ABSTRACT

Title: EFFECTS OF INEQUALITY AND TRANSMISSIVITY IN A COMMON POOL AQUIFER – THEORY, EXPERIMENTAL EVIDENCE AND POLICY IMPLICATIONS

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The early literature on common pool resources focused on the race for appropriation among users and its damaging effects on the resource stock and on the aggregate welfare of all users. The differential game framework was widely used to examine each user's actions under non-cooperative management and to assess the losses from deviating from an optimal resource management under cooperation. Interest has recently shifted toward the effects of characteristics such as heterogeneity among users and level of commonality in the resource on the use of common resources.

This article is interested in combining both effects: I consider a dynamic model of a common pool aquifer with a finite transmissivity used by two farmers with dissimilar efficiencies. I unravel the players' behavior under different strategies and estimate their respective profits in order to evaluate the welfare effects of inequality and transmissivity.

Solving for the aggregate profit of all players allowed me to revisit of a widespread result found in the common pool resource literature, which is that well enforced property rights are always associated with higher profitability; indeed, in the case of highly unequal players I reach a rather counterintuitive result as increasing transmissivity is proven to increase the overall profits. Such a result was never established in the literature at hand.

However, on the distributional aspect, the model shows that the benefits of less efficient users always suffer from more transmissivity, even when the inequality is high enough to generate a raise in aggregate profits.

For the validation of my theoretical results I carried out a series of experiments in the experimental laboratory at the Department of Agricultural and Resource Economics with volunteer subjects recruited from the University of Maryland. I used four experimental treatments. In the first two treatments the transmissivity is infinite; the players are highly differentiated in one treatment and identical in the other. The last two treatments are a replica of the earlier ones but with no transmissivity.

The laboratory data were compared to the theoretical solution following four benchmark paths: the social optimum, the subgame perfect equilibrium, the semi-myopic, and the myopic. The results show that the decisions of a significant share of players follow the myopic path. All the theoretical findings were corroborated by the experimental results including the increasing effect of transmissivity in the presence of users highly unequal.

In Chapter 5 on policy implications, I try to extend the analysis on the combined (or individual) effects of transmissivity and inequality on the aquifer use to the case when the possibility of communication between users, or the existence of a central agency, allows the emergence of alternative resource management modes.

The first mode corresponds to the case of social optimum resource management; when users coordinate their actions to maximize the benefits to the community from the aquifer. The second mode of management corresponds to the case where, from a certain round, only one user, a priori the most effective, is allowed to use the resource, while the other user abandons extraction activities for the remaining duration of the game.

Keywords: Common pool resources, CPR, aquifer, transmissivity, inequality, Subgame perfect equilibrium, SPE, experiment.

EFFECTS OF INEQUALITY AND TRANSMISSIVITY IN A COMMON POOL AQUIFER – THEORY, EXPERIMENTAL EVIDENCE AND POLICY IMPLICATIONS

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Chapter 1 Introduction

1.1 Literature review

The early work on common pool natural resources (hereafter CPR) highlighted the negative effects of common ownership on the use of CPR under a free entry regime such as fisheries (Gordon 1954) or resources with limited access like groundwater (Burt 1964; Brown and Deacon 1972), which results in the decline of their rents or even their destruction (Hardin 1968).

The use of a game theory framework in the analysis of non-cooperative use of CPR provided more substantiation of the tragedy of the commons concept through a better understanding of decision making processes and ensuing evolution of stocks. Khalatbari (1977) showed that in the case of an oil field exploited by non-cooperative players, the seepage between the different sections of the field gave incentive for every player to extract more at any level of stock, leading to rapid field depletion and welfare loss for all players.

A similar result was reached by Eswaran and Lewis (1984), as they used a discrete version of Khalatbari's model and applied a backward induction program to construct the closed loop strategy for every player when they base their decision rule solely on the remaining stock. Backward dynamic programming was similarly used by Levhari and Mirman (1980) in the case of a multi-period fish war between two countries to obtain the decision rule for every player, based on this result they showed that with an infinite horizon a steady state is reached at a lower stock and lower aggregate utility than under a cooperative scheme.

Negri (1989) reaches the same result of lower stocks and smaller profits at the steady state in his study of a finite number of farmers using a common pool aquifer for agricultural production. The same outcome results under the two approaches – feedback strategies or open loop strategies – with the former strategies even more inefficient, as they integrate the extraction cost externality in addition to the appropriation externality incorporated in the open loop strategies. In the same paper, Negri briefly discusses the welfare effects of inequality in land ownership between farmers and concludes that inequality generates more incentives for water conservation.

The repercussions of heterogeneity among users on CPR management and its various welfare effects received more attention in the past decade and have become a focal point of numerous studies (Schlager and Blomquist 1998; Bardhan and Dayton-Johnson 2000). In their study on the effect of inequality in wealth on the voluntary contribution to CPR conservation, Baland and Plateau (1997) show that wealth inequality has an increasing effect on conservation and on overall welfare when the production technology is concave in effort; however, in the cases where contributions are bounded, time-costs are variable, or production exhibits non-convexities, the effect of inequality is ambiguous.

Aggarwal and Narayan (2004) use a two-stage model of a common pool aquifer where players decide on the level of investment in well depth in the first stage, and on the level of use over time in the second stage. The inequality derives from uneven access to the credit market. They show that the inequality effect is U-shaped. First it decreases welfare from aquifer use but, beginning at a certain threshold, more inequality increases welfare. A similar result was found by Dayton-Jhonson and Bardhan (2002) in a two-period game of common fishery exploitation.

Another feature of CPR that did not receive much attention in the early literature is the effect of a limited commonality of the CPR. The assumption of full or complete commonality simplifies the analysis since it allows for a CPR to be represented as a uniform body with the same (relevant) physical or biological properties, where the impacts of the agents' actions on one another are instantaneous. In the case of groundwater, for instance, this would suggest an infinite transmissivity so that the aquifer can be treated as a single cell, with the surface of water at the same level at all times everywhere and such that all farmers face the same extraction cost (Brown and Deacon 1972).

In reality, transmissivity is finite, and the movement of water underground is a relatively slow process, which results in a lag in time and space of pumping effects, as shown in Brozovic et al. (2006), and attenuates the race-for-water predicted by Provencher and Burt (1993). An analogous outcome is observed in the case of near-shore fisheries, where the reduced mobility of species like clams creates conditions where limited property rights might emerge and provides incentives for fishermen to reduce their catch (Janmaat 2005).

1.2 Purpose of the Work and Main Results

In this paper, I address the complex situation where an aquifer with finite transmissivity is exploited by farmers with heterogeneous efficiencies. In my model, presented in the next section, I untangle the players' comportment under different strategies to solve and estimate their decisions and subsequent profits. Using the results found here, I evaluate the welfare effects of inequality and transmissivity.

Special interest is given to the aggregate cumulative profits of all players. This outcome will allow challenging the commonly accepted principle in the CPR literature that well-enforced property rights are always associated with higher profitability. This result is questionable in the case of heterogeneous users. Indeed, resource commonality can have positive effects. Although the negative externalities of competition for the appropriation of more resources persist, a positive externality emerges due to heterogeneity; this externality arises by placing more resources at the more effective user's disposal. Another interesting result is that the gains from making the resource "more common", if any, are not generalized; indeed less-efficient users are always found to lose from increased transmissivity.

For the empirical validation, I start by reworking my model for the discrete case, with multiple rounds. I use the backward solution to derive the players' decisions following different strategies. I reaffirm my theoretical findings, with the continuous model, with an analytical proof for the one round game to, and I follow with the numerical evidence from the 10-round game.

I use laboratory experiments to reproduce the special cases of common pool aquifer use – which are described in the theoretical model – and test for my theoretical findings. I investigate the effects on decision making of a shift in transmissivity in the presence of users with unequal and identical efficiencies under a time-dependent setting.

Walker, Gardner, and Ostrom (1990) pioneered the use of experiments to validate the theoretical assumptions on rent dissipation in CPR. Their simply designed experiment was set up as a one-round game with no communication between subjects. Later experimental studies tried to handle more complex settings of CPR use. Ostrom and

Walker (1991) showed the potential for more efficient extraction from a CPR when communication between appropriators was allowed. Hackett, Schlager, and Walker (1994) closely studied one aspect of the problem – namely, communication – and the prospects for collective action in the presence of heterogeneous users. Herr et al. (1997) considered the case of a resource with a stock effect, and confirmed the time-dependent externality arising from non-cooperative use of a CPR.

In the experimental literature, as in the general literature on CPR there is a clear lack of studies that focus on the combined effects of heterogeneity and (finite) transmissivity on the total and distributional welfare and on the resource conservation; this article is a first step to fill this gap. In this attempt it was possible to draw an interesting first lesson, which states that in the presence of unequal players, making the resource more common might increase total welfare. This result, established theoretically and validated by experimental results, goes against the common wisdom of less profits and lower stocks under open access.

In the last section, I try to extend the analysis on the combined (or individual) effects of transmissivity and inequality on the aquifer use to the case when the possibility of communication between users, or the existence of a central agency, allows the emergence of alternative resource management modes.

The first mode corresponds to the case of social optimum resource management, when users coordinate their actions to maximize the benefits to the community from the aquifer.

Another mode of management corresponds to the case where, from a certain round, only one user, a priori the most effective, is allowed to use the resource, while the other user abandons extraction activities for the remaining duration of the game.

Comparing the profits with a single user to the total profits under non-cooperative management, helps to reveal the conditions, in transmissivity and inequality, under which a player is better off buying out the other player, these conditions are the requirements for water markets to emerge.

Chapter 2 The Continuous Model

This chapter is particularly important for the rest of this dissertation, it introduces the general analytical model, that of an aquifer with a finite transmissivity, exploited by two unequal users with dissimilar efficiencies.

In the case of non-cooperative users, given the continuous extraction game considered in this chapter, I differentiate between open loop and feedback strategies; I begin by solving for the players' decisions and related stocks' evolution, then work out the individual and aggregate cumulative (i.e. over time) profits.

Under both strategies, the results support, in the case of identical users, the widely recognized negative effect of increased externalities and/or open access on aggregate welfare; the results also show that inequality across users has an unambiguous increasing effect on aggregate welfare.

The most interesting result, however, relates to the combined effect of transmissivity and inequality among users; the model shows that with highly unequal users, the aggregate –cumulative– profits increase with transmissivity. At low levels of inequality, the effect is decreasing, similar the case with identical users.

On the distributional side, the analysis shows that the increase in aggregate profits, when it occurs, is not uniform, indeed, the profits of the less efficient user always drop as transmissivity goes up.

Finally, I consider the case with cooperating users, I solve for the players' decisions and the corresponding aggregate cumulative (i.e. over time) profits; I use these outcomes to draw some interesting remarks regarding the effect of inequality and transmissivity on the aggregate profits, and on the difference between this last outcome and the aggregate cumulative profits under non-cooperative management.

2.1 The Model

I consider here the case of a groundwater aquifer commonly owned by two economic agents, *h* and *l*, endowed with fields of identical dimensions both at the same elevation *E* from a given point of reference (surface of the sea). For simplicity, I assume that the fields are of unitary surface sizes. Groundwater operates as a two-cell aquifer, where the surface of the water table beneath each agent is uniform but is not necessarily identical to the surface beneath the next agent (see Figure 1). Given my assumption on surface sizes, the stocks of water in the two compartments of the aquifer are equivalent to the height of the water table vis-à-vis the sea level. I also assume that the aquifer is sufficiently deep so that total depletion in one part or the other cannot take place.

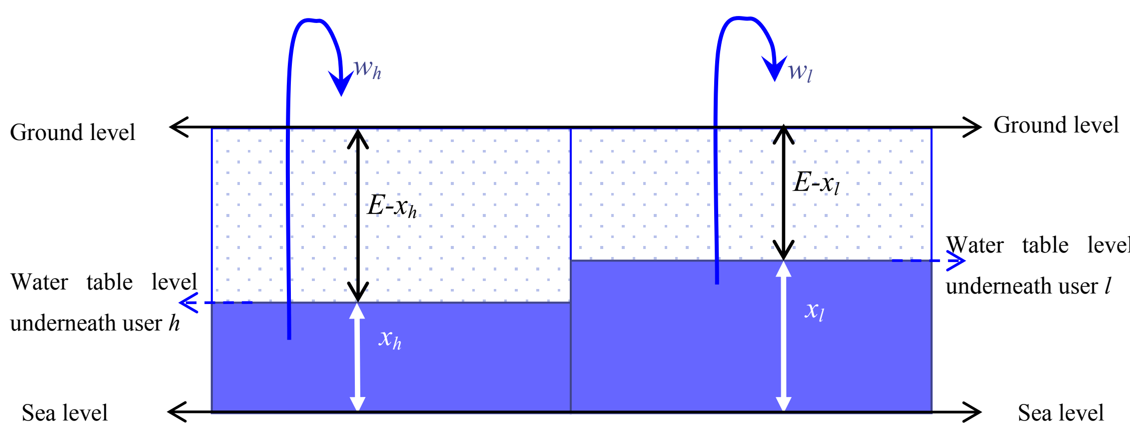

Figure 1 The two cells aquifer

Under the assumption of no seepage between the two compartments, the equation of motion that defines stock adjustment to water use and recharge is given by:[1]

$$\dot{x}_{it} = R_{it} - w_{it}, \qquad i=h,l$$

where $\dot{x}_{it}$ is the change in x_{it}, the level of water table beneath agent i at time t, w_{it} is the rate of water extraction chosen by farmer i at time t, and R_{it} corresponds to the rate of recharge of the aquifer at time t beneath agent i.[2]

Allowing for R_{it} to differ from one cell to another would induce an interesting type of heterogeneity: heterogeneity in endowments. I will not consider this option in the present research. I will only examine the case where the recharge is uniform across farmers, and is exogenous and constant over time. This also means that the model does not consider the percolation of irrigation water. Extracted water is entirely consumed by the crops. This assumption does not affect the results compared to models that assume percolation is proportional to extraction.

When it is "allowed" for water to move from one side of the aquifer to the other, the equation of motion becomes:[3]

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[1] This result is straightforward when w_{it} and R_{it} are continuous and bounded. Between time t and $t+dt$ the change in water stock is given by $x_{it+dt} - x_{it} = R_{it}dt - w_{it}dt$

[2] I assume that every agent can pump water directly underneath his/her field. Therefore the extracted amount is expressed as the equivalent change in the farmer's "own stock".

[3] The subscript $-i$ stands for the other player, i.e. when $i=h$ then $-i$ refers to l and vice versa. The formula in (1) is equivalent to saying that the total change in stock under communication is equal to the change when there is no communication augmented by the variation induced by the transfer (which is positive when water moves from j to i) : $\dot{x}_{it} = \dot{x}_{it}\big|_{s=0} + s\left(x_{-it} - x_{it}\right)$. More formally, given the stock x_{it} at instant t, the stock at time $t+dt$ is given by: $x_{it+dt} - x_{it} = R_{it}dt - w_{it}dt + s\left(\left(x_{-it} + \frac{\left(R_{-it} - w_{-it}\right)}{2}dt\right) - \left(x_{it} + \frac{\left(R_{it} - w_{it}\right)}{2}dt\right)\right)dt$ where

$$\dot{x}_{it} = R - w_{it} + s\left(x_{-it} - x_{it}\right), \qquad i=h,l \qquad (1)$$

where s is the transmissivity or seepage coefficient assumed to be between 0 and infinity.

The transmissivity coefficient is the element of my model that allows control of the level of commonality in the resource's use. It is an indication of the degree of commonality between the two "cells" of the aquifer, and reflects the magnitude by which the different agents impinge on each other.

When s takes a value of 0, the problem corresponds to the case where there is a perfectly impermeable frontier between the two "cells" that water cannot cross. In this case, the problem simplifies to a sole owner optimization problem. As shown in figure 2 the stocks are higher since players have more incentives to conserve the resource in the absence of externality.

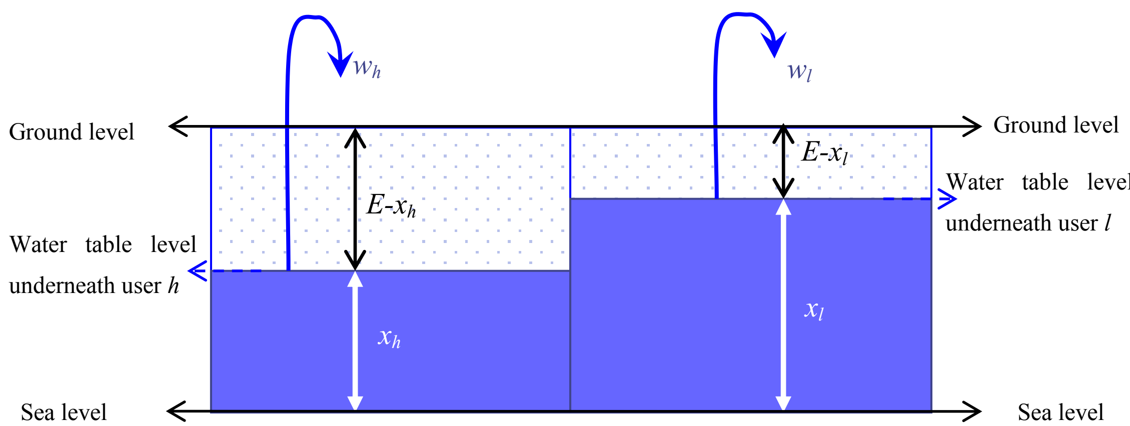

Figure 2 The zero transmissivity aquifer

For drastically large values of s the situation is analogous to an infinite transmissivity aquifer where water circulates from one side to the other instantaneously. If there is a

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$\left(x_{it} + \frac{\left(R_{it} - w_{it}\right)}{2}dt\right)$ is the average stock of agent i between time t and t+dt. Finally, I derive: $\frac{x_{it+dt} - x_{it}}{dt} =$ $R_{it} - w_{it} + s\left(x_{-it} - x_{it}\right) + s\left[\frac{\left(R_{-it} - w_{-it}\right) - \left(R_{it} - w_{it}\right)}{2}\right]dt$ where the last term vanishes as dt tends to zero.

difference in stocks, say $x_{it} > x_{-it}$, then $\dot{x}_{it}$ is negative and $\dot{x}_{-it}$ is positive but both $\dot{x}_{it}$ and $\dot{x}_{-it}$ are of a great magnitude. Therefore, as long as $x_{it} > x_{-it}$ there will be a continuous and rapid decline in x_{it} and an increase in x_{-it} until there is no difference between stocks in the two cells. This case is known as the bathtub aquifer (figure 3).

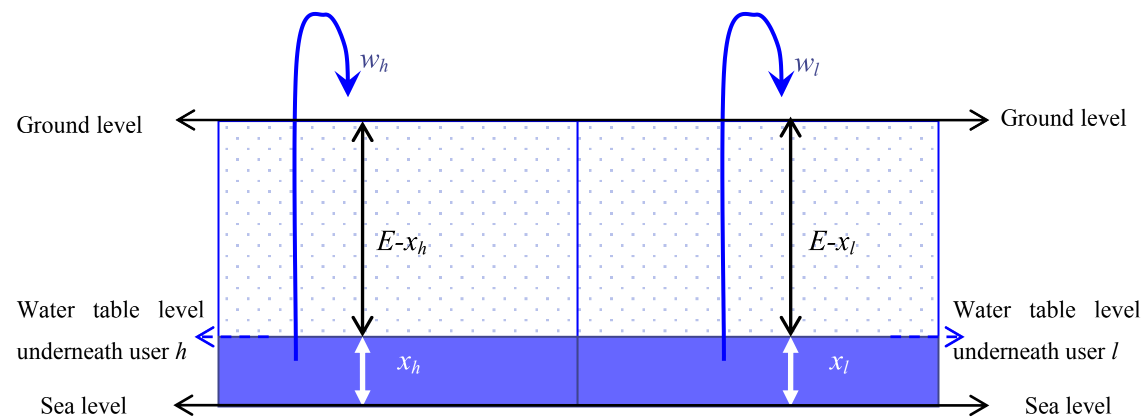

Figure 3 Infinite transmissivity

I assume that there is no storage facility and that all extracted water is used for irrigation purposes. I also assume that the aquifer is the only source of water for both farmers. Agent i uses the extracted water, with proper levels of other inputs, in agricultural production and receives a revenue:

$$NR_{it} = F_i(w_{it}), \qquad i=h,l \qquad (2)$$

where NR_{it} is the revenue net of all expenses other than extraction costs and $F_i(w_{it})$ is agent i's production function, which is strictly concave in water use and $F_i(0)=0$ holds. In the rest of the paper, I will simply assume a quadratic production function of the form:[4]

[4] When the optimal choices are low enough, the quadratic form does not conflict with positive marginal productivity of water.

$$F_i(w_{it}) = a_i w_{it} - \frac{w_{it}^2}{2}. \qquad i=h,l \qquad (3)$$

In the present model, Inequality is introduced under the presumption that different agents might have different efficiencies. The marginal revenue from water at the origin will be used as a proxy for efficiency. When $a_h > a_l$ for any amount of water w_t, player h will derive more profits than player l.

In the model, I will mostly focus on the effects related to level of inequality, as materialized by the difference a_h - a_l; and explore how those effects evolve following mean preserving changes in efficiencies, including the case with identical users with the –same– efficiency, $(a_h + a_l)/2$.

This provides the accurate comparative statics tool, allowing the observed results to be attributed solely to inequality, not to changes in the average efficiency.

When agent i lifts an amount w_{it} from the aquifer, s/he bears the corresponding extraction cost:

$$C_{it}(x_{it}, w_{it}) = w_{it}\, c\left(E - x_{it}\right) \qquad i=h,l \qquad (4)$$

where: c is the cost of lifting one unit of water over one unit of depth. (i.e. the cost is assumed linear)

E is the level of the ground surface, assumed to be identical for both users, and calculated relative to the sea level.

Thus, $E - x_{it}$ is the distance between the surface of groundwater beneath agent i and the ground; and $c\left(E - x_{it}\right)$ is the unitary cost (per unit of water) faced by the farmer at instant t. The assumption here is that the farmer pumps from the nearest point.[5] The cost of extraction varies over time as stocks change and extracted quantities vary.

The net payoff to agent i at time t is given by:

$$NP_{it} = a_i w_{it} - \frac{w_{it}^2}{2} - w_{it} c(E - x_{it}). \qquad i=h,l \qquad (5)$$

Equations (1) and (5), along with the initial level(s) of water completely define the extensive form of my differential game. The objective of agent i is to maximize the sum of her net payoffs:

$$\int_0^T e^{-rt} NP_{it} dt \qquad i=h,l \qquad (6)$$

where T is the time horizon of the control problem, considered infinite for the rest of this section.

2.2 The Game

First, I will focus on the non-cooperative case where I assume that the two players do not communicate with each other and cannot engage in any form of cooperation.

The sole objective of every player is to maximize the sum of her discounted net benefits with no consideration for the impacts of her actions on her neighbor's payoff. Her strategy is, however, motivated by her beliefs about the characteristics of her rival's strategy and course of action.

[5] Again, more formally, the actual total cost between instant t and instant $t+dt$ is given by: $\int_t^{t+dt} C_{it}(x_{it}, w_{it}) dt = \int_t^{t+dt} w_{it} c\left(h - \left(x_{it} + \frac{\dot{x}_{it}}{2} dt\right)\right) dt$, which justifies (4).

I assume that a central authority does not exist or, if it exists, does not intervene in any way to stop or restrain competition among players to establish higher revenues from the aquifer or safeguard the water table.

The levels of water in the two sides of the aquifer are given at the start of the game. The other characteristics of the aquifer, recharge and transmissivity, are also known to both players with certainty.

Finally, I assume that both players have the same discount rate ($r > 0$) and that they maximize their benefits over the entire extraction-game horizon (supposed infinite). Every player knows her efficiency and her rival's efficiency.

The maximization problem of player i is given by:

$$\underset{w_{it}}{Max}\int_0^\infty e^{-rt}\left(a_i w_{it}-\frac{w_{it}^2}{2}-w_{it}\,c\left(E-x_{it}\right)\right)dt \qquad i=h,l \qquad (6)$$

subject to: $\dot{x}_{it}=R-w_{it}+s\left(x_{-it}-x_{it}\right) \qquad i=h,l$

$w_{ht},w_{lt}\geq 0$ (control variables are non-negative).[6]

$x_{ht},x_{lt}\leq E$ (the level of water cannot rise above the ground).[7]

x_{h0} and x_{l0}, the stocks at $t = 0$, are given.

Even with the assumption that there are no threats and that both players have perfect knowledge regarding the stock of water on both sides, this game can still have more than one Nash equilibrium depending on the players' strategies.[8] Indeed, every player's

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[6] This condition is rather undemanding in the non-cooperative game.

[7] Requires some conditions on the efficiency and recharge.

[8] Assuming no threats rules out trigger strategies. Perfect knowledge requirements depend on the strategy of the game. I introduce this assumption to rule out mixed strategies.

decision is dictated by her own strategy of extraction, which expands the entire game horizon: $\mathrm{K}_i = \{w_{it}, t \in [0,T]\}$. In the next section, I will present two particular kinds of Nash equilibrium, the open loop equilibrium and the closed loop equilibrium. This will help illustrate the issues that this research projects intends to explore.

In the open loop equilibrium, every player observes the levels of water at the beginning of the game (x_{h0}, x_{l0}) and develops her strategy of extraction for the entire game. Her strategy becomes: $\mathrm{K}_i = \{w_{it} = w_{it}(t), t \in [0,T]\}$. This approach presupposes that every player takes her opponent's actions as given (as a function of time) and none react to any deviation in the opponent's behavior. Every player picks an extraction strategy at the start of the game and commits to it for the entire time horizon.

Under the open loop approach, the entire game takes a static form where the evolution of stocks is irrelevant and all actions are determined at the beginning. By assuming that the opponent will not base her intensity of extraction on the levels of stock, every player will end up herself lifting water at a lesser rate and the outcome for both players is higher.[9]

Contrary to the open loop outcome, the closed loop equilibrium has a dynamic form since actions are made following a rule based on the in-progress state of the world. The player's decision is wholly tied to the actual levels of water stock. She constantly observes the state variable and, based on her observations, decides the amount of water to extract. Her strategy can be written as $\mathrm{K}_i = \{w_{it} = f_i(x_i(t), x_{-i}(t)),\ t \in [0,T]\}$. where f_i is a stationary function.

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[9] More on the topic in Clemhout and Wan (1994).

In the rest of this section, I will analyze the evolution of water extraction under the open loop equilibrium with a finite transmissivity and the closed loop equilibrium with an infinite transmissivity;[10] for both strategies the primary interest is to derive the cumulative profits, defined as the discounted sum over time of individual users profits, as well as the aggregate cumulative profit, by summing up the cumulative profits (over time) for both players.[11]

The other statistics that will be included in the present analysis are the net individual profits and the aggregate profit (summing the individual profits) at the steady state; these statistics are more pertinent when the concern is also about the survival of all users, resources are generally more abundant (at least relatively) early and rarer in the later stages of the game. The analysis at the steady state is also a means to draw some preliminary results that are more difficult to establish with cumulative profits.

Finally, I will extend the analysis to the case where the two users coordinate their extraction decisions to maximize the Social Optimum, defined as the cumulative profits from the aquifer.

2.2.1 The Open Loop Game

In this subsection, I start by spelling out the optimality conditions under an open loop strategy, for my model; I then report the optimal path and the actual solution given the conditions in stock at the start of the game, the step-by-step mathematical analysis is presented in Appendices A and B. I also derive the system at the steady state, and use the

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[10] The actual full solution for the general case, with a finite transmissivity, was not possible for feedback strategies due to computing limitations.

[11] In what follows, cumulative (profits) will be used to refer to the sum (of profits) over time, while aggregate (profits) is used when summing profits across users.

computed (aggregate and individual) cumulative profits to derive my observations regarding the combined effects of transmissivity and inequality, under non-cooperative management.

2.2.1.1 The Optimality Conditions for the Open Loop Nash Equilibrium

In the open loop equilibrium, the present value Hamiltonian of player i is given as:[12]

$$H_{it}^{p} = e^{-rt}\left(a_i w_{it} - \frac{w_{it}^2}{2} - w_{it} c\left(E - x_{it}\right)\right) + \lambda_{it}^{p}\left(R - w_{it} + s(x_{-it} - x_{it})\right) \qquad i=h,l \quad (8)$$

$$+ \mu_{it}^{p}\left(R - w_{-it} + s(x_{it} - x_{-it})\right)$$

where λ_{it}^{p} is the co-state variable (evaluated at time 0) associated with the state variable x_{it}, her own stock, and μ_{it}^{p} is the co-state variable (also evaluated at time 0) associated with the state variable x_{-it}, her neighbor's stock.

Under the open-loop solution, player i solves for –and commits to– the optimal profile over time of control variables w_{it} to maximize her cumulative profits, over time, given the stocks' evolution ($\dot{x}_{it}$ and $\dot{x}_{-it}$ in (7)) and the conditions at the start.

With an interior solution, w_{it} must satisfy the first order condition:

$$\text{C1} \qquad 0 = \frac{\partial H_{it}^{P}}{\partial w_{it}} = e^{-rt}\left(a_i - w_{it} - c\left(E - x_{it}\right)\right) - \lambda_{it}^{P}. \qquad i=h,l \qquad (9.a)$$

Following the Pontryagin's Maximum Principle along the optimal path, the evolution over time of the co-state variables (λ_{it}^{P} and μ_{it}^{P}) fits the conditions:

$$\text{C2} \qquad \dot{\lambda}_{it}^{P} = -\frac{\partial H_{it}^{P}}{\partial x_{it}} = -w_{it}\, c + s\lambda_{it}^{P} - s\mu_{it}^{P} \qquad i=h,l \qquad (9.b)$$

---

[12] The Hamiltonian is similar to that of an unconstrained problem under the assumption that the optimal solution satisfies the conditions on state and control variables.

C3 $\qquad \dot{\mu}_{it}^{P} = -\dfrac{\partial H_{it}^{P}}{\partial x_{-it}} = -s\lambda_{it}^{P} + s\,\mu_{it}^{P}\,. \qquad\qquad i=h,l \qquad\qquad (9.c)$

In addition to the equations of motion in both stocks (1).[13]

The first condition of optimality requires every player to use her stock to the point where the (present) marginal benefit from one more unit of water lifted from (his side of) the aquifer and used in agricultural production (given the no storage assumption) is equal to the (present) shadow-value of one more unit of water in the aquifer.

Conditions C2 and C3 reflect the transition over time of the (present) shadow-values (for player *i*) of stocks x_{it} and x_{-it}. The estimation of $\dot{\mu}_{it}^{P}$ reflects the assumption that player *i* infers his/her opponents actions only as a function of time and not stock level.[14]

It can be shown that the functions in the Hamiltonian meet the sufficiency conditions and that the solution is unique since H_{it} is strictly concave in w_{it} (See Appendix A for the proof).

Replacing λ_{it}^{P} and μ_{it}^{P} by their current values, respectively, λ_{it} and μ_{it} given by:

$$\lambda_{it}^{P} = e^{-rt}\lambda_{it} \qquad\qquad i=h,l$$

$$\mu_{it}^{P} = e^{-rt}\mu_{it} \qquad\qquad i=h,l$$

and equations C1, C2 and C3 become:

C1 $\qquad 0 = a_i - w_{it} - c\left(E - x_{it}\right) - \lambda_{it} \qquad\qquad i=h,l \qquad\qquad (9.a)'$

---

[13] The conditions on w_{it} , w_{-it}, x_{it} and x_{-it} are not included under the assumption that the optimal solution meets those conditions for an appropriate choice of a_h, a_l, and *R*.

[14] That is, $\dfrac{\partial w_{-it}}{\partial x_{-it}} = 0$ is assumed for player *i=h,l*.

C2 $\dot{\lambda}_{it} = r\lambda_{it} - w_{it}c + s\lambda_{it} - s\mu_{it}$ $i=h,l$ (9.b)’

C3 $\dot{\mu}_{it} = -s\lambda_{it} + (r+s)\mu_{it}$ $i=h,l$ (9.c)’

2.2.1.2 Solving for the Optimal Open Loop Path

The values of w_{it} and w_{-it} [15] along the optimal path are derived from the optimality condition C1, and substituted in the transition and the motion equations, to obtain a system of linear first order differential equations that can be rewritten in a matricial form as follows:

$$\dot{V} = A.V + B \tag{10}$$

where $V = [x_{it}, x_{-it}, \lambda_{it}, \lambda_{-it}, \mu_{it}, \mu_{-it}]$, $B = [R - a_i + cE,\ R - a_{-i} + cE,\ c^2E - ca_i,\ c^2 E - ca_{-i},\ 0,\ 0]$, and

$$A = \begin{bmatrix} -(c+s) & s & 1 & 0 & 0 & 0 \\ s & -(c+s) & 0 & 1 & 0 & 0 \\ -c^2 & 0 & (r+s+c) & 0 & -s & 0 \\ 0 & -c^2 & 0 & (r+s+c) & 0 & -s \\ 0 & 0 & -s & 0 & (r+s) & 0 \\ 0 & 0 & 0 & -s & 0 & (r+s) \end{bmatrix}.$$

The general solution to equation (10) is totally defined by the eigenvalues (and eigenvectors) of matrix A (Simon and Blume, 1994, p 678-81) derived from Det[A- α $I_{6\text{x}6}$] =0, where $I_{6\text{x}6}$ is the identity matrix and Det[A- α $I_{6\text{x}6}$] is the determinant of [A- α $I_{6\text{x}6}$] that can be written factorized as follows:

$Det[A- \alpha I_{6x6}] = EQ_1(\alpha)\ EQ_2(\alpha)$

where: $EQ_1(\alpha) = \left((r+s-\alpha)c^2 - (c+\alpha)\left((r+s+c-\alpha)(r+s-\alpha) - s^2\right)\right)$, and

---

[15] Explicitly, $w_{it} = a_i - c(E - x_{it}) - \lambda_{it}$ for player $i=h,l$.

$$EQ_2(\alpha) = \left((r+s-\alpha)c^2 - (c+2s+\alpha)\left((r+s+c-\alpha)(r+s-\alpha)-s^2\right)\right)$$

As detailed in Appendix B.1.1, matrix A has two eigenvalues, α_1, the negative root of equation $EQ_1(\alpha) = 0$, and α_2, the negative root of $EQ_2(\alpha) = 0$, with: [16]

$$\alpha_1 = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2} \cos\left(\frac{\theta_1 + 2\pi}{3}\right), \text{ where } p_1 = \frac{1}{3}\left(3c\,r + r^2 + 3c\,s + 2r\,s + 4\,s^2\right) \text{ and}$$

$$\theta_1 = ArcCos\left(\frac{q_1/2}{\sqrt{p_1^{\,3}/27}}\right) \text{ with } q_1 = \frac{1}{27}\left(\ 2s^2(9\,c + 6r + 8\,s) - r(9\,c(r+2\,s) + 2\,r\,(r+3\,s))\right)\text{; and}$$

$$\alpha_2 = \frac{2(r+s)}{3} + 2\left(\frac{p_2}{3}\right)^{1/2} \cos\left(\frac{\theta_2 + 2\pi}{3}\right) \text{ where } p_2 = \frac{1}{3}\left(r^2 + 6r\,s + 12s^2 + 3c(r+3s)\right) \text{ and}$$

$$\theta_2 = ArcCos\left(\frac{q_2/2}{\sqrt{p_2^{\,3}/27}}\right) \text{with } q_2 = -\frac{1}{27}\left(2\,r\,(r+3s)(r+6s) + 9c\left(r^2 + 6\,r\,s + 6\,s^2\right)\right).$$

The corresponding eigenvectors (see Appendix B.1.2) are respectively v_1 and v_2:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ c+\alpha_1 \\ c+\alpha_1 \\ \dfrac{s\,c^2}{(r+s+c-\alpha_1)(r+s-\alpha_1)-s^2} \\ \dfrac{s\,c^2}{(r+s+c-\alpha_1)(r+s-\alpha_1)-s^2} \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -1 \\ c+2s+\alpha_2 \\ -(c+2s+\alpha_2) \\ \dfrac{s\,c^2}{(r+s+c-\alpha_2)(r+s-\alpha_2)-s^2} \\ -\dfrac{s\,c^2}{(r+s+c-\alpha_2)(r+s-\alpha_2)-s^2} \end{bmatrix}$$

and the general solution is hence derived as:

$$V_P = c_1\, e^{\alpha_1 t}\, v_1 + c_2\, e^{\alpha_2 t}\, v_2\,,$$

where c_1 and c_2 are two arbitrary constants, that will be determined so as to satisfy the boundary conditions (relatively to the stock levels). It should mentioned here that in the

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[16] The other roots are positive and do not carry an economically relevant meaning and certainly do not correspond to an optimum solution to the objective maximization problem.

case where s=0 both equations combine ($EQ_1(\alpha) = EQ_2(\alpha)$) and Det[$A$- α $I_{6\text{x}6}$] =0 has only one (quadratic) root.[17]

The solution to the original linear system of differential equations ($\dot{V} = [A].V + B$) is simply given by:

$$V = V_P - [A]^{-1}.B = c_1\, e^{\alpha_1 t}\, v_1 + c_2\, e^{\alpha_2 t}\, v_2 - [A]^{-1}.B \tag{11}$$

Given x_{i0} and x_{-i0}, the state of the stocks at time 0, it is possible to solve for the unique pair of constants, c_1 and c_2:

$$c_1 = \frac{1}{2}\left((x_{i0} + x_{-i0}) - \left(2E + \frac{-(a_i + a_{-i})r + (c + 2r)R}{c\,r} + \frac{R}{r + 2s} \right) \right) \tag{12.a}$$

$$c_2 = \frac{1}{2}\left((x_{i0} - x_{-i0}) - \left(- \frac{(a_i - a_{-i})r(c + 2s)}{2rs(r + 2s) + c(r^2 + 4rs + 2s^2)} \right) \right) \tag{12.b}$$

As mentioned earlier, the interest here is in looking into the effects of heterogeneity as it relates to efficiency of production of both players, it is therefore consistent to consider that the stocks of water, in both compartments of the aquifer, are equal at the start of the game: $x_{i0} = x_{-i0} = x_0$.

Failing to make this assumption would introduce another type of heterogeneity (heterogeneity in initial stock) that is beyond the scope of this research work.

2.2.1.3 The Open Loop Solution

The stock level of player i (i=h,l) can be derived from (11) as:

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[17] In this case, the solution still has the same exponential form, but the math further simplifies to show that decisions and stock of player i are not affected by those of her opponent $-i$, which is consistent with the extraction game with zero transmissivity.

$$x_{it} = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + E - \frac{a_i + a_{-i}}{2c} + \frac{R}{c} + \frac{R}{2r} + \frac{R}{2r+4s} - \frac{a_i - a_{-i}}{2} F(s) \quad (13.a)$$

where $F(s) = \dfrac{r\,(r+2s)}{2rs\,(r+2s) + c\,(r^2 + 4rs + 2s^2)}$, a nonnegative function of s.

Substituting the value of λ_{it}, provided by Equation (11), in condition C1 on the optimal level of water use (from Equation (9.a)') yields:

$$w_{it} = R - c_1\,\alpha_1 e^{\alpha_1 t} - c_2\left(2s + \alpha_2\right)e^{\alpha_2 t} + (a_i - a_{-i})\,s\,F(s) \quad (13.b)$$

Given the values of x_{it} and w_{it} and the condition on stocks at the start of the game, $x_{h0} = x_{l0} = x_0$, it can be established that following the open loop game, at any time t:[18]

$0 < (w_{it} - w_{-it})(a_i - a_{-i})$, and

$0 > (x_{it} - x_{-it})(a_i - a_{-i})$

The efficient user extracts more water and her stock is lower than that of her partner, therefore the necessary conditions in equation (6) can be simplified as:

$w_{lt} \geq 0$ & $x_{lt} \leq E$.

The computed values of x_{it} and w_{it} are used (Equation (5)) to obtain NP_{it}, the net payoff of player i at time t; after integration over time (as presented in Equation (6)), the cumulative profits of player i, noted $V^{s,OL}_{a_i,a_{-i}}\left(x_0, x_0\right)$ hereafter, are given as:

[18] The same result will be established at the steady state, with an interior solution, but with no conditions on the stocks at time zero.

$$V^{s,OL}_{a_i,a_{-i}}(x_0,x_0) = \frac{R^2\left(2c(r+s)+r(r+2s)\right)}{2r^2(r+2s)} - \frac{\alpha_1(2c+\alpha_1)}{8(r-2\alpha_1)}\left(\frac{a_i+a_{-i}}{c} - 2E - \frac{2R}{c} - \frac{R}{r} - \frac{R}{r+2s} + 2x_0\right)^2$$

$$+\frac{cR}{2}\left(\frac{r(r+2s)-\alpha_1(r+s)}{r(r+2s)(r-\alpha_1)}\right)\left(\frac{a_i+a_{-i}}{c} - 2E - \frac{2R}{c} - \frac{R}{r} - \frac{R}{r+2s} + 2x_0\right) + P_1(s)\frac{(a_i-a_{-i})^2}{2}$$

$$+(a_i-a_{-i})\left(\frac{R\left(cr\left(r^2+4rs+2s^2\right)+2rs(r+2s)(r-\alpha_2)-c(r+s)(r+4s)\alpha_2\right)}{2r\left(2rs(r+2s)+c\left(r^2+4rs+2s^2\right)\right)(r-\alpha_2)}\right)$$

$$+P_2(s)(a_i-a_{-i})\left(\frac{a_i+a_{-i}}{c} - 2E - \frac{2R}{c} - \frac{R}{r} - \frac{R}{r+2s} + 2x_0\right) \qquad i=h,l \qquad (14)$$

with $P_1(s)$ and $P_2(s)$ two nonnegative functions of s,[19] defined respectively as:

$$P_1(s) = -\alpha_2\frac{(r+2s)^2\left(2cr\left(r^2+4rs+2s^2\right)-2c(r+2s)^2\alpha_2+r(r-\alpha_2)\left(4s(r+2s)+r\alpha_2\right)\right)}{4\left(2rs(r+2s)+c\left(r^2+4rs+2s^2\right)\right)^2(r-2\alpha_2)(r-\alpha_2)}, \text{ and}$$

$$P_2(s) = \frac{-\alpha_1(r-\alpha_1)\left(2sc(r+2s)+c(r+2s)(c+2s+\alpha_2)\right)-\alpha_2\,rc(r+2s)^2+\alpha_1\alpha_2\left(r^2+4rs+2s^2\right)}{4\left(2rs(r+2s)+c\left(r^2+4rs+2s^2\right)\right)(r-\alpha_1)(r-\alpha_1-\alpha_2)}.$$

I then define $V^{OL}(a_h,a_l,x_0,s)$, the sum of aggregate cumulative profits of both players following the open loop path:

$$V^{OL}(a_h,a_l,x_0,s) = V^{s,OL}_{a_h,a_l}(x_0,x_0) + V^{s,OL}_{a_l,a_h}(x_0,x_0) \qquad (15)$$

$$V^{OL}(a_h,a_l,x_0,s) = \frac{R^2}{r^2}\left(c+r+\frac{cr}{r+2s}\right) - \frac{\alpha_1(2c+\alpha_1)}{4(r-2\alpha_1)}\left(\frac{a_h+a_l}{c} - 2E - \frac{2R}{c} - \frac{R}{r} - \frac{R}{r+2s} + 2x_0\right)^2$$

$$+cR\left(\frac{r(r+2s)-\alpha_1(r+s)}{r(r+2s)(r-\alpha_1)}\right)\left(\frac{a_h+a_l}{c} - 2E - \frac{2R}{c} - \frac{R}{r} - \frac{R}{r+2s} + 2x_0\right) + P_1(s)(a_h-a_l)^2$$

[19] See Appendix B.1.4

2.2.1.4 The Steady State

This section focuses on the steady state. In equation (11), as t tends to infinity, the two first terms in the RHS vanish ($e^{\alpha_1 t}$ and $e^{\alpha_2 t}$ converge to zero given the negative sign of α_1 and α_2), and the system V converges continuously toward V^{SS}, the system at the steady state, such that:[20]

$$V^{SS} = -[A]^{-1}.B \tag{16}$$

The subsequent stock of player i, at the steady state, and level of use (more details in Appendix B.1.3) are, respectively:

$$x_i^{SS} = E - \frac{(a_i + a_{-i})}{2c} + \frac{R}{c} + \frac{R}{2r} + \frac{R}{2r+4s} - \frac{(a_i - a_{-i})}{2} F(s) \qquad i=h,l \tag{17}$$

$$w_i^{SS} = R + (a_i - a_{-i})\, s\, F(s) \qquad i=h,l \tag{18}$$

Where $F(s)$, as defined before (13.a), a nonnegative function of s.

I can already derive the following consequences of inequality:

$$0 < (w_i^{SS} - w_{-i}^{SS})(a_i - a_{-i}),$$

which indicates that the agent with higher efficiency uses more water at the steady state; a higher transmissivity also stimulates greater transfer of water towards the more efficient agent, for any given level of inequality $(a_i - a_{-i})$.[21]

$$(x_i^{SS} - x_{-i}^{SS})(a_i - a_{-i}) < 0,$$

[20] An alternative way to derive the steady state is by recognizing that it corresponds to the situation where the system stops moving, the motion and transition equations are set to zero: $\dot{x}_{it} = \dot{\lambda}_{it} = \dot{\mu}_{it} = 0$, while the optimality conditions continue to hold.

[21] Note that $w_i^{SS} - w_{-i}^{SS} = 2(a_i - a_{-i})s\,F_{SS}(s)$ is increasing in s (in absolute value).

which shows that the efficient agent endures higher extraction rates since her stock is the lowest (if $a_h > a_l$ then $x_h^{SS} < x_l^{SS}$).[22] This also means, since the efficient agent uses more water, that her extraction costs are higher.

However, the difference in efficiency has no effect on the average stock of water in the aquifer $\left(\left(x_h^{SS} + x_l^{SS}\right)/2\right)$. The average stock is decreasing in transmissivity, which is consistent with the standard concept that more access to the resource leads to its physical depletion.

The net profit of player *i* at the steady state, noted $NP_{a_i,a_{-i}}^{s,SS-OL}$, is given by:

$$NP_{a_i,a_{-i}}^{s,SS-OL} = \frac{R^2}{2} + \frac{c R^2 (r+s)}{r(r+2s)} + R P_3(s)(a_i - a_{-i}) + \frac{P_4(s)}{2}(a_i - a_{-i})^2 \qquad i=h,l \quad (19)$$

with *P₃(s)* and *P₄(s)* two nonnegative functions of *s*, defined respectively as $P_3(s) = \frac{s(2c(r+s)+r(r+2s))}{2rs(r+2s)+c(r^2+4rs+2s^2)}$ and $P_4(s) = \frac{rs^2(r+2s)(2c(r+s)+r(r+2s))}{\left(2rs(r+2s)+c(r^2+4rs+2s^2)\right)^2}$.

Finally, the aggregate profit at the steady state is obtained by summing up the profits of the two players:

$$TNP^{SS-OL}(a_h, a_l, s) = R^2 + \frac{2 c R^2 (r+s)}{r(r+2s)} + P_4(s)(a_h - a_l)^2 \qquad (20)$$

2.2.1.5 Preliminary Results under Open Loop Equilibrium

***Result OL1**: Under Open Loop, the aggregate profits from the CPR for identical players are decreasing in the level of transmissivity.*

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[22] The overall use at the steady state is, however, unaffected by inequality since $w_h^{SS} + w_l^{SS} = 2R$, while the average stock $\frac{(x_h + x_l)}{2}$ at the steady state is only a function of the average efficiency $\frac{(a_h + a_l)}{2}$ and s.

The first two terms on the RHS of equation (20) correspond to the total profits at the steady state, when both players display the same levels of efficiency. Indeed, for identical players ($a_h = a_l = a$) equation (20) suggests:

$$TNP^{SS-OL}(a,a,s) = R^2 + \frac{2cR^2(r+s)}{r(r+2s)}$$

Thus, $TNP^{SS-OL}(a,a,s)$ is a function only of s and, as portrayed in figure 4.a, transmissivity has a steady diminishing effect of on the total profits at the steady state for identical players.[23] This is due to the fact that the only effect of a higher transmissivity for identical agents is to intensify the race to the bottom, decreasing the stocks of water (for both agents) and resulting in higher extraction costs.

The same observation on the effect of transmissivity is valid with cumulative profits. With identical users, the change in aggregate cumulative profits, as transmissivity increases from zero to infinity, is given by:[24]

$$\begin{aligned} V^{OL}(a,a,x_0,\infty) - V^{OL}(a,a,x_0,0) = -\frac{1}{8(3c+r)}\Bigg(& PQ_1 + PQ_2\left(\frac{a}{c} - E - \left(\frac{1}{c}+\frac{1}{r}\right)R + x_0\right) \\ & + PQ_3\left(\frac{a}{c} - E - \left(\frac{1}{c}+\frac{1}{r}\right)R + x_0\right)^2\Bigg) \end{aligned}$$

where PQ_1, PQ_2 and PQ_3 are three positive variables (See Appendix B.1.5.1 for details).

From the formula it can be concluded that the difference holds a negative sign when the stock at the start is at the maximum, E, given that the conditions for interior solution

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[23] Its first derivative with respect to transmissivity is $\frac{\partial TNP^{SS-OL}(a,a,s)}{\partial s} = \frac{-2cR^2}{(r+2s)^2} < 0$.

[24] For simplification I use $V^{OL}(a,a,x_0,\infty)$ to refer to the limit of $V^{OL}(a,a,x_0,s)$ as s approaches infinity.

(with zero transmissivity) entail $\left(\frac{a}{c}-\left(\frac{1}{c}+\frac{1}{r}\right)R\right)$ positive.[25] In Appendix B.1.5.1, I provide the proof that the difference is always negative, for any level of stock at the start that corresponds to an interior solution; which shows the decreasing effect from a non-incremental increase in transmissivity on the aggregate cumulative profits of identical users.

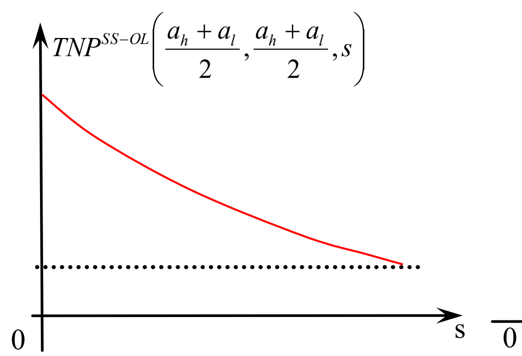

Figure 4.a Transmissivity effect under equality

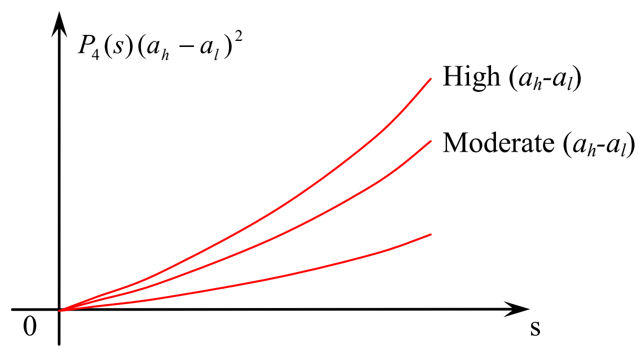

Figure 4.b Inequality component.

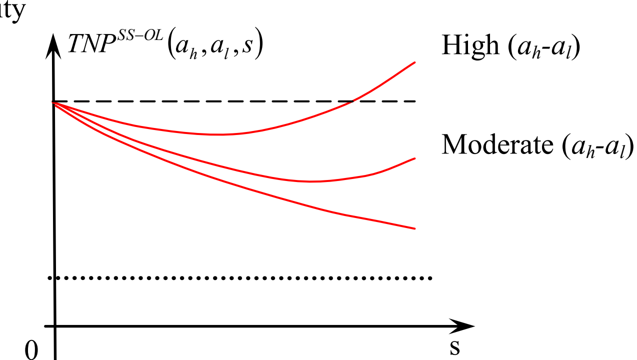

Figure 4.c Combined effect of transmissivity and inequality

Figure 4: Individual and Combined effects of transmissivity and inequality

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[25] In the general case, with s=0, the condition on stock (the level of water cannot rise above the ground) at the steady state translates as $\left(\frac{a_l}{c}-\left(\frac{1}{c}+\frac{1}{r}\right)R\right)$.

Result OL2*: Under Open Loop, inequality has an increasing effect on the aggregate profits from the CPR.*

At the steady state, the last term on the RHS of equation (20), depicted in figure 4.b, is positive and increasing in s and in the degree of inequality.[26] It shows that inequality generates extra payoffs that are proportional in magnitude to the square of ($a_h - a_l$), and that more transmissivity boosts those additional benefits. For instance, when there is no transmissivity ($s = 0$), $P_4(s)$ is equal to zero and the benefits from inequality vanish.

A similar increasing effect of inequality is depicted for the aggregate cumulative profits from the Open Loop game; based on the computed formula for aggregate cumulative profits (15) I obtain a net effect of inequality as follows:

$$V^{OL}(a_h, a_l, x_0, s) - V^{OL}\left(\frac{a_h + a_l}{2}, \frac{a_h + a_l}{2}, x_0, s\right) = P_1(s)(a_h - a_l)^2$$

with $P_1(s)$, as introduced in (14), a nonnegative function of s.

Result OL3*: In the case of highly unequal players, transmissivity has an increasing effect on aggregate profits.*

The first derivative of aggregate profits at the steady state with respect to transmissivity is given by:

$$\frac{\partial TNP^{SS-OL}(a_h, a_l, s)}{\partial s} = \frac{-2cR^2}{(r+2s)^2} + (a_h - a_l)^2 P_4'(s) \tag{21}$$

---

[26] Note that $P_4'(s) = \dfrac{2cr^2 s(r(r+2s)^3 + c(2r^3 + 9r^2 s + 16rs^2 + 10s^3))}{\left(2rs(r+2s) + c(r^2 + 4rs + 2s^2)\right)^3}$.

where the first term is negative, while the second part is strictly positive and increasing (in magnitude) in the level of inequality.[27]

A marginal increase in transmissivity has two opposite effects. It diminishes overall benefits due to amplified externalities (the first RHS term), as better circulation of water has the effect of decreasing the (average) stock at the steady state resulting in higher extraction costs for both players.

On the other hand, it has a welfare increasing effect due to a greater benefits generated when allowing the efficient player to have more access to his/her neighbor's stock (second RHS term). Indeed, as transmissivity increases so does the difference in the level of water use between the two players (in favor of the efficient solution) which translates into higher revenues from water use.

Regarding the effect (of a non-incremental increase in transmissivity) on aggregate cumulative profits, a detailed analysis is presented in Appendix B.1.5.2; for the two extreme cases when with the stock at the start is at the maximum, E, and when it is a the minimum level for an interior solution. In both cases, the effect displays the same tendency as established at the steady state, and depends, in the same way, on the average efficiency combined with the level of inequality. At higher levels of average efficiency, and higher levels of inequality, the effect is increasing. When the average efficiency is low and/or inequality is not high enough, a non-incremental increase in transmissivity causes a drop in aggregate cumulative profits.

[27] This is simply the derivative in the case of symmetric players,

***Result OL4**: Transmissivity always decreases profits for less efficient players and, in the case of a high inequality between players, increases benefits for the more efficient ones.*

The effect of a marginal increase in transmissivity on the individual profits of player i,[28] at the steady state, is given by:

$$DNP^{s,SS-OL}_{a_i,a_{-i}} = \frac{\partial NP^{s,SS-OL}_{a_i,a_{-i}}}{\partial s} = -\frac{c\,R^2}{(r+2\,s)^2} + R\,P_3'(s)(a_i - a_{-i}) + \frac{P_4'(s)}{2}(a_i - a_{-i})^2 \qquad i=h,l$$

where $P_3'(s)$ and $P_4'(s)$ are respectively, the derivatives of $P_3(s)$ and $P_4(s)$, and hold a positive sign everywhere.

Using the change of variables introduced in Appendix B.1.5.2 ($a_h = \bar{a} + \varepsilon$ and $a_l = \bar{a} - \varepsilon$), the effect on the less efficient user can be rewritten as:

$$DNP^{s,SS-OL}_{\bar{a}-\varepsilon,\bar{a}+\varepsilon} = -\frac{c\,R^2}{(r+2\,s)^2} - 2\,R\,P_3'(s)\,\varepsilon + 2\,P_4'(s)\,\varepsilon^2$$

The marginal effect of an increase in transmissivity holds a negative sign at the origin (at ε=0, with identical users); at the highest level of inequality with a non-negative solution w_l^{SS} in equation (18), $\varepsilon = \varepsilon_{maxW}$, the marginal effect is nil for the less efficient user:

$$DNP^{s,SS-OL}_{\bar{a}-\varepsilon_{mazX},\bar{a}+\varepsilon_{mazX}} = 0$$

Given that the marginal effect on the less efficient user is quadratic in ε, with a strictly positive coefficient in ε^2, it is safe to conclude that the effect is negative for any level of inequality between 0 and ε_{maxW}.

[28] Individual profits are provided in Equation (19).

For the efficient user, the marginal effect of an increase in transmissivity still holds a negative sign at the origin (at ε=0, with identical users); however, for $\varepsilon = \varepsilon_{maxW}$, the marginal effect is strictly positive:

$$DNP^{s,SS-OL}_{\bar{a}+\varepsilon_{mazX},\bar{a}-\varepsilon_{mazX}} = 2RP_3'(s)\,\varepsilon_{mazX}$$

Result OL4 provides a clarification with regard to the interpretation of Result OL3, the increase in profits for highly unequal users, when it occurs, is not generalized, it is entirely taken by the efficient user, while the less efficient one sees her profits drop.

2.2.2 The Closed Loop Game

In this subsection, I will try to expand the analysis and solve for the players' decisions, throughout the exploitation horizon, under feedback strategies. Due to the limitations of computer capacity (see Appendix F for full description of computing resources used in this research), I only present the complete solution in the case of infinite transmissivity; for finite levels of transmissivity, I only develop the general form of the solution in Appendix B.2.1.

I use the Bellman Equation to derive the optimality conditions regarding the individual players' decisions then I solve for the optimal path under feedback strategies and the related steady state. The computed (aggregate and individual) cumulative profits are then used to confirm the previously obtained results, under Open Loop strategies.

2.2.2.1 The Closed Loop Nash equilibrium with Infinite transmissivity

Given the assumption of infinite transmissivity and the equal (across compartments) stock of water x_0 in the aquifer at time t = 0, I conclude that the level of water is –all the time– the same in both parts of the aquifer: $x_{ht} = x_{lt} = x_t$.

The condition on mass conservation dictates that, in a perfectly transmissive aquifer, the stock evolution follows a simplified equation of motion:

$$\dot{x}_t = R - \frac{w_{ht} + w_{lt}}{2} \tag{22}$$

Let $V^{CL,\infty}_{a_i,a_{-i}}(x_t)$ designate player *i's* maximal present value when the stock of water in the aquifer is x_t, the Bellman equation corresponding to the dynamic maximization problem is given by:

$$rV^{CL,\infty}_{a_i,a_{-i}}(x_t) = \max_{w_{it}}\left\{a_i w_{it} - \frac{w_{it}^2}{2} - c\,w_{it}(E - x_t) + \left(R - \frac{w_{it} + w_{-it}}{2}\right)Vx^{CL,\infty}_{a_i,a_{-i}}(x_t)\right\} \quad i=h,l \tag{23}$$

where $Vx^{CL,\infty}_{a_i,a_{-i}}(x_t)$ is the first derivative of the value function $V^{CL,\infty}_{a_i,a_{-i}}(x_t)$.

2.2.2.2 Solving for the Closed Loop Nash equilibrium

Solving for the maximal present value function, for the individual player, provides the optimal path and cumulative profits for any given level of stock x_t, particularly at the start of the game, when $x_t = x_0$.

The optimal level of extraction for player *i* (when she chooses to extract) follows:

$$w_{it} = a_i - c(E - x_t) - \frac{Vx^{CL,\infty}_{a_i,a_{-i}}(x_t)}{2} \quad i=h,l \tag{24}$$

substituting the new expressions of w_{it} in (23) I obtain:

$$\begin{aligned} rV^{CL,\infty}_{a_i,a_{-i}}(x_t) = {} & \frac{(a_i - cE)^2}{2} + \left(R + cE - \frac{(a_i + a_{-i})}{2}\right)Vx^{CL,\infty}_{a_i,a_{-i}}(x_t) + \frac{\left(Vx^{CL,\infty}_{a_i,a_{-i}}(x_t)\right)^2}{8} \\ & + \frac{Vx^{CL,\infty}_{a_i,a_{-i}}(x_t)\,Vx^{CL,\infty}_{a_{-i},a_i}(x_t)}{4} + c(a_i - cE)x_t - c\,x_t\,Vx^{CL,\infty}_{a_i,a_{-i}}(x_t) - c^2\frac{x_t^{\,2}}{2} \quad i=h,l \end{aligned} \tag{25}$$

From the last result, it can be concluded that the solution to player *i* dynamic programming problem is polynomial of second order in x_t:

$$V^{CL}_{a_i,a_{-i}}(x_t) = \beta_{0i} + \beta_{1i}\, x_t + \beta_{2i}\, x_t^{\,2} \qquad i=h,l \qquad (26)$$

Substituting the new formula for individual cumulative profits for players *h* and *l*, $V^{CL,\infty}_{a_h,a_l}(x_t)$ and $V^{CL,\infty}_{a_l,a_h}(x_t)$ (and the subsequent $Vx^{CL,\infty}_{a_h,a_l}(x_t)$ and $Vx^{CL,\infty}_{a_l,a_h}(x_t)$) in equation (25) generates a system of 6 equations in 6 unknowns: β_{0h}, β_{1h}, β_{2h}, β_{0l}, β_{1l}, and β_{2l}. I solve for all the betas[29] and obtain finally the analytical expression for the value function $V^{CL,\infty}_{a_i,a_{-i}}(x_t)$**:**

$$\begin{aligned}
V^{CL,\infty}_{a_i,a_{-i}}(x_t) = {} & \frac{PQ_{V1}}{2} + \frac{PQ_{V2}}{2}\left(\frac{a_i + a_{-i}}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_t\right) \\
& + PQ_{Vi1}(a_i - a_{-i}) + \frac{PQ_{V3}}{2}\left(\frac{a_i + a_{-i}}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_t\right)^2 \qquad i=h,l \qquad (27) \\
& + \frac{PQ_{V4}}{2}(a_i - a_{-i})^2 + PQ_{Vi2}(a_i - a_{-i})\left(\frac{a_i + a_{-i}}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_t\right)
\end{aligned}$$

where PQ_{V1}, PQ_{V2}, PQ_{V3}, PQ_{V4}, PQ_{Vi1} and PQ_{Vi2} are all positive values, function only of r, c and R (See Appendix B.2.1).

Deriving $V^{CL,\infty}_{a_i,a_{-i}}(x_t)$ and replacing in (24) provides the decision rule for player *i*:[30]

$$\begin{aligned}
w^{CL}_{it}(x_t) = {} & a_i - c(E - x_t) - \frac{PQ_{V2}}{4} \\
& - \frac{PQ_{V3}}{2}\left(\frac{a_i + a_{-i}}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_t\right) - \frac{PQ_{Vi2}}{4}(a_i - a_{-i})
\end{aligned} \qquad (28)$$

---

[29] Obviously, there is more than one set of solutions; however, only one set is stable and holds a meaningful economic sense, it is therefore the only one kept.

[30] Can be rewritten as:

$$\begin{aligned}
w^{CL}_{it}(x_t) = {} & \frac{1}{3}\left(c - r + \sqrt{c^2 + 4cr + r^2}\right)\left(\frac{a_i + a_{-i}}{2c} - h - \frac{R}{c} - \frac{R}{2r} + x_t\right) \\
& + (a_i - a_{-i})\frac{\left(c^2 + 7cr + 8r^2 + c\sqrt{c^2+4cr+r^2}\right)}{2\left(5c^2 + 12cr + 8r^2\right)} + \frac{\left(c^2 + 9cr + 26r^2 + (c - 2r)\sqrt{c^2+4cr+r^2}\right)R}{6r(c + 4r)}
\end{aligned}$$

Finally, the aggregate cumulative profits with a closed loop game, denoted $V^{CL}(a_h, a_l, x_0, \infty)$[31], sum up to:

$$V^{CL}(a_h, a_l, x_0, \infty) = PQ_{V1} + PQ_{V2}\left(\frac{a_h + a_l}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_0\right) + PQ_{V3}\left(\frac{a_h + a_l}{2c} - E - \frac{R}{c} - \frac{R}{r} + x_0\right)^2 + PQ_{V4}(a_h - a_l)^2 \tag{29}$$

2.2.2.3 The Steady State in the Closed Loop game

At the steady state, the stock is constant and $\dot{x}_t = 0$; using the values of w_{ht} and w_{lt} (from (28)) in (22) provides the stock at the steady state:

$$x^{SS-CL} = E - \frac{(a_h + a_l)}{2c} + \frac{R}{c} + \frac{R}{r} - \frac{\left(c + 3r + \sqrt{c^2 + 4cr + r^2}\right)R}{2r(c + 4r)} \tag{30.a}$$

The corresponding extraction decisions are as follows:

$$w_i^{SS-CL} = R + (a_i - a_{-i})\frac{\left(c^2 + 7cr + 8r^2 + c\sqrt{c^2 + 4cr + r^2}\right)}{2\left(5c^2 + 12cr + 8r^2\right)} \qquad i=h,l \tag{30. b}$$

The system specifications at the steady state reveal some direct consequences of inequality:

$$0 < (w_i^{SS-CL} - w_{-i}^{SS-CL})(a_i - a_{-i})$$

Similar to under Open Loop, the agent with higher efficiency uses more water at the steady state; a higher transmissivity stimulates greater transfer of water towards the more efficient agent, the increase is proportional to the level of inequality $(a_i - a_{-i})$.[32]

---

[31] Clearly, the aggregate cumulative profits are given, at any time *t*, by:

$$V^{CL}(a_h, a_l, x_t, \infty) = V^{CL}_{a_h, a_l}(x_t) + V^{CL}_{a_l, a_h}(x_t)$$

However, the difference in efficiency has no effect on the stock of water in the aquifer at the steady state, x^{SS-CL}.

Given the stock and the extraction levels at the steady state, it is possible to compute the individual net profit of each user *i* at the steady state, denoted $NP_{a_i,a_{-i}}^{SS-CL}$, as follows:

$$NP_{a_i,a_{-i}}^{SS-CL} = \frac{PQ_{CL1}}{2} + PQ_{CL2}\left(a_i - a_{-i}\right) + \frac{PQ_{CL3}}{2}\left(a_i - a_{-i}\right)^2 \qquad i=h,l \quad (31)$$

where PQ_{CL1}, PQ_{CL2} and PQ_{CL3} are three positive variables (see Appendix B.2.3).

Finally, the aggregate profits at the steady state, with an infinite transmissivity, are given by:

$$TNP^{SS-CL}\left(a_h, a_l\right) = PQ_{CL1} + PQ_{CL3}\left(a_h - a_l\right)^2 \qquad (32)$$

2.2.2.4 Preliminary Results under Closed Loop Equilibrium

Since the solution for the closed loop game was only provided for an aquifer with infinite transmissivity, it will not be possible to test the effect of a marginal change in transmissivity; however, it is always possible to test for the non-marginal effect when transmissivity shifts from zero to infinity. Keeping in mind that for a zero transmissivity aquifer, both feedback and open loop strategies converge to a single cell maximization problem provided, the solution with zero transmissivity is obtainable from the – already available – solution with Open Loop strategies, by computing the limits as s tends to zero.

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[32] $w_i^{SS} - w_{-i}^{SS} = \left(a_i - a_{-i}\right)\left(\frac{c^2 + 7cr + 8r^2 + c\sqrt{c^2 + 4cr + r^2}}{5c^2 + 12cr + 8r^2}\right)$.

Result CL1*: Under Closed Loop, the aggregate profits from the CPR for identical players with an infinite transmissivity are lower than with zero transmissivity.*

To verify this result for a non-marginal increase in transmissivity, at the steady state, it suffices to compare, for identical users, the benefits with an infinite transmissivity (in equation (32)), to the benefits from the OL game (equation (20)) with zero transmissivity (s=0):

$$TNP^{SS-CL}(a,a)-\lim_{s\to 0} TNP^{SS-OL}(a,a,s)=PQ_{CL1}-\lim_{s\to 0}\left(R^2+\frac{2cR^2(r+s)}{r(r+2s)}\right)$$
$$=-\frac{cR^2\left(c+3r+\sqrt{c^2+4cr+r^2}\right)}{r(c+4r)}$$

The negative sign is indicative of the loss from making the CPR more common under feedback strategies, the losses to identical users are increasing, in magnitude, in the rate of recharge and in the cost of extraction.

The same approach is adopted to show negative effect of a non-marginal increase in transmissivity on the aggregate cumulative profits, the related difference is given by:[33]

$$V^{CL}(a,a,x_0,\infty)-V^{CL}(a,a,x_0,0) = -\left(PQ_{R11}+PQ_{R12}\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)\right.$$
$$\left.+PQ_{R13}\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)^2\right) \qquad (33)$$

where PQ_{R11}, PQ_{R12} and PQ_{R13} are three positive variables (See Appendix B.2.4.1).

In Appendix B.2.4.1, I provide the proof that the difference is always negative.

[33] The Cumulative profits with zero transmissivity are taken, as discussed before, from the Open Loop Game.

Result CL2*: Under Closed Loop, inequality has an increasing effect on the aggregate profits from the CPR.*

The result is easily established at the steady state, from equation (32) I can derive that the extra payoffs generated by inequality are proportional in magnitude to the square of $(a_h - a_l)$:

$$TNP^{SS-CL}(a_h, a_l) - TNP^{SS-CL}\left(\frac{a_h + a_l}{2}, \frac{a_h + a_l}{2}\right) = PQ_{CL3}\,(a_h - a_l)^2$$

The marginal effect of transmissivity on inequality related gains is difficult to predict since the closed loop problem is only solved for an aquifer with an infinite transmissivity.

Regarding the aggregate cumulative profits from the Closed Loop game, the same increasing effect of inequality is discernible; the computed formula for aggregate cumulative profits (29) suggests:

$$V^{CL}(a_h, a_l, x_0, \infty) - V^{CL}\left(\frac{a_h + a_l}{2}, \frac{a_h + a_l}{2}, x_0, \infty\right) = PQ_{V4}\,(a_h - a_l)^2$$

Result CL3*: Under Closed Loop, and in the presence of a relatively high ratio r/c, making the resource more common has an increasing effect on the aggregate profits of highly unequal players.*

In the case of unequal users, the effect of a non-marginal increase in transmissivity, from zero to infinity, on the aggregate profits at the steady state is given by:

$$TNP^{SS-CL}(a_h, a_l) - TNP^{SS-OL}(a_h, a_l, 0) = -\frac{c R^2 \left(c + 3r + \sqrt{c^2 + 4cr + r^2}\right)}{r(c + 4r)} + PQ_{CL3}\,(a_h - a_l)^2$$

The difference shows, as discussed earlier, that the effect is strictly negative for identical users; it also shows that the effect is increasing in the level of inequality. In order to determine the effect of transmissivity on highly unequal users it suffices to check at the highest acceptable level –with an interior solution– of inequality.

