### ABSTRACT

Title of dissertation:	ANTIDUMPING EFFETS IN THE PRESENCE OF COLLUSION IN AN UPSTREAM MARKET: THE CASE OF U.S. FROZEN SHRIMP IMPORT FROM THAILAND
	Ravissa Suchato, Doctor of Philosophy, 2009
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Many studies have shown a relationship between antidumping duty and collusion. These studies, however, only focus on collusion in output (downstream) market, i.e. collusion between import competing firms and exporters, or among import competing firms. This dissertation explores how the antidumping duty on downstream goods can affect collusive behavior in an upstream market of exporters whom are subjected to the duty.

Bertrand duopoly model with infinite periods is developed to examine the effect of the antidumping duty on collusive behavior. Under a set of discount rate, whether is influenced by a tariff or the antidumping duty, the exporters will fully cooperate. The unaffected rate might be due to the linearity in input supply and output demand assumptions. Although the discount rate is not sufficiently high enough to support the full cooperation, the collusive behavior is still feasible through self-enforcing agreement. With future period self-enforcing agreement, under the antidumping duty, the full cooperation in the initial period that is feasible under a set of the discount rate is called "the restricted full cooperation". The set under free trade that supports the full cooperation is smaller than the one supporting the restricted full cooperation. Therefore, the antidumping duty on downstream goods is pro-collusive in the upstream market.

The theoretical result is tested by using Thai shrimp industry data during 1996-2009; the industry has been subjected to the U.S. antidumping duty since 2005. 2SLS is employed to estimate a system of Thai fresh shrimp supply, the U.S. demand for Thai frozen shrimp, and the mark up equations. Using comparative static in supply approach, with an interaction between fresh shrimp price and rainfall as a supply rotator, the empirical results confirm that the antidumping duty increases the degree of collusion among the exporters in Thai shrimp market at 1 % significant level.

### Antidumping Effects in the Presence of Collusion in an Upstream Market The case of U.S. frozen shrimp imports from Thailand

by

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### Chapter 1

### Introduction

#### 1.1 Statement of problem

Thailand has been the world's leading exporter of shrimp for several years and held 25% of the world market share in 2001 (FAO 2001). Shrimp is considered significant to the Thai economy since shrimp makes up over a quarter of its' total food exports. Although there are a lot of shrimp exporters in Thailand that export shrimp all over the world, about 50% of Thai shrimp export belongs to only three agribusiness companies (Aqua culture Asia Facific, 2005). In 2005, the U.S. imposed an antidumping duty on Thai frozen shrimp, the U.S. largest shrimp supply, along with other five shrimp exporting countries<sup>1</sup>. Dumping is legally defined as when imports are sold at less than "normal value" and when these imports cause injury to domestic industries in the importing country. In addition to the duty, the exporters are required to post a continuous bond (C-bond) which is calculated from the exporter's total export value from the previous year and the current antidumping duty. After the imposition of the antidumping duty, a lot of small and meduim exporters (processors) were shut down (shrimpnews, 2006) which lead to a more concentrated Thai shrimp export sector. The four-firm concentration ratio  $(CR_4)$ of Thai shrimp exporters increases from 38.88 percent in 2004 to 42.42 and 65.23

<sup>&</sup>lt;sup>1</sup>Brazil, China, Ecuador, India, and Vietnam

percent in 2005 and 2007 respectively. This raises the question of whether Thai shrimp exporters (processors) collude among each other and have market power at the domestic fresh shrimp procurement level and whether the imposition of the antidumping duty influences the degree of collusion among the Thai shrimp exporters (processors) in Thai shrimp Industry.

Thai shrimp and international trade have been the focus of many studies (Krasachat(1987), Krasachat and Manerat (1991), Samapat (1994), Ling (1996), Malisorn (1996), Iamlaor(1998), Raubrhoo (2002), Potathanapong(2002)); however, the structure of the Thai shrimp industry and the existence of the exporters' (processors') collusion has never been analyzed. With collusion and market power, when any trade policy is imposed, the exporters (processors) can pass through the effect of the policy to their input supply. By ignoring the existence of collusion and market power, the analysis of trade policy effect on Thai shrimp export would be biased.

