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# Proportionality in middle-school mathematics textbooks

Gwendolyn Joy Johnson
*University of South Florida*

# Proportionality in Middle-School Mathematics Textbooks

by

Gwendolyn Joy Johnson

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Major Professor: Denisse R. Thompson, Ph.D.
Richard Austin, Ph.D.
Helen Gerretson, Ph.D.
Elizabeth Shaunessy, Ph.D.

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ABSTRACT

Some scholars have criticized the treatment of proportionality in middle-school textbooks, but these criticisms seem to be based on informal knowledge of the content of textbooks rather than on a detailed curriculum analysis. Thus, a curriculum analysis related to proportionality was needed.

To investigate the treatment of proportionality in current middle-school textbooks, nine such books were analyzed. Sixth-, seventh-, and eighth-grade textbooks from three series were used: *ConnectedMathematics2* (CMP), Glencoe's *Math Connects*, and the University of Chicago School Mathematics Project (UCSMP). Lessons with a focus on proportionality were selected from four content areas: algebra, data analysis/probability, geometry/measurement, and rational numbers. Within each lesson, tasks (activities, examples, and exercises) related to proportionality were coded along five dimensions: content area, problem type, solution strategy, presence or absence of a visual representation, and whether the task contained material regarding the characteristics of proportionality. For activities and exercises, the level of cognitive demand was also noted.

Results indicate that proportionality is more of a focus in sixth and seventh-grade textbooks than in eighth-grade textbooks. The CMP and UCSMP series focused on algebra in eighth grade rather than proportionality. In all of the sixth-grade textbooks, and some of the

seventh- and eighth-grade books, proportionality was presented primarily through the rational number content area.

Two problem types described in the research literature, *ratio comparison* and *missing value*, were extensively found. However, *qualitative* proportional problems were virtually absent from the textbooks in this study. Other problem types (*alternate form* and *function rule*), not described in the literature, were also found.

Differences were found between the solution strategies suggested in the three textbook series. Formal proportions are used earlier and more frequently in the *Math Connects* series than in the other two. In the CMP series, students are more likely to use manipulatives.

The Mathematical Task Framework (Stein, Smith, Henningsen, & Silver, 2000) was used to measure the level of cognitive demand. The level of cognitive demand differed among textbook series with the CMP series having the highest level of cognitive demand and the Math Connects series having the lowest.

# CHAPTER 1: INTRODUCTION

For decades, there have been criticisms of the U.S. educational system and dissatisfaction with U.S. students' levels of achievement, particularly in mathematics and science. Educators and researchers have recently come to believe that one way to improve mathematics education is to improve the curriculum (Tarr et al., 2008; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Ball and Cohen (1996) expressed the importance of curriculum:

> Unlike frameworks, objectives, assessments, and other methods that seek to guide curriculum, instructional materials are concrete and daily. They are the stuff of lessons and units, of what teachers and students do….Not only are curriculum materials well positioned to influence individual teachers' work but, unlike many other innovations, textbooks are already "scaled up" and part of the routine of schools. They have "reach" in the system. (p. 6)

Schmidt and colleagues (2001) agreed that curriculum is important, stating "Curriculum is at the very center of intentional learning in schools, specifying content and directing students in their efforts to understand mathematics and science" (p. xix). Valverde et al. pointed out that curriculum bridges the gap between content standards and classroom practice; they stated that textbooks can serve "as a mediator between policy and pedagogy" (p. 171).

Mathematics education at all levels is important, but the middle grades may be crucial since "During this time, many students will solidify conceptions about themselves as learners of mathematics – about their competence, their attitude, and their interest and motivation. These conceptions will influence how they approach the study of mathematics in later years" (National Council of Teachers of Mathematics [NCTM], 2000, p. 211).

Although the middle-school mathematics curriculum contains many important concepts, one of the most pervasive is that of proportionality. An understanding of proportionality is essential for comprehending high-school and college-level mathematics. Proportionality is also an area of mathematics with numerous applications to real life. These applications include map reading and calculating the "better buy" given two or more purchasing options, as well as many other useful skills. Proportionality is also closely related to many topics within the middle-school mathematics curriculum, such as algebra, fractions, measurement, percent, and rates and ratios. Specifically, proportionality is ideally suited to provide two types of connections for which the NCTM has called: connections between school and real-life mathematics and connections between mathematical concepts (NCTM, 1989, 2000). Thus, the NCTM has stated that proportionality should be "an integrative theme in the middle-grades mathematics program" (NCTM, 2000, p. 212).

The development of proportional reasoning is considered one of the most important goals of the middle-school mathematics curriculum. It represents a significant shift from additive reasoning, in which *amounts* are compared, to multiplicative reasoning, in which *percents* are compared. Signifying the magnitude of this shift, Lesh, Post, and Behr (1988) referred to proportional reasoning as a "watershed" concept.

Similarly, Smith (2002) described the importance and complexity of proportionality in this way:

> No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality. These ideas…are the first place in which students encounter numerals like "3/4" that represent relationships between two discrete or continuous quantities, rather than a single discrete ("three apples") or continuous quantity ("4 inches of rope") (p. 3).

## Statement of the Problem

Proportionality is a topic with which many children and adults struggle. In fact, Lamon (2007) estimated that "more than 90% of adults do not reason proportionally" (p. 637). Proportionality is related to many of the most difficult topics or "trouble spots" in the middle-school mathematics curriculum, such as equivalent fractions, long division, place value and percents, measurement conversion, and ratio and rates (Lesh et al., 1988). Some of students' difficulty with proportionality is that it requires multiplicative reasoning, which is different from the additive reasoning learned in elementary school. Making this shift is so difficult that students often revert back to additive reasoning on hard problems, even when they are capable of multiplicative reasoning (Karplus, Pulos, & Stage, 1983).

Some of students' difficulty with proportionality is due to the complexity of the topic and the shift from additive to multiplicative reasoning, but the way proportionality is treated in textbooks may also account for some of students' struggles. In most classes, textbooks are used on a daily basis (Grouws, Smith, & Sztajn, 2004), so it is reasonable to assume they affect student understanding. In fact, at least one study has indicated a correlation between the curriculum used in a class and students' understanding of proportionality (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998). However, although Ben-Chaim et al. claimed that there are significant differences in the treatment of proportionality between the two curricula used in their study, these differences have not been described in detail.

Researchers have criticized middle-school curricula for failing to help students recognize the differences between additive and multiplicative relationships, for placing too much emphasis on procedural rather than conceptual understanding of proportionality, and for paying inadequate attention to proportionality (Cramer, Post, & Currier, 1993; Post, Cramer, Behr, Lesh, & Harel, 1993; Watson & Shaughnessy, 2004). These criticisms seem to be based on researchers' general sense of the treatment of proportionality in textbooks rather than on actual research. In fact, there is a remarkable lack of research on the treatment of proportionality in textbooks. A search of the Education Full Text and ERIC databases produced no documents that reported the results of a curriculum analysis related to proportionality. Two studies contained curriculum analyses relevant to the current study. In a report on teachers' solution strategies, Fisher (1988) reported results from an informal look at proportionality in secondary textbooks, but the textbooks used were published in the 1970s and 1980s and the investigation was not detailed. The American Association for the Advancement of Science (AAAS) conducted an evaluation of contemporary middle-school textbooks, but did not focus on proportionality (AAAS, 2000). Therefore, it appears that an analysis of the treatment of proportionality in middle-school mathematics textbooks is needed.

## Purpose of the Study

The purpose of the study was to describe how proportionality is treated in contemporary middle-school mathematics textbooks. Six aspects of this treatment were investigated. First, the NCTM has stated that “curricular focus and integration” can be achieved if proportionality is used as “an integrative theme in the middle-grades mathematics program” (2000, p. 212). However, there has been no curriculum analysis conducted to determine whether proportionality is used this way in textbooks. The NCTM stated that proportionality can become “an integrative theme” if it is emphasized in “many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability” (p. 212). Therefore, there was a need to investigate the extent to which textbooks highlight the proportionality aspect of various topics, such as algebra, measurement, percent, probability, ratios and rates, rational numbers, scale factors, and similar figures.

Second, because researchers have suggested that students should be exposed to a wide range of proportional situations, one goal of the study was to determine the extent to which textbooks include various types of proportional problems. Three types of proportional problems have been identified in the research literature (Cramer et al., 1993); these will be described in Chapter 2. Additionally, a pilot study conducted by the researcher revealed that several types of proportional problems exist in textbooks that have not been identified in the research literature.

Third, students often use additive reasoning in situations where multiplicative reasoning is required and vice versa. The result is that students seem not to know when to apply proportional reasoning. Therefore, there was a need to examine the extent to which textbooks point out differences between the two types of reasoning and the extent to which textbooks illustrate situations where each type is appropriate and inappropriate.

Fourth, research findings have indicated that some solution methods seem more natural to students than others (Karplus et al., 1983). For example, the use of proportions and cross multiplication is traditionally emphasized in textbooks, but is highly symbolic. Therefore, there was a need to analyze the solution strategies presented in textbooks. Fifth, because problems with higher levels of cognitive demand may lead to better conceptual understanding, it was important to note the level of cognitive demand (Boston & Smith, 2009; Stein, Smith, Henningsen, & Silver, 2000) of tasks. Finally, because visual representations may lead to conceptual understanding (e.g., Martinie & Bay-Williams, 2003a), it was also important to note the extent to which textbooks use visual representations to express ideas related to proportionality.

## Research Questions

The purpose of the study was to investigate the following six research questions.

1. To what extent is proportionality emphasized in the treatment of various content areas within mathematics, such as algebra, data analysis/probability, geometry/ measurement, and rational numbers? How does this vary among grade levels and textbook series?
2. Among the tasks related to proportionality in middle-school mathematics textbooks, which problem types (e.g., missing value, ratio comparison, qualitative) are featured most and least often? How does this vary among mathematical content areas, grade levels, and textbook series?

3. Which solution strategies (e.g., building up, unit rate, proportion) to tasks related to proportionality are encouraged by middle-school mathematics textbooks? How does this vary among grade levels and textbook series?
4. What level of cognitive demand (Boston & Smith, 2009; Stein et al., 2000) is exhibited by the proportional exercises in middle-school textbooks? How does this vary among mathematical content areas, grade levels, and textbook series?
5. To what extent are visual representations used in middle-school mathematics textbooks to illustrate concepts related to proportionality? How does this vary among grade levels and textbook series?
6. To what extent are the characteristics of proportional situations pointed out in middle-school mathematics textbooks? How does this vary among grade levels and textbook series?

## Funding by the National Science Foundation

Concerns about U.S. students' mathematics achievement were raised throughout the twentieth century and culminated in *A Nation at Risk*: *The Imperative for Educational Reform* (National Commission on Excellence in Education, 1983) which stated, "Our Nation is at risk….The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people" (p. 5). According to Senk and Thompson (2003), levels of achievement that are seen as inadequate result in calls for changes in the curriculum. Senk and Thompson stated the following: "Invariably, concerns about student outcomes in mathematics give rise to recommendations about what to teach in schools and how to teach it. Advocates of reform often attempt to influence classroom practice, and hence, student achievement, by means of changes in textbooks" (p. 4).

In response to these concerns about mathematics achievement, the NCTM issued several "Standards" documents between 1989 and 1995 that described some of the changes that were needed in mathematics education, including changes to the curriculum. Another effect of the concerns about mathematics achievement was the National Science Foundation (NSF) funding of curriculum development projects. According to Senk and Thompson (2003), "By 1991, the NSF had issued calls for proposals that would create comprehensive instructional materials for the elementary, middle, and high schools consistent with the calls for change in the *Curriculum and Evaluation Standards* [NCTM, 1989]" (pp. 13-14). Five curriculum development projects funded by the NSF produced textbooks for middle-school students (Senk & Thompson).

Scholars have described the differences between NSF-funded and traditional curricula (e.g., Robinson, Robinson, & Maceli, 2000). According to Robinson et al., NSF-funded curricula "represent a profound departure from traditional textbooks of the past, both in content and in format" (p. 113). Robinson et al. state that NSF-funded curricula are more likely than traditional curricula to use problem-solving contexts and to portray mathematics as a unified discipline by making connections between topics.

Researchers have also investigated the effects of NSF-funded curricula on student achievement (Ridgway, Zawojewski, Hoover, & Lambdin, 2003; Tarr et al., 2008). A few detailed comparisons of NSF-funded and traditional curricula have been conducted. For example, Hodges, Cady, and Collins (2008) examined the ways fractions are represented in two NSF-funded and one traditional textbook. However, few in-depth, rigorous curriculum analyses have been conducted to determine how NSF-funded curricula cover various mathematical topics or how their treatment differs from that of traditional textbooks.

## Definition of Terms

In this section, the meanings of some important terms are discussed. The terms defined in this section include the following: *additive reasoning*, *multiplicative reasoning*, *fraction*, *rational number*, *measure space*, *proportionality*, *proportional reasoning*, *rate*, *ratio*, and *task*. This section includes not only a definition of each of these terms, but also a broader discussion of their meanings and usage. Therefore, the sections that follow are lengthier than simple definitions would be.

### *Additive and Multiplicative Reasoning*

*Additive reasoning* is one of the first types of mathematical reasoning learned by young children. It consists of skills related to counting, adding, joining, subtracting, separating, and removing (Bright, Joyner, & Wallis, 2003; Lamon, 2007; Post et al., 1993). *Multiplicative reasoning* refers to reasoning about multiplication, division, linear functions, ratios, rates, rational numbers, shrinking, enlarging, scaling, duplicating, exponentiating, and fair sharing (Lamon, 2007). Behr, Harel, Post, & Lesh (1992) described the differences between additive and multiplicative reasoning in this way:

> As early as possible, children should be brought to understand that the change in 4 to get to 8 (or the difference between 4 and 8) can be defined in two ways: additively (with an addition or subtraction rule) or multiplicatively (with a multiplication or division rule) (p. 316).

Bright et al. (2003) provided the following contrasts: “Proportional or multiplicative reasoning is in contrast to additive reasoning. Additive reasoning involves using counts – for example, sums or differences of numbers – as the critical factor in comparing quantities. Multiplicative or proportional reasoning involves using ratios as the critical factor in comparing quantities” (p. 166).

There is some disagreement about the extent to which multiplicative reasoning should be taught as a natural extension of additive reasoning. From a constructivist, student-centered point of view, Steffe (1994) viewed multiplicative reasoning as an extension of additive reasoning and assumed that “whatever multiplication and division might become for children, the operations would be constructed as modifications of their number sequences” (p. 13). In contrast, Confrey (1994) believed that too much emphasis had been put on the additive roots of multiplication and stated, “the majority of the current approaches to multiplication…rely too exclusively on a model of multiplication as repeated addition with an underlying basis in counting” (p. 291). She pointed out that although some uses of multiplication are related to addition, others, such as sharing and dividing symmetrically, are not.

*Fractions and Rational Numbers*

Rational numbers are numbers that can be written in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* is not equal to zero. For example, $\frac{1}{2}$, 0.5, and 50% are all rational numbers. Fractions are numbers that are written in the form $\frac{a}{b}$. For example, $\frac{1}{2}$ is a both a fraction and a rational number; 0.5 and 50% are rational numbers but not fractions. Not all fractions are rational numbers. For example, $\frac{\pi}{2}$ is not rational even though it is written in fractional form (Lamon, 2007). Thus, many numbers are both fractions and rational numbers, such as $\frac{3}{4}$, $\frac{\sqrt{4}}{3}$, and $\frac{2.1}{4.1}$ (Lamon). Other numbers are fractions but not rational numbers and still other numbers are rational numbers but not written in fractional form. Further complicating the issue is that rates and ratios can also be written in fractional form. Thus, a "fraction" can be considered one way of writing a number, which may or may not be rational. The fractional method of writing a number points out comparisons. For example, $\frac{3}{4}$ compares the numbers three and four and also compares $\frac{3}{4}$ to one. It is this comparison aspect of fractions that relates to proportionality. Researchers agree that all rational numbers, including fractions, are closely related to proportionality. For example, Lamon stated "Educators are accustomed to mentioning rational numbers, ratios, and proportions together in the same phrase simply because the terms are so close mathematically and psychologically" (p. 640).

*Measure Space*

The notion of measure spaces was developed by the French psychologist Gerard Vergnaud. Cramer et al. (1993) provided the following example of measure spaces: "If 3 U.S. dollars can be exchanged for 2 British pounds, then at this rate 21 U.S. dollars can be exchanged for 14 British pounds" (p. 161). The two measure spaces in this example are U.S. dollars and British pounds. The concept of "measure space" helps researchers discuss two methods of solving problems: *Between* and *Within* strategies. Using a *Between* strategy, a person would compare the number of U.S. dollars to the number of British pounds. Using a *Within* strategy, a person would determine the ratio between 21 U.S. dollars and three U.S. dollars.

*Proportionality*

Mathematical relationships that meet two criteria are said to be “proportional” and thus have the characteristic of “proportionality.” These two criteria are the following: (a) the rate of change is positive and constant and (b) the $y$-intercept is zero (Cramer et al., 1993; Lamon, 1999). Both of these characteristics are described below.

In proportional situations, the rate of change is constant. In other words, a change of a certain amount in the independent variable produces a consistent change in the dependent variable. This implies that the slope is constant and that a graph of the relationship will feature a straight line. Lamon (1999) provided the following examples:

> In the midst of change, important relationships can remain constant. For example, whether you are mixing a quart of lemonade or a gallon, the ratio of scoopfuls of drink mix to pints of water is the same. If you buy grapes, although the amount you pay increases if you buy 3 pounds instead of 2 pounds, the rate $1.19 per pound remains constant (p. 187).

In proportional situations, the $y$-intercept is zero, meaning that a value of zero for the independent variable is associated with a value of zero for the dependent variable. Because the $y$-intercept is zero, the familiar equation for a line, $y = mx + b$, simplifies to $y = mx$ (Cramer et al., 1993; Lamon, 2007). It is important to note that the equation $y = mx$ involves only multiplication, not addition. Thus, proportional relationships are always multiplicative in nature (Cramer et al.).

*Proportional Reasoning*

Lamon (1999) provided the following description of proportional reasoning:

> Proportional reasoning results after one has built up competence in a number of practical and mathematical areas….It draws on a huge web of knowledge. So…we cannot say in a very concise way what proportional reasoning is, nor can we say exactly how a person learns to reason proportionally….One way to define proportional reasoning is to say that it is the ability to recognize, to explain, to think about, to make conjectures about, to graph, to transform, to compare, to make judgments about, to represent, or to symbolize relationships. (p. 5)

Although proportional reasoning is difficult to define, there is no doubt it is important. Since Piaget's research, proportional reasoning has been considered a watershed concept "which ushers in a significant conceptual shift from concrete operational levels of thought to formal operational levels of thought" (Lesh et al., 1988, p. 97).

The term *proportional reasoning* is closely related to two other terms: *proportionality* and *multiplicative reasoning*. According to Lamon (2007), the terms *proportionality* and *proportional reasoning* are often used interchangeably. However, there is a distinction between the two. *Proportionality* is a mathematical characteristic of some quantitative situations, whereas *proportional reasoning* is a type of reasoning and thus a cognitive function of the mind.

Multiplicative reasoning and proportional reasoning are closely related to each other, but the distinction between them is not completely clear. In *Principles and Standards for School Mathematics,* the NCTM indicated that multiplicative reasoning is a prerequisite for or less-advanced form of proportional reasoning. NCTM (2000) stated, "The focus on multiplicative reasoning in grades 3-5 provides foundational knowledge that can be built on as students move to an emphasis on proportional reasoning in the middle grades" (p. 144). Thus, although proportional reasoning is closely related to both proportionality and multiplicative reasoning, there are distinctions between the terms.

*Rates and Ratios*

Rates and ratios are both comparisons of two quantities. Various authors have suggested different distinctions between rates and ratios. Some scholars believe that a *rate* compares quantities from two different measure spaces, such as "miles per hour" and that a *ratio* compares quantities from the same measure space, such as 20 children in one class to 18 children in another (Lamon, 2007).

Other scholars have offered different distinctions between rates and ratios. According to Kaput and West (1994), a *ratio* arises from a situation in which the variables are discrete, such as two pencils for 10 cents, 4 pencils for 20 cents, and so on. Conversely, a *rate* arises from a situation in which the variables are continuous, such as a mixture of paint or orange juice. Because no consensus has been reached, the definitions of *rate* and *ratio* have not been firmly established. Lamon (1999) stated the following:

> Everyday language and usage of rates and ratios is out of control. The media have long employed ratios and rates and the language appropriate to ratios and rates in many different ways, sometimes inconsistently, sometimes interchangeably. Students are exposed to less-than-correct usage and terminology, and it is no easy task to reconcile precise mathematical ideas with informal, colloquial usage" (p. 165).

### *Task*

According to Doyle (1979, 1983), earlier studies of curriculum focused on the curriculum in broad terms, such as the percentage of the school day devoted to mathematics, but, as a result of cognitive theories of psychology, attention to individual tasks grew. Doyle (1979) defined a *task* as consisting of two elements: a *goal* and a set of operations necessary to achieve that goal. According to Doyle, in a *learning task*, "the goal is to be able to display a capability that does not presently exist" (p. 4). Doyle (1983) described four categories of academic tasks: memory tasks; procedural or routine tasks; comprehension or understanding tasks; and opinion tasks.

Recently, the analysis of tasks has been conducted in teacher education (Stein et al., 2000). Stein et al. developed a task-analysis framework with categories similar to those described by Doyle (1983). The attention to individual tasks rests on the assumption that "instructional tasks form the basis of students' opportunities to learn mathematics" (Stein et al., p. 3).

Analysis of tasks has also been conducted in curriculum analysis (Jones, 2004; Jones & Tarr, 2007). Jones used the following definition of *probability task*: "an activity, exercise, or set of exercises in a textbook that has been written with the intent of focusing a student's attention on a particular idea from probability" (p. 10). A similar definition of "task" was used in this study, but the author is interested in proportionality rather than probability.

## Significance of the Study

Student achievement is affected by many factors including students' motivation and parental support, teachers' beliefs and knowledge, and educational policies such as standardized testing. Another strong determinant of what students learn is the curriculum (Hirsch, 2007). Both national and international studies have shown that textbooks have a significant impact on the content that is covered in mathematics classes (Grouws et al., 2004; Schmidt et al., 2001; Valverde et al., 2002). For example, according to data obtained through the National Assessment of Educational Progress, in 2000, 72% of eighth-grade students did problems from their mathematics textbook "almost every day" (Grouws et al.). A variety of education organizations have expressed the opinion that the mathematics curriculum is of vital importance (AAAS, 2000; NCTM, 1989). In spite of the importance of the mathematics curriculum, there have been few recent studies of textbooks' content. Most recent research related to curriculum has focused on teachers' implementation of it or learning from it. While these studies provide valuable insights into the instruction that occurs in classrooms, there is also a need to delve into the subject of what textbooks actually contain.

The mathematics curriculum at all levels of education affects student learning, but the middle-school curriculum, in particular, is in need of attention (AAAS, 2000). One of the primary goals of the middle-school mathematics curriculum is students' development of proportional reasoning. For example, in the *Principles and Standards for School Mathematics*, the NCTM stated that reasoning proportionally is "an important integrative thread that connects many of the mathematics topics studied in grades 6-8" (NCTM, 2000, p. 217). In both that document and the *Curriculum Focal Points* (NCTM, 2006), the NCTM offered suggestions regarding what middle-school students should learn about proportional reasoning. For example, the NCTM suggested that because proportions are highly symbolic, they may not be the most natural way for students to solve problems (NCTM, 2000). Therefore, students should develop other solution strategies in addition to proportions. However, because curriculum materials related to proportional reasoning have not been analyzed in depth, the degree to which NCTM suggestions are currently followed by curriculum developers is not clear. Furthermore, because suggestions offered by the NCTM regarding proportional reasoning are meant only to be general guidelines, curriculum developers have a great deal of latitude and may choose to present proportional reasoning in a wide variety of ways. For example, the percentage of questions of missing-value and comparison types is left to the discretion of curriculum developers. Because of this discretion, it is likely that proportional reasoning is treated differently in various curricula. In fact, the development of students' proportional reasoning was a major goal of one of the curricula selected for this study (*Connected Mathematics*), and research indicates that students studying from this curriculum have a better understanding of proportionality than students using other curricula (Ben-Chaim et al., 1998). However, little specific information regarding the differences of the treatment of proportionality in various curricula is available; thus, curriculum developers do not know which aspects of the *Connected Mathematics* curriculum are successful.

Information regarding the treatment of proportional reasoning in textbooks could help curriculum developers make improvements to their treatment of the topic. For example, curriculum developers may inadvertently emphasize proportional reasoning in some content areas more than others, limiting the extent to which proportionality serves as a connective thread as encouraged by the NCTM. Curriculum developers may also inadvertently place a higher emphasis on some problem types while neglecting others. An in-depth analysis of their materials may help them recognize these imbalances. The information provided by this study could help states and school districts select textbooks that best meet the needs of their middle-school students. Information about the strengths of textbooks could help teachers maximize the effects of those strengths and information about their weaknesses could help teachers make modifications to the curriculum when necessary.

## Summary

Textbooks affect what is covered in mathematics classes and what students learn. Therefore, careful analyses of the contents of textbooks are important. The middle-school curriculum, in particular, has been the subject of recent interest (e.g., AAAS, 2000). One of the primary goals at this level is the development of proportional reasoning, which has been called a “watershed” achievement (Lesh et al., 1988). Because proportionality can be a challenging concept, textbook authors must do all they can to help students understand it. Researchers have identified three main types of proportional reasoning questions, which are described in the next chapter; it seems reasonable to assume that textbooks should contain all three types. Because the NCTM encourages middle-school educators to use proportional reasoning as “an integrative theme” (NCTM, 2000, p. 212), the proportionality inherent in algebra, measurement, rational numbers, and probability should be highlighted. Because students often use additive reasoning in situations where proportional reasoning is called for, and vice versa, textbooks should help students understand the differences between the two types of reasoning and when each is required. Researchers claim to have identified several weaknesses in the coverage of proportionality in middle-school textbooks; however, these criticisms do not seem to be based on a thorough curriculum analysis. This study provides the data necessary for a better understanding of the treatment of proportionality in three contemporary middle-school textbook series.

# CHAPTER 2: LITERATURE REVIEW

In Chapter 3, a framework that was used to analyze the presentation of proportionality in middle-school mathematics textbooks is presented. This framework was developed by the author based on research literature regarding the mathematical nature of proportionality as well as research literature on how students develop proportional reasoning. The purposes of this chapter are to discuss the literature on which the framework is based, to discuss other scholars' criticisms of the treatment of proportionality in textbooks, and to discuss research methods that have been used in other curriculum analyses. This presentation of the research literature and discussion of curriculum analysis methodology provides the reader with an understanding of why the study was necessary and the methodology that was used.

This chapter is divided into eight sections. First, because the NCTM has stated that proportionality should be "an integrative theme in the middle-grades mathematics program" (NCTM, 2000, p. 212), the various content areas in which proportionality appears is discussed. Second, the problem types that have been identified in the research literature are discussed. Third, because researchers have suggested that some solution strategies contribute more toward conceptual understanding than do others, a discussion of solution strategies is provided. Fourth, common errors and common misunderstandings about proportionality will be discussed. Fifth, methods other researchers have used to analyze textbooks are discussed. In the sixth section, the literature on proportional reasoning is summarized. In the seventh section, questions that remain unanswered by previous research are identified, and in the final section, a summary of the pilot study conducted by the researcher is provided. Together, these eight sections should prepare the reader to see connections between the research literature, the pilot study, and the framework presented in Chapter 3.

## Content Areas

Many middle-school textbooks contain a lesson titled "Proportional Reasoning." However, proportionality cannot be confined to a single lesson or even a single chapter. Rather, proportionality is a fundamental characteristic of mathematical relationships that appears in several areas of mathematics. Because the NCTM encourages middle-school educators to use proportionality as a connective thread that runs through the curriculum (NCTM, 2000), proportionality should be emphasized in the teaching of several different areas of mathematics. Emphasizing the proportionality that is inherent in various content areas may help students see mathematics as a coherent discipline. These content areas and their connection to proportionality are the subject of this section.

### *Algebra*

One of the content areas closely connected to proportionality is algebra (Cramer et al., 1993; Karplus et al., 1983; Martinie & Bay-Williams, 2003b; NCTM, 2000; Seeley & Schielack, 2007). Some scholars view an understanding of proportionality as a prerequisite to success in algebra. For example, Post et al. stated that proportionality is related to "prealgebra understandings." Whereas these scholars viewed an understanding of proportionality as a prerequisite to success in algebra, Cramer et al. (1993) saw an even tighter mathematical connection between algebra and proportionality. They explained that the connection between algebra and proportionality is that "proportional relationships can be expressed through a rule with the form of $y = mx$" (p. 168). Expressing a proportional relationship in the form $y = mx$ highlights the connections to slope and graphing. Telese and Abete (2002) showed how a lesson related to nutrition could be used to teach middle-school students about algebra and proportionality. Their students found a linear relationship between grams of fat and calories from fat. One of the students stated "You multiply the fat by 9" (p. 9) and expressed this as $y = 9x$. Examples like these show how linear relationships are closely related to multiplication. Proportionality is usually highlighted in algebra lessons in one of two ways: through attention on constant slope or through rate problems. An example of a rate problem is this exercise from a sixth-grade textbook: "A marathon is approximately 26 miles. If Joshua ran the marathon in 4 hours at a constant rate, how far did he run per hour?" (Day et al., 2009a, p. 318).

### *Data Analysis and Probability*

Data from the 1996 National Assessment of Educational Progress (NAEP) indicate that, although students' understanding of data analysis and probability was increasing, "when proportional reasoning is required, students often experience difficulty on items involving reasoning about data, graphs, and chance" (Zawojewski & Shaughnessy, 2000, p. 266). At least three topics within the data analysis and probability standard can be connected to proportionality: arithmetic mean, misleading graphs, and probability.

Watson and Shaughnessy (2004) described an assessment of students' proportional reasoning that was related to arithmetic mean. They presented students with several pairs of bar graphs; each graph illustrated a class's scores on a separate assessment. On the basis of the graphs, students were asked to determine which class did better. In one pair of graphs, one set of data represented a larger class than the other. To someone using additive reasoning, the larger class appeared to have done better, but to someone using proportional reasoning, the smaller class did better. Watson and Shaughnessy found that additive reasoning was common among elementary- and middle-school students and even extended into ninth grade.

Proportional reasoning is also needed when analyzing graphs; Spence and Krizel (1994) explain why:

> The most commonly seen graphs are pie and bar charts…the pie chart sometimes appears as a disk and bars are often substituted by boxes or cylinders….Thus, the apparent dimensionality of graphical elements can be one, two, or three, even though the represented numerical quantities and proportions are unidimensional….The use of area and volume…can introduce perceptual distortion. (p. 1193)

For example, the “pie slices” in pie charts should be two-dimensional, but are often drawn in three dimensions. Modifications like these can mislead the reader; proportional reasoning is required to correctly interpret these graphs.

Another content area in which proportional reasoning is required is probability. For example, Watson and Shaughnessy (2004) stated, “Proportional reasoning is fundamental to making connections between populations and samples drawn from those populations; in the later grades, it provides a basis for statistical inference” (p. 104). For example, proportional reasoning is needed when deciding whether a die or coin is “fair” by comparing experimental and empirical data.

*Geometry and Measurement*

Within geometry and measurement, proportional reasoning is most evident in the topics of size changes and similar figures. According to the NCTM, “It is important that middle-grades students understand similarity, which is closely related to their more general understanding of proportionality” (2000, p. 245). Several educators have published activities related to size changes and similar figures that are designed to promote or assess proportional reasoning (Che, 2009; Frost & Dornoo, 2006; Moss & Caswell, 2004). Che states that she posts pictures of giant pencils on the walls of her classroom and asks questions like “If a giant uses pencils of this size, what can you find out about the giant?” and “How many times larger than you is the giant?” (p. 405).

Frost and Dornoo (2006) discussed some of the issues that arise when helping middle-school students understand size changes. For example, the phrase “twice as big” could have several different meanings. They explained how students can construct squares on geoboards and discover that doubling the lengths of the sides doubles the perimeter but not the area, a common misunderstanding mentioned in the *Curriculum and Evaluation Standards* (NCTM, 1989).

In another example of proportionality related to size changes, Moss and Caswell (2004) had fifth- and sixth-grade students construct dolls whose measurements were in proportion to the average measurements of students in the class. They found that students’ everyday knowledge of percents help them understand size changes. For example, they found that students easily understood statements like “She is about 75% as tall as he is” (p. 70).

Because pi is the ratio of the circumference of any circle to its diameter, reasoning about pi and circles is another area of geometry and measurement related to proportionality (NCTM, 2000). Converting between units of measure is another application of proportionality. Additionally, because perimeter is related proportionally to side lengths but area is not, textbooks often use the topics of area and perimeter to discuss the differences between proportional and nonproportional relationships.

### *Rational Numbers*

There has been an enormous amount of literature published on rational numbers. However, not all of the literature on rational numbers is relevant to a study of proportionality. The purpose of this section is not to review all of the literature on rational numbers, but instead to demonstrate the connection between proportionality and rational numbers. Rational numbers are interpreted in different ways, depending on the context (Behr et al., 1983). Three of the interpretations of rational numbers are closely related to proportionality: fractions, decimals, and percent.

Common in middle-school textbooks are tasks in which students are asked to find equivalent fractions. Post et al. (1993) described the tight connection between equivalent fractions and proportionality problems; they stated, “There is a great deal of similarity between the numerical procedures used in manipulating and finding equivalent fractions and numerical procedures necessary to solve missing value and numerical comparison problems. In both cases relationships are multiplicatively based” (p. 333). In some equivalent-fraction tasks, students are given three numbers and asked to find a fourth. For example, NCTM (2000) posed the problem “A baseball team won 48 of its first 80 games. How many of its next 50 games must the team win in order to maintain the ratio of wins to losses?" (p. 256). To solve the problem, students must find a fraction equivalent to $\frac{48}{80}$ that has a denominator of 50. This is an example of a missing value problem. In other equivalent-fraction tasks, students are given only two numbers and asked to find the other two. For example, students could be asked to find a fraction equivalent to $\frac{2}{6}$. This is *not* a missing value problem as defined by Lamon (2007). In fact, this type of problem is not explicitly discussed in the literature on proportional reasoning.

In other fraction problems, students are asked to compare the size of two or more fractions. For example, a problem on the fourth administration of the NAEP asked students to identify the larger of $\frac{3}{5}$ and $\frac{2}{6}$. About two-thirds of the seventh grade students correctly identified $\frac{3}{5}$ as the larger fraction (Kouba, Carpenter, and Swafford, 1989). These problems are closely related to proportionality; if fact, they fit into one of the three types of proportional problems, the ratio comparison type. To solve the problem, students must not only view $\frac{3}{5}$ and $\frac{2}{6}$ as units but must also then compare the size of those units.

Decimals are closely related to fractions. For example, 0.6 is equivalent to $\frac{6}{10}$ because both mean "six tenths." U.S. middle-school students have traditionally had difficulty converting between decimals and fractions. For example, on the fourth administration of the NAEP, only 60% of the seventh graders tested could correctly write $\frac{6}{100}$ as a decimal (Kouba et al., 1989). Although rational numbers are one of the content areas traditionally associated with proportional reasoning (Behr et al., 1992; Lamon, 2007), decimals are discussed in the research literature far less often than are fractions.

Another rational-number concept closely connected to proportionality is percent (Dole, 2000; Lamon, 1994, 1999; Martinie & Bay-Williams, 2003b; Moss & Caswell, 2004; Parker & Leinhardt, 1995). In a comprehensive literature review of percent, Parker and Leinhardt pointed out that although percent appears to be a simple, straightforward concept, it can be “ambiguous and subtle” (p. 422) due to the fact that “percent changes personalities as it takes on different roles” (p. 423). Perhaps because of these multiple personalities of percent, middle-school students seem to lack an understanding of percent and do relatively poorly on NAEP items involving the concept (Dole, 2000).

The various interpretations of percent include the following: (a) percent as a number, (b) percent as an intensive quantity, (c) percent as a fraction or ratio, and (d) percent as a statistic or function (Parker and Leinhardt, 1995). Within “percent as a fraction or ratio,” there are several more meanings of percent. When a percent functions as a fraction, “the size of a subset is compared to the size of the whole set of which it is a part” (Parker & Leinhardt, p. 440). According to Parker and Leinhardt, this interpretation of percent is the one most prominently featured in textbooks. Parker and Leinhardt give the example of a political candidate who receives 35% of the votes; the whole set is the total number of votes and the subset is the number of votes received by the candidate. In addition to functioning as a fraction, a percent can also be a ratio. Parker and Leinhardt stated that a percent functions as a ratio when “it is used to describe a comparison involving different sets, different attributes of the same set, or the change in a set over time” (p. 440). They give the example of a comparison between the number of students in School A to the number of students in School B and the example of a comparison between the length of a board and its width.

### *Summary of Content Areas*

The NCTM has encouraged teachers of mathematics to point out connections between various mathematical topics and content areas. Because proportionality plays a major role in each of the content areas previously described, it provides a unique opportunity to illustrate the connections the NCTM has recommended. As Seeley and Schielack (2007) stated, “The development and study of ratios, rates, and proportional relationships is a particularly noteworthy example of how ideas from Number and Operations, Algebra, Geometry, Measurement and Data Analysis and Probability can connect through their mathematical commonalities” (p. 141). However, in order to serve this connective role, the proportionality inherent in various content areas must be made explicit to students. Therefore, it seems reasonable to assert that proportionality should be mentioned in textbooks’ coverage of various mathematics content areas.

## Problem Types

Many mathematical concepts have multiple meanings and are used in a variety of situations. For example, the concept of multiplication has a variety of meanings, including repeated addition, the area of a rectangle, or the number of combinations of two or more quantities. Similarly, proportionality is inherent in a wide variety of situations and various types of proportional problems require different types of reasoning and solution methods. Therefore, it is important to understand the various types of problems that require proportional reasoning.

Researchers generally agree that there are three main types of proportional problems: missing value, ratio comparison, and qualitative (Behr et al., 1992; Ben-Chaim et al., 1998; Cramer et al., 1993; Singh, 2000), although some scholars (e.g., Lamon,

2007) recognize two main types (*missing value* and *ratio comparison*) and consider qualitative problems to be a subtype within these two main types. Because *missing value*, *ratio comparison*, and *qualitative* problems are the three main types of tasks that involve proportionality, each of these types is discussed in this section.

*Missing Value Problems*

According to Lamon (2007), "A missing-value problem provides three of the four values in the proportion $\frac{a}{b} = \frac{c}{d}$ and the goal is to find the missing value" (p. 637). *Missing value* problems can be either purely numerical or can be stated in the form of a word problem. Although the above quotation by Lamon mentioned a proportion, there are other methods of solving *missing value* problems as discussed in a later section.

Karplus et al. (1983) offered this example of a *missing value* problem: "A car is driven 175 km in 3 hours. How far will it travel in 12 hours at the same speed?" (p. 220). In this example, the three known values are 175 kilometers, 3 hours, and 12 hours. The missing value is the number of miles traveled in 12 hours. Another example of a *missing value* problem is the well-known Mr. Tall/Mr. Short problem (Karplus, Karplus, Formisano, & Paulson, 1978). In this task, participants are shown two stick figures, one short and one tall. Each stick figure is accompanied by a stack of buttons showing his height. The height of Mr. Short is also given in paper clips and the task is to calculate the height of Mr. Tall in paper clips.

Missing value problems can be very simple or quite complicated. Lo (2004) provides this example that involves time, fractions, and area: "If it took Jane $\frac{3}{4}$ hour to paint a wall that was 12 ft by 12 ft, how long will it take to paint another wall that is 15 ft by 16 ft?" (p. 267). McKenna and Harel (1990) pointed out that missing value problems can be worded in two different ways. One way of wording a missing value problem is to have only one measure space in each sentence, as in “Matthew bought 6 lb of candy and Kim bought 10 lb of the same kind. If Matthew paid $12, how much did Kim pay?” The other way of wording a missing value problem is to mention both measure spaces in a single sentence, as in “If 6 lb of candy cost $12, how much does 10 lb of candy cost?” (McKenna & Harel, p. 589).

### *Ratio Comparison Problems*

In a ratio comparison problem, four values are given. These values form two ratios. The task is to determine which ratio is greater or whether the two ratios are equivalent. This type of problem has been referred to by several different names including “comparison” (Lamon, 2007), “numerical comparison” (Cramer et al., 1993; Singh, 2000), and “ratio comparison” (Karplus et al., 1983). These authors appear to be referring to roughly the same type of problem, however, Lamon includes qualitative problems in this category whereas other authors distinguish between ratio comparison and qualitative problems.

Karplus et al. (1983) offered this example of a ratio comparison problem: “Car A is driven 180 km in 3 hours. Car B is driven 400 km in 7 hours. Which car was driven faster?” (p. 220). Note that this problem is equivalent to asking students to compare the ratios 180 : 3 and 400 : 7. Singh (2000) provided this example: “Pele and Maradona were great soccer players. Pele scored 300 goals in 400 matches, while Maradona scored 400 goals in 500 matches. Who had a better scoring record, Pele, Maradona, or do they have the same scoring record?”

Another example of this type of problem is Noelting's (1980a) orange juice problems. Each task featured two pitchers of orange juice with different numbers of cups of water and orange juice concentrate. Each pitcher of juice contained some ratio of concentrate to water; the task was to compare the two ratios. As a final example, Karplus et al. (1983) showed a picture of 7 girls clustered around three pizzas and three boys clustered around 1 pizza and asked "Who gets more pizza, the girls or the boys?" This problem is equivalent to asking students to compare the ratios 7 : 3 and 3 : 1.

*Qualitative Problems*

In a task that requires qualitative proportional reasoning, a numerical answer is not desired or possible. Some researchers distinguish between two types of qualitative proportional tasks: *qualitative prediction* tasks and *qualitative comparison* tasks (Cramer et al., 1993). In *qualitative prediction* tasks, students compare a past or present situation to a future one. Cramer et al. provided this example of a *qualitative prediction* task: "If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, (d) not enough information to tell" (p. 166). Another example is provided by Lamon (2007): "Yesterday you shared some cookies with some friends. Today, you share fewer cookies with more friends. Will everyone get more, less, or the same amount as they received yesterday?" (p. 630).

