ABSTRACT

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The first chapter derives an empirically testable set of propositions on the determinants of environmental aid as a non-market solution for trans-border pollution. The donor country balances environmental benefits against the social costs of aid which results from possible erosion of competitiveness in the export market. Using the panel data for environmental aid from OECD countries to China, it is shown that trade competition significantly reduces types of environmental aid that enhance the competitiveness of China. As the scope of environmental aid that improves China's energy efficiency is limited by trade competition, the change in composition of bilateral environmental aid may reflect a means by which a solution to the trans-border pollution issue can be found.

The second chapter shows that the dynamic properties of the pollution-income relationship under an optimal pollution tax depends on three key factors, namely the

degree of temporal and inter-temporal flexibility in consumption and the elasticity of substitution among production inputs. This paper derives general conditions for eluding the limits to growth showing that they require rather stringent assumptions which the existing literature has failed to identify.

Finally, the third chapter examines environmentally sustainable growth with reference to climate change assuming two final outputs and two factors of production, accounting for both pollution flow and stock effects. If the elasticity of marginal utility of consumption is greater than one, an optimal pollution tax ensures sustainable growth without any further government intervention. Otherwise, either a high temporal elasticity of substitution in production or consumption is required for sustainability. Even a suboptimal pollution tax may allow sustainable development provided the tax time profile meets certain conditions that are developed and described in this paper.

THREE ESSAYS IN SUSTAINABLE DEVELOPMENT

By

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Dedication

This dissertation is dedicated to my grandparents, parents, my wife Nara and my new born daughter Grace.

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Chapter 1: Strategic Bilateral Environmental Aid and Trans-border Pollution

1. Introduction

Trans-border pollution poses a serious challenge to environmental cooperation among sovereign states. Only a very small number of binding multilateral international environmental agreements have remained successful. Unlike multilateral environmental agreements, however, bilateral environmental aid to reduce trans-border pollution has been much more persistent and successful (Hicks et al., 2008).

This paper is concerned with bilateral environmental official development assistance (EODA) between developed donor and developing recipient countries in the presence of trans-boundary pollution. This paper also considers the fact that the recipient country may potentially increase competition with donor countries in the export markets in part as a result of EODA. This tradeoff may cause donor countries to direct their EODA strategically. This paper provides a theoretical and empirical analysis of such strategic behavior. The empirical analysis employs a new data set developed by the Project Level Aid Database (PLAID) on bilateral EODA for the period from 1985-2008 (Hicks et al., 2008).

The existing literature has recognized strategic behavior among donor countries in the context of general official development assistants (GODA) which consists mostly of non-environmental aid. Many studies find that GODA is merely an instrument for donor countries to enhance their trade penetration in the recipient countries (i.e., Silva and Nelson, 2012; Martínez-Zarzoso et al., 2009; Nowak-Lehmann et al., 2009; Johansson and Pettersson, 2009).

Unlike GODA, however, the donor country experiences a direct environmental benefit from EODA if it is associated with reductions in trans-boundary pollution, in addition to trade effects caused by changes in trade competitiveness. We focus on the relative export performance of both donor and recipient countries and examine two types of EODA: (i) EODA that induces energy efficiency, and (ii) "pure" EODA, which consist of the transfer of technology that reduces pollution emissions without affecting production efficiency (i.e., air purification technology).[1]

Theoretical studies postulate that EODA may enhance the welfare of the donor and recipient countries by inducing a reduction of trans-boundary pollution (i.e., Chambers and Jensen, 2002; Hatzipanayotou et al., 2002; Chao and Yu, 1999; Copeland and Taylor, 1995) .Yet these papers do not examine the strategic role of the government in determining the composition and levels of EODA, nor do they provide insight into identifying the ways in which certain characteristics of the donor and recipient countries affect the amount of EODA granted.

Little empirical research has been done on this topic. One exception is the research conducted by Hicks et al. (2008), which provides detailed data on the time profiles of both multilateral and bilateral EODA and examines its determinants. Although Hicks et al. (2008) acknowledge the role of international trade in the allocation of

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[1] In this paper, we do not explicitly consider the case in which the donor promotes exports by offering environmental aid. The rationale for this decision is that as long as forceful import from the donor country does not comprise a large share of the total import of the recipient from the donor country, the market mechanism may induce the recipient country to adjust its import from the donor country. Moreover, numerous recent studies on ODA find that "Aid-for-Trade" has actually promoted the recipient country's exports (see, Hühne et al., 2014; Helble et al., 2013). Consequently, we focus instead on the effect of environmental aid on bilateral trade competitiveness by considering relative export flows of both donor and recipient countries.

multilateral environmentally related aid, they do not examine the causal relationships between change in trade competitiveness and EODA. The present study is thus the first to provide an empirical estimation of the determinants of bilateral EODA in the presence of trans-boundary pollution.

This study uses a multistage game to examine the aid policy of the donor country. The theoretical framework of this paper incorporates factors affecting the donor country's export competitiveness vis-à-vis the recipient country as well as damages from trans-border pollution.

This paper then derives and empirically estimates a set of testable hypotheses on the determinants of bilateral EODA from developed donors to China using PLAID's well-defined taxonomy of bilateral EODA from. Although many countries receive bilateral EODA from developed nations, China is a natural candidate for our theoretical and empirical investigation for the following reasons. First of all, China has not only emerged as the largest merchandise exporter in the world, competing with developed countries in technology-intensive products, but it has also become one of the countries that is most responsible for trans-border air pollution, raising serious regional environmental concerns.[2] Secondly, a large number of air pollutants move from China into other countries or have global implications.[3] Thirdly, despite the disappointing failure of numerous bilateral and multilateral negotiations with

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[2] For insight into China's successful industrial transition in recent decades, see Rodrik (2006) and Cui and Syed (2007).

[3] Trans-border pollution from China is known to cause serious environmental problems in nearby countries as China's economy continues to expand. Each spring, fierce dust and sand storms take place in the Gobi Desert across northern and western China. As the dust and sand are blown eastward by westerly winds, they pick up air pollution particles, particularly over heavily industrialized areas in northeast China such as Shenyang, and carry them farther east into South Korea and Japan. They even move farther east to reach the west coast of the United States. According to emission researchers, half of the world's man-made mercury emissions come from Asia, with China being the main source (UNEP, 2013).

pollution importing countries to resolve trans-border pollution, EODA to China has persisted over recent decades. While GODA from developed countries to China has declined rapidly since the mid-1990s, EODA to China has remained steadily remained at relatively high. China's data on EODA from OECD countries provides the opportunity to examine a set of theoretical propositions on the nature and determinants of EODA.

2. The Basic Model and Testable Hypothesis

2.1. Theoretical Framework

The analysis considers a multi-stage model of EODA and trade competition between the donor and recipient countries. We assume that the donor has already accumulated a set of technologies that abates pollution.

- In the first stage, the government of the donor announces whether or not to arrange EODA to transfer technology to the country that emits pollution across a national boundary. If the government decides to offer aid, it has to purchase the technology from the technology-developing domestic firm.
- In the second stage, the representative firm in the recipient country decides on the amount extent to which abatement technologies should be developed.
- In the final stage, firms from both countries engage in Cournot competition in the output market; likewise profits and pollution levels are realized.

The aid decision of the donor government takes into account the effect the abatement technology being transferred has on the development of in abatement technology in the recipient country and the profit of the home industry caused by changes in trade. The aid is not offered if the recipient country is reducing investment in abatement technology by more than the amount of aid.

Production and pollution emissions

We assume that each country produces a dirty good and a clean good. A dirty good emits pollution from its production process while clean good does not. There exists a representative firm in the dirty sector in each of the two countries, the donor country (N) and the recipient country (S). The two countries compete in the world export market for the dirty good.[4] Let p denote the world price of the dirty good. The price of the clean good is normalized to unity.

Let the production and demand for the dirty good of country i be denoted as y_i and $d_i(p)$ with $d'_i(p) < 0$ for $i = N, \mathrm{S}, \mathrm{W}$ where W denotes the third country (or the rest of the world) that does not produce output y.[5] The output price adjusts to clear the market instantaneously so that $y = y_N + y_S = d_N(p) + d_S(p) + d_W(p)$.[6] Throughout the paper, we maintain the following assumption which guarantees the existence of Cournot equilibrium in the output market:

Assumption 1: *Let* $p(y)$ *with* $p'(y) < 0$ *denote the inverse world demand for the output. Then* $yp'(y)$ *is declining in* y *or* $p'(y) + yp''(y) \leq 0$ *for any* $y > 0$.

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[4] Some dirty goods and services such as electricity and transportation service are mostly non-tradable. But they constitute important intermediate inputs for dirty manufactured tradable goods. As the donor country is concerned about the effect on competitiveness of environmental aid in the dirty industry, it is quite innocuous to assume that the dirty good is tradable in our model economy.

[5] Throughout the paper we use the prime sign to denote the first derivative and double prime to denote the second derivative

[6] When $d_i(p) = 0,\ i = N, \mathrm{S}$ two countries compete in the third country only as in Barrett (1994).

The production of one unit of output by firm i emits x_i units of pollution. The pollution of firm i depends on the level of pollution abatement technology K_i so that for $i = N, S$:

$$x_i = x(K_i) \text{ with } x'(K) < 0 .$$

If there exist economies of scale in abatement technology, we have $x''(K) < 0$. Otherwise, $x''(K) \geq 0$.

The government of country i imposes a pollution tax, τ_i, per unit of pollution and the marginal environmental cost of increasing one unit of output is defined as $\tau_i x_i$.

Costs and profits in the output market

The marginal production cost is constant with respect to the level of the dirty output. However, it is a decreasing function of the level of abatement technology (i.e. capital). The marginal cost of the dirty good of representative firms in each country is equal to the marginal production cost (c_i^m) plus the marginal environmental cost ($\tau_i x_i$). More explicitly,

$$(1) \qquad c_i = c_i^m(K_i) + \tau_i x_i(K_i) .$$

For simplicity, we assume that the level of abatement technology is measured by the clean output. We also assume that the marginal cost is continuously differentiable function of K_i. The profit of each firm i is,

$$(2) \qquad \pi_i(y_i, y_j; K_i) = \left(p(y) - c_i(K_i)\right) y_i - K_i \text{ for } i, j = N, S .$$

Assumption 1 with constancy of marginal cost for any level of output assures that for any level of K_N and K_S, Cournot equilibrium exists and is unique (Novshek, 1985;Gaudet and Salant, 1991). In addition, the outputs of two firms are strategic substitutes.

Environmental damage from pollution

Let x_{SN} denote the level of trans-border pollution that originates from S and affects N. The clean environmental stock of N can be expressed as:

$$A_N = B_N - x_N y_N - x_{SN} y_S , \tag{3}$$

where B_N is the maximum available environmental stock in the absence of pollution and $x_{SN} \leq x_S$. The environmental stock is measured in terms of monetary unit.

2.2 Description of the Three-Stage Game of Technology Transfer

Two countries engage in a non-cooperative game to reduce trans-border pollution by transferring environmental technology. The strategic aid decision of the donor is described as the three-stage game under complete information. Let K_N be the historically given stock of environmental technologies (i.e., number of patents) of the donor country that have become available prior to the aid decision. The abatement technology of the donor country N is superior to that of polluting country S so that $K_S < K_N$. The polluting country S is assumed to opt for a technology transfer for free from the potential donor.

In the first stage, the donor country decides on the amount of aid, k, to be given to the polluting country, S.[7] In the second stage, the aid-receiving firm decides upon the total level of abatement technology, K_S, to maximize profit in the output market. The best response of the recipient firm in the second stage is rationally anticipated and

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[7] We note that transferring k does not reduce K_N.

taken into account by the donor country. It must meet the constraint that $dK_S / dk \geq -1$, since otherwise EODA does not reduce trans-border pollution. The pollution level in the recipient country is represented as $x_S = x_S(K_S + k)$. Let $c_S^m(K_S + k)$ denote the marginal production cost of the representative firm in the aid-receiving country, S, when it received aid $k \leq K_N$.

The profit of the polluting firm in S becomes,

(4) $$\pi_S(y_N, y_S; k, K_S) = \left[p(y_N + y_S) - c_S(K_S + k, \tau_S)\right] y_S - C(K_S),$$

where $c_S(K_S + k, \tau_S) = c_S^m(K_S + k) + \tau_S f(K_S + k)$ and $C(K_S)$ being the cost of developing domestic technology with $C'(K_S) > 0$ and $C''(K_S) < 0$. In the third stage, the two firms compete in the output market, and an equilibrium price and profits are determined. The pollution emissions of the two firms are also determined.

The Cournot equilibrium output of firm $i = N, S$ can be written as $y_i(c_N(K_N, \tau_N), c_S(K_S + k, \tau_S))$, while the corresponding equilibrium output price and profits are denoted as $p\left(c_N(K_N, \tau_N), c_S(K_S + k, \tau_S)\right)$ and $\pi_i\left(c_N(K_N, \tau_N), c_S(K_S + k, \tau_S)\right)$, respectively. Then, the optimal strategic choice of developing K_S of the aid-receiving firm is given as follows:

$$K_S = Arg \max \pi_S\left(c_N(K_N, \tau_N), c_S(K_S + k, \tau_S), K_S\right).$$

Since the marginal costs of the two firms, c_N and c_S, depend on the degree of environmental regulations, τ_S and τ_N, the K_S level of the aid receiving firm can be written as,

$$K_S = h(k; K_N, \tau_S, \tau_N).$$

Then, given the technological capability of the donor, K_N and the environmental regulations of each country, τ_N and τ_S, the equilibrium price of the dirty output, profits of two representative firms from the North and South, and level of trans-border pollution from the South depend on the size of aid, k.

We denote Cournot equilibrium output price and profits as $p\left(y_N, y_S\right)$ and $\pi_i\left(y_N, y_S\right)$ for $i = N, S$, respectively where y_i can be expressed as $y_i\left(c_N(K_N,\tau_N), c_S(k;K_N,\tau_S,\tau_N)\right)$ with $c_S(k;K_N,\tau_S,\tau_N) = c_S(k + h(k;K_N,\tau_S,\tau_N),\tau_S)$.

2.3 Strategic Objective of the Donor Government

Although the government of the donor country decides on the scope of bilateral EODA to reduce trans-border pollution, it has to consider the cost of aid in the form of the possible erosion of competitiveness of domestic firms in the export market.[8] Following the literature on strategic trade and environmental policy, we assume that the donor government maximizes domestic welfare, which is given as,

$$W_N = \pi_N + \alpha_N A_N, \tag{5}$$

where α_N represents the political weight on maintaining a clean environment.

Throughout the paper, we assume that the political weight on the environment for each country is known to the other.[9]

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[8] We ignore the consumer surplus as it is dispersed among a numerous number of unidentified individual consumers and does not represent organized pressure for the government. We also ignore the effect of aid on the non-tradable sector of the donor country, as the volume of aid is considerably smaller than the typical non-tradable sector of the developed economy. Empirical measures for competitiveness, which takes into account political pressure from reduced market share in the export market, will be discussed in the next section.

[9] Imperfect information regarding the political weight of the aid-recipient country can trigger incentives for a reputation for the building of excessive pollution on the part of the recipient country (see Chambers and Jensen, 2002).

2.4 Equilibrium with Technology Transfer

Let us define the value of environmental aid (VEA) as the welfare gain of the donor country when it offers aid to the polluting country. It consists of changes in domestic profits ($\Delta\Pi$) and environmental benefits resulting from the reduced pollution emissions of the recipient country (EB) that result from bilateral EODA. That is,

$$VEA = \Delta\Pi + EB. \tag{6}$$

2.4.1 The Welfare Effect of EODA for the Donor Country

The effect of aid on the domestic profit of the donor country becomes:

$$\begin{aligned} \Delta\Pi = {} & \pi_N\left[y_N\left(c_N(K_N,\tau_N),c_S(k;K_N,\tau_S,\tau_N)\right),y_S\left(c_N(K_N,\tau_N),c_S(k;K_N,\tau_S,\tau_N)\right)\right] \\ & -\pi_N\left[y_N\left(c_N(K_N,\tau_N),c_S(0;K_N,\tau_S,\tau_N)\right),y_S\left(c_N(K_N,\tau_N),c_S(0;K_N,\tau_S,\tau_N)\right)\right], \end{aligned} \tag{7}$$

where the equilibrium profit of the donor country is given as $\pi_i\left(y_N, y_S\right)$ with $y_i\left(c_N(K_N,\tau_N),c_S(k;K_N,\tau_S,\tau_N)\right)$ and $\partial\pi_i / \partial y_i = 0$ for $i = N, S$. Furthermore,

$$\frac{\partial\pi_N}{\partial k} = \left(\frac{\partial\pi_N}{\partial y_N}\frac{\partial y_N}{\partial c_S} + \frac{\partial\pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S}\right)\frac{\partial c_S}{\partial k} = \frac{\partial\pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S}\frac{\partial c_S}{\partial k}. \tag{8}$$

Since $\partial\pi_N / \partial y_S < 0$ and $\partial y_S / \partial c_S < 0$ by Assumption 1, the sign of $\partial\Delta\Pi / \partial k$ (or $\partial\pi_N / \partial k$) depends on $\partial c_S / \partial k$. When k varies continuously, we have,

$$\frac{dc_S(k)}{dk} = \frac{dc_S^m(k)}{dk} + \tau_S x'\left(\frac{\partial K_S(k;K_N,\tau_S,\tau_N)}{\partial k} + 1\right).^{10} \tag{9}$$

It follows that unless $dc_S^m(k)/dk = 0$ and $\tau_S = 0$, the marginal cost of the representative firm in S decreases in the volume of aid k. As a result, the donor's

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[10] Recall that the marginal cost of the firm in the recipient country is $c_S(K_S+k,\tau_S) = c_S^m(K_S+k) + \tau_S x(K_S+k).$

output and profit decreases in k under Assumption 1 on strategic substitutes. Therefore, the forgone profit, $\Delta\Pi$, can be interpreted as the social cost of aid.

Without loss of generality, let the trans-border pollution originating from the aid-receiving country be given as $x_{SN} = \beta_{SN} x_S$ where $\beta_{SN} \geq 0$ reflects factors that affect the magnitude of trans-border pollution, such as direction of wind or geographical distance from the polluting country. Then, by using Equation (3), (5) and (7) the environmental benefit (*EB*) of the donor country becomes:

(10)

$$\begin{aligned} EB = & -\alpha_N \beta_{SN} \left[y_S \left(c_N(K_N, \tau_S, \tau_N), c_S(0; K_N, \tau_S, \tau_N) \right) \left[x\left(K_S(k; K_N, \tau_S, \tau_N)) + k \right) - x(K_S(0; K_N, \tau_S, \tau_N)) \right] \right] \\ & -\alpha_N \beta_{SN} x\left(K_S(k; K_N, \tau_S, \tau_N)) + k \right) \left[y_S \left(c_N(K_N, \tau_N), c_S(k; K_N, \tau_S, \tau_N) \right) - y_S \left(c_N(K_N, \tau_N), c_S(0; K_N, \tau_S, \tau_N) \right) \right], \end{aligned}$$

Equation (9) implies that the marginal effect of EODA on environmental benefit is given as,

(11)

$$\frac{\partial EB}{\partial k} = -\alpha_N \beta_{SN} \left(\begin{array}{l} \dfrac{\partial y_S}{\partial c_S} \dfrac{\partial c_S}{\partial k} x\left(K_S(k; K_N, \tau_S, \tau_N)) + k \right) \\ + y_S \left(c_N(K_N, \tau_S, \tau_N), c_S(0; K_N, \tau_S, \tau_N) \right) x'\left(K_S(k; K_N, \tau_S, \tau_N)) + k \right) \left(\dfrac{\partial K_S}{\partial k} + 1 \right) \end{array} \right),$$

Since $\partial c_S / \partial k \leq 0$ and $\partial y_S / \partial c_S < 0$ under Assumption 1, the first term, which represents *output effect*, is negative, implying that EODA may induce the recipient country to expand the production of dirty output, thereby increasing the emission volume. The second term $y_S(\bullet) x'(\bullet) \left(\dfrac{\partial K_S}{\partial k} + 1 \right) > 0$ represents positive *emission effect*, which represents the environmental benefit obtained from EODA at the given level of the dirty output. Since EODA causes profit loss in the export market, the following proposition is immediate.

Proposition 1 (condition for existence of EODA). *EODA exists only if the output effect outweighs the emission effect for some* $k > 0$.

Proof: *In the text.*

Proposition 1 implies that if the *output effect* outweighs the *emission effect* for any level of $k > 0$, the potential donor country does not consider bilateral environmental aid.

Regarding the sufficient condition for the existence of EODA, let us assume that the marginal profit loss from EODA converges to 0 as the aid volume decreases to 0. For example if the output demand is linear in price and marginal production cost of the recipient firm is represented as $c_S^m(k) = f(k)c_0$ with $\lim_{k \to 0} f(k) = 1$ and $f'(k) < 0$ for $k > 0$, the condition is satisfied when $\tau_S = 0$.[11] Then, if there exists a positive interval of k over which the *emission effect* outweighs the *output effect*, the environmental aid emerges, increasing the welfare of the donor country. Figure 1 illustrates the case that optimal aid k^* exists. We state the following Corollary to Proposition 1.

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[11] When the demand is given as $p = a - by$, the Cournot equilibrium output becomes $y_i = \left(a + c_j - 2c_i\right)/3$ for $i, j = N, \mathrm{S}$ where c_i denotes the marginal cost of firm i. Then from (8) and (9), $\frac{\partial \pi_N}{\partial k} = \frac{2}{3} b \frac{\partial c_S}{\partial k} f'(k)$ and the condition is satisfied. As an example, we can consider $f(k) = 1 - k^2$ for $0 \le k \le 1$ where the maximum level of k is normalized to 1.

Corollary 1. Assume that $\lim_{k \to 0} \frac{\partial \Pi_N}{\partial k} = 0$. The necessary and sufficient condition for the existence of EODA is that $\frac{\partial EB}{\partial k} > 0$ for $k \in \left[0, \tilde{k}\right]$ with $\tilde{k} > 0$.

Proof: *In the text.*

Alternatively, $\frac{\partial EB}{\partial k}$ can be written as,

$$\text{(11')} \qquad \frac{\partial EB}{\partial k} = -\alpha_N \beta_{SN} \left(\left(\frac{y_S x(\bullet)}{k} \right) \left(\varepsilon_{y_S c_S} \varepsilon_{c_S k} + \varepsilon_{xk} (1 + \frac{\partial K_S}{\partial k}) \right) \right),$$

where $\varepsilon_{y_S c_S}$ is the elasticity of dirty output of the recipient country with respect to marginal cost of the recipient country; $\varepsilon_{c_S k}$ is the elasticity of marginal cost of the recipient country with respect to environmental aid ; and ε_{xk} is the elasticity of emission per unit of output with respect to environmental aid.[12]

Since $\partial y_S / \partial c_S < 0$, and $\partial c_S / \partial k \leq 0$ under Assumption 1 and $x'(K) < 0$, it follows that $\varepsilon_{y_S c_S} < 0$, $\varepsilon_{c_S k} \leq 0$ and $\varepsilon_{xk} < 0$. The product term $\varepsilon_{y_S c_S} \varepsilon_{c_S k}$ represents the elasticity of the dirty output of the recipient country with respect to aid, while the second term $\varepsilon_{xk} \left(1 + \frac{\partial K_S}{\partial k} \right)$ represents the elasticity of emission with respect to aid which in general depends on the response of the recipient country to aid, $\partial K_S / \partial k$. The output elasticity is positive while the emission elasticity is negative. We find that $\partial EB / \partial k > 0$ if and only if $\varepsilon_{y_S c_S} \varepsilon_{c_S k} + \varepsilon_{xk} \left(1 + \frac{\partial K_S}{\partial k} \right) < 0$. In other words, the environmental benefit to the donor country increases in EODA if emission is

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[12] See Appendix for derivation.

sufficiently elastic so as to be reduced to offset the effect of an increase in dirty output. The following remark for Proposition 1 is immediate.

Remark 1. *If the absolute value of the elasticity of pollution emission in the recipient country with respect to EODA is always larger than the elasticity of output of the recipient country with respect to EODA, the donor country does not transfer pollution abatement technology through EODA.*

2.4.2 Trade Competitiveness and the Size of EODA

Since both the forgone profit from offering EODA ($\Delta\Pi$) and the environmental benefit (EB) to the donor country depend on the amount of EODA, k, the socially optimum size of aid, $k*$, is determined at the level that maximizes the value of environmental aid (*VEA*), which can be represented as the function of forgone profit and environmental benefit to the donor country. More explicitly,

(12) $$k^* = Arg\max VEA\left[k;\Delta\Pi, EB\right],$$

subject to $k \geq 0$.

Assuming an interior solution, $k = k* > 0$,

(13) $$\frac{\partial VEA}{\partial k} = \frac{\partial \Delta\pi_N}{\partial k} + \frac{\partial EB}{\partial k} = \frac{\partial \pi_N}{\partial c_S}\frac{\partial c_S}{\partial k} + \frac{\partial EB}{\partial k} = 0.$$

Equation (13) states that the marginal cost of aid needs to be balanced against the marginal environmental benefit. We assume that the second-order condition for interior maximization is also satisfied.