The link between an antidumping and collusion has been a focus in the antidumping literature (Staiger and wolak (1992), Prusa (1992), Veugelers and Vandenbussche (1999), Hartigan (2000), Zanardi (2004), Davies and Liebman (2006)). This literature has only considered the possibility that an antidumping duty induces international collusion in a downstream market<sup>2</sup>; specifically, the literature considers collusion between domestic firms in a country that imposes the antidumping duty and foreign competitors from a country subjected to the antidumping duty, or the antidumping duty enhances the market power of the domestic firms due to the

 $<sup>^2\</sup>mathrm{A}$  downstream market refers to an output market while an upstream market refers to an input market.

constraint placed upon the foreign competitors. Since the presence of foreign competition exerts market discipline upon the domestic firms, introducing the antidumping duty reduces this disciplinary effect. Under the presence of the antidumping law, the foreign competitors will commit to reduce their export in exchange for a commitment by the domestic firms in the importing country not to file the antidumping duty. With import reduction, as a result, the antidumping duty allows the domestic firms to exploit market power in the downstream market.

This dissertation instead examines the impact of antidumping duties on collusion in upstream markets. To the best of my knowledge, this is the first study to examine such a link. A policy maker should be aware of a result of an antidumping duty on collusion in an upstream market. Ignoring this effect might result to miscalculation in the duty level. Moreover, this is also the first study to investigate the industry structure and the existence of collusion within the Thai shrimp industry.

### 1.2 Objective and hypothesis

The objective of this research is to develop a theoretical model which can explain how the antidumping duty influences collusive behavior in the upstream market. The model hypothesizes that, with perfect information, the antidumping duty supports more collusion among the exporters in the upstream market. Later on, we develop an empirical model to examine collusive behavior in Thai shrimp market and test our hypothesis whether the antidumping duty facilitates collusive behavior in the upstream market.

### 1.3 Organization of the study

Chapter 2 provides Thai shrimp industry background and a literature review on the impact of the antidumping duty on collusion and market power. In chapter 3, the theoretical model is developed. Chapter 4 provides an empirical application of the model to Thai shrimp industry and discusses the empirical results. Summary and conclusion are in Chapter 5.

### Chapter 2

### Thai shrimp industry and literature review

### 2.1 Thai shrimp industry

World shrimp production rapidly increased from 0.41 to 1.7 million metric tons between 1950 and 1980 and from 1.6 million metric tons in 1981 to 6.1 million metric tons in 2006 (FAO, 2007). The share of shrimp production from developing countries such as Thailand, China, Vietnam, Ecuador, Indonesia, and India increased consistently while the shrimp production of the main consumers- - the US, EU, and Japan- - steadily declined. The total consumption of these main consumers is around 60% of world production and 80% of the world export. Thailand is the world's major shrimp exporter. In 2001, 80% of Thai shrimp production was exported to the world market, that being 255,717 tons worth \$2.23 billion dollars (table 2.1 and 2.2). In 2005, as illustrated in table 2.1 and 2.2, and figure 2.1, shrimp exports are comprised of 57% chilled and frozen products and 43% prepared and preserved products. Thailand's primary shrimp markets are USA (receiving 50% of total Thai shrimp exports) and Japan, while its secondary markets include Singapore, Canada, China, Australia, the EU, and Taiwan, in that order (Thai Department of Customs, 2007).

		Lade .	z.1: 1 nai	snrimp ex	1able 2.1: 1 nai snrimp export 1999-2001: ton	101:10U2-			
								Qua	Quantity:ton
	1999	2000	2001	2002	2003	2004	2005	2006	2007
chilled and frozen shrimp	142, 320	42,320 144,614 144,614	144,614	99,223	118,917	99,223 118,917 122,502 157,985 178,237	157,985	178,237	195,318
prepared and preserved	104, 101	111,103	111,103		115, 179	112,504  115,179  118,392  121,361  158,627	121,361	158,627	161,464
total	total 246,421 255,717 255,717 211,727 234,096 240,894 279,346 336,864 356,782	255,717	255,717	211,727	234,096	240,894	279, 346	336,864	356,782
Source: Information and Communication Technology Center with Cooperation of Thai Customs Department	ommunica	tion Techr	nology Cei	nter with (	Cooperatio	on of Thai	Customs	Departme	ent