In *qualitative comparison* tasks, students compare two past situations or two current situations. Cramer et al. (1993) provided this example of a *qualitative comparison* task: "Mary ran more laps than Greg. Mary ran for less time than Greg. Who was the faster runner? (a) Mary, (b) Greg, (c) same, (d) not enough information to tell" (p. 166). One advantage of qualitative problems is that students cannot answer them using a memorized procedure. Therefore, some scholars believe they are a better assessment of a student's proportional reasoning than are quantitative problems. For example, Singh (2000) stated that qualitative problems "require students to make comparisons that cannot be obtained just by solving an equation. In contrast to [quantitative] tasks, qualitative tasks require students to understand the meaning of proportions….Given the importance of this type of thinking, qualitative prediction and comparison should have a recognized place in mathematics curriculum" (p. 595).

### *Summary of Problem Types*

Three types of proportional problems have been discussed in the research literature: *missing value*, *ratio comparison*, and *qualitative types*. To the author's knowledge, no researcher has specifically stated that all three types of problems should appear in middle-school textbooks. However, scholars have stated their belief that proportional reasoning develops as students are exposed to a variety of proportional situations. Therefore, it seems reasonable to conclude that all three types of problems should appear in middle-school textbooks.

## Solution Strategies

In the previous section, the differences between missing value and ratio comparison problems were described. Because these problem types are quite different, they are solved using different methods or strategies. Researchers have identified three ways students solve missing value problems and two ways they solve ratio comparison problems.

### *Missing Value Problems*

Missing-value proportional problems generally require both multiplication and division as part of their solution process. This leads to three general solution methods: a) the multiplication can be performed first and the division second, b) the division can be performed first and the multiplication second, or c) they can be performed more or less simultaneously. Several authors have described these three general methods (Karplus et al., 1983; Noelting, 1980a, 1980b).

#### *Building Up (Factor of Change) Strategy*

Two similar strategies fit under the umbrella of the *building up* strategy. The first, “repeated addition” (Tourniaire, 1986) involves only addition and not multiplication. Tourniaire discussed this problem: “There are two mixtures of orange juice and water. One is made with 2 glasses of orange juice and 4 glasses of water. The other is made with 6 glasses of orange juice. How much water should be used to get the same taste?” (p. 404). Tournaire stated that a repeated adder could say ‘I need 4 glasses of water for the first 2 glasses of orange juice, 4 more for the next 2 is 8, and 4 more is 12’” (p. 406). This strategy is often used in conjunction with a “ratio table” (Lamon, 1999). Scholars recognize that this approach is less sophisticated than other methods, but also that it can be successfully used by elementary-school students and can provide a bridge to true proportional thinking (Lamon; Parker, 1999; Tournaire).

Other versions of the *building up* strategy involve simple forms of multiplication, such as doubling. When the solution strategy involves multiplication, it is sometimes referred to as the "factor of change method" (Cramer et al., 1993) but is more often called the "building up strategy" (Heinz & Sterba-Boatwright, 2008; Kaput & West, 1994; Lamon, 1994). Cramer et al described this method:

> A student using the *factor of change* method might reason as follows: 'If I want twice as many apples, then the cost will be twice as much.' The factor of change method is a 'How many times greater' approach and is equivalent to finding the multiplicative relationship within a measure space (p. 168).

Because the *building up* strategy involves finding the multiplicative relationship *within* a measure space, it is equivalent to the "within" measure space strategy described by Karplus et al. (1983). Students who have not received instruction in solving proportional problems seem to naturally and successfully use the *building up* strategy (Lamon, 1993b). Some scholars believe that success with the *building up* strategy does not demonstrate that a student has learned to reason proportionally. For example, Piaget referred to use of the *building up* strategy as "pre-proportionality."

*Unit Rate Strategy*

Cramer et al. (1993) described the unit rate approach in this way:

> This approach is characterized by finding the multiplicative relationship *between* measure spaces. The unit rate is found through division. For example, if 3 apples cost 60 cents, find the cost of 6 apples. The cost for 1 apple is found by dividing:

> 60 cents ÷ 3 apples = 20 cents per apple. This unit rate is the constant factor that relates apples and cost. To find the cost of 6 apples, you simply multiply 6 apples by 20 cents per apple (p. 167).

Many authors, including Cramer et al., associate the unit rate strategy with situations in which a person is purchasing several units of the same item for a given price. In this case, the unit rate is the price per unit. Although often associated with situations involving purchases, the unit rate strategy can be used in a variety of situations. For example, Singh (2000) showed how the Mr. Tall/Mr. Short problem can be solved through a unit rate strategy. As another example of the unit rate strategy in a non-money situation, Telese and Abete (2002) said that if a three-ounce serving of hamburger has 21 grams of fat, the unit rate is seven grams of fat per ounce of hamburger.

As Cramer et al. (1993) pointed out, the unit rate strategy involves finding the multiplicative relationship *between* measure spaces. Thus, it is equivalent to the "between" measure space strategy described by Karplus et al. (1983). This strategy or very similar strategies have been called the "unit ratio" approach by Lo (2004) and the "unit measure" strategy by Rupley (1981).

Several researchers have found that many students use the unit rate strategy and are generally successful with it (Ben-Chaim et al., 1998; Cramer et al., 1993; Post et al., 1993; Rupley, 1981). Ben-Chaim et al. found that students using *Connected Mathematics* were much more likely than other students to use the unit rate approach on some problems and were very successful in its use. Rupley found that ninth-grade students instructed in the use of the unit measure strategy had a success rate on proportion word problems that was similar to the success rate of the eleventh-grade students in the study.

Rupley also found that students invent this strategy on their own before receiving instruction in proportionality and therefore suggested that this method be taught prior to other strategies.

Although students seem to be successful when using the unit rate strategy, this strategy also has disadvantages. For example, it is much easier to use when the first number in the problem is smaller than the second (Rupley, 1981). For example, consider this problem: “The total length of 7 identical pieces of pipe is 28 feet. The length of 5 pieces of this pipe is how much?” (Rupley, p. 2). Rupley believed this is a fairly easy problem because the first step, using the unit rate method, is to divide 28 by seven. The problem would have been more difficult if it had read “The total length of 28 identical pieces of pipe is 7 feet. The length of 5 pieces of this pipe is how much?” This problem would have involved dividing seven by 28. Thus, the size of the numbers can influence students’ success with the unit rate method. However, Rupley found this effect only when students invented the unit rate method on their own; once students received instruction in this method, the order of the numbers did not significantly influence their success with the unit rate method.

Another disadvantage of the unit rate method is that once students have divided, they can use additive reasoning rather than multiplication to solve the problem. For example, in the problem above, once students have divided 28 by seven and determined that each piece of pipe is four feet long, they could add four five times rather than multiplying four and five. Therefore, Singh (2000) cautioned that use of the unit rate method does not ensure that students are using multiplicative reasoning.

*Proportion Strategy*

Another solution method involves the writing of a proportion. After an informal review of textbooks, Fisher (1988) reported that the proportion strategy is the most common strategy presented. Although the proportion method is a powerful problem-solving tool (Behr et al., 1983), it has advantages and disadvantages, which are described in the following paragraphs.

An advantage of the proportion strategy is that it can be applied to all missing value problems with more or less equal ease. In other words, students are less affected by the specific numbers in the problem when they use proportions than they are when they use the unit rate method (Rupley, 1981).

A disadvantage of the proportion strategy is that it is a procedure that some students perform mechanistically; they may arrive at correct answers but not develop proportional reasoning or understand why the procedure works. For example, Lamon (1999) wrote that when faced with a proportional-reasoning problem, "you may be tempted to use an equation of the form $\frac{a}{b} = \frac{c}{d}$, but using those symbols is not reasoning" (p. 5). Heinz & Sterba-Boatwright (2008) found that preservice elementary teachers could successfully set up proportions but had little conceptual understanding of them. They stated, "Students have learned how to arrange the four quantities in a proportion….However, the students did not seem to be attending to the fact that a proportion is more than just a structural way to organize four quantities" (p. 529). Because the use of proportions seems to not foster conceptual understanding, researchers have suggested they be less heavily emphasized (Karplus et al., 1983).

Proportions can be solved several different ways. Two of the most common are the "equivalent fraction" method and the cross-multiply-and-divide algorithm. The fraction strategy is one method of solving a proportion. It has been referred to as the "fraction" strategy (Cramer et al., 1993) or the "equivalent fraction method" (Curcio & Bezuk, 1994). Cramer et al. stated that rate pairs are treated as fractions and that the multiplication rule for generating equivalent fractions is used. Cramer et al. provided this example: $If\ \frac{3}{60} = \frac{6}{?}\ then\ \frac{3}{60} \times \frac{2}{2} = \frac{6}{120}$.

Another method of solving proportions is the cross-multiply-and-divide algorithm. This is the traditional method of solving proportions and is quite common in textbooks. However, when students are given a choice of what strategy to use, very few choose to cross multiply and divide (Karplus et al., 1983). Some educators have suggested that teaching students to use cross multiplication results in lower success rates than teaching students to use the unit rate method. For example, Lamon (1993b) stated, "The traditional, algebraic cross-multiply-and-divide algorithm for solving missing-value proportion problems is used meaningfully by very few students in spite of the fact that it has been the main strategy used in textbooks for many years" (p. 152). The NCTM concurs that "Other approaches to solving proportions are often more intuitive and also quite powerful" (2000, p. 221). The consensus seems to be that cross multiplication should be taught along with other methods of solving proportion problems and that students should understand why it works rather than simply memorizing the procedure.

*Ratio Comparison Problems*

As described above, there is a considerable body of literature regarding solution strategies for *missing value* problems. There is far less research literature on solution strategies for ratio comparison problems. Lamon (1993a, b) and Noelting (1980a, 1980b) are two of the few researchers who have focused on solution strategies for ratio comparison problems. Lamon (2007) stated that children "spontaneously, before instruction in rational numbers" use the process she called "norming" (p. 644). Lamon (2007) discussed a problem in which seven girls share three pizzas and three boys share one pizza. Students must decide "Who gets more pizza, a girl or a boy?" Lamon stated that children use the norming process reason in this way: "If the pizza were served so that there was always 1 pizza for 3 people [as there is for the boys], the first group [girls] would actually get more because they could have fed 2 more people" (p. 644).

Noelting (1980a, b) described two methods of comparing ratios; he called these the *Between* and *Within* strategies. Noelting asked participants to compare two pitchers of orange juice, each made with different amounts of orange juice concentrate and water. According to Noelting, the *Within* strategy was used by participants who compared the amounts of orange juice and water within each recipe first and then compared these two ratios. The *Between* strategy was used by participants who compared the two amounts of orange juice and the two amounts of water.

Lamon (1993a) also discussed *Between* and *Within* strategies. Using her pizza example, she stated "One way to relate the spaces is with the *between strategy,* a comparison of number of children to the corresponding number of pizzas.…A second option is to think about children and pizzas separately, in which case quantities would be related using a *within strategy*" (p. 140). Lamon and Noelting may have had different understandings of these strategies since Noelting's *Between* strategy compared two like objects and Lamon's did not. Thus, there appears to be inconsistency in the research literature in the use of the terms *Between* and *Within*.

### *Summary of Solution Strategies*

Solution strategies for *missing value* problems are more often discussed in research literature than are solution strategies for *ratio comparison* problems. The traditional method of solving a *missing value* problem is through the use of a proportion and the cross-multiply-and-divide algorithm. However, this method is highly symbolic and not easily understood conceptually. Less symbolic methods, such as the *building up* and *unit rate* strategies are less efficient, but may be more easily understood by students.

## Common Errors

Students have predictable error patterns when working with proportional situations. Understanding these common errors may enable curriculum authors to design problems and activities that help students recognize and avoid them. One common error is that students often use additive reasoning when proportional reasoning is needed (Bright et al., 2003; Singh, 2000). Another common error is that students often attempt to apply proportional reasoning to situations in which it is not appropriate (Heinz & Sterba-Boatwright, 2008; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005; Van Dooren et al., 2009). Examples of these errors are provided in the following paragraphs.

*Inappropriate Application of Additive Reasoning*

As described in Chapter 1, there is an important difference between additive and multiplicative (proportional) reasoning. Additive reasoning is developed in early elementary school and many students have difficulty making the transition to multiplicative reasoning in late elementary or middle school. Thus, one mistake students make when working with proportional situations is to attempt to apply additive reasoning (Bright et al., 2003; Che, 2009; Karplus et al., 1983; Lamon, 2007; Parker, 1999; Singh, 2000). Post et al. (1993) pointed out that this error is an attempt to apply previous understandings to new situations; they stated:

> Two types of relationships exist between any two numbers: additive and multiplicative. Additive considerations based on variations of the counting theme (count up, count back, skip count up, skip count back) dominate mathematics prior to the introduction of rational number concepts. We know that this additive baggage is difficult for children to modify when new content domains require multiplicative, rather than additive conceptualizations (p. 333).

Bright et al. (2003) posed the following problem to 14 eighth-grade students:

> A farmer has three fields. One is 185 feet by 245 feet, one is 75 feet by 114 feet, and one is 455 feet by 508 feet. If you were flying over these fields, which one would seem most square? Which one would seem least square? Explain your answers (p. 167).

Five of the 14 students used the *difference* between length and width as the determining factor for “squareness,” an error that reflects the use of additive reasoning.

Lamon asked readers to consider this problem: “For your party, you had planned to purchase 2 pounds of mixed nuts for 8 people, but now 10 people are coming. How many pounds should you purchase?” (p. 650). A person who reasons additively might state that because the number of people increased by two, the pounds of nuts should increase by two. However, as Lamon points out, adding two people and two pounds means that the people : pound ratio would be 1 : 1, which is not correct.

Although most researchers view additive reasoning as an error when multiplicative reasoning is needed, Parker (1999) and Tourniare (1986) pointed out that additive reasoning is a natural transition to multiplicative reasoning. Parker stated, “Students who have not yet encountered the formal study of proportions…frequently use an additive building-up strategy to solve problems. The use of an additive strategy can be a natural transition to a multiplicative strategy” (p. 286). Parker showed how building-up strategies can be based on either addition or multiplication, thereby providing scaffolding for the transition between the two types of reasoning. Tournaire suggested that proportional reasoning can be taught in elementary school provided that instruction built on “the additive techniques naturally used by students” (p. 411). Thus, although additive strategies are sometimes seen as an error in proportional situations, they can also be seen as a starting place for instruction about proportionality.

### *Inappropriate Application of Proportional Reasoning*

According to the *Curriculum Focal Points* (NCTM, 2006), seventh-grade students should be able to distinguish between situations that are proportional and those that are not proportional. However, research findings have shown that both students and teachers have difficulty distinguishing between proportional and non-proportional situations

(Heinz & Sterba-Boatwright, 2008; Van Dooren et al., 2005; Van Dooren et al., 2009). A common example of the difference between proportional and non-proportional situations is the doubling of the dimensions of a rectangle. If both the length and width of a rectangle are doubled, the perimeter is also doubled. This is because the relationship between length, width, and perimeter is proportional. However, if the length and width of a rectangle are doubled, the area is *not* doubled. This is because the relationship between length, width, and area is not proportional.

As Van Dooren et al. (2005) pointed out, many of the situations children encounter in real life are proportional, as in the following examples: "4 handfuls of sand to fill one bucket, so 12 handfuls to fill three buckets; one toy car has four wheels, so two toy cars have eight wheels; and so forth" (p. 59). Van Dooren et al. also pointed out that the elementary-school curriculum is centered around proportional situations. They stated:

> Much attention is paid to proportional relations because of their wide applicability and usefulness not only for understanding numerous everyday life situations but also many problems in mathematics and science. This begins already in Grade 2 of elementary school when children learn how to multiply and divide….[and solve] word problem such as "1 kg of apples costs 2 euro. How much do 3 kg of apples cost?" (p. 60).

Van Dooren et al. speculated that the prevalence of proportional situations in everyday life and school mathematics results in students incorrectly using proportional reasoning to solve problems like "A shop sells 312 Christmas cards in December. About how many do you think it will sell altogether in January, February and March?" (p. 61).

In some mathematical relationships, when the value of one variable increases, the value of another variable decreases. This is called *inverse variation*. Thus, proportional reasoning cannot be used to correctly solve these problems. Fisher (1988) asked teachers to solve this problem: "If it takes 9 workers 5 hours to mow a certain lawn, how long would it take 6 workers to mow the same lawn?" (p. 160). Fisher found that many secondary-education teachers attempt to apply proportional reasoning to inverse relationships and then fail to notice that their answers are unreasonable.

Because students have difficulty determining whether a problem can be solved with proportional reasoning, it seems reasonable to state that textbooks should help students learn to determine which situations require proportional reasoning and which do not. The extent to which recent textbooks contain material intended to help students discriminate between proportional and nonproportional situations is not clear.

In this section, errors that students often make when dealing with proportionality have been discussed. In the next section, curriculum analysis methodology is discussed.

## Curriculum Analysis Methodology

The previous sections suggest that an analysis of middle-school textbooks would be helpful in determining how proportionality is covered. Before researchers begin a curriculum analysis, they should understand the criticisms that have been aimed at current curricula, frameworks that have been used to analyze curricula, and methods that have been used in other curriculum analyses. Therefore, in this section, the following three topics are discussed: (a) methods of selecting textbooks for analysis, (b) methods of measuring the challenge inherent in tasks, and (c) common features of curriculum analysis.

*Textbook Selection*

According to a 2004 report by the National Research Council (NRC), "the conduct of a content analysis requires identifying either a set of standards against which a curriculum is compared or an explicitly contrasting curriculum" (p. 74). State content standards provide a general outline of what should be taught at various grade levels, but do not provide details regarding problem types and solution strategies that should be presented or other factors researchers may look for. Thus, rather than comparing curricula to standards, curricula are often compared to each other, as recommended by the NRC.

*Funding by the National Science Foundation*

In an attempt to improve student achievement in mathematics and science, the National Science Foundation provided funds to develop curricula that would offer creative alternatives to traditional curricula. Tarr et al. (2008) provide background on this occurrence:

> In response to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and in an effort to influence and strengthen the quality of U.S. mathematics textbooks, the National Science Foundation (NSF) has invested an estimated $93 million in K-12 mathematics curriculum development efforts (NRC, 2000). Curriculum development teams…worked together to produce mathematics textbooks that embodied "standards-based" characteristics, including active engagement of students, a focus on problem solving, and attention to connections within mathematical strands as well as to real-life contexts. (p. 248)

Because curriculum development projects that received NSF funding attempted to develop curricula that differed from existing curricula, it is reasonable to expect that the content and methods of presentation in NSF-funded textbooks differs from that of commercially-generated textbooks. In fact, Robinson et al. (2000) stated that NSF-funded curricula “represent a profound departure from traditional textbooks of the past, both in content and in format” (p. 113). One textbook series, developed by the University of Chicago School Mathematics Project, is considered neither NSF-supported nor commercially-generated (NRC, 2004).

*Selecting Textbooks*

Researchers who analyze mathematics textbooks and their effects on achievement generally use two criteria for selecting textbooks: they attempt to select widely-used series and both NSF-funded and non-NSF-funded curricula (Hodges et al., 2008; Johnson, Thompson, & Senk, 2010; Tarr et al., 2008). For example, Hodges et al. selected Glencoe’s *Math Connects* because it was “a popular textbook in the middle schools with which we were familiar” (p. 79) and selected *Connected Mathematics* and *Math Thematics* because they are NSF-funded series. Similar, Tarr et al. chose textbook series with “significant market share” (p. 253) and also chose both traditional and NSF-funded textbooks. Tarr et al. used the Glencoe publication as the non-NSF-funded curriculum and *Connected Mathematics* as the alternative curriculum.

Although researchers attempt to select textbooks with significant market share, determining which textbooks are most widely used can be difficult. The 2000 Mathematics and Science Education Survey has been used to estimate market share (Jones, 2004; Tarr et al., 2008), but the data used in the survey is more than a decade old. Additionally, there has been reorganization among the publishers in the past decade, complicating the use of the 2000 Survey.

*Measuring the Challenge Inherent in Tasks*

One feature of textbooks some researchers have sought to analyze is the level of challenge inherent in tasks. In this section, the author will describe how researchers have attempted to measure the level of challenge.

In the 1990s, the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project drew attention to the importance of the level of challenge offered to students through the problems on which they worked. The QUASAR project was motivated by the observation that "conventional mathematics instruction has placed a heavier emphasis on memorization and imitation than on understanding, thinking, reasoning, and explaining" (Silver & Stein, 1996, pp. 477-478). Silver and Stein encouraged increased "use of tasks that require students to construct meaning and/or to relate important mathematical concepts to symbols, rules, and procedures" (p. 481). There is some evidence that, internationally, tasks in mathematics textbooks offer little challenge. Valverde et al. (2002) believe this situation represents a paradox; they stated "At a time in which school mathematics and science are deemed by many countries to be among the most socially valued curricular areas…the most prevalent expectations for student performance promoted by textbooks were remarkably basic and comparatively undemanding" (pp. 136-138).

Increasing the level of challenge offered by tasks in textbooks requires a method of describing the level of challenge. Researchers have used various methods to do so; these methods are described below.

*Common Methods of Measuring Challenge*

Yan and Lianghuo (2006) described several ways of measuring the challenge inherent in tasks. One way is to distinguish between “routine problems” and “non-routine problems.” Yan and Lianghuo defined a non-routine problem as “a situation that cannot be resolved by merely applying a standard algorithm, formula, or procedure, which is usually readily available to problem solvers” (p. 613). However, most problems can be solved through the use of some algorithm or procedure and determining which algorithms and procedures are “standard” and “readily available to problem solvers” can be difficult.

According to Yan and Lianghuo (2006), the level of challenge offered by tasks can also be measured by distinguishing between “traditional” and “non-traditional” problems. However, this requires careful definitions of these words. Yan and Lianghuo described four sub-types that collectively defined a “non-traditional” problem. Because other researchers have defined “non-traditional” differently, the meanings of these words are not clear.

According to Yan and Lianghuo (2006), the level of challenge offered by tasks can also be measured by distinguishing between open-ended and close-ended problems; “an open-ended problem is a problem with several or many correct answers. Correspondingly, a close-ended problem is a problem that simply has only one answer” (Yan & Lianghuo, p. 613). However, although open-ended tasks often offer more challenge than close-ended tasks, this is not always the case. Therefore, distinguishing between these two types of tasks is not sufficient when attempting to measure the level of challenge inherent in a task.

The previous paragraphs illustrate that describing the challenge inherent in tasks can be difficult and requires careful definitions. Therefore, researchers have frameworks that organize tasks into levels based on the cognitive demand required to complete the task. One such framework is the Mathematical Task Framework.

*The Mathematical Task Framework*

A tool that has been used to measure what is expected of students is the Mathematical Task Framework (Stein et al., 2000), which describes levels of cognitive demand. According to Stein et al., "cognitive demand" refers to "the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 11). Smith et al. described four levels of cognitive demand which are summarized in the following paragraphs.

Stein et al. (2000) described two levels of tasks that require "low" cognitive demand. These are *Memorization* tasks and *Procedures Without Connections* tasks. According to Stein et al., a *Memorization* task "involves either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae or definitions to memory" (p. 16). In a *Memorization* task, students simply reproduce previously seen material. In order to classify an exercise as a *Memorization* task, the researcher would need to know to what content students had been exposed. For example, Stein et al. gives this example of a *Memorization* task: "What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?" (p. 13). However, the classification of this as a *Memorization* task implies that students had previously memorized these facts. If students had never before seen the decimal and percent equivalents for these fractions, this would not be a *Memorization* task.

The second level in the Mathematical Task Framework includes tasks that require *procedures without connections to understanding, meaning, or concepts*, which is usually shortened to *Procedures Without Connections*. In this type of task, "use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task" (Stein et al., 2000, p. 16). Again, to place a task in this category, a researcher would need to be familiar with the prior instruction. Stein et al. gives this example of a *Procedures Without Connections* task: "Convert the fraction $\frac{3}{8}$ to a decimal and a percent" (p. 13). The procedure for doing so is to divide 3 by 8; placing this task in this category is appropriate only if students are familiar with that procedure.

Stein et al. (2000) described two levels of tasks that require "high" cognitive demand. These are *Procedures With Connections* tasks and *Doing Mathematics* tasks. According to Stein et al., in a *Procedures With Connections* task "Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding" (p. 16). These types of tasks often involve graphical representations. For example, asking students to represent $\frac{3}{5}$ on a 10-by-10 grid is a *Procedures With Connections* task (Stein et al.). In order to complete the task, students must reason that a 10-by-10 grid contains 100 squares; thus, $\frac{3}{5}$ would need to be converted to a fraction with 100 as the denominator.

The highest level in the Mathematical Task Framework is titled *Doing Mathematics*. This type of task requires "students to explore and understand the nature of mathematical concepts, processes, or relationships" (Stein et al., 2000, p. 16). Stein et al. provides this example: "Shade 6 small squares in a 4 × 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of area that is shaded, and c) the fractional part of area that is shaded" (p. 13). This is considered a *Doing Mathematics* task because students are provided with little guidance regarding how to proceed. It is up to the student to access relevant knowledge.

According to Jones (2004), classifying tasks according to their level of cognitive demand was more difficult than classifying tasks along other dimensions. Also, the reliability of this coding was lower than the reliability of coding along other dimensions. Jones compared his coding to that of two other raters; his coding of levels of cognitive demand matched the first rater's on 77% of the tasks and matched the second rater's on 73% of the tasks. He also coded 60 tasks twice with a time delay between and found that his earlier codes matched the later codes 88% of the time.

### *Common Features of Curriculum Analysis*

Studies of mathematics textbooks generally focus on a single content area, such as data analysis (Cai, Lo, & Watanabe, 2002), probability (Jones, 2004) or reasoning and proof (Johnson et al., 2010). Despite the varying content areas, researchers conducting curriculum analyses generally look for certain aspects of the treatment of their topic. These common features of curriculum analysis are discussed in the following paragraphs. Then, to familiarize the reader with curriculum analysis methods, two case studies in curriculum analysis are described. Although the research projects discussed below do not relate to proportional reasoning, they do show how curriculum analysis is conducted.

Although the procedures used in curriculum analysis vary from study to study, there are some commonly-used methods, which are described below.

*Number of Pages or Lessons*

In some curriculum analyses, researchers attempt to determine the approximate percentage of a textbook that is devoted to a certain topic. This is often done by counting pages or lessons. For example, Jones (2004) counted both the number of pages and the number of lessons devoted to probability. He then calculated percentages of textbooks devoted to probability. This method allows researchers to compare textbooks from various time periods or various publishers. However, this method is not always feasible. Some mathematical topics are spread throughout the book, making this type of estimate much more difficult. For example, Johnson et al. (2010) investigated the treatment of proof-related reasoning. Rather than examine every page in the textbooks, they restricted their analysis to three topic areas that they thought were most likely to contain proof-related reasoning. Because they did not examine every page in the textbooks, they could not arrive at an estimate of the percentage of pages in the textbook that dealt with proof-related reasoning. Similarly, as an "integrative thread that connects many of the mathematics topics studied in grades 6-8" (NCTM, 2000, p. 217), proportionality should appear throughout middle-school textbooks. Researchers could choose to examine every page in middle-school textbooks, or they could choose to focus on sections that are likely to relate to proportionality, such as sections on algebra, rational numbers, similar figures, and probability. Making this latter choice would allow researchers to devote more attention to these sections but would preclude estimating the percentage of pages devoted to the subject.

*Problem Types*

When conducting curriculum analyses, researchers often note the various types of problems contained in textbooks. Problem types are often supported by research literature. For example, the literature on mathematical proof indicated that students often confuse specific examples of a property with a general proof or argument that applies to all instances of the property. For this reason, Johnson et al. (2010) defined "specific" and "general" problem types. Similarly, Jones (2004) relied on previous research suggesting that children's probabilitistic reasoning consisted of four constructs. He counted the number of tasks that related to each of the four constructs and then calculated the percentage of probability tasks that related to each of the four constructs. As with other curriculum analyses, research on the treatment of proportionality should include attention to problem types. Because the research literature on proportional reasoning suggests that there exists three problem types (missing value, comparison, and qualitative), a curriculum analysis on proportionality should include attention to these problem types.

*Visual Representations*

Educators believe that "using visual representations, such as symbols, drawings, and graphs, helps middle-school students reason about and understand mathematics. These representations support students' learning and help them communicate their mathematical ideas" (Hodges et al., 2008). Specifically, research has shown that middle-school students understand some representations of rational numbers better than others (Martinie & Bay-Williams, 2003a). Research findings indicate that the representations used affect students' understanding of rational numbers. For example, NAEP data indicate that interpreting fractions and decimals as locations on a number line is more difficult for students than interpreting fractions as part of geometric regions (Kouba, Zawojewski, & Struchens, 1997).

*Examples, Exercises, and Tasks*

Researchers who conduct curriculum analyses make decisions about how to segment the curriculum into manageable pieces. Several researchers have defined blocks they call “tasks” (Jones, 2004; Jones & Tarr, 2007; Valverde et al., 2002). Looking at these authors’ sample applications of their coding schemes reveals that a “task” typically consists of one example or one exercise. Other researchers have focused on statements of properties in the “narrative” portions of textbooks and individual exercises in the exercise sets of textbooks (e.g., Johnson et al., 2010). In this study, an “example” was defined as a problem or question that is labeled as an example and that is solved or answered in the lesson in which it appears. An “exercise” was defined as a problem or question that appears in an exercise set and is not solved or answered. Exercises may contain multiple parts, as explained in a later section. In this report, the word “task” refers to an activity, example, or exercise.

*Example of Curriculum Analysis: Reasoning and Proof*

Johnson et al. (2010) analyzed the treatment of proof-related reasoning in high-school textbooks. The researchers began by conducting a literature review to familiarize themselves with major developments in the field. This literature review affected the framework they developed. For example, research findings indicated that many students and teachers do not understand the difference between an empirical argument based on specific examples of a phenomenon and a logical argument that applies to *all* cases of the phenomenon. The researchers later built into the framework a distinction between the two types of arguments.

Once the literature review was complete, the researchers consulted standards documents published by the NCTM. The researchers noted what the standards documents said that students should be able to do. Because some expectations that appear in standards documents are difficult to define precisely, only some of these expectations were built into the researchers' framework. The researchers made decisions about which expectations were likely to appear in textbooks in a measureable manner; the standards-based expectations they felt could be defined in a precise and measureable way were built into the curriculum analysis framework. On the basis of these expectations, the researchers developed a short list of categories or "codes" they could apply to the examples and exercises in the textbooks.

The researchers chose six textbooks series to analyze. They based the selection on their knowledge of which series are frequently used and their desire to compare commercial series to curriculum development projects. They included a total of 20 textbooks. They did not analyze teacher's editions or supplemental materials.

The researchers then developed a list of key words associated with reasoning and proof. They searched both the table of contents and indices for these key words. On the basis of this search, the researchers chose three content areas that seemed to contain the most material related to reasoning and proof. Rather than conducting a curriculum analysis on every lesson in each textbook, they focused on lessons in the three content areas they had identified.

The researchers then examined the examples and exercises in the textbooks and compared them to the categories they had developed. Because they found some examples and exercises that did not fit well into the existing categories, new categories were added. As the researchers examined additional lessons, they continually reviewed the framework to ensure their categories adequately represented the types of reasoning presented in the examples and required in the exercises.

The researchers found that developing a curriculum analysis framework to examine the reasoning and proof in high-school textbooks required several revisions. The researchers often initially disagreed about which category an example or exercise most closely resembled. In the end, however, they did develop a framework that could be used reliably.

*Example of Curriculum Analysis: Probability*

Jones (2004) developed a framework to analyze the treatment of probability in middle school textbooks. Because he wished to compare contemporary textbooks to older ones, he chose two textbook series from each of four time periods. Within each time period, the researcher chose one “popular” and one “alternative” series. He did not analyze teacher’s editions or supplemental materials. Once the researcher had chosen textbooks, he analyzed the textbooks for their treatment of probability.

One of the research questions posed by Jones (2004) related to the extent to which middle-grades textbooks incorporate probability. He answered this question by counting the number of pages that contained probability tasks and comparing it to the number of pages in the textbook. This allowed him to state whether the percentage of pages devoted to probability has changed in the past few decades. He also identified probability “constructs,” or topics within probability such as theoretical probability, independence, experimental probability, sample space, etc. and determined the percentage of analyzed tasks that related to each of the constructs.

Another research question posed by Jones (2004) related to “the nature of the treatment of probability topics” (p. 7). To answer this question, Jones noted which manipulatives were used, whether tasks were new or repetitions of previously-encountered tasks, and the level of cognitive demand required by tasks.

The third and final research question posed by Jones (2004) concerned the “structure” and “components” of probability lessons. He defined “components” as definitions, worked examples, and oral or written exercises. He also reported on lesson sequences, or the organization of lessons related to probability.

*Summary of Curriculum Analysis Methodology*

Although every curriculum analysis study utilizes different research methods, many have similarities. In selecting textbooks for their study, researchers often include both NSF-funded (“*Standards*-based”) and non-NSF-funded (“traditional”). Some researchers attempt to measure the challenge inherent in tasks; one method that has been used (Jones, 2004) is the Mathematical Task Framework (Stein et al., 2000). In some curriculum analyses, page numbers devoted to a given topic are counted. This allows the researcher to state the percentage of the textbook devoted to the topic. Researchers conducting a curriculum analysis regarding a given topic, such as proof or probability, generally do not analyze the entire textbook but instead analyze the lessons most closely related to that topic.

## Summary of Literature Review

An understanding of proportionality, or “proportional reasoning,” represents an important transition from elementary, additive ways of thinking to more advanced, multiplicative modes of thought. However, successfully completing this transition can be difficult. Part of this difficulty is likely due to the complexity of this type of reasoning as it involves comparisons of ratios, which are themselves comparisons of quantities.

Researchers have identified three problem types related to proportionality: *missing value*, *ratio comparison*, and *qualitative*. In a *missing value* problem, students are given three out of four values that form a proportion and are asked to find the fourth. In a *ratio comparison* problem, students are asked to compare or order ratios or fractions. To solve a *qualitative* problem, students must think about which values are bigger or smaller without being given numerical quantities.

Proportionality is a mathematical characteristic of a wide variety of situations and could, therefore, appear within textbooks in a variety of content areas. Proportionality could appear in lessons on algebra through discussions of patterns, slope, and *y*-intercept. Proportionality is likely present in some lessons on probability, statistics, and data analysis. It could appear in lessons on geometry in the guise of similar figures or in measurement in the form of measurement conversions. Proportionality is often closely associated with rational numbers (e.g., Behr et al., 1992; Lamon, 2007).

Methods of solving *missing value* problems have been widely discussed and include the *building up* strategy, *proportions*, and the *unit rate* method. Students seem to be able to discover the *building up* and *unit rate* strategies on their own; thus, they may provide a bridge to true proportional reasoning. The proportion strategy is more advanced and some researchers have suggested that students use it mechanistically without true understanding.

Researchers have discussed errors that students commonly make. These include using additive reasoning when multiplicative reasoning is needed and vice versa. Elementary-school mathematics focuses on additive reasoning, and some scholars believe students have difficulty leaving this “additive baggage” behind (Post at al., 1993, p. 333).

Despite the large body of research on proportional reasoning and rational numbers, there are still unanswered questions. Existing research has focused on understanding the mathematics inherent in these problems or understanding children’s thinking about them. No rigorous analysis of the treatment of proportionality in textbooks has been conducted.

## Unanswered Questions

Because proportionality is difficult for many students, it is important that its coverage in textbooks is the best that it can be. Because no recent curriculum analyses of the treatment of proportionality have been conducted, it is unknown how textbooks cover the topic. A curriculum analysis is needed. This research could help curriculum authors improve how they present proportionality to students and could help teachers improve their use of existing textbooks.

When conducting an analysis of curriculum materials related to proportionality, researchers should look for several factors. First, because the NCTM has stated that proportionality is a connective thread that runs throughout the middle-school curriculum, proportionality should be emphasized in several content areas including algebra, data analysis, geometry and measurement, percent, and probability. It is not necessary that proportionality be emphasized equally in each of these content areas, but it should be mentioned in lessons in each content area.

Second, three main types of problems have been identified (*missing value*, *ratio comparison,* and *qualitative*) and textbooks should contain all three. Again, it is not necessary that one-third of the examples and exercises deal with each type of problem, but each type of problem should be represented.

Third, researchers should examine the solution strategies encouraged by textbooks. Proportions have been the traditional method of answering proportional questions, but many researchers believe that this method is less intuitive than others. Researchers believe that the *building up* strategy can provide a scaffold from additive to multiplicative reasoning and that the *unit rate* method seems more natural to students than setting up a proportion and that many students are more successful with the unit rate method (Cramer et al., 1993).

A fourth factor that researchers should look for is attention to common errors. Many students use additive reasoning when multiplicative reasoning is required and vice versa. Educators have criticized textbooks for inadequately pointing out the differences between proportional and nonproportional situations (Cramer et al., 1993). However, these criticisms are more than a decade old. The extent to which current textbooks point out the differences between proportional and nonproportional situations in unknown.

Fifth, researchers should investigate the level of cognitive demand required by examples and exercises that involve proportionality. There are large differences between "lower-level" and "higher-level" tasks (Stein et al., 2000). These types of tasks are likely to foster different kinds of knowledge and skills. Finally, because students have more difficulty with some graphical representations of rational numbers than with others (Kouba et al., 1997), researchers should determine which types of graphical representations are used.

## Summary of the Pilot Study

As part of the refinement of the framework for analyzing the treatment of proportionality in textbooks, the researcher applied the framework to two sixth-grade textbooks, one from the *Math Thematics* series (Billstein & Williamson, 2008) and the other published by McDougal Littell (Larson, Boswell, Kanold, & Stiff, 2004). A summary of the pilot study is provided below and details are located in Appendix A.

Of the three problem types described in the literature review, many examples of *missing value* and *ratio comparison* problems were found. Less than 1% of the tasks were of the *qualitative* problem type. The researcher found tasks related to proportionality that could not be classified as any of the three types found in the research literature. For example, putting a fraction into lowest terms cannot be classified as any of the three problem types.

The researcher found tasks related to proportionality in all of the content areas described in the literature review. More than half of the tasks were found in the rational number content area, reflecting the middle-school curriculum's emphasis on fractions. The *Math Thematics* textbook also emphasized proportionality in the geometry/ measurement content area whereas in the traditional textbook, less than 10% of the proportional exercises were in this content area.

The researcher found very few tasks that could help students understand the differences between proportional and non-proportional situations. Textbooks could help students understand these differences by pointing out the characteristics of proportional situations: they have a constant rate of change and a *y*-intercept of zero. In the two textbooks combined, less than 4% of the exercises related to proportionality pointed out these characteristics.

# CHAPTER THREE: METHODS

The purpose of this chapter is to describe the methods used in the study. The chapter contains six sections. The first discusses how the textbooks used in the study were selected, the second discusses how lessons from those textbooks were selected, the third describes how tasks were selected and counted, the fourth describes the curriculum analysis framework, the fifth discusses reliability, and the sixth presents a summary of the methods used.

## Textbook Selection

Sample selection occurred in three stages. First, of the wide variety of textbook series available for middle-school students, three widely-used series were selected. Second, from the dozens of lessons in each textbook, lessons that were most closely related to proportionality were selected for the study. Third, from each of the selected lessons, tasks (activities, examples, and exercises) related to proportionality were selected and coded. The textbook-selection criteria are described in this section. Subsequent sections describe the lesson-selection and task-selection processes.

Three contemporary middle-school textbook series were selected: *Math Connects: Concepts, Skills, and Problem Solving*, published by Glencoe McGraw Hill, the third edition of books developed by the University of Chicago School Mathematics Project (UCSMP) which are titled *Pre-transition Mathematics*, *Transition Mathematics*, and *Algebra*, and the second edition of books developed by the Connected Mathematics Project (CMP). These textbooks were selected because are all widely-used. One is NSF-funded (CMP) and two are not (UCSMP and *Math Connects*). Details about selection criteria and information about each textbook series is provided below.

### *Selection Criteria*

Two criteria were used to select textbook series for the study. First, only textbooks from widely-used series were selected. Textbook publishers consider market share data to be proprietary information; thus, this information was unavailable. However, other sources suggested that the three series chosen are widely used (Hodges et al., 2008; Huntley, 2008; Jones & Tarr, 2007; Lappan, Phillips, & Fey, 2007; Ridgway, Zawojewski, Hoover, & Lambdin, 2003; Tarr et al., 2008).

Second, because the researcher was interested in obtaining varying pictures of the treatment of proportionality, she chose series with different histories and goals. Curriculum projects that received funding through the National Science Foundation (NSF) have generally been heavily influenced by the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the *Principles and Standards for School Mathematics* (NCTM, 2000); thus, they are often called *Standards*-based curricula (Robinson et al., 2000). *Standards*-based curricula “offer an approach to mathematics teaching and learning that is qualitatively different from conventional practice in content, priorities, organization, and approaches” (Hirsch, 2007, p. 1). Thus, in order to obtain varying pictures of the treatment of proportionality, the researcher chose one textbook series that was NSF-funded and *Standards*-based (CMP), one series that was not NSF-funded (*Math Connects*), and one series that was not NSF-funded but was designed as an alternative to traditional curricula (UCSMP).

## *Connected Mathematics2 (CMP)*

One of the three textbook series chosen for this study was the CMP curriculum. The following paragraphs describe the goals and format of the CMP curriculum.

### *Goals of the CMP Curriculum*

The authors of the CMP curriculum provide this background:

> The Connected Mathematics Project (CMP) was funded by the National Science Foundation between 1991 and 1997 to develop a mathematics curriculum for grades 6, 7, and 8. The result was Connected Mathematics 1 (CMP 1), a complete mathematics curriculum that helps students develop understanding of important concepts, skills, procedures, and ways of thinking and reasoning in number, geometry, measurement, algebra, probability, and statistics. In 2000, the National Science Foundation funded a revision of the CMP materials. (Lappan et al., 2007, p. 67)

Proportionality is a central focus of the CMP curriculum. In fact, "the highest priority goal for the seventh-grade *Connected Mathematics* curriculum is the development of proportional reasoning" (Ridgway et al., 2003, p. 213). The development of *CMP* materials was guided by research literature on students' proportional reasoning and understanding of rational numbers (Lappan et al., 2007). One would therefore expect the treatment of proportionality in these materials to be different from that of textbooks with different goals and whose development was not guided by this research. Although there is no published information regarding the market penetration of the second edition, the first edition of *Connected Mathematics* was widely used (Hodges et al., 2008; Huntley, 2008; Lappan et al., 2007; Ridgway et al.).

