter knowledge and general pedagogical knowledge (Marks, 1990; Shulman, 1986), it is the view of Shulman, and others, that PCK builds on other forms of professional knowledge (e.g., content knowledge) and therefore is a critical element in the knowledge base of teaching (Rowan, Schilling, Ball, & Miller, 2001). Hutchison (1997) acknowledged the documented CK limitations among preservice and inservice teachers; however, in this study she explored the tie that such weaknesses have to subsequent PCK. Hutchison's case study, Jeannie, involved a preservice elementary teacher who entered her methods course with a procedural-only knowledge of elementary mathematics. Qualitative analysis revealed that although Jeannie strongly desired to be a good teacher, her limited CK resulted in a sporadic and unconnected PCK. Further research is needed to determine effective ways to bridge the gap between a teacher's content knowledge and the pedagogical content knowledge needed for teaching.

In certain instances preservice teachers' PCK has shown limited development even in spite of limited CK. Simon and Blume (1996) conducted a whole-class constructivist teaching experiment examining how mathematical justification, a facet of PCK, could develop within a methods course for prospective elementary teachers. It was reported that participants possessing limited conceptual understandings were hindered in their sense-making of various arguments presented as well as in their ability to accept valid justifications; however, classroom norms regarding presenting, listening, and evaluating mathematical justifications were established by all participants. Being able to justify mathematical responses helps promote and reinforce meaningful understanding within students and builds schemas of students' thinking within the mind of the teacher. Rhine (1998) goes as far as to suggest that increased achievement may be attained if teachers learn about students' thinking from a variety of sources.

Promoting an Awareness of Student Cognition

Knowledge of student thinking is but one component of a teacher's pedagogical content knowledge (Shulman, 1986), and in Shulman's view includes a knowledge of common conceptions, misconceptions, and difficulties that students encounter when learning particular concepts. Shulman (1986) goes on to say that, "The study of student misconceptions and their influences on subsequent learning has been among the most fertile topics for cognitive research (p. 10). Based on their limited teaching experiences, it would not be surprising that preservice teachers lack an understanding of how students think regarding the mathematics they learn. Research confirms this. Even and Tirosh (1995) studied 162 prospective secondary mathematics teachers in the last stage of their formal preservice training. The study investigated how the preservice teachers responded to questions dealing with hypothetical students' difficulties with concepts involving functions and undefined mathematical operations. Through questionnaires and follow-up interviews, Even and Tirosh found that although most of the subjects were able to find the errors in the students' work and provide appropriate rules or definitions to support their answers, they were sadly lacking in the ability to analyze the student's thinking, provide coherent reasons as to why the student gave the answer they did, and explain the concept(s) to the student – other than providing a rule or definition. Results such as these should strengthen the resolve of teacher educators about the importance of addressing student thinking with their preservice teachers (Ball, Lubinski, & Mewborn, 2001). Graeber (1999) further strengthens that point by stating: "If preservice teachers understand that instructional decisions can be guided by what is known about children's understanding, they may be more motivated to pursue understanding of the children's understanding" (p. 195).

Because knowledge of student thinking does not appear to be sufficiently gained by preservice teachers during their coursework, one would be left to assume that such knowledge is attained through interacting with students in the classroom setting. Research does not back up such a claim (Ball et al., 2001; Ma, 1999). The realization of the need for teachers to understand how and why students think the way they do has been slow to develop. Research pertaining to knowledge of student thinking is still in its infancy. In mathematics education, it gained prominence through the work of two extensive research-informed professional development projects that investigated how informing teachers about how children thought about specific mathematical concepts would change the teachers' beliefs and instructional practices and influence student achievement: Cognitively Guided Instruction (CGI) at the University of Wisconsin Madison, and Integrating Mathematics Assessment (IMA) at the University of California, Los Angeles (Rhine, 1995). Each project designed professional development models based on educational research.

A precursor to these projects, Carpenter, Fennema, Peterson, and Carey (1988) investigated how teachers' knowledge of and beliefs about their students' thinking are related to student achievement. They used questionnaires and an interview with 40 first grade teachers and found that the teachers had an informal knowledge about the mathematical thinking of their students, but it was not organized in such a way as to inform classroom instruction. Follow-up research brought the beginnings of the CGI project, under the initial guidance of Carpenter, Fennema, Peterson, Chiang, and Loeff (1989). CGI sought to investigate how incorporating research-based materials into a professional development program would assist teachers in organizing their knowledge of student thinking and in turn influence student achievement. Initial CGI studies focused on addition and subtraction word problems with multiplication and division being included within later studies (e.g., Fennema, 1996). For the Carpenter et al. (1989) study, 40 first grade teachers participated in the study. Half ($n = 20$) were randomly assigned to the treatment group and participated in a 4-week summer workshop designed to familiarize the teachers with research findings on how young children think about and develop solutions strategies for addition and subtraction and to give them an opportunity to plan instruction based on that knowledge. Subsequent classroom observations of teachers receiving the CGI training revealed that they spent significantly more time on word problems than on number facts – a focus of the control teachers. The CGI teachers posed more problems to their students, focused more on the thought processes of their students than on their answers, and knew more about how individual students' solved problems.

This increased awareness and knowledge of students' thinking resulted in higher levels of achievement in problem solving as compared to the students of teachers without this knowledge (extensive tables provided means, standard deviations, and between-groups *t* tests). Follow-up CGI studies by Carpenter and Fennema (1992) and Fennema, Franke, Carpenter, and Carey (1993) reported similar results.

Fennema et al. (1996) performed a subsequent 4-year longitudinal study examining the changes of 21 primary grade teachers who participated in CGI professional development. By the end of the mixed-methods study (observations, interviews, paper-and-pencil instruments, informal interactions, and supportive descriptive statistics), the instruction of 90% of the teachers had become more cognitively guided with the focus of engaging students in authentic problem solving. The substantial gains in students' problem-solving performance as well as teachers' understanding of students' concepts appeared to be related directly to changes in teachers' use of research-informed instruction. What was striking was that this shift in emphasis from skills to concepts and problem solving did not result in a decline in performance on measures of computational skills. It should also be noted that it is hardly a trivial matter to be able to convince teachers to focus on concepts and problem solving rather than on computational skills. These results also have significance to the field of teacher education.

The IMA program, guided by findings regarding effective professional development, identified four elements it believes to be critical in supporting effective instruction: (a) Teachers need a deep understanding of the mathematics they teach, (b) teachers need a deep understanding of the ways that children learn mathematics, (c) they need to support pedagogies that elicit and build upon students' thinking, and (d) teachers need to engage in analytic reflection of their practice (Gearhart et al., 1999). The primary goal of the IMA professional project was to bridge developmental research and practice by helping teachers interpret student cognitions as they made sense of challenging mathematics (specifically fractions). Initial IMA research compared two groups of teachers using the same activity-based, reform minded curriculum (Rhine, 1998). One group received professional development emphasizing the understanding of student thinking. The second group met monthly to collaborate and provide support while preparing for and teaching the unit on fractions. Gearhart et al. (1996) found that the teachers receiving the IMA training provided their students with more opportunities to be engaged with substantive activities involving fractions than did the second group. Gearhart and Saxe (1999) continued the development and investigation of the Integrating Mathematical Assessment (IMA) professional development program by leading a second research team in measuring the impact of professional development upon student's opportunities to learn while studying fractions. Three groups of elementary teachers ($n = 21$) volunteered to participate in the study. Nine teachers received IMA professional development, seven teachers (called the "Support" group) were given the opportunity to build a supportive community of like-minded colleagues, and five teachers committed to teaching with skills-based textbooks. The first two groups of teachers used a problem-solving curriculum. Data from videotapes of classroom instruction and field notes were coded and analyzed. Detailed rubric-like rating scales were used to measure integrated assessment, conceptual issues related to problem solving, and opportunity to gain understanding of concepts linked to uses of numeric representations. A hierarchical linear model (HLM) was fit to student pretest-posttest scores. The HLM along with qualitative analysis revealed mixed results, but overall showed that the problem-solving curriculum (the IMA and Support groups) provided students greater opportunities to engage conceptually with the ideas related to fractions than the skill-based textbooks. Another key finding was that using a curriculum built around assessment of student thinking, as is the reform-based, was more likely to positively affect students' opportunity to learn. Saxe, Gearhart, and Nasir (2001) also researched the effectiveness of IMA. Their methods were very similar as Gearhart and Saxe (1999) in that they elicited volunteers ($n = 23$) who were placed in the same three groups (IMA, Support, and Traditional); however, the 2001 study was purely quantitative in nature. A paper-and-pencil test was used to achieve measures of both computational and conceptual performance. The ANCOVA on the *conceptual* scale revealed a main effect for GROUP $F(2,18) = 7.21$, $p < .005$) followed by a Tukey-HSD post hoc test found the IMA means were greater than the Supported and the Traditional groups. The ANCOVA on the *computational* scale did not reveal an effect for GROUP at conventional levels of significance ($p < .05$); however, although the students in the IMA groups did outperform the other two groups on the computational items (not significantly though), the students in the Traditional groups showed greater achievement on computational items than the students in the Supported groups. These findings indicate that to take full advantage of reform curriculum teachers may well need further support (e.g., IMA) than simply collaborative help of colleagues.

The findings from IMA research would appear to be encouraging to proponents of reform-based professional development; however, one limitation related to IMA research is the extensive use of volunteers. Admittedly more difficult, random assignment of teachers would yield more reliable measures of the program's overall effectiveness.

Rhine (1998) summarized the findings from CGI and IMA research by suggesting that, "teachers' engagement with educational research into students' thinking provides the catalyst that reorients teachers towards the importance of integrating assessment of students' thinking into their instruction" (p. 30). After examining the research conducted by CGI and IMA, it was not apparent that either research program specifically addressed misconceptions regarding the mathematical content they investigated; however, because misconceptions are prevalent within mathematics and confound aspects of student (and teacher) thinking, it would seem advisable to include a discussion of them within any training program designed to improve knowledge of student thinking.

Swafford, Jones, and Thornton (1997) appeared to build upon the CGI research by employing an intervention program for elementary teachers designed to enhance not only teachers' knowledge of research-based findings regarding student cognition (specifically geometry and the van Hiele levels[3]) but also their content knowledge (in geometry). The researchers used multiple measures to analyze the changes in teacher content knowledge and instructional strategies brought about by the intervention of a 4-week summer session and six half-day seminars during the academic year. The emphasis during the sessions was about 85% geometry content and 15% research findings regarding student cognition and the van Hiele levels of geometric thought. The researchers found that teachers experienced a significant, $t(49) = -5.5$, $p < .001$, pretest-posttest gain in geometric CK, 72% of the teachers increased by at least one van Hiele level with more that 50% of the teachers increasing by two levels. This new found knowledge translated into several

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[3] See Swafford, Jones, & Thornton (1997) p. 469 for more information regarding the van Hiele levels of geometric understanding.

important classroom behaviors. Lesson-plan analysis and classroom observations following intervention revealed the teachers now spent more time and more quality time on geometry instruction and possessed the confidence to provoke and respond to higher levels of student thought. Gaining confidence in the teaching of mathematics was also reported in qualitative research conducted by Lowery (2002). She sought to understand how preservice elementary teachers construct CK and PCK while participating in a content-specific methods course that had immediate access to school-based experiences. The intervention provided a unique combination of methods instruction focusing on content knowledge with direct access to field experiences. This setting facilitated the blending and enhancement of CK and PCK in a situated-learning context. Analysis of multiple data sources (e.g., various written assignments, reflection journals, portfolios, and interviews) found that preservice teachers constructed CK while thoughtfully preparing lesson plans and during debriefings regarding classroom teaching experiences, and exhibited developing PCK by adapting real-time teaching, planned activities, and follow-up lessons in response to the needs of students. These results would seem to imply that interventions designed to enhance CK and PCK have greater positive impacts than only addressing one of those knowledge types. Even and Tirosh (1995) echo support for teachers learning about such constructs as student thinking: “To make appropriate decisions for helping and guiding students in their knowledge construction certainly requires an understanding of student ways of thinking” (p. 3).

Measuring Pedagogical Content Knowledge

About thirty years ago the mathematics research community concluded that it could find no important relationship between teacher knowledge and student learning

(Eisenberg, 1977; General Accounting Office, 1984; School Mathematics Study Group, 1972 as cited in Fennema & Franke, 1992). An important distinction between then and now is how teachers' knowledge was defined. These studies defined it as the number of university-level mathematics courses successfully completed. Also, these studies did not attempt to measure what the teachers knew about the mathematics they were teaching or precisely what content was covered in the mathematics course they took (Fennema & Franke, 1992). Much has changed in the past 20 years – especially in the area of research on teacher knowledge. Currently, researchers are not so concerned with what mathematics courses teachers took in college as much as with what mathematical knowledge is needed to teach, can such knowledge be empirically quantified, and what are the relationships between this mathematical knowledge for teaching (i.e., PCK) and student achievement. This research paradigm is in its infancy and is still being formulated, and as such, very little research exists on measuring PCK and its effects on student achievement; however, the implications of such research are far reaching and thus merit some discussion. Piloting of an instrument to be used to measure PCK began in 2001[4]. Hill, Schilling, and Ball (2004) reported that although their findings are only preliminary, have not been replicated, and are based on exploratory (albeit extensive) factor analysis, there is reason to believe that teachers' content knowledge is at least somewhat domain specific (e.g., number, operations, patterns, functions, and algebra). A conclusion worth noting was that from a measurement perspective, the results support constructing separate scales to represent and measure different knowledge types for teaching (e.g., CK and PCK). This research was followed up by Hill, Rowan, and Ball

[4] see Rowan, B., Schilling, S. G., Ball, D. L., & Miller, R. (2001). *Measuring teachers' pedagogical content knowledge in surveys: An exploratory study.* Unpublished manuscript, University of Michigan, Ann Arbor.

(2005). Their study explored whether and how teachers' pedagogical content knowledge contributed to increased student achievement in mathematics. A mixed-model methodology was used and key student- and teacher-level covariates were controlled for. The results of the study indicate that teachers' PCK was a significant predictor of students' learning of mathematics. The authors were quick to mention that "the analyses performed involve clear limitations, including small sample of students [1,190 first graders, 334 first-grade teachers, 1,773 third graders, and 365 third-grade teachers], missing data, and a lack of alignment between our measure of teachers' mathematical knowledge and student achievement" (p. 399). With that being said, the strongest and most robust effect was that of the teacher content knowledge variable on students' achievement. The results of this study, as well as others discussed, point to the ongoing need of analyzing the practice of knowledgeable teachers as well as their content knowledge in the hopes of improving student learning.

Knowledge of and Learning about Area & Perimeter

The previous portion of the review of literature looked at CK and PCK from a generally content-neutral perspective. Research involving young children (e.g., first or second grade) or focusing on how measurement concepts develop during school years will not be components of this research study and hence not a focus of this review of literature. Instead, the next major section will present and discuss literature examining the ongoing struggles students have with concepts related to area and perimeter, common misconceptions regarding area and perimeter, how they relate to instruction and learning, why students (and teachers) struggle with understanding area and perimeter concepts, how traditional instructional strategies tend to confound learning, and then conclude with a look at innovative instructional strategies and why they have been successful. Perfecting the craft of teaching is a life-long endeavor and can be furthered by examining misconceptions surrounding subtleties of assumed mathematical concepts, the mathematics (or lack thereof) that underlies such struggles, and what can be done to intervene and break the cycle of misconception breading misconception (Ball, Lubienski, & Mewborn, 2001; Ma, 1999; Stoddart et al., 1993).

Students' Difficulties with Area and Perimeter

"Measurement is an enterprise that spans both mathematics and science yet has its roots in everyday experience" (Lehreh, 2003). The practical side of measurement, for example area and perimeter, has become an increasingly important component of many school mathematics curricula; however, neither the practical nature of such concepts nor increased emphasis has translated into mastery of basic skills or deeper conceptual understanding regarding area and perimeter (Kenney & Kouba, 1997; Martin & Strutchens, 2000). One ongoing source documenting students' difficulties regarding area and perimeter has been the mathematics assessment of the National Assessment of Educational Progress (NAEP). First administered in 1972-73, the results of several NAEP exercises involving measurement revealed pervasive misunderstandings of basic concepts (Hiebert, 1981). For example, when responding to the question in Figure 1, only 28% of 9-year-olds answered it correctly. Hiebert stated that this, along with other similar results, time the fourth NAEP assessment of mathematics was administered, 14 years later, one indicates that many students do not understand the fundamental meaning of area. By the might assume that significant progress towards remedying such a shortcoming would

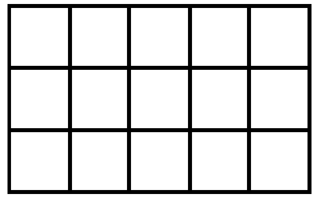


Figure 1. Measurement exercise very similar to one asked in the 1972-73 NAEP.

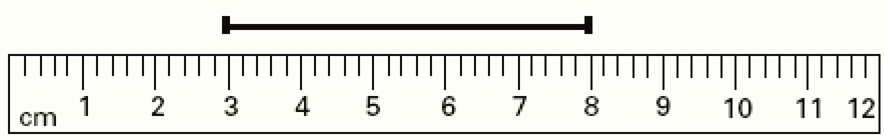
have been reached. Sadly, that was not the case. A little over half of the seventh graders tested could correctly calculate the area of a rectangle labeled with both the length and width (Kouba et al., 1988). More disappointing, even shocking, was that only a little over 10% of the 7th-grade students could find the area of a square when given the length of one side and the fact that the figure was a square. The 1992 NAEP mathematics assessment showed some progress in basic area computation with 65% of the eighth graders tested correctly answering: "A rectangular carpet is 9 feet long and 6 feet wide. What is the area of the carpet in square feet?" (Kenney & Kouba, 1997, p. 153). A mathematics assessment of NAEP conducted in 1996 revealed a significant drop in eighth graders' performance on items involving basic area computation. Only 44% could identify the correct numerical expression for the area of a given geometrical figure (Martin & Strutchens, 2000). An item appearing on the 2003 NAEP asked eighth graders to determine which of four numerical expressions would represent the area of a rectangle whose side measures were given; less than half (48%) answered the question correctly

(National Center for Education Statistics [NCES], 2003). The 2005 NAEP mathematics assessment revealed that only 38% of high school seniors could use a centimeter ruler to measure the appropriate lengths of a pictured parallelogram and correctly compute its area (NCES, 2005). It is worth noting that when comparing similar area questions on the various NAEP assessments, students did notably better when asked to compute the area of a rectangle described with words as opposed to the area of a pictured rectangle. Possibly the visual cues are distracting and cause confusion among students. The 2007 administration of the NAEP mathematics assessment reveals that, while some progress has been made, 4^{th} and 8^{th}-grade students are still struggling with concepts related to area. For example, one problem from the 4^{th}-grade exam gave the dimensions of a room (i.e., 12 feet wide by 15 feet long) and asked students how many square feet of carpet would be needed to cover the floor. Only 42% correctly answered the problem. An interesting side-note was that the most common incorrect response was "27" – which suggests confusion exists between concepts involving finding area and perimeter. The research conducted in this study examined aspects of these possible phenomena.

NAEP assessments also reveal students struggle with fundamental concepts regarding length and perimeter. For example, the results of an item in the 1985-86 NAEP revealed that only 14% of the third graders and 49% of the seventh graders who responded to the question in Figure 2 gave the correct answer of 5 cm (Lindquist & Kouba, 1989). These deficiencies have also been reported more recently. In the 1996 NAEP mathematics assessment, only 22% of 4^{th}-grade and 63% of 8^{th}-grade students, who responded, could correctly determine the length of an object pictured above a ruler when the end of the object and ruler were not aligned (Martin & Strutchens, 2000). A very similar question on the 2003 NAEP, pictured in Figure 2, produced equally troubling results with only 20% of the fourth graders correctly answering the item (NCES, 2003). On the 2005 administration, eighth graders continued to struggle with perimeter concepts with only 40% correctly determining the length of a rectangular playground whose perimeter and width were given (NCES, 2005). Even as recently as 2007, only 43% of 4th-grade students could correctly find the perimeter of a stop sign given that it has eight sides, the length of each side, and told that perimeter was the "distance around" (NCES, 2007). Difficulties with the concept of length may be one factor contributing to students' poor understanding of perimeter, which is a special application of length.

Lindquist and Kouba (1989) report that in the fourth NAEP mathematics

Item	Percent Responding[a] Grade 3	Grade 7



How long is this line segment?[b]		
3 cm	4	1
5 cm *	14	49
6 cm	31	37
8 cm	30	9
11 cm	6	2
I don't know.	15	2

[a] The response rate was .80 for grade 3 and .97 for grade 7.
[b] An actual centimeter ruler was pictured.
* Indicates correct response.

Figure 2. Percentage of students in grades 3 and 7 responding to a NAEP item.

assessment, 17% of 3rd-grade and 46% of 7th-grade students who responded successfully found the perimeter of the rectangle in Figure 3. Poor performance by third graders on this item may not be that surprising because perimeter is still a relatively new concept at that age; however, the performance by seventh was also less than adequate. Some improvement in performance appears in 1996 on the sixth NAEP mathematics assessment when 46% of the 4th-grade students who responded could correctly calculate how many feet of fencing would be needed to go around a rectangular garden (Kenney & Kouba, 1997). The garden was pictured and labeled similarly to Figure 3. Eighth graders were not asked that perimeter problem. A different sort of perimeter problem was asked on the 1996 NAEP when fourth graders were asked to use a ruler to draw a figure with a given perimeter (Martin & Strutchens, 2000). Interestingly enough, only 19% of those who responded could draw a correct figure. The nontraditional format of this problem seemed to cause significant difficulties for the fourth graders.

It would appear the instruction students have been receiving regarding area,

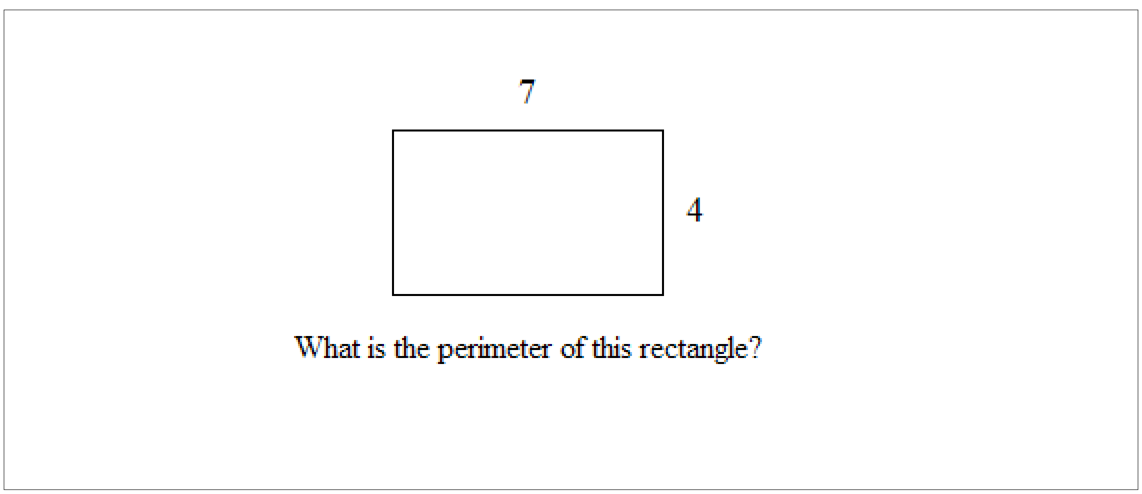


Figure 3. Item from the fourth NAEP.

perimeter, and length is developing an incomplete conceptual understanding of these concepts (Kamii & Clark, 1997; Martin & Strutchens, 2000). The high percentage of incorrect responses alone should be cause for alarm; however, even more troubling are the misconceptions students have regarding area and perimeter.

Prevalent Misconceptions Regarding Area and Perimeter

Perimeter is the length around the outside of a figure (for a rectangle, it would be the sum of the lengths of the sides of a figure), and area is a measure of how much two-dimensional space a figure occupies. Because the calculations of both measures involve the sides of the figures, someone lacking a conceptual understanding of area and perimeter could encounter many problems and difficulties (Ma, 1999). Such errors evolve into knowledge gaps which if left unchallenged manifest themselves as misconceptions – exhibited by students while working problems involving area and perimeter (Hirstein et al., 1978; Wilson & Rowland, 1993) and by teachers while attempting to explain the concepts (Menon, 1998; Reinke, 1997; Simon & Blume, 1994a). The literature discusses many misconceptions regarding area and perimeter. Some are general in nature (e.g., confusing area and perimeter), and others are more focused (e.g., area and perimeter are directly related in that one determines the other). Some misconceptions, such as transitivity (Hiebert, 1984) and conservation (Piaget, Inhelder, & Szeminka, 1981), are more common among young children, although others (e.g., confounding linear and square units) are held by both students and even teachers (Tierney, Boyd, & Davis, 1986). It is this last type of misconception (i.e., those reported to be held by both students and teachers), that will be the focus of this section of the literature review and the proposed research.

Confusing Area and Perimeter

The misconceptions which are held by students, as well as pre- and inservice teachers, are not always mutually exclusive. For example, students often confuse area and perimeter (Hirstein, Lamb, & Osborne, 1978; Kouba et al., 1988), but that confusion can take different forms. In some instances students perform the wrong algorithm by multiplying dimensions that should be added (Kenney & Kouba, 1997), while at other times they focus on the wrong unit of measure (i.e., linear versus square or vice versa) (Carpenter, Cobrun, Reys, & Wilson, 1975; Chappell & Thompson, 1999). In regards to responses to NAEP items, it appears students commonly calculate area in response to a perimeter problem, and vice versa (Kouba, et al., 1988; Kenney & Kouba, 1997; NCES, 2007). Kouba et al. (1988) conclude that the most plausible explanation is that students lack a conceptual understanding of these concepts. Kenney and Kouba (1997) speculate that the items themselves can provide visual cues that may initiate area and perimeter confusion. For example, if a grid is used with the figure then the students may be cued to focus on area even if the question deals with perimeter. Visual cues have been reported by other researchers as contributing to area and perimeter confusion. Wilson and Rowland (1993) discuss findings where students tend to focus on one dimension of a figure (typically the longest one), and Carpenter et al. (1975) explain the tendency for children to judge area strictly on the basis of physical appearance. For example, when attempting to compare different sized rectangular regions in order to find two with the same area, students will choose the shape because they say it is the most similar to the other one, without out any mention of counting or calculating units to do the comparison. Confusions between area and perimeter still persist as evidenced by student performance on measurement items in the 2007 NAEP (NCES, 2007).

Researchers have found that preservice teachers are also prone to confusing area and perimeter. Reinke (1997) asked 76 preservice elementary teachers to explain in writing how they would find the perimeter and area of the shaded shape illustrated in Figure 4. When explaining how they would find the perimeter, approximately 22% of the subjects worked the problem exactly as they would if they were finding area. The preservice teachers performed better when explaining how they would find area, however, an interesting finding was that there were three instances of subjects using degrees for finding area and perimeter. Apparently, knowledge of a circle containing 360 degrees evoked references to the semicircle containing 180 degrees. Possibly the word "containing" (used to describe the figure) implies covering, but the lack of qualitative data (e.g., follow-up interviews) leaves the reader to only speculate the reasoning and conceptions behind the preservice teachers' responses. Tierney et al. (1986) provided such data when reporting findings from the research conducted within a mathematics content course for preservice teachers. The students' responses made in class, along with their journal writings, revealed many misconceptions regarding area and perimeter.

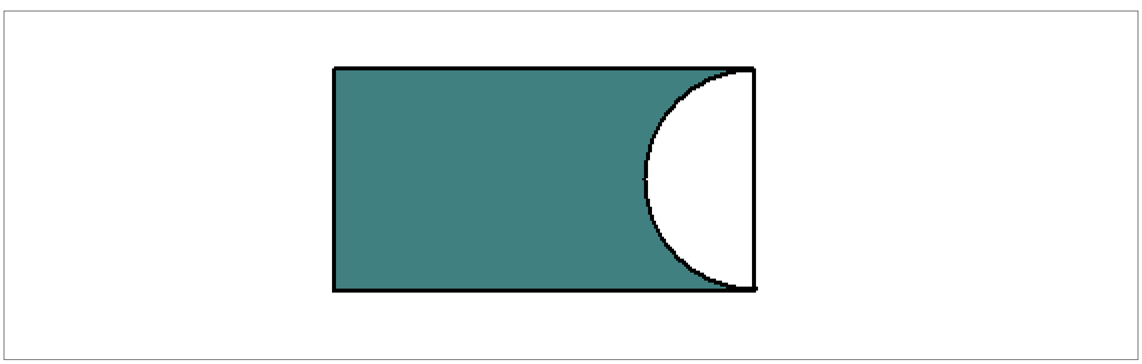


Figure 4. Diagram shown to preservice teachers.

Tierney et al. (1986) found that many preservice teachers equate finding area to finding a number, but too often any number will do. One participant was observed counting the pegs around a figure to find its area. When questioned, they replied that their method seemed to generate a reasonable number. Another wrote that area never seemed like a real concept to her because there was no tool for measuring it. A major difficulty for these preservice teachers was that they would often confuse what exactly they should count in order to find area, and they would have the same problem when attempting to count something to calculate perimeter. A plausible explanation for the confusion of area and perimeter is that conceptions regarding the use and meaning of appropriate units for finding area and perimeter are muddied at best.

Linear Verses Square Units

The unit of measure functions as a conceptual bridge connecting an object and the number used to represent its size. Hiebert (1981) states, “The concept of a unit is a central, unifying idea underlying all measurement” (p. 38); however, traditional instruction does not recognize that the concept of a square unit presents difficulties for students. In addition, knowledge about the square unit (and linear as well) is typically assumed to be ascertained from instruction on finding area (Simon & Blume, 1994). To understand concepts of measurement, the basic properties of units must first be explored and understood. To apply the appropriate unit of measure, the students must decipher what attribute is being measured (Wilson & Rowland, 1993). For example, if measuring length, then a linear unit such as a centimeter or an inch is needed. If area is the desired measurement, then a two dimensional unit such as a square would be appropriate. When these ideas are not understood, then errors are made and misconceptions develop.

Researchers have found that students often confuse linear and square units (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998). While interviewing students Hirstein et al. (1978), found point-counting in place of applying linear or square units to be a common misconception. The fact that 37% of seventh graders answered 6 to the question previously shown in Figure 2 (arrived at by counting the numbers as opposed to the linear units) reveals the confusion that can arise when fundamental ideas regarding units are not understood (Kamii, 2006). Sometimes it is hard to distinguish if students are confusing area and perimeter, linear and square units, or both. Chappell and Thompson (1999) asked sixth, seventh, and eighth graders to construct a figure with a perimeter of 24 units. Figure 5 is an example of what can occur when students have misunderstandings regarding units of measure.

Another difficulty can arise if students believe that units must be single, discrete, and/or whole entities; therefore, fractions of units tend to get ignored or counted as whole (Hiebert, 1981; Lehrer, 2003). For example, when finding the area of an irregular figure (e.g., a footprint), not counting or compensating for partial units results in an incorrect area. It also appears that calculating the area for regular and semi-regular figures is problematic. The 1996 NAEP reported that only 12% of eighth-grade students could correctly determine the number of square tiles needed to cover a region of given dimensions (Martin & Strutchens, 2000). Too often students understand square units simply as something to be counted rather than as a subdivision of a plane (Lehrer, 2003). Such difficulties are often the result of children not being able to conceptualize the constructing of what Reynolds and Wheatley (1996) refer to as “a unity” (p. 564). A unity can be thought of in base-ten terms. It is a single unit comprised of smaller units.

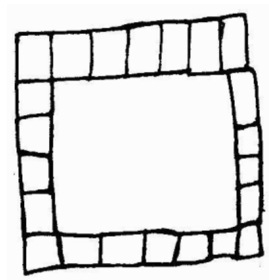


Figure 5. Student's constructed response for a figure having a perimeter of 24 units.

For example, a rectangle that is 10 inches long by 4 inches wide has a unity (or area) of 40 square inches. The rectangle could also be partitioned into four 2 × 5 regions each having a unity of 10 square inches. Although somewhat of an abstract concept, Reynolds and Wheatley used case studies involving four fourth-grade students to report that developing an understanding of and being able to use a unity is a fundamental component of children's meaningful construction of area. The notion of partitioning an area into regions and iterating units has also been investigated by Battista, Clements, Arnoff, Battista, & Borrow, 1998). Battista et al. looked at how students structure and enumerate two-dimensional rectangular arrays (i.e., rows or columns of square units). They found that the array structure, that is often taken for granted by teachers as somewhat obvious to students, is not an intuitive notion. The second graders studied progressed through various levels of sophistication in their understanding of structuring arrays. The importance of each student personally constructing arrays in various settings was stressed. The process of constructing arrays and understanding how and why they can represent area is crucial for the formula A = L × W to be understood conceptually (Battista et al., 1998).

A possible explanation for why teachers might give important concepts such as arrays only cursory attention is that they may possess only a shallow understanding of them. For example, a common misconception among teachers, especially elementary, is that perimeter is two-dimensional. A belief that has been justified by statements such as, "the perimeter of a rectangle has both length and width" (The Conference Board of Mathematics [CBMS], 2001, p. 22). When discussing teachers' understandings regarding area and perimeter the CBMS state:

> Many teachers who know the formula $A = L \times W$ may have no grasp of how the linear units of a rectangle's length and width are related to the units that measure its area or why multiplying linear dimensions yields the count of those units. (p. 22)

Baturo and Nason (1996) studied student teachers' content knowledge and pedagogical content knowledge regarding the domain of area measurement. They conducted qualitative research involving clinical interviews and reported that their subjects had acquired skills for performing the basic algorithms for calculating area and perimeter. Although these skills would most likely allow them to function adequately in society, the subject matter knowledge of the student teachers would extremely limit their ability to scaffold learners in developing meaningful understandings of these concepts. Although Baturo and Nason's (1996) results provide great insight into how and what preservice teachers think about the teaching and learning of area and perimeter, their research did not involve any intervention with the goal of improving the subjects "rather impoverished" (p. 261) understanding of these concepts.

Perceived Relationships Between Area and Perimeter

It is very common for students to think that all rectangles of a given area have the same perimeter or that all rectangles of a given perimeter have the same area (Carpenter, et al., 1975; Hart, 1984; Lappan, 1998; Walter, 1970), as well as exhibit difficulties justifying their reasoning regarding the misconception (Chappell & Thompson, 1999). Woodward and Byrd (1983) posed a question to 258 eighth-grade students at two different schools in Tennessee (129 from each). The gist of the question involved a story problem where a farmer had 60 feet of fence and wanted to construct as large a rectangular garden possible. The story continues by saying that the farmer drew out five possibilities for the garden. Pictures of an 8 × 22, 10 × 20, 15 × 15, 5 × 25, and 2 × 28 rectangle were provided for the students to view. The students were then asked to check which statement they believed to be true. The first five choices involved selecting one of the five rectangles as the biggest, and the last choice was that the gardens were all the same size. The researchers were somewhat concerned that only 55 of 258 (21%) answered the question correctly while 157 (61%) said the gardens were the same size. The results spurred Woodward and Byrd to ask two sections of a mathematics course for prospective elementary teachers the same question. The preservice teachers were also asked to justify their responses. Almost two thirds of all the preservice teachers said the gardens were the same size. Some of the justifications they provided include: “All of them equal 60,” and “They are all the same size since their perimeter is 60 ft. The area is arranged differently” (p. 345). It would appear likely that these preservice teachers received insufficient instruction regarding area and perimeter.

Fuller (1996) compared the pedagogical content knowledge of 26 preservice elementary teachers and 28 experienced elementary teachers. One of the items in her research-designed survey involved asking a question very similar to the above garden question; however, the item concluded with a statement along the lines of, after considering the problem the farmer concludes that it (i.e., building different sized gardens) doesn't really matter because all the pens will have the same perimeter – 60 feet. The pre- and inservice teachers were then asked to: (a) Explain why the farmer made the concluding statement, (b) How would you respond to their solution? and (c) Explain. Fuller reported that only one teacher, an experienced one, provided a response that was correct both procedurally and conceptually. Most of the other pre- and inservice teachers attempted conceptual responses, with the majority of preservice teachers arriving at answers that lacked specific mathematical content as well as appropriate supporting pedagogy. The vague qualitative reporting of this study left the reader guessing as to the subjects' specific mathematical and pedagogical strengths and weaknesses regarding the area and perimeter items.

A minor difficulty that is related to the before-mentioned misconception is dealing with area and perimeter of irregular shapes. In these shapes students appear to set aside their fundamental concepts of conservation of area and the unit of measure (Maher & Beattys, 1986). About 25% of the seventh graders who took the fourth NAEP indicated that the area of a rectangle could not be determined once the rectangle was separated and reformed into a different shape (Kouba et al., 1988). It could be argued that the students had difficult with conservation of area, but based on the students' responses the researchers felt it was more plausible that they lacked a conceptual understanding of area. In the 2004 administration of the Long-Term Trend (LTT) NAEP, only 32% of seventeen-year-old students could correctly find the area of an L-shaped region (Rutledge, Kloosterman, & Kenney, 2009). Finding areas of irregular shapes that are not made up of polygons (e.g., a figure resembling a fried egg) is also difficult for children (Lindquist & Kouba, 1989). Lehrer (2003) investigated the strategies used by younger children when asked to find the area of the figure resulting from tracing their hand on a piece of grid paper. He found that children tended to organize units in ways that would keep within the boundary of closed figures, and that would result in using units that resemble the space they were trying to fill (e.g., triangles for triangle gaps) even if that meant using different units for the same figure. Lehrer reported that less than 20% of the students studied believed that identical units of measure must be used while covering an irregular figure. Preservice teachers have also been found to have similar difficulties with irregular shapes (Maher & Beattys, 1986; Tierney et al., 1990). Tierney found that preservice elementary teachers would often try to reconcile the application of the *length* × *width* formula with calculating the area of irregular shapes. The subjects did not seem to question the appropriateness of the formula but rather communicated a sense of familiarity with it and thus attempted to apply it.

The second major misconception involving a presumed relationship between area and perimeter is best illustrated with the following scenario:

> Imagine that one of your students comes to class very excited. He tells you that he has figured out a theory that you never told to the class. He explains that he has discovered that as the perimeter of a closed figure (e.g., square or rectangle) increases, the area also increases. He shows you a picture (see Figure 6) as proof of his new theory. How would you respond to this student?

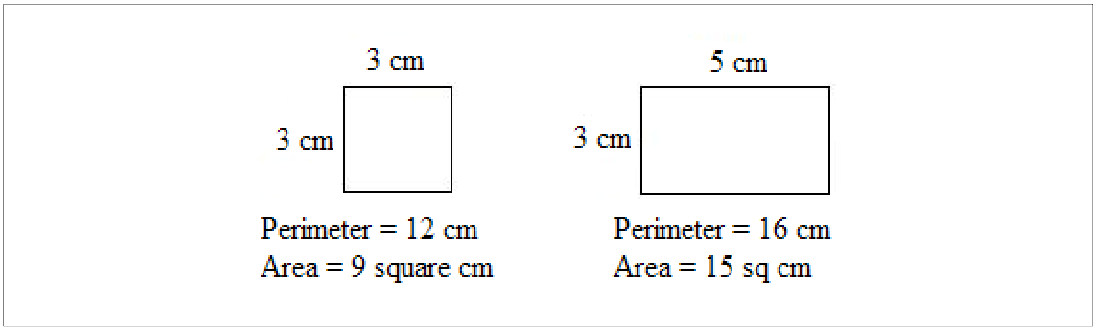


Figure 6. Student's example to "prove" his theory that increasing perimeter also increases area.

The scenario just presented illustrates a very common misconception regarding area and perimeter. Namely, that increasing the perimeter of a figure will always increase the figure's area and vice versa. The perceived direct relationship between area and perimeter is believed by both students and teachers (Lappan et al., 1998; Reinke, 1997; Ma, 1999). Ferrer et al. (2001) write that of the many difficulties students have regarding area and perimeter the nonconstant relationship between these concepts is one of the hardest to grasp. Lappan et al (1998), in their instructional book for teachers on two-dimensional measurement *Covering and Surrounding*, take a whole chapter to address the subtleties of the misconception that perimeter determines area. In spite of the awareness that students struggle with that specific relationship, the only research found that examines the misconception was conducted with preservice teachers. From a teacher's perspective, there are three aspects to the scenario presented above. The first concerns the specific content knowledge regarding perimeter and area and the proposed relationship (i.e., the mathematical substance of the student's claim), the second entails the mathematical knowledge regarding justification (i.e., ideas of theory and proof), and the third is the pedagogical content knowledge involving an appropriate response to the student's proposed theory. Different researchers have posed similar versions of this scenario to preservice teachers.

Ball (1988) interviewed 14 secondary mathematics majors and 26 preservice elementary teachers for their reactions to the student's proposed area and perimeter theorem shown in figure 6. More than a third of all the teachers (44% of the secondary majors, 35% of the elementary) expressed that they were impressed with the student's work and accepted the substance of the claims with little question or reflection. Only 20% of the prospective teachers knew that the student's claim was mathematically incorrect. Many of the teacher candidates (43%) indicated they were unsure whether there was a direct relationship between area and perimeter.

Ma (1999) presented the same question as Ball (1988) to a group of U.S. and Chinese preservice teachers. The immediate reactions to the student's claim were similar for the teachers in both groups. Most of the teachers indicated that they had not heard of this "new theory" before. Similar proportions of U.S. and Chinese teachers accepted the student's theory immediately. All the teachers knew what area and perimeter meant and most could calculate them; however, their strategies for exploring the theory and their responses to the student diverged significantly. Only the findings regarding the U.S. teachers will be discussed. Of the 23 U.S. teachers questioned, two simply accepted the student's claim without question. Among the 21 teachers who suspected that the student's claim was true, five indicated that they would need to consult a textbook before they could respond to the student, 13 proposed a strategy of calling for more examples from the student, and three actually investigated the problem mathematically. Only one U.S. teacher successfully arrived at the correct solution of presenting a counterexample. Even when the U.S. teachers mentioned specific strategies for approaching the problem, the strategies were not based on careful mathematical thinking. They did not consider a systematic way to examine the various cases. Rather, the U.S. teachers proposed a strategy based on the idea that a mathematical claim should be explored and proved by working through a large number of examples. This misconception, as Ma puts it, was shared by many of the U.S. teachers and would likely mislead and confuse a student. Howe (1999), who reviewed Ma's book *Knowing and Teaching Elementary Mathematics*, makes a compelling statement that summarizes his feelings on the U.S. teachers' treatment of the relationship between area and perimeter:

> For me, perhaps the most discouraging aspect of working on K-12 educational issues has been confronting the fact that most Americans see mathematics as an arbitrary set of rules with no relation to one another or to other parts of life. Many teachers share this view. A teacher who is blind to the coherence of mathematics cannot help students see it. (p. 885)

Students' Justification of Responses

The ability to reason is an essential component of learning to do mathematics. Being able to justify one's response is an important reasoning skill and is fundamental in developing a conceptual understanding of mathematics and facilitating its making sense (Ma, 1999; NCTM, 2000). It would be unrealistic to expect most students to develop reasoning skills without a proficient teacher, who possesses such skills, guiding the process. Research indicates that many teachers lack such skills. When Woodward and Byrd (1983) asked prospective elementary teachers to justify their answers to a problem involving area and perimeter, the responses given were shallow in content, were basically restatement of their answer, and involved little or no meaningful mathematical investigations. An alarming finding from Ball's (1988) research involving prospective teachers' understanding of mathematics was regarding their knowledge of justification and pedagogical content knowledge. When asked how they would respond to a student who claimed he had discovered a new (albeit incorrect) theorem, the vast majority, 92% of the elementary and 86% of the secondary prospective teachers, concentrated entirely on the substance of the student's claim and made what they (the preservice teacher) knew about the relationship between area and perimeter the focal point of their response. They provided no meaningful discussion of the student's approach to justify his mathematical claim; instead, they put all their effort into deciphering whether he was right or wrong. Expanding upon Ball's (1988) work, Ma (1999) reported that a lack of meaningful content knowledge regarding a proposed relationship between area and perimeter prohibited the vast majority of U.S. teachers involved in the study from engaging in any constructive conversation with potential students.

Teachers' inadequate ability to effectively question students' mathematical claims as well as to offer clear justifications for mathematical arguments is predictably evident in students' work (Lappan et al., 1998; Martin & Strutchens, 2000). When students are asked to provide written explanations or justifications of answers to constructed-response questions, even a lower-level task becomes more difficult and their performance decreases (Kenney & Kouba, 1997; Strutchens, Harris, & Martin, 2001). Being able to provide real-world applications of mathematical concepts is evidence that students are making sense of the mathematics and developing conceptual understanding (NCTM, 2000). Chappell and Thompson (1999) found that middle school students have difficulties in generating practical application problems for even common measurement concepts as area and perimeter.

Apparently pre- and inservice teachers' levels of knowledge, understanding, and reasoning regarding many concepts surrounding area and perimeter are extremely lacking. One can only assume that if preservice teachers have such misconceptions then their future students will as well. A disadvantage of much of the current elementary mathematics curricula is that problems involving the misconceptions discussed in the previous sections are not part of the instructional discussion – for the teacher or the students. Today's traditional instruction in area and perimeter does not appear to be reversing the poor performance trend nor aiding in revealing or resolving the previously discussed shortcomings and misconceptions. This second major section of the literature review concludes with first examining why there are pervasive misunderstandings regarding area and perimeter and lastly by presenting some innovative instructional strategies to improve the teaching and learning of these concepts.

Likely Causes of Area and Perimeter Misconceptions

Based on the literature addressing these misconceptions, it would appear that a conceptual understanding of fundamental concepts regarding area and perimeter, by both students and teachers, is severely lacking and restricted (Fuller, 1996; Menon, 1998; Reinke, 1997; Woodward & Byrd, 1983). Exploring some of the most probable causes of these difficulties would be a logical first step before offering recommendations for necessary interventions.

Unfocused Curriculum

The goal of elementary mathematics needs to be that of building a firm foundation on which ongoing mathematical learning can be built and understood (NCTM, 2000). The curriculum should only be a part of that foundation, and teachers need to have the confidence and ability to circumvent and supplement when necessary (Ma, 1999). Information collected from the Third International Mathematics and Science Study (TIMMS) revealed that fourth grade students in the U.S. encounter a mathematics curriculum that is unfocused, contains many more topics, and possesses little coherence as compared to those of other countries that significantly outperformed our students (Valverde & Schmidt, 1997). Data collected from a national random sample of teachers in TIMMS indicate that the majority of them are attempting the overwhelming task of covering all the material in the textbook. Consequently, the mathematics contained within our textbooks receives shallow and terse treatment (Valverde & Schmidt, 1997). For example, although an important purpose of measurement is to compare things that cannot be compared directly, the idea of comparison is either absent or casually mentioned within textbook instruction of measurement (Kamii & Clark, 1997). Sometimes a textbook's treatment of measurement topics can indirectly confuse students. A second grade mathematics textbook by Harcourt, Inc. (2004) deals with congruent shapes by encouraging teachers to instruct students that "you know these squares are congruent because both squares have exactly three dots on each side" (p. 345). A process of counting dots to determine side lengths of polygons would most likely cause confusion for students later when learning about perimeter and counting linear units.

Effective instruction of area and perimeter needs to present two perspectives, the static and dynamic (Baturo & Nason, 1996). The static perspective equates area with a number representing the amount of space or surface that is enclosed by a boundary. The dynamic perspective focuses on the relationship between the perimeter and area of a figure, that is, as the perimeter approaches that of a line segment, the area approaches zero. However, the dynamic perspective is rarely examined in the typical textbook (Baturo & Nason, 1996); hence, misconceptions regarding relationships between area and perimeter can develop and go unchecked (Ball, 1988; Woodward & Byrd, 1983). It has been suggested that the learning of area and perimeter could be more coherent and conceptual if the concepts were examined simultaneously (Chappell and Thompson, 1999; Hiebert & Lefevre, 1986; Simon & Blume, 1994a). Scope and sequence of mathematical topics is important to instruction; however, knowledge of how students learn and what they find difficult must also be considered while implementing any curriculum. Outhred and Mitchelmore (2000) found that children learn and conceive about area differently and have been documented as progressing through developmental levels while grasping the concept. To facilitate this progression they recommend the curriculum introduce the concept of area early on by having the students think of area measurement as the act of covering a region with a fixed unit, and then investigate rectangular covering within that context of area measurement later discovering or deriving the area formula. Baturo and Nason (1996) concluded, after studying preservice teachers' understanding of area, that if preservice teachers are to be expected to teach measurement concepts such as area and perimeter from a conceptual perspective then they need to experience as students a more focused and dynamic curriculum complete with many concrete measuring experiences such as covering regions with units of area.

Ineffective Instruction

The curriculum alone cannot be blamed for the ongoing struggles many students have with mathematical achievement nor can it be expected to bring about necessary reform. There are many elements that merge together during the act of teaching, a few of the prominent ones are: the abilities and prior understandings of the students, the teacher's knowledge (both content and pedagogical), the curriculum, and instructional strategies. To assume that all teachers are sufficiently prepared to teach elementary mathematics concepts such as area and perimeter would be a mistake. Tierney et al. (1986) found that when they asked prospective elementary teachers what they would teach a ten year old child about area, 80% of them drew a rectangle and wrote "L × W" near it. Such a simplistic view reflects poorly on their prior training. Along with student performance data, the 1992 and the 1996 mathematics assessment of NAEP gathered data regarding teachers' reported exposure to mathematics content areas. Lindquist (1997) reported the 1992 NAEP found that ten percent of fourth-grade teachers indicated they have received little or no exposure to measurement concepts. Four years later that same category had grown to 13% (Grouws & Smith, 2000). Such trends do not bode well for improving the teaching and learning of measurements concepts such as area and perimeter.

Many of the instructional practices traditionally employed when teaching measurement may actually be contributing to students' lack of conceptual understanding regarding concepts such as area and perimeter. Typical instruction too often treats measurement as a mere empirical procedure requiring little or no logical reasoning (Kamii, 2006; Kamii & Clark, 1997). For example, lining up paper clips along an object and counting them is an empirical procedure that can be done without giving much thought to the meaning of a linear unit of measurement. The students' responses depicted in Figure 2 (see p. 47) are most likely the result of having learned only empirical procedures. In contrast, instruction should be rich in activities involving both transitive reasoning (the mental ability to compare two lengths using a third item) and unit iteration, which involves mentally constructing a part-whole relationship between the total length of a figure and the length of a smaller object (e.g., a linear or square unit) (Kamii & Clark, 1997; Van de Walle, 2007).

Over Emphasis on Procedural Knowledge

A common result of these forces, ineffective instruction and an inadequate curriculum, is the fostering of a counterproductive, procedural-based knowledge (Kouba et al., 1988), rather than a well-connected, conceptual understanding. It is important for those involved in education, especially teacher education, to be aware of the signs of procedural-based knowledge as well as how to counteract it. There is tendency for many teachers to focus their instruction on arriving at an answer rather than on the conceptual development of measurement ideas (Baturo & Nason, 1996; Kamii, 2006). It is not likely that teachers plan their instruction to emphasize procedural knowledge of such concepts as area and perimeter. Often they may not be aware that they lack either the knowledge or the analytical ability to teach conceptually (Hershkowitz & Vinner, 1984). Tierney et al. (1986) found that a high proportion of preservice elementary teachers lack the necessary understanding of area concepts to support their teaching of it even with the aid of a reasonable textbook. This lack of understanding is dangerous in that teachers who have poor conceptual understanding of mathematics will feel more comfortable teaching just for procedural knowledge, and so will be unable and/or unwilling to engage students in problems requiring them to think deeply (Menon, 1998). Procedural knowledge can also be reinforced indirectly. For example, activities involving using wooden squares to cover figures and calculate their area may actually predetermine the task by allowing students to construct rectangular arrays and count the squares without relating the count to area or comprehending the squares as units of area (Outhred & Mitchelmore, 2000). The same researchers also found that representation through drawing was a better alternative in some settings to concrete manipulatives in promoting conceptual understanding of area measurement. Other times the instruction can directly result in emphasizing the procedural side of mathematics to the neglect of the conceptual.

Based on error patterns of responses to NAEP measurement items, Kouba et al. (1988) stated it appears likely that students have been exposed to procedures (e.g., area formulas) before developing a conceptual understanding. Too often area units are not applied to measure area; instead, the practice is to obtain two measures (typically length and width) and insert them into the often over-used formula, $A = L \times W$ (Nunes, Light, & Mason, 1993). However, the procedure of multiplying two linear measures is conceptually far removed from the notion of area (Outhred & Mitchelmore, 2000). Children have difficulty interpreting the results of the procedure (Kenney & Kouba, 1997), and many elementary students do not perceive the resulting product as a measurement (Lehrer, 2003). Many prospective elementary teachers do not have a clear understanding of why multiplying the length and width of a rectangle is an appropriate method to determine its area (Simon & Blume, 1994a). A formula-based approach to the teaching and learning of area and perimeter will not achieve the goal of conceptual understanding (Hiebert & Lefevre, 1986; Lehrer, 2003; Woodward & Byrd, 1983).

Helping students conceptualize measurement ideas is not an easy undertaking because most are operating at the holistic level (the lowest) of the van Hiele levels of geometric thought (Strutchens & Blume, 1997). Developing a fundamental understanding of both the array structure and unit iteration are central to conceptualizing area measure (Kamii & Clark, 1997; Simon & Blume, 1994a). Wilson and Rowland (1993) developed a research-based instructional sequence that would facilitate that. They propose that the following steps be used for learning to measure length, area, volume, or any other system of measurement: "(a) Identify the property to be measured, (b) Make comparisons, (c) Establish an appropriate unit and process for measuring, (d) Move to a standard unit of measurement, and (e) Create formulas to help count units" (p. 185). There are fundamental components that contribute to a student's conceptual understanding of the measurement process (viz., perception, representation, conservation, transitivity, and unit iteration), but these very skills are also developed through measuring (Wilson & Rowland, 1993). This dilemma suggests the importance of being aware of and planning for student abilities and difficulties as they engage in innovative and meaningful activities. Students, as well as prospective teachers, need to be active participants in the process of their mathematical growth and accept the intellectual challenge of learning conceptually (Baturo & Nason, 1996; NCMT, 2000).

Innovative Instructional Strategies

Refine the Focus

The textbook should not have to be the focal point for every mathematics lesson. Following research-based instructional strategies, such as outlined by Wilson & Rowland, (1993), teachers are free to incorporate unique and inviting learning activities; for example, finding the area of a figure resembling a fried egg (Casa, Spinelli, & Gavin, 2006), using broken ruler to resolve misconceptions about measuring length (Wilson & Rowland, 1993), or using a potato and a stamp pad to create and find the area of irregular figures (Johnson, 1986). The teacher can also supplement an existing curriculum with books and publications specially designed for specific mathematical concepts. Moyer (2001) used children's literature to help fourth grade students differentiate between the mathematical concepts of perimeter and area. Although confusing linear and square units is a common difficulty for students, the majority of students in this study had no difficulty with this distinction. Moyer also reported that many students demonstrated confidence while explaining before the class how they determined the perimeter and area for the figures they had constructed. Other publications can actually replace sections or chapters of the required textbook. For example, the publication *Covering and Surrounding* (Lappan et al., 1998) is an extensive textbook unit specifically designed for 6^{th}-8^{th} graders to investigate numerous measurement concepts, specifically area and perimeter.

Occasionally, important topics are neglected within a curriculum. If teachers are aware of such concepts, they can implement the curriculum accordingly. The concept of conservation of area is considered by many to be fundamental to understanding area measurement (Beattys & Mahler, 1985; Piaget et al., 1981). Despite its importance, conservation of area is not emphasized in the school curriculum. Kordaki (2003) found that fourteen year olds, interacting in a computer environment, were able to explore successfully and develop the conservation of area concept from three different perspectives.

Refining the focus within teacher education has also been an area of ongoing discussion (CBMS, 2001; NCTM, 1991, 2000). In teacher education, a topic receiving increased attention is knowledge of student thinking. So often the teaching and learning of mathematics focuses on the act of doing mathematics (Ma, 1999). Teachers need to be aware of how their students think about various mathematics concepts (e.g., area and perimeter), what they find difficult and why, and the misconceptions that are prevalent within the subject matter (Ball & Bass, 2000; Lehrer, 2003; Simon & Blume, 1994a). Gaining such knowledge as preservice teachers, so that student thinking becomes an instructional focus, would be very beneficial to their future teaching and their students' conceptual understanding (Ball et al., 2001; Swafford et al., 1997).

Integrate Innovative Learning Tools

There is little doubt that technology has impacted the teaching of mathematics. It is beyond the scope of this study to discuss all the technologies (e.g., graphing calculators) that can be used to enhance the learning of mathematics. This section will provide a brief overview of some of the technologies being used while focusing on the teaching and learning of area and perimeter. Several of the ideas presented here will be delineated in greater detail in Chapter 3. Visual cues are critical in developing spatial sense and therefore in the study of geometry (Clements & Battista, 1992). Without appropriate feedback, visual cues have been found to contribute to student errors when solving area and perimeter problems (Kenney & Kouba, 1997; Kouba et al., 1988). Incorporating a computer-based environment into the learning of measurement has been shown to improve student performance on these concepts (Clements & Sarama, 1997; Noss, 1987). Specifically, the teaching and learning of area and perimeter has been enhanced through several computer-based tools: Logo (Binswnager, 1988), Geometer's

Sketchpad (Stone, 1994), and a specially designed microworld (Kordaki, 2003).

The previous two sections are by no means exhaustive, but do give insight into the possibilities. With a little experience and creativity, the goals and objectives of mathematics textbooks can provide launching points for investigations that challenge students, confront misconceptions, and encourage the sharing and justifying of problem-solving strategies and solutions (Bray, Dixon, & Martinez, 2006; Chappell & Thompson, 1999; Reinke, 1997).

Enhancing Mathematics Teacher Education with Technology

It is a common notion that teachers tend to teach as they were taught (Goodlad, 1984; NCTM, 1989; Barron & Goldman, 1994), and it is apparent from decades of research and testing that traditional methods of instruction, both for students and for preservice teachers, regarding many mathematical topics (e.g., area and perimeter) are not producing the desired results (CBMS, 2001; Mathematics Association of America, 1991). The research findings regarding pre- and inservice teachers' understandings regarding concepts such as area and perimeter are valuable in informing both teacher educators and professional developers; however, minimal research has been conducted to examine best-practices to address these deficiencies. What is lacking from the research is specific recommendations for innovative interventions within teacher education, as well as professional development, to better equip teachers to correct the previously mentioned misconceptions and stop the perpetual cycle of teachers passing on, both directly and indirectly, their misunderstandings to their students. Ma (1999) states, "To empower students with mathematical thinking, teachers should be empowered first" (p. 105).

A specific form of technology-based instruction will be presented as a means to empower teachers. The literature discussed in the next several sections will be somewhat focused in that many areas of technology will not be reviewed. For example, hand-held technologies, information and communication technologies, computer literacy, or attitudes and beliefs about technology are not the focus of this study; hence, will not be mentioned in great detail, if at all, in the review of literature. What will be discussed is recommendations and guidelines pertaining to how and why to incorporate technology into the mathematics education of prospective teachers, anchored instruction and its connections to mathematics instruction, and research pertaining to microworlds.

The Need for Technology Infusion within Teacher Education

Our schools seem destined to position themselves to be able to incorporate more technology into classroom activities. The NCTM (2000) stated that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances learning" (p. 24). The 1998 International Society for Technology in Education (ISTE) survey on technology use in teacher education reported that the typical K-12 classroom in the United States contains one computer for every five students. A 2005 *Education Week* report indicated the student to Internet-connected computer ratio had improved to 4:1. That ratio is not ideal for a personal and interactive technology-based learning environment, which implies teachers will need creative methods to effectively integrate various forms of technology into the teaching and learning of mathematics (NCTM, 2000). The envisioned benefits of technology, especially upon the teaching and learning of mathematics, have been slow to realize, but a growing number of research studies have found that integrating technology into the learning of mathematics can positively influence achievement, stimulate and enhance spatial visualization skills, and promote a more conceptual understanding of mathematics for students and teachers (Boers-van Oosterum, 1990; Dunham & Thomas, 1994; Groves, 1994; Rojano, 1996; Sheets, 1993). Our constantly evolving and global marketplace demands cutting-edge technology; therefore, our schools can expect to be called upon to contribute to preparing students to meet both the real and the perceived technological needs of such a society. A report by the Congressional Office of Technology Assessment (OTA, 1995) found that only 3 percent of the teacher education graduates indicated they were “very well prepared” to teach with technology. To be ready to enter the technological classrooms of tomorrow, prospective teachers need course instruction in both content and pedagogy to function effectively in these newly forming instructional environments (Cooper & Bull, 1997; Glenn, 2000; Kersaint & Thompson, 2002; Timmerman, 2004); however, it has become apparent that many prospective teachers do not possess the necessary knowledge or experience to meet these demands (Milken Exchange on Education Technology [MEET], 1999; OTA, 1995; Pellegrino & Altman, 1997; Thompson, 2000).

After completing a comprehensive review of the literature regarding information technology and teacher education, Willis and Mehlinger (1996) concluded:

> Most preservice teachers know very little about effective use of technology in education and leaders believe there is a pressing need to increase substantially the amount and quality of instruction teachers receive about technology. The idea may be expressed aggressively, assertively, or in more subtle forms, but the virtually universal conclusion is that teacher education, particularly preservice, is not preparing educators to work in a technology-enriched classroom. (p. 978)

In fact many observers and researchers are suggesting that integration and infusion are not strong enough words for the type of technology use that should be espoused by teacher education (Thompson, 2000). Research indicates that too many teacher education programs have focused on the technology rather than the curriculum (Cooper & Bull, 1997). The prevalence of stand-alone information technology (IT) courses bears out that fact. Stand-alone courses are often needed to supplement a lack of basic skills, but such courses are not preparing preservice teachers to enhance teaching and learning through meaningful and contextual technology integration (Strudler, Quinn, McKinney, & Jones, 1995). A report by the OTA (1995) found, “Much of today’s educational technology training tends to focus on the mechanics of operating new machinery, with little about integrating technology into specific subjects” (p. 25). It is no longer sufficient to teach *about* technology; instead preservice teachers need to be learning how to teach effectively *with* technology (MAA, 1991; Pellegrino & Altman, 1997; Timmerman, 2004).

Recommendations and Guidelines for Effective Technology Integration

Teaching with technology requires instructional planning that contemplates technology as a tool rather than an add-on, something many teacher education programs are not preparing preservice teachers to do (OTA, 1995). Recommendations have been put forth that would promote and guide the technology training of preservice teachers. The research proposed in this study makes every attempt to incorporate as many of the guidelines discussed as is appropriate. The fact that many preservice teachers have not personally experienced technology integration as school students, gives rise to the need for faculty to be encouraged to model effective use of technology within their courses (ISTE, 2000, 2008; MEET, 1999). Although modeling appropriate use of technology is a step in the right direction, the OTA report makes it clear that preservice teachers need more. “They must see technology used by their instructors, observe uses of technological tools in classrooms, and practice teaching with technologies themselves if they are to use these tools effectively in their own teaching” (OTA, 1995, p. 185).

Connors (1997) extends the recommendation by suggesting that teacher preparation and enhancement courses need to model appropriate technology that prospective and experienced teachers can use to promote meaningful learning of the mathematical content that will be taught in the classroom. Such an integration of educational technology is anything but trivial (Timmerman, 2004). Effectively integrating technology into mathematics instruction requires acquiring new knowledge, as well as deepening current understandings, regarding both how and why to use technology in meaningful ways. Dexter, Anderson, and Becker (1999) explain how the newly acquired knowledge must be carefully woven together with the content and demands of the curriculum, classroom management, and existing knowledge of subject matter and pedagogy. The key to successful learning with technology rests in the teacher and not the technology. Although the educational technologies available today are flexible and powerful, they can never replace an effective teacher – nor can they realize full potential without one. Schwab (2000) succinctly captures this thought by stating, “In the hands of a poor teacher it [technology] is a useless tool; in the hands of a good teacher it is a powerful tool” (p. 152). Research-based guidelines have been disseminated to facilitate the equipping of preservice teachers with the necessary knowledge to make good use of educational technologies.

A synthesis of research conducted by Kathleen Heid (1997) offers four principles to guide the use of technology in mathematics education. The first focuses on the value of student-centered learning and the teacher's role in fostering that. Technology has been shown to help in transitioning the teacher into their new role as facilitator (Simonsen & Dick, 1997). This constructivist view is new to some and difficult for others. Indeed, many teachers' instructional methods probably fall somewhere between constructivism (learner-centered) and objectivism (content-centered). Hannafin, Burruss, and Little (2001) refer to this middle ground as "instructivism" (p. 132). Researchers do not propose that teachers abandon active classroom management and allow students complete control of their learning (Clements, 1999; Hannafin, Burruss, & Little 2001), neither do they suggest that software should be the controlling force in the learning process (Jonassen, Carr, & Yueh, 1998). There is little doubt that balancing control issues within a technology-rich classroom is an ongoing and ever-evolving challenge. The second principle involves giving students opportunities to function as a mathematician (e.g., to conjecture, explore, conduct trial and error, and perform hypothesis testing). Technology is thought to provide just such opportunities. Microworlds, which will be discussed later, are a prime example. The third principle suggests that teachers need to provide for and facilitate students' opportunities to reflect upon the mathematics they have encountered. This type of cognitive activity is not easy, but is a valuable part of a technology-based learning experience (Heid, 1997). The last principle is the idea that in an interactive, technology environment the teacher must assume and provide for constant access to the technology. In this setting the teacher takes on an interesting and powerful role in accomplishing what no textbook or worksheet can; to facilitate the computer in the connection of multiple representations (Clements, 1999; Heid, 1997). As will be seen in later sections, there are exciting Internet-based learning environments that can greatly assist the teacher in that new role.

Researchers from the Curry Center for Technology and Teacher Education at the University of Virginia and the University of Wisconsin, have devised five guidelines that reflect what they believe to be appropriate uses of technology in mathematics education: (a) introduce technology in context, (b) address worthwhile mathematics with appropriate pedagogy, (c) take advantage of technology, (d) connect mathematics topics, and (e) incorporate multiple representations (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000). A brief discussion of the guidelines will help to clarify the role and purpose of each. The first guideline, introduce technology in context, suggests that the features of technology should be introduced and illustrated in the context of meaningful content-based activities. In other words, the purpose of technology integration should be to enhance the teaching and learning of mathematics as opposed to using mathematics to teach about technology. The second guideline, address worthwhile mathematics with appropriate pedagogy, encourages incorporating technology-based activities that support sound curricular content and not the development of activities merely because the technology makes them possible. The technology used should support and facilitate conceptual development, exploration, reasoning, and problem solving, as encouraged by the NCTM (1991, 2000). The third guideline recommends that activities take advantage of technology and explore topics well beyond what could be done by hand. The fourth guideline states that technology-enhanced activities should facilitate mathematical connections between topics in the curriculum and to real-world contexts whenever possible. The last guideline involves incorporating multiple representations. Mathematics educators should encourage technology integration that aids students in making connections (e.g., graphical, numerical, and pictorial) between multiple representations of mathematical concepts within problem solving situations (Jiang & McClintock, 2000).

Near the turn of the century, the NRC (2000) conducted a synthesis of research on cognition and learning and within that presented four components deemed essential for the development of effective learning environments: community, learner, knowledge, and assessment. The learner, knowledge, and assessment-centered aspects of the learning environments described by the NRC are all essential, yet coexist, and are dependent upon, the facilitation of a community of learners; where learners and knowledge are honored and where participation, communication, and collaboration are fostered. Hovermill's (2003) research highlighted how profound learning environments can result when technology instruction integrates all the components of the NRC's effective learning environment. Shamatha, Peressini, and Meymaris (2004) strengthened and extended Hovermill's work by providing classroom teachers with a model to guide their technology integration. Their work involving content-based technology integration also provides specific examples demonstrating how various technology-supported mathematics activities exemplify all facets of an effective learning environment proposed by the NRC.

The last set of guidelines that will be discussed emerge from a meta-analysis conducted by Robert Marzano (1998) in which instructional techniques were identified as having a statistically significant impact upon student achievement. Empirical evidence supports the use of these four instructional techniques selected from Marzano's work and provides a model to guide the technology-based instructional strategies proposed in this study. These ideas will be further developed in Chapter 3. Marzano (1998) revealed that the following instructional techniques had an average effect size (ES) greater than one. The reader should keep in mind that an effect size of one corresponds to an average percentile gain of 34% in student achievement. The first technique, representing new knowledge in graphic/nonlinguistic formats, finds its roots in cognitive psychology which states that our brains store knowledge using both words and images. An ability to visualize discriminately is a vital skill that needs to be developed for the successful learning of geometry (Clements & Batista, 1992). Unfortunately, research indicates that such visualization is extremely difficult for students (Dede, 2000). Visual limitations exist in varying degrees across students and can lead to conflicts between visual evidence and information gained from other sources (Triadafillidis, 1995). Computer-based technologies are an ideal medium for minimizing these limitations and conflicts and facilitate the visualization of mathematical concepts (Noss, 1987, 1988; Clements, Sarama, & Battista, 1998).

A second instructional technique is using manipulatives to explore new knowledge and practice applying it. Marzano (1998) found that overall; the use of manipulatives is associated with an average percentile gain of 31 points (ES .89); however, the use of computer simulations as manipulatives produced the highest effect size of 1.45, indicating a percentile gain of 43 points. When a computer simulation assumes the role of a cognitive tool, as opposed to simply modeling a phenomenon, it becomes a microworld – which will be discussed in detail later. Generating and testing hypothesis about new knowledge is a third effective instructional technique identified by Marzano. The implication from the research is that the greatest benefits regarding this technique are gained when the computer-based explorations are guided by an expert teacher in a meaningful way (Clements & McMillen, 1996).

The last pertinent technique discussed from Marzano's analysis is an instructional sequence involving the demonstration of new concepts to students in a rather direct fashion and then having the students apply the concepts, generalizations, and principles to new situations. Technology is not a panacea, and guidelines to implement technology will only be successful to the extent to which they are implemented within a proven and meaningful learning environment. Indeed, it would seem prudent to integrate the four instructional strategies just discussed into any computer-based learning environment in order to maximize student achievement (Cholmsky, 2003). Although Marzano's meta-analysis is valuable to the field of education and very thorough in regards to classroom students and their learning, it includes no mention of effective instructional strategies for training preservice teachers. This is an area ripe for investigation.

As previously mentioned, the dynamic learning environments afforded by today's technologies have been shown to stimulate and promote a conceptual understanding of mathematics within preservice teachers (Keller & Hart, 2002; Wetherill, Midgett, & McCall, 2002). It is only through proper teacher mediation that technology can become a tool to enhance learning (Clements, Sarama, & Battista, 1998). If this is true, then maintaining the current status quo in regards to teaching, and learning to teach, mathematical concepts such as area and perimeter will not bring about the much needed improvements. Technology should not be just another means to disseminate information. With properly trained teachers, it can and needs to be used to develop critical and reflective thinking (Jonassen, Carr, & Yueh, 1998).

The Concept and Possibilities of Anchored Instruction

The anchored instruction model of learning was developed and tested by a team of prolific researchers who derived their insights from the work of Dewey (1933) and Hanson (1970). They worked out of the Learning Technology Center (LTC) at Vanderbilt University and when they published as a team, the group referred to them selves as the Cognition and Technology Group at Vanderbilt (CTGV). The group had concerns with traditional instruction and sought ways to build upon and incorporate preferred constructivist approaches in hopes of developing a more useful knowledge among participants (Bransford, Sherwood, Hasselbring, Kinzer, & Williams, 1990a).

Cognitive psychologists claim that meaningful knowledge is formed when small chunks of information are woven together within a contextual framework (Klock, 2000). Anchored instruction seeks to scaffold just such a framework. Anchored instruction is grounded in and derived from constructivist theories of knowledge and is a specific application of situated cognition. It is a research-based paradigm for examining learning through technology-assisted problem solving. Anchored instruction is similar to case-based learning, although the stories presented are meant to be “explored and discussed rather that simply read or watched” (CTGV, 1992a, p. 249). It is also similar to problem-based learning, but not as open-ended. Bauer, Ellefsen, and Hall (1994) describe anchored instruction as “a model that emphasizes the creation of an anchor of focus [typically, technology-based] around which instruction can take place” (p. 131). Videodiscs, the anchor chosen by the Vanderbilt Group, have often been used to provide an environment to anchor instruction and problem solving to a meaningful context. Each videodisc contains a story organized around an authentic problem-solving task that emphasizes in-context learning that is constructivist or generative in nature (Bransford et al., 1990a; CTGV, 1992a) and emphasizes the importance for students to experience the advantages of apprenticeship learning (Brown, Collins, & Duguid, 1989).

Goals and Uses of Anchored Instruction

The CTGV asserts that traditional curricula focused on memorizing and recalling facts and often introduced different ideas in different contexts – even if those ideas could be meaningfully connected (Bransford et al., 1990b). To combat this weakness the CTGV, under the leadership of John Bransford, established many challenging goals – chief among them was finding a way to address the problem of *inert knowledge* (Baumbach, Brewer, & Bird, 1995; CTGV, 1990; 1992a; 1992b; 1993), which often results from the traditional instruction presented in school (Whitehead, 1929). According to Whitehead, inert knowledge is knowledge that can usually be recalled when explicitly asked to, but is not spontaneously recalled in problem-solving situations even though it is relevant. According to the CTGV (1990), "The major goal of anchored instruction is to let students experience the changes in their perception and understanding of the anchor as they view the situation from multiple points of view" (Bransford et al., 1990b, p. 394). Another goal of anchored instruction is to allow students and teachers to experience cooperatively the kinds of problems and opportunities that experts in various areas encounter (CTGV, 1990, 1992b). The potential of technology to provide representations that can connect mathematical learning to authentic human experience should not be overlooked (Kaput, 1994).

Before attempting to meet the desired goals of anchored instruction, key decisions regarding the choice and use of the anchor must be made. The decision points that follow, respectively, are based on the research of McLarty et al. (1990), and have been instrumental in informing the design of the proposed study: (a) choosing an appropriate anchor, (b) developing shared expertise around the anchor, (c) expanding the anchor, (d) using knowledge as a tool, (e) allowing student exploration, and (f) sharing what was learned from the anchored instruction. The CTGV (1993) maintained that computer simulations, films, videos, and printed materials all can serve as appropriate anchors. It is advantageous for the anchor to be interactive, dynamic, and to be stimulating both visually and spatially (CTGV, 1992a). Once the anchor has been selected, it is important for users to have multiple experiences with the anchor from varying perspectives. Baumbach, Brewer, and Bird (1995) suggest that such activities will encourage students to develop expertise on various aspects of the anchor. As their knowledge of the anchor develops, students can be encouraged to assume greater responsibility for their learning. Once the teacher and the students have developed a shared expertise around the anchor, phase three can be initiated. Now the students can expand the anchor by using their expertise to solve problems requiring the use of the anchor (Bauer et al., 1994). Promoting and refining students' problem-solving skills are essential to success during this phase. In phase four students are allowed greater freedom to plan and conduct their own solution strategies by exploring the anchor. Having the ability to explore the same domain from multiple perspectives is a primary goal of anchored instruction (CTGV, 1992a). Although there are some minor discrepancies regarding certain aspects of the first four phases, it is agreed that learning activities centered around anchored instruction need to culminate with students sharing what they have learned (Bauer et al., 1994; Baumbach et al., 1995; McLarty et al., 1990). Students are encouraged to compare their work with each other and with the teacher or other experts who are present. The dynamic and interactive learning environments that result from attempting to meet the goals of anchored instruction have produced diverse research on the instructional model.

Highlighted Research on Anchored Instruction

The relatively slim body of research encompassing anchored instruction should not detract from its contribution to the study of teaching and learning. The research paradigm of anchored instruction is a relatively new phenomenon, dating back to the late 1980s. Early research conducted by the CTGV indicated that anchored instruction seemed to help students develop rich, organized knowledge structures plus promote long-term retention and spontaneous use of vocabulary (Bransford et al., 1990b). The CTGV later found that fifth graders can become very good at complex problem formulation on tasks similar to those experienced during anchored instruction (CTGV, 1992a). The research group felt that situating the learning experience in meaningful contexts was the key for anchored instruction to facilitate students acquiring knowledge of problem solving strategies as well as knowledge of content that was non-inert.

Following the earlier research studies involving general education fifth graders, anchored instruction has been studied in various settings, including middle-grade science (Goldman, et al., 1996); several studies involving students with disabilities, including: literacy and social studies (Kinzer, Gabella, & Rieth, 1994), effects of media attributes, (Shyu, 1999), social studies (Glaser, Rieth, Kinzer, Coldburn, & Peter, 2000), general education (Bottge, Heinrichs, Mehta, & Hung, 2002), remedial math and pre-algebra (Bottge, Heinrichs, Chan, & Serlin, 2001), mathematical problem solving and transfer (Serafina & Cicchelli, 2003), and procedural math skills (Bottge, Heinrichs, Chan, Mehta, & Watson, 2003). While results from these studies were mixed, there were many positive findings and subsequent helpful recommendations. It appears that the majority of school research on anchored instruction conducted in the past ten years involved, in some way, students with learning disabilities. A plausible explanation for this involves one of the disadvantages of incorporating anchored instruction into the traditional, general education classroom. Implementing anchored instruction is a time consuming proposition. The standardized curriculum found in most of the general education mathematics classes, along with applicable high-stakes tests, produces apprehension among many teachers who feel pressure to cover an unreasonable amount of content and thus settle on lecturing as their primary means of dispensing information (Oliver, 1999). Ironically, one of the biggest detriments to higher-order thinking, a goal of anchored instruction, seems to be a standardized curriculum. Fortunately, for most higher education, the curriculum is not so rigidly defined, and offers a fertile soil for research on anchored instruction, as is the case with my study which will investigate the influence of anchored instruction upon preservice teachers' content knowledge and knowledge of student thinking regarding area and perimeter. Very little research has investigated the use of this instructional method with preservice teachers and even less has involved topics in mathematics.

Early research on anchored instruction explored possible applications within teacher education. One study compared whether anchored instruction could promote reflective thinking among preservice teachers about teaching practices. McIntyre and Pape (1993) had one group of K-6 preservice teachers ($n = 16$) view videodiscs of expert teaching practices as part of their instruction while the other group received typical methods instruction without any video-based instruction. Pre- and posttests (findings limited by small sample size), student logs and progress reports, and student interviews revealed an overall positive attitude from a majority of students receiving anchored instruction. These students appeared to be more descriptive in their analysis of critical classroom events and were better able to support their claims. Student interviews indicated that the interactive videodiscs resulted in more and deeper reflection of classroom activities. The role of anchored instruction in improving preservice teachers' learning about instructional practices has also been examined in the domain of educational technology. Bauer, Ellefsen, and Hall (1994) were interested in determining whether using anchored instruction would help preservice teachers learn how to use a variety of technologies and also the extent to which students could envision applying the model in their future teaching. A variety of data sources were used, including videotaped observations and interviews, student-produced projects, and information provided by instructors. Researchers found that students did learn to incorporate a variety of educational technologies while using the *Oregon Trail* software as an anchor. Student achievement on assigned projects was superior to previous semesters in thoroughness and overall quality. Most of the students interviewed indicated that they felt the anchored instruction approach was worthwhile to learn and that they anticipated using some form of the model in their future teaching; however, a longitudinal study would be needed to determine if exposure to the model would have any impact on the future teaching practices of the participants.

Bauer (1998) replicated his previous research with a larger sample size (n = 48) and reported similar results as before. Kariuki and Duran (2004) expanded upon Bauer's research when they conducted a semester-long case study involving a cohort group of 22 preservice teachers. They used anchored instruction as a means to integrate a curriculum development course with an educational computing class. Participants not only learned about technology applications for the classroom, but they applied their knowledge by developing instructional units to share with an eighth grade student from a local middle school with whom they were paired. Feedback from the participants was overwhelmingly positive. The findings showed that anchored instruction was an effective way to both learn about educational technology tools while at the same time integrating technology into instructional practices – at least in a one-on-one setting.

Only one study was found investigating the use of anchored instruction in a mathematics course for preservice teachers. Kurz and Baterelo (2004) used case study methods to investigate four female preservice teachers (two secondary and two elementary) who volunteered to participate in a mathematics-based technology integration course. The study focused on whether the subjects could determine the significance of using anchored instruction with their future students and if they envisioned student learning and mathematical growth using anchored instruction. To different degrees, the participants expressed optimism about the utilization of anchored instruction and were able to describe salient features of the model that support student learning and growth. Given the fact that previously discussed research indicates many preservice teachers possess similar mathematical shortcomings as their students, it would seem the hypothetical context investigated by Kurz and Batarelo (i.e., studying how preservice teachers *envision* student learning and mathematical growth using anchored instruction) could have been more meaningful if grounded in examining first-hand how preservice teachers themselves learned and grew mathematically through experiencing anchored instruction. Developing knowledge within students and teachers that is conceptually anchored is strongly recommended (CBMS, 2001; NCTM, 1991, 2000). The potential impact of anchored instruction upon preservice teachers' specific content knowledge and knowledge of student thinking has been virtually unexplored and is ripe for investigation.

At the time the CTGV were doing their initial research and formulating their fundamental ideas regarding anchored instruction, it was determined that computer technology was not yet widespread enough, nor affordable, for it to be universally accessible to serve as the anchor for the model; thus, the videodisc was decided upon to fill that role. However, since that time the microcomputer, along with Internet access, have become commonplace for both higher education and the school classroom. The continued advancements in computers, software, and programming languages and platforms (e.g., Java) have allowed other learning environments to develop that share theoretical underpinnings with anchored instruction. Logo and other more dynamic and interactive microworlds represent prime examples.

Microworlds

The purpose of this portion of the literature review is to acquaint the reader with microworlds, explain their distinguishing design features, discuss some popular computer-based geometry microworlds, provide highlights from research involving computer microworlds and students, and then focus on research incorporating microworlds into preservice teacher education. The literature reviewed regarding preservice teachers will focus primarily on microworlds designed to function as online

Java applets, as opposed to general software (e.g., Geometer's Sketchpad, [Jackiw, 1995]), simulations (e.g., SimCity), or games (e.g., Math Blaster Mystery, [David & Associates, 1994]). According to Rieber (1994, p. 229), "Simulations start to become microworlds when they are designed to let a novice begin to understand the underlying model." The various aspects of a microworld's underlying model are the topic of discussion in the next section.

Microworlds: Defined and Described

The power of a microworld lies not necessarily in what it can *do*, but rather in its constructivist environment designed to motivate (and indirectly guide) the user to explore ideas and relationships, and resolve conflicts between prior knowledge and newly encountered information (Papert, 1980; Rieber, 2004). According to the Piagetian principle of equilibrium, this cognitive conflict (referred to as disequilibrium), is necessary for meaningful learning to occur (Hogle, 1995). A well-designed microworld will foster these learning conflicts.

The epistemology underlying microworlds is known as constructivism (Jonassen, 1991). Seymour Papert (1980) coined the term *microworld* over twenty years ago. He defined it as:

> . . . a subset of reality or a constructed reality whose structure matches that of a given cognitive mechanism so as to provide an environment where the latter can operate effectively. The concept leads to the project of inventing microworlds so structured as to allow a human learner to exercise particular powerful ideas of intellectual skills. (p. 204)

Microworlds do not have to be computer-based. For example, a kitchen or a child's chemistry set can function as a microworld. Papert made it clear that the concept of a microworld was not new and was actually related to the longstanding notions and uses of mathematical manipulatives (e.g., Cuisenaire rods). David Jonassen (1996) describes a microworld as a "constrained problem space that resembles existing problems in the real world" (p. 237). The very nature of a microworld presents problems that are inherently interesting; therefore, encouraging the user to generate their own problems and test hypotheses for solving it.

Many definitions have been posited over the years, but perhaps the most elegant comes from Clements (1989): "A microworld is a small playground of the mind" (p. 86). In the next section we consider various defining characteristics of a microworld which support opportunities to learn while exploring a microworld's playground.

Characteristics of a Microworld

Clear distinctions between characteristics that define a microworld and the principles that guide their design are not always evident; however, because the microworlds used in this study were (for the most part) already conceived and designed prior to my implementation, the focus of this section will be on the salient features necessary for a microworld to be able to function as a meaningful learning environment.

The characteristics that follow are presented as a confluence of valuable points of view. Although the guidelines are open to various interpretations (e.g., instructional designers, constructivists, or instructivists), they are meant to provide a sort of filter to help identify microworlds worthy of integrating into instruction. The focus will be on how the microworld functions (i.e., their use), as opposed to how it is structured (i.e., their design). L. P. Rieber has been researching and writing about microworlds for almost twenty years. Based on a synthesis of his own and that of others in the field, Rieber (2004) presented the following definition of a microworld:

> Therefore, a microworld must be defined as the interface between an individual user in a social context and a software tool possessing the following five functional attributes: (a) It is domain specific; (b) it provides a doorway to the domain for the user by offering a simple example of the domain that is immediately understandable by the user; (c) it leads to activity that can be intrinsically motivating to the user – the user wants to participate and persist at the task for some time; (d) it leads to immersive activity best characterized by words such as play, inquiry, and invention; and (e) it is situated in a constructivist philosophy of learning. (p. 588)

Rieber continues by stating that for a microworld to be domain specific implies an appropriate treatment of curricular content and careful attention to pedagogical recommendations for how the domain, such as mathematics, should be taught. Hoyles (1991) explains that in order for investigation within a microworld to be meaningful the learning domain must "connect" with the user's initial conceptions of how the model should work. In other words, the microworld should be able to meet the user where they are. Connecting with pupil conceptions is complex. Learning within a microworld is a very personal experience and what is meaningful can be relative. Rieber (1992) interprets *meaningfulness* as the degree to which a student can link new ideas to prior knowledge. The success of a microworld in opening the doorway to exploring a new domain hinges on its ability to connect with (and then expand) the user's prior knowledge. Such a connection is also considered among the most important determinants of learning

(Ausubel, 1968).

Once the door to a specific content domain has been opened, it is critical that the microworld continue to motivate the user to persist at his or her exploration. It was Benjamin Bloom who said, “Under favorable learning conditions almost all students can learn well” (1977, p. 22). The ability of a microworld to allow for self-correction by providing graphic and quick feedback (Hogle, 1995) combined with linked, interactive representations (Sinclair, 2005) is a valuable tool to help address Bloom’s concerns and increase the opportunity to learn for all. Although the inherent scaffolding features of the microworld’s environment are important, and can aid in understanding mathematics, a qualified and knowledgeable teacher functions as the virtual glue holding all the elements of a meaningful microworld learning environment together. Indeed, “The teacher’s role is critical in supporting and challenging student learning while at the same time modeling the learning process with the microworld” (Rieber, 2004, p. 588). There are many important and interrelated parts operating within a microworld learning environment (e.g., the curriculum, the microworld, the teacher, and the student), and in the works of Reeves (1999), “It is time to assign cognitive responsibility to each part of the learning system that does it best” (p. 7). Working in a microworld does not guarantee learning any more than sitting inside of a library does; however, a microworld situated within a carefully constructed environment can be a valuable cognitive tool to facilitate the learning of mathematics. The concepts within Geometry provide an excellent backdrop for the integration of a microworld tool.

Since the 1980s many other microworlds have become available; however, there are four computer-based microworlds that specifically deal with geometry. They are

Logo, Geometric Supposer (including superSupposer), Cabri Geometry (including Cabri II), and Geometer's Sketchpad. It is important to distinguish the different levels of interaction experienced by the user while exploring within these microworlds. It is outside the scope of this review of literature to discuss thoroughly all the distinguishing features, specific functionality, and instructional uses of those software titles. I will instead summarize the findings involving the influences and impacts of the software upon the teaching and learning of geometry in the school classroom.

Static Geometry Software

Papert's ideas on microworlds evolved from his participation, along with a team from the Massachusetts Institute of Technology, in the development of the programming language that became known as Logo, derived from the Greek word meaning "thought" or "idea" (Rieber, 2004). Appearing in the early 1980s, Logo is one of the earliest static construction environments. The term *static* refers to the type of interaction that occurs between the user and the software. A static environment does not allow the user to manipulate an object directly (referred to as "dragging") and simultaneously observe the effects of that manipulation. This limitation is a prime distinguishing characteristic between static and dynamic software. Despite this limitation there is a considerable amount of research on Logo and results have been very positive. Logo is a programming language and that fact has allowed for updated versions over the years. The primary focus of Logo geometry is properties of two-dimensional shapes and measurement. Research on Logo goes back almost twenty years, and the findings are extensive. The primary focus of this study only warrants a summary of major themes.

Early versions of Logo required students to write basic code to control the movement of a turtle-shaped icon on the screen. Although the code was straightforward, it proved problematic to some young children (Clements & Batista, 1989; Hoyles, Noss, & Adamson, 2002). Turtle Math, a successor of Logo, has greatly reduced the obstacle. For an example of how far the evolution of Logo has progressed, please visit [http://nlvm.usu.edu/en/nav/frames_asid_178_g_3_t_1.html] to experience an Internet version. In spite of some problems with children writing the code, programmers and researchers see great value in the coordinated action of writing symbols (code) and seeing the resulting drawing (Clements & Sarama, 1997). Studies found that students who learned geometry with Logo outperformed the control students on concepts involving angle conservation and angle measure (Noss, 1987) as well as understanding shapes and their components, and describing paths through a map (Clements & Batista, 1989; Clements et al., 1998). One of the most significant findings involves Logo's facilitation of higher levels of geometric thought. Currently, the best description of students' geometric thought regarding two-dimensional shapes is the van Hiele theory. According to this theory, students move through several qualitatively different levels of geometric thinking (Clements & Batista, 1992). The five levels are: (a) level 0 – pre-recognition, (b) level 1 – visual, (c) level 2 – descriptive/analytic, (d) level 3 – abstract/relational, and (e) level 4 – formal axiomatic (this level is required for doing proof). Advancing from one level to the next does not occur naturally in children and requires systematic nurturing (Dix, 1999). Research has shown interactions with Logo can help children (Clements & Meredith, 1993; Glass & Deckert, 2001) and middle school students (Clements & Sarama, 1997) progress into their next van Hiele level. A positive feature of Logo is its inherent ability to reflect individually the user's level of geometric thinking

(Clements & Batista, 1994). Such tailored instruction is very important when attempting to create a student-centered learning environment. Lastly, Clements and Sarama (1997) reported on a very interesting study where the Logo students not only outperformed traditionally-taught students but also another control group of students taught the same content but used concrete manipulatives. An apparent implication here is for teachers to be aware of the strengths and weaknesses of the various learning-support media at their disposal.

Besides the mathematical learning advantages of Logo, certain social benefits have been reported. Students working cooperatively with Logo showed enhanced, specific problem-solving skills such as conflict resolution (Clements & Nastasi, 1999), and displayed sustained enthusiasm for collaborative work resulting in improved communication skills (Yelland, 2002). Logo seems to foster a cooperative environment where both cognitive and social conflicts could be resolved. It is worth noting that the teacher played a crucial role in mediating this process through facilitating appropriate discussion of the activities. Logo activities were found to be most meaningful and beneficial when they were integrated into the existing curriculum and not used as an add-on (Clements & Sarama, 1997). In conclusion, and on a different note, although the research regarding Logo with school children is extensive and well-reported, there is relatively little (if any) that examines the influences of a Logo learning environment upon the mathematical understandings of preservice elementary teachers or their reflective considerations of future instructional strategies in light of such interactions. Although Logo's primary focus is two-dimensional shapes and is used mostly with younger students, the microworld discussed next is geared towards older students.