The maximum inequality depends on the average efficiency, for low values of $\bar{a}$ the maximum inequality is given by ε_{MaxX} with zero transmissivity ($\bar{a}-(c+r)R/r$), when $\bar{a}$ is higher than $\bar{a}_{maxCL}$, the maximum inequality is $\varepsilon_{\mathrm{maxCL}}$,[34] derived from the condition on the positive rate of extraction by the less efficient user, with an infinite transmissivity. In this last case, the effect of an increase in transmissivity on aggregate profits when inequality among users is at the highest level ($\varepsilon_{\mathrm{maxCL}}$) is as follows:

$$TNP^{SS-CL}\left(a+\varepsilon_{\max CL},a-\varepsilon_{\max CL}\right)-TNP^{SS-OL}\left(a+\varepsilon_{\max CL},a-\varepsilon_{\max CL},0\right)=\frac{(-4c^3+5cr^2+4r^3)R^2}{r(3cr+4r^2+2c\sqrt{c^2+4cr+r^2})}$$

The difference follows the sign of $(-4c^3+5cr^2+4r^3)$ and has only one real root, $(r/c)^*$ =0.71; the difference is hence positive if (and only if) the ratio r/c is higher than $(r/c)^*$.

For lower levels of average efficiency ($\bar{a} < \bar{a}_{maxCL}$), a somewhat analogous result emerges, the effect is always negative for low levels of r/c; for r/c higher than $(r/c)^*$, the effect of

[34] $\varepsilon_{\mathrm{maxCL}}=\frac{(5c^2+12cr+8r^2)R}{c^2+8r^2+c(7r+\sqrt{c^2+4cr+r^2})}$ is the maximum level of inequality corresponding to a strictly positive level of extraction by the less efficient user at the steady state, under closed loop ($w_l > 0$), for s infinite. $\bar{a}_{maxCL}=\frac{(4c^3+33c^2r+60cr^2+32r^3)R}{r(3c^2+17cr+16r^2+c\sqrt{c^2+4cr+r^2})}$ is the level of efficiency at which $\varepsilon_{\mathrm{maxCL}}$ equals $\varepsilon_{\mathrm{maxX}}$ with zero transmissivity.

an increase in transmissivity is negative for low levels of average efficiency and is positive for $\bar{a}$ higher than $\bar{a}_{maxX}$, such that:[35]

$$TNP^{SS-CL}\left(a_{\max X}+\varepsilon_{\max X}, a_{\max X}-\varepsilon_{\max X}\right)-TNP^{SS-OL}\left(a_{\max X}+\varepsilon_{\max X}, a_{\max X}-\varepsilon_{\max X}, 0\right)=0$$

In summary, with feedback strategies, the increasing effect of a non-incremental increase in transmissivity, on aggregate profits at the steady state for highly unequal users, is only conceivable in the case of low costs of extraction (*c* low) combined with a low valuation of future gains and avoided future losses (high *r*).

Regarding the aggregate cumulative profits, the effect of a non-marginal increase in transmissivity on the profits of unequal players is given by:[36]

$$\begin{aligned}V^{CL}\left(a_h, a_l, x_0, \infty\right)-V^{CL}\left(a_h, a_l, x_0, 0\right) = -\Bigg(& PQ_{R11}+PQ_{R12}\left(\frac{a_h+a_l}{2c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)\\ & +PQ_{R13}\left(\frac{a_h+a_l}{2c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)^2\Bigg)+PQ_{R31}\left(a_h-a_l\right)^2\end{aligned}$$

where PQ_{R31} is a positive variable.

For identical users the effect of an increase in transmissivity is negative (as established in Result CL1) but the shift in aggregate cumulative profits is increasing in the level of inequality. To investigate the conditions for an overall increasing effect on the aggregate

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[35] $\bar{a}_{maxX}=\frac{R}{2}\sqrt{\frac{c^2+3cr+c\sqrt{c^2+4cr+r^2}}{PQ_{CL3}\, r(c+4r)}}+\frac{(c+r)R}{r}$, is always positive and higher than $\frac{(c+r)R}{r}$, the minimum efficiency, however, $\bar{a}_{maxX}$ is lower than $\bar{a}_{maxCL}$, if and only if, $(-4c^3+5cr^2+4r^3)$ is positive.

[36] The Cumulative profits with zero transmissivity are taken, as discussed before, from the Open Loop Game.

cumulative profits for highly unequal users, it suffices to check the sign of the effect when inequality is at the maximum acceptable level.

In Appendix B.2.4.2, I show that, with feedback strategies, the effect of a non-incremental increase in transmissivity on aggregate cumulative profits of highly unequal users is analogous to the effect on aggregate profits at the steady state. A positive increase in aggregate cumulative profits is only possible in the case of highly unequal users with a high average efficiency, engaging in an extraction game with limited extraction costs, as evidenced by a quotient *r/c* higher than (*r/c)**.

The main singularity with cumulative profits resides in the fact that the conditional quotient (*r/c)** depends on the stock at the start of the game.[37]

Result CL4: *Under Closed Loop, transmissivity always decreases profits for the less efficient player and, in the case of a high inequality between players, the increase in benefits for the more efficient ones is only observed in the presence of a relatively high ratio r/c.*

The effect of a non-marginal increase in transmissivity, from zero to infinity, on the individual profits of player *i*, at the steady state, is given by:[38]

$$NP^{SS-CL}_{a_i,a_{-i}} - \lim_{s\to 0} NP^{s,SS-OL}_{a_i,a_{-i}} = -PQ_{R41} + PQ_{CL2}\left(a_i - a_{-i}\right) + \frac{PQ_{CL3}}{2}\left(a_i - a_{-i}\right)^2 \quad i{=}h,l \qquad (34)$$

where PQ_{R41} is a positive variable.[39]

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[37] (*r/c)** varies from 0.364266 when x_0 is set at the maximum level, E, to 0.254542 when x_0 is set at the minimum stock for an interior solution.

[38] With an infinite transmissivity, equation (31) provides the closed loop solution for the individual profits at the steady state; for *s*=0, the solution is derived from the open loop game solution in Equation (19).

The effect on the less efficient user can be rewritten as:

$$NP^{SS-CL}_{\bar{a}-\varepsilon,\bar{a}+\varepsilon} - \lim_{s\to 0} NP^{s,SS-OL}_{\bar{a}-\varepsilon,\bar{a}+\varepsilon} = -PQ_{R41} - 2\,PQ_{CL2}\,\varepsilon + 2\,PQ_{CL3}\,\varepsilon^2$$

The effect holds a negative sign at the origin (at ε=0, with identical users), and at the highest level of inequality; for instance, when $\bar{a}$ is higher than $\bar{a}_{maxCL}$, the effect at maximum inequality ($\varepsilon = \varepsilon_{maxCL}$) is ε_{maxCL}, the decreasing effect is straightforward:[40]

$$NP^{SS-CL}_{\bar{a}-\varepsilon_{\max CL},\bar{a}+\varepsilon_{\max CL}} - \lim_{s\to 0} NP^{s,SS-OL}_{\bar{a}-\varepsilon_{\max CL},\bar{a}+\varepsilon_{\max CL}} = -\frac{(2\mathrm{c}+\mathrm{r})}{2r} R^2$$

When $\bar{a}$ is lower than $\bar{a}_{maxCL}$, the maximum inequality is lower than ε_{maxCL}, therefore, the increase in transmissivity as always a decreasing effect for the less efficient user.[41]

For the effeicient user the effect is as follows:

$$NP^{SS-CL}_{\bar{a}+\varepsilon,\bar{a}-\varepsilon} - \lim_{s\to 0} NP^{s,SS-OL}_{\bar{a}+\varepsilon,\bar{a}-\varepsilon} = -PQ_{R41} + 2\,PQ_{CL2}\,\varepsilon + 2\,PQ_{CL3}\,\varepsilon^2$$

The effect is negative at the origin, but the sign at higher levels of inequality is ambiguous, depending on the ratio *r/c*; indeed for $\varepsilon = \varepsilon_{maxCL}$, the effect is as follows:

$$NP^{SS-CL}_{\bar{a}-\varepsilon_{\max CL},\bar{a}+\varepsilon_{\max CL}} - \lim_{s\to 0} NP^{s,SS-OL}_{\bar{a}-\varepsilon_{\max CL},\bar{a}+\varepsilon_{\max CL}} = \frac{(-12c^3+44c^2r+81cr^2+36r^3)R^2}{2r(2c^2+15cr+12r^2+4c\sqrt{c^2+4cr+r^2})}$$

---

[39] $PQ_{R41} = \frac{(2c+r)R^2}{2r} - \frac{PQ_{CL1}}{2} = \frac{c(c+3r+\sqrt{c^2+4cr+r^2})R^2}{2r(c+4r)}$

[40] By definition, at the highest level of inequality the less efficient user has a zero level of use at the steady state when *s* infinite, and the effect is $-\lim_{s\to 0} NP^{s,SS-OL}_{\bar{a}-\varepsilon_{\max CL},\bar{a}+\varepsilon_{\max CL}} = -\frac{(2\mathrm{c}+\mathrm{r})}{2r} R^2$. When

[41] The effect is quadratic in ε, with a strictly positive coefficient in ε^2, in addition, the effect is negative at the origin and at $\varepsilon = \varepsilon_{maxCL}$, thus, the effect is negative for any value of ε between zero and ε_{maxCL}.

The difference follows the sign of $(-12c^3 + 44c^2r + 81cr^2 + 36r^3)$ and has only one real root, *(r/c)** =0.20; the difference is hence positive if (and only if) the ratio *r/c* is higher than *(r/c)**.

The condition on *r/c* for the efficient user to benefit from making the resource more common is less demanding than the condition for aggregate profits (*r/c* higher than 0.71); indeed, the increase in aggregate profits requires the efficient user, not only to benefit from the increase in transmissivity, but for his/her benefits to be high enough so as to outweigh the losses incurred by the less efficient user.

The same analysis, completed with individual cumulative profits, leads to the same result, where the less efficient user sees his/her profits shrink as transmissivity goes up, while the efficient user suffers lower losses when inequality is low or moderate, and can even benefit from the shift in transmissivity when high inequality is combined with a high ration *r/c*.[42]

2.3 The Social Optimum

I will present in this section the solution for the optimal extraction path when both players coordinate their decisions to maximize the sum of their cumulative (over time) profits. The objective function under Social Optimum management is as follows:

$$\int_0^\infty e^{-rt}\left\{\left(a_h\, w_{ht} - \frac{w_{ht}^2}{2} - w_{ht}\, c\left(E - x_{ht}\right)\right) + \left(a_l\, w_{lt} - \frac{w_{lt}^2}{2} - w_{lt}\, c\left(E - x_{lt}\right)\right)\right\} dt \qquad (35)$$

[42] The condition is for *r/c* to be higher than *(r/c)**, that varies from 0.37 when x_0 is set at the maximum level, E, to 0.09 when x_0 is set at the minimum stock for an interior solution.

The pair *h-l* applies the Pontryagin's Maximum Principle, and obtains the current value Hamiltonian[43]:

$$H_t = \left(a_h\, w_{ht} - \frac{w_{ht}^2}{2} - w_{ht}\, c\left(E - x_{ht}\right)\right) + \left(a_l\, w_{lt} - \frac{w_{lt}^2}{2} - w_{lt}\, c\left(E - x_{lt}\right)\right)$$
$$+ \lambda_{ht}\left(R - w_{ht} + s\,(x_{lt} - x_{ht})\right) + \lambda_{lt}\left(R - w_{lt} + s\,(x_{ht} - x_{lt})\right)$$

Where λ_{ht} (respectively λ_{lt}) denotes the co-state associated with player *h* stock x_{ht} (respectively her neighbor's stock x_{lt}), it reflects the shadow price value attributed by the pair *h-l* at time t, to a marginal change in x_{ht} (respectively in x_{lt}).[44]

Along the optimal path the following conditions need to be satisfied[45]:

C1 $$0 = \frac{\partial H_t}{\partial w_{it}} = a_i - w_{it} - c\left(E - x_{it}\right) - \lambda_{it} \qquad i=h,l \qquad (36.a)$$

C2 $$\dot{\lambda}_{it} = r\lambda_{it} - \frac{\partial H_t}{\partial x_{it}} = r\lambda_{it} - w_{it}c + s\lambda_{it} - s\lambda_{-it} \qquad i=h,l \qquad (36.c)$$

In addition to the equations of motion in both stocks.[46]

2.3.1 Solving for the Social Optimal Path

Deriving w_{it} and w_{-it} from C1 and replacing in C2 and in the transition equations allows to finally obtain a system of linear first order differential equations that can be written as:

$$\dot{V}_2 = [A_2].V_2 + B_2 \qquad (37)$$

[43] Again, the Hamiltonian is similar to that of an unconstrained problem; the assumption made here is that all optimal solution satisfies the conditions on state and control variables.

[44] The same solution is reached when using the Bellman Equation.

[45] The conditions on w_{it}, w_{-it}, x_{it} and x_{-it} are not included under the assumption that the optimal solution meets those conditions for an appropriate choice of a_i, a_{-i} and R.

[46] Equation 1.a: $\dot{x}_{it} = R - w_{it} + s\left(x_{-it} - x_{it}\right)$ for player *i=h,l*.

where $V_2 = [x_{it}, x_{-it}, \lambda_{it}, \lambda_{-it}]$, $B_2 = [R - a_i + cE,\ R - a_{-i} + cE,\ c^2E - c\,a_i,\ c^2E - c\,a_{-i}]$, and

$$[A_2] = \begin{bmatrix} -(c+s) & s & 1 & 0 \\ s & -(c+s) & 0 & 1 \\ -c^2 & 0 & (r+s+c) & -s \\ 0 & -c^2 & -s & (r+s+c) \end{bmatrix}.$$

The general solution to equation (37) is totally defined by the eigenvalues (and eigenvectors) of matrix A_2 (derived from Det[A_2- α $I_{4\text{x}4}$] =0, where $I_{4\text{x}4}$ is the identity matrix and Det[A_2- α $I_{4\text{x}4}$] is the determinant of [A_2- α $I_{4\text{x}4}$] that can be factorized as follows:

$$\text{Det}[A_2 - \alpha\, I_{4\text{x}4}] = EQ_3(\alpha)\ EQ_4(\alpha)$$

where $EQ_3(\alpha) = \left(r(\alpha+c) - \alpha^2\right)$, and $EQ_4(\alpha) = \left((2s+c)(2s+r) + 2sc + \alpha r - \alpha^2\right)$.

Only the negative eigenvalues of matrix A_2 are considered here; α_3, the negative root[47] of equation $EQ_3(\alpha) = 0$, and α_4 the negative root of $EQ_4(\alpha) = 0$, are given by:[48]

$$\alpha_3 = \frac{1}{2}\left(r - \sqrt{r}\,\sqrt{4c+r}\right) \text{ and,}$$

$$\alpha_4 = \frac{1}{2}\left(r - \sqrt{r+4s}\,\sqrt{4c+r+4s}\right).$$

The corresponding eigenvectors (proof in Appendix B.3.1) are respectively v_3 and v_4:

$$v_3 = \begin{bmatrix} 1 \\ 1 \\ c+\alpha_3 \\ c+\alpha_3 \end{bmatrix}, \qquad v_4 = \begin{bmatrix} 1 \\ -1 \\ c+\alpha_4+2s \\ -c-\alpha_4-2s \end{bmatrix}.$$

[47] The other roots are strictly positive and do not carry an economically relevant meaning and certainly do not correspond to an optimum solution to the objective maximization problem.

[48] For the extreme case with no transmissivity, s=0, both equations combine ($EQ_4(\alpha)$=$EQ_3(\alpha)$) and Det[A_2- α I_{4x4}]=0, has only one (quadratic) root, α_3.

The general solution hence derived is:

$$V_{2P} = c_3 e^{\alpha_3 t} v_3 + c_4 e^{\alpha_4 t} v_4$$

Where c_3 and c_4 are two arbitrary constants (to be determined so as to satisfy the boundary conditions).

The solution to the original linear system of differential equations ($\dot{V}_2 = [A_2].V_2 + B_2$) is simply given by:

$$V_2 = V_{2P} - [A_2]^{-1}.B_2 = c_3 e^{\alpha_3 t} v_3 + c_4 e^{\alpha_4 t} v_4 - [A_2]^{-1}.B_2 \tag{38}$$

Finally, the boundary conditions, i.e. the level of stocks at time zero, x_{i0} and x_{-i0}, arrange for a unique solution for the pair of constants, c_3 and c_4:

$$c_3 = \left(\frac{x_{i0} + x_{-i0}}{2} - E + \frac{a_i + a_{-i}}{2c} - \frac{R}{c} - \frac{R}{r} \right) \tag{39.a}$$

$$c_4 = \left(\frac{x_{i0} - x_{-i0}}{2} + \frac{(a_i - a_{-i})(r + 2s)}{4s(r+2s) + 2c\,(r+4s)} \right) \tag{39.b}$$

2.3.2 The Social Optimum Solution

Equation (38) provides the stock level of player i:

$$x_{it} = c_3 e^{\alpha_3 t} + c_4 e^{\alpha_4 t} + E - \frac{(a_i + a_{-i})}{2c} + \frac{R}{c} + \frac{R}{2r} - \frac{(a_i - a_{-i})(r+2s)}{4s(r+2s) + 2c(r+4s)} \quad i=h,l \tag{40.a}$$

Substituting the value of λ_{it}, provided by Equation (38), in condition C1 on the optimal level of water use (from Equation (36.a)) yields:

$$w_{it} = R - c_3 \alpha_3 e^{\alpha_3 t} - c_4 (2s + \alpha_4) e^{\alpha_4 t} + \frac{(a_i - a_{-i}) s (r+2s)}{2s(r+2s) + c(r+4s)} \qquad i=h,l \tag{40.b}$$

The computed values of x_{it} and w_{it} are used (Equation (5)) to obtain NP_{it}, the net payoff of player i at time t; after integration over time (as presented in Equation (6)), the cumulative profits of player i, given the same level of stocks at the start (x_0), noted $V^{s,SO}_{a_i,a_{-i}}(x_0,x_0)$ hereafter, are given as:

$$\begin{aligned}V^{s,SO}_{a_i,a_{-i}}(x_0,x_0) \;=\; & \frac{(2c+r)R^2}{2r^2}+\frac{P_6(s)}{2}(a_i-a_{-i})^2+\frac{cR}{r}\left(\frac{a_i+a_{-i}}{2c}-E-\frac{R}{c}-\frac{R}{r}+x_0\right)\\ &+(a_i-a_{-i})\left(P_5(s)\left(\frac{a_i+a_{-i}}{2c}-E-\frac{R}{c}-\frac{R}{r}+x_0\right)+\frac{R}{2r}+\frac{4Rcs(r+2s)}{r^2\left(r+\sqrt{r+4s}\sqrt{4c+r+4s}\right)^2}\right) \\ &+\frac{c^2}{2c+r+\sqrt{r}\sqrt{4c+r}}\left(\frac{a_i+a_{-i}}{2c}-E-\frac{R}{c}-\frac{R}{r}+x_0\right)^2\end{aligned} \quad (41)$$

with $P_5(s)$ and $P_6(s)$ two nonnegative functions of s (See Appendix B.3.2 for details).[49]

The aggregate cumulative profits – of both players – under social optimum management, denoted $V^{SO}(a_h,a_l,x_0,s)$ hereafter, is derived as:[50]

$$\begin{aligned}V^{SO}(a_h,a_l,x_0,s)= & \frac{(2c+r)R^2}{r^2}+\frac{2cR}{r}\left(\frac{a_i+a_{-i}}{2c}-E-\frac{R}{c}-\frac{R}{r}+x_0\right)\\ &+P_6(s)(a_i-a_{-i})^2+\frac{2c^2}{2c+r+\sqrt{r}\sqrt{4c+r}}\left(\frac{a_i+a_{-i}}{2c}-E-\frac{R}{c}-\frac{R}{r}+x_0\right)^2\end{aligned} \quad (42)$$

2.3.3 The Steady State under Social Optimum management

Under social optimum management, as t continues to increase, the system moves toward a steady state, specified as follows:[51]

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[49] $P_6(s) = \dfrac{(r+2s)^2}{2r^2\sqrt{r+4s}\sqrt{4c+r+4s}+4r\left(r^2+4rs+8s^2+2c(r+2s)\right)}$, for $P_5(s)$ see Appendix B.3.2.

[50] By summing up $V^{s,SO}_{a_h,a_l}(x_0,x_0)$ and $V^{s,SO}_{a_l,a_h}(x_0,x_0)$.

[51] From equation 35.a; given the sign of α_3 and α_4, as t tends to infinity, $e^{\alpha_3 t}$ and $e^{\alpha_4 t}$ converge to zero.

$$V_2^{SS} = -[A_2]^{-1}.B_2 \tag{43}$$

The subsequent stocks and players' decisions are, respectively:

$$x_i^{SS-SO} = E - \frac{(a_i + a_{-i})}{2c} + \frac{R}{c} + \frac{R}{r} - \frac{(a_i - a_{-i})(r+2s)}{4s(r+2s)+2c(r+4s)} \qquad i=h,l \tag{44}$$

$$w_i^{SS-SO} = R + \frac{s(a_i - a_{-i})(r+2s)}{4s(r+2s)+2c(r+4s)} \qquad i=h,l \tag{45}$$

Some consequences of inequality already established under non-cooperative use seem to hold when users engage in social optimum management:

$$0 < (w_i^{SS} - w_{-i}^{SS})(a_i - a_{-i})$$

Which shows that the efficient agent uses more water at the steady state; higher levels of transmissivity and/or inequality stimulate greater transfer of water towards the more efficient agent.

$$(x_i^{SS} - x_{-i}^{SS})(a_i - a_{-i}) < 0,$$

which shows that the stocks are more depleted on the efficient user side, who endures higher rates and, given the previous result on the extraction decisions, higher costs of extraction.

As under non-cooperative use, the steady state under Social Optimum also shows that the difference in efficiency has no effect on the average stock of water in the aquifer $\left(\left(x_h^{SS} + x_l^{SS}\right)/2\right)$. However, the average stock is not affected by transmissivity, contrasting with the non-cooperative case, where more access to the resource seems to provoke its physical depletion.

The net profit of player *i* at the steady state, noted $NP^{s,SS-SO}_{a_i,a_{-i}}$, is given by:

$$NP^{s,SS-SO}_{a_i,a_{-i}} = \frac{R^2}{2} + \frac{c\,R^2}{r} + \left(a_i - a_{-i}\right)\frac{R\,s\left(2\,c\left(r+s\right)+r\left(r+2\,s\right)\right)}{r\left(2\,s\left(r+2\,s\right)+c\left(r+4\,s\right)\right)} + \left(a_i - a_{-i}\right)^2 \frac{s^2\left(r+2\,s\right)\left(2\,c+r+2\,s\right)}{2\left(2\,s\left(r+2\,s\right)+c\left(r+4\,s\right)\right)^2} \tag{46}$$

Finally, summing up the profits of the two players provides the aggregate profit at the steady state:

$$TNP^{SS-SO}\left(a_h, a_l, s\right) = R^2 + \frac{2\,c\,R^2}{r} + \left(a_h - a_l\right)^2 \frac{s^2\left(r+2\,s\right)\left(2\,c+r+2\,s\right)}{\left(2\,s\left(r+2\,s\right)+c\left(r+4\,s\right)\right)^2} \tag{47}$$

2.3.4 Preliminary Results under Social Optimum management

Result SO1*: Inequality increases the aggregate profits from the Social Optimum management of a CPR.*

Similar to the non-cooperative game, inequality is shown to boost aggregate profits, both, cumulative and at the steady state, under Optimum management; the extra payoffs from inequality are proportional to the square of $(a_h - a_l)$.

Indeed, for aggregate cumulative profits, equation (42) shows that:

$$V^{SO}\left(a_h, a_l, x_0, s\right) - V^{SO}\left(\frac{a_h + a_l}{2}, \frac{a_h + a_l}{2}, x_0, s\right) = P_6(s)\,\left(a_h - a_l\right)^2 > 0 \tag{48}$$

Similarly, at the steady state, the difference in total profits is strictly positive:

$$TNP^{SS-SO}\left(a_h, a_l, s\right) - TNP^{SS-SO}\left(\frac{a_h + a_l}{2}, \frac{a_h + a_l}{2}, s\right) = \left(a_h - a_l\right)^2 \frac{s^2\left(r+2\,s\right)\left(2\,c+r+2\,s\right)}{\left(2\,s\left(r+2\,s\right)+c\left(r+4\,s\right)\right)^2} \tag{49}$$

Result SO2*:* F*or unequal players, transmissivity increases the aggregate profits from the Optimum management of a CPR.*

For identical players, the extraction decision does not depend on transmissivity, implying that transmissivity has no effect on the aggregate profits, cumulative or at the steady state, of identical users.[52]

With unequal users, as transmissivity increases, users adjust their extraction decisions; with more stocks available (in time) to the efficient user, she extracts a higher quantity, while the less efficient user lowers her decision by the same amount. The aggregate profits increase while the effect on individual users is mixed, as will be discussed in the next result.

The extra payoffs from increased transmissivity, in aggregate cumulative profits and in aggregate profits at the steady state are proportional to the square of ($a_h - a_l$), as confirmed by the corresponding derivatives:

$$\frac{\partial TNP^{SS-SO}(a_h, a_l, s)}{\partial s} = (a_h - a_l)^2 \frac{2cs\left((r+2s)^3 + 2c\left(r^2 + 3rs + 4s^2\right)\right)}{\left(2s(r+2s) + c(r+4s)\right)^3}, \text{ and}$$

$$\frac{\partial V^{SO}(a_h, a_l, x_0, s)}{\partial s} = (a_h - a_l)^2 P_6'(s)$$

$P_6'(s)$ is the derivative of $P_6(s)$ and is a nonnegative functions of s (See Appendix B.3.2).

Finally, to draw the comparison with the non-cooperative case, the increasing effect of higher inequality on aggregate profit, under optimum management, is not conditional on a high level of inequality, it occurs at any level of inequality.

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[52] For identical users, and given the assumption of equal stocks at the start of the game, c_4 (equation 34.b) becomes null and w_{it} (equation 35.b) simplifies as $w_{it} = R - c_3\,\alpha_3\,e^{\alpha_3 t}$, and is not a function of s. Practically, this means that at any given time t, the users make the same decisions ($w_{it} = w_{-it}$), keeping the stocks at equal levels all the time, there are no externalities involved and the change in stocks is only affected by the players' own actions.

Result SO3: *Under Optimum management, transmissivity always increases profits for the efficient user and always decreases the benefits for the less efficient one.*

As discussed in Result SO2, with identical users, there are no aggregate or distributional effects that arise from an increased transmissivity.

For unequal users under optimum management, making the resource more common comes with extra benefits for the pair of unequal users *h-l*; however, those benefits from increased transmissivity are not widespread, indeed the less efficient user always endures some losses. The overall benefits for the group are indicative of larger benefits made by the efficient user.

2.3.5 Other Results under Social Optimum management

In addition to results SO1-3, other very interesting results on the effects of transmissivity and inequality on the difference between Social Optimum and the Subgame Perfect Equilibrium are presented and verified in Chapter 5:

2.4 Conclusion

The main purpose of this chapter is to illustrate the importance of a comprehensive analysis that includes inequality between the CPRs users and the degree of transmissivity in (or free access to) the resource in depicting the effects on welfare and on stock levels.

The commonly accepted principle of rapid decline in stocks and welfare loss as the CPR's commonality increases is reproduced in my model for the case where the users are identical or slightly unequal. However, in the presence of a large inequality between players, the gains resulting from the availability of more reserves for use by efficient users of the CPR can be important enough to offset the loss from free-riding behavior. Under such a condition, making the resource more common can result in a net total welfare gain.

It should be pointed out, however, that the outcome is still suboptimal as the net total profit could be improved if the users were to cooperate. In addition, in my case the difference between net total profit under cooperation and net total profit under non-cooperative use is found to increase with the level of inequality and the degree of transmissivity. There is also a need to emphasize that net welfare gains generated by more commonality in the resource is not generalized: in fact, the least efficient player sees her profits erode even (or especially) when the effect on the total profit is positive.

In the model, I assume infinite transmissivity to establish some of the results in the Open Loop game and to derive the closed form solutions for the Closed Loop game; but this assumption is not a necessary condition for the results to hold, for instance, for the Open Loop, some of the results with cumulative profits and all results at the steady state are verified for with a finite level of transmissivity.

However, there is another rationalization to this assumption, given the wide range of transmissivity values even among aquifers categorized as good aquifers; In fact, according to Fetter (2001), the hydraulic conductivity can vary between 10^{2} cm/s and 1 cm/s in well sorted gravel aquifers and between 10^{-1} cm/s and 10^{-3} cm/s in well-sorted sands aquifers.[53] This doesn't contradict that the so called "bath-tub model" is an

[53] A similar classification is provided by Bear (1972) provides hydraulic conductivity for good aquifers in the range of 10^{2} cm/s to 10 cm/s for well sorted gravel aquifers, and between 1 cm/s to 10^{-2} cm/s for well sorted sand or sand & gravel aquifers.

unrealistic depiction of the hydraulic flow in aquifers, even with very high levels transmissivity; but it allows in the present study to emulate the high range of transmissivity levels in nature, it also has the advantage of simplifying some of the theoretical results that would hold even for s finite.

Based on my analysis I can already establish some key policy implications:

- *The incentive for the state/government to intervene is stronger given the higher gap in welfare between the cooperative and* non-cooperative *paths;*

The gains from cooperation computed as the aggregate profits following the SPE deducted from the profits under cooperation are increasing in transmissivity and in the difference in efficiency. The losses from cooperation are therefore larger for unequal users, providing more incentive for a central planner to put in place policies that would shift water extraction toward welfare maximizing levels.

- *The adverse distributional effects for the less efficient players may create environments where certain policies less acceptable to identical players may be acceptable to extremely unequal players.*

It would therefore be very interesting to extend this study to examine the case where there is a central planner that intervenes to enforce policies that aim at enhancing total welfare from the aquifer, and to investigate the effects of inequality and transmissivity on not only the outcome of this game but also the planner's policy choices.

In a following chapter, I consider the case for water markets where a user buys out her or his/her partner; with identical players such contracts have no economically sound reason to materialize, especially for low or moderate costs of extraction, but for unequal players the less efficient player is more likely to accept his or her partner's offer to stop using water so that the efficient player can be the sole user.

Other policies can include payments by the government to users to discontinue their extractions from the aquifer or shift production to less water intensive crops, in the presence of unequal players such policies will involve less money transfers and generate more profits.

I must clarify at this point that the results obtained in this study are, for the most part, triggered by the choice of the type of inequality, and the type of production function; the same conclusions would not necessarily hold with a different model.

For example, when considering a slight change in our model, where the two users have identical linear coefficient but different quadratic coefficients, the results do not reflect the same increasing effect of inequality on aggregate profits or increasing effect of transmissivity with highly unequal users (See Appendix B.4.1). I use another variation of the model, where users have the same production function but have different natural capital, as the rate of recharge is not the same in both compartments of the aquifer, and I show that inequality generates extra payoffs that are decreasing in transmissivity (See Appendix B.4.2). [54]

This does not affect the lesson from this study that good management of natural resources requires taking into account the effects of inequality and the degree of commonality of the resource.

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[54] Appendix B.4.3 presents the conditions for an increasing effect of inequality at the steady state, in the case of a general production function, and where the inequality is either intrinsic or extrinsic.

Chapter 3 The Discrete Model

This chapter will be devoted to introduce the discrete version for the model of a CPR aquifer with a finite transmissivity and unequal users.

There will be presented the adjusted general framework for the extraction game, highlighting the differences with the dynamic model; backward dynamic programming is then used to solve for the players' decisions, following the different strategies, namely, the non-cooperative game, the myopic game and the social optimum management.

Another novelty of this essay is to consider a new strategy, when each player makes her decisions rationally, but acts under the impression that her partner is myopic.

The results presented in the previous chapter, for the non-cooperative game (Results OL1-4 and CL1-4), will be confirmed, with analytical proof for the one-round game, and numerical evidence for the 10-round game.

The model's numerical specifications and the dynamic solutions presented in this chapter will set the framework for the experimental validation that will be presented in the next chapter.

3.1 The Discrete Model with Unequal Players

I use a discrete game form adapted from the continuous model presented in the previous Chapter. The general structure is similar to the model developed by Gardner, Moore, and Walker (1997). Some necessary modifications were introduced to permit a broader framework for analysis, where I allow for inequality among users and test for different levels of transmissivity.

I consider an aquifer with the same physical specifications as described in the previous chapter, used by two agents *h* and *l*, as portrayed in Figure 1.

During round *r*, agent *i* chooses the quantity of water w_{ir} to extract from the aquifer and uses it in agricultural production to receive the revenue net of all costs except water extraction:[55]

$$NR_{ir}(w_{it}) = a_i w_{ir} - w_{ir}^2 / 2 , \qquad i=h,l \quad (3.1)$$

where a_i is, as defined before, the marginal productivity at the origin (w_{ir} = 0) and a_h taken higher or equal to a_l ($a_h \geq a_l$).

The equation of motion that defines stock adjustment to water use and recharge is given by:

$$x_{ir+1} = x_{ir} - w_{ir} + s\left(\left(x_{-ir} - w_{-ir}\right) - \left(x_{ir} - w_{ir}\right)\right) + R , \qquad i=h,l \quad (3.2)$$

where x_{ir} (respectively x_{ir+1}) represents the level of the water table beneath agent *i* at the beginning of round *r* (respectively *r*+1), *s* is the transmissivity or seepage coefficient assumed to be between 0 and ½, and *R* corresponds to the rate of recharge.

The transmissivity coefficient plays the same role as in the dynamic model; it continues to represent the level of commonality in the CPR, but with a small distinction. In the current model, *s*=½ is the value of *s* that corresponds to an infinite transmissivity, where

[55] As before, the extracted amount is expressed in this model as the equivalent change in stock.

the stocks in the following round (r+1) are always equal, regardless of the players' extraction decisions w_{hr} and w_{lr}.[56]

But, s=0 is still analogous to the case where there is a perfectly impermeable frontier between the two "cells" that water cannot cross.

When agent i lifts an amount w_{it} from the aquifer, she faces an extraction cost:

$$C_{ir}(w_{ir},x_{ir},w_{-ir},x_{-ir}) = w_{ir}\, c\left(E - \frac{x_{ir} + (1-s)(x_{ir} - w_{ir}) + s\,(x_{-ir} - w_{-ir})}{2}\right), \qquad i{=}h,l \quad (3.3)$$

where the quantity $\left(E - \frac{x_{ir} + (1-s)(x_{ir} - w_{ir}) + s\,(x_{-ir} - w_{-ir})}{2}\right)$ is the average depth at which agent i extracts water during round r; and c, strictly positive, denotes the cost to lift a unit of water per unit of depth.

The net payoff to agent i (h or l) during period r is therefore:

$$NP_{ir}(w_{ir},x_{ir},w_{-ir},x_{-ir}) = a_i w_{ir} - \frac{w_{ir}^2}{2} - w_{ir}\, c\left(E - \frac{x_{ir} + (1-s)(x_{ir} - w_{ir}) + s\,(x_{-ir} - w_{-ir})}{2}\right) \quad (3.4)$$

Equations (3.2) and (3.4) define completely the extensive form of my differential game.

3.2 The Unequal Discrete Game

The solution to the problem defined by equations (3.2) and (3.4) will depend on the players' strategy and their assumptions regarding their partner's strategy. I will consider four situations:

[56] The difference in the levels of s stems from the fact that in the dynamic model, it affects stocks through the rates of change (in stocks), while in the discrete model it affects the –future– stocks directly.

- The case where each user attempts to maximize the sum of her own profits from the resource throughout the duration of the game. In this case, each player supposes that her partner strategizes in the same manner. This case is similar to the non-cooperative game cosidered in the continuous model.
- The scenario where the decisions of both appropriators are aimed to maximize the sum of their cumulative profits from the resource throughout the game; similar to the Social Optimum in the previous chapter.
- The instance where each user tries to maximize her profits from the resource during the current round and disregards the effects on the following rounds. Here again the player assumes that her counterpart will behave in the same manner. This scenario was not considered in the continuous model.
- The case where each user tries to maximize the sum of her own profits from the resource throughout the duration of the game, presuming that her partner is interested solely in maximizing her profits from the current round and disregards the effects on the following rounds.

The last case presents a unique property, as each player has a poor view of his/her partner's strategy, indeed, in the first three cases, each player trusts her partner in the extraction game to adopt the same approach as that she adopts herself. In the latter case, each player thinks she is rational in considering present and future earnings, while her rival is myopic and only values immediate profits. The result is, as will be presented later, that the aggregate cumulative profits are higher if compared to those from the myopic case; but are lower than the profits from the first case, with rational players believing their partners to be rational as well.

3.2.1 The Non-cooperative Game

First, I will focus on the non-cooperative case, where I assume that the two players do not communicate with each other and cannot engage in any form of cooperation. The sole objective of each player is to maximize the sum of her discounted net benefits, with no consideration for the impacts of her actions on her neighbor's payoff. Her strategy is, however, motivated by her beliefs about her rival's strategy and path of action.

I assume that a central authority does not exist or, if it does exist, that it does not intervene in any way to stop or restrain the externalities from non-cooperative use, to establish higher revenues from the aquifer or to safeguard the water table. The level of water in each side of the aquifer is given at the start of the game. Also, both players know the other characteristics of the aquifer – recharge and transmissivity – with certainty.

I consider that both players have the same discount rate and that they maximize their benefits over the entire game duration. T, the number of rounds, is assumed finite (equates 10 for the numerical example). For practical reasons I will present here only the results for the case with no discounting; the results stand when the discount rate is strictly positive.

Under such conditions, the maximization problem of player *i* is given by:

$$\underset{w_{ir}}{Max} \sum_{r=1}^{r=T} NP_{ir}(w_{ir}, x_{ir}, w_{-ir}, x_{-ir}) \qquad i=h,l \tag{3.5}$$

subject to: $x_{ir+1} = x_{ir} + (1-s)(x_{ir} - w_{ir}) + s(x_{-ir} - w_{-ir}) + R$

$w_{ir} \geq 0$ (extraction decisions are non-negative)[57]

$x_{ir} \leq E$ (the level of water cannot rise above the ground)[58]

$x_{i0} = x_{-i0} = x_0$, the stocks at $t = 0$, are equal and given.

Finally, I also assume that there are no threats and that both players have perfect knowledge regarding the stock of water on both sides, their efficiency, and their rivals' efficiency.[59] Under such conditions, the maximization problem of player i has a unique subgame perfect equilibrium (SPE) corresponding to the case where all players act rationally and selfishly. To obtain the SPE characteristics, I solve for the system of equations combining equations (3.2) and (3.4) over time via the backward dynamic programming approach developed by Levhari and Mirman (1980) and Eswaran and Lewis (1984).[60] The solution in the case of groundwater use in a laboratory is provided in Gardner et al. (1997) and Herr et al. (1997) for identical players, perfect transmissivity, and no recharge.

3.2.1.1 The Subgame Perfect Equilibrium dynamic programming

In the last round (noted T) and knowing that the aquifer has no future residual value to both players the maximization problem of each player simplifies as:

$$\underset{w_{iT}}{Max}\, NP_i(w_{iT}, x_{iT}, w_{-iT}, x_{-iT}) \qquad i=h,l \quad (3.6)$$

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[57] This condition is rather undemanding in the non-cooperative game.

[58] This is a superfluous condition in the case of non-cooperative users.

[59] Assuming no threats rules out trigger strategies. Perfect knowledge requirements depend on the strategy of the game. I introduce this assumption to rule out mixed strategies.

[60] However, if I use a two-period horizon I would not need much to go through. I will just rewrite the whole system as one single equation.

Assuming an interior solution, the optimal extraction decisions in $i=h,l$ the last round must satisfy the simultaneous equations:

$$\frac{\partial NP_i(w_{iT},x_{iT},w_{-iT},x_{-iT})}{\partial w_{iT}}=a_i-w_{iT}-c\left(E-\frac{x_{iT}+(1-s)(x_{iT}-2w_{iT})+s\left(x_{-iT}-w_{-iT}\right)}{2}\right)=0, \quad (3.7)$$

The extractions decisions satisfying the above system of equations are linear in the stocks:

$$w_{iT}^{*}=\alpha_{1T}(E-x_{iT})+\alpha_{2T}(E-x_{-iT})+\alpha_{3Ti}, \qquad i=h,l \quad (3.8)$$

where α_{1T}, α_{2T} and α_{3Ti} are given by:

$$\alpha_{1T}=-\frac{1}{3}-\frac{c}{2+2\,c-c\,s}+\frac{2-c}{6+6\,c-9\,c\,s},$$

$$\alpha_{2T}=\frac{1}{3}-\frac{c}{2+2\,c-c\,s}-\frac{2-c}{6+6\,c-9\,c\,s}, \text{ and}$$

$$\alpha_{3Ti}=\frac{a_i+a_{-i}}{2+2\,c-c\,s}+\frac{a_i-a_{-i}}{2+2\,c-3\,c\,s}. \qquad i=h,l$$

Replacing for the values of w_{hT}^{*} and w_{lT}^{*} in equation (3.4) I get that the stock value to player i at the last round (equal to the net profit for the last round) is quadratic in the stock levels:

$$\begin{aligned}V_{iT}(x_{iT},x_{-iT})=D_{1T}(E-x_{iT})^2+D_{2T}(E-x_{jT})^2+D_{3T}(E-x_{iT})(E-x_{-iT})\\+D_{4Ti}(E-x_{iT})+D_{5Ti}(E-x_{-iT})+D_{6Ti}\end{aligned} \qquad i=h,l \quad (3.9)$$

where D_{1T}, D_{2T} and D_{3T} depend on c and s, while D_{4Ti}, D_{5Ti} and D_{6Ti} depend on a_i and a_{-i} as well:

$$D_{1T}=\frac{c^2\,(1+c-c\,s)\left(4-2s+4c-6cs+cs^2\right)^2}{2\,(2+2c-cs)^2\,(2+2c-3cs)^2};$$

$$D_{2T}=\frac{c^2\left(1+c-c\,s\right)s^2\left(2-c\,s\right)^2}{2\left(2+2c-c\,s\right)^2\left(2+2c-3c\,s\right)^2}$$

$$D_{3T}=\frac{s\,c^2\left(2-c\,s\right)\left(1+c-c\,s\right)\left(4-2s+4c-6c\,s+c\,s^2\right)}{\left(2+2c-c\,s\right)^2\left(2+2c-3c\,s\right)^2}$$

$$D_{4\mathrm{T}i}=-\frac{\mathrm{c}\left(1+\mathrm{c}-\mathrm{c\,s}\right)\left(4-2\mathrm{s}+4\mathrm{c}-6\mathrm{cs}+\mathrm{cs}^2\right)}{\left(2+2\mathrm{c}-\mathrm{cs}\right)\left(2+2\mathrm{c}-3\mathrm{cs}\right)}\left(\frac{a_i+a_{-i}}{2+2c-c\,s}+\frac{a_i-a_{-i}}{2+2c-3c\,s}\right)\qquad i=h,l$$

$$D_{5\mathrm{T}i}=-\frac{\mathrm{c}\left(1+\mathrm{c}-\mathrm{c\,s}\right)s\left(2-\mathrm{cs}\right)}{\left(2+2\mathrm{c}-\mathrm{cs}\right)\left(2+2\mathrm{c}-3\mathrm{cs}\right)}\left(\frac{a_i+a_{-i}}{2+2c-c\,s}+\frac{a_i-a_{-i}}{2+2c-3c\,s}\right)\qquad i=h,l$$

$$D_{6\mathrm{T}i}=\frac{1}{2}\left(1+\mathrm{c}-\mathrm{c\,s}\right)\left(\frac{a_i+a_{-i}}{2+2c-c\,s}+\frac{a_i-a_{-i}}{2+2c-3c\,s}\right)^2\qquad i=h,l$$

At any given round *r*, every player uses the resources weighing in her immediate profits from the extracted water and her benefits from the stock in the subsequent rounds (null in the last round) she solves the following problem:

$$\underset{w_{ir}}{Max}\left\{NP_i(w_{ir},x_{ir},w_{-ir},x_{-ir})+V_{ir+1}(x_{ir+1}(w_{ir},x_{ir},w_{-ir},x_{-ir}),x_{-ir+1}(w_{-ir},x_{-ir},w_{ir},x_{ir}))\right\}\qquad(3.10)$$

$V_{ir+1}(x_{ir+1},x_{-ir+1})$ refers to the stock value for player *i* in the next round (*r+1*), i.e. her profits from the stock (x_{ir+1},x_{-ir+1}) when there are *T-r+1* rounds left in the game.

The decisions by both players, *h* and *l*, need to satisfy the optimality condition:

$$\frac{\partial NP_i(w_{ir},x_{ir},w_{-ir},x_{-ir})}{\partial w_{ir}}+\frac{\partial V_{ir+1}(x_{ir+1},x_{-ir+1})}{\partial x_{ir+1}}\frac{\partial x_{ir+1}}{\partial w_{ir}}+\frac{\partial V_{ir+1}(x_{ir+1},x_{-ir+1})}{\partial x_{-ir+1}}\frac{\partial x_{-ir+1}}{\partial w_{ir}}=0\qquad(3.11)$$

Every player extracts to the point where her extra profit from the current use of one more unit of water equals her losses from the subsequent decrease in stocks (hers and her neighbor's) in the following round.

If I assume that the value of groundwater in the next round $n=r+1$ is quadratic in the level of stocks during that round and can be written as:

$$\begin{aligned} V_{in}(x_{in}, x_{-in}) = {} & D_{1n}(E-x_{in})^2 + D_{2n}(E-x_{-in})^2 + D_{3n}(E-x_{in})(E-x_{-in}) \\ & + D_{4ni}(E-x_{in}) + D_{5ni}(E-x_{-in}) + D_{6ni} \end{aligned} \qquad i=h,l \qquad (3.12)$$

Solving for the optimality condition expressed in (3.11) shows that the decisions of players *h* and *l* during round *r* are, as established for the last round, linear in the stocks:

$$w_{ir}^* = \alpha_{1r}(E-x_{ir}) + \alpha_{2r}(E-x_{-ir}) + \alpha_{3ri} \qquad i=h,l \qquad (3.13)$$

where α_{1r}, α_{2r} and α_{3ri}, provided in Appendix C.1, are functions of D_{1n}, D_{2n}, D_{3n}, D_{4ni}, D_{5ni} and D_{6ni}.

Replacing for the values of optimal extraction decisions in (3.10) I conclude that the stock value function at round r for player *i* is equally quadratic in both stocks:

$$\begin{aligned} V_{ir}(x_{ir}, x_{jr}) = {} & D_{1r}(E-x_{ir})^2 + D_{2r}(E-x_{-ir})^2 + D_{3r}(E-x_{ir})(E-x_{-ir}) \\ & + D_{4ri}(E-x_{ir}) + D_{5ri}(E-x_{-ir}) + D_{6ri} \end{aligned} \qquad i=h,l \qquad (3.14)$$

where D_{1r}, D_{2r}, D_{3r}, D_{4ri}, D_{5ri} and D_{6ri} (See Appendix C.1) are function of the decision coefficients from the current round, α_{1r}, α_{2r} and α_{3ri}, and the coefficients of the value function from the next round, D_{1n}, D_{2n}, D_{3n}, D_{4ni}, D_{5ni} and D_{6ni}.

Given the value function at the last round (*T* or 10) and the relationships established above it is easy to derive the extraction decisions, the stock levels, and the profits at every round for both players.

3.2.2 The Social Optimum, the Myopic, and the Semi-myopic Strategies

3.2.2.1 The Social Optimum

In the Social Optimum (also referred to as the cooperative) case, the two players coordinate their extraction decisions in order to maximize their collective cumulative profits. The maximization problem thus becomes:

$$\underset{w_{hr},w_{lr}}{Max}\sum_{r=1}^{r=T}\left(NP_{hr}(w_{hr},x_{hr},w_{lr},x_{lr})+NP_{lr}(w_{lr},x_{lr},w_{hr},x_{hr})\right), \tag{3.15}$$

subject to the same conditions as in equation (3.5). Similar to the non-cooperative game, the maximization problem for players under Social Optimum management has a unique solution obtained using the backward dynamic programming approach.

3.2.2.1.1 The Social Optimum dynamic programming

In the last round (noted T) and assuming that the aquifer has no future residual value the maximization problem of each player becomes:

$$\underset{w_{iT},w_{-iT}}{Max}\left(NP_i(w_{iT},x_{iT},w_{-iT},x_{-iT})+NP_i(w_{-iT},x_{-iT},w_{iT},x_{iT})\right) \qquad i=h \text{ or } l \tag{3.16}$$

In the case of an interior solution, the optimal extraction decisions in the last round must satisfy the simultaneous conditions:

$$\frac{\partial\left(NP_i(w_{iT},x_{iT},w_{-iT},x_{-iT})+NP_{-i}(w_{-iT},x_{-iT},w_{iT},x_{iT})\right)}{\partial w_{iT}}=$$

$$a_i-c\left(E-x_{iT}\right)-w_{iT}-\frac{c\,s\left(x_{iT}-x_{-iT}\right)}{2}-c\left((1-s)w_{iT}+s\,w_{-iT}\right)=0, \qquad i=h,l \tag{3.17}$$

The extractions decisions satisfying the above system of equations are linear in the stocks:

$$w_{iT}^{SO}=\alpha_{1T}^{SO}(E-x_{iT})+\alpha_{2T}^{SO}(E-x_{-iT})+\alpha_{3Ti}^{SO}, \qquad i=h,l \tag{3.18}$$

where α_{1T}^{SO}, α_{2T}^{SO} and α_{3Ti}^{SO} given by:

$$\alpha_{1T}^{SO} = -\frac{1+3c}{4(1+c)} + \frac{1-c}{4(1+c-2cs)};$$

$$\alpha_{2T}^{SO} = \frac{1-c}{4(1+c)} - \frac{1-c}{4(1+c-2cs)} \text{ and}$$

$$\alpha_{3Ti}^{SO} = \frac{a_i + a_{-i}}{2(1+c)} + \frac{a_i - a_{-i}}{2(1+c-2cs)} \qquad i=h,l$$

Replacing for the values of w_{hT}^{SO} and w_{lT}^{SO} in equation (3.17) I get that the stock value at the last round (equal to the net aggregate profit for the last round) is quadratic in the stock levels:

$$V_T^{SO}(x_{iT}, x_{-iT}) = D_{1T}^{SO}(E - x_{iT})^2 + D_{2T}^{SO}(E - x_{-iT})^2 + D_{3T}^{SO}(E - x_{iT})(E - x_{-iT})$$
$$+ D_{4Ti}^{SO}(E - x_{iT}) + D_{4T-i}^{SO}(E - x_{-iT}) + D_{6T}^{SO} \qquad i=h,l \qquad (3.19)$$

where D_{1T}^{SO}, D_{2T}^{SO} and D_{3T}^{SO} depend on c and s, while D_{4Ti}^{SO} and D_{6T}^{SO} depend on a_i and a_{-i} as well:

$$D_{1T}^{SO} = \frac{c^2\left(2+2c-2s-4cs+s^2+cs^2\right)}{4(1+c)(1+c-2cs)};$$

$$D_{2T}^{SO} = \frac{c^2\left(2+2c-2s-4cs+s^2+cs^2\right)}{4(1+c)(1+c-2cs)};$$

$$D_{3T}^{SO} = \frac{c^2 s(2-s-cs)}{2(1+c)(1+c-2cs)} \text{ and}$$

$$D_{4\mathrm{Ti}}^{\mathrm{SO}} = -c\left(\frac{a_i + a_{-i}}{2(1+c)} + \frac{(a_i - a_{-i})(1-s)}{2(1+c-2cs)}\right) \quad i=h,l$$

$$D_{6T}^{SO} = \frac{(a_i + a_{-i})^2}{4(1+c)} + \frac{(a_i - a_{-i})^2}{4(1+c-2cs)}.$$

At any given round r, the players' decisions need to take into consideration the immediate aggregate profits from the extracted water and the benefits from the stocks in the subsequent rounds (null in the last round), they solve the following problem:

$$\underset{w_{ir},w_{-ir}}{Max}\left\{\left(NP_i(w_{ir},x_{ir},w_{-ir},x_{-ir})+NP_{-i}(w_{-ir},x_{-ir},w_{ir},x_{ir})\right)+ V_{r+1}^{SO}(x_{ir+1}(w_{ir},x_{ir},w_{-ir},x_{-ir}),x_{-ir+1}(w_{-ir},x_{-ir},w_{ir},x_{ir}))\right\} \quad i=h,l \quad (3.20)$$

$V_{r+1}^{SO}(x_{ir+1},x_{-ir+1})$ denotes the stock aggregate value in the next round ($r+1$), i.e. the aggregate cumulative profits from the stock (x_{ir+1},x_{-ir+1}) when there are T-$r+1$ rounds left in the game.

The decisions by players h and l, need to satisfy the optimality conditions:

$$\frac{\partial NP_i(w_{ir},x_{ir},w_{-ir},x_{-ir})}{\partial w_{ir}}+\frac{\partial NP_{-i}(w_{-ir},x_{-ir},w_{ir},x_{ir}))}{\partial w_{ir}} +\frac{\partial V_{r+1}^{SO}(x_{ir+1},x_{-ir+1})}{\partial x_{ir+1}}\frac{\partial x_{ir+1}}{\partial w_{ir}}+\frac{\partial V_{r+1}^{SO}(x_{ir+1},x_{-ir+1})}{\partial x_{-ir+1}}\frac{\partial x_{-ir+1}}{\partial w_{ir}}=0 \quad i=h,l \quad (3.21)$$

Every player extracts to the point where the sum of extra profits to both users, from her current use of one more unit of water, equals the loss, to the two users, from the subsequent decrease in stocks (of both users) in the following round.

If I assume that the value of groundwater in the next round $n=r+1$ is quadratic in the level of stocks during that round and can be written as:

$$V_n^{SO}(x_{in},x_{-in})=D_{1n}^{SO}(E-x_{in})^2+D_{2n}^{SO}(E-x_{jn})^2+D_{3n}^{SO}(E-x_{in})(E-x_{-in}) +D_{4ni}^{SO}(E-x_{in})+D_{4n-i}^{SO}(E-x_{-in})+D_{6n}^{SO} \quad i=h \text{ or } l \quad (3.22)$$

Solving for the optimality condition expressed in (3.21) shows that the decisions of players h and l during round r are, as established for the last round, linear in the stocks:

$$w_{ir}^{SO} = \alpha_{1r}^{SO}(E - x_{ir}) + \alpha_{2r}^{SO}(E - x_{-ir}) + \alpha_{3ri}^{SO} \qquad i=h,l \quad (3.23)$$

where α_{1r}^{SO}, α_{2r}^{SO} and α_{3ri}^{SO} provided in Appendix C.1, are functions of D_{1n}^{SO}, D_{2n}^{SO}, D_{3n}^{SO}, D_{4ni}^{SO}, D_{4n-i}^{SO} and D_{6n}^{SO}.

Replacing for the values of optimal extraction decisions in (3.20) I conclude that the stock value function at round r for player i is equally quadratic in both stocks:

$$V_{ir}(x_{ir}, x_{jr}) = D_{1r}^{SO}(E - x_{ir})^2 + D_{2r}^{SO}(E - x_{-ir})^2 + D^{SO}(E - x_{ir})(E - x_{-ir})$$

$$+ D_{4ri}^{SO}(E - x_{ir}) + D_{4r-i}^{SO}(E - x_{-ir}) + D_{6ri}^{SO} \qquad i=h,l \quad (3.24)$$

Where D_{1r}^{SO}, D_{2r}^{SO}, D_{3r}^{SO}, D_{4ri}^{SO}, D_{4r-i}^{SO} and D_{6r}^{SO} (See Appendix C.1) are function of the decision coefficients from the current round, α_{1r}^{SO}, α_{2r}^{SO} and α_{3ri}^{SO}, and the coefficients of the value function from the next round, D_{1n}^{SO}, D_{2n}^{SO}, D_{3n}^{SO}, D_{4ni}^{SO}, D_{4n-i}^{SO} and D_{6n}^{SO}.

Given the value function at the last round (T or 10) and the relationships established above it is possible to derive the extraction decisions, the stock levels, and the social optimum profits at every round.

3.2.2.2 The Myopic path

For the myopic path, every player maximizes her profits for the current round, and the maximization problem of player i is given by:

$$\underset{w_{ir}}{Max}\, NP_{ir}(w_{ir}, x_{ir}, w_{-ir}, x_{-ir}), \qquad i=h,l \quad (3.25)$$

subject to the same conditions as in equation (3.5). The solution to the myopic problem is forward and is similar to the solution at the last round of a non-cooperative game.

3.2.2.3 The Semi-myopic programming

Finally, under the semi-myopic path, each player maximizes her profits for the current round. The maximization problem of player *i* is given by:

$$\underset{w_{ir}}{Max}\sum_{r=1}^{r=T} NP_{ir}(w_{ir}, x_{ir}, w_{-ir}^{m}, x_{-ir}), \qquad i=h,l \quad (3.26)$$

subject to the same conditions as in equation (5). w_{-ir}^{m} reflects the belief of every user that her partner uses the resource following a myopic path. The solution to the semi-myopic problem is also obtained by backward dynamic programming approach. (The solution is available but not included in the present draft).

In number of economic experiments studying CPRs, the participants' decisions do not reflect a strategic behavior as defined under the SPE; the introduction of a Semi-Myopic path here is a means to investigate if the deviations from the SPE can be explained by participants' beliefs not matching those following the SPE. Under the SPE, participants assume that their partners are strategic users; the semi-myopic path addresses the case where the players are strategic, but believe that the other users are myopic users.

For a resource with zero transmissivity (s=0), the Social Optimum, the SPE, and the Semi-Myopic, strategies are equivalent and will be referred to as maximizing behavior.

3.3 Theoretical Results Following the Non-Cooperative Game

I now present basic results concerning the net profits, over time, for the different treatments; these results were obtained by assuming that all users behave following the subgame perfect equilibrium. In this section, I provide the analytical proof for all results in the case of a one-period non-cooperative game; for the ten-round SPE, I present only the numerical evidence of my findings in the next section.

Individual and Aggregate profits in the one round non-cooperative game

In the case of a one round game the maximization problem of players *h* and *l* is equivalent to that at the last period of a multiple rounds game, with the slight difference that the level of stock *x* is now taken as the same for both players. From equation (3.8) I obtain the extraction decisions and the net individual profits:

$$w_i^* = \frac{a_i + a_{-i} - 2c(E-x)}{2+2c-cs} + \frac{a_i - a_{-i}}{2+2c-3cs}, \qquad i=h,l \quad (3.27)$$

$$NP_i(w_i^*, x, w_{-i}^*, x) = \frac{1}{2}(1+c-cs)\left(\frac{2c(E-x)-(a_i+a_{-i})}{(2+2c-cs)} - \frac{a_i - a_{-i}}{2+2c-3cs}\right)^2 \qquad i=h,l \quad (3.28)$$

For convenience, in the rest of the section I will note $NP^s_{a_i,a_{-i}}(x,x)$ the net profit of player *i* when transmissivity equals s, and $V^s_{a_h,a_l}(x,x)$ the sum of net profits to players *h* and *l*, computed as:

$$V^s_{a_h,a_l}(x,x) = (1+c-cs)\left(\left(\frac{2c(E-x)-(a_h+a_l)}{2+2c-cs}\right)^2 + \left(\frac{a_h-a_l}{2+2c-3cs}\right)^2\right) \quad (3.29)$$

Result NC1*: Inequality increases the aggregate cumulative profits from the common-pool resource.*

Our theoretical model shows that the aggregate (summing up across both players) cumulative (over the entire duration of the game) net profits are increasing in transmissivity and in the difference in efficiency ($a_h - a_l$); the same observation applies for the social optimum and myopic paths.

This can be shown by comparing the $V^s_{a_h,a_l}(x_0,x_0)$, the aggregate cumulative profits of players *h* and *l* when their respective efficiencies are a_h and a_l, with initial stock x_0, and *s* is the level of transmissivity, to the aggregate cumulative profits of identical players with an efficiency that equals the average efficiency of players *h* and *l*, with the same initial stock x_0, and the same level of transmissivity; the difference is given by:

$$V^s_{a_h,a_l}(x_0,x_0) - V^s_{\frac{a_h+a_l}{2},\frac{a_h+a_l}{2}}(x_0,x_0) = \frac{(1+c-c\,s)}{(2+2c-3c\,s)^2}(a_h-a_l)^2 \tag{3.30}$$

Inequality generates extra payoffs, as evidenced by the positive sign on the RHS, that are proportional in magnitude to the square of $(a_h - a_l)$ and are increasing in transmissivity.

This result is in concordance with the findings of Negri (1989) and Baland and Plateau (1997), but differs from that of Aggarwal and Narayan (2004) and Dayton-Jhonson and Bardhan (2002), as it suggests an increase in aggregate cumulative profits even at low levels of inequality.

Result NC2*: Transmissivity decreases the aggregate cumulative profits from the common-pool resource for identical or slightly unequal players.*

In the case of identical players, I reach result that increasing the transmissivity leads to lower net profits as evidenced by the negative derivative of $V^s_{a,a}(x_0,x_0)$ w.r.t. *s*:

$$\frac{\partial V^s_{a,a}(x_0,x_0)}{\partial s} = -\frac{4c^2\,s\,(a-c(E-x_0))^2}{(2+2c-c\,s)^3}$$

The commonly accepted premise that profits are lowered and stock rapidly depleted as a resource becomes more common (Gordon 1954; Burt 1964; Hardin 1968; Brown and Deacon 1972; Eswaran and Lewis 1984) is corroborated in my model in the case of identical or slightly unequal players.

Result NC3*: In the case of highly unequal players, higher transmissivity tends to increase the aggregate cumulative profits.*

In the case of unequal players, the two opposite effects of transmissivity pointed out in Result NC 1 and 2 are put simultaneously at play. In fact, the marginal change in the aggregate profits following a change in transmissivity equals:

$$\frac{\partial V^{s}_{a_h,a_l}(x_0,x_0)}{\partial s}=\frac{c(4+4c-3cs)}{(2+2c-3cs)^3}(a_h-a_l)^2-\frac{c^2 s(a_h+a_l-2c(E-x_0))^2}{(2+2c-cs)^3} \qquad (3.31)$$

When the difference in efficiency is higher than Δ_{NC3}, the increasing effect (first term on the right hand side) outweigh the losses from more non-cooperative use, and I reach the counterintuitive result of higher profits from a marginal increase in transmissivity, where:

$$\Delta_{NC3}=\sqrt{\frac{cs}{4+4c-3cs}}\left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2}(a_h+a_l-2c(E-x_0))$$

Increasing the aquifer's transmissivity has two effects: the first effect is to give more incentive for free-riding for all users, which diminishes profits and exacerbates the race to the bottom. The second effect – positive this time – is that the efficient user is granted more access to more stocks. In the presence of strong inequality between the two players, the second effect is stronger and outweighs the first. This result is established here only in the case of two players and does not necessarily hold as the number of users increases.

The potential positive effects of heterogeneity in the provision of public goods (Olson 1965), limiting resource degradation and total welfare at the steady state (Negri 1989), or improving aggregate profits from a common pool resource (Baland and Plateau 1997) are well known, other studies (Varughese and Ostrom 2001, Poteete and Ostrom, 2004) emphasize the effect of heterogeneity on the collective management of the commons. My result differs in that it suggests that when a large inequality between users prevails, the benefits from common access may be of such magnitude as to overcome the losses due to the CPR non-cooperative use. In such a case, property rights enforcement becomes detrimental. In my model, imposing – if such a thing were even possible – a zero transmissivity to eliminate the circulation of water between the aquifer's two compartments would reduce the total profits for highly unequal users.

The same mechanics are at play when I consider a non-marginal change in transmissivity as I move from a zero transmissivity resource (s=0) to a resource with infinite transmissivity (s=½):

$$V^{1/2}_{a_h,a_l}(x_0,x_0)-V^{0}_{a_h,a_l}(x_0,x_0)=\left(\frac{c(16+7c)}{4(1+c)(4+c)^2}(a_h-a_l)^2-\frac{c^2(a_h+a_l-2c(E-x_0))^2}{4(1+c)(4+3c)^2}\right).$$

High inequality leads to higher benefits as will be shown in the numerical examples and tested for in the experiment.

An interesting near result that I will not test for in my experiment is that, for any strictly positive level of inequality, at low levels of transmissivity, a marginal increase in transmissivity has always an increasing effect on total net benefits:

$$\left.\frac{\partial V^{s}_{a_h,a_l}(x_0,x_0)}{\partial s}\right|_{s=0}=\frac{c(4+4c)}{(2+2c)^3}(a_h-a_l)^2>0.$$

Result NC4: *Transmissivity always decreases the cumulative profits for less efficient players and, in the case of a high inequality between players, increases the cumulative profits for the more efficient ones.*

The derivative of a player's net profit with regard to transmissivity can be written as:

$$\frac{\partial NP^{s}_{a_i,a_{-i}}(x_0,x_0)}{\partial s} = A_i\left(\frac{(a_i-a_{-i})(2+2c-cs)^2(4+4c-cs)}{2c^2(2+2c-3cs)^2} - s\left(\frac{a_i+a_{-i}}{2c}-(E-x_0)\right)\right), \quad i=h,l$$

where: $$A_i = \frac{2c^4}{(2+2c-cs)^3}\left(\frac{2a_i(1+c-cs)-a_{-i}cs}{c(2+2c-3cs)}-(E-x_0)\right) > 0.^{61} \qquad i=h,l$$

It follows that for the less efficient player the derivative has a negative sign in all configurations since both terms between brackets on the RHS are negative.

For the more efficient user the derivative is negative for small levels of inequality and positive when the difference in efficiency is higher than Δ_{NC4}:

$$\Delta_{NC4} = \frac{cs}{4+4c-3cs}\left(\frac{2+2c-3cs}{2+2c-cs}\right)^2\left(a_h+a_l-2c(E-x_0)\right)$$

This last result shows that individual welfare effects are not always the same as the aggregate welfare effects. Indeed, for the less efficient player, higher transmissivity always translates into lower profits; for more efficient players, the effect is positive if inequality is higher than a certain threshold and negative otherwise. Also, predictably, the threshold for net gains for the most efficient player is lower than that for an aggregate cumulative net welfare gain.[62]

Similar welfare effects ensue from a non-marginal change in transmissivity that allows for a shift from a resource with zero transmissivity (s=0) to a perfectly transmissive resource (s=½), in this case I obtain

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[61] Notice that: $0 < \frac{a_l}{c}-(E-x_0) \le \frac{2a_i(1+c-cs)-a_{-i}cs}{c(2+2c-3cs)}-(E-x_0) \le \frac{a_h}{c}-(E-x_0).$

[62] $\frac{\Delta_{NC3}}{\Delta_{NC4}} = \sqrt{\frac{4+4c-3cs}{cs}}\sqrt{\frac{2+2c-cs}{2+2c-3cs}} > 1$

$$NP^{1/2}_{a_i,a_{-i}}(x_0,x_0)-NP^{0}_{a_i,a_{-i}}(x_0,x_0)=\Gamma_i\,\Delta_i\,,\qquad\qquad i=h,l$$

where: $$\Gamma_i=\left(\frac{(a_i-a_{-i})\left(-(8+5c)\sqrt{2}+8\sqrt{1+c}\sqrt{2+c}\right)}{4\sqrt{1+c}\,(4+c)}+\frac{c\left(a_i+a_{-i}-2c(E-x_0)\right)}{2\sqrt{2}\sqrt{1+c}\,(4+3c)}\right)$$ and

$$\Delta_i=\left(\frac{(a_i-a_{-i})\left((8+5c)\sqrt{2}+8\sqrt{1+c}\sqrt{2+c}\right)}{4\sqrt{1+c}\,(4+c)}-\frac{c\left(a_i+a_{-i}-2c(E-x_0)\right)}{2\sqrt{2}\sqrt{1+c}\,(4+3c)}\right).$$

For the more efficient player the first term Γ_h is positive[63] while the sign of the second term depends on the extent of inequality (a_h-a_l).

For player *l* the second term Δ_l is negative while the first term can be shown to be positive for all values of a_l that satisfy the interior solution conditions.[64]

3.4 Numerical Illustration for the Ten-round Non-Cooperative Game

3.4.1 Numerical specifications and solutions

I consider two levels of inequality: the case where the two players are highly unequal and the case where they are identical. In the unequal setting, I consider a player with a high efficiency a_h = 20, sharing the CPR aquifer with a player with a low efficiency a_l = 10. In the setting with identical players I use a_h = a_l = 15; the efficiency in the identical case is

[63] Note that $\left(-(8+5c)\sqrt{2}+8\sqrt{1+c}\sqrt{2+c}\right)$ is positive (multiply by $\left((8+5c)\sqrt{2}+8\sqrt{1+c}\sqrt{2+c}\right)$ to obtain $2c(16+7c)$)

[64] It is easy to notice that Γ_l is increasing in a_l , since for the smallest acceptable value of a_l , $a_{l\ min}$ = $\frac{c(a_h+(4+c)(h-x))}{4+2c}$, Γ_l is given by: $\Gamma_l=\left((4+3c)\sqrt{2}-4\sqrt{1+c}\sqrt{2+c}\right)\left(a_h-c(E-x)\right)/\left(4(4+c)\sqrt{1+c}\right)$ strictly positive given the positive sign of $\left((4+3c)\sqrt{2}-4\sqrt{1+c}\sqrt{2+c}\right)$, to show this last statement it suffices to multiply by $\left((4+3c)\sqrt{2}+4\sqrt{1+c}\sqrt{2+c}\right)$ and obtain $2c^2$.

set equal to the average efficiency from the unequal setting to control for the effect of inequality.

Regarding transmissivity, I consider the two extreme situations: the case where the resource displays zero transmissivity ($s = 0$), and the case where it has an infinite transmissivity, which in the discrete case translates as s = ½.

Matching together the two levels of inequality and the two degrees of transmissivity yields four cases to investigate:

- The Unequal Infinite Transmissivity case: $a_h = 20$, $a_l = 10$ and s = ½,
- The Identical Infinite Transmissivity case: $a_h = 15$, $a_l = 15$ and s = ½,
- The Unequal Zero Transmissivity case: $a_h = 20$, $a_l = 10$ and s = 0, and
- The Identical Zero Transmissivity case: $a_h = 15$, $a_l = 15$ and s = 0.

The other parameters of the game are kept constant, the stocks on the first round are set at 73($x_{h0} = x_{l0} = 73$), while the elevation of the ground is E=100. The unitary cost of extraction is taken $c = 0.15$, and the aquifer recharge is R=3.

The numerical solutions for players' decisions, for the different cases, are regrouped in tables 1 to 4, following the different paths. The solutions are based on the algorithms described earlier.