CMP is a problem-centered curriculum (Lappan et al., 2007), meaning that the "narrative" sections set up one or more mathematical situations for students but generally do not explain how to solve problems. Because the focus is on student solution of problems, CMP texts contain far fewer "examples" than the other two series selected for the study.

*Format of the CMP Curriculum*

The CMP curriculum consists of eight modules at each of the three grade levels. The modules are usually purchased as separate small booklets. Thus, there is no single CMP "textbook" for each grade level. However, the modules are also available bound together and it was this format that the researcher used during this study. Therefore, in this study, a CMP "textbook" refers to the set of modules for a given grade level.

Four sixth-grade modules were included in the study: *Bits and Pieces I* (Lappan et al., 2009a), *Bits and Pieces II* (Lappan et al., 2009b), *Bits and Pieces III* (Lappan et al., 2009c), and *How Likely Is It?* (Lappan et al., 2009d). Six seventh-grade modules were included: *Variables and Patterns* (Lappan et al., 2006a), *Stretching and Shrinking* (Lappan et al., 2006b), *Comparing and Scaling* (Lappan et al., 2006c), *Moving Straight Ahead* (Lappan et al., 2006d), *Filling and Wrapping* (Lappan et al., 2006e), and *What Do You Expect?* (Lappan et al., 2006f). Three eighth-grade modules were included: *Thinking with Mathematical Models* (Lappan et al., 2006g), *Say It with Symbols* (Lappan et al., 2006h), and *Samples and Populations* (Lappan et al., 2006i). A list of the Investigations included from each of these modules is provided in Appendix B. Each Investigation was counted as one lesson.

## *Glencoe Math Connects*

Glencoe's most recent publications for middle-school mathematics were published in 2009 and are titled *Math Connects: Concepts, Skills, and Problem Solving*. The *Math Connects* books are labeled "Course 1" (sixth grade), "Course 2" (seventh grade), and "Course 3" (eighth grade). *Math Connects* replaced Glencoe's previous middle-school mathematics publications which were titled *Mathematics: Applications and Connections* (published 1998-2001) and *Mathematics: Applications and Concepts* (published 2004-2006). Glencoe's middle-school textbooks are widely used (Hodges et al., 2008; Jones & Tarr, 2007; Tarr et al., 2008).

### *Goals of the Math Connects Curriculum*

According to a Glencoe publication (Macmillan/McGraw Hill/Glencoe, 2009), the *Math Connects* textbooks are based on the NCTM's *Curriculum Focal Points* (NCTM, 2006), process standards (NCTM, 2000), and research on problem solving, reasoning, representation, discourse, reading, and writing. According to the Glencoe publication, the *Math Connects* curriculum "offers a balanced approach of real-world applications, hands-on labs, direct instruction, writing exercises, higher-order thinking and practice that enables students to develop both conceptual understanding and procedural knowledge" (p. 1).

### *Format of the Math Connects Curriculum*

Each of the three *Math Connects* books contains five units. Each unit is divided into two or three chapters. Each of these chapters contains between seven and 10 lessons. Thus, each *Math Connects* textbook contains about 125 lessons. Each lesson contains about a page of narrative and several worked examples. Thus, the *Math Connects* books contain more narrative and worked examples for students to read than do the CMP textbooks. Some lessons are titled “Explore” or “Extend” and are meant to introduce or extend another lesson. Each chapter contains between one and four “Explore” and “Extend” lessons. The “Explore” and “Extend” lessons generally consist of an activity, usually with multiple steps.

*University of Chicago School Mathematics Project (UCSMP)*

Thompson and Senk (2001) provided the following background regarding the UCSMP textbooks:

> The University of Chicago School Mathematics Project (UCSMP) is a research and curriculum development project that was begun in 1983, prior to the development of the NCTM *Standards*….Although UCSMP work was begun prior to the publication of the Standards, continued work has been influenced by the *Standards* movement and the discussions about reform that have resulted from this movement. (p. 59)

The middle-school UCSMP books are titled *Pre-Transition Mathematics* (sixth grade), *Transition Mathematics* (seventh grade), and *Algebra* (eighth grade). The third edition of these textbooks, published by Wright Group/McGraw Hill in 2008 and 2009, was used in the study. These middle-school textbooks are part of UCSMP’s line of secondary-school textbooks and are meant to provide a transition to the high-school curriculum (Usiskin, 2007). The UCSMP textbooks are built on four key program features: a wider scope of mathematical content than found in traditional programs, an emphasis on real-world applications, use of technology, and a focus on four dimensions of understanding: skills, properties, uses, and representations (McConnell et al., 2009).

Some studies of middle-school mathematics textbooks have excluded algebra textbooks from their sample (Jones, 2004; Jones & Tarr, 2007). However, because UCSMP intends for their algebra textbook to be used by at-grade-level eighth-grade students, it is considered a middle-school textbook and was therefore included in the study.

### *Summary of Textbook Selection*

The information presented above illustrates that the three textbook series selected for this study have different histories, goals, and emphases on proportionality. These three textbook series were chosen because they were expected to provide different pictures of how proportionality is treated in middle-school textbooks.

## Lesson Selection

Once the three textbook series were selected, the researcher identified lessons most closely related to proportionality. These lessons were selected only from the main part of the textbooks. Excluded sections of the textbooks are described below.

### *Excluded Sections of Textbooks*

To allow the researcher to focus on the main content of each textbook, certain sections of textbooks were excluded from the study. These included introductory material designed to prepare students for the main content of the textbook as well as glossaries, indices, and standardized-test preparation sections that appeared in the back of the textbooks.

The CMP books have unit reviews, called "Looking Back and Looking Ahead," at the end of each unit. These sections are each approximately three pages long and were excluded from the study. Some units also have one or two Unit Projects, each of which is about two pages long. The Unit Projects were excluded.

The Glencoe *Math Connects* books have sections labeled “Start Smart” in the beginning of each textbook. The “Start Smart” sections were excluded from the study. Additionally, each chapter in the Glencoe *Math Connects* books has a one-page review for checking prerequisite skills. These were also excluded from the study. A few of the lessons in each *Math Connects* book are labeled “Problem Solving Investigation.” Although some of these Investigations contain exercises related to proportionality, their focus is on problem solving strategies rather than proportionality and they were excluded from the study. Mid-chapter quizzes, study guides, and practice tests were also excluded. The Glencoe *Math Connects* books also have sections titled “Looking Ahead to Next Year” at the back of the textbooks. These sections were excluded from the study. The Glencoe *Math Connects* books also have Student Handbooks at the back of the textbooks that include extra practice, preparation for standardized tests, a “Concepts and Skills Bank,” a glossary, selected answers, and an index. These Student Handbooks were excluded. Finally, most lessons in *Math Connects* textbooks begin with a short section labeled either “Get Ready for the Lesson” or “Mini Lab.” These sections each contain one to eight questions that usually focus on lower-level skills that are prerequisites to the lesson. These sections were not analyzed.

In the UCSMP textbooks, each chapter contains sections titled “Projects,” “Summary and Vocabulary,” “Self-Test,” and “Chapter Review.” These sections were excluded. The UCSMP books also contain sections titled “Games,” “Selected Answers,” “Glossary,” and “Index” that were excluded from the study.

*Lesson Selection Criteria*

In this section, the criteria used to select lessons are described. The criteria are grouped by content area.

*Included Algebra Lessons*

Patterns, sequences, and functions lead into the concept of slope; a constant slope is one of the hallmarks of proportional situations. Thus, lessons on patterns, sequences, and functions were included in the study. Lessons on slope were also included in the study. Some lessons on functions do not contain the word "slope," but use instead words like "rate," "rate of change" or "function rule." Lessons such as these were included in the study. Lessons on direct variation were also included. Lessons that related to solving proportions were included. Lessons on using graphs and tables to express linear relationships were included as were lessons on the differences between linear and non-linear functions. Lessons on writing equations to correspond to a data table or verbal description were included in the study.

*Excluded Algebra Lessons*

Lessons on negative numbers and absolute value were excluded from the study. Lessons on evaluating expressions and equivalent expressions were also excluded. Lessons on domain and range were excluded from the study as were lessons on expressions and equations. Lessons on the commutative and distributive properties were excluded. Lessons on order of operations, simplifying algebraic expressions and solving algebraic equations were also excluded, unless they related to solving proportions. Lessons on systems of equations were also excluded.

*Included Data Analysis Lessons*

Lessons on circle graphs generally involve percent or proportions. For example, an example in the eighth-grade *Math Connects* textbook states that 12 out of 30 countries won between 1 and 5 medals in the 2006 Winter Olympics. To make a circle graph, students must determine how many degrees (out of 360) correspond to 12 out of 30 countries. This could be solved through the use of a proportion such as $\frac{12}{30} = \frac{x}{360}$. Therefore, lessons on circle graphs were included. Additionally, lessons on using data, ratios, or probability to make predictions were included in the study.

*Excluded Data Analysis Lessons*

Lessons on bar graphs, box plots, histograms, line plots, and stem-and-leaf graphs were excluded from the study. Lessons on interpreting line graphs were excluded unless they contained a discussion of slope. Lessons on the use of tables as a problem-solving strategy were excluded from the study. Lessons on the parts of graphs, such as titles, scales, axes, and intervals, were excluded.

Probability and statistics are included in the data analysis standard. However, lessons on basic probability including tree diagrams, outcomes, and sample spaces were excluded. Lessons on basic statistics, such as mean, median, mode, and variability, were also excluded.

*Included Geometry Lessons*

Lessons on area, perimeter, or circumference were included only if they contained material regarding how these quantities are affected when the side lengths or the radius is changed. Similarly, lessons on volume or surface area were included only if they contained material regarding how these quantities are affected when side lengths are changed. Lessons on dilations, similar figures, and scale drawings were also included.

Converting from one unit of measurement to another requires multiplication or division; proportions and/or ratio tables are often used. Thus, lessons on measurement conversion were included.

*Excluded Geometry Lessons*

Lessons on the coordinate plane and ordered pairs were excluded from the study. Lessons on translations, reflections, and rotations were excluded. Lessons on types of angles (acute, right, and obtuse) were excluded from the study as were lessons on angle relationships (complementary, supplementary, and vertical). Lessons on the sum of the angles in polygons were excluded as were lessons on constructions with straightedge, compass, and protractor.

Lessons designed to familiarize students with units of measurement were excluded unless they also involved conversions between units. Lessons on elapsed time or temperature were also excluded.

Lessons on area of triangles and parallelograms usually focus on identifying the base and height and using formulas. Some lessons are intended to help students formulate or understand the origin of these formulas, but these lessons are not directly related to proportionality. Therefore, lessons on area of triangles and parallelograms were excluded from the study.

Some lessons on congruent and similar figures focus on definitions of these terms and help students identify corresponding parts. Because these skills do not involve proportionality, lessons on congruent and similar figures were not included unless they also involved calculating lengths of sides of polygons.

*Lessons on Integers*

Although integers are rational numbers, they are less obviously comparisons between quantities than are fractions or ratios. For example, the fraction $\frac{2}{3}$ compares the quantities 2 and 3. The integer 5 can be written as $\frac{5}{1}$, which compares 5 to 1, but the integer 5 is usually thought of as a single quantity rather than a ratio. Thus, most lessons on integers were excluded. In middle school textbooks, lessons on integers generally involve computation with integers, divisibility, order of operations, real-life uses of integers, or rounding integers. These lessons were excluded from the study.

*Included Decimal Lessons*

"Comparing" decimals means to decide which of two decimals is larger. "Ordering" decimals means to arrange three or more decimals from least to greatest or vice versa. The skills of comparing and ordering decimals are related to proportionality. For example, when comparing 2.78 and 2.81, a person could compare two mixed numbers, $2\,\frac{78}{100}$ and $2\,\frac{81}{100}$ or two fractions, $\frac{278}{100}$ and $\frac{281}{100}$. Because this involves ratio comparison, lessons on comparing and ordering decimals were included in the study. Rounding decimals also involves ratio comparison. For example, when rounding 99.96 to the nearest tenth, one compares 99.9, 99.96, and 100.0, or $99\frac{9}{10}$, $99\frac{96}{100}$, and 100. Thus, lessons on rounding decimals were included. Lessons on converting between decimals and fractions were also included.

*Excluded Decimal Lessons*

Lessons on "representing decimals" usually focus on place value and various ways of writing decimals. For example, when explaining the meaning of 1.65, a textbook may point out that the one is in the ones place, the six is in the tenths place, and the five is in the hundredths place. Textbooks often use 10-by-10 grids, base-ten blocks, place value charts, and money to explain the meaning of decimals. For example, 1.65 can be represented by one dollar bill, six dimes, and five pennies. These lessons have little to do with proportionality and were excluded.

Lessons on adding, subtracting, multiplying, and dividing decimals were excluded. Lessons on estimating sums, differences, and quotients of decimals were excluded from the study. Although lessons on estimation involve rounding, which is included in the study, lessons on estimation usually assume students know how to round and instead focus on computation procedures.

*Included Fraction Lessons*

Lessons on equivalent fractions, mixed numbers, improper fractions, and simplifying fractions were included in the study. Lessons on comparing and ordering fractions were included. Rounding fractions involves ratio comparison. For example, rounding $2\frac{3}{8}$ to the nearest half involves comparing 2, $2\frac{3}{8}$, and $2\frac{1}{2}$. Thus, lessons on rounding fractions were included.

Ratios can be written as fractions and can be simplified as fractions. For example, the ratio 2:6 can be written $\frac{2}{6}$ and simplified as either 1:3 or $\frac{1}{3}$. Thus, lessons on ratios were included in the study. Lessons on writing and solving proportions were also included.

*Excluded Fraction Lessons*

Lessons on understanding fractions, such as explanations of what the numerator and denominator represent, were excluded from the study. Lessons on greatest common factor and least common multiple were excluded.

Lessons on fraction computation were excluded from the study. Although adding and subtracting fractions involves finding equivalent fractions, which was included in the study, lessons on adding and subtracting fractions usually assume students are familiar with equivalent fractions and focus instead on procedures for computation. Lessons on estimating sums, differences, products, and quotients of fractions were also excluded. Although lessons on estimation involve rounding, which was included in the study, lessons on estimation usually assume students know how to round and instead focus on computation procedures.

*Included Percent Lessons*

Lessons on converting between fractions, decimals, and percents were included in the study. Proportions can be used to find the percent of a number. For example, to calculate 52% of 298, one could use the proportion $\frac{52}{100} = \frac{\quad}{298}$. Thus, lessons on finding the percent of a number were included in the study. To *estimate* 52% of 298, one could round 52% to 50% and use the proportion $\frac{50}{100} = \frac{\quad}{298}$. Thus, lessons on estimating with percents were also included.

*Excluded Percent Lessons*

Many middle-school textbooks include a lesson on using algebraic equations to solve percent problems. Lessons in which students are taught to use an equation to solve percent problems focus on helping students put numbers in their proper places in the equation and on solving the equation. Because these skills are not related to proportionality, lessons on using algebraic equations to solve percent problems were excluded.

Many middle-school textbooks include a lesson on percent increase and percent decrease, sometimes called “percent of change.” These lessons usually involve the use of a formula, $percent\ of\ change = \frac{amount\ of\ change}{original\ amount}$. Similarly, lessons on interest usually involve the formula $I = prt$. Use of these formulas does not require proportional reasoning. Therefore, lessons on percent of change or interest were excluded.

*Summary of Lesson Selection Criteria*

Table 1 summarizes the types of lessons that were included and excluded in each of the content areas.

Table 1

*Topics of Included and Excluded Lessons*

| Content Area | Included Topics | Excluded Topics |
|---|---|---|
| Algebra | methods of solving proportions | solving equations and systems |
| | constant slope (rate of change) | the distributive property |
| | function rules | order of operations |
| Data Analysis | circle graphs | interpretation of graphs |
| | using probability to make predictions | parts of graphs |
| | | problem-solving strategies |
| Geometry | area, perimeter, and circumference | the coordinate plane |
| | similar figures and scale drawings | transformations |
| | measurement conversion | types of angles |
| Decimals | comparing and ordering decimals | representing decimals |
| | rounding decimals | computation with decimals |
| | converting decimals to fractions | |
| Fractions | equivalent fractions | greatest common factor |
| | improper fractions, mixed numbers | least common multiple |
| | comparing and ordering fractions | computation with fractions |
| Percents | converting percents to fractions | equations and percent problems |
| | finding the percent of a number | percent of change |

*Number of Lessons Included in the Study*

Prior to beginning the study, the researcher examined each of the textbooks and developed a preliminary list of 200 lessons to be included in the study. During the study, closer examination of these lessons revealed that 35 of the lessons on the preliminary list did not focus on proportionality to an extent that would warrant inclusion in the study. Therefore, the sample included 165 lessons. Because *Math Connects* textbooks contain more lessons than textbooks from the other series, more *Math Connects* lessons were included in the study. Of the 165 lessons, 28 (17%) were from CMP, 91 (55%) were from *Math Connects*, and 46 (28%) were from UCSMP. Table 2 presents the number and percent of lessons from each textbook.

Table 2

*Number and Percent of Lessons Included in the Study From Each Textbook*

| | *CMP* | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number of Lessons Included | 9 | 15 | 4 | 33 | 32 | 26 | 21 | 19 | 6 |
| Number of Lessons in Book | 34 | 34 | 35 | 133 | 125 | 122 | 106 | 105 | 108 |
| Percent of Lessons Included | 26 | 44 | 11 | 25 | 26 | 21 | 20 | 18 | 6 |

Of the 165 lessons, 35% were in the Algebra content standard. The rational number and geometry/measurement content standards were represented approximately equally, with each accounting for about 26% of the lessons included. Less than 13% of the lessons were in the Data Analysis/Probability content standard.

### *Grade Levels of Lessons Included in the Study*

More lessons were coded from the sixth- and seventh-grade textbooks than from the eighth-grade books. One reason could be that the eighth-grade textbooks focused on algebra. This was especially true in the CMP and UCSMP series. In most of the sixth- and seventh-grade textbooks, more than 20% of the lessons were related to proportionality; in the eighth-grade books, less than 20% of the lessons were related to proportionality.

A larger number of lessons were coded from the *Math Connects* series than from the other two series. At the sixth- and seventh-grade levels, this reflects the fact that the *Math Connects* textbooks contain more lessons than the sixth- and seventh-grade books from the other series. Similar percentages of lessons were coded from each of the sixth- and seventh-grade books. For example, between 20 and 26% of the lessons in each sixth-grade textbook were coded. At the eighth-grade level, a higher percentage of lessons were coded from the *Math Connects* textbook than from the other series. This reflects the fact that the eighth-grade CMP and UCSMP textbooks contain more of a focus on algebra (and less of a focus on proportionality) than does the eighth-grade *Math Connects* book.

## Task Selection and Counting

Three types of tasks were included in the study: activities intended to be completed in class, examples with worked-out solutions, and exercises intended to be completed at home. Each of the tasks in the selected lessons were read and considered, but only tasks related to proportionality were coded according to the curriculum analysis framework. The following sections describe how tasks were selected from each of the lessons included in the study.

## *Algebra Tasks*

Three types of algebra tasks involve proportionality. Some tasks related to patterns and sequences, some related to function rules, and some tasks related to graphing involve proportionality. However, not all tasks related to these topics involve proportionality. Details are provided below.

### *Tasks Related to Patterns and Sequences*

Tasks related to patterns typically show a visual pattern and ask the student to draw the next item in the pattern or predict how many blocks or tiles would be used to make either the next item or an item further along in the pattern. Sequences consist of numbers rather than visual patterns, but the task is generally the same – to predict the next number or a number further along in the sequence. Sequences that were presented as lists of numbers were *not* coded. An example of a task that was *not* coded states, "Complete the pattern: 5, 11, 17, 23,□ ,□, □." (Day et al., 2009a, p. 27). Some tasks related to patterns and sequences ask students to write a function rule. These tasks are directly related to the concept of slope and were coded.

### *Tasks Related to Function Rules*

Tasks related to function rules typically have one of two goals. Some tasks ask students to create a function table based on a given function rule. These tasks require students to substitute numbers for a variable and compute with these numbers. Because these tasks require only computation, they were not coded. An example of an exercise that was not coded appears in Figure 1.

Copy and complete each function table.

| Input ($x$) | Output ($x + 3$) |
|---|---|
| 0 | □ |
| 2 | □ |
| 4 | □ |

(Day et al., 2009a, p. 51)

*Figure 1*. Function rule exercise not related to proportionality

Other tasks require students to do the opposite, to identify a function rule that matches a given table. These tasks require students to notice a pattern and to consider rate of change. Therefore, tasks that require students to identify a function rule to match a given table were coded. An example of an exercise that was coded appears in Figure 2.

Find the rule for each function table.

| $x$ | □ |
|---|---|
| 2 | 4 |
| 5 | 10 |
| 8 | 16 |

(Day et al., 2009a, p. 51)

*Figure 2*. Function rule exercise related to proportionality

*Tasks Related to Graphing*

Tasks related to graphing positive and negative integers on a number line were not coded. Tasks that focused on graphing ordered pairs were not coded. Tasks related to line graphs were coded only if a discussion of slope or rate of change was present.

*Data Analysis Tasks*

Two topics related to data analysis and probability require proportional reasoning: (a) the construction of circle graphs and (b) the use of circle graphs or probability to make predictions. Tasks related to the construction of circle graphs were coded, as in the exercise in Figure 3.

> In Mr. Numkena's class, 15 students ride the bus to school, 4 walk to school, and 5 ride in a car.
>
> a. How many sectors should a circle graph of this information contain?
> b. Determine the measure of the central angle for each sector.
> c. Complete a circle graph for the data (McConnell et al., 2009, p. 297).

*Figure 3*. Data analysis exercise related to proportionality

Tasks related to the use of circle graphs to make a prediction were also coded. For example, an exercise that was coded in the pilot study read "The circle graph shows the types of hits by Cal Ripkin, Jr., in one season. In Exercises 12-15, predict the number of hits he might have gotten in 4 seasons" (Larson et al., 2004, p. 90). The graph showed that Ripkin hit 124 singles in one season; to complete the exercise, students must solve a *missing value* problem and compute the number of singles he would be likely to hit in four seasons. Tasks in which students were asked simply to compare sizes of the sections of a circle graph were not coded. Tasks related to bar graphs, line plots, and stem-and-leaf plots were not coded. Tasks in which students used probability to make predictions were coded. An exercise that was coded read "A random survey of people at a mall shows that 22 prefer to take a family trip by car, 18 prefer to travel by plane, and 4 prefer to travel by bus. If 500 people are surveyed, how many should say they prefer to travel by plane?" (Day et al., 2009c, p. 656).

### *Geometry/Measurement Tasks*

Tasks in which students were asked to convert between units of measurement were coded. For example, a student could convert four yards to feet by solving the proportion $\frac{3\ feet}{1\ yard} = \frac{?}{4\ yards}$. However, tasks in which students are asked to add or subtract measurements were not coded. Although adding and subtracting measurements often requires measurement conversion, the focus of addition and subtraction problems is generally not on the conversion.

One way textbooks hint at when proportional reasoning is appropriate is to point out that the relationship between side lengths and perimeter is proportional but the relationships between side lengths and area and volume are not proportional. Tasks that discussed what happens when side lengths or radii are doubled and tripled were coded. For example, in the pilot study, the following exercise was coded: “Use the expression $\pi r^2$ to write an expression for the area of a circle when the radius is doubled” (Larson et al., 2004, p. 495). Area and perimeter tasks were coded only if they were directly related to proportionality in this way. Tasks in which students simply calculate area or perimeter were not coded. The same reasoning applied to lessons and tasks on surface area and volume: they were only coded if students are asked to consider the consequences of changing side lengths. For example, in the pilot study, the following exercise was coded: “How does the volume of a cube change when its side length doubles?” (Larson et al., 2004, p. 616). Tasks on size changes were coded, as in this seventh-grade exercise: “A model of a car is $\frac{1}{16}$ actual size. The actual car’s rearview mirror is 12 inches long. How long is the model’s mirror?” (Viktora et al., 2008, p. 475).

*Rational Number Tasks*

Tasks that involved combining two numbers to form a fraction or ratio were considered to not involve proportionality. The example in Figure 4 was considered *not* to involve proportionality.

> **Example 1 Writing a Ratio in Different Ways**
>
> In the orchestra shown above, 8 of the 35 instruments are violas. The ratio of the number of violas to the total number of instruments, $\frac{Violas}{Total\ instruments}$, can be written as $\frac{8}{35}$, as 8 : 35, or as 8 to 35 (Larson et al., 2004, p. 374).

*Figure 4*. Ratio example *not* related to proportionality

Lessons that involved conversion between improper fractions and mixed numbers were analyzed, but some of the examples and exercises in these lessons did involve proportionality. For example, if the examples in such a lesson instructed students to convert an improper fraction into a mixed number by dividing the numerator by the denominator, proportional reasoning was not involved.

Lessons that involved finding the percent of a number were analyzed, but the examples in them were coded only if a proportion strategy was used to solve them. Many textbooks instruct students to change percents into decimals or common fractions in order to solve percent problems. For example, one can find 75% of 60 by multiplying $\frac{3}{4}$ by 60. This does not require proportional reasoning. Therefore, examples that suggest this method of solving percent problems were not coded. However, students may solve exercises in these sections with the use of a proportion, such as $\frac{75}{100} = \frac{x}{60}$. Therefore, most of the exercises in lessons on finding the percent of a number were coded. Exercises that instruct students to use the formula $I = Prt$ to find simple interest were not coded.

*Counting Tasks*

Researchers who conduct curriculum analyses must make decisions concerning methods by which tasks are counted. One issue that researchers face is that the narrative sections of many lessons contain problems for students to work or questions for them to answer. Whether these tasks should be included in the analysis of the narrative sections or the analysis of the exercise sets is a decision researchers must make. Another issue is that exercises typically contain several parts, which are often labeled a, b, c, etc. Some researchers count these as separate exercises and others do not. Decisions regarding these questions for the current study are described below.

*Connected Mathematics2 (CMP)*

CMP textbooks are different from most middle-school textbooks in several ways. For instance, the CMP textbooks contain few, if any, “worked examples.” Modules are divided into “Investigations” and each “Investigation” contains approximately two paragraphs of text followed by several “Problems.” Problems in the investigations are intended to be completed in class and were, therefore, coded as activities. The tasks in homework sets were coded as exercises.

In the Investigations, each Problem contains parts that are lettered (typically A through C). Some of the lettered parts contain numbered subparts (typically 1 and 2). In the Investigations, the numbered subparts are often quite different from each other. For example, in a sixth-grade problem, students are shown a floor plan of a bumper-car facility. Subpart 1 asks students to calculate the area and perimeter of the floor plan. In subpart 2, students are asked whether they think the area or perimeter is a better descriptor of the size of the facility. These subparts are quite different from each other. Thus, in the study, each numbered subpart was counted as a separate exercise.

In addition to the Problems in the Investigations, CMP textbooks contain exercises in three types of sets: Applications, Connections, and Extensions. Based on the numbering of the exercises, it seemed that these sections were intended to comprise one exercise set. Thus, no distinction was made between exercises in each of the three sections within each exercise set. Exercises in the exercise sets were numbered and often contained several lettered parts. These lettered parts are usually closely related to each other. Thus, each numbered exercise was counted as one exercise even if it contained several lettered subparts.

*Glencoe Math Connects*

Examples in *Math Connects* textbooks are clearly labeled and numbered. They contain problems for students to work that are labeled “Check Your Progress.” These problems were not analyzed. Some examples are labeled “Real-World Example” or “Test Example.” The numbering of these examples indicates that they are part of the regular narrative of the lesson; therefore, these examples were analyzed.

Exercises in *Math Connects* textbooks are clearly numbered and generally do not have multiple parts. Exercise sets are divided into five sections: “Check Your Understanding,” “Practice and Problem Solving,” “Test Practice,” “Spiral Review,” and “Get Ready for the Next Lesson.” Based on the numbering of the problems, it seemed that the five sections were intended to comprise one exercise set. Additionally, the two largest sections, “Check Your Understanding” and “Practice and Problem Solving,” seemed similar to each other. The other three sections typically contained small numbers of exercises. Thus, the five sections within each exercise set were combined and analyzed as one exercise set.

The "Explore" and "Extend" lessons in the *Math Connects* textbooks do not have examples and exercises that are labeled and numbered like the examples and exercises in the regular lessons. However, some of these lessons contain material related to proportionality, so these lessons were analyzed. Each of these lessons contains one or more activities as well as question sets labeled "Analyze the Results." These questions were considered to be activities.

*UCSMP*

Examples in the narrative sections of UCSMP textbooks are clearly labeled. Some examples are marked "Guided" and leave some of the work for students to complete. Because the "Guided" examples are similar to the other examples, no distinction between the two was made.

Most narrative sections contain a short question or problem labeled "QY," which stands for "quiz yourself." These problems seem similar to the "Check Your Progress" problems in the narrative sections of the Glencoe *Math Connects* textbooks and were, therefore, not analyzed. Some narrative sections also contain a few questions called "Mental Math." These questions were not analyzed.

Exercises in UCSMP textbooks are clearly numbered and often have two, three, or four lettered parts. As in the *ConnectedMathematics2* textbooks, lettered parts are usually closely related to each other. For example, in the *Pre-Transition Mathematics* textbook, part a) of an exercise asks, "If you leave a $2 tip for a meal that cost $10, have you left at least 15%?" and part b) asks "What if you leave a $2 tip for a meal that cost

$15?" Thus, each numbered exercise was counted as one exercise even if it contained several lettered subparts.

Exercise sets in UCSMP textbooks are divided into four sections: "Covering the Ideas," "Applying the Mathematics," "Review," and "Exploration." Based on the numbering of the problems, it seems that the sections are intended to comprise one exercise set. Thus, no distinction was made between exercises in the four sections within each UCSMP exercise set.

*Summary of Task Selection and Counting*

Many of the tasks in the selected lessons were related to proportionality, but many others were not. Only the tasks related to proportionality were coded. To determine which tasks would be coded, the researcher used the task selection criteria previously described. In each of the four content areas, detailed task selection criteria have been described.

Researchers who conduct curriculum analysis must also decide how to count tasks. The diversity of numbering systems used in textbooks can make counting tasks difficult. In this study, methods of counting tasks varied between series and between sections of textbooks, as previously described.

*Number of Tasks Included in the Study*

In all, 4,563 tasks were coded. This number includes 421 activities, 358 examples, and 3,784 exercises. Sixth and seventh-grade textbooks contained more tasks related to proportionality than did eighth-grade books. The CMP and UCSMP textbooks contained a similar number of tasks related to proportionality (965 and 832, respectively). The *Math Connects* textbooks contained a far greater number of tasks related to proportionality

(2,766). Because the total number of tasks in each textbook is unknown, it is not possible to report the percentage of tasks from each book that was related to proportionality.

### *Grade Levels of Selected Tasks*

The sixth- and seventh-grade textbooks in the sample contained more tasks related to proportionality than did the eighth-grade textbooks. Across series, 34% of the tasks coded were from sixth-grade textbooks, 42% were from seventh-grade textbooks, and 24% were from eighth-grade textbooks. The percentage of tasks at each grade level varied by series, as explained below.

In the CMP and UCSMP series, a far greater number of tasks were coded from the sixth- and seventh-grade textbooks than from the eighth grade book. In the *Math Connects* series, there were slightly fewer proportionality-related tasks in the eighth-grade textbook than in the sixth- and seventh-grade books, but the decrease between seventh and eighth grades was smaller in the *Math Connects* series than in the other two.

Figure 5 shows the percentage of tasks from each grade level for each textbook series. Figure 5 shows that, in all three series, the seventh-grade textbook contained more tasks related to proportionality than either the sixth- or the eighth-grade textbook. In the CMP and UCSMP series, the focus on proportionality drops off sharply between seventh and eighth grades. The *Math Connects* series had the smallest difference between the grade levels.

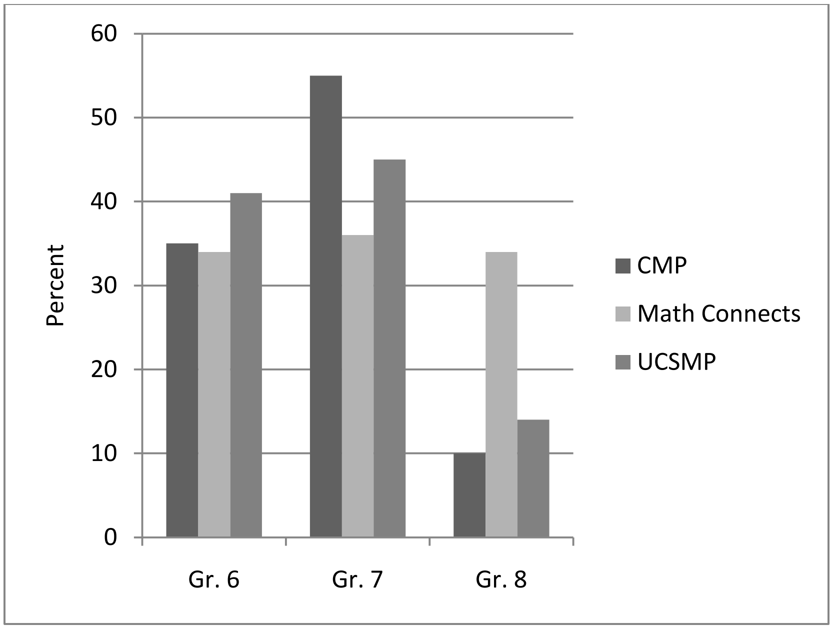


*Figure 5*. Percentage of tasks related to proportionality at each grade level
Note: 1,628 sixth-grade, 1,898 seventh-grade, and 1,037 eighth-grade tasks were coded.

## Framework for Curriculum Analysis

To analyze the activities, examples, and exercises in each textbook, the researcher used a framework that she developed based on research literature. The framework was tested and modified based on a pilot study (Appendix A). The framework involves analyzing tasks along several dimensions. Each dimension involves several categories. In the paragraphs below, the researcher describes the dimensions along which tasks were coded and the categories that were used. Then, examples of tasks that fit into each of the categories are provided.

The author's framework differs from that used in other studies in two ways. First, in some curriculum analyses, only activities and exercises were analyzed (Jones, 2004; Jones & Tarr, 2007). However, this practice ignores the examples in the narrative portion of textbooks. Second, in other curriculum analyses, different frameworks were used to examine the narrative and the exercise sets (e.g., Johnson et al., 2010). However, in this study, the framework used to analyze the narrative and the exercise sets was the same, with one exception. This exception is that the level of cognitive demand was coded for the exercises, but not the examples, for the reasons stated in a later section. Thus, each example and activity was analyzed along five dimensions: content area, problem type, solution strategy, characteristics of proportionality, and the presence or absence of a visual representation. Each exercise was analyzed along six dimensions: the five listed above and the level of cognitive demand.

### *Content Area*

Proportionality appears in many areas of mathematics. Thus, the content area of each task was noted. Each task was coded as primarily relating to algebra, data analysis/probability, geometry/measurement, or rational numbers.

Determining the content area of tasks was generally clear and straightforward. However, coding some tasks according to content area presented minor difficulties. For example, it was difficult to decide whether tasks related to ratios and/or rates should be coded as *algebra* or *rational number*. To decide whether a task should be coded as *algebra* or *rational number*, the researcher attempted to determine which content area was the focus on the lesson. For example, in the seventh-grade CMP Investigation “Walking Rates,” a task reads, “Hoshi walks 10 meters in 3 seconds. What is her walking rate?” (Lappan et al., 2006d, p. 12). This could have been coded as a *rational number* task (because the answer is a decimal) but the previous two pages of the textbook discussed linear equations and asked students to write an equation to match the graph of a line. Because these are algebra topics, the rate exercise was coded as an *algebra* task.

Because proportions are used in all four content areas, some tasks related to proportions were difficult to code according to content area. For example, in the seventh-grade CMP module "Stretching and Shrinking," a task reads, "Find the value of *x* that makes the fractions equivalent: $\frac{5}{2} = \frac{x}{8}$" (Lappan et al., 2006b, p. 85). Some scholars may argue that, because the task contains a variable, it should be coded as *algebra*. Others may argue that the module focused on geometry and measurement. However, because the task specifically mentioned fractions, it was coded as *rational number*.

## *Problem Type*

The literature review suggested the existence of three problem types: *missing value*, *ratio comparison*, and *qualitative*. The pilot study indicated that tasks of these problem types can indeed be identified in middle-school mathematics textbooks. However, there are many tasks in middle-school mathematics textbooks that do not fit neatly into one of these types. Thus, six codes for problem type were used: the three from the literature and *alternate form*, *function rule*, and *other*. Each of these six types is discussed below.

### *Missing Value*

In a *missing value* task, three values are given and students are asked to find the fourth. Missing value tasks are found in every content area within mathematics. Missing value tasks often appear as rate or ratio problems; many of these were coded as *algebra*. The following is an example of a seventh-grade *missing value* task that was coded as *algebra*: "A pizza shop sells 25 pizzas each hour. Find the number of pizzas sold after 1, 2, 3, and 4 hours" (Day et al., 2009b, p. 65).

In the data analysis/probability content area, missing value tasks often appear when students are asked to use data to make a prediction as in this seventh-grade exercise: “A lake has 10,000 fish. When a fisherman scoops up his net, he catches 500 fish. Suppose 150 of the 500 fish in his net are salmon. How many salmon do you predict are in the lake?” (Lappan et al., 2006f, p. 45).

In the geometry/measurement content area, *missing value* tasks are related to similar figures, measurement conversion, or scale models, as in this seventh-grade exercise: “A boat is 40 feet long. A scale model of the boat is 18 inches long. If the scale model is 5 inches wide, how wide is the actual boat?” (Viktora et al., 2008, p. 605).

Many *missing value* tasks are in the rational number content area. Some rational-number tasks that were coded with the problem type *missing value* involved equivalent fractions, as in this sixth-grade exercise: “Replace each $\square$ with a number so the fractions are equivalent. $\frac{3}{8} = \frac{}{24}$” (Day et al., 2009a, p. 206). Other *missing value* tasks involved finding the percent of a number. For example, calculating 75% of 120 could be accomplished by solving the proportion $\frac{75}{100} = \frac{x}{120}$. Therefore, tasks in which students find the percent of a number were coded with the *missing value* problem type.

*Ratio Comparison*

In a *ratio comparison* problem, students are given two fractions or ratios and are asked whether they are equivalent or which is greater. Most *ratio comparison* problems are related to rational numbers. For example, students can be shown two fractions and asked whether the fractions are equivalent, as in this sixth-grade exercise: “Decide whether the statement is *correct* or *incorrect*. $\frac{1}{3} = \frac{4}{12}$” (Lappan et al., 2009a, p. 28).

Because decimals are mathematically equivalent to fractions, tasks in which students are asked to compare the size of decimals were also coded, as in Figure 6.

| For each pair of numbers, find another number that is between them. | |
|---|---|
| **1.** 0.8 and 0.85 | **2.** 0.72 and 0.73 |
| **3.** 1.2 and 1.205 | **4.** 0.0213 and 0.0214 |
| | (Lappan et al., 2009a, p. 40). |

*Figure 6*. Task of problem type *ratio comparison*

Tasks that involved rounding fractions or mixed numbers were also coded as ratio comparison, as in this sixth-grade exercise: "Marina is making birthday cards. She is using envelopes that are $6\frac{3}{4}$ inches by $4\frac{5}{8}$ inches. To the nearest half inch, how large can she make her cards" (Day et al., 2009a, p. 252). This was coded as *ratio comparison* because it involves comparing $6\frac{3}{4}$ to $6\frac{1}{2}$. *Ratio comparison* problems also appear in lessons on geometry. For example, if students are shown two polygons that have measurements marked and are asked whether the polygons are similar, students must use the measurements to make ratios and then compare those ratios.

*Qualitative*

In *qualitative* tasks, no numbers are provided. Thus, students must think about relationships rather than quantities, as in the seventh-grade exercise in Figure 7.

| In which situation will the rate $\frac{x\ feet}{y\ minutes}$ increase? Give an example to explain your reasoning. | |
|---|---|
| **a.** *x* increases, *y* is unchanged | **b.** *x* is unchanged, *y* increases |
| | (Day et al., 2009b, p. 292) |

*Figure 7*. Exercise of problem type *qualitative*

*Alternate Form*

*Alternate form* tasks involve converting between decimals, fractions, and percents or putting fractions and ratios into lowest terms. For example, a textbook could ask students to write the decimal 2.04 as a mixed number in simplest form. *Alternate form* tasks are similar to *missing value* tasks; the difference is that in an *alternate form* task, students are given two numbers and are asked to compute two other numbers. In a *missing value* task, students are given three out of the four numbers in a proportion and are asked to find the fourth.

Tasks that require students to convert a division problem into a fraction or mixed number were not coded. For example, a task could ask students to convert $17 \div 3$ into a mixed number. If the task had asked students to convert $\frac{17}{3}$ into a mixed number, it would be coded as an *alternate form* task. However, when students are asked to think about division problems like $17 \div 3$, they are often asked to think about divisors, dividends, and remainders rather than fractions.

*Function Rule*

In a *function rule* task, students write an expression, equation or description of a function based on data or a table as in the sixth-grade exercise in Figure 8.

Find the rule for each function table.

| ***x*** | □ |
|---|---|
| 6 | 3 |
| 22 | 11 |
| 34 | 17 |

(Day et al., 2009a, p. 51)

*Figure 8*. Exercise of problem type *function rule*

In some function rule tasks, the relationship between variables was not proportional or even linear. Tasks in which the relationship was not proportional were coded because they encouraged students to think about rate of change which may help them understand the characteristics of proportional situations.