Table 2.1: Thai shrimp export 1999-2007:ton

							ŗ	Value:\$million	nillion
	1999	2000	2001	2002	2003	2004	2005	2006	2007
chilled and frozen shrimp 1,501 1,236 1,236	1,501	1,236	1,236	802	865	809	939	1,128 $1,234$	1,234
prepared and preserved	1,182	991	991	923	865	863	836	1,146	1,145
total	2,684	2,226	2,226	1,725	1,730	total $2.684$ $2.226$ $2.226$ $1.725$ $1.730$ $1.672$ $1.775$ $2.275$ $2.379$	1.775	2,275	2.379

Source: Information and Communication Technology Center with Cooperation of Thai Customs Department

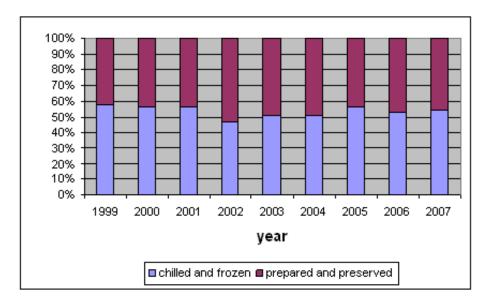


Figure 2.1: Share of Thai shrimp export

### 2.1.1 Detail of Thai shrimp production

Official recording of Thai shrimp production started in early 1970's (Shrimp News International 2007)<sup>1</sup>. Black tiger prawn (*Penaeus Monodon*) was first introduced in 1974 by the Department of Fisheries; however, this shrimp did not become popular until 1983 when a Taiwanese company came to survey a possibility of establishing a shrimp food factory and an aquaculture breeding training center (Manarungsan et al.2005). Early shrimp farming in Thailand was mostly by an extensive system which modifies mangrove forests into large trapping ponds for shrimp. This is because the mangrove forest was a concentrating ground for small aquatic fauna such as juvenile fish which is a great source of shrimp feed, so that shrimp farmers did not have to provide any shrimp feed during the culturing period.

Source: table 2.1

<sup>&</sup>lt;sup>1</sup>http://www.shrimpnews.com/About.html

Shrimp ponds were comparatively large compared to other systems, ranging from 5 to 10 ha (Suchato 2003). However, this extensive system has the smallest yield, and shrimp size is very small.

In 1980, the semi-intensive system was introduced to Thai shrimp farming through the Aquaculture Development Project, a U.S.\$33.1 million soft-loan<sup>2</sup> from the Asian Development Bank (Menasveta 1995). The semi-intensive system ponds could be developed through intensification of existing extensive ponds. There are some modifications needed to be done such as clearing and leveling the pond bottom, digging a canal in the pond bottom, converting from tidal water exchange to pumped water exchange, controlling stocking density, using more fertilizers, using fish toxicants to control fish intruders, and providing shrimp feed etc. Therefore this system has higher yield than the extensive one.

Shrimp farms were occupying some 35,200 ha in 1983 but increased to 40,000 ha in 1985. However, the black tiger prawn was not internationally accepted until 1987 when its demand dramatically increased. The high demand for shrimp and the prospect of high profit have been the main driving forces for farmers to engage in shrimp farming. As a consequence of the growing demand in Japan and other markets, Thai shrimp production increased to 10,544 tons from a land area of 51,200 ha (Manarungsan et al. 2005).

Around 1995, Thai shrimp farming adopted an advanced technology for the intensive system which is very similar to Taiwanese shrimp farming methods. With this new method, the shrimp production rate increased about 5-10 times of that from

<sup>&</sup>lt;sup>2</sup>It is a loan with a below-market interest rate.