From the Cournot equilibrium condition, we have,

$$p + y_N p'(y) = c_N \text{ and } p + y_S p'(y) = c_S,$$

which implies that $p'(y) = \frac{c_N - c_S}{y_N - y_S}$. Then, from Equation (8),

$$\frac{\partial \pi_N}{\partial c_S} = \frac{\partial \pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S} = \left(\frac{c_N - c_S}{y_N - y_S}\right) y_N \frac{\partial y_S}{\partial c_S} > 0,$$

From Equation (13) it follows that,

(13’) $$\frac{\partial VEA}{\partial k} = \left[\left(\frac{c_N - c_S}{y_N - y_S}\right) y_N \frac{\partial y_S}{\partial c_S}\right]\frac{\partial c_S}{\partial k} + \frac{\partial EB}{\partial k} = 0.$$

Let us define the elasticity of the cross-country output difference (with respect to the cost difference) as,

$$\Psi = \left(\frac{d(y_N - y_S)}{(y_N - y_S)}\right) / \left(\frac{d(c_N - c_S)}{(c_N - c_S)}\right) > 0.$$

Ψ measures the proportional change in the output differential between the two countries with respect to one percent change in the cost differential between them. It is a measure of the capacity of the recipient country to increase its competitiveness in relation to the donor country. For example, high Ψ implies that a small decrease in cost differential caused by the small decrease in c_S will cost a large increase in output differential caused by the expansion of y_S. Therefore, Ψ can be regarded as an indicator of bilateral trade competitiveness of the recipient country against the donor country. It remains always positive since Assumption 1 implies that $\partial y_S / \partial c_S < 0$. As Ψ increases, the marginal social cost of environmental aid increases.

In the case of linear demand, Ψ becomes a positive constant. Equation (13’) can now be written as,

(14) $$\frac{\partial VEA}{\partial k} = y_N \Psi \frac{\partial c_S}{\partial k} + \frac{\partial EB}{\partial k} = 0.$$

Letting k^* denote the value maximizing solution for Equation (14), we have,

(15) $$k^{*} = F(\Psi; \beta_{SN}, \alpha_N, \tau_N, \tau_S, K_N, \Gamma),$$

where β_{SN} is the effect of trans-border pollution; α_N is the social preference for a clean environment of the donor country; τ_N and τ_S are the stringency of environmental regulation of the donor and recipient country respectively; and Γ represents a vector of variables that affect incentives for EODA. It may include macro-economic shocks or structural trend of ODA between the donor and recipient countries. The following comparative static results are derived from the previous analysis. Define the value function,

(16) $$z(k,\Psi) = y_N \Psi \frac{\partial c_S}{\partial k} + \frac{\partial EB}{\partial k}.$$

Then, $z(k^{*},\Psi) = 0$ from Equation (14). Using the second order condition for interior maximum of *VEA*, we know that $\partial z(k,\Psi)/\partial k < 0$. Also, we have that $\partial z(k,\Psi)/\partial \Psi = y_N \frac{\partial c_S}{\partial k} < 0$. Totally differentiating value function, z, we have,

$$\frac{dk^{*}}{d\Psi} = -\frac{(\partial z(k,\Psi)/\partial \Psi)}{(\partial z(k,\Psi)/\partial k)} < 0.$$

The inequality implies that the optimal level of aid is negatively affected by the capacity of the recipient country to increase its competitiveness in relation to the donor country.

We now summarize the previous comparative static analysis in the following proposition.

Proposition 2. *EODA and* Ψ *are inversely related. If the demand is linear in price, then* Ψ *becomes a fixed parameter and there exists a negative causal relationship between* Ψ *and EODA.*

Proof: In the text.

2.4.3 Transferring "pure" Abatement Technology

So far we have considered EODA to transfer energy efficient pollution abatement technology (EET) that may affect the marginal production cost of the firm in the recipient country. However, if the abatement technology does not make preventive changes in the upstream process to save resources, it is not likely to affect the marginal production cost.[13] This type of technology may include air purification technology and, educational know-how for waste management to reduce air pollution emissions and infrastructure management for environmental cleaning, among other types of technology. Let us denote such technology as "pure" environmental technology (PET) and denote k_p as the amount of PET transfer through bilateral EODA.

We begin the analysis from the case where the recipient country does not regulate pollution so that $\tau_S = 0$. Since pollution is not regulated in the recipient country, the development of abatement technology by the recipient country is always in the form of EET. Therefore, the recipient country does not take into consideration of EODA that consists of PET in the second stage of the game (i.e., $\partial K_S / \partial k_p = 0$).[14] That is, when the donor country transfers PET only, the marginal cost of the recipient country can be written as,

$$c_S^m(K_S(0)+k_p) = c_S^m(K_S(0)).$$

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[13] Greaker (2003) presents an example of preventive abatement technology, which requires large, fixed costs. It is viable only if the accompanying production technology exhibits substantial scale economies.

[14] For example, the number of patents to save non-renewable resources may not be affected by the number of patents to purify the dirty air.

That is, if $\tau_S = 0$, the marginal cost of the recipient country is not affected by transferring PET from the donor country, so that $\partial c_S / \partial k_p = 0$. This implies that the forgone profit of the donor country is $\Delta\Pi = 0$. Thus, the donor's decision to transfer PET depends solely on the environmental benefit of the donor country, which is always positive since there is no *output effect*. More explicitly, Equation (10) becomes,

$$EB =$$
$$-\alpha_N \beta_{SN} \left\{ y_S \left(c_N(K_N, 0, \tau_N), c_S(0; K_N, 0, \tau_N) \right) \left[x\left(K_S(0; K_N, 0, \tau_N) + k_p \right) - x\left(K_S(0; K_N, 0, \tau_N) \right) \right] \right\} > 0.$$

The following proposition is immediate.

Proposition 3. *In the absence of environmental regulation in the recipient country, bilateral EODA to transfer PET unambiguously enhances welfare level of the donor country.*

Proof: In the text.

Suppose now that the government in the donor country decides whether it should transfer PET or EET through bilateral EODA with a given government budget. Then, the decision to transfer PET or EET will hinge on the marginal value of aid to transfer each type of abatement technology. For comparison, let us assume that both types of technology are measured in monetary terms.

The marginal value of transferring PET through bilateral EODA can be written as,

$$\frac{\partial EB}{\partial k_p} = -\alpha_N \beta_{SN} \left(y_S \left(c_N(K_N, 0, \tau_N), c_S(0; K_N, 0, \tau_N) \right) x'\left(K_S(0; K_N, 0, \tau_N)) + k_p \right) \right) > 0 .^{15}$$

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[15] Marginal value of transferring PET through bilateral EODA is positive since $\partial K_S / \partial k_p = 0$.

As in the previous analysis, let us define the marginal value of environmental aid to transfer PET as the value function $z_p(k_p, \Psi)$ where $z_p(k_p, \Psi) = \frac{\partial \Delta \Pi}{\partial k_p} + \frac{\partial EB}{\partial k_p} = \frac{\partial EB}{\partial k_p}$.

Then, by using Equation (13), the difference of the marginal value of environmental aid from transferring EET and PET can be written as,

$$G = z(k, \Psi) - z_p(k_p, \Psi) = \frac{\partial \Delta \Pi(k)}{\partial k} + \frac{\partial EB(k)}{\partial k} - \frac{\partial EB(k_p)}{\partial k_p}. \tag{17}$$

Given $\tau_S = 0$ and by Equations (11), (16) and (17) we have that

$$\begin{aligned} G\big|_{\tau_S=0} = {} & y_N \Psi \frac{\partial c_S}{\partial k} - \alpha_N \beta_{SN} \frac{\partial y_S}{\partial c_S} \frac{\partial c_S}{\partial k} x\left(K_S(k; K_N, 0, \tau_N)) + k\right) \\ & -\alpha_N \beta_{SN} y_S\left(c_N(K_N, 0, \tau_N), c_S(0; K_N, 0, \tau_N)\right) x'\left(K_S(k; K_N, 0, \tau_N)) + k\right)\left(\frac{\partial K_S}{\partial k} + 1\right) \\ & +\alpha_N \beta_{SN} y_S\left(c_N(K_N, 0, \tau_N), c_S(0; K_N, 0, \tau_N)\right) x'\left(K_S(0; K_N, 0, \tau_N)) + k_p\right), \end{aligned} \tag{17'}$$

where the third term in the bracket on the right hand side of Equation (17') represents the difference in the *emission effect* caused by transferring PET and EET through bilateral EODA.

Since $\partial c_S / \partial k \leq 0$, and $\partial y_S / \partial c_S < 0$ under Assumption 1, the sign of G evaluated at $k = k_p$ depends on this relative magnitude of the *emission effect* from transferring two different technologies.

To determine the sign of G at $k = k_p$ we need further information. In particular, the sign of G depends on the response of the firm in the recipient country in terms of developing domestic abatement technology upon receiving EODA (i.e. $\partial K_S / \partial k$) and property of abatement technology (i.e. $x''(K)$). It can be shown from (17') that if the development of abatement technology in the recipient country decreases in EODA (i.e. $\partial K_S(k)/\partial k < 0$), and if an increasing returns to scale prevails in abatement technology (i.e., $x''(K) < 0$), then the *emission effect* of transferring PET dominates the *emission effect* of EET: thus, $G < 0$.

We now turn to the case when $\tau_S > 0$ under the assumptions that $x''(K) < 0$ and $\partial K_S(k)/\partial k < 0$ so that the sign of G evaluated at $k = k_p$ is negative when $\tau_S = 0$. From the definition of $z(k, \Psi)$ and $z_p(k_p, \Psi)$, we know that $G < 0$ is continuous in τ_S. Then we can rewrite G as,

$$G(\tau_S) = z(k, \Psi) - z_p(k_p, \Psi). \tag{18}$$

If $G < 0$ is continuous in τ_S, there exists a small positive interval, $(0, \hat{\tau}_S)$, such that for any τ_S in the interval, $G(\tau_S) < 0$. In other words, for small enough levels of pollution tax in the recipient country, the marginal value of bilateral environmental aid is greater for PET than that of EET as long as the transfer of PET reduces larger amounts of trans-border pollution than EET. We summarize the analysis in the following proposition.

Proposition 4. *Assume that there exist economies of scale in pollution abatement technology and the development of new pollution abatement technology in the recipient country does not increases as a result of EODA. Then for a sufficiently small level of pollution tax in the recipient country, the marginal value of EODA to transfer PET is greater than that of EODA to transfer EET*.

Proof: In the text.

Proposition 4 examines the conditions under which the donor country chooses PET over EET as a type of environmental technology to transfer through EODA. The donor offers a greater volume of aid to transfer PET than when the aid consists of EET only.

3. Empirical Analysis

In this section, we test the empirical implication of Proposition 2. Equation (15) implies that the optimum amount of EODA from the donor country to China depends on, Ψ which affects the donor country's domestic profit, $\Delta\pi_{Nt}$ given other variables such as the magnitude of trans-border pollution from China to donor, TB_{cN} , that affects the environmental benefit, EB, of the donor country.

$$EODA_{Nc} = f(\Psi; TB_{cN}, \Omega), \tag{19}$$

where Ω denotes a vector of control variables including unobserved country-specific preference toward a cleaner environment, α_N; the pollution regulation measure of China, τ_S and the donor country, τ_N; the technological capability of the donor country that enlarges the set of feasible aid to the polluting country, K_N and structural trend that controls for macro-economic shocks.

3.1. Data

3.1.1 Environmental ODA

The Project-Level Aid Database (PLAID) provides project-based EODA data with detailed aid contents. This database is regarded as the most recent and consistent data source for cross-country ODA grants in general.[16] Data on EODA to China as a means of technology transfer are quite limited in comparison with GEODA that

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[16] The PLAID not only offers disaggregate aid data from every project by multiple policy objects but also provides information on whether each of the EODA projects involved technical cooperation or whether the aid project was intended to aid the energy or industry sectors.

includes non-environmental aid. In fact, only 19 countries have made an effort to transfer their technology and know-how by means of EODA since 1985. Using PLAID on bilateral EODA flows, a panel of yearly aggregate EODA flow for each donor country (19 OECD countries) can be constructed from 1985 to 2008.

3.1.2 Historical Trend of Bilateral Environmental Aid to China

Figure 2 below shows the sum of all bilateral flows of ODA including environmental ODA from 19 OECD countries to China. GODA, which consists largely of dirty aid, has been decreasing since the mid-1990s, as China's economic growth has steadily continued each decade, beginning in the 1980s. In contrast with the declining dirty aid, however, the volume of EODA has increased, even since the early 2000s in which China emerged as a leading exporter in the global market.

3.1.3 Measuring the Possible Loss in Profits

As shown earlier in the previous section, the possible loss in profits depends on the elasticity of the cross-country output difference with respect to the cost difference (Ψ). In order to obtain a more objective measure of the possible loss in profits in the output market, we construct two different measures.

Given the domestic demands of the donor and recipient countries, Ψ depends on the net export flow of the recipient country to the donor country or any other trade-related measures of the competitiveness of the recipient country in the export market.

For this reason, we first consider the net import of the donor country from China. As the elasticity of the cross country output difference with respect to the cost difference depends on output difference, we expect that this elasticity increases with the net import of the donor country from China. The magnitude of the net import can then reflect the threat of forgone profits from offering environmental aid.[17] We use the net import of the donor country from China as a share of the donor's GDP, which can be obtained from the Direction of Trade (DOT) database of the International Monetary Fund (IMF) (IMF, 2010).[18]

In addition, as Ψ is affected by the export competition between each of the donor countries and China, we take the ratio of the market share of China in the donor's market and the market share of the donor in China's market. For expositional convenience we denote this variable as the "relative market share" of China. It can be written as,

$$relmktshare_{cNt} = Log\left(\frac{Export_{cNt}}{\sum_i Export_{cit}} / \frac{Export_{Nct}}{\sum_i Export_{Nit}} \right),$$

where subscripts N, c and t represent donor countries, China, and time, respectively $Export_{cNt}$ refers to the total export volume of China to the donor country at year t; $Export_{Nct}$ is the total export volume from the donor country to China at year t; $\sum_i Export_{cit}$ refers to the total export volume from China to the world and $\sum_i Export_{Nit}$ is total export volume from the donor country to the World. The underlying premise is that if the marginal cost of China's exporting firm decreases

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[17] We implicitly assume that the recipient firm can utilize new environmental technology more efficiently when it has a greater market share in the global market. We can also extend the representative firm model in the previous section by incorporating firm heterogeneity. The greater net trade surplus of the recipient country then implies a greater number of incumbent recipient firms that can benefit from EODA. In addition, EODA may stimulate the new entry of firms in the recipient country.

[18] IMF DOT database provides each country's import and export amount of all goods against China. After calculating net trade balance we divide it with respect to each donor country's nominal GDP in order to take into account the size of each country.

relative to firms in the donor countries, China's relative export performance in comparison with that of the donor country in the global market improves.

Controlling for bilateral trade complementarity

The estimation bias from using both measures of trade competitiveness as proxies for the social cost of aid may be affected by the possible complementary bilateral trade relationship between the donor and China. In fact, in the presence of the vertical international division of labor between certain donor countries and China, estimation results are likely to be biased if we do not control for the trade relationship between donors and China. Hoekman et al. (2002) provides trade complementarity index that contain useful information on how well the structures of a country's imports and exports match. The index is used to control for a possible complementary bilateral trade relationship between the donor and China.[19]

3.1.4 Trans-border Air Pollution Measure

Yellow sand (also known as yellow dust) storms from China have attracted popular attention since 1990 as one of the major trans-border air pollutants blowing to other countries from China. The frequency of these storms has recently increased, damaging Pacific-based countries. It is difficult to obtain scientifically accurate and direct measures of the trans-border content of air pollutants such as yellow dust that cross border. In this paper, the yearly average SO2 emission level of China is adopted as SO2 emissions associated with this dust storms negatively affect the quality of soil, biomass and the respiratory system of human body (i.e. Griffin, et al. 2001).

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[19] See Michaely (1996) for details.

It should be noted that China is located in the middle latitudes between 30 and 60 degrees. Since this region is under the influence of westerly winds, which blow from west to east, pollutants from China also move from west to east.[20] Accordingly, the wind-weighted distance is adopted in the estimation model. Figure 3 shows the direction of westerly wind blowing from west to east.

Damage from trans-border pollution is measured taking into consideration the direction of the wind blowing from China as follows:

$$TB_{cN} = \beta_{cN} x_c ,$$

where $\beta_{cN} > 0$ is the wind-weighted distance between China and each donor country, and x_c is the air pollution content (i.e., level of SO2 emissions) in China that can cross the border of China. For example, for countries located west of China (i.e., European countries), we take the sum of the distance from China to the US and from the US to each donor country as follows:

$$\beta_{cN} = \beta_{c,USA} + \beta_{USA,N*},$$

where $N*$ denotes the OECD member country located in the European region.[21]

We note that the geographical distance differs from the wind-weighted distance. Some European countries are located rather close to China but, are not influenced by

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[20] China-borne air pollutants travel all the way to the western part of the U.S. due to westerly wind (see for example, http://www.businessinsider.com.au/californians-hacking-up-lungs-due-to-china-pollution-2013-1 for a recent news report).

[21] For Australia and New Zealand, we only take physical distance and do not consider the direction of the westerly winds. The distances between countries are retrieved from www.distancefromto.net

the westerly movement of local pollutants. If, however, they maintain a close economic partnership with China, they may have an incentive to offer environmental aid to cope with global pollution.

3.1.5 Other Control Variables

As a base model, we select the following set of control variables to test Proposition 2.

1) Degree of the environmental regulation in China (τ_c) and in the donor country (τ_N): In order to capture the degree of environmental regulation, a number of international environmental agreements including not only those that have been signed or ratified but also those that have entered into effective force are adopted for the estimation. The data for different degrees of international agreements over time are taken from Mitchell (2012).

2) Technological capability of the donor country (K_N): Technological capability for reducing air pollution is not well documented in the literature. However, OECD (2010) documents the number of filed patents that specifically aims to reduce air pollution. We take this measure to gauge the technological capacity to control local air pollution.

3) Country-specific social preference for a clean environment α_N : The social preference of the donor country measures the trade-off between environmental quality and forgone profits. We disaggregate this effect into time varying and time constant effects so that $\alpha_{Nt} = \alpha_N + \delta_{Nt}$ where α_N is unobserved donor specific preference measure that does not change over time and δ_{Nt} is the time varying donor specific preference measure. Following the literature, we use gross national income per capita

as a proxy measure to capture this effect (i.e., Alesina and Dollar, 2000; Burnside and Dollar, 2004; Hicks et al. 2008).

4) Time varying structural effect: We also control for time varying structural effects by including (i) total bilateral ODA to China (ii) time dummies for all sample years and (iii) per capita income of China.

3.2 Base Model Equation

After log linearization, Equation (19) is specified follows;

$$\begin{aligned} (20)\quad LnEODA_{Nct} &= \varsigma_1 Comp_{cNt} + \varsigma_2 LnTB_{cNt-1} + \varsigma_3 LnTCI_{Nct} + \varsigma_4 Ln\tau_{ct-1} + \varsigma_5 Ln\tau_{Nt-1} \\ &+ \varsigma_6 LnM_{Nt-1} + \varsigma_7 LnTech_{Nt-1} + \varsigma_8 LnM_{ct-1} + \varsigma_9 LnODA_{Nct} + \alpha_N + \eta_t + \varepsilon_{Nt}, \end{aligned}$$

where subscripts N , c and t represent the donor countries, China, and time, respectively and $\varsigma_j\,(j=1,...,9)$ are fixed parameters. Thus, $EODA_{Nct}$ is the annual aggregate level of environmental ODA from the individual donor country to China in year t; $Comp_{Nct}$ is the trade competitiveness measures of China against each donor in year t; TB_{cNt-1} is the trans-border air pollution measure from China to the donor country in year $t-1$; TCI_{Nct} is the trade complementarity index between each donor country and China in year t; τ_{ct-1} and τ_{Nt-1} are the degree of environmental regulations for China and each donor county in year $t-1$; M_{Nt-1} is the GNI per capita for each donor country in year $t-1$; $Tech_{Nt-1}$ refers to the accumulated number of patents for reducing air pollution for each donor country in year t; M_{ct-1} is the GNI per capita for China in year $t-1$; ODA_{Nct} is the aggregate volume of ODA from all OECD countries to China in year t; α_N references the unobservable donor country effect that includes cultural, historically driven preferences for a cleaner environment that are time-fixed; η_t is the time effect common to all donor countries and ε_{Nt} is a random disturbance with the usual desirable properties.[22]

Given these control variables as specified in Equation (20), Proposition 2 implies that the estimated coefficient is such that $\varsigma_1 < 0$ and is statistically significant.

To test the hypothesis of Proposition 2, we first run OLS, country fixed, and random effects models for the base specification (Equation (20)). The results for OLS, country, and random and fixed effects are presented in columns 1, 2 and 3, respectively. If the equation (20) is correctly specified so that there are neither endogeneity issues nor measurement errors, we have efficient and consistent estimators. Although the estimated coefficients for net import and trans-border pollution are statistically significant and have expected signs as shown in the first two rows of Table 2, thereby confirming our prediction from Proposition 2, the estimation results may be biased for a number of reasons. In particular, potential econometric issues, such as the possible endogeneity of trade competitiveness measures should be addressed.

3.3 Estimation Strategy: Tackling Endogeneity

Our estimation strategy is geared toward solving the endogeneity from reverse causality. Environmental ODA is comprised of transferring PET which affects the marginal cost of the dirty outputs in China less severely (PEODA) and transferring abatement technology, which affects the marginal production cost of the dirty outputs

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[22] We use one year lagged values of income, degree of environmental regulations and abatement technology stock control variables to avoid simultaneity.

in China more severely (EEODA). When EEODA is offered to China from the donor country, it may increase China's export performance against the donor country given that China and the donor country are competing for the dirty good market share, and therefore biases estimates of the trade competitiveness measure of China. Thus, the estimates of the trade competitiveness measures of China cannot be taken seriously as evidence of causality.

We construct instruments for trade competitiveness measures. Our key idea for instrumentation is to model the variation of trade competitiveness measures by considering the relative size of the donor's home market of the goods that donors are importing from China. Our assumption is that the China's export volume to the donor country (with which she potentially competes) depends on the donor country's market size.

One of the proxy variables for relative market size is the relative country size of the donor and China. The larger the donor is relative to the recipient, the more sales the recipient is likely to make; thus, the ratio of the donor population to that of China may be a good proxy for relative market size. In addition, the relative size effect is likely to be particularly pronounced when China is less resistant to trade with donors. We capture this channel by including the interaction between the relative size of the population and such trade resistant variables as physical distance, past colonial history and free trade regional dummies.

The information content of our instrument regarding the trade competitiveness measures of China over the donor can be examined by depicting the relationship between the actual and fitted competitiveness measures using instrumental variables.

To do this we first run the following equation for both types of trade competitiveness measures used.

$$\begin{aligned} Comp_{cNt} &= \psi_1 \log\frac{Pop_{Dt}}{Pop_{ct}} + \psi_2 \log\left(\frac{Pop_{Dt}}{Pop_{ct}} \times dist_{cD}\right) + \psi_3 \log\left(\frac{Pop_{Dt}}{Pop_{ct}} \times colony\right) + \\ &\psi_4 \log\left(\frac{Pop_{Dt}}{Pop_{ct}} \times Euro\right) + \psi_4 \log\left(\frac{Pop_{Dt}}{Pop_{ct}} \times Asia\right) + \Lambda\chi_t + \phi_{cDt} \end{aligned} \qquad (21)$$

where $Comp_{cNt}$ is the trade competitiveness measures of China against the donor country; Pop_{ct} is the size of population of China in year t; Pop_{Dt} is the size of the population of the donor country D in year t; $dist_{cD}$ is the physical distance between China and donor country D; $colony$ is the dummy variable that is equal to1 if China had been a colony of donor country D; $Euro$ and $Asia$ are regional dummies that capture cultural difference among regions; and χ_t is the vector of year dummies from 1985 to 2008.

After controlling the effects of other covariates included in Equation (21) the relationships between actual and fitted trade competitiveness measures of China are positive and statistically significant.[23] Our instrument appears to contain a non-negligible amount of exogenous information about both measures of trade competitiveness of China.

3.4 Estimation Results

A. The basic IV results

We now present estimates for the fixed country effects using instrumental variables. In models 1 and 3 of Table 3, we present estimation results of the second stage of the

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[23] The coefficient is 0.021 and is statistically significant at 10% confidence level for the relative market share of China against donor. Also the coefficient is 0.027 and is statistically significant at 10% confidence level for the net import of the donor country from China as a share of their GDP.

instrumental variable specification, which is representative of the results that we obtained more broadly. The equations are reasonably specified, as many of the standard covariates show expected signs and statistical significance. In particular, the estimated coefficients for both trade competitiveness measures of China over the donor country is negative and statistically significant at 1% level which confirm our prediction from Proposition 2 and assure that EODA is indeed strategically distributed to China as it decreases the trade competitiveness measures of the recipient country against the donor.

B. Validity of instruments: exclusion restriction

Let us now turn to possible concerns about our instruments. First of all, do they satisfy the exclusion restriction; that is, are they plausibly exogenous? In our framework, the relative population may be correlated with bilateral EODA in other ways, rather than through the trade competitiveness measures of China. For example, the donor may wish to "influence" China through bilateral EODA which can be captured by the relative population size of the donor country in comparison with China. In this case the exclusion restriction may not be satisfied (i.e. Rajan and Subramanian, 2008).

One way to examine whether our relative population variable passes the exclusion restriction is to simply include the variable directly in the second stage. As shown in Table 4, the estimated coefficients of both trade competitiveness measures of China are not significantly altered and we do not find a consistent pattern of the relative population variable being significant.

A second check is to see whether our instrument passes over identification tests. For this reason, we perform a Hansen test for over identification for model 1 and 3. The null hypothesis for this test is that the over-identification restrictions are valid. As reported in Table 3, the p-value for Hansen J statistics rejects the null hypothesis, increasing our confidence that our instrument set is appropriate.

3.5 Robustness Checks

Omitted variable bias

Omitted variable bias is an important potential issue in the specification of Equation (20). If a relevant variable is omitted, it will be absorbed in the error term, which leads to biased and inconsistent estimates. The panel estimation model with country fixed effects will account for time invariant omitted variables. Time varying omitted variables make up the major challenging issue. If a time varying omitted variable, such as the natural capital of the donor country, negatively correlates with net import volume from China, but positively correlates with EODA, then the coefficient of the net import as a share of GDP will be biased downward. We employ the Altonji (2005) methodology, which is known as Added Controls Approach (ACA), where we control for several other variables and see whether the coefficient of interest changes.