*Other*

Any task that did not fit neatly into the five categories previously described was coded *other*. For example, in a sixth-grade exercise, students were given the quantities 45 miles, 22.5 miles per hour, and 2 hours. Students were asked to “Write the four multiplication and division facts that relate the numbers or quantities” (McConnell et al., 2009, p. 403). This exercise is related to proportionality because it involves the rate 22.5 miles per hour, but the exercise does not fit into any of the other problem type categories. Thus, it was coded with the problem type *other*.

## *Solution Strategy*

Most of the worked examples that appear in textbooks are solved using a specific solution strategy. Some activities and exercises suggest that students use a specific solution strategy. The solution strategy suggested by each task was noted. The research literature suggested the existence of three solution strategies: the *building up*, *proportion*, and *unit rate* strategies. The pilot study suggested the need for two additional codes for solution strategy: *decimal* and *manipulative*. The codes that were used for the solution strategy dimensions are described below.

*Building Up*

The *building up* strategy can be based on either addition, in which a quantity is repeatedly added, or multiplication, in which doubling or tripling is used. Construction of a function table is one method of using the *building up* strategy, as in this seventh-grade exercise: "Jonas downloads 8 songs each month onto his digital music player. Make a function table that shows the total number of songs downloaded after 1, 2, 3, and 4 months. Then identify the domain and range" (Day et al., 2009b, p. 65). This task is related to proportionality because a proportion could be used to find the number of songs at the end of each month, as in $\frac{8}{1} = \frac{x}{2}$. It was coded with the *building up* strategy because the function table encourages students to repeatedly add eight.

*Decimal*

The pilot study (Appendix A) established a need for a code for the *decimal* solution strategy. In a task coded with the *decimal* solution strategy, students are encouraged to convert fractions or percents into decimals, as in this sixth-grade exercise: "Use decimal equivalents to order $\frac{2}{3}$, $\frac{7}{10}$, and $\frac{3}{5}$ from greatest to least" (McConnell et al., 2009, p. 113).

*Manipulatives*

The pilot study (Appendix A) established a need for a code for the *manipulative* solution strategy. Some textbooks instruct students to draw pictures and/or use manipulatives, such as fraction strips, to help them answer questions. Thus, *manipulatives* was used as a code for the solution strategy dimension.

*Proportion*

Proportions are one of the solution strategies discussed in the research literature. The example in Figure 9 demonstrates the use of the *proportion* strategy.

**An adult African elephant can be 30 feet long and 11 feet high at the shoulder. Estimate the length of a baby elephant that is 3 feet high at the shoulder.**

**Solution** Compare lengths on the adult with the corresponding lengths on the baby. Set up a proportion by forming two equal ratios. Let *x* be the length of the baby. Since the elephants are similar, the ratios are equal.

$$\frac{height\ of\ adult}{height\ of\ baby} = \frac{length\ of\ adult}{length\ of\ baby}$$

$$\frac{11}{3} = \frac{30}{x}$$

$$11x = 90$$

$$x = \frac{90}{11} \approx 8.2$$

We estimate that the baby elephant is slightly over 8 feet long (Brown et al., 2009, p. 310).

*Figure 9*. Example with *proportion* solution strategy

*Unit Rate*

A task was coded as *unit rate* only if students were instructed to use a unit rate strategy to solve a problem. Tasks that ask students to state the unit rate or to convert a rate into a unit rate were *not* coded because in these situations, the unit rate was not used as a solution strategy. For example, in the pilot study, a sixth-grade example instructed students to “Write the Space Station’s average rate of $\frac{15\ mi}{3\ sec}$ as a unit rate” (Larson et al., 2004, p. 379). This exercise was *not* coded with the *unit rate* strategy because students were instructed to simply compute the unit rate.

*Other*

Tasks in which a solution strategy was suggested that did not fit neatly into one of the above categories were coded as *other*. Many of the tasks coded with the solution strategy *other* were also coded with the problem type *function rule*. Because the *function rule* problem type has not been discussed in the research literature, solution strategies for this type of problem have also not been discussed. Some tasks related to converting between fractions, decimals, and percents were also coded with the solution strategy *other*.

*No Solution Strategy*

Tasks in which no solution strategy was suggested were coded as *no solution strategy*. Most worked examples offered a solution strategy but most activities and exercises simply posed a question or problem and did not offer a solution strategy. Thus, most activities and exercises were coded as *no solution strategy*. Because the CMP series did not contain worked examples, almost all of the CMP tasks were coded as *no solution strategy*.

*Characteristics of Proportional Reasoning*

Many students have difficulty identifying situations in which proportional reasoning is appropriate (Singh, 2000; Van Dooren et al., 2005, 2009; Watson & Shaughnessy, 2004). Therefore, whether or not each task pointed out whether proportional reasoning is appropriate was noted. There are two methods textbook authors use to help students recognize when proportional reasoning is appropriate. One is by pointing out the characteristics of proportional situations. These characteristics are that the rate of change is constant and the $y$-intercept is zero. The other is by considering the effects of doubling (or tripling or quadrupling) one of the variables. In proportional relationships, if one quantity doubles, the other does also.

One way textbook authors help students recognize situations in which proportional reasoning is or is not appropriate is by discussing enlargements of rectangles. When a rectangle is enlarged, the perimeter increases by the same percentage as the side lengths, but the area does not. This is because perimeter is related proportionally to side lengths whereas area is not. The same reasoning applies to the volume of three-dimensional objects. Students may incorrectly think that doubling the radius of a cylinder doubles the volume.

### *Visual Representation*

The researcher noted whether each task was accompanied by a visual representation. Although some researchers have considered “representations” to include real-world contexts, pictures, written language, manipulatives, and symbols (Hodges et al., 2008), the focus in this study was on visual representations, such as charts, diagrams, graphs, illustrations, and pictures. Pictures that were merely decorative and not intended to foster students’ understanding of mathematics were not coded as a visual representation. When a visual representation was present, the researcher entered a short verbal description of the representation into the coding sheet.

### *Level of Cognitive Demand*

The level of cognitive demand of exercises was noted. Consistent with Jones (2004) and Stein et al. (2000), each exercise was coded as *Memorization*, *Procedures Without Connections*, *Procedures With Connections*, or *Doing Mathematics*. As was explained in Chapter Two, classifying exercises into these categories requires knowing what procedures students have been taught. Exercises were classified into these categories by comparing them to the examples in the lesson. For example, if an exercise was similar to an example in the narrative, the exercise was likely to be coded *Memorization* or *Procedures Without Connections*. It would be much more difficult to classify examples in the narrative according to their level of cognitive demand because doing so would require knowing what procedures students had been taught in previous lessons. Therefore, the researcher classified activities and exercises, but not examples, according to their level of cognitive demand.

*Memorization*

According to Stein et al. (2000), *Memorization* tasks "involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formula, or definitions to memory…[Memorization tasks] are not ambiguous – such tasks involve exact reproduction of previously seen material" (p. 16). Thus, an exercise was coded as *Memorization* if it required no computation and if the answer could be easily obtained by reading the narrative section of the lesson. For example, tasks in which students were asked for decimal equivalents of common fractions and percents were coded as *Memorization*.

*Procedures Without Connections*

The second level of task in the framework designed by Stein et al. (2000) is called *procedures without connections to understanding, meaning, or concepts*, which they shorten to *Procedures Without Connections.* These tasks "are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. [They] require limited cognitive demand for successful completion" (Stein et al., p. 16). For example, tasks in which students were asked to solve a straightforward *missing value* task, such as $\frac{1}{2} = \frac{3}{x}$, were coded as *Procedures Without Connections*. More complex tasks were also coded as *Procedures Without Connections* if students had been shown via a worked example exactly how to complete the task. For example, in an eighth-grade UCSMP lesson titled "Rates," an example illustrates how to calculate average speed. An exercise states, "A very fast runner can run a half-mile in 2 minutes" (Brown et al., 2009, p. 267). Students are then instructed to find the average rate in miles per minute. This was coded as *Procedures Without Connections* because an example illustrated virtually the same skill required in the exercise.

*Procedures With Connections*

According to Stein et al. (2000), *Procedures With Connections* tasks "focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas" (p. 16). Stein et al. also state that *Procedures With Connections* tasks "are usually represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations" (p. 16). For example, an exercise from the pilot study read "Three blue rhombuses make one yellow hexagon. Use seven blue rhombuses to make $\frac{7}{3}$ yellow hexagons. Use division to write $\frac{7}{3}$ as a mixed number" (Billstein & Williamson, 2008, p. 44). The exercise was coded as *Procedures With Connections* for two reasons. First, manipulatives (pattern blocks) were required. Second, use of the pattern blocks was intended to help students understand why the procedure for converting fractions to mixed numbers works. In a *Procedures Without Connections* task, students apply a procedure and may have no understanding why it works; a task designed to help students develop a deeper understanding of procedures was coded as *Procedures With Connections*. Tasks in which students were required to explain their reasoning were usually coded as *Procedures With Connections*.

*Doing Mathematics*

According to Stein et al. (2000), *Doing Mathematics* tasks "require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example" (p. 16). Tasks coded as *Doing Mathematics* often had multiple parts and required an explanation of students' reasoning. They also often pointed out connections between mathematical topics. For example, the exercise in Figure 10 points out that multiplicative reasoning can be used in purchasing decisions as well as in tasks related to similar figures.

While shopping for sneakers, Juan finds two pairs he likes. One pair costs $55 and the other costs $165. He makes the following statements about the prices.

"The expensive sneakers cost $110 more than the cheaper sneakers."

"The expensive sneakers cost three times as much as the cheaper sneakers."

**a.** Are both of his statements accurate?
**b.** How are the comparison methods Juan uses similar to the methods you use to compare the sizes and shapes of similar figures?
**c.** Which method is more appropriate for comparing the size and shape of an enlarged or reduced figure to the original? Explain.

(Lappan et al., 2006b, p. 16)

*Figure 10*. Exercise of the *Doing Mathematics* level of demand

*Summary of the Framework*

In summary, examples were coded along five dimensions: content area, problem type, characteristics of proportionality, solution strategy, and visual representation. Activities and exercises were coded along six dimensions: the five previously listed and the level of cognitive demand. Table 3 presents the codes associated with each of these dimensions.

Table 3

*The Curriculum Analysis Framework*

| Dimension | Categories |
|---|---|
| Content Area | Algebra |
| | Data Analysis/Probability |
| | Geometry/Measurement |
| | Rational Numbers |
| Problem Type | Alternate Form |
| | Function Rule |
| | Missing Value |
| | Ratio Comparison |
| | Qualitative |
| | Other |
| Characteristics | Yes |
| | No |
| Solution Strategy | Building Up |
| | Decimal |
| | Manipulatives |
| | Proportion |
| | Unit Rate |
| | Other |
| | No Strategy |
| Visual Representation | Yes |
| | No |
| Level of Cognitive Demand* | Memorization |
| | Procedures without connections |
| | Procedures with connections |
| | Doing mathematics |

*The level of cognitive demand was noted for activities and exercises, but not for examples.

### *Narrative Text*

The basic unit of analysis was the *task*, defined as "an activity, exercise, or set of exercises in a textbook that has been written with the intent of focusing a student's attention on a particular idea" (Jones, 2004, p. 10). However, narrative text, intended for students to read, can also focus a student's attention on a particular idea. Because the number of narrative passages that explicitly discuss proportionality was small, they were not analyzed quantitatively. Instead, the researcher noted where they occurred and their content.

## Reliability

This section is divided into three parts. In the first, the procedures used to monitor reliability are described. In the second, the reliability of the task selection is described. In the final part of this section, the reliability of the coding is discussed.

### *Reliability Procedures*

To ensure reliability of the coding, the researcher employed a check-coding method in which the researcher was assisted by a fellow doctoral student in mathematics education. The doctoral student had completed all coursework, passed comprehensive exams, and was working on his own dissertation. The doctoral student had read several drafts of the researcher's dissertation proposal and was thus familiar with the goals and procedures of the study.

The researcher and doctoral student met 13 times during the months in which the coding took place (November and December of 2009 and January of 2010). Each meeting lasted one to two hours. To ensure that the researcher and doctoral student shared a common understanding of the framework, during the first six meetings, they discussed any tasks on which there was disagreement. After the sixth meeting, the researcher felt that there was a common understanding of the framework; thereafter, tasks were not discussed individually. The doctoral student coded several lessons from each textbook. In all, he coded 26 of the 165 lessons included in the study (15%). The doctoral student recorded his coding data in a spreadsheet that he emailed to the researcher after each meeting. The researcher also kept handwritten notes regarding the substance of each meeting and transferred these notes to a typed Reliability Log within a day or two of each meeting.

*Lessons Coded by the Doctoral Student*

The researcher used a purposeful selection process to choose the lessons for the doctoral student to code. In selecting lessons to be coded by the doctoral student, the researcher used several criteria. First, at least 10% of the lessons from each textbook were coded. Second, lessons from all of the content areas were selected. In most textbooks, lessons from two or three different content areas were selected. Table 4 presents the number of lessons from each textbook included in the study and the number and percent of lessons that were coded by the doctoral student.

Table 4

*Number and Percent of Lessons that were Double Coded*

| | CMP | | | *Math Connects* | | | *UCSMP* | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number of Lessons Double Coded | 3 | 2 | 1 | 6 | 4 | 3 | 4 | 2 | 1 |
| Percent of Lessons Double Coded | 33 | 13 | 25 | 18 | 13 | 12 | 19 | 11 | 17 |
| Total Lessons Coded | 9 | 15 | 4 | 33 | 32 | 26 | 21 | 19 | 6 |

*Reliability Training*

The researcher and doctoral student began meeting on November 6, 2009 to code lessons for the study. During the month of November, the researcher and the doctoral student met five times and coded nine sixth-grade lessons. Although reliability percentages had not been calculated at that point, the researcher had some concerns about the level of reliability. After consulting the Chair of the dissertation committee, the researcher developed a Training Module consisting of 13 exercises from the textbooks in the study. The Training Module can be found in Appendix C. Working together, the researcher and doctoral student coded the exercises in the Training Module. Once the Training Module was completed, a 13-item Reliability Test was administered. The Reliability Test can be found in Appendix D. On the Reliability Test, the researcher and doctoral student disagreed on one code for content area, one code for problem type, and two for level of cognitive demand. Thus, on the reliability test, the reliability for content area and problem type was 92% and the reliability for cognitive demand was 85%. Reliability regarding content area and solution strategy were both 100%.

*Reliability of Task Selection*

Reliability of task selection refers to the extent to which the researcher and doctoral student selected the same tasks to code; i.e., considered the same tasks to involve proportionality. For each textbook, two percentages associated with this type of reliability can be calculated: a) the percent of tasks coded by the doctoral student that were also coded by the researcher, and b) the percent of tasks coded by the researcher that were also coded by the doctoral student. The following paragraphs describe both types of reliability for each textbook.

*Reliability of Task Selection in CMP*

Three lessons were selected from the sixth-grade CMP textbook for check-coding. From these lessons, 123 tasks were coded by both the researcher and doctoral student, 10 were coded only by the researcher, and 16 were coded only by the doctoral student.

Two lessons were selected from the seventh-grade CMP textbook for check-coding. From these lessons, 51 tasks were coded by both the researcher and doctoral student, nine were coded only by the researcher, and six were coded only by the doctoral student.

One lesson was selected from the eighth-grade CMP textbook for check-coding. From this lesson, 37 tasks were coded by both the researcher and doctoral student, nine were coded only by the researcher, and three were coded only by the doctoral student.

Combining the data from the three CMP textbooks, 211 tasks were coded by both the researcher and doctoral student, 28 were coded only by the researcher, and 25 were coded only by the doctoral student. Of the 239 tasks coded by the researcher, 211 were also coded by the doctoral student. Thus, 88% of the tasks coded by the researcher were also coded by the doctoral student. Of the 236 tasks coded by the doctoral student, 211 were also coded by the researcher. Thus, 89% of the tasks coded by the doctoral student were also coded by the researcher.

*Reliability of Task Selection in Math Connects*

Six lessons were selected from the sixth-grade *Math Connects* textbook for check-coding. From these lessons, 111 tasks were coded by both the researcher and doctoral student, eight were coded only by the researcher, and one was coded only by the doctoral student.

Four lessons were selected from the seventh-grade *Math Connects* textbook for check-coding. From these lessons, 99 tasks were coded by both the researcher and doctoral student, none were coded only by the researcher, and three were coded only by the doctoral student.

Three lessons were selected from the eighth-grade *Math Connects* textbook for check-coding. From these lessons, 86 tasks were coded by both the researcher and doctoral student, four were coded only by the researcher, and none were coded only by the doctoral student.

Combining the data from the three *Math Connects* textbooks, 296 tasks were coded by both the researcher and doctoral student, 12 were coded only by the researcher, and four were coded only by the doctoral student. Of the 308 tasks coded by the researcher, 296 were also coded by the doctoral student. Thus, 96% of the tasks coded by the researcher were also coded by the doctoral student. Of the 300 tasks coded by the doctoral student, 296 were also coded by the researcher. Thus, 99% of the tasks coded by the doctoral student were also coded by the researcher.

*Reliability of Task Selection in UCSMP*

Four lessons were selected from the sixth-grade UCSMP textbook for check-coding. From these lessons, 57 tasks were coded by both the researcher and doctoral student, eight were coded only by the researcher, and one was coded only by the doctoral student.

Two lessons were selected from the seventh-grade UCSMP textbook for check-coding. From these lessons, 37 tasks were coded by both the researcher and doctoral student, five were coded only by the researcher, and none were coded only by the doctoral student.

One lesson was selected from the eighth-grade UCSMP textbook for check-coding. From this lesson, 12 tasks were coded by both the researcher and doctoral student, three were coded only by the researcher, and none were coded only by the doctoral student.

Combining the data from the three UCSMP textbooks, 106 tasks were coded by both the researcher and doctoral student, 16 were coded only by the researcher, and one was coded only by the doctoral student. Of the 122 tasks coded by the researcher, 106 were also coded by the doctoral student. Thus, 87% of the tasks coded by the researcher were also coded by the doctoral student. Of the 107 tasks coded by the doctoral student, 106 were also coded by the researcher. Thus, 99% of the tasks coded by the doctoral student were also coded by the researcher.

*Summary of Reliability of Task Selection*

Table 5 summarizes the number of exercises coded by the researcher and the doctoral student for each series. Of the two types of reliability of task selection, the more important is the percent of tasks coded by the doctoral student that were also coded by the researcher. A high percentage indicates that the researcher did not miss many tasks related to proportionality. This percentage is referred to as the "Reliability of Task Selection" and is reported in the bottom line of Table 5.

Table 5

*Reliability of Task Selection by Textbook Series*

| | CMP | Math Connects | UCSMP |
|---|---|---|---|
| Number of Tasks Coded by Both | 211 | 296 | 106 |
| Number of Tasks Coded Only by the Researcher | 28 | 12 | 16 |
| Number of Tasks Coded Only by Doctoral Student | 25 | 4 | 1 |
| Reliability of Task Selection (Percent)* | 89 | 99 | 99 |

*Note: For example, for CMP, the doctoral student coded 236 exercises. 211 of these were also coded by the researcher. 211 ÷ 236 = 0.894    0.894 was rounded to 89%

### *Reliability of Coding*

Each example related to proportionality was coded along five dimensions: content area, problem type, solution strategy, characteristics of proportionality, and the presence or absence of a visual representation. Each activity and each exercise was coded along these five dimensions and also the level of cognitive demand. For four of these dimensions, the codes to be assigned were obvious and the researcher and the doctoral student agreed on almost every item. These dimensions were content area, solution strategy, characteristics of proportionality, and the presence or absence of a visual representation. For two dimensions, the necessary codes were less obvious and the researcher and doctoral student disagreed more frequently. These dimensions were problem type and the level of cognitive demand. Thus, reliability related to these dimensions is discussed in this section.

#### *Reliability of Problem Type Coding*

Codes for problem type included *alternate form*, *function rule*, *missing value*, *ratio comparison*, *qualitative*, and *other*. Over all textbooks and series, the problem type codes matched for 540 of the 613 tasks coded by both researchers (88%). As indicated in Table 6, reliability regarding problem type ranged from a low of 75% in the CMP sixth-grade book to a high of 100% in the *Math Connects* sixth-grade book. Reliability of the problem type coding was lowest in the CMP series (78%) and high in both *Math Connects* and UCSMP (94% and 92%, respectively).

Table 6

*Reliability of Problem Type Coding by Series and Grade*

| | CMP | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number Tasks w/ Agreement | 92 | 42 | 31 | 111 | 94 | 72 | 54 | 33 | 11 |
| Total Number Tasks Coded* | 123 | 51 | 37 | 111 | 99 | 86 | 57 | 37 | 12 |
| Percent Agreement | 75 | 82 | 84 | 100 | 95 | 84 | 95 | 89 | 92 |

*Refers to the number of tasks coded by both the researcher and the doctoral student

Most of the difficulty with coding the problem type dimension related to the code of *other*. It was often difficult to tell whether to code a task as *other* or to assign a specific problem type code. Many tasks were mathematically equivalent to a *missing value* or *ratio comparison* task but had cosmetic features that made them appear different. For example, in an eighth-grade *Math Connects* exercise, students were given coordinates of the three vertices of a triangle and were asked to find the vertices after a dilation with a given scale factor and then graph the original and new triangle. The researcher coded this with the problem type *other* because it did not neatly fit into the categories and the doctoral student coded it with the problem type *missing value* because it involved finding missing values.

*Reliability of Level of Cognitive Demand Coding*

The Mathematical Task Framework (Stein et al., 2000) was used to classify each activity and each exercise by level of cognitive demand. As indicated in Table 7, reliability regarding level of cognitive demand of the exercises ranged from a low of 52% in the eighth-grade *Math Connects* textbook to a high of 90% in the seventh-grade CMP textbook. Over all textbooks and series, the reliability regarding cognitive demand was 72%. Reliability was lowest in the CMP series (65%) and highest in the UCSMP series (76%). Most of the difficulty with coding the cognitive demand dimension related to the two middle levels (the codes of *Procedures Without Connections* and *Procedures With Connections*).

Table 7

*Reliability of Cognitive Demand Coding by Series and Grade*

| | *CMP* | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number Tasks w/ Agreement | 52 | 27 | 17 | 89 | 66 | 39 | 37 | 27 | 6 |
| Total Number Tasks Coded* | 98 | 30 | 20 | 103 | 86 | 75 | 44 | 37 | 10 |
| Percent Agreement | 53 | 90 | 85 | 86 | 77 | 52 | 84 | 73 | 60 |

*Refers to the number of activities plus the number of exercises

Jones (2004) also used the Mathematical Task Framework (Stein et al., 2000) to measure the level of cognitive demand of tasks found in textbooks. He also found that the level of cognitive demand was, in some cases, difficult to determine. He reported reliabilities of 73% to 88%. He also reported the percentage of tasks on which the two raters were within one level of cognitive demand. Applying this strategy to the current study would result in a reliability of 100%. On each task for which the two codes disagreed, the difference was only one level of cognitive demand.

*Reliability for CMP Textbooks*

Cognitive demand was difficult to code in the CMP textbooks because the series contains no labeled examples, making it difficult to determine what procedures students had learned. For example, in an exercise in the sixth-grade textbook, students must find a fraction between $\frac{1}{8}$ and $\frac{1}{4}$. The researcher coded this exercise as *Procedures Without Connections* because there are simple procedures that can be followed to solve the problem. The doctoral student coded the exercise as *Procedures With Connections* because the procedures for solving this type of problem had not been presented to students.

*Reliability for the Math Connects Grade 8 Textbook*

In the *Math Connects* Grade 8 textbook, 75 exercises were coded by both the researcher and the doctoral student. Initially, the two sets of codes were identical for 29 of these exercises, resulting in an initial reliability of 39%. To investigate why the reliability was low, the researcher looked at the tasks on which the two raters disagreed. In a lesson titled "Direct Variation," the researcher decided that she had incorrectly coded the level of cognitive demand for 10 exercises. In these exercises, students were given a set of *x* and *y* values, asked to determine whether the linear function was an example of direct variation, and asked to state the constant of variation. The researcher originally coded these exercises as *Procedures With Connections* because they required several steps. However, examples in the narrative clearly showed students how to solve problems of this type. Therefore, the codes were changed to *Procedures Without Connections* because the use of a procedure was evident based on prior instruction and because the exercises required no explanation. Correcting this error meant that the two sets of codes were identical for 39 of the 75 exercises, resulting in a reliability for the *Math Connects* Grade 8 textbook of 52%.

Reliability of the cognitive demand dimension was low in another lesson from this textbook. In a lesson titled “Indirect Measurement,” two examples show students how to obtain measurements from a diagram and set up and solve a proportion to obtain a measurement that might be difficult to obtain directly, such as the distance across a lake. Fifteen exercises follow the examples and are very similar to the examples. Because the procedure for solving the problems was clearly illustrated in the examples, the researcher coded all 15 exercises as *Procedures Without Connections*. The doctoral student believed that the presence of visual representations and word problems elevated the level of cognitive demand to *Procedures With Connections*. Thus, the two raters disagreed on all 15 exercises. The doctoral student’s coding of this lesson may have been affected by his prior experience with Norman Webb’s framework for depth of knowledge in which the presence of visual representations and word problems is said to increase the depth of knowledge. Because this is not consistent with the Mathematical Task Framework (Stein et al., 2000), the researcher’s codes were used.

*Reliability for the UCSMP Grade 8 Textbook*

Reliability of the coding of the cognitive demand dimension was also low in the UCSMP *Algebra* textbook (60%). However, only one lesson was coded by the doctoral student and only 10 tasks from this lesson were coded by both the researcher and the doctoral student. Had more tasks been double coded, reliability may have been higher.

*Reliability of Coding Over Time*

To determine whether reliability increased, decreased, or remained constant over the three months that coding occurred, the 26 lessons that were coded by the doctoral student were separated into three sets. One set consisted of nine lessons that were coded by the doctoral student between November 18, 2009 and January 2, 2010. A second set consisted of 12 lessons that were coded by the doctoral student between January 2, 2010 and January 10, 2010. The third set, with five lessons, was coded by the doctoral student between January 10, 2010 and January 18, 2010. Reliability was calculated for each of the three sets. For lessons that were discussed on more than one occasion, the last date was used to classify the lesson into one of the three sets.

For the first set of lessons, the reliability of the Problem Type coding was 89%, for the second set it was 84% and for the third set it was 86%. Thus, it appears that the reliability of coding the problem type dimension did not significantly change over the three months in which coding took place.

*Summary of Reliability of Coding*

Coding of four of the six dimensions presented no difficulties. However, coding the problem type and level of cognitive demand dimensions was, in some cases, more difficult. Reliability of problem type codes was above 90% in five of the nine textbooks. Overall, reliability of problem type codes was 88% and was lowest in the CMP series. Coding the level of demand was the most challenging. Reliability of these codes ranged from 52% to 90% and was 72% overall.

## Summary of Research Methods

In this chapter, the researcher described the methods that were used to select a sample of textbooks and lessons, described how examples and exercises would be counted and grouped, described the framework that was used to categorize examples and exercises, and discussed reliability measures that were employed.

Three contemporary middle-school textbooks series were selected for the study: Connected Mathematics2, Glencoe's *Math Connects*, and books from the University of Chicago School Mathematics Project. These series were selected because all are widely used and because their treatment of proportionality was expected to vary in significant ways. Lessons from those textbooks were selected on the likelihood of their inclusion of material related to proportionality. The selected lessons included most, but not all, of the material related to proportionality in the textbooks. Every task in the selected lessons was analyzed, but only tasks that involved proportionality were coded. Three types of tasks were analyzed: activities meant to be completed in class, examples that contained worked-out solutions, and exercises meant to be completed as homework. Examples related to proportionality were coded along five dimensions: content area, problem type, solution strategy, visual representation, and whether the example highlights whether proportional reasoning is appropriate in the given situation. Activities and exercises were coded along six dimensions: the five listed above and the level of cognitive demand. A pilot study was conducted with two sixth-grade textbooks. The pilot study demonstrated the reliability with which the framework can be applied and the usefulness of the results. The pilot study and its findings are described in Appendix A.

# CHAPTER 4: RESULTS

The purpose of this study was to investigate the nature of the treatment of proportionality in contemporary middle-school textbooks. Specifically, the study was designed to answer the following six questions:

1. To what extent is proportionality emphasized in the treatment of various content areas within mathematics, such as algebra, data analysis/probability, geometry/ measurement, and rational numbers? How does this vary among grade levels and textbook series?
2. Among the tasks related to proportionality in middle-school mathematics textbooks, which problem types (e.g., missing value, ratio comparison, qualitative) are featured most and least often? How does this vary among mathematical content areas, grade levels, and textbook series?
3. Which solution strategies (e.g., building up, unit rate, proportion) to tasks related to proportionality are encouraged by middle-school mathematics textbooks? How does this vary among grade levels and textbook series?
4. What level of cognitive demand (Boston & Smith, 2009; Stein et al., 2000) is exhibited by the proportional exercises in middle-school textbooks? How does this vary among mathematical content areas, grade levels, and textbook series?
5. To what extent are visual representations used in middle-school mathematics textbooks to illustrate concepts related to proportionality? How does this vary among grade levels and textbook series?

6. To what extent are the characteristics of proportional situations pointed out in middle-school mathematics textbooks? How does this vary among grade levels and textbook series?

To answer these questions, three contemporary, widely-used textbooks series were analyzed: Connected Mathematics2 (CMP), Glencoe's *Math Connects*, and the sixth-, seventh- and eighth-grade textbooks from the University of Chicago School Mathematics Project (UCSMP).

The unit of analysis was the *task*. Three types of tasks were analyzed: *activities* intended to be completed in class, *examples* with worked-out solutions, and *exercises* intended to be completed as homework. The researcher analyzed 165 lessons that contained 4,563 tasks related to proportionality. Information on the number and percent of lessons included in the study from each textbook was provided in Chapter 3. In most books, between 11% and 26% of the lessons in the textbook were included in the study. A higher percentage of lessons (44%) from the seventh-grade CMP textbook was included, due to this textbook's focus on proportionality.

More lessons and tasks from the *Math Connects* textbooks were included than from the other two series. This does not necessarily mean that *Math Connects* textbooks have more of a focus on proportionality than do the other two series. Rather, it is a reflection of the fact that *Math Connects* textbooks have more overall lessons and tasks than do the other two series. Similarly, more exercises were coded from the CMP series than from the UCSMP series. This reflects the fact that CMP exercise sets tend to contain a larger number of exercises than do UCSMP textbooks. CMP exercise sets typically contain about 40 exercises whereas UCSMP sets typically contain about 20 exercises.

The problems in the Investigations in the CMP textbooks were considered to be activities because they are intended to be done in class. Exercise (homework) sets in all textbooks were considered to be exercises. Overall, 9% of the tasks were in-class activities, 8% were examples, and 83% were exercises. In the *Math Connects* and UCSMP textbooks, about 3% of the tasks were activities, about 10% were examples, and about 87% were exercises. In the CMP textbooks, 34% of the tasks were problems in the Investigations and 65% were exercises found in the homework sets. The CMP textbooks did not have labeled examples. Table 8 shows the number of activities, examples, and exercises coded from each series and each textbook.

Table 8

*Number of Tasks Related to Proportionality by Series and Grade*

| | *CMP* | | | | *Math Connects* | | | | UCSMP | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | total | 6th | 7th | 8th | total | 6th | 7th | 8th | total |
| Activities | 103 | 192 | 34 | 329 | 32 | 26 | 9 | 67 | 13 | 9 | 3 | 25 |
| Examples | 0 | 0 | 0 | 0 | 81 | 82 | 82 | 245 | 48 | 49 | 16 | 113 |
| Exercises | 234 | 336 | 66 | 636 | 833 | 891 | 730 | 2454 | 284 | 313 | 97 | 694 |
| Total Tasks | 337 | 528 | 100 | 965 | 946 | 999 | 821 | 2766 | 345 | 371 | 116 | 832 |

## Content Areas

The first research question related to the content areas within mathematics through which proportionality is presented. Overall, 53% of the tasks related to proportionality were in the rational number content area, 21% were in geometry/measurement, 20% were in algebra, and 6% were in data analysis/probability. In general, the percentage of activities, examples, and exercises in each content area were similar to the percentage of tasks in each content area. For example, 20% of the tasks were related to algebra and approximately 20% of the activities, examples, and exercises were related to algebra. There were two exceptions to this rule. In the geometry/measurement content area, there were more activities and fewer examples and exercises; in the rational number content area, there were fewer activities and more examples and exercises. The content areas of tasks related to proportionality varied by textbook series and by grade level, as explained in the following sections.

### *Content Areas and Textbook Series*

In all three textbook series, a higher percentage of the tasks related to proportionality were from the rational number content area than any other. In all three series, between 48% and 56% of the tasks related to proportionality were from the rational number content area. In the *Math Connects* and UCSMP series, the percentages of tasks in the geometry/measurement and algebra content areas were approximately equal. In the CMP series, more tasks related to proportionality were in geometry/measurement than in the algebra content area. In all three series, few tasks related to proportionality were from the data analysis and probability content area. In all three series, between 4% and 8% of the tasks related to proportionality were from the data analysis and probability content area. This information is detailed in Table 9.

Table 9

*Percentage of Tasks in Each Content Area by Series*

| | Algebra | Data Analysis | Geometry | Rational Number |
|---|---|---|---|---|
| CMP (*n* = 965) | 15.7 | 7.9 | 26.1 | 50.2 |
| *Math Connects* (*n* = 2,766) | 19.4 | 5.6 | 19.1 | 55.9 |
| UCSMP (*n* = 832) | 24.8 | 4.1 | 22.4 | 48.3 |

## *Content Areas, Textbook Series, and Grade Levels*

In all of the sixth-grade textbooks, a higher percentage of the tasks related to proportionality were in the rational number content area than any other. The *Math Connects* and UCSMP textbooks continued the focus on rational numbers through seventh grade. The *Math Connects* textbooks continued the focus on rational numbers through eighth grade as well, but the focus in the eighth-grade CMP and UCSMP textbooks shifted to algebra.

### *Content Areas of Sixth-Grade Textbooks*

Combining the data from all three series, 66% of the sixth-grade tasks related to proportionality were in the rational number content area. In each of the three sixth-grade books, more than 55% of the tasks were in the rational number content area. As indicated in Figure 11, the sixth-grade CMP textbook had a higher percentage of tasks related to rational numbers than the other two sixth-grade textbooks. Other than the focus on rational numbers, there was little content similarity between sixth-grade textbooks. The sixth-grade CMP textbook contained tasks in the data analysis and probability content area but virtually no tasks related to proportionality in the algebra and geometry/measurement content areas. The *Math Connects* and UCSMP sixth-grade books had more tasks related to proportionality in the algebra and geometry content areas than in data analysis.

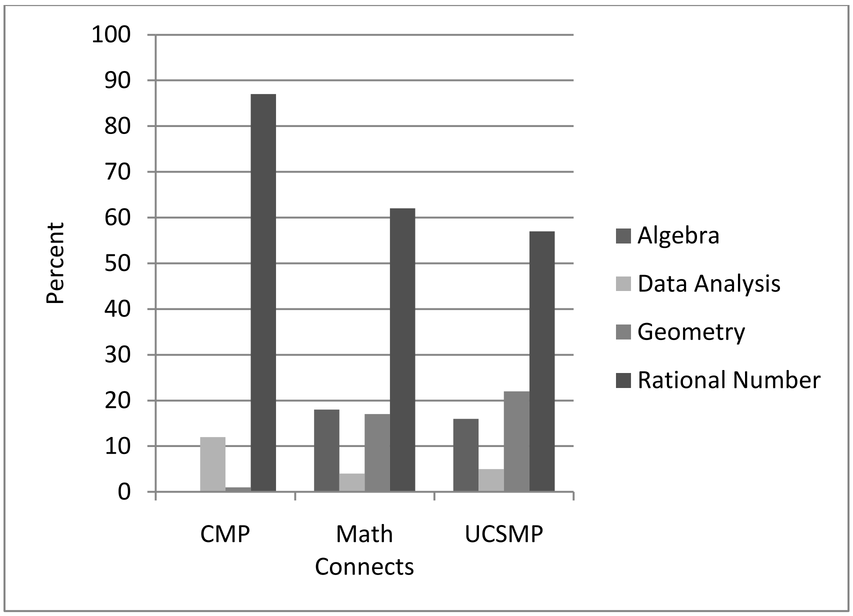


*Figure 11*. Percentage of Sixth-Grade Tasks in Each Content Area

In the sixth-grade CMP curriculum, a higher percentage of tasks related to proportionality were in the rational number content area than in any other. The sixth-grade CMP curriculum contained eight modules, three of which focused on rational numbers. In the first module on rational numbers, students compare and order fractions and decimals and convert among fractions, decimals, and percents. In the second, students learn to estimate and compute with fractions. The third module related to rational numbers covered computation with decimals and algorithms for solving percent problems.

In the sixth-grade *Math Connects* textbook, a higher percentage of tasks related to proportionality were in the rational number content area than in any other. Of the 12 chapters in the textbook, five focused on rational numbers. The sixth-grade *Math Connects* book contained a lesson titled “Ratios and Rates.” Most of the tasks in this lesson were determined to be in the rational number content area.

As in the other two sixth-grade textbooks, in the UCSMP textbook, a higher percentage of tasks related to proportionality were in the rational number content area than in any other. Of the 13 chapters in the textbook, two focused on rational numbers (“Some Uses of Decimals and Percents” and “Using Multiplication”). The chapter “Some Uses of Integers and Fractions” also contained lessons related to rational numbers. The sixth-grade UCSMP textbook had a higher percentage of tasks related to geometry than did the other two sixth-grade textbooks. The UCSMP textbook used rulers and measurement as a way to help students understand fractions, mixed numbers, and decimals.

*Content Areas of Seventh-Grade Textbooks*

As indicated in Figure 12, in the *Math Connects* and UCSMP series, the seventh-grade book continued to cover proportionality primarily through the rational number content area. In the seventh-grade *Math Connects* and UCSMP textbooks, more than 55% of the proportional tasks were in the rational number content area. Unlike the other two series, in the seventh-grade CMP textbook, a higher percentage of tasks related to proportionality were in the geometry/measurement content standard than any other.

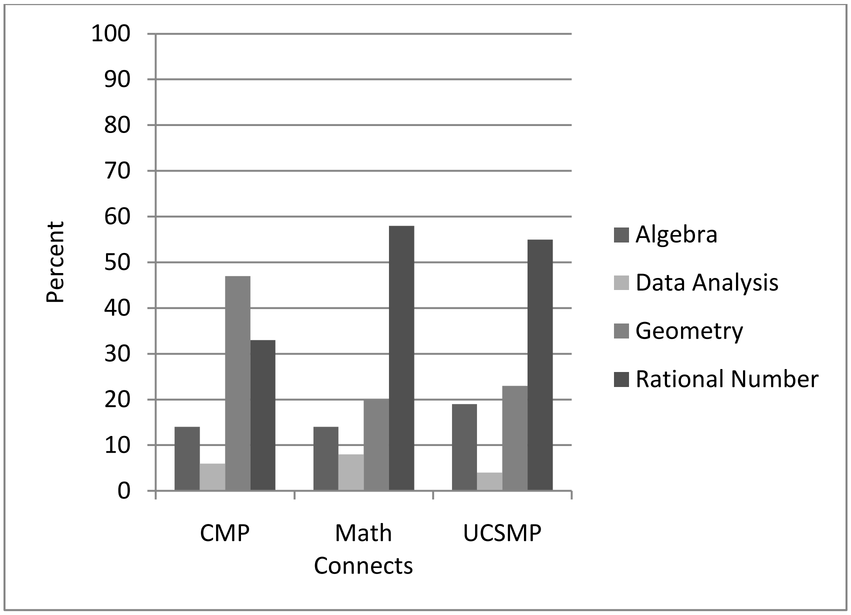


*Figure 12*. Percentage of Seventh-Grade Tasks in Each Content Area

*Seventh-grade CMP*. In the seventh-grade CMP curriculum, more tasks related to proportionality were in the geometry/measurement content area than in any other. The seventh-grade CMP curriculum contained eight modules, two of which focused on geometry and measurement. The module "Stretching and Shrinking" contained tasks related to similar figures and scale factors. In the module "Filling and Wrapping," students "investigate the effects of varying dimensions of rectangular prisms and cylinders on volume and surface area" (Lappan et al., 2006b, p. 4). In "Filling and Wrapping," students learn that the surface and volume of a prism or cylinder are not related proportionally to the linear dimensions. For example, a task asks students to "Describe what happens to the surface area of a cube when the edge lengths are doubled, tripled, quadrupled, and so on" (Lappan et al., p. 68).

*Seventh-grade Math Connects*. In the seventh-grade *Math Connects* book, a higher percentage of tasks related to proportionality were in the rational number content area than in any other content area. Specifically, 58% of the tasks related to proportionality were found in the rational number content area. Of the 12 chapters in the textbook, only three focused specifically on rational numbers (“Fractions, Decimals, and Percents;” “Applying Fractions;” and “Applying Percents”). None of the lessons from “Applying Fractions” were coded because they focused on computation with fractions.

As explained in Chapter 3, tasks related to rates and ratios could be coded as either algebra or rational number. The seventh-grade *Math Connects* book contained a Chapter titled “Ratios and Proportions” that contained lessons titled “Ratios,” “Rates”, and “Fractions, Decimals, and Percents.” Most of the tasks in these lessons were determined to be in the rational number content area. Thus, the seventh-grade *Math Connects* book contained a large percentage of tasks related to rational numbers because a large number of rational-number tasks related to proportionality were found in three chapters: “Fractions, Decimals, and Percents;” “Applying Percents;” and “Ratios and Proportions.”

*Seventh-grade UCSMP.* As in the *Math Connects* book, the seventh-grade UCSMP textbook featured a higher percentage of tasks related to proportionality in the rational number content area than in any other content area. Specifically, 55% of the tasks related to proportionality were found in the rational number content area. Of the 12 chapters in the textbook, only two focused specifically on rational numbers (“Representing Numbers” and “Patterns Leading to Division”). These two chapters contained the vast majority of the tasks related to proportionality. Of the 313 exercises in this textbook, 221 were in one of these two chapters that focused on rational numbers.