Geometric Supposer (1993) is one of the best-known geometry microworlds. It is a static modeling tool used for making and testing conjectures in geometry through manipulating geometric objects and exploring the relationships within and between these objects (Schwartz, 1993). Jonassen (1996) writes, “Geometric Supposer supports the learning of geometry by enabling the students to inductively prove relationships among objects” (p. 246). Its designers have found that, besides promoting the development of geometric concepts by allowing constructions to develop in a direct way, students exhibit a positive attitude towards learning those concepts with Supposer. Clements and Battista (1992) report that there have been numerous studies aimed at improving students’ proof skills through traditional approaches, almost all have been unsuccessful. At that time, they concluded that new learning environments were needed to encourage students to make conjectures and generalizations that would promote both inductive and deductive thinking. Supposer has made great strides in accomplishing just that. Hölzl (1981) explains that students struggle with the rigid nature in which diagrams are presented in traditional geometry textbooks. Supposer’s capability to produce many variations of a single diagram very quickly is one remedy to that problem (Yerushalmy & Houde, 1986). After working with Supposer, students reported a deeper understanding of the role and limitations of diagrams (Yerushalmy & Chazan, 1993). Spending time in the Supposer environment facilitates students’ acquiring of effective problem-solving strategies for analyzing problems, conjectures, and proof. Such students have even reported coming to understand more deeply and personally the value of formal proof in mathematics (Wilson, 1993). The Geometric Supposer has been shown to have the capacity to change how students think and feel about geometry, but these results are not guaranteed or automatic.

The attitude of the teacher and how they implement the Supposer are crucial to its success. Wilson (1993) continues by stating that although the Supposer can be used with traditional instruction as a sort of digital blackboard by a lecturing teacher, its design lends itself to a more open-ended approach. That open-ended approach offers the teacher the opportunity to integrate inductive reasoning back into the classroom. For this to be accomplished, the roles of teacher and student need to be altered. Yerushalmy and Houde (1986) liken the desirable learning environment to that of a typical science class. The scientific process becomes the primary focus, and teacher and student collaborate on collecting data, making conjectures, and looking for counterexamples or generalizations. These changes are not easy and the process is slow, but as seen above the learning dividends outweigh the initial investment of time and effort.

Dynamic Geometry Software

Although pioneering software packages such as Logo and Geometric Supposer made great strides towards achieving the technology recommendations of the NCTM and other interested parties, it was not until the development of software like *Geometer's Sketchpad* and *Cabri Geometry* that spatial concepts were "brought to life" (Dix, 1999, p. 5). Both of these software titles are relatively new to the classroom. Geometer's Sketchpad was released around 1991 and Cabri around 1992; therefore, the volume of research is much less than what exists for Logo or Supposer. There are many articles and conference proceedings for both software programs that primarily discussed suggestions for implementation and interesting activities, but most presented no research framework. This informal finding caused me to wonder if the research is just dragging behind the innovation or if implementation is being done despite an apparent hollow research foundation. It was Kaput who helped put my reflections in perspective by pointing out that research did not bring about the invention of the automobile. It was the result of necessity and progress. Necessity and progress have served as catalysts to facilitate a gradual integration of technology into the teaching and learning of mathematics. Organizations such as the NCTM (2000) suggest that interactive geometry software can be used to enhance student learning, and the results presented, along with those that directly follow, appear to bolster that claim.

Teaching and Learning Mathematics with Microworlds

Microworlds, functioning as cognitive tools (i.e., technologies that support thinking processes during problem-solving and learning), have been shown to assist in the learning of powerful and fundamentally different mathematics (Jonassen & Reeves, 1996; Pea, 1986), enhance student thinking (Lederman & Niess, 2000), support cognitive processes such as logical reasoning and hypothesis testing (Lajoie, 1993), provide specific feedback appropriate to guide in the learning of new material (Roblyer & Edwards, 2000), and encourage the exploration of mathematical ideas (Jensen & Williams, 1993).

It is important to realize that a true computer microworld is not meant to be a panacea functioning in isolation from social interactions with peers and teachers. Although microworlds are a constructivist invention, they can also be a tool for supporting goal-orientated environments in which learning occurs through discovery and exploration (Rieber, 1992). Rieber explains that one way to reach this compromise is by incorporating aspects of guided discovery into the learning activity which would naturally be constrained by the boundaries imposed by a particular microworld. The research presented on microworlds will attempt to strike a balance between describing the salient features of the microworld(s) involved in the study along with an appropriate discussion of the instructional strategies implemented. The most common use of microworlds among successful research studies involves embedding microworlds within a carefully planned curriculum unit, as opposed to treating them as a curricular add-on or as a medium to enhance traditional teacher-lead instruction.

Computer Microworlds in the K-12 Setting

There is limited research beyond the specific applications and domains of popular microworld software such as Logo and Geometer's Sketchpad; the most likely reason being the relatively recent affordability (desktop computers only fell under $1000 in late 1997) and resulting availability of the microcomputer within today's school setting. Initial studies seemed to focus on how students interacted with the microworld as well as the various solution strategies produced. The majority of this research did not attempt to embed the microworld within instructional units based on the curricula found at the school. For example, Steffe and Wiegel (1994) focused on children's transformation of their cognitive play activity into independent mathematical activity while interacting within two different types of microworlds (discrete and continuous). Two case studies involving four third-grade students found that although the microworlds captivated the children's interest and functioned as pathways to mathematical activity, independent mathematical activity was generally initiated by teacher intervention.

Clements, Battista, Sarama, and Swaminathan (1997) investigated the application and development of spatial thinking in an instructional unit on geometric motions and area. This was some of the earliest research to embed the use of microworlds within a specifically designed instructional unit. Observational data and results from paper-and-pencil assessments (including the Wheatley Spatial Ability Test) found that the three third-grade classes showed significant growth in spatial competence although the microworld-based activities motivated and aided the students in building more sophisticated and systematic problem-solving strategies. It is worth noting that although Clement's et al. notes the detrimental affects of isolating curriculum development, classroom teaching, and mathematics education research the role of the teacher within the instructional unit of this research study was not delineated nor were any teacher interventions discussed in conjunction with student comments. The reader is left to wonder if the instructional units were designed with the intent of being "teacher-proof."

Research involving microworlds and school-age children conducted since the late 1990s seems to be following similar frameworks. Healy and Hoyles (1999) conducted case studies of 12-13 years olds using Logo-based microworlds. They provided detailed accounts of how student interaction with microworlds resulted in their adopting different problem-solving strategies incorporating visual and symbolic reasoning in varying degrees. What was absent from the rich description was any account of the teachers' role during the tasks. This omission is curious because the researchers concluded that it is critical that computer use be carefully integrated into instruction and not be a supplemental add-on. It is not apparent if the researchers are envisioning the microworld as a purely self-directed discovery environment. Stohl and Tarr (2002) seemed to echo this sentiment of integrated instruction. They claim that the microworld, *Probability Explorer* (designed by Stohl), although leading to growth in students' ability to make appropriate statistical inferences, is not a panacea for probability instruction. What is critical, they argue, is for teachers to possess a growing understanding of students' reasoning about such topics; however, in their study the researchers designed the instructional program and functioned as classroom teacher. So the reader is left to wonder how well a typical teacher could foster students' probabilistic reasoning with an instructional unit integrating *Probability Explorer*. Kordaki (2003) conducted qualitative research examining the effect of computer microworlds on 9th grade students' strategies regarding the concept of conservation of area. It focused on their learning processes and not on learning outcomes. Log files which recorded students' interactions with the microworlds (i.e., electronic snapshots of students' drawings and audio recordings of all verbal interactions) along with field notes of the researcher showed students exhibiting a flexible and broad view of appropriate solution strategies; however, no information regarding the interventions of the teacher was provided. It would seem beneficial for a research study whose focus is on the learning processes of students to include some mention of the teacher's role within the microworld learning environment.

It would appear that a limitation with much of the research presented in this last section is the absence of discussion related to the role, and impact of the classroom teacher within a microworld-based instructional/exploratory unit. Although tasks and units of discovery that promote independent learning are definitely valuable, one would certainly surmise that a qualified teacher would be able to add support, guidance, and depth to such learning environments. It would be helpful to know if certain qualifications (content or technology-related) are needed for a teacher to implement the various instructional units described in the previous research studies. The research I propose will be providing not only a detailed description of the instructional units and the microworlds integrated into each, but also an explanation of the instructor's role within the instructional setting. It must be noted the research dynamics will be different as the proposed study will be conducted in the context of a teacher education college methods course. In the concluding section of this literature review, the role of the instructor will be one facet examined while reporting on the research that has investigated the use of microworlds within teacher education courses.

Microworlds and Teacher Education

A new technology discussed in this section, and incorporated into this research, is the Internet- or web-based microworld (also known as online or Java applets). This technology is very new and dynamic in the sense that it is evolving along with the Internet. Because of the young age of the Internet (the first commercial web browser was only released in 1994), educational research based on its technologies is also in its early stages, with the vast majority of it surfacing after 1998. The amount of research within this domain is growing but currently very limited. The foci of research involving microworlds and teacher education fall along a continuum involving aspects of the affective domain (Timmerman, 1999) and knowledge types (Keller & Hart, 2002; Wetherill, Midgett, & McCall, 2002), with other research examining specific mathematical content (e.g., fractions – Chinnappan, 2000; and the mathematics of change, Bowers & Doerr, 2001). Another important consideration while evaluating the research is the platform on which the microworld will be running. For example, some of the microworlds investigated are installed and run locally from the user's computer (Bowers & Doerr, 2001; Chinappan, 2000; Timmerman, 1999); however, others are online applets, reside on the Internet, and can be accessed on any computer through an Internet browser (Keller & Hart, 2002; Wetherill, Midgett, & McCall, 2002). Although the foci of the research and the type of microworld used varies, it is widely agreed upon that mathematics teachers, not the tools of technology, are the catalysts to bring about a meaningful learning of mathematics with technology (Kaput, 1992; NCTM 1991, 2000; Willis & Mehlinger, 1996). Garofalo, Drier, Harper, and Timmerman (2000) provide five guidelines (discussed earlier) for technology-based activities designed to help reexamine and deepen understandings of mathematics. All the research found pertaining to web-based microworlds and preservice teachers involved exploring mathematics that pre- and inservice teachers will be responsible for teaching. Browning and Klespis (2000) question this approach, at least in regards to secondary teachers, and instead suggest that in order for preservice teachers to experience and understand the impact of technology upon the learning of mathematics, the concepts must be new and on their level. Although this approach would appear a possible alternative for secondary mathematics majors, it does not fit as well for preservice elementary teachers, which is the focus of my study. Integrating technology into instruction can take on many forms; however, there is consensus that the most effective learning within technology-rich environments occurs within the specific content area which the technology will be used (Bull, 1997; National Governors' Association, 1991). The research that follows addresses this recommendation to different degrees.

The four studies discussed in this section involve software-based microworlds and provide examples of the degrees to which technology can be integrated within a methods course for teachers. Tzur and Timmerman (1997) conducted a teaching experiment with a master’s level course (taught by the first author) containing 12 elementary teachers and case studies with three of the teachers. The Sticks microworld was incorporated within instructional sessions based on conceptions identified in research on children’s learning of the “invert-and multiply” algorithm for fractions. Over the course of the semester the researchers were able to use research on stages of children’s learning about fractions to organize observations of teachers’ knowledge and to devise situations that promote teachers’ understanding. Neither the findings nor the discussion make it clear to what degree the researchers felt that knowledge of student thinking, the microworld, or the instructional sequence and materials contributed to the gains stated.

Chinnappan (2000) examined preservice elementary teachers’ understanding and representation of fractions in a microworld environment. The study was limited in scope. Eight volunteer preservice elementary teachers met individually with the instructor, who was the investigator, for approximately two hours. The interview sessions consisted of an orientation of the software (*JavaBars*) and solving two fraction problems, first without the aid of the microworld and then with. Qualitative analysis of the participants’ knowledge base suggests that they built up a minimum level of content knowledge of fractions. Analysis of their pedagogical content knowledge growth revealed the participants were more concerned with solving problems than thinking about difficulties students might have solving the same problems. The preservice teachers did not exhibit skills at using the microworlds to provide different and pedagogically powerful solutions or representations to the given problems. One might conclude that the relative short contact time with the microworld combined with a lack of appropriate or motivating context could be a cause of the lack of pedagogical growth. Another explanation could be the inexperience of the participants. Livingston and Borko (1990) reported that novice teachers tend to focus on the content and the task at hand while the focus of an expert teacher is more often on the students.

Timmerman (1999) addressed an apparent limitation of Chinnappan's (2000) study by extending contact time with the microworld. This research had a similar methodology to Tzur and Timmerman (1997). Here Timmerman conducted a phenomenological study involving 12 elementary school teachers enrolled in a 16-week master's level mathematics teacher education course that involved learning various number concepts while using computer microworlds. Over the course of the semester, the conceptions of three teachers were studied, but this study focused on two of them. The subjects of the case studies had different motivations towards and backgrounds in mathematics. Field notes, audio-tape interviews, a collection of reflective journals and final projects, classroom observations of the teachers, and pre- and post-course attitude surveys revealed that although the teachers enjoyed the control they had over their own learning with the applets, they could not shift their teaching style from teacher-controlled to one allowing for student independence and freedom to explore and learn about fractions while interacting with the microworlds (*Toys* and *Sticks*). In this study the teachers ended up not using the microworlds as part of instruction on fractions because of the lack of control they had over the environment – even though they acknowledged having difficulty generating conceptual explanations for some basic operations involving fractions (e.g., the division algorithm). It also became evident that personal learning preferences and styles influence the process of teachers learning in technology-rich environments. Although the reporting was rich, details regarding the instructional sequencing were very limited.

Bowers and Doerr (2001) seemed to strike an informative balance with their reporting. They acknowledge that students in a mathematics education course are simultaneously learners and teachers in transition. In their study they analyzed the interrelations between prospective and practicing secondary mathematics teachers' learning of the mathematics of change and their developing understanding of how to teach effectively such concepts. The semester-long study took place at two different universities with a total of 26 participants situated in similar courses designed around a microworld software environment called MathWorlds. The instructional sequence was designed to facilitate the participants' revisiting of prior knowledge from a student's perspective and then engage them as reflective teaching practitioners. Qualitative analysis of written work on problem-solving assignments, reflective journals, and the instructor/researchers' daily teaching journal found that the participants who experienced perturbations as both student and teacher came to develop an appreciation for the value of conceptual explanations and explorations with technology. The value of viewing participants in the dual roles was confirmed as some of the participants developed mathematical insights as they created, taught, and reflected on mathematical lessons although others' most powerful pedagogical insights emerged as they were assuming the role of mathematics students. Viewing preservice teachers in their dual roles as student and teacher and designing activities that stimulate both roles appear as a valuable way of integrating technology in such a way as to help address the demands of balancing content and pedagogy within a mathematics methods course. There is another emerging technology which after closer examination seems even better equipped to facilitate this balancing act.

This review of the literature concludes with research pertaining to Internet-based microworlds. Technologies residing within the Internet comprise an evolving world of knowledge and potential tool for education. Research on such a dynamic domain must be on the cutting edge in both theory and application. In light of the emerging state of Internet-based microworlds, it would seem appropriate to include a discussion of the prominent findings from the two studies found which have and continue to investigate this technology, even though these findings are preliminary. Both studies utilize online applets and activities located at the Illuminations website developed in association with the NCTM and currently found at: http://illuminations.nctm.org/. These studies investigated the influence of applet-based instructional materials on both teacher knowledge (content and pedagogy) and student learning. Based on the success of the Illuminations-based professional development, Wetherill, Midgett, and McCall (2002) designed a two-part qualitative study on the impact of the NCTM Illuminations applets and support materials on teacher knowledge of mathematics content and pedagogy, instructional planning, and students' learning of fractions. From a group of thirty middle-grade teachers who participated in a summer professional development project centered on the resources contained at the illuminations website, three teachers were identified to participate in this two-part study. Data were collected from videotaped lessons, videotaped interviews with the teachers, and teachers' written reflections. Early findings from phase one were encouraging. A paired t-test from the 30 original participating teachers (including the three for this study) showed significant growth in teachers' ability to explain concepts. Other preliminary findings indicate that the fraction applet provided teachers opportunities to develop new insights into their own knowledge as well as their students' understandings of and misconceptions regarding the relationships of fractions. Data from phase one also showed that the fraction applet enabled both teachers and students to visualize mathematical relationships and hence deepen their understandings of fractions. The second phase will continue studying the subjects in the first phase to collect formative data on the design of the applet-based resources. What was lacking in the reporting of phase one was specific information regarding the instructional materials used in the study. It is possible such information will be forthcoming in the formative research involved with phase two.

Another study presenting preliminary findings regarding the use of applets found on the Illuminations website comes from Keller and Hart (2002). Their three phase study (two of which have been completed) evaluated curriculum-embedded applets for isometric drawings to develop preservice elementary teachers' spatial visualization skills. A set of online instructional tasks were created that would engage the preservice teachers in using the applet to develop their spatial visualization skills in the role of a student and then apply that knowledge by filling the role of a future teacher designing lessons involving isometric drawings. Paper and pencil tests and videotaped sessions from phase one suggest that the applet-based instructional materials improved the preservice teachers' visualization skills in the five targeted categories. Results from the second phase suggest that the instructional materials enhanced the preservice teachers' (n = 320) pedagogical content knowledge as evidenced by their increased awareness of certain teaching and learning issues related to isometric drawings. As in the previous study, no specifics were provided regarding the content of the instructional materials or the role of the various instructors. Until formative findings are presented, one can only speculate as to the potential effects or influences of web-based microworlds on the knowledge and skills of pre- and inservice teachers and the resulting impact upon student learning.

Summary of the Literature Review's Salient Points and How they Informed this Study

It becomes clear from reviewing the research that many preservice teachers, even those who possessed a strong mathematics background or at least expressed confidence about their content knowledge, exhibit a very limited pedagogical content knowledge as noted by an inability to provide conceptual explanations (Borko et al., 1992), being baffled by students' questions (Meredith, 1993), and routinely being unable to anticipate students' difficulties or diagnose and address their misconceptions (Mapolelo, 1993). The expert teacher on the other hand has been shown to possess a more conceptually-grounded understanding of many mathematical topics (Fuller, 1993), displays an appropriate balance of procedural and conceptual knowledge (Hiebert & Carpenter, 1992), uses technology to promote conceptual understanding (Mitchell & Williams, 1993), and tends to focus on the student instead of the content (Livingston & Borko, 1990). A novice teacher progressing along the continuum to becoming expert is clearly advantageous and every effort should be made to accelerate that progression. The progression is multi-faceted. Clearly, a teacher's content knowledge will be an integral part of their teaching, and a lack thereof will very likely affect the quality of instruction (Grossman, Wilson, & Shulman, 1989) and ultimately student learning (Fennema & Franke, 1992). Research suggests that preservice teachers can benefit from revisiting their mathematical knowledge in appropriate and meaningful contexts (Ball & Bass 2000), and that pedagogical content knowledge and content knowledge should be developed simultaneously (Good & Grouws, 1987; Stacey et al., 2001). One might assume that many aspects of PCK (e.g., a knowledge of student thinking) naturally develop while performing the act of teaching. Researchers have found too often this is not the case (Ball et al., 2001; Ma, 1999). Methods classes have shown to offer a very suitable environment for the development of preservice teachers' mathematical content knowledge and pedagogical content knowledge (Ball, 1990; McGowen & Davis, 2002; Quinn, 1997; Simon & Blume, 1996; Stoddart, Connell, Stofflett, & Peck, 1993); however, links between how and to what extent CK and PCK, regarding specific mathematics topics, can develop within a methods course are lacking as well as are attempts to establish how dependent PCK may be upon CK. This research seeks to add to the body of knowledge about the relationships and potential dependencies between CK and PCK (specifically, knowledge of student thinking), and how these two can develop within a specially structured methods course.

There is extensive research on students' understandings regarding measurement concepts such as area and perimeter, and the results have consistently shown that large percentages of students struggle with the most fundamental skills and concepts (Hiebert, 1981; Kenney & Kouba, 1997; Kouba et al., 1988; Lindquist & Kouba, 1989; Martin & Strutchens, 2000). Not only are many students not learning the skills necessary to solve even the most basic problems involving area and perimeter, but it appears they are also at the same time developing misconceptions regarding these ideas (Hiebert, 1984; Hirstein et al., 1978; Piaget, Inhelder, & Szeminka, 1981; Wilson & Rowland, 1993). Repeated exposures to procedural-oriented curricula materials and instructional strategies have not been able to address adequately the documented deficiencies regarding area and perimeter (Kamii & Clark, 1997; Martin & Strutchens, 2000). The fact that researchers have found teachers possess many of the same misconceptions regarding area and perimeter as students do is cause for alarm (Ball, 1988; Ferrer et al., 2001; Fuller, 1996; Lappan et al., 1998; Maher & Beattys, 1986; Ma, 1999; Menon, 1998; Reinke, 1997; Simon & Blume, 1994a; Tierney et al., 1986). Although non-traditional instructional strategies have been successful in remediation of student difficulties and developing a more conceptual understanding of area and perimeter (Casa, Spinelli, & Gavin, 2006; Johnson, 1986; Lappan et al., 1998; Moyer, 2001; Wilson & Rowland, 1993), very little research has been conducted to investigate ways to address the deficiencies preservice elementary teachers have shown towards these concepts. It would seem reasonable that if teachers possessed a more conceptual understanding of area and perimeter, they would be better able to compensate for a mediocre curriculum and more prepared to deal with student difficulties. Further research is needed to explore ways to intervene in and challenging preservice elementary teachers' knowledge related to the area and perimeter misconceptions identified by the literature. This research examined what preservice elementary teachers understand about area and perimeter (i.e., their content knowledge) and how they might approach student difficulties regarding these concepts (i.e., their knowledge of student thinking) – both before and after innovative intervention.

Integrating technology into the learning of mathematics has been shown to positively influence achievement, stimulate and enhance spatial visualization skills, and promote a more conceptual understanding of mathematics for students and teachers

(Boers-van Oosterum, 1990; Dunham & Thomas, 1994; Groves, 1994; Rojano, 1996; Sheets, 1993). To be ready to enter the technological classrooms of tomorrow, prospective teachers need content-specific instruction with the appropriate pedagogical support needed for these newly forming instructional environments (Cooper & Bull, 1997; Glenn, 2000; Kersaint & Thompson, 2002; Timmerman, 2004); however, it has become apparent that many prospective teachers do not possess the necessary knowledge or experience to meet these demands (MEET, 1999; OTA, 1995; Pellegrino & Altman, 1997; Thompson, 2000; Willis & Mehlinger, 1996). It is strongly recommended that appropriate technology integration be modeled for and experienced by prospective teachers (Connors, 1997; ISTE, 2000, 2008; MEET, 1999; NCTM, 2000; OTA, 1995; Timmerman, 2004), preferably within contexts that help simulate future classroom experiences (Clements, 1999; Heid, 1997; Thompson, 2000). One such instructional strategy that can accommodate the technology, content, and pedagogy needs of preservice teachers is anchored instruction. Anchored instruction with preservice teachers has been shown to promote reflective thinking (McIntyre & Pape, 1993), help with incorporating appropriate technology integration (Bauer, 1998), develop instructional units (Kariuki & Duran, 2004), and determine the significance of integrating technology into the teaching of mathematics (Kurz & Baterelo, 2004). The last study mentioned is the only one found examining the benefits of preservice teachers learning about and preparing to teach mathematics through anchored instruction. This is certainly an area ripe for further study. This research provided valuable insights into the possibilities of web-based microworlds serving as a technology delivery medium for anchored instruction.

CHAPTER 3

METHODS

Prospective mathematics teachers learn about pedagogical content knowledge when their instructors model activities, introduce tools such as manipulatives and technology, and discuss literature about how students learn certain mathematical concepts and about student misconceptions. (MSEB, 1996, p.6)

Introduction

This study uses quantitative and qualitative methods in an attempt to accomplish three goals: (a) to further understand preservice elementary teachers' (PST's) cognitions of area and perimeter and how they change and develop through intervention, (b) to examine the interplay between PSTs' content knowledge and their knowledge of student thinking, and (c) to examine the use of anchored instruction that integrates the use of web-based microworlds designed for exploring perimeter and area, as a potential learning environment for influencing PSTs' content knowledge and knowledge of student thinking. These goals are motivated by the need to address PSTs' mathematical deficiencies, specifically relating to area and perimeter. Although these goals are specific, they fall under an overarching purpose for preservice teachers, which is to develop contextual content knowledge and pedagogical content knowledge side by side while simulating future classroom scenarios and teacher-student exchanges. The teacher development experiment (TDE) provides a method for studying teacher development (Simon, 2000), and has shown to be a valuable approach for studying prospective elementary teachers' understandings regarding the area of a rectangular region (Simon & Blume, 1994a).

The theory (or models of learning) advanced by this study should not be viewed as static but rather as an "ever-developing entity" (Glaser & Strauss, 1975, p. 32), and as such open to ongoing modification by the researcher as well as other scholars. In as much, the data presented in this study were not designed to "prove" theory or present unquestionable relationships within the data. Rather the goals of this TDE were to appropriately illuminate concepts (Goodman, 1984), develop and describe models of interventions that promote mathematical growth (Simon, 2000), blur the line between theory and practice (Cobb, 2000), and provide a basis for further discussion and research.

Research Questions

The primary research question for this study is, *"In what ways do PSTs' content knowledge and pedagogical content knowledge, related to area and perimeter, change as a result of experiencing anchored instruction integrated with web-based microworlds, designed for investigation of area and perimeter?"*

In particular:

1. What is the PSTs' content knowledge regarding area and perimeter prior to involvement in the teaching episodes?

2. What is the PSTs' knowledge of student thinking regarding area and perimeter prior to involvement in the teaching episodes?
3. How does the PSTs' content knowledge regarding area and perimeter change, if at all, during the course of this study?
4. How does the PSTs' knowledge of student thinking regarding area and perimeter change, if at all, during the course of this study?
5. In what ways, if at all, is the PSTs' knowledge of student thinking regarding area and perimeter related to their content knowledge of those same concepts?

Setting

The context of this study was a mathematics methods course for elementary education majors at a small, liberal arts college in the southeastern United States. The study involved the use of an intact group of PSTs (*n* = 12). The PSTs were enrolled in a methods course that met twice a week for 75 minutes per class. To facilitate the technology component of this study, the class took place in a small computer lab. The lab was equipped with an instructor computer connected to a projector and to the Internet. Each student had their own computer, with Internet access, as well as ample desk space for working and note taking. The PSTs enrolled in this course were juniors and seniors who were working towards state certification as elementary school teachers of grades K-6. Typically, PSTs enrolled in this course will have completed their mathematics requirements (i.e., courses in College Algebra, Probability and Statistics, and Liberal Arts Mathematics). The small class size is in keeping with similar teaching experiments (Borasi, 1994; Leavy, 2006; McClain, 2003; Simon & Blume, 1994a, 1994b, 1996). The study occurred at the college where the researcher is a full-time mathematics professor, who has taught the elementary-level mathematics method course over nine times prior to conducting this study.

According to Simon (2000), it is appropriate to conduct a TDE within the distinct learning community of the PSTs. Because the setting is a small liberal arts college (student enrollment is approximately 600), the researcher typically knows the students who enroll in the only section of the mathematics methods course for elementary education majors, as he is the primary instructor for other required courses they take (e.g., College Algebra, Liberal Arts Mathematics, and Technology in Education). By the time students appear in the elementary mathematics methods course, the researcher/instructor is aware of many of their mathematical strengths and weaknesses.

Information obtained in the pre-study questionnaire and results from the pretest were factors in asking four preservice teachers to participate as case studies adapted for this study. The case subjects' selection was based on: (a) response patterns on their questionnaire, (b) the overall score and mathematical substance of their responses to similar items on the pretest, and (c) and the potential of those responses to facilitate future interviews and interventions, data mining, case study construction, and subsequent model building of mathematical knowledge.

Description of the Methods Course

The methods course in which this study occurred is required for all elementary education majors. The course is conducted from a constructivist learning perspective. Students are actively involved using manipulatives (both concrete and web-based) to assist in constructing understanding of mathematical concepts. They often work in small cooperative groups which encourages sharing and justifying of ideas. The course syllabus (Appendix B) presents the purpose of the course as follows:

> The purpose of this course is to provide opportunities for preservice teachers to examine and build upon their understandings of various mathematics topics, and to construct a vision of teaching and learning mathematics that considers the goals and the assumptions of the current reform movement in mathematics education. Content, methods, and materials for teaching elementary school mathematics will be examined cooperatively.

The preservice teachers are involved in a variety of activities. These include lectures, demonstrations, summarizing journal articles, preparing lesson plans, viewing, reflective writing, and discussing online videos of reform-based teaching episodes, mathematical error analysis of elementary students, question and answer sessions, and numerous problem-solving situations including discussion of applications for teaching.

The textbook used in the course is *Elementary and Middle School Mathematics: Teaching Mathematically, Sixth Edition* by John A Van de Walle (2007). Typically, the textbook is used as a guide while the following mathematical objectives and pedagogy are addressed: (a) develop understanding in mathematics, (b) teaching through problem-solving, (c) build assessment into instruction, (d) teach mathematics equitably to all children, (e) integrate technology and school mathematics, (f) extend early number concepts and number sense, (g) develop meaning for the operations, (h) support understanding of basic facts, (i) increase whole-number place value and whole number computation, (j) promote estimation skills, (k) concepts and computation with fractions, and (l) concepts of measurement. Concepts involving area and perimeter (the focus of this study) do not appear in the Van de Walle text until chapter 20. Although the author admits, "Area and perimeter (the distance around a region) are continually a source of confusion for students" (p. 386), the textbook only provides two brief activities and a total of one page of text to address area and perimeter misconceptions. The treatment that area and perimeter receive in the course text lends further credence as to why research is needed to help with devising instructional methods to integrate seamlessly and efficiently elementary mathematics content with the appropriate pedagogy – especially for methods courses already crowded with an abundance of topics to cover.

The Microworlds

Technology is one tool espoused by many to enhance the teaching and learning of mathematics (ISTE, 2000, 2008; Marzano, 1998; MEET, 1999; NCTM, 2000; NRC, 2001). As mentioned earlier, geometric microworlds, specifically designed for the exploration of area and perimeter concepts, were utilized within the teaching episodes to facilitate and motivate deep and extended exploration of the concept(s) and misconception at hand. After considerable Internet searching, comparing, and experimenting (both personally and with students in my methods classes), two well-designed microworlds were selected for this study – *Shape Builder* and an Explore Learning *Gizmo*. The microworlds facilitated four specific instructional techniques established as "effective" by a meta-analysis conducted by Marzano (1998). One of these interactive microworlds (see Figure 7) was conceptualized and designed by *ExploreLearning,* and is located at: http://www.explorelearning.com/ (2010). The *ExploreLearning* microworld, called a "Gizmo" by the company, is actually an

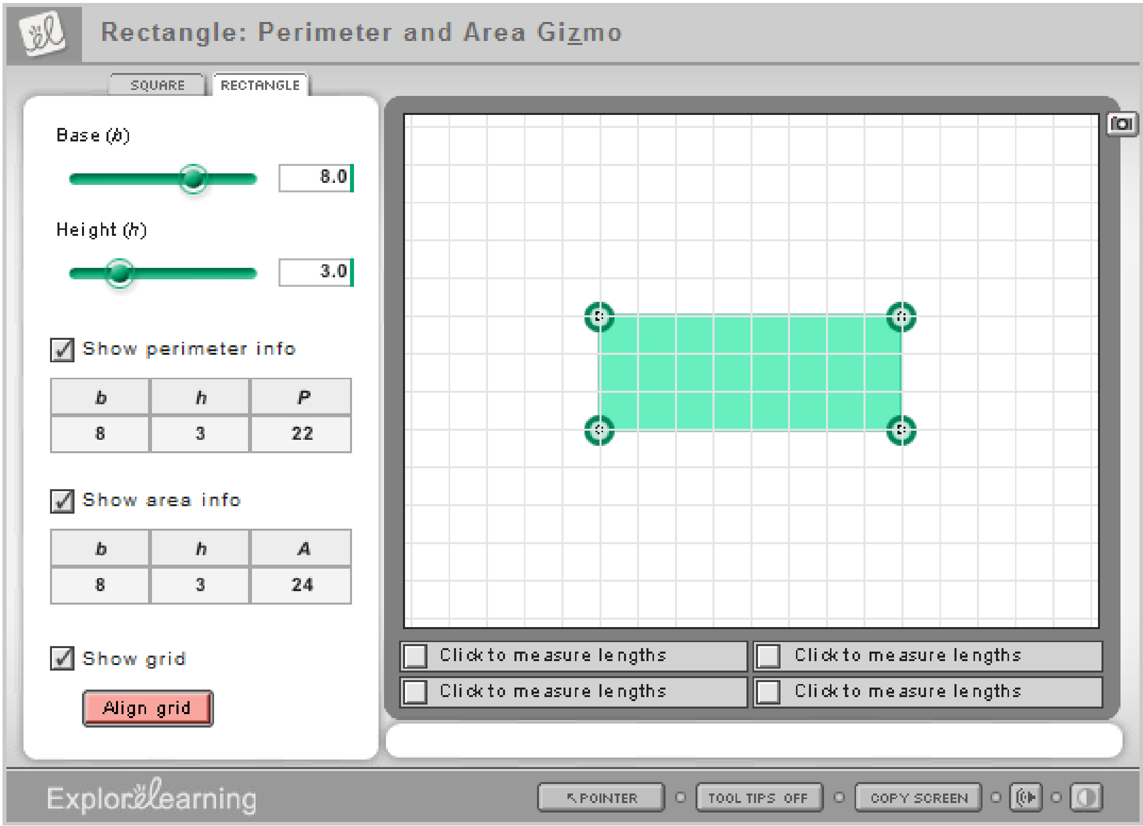


Figure 7. Screenshot of perimeter and area microworld with several options selected. (Copyright © 1999-2010 ExploreLearning. All rights reserved. Used by permission.)

interactive website that allows the user to "grab" the corner of either a square or rectangle (user selects), stretch or shrink it by moving the mouse, and then observe the resulting effect upon the shape's area and perimeter as revealed in tables. Dynamic and real-time feedback allows for the exploration of the misconception that increasing a shape's perimeter will always increase its area. The size of the square or rectangle can also be controlled by directly entering numbers (decimals allowed) for the base and height. Various options can be turned on or off to allow for feedback or for discovery exploration. The "Show grid" feature is a pedagogical tool to help visualize and connect the concepts of area and square units. The picture icon (upper left corner) allows the user to “Copy” the current square or rectangle exactly as pictured and “Paste” it into Word, or any word processor, as a picture.

The other microworld used in this study was developed through a cooperative effort and with the support of the Shodor Education Foundation, Inc. The researcher worked with a programmer to design a microworld that supports the exploration and hypothesis testing of issues related to content knowledge and knowledge of student thinking. The original applet, called *Shape Explorer*, can be seen in Figure 8. Shodor incorporated many of the features from the microworld used for this study into their newest version, called *Shape Builder*. It was released after this study was completed, and

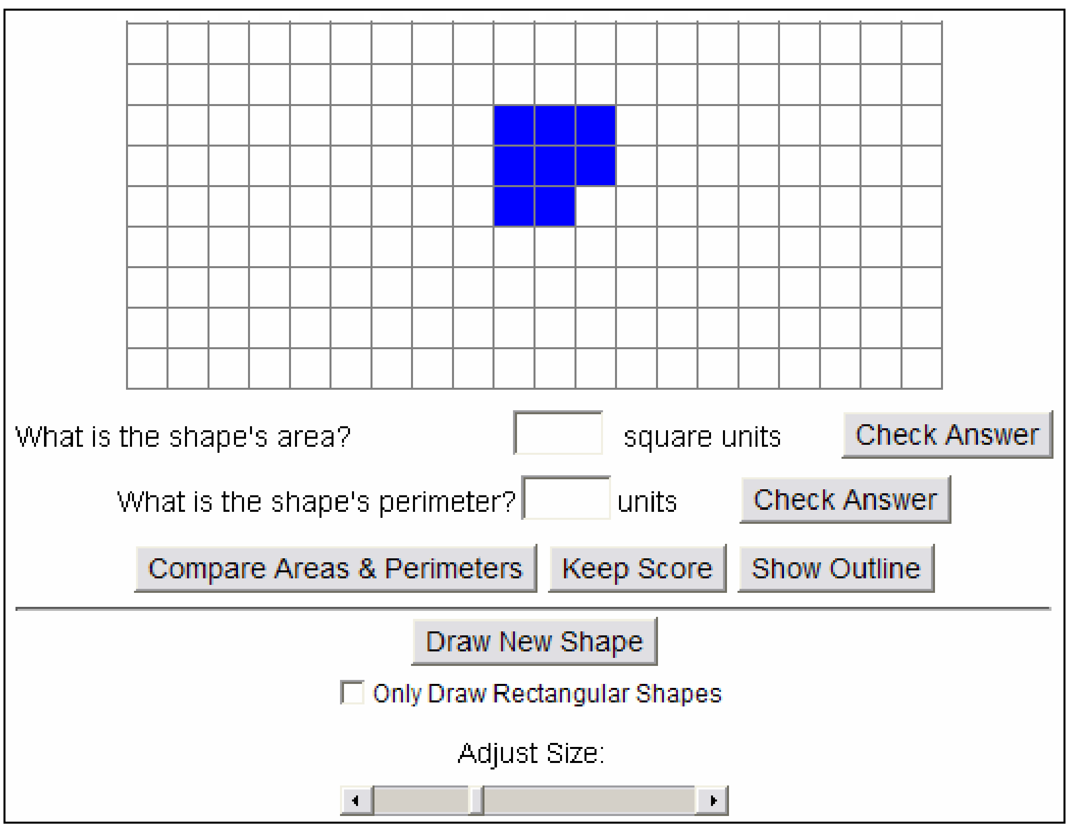


Figure 8. Screenshot from Shape Explorer microworld website. (Reprinted with permission from: http://www.shodor.org/interactivate/activities/ShapeExplorer/, copyright 1997-2010, The Shodor Education Foundation, Inc.)

is found at http://www.shodor.org/interactivate/activities/ShapeBuilder/ (2010). The redesigned microworld that was used in this study is shown in Figure 9. It is also called *Shape Builder*, and can be found at: http://www.shodor.org/~pjacobs/restored/ shapebuilder/. However, because of major Internet-platform upgrades at Shodor, that microworld is no longer supported. The microworld has two modes, *Auto Draw Shape* and *Create Shape*. When the radio button next to the "Auto Draw Shape" mode is selected, the microworld will automatically create random shapes – both irregular

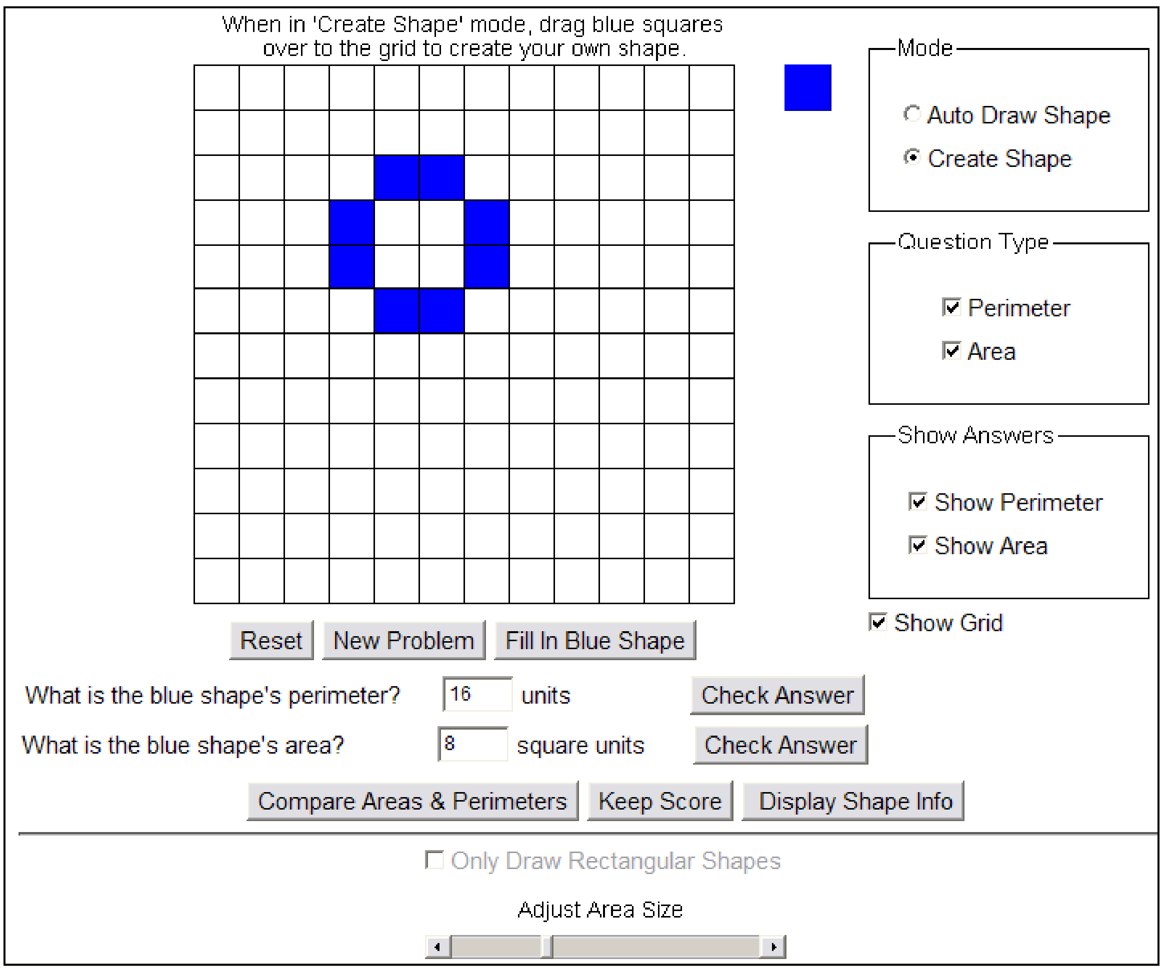


Figure 9. Screenshot from the revised Shape Builder microworld website. (Reprinted with permission from: http://www.shodor.org/~pjacobs/restored/shapebuilder/, copyright 1997-2010, The Shodor Education Foundation, Inc.)

(Figure 10) as well as rectangular (Figure11). The complexity of the shape is determined by how far to the right the slide bar under the “Adjust Area Size” is moved. The user may select to have the microworld ask for perimeter or area or both. Being able to make calculations involving irregular shapes is an option that helps address a major area and perimeter weakness among school students and teachers alike, as presented in chapter 2.

When in the “Create Shape” mode, the user may drag small, blue squares onto a grid, create shapes, enter a guess for the shape’s area or perimeter or both, click the “Check Answer” button, and receive immediate feedback regarding their response. The user can also have the microworld compute the area and perimeter of the shape in real time. In either mode, the microworld will let the user know if they have entered in the correct answer for perimeter and/or area, and after two wrong attempts the microworld will give the correct answer. The microworld tracks and can display the accuracy of correct and wrong responses by clicking the “Keep Score” button. It will also give an error message if the user attempts to create a disconnected shape (Figure 12). A pedagogical feature that was added at the request of the researcher is the “Fill in Blue Shape” button (see Figure 13). This option allows the user to create the outline of a shape (Figure 9), just as one could do with a manipulative such as color tiles, but then fill it in by pressing the “Fill in Blue Shape” button and watch the microworld change the calculation for the area but leave the perimeter the same (compare Figure 9 with Figure 12). Such a feature could help students in addressing the misconception that figures with the same perimeter must have the same area and vice versa. Another way in which the Shape Builder microworld can facilitate the development of conceptual knowledge is the “Compare Areas & Perimeters” feature. This feature keeps track of checked answers and

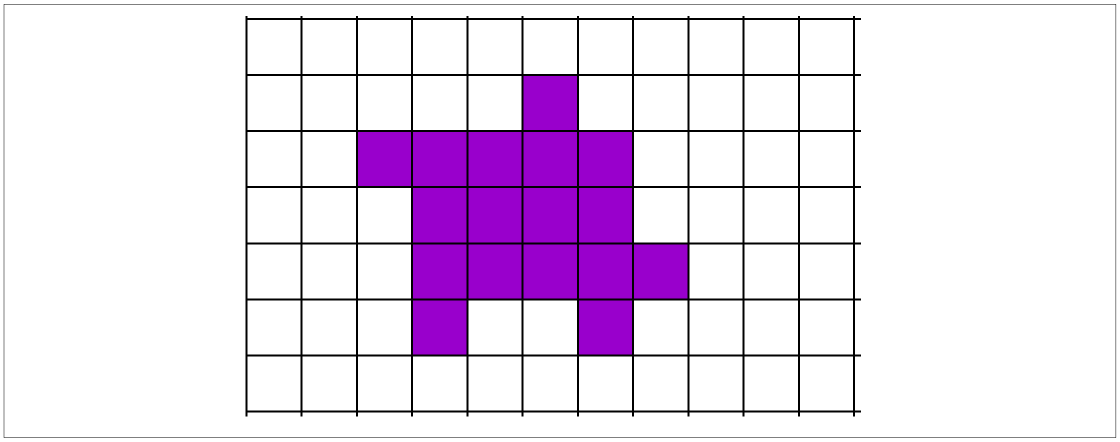


Figure 10. Shape Builder screenshot of shape automatically generated while the "Only Draw Rectangular Shapes" box is unchecked.

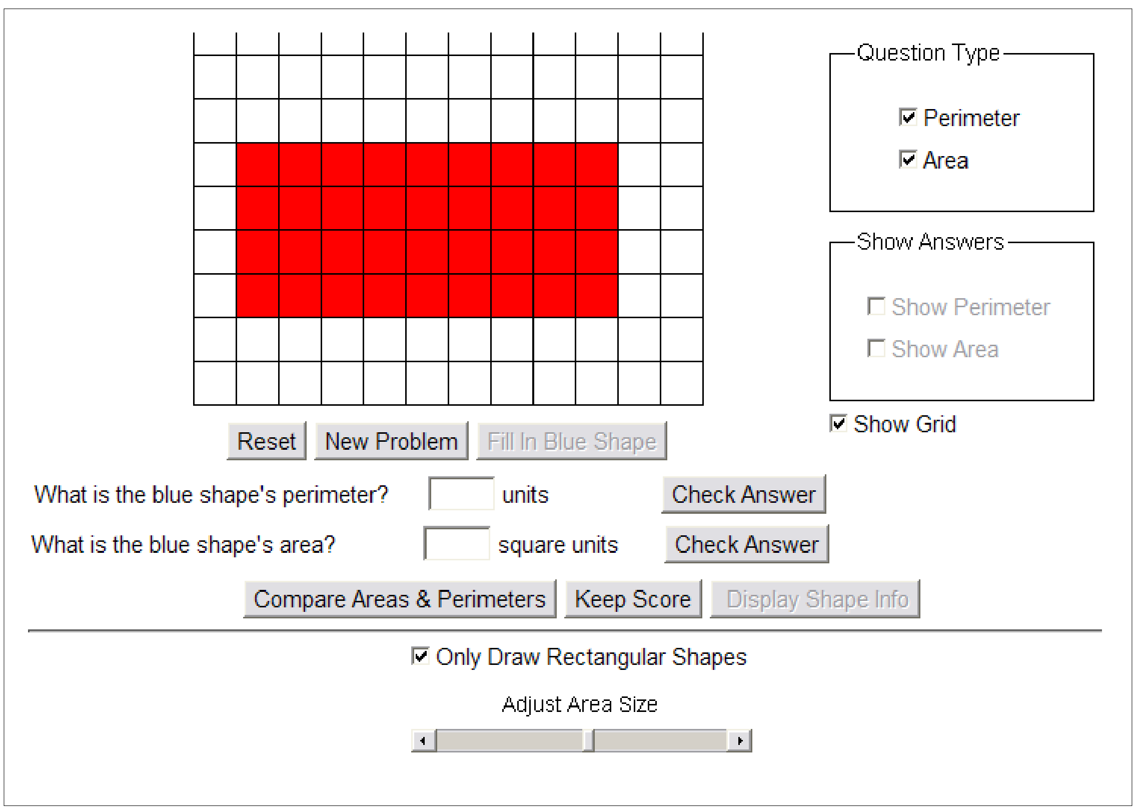


Figure 11. Shape Builder screenshot of a rectangular shape automatically generated by the microworld while in "Auto Draw Shape" mode.

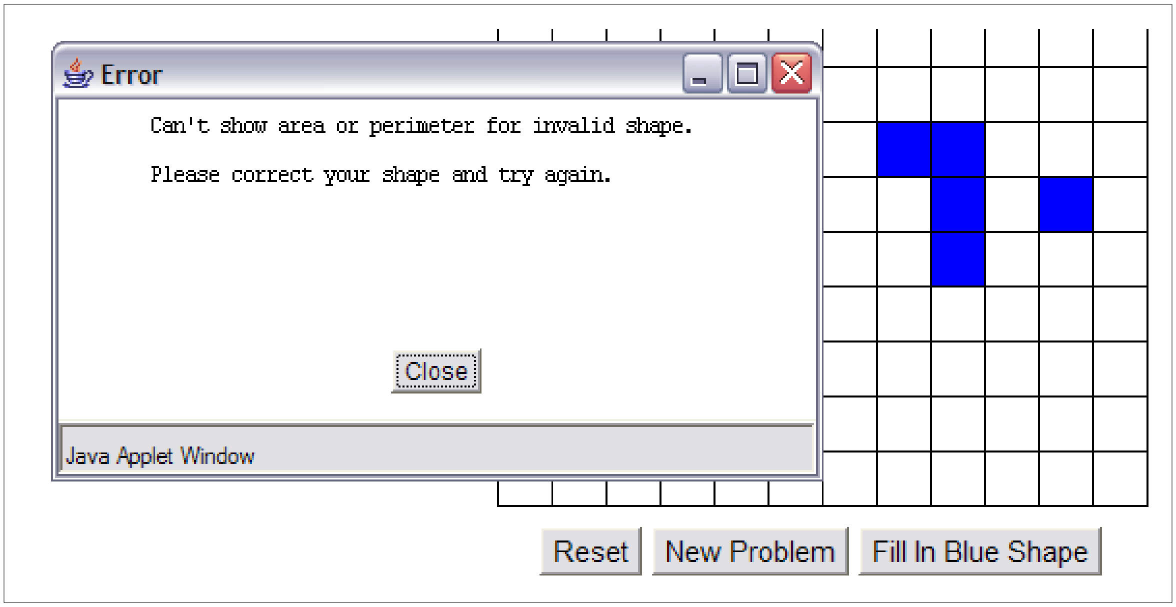


Figure 12. Screenshot from Shape Builder showing error message when an invalid shape is created.

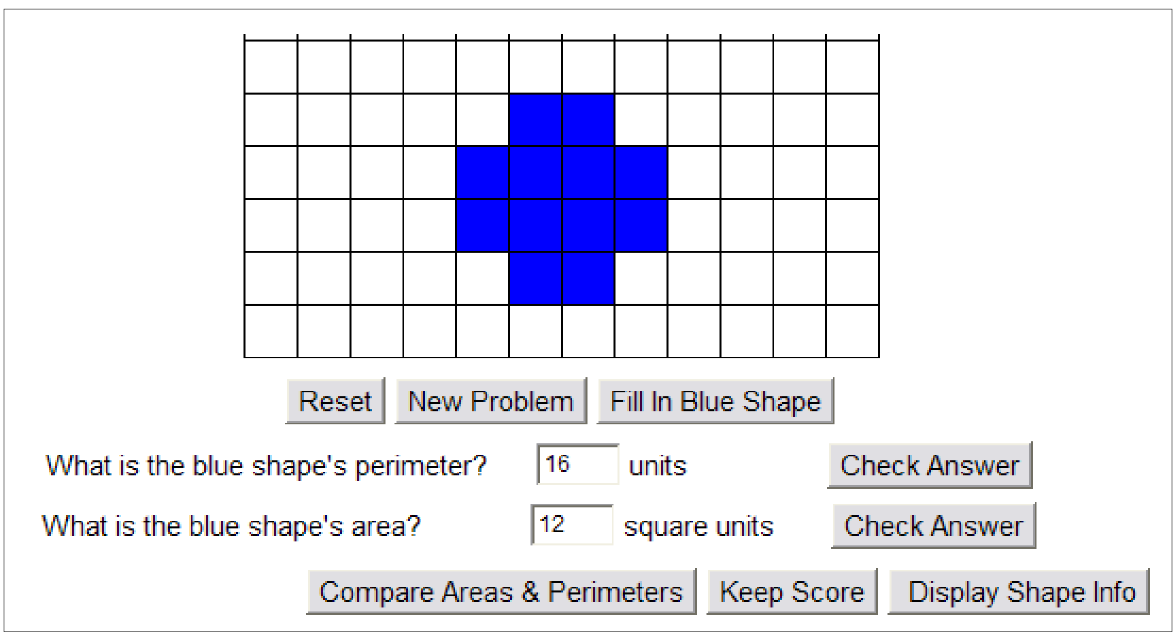


Figure 13. Screenshot from the Shape Builder microworld after the "Fill in Blue Shape" button was pressed with the shape shown in Figure 9.

allows the user to “Compare Areas & Perimeters” of various shapes. Such an option assists in dispelling the common misconceptions that increasing a shape’s perimeter will always increase its area and vice versa. Another feature that helps dispel misconceptions while users are exploring is “Display shape Info.” This feature keeps track of area and perimeter as users make changes to a shape while in “Create Shape” mode. Both microworlds allow for dynamic interaction and real-time feedback which are crucial to the implementation of anchored instruction and the development and enhancing of conceptual understanding of concepts related to area and perimeter. These microworlds possess the necessary options to facilitate the building of a conceptually sound content knowledge of area and perimeter as well as specific tools to allow for hypothesis testing to help address the difficulties and misconceptions regarding area and perimeter as discussed in the literature.

The Intervention

An important feature of a teaching experiment resides in the activities and situations used for the purpose of understanding the mathematical knowledge and growth of the PSTs (Cobb & Steffe, 1983; Simon, 2000; Steffe & D’Ambrosio; 1996). Both the PSTs (preservice teachers) and the instructor/researcher are involved in the active learning environment which is at the core of a teaching episode. In this study, the PSTs learned about elementary mathematics and how classroom students think about elementary mathematics, and the professor learned about the value of the planned teaching episodes in affecting the preservice teachers’ mathematical understandings, their knowledge of student thinking, and the value of these experiences to an already crowded elementary mathematics method's curriculum. The elements of the teaching episodes align with and reinforce many of the objectives of the methods course that include: interactive learning environments, cooperative group activities, round-table like discussions, exploratory learning, the blending of content and pedagogy, technology integration, and examples of theory meeting practice.

In lieu of a formal and complete pilot study, the researcher engaged in piloting the various instruments and interventions that were used in this teacher development experiment. Steffe and Thompson (2000) strongly recommend that:

> Any researcher who hasn't conducted a teaching experiment independently, but who wished to do so, should engage in exploratory teaching first. It is important that one become thoroughly acquainted, at an experiential level, with students' ways and means of operating in whatever domain of mathematical concepts and operations are of interest (p. 275).

Towards that end, various aspects of the proposed study were piloted beginning in the spring semester of 2004 and concluding the fall 2006 semester including: (a) the pre-study questionnaire, (b) the items and format of the area and perimeter pre-, post-, and follow-up tests, (c) the development and refinement of the scoring rubrics for the area and perimeter tests, (d) the framework and classroom testing of the teaching episodes, and (e) interview protocols. All the pilot work done for this study was conducted in various sections of the researcher's mathematics methods courses for elementary teachers. Details of the different piloting sessions are found in Appendix A. The major decisions resulting from piloting are presented within the appropriate section.

A similar version of the format used for the teaching episodes was piloted in the fall of 2006. The pilot informed the actual teaching episodes in the following ways: (a) There needed to be a separate orientation session (a few weeks before the formal study would begin) to acquaint the PSTs with the two microworlds used in this study; (b) there needed to be a clear transition within the teaching episodes between the PSTs thinking as learners of mathematics and as future teachers of mathematics; and (c) each teaching episode needed to comprise two class sessions – one for individual problem solving and opportunities to reflect upon their written responses and another for cooperative work, whole-class discussion, and subsequent reflection writings.

Analysis of PSTs' work from the piloted teaching episode revealed that most of them were currently at a novice stage in their application of both content knowledge and knowledge of student thinking. They spent minimal time analyzing the mathematics of the problem; hence, they initially overlooked mathematical subtleties of the problem – a valuable skill of experienced and effective teachers. For some PSTs the microworld did not seem to facilitate mathematical or pedagogical growth; however, others indicated signs of growth in both categories (see Appendix A).

Anchored Instruction

Anchored instruction was used to frame the teaching experiment and the subsequent teaching episodes. Anchored instruction is a research-based paradigm for learning through technology-assisted problem solving. It is a "model that emphasizes the creation of an anchor of focus [typically, technology-based] around which instruction can take place" (Bauer, Ellefsen, & Hall, 1994, p. 131). The instructional sequence actively involved preservice teachers in thinking about and planning for how best to address students' misconceptions regarding area and perimeter, such activity provided a motivating and authentic context. Although videodiscs have often been used to provide an environment to facilitate anchored instruction and problem solving within a meaningful context, interactive geometry microworlds, specifically designed for the mathematical content in this study, were used to provide the dynamic environment.

Within the anchored instruction framework, features of Wales and Stager's (1977) "Guided Design" were implemented to provide a model through which preservice teachers were observed, their work examined, and discussions and interviews conducted as they explored and wrestled with concepts individually and cooperatively with peers. The model includes: (a) introducing (verbally) an interesting problem and a general framework (which included a microworld) for solving the problem, (b) providing time for PSTs to generate and test their own strategies, (c) providing PSTs time to work with one or two other PSTs to develop a "group" consensus, and (d) sharing and comparing each group's solution to the strategies used and conclusions attained by an expert (the researcher and the research literature). The above processes are not meant to imply that transforming content knowledge into pedagogical content knowledge occurs in a set of fixed stages, phases, or steps. Instead, teacher education can only attempt to provide preservice teachers with the understanding, performance abilities, and a setting in which to develop the tools they will need to teach effectively.

The Teaching Episodes

The focus of the teaching episodes for this study were the common difficulties and misconceptions classroom students (and teachers alike) have regarding area and perimeter, and what effective intervention might involve. Too often the topics of area and perimeter are presented in isolation of each other (Chappell & Thompson, 1999; Hiebert

& Lefevre, 1986; Simon & Blume, 1994a). One aspect of this study investigated the anticipated merits of interweaving the exposure to both these concepts throughout the teaching episodes. With this said, three teaching episodes were constructed.

Each teaching episode began with a whole-class introduction designed to “set the stage” and motivate the situated learning by presenting the contextual problem that was the focus of that teaching episode. Time was taken at the outset to explain the format of the teaching episode (Wales & Stager’s Guided Design, 1977) and allow for questions to help clarify any directions. Because the concepts being explored (area and perimeter) are assumed to be previously learned, there was not any lecture or content–based, teacher-lead instruction prior to engaging the preservice teachers in individual problem solving. During the teaching episodes the preservice teachers first analyzed and attempted to solve the focus problem (see Figure 14) individually. After the individual work, the students were organized into groups of two or three and allowed time to share their thoughts about the problem and their problem-solving strategies, and then given time to reflect upon

Justin wants to calculate the perimeter of the shape shown in Figure 1. Justin’s method is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those squares.

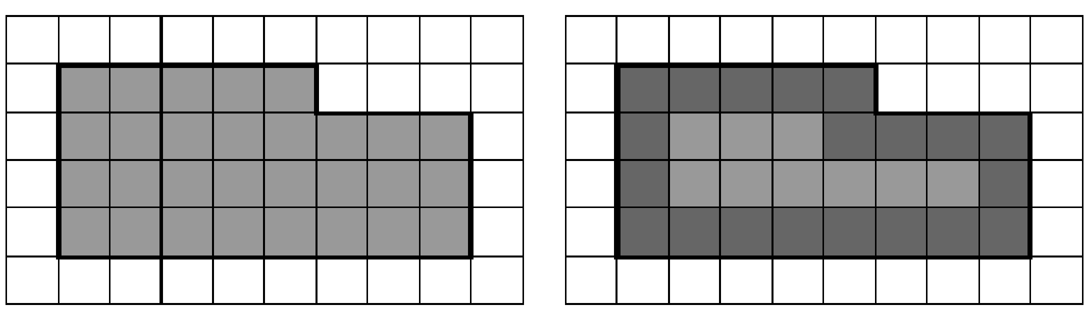


Fig. 1 Fig. 2

Figure 14. The focus problem appearing at beginning of teaching episode 1.

what they have heard and how it had influenced their understandings. Following the cooperative work time, the class came together and the instructor/researcher concluded the teaching episode with a whole-class discussion of the primary concepts and misconceptions addressed by the teaching episode and how the microworld could have been used to provide personal insight and enhance instruction.

Each teaching episode (see Appendix K) was broken up over two class periods. The first class session involved all the individual problem solving and reflection (both with and without the applet), and the second class focused on cooperative work, whole-class discussion, and periods of reflection about both activities. For the first two teaching episodes the microworlds were not made available until after the PSTs had worked on the focus problem for several minutes. Then they were given the next section of the packet and instructed to access the microworld to reevaluate and possibly refine their earlier responses. For the third teaching episode, the preservice teachers had access to the microworlds from the beginning. This was done to determine whether the PSTs considered the microworld (s) as a tool to aid them while problem-solving and when hypothetically interacting with students or viewed it as an add-on (i.e., something used after the majority of the problem-solving was done).

Each teaching episode was self-contained and presented to the PSTs in the form of a *Learning Packet* (see Appendix K). Each packet contained the following:

1. A problem addressing the primary concept(s) and misconception to be explored (see Figure 14),
2. Follow-up questions asking the PSTs about the correctness of the hypothetical student's response, to explain the student's thinking, and then how they would

follow up with the student,

3. Interspersed opportunities for the PST to reflect on their current progress and thinking – writing prompts provided (e.g., “What have you found confusing or difficult about the problem thus far.”),
4. A time to “Share & Compare” where the PSTs got into groups of two or three and discuss their thoughts and findings,
5. A writing time to express shared knowledge in relation to their own previous knowledge prior to the sharing, and
6. A cooperative summary and whole-class discussion of the salient points of the activity and provide the PSTs another opportunity to reflect and summarize how their mathematical understandings, knowledge of student thinking, and potential teaching strategies have changed as a result of the teaching episode.

Because the PSTs were asked to reflect about cognitive issues, as opposed to affective issues (e.g., beliefs), opportunities to reflect are incorporated directly into the context of the teaching episode, as opposed to being placed in a reflection journal and completed outside of class. The timing and placement appeared to help to capture moments of preservice teachers’ insights. The focus problems used in the teaching episodes were a mixture of testing items selected for the study and problems specifically modified to elicit mathematical discussion and contextual pedagogical reflection. In order to facilitate ongoing and retrospective analysis, as required in a teacher development experiment, the three teaching episodes were videotaped. The video tape was used by the researcher for ongoing analysis of the format and carrying out of each teaching episode as well as future analysis of instructor and PST involvement. Each teaching episode encompassed two 70 minute class periods, or one week of the semester.

Modifications to Teaching Episodes

Many of the modifications to the teaching episodes were changes to format. Retrospective analysis of TE 1 resulted in the addition and revision of certain writing prompts to elicit feedback to better establish patterns of novice and/or expert behavior. Near the beginning of teaching episodes 2 and 3 (i.e., “Day 1”) the writing prompt, “What are your initial thoughts regarding Tommy’s method?” (Figure 15) was added to establish a baseline for each PST’s knowledge regarding area and perimeter. It provided a venue to elicit reflective thought regarding PSTs’ initial ideas about the focus problem, without overwhelming them with the specific mathematics inherent to a microworld. Because some of focus problems (e.g., TE 2) could be solved or approached in different ways, a writing prompt similar to the following was added about half-way through

The Setting: Your 5th grade class is studying area, and you challenge them to find the area of one of their footprints. You instruct your students to stand on a piece of paper and trace their shoe, and then individually brainstorm a strategy to find the area of the footprint.

The Situation:

After several minutes one of your students, Tommy, comes up to you and explains his method. He says he would lay a piece of string around the outside of the paper footprint, cut the string to the precise length, form the piece of string into a rectangle, use a ruler to measure the length and width of the rectangle, then find the area of the rectangle. In other words, he believes that the area of the rectangle will be the same as the area of his footprint.”

Figure 15. Focus problem for teaching episode 2.

subsequent teaching episodes, "Other than Tommy's proposed method, what is another way to find the area of a footprint? Can you also think of yet another way to solve the problem?" Analysis of PSTs' responses to teaching episode 1 also revealed modifications were needed to certain writing prompts involving content knowledge (CK) and knowledge of student thinking (KoST.)

Revisions to CK & KoST Writing Prompts

To better discern and decipher PSTs' CK and KoST, two writing prompts were added to Day 1 of TE 2. The content knowledge prompt was, "What mathematical concepts or procedures could be involved with finding the area of a footprint?" Responses like, "I think estimation is involved to an extent because a footprint is not going to be just a standard ("nice") number" allowed for glimpses into PSTs' content knowledge and problem-solving ability. The knowledge of student thinking prompt, "What do you think students might find difficult about finding the area of their footprint? What specifically might be causing their confusion?" was revised for TE 3. Further analysis of teaching episode 1 revealed the PSTs were frequently giving cliché-type responses such as, "The student does not know area and perimeter." Instead, the goal was for the PSTs to reflect upon and consider the educational implications about such things as the curricula and presentation of topics (ideas we had discussed in the whole-class discussion at the end for TE 1), and to encourage them to reflect on personal experiences; thus, revealing more about their mathematical background or beliefs about how students might best learn area and perimeter. To facilitate such reflection, this KoST prompt was rewritten in TE 3 to read, "Do you think many students may have the same incomplete understanding as Jasmine [figure 17]? If so, what do you think may be the cause? When answering, consider the student's mathematical knowledge as well as possible instructional techniques commonly used." Resulting responses such as, "I think the cause could be that too many 'regular' shaped figures are used in the textbooks," and "Maybe students would benefit if area and perimeter were taught together" seemed to justify the change. While these responses provide opinions, they reveal that PSTs were beginning to consider various factors that can influence students' content knowledge and possible instructional techniques to address them.

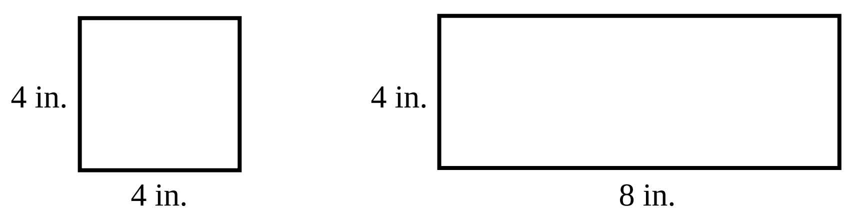
Two important modifications to Day 1 for TE 3 were enacted after analyses of teaching episodes 1 and 2 revealed that when a PST did not fully comprehend the mathematics surrounding or the student's thinking involved with the focus problem, they typically responded "I don't really know," or "I am still unsure about this problem," which provided little insight into their thinking. Therefore, for TE 3 the question "If you are unsure, are you skeptical or do you tend to believe it? Why?" was added to the original writing prompt, "Is Jasmine's 'theory' correct? If no, why not?" This addition increased the amount of content knowledge that could be gleaned from PSTs' responses. The second change was to a prompt addressing KoST. The prompt originally read, "As a teacher, how would you respond to Jasmine's thinking and her proposed theory? What specifically would you say and do?" The phrase, "(even if you are unsure about the mathematics involved)" was added to the end (Figure 16). PSTs who had previously answered, "I don't know" to such prompts would now at least state that they either agreed or disagreed with the student in the problem and occasionally elaborate beyond that. That phrase seemed to allow for more freedom to reflect and hypothesize about how they would respond to future students' thinking.

The Setting:

You have just completed the last scheduled unit on area and perimeter with your 5th grade class. You feel they understand the concepts pretty well. While the students are working at their desks on that day's mathematics homework, one of your students, Jasmine, comes up to you very excited.

The Situation:

Jasmine then tells you that she has figured out a "new theory" that you never told the class about. She explains that she has discovered that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area. She shows you this picture as proof of what she is saying:



perimeter = 16 in.
area = 16 square in.

perimeter = 24 in.
area = 32 square in.

Figure 16. Focus problem for the third teaching episode.

Revisions to Cooperative Work.

The PSTs were asked to work in cooperative groups of three during the second day of each teaching episode. They were then supposed to succinctly share with the groups members their thoughts and ideas about the questions presented in Day 1. There were three "Shared Knowledge" sections – one pertaining to the questions addressing CK, one for KoST and instructional implications, and another for what was learned, and how, by interacting with the microworlds. While each PST took turns sharing, the other group members were to compare what they were hearing with their own understandings about the topic or concept being discussed and to write down what "new" knowledge they had gained. This self-reflective exercise was meant to see if the PSTs could identify their own lack of knowledge and integrate the new knowledge in a meaningful way. After analyzing the responses in these "Shared Knowledge" sections, it became evident that the PSTs were focused on generating lists of factoids as they were given by their group members. Because these sections were designed to organize new knowledge into preexisting schemas of personal knowledge, each of the first three prompts from teaching episode 1 were rewritten to better focus on a specific knowledge type and emphasize the reflective nature of the exercise. For example, the writing prompt from teaching episode 1 that was supposed to address KoST originally read, "What new knowledge did you gain from your group regarding questions 8 & 10?" was rewritten as, "What new knowledge did you gain from your group regarding *student thinking* (see questions __ & __) and *instructional practices* (see questions __ & __)." These changes focused PST's attention on the specific knowledge types in question, however the aspect of personally incorporating what was being heard into their existing knowledge was greatly lacking. PSTs provided comments like, "Sara [a PST] originally solved the problem incorrectly, just like the student did." The word "you" in each prompt was capitalized, "YOU," to remind the PSTs that a personal self-reflection was expected. An examination of the responses to the revised prompts in teaching episode 3 revealed only a slight increase in the quality of responses. While there were a few more meaningful responses along the lines of, "I learned from ______ a different way to disprove Jasmine's theory" [Figure 17], there were still too many shallow comments like, "Make sure Jasmine explains her idea to you." It was assumed that many PSTs may have difficulties reflecting on personal knowledge as well as processing and integrating new knowledge.

One specific prompt - "What new knowledge did you gain [as a result of sharing with your group] regarding the use of the two microworlds (see questions 7, 8, & 11)? Be sure and specify what microworld you are referring to." - that was revised after TE 1 and again after TE 2 still did not produce insightful responses. The purpose of this prompt was to, in part, help evaluate the effectiveness of the microworlds as a tool within the TDE. Instead, the majority of the responses included lists of likes and dislikes or general comments about how the microworld could be used to show Tommy he was wrong. In hindsight, the prompt should have been reworded to get at the idea of how best to use the microworlds with future students to help them uncover and resolve potential misconceptions related to area and perimeter.

Instrumentation

Instruments used in this study are described in this section. For each instrument a brief synopsis of their design, format, and implementation as well as how the pilot study influenced its use are provided.

Pre-Study Survey Questionnaire

The questionnaire (Appendix C) consisted of 23 questions: five multiple choice, thirteen multiple choice followed by a request for further details, and four short-answer constructed-response items. The purpose of the questionnaire was to gather background information about PSTs': (a) extent of exposure to concepts related to area and perimeter, (b) use of concrete manipulatives to learn about area and perimeter, (c) knowledge or use of various forms of technology (specifically computer software or Internet) to assist in the learning or teaching of area and perimeter, (d) confidence regarding their future teaching of area and perimeter, (e) confidence and willingness to use technology while teaching about area and perimeter, and (f) pedagogical choices regarding teaching the fundamental properties of area and perimeter.

Results from piloting this instrument suggest that it was necessary to separate survey items referring to area and perimeter into two different questions and to add more survey items related to previous exposure to technology. In addition the format needed to be standardized (e.g., inclusion of Yes/No boxes) to ensure accurate and uniform completion. The last two survey items were added to address specifically the respondents' present knowledge of student thinking. The categories of information listed above were helpful in establishing baseline measures of the PSTs' content and pedagogical content knowledge of area and perimeter.

Area and Perimeter Tests

The tests used for pre-, post-, and follow-up consisted of 10 constructed-response items (see Appendices D, E, and F, respectively). Before the pretest was administered, each PST was assigned a number (1-12). Each test contained a cover page which had a space for the PSTs' name, classification, and gender. Different colored paper was used for each five-question test. After each test was administered, the PSTs' number was written on the cover page and at the top of the first page of their test. The cover page was removed and filed so the PST's identity was protected during the scoring and analysis process. The content knowledge (CK) questions (i.e., the first five) were administered and completed prior to the five questions designed to reveal each PST' knowledge of student thinking (KoST). This process helped to minimize the content-knowledge questions biasing the knowledge-of-student-thinking questions.

The sources for the potential testing items included: a searchable database of released items from previous administrations of the National Assessment of Educational Progress (National Center for Educational Statistics [NAEP], 2003; 2005), teacher resources dealing with measurement, and an extensive evaluation of research articles. The items selected for this study along with their respective source(s) appear in Table 1. The goal was to select problems appropriate for pre- or inservice elementary teachers that also addressed the prominent difficulties and misconceptions regarding area and perimeter revealed in the literature, namely:

1. Trouble distinguishing between area and perimeter (Carpenter et al., 1975; Chapel & Thompson, 1999; Hart, 1883; Hiebert, 1981; Kouba et al., 1988; Tierney et al., 1900; Woodward & Byrd, 1983),
2. Confusing linear units and square units (CBMS, 2001; Hart, 1984; Hiebert, 1981; Lappan et al., 1998; Moyer, 2001),
3. The idea that all rectangles of a given area must have the same perimeter and vice versa (Lappan, 1998; Woodward & Byrd, 1983),
4. Wrongly believing that area and perimeter are directly related in that one determines or influences the other (Ferrer et al., 2001; Kennedy et al., 1993; Lappan, 1998; Ma, 1999),
5. Trouble devising real-world contexts for area and perimeter problems (Chappell & Thomspon, 1998),

Table 1

Description of Test Questions Selected for this Study

Pretest[1]	Category	Concept(s) addressed	Source
Item #1	Content knowledge	Perimeter & units	Kenney & Kouba, 1997; Chappell & Thompson, 1999
Item #2[S]	Content knowledge	Area	Chappell & Thompson, 1999
Item #3	Content knowledge	Area, perimeter, & units	Hart, 1984
Item #4[S]	Content knowledge	Linear & square units	Sonnabend, 2004
Item #5	Content knowledge	Area & perimeter	Bassarear, 2005
Item #6	Knowledge of student thinking	Area & units	Sonnabend, 2004; Bassarear, 2005
Item #7	Knowledge of student thinking	Perimeter & units	Bush, 2000
Item #8	Knowledge of student thinking	Perimeter	Bassarear, 2005
Item #9	Knowledge of student thinking	Perimeter & units	Beckmann, 2003
Item #10	Knowledge of student thinking	Area, perimeter, & units	Woodward & Byrd, 1983

Note. [1]Items for the Follow-up Test were structured exactly the same (other than changing the names in the problems) as the Pretest. [S]Item also appears on the Posttest.

Table 1 (Continued)

Description of Test Questions Selected for the Study

Posttest	Category	Concept(s) addressed	Source
Item #1	Content knowledge	Area	Hart, 1984
Item #2[S]	Content knowledge	Area	Chappell & Thompson, 1999
Item #3	Content knowledge	Area, perimeter, & units	Sonnabend, 2004
Item #4[S]	Content knowledge	Linear & square units	Sonnabend, 2004
Item #5	Content knowledge	Area, perimeter, & units	Sullivan & Lilburn, 2002
Item #6	Knowledge of student thinking	Area & perimeter	Bassarear, 2005
Item #7	Knowledge of student thinking	Perimeter & units	Chappell & Thompson, 1999
Item #8	Knowledge of student thinking	Area & perimeter	Menon, 1998
Item #9	Knowledge of student thinking	Area & units	Hart, 1984
Item #10	Knowledge of student thinking	Area & perimeter	Bassarear, 2005

Note: [S]Item also appears on the Posttest. The rest of the posttest items are parallel to the pretest - statistically, in format, and in content.

6. Trouble calculating area and perimeter of irregular shapes (Booker et al., 1986; Bray et al., 2006; Carpenter et al., 1975; Cass et al., 2006; Kouba, 1988), and
7. Difficulties explaining and/or illustrating the methods for their solutions (Ball, 1988; Chappell & Thompson, 1999; Woodward & Byrd, 1983).

Analyses of the pilot data revealed that these seven difficulties could be condensed into three broad analysis strands that would serve as an organizing framework for test responses: (a) distinguishing between area and perimeter, (b) units of measure, and (c) perceived relationships between area and perimeter. All three of these strands address, to different degrees, aspects of content knowledge and knowledge of student thinking. To be considered for use in piloting sessions and for final inclusion within the assessment instruments, each question needed to meet the following criteria:

1. The problem was appropriate for pre- and inservice elementary teachers.
2. The problem addressed some form of the common difficulties or misconceptions regarding area and perimeter presented in the literature.
3. The problem was already formatted as a constructed response item or could be easily modified to fit that format.
4. The problem was already written in the context of a teacher addressing a student or students experiencing difficulties with area and perimeter or could easily be modified to accommodate that perspective.
5. The problem lent itself to the PST explaining their solution process and/or the thinking of the hypothetical student presented in the item, and facilitated an opportunity for the PST to respond how they follow up with hypothetical student or students.

6. No manipulatives or technologies were required to solve the problem.

The area and perimeter assessment administered as part of the pilot study contained 15 problems. To provide more time for PSTs to respond and to encourage thoughtful reflection, tests used in this study were shortened to 10 items. The items currently found on the pre-, post-, and follow-up tests for this proposed study were chosen because they: (a) were interesting and challenging enough to produce rich and diverse written responses, (b) were deemed best suited by the researcher to meet the goals of this study, and (c) met necessary guidelines based on descriptive statistics (i.e., mean scores, standard deviation, corrected item-total correlation, and various Cronbach alpha values). The potential to illicit a range of thoughtful responses was very important in the item-selection process because of the nature of the qualitative analysis that followed. The reader is referred to the last section of Appendix A for more details regarding the refinement of these testing instruments.

Validity of Testing Instruments

Test validity refers to the extent to which an instrument measures what it intends to measure. Specifically, it refers to “the appropriateness, meaningfulness, and usefulness of specific inferences made from test scores” (American Psychological Association, 1985, p. 8). This definition highlights the fact that test scores by themselves are neither inherently valid nor invalid. It is the inferences that are made from the test scores that must be established as either valid or invalid (Gall et al., 1996). Evidence then must be provided to support any inferences about scores resulting from administering a test. Three types of evidence are commonly examined to support the validity of an assessment instrument: (a) content-related, (b) construct-related, and (c) criterion-related (American Educational Research Association, American Psychological Association & National Council on Measurement in Education, 1999).

There are two main considerations for establishing content-related evidence for a test. First, attention must be paid to ensure a student's response to a given assessment instrument reflects that student's knowledge of the content area that is of interest (Moskal & Leydens, 2000). To ensure that this criterion is met, the researcher, aided by a second scorer, revised the instruments to clarify and minimize confusions related to language and choice of words used in the item that might interfere with the instrument's ability to measure a PST's knowledge about area and perimeter. Secondly, content-related evidence is also concerned with the extent to which the items on a test represent the conceptual domain that it is designed to measure (Gall et al., 1996). Evidence for content validity is established because the questions used for the pre-, post, and follow-up tests were all drawn from extant literature pertaining to the teaching and/or learning of area and perimeter (see Table 1, p. 141).

Criterion-related evidence supports the extent to which performance on a given task may be generalized to other, more relevant activities (Rafilson, 1991). The items used for the testing instruments in this study are based on research literature investigating various degrees and types of knowledge possessed by students, PSTs, and teachers. The two selected for this study, content knowledge and knowledge of student teaching, are considered indispensable to a meaningful learning and effective teaching of mathematical concepts such as area and perimeter (Ball, 1991, 2003; Ball & Bass, 2000; Hill et al., 2004; Shulman, 1986). The scoring rubrics used to assess the tests also exhibit criterion-related validity because the scoring criteria address the components of the assessments activity (the tests) that are directly related to future practices within the teaching profession (i.e., the need for content knowledge and knowledge of student thinking) (Moskal & Leydens, 2000).

Construct-related evidence focuses on the extent to which a test can be shown to assess the particular hypothetical construct(s) that it claims to measure (Gall et al., 1996). Two constructs this study attempts to measure are content knowledge and knowledge of student thinking, as pertaining to area and perimeter. Such constructs are internal and not directly observable. It is important therefore that any assessment attempting to measure such a construct considers, requests, and then examines both the product (i.e., the answer) as well as the process (i.e., the explanation) (Moskal & Leydens, 2000). The tests used in this study did just that. Although the PSTs were asked to answer several closed-ended questions (e.g., "Is this student right or wrong?" or "What is the area of this shape?"), such questions were followed up by asking for an explanation of their thinking or for what they feel the student in the question was thinking. The holistic scoring rubrics used to grade the tests contain criteria that address both the product and the process of the testing items. No single item of evidence is sufficient to establish construct validity (Gall et al., 1996); therefore, the quantitative and qualitative results from the testing instruments served as supporting evidence (along with other qualitative data) to help explain the degree and type (procedural vs. conceptual) of mathematical and pedagogical growth among this study's PSTs.

Procedures

In order to answer the research questions, data were collected regarding the PSTs' developing understandings related to content knowledge and knowledge of student thinking regarding concepts of area and perimeter. Some data were collected from the entire class while other information (e.g., semi-structured interviews) were unique to the case subjects. Anchored instruction involving teaching episodes situated around students' misconceptions regarding area and perimeter supported the TDE methodology for this study. The Guided Design model (Wales & Stager, 1977), integrated with Marzano's (1998) instructional recommendations, provided sustained opportunities to gather data necessary to answer the study's research questions. When using an emergent methodology, such as this teacher development experiment did, these sustained opportunities of contact with the PSTs are important to generate multiple data sources. When data sources are triangulated to reveal a pattern of theme, there is greater confidence and trustworthiness that the apparent theme is not the coincidental result of a particular form of data (Simon, 2000; Tobin, 2000).

Data Collection

The mixed-methods approach generated both quantitative (e.g., pre-study questionnaire, and area and perimeter tests) and qualitative data (e.g., interviews, Teaching Episodes packets). All the data were gathered within the researcher's Methods of Teaching Elementary Mathematics course occurring in the fall semester, 2007. The PSTs were the 12 preservice elementary teachers who signed up for the class. The course lasted for 15 weeks, and students are only allowed two absences during the course.

The study lasted five weeks and involved approximately ten classroom contact hours as described below:

Week 1: Dispensed and collected the pre-study questionnaire.

Week 2: Administered pretest; based on questionnaire and informal results of pretest, four PSTs were purposely selected for in-depth study as particular cases.

Week 3: Results from the pretest were used to inform semi-structured interviews with the four selected for case study.

Week 5: Conducted “Microworld Orientation” designed to allow PSTs time in class for directed use of the two microworlds that were integrated into the teaching episodes as part of the anchored instruction.

Week 7: Conducted the first teaching episode.

Week 8: Conducted the second teaching episode.

Week 10: Conducted the third teaching episode.

Week 11: Administered posttest; results from posttest were used to inform semi-structured interviews with the four case-study subjects.

Weeks 12 & 13: Conducted second round of semi-structured interviews

Week 15: Administered unannounced follow-up test as part of in-class final exam.

It is common for larger and more extensive teaching experiments to last an entire semester (Leavy, 2006; Simon & Blume, 1994, 1996); however, such studies often investigate broad constructs (e.g., Statistical inquiry – Leavy; Multiplicative relationships & justification – Simon & Blume). Although this study represents a brief intervention, it is in keeping with other similar teaching experiments which studied specific mathematical content (Borasi, 1994; Komerek & Duit; 2004; McClain, 2003).

Whole-Group Data

Pre-Study Questionnaire

The pre-study questionnaire was administered during class time to all the PSTs. Students were instructed to answer each question to the best of their memory and to be as specific as possible (i.e., provide personal situations or supportive examples) when asked for opinions regarding technologies as well as when responding to hypothetical pedagogical questions. All students were present when the questionnaire was administered.

Microworlds' Orientation Session

Before the study began, class time was used to orient the PSTs regarding the two microworlds that were used in this study. One problem was selected for each microworld that highlighted the important features of that microworld (see Appendix M). The researcher modeled the various features of each microworld without specifically discussing the pedagogical benefits of certain features. The PSTs were then given an opportunity to use each microworld while engaged in solving the two chosen problems. Neither of these problems was used in any part of the actual study, and they did not involve any of the misconceptions under scrutiny in this study. One student was absent for the orientation and a time was scheduled the same week for her to work through the orientation in my office while I supervised. The PSTs' responses were analyzed for evidence of novice and/or expert teacher characteristics.

The second observer was present during the orientation session, and the session was video taped. Shortly after the orientation session, the researcher and the second observer meet, discussed the session, compared notes, and agreed that nothing occurred during the orientation session that would bias any aspect of the study. The second observer was the current Dean of Academic Affairs at the institution where the researcher was employed full time. She holds a Ph.D. in Instruction and Curriculum and has vast experience with the elementary curriculum and preservice teachers. The second observer and I met once over the summer, and had several email correspondences, to discuss various aspects of this study, especially methodology, as well as her role as second observer. The observer protocol (Appendix L) and the format of the teaching episodes were discussed.

Administering Area and Perimeter Tests

The pre-, post-, and follow-up tests were taken by all PSTs and were administered during class time. Only one test was not taken as scheduled (a follow-up test), and that was made up under supervision. Each test was comprised of five content knowledge (CK) questions and five questions pertaining to the PSTs' knowledge of student thinking (KoST). Before responding to any items, each PST was given the first half of the test (i.e., the content knowledge questions) and asked to complete its cover page. The PSTs were asked to turn to the first page of the test and the researcher read aloud the instructions. A brief description of the two categories of questions (i.e., CK and KoST) was presented and the PSTs were informed that they would be functioning first as a student/learner and then as a prospective teacher and to think, analyze, and respond accordingly. The PSTs were encouraged to ask questions regarding the format of the test or what was being asked of them. There were no significant questions or discussion that ensued. The instructor/researcher was available during the exams to address questions related to test or item format, but no mathematical assistance was given. The pilot study revealed that one hour would be sufficient to complete each testing session. The PSTs were encouraged to complete the first half of the test (content knowledge) in approximately 25 minutes. When they finished the first half, it was collected and the second half of the test (knowledge of student thinking) was provided for which 35 minutes was scheduled. The one hour proved sufficient for most; however, because the computer lab where we were conducting class was available for the period that directly followed our methods course, a few students needed and took 5-10 minutes to finish their test. Testing times are provided in Chapter 4. PSTs were instructed to raise their hand when they completed each portion of the test so the researcher could document stop-time. The PSTs were instructed that after finishing the entire test, they were to sit quietly and wait (most read a book) until the end of class time. Each PST’s start and stop times for each portion of each test was documented on a spreadsheet. This information was used during the analysis stage. The above process was completed for the pre-, post-, and follow-up tests.

Data from Teaching Episodes

Both the instructor/researcher and the second observer kept field notes during each teaching episode. The instructor/researcher documented pertinent observations of and conversations with PSTs (especially the case subjects, described later) that occurred during the teaching episodes. Special effort was made to document whether the behavior or conversation was focused on mathematical content (i.e., area and perimeter) or aspects of pedagogical content knowledge (specifically, knowledge of student thinking). The second observer had an observer’s protocol sheet (Appendix L) that helped to focus and organize her observation activity. Debriefing time was scheduled for the researcher and observer following each teaching episode.

While engaged in each teaching episode, every PST completed a *Learning Packet* (Appendix K). They were asked to provide written responses to questions and prompts pertaining to aspects of mathematical content knowledge related to area and perimeter and their knowledge of student thinking regarding contextual situations involving those same concepts, reflective activities throughout the episode focusing on current and evolving understanding, perceived and realized benefits of exploring concepts with the microworlds, and how the cooperative work influenced their mathematical and pedagogical understandings.

PSTs' Roles

This study matches the multi-level focus encouraged by and provided for the TDE. There were two levels of participants in this study, the researcher/teacher educator, and the preservice teachers. There were also two levels of curricula being explored: the teacher education curricula and the students' mathematics curricula. This study implemented a unique instructional approach for learning about area and perimeter concepts. It addressed concerns and recommendations of the research literature for both teacher education and the teaching and learning of elementary mathematics. Specifics about the teaching episodes will be presented later in this chapter. Not only did the researcher function in a dual role during this study, but so did the PSTs. Preservice teachers enrolled in a mathematics education course are simultaneously learners and teachers in transition (Bowers & Doerr, 2001). As learners, they have opportunities to investigate and construct new thoughts about seemly familiar mathematics and about ways that others might learn the same concepts. As teachers in transition, they are contemplating how their learning experiences and understandings in mathematics will relate to and prepare them for future experiences as teachers in their own classrooms. This dual role served as a backdrop for rich and meaningful explorations into the development of the PSTs' CK and KoST.

Case-Subjects: Selection and Data Collection Process

Four PSTs, two scoring at or near the bottom on the pretest and two scoring at or near the top were identified as case subjects for in-depth examinations. The quality of their responses on the pretest, as opposed to some predetermined score, was of primary consideration. This purposeful sampling was designed to facilitate "information-rich cases" (Patton, 2002, p. 46), whose in-depth study as particular cases assisted in providing readers with an insider's perspective. Typically, a holistic case study collects, analyzes, and reports upon social and affective components of the environment or setting being investigated. Although the researcher admits it is practically impossible to study mathematical learning in a vacuum apart from these variables, they were not a primary focus in the collection, analysis, or reporting stages of this study. Certain data collection procedures were unique to the case subjects; there were two, semi-structured interviews, and their behavior was a primary focus of observation, intervention, and interaction during the teaching episodes.[3] The interview data served an important role in the pattern matching for test scoring as well as expert/novice coding.

All interviews were videotaped and the audio was transcribed. The video camera was focused on the portion of the desk where the case subject was working. That allowed for capturing the case subject's moments of reflection and problem-solving activity. The

[3] *Intervention* is used to denote action by the researcher designed to further a preservice teachers' learning. *Interaction* refers to communication (usually two-way) between the researcher and the preservice teacher.

video proved valuable during instances when the researcher pointed or made reference to a case subject's drawing or work. Two of the four baseline interviews were double-coded with the expert/novice coding sheets by the same secondary scorer mentioned earlier. This process of pattern matching is a useful validity tool (Gall et al., 1996; Yin, 1994), and helped ensure reliable coding of patterns and identification of possible themes. Before interview transcripts were finalized, the videotapes were watched in entirety to allow for additional comments to be inserted providing any necessary context (e.g., "At this time, the preservice teacher pointed to the 2×7 rectangle she had drawn."). When necessary, the appropriate videotape was consulted during the coding process; thus providing an additional quality-check to help validate analysis.