Studies have shown that several factors may directly or indirectly affect GODA. Despite the fact that the motivation for bilateral EODA is different from GODA, factors that affect GODA may also affect decision of donor countries' government to transfer bilateral EODA. We address this issue by including several time varying variables each of which has been argued to be an important determinant of bilateral aid from the literature (i.e. Lumsdaine, 1993; Alesina and Dollor, 2000; Hicks et al., 2008). We choose degree of democracy of the donor, the volume of natural capital of the donor, the donor's domestic pollution level, the population size for both the donor and China; the population density of the donor; the donor's public expenditure on R&D to protect the environment, and 19 OECD donor's total environmental aid. We add a set of variables representing each of the determinants listed above in sequence into the fixed country effect estimation presented in Table 3 to test the robustness of the variable of interest. Table 5 shows the coefficients of both trade competitiveness measures of China and trans-border pollution measure as each control is added. An increase in the adjusted R-squared relative to the base estimations implies that inclusion of the additional set of controls raises the explanatory power of the model. If the coefficient of both trade competitiveness measures and the trans-border pollution measure retains the sign and significance, the estimated coefficients are stable and robust in the face of the additional regressors.

Table 5 shows that the coefficients of both measures of trade competitiveness of China and the trans-border pollution measure are largely unaffected by the additional control variables. The estimated coefficients for both measures of trade competitiveness have negative signs and statistically significant at 1 % level. In particular, variables such as OECD's total environmental ODA, population of the donor country and donor country's domestic pollution raise the adjusted R squared of both model 1 and model 3 of Table 3. For some variables such as openness, degree of democracy and the population density of the donor country, the adjusted R squared is raised for one of the models, while the adjusted R squared for the other models remained the same. Considering the potential controls presented in Table 5, we can conclude that the results in Table 3 are robust to potentially omitted variables that correlate with these sets of variables.

Dominance test

Easterly (2004) contends that many cross-sectional regression results are driven by a small number of outlier observations. In order to address this issue, we drop both the top 1% and the bottom 1% observations of the dependent variable (log environmental ODA) and the variable of interest (trade competitiveness measures) and re-estimate the country fixed effects with year dummies in Table 3. The results are presented in Table 6. The signs of the coefficients for both trade competitiveness measures of China are negative and have at least 5% level of significance for all of the sample alterations for fixed country effects with IVs. These results imply that the base results are not driven by the outliers.

Misspecification

Aside from the possibility of reverse causality, econometric specification of the base model (Equation (20)) may produce errors due to misspecification. In particular, since the aid data consist of country-specific time series data, these data may be serially correlated and the correlated part of the error term can affect the effects of covariates of major explanatory variables.

In the presence of serial correlation, one possible means of tackling this issue is to recognize the clusters involved in the panel regression and to correct the standard errors accordingly. However, this procedure treats the omitted dynamics detected by the diagnostic test as a problem rather than as an invitation to re-specify the model to include the omitted dynamics in the estimated part of the model and thus to exploit this additional information in estimation. This argument has recently been strongly supported by King and Roberts (2012) in a study of robust standard errors. Accordingly, a potentially more interesting solution is to estimate a dynamic panel model.

We consider a dynamic panel model where the country-specific dependent variable is influenced by its own lagged value. We use the system GMM estimators first proposed by Arelloano and Bover (1995) and Blundell and Bond (1998). Instead of using the seemingly exogenous instruments, the Arellano-Bond system GMM estimator uses lagged values of the trade competitiveness measures. This approach makes endogenous variables pre-determined, and they are, thus, not correlated with the error term in the above equation. The system GMM uses first-differences to transform Equation (20). Through this transformation, the fixed country-specific effect is removed since it does not vary with time.[24]

After log linearization for dynamic panel estimation, we have,

$$\begin{aligned} LnEODA_{Nct} = {} & \beta_1 LnEODA_{Nct-1} + \beta_2 LnComp_{cNt} + \beta_3 LnTB_{cNt-1} + \beta_4 LnTCI_{Nct} + \\ & \beta_5 Ln\tau_{ct-1} + \beta_6 Ln\tau_{Nt-1} + \beta_7 LnM_{Nt-1} + \beta_8 LnTech_{Nt-1} + \beta_9 LnM_{ct-1} \\ & \beta_{10} LnODA_{ct} + \alpha_N + \gamma_t + \mu_{Nt}, \end{aligned} , \tag{22}$$

---

[24] The system GMM is known to have the potential for obtaining consistent parameter estimates even in the presence of measurement error and endogenous right-hand-side variables. For validity, using system GMM can be tested in the GMM framework, for example by the use of Hansen tests of over-identifying restrictions.

where $EODA_{it-1}$ denotes a one-year lag of the dependent variable.

The estimation results for Equation (22) are presented in Table 7. The estimation results re-confirm Proposition 2. In particular, we find that EODA is negatively affected by China's relative trade competitiveness measures against OECD donors. Moreover, the lagged dependent variable is statistically insignificant implying that there are no significant dynamic effects. The Hensen test indicates that the instruments are exogenous, and no second order correlation is found.

4. Extension: Categories of Environmental Aid

Proposition 4 implies that in the cases in which Chinese firms are regulated by its government, the net benefit of aid to transfer PET is greater than that of transferring EET. This fact may imply that China's trade competitiveness measures may negatively affect the volume of bilateral environmental aid more if it contains only EET rather than PET. In this section, we extend our empirical analysis by employing disaggregated environmental ODA data to test and confirm Proposition 2 using the empirical implication of Proposition 4.

4.1 Decomposition of Environmental Aid

As PLAID codes each component of bilateral environmental aid by aid-receiving sectors, we can disaggregate EODA into two different categories, namely, aid that affects production costs more directly and aid that does not directly affect production costs. Table 1 presents environmental aid by its receiving sectors. It is assumed that if environmental aid was transferred to such sectors as *Transport and storage, Energy generation and supply, Agriculture, forestry and fishing production,* and *Industry, mining and construction*, such EODA is supposed to affect marginal production cost of the aid-receiving firms more directly, and is classified as energy efficiency enhancing environmental ODA (EEODA), which transfers abatement technologies other than PET. All other types of environmental aid are assumed not to directly affect the marginal production costs of the individual polluting firms and are classified as pure environmental ODA (PEODA), which transfers PET. PEODA includes types of aid that can be treated as public inputs such as the general education of technical personnel, the improvement of air purification technology and health services.

Figure 4 shows that EEODA seems to be more volatile reflecting cyclical business conditions. In fact, EEODA has declined more rapidly than PEODA over time since the mid-1990s, confirming the conjecture that EEODA tends to decline as China becomes more competitive in the global export markets.[25] Apparently, the volume of PEODA does not co-move with EEODA. Due to the public good character of PEODA, factors affecting aid effectiveness, such as the stringency of China's regulatory measures, are likely to influence PEODA more seriously than EEODA.

4.2 Effect of the Trade Competitiveness Measures of China on PEODA and EEODA

One of the efficient ways of comparing the slopes of the trade competitiveness measures of China for different dependent variables is to estimate both equations simultaneously, assuming that the error terms for both equations are correlated.

Then Equation (20) then becomes,

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[25] It is worth noting that the intellectual property rights of environmental technology have never been strictly enforced in China, while the number of patent filings for environmental technology by patent holders, such as the Japanese, American, or European companies in Africa and other developing countries, has remained quite modest until the recent past.

$$
\begin{aligned}
(23)\quad LnPEODA_{Nct} &= \nu_1 LnComp_{cNt} + \nu_2 LnTB_{cNt-1} + \nu_3 LnTCI_{Nct} + \nu_4 Ln\tau_{ct-1} + \nu_5 Ln\tau_{Nt-1} \\
&\quad + \nu_6 LnM_{Nt-1} + \nu_7 LnTech_{Nt-1} + \nu_8 LnM_{ct-1} + \nu_9 LnODA_{Nct} + \alpha_N + \theta_t + \rho_{Nt},
\end{aligned}
$$

$$
\begin{aligned}
(24)\quad LnEEODA_{Nct} &= \omega_1 LnComp_{cNt} + \omega_2 LnTB_{cNt-1} + \omega_3 LnTCI_{Nct} + \omega_4 Ln\tau_{ct-1} + \omega_5 Ln\tau_{Nt-1} \\
&\quad + \omega_6 LnM_{Nt-1} + \omega_7 LnTech_{Nt-1} + \omega_8 LnM_{ct-1} + \omega_9 LnODA_{Nct} + \alpha_N + \lambda_t + \xi_{Nt}.
\end{aligned}
$$

We jointly estimate Equation (23) and (24) using three-stage least square estimation method with an auxiliary equation for the trade competitiveness measures of China. Based on our analysis, we expect ν_1 and ω_1 to be negative and $|\nu_1| < |\omega_1|$. The results are presented in Table 3 model 2 and model 4.

The estimated coefficients of both trade competitiveness measures of China for both EEODA and PEODA are negative. Also, as expected from the theoretical model, the estimated coefficient for EEODA in the system is greater in absolute value than that for PEODA.[26] Results from a formal test to check whether the two coefficients are different ensure that the effect of both trade competitiveness measures of China in each of the PEODA and EEODA equations are not the same.[27] In fact, this finding not only confirms our theoretical prediction but also renders one possible explanation as to why EEODA from each donor country has decreased over the past three decades while PEODA has been increasing steadily.

Since the profit loss from PEODA is smaller than from EEODA, PEODA is likely to increase more elastically with trans-border pollution if both types of aid are equally

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[26] In the previous section, we explain that in the presence of environmental regulations in China, PEODA can have a less significant effect on the marginal cost of China's exporters than EEODA.

[27] Two equations (model 2 and model 4) were estimated jointly using country fixed three stage least square to test the null hypothesis that the coefficient of the trade competitiveness measure is equal. The P-value of Chi squared test statistics (cross equation restriction) that the effect of each of the trade competitiveness measure is equal in both model 2 and 4 is 0.016 and 0.009 implying that the difference in coefficients of the trade competitiveness measures of China are statistically significant at least at 5% level.

effective at reducing trans-border pollution. The estimation results show that PEODA is more strongly influenced by trans-border pollution than EEODA.[28]

5. Concluding Comments

This paper derives an empirically testable set of propositions related to the determinants of environmental aid as a non-market solution for trans-border pollution. The donor country balances environmental benefits against the costs of aid of the donor country that result in the potential erosion of their export competitiveness in the global market of the donor country. The bilateral trade balance of the donor country against the recipient country is shown to significantly affect the volume and composition of bilateral EODA.

The flow of bilateral EODA from OECD countries to China in recent decades reflects the effects of trade competition and trade volume on environmental bargaining. Various types of sensitivity analysis, including an added control approach and dominance test, support the paper's theoretical proposition that trade competition adversely affects environmental aid.

In addition, the decomposition of environmental aid into energy efficiency improving aid and pure environmental aid enables the estimation of simultaneous equations with which to test proposition 2. The estimation result implies that trade competition or threat of competition significantly reduces EEODA that potentially enhances the export competitiveness of the recipient country. Unlike EEODA, however, PEODA is relatively less affected by China's trade competitiveness.

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[28] The P-value of Chi squared test statistics (cross equation restriction) that the effect of trans-border pollution measure is equal in both model 2 and 4 is 0.000 and 0.000 implying that the difference in coefficients of the trade competitiveness measures are statistically significant at least at 1% level.

The empirical analysis of this paper suggests that bilateral EODA as a non-market solution has a limited role. Despite increasing trans-border pollution and trade volume between the donors and China, EEODA have slowed down. However, PEODA is increasing over time. This trend may reflect that donor countries are reconciling growing environmental concerns with equally growing concerns about trade competitiveness with respect to China. As the scope of environmental aid that improves China's energy efficiency is limited by trade competition, the change in composition of bilateral EODA may reflect a means by which a solution to the trans-border pollution issue can be found.

Figure 1. Condition for the existence of EODA

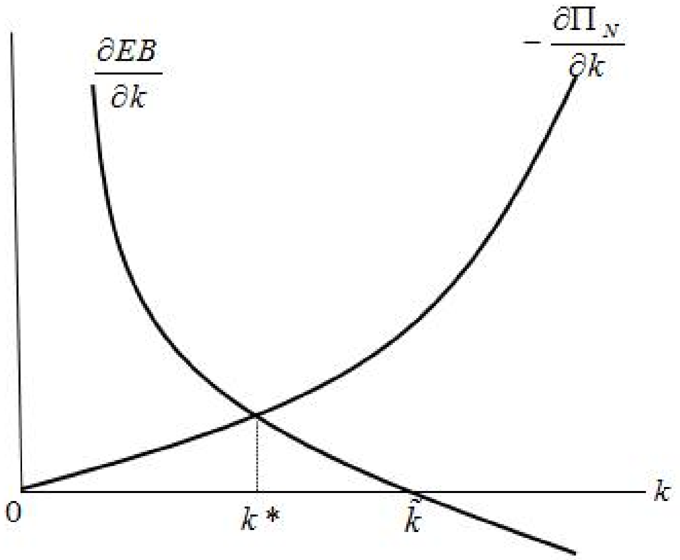


Table 1. Classification of sectors that receive two different types of environmental aid

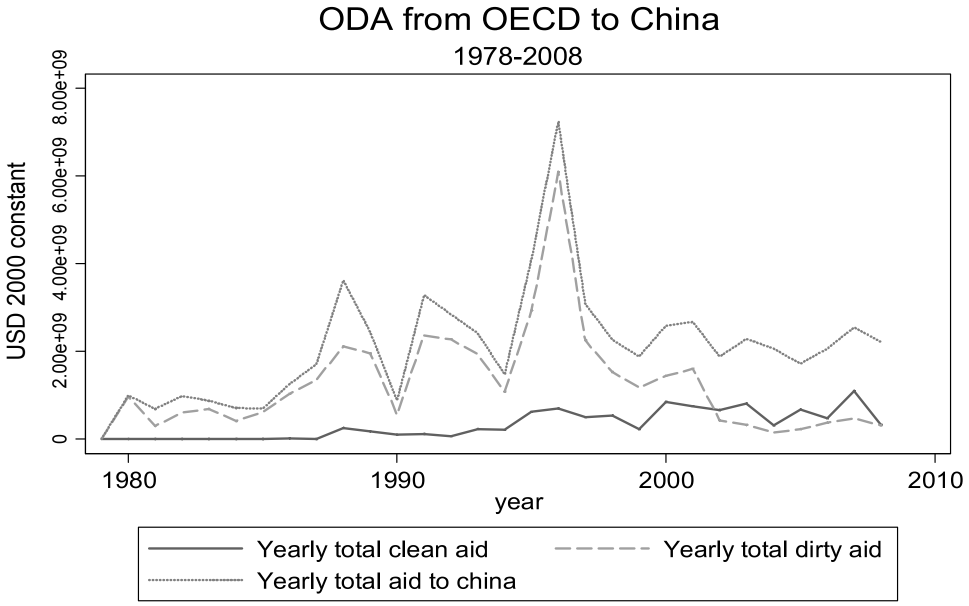
Type of Environmental Aid

Aid receiving sectors

EEODA	**PEODA**
Transport and storage	Education
Energy generation and supply	Health
Agriculture, forestry and fishing production	Population policies / programs and reproductive health
Industry, mining and construction	Water sanitation
	Government and civil society
	Conflict prevention and resolution , peace and security
	Banking and financial services
	Business and other services
	Communication and media
	Banking and financial services

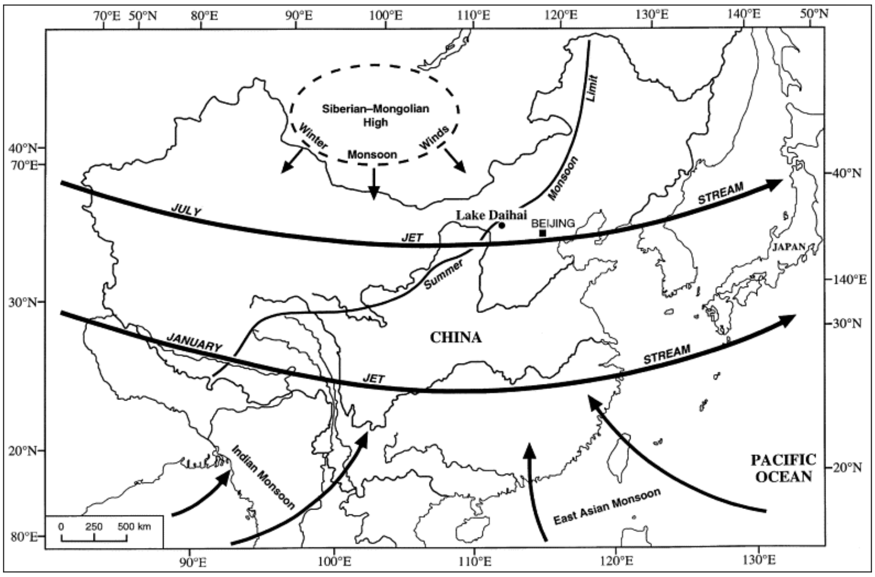
Source: AidData (2010)

Figure 2. ODA from 19 OECD donors to China



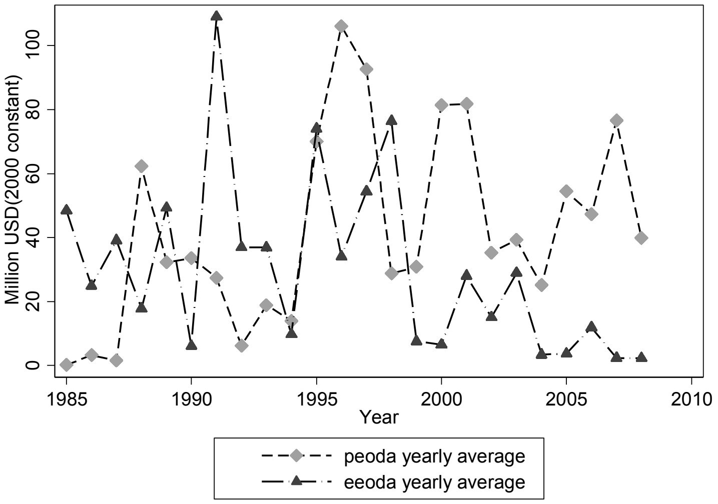
Source: PLAID, OECDdata, and author's calculation

Figure 3. Direction of Westerly wind



Source: Xiao et al. (2004)

Figure 4. Yearly Average of PEODA and EEODA from 19 OECD donors to China, 1985-2008.



Source: PLAID, OECDdata, and author's calculation

Table 2. OLS , Random effects and Fixed effects without using IVs.

	OLS	Country RE	Country FE
Dependent variable	Log (Environmental ODA)		
Net import of the donor (% GDP)	-0.238***	-0.544***	-0.630***
	[0.065]	[0.084]	[0.110]
Log(So2 emissions in China*wind weighted distance, year lagged)	1.250***	1.565**	4.469**
	[0.351]	[0.796]	[1.848]
Log(Trade complementarity index)	0.802	0.390	1.248
	[2.683]	[1.589]	[2.348]
Log(Cumulative number of ratified environmental regulation entered into force, Donor country, year lagged)	1.509***	1.682	1.722
	[0.675]	[1.402]	[2.864]
Log(Cumulative number of ratified environmental regulation entered into force, China, year lagged)	-0.457	-1.309	-0.625
	[0.444]	[3.544]	[0.394]
Log(GNI per capita, Donor country, year lagged)	1.873***	2.589***	2.657***
	[0.343]	[0.883]	[1.111]
Log(Cumulative number of filed patents regarding air pollution, year lagged)	0.010	0.025	0.499*
	[0.071]	[0.565]	[0.281]
Log GNI per capita, China (year lagged)	-1.868	-0.555	-0.859
	[1.214]	[1.284]	[1.708]
Log (19 Donor's total aid to china)	0.953***	0.964***	0.826***
	[0.059]	[0.064]	[0.150]
Breusch and Pagan LM test (P- value)		0.00	
Hausman test (P value)			0.01
Number of observations	245	245	245
Number of countries	19	19	19
R-squared	0.64	0.67	0.58

Note: 1) *,**, *** denotes significance level in 10, 5 and 1% respectively 2) Robust standard errors are in brackets.

Table 3. Country Fixed Effects with IV.

	Model1	Model2		Model3	Model4	
Dependent variable	LnEODA	LnPEODA	LnEEODA	LnEODA	LnPEODA	LnEEODA
Net Import of the donor (% GDP)	-0.470***	-0.376***	-0.542***			
	[0.080]	[0.081]	[0.197]			
Relative export share of China				-3.123***	-1.648**	-3.467**
				[0.715]	[0.750]	[1.479]
Log(SO2 china*wind weighted distance) (year lagged)	3.058**	3.384**	1.608**	2.294***	2.822**	2.269**
	[1.458]	[0.190]	[0.762]	[0.805]	[1.159]	[0.917]
Log (Trade complementarity index)	2.067	3.739	4.141	2.243	1.158	1.051
	[1.662]	[2.784]	[3.013]	[2.675]	[4.464]	[3.316]
Log(Cumulative number of ratified environmental regulation entered into force, Donors) (year lagged)	1.264	3.036	2.674	1.919	3.346	2.048
	[2.145]	[3.026]	[5.538]	[2.443]	[3.006]	[5.718]
Log(Cumulative number of ratified environmental regulation entered into force, China) (year lagged)	0.873	0.731	1.794	1.880	2.900	1.455
	[0.724]	[0.914]	[0.826]	[8.085]	[8.881]	[1.215]
GNI per capita, Donors (year lagged)	2.606**	2.783***	2.467	1.764*	2.294***	0.714
	[1.034]	[1.288]	[2.161]	[1.001]	[0.805]	[0.994]
Log(number of filed patents regarding air pollution) (year lagged)	0.152	0.101	0.267	0.467	0.427	0.823
	[0.443]	[0.600]	[0.553]	[0.401]	[0.562]	[0.543]
Log (GNI per capita, China) (year lagged)	-1.012	-0.422	-1.374*	-2.402	-1.752*	-0.711**
	[1.077]	[0.592]	[0.824]	[2.675]	[0.966]	[0.283]
Log (19 Donor's total aid to china)	0.940***	0.921***	0.907***	0.787***	0.776***	0.711**
	[0.117]	[0.138]	[0.128]	[0.112]	[0.117]	[0.283]
Year dummies	Yes	Yes	Yes	Yes	Yes	Yes
Hansen test (P- value)	0.38			0.32		
Chi sq-Test on competitiveness slope H0: Slopes are equal (P-value)		0.000			0.002	
Observations	245	112	112	245	112	112
Number of countries	19	19	19	19	19	19
Adjusted R-squared	0.20	0.58		0.14	0.47	

Note: 1) *,**, *** denotes significance level in 10, 5 and 1% respectively. 2) List of countries is provided in the appendix. 3) Overall system adjusted R-squared value for estimation models (model 2 and 4) are reported. 3) Robust standard errors in bracket.

Table 4. Validity of instrumental variable

	Model5	Model6
Dependent variable	LnEODA	LnEODA
Net Import of the donor (% GDP)	-0.460**	
	[0.207]	
Relative export share of China		-3.367**
		[1.535]
Log (relative population of Donor over China)	5.214	1.202
	[9.944]	[3.683]
Number of countries	19	19
Observation	245	245

Note: 1) The specification is exactly as in table 2, model 1 and model 3, except for the addition of log of relative population of Donor over China. 2) Robust standard errors in bracket.3) * significant at 10%; ** significant at 5%..

Table 5. Added controls approach

	Coefficient of net import	Adjusted R squared	Coefficient of relative market share	Adjusted R squared
Base	-0.470*** [0.080]	0.20	-3.123*** [0.715]	0.14
Democracy (Log polity 4)	-0.721** [0.323]	0.21	-3.035*** [0.648]	0.14
Natural capital (Log fresh water per capita)	-0.954** [0.377]	0.20	-2.787*** [0.648]	0.17
Donor pollution (Donor's Log Co2 emission)	-0.719** [0.317]	0.22	-3.282*** [0.680]	0.15
Population size (Donor and china Log population size)	-0.460** [0.218]	0.21	-2.954*** [0.617]	0.15
Population density (Donor's population density)	-0.515*** [0.224]	0.21	-2.269*** [0.578]	0.15
Government Expenditure (Log public spending on the R&D to protect the environment)	-0.483** [1.95]	0.22	-3.361*** [0.874]	0.16
OECD total Environmental aid	-0.529** [0.236]	0.20	-3.777*** [1.811]	0.13
Openness (Log total export +import/ GDP)	-0.575*** [0.242]	0.20	-4.470*** [2.000]	0.14

Note: 1) * significant at 10%; ** significant at 5%; *** significant at 1% 2) Robust standard errors in bracket.

Table 6. Extreme Observation Dominance Fixed country effects with IV

A. Net import over GDP

Dominance Test	Net import over GDP		
	Bottom 1% dropped	Top 1% dropped	Top 1% and bottom 1% dropped
Observation dropped of log Environmental ODA	-0.595*** [0.157]	-0.759*** [0.195]	-0.357*** [0.135]
Observation dropped of Net import / GDP	-0.445** [0.202]	-0.567** [0.267]	-0.597** [0.230]

Note: Significance at *10%;**5%; 1%, Robust standard errors used.

B. Relative market share of China over donor

Dominance Test	Relative market share of China over donor		
	Bottom 1% dropped	Top 1% dropped	Top 1% and bottom 1% dropped
Observation dropped of log Environmental ODA	-3.151*** [0.947]	-3.462*** [0.992]	-3.617** [0.874]
Observation dropped of Relative market share	-2.607** [1.149]	-4.291** [0.205]	-3.654*** [1.032]

Note: Significance at *10%;**5%; 1%, Robust standard errors used.