*Content Areas of Eighth-Grade Textbooks*

As indicated in Figure 13, in the eighth-grade CMP and UCSMP textbooks, more of the proportional tasks were in the algebra content standard than any other, with more than 70% of the proportional exercises in the algebra standard. This is likely due to the fact that the CMP and UCSMP textbooks focus on algebra. In the eighth-grade *Math Connects* textbook, a higher percentage of the tasks related to proportionality were in the rational number content standard than any other.

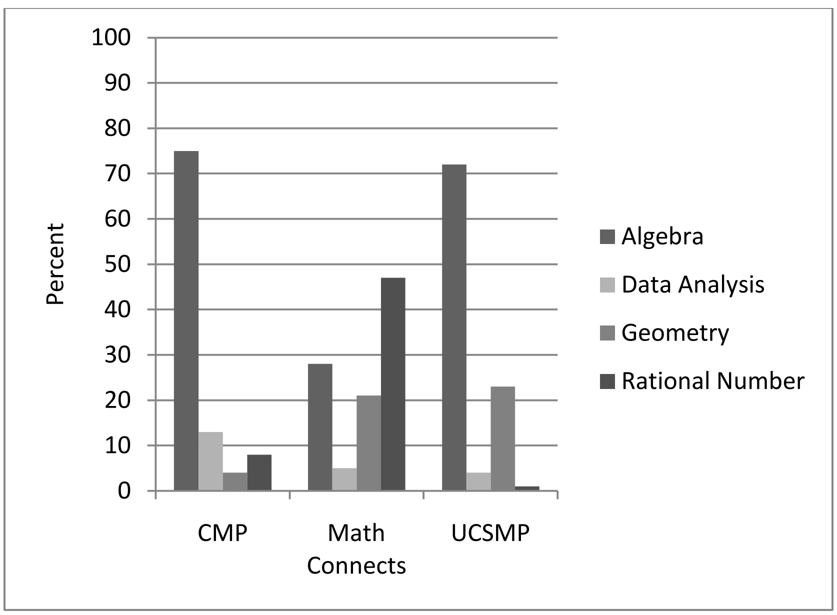


*Figure 13*. Percentage of Eighth-Grade Tasks in Each Content Area

All of the eighth-grade texts provided an explanation of “direct variation.” Only the CMP textbook had a lesson devoted to inverse relationships; many of the tasks in this lesson were related to proportionality.

*Eighth-grade CMP*. Four Investigations in the eighth-grade CMP textbook were coded. They were titled “Linear Models and Equations,” “Inverse Variation,” “Looking Back at Functions,” and “Choosing a Sample From a Population.” The Investigation titled “Inverse Variation” discussed equations and graphs of direct variation and inverse variation relationships. It stated, “A relationship between variables *x* and *y* is a direct variation if it can be expressed as $y = kx$, where *k* is a constant” (Lappan et al., 2006g, p. 58) and “The relationship between two non-zero variables, *x* and *y*, is an inverse variation if $y = \frac{k}{x}$, or $xy = k$” (Lappan et al., p. 49). In the eighth-grade CMP textbook, more tasks were coded from the “Inverse Variation” Investigation than from any other Investigation.

*Eighth-grade Math Connects*. The *Math Connects* book had a lesson on direct variation which informed students, “When the ratio of two variable quantities is constant, their relationship is called a **direct variation**. The constant ratio is called the **constant of variation**” (Day et al., 2009c, p. 487). The direct variation relationship was described in words, with a graph, and with the equations $k = \frac{y}{x}$ and $y = kx$. However, many of the *Math Connects* lessons related to rational numbers rather than algebra; one chapter focused on rational numbers and another on percent. A wide variety of other topics related to proportionality are also covered, such as ratios, rates, rate of change, proportions, similar figures, dilations, sequences, circle graphs, probability, and statistics.

*Eighth-grade UCSMP*. In the eighth-grade UCSMP text, in a lesson titled “Slope-Intercept Equations for Lines,” students are told the following:

> A special case of a linear equation occurs when the *y*-intercept is at the origin. Then the *y*-intercept is 0 and $y = mx + 0$ becomes $y = mx$. This means that *y* is a constant multiple of $x$….When *y* is a constant multiple of *x* it is said that *y* varies

directly as *x*. This situation is called direct variation (Brown et al., 2009, pp. 352-353).

Other lessons in the UCSMP textbook focused on rates, ratios, and similar figures. More tasks were coded from a lesson titled “Proportions” than from any other lesson.

## Summary of Findings Related to Content Area

As indicated in Figure 14, in all three textbook series, a higher percentage of the tasks related to proportionality were in the rational number content area than in any other. This may reflect the fact that the framework included rational-number tasks that other scholars might not consider to be closely related to proportionality, such as finding a percent of a number. In all three textbook series, a lower percentage of the tasks related to proportionality were in the data analysis and probability content area than in any other. This is perhaps not surprising given the emphasis on algebra, geometry, and rational numbers in the *Curriculum Focal Points* for sixth, seventh, and eighth grades (NCTM, 2006).

Figure 14 also shows differences among the three series. The CMP series featured a higher percentage of tasks related to proportionality in the data analysis/probability and geometry/measurement content areas than did the other two series. The *Math Connects* series featured a higher percentage of tasks related to proportionality in the Rational Number content area than did the other two series. The UCSMP series featured a higher percentage of tasks related to proportionality in the algebra content area than did the other two series.

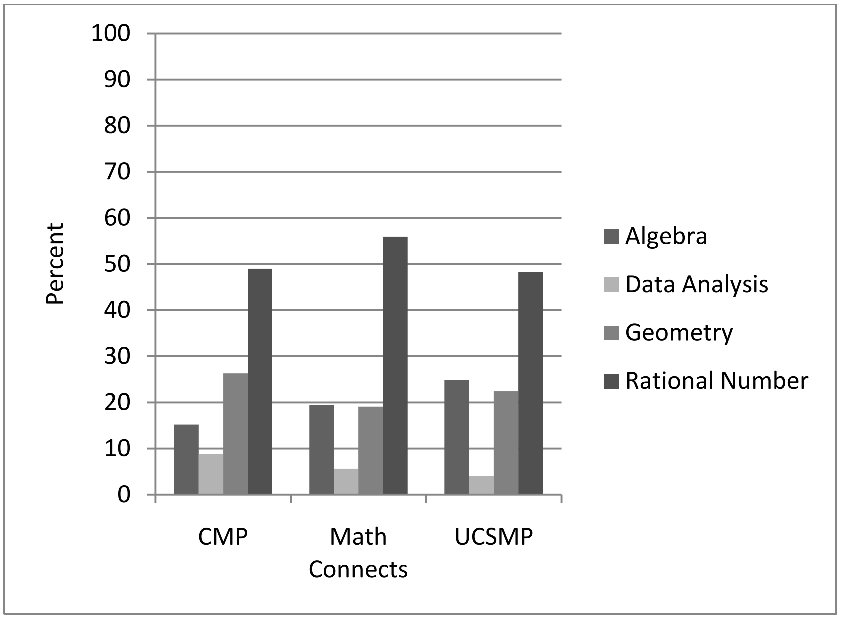


*Figure 14*. Percentage of tasks in each content area by series

## Problem Type

The second research question concerned the problem types of tasks related to proportionality. Three problem types were identified in the research literature: *missing value*, *ratio comparison*, and *qualitative*. Two others were identified in the course of a pilot study: *alternate form* and *function rule*. (See Appendix D.) Overall, 39% of the tasks were *missing value*, 24% were *alternate form*, 20% were *ratio comparison*, 6% were *function rule,* less than 1% were *qualitative,* and 11% were coded as *other*. Large differences in the problem types of tasks existed among content areas, textbook series and grade levels, as explained in the following sections.

### *Problem Type and Content Areas*

Problem types were related to content area. For example, the *alternate form* problem type is closely related to the rational number content area since *alternate form* tasks involve finding equivalent fractions or converting between fractions, decimals, and percents. The *ratio comparison* problem type is also related to rational numbers as ratio comparison is involved in comparing the magnitude of fractions, decimals, percents, and ratios. The *function rule* problem type is naturally associated with the algebra content area. The *missing value* problem type is related to a variety of topics such as equivalent fractions, rates and ratios, similar figures, and probability. Thus, the *missing value* problem type is related to all of the content areas.

### *Problem Type and Textbook Series*

As indicated in Table 10, in the CMP series, *missing value* and *ratio comparison* tasks were common as were tasks coded with the problem type *other*. The relatively high percentage of tasks coded as *ratio comparison* may reflect the sixth-grade textbook's focus on rational numbers; many of the tasks in this textbook asked students to compare fractions. The relatively high percentage of tasks coded as *other* reflects the fact that many CMP tasks are fairly involved, contain multiple parts, and are nonroutine.

In the *Math Connects* series, *alternate form* and *missing value* tasks were common. The relatively high percentage of tasks coded as *alternate form* may reflect the *Math Connects* focus on rational numbers; many of the tasks in this series asked students to find equivalent fractions or to convert between fractions, decimals, and percents.

In the UCSMP series, *missing value* tasks were much more common than any other problem type. *Missing value* tasks were more common in the UCSMP series than in any other series. In the UCSMP series, *missing value* tasks were especially common in the eighth-grade textbook where they appeared in lessons on rates, ratios, proportions, and similar figures. *Function rule* problems were rare in the UCSMP series.

Table 10

*Percentage of Tasks of Each Problem Type by Series*

| | AF | FR | MV | RC | Other |
|---|---|---|---|---|---|
| CMP (*n* = 965) | 14 | 10 | 26 | 28 | 21 |
| *Math Connects* (*n* = 2,766) | 29 | 6 | 40 | 18 | 7 |
| UCSMP (*n* = 832) | 22 | <1 | 49 | 15 | 14 |

Note: AF means *alternate form*, FR means *function rule*, MV means *missing value*, and RC means *ratio comparison*.

### *Problem Type and Grade Levels*

All of the problem types either steadily increased in frequency from sixth to seventh to eighth grade or steadily decreased in frequency. The two problem types associated with rational numbers, *alternate form* and *ratio comparison*, decreased as the grade levels increased. The other two problem types, *function rule* and *missing value*, increased as the grade levels increased.

The percentage of tasks of the *alternate form* and *ratio comparison* problem types decreased as the grade level increased. As illustrated in Figure 15, the *alternate form* problem type decreased from 33% in sixth grade to 22% in seventh grade and 15% in eighth grade. The *ratio comparison* problem type decreased from 31% in sixth grade to 15% in seventh grade and 12% in eighth grade.

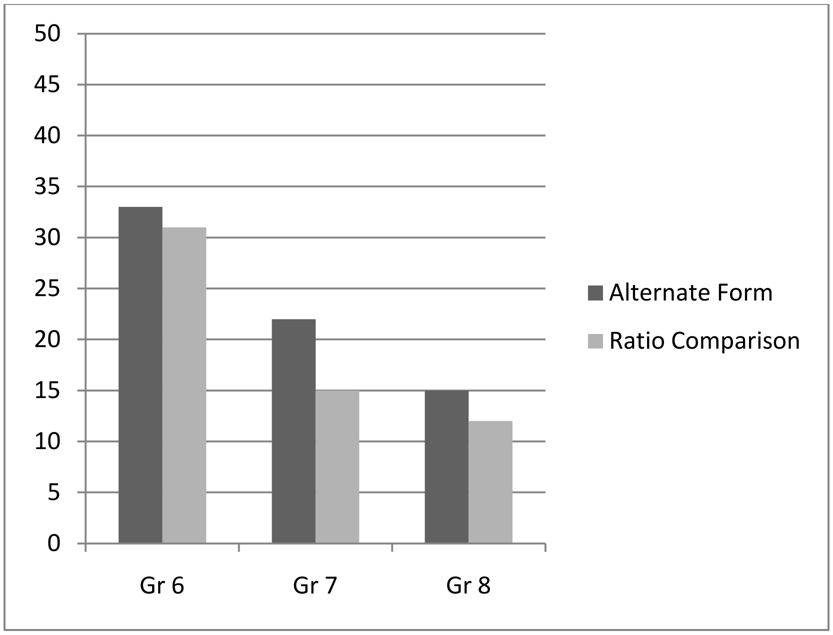


*Figure 15*. Percentage of tasks of types *alternate form* and *ratio comparison*
Note: 1,628 sixth-grade, 1,898 seventh-grade, and 1,037 eighth-grade tasks were coded.

The percentage of tasks of the *function rule* and *missing value* problem types increased with grade level. As illustrated in Figure 16, the *function rule* problem type increased from 4% in sixth grade to 6% in seventh grade and 7% in eighth grade. The *function rule* problem type is, naturally, associated with the algebra content area. Thus, the fact that its frequency increased with grade level seems reasonable.

As illustrated in Figure 16, the *missing value* problem type increased from 29% in sixth grade to 43% in seventh grade and 46% in eighth grade. The *missing value* problem type appeared in all four content areas; thus, it seems reasonable that it was common at all three grade levels. The *missing value* problem type was especially common in the eighth-grade UCSMP textbook, in which 71% of the tasks were coded as *missing value*.

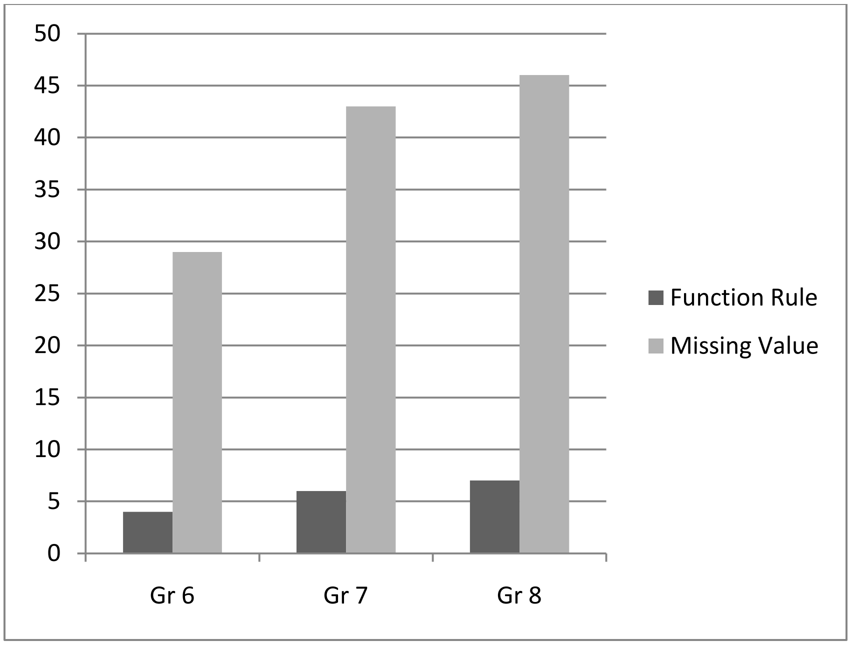


*Figure 16*. Percentage of tasks of types *function rule* and *missing value*
Note: 1,628 sixth-grade, 1,898 seventh-grade, and 1,037 eighth-grade tasks were coded.

*Problem Type, Textbook Series, and Grade Levels*

As indicated in Table 11, in the CMP textbooks, the most frequent problem type shifted from *ratio comparison* in sixth grade to *missing value* in seventh grade and *function rule* in eighth grade. This may reflect the sixth-grade focus on rational numbers and the eighth-grade focus on algebra. Tasks classified as "other" were common in the CMP seventh- and eighth-grade textbooks, perhaps indicating that the CMP tasks were difficult to classify. In the *Math Connects* textbooks, the most frequent problem type shifted from *alternate form* in sixth grade to *missing value* in seventh and eighth grades. In the UCSMP textbooks, the most frequent problem type was *missing value* throughout all three grades. The *missing value* problem type in UCSMP textbooks is discussed in the following section.

Table 11

*Percentage of Tasks of Each Problem Type by Series and Grade*

| | CMP | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Alternate Form | 32 | 5 | 0 | 37 | 30 | 19 | 24 | 25 | 4 |
| Function Rule | 0 | 10 | 40 | 8 | 5 | 4 | 0 | 1 | 0 |
| Missing Value | 14 | 37 | 14 | 27 | 47 | 46 | 49 | 43 | 71 |
| Ratio Comparison | 52 | 15 | 10 | 26 | 15 | 14 | 22 | 13 | 3 |
| Other | 0 | 33 | 33 | 2 | 3 | 17 | 6 | 19 | 22 |
| Number of Tasks | 337 | 528 | 100 | 946 | 999 | 821 | 345 | 371 | 116 |

### *Missing Value Tasks in UCSMP*

*Missing value* tasks were common in most of the textbooks in the study and particularly in the UCSMP textbooks. The UCSMP sixth-grade textbook contained two chapters ("Using Multiplication" and "Ratio and Proportion") that had an especially large number of *missing value* tasks. The chapter "Using Multiplication" contained many tasks related to percentages, most of which were coded as *missing value*. For example, one exercise instructed students to calculate 36% of 50. This was coded as *missing value* because it could be solved with the proportion $\frac{36}{50} = \frac{x}{100}$. The chapter "Ratio and Proportion" contained exercises in which students were asked to simply solve a proportion, such as $\frac{3}{5} = \frac{12}{m}$ or to solve word problems such as "Amanda can type 7 words in 6 seconds. If she continues working at this rate, how many words can she type in 3 minutes?" (McConnell et al., 2009, p. 491).

*Missing value* tasks appeared in the seventh-grade UCSMP textbook in many of the same ways that they did in the sixth-grade book: through percentages and lessons called “Proportions” and “Proportional Thinking.” The seventh-grade textbook also contained a lesson titled “Similar Figures,” which was not present in the sixth-grade textbook. The seventh-grade textbook contained chapters that point out when multiplication and division are useful, called “Multiplication in Geometry,” “Multiplication in Algebra,” and “Patterns Leading to Division.” All three of these chapters contained large numbers of *missing value* tasks.

Almost all of the missing value tasks in the eighth-grade UCSMP textbook appeared in a chapter titled “Division and Proportions in Algebra.” Lessons in this chapter were related to rates, ratios, proportions, and similar figures. All of these lessons contained large numbers of *missing value* tasks. For example, all of the 13 exercises that were coded from the “Rates” lesson were coded as *missing value*.

### *Qualitative Tasks*

According to Cramer et al. (1993), qualitative tasks “contain no numerical values but require the counterbalancing of variables in measure spaces” (p. 166). Cramer et al. provided this example: “If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, or (d) not enough information to tell” (p. 166). Although qualitative tasks have been discussed in the research literature, they were virtually absent from the textbooks in this study. Two percent of the tasks related to proportionality in the eighth-grade CMP textbook were coded as *qualitative*; in all other texts, less than one percent of the tasks related to proportionality were coded as *qualitative*. The tasks in the eighth-grade CMP textbook that were coded as *qualitative* were similar to the following: "Suppose the time *t* in the equation $d = rt$ is held constant. What happens to the distance *d* as the rate *r* increases?" (Lappan et al., 2006g, p. 60).

## Solution Strategy

The third research question concerned the solution strategies suggested for solving tasks related to proportionality. The codes for solution strategy included the *building up* strategy in which one of the quantities is doubled or tripled, the *decimal* strategy in which a fraction or percent is converted to a decimal, the *manipulatives* strategy in which students are instructed to use manipulatives or pictures, the *proportion* strategy, and the *unit rate* strategy.

Most of the tasks included in the study did not suggest a solution strategy. Across all series and grade levels, solution strategies were suggested in 14% of the tasks. Solution strategies were suggested more often in examples than in exercises.

The frequency with which solution strategies were suggested varied by series and by grade level. In CMP texts, solution strategies were suggested in 10% of the tasks. In *Math Connects* texts, solution strategies were suggested in 14% of the tasks. In UCSMP texts, solution strategies were suggested in 22% of the tasks. In the sixth-grade texts, solution strategies were suggested in 15% of the tasks. In the seventh-grade texts, solution strategies were suggested in 13% of the tasks. In the eighth-grade texts, solution strategies were suggested in 17% of the tasks. As indicated in Table 12, tasks in UCSMP textbooks were more likely than tasks from other series to suggest a solution strategy.

Table 12

*Number and Percentage of Tasks With a Solution Strategy*

| | *CMP* | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number of Tasks with SS* | 43 | 50 | 0 | 103 | 126 | 151 | 94 | 67 | 24 |
| Percent of Tasks with SS | 13 | 9 | 0 | 11 | 13 | 18 | 27 | 18 | 21 |
| Number of Tasks in Book | 337 | 528 | 100 | 946 | 999 | 821 | 345 | 371 | 116 |

*SS stands for "solution strategy."

### *Solution Strategy and Grade Level*

One might expect to see the strategies that are less formal and more likely to foster conceptual understanding at lower grade levels and more symbolic methods at higher grade levels. Previous research suggested that students find the *building up* method to be more natural than the *proportion* strategy (Lamon, 1999; Parker, 1999; Tournaire, 1986). Therefore, one might expect to see the *building up* strategy in sixth-grade textbooks. However, as indicated in Table 13, the *building up* strategy was relatively rare at all three grade levels.

The *manipulative* strategy, in which students are encouraged to use manipulatives or pictures, is less symbolic and less formal than other methods; one might expect to find the *manipulative* strategy more often at lower grade levels. This was the case in the textbooks in this study; the *manipulative* strategy decreased in frequency as the grade level increased. In contrast, the *proportion* strategy is quite symbolic; one might therefore expect to find it more frequently at higher grade levels. This was the case in the textbooks in the study; although it was about equally common in sixth and seventh grades, it was much more common in the eighth-grade textbooks.

Table 13

Percentage of Tasks With Each Solution Strategy by Grade

| | Grade 6 | Grade 7 | Grade 8 |
|---|---|---|---|
| Building Up | 5 | 7 | 0 |
| Decimal | 6 | 9 | 8 |
| Manipulative | 26 | 21 | 8 |
| Proportion | 29 | 28 | 52 |
| Other | 27 | 25 | 30 |
| n* | 240 | 243 | 175 |

*For this table, *n* was the number of tasks in which a solution strategy was suggested.

*Solution Strategy, Textbook Series, and Grade Level*

The frequency with which specific solution strategies were suggested varied by textbook series. As indicated in Table 14, the CMP texts tended to suggest the *manipulative* strategy whereas the *Math Connects* and UCSMP texts tended to suggest the *proportion* strategy. In the *Math Connects* and UCSMP texts, about 30% of the tasks were coded with the solution strategy *other*, suggesting that the solution strategies in these series did not fit neatly into the framework. In none of the eighth-grade CMP tasks that were coded was a solution strategy suggested. The *building up* strategy, which has been identified in the research literature as a useful tool for transitioning to true proportional reasoning, was found in the seventh-grade CMP textbook and in the sixth-grade *Math Connects* textbook, but was rare in the other seven texts.

Table 14

Percentage of Tasks With Each Solution Strategy by Textbook Series and Grade Level

| | CMP | | | | *Math Connects* | | | | UCSMP | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | overall | 6th | 7th | 8th | overall | 6th | 7th | 8th | overall |
| Building Up | 0 | 22 | 0 | 12 | 10 | 3 | 0 | 4 | 3 | 2 | 0 | 2 |
| Decimal | 7 | 4 | 0 | 5 | 0 | 6 | 9 | 6 | 13 | 18 | 4 | 14 |
| Manipulative | 70 | 42 | 0 | 55 | 16 | 21 | 9 | 15 | 18 | 5 | 0 | 11 |
| Proportion | 2 | 26 | 0 | 15 | 41 | 27 | 52 | 41 | 28 | 22 | 25 | 32 |
| Other | 7 | 4 | 0 | 5 | 34 | 31 | 29 | 31 | 29 | 28 | 38 | 30 |
| n* | 43 | 50 | 0 | 93 | 103 | 126 | 151 | 380 | 94 | 67 | 24 | 185 |

*For this table, *n* was the number of tasks in which a solution strategy was suggested.

### *The Proportion Strategy*

Proportions are a common and traditional method of solving proportional problems; thus, the proportion strategy is of particular interest. As shown in Table 14, the frequency with which proportions were used varied considerably with textbook series. The percentage of tasks in which proportions were used varied most in the sixth-grade textbooks. Proportions were common in the sixth-grade *Math Connects* textbook, with 41% of the solution strategies featuring a proportion. Proportions were less common in the sixth-grade UCSMP textbook, with 28% of the solution strategies featuring a proportion. Proportions were rare in the sixth-grade CMP book, with only 2% of the solution strategies featuring a proportion.

The following is an illustration of a type of task that can be presented either through the use of a proportion or through the use of a visual aid. Common in the sixth-grade books were tasks in which students found fractions equivalent to a given fraction, such as converting $\frac{1}{4}$ to $\frac{25}{100}$. The sixth-grade *Math Connects* textbook contains a two-page "Math Lab" that shows students how to use red and blue counters to illustrate equivalent fractions. The Lab contains two activities with a total of eight "Check Your Progress" exercises in which students are instructed to use counters to name fractions equivalent to given fractions. The next lesson in the *Math Connects* textbook is intended to help students use proportions to find equivalent fractions. The five-page lesson contains four examples and 49 exercises. Twenty-four proportions appear in the lesson. Thus, although equivalent fractions are presented first through the use of manipulatives, proportions seem to be the preferred solution strategy.

By way of contrast, the sixth-grade CMP Investigation "Sharing and Comparing with Fractions" is also intended, in part, to help students find equivalent fractions. This is accomplished primarily through the use of pictures of manipulatives (fraction strips) and number lines. No complete proportions appear in the in-class portion of the Investigation, although in six tasks students are asked to insert one of the signs <, >, or = between two fractions. The exercise set of the investigation contains four complete proportions and in 12 tasks students are asked to insert one of the signs <, >, or = between two fractions. In several tasks, students are asked to represent their ideas visually, as in "Describe, in writing or with pictures, how $\frac{7}{3}$ compares to $2\frac{1}{3}$" (Lappan et al., 2009a, p. 29). Thus, although proportions appear in the CMP textbook, manipulatives and visual representations are encouraged.

### *The Unit Rate Strategy*

All of the textbook series in this study contained material about the definition of a unit rate and procedures for calculating a unit rate. However, very rarely was the *unit rate* method suggested as a means for completing a task. As a reminder to the reader, a task was coded with the *unit rate* strategy only if a unit rate was used as a means for solving a larger problem. The following is an example of an exercise coded with the *unit rate* solution strategy: “A French bakery sells its famous chocolate cake in three sizes: 15 cm, 20 cm, and 25 cm in diameter. The cakes are the same height and cost $8, $16, and $20, respectively. Which size gives the most cake per dollar? Which size gives the least cake per dollar?” (McConnell et al., 2009, p. 536). Tasks in which students were instructed to simply compute a unit rate were not coded. For example, an exercise that read “Estimate the unit rate if 12 pairs of socks sell for $5.79” (Day et al., 2009b, p. 290) was not coded because the unit rate was not used to solve a larger problem.

In five of the nine textbooks in the study, in no task was the *unit rate* strategy explicitly suggested. In all of the textbooks in the study, the *unit rate* strategy was explicitly suggested in less than 1% of the tasks. However, for many tasks, the *unit rate* method may have been applied or assumed. For example, in a seventh-grade *Math Connects* lesson titled “Rates,” students are shown four examples of situations in which unit rates are used. The exercise set following these examples contains 46 exercises similar to the examples. In might be reasonable to assume that students would follow the examples and use the *unit rate* strategy for most of the exercises. However, in none of the exercises was the *unit rate* strategy explicitly suggested. Thus, the exercises were coded as *no solution strategy*.

### *Other Solution Strategies*

About 30% of the tasks in the *Math Connects* and UCSMP textbook series were coded with the solution strategy *other*. This suggests that the framework used to code solution strategies was not adequate to describe many of the solution strategies suggested in the textbooks. Virtually all of the tasks coded with the solution strategy *other* were examples rather than exercises, as exercises generally did not suggest any solution strategy. In some of the examples coded as *other*, students were instructed to compare decimals by lining up the decimal points and noticing the location of the digits that differed. Other examples coded as *other* instructed students to round decimals using a similar place-value procedure. Other examples coded as *other* instructed students to compare fractions by using a common denominator.

## Level of Cognitive Demand

The fourth research question concerned the level of cognitive demand (Stein et al., 2000) of tasks related to proportionality. According to the framework designed by Stein and colleagues, *Memorization* tasks "involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory" (Stein et al., p. 16). *Procedures Without Connections* tasks "are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task" (Stein et al., p. 16). Tasks at the level *Procedures With Connections* "require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly" (Stein et al., p. 16).

Tasks at the highest level, *Doing Mathematics*, "require complex and nonalgorithmic thinking" (Stein et al., p. 16).

One of the ways to determine the level of cognitive demand of a task is to notice whether students had recently seen a worked-out example similar to the task. Thus, the researcher coded the level of cognitive demand only of exercises, not of activities and examples. Coding the level of cognitive demand was especially difficult in the CMP series which has no labeled examples.

Far less than 1% of the exercises coded were at the lowest level of cognitive demand, the *Memorization* level. This was true in all three textbook series. Overall, 68% of the tasks were *Procedures Without Connections*, 25% of the tasks were *Procedures With Connections*, and 7% of the tasks were *Doing Mathematics*. The level of cognitive demand varied by textbook series and by grade level, as explained in the following sections.

### *Level of Cognitive Demand and Content Area*

The level of cognitive demand varied according to the content area of the exercise. As indicated in Table 15, most of the exercises in the rational number content area were coded as *Procedures Without Connections*. Thus, exercises in the rational number content area had the lowest levels of cognitive demand. Exercises in the data analysis and probability content area had the highest levels of cognitive demand.

Table 15

*Percentage of Exercises at Each Level of Cognitive Demand by Content Area*

| | Algebra | Data Analysis | Geometry | Rat. Number |
|---|---|---|---|---|
| Procedures Without Connections | 57 | 38 | 56 | 79 |
| Procedures With Connections | 34 | 47 | 29 | 20 |
| Doing Mathematics | 9 | 16 | 15 | 2 |
| Total Number of Exercises | 734 | 212 | 765 | 2062 |

*Level of Cognitive Demand and Textbook Series*

As indicated in Table 16, the level of cognitive demand in the *Math Connects* and UCSMP textbooks was similar, with about 70% of the tasks being at the level *Procedures Without Connections*, about 25% of the tasks being at the level *Procedures With Connections*, and about 5% of the tasks at the level *Doing Mathematics*. The level of cognitive demand in the CMP textbooks was much higher, with 43% of the tasks being at the level *Procedures Without Connections*, 36% of the tasks being at the level *Procedures With Connections*, and 21% of the tasks at the level *Doing Mathematics*.

Table 16

*Percentage of Exercises at Each Level of Demand by Textbook Series*

| | *CMP* | *Math Connects* | UCSMP |
|---|---|---|---|
| Procedures Without Connections | 43 | 75 | 65 |
| Procedures With Connections | 36 | 22 | 29 |
| Doing Mathematics | 21 | 3 | 7 |
| Total Number of Exercises | 636 | 2454 | 694 |

*Cognitive Demand in CMP Textbooks*

As indicated in Table 16, the level of cognitive demand of tasks related to proportionality was higher in the CMP series than in the other two textbook series. Many CMP tasks ask students to create or design a mathematical or real-life situation. This involves nonalgorithmic thinking, which is one of the hallmarks of a *Doing Mathematics* task (Stein et al., 2000). For example, a sixth-grade CMP exercise related to probability asks students to “Make a spinner and a set of rules for a fair two-person game. Explain why your game is fair. Make a spinner and a set of rules for a two-person game that is not fair. Explain why your game is not fair” (Lappan et al., 2009d, p. 47). Additionally, many of the CMP exercises ask students to make and justify conjectures and to use drawings and examples in their response. For example, a seventh-grade CMP task on similar figures read, “Suppose rectangle A is similar to rectangle B and to rectangle C. Can you conclude that rectangle B is similar to rectangle C? Explain. Use drawings and examples to illustrate your response” (Lappan et al., 2006b, p. 54).

*Level of Cognitive Demand and Grade Level*

The level of cognitive demand increased across the three grade levels. As illustrated in Figure 17, the percentage of tasks at the level *Procedures Without Connections* decreased through the grade levels. Specifically, the percentage of tasks at the level *Procedures Without Connections* decreased from 73% in sixth grade to 68% in seventh grade and 59% in eighth grade. The percentage of tasks at the higher levels of cognitive demand increased between sixth and eighth grades.

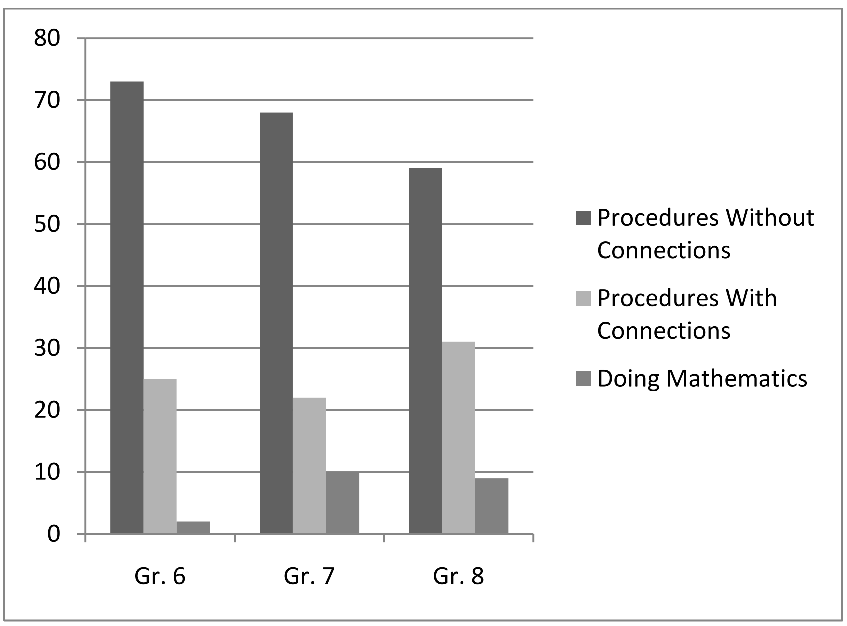


*Figure 17*. Percentage of tasks at each level of cognitive demand by grade

The increase in cognitive demand at higher grade levels was most noticeable in the CMP textbooks. In the CMP textbooks, the percentage of tasks at the *Procedures Without Connections* level decreased steadily from 55% in sixth grade to 42% in seventh grade and 11% in eighth grade. The percentage of tasks at the *Doing Mathematics* level increased steadily from 4% in sixth grade to 28% in seventh grade and 40% in eighth grade. Similar trends were seen in the *Math Connects* and UCSMP series, but in these series, the difference between sixth and eighth grades was smaller.

## Visual Representation

The fifth research question concerned the visual representations that appear in textbooks in tasks related to proportionality. Overall, 28% of the tasks related to proportionality involved a visual representation such as a table, graph, or picture of manipulatives. This percentage varied with textbook series and grade level, as explained in the sections below. Activities were more likely than examples or exercises to have a visual representation; 52% of the activities had one compared to 33% of the examples and 25% of the exercises. The two most common visual representations were tables and similar figures. Tables were used for a variety of purposes, such as showing the equivalence of various fractions, decimals, and percents, or for showing a relationship between an $x$ and $y$ variable.

### *Visual Representation and Textbook Series*

Visual representations were more common in the CMP textbooks than in the other series. In the CMP textbooks, 45% of the tasks were accompanied by a visual representation. This number was 26% for the *Math Connects* textbooks, and 17% for UCSMP. In the CMP series, tasks in Investigations and exercise sets were about equally likely to contain a visual representation. In the *Math Connects* and UCSMP series, activities were more likely to contain visual representations than were examples and exercises. For example, in the UCSMP series, 76% of the activities included a visual representation compared with 15% of the examples and 15% of the exercises.

*Visual Representations in CMP Textbooks*

Because visual representations were more common in CMP textbooks than in the other two series, the researcher paid special attention to the visual representations in CMP textbooks. The sixth-grade CMP textbook focuses on rational numbers; thus most of the visual representations are related to rational numbers. Many of the tasks in the sixth-grade module "Bits and Pieces I" contain a number line on which students place various decimals and fractions. Other tasks in the same module refer to a table of data and ask students to use the data to calculate percentages. The sixth-grade curriculum also contains a module on probability called "How Likely Is It?" This module contains many tasks that include a picture of a spinner.

The seventh-grade CMP curriculum contains a module titled "Stretching and Shrinking," which includes tasks related to similar figures. Many of these tasks include a visual representation of similar figures. Another seventh-grade module, "Comparing and Scaling," includes tasks in which students are given a set of data and are asked to use fractions, decimals, and percents to compare quantities. The data are often presented in a table.

The eighth-grade CMP textbook focuses on algebra and thus contains visual representations of functions, such as function tables and graphs. More than half of the tasks coded from the eighth-grade CMP textbook contained a visual representation, most of which were either a table or a graph. Quite a few of the eighth-grade tasks contain both a table and a graph. Tables were used for two distinct purposes. Some tables contained data from a real-life situation. Other tables were function tables that showed a relationship between $x$ and $y$ variables.

*Visual Representation and Grade Level*

The percentage of tasks with a visual representation increased through the grade levels, with 35% of the sixth-grade tasks, 60% of the seventh-grade tasks, and 67% of the eighth-grade tasks accompanied by a visual representation. As indicated in Table 17, in the CMP and *Math Connects* series, the frequency of visual representations increased with grade level. For example, in the CMP series, 32% of the sixth-grade tasks, 49% of the seventh-grade tasks, and 63% of the eighth-grade tasks were accompanied by a visual representation. In the UCSMP series, the percentage of tasks with a visual representation increased between sixth and seventh grades and then decreased slightly in eighth grade.

Table 17

*Visual Representation by Textbook Series and Grade*

| | *CMP* | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number of Tasks with VR* | 109 | 253 | 70 | 204 | 235 | 278 | 32 | 82 | 23 |
| Percent of Tasks with VR | 32 | 49 | 63 | 22 | 24 | 34 | 9 | 22 | 20 |
| Number of Tasks in Book | 337 | 528 | 100 | 946 | 999 | 821 | 345 | 371 | 116 |

*VR stands for “visual representation.”

The specific visual representations that were used also varied with grade level. Number lines and 10-by-10 grids were used to convey concepts related to rational numbers and were, thus, common in sixth grade textbooks. Similar figures were common in seventh-grade textbooks, and tables and graphs were common in eighth-grade texts.

## Characteristics of Proportional Situations

The sixth research question related to the characteristics of proportional situations and the extent to which tasks in textbooks point out when proportional reasoning is appropriate. Overall, 9% of the tasks related to proportionality pointed out the characteristics of proportional situations or whether proportional reasoning was appropriate in a given situation. Activities were more likely to cover this content than examples and exercises. Nineteen percent of the activities pointed out the characteristics of proportional situations as did 7% of the examples and 8% of the exercises.

### *Characteristics and Textbook Series*

CMP textbooks contained a higher percentage of tasks that pointed out the characteristics of proportional situations than did the *Math Connects* and UCSMP series. Seventeen percent of the CMP tasks covered this content as did 7% of the *Math Connects* tasks and 9% of the UCSMP tasks. As indicated in Table 18, in most of the textbooks, less than 20% of the tasks pointed out the characteristics of proportionality. In this respect, the eighth-grade CMP textbook was much different from the others in the study; 62% of the tasks in this textbook pointed out the characteristics of proportionality or discussed the appropriateness of proportionality.

Table 18

*Number and Percentage of Tasks that Point Out the Characteristics of Proportionality*

| | *CMP* | | | *Math Connects* | | | UCSMP | | |
|---|---|---|---|---|---|---|---|---|---|
| | 6th | 7th | 8th | 6th | 7th | 8th | 6th | 7th | 8th |
| Number of Tasks with Char* | 3 | 96 | 62 | 34 | 30 | 127 | 15 | 56 | 3 |
| Percent of Tasks with Char | 1 | 18 | 62 | 4 | 3 | 15 | 4 | 15 | 3 |
| Number of Tasks in Book | 337 | 528 | 100 | 946 | 999 | 821 | 345 | 371 | 116 |

*Note: "Char" refers to tasks that point out the characteristics of proportionality

*Characteristics of Proportionality in CMP Textbooks*

In the sixth-grade CMP textbook, there are a few tasks related to the differences between additive and multiplicative reasoning when percents are introduced. For example, students are provided with this information:

> During a recent year, Yao Ming made 301 out of 371 free-throw attempts and Shaquille O'Neal made 451 out of 725 attempts. It is hard from these raw numbers to tell who was better at free throws. But in sports, the announcers give these raw numbers as percents (Lappan et al., 2009a, p. 56).

Students then use a visual representation to estimate the percent of shots each player made. Students are then asked "Alisha says that it is easy to tell who has the best free-throw record. She says, 'Yao Ming made 301 free throws and Shaquille O'Neal made 451. So, Shaquille O'Neal has the better record!' Do you agree? Why or why not?" Lappan et al., p. 56). Students using additive reasoning would reason that 451 is greater than 301 and agree with Alisha. Students using multiplicative reasoning would realize that fractions or percents are necessary to compare the players.

Both the seventh- and eighth-grade CMP textbooks contained a relatively large percentage of tasks that pointed out the characteristics of proportionality. In general, the seventh-grade textbook accomplishes this through geometry/measurement tasks and the eighth-grade textbook accomplishes this through algebra tasks. The seventh-grade CMP textbook contained many tasks that prompt students to consider what happens to the side lengths, perimeter, and area of a shape that undergoes a size change. These tasks could help students realize that the side lengths and perimeter increase by the same percentage as the size change, but that the area does not. This could help students understand that the relationship between size-change factor and perimeter is proportional, but the relationship between size-change factor and area is not proportional. The seventh-grade CMP textbook contains quite a few tasks that point out that a figure enlarged by a scale factor $s$ will have its perimeter increased by a factor of $s$, its area increased by $s^2$, and its volume increased by $s^3$. The seventh-grade CMP textbook also has several tasks that explicitly address the issue of additive versus multiplicative reasoning, such as the question "How do you decide when to compare numbers using rates, ratios, or percents rather than by finding the difference of two numbers?" (Lappan et al., 2006c, p. 67).