The first semi-structured interview with each case subject was conducted within ten days following the pretest and before the first teaching episode (which began approximately one month after the pretest). All four first interviews were completed within two and a half weeks following the pretest. To reduce the likelihood that PSTs' memory failures would impact the results of the interviews, the PSTs were shown their own work while answering interview prompts. For the first interview, responses from the questionnaire and pretest served as a basis for interview protocols. Questions and probes were designed to clarify responses from those instruments and help gain an understanding of the subject's current content knowledge and knowledge of student thinking as related to area and perimeter. Probes consisted of statements such as: "I want to show you your response to question ___." "Would you please tell me what you were thinking about when you wrote this?", "What do you mean?", "Can you give me an example?", "Why do you think a student would say that?", or "How would you respond to a student who had such a misunderstanding?" Clarifying questions drove the interview protocol, but there were also times where unstructured (or unplanned) follow-up questions proved necessary. While piloting interview protocols with PSTs, the need for such a semi-structured approach was reinforced. On two different occasions an interviewee was asked to explain what exactly the perimeter of a shape is. Responses included, "It is the area of the outside," and "the area around the figure." These statements elicited further probing where it was determined that one respondent actually did understand perimeter but simply misspoke, but the other preservice teacher was truly confused and lacked a conceptual understanding of the measure. Purposeful questions were avoided during the first interview as they could result in a teaching situation and as such potentially bias the interviewee's posttest score. Before the second interview and during the three teaching episodes, the instructor/researcher observed, interacted with (in more of a clarifying manner), and took field notes of meaningful activities, taking special note of the investigative processes, hypotheses tested, and reasons offered for various insights and interpretations of the four case PSTs.

The second interview involved the same four case subjects and occurred after the posttest and during weeks 12 and 13 of the semester. This interview included direct, follow-up contact with the case subjects. The initial protocol consisted of clarifying questions based on posttest responses, but also included some purposeful questions (e.g., "What do you think students would find difficult about learning . . .", and "What would you say or do to help them understand?") were included. Two purposely-selected tasks (see Appendix N) were also integrated to further assist with collection of data measuring growth, or lack thereof, of content and pedagogical content knowledge. Observing the preservice teachers analyze, problem solve, and respond to real-time questioning regarding a previously unseen problem added valuable information to each subject's case record. There were no significant clarifications needed for any interview episodes before the follow-up test was administered. Each of the first and second interviews were approximately 45 minutes to an hour in duration.

Case subject data were also collected during the teaching episodes. All teaching episodes were videotaped, and both the researcher and second observer kept field notes to document significant individual and group behaviors, responses to classmates, and responses to researcher interventions. The researcher looked for opportunities to interact with all PSTs – especially the case subjects. These opportunities were used as an attempt to document what might not have been captured in the learning packet or on video tape, or to clarify observed behavior. In other words, case subjects were often asked, "What are you thinking?" or "Why did you do that?" while they were solving the problems presented in the teaching episodes.

Data Analysis

The emergent and unpredictable nature of a teacher development experiment requires a flexible analysis scheme. The analysis method use in this TDE was adopted from a grounded theory approach and its constant comparative method of analysis (Glaser & Strauss, 1975). The TDE involves two important levels of data analysis: the ongoing analysis, which occurred during the teaching episodes with the preservice teachers and between the teaching episodes as a personal reflection activity, and the retrospective analysis, which focused on the entire TDE or a subset of those data considered to be a useful unit of analysis (Simon, 2000). Simon explains how the ongoing analysis is the basis for spontaneous and planned interventions with the preservice teachers; these interactions helped gather additional information, test hypotheses, and promote further mathematical and pedagogical development. A key aspect of ongoing analysis is the iterative process of generating and modifying models of student development. For this study, that involved models of the PSTs' content knowledge and knowledge of student thinking, how they develop and how they may interact.

The retrospective analysis, according to Simon (2000), involves a reexamination of a larger body of data. This could be the entire TDE to date or a subset of those data (e.g., a baseline and follow-up interview with a case subject) that is considered to be a useful unit of analysis. This analysis involves a careful structured review of all the relevant data of the TDE for the purpose of continuing to develop and refine explanatory models of the preservice teachers' mathematical and pedagogical development.

Simon conveys that the development of explanatory models of preservice teachers' mathematical and pedagogical development is a hallmark of the TDE. These descriptive and illuminating models begin to appear and take shape during the ongoing analysis; however, it is during the retrospective analysis that the models begin to stabilize and can be articulated more fully. The TDE methodology, supported by anchored instruction and the Guided Design model, directed and informed the ongoing interventions and interactions between the PSTs and the researcher; thus, providing continued opportunity to collect data and refine hypotheses regarding individual and group development pertaining to content knowledge and knowledge of student thinking, and to permit finding answers to the five research questions of this study.

Scoring Rubrics for Area and Perimeter Tests

The overall scheme and initial criteria used for both the content knowledge and knowledge of student thinking holistic scoring rubrics were directly adopted from Cai, Lane, and Jakabcsin (1996), and informed and influenced by Thompson and Senk (1998) and to a lesser extent by a “focused holistic scoring point scale” (Randall, Lester, & O’daffer, 1987). Research conducted by Hill, Schilling, and Ball (2004) supports the decision to use separate zero to four-point scale rubrics for measuring content knowledge and knowledge of student thinking (see Appendix H). The reader should keep in mind that the language used in the scoring rubrics to describe a PST’s quality of response (e.g., “inferior” or “model”) is intended for a context involving preservice teachers. As a result of the scoring-training process and many pilot sessions, tables were created to delineate succinctly each item’s major concept(s) and potential misconception (see Appendix I) and to help differentiate a response emphasizing procedures from one focusing on understanding as well as responses teetering between scores. As reflected in the rubrics, a key distinguishing scoring factor is the presence and degree of conceptual understanding, versus procedural, in the PST’s response. The dividing line between unacceptable, acceptable, and model responses rests in that construct.

Reliability of the Data

The reliability of test scores refers to the consistency, stability, and precision of test scores (Gall, 1996). On a reliable test a student would expect to receive the same score regardless of when the student completed the test, when it was scored, or who scores it (Moskal & Leydens, 2000). There are four general classes of reliability estimates: (a) internal consistency reliability, (b) test-retest reliability, (c) parallel-forms reliability, and (d) inter-rater reliability (Gall et al., 1996). The following four sections will present the extent to which this study addresses each of these reliability measures.

Internal Consistency Reliability

This form of test-score reliability is used to judge the consistency of results across items on the same test. Essentially, you are comparing test items that measure the same construct (e.g., area or perimeter) to determine if they yield similar results. When a test taker answers similar questions in similar ways, that is an indication that the test has internal consistency. Cronbach's coefficient alpha is one method used to measure internal consistency when items are not scored dichotomously (e.g., right or wrong) but rather given a range of scores. Because the items used for the tests in this study were scored on a scale of zero to four, Cronbach's alpha is an appropriate measure of reliability for this study's test items. The Cronbach's alpha for the three pilot sessions were .82, .73, and .63, respectively, meeting the criteria for internal consistent reliability (Nunnally, 1978). The third Cronbach's alpha is low because four of the ten items on the test had negative corrected item-total correlation. None of those problems appeared on any future tests in this study.

For the actual study, it is necessary to discuss not only Cronbach's alpha for the entire pre-, post-, and follow-up test, but also for the CK and KoST subtests. Recall that each 10-question test was split into a five-question CK subtest and a five-question KoST subtest. Table 2 reveals three low Cronbach's alphas (.37, .48, and .54) that warrant explanation. There are two important factors that can negatively influence reliability: a limited number of items or limited variability in the scores of those items. In this circumstance, both factors are present and result in less than desirable Cronbach's alpha for certain parts of each test (see Table 2). The limited number of items in each subtest (n = 5) is one potential culprit for the low alpha coefficients; however, after careful analysis, it was found that each subtest possessing a low Cronbach's alpha also contained a test item having limited variability in its scores. For example, item 10 on the pretest (same item was problematic on the follow-up test) proved to be the easiest question of any item on any tests (mean of 2.75, SD of only 0.45). What made this item even more troubling to reliability was the fact that PSTs who scored low on various other test items scored equally well on question #10 as those who scored well on those same items. That same situation was present for the other subtests with the low Cronbach's alpha. Although the complete cause of the low Cronbach's alpha is not entirely known, a partial explanation includes the limited number of items and a small number of problematic test questions. The overall Cronbach's alpha for the pre-, post-, and follow-up tests were strong (.75, .75, and .76, respectively) indicating that the testing instruments produced a majority of scores that had an acceptable level of internal consistency (Nunnally, 1978). Caution however must be taken when drawing conclusions with measures derived from the three subtests with the low Cronbach's alpha.

Table 2

Cronbach's Alpha for Pre-, Post-, and Follow-up Tests

	Cronbach's Alpha		
	CK subtest	KoST subtest	Overall
Pretest	.75	.<u>37</u>	.747
Posttest	.<u>48</u>	.66	.752
Follow-up	.64	.<u>54</u>	.761

Inter-Rater Reliability: Training and Scoring

Whenever human beings are involved in a measurement process, careful consideration must be made to establish the reliability and consistency in the scoring of the items on an assessment. In an effort to measure the extent to which the researcher consistently and reliably applies the scoring rubrics to the testing instruments, 27 (out of an available 81) area and perimeter tests (each containing 15 items) were double-scored and used for training purposes. Before any scoring was done by the second scorer, a lengthy training session was conducted. The second scorer holds a Ph. D. in Curriculum and Instruction with a concentration in mathematics education and has considerable experience with elementary mathematics content and pedagogy. The second scorer double-scored 5 of the 12 pretests (or roughly 30%) and 4 of the 12 posttests.

As part of this effort, the results from the inter-rater reliability process resulted in clarifications made to the language of the holistic scoring rubrics, the addition of supplemental grading sheets (see Appendix I), and improvements in item format and wording – including the elimination of several items. These revised rubrics were used to score all subsequent test papers, and high scoring reliability was achieved throughout. The training and scoring sessions for the first batch of 27 tests had an inter-rater reliability of 94%. The second and third scoring sessions had a slight drop in inter-rater reliability, 88% and 86%. These two subsequent scoring sessions involved only four 10-item tests, which may help to explain the slight drop in inter-rater reliability. Also, the test used for the third pilot contained four problems which had negative corrected item-total correlation. These problems were removed from consideration for this study.

Rubric Scoring and Coding Training

Before the pretests were scored, the researcher purposely selected two pilot test papers, which reflected a wide range of responses, to be used for a training session. The primary purpose of this session was to reacquaint the scorers with both the scoring rubrics (Appendix H) and the supplemental grading sheets (Appendix I). Discussion occurred after each test was independently scored. Among other things, this allowed the researcher to clarify the phrase “limited insight” as they appeared on the KoST scoring rubric. This training session took place approximately a week and a half before the training for pretest scoring was scheduled to occur.

There were two training sessions that preceded the formal double-scoring of 5, ten-question pretests. After perusing all the pretests, the researcher purposely selected two pretests (one that appeared strong and another that appeared weak) that appeared to provide a wide range of response patterns. The researcher and the second scorer independently scored the same training paper. There was agreement on nine out of ten items for the first training test. The one disagreement was on question #4, which appeared as the same numbered question for the pre-, post-, and follow-up tests. It proved to be one of the most difficult problems both to answer and to score. The second training pretest was handled in the same manner. Subsequent discussion of that test’s scoring resulted in more clearly defining a score of “1” as possessing “no clear conceptual understanding” of the problem, its underlying misconceptions, or of the student’s thinking portrayed in the item. For example, one of the test items asked the PSTs to “Present a real-world situation (or story problem), appropriate for 4th or 5th graders, in which they would need to find the area of a specific region.” One PST’s response was, “We need to find the area of a fence we are going to build for our pet turtles. Two sides of the fence will be 12 inches. The other two sides will be 8 inches. It will look like this: (a rectangle was drawn and all four sides were appropriately labeled). What is the area of this yard?” One scorer gave this response a 2 and the other gave it a 1. During the discussion, each scorer could be convinced (based on the rubric) to change their score. After further examination, it was decided that the response was conceptually incomplete and very weak (e.g., her comment, “area of a fence”). The fact that all four sides of the rectangle were labeled also left us wondering if the PST was actually thinking about perimeter instead of area. The lack of conceptual understanding provided a meaningful dividing line between a score of 1 and a score of 2. It was agreed this item should be scored a 1. To help reduce similar confusion on future tests, this problem was revised to include the statement, “Provide the solution to your problem.” Following this clarification, there was agreement on all ten of the scores awarded. This was an important clarification that helped in distinguishing whether an item deserved a score of 1 or a score of 2. There were no other significant changes to the scoring rubrics, the supplemental grading sheets, or the manner in which they were applied as a result of the training sessions.

The 5 pretest papers that were formally double-scored were purposely selected based on an informal examination of the quality and depth of responses (both strong and weak). The goal was to provide scoring opportunities that would span a potential range of scores across a diversity of knowledge and understanding. Before any scoring was done, the researcher and second scorer agreed to grade the same problem for each test before moving on to the next problem. Two tests were double-scored and the results discussed before scoring the other three pretests. The final pretest that was double scored included scores ranging from a 1 to a 4. In spite of that, there was 80% agreement on the scoring of the items. The double scoring of these five pretests produced no clarifications to the scoring rubrics or the scoring process. The end result was an inter-rater reliability of 94%.

There is an interesting side-note regarding the scoring of the pretests. One of the pretests that the researcher scored received a very low score (in the bottom 25%). Since this PST was also one of the case subjects, extra measures were taken to establish reliable baseline knowledge; therefore, the second scorer was asked to double score the test. Although the test was scored well after the double-scoring session had concluded (two months for the researcher and four months for the second scorer), there was 100% initial agreement on the scoring of the 10 items.

The double-scoring training of the posttest proceeded in similar fashion as the pretests. The first training test was purposely selected based on the PST's pretest score, which was in the middle of the distribution, and the fact that the responses appeared substantial enough to potentially elicit a range of scores. Because the researcher also served as the instructor for the course, there was a potential that my expectations as the instructor might influence how I scored the test items. To limit this bias, I made a conscience effort to focus on the scoring rubrics and the supplemental grading sheets during the scoring process and not take into account my experiences as the instructor.

The first training test was scored independently and the results were discussed. Initial agreement was only 50%, although disagreement never differed by more than one number. It was discovered that the second scorer was relying too heavily on the supplemental grading sheet, as opposed to focusing on the rubric and grading the responses holistically. After correcting that, two more tests were purposely selected based on pretests scores (one high and one low). The weaker tests had scores ranging from a 1 to an almost 4, and the better test had scores ranging from 2 up to 4. Initial agreement for each test was 80%, with no scores differing by more than one. Strong agreement on these varying responses provided evidence for the reliability of the scoring process.

The posttests of the four case subjects were purposely selected for the formal round of double scoring. There were two reasons for this. First, the pretest results corroborated that the case subjects, as anticipated, comprised two weak and two strong students – relative to the rest of the class, therefore providing, theoretically, a wide range of responses to score. Secondly, since a significant portion of analysis would be based upon the posttest scores of the case subjects, an extra level of reliability of their scores was warranted. It was decided that all four tests would be independently scored and that the same item for each test would be scored consecutively and that the order of the tests would be changed after each item, to avoid a specific test setting an unintentional standard against which the other tests might be measured. For the first three posttests scored there was an 80% initial agreement rate and a 90% agreement on the fourth. The inter-rater reliability for the four posttests scored was 94%. The high level of agreement, and the consistency in scoring differences, gives the researcher confidence that the scoring process yields a reliable measure of the PSTs' CK and KoST in relation to area and perimeter.

Expert/Novice Coding: Development, Training, and Usage

The Expert/Novice Coding Sheets (Table 3) were used to examine the PSTs' content knowledge and their knowledge of student thinking. They were used to identify evidence of expert and novice language. The coding sheets are based on extant literature

Table 3

Coding Sheets to Help Categorize Novice versus Expert Preservice-Teacher Behavior, within the Context of this Study

	Novice	Expert	Source
Knowledge Structures	(1a) Sparse, lacking, vague and/or disconnected (fragile[a])	(1b) Substantial amounts; richly interconnected and hierarchical	Dufresne, Leonard, & Grace, (nd)
	(17a)[b] Contradict own response (written and/or verbal)	(refer to 1b)	(emerged during the study)
	(2a) Exhibit little knowledge of misconceptions or concepts most difficult for students	(2b) Possesses an awareness of common student errors and misconceptions	Livingston & Borko (1990)
	(14a)[b] Tendency to over generalize	(14b)[b] Realizes limitations to generalizing	(emerged during the study)
	(15a)[b] Incorrect mathematical computations and/or procedures	(15b)[b] Correct, precise, & conceptually strong mathematical procedures & work	(emerged during the study)
Problem Solving	(3a) Typically consider only one way of solving a problem	(3b) Often able to find more than one way to solve a problem	Dufresne, Leonard, & Grace, (nd)
	(4a) Tend to skip the analysis stage when problem solving	(4b) Carefully analyze a problem <u>before</u> and/or while solving it	LaFrance (1989); Chi, Glaser, & Farr (1988)
	(5a) Are slower and prone to making errors	(5b) Perform faster than novices at domain-specific skills - usually with less errors	Chi, Glaser, & Farr (1988)
	(6a) Respond to superficial features of a problem	(6b) Initially try categorizing a problem and apply appropriate mathematical principles	LaFrance (1989); Niemi (1997); & Chi, Glaser, & Farr (1988)

Note. [a]Specifically refers to a changing/vacillating response. [b]Identified category that emerged during the study.

Table 3 (Cont.)

Coding Sheets to Help Categorize Novice versus Expert Preservice-Teacher Behavior, within the Context of this Study

	Novice	Expert	Source
Representations	(7a) Poorly formed and/or unrelated representations	(7b) Able to generate contextual and even multiple representations	Dufresne, Leonard, & Grace, (nd); Livingston & Borko, 1990
	(7a-)[b] Neglect to use representations		(emerged during the study)
Justification	(8a) Are often unable to explain why their answers are correct	(8b) Can explain why their answers are correct	Dufresne, Leonard, & Grace, (nd)
Instructional Strategies	(9a) Primarily procedural in content and application	(9b) Presents clear & complete conceptual explanations	Ball & Wilson, (1990); Leinhardt & Smith (1985); Fuller, (1996)
	(10a) Tend to focus on the content	(10b) Primary focus is the student	Livingston & Borko, 1990
	(11a) Primary concern is performance and getting right answers	(11b) Focuses on developing conceptual understanding	Livingston & Borko, 1990
	(12a) Fail to incorporate learning tools, such as manipulatives, where appropriate	(12b) When appropriate, incorporates learning tools, such as manipulatives	Eisenhart et al., 1993
	(13a) Fail to incorporate technology, when appropriate, to promote a focus on understanding	(13b) When appropriate, incorporates technology to promote understanding of content and processes	Mitchell & Williams, (1993); Marzano, (1998)
	(16a)[b] Present incorrect, incomplete, or inadequate explanations	(refer to 9b)	(emerged during the study)

Note. [b]Identified category that emerged during the study.

that addresses behaviors of pre- and inservice teachers that had been categorized as either novice or expert. “Behavior” is taken to mean written communication (e.g., pre-, post-, and follow-up tests and the Teaching Episodes), and verbal interaction (e.g., interview transcripts or comments made during the Teaching Episodes). The coding sheets are by no means all-inclusive. For example, several expert-novice categories presented in the literature dealt with classroom teachers interacting with their students (e.g., Experts are more apt to correct student performance while novices tend to correct student behavior (Mitchell & Williams, 1993), and would not be compatible with this study. The categories that were chosen were considered to be most appropriate for the context, instruments, and PSTs (i.e., preservice teachers) of this study. There is no significance associated with the numbering of the codes.

The coding sheets provide structure while analyzing various forms of data (e.g., pre-, post-, and follow-up tests) for evidence of the PSTs’ current-knowledge levels as well as to determine any growth that might have occurred as a result of the various interventions (e.g., Teaching Episodes and semi-structured interviews). The numbering sequence (e.g., 1*a* and 1*b*) was used during the coding process and reference to these codes will occur while reporting findings. Certain codes aligned very well with aspects of both CK and KoST, and helped to quantify and qualify the amount and type of respective knowledge present at different times throughout the study. For example, codes involving knowledge structure (e.g., 1*a*/1*b*) and explanatory framework (e.g., 8*a*/8*b*, 15*a*/15*b*, and 16*a*/9*b*) help to explain PSTs’ CK. Codes that described the PSTs’ understanding of children’s thinking (e.g., 2*a*/2*b*) and their ability to address shortcomings and misconceptions (e.g., 7*a*/7*a*-/7*b*, 12*a*/12*b*, and 13*a*/13*b*) were used to clarify PSTs’ levels of KoST.

The coding sheets were used to categorize the PSTs' responses and identify response patterns that emerged into new codes. New codes identified within the sub-category of "Knowledge Structures" include: (a) contradicts own response (written and/or verbal), (b) tendency to over generalize, and (c) incorrect mathematical computations and/or procedures. A new code that emerged within the "Representation" sub-category was "neglected to use representations," and a new code within the "Instructional Strategies" was "presents incorrect, incomplete, or inadequate explanations."

To balance out the holistic nature of the scoring rubrics and provide a broader representation of each PST's knowledge, the Expert/Novice coding sheets were applied in a more analytic nature. When scoring the pre-, post-, and follow up tests, "model" responses were not often found. Something as minor as leaving off the appropriate unit was grounds for assigning a score of 3 (acceptable) as opposed to a 4 (model); thus, some very good responses were assigned a 3. The Expert/Novice coding was completed on a more part-by-part basis. Each test question contained multiple parts, and thus the opportunity to assign multiple codes to the same question existed. For example, within one question a PST might perform one calculation correctly (thus earning a code of 15*b*) but another incorrectly (thus a 15*a*). In that same question, an explanation for one part might be completely procedural (thus earning a 9*a*) while a conceptual explanation might be provided in another part of the same question (thus a code of 9*b* would be assigned). In addition, a single question might contain two incorrect computations or two separate procedurally-based explanations. In such instances, the same code was applied multiple times (e.g., two 15*a*'s or two 9*a*'s).

To establish reliability for the coding process, the researcher and second scorer (the same one who double scored the pre- and posttest) completed an extensive training program similar to what was done for the rubric-scoring training. Two pretests were purposely selected to provide a range of responses to code. It should be noted that the researcher selected to double code all four of the case subjects' pretests and two of their pretest interviews. That provided a broad range of responses as well as added reliability to the baseline analysis of the case subjects' knowledge.

The training sessions helped the researcher to refine the coding instruments. The following changes were made. For example, 4 new codes were added to the coding sheets: (a) 14*a* - a novice tendency to over-generalize solution strategies, (b) 14*b* – the expert understands and recognizes the limitations to generalizing, and (c) 15*a* – while the literature discussed procedurally-based, vague, disconnected, and conceptually weak aspects of the novice's knowledge structures, there was found no category specifically mentioning that the novice often displays an incorrect understanding of mathematical content (although it does seem obvious), and 15*b* – the expert displays a thorough conceptual understanding of mathematical content. Other codes were revised to support an item-by-item coding, rather than a generalized comment related to teaching tendencies. For example, code 12*a* originally read, "Less likely to incorporate learning tools such as manipulatives." To better fit the coding process, it was revised to read, "Fails to incorporate learning tools, such as manipulatives, when appropriate." Two observations were made during the first training session: (a) certain codes (especially 4*a*, 4*b*, 5*a*, 5*b*, 10*a*, & 10*b*) might not be applicable to both the written tests and interview transcripts, and (b) there where instances where a response contained both novice and expert characteristics. For example, one PST's response possessed several features of an expert knowledge structure; however, that same response also contained an obvious conceptual error. Since that PST was a case subject, the interview transcript was consulted and it was concluded that the PST actually did posses expert knowledge regarding the question; hence, that response did not receive a novice code of 1*a*. Interview transcripts were only available for case subjects; therefore, their responses allowed for member-checking and hence greater reliability.

During the second training session, conversation between scorers established that another code needed to be added to the coding sheets; a novice code of 16*a* was added to apply to incorrect instruction and/or explanation. It was decided that code 9*b* could function as the expert's opposing code to 16*a*. There also appeared strong relationships between certain codes. For example, a code of 2*b*, 3*b*, or 9*b* was almost always accompanied by a code of 1*b*. The following problem (see Appendix D, problem 1) provides a helpful example of the type of responses that would elicit different codes. The PSTs were provided a 10 × 10 grid including the statement beneath it that each grid-square represented "1 square unit." PSTs were first asked, "On the grid provided, draw a polygon that has a perimeter of 24 units." The second part of the problem asked: "How would you help a 5th grader understand that the polygon you drew really does have a perimeter of 24?" One PST drew a 6 × 6 square on the grid and provided the following response for the second part: "b/c 24/4 = 6. It might help to count out each square individually." That response received a "1*a*" for a sparse and disconnected knowledge structure, a "2*a*" for not drawing clear distinctions between linear and square units, (especially because the polygon was drawn on a grid), and a "9*a*" for a procedural explanation that would not aid understanding. Contrast that with the following response given for the same question by another PST, who drew a 5 × 7 rectangle on the grid, and followed up with this explanation, "Count the units on the outside all the way around the rectangle. Make sure they count the outside edge of the boxes, using linear units instead of the boxes themselves. When we add up those edges (7+7+5+5), we will get 24." That response received a "1*b*" for richly connecting perimeter to linear units; a "2*b*" for plainly addressing the common misconception regarding linear and square units; and a "9*b*" for clearly delineating a conceptual explanation. The third training session involved coding 5 KoST questions. Nothing occurred that required any revisions to the coding sheets.

The only revision to the coding sheets occurred during the first session's discussion of the pretest and its interview transcript. It was noted while examining an interview transcript and comparing it to the pretest that one PST would quite readily change his/her mind and vacillate between responses after just a basic interview prompt, such as, "Would you please provide further explanation, and possibly clarify, what you were thinking when you wrote this." Based on that finding the novice code 17*a*, which states, "Contradict own response (written and/or verbal)" was added. The expert code 1*b*, which refers to a sound CK, functioned as the contrasting code to 17*a*.

Of the total 103 codes applied to the three pretests, there was initial agreement on 79 (77%). We had very strong agreement (98%) on identifying whether a specific response was novice or expert in nature. The vast majority of disagreements were related to which specific novice or expert code should be awarded (e.g., I would code something 10*a* and the second scorer would code the same response as 11*a*), as opposed to one of awarding a novice code and the other awarding an expert code to the same response. Out of a total of 103 codes applied during the training sessions, that sort of disagreement occurred only twice. Those were resolved after agreeing that any code applied must be done in light of the whole response to avoid attributing undue significance to any one part of a PST's response. Other disagreements were discussed until strong consensus was reached. In summary, following discussion consensus was reached on 101 out of 103 codings representing 98% agreement for the training sessions.

Following the training sessions, two pretests were formally double-coded and pattern matching was performed through examining their respective interview transcripts. For the two double-coded pretests, there was initial agreement on 47 out of 64 codes (73%). Clarifying how certain codes (e.g., 9*a*, 10*a*, & 11*a*) were applied improved agreement to 96%. All but one of the disagreements were of the novice type (i.e., either a different novice code or an extra novice code was applied). The one novice/expert disagreement was resolved when the second scorer consulted the interview transcript during the pattern matching and realized that the PST was not "expert" in their response. The agreement on the pattern matching was 97% (i.e., every code, except one, that was applied to the pretest was confirmed by the transcript), and agreement on new codes applied while reviewing the transcripts were 11 out of 17 (67%). One explanation for the slightly lower agreement was that the researcher consistently applied a code of 4*a* to a transcript every time (n = 3) the case subject remarked, "Oh, I guess I did not read the problem very carefully," whereas the second scorer chose not to code such comments. All other pattern-matching disagreements involved different selections of novice codes.

The high levels of agreement provided the researcher confidence that the coding process could be done reliably. That reliability was valuable in constructing the PST's content knowledge and knowledge of student thinking related to area and perimeter.

Validation of Anchored-Instruction Intervention

To ensure that the Anchored Instruction framework was used with fidelity, experts who were familiar with this approach were asked to provide an *expert review* of various aspects of the study's conceptual framework. Four doctoral candidates, from the field of instructional technology, agreed to examine and evaluate four aspects of this study's conceptual framework: (a) the researcher's operational definition of anchored instruction, (b) the degree to which the anchor of choice (situated within the Teaching Episodes) captured the essence and addressed the goals of an "anchor" as expressed by the designers of Anchored Instruction, (c) the degree to which the design principles of Anchored Instruction were addressed by the materials of this study, and (d) the degree to which PSTs in this study experienced Anchored Instruction.

Each expert reviewer received an email explaining the review process. There were several files attached to the email: (a) an overview of the study, (b) a summary of the study's conceptual framework, (c) a document containing a literature-based summary of the qualities of Anchored Instruction, (d) information on, including hyperlinks to, the two microworlds integrated into the instructional sequence, (e) all three teaching episodes, and (f) the Anchored Instruction Assessment Survey (Appendix O). The survey instrument contained four sections consisting of an explanation for each component of the conceptual framework that was to be reviewed followed by a Likert-scale checklist. Each reviewer took about a month to work through the materials and return his/her completed survey instrument. The results are summarized in Table 4.

Table 4

Results from Assessment Survey of Anchored Instruction (n=4)

Construct being reviewed	Strongly Agree	Agree	Disagree	Strongly Disagree
I. Definition of Anchored Instruction	**3**	**1**		
II. Selection for the anchor	**3**	**1**		
III. 8 Design Principles:				
1. Choosing an appropriate anchor	**3**	**1**		
2. Possess a generative learning environment		**4**		
3. Developing shared expertise around the anchor	**3**	**1**		
4. Expanding of the anchor	**2**	**2**		
5. Using knowledge as a tool	**1**	**3**		
6. Merging of the anchor	**3**	**1**		
7. Allowing student exploration	**4**			
8. Provide opportunity for PSTs to share new knowledge	**3**	**1**		
IV. PSTs should experience anchored instruction	**2**	**2**		

Cross-Case Analysis

Answering each of the five research questions involved, to different degrees, cross-case analysis. For the non-case subjects, their responses to the problems on the area and perimeter tests, as well as items within the teaching episode packets served as a means to conduct cross-case analysis and comparison. Yin (1984) advocates a process that has been referred to as *replication* (Miles & Huberman, 1994). The analysis process typically involves studying in-depth cases and then examining successive cases (less in-depth) to see whether the patterns found match those in the case subjects. This cross-case comparison helped present a wider view of the data and facilitate a more comprehensive examination of mathematical and pedagogical change, when it occurred. Including data from all the PSTs within the constant comparison analysis helped to support the findings from the case subjects.

PSTs' Pre-Intervention CK and KoST

In order to answer research questions one and two, it was necessary to establish the PSTs' pre-intervention content knowledge (CK) and knowledge of student thinking (KoST); Their written responses to the pre-study questionnaire, the 10-item area and perimeter pretest, and the case-subjects' baseline interviews were analyzed. The expert/novice coding sheets were applied to the pre-study questionnaire, pretest, and the baseline interviews. How the assigned codes were used in analysis and in the reporting of findings is described later in this section. The bulk of pre-intervention findings were drawn from analysis of the PSTs' written responses to the 10 pretest items. Analysis of the pretest items was done from three perspectives.

Analysis of Pretest Written Responses

First, the PSTs' responses to the pretest items received a score from 0 to 4 based on the researcher-created holistic scoring rubrics (see Appendix H) developed from criteria established by Cai, Lane, and Jakabcsin (1996); thus, each PST's test received an overall score ranging from 0-40. As described in the instrumentation section, the criteria of the scoring rubrics incorporate distinguishing characteristics of both novice and expert mathematics teachers obtained from the literature (e.g., novice teachers focus on the content at hand while expert teachers continually consider the various needs of the students) so that each score actually represents a location on a theoretical continuum from novice to expert. For example, procedural versus conceptual responses were addressed, and procedural-laden responses ended up with a score of two or lower. Although each item contained two, three, or four parts (see Appendix D), both the closed- and open-ended parts received one overall score. The pretest contained 10 total items – five addressing content knowledge (CK) and five dealing with knowledge of student thinking (KoST). Each test generated an overall score, which ranged from 0-40. The mean and standard deviation for the overall score were calculated and discussed. A test scoring in the range of 0-20 was considered "unacceptable" and "mediocre," and test scores ranging from 21-40 were "acceptable" with the possibility of being deemed "model." A test receiving a score of 40 would imply every response to be model. Piloting revealed that tests receiving overall scores in the 20's often contained one or two model responses. Pilot scoring of 65 tests resulted in a mean score of 17.9 with a low score of 8 and a high of 25.

The total pretest score served as a baseline indicator that was later used in growth curve analysis – a quantitative approach to display the change, if any, in the PSTs' mathematical knowledge. Total scores from the pretest also functioned as the first time-point recording in the growth-curve analysis. The total test score was also a factor in the purposeful selection of four PSTs for in-depth study. Among the four who were selected, two scored at or near the bottom on the pretest and two scored at or near the top. The quality of their responses, as opposed to some predetermined score, was of primary importance in case subject selection. This criterion is discussed in greater detail in the sections addressing research questions three and four, where a more detailed explanation of how the PSTs' mathematical change, if any, was observed, analyzed, displayed, and discussed.

The second, more focused, perspective that was used to gain insight into the PSTs' pre-intervention levels of CK and KoST involved the preservice teachers' pretest scores on the five CK questions and five KoST questions. They were analyzed and discussed as *sub-tests* within each test. The scores on these sub-tests can range from 0-20. Descriptive statistics (mean and standard deviation) for each PST's CK and KoST pretest score were analyzed and reported. Examining descriptive statistics for sub-tests scores within the 65 piloted tests revealed no consistent or statistically significant trends. Frequencies of rubric scores were presented and any score-patterns for the CK and KoST items were discussed.

Frequencies from expert/novice codings of the pretest responses were presented and discussed as a means to help establish baseline measures of the PSTs' CK and KoST. Transcripts of the first interviews were used as a form of pattern matching with the analysis of pretest responses. Based on the actual definitions of both CK and KoST, responses receiving certain codings were more informative than others. For example, CK involves: (a) an organization of facts and concepts; thus, analysis surrounding responses receiving codes 1*a*/1*b* and 15*a*/15*b* would be helpful, and (b) an explanatory framework; therefore, responses receiving 8*a*/8*b* and 16*a*/9*b* would be useful. For KoST, it involves: (a) understanding children's thinking about content areas, so for this study responses receiving codes of 2a and 2b were valuable, and (b) appropriately addressing any shortcoming or misconceptions; hence, responses receiving 7*a*-/7*a*/7*b*, 12*a*/12*b*, and/or 13*a*/13*b* were considered carefully.

A qualitative examination of the PSTs' responses on the questionnaire, the 10 pretest items, and the first interview with the case subjects comprises the third perspective used to describe the preservice teachers' CK and KoST prior to involvement with the anchored instruction. As the data analysis of the questionnaire and pretest proceeded, three broad categories of responses were identified. They are: (a) distinguishing between area and perimeter, (b) units of measure, and (c) perceived relationships between area and perimeter. These broad categories were used to help organize themes within the responses containing findings needed to answer research questions one through four. The cross-case analysis began by examining the PSTs' written responses to the pretest items and comparing them to the coding sheets of difference patterns between novice and expert preservice and classroom teachers (see Table 3, p. 166). Another component of the analysis of the pretest responses was the integration of certain aspects of Liping Ma's (1999) four levels of understanding that teachers can exhibit as they explore a new idea presented to them by a student. They are, in order: (a) Disproving the claim, (b) Identifying the possibilities, (c) Clarifying the conditions, and (d) Explaining the conditions. A category of understanding (Justifying an invalid claim) was designated as "Level 0" for purposes of this study. It was not designated as a level by Ma, since it was not deemed successful.

Analysis of the First Interview

The transcripts from the first interview with the case subjects were used to pattern match the codes assigned to the pretest responses and as a source to aid in triangulating data. All interviews were videotaped and the audio was transcribed. Two of the four baseline interviews were double-coded, using the expert/novice coding sheets (see Table 3), by the same secondary scorer mentioned earlier. This functioned as a sort of pattern matching (Gall et al., 1996) to help ensure reliable coding of patterns and identification of expert/novice themes. Before interview transcripts were finalized, the videotapes were watched in entirety to allow for additional comments to be inserted that added necessary context (e.g., "The preservice teacher pointed to the 2 × 7 rectangle at this time."). The videotapes were available during the coding process which provided an additional quality-check to help validate analysis.

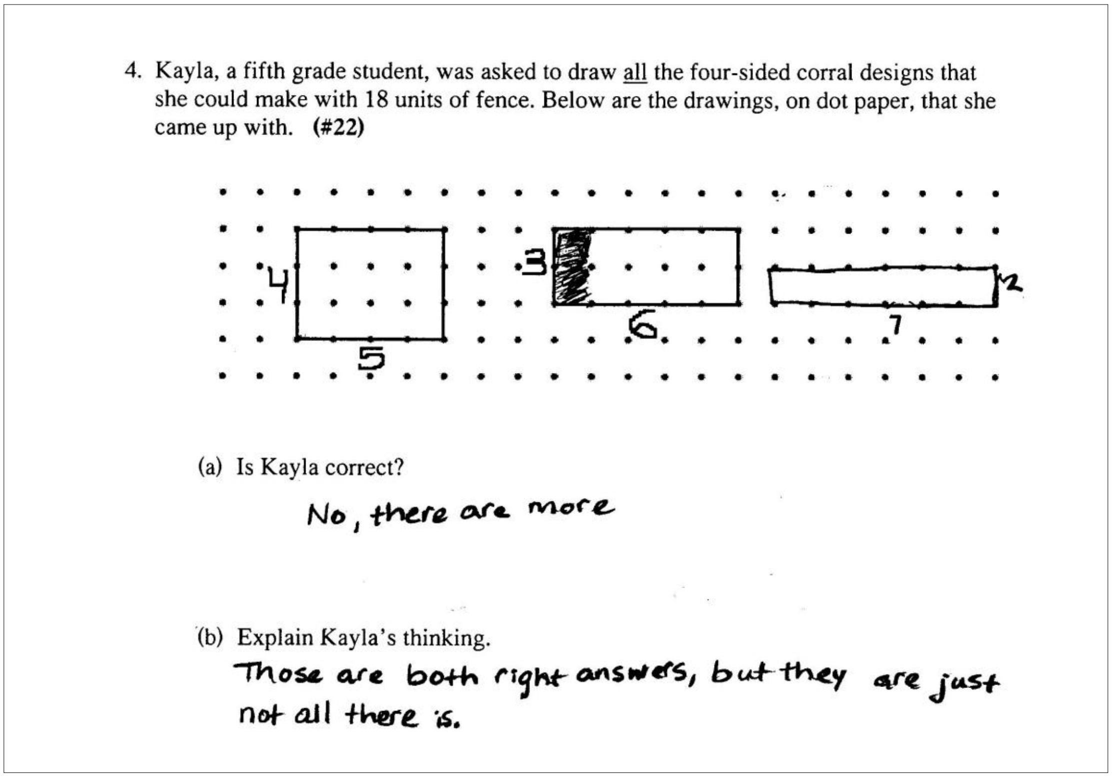
Transcripts from the first (baseline) interview were analyzed in similar fashion as the pretest responses. Meaningful interview passages were compared to the coding sheets of difference patterns between novice and expert teachers (Table 3), and to prior responses on the pretest, looking for previously identified themes or emerging ones. Each case-subject interview contributed to ongoing collection of data regarding their CK and KoST, thus providing another means to triangulate the data, hence adding credibility and strengthening confidence in subsequent conclusions (Patton, 2002). While analyzing and coding PSTs' responses to test items and teaching episodes prompts, the interview transcripts provided a means to substantiate, or even refute, claims and/or identified patterns. If a test response was unclear and difficult to score or code, being able to address that response during a follow-up interview proved valuable and lent credence to the final score or code awarded. A good example of this process occurred while evaluating the substance of a preservice teacher's response to a piloted item (Figure 17).

Initial evaluation concluded that the response contained questionable content knowledge (see the preservice teachers' improper labeling of a "2 × 7"rectangle they drew in Figure 17) and a limited knowledge of student thinking. This question, along with the student's response, was included as part of the protocol for a follow-up, semi-structured interview. After the interview was completed and transcribed, ongoing analysis revealed that a lack of appropriate scrutiny during the problem-solving stage was the major reason for the deficient response and not a genuine lack of understanding as was first thought. Below is a portion of the transcribed interview:

(**I** = instructor; **S** = student)

> **I:** I want to ask you what you think the "4" and the "5" written by Kayla's first rectangle mean.
>
> **S:** The way that she is thinking is about the outside. For instance, the 18 units of fence would mean that "4" would be 4 feet.
>
> **I:** I'm curious; could you point to and count off the 4 feet?
>
> **S:** Oh yes, each dot represents one of the (pause), although if you use the space in between, then it wouldn't really be (pause again). Oh, she was just connecting the dots and to her each dot represented a unit - and the same for the 5 also.
>
> **I:** So how about the rectangle you drew? If you put that up on a board to show students, how would you explain the dimensions of what you drew? Is that rectangle 7 × 2?
>
> **S:** No, it would actually be 6 × 1.

So it was determined that the PST's inadequate knowledge of student thinking was more likely a result of inadequate analysis as opposed to limited content knowledge.


4. Kayla, a fifth grade student, was asked to draw <u>all</u> the four-sided corral designs that she could make with 18 units of fence. Below are the drawings, on dot paper, that she came up with. **(#22)**

(a) Is Kayla correct?

No, there are more

(b) Explain Kayla's thinking.

Those are both right answers, but they are just not all there is.

Figure 17. Piloted item used in follow-up interview for pattern matching.

A similar situation occurred during analysis of the pretest responses for the full study. One of the case subjects (Grace) provided an incomplete and shallow response to two items near the end of the pretest. They were both scored as "inferior." Both the responses were topics of discussion for her first interview. It was then that she shared how she ran out of time while answering those two items. Given the opportunity, she was able to complete her responses, without any help or prompting, and provide a more accurate picture of her true understanding regarding the concepts and misconceptions contained in the items.

These processes provided a descriptive notion of the level of expertise regarding content knowledge and knowledge of student thinking possessed by the preservice teachers prior to intervention. Claims regarding the four case subjects selected were analyzed and evaluated further in subsequent interviews as well as with cross-case analysis of the non-case subjects, as described earlier.

PSTs' Emergent and Post-Intervention CK and KoST

To be able to answer research questions three and four, it was necessary to ascertain in what ways the preservice teachers' content knowledge (CK) and knowledge of student thinking (KoST) changed, if at all, throughout the course of the study.

Emergent Knowledge: The Teaching Episodes

The teaching episodes (TEs) comprise the primary means of intervention for this study; therefore, the findings from the TEs embody the PSTs' emergent knowledge. All three teaching episodes were videotaped. The videotapes were watched before any coding was performed and were used as a reference to inform and support ongoing and retrospective analysis. Repeated viewing and analysis of the whole-class discussions proved helpful in providing context and supportive data for the non case-subjects. Because research questions three and four specifically addressed CK or KoST, each writing prompt from the three teaching episodes was identified as focusing on CK, KoST, or the use of microworld(s) within the TE (an application of KoST). The subsequent PSTs' responses were then analyzed in much the same fashion as the pre-, post-, and follow-up tests. The expert/novice coding sheets were applied to each response and, when necessary, pattern matching was performed for the 4 case-subjects through analyzing interview transcripts. Interventions by the researcher during the teaching episodes also provided opportunities to pattern match data identified during reflective analysis involving the researcher's field notes and reflection journal.

The numerous data samples collected and the analysis conducted were valuable in helping to generate rich description of how the PSTs' content knowledge and knowledge of student thinking changed throughout the study, hence answering research questions three and four from a qualitative perspective.

Post-Intervention Knowledge

The data analysis for the post- and follow-up tests was conducted in similar fashion as for questions one and two. Regarding the pre-, post-, and follow-up tests, the following were calculated, analyzed, and discussed: (a) descriptive statistics of the total and sub-test scores, (b) rubric-score frequencies, (c) expert/novice coding totals, and (d) individual expert/novice code frequencies. However, additional analysis was also conducted. Expert/novice coding totals for CK and KoST, as well as regression equations and graphs for total score and CK and KoST sub-test scores, were presented and discussed. In order to present the PSTs' emergent knowledge, the three TEs (involving the anchored instruction intervention) were the focal point of the qualitative cross-case analysis, supplemented (i.e., supported or refuted) with the PSTs' written responses to the post- and follow-up tests, and the case-subjects' second interview. The second and final interview involving the four case subjects followed the last of three teaching episodes, and was analyzed in the same manner as the pretest (baseline) interview.

Regression Analysis of Tests Scores

The second way that potential mathematical change was investigated involved regression analysis of mean scores from the pre-, post-, and follow-up tests. "The very notion of learning implies growth and change" (Willett, 1988, p. 345); However, quantitative measurements of change have proven controversial, with some seeing its value (Rogosa, Brandt, & Zimowski, 1982; Willett, 1988; Zimmerman & Williams, 1982), and others who are suspect (Gall et al., 1996; Linn & Slinde, 1977; Lord, 1956).

The approach taken in this study involved an adaptation of the difference score (i.e., gain score). The PSTs' total scores on the pre-, post-, and follow-up tests were used as the dependent variable, and the corresponding points in time (i.e., pre-, post- and follow-up) functioned as the independent variable to construct individual growth curves. The test scores can be thought of as "points in time" or repeated measures, and a regression line was fit to those points. Significance of any growth, or lack thereof, was explained and supported by qualitative measures (e.g., the PSTs' written responses on the area and perimeter tests and the problems posed during teaching episodes, students' written reflections, observations, and field notes during the teaching episodes, and the interviews of the case subjects). Change related to the specific components of CK and KoST (as described in their definitions) were analyzed and reported in much the same fashion as was done in answering research questions one and two. The presentation of the regression lines and equations for each participant's CK, KoST, and total test score provided a visual confirmation of any change. Although the teaching episodes provided a picture of the PSTs' emerging growth related to CK and KoST, the posttest and second interviews were the primary data sources for documenting more immediate growth (or lack thereof). The follow-up test was more a measure of retention as well as a means of confirming and/or illustrating the growth (or lack thereof) delineated by the triangulation of the previously mentioned data sources. This simplified approach assisted in presenting a second perspective on the mathematical growth of the PSTs and contributed to answering research questions three and four.

Relationships Between CK and KoST

To answer research question 5, it was necessary to examine potential relationships that might exist between CK and KoST (e.g., Does KoST increase as CK increases?) - as related to area and perimeter in general, and more specifically units of measure and perceived relationships. Two approaches were used to answer this question. The first involved an analysis of quantitative data. The three correlation coefficients for CK and KoST at the three time-points (i.e., pre-, post-, and follow-up) were calculated and discussed. CK and KoST sub-test scores for the pre-, post-, and follow-up tests (e.g., Table 14, p. 256) and summary tables of expert/novice codings (e.g., Table 16, p. 261) were analyzed and patterns were noted and examined (e.g., 9 of the 12 PSTs showed increases in their CK or KoST, but only 6 showed increases in both), and appropriate regression graphs (created to help answer research questions 3 and 4) were presented. One goal was to identify and describe CK-KoST relationships that surfaced primarily due to the intervention (i.e., from pre- to posttest), and since the follow-up test is more a measure of retention, its results were not weighted as heavily. During analysis it was concluded that a change of ±3 points (range 0 to 20) from a PST's pretest sub-test score (CK or KoST) to their posttest sub-test score (CK or KoST) was a necessary criterion to assist in identifying and deciphering CK-KoST relationships (e.g., increased CK and KoST, and static CK with increased KoST). That number (±3) represents a 15% change and helped to rule out trivial and inconsistent patterns or weak relationships. It should be kept in mind that the goal of answering research question 5 was not to look for or attempt to establish statistical significance within or among CK and KoST data, but rather to discover and then describe CK-KoST relationships that could be collaborated through different sources (e.g., responses to tests and TEs, and interview transcripts).

The second aspect to answering research question 5 involved two comprehensive analysis strands, devised around the area and perimeter concepts/misconceptions central to this study (see Table 5), which helped to focus and guide further analysis necessary to illuminate and describe patterns identified during quantitative analysis. The two analysis strands are (a) units of measure (i.e., linear and square units), and (b) the perceived relationships between area and perimeter (i.e., that equal perimeters must result in equal areas and vice versa, and the belief that a direct relationship exists between area and perimeter in that increasing/ decreasing one will have the effect of increasing/ decreasing the other). These analysis strands formed the basis for the topics of inquiry across various time-points (i.e., across teaching episodes and from pretest to posttest, and to a lesser degree the follow-up test). Answering research question 5 followed similar paths as used to answer research questions 1-4: (a) Case subjects were the primary focus of the comparative analysis, because their responses received appropriate pattern matching through two semi-structured interviews, and (b) Any discussion of CK-KoST relationships focused on the pre- and posttest findings, since the follow-up test has implications more for retention. The comparative analysis was supported with appropriate findings from the non-case subjects. An example of how the descriptive statistics and the analysis strands functioned together will be presented next.

What follows is a theoretical example of the analysis processes just described. If a PST's responses concerning issues of CK regarding area and perimeter were consistently scored and determined to be weak and of novice standing (based on rubric scoring and

Table 5

Corresponding Test Items for Comparative Analysis for Answering Research Question Five

"Units of Measure" Analysis Strand

Source:	Pretest/Follow-up*	TE 1	TE 2	TE 3	Posttest
CK	1, 3, 4	2, 3, 4, 6^{M}			1, 3, 4
KoST	6, 7, 9	5, 7^{M}, 8^{M}, 10, 11^{M}			7, 9

"Perceived Relationships between Area and Perimeter" Analysis Strand

Source:	Pretest/Follow-up*	TE 1	TE 2	TE 3	Posttest
CK	5		2, 3, 4, 5, 7^{M}, 9	2, 3, 4, 6^{M}	5
KoST	8, 10		6, 8^{M}, 10, 12, 13^{M}	5, 7, 8, 10^{M}	6, 8, 10

Note. CK = content knowledge; KoST = knowledge of student thinking; and TE = teaching episode. ^{M}The question encouraged the use of a microworld. *Follow-up test contains same problems as pretest.

Table 3, p. 166), and such a finding received substantiation by a second data source (e.g., the teaching episode) or better yet a third (e.g., an interview), then logical progression should proceed to an analysis of that PST's handling of KoST questions addressing similar concepts. For example, if a PST continually confused area and perimeter concepts (e.g., linear versus square units) while addressing questions related to CK of area and perimeter, and that same PST also exhibited a limited, or even inaccurate, knowledge of how to best deal with a hypothetical student struggling with similar concepts, then a strong possibility would be that the PST's CK was influencing their ability to effectively respond to a student and their thinking. Also, it should be mentioned that each KoST question is designed to focus on a common misconception regarding area and perimeter concepts (i.e., CK). In other words, it was hypothesized that if a PST was unable to perceive the misconception presented in the problem (i.e., fallible CK), they would typically present inferior methods of dealing with students exhibiting the same misconception (i.e., inferior KoST). It was conjectured that a substantial CK of area and perimeter was necessary for preservice teachers to be able to meaningfully and conceptually address student misconceptions regarding those concepts (i.e., a well-developed KoST is dependent upon robust CK).

A goal and challenge was to associate and provide an explanation for a PST's performance on KoST questions to their performance on CK questions addressing similar concepts (or misconceptions), as opposed to another shortcoming possibly not directly related to CK (e.g., carelessness or running out of time). Equally important was the investigation and description of various relationships between KoST and CK that were identified. For example: (a) an increase in CK from pre- to posttest accompanied by a static or decreasing KoST, or (b) a static CK from pre- to posttest while the KoST increased. Such relationships, when discovered, were analyzed, patterns compared and categorized, and narrative written in an attempt to explain and clarify any counterintuitive results (e.g., an increased KoST with decreased CK). When multiple data sources substantiated a relationship between CK and KoST, rich description was used in an attempt to illuminate such relationships.

Limitations of this Study

As in all research possessing a qualitative element, the quality of a teacher development experiment will be directly dependent upon the knowledge, skills, and interactive abilities of the researcher (or researchers). As such, the researcher functioned as an “instrument” in the study. Additionally, the TDE contains an additional layer – teaching. The overall goal of that teaching was to promote mathematical development within the PSTs, which puts added importance upon the competencies of the researcher (beyond the usual involving observation, questioning, and data management). According to Simon (2000), preparing to conduct TDE research combines two difficult processes: learning to conduct research while simultaneously learning to teach in ways appropriate for the TDE. These challenges, along with certain inherent aspects of this study, contributed to the following limitations:

1. Although the overall Cronbach’s alpha for the three tests (pre-, post-, and follow-up) were satisfactory (i.e., $\geq .75$), certain five-question sub-tests (e.g., pretest KoST, posttest CK, and follow-up KoST) were less than satisfactory. They were accounted for and discussed previously in Chapter 3.

2. The TDE is dependent on the researcher's ability to promote development (Simon, 2000).

3. The researcher's role as instructor of the teaching episodes could bias the validity of certain qualitative elements of this study, but such bias was minimized by the presence and feedback of a second observer.

CHAPTER 4

FINDINGS

The purpose of this study was to examine levels of knowledge in the context of anchored instruction with geometry microworlds upon preservice elementary teachers' (PSTs') content knowledge and knowledge of student thinking related to area and perimeter. In particular, it focused on their understandings, misconceptions, written and verbal explanations of that knowledge, and achievement on written area and perimeter tests – within the context of a mathematics methods course for PSTs.

The primary research question examined by this study was, *"In what ways does PSTs' content knowledge and pedagogical content knowledge, related to area and perimeter, change as a result of experiencing anchored instruction integrated with web-based microworlds, designed for investigation of area and perimeter?"* In particular:

1. What is the PSTs' content knowledge regarding area and perimeter prior to involvement in the teaching episodes?
2. What is the PSTs' knowledge of student thinking regarding area and perimeter prior to involvement in the teaching episodes?
3. How does the PSTs' content knowledge regarding area and perimeter change, if at all, during the course of the study?

4. How does the PSTs' knowledge of student thinking regarding area and perimeter change, if at all, during the course of the study?
5. In what ways, if at all, is the PSTs' knowledge of student thinking regarding area and perimeter related to their content knowledge of those same concepts?

This chapter consists of results that are presented in three distinct sections. The first major section answers research questions 1 and 2 by discussing the results pertaining to pre-intervention content knowledge (CK) and knowledge of student thinking (KoST). Descriptive statistics and qualitative analysis of the pre-study questionnaire, pretest, the first interview with the case subjects, and microworlds' orientation session comprise the pre-intervention results. The second major section presents findings taken from the post- and follow-up tests, the three teaching episodes, and the second interview with the case subjects. By comparing and discussing these findings to the (PSTs') pre-intervention CK and KoST, research questions 3 and 4 are addressed. Chapter 4 concludes by discussing results pertaining to possible relationships between CK and KoST as deciphered within predetermined content strands taken from the pre-, post-, and follow-up tests, and the three teaching episodes.

Selection of Case Subjects

Using the selection process described in Chapter 3, the following four case subjects were identified.

Case-Subject Jackie

Jackie is a very diligent student who earns good grades (see Table 6). The researcher was Jackie's instructor for two of her mathematics courses. Mathematics does not come naturally to Jackie, and she would be the first to admit that. Jackie is an inquisitive person and not ashamed to admit it when she is confused about a concept, nor was she afraid to ask a question in class or after class. In the Survey Questionnaire (Appendix C) Jackie indicated she had studied area and perimeter in high school as well as college (see Table 6). When given the choices "apprehensive, confident, or very confident" in regards to how confident she was about teaching area and perimeter to elementary-age children, she replied "confident." She also wrote, "I have never taught any mathmatic [*sic*] concepts to children . . . so that is why I am not 'very confident.' I have had great tudors [*sic*] who have gave me some tips on how to teach it."

The researcher also taught Jackie in a technology course designed for preservice teachers. She proved quite capable with concepts and applications surrounding technology integration. When asked in the questionnaire about her opinion on using technology to help elementary students learn about area and perimeter, she wrote, "I love it. Elementary students are so connected to the computer these days."

Jackie is well liked and has many friends within her elementary education cohort and throughout the campus. She also holds leadership positions within the student body as well as her college Greek organization. She is socially confident and very eager to participate in class discussions. Jackie has an open mind to both content and pedagogical issues related to education and the study of teaching. She was not only willing to be a case subject but expressed excitement at the opportunity. At times during interviews and class discussions Jackie could get verbose, and this would tend to dilute her responses.

Case-Subject Brianna

Brianna is a very conscientious student who performs very well academically (see Table 6). Instead of taking Liberal Arts Mathematics for her third mathematics course,

Table 6

Case-Subject Data

	Jackie	**Brianna**	**Larry**	**Grace**
		Academic background		
GPA	3.07	3.33	2.21	4.0
College algebra	B	A	C	A
Liberal arts math	B		C	A
Prob. & Stats.	C	A	D (C 2nd time)	A
Pre-calculus		B		
		Exposure and confidence related to area & perimeter		
HS geometry	X		X	X
Other HS math courses	X	X	X	
College math courses	X		X	X
Involved manipulatives	X		X	
Involved technology				
Confidence level to teach concepts	"confident"	"apprehensive"	"apprehensive"	"apprehensive"

Note. All data was current through their junior year.

she took Pre-Calculus, a course not typically taken by elementary education majors. The researcher was Brianna's instructor for both College Algebra and Pre- Calculus. When asked why she signed up for Pre-Calculus, she said that she has always enjoyed math. Brianna was quiet during class, did not ask many questions, and was uncomfortable when called on to respond. Brianna is a very careful thinker, who would often take 10-20 seconds to ponder a question before giving a response.

Despite Brianna's strong mathematics background, she indicated she was "apprehensive" about teaching area and perimeter to elementary-age children. When asked why she felt that way, she replied, "I have never taught these things before." Brianna wrote that she had never been exposed to any instructional technologies while learning about area and perimeter. When asked her opinion on using technology to assist elementary students in learning about area and perimeter, she responded "I think it would be beneficial to use technology when teaching about area and perimeter, to help students understand the concepts more. However, I don't think technology should take the place of the teacher."

Case-Subject Larry

Larry was the only male student in the Methods of Teaching Elementary Mathematics course. He is an exceptional athlete and a very successful soccer player for the college. Academically, Larry struggles (see Table 6). He often appeared overwhelmed with his course work; he would forget about assignments, and the depth of his work was average at best. The researcher was Larry's instructor for College Algebra and Liberal Arts Math. Larry is a fun-loving guy, enjoyable to talk to, and well-liked. He does not enjoy mathematics and must work very hard to earn a passing grade. Larry would not seek assistance and rarely asked questions in class. Tests and in-class projects would overwhelm him, and he frequently did not perform well on them.

In the Survey Questionnaire Larry indicated he would be "apprehensive" about teaching area and perimeter to elementary-age children. He wrote, "I would have to brush up on the topic a little more before I taught it." Larry was not aware of any technology that could aid in the teaching and learning of area and perimeter, but seemed open to its possibilities. When asked his opinion on using technology to help elementary students in learning about area and perimeter, he responded, “I think it is a great way to assist students in learning. It can do many things that cannot be done in the classroom. It makes students think on their own.”

Case-Subject Grace

Unlike the other case subjects, Grace was not a “traditional” college student. After raising a family and working as an administrative assistant at the college where this study took place, Grace decided it was time for a career change, and at the age of 52 she enrolled in the school of education. Grace was an amazing student. She maintained a 4.0 GPA her entire college career (see Table 6). I was her instructor for College Algebra and Liberal Arts Math. Grace is quiet, humble, and unassuming but was not afraid to ask a question in class and was thoughtful when responding to questions during class.

In the Survey Questionnaire Grace indicated she had studied area and perimeter in her high school Geometry class, and did not recall any other exposure to those concepts since that time (Table 6). When asked whether she was “apprehensive, confident, or very confident” in regards to her feeling prepared to teach area and perimeter to elementary-age children, she replied “apprehensive,” because “I have no experience in current methods.” Grace said she had no past experiences with either concrete manipulatives or educational technologies (i.e., software or the Internet) while learning about area and perimeter. When asked her opinion on using technology to help elementary students in learning about area and perimeter, she responded, “I think it would be helpful – keep attention and provide different types of visuals. Web-based technologies provide a vast array of tools for assisting teaching; much more varied than a teacher could supply otherwise. And students are comfortable with and adept at using them."

Research Questions 1 and 2: PSTs' Pre-Intervention CK and KoST

The findings in this section address the following research questions: What is the preservice teachers' (1) content knowledge (CK) and (2) knowledge of student thinking (KoST) regarding area and perimeter prior to involvement in the teaching episodes? The pre-intervention data came from the pre-study survey questionnaire (Appendix C), pretest (Appendix D), the first interviews with the case subjects, and microworlds orientation (Appendix M). Findings were extracted from the PSTs' written responses to the questionnaire and the pretest, and from transcripts from the first interview. Descriptive statistics were performed on the resulting scores as well on the expert/novice coding applied to all the PSTs' written responses. Descriptive statistics will be presented first, followed by qualitative findings meant to support and illuminate the descriptive results.

Pretest Levels of CK and KoST

As described in Chapter 3, findings involving CK and KoST, will address key components of their definitions. For CK that involves: (a) the amount and organization of facts and concepts, and (b) the ability to explain that knowledge in meaningful ways, and KoST entails: (a) organizing CK in a way that would enable a teacher to understand children's thinking, and (b) appropriately addressing any shortcomings or misconceptions. This will be the case for the first four research questions.

Descriptive Statistics for Rubric Scorings of Pretest Items

Results from the PSTs' scores on the pretest showed an overall mean of 21.25, and a standard deviation of 4.97 (see Table 7). The data appeared relatively normally distributed with skewness and kurtosis values of .08 and -.65 respectively. Jackie scored a 13 on the pretest, which was the lowest overall score. Her score of 4 on the CK subtest was the lowest in this category and was almost two standard deviations below the mean of 10.58. Brianna received the highest score of 30, and Grace was second at 27. The results indicated that the two easiest questions on the pretest, each with a mean of 2.75, were three (SD = .75) and ten (SD = .45), and the hardest question was four (M = 1.58; SD = .9). Jackie scored a 1 on question 3, a 1 on four, and a 3 on ten (see Table 10).

Table 7

Descriptive Statistics for Pretest

	Pretest		CK (items 1-5)		KoST (items 6-10)	
	Mean	SD	Mean	SD	Mean	SD
	21.25	4.97	10.58	3.48	10.67	2.10

PST*	Pretest Score	CK Items (1-5)	KoST Items (6-10)
#1	25	12	13
Grace (#2)	27	16 (high)	11**
#3	23	13	10
#4	18	8	10
#5	16	8	8
#6	24	14	10
Jackie (#7)	13 (low)	4 (low)	9
Brianna (#8)	30 (high)	14	16 (high)
#9	18	8	10
#10	23	12	11
#11	21	10	11
Larry (#12)	17	8	9

Note. Pretest scores range from 0 to 40. A score of 40 indicates a model response for all 10 items.
*PST = preservice teacher (i.e., study participant). **PST ran out of time and did not finish.

Jackie's score frequencies on the 10-items indicate the majority of her knowledge was categorized as "unacceptable" (a rubric score of 1), according to the criteria for the scoring rubrics (Appendix H). She scored four 1s and a 0 for the five content knowledge questions. These questions were designed to evaluate the PSTs' knowledge and understanding of basic area and perimeter ideas (i.e., draw a polygon that has a perimeter of 24, find the perimeter of an irregular polygon, and how, as a teacher, would you explain the concepts of linear and square units). Jackie performed better on the five knowledge of student thinking questions (items six through ten) earning one "acceptable" response. Jackie's higher KoST score may be due in part to her ability to relate to the students in the problems and correctly predict their struggles because, admittedly, she shares many of the same difficulties. Interview excerpts revealed that while Jackie may be aware of certain aspects of the misconceptions students possess, her ability to effectively intervene and her overall pre-intervention KoST is fragile at best. Contrast that with Brianna who only received one score of 2 for her entire pretest; the rest of her scores were "acceptable" (i.e., 3s or 4s). Of the 120 scores assigned on the pretest items, there were only seven scores of 4 awarded and only one on the KoST subtest (Brianna). Grace was the only PST who received more than one score of 4 (both came on the CK subtest).

Descriptive Statistics for Expert/Novice Codings for Pretest

Identifying examples of expert/novice behavior (Table 3, page 167) within the PSTs' work was another way to establish and describe their pre-intervention levels of CK and KoST. Table 9 displays the total frequencies of novice ("a") and expert ("b")

Table 8

PSTs' Pretest Item Rubric Scores and Frequencies

	Item		(CK: 1-5)					(KoST: 6-10)					Score Frequencies		
PST*	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4
#1	1	4	3	2	2	3	3	2	2	3		1	4	4	1
Grace	4	4	3	2	3	3	2	2	2	2			5	3	2
#3	2	4	3	2	2	3	1	2	2	2		1	6	2	1
#4	1	1	3	2	1	2	1	3	1	3		5	2	3	
#5	1	3	2	0	2	2	1	1	1	3	1	4	3	2	
#6	2	4	3	2	3	1	3	1	2	3		2	3	4	1
Jackie	1	1	1	1	0	2	1	1	2	3	1	6	2	1	
Brianna	3	3	3	3	2	3	3	3	4	3			1	8	1
#9	2	1	3	0	2	3	1	2	1	3	1	3	3	3	
#10	3	1	4	2	2	3	3	1	1	3		3	2	4	1
#11	1	3	3	2	1	2	1	2	3	3		3	3	4	
Larry	2	2	2	1	1	3	1	1	2	2		4	5	1	
										Totals	3	32	39	39	7

Note. Rubric scores range from 0 to 4. A score of 4 indicates a model response, 1 is unacceptable, and 0 indicates no response. *PST = preservice teacher (i.e., study participant) behavior as seen in each PST's pretest responses. Jackie and Larry displayed considerably more novice tendencies than did Brianna or Grace. Jackie's responses produced the highest frequency of novice-teacher codings (50) and the second lowest (4) number of expert-teacher traits. Brianna received the highest number of expert codings (21) followed by Grace with 20. Because the pretest was given prior to the microworld orientation and because all the PSTs indicated in their survey questionnaire that they had no prior exposure to learning mathematics with technology, codes 13*a* and 13*b* were not

Table 9

Expert/Novice Coding Frequencies for Pretest

	Total Score		**CK** (items 1 – 5)		**KoST** (items 6 – 10)	
	Mean for *a*	Mean for *b*	Mean for *a*	Mean for *b*	Mean for *a*	Mean for *b*
	34.6	10.3	15.8	4.1	18.8	6.2
SD	8.7	6	5	3.3	5.4	3.4
PST	*a* Sum	*b* Sum	*a* Sum	*b* Sum	*a* Sum	*b* Sum
#1	34	9	16	3	18	6
Grace	27	20	11	11	16	9
#3	45	10	14	4	31	6
#4	36	7	20	1	16	6
#5	43	**3**	21	1	22	2
#6	26	16	10	8	16	8
Jackie	**50**	4	26	0	24	4
Brianna	**23**	**21**	13	6	10	15
#9	27	9	12	4	15	5
#10	27	13	10	7	17	6
#11	37	7	18	2	19	5
Larry	40	5	18	2	22	3

Note. An *a* signifies a novice response and *b* signifies an expert response (see Table 2).
For total score: Min. *a* Sum = 23, Max. *a* Sum = 50, Min. *b* Sum = 3, and Max. *b* Sum = 21.

assigned to any of the pretest responses.

Table 10 presents frequencies of individual codes identified on the pretest. This allows the comparison of frequencies among case subjects and the class- frequency averages for each code. Jackie's and Larry's relatively high frequency of code 1*a* indicate an amount and organization of CK that is sparse, lacking, and/or disconnected (i.e., fragile). A high frequency of code 2*a* signals a PST exhibits little knowledge of misconceptions or concepts most difficult for students and would point to insufficient levels of KoST. Jackie and Larry had higher than average frequencies of *2a* while Jackie and Brianna were much lower. The ability to explain one's knowledge about mathematics is an important facet of CK, and codes *8a*/*8b*, *9a*/*9b*, and *16a* reflect that. The low frequency of code *8b* for Jackie and Larry is an indicator that they struggle when trying to explain their responses. Grace and Brianna had higher frequencies of code *9a* which would signify a tendency to be procedural when explaining how to do and think about mathematics. Codes *7a-*, *7a*, and *7b*, which involve the effective use of representations (or neglecting a representation, as in 7*a*-), are important because they indicate if a PST

Table 10

Novice/Expert Specific-Code Frequencies from Case Subjects' Pretest

Code	1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	6*a*	7*a*-	7*a*	7*b*	8*a*	8*b*	9*a*	9*b*	10*a*	10*b*	11*a*	11*b*	12*a*	12*b*	13*a*	14*a*
Grace	6	3	3	3	0	3	0	1	3	0	2	1	4	5	1	0	1	2	1	1	2	4	0
Jackie	10	0	8	1	0	0	1	1	3	1	1	3	1	3	0	0	0	0	1	3	4	11	2
Brianna	6	1	2	7	1	1	1	0	3	0	2	0	6	6	2	0	1	2	0	1	0	2	0
Larry	8	0	5	3	0	0	1	0	3	1	1	0	1	4	0	0	0	0	0	3	5	9	0
class avg	7.8	1.2	4.6	3.5	.7	.6	.3	.5	2.8	1.3	1.2	1.1	2.1	3.5	.7	.2	1.6	.8	.2	1	2	7.1	.3
SD	3.1	2.1	5.1	3.6	1.9	.7	.7	1.3	4.2	2.2	1.9	1.4	2.9	4.9	1.2	.4	2.2	1.2	.4	1.7	2.5	6.2	.9

Note. There were no codes of 4*b*, 5*a*, 5*b*, 6*b*, 10*b*, 13*a*, 13*b*, 14*b*, or 15*b* assigned for the pretest.

understands and appreciates appropriate means to addresses the shortcomings and misconceptions of students – a crucial component of one's KoST. As stated in Table 3, code *16a* is assigned to a response that presents incorrect, incomplete, or inadequate explanations. A frequency rate above the mean (as in the case of Jackie and Larry) identifies a deficient CK. The discussion of the PSTs' pre-intervention CK and KoST will now transition into presenting findings that expound on the descriptive statistics.

Describing PSTs' Pre-intervention CK and KoST

The findings presented in these next several sections answer research questions one and two and are organized under three major categories: (a) Distinguishing between area and perimeter, (b) Units of measure, and (c) Perceived relationships between area and perimeter. As anticipated prior to intervention, the PSTs' KoST pertaining area and perimeter was relatively underdeveloped. KoST is an application of one's CK, and each PST possessed an incomplete CK regarding these concepts. Because of the important role CK plays in the organization of KoST, greater emphasis was placed on the analysis of the PSTs' CK in order to understand the quantity and quality of their CK and their lack of pre-intervention KoST.

Distinguishing Between Area and Perimeter

Although area and perimeter are used for different applications, they do have similarities. It is those similarities which make these concepts susceptible to confusion. Although each measure involves a calculation with sides, area and perimeter also require attention to their appropriate unit (i.e., linear or square). These concepts are intrinsically linked, and a PST with a profound CK and KoST realizes the importance and value of incorporating linear and square units within discussions involving area and perimeter.

Procedural versus conceptual CK. According to the survey questionnaire responses, the majority of the PSTs seemed to equate “teaching” about area and perimeter with describing a basic procedure for finding their measure. They focused on explaining “how to” find an answer. Of all the PSTs, Grace was the only one to attempt to differentiate between area and perimeter by discussing dimensions. Most PSTs addressed the concepts of area and perimeter without any discussion about their appropriate units. Perimeter was defined as the length around the outside of a shape - found by adding up the sides. Larry and Brianna defined area to be the “room” or “space inside a shape.” Jackie simply wrote “b × h.” Brianna also included that area is found by “multiplying the length by the width.” During the first interview, Jackie and Brianna were asked about their formula-based approach to finding area. They were asked how they would find the area of an irregular shape, like Figure 18, and if the formula “base times height” would produce the area of that shape. Both indicated “no” and after some time, proceeded to break the shape into triangles or triangles and a rectangle and said that formulas could then be used. I also asked Brianna if she thought it were possible that there existed a

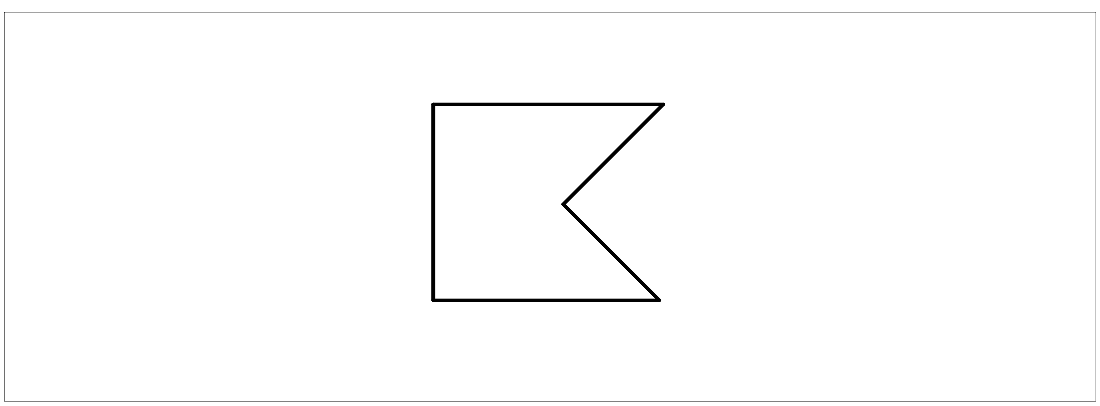


Figure 18. Figure introduced during first interview with case subjects.

shape whose area could not be found with a formula. She said, “Yes, but I can’t think of one.” Grace incorporated the idea of dimension when discussing area. She wrote, “The area is the space inside a 2 or 3-dimensional shape.” During our first interview, I asked her to elaborate on her response and she drew a square and a circle as representations for 2-dimensional shapes. When asked to clarify what she meant by the area of a 3-dimensional shape, she said, “Like, if it was a sphere; there is area within a sphere. So it’s the space within the perimeter.” Even though it appeared Grace confused area and volume, her correct mention of area being 2-dimensional was significant as she was the only PST to do so. No other PST wrote about perimeter being a one-dimensional concept.

Most PSTs were bound, even handicapped, by a dependency on formulas for both solving and explaining problems involving area and perimeter. Such a dependency might help explain why on the survey questionnaire 8 out of 12 PSTs indicated they were “apprehensive” about teaching area and perimeter concepts to elementary children. Question 6 from the pretest provided the context to investigate the PSTs’ CK regarding the area formula (A = L × W), which was done during the first interview with the case subjects. In this problem, a student (Pete) correctly calculated the perimeter of a 3cm × 6 cm rectangle (included in problem), but is confused about what exactly the 18 represents. PSTs were asked to respond to Pete’s confusion. As a result of less than adequate responses, it seemed appropriate to further investigate the PSTs CK regarding the area formula. The first interview with the case subject provided an opportunity to do that. In order to explore the case subjects’ understanding of a very common area formula (A = L × W), the question, “Why does multiplying length times width produce the area of a rectangle?” was asked during the first interview. A conceptual explanation of this question should involve a discussion of arrays (i.e., rows and columns of square units). Their responses revealed different levels of knowledge and understanding regarding common procedures used to find area.

Jackie gave a candid answer, “To be honest with you, I just know that you multiply the base times the height and you’ll get the area. I have no idea why.” When asked about the grid of 18 boxes she drew on the 3 × 6 rectangle in question 6, Jackie talked about how she remembered the square unit that was given in question 1: “I kind of just thought maybe I would try this. It did come out to 18, so I guess that could be a way to show by doing square units.” Larry seemed to grasp that the dimensions of the rectangle (3 × 6) could be used to insert the correct number of boxes (he called them centimeters squared) along the length and width and that the L × W formula was a shortcut to add up the 18 boxes that could fit inside. Larry did not visualize or grasp the row-by-column structure of the rectangle but instead saw the square units as simply something to be counted. Contrast that explanation with Brianna who described the rectangle as possessing “6 rows and 3 columns” (she confused rows and columns) and Grace who discussed that the rectangle is comprised of “6 columns with each containing 3 units.” While this language needs some refining, a realization of the array structure is a significant part of a foundation upon which conceptual knowledge and instructional strategies can be built. On the contrary, it was apparent Jackie did not comprehend the row-by-column structure in the 3 × 6 array. It is therefore not surprising then that she does not understand why the multiplication formula enumerates the units in the array.

Perceived student difficulties. The PST’s pre-intervention KoST was revealed in their responses to the last prompt on the survey questionnaire. They were asked, “What do you think elementary students may find difficult regarding the learning of area and perimeter?" Because all four case subjects were juniors, they would have had limited opportunities to interact with elementary students. The 12 responses were varied, but the majority of responses (9 out of 12) were along the lines of "students would most likely confuse the two," (Larry), and "have difficulty differentiating what formula to use" (Jackie). In contrast, Brianna and Grace touched on difficulties that went beyond a surface-level comment. Brianna thought that when students were presented with a rectangle with only the length and width given and asked to find the perimeter, they might get confused and only add the two sides, "since for area you multiply the length of only two sides."

Grace's response to the last question on the questionnaire showed elements of both novice and expert understanding. After providing a thoughtful response regarding perimeter for the next to last question, which focused on understanding the boundary properties of the concept, Grace indicated that "the different formulas" would be most difficult for students. However, during the interview when discussing what students might find difficult about area, Grace commented that, "I think that kids a lot of times forget that they are working with a 2-dimensional or 3-dimensional shape, and their answer might reflect a squared unit when it is supposed to be a cubed unit or vice versa." Grace was the only one who specifically discussed square units along with area at any time during the questionnaire. She did not provide any drawings to support her conceptual approaches to these questions. While Grace's responses included aspects of conceptual understanding, the majority of PSTs indicated that "getting the right answer" would be the primary source of difficulty for students, as opposed to understanding the concepts.

Distinguishing the Correct Unit of Measure

The major categories "Distinguishing between Area and Perimeter" and "Units of Measure" are not mutually exclusive, so there will be some overlap while discussing each. Several problems on the pretest addressed various aspects regarding units of measure. Because of the fundamental importance of units of measure, a greater amount of reporting will be devoted to it.

Confusing the measure with its unit. The first question on the pretest asked the PSTs to, on the grid provided, "draw a polygon that has a perimeter of 24 units" (Figure 19). The word "linear" was purposely left off so as not to bias or influence PSTs' responses. The second part of this question asks, "How would you help a 5th grader understand that the polygon you drew really does have a perimeter of 24?" Brianna and Grace had no problem with this question and justified their solution by similarly explaining that adding the lengths of all four sides of their rectangle would produce a

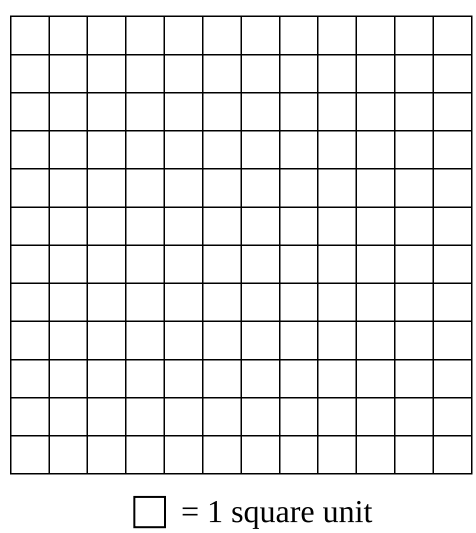


Figure 19. Grid included as part of question 1 on the pretest.

perimeter of 24 units. Compared with Brianna and Grace, Jackie and Larry were not as confident or successful. They were not alone, as this problem proved difficult for the majority of PSTs. Eight out of 12 PSTs provided a response that addressed, to different degrees, concepts related to area. A common shape drawn was a 6 × 6 square, which does have a perimeter of 24; however, the justification provided by several for the second part included shading the inside of their shape. It was difficult to discern whether they were claiming the inside or the boundary as the perimeter. Others, including Jackie and Larry, were confusing area and perimeter along with linear and square units (Figure 20). Jackie drew a 3 × 8 rectangle. There was a dot inside each box of her rectangle indicating she apparently touched each box as she counted them. The explanation revealed her confusion, "To be honest . . . I have no idea if the polygon I drew represents a perimeter of 24. But I guess I would show them that each box is 1 unit and in the box there is 24

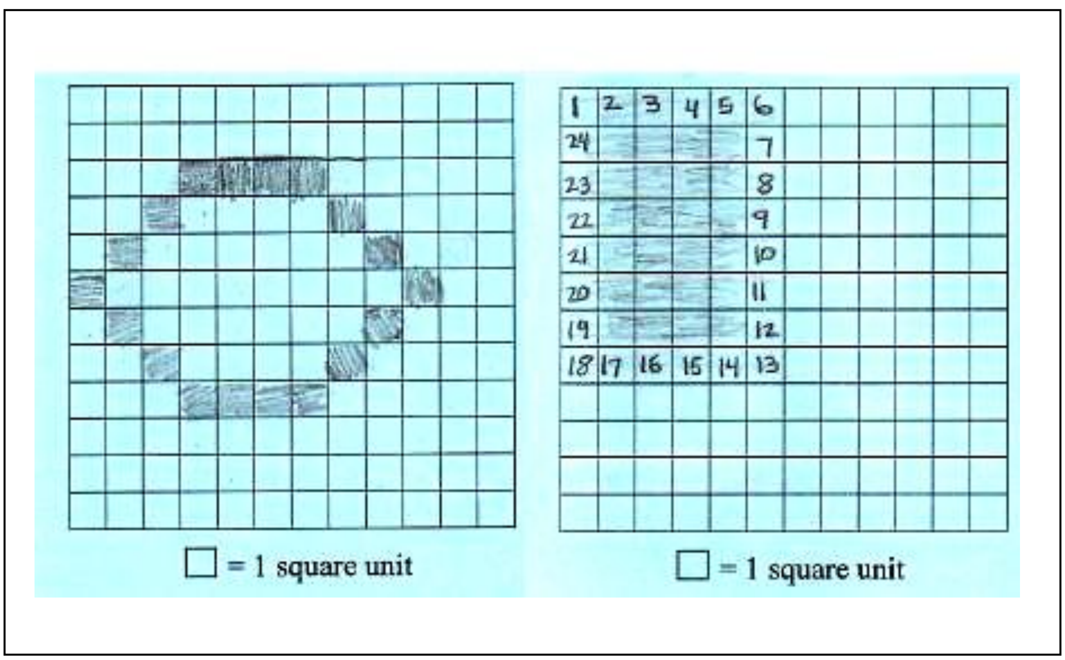


Figure 20. Samples of students' responses to question 1 on the pretest.

units?" During the interview Jackie indicated that after reexamining the figure, she claimed her rectangle might have an area of 24. Jackie's knowledge about area, perimeter, linear, and square units is disconnected. It is common for someone lacking a conceptual understanding of these concepts to wrongly believe square units are simply something to be counted rather than a subdivision of a plane (i.e., an area) (Battista et al., 1998). Jackie displayed this thinking when she responded to a question about how she determined her answer to question one on the pretest. She said, "I don't know, because my first approach was to count the boxes and then draw a line around the boxes."

Larry drew a 6 × 6 square and placed one dot in each box along the perimeter of the square. Larry wrote, "24/4 = 6; It might help to count out each square individually." The following interview portion reveals Larry's confusion about perimeter and units (T = teacher/researcher and L = Larry):

T: Can you tell me why you divided 24 by 4?
L: [Takes 10 seconds to reread problem and then 8 more seconds to think] I was thinking 24 because there are 6 squares on one side, so 6 times 4 is 24 – err, I'm sorry, uh yes. And then I took 24 and divided it by 4 to show that there are 6 sides. I think I may have been confused on this one. Maybe what I was thinking was it might help the student to count out each individual square to see if there are 6 squares on one side, six squares on this side, six squares, and six squares and adding those four together and it comes to 24.
T: So if you count up all the squares along the outside you are going to get 24?
L: [2 sec pause] Yea.
T: Would you please show me? I'm curious.
L: [Larry touches and counts the squares along the outside of the square he drew] It's just 6 on each side. I count the 6 along the top right here, so that's 6, then I counted these 6 along the side; I guess you count this corner one twice, because it wouldn't make sense if you did 6 and then – hmm, I don't know, I guess I'm just confused here.
T: Does this question involve perimeter or area?
L: [5 sec pause] Perimeter
T: You said it might be helpful to count out each square individually. What exactly do you mean by that?
L: Yea, I don't know what I was thinking here, because if each side measures 6 . . . [pause, then just stops]

It is very possible that the grid, or the hint provided below the grid, served as a visual cue prompting Jackie and Larry to think about area and/or square units. It is also possible Larry made a common error in conceptualizing perimeter as 2-dimensional. Either way, thinking patterns such as Jackie's or Larry's are evidence of low-level measurement reasoning, where consistent unit iteration is performed howbeit the wrong unit (Battista, 2006). Jackie's and Larry's rules-oriented approach to area and perimeter, inability to consistently focus on the correct unit of measure, and tendency to respond to superficial features of a problem indicate a fragile and novice understanding of these concepts.

Knowledge regarding irregular shapes. Question three from the pretest (Figure 21) provided insights about how the PSTs dealt with area and perimeter as well as units of measure of an irregular shape. The two PSTs who produced the shapes shown in Figure 20 had no apparent trouble solving this problem. The only mistake was one of

3. (a) What is the area and perimeter of Figure A? (All corners are right angles.)

   (b) Explain, as you would to a fourth grader, how you arrived at both your answers.

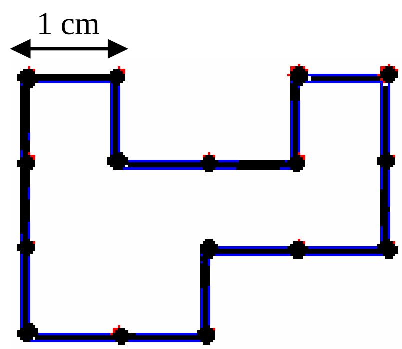


Fig. A

Figure 21. Problem 3 from the pretest.

them left off the “square” on the units for area. Only Jackie wrongly calculated the area while three out of the 12 PSTs (including Jackie and Larry) wrongly calculated perimeter. Several of the PSTs responded that they could figure out the area of the irregular figure in problem 3, but explanations revealed they lacked a strong conceptual understanding of square units. One PST gave the correct answer of “8 cm^2;” however, her explanation reveals her sparse awareness of square units, “I divided each section into perfect squares. The area of a square is s^2. So $1^2 = 1$ square; count up the squares to = 8 cm^2.” Larry also identified the area as 8 cm^2, and although he had partitioned the figure into 8 squares (a conceptual approach), his explanation was confusing and would not produce his answer: “Get the # of units on the length & the width & multiply.” A subsequent interview revealed Larry had an impoverished understanding of a square unit:

> **T**: I think I follow how you got the area. I just want to make sure. Would you recount what you did, or how you came up with your answers?
> **L**: I don’t think I used an equation on this. I just boxed it off. You put those little dots there, so I just drew lines and made boxes and counted the boxes. It’s 8, so it would be 8 centimeters squared.
> **T**: You said 8 centimeters squared. Is there a reason why the area is centimeters square, and the perimeter is centimeters? Is that meaningful?
> **L**: I was trying to think if it was something meaningful, or if it was just something I was always taught to do. I don’t think I can really tell you, to be honest, why it’s squared, except for the fact that that is the way I was told to do it.

The Microworld Orientation Session (Appendix M), which occurred almost one month after the pretest and one month prior to the first teaching episode, provided another example of Larry’s and Jackie’s difficulties with irregular shapes. When Larry encountered the first of two problems presented during the session (Figure 22), he just stared back and forth between his computer monitor and the four writing prompts related

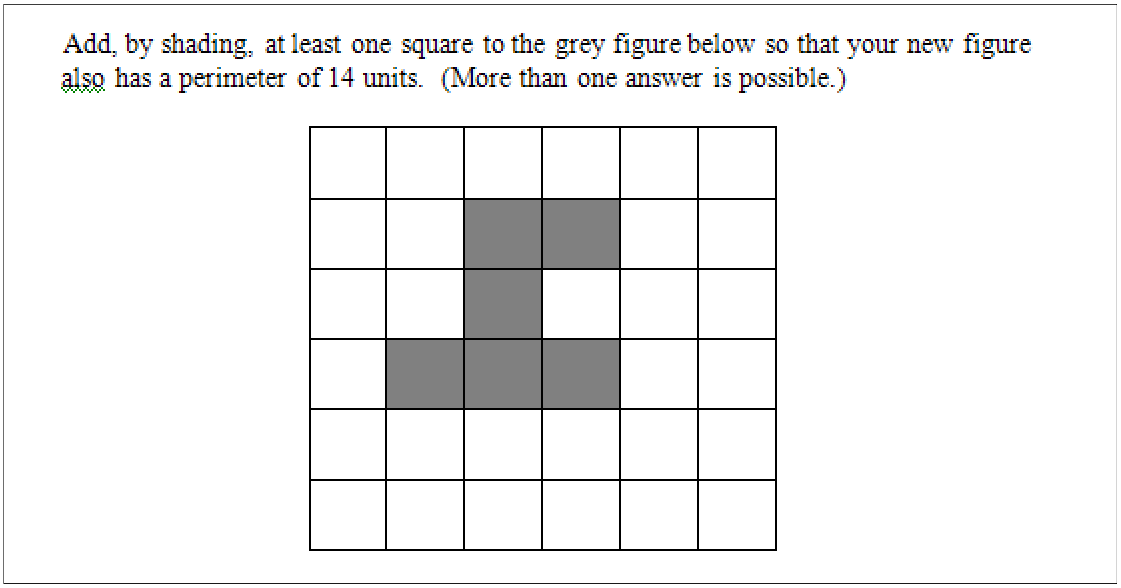


Figure 22. First problem presented in the microworlds' orientations session.

to the problem. Larry never created any figures in Shape Builder nor did he explore any of its features. Larry would often focus on only one way to solve a problem, a behavior of a novice teacher, and also had great difficulty imagining and testing hypotheses – even with the microworld (MW) tools.

Jackie's work with irregular shapes (first on pretest #3 and then on the MW orientation session) exposed a noticeable lack of CK regarding area and perimeter. On the pretest item (see Figure 21), Jackie wrote, "4 × 3 = 12" for her answer for the shape's area. Apparently she ignored the concavity of the shape and simply applied the area formula to the length and the width. I asked Jackie about this during our interview:

> **T:** For area, I see that you multiplied 4 times 3 to find the area. Tell me more about those numbers. Why did you do that and where did they come from?
>
> **J:** I specifically remember doing ones like these, but it's been a long time, so I had no idea. But what I kind of did again is that I broke this into shapes and you had the dots which made it kind of easier. So I kind of just broke it up like this in order to show you, so we knew that this [the labeled segment] was 1 centimeter, so I just kind of assumed because they all look like they have the same amount – length of side, so I just said 1, 1, 1, 1 [pointing across the top

of the shape] and I added it up to 4 and then I did the same thing for the right side; I went down 1, 2, 3, this is another one, [segment drawn in] so this is the main number for this side. It was the same for the bottom, and the other side [the left side]. I mean, I just brought everything down which, I don't know if you can do that, but I just took a guess and that's how I got 4 times 3 is 12.

T: So, might you be including area that's not part of the original shape?

J: Exactly, now that I see it again for the second time I realize that I just added more area, probably to the shape.

T: So, for shapes like this there's not a formula per se?

J: No

Jackie gave an answer of 14 for the perimeter, which is the perimeter of the 3 x 4 rectangle she built around Figure A, but not the perimeter of Figure A.