Table 7. System GMM estimation

	Model7	Model8
Dependent variable	LnEODA	LnEODA
Estimation method	One-step System GMM	One-step System GMM
Net import of the donor (% GDP)	-0.385**	
	[0.154]	
Relative export share		-1.023*
		[0.580]
Log(so2 china*wind weighted distance) (year lagged)	0.522***	0.275**
	[0.191]	[0.105]
Log (Trade complementarity index) (year lagged)	-0.854	-0.205
	[2.550]	[4.064]
Log(Cumulative number of ratified environmental regulation entered into force, Donors, year lagged)	0.916	0.660
	[5.040]	[2.827]
Log(Cumulative number of ratified environmental regulation entered into force, China, year lagged)	1.760***	2.034***
	[0.671]	[0.591]
Log (GNI per capita, Donors, year lagged)	0.382	0.558
	[1.268]	[0.907]
Log(number of filed patents regarding air pollution, year lagged)	0.135	0.701*
	[0.525]	[0.398]
Log (GNI per capita, China, year lagged)	0.769***	0.875***
	[0.206]	[0.104]
Log (19 Donor's total aid to china)	-0.896	-0.701
	[0.957]	[0.612]
EODA (Lagged)	-0.227	-0.074
	[0.188]	[0.875]
Year dummies	Yes	Yes
Arellano-Bond test for AR(1) (P-value)	0.081	0.340
Arellano-Bond test for AR(2) (P-value)	0.869	0.461
Hansen test (P- value)	0.63	0.21
Observations	211	211
Number of countries	18	18

Note: * significant at 10%; ** significant at 5%; *** significant at 1% 2) Robust standard errors in bracket.3) Sixth lag of endogenous variables used as instruments.

Appendix

Derivation of Equation (10')

$$\frac{\partial EB}{\partial k}=-\alpha_N\beta_{SN}\left(\begin{array}{l}\dfrac{\partial y_S}{\partial c_S}\dfrac{\partial c_S}{\partial k}x\big(K_S(k;K_N,\tau_S,\tau_N))+k\big)\\ +y_S\big(c_N(K_N,\tau_S,\tau_N),c_S(0;K_N,\tau_S,\tau_N)\big)x'\big(K_S(k;K_N,\tau_S,\tau_N))+k\big)(\dfrac{\partial K_S}{\partial k}+1)\end{array}\right)$$

$$=-\alpha_N\beta_{SN}\left(\frac{\partial y_S}{\partial c_S}\frac{c_S}{y_S}\frac{\partial c_S}{\partial k}\frac{y_S}{c_S}\frac{k}{k}x(\cdot)+y_S(\cdot)\frac{x}{x}\frac{k}{k}\frac{\partial x}{\partial k}\left(\frac{\partial K_S}{\partial k}+1\right)\right)$$

$$=-\alpha_N\beta_{SN}\left(\frac{\partial y_S/y_S}{\partial c_S/c_S}\frac{\partial c_S/c_S}{\partial k/k}y_S\frac{x(\cdot)}{k}+y_S\frac{x(\cdot)}{k}\frac{\partial x/x}{\partial k/k}\left(\frac{\partial K_S}{\partial k}+1\right)\right)$$

$$=-\alpha_N\beta_{SN}\left(y_S\frac{x(\cdot)}{k}\left(\varepsilon_{y_S c_S}\varepsilon_{c_S k}y_S+\varepsilon_{xk}\left(\frac{\partial K_S}{\partial k}+1\right)\right)\right).$$

Derivation of Equation (17)

$$z(k,\Psi)-z_p(k_p,\Psi)$$

$$=\frac{\partial\Delta\Pi(k)}{\partial k}+\frac{\partial EB(k)}{\partial k}-\frac{\partial\Delta\Pi(k_p)}{\partial k_p}-\frac{\partial EB(k_p)}{\partial k_p}$$

$$=\frac{\partial\pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S}\frac{\partial c_S}{\partial k}-\alpha_N\beta_{SN}\left(\begin{array}{l}\dfrac{\partial y_S}{\partial c_S}\dfrac{\partial c_S}{\partial k}x\big(K_S(k;K_N,0,\tau_N))+k\big)\\ +y_S\big(c_N(K_N,0,\tau_N),c_S(0;K_N,0,\tau_N)\big)x'\big(K_S(k;K_N,0,\tau_N))+k\big)(\dfrac{\partial K_S}{\partial k}+1)\end{array}\right)$$

$$-\frac{\partial\pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S}\frac{\partial c_S}{\partial k_p}+\alpha_N\beta_{SN}\left(\begin{array}{l}\dfrac{\partial y_S}{\partial c_S}\dfrac{\partial c_S}{\partial k_p}x\big(K_S(k_p;K_N,0,\tau_N))+k\big)\\ +y_S\big(c_N(K_N,0,\tau_N),c_S(0;K_N,0,\tau_N)\big)x'\big(K_S(k_p;K_N,0,\tau_N))+k_p\big)(\dfrac{\partial K_S}{\partial k_p}+1)\end{array}\right)$$

Since $\frac{\partial c_S}{\partial k_p}=0$ and $\frac{\partial x}{\partial k}=\frac{\partial x}{\partial k_p}$

$$=\frac{\partial\pi_N}{\partial y_S}\frac{\partial y_S}{\partial c_S}\frac{\partial c_S}{\partial k}-\alpha_N\beta_{SN}\left(\frac{\partial y_S}{\partial c_S}\frac{\partial c_S}{\partial k}x\big(K_S(k;K_N,0,\tau_N))+k\big)\right)+y_S\frac{\partial x}{\partial k}\left(\frac{\partial K_S}{\partial k_p}-\frac{\partial K_S}{\partial k}\right).$$

Proof of Proposition 4.

From Equation (17') we have $G\big|_{\tau_S=0}<0$ if

$$x'\left(K_S(k;K_N,0,\tau_N))+k\right)\left(\frac{\partial K_S}{\partial k}+1\right)-x'\left(K_S(0;K_N,0,\tau_N))+k_p\right)>0.$$

Let us denote $K_S(k;K_N,0,\tau_N))+k=K^U$ and $K_S(0;K_N,0,\tau_N))+k_p=K^L$

Since $-1<\frac{\partial K_S}{\partial k}<0$ we know that at $k=k_p$, $K^U<K^L$.

Also since $x''(K)<0$ we have that $\left|x'\left(K^U\right)\right|<\left|x'\left(K^L\right)\right|$.

Then since $x'\left(K^U\right)>x'\left(K^L\right)$ and since $\frac{\partial K_S}{\partial k}+1>0$ we have that

$$x'\left(K^U\right)\left(\frac{\partial K_S}{\partial k}+1\right)>x'\left(K^L\right).$$

Table A1-Data description and source

Variable	Description	Source
EODA	Environmental ODA(Million US $)	aidData.org (2010)
PEODA	Log Less cost affecting environmental ODA amount (Million US $)	aidData.org (2010)
EEODA	More cost affecting Environmental Technology ODA amount (Million US $)	aidData.org (2010)
Comp	Net import of the donor country from China/GDP	IMF Direction of Trade (DOT) (2010)
	Market share of China in the donor country / Donor's market share in China	IMF Direction of Trade (DOT) (2010)
TCI	Log Trade complementary Index	UN comtrade & Author's calculation
TB_{cN}	Transborder pollution from China (Wind-weighted distance * SO2 emissions in china)	Smith et al.(2011) & Author's calculation
τ_c	China's cumulative number of international environmental treaties that are ratified and entered into force	Mitchell (2012).
τ_N	Donor's cumulative number of international environmental treaties that are ratified and entered into force	Mitchell (2012)
M_c	China's GNI per capita	OECD (2010)
M_N	Donor's GNI per capita	OECD (2010)
$Tech_N$	Air pollution reducing technology patent count cumulative	OECD (2010)
ODA_{Dc}	Sum of all ODA for 19 OECD donor countries	aidData.org (2010)
Natural capital	Log fresh water per capita	OECD (2010)
Pop_{ct}	China's population	OECD (2010)
Pop_{Dt}	Donor's population	IMF Direction of Trade (DOT) (2010)
$dist_{cD}$	Distance between China and the donor country	International Environmental Agreements Database Project (Version 2010.3)

Democracy	Degree of political freedom index (polity 3)	http://mailer.fsu.edu/~whmoore/garnet-whmoore/polity/
Government Expenditure	Public spending on the R&D to protect the environment)	OECD (2010)
OECD total Environmental aid	Sum of 19 OECD donor's environmental aid	aidData.org (2010)
Openness	Log total export +import/ GDP	OECD (2010)
Population density	Log donor's population density	OECD (2010)

List of countries
Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Korea, Netherland, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, United Kingdom, U.S.A.

Table A2-Summary statistics

Variable	Mean	Std.Dev	Min	Max
LogEODA	15.05	2.44	4.74	20.45
LogPEODA	14.71	2.62	4.74	20.45
LogEEODA	13.77	2.62	6.97	20.10
Net import of Donor country from China / GDP	-0.05	1.10	-9.01	6.59
Relative market share of China	-0.66	0.79	-2.9	1.69
Log TCI	4.24	0.09	3.78	4.42
Log (Cumulative environmental regulations entered into force, donor)	3.77	0.36	2.77	4.36
Log (Cumulative environmental regulations entered into force, China)	3.31	0.12	2.99	3.43
Log (So2emission * Wind distance)	19.49	0.64	17.21	20.35
Log (GNI per capita, donor)	9.93	0.57	7.75	11.37
Log (GNI per capita, China)	8.67	0.57	7.72	9.64
Log (relative population of the donor)	0.33	0.42	0.10	2.55
Log (fresh water per capita)	2.11	1.27	0.39	4.97
Log (Population, donor)	4.02	5.14	1.18	27.32
Population, China	7.09	0.07	6.95	7.18
Log (Sum of 19 OECD donor's total ODA)	17.23	1.87	11.31	22.46
Degree of political freedom index (polity 3)	0.48	0.09	0.22	0.71
Public spending on the R&D to protect the environment as a share of GDP(%)	2.52	1.50	0.14	15.99
OECD total Environmental aid	19.85	0.85	15.69	20.81
Log ((total export +import)/ GDP)	3.41	0.48	2.08	4.52
Log (population density)	4.20	1.47	0.71	6.22

Chapter 2: Pollution-Income Dynamics

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1. Introduction

We examine the relationship between pollution and income in a dynamic general equilibrium framework with endogenous growth in a multi-output context. Previous theoretical literature has assumed a single final good thus ignoring the output composition effect and has often modeled production using a Cobb-Douglas specification (i.e., López, 1994; Stokey, 1998; Andreoni and Levinson, 2001; Johansson and Kriström, 2007). However, empirical evidence shows that the structure of consumption, not merely its level, is important in affecting the pollution-income relationship (Grossman and Krueger, 1995), and the Cobb-Douglas specification is often rejected (Chirinko, 2008). Figueroa and Pasten (2013) is one of the few analyses that allow for more general functional forms for consumer preferences and production functions. While their analysis constitutes an important generalization of earlier models in several respects, it is static in the sense that output expansion is exogenous and it still considers only one final consumer good, thus neglecting the output composition effect.

The most important conclusion of the existent theoretical literature is that the so-called environmental Kuznets process (EKC), where pollution first increases with income but beyond a certain income level it secularly declines, constitutes a plausible description of the pollution-income relationship. That is, the limits to growth would in this case be overcome. Below we show that this optimistic conclusion requires rather stringent assumptions often ignored by the literature. Consumer preferences both temporal and inter-temporal, and/or production technologies must have a high degree of flexibility for an EKC to be a relevant for an economy that taxes pollution optimally.

2. The Model

The economy produces two goods: a clean and a dirty one. The dirty-good production generates pollution as a byproduct while production of the clean good involves no pollution. Let k denote the total man-made composite clean input available at time t. The composite input includes human and physical capital. Henceforth, we refer to k as capital, which is momentarily distributed between the clean industry and dirty industry. Let k_d denote the amount of capital employed in the dirty sector. The flow of pollution from the dirty sector is represented by x. Following López (1994), and Copeland and Taylor (2005), we regard pollution as a factor of production, its price being determined by a pollution tax. Let $F(k_d, x)$ represents the production technology of the dirty-good sector, which is characterized by the constant elasticity of substitution (CES) function,

$$(1) \qquad y_d = F\left(k_d, x\right) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha) x^{-\frac{1-\omega}{\omega}}\right]^{-\frac{\omega}{1-\omega}},$$

where ω represents the elasticity of substitution between capital and pollution. The dirty sector produces only final goods. The output of the clean-good sector is assumed to depend only on the capital input and is governed by the linear technology

$$(2) \qquad y_c = A(k - k_d).$$

This sector produces the final good and new capital. If we normalize the price of the clean good to unity ($p_c = 1$), the economy's budget constraint is,

(3) $$\dot{k} = A(k - k_d) + pF\left(k_d, x\right) - c - \delta k\,,$$

where $p \equiv p_d / p_c$ is the relative price of the dirty good, $c \equiv c_c + pc_d$ is the total-consumption expenditure expressed in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk/dt$ is the net capital accumulation. The sum of the first two terms on the right-hand side on (3) represents the income of the economy expressed in units of the clean good. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption), also in units of the clean good.[29]

The consumer's indirect utility function is

$$u = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a},$$

where c denotes the total-consumption expenditure, $e(1,p)$ is the unit (dual) expenditure function or cost-of-living index, and a is a parameter that is equal to the elasticity of marginal utility (*EMU*).[30] The indirect utility function is assumed to be increasing and strictly concave in c.

The consumer's underlying preferences are described by a CES utility function so that the unit expenditure function is given as

$$e(1,p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the consumption elasticity of substitution between a dirty good and clean good, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. Consumer demand for the clean good c_c and dirty good c_d can be retrieved from the indirect utility function using

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[29] We assume that investment in capital is irreversible. Once the economy builds capital, it cannot be transformed back into consumption goods.

[30] If $a < 1$ we adopt a positive utility scale such that $0 < u < \infty$, while we scale the utility index to $-\infty < u < 0$ when $a > 1$.

Roy's identity. The optimal level of c is determined by the inter-temporal optimization, as detailed below. We assume for analytic convenience that the environmental damage is separable with consumption in consumer welfare, and can be represented as $v(x)=\frac{x^{1+\eta}}{1+\eta}$, where η>0 is a fixed parameter. Then the consumer's instantaneous welfare is

$$U \equiv \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta},$$

where a is the elasticity of marginal utility of income (*EMU*).

We assume that the discount rate ρ is fixed. When the government regulates pollution emissions in an optimal way, the competitive economy behaves "as if" it maximizes the present discounted value of the utility function,

$$\int_0^\infty \left\{\frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta}\right\}\exp(-\rho t)dt,$$

subject to the budget constraint (3), and the initial condition $k=k_0$. The consumer chooses the levels of c and x at each point in time. The government imposes a pollution tax in a socially optimal way and reimburses the tax revenue in a lump-sum way to the consumer. The above optimization implies the following current-value Hamiltonian function,

$$H=\frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} + \lambda\left[A(k-k_d)+pF(k_d,x)-c-\delta k\right],$$

where λ is the shadow price of capital (also equal to the marginal utility of consumption).

The following first-order conditions to the optimization program are necessary:

(4) $$e(1,p)^{a-1}c^{-a}=\lambda\,,$$

(5) $$pF_1\left(k_d,x\right)-A=0\,,$$

(6) $$-v_1(x)+\lambda pF_2(k_d,x)=0\,,$$

(7) $$\frac{\dot{\lambda}}{\lambda}=-\left[A-\rho-\delta\right]=-M\,,$$

(8) $$\dot{k}=A(k-k_d)+pF(k_d,x)-c-\delta k\,,$$

(9) $$\lim_{t\to\infty}\lambda k\,(t)e^{-\rho t}=0,$$

where a subscript number reflects the first derivative with respect to the corresponding argument in functions of more than one variable. The optimal pollution tax is equal to the marginal rate of substitution between the pollution and consumption expenditure, and is thus $\tau\equiv v_1(x)/\lambda$.

Using Roy's identity, we can derive the consumer demand for the dirty good from the indirect-utility function, which is $c_d=\frac{\gamma_d p^{-\sigma}c}{\gamma_c+\gamma_d p^{1-\sigma}}$. We then have the following market clearing condition for the dirty good:

(10) $$F\left(k_d,x\right)=\frac{\gamma_d p^{-\sigma}c}{\gamma_c+\gamma_d p^{1-\sigma}},$$

while the rate of growth of production of the dirty good is

(11) $$\hat{F}\left(k_d,x\right)=S_k\widehat{\left(\frac{k_d}{x}\right)}+\hat{x}\,.$$

Noting that the market for the dirty good must clear at all points in time, it follows that the growth-rate of production and demand for the dirty good must be equal (i.e., $\hat{c}_d = \hat{F}(k_d, x)$). Hence, using Lemma 2, (1) and (11), we arrive at

$$zp̂ + S_k \widehat{\left(\frac{k_d}{x}\right)} + \hat{x} = \frac{M}{a} \quad , \tag{12}$$

where $z \equiv \frac{s(p)}{a} + (1 - s(p))\sigma > 0$.

From (5), we also have that $\hat{p} + \hat{F}_1(k_d, x) = 0$,

$$\hat{p} - \frac{1}{\omega}(1 - S_K)\widehat{\left(\frac{k_d}{x}\right)} = 0 . \tag{13}$$

Finally, differentiating the (6) with respect to time, we obtain

$$-\eta\hat{x} + \hat{p} + \frac{1}{\omega} S_K \widehat{\left(\frac{k_d}{x}\right)} = M . \tag{14}$$

The equation system (12), (13), and (14) simultaneously solves for the three endogenous variables $\hat{p}$, $\widehat{\left(\frac{k_d}{x}\right)}$, and $\hat{x}$,

$$\hat{p} = \frac{\frac{M}{\omega}(1 - S_k)\left[\frac{\eta}{a} + 1\right]}{|W|} \geq 0 \tag{15}$$

$$\widehat{\left(\frac{k_d}{x}\right)} = \frac{M\left[\frac{\eta}{a} + 1\right]}{|W|} > 0 \tag{16}$$

$$\hat{x} = \frac{\frac{M}{\omega}T(p)}{|W|} \quad , \tag{17}$$

where $T(p) \equiv \frac{1}{a}[1-s(p)(1-S_k)] - S_k\omega - (1-s(p))(1-S_k)\sigma$,

$|W| \equiv \frac{1}{\omega}\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k$, $s(p)=\frac{pc_d}{pc_d+c_c}$ and

$$S_k = \alpha\left[(1-\alpha)\left(\frac{k_d}{x}\right)^{\frac{1-\omega}{\omega}}+\alpha\right]^{-1}.[31]$$

Equation (15) implies that the price of dirty goods continuously increases over time if the economy has sufficiently strong growth potential ($A > \rho+\delta$ or, equivalently, if $M > 0$). This is partly due to the fact that, under optimal regulation, the price of the dirty good depends on the marginal social cost of pollution $(v_1(x)/\lambda)$, which, at the given level of pollution, is increasing over time as λ falls. The increasing price of the dirty good induces consumers to increase the clean-good–dirty-good consumption ratio. This triggers a structural change in production and leads to the output-composition effect, where production of the dirty good declines relative to that of the clean good.

Equation (16) shows that the so-called technique effect takes place along the optimal-growth path. Thus, (15) and (16) imply that closed economy must rely on both the output composition and technique effects as a way to counter the scale effect caused by positive economic growth. The net result which is described by (17) is, in general, ambiguous and critically dependent on the dynamics of $s(p)$ and $S_k(p)$, in

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[31] The share of the dirty good in the consumer budget, $s(p)$, is an increasing (decreasing) function of p if $\sigma < 1$ $(\sigma > 1)$. The factor share of capital in the production of dirty goods, $S_k(p)$, is increasing (decreasing) in p if $\omega > 1$ $(\omega < 1)$.

addition to the consumption elasticity of substitution, σ, the production elasticity of substitution, ω, and *EMU*.

An important issue is whether the dynamic path described by (15) to (17) allows for a positive rate of consumption growth. Proposition 1 below shows that this is indeed the case.

Proposition 1: (i) *The growth rate of real consumption expenditure is:* $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$, *where* $\hat{p}$ *is given by (14).* (ii) *The rate of growth of real consumption remains positive throughout the equilibrium path for any positive* ω *and* σ.

Proof*: See Appendix.*

3. Conditions for an EKC

Sufficient conditions for the emergence of EKC can be summarized as follows.

Proposition 2: *Assume* $a > 0$, *then pollution emissions increase over a certain interval of time before eventually declining if any of the following three conditions are satisfied.*

(i) If $\sigma < Min\left[1, \frac{1}{a}\right]$ *and* $\omega > Max\left[1, \frac{1}{a}\right]$,

(ii) If $\sigma > Max\left[1, \frac{1}{a}\right]$ *and* $\omega < Min\left[1, \frac{1}{a}\right]$,

(iii) If $\sigma > \frac{(1/a) - \alpha}{1 - \alpha}$ *and* $\omega = 1$.

Proof*: See Appendix.*

Proposition 2 states that if the economy lacks flexibility in both consumer preferences and input substitution, an EKC is not feasible. If *EMU* is less than one the likelihood for the emergence of an EKC is low. In this case at least one of the elasticities of substitution must be much greater than one. The popular specification where both consumer preferences and production technologies are Cobb-Douglas may be consistent with an EKC process only if the *EMU* is greater than one, which is precisely the assumption made by most of the EKC literature (i.e., Stokey, 1998).

4. Conclusion

This paper examines the scale, composition, and technique effects of economic growth on pollution emission growth. This paper shows that the limits to growth can be eluded through a Kuznets-type process only if there is a sufficient degree of substitution flexibility in either production technology or consumer preferences. The flexibility requirements are more demanding the lower is the *EMU*. If an economy is endowed with such flexibility, then economic growth can be sustained at positive levels while pollution falls over the long run. If the elasticity of substitution between the dirty inputs and the clean inputs is much less than unity, as often reported in the empirical literature, the feasibility of sustainable growth under optimal pollution tax hinges greatly on the size of the output composition effect, an effect that has been consistently neglected in the theoretical literature.

Appendix

Proof of Proposition 1

(i) By Roy's identity, the demand for the dirty good $c_d = \frac{c}{e(1,p)} e_2(1,p)$. Using

Shephard's lemma, $\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$.

Therefore, $\widehat{\left(\frac{c}{e}\right)} = \hat{c} - \hat{e} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$.

(ii) The real consumption grows over time if $\hat{p} < M / s(p)$. Using (15) this inequality holds if

$$\hat{p} = \frac{(1/\omega)M(1-S_k)\left[(\eta/a)+1\right]}{(1/\omega)\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k} < M/s(p).$$

Rearranging this inequality we have the following;

(A1) $(1-S_k)\left(\frac{\eta}{a}+1\right)s(p) < \left[(1-S_k)(1+z\eta)\right] + S_k + \eta S_k \omega$.

Since, $(S_k + \eta S_k \omega) > 0$ and $z \equiv \frac{s(p)}{a} + (1-s(p))\sigma$, (A1) is satisfied if

(A2) $\frac{\eta s(p)}{a} + s(p) < 1 + \frac{\eta s(p)}{a} + (1-s(p))\sigma\eta$.(A2) holds if $0 < (1-s(p))(1+\sigma\eta)$, which is always true for $0 < s(p) < 1$. Thus, we have $\hat{p} < (M/s(p))$ and hence consumption growth is positive for all finite $\sigma > 0$ and $\omega > 0$. Q.E.D.

Proof of Proposition 2

We note that T changes continuously in time.

(i) If $\omega > Max\left[1, \frac{1}{a}\right]$, S_k increases to 1 over time. Since $T\big|_{S_k=1} = \frac{1}{\omega a} - 1 < 0$ and

$T\big|_{S_k=0} = \left(\frac{1}{a} - \sigma\right)(1-s(p))$, EKC can emerge as long as $\sigma < Min\left[1, \frac{1}{a}\right]$.

(ii) If $\omega < Min\left[1, \frac{1}{a}\right]$, S_k decreases to 0 over time. Since $T\big|_{S_k=1} = \frac{1}{a} - \omega > 0$ and $T\big|_{S_k=0} = (1-s(p))\left(\frac{1}{a} - \sigma\right)$, we have $T\big|_{S_k=0} = \left(\frac{1}{a} - \sigma\right) < 0$ if either $1 < \frac{1}{a} < \sigma$ or $\frac{1}{a} < 1 < \sigma$.

(iii) If $\omega = 1$, S_k remains equal to some value $0 < \alpha < 1$.

When $0 < a < 1$ then $\sigma > \frac{(1/a) - \alpha}{1-\alpha} > 1$and $s(p)$ decreases to 0 overtime. EKC can emerge since $T\big|_{S_k=\alpha, s(p)=0} = \frac{1}{a} - \alpha - (1-\alpha)\sigma < 0$ and $T\big|_{S_k=\alpha, s(p)=1} = \alpha\left(\frac{1}{a} - 1\right)$>0.

When $a > 1$ then $1 > \sigma > \frac{(1/a) - \alpha}{1-\alpha}$ and $s(p)$ decreases to 0 overtime. Since $T\big|_{S_k=\alpha, s(p)=1} = \alpha\left(\frac{1}{a} - 1\right) < 0$ and $T\big|_{S_k=\alpha, s(p)=0} = \frac{1}{a} - \alpha - (1-\alpha)\sigma < 0$. The pollution level can either monotonically decrease over time, or EKC can emerge. Q.E.D.