Table 1: The Numerical solution for the 10-round game following the (SPE) path

	Round	Decisions		Stocks		Profits			Stock Values		
Case	r	w_{hr}	w_{lr}	x_{hr}	x_{lr}	NP_{hr}	NP_{lr}	$NP_{h+l\,r}$	V_{hr}	V_{lr}	$V_{h+l\,r}$
The Unequal Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	9.8	4.0	73.0	73.0	103.2	13.7	117.0	810.5	56.4	866.9
	2	9.7	3.6	69.1	69.1	97.2	11.1	108.3	707.2	42.7	749.9
	3	9.7	3.3	65.4	65.4	91.8	8.8	100.6	610.0	31.6	641.6
	4	9.6	3.0	62.0	62.0	86.8	6.9	93.7	518.2	22.8	541.0
	5	9.7	2.7	58.6	58.6	82.2	5.3	87.6	431.4	15.8	447.3
	6	9.8	2.4	55.5	55.5	78.0	4.0	82.0	349.2	10.5	359.7
	7	9.9	2.0	52.4	52.4	73.9	2.8	76.8	271.2	6.5	277.7
	8	10.1	1.7	49.4	49.4	69.9	1.9	71.8	197.3	3.7	201.0
	9	10.4	1.4	46.5	46.5	65.8	1.2	67.0	127.4	1.8	129.1
	10	10.7	1.1	43.6	43.6	61.5	0.6	62.1	61.5	0.6	62.1
The Identical Infinite Transmissivity (a_h = 15, a_l = 15, s = ½)	1	6.9	6.9	73.0	73.0	48.2	48.2	96.3	318.5	318.5	637.1
	2	6.7	6.7	69.1	69.1	43.6	43.6	87.1	270.4	270.4	540.8
	3	6.5	6.5	65.4	65.4	39.5	39.5	78.9	226.8	226.8	453.6
	4	6.3	6.3	62.0	62.0	35.8	35.8	71.5	187.4	187.4	374.7
	5	6.2	6.2	58.6	58.6	32.4	32.4	64.8	151.6	151.6	303.2
	6	6.1	6.1	55.5	55.5	29.3	29.3	58.6	119.2	119.2	238.4
	7	6.0	6.0	52.4	52.4	26.4	26.4	52.9	89.9	89.9	179.8
	8	5.9	5.9	49.4	49.4	23.7	23.7	47.5	63.5	63.5	126.9
	9	5.9	5.9	46.5	46.5	21.1	21.1	42.3	39.7	39.7	79.4
	10	5.9	5.9	43.6	43.6	18.6	18.6	37.2	18.6	18.6	37.2
The Unequal Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	1	5.2	1.2	73.0	73.0	67.0	6.2	73.2	654.6	135.6	790.1
	2	5.6	1.6	70.8	74.8	69.6	8.6	78.2	587.5	129.4	716.9
	3	6.1	2.1	68.2	76.2	71.2	10.8	82.1	517.9	120.8	638.8
	4	6.5	2.5	65.2	77.2	71.8	12.9	84.7	446.7	110.0	556.7
	5	7.0	3.0	61.6	77.6	71.3	14.7	86.0	374.8	97.1	471.9
	6	7.4	3.4	57.7	77.7	69.6	16.0	85.6	303.5	82.5	386.0
	7	7.9	3.9	53.3	77.3	66.6	16.9	83.5	233.9	66.5	300.4
	8	8.3	4.3	48.4	76.4	62.2	17.2	79.3	167.3	49.6	216.9
	9	8.8	4.8	43.1	75.1	56.3	16.8	73.1	105.1	32.4	137.5
	10	9.2	5.2	37.3	73.3	48.8	15.6	64.5	48.8	15.6	64.5
The Identical Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	1	3.2	3.2	73.0	73.0	28.9	28.9	57.8	345.1	345.1	690.1
	2	3.6	3.6	72.8	72.8	32.0	32.0	64.0	316.2	316.2	632.3
	3	4.1	4.1	72.2	72.2	34.5	34.5	69.1	284.2	284.2	568.4
	4	4.5	4.5	71.2	71.2	36.5	36.5	72.9	249.6	249.6	499.3
	5	5.0	5.0	69.6	69.6	37.7	37.7	75.4	213.2	213.2	426.3
	6	5.4	5.4	67.7	67.7	38.1	38.1	76.2	175.5	175.5	351.0
	7	5.9	5.9	65.3	65.3	37.6	37.6	75.3	137.4	137.4	274.8
	8	6.3	6.3	62.4	62.4	36.2	36.2	72.3	99.7	99.7	199.5
	9	6.8	6.8	59.1	59.1	33.6	33.6	67.3	63.6	63.6	127.1
	10	7.2	7.2	55.3	55.3	29.9	29.9	59.9	29.9	29.9	59.9

Table 2: The Numerical solution for the 10-round game following the myopic path

	Round	Decisions		Stocks		Profits			Stock Values		
Case	r	w_{hr}	w_{lr}	x_{hr}	x_{lr}	NP_{hr}	NP_{lr}	$NP_{h+l\,r}$	V_{hr}	V_{lr}	$V_{h+l\,r}$
The Unequal Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	14.7	5.0	73.0	73.0	115.5	13.6	129.1	749.5	36.4	785.9
	2	13.7	4.1	66.2	66.2	101.5	9.0	110.5	634.0	22.8	656.8
	3	12.9	3.3	60.2	60.2	90.0	5.9	95.9	532.5	13.8	546.3
	4	12.3	2.6	55.1	55.1	80.7	3.7	84.3	442.5	7.9	450.5
	5	11.7	2.0	50.7	50.7	73.0	2.2	75.2	361.8	4.3	366.1
	6	11.1	1.5	46.9	46.9	66.7	1.2	67.9	288.9	2.1	290.9
	7	10.7	1.1	43.5	43.5	61.4	0.6	62.0	222.2	0.9	223.1
	8	10.3	0.7	40.7	40.7	57.0	0.2	57.3	160.8	0.3	161.1
	9	10.0	0.3	38.2	38.2	53.4	0.1	53.5	103.7	0.1	103.8
	10	9.7	0.0	36.0	36.0	50.3	0.0	50.3	50.3	0.0	50.3
The Identical Infinite Transmissivity (a_h = 15, a_l = 15, s = ½)	1	9.8	9.8	73.0	73.0	52.1	52.1	104.1	268.1	268.1	536.3
	2	8.9	8.9	66.2	66.2	42.8	42.8	85.5	216.1	216.1	432.1
	3	8.1	8.1	60.2	60.2	35.5	35.5	70.9	173.3	173.3	346.6
	4	7.4	7.4	55.1	55.1	29.7	29.7	59.4	137.8	137.8	275.7
	5	6.8	6.8	50.7	50.7	25.1	25.1	50.2	108.2	108.2	216.3
	6	6.3	6.3	46.9	46.9	21.4	21.4	42.9	83.1	83.1	166.1
	7	5.9	5.9	43.5	43.5	18.5	18.5	37.0	61.6	61.6	123.2
	8	5.5	5.5	40.7	40.7	16.2	16.2	32.3	43.1	43.1	86.2
	9	5.1	5.1	38.2	38.2	14.2	14.2	28.5	26.9	26.9	53.9
	10	4.9	4.9	36.0	36.0	12.7	12.7	25.4	12.7	12.7	25.4
The Unequal Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	1	13.9	5.2	73.0	73.0	110.6	15.4	126.0	529.7	105.5	635.2
	2	12.5	4.9	62.1	70.8	89.2	13.8	102.9	419.1	90.1	509.2
	3	11.2	4.6	52.7	68.9	72.4	12.4	84.8	330.0	76.4	406.3
	4	10.1	4.4	44.5	67.3	59.2	11.3	70.5	257.6	64.0	321.6
	5	9.2	4.2	37.3	65.9	48.8	10.4	59.2	198.4	52.7	251.1
	6	8.4	4.1	31.1	64.6	40.6	9.6	50.2	149.6	42.3	191.9
	7	7.7	3.9	25.7	63.5	34.1	8.9	43.0	109.0	32.7	141.7
	8	7.1	3.8	21.0	62.6	28.9	8.4	37.3	74.9	23.8	98.7
	9	6.6	3.7	16.9	61.8	24.7	7.9	32.6	46.0	15.4	61.5
	10	6.1	3.6	13.4	61.1	21.3	7.5	28.9	21.3	7.5	28.9
The Identical Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	1	9.5	9.5	73.0	73.0	52.1	52.1	104.3	275.8	275.8	551.5
	2	8.7	8.7	66.5	66.5	43.2	43.2	86.5	223.6	223.6	447.3
	3	7.9	7.9	60.8	60.8	36.2	36.2	72.3	180.4	180.4	360.8
	4	7.3	7.3	55.9	55.9	30.5	30.5	61.1	144.2	144.2	288.5
	5	6.7	6.7	51.6	51.6	26.0	26.0	52.1	113.7	113.7	227.4
	6	6.2	6.2	47.9	47.9	22.4	22.4	44.8	87.7	87.7	175.3
	7	5.8	5.8	44.6	44.6	19.5	19.5	38.9	65.3	65.3	130.5
	8	5.5	5.5	41.8	41.8	17.1	17.1	34.2	45.8	45.8	91.6
	9	5.1	5.1	39.3	39.3	15.1	15.1	30.3	28.7	28.7	57.4
	10	4.9	4.9	37.2	37.2	13.5	13.5	27.1	13.5	13.5	27.1

Table 3: The Numerical solution for the 10-round game following the semi-myopic path

	Round	Decisions		Stocks		Profits			Stock Values		
Case	r	w_{hr}	w_{lr}	x_{hr}	x_{lr}	NP_{hr}	NP_{lr}	$NP_{h+l\ r}$	V_{hr}	V_{lr}	$V_{h+l\ r}$
The Unequal Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	10.1	4.2	73.0	73.0	104.6	13.9	118.5	808.1	55.4	863.5
	2	9.9	3.8	68.9	68.9	97.7	11.0	108.7	703.5	41.5	745.0
	3	9.8	3.4	65.1	65.1	91.6	8.7	100.3	605.8	30.5	636.3
	4	9.7	3.0	61.5	61.5	86.3	6.7	93.0	514.2	21.8	536.0
	5	9.7	2.6	58.2	58.2	81.6	5.1	86.7	427.9	15.1	443.0
	6	9.8	2.3	55.0	55.0	77.3	3.8	81.1	346.3	10.0	356.3
	7	9.9	2.0	52.0	52.0	73.2	2.7	75.9	269.0	6.2	275.2
	8	10.1	1.7	49.1	49.1	69.3	1.8	71.1	195.8	3.5	199.3
	9	10.3	1.3	46.2	46.2	65.4	1.1	66.5	126.5	1.7	128.2
	10	10.7	1.0	43.4	43.4	61.1	0.6	61.7	61.1	0.6	61.7
The Identical Infinite Transmissivity (a_h = 15, a_l = 15, s = ½)	1	7.1	7.1	73.0	73.0	48.8	48.8	97.7	316.5	316.5	633.0
	2	6.8	6.8	68.9	68.9	43.7	43.7	87.4	267.7	267.7	535.4
	3	6.6	6.6	65.1	65.1	39.3	39.3	78.5	224.0	224.0	447.9
	4	6.3	6.3	61.5	61.5	35.4	35.4	70.7	184.7	184.7	369.4
	5	6.2	6.2	58.2	58.2	31.9	31.9	63.8	149.4	149.4	298.7
	6	6.0	6.0	55.0	55.0	28.8	28.8	57.7	117.4	117.4	234.9
	7	5.9	5.9	52.0	52.0	26.0	26.0	52.0	88.6	88.6	177.2
	8	5.9	5.9	49.1	49.1	23.4	23.4	46.8	62.6	62.6	125.2
	9	5.8	5.8	46.2	46.2	20.9	20.9	41.7	39.2	39.2	78.5
	10	5.9	5.9	43.4	43.4	18.4	18.4	36.7	18.4	18.4	36.7

NB: With zero transmissivity (s=0) the decisions under the Semi-myopic are identical to those from SPE and Social Optimum.

Table 4: The Numerical solution for the 10-round game following the social optimum path

	Round	Decisions		Stocks		Profits			Stock Values		
Case	r	w_{hr}	w_{lr}	x_{hr}	x_{lr}	NP_{hr}	NP_{lr}	$NP_{h+l\ r}$	V_{hr}	V_{lr}	$V_{h+l\ r}$
The Unequal Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	8.0	0.0	73.0	73.0	93.2	0.0	93.2	922.3	14.2	936.5
	2	8.4	0.0	72.0	72.0	95.1	0.0	95.1	829.1	14.2	843.3
	3	8.9	0.0	70.8	70.8	96.4	0.0	96.4	734.0	14.2	748.2
	4	9.3	0.0	69.3	69.3	97.0	0.0	97.0	637.6	14.2	651.8
	5	9.8	0.0	67.7	67.7	96.8	0.0	96.8	540.6	14.2	554.8
	6	10.3	0.3	65.8	65.8	95.8	1.1	96.9	443.8	14.2	458.0
	7	10.7	0.7	63.5	63.5	93.6	2.6	96.2	348.0	13.1	361.1
	8	11.2	1.2	60.8	60.8	90.1	3.5	93.7	254.4	10.5	264.9
	9	11.6	1.6	57.7	57.7	85.3	3.8	89.1	164.3	6.9	171.2
	10	12.1	2.1	54.1	54.1	79.0	3.2	82.2	79.0	3.2	82.2
The Identical Infinite Transmissivity (a_h = 15, a_l = 15, s = ½)	1	3.2	3.2	73.0	73.0	28.9	28.9	57.8	345.1	345.1	690.1
	2	3.6	3.6	72.8	72.8	32.0	32.0	64.0	316.2	316.2	632.3
	3	4.1	4.1	72.2	72.2	34.5	34.5	69.1	284.2	284.2	568.4
	4	4.5	4.5	71.2	71.2	36.5	36.5	72.9	249.6	249.6	499.3
	5	5.0	5.0	69.6	69.6	37.7	37.7	75.4	213.2	213.2	426.3
	6	5.4	5.4	67.7	67.7	38.1	38.1	76.2	175.5	175.5	351.0
	7	5.9	5.9	65.3	65.3	37.6	37.6	75.3	137.4	137.4	274.8
	8	6.3	6.3	62.4	62.4	36.2	36.2	72.3	99.7	99.7	199.5
	9	6.8	6.8	59.1	59.1	33.6	33.6	67.3	63.6	63.6	127.1
	10	7.2	7.2	55.3	55.3	29.9	29.9	59.9	29.9	29.9	59.9

NB: With zero transmissivity (s=0) the decisions under the Social Optimum are identical to those from SPE and Social Optimum.

3.4.2 Discussion of numerical results for the Ten-round Game

The numerical results presented in Tables 1-4 were used to construct Figures 5, 6 portraying respectively the players' decisions and the stock evolution, and Figures 7, 8 and 9 on the players' individual, aggregate and aggregate cumulative profits, following the different paths.

In the case of unequal users with an infinite transmissivity (figure 5.a), the Myopic solution involves higher levels of extraction in the early rounds, for both users, that decrease steadily over time.

With the Social Optimum solution, the opposite trend is observed, where the efficient user starts with lower levels of extraction and the less efficient user only starts to extract on the sixth round, indicating that the solution, in this specific case, is a corner solution. The extraction decisions under Social Optimum increase over time and, on the seventh round, compare to those following the Myopic path; in the following rounds the Social Optimum involves higher extractions by both users than the respective levels of extractions with Myopic users.

The figures also show the substantial discrepancy between the extraction decision of unequal users, the efficient player average level of extraction is 5.67 times that of the less efficient user for the Myopic solution, and 17 times for the Social Optimum.

Regarding the SPE, and the Semi-Myopic paths, the figure shows that both solutions are quasi-equivalent for unequal users; for the efficient player, the two paths involve relatively steady levels of use over the entire game.

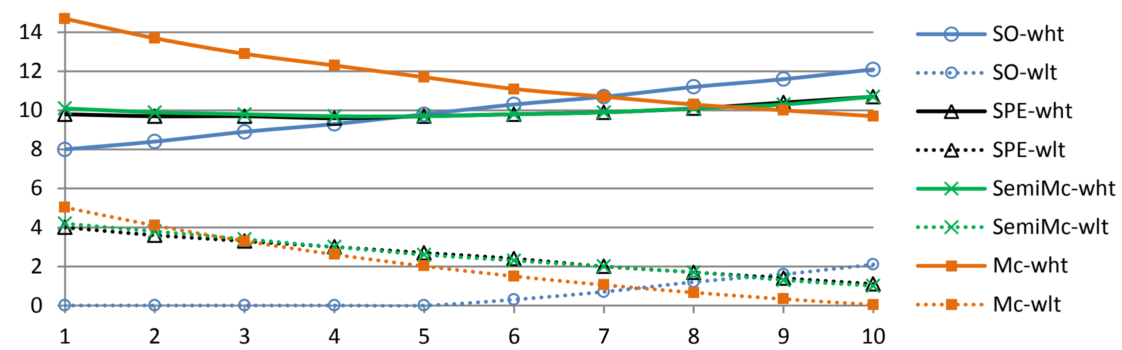

Figure 5.a Extraction decisions in the ten-round game with infinite transmissivity and unequal users

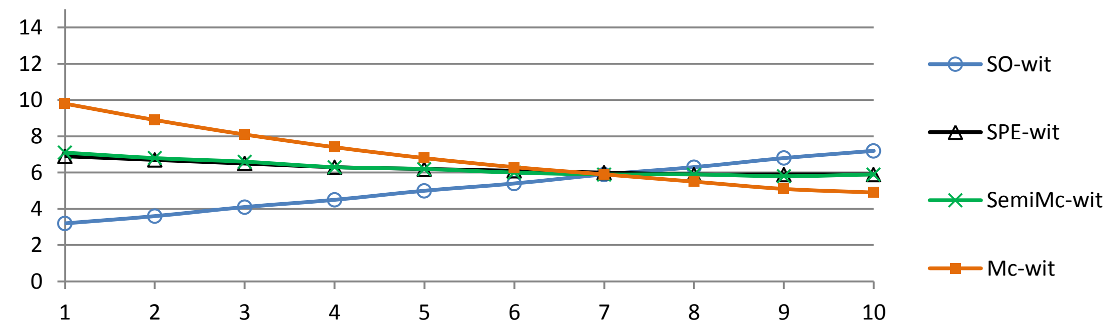

Figure 5.b Extraction decisions in the ten-round game with infinite transmissivity and identical users

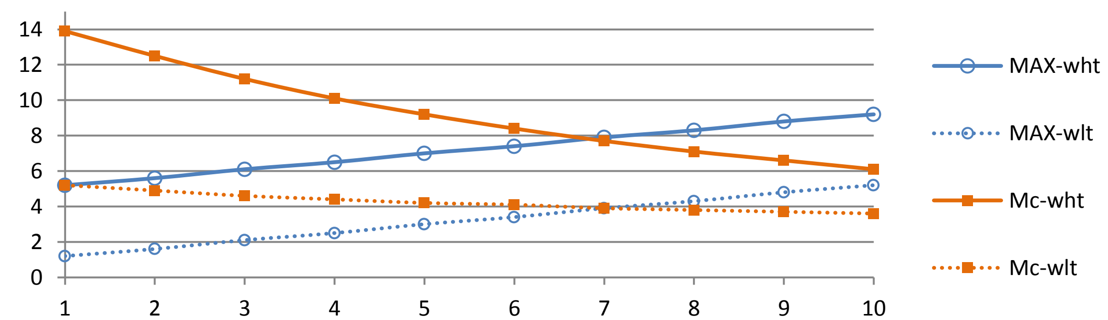

Figure 5.c Extraction decisions in the ten-round game with zero transmissivity and unequal users

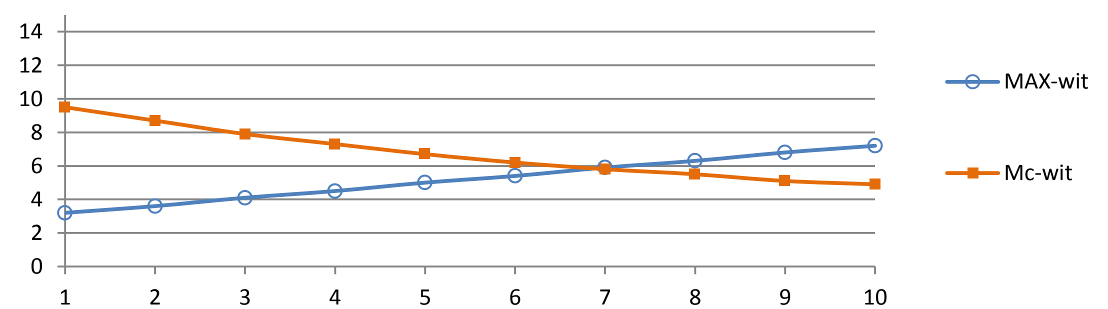

Figure 5.d Extraction decisions in the ten-round game with zero transmissivity and identical users

Figure 5: Extraction decisions in the ten-round game

For the less efficient player, the figures show a tendency to slowly reduce the levels of extraction, they are lower than the levels from the Myopic path in the early rounds, but become higher for the rest of the game, given the sharper drop in the Myopic levels of use.

In the case of identical users with an infinite transmissivity (figure 5.b), the same trends reported in the previous case for the efficient user are maintained for the identical users. In the early rounds, the extraction decisions under Social Optimum are lower than those under SPE, marginally lower than the Semi-Myopic, while the Myopic solution provides the highest levels of extraction.

At the seventh round, the extraction decisions following all solutions merge (around 5.9), and the order is reversed in the following rounds, the Optimum solution involves higher extractions, than the extractions under SPE and Semi-Myopic, both virtually equal and higher than the extractions following the Myopic path.

For every round, the extraction decision for the identical users –with an infinite transmissivity– matches the average decision for the same round, by the two unequal users, from the previous case with unequal users; however, this remark does not hold for the decisions following the Social Optimum, since the solution for unequal users is a corner solution.

In the cases with zero transmissivity, the Social Optimum, the SPE and the Semi-Myopic merge together, and the related solution (to all three strategies) will be referred to as the Maximum path. With unequal users (figure 5.c) and with identical users (figure 5.d), the Maximum solution involves lower extraction at the start that increase steadily in the following rounds, similar to the pattern recorded with the Social Optimum path in the

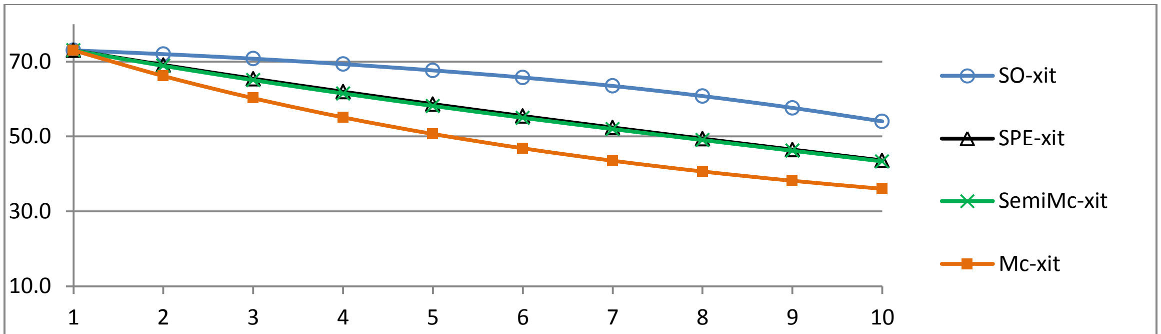

Figure 6.a Stock evolution in the ten-round game with infinite transmissivity and unequal users

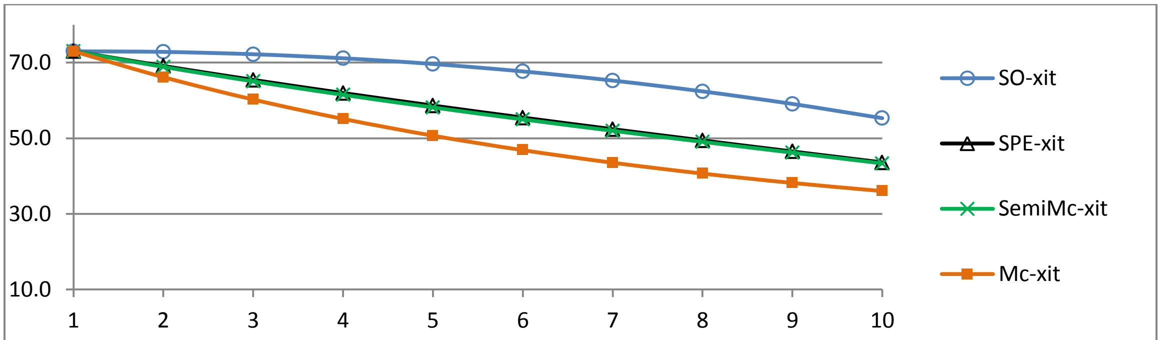

Figure 6.b Stock evolution in the ten-round game with infinite transmissivity and identical users

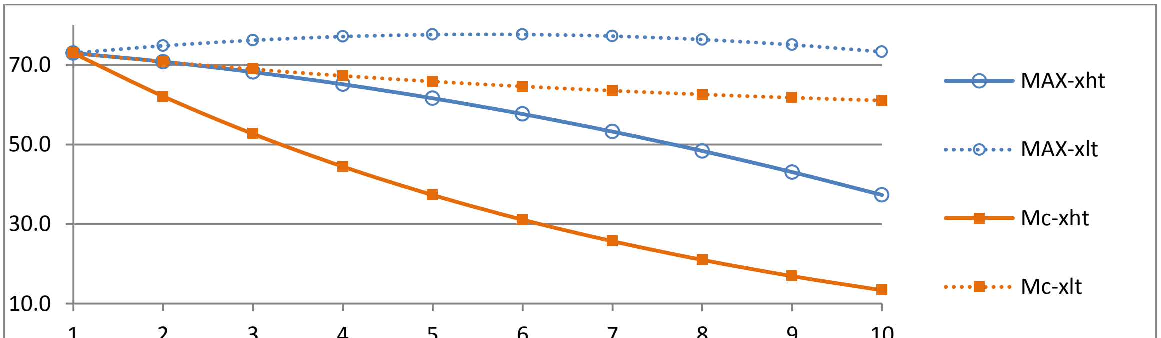

Figure 6.c Stocks evolution in the ten-round game with zero transmissivity and unequal users

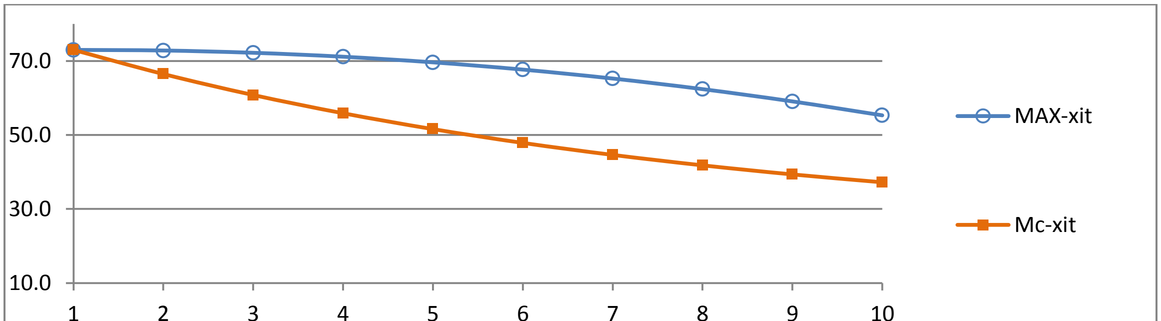

Figure 6.d Stocks evolutionin the ten-round game with zero transmissivity and identical users

Figure 6: Stock evolution in the ten-round game

previously discussed cases, with infinite transmissivity. On the other hand, with zero transmissivity, the Myopic path is connected with higher extraction levels at the start that decrease gradually in the following rounds (figures 5.c & 5d) and, on the seventh round,fall slightly below the extraction levels following the Maximum path, but the discrepancy between the two paths widens in the following rounds.

In the case with unequal users and zero transmissivity (figure 5.c), the increase in the extraction levels following the Maximum solution is uniform across users, while the decrease in the extraction levels following the Myopic path is greater for the efficient user, in absolute and relative terms. The average decisions by unequal users following one path –Myopic or Maximum– equate those by identical users following the same path.

Regarding the evolution of stocks, Figures 6.a and 6.b show that the stocks are the same with infinite transmissivity for the Myopic, the Semi-Myopic and the SPE paths, this is in agreement with the previously established fact that the level of extraction by identical users matches the average extraction by unequal users. For the Social Optimum, the stocks are slightly higher for identical users with an infinite transmissivity.

Figures 6.a and 6.b also confirm that the stocks are always declining, in accord with the fact that the rates of extraction are higher than the rate of renewal. The stock at the start is fixed at 73 for all four paths, but the cumulative rates of extraction are higher under the Myopic regime, and lower under the Social Optimum, the cumulative rates of extraction following the SPE and the Semi-Myopic are very close and lay in the middle.

At the end of the game, Myopic users end up with a stock at 36.0, the users following the Semi-Myopic and the SPE paths finish the game with stocks respectively at 43.6 and 43.4, and the highest stock at the end is related to the Social Optimum path at 54.1.

With zero transmissivity, Figure 6.c shows dissimilarity in the evolution of stocks following the Social Optimum in the case of unequal players, the less efficient user extraction levels are lower than the rate of renewal, the Maximization solution involves building the stock levels to gain from lower costs of extraction. However, following the Myopic path, the levels of extraction by less efficient user are higher than the extraction rates and the stock levels are always lower than those with the Maximization path. For the efficient user the stock is always declining following both paths, the Maximization and the Myopic, the cumulative levels of extraction are higher following the Myopic path leading to lower levels of stock.

For identical users, the level of stock is relatively stable in the early rounds following the Maximization path (Figure 6.d), then it starts to decline steadily in the rest of the game; with Myopic users, the drop in the level of stock is high throughout the game especially in the early rounds. At the end of the game, the stock is at 37.2 for Myopic users and 55.3 for Maximizing users.

In terms of individual profits, figure 7.a shows that with unequal users and infinite transmissivity, the profits for the efficient user from the Myopic extraction game are always decreasing, while those following the Social Optimum are increasing gradually in the first half of the game then decrease slowly in the second half; in the first two rounds, the individual profits –for the efficient user– are higher following the Myopic path, but starting on the third round, the Social Optimum guarantees higher profits, and the gap even widens in the next rounds. The SPE and Semi-Myopic paths engender practically the same profits for the efficient user, progressively decreasing over time and always at halfway between the profits from Social Optimum and those from the Myopic path.

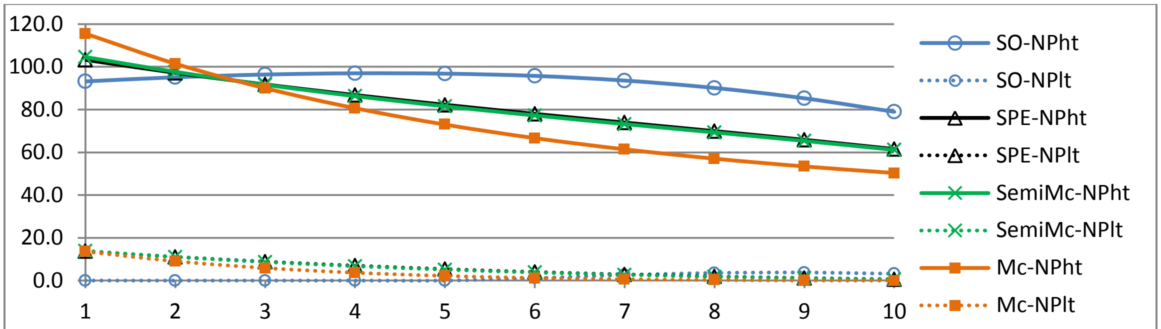

Figure 7.a Individual profits in the ten-round game with infinite transmissivity and unequal users

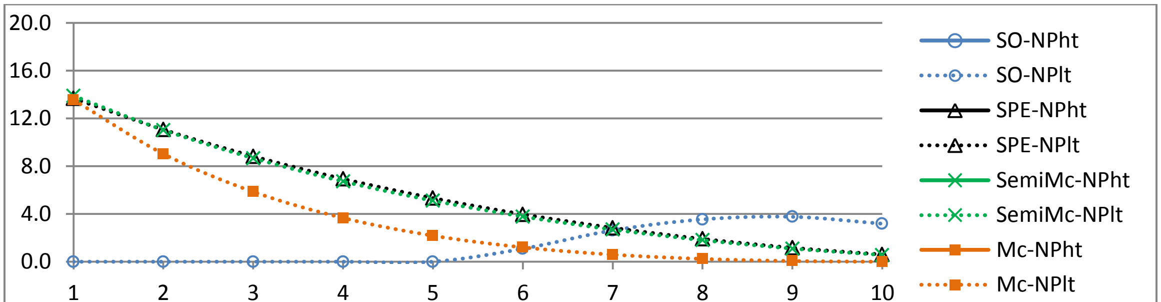

Figure 7.b Individual profits in the ten-round game with infinite transmissivity and unequal users

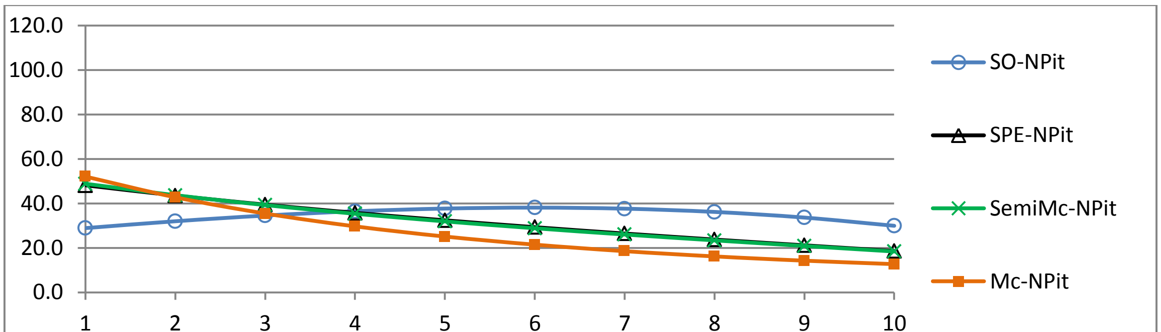

Figure 7.c Individual profits in the ten-round game with infinite transmissivity and identical users

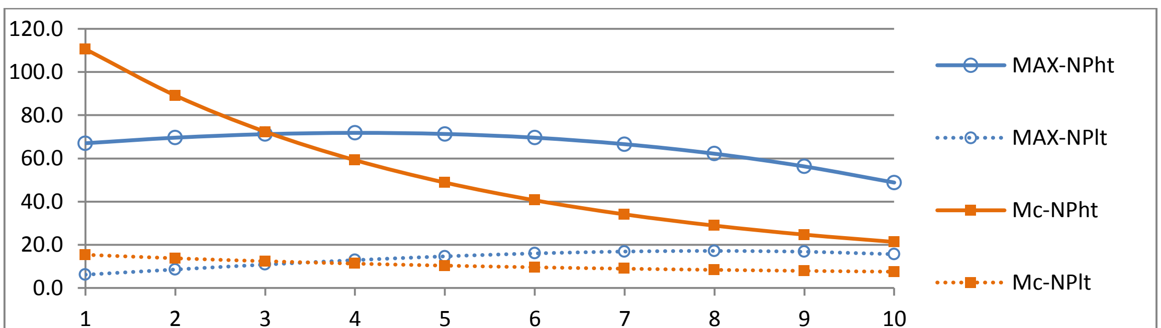

Figure 7.d Individual profits in the ten-round game with zero transmissivity and unequal users

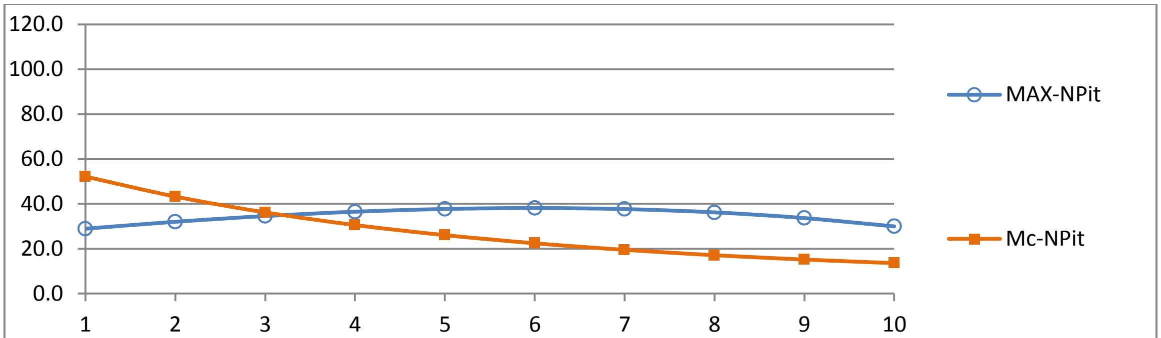

Figure 7.e Individual profits in the ten-round game with zero transmissivity and identical users

Figure 7 Individual profits in the ten-round game

As a result, as displayed in Figure 9, the cumulative profits for the efficient user are higher under the Social Optimum, and lower when the Myopic extraction prevails.

In the same setting, with unequal users and infinite transmissivity, Figure 7.b shows that the SPE and Semi-Myopic paths engender practically the same profits for the less efficient user that are always higher, even in the early rounds, than the profits from the Myopic use. The corner solution following the Social Optimum, as pointed out earlier, implies zero profits in the 5 first rounds. Despite the less efficient user's higher profits, in the last rounds, from Social Optimum, the cumulative profits –from Social Optimum– fall below those under SPE and Semi-Myopic paths, and even those from the Myopic path (figure 9). This is an important result, as it shows that the Social Optimum does not benefit the less efficient user.

Finally, Figure 8.a shows that, for unequal users with infinite transmissivity, the overall evolution of aggregate profits is greatly influenced by the efficient user's profits, and most of the observations previously outlined for the efficient user apply for aggregate profits, for all four considered extraction strategies. This is especially true for the aggregate cumulative profits, as shown in figure 9, where the aggregate cumulative profits for unequal users –with infinite transmissivity– are lower following the Myopic path and higher for the Social Optimum, while the SPE and Semi-Myopic paths are in the middle.

For identical users with infinite transmissivity, the individual (and aggregate) profits are declining following the Myopic path and progressively decreasing following the SPE and Semi-Myopic paths (figure 7.c). The profits from the Social Optimum are lower in the first rounds, but they increase progressively in the early rounds to equate the profits following the Myopic path and the SPE and Semi-Myopic paths, respectively on the third and fourth rounds, then exceed them for the rest of the game, despite the slowly decreasing tendency of Social Optimum profits in the last rounds. The cumulative aggregate profits for identical users –with infinite transmissivity– replicate the same trend observed in the previous case with unequal users (figure 9), but are lower in value.

With zero transmissivity, Figure 7.d shows that the profits following the maximizing path tend to increase in the early rounds, a little sharper, in relative terms, for the less efficient user, then decrease in the last rounds, especially for the efficient user.

The profits following the Myopic path, on the other hand, present a tendency to decrease during the entire game, at a stronger rate in the early rounds for the efficient user.

The aggregate profits (for the two paths) are heavily influenced by the efficient user's profits and present the same evolution (Figure 8.c).

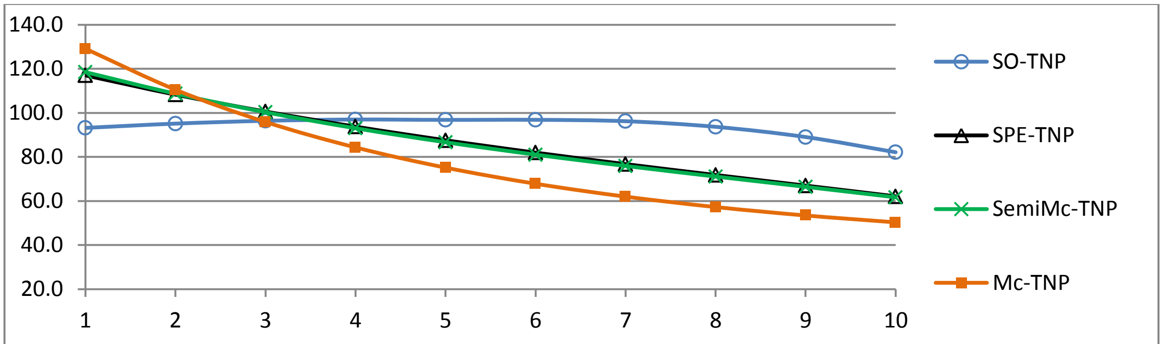

Figure 8.a Aggregate profits in the ten-round game with infinite transmissivity and unequal users

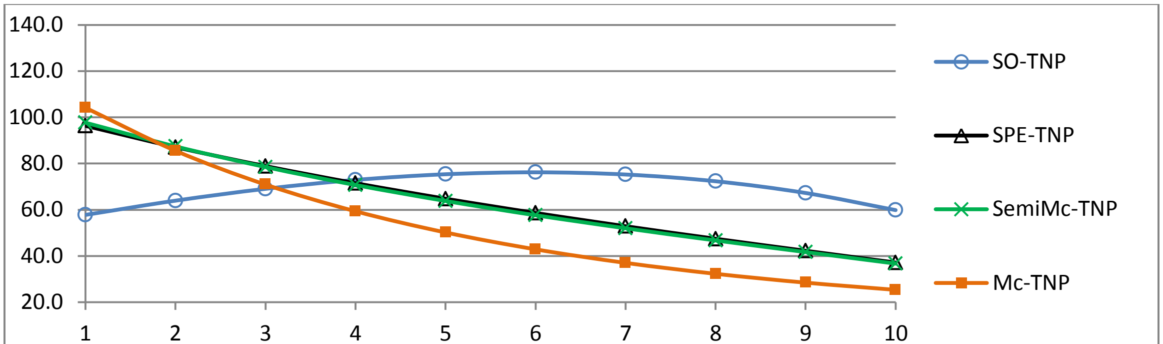

Figure 8.b Aggregate profits in the ten-round game with infinite transmissivity and identical users

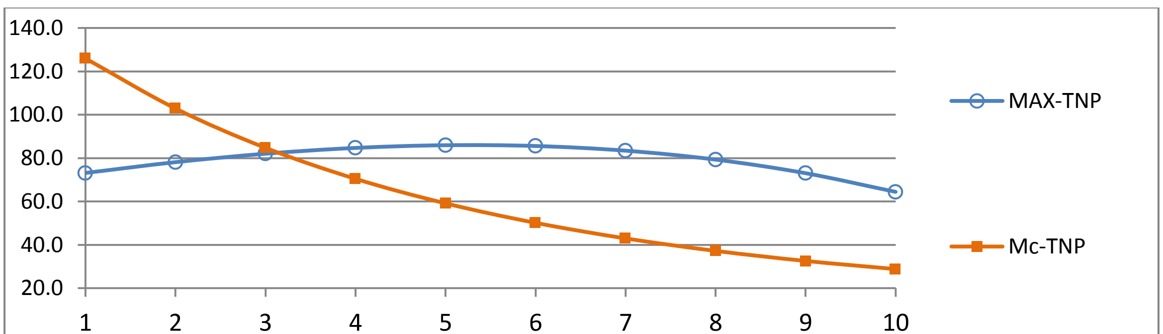

Figure 8.c Aggregate profits in the ten-round game with zero transmissivity and unequal users

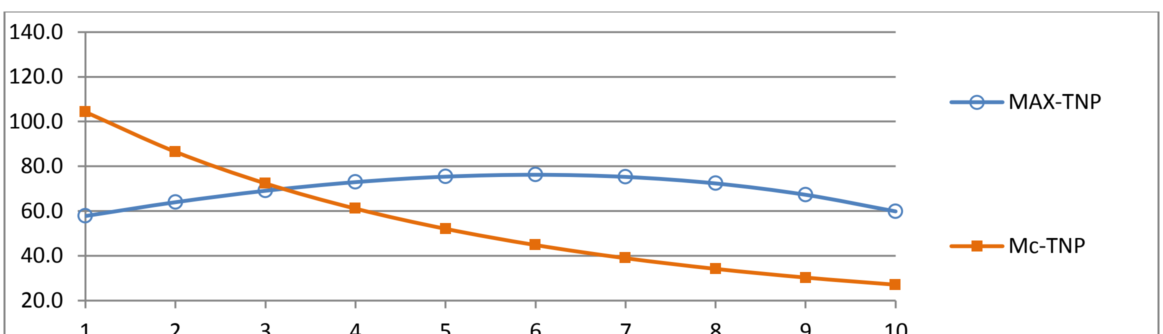

Figure 8.d Aggregate profits in the ten-round game with zero transmissivity and identical users

Figure 8: Aggregate profits in the ten-round game

For identical users –with infinite transmissivity–, figures 7.e shows that the profits follow the same tendency for the efficient user in the previous case, the profits resulting from a Myopic use are decreasing, at a greater rate in the early rounds, while the profits from the Maximizing use increase slowly to reach a maximum (at the sixth round) then decrease slowly.

Regarding aggregate cumulative profits, figure 9 shows that the Maximizing path leads to the same profits for identical users as in the case with identical users with infinite transmissivity following the Social Optimum. The Myopic solution brings about higher profits for identical users than in the case of Myopic identical users with infinite transmissivity. Further results will be discussed in the next subsection that looks especially into validating the theoretical results SPE1-4.

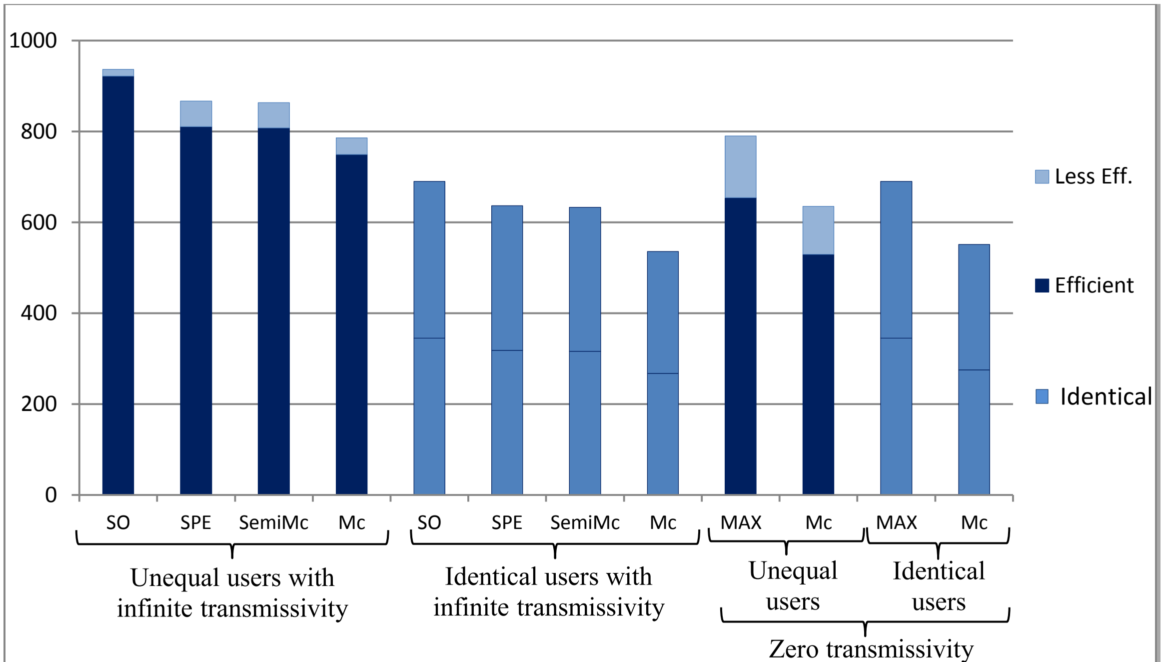

Figure 9: Aggregate cumulative profits in the ten-round game

3.4.3 Numerical validation of results SPE 1-4 for the 10-round game

***Result SPE1**: Inequality increases the aggregate cumulative profits from the common-pool resource.*

In the case of an infinite transmissivity (s = 1/2), table 1 shows that aggregate cumulative profits from the resource, following the SPE path, come to 866.9 ($\sum_{t=1}^{10} NP_{ht} = 810.5$ and $\sum_{t=1}^{10} NP_{hl} = 56.4$) for unequal users, while the aggregate cumulative profits for identical users are only 637.0 ($\sum_{t=1}^{10} NP_{it} = 318.5$ for every user).

Under the assumption of zero transmissivity (s = 0), the aggregate cumulative profits from the resource with unequal users sum up to 790.2 ($\sum_{t=1}^{10} NP_{ht} = 654.6$ and $\sum_{t=1}^{10} NP_{hl} =$ 135.6) and only reach 690.2 for identical users (345.1 each).

***Result SPE2**: Transmissivity decreases the aggregate cumulative profits from the common-pool resource for identical or slightly unequal players.*

As pointed out in the above paragraph the aggregate cumulative profits from a resource exploited by identical users diminishes from 690.2 to 637.0 as I move from a zero-transmissivity resource to a resource with infinite transmissivity.

***Result SPE3**: In the case of highly unequal players, higher transmissivity tends to increase the aggregate cumulative profits.*

When transmissivity is infinite (s = 1/2) the aggregate cumulative profits from a resource used by unequal appropriators is 866.9, with a zero-transmissivity resource (s=0), the same profits are reduced to 790.2.

Result SPE4*: Transmissivity always decreases the cumulative profits for less efficient player and, in the case of a high inequality between players, increases the cumulative profits for the more efficient ones.*

As confirmed in Result SPE2, in the case of identical players transmissivity decreases the cumulative profits of both players, when the resource is used by unequal appropriators transmissivity decreases the profits of the lower efficiency player from 135.6 when s=0 to 56.4 for s=1/2. For the user with high efficiency, the opposite trend prevails, and cumulative profits increase from 654.6 to 866.9.

3.5 Conclusion

This chapter is key to understanding the rest of this research project; it introduces the discrete version of the model of a CPR aquifer with a finite transmissivity and unequal users.

Backward dynamic programming was then used to solve for the players' decisions, following the non-cooperative game, the myopic game, the semi-myopic game, and the social optimum management.

The semi-myopic game refers to a new strategy, introduced in this essay, when each player makes her decisions rationally, but acts under the impression that her partner is myopic.

For the one-round game, the analytical proof is provided to validate the first four results established in the previous chapter, for the non-cooperative game:

- ***Result SPE1****: Inequality increases the aggregate cumulative profits from the common-pool resource.*
- ***Result SPE2****: Transmissivity decreases the aggregate cumulative profits from the common-pool resource for identical or slightly unequal players.*
- ***Result SPE3****: In the case of highly unequal players, higher transmissivity tends to increase the aggregate cumulative profits.*
- ***Result SPE4****: Transmissivity always decreases the cumulative profits for less efficient players and, in the case of a high inequality between players, increases the cumulative profits for the more efficient ones.*

Those results are confirmed, with numerical evidence, for the 10-round game; the numerical specifications used here and the related dynamic solutions will set the framework for the experimental validation that will be presented in the next chapter.

The same discrete model is applied to establish the theoretical results in Chapter 5 devoted to policy analysis.

In the model, I assume infinite transmissivity for the numerical validation in the discrete model, but the proof can be established even with finite transmissivity (s less than ½ in this case); this choice is solely dictated by the fact that the numerical example presented is also replicated in the experimental design, where the assumption of infinite transmissivity facilitates participants' understanding of the game.

Chapter 4 The Experimental Validation for the Extraction Game

4.1 The Experimental Design

4.1.1 Parameterization

In my experimental design, I use four treatments corresponding to the four cases previously presented in the numerical validation in Chapter 3:

- The Unequal Users - Infinite Transmissivity treatment: $a_h = 20$, $a_l = 10$ and $s = ½$,
- The Identical Users - Infinite Transmissivity treatment: $a_h = 15$, $a_l = 15$ and $s = ½$,
- The Unequal Users - Zero Transmissivity treatment: $a_h = 20$, $a_l = 10$ and $s = 0$, and
- The Identical Users - Zero Transmissivity treatment: $a_h = 15$, $a_l = 15$ and $s = 0$.

The four treatments are the matching of two levels of inequality (highly unequal and identical players) and two levels of transmissivity (Infinite and zero transmissivity) which will allow investigating the effects of inequality and transmissivity. For all other parameters of the game, I use the same specifications as in Chapter 3, constant across treatments:

- $x_{h0} = x_{l0} = 73$, the stocks at the start of round one,
- $E = 100$, Elevation of the fields' surface,
- $c = 0.15$, Cost to lift a unit of water per unit of height, and
- $R = 3$, the aquifer natural recharge, uniform across users.

4.1.2 The Experiment

The experimental sessions were held in the experimental laboratory of the Department of Agricultural and Resource Economics at the University of Maryland. Volunteer subjects were recruited from undergraduate classes in economics, business, and civil engineering. They were informed that they would participate in an experiment where they would be asked to make economic decisions and receive payments based on their decisions and those of other participants. They were also informed of the average length of a session.

I ran seven experimental sessions. Each session was dedicated entirely to one of the four treatments presented earlier. For treatments with infinite transmissivity, at the beginning of each session the subjects were randomly assigned into different groups (of two each). Each subject was informed of her own efficiency, a_i, and, when applicable, that of her partner's, a_{-i}. The subjects were also informed of the other parameters that define the game: s, c, E, x_{i0}, x_{-i0}, and R, the rate of renewal. The efficiencies and pairings of the players were kept secret, as were the individual decisions made throughout the experiment.

Instructions were read aloud while subjects followed on the instructions text that they were provided, and formulas for production and cost were given (See appendix D for the full instructions text). In each round, participants were asked to choose (simultaneously) the quantities to be extracted from a given range [0, a_i] (Figure 10.a).

Once the two partners made their decisions, they were informed of the total order (per group) and the subsequent costs (Figure 10.b). At the end of each round they were updated on their earnings, in laboratory dollars, and on the level of water in the aquifer available at the beginning of the next period (to evaluate the cost).

Figure10.a Decision table from the Extraction game experiment

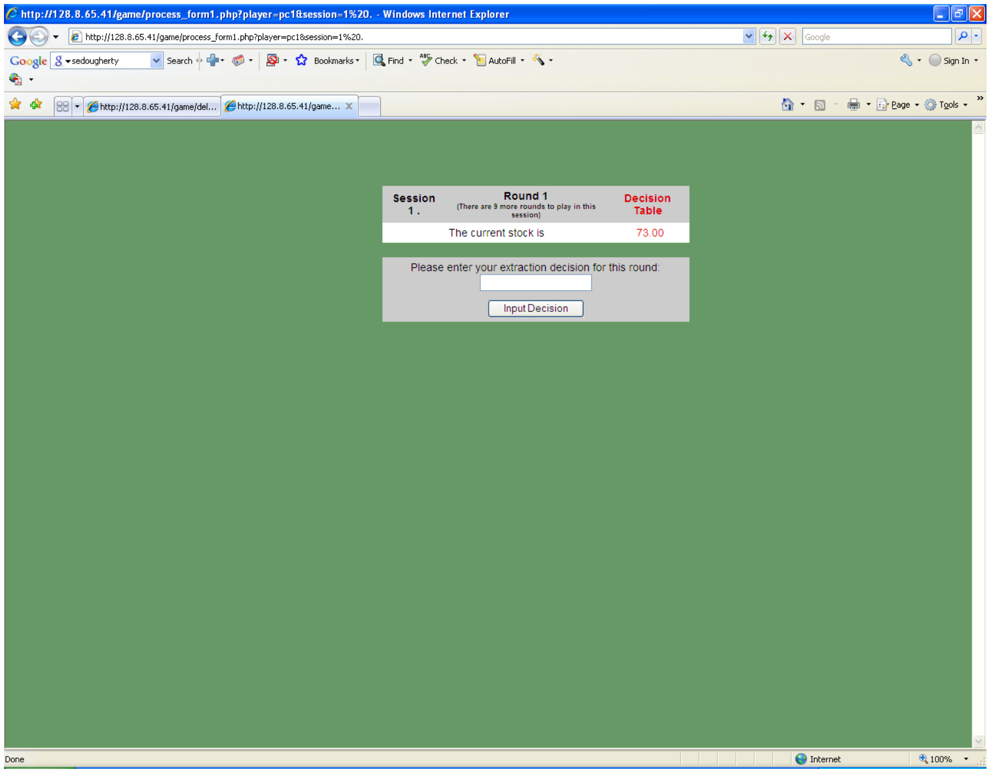

Figure10.b Results table from the Extraction game experiment

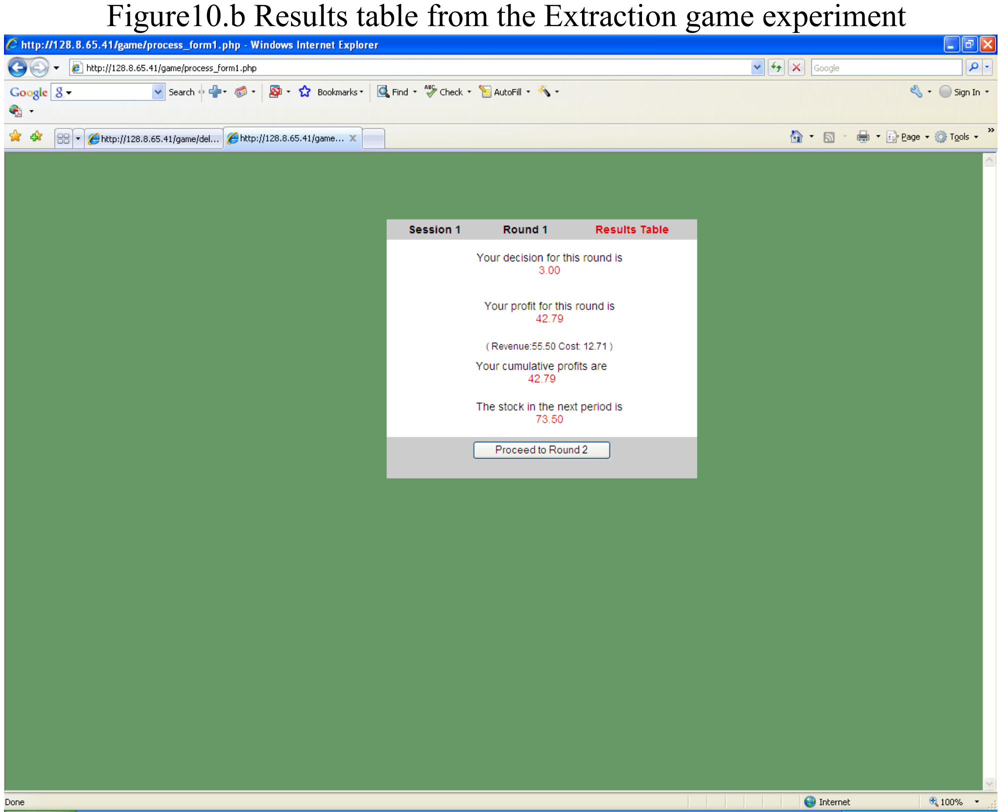

Figure 10: Decision and Results tables from the Extraction game experiment

The participants were awarded $10 ($5 in the early sessions) for their participation and were given the conversion rate that would be used for each type to convert the computer dollars earned during the experiment to real dollars. At the end of the experiment they were given, privately, their earnings in cash.

Every experimental session consisted of four series of 10 rounds each and lasted 15 and 20 minutes.[65] For infinite-transmissivity resource treatments, during the first two sessions subjects were matched with the same partner, and during the two last sessions they were matched with another partner with the same efficiency (a_{-i}) as their previous partner (therefore they always participated in the same treatment). This way I got more data for my (in-between) analysis and I can expect more power for the statistical tests. The subjects were informed of this (random) change of partner.

Table 5: Summary of experimental sessions.[66]

Treatment	Experimental Sessions	Type	Subjects	Observations	Average profits per subject
Unequal Users - Infinite Transmissivity	4	*h*	12	41	$31.4
		l	12	41	$30.2
Identical Users - Infinite Transmissivity	2		24	76	$28.1
Unequal Users – Zero Transmissivity	1	*h*	12	48	$29.3
		l	12	47	$29.1
Identical Users – Zero Transmissivity	1		20	79	$28.2

To help participants make their decisions, they were invited to use the *decision support window* that was provided to them on screen (See Figure 11). Based on the current stocks

[65] The total length of experimental sessions was between one and a half hour and two hours.

[66] The average profits reported in the Table 5 include the show-up fees.

(x_{ir}, x_{-ir}), their decision, w_{ir}, and their assumption about their partner's decision, w_{-ir}, the players were able to compute their hypothetical profits for the current round, the stocks at the start of the next round, and the ensuing hypothetical profits of their partners.

Figure 11: Support Sheet from the Extraction game experiment

1- Enter the updated stock for this round as provided in the explorer program

The current stock 73.00

2- Enter your decision and your "best guess" for your partner's decision

Your decision 3.00

The other person's decision 4.00

3- Outcomes and next round stock

Your earnings for this round are: **27.56** (revenue: 40.50 and cost: 12.94)

The other person's earnings for this round a **34.75** (revenue: 52.00 and cost: 17.25)

The stock in next period : **72.50**

4- Tables of Extended Outcomes and next round stocks

Your earnings table

		Your decision				
		2.00	2.50	**3.00**	3.50	4.00
The other person's decision	3.00	19.53	23.73	27.68	31.35	34.75
	3.50	19.49	23.69	27.62	31.28	34.68
	4.00	19.45	23.64	**27.56**	31.22	34.60
	4.50	19.41	23.59	27.51	31.15	34.53
	5.00	19.38	23.55	27.45	31.08	34.45

The other person's earnings table

		The other person's decision				
		3.00	3.50	**4.00**	4.50	5.00
Your decision	2.00	27.79	31.48	34.90	38.05	40.94
	2.50	27.73	31.41	34.83	37.97	40.84
	3.00	27.68	31.35	**34.75**	37.88	40.75
	3.50	27.62	31.28	34.68	37.80	40.66
	4.00	27.56	31.22	34.60	37.72	40.56

Next round stock table

		Your decision				
		2.00	2.50	**3.00**	3.50	4.00
The other person's decision	3.00	73.50	73.25	73.00	72.75	72.50
	3.50	73.25	73.00	72.75	72.50	72.25
	4.00	73.00	72.75	**72.50**	72.25	72.00
	4.50	72.75	72.50	72.25	72.00	71.75
	5.00	72.50	72.25	72.00	71.75	71.50

NB: The spreadsheet was protected and subjects were only allowed to modify the yellow cells.

4.2 Analysis of Experimental Results

4.2.1 Analysis of Individual Behavior

For the analysis of data from the experiment, I proceed here with a primary phase, inspired by the work of Herr et al. (1997), where experimental data are measured up against markers corresponding to the theoretical predictions following the different paths. I will use as markers sequences of decisions over the 10-rounds, based on the decisions following the four paths previously introduced (the Social Optimum, the SPE, the Semi-Myopic and the Myopic). For every treatment, I will compare the participants' extraction decisions from every experimental treatment to markers corresponding to the same experimental settings in terms of transmissivity, inequality and, when appropriate, the type of player. The objective of this first step is to get a better perception of individual subjects' behavior during the experiment.[67]

Table 6 recaps the individual extraction decisions following the different paths, as defined in the theoretical section.[68] The figures in Table 6 were used in Figures 5a-d presented and discussed in the Chapter 3. In summary, Table 6 illustrates a tendency to conserve in the early stages and to extract more in the last rounds, following the social optimum path. The Myopic path presents the opposite trend, as extractions are high in the first round and drop continuously over the entire duration of the game.

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[67] Theoretical Results SPE 1-4 are based on the assumption that players will act following the SPE. I therefore need to test that hypothesis before trying to test for the changes in subjects' behavior from one treatment to another.

[68] For the two last treatments, I will only have two markers — maximum and myopic — since they correspond to the case with zero transmissivity. In that case, the Social Optimum, the SPE and the Semi-myopic paths converge.

Table 6: Players' theoretical decisions following the Social Optimum, SPE, Semi-Myopic and Myopic paths

	Round	Social Optimum		SPE		Semi-Myopic		Myopic	
	r	w_{hr}	w_{lr}	w_{hr}	w_{lr}	w_{hr}	w_{lr}	w_{hr}	w_{lr}
Unequal users - Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	8.0	0.0	9.8	4.0	10.1	4.2	14.7	5.0
	2	8.4	0.0	9.7	3.6	9.9	3.8	13.7	4.1
	3	8.9	0.0	9.7	3.3	9.8	3.4	12.9	3.3
	4	9.3	0.0	9.6	3.0	9.7	3.0	12.3	2.6
	5	9.8	0.0	9.7	2.7	9.7	2.6	11.7	2.0
	6	10.3	0.3	9.8	2.4	9.8	2.3	11.1	1.5
	7	10.7	0.7	9.9	2.0	9.9	2.0	10.7	1.1
	8	11.2	1.2	10.1	1.7	10.1	1.7	10.3	0.7
	9	11.6	1.6	10.4	1.4	10.3	1.3	10.0	0.3
	10	12.1	2.1	10.7	1.1	10.7	1.0	9.7	0.0
Identical Users - Infinite Transmissivity (a_h = 15, a_l = 15, s = ½)	1	3.2	3.2	6.9	6.9	7.1	7.1	9.8	9.8
	2	3.6	3.6	6.7	6.7	6.8	6.8	8.9	8.9
	3	4.1	4.1	6.5	6.5	6.6	6.6	8.1	8.1
	4	4.5	4.5	6.3	6.3	6.3	6.3	7.4	7.4
	5	5.0	5.0	6.2	6.2	6.2	6.2	6.8	6.8
	6	5.4	5.4	6.1	6.1	6.0	6.0	6.3	6.3
	7	5.9	5.9	6.0	6.0	5.9	5.9	5.9	5.9
	8	6.3	6.3	5.9	5.9	5.9	5.9	5.5	5.5
	9	6.8	6.8	5.9	5.9	5.8	5.8	5.1	5.1
	10	7.2	7.2	5.9	5.9	5.9	5.9	4.9	4.9
Unequal Users - Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	1			5.2	1.2			13.9	5.2
	2			5.6	1.6			12.5	4.9
	3			6.1	2.1			11.2	4.6
	4			6.5	2.5			10.1	4.4
	5			7.0	3.0			9.2	4.2
	6			7.4	3.4			8.4	4.1
	7			7.9	3.9			7.7	3.9
	8			8.3	4.3			7.1	3.8
	9			8.8	4.8			6.6	3.7
	10			9.2	5.2			6.1	3.6
Identical Users - Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	1			3.2	3.2			9.5	9.5
	2			3.6	3.6			8.7	8.7
	3			4.1	4.1			7.9	7.9
	4			4.5	4.5			7.3	7.3
	5			5.0	5.0			6.7	6.7
	6			5.4	5.4			6.2	6.2
	7			5.9	5.9			5.8	5.8
	8			6.3	6.3			5.5	5.5
	9			6.8	6.8			5.1	5.1
	10			7.2	7.2			4.9	4.9

NB: With zero transmissivity (s=0) the decisions under the Social Optimum are identical to those from SPE and Semi-Myopic.

The same decreasing tendency, but less pronounced, is observed with identical and less efficient users of an infinite transmissivity resource, following the Semi-Myopic and SPE paths. In the case of a resource with infinite transmissivity used by unequal players, under both strategic and semi-myopic regimes the more efficient player tends to decrease extraction slightly, in the early rounds, before increasing it moderately in the final rounds.

At first, I start by comparing the extraction decisions data, by the laboratory subjects, to the theoretical results (in Table 6) following the four strategies, used here as unconditional markers. Next, the laboratory extraction decisions will be judged against the extraction decisions obtained based on the decisions rules following the different paths.

As established for all considered strategies (See Chapter 3), with an interior solution, the theoretical extraction decisions are linear functions of current level(s) of stock, where coefficients (for a given path) depend on the current round. The decisions based on the current level of stock will be referred to as conditional markers.

This analysis will be carried by sequence of 10-rounds. A first analysis of experimental data shows a significant correlation between the players' decisions and the corresponding round (when the decision is made) while the stock level at the start of the round does not have any significant effect.[69]

When the dummies for the sequences of ten rounds are included in regression; the stock level is shown to have a strong effect on the players decisions; in addition, out of the 329

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[69] The other variables included in the regression are efficiency and transmissivity.

sequences used in the regression (2 sequences eliminated for collinearity) 300 have are statistically significant at the 1% level (p<0.01).

4.2.1.1 Best Describing Unconditional Marker

In each experimental session, subjects participated in four 10-round sequences. The decisions made by each player in every round of every sequence were compared to the Social Optimum, SPE, Semi-Myopic and Myopic paths decisions corresponding to the same treatment and for the same player's type *h* or *l* (see Table 6).

For each sequence, the sum of square deviations (SSD) of all rounds was calculated for every marker; the marker that corresponds to the lowest SSD was recognized as the best describer of the player's behavior for that sequence.[70]

The results from this first step helped compile the partition of players by best describing unconditional marker, presented in Table 7;[71] the table also displays the average (across players) SSD for each unconditional marker.

Analysis of the results in Table 7 shows that myopic behavior is clearly the most common best-describing marker; indeed 56% of all sequences, across all four treatments, are best described by the myopic behavior. However, in the case of unequal users with zero transmissivity, a -slim- majority of efficient users follows the maximizing path.

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[70] For example, for the Myopic –unconditional– marker I evaluate the SSD for player *i* during sequence *S* as: $SSD_{i,S}^{Myopic} = \sum_{r=1}^{10} \left(w_{ir}^{S} - w_{ir}^{Myopic}\right)^2$, where w_{ir}^{S} denotes the decision taken by player *i*,on round *r* of the considered sequence *S*; the Best Describing Unconditional Marker is the marker corresponding to the minimum of $SSD_{i,S}^{Myopic}, SSD_{i,S}^{Semi-Myopic}, SSD_{i,S}^{SPE}, and\ SSD_{i,S}^{Social\ Optimum}$.

[71] In fact, the result is the partition of 10-round sequences by best describing marker, the use of partition of players is a reminder that the calculations and following analysis are carried by type of player.

Table 7 Partition of Players per Best Unconditional Describing Marker & Average SSD.

				Partition of players per best describing marker				Average Sum of Square Deviations			
Treatment	Sequence	Type	Obs.	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic
Unequal users & Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	*h*	12	8.3%	25.0%	16.7%	50.0%	161.4	130.9	**129.5**	174.0
		l	12	0.0%	25.0%	33.3%	41.7%	135.5	36.1	**35.5**	44.0
	2	*h*	11	9.1%	18.2%	18.2%	54.5%	163.1	134.3	**132.6**	168.7
		l	11	0.0%	27.3%	27.3%	45.5%	106.0	23.8	**23.5**	32.0
	3	*h*	7	0.0%	28.6%	0.0%	71.4%	185.0	149.1	**147.1**	182.6
		l	7	0.0%	42.9%	0.0%	57.1%	98.2	17.8	**17.4**	23.4
	4	*h*	11	27.3%	0.0%	9.1%	63.6%	161.8	135.3	**133.7**	172.8
		l	11	0.0%	36.4%	0.0%	63.6%	106.0	20.3	**19.8**	26.7
	Total per type	*h*	41	12.2%	17.1%	12.2%	58.5%	166.0	136.1	**134.5**	173.7
		l	41	0.0%	31.7%	17.1%	51.2%	113.3	25.4	**25.0**	32.6
	Total		82	6.1%	24.4%	14.6%	54.9%	139.6	80.8	**79.7**	103.2
Identical users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	1		22	27.3%	18.2%	4.5%	50.0%	96.4	48.2	**48.0**	60.4
	2		24	12.5%	12.5%	12.5%	62.5%	120.4	48.0	46.3	**41.4**
	3		12	16.7%	16.7%	8.3%	58.3%	119.1	50.3	49.0	**47.0**
	4		18	0.0%	11.1%	5.6%	83.3%	124.3	42.9	41.1	**30.6**
	Total		76	14.5%	14.5%	7.9%	63.2%	114.2	47.2	46.0	**45.2**
Unequal users & Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	1	*h*	12		41.7%		58.3%		149.3		**135.0**
		l	12		33.3%		66.7%		49.3		**29.2**
	2	*h*	12		33.3%		66.7%		127.1		**97.9**
		l	11		36.4%		63.6%		37.2		**22.7**
	3	*h*	12		50.0%		50.0%		**87.4**		131.9
		l	12		58.3%		41.7%		30.8		**21.0**
	4	*h*	12		83.3%		16.7%		**73.7**		173.4
		l	12		50.0%		50.0%		29.9		**19.3**
	Total per type	*h*	48		52.1%		47.9%		**109.4**		134.5
		l	47		44.7%		55.3%		36.8		**23.1**
	Total		95		48.4%		51.6%		**73.5**		79.4
Identical users & Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	1		19		21.1%		78.9%		92.9		**47.7**
	2		20		45.0%		55.0%		75.3		**54.2**
	3		20		50.0%		50.0%		86.9		**79.6**
	4		20		55.0%		45.0%		**80.4**		86.0
	Total		79		43.0%		57.0%		83.8		**67.1**

NB: With zero transmissivity (*s*=0) the decisions under the Social Optimum are identical to those from SPE and Semi-Myopic.