During the last part of the Microworld Orientation Session's planned activity with the Shape Builder microworld (see Figures 8-13), the PSTs were asked to comment on any particular features of the microworld that they saw as potentially helpful for the teaching/learning of area and perimeter. Jackie said she found the microworld "very helpful." "The whole concept of area was clarified for me. I have trouble with irregular shapes, so the Shape Builder allows me to see what is going on and to see relationships." I observed Jackie interacting with the microworld and verbalize some of her frustrations. She struggled with the perimeter of irregular shapes. She said she was not sure if counting the outside segments would give the perimeter. After replicating Figure 22 in Shape Builder and experimenting with it, Jackie indicated that she found it interesting that if she dragged a square onto the working grid (a feature in *Create Mode*) and placed it in the hole on the right side of the shape (Figure 21) that the number of countable, outside segments went from 3 to 1; hence, she concluded that counting the outside segments was the correct way to find the perimeter. It appeared that this was the first time she had decided, on her own, that counting dots (i.e., the endpoints of a linear unit) was not the correct way to find perimeter. The knowledge now seemed to be personalized.

Unlike Larry and Jackie, Brianna and Grace not only presented entirely correct responses to question 3 on the pretest, they also provided clear explanations of their methods. Both used a procedural approach involving dividing the figure (see Figure 21) up into rectangles and squares and adding the smaller areas; however, Grace went a step further and provided a second way to solve the problem. Grace displayed an understanding of conservation of area by explaining (and drawing) how the top 2 square cm on the corners of the figure could be moved to the "hole" in the bottom right; thus, forming a 2 x 4 rectangle.

Creative in problem solving. Being able to solve a problem in more than one way is a trait of an expert teacher. Grace displayed this trait when solving question 3 on the pretest and was the only PST to do so. Her problem solving lead to a planned follow-up with the other three case subjects during the first interview to see if they could also solve question #3 in a different way than they did on the pretest. Larry was unsuccessful. During the interview, it became increasingly evident that he could not intelligently talk about area, perimeter, and units of measure. Larry was unable to consistently identify what attribute was being measured (i.e., one or two-dimensional). In contrast, Jackie was able to find the area of Figure A in problem 3 using another approach:

T: Can you think of another way that could be used to find the area of this shape, since you are kind of stuck without a formula?
J: Um, without kind of looking at it, because I don't know if that would be the area, but these are the units within the shape [pointing to one of the boxes within the shape].
T: OK, and how many do you get when you count those up?
[Jackie uses a pencil to partition Fig. A into 8 squares]
J: 8. So that could be a way.
T: You got two different answers, right?

J: Yes, they're both different, but that would be getting rid of those boxes that aren't really there.
T: Is that kind of bothering you?
J: Oh yeah [recounting the actual squares], that would make perfect sense.
T: Does that make more sense?
J: Yes
T: And those boxes, I guess, we would try to describe those as being… [Jackie interrupts]
J: Units, or something. Each box represents one unit.

This was the first time Jackie was able to think beyond her initial response and problem solve in real-time. However, her initial overgeneralization (i.e., the use of formulas), along with her inability to coherently explain how she arrived at her answers, are examples of novice teacher's thought processes. Jackie's final statement, "Each box represents one unit" is also lacking complete understanding. Contrast that with Brianna's "ah ha" moment that occurred during an interview:

T: Can you think of any other way to find the area of that shape besides using the length-times-width formula?
B: [35 sec. pause] Well, if you broke it up into little squares by drawing dotted lines (student partitions shape into 8 squares) and added up all the squares. You have 8 squares, and then, I don't know, you would multiply that by 2 to get 16, but I don't know [sort of mumbling and trailing off].
T: For area or perimeter? Are you doing area?
B: Area. Oh ok. OH! So that would be right. 1, 2, 3, 4, 5, 6, 7, 8 (student points to and counts the 8 partitioned squares on the inside).
T: So each one of those squares represents what?
B: One square centimeter.
T: So for this problem could students figure out the area and perimeter without formulas?
B: Well if you are given that that the one segment shown as 1 centimeter, then I guess you could figure it out.

Brianna's response of "One square centimeter" could be considered a more conceptual way to refer to a square unit, as opposed to cm^2 which has procedural undertones (i.e., $cm \times cm = cm^2$).

Ability to explain and illustrate units of measure. The depth of the PSTs' CK regarding area and perimeter (units in this case) was observed whenever they were asked to explain concepts, as was the case with question 4 from the pretest. Statistically the most difficult question on the pretest, it provided insight into why some PSTs were having trouble consistently finding correct areas and perimeters as well as coherently explaining various aspects of these concepts (e.g., linear and square units). The problem asked the PSTs, "As a teacher might, how would you explain the concepts of *linear units* and *square units* to a 5th grader? Stress the differences in the concepts. Include a practical example of each (i.e., how they're used in the real world)." Jackie replied, "I don't know what this is either, sorry!" She then made an attempt, "Linear – units that cannot be measured. Square – units can be measured." Even though Jackie referred to the example of a square unit that was presented as part of question 1, that did not seem to inform her response to question 4. For practical examples, Jackie provided, "Linear – you cannot measure air. Square – you can measure a wall?" During the interview she remarked, "Ok, this is the question I had the most trouble out of any in this survey, because I really have no idea what a linear unit is." Jackie was not able to clarify her ideas much during the interview other than referring back to the square unit given in question 1 and mentioning how she thought maybe those could be used to measure a flat surface like a wall.

Larry explained a linear unit as, "a measurement of one side of an object" and illustrated his idea by circling the entire side of a rectangle, which classifies as very low measurement reasoning (Battista, 2006). Larry admitted he was very unsure about linear units but was "kinda sure" about square units. He defined a square unit as "a measurement representing a whole square within a shape or object." Similar to his work in problem 3, Larry illustrated square units by drawing boxes inside a rectangle. He appeared to think of square units as something to be counted, but his understanding of square units as a subset of a plane is unsettled. Instead of explaining the distinguishing characteristics of linear and square units and providing classroom-useful practical examples, Larry and Jackie (and most PSTs), simply explained how they are used (i.e., linear units are used with perimeter and square units with area). Brianna was able to more coherently and accurately distinguish between linear and square units, but when asked to illustrate her ideas the results were less than complete. Her diagram of a linear unit was a line segment which she labeled as "4 cm," and for a square unit she drew a 2 cm × 2 cm square. It was difficult to ascertain if she was implying that the 4 cm segment is made up of linear units and that square units would be used to measure the area of the 2 × 2 square or if she really thought of her diagrams as discrete units. Her previous work would indicate the later, but her understanding of these concepts is clouded at best. Of the two PSTs who described linear units as one-dimensional and square units as two-dimensional, only Grace provided enough information to establish her explanation as classroom-useful. Her explanation on the pretest focused on telling how the units are used rather than describing their properties. A portion of Grace's interview revealed a relatively solid understanding of these concepts, but provided evidence that she might not be able to explain them to elementary students in a meaningful way:

> **T**: Could you draw or show me what one linear unit might look like? This question is talking about linear units and square units. Would it be possible for you to illustrate those concepts?
>
> **G**: Yes, [Grace draws a square to the left of her writing]. When you are measuring a side of a square or a rectangle, you are measuring a linear measurement [she darkens the top of the square and draws 4 evenly-spaced tick marks]. So, say these [she points to one of the segments] are the units,

you would say that this side of this square has five linear units. It's one straight line in one dimension.

T: Ok, how about a square unit? What might that look like?

G: A square unit would be one that has 2 dimensions. It has a length and a width [Grace draws in tiny square in the upper right-hand corner of the same square she used to discuss linear units], and that would be what you would find for the area, so this would be the area – this unit right here would be squared, because it has 2 dimensions; the area of that [the tiny square] right there.

U*tilizing drawings.* An important aspect of a teacher's CK is the ability to explain concepts in meaningful ways (i.e., their explanatory framework), facilitated by effective communication. Incorporating suitable drawings is one important aspect of successful explanations. The extent of this facet of the PSTs' CK was evident when they were given the opportunity to hypothesize about future teaching.

PSTs were asked to respond to the prompt, "What would you do to help future students better understand area and perimeter?" Although 9 out of 12 PSTs made reference to drawing a picture or bringing in objects for display, only four provided drawings to represent their ideas. The ineffective use or lack of drawings to assist in problem solving or to clarify explanations is evidence of CK that lacks a well-developed explanatory framework, which turned out to be an all too common theme found within the PSTs' work. Larry was the only case subject to provide drawings to support his response; however, the drawings were sloppy and the response was incomplete, providing further evidence of insufficient CK. During the interview he was not able to elaborate upon his limited response regarding perimeter. When asked about his partially complete drawing of a rectangle with the square units drawn across the top row, his response revealed some recognition of the value of using grid paper when teaching about area. He said, "You could count across and count down and then you could multiply that and get that area," illustrating that he viewed the formula as a short-cut for finding area.

An emphasis on procedures was also evident in Jackie's and Brianna's responses. Although they suggested drawing pictures and bringing in objects to help students in understanding area and perimeter, their purpose for doing so was to assist in the explanation of how to use the formulas. Also, one of the objects that Jackie recommended was a cereal box, which could be useful for surface area or volume but quite confusing for discussing area. Alternatively, Grace seemed more concerned with a conceptual approach and thought it would be meaningful for students to see shapes drawn on a grid with the outside boundary "brightly colored" to highlight the perimeter. Regarding area, she recommended using a grid and highlighting the inside. However, Grace made no reference to discussing units for either perimeter or area. Her concerns with helping students understand area and perimeter were evident during our first interview. When asked why she would use a grid with the students, she responded, "It can help you show students the units that you are looking for." She then went on to elaborate on how both the "units of perimeter" could be traced and highlighted by going around the outside of a 2 × 2 square. She correctly called each outside edge of a square a "unit of length," although she never used the terms "linear" or "square" to describe the different units. A lack of realization of the profound importance of discussing units when teaching about area and perimeter limited the effectiveness of the PSTs' explanations here and throughout the study.

Several test questions (e.g., #'s 4, 5, 6, & 8 on the Pretest, #'s 1, 4, 6, 8, 9, & 10 on the posttest, and #'s 4, 5, 6, & 8 on the follow-up) were included with the expectation that PSTs would include appropriate drawings to clarify and support their explanation as well as to assist in effectively addressing student difficulties and misconceptions. Table 11 shows the PSTs' use of drawings for the four pretest questions in which the problem was written with the expectation that drawings should be used to effectively communicate a thorough response. Out of 48 potential opportunities (12 PSTs × 4 problems), only five drawings were provided that accompanied a meaningful and correct response. Question #4, which appeared on the pre-, post- and follow-up test was statistically the most difficult (Mean of 1.58, 2.33, and 2.33 respectively; range 0 to 4). That question asked the PSTs to "As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5th grader?" Most PSTs indicated that conceptualizing and explaining linear and square units was very difficult for them; however, only one of the 12 PSTs even attempted to draw a figure as a means to help visualize and/or explain these difficult concepts. Even when the PSTs were struggling to express themselves meaningfully, they would not provide a drawing to visualize the concepts or aid in the effective communication of their ideas. These traits reveal a novice level of problem solving. As the table 11 indicates, other times PSTs would suggest or refer to making a

Table 11

Pre-Intervention Use of Drawings

						Pretest Items						
PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry
#4 (U)											"X"	
#5 (R)	"X"	X		x					"X"	"X"		"X"
#6 (U)		*					X	*	*			*
#8 (R)	"X"		X	X		x		X	x	x	x	

Note. U = dealt with units, R = dealt with perceived relationships; * = suggested a drawing but did not draw it; X = used appropriate drawing; x = used a drawing inappropriate for teaching/learning; "X" = drawing did not facilitate a meaningful or correct response.

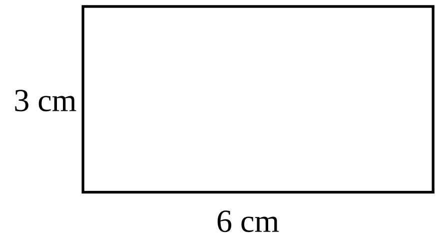
drawing, but would not actually draw one. Higher performing PSTs, (e.g., Grace, Brianna, and #6) would often provide a thorough written response, complete with accurate and informative mathematics, but void of supportive drawings. The PSTs' limited CK left them ill-prepared to construct a meaningful drawing, as was the case with question 4, while other times the PSTs were careless and drew rectangles that were not to scale and thus were not helpful in facilitating a correct response. Even though the PSTs would often write of how helpful visuals were for both themselves and students, supportive diagrams and meaningful representations were often absent from their explanations.

Responding to Student's Misunderstandings Regarding Units of Measure

The findings presented in this next section address the PSTs' understanding of, and how they indicated they would respond to, student difficulties and misconceptions, specifically regarding units of measure. These facets of the PSTs' KoST are manifestations of the organization of their CK and how well it enables them to understand children's thinking and subsequently respond appropriately.

The importance of units in explanations. Mathematical procedures, while effective at producing answers, typically do not inherently convey conceptual understanding of a construct. The area formula for a rectangle is a prime example of this. The PSTs' realization of the importance of connecting area with its appropriate unit was revealed in question 6 of the pretest (Figure 23). It asked PSTs if a student's answer of 18, for the area of a 3cm x 6cm rectangle, was "correct and complete." All 12 PSTs indicated that 18 was the correct calculation for the area of the rectangle, and only Brianna did not make any mention that Pete's answer was not complete because he forgot

6. Pete, a 5^{th} grader, calculates the area of the rectangle below. He arrives at an answer of 18.



(a) Is Pete's answer correct and complete?

(b) Explain why or why not.

(c) After performing the calculation, Pete comes up to you looking puzzled and asks what exactly the "18" represents or means. Respond, as a teacher might, to Pete's question?

Figure 23. Question 6 from the pretest.

to include the right unit. While attempting to respond to Pete's confusion about the meaning of the 18, explanations included: square units, units^2, small squares, square footage, 1cm x 1cm boxes, little squares, and centimeters. PST #4 attempted to explain the meaning of the 18 by writing, "Think of stuffing air into the rectangle. You have 18 cm to fill up." Only four out of 12 PSTs (one case subject) correctly identified "sq. cm." (or cm^2) as the appropriate unit missing from Pete's answer. Jackie indicated the correct answer was "18 cm." Larry said that "units^2" needed to be added to the 18, and Grace correctly commented that "the unit cm^2 needed to be included because he is using 2 dimensions." Part (c) addressed the PSTs' KoST regarding problem solving and use of representations. There were two anticipated avenues in which to approach Part (c). One possibility was to realize that the 3 x 6 rectangle has both an area AND perimeter of 18 and that Pete may have actually performed a perimeter algorithm. This realization should have evoked a response asking Pete to explain how he arrived at his answer as well as delineating the differences between area and perimeter and their meaning - even when they are represented by the same number. This same problem appeared on the pretest and follow-up test, and no PST ever mentioned that the rectangle also had a perimeter of 18, which would definitely cause students confusion and provide a "teachable moment." On the pretest, PST #6 discussed the fact that the 3×6 rectangle had a perimeter of 18, but that was only because she misread the problem and thought the student was supposed to be calculating perimeter.

Another avenue to approach Pete's confusion was to appreciate that simply applying the formula $L \times W$ does not directly help students conceptually understand what the answer represents; hence, a discussion about square units would be in order. Most of the PSTs stated they would show and/or explain what the 18 represented, but none, other than the case subjects, recommended gridding off the rectangle to expose the square units (i.e., centimeters). Jackie wrote how she would tell Pete that the 18 represents how many "centimeters" (as opposed to square cm.) are on the inside of the box. She also drew a 3 cm × 6 cm grid inside the rectangle but failed to mention the significance of the grid or how it could be helpful to student understanding. It is possible that Jackie simply confused cm with square cm. Larry, Brianna, and Grace realized the importance of a visual aid (i.e., a grid) to help explain the square units that were left off Pete's answer, but only Brianna, and to a greater degree Grace, placed an emphasis on understanding the meaning of the 18. Brianna wrote, "I would help by drawing the rectangle on a grid to represent the 18 square units inside the rectangle." Grace suggested, "Show him a grid of the rectangle and how 18 individual cm^2 fit into the rectangle; completely covering the area of the figure." While all three recommended using a grid or graph paper to represent the square units, yet again, none of the case subjects both drew AND adequately discussed an appropriate representation – evidence of incomplete CK and an inadequate KoST. As described in the previous section on PSTs' drawings, the absence of supportive drawings was an all too common occurrence.

Focused on solving, or diagnosing & responding. The majority of PSTs in this study tended to focus on solving the problem (i.e., finding an answer), to the neglect of diagnosing student thinking. This was very evident in questions 7 and 9 from the pretest. The PSTs' CK and KoST were both involved in answering these questions.

Question 7 on the pretest investigates the PSTs' understanding of linear measure in calculating perimeter, as well as their ability to diagnose a common student misconception regarding linear measure (i.e., point-counting). Point counting is the process of counting points around the perimeter of a shape in order to determine the shape's dimensions and thus its perimeter. The problem is shown in Figure 24, and the three questions related to the problem were: (a) Is Kayla's answer correct and complete? Explain your answer, (b) Explain what is correct and incorrect regarding Kayla's thinking, as evident in her work, and (c) As a teacher, how would you respond to Kayla? What precisely would you say and do? Larry interpreted the problem as though Kayla must use all 18 units of fence to build only one pen; therefore, his analysis of Kayla's work and her thinking resulted in Larry's suggesting that Kayla "read the question more carefully." During our interview, Larry contradicted himself and said his response to part (a) was wrong and that Kayla's drawing would be satisfactory and that "she's on the right track." Later during the interview, he contradicted his pretest responses again when he said, "Her numbers are right, but she did not draw them right. Each side needs one more."

Kayla, a fifth grade student, was asked to draw <u>all</u> the four-sided dog-pen designs that she could make using 18 units of fence for each design. Below are the drawings, on dot paper, that she came up with.

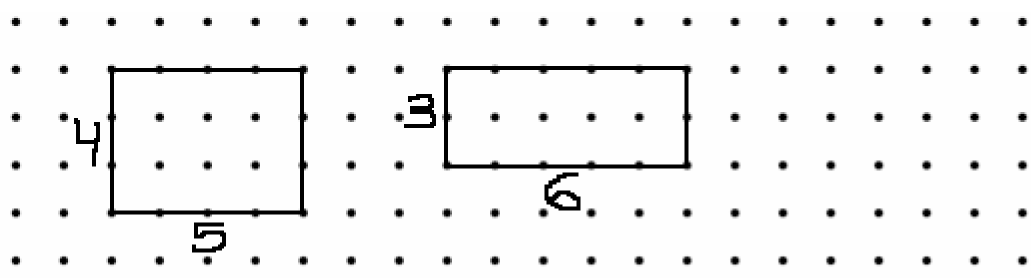


Figure 24. Question 7 on the pretest.

Larry never took any time to try to analyze why Kayla did not draw her dog pens correctly. His overall CK and KoST up to this point can be characterized by his comment, “I don’t know what I was thinking on this problem. I’m just kind of figuring it out as I go.” Larry’s frequent contradictions of himself are a strong indicator of an unstable CK.

Based on their pretest responses, Jackie and Grace interpreted the problem as involving area instead of perimeter. They both indicated the 4 x 5 rectangle would use more fence (20) than was allowed (18). During our interview, Jackie struggled with trying to explain Kayla’s thinking. Early on she did realize that the problem and the term “18 units of fence” dealt with perimeter instead of area. Jackie also eventually figured that Kayla was counting dots, instead of linear units, to determine perimeter but apparently found no problem in that: “She [Kayla] counted the dots and thought she was doing the perimeter and she did it. She got 18 by using that.” Jackie’s content knowledge is sparse and fragile (she often contradicts herself) and that appears to hinder her ability to effectively diagnose student thinking and identify misconceptions (e.g., point-counting). These are both traits of a novice teacher. While Grace began the problem with the same incorrect assumptions as Jackie, her comprehension of Kayla's thinking was much more acute. In her response to part (b), Grace correctly identified Kayla's measurement misconception: "She is counting the dots, not the lines." During our interview we addressed Grace's wrong assumption that question 7 dealt with area rather than perimeter.

In contrast to Jackie and Larry, Grace would not become flustered after realizing her thinking was incorrect. Expert teachers are able to carefully analyze a problem before and/or while solving it. Grace displayed this often. She would pause, reread the problem, gather her thoughts, explain where she had gone wrong and why, and then continue on with her work or explanation. Grace responded to the first interview probe by reasoning that the problem: "Is more about perimeter, I would say, and what she's [Kayla] counting are the dots. She doesn't understand that the unit is between the dots." This response reflected a change in thinking from her pretest. Grace continued to redraw Kayla's "dog pens" to the correct size. "She was thinking perimeter. She just didn't count the units correctly." Near the end of the interview Grace correctly identified that Kayla forgot to include an "8 by 1" and a "2 by 7" rectangle as possible dog pens.

Brianna was the only case subject who correctly interpreted the problem as involving perimeter, that Kayla's rectangles were missing a unit of length on each side, and that Kayla was confusing dots with units. During our interview Brianna explained Kayla's thinking: "I guess she was confused with the dots. She thought each dot represented a unit, but it it's really from one dot to another dot that is one centimeter – or one unit.” Brianna would often made good use of her strong mathematics background. At times she would quietly think for 30 or more seconds before making, what was usually, an insightful comment. I then asked her if Kayla had drawn all the possible pens that would use 18 units of fence. She had not offered any information about this on her pretest, but she thought for a second and said, without any written calculations or drawings, Kayla could have done an 8 by 1 and a 7 by 2. Of the other 8 PSTs, only three were able to decipher that question 7 referred to perimeter and not area. In regards to KoST, Brianna was one of only two (and the only case subject) to appropriately respond to Kayla’s thinking when she stated that, “I would show her that the dots do not actually represent units, but the distance from one dot to the next represents a unit.” A model response would have included the word “linear” in the response.

Another finding regarding question 7 involved the term “18 units of fence.” The phrase brought to light a certain degree of disconnect between the preservice teachers’ thinking regarding classroom mathematics and the real world. Several PSTs indicated that they thought Problem 7 was poorly written and that using the word “units” (which was by design) in conjunction with fence was confusing; however, many of these same PSTs used the idea of enclosing something with fence to illustrate the concept of perimeter when they responded to other pretest questions. Thus, it can be assumed that many are unsure which attribute to measure, and which unit to use, when calculating area or perimeter.

The last problem from the pretest that explores the PSTs pre-intervention CK and KoST regarding units of measure is question 9 (Figure 25). Similar to question 7, this problem produced valuable findings related to the PSTs’ intervention choices when

9. Jose wants to calculate the perimeter of the shape shown in Figure 1. Jose's method is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those squares.

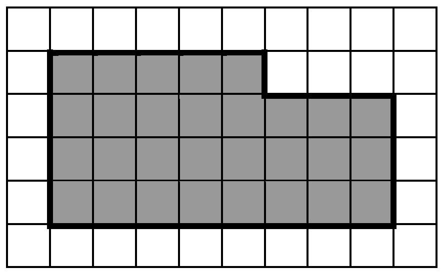


Fig. 1

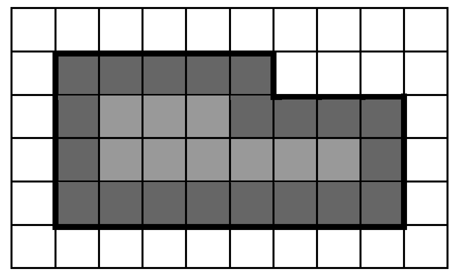


Fig. 2

(a) Is Jose's method correct? If no, what would Jose's method produce for the perimeter of Fig. 1, and if necessary, state what is the correct answer?

(b) Explain why or why not.

(c) As a teacher, how would you respond to Jose's thinking and his method? What specifically would you say and do?

Figure 25. Question 9 from the pretest.

responding to erroneous student thinking. Question 9 provided a useful variety of data as it was also the focus problem for teaching episode 1. In it, PSTs are asked to verify an untraditional approach for finding the perimeter of irregular shape. There were two aspects to correctly addressing problem 9. First, PSTs had to decipher the legitimacy of Justin's method, and secondly, prescribe an approach to address his thinking. Ten out of 12 indicated Justin's method was wrong. However, explanations involving how to respond to Justin took different paths. A common explanation provided for why Justin's method was wrong was he did not count the corner boxes twice. There were six PSTs, including Larry, who responded this way. These PSTs focused exclusively on the correctness of Justin's method and whether he was right or wrong. They spent no time discussing the mathematics undergirding his approach (i.e., using square units to determine perimeter). As is common with novice teachers, they tend to respond to faulty student thinking by simply reiterating what they know about the topic at hand, rather than investigating the student's thinking and what lead up to the their claim. Larry's response, and subsequent interview follow-up, illustrates this point. Larry said Justin's method was incorrect: "You have to make sure to count the corners twice if you do it that way." I asked Larry how he might respond to Justin, his method, and his thinking. Larry said:

> I mean if that helps him, I think shading and counting the boxes, might help him, but he needs to do it the right way if he is going to do it. Right now he's not coming up with the right answer. I guess if you explain perimeter and how each side, you know this is a side of 8 [counting along the bottom of Fig.2] and a side of 4 [counting the left side of Fig. 2], and add that up accordingly, and go through it and count everything out. Just show them both ways and how they both work. And help him work through it a little easier, so he knows he needs to count the corners twice for each side and he understands why.

Larry was able to correctly determine the perimeter of Figure 1; however, his comprehension of Justin's thinking was inadequate and his subsequent instructional recommendations would confuse classroom students. Perturbations can lead to a stronger understanding and more flexible content knowledge, but only if the cause of the dissonance is actively investigated and the misconceptions identified and addressed. An important finding resulting from question 9 was that none of the six PSTs who focused solely on the *rightness* of Justin's method explored to see if Justin's method worked for different shapes (i.e., look for a counterexample). Regarding units of measure, Larry's content knowledge was limited in scope and his knowledge of student thinking was narrow in focus.

Jackie's responses to Justin's erroneous method revealed the fragile nature of her content knowledge. Initially, it appeared that Jackie seems to grasp the error in the student's method. She indicated that Justin's method is not correct, because "Justin is not determining the perimeter, but the area." However, she could not analyze Justin's thinking much past that. Apparently the insights gained during previous interview dialogues had not been incorporated into her evolving knowledge, or they were never actually learned at the time. During an interview Jackie seems to incorporate various elements of different problems, but without any systematic approach:

T: Since you said Justin's method was not correct, what would the correct answer be for the perimeter for Figure 1?
J: Um let's see. [Jackie takes several seconds to look over the problem.]
T: Tell me what you are counting, what you are thinking.
J: Well, I was going back to what we were doing before with the problem back here [student refers back to problem #7]. The thing is this shape goes back down, like that [tracing over the one unit drop along the top]. It's not a typical shape. So, I'm thinking more, you know sometimes they can break shapes up. I don't know where I was coming from though. I just remember doing that. So there is 5 on this side right here [referring to the left side of Fig. 1].
T: Five what?
J: Dots, well, we're trying to figure out the perimeter of this? Ok, yes, the dots [Jackie draws 5 dots up left side of Fig. 1]. There are 6 [student draws in dots along the bottom of Fig. 1] down here, 6 up there [student draws in 6 dots along the first part of the top] and 5 for this side [student draws in a line down through Fig. 1 and labels it 5]. And then you could do this one too (student points to what she labels as a 4 × 4 square within Fig. 1], but kinda where I get confused too, like figuring out, do I just do the perimeter of this one [student refers to the outside of the "4 × 4" part], and then the perimeter of this one [student traces around the "5 × 6" rectangle], and add those two together to get the full object? Or, do I do a different way of doing it? Like do I, you know how before I had kind of added extra units, but that would be for the area. So… [Jackie unable to finish thought]

The idea of counting dots, instead of linear units, to find perimeter was contained in question 7. When we addressed that problem, Jackie indicated that such a method was wrong. Two problems later however Jackie implemented the exact same method (Figure 26) in an attempt to find the perimeter of Figure 1 in question 9. Jackie's final answer for the perimeter of Fig. 1 was 22 + 16, although she was not sure it was right. Jackie actually contradicted herself two different times while explaining her thoughts on this problem, and even had trouble remembering the details regarding Justin's method. It is obvious that Jackie's CK regarding area, perimeter, and units of measure is fragile and disconnected which negatively affects her explanatory framework and her ability to appropriately address the shortcomings of students (her KoST). Larry and Jackie, as well as others, struggled with conceptualizing perimeter and what it measures. This reflected poorly on their CK.

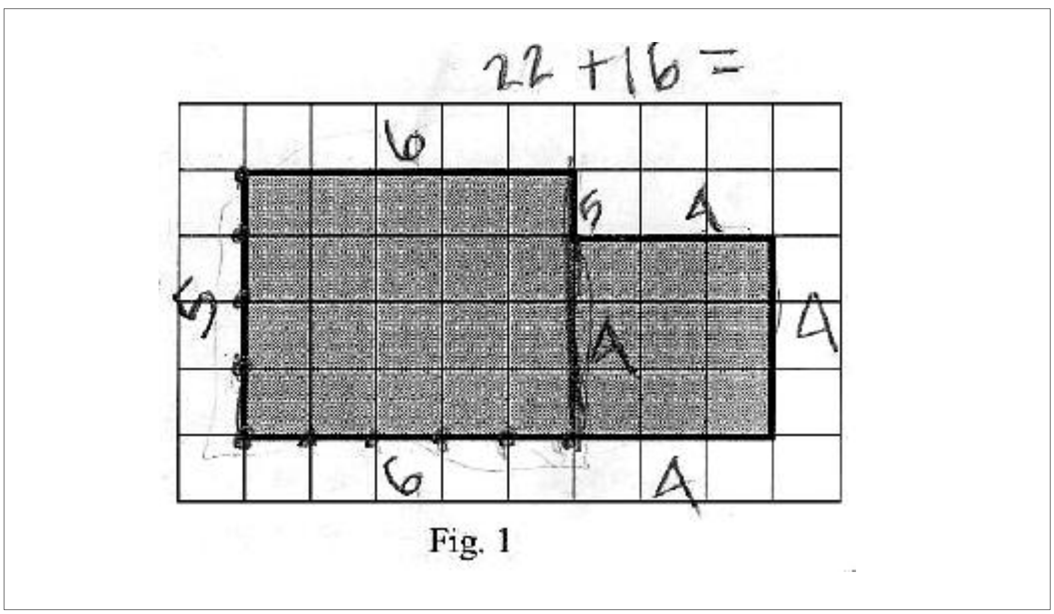


Figure 26. Jackie's method to find the perimeter of Fig. 1 (part of problem 9).

Grace also struggled to diagnose Justin's thinking, but in different ways and not to the same degree as Larry and Jackie. Grace knew Justin's method contained mathematical inconsistencies: "The squares shaded are 2 dimensional; he should be counting the lengths of the outside boundaries of the shape." However, Grace must not have investigated Justin's thinking thoroughly: "I would say – even though you get the right answer this time; it may not work in all situations." During her interview, Grace was given the opportunity to revisit Justin's method. She correctly figured the perimeter to be 24, but became confused and frustrated when Justin's method produced a perimeter of 21. She was not able to reconcile the discrepancy. In the end, Grace decided that even if Justin's method did work sometimes, it is not a helpful method for students to use since it did not work all the time: "You don't get the correct answer in this problem." It appeared Grace had a good amount of CK regarding units of measure (e.g., knew about dimensions), but struggled using it consistently to diagnose student thinking and therefore could not adequately address certain student misconceptions regarding theses concepts.

Brianna earned a score of 4 (a model response) for her clear explanation of Justin's thinking as well as her suggestions for how to assist him: "He's confusing linear and square units, and the perimeter you have to use linear units and he's using the square units. I would explain the difference between linear and square units, and that the shaded boxes are square units." Brianna did not stop after diagnosing Justin's faulty method. She also explained how she would step through the problem with Justin and count the lengths of each side to get the perimeter and then compare that to the number you would get doing it Justin's way. While Brianna's intervention with Justin should help to clear up his confusion, it is always more meaningful when students are actively involved in their education. A thorough KoST would have also included such an approach with Justin. Brianna's methods (i.e., "show and tell") are all too common with this study's PSTs, especially those who indicated that that is how they were taught, as Brianna did in her questionnaire. Brianna's pre-intervention CK regarding units of measure was sufficient to get correct answers, but it was very procedural in nature and application. Her CK was organized enough to allow her to diagnose many student difficulties; however, the focus of her explanatory framework was more about getting correct answers than it was about developing conceptual understanding, which is not the goal of a more expert KoST. At this point Brianna was focused on "how" than about "why." Brianna's strong mathematical foundation translated into very teacher-centered approaches. She was not alone in this tendency. Unfortunately, it was found that PSTs who indicated they would allow students the opportunity to personally work through the various mathematical concepts was uncommon, and encouraging students to investigate further with manipulatives or technology was almost nonexistent.

Perceived Relationships Between Area and Perimeter

The perimeter and area of a figure are two different measures. The perimeter is a measure of the length of the boundary of a figure, whereas the area is a measure of how much space a figure occupies. In the case of a rectangle, the calculations of both measures are related to the sides of the figure. These similarities provide the setting for two classic misconceptions involving the area and perimeter of a rectangle: (1) That increasing the perimeter of a rectangle will always increase its area (i.e., the *direct-relationship* misconception), and (2) Rectangles that have the same perimeter measurement will also have the same area, and vice versa (i.e., the *fixed-relationship* misconception). The first misconception appeared in question 8 of the pretest and took the form of a classroom scenario. The second misconception was contained in question 10 and involved a problem-solving situation.

Knowledge of the direct-relationship misconception. Question 8 on the pretest presents the PSTs with a special case involving area and perimeter. The scenario is as follows: "Jasmine [a hypothetical 5th grade student] claims that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area." The PSTs are then asked: (a) Is she correct? (b) Explain why you agree or disagree with Jasmine's thinking., and (c) How might you as a teacher respond to Jasmine? What specifically would you say and do? Ma's (1999) research, involving a very similar problem, aided in the analyses of the case subjects' responses and characterizations of their levels of understanding related to relationships involving area and perimeter. When this problem has been used by other researchers, it typically includes two rectangles (a 4×4 and a 4×8) complete with area and perimeter calculations provided by the hypothetical student as "proof" of their claim. Question 8 did not provide such rectangles in an attempt to not influence the PSTs' responses.

There are two major aspects to this scenario involving the direct-relationship misconception: (a) the PSTs' reaction to the claim (related to CK), and (b) the PSTs' response to the student (related to KoST). Because these findings are pre-intervention, not only was the PSTs' CK relatively underdeveloped but their KoST was even more so. The KoST findings regarding the direct-relationship misconception were sparse and therefore will be interspersed within the CK findings. Four out of the 12 PSTs, including Larry and Jackie, indicated that Jasmine's claim was correct. Their explanations tended to be built on the incorrect assumption that increasing the perimeter of a rectangle must increase both dimensions and thus the area, and were similar to: “Because the longer the perimeter, the longer the sides, and the more area the box will have.” Jackie and Larry provided no mathematical examples or pictures to support their response – evidence of inferior CK and KoST. Their lack of understanding regarding the mathematics surrounding the student’s claim left them ill-equipped to engage the student in any meaningful discussion regarding that claim. The other two PSTs attempted to justify the invalid claim by providing sample rectangles, including diagrams and calculations, illustrating that an increased perimeter did in fact result in an increased area. They correctly identified the student’s claim as a mathematical relationship; however, they failed to notice that the perimeter of a rectangle can increase as two of the sides of the rectangle decrease in length. The 4 PSTs, who said the claim was true, thought an appropriate response to Jasmine should involve praise and an example or two illustrating her claim:

> I would take simple measured boxes (1×2 cm and 2×4 cm). I would calculate the perimeters of both (6 cm and 12 cm), then calculate the areas: $1 \times 2 = 2$ cm^2 and $2 \times 4 = 8$ cm^2, then show the relationship that the larger perimeter is also the larger area.

Larry’s response to Jasmine was simply to “Tell her she did a good job.” During our interview I asked Larry if he could give me an example that would illustrate or support Jasmine’s claim. He referred back to question 5 on the pretest: “If each of the dimensions of a 2×4 rectangle is tripled, what is the relationship between the original and the enlarged?” “I’d just kind of show her that the 6×12 has the greater perimeter and it’s obvious that the rectangle is a lot bigger [than the 2×4].” By simply justifying the student’s invalid claim and not investigating any other possibilities, Larry displayed a limited knowledge of this area/perimeter relationship (i.e., Level 0) according to Ma (1999). Larry’s responses were often brief and incomplete. He exhibited little knowledge of the direct-relationship misconception or surrounding concepts, he neglected to use representations, and his primary concern was getting, what he thought to be, right answers. These are all characteristics of a novice teacher and an underdeveloped KoST.

Jackie agreed with Jasmine’s claim but also added, “It all depends.” Her explanation revolved around the idea that “the bigger the object is, then the more area it takes up.” I tried to guide Jackie into summarizing Jasmine’s claim into some sort of mathematical property or rule, with the thought that might make it easier for her to decipher the validity of the student’s claim.

> **T:** Now regarding Jasmine’s claim, can you restate her claim in your own words just so I know that you understand what she came up with?
>
> **J:** [Student rereads problem] The question says, Jasmine claims that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area. I put yes, because it’s, but I wasn’t really sure about this, so, my thinking, initially, kind of going back to the rectangle problem when you triple it and you get the greater area [Question5]. I said, yes, because the bigger the object is the more area it takes up. That was kind of my reasoning. And I said, sometimes the side of something is a large number, but the width is small. So, sometimes the ones that appear smaller have the bigger area. I don’t know if that’s confusing though.
>
> **T**: If it’s longer, will it always have more area?
>
> **J:** No, not always, but say this is like 15 and then 2 [student draws a 2×15 rectangle, call it #3] or something like that. And then this one was 4 times 4 [student draws a 12×12 rectangle, call it #4]. I don’t know, sometimes though the opposite can happen. A child will look at this [rectangle #3] and think, oh, 15, that’s definitely bigger, but this one [pointing to rectangle #4] is really the one that’s bigger. Does that make sense? I don’t know. I’m not drawing really correct illustrations here.

In Jackie's rather long explanation of her thinking on this problem, she correctly identifies a tangential misconception students' have regarding relationships between area and perimeter. It is common for students to think, when comparing rectangles, that the one with the longest side will usually have the greater area. We continued:

> **T:** So, do you think that is what Jasmine was claiming? That the rectangle with the longest side will have the greater area or is that just part of her claim?
> **J:** I would say more part, because now I'm understanding this question a little bit better.
> **T:** She's thinking that as you increase the perimeter….
> **S:** But I do agree with her. I think that when you increase the perimeter you do have greater area, because it's bigger, a bigger object.
> **T:** And that would always be the case?
> **J:** Yes, but that is what I was thinking for Part (a), I think; more like that. I don't know where that [what the student originally wrote for Part (b)] came from.

I then explored her pedagogical content knowledge regarding her response to part (c).

> **T:** In Part (c) you mentioned that you would try to bring in actual rectangular objects. I like that idea. How would you go about determining the perimeter of objects you brought in?
> **J:** I was thinking measuring them with an actual ruler or something, but that's probably more along the lines I was thinking of, but seeing some of those manipulatives too, those would be really helpful for figuring out if you had like the smaller rectangle with the rubber bands, the geoboard I think it is, the rubber bands, and then you did a bigger one and show that there's way more; if you put little, for the units, the square units in it, you could show that the bigger, the more perimeter, the bigger the sides the more area there is in it. So I think I'm just becoming acquainted with what's out there to use, too. But, if you want to be really old fashioned, you can use a ruler.

Jackie actually gave the previous response without pausing. This is an example that, up to this point in the study, characterizes Jackie as a learner – her tendency to ramble in her responses to the point where she digresses away from the original question. The conclusion of our interview related to question 8, reveals Jackie's inability to keep her previous and emerging thoughts organized while engaged in a learning situation:

T: So if you had the geoboard what would you be counting to find the perimeter?
J: I was thinking about the . . . [students draws a 4 × 4 “geoboard” in the margin of her pretest while talking]. Look, like this, they have the little dots, the pegs, and they are kind of even and I was thinking back to those units again [student connects the dots to make rectangles inside her geoboard. See this is your object. You can do this again, and then do this with a smaller one [another rectangle].
T: So, if you are going to try to calculate the perimeter of one of those shapes, what would you be counting to try to figure out their perimeters?
J: The dots? Back to the dots. [student laughs out loud]

Apparently Jackie now thinks Kayla’s thinking from before was correct (Figure 26). As was often the case with Jackie, even an initial correct mathematical response would be found to be built on a fragile conceptual understanding of the concepts at hand. Jackie was not able to successfully justify the student’s invalid claim, which is the lowest knowledge level established by Ma (1999) for measuring understanding related to this misconception. Jackie had difficulties explaining her ideas, which resulted in poorly structured interventions with potential students regarding their struggles in the pretest questions. Her CK was insufficient and unorganized, which appeared to impede her KoST and hamper her ability to diagnose and appropriately respond to student thinking.

Investigating a student’s claim – CK informing KoST. The responses of four PSTs (including Jackie and Larry) regarding the direct-relationship misconception, contained in question 8, indicated they had not completely examined the student’s claim. They stopped after explaining why the claim could work and did not investigate the cases in which it would not work. Providing a counterexample was the most straightforward way to disprove Jasmine’s claim. The other eight PSTs indicated that Jasmine’s claim was incorrect. Of those, five PSTs (see Table 12) said Jasmine’s claim was incorrect but their explanation and/or counterexamples did not directly disprove the claim; for

Table 12

Investigating an Erroneous Student Claim (Pre-Intervention)

Number of PSTs (*n* = 12)	Agreed with the student	Provided appropriate counterexample	Investigated the claim	Ma's "Level of Understanding" attained
4 (including Larry & Jackie)	Yes	No	No	Level 0
1	No	No	No	In-between Level 0 & 1
4 (including Grace)	No	No	Yes	Closer to Level 1 than Level 0
3 (including Brianna)	No	Yes	Yes*	Level 1

Note. *Implied, but did not provide examples that student's claim could work.

example, "You can have two objects with the same perimeter and not the same area." Such a counterexample would disprove the existence of direct relationship between area and perimeter but would not directly address the claim which revolves around increasing the perimeter. It is possible that question 10 influenced some of the PSTs' thinking, as was the case with Grace. On her pretest Grace indicated that Jasmine's claim was incorrect, but the explanation justifying her position did not make mathematical sense. Grace's recommendation for how she would respond to Jasmine and her thinking (e.g., "I would give her examples of two rectangles which disprove her theory.") was uncharacteristically shallow, teacher-centered, and focused on getting the right answer. It was noticeable that Grace had done a lot of erasing while answering this question. It also seemed uncharacteristic that she did not provide any diagrams to support her response. The reasons behind these occurrences and her poor applications of her KoST became evident during the interview:

> Actually, I [Grace] said she was correct at first. Because I was thinking that if you were looking at a fence and you're going to have this much area around the fence, then if you have a longer fence, you are going to have more area. But, then I got to the end of the test and saw the last question [#10] and all the perimeters were the same, but the areas were not the same. So I thought my thinking is wrong here, so I went back to this one and then ran out of time. But, what I said is that she was incorrect because the greater the difference in the length of the dimensions, the smaller the area. Even if the perimeters are the same.

Grace ran out of time, but her abbreviated response revealed she had begun to explore the relationship: "The greater the difference in length of dimensions, the smaller the area – even if perimeters are the same." She did not have enough time to provide a meaningful intervention with the student beyond: "I would give her examples of two rectangles which disprove her theory." Grace's first response indicated that she was in the process of discovering that a square is the rectangle with the largest area, an idea she would develop more fully later in the study. That is a relatively high level of understanding related to this problem (Ma, 1999). However the student's claim was not based on the perimeter remaining the same, and when Grace was made aware of this she was not able to make any significant progress in disproving Jasmine's claim. Her attempt to disprove Jasmine's claim indicated she was approaching a *Level 1* understanding of the relationships between area and perimeter (Ma, 1999). Evidently she was slightly embarrassed by her inability to sort through the elements of this problem. Her CK regarding perceived relationships was incomplete. Since her initial thoughts were wrong about this question, she ran short of time on the pretest; however, as will be shared in later sections, Grace had only begun to fully investigate the possible conditions involving this problem. Grace was not generally satisfied with leaving mathematical conflicts unresolved. The last thing she said regarding question 8 was, “I have been thinking about this problem for the last couple days, but did not really have time to play with it, but it was really bugging me.”

Three PSTs indicated that Jasmine’s claim was incorrect, and also mathematically investigated the claim in an appropriate manner. Their explanations were similar to Brianna’s: “There are many times when a rectangle has a smaller perimeter than another rectangle but has a larger area.” Of those seven, two presented counterexamples involving irregular shapes – the question specifically mentioned rectangles, and another’s “counter-example” involved two rectangles with equal perimeters having different areas - the claim involved *increasing* the perimeter. The remaining three, including Brianna, provided an appropriate counterexample to disprove the student’s claim as “always” being true. By using words such as “many times,” sometimes,” and “it depends,” these three acknowledged the fact that the student’s claim might hold under certain circumstances; however, because they did not provide suitable examples or explanations, they did not fully attain Ma’s *second level of understanding* (1999).

The three higher levels of understanding (Ma, 1999) went unexplored by PSTs. There are three possibilities to identify when the perimeter of a rectangle is increased: (a) the area can increase, (b) it can decrease, or (c) it may stay the same. The majority of the PSTs only discussed the first possibility. Three provided correct examples of the second possibility, but did not acknowledge that Jasmine’s claim could hold in some circumstances. None of the PSTs mentioned or discussed the third possibility. Besides identifying or displaying one of the three previously mentioned possibilities, none of the PSTs revealed the two higher levels of understanding: (a) clarifying the conditions under which these possibilities held (Level 3), and (b) explaining why some conditions supported the student's claim and why other conditions did not (Level 4). The PSTs in this study simply stopped exploring the problem after arriving at one of the three possibilities, assuming they had adequately answered the question. Although 8 of 12 PSTs provided diagrams to support their explanations, only two of them were suitable for classroom use. Even though Brianna's explanation of why she disagreed with Jasmine's claim was incomplete, she was one of three, and the only case subject, to fully reach a "level 1" understanding as explained above. Larry and Jackie were functioning at a "level 0," and Grace was in between a level 0 and a level 1. Brianna's diagnosing of the student's thinking was partially successful in that she was able to understand that Jasmine's claim was not always true; however, her partial CK regarding all the relationship possibilities resulted in an incomplete intervention of Jasmine's misconception by Brianna, and revealed a less than thorough KoST regarding this misconception. No PST suggested engaging the student in exploring the truth of her claim. Instead, their responses indicated they would "show" or "explain" the answer by providing specific examples.

Knowledge regarding the fixed-relationship misconception. The last question of the pretest addressed the second and final prominent misconception related to perceived relationships between area and perimeter – the notion that rectangles with the same perimeter measurement will also have the same area (and vice versa). The question also investigated the PSTs' knowledge regarding the subtle, hierarchical relationship that would include a square as a type of rectangle.

The question states, "Mr. Jones purchased 60 feet of fence to enclose his garden. He wanted the garden to have a rectangular shape. He also wanted to have the most space possible for his garden. He drew out several possibilities, which are shown below." Five rectangular gardens are pictured (an 8 × 22, a 10 × 20, a 15 × 15, one 5 × 25, and a 2 × 28). PSTs were asked whether one specific garden is the biggest, or if they are all the same size, and to explain their selection. Question 10 (see Appendix D) was the overall easiest question on the pretest. It had a mean of 2.75 (range 0-4) and a standard deviation of only 0.45. The implications of this question's scoring statistics are discussed in Chapter 3 and in the limitations section. The potential misconceptions for question 10 were: (a) assuming that because all the gardens had the same perimeter, they would have the same area, (b) predicting the greatest area based solely on appearance, and (c) not recognizing and/or acknowledging that squares are also, by definition, rectangles. Every PST calculated the area for each garden, and chose Garden 3 (the 15 × 15 square) as the garden with the greatest area on those calculations; however, because no PST justified their response by stating that squares ARE rectangles, no maximum score of 4 awarded. The fragile nature of Larry's and Jackie's CK was evident when asked during an interview about their selection of a square (Garden 3) when Mr. Jones wanted "a rectangular shape" for his garden. Larry said, "That wouldn't be right then. If he wants a rectangle, then it needs to be one of the other four." Jackie replied, "That's a problem. I did not read that part. That [Garden 3] is not really a rectangle."

Grace and Brianna were more confident of their responses. Grace mentioned that she was running out of time on this problem and was not able to carefully consider all parts of the question. I asked Grace if all the gardens were rectangles. She replied, “They are all rectangles. Well, no . . . One is a square, but . . . I don’t think it’s a problem.” Grace was not sure about this, but she was not willing to give up on her hunch. I asked her, “Mr. Jones wanted a rectangular shaped garden, right?” After a 15 second pause, Grace replied, “I don’t know that it is a problem. To me it’s a square, but . . .” Grace’s justification for selecting Garden 3, “The closer to equal the dimensions; the greater the area” continues to build on her emerging idea that squares are the rectangle with the greatest area, although she did not say it directly. Brianna’s CK was the strongest of the four case subjects. When asked if selecting Garden 3 would be a problem because it was a square, she confidently replied, “No, because a square is a rectangle.”

Part (c) of question 10 asked the PSTs, “Which incorrect statement [e.g., ‘Garden 1 is the biggest garden.’] do you think would most often be selected by 4th or 5th graders? <u>Please explain your choice</u>. What might they be thinking?” This question helped reveal the PSTs’ KoST regarding the misconceptions present within this problem. Only four PSTs (no case subjects) identified the choice, “The gardens are all the same size” as the most common error that would be made by elementary students. That choice would characterize a student who thought that a specific perimeter can have only one area – the primary misconception addressed by the problem. Those four explained their selection along similar lines, “Because all the gardens have the same perimeter students would expect them to have the same area.” The majority however, including Larry, Jackie, and Brianna, identified Garden 5 as the most probable to be selected by elementary students for similar reasons as given by Brianna: “They might think that Garden 5 is the biggest, because it has the longest length." Basing the area of a shape on its appearance is a common misconception among elementary students (PSTS as well), but it was not the primary misconception of this problem. Grace apparently ran out of time and left part (c) blank. During the first interview, she was given the chance to offer a response. Grace thought about it for a minute and a half before saying, "I still don't know what they [the students] would say." She did however rule out choice 6 (The gardens are all the same size) by saying "I don't think that they would think that they were all the same size;" however, research has shown that the responses and explanations offered by many students (and even preservice teachers) indicate they do think choice 6 is viable. The fact that only four out of 12 expressed an awareness of this student tendency indicates the majority of the PSTs were not sensitive to the fixed-relationship misconception.

In sum, the CK and KoST for the four case subjects has been presented, discussed, analyzed, compared and contrasted. The strengths of Grace (her ability to carefully process information coupled with the desire to help students understand) and Brianna (her strong mathematical background and sharp attention to detail) have been contrasted with the fragile understandings of Jackie and Larry. A reflective statement made by Jackie near the end of our first interview aptly summarized the struggles that she, Larry, and other PSTs experienced prior to the study's intervention:

> I think my biggest problem is I just don't know why things are the way they are. I just kind of have this knowledge of formulas and a few concepts that I've learned here and there, and I think that some of them are mixed up.

Research Questions 3 and 4: PSTs' Emergent and Post-Intervention CK and KoST

The findings in the next several sections address research questions 3 and 4: How does the PSTs' content knowledge (CK) and knowledge of student thinking (KoST) regarding area and perimeter change, if at all, during the course of the study?

The emerging and post-intervention data came from (as they occurred in chronological order) the three teaching episodes (Appendix K), posttest (Appendix E), the second interview with the four case subjects, and the follow-up test (Appendix F). Findings were extracted from the PSTs' written responses to the three teaching episodes (TEs), the post- and follow-up tests, and from transcripts from the second interview. Descriptive statistics will be presented first, followed by qualitative findings meant to support and illuminate the descriptive results.

The first major category of findings deals with concepts surrounding units of measure (e.g., linear and square units). This category contains several sections of findings examining the PSTs' understandings regarding units of measure (i.e., their CK) as well as their ability to respond to hypothetical students who are struggling or have misconceptions concerning those concepts (i.e., their KoST). The PSTs' CK, prior to, during, and after the intervention, will be the focus of the first several sections of findings, and address research question 3. Findings for those sections were primarily taken from the pre-study questionnaire, the microworld orientation session, the post-, and follow-up tests, the second interviews, with brief references to teaching episode 1 (TE 1). There will then be a transition to the next major category of sections focusing on findings related to the PSTs' KoST; thus, addressing research question 4. Emergent findings from TE 1 will be presented and supported by relevant findings from the post- and follow-up tests, and second interview. In each section, *change* was examined by looking back and comparing to the PSTs pre-intervention CK and/or KoST that was presented while answering research questions 1 and 2.

The second major category of findings relates to *Perceived Relationships between Area and Perimeter* and will examine the PSTs' CK and KoST regarding the fixed-relationship and direct-relationship misconceptions. Findings will be presented in similar fashion as they were for *Units of Measure*. One major difference is that this second major category will involve findings from two teaching episodes – TE 2 and TE 3.

The findings related to *emergent* CK and KoST were extracted from the PSTs' responses to the numerous writing prompts contained within this study's intervention – the three teaching episodes. A very brief synopsis of this study's framework will help explain the intervention and set the stage for the discussion of findings that will answer research questions 3 and 4.

A Teacher Development Experiment

The intervention for this study was couched within a teacher development experiment. A dynamic of the teacher development experiment (TDE) is the opportunity to perform the role of instructor and researcher simultaneously while attempting to promote development (teaching) within the preservice teachers as both students and future teachers all taking place within a cycle of interaction and reflection (Simon & Tzur, 1999). Whole-class interaction for this study took the form of three individual teaching episodes (see Appendix K). The most prolonged individual interaction occurred during the second of two planned interviews with the four case subjects. The goal of this

TDE was to contribute to teacher educators' understanding of how preservice teachers resolve conflicts and deficiencies in their current content knowledge (CK) and knowledge of student thinking (KoST), as related to area and perimeter, and how they endeavor to incorporate new knowledge (preferably conceptual rather than procedural).

The major components of this TDE were the three teaching episodes. Anchored instruction (anchored on major misconceptions surrounding area and perimeter) provided the scaffold for each teaching episode and two specifically designed microworlds were intended to offer support and motivation for the PSTs as they explored concepts and tested hypotheses. I conjectured that the microworlds would provide a fertile "playground" to facilitate the exploration of documented misconceptions, as well as certain profound subtleties, related to area and perimeter.

Emergent Levels of CK and KoST

As explained in Chapter 3, findings involving CK and KoST will involve addressing key components of their definitions. For CK that includes: (a) the amount and organization of facts and concepts, and (b) the ability to explain that knowledge in meaningful ways. For KoST that entails: (a) organizing CK in a way that would enable a teacher to understand children's thinking, and (b) appropriately addressing any shortcomings or misconceptions. This was true for research questions 1 and 2 and will again apply to answering of research question 3 and 4.

Identifying examples of expert/novice behavior (Table 3, page 166) within the PSTs' work was an important aspect in describing their emergent levels of CK and KoST. Table 13 (p. 251) displays the total frequencies of novice ("a") and expert ("b") behavior as seen in each PST's teaching episode responses. It contains frequency

Table 13

Expert/Novice Coding Totals from Teaching Episodes

Teaching Episode 1

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	**3**	1	2	1	3	**0**	3	0	2	2	0	0	**1.4**	**1.2**
CK *b* Sum	3	3	2	**4**	2	3	**0**	4	3	2	4	3	**2.8**	**1.1**
KoST *a* Sum	**13**	3	7	6	7	5	9	**0**	11	12	4	12	**7.4**	**4.1**
KoST *b* Sum	**1**	15	10	7	7	10	3	**16**	4	5	9	2	**7.4**	**4.8**
***a* Sum**	**16**	4	9	7	10	5	12	**0**	13	14	4	12	**8.8**	**4.9**
***b* Sum**	4	18	12	11	9	13	**3**	**20**	7	7	13	5	**10.2**	**5.4**

Teaching Episode 2

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	3	**0**	**7**	7	4	4	6	0	2	3	6	5	**3.9**	**2.4**
CK *b* Sum	1	6	1	**0**	1	4	3	**8**	6	4	2	1	**3.1**	**2.5**
KoST *a* Sum	9	5	9	12	14	5	18	**3**	9	10	**18**	14	**10.5**	**4.9**
KoST *b* Sum	2	9	8	9	**1**	11	4	**24**	8	7	1	2	**7.2**	**6.4**
***a* Sum**	12	5	16	19	18	9	**24**	**3**	11	13	24	19	**14.4**	**6.8**
***b* Sum**	3	15	9	9	**2**	15	7	**32**	14	11	3	3	**10.3**	**8.4**

Teaching Episode 3

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	1	2	**0**	absent	1	2	4	0	0	**5**	3	3	**1.9**	**1.7**
CK *b* Sum	**0**	1	4		1	2	0	**5**	3	4	2	2	**2.2**	**1.7**
KoST *a* Sum	**21**	**4**	9		9	4	11	7	13	5	12	19	**10.4**	**5.7**
KoST *b* Sum	**2**	16	13		5	**22**	4	18	7	5	11	2	**9.5**	**6.9**
***a* Sum**	**22**	**6**	9		10	6	15	7	13	10	15	22	**12.3**	**5.8**
***b* Sum**	**2**	17	17		6	**24**	4	23	10	9	13	4	**11.7**	**7.7**

Note. An *a* signifies a novice response and *b* signifies an expert response (see Table 3). Bold sums represent a Min or Max.

totals of novice and expert behaviors, as indicated by *a* and *b*, respectively, as they were coded in the three teaching episodes (TEs). As might be expected, the frequency patterns present in the TEs are very similar as those observed in the pre-, post-, and follow-up tests. Each TE contained 12-14 writing prompts and each prompt was categorized as predominantly addressing content knowledge (CK) or knowledge of student teaching (KoST), thus accounting for the frequencies CK *a*, CK *b*, KoST *a*, and KoST *b*. The PSTs found TE 1 the easiest of the three to decipher. All of the PSTs, except Jackie, found interpreting the mathematical correctness of the student's method to be rather straightforward. Because of that, Jackie received no CK *b* codes and the other PSTs had relatively similar CK *b* frequencies. While the PSTs performed pretty well with the CK questions related to TE 1, their inability to explain that knowledge along with a limited capacity to apply their CK and adequately address the struggling student in the TE resulted in much higher novice frequencies related to KoST. Brianna and Grace had the highest KoST *b* (i.e., expert) frequencies by a relatively large margin and this was reflected in the substance of their responses, as will be seen later. It is worth noting that Brianna was not assigned a single novice code for her TE 1 responses, and Grace received the second lowest total of four.

Teaching episode 2 (Figure 15, page 134) required the PSTs to grapple with two relatively difficult concepts. One was the misconception that a fixed perimeter (i.e., the piece of string) can have only one area (i.e., the desired area of the footprint). The second involved a correct method to find/estimate the area of a footprint (an irregular shape). Several became fixated with finding the area of the footprint rather than on the misconception contained within the students' method – the focus of the TE. That accounted for some PSTs' (e.g., #3, 4, 5 & 11) dramatic increase in the number of novice codes received, especially those relating to KoST. TE 2 was probably the most difficult for two reasons: (a) the potential distraction of finding the area of the footprint, and (b) because of where it fell within the intervention; there was still considerable instruction and learning to take place. The mean number of novice codes assigned was highest for this TE. There was a lot of mathematics involved with TE 2, and Brianna excelled. She was able to effectively apply her strong mathematical CK, and because of that she earned the highest number of expert codes (32) and the lowest number of novice (3). Grace was second in both areas with 15 and 5, respectively. Jackie and Larry ranked first and second in receiving the most novice codes, and while Jackie improved slightly over TE 1 by receiving more expert codes, Larry continued to perform near the bottom of the class.

Teaching episode 3 involved the PSTs investigating a very common, and elusive, misconception regarding a perceived relationship between area and perimeter. The class averages for novice and expert codes were relatively equal to the previous 2 TEs, with a slight increase in expert levels of KoST. Brianna and Grace ranked second and third in overall frequency of expert responses, and this was primarily accounted for by strong performance in the KoST category. During all the teaching episodes, and this one particularly, Larry was observed just staring at the work in front of him for several minutes. This lack of activity (e.g., exploring with the microworlds) accounted for the high frequency of novice codes, especially regarding his KoST. Jackie's improvements are not readily evident in Table 13. Jackie does not seem to respond well initially to new material, as was the case with the unique nature of each teaching episode. In the qualitative section, it will be shown that when engaged in conversation about mathematical content and students' thinking, Jackie was better able to clarify and present her understanding about the concepts being discussed.

Comparisons of Pre-, Post-, and Follow-up Levels of CK and KoST

Descriptive statistics are presented to provide an overall view of the changes in CK and KoST that were measured following the three teaching episodes. Because the posttest occurred one week after the third and final whole-class intervention (i.e., TE 3), the posttest is more of an "immediate" measure of growth (or lack thereof). The only intervention that occurred after the posttest was the second interview with the four case subjects. That interview involved both planned and unplanned teaching opportunities. The follow-up test is better thought of as a measure of retention; however, since it was the same test as the pretest, there is value in comparing responses – especially for the case subjects, in light of their second interview. With that in mind, the posttest will receive a thorough and in-depth analysis with responses from the follow-up test being used as confirmation that what was indicated as "learned" on the posttest (and during the second interview) was retained. The significance of scores and written responses on the posttest, with appropriate data from the follow-up test, will then be delineated by discussing results from the teaching episodes as well as vignettes from the second interviews with the case subjects. This triangulation will provide a rich description of the how the PSTs' (primarily the case subjects') CK and KoST changed throughout the course of this study.

The posttest (Appendix E) was given to all 12 PSTs on October 30, 2007 – one full week after the completion of the third and final teaching episode. The pre-, post-, and follow-up tests all consisted of 10 items. The first 5 were intended to focus on CK and questions six through ten were designed to elicit KoST responses along with CK. The mean time to complete the posttest was 64.2 minutes – almost 10 minutes greater than the pretest. The mean completion times for the CK and KoST subtests were 25.2 and 39 minutes, respectively. That is an increase of almost 8 minutes for the KoST subtest. The least amount of time spent on the posttest was 45 minutes and the longest was 85 minutes – by Jackie. She asked for an extra 10 minutes to complete the KoST section, and at the end of the posttest she wrote, "Yay Mr. Kellogg . . . I understood all of them!" Larry took only 51 minutes to complete the posttest, Grace required 70, and Brianna took 80. Although Brianna methodically worked through the test, it appeared to the researcher that Larry was concerned with just getting done. PST #1 had the shortest completion time, and she also was the only PST to have a lower score on the posttest than on the pretest.

PSTs' scores on the posttest showed an overall mean of 28.25, a standard deviation of 4.0 (see Table 14). The scores appeared to have a relatively normal distribution with skewness and kurtosis values of 0 and -1.2, respectively. The kurtosis value, while slightly platykurtic, is within acceptable ranges. The follow-up test was administered on December 11, 2007. The mean for the follow-up test was 27.83 (*SD* = 4.3). Skewness and kurtosis values were acceptable at -0.07 and -1.3, respectively. Although he showed slight improvement over his pretest score, Larry's score of 20 was the lowest total score and was over two standard deviations below the mean. He had the lowest scores on the CK and KoST subtests as well. Grace shared the highest overall score of 33 and the highest KoST subtest score (17) with PST #6.

Table 14

Descriptive Statistics for Pre-, Post-, and Follow-up Tests

Total Score

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	Mean	*SD*
Pretest	25	27	23	18	16	24	**13**	**30**	18	23	21	17	21.25	4.97
Posttest	23	**33**	25	31	27	**33**	28	31	29	31	28	**20**	28.25	4.00
Follow-up	25	33	27	29	23	32	25	**34**	31	30	24	**21**	27.83	4.26

Content Knowledge (CK)

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	Mean	*SD*
Pretest	12	**16**	13	8	8	14	**4**	14	8	12	10	8	10.58	3.48
Posttest	12	16	13	**17**	12	16	13	16	16	15	14	**11**	14.25	2.00
Follow-up	11	17	15	13	12	17	11	**18**	16	17	12	**10**	14.08	2.87

Knowledge of Student Thinking (KoST)

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	Mean	*SD*
Pretest	13	11*	10	10	**8**	10	9	**16**	10	11	11	9	10.67	2.10
Posttest	11	**17**	12	14	15	**17**	15	15	13	16	14	**9**	14.00	2.41
Follow-up	14	**16**	12	**16**	11	15	14	**16**	15	13	12	**11**	13.75	1.91

Note. Posttest total scores range from 0 to 40. A score of 40 indicates a model response for all 10 items. CK & KoST subtest scores range from 0 to 20.
*PST ran out of time and did not completely finish two problems. Min. and Max. scores are in bold.

The results indicated that the easiest problem on the posttest was question 5 (M = 3.25; SD = 0.62). The misconception being tested is that a fixed perimeter will have only one area - the very same misconception investigated in teaching episode 2, which proved difficult for most PSTs. The hardest item was once again question 4 (mean = 2.33; SD = .78), which asked PSTs to explain and differentiate between linear and square units. The exact same question appeared on the pre-, post-, and follow-up tests, and was statistically the most difficult each time. On this question, both Jackie and Larry received a score of 2 (inferior), Grace scored a 3 (acceptable) and Brianna earned a model score of 4.

Examining the change in total scores from the pre- and posttest revealed positive growth for 11 out of 12 PSTs. The posttest mean of 28.25 represents an impressive 33% increase over the pretest average score. Grace showed an increase of 22%, Brianna 3%, Larry18%, and Jackie's posttest score of 28, while still below the mean, was an increase of 115% over her pretest score. This was largely due to an increase in her CK subtest score from 4 to 13. Every PST's CK subtest score either grew (n = 9) or remained unchanged (n = 3). The KoST subtest scores showed strong improvement as well. The range of increase was from 2 points (20%) to 7 points (70%). Two PSTs' KoST subtest scores (#1 and Brianna) decreased slightly by 2 and 1 point, respectively. The largest score difference between a CK subtest and KoST subtest was three. The total score percent increase of 33% was well balanced between a CK score increase of 35% and a KoST increase of 31%. Results from the follow-up test lend credence to the statistical evidence that knowledge gained during the study and demonstrated on the posttest was retained. The group mean decreased from 28.25 to 27.83 (-1.5%) from post- to follow-up test. Means from the CK and KoST subtests were basically unchanged. The greatest individual drop between posttest to follow-up was 4 points (-14%) by PST #11, while the greatest increase was 3 points (+10%) by Brianna. These changes can be depicted by the use of regression lines and will be presented at the end of the qualitative analysis section.

Changes in Rubric-Score Frequencies

Examining posttest score frequencies of the PSTs (Table 15) revealed several noteworthy results. On the pretest Jackie received seven unacceptable scores (one 0 and six 1s); however, on the posttest she did not receive any such scores, and while she only achieved one acceptable score of 3 on the pretest, her responses earned 8 such scores on the posttest. Larry did not experience the same success. On the pretest, Larry received 9 unacceptable scores (four 1s and five 2s), and on the posttest Larry received the highest number of unacceptable scores (8; two 1s and six 2s). The entire class decreased their total number of unacceptable scores (0s, 1s, and 2s) from 74 on the pretest to only 35 on the posttest. Grace was the only PST who received all acceptable scores (seven 3s and three 4s). Model responses rose sharply for the posttest. There were only seven 4s assigned on the pretest but 19 on the posttest. There were only three PSTs who did not receive any scores of 4 on their posttest, two of whom were Jackie and Larry.

Changes in Expert/Novice Frequency Totals

Comparing frequencies of expert/novice behavior (see Table 3, page 167) as identified within the PSTs' work (written and verbal) throughout the study was another way to portray the changes that occurred in the PST's CK and KoST. Table 18 presents frequency totals of novice and expert behaviors, as indicated by *a* and *b*, respectively, as they were calculated from the pre-, post-, and follow-up test. Each test consisted of 10

Table 15

PSTs' Pre-, Post-, & Follow-up Test Rubric-Score Frequencies

	Pretest					**Posttest**					**Follow-up Test**				
PST	***0***	***1***	***2***	***3***	***4***	***0***	***1***	***2***	***3***	***4***	***0***	***1***	***2***	***3***	***4***
#1		1	4	4	1		2	3	5					10	
Grace			5	3	2				7	3			1	5	4
#3		1	6	2	1		1	4	4	1			4	5	1
#4		5	2	3				1	7	2		1	1	6	2
#5	1	4	3	2				4	5	1			7	3	
#6		2	3	4	1			1	5	4			1	6	3
Jackie	1	6	2	1				2	8			1	4	4	1
Brianna			1	8	1			2	5	3				6	4
#9	1	3	3	3				3	5	2			2	5	3
#10		3	2	4	1			1	7	2			2	6	2
#11		3	3	4				3	6	1		1	4	5	
Larry		4	5	1			2	6	2			3	3	4	
Totals	3	32	39	39	7		5	30	66	19		6	29	65	20

Note. The questions for the pretest and follow-up test were exactly the same (other than changing student names).

questions. The first five focused on CK and the second five questions added KoST parts. The pretest and follow-up test were identical in content and presentation, and the posttest contained items that parallel the pre- and follow-up tests. Examining class means, we can see there were improvements from pretest to posttest. There were fewer novice codes assigned and the number of expert codes increased by over three-fold (from 10.3 to 31.3). Of all the PSTs, Jackie's knowledge levels made the greatest positive change. On the pretest she received by far the most novice codes (50, which was almost 16 above the mean), while her CK did not earn any expert codings and her responses related to KoST received only 4. On the posttest Jackie was able to decrease the frequency of novice CK responses (from 26 down to 14) and increase those earning expert codes (from 0 to 10). Jackie's responses on the posttest reflecting her KoST received a total of 19 expert codes – up from only four on the pretest. Apparently the various interventions helped Jackie to both increase and organize her CK in ways that enabled her to more appropriately respond to student difficulties and misconceptions (i.e., her KoST).

There was a decrease in the frequency of their novice codes for both Grace and Brianna from pretest to posttest. This change remained stable through the follow-up test. Brianna had the highest combined frequency of expert codes (led by her strong CK) for the posttest, along with the lowest number of novice codes. For the posttest, Grace was second in each respective category. On the follow-up test, Brianna's KoST received slightly fewer expert codes than did Grace (who had the most), due primarily to Brianna neglecting to include appropriate diagrams with her responses. Larry did increase the number of expert codes received from pretest to posttest (from 5 to 15); however, his

Table 16

Expert/Novice Coding Totals for Pre-, Post-, and Follow-up Tests

Pretest

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	16	11	14	20	21	10	26	13	12	10	18	18	**15.8**	**5**
CK *b* Sum	3	11	4	1	1	8	0	6	4	7	2	2	**4.1**	**3.3**
KoST *a* Sum	18	16	31	16	22	16	24	10	15	17	19	22	**18.8**	**5.4**
KoST *b* Sum	6	9	6	6	2	8	4	15	5	6	5	3	**6.3**	**3.4**
***a* Sum**	34	27	45	36	43	26	50	23	27	27	37	40	**34.6**	**8.7**
***b* Sum**	9	20	10	7	3	16	4	21	9	13	7	5	**10.3**	**6**

Posttest

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	14	7	13	11	20	11	13	6	14	10	12	17	**12.3**	**3.9**
CK *b* Sum	11	17	10	12	8	11	10	19	9	11	15	7	**11.7**	**3.6**
KoST *a* Sum	22	12	22	18	13	11	18	11	22	13	19	29	**17.5**	**5.6**
KoST *b* Sum	15	26	15	21	23	26	19	25	14	25	20	8	**19.8**	**5.7**
***a* Sum**	36	19	35	29	33	22	31	17	36	23	31	46	**29.8**	**8.4**
***b* Sum**	26	43	25	33	31	37	29	44	23	36	35	15	**31.4**	**8.4**

Follow-up Test

PST	#1	Grace	#3	#4	#5	#6	Jackie	Brianna	#9	#10	#11	Larry	**Mean**	**SD**
CK *a* Sum	15	8	12	19	14	12	18	12	15	10	18	17	**14.2**	**3.5**
CK *b* Sum	8	19	14	8	13	11	6	14	11	15	9	4	**11**	**4.2**
KoST *a* Sum	15	8	18	9	16	5	11	11	13	16	11	19	**12.7**	**4.2**
KoST *b* Sum	13	21	9	18	10	23	12	15	12	10	13	7	**13.6**	**4.9**
***a* Sum**	30	16	30	28	30	17	29	23	28	26	29	36	**26.8**	**5.7**
***b* Sum**	21	40	23	26	23	34	18	29	23	25	22	11	**24.6**	**7.4**

frequency of novice codes also increased (from 40 to 49). Larry received the most novice codes as well as the fewest expert codes for both the posttest and the follow-up test. During our first interview, Larry often indicated that he did not have a firm understanding of the concepts at hand, and that he often would "make things up as he went along." His relative quick completion time on each test, combined with his brief (often unclear) responses, indicated that Larry was more interested in completing the tests than doing a thorough job.

Changes in the Frequency of Specific Expert/Novice Codes Assigned

Table 17 shows many of the specific codes that comprised the totals that were just discussed in Table 16. This table also reveals strengths and weaknesses of various PSTs. The case subjects were the focus of this table because their coded responses could be verified through the second interview. Certain codes, because they required a high level of expertise (e.g., 1*b* and 9*b*), were not assigned very often. Specific codes aligned very well with aspects of CK and KoST, and were used to compare the amount and type of respective knowledge present at the pre-, post, and follow-up test. For example, codes involving knowledge structure (1*a*/1*b*) as well as explanatory framework (8*a*/8*b*, 15*a*/15*b*, and 16*a*/9*b*) provide feedback related to PSTs' CK. Codes that described a PSTs' understanding of children's thinking (e.g., 2*a*/2*b*) as well as their ability to address shortcomings and misconceptions (e.g., 7*a*/7*a*-/7*b*, 12*a*/12*b*, and 13*a*/13*b*) were useful in clarifying PSTs' levels of KoST. For example, the change in Jackie's CK from pretest to posttest can be partially explained by the fact she received the novice codes of *1a*, *8a*, and 16*a* a total of 10, 3, and 16 times respectively, but the frequencies of those codes were reduced on the posttest to 5, 0, and 10 respectively. In addition to the reduction of

Table 17

Expert/Novice Coding Frequencies for Case Subjects from Pre-, Post-, and Follow-up Tests

Pretest

Code	1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	6*a*	7*a*-	7*a*	7*b*	8*a*	8*b*	9*a*	9*b*	10*a*	10*b*	11*a*	11*b*	12*a*	12*b*	13*a*	14*a*	15*a*	15*b*	16*a*	17*a*
Grace	6	3	3	3	0	3	0	1	3	0	2	1	4	5	1	0	0	1	2	0	1	0	1	2	0	4	0
Jackie	10	0	8	1	0	0	1	1	3	1	1	3	1	3	0	0	0	0	0	0	1	0	3	4	0	11	2
Brianna	6	1	2	7	1	1	1	0	3	0	2	0	6	6	2	0	0	1	2	0	0	0	1	0	0	2	0
Larry	8	0	5	3	1	0	1	0	4	2	1	1	1	4	0	0	0	2	0	0	0	0	3	5	0	9	0
Class Avg	7.8	1.2	4.6	3.5	0.7	0.6	0.3	0.5	2.8	1.3	1.2	1.1	2.1	3.5	0.7	0.2	0	1.6	0.8	0	0.2	0	1.0	2.0	0	7.1	.3
Class SD	3.1	2.1	5.1	3.6	1.9	0.7	0.7	1.3	4.2	2.2	1.9	1.4	2.9	4.9	1.2	0.4	2.2	1.2	0.4	1.7	2.5	6.2	0.9	3.1	2.1	5.1	3.6

Posttest

Code	1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	6*a*	7*a*-	7*a*	7*b*	8*a*	8*b*	9*a*	9*b*	10*a*	10*b*	11*a*	11*b*	12*a*	12*b*	13*a*	14*a*	15*a*	15*b*	16*a*	17*a*
Grace	0	8	0	9	0	1	0	0	5	1	1	0	9	2	3	0	4	0	5	2	2	4	0	1	1	7	0
Jackie	5	3	0	9	0	0	0	0	1	2	5	0	6	1	0	1	3	4	2	4	0	4	0	2	0	10	0
Brianna	0	9	0	9	0	0	0	0	3	0	4	0	7	2	6	2	3	1	3	3	1	4	0	2	2	3	0
Larry	8	0	3	7	0	0	1	0	4	2	2	1	5	3	0	3	0	6	0	3	1	4	0	1	0	10	0
Class Avg	3.4	5	1.1	7.8	0	0.4	0.1	0	3.1	1.3	3.3	1	6.7	2.3	1.4	1.9	2.4	2.3	2.6	3.3	0.8	3.8	0.1	1.4	0.5	7.7	0
Class SD	2.1	2.9	1.7	3.7	0	1	0.3	0	2.5	1.3	2.4	1.2	3.5	3.2	1.3	2.9	3.1	4	2.6	5.1	2	5.6	0.3	1.7	1.3	4.3	0

Follow-up test

Code	1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	6*a*	7*a*-	7*a*	7*b*	8*a*	8*b*	9*a*	9*b*	10*a*	10*b*	11*a*	11*b*	12*a*	12*b*	13*a*	14*a*	15*a*	15*b*	16*a*	17*a*
Grace	1	7	1	7	0	3	0	1	3	1	3	0	8	0	5	2	2	0	4	2	1	3	1	1	0	4	0
Jackie	6	0	2	6	0	0	0	0	1	1	5	0	5	2	0	2	1	1	1	2	0	3	0	4	0	9	0
Brianna	1	7	1	7	0	0	0	0	2	1	2	1	9	5	1	2	1	3	2	2	0	3	1	1	0	4	0
Larry	6	2	6	2	0	1	0	0	4	1	1	1	4	1	0	3	0	4	0	2	1	3	1	0	0	8	0
Class Avg	4.1	3.5	2.2	5.8	0	0.9	0.1	0.1	2.2	0.8	3.3	0.9	6.8	2.5	0.9	1.5	0.9	2.5	1.8	2.3	0.4	2.9	0.8	1.2	0	6.8	0
Class SD	3.2	2.5	3.8	4.9	0	1.3	0.3	0.3	2.8	1.3	3.4	1.9	2.3	5.3	1	3	1.7	2.4	1.9	4.8	1	5.6	2.2	1.3	0	3.9	0

Note. There were no codes of 4*b*, 5*a*, 5*b*, 6*b*, or 14*b* assigned for any test; 13*b* was assigned only 5 times (4 on the post- and 1 on the follow-up).

Jackie's novice codes on the posttest, there were increased frequencies in the expert categories. Take for example those reflecting her KoST. Jackie received 1 code of 2*b* on the pretest and 9 such codes on the posttest; similarly, the frequency of 7*b* increased from 1 to 5, from pre- to posttest. Jackie's frequencies within these various categories remained fairly constant on the follow-up test.

Brianna and Grace strengthened their CK as evident by the fact that they received no codes of 1*a* or 8*a* on the posttest and received relatively high numbers of codes 8*b* and 9*b*. Their increases in KoST can be seen by the higher than average frequencies of codes 2*b*, 10*b* and 11*b*. A significant change regarding Brianna can be seen by examining the codes 9*a* and 9*b*. Brianna has a strong mathematics background and tended to be very procedural in her problem solving, explanations, and how she indicated she would interact with students, indicated by the high rate of code 9*a* on the pretest. Throughout the teaching episodes there was a noticeable shift in Brianna's approach to viewing, doing, and explaining mathematics. She consciously made efforts to think more conceptually, which was evidenced by the decrease in 9*a*s assigned and the increase in 9*b*s she received. Larry on the other hand continued to struggle with the mathematics contained in the study as well as explaining his ideas (see the high rates of codes 1*a*, 7*a*-, and 16*a*). He also showed little, if any, improvement in how he contemplated and addressed student thinking (see codes 2*a*, 2*b*, and 11*b*). Tables of expert/novice codes revealed response patterns within individuals, as well as within the entire class. For example, the relatively low frequency of code 7*b* (i.e., the ability to generate appropriate representations) showed a notable gap in the PSTs' KoST, because they apparently did not realize the importance of diagrams presenting conceptual explanations of mathematical concepts. This tendency was repeated by a low rate of code 12*b* (i.e., the appropriate use of manipulatives) and the total absence of code 13*b* (i.e., the appropriate integration of technology to promote understanding) on any test. The PSTs' oversight of incorporating technology is somewhat troubling given the tremendous focus placed upon the two microworlds used in this study.

Linear Regression Involving CK and KoST, and Total Test Scores

The last quantitative measures used to illustrate and help describe the PSTs' change in knowledge that occurred during this study were regression lines fitted to each PST's pre-, post-, and follow-up CK, KoST, and total test scores (Table 18). R^2 values were included as an indication of how well the regression line fits the test scores. The

Table 18

Regression Equations for PSTs' CK, KoST, and Total Score

	CK Scores		***KoST Scores***		**Total Scores**	
PST	regression eq.	R^2	regression eq.	R^2	regression eq.	R^2
#1	y = -.5x + 12.2	.75	y = .5x + 12.2	.11	y = 24.3	0
Grace	y = .5x +15.8	.75	y = 2.5x + 12.2	.60	y = 3x + 25	.75
#3	y = x + 12.7	.75	y = x + 10.3	.75	y = 2x + 21	.99
#4	y = 2.5x + 10.2	.31	y = 3x + 10.3	.96	y = 5.5x + 15	.62
#5	y = 2x + 8.7	.75	y = 1.5x + 9.8	.18	y = 3.5x + 15	.40
#6	y = 1.5x + 14.2	.75	y = 2.5x + 11.5	.48	y = 4x + 21.7	.66
Jackie	y = 3.5x + 5.8	.55	y = 2.5x + 10.2	.60	y = 6.5x + 10	.57
Brianna	y = 2x + 14	.99	y = 15.7	0	y = 2x + 27.7	.92
#9	y = 4x + 9.3	.75	y = 2.5x + 10.2	.99	y = 6.5x + 13	.86
#10	y = 2.5x + 12.2	.99	y = x + 12.3	.16	y = 3.5x + 21	.65
#11	y = x + 11	.25	y = .5x + 11.8	.11	y = 1.5x + 21.3	.18
Larry	y = x + 8.7	.43	y = x + 8.7	.75	y = 2x + 15.3	.92
Class	y = 1.8x + 11.2	.71	y = 1.5x + 11.3	.69	y = 3.3x + 19.2	.70

closer their value is to 1 the better the regression line fits the data. The mean R^2 for CK, KoST, and total score was .67, .47, and .55 respectively, while the median was .75, .54, and .66, respectively. Due to small sample size (N = 12), the mean was more volatile to extreme R^2 values. An R^2 of 0 occurred twice (once for KoST and once for total score), and in both instances there was no change in the PST's from pretest to follow-up test. The several lower/weaker R^2 values for KoST scores can be partially explained by the six instances where a follow-up KoST score was lower than the posttest score (average decrease was 3.25). Compare that with the four instances where CK had a lower follow-up score than posttest (average decrease 2.25). The slope of the class' CK regression line (1.8) indicates the estimated average change for the PSTs' CK regarding area and perimeter increased by 1.8 points (range 0-20) from pretest through follow-up test. The slope of class' KoST was 1.5. Of the 12 CK regression equations, nine had R^2 values which explained more than 50% of the variance, whereas six of the KoST equations had R^2 values > 50%. The regression lines for the case subjects' CK and KoST (Figure 27) and total score (Figure 30), along with those of the other eight PSTs (Figures 28, 29, 31, & 32), appear below to provide comparisons as well as to demonstrate each individual's change in CK, KoST, and total knowledge that occurred throughout the study.

Describing the Change in PSTs' CK and KoST

The first category of findings used in answering research questions 1 and 2, *Distinguishing between area and perimeter*, was not as clearly discernable in the findings from the intervention or post-intervention stages of the study. This would most likely be due to the very nature of the intervention. That first category became apparent in the findings from the pre-study Survey Questionnaire. The PSTs were specifically asked to

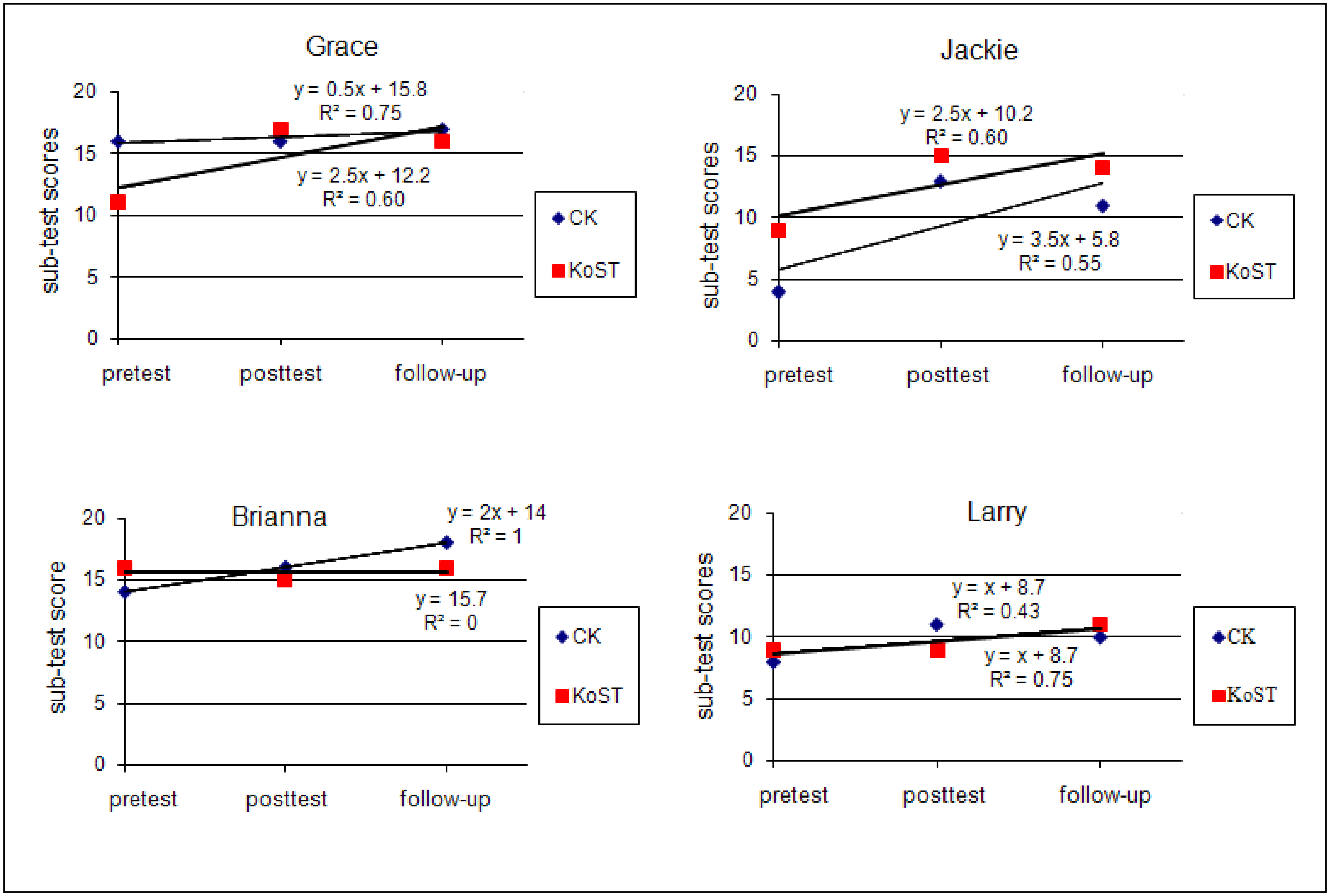


Figure 27. Regression lines and equations for change in case subjects' CK and KoST.

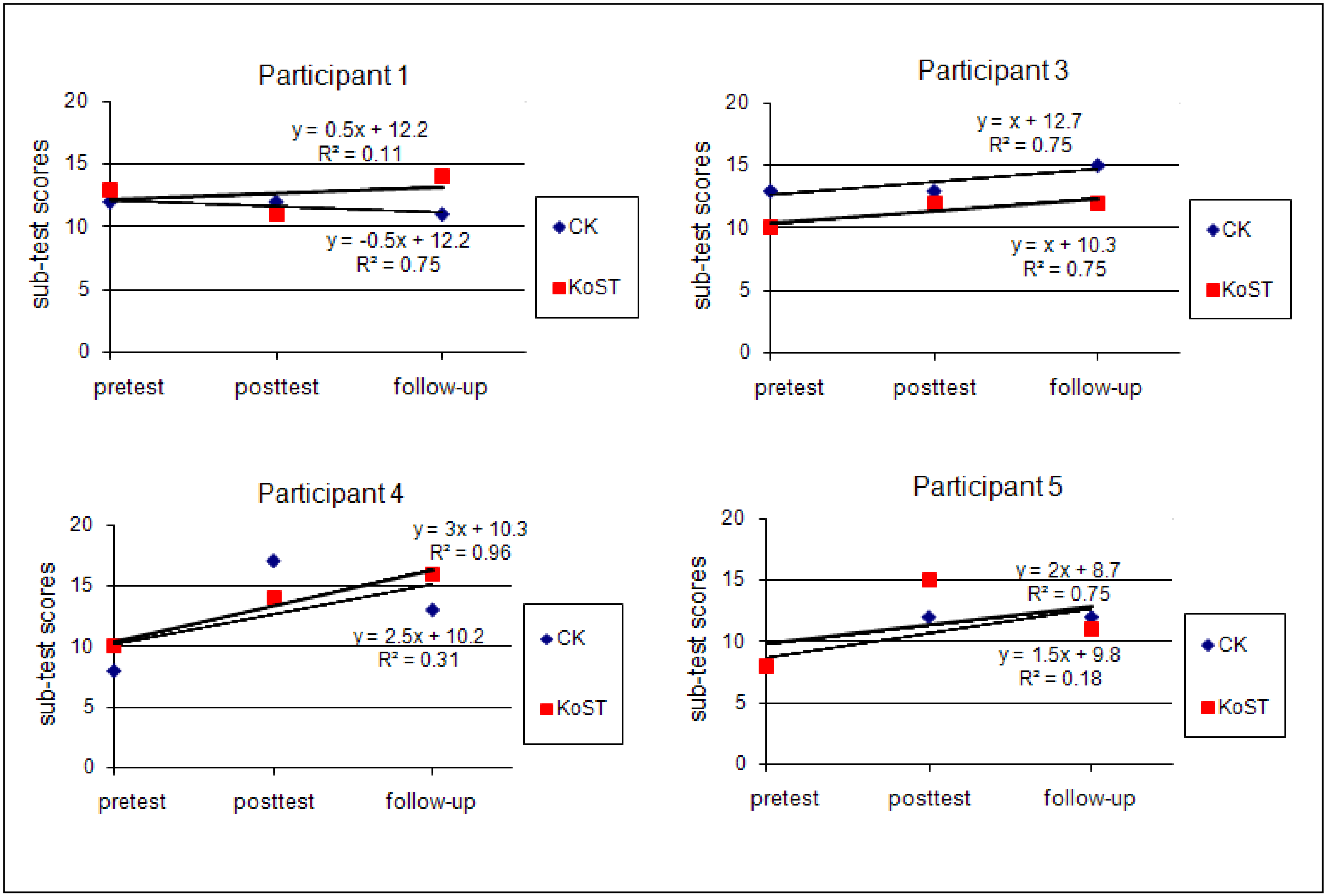


Figure 28. Regression lines and equations for PSTs' CK and KoST.