Chapter 3: Environmental Sustainability with a Pollution Tax

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1. Introduction

This paper examines the feasibility of environmentally sustainable economic growth in a dynamic general equilibrium framework of a closed economy with two final outputs and two factors of production. It explicitly accounts for both pollution flow effects and the existence of irreversible thresholds affecting the stock of renewable natural resources (i.e., the stock of clean air in the upper atmosphere). The paper highlights the important role played by two key facets of consumer preferences, namely the temporal substitution among final goods of diverse environmental impacts (represented by their elasticity of substitution) and the inter-temporal substitution of consumption (represented by the elasticity of marginal utility of consumption, EMU). If the EMU is greater than one, an optimal pollution tax ensures sustainable growth even if the elasticity of substitution in production between clean and dirty inputs and in consumption between clean and dirty consumer goods are well below one without requiring any further government intervention. If the EMU is less than one, sustainable growth is still feasible but requires much more demanding conditions: either temporal elasticity of substitution must be substantially greater than one.

This paper finds further that even a suboptimal pollution tax may allow sustainable development as long as the tax time profile meets certain plausible conditions that are developed below. Finally, numerical simulation results in section 8 demonstrates that if the pollution tax used as the sole policy instrument to prevent climatic disaster is well designed, it may only modestly affect the rate of economic growth.

The paper assumes that there exists a threshold level of the stock of the renewable natural resource which, if crossed, may drastically and irreversibly harm human health with the utility of the representative consumer falling to minus infinity (Cropper, 1976; Keller et al., 2004; Nævdal, 2006; Nævdal and Oppenheimer, 2007; Leizarowitz and Tsur, 2012). However, as long as such stock is above this threshold, human welfare is only affected by the flow levels of pollution emissions gradually.[32] The paper explores the properties of a pollution tax for sustainable development under the resource stock constraint and identifies a family of growth paths (including suboptimal paths as well as an optimal one) each of which guaranteeing environmentally sustainable economic growth.

The theoretical literature on growth and the environment over the last few decades has provided significant insights regarding the role of institutional and policy conditions in supporting environmentally sustainable economic growth (i.e., Bovenberg and Smulders, 1995; Bovenberg and de Mooij, 1997; Stokey, 1998; Brock and Taylor, 2010; Golosov et al., 2011; Acemoglu et al., 2012). Despite substantial progress in modeling, the existing growth theoretic literature still relies on certain restrictive models and assumptions that often fail to persuade many (especially

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[32] Consider the case of climate change gases; the emission flows consist not of one gas but of a cocktail of pollutants, including pollutants of mostly local effects (i.e., carbon monoxide), of local and global climatic effects (i.e., soot), and mainly global effects (i.e., carbon dioxide). The latter two pollutants accumulate in the upper atmosphere, thus affecting the stock of "clean air" over time. The effect of these flows is to cause health and other detrimental effects gradually over time while the stock accumulation effect is of little immediate effect as long as certain threshold stock levels are not surpassed. If the stock of CO2 in the atmosphere increased marginally from 250 parts per million (*ppm*), there is little consequence for human life. However, if it surpasses, for example, 650 *ppm*, the potential catastrophic effects of the stock accumulation may be felt.

environmentalists and ecologists) of the idea that persistent positive economic growth over the long run may eventually be consistent with an improving environment, thus preventing environmental catastrophe. The present paper is mainly inspired by and related to the landmark studies by Stokey (1998) and Acemoglu et al. (2012). It generalizes their findings in several respects by highlighting the role of a variety of features of consumer preferences and producer technologies, demonstrating that, contrary to the conclusion of most studies, elastic production and/or consumer choices are not necessary conditions for sustainable economic growth.

Most existing growth models assume one final good, which precludes the existence of an output composition effect, often considered important by empirical analyses (i.e., Grossman and Krueger, 1995; Cole and Elliot, 2003). A model with two final goods and two factors of production, as the one we developed below, may be considered isomorphic to existing models which assume one final good produced using two inputs one of which is a composite input in turn produced with another clean input and a dirty one (as in Acemoglu et al., 2012). However, this is not necessarily the case; a model that explicitly recognizes more than one final good where both endogenous savings and technological change are sources of economic growth, as the one developed below, brings to the forefront peculiarities of consumer conditions, in particular the role of the *EMU* vis-à-vis the temporal elasticity of substitution either among consumption goods or factors of production. We show that the relationship between *EMU* and the temporal elasticity of substitution in consumption or production plays a key role in sustainable development, an insight lost in models that assume a single final good with technological change as the primary source of economic growth.[33]

Standard growth models that allow for savings as a source of growth often assume that the value of *EMU* is greater than one, an assumption that has been criticized by prominent authors on conceptual grounds (i.e., Aghion and Howitt, 1997; Ogaki and Reinhart 1998). Additionally, the empirical evidence regarding the size of *EMU* is mixed; some recent studies tend to contradict this assumption (i.e., Ogaki and Reinhart 1998; Vissing-Jørgensen and Attanasio, 2003; Layard et al., 2008). We thus relax this assumption and consider sustainable development alternatively considering levels of *EMU* above or below one.

Most existing models assume either unitary or highly elastic substitution between man-made and environmental factors of production (i.e., Stokey, 1998; Acemoglu et al., 2012).[34]This assumption has been challenged by environmentalists claiming that natural capital (i.e., the environment) and man-made capital are complements rather than substitutes (Daly, 1992). Moreover, to some degree, a number of empirical studies seem to support the claims made by environmentalists, concluding that factor input substitution is indeed substantially less than one (i.e., Field and Grebenstein, 1980; Kemfert and Welsch, 2000; van der Werf, 2008; Hassler et al., 2012).

Empirical studies report stronger substitution between clean and dirty consumer goods than among factors of production, often obtaining elasticity of substitution

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[33] See Baylis, Fullerton and Karney (2013) for the importance of considering at least two final goods and two productive inputs in examining the effects of unilateral carbon policy in a static equilibrium model.

[34] Recent growth theoretic studies do allow for factor input complementarities but in the context of non-renewable resources; their depletion is assumed to induce endogenous innovation (Bretschger and Smulders, 2012; Peretto, 2009). The focus on non-renewable resources, however, prevent consideration of the possibility of catastrophic and irreversible losses of renewable natural resources such as the atmospheric stock of clean air, a central focus of the present paper.

estimates well above 3 for consumer goods. Consequently, it appears that the scope for substitution between clean and dirty goods by consumers is greater than the substitution potential among inputs by producers, a feature that we explicitly consider in this study (i.e., Lin et al., 2008; Galarraga et al., 2011).[35]

Clean input-augmenting exogenous technological change is often assumed (i.e., Stokey, 1998; Brock and Taylor, 2010). However, recent studies have emphasized the endogenous nature of technological change; for example, Acemoglu et al. (2012) allows for endogenous technological change, showing that targeted research subsidies may transform pollution-augmenting technological change into clean input-augmenting technological change as long as the elasticity of substitution between the clean and dirty inputs is much greater than one. Otherwise, targeted research subsidies are impotent to affect the structure of technological change. We consider exogenous technological change allowing alternatively for various types of it (neutral, pollution-augmenting and/or clean input-augmenting), an assumption that simplifies the analysis considerably. In view of the point made by Acemoglu et al. (2012) regarding the impotency of research subsidies when the elasticity of substitution between clean and dirty inputs is less than one, the assumption of exogenous technological change is innocuous, given that we focus mostly on cases where this elasticity is in fact less than one. Moreover, as Golosov et al. (2011) show, whether technological change is endogenous or exogenous is irrelevant in deriving an optimal disaster-avoiding pollution tax.

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[35] Moreover, studies have shown that the consumers' flexibility with regards to clean goods is highly responsive to increased information and public education on the pollution content of the various consumer goods, as well as to eco-labeling (Kotchen and Moore, 2007). This is in sharp contrast with the reported lack of responsiveness to these interventions by manufacturing firms (Banerjee and Solomon, 2003).

The standard neoclassical growth model of sustainable development has been criticized by environmentalists mainly on the grounds that man-made and natural capital are not likely to be strong substitutes in production, as assumed by most neoclassical growth models (i.e., Daly, 1992) and that there is excessive optimism regarding the role of technological change (i.e., Vollebergh and Kemfert, 2005).

The fact that we show that environmental sustainability accompanied with positive and persistent economic growth can be achieved in economies where natural and man-made capital have low elasticity of substitution, and that it may proceed under any type of technological change, constitutes an important response to the above critiques.

2. Framework of the Analysis

The economy produces two goods: a clean good and a dirty one. The dirty good sector includes traditional manufacturing industries and primary industries that generate air and/or water pollution as a byproduct of their production processes. The clean good sector includes services and other goods that generate little or no pollution.

Production.—Let k denote the total man-made composite input available at time t in the economy. This composite input includes human capital as well as other more tangible forms of capital. Henceforth, we refer to k as "capital", which is momentarily distributed between the clean industry and the dirty industry. Let k_d denote the amount of capital employed in the dirty industry. The flow of pollution from the dirty sector is represented by x. Following Cropper and Oates (1992), López (1994), and Copeland and Taylor (2004), we consider pollution as a factor of production directly. The output of the dirty good is:

(1) $$y_d = A_d F(k_d, bx).$$

The parameter A_d denotes total factor productivity with proportional growth rate, $\dot{A}_d / A_d \equiv g_d \geq 0$ and $b > 1$ is a factor-augmenting technological factor with $\dot{b}/b \equiv \zeta \geq 0$.

The dirty sector produces only a final consumer good. F is a Constant Elasticity of Substitution (CES) function, and it is given as follows:

$$F(k_d, bx) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)(bx)^{-\frac{1-\omega}{\omega}} \right]^{-\frac{\omega}{1-\omega}},$$

where ω is the elasticity of substitution between capital and pollution and α is a fixed distribution coefficient.

The output of the clean good is assumed to depend only on the capital input and is governed by the linear production technology, as follows:

(2) $$y_c = A_c(k - k_d).$$

where the parameter A_C is the return to capital in the clean sector and k is the total stock of capital in the economy at a point in time. The clean sector produces a final consumer good as well as new capital (or investment). Mostly for the sake of reducing notational clutter, we focus primarily on pollution-augmenting and neutral technological change. Later in the paper, however, we show that the results remain mostly unchanged by considering capital-augmenting technological change.

We consider two sources of economic growth, technological change and capital accumulation. We specify the various types of technological change below. Here we focus on capital accumulation using the budget constraint of the economy. If we normalize the price of the clean good to unity (i.e., $p_c = 1$), the economy's budget constraint can be written as:

$$(3) \qquad \dot{k} = A_c(k - k_d) + pA_d F(k_d, bx) - c - \delta k,$$

where $p \equiv p_d / p_c$ is the relative price of dirty goods, $c \equiv c_c + pc_d$ is the total consumption expenditure expressed in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk / dt$ is the net capital accumulation. The sum of the first two terms on the right-hand side of Equation (3) represents the income of the economy expressed in units of clean goods. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption), which is also expressed in units of the clean good.[36]

Stock of clean air.—Economic activity releases pollution flows into the atmosphere. A portion of the pollution emissions are removed by nature's revitalization processes but another portion of them remains as a stock that accumulates in the upper atmosphere. Pollution emissions (whether they accumulate in the atmosphere or rapidly dissipate) have instantaneous direct negative effects on welfare. In addition, the fact that a portion of the emissions accumulates in the upper atmosphere causes very gradual and subtle changes in climate, which may have negligible direct effects on welfare unless such accumulations reach a threshold level at which point catastrophic events may be triggered, causing massive welfare losses.

Thus, pollution reduces the stock of clean air, so that the changes in the stock of clean air are the net result of two forces, the natural purification rate of pollution and

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[36] We assume that the investment in capital is irreversible. Once the economy builds capital, it cannot be transformed back into consumption goods; capital can be reduced over time only by allowing it to depreciate.

the flow emission of pollution. Following most of the literature we assume a constant rate of environmental regeneration (i.e., Aghion and Howitt, 1997; Acemoglu et al., 2012). Denote the stock of clean air in the upper atmosphere as E, the threshold of minimal stock of clean air below which an environmental catastrophe occurs as $\underline{E}$, the pristine stock level by $\bar{E}$, and let $0 < \psi < 1$ be the constant rate of natural atmospheric purification. Then we have:

(4) $$\dot{E} = \psi E - x \text{ for } \underline{E} \leq E < \bar{E}.$$
$$= -x \qquad \text{for } E < \underline{E}.$$

For future reference we note that by integrating (4) within the specified boundaries we obtain:

(4’) $$E(t) = \exp(\psi t)\left(E_0 - \int_0^t x(\upsilon)\exp(-\psi\upsilon)d\upsilon\right)$$

For $E(t) \geq \underline{E}$; E_0 is the initial, predetermined level of the stock of clean air.

Consumption and welfare.—The welfare function of the representative consumer is comprised of two parts, a utility derived from the consumption of goods and the disutility generated by pollution. We represent the utility derived from the consumption of goods by an indirect utility function as follows:

$$u = \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a},$$

where c denotes the total consumption expenditure, $e(1,p)$ is the unit (dual) expenditure function or cost-of-living index, and $a > 0$ is a parameter equal to EMU. If $a < 1$, we adopt a positive utility scale such that $0 < u < \infty$, while we scale the utility index to $-\infty < u < 0$ when $a > 1$. Of course, a special case of the above specification occurs when $a = 1$, in which case we obtain the often-used logarithmic specification, $u = \ln[c / e(1, p)]$. The indirect utility function is assumed to be increasing and strictly concave in the real consumption level, $c / e(1, p)$.

We assume that the consumer's underlying preferences for goods are described by a CES utility function, so that the unit expenditure function is:

$$e(1, p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where σ is the consumption elasticity of substitution between the dirty and clean goods, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. The indirect utility function defined above presumes homothetic preferences. Consumer demand for the clean good c_c and dirty good c_d can be retrieved from the indirect utility function using Roy's identity. The optimal level of c is determined by the inter-temporal optimization (as detailed below).

The second part of the welfare function corresponds to the disutility generated by pollution. Let $v(x; E)$ denote the environmental damage function, which is assumed to be increasing and convex in the level of pollution, x. We assume that the environmental damage function is:

$$v(x; E) = \frac{x^{1+\eta}}{1+\eta} \text{ if } E \geq \underline{E} ,$$

$$= \infty \quad \text{ if } E < \underline{E} .$$

Also, $\eta > 0$ denotes the elasticity of marginal damage caused by pollution and is assumed to be a fixed parameter. Therefore, the consumer's total welfare function is:

$$U(c,x;E) \equiv \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} \text{ when } E \geq \underline{E}$$

$$\equiv -\infty \qquad\qquad \text{when } E < \underline{E} \,.$$

Assuming a fixed pure time discount rate (ρ) and socially optimal intervention, the competitive economy is modeled "as if" it maximizes the present discounted value of the utility function:

$$\int_0^\infty U(c,x;E)\exp(-\rho t)dt,$$

subject to the budget constraint (i.e., Equation (3)), clean air stock level constraint $E \geq \underline{E}$ (Equation (4)) and the initial conditions $k = k_0$ and $E = E_0$. In other words, the competitive behavior of the representative consumer and producer under optimal pollution tax and lump-sum reimbursement is described by the choices of the optimal levels of c and x at each point in time.

We assume that both goods are always produced, which implies that $k_d(t) < k(t)$ for all t. Thus, the current value Hamiltonian function assuming an interior solution is:

$$H_E = U(c,x,E) + \lambda\left[A_C(k-k_d) + pAF(k_d,bx) - c - \delta k\right] + \mu\left[\psi E - x\right] + \phi\left[E - \underline{E}\right]$$

where λ and μ denote co-state variables each representing the shadow price of man-made capital and natural capital, respectively while $\phi \geq 0$ is a time-varying Lagrange multiplier associated with the stock constraint.

Analytical Strategy.—We assume that the economy maximizes H_E subject to the market equilibrium conditions for the final goods to be introduced later in the next section. This means that in addition to the usual endogenous variables of the optimal control problem we need to solve for the endogenous market prices. Using the system of necessary conditions for dynamic optimization (Maximum principle and Kuhn-Tucker conditions) and the said market clearing conditions, we may in principle solve for seven endogenous variables $(c, k_d, x, p, \lambda, \mu, \phi)$ at each point in time. While the analysis of the original problem is extremely complex given the fact that the utility function is discontinuous at $E = \underline{E}$, the dynamic optimization process can be examined in a more tractable way if the shadow price of the stock of pollutant, ϕ, is zero (that is, if the stock constraint is not binding).

We therefore use the following strategy: First, we solve the model of dynamic optimization and market equilibrium using as a maintained assumption that $\phi = 0$, that is, that the stock of clean air remains above $\underline{E}$ throughout all time. Next, we analyze the conditions under which, given the solution derived from the first step, the constraint $E(t) \geq \underline{E}$ is satisfied for all *t* given initial stock levels of the natural and man-made assets, E_0 and K_0. Thus, the first part of the solution is obtained by maximizing H_E (subject to the relevant market clearing conditions) with $\phi = 0$ and the second part examines whether or not this solution satisfies the stock constraint.

Under our stated assumptions on preferences and production technology, H_E is strictly concave with respect to state and control variables, and the necessary conditions become sufficient. In fact there exists a unique solution for the optimal control problem.[37] In the subsequent sections, we also characterize the conditions for the clean air stock to remain above the threshold level. If the optimal path of

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[37] We note also that the Inada condition is satisfied. In other words, for any $a > 0$ our utility scale guarantees that $\lim_{c \to 0} U_x(c, x, E) = \infty$ for any finite x and $\underline{E} \leq E$.

emissions obtained by maximizing H_E does not permit the stock of clean air to fall below the critical threshold at any point in time then it constitutes an optimal solution for the original problem of dynamic market equilibrium with stock constraint.

Definition of Sustainable Economic Growth.—It is now necessary to define what we mean by "sustainable economic growth".

Definition 1: *We say that sustainable growth is possible if, at some point along the growth process, the economy is able to continue growing indefinitely while pollution emissions permanently decline and the stock of natural capital never falls below the critical threshold level.*

Therefore, sustainability requires that there exists a finite time, $T \geq 0$, such that at any time $t > T$, $\hat{x} < 0$,which implies that $\lim_{t \to \infty} \hat{x} \leq 0$, and that $E(t) \geq \underline{E}$ for all t.[38]

Additional Considerations.—Here we establish some basic properties of the consumption and factor shares which are essential for the ensuing analysis. The budget share of the dirty final good in the consumption expenditure for the CES utility function is $s(p) = \dfrac{\gamma_d}{\gamma_c p^{\sigma-1} + \gamma_d}$ and the factor share of the clean input in the cost of production of the dirty good for a CES production function is $S_k(k_d / bx) = \alpha \left[(1-\alpha) \left(\dfrac{k_d}{bx} \right)^{\frac{1-\omega}{\omega}} + \alpha \right]^{-1}$. Of course, the share of the dirty input in the cost of production of the dirty final good is $1 - S_k$. Then we have the following remark:

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[38] A similar notion has been adopted by several authors, including Stokey (1998) and Brock and Taylor (2010). This concept of sustainable growth conforms to the concept of sustainable development in Arrow et al. (2010).

Remark 1: *The share* $s(p)$ *is an increasing (decreasing) function of* p *if* $\sigma < 1(\sigma > 1)$. *The share* $S_k(k_d / bx)$ *is increasing (decreasing) in* k_d / bx *if* $\omega > 1(\omega < 1)$.

Remark 1 is important for subsequent analysis because it allows us to predict the evolution of $s(p)$ and $S_k(k_d / bx)$ over time if we know the dynamics of p and k_d / bx, on the basis of the size of the elasticity of substitution. As shown below the dynamics of these shares are key factors determining the sustainability (or lack of sustainability) of the economy.

Assumptions.—We make the following assumptions:

Assumption 1: *The clean sector of the economy is sufficiently productive so that the marginal return to capital* (A_c) is *higher than the marginal opportunity cost of capital* ($\rho + \delta$)*; hence,* $M \equiv A_c - \rho - \delta > 0$.

Assumption 2: *Technological change can be pollution-augmenting occurring at an exogenous rate* $\zeta \geq 0$ *and/or neutral, raising the total factor productivity of the dirty sector at an exogenous rate* $g_d \geq 0$. *However, the rate of technological change is bounded from above as follows:* $\zeta + g_d \leq min\{M, M / a\}$.

Assumption 1 is a necessary condition for the economy to be able to accumulate capital over time. Meanwhile, Assumption 2 implies that all exogenous technological changes are concentrated in the dirty industry. The assumption of dirty input (pollution)-augmenting technological change in the context of endogenous technological change is consistent with the so-called laissez-faire or market solution arising when the government does not intervene to subsidize research and development to increase the productivity of the clean inputs (i.e., Acemoglu et al., 2012). In section Ⅴ, we relax this assumption by also allowing for capital-augmenting technological change.

Assumption 2 also places a limit on the speed of technological progress. As we shall show below, this limit is necessary for technical reasons. It assures that the net effect of the two primary sources of growth, namely capital accumulation and technological change, is pollution-increasing while the technique and composition effects are pollution-reducing. If this assumption is not satisfied then we would obtain that the direct effect of economic growth (i.e., the factor accumulation-cum-technological change effect) would be pollution-reducing while the technique and composition effects would be pollution-increasing. This baffling condition would in fact render the analysis of sustainable development meaningless. If the direct effect of economic growth were to lower pollution then we would have sustainable development even in the absence of a pollution tax and, hence, in the absence of technique and composition effects.

3. Optimality and Market Clearing Conditions

Optimality Conditions.—The first-order necessary conditions for maximization of the Hamiltonian function imply that the marginal utility of consumption must be equal to the shadow price of capital, λ :

$$e(1,p)^{a-1}c^{-a}=\lambda. \tag{5}$$

Meanwhile, along the optimal path the well-known no arbitrage condition must be satisfied:

$$\frac{\dot{\lambda}}{\lambda}=-\left[A_c-\rho-\delta\right]\equiv -M. \tag{6}$$

There are two additional conditions for optimality as follows: first, the marginal value product of capital should be equal across the two sectors; second, firms equalize the marginal value product of pollution to the optimal pollution tax. Therefore, assuming an interior solution, we have:

$$pA_d \frac{\partial F(k_d, bx)}{\partial k_d} - A_c = 0, \tag{7}$$

$$pA_d \frac{\partial F(k_d, bx)}{\partial x} - v'(x)/\lambda = 0 \tag{8}$$

Equation (7) indicates that in equilibrium the marginal value product of capital should be equalized across the two sectors. Equation (8) says that the optimal pollution tax, which is equal to the marginal rate of substitution between pollution and consumption, $\tau \equiv v'(x)/\lambda$, is equalized to the marginal value product of pollution. Finally, the savings should be equal to the net investment at each moment of time, so that we have Equation (3) as an additional first order condition. Moreover, we have the standard transversality condition, $\lim_{t \to \infty} \lambda k(t) e^{-\rho t} = 0$.

Market clearing conditions.—In Appendix we show that the rate of growth of the consumer demand for dirty goods is:

$$\hat{c}_d = \frac{1}{a} M - \left[\frac{s(p)}{a} + (1 - s(p))\sigma \right] \hat{p}. \tag{9}$$

A circumflex above the symbol reflects its corresponding rate of growth. In addition, the rate of growth of production of the dirty goods is:

$$\hat{y}_d = g_d + \hat{F}(k_d, bx) = g_d + S_k \widehat{\left(\frac{k_d}{bx} \right)} + \widehat{(bx)}. \tag{10}$$

Because the dirty goods are used for consumption only, market equilibrium requires that $y_d = c_d$ at all points in time. Furthermore, once the dirty goods market is cleared, the market for the clean goods automatically clears because the current savings are equal to the current investment, as stipulated in Equation (3). Therefore, the relative price of dirty goods must adjust endogenously over time to allow for such equilibrium to persist. Along the equilibrium path, the growth rate of production and demand for the dirty good must be equal, so that $\hat{y}_d = \hat{c}_d$.

4. Dynamic Equilibrium

The Conditions.—Using Equation (9) and Equation (10), we obtain:

(11) $$z\hat{p} + S_k \overset{\wedge}{\left(\frac{k_d}{bx}\right)} + \hat{x} = \frac{M}{a} - g_d - \zeta$$

where $z \equiv s(p)\frac{1}{a} + (1 - s(p))\sigma > 0$ (also recall that $\zeta \equiv \dot{b}/b$ and $g_d \equiv \dot{A}_d / A_d$). The function z corresponds to the weighted average of the inter-temporal elasticity of substitution ($1/a$) and the temporal elasticity of substitution, using the budget shares as weighting factors.

From Equation (7), we have $\hat{p} + \hat{A}_D + \hat{F}_1(k_d, bx) = 0$, which given the CES production function implies that:

(12) $$\hat{p} - \frac{1}{\omega}\left(1 - S_K\right)\overset{\wedge}{\left(\frac{k_d}{bx}\right)} = -g_d.$$

Finally, in Appendix we show that using Equation (8) the following expression follows:

(13)
$$\hat{p} + \frac{1}{\omega} S_K \widehat{\left(\frac{k_d}{bx}\right)} - \eta \hat{x} = M - g_d - \zeta .$$

This states that the rate of increase of the private marginal revenue of the dirty input, $\hat{p} + \frac{1}{\omega} S_K \widehat{\left(\frac{k_d}{bx}\right)} + \zeta + g_D$, is equal to the rate of increase of the input price, which in turn equals rate of increase of the pollution tax, $\hat{\tau} = \eta \hat{x} + M$.