Table 7 also shows that the share of sequences following the myopic path is higher with infinite transmissivity, but this observation is not valid -in treatments with unequal users- for the less efficient users.

The myopic path is followed by strategic behavior (along SPE) for treatments with transmissivity, and by the maximizing[72] behavior for treatments without transmissivity. However, when I recreate Table 7 without the myopic marker, in order to accurately compare the three other paths, for the treatments with infinite transmissivity, the Semi-Myopic behavior emerges as the best-describing path (Table 8).

Table 8: Partition of Players per Best Unconditional Describing Marker without Myopic path

Treatment	Sequence	Type	Obs.	Social Optimum	Subgame Perfect Eq.	Semi-Myopic
Unequal users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	Total per type	*h*	41	12.20%	17.10%	70.70%
		l	41	0.00%	31.70%	68.30%
	Total		82	6.10%	24.40%	69.50%
Unequal users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	Total		76	14.50%	14.50%	70.00%

A look at the sum of square deviations in Table 7 confirms the previous findings in the case of treatments with identical players, as the sum of square deviations from the myopic path is found to be lower. For treatments with unequal players, the lowest deviations correspond to the semi-myopic path for a zero transmissivity resource; for an infinite transmissivity resource, both the partition table and the square deviations show that the maximizing path is the best describing marker for efficient players and the myopic path is the best one for less efficient players.

4.2.1.2 Best Describing Conditional Marker

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[72] In the case where s = 0, the semi-myopic, strategic (SPE) and social optimum (SO) paths are one and the same and correspond to the single user maximization path.

In this section, an exercise similar to the one conducted for unconditional markers is conducted using the conditional markers for the Social Optimum, SPE, Semi-Myopic and Myopic paths. The conditional markers are calculated using the same approach described in the theoretical section, readjusted to the actual levels of stock and to the number of rounds left in the game.

For example, in the fifth round the maximization is reviewed for the last six rounds using the actual level of stocks at the beginning of round five. This of course implies that the calculations were done for every player at every round for every sequence, instead of one table per treatment as in the previous subsection.[73]

As shown in Table 9, taking into consideration the actual stocks emphasizes even further the share of players following the –conditional– myopic path, especially for treatments with transmissivity, where 67% of all sequences for identical players and 74% of sequences for unequal players are best described by the myopic marker. In treatments with zero transmissivity, 44% of sequences with identical players and 48% of sequences with unequal players are best described by the conditional maximizing marker (the rest of the sequences follow the myopic path). The effect of transmissivity on the share of sequences best described by the myopic marker is greater than with unconditional markers.

[73] Thirty-eight or more tables with nine rounds each were calculated for every treatment (since the stock at the first round is fixed at 73 for all treatments there was no difference for this round from the unconditional markers). To do so I took advantage of the fact that, in the case of interior solutions, the solution takes the form: $w_{ir} = \alpha_{1r}(E - x_{ir}) + \alpha_{2r}(E - x_{-ir}) + \alpha_{3ri}$.

Table 9: Partition of Players per Best Conditional Describing Marker & Average SSD

				Partition of players per best describing marker				Average Sum of Square Deviations			
Treatment	Sequence	Type	Obs.	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic
Unequal users & Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	1	*h*	12	25.0%	8.3%	16.7%	50.0%	181.2	171.8	**170.7**	247.7
		l	12	0.0%	0.0%	8.3%	91.7%	136.3	34.5	33.4	**29.1**
	2	*h*	11	27.3%	0.0%	18.2%	54.5%	188.3	175.1	**173.5**	239.0
		l	11	0.0%	9.1%	9.1%	81.8%	107.0	15.6	15.1	**12.4**
	3	*h*	7	28.6%	0.0%	0.0%	71.4%	213.5	202.5	**200.8**	274.9
		l	7	0.0%	14.3%	0.0%	85.7%	93.8	5.1	4.5	**2.1**
	4	*h*	11	27.3%	0.0%	0.0%	72.7%	188.5	177.6	**176.2**	245.1
		l	11	0.0%	9.1%	0.0%	90.9%	103.1	11.5	10.8	**8.2**
	Total per type	*h*	41	26.8%	2.4%	9.8%	61.0%	190.5	179.5	**178.1**	249.3
		l	41	0.0%	7.3%	4.9%	87.8%	112.3	18.3	17.5	**14.4**
	Total		82	13.4%	4.9%	7.3%	74.4%	151.4	98.9	**97.8**	131.8
Identical users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	1		22	18.2%	18.2%	9.1%	54.5%	125.9	66.3	**66.0**	88.8
	2		24	12.5%	12.5%	12.5%	62.5%	157.0	72.3	**70.5**	70.6
	3		12	16.7%	8.3%	8.3%	66.7%	143.8	57.3	55.8	**55.0**
	4		18	0.0%	11.1%	0.0%	88.9%	157.5	51.4	49.1	**31.1**
	Total		76	11.8%	13.2%	7.9%	67.1%	146.0	63.3	**61.8**	64.1
Unequal users & Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	1	*h*	12		41.7%		58.3%		**182.2**		186.2
		l	12		33.3%		66.7%		58.2		**32.4**
	2	*h*	12		33.3%		66.7%		161.7		**152.5**
		l	11		36.4%		63.6%		44.7		**32.9**
	3	*h*	12		50.0%		50.0%		**106.4**		196.4
		l	12		58.3%		41.7%		36.6		**29.5**
	4	*h*	12		75.0%		25.0%		**84.7**		262.2
		l	12		58.3%		41.7%		35.8		**32.0**
	Total per type	*h*	48		50.0%		50.0%		**133.7**		199.3
		l	47		46.8%		53.2%		43.8		**31.7**
	Total		95		48.4%		51.6%		**89.3**		116.4
Identical users & Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	1		19		21.1%		78.9%		114.0		**64.9**
	2		20		50.0%		50.0%		89.5		**86.9**
	3		20		50.0%		50.0%		**100.3**		110.9
	4		20		55.0%		45.0%		**94.8**		114.4
	Total		79		44.3%		55.7%		99.5		**94.6**

NB: With zero transmissivity (*s*=0) the decisions under the Social Optimum are identical to those from SPE and Semi-Myopic.

Similarly to the analysis with conditional markers, when I drop the myopic path and reconstruct Table 9 with only three markers, the semi-myopic behavior comes out as the new most common Best Describing Conditional Marker.[74]

Table 9 also shows that, the lowest SSD is always related to the conditional myopic for less efficient players, with only 14.4 in average with an infinite transmissivity and 31.7 with zero transmissivity. On the other hand, for the efficient users, the lowest SSD is related to the conditional Semi-Myopic (178.1 in average) with an infinite transmissivity, and to the Maximizing path with zero transmissivity. For identical users, the lowest SSD switches from that with conditional Semi-Myopic (61.8 in average) with an infinite transmissivity, to that with the Myopic path with zero transmissivity (94.6 in average).

Table 10: Partition of players' per Best Conditional Describing Marker & SSD

				Partition of players per best describing marker				Average Sum of Square Deviations			
Treatment	Type	Sequence	Obs.	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic
Unequal users & Infinite Transmissivity	*h*	1+2	23	26.2%	4.2%	17.5%	52.3%	184.8	173.5	**172.1**	243.4
		3+4	18	28.0%	0.0%	0.0%	72.1%	201.0	190.1	**188.5**	260.0
	l	1+2	23	0.0%	4.6%	8.7%	86.8%	121.7	25.1	24.3	**20.8**
		3+4	18	0.0%	11.7%	0.0%	88.3%	98.5	8.3	7.7	**5.2**
Identical users & Infinite Transmissivity		1+2	46	15.4%	15.4%	10.8%	58.5%	141.5	69.3	**68.3**	79.7
		3+4	30	8.4%	9.7%	4.2%	77.8%	150.7	54.4	52.5	**43.1**
Unequal users & Zero Transmissivity	*h*	1+2	24		37.5%		62.5%		172.0		**169.4**
		3+4	24		62.5%		37.5%		**95.6**		229.3
	l	1+2	23		34.9%		65.2%		51.5		**32.7**
		3+4	24		58.3%		41.7%		36.2		**30.8**
Identical users & Zero Transmissivity		1+2	39		35.6%		64.5%		101.8		**75.9**
		3+4	40		52.5%		47.5%		**97.6**		112.7

[74] Without the Conditional Myopic Marker, the shares of players following the Conditional SPE and Social Optimum paths stay the same as in Table 9, while the share of players following the Conditional Semi-Myopic path is the sum of those following the Conditional Semi-Myopic and Myopic paths in table 9.

Table 10, obtained by combining the results from the two first sequences and the two last sequences, suggests a noticeable learning effect in the case with zero transmissivity; the share of players with the conditional maximum marker as best describing marker increases from 37.5% in the first sequences for efficient users (respectively 34.9% for less efficient users and 35.6% for identical users) to 62.5% (respectively 58.3% for less efficient users and 52.5% for identical users) in the last sequences.

However, in treatments with infinite transmissivity, the players' seem to be more inclined to follow the Conditional Myopic path in the last sequences than in the first two sequences, especially for the efficient and identical users.

4.2.2 A Treatment Effect on the First Round

The difficulty in analyzing the changes in players' decisions from one treatment to another resides in the fact that the stocks at every round depend on the players' decisions in the previous rounds and are different from one treatment to another. Therefore, the decisions cannot be compared. For the first round, however, all stocks in the different treatments are set to the same level ($x_{h0}=x_{l0}=x_0=73$). Figure 12 provides the results from the experiment together with the theoretical predictions.

The results from the first round show that with infinite transmissivity, the average laboratory decision in the treatment with unequal users (8.24 the mean of 4.84 and 11.65, the average laboratory decisions on the 1^{st} round of respectively less efficient and efficient users) is practically the same as in the treatment with identical users (8.37). With zero transmissivity, I witness a substantial and statistically significant decrease in the players' decisions on the first round, the average decision drops by around two units for both unequal players (6.28, the mean of 3.98 and 8.59) and identical players (6.30).

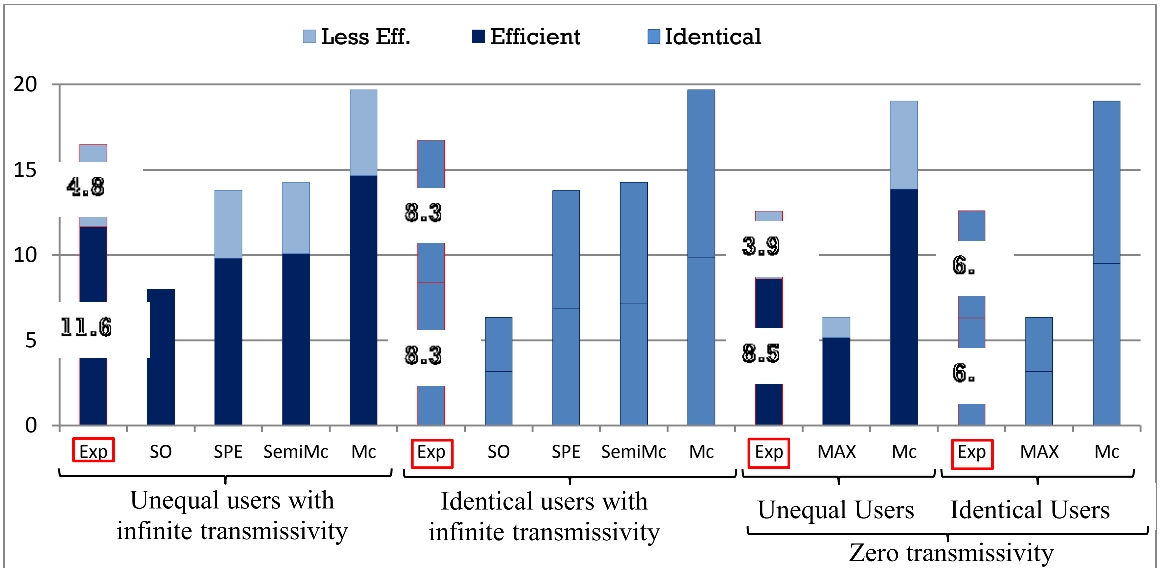

Figure 12: Theoretical and Average Laboratory Decisions on the First Round

A t-test shows that –infinite– transmissivity is associated with a 2.02 increase in the players' average decision during the first round, with a 95 percent confidence interval [1.33-2.71]; a one sided t-test shows that the probability of the increasing effect of transmissivity on the average decision on the first round ($\beta_{transmissivity} \geq 0$) is p-value = 0.99999967.

For the treatment with unequal users and zero transmissivity, there is a substantial gap in the players' average decision (4.61) between high-efficiency players (8.59 in average) and low-efficiency players (3.98 in average); but the difference is even greater (6.81) with infinite transmissivity, since the increase in the level of use by the high-efficiency player (the average decision increases by 3.06, from 8.59 to 11.65) is more than three times that by the low-efficiency player (an average increase of 0.86, from 3.98 to 4.84).

The aforementioned remarks, based on experimental data, replicate qualitatively the predictions following the SPE path, even though the actual laboratory extraction decisions are -on average- higher than the decisions following the SPE path (see Figure

12). Indeed, an important percentage of players opted, on the first round, for extraction levels that are closer to the myopic path.

Table 11: First round Partition per best describing marker & Mean squared deviations

			Partition of players per best describing marker				Mean Square Deviations			
Treatment	Type	Obs.	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic	Social Optimum	Subgame Perfect Eq.	Semi-Myopic	Myopic
Unequal users & Infinite Transmissivity	h	47	19.1%	0.0%	21.3%	**59.6%**	32.5	22.1	**21.1**	27.2
	l	47	2.1%	12.8%	0.0%	**85.1%**	26.6	2.6	2.3	**1.7**
	$h+l$	94	10.6%	6.4%	10.6%	**72.3%**	29.6	12.4	**11.7**	14.5
Identical users & Infinite Transmissivity		94	20.2%	6.4%	12.8%	**60.6%**	36.1	10.4	**9.8**	10.2
Unequal users & Zero Transmissivity	h	48		**62.5%**		37.5%		**30.2**		46.3
	l	47		44.7%		**55.3%**		12.0		**6.1**
	$h+l$	95		**53.7%**		46.3%		**21.2**		26.4
Identical users & Zero Transmissivity		79		**57.0%**		43.0%		20.0		**20.0**

Table 11 provides the mean squared deviation of experimental first round data from the extraction decisions (for the first round) following the different theoretical paths, for the same settings (in transmissivity and inequality), and the same type of player. The table also provides the partition of players by best describing marker for the different treatments, based, in this instance, solely on the path corresponding to the lowest squared deviation on the first round. The table shows that a majority of players follow the myopic path in treatments with infinite transmissivity, especially for low-efficiency players (even with zero transmissivity).

Regarding profits, Figure 13 shows that the individual and aggregate profits on the first round are higher in treatments with infinite transmissivity -for all players, in comparison to the profits with zero transmissivity. For example, with an infinite transmissivity, the efficient user earns 101 computer dollars in average, on the first round, versus 84 computer dollars in the treatment with zero transmissivity; this effect is consistent with the higher extraction levels with infinite transmissivity. The figure also reveals that treatments with identical players generate lower profits even though the extracted quantities are the same as in treatments with unequal users: this result is valid with and without transmissivity. These findings follow the predictions from the SPE path solution.

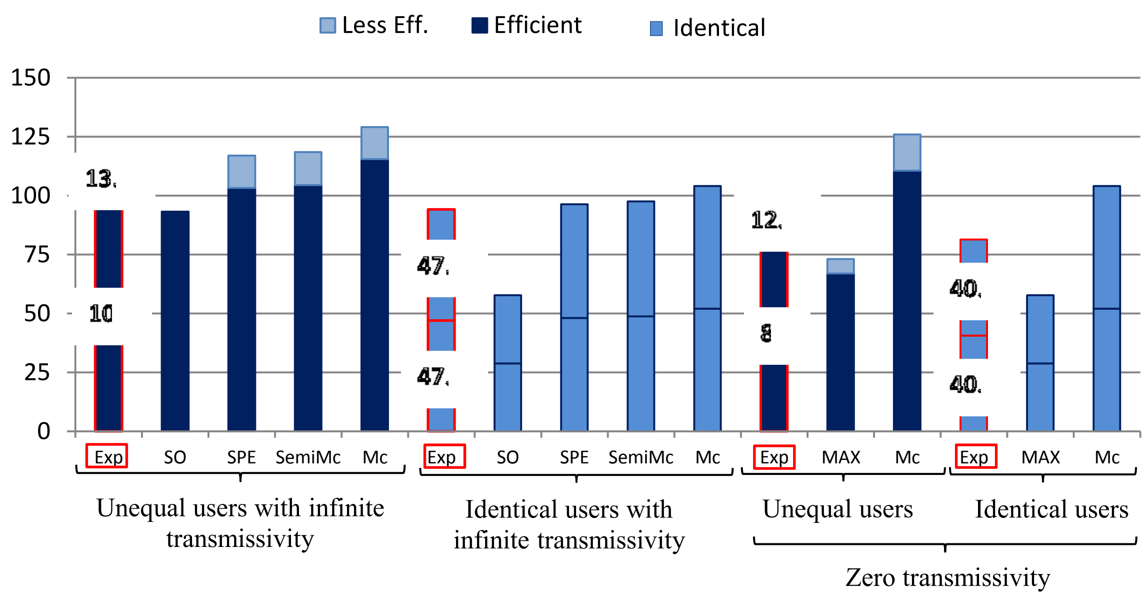

Figure 13: Theoretical and Average Laboratory Profits on the First Round

For treatments with unequal users, high-efficiency players capture most of the increase in aggregate profits (in value and in percentage) from transmissivity. This last outcome is mainly due to the fact that the less efficient players' decisions are closer to those of the myopic path.

4.2.3 Testing the Hypotheses

Contrary to the previous test where I considered solely the first round, in this section I will consider the aggregate cumulative (from all ten rounds) earnings of the two players within each pair, as this represents the appropriate statistic to use to test the theoretical Results SPE1-3 presented in Chapter 3.3 (and validated with a numerical example in 0).

Each group provided one data point per every 10-round sequence – a total of 38 or more statistics per treatment. For the distributional effects in Result SPE4, I used the cumulative earnings over the 10-round sequence for player h and l separately.

Table 12: Experimental average cumulative profits per 10-round game

	Cumulative Profits for player h	Cumulative Profits for player l	Aggregate Cumulative Profits (h+l)	Observations
Unequal users & Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	690.2	49.2	739.4	41/41
	(104.0)	(38.1)	(78.0)	
Identical users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	266.9	266.9	533.9	38/38
	(27.0)	(27.0)	(54.0)	
Unequal users & Zero Transmissivity (a_h = 20, a_l = 10, s = 0)	581.6	111.9	693.6	47/47
	(52.3)	(18.1)	(64.3)	
Identical users & Zero Transmissivity (a_h = 15, a_l = 15, s = 0)	291.4	291.4	582.8	39/39
	(32.2)	(32.2)	(64.4)	

Table 12 provides the average cumulative earnings per sequence per treatment and per type of players and, between brackets, the corresponding standard deviations.

The experimental results confirm the theoretical conclusions as illustrated in Figure 14, where I depict the changes in profits from one treatment to another:

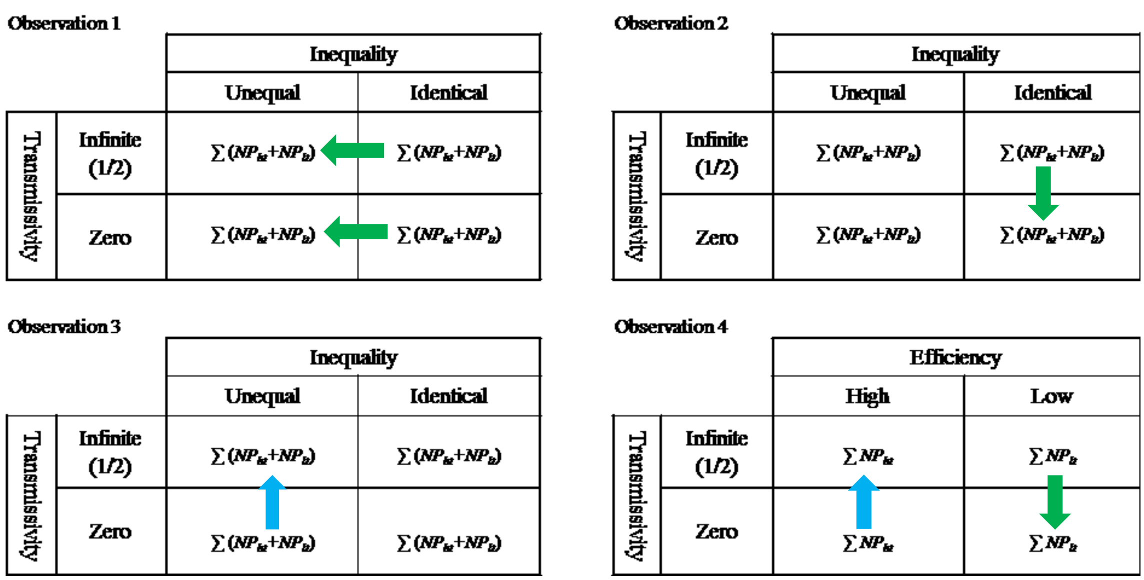

Figure 14: Experimental observations on aggregate and individual cumulative profits

The green arrows indicate an increase in profits while the blue arrows denote an increase under the condition of high inequality between players and a decrease otherwise.

Observation 1*: Inequality increases the aggregate cumulative profits from the common-pool resource.*

For treatments with zero transmissivity, Table 12 shows that the aggregate cumulative profits (from a 10-round game) increase from 582.8 computer dollars with identical players to 693.6 with unequal players, an increasing effect of inequality of around 19%. With infinite transmissivity, the effect of inequality is even greater (in absolute and relative terms) as the average aggregate cumulative profits expands from 533.9 with identical users to 739.4 with unequal users, a net growth of 38.5%.

Combining all experiments (zero/infinite transmissivity), the average aggregate cumulative profits increases from 558.6 for identical players to 714.9 for unequal users (mean difference = 156.2, n = 165, two sample Wilcoxon rank sum test, P = 0.0000).

Observation 2*: Transmissivity decreases the aggregate cumulative profits from the common-pool resource for identical or slightly unequal players.*

The average aggregate cumulative profits for unequal users falls from 582.8 with zero transmissivity to 533.9 with infinite transmissivity, an 8.4% decrease that is in line with the predicted 7.7% following the SPE solution.[75] The drop in profits is substantial and statistically significant (mean difference = 49.0, n = 77, two sample Wilcoxon rank sum test, P = 0.0000), even though the actual profits are noticeably lower than with the SPE.

[75] As presented in Table 1, the aggregate cumulative profits of identical users following the SPE fall from 690.1 with zero transmissivity to 637.1 with infinite transmissivity.

The higher share of players following the –conditional– myopic path with infinite transmissivity (67.1% in Table 9), than with zero transmissivity (55.7%), as evidenced in the first part of this analysis, is also an element that contributed to this result.

Observation 3*: In the case of highly unequal players, higher transmissivity tends to increase the aggregate cumulative profits.*

As indicated in Table 12, for treatments with unequal players, the average aggregate cumulative profits witnesses a moderate, but statistically significant, increase following a move from a perfectly impermeable resource with zero transmissivity to a perfectly transmissive resource. The average aggregate profit is 693.6 with s=0 and 739.4 when s=1/2 (mean difference = 45.84, n = 88, two-sample Wilcoxon rank sum test, P = 0.0002), an increase in the range of 6.6%.

Observation 4*: Transmissivity always decreases the cumulative profits for less efficient player and, in the case of a high inequality between players, increases the cumulative profits for the more efficient ones.*

For treatments with unequal players, a move from the no-transmissivity regime to an infinite transmissivity resource engenders an 18.7% increase in the average cumulative profits for the high-efficiency users, from 581.6 to 690.2, at the same time, the less efficient users see their average cumulative profits drop by 56% (from 111.9 to 49.2).

In treatments with identical users, transmissivity has the same decreasing effect on the individual cumulative profits, as discussed in Observation 2, with aggregate profits.

4.2.4 Efficiency Across Treatments

Efficiency of a given path, expressed as a percentage, is obtained by dividing the profits achieved following that path by the profits following the social optimum path. The efficiency results from the laboratory experiment are presented in the table 13, along with the efficiency results from the strategic, semi-myopic, and myopic paths.

The experimental data show that there is a significant increase in efficiency, with regard to aggregate cumulative profits, as I move from a resource with infinite transmissivity to a resource with zero transmissivity. The gain in efficiency amounts to 7% for treatments with identical players (from 77.4% to 84.4%), and approaches 9% in the case of unequal players (from 79.0% to 87.8%).

This last result with unequal users is in fact the accumulation of two opposite effects. For treatments with infinite transmissivity, the efficiency in individual profits is particularly high for low efficiency players, 346.2% in average, a reminder that the social optimum entails for the less efficient user to reduce his/her extraction decisions in order to increase the aggregate –cumulative– profits, and consistent with the high theoretical efficiencies following the strategic, semi-myopic, and myopic paths. In a setting with zero transmissivity, the social optimum (maximum path) is directly linked to the users' own profits and the efficiency levels, from the experiment and from the other theoretical markers, are back to normal levels below 100%,

For the high efficiency user (in treatments with unequal users) the gain in efficiency in individual cumulative profits is even higher and reaches 14.1% (from 74.8% with infinite transmissivity to 88.9% with zero transmissivity).

The table also shows that in both settings (zero/infinite transmissivity) the efficiency is higher with unequal players; the P value from the Mann-Whitney test is 0.02 with infinite transmissivity and 0.00 with zero transmissivity.

Table 13: Average Experimental & Theoretical efficiencies

		Experiment		Theoretical efficiency		
Treatment	Type	Efficiency	Standard Deviation	Subgame Perfect Eq.	Semi-Myopic	Myopic
Unequal users & Infinite Transmissivity	*h*	**74.8%**	11.3%	87.9%	87.6%	81.3%
	l	**346.2%**	268.4%	396.9%	390.1%	256.3%
	h+l	**79.0%**	8.3%	92.6%	92.2%	83.9%
Identical users & Infinite Transmissivity		**77.4%**	7.8%	92.3%	91.7%	77.7%
Unequal users & Zero Transmissivity	*h*	**88.9%**	7.9%	100.0%	100.0%	80.9%
	l	**82.6%**	13.3%	100.0%	100.0%	77.8%
	h+l	**87.8%**	8.1%	100.0%	100.0%	80.4%
Identical users & Zero Transmissivity		**84.4%**	9.3%	100.0%	100.0%	79.9%

For the treatments with infinite transmissivity, the efficiency results from the laboratory experiment are found to be always lower than the efficiency results from the SPE path (for identical, efficient and less efficient players), and even lower than the results from the Myopic path with unequal players (79.0% efficiency in aggregate profits from the experiment versus 83.9% following the myopic), while treatments with a zero transmissivity perform better than the Myopic path.

4.3 Conclusion

The experimental part was successful in providing the needed empirical substantiation for all theoretical results; indeed, all four results based on the non-cooperative management were corroborated with strong statistical significance.

The analysis of best conditional and unconditional indicators shows that a great proportion of subjects follow the myopic path, particularly with an infinite-transmissivity resource; when there is no transmissivity, the maximizing behavior stands as the best describing marker.

The analysis with the sum of square deviations favors the semi-myopic behavior in the case of an infinite-transmissivity resource, both with conditional and unconditional markers; with no transmissivity, the lowest sum of square deviations relate to the maximizing behavior.[76]

In concordance with the two previous results, transmissivity is shown to have a significant effect on efficiency; the data shows that treatments with an an infinite-transmissivity resource where the players' behavior is best described by myopic and semi myopic strategies, are less efficient than treatments with a zero-transmissivity resource, where the maximizing behavior is more common.

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[76] All three paths, Social Optimum, SPE and semi-myopic converge to the maximizing behavior as *s* tends to zero.

Chapter 5 The Policy Implications

5.1 Introduction

In this section I use the same discrete model introduced in Chapter 3.1 to extend the analysis on the combined (and individual) effects of transmissivity and inequality on the aquifer use to the case when the possibility of communication between users, or the existence of a central agency, allows the emergence of alternative resource management modes.

The first mode corresponds to the case of social optimum resource management; as described before, under this mode of management the resource users coordinate their actions to maximize the benefits to the community from the aquifer.

In the model, the only benefits considered are the players' profits from water user, and the social optimum management would translate into maximizing the aquifer users' aggregate profits, over the entire duration of the game.

Another mode of management corresponds to the case where, from a certain round, only one user, a priori the most effective, is allowed to use the resource, while the other user abandons extraction activities for the remaining duration of the game.

Comparing the profits with a single user to the aggregate profits under a non-cooperative use, helps to reveal the conditions, in transmissivity and inequality, under which a player is better off buying out the other player, these conditions are the requirements for water markets to emerge. Indeed, the exclusive use of the aquifer is only desirable for a –rational– player if her/his profits under this mode of CPR management match or exceed her/his own profits from the collective use, after fully compensating the partner for her/his profits from the collective use.

For example, in the case of a resource with zero transmissivity, the decisions and profits of any –rational– player are the same under single user management as under non-cooperative mode, and therefore, he/she has no incentive to seek the exclusive use of the aquifer, when there are no additional earnings to expect and no externalities to prevent.

Our analysis will be conducted in three stages, first I will provide the backward solutions to the multi-period game under Single User management; the solution for the Social Optimum management is already presented in Chapter 3.2.2.1. I will derive some important observations with analytical evidence for the one period game and numerical examples from a multi-period game. Finally, for the case of water markets, an experimental session will be organized to provide empirical validation to the theoretical observations.

5.2 The Single User & Water Markets Emergence

Under Single User management only one player is granted access to both compartments of the aquifer, she manages her extraction decisions in order to maximize her cumulative (over time) profits. I present basic results concerning the net cumulative profits time, for the single user management, and compare them to the aggregate (both users) cumulative profits under non-cooperative management to reveal the conditions, in inequality and transmissivity, under which institutions for water markets can emerge. I also provide the analytical proof for all results in the case of a one-period game; for the ten-round game I present only the numerical evidence for some interesting results, using the same parameterization that was used for the non-cooperative game. Finally I present the results from the experimental sessions carried (using the same parameterization) to validate some of the theoretical results.

5.2.1 The Multi-period Single User Path

The maximization problem for player *i*, the single user, granted exclusive access to the aquifer, is obtained by incorporating the condition on her parner's (player -*i*) level of use, set to zero ($w_{-ir} = 0$ throughout the duration of the game), in the general maximization problem presented in Equation 3.5 in Chapter 3.2.1, to get the new maximization problem:

$$\underset{w_{ir}}{Max} \sum_{r=1}^{r=T} NP_{ir}(w_{ir}, x_{ir}, 0, x_{-ir}), \tag{5.1}$$

subject to: $x_{ir+1} = x_{ir} + (1-s)(x_{ir} - w_{ir}) + s(x_{-ir} - 0) + R$

$w_{ir} \geq 0$ (the extraction decisions of player *i* are non-negative)

$x_{ir} \leq E$, $x_{-ir} \leq E$ (the level of water cannot rise above the ground)

$x_{i0} = x_{-i0} = x_0$, the stocks at $t = 0$, are equal and given.

where $NP_{ir}(w_{ir}, x_{ir}, 0, x_{-ir})$, the net payoff to agent *i* on round *r*, is given by:

$$NP_{ir}(w_{ir}, x_{ir}, 0, x_{-ir}) = a_i w_{ir} - \frac{w_{ir}^2}{2} - w_{ir}\, c\left(E - \frac{x_{ir} + (1-s)(x_{ir} - w_{ir}) + s(x_{-ir} - 0)}{2}\right) \tag{5.2}$$

The solution to the single user maximization problem is obtained using the backward dynamic programming approach; in the last round (noted T) and assuming that the aquifer has no future residual value the maximization problem becomes:

$$\underset{w_{iT}}{Max}\, NP_i(w_{iT}, x_{iT}, 0, x_{-iT}) \tag{5.3}$$

In the case of an interior solution, the optimal extraction decision in the last round must satisfy the first order condition:

$$\frac{\partial NP_i\left(w_{iT}, x_{iT}, 0, x_{-iT}\right)}{\partial w_{iT}} = a_i - c\left(E - x_{iT}\right) - w_{iT}\left(1 + c - c\,s\right) - \frac{c\,s\left(x_{iT} - x_{-iT}\right)}{2} = 0 \qquad (5.4)$$

The extraction decision satisfying the last equation is linear in the stocks:

$$w_{iT}^{SU} = \alpha_{1T}^{SU}(E - x_{iT}) + \alpha_{2T}^{SU}(E - x_{-iT}) + \alpha_{3iT}^{SU}, \qquad (5.5)$$

where α_{1T}^{SU}, α_{2T}^{SU} and α_{3Ti}^{SU} given by:

$$\alpha_{1T}^{SU} = -\frac{c(2-s)}{2(1+c-c\,s)}; \quad \alpha_{2T}^{SU} = -\frac{c\,s}{2(1+c-c\,s)} \text{ and}$$

$$\alpha_{3iT}^{SU} = \frac{a_i}{(1+c-c\,s)}.$$

Replacing for the value of w_{iT}^{SU} in equation (5.3) I get that the stock value at the last round (equal to the net profit for the last round) is quadratic in the stock levels:

$$V_T^{SU}(x_{iT}, x_{-iT}) = D_{1T}^{SU}(E - x_{iT})^2 + D_{2T}^{SU}(E - x_{jT})^2 + D_{3T}^{SU}(E - x_{iT})(E - x_{-iT})$$

$$+ D_{4iT}^{SU}(E - x_{iT}) + D_{5iT}^{SU}(E - x_{-iT}) + D_{6iT}^{SU} \qquad (5.6)$$

where D_{1T}^{SU}, D_{2T}^{SU} and D_{3T}^{SU} depend on c and s, while D_{4iT}^{SU}, D_{5iT}^{SU} and D_{6iT}^{SU} depend on a_i as well:

$$D_{1T}^{SU} = \frac{c^2\,(2-s)^2}{8\,(1+c-c\,s)};$$

$$D_{2T}^{SU} = \frac{c^2\,s^2}{8\,(1+c-c\,s)};$$

$$D_{3T}^{SU} = \frac{c^2\,s(2-s)}{4\,(1+c-c\,s)};$$

$$D_{4iT}^{SU} = -\frac{a_i\, c\,(2-s)}{2\,(1+c-c\,s)};$$

$$D_{5iT}^{SU} = -\frac{a_i\, c\, s}{2\,(1+c-c\,s)} \text{ and}$$

$$D_{6iT}^{SU} = \frac{a_i^{\,2}}{2\,(1+c-c\,s)}.$$

At any given round *r*, the player's decision need to take into consideration the immediate profits from the extracted water and the benefits from the stocks in the subsequent rounds (zero in the last round), she solves the following problem:

$$\underset{w_{ir}}{Max}\left\{\left(NP_i(w_{ir}, x_{ir}, 0, x_{-ir})\right) + V_{r+1}^{SU}(x_{ir+1}(w_{ir}, x_{ir}, 0, x_{-ir}), x_{-ir+1}(0, x_{-ir}, w_{ir}, x_{ir}))\right\} \tag{5.7}$$

$V_{r+1}^{SU}(x_{ir+1}, x_{-ir+1})$ denotes the stock value in the next round (*r+1*), i.e. the profits from the stock (x_{ir+1}, x_{-ir+1}) when there are *T-r+1* rounds left in the game.

The decision by the single user needs to satisfy the optimality condition:

$$\frac{\partial NP_i(w_{ir}, x_{ir}, 0, x_{-ir})}{\partial w_{ir}} + \frac{\partial V_{r+1}^{SU}(x_{ir+1}, x_{-ir+1})}{\partial x_{ir+1}}\frac{\partial x_{ir+1}}{\partial w_{ir}} + \frac{\partial V_{r+1}^{SU}(x_{ir+1}, x_{-ir+1})}{\partial x_{-ir+1}}\frac{\partial x_{-ir+1}}{\partial w_{ir}} = 0 \tag{5.8}$$

The player extracts to the point where her extra profits, from her current use of one more unit of water, equals the loss from the subsequent decrease in stocks in the following round.

If I assume that the value of groundwater in the next round *n=r+1* is quadratic in the level of stocks during that round and can be written as:

$$V_n^{SU}(x_{in}, x_{-in}) = D_{1n}^{SU}(E - x_{in})^2 + D_{2n}^{SU}(E - x_{-in})^2 + D_{3n}^{SU}(E - x_{in})(E - x_{-in})$$
$$+ D_{4in}^{SU}(E - x_{in}) + D_{5in}^{SU}(E - x_{-in}) + D_{6in}^{SU} \tag{5.9}$$

Solving for the optimality condition expressed in (5.6) shows that the decision during round r is, as established for the last round, linear in the stocks:

$$w_{ir}^{SU} = \alpha_{1r}^{SU}(E - x_{ir}) + \alpha_{2r}^{SU}(E - x_{-ir}) + \alpha_{3ir}^{SU} \tag{5.10}$$

where α_{1r}^{SU}, α_{2r}^{SU} and α_{3ir}^{SU}, provided in Appendix E.1, are functions of D_{1n}^{SU}, D_{2n}^{SU}, D_{3n}^{SU}, D_{4ni}^{SU}, D_{5ni}^{SU}, and D_{6ni}^{SU}.

Replacing for the values of optimal extraction decisions in (5.7) I conclude that the single user stock value function at round r for player i is equally quadratic in both stocks:

$$\begin{aligned} V_r^{SU}(x_{ir}, x_{jr}) = {} & D_{1r}^{SU}(E - x_{ir})^2 + D_{2r}^{SU}(E - x_{-ir})^2 + D_{3r}^{SU}(E - x_{ir})(E - x_{-ir}) \\ & + D_{4ir}^{SU}(E - x_{ir}) + D_{5ir}^{SU}(E - x_{-ir}) + D_{6ir}^{SU}, \end{aligned} \tag{5.11}$$

where D_{1r}^{SU}, D_{2r}^{SU}, D_{3r}^{SU}, D_{4ri}^{SU}, D_{5ri}^{SU}, and D_{6ri}^{SU} (See Appendix E.1) are function of the decision coefficients from the current round, α_{1r}^{SU}, α_{2r}^{SU} and α_{3ir}^{SU}, and the coefficients of the value function from the next round, D_{1n}^{SU}, D_{2n}^{SU}, D_{3n}^{SU}, D_{4ni}^{SU}, D_{5ni}^{SU}, and D_{6ni}^{SU}.

Given the value function at the last round (T or 10) and the relationships established above it is possible to derive the extraction decisions, the stock levels, and the social optimum profits at every round.

5.2.2 Analytical Evidence for the One Round Single User Game

In the one round game, player i, the single user, is faced with the same problem as at the last period of a multi-period single-user maximization problem, with an additional condition on the levels of stock at the start of the round, that need to be the same for both compartments of the aquifer. Denoting x the level of stock at the start of the one round game, and replacing ($x_{iT} = x_{-iT} = x$) in Equations (5.5) and (5.2), provides the extraction decision and net profit, for an interior solution, for the one round game single user game:

$$w_i^{SU} = \frac{a_i - c\,(E - x)}{1 + c - c\,s} \text{ and} \tag{5.12}$$

$$NP_i(w_i^{SU}, x, 0, x) = \frac{(a_i - c\,(E - x))^2}{2(1 + c - c\,s)}. \tag{5.13}$$

For convenience, I will note $NP_{a_i, a_{-i}}^{s,\,SU}(x, x)$ the net profit of player i under single user management when transmissivity equals s, which in this case (single user) equates the value of the stock (noted $V_{a_i, a_{-i}}^{s,\,SU}(x, x)$).

5.2.2.1 Welfare Effects of Inequality and Transmissivity on the Single User

From equations (5.12) and (5.13) it can be easily proven that when the single user is the more efficient player, she drafts more water from the aquifer and generates greater profits, than in the case with the less efficient user as sole user of the CPR.[77] In most of the current analysis, I will focus on the case where the single user is the more efficient, but for the sake of completeness, I will treat the general case, where the single user can be either one of the two players.

Result SU1*: Inequality increases the -single user's- profits from the Common Pool Resource with an efficient single user.*

The increasing effect of inequality can be easily validated by comparing the benefits in the case with unequal players, where the efficient player is the single user of the resource (a scenario that will be referred to as efficient single user hereafter), to the benefits in the case with identical players under single user management, where both unequal and identical players have the same average efficiency:

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[77] The result regarding profits can be easily extended for the multi-period case, keeping in mind that for any level of use of water the more efficient player makes more profits.

$$NP^{s,SU}_{a_h,a_l}(x,x) - NP^{s,SU}_{\frac{a_h+a_l}{2},\frac{a_h+a_l}{2}}(x,x) = \frac{(a_h - a_l)}{2(1+c-cs)}\left(\frac{3a_h + a_l}{4} - c(h-x)\right). \tag{5.14}$$

The quantity on the right side is always positive, which shows that inequality has a positive effect on the profits from single user management, as long as the efficient player is the single user.

For more clarity on the role of inequality, I will introduce two parameters, $\bar{a} = \frac{a_h+a_l}{2}$, the average efficiency of players *h* and *l*, and $\varepsilon_i = a_i - \bar{a}$, the efficiency deviation of player *i*, i.e. the difference between her efficiency and the average.

The efficiency deviation can be rewritten as $\varepsilon_i = \frac{a_i - a_{-i}}{2}$ to show that $\varepsilon_{-i} = -\varepsilon_i$; The absolute value of ε_i, that will be denoted ε hereafter, equals the efficiency average deviation $(a_h - a_l)/2$, and will be used as a proxy for the level of inequality.

When only the efficient user is allowed to extract from the aquifer, the marginal change in her profits following a change in the level of inequality is always positive:

$$\frac{\partial NP_h(w_h^{SU}, x, 0, x)}{\partial \varepsilon} = \frac{(\bar{a} + \varepsilon - c\ (E-x))}{(1+c-cs)}$$

The profit from single user management with the (respectively the less) efficient single user, as shown in Equation 5.13, is increasing in her own efficiency, which is increasing (respectively decreasing) in inequality for any mean preserving change in efficiencies.

Result SU2: *Transmissivity increases the profits from the Common Pool Resource under single user management.*

Higher transmissivity translates into more availability of water to every user from the compartment beneath the other user, which in the case of exclusive access to a single user leads to higher extractions from the aquifer and higher profits as shown by the positive sign of both derivatives:

$$\frac{\partial w_i^{SU}}{\partial s} = c\,\frac{\left(a_i - c\,(E-x)\right)}{\left(1+c-c\,s\right)^2}, \text{ and}$$

$$\frac{\partial V_{a_i,a_{-i}}^{s,SU}(x,x)}{\partial s} = \frac{c}{2}\left(\frac{a_i - c(E-x)}{1+c-c\,s}\right)^2$$

The positive sign of the derivative of profits w.r.t. transmissivity is not contingent on the efficiency of the single user, but it is worth noticing that the effect is stronger (in magnitude) when the player with exclusive access is the efficient one.

Evidently, the same increasing effect on the profits of single user is observable with a non-incremental change in transmissivity, from zero to infinity (s = ½), as evidenced by the always-positive sign of the change in profits:

$$NP_i(w_i^{SU},x,0,x)\Big|_{s=1/2} - NP_i(w_i^{SU},x,0,x)\Big|_{s=0} = \frac{\left(a_i - c\,(E-x)\right)^2}{(2+c)} - \frac{\left(a_i - c\,(E-x)\right)^2}{2(1+c)}$$
$$= \frac{c\left(a_i - c\,(E-x)\right)^2}{2(1+c)(2+c)}$$

5.2.2.2 Effects on the difference between Single User and Non-cooperative management

In this part I attempt to analyze the difference between the profits under single user management and the aggregate profits under non-cooperative extraction game; the goal is to determine the effect of inequality and transmissivity on the difference between the profits under both modes of management. This analysis is essential in defining the conditions under which it is possible for the single user to fully compensate the other user, for leaving the extraction game, and be left with greater profits for herself, than her expected profits under non-cooperative extraction. This is particularly true as the present set-up does not take into consideration the transaction costs and other expenses to enforce the agreements.

Under such conditions on inequality and transmissivity and in the presence of a water market institution, a framework that allows the players to communicate and make binding agreements, it is possible for the two players to reach an understanding that would grant one player the exclusive use of the aquifer, in exchange of an income transfer to the player exiting the resource.[78]

In the case of an interior solution[79] the difference between the single user profits and the aggregate profits under non-cooperative extraction game, that I assume follows the SPE, is given by:

$$\Delta V_{a_i,a_{-i}}^{s,SU-SPE}(x,x) = \frac{2c^2s^2-(2+2c-3cs)^2}{2(2+2c-cs)^2(1+c-cs)}\left(\frac{a_i+a_{-i}-2c(E-x)}{2}\right)^2$$

$$+\frac{(a_i-a_{-i})}{2(1+c-cs)}\left(\frac{a_i+a_{-i}-2c(E-x)}{2}\right)-\frac{(2+2c-2cs)^2+cs(4+4c-5cs)}{8(2+2c-3cs)^2(1+c-cs)}(a_i-a_{-i})^2 \quad (5.15)$$

Result SU3*: In the case with a high transmissivity (s > 0.453) and high costs of extraction* $\left(c > \frac{2}{-2+(3+\sqrt{2})\,s}\right)$*, a single user generates more profits than the aggregate profits of identical users in the non-cooperative extraction game.*

---

[78] In the case of water markets where the contracts involve the amounts to extract rather than the right to the exclusive use of the aquifer, there is always potential for additional gains from coordinating the extraction decisions to maximize the aggregate profits, following the Social Optimum path. In that case, the difference between individual and aggregate profits following the Social Optimum and those following the SPE would be the appropriate measures to assess the gains and individual potential losses from water markets, and the –set of– income transfers for the agreements to be acceptable to both players.

[79] The binding condition for an interior solution is the one related to non-cooperative management, more constraining with regard to the level of inequality.

In the case with zero transmissivity (s = 0), the difference function in Equation 5.15 is strictly negative, regardless of the level of inequality or the value of unitary cost; the loss matches in magnitude the full profits of the player leaving the resource:

$$\Delta V_{a_i,a_{-i}}^{0,SU-SPE}(x,x) = -\frac{\left(a_{-i} - c(E-x)\right)^2}{2+2c}$$

Since there are no externalities involved, the extraction decision under single user management, for the player still using the aquifer, is the same as her decision following the SPE, and she receives the same profit. The only change from the move (from SPE) to Single User management is related to the –other– player leaving the extraction game.

As pointed out in Result SU1, increasing transmissivity leads to an improved access to the aquifer and more availability of water to the single user, the single user adjusts by increasing her extraction decisions from the aquifer, for which she earns greater profits, this is true for unequal and identical users.

At the same time, for identical users following the SPE, Result SPE2 states that the increase in the level of transmissivity engenders more free riding behavior and thereby a drop in aggregate profits.

With identical users, the difference between the profits from Single User management and the aggregate profits following the SPE simplifies as:

$$\Delta V_{a,a}^{s,SU-SPE}(x,x) = \frac{2c^2s^2 - (2+2c-3cs)^2}{2(2+2c-cs)^2(1+c-cs)}\left(a - c(E-x)\right)^2 \tag{5.16}$$

For a transmissivity level higher than s_{hSing}= 0.453, and a unitary cost (c) higher than c_{hSing}= $\frac{2}{-2+(3+\sqrt{2})\,s}$, the numerator in equation (5.16) is positive, which indicates that more profits are generated under single user management.

High levels of transmissivity trigger large physical externalities, combined with high costs of extraction, they engender substantial financial externalities; to the point where it becomes possible for (even) the identical single user to gain more profits than the aggregate profits of the two non-cooperating identical players, following the SPE. The combined effects described in Results SU1 and SPE2, culminate into reversing the observation at s = 0.

Result SU4: *The effect of inequality on the difference between the profits under Single User management and the aggregate profits from the non-cooperative game has an overall inverted U-shaped pattern. The difference between the profits from the two regimes in increasing in inequality at low levels of inequality, it reaches a maximum with a positive sign, then starts to diminish in inequality; the difference becomes zero at levels of inequality higher than* $\varepsilon_{\mathrm{SPE}}^{\mathrm{Max}} = \dfrac{2+c(2-3s)}{2+c(2-s)}\left(\bar{a}-c(E-x)\right)$.

Replacing with the alternative expressions for a_i and a_{-i}, respectively $\bar{a}+\varepsilon_i$ and $\bar{a}-\varepsilon_i$, in Equation 5.15, the difference between the single user profits and the aggregate profits under the non-cooperative extraction game becomes:

$$\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x) = \frac{2c^2s^2-(2+2c-3cs)^2}{2(2+2c-cs)^2(1+c-cs)}\left(\bar{a}-c(E-x)\right)^2$$

$$+\ \varepsilon_i\,\frac{\left(\bar{a}-c(E-x)\right)}{(1+c-cs)}-\frac{(2+2c-2cs)^2+cs(4+4c-5cs)}{2(2+2c-3cs)^2(1+c-cs)}\,\varepsilon_i^{\,2} \qquad (5.17)$$

Equation 5.17 shows that the difference is a quadratic function of ε_i, with a negative coefficient; it follows that the difference has a unique maximum (See Figure 15).

The difference function $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ reaches its maximum at $\varepsilon^{\text{Max}}_{\text{SU-SPE}}$, the unique root of equation $\partial\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)/\partial\varepsilon = 0$, where:

$$\varepsilon^{\text{Max}}_{\text{SU-SPE}} = \frac{(2+2c-3cs)^2(\bar{a}-c(E-x))}{(2+2c-2cs)^2+cs(4+4c-5cs)} \quad \text{, and}$$

$$\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon^{\text{Max}}_{\text{SU-SPE}},\bar{a}-\varepsilon^{\text{Max}}_{\text{SU-SPE}}}(x,x) = \frac{8\,c^2(1+c-cs)s^2(\bar{a}-c(E-x))^2}{(2+2c-cs)^2\left((2+2c-2cs)^2+cs(4+4c-5cs)\right)}$$

The positive sign of $\varepsilon^{\text{Max}}_{\text{SU-SPE}}$ (keeping in mind that $0 \le s \le 1/2$) confirms that the maximum is reached with player h –the high efficient user– as the sole user under single user management.

In the following few paragraphs I will limit the analysis to the case where single user management (in the difference function $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$) involves player h having exclusive access and player l exiting the resource.

The maximum (at $\varepsilon^{\text{Max}}_{\text{SU-SPE}}$) is strictly positive, which suggests that equation $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)=0$ has two roots, given the negative coefficient in ε_i^2; $\varepsilon^{\text{Max}}_{\text{SPE}} = \frac{2+c(2-3s)}{2+c(2-s)}(\bar{a}-c(E-x))$ is the larger root, and corresponds to the level of inequality at which the extraction decision of player l, the less efficient user, following the SPE interior solution, is $w_l^{SPE} = 0$.

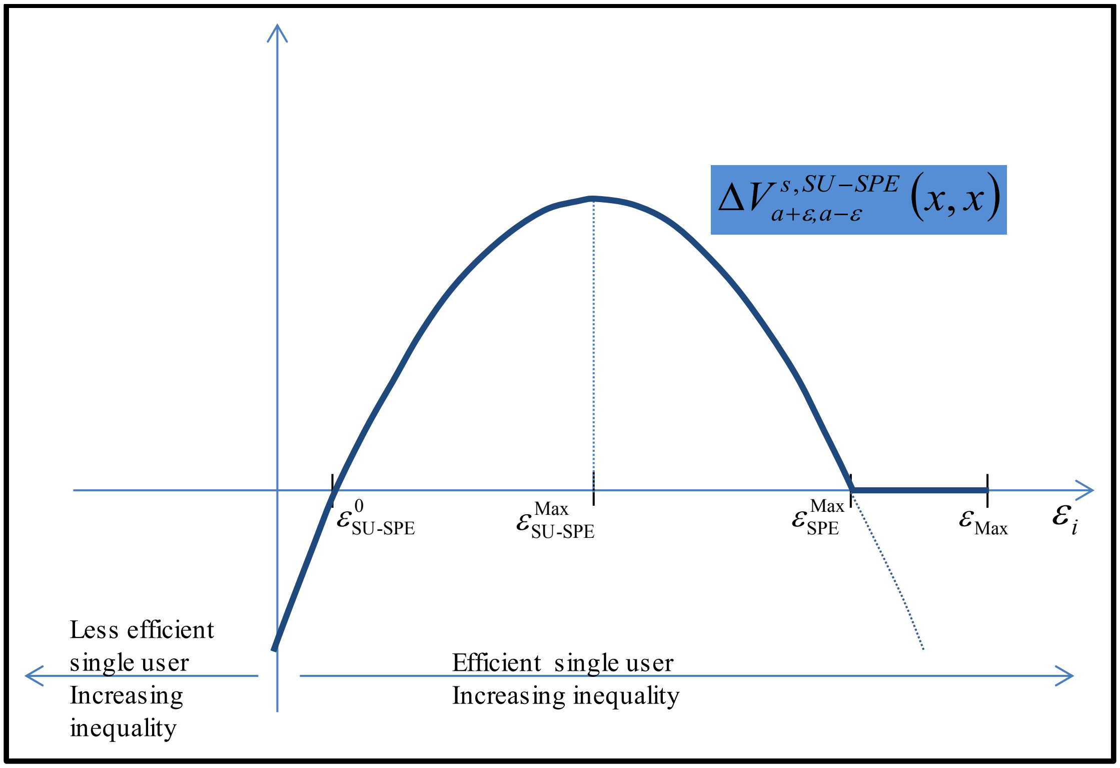

Figure 15: Effect of inequality on the difference in profits between single user and SPE

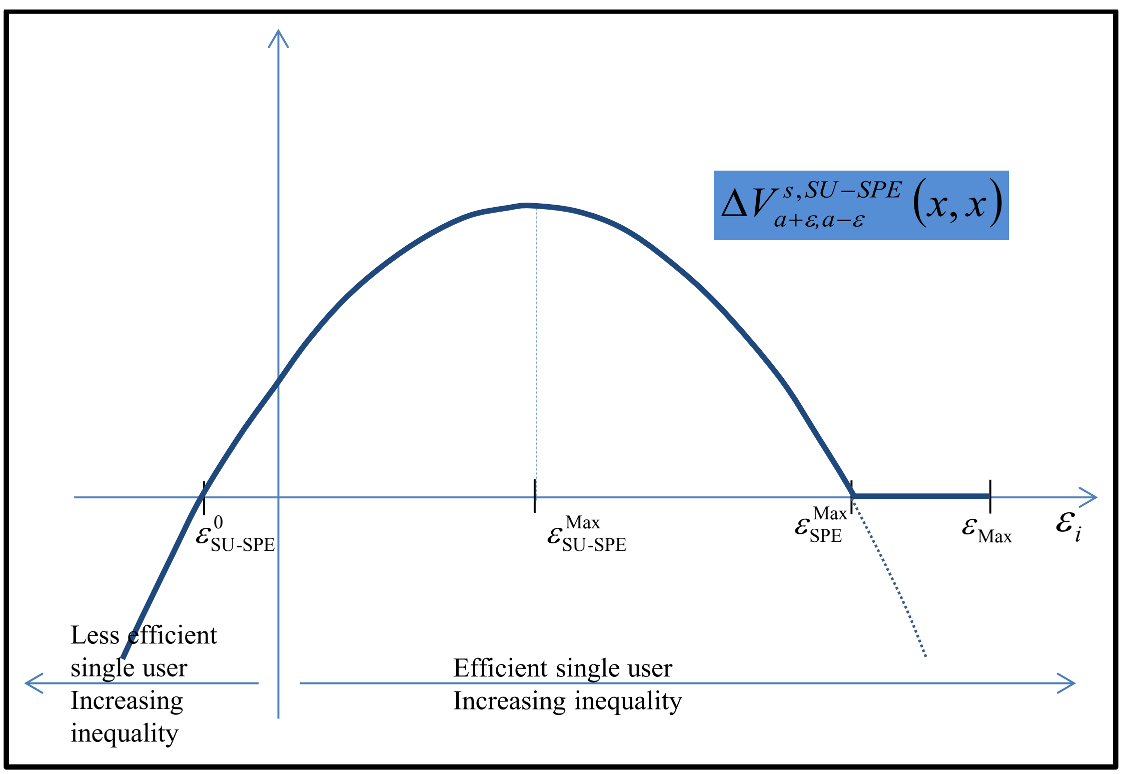

Figure 16: Effect of inequality on the difference in profits between single user and SPE in the case with high transmissivity and high unitary cost

At levels of inequality higher than $\varepsilon_{\text{SPE}}^{\text{Max}}$, the solution under non-cooperative management (following the SPE) is a corner solution that entails player l to not extract from the aquifer, $w_l^{SPE} = 0$, while player h makes her extraction decisions as the "De facto" sole user, with ε > $\varepsilon_{\text{SPE}}^{\text{Max}}$ the difference between profits becomes: $\Delta V_{\bar{a}+\varepsilon,\bar{a}-\varepsilon}^{s,SU-SPE}(x,x) = 0$.

The smaller root, denoted $\varepsilon_{\text{SU-SPE}}^{0}$, is given by:

$$\varepsilon_{\text{SU-SPE}}^{0} = \frac{2+2c-3cs}{2+2c-cs}\left(\frac{(2+2c-3cs)^2-2c^2s^2}{(2+2c-2cs)^2+cs(4+4c-5cs)}\right)(a-c(E-x))$$

The sign of $\varepsilon_{\text{SU-SPE}}^{0}$ is positive when ($s<s_{hSing}$) and/or ($c<c_{hSing}$) as presented in Figure 15.[80] For higher levels on transmissivity ($s>s_{hSing}$) combined with a high unitary cost ($c>c_{hSing}$), the difference between the aggregate profits is positive even with zero inequality, as already discussed in Result SU3 ($\Delta V_{\bar{a},\bar{a}}^{s,SU-SPE}(x,x) > 0$), and the smaller root is negative as presented in Figure 16.

In the case of Single User management, increasing inequality (ε) -with player h as single user- increases the profits from the Common Pool Resource, as discussed in Result SU1:

$$\frac{\partial V_{a+\varepsilon,a-\varepsilon}^{s,SU}(x,x)}{\partial \varepsilon} = \frac{a+\varepsilon-c(E-x)}{1+c-\text{cs}}$$

Result SPE2 establishes the same increasing effect of inequality on the aggregate profits under Non-cooperative of management, for players following the SPE path:

$$\frac{\partial V_{a+\varepsilon,a-\varepsilon}^{s,SPE}(x,x)}{\partial \varepsilon} = \frac{8(1+c-cs)\varepsilon}{(2+2c-3\text{cs})^2}$$

[80] s_{hSing}= 0.453, and c_{hSing}= $\frac{2}{-2+(3+\sqrt{2})\,s}$ as introduced in Result SU3.

At low levels of inequality ($\varepsilon < \varepsilon_{\text{SU-SPE}}^{\text{Max}}$), the derivative of $\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ w.r.t inequality, the effect of an incremental change in inequality on the profits of the Single User are higher than its effect on the aggregate profits following the SPE; for instance, at the origin, the effects are as follows:

$$\left.\frac{\partial V_{a+\varepsilon,a-\varepsilon}^{s,SU}(x,x)}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{a-c(E-x)}{1+c-\text{c}\,\text{s}} \text{ , and } \left.\frac{\partial V_{a+\varepsilon,a-\varepsilon}^{s,SPE}(x,x)}{\partial \varepsilon}\right|_{\varepsilon=0} = 0 \cdot$$

At the maximum ($\varepsilon = \varepsilon_{\text{SU-SPE}}^{\text{Max}}$) the two effects are equal and cancel one another out and $\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x) = 0$; for higher levels of inequality, the effect of an incremental change in inequality on the profits of the Single User drops below the effect on the aggregate profits following the SPE and the derivative of $\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ w.r.t inequality is negative. At the maximum level of inequality with an interior solution, $\varepsilon_{\text{SU-SPE}}^{\text{Max}}$, the right derivative is zero, while the left derivative is strictly negative:

$$\lim_{\varepsilon_i \to \varepsilon_{\text{SPE}}^{\text{Max}}} \frac{\partial \Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)}{\partial \varepsilon_i} = -\frac{4cs\,(a-c(E-x))}{(2+2c-cs)\,(2+2c-3cs)}$$

A fundamental result from the above discussion is that, for any set of parameters of the game ($\bar{a}$, h, x, c, s) there exists a level of inequality leading to a higher outcome with a single user, and allowing therefore for a water market to emerge. Following on the previous assumption that there are no transaction costs, or only negligible costs, it is conceivable for any two users with an inequality level that falls in the interval [$\varepsilon_{\text{SU-SPE}}^{0}$, $\varepsilon_{\text{SU-SPE}}^{\text{Max}}$] to engage in negotiations, and reach an agreement, where one player exits the extraction game -in exchange of an agreed payment, while the other player obtains the exclusive use of the aquifer.

Given the positive sign of difference in gains $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$, it is possible for player *h* to fully compensate the other user for forsaken profits from the non-cooperative game, if the latter were to decide to leave the extraction, while maintaining (after payment of the agreed compensation) a net profit that equals or exceeds his/her own profits following the SPE.

The shape of the difference in gains $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ is very indicative of the "unnecessary nuisance" that player *l* is causing to player *h*, and the value of $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ can be viewed as a window of opportunities for both players to reach an agreement allowing player *h* exclusive use of the aquifer. At low levels of inequality $\left(\varepsilon < \varepsilon^{\text{Max}}_{\text{SU-SPE}}\right)$, the difference is negative showing that there are no prospects for rational users to reach any agreement regarding Single Use management; at higher levels of inequality $\left(\varepsilon < \varepsilon^{\text{Max}}_{\text{SU-SPE}}\right)$, any money transfer –from player *h* to player *l*– in amount (higher than zero and) lower than $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ to leave extraction, presents an opportunity for profits for both users, in that regard, $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ is the extent of the window for trade.

Finally, in the specific case where high transmissivity ($s>s_{hSing}$) is combined with a high unitary cost ($c>c_{hSing}$), the smaller root is negative (Figure 16), and the difference between profits under the two modes of management, $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ is positive at low levels of inequality ($\varepsilon < \left|\varepsilon^{0}_{\text{SU-SPE}}\right|$) for both users and the two players can play either role, seller or buyer. The financial externalities are so high, that even player *l*, is able, if provided exclusive use of the aquifer, to remunerate player *h* for forgone earnings following the SPE, and secure more net profits for herself than following the SPE.

Result SU5*: The derivative with regard to transmissivity of* $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$*, the difference between the profits under Single User management and the aggregate profits from the non-cooperative game, has an overall inverted U-shaped pattern. The derivative is positive and increasing in inequality at low levels of inequality, it reaches a maximum, then starts to decrease and becomes negative at higher levels of inequality.*

In this part I attempt to analyze the effect of transmissivity on $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$, by analyzing its derivative with regard to *s*, that will be denoted $\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$:

$$\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x) = \frac{\partial \Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)}{\partial s}$$

In the case of identical players the derivative of $\Delta V^{s,SU-SPE}_{a,a}(x,x)$ w.r.t *s*, $\Delta_s V^{s,SU-SPE}_{a,a}(x,x)$ becomes:

$$\Delta_s V^{s,SU-SPE}_{a,a}(x,x) = \frac{c(2+2c-cs)^3 + 8c^2 s(1+c-cs)^2}{2(2+2c-cs)^3(1+c-cs)^2}(a-c(E-x))^2 \tag{5.18}$$

The derivative's positive sign (keeping in mind that *s* <1/2) is explained by the benefit to the single user from a marginal increase in the level of transmissivity (result SU2), augmented by the net "savings" in terms of avoided additional externalities from the non-cooperative extraction game with identical users; as established in Chapter 3.3, higher transmissivity has a strictly negative effect on the aggregate profits of non-cooperative identical users.[81]

With unequal players, the derivative is as follows:

[81] Result SPE1: Inequality increases the aggregate cumulative profits from the common-pool resource.

$$\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x) = \frac{c(2+2c-cs)^3+8c^2 s(1+c-cs)^2}{2(2+2c-cs)^3(1+c-cs)^2}(\bar{a}-c(E-x))^2 + c\,\varepsilon_i \frac{\bar{a}-c(E-x)}{(1+c-cs)^2}$$
$$-c\left(\frac{3(2+2c-2cs)^3+5cs(2+2c-3cs)^2+7c^2s^2(2+2c-3cs)+3c^3s^3}{2(1+c-cs)^2\,(2+2c-3cs)^3}\right)\varepsilon_i^{\,2} \quad (5.19)$$

Equation 5.19 shows that the derivative is a quadratic function of ε_i, with a negative coefficient; it follows that the derivative of the difference also has a unique maximum (Green line in Figure 17).

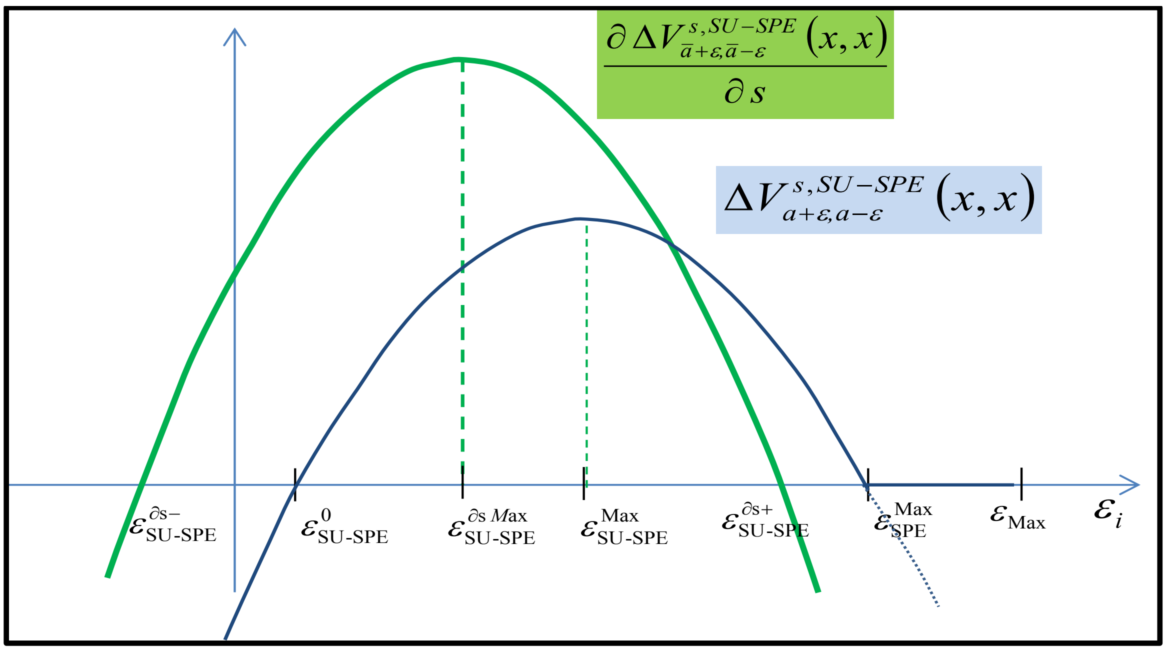

Figure 17: Effect of inequality on the derivative of the difference in profits between single user and aggregate SPE profits

The derivative function $\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$ reaches its maximum at $\varepsilon^{\partial s\,\text{Max}}_{\text{SU-SPE}}$, the unique root of equation $\frac{\partial}{\partial\varepsilon}\frac{\partial\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)}{\partial s}=0$, where:

$$\varepsilon^{\partial s\,\text{Max}}_{\text{SU-SPE}} = \frac{(2+2c-3cs)^3(\bar{a}-c(E-x))}{3(2+2c-2cs)^3+20cs(1+c-2cs)^2+34c^2s^2(1+c-2cs)+15c^3s^3}$$

Given the positive sign of $\Delta_s V_{a,a}^{s,SU-SPE}(x,x)$, it is evident that the maximum of $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ (at $\varepsilon_{\text{SU-SPE}}^{\partial s\ \text{Max}}$) is strictly positive; it is also clear, taking into account the negative coefficient in ε_i^2, that $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ has two roots, of opposite signs, that will be denoted $\varepsilon_{\text{SU-SPE}}^{\partial s-}$ for the smaller root, and $\varepsilon_{\text{SU-SPE}}^{\partial s+}$ for the larger root. Appendix E.2 provides the actual values for the maximum and the two roots of $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$,[82] with of $\varepsilon_{\text{SU-SPE}}^{\partial s+}$, and supporting proof for the aforementioned results; the Appendix also shows that, as suggested in the figure: $\varepsilon_{\text{SU-SPE}}^{\partial s-} < \varepsilon_{\text{SU-SPE}}^{0}$, $\varepsilon_{\text{SU-SPE}}^{\text{Max}} < \varepsilon_{\text{SU-SPE}}^{\partial s+}$, and $\varepsilon_{\text{SU-SPE}}^{\partial s+} < \varepsilon_{\text{SPE}}^{\text{Max}}$.

The key result from this discussion is that for any levels of inequality lower than $\varepsilon_{\text{SU-SPE}}^{\partial s+}$, the derivative $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ is positive and the increasing effect of transmissivity on the difference $\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$, as established at the origin (Equation 5.18), continues to hold. For levels of inequality higher than $\varepsilon_{\text{SU-SPE}}^{\partial s+}$, but lower than $\varepsilon_{\text{SPE}}^{\text{Max}}$, $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ is negative, and an incremental increase transmissivity decreases the actual gains from moving to the Single User management. When inequality is higher than $\varepsilon_{\text{SPE}}^{\text{Max}}$, the difference is zero ($\Delta V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)=0$) and the incremental increase in transmissivity has no effect.

To understand the effect of transmissivity it suffices to rewrite $\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x)$ as:

$$\Delta_s V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU-SPE}(x,x) = \frac{\partial V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU}(x,x)}{\partial s} - \frac{\partial V_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}^{s,SU}(x,x)}{\partial s}$$

[82] $\Delta_s V_{\bar{a}+\varepsilon_{\text{SU-SPE}i}^{\partial s\ \text{Max}},\bar{a}-\varepsilon_{\text{SU-SPE}}^{\partial s\ \text{Max}}}^{s,SU-SPE}(x,x)$, $\varepsilon_{\text{SU-SPE}}^{\partial s-}$, and $\varepsilon_{\text{SU-SPE}}^{\partial s+}$.