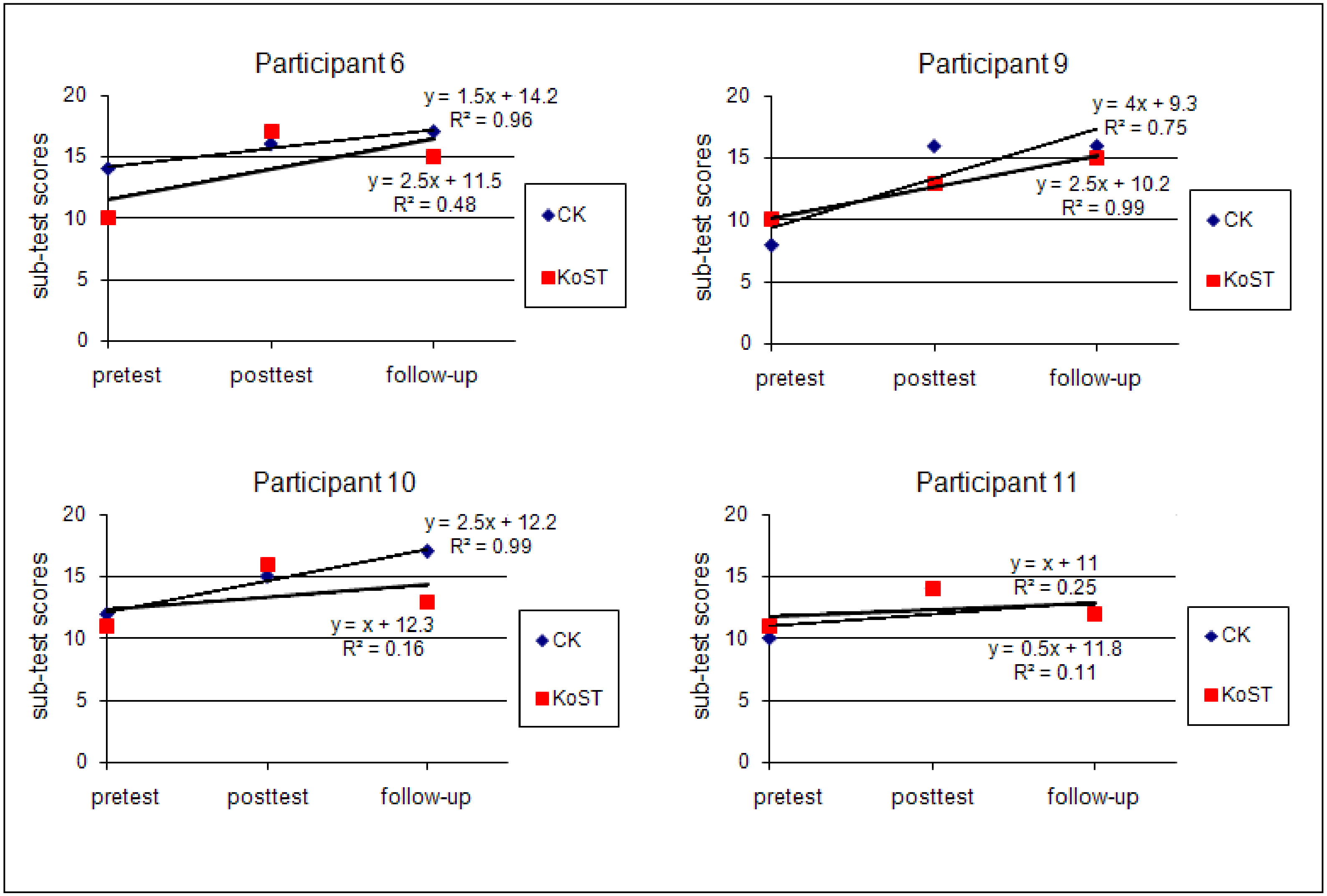


Figure 29. Regression lines and equations for change in PSTs' CK and KoST.

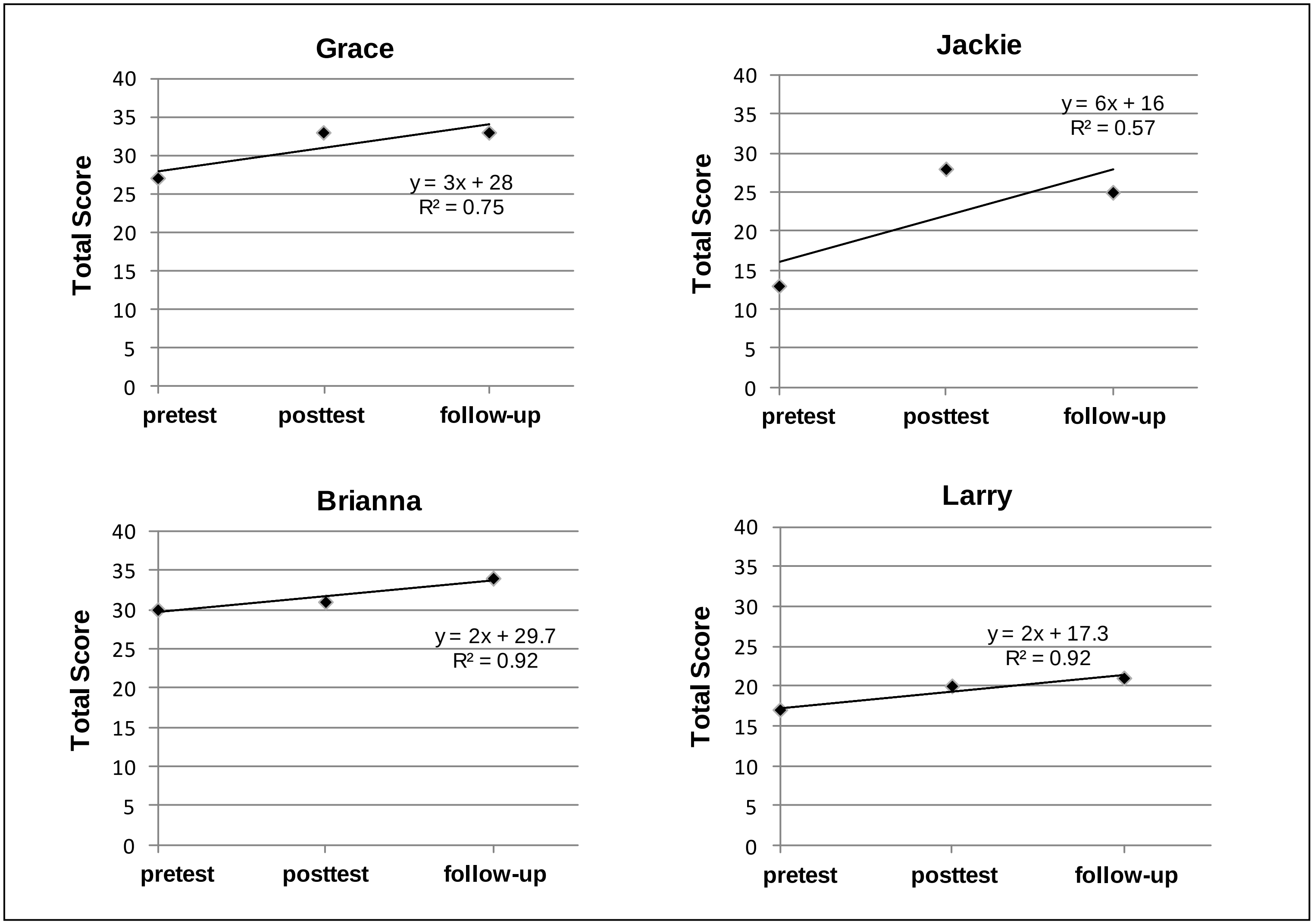


Figure 30. Regression lines and equations for each case subject's total score.

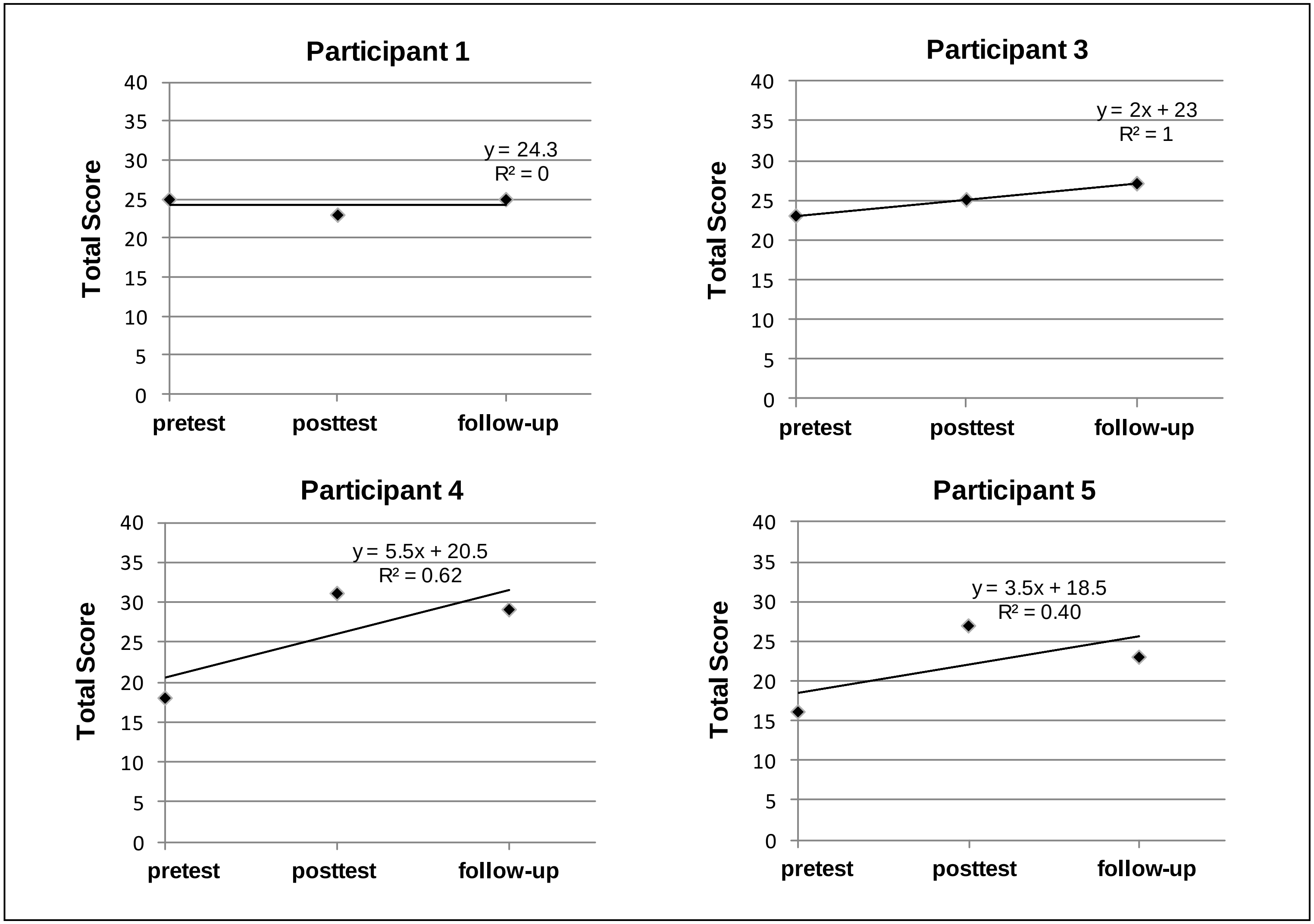


Figure 31. Regression lines and equations for each PST's total score.

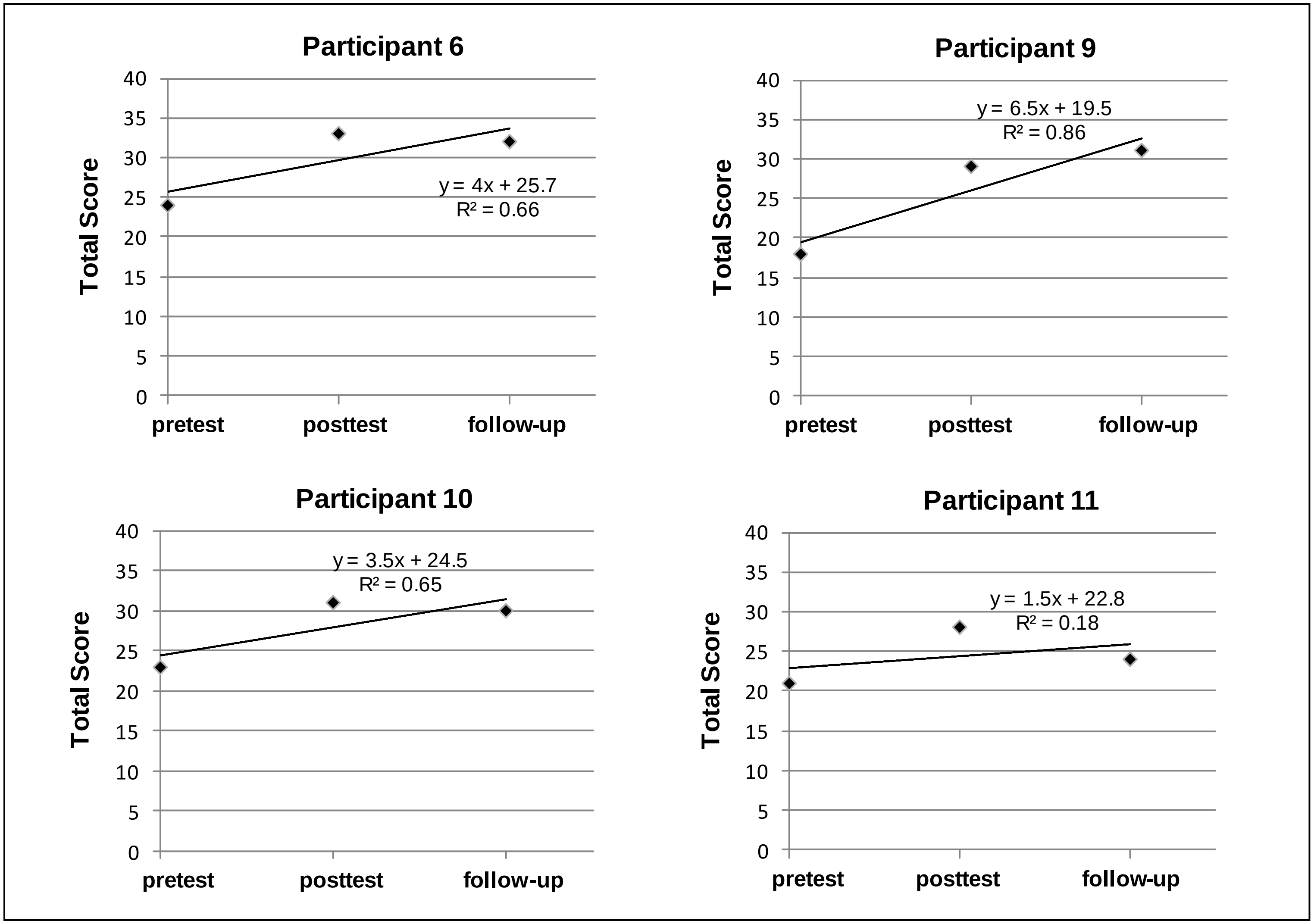


Figure 32. Regression lines and equations for each PST's total score.

discuss and explain their current notions and understandings regarding area and perimeter. The very act of working through the pretest, being interviewed (for the case subjects), and then receiving content instruction prior to the first teaching episode seemed to resolve many of the glaring confusions regarding distinguishing whether a problem involved working with area or perimeter. For example, there were no responses similar to Jackie's answer to the first question on the pretest: "To be honest I have no idea if the polygon I drew represents a perimeter or area of 24." However, any meaningful findings regarding the category of "Distinguishing between Area and Perimeter" were integrated within the two major categories of knowledge used to answer research questions 3 and 4: (a) Units of measure, and (b) Perceived relationships between area and perimeter.

Findings from the three teaching episodes, interview vignettes, posttest, follow-up test, and classroom observations will be presented in the next several sections. Because the teaching episodes (TEs) comprise the primary means of intervention for this study, findings from the TEs embody emergent knowledge. Findings from the posttest represent post-intervention knowledge, and are supported by findings from the follow-up test, an indication of retention. The writing prompts contained within the TEs were written to provide a progressive learning experience. By design, the TEs allowed each PST to create their own personal learning trajectory. Because of this, findings presented in the emergent-knowledge sections were not directly compared to findings from specific test items (i.e., in a pre-post comparison method). The results from the TEs function as a bridge between the pretest and posttest, and indicate levels of change that were discussed as a continuum of change resulting from the intervention (i.e., from TE 1 through TE 3). Therefore whenever possible, discussions began with appropriate findings from a teaching episode and then be expanded upon and/or supported with (i.e., triangulated) select problems from the post- and follow-up tests. The questions on the posttest were parallel to the pretest in difficulty, content (e.g., area, perimeter, linear, and/or square units), and misconception(s) addressed. The questions on the follow-up test were identical to the pretest. The majority of the findings and subsequent discussion regarding the posttest focused on the questions that parallel those presented while answering research questions one and two.

In order to make the answering of research questions 3 and 4 more apparent, findings were presented as predominantly addressing either the CK (the focus of question 3) or the KoST (the focus of 4) of the PSTs. By their very nature, CK and KoST interact with each other and are therefore not mutually exclusive. At times it was both impossible and impractical to completely separate certain CK and KoST findings. Also, not every category of findings (e.g., “Knowledge regarding irregular shapes”) addressed both CK and KoST or contained pre-intervention, emergent, and post-intervention findings. Emergent findings were limited in scope by the content contained within the three TEs; however, each appropriate category of findings contained some form of comparison (i.e., pre- to post-, or pre- to follow-up, with emergent findings strategically inserted) in order to document change in CK and/or KoST. The findings regarding units of measure and perceived relationships in entirety provided a useful comparison of the PSTs’ (especially the case-subjects’) pre-intervention CK and KoST with their emergent and post-intervention CK and KoST to assist in answering research questions 3 and 4. The fact that units of measure (i.e., linear and square units) are fundamental to area and perimeter resulted in their findings being interspersed throughout the pre-, post-, and follow-up tests as well as the planned intervention (i.e., the TEs); therefore, it was not possible to parcel the categories of findings regarding units of measure in the same fashion as it was with the findings on perceived-relationships. Research question 3 specifically deals with changes in PSTs' CK, and answering it began by examining findings regarding concepts of area and perimeter surrounding linear and square units.

Changes in CK Regarding Units of Measure

When considering rectangles (the primary shape discussed in this study), determining area and perimeter involves calculations with the lengths of sides. A conceptual understanding of area and perimeter needs to equip the student and teacher alike with the knowledge to more consistently perform the correct measurement. While each measure involves a calculation with sides, area and perimeter also require attention to their appropriate units (i.e., linear or square). These concepts are intrinsically linked, and a profound CK and KoST should always include appropriate mention of linear and square units when discussing area and perimeter. Because of the fundamental importance of units of measure, a considerable amount of reporting will be devoted to this category.

Findings relevant to the PSTs' change in knowledge related to units of measure came from TE 1, TE 2, the post- and follow-up tests, observations by the researcher/instructor and second observer, and the second interview with the case subjects. The first interview with each case subject was designed to only gather information to help establish a baseline of their CK and KoST; therefore, the first intentional intervention came on November 2, 2007 with the presentation of TE 1 (see Figure 14, p. 130). Teaching episode 1 commenced with a 15 minute, instructor-lead discussion regarding units of measure. Linear, square, and cubic units were taught along with their appropriate measurement (perimeter, area, and volume). These central, unifying ideas undergird all measurement. Visual representations for each unit were presented to help develop a conceptual understanding of how shapes are comprised of the various units used to measure them. The instructor/researcher purposely used diagrams when teaching about units to model effective instruction; however, the instructor/researcher did not specifically tell the PSTs that they should follow suit in their personal responses.

Confusing the measure with its unit. Since teaching episode 1 (TE 1) was the first phase of the planned intervention, it provided emergent findings related to the PSTs' CK regarding the measures of area and perimeter and their understandings of the appropriate unit for each. TE 1 was designed to provide the PSTs an opportunity to investigate ideas surrounding area and perimeter and linear and square units. There were 3 primary concepts at work within TE 1: (a) perimeter involves linear not square units (CK), (b) finding the perimeter of an irregular shape (CK), and (c) comprehending, explaining, and addressing Justin's thinking (KoST), which will be examined later. The PSTs' CK was investigated by asking them: (1) What perimeter Justin's method would produce and if his method was mathematically correct, (2) If Justin's method was incorrect, what the correct perimeter would be, and (3) Explain, mathematically speaking, what is correct or incorrect about Justin's method. Justin's method produced a perimeter of 20 square units, although the correct perimeter of the irregular shape is 24 linear units. PSTs' responses to this TE fell into one of four groups.

This first group of two PSTs initially thought Justin's method was correct. Out of 12 PSTs only Jackie and one other PST did not initially conclude Justin's method to be incorrect. The other PST (#9) who initially thought Justin's method was correct wrote, "This method may not necessarily be the best, but in this situation he came up with the answer he needed." She indicated Justin's method would produce "20 units" for the perimeter. Her response was interesting because after indicating that Justin's method would produce the right answer she then went on to explain why it was wrong, "Justin is counting the square units that are shaded. He really only needs the linear units. He does understand that perimeter is only the 'outside part' of the figure." Initially, it would appear this PST was careless in her analysis of the question, Justin's method or both. That would be an example of a novice teacher's approach to problem solving. Later in the TE after exploring with the Shape Builder microworld this PST wrote, "My first response, I'd add some information to it. His [Justin's] answer will be incorrect because if he only counts the squares, he'll get 20 units, whereas, the perimeter itself is 24 units, since the corners get counted twice ☺." The last part of her quotation, since the corners get counted twice, is troubling because it seems to put the focus on trying to make Justin's method work as opposed to correcting his erroneous method and focusing on using the correct unit, linear in this case, for the appropriate measure (i.e., perimeter).

Jackie wrote, "I believe Justin's method will produce a correct answer." Jackie treated the shape as though it were a 4 × 9 rectangle, with a perimeter of "9 + 9 + 4 + 4 = 26." Obviously, Jackie was initially confused by this problem. She went on to explain her thinking, "Justin's method is correct because he counted the square units on the outside of the shape. Here Jackie is performing an iteration to calculate perimeter; albeit, she iterated the wrong unit. Jackie, just as Justin did, incorrectly applied her CK within a problem-solving situation. Later during the same session, after reflecting on her ideas (with the aid of the Shape Builder MW), Jackie wrote, "I believe that Justin counted the boxes around the shape instead of counting the sides around the shape. That would change my response completely." The choice of the PSTs' vocabulary when explaining their ideas (e.g., Jackie's use of the words "boxes" and "sides" instead of square and linear units) was seen often within the findings as a dividing line between novice and expert responses. So after initial difficulties, it appeared Jackie had resolved her confusion to a greater degree than the other PST. Once Jackie and the other PST realized their initial thoughts about the focus problem were wrong, that meant all 12 PSTs were able to (although at different times and to different degrees) decipher Justin's method as incorrect.

There were three PSTs in the next category of responses. These PSTs realized that Justin's method was incorrect, but subsequent explanations focused unproductively on Justin's method – either what would have to be done in order to make his method work, or trying to over-analyze it instead of simply explaining why it was wrong. For example, one PST wrote, "The corner boxes [of Figure 2 of the focus problem], which I have marked, with an "X" above, have two edges that must be counted in order to get the perimeter correct." Although this compensation method may work for this figure, it will not for other irregular shapes and is basically unproductive.

There were three PSTs (Larry was one) who, although they indicated Justin's method was incorrect and were also able to find the correct perimeter, used either unclear or unproductive language in their explanations. Words such as "squares," "boxes," "sides," and "lines" were common in their responses. For example, Larry, gave some consideration to discussing the error of Justin's method, but his vague vocabulary left much to be desired, “Justin’s method is incorrect because he is counting the actual squares not the perimeter outside the shape. It is the black line around the outside of the shape.” While that might be an acceptable explanation by a fourth grader, it is not acceptable language for a teacher. During the second interview with Larry, after weeks of intervention, we discussed his responses to the TE 1. He was given an opportunity to clarify his vague choice of words regarding Justin’s method. Larry responses, “He’s got the right idea, with counting the ones on the outside, but it’s not the whole square that you count. It’s just the outside boundary line of each square.” Larry’s CK was still either lacking or unorganized which affected his ability to use meaningful and appropriate vocabulary when discussing mathematics with elementary children. All eight of the previously mentioned PSTs avoided the important discussion involving terms, such as linear and square units, and how Justin was using square units to measure perimeter. The last category of responses more effectively communicated these ideas.

There were four PSTs (including Grace and Brianna) whose responses incorporated, to different degrees, the concepts of perimeter and linear and square units, and an accurate and meaningful explanation of the errors of Justin’s method. One PST (#5) wrote, “Justin is thinking in terms of square units instead of linear units.” However, in a subsequent reflective writing prompt the same PST wrote, “I am very unsure of how I answered this problem because I am still struggling with the concepts of area and perimeter.” Several writing prompts later, after having time to explore with the Shape Builder microworld, she wrote, “The Shape Builder microworld provides the answer to the perimeter, so now I know more about the problem and how to work with it.” Brianna’s explanation about Justin’s method accurately represents the more confident and coherent CK held by the other three PSTs, “Justin’s method is incorrect, because he is measuring square units instead of linear units. Perimeter is the outside boundary of the shape, and must be found by using linear units.” Another PST added that Justin had “mixed area with perimeter,” and Grace added that Justin was using “2-dimensional units, rather than the 1-dimensional linear units that make up the actual perimeter of the shape.” These explanations represent a CK possessing a strong explanatory framework.

These findings were early on in the intervention process, and several of the PSTs who had incomplete, unorganized, or unproductive explanations in the first half of TE 1 were making positive strides near the end, as will be seen when discussing their KoST regarding the student presented in TE 1.

Findings related to the category, *Confusing the measure with its units*, were also observed in the PSTs’ responses to the first problem appearing on the follow-up test (Note: the pretest and the follow-up tests contained the same problems in the same order). As reported when discussing the pretest (see Figure 22, p. 214), the PSTs had considerable difficulty with drawing a polygon (on a grid provided) that had a perimeter of 24 units and then explaining how they knew they were correct – the two parts of problem #1. Eight out of 12 PSTs provided diagrams and/or explanations that addressed, to different degrees, concepts related to area, and the scores reflected the confusion. There were five scores of 1 (range 0 to 4), four scores of 2, two who earned a score of 3, and one model response of 4 (Grace). Results from the same item appearing on the follow-up test were much better.

The mean score for problem #1 increased from 1.92 on the pretest to 2.83 on the follow-up. Overall, there were three scores of 2 awarded, eight scores of 3, and one model response of 4 (Brianna). Not only did the scores improve, but so did the depth of the responses. Three PSTs correctly drew an irregular polygon that had a perimeter of 24. Six responses included justifications of their shape using language similar to, "outside edge," "border," and "line segments" for descriptions about perimeter. Three PSTs were even more precise by explaining that the perimeter of their shape could be found by counting the outside linear units. CK containing rich dialogue such as this was, for the most part, noticeably absent from the PSTs' pretest responses. Larry and Grace were two of the three earning a score of 2 on item one of the follow-up. Larry drew a 6 × 6 square, which does have a perimeter of 24, but his response to the second part of the problem (How would you help a 4th grader understand that the polygon you drew really does have a perimeter of 24?) was simply, "Count out the individual lines." Larry was not feeling well when he took the follow-up test, but one would still hope for greater detail and explanation. At this point, all that can be surmised about Larry's CK regarding perimeter and its appropriate unit of measure is that it is lacking.

Grace made what appeared to be a careless mistake and drew a 4 × 6 rectangle, which has an area of 24. The reason it appeared to be careless was because her explanation for part 2 implied she drew a rectangle that had a perimeter of 24. She wrote as justification, "Count each unit length around the border of the polygon and find that it has 24 units in length." She correctly contrasted between linear and square units, albeit did not use the term "linear." Had she drawn a correct picture, she would have earned a 4 for her response. Grace's pre-intervention CK could be summarized as most often correct but possessing a limited ability to explain. This response, as well as more in the coming pages, will reveal that Grace's explanatory framework grew in both scope and depth.

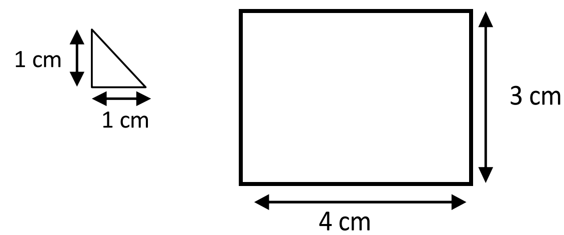
There were eight PSTs who earned a score of 3 for their work on the first problem of the follow-up test. Their responses revealed slight differences in their understandings related to units of measure, and in an ability to explain their ideas. All eight drew a correct shape but their subsequent justification was either not directly connected to their picture or contained vague references. For example, one PST wrote, “Perimeter measures the linear units around the outside of the polygon, not the square units,” but there were no specifics relating her explanation to the shape she drew; thus, her response would not be helpful to a 4th grader. Jackie was also in this group and her response contained vague language, “I would show them how to count the edges of the shape,” accompanying that response were clearly labeled numbers on her shape correctly explaining and showing how to count the edges (linear units). Since the follow-up test occurred after all the intervention, Jackie’s response might be considered less than adequate; however, when compared to what she wrote on her pretest regarding the same question, “To be honest . . . I have no idea if the polygon I drew [a 3 × 8 rectangle] represents a perimeter of 24,” it is evident that Jackie’s CK had indeed increased beyond her disconnected and fragile knowledge of area and perimeter and linear and square units.

The only “model” response to this question came from Brianna. Brianna possesses a strong mathematics background, but pre-intervention explanations often lacked specifics (e.g., meaningful language) necessary for elementary children. Her pretest response to the same question earned a 3, because it was less than thorough and did not include any mention of linear units. On the follow-up test she drew the same picture as on the pretest (a 5 × 7 rectangle), but now it was clearly evident that she saw the need to discuss units when explaining about finding perimeter, “Count the units on the outside all the way around the rectangle. Make sure you count the outside edge of the boxes, using linear units, instead of the boxes. When you add up the sides, 7 + 7 + 5 + 5, you will get 24." This is just one example of how Brianna's CK, and especially her explanatory framework, appeared to be reaching similar levels as her mathematical knowledge.

Manifestations of the PSTs' CK (i.e., procedural versus conceptual) were often displayed through their solution strategies and subsequent explanations to post- and follow-up test items. Problem 1 from the posttest (Figure 35) illustrates this facet of the PSTs' CK and specifically relates to their understandings involving units of measure.

Procedural versus conceptual CK. Problem 1 was meant to be relatively easy so the PSTs could ease into the posttest and gain some confidence. The primary concepts involved realizing that the wording "to completely cover" implied area and then recognizing/remembering the area relationship between a triangle and a rectangle half. The expectation was that the PSTs would quickly calculate the area of the rectangle to be 12, or better yet visually recognize the rectangle comprised a 3 × 4 array of squares (or square units), and then see the one-half relationship (or better yet draw it) to calculate the answer of 24 triangles. While 11 out of 12 PSTs (including all 4 case subjects) got the correct answer, the different methods used, along with the responses given to part (b), revealed various degrees of CK. Five PSTs drew in a 3 × 4 array of squares inside the rectangle (Larry was the only case subject) and of those only two (no case subjects) showed the one-half relationship by dividing the 12 squares into 24 triangles and thus arriving at their answer. Such a method typically produced conceptual responses similar to: "If 1 square unit is made up of 2 triangles and there are 12 square units in the

1. (a) How many triangles, like the one shown below, will it take to completely cover the rectangle shown?



(b) As a teacher might, clearly explain how you arrived at your answer?

Figure 33. Problem 1 from the posttest.

rectangle, we multiply 12 × 2 and we get the answer 24.” Although Larry came up with the right answer to problem 1 and also drew an array of squares inside the rectangle (i.e., conceptual groundwork), his explanation does not connect the area of the rectangle with the area of the triangle, or emphasize the one-half relationship. It reveals a limited ability to communicate appropriately as teacher: “Just fill in the rectangle with gridlines. Each square contains 2 triangles.” A common thread to most of Larry’s “explanations” was an underlying motivation to simply get right answers and tell students how to get right answers, as opposed to developing conceptual understanding.

Two other PSTs used the triangle given and drew another triangle on top of it; thus, producing a square and illustrating the one-half relationship. From there they provided a conceptual response focusing on the one-half relationship. Jackie’s response was a blend of conceptual and procedural ideas. She indicated during the second interview: “This was the one I had the hardest time with.” Her response to the problem began with, “I don’t know how to do this problem, but . . .” That revealed she still possessed a fragile confidence in her own CK; however, what she wrote next is evidence that her CK was truly becoming more organized: “I looked at the area for both shapes and saw the triangle’s area was ½ and the square’s area was 12.” This was the conceptual part of her response, albeit a little vague. But instead of continuing by filling in a grid of square centimeters and dividing them in half, she said, “So I divided .5 into 12 and got 24 . . . which I assume would be the answer.” The fact that Jackie did not “grid-in” the rectangle is evidence that, at this point in the study, she was still unaware of the conceptual value of the array structure of a rectangle’s square units. The following vignette from our second interview reveals Jackie possessed more CK than she was able to consistently apply and effectively communicate. I wanted to determine how much conceptual understanding was supporting her procedural knowledge.

T: How did you know that the area of the rectangle was 12?
J: I did 4 times 3 for the area of the rectangle.
T: And why does that produce area, multiplying 4 times 3?
J: Because that’s how many units are inside. Because, if you were picturing it, this is how I was thinking it [drawing horizontal lines in rectangle]. I was picturing one, two, three [counting] columns, and then [drawing vertical lines in rectangle] one, two, three. This is how I viewed it. I put it in terms of square units [she draws in a grid]. So I guess I could show my students that way, [pointing to the rectangle], and this will give you twelve. That’s how I figured it out.
T: So the formula is basically the short cut for summing up all the rows and columns?
J: Yeah, for summing up all the rows and columns. And then for the area of the triangle I did 1 times 1 divided by - I know that to find the area of a triangle you use a formula. You go 1 times 1 divided by ½ or - and so I just did .5, and cause that’s like a whole other field explaining that, so then I did 12 divided by .5 and you can see I did some division work on the side, with the decimal I just brought it over and then I got 24.
T: Ok, and then at the end you said that you “assumed” that it was right.
J: I assumed it would be the answer, but I wasn’t completely confident. I felt confident about this test and I thought like that it was a pretty good way, like it could be, but I wasn’t 100%. This was the only one I was kind of iffy on.
T: Can you think of a way to verify your answer now, or is it still one that’s got you a little puzzled?

J: I don't know how to verify it, no.
T: Ok, well, what does this little square represent (pointing to a square inside the grid]? This is one of the twelve, so what could you actually call this? This is one . . . ?
J: One twelfth?
T: Oh yes, very good. I was thinking simpler, like one square centimeter, and the total area is twelve square centimeters.
J: Ok.
T: What if I drew a diagonal through one of the squares inside the rectangle?
J: Ok, OH! Then I could just do that for all of them [laughing, and starts to draw in diagonals inside each square unit].
T: Each one of these shapes [pointing to one of the triangle drawn in] would be?
J: Umm (5 second pause)
T: Just like the triangle given in the problem, right?
J: Right, yeah.
T: And what's the formula for the area of a triangle?
J: Base times the height divided by two.
T: Why do we divide by two?
J: Because it's half, oh, yeah, ok, I see, yeah.
T: So, you could have actually just drawn out the rest of the square centimeters.
J: But, I'm still right?
T: Yes, you are still right; you're very right.
J: Oh!
T: You did it mathematically – procedurally.
J: Yeah.
T: I'm just showing you the relationship between the shapes and a more conceptual way to get the answer.
J: Ok.
T: Is that "cool?"
J: Yeah, that's really "cool."
T: And that would be a good way to verify it for your students, and they could see the twenty-four triangles.
J: Yeah, and that would be a really good way, especially since I was thinking in my head about the rows and columns.
T: Yes, that's why I was so surprised at your lack of drawings, because you are such a visual person, and you went away from that. I saw that you started to draw something inside the rectangle. Do you remember that?
J: Yeah, oh yeah.
T: You remember that? That you started to draw something there?
J: Oh yeah. I thought about it, but I didn't know I could do that.
T: Does that seem mathematically ok in your head?
J: Yeah, I love that. Yeah.

It took considerable prodding to lead Jackie into discovering the one-half relationship and a more conceptual solution strategy. It appears however that the above conversation was meaningful to Jackie. On the follow-up test (6 weeks later), Jackie used an array structure in response to a hypothetical student who calculated 18 for the area of a 3 cm × 6 cm rectangle that was given, but who indicated that he did not understand what the 18 represented or meant (Figure 23, p. 224). When prompted, "How would you respond to this student's apparent confusion?" Jackie wrote, "I could demonstrate the area of 18 by drawing the square units [which she did] and having the student count them." Contrast that with what she wrote for the same question on the pretest: "I would say the '18' represents how many cm's are on the inside of the box." That response characterized Jackie's pretest CK about units of measure where she was unsure which unit (linear or square) was used for which measure (perimeter or area). Her apparent growth during the second interview and her response to the above question on the follow-up test are a significant improvement from her CK displayed during the first interview. There she was asked, "Why does multiplying length times width produce the area of a rectangle?" she responded, "To be honest with you, I just know that you multiply the base times the height and you'll get the area. I have no idea why." It appeared that as Jackie's CK developed and became better organized there was a more stable foundation from which her explanatory framework could better support her KoST.

Brianna's method and explanation was representative of those who took a purely procedural approach to solving problem 1. Brianna correctly answered part (a) through straight calculations involving formulas. Her response to part (b), which involved explaining "as a teacher might" how she arrived at her answer, was equally procedural:

I found the area of the rectangle by multiplying 4 × 3, which gave me 12. I know

> that the area of a triangle is ½ base × height. Since base and height are 1, the area would be ½. Then I divided the area of the rectangle by the area of the triangle, 12 ÷ ½, which is the same as 2 × 12 and will give me 24. So I know there are 24 triangles in the rectangle.

Procedurally, Brianna gave a clear and precise explanation, although such explanations fall short in developing conceptual understanding among students. Her lack of any mention of appropriate units is less than acceptable. There is evidence however that Brianna did not conclude the study with a strictly procedural-based CK, which would characterize a novice teacher. During her second interview, Brianna and I discussed her work on problem 1 on the posttest. I asked her, "Brianna, what if a student said to you that they did not understand or follow all the mathematics in your explanation. Can you think of a way to help that student visualize and better understand the answer you came up with?" She thought for several seconds and replied, "I guess I could draw it out [She continues to draw a 1 × 1 square next to the 3 × 4 rectangle and then divides the square into two triangles]. So, there are two triangles inside and each triangle is half the square." Brianna then went on to begin partitioning up the 3 × 4 rectangle into 1 × 1 squares and dividing each square into two triangles while she explained the relationship between the area of the rectangle (12) and the number of triangles inside the rectangle (24).

Brianna's initial bent towards procedural solutions and explanations was also evident in her work with irregular figures. Her method for and explanation of problem 3 on the pretest (Figure 21, p. 212) was procedural and formula-driven. To find the area of a relatively easy irregular figure, she divided it up into squares and rectangles and applied the appropriate formulas. When faced with the same problem on the follow-up test, she partitioned the figure into square units (using dotted lines), a conceptual approach, and concluded her explanation with: “We add up all the boxes to get 8 cm^2.” Brianna’s more blended post-intervention CK was also evident in how she responded to “students” struggling to make meaning of mathematical procedures. This is illustrated by her response to the student in problem 6 on the follow-up test (Figure 23, p. 224) who was struggling to make sense of what the answer (i.e., the number) to the area of rectangle really meant. Brianna said, “I would make sure he understood what square units are [square centimeters would have been better] and when we find area we use square units. I would <u>divide the rectangle up</u> to show him that when we count up the squares inside the rectangle, we are finding the area.” Although she did not draw in the grid, Brianna’s reference to that conceptual idea showed how to effectively address a student’s mathematical difficulty, and demonstrated her developing KoST.

Throughout the study both Brianna and Grace performed relatively well. One somewhat noticeable difference was in their explanations. While Brianna was very mathematical and procedural, Grace more often than not made obvious attempts to conceptually explain her ideas and methods. For example when solving problem one on the posttest (see Figure 33), even when Grace did not include any drawings, as was a consistent finding in her responses, she provided a very conceptual explanation that highlighted making use of a helpful representation (grid paper in this case):

> The rectangle contains 12 cm^2, in other words, 12 – 1cm squares will fit in the rectangle (put a cm^2 grid over the rectangle to illustrate). Then <u>show</u> each cm square can be divided in half to look like the triangle given. So there are 24 triangles – twice as many as the number of squares.

Grace's response would have earned a 4 had she included a diagram. The thoroughness, conciseness, and clarity of her response illustrate how her CK became well organized during the study.

Knowledge regarding irregular shapes. Finding the area and perimeter of an irregular shape has been shown to pose various difficulties for students and teachers alike (Rutledge, Kloosterman, & Kenney, 2009; Tierney et al., 1990). Question 3 on the pretest (Figure 21, p. 212), and again on the follow-up test, asked the PSTs to find the area and perimeter of an irregular figure and then explain "as you would to a fourth grader" how you arrived at both your answers. On the pretest Larry correctly found the area but not the perimeter. He got both correct on the follow-up test. A comparison of Larry's explanations (that were supposed to be meaningful to a 4th grader) reveals a minimal explanatory framework which does nothing to bolster his limited CK. First, from his pretest: to find Area - "Get the # of units on the length and width and multiply," and for perimeter – "Count out each unit around the shape." Now, from the follow-up test: for Area – "Divide it into sections and count how many squares you have in the shape," and to find perimeter – "Count the outermost lines going around the shape." In summary, Larry's construction of a 3 × 4 array inside the rectangle, to help visualize the area, involved making inferences about the shape and is a higher level of measurement reasoning than before the intervention (Battista, 2006). So, although Larry showed some progress regarding concepts related to area, his explanations (such as those presented above) are still lacking and would be confusing in any classroom setting. Although certain mathematical aspects of Larry's responses improved throughout the study, the quality and depth of his explanations revealed an overall shallow CK ill-equipped to support a robust and classroom-useful KoST.

Even before the intervention, Grace had a relatively solid understanding of the major concepts being discussed in this study; however, her explanatory framework (especially regarding units of measure) at times was unorganized and she would struggle trying to clearly communicate her thoughts, as a teacher would need to do. This is illustrated by comments made during our first interview, such as: “I’m not sure how I would explain this to children,” “Oh here, I’m getting confused again,” and “I guess I don’t know what I’m talking about.” Such comments were almost nonexistent in Grace’s responses in the TEs and second interview. Contrast Larry’s work for problem 3 on the pre- and follow-up tests with Grace’s. Larry began the problem each time with a conceptual approach (i.e., he partitioned the irregular figure into square units), but his meager explanations nullified any benefit to that approach. Grace did not pursue a conceptual approach for finding the area in problem 3, either on the pretest or follow-up; however, she not only solved it correctly both times, but also offered two different solution strategies for finding the area (one involving conservation). Solving a problem in more than one way is a trait of an expert teacher and was one quality of her CK that distinguished her response from other PSTs. As compared to her pretest, Grace’s explanations (part b of problem 3) increased in detail, organization, and clarity.

Creative in problem solving. Being able to solve a problem in more than one way is an example of an application of an organized CK and is also a trait of an expert teacher. This category of findings originated after Grace, without prompting, solved question 3 on the pretest in more than one way. Because such problem solving characterizes expert teachers, the other three case subjects were given an opportunity, during the first interview, to solve question 3 differently than they did on the pretest. Larry was not able to solve the problem another way. While talking with Larry it became evident that he could not intelligently talk about area, perimeter, and units of measure, because Larry was unable to consistently identify what attribute of the figure was being measured (i.e., one or two-dimensional). Jackie, after a few exchanges, was able to see that partitioning the figure into square units would produce its area – although she described the square centimeters as, “Each box represents one unit.” Brianna took about 35 seconds to consider the task and after momentarily calculating perimeter, got herself back on track and suggested breaking up the figure into 8 “square centimeters.”

Teaching episode 2 (Figure 15, p. 133) provided the next setting for a planned opportunity to investigate the PSTs’ ability to solve a problem in more than way. The second part of TE 2, which is relevant to this discussion, involved the PSTs finding a correct method to find/estimate the area of a student’s footprint drawn on top of 1 cm grid paper. Question 5 from TE 2 asked the PSTs, “What is one way (other than Tommy’s) to figure out how much area the footprint covers? Can you also describe a second?” The purpose of the second question was to continue in ascertaining whose CK possessed expert tendencies, such as being able to solve a problem in more than one way, which is a trait of an expert teacher. One specific response to the second part of this question offers a humorous side note and a reminder of the importance of clear communication in assessment. In response to the writing prompt, “Can you also describe a second,” one of the higher-achieving PSTs responded, in all seriousness, “A second is a very small amount of time.”

There were four PSTs (including Larry) who similarly indicated that they had no idea how to solve this problem. Ironically, one of the four did some creative sketch work (see Figure 34) on the copy of the footprint provided and came up with a very good approximation for the area of the footprint ("Area ≈ 18.75"). As was the pattern with most unsuccessful responses in this study, there were no sketches at all from the

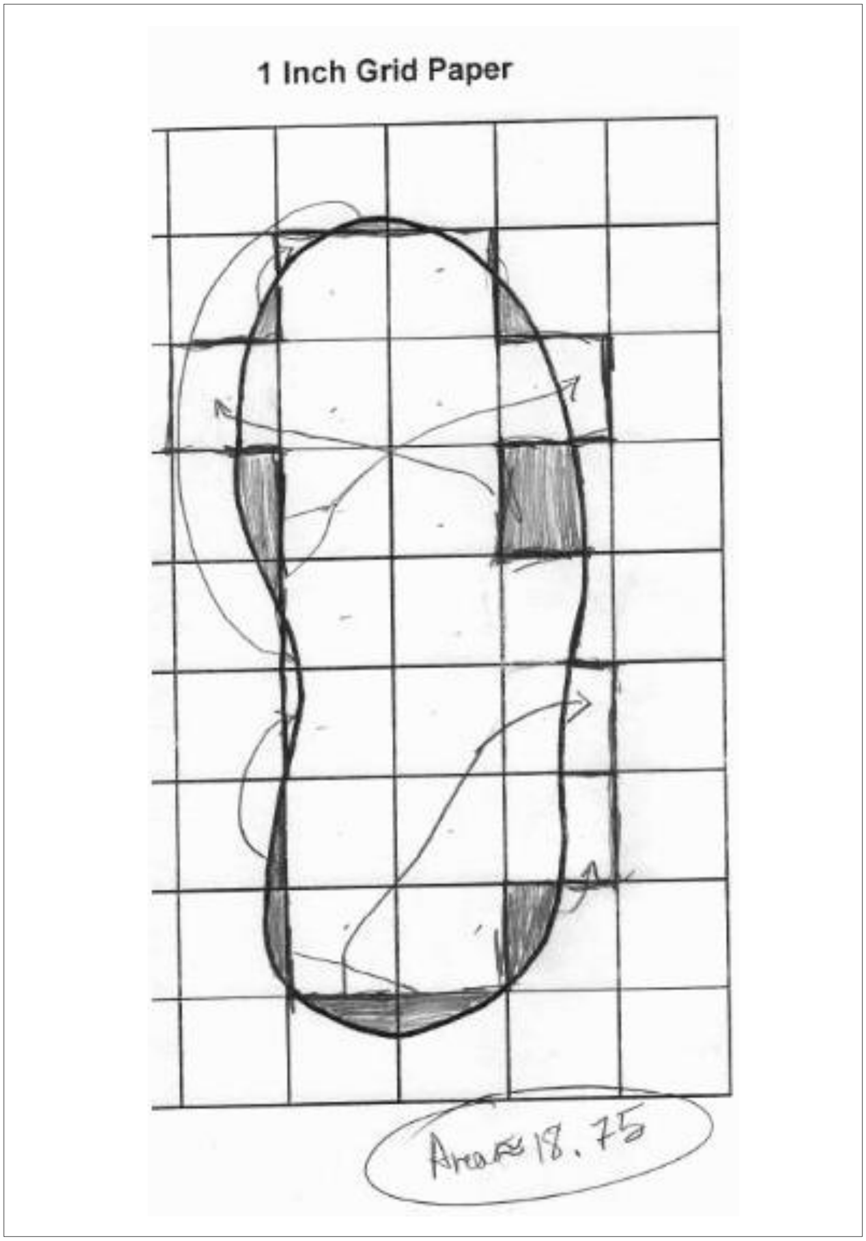


Figure 34. PST's sketch

other three who indicated they did not know how to solve the problem. Up to this point, a lack of productive exploring, and even initiative, had characterized Larry's problem solving. Three other PSTs (including Jackie) offered vague methods with no final answer. Jackie was the only one of these three to do any work on the paper footprint. She numbered the eight complete square units inside the footprint and then basically stopped. This brings up two related facts regarding area that caused confusion for many in the class: (a) a figure can contain partial/incomplete square units, and (b) a figure can have a decimal area. Several of those previously mentioned indicated that it was interacting with the *Gizmo* microworld that opened their eyes to both of these possibilities. Of the five remaining PSTs, two offered very good methods for approximating the area of the footprint, but they did not actually apply their method and get an approximation.
One suggested cutting out all the square and parts of squares and forming a rectangle and then using the L × W formula.

Brianna, who was the other, actually offered two solution strategies. One involved adding up the whole and partial squares and the other was to estimate the height and width and multiply them. It spoke well of Brianna's problem-solving abilities to offer two realistic strategies, but she did not apply either. She made no sketches and offered no estimations. Literally, the question only asks for a strategy, but four of the five PSTs who came up with a strategy also continued the progression and arrived at an estimation. Two others recommended an approach similar to Brianna where they approximated the length and the width and multiplied them. Their approximations were 21.25 and 22.5. Only Grace addressed all the CK components of this TE, and did so in expert fashion. Her two strategies were: (a) "cut out the pieces and fit them into a grid and count the approximate number of square inches," and (b) the other involved moving partial square inches together to form wholes and then adding them up. Grace said her second method would produce an area of between 18.5-18.75 square inches. That work points out a unique difference between the structure of Brianna's and Grace's CK. Both are excellent students and both have performed relatively well throughout the study. The footprint problem involved more creative problem solving than detailed mathematics, and that appeared to be a strength for Grace. Grace was 54 years old when this study was conducted. Her high school geometry course was far in her past. The pre-, post-, and follow-up tests, because of time constraints, tended to be more mathematical (as opposed to exploratory) which favored Brianna.

Ability to explain and illustrate units of measure. Problem 4, which appeared on the pre-, post, and follow-up tests, offered a good opportunity to further investigate any changes in knowledge regarding units of measure (and the ability to explain that knowledge) that occurred from pretest through the follow-up test. Problem 4 asked the PSTs, "As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5th grader? Stress the differences in the concepts." This same question appeared on the pre-, post-, and follow-up tests, and proved to be the most difficult problem for the PSTs. A model response would involve: (a) linking a linear unit to perimeter and a square unit to area, (b) illustrating a discrete linear and square unit, and (c) clearly explaining these concepts without confusing language such as "lines" or "boxes." Although problem 4 was statistically the most difficult problem on the posttest, there was only one PST who received an unacceptable score of 1 for her posttest response, which consisted of: "linear units represent perimeter and square units represent area." The lack of appropriate diagrams was especially noticeable on this problem and contributed to an overall less than acceptable conveyance of these relatively elusive concepts. Only 4 out of 12 PSTs (Jackie, Larry, and Brianna were three of them) incorporated any diagrams as part of their explanation, and there were only 4 (Jackie was one) who did so on the follow-up test. Both Larry and Jackie earned a 1 for her pretest response to this problem, but improved on that dramatically by providing an appropriate diagram for a linear and a square unit (i.e., a shaded square for a square unit and a line segment for a linear unit) on both her posttest and follow-up test; however, she did not connect linear units to perimeter or area to square units on either of those tests. She also called square units "boxes" on both tests and she called linear units "lines" on the posttest. On the follow-up test Jackie correctly referred to linear units as "line segments," but because of the shortcomings mentioned earlier only received a score of 2 for her post- and follow-up test responses.

Larry's responses on his pretest, and subsequent interview, revealed he was very confused regarding linear units and only slightly more knowledgeable regarding square units. During the intervention Larry did however show some growth in his understanding of units of measure. In his response to posttest question 4 he described linear units as "counted line segments," and square units he called "actual squares." His accompanying diagram for a linear unit was a square and he said, "The bold outline of this square is a linear unit." During our interview he clarified that he meant only one side of the square would represent a linear unit. For square units, Larry drew a 2 × 3 array of square units with 4 of the 6 shaded in. He explained how each shaded square represented a square unit. Just like Jackie, Larry neglected to connect linear units to perimeter and square units to area. That combined with the initial unclear diagram for linear units earned Larry a 2 for his posttest response. Larry's work on problem 4 for the follow-up test was a retreat to his pretest quality. He explained square units as "when you are counting squares," and linear units to be "when you count lines." These are unacceptable responses and of no classroom use. It should probably be noted that Larry reported not feeling well during the follow-up test, which may help to explain his relatively quick completion time.

Grace continued to improve on her ability to explain mathematical concepts and on problem 4 on the posttest, she did a good job of differentiating between linear and square units by using words such as "one-dimensional" to describe linear units and "two-dimensional" for square units. She also provided sound practical uses for each unit. Grace's definitions lacked mathematical precision (e.g., no mention of linear units being line segments), and combined with the fact that Grace never included any diagrams to clarify or strengthen her responses resulted in her not receiving a score higher than 3. Brianna, on the other hand, earned a 4 for her posttest response (the only 4 given for this problem), because her diagrams were mathematically correct and pedagogically useful. This is an improvement over her pretest CK regarding units, as diagrams provided during our first interview illustrated she was unclear about the precise nature of a discrete linear and square unit. Her inconsistency with diagrams surfaced on the follow-up test as she only received a 3 for this problem, because she forgot to include appropriate diagrams. One detractor from both her posttest and follow-up test responses was her choice of words. Brianna used the word "line" when describing linear units and that is technically incorrect. The mathematical vocabulary (or at least the choice of words) employed within the PSTs' responses throughout this study often had negative scoring implications. More importantly, a limited mathematical vocabulary hindered the PSTs' ability to respond appropriately to students' difficulties and misconceptions.

Given the relative difficulty of problem 4, one might expect the PSTs to make every effort to thoroughly communicate their ideas. This however was not the case. Using appropriate vocabulary (e.g., saying square cm to describe area when cm are given) was definitely the exception throughout the study for most PSTs. When asked to "Explain mathematically what is correct or incorrect about Justin's method," only 3 of the 10 PSTs (Brianna being one) who identified Justin's method as incorrect were able to explain precisely that Justin used "square units" to measure perimeter instead of "linear units." Grace used the term "2-dimensional units" instead of the more common square units, but she did use "one-dimensional linear units" to describe what makes up the perimeter. Grace was the only PST who consistently used the terms "1- and 2-dimensional" when referring to linear and square units, respectively. Larry said, "Justin's method is incorrect because he is counting the actual squares, not the perimeter outside the shape." This tendency of referring to linear and square units in terms of how they are used (i.e., in finding perimeter and area) as opposed to describing their distinguishing properties, was common among PSTs possessing an incomplete CK about these concepts.

In addition to using clear and precise language, integrating diagrams (and other representations) can help improve communication and foster conceptual understanding of mathematical concepts. The lack of PSTs providing diagrams to support and illustrate their explanations was troubling. That behavior contributed to poorly communicated and insufficient explanations. The word "explain" means to "give details" and "to make clear," but it appeared that to many of the PSTs in this study providing appropriate diagrams was not at the forefront of importance when explaining. It will be seen how this belief interfered with the PSTs' capacity to consistently and effectively apply their CK in order to respond to students' questions and their thinking.

Utilizing drawings. An important aspect of one's CK, especially a teacher, is the ability to explain concepts in meaningful ways (i.e., their explanatory framework). Such explanations involve effective communication. Incorporating suitable drawings is one important aspect of successful explanations. The extent of this facet of the PSTs' CK was evident when they were given opportunities to provide diagrams to support or add precision to a mathematical response or to add necessary context or to clarify when asked to respond to a hypothetical student's difficulty or misconception. Table 19 reveals the progression of PSTs' use of drawings as the study continued. Out of 48 potential opportunities (12 PSTs × 4 problems) to use drawings on the pretest, 16 (33%) drawings were attempted, but there were only five (10%) that accompanied a meaningful and correct response. The rate of drawings provided increased for the posttest. There were 72 reasonable opportunities (12 PSTs × 6 problems) to incorporate a drawing, 42 (58%) drawings were provided, and of those, 27 (38%) assisted in achieving a correct response. That is an increase of 28% over the pretest. The follow-up test, which contained the exact same questions as the pretest, showed an increased use of drawings over the pretest. Out of the same 48 opportunities, drawings were used 29 times (60%), and 19 of those (40%) were successful in facilitating an acceptable response. That is a 30% increase over the pretest rate and a negligible 2% increase over the posttest.

A prime example of that fact is how the PSTs dealt with question #4, which appeared on the pre-, post-, and follow-up tests, and was statistically the most difficult item in the study (mean of 1.58, 2.33, and 2.33, respectively; range 0 to 4). That question

Table 19

Use of Drawings Throughout the Study

	Pretest Items				**Posttest Items**						**Follow-up Items**			
PST	4 (U)	5 (R)	6 (U)	8 (R)	1 (U)	4 (U)	6 (R)	8 (R)	9 (U)	10 (R)	4 (U)	5 (R)	6 (U)	8 (R)
#1		"X"		"X"	X			X	X	x		X		X
Grace		X	*					*		*			X	X
#3				X	"X"		X	X	X	X	X	*		X
#4		X		X	X		x	x	x				*	X
#5						x				X	X	"X"	X	
#6				X	X		X	X		X		X	X	X
Jackie			X			"X"	x	X	X	X	"X"	X	X	X
Brianna			*	X		X	X	X			X	X		X
#9		"X"	*	x		x	X	X		"X"		"X"		X
#10		"X"		x	X	"X"	X			X	X	X	*	X
#11	"X"			x	X		X		X	x		"X"	X	X
Larry		"X"	*		X	x			X			x	*	

Note. U = dealt with units, R = dealt with perceived relationships; * = suggested a drawing but did not draw it; X = used appropriate drawing; x = used a drawing inappropriate for teaching/learning; "X" = drawing did not facilitate a meaningful or correct response.

asked the PSTs to "As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5th grader?" Most PSTs indicated that conceptualizing and explaining linear and square units was very difficult for them; however, on the pretest only one of the 12 PSTs attempted drawings, albeit inaccurate, as a means to help visualize and/or explain these difficult concepts. As evidenced in Table 19, the use of drawings increased for question #4 from the pretest levels, but the occurrence of meaningful and accurate drawings was very low – 1 out of 6 for the posttest and 2 out of 5 for the follow-up. It was very common for PSTs to use the word "line" to describe a linear unit, and then also draw a line, as Brianna did on the follow-up test. At other times, PSTs would draw things such as a 12 inch ribbon (not to scale) when describing linear units. The discrete nature of the concept of a unit was not consistently evident.

An apparent pattern in Table 19 was that certain PSTs tended to use drawings more consistently than others. For example, following the pretest both Jackie and Brianna began incorporating drawings in their responses on a more regular basis, whereas Grace and Larry did not. The use of drawings was not directly connected to performance. Grace was one of the top performers in the study, but barely ever used drawings to communicate her ideas, but PST #6, another top performer, effectively used drawings on the post- and follow-up tests. Some weaker PSTs increased in their successful use of drawings (e.g., Jackie), while other low performing PSTs' (e.g., #5 and Larry) use of drawings was inconsistent. On the entire pretest Jackie only provided one (rather vague) diagram to help support her explanations. For the posttest however, Jackie included 19 appropriate diagrams. That awareness of the importance of including representations when explaining mathematical principles and relationships showed a significant increase in her KoST. One possible explanation for the lack of drawings by certain PSTs might be that of necessity. For many of the pre-, post, and follow-up questions, certain higher-performing PSTs (e.g., Grace) did not seem to need sketches or diagrams in order to facilitate a successful answer; however, when faced with an elusive problem, Grace would use sketches. For example, TE 2 asked the PSTs for their thoughts on finding the area of a footprint traced on square-inch grid paper, and while that task was not the primary focus of TE 2, it proved very motivating and equally challenging. Only 4 out of 12 PSTs were even able to provide any meaningful sketches in an attempt to approximate the area of the footprint, and Grace was one of them. As a matter of fact, she was one of only two who arrived at a very accurate approximation of between 18.5 and 18.75 square inches. Grace's sketch was very similar to Figure 34, but hers included a numbering of the full and partial square inches. So for some (e.g., Grace and Brianna), not consistently using diagrams did not appear to be due to a lack of CK. Another example of this arose during the posttest. There were two questions on the posttest (#s 9 and 10) in which drawings were expected and yet Brianna did not provide any. During her second interview, she was asked about her lack of drawings. Although Brianna did not provide a reason for not including drawings, whenever one was requested she rather easily provided useful and meaningful drawings. As was true on the pretest, there were times on the post- and follow-up tests when the PSTs' limited CK left them ill-prepared to construct a meaningful drawing. That was the case with question 4. Other times the PSTs were careless and drew rectangles that were not to scale and thus did not facilitate a correct response. Although an increase in the use of diagrams was noticeable for many PSTs, there were numerous missed opportunities, which in reality, translate into a lack of realization of the importance of drawings in communicating and clarifying mathematical concepts. Both the increased usage and the missed opportunities reveal varying degrees of change in this facet of the PSTs' explanatory framework (part of their CK), which plays into their ability to successfully respond to student shortcomings and/or misconceptions (a facet of their KoST).

Responding to Student's Misunderstandings Regarding Units of Measure

The findings in these next several sections primarily address research question 4, by focusing on how the PSTs' KoST changed during the study. The two primary facets of KoST are: (a) the organization of CK so as to enable a teacher to understand children's thinking – the diagnosing aspect, and (b) appropriately addressing student difficulties and misconceptions – the intervention.

Focused on solving, or diagnosing & responding – emergent CK & KoST. The emergent findings presented in the next rather detailed section continues to examine the PSTs' understandings regarding units of measure (i.e., their CK), but now the focus will be on how they indicated they would respond to student difficulties and misconceptions, specifically regarding units of measure (i.e., their KoST). These facets of the PSTs' KoST are manifestations of the organization of their CK. An expert KoST would enable a PST to understand children's thinking and then respond appropriately to difficulties by focusing on the student's understandings instead of the content and getting right answers. These findings came primarily from the three teaching episodes (TEs), and include a discussion on the impact of the microworlds (MWs) upon the PSTs' CK and KoST. It will be shown how several PSTs had a misguided focus which lead them to work on secondary aspects of certain TEs, while not giving enough attention to diagnosing the student's erroneous thinking and adequately responding to that student. Certain emergent findings taken from the TEs (e.g., PSTs' use of MWs) have no parallel pre-intervention findings to compare to; however, such findings still contribute to answering research questions 3 and 4 as they illuminate the PST's CK and KoST. An in-depth look at emergent findings related to KoST will begin by revisiting TE 1.

Teaching episode 1 (Figure 14, p. 130) involved a student (Justin) using square units in an attempt to devise an alternative method to find the perimeter of an irregular figure. As discussed previously, only Jackie thought Justin's erroneous method to be viable. Writing prompt 5 asked the PSTs, "As a teacher, how would you respond to Justin's thinking and his method? What specifically would you say and do?" Jackie wrote, "I would agree with Justin's method because he found the perimeter by calculating the square units around the sides." That type of writing prompt provided insight into the PSTs' KoST and was useful in examining how these future teachers indicated they would respond to the student and his/her thinking. Jackie's knowledge, both her CK and KoST, did not remain dormant during the teaching episodes. Her responses to questions 6 and 7 from TE 1 indicated she realized Justin's method was incorrect, albeit after interacting with the Shape Builder microworld (MW): "I now believe that Justin counted the boxes around the shape instead of counting the sides around the shape." Jackie's revised response to writing prompt 5 was teacher-centered and focused on telling Justin how to get the correct answer. While Jackie's realization about Justin's incorrect method strengthened her CK regarding appropriate units for perimeter, her mathematical vocabulary left much to be desired. Her reference to square units as "boxes" and linear units as "sides" revealed a weak explanatory framework, another facet of CK.

The PSTs' reactions to Justin's method and his thinking involved various responses with common themes, which helped to paint a picture of their current KoST. Generally their responses involved: (a) praising him for realizing perimeter was around the outside, (b) trying to modify Justin's method to produce a correct answer, (c) asking him to explain his method, (d) teacher-centered activity (e.g., "I would explain" or "I would show") involving re-explaining what perimeter is, or (e) systematically walking Justin through his method and pointing out that it would not arrive at the right answer. Larry's response characterized those whose response addressed parts a & b: "I would tell him that he is doing a good job in trying to make sense of it visually, but he needs to understand that counting squares will leave him coming up with a short answer." Larry's response (and those like his) falls short because instead of addressing the fundamental misconception surrounding Justin's method (i.e., using square units to measure and calculate perimeter), he focused on explaining how Justin's method might work if it were modified, besides the fact that the modification was mathematically incorrect. Unlike Larry, Grace indicated she would respond to Justin through a teacher-centered approach involving a detailed explanation of what perimeter is ("a 1-dimensional linear measurement") as well as how to calculate it ("Count the segments of the line that borders the shape"). Grace did not include a discussion of units with her explanation; however, after interacting with the Shape Builder MW, Grace amended her previous response to include diagrams and meaningfully directed questions to help Justin conceptualize and clarify the differences between perimeter and area.

Several other PSTs were very creative in offering alternative illustrations to help Justin better understand perimeter (e.g., fences, pieces of string), but only two PSTs (one being Brianna) actually discussed the most-likely cause of Justin's incorrect method, his confusion with linear and square units, and why one measures perimeter and the other measures area. Brianna's response involved acknowledging the correct aspect of Justin's method (i.e., perimeter is the measure of a shape's outer boundary), explaining the error in his method, showing (with diagrams) the differences between linear and square units and why linear units should be used, and concluded by having Justin rework the problem to see if he understood. Brianna was able to apply her CK and customize her response to appropriately address Justin's method and his thinking. This type of focus on the student, while promoting conceptual understanding, earned Brianna expert codes for her KoST.

Near the end of the individual work for TE 1, after the PSTs had opportunities to investigate the problem with the Shape Builder MW and reflect on their previous responses, a writing prompt asked them, "As a result of seeing Justin's method and apparent confusion, how would you follow up with the entire class about the concepts that surround this classroom episode?" A majority of PSTs (9 out of 12) again responded with teacher-centered suggestions; however this time many said they would incorporate the microworld into their explanation. Larry's response, while containing technology, lacked mathematical and instructional specifics: "I would probably project the microworld onto the screen and explain with a laser pointer how to come up with the solution." Jackie's method involved several more incremental steps and tried to place a stronger emphasis on student understanding; however, because it lacked a thorough discussion of linear and square units it too digressed into a show-and-tell approach to finding the correct answer. Teacher-lead discussions emphasizing how to get the correct answer dominated these responses. Of the remaining three PSTs, one (#3) presented a very clever use of the Shape Builder MW to help the students better understand why Justin's method was wrong, but again the focus was on finding the solution. Grace offered vague ideas involving discovery-type activities for the students to do on the MW, but did not indicate she would summarize the concepts of linear and square units. Only Brianna used the MW and its features to guide the students in discovering for themselves that Justin's method was wrong and why it was wrong – more evidence that Brianna was slowly moving away from purely procedurally-based approaches to where she was applying her CK in ways that bolstered her KoST.

Although a more thorough discussion regarding TE 2 will be presented in later sections, TE 2 contained specific findings related to the PSTs' focus while diagnosing student's methods, and offered a prime example of how a wrong focus by PSTs can result in poor diagnosing of student misconceptions and missed opportunities to address those difficulties. Teaching episode 2 (Figure 15, p. 133) involved a situation in which a 5th grade class is studying area, and they are challenged to find the area of one of their footprints. Their teacher instructs them to stand on a piece of paper and trace their shoe, and then individually brainstorm a strategy to find the area of the footprint. After several minutes one of the students, Tommy, comes up and explains his method. He says he would lay a piece of string around the outside of the paper footprint, cut the string to the precise length, form the piece of string into a rectangle, use a ruler to measure the length and width of the rectangle, then find the area of the rectangle. In other words, he believes that the area of the rectangle will be the same as the area of his footprint. TE 2 required the PSTs to grapple with two relatively difficult concepts. One was the misconception that a fixed perimeter (i.e., the piece of string) can have only one area (i.e., the desired area of the footprint). The second involved a correct method to find/estimate the area of a footprint (an irregular shape). Each PST was provided with two copies of a footprint drawn on 1-inch grid paper as well as blank pieces of the 1-inch grid paper. Findings showed that the PSTs who struggled most throughout TE 2 were also the ones who excessively focused on trying to find the area of the footprint (i.e., what they thought "solving the problem" involved), and as a result paid too little attention to dissecting Tommy's method and the misconception behind it.

As was the case with Jackie, it appeared that several PSTs had difficulty in translating Tommy's method into a concept that could be verified or disproved. She wrote early on in TE 2, "At this point I don't know what to do next, because I don't really know how to find the area of a footprint." Another PST wrote "To be honest, this problem has stumped me ☺. I don't really know how to solve this problem, but I think that Tommy's method will work." Even though this PST indicated that she felt estimation would be needed to find the area of a footprint, she did not attempt any sketches and did not draw anything on the footprint copies. Other PSTs realized Tommy's method was an incorrect generalization but still struggled in responding clearly and succinctly to Tommy's thinking. For example, Larry had figured out mid-way through day 1 that Tommy's method was wrong, "I would show him (by using his method with the string) that the perimeter can be equal but the area can be different;" however, his writings indicated that he felt he could not address Tommy's thinking without first figuring out how to find the area of the footprint, which he never did. That was certainly not the case since Brianna, and three other PSTs, were able to correctly diagnose the inconsistencies in Tommy's method while not expressing confidence about finding the area of the footprint. During the first day of TE 2 another PST, call her Stephanie, had apparently stumbled upon the misconception behind Tommy's method when she wrote, "The fact that two objects have the same perimeter does not automatically mean that they will have the same area," but from that point, the focus of her writings turned to finding the area of the footprint. At some time during the TE that same PST produced the sketch in Figure 36, which is a very close estimate to the area of the footprint; however, five different times while completing the remainder of the writing prompts she wrote, "I don't know how to find the area of the footprint."

Overall, a preoccupation with finding what the PSTs judged as "the answer" to the TE not only hindered their ability to properly diagnose and address Tommy's thinking, but it also limited their meaningful interaction with the Shape Builder MW, which incidentally could have been used to build a very close replica of the footprint and approximate its area. Jackie reported: "I don't think they [the microworlds] really helped me with this problem [TE 2]. At this point, I am still confused on what the *right way* [italics added] is to figure out the area of the foot." Rather than assessing the student's thinking, this PST was focused on determining the answer for herself. This is an example where the PSTs were over-engaged in their role as a learner (i.e., problem solver) to the neglect of their role as a teacher (i.e., to diagnose and instruct).

Findings related to the PSTs' focus while diagnosing student thinking will continue by examining their use of and recommendations regarding the MWs integrated into this study. Such findings contribute to answering research questions 3 and 4 as they illuminate the PSTs' CK and KoST; CK, because the MWs facilitated various self-proclaimed "ah-ha" moments for the PSTs, and KoST, because the MWs are an effective tool to facilitate the application of one's CK to appropriately respond to a struggling student or facilitate a meaningful whole-class discussion.

Microworlds' impact on PSTs' knowledge. Each TE presented a classroom-based scenario focused on a documented misconception regarding area and perimeter. Each began with questions related to CK, and then would transition into KoST. Interacting with the MWs came at different times during the TEs, and was accompanied with opportunities to reflect upon earlier writings regarding the PSTs' CK and KoST. This progression proved valuable to several PSTs in each of the teaching episodes. The two MWs utilized in this study possessed specially-designed features that would allow for and facilitate the exploration and hypothesis testing of the student's thinking described in the TE. There were many comments such as, "After I used the microworld, I saw the error in the student's thinking" that indicate various forms of learning occurred while PSTs interacted with the MWs.

The first teaching episode (see Figure 14, p. 130) focused on misconceptions involving area and perimeter and linear and square units. For this teaching episode, the students were only given access to the Shape Builder MW, as its features matched well the concepts related to the focus problem. A unique aspect of this MW is its presentation of area and perimeter as well as linear and square units simultaneously. This feature did prove to be a perturbation for some; however, two of the more "expert" PSTs (one of them Brianna – a case subject) commented on this potential confusion and offered a pedagogically-sound recommendation. They both thought it would be helpful if Shape Builder had a feature that could be turned on and off and would darken the outside edges (i.e., linear units) of any shape on the grid, hence making the perimeter stand out from the shape's area. From a mathematical perspective, the focus problem presented in TE 1 was the easiest of the three to decipher. All 12 PSTs correctly indicated that Justin's method was wrong and they also were able to find the correct perimeter of the figure. This should have allowed for the PSTs to more freely explore with Shape Builder as well as to better focus on the student's thinking and subsequent instructional strategies, as opposed to solving the problem. The hope was that the PSTs would recognize that the primary confusion of Justin was that he used square units to calculate perimeter; thus, the misconception centered on units of measure. The PSTs' interaction with Shape Builder produced various learning paths and outcomes. Table 20 reveals the case subjects' usage of MWs ranged from a means to confirm CK, to a tool to investigate the student's thinking. While case subjects were the focus for Table 20, because their responses could

Table 20

Findings Related to Microworld Usage & Benefits

	Grace			**Jackie**			**Brianna**			**Larry**		
	TE 1	TE 2	TE 3	TE 1	TE 2	TE 3	TE 1	TE 2	TE 3	TE 1	TE 2	TE 3
Used mostly to confirm answers*								X			X	
Used also for exploration*	X		X	X		X	X	X				X
Saw value for personal learning*			X	X		X		X	X	X		X
Saw value for instruction*	X	X	X	X			X	X	X	X	X	X
Facilitated a more thorough CK*	X		X		X			X	X	X		
Facilitated a more thorough KoST*	X		X	X			X		X	X		

Note. *Based on written responses found in TEs. For TE 1 and TE 2, the MWs were not available until after the PSTs had already worked on the problem.

be corroborated during the second interview, their positions were representative of various subsets of the PSTs. In part, the range of reactions is illustrated by the responses to a writing prompt which asked the PSTs, “In what ways, if any, did interacting with the microworld help you better understand the ideas surrounding this problem and Justin’s thinking?” One of the weaker students wrote, “The microworld helped me to verify that my answer was correct” (even though it actually was incomplete and limited in scope and depth); whereas, a stronger-performing student seemed to realize the intended purpose of the activity and its accompanying MW when she wrote, “Definitely yes! I understand why Justin shaded in the squares and counted them to find the perimeter. As I drew the figure in the microworld, I was beginning to think I was thinking the way he did!” This quotation reveals how the PSTs’ KoST grew as a result of interacting with the MW. A teacher cannot help a struggling student until they can understand what they are thinking.

The way in which the PSTs indicated they would address the entire class as a result of becoming aware of Justin’s thinking paralleled their overall progress to that point in the study (i.e., pretest score and teaching episode codings) and reveals PSTs’ levels of KoST. About half the PSTs indicated they would use Shape Builder and project an exact replica of Justine’s diagrams up on a screen in front of the class and walk the students through Justin’s method (several said they would not mention Justin’s name) and point out what is wrong with the method and what the right answer is. This tendency of teaching in order to enable students to get right answers, in contrast to focusing on conceptual understanding, is a trait of a novice teacher. Contrast that with the instruction suggested by several other PSTs. For these the focus was on identifying and distinguishing between linear and square units and how this would enable the students to ascertain that Justin's method was incorrect. Two particular responses (one of them being Grace) tend to substantiate that interacting with the microworld helped to stimulate creative and conceptual instruction strategies. The first involved using a feature of Shape Builder to help drive home a fundamental difference between area and perimeter.

The recommendation was to use the "Create Shape" mode to build a square but leave the center "hollow," and have the "Show Perimeter" and Show Area" boxes checked. The square would look similar to Figure 9 (p. 122). Then use the "Fill Blue Shape" feature, which would completely fill the square with square units. The "ah ha" moment for the student occurs when they click the "Fill Blue Shape" button and the area number changes but the perimeter number does not; thus, illuminating the concept for them that the area is the inside of a shape and comprises square units while the perimeter is represented by the outside boundary of a shape. A second PST suggested an instructional strategy that was straightforward and illuminating. The recommendation would not only show a major inconsistency with Justin's method, but it also emphasized an understanding of linear versus a square units. Grace recommended creating a 1 × 2 rectangle in Shape Builder (i.e., a rectangle made up of two squares); hence, there would be nothing to shade (the major aspect of Justin's incorrect method for finding perimeter), and then it would be plain to see the shape had an area of two square units and a perimeter of six outside edges (i.e., linear units). The fact that these two PSTs ventured away from simply creating the figures presented in the focus problem and came up with two totally different instructional strategies reveals the flexibility and subtle power of a microworld. The intended instruction would not only help classroom students see the error of Justin's proposed method but also experience a conceptual approach to learning fundamental concepts of area and perimeter (i.e., linear and square units).

The relative difficulty of teaching episode 2 (Figure 15, p. 133) resulted in more extensive investigating with both of the microworlds available in this study (Shape Builder and Gizmo) as well as some meaningful learning outcomes. One PST commented that the “Compare Areas and Perimeters” feature of Shape Builder helped her realize “that she, like Tommy, was over-generalizing that the 18” string could have only one area. I think the string distracted me from realizing sooner that perimeter does not determine area.” Another PST, who had already found several counterexamples to Tommy’s solution strategy, was exploring with the Gizmo MW (Figure 7, p.120) when she indicated that she found a “shape” that had a perimeter of 18” but an area of 0 (i.e., a line segment). Although that is somewhat of an extreme counterexample (and not a 2-dimensional shape) of the TE’s primary misconception (i.e., a fixed perimeter can have only one area), it does show the facilitative nature of a well-constructed microworld to stimulate growth in CK. Along these lines, several PSTs went to great lengths to list many rectangles (including ones with decimal dimensions) that had a perimeter of 18, but having different areas, thus effectively disproving Tommy’s method. No one, however, wrote about how the Gizmo MW could be a jumping point for a discussion that there are actually an infinite number of rectangles that have a perimeter of 18”. Only six PSTs were able to establish that Tommy’s method would not necessarily work, hence for TE 2 there were limited findings on the microworlds facilitating content learning or informing instructional strategies. Five of the six PSTs who successfully diagnosed Tommy’s misconception specifically wrote about when their epiphany occurred. Of those, only two indicated the MWs were instrumental, while three discussed how “playing around” with the string helped them the most.

Results regarding the ways in which the PSTs' CK and KoST changed through interactions with the MWs will conclude with an interesting finding related to their opinions concerning learning with, versus teaching with, the MWs. For TE 1 and TE 2, the MWs were not introduced into the session until half way through Day 1. For TE 3 (Figure 16) the PSTs were instructed that they could access either microworld right from the outset. For the first TE (the easiest of the three) the vast majority of the PSTs (11 out of 12) indicated they found the microworld helpful to their understanding of the problem as well as Justin's thinking. They also explained that they would use the microworld as an instructional tool in a whole-class discussion of Justin's misconception. A similar majority (10 out of 12) indicated they believed classroom students would benefit from personally interacting with the MW in a structured context. However, an unexpected trend developed as the mathematical content of the teaching episodes got progressively more difficult and the hypothetical students' thinking was increasingly more elusive.

Although the number of PSTs who indicated they learned with and/or saw benefits of personally interacting with the microworlds was a strong majority (8 for TE #2 and 11 for TE #3), fewer (five from TE #2 and six from TE #3) said they would incorporate the microworlds when instructing future students about the concepts presented in the TEs, even though the same PSTs admitted those future students would most likely possess similar misconceptions as the hypothetical students presented in the teaching episodes. These beliefs indicate an incomplete application of the PSTs' KoST. The PSTs in TE 3 who did suggest incorporating MWs did so in very teacher-centered ways, evidenced by comments such as, "I would show . . . ." or "I would use the microworld to explain . . .” The MWs were often seen as a means to simply verify answers and/or display visual representations, and some even viewed the technology as a potential nuisance or distraction, as one PST remarked, “The microworlds were beneficial, but I do not believe they should take away from classroom instruction.” The same PST wrote just a few pages earlier, “I used the microworlds to do a little searching and analyzing and came to the conclusion she (Jasmine) is <u>very</u> mistaken and should be clarified.” Brianna, a case subject, wrote, “I did not know, before interacting with the microworlds, that Jasmine’s theory was incorrect. But using the microworlds, particularly Shape Builder, helped prove that it was wrong and helped me visualize the concept;” however, neither of these PSTs recommended that students spend any time interacting with the microworlds as part of their instructional strategies. A similar contradiction appeared when only two PSTs from TE #2 and three from TE #3 (of the eight and 11 respectively who indicated they learned from the microworlds) wrote that they would allow time for the students to personally use the microworlds to explore the concepts surrounding the teaching episodes. Apparently, the majority of PSTs felt the microworlds were a valuable learning tool for themselves but not for their future students. There seems to be evidence that indicates that the low occurrence of suggested MW usage from the TEs was not due in entirety to the newness of the technology.

Table 21 shows that of the questions whose design and content could have easily facilitated discussions involving the use of a MW, only a couple elicited such responses from the PSTs. Even questions 8 and 9 from the follow-up test, which formed the basis for TEs 3 and 1 respectively (where MWs were used extensively), received very few references to using MWs to help instruct a struggling student. Apparently, it takes time

Table 21

Instructional Recommendations for Microworlds

	Posttest Items				**Follow-up Items**		
PST	6 (U)	7 (U)	8 (R)	10 (R)	5 (R)	8 (R)	9 (U)
#1	X					X	
Grace	A		A				
#3		A	A				
#4	A	A		A			
#5	X			D		X	
#6	A	A	A	X		D	
Jackie	A	A	A				
Brianna			A	A			
#9				A			
#10	A	A		A			
#11	X	A					
Larry	A		A	A			

Note. U = dealt with units, R = dealt with perceived relationships; D = written response included pictures that looked like images from a microworld (MW); X = recommended using a MW w/o being prompted; A = recommended using a MW in response to a writing prompt at the end of the posttest.

and many experiences for a microworld to become an extension of and tool for one's thinking. Once the personal integration of microworld-thinking has begun to take root, then a vision for its integration into instruction can begin to take form.

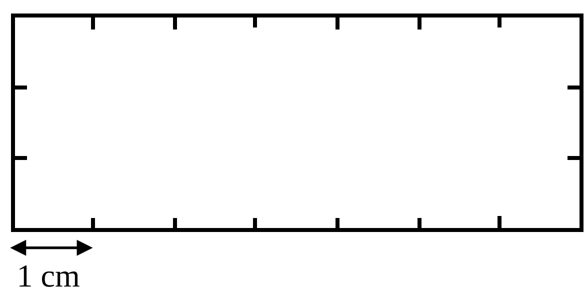
Realizing the importance of units in explanations. Results from question 9 on the posttest (Figure 37) helped in describing the change in PSTs KoST as it relates to units of measure. Question 9 addresses similar concepts as question 6 from the pretest, and the PSTs' responses were compared for signs of growth. Both questions present a problem centered on a figure and a scenario in which a discussion of units, by the PST, would be needed to clarify the difficulties of the hypothetical student. Question 6 presented a student who correctly found the area of a rectangle (i.e., 18) but is confused about what the number 18 actually represented (i.e., the number of square units), while question 9 involved a student who calculated the area of a 3 × 7 rectangle to be "20 square cm." The perimeter would be 20 but "cm" would be the correct unit. To be successful with question 9, the PSTs needed to do two things. First, realize that the student's answer of 20 is the number of centimeters in the perimeter of the rectangle; therefore, the student is apparently confusing area with perimeter. Second, an appropriate intervention would involve combinations of the following: (a) asking how the student arrived at their answer of 20 so that an appropriate follow-up could ensue, (b) construct a 3 × 7 array within the rectangle to visualize the 21 square units – the area, (c) review what is involved with finding area and perimeter, and (d) have the student then compute both the perimeter and area to compare. The scores on this problem indicated it was the

9. A student calculates the area of the rectangle shown to be 20 square cm.

(a) Is the student correct?

If not what is the correct answer?

How did you figure your answer?



(b) What do you think the student was thinking to arrive at their answer?

(c) As a teacher, what specifically would you say or do to help clear up any possible confusions the student might have?

Figure 35. Question 9 from the posttest.

5th hardest problem on the posttest. There were five 2s, five 3s, and only two 4s. The primary cause for the lower scores was a wrong focus, which then lead to an incomplete and, subsequently, ineffective intervention. Take for example Brianna, who only scored a 2 on this question. She focused on the belief that the student simply used the wrong formula (i.e., followed the wrong procedure). So Brianna's intervention was: "I would explain that to find the area of a rectangle, by using the formula length × width, we must first find the length and the width." As was representative of the weaker responses, there was no discussion of linear and square units. Another aspect lacking from the weaker responses was the inclusion of a diagram to aid in a conceptual explanation. The student's answer of 20 (the perimeter of the rectangle) should also have initiated a conceptual explanation of the student's error by comparing it to a 3 × 7 array, which represents the area and could have been drawn inside the rectangle.

During our second interview, I asked Brianna: "What if the student has a hard time seeing why the 21, that the formula produces, is the correct answer?" Brianna, without hesitation, replied: "So, divide the shape up using grid lines to reveal the square units." She answered so quickly and confidently that I am not sure why she did not just include that in her initial response. Larry actually did draw in the 3 × 7 array, but after that basically said the student got confused and did perimeter instead of area. His intervention was simply, "Just review what area and perimeter are again." During our interview I asked him about why he drew the grid of squares inside the rectangle. To my surprise, he replied, "I don't know. I just did it to make sure? I don't know." It is possible that Larry constructed the array "to make sure" that the area actually was 21, but apparently that approach was not seen as valuable to the struggling student. Jackie also constructed the 3×7 array and according to our interview used that method to find the area and conclude that the student's answer of 20 was wrong. Jackie diagnosed that the student used the word unit to calculate area, but did not include that dialogue or any mention of her array in her planned intervention. Jackie's CK continued to become better organized to assist in her diagnosing of student difficulties and misconceptions, but her ability to process the implications of the students' errors and respond accordingly needs further intervention.

Before discussing Grace's response, there was one more comment made by three PSTs that bears mentioning. Three different times it was brought up that a PST felt the "tick marks" included on the rectangle were confusing and should be removed. Brianna was one of the three, so I asked her during her interview if she saw any value in the apparent confusion caused by the tick marks on the rectangle. After a 15 second pause, she responded, "I don't know." It is curious that she indicated in her response to the question that, "The student was confused with area and perimeter," but she could not conceive that the tick marks would most likely produce the perturbation that should have served as a valuable assessment tool. This portion of the discussion of PSTs' KoST will conclude with a brief examination of Grace's response. Grace received a 4 for her answer. She used a formula (3 cm × 7 cm = 21 sq cm) to calculate the area, and followed with: "The student may have been thinking perimeter, because the perimeter is 20 cm." Her intervention included three of the four recommendations listed earlier. She did include that she would ask the student how he/she came up with the answer – only two PSTs did. As has been seen in other responses by Grace involving units, she did not draw in the array to illustrate the square units; however, she indicated that she would do just that as part of her response to the student. Battista’s (2006) highest level of measurement reasoning is that of making inferences about numerical measurements of objects (e.g., as if the array has fallen into the background and is considered already complete). It cannot be said for certain that this applies to Grace, but it would help to explain why she has continually used arrays in her discussions while seldom including drawings of them.

I will conclude this discussion of the PSTs’ post-intervention knowledge regarding units of measure by highlighting findings of the PSTs’ responses (specifically the case subjects) to question 9 on the follow-up test (Figure 36) while at the same time

9. Jose wants to calculate the perimeter of the shape shown in Figure 1. Jose’s method is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those squares.

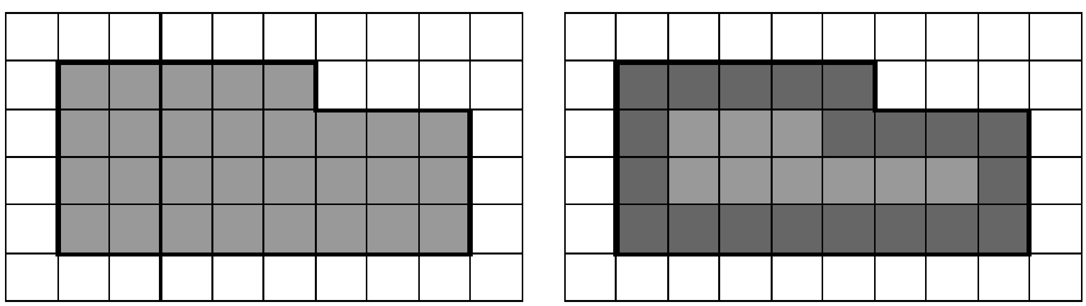


Fig. 1 Fig. 2

(a) Is Jose’s method correct? If no, what would Jose’s method produce for the perimeter of Fig. 1, and if necessary, state what is the correct answer?

(b) Explain why or why not.

(c) As a teacher, how would you respond to Jose’s thinking and his method? What specifically would you say and do?

Figure 36. Question 9 from the follow-up test.

comparing them to the other two instances in which they faced the same problem (in the pretest and TE 1). The findings for question 9 will focus on the case subjects, since their second interview, which was structured to be a learning experience, occurred after the posttest and a month before the follow-up test. Since question 9 spans the timeline of the study, the findings surrounding it are a good representation of the case subjects' knowledge regarding units of measure. Question 9 is one of only 2 test questions that appeared on both the pretest and the follow-up test, as well as being features in a TE (i.e., before, during, and after the intervention). The other one is #8, which will be discussed in the "Perceived Relationships" section to follow.

The knowledge necessary to formulate methods to solve problems in mathematics draws on one's CK related to that subject; being able to apply that knowledge as a teacher in order to understand student's methods of solving problems (especially when unconventional) draws on one's KoST. An examination of the PSTs' scores (range = 0-4) of question 9 on the follow-up test reveals some change in both CK and KoST. When this same problem was asked on the pretest the scores indicated that it was the second hardest item on the test (M = 1.92; SD = 0.9). The only scores above a 2 were one 3 and a 4 received by Brianna. On the follow-up test the mean climbed to 3.17 (SD = 0.84), which was the second highest mean on the test. Although it might be expected that most PSTs would make progress in their understanding of this problem's concepts (CK), based simply on repeated exposure to the problem, there was marked improvement in how several indicated they would respond to the student and his confusion (KoST). Jackie is a prime example of this. On the pretest, Jackie barely earned a 2 by providing the diagnosis: "Justin is not determining the perimeter but the area." Her response to the student involved only clarifying the differences between area and perimeter – no mention of units. During the first interview I asked Jackie about Justin's (Note: the students' names were changed from the pretest to the follow-up test) method and how she would clarify area and perimeter for him. She had no clear idea of why Justin might come up with such a method, and her clarification of how to find perimeter digressed into an explanation involving point-counting (instead of linear units). A fragile and unorganized CK left Jackie with no foundation from which to respond effectively to the student's misconception. When this question surfaced again as the focus problem for TE 1, Jackie initially responded by saying, "I believe Justin's method is correct because he counted the square units on the outside of the shape." While Jackie's mathematical vocabulary had expanded (i.e., correct use of "square units"), her understanding of perimeter and linear units (her CK) was still sparse and disconnected.