Solution of the dynamical system.—In Appendix, we show that the dynamical system of Equations (11), (12), and (13) solves for the equilibrium growth rates of $\hat{p}$, $\widehat{\left(\frac{k_d}{bx}\right)}$ and $\hat{x}$ as follows:

(14)
$$\hat{p} = \frac{1}{|W|\omega}\left(\left[M(1-S_k)\left(\frac{\eta}{a}+1\right)\right] - g_d\left[(1-S_k)(\eta+1) + \omega S_k\left(\eta + \frac{1}{\omega}\right)\right] - \zeta\left[(1-S_k)(\eta+1)\right]\right),$$

(15)
$$\widehat{\left(\frac{k_d}{bx}\right)} = \frac{1}{|W|}\left[M\left(\frac{\eta}{a}+1\right) + g_d \eta (z-1) - \zeta(\eta+1)\right] > 0,$$

(16)
$$\hat{x} = \frac{1}{|W|\omega}\left\{M\left(\frac{1}{a} - z(1-S_k) - \omega S_k\right) + g_d(z-1) + \zeta\left(z(1-S_k) + \omega S_k - 1\right)\right\},$$

where $|W| \equiv \frac{1}{\omega}\left[(1-S_k)(1+z\eta) + S_k\right] + \eta S_k > 0$.

Using Equation (16) we can decompose the dynamics of pollution flows into four partial effects, as follows:

(16’)
$$\hat{x} = \frac{1}{\omega |W|}[\varepsilon_k + \varepsilon_t + \varepsilon_s + \varepsilon_c],$$

where $\varepsilon_k \equiv M / a > 0$ is the *pure capital increasing effect*; $\varepsilon_t \equiv -(\zeta + g_D) < 0$ is the *pure technological change effect*; $\varepsilon_s \equiv -\omega S_k[M - \zeta] < 0$ is the *technique effect*; and $\varepsilon_c \equiv z\{S_K g_D - (1 - S_k)[M - \zeta - g_D]\}$ is the *output composition effect.*

The pure capital effect and technological change effect constitute the two primary sources of economic growth. Meanwhile, the technique and output composition effects are dependent on the primary sources of growth. The pure capital scale effect, *ceteris paribus*, increases pollution while the pure technological change effect reduces pollution because it reflects the fact that the effective dirty input may rise over time without necessarily increasing pollution. Assumption 2 guarantees that the net direct effect of economic growth, $\varepsilon_k + \varepsilon_t$, is pollution-increasing.

Expanding income due to the two primary sources induces an increase of the pollution tax due to the fact that the marginal utility of consumption, λ, falls as $M > 0$. This means that the relative price of the dirty input (pollution) increases over time which, in turn, triggers a technique or input substitution effect that has a pollution-reducing effect. The tax increase also causes an output composition effect by raising the cost of production and, hence the relative price, of the dirty good which in turn induces consumers to substitute consumption of dirty goods with clean goods and, hence, reduce pollution.

Pollution-augmenting technological change weakens both the technique and composition effects. Assumption 2 assures that although technological change only partially mitigates these effects, it cannot reverse them. The increase of the productivity of pollution due to technological change counters the effect of the increased pollution tax because the relative price of effective pollution increases less, causing the incentives to substitute pollution with clean inputs to weaken. Similarly, the increased productivity associated with technological change attenuates the cost increase of the dirty goods caused by the pollution tax. This, in turn, reduces the price increase of the dirty goods and, hence, weakens the consumers' incentives to substitute dirty goods with clean ones.

The optimal pollution tax dynamics.—Finally, we derive the dynamics of the optimal pollution tax that is consistent with the system (14) to (16). Noting that $\tau = v'(x)/\lambda$ we have that $\hat{\tau} = \eta\hat{x} + M$. Therefore, using Equations (8), (13) and (15) we can derive the rate of change of the pollution tax over time:

$$\hat{\tau} = \frac{\eta}{|W|\omega}\left(\left(\frac{1}{a}+\frac{1}{\eta}\right)M - (\zeta + g_d) + zg_d + ((1-S_k)z + \omega S_k)\zeta\right) > 0.$$

By Assumption 2, $M/a \geq \zeta + g_d$, which means that the pollution tax increases continuously along the optimal path. While the tax increases over time, the share of the pollution tax costs on the total value of consumption, $\tau x/c$, may eventually decline along the optimal path.

Suboptimal pollution paths.—The fact that we can obtain an explicit and tractable solution for the optimal rates of change of pollution and the other relevant variables show that, with enough information regarding the key parameters considered, this part of the solution is relatively easy to obtain for a government or planner. But this is, of course, not a complete solution; in order to obtain a complete solution we need to solve for the *initial* values of the endogenous variables (p , k_d/bx , x and, therefore, τ) in addition to their optimal rates of change as provided by (14) to (16). In fact, determining such initial values is extremely complex, not only for analysts but also for governments. Fortunately, as can be seen though an inspection of equations (14) to (16), the optimal rates of change of the variables are *not* dependent on the initial values of such variables.

This characteristic of the dynamical solution is very important because, as we shall see below, it allows us to determine the maximal critical initial level of pollution that assures that the stock of clean air will never fall below the catastrophic threshold. An imperfect government that is unable to ascertain the optimal initial values of the endogenous variables could still determine such a critical level and its job would be reduced to ensuring that the initial pollution level is below the critical point and from then on follows the myopic growth rule dictated by equation (16). The result would be a suboptimal rule, implying higher pollution levels than the optimum at all points in time, but one that assures sustainable and positive economic growth thus preventing environmental disaster. Section 6 deals with these issues.

5. Economic Growth

An important issue is whether the dynamic path described by Equations (14) to (16) implies a positive rate of consumption growth despite that the pollution tax is continuously increasing. The following proposition shows that this is indeed the case:

Proposition 1: (i) *The growth rate of real consumption expenditure is:* $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$, *where* $\hat{p}$ *is given by (14).*(ii) *The rate of growth of real consumption remains positive throughout the equilibrium dynamic path for any positive* ω *and* σ .(iii) *If either input substitution or consumption substitution is elastic* (*if* $\omega > 1$ *or* $\sigma > 1$), *but not both, the rate of growth of real consumption* *converges from below towards a rate* M / a. *If both* $\omega > 1$ *and* $\sigma > 1$ *, then the growth rate of real consumption converges to* $(1/a)\left(M + g_d\right)$.*(iv) If* $\omega < 1$ *and* $\sigma < 1$*, then the rate of growth of real consumption converges from above towards a rate* $\left((1+\eta)/(a+\eta)\right)(\zeta + g_d) < M / a$.

Proof: See Appendix.

Proposition 1 demonstrates that the dynamic equilibrium path described by Equations (14) to (16) is associated with a positive rate of growth of real consumption regardless of the size of the elasticity of substitution. However, the economy's growth rate is below its potential as a consequence of the fact that the optimal pollution tax forces the relative price of dirty goods to continuously increase over time. This, in turn, increases the cost of living for consumers, implying that economic growth must be partially sacrificed. However, as shown in Remark 1, if $\sigma > 1$, the share of the dirty goods in the consumption bundle declines, and if $\omega > 1$, the share of the clean input in production increases. In either of these cases the sacrifice of the growth rate vis-à-vis its potential level becomes progressively smaller beyond a certain point in time. That is, the growth rate of the economy approaches in the long run its maximum potential rate, which in this case is equal to M / a in the absence of neutral technological progress in the dirty sector.

The fact that when $\sigma > 1$ or $\omega > 1$ the convergence (or long run rate of growth) of the economy is not affected by the rate of pollution-augmenting technological change might seem surprising. The reason for this fact is that, in this case, the consumer budget share of pollution and/or the share of pollution in the cost of production approaches zero.[39] That is, pollution-augmenting technological change becomes irrelevant for economic growth over the long run because the share of the dirty input in the production of the dirty goods and/or the share of dirty final goods constitute a negligible fraction of the economy.

Furthermore, from Remark 1 it follows that if $\omega < 1$ and $\sigma < 1$, the share of the dirty input (pollution) in the cost of production increases over time and the share of dirty goods in the consumer budget increases over time, both converging to 1. Therefore, in such a case the technological change becomes the key determinant of the convergence rate of economic growth. Conversely, because the share of the clean goods approaches zero, the capacity of the economy to expand such goods becomes increasingly irrelevant for economic growth. This means that in the inelastic case the economy's growth rate declines and becomes increasingly dependent on the rate of technological change and less dependent on the rate of capital accumulation as the shares of the dirty input and dirty final output increase over time. Moreover, Assumption 2 implies that the growth rate of the economy converges to a lower level than in the elastic case.

The following corollary to Proposition 1 summarizes the results discussed in the previous two paragraphs:

Corollary 1: *Economies characterized by elastic producer and/or consumer choices tend to grow more rapidly and converge towards higher secular growth rates than economies exhibiting inelastic producer and consumer choices.*

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[39] This is true if $\sigma > 1$ but $\omega < 1$ because in this case the consumption share of the dirty goods approaches zero and hence the participation of the dirty goods in the economy becomes negligible in the very long run. Furthermore, if $\sigma < 1$ but $\omega > 1$ the share of pollution in production of the dirty goods approaches zero, meaning that in the very long run the participation of pollution as an input becomes negligible.

6. Conditions for Sustainable Growth (assuming that $\phi = 0$)

We first consider the case when *EMU* is greater than one, as assumed by standard sustainable growth models. In this study we will also consider the case when *EMU* is less than one, in light of the fact that some recent studies have shown that the *EMU* may reach levels below one, contrary to what has previously been assumed to be the case (i.e., Attanasio and Browning, 1995; Vissing-Jørgensen, 2002). Although the analysis is conducted under the assumption of pollution-augmenting and neutral technological progress in the dirty sector, the results hold under the more general assumptions on technological changes, including capital-augmenting technological progress in the dirty sector. This is shown in Appendix.

A. The Case When EMU is Greater Than One

A consequence of allowing $a > 1$ is that the rate of economic growth is slower than in a case where $a < 1$. In other words, the scale effect is less powerful and, hence, *ceteris paribus*, pollution emissions will tend to grow more slowly as the economy grows. This makes the conditions for sustainability much weaker than in a case where $a < 1$. From Proposition 1 and Equation (16), the following proposition emerges:

Proposition 2: *Suppose* $a > 1$*, technological change is either pollution-augmenting and/or neutral or non-existent, and that Assumptions 1 and 2 hold. Then, if either* σ *and/or* ω *is positive, an optimal pollution tax is sufficient to induce sustainable development.*

Proof: See Appendix.

Therefore, the conditions for sustainable development are extraordinarily weak in the case where $a > 1$. In this case, a society's willingness to pay for a marginal reduction of pollution increases rapidly with income. The growth effect then becomes relatively weak vis-à-vis the case where $a < 1$. Even when both consumption and input elasticity of substitution are less than one, sustainable development arises.

The intuition of this important result is as follows: assuming that $\sigma < 1$ and $\omega < 1$, and using Equation (8) (noting that $s \to 1$ in the long run) and the expression for $\widehat{\left(\frac{c}{e}\right)}$ in Proposition 1, the secular or long run rate of growth of real consumption is found to be equal to the growth rate of dirty consumer goods. The rationale for this result is that in the long run, the clean consumption goods become a negligible fraction of total consumption and, hence, the rate of growth of total consumption is given by the rate of growth of the dirty consumption goods only. This, in turn, implies that the rate of long run growth of the dirty output is also equal to the long run growth rate of real consumption. Therefore, using part *(iii)* of Proposition 1, it follows that:

$$\widehat{\left(\frac{c}{e}\right)}^{\infty} = \hat{c}_d^{\infty} = \hat{y}_d^{\infty} = \frac{1+\eta}{a+\eta}(\zeta + g_d) < \zeta + g_d,$$

where the ∞ superscript denotes the long run values (i.e., $\hat{y}_d^{\infty} \equiv \lim_{t\to\infty} \hat{y}_d$).[40] We note from the above expression that since $a > 1$, the long run rate of growth of the dirty good is less than the growth rate of technological change. On the other hand, from Equation (9) it follows that since over the long run $S_k \to 0$ (because $\omega < 1$) then $\hat{y}_d^{\infty} = \zeta + g_d + \hat{x}^{\infty}$. Hence, $\hat{x}^{\infty} < 0$. If $a > 1$, then the economy's growth rate is low

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[40] We note that $\hat{c}_d^{\infty} = (1/a)[M - \hat{p}^{\infty}]$, where $\hat{p}^{\infty} = (a/1+a)[(M/a - \zeta - g_d)\eta + (M - \zeta - g_d)]$. In addition, since the market equilibrium condition implies that $\hat{c}_d^{\infty} = \hat{y}_d^{\infty} = \zeta + g_d + \hat{x}^{\infty}$, we have; $\hat{x}^{\infty} = (1/a)[M - \hat{p}^{\infty}(\zeta)] - \zeta - g_d$
$= ((1+\eta)/(a+\eta))(\zeta + g_d) - \zeta - g_d < 0$.

enough to have a smaller impact on pollution. This, in turn, means that an optimal pollution tax is sufficient to cause pollution to decrease over the long run, even if the economy is wholly inelastic.

Of course, while sustainability is in this case attained, the rate of economic growth of the economy remains positive; however, if both $\sigma < 1$ and $\omega < 1$, this rate can be quite low and may be below the rate of technological change. In other words, the sacrifice in terms of economic growth imposed by environmental sustainability is, in this case, large and permanent. However, this is not the case when either producers or consumers exhibit higher rates of flexibility. As shown in the proof of Proposition 2, if $\sigma > 1$ and/or $\omega > 1$, then sustainable growth also arises. Moreover, in such cases part (ii) or (iii) of Proposition 1 apply, meaning that the long run rate of growth of real consumption is M / a, which is of course greater than the long run rate of growth prevailing when both $\sigma < 1$ and $\omega < 1$ ($M / a > \frac{1+\eta}{a+\eta}(\zeta + g_d)$). In other words, in such a case, the growth rate sacrifice in terms of environmental sustainability is much smaller and is merely temporary.

The reason why this important result is missed by the standard growth theoretical models is that they drastically limit the consumer's role in the economy by assuming only one final good. Proposition 2 arises because the growth rate of the consumption of the dirty goods dictates the long run rate of growth of real consumption, which is sufficiently slow to permit pollution to eventually start falling within a finite period of time. Therefore, we are able to derive this considerably important new insight by explicitly allowing for more than one type of consumer good. If the *EMU* of consumption is greater than one, then the sustainable economic growth is effectively a natural condition, provided an optimal environmental tax is implemented.

B. Capital-Augmenting Technological Progress

We now introduce capital-augmenting technological progress in the dirty sector to demonstrate the robustness of our results in Proposition 2. In the case of capital-augmenting technological change affecting the dirty sector, we simply augment capital by factor, n, with $\dot{n} / n = \theta > 0$.

Corollary 2: *Suppose* $a > 1$*, technological change in the dirty sector augments any factor of production (and/or is neutral or non-existent), and that Assumptions 1 and 2 hold. Then, if either* σ *and/or* ω *is positive, an optimal pollution tax is sufficient to induce sustainable development.*

Proof: See Appendix.

Corollary 2 implies that progressively higher optimal pollution tax along the growth path induces sustainable growth under any type of exogenous technological changes. Corollary 2 also implies that when $\omega < 1$ and $\sigma < 1$, the necessary and sufficient condition for sustainable growth is that EMU is greater than one.

When the technical elasticity of substitution between clean and dirty inputs is greater than one, capital-augmenting technological change decreases the relative price of dirty goods even under the rising pollution tax. Since the expenditure share of dirty goods increases when the consumption elasticity of substitution is greater than one, the flexibility requirement in the production of dirty goods under capital-augmenting technological change becomes more stringent than in its absence. On the other hand, if the consumption elasticity is less than one, the presence of capital-augmenting technological progress makes it easier to achieve sustainable growth than its absence would.

Finally, it can be shown that if the capital-augmenting technological progress takes place not only in the dirty sector but also in the clean sector, together with pollution-augmenting technological progress, sustainable growth occurs under an optimal pollution tax. Thus, as long as EMU is greater than one, sustainable growth occurs under an optimal pollution tax for any type of exogenous technological progress.

C. The Case When EMU is Less Than One

Here, we demonstrate that when $a < 1$ the conditions for sustainable economic growth are more demanding than in the previous case. This section will first characterize the output composition effect and will then look into the input substitution (or technique) effect.

The output composition effect.—The composition effect works when consumers substitute dirty goods with clean goods in the face of the rising relative price of the dirty goods. Here we consider the case when the consumption elasticity of substitution is strictly greater than 1, but the production elasticity of substitution is less than 1. In this case, the feasibility of sustainable growth relies exclusively on consumer flexibility. Using Remark 1, it follows that the factor share of the clean input in the output value of the dirty final goods, S_k, converges to zero (and concomitantly, the share of the dirty input converges to 1). The fact that the relative price of dirty goods continuously increases over time means that consumers substitute dirty goods with clean ones.

Therefore, assuming that $\sigma > 1$ and $\omega < 1$, then the limit to Equation (16) is:

$$\lim_{t\to\infty} \hat{x} = \frac{M\left(\frac{1}{a}-\sigma\right)-(\zeta+g_d)(1-\sigma)}{(1+\sigma\eta)}. \tag{17}$$

From Equation (17) it follows that $\lim_{t\to\infty} \hat{x} < 0$ if and only if

$$\sigma > \frac{\frac{M}{a}-(\zeta+g_d)}{M-(\zeta+g_d)} \equiv d(M,a;\zeta,g_d) > 1.$$

The threshold level, $d(M,a;\zeta,g_d)$, above which sustainable growth becomes possible, is increasing in ζ and g_d respectively. As a consequence of technological change in the dirty sector, sustainable growth becomes more difficult. The threshold level reduces to $1/a$ in the absence of any form of technological progress. The following lemma summarizes the previous results:

Lemma 1 (on the role of the composition effect): *Suppose that technological progress is pollution- augmenting and/or neutral or non-existent, and that Assumptions 1 and 2 hold. If* $a<1$*, then* $\omega<1$ *does not preclude sustainable economic growth if and only if* σ *is greater than a threshold level exceeding one* (*i.e.,* $\sigma > d(M,a;\zeta,g_d) > 1$).

Lemma 1 underlines the importance of the composition effect in circumventing the case of an inelastic production technology. All of the previous analyses have assumed a single final good, and hence have ignored the output composition effect, concluding that a flexible production technology ($\omega \geq 1$) is a necessary condition to allow for sustainable development. Lemma 1 shows that this is not true as long as consumer preferences are sufficiently flexible ($\sigma > d(M,a;\zeta,g_d) > 1$). Remarkably, sustainable growth under an optimal pollution tax may occur even if the production function of dirty goods is Leontief ($\omega = 0$); that is, even if clean and dirty inputs are complements rather than substitutes. Also, the absence of technological change means that $d(M,a;\zeta,g_d) = 1/a$ and thus the condition for sustainable development is not qualitatively affected.

A sufficient condition for the share of dirty consumption goods to approach zero in the long run is that $\sigma > 1$ when $\omega < 1$, so that the relative price of dirty goods increases over time. It might seem surprising that this condition is not sufficient for sustainable development. This is the case because the share of dirty goods approaching zero does not necessarily imply that the rate of growth of the demand for (and hence supply of) the final dirty goods will become negative. In fact, the growth rate of dirty goods continues to be positive over the long run if the economy's growth rate is sufficiently rapid, and may even surpass the rate of pollution-augmenting technological change, in which case pollution will continue to increase in the long run. Lemma 1 shows that only when the elasticity is sufficiently large ($\sigma > d(M,a;\zeta,g_d) > 1$) will the consumption of dirty goods (and hence the production of dirty goods) grow at a rate that is below the pollution-augmenting technological change, thus leading to a reduction of pollution levels.[41]

The input substitution or technique effect.—We will now consider the case when the technical elasticity of substitution between the two inputs is strictly greater than one, while the consumption elasticity of substitution is less than one but still positive. In this case, the cost share of the clean input approaches one, while the share of the dirty

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[41] Given that $\omega < 1$, which implies that $\lim_{t\to\infty} S_k = 0$, it follows from (10) that the rate of growth of the dirty good production over the long run is equal to the growth rate of effective pollution, $\hat{y}_d^{\infty} = \hat{x} + \zeta$. Hence, if $\hat{y}_d^{\infty} = \hat{c}_d^{\infty} > \zeta$ then $\hat{x} > 0$, where a superscript ∞ denotes long run levels.

good in the consumer budget also approaches one. The feasibility of sustainable growth depends solely on technique effect. From Equation (16) we have:

$$\text{(18)} \qquad \lim_{t\to\infty} \hat{x} = \frac{\left(\frac{M}{a} - \zeta\right) - \omega\left(M - \zeta\right) + g_d(\sigma - 1)}{1 + \omega\eta}.$$

The first term of the numerator of Equation (18) represents the technique effect resulting from a change in the relative factor costs of production. The optimal pollution tax causes the pollution input to become increasingly expensive. In addition, if the elasticity of substitution between the clean and the dirty input is greater than one, the pollution input is gradually substituted with capital, causing its share to converge to zero. The second term of the numerator (which is positive) captures the productivity effect of pollution, an effect that makes it more difficult to achieve sustainable growth over the long-term. The third term represents the effect of growth of total factor productivity in the dirty sector, which reduces pollution growth when $\sigma < 1$. It follows that sustainable growth only becomes possible if the technique or substitution effect outweighs the technological change effect. This condition is satisfied if $\omega > d(M, a; \zeta, 0) > 1$ where

$$d(M, a; \zeta, 0) = \frac{\left(\frac{M}{a} - \zeta\right)}{M - \zeta}.$$

Consequently, if $a < 1$, a Cobb-Douglas production function ($\omega = 1$) is not consistent with sustainable development when $g_d = 0$. As we demonstrate below, the standard growth models have almost always assumed Cobb-Douglas production functions, and are therefore able to conclude that growth is sustainable only because they assume that the *EMU* is greater than one. The following lemma summarizes these findings.

Lemma 2 (on the technique or input composition effect): *Suppose that technological progress is pollution-augmenting and that Assumptions 1 and 2 hold. If* $a < 1$*, then* $\sigma < 1$ *does not preclude sustainable economic growth if an optimal pollution tax is implemented and* ω *is greater than a threshold level,* $d(M, a; \zeta, 0)$ *that exceeds one.*

In our model (unlike, for example, the model in Acemoglu et al., 2012) capital (i.e., the clean input) is expanding in a growing economy and, moreover, the rate of economic growth is endogenous. Hence, even if technological change is only pollution-augmenting and concentrated in the dirty sector (as we assume), the capital-to-effective pollution ratio (k_d / bx) may increase without requiring so rapid an increase of the pollution tax as to smother economic growth. This follows because the technique effect does not rely exclusively on the pollution tax, but is reinforced by the capital growth effect. Therefore, if the elasticity of substitution between capital and pollution is greater than the threshold level, then the substitution effect may dominate the expansion effect within the dirty sector and pollution will begin decreasing at some finite time along the growth path. Combining Lemmas 1 and 2, we obtain the following proposition:

Proposition 3: *Suppose that technological change is pollution-augmenting and Assumptions 1 and 2 hold. If* $a < 1$*, then sustainable growth is feasible if an optimal pollution tax is implemented and either* ω *or* σ *is greater than the threshold level,* $d(M, a; \zeta, 0)$, *which exceeds one.*

Proof: See Appendix.

Proposition 3 demonstrates that even if technological progress benefits only the dirty sector and is biased toward the dirty input in a pollution-augmenting fashion, and if the *EMU* is less than one, then an optimal pollution tax may be sufficient to induce environmental sustainability if either the consumer's preferences or the producer's technologies exhibit sufficient flexibility. From Proposition 1, it follows that this occurs while the economy's growth rate is positive throughout the full adjustment path. Moreover, since environmental sustainability requires that either $\sigma > 1$ or $\omega > 1$, Proposition 1 clearly shows that economic growth is lowered in the short run but that the economy's growth rate gradually recovers towards its potential rate over the long run. Therefore, the optimal pollution tax alone can lead to sustainable growth without requiring further policy interventions (such as subsidies directed at transforming technological change from pollution-augmenting to clean sector or clean input augmenting).

7. Stock Effects: Conditions for Avoiding an Environmental Disaster

In this section we analyze the conditions under which the solution for the dynamical system developed in the previous sections is indeed consistent with avoidance of environmental disaster at any point in time. Assuming that the dynamic path of pollution is defined by equation (16), we find that for any given initial level of clean air stock there exists a corresponding critical level of initial emission flow such that if the initial value of pollution emissions is less than such critical level, the clean air stock remains at all times above a minimal threshold level that prevents environmental disaster. Otherwise, if the initial pollution level is above the critical level, then the clean air stock falls below the threshold level and catastrophic environmental disaster will eventually ensue. The intuition behind this result is that since equation (16) gives the (optimal) rate of change of pollution for all times, then the full path of pollution is entirely determined by the initial level of pollution. The question is whether along this path the stock of clean air ever reaches the catastrophic level. If we find the initial (critical) level of pollution that in conjunction with (16) causes a pollution path that exactly avoids reaching such a catastrophic stock level, then any other pollution path following the same rate of change established by (16) but starting from a lower pollution level will also avoid catastrophe.

In order to identify such a critical level of initial emissions, we first note that for any given initial level of man-made capital, the system of equations (14) to (16) yields a unique optimal growth path for p, (k_d / x) and x.[42] In particular, we can define the pollution level at a point in time as:

$$x(t) = x_0 \int_0^t \exp(g(\upsilon)\upsilon) d\upsilon,$$

where $g(\upsilon)$ is the rate of change of pollution at time υ, which is a function of all parameters and the predetermined variable, k_0. As we show below, the effect of the initial clean air stock on $x(t)$ occurs entirely through its effect on x_0. In addition, the stock of clean air at any point in time is given by Equation (4'). Hence, we can define the unique path of pollution emission flows and stock of clean air as conditional

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[42] We note that the system of equations (14), (15) and (16) can be represented as a system of autonomous differential equations $\dot{p} = \Theta(S_k, s(p), p)$, $\dot{(k_d / bx)} = \Gamma(S_k, s(p), (k_d / bx))$ and $\dot{x} = \Phi(S_k, s(p), (k_d / bx), x)$. Since $\Theta(\cdot)$, $\Gamma(\cdot)$ and $\Phi(\cdot)$ are all continuously differentiable functions, there exists a unique solution for each set of initial values. We also note that the solution for emission, x, constitutes an optimal control for dynamic optimization in the absence of stock constraints. The initial level of emission is determined endogenously within the system. Likewise, initial values of k_d and therefore p are all endogenously determined within the system.

functions of the (endogenous) initial value of pollution as well as of the (predetermined) initial stocks of clean air and natural capital as follows:

$$x(t) = G(t, x_0; k_0, \chi) \text{ and } E(t) = J(t, x_0; k_0, E_0, \chi),$$

where the function $J(t, x_0; k_0, E_0, \chi)$ is defined in (4’) and $\chi = (a, \sigma, \omega)$ denotes a vector of structural parameters. Also, we have that $x(0) = G(0, x_0; k_0, \chi) = x_0$ and $E(0) = J(0, x_0; k_0, E_0, \chi) = E_0$ by the fixed point theorem. From Equation (4’) it is clear that unless the pollution emissions $x(t)$ eventually starts falling over time the stock constraint, $E(t) \geq \underline{E}$ for all $t \geq 0$, cannot be satisfied.