The first term on the right hand is always positive (Result SU2) while the sign of the second term is negative for low levels of inequality and positive for higher levels (Result SPE3).

Another aspect of the effect of the transmissivity is its influence on the general shape of the difference function ($\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$), indeed, the corresponding roots ($\varepsilon^{0}_{\text{SU-SPE}}$ and $\varepsilon^{\text{Max}}_{\text{SPE}}$) and maximum ($\varepsilon^{\text{Max}}_{\text{SU-SPE}}$) are all decreasing in the level transmissivity.

In the rest of this discussion the analysis is shifted toward the effect of a non-marginal change in transmissivity, from a zero-transmissivity resource (s=0) to a infinite-transmissivity resource (s=1/2). With a zero-transmissivity, the single user management is not a plausible option since it results in one user giving away her benefits with no gains to the other user. In the analysis of a non-marginal change in transmissivity I will rather compare the profits from an infinite-transmissivity resource used by an efficient player to those from a zero-transmissivity resource used by maximizing players.

The benefits of identical players are always harmed from the considered non-marginal shift in transmissivity; for unequal players however, the single efficient user management generates a higher profit than the aggregate profits for the two users with a zero-transmissivity resource, for any level of inequality higher than $\varepsilon^{NM}_{\text{SU-SPE}}$ given by:

$$\varepsilon^{NM}_{\text{SU-SPE}} = \frac{a-c(E-x)}{1+c+\sqrt{2c+c^2}}$$

With a high unitary cost of extraction (c higher than $4/\sqrt{3}$), $\varepsilon^{NM}_{\text{SU-SPE}}$ falls within the interval where single user management brings higher profits than non-cooperative management (with an infinite-transmissivity resource), and single user management is indeed a better option to generate higher profits.

When c is low, moderate levels of inequality (higher than $\varepsilon^{NM}_{\text{SU-SPE}}$ but lower than $\varepsilon^{0}_{\text{SU-SPE}}$) are still associated with higher profits under single user management for an infinite-transmissivity resource, but the aggregate profits under non-cooperative management (with an infinite-transmissivity resource) are even higher. The low cost of extraction makes it possible for the gains from more availability of water to the efficient user with an infinite-transmissivity resource to outweigh the losses from the non-cooperative use even at moderate levels of inequality, where Single User management generates less profits than the SPE outcome. For high levels of inequality, single user management is reestablished as the more profitable management.

The main result from this discussion is that for highly unequal players, using an infinite-transmissivity resource, single efficient user management carries out more profits than the maximum aggregate (to the two users) profits from a zero-transmissivity resource.

5.2.2.3 Water Conservation Under the Single User

Result SU6*: The aggregate amount of water used by the two users under non-cooperative management is higher than the Single User use, but the savings in water use are decreasing in transmissivity.*

The extraction decision of player *i*, following the SPE, is given by (Equation 3.27):

$$w_i^{SPE}=\frac{a_i+a_{-i}-2c(E-x)}{2+2c-cs}+\frac{a_i-a_{-i}}{2+2c-3cs}, \qquad i=h,l$$

Given the extraction decision following the Single User path (Equation 5.12), the savings in –aggregate– water use resulting from the switch from SPE to Single User management, with player *i* as sole user, are computed as follows:

$$(w_i^{SPE}+w_{-i}^{SPE})-w_i^{SU}=\frac{(2+2c-3cs)\,(a_i+a_{-i}-2c(E-x))}{2(2+2c-cs)\,(1+c-cs)}-\frac{a_i-a_{-i}}{2+2c-2cs} \tag{5.20}$$

Player *i*, with exclusive access to the resource under single user management, extracts more water than she does under non-cooperative management, but there is a net reduction in water use as the amount used by her partner is taken into consideration.[83] Indeed, the savings in Equation 5.20 are decreasing in the level of inequality, and completely vanish as inequality reaches the level ($\varepsilon=\varepsilon_{\mathrm{SPE}}^{\mathrm{Max}}$) at which player *l* is deterred entry to the non-cooperative game.[84]

Increasing the level of transmissivity has an increasing effect on the aggregate extractions by the non-cooperative users; the same increasing effect, and even stronger, is observed in the case of a single user; this leads to a net decrease in water savings from single user management at higher levels of transmissivity, as suggested by the sign of the derivative with regard to *s*:

$$\frac{\partial\left((w_i^{SPE}+w_{-i}^{SPE})-w_i^{SU}\right)}{\partial s}=-c^2\,s\frac{(4+4c-3cs)\,(a_i+a_{-i}-2c(E-x))}{2(2+2c-cs)^2\,(1+c-cs)^2}-\frac{c(a_i-a_{-i})}{2\,(1+c-cs)^2}$$

[83] The difference in Player *i* level of use, $w_i^{SU}-w_i^{SPE}=\frac{cs}{(1+c-cs)}\left(\frac{\varepsilon}{2+2c-3cs}-\frac{a-c(h-x)}{2+2c-cs}\right)$, is strictly increasing in the level of inequality, it is negative at the origin ($\varepsilon=0$), and equals zero when $\varepsilon=\varepsilon_{\mathrm{SPE}}^{\mathrm{Max}}$.

[84] When the solution to the maximization problem is a corner solution.

5.2.3 Numerical Illustration for the Ten-round Single User Game

5.2.3.1 Numerical specifications and solutions

I use the same numerical specifications presented in Chapter 3.4.1, with two levels of inequality: unequal players (a_h = 20, a_l = 10) and identical players (a_h = a_l = 15); and for transmissivity, I consider the two extreme situations: zero transmissivity (s = 0), and infinite transmissivity (s = ½). For completeness, with unequal players, I will consider the two assumptions regarding the Single User, the case with player *h* as Single User and the case with player *l*.

Matching together the two levels of inequality, the two degrees of transmissivity, and the two assumptions regarding the Single User (when relevant i.e. for unequal players), yields six cases to investigate:

- The Player *h* Single User - Unequal Infinite Transmissivity case: $\boldsymbol{a_h}$ **= 20**, a_l = 10 and s = ½, with player *h* as Single User.
- The Player *l* Single User - Unequal Infinite Transmissivity case: a_h = 20, $\boldsymbol{a_l}$ **= 10** and s = ½, with player *l* as Single User.
- The Identical Infinite Transmissivity case: a_h = 15, a_l = 15 and s = ½.
- The Player *h* Single User - Unequal Zero Transmissivity case: $\boldsymbol{a_h}$ **= 20**, a_l = 10 and s = 0, with player *h* as Single User.
- The Player *l* Single User - Unequal Zero Transmissivity case: a_h = 20, $\boldsymbol{a_l}$ **= 10** and s = 0, with player *l* as Single User.
- The Identical Zero Transmissivity case: a_h = 15, a_l = 15 and s = 0.

Table 14: The single user path with infinite transmissivity

Case	Round	Decisions		Stocks		Profits		Stock Values
	t	w_{ht}	w_{lt}	x_{ht}	x_{lt}	NP_{ht}	NP_{lt}	$V_{h+l\ t}$
Player *h* Single User - The Unequal Infinite Transmissivity (**a_h = 20**, a_l = 10, s = ½)	1	8.2		73.0		95.0		**931.5**
	2	8.7		71.9		96.6		836.5
	3	9.1		70.5		97.5		739.9
	4	9.6		69.0		97.7		642.4
	5	10.0		67.2		97.2		544.7
	6	10.5		65.1		95.8		447.5
	7	10.9		62.9		93.6		351.7
	8	11.4		60.4		90.4		258.1
	9	11.8		57.7		86.4		167.6
	10	12.3		54.8		81.3		81.3
Player *l* Single User - The Unequal Infinite Transmissivity (a_h = 20, **a_l = 10**, s = ½)	1		2.5		73.0		11.6	**190.1**
	2		3.0		74.7		13.7	178.4
	3		3.4		76.2		15.8	164.7
	4		3.9		77.5		17.6	149.0
	5		4.3		78.6		19.3	131.3
	6		4.8		79.4		20.8	112.0
	7		5.2		80.0		21.9	91.2
	8		5.7		80.4		22.8	69.3
	9		6.1		80.6		23.2	46.5
	10		6.6		80.5		23.3	23.3
Player *h* or *l* Single User - The Identical Infinite Transmissivity (**a_h = 15**, a_l = 15, s = ½)	1	5.4		73.0		43.4		**489.4**
	2	5.8		73.3		45.9		446.0
	3	6.3		73.4		48.0		400.1
	4	6.7		73.2		49.6		352.1
	5	7.2		72.9		50.8		302.5
	6	7.6		72.3		51.5		251.7
	7	8.1		71.5		51.5		200.2
	8	8.5		70.4		51.0		148.7
	9	9.0		69.1		49.8		97.7
	10	9.4		67.6		47.9		47.9

Table 15: The single user path with zero transmissivity

	Round	Decisions		Stocks		Profits		Stock Values
Case	t	w_{ht}	w_{lt}	x_{ht}	x_{lt}	NP_{ht}	NP_{lt}	$V_{h+l\ t}$
Player h Single User - The Unequal Zero Transmissivity (<u>a_h = 20</u>, a_l = 10, s = 0)	1	5.2		73.0		67.0		**654.6**
	2	5.6		70.8		69.6		587.5
	3	6.1		68.2		71.2		517.9
	4	6.5		65.2		71.8		446.7
	5	7.0		61.6		71.3		374.8
	6	7.4		57.7		69.6		303.5
	7	7.9		53.3		66.6		233.9
	8	8.3		48.4		62.2		167.3
	9	8.8		43.1		56.3		105.1
	10	9.2		37.3		48.8		48.8
Player l Single User - The Unequal Zero Transmissivity (a_h = 20, <u>a_l = 10</u>, s = 0)	1		1.2		73.0		6.2	**135.6**
	2		1.6		74.8		8.6	129.4
	3		2.1		76.2		10.8	120.8
	4		2.5		77.2		12.9	110.0
	5		3.0		77.6		14.7	97.1
	6		3.4		77.7		16.0	82.5
	7		3.9		77.3		16.9	66.5
	8		4.3		76.4		17.2	49.6
	9		4.8		75.1		16.8	32.4
	10		5.2		73.3		15.6	15.6
Player h or l Single User - The Identical Zero Transmissivity (<u>a_h = 15</u>, a_l = 15, s = 0)	1	3.2		73.0		28.9		**345.1**
	2	3.6		72.8		32.0		316.2
	3	4.1		72.2		34.5		284.2
	4	4.5		71.2		36.5		249.6
	5	5.0		69.6		37.7		213.2
	6	5.4		67.7		38.1		175.5
	7	5.9		65.3		37.6		137.4
	8	6.3		62.4		36.2		99.7
	9	6.8		59.1		33.6		63.6
	10	7.2		55.3		29.9		29.9

- The other parameters of the game are kept constant, the stocks on the first round are set at 73 ($x_{h0} = x_{l0} = 73$), while the elevation of the ground is E=100. The unitary cost of extraction is taken $c = 0.15$, and the aquifer recharge is R=3.
- The numerical solutions for the Single User' decisions are displayed in table 14 for the cases with infinite transmissivity and table 15 for the cases with zero transmissivity. With infinite transmissivity, the solution for the Single User decisions are based on the algorithms described earlier; with zero transmissivity, the Single User decisions, earnings and costs are the same as following the maximizing path, even though the partner leaves the extraction game, the decisions of the player with exclusive access are not affected, as there were no externalities involved.

5.2.3.2 Discussion of numerical results for the Ten-round Game

The numerical results presented in Table 14, together with other data from Tables 1-2 and 4, are used to construct figures 18 and 19 portraying respectively the players' decisions and the stock evolution, and figures 20, 21 and 22 representing the players' individual, aggregate and aggregate cumulative profits, following the Single User, the Social Optimum, the SPE and the Myopic paths. The last three paths were already discussed in Chapter 3.4.2 and are included only for comparison, in addition, the analysis is limited to the case with infinite transmissivity; with zero transmissivity the Single User decisions and profits are the same as from the maximizing path, discussed in Chapter 3.4.2.

In the case of unequal users, with player *h* as Single User (red solid line SU-wht in figure 18.a), the decisions are very close to those following the Social Optimum (for player *h*), the extraction decisions are moderate in the early rounds (8.2 on the first round) but increase gradually throughout the game to reach 12.3 on the last round; with player *l* as Single User (red dashed line SU-wht), the decisions are substantially higher than those following the Social Optimum (for player *l*), the extraction starts at lower levels and increases steadily, but the actual values are much lower than those with player *h*, and the difference is maintained constant at 5.7.

In the case with identical players, the Single User decisions (red solid line SU-wit in figure 18.b) reproduce the same increasing evolution as the decisions following the Social Optimum, the moderately low levels of extraction in early rounds (5.4 on the first round) rise gradually to reach higher levels in the last rounds (9.4 on the 10^{th} round), however, the decisions under single user are always 2.2 units higher than those from the Social Optimum.

Regarding the evolution of stocks, with player *h* as single user, figure 19.a shows that the stock levels are practically the same for the Single User and the Social Optimum paths, this is in agreement with the previous observation on the uniformity of decisions of player *h* following the two paths, given the low levels of extraction by player *l* following the Social Optimum (no extraction in the first five rounds). With player *l* as single user, the figure indicates the opposite trend, especially in the early rounds, where the stock levels increase, as of consequence of rates of use that are even lower than the rate of recharge, before stabilizing in the last rounds.

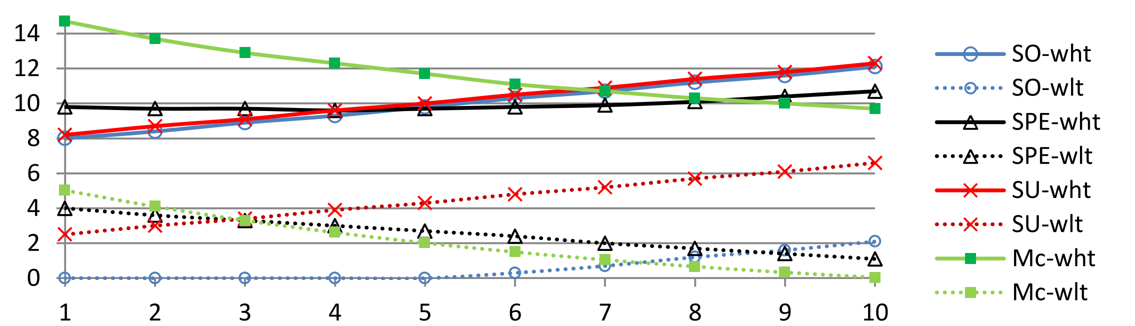

Figure 18.a Single User (SU) Extraction decisions in the ten-round game with unequal users (s=1/2)

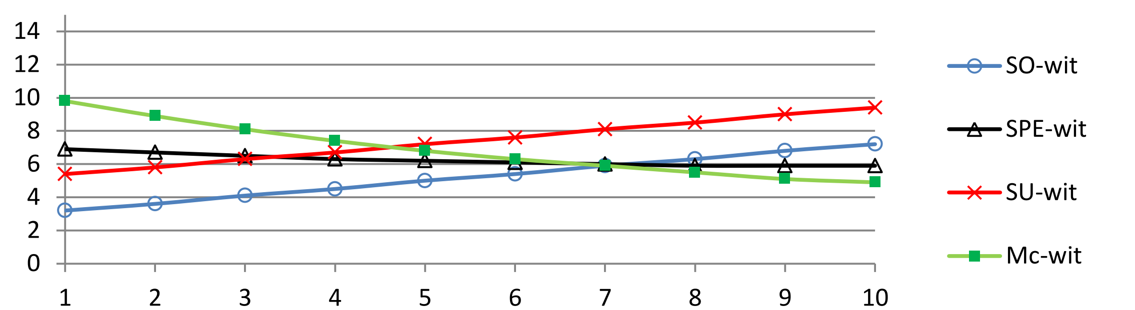

Figure 18.b Single User (SU) Extraction decisions in the ten-round game with identical users (s=1/2)

Figure 18: Single User (SU) Extraction decisions in the ten-round game

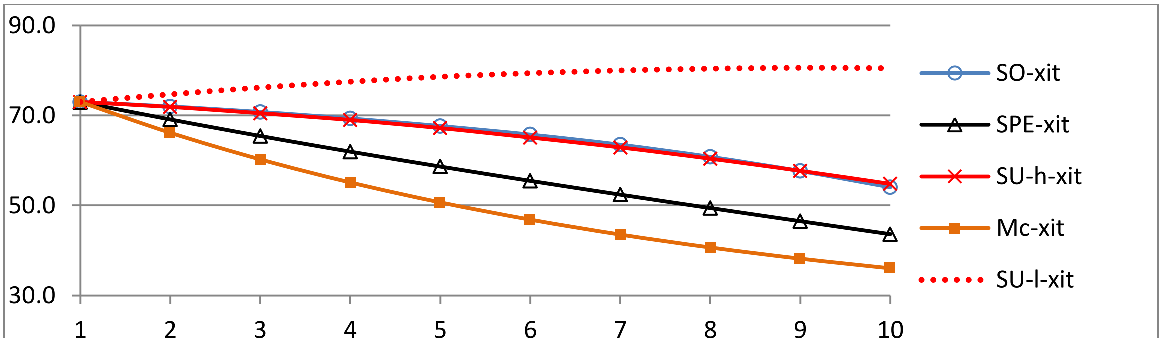

Figure 19.a Single User (SU) Stock evolution in the ten-round game with unequal users (s=1/2)

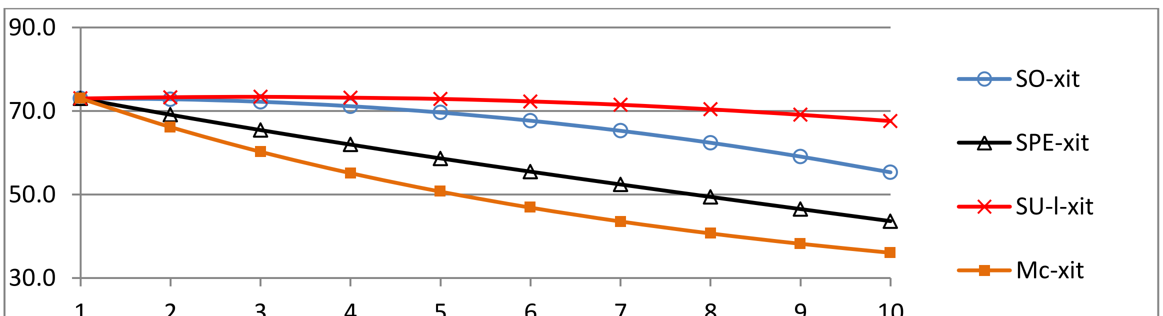

Figure 19.b Single User (SU) Stock evolution in the ten-round game with identical users (s=1/2)

Figure 19: Single User (SU) Stock evolution in the ten-round game

With identical users, figure 19.b shows that there is little evolution in stock levels under Single User management, the stocks increase slowly in the two first rounds then start decreasing at a moderate rate in the following rounds, on the 10th round the stock is at 67.6, much higher than the stock level from any other path, including the Social Optimum that shows a 55.3 stock level on the last round.

In terms of individual profits, with unequal users, figure 20.a shows that with player *h* as single User, the profits are practically equal to those following the Social Optimum; they are increasing gradually in the first half of the game to reach a maximum of 97.7 on the fourth round, and then decrease slowly in the second half. When player *l* is the Single User, the profits are much lower; they start at 13.6 on the first round and increase at a slow rate during the course of the game.

For identical users, figure 20.b shows that the Single User profits are higher than following the Social Optimum, even though they present the same tendency over the duration of the game. The –Single User– profits increase gradually in the first half of the game, the maximum is reached on rounds 6 and 7 at about 51.5, and then the profits decrease slowly in the second half.

Finally, in figures 21.a and 21.b, I compare the –individual– profits from Single User to the aggregate profits from the other paths, this of course is justified by the fact the aggregate profits under Single User management are the same as the individual profits of the Single User.

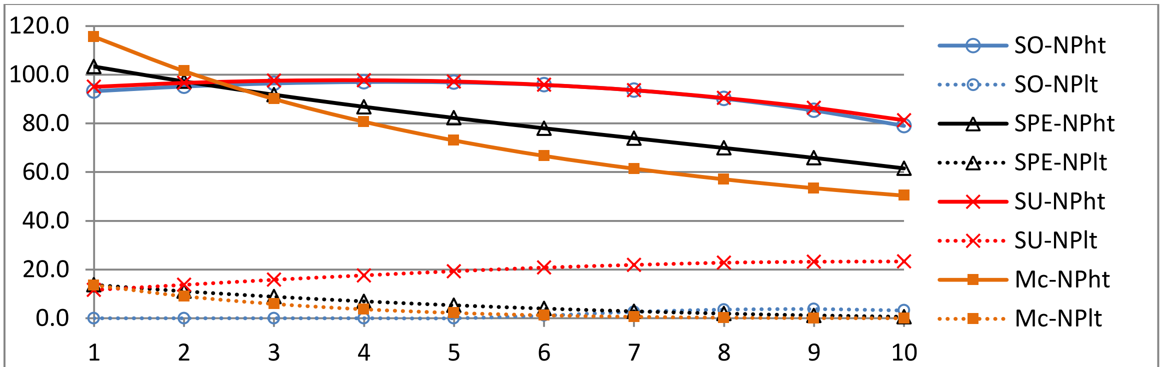

Figure 20.a Single User (SU) Individual profits in the ten-round game with unequal users (s=1/2)

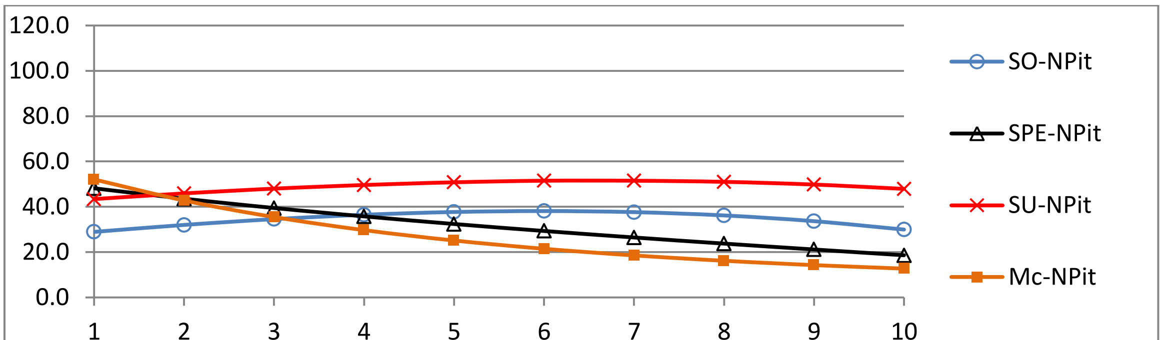

Figure 20.b Single User (SU) Individual profits in the ten-round game with Identical users (s=1/2)

Figure 20: Single User (SU) Individual profits in the ten-round

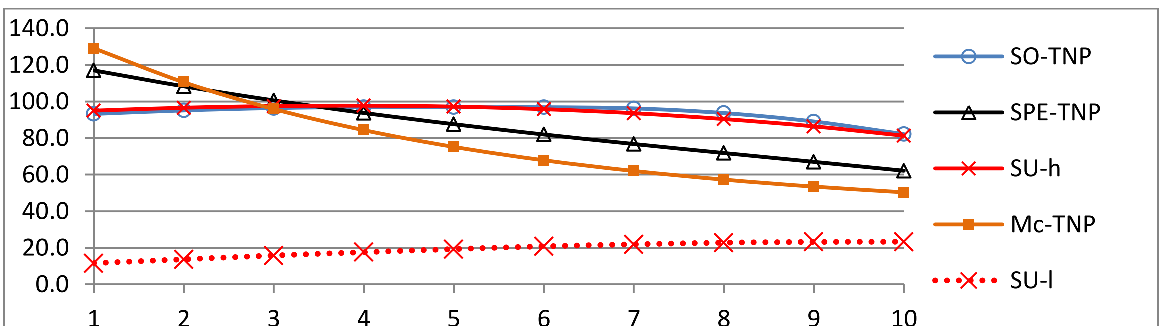

Figure 21.a Aggregate profits in the ten-round game with unequal users (s=1/2)

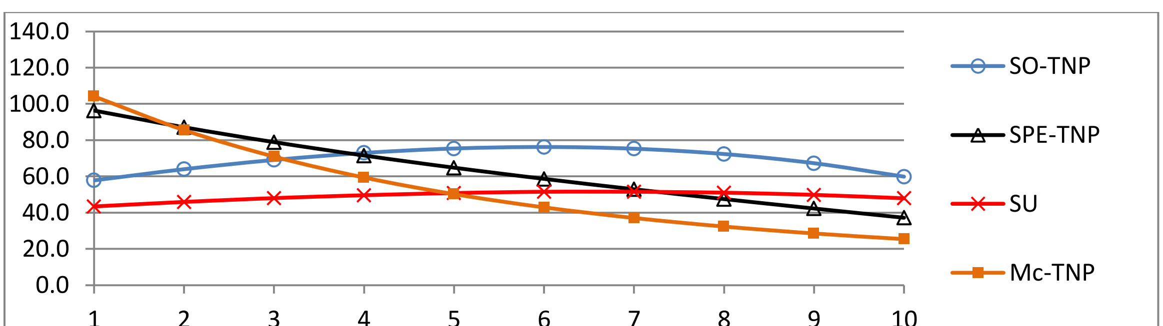

Figure 21.b Aggregate profits in the ten-round game with identical users (s=1/2)

Figure 21: Aggregate profits in the ten-round game – including Single User

With unequal users, figure 21.a shows that with player *h* as Single user, the profits are practically equal to the aggregate profits following the Social Optimum, they increase slowly to reach a maximum of 97.7 on the fourth round then decrease slowly in the following rounds. More importantly, the figure shows that aggregate profits following the SPE and the Myopic paths are only higher than the Single User profits in the two first rounds. With player *l* as Single User, the results are much less interesting as the aggregate profits are much lower than those from the other paths, at any given round.

For identical users (figure 21.b), the profits for the Single User are always lower than the aggregate profits following the Social Optimum path, they are lower than those from the SPE game for all seven first rounds. With regard to the Myopic path, the Single User profits are substantially below the profits of myopic users in the first four rounds (on the first round the aggregate profits following the myopic path equal 104.1 while the Single User only earns 43.4) but are higher in the last five rounds.

Because of the evolution of Single User profits, the resulting cumulative profits are higher than those following the SPE and Myopic paths in the case of unequal users, when player *h* is the Single User, but much lower when player *l* is the user benefiting from exclusive access to the resource. With identical users, the data shows that the cumulative profits from Single User management are lower than those following the SPE or even those with myopic users; given that the level of transmissivity is at the highest, one can only assume that this result stems from a not high enough unitary cost *c*.

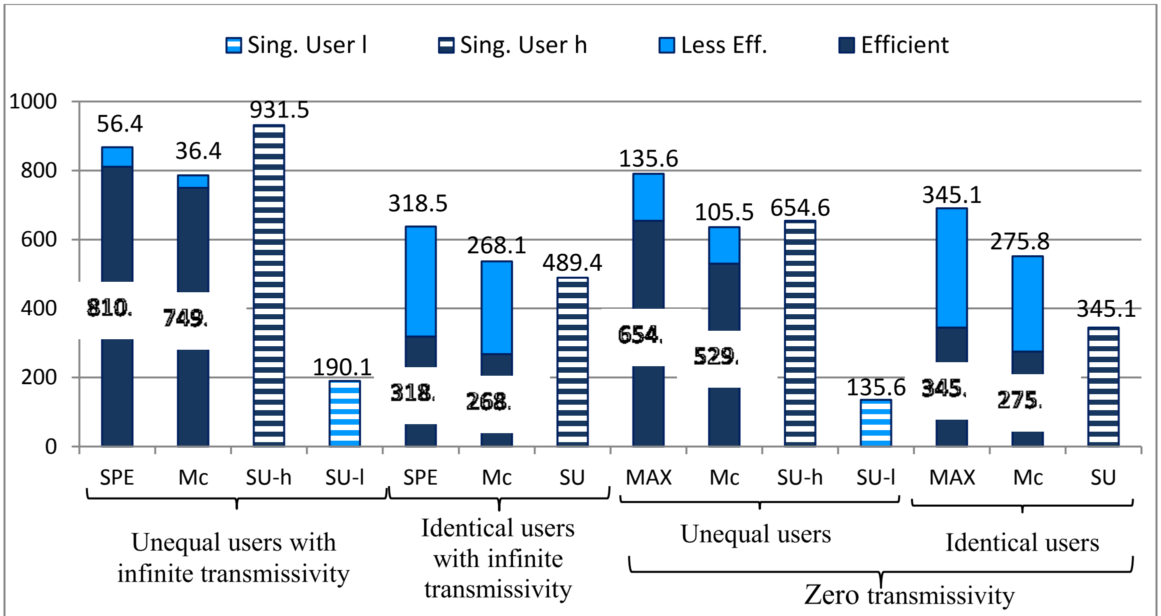

Figure 22: Aggregate cumulative profits in the ten-round game

For the case with zero transmissivity, the figure shows that cumulative profits of unequal users following the myopic path are below those from Single User management, when player h is the Single User, this result does not hold any economic significance, since nothing prevents the players from following the same path, with or without exclusive access or right of use of the resource, given the zero transmissivity, it is not clear how the Single User status would affect the player's behavior.

5.2.3.3 Numerical validation of results SU1-6 for the 10-round game

Result SU1*: Inequality increases the -single user's- profits from the Common Pool Resource with an efficient single user.*

In the case of a resource with infinite transmissivity (s = 1/2), Table 14 shows that the cumulative profits of player h as single user amount to \$931.5 with a high inequality (a_h

= 20, a_l = 10),[85] while the cumulative profits from single user management are only \$489.4 with identical players ($a_h = a_l = 15$).

With zero transmissivity ($s = 0$), the results (Table 15) show that with unequal users, the cumulative profits of player h as single user add up to \$654.6, those for the identical single user come shorter, at \$345.1.

Result SU2: *Transmissivity increases the profits from the Common Pool Resource under single user management.*

In the case of unequal users, with player h ($a_h = 20$) as single user, the aggregate profits increase from \$654.6 to \$931.5 as transmissivity shifts from 0 to ½, at the same time, for identical users, the profits of the single user grow from \$345.1 to \$489.4.

Result SU3: *In the case with a high transmissivity ($s > 0.453$) and high costs of extraction* $\left(c > \frac{2}{-2+\left(3+\sqrt{2}\right)s}\right)$, *a single user generates more profits than the aggregate profits of identical users in the non-cooperative extraction game.*

This result cannot be validated with the specifications for the 10-rounds game, but an example is provided in Appendix E.3 for the one-round game.

Result SU4: *The effect of inequality on the difference between the profits under Single User management and the aggregate profits from the non-cooperative game has an overall inverted U-shaped pattern.*

[85] Referred to as "Player *h* Single User - The Unequal Infinite Transmissivity" in the table.

Since I only use two levels of inequality, the 10-periods solution cannot validate the nonlinear effect of inequality; however, given that one of the two instances involves identical users, the data can be used to show the increasing effect of inequality.

In the case of an infinite transmissivity (s = 1/2), for identical users ($a_h = a_l = 15$), the difference between the aggregate profits, under single user management (by either one of the identical users), and the aggregate (both users) cumulative profits, following the SPE path, is equal to –\$147.7 (from Figure 22, obtained as \$489.4 minus twice \$318.5). For unequal users ($a_h = a_l = 15$), the difference between the aggregate profits, under single user management by player h, and the aggregate cumulative profits, following the SPE path, is \$64.6 (from Figure 22, obtained as \$931.5 minus the sum of \$810.5 and \$56.4). This last figure is evidence of the prospect for net gains from the switch from non-cooperative to single user management, in the case of unequal users and infinite transmissivity, that is of course on condition that player h is the one granted exclusive access to the resource.

The increasing effect of inequality can also be shown to hold with zero transmissivity (s = 0). As previously supported, the move from the non-cooperative management (equivalent to the maximizing path when s = 0) to the Single User management, with player h as single user, results in net losses equaling the profits of player l following the maximizing path, since there are no involved benefits (in terms of change of behavior or avoided externalities) to player h. From Table 1 it can be concluded that the difference in profits is –\$345.1 with identical users and –\$135.6 with unequal users.

Result SU5*: The derivative with regard to transmissivity of* $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$*, the difference between the profits under Single User management and the aggregate profits from the non-cooperative game, has an overall inverted U-shaped pattern.*

As discussed before, since I only use two levels of inequality, one of them been the identical users, the 10-periods solution can only to check the increasing effect of inequality, keeping in mind that the derivative is negative at high levels of inequality.[86]

First, for unequal appropriators, the difference between the cumulative profits under single user management (for player *h*) and the aggregate cumulative profits, following the SPE path, is –\$135.6 with zero transmissivity and \$64.6 with infinite transmissivity; the increase from the shift in the level of transmissivity is therefore \$200.2.

For identical users, the difference between the cumulative profits under single user management and the aggregate cumulative profits, following the SPE path increases from –\$345.1 with zero transmissivity to –\$147.7; the increase from the shift in the level of transmissivity is therefore \$197.4.

Result SU6*: The aggregate amount of water used by the two users under non-cooperative management is higher than the amount used by the Single User, but the savings in water use are decreasing in transmissivity.*

For the unequal users, the savings in –cumulative– water use from single user management, when compared to the aggregate –cumulative– water following the SPE is 31.8 (total use of player *l* in Table 1) with zero transmissivity, and decreases to 21.8 with

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[86] At levels between the root of the derivative and $\varepsilon^{\text{Max}}_{\text{SPE}}$.

an infinite transmissivity (obtained as the sum of 99.4, the total use by Player *h*, and 25.1, the total use of player *l* in Table1, minus 102.7, the total use of player *h* in table 14).

The same decreasing trend is observed with identical users, as the savings in aggregate water use associated with the shift from the non-cooperative management to single user management, decrease from 51.9 (total use by the identical user in Table 1) with zero transmissivity, to 50.4 with an infinite transmissivity (twice 62.3, the use by each identical player in Table1, minus 74.1, the total use in table 14).

5.3 The Social Optimum

To reach the Social Optimum the players' extraction decisions are coordinated in order to maximize their aggregate cumulative profits. I will present basic results concerning the net profits, over time, for the social optimum treatment. I provide the analytical proof for all results in the case of a one-period game; for the ten-round game I present only the numerical evidence of my findings, using the same parameterization used for the non-cooperative game and the Single User solution.

5.3.1 The Multi-period Social Optimum Path

The maximization problem is given by:

$$\underset{w_{hr},w_{lr}}{Max}\sum_{r=1}^{r=T}\left(NP_{hr}(w_{hr},x_{hr},w_{lr},x_{lr})+NP_{lr}(w_{lr},x_{lr},w_{hr},x_{hr})\right), \tag{5.21}$$

subject to: $x_{ir+1}=x_{ir}+(1-s)(x_{ir}-w_{ir})+s\left(x_{-ir}-w_{-ir}\right)+R$

$w_{ir}\geq 0$ (extraction decisions are non-negative)

$x_{ir}\leq E$ (the level of water cannot rise above the ground)

$x_{i0} = x_{-i0} = x_0$, the stocks at $t = 0$, are equal and given.

Similar to the non-cooperative game, the solution to the Social Optimum problem is unique and is obtained using the backward dynamic programming approach. The solution is presented in Chapter 3.2.2.1; numerical results are presented later in this section to substantiate the theoretical findings.

5.3.2 Analytical Evidence for the One Round Game

In the one round game, players *h* and *l* face the same maximization problem as at the last period of a multiple rounds Social Optimum solution, with an additional condition on the level of stock *x,* taken now as the same for both players. Equations (3.18) and (3.19) in 3.2.2.1.1 provide the extraction decisions and the net profits under Social Optimum for an interior solution:

$$w_i^{SO} = \frac{a_i + a_{-i} - 2c(E-x)}{2+2\,c} + \frac{a_i - a_{-i}}{2+2\,c-4\,c\,s} \qquad i=h,l \quad (5.22)$$

$$NP_i(w_i^{SO}, x, w_{-i}^{SO}, x) = \frac{\left(a_i - c\,(E-x)\right)}{2+2\,c}\left(a_i - c(E-x) + \frac{c\,s\,\left(a_i - a_{-i}\right)}{\left(1+c-2\,c\,s\right)}\right) \qquad i=h,l \quad (5.23)$$

Since I only consider the current round, there is only one condition for an interior solution: the extraction of player *l* in equation (5.22) needs to be non-negative; this condition is only satisfied when inequality, measured by the efficiency average deviation ε, is below a certain limit ε_{SO}^{Max} given as:

$$\varepsilon_{SO}^{Max} = \frac{1+c-2\,c\,s}{1+c}\left(\frac{a_h + a_l}{2} - c\left(E-x\right)\right)$$

At higher levels of inequality, social optimum entails player *l* leaving the extraction game while player *h* acts as the sole user of the CPR.

It is important to notice at this point that the level of inequality that would deter entry to the resource, to the less efficient user, in the non-cooperative game (ε_{SPE}^{Max}), is higher than the level that would deter her entry under cooperation; this suggests that in the present model it is never socially beneficial to maintain a user who is unable to compete.[87]

For convenience, I will note $NP_{a_i,a_{-i}}^{s,\,SO}(x,x)$ the net profit under Social Optimum of player i when transmissivity equals s, and $V_{a_h,a_l}^{s,\,SO}(x,x)$ the sum of net profits to players h and l, computed as:

$$V_{a_h,a_l}^{s,\,SO}(x,x)=\frac{1}{4}\left(\frac{\left(a_h+a_l-2c(E-x)\right)^2}{1+c}+\frac{\left(a_h-a_l\right)^2}{1+c-2cs}\right). \tag{5.24}$$

It can be easily derived from Equations (5.22) and (5.23) that, under cooperation, the efficient player uses more water and earns more profits. As shown in the previous section, this observation holds even for a high level of inequality inducing a single efficient user solution to the social optimization problem.

5.3.2.1 Welfare Effects of Inequality and Transmissivity on the Social Optimum

Result SOD1*: Inequality increases the aggregate cumulative profits from the Social Optimum management of a CPR.*

In the case of an interior solution, the total net profit is given by $V_{a_h,a_l}^{s,\,SO}(x,x)$ in Equation 5.24, and is composed of two terms, the first term depends on the average efficiency and the last term depends on, and is increasing in, the level of inequality; more formally, the

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[87] $\varepsilon_{SO}^{Max} = \dfrac{1+c-2cs}{1+c}\left(a-c(E-x)\right) < \varepsilon_{SPE}^{Max} = \dfrac{2+2c-3cs}{2+2c-cs}\left(a-c(E-x)\right)$

increasing effect of inequality can be shown by comparing the total net profit for unequal players to that of identical players with the same average efficiency:

$$V^{s,SO}_{a_h,a_l}(x,x) - V^{s,SO}_{\frac{a_h+a_l}{2},\frac{a_h+a_l}{2}}(x,x) = \frac{(a_h - a_l)^2}{4(1+c-2cs)} \; . \tag{5.25}$$

Inequality generates extra payoffs that are proportional in magnitude to the square of (a_h - a_l) and are increasing in transmissivity. The same result holds when considering a marginal increase in inequality, as evidenced by the positive derivative of the social optimum outcome w.r.t ε:

$$\frac{\partial V^{s,SO}_{a+\varepsilon,a-\varepsilon}(x,x)}{\partial \varepsilon} = \frac{2\,\varepsilon}{1+c-2cs} \; .$$

When inequality is high and Social Optimum corresponds to the single efficient user solution, the increasing effect of inequality is still valid as reported in Result SU1.

This is an interesting result as it shows that inequality affects positively the social optimum, but it also raises an interesting question regarding its effect on the incentive to shift from non-cooperative to social optimum management. Since inequality has the same increasing effect on aggregate profits in the non-cooperative game, the full answer is only achievable through further analysis as will be provided in Result SOD4.

Result SOD2*: For unequal players, transmissivity increases the aggregate cumulative profits from the socially optimal management.*

For identical players, the level of use and individual net profits are not affected by transmissivity (equations 5.22 & 5.23), therefore, the aggregate benefits are the same for any level of transmissivity s between 0 and ½.

For unequal players, higher transmissivity translates, under social optimum management, in an increased level of water used by the more efficient player, at the same time, the less efficient user decreases her use of water in the same amount. The larger share of water available to the more efficient user leads to higher aggregate profits as can be shown by its positive derivative:

$$\frac{\partial V^{s,SO}_{a_h,a_l}(x,x)}{\partial s} = \frac{c(a_h - a_l)^2}{2(1+c-2cs)}$$

Result SU2 shows that the increasing effect of transmissivity under Social Optimum management is maintained even when the solution is not interior and the efficient player is the sole user.

Result SOD3*: Under Social Optimum management, transmissivity increases the efficient player profits and decreases those of the less efficient user.*

The derivative of player *i* net profit w.r.t. transmissivity is given by:

$$\frac{\partial NP^{s,\,SO}_{a_i,a_{-i}}(x,x)}{\partial s} = c(a_i - a_{-i})\frac{(a_i - (E-x))}{2(1+c-2cs)^2} . \quad i{=}h,l$$

The derivative has a negative sign for the player *l* and a positive sign for the player *h*; the overall increasing effect of transmissivity on the sum of profits (Result SOD2) follows from the stronger effect on player *h*.

This result shows that individual welfare effects do not necessarily follow aggregate welfare effects. Indeed, for player *h*, higher transmissivity always translates into higher profits, while its effect on player *l* is opposite in sign and lower in magnitude; as a result, the overall effect on aggregate welfare is a net gain.

In the case of high inequality triggering a single efficient user solution to the Social Optimum maximization problem, the effect on player *h* is still increasing, for player *l*, it has no effect.[88]

5.3.2.2 Effects on the Difference Between Social Optimum and Non-Cooperative Management

This part is dedicated to the analysis of the difference between the players' aggregate and individual profits following the move from a non-cooperative extraction game to a social optimum management, the goal is to determine the effect of inequality and transmissivity on this difference and thereby on the players' motivation to move toward a socially optimal use. In the case of an interior solution, the difference in total profits under Social Optimum management and following the SPE path is as follows:

$$\Delta V_{a_h,a_l}^{s,SO-SPE}(x,x)=\frac{c^2 s^2}{4(1+c)}\left(\frac{a_h+a_l-2c(E-x)}{2+2c-\mathrm{cs}}\right)^2+\frac{c^2 s^2}{4(1+c-2\mathrm{cs})}\left(\frac{a_h-a_l}{2+2c-3\mathrm{cs}}\right)^2 \quad (5.26)$$

Result SOD4*: With an interior solution, inequality has an increasing effect on the difference between aggregate cumulative profits under Social Optimum and those from the non-cooperative game.*

At levels of inequality allowing for an interior solution, inequality has the same increasing effect on aggregate profits under the non-cooperative and the socially optimal solutions, but its effect is stronger under social optimum management.

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[88] For the general case, the Result SOD3 can be rewritten as "the effect of a (non-marginal) change in transmissivity is increasing for the efficient player, and non-increasing for the less efficient".

The second term in Equation (5.26), strictly positive, is in fact the difference between the net benefit from moving to the Social Optimum for unequal players and that for identical players with the same average efficiency:

$$V^{s,SO-SPE}_{a_h,a_l}(x,x) - V^{s,SO-SPE}_{\frac{a_h+a_l}{2},\frac{a_h+a_l}{2}}(x,x) = \frac{c^2 s^2}{4(1+c-2cs)}\left(\frac{a_h-a_l}{2+2c-3cs}\right)^2$$

This result is also valid for a marginal increase in inequality since the derivative of $\Delta V^{s,SO-SPE}_{a_h,a_l}(x,x)$ w.r.t ε, the efficiency average deviation, is always positive:

$$\frac{\partial \Delta V^{s,SO-SPE}_{\bar{a}+\varepsilon,\bar{a}-\varepsilon}(x,x)}{\partial \varepsilon} = \frac{2c^2 s^2 \varepsilon}{(2+2c-3cs)^2 (1+c-2cs)}.$$

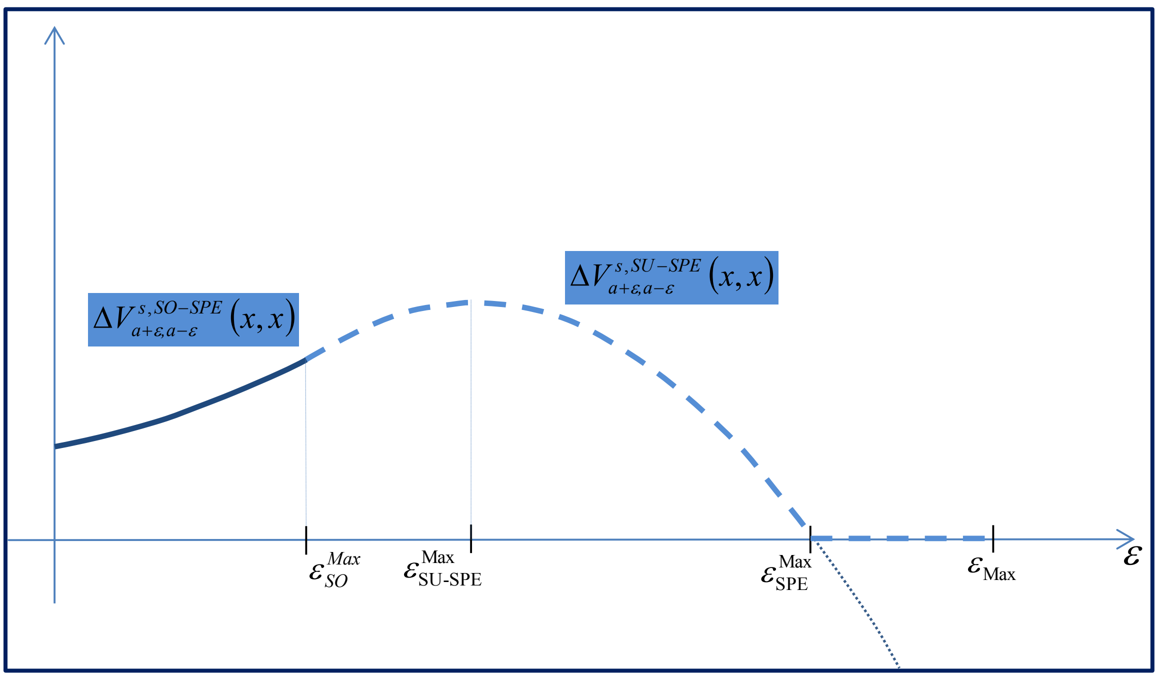

Figure 23: Effetct of inequality on the difference in aggregate profits between Social Optimum and SPE

For a level of inequality superior to ε^{Max}_{SO}, the Social Optimum solution is reached when the less efficient user suspends her extraction and the difference in profits follows the trend already established for the difference between the Single Efficient User and non-cooperative management, it is increasing in inequality ($\varepsilon_{SO}^{Max} < \varepsilon_{SU-SPE}^{Max}$), reaches the maximum, then starts to diminish and vanishes when inequality reaches ε_{SPE}^{Max}, the level of inequality at which the less efficient user quits the non-cooperative extraction game.

Result SOD5*: With an interior solution, transmissivity has an increasing effect on the difference between aggregate profits under Social Optimum and those under non-cooperative management.*

$V_{a_h,a_l}^{s,SO-SPE}(x,x)$, that will be referred to as the SO difference hereafter, and its derivative w.r.t. the level of transmissivity, both equal zero with a zero-transmissivity resource (s=0). With $s > 0$, the transmissivity has an increasing effect on both terms of $V_{a_h,a_l}^{s,SO-SPE}(x,x)$ (Equation 5.26), as shown by the derivative:

$$\frac{\partial \Delta V_{a_h,a_l}^{s,SO-SPE}(x,x)}{\partial s} = \frac{c^2 s\left(a_h + a_l - 2c(E-x)\right)^2}{(2+2c-\mathrm{c}\,\mathrm{s})^3} + 2c^2 s\left(\frac{2(1+c-2\mathrm{c}\,\mathrm{s})^2 + 6cs(1+c-2\mathrm{c}\,\mathrm{s}) + c^2 s^2}{(1+c-2\mathrm{c}\,\mathrm{s})^2(2+2c-\mathrm{c}\,\mathrm{s})^3}\right)\left(\frac{a_h - a_l}{2}\right)^2$$

The effect on the first term replicates a previous result, result SPE2, stating that externalities from the noo-cooperative (removed under social optimum) increase with the transmissivity in the case of identical users.

The effect on the second term implies that the benefits from inequality are increasing with transmissivity, as the players move from non-cooperative to socially optimal management. The gains from inequality under Social Optimum (Result SOD1) equal those under SPE (Result SPE1) with a zero-transmissivity resource, and the difference increases as more water becomes available to player h with higher levels of transmissivity.

For higher levels of inequality (higher than ε_{SO}^{Max}), the solution to the Social Optimum maximization problem is no longer interior, and the socially optimal outcome requires player *l* to exit the extraction game, leaving player *h* as the single user, the effect of transmissivity on the SO difference is as described in the previous section (Result SU5).

Result SOD6*: With an interior solution, the efficient player profits increase following the move toward the Social Optimum while the effect on the profits of the less efficient player is increasing for a the level of inequality lower than* $\varepsilon^{0}_{\text{SO-SPE}}$*, and decreasing otherwise.*

This result relates to the distributional effects of cooperation. Figure 24 displays the difference in individual profits between social optimum and non-cooperative use (line in blue, solid for interior solution and dashed otherwise). For identical players, the figure shows the – obvious – net gains from switching to the social optimum.

With an interior solution for the Social Optimum (which entails that the solution for the SPE is also interior), the difference between player *i* individual profits following the social optimum and those from the non-cooperative use, denoted as $NP^{s,SO-SPE}_{a+\varepsilon_i,a-\varepsilon_i}(x,x)$, is given as:[89]

$$\begin{aligned} NP^{s,SO-SPE}_{a+\varepsilon_i,a-\varepsilon_i}(x,x) = {} & \frac{c^2 s^2 (a-c(E-x))^2}{2(1+c)(2+2c-cs)^2} + \frac{1}{2}\frac{c^2 s^2}{(2+2c-3cs)^2(1+c-2cs)}\varepsilon_i^2 \\ & + \frac{3c^2 s^2 (1+c-cs)(a-c(E-x))}{(1+c)(2+2c-cs)(1+c-2cs)(2+2c-3cs)}\varepsilon_i \end{aligned} \quad i=h,l \quad (5.27)$$

[89] For an interior solution, $NP^{s,SO-SPE}_{a+\varepsilon_i,a-\varepsilon_i}(x,x) = NP^{s,SO}_{a+\varepsilon_i,a-\varepsilon_i}(x,x) - NP^{s,SPE}_{a+\varepsilon_i,a-\varepsilon_i}(x,x)$.

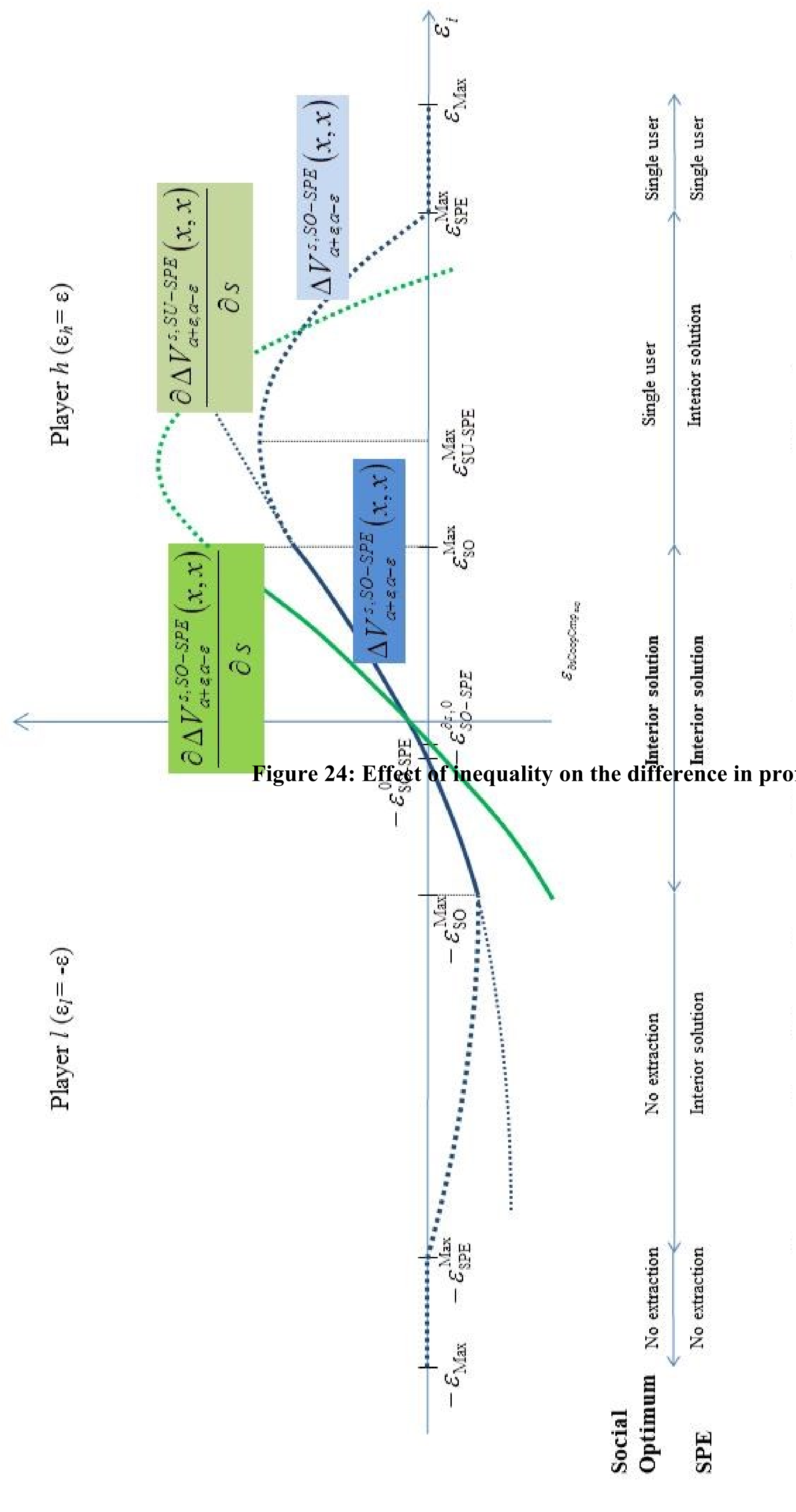

Figure 24: Effect of inequality on the difference in profits between Social Optimum and SPE

For unequal users, the gains are increasing in the level of inequality for the efficient player; for the less efficient user, the effect is opposite, the gains are decreasing in inequality and beyond a certain level of inequality ($\varepsilon^{0}_{\text{SO-SPE}}$) she starts to sustain losses from a move toward the social optimum, where:[90]

$$\varepsilon^{0}_{\text{SO-SPE}} = \frac{(2+2c-3cs)}{(2+2c-cs)} \frac{(1+c-2cs)(a-c(h-x))}{3(1+c-cs)+\sqrt{8(1+c-cs)^2+c^2s^2}}$$

This is an interesting result as it shows that with a level of inequality high enough, the social optimum is not a welfare improving strategy for the less efficient user; and that her own interest collides with the social optimum. In such a case, the less efficient user will not depart voluntarily from the non-cooperative use, and the social optimum is only reachable if enforceable through a command and control policy, or via agreeable transfers of revenues by the efficient user to compensate the less efficient one.

This result does not contradict with the fact that the social optimum is the best strategy if the aim is to maximize the aggregate profits, it merely shows that under inequality, it is not necessarily the best for each and everyone involved in the extraction game.

For higher levels of inequality where the socially optimal path is associated with the efficient player as Single User, the effect of inequality is generally similar to that described with aggregate profits. For the less efficient user, the difference is equal in magnitude to her benefits under non-cooperative use, which are decreasing in inequality.

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[90] $-\varepsilon^{0}_{\text{SO-SPE}} =$ This is the only root of eq 5.27, with an absolute value that falls in the interval with interior solutions: $\left[0, \varepsilon^{\text{Max}}_{\text{SO}}\right]$.

Result SOD7: *With an interior solution, transmissivity has an increasing effect on the profits of the efficient player; for the less efficient player the effect is positive with low levels of inequality and negative for levels higher than* $\varepsilon_{SO-SPE}^{\partial s,0}$.

With an interior solution (See solid line in gree in figure 24), the derivative with regard to transmissivity of the difference in individual profits of player i, $NP_{a+\varepsilon_i,a-\varepsilon_i}^{s,SO-SPE}(x,x)$, is given by:

$$\frac{\partial NP_{a+\varepsilon_i,a-\varepsilon_i}^{s,SO-SPE}(x,x)}{\partial s} = PQ_{SOD7a} + PQ_{SOD7b}\varepsilon_i + PQ_{SOD7c}{\varepsilon_i}^2$$

Where where PQ_{SOD7a}, PQ_{SOD7b}, and PQ_{SOD7c}, are three positive variables given by:

$$PQ_{SOD7a} = \frac{2c^2 s(a-c(h-x))^2}{(2+2c-cs)^3}$$

$$PQ_{SOD7b} = \frac{3c^2 s((2+2c-3cs)^3 + 2cs(2+2c-3cs)^2 + c^2s^2(2+2c-4cs))(a-c(h-x))}{(2+2c-cs)^2(1+c-2cs)^2(2+2c-3cs)^2}$$

$$PQ_{SOD7c} = \frac{c^2 s(2(1+c-2cs)^2 + 6cs(1+c-2cs) + c^2s^2)}{(1+c-2cs)^2(2+2c-3cs)^3}$$

The effect of an increase in transmissivity on the difference between individual profits following the Social Optimum and the SPE is always positive for identical users (ε_i=0). With unequal users, increasing transmissivity has a positive effect on the efficient user (ε_h>0) difference of profits, for the less efficient user, the effect is positive with low levels of inequality and negative with levels higher than $\varepsilon_{SO-SPE}^{\partial s,0}$ (ε_l<- $\varepsilon_{SO-SPE}^{\partial s,0}$), where:

$$\varepsilon_{SO-SPE}^{\partial s,0} = \frac{4(1+c-2cs)^2(2+2c-3cs)^2(a-c(h-x))}{(2+2c-cs)(\sqrt{PQ_{SOD7d}}+PQ_{SOD7e})}$$

Where where PQ_{SOD7d} and PQ_{SOD7e}, are two positive variables given by:

$$PQ_{SOD7d} = 8(2+2c-3cs)^6 + 32cs(2+2c-4cs)^5 + 220c^2s^2(2+2c-4cs)^4 + 588c^3s^3(2+2c-4cs)^3 + 720c^4s^4(2+2c-4cs)^2 + 384c^5s^5(2+2c - 4cs) + 73c^6s^6$$

$$PQ_{SOD7e} = 3(2+2c-3cs)^3 + 6cs(2+2c-3cs)^2 + 6c^2s^2(1+c-2cs)$$

It can be shown that $-\varepsilon_{SO-SPE}^{\partial s,0}$ is the only root of equation $\partial NP_{a+\varepsilon_i,a-\varepsilon_i}^{s,SO-SPE}(x,x) \big/ \partial s = 0$, with an absolute value that falls in the interval with interior solutions: $\left[0,\varepsilon_{\mathrm{SO}}^{\mathrm{Max}}\right]$.

5.3.2.3 Water Conservation Under the Social Optimum

Result SOD8: *With an interior solution, the total amount of water used by the two users is higher under non-cooperative management, and the savings from Social Optimum are increasing in transmissivity.*

Given the extraction decision following the Social Optimum path (Equation 5.22) and the extraction decision following the SPE (Equation 3.27), the savings in –aggregate– water use resulting from the switch from SPE to Social Optimum are as follows:

$$(w_h^{SPE} + w_l^{SPE}) - (w_h^{SO} + w_l^{SO}) = \frac{cs}{1+c}\left(\frac{a_h + a_l - 2c(E-x)}{2+2c-\mathrm{cs}}\right)$$

This result is very interesting as it shows that the level of stocks under social optimum, are higher, even after the last round. In a multi-period game, conservation is also driven by future benefits in the form of lower costs of extraction in the following rounds; in the one-period game, current savings in the costs of extraction are the sole motivation for conservation.

The effect of an increase in transmissivity on savings is increasing as shown by the positive sign of:

$$\frac{\partial\left(w_h^{SPE}+w_l^{SPE}-w_h^{SO}-w_l^{SO}\right)}{\partial s}=\frac{2c\ \left(a_h+a_l-2c\left(E-x\right)\right)}{\left(2+2c-\mathrm{cs}\right)^2}$$

The positive effect of transmissivity is driven by the increasing effect of transmissivity on water use in the non-cooperative game.

5.3.3 Numerical validation for the Ten-round Social Optimum Game

For the numerical validation, I use the same specification used in Chapter 3.4.1, then in the laboratory experiment in Chapter 4.1.1. The numerical solutions for players' decisions and profits, following the SPE and the SO are regrouped in (respectively) Table 1 and Table 4.

Result SOD1*: Inequality increases the aggregate cumulative profits from the Social Optimum management of a CPR.*

With infinite transmissivity, Table 4 shows that the aggregate cumulative profits under Social Optimum are $936.5 for unequal users and $690.1for identical users.

With zero transmissivity, Table 1 shows that aggregate cumulative profits under Social Optimum (following the maximizing path in this case with *s*=0) is $790.1 for unequal users and $690.1 for identical users.[91]

Result SOD2*: For unequal players, transmissivity increases the aggregate cumulative profits from the socially optimal management.*

For unequal users, the aggregate cumulative profits under Social Optimum increase from $790.1 with zero transmissivity to $936.5 with infinite transmissivity (s= to ½).

[91] The solution for Social Optimum with unequal users is not interior, player *l* extraction decisions are set to zero on the first five rounds, but this does not affect any of the results.

Following the Social Optimum, the profits of identical users stay unchanging (at $690.1) as transmissivity shifts from 0 to ½.

Result SOD3*: Under Social Optimum management, transmissivity increases the efficient player profits and decreases those of the less efficient user.*

For unequal users, the individual profits of player *h* following the Social Optimum increase from $654.6 to $922.3 as the level of transmissivity goes from 0 to ½. At the same time, the profits of player *l* fall from $135.6 to $14.2.

Under Social Optimum, the individual profits of identical players stay unchanging (at $345.1) as the level of transmissivity shifts from 0 to ½.

Result SOD4*: With an interior solution, inequality has an increasing effect on the difference between aggregate cumulative profits under Social Optimum and those from the non-cooperative game.*

For unequal users, Table 4 shows that, with an an infinite-transmissivity resource, the aggregate cumulative profits from the Social Optimum amount to $936.5, while those following the SPE only add up to $866.9, resulting in a $69.6 difference between Social Optimum and non-cooperative use.

For identical users, the aggregate cumulative profits under Social Optimum are $690.1, and $637.1 under non-cooperative management, resulting in a $53.0 difference between Social Optimum and the non-cooperative use, lower than the $69.6.

Result SOD5*: With an interior solution, transmissivity has an increasing effect on the difference between aggregate profits under Social Optimum and those under non-cooperative management.*[92]

For unequal users, the difference in aggregate cumulative profits between Social Optimum and non-cooperative management increases from $0 with no transmissivity (*s*=0), to $69.6 when *s* reaches 1/2. At the same time, the difference in aggregate profits for identical players increases from $0 to $53.0.[93]

Result SOD6*: With an interior solution, the efficient player profits increase following the move toward the Social Optimum while the effect on the profits of the less efficient player is increasing for a the level of inequality lower than* $\varepsilon^{0}_{\text{SO-SPE}}$*, and decreasing otherwise.*

For identical users, the individual profits increase from $318.6 under SPE to $345.1 under social optimum.

In the case of unequal users, with an infinite-transmissivity resource, the total profits for player *h* increase from $654.6 under SPE to $922.3 under Social optimum, for player *l*, the total profits fall from $135.6 under SPE to $14.2 under the Social optimum.

Result SOD7*: With an interior solution, transmissivity has an increasing effect on the profits of the efficient player; for the less efficient player the effect is positive with low levels of inequality and negative for levels higher than* $\varepsilon^{\partial s,0}_{SO-SPE}$.

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[92] The solution for Social Optimum with unequal users is not interior, player *l* extraction decisions are set to zero on the first five rounds, but this does not affect any of the results.

[93] With *s* = 0, the non-cooperative and Social Optimum merge with the maximizing path.

With a zero-transmissivity resource, there is no loss or gain from switching from non-cooperative use to the social optimum, the two paths are one at $s = 0$.

With an infinite-transmissivity resource, identical users secure a \$26.5 net –individual– gain following the switch from the SPE to social optimum; for unequal users, player h makes a net gain of \$267.8 from the switch in the CPR management, while the less efficient user suffers a net loss of –\$121.3.

Result SOD8*: With an interior solution, the total amount of water used by the two users is higher under non-cooperative management, and the savings from Social Optimum are increasing in transmissivity.*

For unequal users, the savings in water use from social optimum (vs. non-cooperative) management increase from 0 when s=0, to 18.5 (106, the sum of 100.2 and 5.7, minus 124.5, the sum of 99.4 and 25.1) when s=1/2.

For identical users, the savings in water use from social optimum increase from 0 when s=0, to 20.7 (twice 51.9 minus twice 62.3) with an infinite-transmissivity resource.

5.4 Experimental validation

5.4.1 Parameterization

In this part, I only use two treatments with an infinite-transmissivity resource, the case(s) with a zero-transmissivity resource were already considered in the previous experiment, with no buy-out alternative; I assume that the results are still relevant since, players have no incentive to take advantage of the buy-out alternative; there are no externalities to avoid with a zero-transmissivity resource.

Regarding the level of inequalities, I carry on with the same cases considered in the previous set of experiments of extraction game –with no buy-out– and in the numerical examples; I consider two levels of inequality: a treatment with unequal players and another with identical ones. In the treatment with unequal players, one player displaying a high efficiency $a_h = 20$ is paired with a player with low efficiency $a_l = 10$. In the treatments with identical players I use $a_h = a_l = 15$, to preserve the average efficiency in the two treatments.

5.4.2 The Experiment

The experimental sessions were held in the experimental laboratory of the Department of Agricultural and Resource Economics at the University of Maryland. Volunteer subjects were recruited from undergraduate classes in economics, business, and civil engineering. They were informed that they would participate in an experiment where they would be asked to make economic decisions and receive payments based on their decisions and those of other participants. They were also informed of the average length of a session.

I ran two experimental sessions, one session dedicated to the treatment with identical players and the second to the treatment with unequal users. At the beginning of each session, the subjects were randomly assigned into different groups (of two each). Each subject was informed of her own efficiency, a_i, and that of her partner's, a_{-i}. The subjects were also informed of the other parameters that define the game: s, c, E, x_{i0}, x_{-i0}, and R, the rate of renewal. The efficiencies and pairings of the players were kept secret, as were the individual decisions made throughout the experiment.

Instructions were shown on the screen of the computer (See figures 25a-c), and formulas for production and cost were given. In each round, participants were asked to make (simultaneously) three choices:

- Their OFFER, to “buy out” their partner; it represents the amount of money that the player is willing to pay to his/her partner in exchange of this latter stopping his/her use of the stock during the current round and all following rounds.
- Their DEMAND, the amount of money for which the player would be willing to leave the stock, it represents the price that the partner needs to pay to the player in order to accept to depart from the extraction stock during the current round and all following rounds.
- The quantities to be extracted from a given range [0, a_i], if no transaction is passed.
- A transaction takes place when, at least,[94] one OFFER by one player is larger than the DEMAND (price) set by her partner in the group; the transaction amount is then set equal to the average of the OFFER and DEMAND, it is deducted from the buyer whose OFFER was accepted and awarded to the seller, who made the DEMAND.

[94] When the demand of every player within the group is satisfied by the offer by her partner in the group, then priority is given to the transaction that generates the highest value i.e. the transaction with the larger difference between offer and demand.

Figure 25.a Decision table from the Buy- out experiment

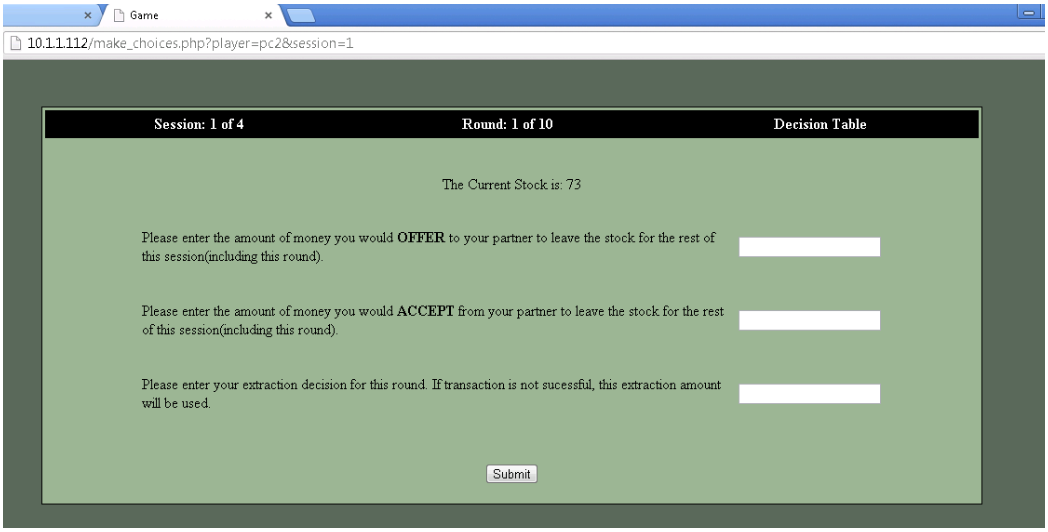

Figure 25.b Results table from the Buy- out experiment

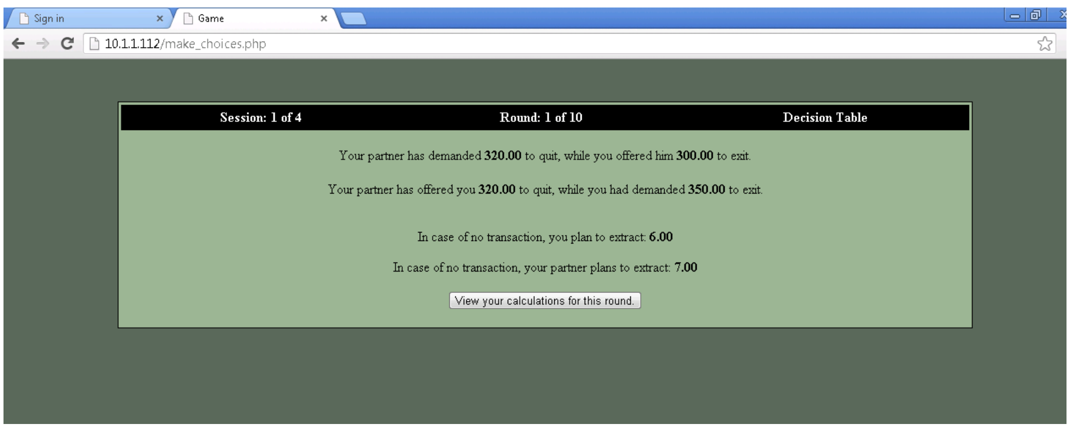

Figure 25.c Transaction Results table from the Buy- out experiment

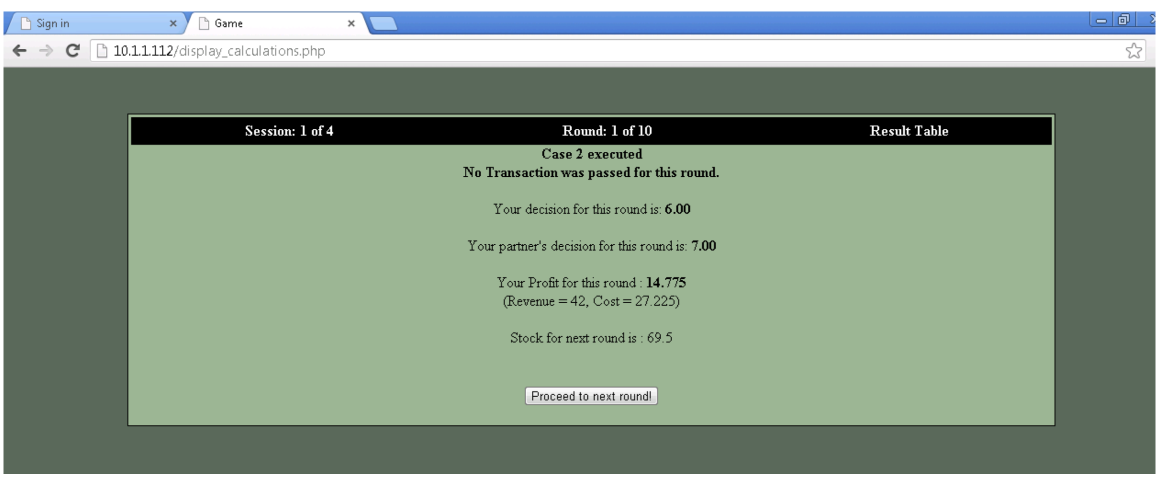

Figure 25: Decision, Results and Transaction results tables from the Buy- out experiment

- Once the two partners have entered their decisions, they were informed on their partner's decisions (See figure 25.b); they were also informed whether a transaction was passed or not (See figure 25.c). If a transaction was passed, the seller received the transaction price and waited for the next session while the buyer was asked to re-enter her decision for the current round, and continue the extraction game as a sole user for the rest of the session.
- When no transaction was passed, the extraction decisions were carried, and the players were updated on their earnings, in laboratory dollars, and on the level of water in the aquifer available at the beginning of the next period.