The intervention contained in TE 1 (exploring with the microworld, small-group sharing, whole-class discussion) resulted in Jackie realizing the error in Justin's method; however, her response to Justin and his thinking was primarily focused on helping Justin get the right answer: "I would explain to Justin not to count the squares around the shape, but count the sides of the boxes around the shape, which is a common trait among novice teachers. That response, making use of the words "squares" and "sides" is unclear and reveals a KoST that was still unprepared to address student shortcomings in meaningful ways. She made this comment before the small-group sharing and whole-class discussion which Jackie indicated she enjoyed and learned much from. During our second interview I asked Jackie about her choice of words in the preceding quotation and at this point she said, "I meant that he shouldn't be counting the square units, he should be counting the linear units around the outside.” This is the same sort of precise language that she used while answering question 9 on the follow-up test (the same question). She also added that a proper response to the student should involve “clarification on square and linear units and when to use them. He has the right idea about perimeter being around the shape.” These responses earned Jackie a 4 (model response) on this question – the only one she received throughout the study.

Larry and Jackie entered the study with similar weaknesses in their CK and KoST regarding units of measure. While Jackie made marked improvements in both knowledge types, Larry appeared to make little progress in either category. Larry identified Justin’s method as incorrect all three times, but his explanation for why it was wrong and his recommended intervention are representative of why Larry ranked in the bottom third in every statistical measure in this study. His focus started out, and remained on, getting the right answer – to the neglect of developing understanding. On the pretest Larry explained the reason Justin’s method was wrong was because, “You have to make sure to count the corners twice if you do it that way.” Even that does not “fix” Justin’s method. A positive aspect of Larry’s response to Justin and his thinking was that he indicated he would tell Justin his method “is not the best for solving the problem. Adding up each side is much easier and more efficient.” In any of his pretest responses there was no discussion of linear versus square units or even area and perimeter. Larry’s pre-intervention CK was limited in scope and his KoST was narrow in focus. He showed some growth during the beginning stages of TE 1 when his explanation of why Justin’s method was wrong departed from his former by including concepts related to perimeter: “Justine’s method is incorrect because he is counting the outside square, not the perimeter outside the shape.”

However, Larry's response for how he would address Justin's thinking once again reverted back to focusing on exactly why Justin's method would not produce the correct answer, as opposed to speaking to and clarifying the concepts surrounding Justin's difficulties. We addressed this question in our second interview, and even with prompting Larry would not thoughtfully discuss what precisely Justin might be confusing and what as a teacher he should do as a teacher. After not getting a meaningful response, I would refocus the discussion and offer Larry meaningful suggestions. I was troubled when Larry's response on the follow-up exam to this same question included nothing from our interview. Larry had even gone back to his pretest explanation for why Justin's method was wrong and his intervention strictly focused on trying to help Justin make his method work. Larry was able to correctly calculate the perimeter of the irregular shape on the pretest, in TE 1, and on the follow-up test. Overall though, his understanding regarding the concepts surrounding units of measure (his CK) was both sparse and disconnected, which resulted in a lack of awareness and appreciation of what would constitute an effective intervention for a struggling student (his KoST).

Brianna and Grace entered the study with somewhat similar levels of CK and KoST. Brianna possessed stronger mathematics than Grace, but Grace was prone to be more conceptual in her approaches than Brianna who opted for procedural. On the pretest however, their performances on question 9 were not similar. Grace only scored a 2. She indicated Justine's method was not correct, adequately explained why (i.e., he used 2-dimensional instead of 1-dimensional units), but then somewhat contradicted herself by indicating in her intervention that "even though you get the right answer this time, it may not work in all situations." During our first interview, Grace became frustrated and confused when she could not reconcile her pretest response. She did seem to know what unit should be counted to find the perimeter, but was not sure why she wrote that Justin's method worked in this instance. When she faced the problem again during TE 1 she again indicated that Justin's method was wrong, only this time she more clearly explained why indicating Justin's method would produce 20 square units instead of the correct answer of 20 linear units. It is not known how Grace resolved her pretest difficulties with this problem, other than that she indicated several times during the study how she would spend time outside of class thinking about certain problems that had given her difficulties. Grace's CK regarding this problem had apparently stabilized.

While responding ("as a teacher") to Justin's thinking and his method, she definitely improved on her pretest response. She wrote, "I would explain that the perimeter of a shape is a 1-dimensional linear measurement and that Justin should be counting line segments." This is a more organized KoST than shown in the pretest, but it still was lacking in thoroughness. Grace did not specifically mention linear and square units or work in the concept of area in case Justin might be confusing those concepts as well. There is a chance that Justin could have been recently finding the area of shapes drawn on grid paper and was blending his ideas together. Realizing the benefit of and providing appropriate diagrams illustrating linear and square units would also have illustrated a more complete KoST. Overlooking the value of providing diagrams as part of a thorough explanation was a missing component of most PSTs' KoST. The changes in Grace's knowledge related to units of measure reached a plateau during TE 1 as her responses to question 9 on the follow-up test added no new information.

The results related to change in the PSTs' CK and KoST related to units of measure concludes a summary of Brianna's knowledge of these concepts. Brianna was the only PST to earn a 4 on question 9 on both the pretest and the follow-up test. Qualitatively speaking, her responses to this question actually improved. Based on criteria established for the rubric scoring, she received a 4 on the pretest; however, her responses were not entirely thorough or complete. For example, when explaining why Justin's method was wrong she focused on why it does not work rather than pointing out that he used square units for a linear measurement (perimeter). That represented a weak explanatory framework for her CK. Her response to the part addressing KoST made it clear however that her CK was organized and enabled her explain to Justin's thinking and prescribe an appropriate response – involving linear and square units and a nice explanation/definition of perimeter. Integrating area and some diagrams would have made for a model response. Brianna improved on her pretest response by including useful diagrams in her responses for TE 1. This was a positive change for her KoST. Her CK was equally substantial and interconnected throughout TE 1, and her model score of 4 on the follow-up test revealed she had retained her knowledge about units of measure.

Throughout the study, a proper treatment of units was critical to forming a proper foundation to discuss other pertinent concepts related to area and perimeter (e.g., perceived relationships). Near the end of the study (e.g., post- and follow-up test), not including the appropriate units with responses was the primary reason more model responses of 4 were not assigned. It seems unlikely for teachers to build within students a conceptual understanding of area and perimeter without being able to coherently discuss linear and square units.

Knowledge Regarding Perceived Relationships

The exhaustive reporting regarding *units of measure* was necessary given their fundamental and unifying properties. This next major section deals with *perceived relationships* between area and perimeter and addresses a more self-contained class of difficulties and misconceptions. There are primarily two relationships between area and perimeter that students and PSTs (and even teachers) are reported to mistakenly suppose as true. The first provides the setting for TE 2 and involves the belief that a fixed perimeter can have only one area (and vice versa). The second, and slightly more elusive, misconception forms the basis for TE 3. It involves the belief that there exists a direct relationship between perimeter and area, that is, as the perimeter of a shape increases its area must also increase (and vice versa). This misconception can also be stated as, if the perimeter of a shape decreases, its area will always decrease. The next several sections present findings regarding the PSTs' CK and KoST related to these erroneous relationships.

Emergent CK of the fixed-relationship misconception. The next section will continue answering research question 3 by presenting findings related to the PSTs' CK regarding perceived relationships and how that knowledge changed as a result of the intervention of TE 2 and to a lesser degree the second interview, which only pertains to the case subjects. The PSTs' understandings related to the fixed-relationship misconceptions will be investigated through the findings extracted from TE 2, the post- and follow-up tests, and the second interview.

There was no formal instructor-lead introduction to TE 2 (Figure 15, p. 133), other than to make sure the PSTs understood the elements of the scenario presented and to motivate them with the many benefits of the upcoming classroom scenario. They were also reminded to, when appropriate, include in their responses the very same things a teacher might put on a chalkboard while teaching about these ideas. Teaching episode 2 required the PSTs to grapple with two relatively difficult concepts. First, and primarily, was the misconception that a fixed perimeter can have only one area. This misconception was somewhat concealed within the hypothetical student's (Tommy's) method to find the area of his footprint, which involved taking a piece of string, measuring around the footprint he had traced on grid paper, precisely cutting the piece of string, and then forming the string into a rectangle and computing the area of the rectangle as the area for his footprint. The second involved a correct method to find/estimate the area of a footprint (an irregular shape). Contemplating and then discussing the mathematics behind Tommy's method for finding the area of his footprint constituted the CK portion of TE 2.

There were three primary concepts at work within TE 2: (a) The string represents the perimeter of the footprint, (b) The string could be formed into many rectangles (or even other shapes) each having different areas; thus, Tommy's method was not reliable (although it is possible he could form a rectangle that was a good approximation of the footprint's area), and (c) The area of the footprint must be approximated (includes the ideas of an irregular shape, partial square units, and a decimal area measure). A model response would successfully address all three. It initially appeared that all 12 PSTs correctly surmised that the 18 inch string represented a perimeter measure; however, a response by Jackie later in the TE (which will be shared) casts doubt on that conclusion. The PSTs' reactions to Tommy's method varied. Two believed the method would produce the correct area. One of those even wrote, "With an irregular shape like this, there are not many different ways to come up with the area of this shape." That statement reveals a limited CK of irregular shapes. Two were leaning towards no, but their justifications were either unclear or faulty. Of these four, one eventually realized, through exploring with a microworld, that Tommy's method was an incorrect overgeneralization. The remaining eight PSTs correctly determined that Tommy's method was not reliable. The thoroughness and insight of their explanations revealed varying levels of understanding. For example, two of them appeared to grasp Tommy's misconception but failed to provide meaningful explanations and/or diagrams as evidence. Jackie was one, and she wrote, "This string can be used to make many different shapes that will have different areas." Of the three TEs, TE 2 was the only one in which Jackie correctly diagnosed the student's thinking on her own. It also represented the first time during the intervention process that she was able to correctly and clearly communicate the reasons behind her thinking. That represented positive growth in both Jackie's CK and KoST. In this instance, Jackie seemed to possess the CK necessary to successfully diagnose Tommy's erroneous thinking, and she was able to provide a reasonable justification; however, there was still ample evidence to the incompleteness of Jackie's CK.

After the PSTs decided whether Tommy's method was correct or not, they were asked to, "Explain, mathematically speaking, what is correct or incorrect about Tommy's method. Instead of just building on and possibly clarifying what she said earlier about "different shapes that will have different areas," her surmise of Tommy's method was vague: "he is confusing perimeter and area." During our second interview when that response was brought up, Jackie took several seconds to reflect and then responded, "So, he [Tommy] assumed with that length of perimeter [i.e., the string], he would get the same area – no matter what shape he made." That was a very good summary of the common misconception found in Tommy's work. Eventually, Jackie tied together the major aspects of TE 2, but as was often the case, her explanations were initially confusing and would not be meaningful to classroom students. Jackie's CK regarding certain facts and concepts related to relationships between area and perimeter, specifically the fixed-relationship misconception, had increased, however her ability to clearly explain her knowledge had not developed to the same extent.

As was the case with TE 1, Brianna did not appear to struggle with diagnosing the student's (Tommy's) method. In each of the first two TEs, she was able to coherently explain the mathematical mistakes the students had made. Regarding Tommy's method in TE 2, Brianna responded, "No, his method will not produce the correct answer. He fails to understand that not all shapes with the same perimeter will also have the same area." Although Brianna also provided four, properly-scaled diagrams showing how a perimeter of 18 could have different areas, she did not acknowledge that Tommy's method *could,* if he formed the right rectangle, produce a reasonable approximation of the footprint's area. Up to this point in the intervention, Brianna had shown a tendency to view these classroom scenarios involving student thinking as something that must be always right or always wrong. That aspect of her CK was still limited in scope. She was hardly alone.

There were five PSTs who acknowledged that Tommy's method <u>could</u> produce a correct answer. Their responses were similar to Grace and Larry. Grace wrote, "No, *not necessarily* [emphasis added]. The perimeter of a shape is related to the area, but the total perimeter will not give you a definite area, because you have to know the dimensions." Grace implied that Tommy's method could produce the correct area, and that represents growth over her pretest handling of student's erroneous claims, where she simply focused on producing a counterexample. Grace's response might be a little technical for the classroom and her lack of diagrams minimized its overall effectiveness. Overall, as was the case with TE 1, Grace strove to make her explanations thorough and appeared to focus on helping the student understand the concepts being discussed.

It was the exception to see Larry's work in a group containing "better responses." Larry seemed to grasp the mathematical concepts intertwined in TE 2, as evidenced in his writing: "No, it depends on the size rectangle that he makes. A 7 × 2 rectangle and a 5 × 4 have different areas but equal perimeter of 18." His explanation had expert qualities. It was organized and included examples. These explanations represent a relative higher level of understanding, and for Larry that was significant. Although he eventually diagnosed the student's thinking in TE 1 correctly, his explanations were vague, confusing, and even mathematically incorrect at times; however, in TE 2 Larry showed signs of beginning to organize his CK in ways that produced coherent explanations.

As was just described, research question 3 was addressed in part by presenting evidence of growth in various aspects of the PSTs' CK from TE 1 to TE 2; however, not all the subtleties of TE 2 were addressed. No PST was able to suggest which rectangle, with a perimeter of 18, would most closely represent the area of the footprint. A correct answer would be a 3 × 6 rectangle. To be able to do that, they would need to be able to decipher a way to approximate the area of the footprint – the last CK question for TE 2.

Before presenting findings from the posttest, to help portray post-intervention knowledge, specifics related to the instructor-lead, whole-class discussions from TE 2 will be shared. This is done to add context for future evaluations of findings regarding

PSTs' CK regarding the fixed-relationship misconception. At the conclusion of TE 2, a whole-class discussion was held to provide PSTs the opportunity to share learning experiences and other personal reflections regarding the TE. The instructor/researcher facilitated the discussion and had prepared material to present and spark class discussion. The purpose of these summaries was to clarify the major misconception(s) presented within each TE and address pertinent and tangential concepts. For TE 2, Tommy's method was restated as a mathematical claim (i.e., "A fixed perimeter can have only one area.") to model for the preservice teachers how to rephrase a student's claim into something that can be explored and tested. During our discussion, it was brought out that Tommy's method/claim was incorrect and that the *Gizmo* microworld allowed a couple PSTs to realize that there were actually an infinite number of rectangles possible that could have a perimeter of 18. The dimensions would be decimal numbers, and this was quite eye-opening for most of the PSTs.

Post-intervention CK of the fixed-relationship misconception. Problem 10 on the pretest addressed the misconception that a specific perimeter can have only one area. That problem had the highest mean score (M = 2.75, SD = 0.6) for the test; however, as discussed while answering research questions 1 and 2, the PSTs' knowledge regarding that misconception was incomplete; to recap: (a) Only three PSTs (no case subjects) perceived that students would tend to believe the misconception that equal perimeter implies equal area, (b) during interviews, Larry and Jackie changed their initial pretest answer by indicating that squares were not rectangles, (c) Grace was unsure but leaned towards the idea that squares are rectangles, and (d) Brianna was confident in the fact that a square was also a rectangle.

Problem 5 on the posttest parallels the concepts contained in question 10 from the pretest. In problem 5, the PSTs were asked, "A certain rectangle has a perimeter of 16 cm; (a) What might its area be? (b) Explain how you arrived at your answer, and (c) Are there other correct responses? If so, explain what they are." There were four concepts surrounding this problem: (a) the misconception that there was only one possible area, (b) a 4 × 4 square is one of the possible rectangles, (c) there are actually an infinite number of rectangles with a perimeter of 16 cm, and to a lesser degree, (d) using the semi-perimeter to assist in more quickly finding possible rectangles.

Question 5 had the highest mean on the posttest (*M* = 3.25; *SD* = 0.6). There was only one score below a 3 on this question (PST #5), and it appeared to be due to the fact that she interpreted that question as looking for a rectangle whose perimeter and area were 16. Nine of the 12 PSTs included a 4 × 4 shape in their list of possible rectangles with a perimeter of 16, but only one (PST #5) specifically mentioned that "the square is a type of rectangle." There was not an opportunity to follow up with the other eight to be sure that they included the 4 × 4 because they knew it was a rectangle. Larry was the only case subject not to include a 4 × 4 shape in his list of possible rectangles; however, the fact that he included three rectangles seems to indicate he gained an understanding of the fixed-relationship misconception. During our interview, it was obvious that his CK regarding the hierarchical nature of quadrilaterals was not organized enough for him to accommodate a square as a rectangle. After walking him through the classification process, it was still unclear if Larry grasped the hierarchical nature of this classification:

T: Does a square satisfy the properties of a rectangle?
L: Yes, so a rectangle is a square.
T: Are you sure?
L: Um, yeah, a square is a rectangle. It's confusing.

Jackie used a trial-and-error approach in finding different rectangles with a perimeter of 16, as did nine other PSTs. Her success in generating two (a 4 × 4 and a 3 × 5) indicated she did not hold to the fixed-relationship misconception. A downside to her response was that neither of her rectangles was scaled appropriately. During our interview, she seemed confident that the 4 × 4 shape belonged as a possible rectangle; however, she had considerable difficulty comprehending how the semi-perimeter could be used as a "short-cut" to find rectangles with a perimeter of 16 (i.e., find two numbers whose sum was eight). Jackie often needed repeated exposure to concepts before she could assimilate them into her current CK. Her realization that a square can be included in a list of rectangles illustrates positive change from her pre-intervention knowledge.

There were three PSTs (Grace, Brianna, and #10) who successfully deduced that there were an infinite number of rectangles (including the square) with a perimeter of 16 cm. Grace and Brianna's methods for finding their possible rectangles showed an ability to recall prior class discussions and microworld experiences and incorporate that knowledge into their explanatory framework – evidence of a maturing CK. They both included squares in their list of possible rectangles, thus acknowledging the hierarchical relationship between squares and rectangles. Brianna used a semi-perimeter method to find possible rectangles and listed all the whole-number possibilities (i.e., 1 × 7, 2 × 6, 3 × 5, and 4 × 4). She also provided appropriately scaled rectangles as well. While answering part c, Brianna said, "We can find many other sets of numbers that add up to 8 by using decimals. For example, we can use 1.5 and 6.5." Other than leaving off the units from her rectangles (i.e., cm^2), Brianna provided the most thorough response. Grace's method involved starting with a width of 1 cm, then found the necessary length (i.e., 7), and she continued this process up to the 4×4 square. She wrote that many other rectangles could be generated because "The dimensions could incorporate fractions." It would have been a model response if Grace had explained why her method worked (i.e., she was employing the semi-perimeter), and even more importantly if she had included useful pictures of her rectangles. The lack of incorporating diagrams into her explanations is a significant shortcoming in her CK. The continual absence of appropriate, supportive diagrams was an indicator that these PSTs did not truly comprehend what is typically involved in providing conceptual explanations that are meaningful to students.

Emergent CK of the direct-relationship misconception. The section that follows will aid in answering research question 3 by presenting findings related to the PSTs' CK regarding the fixed-relationship misconception – a slightly more elusive misconception than contained within either TE 1 or TE 2. These findings were extracted from TE 3, the post- and follow-up tests, and the second interview. The gist of this misconception is that there exists a direct relationship between perimeter and area, that is, as the perimeter of a shape increases/decreases its area must also increase/decrease (and vice versa). The focus problem for teaching episode 3 (Figure 16, p. 136) will provide the setting for the PSTs' emerging CK of the direct-relationship misconception. TE 3 began with four questions related to the PSTs' CK (their reaction to the claim), and then transitioned into examining their KoST (their reaction to the student). For this last TE, the PSTs were instructed they could interact with either microworld from the outset. Five of the 11 PSTs (one was absent) indicated they used the microworld(s) immediately to investigate the student's (Jasmine) claim, including Jackie, Larry, and Grace. Their reactions to the claim resulted in four categories.

The first category contained two PSTs, and they accepted the student's claim as correct. One of them provided two examples which, in the PST's mind, established that "the student's theory is technically accurate." The other PST, Jackie, went right to the microworlds and to "test the student's theory." While admittedly unsure, Jackie indicated, "I do think she is on the right track. I think she needs to test her theory more to be 100% confident." As was common with many PSTs while examining the various student claims in this study, they apparently believed that if enough examples are presented then the claim can be either proved or disproved. This belief can be seen in a comment made by Jackie during our second interview. I asked Jackie what her plan was when she used the microworlds to investigate Jasmine's claim. She responded. "I tried a prove-her-wrong kind of thing, but I just don't think I tried enough examples." A limited background in mathematics led most of these PSTs to where they viewed the role of examples as a way to *prove* something, rather than just an *illustration* of a numerical relationship. They did not, or possibly cannot, appreciate the need for a mathematical argument in such cases. Jackie also wrote, "It just seems kind of obvious that if an object takes up more space, it probably is bigger." Comments such as these are based on common sense, rather than mathematics. At this point in the TE, Jackie is functioning below a Level 0, since she did not even attempt to justify the student's invalid claim. This is the same level she performed at when this misconception was presented on the pretest in Question 8.

The second category involves two PSTs who initially accepted the claim but very soon after changed their minds. Both indicated that while exploring with a microworld they found a counterexample to Jasmine's claim. One PST's strategy was to present examples where one perimeter had several different areas: “I found that a rectangle can have a perimeter of 40 but the area could be 96, 99, or 100 and possibly more.” These examples do not directly address Jasmine’s claim which involves “increasing” the perimeter. Larry was the other PST in this category. After “playing with the *Gizmo* microworld,” Larry wrote, “I changed my mind. She is incorrect. You can have a . . . I don’t know how to explain this!” Larry proceeded to provide a 3 × 3 square, which he indicated had a “P = 12” and an “A = 9,” and a second 1 × 6 rectangle, which had a “P = 14” and an “A = 6.” While Larry’s explanation would be insufficient for the classroom, his understanding has progressed from where it was prior to any intervention. On the pretest and in the interview, Larry was only able to attain a Level 0 (i.e., he justified the student’s invalid claim), but in the early stages of TE 3 his disproving of the claim, by providing a counterexample, had moved him to a Level 1 understanding (Ma, 1999).

The third category of responses identified were those who thought the claim was incorrect from the onset and offered at least one appropriate counterexample. Their counterexamples were all very similar in that the second rectangle provided had a much smaller width and a much longer length than the first (e.g., first rectangle would be a 4 × 4 and the second would be a 1 × 11), which would result in a larger perimeter but a smaller area. Of the four who applied this approach, there were two who also explained a key failure in Jasmine’s claim – that of over-generalizing. This observation characterizes an expert teacher and is represented by Grace who wrote, “I know that her thoughts are based on one example.” Although both these PSTs realized Jasmine’s error right away, one of them stopped after simply disproving the claim; therefore, she only achieved a Level 1 understanding. Grace, however, continued to explore various relationships between area and perimeter and discovered two separate conditions for area-perimeter relationships that elevated her understanding to a Level 3. That represented a marked increase from Grace's pretest Level of understanding regarding the same misconception (Table 22). Grace also provided evidence of expert-like analysis and problem solving.

Table 22

Investigating an Erroneous Student Claim

		Pretest Results (Question 8)		
Number of PSTs (N = 12)	Agreed with the student	Provided appropriate counterexample	Investigated the claim	Ma's "Level of Understanding" attained
4 (including Larry & Jackie)	Yes	No	No	Level 0
2 (including Grace)	No	No	No	In-between Level 0 & 1
3	No	No	Yes	In-between Level 0 & 1
3 (including Brianna)	No	Yes	Yes, but insufficiently	Level 1
N = 11*		**Emergent Results (TE 3)**		
2 (including Jackie)	Yes	No	No	Level 0
2 (including Larry)	Initially Yes, then No	Yes	No	Level 1
3	No	Yes	No	Level 1
3 (including Brianna)	No	Yes	Yes**	Level 2
Grace	No	Yes	Yes**	Level 3^

Note. *One PST (#4) was absent for TE 3. **These PSTs acknowledged the condition that Jasmine's claim could be true. ^Clarified certain conditions of the area-perimeter relationship.

The first writing prompt of TE 3 asked PSTs, "What was the first thing you did after reading through this situation?" The vast majority of responses were along the lines of "I double checked Jasmine's calculations" or "I went to the microworlds to try out her theory." Expert teachers are expert problem solvers (see Table 3, p. 166). They are able to effectively analyze mathematical problems, as well as student thinking, by recalling past knowledge, incorporating new knowledge, and organizing both in a way that facilitates application to new settings. Grace's response to the first writing prompt indicated the problem-solving component of her CK was maturing in the way just described: "I began to recall that in sessions in the past this type of thinking has been proven false. The perimeter and area are related but not in this way."

There were three PSTs (including Brianna) who comprised the final category of understanding related to this misconception. As in the previous category, both PSTs supplied an appropriate counterexample to refute Jasmine's claim; however, unlike any previous PSTs they acknowledged that Jasmine's claim could be correct: "In a majority of instances she would be correct, but it does not hold true all the time." Brianna's response was very similar, and this acknowledgment would move these two PSTs into the second level of understanding (Ma, 1999). This transition marked growth for Brianna who had moved from a level 1 to a Level 2 (see Table 22). The supportive explanation behind her approach bears reporting:

> Although it does seem logical, it is incorrect. Jasmine is correct in understanding what perimeter and area are. She calculated them correctly in her example, but she is incorrect in thinking that area and perimeter are related like that. Also, in her theory she only gave one example. She fails to try other rectangles and see

if it [her claim] works for every one.

Brianna presented a balanced approach involving praise and corrective instruction. Her specific mention of Jasmine over-generalizing is an example of expert CK. Brianna was one of the few who had success deciphering Jasmine's claim without, by her own admittance, consulting either microworld. It is somewhat surprising that only two PSTs included in their response that Jasmine's claim was sometimes true, especially since the focus problem for the TE included a specific example as "proof" to illustrate her claim.

Teaching episode 3 concluded with an instructor-lead, whole-class discussion. This session began with a detailed discussion built around Ma's levels of understanding (1999) as they related to Jasmine's claim. Questions such as, "Is Jasmine's 'theory' always, sometimes, or never correct?" were raised and discussed. The appropriate role of examples and counterexamples was discussed. The various numerical relationships between perimeter and area were investigated and specific examples were elaborated upon. There was also time spent explaining why some conditions supported Jasmine's claim and why other conditions did not. The idea of a fixed perimeter having an infinite number of possible areas was reiterated during this whole-class discussion. Another concept shared during the extensive summary of TE 3 was that a square could be included in any list of possible rectangles having a specific perimeter. Overall, the PSTs were provided with the information necessary to achieve a Level-4 response on future questions addressing the misconception that there exists a direct relationship between perimeter and area. It was conceded that PSTs would not have enough time on the post- or follow-up test to fully develop the various levels of understanding related to this misconception (e.g., Grace reached Level 3 during TE 3 but fell back to Level 1 on posttest), but simply mentioning the various possibilities would be significant. Details from the TE summaries are shared to help the reader appreciate the depth of the intervention and also realize the extent of knowledge (both CK and KoST), including appropriate language, made available to the PSTs. The anticipation was that this knowledge would be apparent in their post- and follow-up test responses.

Post-intervention CK of the direct-relationship misconception. Question 6 on the posttest addressed the direct-relationship misconception and also presented the first opportunity for the PSTs to share what they had gleaned from the in-depth summary of TE 3. Statistically, this question had the second lowest mean on the test (2.58, SD = .9). Five responses that received scores of 3 would have received a 4 had the PSTs included appropriate units with their examples (including Jackie, Grace, and Brianna). The question read, “Stacey claims that whenever you compare two rectangles, the one with the smaller perimeter will always have the smaller area.” The two follow-up questions relating to CK were: (a) “Is she correct? If you are unsure, are you skeptical or do you tend to believe her? Why?” and (b) “Explain why you agree or disagree with Stacey’s thinking.” One difference between this question and pretest #8 and TE 3 is that those questions used the word *larger* instead of *smaller*; however, the direct-relationship claim would be examined and discussed in much the same way. Another difference for question 6 was that no example (i.e., student work) was provided as “proof” of the student’s claim, as was the case for TE 3.

Interestingly, the responses aligned very similarly as they did in TE 3, both in what were said and by whom (see Table 23). Once again, four categories of responses were evident: (a) accepted the claim (n = 2), (b) rejected claim without counterexample

Table 23

Investigating an Erroneous Student Claim: Throughout the Study

Number of PSTs (N = 12)	Agreed with the student	Provided appropriate counterexample	Investigated the claim	Ma's "Level of Understanding" attained
		Pretest Results (Question 8)		
4 (including Larry & Jackie)	Yes	No	No	Level 0
2 (including Grace)	No	No	No	In-between Level 0 & 1
3	No	No	Yes	In-between Level 0 & 1
3 (including Brianna)	No	Yes	Yes, but insufficiently	Level 1
N = 11*		**Emergent Results (TE 3)**		
2 (including Jackie)	Yes	No	No	Level 0
2 (including Larry)	Initially Yes, then No	Yes	No	Level 1
3	No	Yes	No	Level 1
3 (including Brianna)	No	Yes	Yes**	Level 2
Grace	No	Yes	Yes**	Level 3^
N = 12		**Post-Intervention Results**		
2 (including Larry)	Yes, w/o ample justification	No	No	below Level 0
1	No, but w/o counterexample	No	No	In-between Level 0 & 1
6 (including Jackie & Grace)	No	Yes	No	Level 1; Jackie close to Level 2
3 (including Brianna)	No	Yes	Yes**	Level 2^

Note. *One PST (#4) was absent for TE 3. **These PSTs acknowledged the condition that Jasmine's claim could be true. ^Clarified certain conditions of the area-perimeter relationship.

(*n* = 1), (c) rejected claim with counterexample (*n* = 7), and (d) both opposed and supported the claim with examples (*n* = 2). There were four PSTs (including Larry and Jackie) who moved to a different level of understanding from where they were in TE 3. By accepting the student's claim without justification, Larry's response did not even attain the lowest level of zero. That represents a step backwards from the Level 1 he eventually reached in TE 3. For posttest question 6, Larry originally wrote "No" that Stacey's claim was not correct, before he scribbled it out and wrote "Yes." Larry even provided a response to Part (b): "Just because the perimeter is smaller does not mean that the area is smaller" before he crossed it out and wrote, "I agree because the perimeter is the measurement around the outside of the shape. If that is small then the area must be small." Neither of his explanations addressed the pertinent aspects of the student's claim. Larry's responses occurred just one week after the completion of TE 3, where we spent over three hours addressing this misconception. Larry's flip-flop was brought up during his second interview, which followed the posttest. Our conversation follows:

> **T:** Do you recall anything about TE 3?
> **L:** I was thinking about that [TE 3] when I was doing this [posttest #6]. That's why I was kind of, at first. I was kind of like, well, yeah. Then I thought about it, and I think that's why I got confused. I don't know. I probably contradicted myself. I don't know what I'm doing here. It sounds like I'm kind of going back and forth and I don't have an answer.
> **T:** Well, what if you were going to disprove her claim, what would you try to do?
> **L:** Try to make two rectangles, one with a [4-second pause]. I always get this – I can never, um [4 second pause], one with a greater perimeter and a smaller area or a . . . I don't know what I am trying to say.

As we continued discussing this question, it became apparent that Larry still had trouble rephrasing student's claims into a mathematical statement that could then be verified or disproved. That, combined with Larry's novice-like tendency to change his answers on apparent whims revealed a still fragile CK.

Jackie's understanding of the direct-relationship misconception in TE 3 was rated below a Level 0, because she accepted the student's erroneous claim without any justification. On the posttest however, her response to question 6 revealed positive change in several aspects of her CK. First, instead of just writing generalities about the various concepts involved, she investigated the problem mathematically. That resulted in a classroom-appropriate counterexample. She did not stop there. In her explanation, she said, "Although this [Stacey's claim] may be true for some problems, it is not true for all." While she did not include a specific example supporting Stacey's claim, the mere mention of that possibility borders on a Level-2 response and represents a wider perspective in Jackie's consideration of Stacey's claim, rather than simply disproving it. Jackie's response also reveals an expert-teacher trait of realizing the limitation of Stacey over-generalizing by presenting only one example as "proof" of her claim.

Three other PSTs switched level; one moved from a Level 0 to a Level 1 and the other went from a Level 1 to a Level 0. Grace dropped to a Level 1, because she did not acknowledge that the student's claim could be true nor did she provide any evidence of investigating the relationships between area and perimeter, as she did in TE 3. Examining the content of the other responses revealed none had made any significant progress and had remained at the same level of understanding as in TE 3; Brianna again reached a Level 2, but no further. Given the thorough discussion following TE 3 that had occurred just a week before the posttest, it was somewhat surprising that no PST, other than Jackie, was able to incorporate and organize that discussion into their CK in order to facilitate a move to a higher level of understanding on the posttest.

The last item containing findings relevant to the PSTs' post-intervention CK of the direct-relationship misconception is question 8 from the follow-up test. The significance of this question is its representative nature of the case subjects' CK regarding perceived relationships. Question 8 is representative, because it is one of only 2 test questions that appeared on both the pretest and the follow-up test, as well as being featured in a TE (i.e., before, during, and after the intervention). The question read, "Madison claims that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area. The two questions relating to CK were: (a) Is she correct? and (b) Explain why you agree or disagree with Madison's thinking. As was the case with posttest question 6 (unlike TE 3), question 8 did not provide any example as "proof" of the student's claim. That would imply that any PST who supported the claim, as a possible condition, would have to provide their own appropriate example.

A careful examination of the PSTs' responses revealed the same four categories of responses as were found in TE 3 and posttest question 6, with a few variations: (a) accepted the claim (n = 1), (b) rejected claim without appropriate counterexample (n = 2), (c) rejected claim with counterexample (n = 5), and (d) both opposed and supported the claim (n = 4). These similar findings would seem to suggest that once a PST arrived at a certain level of understanding regarding the direct-relationship misconception, they did not expand very much on that understanding or venture beyond their CK *comfort zone*, if you will. Larry continued his posttest retreat from the Level 1 understanding he achieved during TE 3 by again accepting the student's claim without any justification. Larry's explanation for his stance involved shallow mathematical thinking, only considered the most obvious of possibilities, and appeared to involve no significant investigation: "The more you have for a perimeter means that you will have more area boxes on the inside." The absence of a mathematically meaningful justification means Larry again did not even reach a Level 0 understanding of these relationships.

The third category of responses represents those PSTs who disproved the claim and provided an appropriate counterexample. The five PSTs in this category (including Brianna) disproved the student's claim in very similar ways. They first provided a shape that was very close in dimensions to a square. Their second rectangle was always very long and narrow. This would produce a perimeter greater that the first with a smaller area, thus disproving the claim. The explanations supporting this counterexample were similar in content to Brianna's: "I disagree, because there are many cases when you can have a shape w/ a greater perimeter that has a smaller area." While Brianna hinted at the possibility of examples that supported the student's claim (e.g., use of the word "many" and not "all"), she did not specifically mention that possibility; therefore, she dropped from a Level 2 understanding, which she had during TE 3 and had also displayed on the posttest, to a Level 1 on the follow-up test. The testing situation seemed to promote Brianna's documented tendency to focus on answering the question (albeit often very well) without considering or investigating other possibilities; however, in past situations when she was questioned about certain limited responses, as during an interview, she almost always was able to provide added depth and insight.

Grace was also in category three and the fact that she only provided a counterexample, and no supportive example, resulted in her once again attaining a Level 1; however, her explanation entered the realm of a higher level of understanding: "I disagree, because although the perimeter & area have a relationship, it is not this one. The closer the dimensions are in length, the larger the area, even though the perimeter stays the same." Grace's explanation enters the Third Level of Understanding, that of *clarifying the conditions* (Ma, 1999). Grace argued that with the same perimeter there are many rectangles whose pairs of addends can make the same sum. She also implied that when these pairs of addends become factors, as in calculating the area of the rectangle, they will produce different products. Finally, Grace uses the fact that the closer in value the two factors are, then the larger the product; hence, for a given perimeter, the square is the rectangle with the largest area. Grace had informally brought this idea up in her first interview, but now it appeared she had refined and organized it and is able to present it coherently. This represents a positive change in Grace's CK regarding perceived relationships and in her ability to synthesize and explain information.

Another PST, call her Audrey (PST #1), showed strong positive growth regarding this misconception. Up to this point, Audrey had never attained higher than a Level 0 on any response related to the direct-relationship misconception. On the follow-up test, she attained a Level 2. She provided both an example that supported the claim and one in which the perimeter remained the same but the areas changed. While the second example does not directly address the student's claim of increasing the perimeter it still refutes that a direct-relationship exists between perimeter and area.

The fourth category of responses involved those who both supported and refuted the student's claim. There were three PSTs in this category and each one provided an appropriate counterexample but failed to include a supportive example. Instead, each made reference to the possibility of the student's claim holding by providing explanations similar to: "It <u>may be</u> true in some cases, but area and perimeter are not directly related.

So, you cannot assume what is true in one case is true in another." Only Jackie and one other PST (#6) maintained their Level 2 understanding from posttest to follow-up test. As will be presented in the KoST section, Jackie's CK had become much more stable and organized. She was now able to clearly and concisely present and explain various concepts related to area and perimeter.

Overall, the class showed improvement from their first exposure to the direct-relationship misconception (i.e., pretest, question 8); however, there were still two more levels of understanding that went basically unexplored (Ma, 1999). First, there are three possibilities to identify when the perimeter of a rectangle is increased: (a) the area can increase, (b) it can decrease, or (c) it may stay the same. The majority of the PSTs only discussed the first two possibilities. Beyond identifying or displaying one of the three previously mentioned possibilities, none of the PSTs reached the two higher levels of understanding: (a) clarifying the conditions under which these possibilities held, and beyond that (b) explaining why some conditions supported the student's claim and why other conditions did not. Table 24 summarizes the approaches used by the PSTs as they responded to questions addressing the erroneous direct-relationship misconception. For the most part, the PSTs in this study simply stopped exploring after discussing their initial reaction. Many of these PSTs did not appear self-motivated to delve far beyond providing one possibility to the stated question, very often the same one they had given in the past similar situations. Instead of investigating the various possibilities surrounding this misconception, the majority would give the same, or a very similar, answer as they had previously and continued to operate within their *CK comfort zone*. For example, once many realized that there was not a direct-relationship between perimeter and area, which

Table 24

Reactions to Student's Claim of a Direct Relationship (N = 12)

	Pretest		Posttest		Follow-up	
Reaction:	N	%	N	%	N	%
Simply accepted the claim	4	33	2	17	1	8
Rejected claim without investigation	2	17	1	8.3	1	8
Rejected claim and investigated mathematically	6	50	9	75	10	84

could be acceptably shown with a single counterexample, they were satisfied with this degree of investigation even though they had been made aware that there were more possibilities that could be discussed, or at least mentioned. Due to time constraints, it would be unrealistic to expect any PST to expand upon, or even duplicate, responses provided in TEs, while working on a pre-, post-, or follow-up test.

Emergent KoST related to the fixed-relationship misconception. It was important to lay the foundation with research questions 1 and 2, and then examine the change in the PSTs' CK (research question 3) as those facets of knowledge are instrumental in informing and facilitating an effective knowledge of student thinking (KoST). The PSTs' KoST related to perceived relationships represents the last major category of findings associated with answering research question 4.

The findings presented in these next final sections address the PSTs' understanding of and more importantly how they indicated they would respond to student difficulties and misconceptions (i.e., their KoST), specifically regarding the fixed-relationship misconception. The findings will be presented and discussed in much the same way as it was in the previous CK section, with an emergent perspective gained from examining responses from the teaching episodes followed by pertinent questions from the post- and follow-up tests, and excerpts from the second interview with the case subjects.

Each teaching episode began with questions related to CK, and then would transition into KoST. Interacting with the microworlds came at different times during the TEs, but always allowed for CK and KoST to be reexamined and reflected upon. This progression proved valuable to several PSTs in each of the teaching episodes. To recap, TE 2 (Figure 15, p. 133) required the PSTs to grapple with two relatively difficult concepts. First, and primary, was the misconception that a fixed perimeter can have only one area. That misconception was somewhat obscured within a hypothetical student's [Tommy's] method to find the area of his footprint, which involved taking a piece of string, measuring around the footprint he had traced on grid paper, precisely cutting the piece of string, and then forming the string into a rectangle and computing the area of the rectangle as the area for his footprint. Just as in TE 1, the PSTs' knowledge and understanding related to the concepts and misconceptions surrounding TE 2 were positively influenced after interacting with a microworld. During the reflection opportunity for the CK questions, two PSTs (Jackie and #4) indicated their initial and wrong understanding regarding Tommy's string method changed after working with the Gizmo microworld. Other than those accounts, each PST began the KoST questions for TE 2 with the same level of understanding as revealed in the CK section.

There were three primary questions that were designed to address the PSTs' KoST: (a) #6. "As a teacher, how would you respond to Tommy's thinking and his strategy? What specifically would you say and do?" (b) #10. "What do you think students might find difficult about finding the area of their footprint? What specifically could confuse them?" and (c) #12. "As a result of seeing Tommy's method and his apparent lack of complete understanding regarding the perceived direct relationship between perimeter and area, how would you follow up with the entire class about the concepts that surround this classroom episode?" Findings related to questions 6 and 12 are similar in that they involve a more pedagogical aspect of the PSTs' KoST, and thus will be presented together. First though, the responses to question 10 will be examined.

Question 10 of TE 2 required the PSTs to apply their CK to the realm of analyzing student thinking. They are not yet asked how they would intervene; rather, the question is concerned with their comprehension of what students might find difficult related to the area of a footprint. There were two PSTs whose response was similar to Larry: "There is no real formula that a student can bank on to know <u>for sure</u> that their answer is right." Such a response reveals very little understanding of student thinking. Larry did not discuss (a) why the absence of a formula would be problematic, or (b) what specific mathematical features of the footprint would not accommodate the direct use of a formula to find its area. It was common for Larry, as with a novice teacher, to focus solely on content and applying a procedure to get the right answer, as opposed to, examining specific properties of the footprint that student could find difficult. Larry made a comment about the footprint problem during his interview that fairly summarized his approach to problem solving, "Like when I saw the weird shape, I was like, 'What am I gonna do now? I can't just do b × h.'"

The majority of responses focused on the irregular shape of the footprint. That observation was an extension of Larry's response, because it offered a specific explanation to why a formula could not be used, as well as a more appropriate mathematical reason as to why students might initially struggle with finding the area of a footprint. Both Jackie and Grace answered along those lines. Grace simply wrote, "The irregularity of the shape." This response is incomplete, because it does not address the second part of the question by providing specifics. Grace's response is correct, but the simplistic nature of it does not reveal if she knows why the irregular shape would confuse students, either mathematically (because there is no formula for the shape) or pedagogically (because textbooks do not typically present such shapes). The first part of Jackie's response was similar to Grace's, but Jackie offered more. She wrote, "The grid could also be confusing to students" [The footprint was drawn on 1-inch grid paper]. That is an interesting comment, especially since Jackie recommended earlier that counting the whole and ½ boxes was one method to find the area. The grid is actually needed to help with approximating the area of the footprint. It also provides the context and representation for a discussion regarding partial units and approximating area, which an expert teacher would have realized. Such discussions would inherently place the focus on the students and their understanding, rather than on using procedures to find answers.

There were four PSTs who offered more than one issue they felt students might struggle with. Three of the responses were of a more dependent relationship between the irregular shape causing the problem that no formula would directly give the area of the footprint. Brianna also suggested two issues students might struggle with. One was the irregular nature of the shape, but the second involved the unit measure of the footprint. She wrote, "They may leave many sections out, because they are not full squares, or they may count each part of a square as a whole one, and have too many." This is a significant quotation, since these precise student difficulties have been cited by other researchers (e.g., Hiebert, 1981; Lehrer, 2003). It appeared Brianna's relatively strong CK surrounding TE 2 enabled her KoST to perform in powerful ways.

The purely mathematical perspective of Tommy's thinking was addressed in questions 1-5. Questions 6 and 12 looked at how the PSTs would specifically address Tommy's thinking and subsequently Tommy's class. Question 6 read, "As a teacher, how would you respond to Tommy's thinking and his strategy? What specifically would you say and do?" A thorough response to Tommy, as well as his class, would have included: (a) a discussion of why his "string method" would not be reliable (i.e., the direct-relationship misconception), (b) an exploration of Tommy's method using either the string, or a microworld, or both, and (c) some mention of at least one reasonable strategy to find the area of the footprint. Notable responses by the PSTs to Tommy's thinking and his strategy fell into three categories: (a) offered only explanation, (b) engaged in exploration, or (c) a combination of explanation and exploration. There were three PSTs (#'s 1, 5, & 11) who offered no meaningful intervention with either Tommy or the class. Their explanations were based on the fact that since they did not know how to find the area of the footprint they did not know what to say to Tommy or the class. It is interesting that, based on her responses to questions 1-5, one of these PSTs (#5) had a decent understanding of the misconception surrounding Tommy's method, but apparently did not view that as information worth sharing with Tommy or his class.

There were 7 other PSTs who indicated their intervention with Tommy would involve either some sort of an explanation or exploring the situation with the student, but not both. Generally, an explanation began with "I would show him . . ." or "I would give him . . ." After that, there were three primary approaches: (a) show examples with the string (n = 3), (b) use the MW to show examples (n = 2), or (c) use hand-drawn shapes (n = 1). Jackie's intervention involved an explanation that did not involve string or a MW. She wrote, "I would go over how to deal with regular and irregular shapes, by comparing and contrasting how to do the area." While this would be helpful, it would be meaningful only after debugging Tommy's method, which Jackie was not be able to do because at this point in the TE she could not articulate the misconception surrounding Tommy's method. Jackie's suggested intervention with Tommy marks a slight improvement from the vague and mathematically confusing response offered to the student in TE 1. Up to this point, Jackie's comprehension of the erroneous mathematics behind Tommy's method was unorganized and incomplete, and that is reflected in her inability to effectively address the student's difficulties (an aspect of KoST).

As was the case with TE 1, it was not until after interacting with a MW (the *Gizmo* in this instance) that Jackie realized the error in the student's thinking. For TE 2, Jackie wrote, "I think I understand now that Tommy believes that if he can form a shape he recognizes it [finding the area of the footprint] will be easier, but there are many shapes he could form with 18 inches of string, all with different areas." Even though Jackie was finally able to comprehend the misconception surrounding Tommy's method, that knowledge did not translate into meaningful instruction with Tommy's class. Question #12 on TE 2 asked the PSTs, "As a result of seeing Tommy's method and apparent lack of complete understanding, how would you follow up with the entire class about the concepts that surround this classroom episode?" Interestingly enough, while Jackie admitted earlier that the MWs aided her in understanding Tommy's error, she did not think they would benefit Tommy's class: "I don't know if I would use the microworld much to show them anything b/c it didn't help me at all." In light of that, Jackie basically wrote that she would review area and perimeter with Tommy's class. During the second interview, that rather shallow response was partially explained. Jackie said, "I would not be 100% confident doing the footprint problem with elementary students. I don't know why, but I wouldn't." Overall, Jackie's CK was still fragile, and even though she at times was able to experience some success (e.g., diagnosing student errors in both TE 1 and 2), she was not able to organize that new-found CK in ways that would enable her to meaningful respond to individual students or an entire class.

Larry's CK regarding Tommy's misconception was stronger than Jackie's, in that he had correctly identified the flaw in Tommy's proposed method; however, even with that knowledge his suggested intervention did not thoroughly address that misconception with the student. Larry indicated he would use the 18" string to show Tommy that "the perimeter can be equal but the area can be different;" however, he did not provide any justification (i.e., further explanation and/or diagrams) of his strategy. During the interview, Larry was asked to provide some specifics regarding his initial response. After contemplating for a while, Larry was able to provide two rectangles (a 2×7 and a 3×6) as proof that two rectangles could have the same perimeter but different areas; hence, disproving Tommy's method. Larry offered nothing more, and it appeared that Larry's primary goal while working with Tommy would be to prove his string method wrong. Larry gave the impression that once Tommy saw him make a couple different rectangles with the piece of string, then he would almost immediately and completely understand why his method was wrong. Larry's approach to the student in TE 2 showed growth from his recommendations in TE 1, where he focused on modifying the student's erroneous method in order to produce a correct answer. Larry left question 12 blank on TE 2, so during our second interview I asked him how he would follow up with Tommy's entire class. The focus of Larry to find the correct answer was once again evident as he described how he would use various methods (e.g., cut up the footprint into squares) to help the students find the area of the footprint. There was no mention of addressing Tommy's misconception that he had identified earlier in the TE. Even though Larry's CK had experienced some growth throughout the study, this novice pattern of responding to students' difficulties by helping them find right answers reveals his KoST was still quite insufficient and had not changed much up to this point in the study.

Two of the remaining five PSTs (#3 and #4) indicated they would use the microworlds with Tommy as well as with the class, but they were very unclear in what precisely they would do and what they were hoping to accomplish. Three other PSTs (#'s 6, 9, & Grace) stated that they would investigate with the piece of string to help Tommy see the error in his method. One PST (#9) suggested a teacher-centered approach of showing Tommy two rectangles (one 2×7, another 3×6) that had a perimeter of 18 inches but different areas. There was no supportive explanation. Again, it appears the PST thought it was obvious what the examples would accomplish. She used a similar approach with the entire class, only this time she incorporated the MW. Two others (#6 and Grace) suggested guided exploration for Tommy (#6 also recommended using a *geoboard*) with the expectation that he would discover the inconsistent nature of his method; however, PST #6 was not as confident when addressing the entire class. She wrote, "I don't think at the point I can totally explain to the students why the same perimeter does not equal the same area, but I would show them either of the applets. I would use the same perimeter to make different areas and vice versa." It seemed PST #6 did not quite grasp the role or value of counterexamples (which she had provided earlier in the TE) when addressing erroneous student methods or claims. A more extensive examination regarding the PSTs' experiences with and recommendations regarding MW uses was covered in an earlier section titled, *Microworlds' impact on PSTs' knowledge*.

Grace also suggested guided exploration for Tommy: "Ask him if there are other rectangles he could make with his string and what would the areas be. He would discover his own counterexamples." Under the right circumstances this student-centered approach could be effective; however, based on Grace's strong CK regarding Tommy's misconception (considering the fact that she was one of the few able to articulate a very good strategy to approximate the area of the footprint), it would have seemed logical to include some mention of these while working with Tommy. Grace again focused on Tommy's misconception, and did not discuss the area of the footprint, during her response to the class. She wrote how, "Using either applet and showing the changes of area when the perimeter stays the same, will give the students the experiences they need to help them develop their understanding of the perimeter/area relationship." Her somewhat vague response left me wondering if she planned on including certain specifics she had discussed earlier in the TE (i.e., dimensions of appropriate rectangles) to help clarify the response. Either way, Grace's KoST, and intervention strategies, had advanced from her teacher-centered intervention in TE 1, which did not involve any manipulatives.

The final two PSTs (#10 & Brianna) responding to Tommy with a combination of explanation and investigation. PST #10 was more student-centered. She had Tommy investigate predesigned rectangles while interjecting thoughtful questions throughout the process, but all that changed when responding how she would address Tommy's class. Strangely enough, PST #10 wrote, "I am not sure how to resolve the problem." Apparently, the interaction with the MW that occurred between questions 6 and 12 had completely altered her focus away from the misconception, which she addressed with Tommy individually, and toward an unwarranted emphasis with the class on finding the area of the footprint. It is possible that this PST (and maybe others) view the MWs as *technological algorithms*, whose primary purpose is to confirm or help find answers.

Brianna incorporated praise and a scaffolding explanation with Tommy that summarized the direct-relationship misconception:

> I would explain that he was right when he used the string to measure the line drawn for the footprint. This is called perimeter. However, we can't just use that string to make a rectangle and measure its area. Just because two things have the same perimeter does not mean they will also have the same area. I would then give examples of rectangles with the same perimeter but different areas.

Brianna had provided several examples earlier in the TE, so it was clear she could accomplish her recommendation. Brianna's student intervention for both TE 1 and TE 2 involved appropriate diagrams and addressed and clarified the student's misconception; however, her approach in TE 1 had the student actively involved with solving problems while in TE 2 she proposed a less-effective teacher-centered approach. On the other hand, a teacher-centered approach did not dominate her proposed instruction for Tommy's class. Brianna gave by far the most thorough response to question 12. Her whole-class intervention involved: (a) explaining the concept behind Tommy's method and why it would not work, (b) having the students draw a rectangle with a perimeter of 18, find its area, and then call on them so everyone could see there are different possibilities, (c) having students go to computers and use the Gizmo to find as many rectangles as possible with a perimeter of 18, and notice the many different areas that are possible. For TE 1, Brianna also suggested a discovery-learning approach with the class to help them understand why the student-proposed method was wrong. The difference was that for TE 1 the learning about the important concepts at play (i.e., differences between linear and square units) was secondary to debugging the student's method. For TE 2, Brianna's student-centered and well-thought response included multiple representations of the key concepts, and summarized the ongoing, positive changes occurring to Brianna's KoST.

It was apparent that those PSTs who were not able to explore the problem deeply on their own also had difficulty intervening with Tommy in meaningful ways; whereas, those with a better understanding of the mathematics surrounding the TE (e.g., Brianna) were more confident and adept at engaging both the student and the entire class in a discussion of the misconception as well as clarifying the major concepts surrounding it.

Post-intervention KoST of the fixed-relationship misconception. Pretest question 10 examined the PSTs pre-intervention KoST regarding the fixed-relationship misconception and those findings were previously discussed in detail. To recap the pertinent findings: (a) When presented with the opportunity, no PST expressed an understanding of the fact that squares are a special classification of rectangles (Grace & Brianna did so during the second interview), (b) Only four PSTs (and no case subjects) expressed an awareness of the misconception commonly held by elementary students that equal perimeters must have equal areas, and vice versa.

Question 8 on the posttest parallels the concepts presented in pretest question 10.

It read: “A student comes to you and says that he/she was able to draw several different rectangles that, according to the area formula, have an area of $36in^2$, but the student was a little surprised when the rectangles did not all look the same size.” The three follow-up questions were: (a) Are the student’s results mathematically reasonable? (b) As a teacher might, explain the reasons for your answer to Part (a), and (c) Why do think the student was surprised by their results? What specifically would you say and do in response to this student’s thinking? The KoST component of this question was primarily Part (c), but before presenting those findings, certain relevant findings from Part (b) bear mentioning. A documented shortcoming of the PSTs throughout the study had been the lack of including appropriate drawings to support explanations; however, for question 8 of the posttest six out of 12 PSTs (including Jackie and Brianna) included useful drawings to enhance their explanations and another three PSTs (including Grace and to a lesser degree Larry) included a table of the factors of 36 that would also help support their explanation. Included within those drawings and tables were eight individual instances where PSTs (including all four case subjects) included a 6x6 square as an appropriate rectangle with an area of 36. Both of these findings are marked increases over pre-intervention findings.

Responses to Part (c) of question 8 of the posttest provided two main categories of findings regarding the PSTs’ KoST regarding the fixed-relationship misconception; those who appeared to grasp the misconception and those who were able to effectively articulate the intricacies of the misconception. There was only one PST (#3) who showed no evidence of understanding the misconception behind the student’s confusion. The responses of three PSTs (represented by Jackie) resulted in uncertainty as to the extent to which they completely grasped the various elements of the misconception. Jackie and another PST (#1) included properly scaled and correctly labeled rectangles that showed several different rectangles could all have an area of 36; however, their subsequent explanations would not have resolved the misconception among classroom students. For example, Jackie wrote, "They [the rectangles] look different but contain same amount of space." In response to Part (c), Jackie wrote, "I think it is important for teachers to have students do many different shapes w/ the same and different areas so that they can see connections." Jackie never explicitly wrote that the student might be confused because it seems logical to expect rectangles of the same area to have the same perimeter even though that is not actually true. Though Jackie did address several issues related to the misconception (the unspoken question) it would have been wise to inquire of the student why s/he was surprised by their results. That way she could have customized her examples and drawings to specifically address the student's concerns. Jackie's post-intervention KoST had progressed from her previous levels in that she now rather consistently diagnosed incorrect student thinking; however, she continued to struggle with providing lucid explanations of those diagnoses as well as with including appropriate mathematical language.

The second category of findings involves eight PSTs who apparently grasped the misconception the student was struggling with, but specific wording and suggested intervention separated the "better" responses from the "best" ones. Four of these PSTs (including Larry and Grace) failed to completely articulate the misconception. Their explanations were similar to Grace's: "By showing these examples (a 1×36, 2×18, 3×12, 4×9, 5×7.2, and 6×6), it can be seen that many rectangles can have the area of 36 in^2 but have different dimensions and look different." Grace, like the other three in this group, failed to use the word *perimeter* while explaining the misconception. That is fairly significant since the misconception being discussed involves area <u>and</u> perimeter. Grace's response signified growth from her pre-intervention KoST. While being interviewed regarding her pretest KoST (discussed above), Grace admitted, "I still don't know which concepts would give them the most trouble;" however, for question 8 on the posttest she stressed conceptual methods and was very clear on how she would approach the student struggling. She was also very confident about why students might have such a misconception: "Students tend to think that the area in a rectangle is going to be different when the dimensions are different." Grace did not express such awareness before the intervention.

The other four PSTs (including Brianna) used the word *perimeter* while explaining why the student might be confused. Brianna's responses were the best and are representative of the others: "Figures with the same area may look different, because they have different perimeters. Many students correlate one area to one shape with one perimeter. We can have the same amount of space inside two objects yet they can have different shapes." She then referred to five different rectangles she had drawn to scale and labeled correctly, which all had an area of 36 but different perimeters. Brianna did not express knowledge of the fixed-relationship misconception before the intervention nor had she clearly explained how students might think about the fixed relationship. Both of these are evidence of a maturing KoST. Her ability to apply it will be seen next.

The final distinction that elevated certain PSTs' KoST regarding the fixed-relationship misconception was their suggested intervention for the confused student. It was common for PSTs to simply refer back to the rectangles they had previously drawn when suggesting an intervention for the confused student; however, there were five PSTs (#6, #11, Larry, Grace, & Brianna) whose suggested intervention would promote (to varying degrees) a conceptual understanding of the fixed-relationship misconception. There were three different recommendations to help the student better understand how different-shaped rectangles could still have the same area. PSTs 6 and 11 similarly proposed, "Have the student cut out square inches and create the rectangles to <u>see</u> that they have the same area." Grace and Larry thought it would help the student if the various rectangles were drawn on grid paper. Grace added, "That way he could <u>count the square inches</u>." Brianna's intervention was the most thorough. She included a detailed explanation about the misconception and why it was not correct – including language that would be meaningful to students. The activity she suggested to promote understanding involved: "Fill the different rectangles on a grid with pattern blocks. Have students count them and see that they have the number of blocks inside them but they look different." During our second interview, Brianna clarified that "the different rectangles" were those she had drawn earlier in question 8 which all had an area of 36 but different perimeters. These statements reveal a rather significant change in Brianna's pre-intervention KoST, which was very procedural and designed to help students overcome their weaknesses and get the right answer. Now Brianna incorporated activates that focused on the students understanding the mathematical concepts. That represents a rather robust KoST.

Emergent KoST of the direct-relationship misconception. The PSTs' CK regarding the direct relationship was previously examined and it was shown that many experienced growth in their levels of understanding (Ma, 1999). The findings presented in these next final sections address the PSTs' understanding of, and more importantly how they indicate they would respond to, student difficulties and misconceptions (i.e., their KoST). The PSTs' KoST regarding perceived relationships will now examine the second, and slightly more elusive, misconception. The gist of this misconception is that there exists a direct relationship between perimeter and area, that is, as the perimeter of a shape increases/decreases its area must also increase/decrease (and vice versa). The focus problem for TE 3 (Figure 16, p. 136) provided the setting for findings related to the PSTs' emerging KoST of the direct-relationship misconception. TE 3 began with four questions related to the PSTs' CK (their reaction to the claim), and then transitioned into examining their KoST (their reaction to the student). For this last TE, the PSTs were instructed they could interact with either microworld from the outset.

There are two questions from TE 3 that provided useful findings to investigate the PSTs' KoST: (a) #5 – "As a teacher, how would you respond to Jasmine's thinking and her proposed theory? What specifically would you say and do (even if you are unsure about the mathematics involved?)" and (b) #10 – "As a result of seeing Jasmine's theory and apparent lack of complete understanding, how would you follow up with the entire class about the concepts that surround this classroom episode? Remember to share specific examples and representations (possibly from a microworld) just as you would in the classroom." Findings related to questions 5 and 10 are similar in that they involve a more pedagogical aspect of the PSTs' KoST, and thus will be presented together.

Questions 5 and 10 looked at how the PSTs would specifically address Jasmine's thinking and subsequently Jasmine's class. A thorough and model response to these questions would have included: (a) a discussion of why Jasmine's "theory" would not always be true (addressing Jasmine’s over-generalization), including appropriate counterexamples, hence disproving the direct-relationship misconception, (b) investigating (or at least mentioning) the various relationships surrounding Jasmine’s method. For example, her *theory* does hold under certain circumstances (i.e., if both dimensions are increased), and (c) allowing for students to explore these relationships with either MW (the *Gizmo* would be preferable).

By the time the PSTs reached these KoST questions, all but two of them (Jackie and #1) had come to the conclusion that Jasmine’s “theory” was not always correct, and had already provided counterexamples to illustrate their position. Consequently, the two PSTs that were not able to debunk Jasmine’s theory were not able to offer any meaningful intervention to help Jasmine or her class. It is not that surprising that Jackie and PST #1 thought that Jasmine’s theory was correct since they were two of the three PSTs who thought the same way when this misconception appeared on the pretest, and there had been no formal intervention up to TE 1. Even though Jackie was not able to address the mathematical aspects of Jasmine’s thinking, she still displayed some positive applications of her KoST. First, she offered the student praise for, “her excitement in trying to discover more about math. The NCTM Standards encourage students to reason and make connections.” Second, and more important, she wrote, “I would tell her to test her theory with some more problems. You can’t be too sure w/ just 1 try.” Jackie recognized the danger of over-generalizing when making mathematical claims and that was significant as it is a characteristic of an expert teacher (Table 3, p. 166). What is somewhat puzzling is why Jackie did not take her own advice and test Jasmine’s theory out on one of the MWs. A possible answer to that question, which also exposes what was a common view of and approach to using the MWs, was made apparent during the second interview with Jackie. I asked her about her lack of progress on deciphering Jasmine's thinking in TE 3 and why she did not try investigating with the MWs. She replied, "If a student doesn't really know the concept, then no matter what you do to help them, you know, no matter what resources or what materials, or games, or anything you give them, it is not going to help them if they don't know what they are looking for. I still think they [the MWs] are beneficial, but maybe it's necessary to explain the concept to her first and show her through examples." Jackie's admitted over-exposure to show-and-tell teaching approaches seems to have affected her belief in what students are capable of doing on their own as well as how she herself approaches problem solving.

While Jackie's CK appeared to change and grow after repeated exposure to area and perimeter concepts, her KoST struggled adapting throughout the intervention. Jackie had difficulty "thinking on her feet" and was often unable to work through various mathematical scenarios, which left her ill-equipped to respond to student difficulties. Jackie's suggested student interventions often focused only on big ideas (e.g., clarifying area and perimeter), even when those ideas were not helpful in resolving the current misconception. Her choices of mathematical language often confused and muddied her attempts at explaining concepts – even those concepts she seemed to understand. She did not appear to learn well on her own, but rather indicated several times how the small-group and whole-class sessions were very helpful. Jackie put forth a lot of effort throughout the intervention and was very engaged during both interviews. Her increased posttest scores revealed that her hard work was not in vain. Jackie's intense desire to be a successful teacher also translated into moments of pedagogical clarity. For example, a comment made by Jackie during a teaching episode involved her belief that it might help students resolve area and perimeter conflicts if the concepts were studied simultaneously. Her view displayed relative expert pedagogical KoST, shared by several researchers (Chappell & Thompson, 1999; Hiebert & Lefevre, 1986; Simon & Blume, 1994a).

The focus will now turn to the recommendations of the other nine PSTs who did realize the student's theory was incorrect. Their instructional strategies, both with Jasmine and her class, divided along lines of teacher-centered versus student-centered, with approaches involving hand-drawn examples and/or the use of a MW. The first category involves those who suggested very teacher-centered activities. There were four PSTs in this group (including Larry and Brianna), and generally, their explanations contained assertions that would begin with "I would show her examples . . ." or "I would tell her that . . ." All four PSTs wrote how they would make sure Jasmine realized her theory would not work all the time. Two PSTs (Larry and #5) indicated they would use the *Gizmo* MW with Jasmine, and the class, to help them see inconsistencies in her proposed theory. PST #5 included specific details about the types of examples she would use as well as the accompanying explanations she would use. Larry provided neither. He was vague with Jasmine: "I would set up a bunch of examples," and for the class: "Project the Gizmo up in front of the class and show the students that just b/c the perimeter is greater does not mean that the area is also." Larry gave a very similar, and equally vague, response as in TE 1. It is a little surprising that Larry did not consider it important to provide more information, given the thorough summaries provided for TE 1 and TE 2 – what appropriate student intervention should involve.

Larry's performance was erratic throughout the study. He often appeared confused or distant during discovery-learning sessions, and did not seem interested in exploring concepts which he struggled with. Once he seemed to grasp a concept (i.e., TE 2), he rarely ventured beyond that knowledge. At times he appeared distracted by the MWs and wrote several times how he “figured things out better by hand.” When he did use MWs in his responses, the goal was to accelerate the viewing of many examples – to more efficiently arrive at an answer. He continually appeared content with simply getting what he thought to be “the right answer,” and that CK facilitated a KoST that was satisfied with responding to student shortcomings in an attempt to guide them to get right answers. Larry’s explanations were often tied to formulas and procedures, and involved teacher-centered behavior. They frequently lacked meaningful and classroom-useful diagrams. Larry’s responses would incorporate instructional aids at times (e.g., grid paper); however, it would often be the same ones and many times the reason for the aid was unclear. Overall, finishing problems and generating answers appeared to take precedent during the intervention over gaining personal insights and knowledge necessary to develop conceptual understanding within future students.

Brianna and PST #10 were the other two who proposed teacher-centered interventions. PST #10 incorporated thoughtful and directed questions with Jasmine while sharing examples that would lead her to find the error with her theory. Brianna’s response to Jasmine involved presenting counterexamples for her to calculate the area and perimeter of in hopes she would realize the error of her theory. Brianna was the only PST to go one step further with Jasmine and formally acknowledge that her theory could be true, she wrote, “Even though sometimes it does work out, it does not always.” Brianna did not provide the specific examples she referred to in her explanation. Her intervention with the entire class was very similar in content, although she did suggest using the Shape Builder MW to present the various examples. As was common with Brianna, she directed and/or guided the instruction, whether working with one student or an entire class. In that aspect, her KoST was very narrow in focus and application.

Brianna's strong mathematics background powered her CK and allowed her to grasp every misconception within the TEs and to be very thorough and accurate in her prescribed activities. Her ample CK initially interfered with her ability to see the need to include diagrams to help students understand her ideas; however, the frequency of quality diagrams increased From TE 2 right through the follow-up test. That strong CK likely facilitated Brianna's propensity to control the learning environment. In all three TEs, Brianna indicated that she would direct the learning during the interventions (both with individual students and with a class). She often had students investigating with MWs, but with predesigned problems. Her instructional strategies gradually evolved from teacher-centered, with students receiving instruction, to teacher-directed, with students participating more in their learning. Absent however were frequent opportunities for students to interact with her (through assessment questions) or explore on their own. Only in TE 2 did Brianna indicate she would allow students to work independently with a MW, even then it was on a predetermined problem. Brianna was modest and relatively quiet. During her second interview I informed her that several PSTs wrote how they learned a lot when they were in her small group; that she always had clever ways to look at and explain things. Brianna's response to that was a genuine, "Really?" Her lack of confidence in certain social/teaching situations may help to explain her teacher-centered tendencies and her incomplete KoST.

The final group of five PSTs (including Grace) represents those who, to varying degrees, encouraged both Jasmine and her class to explore the concepts surrounding the direct-relationship misconception. For three of these PSTs, it was interesting how two (#3 and #11) suggested more teacher-directed approaches with Jasmine, but more discovery-based with the entire class, and the other (#9) was more student-centered working with Jasmine but teacher-directed with the class. The discovery activities typically involved the student(s) finding several rectangles that have the same area and then comparing their perimeters to see that the larger perimeter does not always have the larger area; hence, refuting the “always” aspect of Jasmine’s theory. All of these three recommended using MWs, but they thought hand-drawn examples would be more meaningful with Jasmine while MWs would be more appropriate when working with the class.

Grace and PST #6 were the only two to accomplish all three KoST objectives established at the beginning of this section: (a) they addressed the misconception, (b) they encouraged investigation to discover other relationships, and (c) they realized the value of the MWs in that investigation. PST #6, who was one of the top achievers in the study, promoted exploratory methods for both Jasmine and her class. She explained how Jasmine’s theory worked for one example and then suggested asking Jasmine (and the class) if she/they could find two rectangles where the theory does not work. She concluded by writing, “I could let them use the Gizmo to see if they can find any other relationships.” She was the only PST who encouraged this level of exploration both for Jasmine and for the class. PST #6 displayed a pedagogically-powerful KoST. These misconceptions facilitate discovery learning and a responsive KoST would recognize that as appropriate intervention. Grace went one step further and shared two specific area-perimeter relationships she would guide the students into discovering: (a) If you increase one dimension of a rectangle but decrease the other, it will result in a smaller area, and (b) If you leave one dimension of a rectangle fixed and increase the other dimension, that will always increase the area (i.e., Jasmine's theory). Grace expounded on her ideas during the second interview. Regarding her planned intervention with Jasmine, "The Gizmo would allow her [Jasmine] to see that the greater the difference between the dimensions of the rectangles, the lesser the area – up to a point. It does not always happen." Grace also commented, "As the difference between the dimensions decreases, where the numbers get closer together, the area will increase up to a square which has the greatest area." It was just the possession of that CK that showed how Grace had grown through the intervention, but it was her sharing of that CK with Jasmine and the class that revealed her KoST had equally matured. Grace wrote how, after giving the students an opportunity to investigate Jasmine's theory, she would systematically show (using the *Gizmo* MW) and explain with the students the various conditions that influence whether the area increases or decreases, "in the same way I explored and discovered those same conditions." That last quotation draws together several aspects of Grace's KoST and her desire to understand mathematics, how students think about it, and how she can help them understand it better.

Throughout the intervention, Grace would often call me over to see her computer and what she was working on. She would ask questions, because she had a genuine desire to understand the concepts we were covering. She wanted to be prepared to teach them well. Grace is somewhat of a perfectionist, as her 4.0 GPA testifies. Early on in the study, Grace appeared to know more than she would write in her responses. That became evident during the first interview. Once Grace became aware of how thorough communication was necessary to promote understanding of mathematical principles, her responses changed to include greater specificity. Her desire to understand mathematical concepts did not end when class ended. Grace's first exposure to the direct-relationship misconception came on the pretest (question 8). It was during our first interview that her internal drive to better understand the mathematics she would have to teach became evident. We were discussing her thoughts on question 8 and the proposed direct relationship between area and perimeter. She shared how she had "been thinking about this problem for the last couple days," and she found that "the rectangles that have dimensions that are closer to being equal have more area." Grace is of course referring to the idea that, for quadrilaterals, a square maximizes area. Grace was not generally satisfied with leaving mathematical conflicts unresolved, and the fact that she was thinking about and working on a problem outside of class was evidence of that. It also helps to explain how she was able to make such noticeable improvements on the same misconception when it resurfaced in TE 3. Grace's desire for her students to have a conceptual understanding of mathematics has been shared numerous times. It was apparent in the application of both her CK and KoST, which strived to clearly communicate mathematical ideas so that students would understand them.

Post-intervention KoST of the direct-relationship misconception. The findings in this section concludes the discussion regarding perceived relationships (specifically the direct relationship), and finishes addressing research question 4, which was concerned with how the PSTs' KoST had changed during the course of this study.

Pretest question 8 examined the PSTs pre-intervention KoST (and CK) regarding the direct-relationship misconception, and those findings were previously discussed in detail. To recap the pertinent findings: (a) 4 out of 12 PSTs (including Larry & Jackie) agreed with the student's erroneous claim, which rendered applications of their KoST ineffective, (b) Of the nine PSTs who disagreed with the student's claim, only three (including Brianna) provided appropriate counterexamples in their response to the student. (c) Only one PST (#10) included any discovery-type activities in her response to the struggling student. The suggested intervention by the other 11 PSTs was completely teacher-centered.

Question 6 on the posttest parallels the concepts presented in pretest question 8. It reads: "Stacey claims that whenever you compare two rectangles, the one with the smaller perimeter will always have the smaller area." The follow-up questions that touch on KoST were: (b) Explain why you agree or disagree with Stacey's thinking." and (c) As a teacher, how would you respond to Stacey? What specifically would you say and do (even if you are unsure about the mathematics involved)? One difference between this question and pretest #8 and TE 3 is that those questions used the word *larger* instead of *smaller*; however, the direct-relationship claim would be examined and discussed in much the same way. Another difference for question 6 was that no example (i.e., student work) was provided as "proof" of the student's claim, as was the case for TE 3. An implication of that last statement was that if a PST acknowledged that Stacey's claim could be true, they would have to supply their own example.

A thorough and model response, revealing the PSTs' KoST should include all or most of the following: (a) an acknowledgement that Stacey's claim is not "always" true, followed by an explanation detailing why and including appropriate counterexamples;

hence, disproving the direct-relationship misconception, (b) mention of Stacey's potential over-generalization (i.e., that she generated her claim after only a few, or even one, example), (c) investigating (or at least mentioning) the various area-perimeter relationships surrounding Stacey's claim. For example, her *theory* does hold under certain circumstances (i.e., if both dimensions are increased), and (d) recommending students to explore those relationships with either MW (preferably the *Gizmo*). It should be mentioned that, due to time constraints, it would be unrealistic to expect any PST, while working on the post- or follow-up test, to expand upon or even duplicate the extent of the responses provided in the TEs.

Descriptive statistics for posttest question 6 indicated it was the second hardest item on the test. That was evident by the fact that two PSTs (Larry and #1), agreed that Stacey's erroneous claim was correct. That is a slight improvement over TE 3, where 4 initially agreed with the claim. Larry originally disagreed with Stacey's claim, but then changed to agreeing with her. Larry's final answer regarding how he would respond to Stacey was, "I would tell her great that she is thinking correctly. But make sure she tries to disprove her method." Larry's response is somewhat ironic, because nowhere on his paper did he attempt any diagrams or examples – either proving or disproving Stacey's claim. What made Larry's comments hard to understand was that they came just one week after TE 3, where we had spent three class hours addressing the misconception. On the follow-up test, Larry never wavered as he once again agreed with the student and their flawed claim regarding a direct relationship between perimeter and area. It was apparent that Larry's CK was still much unorganized, and he has trouble remembering ideas recently discussed. Obviously, Larry would be unable to engage a struggling student in any meaningful dialogue regarding these concepts, as he himself is confused about them. His ability to understand and then respond to student's misconceptions (i.e., his KoST) is continually derailed by his insufficient CK. Any progress Larry seemed to make during the planned intervention appeared to be short lived.

The responses to question 6 from the other 10 PSTs formed three categories of findings: (a) those who only disagreed with Stacey's claim (*n* = 3), (b) those who disagreed with the claim but made some reference to the fact that it could work (*n* = 5), and (c) those who refuted the claim and also explained or illustrated when the claim would hold (*n* = 2). All three PSTs in the "only-disagreed" group provided suitable counterexamples to Stacey's claim. One PST suggested having the student "run several more trials using a variety of numbers." That showed an awareness of the limitations of over-generalizing. Another PST wrote, "I would pull up the SB [Shape Builder MW] and let her draw some random examples." The implication of active-student learning was positive, but the lack of specificity left the intervention inconclusive. Overall, their suggested responses to Stacey were more teacher-centered, and similar in content, because each narrowly focused the discussions surrounding only counterexamples.

There were a total of seven PSTs who indicated that they both disagreed with and could correctly support Stacey's claim. That was double the amount (*n* = 3) who reached this level of understanding and effective student involvement during TE 3. Five of the PSTs (including Jackie and Grace), while alluding to the possibility that the claim could hold, did not provide any specific examples (i.e., diagrams or dimensions) which would be meaningful in helping Stacey understand more about the misconception. They did provide either a picture or table of dimensions of their counterexample to Stacey's claim, but not much beyond that. Somewhat surprisingly, Grace's response to Stacey was vague and relatively short: "I would ask her to show me her thinking and direct her toward discovering a non-example." While inquiring to better understand the student's thinking is wise, Grace provided no details regarding how she would "direct her," nor did she explain how she would follow up with Stacey to assist in resolving the misconception, exploring the other possibilities she had mentioned earlier, or how she would help Stacey reconstruct her knowledge.

The same misconception appeared on the follow-up test, and this time Grace only disproved the claim; however, she did a much better job explaining the condition that would make it false and her response to the student was coherent and included drawings of her counterexample. Two others (Jackie and #4) of these same five mentioned they would caution Stacey about basing her claim on only one example: "First, I would ask her to prove her theory to me providing more than 1 example" (Jackie). During our second interview, Jackie expanded on that thought: "It would be a pretty absolute statement to make with only one example." These five PSTs also acknowledged that Stacey's claim could be true, as represented by Jackie: "Although this may be true for some problems, it is not true for all." Jackie's response to Stacey, while very student-centered, did not initially discuss other possible conditions: "I would propose her theory to the class. Then I would play devil's advocate and prove why her theory is wrong." During her second interview Jackie elaborated more on her proposed intervention to include conditions beyond just proving the theory wrong:

> I think I would have her come up in front of the class and present her theory so the class could see what she meant. Then I would have her ask the class what they

> thought about it; have them work on the problem, and have them raise their hand if they proved her theory wrong, or raise their hand if they proved it right.

On the basis of her CK regarding this misconception Jackie realized that the student was wrong, and also pinpointed the source of the erroneous thinking. The context and level of student involvement recommended by Jackie revealed her KoST was beginning to incorporate the ideas and practices that had been discussed during the TEs. These were noticeable differences from her pre-intervention awareness and application of such pedagogy. Jackie showed she had retained much of her KoST when faced with the same misconception on the follow-up test. There she gave the same basic response as on the posttest, but was even more clear about how she would respond to the struggling student.

There were two PSTs (Brianna and #10) who went one step beyond simply providing a counterexample to Stacey's claim and acknowledging that the claim could hold. These PSTs informally explained or illustrated one condition that would support the student's claim. For example, Brianna wrote, "long, skinny rectangles may have a larger perimeter, but will have a smaller area than many rectangles with a smaller perimeter." PST #10 provided a diagram of a 3×7 and a 1×11 rectangle that illustrated Brianna's idea. Brianna indicated that she would have the student provide examples supporting her claim Gathering background information on a struggling student's thinking is a wise first step when intervening for the purpose of reconstructing that student's knowledge. The parallel item on the follow-up test (question #8) revealed some concepts regarding this misconception were not retained by these two PSTs. Both of them neglected to even mention the possibility of the student's claim working under certain conditions.

It was somewhat unexpected that only three PSTs made reference to incorporating

MWs while working with Stacey. The direct-relationship misconception had just been personally investigated and corporately discussed the previous week, and it provides a prime opportunity to explore the various concepts with the *Gizmo* MW. The user can quickly and easily drag the corner of a rectangle to produce countless different rectangles, while watching the area and perimeter measurements change in real time. The instant feedback would be very valuable for a student and support the various conditions surrounding this misconception. A well-developed KoST would have realized the benefits of the MW to aid a struggling student.

Other findings from the follow-up test indicated that there were signs of continued growth regarding certain PSTs' understandings related to this misconception. Only one PST (Larry) agreed with the claim on the follow-up test, as compared to four on the pretest and two on the posttest. PST #1, who was the other PST on the posttest to agree with the student's invalid claim, experienced positive changes in both her CK and KoST on the same question on the follow-up test.

Research Question 5: Identifying and Describing CK-KoST Relationships

This time I understood, so I felt I could do that. Now that I understand, I thought that would be a good way to go. (Jackie, following the posttest, discussing on how she would address a student's erroneous thinking)

The findings in this next section address the fifth and final research question: In what ways, if at all, is the PST's knowledge of student thinking (KoST) regarding area and perimeter related to their content knowledge (CK) of those same concepts. This study operated under the somewhat logical assumption that CK and KoST are interrelated; further more, possessing a robust KoST would be dependent upon possessing at least an adequate CK. Answering research question 5 involved examining the various relationships that might exist between CK and KoST. The case subjects were the focus of this research question, because their interview findings were necessary to triangulate with other data (i.e., tests and teaching episodes). The relationships explored were associated with area and perimeter in general, and more specifically, units of measure and perceived relationships. The answering of research question 5 involved two components. First, were quantitative findings involving: (a) the correlation coefficients for CK and KoST at the three time-points (i.e., pre-, post,- and follow-up), (b) CK-KoST relationships as seen in both the rubric scoring of responses to pre-, post-, and follow-up test items (e.g., Table 14, p. 256) and the summary tables of expert/novice codings (e.g., Table 16, p. 261), and (c) appropriate regression graphs (previously used to answer research questions 3 and 4). The second element was more descriptive and entailed elaborating on the initial relationships identified by the quantitative analysis. Two comprehensive analysis strands, devised and organized around the area and perimeter concepts/misconceptions central to this study (Table 5, p. 188), helped guide the presentation of the qualitative findings and answer research question 5. These strands tracked parallel items (e.g., CK related to units of measure) from the pre-, post-, and follow-up tests, and the three teaching episodes. The goal and challenge of answering research question 5 was to ascertain and then describe how, if at all, KoST and CK are related within the context of this study.

Identifying CK-KoST Relationships

As was shown while answering research questions 3 and 4, the majority of PSTs exhibited some sort of increase in their CK (75%), KoST (also 75%), or both (58%) from pretest to posttest. A reexamination of the regression lines (Figures 27-29, pp. 267-269) revealed that a positive relationship (i.e., correlation) existed for those same PSTs, as seen in the positive slopes. Subsequent calculations of the correlation coefficients for KoST and CK at the three time points confirmed the existence of some relationship: (a) pretest, $r = .53$, (b) posttest, $r = .64$ (significant at the .05 level [two-tailed]), and (c) follow-up, $r = .57$. Not completely surprisingly, these values are moderate to strong. The lower variability, small number of sub-test items ($n = 5$), and the presence of one, possibly two, poorer-measuring question helps explain the lower correlations for the pre- and follow-up tests. Before discussing the one viable relationship uncovered, there are other results worth mentioning, although none involved more than two PSTs.

Grace and PST #6 showed an initial similarity involving a static CK and an increasing KoST. That result would seem like an illogical relationship. One would think that in order for KoST (an application of CK) to increase, a PSTs' CK would also have to be increasing. After closer examination, their CK was static because it was initially very high. Grace and #6 had the highest and second higher scores respectively on the pretest CK sub-pretest. Their CK was more than adequate to support an increase in their KoST, which for them involved incorporating effective instructional methods into an already receptive framework. There were no other descriptive indicators warranting further investigation of this result.

A second observation involved the other two case subjects - Brianna and Larry.

There were slight increases in their CK but no discernable increase in the KoST. It might seem that this result would warrant further discussion; however, when the scores of these PSTs were more closely examined, the need for further investigation was sufficiently diminished. Larry, the overall weakest performing PST, concluded the pretest with the second lowest score on both the CK and KoST sub-tests, and he made very little measurable change throughout the study. Brianna completed the posttest with the second highest CK sub-test score and the highest KoST sub-test score; therefore, the fact that Brianna's KoST did not increase substantially from pretest to posttest was not surprising. A result of these facts was the lack of *numerical trails* to investigate further.

Delving deeper into the KoST and CK sub-test scores from the pre- and posttest, and applying the ±3-point criterion established and described in Chapter 3, revealed several patterns that formed the basis for the findings that will assist in answering research question 5. There were six PSTs (Jackie, #4, #5, #9, #10, and #11) who experienced a discernable increase in both their CK and their KoST – the "increased CK–increased KoST" group (labeled, ↑CK - ↑KoST). Jackie, for reasons given in Chapter 3, will be the focus of the findings regarding this group. Every member in the group had both their KoST and their CK sub-test scores increase by at least 3 points from pretest to posttest (range of increase 3-9). All six of the PSTs in the ↑CK - ↑KoST group also saw increases from pretest to posttest in the frequency of expert codings assigned to both their CK and KoST. There are other common traits within the group, that will be presented later, that help confirm Jackie as a fair representative for the group. At this point the identified relationship is mostly numerical. The goal now is to attempt to uncover and explain the character of those numbers.

Describing CK-KoST Relationships

Two comprehensive analysis strands were devised to organize the area and perimeter concepts/misconceptions central to this study (Table 5, p. 188). They helped to focus and guide the analysis necessary to ascertain the role, if any, that a PST's CK plays in their ability to understand, analyze, and respond to hypothetical students' thinking (i.e., their KoST). The two analysis strands are (a) units of measure (i.e., linear versus square), and (b) the presumed relationships between area and perimeter (i.e., that equal perimeters must result in equal areas and vice versa, and the belief that a direct relationship exists between area and perimeter in that increasing (or decreasing) one will have the effect of increasing (or decreasing) the other. These analysis strands formed the basis for the topics of inquiry across various time-points (i.e., across teaching episodes and from pretest to posttest to follow-up). A case subject was the primary focus of the comparative analysis, because her responses received appropriate pattern matching through the two semi-structured interviews. She was also representative of the prominent CK-KoST relationship patterns identified in the previous section (e.g., Jackie – increase in KoST [+6] with increase in CK [+9]).

The Increased CK–Increased KoST (↑CK - ↑KoST) Relationship

There were six PSTs (Jackie, #4, #5, #9, #10, and #11) who experienced a discernable increase in both their CK and their KoST – the "increased CK–increased KoST" group. Every member in that group had both their KoST and their CK sub-test scores increase by at least 3 points from pretest to posttest (range of increase 3-9). All six of the PSTs in the group also saw increases from pretest to posttest in the frequency of expert codings assigned to both their CK and KoST. All but one PST in the ↑CK - ↑KoST group scored below the mean on the CK sub-test and all six scored very close to the KoST sub-test mean. As described earlier, various descriptive statistics placed the behavior of Jackie's CK and KoST into this group, and established her as a model PST to represent the group. The fact that Jackie was a case subject allows for additional sources (e.g., the first and second interviews) to help document and explain possible relationships that exist between her CK and KoST. The purpose of the following sections is not just to present examples of the increases in CK and KoST, as that was done while answering research questions 3 and 4, but rather to establish baseline relationships between CK and KoST, to describe how they changed through intervention, and to discern in what ways CK and KoST interact with each other.

↑*CK* - ↑*KoST relationship prior to intervention*. The comparative analysis began with a condensed recap of pretest performance and a description of how Jackie's pre-intervention CK informed her KoST regarding units of measure and perceived relationships. Problems 1, 3, and 4 from the pretest focused on basic CK regarding units of measure and 5 addressed perceived relationships, while corresponding KoST problems were numbers 6, 7, and 9 for units of measure and 8 and 10 for perceived relationships. It was apparent from the CK problems that Jackie was lacking an understanding of fundamental concepts surrounding area and perimeter (i.e., which unit should be used to calculate each, and how area and perimeter relate to each other), and she knew it. Subsequent probing would reveal just how much Jackie did not know, had forgotten, or likely a combination of both. One problem asked her to "On the grid provided, draw a polygon that has a perimeter of 24 units." The follow-up question asked how she would explain her answer to a 5th grader. Jackie drew a 3 × 8 rectangle, which has an area of 24 square units. So it is obvious she had forgotten, or does not understand, what the concept of perimeter means. Her explanation to the hypothetical student bears repeating, “To be honest . . . I have no idea of the polygon I drew represents a perimeter of 24. But I guess I would show him that each box is 1 unit and in the box there is 24 units?” Granted, Jackie is admittedly confused but the parallel trend from meager CK to an inappropriate explanation to a student (an aspect of her KoST) is telling. She not only initially confuses perimeter with area, but her explanation adds more contradictory information by introducing the vague term “box” (a 2-dimensional concept at best, or a 3-dimensional at worst) while supposedly explaining to a student about perimeter (a 1-dimensional measurement). Basic relationships between area and perimeter also involve dimensions. Question 5 on the pretest asked PSTs, “If each dimension of a 2 × 4 rectangle is tripled, what is the relationship between the original and the enlarged figure?” Jackie misread the problem to involve triangles, thus was unproductive describing the relationships. Others in the ↑CK - ↑KoST group were able to understand the perimeter would be tripled, but none realized the 2-dimensional aspect of area would cause the area to be increased by a factor of 9. Not appreciating the fact that area is a 2-dimensional concept would often cause conflict within the PSTs’ CK.

Prior to intervention, the majority of PSTs in the study were not able to coherently explain or illustrate the concepts of linear and square units. Her first interview confirmed Jackie’s fragile CK as she continually confused area and perimeter concepts, which routinely resulted in confusing the meaning and use of linear and square units. Jackie’s fragile CK would also cause her to wrongly apply procedural methods, followed by procedural explanations even when inappropriate. For example, problem 3 on the pretest

(Figure 21, p. 212) asked the PSTs to find the area and perimeter of an irregular shape. Jackie was not able to draw from a CK that included an understanding of linear and square units, and that caused her to apply erroneous methods to find perimeter and area of the irregular shape. The other PSTs in this group were able to find the correct area and perimeter in problem 3, but their incorrect treatment of the appropriate unit for each measure lead to nonconceptual explanations and misapplying the b × h formula to situations where it was not needed or helpful. As the pretest continued, the PSTs faced problems which required them to more directly apply their KoST.

Jackie knew, and stated often, that multiplying base times height would give the area of a rectangle. Yet, further probing revealed a lack of understanding about the common formula. I asked Jackie why multiplying base times height produces the area of a rectangle. She replied, “To be honest, I just know that you multiply the base times the height and you’ll get the area. I have no idea why.” That procedural and incomplete CK continued to leave its mark on how Jackie responded to struggling students. Problem 6 on the pretest (Figure 23, p. 224) asked the PSTs to respond to a student who correctly found the area of a 3 × 6 rectangle to be 18, but indicated he did not understand what exactly the 18 represented. Jackie attempted a conceptual approach by drawing a 3 × 6 array of squares inside the rectangle, but her subsequent explanation of calling the 18 “cm’s” not only is incorrect but would be very confusing since the 3 × 6 rectangle would have a <u>perimeter</u> of 18 cm. Several PSTs in the ↑CK - ↑KoST group incorrectly used cm as a unit for measuring area.

Conflicting ideas about area and perimeter, linear and square units, and perceived relationships also produced incomplete diagnoses of student misconceptions and ineffective instructional suggestions regarding these concepts. The last problems on the pretest that specifically addressed perceived relationships and units of measure were 8 and 9 (Figure 25, p. 230), respectively. Each problem centered on a student proposing either an erroneous claim (#8) or solution method (#9). In problem 8 Jasmine, the student, claimed “that whenever you compare two rectangles, the one with the grater perimeter will always have the greater area.” The student’s claim is correct or incorrect depending on how the rectangle’s dimensions are changed. Sketching out various rectangles can often lead to, at the very least, a counterexample to the claim. Jackie did not attempt any sketches and did not offer any evidence of fully comprehending the claim, and as a result offered nothing but vague suggestions for how to respond to Jasmine: “demonstrate how to determine area and perimeter and have her see the results.” Problem 9 involved a student proposing an erroneous method to find the perimeter of an irregular shape (drawn on a grid) by counting the number of square units. Because Jackie had an insufficient understanding regarding units of measure, her diagnosis and intervention had an improper focus. Again, Jackie wrote that she would, “Have him understand the differences between area and perimeter.” The student actually seemed to understand perimeter. His confusion involved using the wrong unit (i.e., square unit) to measure perimeter. In both instances, Jackie’s CK did not appear to provide the necessary foundation for which to explore, diagnose, and then respond to the student and their thinking.