Let χ^* denote the set of $\chi = (a, \sigma, \omega)$ which guarantees eventual decline of pollution emissions, and are the parameters that satisfy the conditions established by either Propositions 2 or 3. Then for any χ in χ^*, and man-made stock of capital, we can define the *admissible* set, $D(\chi, k_0)$ of initial values of clean air stock and flow level of pollution which assures sustainable growth. Thus,

$$D(\chi, k_0) = \left\{ (x_0, E_0) \middle| J(t, x_0; k_0, E_0, \chi) \geq \underline{E}, \text{ for all } t > 0 \right\}.$$

Given the initial level of clean air, E_0, the set $D(\chi, k_0; E_0)$ of initial levels of flow pollution that an economy can emit while maintaining the stock of clean air above the threshold level is bounded above and closed. This is so because the function $J(t, x_0; k_0, E_0, \chi)$ is continuous as shown by (4’) and is also bounded from above. There exists the maximal element, $x_0^c(E_0)$ of the set $D(\chi, k_0; E_0)$, above which an environmental disaster occurs. We define $C(\chi, k_0) = \left\{ E_0, x_0^c(E_0) \middle| \underline{E} \leq E_0 \right\}$, which constitutes the boundary or envelope of the set $D(\chi, k_0)$.

Alternatively, we note that for any eventually declining pollution path, there exists a time $T \geq 0$ after which pollution decreases in a monotonic way. It follows that there exists a critical turnaround time $t^* > T$ such that

$$(19) \qquad x(t^*) = G(t^*, x_0^c; k_0, \chi) = \psi \underline{E},$$

$$(20) \qquad E(t^*) = J(t^*, x_0^c; k_0, E_0, \chi) = \underline{E},$$

where x_0^c is the *maximum* initial level of pollution emissions that corresponds to any given $E_0 > \underline{E}$ consistent with avoiding environmental disaster and t^* is the critical turnaround time at which the stock of clean air reaches the minimum level necessary to avoid a catastrophe. The two equations (19) and (20) solve for the two endogenous variable, $x_0^c = x_0^c(E_0; k_0, \underline{E}, \chi, \psi)$ and $t^* = N(E_0; k_0, \underline{E}, \chi, \psi)$. [43]

Figure 1 illustrates the previous analysis. The thick curve, denoted as C, is the envelope of set D as defined above. Therefore, C provides an envelope for all trajectories of x as a function of E_0 that satisfy the constraint $E(t) \geq \underline{E}$ at all times, which is called set D in Figure 1. By contrast, any trajectory that is outside (above) the envelope C, denoted as a complement of set D (set D^c) in Figure 1 (which is shaded), reaches an environmental catastrophe. Figure 1 shows the particular case where pollution emissions follow an inverted U-shaped pattern where the envelope C reaches $\underline{E}$ at the turnaround time t^*. The uniqueness property of the adjustment paths guarantees that any two different trajectories starting from different initial positions move in parallel and never cross each other. Hence, any trajectory starting

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[43] Section 8 presents an explicit solution of these endogenous variables in a Cobb-Douglas economy.

below $x_0^c(E_0)$ never reaches the catastrophic stock level, while any trajectory starting above C is bound to eventually violate the stock constraint.

In Figure 1 the curve labeled OO represents the optimal trajectory while the curve SS shows an arbitrary suboptimal but sustainable trajectory associated with a suboptimal tax. The tax that underlying trajectory SS satisfies two conditions: first, it is sufficiently high to permit the initial pollution level to be below the critical level (x_0^c) as defined earlier and second, it adjusts over time to allow for an optimal rate of change of pollution according to Equation (16). In general, finding x_0^c is easier and demands much less information than determining the optimal initial pollution level. It must be noted that, as expected, pollution levels within trajectory OO are lower than those within trajectory SS at each point in time.[44]

8. Numerical calibrations

Here we develop a numerical example to obtain further insights into the propositions of this paper.[45] In order to highlight the role of the consumption composition effect, we assume that the clean and dirty inputs are complements (i.e., $\omega = 0$). For simplicity we focus only on pollution-augmenting technological progress. We first calibrate our model only with flow emissions of pollution using parameters based on data from the US economy and check the sustainability condition for the stock constraint later.

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[44] Figure 1 does not illustrate time profiles of pollution emissions for the two trajectories. It can be shown, however, that each level of E is reached at an earlier time along the trajectory OO than SS. Although it appears in the figure that the level of pollution emissions is higher in OO than SS beyond the turnaround level, this is due to the fact that the visual comparison considers indeed different points in time. At each point of time the level of E is higher within trajectory OO than SS.

[45] Here we provide a succinct description of the simulation methodology. For further detail, please check the online resource.

Parameter Choices.—In the recent literature the long-run annual growth rate of the US economy is often assumed to be 2 percent (i.e., Nordhaus, 2007; Weitzman, 2007; Acemoglu et al., 2012). As shown in Proposition 1 above, this corresponds to M / a where $a = EMU$. Since the EMU is assumed to be approximately 2 in the literature, the net return to the capital input, M, is approximately 0.04. We thus assume that the net return to capital is four percent, and examine the feasibility of sustainable growth under varying assumptions of the EMU and temporal substitution parameters in consumption, σ.

Based on recent econometric estimates we alternatively consider values of EMU of 2 and 0.8. (i.e., Ogaki and Reinhart, 1998 and Vissing-Jørgensen, 2002). For $M = 0.04$, the long run growth rate of the economy becomes 5 percent when $EMU = 0.8$, which is much greater than the commonly accepted rate of 2 percent. In spite of this, we perform this simulation to highlight the fact that when EMU is low, the scale effect is much larger and therefore makes sustainable growth more difficult to achieve. In addition, in order to highlight the role of the composition effect, we consider three different values for σ, namely, 4, 2 and 0.8. Finally, we assume that the rate of pollution-augmenting technological progress is $\zeta = 0.005$, the parameter for the elasticity of marginal damage is $\eta = 1$, the ratio A_C / A_D is 1 and the ratio γ_c / γ_d in the unit expenditure function is 0.7.

The pollution emissions path.—Figure 2 provides the growth of pollution emissions over time for various values of *EMU*.[46]

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[46] For illustration purposes, we use a time scale obtained by calibrating the changes in the share of the clean input (labor) of the U.S. manufacturing industry over the past decade. For the detailed procedure, see the online resource.

Panel (a) shows the case when $EMU = 0.8$. If the elasticity of substitution is greater than the threshold level, $\left(\frac{M/a-\zeta}{M-\zeta}\right) \approx 1.28$, implied by Proposition 3, there exists a critical time until which pollution increases monotonically and after which declines over time. This turning point depends on the level of σ. If $\sigma = 4$, the turning point takes place in the year 2069, and if $\sigma = 2$, in 2185. This is due to the fact that the consumption composition effect becomes more effective when σ is larger. Panel (b) depicts the case when $EMU = 2$: if $\sigma = 4$ then pollution begins falling very quickly by the year 2025, but if $\sigma = 2$ or $\sigma = 0.8$ then the turning point occurs during a much later year (2057 and 2178, respectively). Panel (c) illustrates the pollution emissions path for the case when both EMU and σ are less than one, in which case pollution increases in all periods. Given that $\sigma < 1.28$, pollution emissions continue to increase over time for all periods as indicated by Lemma 1. In summary, if $EMU < 1$, sustainable growth requires that the consumption elasticity of substitution is greater than the threshold level. However, as shown in Panel (b), if $EMU > 1$ then economic growth is sustainable even if σ is very low (and $\omega = 0$ as we assumed here). In this case, as predicted by Proposition 2, even highly inelastic consumer preferences and producer technology do not prevent pollution from beginning to decline along the optimal path.

Growth sacrifice caused by the pollution tax.—Finally, Panel (a) in Figure 3 shows the rates of growth of real consumption $\widehat{\left(c/e\right)}$ for $EMU = 0.8$. The rate of economic growth is always positive, although it falls below the potential growth rate over the short run. However, if $EMU > 1$, it recovers towards the potential growth rate over the long run. The growth sacrifices over the short and medium terms are rather small and growth recovers more quickly if the elasticity of substitution is larger. Even when σ is relatively low (i.e., 2), the growth sacrifice is not very large, reaching a maximum value of the order of 0.6 annual percentage points, although the growth rate begins recovering at a much later date than when $\sigma = 4$. The growth sacrifice is large if σ is less than one (i.e.,$\sigma = 0.8$) and, more importantly, and as predicted by Proposition 1, the economy's growth rate converges to a lower but still positive rate of growth over the long run.

Panel (b) of Figure 3 illustrates the case when $EMU = 2$. If $\sigma < 1$, then the long run growth rate remains positive but falls below toward the technological growth rate ($\zeta = 0.005$). However, as predicted by Proposition 1, if $\sigma = 2$ then the rate of economic growth converges to the potential growth rate M / a and, moreover, the growth sacrifice imposed by environmental sustainability is smaller than the previous case and temporary. The maximum reduction of the rate of economic growth is in this case only about 0.5 percentage points. In the short run the growth sacrifice caused by the pollution tax is only 0.2 percentage points, from 2% annual growth when no environmental tax is implemented to about 1.8% when the tax imposed.

Numerical Simulation Considering the Stock Effects.— we now consider the possibility of irreversible disaster assuming Cobb-Douglas utility and production function, and that $EMU > 1$. Although there is no clear consensus on the structure of the carbon cycle, recent scientific studies find that the lifetime of carbon in the air spans a few centuries. According to IPCC (2007), about half of an increase of CO2 will be removed from the atmosphere within 30 years, implying a 1.6 percent regeneration rate of clean air per annum (IPCC, 2007). Then, Equation (19) implies that $x(t^*) = 0.016\underline{E}$.

Given the Cobb-Douglas specification, the cost share of clean input in production, S_k, and the consumer's budget share of the dirty final good, s, are constant. Assuming that service output and labor input are less pollution intensive than manufacturing output and energy intensive input, we use estimates for the share of clean input and clean final goods in world GDP for calibration purposes and set $S_k = 0.5$ and $s = 0.54$ (Guscina, 2006; World Bank, 2012). Using the same values for the other parameters (i.e., $a = 2$; $\zeta = 0.005$; $M = 0.04$; $\eta = 1$), we obtain from Equation (16) that $x(t) = x_0 \exp(-\vartheta t)$, where $\vartheta = 0.0085$, implying that the optimal pollution decreasing rate is equal to 0.85 percent per annum.

Since there is no direct measure to gauge absolutely clean air stock, we construct the so-called relative clean air stock (RCAS) index to represent $E(t)$ in section 7. Let $Carbon_t$ and $Carbon^D$ represent the current global carbon stock in year *t* and the disaster-rendering magnitude of the global carbon stock, both measured in *ppm*. Define $RCAS$ index as follows;

$$E(t) = RCAS(t) = Carbon^D / Carbon_t.$$

For calibration purposes, we assume that the disaster-rendering level of the carbon stock is 650 *ppm*.[47] In addition, we set the initial value (year 2013) and pre-industrial

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[47] Although the disaster-rendering magnitude of the stock of CO2 differs according to various experts, commonly accepted carbon concentration levels lie somewhere between 550 *ppm* and 750 *ppm,* implying a 3 Celsius degree and 4 Celsius degree increase, respectively (i.e., Glasby, 2006; Pearson et al., 2009).

value of global carbon stock level in the atmosphere at 395 *ppm* and 280 *ppm,* respectively (NOAA, 2013). Then the clean air stock index for the pre-industrial level that we consider environmentally pristine is $\overline{E} = 650/280 \approx 2.32$,while the current level and disaster-rendering level of clean air stock are $E_{2013} = 650/395 \approx 1.81$ and $\underline{E} = 650/650 = 1$, respectively.[48]

To solve for the corresponding critical level of emission, x_{2013}^{c} numerically, we first note that using Equation (19),

$$x_{2013}^{c} \exp(-\vartheta t^{*}) = \psi \underline{E} = \psi \tag{21}$$

Also, from Equation (4') and (20), we have,

$$E(t^{*}) = \exp(\psi t^{*})(E_{2013} - \int_{0}^{t^{*}} x_{2013} \exp(-\vartheta t) dt) = \underline{E} = 1 .$$

Using the expression for the pollution emissions in the Cobb-Douglas case, $x(t) = x_0 \exp(-\vartheta t)$ and integrating, it follows that the previous expression can be written as:

$$\exp(\psi t^{*}) \left(E_{2013} + \frac{x_{2013}^{c}}{\vartheta + \psi} (\exp(-(\vartheta + \psi) t^{*}) - 1) \right) = 1 \tag{22}$$

Solving Equations (21) and (22) using numerical methods gives the point for the year 2013 located in the envelope C , which corresponds to $x_{2013}^{c} = 0.043$ and $E_{2013} \approx 1.81$. We then generate the time profiles of pollution emissions and the stock of clean air under alternative scenarios.

We consider four alternative scenarios.

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[48] A pre-industrial level of carbon stock is often considered an environmentally clean air condition (i.e., Acemoglu et al., 2012).

Scenario 1 (Optimistic case): The government is able to reduce emissions by 10 percent below the critical level, x_{2013}^{c}, and the rate of pollution emissions growth is to be regulated optimally according to Equation (16).

Scenario 2 (Sufficient case): The government takes measures to reduce emissions exactly to the critical level, x_{2013}^{c}, and the rate of pollution emissions growth is to be regulated optimally according to Equation (16).

Scenario 3 (Insufficient, late disaster case): The government is unable to reduce pollution emissions to the critical level, x_{2013}^{c}, and allows emission levels 10 percent higher than the critical level, x_{2013}^{c}, while still restricting the rate of pollution emissions growth optimally according to Equation (16).

Scenario 4 (Business as usual, early disaster case): Pollution emissions are 10 percent above the critical level, x_{2013}^{c}, and they grow by 3.1 percent per year, which corresponds to the historical growth rate of carbon emissions over the 2000-2010 time period (Peters et al., 2011).

Table 1 shows the simulation results for the time profiles of $x(t)$ and $E(t)$ under the above scenarios. Under Scenario 1, sustainable development takes place. In this scenario the turnaround point of the clean air stock occurs in 2066, reaching an environmentally pristine condition by 2141. Under Scenario 2, sustainable development is also feasible, as the clean air stock never falls below the threshold level and starts growing in 2130. Under Scenario 3, an environmental disaster is unavoidable; by 2063, the stock of the clean air falls below the threshold level. An environmental disaster occurs despite the assumption that the government is able to regulate emissions growth according to the optimal rate of change. Lastly, under Scenario 4, an environmental disaster occurs by the year 2028.

9. Conclusion

Sustainable development can be achieved under a variety of plausible technological conditions using a pollution tax as the only policy instrument. If the often-used assumption regarding *EMU* being greater than one holds, then sustainable development is almost automatically satisfied as long as either the elasticity of substitution in production or in consumption is positive. An optimal pollution tax profile rules optimal pollution changes over time as defined by our expression (16) and it is sufficiently high to set the initial pollution level below a critical level defined in the text. Even if the initial pollution tax is suboptimal level, sustainable development still takes place as long as the initial tax level is sufficient to set the initial pollution flow less than or equal to its critical level and that the rate of change of the tax over time be at the rate necessary to induce optimal pollution changes over time as defined by equation (16).

Sustainable development mainly becomes an issue when *EMU* is less than one. Sustainability may also occur in this case if consumer preferences between the clean and dirty goods are flexible enough, even if the production technology is highly inflexible. In contrast to the assumption of high producer flexibility made by the standard growth models, the assumption of consumer flexibility required in this case appears to be more adequately supported by empirical studies. This paper has demonstrated that neither strong production substitution nor technological optimism is necessary for environmentally sustainable growth.

Figure 1. The admissible set *D* and the envelop *C* in *E*-*x* space

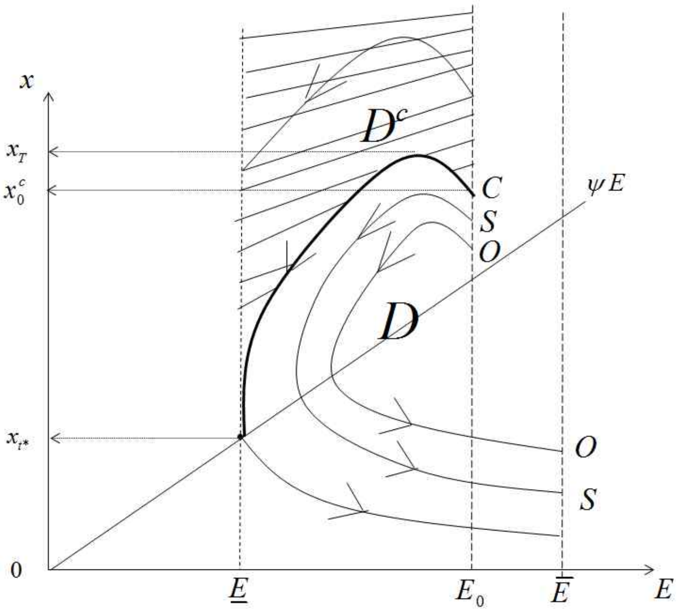
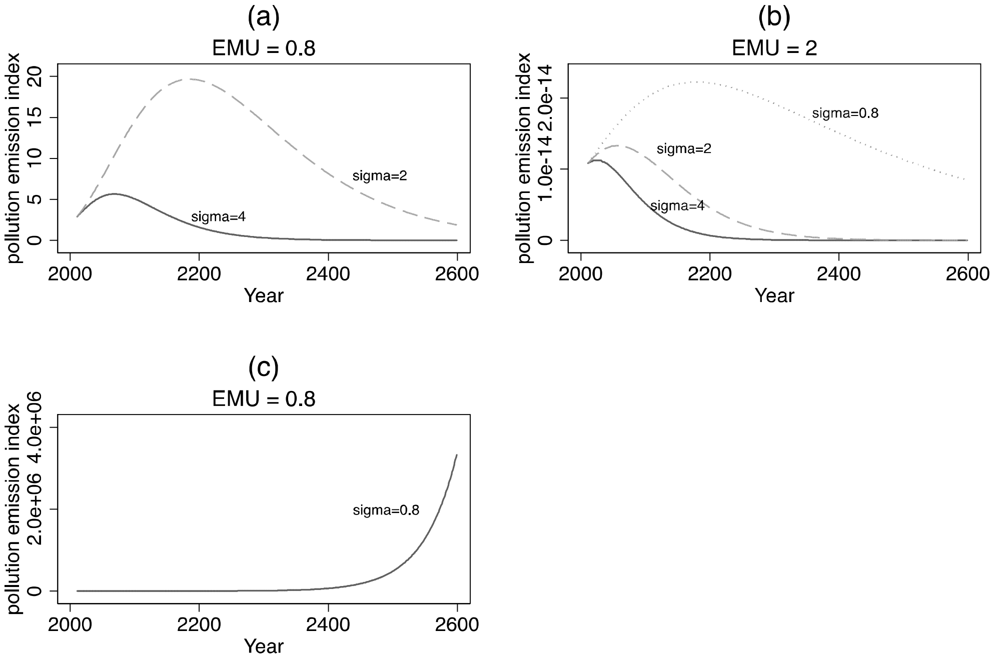
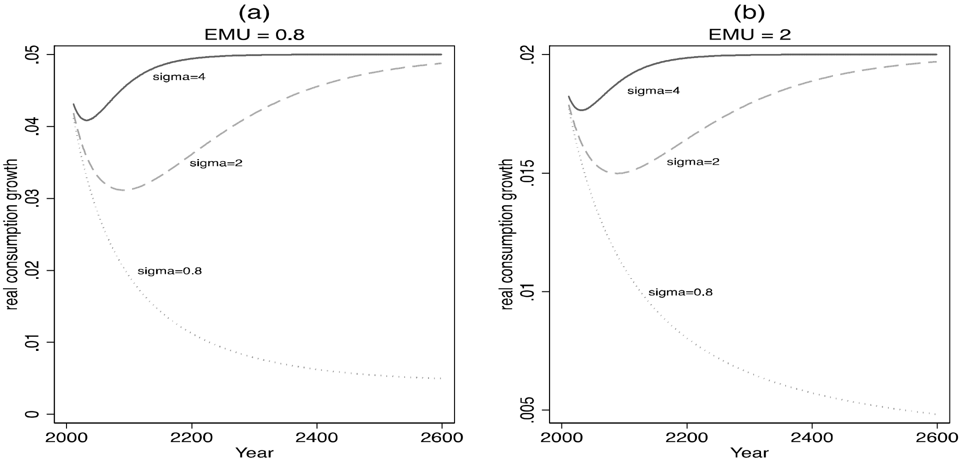


Figure 2. Pollution emissions for different values of σ and *EMU*



Source: Authors' calculation

Figure 3. Real consumption growth rates for different values of σ and *EMU*



Source: Authors' calculation

Table 1. Time path of pollution emissions and clean air stock under different scenarios

	Scenario 1 Optimistic case $(\hat{x} = -0.0085)$		Scenario 2 Sufficient case $(\hat{x} = -0.0085)$		Scenario 3 Insufficient, late disaster case $(\hat{x} = -0.0085)$		Scenario 4 Business as usual, early disaster case $(\hat{x} = 0.031)$	
Year(t)	$x(t)$	$E(t)$	$x(t)$	$E(t)$	$x(t)$	$E(t)$	$x(t)$	$E(t)$
2013	**0.0387**	1.809	**0.0430**	1.809	**0.0473**	1.809	**0.0473**	1.809
2027	0.0343	1.689	0.0381	1.625	0.0419	1.561	0.0706	1.063
2028	0.0340	1.682	0.0378	1.613	0.0416	1.544	**Environmental Disaster**	
2062	0.0255	1.544	0.0283	1.275	0.0311	1.006		
2063	0.0253	1.543	0.0281	1.267	**Environmental Disaster**			
2065	0.0248	1.5425	0.0276	1.252				
2066	**0.0246**	**1.5424**	0.0274	1.244				
2067	0.0244	1.5426	0.0271	1.237				
2129	0.0144	2.057	**0.0160**	**1.000**				
2130	0.0143	2.076	0.0159	1.00003				
2140	0.0134	2.311	0.0146	1.007				
2141	**Pristine condition**		0.0144	1.009				
2240			**0.0062**	2.293				
2241			**Pristine condition**					

Notes: 1) $x(t)$ and $E(t)$ denote the yearly index of pollution emissions and relative clean air stock, respectively. 2) For each scenario, Equation (4') is used to generate $E(t)$ over time starting from the initial year of 2013.

Source: Author calculations.

Appendix

Proofs of propositions and assertions in the text

Derivation of equation (9):

Use Roy's identity to derive the demand for the dirty good from the indirect utility function as follows.

(A1) $$c_d = \frac{c}{e(1,p)} e_2(1,p).$$

Logarithmic time differentiation yields,

(A2) $$\hat{c}_d = \hat{c} + \hat{e}_2(1,p) - \hat{e}(1,p).$$

Totally differentiating both sides of first order condition Equation (5) with respect to time and using Equation (6), we have,

(A3) $$\hat{c} = \left(\frac{a-1}{a}\right)\hat{e} + \frac{M}{a}.$$

The second term of the right-hand side of Equation (A2) can be written as,

(A4) $$\hat{e}_2 = \frac{d \log e_2}{dp}\frac{dp}{dt}.$$

Using the CES utility function we obtain,

(A5) $$\frac{d \log e_2}{dp} = \left(\frac{\sigma}{1-\sigma}\right)\frac{\gamma_d (1-\sigma) p^{-\sigma}}{\gamma_c + \gamma_d p^{1-\sigma}} - \frac{\sigma}{p} = \frac{\sigma}{p}\left(s(p) - 1\right)$$

On the other hand, using Shephard's lemma on the expenditure function $e(1,p)$ we have,

(A6) $$\hat{e}(1,p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}\,.$$

Using Equation (A5) into Equation (A4) and then using (A3), (A4) and (A6) in (A2) we find,

(A7)
$$\hat{c}_d = \left(\frac{1-a}{a}\right)\left[M - s(p)\hat{p}\right] + \sigma(s(p)-1)\hat{p} - s(p)\hat{p}$$

$$= \frac{1}{a}M - \left[\frac{s(p)}{a} + (1-s(p))\sigma\right]\hat{p}.$$

Derivation of Equation (13) :

Logarithmic total differentiation of both sides of the first order condition Equation (8),

(A8)
$$\eta\hat{x} - \hat{\lambda} = \hat{p} + g_D + \hat{b} + \left(F_2(\overset{\wedge}{k_d}, bx)\right).$$

Also, since the function F is CES , we have,

(A9)
$$\left(F_2(\overset{\wedge}{k_d}, bx)\right) = \frac{\alpha}{\omega}\frac{\overset{\wedge}{\left(\frac{k_d}{bx}\right)}}{\left[(1-\alpha)\left(\frac{k_d}{bx}\right)^{-\frac{\omega-1}{\omega}} + \alpha\right]} = \frac{S_k}{\omega}\overset{\wedge}{\left(\frac{k_d}{bx}\right)}.$$

Rearranging (A8) and using (A9) and $\hat{b} \equiv \zeta$, we arrive at

(A10)
$$\hat{p} + \frac{S_k}{\omega}\overset{\wedge}{\left(\frac{k_d}{bx}\right)} - \eta\hat{x} = M - \zeta - g_D.$$

Derivation of equations (14), (15) and (16) :

The system of Equations (11), (12) and (13) in matrix form can be written as,

$$\begin{bmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{bmatrix}\begin{bmatrix} \hat{p} \\ \left(\frac{\hat{k}_d}{bx}\right) \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \frac{M}{a} - g_d - \zeta \\ -g_d \\ M - g_d - \zeta \end{bmatrix}.$$

Using Cramer's rule and noting that the determinant

$$|W| = \begin{vmatrix} z & S_k & 1 \\ 1 & -\frac{1}{\omega}(1-S_k) & 0 \\ 1 & \frac{1}{\omega}S_k & -\eta \end{vmatrix} = \frac{1}{\omega}\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k > 0,$$

we arrive at the solutions that are given in Equations (14), (15) and (16).