The participants were awarded $8 for their participation and were given the conversion rate that would be used, for each type of players, to convert the computer dollars earned during the experiment to real dollars. At the end of the experiment they were given, privately, their earnings in cash. Every experimental session consisted of four series of 10 rounds each.

To help participants make their decisions, they were invited to use the *decision support window* that was provided to them on screen. Based on the current stock (x_t), their decision, w_{it}, and their assumption about their partner's decision, w_{-it}, the players were able to compute their hypothetical profits for the current round, the stocks at the start of the next round, and the ensuing hypothetical profits of their partners.

Figure 26: Support Sheet from the Buy-out game experiment

1- Enter the updated stock for this round as provided in the explorer program

The current stock 73.00

2- Enter your decision and your "best guess" for your partner's decision

Your decision 9.00

The other person's decision 14.99

3- Outcomes and next round stock

Your earnings for this round are: **49.95** (revenue: 94.50 and cost 44.55)

The other person's earnings for this round : **38.31** (revenue: 112.50 and cost 74.19)

The stock in next period : **64.01**

4- Tables of Extended Outcomes and next round stocks

Your earnings table

The other person's decision		Your decision				
		8.00	8.50	**9.00**	9.50	10.00
	13.99	49.00	49.78	50.29	50.53	50.50
	14.49	48.85	49.62	50.12	50.35	50.32
	14.99	48.70	49.46	**49.95**	50.18	50.13
	15.49	48.55	49.30	49.78	50.00	49.94
	15.99	48.40	49.14	49.62	49.82	49.75

The other person's earnings table

Your decision		The other person's decision				
		13.99	14.49	**14.99**	15.49	15.99
	8.00	43.79	41.46	38.87	36.00	32.87
	8.50	43.53	41.19	38.59	35.71	32.57
	9.00	43.27	40.92	**38.31**	35.42	32.27
	9.50	43.01	40.65	38.02	35.13	31.97
	10.00	42.74	40.38	37.74	34.84	31.67

Next round stock table

The other person's decision		Your decision				
		8.00	8.50	**9.00**	9.50	10.00
	13.99	65.01	64.76	64.51	64.26	64.01
	14.49	64.76	64.51	64.26	64.01	63.76
	14.99	64.51	64.26	**64.01**	63.76	63.51
	15.49	64.26	64.01	63.76	63.51	63.26
	15.99	64.01	63.76	63.51	63.26	63.01

5- Calculating the maximum total profits if using the stock alone

Enter the current stock 73.00

Enter the current round 1.00

The maximum total profits for a single user are: 489.4
The best current decision for a single user: 5.389

NB: The spreadsheet was protected and subjects were only allowed to modify the yellow cells.

The window also helped the players to decide on their OFFER by providing them with the maximum total profits for the sole user based on the current round and current stock.

Finally, the players were provided with the actual average cumulative profits (from the entire session) from previous experiments where no transaction was allowed (Table 12 in Chapter 4.2.3).

Table 16: Summary of Buy-out experimental results.

Treatment	Experimental Sessions	Type	Subjects	Observations	Average profits per subject
Unequal users	1	*h*	5	20	\$25.1
		l	5	20	\$27.4
Identical users	1		10	19	\$24.3

5.4.3 Theoretical Analysis of Individual Behavior

5.4.3.1 Single User Versus SPE

I present here the summary of theoretical results when one player exits the extraction game at round *r*, leaving the resource to the other user. The assumption here is that both players use the resource following the SPE until round *r*, at the start of round *r* one player exits the extraction game and the other user becomes the sole user of the resource, following the Single User path, the results from this approach are included in Table 17.

In the case of unequal players, Table 17 presents the –individual and aggregate– cumulative profits when player *l* exits the extraction game at round *r*, and the efficient user becomes the sole user for round *r* and all remaining rounds. The figures show that there are net gains, in aggregate –cumulative– profits, when player *l* exits the resource in any one of the eight first rounds, but the gains are diminishing over time, and, on the 9th and 10th rounds, there is a net loss from the less efficient user exiting the extraction game.

Table 17 also shows that there are net losses, in terms of cumulative aggregate profits, when player *h* exits the resource, in any round; the losses are diminishing (in absolute value) over time. Player *l* makes higher profits following his/her partner's exit, but this latter endures much larger losses.

The same conclusion is reached in the case of identical users, in the current numerical specification, the externalities related to extraction costs are not high enough to make it advantageous to the group to deviate from the SPE toward the Single User path. The player granted exclusive use of the aquifer makes higher profits that decrease as the round when the partner player is postponed; but the increase in the his/her profits are not high enough to overweigh the losses to the player exiting the resource.

Table 17: Gains and losses from Single User versus the SPE path

	Exit Round	Individual cumulative profits when player *-i* exits on round *r*		Aggregate cumulative profits	Nominal individual win/loss from exit		Group win/loss from exit
		V_{ir}	V_{-ir}	*h+l*	*Single user*	*Partner*	*h+l*
Unequal users, player *l* exits ($a_h = 20$, $a_l = 10$)	1	931.50	0.00	931.50	120.97	-56.38	64.60
	2	901.03	13.69	914.72	90.50	-42.68	47.82
	3	876.40	24.76	901.16	65.87	-31.61	34.26
	4	856.80	33.60	890.40	46.27	-22.78	23.50
	5	841.55	40.54	882.09	31.02	-15.83	15.19
	6	830.03	45.87	875.90	19.50	-10.50	9.00
	7	821.70	49.85	871.55	11.18	-6.53	4.65
	8	816.06	52.69	868.75	5.54	-3.68	1.85
	9	811.82	54.60	866.43	1.30	-1.77	-0.47
	10	810.95	55.77	866.72	0.43	-0.61	-0.18
	No exit	810.52	56.38	866.90	0.00	0.00	0
Unequal users, player *h* exits ($a_h = 20$, $a_l = 10$)	1	190.07	0.00	190.07	133.69	-810.52	-676.83
	2	157.72	103.31	261.03	101.35	-707.22	-605.87
	3	131.11	200.53	331.64	74.73	-609.99	-535.26
	4	109.54	292.29	401.83	53.16	-518.23	-465.07
	5	92.43	379.10	471.53	36.06	-431.43	-395.37
	6	79.27	461.34	540.62	22.90	-349.18	-326.28
	7	69.59	539.33	608.92	13.21	-271.19	-257.98
	8	62.93	613.24	676.18	6.56	-197.28	-190.72
	9	58.70	683.15	741.85	2.33	-127.37	-125.05
	10	56.88	748.99	805.86	0.50	-61.54	-61.04
	No exit	56.38	810.52	866.90	0.00	0.00	0
Identical users ($a_h = 15$, $a_l = 15$)	1	489.35	0.00	489.35	170.81	-318.55	-147.74
	2	451.87	48.16	500.03	133.33	-270.38	-137.06
	3	420.33	91.72	512.05	101.78	-226.82	-125.04
	4	394.02	131.17	525.19	75.47	-187.37	-111.90
	5	372.38	166.93	539.31	53.83	-151.62	-97.78
	6	354.91	199.33	554.24	36.37	-119.22	-82.85
	7	341.23	228.64	569.87	22.69	-89.90	-67.22
	8	331.00	255.08	586.09	12.46	-63.46	-51.00
	9	323.47	278.82	602.29	4.92	-39.72	-34.80
	10	319.86	299.96	619.82	1.32	-18.59	-17.27
	No exit	318.55	318.55	637.09	0.00	0.00	0

5.4.3.2 Single User Versus Myopic path

The assumption now is that both players are engaging in a myopic game until the start of round *r*, when one player exits the extraction game and the other user becomes the sole user of the resource, following the Single User path; the results from this approach are included in Table 18.

Table 18 presents the cumulative profits of unequal players, when player *l* exits and player *h* becomes the sole user on round *r* and all remaining rounds. Similarly to the previous discussion with strategic players, the table shows that there are net gains, in aggregate –cumulative– profits, when player *l* exits the resource in any one of the eight first rounds, but the gains are diminishing over time, and, on the 9th round, there is a net loss following the player's exit. The main difference with the previous observations related to the comparison to the SPE path is that the potential profits from tplayer *l* exit are higher when players are myopic.[95]

When player *h* exits the resource, the cumulative aggregate profits always shrink; the losses are diminishing (in absolute value) over time. The profits of player *l*, following the exit, are much lower than the losses to player *h*. Even though the drops in aggregate cumulative profits are lower in comparison to those with strategic users, they are still very important in absolute terms.

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[95] This is true for all first 8 rounds, in the 9th round, the loss from the exit with myopic users is lower than that with strategic users, finally, the exit in the 10th translates into a tiny increase in aggregate cumulative profits for myopic users and into a tiny loss for the strategic users.

Table 18: Gains and losses from Single User versus the Myopic path (s=1/2)

	Exit Round	Individual cumulative profits when player $-i$ exits on round r		Aggregate cumulative profits	Nominal individual win/loss from exit		Group win/loss from exit
		V_{ir}	V_{-ir}	*h+l*	*Single user*	*Partner*	*h+l*
Unequal users, less efficient exits ($\mathbf{a_h = 20}$, $a_l = 10$)	1	931.50	0.00	931.50	181.96	-36.41	145.55
	2	873.00	13.56	886.56	123.46	-22.84	100.62
	3	830.49	22.60	853.09	80.95	-13.81	67.15
	4	800.24	28.47	828.71	50.71	-7.94	42.76
	5	779.36	32.13	811.49	29.82	-4.28	25.54
	6	765.55	34.31	799.87	16.02	-2.09	13.92
	7	757.02	35.52	792.54	7.48	-0.89	6.59
	8	752.29	36.11	788.41	2.75	-0.30	2.46
	9	749.43	36.35	785.77	-0.11	-0.06	-0.17
	10	749.55	36.41	785.96	0.01	0.00	0.01
	No exit	749.54	36.41	785.95	0.00	0.00	
Unequal users, efficient exits ($\mathbf{a_h = 20}$, $a_l = 10$)	1	190.07	0.00	190.07	153.66	-749.54	-595.88
	2	141.09	115.55	256.64	104.68	-633.99	-529.31
	3	105.55	217.01	322.56	69.14	-532.53	-463.38
	4	80.23	307.03	387.26	43.82	-442.51	-398.69
	5	62.65	387.70	450.35	26.24	-361.84	-335.60
	6	50.91	460.68	511.59	14.50	-288.85	-274.35
	7	43.51	527.34	570.85	7.10	-222.20	-215.10
	8	39.25	588.75	628.01	2.85	-160.79	-157.94
	9	37.11	645.80	682.91	0.70	-103.74	-103.04
	10	36.48	699.20	735.68	0.08	-50.34	-50.27
	No exit	36.41	749.54	785.95	0.00	0.00	
Identical users ($\mathbf{a_h = 15}$, $a_l = 15$)	1	489.35	0.00	489.35	221.22	-268.14	-46.92
	2	427.40	52.07	479.47	159.26	-216.06	-56.80
	3	380.55	94.84	475.39	112.42	-173.30	-60.88
	4	345.41	130.30	475.71	77.27	-137.84	-60.57
	5	319.35	159.98	479.33	51.21	-108.16	-56.95
	6	300.36	185.08	485.44	32.22	-83.06	-50.83
	7	286.90	206.53	493.43	18.76	-61.61	-42.84
	8	277.78	225.05	502.82	9.64	-43.09	-33.45
	9	271.66	241.20	512.86	3.52	-26.93	-23.41
	10	269.04	255.45	524.49	0.90	-12.69	-11.79
	No exit	268.14	268.14	536.27	0.00	0.00	

All previous remarks –with identical strategic users– apply to the case with identical myopic users, when one player exits the extraction game, the aggregate cumulative profits drop, indicating that the extra profits generated by a Single identical User are lower than the losses to the player exiting the game; however, the losses from the exit are lower than with strategic players.

5.4.4 Analysis of Experimental Results

Table 19 summarizes the results from the buy-out experiment, the profits are reported as earnings from the use of water from the aquifer, while the actual profits are computed based on profits and gains or losses from the transaction, where trade has occurred.

Table 19: Experimental results from the buy-out game.

Treatment	Transaction Round	Obs.	Buyer cumulative Profits.	Seller cumulative Profits.	Transaction	Offer	Demand	Actual Buyer Profits.	Actual Seller Profits.	Aggregate Profits
Unequal users & Infinite Transmissivity (a_h = 20, a_l = 10, s = ½)	**1**	**7**	**771.2**	**0.0**	**279.6**	**365.0**	**194.3**	**491.5**	**279.6**	**771.2**
	2	1	614.7	10.9	390.0	500.0	280.0	224.7	400.9	625.6
	3	1	730.4	19.6	41.5	50.0	33.0	688.9	61.1	750.0
	4	2	556.7	15.9	59.8	65.0	54.5	496.9	75.6	572.5
	5	1	690.6	25.8	40.0	40.0	40.0	650.6	65.8	716.4
	6	1	662.5	46.3	200.0	200.0	200.0	462.5	246.3	708.8
	7	1	681.7	-25.8	25.0	25.0	25.0	656.7	-0.8	655.8
	8	2	721.7	27.5	25.0	25.0	25.0	696.7	52.5	749.2
	9	1	754.7	38.1	10.0	10.0	10.0	744.7	48.1	792.8
	No trade	**3**	**700.8**	**-14.1**				**700.8**	**-14.1**	**686.7**
Identical users & Infinite Transmissivity (a_h = 15, a_l = 15, s = 1/2)	**1**	**8**	**447.3**	**0.0**	**299.1**	**296.9**	**301.4**	**148.1**	**299.1**	**447.3**
	2	2	308.9	48.0	291.3	305.0	277.5	17.6	339.3	356.9
	4	1	254.7	115.6	185.5	170.0	201.0	69.2	301.1	370.2
	5	2	350.9	124.0	154.0	165.0	143.0	196.9	278.0	474.9
	6	1	403.6	149.3	194.5	199.0	190.0	209.1	343.8	552.9
	8	1	290.2	236.5	85.0	80.0	90.0	205.2	321.5	526.7
	No trade	**8**	**302.4**	**286.0**				**302.4**	**286.0**	**588.4**

For the Unequal treatment, the experimental data, as presented in Table 19, shows that in most sessions (11 out of 20), a transaction was passed in the first four rounds, and only in three instances did the whole session proceed with no transaction been passed. In all transactions the buyer was a player type *h*.

In accordance with the theoretical deductions, the aggregate cumulative profits display a negative correlation with the transaction round. In the same course, the sessions where no transaction was passed generate lower profits (36.33 lower in average) than those where a transaction was passed, but both observations do not hold a strong statistical significance.[96] However, if the comparison is limited to the aggregate profits from the sessions where a transaction was passed on the first round, making the whole game as a Single User game, the difference in profits is significantly higher than in the sessions with no trade.

The data also indicates a visible learning effect during the experiment, as evidenced by the improvement in terms of increased aggregate profits, as the players advance in the game, across the 4 sessions:

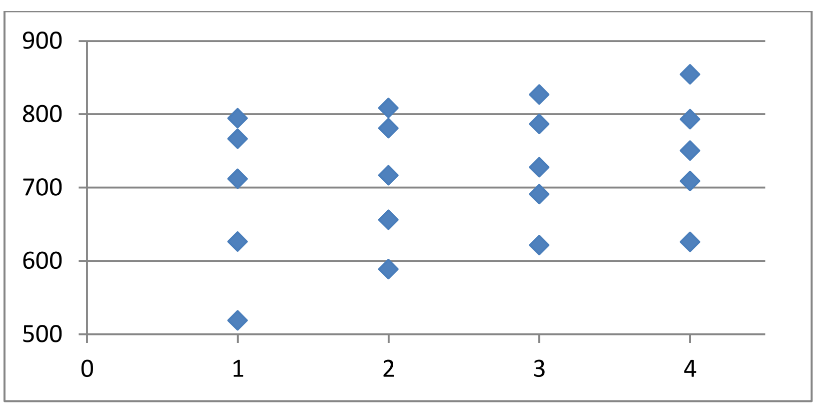

Figure 27: Learning effect with Unequal users

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[96] The average is 686.67for sessions with no transaction versus 723.01for sessions with transactions.

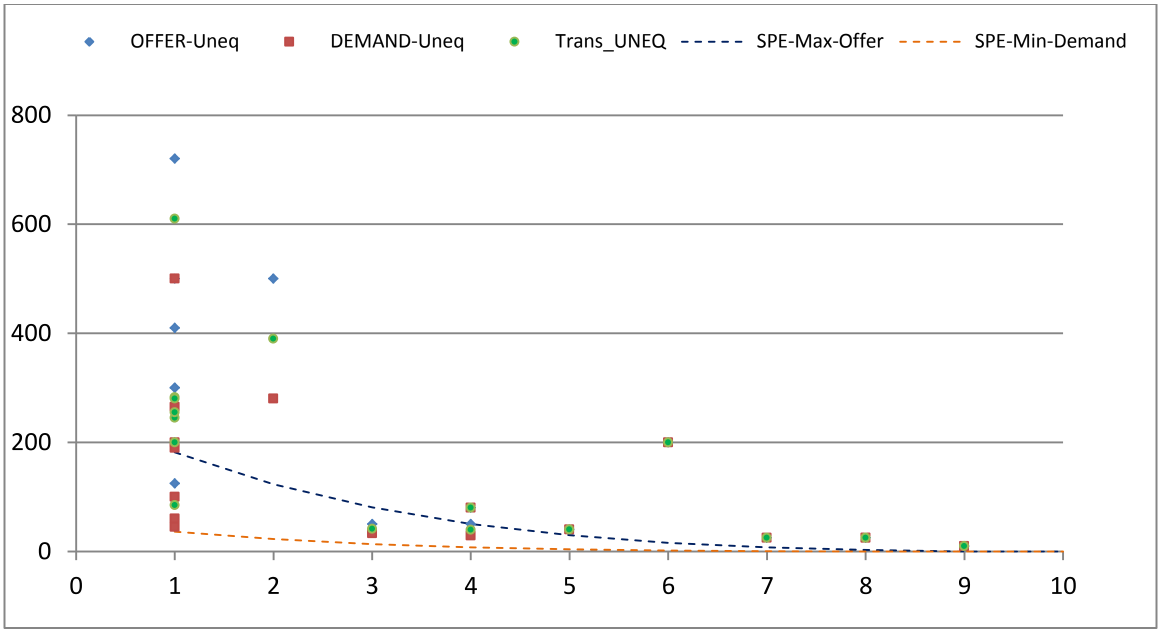


Figure 28. OFFER, DEMAND and values of passed Transaction with unequal users

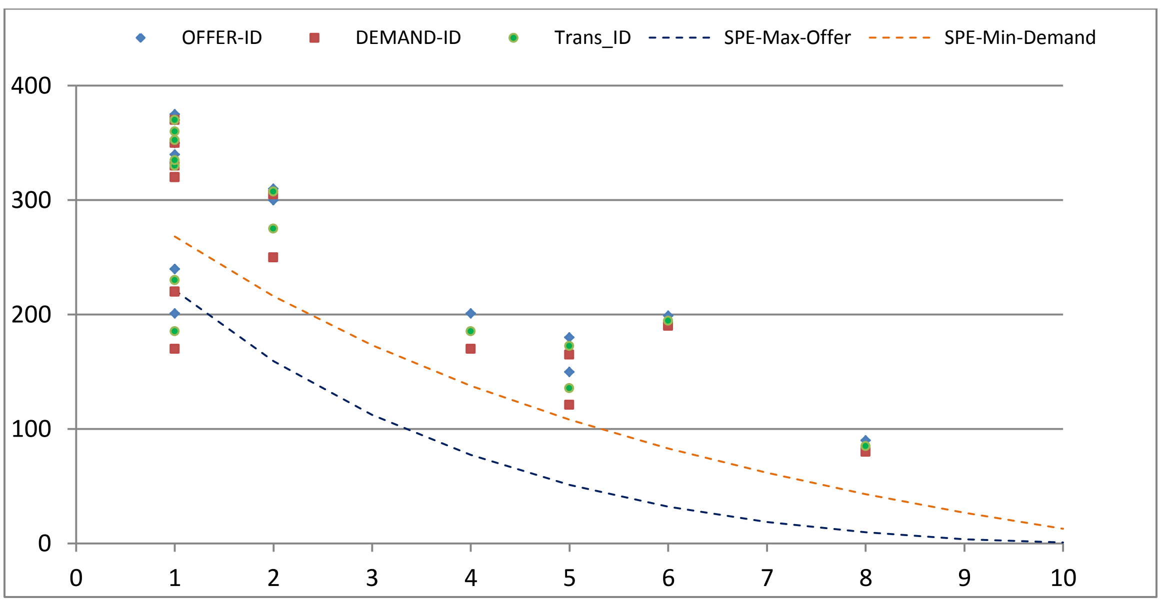


Figure 28.b OFFER, DEMAND and values of passed Transaction with identical users

Figure 28: OFFER, DEMAND and values of passed Transactions from the Buy-Out game

Finally, the data from unequal users shows a clear tendency to overbid by buyers and to overprice by sellers; figure 28.a shows that in most sessions where a transaction was passed, the DEMAND, OFFER and the transaction values, are higher, in some instances much higher, than the maximum offer following the SPE or even the Myopic path. For the seven transactions passed on round one for example, the average OFFER is 365.0 and the average DEMAND is 194.3 (See Table 19), when the maximum OFFER is respectively 120.97 following the SPE (Table 17), and 181.96 following the Myopic path.

This result might be attributed to the complexity of the experiment, and, maybe, the lack of clearness of some instructions; another explanation relates to some unfounded risk aversion behavior by subjects, where players would be willing to overbid, to have the exclusivity over the resource and not depend on other users decisions.

For the identical treatment, and contrarily to the theoretical predictions, the experimental data shows a high occurrence of buy-outs, transactions were indeed passed in a majority of sessions (79%). This result is related to the same tendency to overbid by buyers and overprice by sellers, figure 28.b shows that in 13 (out of 15) of transactions, the transaction value was higher than the maximum OFFER following the SPE. As a result, the actual cumulative profits are significantly higher for sellers than for the buyers:

Figure 29: Individual actual profits with identical users

The experimental data also shows that, just as predicted in theory, with identical users, games where a transaction was passed generate significantly lower aggregate profits than sessions where no transaction is approved; indeed the profits drop from an average of 588.4 computer dollars, for games with no approved transaction, to 446.1 in games with transaction.

5.4.5 Testing the Hypotheses

For results analysis I will only use the data from sessions of the buy-out experiment where the transaction occurred on the first round, this doesn't mean necessarily that the observations that will arise from the discussions do not apply to all the sessions. The motivation from this restriction is the obvious fact that the sessions where the transaction occurred later in the game or did not occur at all are not truly Single User games.

Table 20: Experimental average cumulative profits per 10-round game from all experiments

		Cumulative Profits for player *h*	Cumulative Profits for player *l*	Aggregate Cumulative Profits (*h*+*l*)	Observations
Buy-Out Experiment – Single User Sessions	Unequal users & Infinite Transmissivity ($a_h = 20$, $a_l = 10$, s = ½)	771.2	0.0	771.2	7
	Identical users & Infinite Transmissivity ($a_h = 15$, $a_l = 15$, s = 1/2)	447.3	0.0	447.3	8
Extraction Game Experiment	Unequal users & Infinite Transmissivity ($a_h = 20$, $a_l = 10$, s = ½)	690.2	49.2	739.4	41
	Identical users & Infinite Transmissivity ($a_h = 15$, $a_l = 15$, s = 1/2)	266.9	266.9	533.9	38
	Unequal users & Zero Transmissivity ($a_h = 20$, $a_l = 10$, s = 0)	581.6	111.9	693.6	47
	Identical users & Zero Transmissivity ($a_h = 15$, $a_l = 15$, s = 0)	291.4	291.4	582.8	39

Result SU1: *Inequality increases the -single user's- profits from the Common Pool Resource with an efficient single user.*

The data from the buy-out experiment with unequal users (Table 20) shows that the average cumulative profits of player *h* in the sessions where the transaction was passed on the first round (making player *h* the single user for the whole game) amount to 771.2. In sessions form the buy-out experiment with identical players where a transaction was passed on the first round, the cumulative profits for the buyer average to only 447.3.

With zero transmissivity ($s = 0$), the results (Table 19) show that with unequal users, the cumulative profits of player *h* as single user add up to 581.6, those for the identical single user come shorter, at 291.4.

Result SU2: *Transmissivity increases the profits from the Common Pool Resource under single user management.*

In the case of unequal users, with player *h* ($a_h = 20$) as single user, the aggregate profits increase from 581.6 in the extraction game experiment with zero-transmissivity, to 771.2 in the Single User sessions from the buy out experiment (with $s = ½$); at the same time, for identical users, the profits of the single user grow from 291.4 to 447.3.

Result SU3: *In the case with a high transmissivity ($s > 0.453$) and high costs of extraction* $\left(c > \frac{2}{-2+(3+\sqrt{2})\,s}\right)$, *a single user generates more profits than the aggregate profits of identical users in the non-cooperative extraction game.*

This result cannot be validated with the specifications for the 10-rounds game.

***Result SU4**: The effect of inequality on the difference between the profits under Single User management and the aggregate profits from the non-cooperative game has an overall inverted U-shaped pattern.*

Since I only use two levels of inequality, the experimental results cannot validate the nonlinear effect of inequality; however, given that one of the two treatments involves identical users, the data can be used to show the increasing effect of inequality.

For identical users, the difference between the buyer's cumulative profits in the "single user sessions" (sessions where transaction is passed on the first round) in the buy out experiments, 447.3, and the aggregate cumulative profits from the extraction game experiment, 533.9, is equal to –86.6.

For unequal users, the difference between the player h cumulative profits profits in the single user sessions in the buy out experiments, 771.2, and the aggregate cumulative profits from the extraction game experiment, 739.4, is 31.8, higher than –86.6. The difference is strictly positive, substantiating the experimental evidence of the prospect for net gains from the switch from non-cooperative to single user management, in the case of unequal users and infinite transmissivity, that is of course on the condition that player h is the one granted exclusive access to the resource. The difference is even stronger and more significant when comparing the profits from the single user sessions to those from the no trade sessions (sessions where no transaction was passed) from the buy-out experiment, 686.7.

The increasing effect of inequality can also be shown to hold with zero transmissivity (s = 0). From the extraction game experiment data (with s = 0), it can inferred that the loss from the shift from the non-cooperative extraction to the single user management will result in a loss in the amount of –291.4 with identical users and –111.9 in the case of unequal users with player *h* as single user.

Result SU5: *The derivative with regard to transmissivity of* $\Delta V^{s,SU-SPE}_{\bar{a}+\varepsilon_i,\bar{a}-\varepsilon_i}(x,x)$*, the difference between the profits under Single User management and the aggregate profits from the non-cooperative game, has an overall inverted U-shaped pattern.*

As discussed before, the data from the experiment can only check the increasing effect of inequality, since the derivative is strictly positive at the origin and can be negative at levels of inquality higher than the higher root (and lower than $\varepsilon^{\mathrm{Max}}_{\mathrm{SPE}}$).

First, for unequal appropriators, the data from the extraction game experiment with zero transmissivity shows that difference between the cumulative profits under single user management (for player *h*) and the aggregate cumulative profits, is –111.9 when s=0. The data on cumulative profits from the single user sessions from the buy-out experiment and the aggregate cumulative profits from the extraction game experiment provide the same difference with s=1/2 to be 31.8. The increase from the shift in the level of transmissivity is therefore 143.7.

Following the same approach, for identical users, the difference between the cumulative profits under single user management and the aggregate cumulative profits from non-cooperative use, increases from –291.4 with zero transmissivity to –88.6 (447.3 minus 533.9) with infinite transmissivity; the increase from the shift in the level of transmissivity is therefore 204.8.

The failure to check result SU5 can be attributed, in part, to the high gap between theoretical and experimental data in the case of Single User with unequal players (771.2

in lab versus 931.5 achievable or 83%) while for identical players the buyers' profits are much closer to the theoretical figures (447.3 in lab versus 489.4 achievable or 91%).

Result SU6*: The aggregate amount of water used by the two users under non-cooperative management is higher than the amount used by the Single User, but the savings in water use are decreasing in transmissivity.*

For the unequal users, the cumulative water use in single user sessions from buy-out experiment is 126.4 in average, while the aggregate cumulative use from the extraction game experiment with infinite transmissivity is 127.2 (98.0 for player *h* and 29.6 for player *l*); the savings average to only 1.2 with an infinite transmissivity. With zero transmissivity, the savings from the shift to Single user management are equal to the total use of player *l*, from the extraction game with zero transmissivity, 38.7 in average.

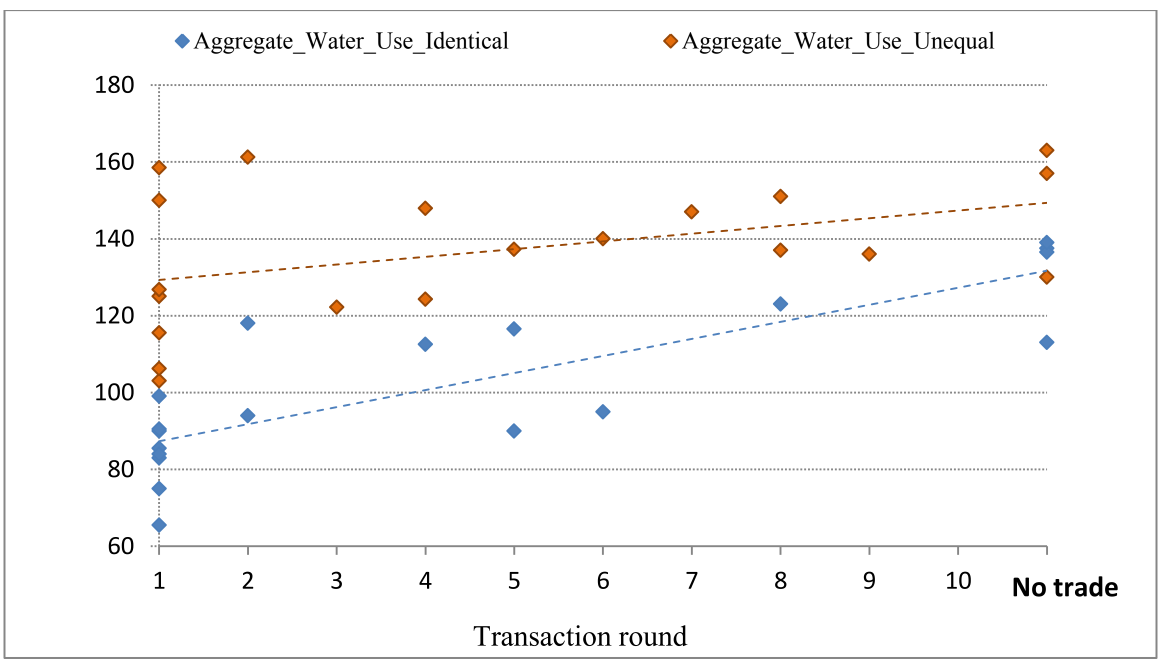


Figure 30: Aggregate water use per 10-round session in the buy out experiment

The same decreasing trend is observed with identical users, as the savings in aggregate water use decrease from 60.4 (total use by the identical user in the extraction game with *s*=0) with zero transmissivity, to 47.3 with an infinite transmissivity (twice the difference between 65.7, the average use identical players in the extraction game experiment with s=1/2, and 84.1 the total use in single user sessions in buy out experiments).

The data from buy-out experiments confirms (Figure 30) that water use is increasing in the transaction round, the tendency is even more significant with identical users.

5.5 Conclusion

This section is of particular importance to this research project, it aims to draw conclusions on the effect of inequality and transmissivity on the performance and implementability of alternative policies –to the non-cooperative use– for the management of CPRs.

The results, based on theoretical evidence and numerical examples show that inequality has a increasing effect on the aggregate profits from the social optimum; transmissivity has an increasing effect on the efficient user profits, and a decreasing but weaker effect on the less efficient player profits, the resultant effect of transmissivity on aggregate profits is therefore welfare increasing.

With regard to the difference between the profits following the social optimum and those generated under a non-cooperative regime, inequality was shown to have an increasing effect on the difference; the same observation holds for the effect of transmissivity when the solution to the social optimum maximization problem is interior; however, the effect is not uniform, indeed, the less efficient user may suffer losses, following the shift from noon-cooperative use to the social optimum, when the level of inequality is higher than $\varepsilon^{0}_{\text{SO-SPE}}$.

This result, somewhat counterintuitive, is very important; number of studies on CPRs management focused on the problem of communication (or lack of it), or the enforceability of agreements, as the main barriers to CPRs users' partaking in a socially optimal type of management that would maximize the aggregate (across users) profits from the CPR; the result related to the effect of transmissivity on the difference between the profits from the Social Optimum and those following the SPE suggests otherwise. Indeed, in the case where inequality between users is higher than a certain threshold, the switch from noon-cooperative use to the social optimum, results in net losses for the less efficient user, in such situation, policies that rely on voluntary participation of resource users to increase the aggregate profits from the CPR are no longer viable, unless they are accompanied by mechanisms for money transfers to compensate the less efficient users, and to ensure the participation of all users in the social optimum.

Finally, it is also shown that the stock(s) of water in the aquifer is best preserved under the social optimum; even as the very definition of social optimum in the model is formulated in terms of profits' maximizing, the solution favors resource conservation.

In the case of single user management, transmissivity has an obvious positive effect on the profits of the single user, and the same observation applies to the effect of inequality when the efficient user is the one granted exclusive use.

Comparing the profits from Single User management to the aggregate (by the two users) profits under non-cooperative management, shows that, for identical users, the difference is only positive in the very special case, where the aquifer is very transmissive and the unitary cost of extraction is very high. The simultaneous conditions on unitary cost and level of transmissivity, when observed, would lead to high financial costs associated with the intensified physical externalities in water extraction, to the point where the use of a single parcel by one of the identical users generates more profits than the aggregate profits of the two –identical– users following the SPE.

With unequal users, inequality has an inverted U-shaped effect on the difference in profits between the two regimes (Single User and SPE); the difference is increasing in the level of inequality, reaches a positive maximum, then decreases until it vanishes at inequality level ε_{SPE}^{Max}, beyond which the solution to the SPE is a corner solution, requiring the less efficient user to exit the extraction game; the SPE path merges with the Single User, with the efficient user (player h in the model) as sole user.

This result is important as it shows that the profits from (and implementability of) a "buy-out" water market are more marked for relatively moderate inequality levels, that correspond to higher –positive– gains from the shift in the mode of management.

Regarding the aquifer exploitation, as one might expect, the results show that Single User management leads to more conservation of the groundwater common pool.

In addition to the theoretical evidence and the numerical examples, a series of experiments were conducted to validate the aforementioned observations in the buy-out market. The results of the experiments carried out at the University of Maryland presented a somewhat mixed set of results.

On the one hand, buy-out transactions were frequently passed with identical users, when the theoretical predictions, following the strategic or myopic paths, suggest that buy-out transactions involve lower profits to the player staying in the game than the aggregate profits following the SPE, for all ten rounds. This theoretical insight was validated by the experimental data, since sessions with identical users where no transaction was passed generated significantly higher profits than sessions with transaction.

With unequal users, experimental results show that sessions where a buy-out transaction was reached are very frequent, and that the corresponding aggregate profits are higher than in sessions where no transaction was passed, the results are significant when considering only the sessions where transactions were passed on the first round.

Chapter 6 Conclusions and Future Work

6.1 Conclusions

The key lesson from this research project is to emphasize the importance of a thorough – prior–knowledge of the CPR's physical characteristics and the users' production technology for an accurate assessment of the present situation, in terms of revenues from the CPR use and related levels of depletion; this knowledge is even more critical in providing better visibility regarding the prospects and means to improve the CPRs' governance.

The complex situation that was considered in the study, where an aquifer with finite transmissivity is exploited by farmers with unequal efficiencies, presented an appropriate framework to revisit and even challenge some of the widely accepted beliefs in CPR literature.

The most striking result from the study relates to the case with highly unequal users, where well-enforced property rights, illustrated in the model with a zero-transmissivity resource, are no longer associated with higher profitability from the resource, even though, the increased profits from higher transmissivity come at a high cost, in even lower profits for the disadvantaged users and more depletion of groundwater stocks.

The model also highlighted another "perverse effect" of high inequality, as it shows that the social optimum solution is no longer associated with an advantageous outcome, compared to the non-cooperative outcome, for all users, and the best interest of less efficient users is not aligned with the community.

Those results are even more important as they indicate that some of the most promoted policies for the efficient governance of CPRs may have very limited success if implemented in the wrong setting.

For example, the model shows that a water market institution will have little impact if the users are somewhat alike in their production capabilities, that is, unless the financial externalities from the resource overexploitation are very high, but even then, the fact that both users can play either role, buyer or seller, can limit the success of the water market.

Another trend in water management is the “aquifer contract”, where all the aquifer users get together, with supervision from the water authority, to agree on a set of rules and agreed change or restraint in water use that would lessen the stress on the resource; the model shows that in the case of highly ineffective users, this approach might fail to win the level of approval among users, that is much needed for it to work.

6.2 Future Work

This work would feel incomplete if some important aspects are not fully included in the analysis; the model offers a good framework to investigate the importance that some parameters can have in shaping the users’ decisions and related outcomes, in profits and stock levels. One of the most significant parameters that deserves attention is certainly the rate of recharge, especially since groundwater exhaustion is more common in areas with limited water resources. The analysis of effects related to the cost of extraction can also be revealing on the real cost of government policies to support farmers.

Another addition to the present study would consist in addressing the obvious criticism to this type of models that consider users’ efficiency as an exogenous parameter, rather than an endogenous item of the model, contingent on the users’ previous decision to invest or not invest in improved technologies. The idea here is to consider the same two-stage approach used in Aggarwal and Narayan (2004), with players deciding on the type of technology in the first stage and on the level of use over time in the second stage. In Aggarwal and Narayan (2004), inequality has a U-shaped effect, decreasing welfare from aquifer use at first, but beginning at a certain threshold, more inequality increases welfare. It would be interesting to see if the theoretical results from the present model would lead to the same conclusions, and eventually, if experimental data would reproduce the theoretical results.

Appendix A Sufficiency Conditions

I reproduce here the maximization problem of player i:

$$\underset{w_{it}}{Max}\int_0^\infty e^{-rt}\left(a_i w_{it}-\frac{w_{it}^2}{2}-w_{it}c\left(E-x_{it}\right)\right)dt$$

Subject to: $\dot{x}_{it}=R-w_{it}+s\left(x_{jt}-x_{it}\right)$

$$\dot{x}_{jt}=R-w_{jt}+s\left(x_{it}-x_{jt}\right)$$

$$w_{it},w_{jt}\geq 0$$

$$x_{it},x_{jt}\leq h$$

x_{i0} and x_{j0} given.

and the corresponding Hamiltonian:

$$H_{it}=\left(a_i w_{it}-\frac{w_{it}^2}{2}-w_{it}c\left(h-x_{it}\right)\right)+\lambda_{it}\left(R-w_{it}+s(x_{jt}-x_{it})\right)+\mu_{it}\left(R-w_{jt}+s(x_{it}-x_{jt})\right)$$

This optimization problem satisfies the Mangasarian sufficiency conditions for an infinite horizon (Seierstad and Sydsaeter, 1987, Theorem 13, p. 234-5) since:

The Hamiltonian is linear in x_{it} and concave in w_{it}, therefore concave in (x_{it}, w_{it}).

The set of possible values for w_{it} (nonnegative) is convex (it can also be add that the set is bounded since any optimal choice needs to satisfy $w_{it}\leq a_i-c\left(h-x_{it}\right)$, given the condition on x_{it} it is possible to derive that $0\leq w_{it}\leq a_i$).

$\dot{x}_{it}$ is linear in w_{it} .

Appendix B Solving for the Continuous Model

B.1 Open Loop Nash Equilibrium

B.1.1 Deriving the Eigenvalues

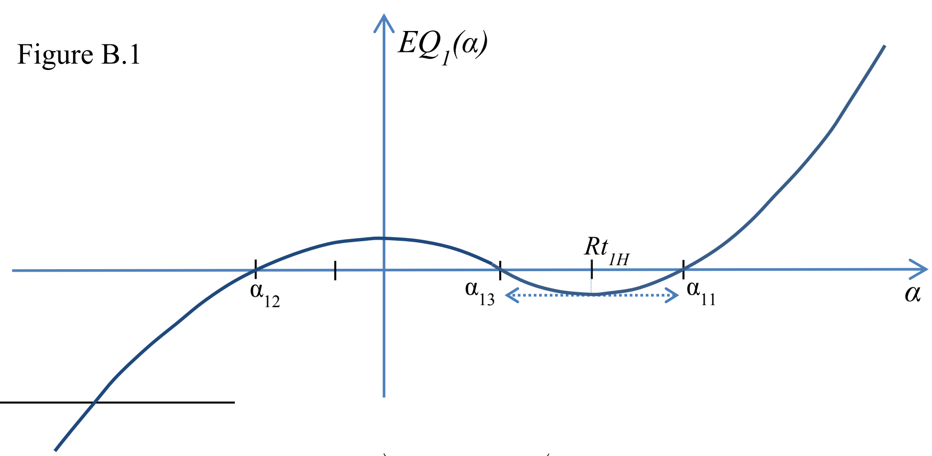
B.1.1.1 The Negative Eigenvalue of Equation *$EQ_1(\alpha)$*

$$EQ_1(\alpha) = \alpha^3 - 2(r+s)\alpha^2 + \left(-c(r+s) + r\,(r+2s)\right)\alpha + c\,r\,(r+2s)$$

$EQ_1(\alpha)$ is a degree three polynomial function of α therefore the equation $EQ_1(\alpha)$ has either one or three real roots. The coefficient for the term in α^3 (the unity) is strictly positive, consequently, the limit of $EQ_1(\alpha)$ as α approaches infinity is infinity and the limit is minus infinity as α approaches minus infinity.

$EQ_1{}'(\alpha)$, the derivative of $EQ_1(\alpha)$, has two roots noted Rt_{1L} and Rt_{1H} with $Rt_{1L}<Rt_{1H}$.[97] It is possible to show that $EQ_1(Rt_{1H})< 0$, given that Rt_{1H} is strictly positive and given the sign of $EQ_1(0)>0$, it is concluded that equation $EQ_1(\alpha)=0$ has three real roots, one negative and two positives.[98]

Figure B.1



[97] $Rt_{1L} = \frac{1}{3}\left(2r + 2s - \sqrt{r^2 + 2\,r\,s + 4\,s^2 + 3\,c\,(r+s)}\right)$ and $Rt_{1H} = \frac{1}{3}\left(2r + 2s + \sqrt{r^2 + 2\,r\,s + 4\,s^2 + 3\,c\,(r+s)}\right)$.

[98] Given the sign of the limit of $EQ_1(\alpha)$ at minus infinity and the sign of $EQ_1(0)$, there is at least one negative root; given the sign of $EQ_1(0)$ and the sign of $EQ_1(Rt_{1H})$ there is at least one root in the interval [0, Rt_{1H}] and given the sign of the limit of $EQ_1(\alpha)$ at infinity there is at least one root strictly higher than Rt_{1H}. Finally, knowing that the number of real roots cannot exceed three, it becomes clear that that there is exactly one root in every specified interval.

To derive the closed form solution of equation $EQ_1(\alpha)$=0, I start by rewriting $EQ_1(\alpha)$ as:

$$EQ_1(\alpha)=\left(\alpha - \frac{2\,(\mathrm{r}+\mathrm{s})}{3}\right)^3 - p_1\left(\alpha - \frac{2\,(\mathrm{r}+\mathrm{s})}{3}\right) - q_1,$$

Where: $p_1=\frac{1}{3}\left(3c\,r+r^2+3c\,s+2r\,s+4\,s^2\right)$, and

$$q_1=\frac{1}{27}\left(\,2\,\mathrm{s}^2\left(9\,\mathrm{c}+6\,r+8\,\mathrm{s}\right)-r\left(9\,\mathrm{c}\left(\mathrm{r}+2\,\mathrm{s}\right)+2\,\mathrm{r}\left(r+3\,\mathrm{s}\right)\right)\right).$$

I operate a first change in variable and continue with the new equation:

$$EQ_{11}(\alpha)=\alpha^3-p_1\,\alpha-q_1.$$

I proceed with a second change of variable by rewriting: $\alpha=w+\frac{p_1}{3w}$ to obtain:

$$EQ_{11}(\alpha)=\frac{1}{w^3}\left(w^6-q_1\,w^3+\frac{p_1^{\;3}}{27}\right).$$

If I note $w_3 = w^3$ then α is solution to $EQ_{11}(\alpha)=0$ if and only if the corresponding w_3 is solution to $w_3^{\;2}-q_1\,w_3+\frac{p_1^{\;3}}{27}=0$ or equivalently: $\left(w_3-\frac{q_1}{2}\right)^2=-\frac{p_1^{\;3}}{27}+\frac{q_1^{\;2}}{4}$.

Given that $-\frac{p_1^{\;3}}{27}+\frac{q_1^{\;2}}{4}<0$ I derive two complex solutions for w_3: $w_{31}=\frac{q_1}{2}+i\sqrt{\frac{p_1^{\;3}}{27}-\frac{q_1^{\;2}}{4}}$ and $w_{31'}=\frac{q_1}{2}-i\sqrt{\frac{p_1^{\;3}}{27}-\frac{q_1^{\;2}}{4}}$

From w_{31} I derive the three solutions[99] to the original equation $EQ_1(\alpha)$=0:

$$\alpha_{11}=\frac{2(r+s)}{3}+\left(\frac{q_1}{2}+i\sqrt{\frac{p_1^{\;3}}{27}-\frac{q_1^{\;2}}{4}}\right)^{1/3}+\frac{p/3}{\left(\frac{q_1}{2}+i\sqrt{\frac{p_1^{\;3}}{27}-\frac{q_1^{\;2}}{4}}\right)^{1/3}}$$

[99] These are the solution corresponding to $(w_{31})^{1/3}$, $(w_{31})^{1/3}e^{i2\pi/3}$ and $(w_{31})^{1/3}e^{i4\pi/3}$, the 3 solutions to $(w)^3=w_{31}$.

$$\alpha_{12} = \frac{2(r+s)}{3} + \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{q_1}{2} + i\sqrt{\frac{p_1^{\,3}}{27} - \frac{q_1^{\,2}}{4}}\right)^{1/3} + \frac{p/3}{\left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{q_1}{2} + i\sqrt{\frac{p_1^{\,3}}{27} - \frac{q_1^{\,2}}{4}}\right)^{1/3}}$$

$$\alpha_{13} = \frac{2(r+s)}{3} + \left(\frac{-1-i\sqrt{3}}{2}\right)\left(\frac{q_1}{2} + i\sqrt{\frac{p_1^{\,3}}{27} - \frac{q_1^{\,2}}{4}}\right)^{1/3} + \frac{p/3}{\left(\frac{-1-i\sqrt{3}}{2}\right)\left(\frac{q_1}{2} + i\sqrt{\frac{p_1^{\,3}}{27} - \frac{q_1^{\,2}}{4}}\right)^{1/3}}$$

w_{31} can be rewritten as $\left(\frac{p_1^{\,3}}{27}\right)^{1/2} e^{i\theta_1}$, with θ_1 given as $\theta_1 = ArcCos\left(\frac{q_1/2}{\sqrt{p_1^{\,3}/27}}\right)$, where ArcCos refers to the inverse function of cosine; and the three solutions become:[100]

$$\alpha_{11} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{\theta_1}{3}\right),$$

$$\alpha_{12} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{\theta_1+2\pi}{3}\right), \text{ and}$$

$$\alpha_{13} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{\theta_1+4\pi}{3}\right)$$

$w_{31'}$ can be rewritten as $\left(\frac{p_1^{\,3}}{27}\right)^{1/2} e^{i(2\pi-\theta_1)}$ and the three solutions corresponding to $w_{31'}$ are shown to be a duplication of the three previous solutions:

$$\alpha_{11'} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{2\pi-\theta_1}{3}\right) = \alpha_{13},$$

$$\alpha_{12'} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{4\pi-\theta_1}{3}\right) = \alpha_{12} \text{ , and}$$

$$\alpha_{13'} = \frac{2(r+s)}{3} + 2\left(\frac{p_1}{3}\right)^{1/2}\cos\left(\frac{6\pi-\theta_1}{3}\right) = \alpha_{11}.$$

I will now show that α_{12} is the negative root of equation $EQ_1(\alpha)=0$.

---

[100] Notice that the three roots are real.

Case1: $q_1 < 0$ (roughly for s small)

$$\theta_1 \in \left[\frac{\pi}{2}, \pi\right] \Rightarrow \begin{cases} \dfrac{\theta_1}{3} \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right] \\ \dfrac{\theta_1 + 2\pi}{3} \in \left[\dfrac{5\pi}{6}, \pi\right] \\ \dfrac{\theta_1 + 4\pi}{3} \in \left[\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right] \end{cases} \Rightarrow \begin{cases} \cos\left(\dfrac{\theta_1}{3}\right) > 0 \\ \cos\left(\dfrac{\theta_1 + 2\pi}{3}\right) < 0 \\ \cos\left(\dfrac{\theta_1 + 4\pi}{3}\right) > 0 \end{cases}$$

In this case it is clear that α_{11} and α_{13} are both strictly positive, and α_{12} is therefore the only negative root.

Case2: $q_1 > 0$ (s high)

$$\theta_1 \in \left[0, \frac{\pi}{2}\right] \Rightarrow \begin{cases} \dfrac{\theta_1}{3} \in \left[0, \dfrac{\pi}{6}\right] \\ \dfrac{\theta_1 + 2\pi}{3} \in \left[\dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right] \\ \dfrac{\theta_1 + 4\pi}{3} \in \left[\dfrac{4\pi}{3}, \dfrac{3\pi}{2}\right] \end{cases} \Rightarrow \begin{cases} \cos\left(\dfrac{\theta_1}{3}\right) > 0 \\ \cos\left(\dfrac{\theta_1 + 2\pi}{3}\right) < 0 \\ \cos\left(\dfrac{\theta_1 + 4\pi}{3}\right) < 0 \end{cases}$$

In such case α_{11} is strictly positive. Also, $\frac{-\sqrt{3}}{2} < \cos\left(\frac{\theta_1 + 2\pi}{3}\right) < \frac{-1}{2}$ and $\frac{-1}{2} < \cos\left(\frac{\theta_1 + 4\pi}{3}\right) < 0$, therefore $\alpha_{12} < \alpha_{13}$ and α_{12} is the negative root.

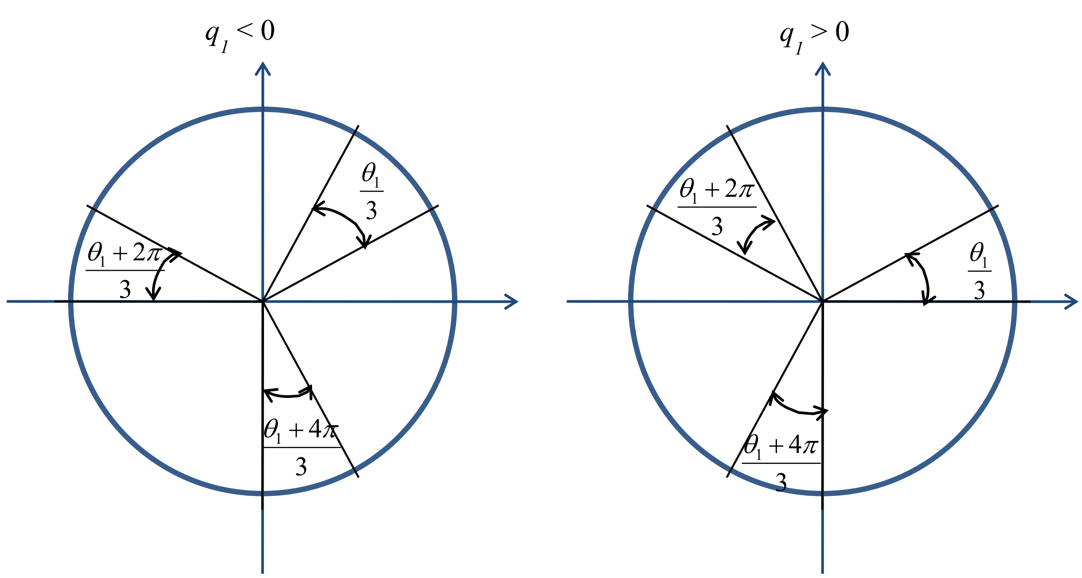


Figure B.2

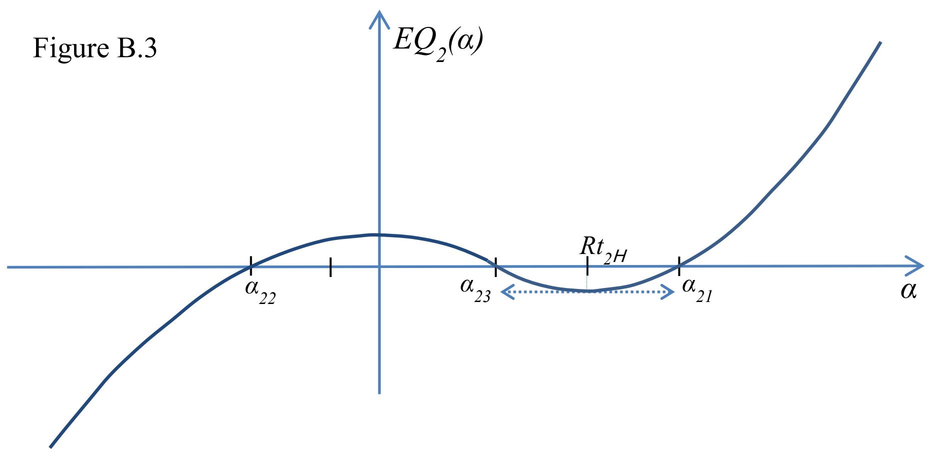
B.1.1.2 The Negative Eigenvalue of Equation $EQ_2(\alpha)$

$$EQ_2(\alpha) = \alpha^3 - 2r\alpha^2 + \left(r^2 - 2rs - 4s^2 - c(r+3s)\right)\alpha + 2rs(r+2s) + c\left(r^2 + 4rs + 2s^2\right)$$

Similarly to the previous analysis for $EQ_1(\alpha)$=0, equation $EQ_2(\alpha)$=0 has either one or three real roots. I also observe the same trends at infinity and minus infinity.

$EQ_2'(\alpha)$=0 has two roots noted Rt_{2L} and Rt_{2H} with Rt_{2L}<Rt_{2H}.[101] I show that Rt_{2H}>0, while $EQ_2(Rt_{2H})$<0. Since $EQ_2(0)$>0 I can conclude that equation $EQ_2(\alpha)$=0 has three real roots, one negative and two positives.

Figure B.3



To derive the closed form solution for the negative root of equation $EQ_2(\alpha)$=0, I start by rewriting $EQ_2(\alpha)$ as:

$$EQ_2(\alpha) = \left(\alpha - \frac{2r}{3}\right)^3 - p_2\left(\alpha - \frac{2r}{3}\right) - q_2,$$

With $p_2 = \frac{1}{3}\left(r^2 + 6rs + 12s^2 + 3c(r+3s)\right)$ and $q_2 = -\frac{1}{27}\left(2r(r+3s)(r+6s) + 9c\left(r^2 + 6rs + 6s^2\right)\right)$.

---

[101] $Rt_{2L} = \frac{1}{3}\left(2r - \sqrt{r^2 + 6rs + 12s^2 + 3c(r+3s)}\right)$ and $Rt_{2H} = \frac{1}{3}\left(2r + \sqrt{r^2 + 6rs + 12s^2 + 3c(r+3s)}\right)$.

I operate a first change in variable and continue with the equation:

$$EQ_{22}(\alpha) = \alpha^3 - p_2\,\alpha - q_2\,.$$

Similarly to the solution for $EQ_{11}(\alpha)$=0, using the change of variable $\alpha = w + \frac{p_2}{3\,w}$ allows to rewrite $EQ_{22}(\alpha)$ as: $EQ_{22}(\alpha) = \frac{1}{w^3}\left(w^6 + q_2\,w^3 + \frac{p_2^{\ 3}}{27}\right)$.

α is solution to $EQ_{22}(\alpha)=0$ if and only if the corresponding w_3, defined as $w_3 = w^3$, is solution to $w_3^{\ 2} - q_2\,w_3 + \frac{p_2^{\ 3}}{27} = 0$ or equivalently: $\left(w_3 - \frac{q_2}{2}\right)^2 = -\frac{p_2^{\ 3}}{27} + \frac{q_2^{\ 2}}{4}$.

Given that $-\frac{p_2^{\ 3}}{27} + \frac{q_2^{\ 2}}{4} < 0$ the two solutions for w_3 are given as:

$$w_{32} = \frac{q_2}{2} + i\sqrt{\frac{p_2^{\ 3}}{27} - \frac{q_2^{\ 2}}{4}} \text{ and } w_{32'} = \frac{q_2}{2} - i\sqrt{\frac{p_2^{\ 3}}{27} - \frac{q_2^{\ 2}}{4}}$$

From w_{32} I derive the three solutions to equation $EQ_2(\alpha)$=0 as follows:

$$\alpha_{21} = \frac{2r}{3} + \left(w_{32}\right)^{1/3} + \frac{p_2/3}{\left(w_{32}\right)^{1/3}}$$

$$\alpha_{22} = \frac{2r}{3} + \left(\frac{-1+i\sqrt{3}}{2}\right)\left(w_{32}\right)^{1/3} + \frac{p_2/3}{\left(\frac{-1+i\sqrt{3}}{2}\right)\left(w_{32}\right)^{1/3}}$$

$$\alpha_{23} = \frac{2r}{3} + \left(\frac{-1-i\sqrt{3}}{2}\right)\left(w_{32}\right)^{1/3} + \frac{p_2/3}{\left(\frac{-1-i\sqrt{3}}{2}\right)\left(w_{32}\right)^{1/3}}$$

w_{32} can be rewritten as $\left(\frac{p_2{}^3}{27}\right)^{1/2} e^{i\theta_2}$, with θ_2 given as $\theta_2 = ArcCos\left(\frac{q_2/2}{\sqrt{p_2{}^3/27}}\right)$; and the three –real– solutions become:

$$\alpha_{21} = \frac{2(r+s)}{3} + 2\left(\frac{p_2}{3}\right)^{1/2}\cos\left(\frac{\theta_2}{3}\right),$$

$$\alpha_{22} = \frac{2(r+s)}{3} + 2\left(\frac{p_2}{3}\right)^{1/2}\cos\left(\frac{\theta_2+2\pi}{3}\right), \text{ and}$$

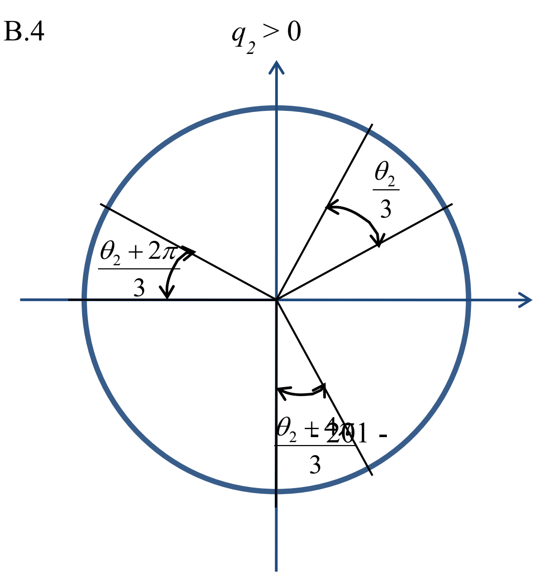
$$\alpha_{23} = \frac{2(r+s)}{3} + 2\left(\frac{p_2}{3}\right)^{1/2}\cos\left(\frac{\theta_2+4\pi}{3}\right)$$

Similarly, $w_{32'}$ is rewritten as $\left(\frac{p_2{}^3}{27}\right)^{1/2} e^{i(2\pi-\theta_2)}$ and the three solutions corresponding to $w_{31'}$ can be shown to be a replication of the previous solutions from w_{32}.

To show that α_{22} is always the negative root of equation $EQ_2(\alpha)$=0 it suffice to notice that q_2 is negative, and conclude that α_{21} and α_{23} are strictly positive.

$$\theta_2 \in \left[\frac{\pi}{2}, \pi\right] \Rightarrow \begin{cases} \frac{\theta_2}{3} \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right] \\ \frac{\theta_2+2\pi}{3} \in \left[\frac{5\pi}{6}, \pi\right] \\ \frac{\theta_2+4\pi}{3} \in \left[\frac{3\pi}{2}, \frac{5\pi}{3}\right] \end{cases} \Rightarrow \begin{cases} \cos\left(\frac{\theta_2}{3}\right) > 0 \\ \cos\left(\frac{\theta_2+2\pi}{3}\right) < 0 \\ \cos\left(\frac{\theta_2+4\pi}{3}\right) > 0 \end{cases}$$

Figure B.4



B.1.2 Deriving the Eigenvectors

If V = [y_1, y_2, y_3, y_4, y_5, y_6] is the eigenvector associated with α, eigenvalue of matrix A, then V needs to satisfy the condition AV = αV, which can be written as a system of six linear equations:

$$-(c+s)\,y_1 + s\,y_2 + y_3 = \alpha\,y_1 \tag{B.1}$$

$$s\,y_1 - (c+s)\,y_2 + y_4 = \alpha\,y_2 \tag{B.2}$$

$$-c^2\,y_1 + (r+s+c)\,y_3 - s\,y_5 = \alpha\,y_3 \tag{B.3}$$

$$-c^2\,y_2 + (r+s+c)\,y_4 - s\,y_6 = \alpha\,y_4 \tag{B.4}$$

$$-s\,y_3 + (r+s)y_5 = \alpha\,y_5 \tag{B.5}$$

$$-s\,y_4 + (r+s)y_6 = \alpha\,y_6 \tag{B.6}$$

The values of y_3 and y_4, as functions of y_5 and y_6, are drawn from (B.5) and (B.6):

$$s\,y_3 = (r+s-\alpha)y_5 \tag{B.5)'}$$

$$s\,y_4 = (r+s-\alpha)\,y_6 \tag{B.6)'}$$

Multiplying (B.1) (B.2) (B.3) and (B.4) by s then replacing for $s\,y_3$ and $s\,y_4$ from (B.5)' and (B.6)' gives the new system (B.1)' – (B.4)':

$$-s(c+s)\,y_1 + s^2\,y_2 + (r+s-\alpha)y_5 = s\,\alpha\,y_1 \tag{B.1)'}$$

$$s^2\,y_1 - s(c+s)\,y_2 + (r+s-\alpha)y_6 = s\,\alpha\,y_2 \tag{B.2)'}$$

$$-s\,c^2\,y_1 \quad s+c)(r+s-\alpha)y_5 - s^2\,y_5 = \alpha(r+s-\alpha)y_5 \tag{B.3)'}$$

$$-s\,c^2\,y_2 + (r+s+c)(r+s-\alpha)y_6 - s^2\,y_6 = \alpha(r+s-\alpha)y_6 \tag{B.4)'}$$

Equations (B.3)' and (B.4)' can be further simplified as:

$$s\,c^2\,y_1 = \delta\,y_5 \tag{B.3)''}$$

$$s\,c^2\,y_2 = \delta\,y_6 \tag{B.4)''}$$

where $\delta = \left[(r+s+c-\alpha)(r+s-\alpha)-s^2\right]$

Multiplying (B.1)’ and (B.2)’ by c^2 and using the last results from (B.3)” and (B.4)” yields:

$$-(c+s)\delta y_5 \quad + s\delta y_6 \quad + (r+s-\alpha)c^2 y_5 - \alpha\delta y_5 \qquad = 0 \qquad \text{(B.1)”}$$

$$s\delta y_5 \quad -(c+s)\delta y_6 \quad + (r+s-\alpha)c^2 y_6 \ - \alpha\delta y_6 \ = 0 \qquad \text{(B.2)”}$$

Adding and subtracting (B.1)” and (B.2)” provides an equivalent system of equations:

$$EQ_1(\alpha)\,(y_5 + y_6) \quad = 0 \qquad \text{(B.1)””}$$

$$EQ_2(\alpha)\,(y_5 - y_6) \quad = 0 \qquad \text{(B.2)””}$$

I differentiate two cases; in the first case, α is the negative root of equation $EQ_1(\alpha) = 0$ and equation (B.1)”” is valid for any pair of variables, while the second equality (B.2”’) is only true if $y_5 = y_6$. This result, together with the previous results in (B.5)’, (B.6)’, (B.3)” and (B.4)” shows that the eigenvector associated with α_1 is any multiple of v_1:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ c+\alpha_1 \\ c+\alpha_1 \\ \dfrac{sc^2}{(r+s+c-\alpha_1)(r+s-\alpha_1)-s^2} \\ \dfrac{sc^2}{(r+s+c-\alpha_1)(r+s-\alpha_1)-s^2} \end{bmatrix}$$

For the case with $\alpha = \alpha_2$, the negative root of equation $EQ_2(\alpha) = 0$, (B.1)”” is only true if $y_6 = -y_5$. Compiling all results shows that the eigenvector in this instance is v_2:

$$v_2 = \begin{bmatrix} 1 \\ -1 \\ c+2s+\alpha_2 \\ -(c+2s+\alpha_2) \\ \dfrac{s\,c^2}{(r+s+c-\alpha_2)(r+s-\alpha_2)-s^2} \\ \dfrac{-s\,c^2}{(r+s+c-\alpha_2)(r+s-\alpha_2)-s^2} \end{bmatrix}$$

B.1.3 The Steady State following the Open Loop path

I rewrite the conditions at the steady state in a Matricial form as follows:

$$A_{ss}\,V_{ss} + B_{ss} = 0$$

Where:

$$A_{ss} = \begin{bmatrix} -s & s & 0 & 0 & 0 & 0 & -1 & 0 \\ s & -s & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & r+s & 0 & -s & 0 & -c & 0 \\ 0 & 0 & 0 & r+s & 0 & -s & 0 & -c \\ 0 & 0 & -s & 0 & r+s & 0 & 0 & 0 \\ 0 & 0 & 0 & -s & 0 & r+s & 0 & 0 \\ c & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & c & 0 & -1 & 0 & 0 & 0 & -1 \end{bmatrix},\ V_{ss} = \begin{bmatrix} x_{it} \\ x_{jt} \\ \lambda_{it} \\ \lambda_{jt} \\ \mu_{it} \\ \mu_{jt} \\ w_{it} \\ w_{jt} \end{bmatrix} \text{ and } B_{ss} = \begin{bmatrix} R \\ R \\ 0 \\ 0 \\ 0 \\ 0 \\ a_i - c\,h \\ a_j - c\,h \end{bmatrix}.$$

The solution is given by:

$$V_{ss} = A_{ss}^{-1}\,(-B_{ss})$$

where A_{ss}^{-1} is the inverse of matrix A_{ss}.

The solution, after simplification, is as follows:

$$
\begin{cases}
x_{it}^{SS} = h - \dfrac{(a_i + a_{-i})}{2c} + R\left(\dfrac{1}{c} + \dfrac{1}{2r} + \dfrac{1}{2r+4s}\right) - \dfrac{(a_i - a_{-i})}{2} \dfrac{r\,(r+2s)}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
x_{-it}^{SS} = h - \dfrac{(a_i + a_{-i})}{2c} + R\left(\dfrac{1}{c} + \dfrac{1}{2r} + \dfrac{1}{2r+4s}\right) + \dfrac{(a_i - a_{-i})}{2} \dfrac{r\,(r+2s)}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
\lambda_{it}^{SS} = \dfrac{cR\,(r+s)}{r\,(r+2s)} + (a_i - a_{-i}) \dfrac{cs\,(r+s)}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
\lambda_{-it}^{SS} = \dfrac{cR\,(r+s)}{r\,(r+2s)} - (a_i - a_{-i}) \dfrac{cs\,(r+s)}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
\mu_{it}^{SS} = \dfrac{cR\,s}{r\,(r+2s)} + (a_i - a_{-i}) \dfrac{cs^2}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
\mu_{-it}^{SS} = \dfrac{cR\,s}{r\,(r+2s)} - (a_i - a_{-i}) \dfrac{cs^2}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
w_{it}^{SS} = R + (a_i - a_{-i})\,s \dfrac{r\,(r+2s)}{2rs(r+2s)+c(r^2+4rs+2s^2)} \\
w_{-it}^{SS} = R - (a_i - a_{-i})\,s \dfrac{r\,(r+2s)}{2rs(r+2s)+c(r^2+4rs+2s^2)}
\end{cases}
$$

B.1.4 Showing the positive sign of $P_1(s)$ and $P_2(s)$

B.1.4.1 The positive sign of $P_1(s)$

To show the positive sign of $P_1(s)$ I start by rewriting the numerator as the product of $-\alpha_2(r+2s)^2$, and $P_{F1N}(\alpha_2)$, where $P_{F1N}(\alpha)$ is a function of α, defined as:

$$P_{F1N}(\alpha) = \left(2cr\left(r^2+4rs+2s^2\right) - 2c(r+2s)^2\alpha + r(r-\alpha)\left(4s(r+2s)+r\alpha\right)\right)$$

Both the denominator and $-\alpha_2(r+2s)^2$ are –always– positive, leading to the conclusion that $P_1(s)$ has the same sign as $P_{F1N}(\alpha_2)$.