It should be noted that Jackie’s use of a 3 × 6 array on problem 6 actually earned her a higher rubric score (for including a conceptual approach), even though the subsequent interview revealed she did not possess the mathematical understanding to make good pedagogical use of the array. Such instances also help to explain how Jackie had a higher KoST score on the pretest (5 points higher) than she did for CK. Initially at times, inferior CK was easier to identify, and score or code, than was inferior KoST. A PST could provide what appeared on the surface as evidence of expert KoST. For example, they might write that students often struggle with certain concepts regarding area and perimeters (e.g., linear and square units), and such an acknowledgment would earn various expert codes (see Table 3, p. 166); however, it could be possible (and many times was) that that same PST did not possess the necessary CK to be able to adequately explain those concepts to the student. It has just been shown how Jackie's incomplete and fragile CK resulted in inadequate and often ineffective response to student's shortcomings and misconceptions (i.e., an equally incomplete KoST). The next section will present findings that demonstrate how Jackie's CK and KoST interacted as a result of the planned intervention.

↑*CK* - ↑*KoST relationship: Emergent findings*. The primary means to strengthen the PSTs' CK and KoST regarding units of measure and perceived relationships were the three teaching episodes (TEs). Teaching episode 1 focused on units and TEs 2 and 3 addressed perceived relationships involving area and perimeter. Tables 25 and 26 (two of 16 such tables consulted while organizing findings for research question 5) provided evidence of the slow transition that Jackie's CK and KoST went through during TE 1. Note the low frequency of *b* (or expert) codings assigned during the TE, but how they increased on the posttest. The progression regarding perceived relationships was even slower to develop. There were many more novice (*a*) codes assigned to responses within TEs 2 and 3 than to TE 1 and also fewer expert (*b*) codes awarded. Table 13 (p. 251)

Table 25

*Sample of Expert/Novice Codings Relevant to Units-of-Measure Analysis Strand (**CK**)*

	Pre-Intervention (Pretest)			Intervention (Teaching Episode 1)				Post-Intervention (Posttest)			Post-Intervention (Follow-up)		
	Q1	Q3	Q4	Q2	Q3	Q4	Q6	Q1	Q3	Q4	Q1	Q3	Q4
1a	1	1	1			1		1		1	1	1	1
1b													
2a	1	1	1		1								
2b							1	1	1	1		1	1
3a													
3b													
4a													
4b													
5a													
5b													
6a	1												
6b													
7a-			1					1					
7a	1												
7b									1	1	1		1
8a		1											
8b								1	1		1	1	
9a		1						1				2	
9b													
10a													
10b													
11a								1	2		1		
11b													
12a										1			1
12b													
13a													
13b													
14a		1											
14b													
15a	1	2		1								1	
15b													
16a	1	2	2						1	2	1		2
17a													
17b													
a Sum	**6**	**9**	**5**	**1**	**1**	**1**	**0**	**4**	**3**	**4**	**3**	**4**	**4**
b Sum	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**2**	**2**	**2**	**2**

Note. An *a* signifies a novice response and *b* signifies an expert response (see Table 3).

Table 26

*Sample of Expert/Novice Codings Relevant to Units-of-Measure Analysis Strand (**KoST**)*

	Pre-Intervention (Pretest)			Intervention (Teaching Episode 1)					Post-Intervention (Posttest)		Post-Intervention (Follow-up)		
	Q6	Q7	Q9	Q5	Q7	Q8	Q10	Q11	Q7	Q9	Q6	Q7	Q9
1a	1	1	1							1		1	
1b									1				
2a	2	1	1	1		1					1		
2b									1	1		1	1
3a													
3b													
4a													
4b													
5a													
5b													
6a													
6b													
7a-													
7a													
7b	1									1	1	1	
8a		1	1										
8b									1	1			1
9a	1	1			1								
9b													
10a													1
10b							1		1			1	
11a					1								
11b							1		1		1		
12a				1						1			
12b													
13a					1		1		1				1
13b													
14a	1	1											
14b													
15a												1	
15b													
16a	1	2	1						1	1	1	1	1
17a													
17b													
a Sum	**6**	**7**	**4**	**2**	**3**	**1**	**1**	**0**	**2**	**3**	**2**	**3**	**3**
b Sum	**1**	**0**	**0**	**0**	**0**	**0**	**2**	**0**	**5**	**3**	**2**	**3**	**2**

Note. An *a* signifies a novice response and *b* signifies an expert response.

reveals similar trends for expert/novice frequencies during the TEs for Jackie and the other group members currently being discussed. The intervention was designed around discovery learning, so PSTs progressed at their own rate. By the time the posttest was given, the PSTs had experienced three TEs, and they had multiple opportunities to refine both their CK and KoST.

There are two emergent findings from TE 1 relevant to the ↑CK - ↑KoST relationship under investigation. The first involves Jackie's inability to diagnose the student's error in the TE. It was the same problem she faced in question 9 on the pretest, which she performed better on. For the TE, she had little problem agreeing with the student's incorrect method involving measuring perimeter with square units. Her wrong diagnosis was based on the fact that she had incorrectly calculated the perimeter of the irregular shape earlier in the problem. Her feeble CK about perimeter and units led her to agree with a student's erroneous method, which resulted in a lost opportunity for a successful intervention. Later on in the TE, when it became evident to her that the student's method was wrong, she had another opportunity to apply her KoST when suggesting how she would follow up with that student's entire class. If you read her lengthy response without being aware of her previous struggles, one might be impressed with her suggestions of bringing up the problem along with the student's method for class discussion, using the Shape Builder microworld (MW) to display the irregular figure in front of the class, and then having students provide reasons why they agreed or disagreed. Jackie did seem more concerned with promoting understanding than simply dismissing the student's method and showing the correct answer. Her approach earned a couple expert codes; however, her desire to promote understanding proceeded no further than her good-sounding instruction strategies. There was no mention of the source of the student's erroneous method (i.e., using square units to calculate area), nor was there any suggestion of reviewing important concepts regarding area and perimeter. In other words, Jackie's well-intentioned response to the class fell short because her CK regarding these concepts were still unorganized and unable to properly inform her KoST. The CK-KoST interactions were very similar for TEs 2 and 3. On her own, Jackie was not able to advance her CK very far during TE 2 and 3. Jackie increasingly explored more about the various misconceptions on her own – especially through the MWs, but she often became distracted with side issues (e.g., finding the area of the footprint in TE 2) that kept her from fully deciphering the misconception so she could properly respond to the student. Through her writings and interviews Jackie indicated that she had gained knowledge "about these ideas" (i.e., CK) and "on how to help students" (i.e., KoST) through the small-group and whole-class discussions embedded within the TEs. Several other PSTs took occasion to share all they felt they had learned throughout the TEs.

Concluding the pre-intervention and emergent findings, there are four results that summarize how Jackie's weak CK affected her KoST: (a) She did not possess the necessary mathematical vocabulary to support explanations, (b) it (her weak CK) interfered with her ability to effectively diagnose student errors and misconceptions, (c) it limited her instruction/intervention to procedural methods and responses, and (d) it hindered her capacity to fully utilize the features and educational benefits of instructional technologies (e.g., microworlds). As testimonies have indicated, the various activities contained within the TEs helped to improve the current status of Jackie, and the others in the ↑CK - ↑KoST group, to where they performed much better on the posttest.

↑*CK* - ↑*KoST relationship, post-intervention.* It was just described how a PST's impoverished CK can and does affect the usefulness of their KoST. What about when these knowledge types appear to mature together? How do the increases interact? The post-intervention improvements in both CK and KoST of the members in Jackie's group have been presented (e.g., Tables 13, 14, 16, 25, and 26), and illustrated (Figure 37), but how and in what ways did they occur? Did they occur in conjunction with each other or were there times of disconnect (i.e., CK improving with KoST lagging behind). There was evidence from the first problem on the posttest that Jackie's, and others', CK had not only increased but that it had also changed. On the pretest when Jackie and others ran into problems that were unfamiliar to them, they would either leave them blank (e.g., PST 5 and problem 4) or what they wrote was incorrect and/or unrelated. On the first problem of the posttest, Jackie began her response with, "I don't know how to do this problem, but . . . ;" however, she continued to work on it and actually got the correct answer. She attempted to solve the problem through a conceptual approach, but in the end resorted to a procedural, formula-based solution. Jackie's increased level of confidence was evident by the comment she wrote at the end of the posttest, "Yay Mr. Kellogg . . . I understood all of them!" Although her actual understanding will be shown to still be incomplete, her self-professed confidence was due to a more stable CK of basic area and perimeter facts and concepts. For example, Jackie (and others) exhibited a new awareness of the discreteness and defining characteristics of linear and square units (see Figure 38). This aspect of her improved CK allowed for better clarity and mathematical vocabulary while unpacking and explaining her ideas. It also facilitated more conceptual solution methods.

Similar to the pretest, problem 3 on the posttest had the PSTs find the perimeter

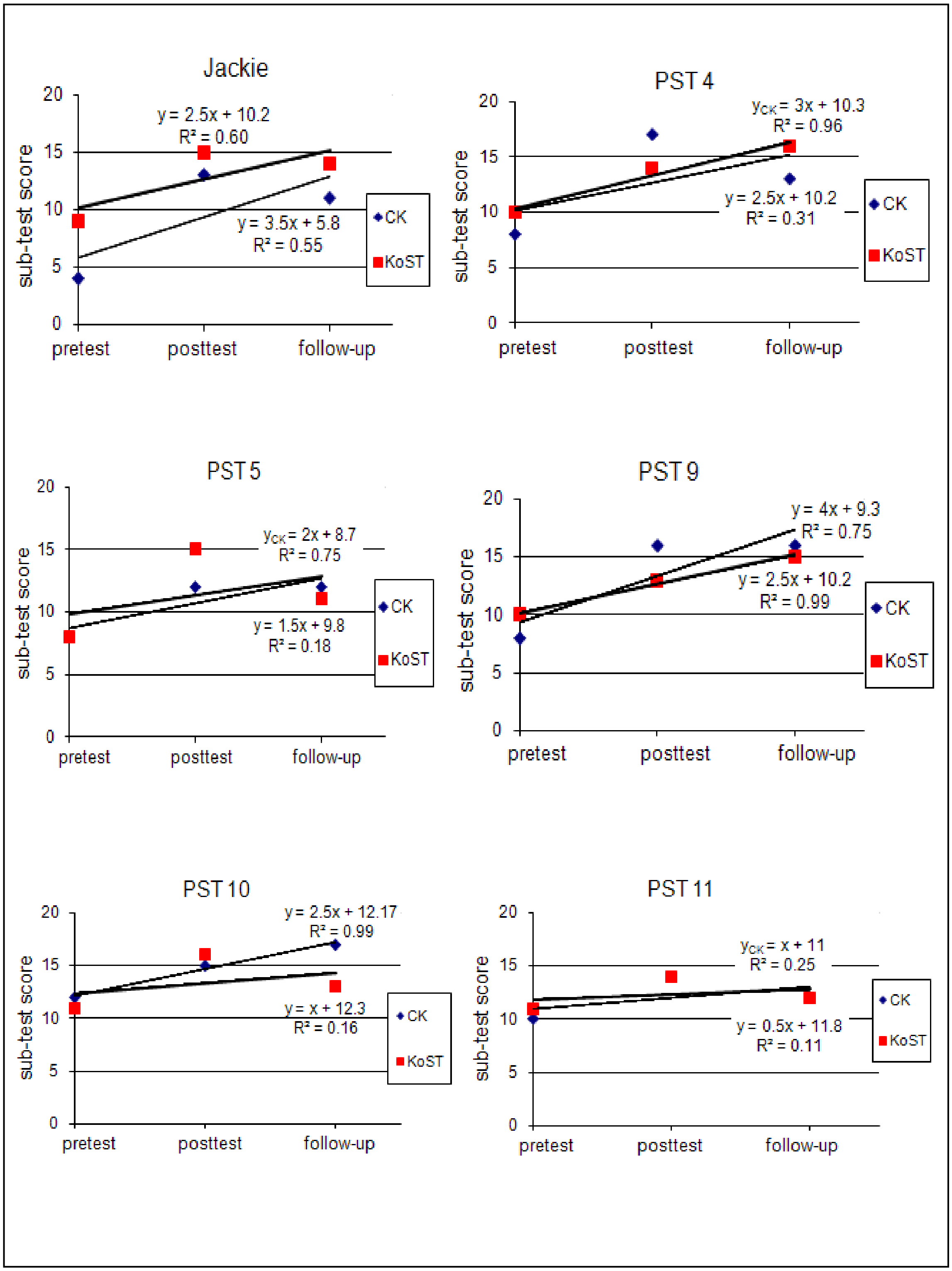


Figure 37. Regression lines and equations for change in case subjects' CK and KoST.

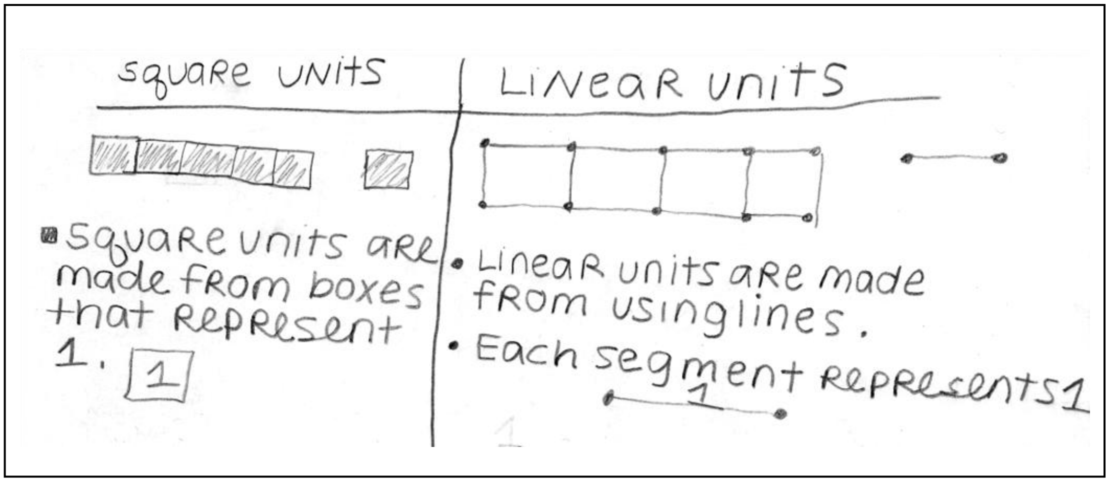


Figure 38. Jackie's posttest explanation of square and linear units.

and area of an irregular figure. On the pretest, Jackie tried to apply formulas to find the perimeter and area and was unsuccessful on both. On the posttest, she focused on the linear units when finding perimeter and on the square units (or the "squares on the inside of the shape," as Jackie called them) when finding area. She calculated both correctly, and her accompanying explanations included helpful diagrams with meaningful dialogue. Jackie's increased understanding of and attention to units of measure also contributed to a more successful handling of area-perimeter relationships. Instead of trying to describe the various relationships presented within the problems (e.g., fixed and direct) with just words, as she did on the pretest, Jackie now supported her responses with ample diagrams. On the pretest, she only provided one (rather vague) diagram while explaining her thoughts and ideas. For the posttest, however, Jackie included 19 appropriate diagrams. That awareness of the importance of including representations when explaining mathematical principles and relationships showed a significant increase in her KoST.

Jackie earned an "acceptable" score of 3 (see Appendix H) for each KoST problem on the posttest. She successfully diagnosed all five of the erroneous student claims and/or solution methods. Jackie not only identified the student's errors in the problems, which in and of itself could result in higher rubric scores and greater frequencies of expert codes assigned, but now her responses were much more organized and addressed concepts central to the problem. For example, on the pretest when a student suggested finding the perimeter of an irregular figure by counting the square units around the inside border of the shape (i.e., Problem 9), Jackie said the method was wrong. Her subsequent response to the student was shallow and involved a basic review of how to find area and perimeter, but included no mention of the appropriate unit for each measure. That would have been meaningful, since the student was using square units (a 2-dimensional concept) to find perimeter (a 1-dimensional concept). A similar problem on the posttest (#7) involved a student (Jose) who was asked to draw a rectangle with a perimeter of 24 units. Figure 39 contains the student's response. Every PST indicated that the student was incorrect, and most (including every member of the ↑CK - ↑KoST group) indicated Jose's primary confusion involved linear and square units. That represented a more powerful CK, and the PSTs' responses to the student benefited because of it. Jackie was again representative of her group, and her intervention with Jose

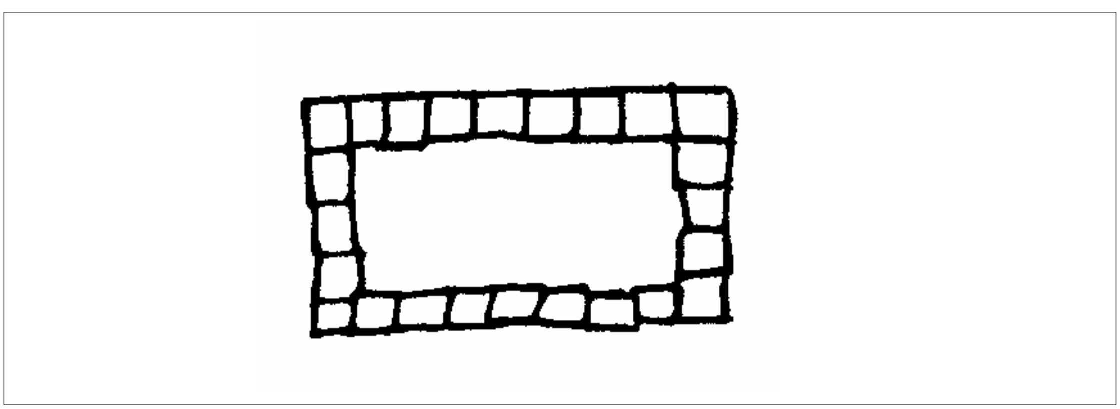


Figure 39. Student's constructed response for a figure with a perimeter of 24 units.

involved: (a) asking him "how he got his answer and why he chose to do it that way," (b) explaining "the difference between linear and square units," and (c) having Jose then find the perimeter of the shape he drew so that he would find out it has a perimeter of "28 linear units." That response characterized a much more classroom-useful KoST, and based on comments made during her second interview, it appeared her KoST regarding these concepts had benefited from an increased CK. Her response regarding Jose's thinking was telling:

> He was thinking that way because those squares are on the outside of the shape, and that would be perimeter. It's the same thing I did at first. It's the same exact thing, and that's why it hit me. I think that's why I knew, because I thought oh, that's what I did.

This new CK-KoST partnership was also evident when dealing with student thinking about perceived relationships.

Problem 6 on the posttest will conclude the findings regarding the ↑CK - ↑KoST group. It addressed the direct relationship misconception, which proved to be relatively troubling to the PSTs. Responses to this misconception have been examined repeatedly throughout this study, and on the pretest the members of Jackie's group handled it poorly. They either agreed with the student (as Jackie did) or they disagreed without providing any counterexamples or meaningful follow-up with the student. Facing it again in TE 3, Jackie initially struggled with the relationship, but by the end seemed to reconcile the student's erroneous claim. TE 3 apparently addressed the majority of the PSTs' shortcomings regarding the direct-relationship misconception to the point where on the posttest their CK had grown from "unacceptable" or "inferior" to "acceptable" (see

Appendix H). That was evident by the fact that many in the group, including Jackie, illustrated with a counterexample why a smaller perimeter will not always produce a smaller area. Jackie, and others, also displayed a deeper understanding of this relationship by recognizing the condition that the student's claim could be true under the right conditions. This CK provided the basis for a meaningful response to the student. Jackie's writings appeared confident: "First, I would ask her to prove her theory to me – providing more than example." Here Jackie acknowledged a common tendency of students to over-generalize after seeing only one example of a mathematical relationship. Jackie continued, "I would then propose her theory to the class and have the class decide if her theory is right or wrong." This student-centered approach was geared towards understanding, rather than simply disproving the student or getting the right answer. Jackie was asked during the second interview about her apparent new level of confidence displayed on the posttest regarding this problem:

> Well, I hit the thing where she has to provide more than one example. You know how before we were saying that it would be a pretty absolute statement for the student to make their claim with only one example. Then I would propose her 'theory' to the class and have them play *devil's advocate*. This time I understand, so I felt I could do that. Now that I understand, I thought that would be a good way to go.

The increase in Jackie's CK had apparently rendered her formerly limited KoST into something meaningful to her and beneficial to students. An examination of Jackie's responses on the follow-up test revealed that these changes were not short-term.

One thing absent from Jackie's posttest (and follow-up) responses to students was the integration of MWs as an instructional tool. Problem 6 (as well as 7, 8, and 10) involved misconceptions, or erroneous claims, that could have been disproved and then explored effectively with the assistance of either of the MWs used in this study; however, references to the MWs were very rare in the PSTs' post- and follow-up test responses. That was most likely due to time constraints. The question at this point in the study was whether the PSTs possessed the CK and the KoST to appreciate and effectively use the MWs as an instructional tool. Part of that question was answered on the posttest. The last question on the posttest asked the PSTs, "For which of the ten problems you just completed would a MW had been useful. Please explain how or why." As stated above, problem 6, 7, 8, and 10 were anticipated results. Jackie mentioned 6b, 7, and 8b. The "b" signified she would use the MW in the part of the question that would compliment her explanation of the student's thinking – more evidence of a maturing KoST.

The changes that occurred in these PSTs' CK and KoST from pre- to post intervention have been described, along with how they appeared to interact. All four of the earlier findings regarding the impact of Jackie's weak CK need to be modified to reflect how a more robust CK had influenced her KoST: (a) It supplied the necessary vocabulary to enhance her explanations, (b) She was much more capable to consistently diagnose student errors and misconceptions, (c) Her explanations now included multiple entry points and tended to focus on conceptual approaches, and (d) It increased her awareness of the benefits of instructional technologies (e.g., microworlds) to help struggling students. In conclusion, the proposed ↑CK - ↑KoST relationship did appear to behave in many of the ways anticipated by the researcher. There appears to be a mutually beneficial interaction between advances in CK and KoST.

CHAPTER 5

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

This study examined the levels of content knowledge and knowledge of student thinking related to area and perimeter of an intact group of preservice elementary teachers' within a framework involving anchored instruction incorporating geometry microworlds. In particular, it focused on their understandings, misconceptions, written and verbal explanations of that knowledge, and achievement on written area and perimeter tests – within the context of a mathematics methods course. In short, this study sought to: (a) further understand preservice elementary teachers' (PSTs') cognitions of area and perimeter and how they change and develop through planned intervention, (b) examine the interplay between PSTs' content knowledge (CK) and their knowledge of student thinking (KoST), and (c) develop a form of anchored instruction involving web-based microworlds designed for exploring area and perimeter. That framework focused on situated problem solving and provided a learning environment for both individuals and cooperative groups, with a goal of influencing the PSTs' CK and KoST.

This chapter contains three sections. The first section presents a summary of the study's findings. The second section describes the conclusions derived from highlighted research findings, and is organized around this study's research question(s). The third section discusses the implications of the research findings for teachers, teacher educators, and future research.

Summary of Findings

This summary is comprised of two main sections. The first section provides a comprehensive look at findings related to all PSTs. This will involve the two main strands of inquiry used throughout the study (i.e., units of measure and perceived relationships). The second focuses on the four case subjects and provides individual learning trajectories, involving: (a) Their knowledge prior to any intervention, (b) Their reactions during the intervention, and (c) The changes in their knowledge following the intervention. These findings taken together addressed this study's research questions.

The primary research question examined by this study was, *"In what ways do preservice elementary teachers' (PSTs') content knowledge and pedagogical content knowledge, related to area and perimeter, change as a result of experiencing anchored instruction integrated with web-based microworlds, designed for the investigation of area and perimeter?"* In particular:

1. What is the PSTs' content knowledge regarding area and perimeter prior to involvement in the teaching episodes?
2. What is the PSTs' knowledge of student thinking regarding area and perimeter prior to involvement in the teaching episodes?
3. How does PSTs' content knowledge regarding area and perimeter change, if at all, during the course of this study?
4. How does the PSTs' knowledge of student thinking regarding area and

perimeter change, if at all, during the course of this study?

5. In what ways, if at all, is the PSTs' knowledge of student thinking regarding area and perimeter related to their content knowledge of those same concepts?

PSTs' Pre-Intervention CK and KoST: Research Questions 1 & 2

As anticipated, prior to intervention the PSTs' KoST pertaining to area and perimeter was relatively underdeveloped. KoST is an application of one's CK, and each PST possessed an incomplete CK regarding these concepts. Because of the important role CK plays in the organization of KoST, greater emphasis was placed on the analysis and reporting of the PSTs' CK in order to understand the quantity and quality of their CK and their lack of pre-intervention KoST.

General CK Regarding Area and Perimeter

Although area and perimeter are used for different applications, they do have similarities. It is these similarities that make the concepts of area and perimeter susceptible to confusion. If someone possess an incomplete or strictly procedural knowledge of area and perimeter, then it is understandable why they could confuse the two. When considering rectangles (the primary shape discussed in this study), determining area and perimeter involves calculations with lengths of sides. A conceptual understanding of area and perimeter better equips both the student and teacher with the knowledge to more consistently perform the correct measurement. Although each measure involves a calculation with lengths of sides, area and perimeter also require attention to their appropriate unit (i.e., linear or square). These concepts are intrinsically linked, and a PST with a profound CK and KoST realizes the importance and value of incorporating linear and square units within discussions involving area and perimeter.

Distinguishing between area and perimeter. Early on in the study it was apparent that most of the PSTs possessed a procedural knowledge of area and perimeter. The majority of them seemed to equate "teaching" about area and perimeter with describing a basic procedure for finding their measure. Most PSTs were bound, even handicapped, by a dependency on formulas. The result of which was a "how to" approach. They seemed completely unaware of the various misconceptions students encounter when working with area and perimeter. Prior to the pretest, the PSTs were asked, "What do you think elementary students may find difficult regarding the learning of area and perimeter?" The 12 responses were varied, but the vast majority of them (9 out of 12) were along the lines of "students would most likely confuse the two," (Larry), and "have difficulty differentiating what formula to use" (Jackie). In contrast, Brianna and Grace touched on difficulties that went beyond a surface-level answer. Although Grace's responses included aspects of conceptual understanding, the majority of PSTs indicated that "getting the right answer" would be the primary source of difficulty for students, in contrast to understanding the concepts.

CK Regarding Units of Measure

The importance of possessing a conceptual understanding of linear and square units cannot be overstated. The unit of measure functions as a conceptual bridge connecting an object and the number used to represent its size. Hiebert (1981) states, "The concept of a unit is a central, unifying idea underlying all measurement" (p. 38). As reported by research with school students (Chappell & Thompson, 1999; Kamii, 2006), it was difficult at times in this study to distinguish if the PSTs were confusing area and perimeter, linear and square units, or both.

Inattention to units. Most of the PSTs addressed concepts of area and perimeter without any discussion about their appropriate units. This oversight contributed to PSTs confusing area with perimeter. The first question on the pretest asked the PSTs to, "draw a polygon that has a perimeter of 24 units" (on a grid that was provided). Eight out of 12 PSTs offered a response that addressed, to different degrees, concepts related to area. Likewise, insufficient attention to units resulted in several of the weaker-performing PSTs (e.g., Jackie and Larry) struggling with various aspects of irregular shapes (especially perimeter). These PSTs often attempted only procedural methods (typically involving a formula) to find the area and perimeter of irregular shapes. Even the higher-performing PSTs (e.g., Brianna and Grace), although more mathematically accurate with their responses, were also very procedural in their approaches to finding area and perimeter. A lack of CK regarding units of measure hindered the PSTs' ability to coherently explain concepts related to area and perimeter.

Ability to explain and illustrate units of measure. Mathematical procedures, although effective at producing answers, typically do not inherently convey conceptual understanding of a construct. The area formula for a rectangle is a prime example of this. Instead of actually explaining the distinguishing characteristics of linear and square units and providing classroom-useful and practical examples, most PSTs (including Larry and Jackie) simply explained how they are used (i.e., linear units are used with perimeter and square units with area). The PSTs' realization of the importance of connecting area with its appropriate unit was revealed in question 6 of the pretest when only four out of 12 PSTs (one case subject) correctly identified "sq. cm." (or cm^2) as the appropriate unit missing from a student's area calculation. PSTs possessing a stronger mathematical knowledge (e.g., Brianna) seemed better able to coherently and accurately distinguish between linear and square units. Overall for those PSTs, their pre-intervention CK regarding these concepts was sufficient to get correct answers, but it was procedural in nature and application. A strong mathematical acuity was not sufficient to facilitate conceptual explanations or the illustrating of ideas regarding units of measure.

Utilizing drawings. An important aspect of a teacher's CK is the ability to explain concepts in meaningful ways (i.e., their explanatory framework) using effective communication. Incorporating suitable drawings is one important aspect of a successful explanation. On the survey questionnaire, the PSTs were asked, "What would you do to help future students better understand area and perimeter?" Although 9 out of 12 PSTs made reference to drawing a picture or bringing in objects for display, only four provided any type of drawing to represent their ideas. Drawings were overlooked while addressing basic as well as more obscure ideas regarding area and perimeter. Most PSTs indicated that conceptualizing and explaining linear and square units was difficult for them; however, only one of the 12 PSTs even attempted to draw a figure as a means to help visualize and/or explain these difficult concepts. Even when the PSTs were struggling to express meaningful thoughts and ideas, as evidenced by their scored responses, they frequently would not resort to a drawing to either help themselves visualize the concept or aid in the effective communication of their ideas. Out of 48 potential opportunities on the pretest (12 PSTs × 4 problems), only five drawings were provided that accompanied a meaningful and correct response. This pattern was also evident when the PSTs tried to explain their thinking regarding certain perceived relationships between area and perimeter. Although 8 of 12 PSTs did provide diagrams to support their explanations, only two of them were suitable for classroom use. On some problems it appeared the PSTs' limited CK left them ill-prepared to construct a meaningful drawing, while other times the PSTs were careless and drew rectangles that were not to scale and thus were not helpful in facilitating a correct response. In all, the ineffective use or lack of drawings to assist in problem solving or to clarify explanations was evidence of CK that lacked a well-developed explanatory framework.

CK Regarding Perceived Relationships Between Area and Perimeter

The perimeter and area of a figure are two different measures. The perimeter is a measure of the length of the boundary of a figure, whereas the area is a measure of how much space a figure occupies. In the case of a rectangle, the calculations of both measures are related to the sides of the figure. These similarities provide the settings for two classic misconceptions involving the area and perimeter of a rectangle: (1) That increasing the perimeter of a rectangle will always increase its area (i.e., the *direct-relationship* misconception), and (2) Rectangles that have the same perimeter measurement will also have the same area, and vice versa (the *constant-relationship* misconception).

When presented with a problem on the pretest containing the direct-relationship misconception, four out of 12 PSTs (including Larry and Jackie) indicated that the student's erroneous claim was correct. Their explanations tended to be based on the incorrect assumption that increasing the perimeter of a rectangle must increase both dimensions and thus the area. Another five PSTs, although they disagreed with the student in the problem, were unable to provide an appropriate counterexample. All nine of these PSTs offered a trivial examination of the student's claim. That reflected low levels of thinking regarding this misconception (Ma, 1999). Only three PSTs (including Brianna) successfully examined other possible relationships beyond their initial explanation. The PSTs' treatment of the fixed-relationship misconception resulted in similar, mostly unsuccessful, results. Problem 10 on the pretest (p. 245) provided an opportunity for PSTs to share their understandings regarding different sized rectangles (i.e., different areas) with the same perimeter. The fact that only four out of 12 expressed an awareness of a common student tendency to erroneously think that equal perimeters will result in equal areas indicates the majority of the PSTs were not aware of the fixed-relationship misconception. Another relationship contained within this problem was that squares are special rectangles. No PSTs acknowledged this hierarchical relationship or considered it relevant enough to discuss it with the student. The PSTs' pre-intervention CK was not sufficiently organized to enable them to consistently understand and diagnose student thinking or appropriately respond to student difficulties.

Pre-Intervention KoST

On the pre-study questionnaire, the majority of the PSTs indicated that "getting the right answer" would be the primary source of difficulty for students when studying area and perimeter, in contrast to understanding the concepts. They were concerned that most elementary students would have difficulties with all the formulas. Before the intervention, many PSTs indicated a lack of confidence in mathematics and having limited experience in diagnosing student thinking related to mathematics; therefore, when faced on the pretest with a problem-solving situation involving erroneous student thinking, the majority of PSTs' in this study tended to focus on solving the problem (i.e., finding the answer), to the neglect of diagnosing the hypothetical student's thinking.

PSTs who focused solely on the *rightness* of the students' work almost always failed to adequately explore the mathematics surrounding the problem or the misconception (i.e., look for a counterexample), or properly diagnose the students' claims. This lack of comprehending the students' thinking resulted in very few PSTs indicating they would allow students opportunities to personally work through the various mathematical concepts of a problem and no PST displayed the wherewithal to encourage students to investigate further with manipulatives or technology.

Summary of Emergent Findings: Impact of Intervention

These findings came primarily from the three teaching episodes (TEs), and include discussing the impact of the microworlds (MWs) upon the PSTs' CK and KoST.

The Teaching Episodes

TE 1: Units of measure. The PSTs performed relatively well with the CK questions related to TE 1. Out of 12 PSTs only Jackie and one other PST did not initially conclude the student's method to be incorrect. However, their inability to explain that knowledge along with a limited capacity to apply their CK and adequately address the struggling student (Justin) in the TE resulted in much higher novice frequencies related to KoST. Jackie's use of the words "boxes" and "sides" instead of square and linear units was seen often within the findings as a dividing line between more expert responses. The majority of PSTs avoided discussing important terms, such as linear and square units, and how Justin was incorrectly using square units to measure perimeter. While many of the PSTs' explanations continued to be weak, their suggested interventions (an aspect of their KoST) began to show improvement. Several PSTs were creative in offering alternative illustrations to help Justin better understand perimeter (e.g., fences, pieces of string), but only two PSTs (one being Brianna) actually discussed the most likely cause of Justin's incorrect method, his confusion with linear and square units, and how one measures perimeter and the other measures area.

TE 2: The fixed-relationship misconception. For several PSTs (e.g., Jackie), their CK regarding certain facts and concepts related to relationships between area and perimeter, specifically the fixed-relationship misconception, had increased; however, their ability to clearly explain their knowledge had not developed to the same extent. As was the pattern with most unsuccessful responses in this study, no sketches were provided from the three PSTs who indicated they did not know how to solve the problem. TE 2 contained specific findings related to the PSTs' focus while diagnosing students' methods, and offered a prime example of how a wrong focus by PSTs can result in poor diagnosing of student misconceptions and missed opportunities to address those difficulties. Findings showed that the PSTs who struggled most throughout TE 2 were also the ones who excessively focused on trying to find the area of the footprint, and as a result paid too little attention to dissecting Tommy's method and the misconception behind it. It appeared that several PSTs (e.g., Jackie) had difficulty translating the student's erroneous method into a concept or rule that could be verified or disproved. It was apparent that those PSTs who were not able to explore the problem deeply on their own also had difficulty responding to the fictitious student in meaningful ways; whereas, those with a better understanding of the mathematics surrounding TE 2 (e.g., Brianna) were more confident and adept at suggesting how best to engage both the student and the entire class in a discussion of the misconception. Overall, a preoccupation with finding what many PSTs judged as "the answer" to TE 2 not only hindered their ability to properly diagnose and address the student's thinking, but it also limited their meaningful interaction with the Shape Builder MW.

TE 3: The direct-relationship misconception. It was common with many PSTs (especially the poorer-performing) that while examining the various student claims in this study, they apparently believed if enough examples were presented then the claim can be either proved or disproved. A limited background in mathematics led most of these PSTs to where they viewed the role of examples as a way to *prove* something, rather than just an *illustration* of a numerical relationship. They did not, or possibly cannot, appreciate the need for a mathematical argument in such cases. Overall, the PSTs attained higher levels of understanding (Ma. 1999) regarding the misconception that there exists a direct relationship between perimeter and area. Table 23 (p. 343) shows that while only one PST achieved a Level 1 understanding (out of 4) during the pretest, 10 out of 12 PSTs reached at least Level 1 during TE 3, including three Level 2s and one level 3.

The PSTs' interactions throughout the study's intervention provided moments of pedagogical clarity – even for those who initially struggled. A comment made by Jackie during a teaching episode involved her belief that it might help students resolve area and perimeter conflicts if the concepts were studied simultaneously. Her view displayed relative expert pedagogical content knowledge, shared by several researchers (Chappell & Thompson, 1999; Hiebert & Lefevre, 1986; Simon & Blume, 1994a).

Impact of Microworld Usage

During the TEs there were many comments such as, "After I used the microworld, I saw the error in the student's thinking" that indicated various forms of learning occurred while PSTs interacted with the MWs. Table 21 (p. 317) reveals the case subjects' usage of MWs ranged from a means to confirm CK to a tool to investigate the student's thinking. When asked in TE 1 whether the MW was helpful in deciphering the focus problem, one PST wrote, "Definitely yes! I understand why Justin shaded in the squares and counted them to find the perimeter. As I drew the figure in the microworld, I was beginning to think I was thinking the way he did!" During TE 2 PST #11 commented that the "Compare Areas and Perimeters" feature of Shape Builder helped her realize "that she, like Tommy, was over-generalizing that the 18" string could have only one area. I think the string distracted me from realizing sooner that perimeter does not determine area." These quotations are just a few of the many examples of how the PSTs' KoST grew as a result of interacting with the MW and also how they were gaining a vision for how to use the MW as a tool to help diagnose student thinking. Findings related to how the PSTs proposed using the MWs with the students presented in the TEs, as well as their classmates, revealed mixed results.

For the first teaching episode (the easiest of the three) the vast majority of the PSTs (11 out of 12) indicated they found the microworld helpful to their understanding of the problem as well as Justin's thinking. They also explained that they would use the microworld as an instructional tool in a whole-class discussion of Justin's misconception. A similar majority (10 out of 12) indicated they believed classroom students would benefit from personally interacting with the MW in a more controlled setting. However, an unexpected trend developed as the mathematical content of the teaching episodes got progressively more difficult and the hypothetical students' thinking was increasingly more elusive. Although the number of PSTs who indicated they learned with and/or saw benefits of personally interacting with the microworlds was a strong majority (8 for TE

#2 and 11 for TE #3), fewer (five from TE #2 and six from TE #3) said they would incorporate the microworlds when instructing future students about the concepts presented in the TEs, even though the same PSTs admitted those future students would most likely possess similar misconceptions as the hypothetical students presented in the teaching episodes. Apparently, the majority of PSTs felt the microworlds were a valuable learning tool for themselves but not for their future students. This trend may be partially explained by the following quotation given by PST #3 near the end of TE 2: “Interacting with microworlds still seems slightly foreign to me, since it was in this class that I received my first opportunity to use an applet. I have found the applets helpful in supporting or refuting theories proposed by students and myself.”

The summary of findings about this study’s intervention will conclude with a quotation from one of the higher-achieving PSTs. During her second interview, Grace provided what she perceived as the value of the area and perimeter misconceptions studied during the intervention (i.e., the focus of this study’s anchored instruction):

> Working through some examples of what kids were thinking when they figured out the problems, and just having all those examples, I think was very beneficial. Instead of just learning the concepts, and how to do them, you need to be challenged. You’re going to be faced with this in your classroom; how are you going to deal with it? That’s what I got out of it – was how to deal with the way the kids might think, and how they might be thinking.

Summary of PSTs’ Post-Intervention CK and KoST

The findings presented in chapter 4 related to research questions three and four were quite extensive. To facilitate cohesion, concise summaries highlighting post-intervention CK and KoST findings will be presented. Readers interested in deeper discussions of any findings presented here are encouraged to reference chapter 4.

Descriptive Findings

The posttest mean of 28.25 represents a 33% increase over the pretest average score of 21.25 (range = 0-40). The entire class decreased their total number of unacceptable scores (0s, 1s, and 2s) from 74 on the pretest to 35 on the posttest. There were seven 4s (model scores) assigned on the pretest, however 19 on the posttest. There were fewer novice codes assigned and the number of expert codes increased by over three-fold (from 10.3 to 31.3). Of all the PSTs, Jackie's knowledge levels showed the greatest positive change. The relatively low frequency of code 7*b* (i.e., the ability to generate appropriate representations) assigned to the PSTs responses revealed a notable gap in their KoST, because they apparently did not realize the importance of diagrams presenting conceptual explanations of mathematical concepts. This tendency was repeated by a very low rate of code 12*b* (i.e., the appropriate use of manipulatives) and the total absence of code 13*b* (i.e., the appropriate integration of technology to promote understanding) on any test. The PSTs' oversight of incorporating technology is somewhat troubling given the tremendous focus placed upon the two microworlds used in this study.

Change in PSTs' CK: Research Question 3

Positive change was seen quantitatively. Table 14 (p. 256) illustrates that the CK for 9 of the 12 PSTs increased from pretest to posttest. The features of the PSTs' CK also changed. Table 16 (p. 261) reveals how the CK of all 12 PSTs experienced increases from pretest to posttest in the number of expert-like characteristics assigned to their written responses. The PSTs' amount and organization of facts and concepts grew and became clarified throughout the study. PSTs showed a greater propensity to include and discuss the correct unit of measure when solving area and perimeter problems. This was evident when working with irregular shapes. On the pretest, confusion regarding what a linear unit was caused several PSTs to incorrectly calculate the perimeter of an irregular figure. That difficulty was almost nonexistent on the post and follow-up tests. Conceptual approaches aided in gaining new knowledge about finding area of irregular shapes and how the focus should be on counting square units instead of formulas.

Procedural versus conceptual knowledge. There was a noticeable shift in the type of CK being displayed, from a procedural, formula-based approach to a more conceptual one. Procedural CK dominated pre-intervention thinking; however, a slow transition to more conceptual approaches began to surface during the teaching episodes and was much more evident during the post and follow-up tests. For example, Brianna's strong mathematics background facilitated predominately procedural responses on the pretest, but during and after the intervention she was more prone to support her procedurally-correct responses with conceptual elements (e.g., she would discuss and illustrate units when explaining answers regarding her area and perimeter).

Ability to explain. Promoting understanding became equally, or in some cases more, important to the PSTs than simply finding the right answer. This new-found appreciation of conceptual understanding helped PSTs solve non-traditional problems like finding the area of a footprint, and more importantly facilitated more powerful explanatory frameworks. The explanations regarding relatively difficult concepts, such as linear and square units, grew in clarity and thoroughness as a result of the PSTs experiencing the interventions. A problem on the follow-up test required drawing a polygon that had a perimeter of 24 and then justifying that response. Six responses included justifications of their shape using language similar to, “outside edge,” “border,” and “line segments” for descriptions about perimeter. Three PSTs were even more precise by explaining that the perimeter of their shape could be found by counting the outside linear units. CK containing rich dialogue such as this was, for the most part, noticeably absent from the PSTs’ pretest responses.

Utilizing drawings. Further evidence of the PSTs’ improved ability to communicate their new-found CK was an increased use of classroom-appropriate drawings in the post- and follow-up tests that helped support an unpacking of the PSTs’ CK when explaining their ideas and solution strategies. Table 19 (p. 300) reveals an increased use of drawings following the intervention. Out of 48 potential opportunities (12 PSTs × 4 problems) to use drawings on the pretest, 16 (33%) drawings were attempted, but there were only five (10%) that accompanied a meaningful and correct response. The rate of drawings provided increased for the posttest. There were 72 reasonable opportunities (12 PSTs × 6 problems) to incorporate a drawing, 42 (58%) drawings were provided, and of those, 27 (38%) assisted in achieving a correct response. Use of drawings on the follow-up test increased very slightly (+2%). An apparent pattern in Table 19 was that certain PSTs tended to use drawings more consistently than others. For example, following the pretest both Jackie and Brianna began incorporating drawings in their responses on a more regular basis, whereas Grace and Larry did not. The use of drawings was not directly connected to performance. Grace was one of the top performers in the study, but barely ever used drawings to communicate her ideas, but

PST #6, another top performer, effectively used drawings on the post- and follow-up tests. Jackie only provided one (rather vague) diagram on the entire pretest to help support her explanations. For the posttest however, Jackie included 19 appropriate diagrams. These findings illustrate how the explanatory-framework component of the PSTs' CK had developed and matured.

CK Regarding Perceived Relationships. The PSTs' understanding of a rather elusive misconception (i.e., the direct-relationship between area and perimeter) grew as evidenced by their progressing within Ma's (1999) *Levels of Understanding* of that relationship. To do so they needed to be able to translate a student's erroneous solution method (or claim) into a mathematical relationship that could then be verified, disproved, or even both. The "both" aspect was a level of understanding that only a few reached (namely Brianna, #6, & #10), where the PSTs explored the various relationships in which a student's proposed method worked and when it would not. For the most part, the PSTs in this study simply stopped exploring after discussing their initial reaction. Many of these PSTs did not appear self-motivated to delve far beyond providing one possibility to the stated question, very often the same one they had given in the past similar situations. Instead of investigating the various possibilities surrounding this misconception, the majority would give the same, or a very similar, answer as they had previously and continued to operate within their *CK comfort zone*. Throughout the study, only one PST (#1) was not able to display some measurable increase in her understanding of the direct-relationship misconception.

Problem 10 on the follow-up test provided an opportunity for PSTs to share their understandings regarding different sized rectangles (i.e., different areas) with the same perimeter. Four out of 12 PSTs on the pretest expressed an awareness of a common student tendency to erroneously think that equal perimeters will result in equal areas, and three PSTs made the same acknowledgement on the follow-up test. This finding indicates the majority of the PSTs were still not perceptive to the fixed-relationship misconception even after intervention. Another relationship contained within this problem was that squares are special rectangles. No PSTs acknowledged this hierarchical relationship on the pretest, but on the posttest nine of the 12 PSTs included a 4 × 4 shape in their list of possible rectangles with a perimeter of 16. Only PST #5 specifically mentioned that "the square is a type of rectangle."

Changes in PSTs' KoST: Research Question 4

The pedagogical component of KoST made it slightly more challenging than CK to isolate, quantify, and describe how it changed during the study. In spite of that, findings showed that the PSTs' ability to apply their CK and appropriately address the shortcomings and misconceptions of students (i.e., their KoST changed in positive ways) grew within the context of this study, in different ways and to varying degrees.

Positive change was seen quantitatively. Table 14 (p. 256) illustrates that the KoST subtest scores for 9 of the 12 PSTs increased from pretest to posttest. The quality of PSTs' KoST also changed. Table 16 (p. 261) revealed how the KoST of all 12 PSTs increased in the number of expert-like characteristics assigned to their written responses from pretest to posttest. Precisely how the KoST changed was also discussed in great detail in chapter 4.

The evolution of most PST's instructional strategies was evidenced by, but not limited to: (a) an increased awareness of common misconceptions students have regarding area and perimeter, (b) a development and restructuring of their mathematical vocabulary (relative to the concepts in this study), (c) a realization of the value of discussing and illustrating individual units of measure when explaining area and perimeter concepts, (d) increased use of drawings when communicating ideas to students, (e) a movement away from procedural and teacher-centered interventions to more conceptual explanations and student-centered activities (e.g., PSTs showed an increased understanding of how and why to integrate MWs to help build conceptual knowledge), and (f) an increased focus on diagnosing student difficulties and less of an emphasis on solving problems and finding answers.

An interesting finding involved the PSTs' KoST and their thoughts regarding the perturbations purposely placed within several test problems. Several noted that certain aspects of various test questions (e.g., Figure 35, p. 318) should be changed or removed so as to "not confuse the students." However, the responses of the PSTs confirmed that it was those very aspects of the problems that served as a catalyst to promote intellectual struggle, reflection, and a new-found understanding regarding a certain concept. Apparently, several PSTs viewed such conflicts as too troublesome for elementary students, unknowingly failing to acknowledge the motivating nature of true problem solving. Similar "complaints" by the PSTs were not expressed while working on the scenarios presented in the TEs. Possibly the timed element of the tests, or the interviews, influenced the PSTs' beliefs regarding the value of such perturbations.

In summary, the planned intervention of this study appeared to play a role in the PSTs becoming more perceptive of subtly difficult mathematics involving area and perimeter (e.g., linear and square units and the fixed- and direct-relationship misconceptions) and better equipped to anticipate and address those difficulties with future students. The PSTs' CK and KoST showed signs of growth, albeit in varying quantities and qualities, after their involvement with the anchored instruction.

Case-Subject Summaries

Four case subjects were identified and examined in-depth to gain insights about the range of knowledge of PSTs in the class. Grace and Brianna represented PSTs with above-average cognitive and mathematical ability, and Jackie and Larry were representative of PSTs possessing average to below-average ability in mathematics and cognitive processes. The case-subjects' learning trajectories that follow involve: (a) Their knowledge prior to any intervention, (b) Their reactions during the intervention, and (c) The changes in their knowledge following the intervention.

Larry's Learning Trajectory

Larry's performance throughout the study was erratic. His CK regarding area and perimeter was sparse in amount and poorly organized at the beginning of the study. Initially, he displayed a rules-orientated approach to area and perimeter, an inability to consistently focus on the correct unit of measure, and a tendency to respond to superficial features of a problem. In addition, he struggled when asked to explain his responses. In fact, during interviews he would often contradict himself.

Larry's limited CK provided an inadequate foundation from which to support his KoST. He was ill-prepared to consistently construct meaningful and/or accurate drawings, which limited the degree to which he could respond to student difficulties. His overall CK and KoST prior to intervention can be characterized by a comment Larry made, "I don't know what I was thinking on this problem. I'm just kind of figuring it out as I go."

Larry often appeared confused or distant during discovery-learning sessions. For example, TE 3 allowed the PSTs to use either MW right from the start. I observed Larry open a MW, create the shapes presented in the focus problem (p. 136), and then stare at the computer screen for several minutes, occasionally glancing at the fist page of the TE. The scenario presented in TE 3 resulted in most PSTs exploring and testing hypotheses in the MWs, but Larry appeared to disengage when there was a need to address concepts he found difficult. When he was able to grasp the mathematical underpinnings of a concept, he rarely ventured beyond that knowledge. At times he appeared distracted by the MWs and wrote several times how he "figured things out better by hand." When he did use MWs in his responses, it was to permit him to view examples quickly so that he could efficiently arrive at an answer. He appeared to be content with getting what he thought to be "the right answer," and this aspect of his CK resulted in his responding to struggling students by attempting to guide them to get right answers.

Larry did not experience great success with the independent-learning component of the TEs. To encourage success during the TEs, it was necessary to continually prod and prompt Larry to continue to explore the concept beyond his initial shallow understanding of the concept(s). The majority of Larry's explanations were often tied to formulas and procedures, and involved teacher-centered behavior. Larry's responses would incorporate instructional aids at times; however, he would often utilize the same ones (e.g., grid paper), and many times the reason for incorporating the aid was unclear. Overall, he placed greater he placed greater precedence on completing the problems and generating answers than on gaining personal insights and knowledge necessary to develop conceptual understanding within future students.

As is common with novice teachers (like Larry), they tend to respond to faulty student thinking by simply reiterating what they know about the topic, rather than investigating the student's thinking and what lead to the erroneous claim (Fuller, 1996; Livingston & Borko, 1990). Larry's ability to understand and then respond to student's misconceptions (i.e., his KoST) was limited by his insufficient CK. Progress made in relation to connecting mathematical concepts in meaningful ways tended to be short lived. Throughout the study he struggled with the mathematics as well as with explaining his ideas. In addition, Larry showed little to no improvement in how he contemplated and addressed student thinking

Grace's Learning Trajectory

At the onset of the study, Grace appeared to possess above-average amounts of CK regarding various aspects of area and perimeter but struggled using it consistently to diagnose student thinking and therefore could not adequately address certain student misconceptions regarding theses concepts. Her strengths included an ability to carefully process information coupled with a strong desire to help future students understand mathematics. In contrast to Jackie and Larry, Grace did not become flustered after realizing her thinking was incorrect. Like expert teachers (Chi, Glaser, & Farr, 1988), Grace was able to carefully analyze a problem before and while solving it. Grace displayed this often. She would pause, reread the problem, gather her thoughts, explain where she had gone wrong and why, and then continue on with her work or explanation.

Throughout the intervention, Grace would often call me over to show me and/or inquire about her work with the MWs. She often explored beyond the basic ideas surrounding the TE’s focus problem, as will be described later while discussing TE 3. It appeared Grace’s CK and KoST grew, and became better organized, as a result of the intervention, even beyond the planned learning. She would ask clarifying questions that reflected a genuine desire to understand the concepts we were being addressed. She wanted to be prepared to teach students well. During the first interview, Grace appeared to know more than she would write in her responses. Once Grace became aware of how thorough communication was necessary to promote understanding of mathematical principles, her responses changed to include greater specificity. As her CK regarding area and perimeter misconceptions became more coherent and organized, she was better equipped to respond to student difficulties in pedagogically powerful ways.

Her desire to understand mathematical concepts did not end when class ended. During our first interview, Grace shared how she had “been thinking about the focus problem in TE 3 for the last couple days,” and that she figured out that “rectangles that have dimensions closer to being equal have more area.” Grace is of course referring to the idea that, for quadrilaterals, a square maximizes area. Grace was not generally satisfied with leaving mathematical conflicts unresolved. Her stated desire for her future students was for them to have a conceptual understanding of mathematics. That was apparent in the application of both her CK and KoST, for which their focus was to clearly communicate mathematical ideas so that students would understand them. The outcomes from the TEs provided empirical evidence that Grace was motivated by and benefited from exploring the student misconceptions presented in the TEs. She thrived within the discovery learning environment and her classmates reported profiting from having her in their cooperative learning groups.

Brianna's Learning Trajectory

Throughout the study, Brianna made good use of her strong mathematics background (e.g., she successfully completed Pre-Calculus), and was careful and precise in her problem solving. It was common for Brianna to quietly think over a question for 30 seconds before making, what was usually, an insightful comment. As one of the three top-performing PSTs (Grace and #6 were the other two), Brianna often provided coherent and thorough written responses, complete with accurate mathematics; however, prior to intervention she struggled when asked to illustrate and explain her ideas conceptually.

Brianna's pre-intervention CK was sufficient to get correct answers, but it was very procedural in nature and application. Her CK was sufficient to allow her to diagnose many of the student difficulties presented; however, her responses tended to focus on getting correct answers rather than on developing conceptual understanding. Prior to the intervention, Brianna was more focused on "how" than "why," which often produced insufficient interventions for students. This illustrated that her KoST was not at the same levels as her CK.

Throughout the teaching episodes there was a noticeable shift in Brianna's approach to viewing, doing, and explaining mathematics. She consciously made efforts to think more conceptually. Brianna would become very engaged in the mathematical challenges of the TEs. Her strong mathematics background continued to power her CK and allowed her to grasp every misconception within the TEs and to be very thorough and accurate in her prescribed activities. Her ample CK appeared to initially interfere with her ability to see the need to include diagrams to help students understand her explanations; however, the frequency of quality diagrams increased from TE 2 right through the follow-up test. That strong CK likely facilitated Brianna's propensity to control the learning environment which at times hindered her instructional recommendations from focusing on the students.

In all three TEs, Brianna indicated that she would direct the learning during the interventions (both with individual students and with a class). She often recommended having students investigate with the MWs, but with predesigned problems. Brianna thoroughly explored within the MW environments, often commenting on interesting nuances. For example, she wrote how she discovered that there are an infinite number of rectangles with different dimensions that could have the same perimeter. Her instructional strategies gradually evolved from teacher-centered, with students receiving instruction, to teacher-directed, where students participating more in their learning. Brianna appeared to benefit from being required, throughout the study, to communicate her mathematical understandings on a level appropriate for elementary students. Near the end of the intervention Brianna was exhibiting the greatest levels of expert-teacher qualities of any PST in the study. Brianna's CK and KoST, especially her explanatory framework, appeared to reach similar levels as her mathematical knowledge.

Jackie's Learning Trajectory

At the onset of the study, Jackie's CK regarding area and perimeter was fragile and disconnected. She was unable to consistently decipher whether problems were addressing area or perimeter, and was unaware of the importance of delineating such ideas as appropriate units of measure. Jackie's CK comprised a very rules-orientated approach, which left her unable to conceptually explain basic area and perimeter concepts or provide practical examples of them, other than how they are used (e.g., linear units are used with perimeter and square units with area).

Interview excerpts revealed that although Jackie was aware of certain aspects of the student misconceptions presented, her lack of CK impeded her ability to diagnose and appropriately respond to faulty student thinking. The fragile nature of her CK was evident as she would often change her initial answer when asked to clarify her thoughts. A reflective statement made by Jackie near the end of her first interview aptly summarized the struggles that she experienced prior to the study's intervention:

> I think my biggest problem is I just don't know why things are the way they are. I just kind of have this knowledge of formulas and a few concepts that I've learned here and there, and I think that some of them are mixed up.

Although Jackie's CK appeared to change and develop after repeated exposure to area and perimeter concepts, her KoST struggled to adapt throughout the intervention. Jackie had difficulty "thinking on her feet" and was often unable to thoroughly work through various mathematical scenarios, and that left her ill-equipped to effectively respond to student difficulties. Jackie's suggested student-interventions often focused on general ideas (e.g., clarifying area and perimeter), even when those ideas were not helpful in resolving the misconception at hand. Her choices of mathematical language often confused and muddied her attempts at explaining concepts to students – even those concepts she seemed to understand. Jackie indicated, and displayed, how interacting with the MWs deepened her understanding of area and perimeter concepts as well as how students think about them; however, she was not able to consistently perceive their relevance to the learning process or provide viable classroom uses for the MWs. Jackie would need repeated exposure and support to enable her to incorporate such tools into her future teaching.

Jackie did not appear to learn best on her own, but rather indicated several times how the small-group and whole-class sessions were very helpful. It was observed that when Jackie was engaged in conversation (e.g., interviews, cooperative work) about mathematical content and students' thinking, she was better able to clarify and present her understanding about the concepts being discussed. Her increased posttest score (115% increase over her pretest) was evidence of her effort throughout the study. Jackie made noticeable gains in her CK related to area and perimeter. These gains appeared to stabilize following the intervention. Jackie's intense desire to be a successful teacher also translated into moments of pedagogical clarity. For example, a comment made by Jackie during a teaching episode involved her belief that it might help students resolve area and perimeter conflicts if the concepts were studied simultaneously. Her view displayed relative expert pedagogical KoST, shared by several researchers (Chappell and Thompson, 1999; Hiebert & Lefevre, 1986; Simon & Blume, 1994a). Following the intervention, Jackie's recommendations for helping struggling students involved a context and level of student involvement that revealed her KoST was beginning to incorporate the ideas and practices that had been discussed during the TEs.

Conclusions

Previous research has shown that preservice elementary teachers (PSTs) have procedural and conceptual shortcomings regarding area and perimeter. The majority of that research focused on revealing and measuring such misconceptions; therefore, little is known about the underlying causes of these misconceptions, how they may interfere with preservice elementary teachers' ability to diagnose and address future students' difficulties, or what alternative instructional methods may help alleviate the area and perimeter misconceptions that PSTs have. This study sought to measure and describe the content knowledge (CK) and knowledge of student thinking (KoST) of an intact group of PSTs both before and after a planned intervention, and then examine possible relationships between their CK and KoST.

Regarding Pre-Intervention CK and KoST

Expert/Novice Differences

Preservice elementary teachers (including student teachers) are obviously considered novices. It was not surprising then that, prior to any intervention, the 12 PSTs in this study displayed many of the same novice tendencies reported in the literature. Researchers have found that the CK acquired by novice teachers is primarily procedural in content and application (Ball & Wilson, 1990; Borko et al., 1992; Fuller, 1996; Simon & Blume, 1994a). Similarly, the majority of PSTs in this study seemed to equate "teaching" about area and perimeter with describing a basic procedure for finding their measure. Most were bound, even handicapped, by a dependency on formulas; the result of which was a "how to" approach for teaching the subject matter. Their procedural CK resulted in a narrow KoST. Many PSTs indicated that "getting the right answer" would be the primary source of difficulty for students, in contrast to understanding the concepts. Their tendency to focus on the mathematical content at hand rather than the student confirms what other researchers have found to be true of novice teachers (Brown & Borko, 1992; Livingston & Borko, 1990; Meredith, 1993).

The PSTs in this study expressed concerns about teaching mathematics. Eight out of 12 indicated they were "apprehensive" about teaching area and perimeter to elementary-age children. Even Brianna, who entered the study with a strong mathematics background, was apprehensive about teaching. Similarly, Borko et al. (1992) reported that novice teachers are very concerned about their limited pedagogical content knowledge and the impact such a shortcoming may have on teaching and learning. The PSTs' lack of confidence and ability regarding the concepts being studied often resulted in their getting bogged down or confused and therefore unable to appreciate or contemplate how students might interact with the same mathematics. These findings taken together suggest that the college mathematics courses taken by PSTs do not inherently promote a conceptual understanding of area and perimeter or instill sufficient confidence to teach elementary children about these concepts.

Basic CK: Units of Measure

As presented in chapter 2, many studies have documented the ways in which novice teachers struggle with the mathematical content they must teach. This was evident on the first problem of the pretest which asked the PSTs to "draw a polygon that has a perimeter of 24 units." Eight out of 12 provided a response that addressed, to different degrees, concepts related to area. Similar confusion has been documented with classroom students (Hirstein et al., 1978; Kouba et al., 1988) and preservice teachers (Reinke, 1997).

Since area and perimeter concepts were not understood conceptually, it was rather easy for many PSTs to confuse area and perimeter along with linear and square units. Instead of these concepts being a part of a web of ideas they were isolated facts which provided a very fragile foundation on which to attempt to problem-solve and diagnose faulty student thinking. Confounding linear and square units is a specific application of area and perimeter confusion, and has been reported among classroom children (Chappell & Thompson, 1999; Lappan et al., 1998; Lehrer, 2003) and teachers as well (CBMS, 2001; Tierney et al., 1986).

The unit of measure functions is a conceptual bridge connecting an object and the number used to represent its size. Hiebert (1981) states, “The concept of a unit is a central, unifying idea underlying all measurement” (p. 38). Although the importance of a teacher possessing a conceptual understanding of linear and square units cannot be overstated, there is little research examining PSTs understandings regarding these concepts or how to improve the teaching of them. This study found that prior to intervention, PSTs often forgot to include or discuss units with their answers and their ability to explain the concepts of linear and square units was sadly lacking. Instead of actually explaining the distinguishing characteristics of linear and square units and providing classroom-useful examples, Larry and Jackie (and most PSTs in this study), simply explained how they are used (i.e., linear units are used with perimeter and square units with area). Other studies have reported that PSTs struggle with explaining concepts related to area and perimeter (Even & Tirosh, 1995; Menon, 1998; Reinke, 1997; Simon & Blume, 1994a), but few have specifically described what was deficient with the subjects’ explanations.

The finding that a common thread to inferior responses by PSTs involved a lack of appropriate drawings to support explanations is important and had not been seen reported in previous studies. This finding was very evident with problems related to units of measure. Drawings were also frequently neglected when the PSTs suggested instructional strategies to use with struggling students. This reflected an underdeveloped KoST. Hiebert and Carpenter (1992) acknowledge that the expert teacher realizes the importance of providing conceptual representations; however, the PSTs in this study, even though they often wrote about how important and helpful visuals are to students, neglected to include supportive diagrams and/or meaningful representations with their explanations. It was not just the poorer-performing PSTs who struggled explaining their ideas and justifying their answers. Brianna and Grace (two, top-performing PSTs) were relatively successful at distinguishing between and appropriately using linear and square units; however, when asked to explain and illustrate these concepts, their responses were deficient. It would seem that possessing mathematical knowledge about area and perimeter does not automatically translate into knowing how best to represent those concepts to elementary children – or possibly even realizing the importance of doing so.

Ability to Diagnose and Respond to Student Thinking

Knowledge of student thinking (KoST) is a component of PCK. Research pertaining to knowledge of student thinking is still in its infancy. Shulman (1986) noted that, "The study of student misconceptions and their influences on subsequent learning has been among the most fertile topics for cognitive research" (p. 10); however, little research could be found examining PSTs' understandings of and reactions to students' misconceptions regarding area and perimeter, and none involving intervention to address PSTs' shortcomings in these areas.

When faced with problem-solving situations involving erroneous student thinking, the majority of PSTs' in this study tended to focus on solving the problem (i.e., finding some perceived "answer"), to the neglect of diagnosing the hypothetical student's thinking. PSTs possessing stronger mathematical competencies were more adept at diagnosing student errors. Such a finding runs counter to research performed by Meredith (1993) who found that preservice elementary teachers specializing in mathematics were often "baffled by learners' difficulties" (p. 332). However, those PSTs successful at diagnosing student errors were often unable to provide coherent explanations that included supportive diagrams. These findings are in keeping with Borko et al. (1992) and Even and Tirosh (1995) who found that PSTs with strong mathematics backgrounds displayed a limited repertoire of instructional representations and were often unable to generate meaningful examples in responses to students' questions. It does not appear that increased mathematics training alone will develop or enhance pedagogical content knowledge. Most PSTs in this study did not possess the necessary knowledge, experience, or both to consistently diagnose student thinking or appreciate what is essential to help children understand the errors in their thinking.

Perceived Relationships between Area and Perimeter

The calculations of both area and perimeter involve the lengths of the sides of the figures, and thus someone lacking a conceptual understanding of area and perimeter could encounter many problems and difficulties (Ma, 1999). These similarities provide the setting for two common misconceptions involving the area and perimeter of a rectangle: (1) That increasing the perimeter of a rectangle will always increase its area (i.e., the *direct-relationship* misconception), and (2) Rectangles that have the same perimeter measurement will also have the same area, and vice versa (i.e., the *fixed-relationship* misconception).

Question 10 on the pretest (see Appendix D) asked whether a fixed perimeter ("60 feet of fence") could have several or only one sized garden (i.e., area). All 12 PSTs correctly concluded that the square garden had the greatest area. This finding differs from previous research done by Woodward and Byrd (1983), who found that 76 out of the 129 PSTs (or 59%), who were asked the same question, thought the gardens would be the same size. Similar percentages of PSTs in both studies (around 30%) expressed at least some awareness of the common student tendency to think that equal perimeters will result in equal areas. This represents a somewhat predictable finding. The PSTs would be successful on the mathematical component of a problem, however they would not be able to apply that knowledge so as to anticipate what students might find difficult or confusing about the same problem. This mindset inhibited many PSTs from systematically investigating an erroneous student claim.

The direct-relationship misconception (the belief that increasing/decreasing perimeter must increase/decrease area) offered the PSTs various learning trajectories to follow and explore. Question 8 on the pretest presented a student who claimed that increasing the perimeter of a rectangle will "always" result in a greater area. Four out of the 12 PSTs (33%), including Larry and Jackie, indicated that the student's claim was correct. Their explanations tended to be built on the incorrect assumption that increasing the perimeter of a rectangle must increase both dimensions and thus the area, and were similar to: "Because the longer the perimeter, the longer the sides, and the more area the box will have." They correctly identified the student's claim as a mathematical relationship; however, they failed to notice that the perimeter of a rectangle can increase as two of the sides of the rectangle decrease in length. Only three PSTs (25%) were able to arrive at a correct solution by presenting an appropriate counterexample. Ball (1988) and Ma (1999) presented a problem very similar to question 8 to elementary preservice teachers (26 and 23, respectively) and reported similar shortcomings. The PSTs' lack of understanding regarding the mathematics surrounding the student's claim affected their KoST in that it left them ill-equipped to engage the student in any meaningful discussion regarding that claim. Most PSTs in this study put all their effort into deciphering whether the student was right or wrong. That hindered the extent to which they investigated the various area-perimeter relationships beyond what they initially found or concluded.

Regarding Relationships between CK and KoST

The PSTs in this study exhibited varying degrees of growth in their CK (75%), KoST (also 75%), or both (58%) from pretest to posttest. It was found in several different contexts throughout the study how a PST's limited CK regarding specific concepts (e.g., units of measure) often left them ill-equipped to explain and illustrate their own thoughts about those concepts and even more incapable of appropriately responding to student shortcomings and misconceptions. This was manifested by a lack or poor use of representations, imprecise mathematical language (e.g., "boxes" instead of square units), and effective intervention strategies. Ma (1999) reported similar findings with the U.S. teachers she studied; however, she conducted no intervention to allow for further findings regarding potential relationships between the two knowledge types. The common trend observed in this study was an increased CK regarding area and perimeter concepts and misconceptions (following intervention) was typically accompanied by a growing use of appropriate drawings and coherent language when providing explanations. Also noted was an increased focus on diagnosing student thinking and suggesting more student-centered interventions - all evidence of a maturing KoST. The apparent dependency of KoST (more broadly PCK) upon CK has been written about by researchers such as Shulman (1986, 1987), Rowan et al. (2001), and Hutchison (1997), but drawing conclusions and making recommendations based on that dependency has proven elusive. There is little research examining relationships between novice teachers' CK, and their cognitions about student thinking (i.e., their KoST) and the interplay of these upon subsequent instructional decisions.

Regarding Anchored Instruction with Web-Based Microworlds

This teacher development experiment (Cobb, 2000; Simon & Tzur, 1999; Simon, 2000) sought to implement and closely observe instructional strategies that aligned with the theoretical underpinnings of anchored instruction (CTGV, 1990, 1991, 1992, 1993) and Shulman's (1987) model for developing pedagogical reasoning. Web-based microworlds provided a research-based technology conduit (Marzano, 1998) to support and aid the learning of area and perimeter misconceptions through various learning settings: independent discovery, and group dynamics between myself (the researcher) and the participants (preservice teachers) and among the participants themselves.

The focus problems for the instructional sequence, which were based on common area and perimeter misconceptions held by elementary students (and teachers), proved to be motivating and provided a range of entry points from which the PSTs could investigate concepts and misconceptions. The PSTs made several comments regarding how they enjoyed learning about what their future students could be expected to struggle with. There were several interesting findings regarding the web-based microworlds (MWs), *Shape Builder* and *Perimeter and Area Gizmo*, specifically selected for this study. The MWs did not consistently promote the type or level of involvement that was anticipated. Throughout the teaching episodes (TEs) the PSTs who struggled the most were also the ones who became preoccupied with some tangential aspect of the TE (e.g., finding the area of the footprint in TE 2), and as a result spent insufficient time analyzing a student's erroneous method and the misconception(s) behind it. For the most part, the PSTs in this study simply stopped exploring after arriving at and discussing their initial reaction. Many of these PSTs did not appear self-motivated to delve far beyond providing one possibility to the stated question. Instead of investigating the various possibilities surrounding a misconception (either with or without the MWs), the majority would give the same, or a very similar, answer as they had previously and continued to operate within their *CK comfort zone*. Similar to PSTs in Chinnappan (2000), this study found that a preoccupation with finding, what the PSTs judged as, "the answer" to the TE not only hindered their ability to properly diagnose and address student thinking, but it also limited their meaningful interaction with the MWs. This finding may be explained in part because several PSTs struggled translating the student's erroneous method or claim into a mathematical conjecture to refute or justify, and they lacked the necessary mathematical details for which to explore with the MWs.

Throughout the intervention, the vast majority of the PSTs commented on how they found specific features of the microworlds helpful to their understanding of the mathematics surrounding the focus-problems as well as facilitating insights regarding the students' thinking. A few of the higher-achieving PSTs displayed evidence of technological pedagogical content knowledge (TPCK) by suggesting specific revisions to the Shape Builder MW that would improve feedback and heighten awareness of distinguishing learning features of the MW. During the early stages of the intervention, the PSTs explained how they would use the microworld as an instructional tool in a whole-class discussion of the student's misconception. A similar majority (10 out of 12) indicated they believed future classroom students would benefit from personally interacting with the MW in a structured context. However, an unexpected trend developed as the mathematical content of the teaching episodes got progressively more difficult and the hypothetical students' thinking was increasingly more elusive.

Although the number of PSTs who indicated they learned with and/or saw benefits of personally interacting with the microworlds was a strong majority, far fewer said they would incorporate the microworlds when instructing future students about the concepts presented in the TEs, even though the same PSTs admitted those future students would most likely possess similar misconceptions as the hypothetical students presented in the teaching episodes. A similar contradiction appeared when of the several PSTs who indicated they learned from the microworlds only a few wrote that they would allow time for the students to personally use the microworlds to explore the concepts surrounding the teaching episodes. These findings concur with research done by Timmerman (1999). In both studies the PSTs did not use MWs as part of suggested instruction even though they acknowledged having difficulties generating conceptual explanations. Apparently, the majority of PSTs concluded the microworlds were a valuable learning tool for themselves but not necessarily for students. Every PST indicated that this study was their first exposure to web-based MWs, which helps to explain their frequent neglect to incorporate them within instructional recommendations. Collectively these results suggest that even though the content of the study was accessible (i.e., area and perimeter) and the technology which was integrated was appropriate for elementary students, there are no guarantees that PSTs will automatically perceive how best to utilize the features of the MWs to promote exploration and a deeper understanding of area and perimeter concepts nor necessarily comprehend the MW as a tool for future teaching.

The instructional sequence for this study was designed to encourage the PSTs to revisit their prior knowledge and consider them as points for reflecting about teaching. The value of viewing the PSTs in these dual roles was confirmed as most of them developed mathematical insights (i.e., a more heightened CK) as they attempted to solve problems that involve area and perimeter misconceptions and address erroneous student claims as they were functioning as students themselves. Their KoST was challenged and enhanced as they reconciled their personal mathematical understandings with what would be necessary and to provide an appropriate explanation and instruction to elementary students.

There was only one study found that investigated the use of anchored instruction in a mathematics course for preservice teachers. Kurz and Baterelo (2004) found that most PSTs who were exposed to anchored instruction expressed optimism that students could learn through such an instructional approach. This research extends their findings by describing how anchored instruction could be successfully integrated into a mathematics course for elementary preservice teachers and by documenting the positive changes to PSTs' CK and their KoST as a result of that intervention.

Implications for Practice

The results of this study, coupled with the knowledge provided from existing research, lead to some implications for teachers and teacher educators. As discussed in the review of literature and the results of this study, students and preservice teachers struggle with many aspects related to area and perimeter concepts and relationships.

Implications for Teachers

Confusions between area and perimeter and linear and square units could be reduced if these topics were introduced and developed in conjunction with each other. Traditionally, in school mathematics area and perimeter are taught in isolation, thus making it difficult to uncover misconceptions until these concepts appear together – typically on a test. These misconceptions (especially involving linear and square units) could function as springboards for engaging in the exploration of area and perimeter. Presenting scenarios involving student misconceptions and erroneous student work (or claims) could motivate students to delve deeper than the surface understanding presented in most textbooks. The very nature of such problem-solving scenarios would encourage reading, explaining, representations, and justifying of responses. These activities would more readily alert the teacher to existing and potential confusions as well as promote various forms of discourse and higher-ordered thinking.

Studying misconceptions would most likely involve the use of manipulatives to help promote conceptual understanding and better visualization of the concepts being explored. Results from previous research along with findings from this study suggest that technology (e.g., web-based MWs) is an effective and dynamic alternative to hand-held manipulatives. The benefits of technology-use include immediate feedback for students, features that promote independent discovery, and the ability to quickly “test” hypotheses. If area and perimeter were taught in tandem, then fewer individual lessons would be needed and time spent on reviewing these concepts would be decreased, because students would have a more connected and conceptual understanding of the subject matter.

Implications for Teacher Educators

Teacher educators must take a greater role in familiarizing teachers with common area and perimeter misconceptions and in providing instructional approaches to address those misconceptions. The 12 PSTs involved in this study were juniors and seniors and had completed all their mathematics requirements. That is, they had received all the subject matter instruction deemed necessary to teach elementary mathematics. However, as discussed earlier, PSTs (and classroom teachers) struggle conceptualizing many of the mathematical concepts (including area and perimeter) they have to teach, and hence have difficulties diagnosing misconceptions and effectively anticipating and addressing student errors – without simply restating rules or procedures. The results from this study suggest that undergraduate teacher education programs must ensure that preservice teachers, elementary and secondary, are fully prepared to be teachers of mathematics including addressing student misconceptions.

Research has documented numerous misconceptions and error patterns that students possess regarding the mathematics they learn. To increase levels of CK and KoST within PSTs, teacher educators must examine their programs to ensure that the misconceptions identified in this and other studies are addressed. It is important to not only examine the mathematical perspective of these misconceptions (e.g., possessing a profound understanding of linear and square units) but also to cultivate various knowledge types (PCK, CK, KoST, TPCK, etc.) simultaneously. For example, although it is important for PSTs to know that increasing the perimeter of a rectangle will not

ALWAYS result in a larger area, it is equally important for them to understand why students would think this and how then to address the misconception. PSTs must be aware of powerful and easily-accessible technologies (e.g., web-based MWs) that can be used to facilitate the exploration and deeper understanding of the mathematics surrounding these misconceptions. These technologies are becoming readily available in most classrooms. PSTs should learn best practices for incorporating them.

Results from the research literature reveal PSTs' mathematical shortcomings when asked to explain and represent their ideas (Borko et al., 1992; Even & Tirosh, 1995; Menon, 1998; Reinke, 1997; Simon & Blume, 1994a). PSTs need many opportunities to present and refine their subject matter knowledge, and instructional strategies. Promoting a community of learners within the methods course that encourages interactive cycles of reflection and cooperative sharing will help strengthen PSTs' new-found ideas and integrate them to form a more coherent understanding of the mathematics they must teach (Bowers & Doerr, 2001; Simon, 2000; Wales & Stager, 1977).

Implications for Future Research

Although this study answers some questions about PSTs' CK and KoST regarding area and perimeter (prior to, during, and following a specially-designed intervention), it leads to new questions. The results appear to show that the planned intervention positively influenced PSTs' personal knowledge about area and perimeter, their understandings of common student misconceptions as well as instructional strategies for responding to student difficulties and erroneous claims; however, only one other study (Kurz & Baterelo, 2004) was found that investigated the use of anchored instruction in a mathematics course for PSTs, and it did not involve any specific content. Further research is needed to help establish the viability of such an instructional approach within a mathematics methods course – not just to instruct in area and perimeter but other content as well. Future research could also help further evaluate various aspects of this study's intervention. For example, what specific aspect(s) of this study had the greatest impact upon PSTs' knowledge – the three tests, the teaching episodes, the anchor (i.e., student misconceptions), the cooperative learning experiences, or the interactions with the MWs? Such questions have not been answered. Multivariate analysis might prove helpful in isolating the strength of the contributing variables to the entire anchored-instructional sequence. For example, there were inconsistencies regarding what the PSTs wrote about the MWs and their personal learning, versus their proposed instructional strategies involving MWs with future students. More research is needed to determine if, or to what degree, the MWs are a valuable component of anchored mathematics instruction with PSTs. Conducting research with interns, possibly a longitudinal study, where they experience anchored instruction similar to this study and then are observed teaching the same concepts within a school setting, possessing the necessary technology, might help provide insight as to how well knowledge of content and instructional strategies gained during anchored instruction transfers to actual classroom practices of PSTs.

Learning about students' area and perimeter misconceptions proved to be motivational to the PSTs in this study. There is a need to examine the extent to which classroom students might also find such learning settings interesting. Researchers could conduct an experimental study with classroom students examining the impact of learning area and perimeter concepts through studying misconceptions. Results from such studies would provide a foundation to extend future research to other content areas. Other questions that need to be addressed include: In what ways would the anchored mathematics instruction need to be altered to be compatible with school students? To what extent would classroom students' CK grow as a result of using anchored instruction with web-based applets? Previous research has shown the benefits of MWs within school settings (Clements & Sarama, 1997; Kordaki, 2003; Lederman & Niess, 2000; Yelland, 2002). Given their aptitude towards technology, it is important to examine differences between PSTs and students' use of MWs.

Another question raised by this study that needs further investigation involves the PSTs' use of drawings while providing written explanations and when making instructional recommendations. It was not clear why the PSTs did not perceive the importance of diagrams when communicating mathematical concepts – especially more difficult ones. Representations, including demonstrating understanding, have been described as a vital part of effective classroom communication (NCTM, 2000), and since the majority of PSTs in this study did not use them, research is needed to investigate the PSTs' use of representations and the importance attributed to them.

In Chapter 3 it was reported how Cronbach's alpha for certain subtests was less than satisfactory. This was most likely due to a combination of one or more of the following: (a) small sample size (n = 12), (b) small number of items on subtest (n = 5), and (c) a couple poorly-written test items (identified through analysis of descriptive statistics). A replication of this study with a much larger sample (including modified test items) could help to mitigate these concerns and help to clarify the extent to which this study's planned intervention influences the CK and KoST of PSTs.

This study represents beginning steps in understanding how to develop anchored instruction useful for a mathematics methods course. There is much more to investigate and much more work to be done. Based on the results of this teaching experiment, I believe there is hope for further development and deeper understanding of the impacts of anchored instruction upon PSTs' content knowledge and knowledge of student thinking.

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APPENDICES

Appendix A: Piloting of Instruments

Timeline and Summary of Piloting Sessions

- **Spring, 2004** – A 16 question (14 open-ended, two multiple choice) area and perimeter assessment was administered. The problems pertained to student difficulties with area and perimeter as presented in the literature. Before the assessments were collected, the preservice teachers were shown four web-based microworlds that appeared appropriate for exploring area and perimeter concepts. Because we were conducting class in a computer lab, the students were then given the chance to review their answers to the assessment and make appropriate changes. They were asked to provide feedback regarding which applets they liked and why. One student, Anna, commented regarding an NCTM Illuminations applet, "I liked how I could see the relationship of doubling the perimeter, but quadrupling the area." During this exploration time, I was able to observe the students interacting within the microworlds and question them on their choices and the features of the applets. Informal analysis revealed that in order to elicit more reflective feedback future assessments would need to ask for greater justification of answers as well as specifically asking the preservice teachers to explain their responses as if they were talking to an elementary student. It was also found that certain questions would have a tendency to bias others.
- **Fall, 2004** – First, near the beginning of the semester a version of the proposed questionnaire was administered to the students in my methods of teaching elementary mathematics course. All of the preservice teachers surveyed indicated

Appendix A (Continued)

that they were not aware of any specific technology that could be used to enhance the teaching or learning of area and perimeter. After examining the student's responses on the questionnaire, the format was changed to be more standardized, check boxes were added, and more opportunities for open-ended responses were included. About a month later, a 13-question pretest was administered. Two subsequent whole-class discussions addressed the area and perimeter misconceptions that were infused into the questions. The number of microworlds now being considered was down to three, from the previous four. Those three applets were used as part of instruction for the whole-class discussion and their effectiveness was evaluated by observation and student reflection. For example Katie reported, "I really like this (the Shodor) website because it gave me a chance to practice area and perimeter and gave me immediate feedback. I was able to instantly see if I was right or wrong in my answer." Two weeks after the pretest a 14 question posttest (similar but not parallel) was administered. Wording of questions was again refined, and the time required to take the test was evaluated. It was concluded that statements such as, "Include appropriate diagrams to illustrate your ideas," or "Illustrate your answer" should be removed from future assessments as they bias future attempts to measure a participant's pedagogical content knowledge. Scoring of the pre- and posttests revealed students did much better on the posttest and further work appeared promising.

- **Spring, 2005** – What had proved to be the six most challenging area and perimeter items from previous assessments were administered to the students in my methods of

Appendix A (Continued)

teaching secondary mathematics course for the purpose of formulating follow-up interviews. Purposeful sampling of two students resulted in the opportunity to design semi-structured interviews to further probe the understandings underlying their responses to the area and perimeter questions.

- **Fall, 2005** – Versions of the pre- and posttest were administered as well as a five-question follow-up test that was incorporated into their final exam at the end of the semester. As a result of this pilot work, more explicit directions were written and the questions focusing on content knowledge and knowledge of student thinking were separated and identified within the test. Lists were written identifying questions that would bias each other as well as one indicating pairs of parallel questions based on content and difficulty.
- **Spring, 2006** – Considerable time was spent in revising items for the area and perimeter assessments. This version included more formal and explicit directions and separate sections were created indicating content knowledge (CK) and knowledge of student thinking (KoST) questions. The revised assessment was administered to a section of students in a course titled, Teaching Elementary School Mathematics at a nearby large southeastern university. These assessments were individually scored by both the researcher and a second scorer using specially designed rubrics (Appendix H). As a result of the difficulties with double scoring the 27 tests, considerable revisions to the scoring rubrics (Appendix G contains examples of earlier versions) and "Supplemental Grading Sheets" (see Appendix I) were created and incorporated into the rubrics to help distinguish between scores bordering

Appendix A (Continued)

between two scores on the rubric. Important results from this scoring session were the breaking down of content- knowledge questions into a part (a) in which correctness would be considered and a part (b) which would decipher the quality of their explanation. The knowledge-of- student-thinking questions were constructed to now have three parts: a part (a) in which the test taker decided if the "student" named in the item correctly solved the problem presented, a part (b) which asked for the test taker to explain the "student's" thinking, and a part (c) asking for how the test taker, thinking and functioning as a teacher, would respond to the student in the hypothetical problem or situation.

- **Summer, 2006** – Revisions from the spring pilot were applied and two different, but similar versions, of the area and perimeter assessment were constructed and administered to two summer sessions of a teaching elementary school mathematics course at the same southeastern university.
- **Fall, 2006** – A five-question, multiple-part area and perimeter assessment was administered in the researcher's methods of teaching elementary mathematics course. These questions were purposely chosen because they had produced the most diverse responses in previously administered assessments, and the researcher wanted to pilot the revised versions of these questions in order to generate interview protocols as well as a follow-up instructional session involving anchored instruction.

Near the end of the semester, an early version of the Teaching Episode format was piloted. The purposes were to: (a) observe how the preservice teachers (PTs) worked through the problem-solving scenario presented, (b) observe how they

Appendix A (Continued)

interacted with the *Explorelearning* microworld and how the applet influenced their problem solving approaches and their instructional suggestions, (c) observe the level and value of PT's cooperative interaction provided by the Teaching Experiments' format, (d) accumulate written data to provide insight into and allow for analysis of the PT's thinking regarding content knowledge and knowledge of student thinking of area and perimeter, and (e) provide visual, audio, and written feedback regarding the current format of the Teaching Episode.

Analysis revealed that most of the PTs were currently at a novice stage in both their treatment of content knowledge and knowledge of student thinking. They spent minimal time analyzing the mathematics of the problem; hence, they initially overlooked mathematical subtleties of the problem – a valuable skill of experienced and effective teachers. For some students the applet did not seem to facilitate mathematical or pedagogical growth; however, others indicated signs of growth in both categories. One of the PT, Kristen, indicated a growth in content knowledge by writing, "Without the Gizmo (applet) I would not have known of anything to say to Pete (the fictitious student presented in the focus problem of the Teaching Episode) because I forgot that an area of 18 meant that 18 square units could fit into the rectangle. When I used the Gizmo I saw for myself that there 18 squares inside." Several PTs tended to focus on the content at hand (e.g., using the correct formula to find area) instead of the student and how to help him conceptually understand the problem. Rebekah, on the other hand, showed some growth in her pedagogical content knowledge when she wrote, "He (Pete) may not understand what a square

Appendix A (Continued)

centimeter is. It might be helpful to show him a grid like the one on the Gizmo website. If he can picture what exactly he is measuring, he will be learning more than just a formula." The reflection sections within the Teaching Episode revealed that the majority of the PTs were thoughtfully involved with the problem-solving scenario and being introspective about their current understandings regarding the problem at hand.

Conclusion: Summary of Design, Training, and Major Results Involving the Instruments as a Result of Piloting

Area & Perimeter Tests

The end result of applying the search criteria presented in chapter 2 was a collection of 28 questions that were then categorized as most appropriate (or easiest to modify) to address either content knowledge (CK) or knowledge of student thinking (KoST) regarding area and perimeter. A total of 35 items (some were various forms of the same problem) were then piloted. Content knowledge problems were amended to ask the participant to perform a calculation or answer a constructed response question and then to explain how they arrived at their answer. The knowledge of student thinking problems typically have three parts: (a) decide if the thinking, solution, method, or claim presented regarding a hypothetical student is correct, (b) justify their response to part (a), and (c) as a teacher explain exactly how they would respond to the mathematical thinking of the hypothetical student or students presented in the problem. A statement similar to,

Appendix A (Continued)

“As a teacher, how would you respond to . . .” was added after the first piloted test and proved more effective at eliciting the desired level of reflection. Many of the problems required slight modifications including the addition of appropriate drawings and grids for the participants’’ drawings. Explicit directions for answering the content knowledge and knowledge of student thinking problems evolved through piloting and were finalized by the fourth and last pilot test.

Training and scoring sessions (discussed later) conducted with the second scorer proved very helpful in strengthening certain test items to be used in future piloting while also eliminating other weaker items. The potential to illicit a range of thoughtful responses was very important in the item-selection process. As the tests were created issues such as posttest sensitivity were considered and planned for. Posttest sensitization can occur when the posttest inadvertently acts as a learning experience in its own right (Gall, Borg, & Gall, 1996). To address this possibility, the posttest will consist primarily of parallel items to the pretest with two items the same as the pretest and one item modified slightly. Because the follow-up test is interested more in retention than growth, it will contain the same items as the pretest.

Rubric Scoring

A zero to four-point scale was utilized and the criteria for the different score levels was initially based on the sub-categories of “mathematical knowledge,” “strategic knowledge,” and “communication” presented in the general holistic scoring rubric of Cai et al., (1996, p. 143). A score of 0, 1, or 2 was considered unacceptable, and a score of 3 or 4 was considered acceptable. The researcher and a secondary scorer were involved in

Appendix A (Continued)

numerous training rounds of scoring and revising to both the rubrics and the format of the testing items (see Appendix A). For example, the first dual scoring session of five area and perimeter tests incorporated the use of holistic scoring rubrics (Appendices G and H) and an anchor paper (Thompson & Senk, 1998). The fact that roughly 35 open-ended questions were going to be piloted and the participants were frequently encouraged to explain and justify their responses produced too many response variations for effective use of anchor papers. Instead the language of the rubrics was gradually refined (see Appendix H) to reflect a conscious effort to separate a procedurally-oriented response from a more conceptually-based one. A score of two became the dividing line to separate a procedural-only response and one demonstrating conceptual understanding of the concepts at hand. That is, the best score that a response lacking conceptual understanding could receive is a two. Later on in the training process tables were created to succinctly delineate each item's major concept(s) and potential misconception (see Appendix I) and help differentiate a response emphasizing procedures from one focusing on understanding. Table 2 contains information on only the items proposed for use in the pre-, post-, and follow-up tests. The item-specific tables supplemented the scoring rubrics and proved especially helpful in scoring a participant's knowledge of student thinking. During the training process, the tables were clarified to improve consistent application and separate procedural-only from conceptual-based responses.

Training & Scoring

The area and perimeter testing instrument was piloted three separate times. Each pilot used a test containing a majority of different questions with one or two problems

Appendix A (Continued)

revised from previous pilots. The first test pilots contained 15 questions which proved difficult to complete in the preferred one-hour time constraint. The second and third tests were shortened to 10 items, but still were producing reliable measures.