Proof of Proposition 1:

(i) The growth rate of real consumption is $\widehat{\left(\frac{c}{e}\right)} = \hat{c} - \hat{e}$. Using Equations (A3) and (A6), it follows that

(A11) $$\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right].$$

(ii) Equation (A11) implies that real consumption grows over time as long as $\hat{p} < \frac{M}{s(p)}$. From Equation (14), we can decompose $\hat{p}$ as follows; $\hat{p} \equiv \hat{p}_0 + \hat{p}_b + \hat{p}_g$.

where $\hat{p}_0 = \dfrac{\frac{M}{\omega}(1-S_k)\left[\frac{\eta}{a}+1\right]}{|W|}$, $\hat{p}_b = \dfrac{-\zeta\left[(1-S_k)(\eta+1)\right]}{\omega|W|}$ and

$\hat{p}_g = \dfrac{-g_d\left[(1-S_k)(\eta+1)+\omega S_k\left(\eta+\frac{1}{\omega}\right)\right]}{\omega|W|}$. Then since $\hat{p}_b < 0$ and $\hat{p}_g < 0$, we find that a sufficient condition for $\hat{p} < \frac{M}{s(p)}$ to hold is, (A12) $$\hat{p}_0 = \frac{(1/\omega)M(1-S_k)\left[(\eta/a)+1\right]}{(1/\omega)\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k} < \frac{M}{s(p)}.$$

Rearranging (A12) we have,

(A13) $$(1-S_k)\left(\frac{\eta}{a}+1\right)s(p) < \left[(1-S_k)(1+z\eta)\right]+S_k+\eta S_k\omega.$$

Since $(S_k+\eta S_k\omega)>0$ and $z \equiv \frac{s(p)}{a}+(1-s(p))\sigma$, (A13) is satisfied if the following inequality holds,

(A14) $$\frac{\eta s(p)}{a}+s(p) < 1+\frac{\eta s(p)}{a}+(1-s(p))\sigma\eta,$$

or, equivalently if $0<(1-s(p))(1+\sigma\eta)$, which is always true for $0<s(p)<1$. Thus, we have $\hat{p}<(M/s(p))$ at any finite point of time and for all finite σ and ω. That is, real consumption growth is positive along the equilibrium dynamic path.

(iii) If $\omega>1$, then $\lim_{t\to\infty} S_k = 1$ and $\lim_{t\to\infty}\hat{p} = -g_d$ for any $\sigma>0$. If $\sigma<1$, $\lim_{t\to\infty} s(p)=0$. Suppose that $\omega<1$ and $\sigma>1$. Then we have $\lim_{t\to\infty} S_k = 0$ and the relative price of dirty goods monotonically increases over time under Assumption 2. It then follows that $\lim_{t\to\infty} s(p)=0$. In either case we find that $s(p)\hat{p}$ approaches to zero. Thus, from (A11) it follows that the growth rate of real consumption converges from below to M/a if either $\omega>1$ or $\sigma>1$, but not both. When $\omega>1$, and $\sigma>1$, then $\lim_{t\to\infty}\hat{p}=-g_d$ and $\lim_{t\to\infty} s(p)=1$. It follows that $s(p)\hat{p}$ converges to $-g_d$ and the consumption growth rate converges to $(M+g_d)/a$.

(iv) If $\omega<1$ and $\sigma<1$, then $\lim_{t\to\infty} S_k = 0$ and $\lim_{t\to\infty} s(p)=1$.

This implies that $\lim_{t\to\infty}\hat{p} = \frac{(1+\eta/a)M-(1+\eta)(\zeta+g_d)}{1+z\eta} > 0$. But since $\lim_{t\to\infty} s(p) = 1$, we have that $\lim_{t\to\infty} z = 1/a$. It follows that $\lim_{t\to\infty}\hat{p} = M - \frac{(1+\eta)(\zeta+g_d)}{1+(\eta/a)}$. Thus, using this expression in (A11) and considering the fact that $\lim_{t\to\infty} s(p) = 1$ we have,

$$\lim_{t\to\infty}\widehat{\left(\frac{c}{e}\right)} = \left(\frac{1+\eta}{a+\eta}\right)(\zeta+g_d).$$

Finally, we show that $s(p)\hat{p}$ is increasing over time, meaning that $\widehat{\left(\frac{c}{e}\right)}$ converges towards the limit from above. Substituting the definitions of $|W|$ and z into Equation (14) we can write,

$$s\hat{p} = \frac{(1+\eta)\left[\frac{1+\eta/a}{1+\eta}M-(\zeta+g_d)\right]+\frac{S_k}{1-S_k}(1+\eta\omega)g_d}{1+\frac{s\eta}{a}+\frac{(1-s)}{s}\sigma\eta+\frac{S_k}{1-S_k}(1+\eta\omega)}.$$

Clearly, this expression is increasing in s and decreasing in S_k. If $\sigma < 1$ it follows that s is increasing over time as p increases. Also, since k_d / bx increases over time, the assumption that $\omega < 1$ implies that S_k is falling. Thus, along the equilibrium growth path $s\hat{p}$ is increasing when g_d is sufficiently small. Hence, we have that $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$ must be falling over time. That is, the rate of growth of real consumption converges to a positive rate $\frac{1+\eta}{a+\eta}\zeta$ from above. In other words, if $\sigma < 1$ and $\omega < 1$, then the rate of economic growth is declining over time. To show that $M/a > \frac{1+\eta}{a+\eta}(\zeta + g_d)$ note that this inequality can be written as $M + \eta M/a > (\zeta + g_d) + \eta(\zeta + g_d)$, which is true under Assumption 2. *QED*

Proof of Proposition 2:

Proposition 1 already shows that the growth rate of real consumption always remains positive for any positive ω and σ. Here we show that positive growth is accompanied by a decreasing level of pollution over the long run, that $\lim_{t\to\infty} \hat{x} < 0$ as long as $a > 1$. We first note from Equation (15) that k_d / bx always increases over time which implies that $\lim_{t\to\infty} S_k = 1$ for $\omega > 1$, and $\lim_{t\to\infty} S_k = 0$ for $\omega < 1$. Then from Equation (14) and Assumption 2, we find that $\lim_{t\to\infty} \hat{p} > 0$ for $\omega < 1$, and $\lim_{t\to\infty} \hat{p} < 0$ for $\omega > 1$.

Case 1: $\omega > 1$ and $\sigma > 1$

We have $\lim_{t\to\infty} s = 1$; $\lim_{t\to\infty} z = 1/a$; $\lim_{t\to\infty} S_k = 1$.

Plugging these values into Equation (16),

$\lim_{t\to\infty} \hat{x} = \frac{1}{(1+\omega\eta)}\left\{\left(\frac{M}{a} - \zeta\right) - \omega\left(M - \zeta\right) + g_d\left(\frac{1}{a} - 1\right)\right\}$. Assumption 2 implies that $\lim_{t\to\infty} \hat{x} < 0$ if $a > 1$. This is also valid if technological change is absent, $\zeta = g_d$=0.

Case 2: $\omega > 1$ and $\sigma < 1$

We have $\lim_{t\to\infty} s = 0$; $\lim_{t\to\infty} z = \sigma$; $\lim_{t\to\infty} S_k = 1$ ·

Plugging these values into Equation (16),

$\lim_{t\to\infty}\hat{x}=\frac{1}{(1+\omega\eta)}\left\{\left(\frac{M}{a}-\zeta\right)-\omega\left(M-\zeta\right)+g_d(\sigma-1)\right\}$. Assumption 2 implies that $\lim_{t\to\infty}\hat{x}<0$.

Case 3: $\omega<1$ and $\sigma>1$

We have $\lim_{t\to\infty}s=0$; $\lim_{t\to\infty}z=\sigma$; $\lim_{t\to\infty}S_k=0$.

Plugging these values into Equation (16),

$$\lim_{t\to\infty}\hat{x}=\frac{1}{(1+\sigma\eta)}\left\{\left(\frac{M}{a}-\zeta-g_d\right)-\sigma(M-\zeta-g_d)\right\}.$$

Since $a>1$, we have $\left(\frac{M}{a}-\zeta-g_d\right)/\left(M-\zeta-g_d\right)<1<\sigma$, and $\lim_{t\to\infty}\hat{x}<0$.

Case 4: $\omega<1$ and $\sigma<1$

We have $\lim_{t\to\infty}s=1$; $\lim_{t\to\infty}z=1/a$; $\lim_{t\to\infty}S_k=0$.

Plugging these values into Equation (16), $\lim_{t\to\infty}\hat{x}=\frac{1}{(1+\eta/a)}\left\{\left(g_d+\zeta\right)\left(\frac{1}{a}-1\right)\right\}<0$.

Case 5: $\omega\neq 1$ and $\sigma=1$

We have $0<s=\beta<1$ and $z=\frac{\beta}{a}+(1-\beta)<1$ for $a>1$. We consider two cases.

If $\omega>1$, then $\lim_{t\to\infty}S_k=1$ and $\lim_{t\to\infty}\hat{x}=\frac{1}{1+\omega\eta}\left\{\left(\frac{M}{a}-\zeta\right)-\omega\left(M-\zeta\right)+g_d(z-1)\right\}$.

Since $z<1$, Assumption 2 implies that $\lim_{t\to\infty}\hat{x}<0$. If $\omega<1$, then $\lim_{t\to\infty}S_k=0$ and

$$\lim_{t\to\infty}\hat{x}=\frac{1}{1+\eta z}\left(M\left(\frac{1}{a}-z\right)-\xi(1-z)+g_d(z-1)\right).$$

Since $\frac{1}{a}<z<1$, we have $\lim_{t\to\infty}\hat{x}<0$ for $a>1$.

Case 6: $\omega = 1$ and $\sigma \neq 1$

Since $0 < S_k = \alpha < 1$, we have ;

$$\lim_{t\to\infty}\hat{p} = (\lim_{t\to\infty}\frac{1}{|W|\omega})\left[(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - g_d(1+\eta)\right]$$

.It follows that

$\lim_{t\to\infty}\hat{p} > (<) 0$ if and only if $g_d < (>) \dfrac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right)}{1+\eta}$. We consider four alternative cases.

(i) $\sigma < 1$ and $g_d < \dfrac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right)}{1+\eta}$.

Since $\sigma < 1$, we have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$.

It follows that $\lim_{t\to\infty}\hat{x} = \dfrac{\left(\frac{1}{a}-1\right)\left(M\alpha + g_d + \zeta(1-\alpha)\right)}{\left((1-\alpha)\left(1+\frac{\eta}{a}\right)+\alpha(1+\eta)\right)} < 0$ for $a > 1$ regardless of magnitude of $g_d > 0$.

(ii) $\sigma > 1$ and $g_d < \dfrac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right)}{1+\eta}$. We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty}\hat{x} = \frac{M(\frac{1}{a}-1) - (1-\alpha)(\sigma-1)(M-\zeta) + g_d(\sigma-1)}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha}$$

. The first term of the numerator is negative, while the sum of second and third term becomes negative since

$$-(1-\alpha)(\sigma-1)(M-\zeta) + g_d(\sigma-1) < -(1-\alpha)(\sigma-1)(M-\zeta) + \frac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right)(\sigma-1)}{1+\eta} < 0$$

(iii) $\sigma < 1$ and $g_d > \dfrac{(1-\alpha)\left((\dfrac{M}{a} - \zeta)\eta + (M-\zeta) \right)}{1+\eta}$ .We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma \cdot$

It follows that $\lim_{t\to\infty} \hat{x} = \dfrac{\left(\dfrac{1}{a} - \sigma\right) M + M\alpha(\sigma-1) + g_d(\sigma-1) + \zeta(1-\alpha)(\sigma-1)}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha} < 0$ for $a > 1$.

(iv) $\sigma > 1$ and $g_d > \dfrac{(1-\alpha)\left((\dfrac{M}{a} - \zeta)\eta + (M-\zeta) \right)}{1+\eta}$. We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that $\lim_{t\to\infty} \hat{x} = \dfrac{\left(\dfrac{1}{a} - 1\right)\left(M\alpha + g_d + \zeta(1-\alpha)\right)}{\left((1-\alpha)\left(1 + \dfrac{\eta}{a}\right) + \alpha(1+\eta)\right)} < 0$ for $a > 1$.

Case 7: $\omega = 1$ and $\sigma = 1$

We always have $0 < S_k = \alpha < 1$, $0 < s(p) = \beta < 1$, and $z = \dfrac{\beta}{a} + (1-\beta) < 1$.

Then $\lim_{t\to\infty} \hat{x} < 0$ if and only if $M\left(\dfrac{1}{a} - (1-\alpha)z - \alpha\right) - \zeta\left(1 - (1-\alpha)z - \alpha\right) < 0$.

Rearranging, we have,

$$M\left(\frac{1}{a} - (1-\alpha)z - \alpha\right) - \zeta\left(1 - (1-\alpha)z - \alpha\right)$$

$$= \left[M\left(\frac{1}{a} - z\right) - \zeta\left(1 - z\right) \right] + (M-\zeta)\alpha(z-1).$$

The first term is negative since $\frac{(M/a-\zeta)}{(M-\zeta)} < \frac{1}{a} < \frac{\beta}{a} + (1-\beta) = z$, and the second term is also negative since $z < 1$. *QED*

Proof of Proposition 3:

(i) First we assume $\omega > 1$. For any $\sigma > 0$, Equation (18) applies with $g_d = 0$,

$\lim_{t\to\infty} \hat{x} = \frac{M\left(\frac{1}{a\omega}-1\right) - \zeta\left(\frac{1}{\omega}-1\right)}{\frac{1}{\omega}+\eta} < 0$ if and only if $\omega > d(M,a;\zeta,0) = \frac{\frac{M}{a}-\zeta}{M-\zeta}$. Since the minimum value of $d(M,a;\zeta,0)$ is $\frac{1}{a} > 1$ for $0 < a < 1$, we have $d(M,a;\zeta,0) > 1$.

(ii) Consider now the case where $\omega < 1$. If $\sigma > 1$, Equation (18) applies with $g_d = 0$,

$\lim_{t\to\infty} \hat{x} = \frac{M\left(\frac{1}{a}-\sigma\right) - \zeta(1-\sigma)}{(1+\sigma\eta)} < 0$ if and only if $\sigma > d(\mathrm{M},\mathrm{a};\zeta,0) = \frac{\frac{M}{a}-\zeta}{M-\zeta} > 1$ for $0 < a < 1$.⊗

Proof of Corollary 2:

If we allow capital-augmenting technological change, $\dot{n}/n = \theta > 0$, in addition to pollution-augmenting and neutral technological change in the dirty sector, the equilibrium growth rates of $\hat{p}$, $\widehat{\left(\frac{nk_d}{bx}\right)}$ and $\hat{x}$ become as follow:

$$(A16)\quad \hat{p} = \frac{1}{|W|\omega}\left(\begin{array}{l}\left[M(1-S_k)\left(\frac{\eta}{a}+1\right)\right] - g_d\left[(1-S_k)(\eta+1) + \omega S_k\left(\eta+\frac{1}{\omega}\right)\right] \\ -\zeta\left[(1-S_k)(\eta+1)\right] - \theta S_k(\eta+\frac{1}{\omega})\end{array}\right),$$

(A17) $\left(\frac{\widehat{nk_d}}{bx}\right)=\frac{1}{|W|}\left[M\left(\frac{\eta}{a}+1\right)+g_d\eta(z-1)-\zeta(\eta+1)+\theta(z\eta+1)\right]>0$

(A18)

$$\hat{x}=\frac{1}{|W|\omega}\left\{M\left(\frac{1}{a}-z(1-S_k)-\omega S_k\right)+g_d(z-1)+\theta S_k(z-\omega)+\zeta\left(z(1-S_k)+\omega S_k-1\right)\right\},$$

where $|W|\equiv\frac{1}{\omega}\left[(1-S_k)(1+z\eta)+S_k\right]+\eta S_k>0$.

We prove Corollary 2 for all different cases of parameter combinations.

Case 1: $\omega>1$ and $\sigma>1$

By Equation (A17) for $\omega>1$, we have $\lim_{t\to\infty}S_k=1$. Plugging this into Equation (A16),

we have; $\lim_{t\to\infty}\hat{p}=-\frac{1}{1+\eta\omega}\left(\eta+\frac{1}{\omega}\right)(g_d+\theta)<0$. It follows that for $\sigma>1$, $\lim_{t\to\infty}s=1$,

and $\lim_{t\to\infty}z=1/a$ .Then Equation (A18) implies;

$\lim_{t\to\infty}\hat{x}=\frac{1}{|W|\omega}\left\{\left(\frac{M}{a}-\zeta\right)-\omega\left(M-\zeta\right)+g_d\left(\frac{1}{a}-1\right)+\theta\left(\frac{1}{a}-\omega\right)\right\}$ .Since

$\left(\frac{M}{a}-\zeta\right)-\omega\left(M-\zeta\right)<0$ for $a>1$ and $\omega>1$, it follows that $\lim_{t\to\infty}\hat{x}<0$.

Case 2: $\omega>1$ and $\sigma<1$

By Equation (A17) for $\omega>1$, we have $\lim_{t\to\infty}S_k=1$. Plugging this into Equation (A16),

we have; $\lim_{t\to\infty}\hat{p}=-\frac{1}{1+\eta\omega}\left(\eta+\frac{1}{\omega}\right)(g_d+\theta)<0$. It follows that for $\sigma<1$, $\lim_{t\to\infty}s=0$

and $\lim_{t\to\infty}z=\sigma$ .Then Equation (A18) becomes;

$\lim_{t\to\infty}\hat{x} = \frac{1}{|W|\omega}\left\{\left(\frac{M}{a}-\zeta\right)-\omega(M-\zeta)+g_d(\sigma-1)+\theta(\sigma-\omega)\right\}$. We find that $\lim_{t\to\infty}\hat{x}<0$

if $\left(\frac{M}{a}-\zeta\right)<\omega\left(M-\zeta\right)$, which is always true for $a>1$.

Case 3: $\omega<1$ and $\sigma>1$

By Equation (A17) for $\omega>1$, we have $\lim_{t\to\infty}S_k=0$. Plugging this into Equation

(A16), we have; $\lim_{t\to\infty}\hat{p}=\frac{1}{\left(\lim_{t\to\infty}|W|\omega\right)}\left(M\left(\frac{\eta}{a}+1\right)-(g_d+\zeta)(\eta+1)\right)>0$. Therefore for

$\sigma>1$, we have that $\lim_{t\to\infty}s=0$ and $\lim_{t\to\infty}z=\sigma$ so that

$\lim_{t\to\infty}\hat{p}=\frac{1}{1+\sigma\eta}\left(M\left(\frac{\eta}{a}+1\right)-(g_d+\zeta)(\eta+1)\right)>0$. Then by Equation (A18),

$\lim_{t\to\infty}\hat{x}=\frac{1}{1+\sigma\eta}\left\{M\left(\frac{1}{a}-\sigma\right)+(g_d+\zeta)\left(\sigma-1\right)\right\}<0$ if and only if

$$\sigma>\frac{\frac{M}{a}-\zeta-g_d}{M-\zeta-g_d}=h_3(\zeta,g_d).$$

For $a>1$, this requirement is automatically satisfied since $h_3(\zeta,g_d)<1$.

Case 4: $\omega<1$ and $\sigma<1$

From Equation (A17) for $\omega>1$, we have $\lim_{t\to\infty}S_k=0$. It follows that $\lim_{t\to\infty}\hat{p}>0$. Since

$\sigma<1$, we have that $\lim_{t\to\infty}s=1$ and $\lim_{t\to\infty}z=1/a$, and therefore

$\lim_{t\to\infty}\hat{p} = M - \frac{1+\eta}{1+(\eta/a)}(\zeta + g_d) > 0$ and $\lim_{t\to\infty}(\widehat{c/e}) = \left(\frac{1+\eta}{a+\eta}\right)(\zeta + g_d)$. By Equation (A18), $\lim_{t\to\infty}\hat{x} = \frac{\left(\frac{1}{a}-1\right)(g_d+\zeta)}{\left(1+(\eta/a)\right)} < 0$ for $a > 1$.

Case 5: $\omega = 1,\ \ \sigma \neq 1$

Since $0 < S_k = \alpha < 1$ we have,

$$\lim_{t\to\infty}\hat{p} = (\lim_{t\to\infty}\frac{1}{|W|\omega})\left[(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - g_d(1+\eta) - \theta\alpha\left(\eta + \frac{1}{\omega}\right)\right].$$

It follows that $\lim_{t\to\infty}\hat{p} > (<)0$ if and only if

$$g_d < (>) \frac{(1-\alpha)\left((\frac{M}{a}-\zeta)\eta + (M-\zeta)\right) - \theta\alpha\left(1+\eta\right)}{1+\eta} = \bar{g}\ .$$

We consider four different sub-cases.

5-1) $\sigma < 1$ and $g_d < \bar{g}$: We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that

$$\lim_{t\to\infty}\hat{x} = \frac{\left(\frac{1}{a}-1\right)\left(M\alpha + g_d + \zeta(1-\alpha) + \theta\alpha\right)}{\left((1-\alpha)\left(1+\frac{\eta}{a}\right) + \alpha(1+\eta)\right)} < 0 \quad \text{for} \quad a > 1 \quad \text{regardless of}$$

magnitude of $g_d > 0$.

5-2) $\sigma > 1$ and $g_d < \bar{g}$: We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty}\hat{x} = \frac{\left[M\left(\frac{1}{a}-1\right)\right] - \left[(1-\alpha)(\sigma-1)(M-\zeta)\right] + \left[(\sigma-1)(g_d+\theta\alpha)\right]}{(1-\alpha)(1+\sigma\eta) + (1+\eta)\alpha}.$$

The first term of the numerator is negative, while the sum of second and third term becomes negative since

$$
\begin{aligned}
&-(1-\alpha)(\sigma-1)(M-\zeta)+(\sigma-1)(g_d+\theta\alpha) < \\
&-(1-\alpha)(\sigma-1)(M-\zeta)+ \\
&\left[\frac{(1-\alpha)\left(\left(\frac{M}{a}-\zeta\right)\eta+(M-\zeta)\right)-\theta\alpha\left(\eta+1\right)+\theta\alpha(1+\eta)}{1+\eta}\right](\sigma-1) < 0.
\end{aligned}
$$

Therefore, $\lim_{t\to\infty}\hat{x} < 0$.

5-3) $\sigma < 1$ and $g_d > \bar{g}$: We have $\lim_{t\to\infty} s(p) = 0$ and $\lim_{t\to\infty} z = \sigma$. It follows that

$$\lim_{t\to\infty}\hat{x} = \frac{\left(\frac{1}{a}-\sigma\right)M + M\alpha(\sigma-1) + g_d(\sigma-1) + \zeta(1-\alpha)(\sigma-1) + \theta\alpha(\sigma-1)}{(1-\alpha)(1+\sigma\eta)+(1+\eta)\alpha} < 0 \text{ for}$$

$a > 1$.

5-4) $\sigma > 1$ and $g_d > \bar{g}$: We have $\lim_{t\to\infty} s(p) = 1$ and $\lim_{t\to\infty} z = 1/a$. It follows that

$$\lim_{t\to\infty}\hat{x} = \frac{\left(\frac{1}{a}-1\right)\left(M\alpha + g_d + \zeta(1-\alpha)+\theta\alpha\right)}{\left((1-\alpha)\left(1+\frac{\eta}{a}\right)+\alpha(1+\eta)\right)} < 0 \text{ for } a > 1.$$

Case 6: $\omega = 1$ and $\sigma = 1$

We always have $0 < S_k = \alpha < 1$, $0 < s(p) = \beta < 1$, and

$z = \frac{\beta}{a} + (1-\beta) < 1$.Equation (A18) implies that $\lim_{t\to\infty}\hat{x} < 0$ if and only if

$M\left(\frac{1}{a}-(1-\alpha)z-\alpha\right)-\zeta\left(1-(1-\alpha)z-\alpha\right)+\theta\alpha(z-1) < 0$. Rearranging terms in the left-hand side, we have,

$$
\begin{aligned}
&M\left(\frac{1}{a}-(1-\alpha)z-\alpha\right)-\zeta\left(1-(1-\alpha)z-\alpha\right)+\theta\alpha(z-1)\\
&=\left[M\left(\frac{1}{a}-z\right)-\zeta\left(1-z\right)\right]+(M-\zeta+\theta)\alpha(z-1)
\end{aligned}
.
$$

The first term is negative since $\frac{(M/a-\zeta)}{(M-\zeta)}<\frac{1}{a}<\frac{\beta}{a}+(1-\beta)=z$, and the second term is also negative since $z<1$. *QED*

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