$P_{F1N}(\alpha)$ has two roots, a negative root noted α_{R2N}, and a positive root, α_{R2P} .given by:

$$\alpha_{R2P} = r\left(\frac{\zeta_{R2.1}}{\zeta_{R2.2}}\right), \text{ and } \alpha_{R2N} = -\left(\frac{2\zeta_{R2.3}}{r^2}\right)\left(\frac{\zeta_{R2.2}}{\zeta_{R2.1}}\right),$$

where ζ_{2sq}, $\zeta_{R2.1}$, $\zeta_{R2.2}$, and $\zeta_{R2.3}$ are all positive quantities as shown below:

$$\zeta_{2sq} = 4c^4(r+2s)^4 + r^2(r^2+4rs+8s^2)^2 + 4cr(r^4+8r^3s+24r^2s^2+48rs^3+32s^4),$$

$$\zeta_{R2.1} = 2cr^2+r^3+8crs+4r^2s+8rs^2+\sqrt{\zeta_{2sq}}\ ,$$

$$\zeta_{R2.2} = r^3+4r^2s+8rs^2+2c(r+2s)^2+\sqrt{\zeta_{2sq}}\ \text{, and}$$

$$\zeta_{R2.3} = 2rs(r+2s)+c(r^2+4rs+2s^2).$$

As displayed in Figure B.5, $P_{F1N}(\alpha)$ holds a positive sign between the two roots and is negative everywhere;[102] therefore, to show the positive sign of $P_{F1N}(\alpha_2)$, it suffices to show that α_2 is higher than the negative root, α_{R2N}.[103]

The proof will conducted in two steps, first I show that α_{R2N} is lower than a given negative variable, $(-4s-c)$, then I show that $(-4s-c)$ is lower than α_2.

Indeed, the difference between α_{R2N} and $(-4s-c)$, is always negative:[104]

$$\alpha_{R2N} - (-4s-c) = -\left(\frac{\zeta_{R2.6}\,\zeta_{R2.4}}{2r^2\ \zeta_{R25}}\right)$$

Where $\zeta_{2.4}$, $\zeta_{R2.5}$, and $\zeta_{R2.6}$ are all positive quantities:

$$\zeta_{R2.4} = r^3+4r(2c+r)s+8(c+r)s^2+\sqrt{\zeta_{2sq}}$$

---

[102] To reach this conclusion it suffices to notice that the function has a negative limit at infinity and at minus infinity; in addition, it takes a positive value at the origin: $P_{F1N}(0) = 2\mathrm{r}(2\mathrm{rs}(\mathrm{r}+2\mathrm{s})+\mathrm{c}(\mathrm{r}^2+4\mathrm{rs}+2\mathrm{s}^2))$.

[103] α_2 is negative, thus lower than the positive root α_{R2P}.

[104] An alternative way would be to remark that $P_{F1N}(-4s-c)$ is positive:

$$P_{F1N}(-4s-c) = cr^2(c+r)+4cr(2c+3r)s+4(2c^2+11cr+2r^2)s^2+32(c+r)s^3$$

$$\zeta_{R2.5} = r^2\left(2c^2+2cr+r^2\right)+8r\left(2c^2+2cr+r^2\right)s+8\left(2c^2+6cr+3r^2\right)s^2+32(c+r)s^3 \\ +(r+4s)\sqrt{\zeta_{2sq}}$$

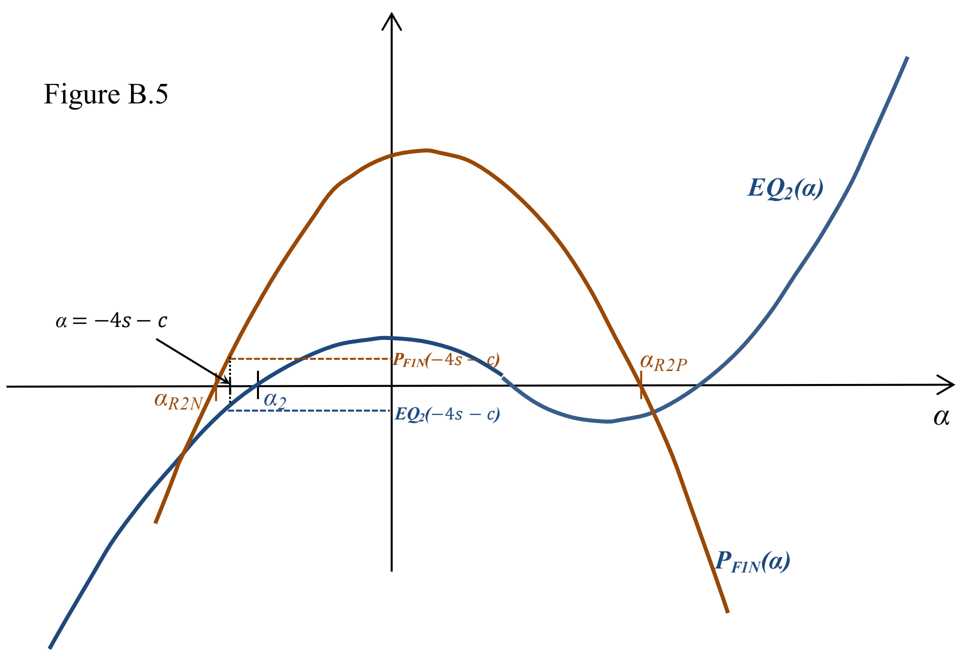
$$\zeta_{R2.6} = 2\left(cr^2\,(c+r)+4cr\,(2c+3r)s+4\left(2c^2+11cr+2r^2\right)s^2+32(c+r)\,s^3\right)$$

To show that that $(-4s-c)$ is lower than α_2, I compute $EQ_2\,(-4s-c)$:

$$EQ_2(-4s-c) = -\left(\left(c^3+6cs(r+5s)\right)+2s\,(r+4s)(r+6s)+c^2\,(r+9s)\right).$$

Since for the negative values of α, $EQ_2(\alpha)$ is only positive between α_2 and 0, the negative sign of $EQ_2\,(-4s-c)$, is evidence that $(-4s-c)$ is lower than α_2.

Figure B.5



B.1.4.2 The positive sign of $P_2(s)$

The denominator of $P_2(s)$ is always positive, while the numerator of $P_2(s)$ can be rewritten as the sum of two functions of α_2, $P_{F2P}(\alpha_2)$ and $P_{F2N}(\alpha_2)$, defined as follows:

$$P_{F2P}(\alpha) = r(r+2s)(-2\alpha_1 s(r-\alpha_1)-c\alpha(r+2s)) + c\left(r^2+4rs+2s^2\right)(-\alpha_1)(r-\alpha_1-\alpha)$$

$$P_{F2N}(\alpha) = r(-\alpha_1)\alpha(r+2s)(r-\alpha_1)$$

To show the positive sign of $P_2(s)$, I start by showing that α_2 is higher than $(-c-2s)$, as evidenced by the negative sign of $EQ_2\ (-c-2s)$:

$$EQ_2(-c-2s) = -c^2(c+r+3s).$$

This result, combined to the fact that $P_{F2N}(\alpha)$ is increasing in α –given the negative sign of α_1–, leads to the result:

$$P_{F2P}(\alpha_2) + P_{F2N}(\alpha_2) \ > \ P_{F2P}(\alpha_2) + P_{F2N}(-c-2s)$$

This last inequality simplifies as I replace for the term on the right to:

$$\begin{aligned} P_{F2P}(\alpha_2) + P_{F2N}(\alpha_2) \ > \ & 2cs(r+2s)(-\alpha_1)(r-\alpha_1) + rc(r+2s)^2(-\alpha_2) \\ & + c\left(r^2+4rs+2s^2\right)(\alpha_1\,\alpha_2) \end{aligned}$$

All three terms on the right side are positive and so is $P_2(s)$.

B.1.5 Analytical Proofs of Preliminary Results

B.1.5.1 Analytical Proof of Result OL1

To establish the negative effect of an increase of transmissivity on the aggregate cumulative profits of identical users I will consider the case of a non-incremental shift in transmissivity, s, from zero to infinity. The effect of an incremental increase in transmissivity is particularly difficult to analyze, given that α_1 and α_2 are complex functions of s.

For simplicity, I note $V^{OL}(a,a,x_0,\infty)$, the limit at infinity of aggregate cumulative profits; and the effect on aggregate cumulative profits, with interior solutions,[105] is computed as:

$$V^{OL}(a,a,x_0,\infty)-V^{OL}(a,a,x_0,0) = \frac{-1}{8(3c+r)}\left(PQ_1 + PQ_2\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right) + PQ_3\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)^2\right)$$

where PQ_1, PQ_2 and PQ_3 are all positive:

$$PQ_1 = \frac{16c^2\,(3c+r)(10c+3r)R^2}{r^2\left(47c^2+52cr+12r^2+(17c+6r)\sqrt{c^2+12cr+4r^2}\right)}$$

$$PQ_2 = \frac{32c^3\,(3c+r)\,R}{r\left(11c^2+16cr+4r^2+(5c+2r)\sqrt{c^2+12cr+4r^2}\right)}$$

$$PQ_3 = \frac{32c^4\,(3c+r)}{\left(2c+r+\sqrt{4cr+r^2}\right)\left((12c+4r)\sqrt{4cr+r^2}+c^2+(7c+2r)\sqrt{c^2+12cr+4r^2}\right)}$$

To demonstrate the negative effect of transmissivity with identical users I will first show that the effect is negative at the lowest and higher acceptable levels of initial stock, that satisfy the conditions for an interior solution; I will then show that the difference is decreasing in the initial level of stock.

At the minimum level of stock x_{Min0},[106] the effect simplifies as:

[105] I only conduct the calculations when the –most stringent- conditions for an interior solution are satisfied for both levels of transmissivity, this does not mean necessarily that the conclusions cannot be extended to the cases with corner solutions.

$$V^{OL}(a,a,x_{Min0},\infty)-V^{OL}(a,a,x_{Min0},0) = \frac{R^2\left(PQ_4-PQ_5\right)}{8cr^2\left(c+\sqrt{c^2+12cr+4r^2}\right)\left(c+2r+\sqrt{c^2+12cr+4r^2}\right)}$$

with PQ_4 and PQ_5 both positive:

$$PQ_4 = 24cr^3+8r^4+2\left(c^3+7c^2r+20cr^2+4r^3\right)\sqrt{4cr+r^2}+4c^2r\sqrt{c^2+12cr+4r^2}$$
$$+2c(c+r)\sqrt{4cr+r^2}\sqrt{c^2+12cr+4r^2}$$

$$PQ_5 = 5c^4+26c^3r+46c^2r^2+\left(5c^3+22cr^2+4r^3+4r^2\sqrt{4cr+r^2}\right)\sqrt{c^2+12cr+4r^2}$$

The difference between PQ_4 and PQ_5 is always negative,[107] implying a negative effect of a non-incremental change in transmissivity.

When the level of stock at the start of the extraction game is at the highest level, E, the difference becomes:

$$V^{OL}(a,a,E,\infty)-V^{OL}(a,a,E,0) = \frac{-1}{8(3c+r)}\left(PQ_1+PQ_2\left(\frac{a}{c}-\left(\frac{1}{c}+\frac{1}{r}\right)R\right)+PQ_3\left(\frac{a}{c}-\left(\frac{1}{c}+\frac{1}{r}\right)R\right)^2\right)$$

The effect is negative since $\left(a/c-\left(R/c+R/r\right)\right)$ is positive.

The derivative of the difference in aggregate cumulative profits from a non-incremental increase in transmissivity, with regard to the initial level of stock x_0, noted $\Delta Vx^{OL}(a,x_0)$, is computed as:

---

[106] $x_{Min0} = \left(E-\frac{a}{c}+\frac{R}{r}+\frac{R}{2c}-\frac{R\sqrt{r}\sqrt{4c+r}}{2cr}\right)$ corresponds to the minimum level of stock with a positive level of use by both users when transmissivity is equal to zero; when s tends to infinity the minimum level of stock at the start is $x_{Miin\infty} = \left(E-\frac{a}{c}+\frac{R}{2r}+\frac{R}{c}+\frac{\left(c-2r-\sqrt{c^2+12cr+4r^2}\right)R}{4cr}\right)$, lower than x_{Min0}.

[107] This result is obtained through lengthy but straightforward calculations.

$$\Delta Vx^{OL}(a,x_0) = \frac{-1}{8(3c+r)}\left(PQ_2 + 2\,PQ_3\left(\frac{a}{c} - E - \left(\frac{1}{c}+\frac{1}{r}\right)R + x_0\right)\right)$$

At the minimum level of stock x_{Min0},[108] the derivative has a negative sign:

$$\Delta Vx^{OL}(a,x_{Min0}) = -\frac{8\,R\,c^4\left(c^3+16c^2\,r+68\,c\,r^2+16r^3\right)}{r\,PQ_6\,PQ_7}$$

where PQ_6 and PQ_7 are two positive quantities:

$$PQ_6 = 11c^3+65c^2\,r+32\,c\,r^2+4r^3+\left(c^2+12\,c\,r+4r^2\right)\sqrt{4\,c\,r+r^2} + \left(5c^2+9\,c\,r+2r^2\right)\sqrt{c^2+12\,c\,r+4r^2}+(7c+2r)\sqrt{4\,c\,r+r^2}\,\sqrt{c^2+12\,c\,r+4r^2}$$

$$PQ_7 = 2c^3+17c^2\,r+20\,c\,r^2+4r^3+\left(c^2+12\,c\,r+4r^2\right)\sqrt{4\,c\,r+r^2}$$

When the initial level of stock is E, the derivative is also negative:

$$\Delta Vx^{OL}(a,E) = \frac{-1}{8(3c+r)}\left(PQ_2 + 2\,PQ_3\left(\frac{a}{c} - \left(\frac{1}{c}+\frac{1}{r}\right)R\right)\right)$$

The derivative with regard to the initial level of stock been linear, I can conclude that $\Delta Vx^{OL}(a,h)$ has a negative sign for any level of stock within the interval of acceptable stocks, given the negative effect at the lowest level of stock, to determine that the effect on aggregate cumulative profits from a non-incremental increase in transmissivity is negative.

---

108 $x_{Min0} = \left(E - \frac{a}{c} + \frac{R}{r} + \frac{R}{2c} - \frac{R\sqrt{4c+r}}{2c\sqrt{r}}\right)$ corresponds to the minimum level of stock with a positive level of use when transmissivity is equal to zero, and is higher than the equivalent level when s tends to infinity.

B.1.5.2 Analytical Proofs of Result OL3

B.1.5.2.1 Proof with aggregate profits at the steady state

If I note $\bar{a}$, the average efficiency, and ε_i, the efficiency deviation of player i, i.e. the difference between her efficiency and the average, the absolute value of ε_i, that will be denoted ε hereafter, equals the efficiency average deviation ($(a_h - a_l)/2$), and will be used as a proxy for the level of inequality. The first derivative of aggregate profits at the steady state with respect to transmissivity becomes:

$$\frac{\partial TNP^{SS-OL}(a_h, a_l, s)}{\partial s} = \frac{-2cR^2}{(r+2s)^2} + 4\varepsilon^2 P_2'(s)$$

The necessary conditions on x_l and w_l, for an interior solution, imply that the level of inequality needs to be less than ε_{MinMax}, the minimum of ε_{MaxX} and ε_{MaxW} , respectively the solutions to x_l=E and w_l=0. ε_{MaxX} and ε_{MaxW} are given by: [109]

$$\varepsilon_{MaxX} = \left(\left(a\,r - (c+r)R\right)(r+2s) + c\,R\,s\right)\left(\frac{2r\,s\,(r+2s) + c\,(r^2+4r\,s+2s^2)}{c\,r^2\,(r+2s)^2}\right) \text{and}$$

$$\varepsilon_{MaxW} = \frac{R}{2}\left(2 + c\left(\frac{1}{r} + \frac{1}{s} + \frac{1}{r+2s}\right)\right).$$

It can be easily shown that ε_{MaxX} is increasing in transmissivity while ε_{MaxW} is decreasing; furthermore, ε_{MaxW} tends to infinity at very low levels of transmissivity, and $\varepsilon < \varepsilon_{MaxX}$ becomes the binding condition while at very high levels of transmissivity, ε_{MaxX} tends to infinity and $\varepsilon < \varepsilon_{MaxW}$ becomes the binding condition.[110]

---

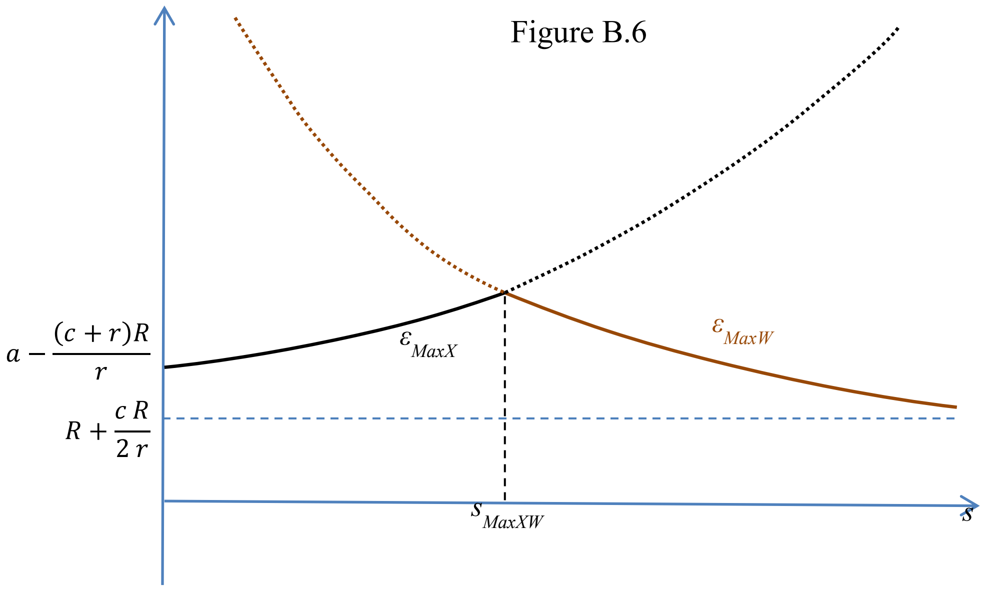
[109] Given the higher efficiency of player h it is clear that the two conditions are sufficient for an interior solution at the steady state.

[110] At the same time $\lim_{s\to\infty} \varepsilon_{MaxW} = R + \frac{Rc}{2r}$ and $\lim_{s\to 0} \varepsilon_{MaxX} = \bar{a} - \frac{(c+r)R}{r}$ positive for a higher than $\bar{a}_{Min} = \frac{(c+r)R}{r}$

.

For a level of transmissivity lower than s_{MaxXW}, where $\varepsilon_{MaxX} = \varepsilon_{MaxW}$, ε_{MinMax}, the highest acceptable level of inequality, equals ε_{MaxX}; as transmissivity exceeds s_{MaxXW}, ε_{MinMax} equals ε_{MaxW}.[111]

Figure B.6



The derivative of aggregate profits with regard to transmissivity is strictly increasing in the level of inequality, for identical users the derivative is always negative:

$$\frac{\partial TNP^{SS-OL}(\bar{a},\bar{a},s)}{\partial s} = \frac{-2cR^2}{(r+2s)^2} \,.$$

For higher levels of transmissivity, the derivative at the highest acceptable level of inequality is strictly positive:

$$\frac{\partial TNP^{SS-OL}(\bar{a}+\varepsilon_{MaxW},\, \bar{a}-\varepsilon_{MaxW},\, s)}{\partial s} = \frac{2cR^2\left(r^2(2c+r)+4r(c+r)s+2(2c+r)s^2\right)}{s(r+2s)\left(2rs(r+2s)+c(r^2+4rs+2s^2)\right)} \,.$$

---

[111] $s_{MaxXW} = r\left(\frac{-\bar{a}\,r+2cR+rR+\sqrt{\bar{a}^2r^2-2\bar{a}\,r^2R+\left(2c^2+r^2\right)R^2}}{4\bar{a}\,r-2(c+2r)R}\right)$ is the unique positive solution to equation $\varepsilon_{MaxX} = \varepsilon_{MaxW}$.

At lower levels of transmissivity, the derivative at the highest acceptable level of inequality has an ambiguous sign:

$$\frac{\partial TNP^{SS-OL}\left(\bar{a}+\varepsilon_{MaxX},\bar{a}-\varepsilon_{MaxX},s\right)}{\partial s}=\frac{8\,s\left(cRs+\left(a\,r-(c+r)R\right)(r+2s)\right)^{2}\,\left(r\,(r+2s)^{3}+c\left(2r^{3}+9r^{2}s+16\,r\,s^{2}+10s^{3}\right)\right)}{c\,r^{2}\,(r+2\,s)^{4}\left(2\,r\,s\,(r+2\,s)+c\left(r^{2}+4\,r\,s+2\,s^{2}\right)\right)}-\frac{2\,c\,R^{2}}{(r+2\,s)^{2}}.$$

The derivative sign for highly unequal users depends on the level of transmissivity; at low levels of transmissivity, the derivative is strictly negative,[112] as transmissivity approaches s_{MaxXW}, the derivative is strictly positive for any level of average efficiency higher than $\bar{a}_{Min0}$, the minimum efficiency for an interior solution when s=0. [113]

In summary, the effect of a marginal increase in transmissivity on aggregate profits depends on the level of transmissivity combined with the level of inequality: at higher levels of transmissivity and higher levels of inequality, the effect (on aggregate profits) is increasing; when transmissivity is low and/or inequality is not high enough, a marginal increase in transmissivity causes a drop in aggregate profits.

A similar result can be reached when considering the effect of a non-incremental increase in transmissivity, shifting from a zero transmissivity to a resource with infinite transmissivity. $\Delta TNP^{SS-OL}\left(a_h,a_l\right)$, the change in aggregate profits at the steady state is given by:

---

[112] $\left.\frac{\partial TNP^{SS-OL}\left(\bar{a}+\varepsilon_{MaxX},\bar{a}-\varepsilon_{MaxX},s\right)}{\partial s}\right|_{s=0}=\frac{-2\,c\,R^{2}}{r^{2}}$

[113] $\bar{a}_{Min0}=\frac{(c+r)R}{r}$ is the minimum individual efficiency for an interior solution (at the steady state) when s=0.

$$\Delta TNP^{SS-OL}(a_h, a_l) = \lim_{s\to\infty} TNP^{SS-OL}(a_h, a_l, s) - TNP^{SS-OL}(a_h, a_l, 0)$$

Assuming an interior solution under both levels of transmissivity I obtain:

$$\Delta TNP^{SS-OL}(a_h, a_l) = \frac{4\varepsilon^2\, r(c+r)}{(r+2r)^2} - \frac{c R^2}{r}$$

$\Delta TNP^{SS-OL}(a_h, a_l)$ is unambiguously negative for identical users and is strictly increasing in the level of inequality. To investigate the sign of $\Delta TNP^{SS-OL}(a_h, a_l)$ with highly unequal users, I distinguish two cases, depending on the range of average efficiency. In the case where the average efficiency is higher than $\bar{a}_{maxOL}$,[114] the condition for an interior solution is given by the limit of ε_{MaxW} at infinity, for which a non-incremental increase in transmissivity prompts a net increase in aggregate profits:

$$\Delta TNP^{SS-OL}(\bar{a} + \varepsilon_{MaxW}, \bar{a} - \varepsilon_{MaxW}) = R^2$$

For a lower average efficiency, the condition for an interior solution is given by the limit of ε_{MaxX} at zero transmissivity, and the effect of an increase in transmissivity reads as:

$$\Delta TNP^{SS-OL}(\bar{a} + \varepsilon_{MaxX}, \bar{a} - \varepsilon_{MaxX}) = \frac{4\, r(c+r)}{(r+2r)^2}\left(\bar{a} - \frac{(c+r)R}{r}\right)^2 - \frac{c R^2}{r}$$

[114] $\bar{a}_{\max OL} = \frac{(3c+4r)R}{2r}$, the level of average efficiency at which, the value of $\varepsilon_{\mathrm{MaxX}}$ with zero transmissivity $\left(\bar{a} - \frac{(c+r)R}{r}\right)$ is higher than the value of $\varepsilon_{\mathrm{MaxW}}$ with an infinite transmissivity $\left(\frac{(c+2r)R}{2r}\right)$.

The sign of $\Delta TNP^{SS-OL}(\bar{a}+\varepsilon_{MaxX},\bar{a}-\varepsilon_{MaxX})$ is still positive for $\bar{a}$ higher than $\bar{a}_{\Delta TNP}$, the lower root of $\Delta TNP^{SS-OL}(\bar{a}+\varepsilon_{MaxX},\bar{a}-\varepsilon_{MaxX})=0$, and negative below.[115]

In summary, the effect of a non-incremental increase in transmissivity on aggregate profits depends on the average efficiency combined with the level of inequality. At higher levels of average efficiency, and higher levels of inequality, the effect is increasing. When the average efficiency is low and/or inequality is not high enough, a non-incremental increase in transmissivity causes a drop in aggregate profits.

B.1.5.2.2 Proof with aggregate cumulative profits

With unequal users, a non-incremental increase in transmissivity, from zero to infinity, brings about a change in aggregate cumulative profits as follows:

$$V^{OL}(a_h,a_l,x_0,\infty)-V^{OL}(a_h,a_l,x_0,0) = \frac{-1}{8(3c+r)}\left(PQ_1+PQ_2\left(\frac{a_h+a_l}{2c}-(E-x_0)-\frac{R}{c}-\frac{R}{r}\right)\right.$$
$$\left.+PQ_3\left(\frac{a_h+a_l}{2c}-(E-x_0)-\frac{R}{c}-\frac{R}{r}\right)^2\right)+PQ_8\,(a_h-a_l)^2$$

where PQ_8 positive and given as:

$$PQ_8=\frac{c(9c+10r)\left((c-r)^2+cr+r^2\right)+3c\,(c+2r)^2\sqrt{r(4c+r)}}{2(c+2r)^2\left((c+2r)^2\sqrt{4cr+r^2}+6c^3+c^2\,r+12cr^2+4r^3\right)}$$

I will first consider the case where the initial stock is at the highest level, E, operating the change of variables introduced earlier ($a_h=\bar{a}+\varepsilon$ and $a_l=\bar{a}-\varepsilon$), the effect becomes:

---

[115] $\bar{a}_{\Delta TNP}=\frac{(4c^2+8cr+4r^2-(c+2r)\sqrt{c(c+r)})R}{2r(c+r)}$ and falls in the interval $\left[\frac{(c+r)R}{r},\frac{(3c+4r)R}{2r}\right]$.

$$V^{OL}(a_h,a_l,E,\infty)-V^{OL}(a_h,a_l,E,0) = \frac{-1}{8(3c+r)}\left(PQ_1+\frac{1}{c}PQ_2\left(\bar{a}-c\left(\frac{1}{c}+\frac{1}{r}\right)R\right)\right.$$
$$\left.+\frac{1}{c^2}PQ_3\left(\bar{a}-c\left(\frac{1}{c}+\frac{1}{r}\right)R\right)^2\right)+4\varepsilon^2 PQ_8$$

As established before, the difference is negative for identical users ($\varepsilon = 0$). Noting that the difference is increasing in ε^2 I will check the sign of the difference at the highest acceptable level of inequality for an interior solution.

When the average efficiency is higher than $\bar{a}_{maxOL}$, introduced in the previous section, the maximum level of inequality for an interior solution is given by the value of ε_{MaxW} for s infinite, and the corresponding effect of an increase in transmissivity is as follows:

$$V^{OL}(a_h,a_l,E,\infty)-V^{OL}(a_h,a_l,E,0) = \frac{1}{8(3c+r)}\left(PQ_9\frac{R^2}{4c^2r^2}-PQ_2\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right)\right.$$
$$\left.-PQ_3\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right)^2\right)$$

where PQ_9 is strictly positive and given by:[116]

$$PQ_9 = (5c+2r)(5c^3-10c^2r-28cr^2-8r^3)+4(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}$$
$$+\ c^2(17c+6r)\sqrt{c^2+12cr+4r^2}$$

[116] Given the three positive quantities PQ_{9a}, PQ_{9b}, and PQ_{9c}, defined as:

$PQ_{9a} = 73c^4+16c^3r+144c^2r^2+96cr^3+16r^4+8(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}\ +(c+2r)^2(7c+2r)\sqrt{c^2+12cr+4r^2}$

$PQ_{9b} = 110c^7+39c^6r+282c^5r^2+990c^4r^3+1216c^3r^4+880c^2r^5+288cr^6+32r^7+c^3(73c+16r)(c+2r)^2\sqrt{r}\sqrt{4c+r}+2r^2(3c+r)(c+2r)^2(7c+2r)\sqrt{c^2+12cr+4r^2}$

$PQ_{9c} = 6050c^{11}+14948c^{10}r+124381c^9r^2+436062c^8r^3+741950c^7r^4+851312c^6r^5+941464c^5r^6+1041632c^4r^7+808352c^3r^8+349184c^2r^9+74624cr^{10}+6144r^{11}+(73c+16r)(c+2r)^2(110c^7+39c^6r+282c^5r^2+990c^4r^3+1216c^3r^4+880c^2r^5+288cr^6+32r^7)\sqrt{r}\sqrt{4c+r}$

PQ_9 can be rewritten as: $PQ_9 = \dfrac{32c^3(3c+r)PQ_{9c}}{PQ_{9a}\ PQ_{9b}}$

At the lowest level of efficiency in the considered category, the effect of making the resource more common is strictly welfare increasing as shown below:

$$V^{OL}\left(\bar{a}_{\max OL}+\varepsilon_{MaxW},\bar{a}_{\max OL}-\varepsilon_{MaxW},E,\infty\right)-V^{OL}\left(\bar{a}_{\max OL}+\varepsilon_{MaxW},\bar{a}_{\max OL}-\varepsilon_{MaxW},E,0\right)=\frac{PQ_{10}\,R^2}{4c^2\,r^2}$$

where PQ_{10} is strictly positive:[117]

$$PQ_{10}=c^4-28c^3r-57c^2r^2-28cr^3-4r^4+2(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}+(5c^3-7cr^2-2r^3)\sqrt{c^2+12cr+4r^2}$$

However, as the average efficiency increases, the benefits from making the resource more common vanish, given the negative sign in the square of average efficiency in the general formula. This result however needs to be interpreted in the context of this analysis and the imposed constraint on inequality, increasing efficiency while keeping the maximum inequality constant translates into lower inequality in relative terms.

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[117] Given the three positive quantities PQ_{10a}, PQ_{10b}, and PQ_{10c}, defined as:

$$PQ_{10a}=c^4+28c^3r+57c^2r^2+28cr^3+4r^4+2(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}+(5c^3+7cr^2+2r^3)\sqrt{c^2+12cr+4r^2}$$

$$PQ_{10b}=13c^8+222c^7r+300c^6r^2+680c^5r^3+1617c^4r^4+1744c^3r^5+856c^2r^6+192cr^7+16r^8+(5c^7+196c^4r^3+455c^3r^4+310c^2r^5+84cr^6+8r^7)\sqrt{c^2+12cr+4r^2}+2c^4(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}+10c^3(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}\sqrt{c^2+12cr+4r^2}$$

$$PQ_{10c}=97c^{13}+4908c^{12}r+66884c^{11}r^2+334416c^{10}r^3+1069281c^9r^4+2126006c^8r^5+2673732c^7r^6+2404796c^6r^7+1772064c^5r^8+1025776c^4r^9+408384c^3r^{10}+100416c^2r^{11}+13568cr^{12}+768r^{13}+2c(3c+r)(c+2r)^2(38c^8+522c^7r+400c^6r^2+680c^5r^3+1617c^4r^4+1744c^3r^5+856c^2r^6+192cr^7+16r^8)\sqrt{r}\sqrt{4c+r}+5c^4(13c^8+366c^7r+1584c^6r^2+5232c^5r^3+9717c^4r^4+9264c^3r^5+4344c^2r^6+960cr^7+80r^8)\sqrt{c^2+12cr+4r^2}+10(3c+r)(c+2r)^2(14c^8+222c^7r+300c^6r^2+680c^5r^3+1617c^4r^4+1744c^3r^5+856c^2r^6+192cr^7+16r^8)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+12cr+4r^2};$$

PQ_{10} can be rewritten as: $PQ_{10}=\dfrac{4c^3\,PQ_{10c}}{PQ_{10a}\,PQ_{10b}}$

When the average efficiency is lower than $\bar{a}_{maxOL}$, the maximum level of inequality for an interior solution is given by the value of ε_{MaxX} for s=0, $\bar{a}-\frac{(c+r)}{r}R$, and the corresponding effect of an increase in transmissivity (at the highest level of inequality) simplifies as:

$$V^{OL}(a_h,a_l,E,\infty)-V^{OL}(a_h,a_l,E,0)=\frac{1}{8(3c+r)}\left(\frac{PQ_{11}}{(c+2r)^2}\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right)^2-PQ_1-PQ_2\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right)\right)$$

where PQ_{11} is given by: $PQ_{11}=4\,PQ_8-\frac{PQ_3}{8c^2\,(3c+r)}$

At low levels of average efficiency the effect of an increase in transmissivity is clearly negative, indeed, for $\bar{a}=R(c+r)/r$,[118] a non-incremental increase in transmissivity results in a net loss in total welfare in the amount of PQ_1, in absolute value; and a marginal increase in average efficiency results in even greater losses. But the positive sign of PQ_{11} suggests that the increase in transmissivity can be welfare increasing at high levels of average efficiency,[119] indeed, at the highest level of average efficiency –in this category–, when $\bar{a}=\bar{a}_{maxOL}$, the effect is strictly positive:

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[118] The lowest level of acceptable average efficiency for an interior solution.

[119] PQ_{11} can be rewritten as $PQ_{11}=\frac{32c^3\,(3c+r)\,PQ_{11c}}{PQ_{11a}\;PQ_{11b}}$, where PQ_{11a}, PQ_{11b}, and PQ_{11c}, defined as:

$$PQ_{11a}=73c^4+16c^3r+144c^2r^2+96cr^3+16r^4+8(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}+(c+2r)^2(7c+2r)\sqrt{c^2+12cr+4r^2}$$

PQ_{11b}

$$+2r^2(3c+r)(c+2r)^2(7c+2r)\sqrt{c^2+12cr+4r^2}$$

$$\begin{aligned}PQ_{11c}=\;&6050c^{11}+14948c^{10}r+124381c^9r^2+436062c^8r^3+741950c^7r^4+851312c^6r^5+941464c^5r^6+1041632c^4r^7\\&+808352c^3r^8+349184c^2r^9+74624cr^{10}+6144r^{11}+(73c+16r)(c+2r)^2(110c^7+39c^6r\\&+282c^5r^2+990c^4r^3+1216c^3r^4+880c^2r^5+288cr^6+32r^7)\sqrt{r}\sqrt{4c+r}\\&+16r^8)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+12cr+4r^2};\end{aligned}$$

$$V^{OL}\left(\bar{a}_{\max OL}+\varepsilon_{MaxX},\bar{a}_{\max OL}-\varepsilon_{MaxX},E,\infty\right)-V^{OL}\left(\bar{a}_{\max OL}+\varepsilon_{MaxX},\bar{a}_{\max OL}-\varepsilon_{MaxX},E,0\right)=\frac{PQ_{10}\,R^2}{4c^2\,r^2}$$

At the highest level of stock at the start, the effect of a non-incremental increase in transmissivity on aggregate cumulative profits displays the same tendency as established at the steady state, and depends, in the same way, on the average efficiency combined with the level of inequality. At higher levels of average efficiency, and higher levels of inequality, the effect is increasing. When the average efficiency is low and/or inequality is not high enough, a non-incremental increase in transmissivity causes a drop in aggregate cumulative profits.

I consider now the extraction game where the initial stock is at the lowest acceptable level for an interior solution; such stock, noted $x_{Min0\varepsilon}$ in the case of unequal users, corresponds to the level with a positive level of use by the less efficient user when transmissivity is equal to zero, and is given by:[120]

$$x_{Min0\varepsilon}=\left(E-\frac{a-\varepsilon}{c}+\frac{R}{r}+\frac{R}{2c}-\frac{R\sqrt{r}\sqrt{4c+r}}{2cr}\right)$$

The corresponding effect of an increase in transmissivity simplifies as:

$$V^{OL}\left(a_h,a_l,Min0\varepsilon,\infty\right)-V^{OL}\left(a_h,a_l,Min0\varepsilon,0\right)=\frac{1}{8c^2(3c+r)}\left(\frac{PQ_{12}}{(c+2r)^2}\varepsilon^2-\frac{PQ_{13}}{r}R\varepsilon-\frac{PQ_{14}}{4r^2}R^2\right)$$

where PQ_{12}, PQ_{13} and PQ_{14} are all positive quantities.[121]

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[120] The solution for the extraction decisions in an Open Loop game show that for any level of stock, the more efficient user extracts more, also, the minimum level of stock with a positive level of use when transmissivity is equal to zero is higher than the equivalent level when s tends to infinity.

[121] $PQ_{12}=73c^4+16c^3r-144c^2r^2-96cr^3-16r^4+8(3c+r)(c+2r)^2\sqrt{r}\sqrt{4c+r}-(c+2r)^2(7c+2r)\sqrt{c^2+12cr+4r^2}$;

For identical users (ε = 0), the difference is negative confirming the decreasing effect of making the resource more common on aggregate cumulative profits. The difference is quadratic ε, and is increasing in ε^2 , therefore, to investigate the effect on high levels of inequality, it will suffice to check the sign of the difference at the highest acceptable level of inequality for an interior solution.

When the average efficiency is higher than $\bar{a}_{maxOL}$, the maximum inequality is given by (the limit of) ε_{MaxW} for s infinite, $R + {cR}/{2r}$, and the corresponding effect of an increase in transmissivity is as follows:

$$V^{OL}\left(\bar{a}+\varepsilon_{MaxW}, \bar{a}-\varepsilon_{MaxW}, Min0\varepsilon, \infty\right) - V^{OL}\left(\bar{a}+\varepsilon_{MaxW}, \bar{a}-\varepsilon_{MaxW}, Min0\varepsilon, 0\right) = \frac{PQ_{15}\, R^2}{16c^2\, r^2\,(3c+r)}$$

PQ_{15} is positive,[122] evidencing the increasing effect on the aggregate cumulative profits of highly unequal users.

When the average efficiency is lower than $\bar{a}_{maxOL}$, the maximum inequality is given by ε_{MaxX} for s=0, $\bar{a} - R(c+r)/r$, and the corresponding effect of an increase in transmissivity is as follows:

$PQ_{13} = 11c^3 + 65c^2r + 32cr^2 + 4r^3 + (c^2 + 12cr + 4r^2)\sqrt{r}\sqrt{4c+r} - (5c^2 + 9cr + 2r^2)\sqrt{c^2 + 12cr + 4r^2} - (7c + 2r)\sqrt{r}\sqrt{4c+r}\sqrt{c^2 + 12cr + 4r^2}$;

$PQ_{14} = 47c^4 + 26c^3r - 118c^2r^2 - 64cr^3 - 8r^4 - 2(11c^3 + 41c^2r + 24cr^2 + 4r^3)\sqrt{r}\sqrt{4c+r} + (-17c^3 + 32c^2r + 26cr^2 + 4r^3)\sqrt{c^2 + 12cr + 4r^2} + 2(5c^2 + 9cr + 2r^2)\sqrt{r}\sqrt{4c+r}\sqrt{c^2 + 12cr + 4r^2}$;

All three quantities can be shown to be positive.

[122] $\mathrm{PQ}_{15} = 2c^4 - 92c^3r - 175c^2r^2 - 84cr^3 - 12r^4 + (22c^3 + 79c^2r + 60cr^2 + 12r^3)\sqrt{r}\sqrt{4c+r} + (10c^3 - 12c^2r - 11cr^2 - 2r^3)\sqrt{c^2 + 12cr + 4r^2} + (2c^2 + 7cr + 2r^2)\sqrt{r}\sqrt{4c+r}\sqrt{c^2 + 12cr + 4r^2}$

$$V^{OL}\left(a_h, a_l, Min0\varepsilon, \infty\right) - V^{OL}\left(a_h, a_l, Min0\varepsilon, 0\right) = \frac{1}{8c^2\left(3c+r\right)}\left(\frac{c^2 PQ_{12}}{\left(c+2r\right)^2}\left(\frac{\bar{a}}{c} - \frac{R}{c} - \frac{R}{r}\right)^2 - \frac{c\,PQ_{13}}{r} R\left(\frac{\bar{a}}{c} - \frac{R}{c} - \frac{R}{r}\right) - \frac{PQ_{14}}{4r^2} R^2\right)$$

At low levels of average efficiency the effect of an increase in transmissivity is clearly negative, indeed, for $\bar{a} = R(c+r)/r$, a non-incremental increase in transmissivity results in a net loss in total welfare in the amount of $PQ_{14}R^2/4r^2$, in absolute value; and a marginal increase in average efficiency results in even greater losses. But the positive sign of PQ_{12} suggests that the increase in transmissivity can be welfare increasing at high levels of average efficiency, indeed, at the highest level of average efficiency –in this category–, when $\bar{a} = \bar{a}_{maxOL}$, the effect is strictly positive:

$$V^{OL}\left(\bar{a}_{\max OL} + \varepsilon_{MaxX}, \bar{a}_{\max OL} - \varepsilon_{MaxX}, E, \infty\right) - V^{OL}\left(\bar{a}_{\max OL} + \varepsilon_{MaxX}, \bar{a}_{\max OL} - \varepsilon_{MaxX}, E, 0\right) = \frac{PQ_{15}\,R^2}{16c^2\,r^2\left(3c+r\right)}$$

In summary, at the minimum level of stock at the start, the effect of a non-incremental increase in transmissivity on aggregate cumulative profits displays the same tendency as established with the maximum stock, and depends on the average efficiency combined with the level of inequality. At higher levels of average efficiency, and higher levels of inequality, the effect is increasing. When the average efficiency is low and/or inequality is not high enough, a non-incremental increase in transmissivity causes a drop in aggregate cumulative profits.

B.2 Solving for the Closed Loop Nash Equilibrium

B.2.1 The General solution with finite transmissivity

In this section, I will try to expand the analysis and unravel players' decisions throughout the exploitation horizon under feedback strategies in the general case, i.e. with s anywhere between zero and infinity. User *i* (similarly user *-i*) is interested in maximizing his net payoff:

$$\int_0^\infty e^{-rt}\left(a_i w_{it} - \frac{w_{it}^2}{2} - w_{it} c\left(h - x_{it}\right)\right) dt \qquad i=h,l$$

Given the stock of water x_0 in the aquifer at time $t = 0$ and the stock evolution given the simplified form of the equation of motion for a perfectly transmissive aquifer:

$$\dot{x}_{it} = R - w_{it} + s\left(x_{-it} - x_{it}\right) \qquad (B_2.1)$$

Let $V_{a_i,a_{-i}}^{CL}\left(x_{it}, x_{-it}\right)$ designate player *i* maximal present value when the stocks of water in the aquifer are (x_{it}, x_{-it}), the Bellman equation corresponding to the dynamic maximization problem -with two state variables- is given by:

$$\begin{aligned} rV_{a_i,a_{-i}}^{CL}\left(x_{it}, x_{-it}\right) = \max_{w_{it}} \Big\{ & a_i w_{it} - \frac{w_{it}^2}{2} - c\, w_{it}\left(h - x_{it}\right) \\ & + \left(R - w_{it} + s\,\left(x_{-it} - x_{it}\right)\right) Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right) \\ & + \left(R - w_{-it} + s(x_{it} - x_{-it})\right) Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right)\Big\} \end{aligned} \qquad i=h,l \quad (B_2.2)$$

$Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right)$ in (B_2.2) is the first derivative of the value function of player *i*, $V_{a_i,a_{-i}}^{CL}\left(x_{it}, x_{-it}\right)$ with regard to her own stock, x_{it}, and $Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right)$ the derivative with regard to her partner's stock, x_{-it}. The optimal level of extraction for player *i* (when she chooses to extract) is as follows:

$$w_{it} = a_i - c\left(h - x_{it}\right) - Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right) \qquad i=h,l \quad (\mathrm{B}_2.3)$$

substituting the new expressions of w_{it} in (R.2) to obtain:

$$\begin{aligned} rV_{a_i,a_{-i}}^{CL}\left(x_{it}, x_{-it}\right) = & \frac{\left(a_i - c\,h\right)^2}{2} - \left(a_i - R - c\,h\right) Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right) \\ & - \left(a_{-i} - R - c\,h\right) Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right) + \frac{Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right)^2}{2} \\ & + Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right) Vx_{-i,-i}^{CL}\left(x_{-it}, x_{it}\right) + c\left(a_{-i} - c\,h\right)x_{it} \\ & - \left(s + c\right)x_{it}\, Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right) + s\; x_{it}\, Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right) \\ & - \left(s + c\right)x_{-it}\, Vx_{i,-i}^{CL}\left(x_{it}, x_{-it}\right) + s\, x_{-it}\, Vx_{i,i}^{CL}\left(x_{it}, x_{-it}\right) + \frac{c^2}{2} x_{it}^{\;2} \end{aligned} \qquad i=h,l \quad (\mathrm{B}_2.4)$$

From the last result it can be concluded that the solution to player i dynamic programming problem is polynomial of second order in x_{it} and x_{-it}:

$$V_{a_i,a_{-i}}^{CL}\left(x_{it}, x_{-it}\right) = \beta_{0i} + \beta_{1i}\, x_{it} + \beta_{2i}\, x_{it}^{\;2} + \beta_{3i}\, x_{-it} + \beta_{4i}\, x_{-it}^{\;2} + \beta_{5i}\, x_{it}\, x_{-it} \qquad i=h,l \quad (\mathrm{B}_2.5)$$

Substituting the new formula for individual cumulative profits for players h and l, $V_{a_h,a_l}^{CL}\left(x_{ht}, x_{lt}\right)$ and $V_{a_l,a_h}^{CL}\left(x_{lt}, x_{ht}\right)$ (and the subsequent derivatives with regard to x_{ht}, $Vx_{h,h}^{CL}\left(x_{ht}, x_{lt}\right)$ and $Vx_{l,h}^{CL}\left(x_{lt}, x_{ht}\right)$, and with regard to x_{lt}, $Vx_{h,l}^{CL}\left(x_{ht}, x_{lt}\right)$ and $Vx_{l,l}^{CL}\left(x_{lt}, x_{ht}\right)$) in equation (R.4) generates a system of 12 non-linear equations in 12 unknowns: β_{0h}, β_{1h}, β_{2h}, β_{3h}, β_{4h}, β_{5h}, β_{0l}, β_{1l}, β_{2l}, β_{3l}, β_{4l}, and β_{5l}. It was not possible for me to solve for the system with s variable.

The system can be further simplified by noticing that the it can be solved in two stages, where the first stage consisted in solving six equations in six unknown: β_{2h}, β_{4h}, β_{5h}, β_{2l}, β_{4l}, and β_{5l}:

$$r\beta_{2h} - \left(c^2 - 4c\beta_{2h} - 4s\beta_{2h} + 4{\beta_{2h}}^2 + 2s\beta_{5h} + 2\beta_{5h}\beta_{5l}\right)/2 = 0,$$

$$r\beta_{4h} - \left(-4(c + s - 2\beta_{2l})\beta_{4h} + 2s\beta_{5h} + {\beta_{5h}}^2\right)/2 = 0,$$

$$r\beta_{5h} - 2(s(\beta_{2h} + \beta_{4h} - \beta_{5h}) + (-c + \beta_{2h} + \beta_{2l})\beta_{5h} + \beta_{4h}\beta_{5l}) = 0,$$

$$r\beta_{2l} - \left(c^2 - 4c\beta_{2l} - 4s\beta_{2l} + 4{\beta_{2l}}^2 + 2s\beta_{5l} + 2\beta_{5l}\beta_{5h}\right)/2 = 0,$$

$$r\beta_{4l} - \left(-4(c + s - 2\beta_{2h})\beta_{4l} + 2s\beta_{5l} + {\beta_{5l}}^2\right)/2 = 0, \text{ and}$$

$$r\beta_{5l} - (s(\beta_{2l} + \beta_{4l} - \beta_{5l}) + (-c + \beta_{2l} + \beta_{2h})\beta_{5l} + \beta_{4l}\beta_{5h})/2 = 0.$$

The solution to this system is then used to solve the 6 other equations in 6 unknowns, in the second stage. But it was not possible to solve for this simplified system in β_{2h}, β_{4h}, β_{5h}, β_{2l}, β_{4l}, and β_{5l}.

B.2.2 The Individual Value Function- Cumulative profits

In equation (27), the individual value functions are given by:

$$\begin{aligned} V_{a_i,a_{-i}}^{CL,\infty}(x_t) = {} & \frac{PQ_{V1}}{2} + \frac{PQ_{V2}}{2}\left(\frac{a_i + a_{-i}}{2c} - h - \frac{R}{c} - \frac{R}{r} + x_t\right) \\ & + PQ_{Vi1}(a_i - a_{-i}) + \frac{PQ_{V3}}{2}\left(\frac{a_i + a_{-i}}{2c} - h - \frac{R}{c} - \frac{R}{r} + x_t\right)^2 \qquad i=h,l \\ & + \frac{PQ_{V4}}{2}(a_i - a_{-i})^2 + PQ_{Vi2}(a_i - a_{-i})\left(\frac{a_i + a_{-i}}{2c} - h - \frac{R}{c} - \frac{R}{r} + x_t\right) \end{aligned}$$

PQ_{V1}, PQ_{V2}, PQ_{V3}, PQ_{V4}, PQ_{Vi1} and PQ_{Vi2} are as follows:

$$PQ_{V1} = \frac{2(c^4 + 20c^3r + 78c^2r^2 + 60cr^3 + 13r^4)R^2}{r^2\left(2c^3 + 23c^2r + 64cr^2 + 25r^3 + (c + r)^2\sqrt{c^2 + 4cr + r^2}\right)}$$

$$PQ_{V2} = \frac{4c(c^2 + 6cr + r^2)R}{r(2c^2 + 9cr + r^2 + (c + r)\sqrt{c^2 + 4cr + r^2})}$$

$$PQ_{V3} = \frac{2c^2}{2c + r + \sqrt{c^2 + 4cr + r^2}}$$

$$PQ_{V4} = \frac{(4c^2 + 22cr + 13r^2 + (4c + 5r)\sqrt{c^2 + 4cr + r^2})}{2r(17c^2 + 44cr + 26r^2 + (8c + 10r)\sqrt{c^2 + 4cr + r^2})}$$

$$PQ_{Vi2} = \frac{3c}{4c + 5r + \sqrt{c^2 + 4cr + r^2}}$$

$$PQ_{Vi1} = \frac{PQNum_{Vi1}}{PQDen_{Vi1}},$$

with: $$PQNum_{Vi1} = \Big(37c^6 + 400c^5 r + 1629c^4 r^2 + 3158c^3 r^3 + 3080c^2 r^4 + 1440cr^5 + 256r^6 + 2c^2\sqrt{c^2 + 4cr + r^2}(6c^3 + 33c^2 r + 45cr^2 + 16r^3)\Big)R$$

$$PQDen_{Vi1} = r(c + 4r)(5c^2 + 12cr + 8r^2)(6c^3 + 33c^2 r + 45cr^2 + 16r^3 + c(c + r)\sqrt{c^2 + 4cr + r^2})$$

It is clear that all six variables are positive and depend only on *c*, *r* and *R*.

B.2.3 The Individual and Aggregate profits at the Steady State

In equation (31), the individual profits of player *i*, at the steady state, amount to:

$$NP^{SS-CL}_{a_i,a_{-i}} = \frac{PQ_{CL1}}{2} + PQ_{CL2}\left(a_i - a_{-i}\right) + \frac{PQ_{CL3}}{2}\left(a_i - a_{-i}\right)^2 \qquad i=h,l$$

PQ_{CL1}, PQ_{CL2} and PQ_{CL3} are as follows:

$$PQ_{CL1} = \frac{\left(8c^2 + 11cr + 4r^2\right)R^2}{c^2 + 6cr + 4r^2 + c\sqrt{c^2 + 4cr + r^2}}$$

$$PQ_{CL2} = \frac{\left(8c^2 + 11cr + 4r^2\right)R}{9c^2 + 17cr + 8r^2 + c\sqrt{c^2 + 4cr + r^2}}$$

$$PQ_{CL3} = \frac{4c^4 + 38c^3 r + 99c^2 r^2 + 96cr^3 + 32r^4 + c^2\left(4c + 5r\right)\sqrt{c^2 + 4cr + r^2}}{4\left(5c^2 + 12cr + 8r^2\right)^2}$$

All three variables are positive, and their values depend only on *c*, *r* and *R*.

B.2.4 Analytical Proofs of Preliminary Results

B.2.4.1 Analytical Proof of Result CL1 with cumulative aggregate profits

Under feedback strategies, the difference in aggregate cumulative profits of identical users, following non-marginal increase in transmissivity from zero to infinity, is given by (33), that writes as follows:

$$V^{CL}(a,a,x_0,\infty)-V^{CL}(a,a,x_0,0) = -\left(PQ_{R11}+PQ_{R12}\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)\right.$$
$$\left.+PQ_{R13}\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)^2\right)$$

where PQ_{R11}, PQ_{R12} and PQ_{R13} are three positive variables given by:

$$PQ_{R11}=\frac{c^2(8c^4+52c^3r+117c^2r^2+128cr^3+28r^4+4(c+r)^2(2c+5r)\sqrt{c^2+4cr+r^2})R^2}{3r^2(c+4r)^2(2c^3+5c^2r+8cr^2+2r^3+2(c+r)^2\sqrt{c^2+4cr+r^2})}$$

$$PQ_{R12}=\frac{2c^3R}{r(c^2+6cr+2r^2+(2c+2r)\sqrt{c^2+4cr+r^2})}$$

$$PQ_{R13}=\frac{4c^4}{\widetilde{PQ}_3}$$

with: $\widetilde{PQ}_3=2(2c+r)(c^2+8cr+2r^2)+2(5c^2+8c\,r+2r^2)\sqrt{r}\,\sqrt{4c+r}+4(2c^2+4cr+r^2)\sqrt{c^2+4cr+r^2}+4(2c+r)\sqrt{r}\,\sqrt{4c+r}\,\sqrt{c^2+4cr+r^2}$,

The proof that the difference is always negative will be conducted in two steps; the first part shows that the difference is negative at the lowest and highest acceptable levels of stock, x_0. The acceptable levels of stock here refer to the levels for which the assumption of interior solution holds.[123]

[123] For equation (33) to express the change in aggregate cumulative profits, the solution needs to be interior at the two considered levels of transmissivity, zero and infinite.

In a second step, I will show that the derivative of $V^{CL}(a,a,x_0,\infty)-V^{CL}(a,a,x_0,0)$ w.r.t x_0 is negative everywhere, therefore, the are no "nonlinear effects" and the difference, confirmed negative on the boundaries, is negative everywhere.

When the stock at start is at the highest acceptable level, E, the quantity $\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)$ simplifies as $\left(\frac{a}{c}-\left(\frac{1}{c}+\frac{1}{r}\right)R\right)$, that needs to be positive for the assumption on interior solution with zero transmissivity to hold;[124] therefore, with $x_0 = E$, all terms between brackets on the RHS of equation (33) are positive and the difference is negative.

When the stock at the start is at x_{Min0},[125] the minimum level that allows for an interior solution, equation (33) delivers:

$$V^{CL}(a,a,x_{Min0},\infty)-V^{CL}(a,a,x_{Min0},0) = -\left(\frac{4\,c^2\,\left(PQ_{R17}\,PQ_{R16}+4\,c^2\,r^{10}\,PQ_{R18}\right)}{3\,r^2\,(c+4\,r)^2\,PQ_{R14}\,PQ_{R15}\,PQ_{R16}}\right)R^2$$

where PQ_{R14}, PQ_{R15}, PQ_{R16}, PQ_{R17} and PQ_{R18} are all positive variables:

$$PQ_{R14} = 2c^5 + 5c^4r + 10c^3r^2 + 25c^2r^3 + 64cr^4 + 16r^5 + (c+4r)(c^3 + 4c^2r + 7cr^2 + 4r^3)\sqrt{r}\sqrt{4c+r} + (2c^4 + 4c^3r + 9c^2r^2 + 32cr^3 + 16r^4)\sqrt{c^2+4cr+r^2} + (2c^3 + 9c^2r + 16r^3)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

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124 For the condition on the level of stock (below the ground) to hold at the steady state, it needs the condition: $a > \left(1+{}^{c}\!/_{r}\right)R$ to hold, a high extraction cost and/or a low discount rate, encourage the users to save water, only a high efficiency will guarantee an interior solution.

125 $x_{Min0} = \left(E-\frac{a}{c}+\frac{R}{r}+\frac{R}{2c}-\frac{R\sqrt{4c+r}}{2c\sqrt{r}}\right)$, as introduced earlier, corresponds to the minimum level of stock with a positive level of use when transmissivity is equal to zero, and is higher than the equivalent level when s tends to infinity.

$$PQ_{R15} = 4c^8 + 20c^7r + c^6r^2 + 72c^5r^3 + 302c^4r^4 + 1204c^3r^5 + 479c^2r^6 + 480cr^7 + 112r^8$$
$$+ (2c^7 + 21c^6r + 72c^5r^2 + 112c^4r^3 + 122c^3r^4 + 63c^2r^5 + 256cr^6$$
$$+ 112r^7)\sqrt{r}\sqrt{4c+r}$$
$$+ (4c^7 + 18c^6r + 38c^5r^2 + 53c^4r^3 + 266c^3r^4 + 799c^2r^5 + 256cr^6$$
$$+ 112r^7)\sqrt{c^2+4cr+r^2}$$
$$+ (2c^6 + 20c^5r + 53c^4r^2 + 60c^3r^3 + 143c^2r^4 + 32cr^5$$
$$+ 112r^6)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

$$PQ_{R16} = 235875c^3r^2 + 85744c^2r^3 + 106928cr^4 + 17920r^5 + r(112195c^3 + 112376c^2r$$
$$+ 71088cr^2 + 17920r^3)\sqrt{r}\sqrt{4c+r} + r(86099c^3 + 142736c^2r + 71088cr^2$$
$$+ 17920r^3)\sqrt{c^2+4cr+r^2} + (115277c^3 + 33096c^2r + 35248cr^2$$
$$+ 17920r^3)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

$$PQ_{R17} = 16c^{14} + 200c^{13}r + 1174c^{12}r^2 + 5644c^{11}r^3 + 24334c^{10}r^4 + 76996c^9r^5 + 166557c^8r^6$$
$$+ 266258c^7r^7 + 390250c^6r^8 + 495512c^5r^9 + 189421c^4r^{10} + (16c^{13} + 272c^{12}r$$
$$+ 1828c^{11}r^2 + 6656c^{10}r^3 + 16946c^9r^4 + 39945c^8r^5 + 94026c^7r^6 + 195054c^6r^7$$
$$+ 297918c^5r^8 + 312419c^4r^9)\sqrt{r}\sqrt{4c+r} + (16c^{13} + 168c^{12}r + 844c^{11}r^2$$
$$+ 3734c^{10}r^3 + 14526c^9r^4 + 44533c^8r^5 + 106720c^7r^6 + 169272c^6r^7$$
$$+ 163072c^5r^8 + 169755c^4r^9)\sqrt{c^2+4cr+r^2} + (16c^{12} + 240c^{11}r + 1354c^{10}r^2$$
$$+ 3948c^9r^3 + 8729c^8r^4 + 21704c^7r^5 + 53216c^6r^6 + 105818c^5r^7$$
$$+ 161597c^4r^8)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

$$PQ_{R18} = 26577573458c^7 + 131922036465c^6r + 172495304928c^5r^2 + 138434950121c^4r^3 + 128515932416c^3r^4 + 53082939728c^2r^5 + 9598879104cr^6 + 702607360r^7 + (9925234423c^6 + 59004648068c^5r + 91864066583c^4r^2 + 47819721080c^3r^3 + 23850635536c^2r^4 + 8193664384cr^5 + 702607360r^6)\sqrt{r}\sqrt{4c+r} + r(51734012060c^5 + 79603798503c^4r + 71706076416c^3r^2 + 34265244080c^2r^3 + 19369302656cr^4 + 3150909440r^5)\sqrt{c^2+4cr+r^2} + r(9659877305c^4 + 35574037832c^3r + 29010706928c^2r^2 + 13067483776cr^3 + 3150909440r^4)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

Now regarding the derivative w.r.t. x_0, the formulation is derived from equation (33):

$$\frac{\partial\left(V^{CL}(a,a,x_0,\infty)-V^{CL}(a,a,x_0,0)\right)}{\partial x_0} = -\left(PQ_{R12}+2\,PQ_{R13}\left(\frac{a}{c}-E-\left(\frac{1}{c}+\frac{1}{r}\right)R+x_0\right)\right)$$

The sign for a stock at the start $x_0 = E$, is always negative; when the stock at the start is x_{Min0}, the derivative can be rewritten as:

$$\left.\frac{\partial\left(V^{CL}(a,a,x_0,\infty)-V^{CL}(a,a,x_0,0)\right)}{\partial x_0}\right|_{x_0=x_{Min0}} = -\frac{4\,c^4\,R}{r\,PQ_{R19}}\left(\frac{c^2+4r^2+4r\sqrt{c^2+4cr+r^2}}{c^2+6cr+2r^2+2(c+r)\sqrt{c^2+4cr+r^2}}\right)$$

where PQ_{R19} is a positive variable, function of r and c, given as:

$$PQ_{R19} = 2c^3 + c^2r + 8cr^2 + 2r^3 + (c^2 + 4cr + 2r^2)\sqrt{r}\sqrt{4c+r} + 2r(2c+r)\sqrt{c^2+4cr+r^2} + 2r\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$$

The derivative of the difference is linear and decreasing in x_0 and its negative sign when $x_0 = x_{Min0}$ shows that the derivative is negative for the entire range of stocks under consideration, the difference is negative at $x_0 = x_{Min0}$ and is decreasing in the level of stock, therefore, the difference is always negative when conditions for an interior solution hold.

B.2.4.2 Analytical Proof of Result CL3 with cumulative aggregate profits

Regarding the aggregate cumulative profits, the effect of a non-marginal increase in transmissivity on the profits of unequal players is given by:[126]

$$V^{CL}(a_h, a_l, x_0, \infty) - V^{CL}(a_h, a_l, x_0, 0) = -\left(PQ_{R11} + PQ_{R12}\left(\frac{a_h + a_l}{2c} - E - \left(\frac{1}{c}+\frac{1}{r}\right)R + x_0\right) + PQ_{R13}\left(\frac{a_h + a_l}{2c} - E - \left(\frac{1}{c}+\frac{1}{r}\right)R + x_0\right)^2\right) + PQ_{R31}\,(a_h - a_l)^2$$

where PQ_{R31} is a positive variable.[127]

The effect is strictly negative for identical users, but it is increasing in the level of inequality. In order to determine the effect of transmissivity on highly unequal users it suffices to check at the highest acceptable level –with an interior solution– of inequality.

B.2.4.2.1 At the Maximum Initial Stock

I will first consider the case where the initial stock is at the highest level, E; operating the change of variables introduced earlier ($a_h = \bar{a} + \varepsilon$ and $a_l = \bar{a} - \varepsilon$), the effect becomes:

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[126] The Cumulative profits with zero transmissivity are taken, as discussed before, from the Open Loop Game.

[127] $PQ_{R31} = \dfrac{c}{4r\left(5c^2+12cr+8r^2\right)^2}\,\dfrac{PQ_{R31a}}{PQ_{R31b}}$ where:

$PQ_{R31a} = 256c^9 + 2464c^8r + 8906c^7r^2 + 15732c^6r^3 + 21093c^5r^4 + 54200c^4r^5 + 122592c^3r^6 + 142272c^2r^7 + 78400cr^8 + 16384r^9 + cr(16c^2 + 102cr + 131r^2)(5c^2 + 12cr + 8r^2)^2\sqrt{r}\sqrt{4c+r} + 4c^5(4c + 5r)(16c^2 + 102cr + 131r^2)\sqrt{c^2+4cr+r^2} + 4cr(4c+5r)(5c^2 + 12cr + 8r^2)^2\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$; and

$PQ_{R31b} = 16c^6 + 102c^5r + 131c^4r^2 + 184c^3r^3 + 480c^2r^4 + 320cr^5 + 64r^6 + r(5c^2 + 12cr + 8r^2)^2\sqrt{r}\sqrt{4c+r} + 4c^4(4c+5r)\sqrt{c^2+4cr+r^2}$.

$$V^{CL}(a_h, a_l, E, \infty) - V^{CL}(a_h, a_l, E, 0) = -\left(PQ_{R11} + PQ_{R12}\left(\bar{a} - c\left(\frac{1}{c} + \frac{1}{r}\right)R\right) + PQ_{R13}\left(\bar{a} - c\left(\frac{1}{c} + \frac{1}{r}\right)R\right)^2 \right) + 4\,PQ_{R31}\;\varepsilon^2$$

The maximum inequality depends on the average efficiency, when $\bar{a}$ is lower than $\bar{a}_{maxCL}$, the maximum inequality is given by ε_{MaxX} with zero transmissivity $(\bar{a} - (c + r)R/r)$, and the corresponding effect is as follows:

$$V^{CL}(\bar{a} + \varepsilon_{MaxX}, \bar{a} - \varepsilon_{MaxX}, E, \infty) - V^{CL}(\bar{a} + \varepsilon_{MaxX}, \bar{a} - \varepsilon_{MaxX}, E, 0) = PQ_{R32}\left(\bar{a} - c\left(\frac{1}{c} + \frac{1}{r}\right)R\right)^2 - PQ_{R11} - PQ_{R12}\left(\bar{a} - c\left(\frac{1}{c} + \frac{1}{r}\right)R\right)$$

where PQ_{R32} is a positive variable.[128]

At the lowest level of acceptable average efficiency level, $\bar{a} = R(c + r)/r$, the (non-incremental) increase in transmissivity results in a net loss in total welfare in the amount of PQ_{R11}, in absolute value; and a marginal increase in average efficiency results in even greater losses. However, the positive sign of PQ_{R32} suggests that the increase in transmissivity can be welfare increasing at high levels of average efficiency.

At the highest level of average efficiency in this category, when $\bar{a} = \bar{a}_{maxCL}$, the effect is given by:

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[128] $PQ_{R32} = 4\,PQ_{R31} - PQ_{R13}$ or:

$PQ_{R32} = 24c^6 + 178c^5r + 329c^4r^2 + 8c^3r^3 - 416c^2r^4 - 320cr^5 - 64r^6 + 3r(5c^2 + 12cr + 8r^2)^2\sqrt{r}\sqrt{4c + r} + 4(6c^5 - 5c^4r - 60c^3r^2 - 112c^2r^3 - 96cr^4 - 32r^5)\sqrt{c^2 + 4cr + r^2}$.

PQ_{R32} can also be rewritten as a product of positive variables.

$$V^{CL}\left(\bar{a}_{\max CL}+\varepsilon_{MaxX},\bar{a}_{\max CL}-\varepsilon_{MaxX},E,\infty\right)$$
$$-V^{CL}\left(\bar{a}_{\max CL}+\varepsilon_{MaxX},\bar{a}_{\max CL}-\varepsilon_{MaxX},E,0\right) = \frac{R^2 F_{R3}(c,r)}{6c^2 r^2 (c+4r)^2}$$

$F_{R3}(c, r)$ is a function of c and r;[129] its sign depends solely on the ratio r/c, and has only one real root, $(r/c)^* = 0.36$. The effect –on aggregate cumulative profits– of an increased transmissivity, at the highest level of inequality, follows the sign of $F_{R3}(c, r)$, it is negative for low ratios r/c, and positive when the ratio is higher than $(r/c)^* = 0.36$.

For the levels of average efficiency higher than $\bar{a}_{maxCL}$, the maximum inequality is $\varepsilon_{\text{maxCL}}$,[130] derived from the condition on the positive rate of extraction by the less efficient user, with an infinite transmissivity, and the corresponding effect from an increase in transmissivity is as follows:

$$V^{CL}\left(\bar{a}+\varepsilon_{MaxCL},\bar{a}-\varepsilon_{MaxCL},E,\infty\right)-V^{CL}\left(\bar{a}+\varepsilon_{MaxCL},\bar{a}-\varepsilon_{MaxCL},E,0\right) = PQ_{R33} -$$
$$PQ_{R12}\left(\bar{a}-c\left(\frac{1}{c}+\frac{1}{r}\right)R\right) - PQ_{R13}\left(\bar{a}-c\left(\frac{1}{c}+\frac{1}{r}\right)R\right)^2$$

where PQ_{R33} is a positive variable.[131]

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[129] $F_{R3}(c,r) = 8c^5 + 99c^4r + 307c^3r^2 + 161c^2r^3 - 72cr^4 - 32r^5 + 3\sqrt{r}\sqrt{4c+r}(c^4 + 9c^3r + 33c^2r^2 + 56cr^3 + 32r^4) - (16c^4 + 79c^3r + 111c^2r^2 + 136cr^3 + 64r^4)\sqrt{c^2+4cr+r^2} - 3c(c^2 + 7cr + 8r^2)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$

[130] $\varepsilon_{\text{maxCL}} = \frac{(5c^2+12cr+8r^2)R}{c^2+8r^2+c(7r+\sqrt{c^2+4cr+r^2})}$ as introduced earlier in Chapter 2.

[131] $PQ_{R33} = 4\,PQ_{R31}\,\varepsilon^2 - PQ_{R11}$ or:

$PQ_{R33} = R^2\big(-2c^5 + 43c^4r + 143c^3r^2 - 235c^2r^3 - 360cr^4 - 96r^5 + 3(c^4 + 9c^3r + 33c^2r^2 + 56cr^3 + 32r^4)\sqrt{r}\sqrt{4c+r} + c(-14c^3 - 19c^2r + 61cr^2 + 24r^3)\sqrt{c^2+4cr+r^2} - 3c\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}(c^2 + 7cr + 8r^2)\big)/(12c^2r^2(c+4r)^2)$

PQ_{R33} can also be rewritten as a product of positive variables.

At the lowest level of average efficiency in the considered category, the sign of the effect of making the resource more common on the aggregate cumulative profits of highly unequal users depends solely on the ratio *r/c*:

$$V^{CL}\left(\bar{a}_{\max CL}+\varepsilon_{MaxCL},\bar{a}_{\max CL}-\varepsilon_{MaxCL},E,\infty\right) \\ -V^{CL}\left(\bar{a}_{\max CL}+\varepsilon_{MaxCL},\bar{a}_{\max CL}-\varepsilon_{MaxCL},E,0\right) = \frac{R^2\,F_{R3}\left(c,r\right)}{6c^2\,r^2\left(c+4r\right)^2}$$

At higher levels of average efficiency, making the resource more common has a negative effect on aggregate cumulative profits; this outcome is rather the consequence of the conditions for an interior solution, since increasing $\bar{a}$ while keeping the maximum inequality fixed ($\varepsilon_{\text{maxCL}}$) translates into lower levels of inequality in relative terms.