To point out the differences between additive and multiplicative reasoning, the seventh-grade CMP textbook includes a Unit Review at the end of the "Comparing and Scaling" module in which students are asked a series of questions designed to illustrate the differences between additive and multiplicative reasoning, such as the following:

> There are 300 students in East Middle School. To plan transportation services for the new West Middle School, the school system surveyed East students. The survey asked whether students ride a bus to school or walk. In Mr. Archer's

homeroom, 20 students ride the bus and 15 students walk….In what ways can you compare the number of students in Mr. Archer's homeroom who are bus riders to the number who are walkers? Which seems to be the best comparison statement? (Lappan et al., 2006c, p. 65).

Ideally, students would state both that five more students ride than walk (additive reasoning) and that 57% of the students ride the bus (multiplicative reasoning.) However, because little guidance is provided, teachers should ensure that students come up with both answers.

The eighth-grade CMP textbook contained many tasks that prompt students to compare linear to non-linear relationships. These were coded as pointing out the characteristics of proportional situations since linearity is one of the characteristics of those situations. However, many of the tasks in the eighth-grade CMP textbook that were coded as pointing out the characteristics of proportional situations were not closely related to proportionality. For example, in the lesson "Looking Back at Functions," students are shown 18 equations and asked which represent functions that are linear, exponential, and quadratic. Although distinguishing between linear and non-linear relationships may be an important part of distinguishing between proportional and non-proportional situations, it is possible that other researchers would not have coded this and similar tasks as pointing out the characteristics of proportional situations. Thus, although the researcher found that 62% of the eighth-grade CMP tasks cover this content, this may be an over-estimate.

In the eighth-grade CMP module "Thinking with Mathematical Models," many tasks ask students to write an equation of the form $y = mx + b$ to match a given data table or graph. At least one of these graphs contains both a line with a *y*-intercept of zero and a line with a positive *y*-intercept. This would be a logical place to ask students to consider the effects of a positive *y*-intercept, but students are not asked to do so.

*Characteristics of Proportionality in Math Connects Textbooks*

All three of the *Math Connects* textbooks contain material designed to help students distinguish between proportional and nonproportional situations. However, in some cases, this material is presented in an abstract, symbolic manner that seems to encourage the use of procedures rather than foster conceptual understanding. For example, the sixth-grade textbook states the following:

> To verify a proportion, you can use cross products. If the product of the *means* equals the product of the *extremes*, then the two ratios form a proportion. In a proportion, the top left and bottom right numbers are the extremes. The top right and bottom left numbers are the means. (Day et al., 2009a, p. 333)

Although this explanation seems abstract and procedural, a subsequent exercise encourages multiple solution strategies; it reads: "Cecil's Pizza offers two large pizzas for $15 and four large pizzas for $28. Describe and use three different ways to determine if the pair of ratios is proportional" (Day et al., 2009a, p. 333).

The seventh-grade *Math Connects* textbook states, "Two quantities are proportional if they have a constant rate or ratio" (Day et al., 2009b, p. 310). The textbook shows that one serving of milk contains 300 milligrams of calcium and that four servings of milk contain 1,200 milligrams of calcium. The textbook uses the proportion $\frac{300\ mg}{1\ serving} = \frac{1{,}200\ mg}{4\ servings}$ to show that the relationship is proportional. The exercise set for this lesson consists of 12 exercises in which students are given two ratios and asked whether the relationship is proportional, such as “A store sells 2 hamsters for $11 and 6 hamsters for $33. Is the cost proportional to the number of hamsters sold? Explain” (Day et al., p. 313). This method of comparing ratios is mathematically correct and will help students distinguish between proportional and non-proportional relationships, but is likely to lead only to procedural understanding and will likely not help students understand the differences between additive and multiplicative reasoning.

In the eighth-grade textbook, two examples are provided in which data is taken from a ratio table and ratios are formed and compared. However, in the exercise set, no ratio tables are provided and in some cases would be impossible to construct due to a lack of information.

*Characteristics of Proportionality in UCSMP Textbooks*

The sixth- and seventh-grade UCSMP textbooks explicitly address the issue of recognizing situations in which proportional reasoning is required. However, the number of tasks devoted to the topic is small. For example, in the sixth-grade textbook, in a lesson titled “Solving Proportions,” four exercises ask students to “Tell whether it would make sense to use proportions to solve the problem. Explain your answer” (McConnell et al., 2009, p. 496). Neither of the two examples in the lesson is related to the appropriateness of proportional reasoning. Thus, if students complete and understand these exercises, they will have considered the issue. However, since students generally do not complete or understand every exercise in a textbook, the issue could easily be missed.

The seventh-grade UCSMP textbook also devotes a small amount of space to the issue. The textbook states:

> There are two parts to **proportional thinking:** (1) the sense to recognize situations in which setting up a proportion is a way to find an answer and (2) the ability to get or estimate an answer to a proportion without solving an equation. Some people believe that proportional thinking is one of the most important kinds of thinking you can have in mathematics (Viktora et al., 2008, p. 596).

An in-class activity in this same lesson presents students with four different situations and asks which of the four calls for proportional thinking.

Few tasks in the eighth-grade textbook are designed to help students distinguish proportional from non-proportional situations. A few exercises are designed to help students realize that the ratio of the areas of similar figures is not proportional to the size change factor and several exercises help students distinguish between linear and non-linear functions.

*Characteristics and Grade Level*

Overall, the percentage of tasks that pointed out the characteristics of proportional situations increased with grade level. Three percent of the sixth-grade tasks covered this content as did 10% of the seventh-grade tasks and 19% of the eighth-grade tasks. However, this varied with textbook series. In the CMP texts, the percentage of tasks that pointed out the characteristics of proportional situations increased steadily with grade level from 1% in sixth grade to 18% in seventh grade to 62% in eighth grade. In the *Math Connects* series, the percentage of tasks that pointed out the characteristics of proportional situations was higher in eighth grade than in either the sixth or seventh grades. In the UCSMP series, the percentage of tasks that pointed out the characteristics of proportional situations was highest in seventh grade.

According to the *Curriculum Focal Points,* (NCTM, 2006), seventh-grade students should "distinguish proportional relationships…from other relationships, including inverse proportionality" (p. 19). In the textbook series in this study, contrasting proportional and inverse relationships was covered in eighth grade rather than seventh. The developers of these series will need to decide whether they wish to comply with the recommendations of the *Curriculum Focal Points* and move their comparisons of proportional and inverse relationships from eighth to seventh grade.

## Overview of Textbook Series

In this section, an overview of the treatment of proportionality in each textbook series is provided. This should help the reader understand the material related to proportionality that students studying from each series experience as they move through middle school.

### *Proportionality in CMP Textbooks*

Each of the textbooks in the CMP series covers proportionality differently; in each grade level, students are presented with a different view of proportionality. All of the CMP textbooks use visual representations and hands-on activities to foster conceptual understanding, but the content areas through which proportionality is presented vary among grade levels. At the sixth-grade level, proportionality is presented through rational numbers and probability. At the seventh-grade level, proportionality is presented through all of the content areas. At the eighth-grade level, it is presented through algebra. Proportions were used less often in the CMP textbooks than in the other series. Some important topics, such as the definition of direct variation, appeared only in exercise sets and were not explicitly addressed in the portion of the textbook that would be covered in class. Thus, students who study from CMP textbooks for three years have the opportunity to see proportionality as a theme that connects content areas. However, these students will have had less practice with formal proportions than students who study from other textbook series.

*The Sixth-grade CMP Textbook*

In the sixth-grade CMP textbook, there is a focus on helping students develop a conceptual understanding of fractions. Students use fraction strips, number lines, and visual representations of fractions to develop understandings of equivalent fractions and the relative size of various fractions. Proportions are used to compare fractions *after* students have had opportunities to develop conceptual understandings of fractions. Similarly, information related to decimals is first presented visually, through the use of hundredths grids and number lines. As previously described, the differences between additive and multiplicative reasoning are hinted at when percents are introduced.

In addition to the focus on rational numbers, the sixth-grade CMP book also presents proportionality through probability; one of the eight modules is related to probability and includes three lessons that involve proportionality. In the lessons on probability, students first conduct hands-on probability experiments. A large number of tasks ask students to predict and analyze the data sets that would result from various spinners.

In summary, the sixth-grade CMP textbook presents proportionality through rational numbers and probability. The visual representations and hands-on activities that are featured in this book are likely to help students develop conceptual understanding of rational numbers and probability. In the sixth-grade CMP book, there are almost no tasks related to proportionality in the algebra and geometry content areas. Thus, there are no lessons on similar figures, rates, ratios, or slope, which are common in most middle-school textbooks. Proportions were used in the sixth-grade CMP textbook only to compare fractions.

*The Seventh-grade CMP Textbook*

Six seventh-grade CMP modules were included in the study. Two related primarily to algebra, two to geometry/measurement, one to rational numbers, and one to data analysis/probability. However, in each of the algebra modules, only one investigation related to proportionality. In one of the geometry/measurement modules, “Stretching and Shrinking: Understanding Similarity,” all five of the investigations were related to proportionality. In the other geometry/measurement module, “Filling and Wrapping: Three-Dimensional Measurement,” two out of the five investigations were related to proportionality. Thus, of the 15 investigations related to proportionality, seven involved geometry and measurement.

The module “Filling and Wrapping” contained an investigation titled “Scaling Boxes” that involved comparing or changing dimensions of three-dimensional solids. This was the only textbook in the study to contain such a lesson. The investigation pointed out that when a solid figure is enlarged by a scale factor, the relationship between the dimensions and the scale factor is proportional, but the relationship between the surface area and scale factor is not proportional and the relationship between the volume and the scale factor is not proportional.

Although one lesson in the seventh-grade book was devoted to the use of proportions, in other lessons, students were instructed to use tables, graphs, and equations to find missing values rather than proportions. Thus, as in the sixth-grade book, proportions were used less often than one might expect.

*The Eighth-grade CMP Textbook*

Only three of the eight modules in the eighth-grade curriculum featured a focus on proportionality and were included in the study. Thus, one could say that proportionality is less of a focus in the eighth-grade book than in the sixth- and seventh grade CMP textbooks.

Of the eight modules in the eighth-grade CMP curriculum, five focus on algebra, two on geometry, and one on data analysis. The geometry covered in the textbook is primarily the Pythagorean Theorem and transformations, which are not closely related to proportionality. Two of the algebra modules as well as the data analysis module contained material related to proportionality and were included in the study. In the data analysis module, only one investigation contained material related to proportionality and was included in the study. Thus, in the eighth-grade CMP textbook, proportionality is presented primarily through the algebra content area. Because of the focus on algebra, there were more tasks of the *function rule* problem type than any other. Because none of the modules focus on rational numbers, none of the tasks were coded with the *alternate form* problem type.

Many of the tasks in the algebra modules ask students to compare linear relationships to inverse or other nonlinear relationships. These tasks were included in the study because linearity is one of the characteristics of proportional situations. Many of the tasks in the algebra modules ask students to translate between tables, graphs, and equations. The definition of inverse variation and the inverse variation equations are highlighted, but the definition of direct variation and the direct variation equation appear only in one exercise in a homework set.

### *Proportionality in Math Connects Textbooks*

Whereas in the CMP series, proportionality was presented differently at each grade level, in the *Math Connects* series, students see a similar view of proportionality at each grade level. All three of the *Math Connects* textbooks cover a great deal of material. Thus, in all three textbooks, proportionality is covered through all four content areas. Although each textbook contains tasks related to proportionality in all four content areas, more tasks are related to rational numbers than any other content area.

The sixth- and seventh-grade *Math Connects* textbooks seem repetitive; much of the material related to proportionality that is covered in sixth grade is also covered in the seventh-grade textbook. For example, both textbooks contain several chapters on decimals, fractions, and percent. Although the eighth-grade textbook contains five chapters on algebra, the book as a whole has much less of a focus on algebra than do the eighth-grade CMP and UCSMP textbooks. The algebra that does appear in the *Math Connects* book is at a lower level than in the eighth-grade CMP and UCSMP textbooks.

#### *The Sixth-grade Math Connects Textbook*

Proportionality in the sixth-grade *Math Connects* textbook is presented primarily through rational numbers; 62% of the tasks related to proportionality were coded as *rational number*. Three of the 12 chapters focus on fractions, decimals, and/or percents. More of the tasks are of the problem *alternate form* than any other problem type. This indicates that many tasks in the sixth-grade book ask students to convert between fractions, decimals, and percents.

The lessons on rational numbers contain a few visual representations, but far fewer than were found in the sixth-grade CMP textbook. The lessons on rational numbers contain large numbers of tasks of the cognitive level of demand *Procedures Without Connections*. These tasks seem repetitive. For example, in the lesson "Representing Decimals," 44 out of the 52 exercises (85%) ask students to translate between word form, standard form, and expanded form of decimals.

The sixth-grade textbook also presents proportionality through algebra; 18% of the tasks related to proportionality were coded as *algebra*. Two of the 12 chapters focus on algebra. One of the chapters on algebra contains a lesson titled "Function Machines," which includes numerous tasks of the *function rule* problem type.

The sixth-grade *Math Connects* textbook has much more material related to formal proportions than does the sixth-grade CMP textbook. The textbook contains a chapter titled "Ratio, Proportion, and Functions." In this chapter, proportions are presented as a way of writing ratios in simplest form. Most of the solution strategies that are suggested involve proportions. However, in very few of these tasks is cross multiplication used to solve the proportion.

*The Seventh-grade Math Connects Textbook*

Proportionality in the seventh-grade *Math Connects* textbook is presented primarily through rational numbers and, to a lesser extent, geometry. Three of the 12 chapters focus on fractions, decimals, and/or percents. Several of the seventh-grade lessons on rational numbers seem similar to lessons in the sixth-grade textbook. For example, both textbooks contain a lesson titled "Simplifying Fractions," both textbooks contain lessons on converting between fractions, decimals, and percents, and both textbooks cover computation with fractions.

The seventh-grade chapter titled “Ratios and Proportions” covers much of the information that was covered in the sixth-grade lesson “Ratio, Proportion, and Functions.” This chapter, in both books, involves algebra. The chapter in the seventh-grade book has lessons titled “Algebra: Solving Proportions” and “Inverse Proportionality.”

Three of the 12 chapters focus on geometry. There is one lesson in the seventh-grade *Math Connects* book on similar figures, but there is far less material on similar figures in this book than in the seventh-grade CMP textbook. Although there are several lessons on three-dimensional solids, there is not a focus on changing the dimensions of these solids as there was in the seventh-grade CMP textbook.

*The Eighth-grade Math Connects Textbook*

Proportionality in the eighth-grade *Math Connects* textbook is presented primarily through rational numbers. Two of the 12 chapters focus on fractions, decimals, and/or percents. As in the sixth- and seventh-grade books, the eighth-grade book contains material related to converting among fractions, decimals, and percents, comparing the magnitudes of rational numbers, and computation with rational numbers. Each of the rational-number lessons contain a large number of exercises which are mostly of a low level of cognitive demand. For example, in the eighth-grade lesson titled “Rational Numbers,” 53 exercises were coded and all but seven of them were at the level *Procedures without Connections*.

Proportionality in the eighth-grade *Math Connects* textbook is also presented through the *algebra* content area. Three of the 12 chapters focus on algebra. One of these contains a lesson titled "Direct Variation" which informs students that "When the ratio of two variable quantities is constant, their relationship is called a direct variation. The constant ratio is called the constant of variation" (Day et al., 2009c, p. 487). This lesson explicitly address the differences between proportional linear functions and nonproportional linear functions.

*Proportionality in UCSMP Textbooks*

As in the CMP series, in the UCSMP series, proportionality is more prominent in the sixth- and seventh-grade textbooks than in eighth grade. This is likely due to the fact that the eighth-grade CMP and UCSMP textbooks focus on algebra. In other ways, the UCSMP series resembles the *Math Connects* series more than the CMP series. For example, the sixth-grade *Math Connects* and UCSMP series both contain a lesson on proportions, which are virtually absent from the sixth-grade CMP textbook.

In the UCSMP textbooks, rates, ratios, and proportions arise from division. For example, the seventh-grade textbook contains a chapter titled "Patterns Leading to Division" which includes four lessons on rates, ratios, and proportions. The UCSMP textbooks emphasize that units like "words per minute" result from dividing a certain number of words by a certain number of minutes and that units such as these can be written as fractions, as in $\frac{\text{words}}{\text{minute}}$. This connection between division and proportionality was much less visible in the other two series.

*The Sixth-grade UCSMP Textbook*

Proportionality in the sixth-grade UCSMP textbook is presented primarily through rational numbers. Two of the 12 chapters focus on fractions, decimals, and/or percents and other chapters also contain lessons on rational numbers. Fractions are first presented with a geometric context that was not present in the other two series; the fraction $\frac{1}{2}$ is compared to the midpoint of a segment and rulers are used to provide a visual representation of other fractions. Decimals are first presented through the use of number lines instead of the hundredths grids that were used in the other two series. Rulers and number lines are shown frequently in the lessons on rational numbers, but few other visual representations are used.

As in the *Math Connects* series, there is a sixth-grade lesson on ratio and proportion. As in the *Math Connects* series, proportions are first introduced as a way to put a ratio in lowest terms. As in the *Math Connects* series, a connection is made between rate tables and proportions. The UCSMP lesson on proportions contains several exercises that check whether students understand the definition of a proportion; in the *Math Connects* series, exercises focus on students' ability to write and solve proportions.

In the sixth-grade UCSMP textbook, ratios and proportions arise from division. The sixth-grade book presents the Rate Model for Division and the Ratio-Comparison Model for Division. The Rate Model for Division states "When two quantities with different kinds of units are divided, the quotient is a rate" (McConnell et al., 2009, p. 401). The Ratio-Comparison Model for Division states "If *a* and *b* are quantities with the same units, then $\frac{a}{b}$ compares *a* to *b*" (McConnell et al., 2009, p. 462).

*The Seventh-grade UCSMP Textbook*

As in the sixth-grade book, proportionality in the seventh-grade UCSMP textbook is presented primarily through rational numbers. Although only one of the 12 chapters focuses on fractions, decimals, and percents, five of the lessons in this chapter were included in the study. Thus, five of the 19 lessons coded from the seventh-grade book were related to fractions, decimals, or percents. Chapters on multiplication and division contained three lessons related to rational numbers. Thus, eight of the 19 lessons coded from the seventh-grade book were related to rational numbers.

As in the sixth-grade book, there are few visual representations of rational numbers. Most of the representations that are present are number lines, not the fraction strips and hundredths grids that are present in other series.

As in the sixth-grade textbook, ratios and proportions arise from division. The seventh-grade book reiterates the Rate Model for Division and the Ratio-Comparison Model for Division that were covered in the sixth-grade textbook. The Ratio-Comparison Model for division is presented identically in both textbooks, but the Rate Model for Division is presented more algebraically in the seventh-grade textbook, which states "If *a* and *b* are quantities with different units, then $\frac{a}{b}$ is the amount of quantity *a* per amount of quantity *b*" (Viktora et al., 2008, p. 563).

*The Eighth-grade UCSMP Textbook*

As in the CMP series, the eighth-grade UCSMP textbook presents proportionality through algebra. In fact, a chapter in the eighth-grade UCSMP textbook is titled "Division and Proportions in Algebra." Some of the material in the eighth-grade book is a repetition of material in the sixth- and seventh-grade books. For example, the sixth- and seventh-grade books both contain a lesson titled "The Rate Model for Division" and rates are covered again in the eighth-grade textbook.

The eighth-grade UCSMP textbook contains less material on the differences between linear and non-linear functions than do the other two series. The eighth-grade UCSMP book has lessons on exponential functions, including a lesson comparing linear increase to exponential growth, but there is less of a focus on this than in the other two series. The UCSMP book also does not cover inverse variation, which is covered in the other two series. Direct variation is defined in a lesson titled "Slope-Intercept Equations for Lines."

## Summary of Results

The purpose of this study was to investigate how the treatment of proportionality in textbooks varies by grade level and textbook series. Results indicate that the amount of attention paid to proportionality varies by grade level; sixth- and seventh-grade textbooks contain considerably more tasks related to proportionality than do eighth-grade textbooks. Results also indicate that the treatment of proportionality in textbooks varies by textbook series. Tasks related to proportionality in CMP textbooks have higher levels of cognitive demand and are more likely to contain a visual representation.

Proportionality is a characteristic of mathematical relationships that appear in a variety of topics within mathematics, such as algebra, data analysis, geometry, and rational numbers. Results of this study indicate that more of the tasks related to proportionality are also related to rational numbers than to any other content area. This likely occurs because lessons on rational numbers typically have large numbers of relatively simple practice problems. In the *Math Connects* series, this focus on rational numbers was present at all three grade levels, whereas in the other two series, the eighth-grade textbook focused on algebra.

Other findings of this study indicate that the *qualitative* problem type is virtually absent from the textbooks in this study as were the *building up* and *unit rate* solution strategies. Textbooks contain material on unit rate, but typically have students simply compute the unit rate rather than use it as a solution strategy.

# CHAPTER 5: DISCUSSION

The purpose of this study was to investigate the treatment of proportionality in three contemporary, widely-used middle-school textbook series: Connected Mathematics2 (CMP), Glencoe's *Math Connects*, and the sixth-, seventh- and eighth-grade textbooks from the University of Chicago School Mathematics Project (UCSMP). The three series were chosen because their treatment of proportionality was expected to be quite different. Results of the study indicate that this was indeed the case. In each of the nine textbooks, the researcher analyzed lessons in which proportionality was a focus, such as lessons on rational numbers, rates, ratios, proportions, similar figures, and certain probability lessons. The researcher compared the treatment of proportionality across grade levels to see whether there is a logical progression as students move from sixth to seventh to eighth grade. According to NCTM's *Principles and Standards for School Mathematics* (NCTM, 2000), proportionality can be used to connect various content areas in mathematics. The researcher noted the content area of each task related to proportionality in order to determine the extent to which proportionality is indeed used in this manner.

Several conclusions based on the analysis of the nine textbooks in this study can be made. Three of these conclusions seem particularly salient and are elaborated upon in the following sections. First, a clear finding of this study is that more of the tasks related to proportionality were in the rational number content area than any other. This was particularly true in the sixth- and seventh-grade textbooks. The *Principles and Standards* *for School Mathematics* state that middle-school students should “develop a deep understanding of rational-number concepts, become proficient in rational-number computation and estimation, and learn to think flexibly about relationships among fractions, decimals, and percents” (NCTM, 2000, p. 212). However, more recent recommendations in the *Curriculum Focal Points* (NCTM, 2006) state that fractions and decimals should be studied earlier, in third and fourth grades. If one follows the newer recommendations, a focus on fractions and decimals in middle school may be inappropriate.

Partly because of the focus on rational numbers in sixth and seventh grades, there were more tasks related to proportionality in the sixth- and seventh-grade textbooks than in the eighth-grade textbooks. The sixth- and seventh-grade CMP and UCSMP textbooks had a much heavier focus on proportionality than did the eighth-grade textbooks in the series. This may be appropriate, given the recommendations in the *Curriculum Focal Points* (NCTM, 2006) which indicate that eighth-grade students should study a variety of algebra topics. Proportionality should be studied in eighth grade, but should be studied through an algebra lens, which was present in some, but not all, of the eighth-grade textbooks in the study.

A third salient finding of this study is that real differences exist in the presentation of proportionality in the three series in this study. Differences can be seen in the content area of the proportional tasks, the solution strategies encouraged by the textbooks, the level of cognitive demand of the tasks, and the ways in which textbooks point out the characteristics of proportionality and the differences between additive and multiplicative reasoning. Although student understanding was not measured in this study, it seems likely that many of the *Math Connects* lessons would encourage procedural understanding and that the CMP series is designed to foster conceptual understanding. Justification of this claim appears in a later section.

## Grade Levels

One of the findings of this study is that, in the textbook series included in this study, proportionality is emphasized in sixth and seventh grades more than in eighth grade. This finding applies to the CMP and UCSMP series more than to the *Math Connects* series. This finding reflects the fact that the eighth-grade CMP and UCSMP textbooks focus more on algebra than other content areas.

### *Sixth Grade*

According to the *Curriculum Focal Points* (NCTM, 2006), sixth-grade students should connect rates and ratios to multiplication and division. The example given is the following: “If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12” (NCTM, p. 18). The *Curriculum Focal Points* suggest that sixth-grade students develop this reasoning by using a multiplication table or simple drawings. Noticeably absent from this recommendation is any mention of formal proportions or cross multiplication. Because one of the three sixth-grade focal points directly relates to proportionality, one would expect proportionality to be a focus of sixth-grade textbooks. This was the case in the textbooks in the study. A fairly large number of lessons and tasks were coded from each sixth-grade textbook. The sixth-grade lessons and tasks that related to proportionality may have been too focused on rational numbers, but at least proportionality was present and common in sixth-grade textbooks.

*Seventh Grade*

According to the *Curriculum Focal Points* (NCTM, 2006), seventh-grade students should develop an understanding of proportionality, including similarity. All three of the seventh-grade textbooks in the study had at least one lesson on similar figures. The seventh-grade CMP textbook had five Investigations related to similar figures. The seventh-grade *Math Connects* book had one lesson titled “Scale Drawings” and another titled “Similar Figures.” The UCSMP textbook included a lesson titled “Proportions in Similar Figures.” The *Curriculum Focal Points* also state that seventh-grade students should:

> use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects….[and] graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships…from other relationships, including inverse proportionality (NCTM, 2006, p. 19).

Thus, one would expect proportionality to be a focus of seventh-grade textbooks. This was the case in the textbooks in the study. In each of the three series, more tasks were coded from the seventh-grade textbook than from either of the other two grade levels. In the CMP series, there were more tasks related to proportionality in the seventh-grade book than in the sixth- and eighth-grade books combined. Furthermore, one would expect to see seventh-grade tasks related to algebra, geometry, and rational numbers. This also was the case with the textbooks in the study. All three seventh-grade textbooks had relatively large percentages of proportional tasks related to each of these three content areas.

*Eighth Grade*

One of the eighth-grade focal points mentions proportionality but seems more focused on other algebraic concepts. The focal point states that eighth-grade students should "recognize a proportion ($y/x = k$ or $y = kx$) as a special case of a linear equation of the form $y = mx + b$, understanding that the constant of proportionality, $k$, is the slope and the resulting graph is a line through the origin" (NCTM, 2006, p. 20). However, the *Curriculum Focal Points* also describes a wide variety of other algebraic concepts to be studied that are not closely related to proportionality. Additionally, the other two eighth-grade focal points are not directly related to proportionality. Thus, one might expect to see proportionality less of a focus in eighth grade than in sixth and seventh grades. This was the case in the textbooks in the study. In each of the three series, fewer tasks were coded from the eighth-grade textbook than from the other two grade levels. The number of tasks coded in the eighth-grade CMP and UCSMP textbooks is sharply lower than the number of tasks related to proportionality in the seventh-grade textbook. The difference is much less dramatic in the *Math Connects* series.

Thus, the CMP and UCSMP series are consistent with the recommendations in the *Curriculum Focal Points* in that they focus on proportionality in sixth and seventh grades and on algebra in eighth grade. However, for this strategy to work, students must understand proportionality by the time they leave seventh grade since there is less material related to it in the eighth-grade textbooks. In contrast, the *Math Connects* series has only slightly less material in the eighth-grade book related to proportionality than in the sixth- and seventh-grade textbooks. This provides the advantage of review for eighth-grade students who have not mastered proportionality, but has the disadvantage of a lesser emphasis on algebra in the eighth-grade *Math Connects* textbook.

## Content Areas

Overall, 55% of the tasks related to proportionality were in the rational number content standard, 20% were in the algebra standard, 22% were in geometry/measurement, and 6% were in data analysis/probability. The large percentage of tasks related to rational numbers is likely a reflection of two factors. First, rational numbers have historically constituted a large part of the middle-school mathematics curriculum. Although the *Curriculum Focal Points* (NCTM, 2006) states that an understanding of rational numbers should be developed in third and fourth grades, the textbooks examined in this study contain many tasks aimed at helping students understand and translate among fractions, decimals, and percents. Thus, the large percentage of tasks related to rational numbers in part reflects a perhaps outdated focus on helping middle-school students understand and translate between various forms of rational numbers.

The large percentage of tasks related to rational numbers is also a reflection of the framework used in this study. The researcher intentionally adopted a broad view of proportionality which includes some tasks that other scholars may not classify as directly related to proportionality. Many of these tasks are related to rational numbers. For example, many tasks in the sixth-grade textbooks asked students to compare the size of two or more decimals. Some scholars may not recognize this as proportional reasoning, but these tasks were included in the study. Thus, the large percentage of tasks related to rational numbers in part reflects the framework.

The low percentage of tasks related to data analysis is likely a reflection of the overall focus on this standard in the middle-school curriculum. Of the nine focal points in the middle-school curriculum, only one is directly related to data analysis. This focal point states that students should use descriptive statistics to compare data sets (NCTM, 2006). Because mean, median, and mode were not included in this study, it is not surprising that only a small percentage of tasks in this study related to data analysis. One of the connections to the seventh-grade focal points states that students should use proportions to make predictions and use percentages in relation to circle graphs. These types of tasks were found in the textbooks in the study and were included in the study.

### *Content Areas of Tasks in Sixth-Grade Textbooks*

All of the sixth-grade textbooks in this study present proportionality primarily through the rational number content area. Specifically, 87% of the tasks in the CMP textbook, 62% of the tasks in the *Math Connects* textbook, and 57% of the tasks in the UCSMP textbook were in the rational number content area. As previously mentioned, this could be due, in part, to the nature of the framework. However, it is clear that all of the sixth-grade textbooks in the study contain a large number of tasks that require students to compare the size of fractions or decimals, to convert between fractions, decimals, and percents, or to find percentages of various numbers. Many scholars may not consider these activities to involve proportional reasoning, but because of the broad definition of proportionality used in this study, these topics were included in the framework.

Two characteristics of the sixth-grade textbooks resulted in this tendency to represent proportionality through rational numbers. First, most of the lessons related to proportionality were also related to rational numbers. For example, in the sixth-grade CMP textbook, nine Investigations were analyzed; six of these focused on rational numbers and the other three were in the data analysis and probability content area. Second, lessons related to rational numbers had a greater number of tasks coded than did lessons in other content areas. For example, in the CMP book, in the Investigation “Moving Between Fractions and Decimals,” 86 tasks were coded whereas in the Investigation “A First Look at Chance,” only 14 tasks were coded. Thus, not only did the sixth-grade textbooks contain a large number of lessons related to rational numbers, but also many of the tasks in the lessons fit into the framework for identifying proportional tasks.

The *Curriculum Focal Points* (NCTM, 2006) state that students should develop an understanding of fractions and decimals in third and fourth grades. The content standards of most states indicate that students should develop an understanding of equivalent fractions in fourth or fifth grade, but that equivalence of fractions, decimals, and percents is studied in sixth and seventh grades (Reys et al., 2006). Thus, whether the sixth-grade focus on rational numbers is appropriate is not clear as mathematics educators are not in agreement regarding the placement of rational numbers.

Geometry and measurement tasks involving proportionality were also common in the sixth-grade *Math Connects* and UCSMP textbooks. In the *Math Connects* textbook, many of these tasks had to do with converting units of measurement. The *Math Connects* textbook contains a large number of exercises similar to 6 yd = ____ ft. The UCSMP textbook contains fewer exercises of this nature.

*Content Areas of Tasks in Seventh-Grade Textbooks*

The *Curriculum Focal Points* (NCTM, 2006) suggest that proportionality in seventh grade should be presented through a variety of content areas. This document states that seventh graders should solve a wide variety of percent problems, learn about similar figures, distinguish proportional relationships from other relationships, including inverse, and use proportions to solve data analysis and probability problems. In all of the seventh-grade textbooks, proportionality was presented through all four content areas. All of the seventh-grade textbooks contained at least one lesson on similar figures. However, content designed to help students distinguish proportional relationships from other relationships was minimal. For example, the seventh-grade *Math Connects* textbook had a one-page "Math Lab" on inverse proportionality and no discussion of when proportional reasoning is appropriate.

A noticeable difference between the seventh-grade textbooks was the focus on geometry in CMP and rational numbers in *Math Connects* and UCSMP. A focus on rational numbers could be appropriate in seventh grade, provided that percents, rates, and ratios constitute the bulk of that focus, rather than decimals or fractions, which should be studied by students in third and fourth grades, according to the *Curriculum Focal Points* (NCTM, 2006). The framework used in this study was not designed to distinguish between the various types of rational numbers, so percentages cannot be reported, but the titles of the lessons provide some insight into their content. The rational number tasks in the seventh-grade UCSMP textbook appear in lessons on a variety of topics including fractions, decimals, percents, rates, and ratios. However, in the seventh-grade *Math Connects* textbook, many of the rational number tasks appear in lessons on simplifying fractions, converting between fractions and decimals, and comparing and ordering fractions and decimals, topics that, according to the *Curriculum Focal Points* should have been studied prior to seventh grade.

### *Content Areas of Tasks in Eighth-Grade Textbooks*

The *Curriculum Focal Points* (NCTM, 2006) suggest that proportionality in eighth grade should be presented primarily through algebra. This was the case in the eighth-grade CMP textbook, in which three of the four Investigations related to proportionality were also related to algebra. The UCSMP textbook also focused on algebra; five of the six lessons related to proportionality were also related to algebra. The *Math Connects* textbook, however, contained a plethora of topics. Twenty-six lessons related to proportionality, covering ratios, rates, rate of change, proportions, similar figures, dilations, indirect measurement, solid figures, sequences, slope, direct variation, graphs of functions, circle graphs, probability, and statistics. Although some of these lessons related to algebra, they may be overshadowed by the mountain of other content.

### *Content Area Progression From Sixth To Eighth Grade*

Given the nature of the mathematics curriculum as described in the *Curriculum Focal Points* (NCTM, 2006), one would expect to see a progression from a focus on rational numbers in late elementary and early middle school to algebra in late middle school. This progression is clear in the CMP textbooks. The CMP textbooks present proportionality primarily through rational numbers in sixth grade, geometry in seventh grade, and algebra in eighth grade. One could perhaps argue that the transition from rational numbers to algebra in the CMP series is too extreme as *none* of the 342 tasks related to proportionality in the sixth-grade book were in the algebra content area and only the seventh-grade book had a significant percentage of tasks related to proportionality in the geometry content area.

The progression from rational numbers to algebra is less dramatic but still present in the UCSMP books. In the UCSMP series, a high percentage of sixth- and seventh-grade tasks are related to rational numbers but almost none are in the eighth-grade *Algebra* text. Not surprisingly, most of the proportional tasks in the eighth-grade *Algebra* text are in the algebra content standard.

The progression is less clear in the *Math Connects* textbooks. The percentage of tasks in the rational number content area decreases little from sixth to eighth grade, the percentage of tasks in the geometry and data analyses content areas remain relatively constant, and the percentage of proportion-related tasks in the algebra standard increases only slightly over the three years.

Research cannot determine whether proportionality should be presented through a different content area in sixth grade than it is in eighth grade. Research findings can describe differences between textbooks and the results of this study indicate that, among tasks related to proportionality, there is a progression from rational numbers to algebra in the CMP and UCSMP series.

### *Proportionality as a Connection Between Content Areas*

The NCTM's *Principles and Standards for School Mathematics* suggest that proportionality can be used as a theme to integrate various topics. The *Principles and Standards* state the following:

> Curricular focus and integration are also evident in the proposed emphasis on proportionality as an integrative theme in the middle-grades mathematics

> program. Facility with proportionality develops through work in many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability (NCTM, 2000, p. 212).

To what degree do the textbooks in this study use proportionality as a connective theme? Because this was not one of the research questions of this study, the study was not designed specifically to answer this question. However, based on the analysis of the textbooks, it appears that the connections between content areas results from the capacity of proportions to solve problems in diverse situations. Lessons that focus on proportions generally incorporate several content areas. Algebra is usually involved in these lessons as students learn the cross-multiplication procedure. Lessons that focus on proportions often include ratio and rate problems as well as geometry problems related to maps or similar figures. Thus, lessons on proportions are one way textbooks use proportionality to connect content areas. Most textbook series, including *Math Connects* and UCSMP, separate content into lessons that each focus on a narrow range of content. For example, the seventh-grade *Math Connects* textbook has separate lessons on ratios, rates, rate of change, solving proportions, scale drawings, similar figures, and probability. This may lead students to believe that the topics have nothing in common. In contrast to this, the CMP seventh-grade textbook, in an investigation titled "Making Sense of Proportions," brings together tasks related to percent, proportions, similar figures, scale factors, indirect measurement, rates, ratios, ratio comparison, probability, and sampling in a single investigation. Having tasks from various content areas in close proximity may help students recognize connections between them.

## Problem Types

There is little discussion in the existing research literature regarding the difficulty of various problem types or the grade levels at which each type should be studied. However, based on this study, some generalizations can be made.

### *Alternate Form and Ratio Comparison*

*Alternate form* and *ratio comparison* problem types tend to accompany a focus on rational numbers. For example, the *alternate form* problem type occurs primarily when students are asked to simplify fractions or convert between fractions, decimals, and percents. Similarly, in the textbooks analyzed, the *ratio comparison* problem type appears primarily when students were asked to compare fractions or decimals or to place fractions and decimals on number lines. According to the *Curriculum Focal Points* (NCTM, 2006), these skills should be studied in third and fourth grades. Therefore, one might hope to see the *alternate form* and *ratio comparison* problem types common in third- to sixth-grade textbooks, and less common in seventh- and eighth-grade textbooks. In fact, in the books in this study, the *alternate form* and *ratio comparison* problem types did in fact decrease between sixth and eighth grades. This could be seen as an encouraging sign as it may indicate that seventh and eighth graders are assumed to have mastered skills like simplifying fractions and converting between decimals, fractions, and percents.

### *Missing Value*

The *missing value* problem type is often associated with rates and ratios as in this example from the seventh-grade *Math Connects* textbook: “Desiree earns $280 in 40 hours. What is her hourly pay rate?” (Day et al., 2009b, p. 287). According to the *Curriculum Focal Points,* sixth-grade students should work on “connecting ratio and rate to multiplication and division" (NCTM, 2006, p. 18). The *missing value* problem type is also associated with similar figures and proportions. According to the *Curriculum Focal Points,* seventh-grade students should work on "developing an understanding of and applying proportionality, including similarity" (NCTM, 2006, p. 19). Therefore, one might expect the *missing value* problem type to be common in sixth- and seventh-grade textbooks and less common in eighth-grade textbooks. However, findings of the study indicate that the *missing value* problem type is more common in seventh- and eighth-grade textbooks than in sixth-grade books.

As explained above, more missing value problems were found in eighth-grade books than one might expect given the recommendations of the *Curriculum Focal Points*. The *missing value* problem type is particularly common in the eighth-grade *Math Connects* textbook, in which 46% of the proportional tasks are *missing value* and in the eighth-grade UCSMP textbook, in which 71% of the proportional tasks are *missing value*.

In the eighth-grade UCSMP textbook, the majority of the *missing value* tasks were found in lessons on rates, ratios, proportions, and similar figures. Given that the *Curriculum Focal Points* places these topics in sixth and seventh grades, one might wonder whether these lessons would be more appropriate in the sixth- and seventh-grade textbooks.

In the eighth-grade *Math Connects* textbook, a wide variety of lessons contain *missing value* tasks including lessons on proportions, similar figures, indirect measurement, scale drawings, and percents. As with the eighth-grade UCSMP series, one might wonder whether these lessons would be more appropriate in the sixth- and seventh-grade textbooks.

*Function Rule*

The *function rule* problem type has not been described in the research literature on proportionality, but was identified by the researcher during the course of a pilot study (Appendix A). In a typical *function rule* task, students are shown a function table and asked to write the function rule. In sixth-grade textbooks, this rule is generally written as an expression, such as $4x$. In eighth-grade texts, this rule is generally an equation, such as $y = 4x$. Although there was some variety in the types of tasks coded with the *function rule* problem type, most seem to be related either to the sixth-grade focal point on algebra which states that students should "use equations to describe simple relationships (such as $3x = y$) shown in a table" (NCTM, 2006, p. 18) or the eighth-grade focal point on algebra which states that "Students translate among verbal, tabular, graphical, and algebraic representations of functions" (NCTM, p. 20). Thus, it appears that function rule tasks should be fairly common at more than one grade level. However, this was not the case in the textbooks studied. Function rule tasks were fairly rare, with less than 6% of the tasks being of this type. In the CMP texts, they were common in seventh- and eighth-grade texts, but virtually absent from the sixth-grade text. In the *Math Connects* series, they were present but rare at all three grade levels. Function rule tasks were virtually absent from all three UCSMP texts.

When functions are written as equations, the slope and $y$-intercept are clear. Thus, *function rule* problems may help students distinguish between proportional and nonproportional relationships. Because the *function rule* problem type has not been described in the research literature, more discussion of this problem type is necessary. If educators decide that the *function rule* problem type is an important part of the middle-school mathematics curriculum, textbook authors will need to give more attention to it.

### *Qualitative*

Many researchers have mentioned *qualitative* proportional tasks as an important part of proportional reasoning (e.g., Ben Chaim et al., 1998; Lamon, 2007). One might expect, therefore, to see qualitative proportion tasks in middle-school textbooks. However, they were virtually absent from the textbooks in this study. Educators should decide whether qualitative proportion tasks are important for middle-school students to encounter; if they are, textbook authors will need to pay them more attention.

## Differences Between Textbook Series

The three textbook series were chosen because the researcher expected their treatment of proportionality to differ. In fact, the three textbook series differ in at least three important ways: (a) the amount of repetition of content among the three grade levels in each series, (b) the degree of emphasis on algebra in the eighth-grade textbook, and (c) the degree to which each series seems designed to foster conceptual understanding. The purpose of this discussion is not meant to offer an opinion on the quality of each series but instead to describe the differences between the series.

### *Amount of Repetition Among Grade Levels*

One of the findings of this study is that content related to rational numbers and proportionality is often repeated in sixth-, seventh- and eighth-grade textbooks. For example, in the *Math Connects* series, all three textbooks contain lessons on rates, ratios, and solving proportions. The content in these lessons is very similar. In all three textbooks, the lesson on ratios and rates begins with one or two examples titled “Write

Ratios in Simplest Form." The exercises on rates and ratios in the sixth-grade book include more visual representations, but the content covered is virtually identical in all three books; in all three books, students are asked to write ratios in simplest form and to compare ratios.