Copies of the 27 tests from the first piloting of the area and perimeter assessment instruments were mailed to the second scorer. Soon after, the first training session occurred and involved familiarizing the second scorer with the goals of the study, the nature and objectives of the area and perimeter tests themselves, and the scoring rubrics for content knowledge and knowledge of student thinking items on the tests (Appendix H). During the first session, the wording of various sections of the rubrics was clarified and the session concluded with some important revisions regarding differentiating specific criteria for certain scores on the rubrics, including the importance of diagrams for responses. It was agreed upon that when scoring the tests we would grade by items (i.e., grade the first problem on all tests before grading the second problem). It was decided that we would completely score all 15 items for two randomly selected tests. We then worked through each item, discussing how and why we arrived at the scores we did. We spent extra time discussing the responses we scored differently. We concluded with a general reminder to focus on conceptual understanding and use that construct in the process of separating acceptable from unacceptable responses – within the range of the rubric criteria. The first session also resulted in making sure all test items clearly separated the types of responses (e.g., correctness, explain your thinking, explain the thinking of the student in the problem).

Before the second training session occurred, both the researcher and the second

Appendix A (Continued)

scorer independently scored all 15 items on three more tests. The purpose of the second training session was to use all disagreements to help clarify the scoring rubrics to improve consistent application of the criteria and to strengthen the testing items through revision of confusing language. The format of items was modified to improve the potential of diverse and rich responses. For example, in addition to asking the participants (preservice elementary teachers) to attempt to explain what they thought the student in a certain problem might have been thinking when making their (incorrect) response, when appropriate the participant was also asked to explain what and why elementary students might have difficulty with a particular question or concept. This change produced a greater range and depth of responses on future piloted tests.

Three more training sessions were conducted. Because of the large number of items being piloted (28), there was a concerted effort to clarify the language of the rubric so as to avoid item-specific rubrics. Each session would involve independently scoring all 15 items for five tests and then comparing all scores and then discussing the modifying of items and rubric revisions. There were several important results of these sessions, including: (a) appropriate units must be included to receive a score of 4, (b) conceptually wrong responses cannot receive a score higher than 2, (c) rubric language was clarified to increase the consistency in distinguishing between a score of 3 and 4 – especially for the Knowledge of Student thinking rubric, and (d) before any future scoring was conducted, the researcher should create tables specifying the concepts and misconceptions being addressed by each item (Appendix I). This proved instrumental to future scoring sessions. Following the construction of the “Concepts and Misconceptions” tables five more tests were double scored. There were two important results from subsequent discussions. First, a score of 4 could be awarded as long as at least one of the items' major concepts and misconceptions was addressed. Second, the rubrics were to be the primary scoring tools with the Concepts and Misconceptions table assisting with responses that were "borderline" between scores on the rubric.

Throughout the scoring of the first 27 tests, repeated revisions and modifications were made to the scoring rubrics and their application. An example of a clarification that arose during the training process involved the criteria for separating a score of 3 from a score of 4. For both rubrics a top score of 4 was reserved for what is termed a model response that demonstrates a thorough understanding of the problem's concept, provides a completely correct response including precise terminology, notation, and execution of algorithms, and provides diagrams or pictures to support/explain the response. A score of 3 also represents a successful or acceptable response and differs from a 4 in that it indicates an essential, nearly complete understanding of the problem's concepts, provides an essentially correct response but may contain minor computational errors, and includes a picture or diagram that may contain minor errors (e.g., not drawn to scale) but offers very little explanation, *or* provides a detailed explanation but no supporting picture or diagram. In an earlier version of the rubrics, the language describing a score of 3 and 4 simply made reference to the inclusion of diagram or picture to support the response. The need to clarify and specify the scoring criteria became evident in several items.

An example illustrating the need to clarify the scoring involved the use, misuse, or omission of an appropriate diagram or picture along with the response is the following item: "If each of the dimensions of a 2 x 4 rectangle is tripled, what is the relationship between the original and the enlarged figures?" After providing a response, the participant is then asked, "As a teacher, how would you present the explanation for how you arrived at your answer to a class of 4th or 5th graders?" It was common for a participant to correctly explain how they arrived at their answer but their often lengthy responses were somewhat confusing, and would certainly be so to a 4th or 5th grader. Therefore, it was decided that a response mathematically and procedurally correct but lacking a diagram that would help a student conceptualize the explanation (or a diagram with insufficient explanation) would receive at best a score of 3. It was important to establish a model response as one procedurally correct and conceptually robust, and including an appropriate diagram or picture to support an explanation geared toward elementary students was deemed necessary.

The training and cooperative revising proved successful. The results from the inter-rater reliability process include: clarifications made in the language of the holistic scoring rubrics, the addition of Concepts and Misconceptions tables, and improvements in item format and wording – including the elimination of several items. These improvements were implemented in the scoring of all subsequent test papers, and high scoring reliability was achieved throughout. The training and scoring sessions for the first batch of 27 tests had a robust inter-rater reliability of 94%. The second and third scoring sessions had a slight drop in inter-rater reliability, 88% and 86%. These two subsequent scoring sessions involved only 10-item tests, which helps to explain the drop in inter-rater reliability. Also, the test used for the third pilot contained four problems which had negative corrected item-total correlation. These problems were removed from consideration for this study.

Appendix B: SYLLABUS FOR METHODS COURSE

METHODS OF TEACHING ELEMENTARY MATHEMATICS

<u>COURSE GOALS</u>

"To Know How and More Importantly to Know Why"

This course focuses on discovering the reasons behind the actions in mathematics.

This course is required in the undergraduate program in Elementary Education. It provides the development of knowledge and skills necessary to prepare students to assume roles as teachers of mathematics in elementary classes. The National Council of Teachers of Mathematics (NCTM) in its Guidelines for the Preparation of Teachers recommends such a course.

The vision of mathematics learning espoused by the NCTM assumes the following: Knowing mathematics means being able to use it in powerful ways. To learn mathematics, Students must be engaged in exploring, conjecturing, and thinking rather than only in rote learning of rules and procedures. Mathematics learning is not a spectator sport. When students construct personal knowledge from meaningful experiences, they are much more likely to retain and use what they have learned. This fact underlies teachers' new role in providing experiences that help students make sense of mathematics, to view and use it as a tool for reasoning and problem solving (*Curriculum and Evaluations Standards for School Mathematics: Executive Summary,* NCTM, March 1989, p. 5*)*.

The purpose of this course is to provide opportunities for preservice teachers to examine and build upon their understandings of various mathematics topics, and to construct a vision of teaching and learning mathematics that considers the goals and the assumptions of the current reform movement in mathematics education. Content, methods, and materials for teaching elementary school mathematics will be examined cooperatively.

As a perspective elementary teacher it is important to:

- Develop a <u>conceptual understanding</u> of the mathematics topics.
- Think about the kinds of mathematics students can learn through the use of multiple representations (i.e., applets, manipulatives).
- Evaluate mathematical activities from the standpoint of a teacher.

Appendix B (Continued)

III. COURSE OBJECTIVES

Upon completion of this course, students will have demonstrated:

1. Knowledge of the major goals and characteristics, including scope and sequence, of elementary school mathematics programs, and aspects of theories of learning as applied to the planning and instruction for the teaching of elementary school mathematics.

2. Knowledge of the current developments in education, including research that may affect the elementary school mathematics curriculum.

3. Knowledge of the properties of a number system and their application in the teaching of elementary school mathematics.

4. Knowledge of pre-number concepts and ideas and their application in the teaching of elementary school mathematics.

5. Knowledge of numeration concepts and principles and their application within the Hindu-Arabic System.

6. Knowledge of the whole number concepts, principles and computational skills (algorithms) and their application in the teaching of elementary school mathematics.

7. Knowledge of number theory concepts and principles and their application in the teaching of elementary school mathematics.

8. Knowledge of rational number (fraction and decimal) concepts, principles and computational skills (algorithms) and their application in the teaching of elementary school mathematics.

9. Knowledge of problem-solving process/strategies and their application in the teaching of elementary school mathematics.

10. Knowledge of and an ability to use the various tools available to the elementary teacher to aid in the effective teaching of elementary mathematics (e.g., traditional concrete manipulatives as well as technological advances, for example, the Internet including various web applets)

Appendix C: PRE-STUDY SURVEY QUESTIONNAIRE

Survey Questionnaire

Name:________________________________ **Classification**: Junior or Senior

Age: (check one) ____ 18-22
____ 23-27
____ 28-32
____ 33-37
____ 38 or older

Gender: ____ Male
____ Female

Please indicate or write in your response to each question below.

1. Did you take a class titled "Geometry" in middle school or high school?

   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

2. Did your Geometry class include doing proof (e.g., two-column proofs)?

   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

3. Did you take any other classes in high school, besides *Geometry*, that included geometry topics?

   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

   If yes, what was the course called?

Appendix C (Continued)

4. If you answered “yes” to 1 or 3, did you learn about <u>area</u>, or, <u>perimeter</u>, or <u>both</u> in your geometry class(es)?

   PLEASE CHECK ONLY ONE BOX.

   ☐ Area

   ☐ Perimeter

   ☐ Both

5. Were there any other high school classes in which you remember studying <u>area</u>?

   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

   If yes, please give details of the class(es).

6. Were there any other high school classes in which you remember studying <u>perimeter</u>?
   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

   If yes, please give details of the class(es).

7. Have you taken MAT 145, *Liberal Arts Mathematics*?
   PLEASE CHECK ONLY ONE BOX.

   ☐ Yes

   ☐ No

   When?

8. If you answered yes to question 6, who was your instructor when you took MAT 145, *Liberal Arts Mathematics*?

Appendix C (Continued)

9. Did you study area, or perimeter, or both in MAT 145, *Liberal Arts Mathematics*?

PLEASE CHECK ONLY ONE BOX.

☐ Area

☐ Perimeter

☐ Both

10. Have you studied area in any other college mathematics courses?

PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, please explain.

11. Have you studied perimeter in any other college mathematics courses?

PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, please explain.

12. Are you currently taking a mathematics course this semester?

PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, please list it.
Who is your instructor for that course?

Appendix C (Continued)

13. Do you remember ever using concrete manipulatives (i.e., square tiles, geoboards) when learning about area? PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, what manipulatives did you use and what do you remember doing with them?

14. Do you remember ever using concrete manipulatives (i.e., square tiles, geoboards) when learning about perimeter? PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, what manipulatives did you use and what do you remember doing with them?

15. Do you remember using any forms of technology (i.e., computer software or the Internet) when learning about area and perimeter?
PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

If yes, what form(s) of technology and what do you remember about the experience?

16. What is your opinion on using technology (e.g., computers and/or the Internet) to help elementary students learn about area and perimeter?

Appendix C (Continued)

17. How confident are you currently about teaching area and perimeter concepts to elementary-age children? PLEASE CHECK ONLY ONE BOX.

- ☐ apprehensive
- ☐ confident
- ☐ very confident

Please share the reasons behind your response.

18. Are you aware of any specific technology currently available to assist elementary teachers in <u>gaining a better understanding of</u> area and perimeter concepts? PLEASE CHECK ONLY ONE BOX.

- ☐ Yes
- ☐ No

If yes, please explain.

19. Are you aware of any specific technology currently available to assist elementary teachers when instructing children regarding the concepts of area and perimeter? PLEASE CHECK ONLY ONE BOX.

- ☐ Yes
- ☐ No

If yes, please explain.

20. If web-based technologies (i.e., Internet activities) were available to help you teach elementary children about area and perimeter would you feel confident using them in your future classroom? PLEASE CHECK ONLY ONE BOX.

- ☐ Yes
- ☐ No

Why or why not?

Appendix C (Continued)

21. If you answered no, what do you think it would take for you to feel more confident in using technology to teach mathematics to your future students?

22. Do you feel web-based technologies (i.e., Internet activities) are an appropriate tool to assist in teaching area and perimeter to elementary children?
PLEASE CHECK ONLY ONE BOX.

☐ Yes

☐ No

Please explain the reason(s) behind your choice.

23. If you had to teach area and perimeter to elementary children tomorrow:

(a) What specifically would you <u>tell</u> them about the concepts?

Perimeter -

Area -

(b) What would you <u>do</u> to help them understand the concepts?

Perimeter –

Area -

24. What do you think your future <u>students</u> may find difficult regarding the learning of:

Perimeter –

Area –

APPENDIX D: AREA AND PERIMETER PRETEST[6]

DO NOT WRITE IN THIS AREA

Student Number: _________

NAME ______________________________

Classification _________________________

Gender _____________________________

[6] The actual tests (pre-, post, and follow-up) had more room to show work than those appearing in the appendices. Typically, there were one or two questions per page.

Appendix D (Continued)

NAME ________________________________

<u>Area and Perimeter Pretest</u>

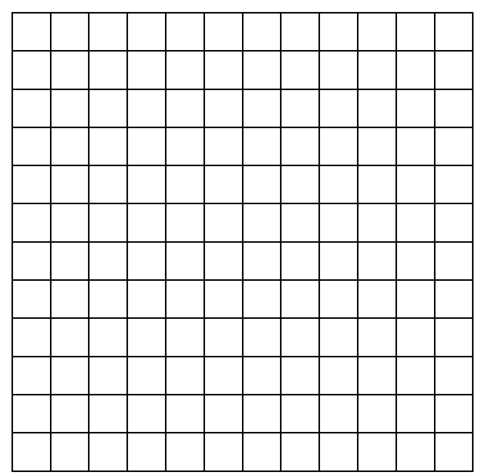
<u>All explanations and diagrams should be appropriate for elementary-age students</u>. **Please do not use a calculator. Be sure your answers include proper units. Write with <u>pencil</u>, and please write <u>legibly</u>.** Feel free to use the back of any page for comments regarding any questions you found confusing and explain why it confused you.

<u>PART I</u>: Content Knowledge (CK)

For questions 1 – 5: (1) Answer each question the best that you can. (2) It is very important to use thorough <u>and</u> detailed explanations to <u>fully</u> represent your knowledge.

1. (a) On the grid below, draw a polygon that has a perimeter of 24 units.

(b) How would you help a 5th grader understand that the polygon you drew really does have a perimeter of 24?



□ = 1 square unit

2. Present a real-world situation (or story problem), appropriate for 4th or 5th graders, in which they would need to find the area of a specified region.

Appendix D (Continued)

3. What is the area and perimeter of Figure A?
(All corners are right angles.)

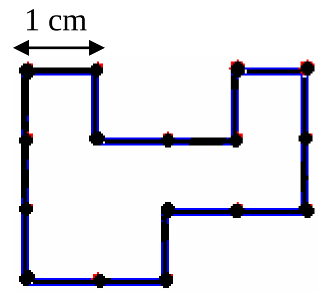


Fig. A

(a) area =

perimeter =

(b) Explain, as you would to a 4th grader, how you arrived at both your answers.

Area

Perimeter

4. As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5th grader? Stress the differences in the concepts. Include a practical example of each (i.e., how they're used in the real world).

5. If each of the dimensions of a 2 x 4 rectangle is tripled, what is the relationship between the original and the enlarged figures?

(a) Your answer?

(b) As a teacher, how would you present the explanation for how you arrived at your answer to a class of 4th or 5th graders?

Appendix D (Continued)

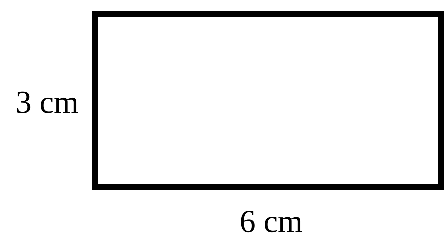
NAME __________________________________

Pretest PART II: Knowledge of Student Thinking (KoST)

For problems 6 - 10, please address the following: **Part (a)** is a short answer – typically a "yes" or "no" response, **Part (b)** asks for you to explain your thinking, and **Part (c)** asks you to explain the student's or students' thinking - from the perspective of a teacher. If there are more than three parts, **please address each part thoroughly and separately.**

6. Pete, a 5th grader, calculates the area of the rectangle below. He arrives at an answer of 18.

(a) Is Pete's answer correct and complete?

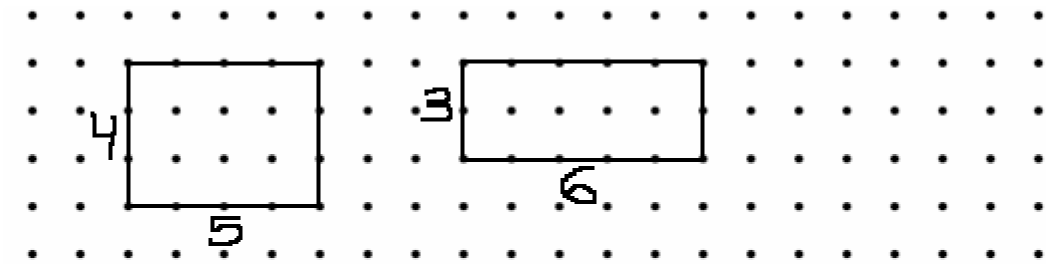


(b) Explain why or why not:

(c) After performing the calculation, Pete comes up to you looking puzzled and asks what exactly the "18" represents or means. As a teacher, how would you respond to Pete's question and his thinking? What specifically would you say and do?

Appendix D (Continued)

7. Kayla, a 5th-grade student, was asked to draw all the four-sided dog-pen designs that she could make using 18 units of fence for each design. Below are the drawings, on dot paper, that she came up with.



(a) Is Kayla's answer correct and complete? Explain your answer.

(b) Explain what is correct and incorrect regarding Kayla's thinking, as evident in her work.

(c) As a teacher, how would you respond to Kayla? What precisely would you say and do?

8. Jasmine claims that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area.

(a) Is she correct?

(b) Explain why you agree or disagree with Jasmine's thinking.

(c) As a teacher, how would you respond to Jasmine? What specifically would you say and do?

Appendix D (Continued)

9. Justin wants to calculate the perimeter of the shape shown in Figure 1. Justin's method is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those squares.

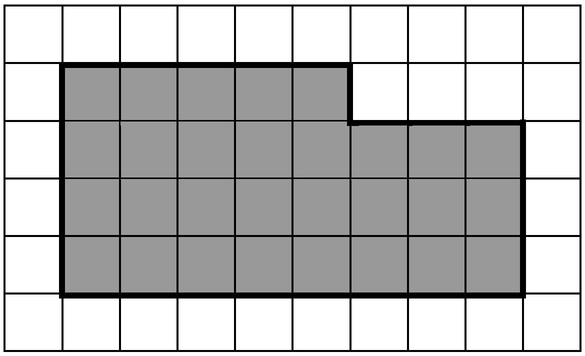


Fig. 1

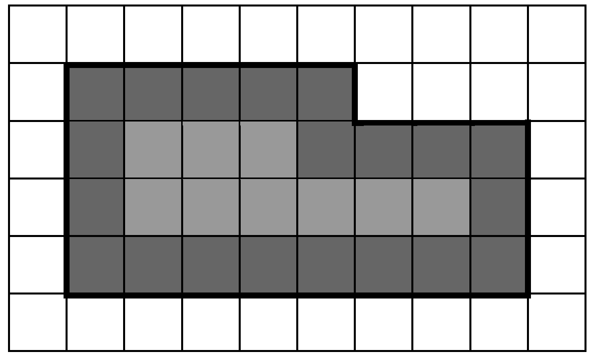


Fig. 2

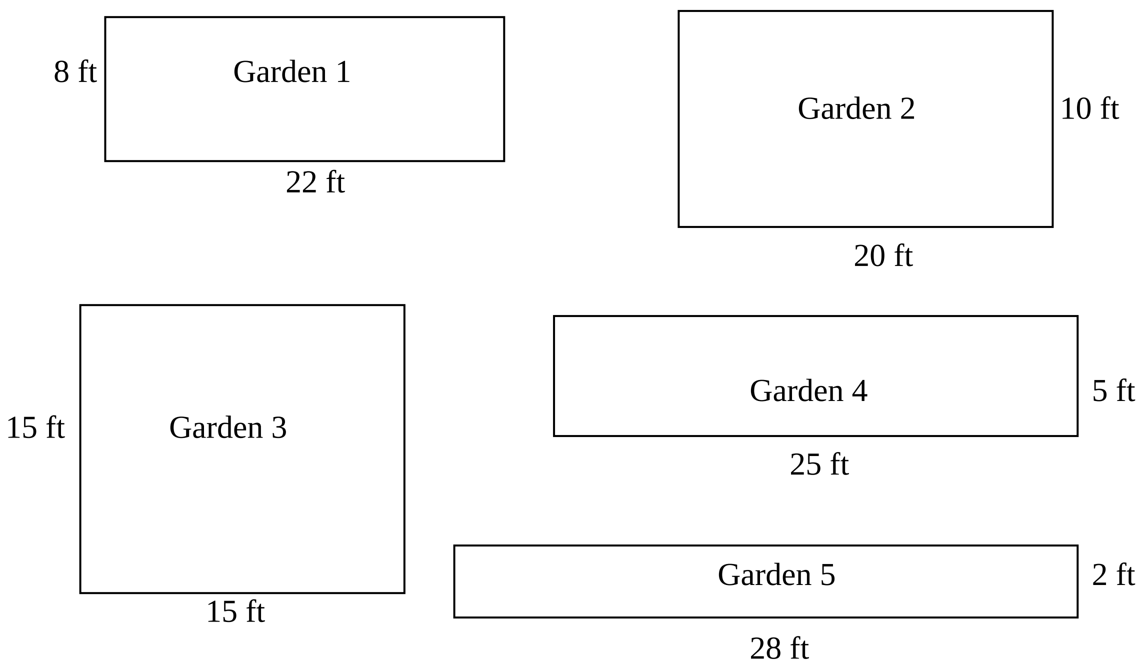
(a) Is Justin's method correct?

(b) Explain why or why not.

(c) As a teacher, how would you respond to Justin's thinking and his method? What specifically would you say and do?

Appendix D (Continued)

10. Mr. Jones purchased 60 feet of fence to enclose his garden. He wanted the garden to have a rectangular shape. He also wanted to have the most space possible for his garden. He drew out several possibilities, which are shown below.



Examine each of Mr. Jones' drawings of his possible garden designs. For Part **(a)** place an "X" beside the numbered statement below that you believe to be true; Part **(b)** explain your selection for Part (a); and Part **(c)** is below.

_____ 1. Garden 1 is the biggest garden.

_____ 2. Garden 2 is the biggest garden.

_____ 3. Garden 3 is the biggest garden.

_____ 4. Garden 4 is the biggest garden.

_____ 5. Garden 5 is the biggest garden.

_____ 6. The gardens are all the same size.

(b) Explanation for Part **(a)**:

Part **(c)**: Which incorrect statement do you think would most often be selected by 4^{th} or 5^{th} graders? What might they be thinking? Please explain your choice.

APPENDIX E: AREA AND PERIMETER POSTTEST

DO NOT WRITE IN THIS AREA
Student Number: _________

NAME ________________________________

Classification ___________________________

Gender _______________________________

Appendix E (Continued)

NAME ___________________________________

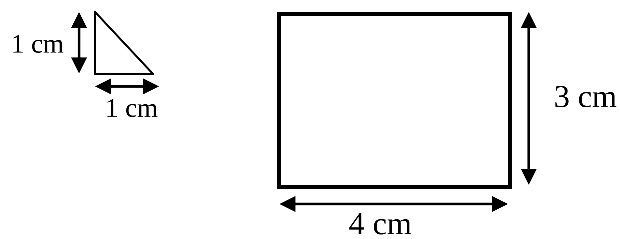
Area and Perimeter Posttest

All explanations and diagrams should be appropriate for elementary-age students. **Please do not use a calculator. Be sure your answers include proper units. Write with pencil, and please write legibly.** Feel free to use the back of any page for comments regarding any questions you found confusing and explain why it confused you.

PART I: Content Knowledge (CK)

For questions 1 – 5: (1) Answer each question the best that you can. (2) It is very important to use thorough and detailed explanations to fully represent your knowledge.

1. (a) How many triangles, like the one shown below, will it take to completely cover the rectangle?



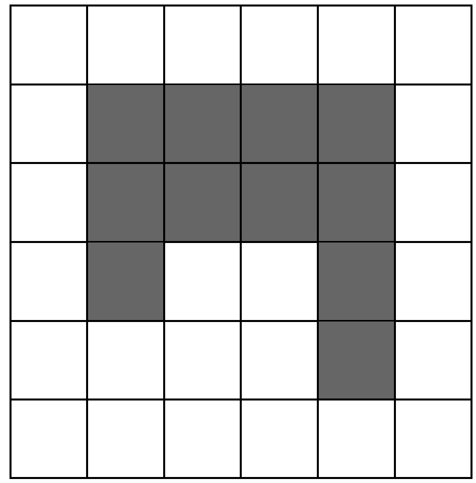
(b) As a teacher might, clearly explain how you arrived at your answer?

2. Present a real-world story problem, appropriate for 4th or 5th graders, in which they would need to find the area of a specified region. Provide the solution to your problem.

Appendix E (Continued)

3. If each individual segment is equal to 1 cm, what is the area and perimeter of the shaded figure?

1 cm



(a) Area = ____________

Perimeter = ____________

(b) As a teacher, explain how you arrived at BOTH your answers, and the meaning of those numbers.

Area:

Perimeter:

4. As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5th grader? Stress the differences in the concepts. Include a practical example of each (i.e., how they're used in the real world).

5. A certain rectangle has a perimeter of 16 cm.

(a) What might its area be?

(b) Explain how you arrived at your answer.

(c) Are there other correct responses? If so, explain what they are.

Appendix E (Continued)

NAME ________________________________

<u>Posttest PART II</u>: Knowledge of Student Thinking (KoST)

For problems 6 - 10, please address the following: **Part (a)** is a short answer – typically a "yes" or "no" response, **Part (b)** asks for you to explain <u>your</u> thinking, and **Part (c)** asks you to explain the student's or students' thinking - from the perspective of <u>a teacher</u>. If there are more than three parts, **please address each part thoroughly and separately.**

6. Stacey claims that whenever you compare two rectangles, the one with the smaller perimeter will always have the smaller area.

(a) Is she correct? <u>If you are unsure</u>, are you skeptical or do you tend to believe her? Why?

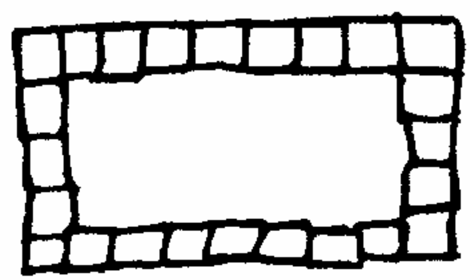
(b) Explain why you agree or disagree with Stacey's thinking.

(c) As a teacher, how would you respond to Stacey? What specifically would you say and do (even if you are unsure about the mathematics involved)?

7. Jose, a fifth grader, was asked to draw a rectangle with a perimeter of 24. Below is his drawing.

(a) Is he correct?

Why?



(b) What does Jose's drawing reveal about his knowledge of perimeter?

(c) As a teacher, how would you respond to Jose and his drawing?

Appendix E (Continued)

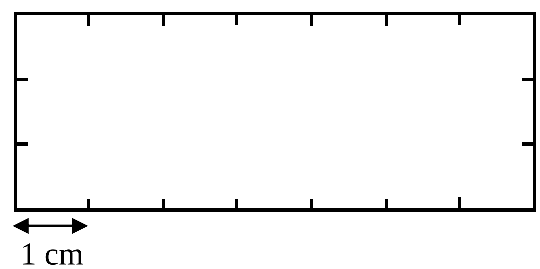
8. A student comes to you and says that he/she was able to draw several different rectangles that, according to the area formula, have an area of 36 in^2, but the student was a little surprised when the rectangles did not all look the same size.

(a) Are the student's results mathematically reasonable?

(b) As a teacher might, explain the reasons for your answer to Part (a).

(c) Why do you think the student was surprised by their results? What specifically would you say and do in response to this student's thinking?

9. A student calculates the area of the rectangle shown to be 20 square cm.



(a) Is the student correct?

If not what is the correct answer?

How did you figure your answer?

(b) What do you think the student was thinking to arrive at their answer?

(c) As a teacher, what specifically would you say or do to help clear up any possible confusions the student might have?

10. Marcus claims that it is only logical that if two different rectangular figures have the same perimeter they must have the same area.

(a) Is Marcus correct? Why?

(b) What do you think Marcus might have been thinking about in order to make his claim?

(c) As a teacher, how would you respond to Marcus' claim and his thinking?

APPENDIX F: AREA AND PERIMETER FOLLOW-UP TEST

DO NOT WRITE IN THIS AREA

Student Number: _________

NAME ________________________________

Classification ___________________________

Gender _______________________________

Appendix F (Continued)

NAME ___________________________________

Area and Perimeter Follow-Up Test

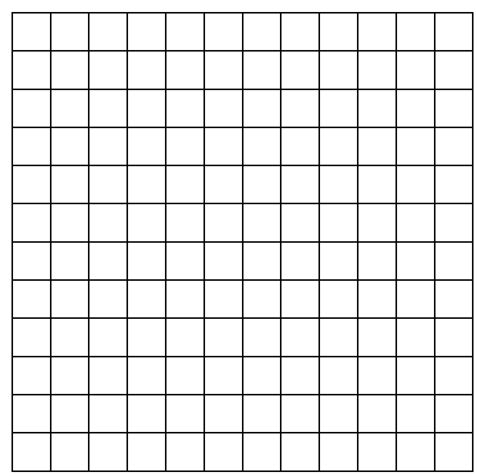
All explanations and diagrams should be appropriate for elementary-age students. **Please do not use a calculator. Be sure your answers include proper units. Write with pencil, and please write legibly.** Feel free to use the back of any page for comments regarding any questions you found confusing and explain why it confused you.

PART I: Content Knowledge (CK)

For questions 1 – 5: (1) Answer each question the best that you can. (2) It is very important to use thorough and detailed explanations to fully represent your knowledge.

1. (a) On the grid below, draw a polygon that has a perimeter of 24 units.

(b) How would you help a 4th grader understand that the polygon you drew really does have a perimeter of 24?



□ = 1 square unit

2. Present a real-world story problem, appropriate for 4th or 5th graders, in which they would need to find the area of a specified region. Provide the solution to your problem.

Appendix F (Continued)

3. What is the area and perimeter of Figure A?
(All corners are right angles.)

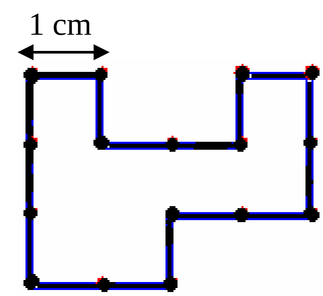


Fig. A

(a) area =

perimeter =

(b) Explain, as you would to a 4^{th} grader, how you arrived at both your answers.

<u>Area</u>

<u>Perimeter</u>

4. As a teacher, how would you explain the concepts of a *linear unit* and a *square unit* to a 5^{th} grader? Stress the differences in the concepts. <u>Include</u> a practical example of each (i.e., how they're used in the real world).

5. If each of the dimensions of a 2 x 4 rectangle is tripled, what various relationships between the original and the enlarged figures should be discussed with a class of 4^{th} or 5^{th} graders?

(a) Your answer?

(b) As a teacher, how would you present the explanation for how you arrived at your answer to a class of 4^{th} or 5^{th} graders?

Appendix F (Continued)

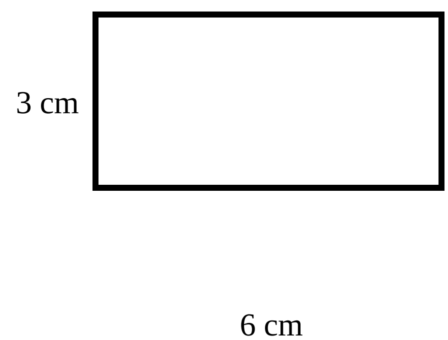
NAME ___________________________________

<u>Follow-up test PART II</u>: Knowledge of Student Thinking (KoST)

For problems 6 - 10, please address the following: **Part (a)** is a short answer – typically a "yes" or "no" response, **Part (b)** asks for you to explain <u>your</u> thinking, and **Part (c)** asks you to explain the student's or students' thinking - from the perspective of <u>a teacher</u>. If there are more than three parts, **please address each part thoroughly and separately.**

6. John, a 4th grader, calculates the area of the rectangle below. He arrives at an answer of 18.

(a) Is John's answer correct and complete?

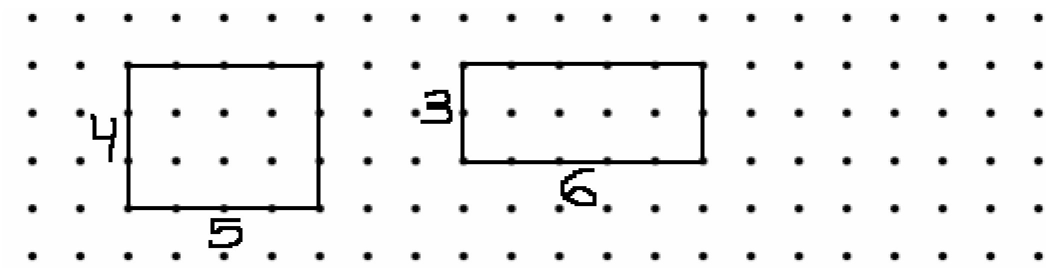


(b) <u>Explain</u> why or why not:

(c) After performing the calculation, John comes up to you looking puzzled and asks what exactly the "18" represents or means. As a teacher, how would you respond to John's question and his apparent confusion?

Appendix F (Continued)

7. Ariel, a 5th-grade student, was asked to draw all the four-sided dog-pen designs that she could make using 18 units of fence for each design. Below are the drawings, on dot paper, that she came up with.



(a) Is Ariel's answer correct and complete? Explain your answer.

(b) Explain what is correct and incorrect regarding Ariel's thinking, as evident in her work.

(c) As a teacher, how would you respond to Ariel? What precisely would you say and do?

8. Madison claims that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area.

(a) Is she correct? If you are unsure, are you skeptical or do you tend to believe her? Why?

(b) Explain why you agree or disagree with Madison's thinking.

(c) As a teacher, how would you respond to Madison? What specifically would you say and do?

Appendix F (Continued)

9. Jose wants to calculate the perimeter of the shape shown in Figure 1. Jose's method is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those squares.

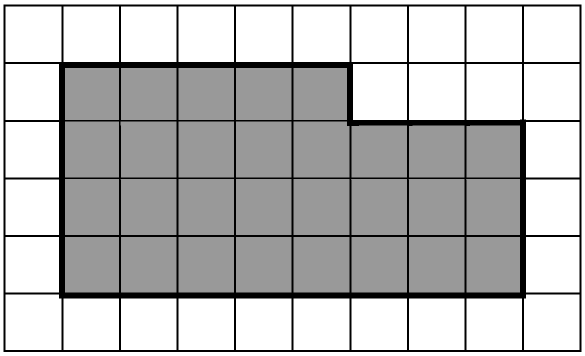


Fig. 1

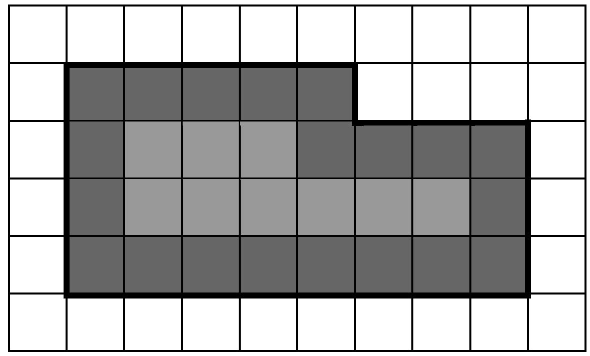


Fig. 2

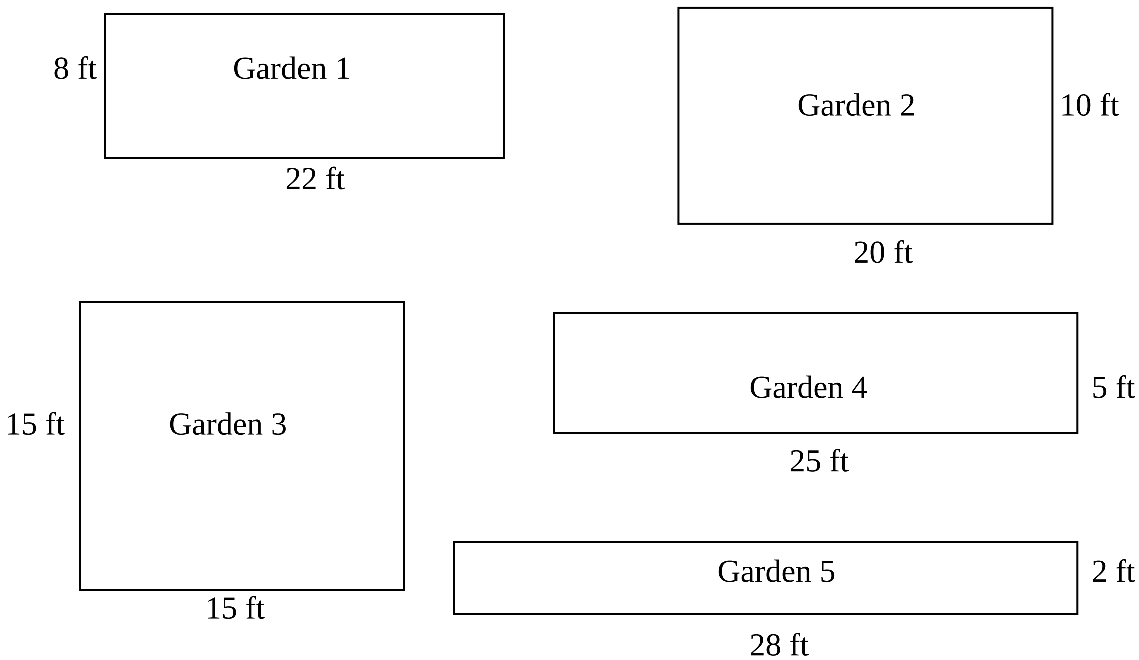
(a) Is Jose's method correct? If no, what would Jose's method produce for the perimeter of Fig. 1, and if necessary, state what is the correct answer?

(b) Explain why or why not.

(c) As a teacher, how would you respond to Jose's thinking and his method? What specifically would you say and do?

Appendix F (Continued)

10. Mrs. Smith purchased 60 feet of fence to enclose her flower garden. She wanted the garden to have a rectangular shape. She also wanted to have the most space possible for her garden. She drew out several possibilities, which are shown below.



Examine each of Mrs. Smith's drawings of her possible garden designs. For Part **(a)** place an "X" beside the numbered statement below that you believe to be true; Part **(b)** explain your selection for Part (a); and Part **(c)** is below.

______ 1. Garden 1 is the biggest garden.

______ 2. Garden 2 is the biggest garden.

______ 3. Garden 3 is the biggest garden.

______ 4. Garden 4 is the biggest garden.

______ 5. Garden 5 is the biggest garden.

______ 6. The gardens are all the same size.

(b) Explanation for Part **(a)**:

Part **(c)**: Which incorrect statement do you think would most often be selected by 4th or 5th graders? Please explain your choice. What might they be thinking and why?

APPENDIX G: PRELIMINARY RUBRICS FOR SCORING AREA AND PERIMETER TESTS

Scoring Rubric for Content Knowledge (CK) Questions

A score of **0, 1,** or **2** should be considered "unacceptable," while a score of **3** or **4** should be considered "acceptable."

0 for no response	**1 = unacceptable**	**2 = inferior/mediocre**	**3 = acceptable**	**4 = complete**
	The response is incomplete or contains many errors. Although some of the conditions of the task may have been addressed, an inadequate conclusion and/or faulty reasoning are present. Shows a very limited understanding of the problem's inherent mathematical concepts and procedures embodied by the task.	Although a correct approach, or even a correct solution, is provided, an essential understanding of the problem's underlying mathematical concepts are lacking. Indicates partial understanding of the problem's inherent mathematical concepts and/or procedures embodied in the task. The response contains errors related to misunderstanding important aspects of the task, misuse of the mathematical procedures, or faulty interpretations of results, and may contain some major computation errors.	An essentially correct response. Response indicates an essential, nearly complete (but less than thorough) understanding of the problem's inherent mathematical concepts & principles. Uses nearly correct mathematical terminology and notations. Computations are generally correct but may contain minor errors	A correct response. Response indicates a thorough and well-connected understanding of the problem's inherent mathematical concepts & principles. (The response may contain minor flaws which do not detract from the demonstration of a thorough understanding. Uses appropriate mathematical terminology and notations. Executes algorithms completely and correctly.

Appendix G (Continued)

Scoring Rubric for Knowledge of Student Thinking (KoST) Questions

A score of **0, 1,** or **2** should be considered "unacceptable," while a score of **3** or **4** should be considered "acceptable."

0 = no response	**1 = unacceptable**	**2 = inferior/mediocre**	**3 = acceptable**	**4 = complete**
	Response provides no, or incorrect, insight into the student's thinking; provides no, or incorrect diagnosis of student error(s). Offers no, or incorrect, examples, explanations, or representations that could serve as constructive feedback. Shows no understanding of the problem's mathematical concepts and principles.	Response provides limited insight into the student's thinking, **OR** provides limited diagnosis of student error(s) when present. Offers incomplete or partially incorrect examples, explanations, **OR** representations that provides constructive feedback Response contains errors related to misunderstandings of important mathematical concepts.	Response provides adequate insight into the student's thinking, **OR** provides adequate diagnosis of student error(s) – when present. Offers appropriate examples, explanations, **OR** representations that provides constructive feedback. Response is mathematically sound. It may contain minor computation errors but no conceptual ones.	Response provides thorough insight into the student's thinking, **AND** provides complete diagnosis of student error(s) – when present. Offers clear and complete examples, explanations, **AND** representations (when appropriate) that provides constructive feedback. Response is mathematically correct and contains no computational or conceptual errors.

Note. KoST includes using explanations focusing on building conceptual understanding.

APPENDIX H: AMENDED RUBRICS FOR SCORING AREA AND PERIMETER TESTS

Scoring Rubric for Content Knowledge (CK) Questions

A score of **0, 1,** or **2** should be considered "unacceptable," while a score of **3** or **4** should be considered "acceptable."

	1 = unacceptable	**2 = inferior/mediocre**	**3 = acceptable**	**4 = model response**
Assign a score of 0 when no meaningful response is provided. *See separate table for pertinent concepts and misconceptions.	The response is incomplete and contains many errors. Although some of the conditions of the task have been addressed, an inadequate conclusion and/or faulty reasoning are present. Very little, if any, conceptual understanding is evident. No concepts* or misconceptions* are adequately addressed. Uses completely incorrect terminology and notations. Provides incorrect and misleading procedures and computations. Includes incorrect and misleading diagrams or pictures.	A partly correct approach or partly completed solution. Indicates a partial understanding of the problem's inherent mathematical concepts* and/or misconceptions*. Mathematical terminology and notations reveal some misunderstandings. Misuses some procedures and contains some major computation errors. Fails to include, make reference to, or acknowledge the value of an appropriate picture or diagram, or it is only minimally helpful. A good explanation following a wrong response or a misunderstanding of the question, or a right response followed by a non-conceptual explanation.	Responds essentially correct to each part. Indicates an essential, nearly complete (but less than thorough) conceptual understanding of the problem's inherent mathematical concept(s)* and/or underlying misconception(s)*. May not contain conceptual errors or any incorrect extraneous information. Uses nearly correct mathematical terminology, notations, and explanations (may omit units). Computations are generally correct but may contain minor errors. Includes a helpful picture or diagram but it contains minor errors, **OR** includes a statement/picture where a picture/explanation would have been more helpful.	Correctly responds to each part of the question in a well-articulated manner. Comprehends the problem's inherent mathematical concept(s)* and underlying misconception(s)*. A plausible response that may contain minor flaws which do not detract from the demonstration of a thorough and conceptual understanding. May not contain conceptual errors or any incorrect extraneous information. Uses precise and complete mathematical terminology & computations and notations (MUST include appropriate units). States & executes algorithms completely and correctly. When necessary, includes diagrams or pictures that support and help to interpret, understand, and conceptualize the response.

Appendix H (Continued)

Scoring Rubric for <u>Knowledge of Student Thinking</u> (KoST) Questions

A score of **0, 1,** or **2** should be considered “unacceptable,” while a score of **3** or **4** should be considered “acceptable.”

	1 = unacceptable	**2 = inferior/mediocre**	**3 = acceptable**	**4 = model response**
Knowledge of Student Thinking - includes an explanation focusing on building conceptual understanding of mathematical content. **Assign a score of <u>0</u> when no meaningful response is provided.** *See separate table for concepts and misconceptions.	A partly correct response that provides <u>no</u>, <u>or incorrect</u>, insight into or diagnosis of the student’s thinking; and/or fails to address a major concept* or misconception.* A correct yes/no response followed by no, or incorrect, examples, explanations, or representations that could serve as constructive feedback. Shows no <u>clear</u> understanding of the problem’s mathematical concepts or the appropriate notation. Includes incorrect and misleading diagrams, pictures, or explanation.	A partly correct, procedurally-based, response that provides <u>limited</u> insight into or diagnosis of student thinking, **OR** addresses a <u>major</u> concept or misconception. Offers incomplete, <u>vague</u>, <u>partially correct</u> , or confusing examples, explanations, **OR** representations in an attempt to provide constructive feedback Contains errors related to: the problem’s concepts or misconceptions, notation, or the question itself. Fails to include, make reference to, or acknowledge <u>either</u> the value of an appropriate picture or diagram, or the <u>major concept</u> behind the question.	A mostly correct and conceptually-based response that provides <u>adequate</u> insight into the student’s thinking <u>and</u> diagnosis of student error(s) – when present, **AND** addresses a <u>major</u> concept <u>or</u> misconception. Appropriate examples, explanations, **OR** representations (1 of 3) that provide constructive or facilitative feedback. Is mathematically sound. It may contain <u>minor</u> computational or notational errors but <u>no conceptual ones</u>. Includes helpful picture or diagram but it may contain minor errors, **OR** response is sufficient but a picture or diagram would have been more helpful.	A completely correct and well-articulated response that: provides <u>thorough</u> insight into the student’s thinking, complete diagnosis of student error(s) – when present, **AND** addresses a <u>major</u> concept <u>and</u> a misconception **-** when both are present. Offers clear, complete, and plausible examples, explanations, **AND** representations (<u>2 of 3</u>) that provide constructive or facilitative feedback. Is mathematically correct and contains no computational, conceptual, or notational errors or omissions. Includes diagrams or pictures that support and help conceptualize the response, when necessary.

APPENDIX I: SUPPLEMENTAL GRADING SHEETS

Explanation of usage: As a result of piloting the scoring rubrics, supplemental grading sheets were created to assist in scoring items (especially the knowledge of student thinking questions) from the pre-, post-, and follow-up tests. The tables that follow summarize the major concepts and misconceptions that each item contains. They are not meant to be stand-alone scoring tools. When a scorer was unsure of which score to award to a certain item or teetering between scores, the supplemental grading sheets (SGS) proved very helpful in deciphering the most appropriate score. The following criteria, which appear in parts of the Content Knowledge and Knowledge of Student Thinking rubrics have been combined for sake of simplifying the explanation, is applied: (a) If a response fails to address either a major concept or a misconception listed in the SGS for that item, then a 1 is the highest score that item can receive, (b) If a response indicates partial or limited understanding of the mathematical content or the "student's thinking, or addresses either a major concept or misconception from the SGS, then that item could at most receive a score of 2, (c) If a response indicates adequate or nearly complete understanding of the mathematical content or the "student's thinking and addresses a major concept or misconception from the SGS, then that item could at most receive a score of 3, (d) If a well-articulated response indicates complete understanding of the problem's mathematical content and the thinking of the student presented in the problem and addresses both a major concept and misconception form the SGS, then that item is considered a model response and can be awarded the highest score possible of 4. These criteria represent part, albeit an important part, of the rubric used in the scoring process.

Appendix I (Continued)

Supplemental Grading Sheet for **Pretest**[1]

#	The question's major concept(s)	Potential major misconception(s)
1	1. Polygon has perimeter of 24 2. Appropriate explanation	1. Perimeter (P) versus area (A) 2. Linear units versus square units
2	1. Plausible context that requires finding area for the stated question	1. Addresses perimeter or volume instead of area
3	1. Correct A & P with correct units 2. Conceptual explanation	1. Applying A & P formulas for rectangle to irregular polygon
4	1. Conceptual differences between linear and square units (not A & P) 2. Good practical examples of each	1. Linear units are two-dimensional and square units are 3-D 2. Confusing which unit is used for which
5	1. Tripling both dimensions 2. Conceptually representing or explaining area increasing 9 times	1. The area will only triple 2. Scaling of "tripled" rectangle – does each side look 3 times larger?
6	1. Area requires square units (e.g., sq cm) 2. The 18 represents how many sq cm are needed to cover the rectangle	1. The rectangle also has an perimeter of 18 (cm) 2. The meaning of the 18 square cm
7	1. Using fence to build pens implies a perimeter measure 2. There are also a 2x7 & 1x8 dog pen possible	1. Understanding of a linear unit 2. Counting dots = finding perimeter 3. A 3x6 rectangle results in the number 18 for area AND perimeter.
8	1. Perimeter can be increased by increasing one dimension & decreasing the other 2. Provide appropriate counter example	1. Increasing perimeter of a rectangle will always increase the area (i.e. a direct relationship exists) 2. Not realizing that increasing perimeter CAN increase area
9	1. Discuss correct method for finding perimeter 2. Distinguishing between linear and square units 3. Explaining why Justin's method of using square units is incorrect.	1. Counting squares to figure perimeter is a correct procedure 2. Must have a formula to calculate perimeter
10	1. Squares ARE rectangles (units not needed for scratch work)	1. Same perimeters will have same areas 2. Basing greatest area on appearance

1 = Follow-up test is exactly the same as the pretest.

Appendix I (Continued)

Supplemental Grading Sheet for **Posttest**

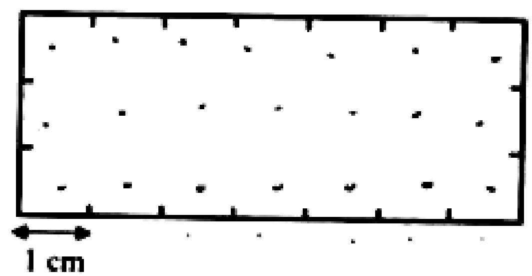
#	The question's major concept(s)	Potential major misconception(s)
1	1. Area relationship between figures 2. "Cover" implies to find area	1. Confusing area & perimeter 2. Confusing linear & square units
2	1. Plausible context that <u>requires</u> finding area for the stated question	1. Addresses perimeter or volume instead of area
3	1. Correct A & P <u>with</u> correct units 2. Conceptual explanation	1. Applying A & P formulas for rectangles and/or squares
4	1. Conceptual differences between linear and square units (not A & P) 2. Good practical examples of each	1. Linear units are two-dimensional and square units are 3-D
5	1. Figuring area from perimeter 2. Infinite possible answers (including a 4 x 4 square)	1. Fixed perimeter implies fixed area
6	1. Perimeter can be increased by increasing <u>one</u> dimension & decreasing the other 2. Provide appropriate counter example	1. Decreasing perimeter of a rectangle will <u>always</u> decrease the area (i.e. a direct relationship exists) 2. Not realizing that decreasing perimeter CAN decrease area
7	1. Linear units for perimeter 2. Rectangle shown has Per of 28	1. Using sq. units to represent Per 2. Confusing area with perimeter
8	1. Several factors of 36 produce the same area 2. Equal areas may have different perimeters (i.e. "look" different)	1. Expecting all rectangles with same area to have same perimeter 2. Figures with equal areas will all look the same (i.e. be the same size)
9	1. Conceptually represent the area	1. Confusing area with perimeter 2. Confusing linear and square units
10	1. Comparing 2 <u>different</u> rectangles with the <u>same</u> perimeter 2. Value of a counter example	1. Figures with same area will have the same perimeter, & vice versa

APPENDIX J: SAMPLES OF TEST ITEMS FROM PILOTING TO ILLUSTRATE SCORING

All samples involve the same Knowledge-of-Student-Thinking type question to illustrate what elements of a response result in different scores.

The following response earned a score of 1 based on the Knowledge of Student Thinking (KoST) rubric. The answer is “partly correct” because parts (a) and (b) were not attempted. It appears the preservice teacher was using dots to possibly count square units, but because nothing was said regarding that, no credit could be awarded.

A student calculates the area of the rectangle shown to be 20 square cm.



(a) Is the student correct?

No!

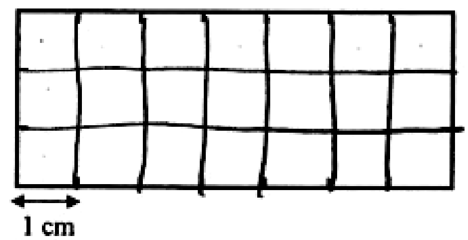
(b) What do you think the student was thinking to arrive at their answer?

(c) As a teacher, how would you respond to them?

Appendix J (Continued)

The following response earned a score of 2 because although part (a) is correct, parts (b) and (c) are only partly correct. The response to part (b) failed to acknowledge that the student most likely came up with an answer of 20 because they were calculating perimeter and not just because they miscounted. The preservice teacher's response to part (c) is procedural in nature (i.e. focuses on using a formula) in contrast to a conceptual approach which would encourage the counting of the square units to find area.

A student calculates the area of the rectangle shown to be 20 square cm.



(a) Is the student correct?

no

(b) What do you think the student was thinking to arrive at their answer?

they simply counted the squares but miscounted

(c) As a teacher, how would you respond to them?

You should multiply 3X7 which is 21 not count the squares

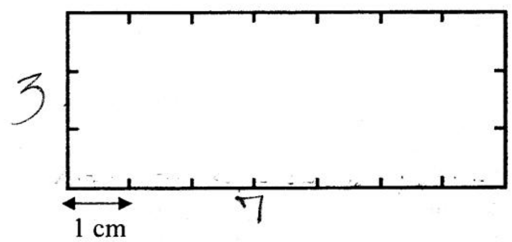
Appendix J (Continued)

The following response earned a score of 3. Part (a) is correct and in part (b) the preservice teacher correctly identified that the student was calculating perimeter as opposed to area (even though they did not specifically write that). The answer to part (c) is what keeps this response from being considered “model.” Again, the focus is on a procedural explanation (i.e. using the LxW formula) which is not best suited or the most meaningful for a student exhibiting misunderstandings. A conceptual approach would involve drawing in the 3x7 grid and revealing the 21 square centimeters and drawing the student’s attention to those square units.

A student calculates the area of the rectangle shown to be 20 square cm.

(a) Is the student correct?

no



(b) What do you think the student was thinking to arrive at their answer?

They were adding the length and the width then multiplying by 2.

(c) As a teacher, how would you respond to them?

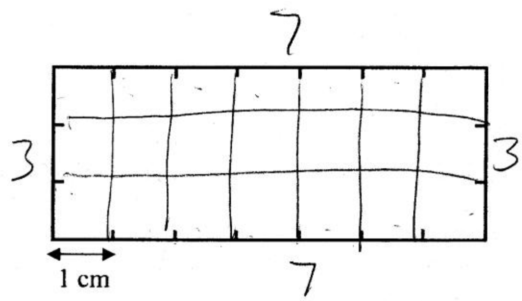
well if the length is 7 or 7 squares and the width is 3 or 3 squoures how would your answer be 20? What is the formula for Area? So LXW = 7X3. = 21

Appendix J (Continued)

The following response was determined to be “model” and earned a score of 4. Part (a) is correct and part (b) correctly states the student was most likely confused area with perimeter. The preservice teacher’s response to part (c) was conceptually orientated and well said. They mentioned the importance of connecting the concept of square units with finding area and also drew in the square units in the rectangle. The response also made a point to differentiate what it means to find area from that of finding perimeter.

A student calculates the area of the rectangle shown to be 20 square cm.

(a) Is the student correct? NO



(b) What do you think the student was thinking to arrive at their answer?

The student was thinking that if you add all 4 sides that would be the area, The student confused Area with Perimeter.

(c) As a teacher, how would you respond to them?

As a teacher I would tell the student that they have to connect the lines and make square units This would make it a whole lot easier to calculate the area (inside). I would also try to drill in their heads that area is counting the inside square units and perimeter is adding the sides.

APPENDIX K: LEARNING PACKETS FOR TEACHING EPISODES

Note: The spacing for teaching episode #1 will be very similar to the one used for the study; however, teaching episodes 2 and 3 will be condensed to save space.

Teaching Episode #1: Units of Measure

While involved with the teaching of elementary mathematics, you will also be continually learning about mathematics – about the subtle notions underlying the structure and concepts as well as what students find difficult about learning and doing mathematics. So in reality, a teacher is also a student.

When however you do assume the role of classroom teacher, you will often be faced with situations in which students produce responses or ask questions that will stretch the limits of your knowledge and understanding of elementary mathematics and how to help students understand it. Today you will encounter one such situation. Taking the time to reflect upon (i.e. ponder or think about) your knowledge and how it impacts your instructional decisions is a necessary and vital part of becoming an effective teacher; therefore, throughout this learning experience you will be asked to pause and reflect upon your current understanding of the problem, questions you are working through, possible misconceptions you may have had regarding the problem's concepts and how you resolved them, and the resulting changes in your knowledge of the concepts at hand. Such activity is vital to developing and maturing into an insightful, responsive, and effective communicator of elementary mathematics.

Appendix K (Continued)

Name: ______________________________________

Teaching Episode #1

The Setting:

You are a fifth grade teacher, and you have just begun a review of basic area and perimeter concepts that your students had explored in fourth grade. You present your students with what you believe will be a rather easy task: **"Calculate the perimeter of the shape in Figure 1."**

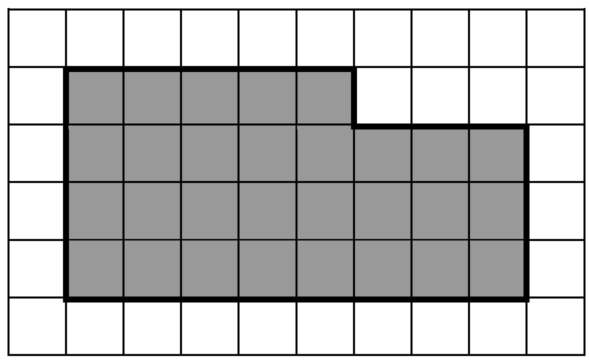


Figure 1

The Situation: One of your students, Justin, shows you his method which is to shade the squares along the outside of the shape, as shown in Figure 2, and then to count those shaded squares.

1. What was the first thing you did after reading through this situation?

Why did you do that?

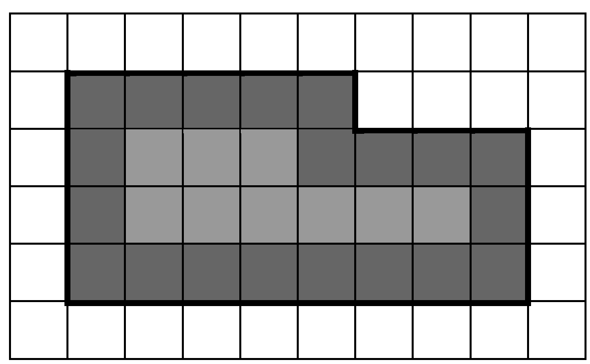
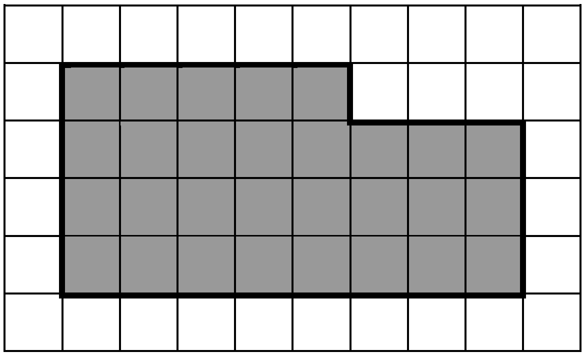


Figure 2

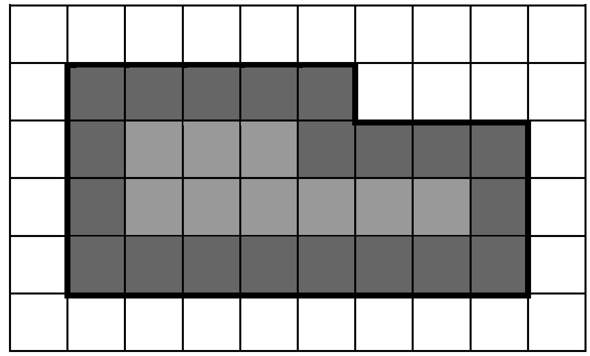
2. What answer will Justin's method produce?

3. Is Justin's method correct? If no, what is the correct answer?

Appendix K (Continued)



(Students asked to find perimeter)



(Justin's method)

4. Explain, mathematically speaking, what is correct or incorrect about Justin's method.

Time to Reflect:

Please take a moment and write down your initial thoughts regarding the problem to this point.

5. As a teacher, how would you respond to Justin's thinking and his method? What specifically would you say and do?

Appendix K (Continued)

Name: ______________________________________ (Day 1 cont.)

<u>Your Investigative Tool</u>: (Researcher will read what appears in quotes. It will not appear on the student's version)

"Being aware of and willing to use various manipulative tools to enhance the teaching and learning of elementary mathematical concepts is a trait of successful teachers. Such tools can be instrumental in deepening your own personal understanding of the mathematical concepts you must teach. Some of these tools can be found on the Internet in the form of Java applets called *microworlds*. They are interactive and designed to help you visualize and analyze the various concepts surrounding today's learning experience, you will have access to such an applet which has been specially designed to explore concepts related to area and perimeter."

Use the microworld to explore patterns, test your hypotheses, and generate helpful representations for your solutions and your explanations. **Include appropriate sketches of your microworld designs to help illustrate and explain your thinking.**

<u>Please begin by following these directions</u>:

1. Open the Internet, and enter the website for EDU 316.
2. Under the "Course Links" section, open the "Area & Perimeter microworlds" folder.
3. Click on the link titled, "Shape Builder microworld."

I would like you to thoughtfully consider your previous responses to questions 1 – 5. As you do so imagine you have the ability to use and display the Internet applet when personally thinking about this problem, when working individually with a student, and when addressing the entire class. After exploring and investigating with the microworld, and the questions below:

6. What, if anything, would you add or revise from your responses to questions **1** – **4**?

Appendix K (Continued)

7. What, if anything, would you add or revise to your response to question **5**?

Time to Reflect:

As you think about yours and Justin's thinking regarding this problem, remember to document (in the ***Time to Reflect*** sections) specific questions and ideas (including false starts) you have thought about and explored with the *microworld*. Share details regarding how you decided what to say and show to Justin, including specific examples to represent how and what you would communicate. For example, you could include statements such as, "While exploring with the ***applet***, I came to realize that my understanding concerning . . . was not completely correct. Originally I thought . . ., but now I realize that . . ." Then describe how the *applet* may have influenced your new understanding – include specific drawings of *applet* designs (or discuss specific features of the *applet*) that helped you.

Please take a moment and address questions 8 - 12:

#8. What do you think students might find difficult about finding the perimeter of the shape shown in Figure 1? What could confuse them?

Appendix K (Continued)

#9. In what ways, if any, did interacting with the microworld help you better understand the ideas surrounding this problem and Justin's thinking? In other words, what did you do and how did it help.

#10. As a result of seeing Justin's method and apparent confusion regarding units of measure, how would you follow up with the entire class about the concepts that surround this classroom episode? Remember, share specific examples and representations (possibly from the microworld) just as you would in the classroom, as well as why you choose what to say and do.

#11. Do you think elementary students could benefit from personally interacting with the microworld while learning about today's concepts? In what ways? **(If you think no, please see #12).**

#12. If you said "no" to #11, please share why, and then tell what instructional tool(s) and/or strategies you feel would be more appropriate for the concepts investigated today.

Appendix K (Continued)

Name: ______________________________________ (Day 2)

Time to Work Together: ***You will now be asked get into cooperative groups.***

13. Who are your "Share & Compare" partners?

a.

b.

Take the next several minutes and have each group member share how they arrived at their solutions for questions **1 – 5** (pp. 2-3) as well as the two questions on page 4 pertaining to *Shape Builder*. As each member shares, the other members should compare what they are hearing with their personal responses. Make notes under the "Shared Knowledge" header to include ideas, insights, and instructional strategies that were not part of, or are extensions of, your responses. Indicate from whom you gained the new ideas and how these ideas have influenced your thinking.

Shared Knowledge

14. What new knowledge did you gain regarding questions 1 – 5 (pp. 2-3)?

Appendix K (Continued)

Shared Knowledge cont.

15. What new knowledge did you gain regarding the two *Shape Builder* questions (#'s 6, 7, & 9) on pp. 4 & 6?

16. What new knowledge did you gain from your group regarding questions 8 & 10?

Appendix K (Continued)

Grand Discussion

17. After the group sharing is done, your instructor will conclude with a brief summary. Again, in the space provided below write down anything presented that added to:

(a) Your understanding of the concepts surrounding today's teaching scenario,

(b) Your knowledge of student thinking and the specific difficulties they can have with area and perimeter, and

(c) Your knowledge of potential teaching strategies to help address student thinking related to these concepts.

Appendix K (Continued)

Teaching Episode #2: Fixed Area & Perimeter

INTRODUCTION: While involved with the teaching of elementary mathematics, you will also be continually learning about mathematics – about the subtle notions underlying the structure and concepts as well as what students find difficult about learning and doing mathematics. So in reality, a teacher is also a student.

When however you do assume the role of classroom teacher, you will often be faced with situations in which students produce responses or ask questions that will stretch the limits of your knowledge and understanding of elementary mathematics and how to help students understand it. Today you will encounter one such situation. Taking the time to reflect upon (i.e. ponder or think about) your knowledge and how it impacts your instructional decisions is a necessary and vital part of becoming an effective teacher; therefore, throughout this learning experience you will be asked to pause and reflect upon your current understanding of the problem, questions you are working through, possible misconceptions you may have had regarding the problem's concepts and how you resolved them, and the resulting changes in your knowledge of the concepts at hand. Such activity is vital to developing and maturing into an insightful, responsive, and effective communicator of elementary mathematics.

Appendix K (Continued)

Name: ___________________________________ (Day 1)

Teaching Episode #2

The Setting: (adopted from Bassarear, 2005, p. 677)

Your 5th grade class is studying area, and you challenge them to find the area of one of their footprints. You instruct your students to stand on a piece of paper and trace their shoe, and then individually brainstorm a strategy to find the area of the footprint.

The Situation:

After several minutes one of your students, Tommy, comes up to you and explains his method. He says he would lay a piece of string around the outside of the paper footprint, cut the string to the precise length, form the piece of string into a rectangle, use a ruler to measure the length and width of the rectangle, then find the area of the rectangle. In other words, he believes that the area of the rectangle will be the same as the area of his footprint." [Each participant will be provided with a copy of a footprint drawn on square-inch grid paper (its perimeter is approximately 18"), two pieces of inch grid paper, and an 18" piece of string.]

1. What was the first thing you did after reading through this situation?

Why did you do that?

2. What are your initial thoughts regarding Tommy's method?

3. Will Tommy's method produce the correct answer? If no, why not?

4. Explain, mathematically speaking, what is correct or incorrect about Tommy's method.

Appendix K (Continued)

5. What is one way (other than Tommy's) to figure out how much area the footprint covers? Try to describe a <u>second</u> way to find the area of the footprint.

(page break)

Time to Reflect:

Please take a moment and write down your current thoughts regarding the mathematics surrounding this problem as well as Tommy's strategy. Has your knowledge and or understandings changed from when you began working on this problem? If so, please share these changes.

6. As a teacher, how would you respond to Tommy's thinking and his strategy? What specifically would you say and do?

(page break)

Name: ____ (Day 1 cont.)

<u>Your Investigative Tool</u>: (Researcher will read what appears in quotes. It will not appear on the student's version)

"Being aware of and willing to use various manipulative tools to enhance the teaching and learning of elementary mathematical concepts is a trait of successful teachers. Such tools can be instrumental in deepening your own personal understanding of the mathematical concepts you must teach. Some of these tools can be found on the Internet in the form of Java applets called *microworlds*. They are interactive and designed to help you visualize and analyze the various concepts surrounding today's learning experience, you will have access to such an applet which has been specially designed to explore concepts related to area and perimeter."

Use the microworld to explore patterns, test your hypotheses, and generate helpful representations for your solutions and your explanations. **Include appropriate sketches of your microworld designs to help illustrate and explain your thinking.**

Appendix K (Continued)

Please begin by following these directions:

1. Open the Internet, and enter the website for EDU 316.
2. Under the "Course Links" section, open the "Area & Perimeter microworlds" folder.
3. Open both microworlds. You will be provided with login information.

Part of becoming a professional educator is becoming proficient at selecting the most appropriate instructional tool(s) for a specific learning outcome. With that in mind, please access either microworld, and thoughtfully consider your previous responses to questions 1 – 6. As you do so imagine you have the ability to use and display the microworlds while personally thinking about this problem, while working individually with a student, and when addressing the entire class. After exploring and investigating with the microworlds, answer the questions below.

7. What, if anything, would you add or revise from your responses to questions **1** – **5**?

8. What, if anything, would you add or revise from your response to question **6**?

(page break)

Now I would like you (functioning as both a learner of mathematics as well as a teacher) to thoughtfully answer questions 9 – 14.

#9. What mathematical concepts are involved with finding the area of a footprint?

#10. What do you think students might find difficult about finding the area of their footprint? What specifically could confuse them?

(page break)

#11. In what ways, if any, has interacting with the microworlds influenced your thoughts related to this problem? How has your thinking changed up to this point; both your personal understandings regarding the concepts in this problem and your knowledge related to Tommy's method? Remember, please be specific and provide examples – be sure and specify what microworld you are referring to.

(page break)

#12. As a result of seeing Tommy's method and apparent lack of complete understanding regarding the perceived direct relationship between perimeter and area, how would you follow up with the entire class about the concepts that surround this classroom episode? Share specific examples and representations (possibly from a *microworld*) just as you would in the classroom. Be sure and tell why you choose what to say and do.

Appendix K (Continued)

#13. Do you think elementary students could benefit from personally interacting with the microworlds while learning about today's concepts? If yes, in what ways? **(If you think "no," please see #14).**

#14. If you said "no" to #13, please share why, and then tell what instructional tool(s) and/or strategies you feel would be more appropriate for the concepts investigated today.

(page break)

Name: ____ (Day 2)

Time to Work Together:

You will now be asked to get into cooperative groups.

15. Who are your "Share & Compare" partners?

a.

b.

Take the next several minutes and have each group member share how they arrived at their solutions for the questions stated in problems 16-18. <u>As each member shares</u>, the other members should compare what they are hearing with their personal responses. <u>Make</u> <u>notes</u> under the "Shared Knowledge" header to include ideas, insights, and instructional strategies that were not part of, or are extensions of, your personal responses. Indicate from whom you gained the new ideas <u>and how these ideas have influenced your thinking</u>.

Shared Knowledge

16. What new <u>mathematical knowledge</u> did you gain regarding questions 1 – 5 on pp. 1-3 and #9 on p. 6?

(page break)

17. What new knowledge did you gain regarding the use of the TWO microworlds (see questions 7, 8, & 11 on pp. 5 & 7)? Be sure and specify what microworld your are referring to. <u>Use</u> "SB" when referring to *Shape Builder* and "Giz" when referring to the *ExploreLearning Gizmo*.

Appendix K (Continued)

18. What new knowledge did you gain from your group regarding *student thinking* (see questions 10 & 13) <u>and</u> *instructional practices* (see questions 6 & 12)?

(page break)

Grand Discussion

19. After the group sharing is done, your instructor will conclude with a brief summary. Again, in the space provided below write down anything presented that added to:

(a) Your understanding of the **<u>mathematical concepts</u>** surrounding today's teaching scenario,

(b) Your knowledge of **<u>student thinking</u>** and the specific difficulties they can have with area and perimeter, and

(page break)

(c) Your knowledge of **<u>potential teaching strategies</u>** to help address student thinking related to these concepts. Please be specific.

<u>Concluding Question</u>: Did you access either microworld outside of class?
If yes, why?

Appendix K (Continued)

Teaching Episode #3: A Direct Relationship?

While involved with the teaching of elementary mathematics, you will also be continually learning about mathematics – about the subtle notions underlying the structure and concepts as well as what students find difficult about learning and doing mathematics. So in reality, a teacher is also a student.

When however you do assume the role of classroom teacher, you will often be faced with situations in which students produce responses or ask questions that will stretch the limits of your knowledge and understanding of elementary mathematics and how to help students understand it. Today you will encounter one such situation. Taking the time to reflect upon (i.e. ponder or think about) your knowledge and how it impacts your instructional decisions is a necessary and vital part of becoming an effective teacher; therefore, throughout this learning experience you will be asked to pause and reflect upon your current understanding of the problem, questions you are working through, possible misconceptions you may have had regarding the problem's concepts and how you resolved them, and the resulting changes in your knowledge of the concepts at hand. Such activity is vital to developing and maturing into an insightful, responsive, and effective communicator of elementary mathematics.

Your Investigative Tools:

Being aware of and willing to use various manipulative tools to enhance the teaching and learning of elementary mathematical concepts is a trait of successful teachers. Such tools can be instrumental in deepening your own personal understanding of the mathematical concepts you must teach. Some of these tools can be found on the Internet in the form of Java applets called *microworlds*. They are interactive and specially designed to help you visualize and analyze the various concepts surrounding today's learning experience. For this teaching episode, you may use either microworld from the outset to explore patterns, test your hypotheses, and generate helpful representations for your solutions and your explanations. **Include appropriate sketches of your microworld designs to help illustrate and explain your thinking.**

Please begin by following these directions:

1. Open the Internet, and enter the website for EDU 316.
2. Under the "Course Links" section, open the "Area & Perimeter applets" folder.
3. Click on and open both microworlds. **You may use them from the beginning.**

Appendix K (Continued)

Name: ________________________________

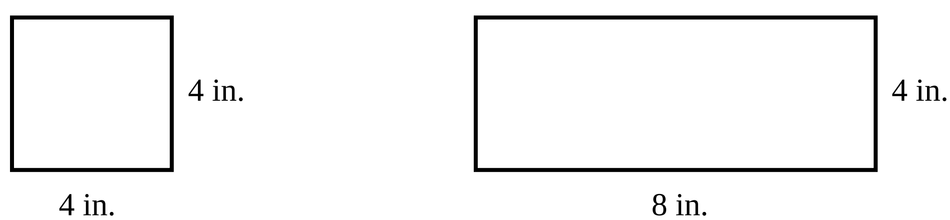
Teaching Episode #3

The Setting:

You have just completed the last scheduled unit on area and perimeter with your 5th grade class. You feel they understand the concepts pretty well. While the students are working at their desks on that day's mathematics homework, one of your students, Jasmine, comes up to you very excited.

The Situation:

Jasmine then tells you that she has figured out a "new theory" that you never told the class about. She explains that she has discovered that whenever you compare two rectangles, the one with the greater perimeter will always have the greater area.
She shows you this picture as proof of what she is saying:



perimeter = 16 in.
area = 16 square in.

perimeter = 24 in.
area = 32 square in.

1. What was the first thing you did after reading through this situation?

Why did you do that?

2. What are your initial thoughts regarding Jasmine's "theory"?

Appendix K (Continued)

3. Is Jasmine's theory correct? If no, why not?
If you are unsure, are you skeptical or do you tend to believe it? Why?

4. Explain, mathematically speaking, what is correct or incorrect about Jasmine's theory.

(page break)

Time to Reflect:

Please take a moment and write down your current thoughts regarding the mathematics surrounding this problem as well as Jasmine's theory. Has your knowledge and or understandings changed from when you began working on this problem? If so, please share these changes.

5. As a teacher, how would you respond to Jasmine's thinking and her proposed theory? What specifically would you say and do (even if you are unsure about the mathematics involved)?

(page break)

Now, if you have not already done so, please access either, or both, of the microworlds available to you. I would like you to thoughtfully consider your previous responses to questions 1 – 5. As you do so, imagine you have the ability to use and display the microworlds while personally thinking about this problem, while working individually with a student, and when addressing the entire class. Include appropriate sketches of your microworld designs to help illustrate and explain your thinking. After exploring and investigating with the microworlds, answer questions 6 & 7.

6. What, if anything, would you add or revise from your responses to questions **1** – **4**?

7. What, if anything, would you add or revise from your response to question **5**?

(page break)

Appendix K (Continued)

Now I would like you to thoughtfully answer questions 8 – 12.

#8. Do you think many students may have the same incomplete understanding as Jasmine? If so, what do you think might be the cause? When answering, consider the student's mathematical knowledge as well as possible instructional techniques commonly used.

#9. In what ways, if any, did interacting with the microworlds help you better understand the ideas surrounding this problem and Jasmine's thinking? In other words, what did you do and how did it help. Remember, please be specific - provide examples. Be sure and share which microworld (and what features) helped with what ideas or concepts.

(page break)

#10. As a result of seeing Jasmine's theory and apparent lack of complete understanding regarding the perceived direct relationship between perimeter and area, how would follow up with the entire class about the concepts that surround this classroom episode? Remember, share specific examples and representations (possibly from a *microworld*) just as you would in the classroom. Be sure and tell me <u>why</u> you choose what to say and do.

#11. Do you think elementary students could benefit from personally interacting with the microworlds while learning about today's concepts? If yes, in what ways?

(If you think "no," please see #12).

#12. If you said "no" to #11, please share why, and then tell what instructional tool(s) and/or strategies you feel would be more appropriate for the concepts investigated today.

(page break)

Appendix K (Continued)

Name: ______________________________________ (Day 2)

Brief A & P Review:

1. How do you find the perimeter of a rectangle?

2. How do you find the area of a rectangle?

Time to Work Together: ***You will now be asked to get into cooperative groups.***

13. Who are your "Share & Compare" partners?

a.

b.

Take the next several minutes and have each group member share how they arrived at their solutions for the questions stated in problems 14-18. As each member shares, the other members should compare what they are hearing with their personal responses. Make notes under the "Shared Knowledge" header to include ideas, insights, and instructional strategies that were not part of, or are extensions of, your personal responses. Indicate from whom you gained the new ideas and how these ideas have influenced your thinking.

Shared Knowledge

14. What new mathematical knowledge did YOU gain regarding questions 1 – 4 on pp. 1 & 2?

(page break)

15. What new knowledge did YOU gain regarding the use of the TWO microworlds (see questions 6, 7, & 9 on pp. 4, & 5)? Be sure and specify what microworld your are referring to. Use "SB" when referring to *Shape Builder* and "Giz" when referring to the *ExploreLearning Gizmo*.

(page break)

Appendix K (Continued)

16. What new knowledge did YOU gain from your group regarding: (a) *student thinking (*see questions 8 & 11) <u>and</u> (b) *instructional practices* (see questions 5 & 10)?

(page break)

17. As a result of hearing the ideas of your group members, what is YOUR current opinion of Jasmine's "new theory?" What is that opinion based on?

(page break)

Grand Discussion

18. After the group sharing is done, your instructor will conclude with a brief summary. Again, in the space provided below write down anything presented that added to:

(a) Your understanding of the **<u>mathematical concepts</u>** surrounding today's teaching scenario,

(b) Your knowledge of **<u>student thinking</u>** and the specific difficulties they can have with area and perimeter, and

(c) Your knowledge of **<u>potential teaching strategies</u>** to help address student thinking related to these concepts. Please be specific.

<u>Concluding Question</u>: Did you access either applet outside of class? If yes, why?

APPENDIX L: SECOND OBSERVER PROTOCOL

During the *Teaching Episode*, please record your observations of instructional activities as well as the activities of the preservice teachers. Please make special note of activity that reflects the preservice teacher's content knowledge regarding area and perimeter as well as their pedagogical content knowledge (specifically, knowledge of student thinking). <u>Please pay careful attention as to</u>: (a) how the preservice teachers' go about making sense of the teaching scenario, (b) how they make use of the *applet* while problem solving, and (c) how they interact cognitively with their peers.

<u>Indicate behavior as focusing on</u>: 1. Content knowledge regarding area & perimeter (CK), or 2. Knowledge of student thinking (PCK)	<u>Personal Insights & Interpretations</u>

APPENDIX M: MICROWORLDS ORIENTATION SESSION

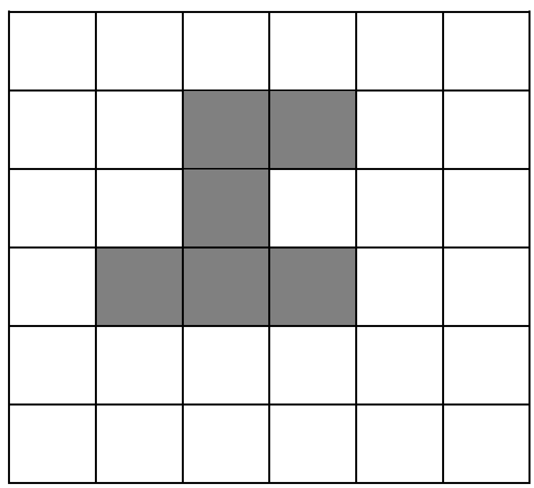
MICROWORLDS ORIENTATION SESSION

NAME: ____

Open the ***ShapeBuilder*** microworld and follow the instructor while you are guided on an overview of the microworld's features.

Use the microworld to help answer question #1. Please document what features you used and which ones help you in solving the problem.

1. Add, by shading, at least one square to the grey figure below so that your new figure also has a perimeter of 14 units. (More than one answer is possible.)



= 1 square unit

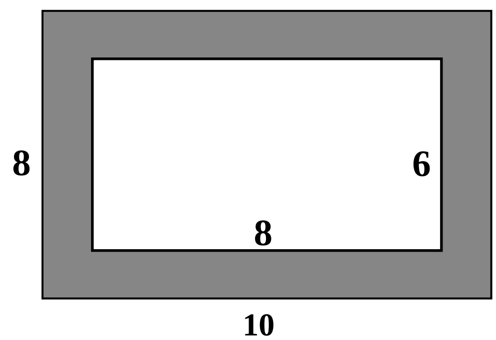
Summary of microworld usage:

Appendix M (Continued)

Now open the area & perimeter microworld from *ExploreLearning*. Once again, please follow the instructor as you are guided through the many features of this applet.

Use the applet to answer question #2. Please document what features you used and which ones help you in solving the problem.

2. What is the area of the shaded region? (Each measure is in inches.)
   Explain how you arrived at your answer.



<u>Summary of applet usage</u>:

APPENDIX N: PURPOSELY SELECTED TASKS FOR FINAL INTERVIEW

Name ______________________________________

Task 1

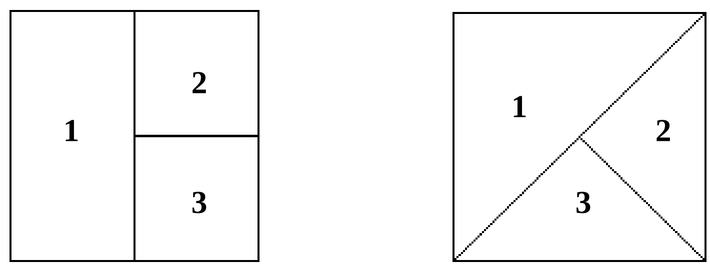
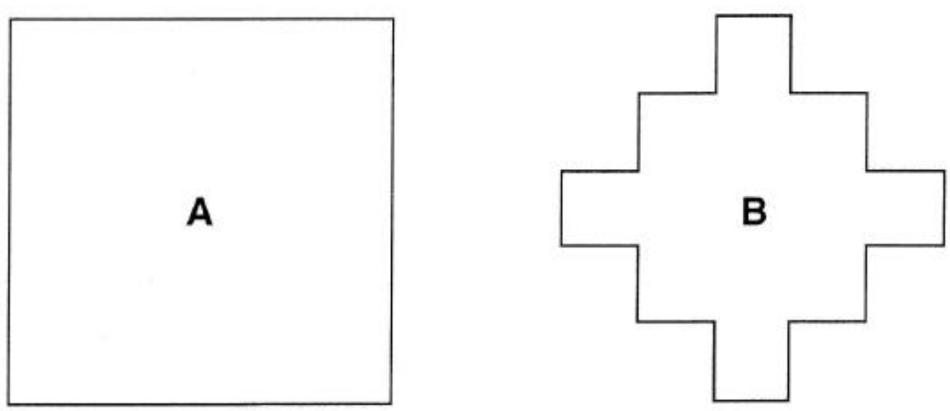


Figure 1 Figure 2

Examine Figures 1 & 2. Assuming Figures 1 & 2 are congruent squares, what relationships do you notice between the rectangles 1, 2, & 3 and the triangles 1, 2, & 3?

Task 2

Given the fact that shape A and shape B have the same length and width, which shape will have the greater perimeter? Why?



APPENDIX O: Anchored Instruction Assessment Survey

What follows is a checklist designed to elicit from you (the expert reviewer) the degree to which you: (1) agree with the definition of anchored instruction (as operationally defined by me, the researcher), (2) are able to identify the elements (design principles) of anchored instruction in my materials, and (3) anticipate that my materials and procedures will cause anchored instruction to happen for my participants.

Below each section of the survey you will have the opportunity to provide qualitative input regarding your selections. For example, answering questions such as: Why? Why not? How might it be improved? Your suggestions for improving my materials and procedures are welcomed and appreciated.

Section 1 ~ **My Definition of Anchored Instruction**

> Text-based *Teaching Episodes* (i.e., a series of three, spanning 6 class periods) will present authentic, problem-solving scenarios anchored around common difficulties and misconceptions elementary students (and teachers alike) have regarding area and perimeter. The Teaching Episodes will be enhanced and supported by two geometry microworlds whose features should promote sustained exploration of each classroom-based scenario from multiple points of view.

Appendix O (Continued)

Please indicate by placing an "X" in the appropriate box that best describes the degree to which you agree with the above definition of Anchored Instruction.

	Strongly Agree	Agree	Disagree	Strongly Disagree
My definition of Anchored Instruction				

If you Disagree or Strongly Disagree with my definition, please share why.

Appendix O (Continued)

Section 2 ~ **The Anchor**

According to the literature, an appropriate "anchor" to use within an anchored instructional setting should:

1. be a macro-contextual video-based anchor capable of random accessibility – videodiscs were chosen by the CTGV (e.g., The Sherlock Project & the Jasper Projects) (Bransford et al., 1989; CTGV, 1990, 1992b, 1992c, 1993). ***"We do not mean to imply that the anchors in anchored instruction must always be based on video. Case-based approaches to instruction provide an excellent illustration of anchored instruction that relies on a verbal (or textual) mode"*** (Bransford, 190b, p. 398). Such approaches have met with great success in business schools. The CTGV felt however that video would provide richer sources of information better suited for school students.
2. develop within a narrative format
3. promote broad transfer (i.e., by promoting an explicit emphasis on analyzing similarities and differences among problem situations, and on bridging to new areas of application, facilitates the degree to which spontaneous transfer occurs (CTGV, 1992)
4. help students notice the features of problem situations that make particular actions relevant. In order to appropriately conditionalize their knowledge, the anchors for instruction must help students focus on the relevant features of the problems they are trying to solve (Bransford et al., 1990a).
5. allow participants to experience the kinds of problems and opportunities that experts in various areas encounter (e.g., classroom teachers interacting with a student who has a misconception related to material being taught) (Goldman et al., 1996).
6. involve complex situations that require students to formulate and solve a set of interconnected subproblems (Bransford, Sherwood, & Hasselbring, 1988)

Appendix O (Continued)

In this study, the "anchor" for instruction will be Teaching Episodes (i.e., a series of three, spanning 6 class periods) which address common difficulties and misconceptions elementary students (and teachers alike) have regarding area and perimeter. (Please refer to the documents included in your packet).

Please indicate by placing an "X" in the appropriate box that best describes the degree to which you feel the anchor (situated within the Teaching Episode) of my study captures the essence and addresses the goals of an "anchor" as expressed by the designers of Anchored Instruction.

	Strongly Agree	Agree	Disagree	Strongly Disagree
My selection for the anchor				

If you Disagree or Strongly Disagree with my choice for an anchor, please share why.

Appendix O (Continued)

Section 3 ~ **Design Principles** (Principles presented by McLarty et al., 1989 & CTGV, 1997)

#1. Choosing an appropriate anchor. (Addressed separately above)

#2. Possess a generative learning format. The anchored environment involves complex situations that create a meaningful context for problem solving.

#3. Developing shared expertise around the anchor. Students (or preservice teachers) need multiple opportunities to view the anchor and be engaged in problem solving. Discussion based upon the shared context of the anchor helps students comprehend and organize the information.

#4. Expanding the anchor. One anchor may not meet all the learning objectives that have been set forth. Students may need more than one experience with the anchor to enable acquiring more balanced information which could facilitate comparisons or contrasts between anchored experiences.

#5. Using knowledge as a tool. The anchor provides students with a meaningful context from which they acquire new information; such opportunities increases student's ability to transfer concepts from one context (e.g., a Teaching Episode) to another (e.g., the actual classroom).

#6. Merging the anchor. The anchor will provide opportunities for using oral language, reading, writing, and participating in other literacy-related skills (e.g., cooperative work and classroom discussion).

#7. Allowing student exploration. Giving students/preservice teachers access to, and opportunities to explore, the elements and concepts surrounding the anchor helps them to develop a sense of expertise. (Examining the microworlds and their features would be encouraged). Realize participants will experience a pre-study orientation session designed to acquaint them with the various features of both microworlds used in this study.

Appendix O (Continued)

#8. Provide opportunity for participants to share what was learned from the anchored instruction. My study addresses this design principle by incorporating features of an instructional model by Wales and Stager's (1997) called Guided Design (see Conceptual Framework document included in your packet)
Please indicate by placing an "X" in the box that best describes the degree to which you feel the design principles of Anchored Instruction are addressed by the materials of my study.

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. Choosing an appropriate anchor				
2. Possess a generative learning format				
3. Developing shared expertise around the anchor				
4. Expanding the anchor				
5. Using knowledge as a tool				
6. Merging the anchor				
7. Allowing student exploration				
8. Provide opportunity for participants to share what was learned from the anchored instruction				

If you Disagree or Strongly Disagree that my materials address a specific design principle, please share your rationale(s) below. (more room was left in the actual survey)

Appendix O (Continued)

Please indicate by placing an "X" in the box that best describes the degree to which you anticipate that my materials and procedures will cause anchored instruction to occur for the participants of my study.

	Strongly Agree	Agree	Disagree	Strongly Disagree
My study's participants will experience anchored instruction				

If you Disagree or Strongly Disagree that my study's participants will experience anchored instruction, please share why.

ABOUT THE AUTHOR

Matthew Kellogg received his bachelor's degree in Mathematics Education from Bob Jones University in 1988. Immediately after graduation, he accepted a job teaching mathematics in northeast PA for grades 8-12. He returned to school at DeSales University in PA to enter the Master's program in Mathematics Education, and completed that degree in 1998. He continued teaching high school mathematics while pursuing his master's degree, and in 2000 accepted a position with Clearwater Christian College (CCC) teaching mathematics, IT, and undergraduate mathematics methods courses for pre-service teachers. Also in 2000, he took an adjunct position with St. Petersburg College (SPC) where he continues to teach various mathematics courses. In 2003, while still employed with CCC and SPC, he entered the Ph.D. program in Mathematics Education at the University of South Florida.

While in the Ph.D. program, Mr. Kellogg continued to teach mathematics, IT, and methods courses at CCC, and also has been published in a technology-education journal, worked closely with elementary and secondary interns, and conducted an in-service session for elementary classroom teachers.