In summary, under feedback strategies, and with the stock at the start (x_0) set at the maximum, the increasing effect of a non-incremental increase in transmissivity on the aggregate cumulative profits of highly unequal users is only conceivable in the case of low costs of extraction (*c* low), combined with a low valuation of future gains and avoided future losses (high *r*). In other terms, the financial externalities from making the resource more common to be limited, for the benefits from inequality to outweigh the losses from non-cooperative use.

B.2.4.2.2 At the Minimum Initial Stock

I consider now the extraction game where the initial stock is set at $x_{Min0\varepsilon}$, the lowest acceptable level for an interior solution;[132] in this situation, the effect of an increase in transmissivity simplifies as:

$$V^{CL}(a_h, a_l, x_{Min0\varepsilon}, \infty) - V^{CL}(a_h, a_l, x_{Min0\varepsilon}, 0) = \frac{PQ_{R34}}{3c^2 r(5c^2+12cr+8r^2)^2}\varepsilon^2 - \frac{PQ_{R35}}{3c^2 r(c+4r)}R\varepsilon - \frac{4c^4\ PQ_{R36}}{r^2(c+4r)}R^2$$

where PQ_{R34}, PQ_{R35}, and PQ_{R36} are all positive variables.[133]

When $\bar{a}$ is lower than $\bar{a}_{maxCL}$, the maximum inequality is given by ε_{MaxX}, and the related effect of an increase in transmissivity simplifies as:

$$V^{CL}(\bar{a}+\varepsilon_{MaxX}, \bar{a}-\varepsilon_{MaxX}, x_{Min0\varepsilon}, \infty) - V^{CL}(\bar{a}+\varepsilon_{MaxX}, \bar{a}-\varepsilon_{MaxX}, x_{Min0\varepsilon}, 0) =$$
$$\frac{PQ_{R34}}{3r(5c^2+12cr+8r^2)^2}\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right)^2 - \frac{PQ_{R35}}{3cr(c+4r)}R\left(\frac{\bar{a}}{c}-\frac{R}{c}-\frac{R}{r}\right) - \frac{4c^4\ PQ_{R36}}{r^2(c+4r)}R^2$$

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[132] $x_{Min0\varepsilon} = \left(E - \frac{a-\varepsilon}{c} + \frac{R}{r} + \frac{R}{2c} - \frac{R\sqrt{r}\sqrt{4c+r}}{2cr}\right)$, as introduced earlier is the minimum stock with a positive level of use by the less efficient user when transmissivity is equal to zero.

[133] $PQ_{R35} = -2c^3 - 5c^2r + 26cr^2 + 8r^3 + (c+2r)(c+4r)\sqrt{r}\sqrt{4c+r} + 2(2c^2+cr-4r^2)\sqrt{c^2+4cr+r^2}$
$-2(c+4r)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$

$PQ_{R34} = 24c^6 + 178c^5r + 329c^4r^2 + 8c^3r^3 - 416c^2r^4 - 320cr^5 - 64r^6 + 3r(5c^2+12cr+8r^2)^2\sqrt{r}\sqrt{4c+r} + 4(6c^5 - 5c^4r$
$- 60c^3r^2 - 112c^2r^3 - 96cr^4 - 32r^5)\sqrt{c^2+4cr+r^2}$

and $PQ_{R36} = PQ_{R36a}/(c^2+6cr+2r^2+2(c+r)\sqrt{c^2+4cr+r^2})PQ_{R36b}$ where:

$PQ_{R36b} = 2c^4 + 7c^3r + 6c^2r^2 + 17cr^3 + 4r^4 + (c+4r)(c^2+r^2)\sqrt{r}\sqrt{4c+r} + (2c^3+2c^2r+cr^2+4r^3)\sqrt{c^2+4cr+r^2} + r(c$
$+ 4r)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$;

$PQ_{R36a} = 2c^4 + 12c^3r + 21c^2r^2 + 32cr^3 + 66r^4 + (2c^3+9c^2r+12cr^2+14r^3)\sqrt{c^2+4cr+r^2}$

At the lowest level of acceptable average efficiency level, $\bar{a} = R(c+r)/r$, the (non-incremental) increase in transmissivity results in a net loss in total welfare;[134] a marginal increase in average efficiency results in even greater losses. However, the positive sign of PQ_{R34} suggests that the increase in transmissivity can be welfare increasing at high levels of average efficiency.

At the highest level of average efficiency in this category, when $\bar{a} = \bar{a}_{maxCL}$, the effect is given by:

$$V^{CL}\left(\bar{a}_{\max CL} + \varepsilon_{MaxX}, \bar{a}_{\max CL} - \varepsilon_{MaxX}, x_{Min0\varepsilon}, \infty\right) - V^{CL}\left(\bar{a}_{\max CL} + \varepsilon_{MaxX}, \bar{a}_{\max CL} - \varepsilon_{MaxX}, x_{Min0\varepsilon}, 0\right) = \frac{R^2 F_{R3b}(c,r)}{6c^2 r^2 (c+4r)^2}$$

where $F_{R3b}(c, r)$ is a function of c and r;[135] its sign depends solely on the ratio r/c, and has only one real root, $(r/c)^* = 0.25$. The effect –on aggregate cumulative profits– of an increased transmissivity, at the highest level of inequality, follows the sign of $F_{R3b}(c, r)$, it is negative for low ratios r/c, and positive when the ratio is higher than $(r/c)^* = 0.25$.

When $\bar{a}$ is higher than $\bar{a}_{maxCL}$, $\varepsilon_{\text{maxCL}}$ becomes the maximum acceptable inequality, and the corresponding effect from an increase in transmissivity is as follows:

$$V^{CL}\left(\bar{a} + \varepsilon_{MaxCL}, \bar{a} - \varepsilon_{MaxCL}, x_{Min0\varepsilon}, \infty\right) - V^{CL}\left(\bar{a} + \varepsilon_{MaxCL}, \bar{a} - \varepsilon_{MaxCL}, x_{Min0\varepsilon}, 0\right) = \frac{R^2 F_{R3b}(c,r)}{6c^2 r^2 (c+4r)^2}$$

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[134] In the amount of $4c^4 R^2 Q_{R36} \big/ r^2 (c+4r)$, in absolute value.

[135] $F_{R3b}(c,r) = 8c^5 + 90c^4 r + 216c^3 r^2 - 101c^2 r^3 - 248cr^4 - 64r^5 + (-2c^4 - 18c^3 r - 11c^2 r^2 + 120cr^3 + 64r^4)\sqrt{r}\sqrt{4c+r} - (16c^4 + 70c^3 r + 73c^2 r^2 + 120cr^3 + 32r^4)\sqrt{c^2 + 4cr + r^2} + (4c^3 + 25c^2 r + 56cr^2 + 32r^3)\sqrt{r}\sqrt{4c+r}\sqrt{c^2 + 4cr + r^2}$

The sign of the effect of making the resource more common on the aggregate cumulative profits of highly unequal users depends solely on the ratio *r/c*:

In summary, when stock at the start is set at the minimum, the increasing effect from making the resource more common on the aggregate cumulative profits of highly unequal users is only possible under the condition of a ratio *r/c* higher than *(r/c)** = 0.25. This result portrays the same requirement drawn in the case of a maximum stock at the start, where the benefits from inequality are only expected to outweigh the losses from non-cooperative use when the financial externalities from making the resource more common are limited, but the condition is less stringent with the minimum stock.

B.2.4.2.3 The Effect of the Starting Stock

In this subsection, the aim is to provide more insight on the previous result, by showing that the effect of an increase in transmissivity on the aggregate cumulative profits is declining in the level of stock at the start, x_0, as evidenced by the negative sign, everywhere, of $\partial x V^{CL}(a_h, a_l, x_0)$, its derivative w.r.t x_0, given by:[136]

$$\partial x V^{CL}(a_h, a_l, x_0) = -PQ_{R12} - 2PQ_{R13}\left(\frac{a_h + a_l}{2c} - E - \left(\frac{1}{c} + \frac{1}{r}\right)R + x_0\right)$$

At the minimum stock, the derivative is always negative and writes as follows:

$$\partial x V^{CL}(a_h, a_l, x_{Min0\varepsilon}) = -\frac{PQ_{R37}R}{3cr(c+4r)} - \frac{4c^3}{PQ_{R38}}(a_h - a_l)$$

[136] $\partial x V^{CL}(a_h, a_l, x_0) = \dfrac{\partial\left(V^{CL}(a_h, a_l, x_0, \infty) - V^{CL}(a_h, a_l, x_0, 0)\right)}{\partial x_0}$

where PQ_{R37} and PQ_{R38} are both positive.[137]

The derivative is linear and decreasing in the level of stock, in addition, it is negative at the minimum acceptable level of stock, therefore the derivative is negative everywhere.[138] This result reinforces the previous result regarding the lesser requirement on the ratio at the minimum stock, for a shift in transmissivity to produce an increase in profits, in comparison to the requirement at the maximum stock.

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[137] $PQ_{R37} = -2c^3 - 5c^2r + 26cr^2 + 8r^3 + (c+2r)(c+4r)\sqrt{r}\sqrt{4c+r} + 2(2c^2 + cr - 4r^2)\sqrt{c^2+4cr+r^2} - 2(c+4r)\sqrt{r}\sqrt{4c+r}\sqrt{c^2+4cr+r^2}$

$PQ_{R38} = (2c + r + 3\sqrt{r}\sqrt{4c+r} + 4\sqrt{c^2+4cr+r^2})(2c^2 + 4cr + r^2 + (2c+r)\sqrt{r}\sqrt{4c+r})$

PQ_{R37} is positive everywhere (for r and c positive).

[138] At the maximum stock, for instance, the derivative is negative for any acceptable level efficiency:

$$\partial x V^{CL}(a_h, a_l, E) = -PQ_{R12} - 2PQ_{R13}\left(\frac{\bar{a}}{c} - \frac{R}{c} - \frac{R}{r}\right)$$

B.3 Solving for the Social Optimum

B.3.1 Deriving the Eigenvectors

If V = [y_1, y_2, y_3, y_4] is the eigenvector associated with α, eigenvalue of matrix A_2, then V needs to satisfy the condition A_2 V = α V, that can be written as a system of four linear equations:

$$-(c+s)\,y_1 \quad +s\,y_2 \quad +y_3 \quad =\alpha\,y_1 \qquad (B_3.1)$$

$$s\,y_1 \quad -(c+s)\,y_2 \quad +y_4 \quad =\alpha\,y_2 \qquad (B_3.2)$$

$$-c^2\,y_1 \quad +(r+s+c)\,y_3 \quad -s\,y_4 \quad =\alpha\,y_3 \qquad (B_3.3)$$

$$-c^2\,y_2 \quad -s\,y_3 \quad +(r+s+c)\,y_4 \quad =\alpha\,y_4 \qquad (B_3.4)$$

The values of y_3 and y_4, as functions of y_1 and y_2, are drawn from (B$_3$.1) and (B$_3$.2):

$$y_3 = (s+c+\alpha)y_1 \quad -s\,y_2 \qquad (B_3.1b)$$

$$y_4 = \quad -s\,y_1 \;+\; (s+c+\alpha)y_2 \qquad (B_3.2b)$$

Substituting the new values for y_3 and y_4 in (B$_3$.3) and (B$_3$.4) and factorizing in y_1 and y_2 gives:

$$y_1\left(-c^2+(r+s+c)(s+c+\alpha)+s^2-\alpha\,(s+c+\alpha)\right)+y_2\left(-s\,(r+s+c)-s(s+c+\alpha)+\alpha\,s\right)=0$$

$$y_2\left(-c^2+s^2+(r+s+c)\,(s+c+\alpha)-\alpha\,(s+c+\alpha)\right)+y_1\left(-s\,(s+c+\alpha)-s(r+s+c)+\alpha\,s\right)=0$$

Adding and subtracting the two last equations provides, after simplification, an equivalent system of equations:

$$\left(r\,(\alpha+c)-\alpha^2\right)\left(y_1+y_2\right) \quad =0 \qquad (B_3.3b)$$

$$\left((2s+c)(2s+r)+2\,s\,c+\alpha\,r-\alpha^2\right)\left(y_1-y_2\right) \; =0 \qquad (B_3.4b)$$

I differentiate two cases, depending on the value of the eigenvalue; When α is the negative root of equation $EQ_3(\alpha) = 0$,[139] the first equality (B_3.3b) is valid for any pair of variables while the second equality (B_3.4b) is only true if $y_2 = y_1$. This result, together with the previous results in (B_3.1b) and (B_3.2b) shows that the eigenvector associated with α_3 is any multiple of v_3:

$$v_3 = \begin{bmatrix} 1 \\ 1 \\ c+\alpha_3 \\ c+\alpha_3 \end{bmatrix}$$

For the case with $\alpha = \alpha_4$, the negative root of equation $EQ_4(\alpha) = 0$, (B_3.3b) is only true if $y_2 = -y_1$. Compiling all results shows that the eigenvector in this instance is v_4:

$$v_4 = \begin{bmatrix} 1 \\ -1 \\ c+\alpha_4+2s \\ -c-\alpha_4-2s \end{bmatrix}$$

B.3.2 Showing the positive sign of $P_5(s)$ and $P_6'(s)$

$P_5(s)$ is a nonnegative function of s defined as: $P_5(s)=\dfrac{PQ5_{SO1}}{PQ5_{SO2}}$

with: $$PQ5_{SO2} = 2\sqrt{r}\left(\sqrt{r}+\sqrt{4c+r}\right)\left(2s(r+2s)+c(r+4s)\right)\left(\sqrt{r}\sqrt{4c+r}+\sqrt{r+4s}\sqrt{4c+r+4s}\right)$$

$$PQ5_{SO1} = 4cr\left(\frac{cr^2(4c+r)+2rs(4c+r)^2+16s^2(2c+r)^2+32s^3(2c+r)+16s^4}{r^2+8rs+8s^2+4c(r+4s)+(r+4s)\sqrt{r}\sqrt{4c+r}}\right)$$
$$+4c\sqrt{r+4s}\sqrt{4c+r+4s}\left(\frac{cr(r+4s)^2+2s\left(r^3+2r^2s+\left(2rs+8s^2\right)\sqrt{r}\sqrt{4c+r}+8s^3\right)}{8s^2+(r+4s)\sqrt{r}\sqrt{4c+r}+r^2}\right)$$

$P_6'(s)$ the derivative of $P_6(s)$ w.r.t s, and is given by: $P_6'(s)=\dfrac{4c(r+2s)PQ6_{SO1}}{PQ6_{SO2}PQ6_{SO3}}$

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[139] $EQ_3(\alpha) = \left(r(\alpha+c)-\alpha^2\right)$, and $EQ_4(\alpha) = \left((2s+c)(2s+r)+2sc+\alpha r-\alpha^2\right)$.

with: $$PQ6_{SO1} = (r+4s)\left(r^2+10rs+8s^2\right)+c\left(6r^2+44rs+32s^2\right) + \left(r^2+2(6c+r)s+8s^2\right)\sqrt{r+4s}\,\sqrt{4c+r+4s}$$

$$PQ6_{SO2} = r\sqrt{r+4s}\,\sqrt{4c+r+4s}\left(6c+r+4s+\sqrt{r+4s}\,\sqrt{4c+r+4s}\right)$$, and

$$PQ6_{SO3} = \left(r^2+8s(c+s)+2r(c+2s)+r\sqrt{r+4s}\,\sqrt{4c+r+4s}\right)^2$$

B.4 Alternative categories of inequality

B.4.1 Inequality in the derivative of marginal productivity

In this sub-section, I use the same continuous model as in Chapter 2, with one variation: the production function of player *i*, given her level of extraction of water w_{it}, is now given by:

$$F_i(w_{it}) = a\,w_{it} - \frac{b_i\,w_{it}^2}{2}. \qquad i=h,l$$

where b_h , $b_l > 0$

In the new setting, the two users have identical linear coefficient but different quadratic coefficients, which leads to inequality in productivity; when $b_h < b_l$, for any amount of water w_t, player *h* will derive more profits than player *l*.

With an interior solution for the corresponding maximization problem, the stocks and extraction decisions at the steady state are given by:

$$x_i^{SS} = E - \frac{a}{c} + \frac{R(r+s)}{r(r+2s)} + \frac{R}{2c}\left(b_i + b_{-i}\right) + \frac{(b_i - b_{-i})\;r\,R(r+2s)\;(c - s(b_i - b_{-i}))}{2c(b_i + b_{-i})\,r\,s\,(r+2s) + 2c^2\,(r^2+4rs+2s^2)} \qquad i=h,l$$

$$w_i^{SS} = R + (b_{-i} - b_i)\frac{r\,R\,s\,(r+2\,s)}{(b_i + b_{-i})\,r\,s\,(r+2\,s) + c\,(r^2 + 4rs + 2s^2)} \qquad i=h,l$$

The aggregate profit of players h and l at the steady state is obtained by summing up the profits of the two players as:

$$TNP^{SS-OL}(b_h, b_l, s) = \frac{(b_h + b_l)}{2}R^2 + \frac{2\,c\,R^2\,(r+s)}{r\,(r+2\,s)} - (b_h - b_l)^2\,\frac{r^2\,R^2\,s\,(r+2\,s)^2\,\,(2\,c + s\,(b_h + b_l))}{2\big((b_h + b_l)\,r\,s\,(r+2\,s) + c\,(r^2 + 4rs + 2s^2)\big)^2}$$

where the last term is negative and increasing, in absolute value, in s and in the degree of inequality. It shows that inequality causes losses that are proportional in magnitude to the square of ($b_h - b_l$), and that more transmissivity enhances those losses.

B.4.2 Inequality in the Natural Capital

In this sub-section, I use the same continuous model as in Chapter 2, where the two users have the same production function:

$$F_i(w_{it}) = a\,w_{it} - \frac{w_{it}^2}{2}. \qquad i=h,l$$

I assume now that the two users have different rates of recharge:

$$\dot{x}_{it} = R_i - w_{it} + s(x_{-it} - x_{it}) \qquad i=h,l$$

where R_h , $R_l > 0$

With the new specification and assuming the solution to the –new– maximization problem implies an interior solution at the steady state, the corresponding stocks and extraction decisions are given by:

$$x_i^{SS} = E - \frac{a}{c} + \frac{(R_i + R_{-i})}{2}\left(\frac{r+s}{r(r+2s)} + \frac{1}{c}\right) + \frac{(R_i - R_{-i})}{2}\left(\frac{c\,(r+s) + r\,(r+2s)}{2rs(r+2s) + c(r^2+4rs+2s^2)}\right) \qquad i=h,l$$

$$w_i^{SS} = \frac{(R_i + R_{-i})}{2} + (R_i - R_{-i})\frac{c\,r\,(r+2s)}{4\,rs(r+2s) + 2c(r^2+4rs+2s^2)} \qquad i=h,l$$

The aggregate profit of players *h* and *l* at the steady state is obtained by summing up the profits of the two players as:

$$TNP^{SS-OL}(b_h, b_l, s) = \left(\frac{R_h + R_l}{2}\right)^2\left(1 + \frac{2c\,(r+s)}{r(r+2s)}\right) + (R_h - R_l)^2\,\frac{c^2\,r\,(r+2s)\,(2c(r+s) + r(r+2s))}{4\left(2rs(r+2s) + c(r^2+4rs+2s^2)\right)^2}$$

In this case, inequality in natural capital has an increasing effect on the aggregate profits, and the extra benefits from inequality are proportional in magnitude to the square of ($R_h - R_l$). On the other hand, transmissivity has a decreasing effect on the profits from inequality, as a consequence, higher transmissivity always decreases the aggregate profits.

B.4.3 Further discussion of the effect of inequality

Extrinsic Inequality

In this part I consider the effects of inequality on the non-cooperative use of an aquifer by two farmers *i* and *j* for a general concave function $f(w)$. The inequality is introduced by considering that the production function of players *i* and *j* is such that:

$$f_i(w_i) = (1+\alpha)f(w_i) \qquad (1.a)$$

$$f_j(w_j) = (1-\alpha)f(w_j) \qquad (1.b)$$

where $0 \leq \alpha \leq 1$.

Given s the transmissivity of water, the stocks of water x_{it} and x_{jt} beneath players i and j evolve following the equations of motion previously established:

$$\dot{x}_{it} = R - w_{it} + s\left(x_{jt} - x_{it}\right) \tag{2.a}$$

$$\dot{x}_{jt} = R - w_{jt} + s\left(x_{it} - x_{jt}\right) \tag{2.b}$$

User i (similarly user j) maximizes his net payoff:

$$\underset{w_{it}}{Max}\int_0^{\infty} e^{-rt}\left(f_i\left(w_{it}\right) - w_{it}c\left(h - x_{it}\right)\right)dt \tag{3}$$

If we denote by λ_{it} the value attributed by player i, at time t to a unit increase in her own stock x_{it}, and μ_{it} the value the same player attributes at time t to a marginal increase in her neighbor's stock x_{jt}, then, at any time t, player i's problem is to pick w_{it} that maximizes the current value Hamiltonian:

$$H_{it} = f_i\left(w_{it}\right) - w_{it}c\left(h - x_{it}\right) + \lambda_{it}\left(R - w_{it} + s(x_{jt} - x_{it})\right) + \mu_{it}\left(R - w_{jt} + s(x_{it} - x_{jt})\right) \tag{4}$$

With an interior solution, w_{it} needs to satisfy:

C1 $$0 = \partial H_{it} \big/ \partial w_{it} = \left(1 + \alpha\right) f'\left(w_{it}\right) - c\left(h - x_{it}\right) - \lambda_{it} . \tag{5}$$

Following the Pontryagin's Maximum Principle along the optimal path, the evolution over time of the co-state variables (λ_{it} and μ_{it}) satisfies the conditions:

C2 $$\dot{\lambda}_{it} = r\lambda_{it} - \partial H_{it} \big/ \partial x_{it} = -w_{it}c + (r + s)\lambda_{it} - s\mu_{it} \tag{6}$$

C3 $$\dot{\mu}_{it} = r\,\mu_{it} - \partial H_{it} \big/ \partial x_{jt} = -s\lambda_{it} + \left(r + s\right)\mu_{it} . \tag{7}$$

In addition to the equations of motion in both stocks.

Player j faces a similar maximization problem and decides to strategize in the same manner, she establishes her own current Hamiltonian H_{jt}:

$$H_{jt} = \left(f_j\left(w_{jt}\right) - w_{jt}c\left(h - x_{jt}\right)\right) + \lambda_{jt}\left(R - w_{jt} + s(x_{it} - x_{jt})\right) + \mu_{jt}\left(R - w_{it} + s(x_{jt} - x_{it})\right) \quad (8)$$

Her optimal path is restricted by the same transition equations (using the current time formulations this time) in stocks and satisfies similar conditions in w_{jt}, $\dot{\lambda}_{jt}$ and $\dot{\mu}_{jt}$:

C4: $$0 = \partial H_{jt} / \partial w_{jt} = (1-\alpha) f'\left(w_{jt}\right) - c\left(h - x_{jt}\right) - \lambda_{jt} \quad (9)$$

C5 $$\dot{\lambda}_{jt} = r\lambda_{jt} - \partial H_{jt} / \partial x_{jt} = r\lambda_{jt} - w_{jt}c + s\lambda_{jt} - s\mu_{jt} \quad (10)$$

C6 $$\dot{\mu}_{jt} = r\mu_{jt} - \partial H_{jt} / \partial x_{it} = -s\lambda_{jt} + (r+s)\mu_{jt} \quad (11)$$

Steady state

The steady state is reached when the system stops moving which means:

$$\dot{\lambda}_{it} = \dot{\mu}_{it} = \dot{\lambda}_{jt} = \dot{\mu}_{jt} = \dot{x}_{it} = \dot{x}_{jt} = 0 \quad (12)$$

From C2, C3, C5 and C6 we derive that at the steady state:

$$\lambda_i = \frac{c(r+s)w_i}{r(r+2s)},\ \mu_i = \frac{c\,s\,w_i}{r(r+2s)},\ \lambda_j = \frac{c(r+s)w_j}{r(r+2s)} \text{ and } \mu_j = \frac{c\,s\,w_j}{r(r+2s)} \quad (13)$$

From C1 we solve for x_i: $$x_i = h + \frac{(r+s)w_i}{r(r+2s)} - \frac{(1+\alpha)f'\left(w_i\right)}{c} \quad (14)$$

Given the equation of motion in 2.a we draw x_j:

$$x_j = x_i + \frac{w_i - R}{s} = h + \frac{\left(r^2 + 3rs + s^2\right)w_i}{rs(r+2s)} - \frac{(1+\alpha)f'\left(w_i\right)}{c} - \frac{R}{s} \quad (15)$$

And given 2.b we solve for w_j: $$w_j = 2R - w_i \tag{16}$$

Finally, using all the previous results we rewrite C4:

$$\partial H_{it} \big/ \partial w_{it} = (1-\alpha) f'(2R - w_i) - (1+\alpha) f'(w_i) + \frac{c(r^2 + 4rs + 2s^2)(w_i - R)}{rs(r+2s)} = 0 \tag{17}$$

If we define the function constrainte(α, w_i):

$$\text{constrainte}(\alpha, w_i) = (1-\alpha) f'(2R - w_i) - (1+\alpha) f'(w_i) + \frac{c(r^2 + 4rs + 2s^2)(w_i - R)}{rs(r+2s)}$$

w_i is therefore defined as: $w_i(\alpha) = \{w_i / \text{constrainte}(\alpha, w_i) = 0\}$. When $\alpha = 0$ the equation *constrainte*(α, w_i) = 0 has a unique solution w_i = R[140].

Completely differentiating the condition at the optimum ($\text{constrainte}(\alpha, w_i) = 0$) we find:

$$\frac{\partial\, constrainte(\alpha, w_i)}{\partial\, \alpha} d\alpha + \frac{\partial\, constrainte(\alpha, w_i)}{\partial\, w_i} dw_i = 0$$

which allows us to derive the derive $d\,w_i \big/ d\alpha$:

$$\frac{d\,w_i}{d\alpha} = -\frac{f'(2R - w_i) + f'(w_i)}{-(1-\alpha) f''(2R - w_i) - (1+\alpha) f''(w_i) + \dfrac{c(r^2 + 4rs + 2s^2)}{rs(r+2s)}} > 0 \tag{18}$$

The aggregate profits of players *i* and *j* at the steady state is given by:

$$\pi_T(\alpha, w_i) = f_i(w_i) - c\,w_i\,(h - x_i) + f_j(w_j) - c\,w_j\,(h - x_j)$$

140 $constrainte(0, w_i) = f'(2R - w_i) - f'(w_i) + \frac{c(r^2 + 4rs + 2s^2)(w_i - R)}{rs(r+2s)}$. *f'* is decreasing therefore for w_i > R *constrainte*(α, w_i) > 0 while *constrainte*(α, w_i) < 0 when w_i < R.

$$\pi_T(\alpha, w_i) = (1+\alpha)f(w_i) + (1-\alpha)f(2R - w_i) - 2R(1+\alpha)f'(w_i) + \frac{(3r^2 + 8rs + 2s^2)cRw_i}{rs(r+2s)} - \frac{2cR^2}{s} - \frac{cw_i^2}{s} \tag{19}$$

The derivative of $\pi_T(\alpha, w_i)$ with regard to α is given by:

$$\frac{d\pi_T(\alpha, w_i)}{d\alpha} = \frac{\partial \pi_T(\alpha, w_i)}{\partial \alpha} + \frac{\partial \pi_T(\alpha, w_i)}{\partial w_i}\frac{dw_i}{d\alpha} \tag{20}$$

$$\frac{d\pi_T(\alpha, w_i)}{d\alpha} = \frac{\left(-2R(1+\alpha)f''(w_i) + \frac{c(-r^2+2s^2)w_i}{rs(r+2s)} + \frac{2cR}{s}\right)(f'(2R-w_i) + f'(w_i))}{-(1-\alpha)f''(2R-w_i) - (1+\alpha)f''(w_i) + \frac{c(r^2+4rs+2s^2)}{rs(r+2s)}} + (f(w_i) - f(2R-w_i) - 2Rf'(w_i)) \tag{21}$$

The change in total welfare at the steady state due to an incremental level of inequality is given by $\frac{d\pi_T(\alpha, w_i)}{d\alpha}$ at the origin, $\alpha = 0$, but for α null w_i equals R and from (21): $\left.\frac{d\pi_T}{d\alpha}\right|_{\alpha=0} = 0$

Given that $\left.\frac{d\pi_T}{d\alpha}\right|_{\alpha=0} = 0$ the sign of the aggregate welfare change can be derived by totally differentiating $\frac{d\pi_T(\alpha, w_i)}{d\alpha}$ with regard to α and calculating $\frac{d^2\pi_T(\alpha, w_i)}{d\alpha^2}$ at the origin (*$\alpha = 0$* and *$w_i = R$*).

To show that:

$$\left.\frac{d^2\pi_T}{d\alpha^2}\right|_{\alpha=0} = \frac{8rs(r+2s)f'(R)}{\left(c(r^2+4rs+2s^2) - 2rs(r+2s)f''(R)\right)^2}\begin{pmatrix} -cR(r^2+4rs+2s^2)f''(R) + 2rsR(r+2s)f''(R)^2 \\ +sf'(R)(2c(r+s) + rs(r+2s)(-f''(R) - Rf'''(R))) \end{pmatrix}$$
(22)

Therefore the change in welfare depends on the sign and magnitude of $Rf'''(R)$; all other terms are positive and favor an increase in welfare for a low level of inequality. If *f*(.)is a quadratic function then $f'''(R) = 0$, and the introduction of a low enough level of inequality leads unequivocally to the increase of aggregate welfare.

For s infinite:

$$\left.\frac{d^2\pi_T}{d\alpha^2}\right|_{\alpha=0} = \frac{8r\,f'(R)}{\left(c-2r\,f''(R)\right)}\left(-R\,f''(R)+\left(\frac{8r\,f'(R)}{c-2r\,f''(R)}\right)\left(c-r\,f''(R)-r\,R\,f'''(R)\right)\right) \qquad (23)$$

Intrinsic Inequality

In this part the inequality is introduced by considering that the production function of players *i* and *j* is such that:

$$f_i(w_i) = f\left((1+\alpha)w_i\right) \qquad (24.a)$$

$$f_j(w_j) = f\left((1-\alpha)w_j\right) \qquad (24.b)$$

An analysis similar to the one done with extrinsic inequality leads to the same conclusion, $\left.\frac{d\,\pi_T}{d\alpha}\right|_{\alpha=0} = 0$, it is therefore the sign of $\left.\frac{d^2\pi_T}{d\alpha^2}\right|_{\alpha=0}$ that will indicate the change in welfare resulting from a low level of inequality.

The formula for s finite is rather complicated I will only present here $\left.\frac{d^2\pi_T}{d\alpha^2}\right|_{\alpha=0}$ for s infinite:

$$\left.\frac{d^2\pi_T}{d\alpha^2}\right|_{\alpha=0} = A\begin{pmatrix}16r^2R\,f'(R)f''(R)^2+8rf'(R)^2\left(c-r\,f''(R)\right)-2R^2f''(R)\left(c^2-8r^2f''(R)^2\right)\\ -2R\left(c\,R+2r\,f'(R)\right)^2 f'''(R)\end{pmatrix}$$
(25)

Appendix C Solving for the Discrete Model

C.1 Subgame Perfect Equilibrium Dynamic Programming

$$\alpha_{1r} = -1 + \frac{c}{2\,c - 8\,D_{1n} + 8\,D_{2n}} + \frac{c^2 + 4\,(D_{1n} - D_{2n}) - c\,(2\,D_{1n} + D_{3n})}{(c - 4\,D_{1n} + 4\,D_{2n})(-2 + 4\,D_{1n} + 2\,D_{3n} + c\,(-2+s) - 4\,D_{1n}\,s + 4\,s\,D_{2n})} + \frac{2 - c\,s}{2(2 + 2c - 4D_{1n} + 2D_{3n} - 3c\,s + 12D_{1n}\,s + 4D_{2n}\,s - 8D_{3n}\,s - 8D_{1n}\,s^2 - 8D_{2n}\,s^2 + 8D_{3n}\,s^2)}$$

$$\alpha_{2r} = \frac{c}{2\,c - 8\,D_{1n} + 8\,D_{2n}} + \frac{c^2 + 4\,(D_{1n} - D_{2n}) - c\,(2\,D_{1n} + D_{3n})}{(c - 4\,D_{1n} + 4\,D_{2n})(-2 + 4\,D_{1n} + 2\,D_{3n} + c\,(-2+s) - 4\,D_{1n}\,s + 4\,s\,D_{2n})} - \frac{2 - c\,s}{2(2 + 2c - 4D_{1n} + 2D_{3n} - 3c\,s + 12D_{1n}\,s + 4D_{2n}\,s - 8D_{3n}\,s - 8D_{1n}\,s^2 - 8D_{2n}\,s^2 + 8D_{3n}\,s^2)}$$

$$\begin{aligned}\alpha_{3ri} = {} & \frac{-\,(a_i + a_{-i})(c - 4\,D_{1n} + 4D_{2n})}{(c - 4\,D_{1n} + 4\,D_{2n})(-2 + 4\,D_{1n} + 2\,D_{3n} + c\,(-2+s) - 4\,D_{1n}\,s + 4\,s\,D_{2n})} \\ & + \frac{a_i - a_{-i}}{2(1 + c - 2D_{1n} + D_{3n}) - (3c - 4(3D_{1n} + D_{2n} - 2D_{3n}))s - 8(D_{1n} + D_{2n} - D_{3n})s^2} \\ & + \frac{4\,D_{1n} + 2D_{3n} - 4\,D_{1n}s + 4D_{2n}\,s}{-2 + 4\,D_{1n} + 2\,D_{3n} + c\,(-2+s) - 4\,D_{1n}\,s + 4\,s\,D_{2n}}R + \frac{(D_{4\mathrm{ni}} + D_{4\mathrm{n\text{-}i}}) - (D_{5\mathrm{ni}} + D_{5\mathrm{n\text{-}i}})}{c - 4\,D_{1n} + 4\,D_{2n}} \qquad i=h,l \\ & + \frac{(2 + c - 4D_{2n} - 2D_{3n})(D_{4\mathrm{ni}} + D_{4\mathrm{n\text{-}i}}) - 2(1 + c - 2D_{1n} - D_{3n})(D_{5\mathrm{ni}} + D_{5\mathrm{n\text{-}i}})}{(c - 4\,D_{1n} + 4\,D_{2n})(-2 + 4\,D_{1n} + 2\,D_{3n} + c\,(-2+s) - 4\,D_{1n}\,s + 4\,s\,D_{2n})} \\ & + \frac{(1-s)(D_{4ni} - D_{4n\text{-}i}) + s\,(D_{5ni} - D_{5n\text{-}i})}{2 - 4\,D_{1n} + 2D_{3n} + c\,(2 - 3s) + 4\,s\,(3\,D_{1n} + D_{2n} - 2\,D_{3n} - 2\,(D_{1n} + D_{2n} - D_{3n})s)}\end{aligned}$$

$$\begin{aligned}D_{1r} = {} & \frac{\alpha_{1r}}{2}(-\alpha_{1r} + c(-2 - \alpha_{1r} + s(1 + \alpha_{1r} - \alpha_{2r}))) + D_{2n}\,(s(1 + \alpha_{1r} - \alpha_{2r}) + \alpha_{2r})^2 \\ & - D_{3n}\,(s(1 + \alpha_{1r} - \alpha_{2r}) + \alpha_{2r})((-1+s)(1 + \alpha_{1r}) - s\,\alpha_{2r}) + D_{1n}\,(1 + \alpha_{1r} + s(-1 - \alpha_{1r} + \alpha_{2r}))^2\end{aligned}$$

$$\begin{aligned}D_{2r} = {} & D_{2n}\,(1 + \alpha_{1r} - s(1 + \alpha_{1r} - \alpha_{2r}))^2 + D_{1n}\,(s\,(1 + \alpha_{1r} - \alpha_{2r}) + \alpha_{2r})^2 \\ & - D_{3n}\,(s(1 + \alpha_{1r} - \alpha_{2r}) + \alpha_{2r})((-1+s)(1 + \alpha_{1r}) - s\,\alpha_{2r}) \\ & + \frac{\alpha_{2p}}{2}(-\alpha_{2r} - c(s(1 + \alpha_{1r} - \alpha_{2r}) + \alpha_{2r}))\end{aligned}$$

$$
\begin{aligned}
D_{3r}= \quad & D_{3n}\left((1+\alpha_{1r})^2-2s\left(1+\alpha_{1r}-\alpha_{2r}\right)^2+2s^2\left(1+\alpha_{1r}-\alpha_{2r}\right)^2+\alpha_{2r}{}^2\right)\\
&+\frac{1}{2}\left(-2\alpha_{1r}\,\alpha_{2r}-4D_{1n}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)\left((-1+s)(1+\alpha_{1r})-s\,\alpha_{2r}\right)\right.\\
&-4D_{2n}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)\left((-1+s)(1+\alpha_{1r})-s\,\alpha_{2r}\right)\\
&\left.-c\left(s\left(\alpha_{1r}-\alpha_{2r}\right)\left(1+\alpha_{1r}-\alpha_{2r}\right)+2(1+\alpha_{1r})\alpha_{2r}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
D_{4ri}= \; & -D_{3n}R\left(1+\alpha_{1r}+\alpha_{2r}\right)-2D_{2n}R\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)+2D_{1n}R\left((-1+s)(1+\alpha_{1r})-s\,\alpha_{2r}\right)+a_i\,\alpha_{1r}\\
&+D_{5ni}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)+D_{4ni}\left(1+\alpha_{1r}-s\left(1+\alpha_{1r}-\alpha_{2r}\right)\right)+\frac{1}{4}\left(c\left(-2+s\right)(1+\alpha_{1r})-c\,s\,\alpha_{2r}\right.\\
&\left.+2\left(-\alpha_{1r}+D_{3n}\left(1+\alpha_{1r}+\alpha_{2r}\right)+2D_{2n}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)+2D_{1n}\left(1+\alpha_{1r}-s\left(1+\alpha_{1r}-\alpha_{2r}\right)\right)\right)\right)\left(\alpha_{3ri}+\alpha_{3r-i}\right)\\
&+\frac{1}{4}\left(c\left(-2+s-2\alpha_{1r}+3s\,\alpha_{1r}-s\,\alpha_{2r}\right)+2\left(-\alpha_{1r}-D_{3n}(1-2s)^2\left(1+\alpha_{1r}-\alpha_{2r}\right)\right.\right.\\
&\left.\left.+2D_{2n}(-1+2s)\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)+2D_{1n}(-1+2s)\left((-1+s)(1+\alpha_{1r})-s\,\alpha_{2r}\right)\right)\right)\left(\alpha_{3ri}-\alpha_{3r-i}\right)
\end{aligned}
$$

$$
\begin{aligned}
D_{5ri}= \; & D_{4ni}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)+D_{5ni}\left(1+\alpha_{1r}-s\left(1+\alpha_{1r}-\alpha_{2r}\right)\right)+2D_{2n}\left((-1+s)(1+\alpha_{1r})-s\,\alpha_{2r}\right)\\
&\left(R-s\,\alpha_{3ri}+(-1+s)\alpha_{3r-i}\right)+D_{3n}\left(-R\left(1+\alpha_{1r}+\alpha_{2r}\right)+\left(1+\alpha_{1r}+2(-1+s)s\left(1+\alpha_{1r}-\alpha_{2r}\right)\right)\alpha_{3ri}\right.\\
&\left.+\left(-2(-1+s)s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)\alpha_{3r-i}\right)+\frac{1}{2}\left(2a_i\,\alpha_{2r}-2\alpha_{2r}\,\alpha_{3ri}-4D_{1n}\left(s\left(1+\alpha_{1r}-\alpha_{2r}\right)+\alpha_{2r}\right)\right.\\
&\left.\left(R+(-1+s)\alpha_{3ri}-s\,\alpha_{3r-i}\right)-c\left(2\alpha_{2r}\,\alpha_{3ri}+s\left(\left(1+\alpha_{1r}-2\alpha_{2r}\right)\alpha_{3ri}+\alpha_{2r}\,\alpha_{3r-i}\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
D_{6ri}= \; & \frac{1}{2}\left(2D_{6ni}+2D_{3n}R^2+2a_i\,\alpha_{3ri}-2D_{3n}R\,\alpha_{3ri}-\alpha_{3ri}{}^2-c\,\alpha_{3ri}{}^2+c\,s\,\alpha_{3ri}{}^2+2D_{3n}s\,\alpha_{3ri}{}^2-2D_{3n}s^2\,\alpha_{3ri}{}^2\right.\\
&-2D_{3n}R\,\alpha_{3r-i}+2D_{3n}\,\alpha_{3ri}\,\alpha_{3r-i}-c\,s\,\alpha_{3ri}\,\alpha_{3r-i}-4D_{3n}s\,\alpha_{3ri}\,\alpha_{3r-i}+4D_{3n}s^2\,\alpha_{3ri}\,\alpha_{3r-i}+2D_{3n}s\,\alpha_{3r-i}{}^2\\
&-2D_{3n}s^2\,\alpha_{3r-i}{}^2+2D_{5ni}\left(-R+s\left(\alpha_{3ri}-\alpha_{3r-i}\right)+\alpha_{3r-i}\right)+2D_{2n}\left(R-s\,\alpha_{3ri}+(-1+s)\alpha_{3r-i}\right)^2\\
&\left.+2D_{1n}\left(R+(-1+s)\alpha_{3ri}-s\,\alpha_{3r-i}\right)^2+2D_{4ni}\left(-R-s\left(\alpha_{3ri}-\alpha_{3r-i}\right)+\alpha_{3ri}\right)\right)
\end{aligned}
$$

C.2 Social Optimum Dynamic Programming

$$\alpha_{1n}^{SO} = \frac{1}{2}\left(-2 + \frac{1}{1+c-\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)} + \frac{1-cs}{1+c-2cs-\left(2D_{1n}^{SO}-D_{3n}^{SO}\right)(1-2s)^2} \right)$$

$$\alpha_{2n}^{SO} = \frac{1}{2}\left(\frac{1}{1+c-\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)} - \frac{1-cs}{1+c-2cs-\left(2D_{1n}^{SO}-D_{3n}^{SO}\right)(1-2s)^2} \right)$$

$$\alpha_{3ni}^{SO} = \frac{1}{2}\left(\frac{a_i + a_{-i} + D_{4n}^{SO} + D_{4n-i}^{SO} - 2\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)R}{1+c-\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)} + \frac{a_i - a_{-i} + (1-2s)\left(D_{4n}^{SO}-D_{4n-i}^{SO}\right)}{1+c-2cs-\left(2D_{1n}^{SO}-D_{3n}^{SO}\right)(1-2s)^2} \right)$$

$$\begin{aligned} D_{1r}^{SO} = \; & \frac{1}{2}\Big(-\alpha_{1r}^{SO2} - \alpha_{2r}^{SO2} - 2D_{3n}^{SO}\left(s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)+\alpha_{2r}^{SO}\right)\left((-1+s)\left(1+\alpha_{1r}^{SO}\right)-s\,\alpha_{2r}^{SO}\right) + 2D_{1n}^{SO}\Big(\left(1+\alpha_{1r}^{SO}\right)^2 \\ & -2s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)^2 + 2s^2\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)^2 + \alpha_{2r}^{SO2}\Big) + c\Big(\alpha_{1r}^{SO}\left(-2+s+(-1+s)\alpha_{1r}^{SO}\right) \\ & -s\left(1+2\alpha_{1r}^{SO}\right)\alpha_{2r}^{SO} + (-1+s)\alpha_{2r}^{SO2}\Big)\Big) \end{aligned}$$

$$D_{2r}^{SO} = D_{1r}^{SO};$$

$$\begin{aligned} D_{3r}^{SO} = \; & -2\,\alpha_{1r}^{SO}\,\alpha_{2r}^{SO} - 4D_{1n}^{SO}\left(s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)+\alpha_{2r}^{SO}\right)\left((-1+s)\left(1+\alpha_{1r}^{SO}\right)-s\,\alpha_{2r}^{SO}\right) - c\Big(s\left(\alpha_{1r}^{SO}-s\,\alpha_{2r}^{SO}\right) \\ & \left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right) + 2\left(1+\alpha_{1r}^{SO}\right)\alpha_{2r}^{SO}\Big) + D_{3n}^{SO}\Big(\left(1+\alpha_{1r}^{SO}\right)^2 - 2s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)^2 \\ & + 2s^2\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)^2 + \alpha_{2r}^{SO2}\Big) \end{aligned}$$

$$\begin{aligned} D_{4ri}^{SO} = \; & -\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)R + a_i\,\alpha_{1r}^{SO} - \left(2D_{1n}^{SO}+D_{3n}^{SO}\right)R\,\alpha_{1r}^{SO} + a_{-i}\,\alpha_{2r}^{SO} + 4D_{4n-i}^{SO}\Big(s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right) \\ & + \alpha_{2r}^{SO}\Big) + 4D_{4ni}^{SO}\left(1+\alpha_{1r}^{SO}-s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)\right) - c\,\alpha_{3ri}^{SO} + \frac{1}{2}\Big(2D_{3n}^{SO}\Big(-R\,\alpha_{2r}^{SO}+\alpha_{2r}^{SO}\alpha_{3ri}^{SO} \\ & + 2s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)\left(\alpha_{3ri}^{SO}-\alpha_{3r-i}^{SO}\right) - 2s^2\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)\left(\alpha_{3ri}^{SO}-\alpha_{3r-i}^{SO}\right) + \alpha_{3r-i}^{SO} + \alpha_{1r}^{SO}\alpha_{3r-i}^{SO}\Big) \\ & -2\left(\alpha_{1r}^{SO}\alpha_{3ri}^{SO}+\alpha_{2r}^{SO}\alpha_{3r-i}^{SO}\right) + 4D_{1n}^{SO}\Big(-R\,\alpha_{2r}^{SO} + \left(1+\alpha_{1r}^{SO}+2(-1+s)s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)\right)\alpha_{3ri}^{SO} \\ & + \left(-2(-1+s)s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)+\alpha_{2r}^{SO}\right)\alpha_{3r-i}^{SO}\Big) + c\Big(s\left(1+\alpha_{1r}^{SO}-\alpha_{2r}^{SO}\right)\left(\alpha_{3ri}^{SO}-\alpha_{3r-i}^{SO}\right) \\ & -2\left(\alpha_{1r}^{SO}\alpha_{3ri}^{SO}+\alpha_{2r}^{SO}\alpha_{3r-i}^{SO}\right)\Big)\Big) \qquad i=h,l \end{aligned}$$

$$\begin{aligned} D_{6r}^{SO} = \; & D_{6n}^{SO} + R\left(-D_{4ni}^{SO}-D_{4n-i}^{SO}+\left(2D_{1n}^{SO}+D_{3n}^{SO}\right)\left(R-\alpha_{3ri}^{SO}-\alpha_{3r-i}^{SO}\right)\right) + \frac{1}{2}\Big(-1+2D_{1n}^{SO}+(-1+s) \\ & \left(c+4s\,D_{1n}^{SO}-2s\,D_{3n}^{SO}\right)\Big)\left(\alpha_{3ri}^{SO2}+\alpha_{3r-i}^{SO\,2}\right) + \left(-\left(c+4D_{1n}^{SO}(-1+s)\right)s + D_{3n}^{SO}\left(1+2(-1+s)s\right)\right) \\ & \alpha_{3ri}^{SO}\,\alpha_{3r-i}^{SO} + \left(a_{-i}+D_{4n-i}^{SO}+s\,D_{4ni}^{SO}-s\,D_{4n-i}^{SO}\right)\alpha_{3r-i}^{SO} + \left(a_i+D_{4ni}^{SO}-s\,D_{4ni}^{SO}+s\,D_{4n-i}^{SO}\right)\alpha_{3ri}^{SO} \end{aligned}$$

Appendix D Experimental Validation for the Extraction Game

D.1 Experiment Instructions (Unequal Users – Infinite Transmissivity)

Welcome

This is an experiment in decision making. Please, do not close your internet explorer during the experiment. If you have any questions during the experiment or if your internet explorer is accidentally logged off, please raise your hand and an experimenter will come to you. You are participating in this experiment on a voluntary basis and you are allowed to stop at any time.

Please, make sure that the number on your computer matches the Id number on the top of this page (that was assigned to you randomly when entering the lab). Your identity will be kept secret during the experiment. During the experiment, you will be invited to fill in and sign a consent form and to report your decisions in a record sheet, the information provided and the decisions made will be kept secret. Please do not engage in any conversation with the other participants.

You will participate in 4 series of 10-rounds each; every 10-rounds sequence will last between 20 and 30 minutes. In every round you will be asked to make an economic decision. During the first two sessions, you will be matched with the same partner, and during the two last sessions you will be matched with another partner with the same type as your previous partner.

You will receive a 5$ for your participation. Additional earnings can be gained during the experiment. Your earnings will depend on your decisions and on the decisions of another participant you will randomly be paired with. If you follow the instructions cautiously and make good decisions you can earn up to thirty dollars. Your earnings will be paid to you, privately, at the end of the experiment, in cash.

The computer screen in front of you shows the current stock. In every round you will be asked to make three decisions. The first decision, noted WP, is your offer to “buy out” your partner; it represents the amount of money that you are willing to pay to your partner in exchange of her stopping her use of the stock during the current round and all following rounds.

The second decision, noted WA, represents your own buy out price; WA is the amount of money for which you would be willing to leave the stock, it represents the price that your partner needs to pay to you in order for you to accept to depart from the extraction stock during the current round and all following rounds.

The last decision is the quantity to extract from the stock if there is no transaction during the current round. A transaction takes place when your offer is larger than the price set by your partner or vice versa - your price is lower than the partner's offer. When the price of every player within the group is satisfied by the offer by her partner in the group, then priority is given to the transaction that generates the highest value i.e. the transaction with the larger difference between price and offer. For all transactions, the transaction cost is set equal to the average of the offer and price, it is deducted from the buyer whom offer was accepted and awarded to the seller.

The extraction decision will only be carried out when there is no transaction. In the case where there is a transaction and your offer is accepted, your earnings will be reduced by the transaction cost and you will be given a chance to review your decision knowing that you are the sole user of the stock. If there is a transaction but this time it is your price that was matched by your partner, you receive the transaction cost and your decision is set to zero for the current and all following rounds.

The net earnings from the extraction for the current round are given by: NP = Revenue – Cost.

The revenue depends solely on your decision and is given by: Revenue = $E\,D - \frac{D^2}{2}$

D is the amount you decide to extract from the stock, D needs to be positive lower than the stock. E is an efficiency coefficient, in every group there are two different players: a high efficiency player with efficiency is E= 20 and a low efficiency player with E= 10.

You are a high efficiency player. [You are a low efficiency player]

The cost of extraction depends on the current level of stocks and on your decision and the decision of your partner: Cost = $C\,D\left(H - X - \frac{SD}{4}\right)$

C=0.15 is the unitary cost of extraction (in dollars per unit of stock per meter). H=100 is the elevation to which the extracted stock needs to be lifted up it also marks the maximum stock allowed, X is the current stock and SD stands for the sum of extraction decisions made by the two players within the same group, $\left(H - X - \frac{SD}{4}\right)$ can be perceived as an average depth of extraction.

The net earnings, in computer dollars, are therefore given by the formula:

$$\mathrm{NE} = \mathrm{E\,D} - \frac{\mathrm{D}^2}{2} - \mathrm{C\,D}\left(H - X - \frac{SD}{4}\right)$$

Given your decision and the decision of your partner the stock evolves in the next round following the equation of motion: $NX = X - \frac{SD}{2} + R$

R= 3 reflects a fixed rate of renewability/recharge of the stock at the beginning of every period. In the case where NX is higher than H then NX is replaced by this later[141]. The stock at the beginning of the first round will always be taken as 73.

To help you with the calculations you have been provided a calculation support box in an excel spreadsheet. At the beginning of every round, make sure to copy the current stock level as provided in the explorer screen, enter the updated value in the appropriate cell of your calculation box. You will then enter your trial decision in the corresponding cell and your speculation regarding your partner's decision in the proper cell. The cells corresponding to the three entries are highlighted in a yellow color as shown in the figure below.

1- Enter the updated stock for this round as provided in the explorer program

The current stock	**90.00**

2- Enter your decision and your "best guess" for your partner's decision

High efficiency player decision	**3.00**
Low efficiency player decision	**2.00**

[141] In mathematical terms and more completely: $NX = inf\left\{X - \frac{SD}{2} + R, H\right\}$

Please note that the stock level in the calculation support box is not updated automatically and needs to be entered for every round, also note that your partner will make his/her own decision that may or may not match your speculation.

Based on the entries you make the calculator box will display your earnings and the earnings of your partner, along with the stock at the beginning of the next period. [Meaningless for the last period].

3- Outcomes and next round stock

The high efficiency earnings for this round are:	**16.13**	(revenue:	19.50	and cost:	3.38	)
The low efficiency earnings for this round are:	**3.75**	(revenue:	6.00	and cost:	2.25	)
The stock in next period :	**89.00**					

The calculator box will also display an earnings-table and future-stock-table for an array of decisions centered on the values that you entered, this table is meant to help refine your choice.

4- Tables of Extended Outcomes and next round stock

High Efficiency player earnings table

		High efficiency type decision				
		1.00	2.00	**3.00**	4.00	5.00
Low efficiency type decision	0.00	6.48	11.90	16.28	19.60	21.88
	1.00	6.45	11.85	16.20	19.50	21.75
	2.00	6.43	11.80	**16.13**	19.40	21.63
	0.00	6.48	11.90	16.28	19.60	21.88
	0.00	6.48	11.90	16.28	19.60	21.88

Low Efficiency player earnings table

		High efficiency player decision				
		1.00	2.00	**3.00**	4.00	5.00
Low efficiency type decision	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	2.45	2.43	2.40	2.38	2.35
	2.00	3.85	3.80	**3.75**	3.70	3.65
	3.00	4.20	4.13	4.05	3.98	3.90
	4.00	3.50	3.40	3.30	3.20	3.10

Next round stock table

		High efficiency type decision				
		1.00	2.00	**3.00**	4.00	5.00
Low efficiency type decision	0.00	91.00	90.50	90.00	89.50	89.00
	1.00	90.50	90.00	89.50	89.00	88.50
	2.00	90.00	89.50	**89.00**	88.50	88.00
	3.00	89.50	89.00	88.50	88.00	87.50
	4.00	89.00	88.50	88.00	87.50	87.00

In every round you can change the entries in the calculation support sheet as many times as you need (note that you only review the decisions since the stock is updated once for every round); there are no limits on the number of trials, however all participants are recommended to make their decisions in no more than 3 minutes.

Once you reach an acceptable choice for your extraction decision, switch to the explorer page and enter your decision then press the accept key, remember, your decision needs to be a nonnegative value below the stock level

Appendix E Policy Implications

E.1 Deriving the Multi-period Path Under Single User management

gfgfg

$$\alpha_{1r}^{SU} = -1 + \frac{-2 + c\,s}{-2 + 4 D_{1n}^{SU} + 2c\left(-1+s\right) + 4s\left(-2 D_{1n}^{SU} + D_{3n}^{SU} + \left(D_{1n}^{SU} + D_{2n}^{SU} - D_{3n}^{SU}\right)s\right)},$$

$$\alpha_{2r}^{SU} = \frac{\left(c + 4\left(D_{1n}^{SU} + D_{2n}^{SU}\right)\left(-1+s\right)\right)s + D_{3n}^{SU}\left(-2 - 4\left(-1+s\right)s\right)}{-2 + 4 D_{1n}^{SU} + 2c\left(-1+s\right) + 4s\left(-2 D_{1n}^{SU} + D_{3n}^{SU} + \left(D_{1n}^{SU} + D_{2n}^{SU} - D_{3n}^{SU}\right)s\right)}, \text{ and}$$

$$\alpha_{3ir}^{SU} = \frac{-a_i + D_{4in}^{SU}\left(-1+s\right) - s\,D_{5in}^{SU} + R\left(2 D_{1n}^{SU} + D_{3n}^{SU} - 2s\,D_{1n}^{SU} + 2s\,D_{2n}^{SU}\right)}{-1 + 2 D_{1n}^{SU} + c\left(-1+s\right) + 2s\left(-2 D_{1n}^{SU} + D_{3n}^{SU} + \left(D_{1n}^{SU} + D_{2n}^{SU} - D_{3n}^{SU}\right)s\right)}.$$

$$\begin{aligned} D_{1r}^{SU} = \; & \frac{1}{2}\Big(-\alpha_{1r}^{SU\,2} + \left(2 D_{1n}^{SU}\left(1-s\right)^2 - 2 D_{3n}^{SU}\left(-1+s\right)s + 2 D_{2n}^{SU}\,s^2\right)\left(1+\alpha_{1r}^{SU}\right)^2 \\ & + c\,\alpha_{1r}^{SU}\left(-2 + s + \left(-1+s\right)\alpha_{1r}^{SU}\right) \end{aligned}$$

$$\begin{aligned} D_{2r}^{SU} = \; & \frac{1}{2}\Big(2 D_{2n}^{SU}\left(1 + s\left(-1+\alpha_{2r}^{SU}\right)\right)^2 - 2 D_{3n}^{SU}\left(1 + s\left(-1+\alpha_{2r}^{SU}\right)\right)\left(s\left(-1+\alpha_{2r}^{SU}\right) - \alpha_{2r}^{SU}\right) \\ & + c\left(s\left(-1+\alpha_{2r}^{SU}\right) - \alpha_{2r}^{SU}\right)\alpha_{2r}^{SU} - \alpha_{2r}^{SU\,2} + 2 D_{1n}^{SU}\left(s + \left(1-s\right)\alpha_{2r}^{SU}\right)^2\Big) \end{aligned}$$

$$\begin{aligned} D_{3r}^{SU} = \; & \frac{1}{2}\Big(-4\left(D_{1n}^{SU} + D_{2n}^{SU}\right)\left(-1+s\right)s - \left(c + 4\left(D_{1n}^{SU} + D_{2n}^{SU}\right)\left(-1+s\right)\right)s\,\alpha_{1r}^{SU} \\ & - 2 D_{3n}^{SU}\left(1+\alpha_{1r}^{SU}\right)\left(-1 + 2\left(-1+s\right)s\left(-1+\alpha_{2r}^{SU}\right)\right) + \Big(-2\alpha_{1r}^{SU} + \\ & 4\left(D_{1n}^{SU} - 2s\,D_{1n}^{SU} + \left(D_{1n}^{SU} + D_{2n}^{SU}\right)s^2\right)\left(1+\alpha_{1r}^{SU}\right) + c\left(-2 + s + 2\left(-1+s\right)\alpha_{1r}^{SU}\right)\Big)\alpha_{2r}^{SU}\Big) \end{aligned}$$

$$\begin{aligned} D_{4ir}^{SU} = \; & -R\,D_{3n}^{SU} + s\,D_{5in}^{SU} - 2R\,s\,D_{2n}^{SU} + a_i\,\alpha_{1r}^{SU} - R\,\alpha_{1r}^{SU}\,D_{3n}^{SU} + s\,\alpha_{1r}^{SU}\,D_{5in}^{SU} - 2R\,s\,\alpha_{1r}^{SU}\,D_{2n}^{SU} \\ & - D_{4in}^{SU}\left(-1+s\right)\left(1+\alpha_{1r}^{SU}\right) - c\,\alpha_{3ir}^{SU} + \frac{1}{2}\Big(-2\alpha_{1r}^{SU} - 4s\left(D_{3n}^{SU}\left(-1+s\right) - s\,D_{2n}^{SU}\right)\left(1+\alpha_{1r}^{SU}\right) \\ & + c\left(s + 2\left(-1+s\right)\alpha_{1r}^{SU}\right)\Big)\alpha_{3ir}^{SU} + 2 D_{1n}^{SU}\left(-1+s\right)\left(1+\alpha_{1r}^{SU}\right)\left(R + \left(-1+s\right)\alpha_{3ir}^{SU}\right) \end{aligned}$$

$$D_{5ir}^{SU}= -R\left(2D_{2n}^{SU}+D_{3n}^{SU}\right)+2RsD_{2n}^{SU}+D_{5in}^{SU}\left(1+s\left(-1+\alpha_{2r}^{SU}\right)\right)+a_i\,\alpha_{2r}^{SU}-R\alpha_{2r}^{SU}D_{3n}^{SU}$$
$$-s\alpha_{2r}^{SU}D_{4in}^{SU}-2Rs\alpha_{2r}^{SU}D_{2n}^{SU}+D_{4in}^{SU}\left(s+\alpha_{2r}^{SU}\right)+\frac{1}{2}\left(-cs+4sD_{2n}^{SU}\left(1+s\left(-1+\alpha_{2r}^{SU}\right)\right)\right.$$
$$-2\alpha_{2r}^{SU}+2c\left(-1+s\right)\alpha_{2r}^{SU}+D_{3n}^{SU}\left(2+4s\left(-1+s+\alpha_{2r}^{SU}-s\alpha_{2r}^{SU}\right)\right)\Big)\alpha_{3ir}^{SU}$$
$$+2D_{1n}^{SU}\left(s\left(-1+\alpha_{2r}^{SU}\right)-\alpha_{2r}^{SU}\right)\left(R+\left(-1+s\right)\alpha_{3ir}^{SU}\right)$$

And,

$$D_{6ir}^{SU}= D_{6in}^{SU}-R\left(D_{4in}^{SU}+D_{5in}^{SU}\right)+R^2\left(D_{1n}^{SU}+D_{2n}^{SU}+D_{3n}^{SU}\right)+\left(a_i+D_{4in}^{SU}+2RD_{1n}^{SU}\left(-1+s\right)\right.$$
$$-sD_{4in}^{SU}+sD_{5in}^{SU}-R\left(D_{3n}^{SU}+2sD_{2n}^{SU}\right)\Big)\alpha_{3ir}^{SU}+\frac{1}{2}\left(-1+2D_{1n}^{SU}+c\left(-1+s\right)+2s\left(-2D_{1n}^{SU}\right.\right.$$
$$\left.\left.+D_{3n}^{SU}+\left(D_{1n}^{SU}+D_{2n}^{SU}-D_{3n}^{SU}\right)s\right)\right)\alpha_{3ir}^{SU\,2}$$

E.2 The effects of transmissivity on the Difference Between Single User profits and SPE Aggregate Profits

The maximum of the derivative of the difference is given by:

$$\varepsilon_{SU-SPE}^{\partial sMax}=\frac{(2+2c-3cs)^3\quad(a-c(h-x))}{3(2+2c-2cs)^3+20cs(1+c-2cs)^2+34c^2s^2(1+c-2cs)+15c^3s^3} \tag{E.1}$$

Given the value of $\varepsilon_{\text{SU-SPE}}^{\partial s\,Max}$ is positive

$$\varepsilon_{SU-SPE}^{\partial s-}=\varepsilon_{SU-SPE}^{\partial sMax}-P\Delta_{SU} \tag{E.2}$$

$$\varepsilon_{SU-SPE}^{\partial s+}=\varepsilon_{SU-SPE}^{\partial sMax}+P\Delta_{SU} \tag{E.3}$$

where $P\Delta_{SU}$, is a positive variable:

$$P\Delta_{SU}=\frac{4\quad PQ_{SU1}\ (2+2c-3cs)(1+c-cs)}{3(2+2c-2cs)^3+20cs(1+c-2cs)^2+34c^2s^2(1+c-2cs)+15c^3s^3}\,\frac{a-c(h-x)}{(2+2c-cs)^2}$$

Where PQ_{SU1} is a positive variable given as:

$$PQ_{SU1=}\sqrt{(2+2c-3cs)(2+2c-cs)}\sqrt{PQ_{SU1a}}$$

PQ_{SU1a} been a positive variable:

$$PQ_{SU1a} = (2+2c-cs)^4 + cs(2+2c-3cs)^3 + 4c^2s^2(2+2c-3cs)^2 + 2c^3s^3(2+2c-4cs) + c^4s^4$$

From the previous calculations it is clear that $P\Delta_{SU}$ is strictly positive, and, consequently, $\varepsilon_{SU-SPE}^{\partial s+}$ is strictly positive; but the negative sign of $\varepsilon_{SU-SPE}^{\partial s-}$ cannot be drawn from (E.3), an alternative approach to show that is to rewrite $\varepsilon_{SU-SPE}^{\partial s-}$ as:

$$\varepsilon_{SU-SPE}^{\partial s-} = -\frac{\varepsilon_{SU-SPE}^{\partial sMax}}{\varepsilon_{SU-SPE}^{\partial s+}}\ \frac{(2+2c-cs)^3 + 8cs(1+c-cs)^2}{(2+2c-cs)^3}\ (a - c(h-x))$$

Now I will try to show that, as suggested in the Figure 17, $\varepsilon_{\text{SU-SPE}}^{\partial s-} < \varepsilon_{\text{SU-SPE}}^{0}$, $\varepsilon_{\text{SU-SPE}}^{\text{Max}} < \varepsilon_{\text{SU-SPE}}^{\partial s+}$, and $\varepsilon_{\text{SU-SPE}}^{\partial s+} < \varepsilon_{\text{SPE}}^{\text{Max}}$.

The proof that $\varepsilon_{\text{SU-SPE}}^{\partial s+} < \varepsilon_{\text{SPE}}^{\text{Max}}$, is provided by computing $\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_{\text{SPE}}^{\text{Max}},\bar{a}-\varepsilon_{\text{SPE}}^{\text{Max}}}(x,x)$ as a negative quantity:

$$\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_{\text{SPE}}^{\text{Max}},\bar{a}-\varepsilon_{\text{SPE}}^{\text{Max}}}(x,x) = -\frac{16c^2(1+c)s(a-c(h-x))^2}{(2+2c-cs)^3(2+2c-3cs)}$$

Given the positive sign of $\varepsilon_{\text{SPE}}^{\text{Max}}$, and the fact that $\Delta_s V^{s,SU-SPE}_{\bar{a}+\varepsilon_{\text{SPE}}^{\text{Max}},\bar{a}-\varepsilon_{\text{SPE}}^{\text{Max}}}(x,x)$ is of positive sign for any level of inequality within the interval $[\varepsilon_{SU-SPE}^{\partial s-}, \varepsilon_{SU-SPE}^{\partial s-}]$, where $\varepsilon_{SU-SPE}^{\partial s-}<0$, it is clear that the only deduction is $\varepsilon_{\text{SU-SPE}}^{\partial s+} < \varepsilon_{\text{SPE}}^{\text{Max}}$.

To show that $\varepsilon_{\text{SU-SPE}}^{\text{Max}} < \varepsilon_{\text{SU-SPE}}^{\partial s+}$ it suffices to notice that:

$$\varepsilon_{SU-SPE}^{\partial s+} - \varepsilon_{SU-SPE}^{Max} = \left(\frac{(a-c(h-x))}{(2+2c-cs)((2+2c-2cs)^2+cs(4+4c-5cs))}\right) PQ_{SU1b}$$

where PQ_{SU1b} is a positive variable given as:

$$PQ_{SU1b} = \frac{4cs(1+c-cs)(2+2c-3cs)^2\left((2+2c-2cs)^2(2+2c)^2+c^4s^4\right)}{PQ_{SU1}\left((2+2c-2cs)^2+cs(4+4c-5cs)\right)+4(1+c)(1+c-cs)(2+2c-3cs)(2+2c-cs)^2}$$

To show that $\varepsilon^{\partial s-}_{\text{SU-SPE}} < \varepsilon^{0}_{\text{SU-SPE}}$, the difference, $\varepsilon^{0}_{SU-SPE} - \varepsilon^{\partial s-}_{SU-SPE}$, is rewritten as a product of positive variables:

$$\varepsilon^{0}_{SU-SPE} - \varepsilon^{\partial s-}_{SU-SPE} = \left(\frac{4(1+c-cs)(2+2c-3cs)}{PQ_{SU4}(2+2c-cs)^2}\right)\left(\frac{c^3s^3PQ_{SU6}+PQ_{SU7}PQ_{SU8}}{(2+2c-2cs)^2+cs(4+4c-5cs)}\frac{(a-c(h-x))}{PQ_{SU8}}\right)$$

where PQ_{SU4}, PQ_{SU6}, PQ_{SU7} and PQ_{SU8} are all positive variables:

$$PQ_{SU4} = 3(2+2c-2cs)^3 + 5cs(2+2c-3cs)^2 + 7c^2s^2(2+2c-3cs) + 3c^3s^3;$$

$$PQ_{SU6} = 16(1+c-cs)^7 + c(1+c)s(31(1+c-2cs)^5 + 152cs(1+c-2cs)^4 + 305c^2s^2(1+c-2cs)^3 + 274c^3s^3(1+c-2cs)^2 + 93c^4s^4(1+c-2cs) + 2c^5s^5);$$

$$PQ_{SU7} = 64(1+c-cs)^5 + 72cs(1+c-2cs)^4 + 263c^2s^2(1+c-2cs)^3 + 319c^3s^3(1+c-2cs)^2 + 133c^4s^4(1+c-2cs) + 4c^5s^5;$$

$$PQ_{SU8} = PQ_{SU1}(2+2c-2cs)^2 + cs(4+4c-5cs)) + 32(1+c-cs)^5 + 72cs(1+c-cs)^4 + 31c^2s^2(1+c-2cs)^3 + 75c^3s^3(1+c-2cs)^2 + 53c^4s^4(1+c-2cs) + 9c^5s^5;$$

E.3 Numerical Example for Result SU3

The numerical specifications used here are built on those in Chapter 3.4.1, fairly modified to substantiate the theoretical result SU3. I consider the case of an infinite-transmissivity resource (s = ½), the highest level of transmissivity is evidently in compliance with the condition on the level of transmissivity (s>0.453) expressed in Result SU3.

The unitary cost *c*, is taken equal to 15, which satisfies the condition on c ($c > \frac{2}{-2+(3+\sqrt{2})\,s}$). The stocks at the start of the one-round game are set at 82 ($x_{h0} = x_{l0} = 82$), and the efficiency of both –identical– players is set to 300 ($a_h = a_l = 300$); while the elevation of the ground is kept at *E*=100 and the aquifer recharge is *R*=3.

Following the SPE, every user extracts 2.45, she receives a net profit in the amount of 25.49; adding up the profits of both users provides the aggregate profit, 50.98. With only one of the two identical users been granted access to the resource, she follows the Single User path as solved for previously and chooses to extract 3.53, for which she receives a net profit of 52.94, higher than the aggregate profit following the SPE.

Appendix F Computing Resources

I used the computing resources available at the computer center at the Department of Agricultural and Resource Economics at UMD:

Processor:	Intel® Xeon ® CPU E5-2690 0 @ 2.90GHz 2.90 GHz (2 processors)
Installed Memory (RAM):	2.00 GB
System type:	64-bit Operating System

For the statistical analysis, I used the version of Stata available at AREC:

Stata/MP 11.2 for Windows (64-bit x86-64)

4-user 6-core Stata network perpetual license:

Serial number:	50110589403
Licensed to:	AREC Dept. U. of Maryland

For the mathematical computations, I used the Mathematica package available at AREC:

Version Number:	8.0.0.0
Platform:	Microsoft Windows (64-bit)

Appendix G Mathematica Program for the Discrete Game

See under supplementary documents.

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