The sixth- and seventh-grade UCSMP textbooks also contain repetition related to ratios and rates: both books feature lessons titled "The Rate Model for Division" and "The Ratio Comparison Model for Division." The examples and exercises in these lessons are somewhat different; the sixth-grade lessons focus on helping students understand the connection between division, rates, and ratios and the eighth-grade textbook contains more tasks with variables. The sixth- and seventh- grade UCSMP books also both contain lessons titled "Proportions." Textbooks at both grade levels cover the definition of a proportion, how to state a proportion in words, and how to determine whether a proportion is true or false.

A content analysis such as this cannot state how much repetition is desirable. To determine the amount of review needed by students would require a study involving measures of student understanding. However, the similarity of the lessons on rates, ratios, and proportions in the *Math Connects* series seems excessive; if a lesson is effective in helping students learn material, it seems unnecessary to repeat virtually the same lesson in sixth, seventh, and eighth grades.

### *Algebra in the Eighth-Grade Textbooks*

Textbook authors cannot emphasize all content to the same degree; they must make choices regarding the content on which to focus at each grade level. The results of this study indicate that authors of eighth-grade textbooks choose between a focus on algebra or proportionality. In the CMP and UCSMP series, a much larger number of tasks were coded in the sixth- and seventh-grade books than in the eighth-grade book. This suggests that the CMP and UCSMP series emphasize proportionality in sixth and seventh grades. Not only do these series have more tasks related to proportionality in sixth and seventh grades, but also, the percentage of tasks related to rational numbers plummets in eighth grade in both series. The topics covered in the CMP and UCSMP eighth-grade books clearly indicate that algebra is the focus of these books. This suggests that the authors of the CMP and UCSMP textbooks expect students to have developed proportional reasoning and an understanding of rational numbers by the end of seventh grade, which allows them time to study algebra in eighth grade. By contrast, the *Math Connects* series continues to feature a focus on both rational numbers and proportionality through eighth grade. As discussed in a previous section, the *Curriculum Focal Points* (NCTM, 2006) seem to suggest that eighth-grade textbooks should focus on many aspects of algebra, of which proportionality is only one part. Thus, given the recommendations, the focus on algebra in the eighth-grade CMP and UCSMP series seems more appropriate than the focus on rational numbers and proportionality in the eighth-grade *Math Connects* textbook, particularly when much of the content related to proportionality in the eighth-grade *Math Connects* textbook is a repetition of material in the sixth- and seventh-grade textbooks.

### *Procedural Versus Conceptual Understanding*

Another difference between the series in this study is that the intent of CMP textbooks seems to be to foster a conceptual understanding of rational numbers and proportionality whereas many of the lessons in the *Math Connects* series seem likely to promote procedural understanding of these topics. Student understanding was not measured in this study; therefore, it is not possible to state definitively whether a series promotes procedural or conceptual understanding. However, a close look at the tasks and lessons of these textbooks clearly suggests that they are likely to result in different types of understanding of proportionality. Three of the dimensions coded in this investigation help justify the claim that the CMP series is more likely than the *Math Connects* series to foster conceptual understanding of proportionality. These dimensions are the solution strategies encouraged, the level of cognitive demand, and the way textbooks distinguish between proportional and non-proportional relationships.

The UCSMP textbooks in some ways resemble the CMP textbooks. For example, both the CMP and UCSMP textbooks encourage conceptual understanding of the distinctions between proportional and non-proportional situations. In other ways, the UCSMP textbooks resemble the *Math Connects* books. For example, in both the *Math Connects* and UCSMP textbooks, more than 30% of the tasks that suggest a solution strategy encourage the use of a proportion. Also, the level of cognitive demand of the tasks in the UCSMP textbooks is closer to that of the *Math Connects* books than the CMP series, at least for the topic of proportionality.

*Solution Strategies*

According to the research literature, informal solution strategies such as the *unit rate* and *building up* methods may be easier for children to understand than the symbolic and procedural use of proportions and cross multiplication (Cramer et al., 1993). One might expect, therefore, that proportionality be introduced through solution strategies that foster conceptual understanding, such as through the use of pictures and manipulatives.

Differences exist in the solution strategies encouraged by the three textbooks in this study.

The CMP textbooks feature a large percentage of tasks that encourage students to use manipulatives and/or pictures and a small percentage of tasks that encourage the *proportion* strategy. Thus, the CMP series seems to encourage a conceptual understanding of proportionality. The *Math Connects* textbooks feature a large percentage of tasks that encourage students to use the *proportion* strategy and thus seem to encourage a procedural understanding of proportionality.

In the *Math Connects* textbooks, the *proportion* strategy was more common than any other at all three grade levels. This use of symbolism before proportional reasoning has been fully developed may encourage only procedural understanding and prevent students from understanding proportionality more conceptually. In the UCSMP textbooks, the *proportion* strategy is also common at all three grade levels, but the *decimal* and *manipulative* strategies are also common, as is encouragement to use technology. In the CMP textbooks, encouragement to use manipulatives or pictures is much more common than the *proportion* strategy. Thus, the CMP textbooks seem to lean strongly toward conceptual rather than procedural understanding. In fact, the CMP textbooks could be criticized for inadequate instruction in procedures; some might argue that the lack of encouragement for the *proportion* and *cross multiplication* strategies leaves students without procedures to efficiently solve routine problems related to proportionality.

*Level of Cognitive Demand*

Helping support the claim that the CMP textbooks are more likely to foster conceptual understanding is their level of cognitive demand. Almost none of the tasks coded were at the *Memorization* level; thus, the lowest level of demand was the *Procedures Without Connections*. In this type of task, a student is asked to follow a simple procedure which has likely been demonstrated. For example, the following is a task from the sixth-grade CMP textbook that was coded as *Procedures Without Connections*: "Find three fractions that are equivalent to $\frac{2}{7}$." (Lappan et al., 2009a, p. 23). In the CMP textbooks, 43% of the coded tasks were at this level compared to 65% of the UCSMP tasks and 75% of the *Math Connects* tasks. The highest level of demand is *Doing Mathematics*. According to Stein et al. (2000), *Doing Mathematics* tasks "require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example" (p. 16). In the CMP textbooks, 21% of the coded tasks were at this level compared to 7% of the UCSMP tasks and 3% of the *Math Connects* tasks. Thus, it is clear that the CMP textbooks had a higher level of demand than either of the other two series. This is not necessarily a benefit of the CMP textbooks; one could perhaps argue that the level of demand is so high that it might frustrate students and their teachers. Also, higher-level tasks take much more time to complete, leaving less time for routine tasks that allow students to practice procedures. However, if level of cognitive demand is associated with conceptual understanding, then it seems likely that the CMP textbooks are more likely than the other two series to foster conceptual understanding, at least with respect to the topic of proportionality.

*Characteristics of Proportional Situations*

Textbooks have been criticized for years for failing to help students understand when proportional reasoning is appropriate. For example, Cramer et al. (1993) stated, "Proportional reasoning abilities are more involved than textbooks would suggest….Students need to see examples of proportional and nonproportional situations so they can determine when it is appropriate to use a multiplicative solution strategy" (p. 169). Recent studies have found that children who have studied proportionality in school apply proportions where they are not appropriate (e.g., Van Dooren et al., 2009). Although researchers seem to recognize this as a problem, there has been little discussion about how to help children understand the characteristics of proportionality or know when to apply proportional reasoning.

Some of the textbooks in this study make an attempt in this direction, as explained in Chapter 4. However, the number of tasks in each textbook that address the characteristics of proportionality is very small; if a teacher chooses to skip a single lesson, students could never see this material. Additionally, the *Math Connects* textbooks address certain issues in only a very procedural way that may not help students develop true proportional reasoning or understand the difference between additive and multiplicative reasoning. Thus, the researcher concludes that although the CMP and UCSMP series have made steps toward helping students understand when proportional reasoning is called for, more attention to this issue is needed.

Based on the solution strategies encouraged, the level of cognitive demand, and the way textbooks instruct students to distinguish between proportional and non-proportional situations, the researcher believes that the CMP series is likely to promote conceptual understanding of proportionality and that the *Math Connects* series is likely to promote procedural understanding of proportionality. The researcher does not claim that one type of understanding is better than the other, just that the textbook series are likely to promote different types of understanding. As shown in Table 19, the UCSMP series in some ways resembles the *Math Connects* textbooks and in other ways resembles the CMP series. The classifications in Table 19 are somewhat arbitrary; for example, “High” and “Low” cognitive demand have not been defined. However the data presented in Chapter 4 indicate that there are substantial differences between the series. Thus, although the classifications have not been defined, the researcher nevertheless believes Table 19 illustrates important differences between the series.

Table 19

*Conceptual and Procedural Aspects of Textbook Series*

| | CMP | *Math Connects* | UCSMP |
|---|---|---|---|
| Common Solution Strategies | Building Up | Proportion | Proportion |
| | Manipulative | | Decimal |
| Level of Cognitive Demand | High | Low/Moderate | Moderate |
| Proportional versus Nonprop.* | Conceptual | Procedural | Conceptual |

*Refers to the method by which students are taught to identify proportional situations

## Conclusion

Textbooks are influential in mathematics classes, affecting the content that is covered and the ways in which that content is presented. As Ball and Cohen (1996) pointed out, because textbooks are widely used in classrooms, they have “reach” in the system and are likely to be an effective way of influencing what happens in mathematics classes. Different curricula have been shown to have differing effects on student achievement including differing effects on students’ understanding of proportionality (Ben-Chaim et al., 1998). Thus, the importance of studying textbooks’ content and presentation of material seems clear.

Students’ development of proportional reasoning is widely recognized as one of the most important goals of the middle-school mathematics curriculum (e.g., Curcio & Bezuk, 1994). This study has shown that there are some similarities between middle-school textbooks’ presentation of proportionality and also important differences. The series in this study were similar in the fact that all of the sixth-grade textbooks focused on helping students understand rational numbers, despite the fact that the *Curriculum Focal Points* states that this understanding should be achieved in third and fourth grades (NCTM, 2006).

The differences between textbook series are striking. For example, proportions are common in the *Math Connects* sixth-grade textbook, but almost absent from the CMP sixth-grade book. Another difference is that the presentation of proportionality in the *Math Connects* textbooks is clearly more procedural than in the other two series, and the CMP series is clearly more conceptual, with its focus on manipulatives and pictures. A third difference is that the level of cognitive demand of the CMP series seems higher than that of the other two series, with over 20% of the CMP tasks classified at the highest level of cognitive demand.

The results of this study show a disconnect between the research literature on proportionality and a sample of middle-school textbooks. The research literature indicates the existence of three problem types: *missing value*, *ratio comparison*, and *qualitative*. Not only were these three problem types insufficient to describe many of the tasks in textbooks, but the *qualitative* type is virtually absent from textbooks. The research literature also indicates that students have difficulty distinguishing between proportional and non-proportional relationships. Although all of the seventh-grade textbooks in the study made an attempt to cover this topic, it was often explicitly addressed in one lesson, and in the case of *Math Connects*, was addressed in a very procedural manner. Finally, the research literature suggests that the *building up* and *unit rate* strategies may help students transition toward proportional reasoning before learning the symbolism of proportions and cross multiplication. However, the *building up* and *unit rate* strategies were virtually absent from most of the textbooks in the study.

One would hope to see evidence of progression and development as students move from sixth to seventh to eighth grades. Some evidence of such a progression was present. In all three series, the focus on rational numbers decreased through the three years. The two problem types associated with rational numbers, *alternate form* and *ratio comparison*, also decreased through the three years. Along with the decreased emphasis on rational numbers, one would expect to see an increased focus on algebra in eighth grade. This occurred in the CMP and UCSMP series, but much less so in the *Math*

*Connects* series. Progression was also evident in the level of cognitive demand, which increased from sixth to seventh to eighth grade in all three series.

In conclusion, some encouraging signs are present, such as the intertwining of algebra and proportionality that was seen in the eighth-grade CMP and UCSMP textbooks. Perhaps in response to recent literature, all of the series attempt to help students understand the difference between proportional and non-proportional situations. In some areas and in some textbooks, improvement is necessary, such as the procedural manner in which the seventh-grade *Math Connects* textbook explains how to compare ratios to determine if a situation is proportional.

Curriculum content analyses, such as the one reported here, cannot answer certain questions, such as "Should students develop an understanding of fractions and decimals in third and fourth grades, as recommended by the *Curriculum Focal Points* (NCTM, 2006), or in the sixth and seventh grades, as the material is covered in the textbooks in this study?" Studies like this one cannot answer the question about when students should learn certain topics, but curriculum content analyses can identify areas in which textbooks differ from recommendations, as this study has done.

## Limitations of the Study

The study has several limitations. The first is that only three textbook series were studied and these series may not be representative of all the middle-school textbooks used in the United States. The second limitation is that researcher's conclusions rest on reliability and validity of the framework used to analyze the textbooks. The third limitation is that textbooks are not the only influence on the mathematical content presented to students. Each of these limitations is described in more detail below.

## *Limited Sample Size*

To allow the researcher to study the selected texts in depth and devote sufficient attention to each, only three textbook series were used. The results may not apply to other textbook series. An attempt was made to choose textbooks that are widely used; however, because market share data are not available, the percentages of U.S. students using the selected series are not known. The researcher also attempted to select textbooks that were authored under different educational philosophies, thereby capturing some of the diversity among middle-school textbooks. However, many middle-school textbooks series are used in the United States, and three series cannot capture all of the diversity. Thus, the limited sample size allowed an in-depth analysis of the selected textbooks, but may not represent the curriculum studied by many middle-school students.

Also, the validity of the researcher's conclusions rested on her ability to select lessons that present an accurate picture of the coverage of proportionality in each textbook. Many curriculum analyses restrict themselves to only certain sections of textbooks (e.g., Johnson et al., 2010; Jones, 2004) and this study did the same. The extent to which these selections provide an accurate portrait of the entire book affects the validity of the conclusions.

## *Limitations of the Framework*

Another limitation of the study relates to the reliability and validity of the framework that was used to code each task. The reliability of the framework refers to the extent to which a researcher unrelated to this study would code tasks in the same way the researcher would. To ensure reliability, another doctoral student coded 16% of the lessons analyzed by the researcher. Two types of reliability were calculated. One reflected the extent to which the researcher and graduate chose the same tasks to code and the other type reflected the extent to which they assigned the same codes. Reliability percentages were reported in Chapter 4.

The validity of the framework refers to the accuracy with which the framework measures important features of proportional examples and exercises in middle-school mathematics textbooks. To ensure validity of the framework, the researcher conducted a thorough literature review on the aspects of proportionality that are taught in middle school and based the framework on the research literature. The aspects of proportionality that the research community has deemed important were incorporated into the framework. The researcher acknowledges that not all content related to proportionality was examined. For example, long division is related to proportionality (Lesh et al., 1988), but was not included in the study. However, the definition of proportionality incorporated in the framework is quite broad and captures the vast majority of topics related to proportionality.

### *Intended Versus Implemented Curriculum*

Scholars interested in curriculum have distinguished between the intended and the implemented curriculum, describing the *intended curriculum* as the set of goals set forth in standards and policy statements and the *implemented curriculum* as what actually is covered in classrooms (Schmidt et al., 2001; Valverde et al., 2002). Valverde et al. offered this explanation:

> The inclusion of a learning goal in the intended curriculum does not guarantee that it will be covered. Including an intention as a goal does not guarantee that the

> opportunity to attain that goal will actually be provided in classrooms – but does greatly increase the probability that it will (p. 8).

In this study, percentages were used in an attempt to describe the relative emphasis on various content areas, problem types, and solution strategies. However, when tasks are implemented, it is likely that they do not all receive equal attention.

Because only student texts were studied, the researcher was unable to account for other influences on classroom instruction. For example, the teacher's edition may suggest activities that teachers choose to use. Teacher's editions were not included in the study. A study of student texts is considered to be a study of the intended curriculum whereas a study of what actually occurs in classrooms would be a study of implemented curriculum. Also, the implementation of textbooks in classes is affected by teacher knowledge, beliefs, and many other factors (Tarr et al., 2008). Therefore, textbooks are far from the only influence on student achievement. Although the significance of this study rests on the assumption that textbooks have some influence on student achievement, the researcher acknowledges that other influences also have an impact on student achievement.

## Recommendations

Two types of recommendations are offered below: recommendations for curriculum developers and recommendations for future research.

### *Recommendations for Curriculum Developers*

The researcher has two recommendations for curriculum developers. First, curriculum developers should consider which content related to proportionality should appear at more than one grade level; content should not be repeated unless there is a purpose behind the repetition. In some of the textbook series in this study, repetition appears in two different forms. One, particularly in the *Math Connects* series, proportionality is presented primarily through rational numbers at all three grade levels. It is not clear whether a conceptual understanding of fractions and decimals should be achieved in third and fourth grades, as the *Curriculum Focal Points* (NCTM, 2006) suggest, or at the sixth-grade level, as the CMP series suggests, but the repetition of rational-number content in sixth, seventh, and eighth grades is likely unnecessary.

The second form of repetition occurs when lessons that are very similar to each other are placed at multiple grade levels. When similar lessons appear at more than one grade level, there should be a logical progression and development through the grade levels rather than mere repetition. As discussed in an earlier section, in the *Math Connects* and UCSMP series, multiple textbooks in each series contain lessons on ratios, rates, and proportions. In some cases, the presentation of the content does not vary significantly from grade level to grade level.

Second, curriculum developers should include more material designed to help students understand the differences between additive and multiplicative reasoning. This material should foster more than just procedural understanding. Most of the textbooks in the study contain material designed to help students understand these differences. Some of the textbooks contain visual representations that may help students develop conceptual understanding of the difference between additive and multiplicative reasoning. An example is the investigation titled "Working with Percents" in the sixth-grade CMP module "Bits and Pieces I." However, in most books, only a few exercises explicitly address the issue of additive versus multiplicative reasoning. In some textbooks, these exercises appear in unit reviews which may or may not be assigned by the teacher.

*Recommendations for Future Research*

The researcher has identified three areas in need of additional research: a framework for measuring cognitive demand, proportional-reasoning problem types, and students' transitions from informal to more symbolic solution strategies.

In this study and others (e.g., Jones, 2004), the framework developed by Stein et al. (2000) was used to measure cognitive demand. However, this framework was originally developed for the context of teacher education rather than curriculum analysis. Not surprisingly, then, its reliability was in some instances lower than would be desired, both in this study and others (e.g., Jones). The level of cognitive demand is an important aspect of tasks and mathematics education researchers need a reliable way to measure it. The development of a framework, or the refinement of an existing one, would be a valuable contribution.

Researchers seem to have accepted without question that there are two or three types of proportional reasoning problems: ratio comparison, missing value, and qualitative problems. Ratio comparison and missing value problems are indeed common in middle-school textbooks. However, not only are qualitative problems virtually absent from textbooks, but there are additional problem types that have not been described in the research literature. This study has provided examples of these additional types and also connected various problem types to content areas. Further investigation of the problem types actually found in textbooks would be beneficial.

A few studies have indicated that certain solution strategies may seem more natural to students than others. For example, the *unit rate* strategy is often used by children who have not had instruction in proportionality (Lamon, 1993). Additional research is needed to discover how informal strategies can help students transition to proportions and cross multiplication. Some of the textbooks in this study used an equivalent-fractions method of solving proportions before introducing cross multiplication; whether the equivalent-fractions method is an effective transition between informal and more symbolic reasoning is unknown.

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# Appendix A:

# Pilot Study

# PROPORTIONALITY IN TWO SIXTH-GRADE MATHEMATICS TEXTBOOKS: A PILOT STUDY

In 1989, the NCTM published *Curriculum and Evaluation Standards for School Mathematics* which, among other things, described thirteen curriculum standards. One of these, called “mathematical connections,” stated that the “mathematics curriculum should include the investigation of mathematical connections so that students can see mathematics as an integrated whole” (NCTM, 2000, p. 84). In 2000, the NCTM published *Principles and Standards for School Mathematics* which proposed certain mathematical topics, including proportionality, be used to help students investigate connections. NCTM (2000) stated that curricular focus and integration are “evident in the proposed emphasis on proportionality as an integrative theme in the middle-grades mathematics program” (p. 212).

In order for proportionality to function effectively as an integrative theme, students must be able to reason proportionally. However, researchers agree that proportional reasoning is a difficult skill to acquire. For example, Karplus, Pulos, and Stage (1983) found that about 30% of the sixth- and eighth-grade students studied never used proportional reasoning and Lamon (2007) estimated that about 90% of adults do not reason proportionally. Researchers believe that part of the difficulty lies in the complexity inherent in this type of reasoning, but also that curriculum resources do not present the topic as well as they could (e.g., Cramer, Post, & Currier, 1993).

To determine how proportionality is treated in middle-school mathematics textbooks, the researcher designed a curriculum analysis framework. In this section, the researcher describes a pilot study that was undertaken to test the framework.

To test a framework that she developed to analyze textbooks' treatment of proportionality, the researcher used several variations of it to analyze the treatment of proportionality in two sixth-grade textbooks: *Middle School Math Course 1* (Larson, Boswell, Kanold, & Stiff, 2004) and *Math Thematics Book 1* (Billstein & Williamson, 2008). Although these books are both published by McDougal Littell, they represent different educational philosophies. *Middle School Math* was not developed with funding by the National Science Foundation (NSF) whereas *Math Thematics* is based on the standards and principles of the NCTM and was developed through funding by the NSF.

## Methods

In the following paragraphs, the researcher will describe how she selected lessons to be included in the pilot study and how she decided which tasks would be coded using the framework. Then, a description of the framework is provided.

### *Lesson Selection*

The researcher selected lessons from each textbook that seemed the most likely to involve proportional reasoning. These lessons were drawn from all content areas, including algebra, data analysis/probability, geometry/measurement, and rational numbers. Within the algebra standard, lessons on function rules, rate of change, slope, and solving proportions were selected for the study. Within the data analysis and probability standard, lessons on circle graphs and using probability to make predictions were selected. Within the geometry and measurement standards, lessons on area, measurement conversions, similar figures, and volume were selected. Within the number and operations standard, lessons on equivalent fractions and lessons on converting between decimals, fractions and percents were selected, but lessons on fraction computation were excluded.

Of the 93 lessons in the *Middle School Math* textbook, 17 (18%) were selected and analyzed. The majority of these lessons were in the rational number content area. Of the 43 sections in the *Math Thematics* textbook, 15 (35%) were selected and analyzed. The largest percentage of these lessons related to geometry and measurement.

*Task Selection*

The researcher analyzed every example and every exercise in the selected lessons. However, not all examples and exercises in these lessons involved proportionality. Only tasks that did involve proportionality were coded. A *task* was defined as a labeled example or an exercise to be completed in class or at home. Items presented as "projects" were not included. In the *Middle School Math* textbook, 485 tasks were coded and from the *Math Thematics* textbook, 827 tasks were coded. This information is summarized in Table A1.

Table A1

*Lessons and Tasks Included in the Pilot Study*

| | Lessons | | | Tasks | | |
|---|---|---|---|---|---|---|
| | Total | Number Included | Percent Included | Total* | Number Included | Percent Included |
| *Math Thematics* | 43 | 17 | 40 | 1389 | 827 | 60 |
| *Middle School Math* | 93 | 17 | 18 | 704 | 485 | 69 |

*The total number of tasks refers to the number of tasks within the selected lessons.

The percentage of lessons coded in the *Middle School Math* textbook was far less than the percentage of lessons coded in the *Math Thematics* textbook. This is due to the fact that the *Middle School Math* textbook includes four chapters (27 lessons) on computation with rational numbers. Because these lessons were excluded from the study, a smaller percentage of lessons was coded.

### *The Framework*

The framework was based on research literature that identified various problem types and solution strategies (e.g., Cramer et al., 1993). Because research has suggested that students have difficulty distinguishing between proportional and nonproportional situations, the framework notes whether tasks in textbooks point out the characteristics of proportional situations. Because the NCTM has stated that proportionality could be used to connect various mathematical topics, the framework looks for proportionality in several different content areas. Each example was coded along five dimensions: problem type, content area, solution strategy suggested, whether a visual representation was present, and whether the example pointed out the characteristics of proportional situations. Each exercise was coded along six dimensions: the five listed above and also the level of cognitive demand (Stein et al., 2000).

#### *Problem Types*

Each task related to proportionality was classified into one of the following problem types: *alternate form*, *function rule*, *missing value*, *ratio comparison*, *qualitative*, and *other*.

*Alternate form problem type.* In tasks of this problem type, students are asked to put a fraction, decimal, or percent into another form. Figure A1 shows an example of this type.

**Write two fractions that are equivalent to $\frac{1}{3}$.**

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ Multiply the numerator and denominator by 2.

$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$ Multiply the numerator and denominator by 3.

**Answer** The fractions $\frac{2}{6}$ and $\frac{3}{9}$ are equivalent to $\frac{1}{3}$ (Larson et al., 2004, p. 228).

*Figure A1*. Example of problem type *alternate form*

Although the problem type *alternate form* was not identified in the research literature, problems of this type involve proportionality and appear in middle-school textbooks. Therefore, the problem type *alternate form* is a necessary part of the framework. These tasks are similar to *missing value* tasks, but in *alternate form* tasks students must find two values, not one.

*Function rule problem type.* In tasks of this problem type, students are asked to find a rule that relates members of one set to members of a second set. For example, in the exercise in Figure A2, students are asked to find a rule that relates the "term number" to the "term." This is equivalent to finding the slope of the function.

**a.** Copy and complete the table.

| Term number | 1 | 2 | 3 | 4 | ? | ? | ? | ? |
|---|---|---|---|---|---|---|---|---|
| Term | 12 | 24 | 36 | 48 | ? | ? | ? | ? |

**b.** How are the term numbers and terms related?

**c.** Write an equation for the rule for the sequence. Use *t* for the term and *n* for the term number.

**d.** Use the equation to find the $30^{th}$ term in the sequence.

*Figure A2*. Exercise of the problem type *function rule*

Although this problem type was not identified in the research literature on proportional reasoning, problems of this type are related to proportionality and appear in middle-school textbooks. Therefore, the problem type *function rule* is a necessary part of the framework.

*Missing value problem type.* In a *missing value* task, students are given three out of four numbers that form a proportion and are asked to find the fourth. This was one of the problem types identified in the research literature as an important part of proportional reasoning.

**Solve the proportion** $\frac{30}{12} = \frac{x}{18}$.

**Solution**

| | |
|---|---|
| $\frac{30}{12} = \frac{x}{18}$ | Write the cross products. They are equal. |
| $540 = 12x$ | |
| $540 \div 12 = x$ | Write the related division equation. |
| $45 = x$ | Divide. |

**Answer** The solution is 45 (Larson et al., 2004, p. 385).

*Figure A3*. Example of the problem type *missing value*

*Ratio comparison problem type.* In a *ratio comparison* problem, students are given two ratios and are asked which is larger or if the ratios are equal. This was one of the problem types identified in the research literature as an important part of proportional reasoning (e.g., Lamon, 2007). Figure A4 shows an example of this type.

**Football** Allen completes $\frac{3}{4}$ of his pass attempts. Mike completes 7 out of every 10 pass attempts. Who has the better record?

**Solution**

Write each ratio as a decimal. Then compare the decimals.

Allen: $\frac{3}{4} = 0.75$ Mike: 7 out of 10 = $\frac{7}{10} = 0.7$

**Answer** Because 0.75 > 0.7, Allen has the better record. (Larson et al., 2004, p. 375)

*Figure A4*. Example of the problem type *ratio comparison*

*Qualitative problem type*. In the research literature, questions like the following were identified as *qualitative*: "If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, (d) not enough information to tell" (Cramer et al., 1993, p. 166). As explained in a later section, no tasks resembling this one was found in either of the two sixth-grade textbooks examined in this pilot study.

*Solution Strategies*

In middle-school mathematics textbooks, most examples are accompanied by a suggested solution strategy. Some exercises also suggest a solution method to students, although most do not. Each task related to proportionality was classified into one of the following solution strategies: *building up, decimals, proportion, unit rate, other,* and *none*. The *proportion* solution strategy was sub-divided into two categories: *proportion: cross multiplication* and *proportion: multiply or divide*.

*Building up strategy*. Figure A5 shows an example of the building up strategy. The example in the textbook featured a picture of a canoe with measurements indicating that the picture was three inches long. The example read “Find the actual lengths that correspond to 1 inch, 2 inches, and 3 inches on the scale drawing. How long is the actual canoe?

**Solution**
Make a table. The scale on the drawing is 1 in : 5 ft. Each inch on the drawing represents 5 feet on the canoe.

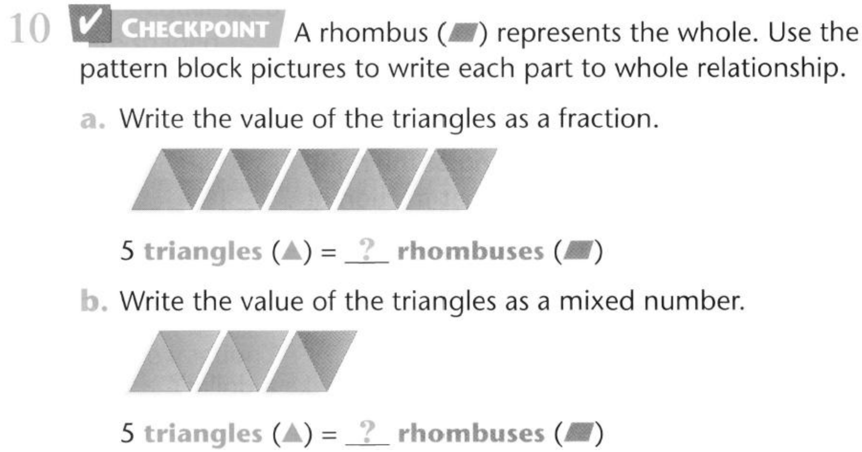
| Scale drawing length (inches) | Length × 5 | Actual canoe length (feet) |
|---|---|---|
| 1 | 1 × 5 | 5 |
| 2 | 2 × 5 | 10 |
| 3 | 3 × 5 | 15 |

**Answer** The actual canoe is 15 feet long. (Larson et al., 2004, p. 68)

*Figure A5*. Example of the solution strategy *building up*

*Decimals.* Tasks in some textbooks instruct students to compare fractions by converting them to decimals. The example in Figure A4 was coded with the solution strategy *decimals*.

*Manipulatives*. Some tasks suggest that students use drawings or manipulatives to compare fractions and mixed numbers. These tasks were coded with the *manipulatives* solution strategy. The exercise in Figure A6 was coded with the solution strategy *manipulatives*.


10 CHECKPOINT A rhombus (▰) represents the whole. Use the pattern block pictures to write each part to whole relationship.

a. Write the value of the triangles as a fraction.

5 triangles (▲) = ? rhombuses (▰)

b. Write the value of the triangles as a mixed number.

5 triangles (▲) = ? rhombuses (▰)

(Billstein & Williamson, 2008, p. 42).

*Figure A6*. Exercise of the solution strategy *manipulatives*

*Proportion.* The example in Figure A7 demonstrates the use of the proportion strategy.

Find the missing term in the proportion $\frac{10}{15} = \frac{x}{12}$.

$15 \cdot x = 10 \cdot 12$

$15 \cdot x = 120$

$x = 120 \div 15$

$x = 8$ (Billstein & Williamson, 2004, p. 401)

*Figure A7*. Exercise of the solution strategy *proportion.*

*Unit rate*. One way to solve missing value problems is through the use of a unit rate. Using the method, students convert the given rate to one that has a denominator of one. For example, to solve the problem in Figure A8, students would divide 3,000 by 60 and 180 by 15.

On average, a Ruby-throated Hummingbird beats its wings about 3000 times in 60 seconds. A Giant Hummingbird beats its wings an average of about 180 times in 15 seconds. Which bird beats its wings faster?

(1 Find the unit rate for the Ruby-throated Hummingbird.
(2 Find the unit rate for the Giant Hummingbird.
(3 Compare the unit rates (Larson et al., 2004, p. 381).

*Figure A8*. Exercise of the solution strategy *unit rate*

A task was coded as *unit rate* only if it suggested that students use a unit rate strategy to solve a problem. Tasks that ask students to state the unit rate or to convert a rate into a unit rate were not coded with the *unit rate* solution strategy because in these situations, the unit rate is not used as a solution strategy. For example, in a sixth-grade lesson titled "Rates," an example instructed students to "Write the Space Station's average rate of $\frac{15\ mi}{3\ sec}$ as a unit rate" (Larson et al., 2004, p. 379). Although "unit rate" is mentioned, this exercise was coded as *no solution strategy* because the unit rate in this example is not used as a solution strategy.

*Other*. Tasks in which a solution strategy is suggested that does not fit neatly into one of the above categories were coded as *other*. For example, consider comparison of fractions, such as determining whether $\frac{5}{6}$ or $\frac{7}{9}$ is larger. This could be done by converting both fractions to decimals, which would be coded with the solution strategy *decimals*. It could also be accomplished by using a common denominator, i.e., converting both fractions to eighteenths. In the pilot study, the researcher found few tasks in which students were asked to compare fractions by using common denominators. Therefore, the framework does not include a code for it.

Tasks in which multiple solution strategies were suggested were also coded as *other*. For example, an exercise in a sixth-grade textbook reads, “Use mental math, paper and pencil, or a calculator to compare the fractions” (Billstein & Williamson, 2008, p. 292). Although in some curriculum analyses, tasks that fit into several categories receive multiple codes, the percent of tasks in which multiple solution strategies were offered was far less than 1%. Thus, in the pilot study, it was not problematic to code these tasks as *other*. When the framework is applied to other series, it may be necessary to assign multiple codes to tasks in which multiple solution strategies are suggested depending on the number of these tasks that is encountered.

## Results

In this section, results related to the following three topics are reported: a) the content area of lessons related to proportionality, b) proportionality in the *Middle School Math* textbook, and c) proportionality in the *Math Thematics* textbook. Comparisons between the two textbooks are made in the Discussion section.

### *Content Area of Lessons Related to Proportionality*

As indicated in Figure A9, in the *Middle School Math* textbook, more lessons related to proportionality were in the Number and Operations standard than any other. In the *Math Thematics* textbook, more lessons related to proportionality were in the Geometry/Measurement standard than any other. Specifically, in the *Middle School Math* textbook, the researcher found 17 lessons related to proportionality: seven (41%) in the Number and Operations standard, four (24%) in the algebra standard, four (24%) in the Geometry/Measurement standard, and two (12%) in the data analysis standard. In the *Math Thematics* textbook, the researcher found 17 lessons related to proportionality: five

(29%) in the Number and Operations standard, three (18%) in the Algebra standard, eight (47%) in the Geometry/Measurement standard, and one (6%) in the Data Analysis/Probability standard.

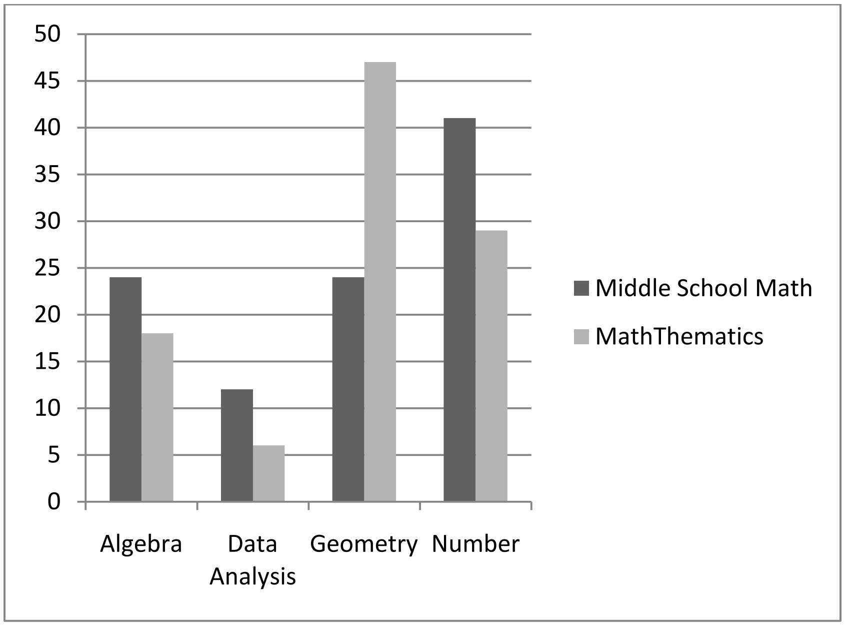



*Figure A9*. Percent of lessons related to proportionality in each content area

*Proportionality in the Middle School Math Textbook*

In this section, the treatment of proportionality in the *Middle School Math* textbook is described. First, the examples related to proportionality from this book are discussed. In a later section, the exercises related to proportionality are described.

*Examples in the Middle School Math Textbook*

In the lessons selected for the study from the *Middle School Math* textbook, the researcher found 54 examples, 41 (76%) of which were related to proportionality. As explained below, very few of the examples pointed out the characteristics of proportionality. *Rational number* was the most common content area, followed by *geometry/measurement*. *Alternate form* and *missing value* were the most common problem types. The solution strategy most-commonly suggested was the *proportion* strategy. Proportions were more often solved through multiplication or division rather than cross products.

*Characteristics of proportionality in Middle School Math examples.* Of the 41 examples related to proportionality in the Middle School Math textbook, three (7%) contained material that could help students realize that proportional reasoning is appropriate in some situations but not others. Two were in a lesson titled “Proportions and Scale Drawings” and one was in a lesson titled “Area of a Circle.” All three examples were designed to point out that area is *not* related proportionally to the lengths of the sides of a rectangle or to the radius of a circle. The two examples from the lesson “Proportions and Scale Drawings” asked students to consider a drawing that was to be used as a scale model for a mural. One example asked students to write a ratio comparing the perimeter of the drawing to the perimeter of the actual mural. The answer provided read “The ratio of the perimeters is the same as the scale” (Larson et al., 2004, p. 389). The other example asked students to write a ratio comparing the area of the drawing to the area of the actual mural. The answer provided read “The ratio of the areas…is the square of the scale” (Larson et al., p. 389). These examples may help students realize that the relationship between the side lengths of a rectangle and its perimeter is proportional whereas the relationship between the side lengths of a rectangle and its area is not proportional.

*Content area of Middle School Math examples.* Of the 41 examples related to proportionality in the *Middle School Math* textbook, 24 (59%) related to rational numbers, eight (20%) to geometry/measurement, six (15%) to algebra, and three (7%) to data analysis/probability

*Problem type of Middle School Math examples.* Of the 41 examples related to proportionality in the *Middle School Math* textbook, 16 (39%) were of the type *alternate form*, 11 (27%) of the type *missing value*, seven (17%) of the type *ratio comparison*, three (7%) of the type *function rule*, three (7%) of the type *other*, and one (2%) of the *qualitative* problem type. All 41 examples clearly fit into one of the six problem-type categories.

*Solution strategy of Middle School Math examples.* Of the 41 examples related to proportionality in the *Middle School Math* textbook, 18 (44%) suggested the *proportion* strategy, three (7%) suggested the *decimals* strategy, two (5%) suggested the *building up* strategy, and 18 (44%) were coded as *other*. None of the examples in the *Middle School Math* textbook suggested solving a problem through the use of manipulatives or the unit rate strategy. Of the 18 examples that suggested the *proportion* strategy, 14 (78%) suggested multiplying or dividing both the numerator and denominator by the same number. Of the 18 examples that suggested the *proportion* strategy, 4 (22%) suggested using cross products.

*Visual representations of Middle School Math examples.* An example was coded as having a visual representation if it included a chart, diagram, graph, or picture that could help a student understand the mathematical concepts. Pictures that seemed purely decorative were not coded. Of the 41 examples related to proportionality in the *Middle School Math* textbook, 12 (29%) were accompanied by a visual representation.

*Exercises in the Middle School Math Textbook*

In the selected lessons of the *Middle School Math* textbook, the researcher found 650 exercises, 447 (69%) of which were related to proportionality. Exercises resembled the examples in several ways: few pointed out the characteristics of proportionality, *rational number* was the most common content area, and few were of the *qualitative* problem type.

*Characteristics of proportionality in Middle School Math exercises.* Of the 447 exercises related to proportionality in the *Middle School Math* textbook, 10 (2%) contained material that could help students realize that proportional reasoning is appropriate in some situations but not others. In all of these exercises, the differences between proportional and non-proportional situations were only hinted at rather than stated explicitly. These exercises were spread throughout the book and related to a variety of content areas.

*Content area of Middle School Math exercises.* Of the 447 exercises related to proportionality, 344 (77%) were in the *rational number* content area, 41 (9%) were in *geometry/measurement*, 36 (8%) were in *algebra*, and 26 (6%) were in *data analysis/probability*. Thus, in the *Middle School Math* textbook, the majority of both the examples and the exercises related to proportionality were in the *rational number* content area. This emphasis on proportionality in the *rational number* content area was even more pronounced in the exercises than it is in the examples.

*Level of cognitive demand of Middle School Math exercises.* Of the 447 exercises related to proportionality in the *Middle School Math* textbook, 10 (2%) were at the level *Memorization*, 362 (81%) were at the level *Procedures Without Connections*, 14 (3%) were at the level *Procedures With Connections*, and 11 (2%) were at the level *Doing Mathematics*. It is not possible to compare exercises to examples because the level of cognitive demand of examples was not coded.

*Problem type of Middle School Math exercises.* Of the 447 exercises related to proportionality in the *Middle School Math* textbook, 165 (37%) were of the type *missing value*, 138 (31%) were of the type *alternate form*, 84 (19%) were of the type *ratio comparison*, 32 (7%) were of the *other* problem type, 18 (4%) were of the *function rule* type, and 10 (2%) were of the *qualitative* problem type. Figure A10 shows a comparison of the problem type of examples and exercises. The figure indicates that, in the *Middle School Math* textbook, problem types were distributed in approximately the same way in examples and exercises. Problem types emphasized in examples were also emphasized in exercises.

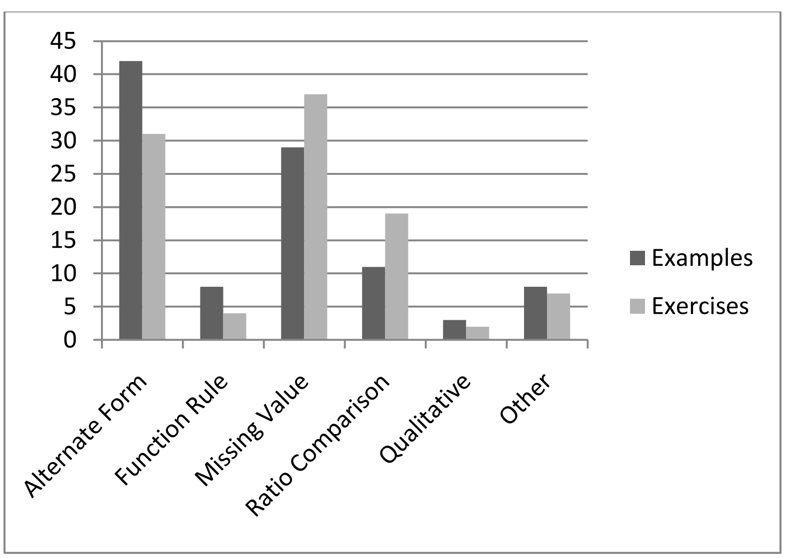



*Figure A10.* Problem type of tasks in the *Middle School Math* textbook

In the research literature, questions like the following were identified as *qualitative*: "If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, (d) not enough information to tell" (Cramer et al., 1993, p. 166). However, the researcher found no exercises similar to this in the sixth-grade *Middle School Math* textbook. Nine exercises were coded as *qualitative*. All of these involved examining a circle or bar graph and estimating the fraction of the population in a given category.

*Solution strategy of Middle School Math exercises.* Of the 447 exercises related to proportionality in the *Middle School Math* textbook, 401 (90%) did not suggest a solution strategy, 40 (9%) suggested the *proportion* strategy, five (1%) were coded as *other*, and one (0.2%) suggested the *unit rate* strategy. None of the exercises in the *Middle School Math* textbook suggested the *building up*, *decimals*, or *manipulatives* strategies.

### *Proportionality in the Math Thematics Textbook*

In this section, the treatment of proportionality in the *Middle School Math* textbook is described. First, the examples related to proportionality from this book are discussed. In a later section, the exercises related to proportionality are described.

#### *Examples in the Math Thematics Textbook*

In the 17 lessons selected for the study from the *Math Thematics* textbook, the researcher found 62 examples, 36 (58%) of which were related to proportionality. Examples in the *Math Thematics* textbook were similar to examples in the *Middle School Math* textbook in that few pointed out the characteristics of proportionality, most were related to rational numbers, and few were of the *qualitative* problem type.

*Characteristics of proportionality in the Math Thematics examples.* Of the 36 examples related to proportionality in the *Math Thematics* textbook, only one contained material that could help students realize that proportional reasoning is appropriate in some situations but not others. This example stated “The snack mix has 10 cashews for every 4 pretzels” (Billstein & Williamson, 2008, p. 368). The example featured a diagram that showed *why* the ratio 10 : 4 is equivalent to the ratio 5 : 2. This example was coded as pointing out whether proportional reasoning is appropriate because it emphasized that every five cashews are paired with two pretzels, thus emphasizing the constant ratio. Because a constant ratio (or rate of change) is one of the characteristics of proportional situations, this example was considered to point out whether proportional reasoning was appropriate.

*Content area of the Math Thematics examples.* Of the 36 examples related to proportionality in the *Math Thematics* textbook, 23 (64%) were in the *rational number* content area, seven (19%) were in *algebra*, five (14%) were in *geometry/measurement*, and one (3%) was in *data analysis/probability*. Thus, in both the *Middle School Math* and *Math Thematics* textbooks, more than 50% of the examples related to proportionality were in the *rational number* content area.

*Problem type of the Math Thematics examples.* Of the 36 examples related to proportionality in the *Math Thematics* textbook, 12 (33%) were of the type *alternate form*, 11 (31%) were of the type *ratio comparison*, eight (22%) were of the type *missing value*, three (8%) were of the type *function rule*, one (3%) was of the *qualitative* problem type, and one (3%) was of the type *other*.

*Solution strategy of the Math Thematics examples.* Of the 36 examples related to proportionality in the *Math Thematics* textbook, 18 (50%) were coded as *other*, six (17%) did not suggest a strategy, five (14%) suggested *manipulatives*, five (14%) suggested the *proportion* strategy, one (3%) suggested the *decimals* strategy, and one (3%) suggested the *cross products* strategy. None of the examples in the *Math Thematics* textbook suggested the *building up* or *unit rate* strategies. A large number of examples were coded with the solution strategy *other* for a variety of reasons. One reason is that the examples in the *Math Thematics* textbook tended to be longer and contain more verbal explanation than examples in the *Middle School Math* textbook. Examples in the *Math Thematics* textbook typically contained several mathematical ideas and were thus harder to classify by solution strategy. In six of the examples, no solution strategy was suggested.

*Visual representation in the Math Thematics examples.* Of the 36 examples related to proportionality in the *Math Thematics* textbook, 12 (33%) were accompanied by a visual representation. This is slightly greater than the percent of examples in the *Middle School Math* textbook that were accompanied by a visual representation. In the *Math Thematics* textbook, many of the examples that included a visual representation were examples that encouraged the use of manipulatives; a picture of the manipulatives was often included to illustrate their use.

*Exercises in the Math Thematics Textbook*

The *Math Thematics* textbooks separate exercises into three types of exercise sets. Some exercises are found in the "Exploration" sections, some are found in the "Practice and Application" sets, and others are found on the "Extra Skills" pages. These three types of exercises were coded separately. In the selected lessons, there were 149 exercises related to proportionality in the Explorations, 323 in the Practice and Application sets, and 220 in the Extra Skills pages, for a total of 692 exercises related to proportionality. Because these sections contained a total of 1,327 exercises, 52% of the exercises in the lessons selected from the *Math Thematics* textbook were related to proportionality.

*Characteristics of Proportionality the Math Thematics exercises.* Of the 692 exercises related to proportionality in the *Math Thematics* textbook, 32 (5%) contained material that could help students realize that proportional reasoning is appropriate in some situations but not others. These exercises included 11 of the 149 exercises in the Exploration sections, 13 of the 323 exercises in the Practice and Application sets, and 9 of the 220 exercises in the Extra Skills pages. Thus, the percentage of exercises that emphasized the characteristics of proportional situations was higher in the Exploration sections than in the other two types of sections. Some of the exercises that pointed out the characteristics of proportional situations involved collecting data, making a scatterplot, drawing a line of best fit, and using this line to make predictions. These were coded as pointing out the characteristics of proportional situations because the line of best fit had a constant slope, which could help students understand that, in proportional situations, the ratio of the variables is constant.

*Content area of the Math Thematics exercises.* Of the 692 exercises related to proportionality in the *Math Thematics* textbook, 430 (62%) were in the *rational number* content area, 183 (26%) were in *geometry/measurement*, 64 (9%) were in *algebra*, and 15 (2%) were in *data analysis*/*probability*. Thus, in both the examples and exercises in the *Math Thematics* textbook, *rational number* was the most common content area.

Examples were more likely than exercises to involve algebra and exercises were more likely to involve geometry and measurement.

*Problem type of the Math Thematics exercises.* Of the 692 exercises related to proportionality in the *Math Thematics* textbook, 260 (38%) were of the type *missing value*, 195 (28%) were of the type *alternate form*, 179 (26%) were of the type *ratio comparison*, 19 (3%) were of the *function rule* type, three (0.4%) were of the *qualitative* problem type, and 36 (4%) were of the *other* problem type. As in the *Middle School Math* textbook, problem types were distributed in approximately the same way in examples and exercises.

In the research literature, questions like the following were identified as *qualitative*: "If Devan ran fewer laps in more time than she did yesterday, would her running speed be (a) faster, (b) slower, (c) exactly the same, (d) not enough information to tell" (Cramer et al., 1993, p. 166). As in the *Middle School Math* textbook, no tasks in the *Math Thematics* book resembled the qualitative tasks described in the literature. Four tasks in the *Math Thematics* book were coded as *qualitative*. Three of these involved estimating the fraction of a circle graph represented by a given sector and one required students to examine an arrangement of rectangles and triangles and estimate the fraction of the figure represented by each shape. Although fractions are quantitative, students needed to use visual estimation skills rather than computation to arrive at these answers; thus, the tasks were coded as *qualitative*.

In the *Math Thematics* textbook, the percentages of examples and exercises of each problem type were similar. Approximately one-third of the tasks were of the type *alternate form*, approximately one-third were of the type *missing value*, and slightly less than one third were of the type *ratio comparison*. In both the examples and exercises, *function rule*, *qualitative*, and *other* tasks were uncommon.

*Level of cognitive demand of Math Thematics exercises.* Of the 692 exercises related to proportionality, none were at the *Memorization* level, 516 (75%) were at the level *Procedures without Connections*, 141 (20%) were at the level *Procedures with Connections*, and 35 (5%) were at the level *Doing Mathematics*. Exercises in the Explorations sections were more likely to be at the level *Procedures with Connections* than exercises in the other two types of exercise sets. Problems in the Extra Skills sets were almost exclusively at the level *Procedures without Connections*.

*Solution strategy of the Math Thematics exercises.* Of the 692 exercises related to proportionality in the *Math Thematics* textbook, 563 (81%) did not suggest a solution strategy, 58 (8%) were coded as *other*, 28 (4%) suggested using manipulatives, 24 (3%) suggested the *proportion* strategy, 14 (2%) suggested using *decimals*, three (0.4%) suggested the *building up* strategy, and two (0.3%) suggested the *unit rate* strategy.

## Discussion

This discussion section is divided into six parts: a) the prevalence of proportionality, b) the extent to which textbooks discuss the appropriateness of proportional reasoning, c) content areas, d) problem types, e) solution strategies, and f) level of cognitive demand.

### *Prevalence of Proportionality*

The extent of each textbook related to proportionality can be analyzed in two different ways: (a) the percentage of *lessons* related to proportionality and (b) the percentage of *tasks* related to proportionality.

*Percentage of Lessons Related to Proportionality*

The percentage of lessons related to proportionality may indicate how integrated proportionality is into the curriculum. A high percentage of lessons related to proportionality may indicate that it is covered throughout the school year whereas a small percentage of lessons related to proportionality indicates that it is concentrated in a few lessons or chapters.

Eighteen percent of the lessons in the *Middle School Math* textbook and 35% of the lessons in the *Math Thematics* textbook were related to proportionality. Thus, the percentage of lessons in the *Math Thematics* textbook related to proportionality was almost twice as high as the percentage in the *Middle School Math* textbook. This indicates that proportionality is spread throughout the *Math Thematics* textbook and is more concentrated in the *Middle School Math* textbook. NCTM (2000) suggested that proportionality be used "as an integrative theme in the middle-grades mathematics program" (p. 212). In order for proportionality to be an integrative theme, it would need to appear in a significant percentage of lessons. The results of this pilot study may suggest that proportionality is used to illustrate connections between mathematical concepts in the *Math Thematics* textbook more than in the *Middle School Math* textbook.

*Percentage of Tasks Related to Proportionality*

As indicated in Figure A11, 53% of the tasks in selected lessons in the *Math Thematics* textbook and 69% of the tasks within selected lessons in the *Middle School Math* textbook were related to proportionality. Thus, although the *Math Thematics* textbook had a much higher percentage of lessons related to proportionality than did the *Middle School Math* textbook, the percentage of tasks related to proportionality within the selected lessons was lower in the *Math Thematics* textbook. This is consistent with the finding discussed above, that proportionality is more spread out in the *Math Thematics* textbook and more concentrated in the *Middle School Math* textbook.

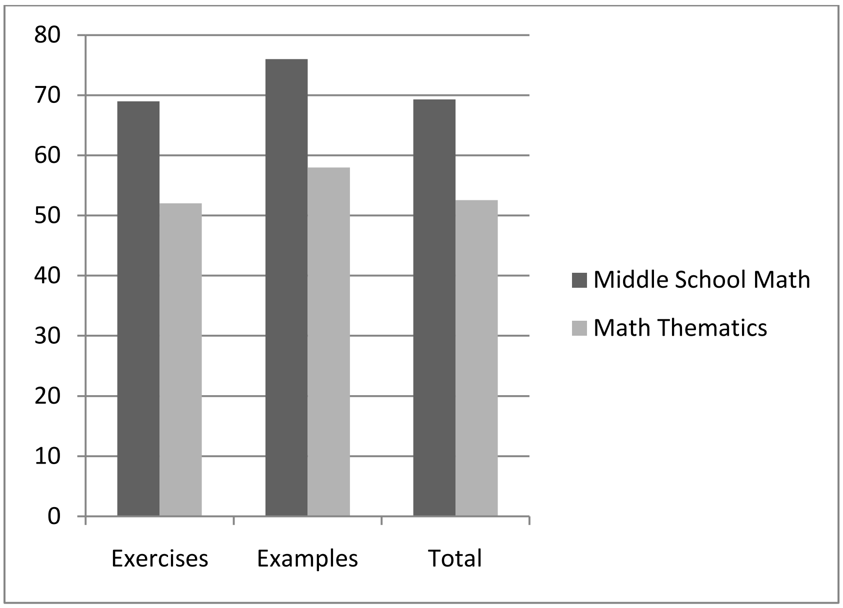



*Figure A11*. Percent of tasks in selected lessons related to proportionality

Calculating the percentage of tasks within an entire book that relate to proportionality would require counting the number of tasks in an entire book, which was not done in this study. However, this percentage can be estimated by multiplying the percent of tasks coded in the selected lessons by the percent of lessons selected. For example, in the *Middle School Math* textbook, 18% of the lessons and 69% of the tasks within them were coded. One can multiply these probabilities to arrive at an estimate that 12% of the tasks in the *Middle School Math* are related to proportionality. Using this method, an estimated 19% of the tasks in the *Math Thematics* textbook are related to proportionality.

*Characteristics of Proportional Reasoning*

Research findings have indicated that both students and teachers have difficulty recognizing situations in which proportional reasoning is or is not appropriate (Cramer et al., 1993; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). Given that many students and teachers attempt to apply proportional reasoning to situations in which it is not appropriate, it might be beneficial for tasks in middle-school mathematics textbooks to point out the characteristics of proportional and non-proportional situations. The researcher has identified three problems with the way textbooks address this issue. First, too few tasks point out the characteristics of proportional situations. Second, in the *Middle School Math* textbook, there was little diversity in the tasks that did point out the characteristics of proportional situations. Third, tasks that did point out the characteristics of proportional situations did so in only subtle ways; there was little explicit discussion of these characteristics.

*Too Few Tasks Point Out the Characteristics of Proportional Situations*

As illustrated in Table A2, only small percentages of the tasks related to proportional reasoning seem to be designed to help students recognize the characteristics of proportionality, which may help them recognize situations in which proportional reasoning is or is not appropriate.

Table A2

Percentages of Tasks Related to the Appropriateness of Proportional Reasoning

| | Examples | Exercises |
|---|---|---|
| *Middle School Math* | 7 | 2 |
| *Math Thematics* | 3 | 5 |

In the two textbooks combined, only 42 of the 1,139 exercises (3.7%) related to proportionality pointed out these characteristics. Thus, there are likely too few tasks that point out the differences between proportional and non-proportional situations.

#### *There is Little Diversity in the Tasks*

In the *Middle School Math* textbook there was little diversity in the examples and exercises that did point out the characteristics of proportional situations. For example, in the *Middle School Math* textbook, all three examples related to this topic involved area of rectangles or circles. There was more diversity in the tasks in the *Math Thematics* textbook. At least one example in the *Math Thematics* textbook used a diagram to illustrate the meaning of a constant ratio and some exercises involved using a line of best fit to point out that the relationship between the variables was constant.

#### *There are Few Explicit Discussions of the Characteristics of Proportional Situations*

Tasks were coded as pointing out the appropriateness or inappropriateness of proportional reasoning even if the issue was touched on very subtly. Students and teachers would have to be very perceptive to recognize some of these tasks as pointing out the characteristics of proportional situations. Very rarely was an explicit discussion of these characteristics seen in either textbook.

### *Content Areas*

In both textbooks, most of the examples and most of the exercises related to proportionality were also related to rational numbers. This likely reflects both the researcher's broad definition of proportionality and the emphasis on rational numbers in the middle-school curriculum.

*Content Area of Examples*

As illustrated in Table A3, the content area of examples in the *Math Thematics* textbook was similar to the content area of examples in the *Middle School Math* textbook.

Table A3

Percent of Examples Related to Proportionality in Each Content Standard

| | Content Area | | | |
|---|---|---|---|---|
| | Algebra | Data Analysis | Geometry | Rational Numbers |
| *Middle School Math* | 15 | 7 | 20 | 59 |
| *Math Thematics* | 19 | 3 | 14 | 64 |

*Content Area of Exercises*

As illustrated by Table A4, most of the proportional exercises in the *Middle School Math* textbook were related to rational numbers. The *Math Thematics* textbook had many proportional exercises that were related to rational numbers but also featured a considerable number of proportional exercises that were related to the *geometry/ measurement* content area.

Table A4

Percent of Exercises Related to Proportionality in Each Content Standard

| | Content Area | | | |
|---|---|---|---|---|
| | Algebra | Data Analysis | Geometry | Rational Numbers |
| *Middle School Math* | 8 | 6 | 9 | 77 |
| *Math Thematics* | 9 | 2 | 26 | 62 |

The large percentage of examples and exercises that were related to rational numbers reflects the researcher's broad definition of "proportional reasoning." Many scholars would not consider conversion between decimals, fractions, and percents to involve proportional reasoning. Because the researcher did include this and similar concepts, the percentage of proportional exercises related to rational numbers may be overstated.

## *Problem Types*

### *Problem Type of Examples*

The problem types of the examples in the two textbooks were somewhat similar. The *Math Thematics* textbook contained a smaller percentage of examples of the type *alternate form* and a larger percentage of examples of the type *ratio comparison*, but the percentage of examples of the other problem types did not vary greatly between the two textbooks.

### *Problem Type of Exercises*

The problem types of the exercises in the two textbooks were also similar. The *Math Thematics* textbook contained a larger percentage of exercises of the type *ratio comparison*, but the percentage of examples of the other problem types did not vary greatly between the two textbooks.

### *Alternate Form Problem Type*

In the *Middle School Math* textbook, almost half of the examples and almost one-third of the exercises were of the *alternate form* problem type. This type of example involves converting between decimals, fractions, or percents, or finding equivalent fractions. Tasks of the type *alternate form* are useful in helping students acquire skills related to rational numbers, but tasks of this type relate almost exclusively to rational numbers and thus do not point out the usefulness of proportional reasoning in algebra, data analysis, geometry, or measurement. Furthermore, this problem type rarely points out why proportional reasoning is appropriate in some situations and not others and rarely includes a visual aid. Thus, although this type of example is important, having almost half of the examples related to proportionality of the type *alternate form* may over-emphasize this problem type.

### *Qualitative Problem Type*

Although *qualitative* was one of the problem types identified in the research literature as an important part of proportional reasoning (e.g., Lamon, 2007), the middle-school textbooks examined contained few examples or exercises of this type. Although no guidelines have been set for the percentage of proportional-reasoning examples that should be of each problem type, given that qualitative problems are one of the major components of proportional reasoning, a textbook in which less than 3% of the proportional-reasoning examples are of the *qualitative* type may not provide students sufficient opportunity to view examples of this type.

## *Solution Strategies*

The results of the pilot study led to three conclusions. One, in some sixth-grade textbooks, the *building up* and *unit rate* strategies are rarely suggested. Two, the solution strategies suggested in examples varies between series. Three, there exist solution strategies that are suggested by some textbooks that have not been described in the research literature.

*The Building Up and Unit Rate Strategies*

Two solution strategies recognized in the literature as fostering conceptual understanding of proportionality are the *building up* strategy (Heinz & Sterba-Boatwright, 2008; Parker, 1999) and the *unit rate* strategy (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998). These strategies seem natural to students, who often use them before they have received instruction in proportionality. Thus, one might expect that they would appear in sixth-grade textbooks and that more formal, algebraic methods, such as proportions and cross-products, might appear in seventh- and eighth-grade textbooks. Based on the two textbooks studied, this is not the case. The *building up* and *unit rate* strategies were virtually absent from the textbooks studied. By not including these solution strategies in sixth-grade textbooks, authors may be missing an opportunity to build students' conceptual understanding of proportionality before transitioning students into formal, algebraic approaches to solving proportional problems.

*The Solution Strategies Suggested Varies Between Series*

The *Math Thematics* textbook contained examples that suggested the use of manipulatives and the *Middle School Math* textbook did not.  In the *Math Thematics* textbook, the examples that suggested the use of manipulatives were generally of problem type *alternate form*. In these examples, students were shown how to use manipulatives to find equivalent fractions or to convert improper fraction to a mixed number. The manipulatives used were generally pattern blocks.

The *Middle School Math* textbook contained a much higher percentage of examples in which a proportion strategy was suggested. When proportions were used in the first half of the *Middle School Math* book, they were solved by multiplying or dividing the numerator and denominator by the same number. Once a section on cross-products had been covered, they were typically used to solve proportions in the examples.

*Solution Strategies That Have Not Been Described*

Several solution strategies are well known, such as the cross-multiply-and-divide method of solving proportions. Other solution strategies have been described in the research literature. However, in this study, a large percentage of the solution strategies were classified as *other* because they have not been recognized as solution strategies. To some extent, this reflects the inclusion in the study of problem types that are not recognized in the research literature. For example, many of the tasks coded with the solution strategy *other* were coded with the problem type *alternate form* or *function rule*, which are not recognized in the research literature. However, some tasks of well-known problem types were also solved by solution strategies that have not been discussed in the literature.

## *Levels of Cognitive Demand*

Textbooks based on the principles of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 2000) are generally believed to have higher levels of cognitive demand than traditional textbooks. For example, Senk and Thompson (2003) stated that Standards-based materials have "far fewer exercises requiring only arithmetic or algebraic computation" (p. 15). However, the level of cognitive demand in the two textbooks studied was similar and most of the exercises in both the *Middle School Math* and *Math Thematics* textbook were at the level *Procedures without Connections*. Given the need for students to practice skills, perhaps this is to be expected.

As indicated in Figure A12, in both the *Middle School Math* and the *Math Thematics* series, most exercises were at the level *Procedures without Connections*. The percentage of exercises at this level in the *Middle School Math* textbook was consistent with the findings of Jones (2004) who found that 75% of the exercises related to probability in a *Middle School Math* textbook are at this level. However, the percentage of exercises at this level in the *Math Thematics* textbook was quite different from Jones's estimate; he reported that 40% of the tasks in a *Math Thematics* textbook were at this level whereas the current study places this percent at 75%. The *Math Thematics* textbooks may be unusual among NSF-funded series in that *Math Thematics* books contain exercise sets labeled "Extra Skills Practice" that feature exercises almost exclusively at the level *procedures without connections*. Other *Math Thematics* textbooks that do not contain these sets of extra practice problems may have higher cognitive demand.

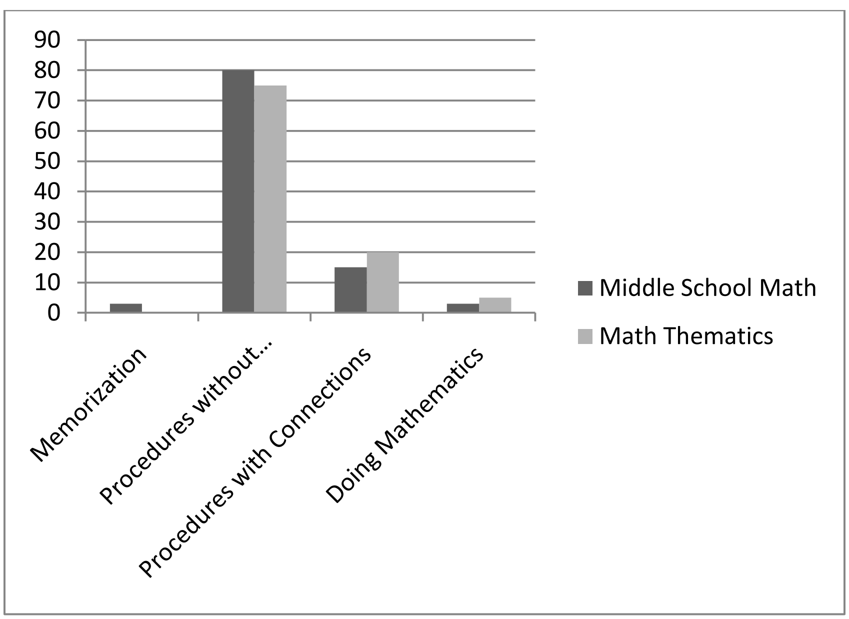



*Figure A12*. Percent of exercises at each level of cognitive demand

## Conclusion

This pilot study demonstrated that an analysis of middle-school textbooks can reveal important information about the textbooks' treatment of proportionality. However, because only two textbooks were used in this pilot study and because only sixth-grade textbooks were analyzed, a more complete study should be conducted to determine whether the results hold true in a larger sample size and for other grade levels. Because one of the problem types and some of the solution strategies discussed in the literature were virtually absent from the two textbooks analyzed, the issues of problem types and solution strategies should be further addressed. Finally, because the findings regarding the level of cognitive demand of tasks in the *Math Thematics* textbook differed dramatically from that reported by Jones (2004), the level of cognitive demand of tasks should be further investigated.

# APPENDIX B:

# LESSONS INCLUDED IN THE STUDY

Table B1

*Grade 6 CMP Investigations Included in the Study*

| Module | Investigation |
|---|---|
| Bits and Pieces I | Fundraising Fractions |
| Bits and Pieces I | Sharing and Comparing with Fractions |
| Bits and Pieces I | Moving Between Fractions and Decimals |
| Bits and Pieces I | Working with Percents |
| Bits and Pieces II | Estimating with Fractions |
| Bits and Pieces III | Decimals – More or Less! |
| How Likely Is It? | A First Look at Chance |
| How Likely Is It? | Experimental and Theoretical Probability |
| How Likely Is It? | Making Decisions with Probability |

Table B2

*Grade 7 CMP Investigations Included in the Study*

| Module | Investigation |
|---|---|
| Variables and Patterns | Rules and Equations |
| Stretching and Shrinking | Enlarging and Reducing Shapes |
| Stretching and Shrinking | Similar Figures |
| Stretching and Shrinking | Similar Polygons |
| Stretching and Shrinking | Similarity and Ratios |
| Stretching and Shrinking | Using Similar Triangles and Ratios |
| Comparing and Scaling | Making Comparisons |
| Comparing and Scaling | Comparing Ratios, Percents, and Fractions |
| Comparing and Scaling | Comparing and Scaling Rates |
| Comparing and Scaling | Making Sense of Proportions |
| Moving Straight Ahead | Walking Rates |
| Filling and Wrapping | Prisms and Cylinders |
| Filling and Wrapping | Scaling Boxes |
| What Do You Expect? | Analyzing Situations Using an Area Model |
| What Do You Expect? | Expected Value |

Table B3

*Grade 8 CMP Investigations Included in the Study*

| Module | Investigation |
| --- | --- |
| Thinking with Mathematical Models | Linear Models and Equations |
| Thinking with Mathematical Models | Inverse Variation |
| Say It With Symbols | Looking Back at Functions |
| Samples and Populations | Choosing a Sample from a Population |

Table B4

*Grade 6 Math Connects Lessons Included in the Study*

| Chapter | Lesson |
|---|---|
| Algebra: Number Patterns and Functions | Function Machines |
| Algebra: Number Patterns and Functions | Algebra: Functions |
| Algebra: Number Patterns and Functions | Algebra Lab: Writing Formulas |
| Operations with Decimals | Comparing and Ordering Decimals |
| Operations with Decimals | Rounding Decimals |
| Fractions and Decimals | Math Lab: Equivalent Fractions |
| Fractions and Decimals | Simplifying Fractions |
| Fractions and Decimals | Mixed Numbers |
| Fractions and Decimals | Comparing and Ordering Fractions |
| Fractions and Decimals | Writing Decimals as Fractions |
| Fractions and Decimals | Writing Fractions as Decimals |
| Operations with Fractions | Math Lab: Rounding Fractions |
| Operations with Fractions | Rounding Fractions and Mixed Numbers |
| Ratio, Proportion, and Functions | Ratios and Rates |
| Ratio, Proportion, and Functions | Math Lab: Ratios and Tangrams |
| Ratio, Proportion, and Functions | Ratio Tables |
| Ratio, Proportion, and Functions | Proportions |
| Ratio, Proportion, and Functions | Algebra: Solving Proportions |
| Ratio, Proportion, and Functions | Sequences and Expressions |
| Ratio, Proportion, and Functions | Proportions and Equations |
| Ratio, Proportion, and Functions | Graphing Proportional Relationships |
| Percent and Probability | Percents and Fractions |
| Percent and Probability | Circle Graphs |
| Percent and Probability | Percents and Decimals |
| Percent and Probability | Experimental and Theoretical Probability |
| Percent and Probability | Making Predictions |
| Percent and Probability | Estimating with Percents |
| Systems of Measurement | Length in the Customary System |
| Systems of Measurement | Capacity & Weight in the Customary System |
| Systems of Measurement | Changing Metric Units |
| Geometry: Angles and Polygons | Similar and Congruent Figures |
| Perimeter, Area, and Volume | Measurement Lab: Area and Perimeter |
| Perimeter, Area, and Volume | Measurement Lab: Circumference |

Table B5

*Grade 7 Math Connects Lessons Included in the Study*

| Chapter | Lesson |
|---|---|
| Introduction to Algebra and Functions | Algebra: Arithmetic Sequences |
| Introduction to Algebra and Functions | Algebra: Equations and Functions |
| Linear Equations and Functions | Measurement: Perimeter and Area |
| Linear Equations and Functions | Functions and Graphs |
| Fractions, Decimals, and Percents | Simplifying Fractions |
| Fractions, Decimals, and Percents | Fractions and Decimals |
| Fractions, Decimals, and Percents | Fractions and Percents |
| Fractions, Decimals, and Percents | Percents and Decimals |
| Fractions, Decimals, and Percents | Comparing Rational Numbers |
| Ratios and Proportions | Ratios |
| Ratios and Proportions | Rates |
| Ratios and Proportions | Rate of Change and Slope |
| Ratios and Proportions | Changing Customary Units |
| Ratios and Proportions | Changing Metric Units |
| Ratios and Proportions | Algebra: Solving Proportions |
| Ratios and Proportions | Math Lab: Inverse Proportionality |
| Ratios and Proportions | Scale Drawings |
| Ratios and Proportions | Spreadsheet Lab: Scale Drawings |
| Ratios and Proportions | Fractions, Decimals, and Percents |
| Applying Percents | Math Lab: Percent of a Number |
| Applying Percents | Percent of a Number |
| Applying Percents | The Percent Proportion |
| Applying Percents | Estimating with Percents |
| Statistics: Analyzing Data | Spreadsheet Lab: Circle Graphs |
| Statistics: Analyzing Data | Using Data to Predict |
| Statistics: Analyzing Data | Using Sampling to Predict |
| Probability | Theoretical & Experimental Probability |
| Geometry: Polygons | Display Data in a Circle Graph |
| Geometry: Polygons | Similar Figures |
| Two- and Three-Dimensional Figures | Circumference of Circles |
| Two- and Three-Dimensional Figures | Graphing Geometric Relationships |
| Geometry and Measurement | Measurement Lab: Changes in Scale |

Table B6

*Grade 8 Math Connects Lessons Included in the Study*

| Chapter | Lesson |
|---|---|
| Algebra: Rational Numbers | Rational Numbers |
| Algebra: Rational Numbers | Comparing & Ordering Rational Numbers |
| Proportions and Similarity | Ratios and Rates |
| Proportions and Similarity | Proportional and Nonproportional Relationships |
| Proportions and Similarity | Rate of Change |
| Proportions and Similarity | Constant Rate of Change |
| Proportions and Similarity | Solving Proportions |
| Proportions and Similarity | Similar Polygons |
| Proportions and Similarity | Dilations |
| Proportions and Similarity | Indirect Measurement |
| Proportions and Similarity | Scale Drawings and Models |
| Percent | Ratios and Percents |
| Percent | Comparing Fractions, Decimals, and Percents |
| Percent | Algebra: The Percent Proportion |
| Percent | Finding Percents Mentally |
| Percent | Percents and Estimation |
| Measurement: Area and Volume | Circumference and Area of Circles |
| Measurement: Area and Volume | Explore Similar Solids |
| Measurement: Area and Volume | Similar Solids |
| Algebra: Linear Functions | Sequences |
| Algebra: Linear Functions | Slope |
| Algebra: Linear Functions | Direct Variation |
| Algebra: Linear Functions | Geometry Lab: Slope Triangles |
| Statistics | Circle Graphs |
| Probability | Experimental and Theoretical Probability |
| Probability | Using Sampling to Predict |

Table B7

*Grade 6 UCSMP Lessons Included in the Study*

| Chapter | Lesson |
| --- | --- |
| Some Uses of Integers and Fractions | Measuring Length |
| Some Uses of Integers and Fractions | Mixed Numbers and Mixed Units |
| Some Uses of Integers and Fractions | Equal Fractions |
| Some Uses of Integers and Fractions | Negative Fractions and Mixed Numbers |
| Some Uses of Decimals and Percents | Decimals for Numbers between Whole Numbers |
| Some Uses of Decimals and Percents | Decimals and the Metric System |
| Some Uses of Decimals and Percents | Converting Fractions to Decimals |
| Some Uses of Decimals and Percents | Decimal and Fraction Equivalents |
| Some Uses of Decimals and Percents | Percent and Circle Graphs |
| Some Uses of Decimals and Percents | Comparing Fractions, Decimals, and Percents |
| Statistics and Displays | Circle Graphs |
| Using Multiplication | Calculating Percents in Your Head |
| Using Multiplication | Using the Percent of a Quantity |
| Using Division | The Rate Model for Division |
| Ratio and Proportion | The Ratio Comparison Model for Division |
| Ratio and Proportion | Proportions |
| Ratio and Proportion | Solving Proportions |
| Ratio and Proportion | Proportions in Pictures and Maps |
| Area and Volume | Area and Operations of Arithmetic |
| Area and Volume | The Circumference and Area of a Circle |
| Area and Volume | Surface Area and Volume of a Cube |

Table B8

*Grade 7 UCSMP Lessons Included in the Study*

| Chapter | Lesson |
|---|---|
| Representing Numbers | Decimals for Numbers between Integers |
| Representing Numbers | Equal Fractions |
| Representing Numbers | Fraction-Decimal Equivalence |
| Representing Numbers | Fractions, Decimals, and Percents |
| Representing Numbers | Using Percents |
| Multiplication in Geometry | The Area Model for Multiplication |
| Multiplication in Geometry | Circles |
| Multiplication in Geometry | The Size-Change Model for Multiplication |
| Multiplication in Algebra | The Rate-Factor Model for Multiplication |
| Multiplication in Algebra | Combining Percents |
| Patterns Leading to Division | The Rate Model for Division |
| Patterns Leading to Division | The Ratio-Comparison Model for Division |
| Patterns Leading to Division | Proportions |
| Patterns Leading to Division | Proportional Thinking |
| Patterns Leading to Division | Proportions in Similar Figures |
| Geometry in Space | Volume of Prisms and Cylinders |
| Geometry in Space | How Changing Dimensions Affects Area |
| Geometry in Space | How Changing Dimensions Affects Volume |
| Statistics and Variability | Representing Categorical Data |

Table B9

*Grade 8 UCSMP Lessons Included in the Study*

| Chapter | Lesson |
|---|---|
| Division and Proportions in Algebra | Rates |
| Division and Proportions in Algebra | Multiplying and Dividing Rates |
| Division and Proportions in Algebra | Ratios |
| Division and Proportions in Algebra | Proportions |
| Division and Proportions in Algebra | Similar Figures |
| Slopes and Lines | Rate of Change |

APPENDIX C:

TRAINING MODULE AND ANSWER KEY

TRAINING MODULE

**Work with the researcher to code the content area, problem type, level of cognitive demand, and solution strategy of the following exercises.**

1. Marissa is 5 feet 6 inches tall. How tall is she in inches?
2. In the first five basketball games, Jamil made 9 out of 12 free-throw attempts. Find the probability of Jamil making his next free-throw attempt.
3. Write $\frac{4}{5}$ as a percent.
4. Insert >, < or = to make a true statement. 0.86 $\frac{7}{8}$
5. Express as a unit rate: 153 points in 18 games
6. Explain how to write a rate as a unit rate.
7. Which measure is greater? Or are the measures the same? Explain.
   One square yard or one square foot
8. Students at Neilson Middle School are asked if they prefer watching television or listening to the radio. Of 150 students, 100 prefer television and 50 prefer radio. Decide whether this statement is accurate: At Neilson Middle School, 1/3 of the students prefer radio to television.
9. A car can average 140 miles on 5 gallons of gasoline. Write an equation for the distance *d* in miles the car can travel on *g* gallons of gas.
10. Jasmin and Janelle are driving to Cincinnati. They think they can average 60 miles per hour for the 310 miles. At this rate, how long will it take them to get to Cincinnati?
    a. Let *t* be the time (in hours) it will take them. Write an equation involving *t* that can answer the question.
    b. Solve your equation. Check the solution in the original equation.
    c. Answer the question with a sentence.
11. There are 640 acres in a square mile. On August 20 and 21, 1910, what may have been the largest forest fire in American history burned three million acres of timberland in northern Idaho and western Montana. About how many square miles is this?
12. Multiple Choice In *t* minutes, a copy machine made *n* copies. At this rate, how many copies per second does the machine make?
    A $\frac{n}{60t}$ B $\frac{60t}{n}$ C $\frac{60n}{t}$ D $\frac{t}{60n}$
13. The Talkalot cell phone company sells a pay-as-you-go phone with 700 minutes for $70.
    a. What is the rate per minute?
    b. What is the rate per hour?

Table C1

Answer Key for Training Module

| Question Number | Content Area* | Problem Type | Solution Strategy | Cognitive Demand** | Visual Repres. | Charact. of Proportionality |
|---|---|---|---|---|---|---|
| 1 | GM | MV | None | Low: Proc. | None | No |
| 2 | DAP | MV | None | Low: Proc. | None | No |
| 3 | RN | MV | None | Low: Proc. | None | No |
| 4 | RN | RC | None | Low: Proc. | None | No |
| 5 | ALG | MV | None | Low: Proc. | None | No |
| 6 | ALG | Other | None | High: Proc. | None | No |
| 7 | GM | Other | None | High: Proc. | None | Yes |
| 8 | DAP | RC | None | High: Proc. | None | No |
| 9 | ALG | FR | None | High: Proc. | None | No |
| 10 | ALG | FR | None | High: Proc. | None | No |
| 11 | GM | MV | None | Low: Proc. | None | No |
| 12 | ALG | Other | None | High: Proc. | None | No |
| 13 | ALG | MV | None | Low: Proc. | None | No |

*DAP = data analysis/probability GM = geometry/measurement RN = rational number

**Low: Proc = Procedures Without Connect. High: Proc = Procedures With Connect.

APPENDIX D:

RELIABILITY TEST AND ANSWER KEY

RELIABILITY TEST

**On your own, code the content area, problem type, level of cognitive demand, and solution strategy of the following exercises.**

**1.** Write $\frac{1}{20}$ as a decimal.

**2.** In the first five basketball games, Jamil made 9 out of 12 free-throw attempts. Suppose Jamil attempts 40 free throws throughout the season. About how many free throws will you expect him to make? Justify your reasoning.

**3.** 720 cm = ____ m

**4.** Order this set of numbers from least to greatest: ¼, 22%, 0.3, 0.02

**5.** The Statue of Liberty stands about 305 feet tall, including the pedestal. Christina wants to build a model of the statue that is at least 20 inches tall. Find the scale if the model is 20 inches tall.

**6.** Find 26% of 360.

**7.** Using a sales tax of 6%, find the total cost for a $2.00 magazine.

**8.** Rewrite $\frac{2}{5}$ using a denominator of 10. Then, write a decimal for each fraction.

**9.** Find the value of *x* that makes the fractions equivalent. $\frac{5}{2} = \frac{x}{8}$

**10.** A school photograph measures 12 centimeters by 20 centimeters. The class officers want to enlarge the photo to fit on a large poster. Can the original photo be enlarged to 60 centimeters by 90 centimeters?

**11.** What fraction of an hour is 3 minutes?

**12.** Translate the English expression “the product of a number and five” into an algebraic expression.

**13.** A 5-pound bag of dog food costs $4.89. A 12-pound bag costs $9.59. Which is the better buy?

Table D1

Answer Key for Reliability Test

| Question Number | Content Area | Problem Type | Solution Strategy | Cognitive Demand | Visual Repres. | Charact. of Proportionality |
|---|---|---|---|---|---|---|
| 1 | RN | AF | None | Low: Proc. | None | No |
| 2 | DAP | MV | None | High: Proc. | None | No |
| 3 | GM | MV | None | Low: Proc. | None | No |
| 4 | RN | RC | None | Low: Proc. | None | No |
| 5 | GM | MV | None | High: Proc. | None | No |
| 6 | RN | MV | None | Low: Proc. | None | No |
| 7 | RN | MV | None | Low: Proc. | None | No |
| 8 | RN | MV & AF | None | Low: Proc. | None | No |
| 9 | RN | MV | None | Low: Proc. | None | No |
| 10 | GM | RC | None | High: Proc. | None | No |
| 11 | RN | MV | None | Low: Proc. | None | No |
| 12 | ALG | FR | None | Low: Proc. | None | No |
| 13 | ALG | RC | None | Low: Proc. | None | No |

*DAP = data analysis/probability   GM = geometry/measurement   RN = rational number

**Low: Proc = Procedures Without Connect.   High: Proc = Procedures With Connect.

# ABOUT THE AUTHOR

Gwendolyn Johnson received a bachelor's degree in Secondary Mathematics Education from Bowling Green State University in 1993 and a master's degree in Business Administration from the same institution in 2000. She taught high school in Toledo, Ohio and Ann Arbor, Michigan from 2000 to 2004. In 2004, she entered the doctoral program in Mathematics Education at the University of South Florida in Tampa. From 2004 to 2010, she taught mathematics methods courses for undergraduate and graduate pre-service teachers. She was also employed by ITT Technical Institute in 2009 and 2010 where she taught mathematics content courses.