)T	$\frac{1}{1}$	1 2010 2.9. Unated without of annul available of internation	(ATCTION
characteristics		level of Intensity	
	Extensive	Semi-intensive	Intensive
Land elevation (m)	$0 \text{ to } +1.4 \text{ MSL}^*$	0 to +1.4 MSL	>+2.0 MSL
Pond size (ha)	>5	1 to 2	1 or less
Aeration	Natural	Water Exchange or mechanical	Natural Water Exchange or mechanical Continual mechanical abd flushing
stocking rate $(PL/m2/crop)$	↓ ℃	5 to 15	20 or more
feed	Natural (No supplement)	Natural + Supplement	Formulated
yield (kg/ha/yr)	100 to 300	600 to 1,800	>6,000
*MSL= Mean Sea Level			

<sup>\*</sup>MSL= Mean Sea Level Source: Menasveta 1995

the semi-intensive system as illustrated in table 2.3. Since then the intensive shrimp farms are also liberated from harvesting wild-caught juveniles as a large number of shrimp fry are produced from hatcheries. It should be noted that the mangrove forests are not required in this new technology. This is because some requirements of the intensive method make the mangrove areas entirely incompatible. For example, shrimp ponds are required to be completely dried to facilitate sanitation and removal of large organic accumulation at the bottom of the ponds after a shrimp crop. Moreover, alkaline water conditions are required for optimal shrimp culture, but the soil in the mangrove areas is high in organic content such as humus, which is acidic in nature and inappropriate for shrimp growth. With this intensive farming technology, however, a lot of shrimp farmers adopting semi-intensive and intensive systems had over used chemicals to prevent diseases in their ponds. Moreover, those with the extensive system also use chemicals as well. Therefore, antibiotics are often found in shrimp residue.

In 2001, the EU, one of the largest shrimp consumers in the world, declared a zero tolerance policy for bacteria residual called *chloramphenicol* that restricted exports of shrimp from Vietnam and China and imposed 100 percent testing on Thai shrimp.

Although the EU was the third largest market for exports of Thai shrimp at that time<sup>3</sup>, the EU's new zero tolerance policy posed only mild threat to Thai shrimp industry, since, in 2002, Thailand widely adopted chemical and antibiotic-

<sup>&</sup>lt;sup>3</sup>In 1996, Thai shrimp exporters were facing a problem of their graduation from the EU's Generalized System of Preferences (GSP) which increased import tariff on Thai shrimp to the EU.

		inp cancer .	9800118 01 1	najoi produceis		
Country	Production	Area	$\mathbf{C}$	ulture system (%	)	Yield
	(Ton)	(Hectare)	Extensive	Semi-intensive	Intensive	(ton/Ha./year)
Thailand	250,000	80,000	5	10	85	3.13
Ecuador	100,000	90,000	50	45	5	1.11
Indonesia	100,000	300,000	80	10	10	0.33
India	70,000	80,000	65	30	5	0.87
Vietnam	50,000	225,000	90	10	0	0.22
Bangladesh	35,000	110,000	60	40	0	0.32
China	35,000	150,000	10	80	10	0.23
Philippines	30,000	50,000	35	50	15	0.6
11	1 0005					

Table 2.4: Shrimp culter systems of major producers in 2002

source: Menasveta 2005

free technology called *Probiotic farming*. Probiotic farming is a bio-technology to grow shrimp by using only necessary and environmentally friendly microorganisms instead of the chemicals or antibiotics. As a result, Thailand claims its shrimp products as the safest and most environmentally friendly in the world compared to its competitors such as China, Vietnam, Indonesia; see table 2.4. Moreover, this new technology has benefited Thai shrimp industry a lot as the US and Japan, later on, have become increasingly concerned about food safety.

Table 2.5: Thai shrimp production by specie

	year	-	uction (to imp specie	/
		black tiger	white	total
2	2000	307,261	0	307,261
2	2001	276,044	0	276,044
2	2002	202,439	60,000	$262,\!439$
20	)03*	$194,\!909$	$132,\!365$	$327,\!274$
20	04*	106,884	$251,\!697$	$358,\!581$
20	)05*	20,055	$374,\!487$	$394,\!542$
20	06*	$3,\!977$	$503,\!207$	$507,\!184$

Source: FAO (2007) and fisheries economics division, Department of Fishery, Thailand (2007) (\*)

In addition, in 2002, the new shrimp species called white shrimp or *Penaeus* vannamei was first introduced in Thailand but not commercially accepted until 2003. Thai shrimp farmers rapidly switched to the white shrimp. As shown in table 2.5, the production of Thai white shrimp was only 60,000 tons while Thailand produced 202,439 tons of black tiger prawns in 2002. The production of white shrimp jumped to 503,207 tons (or about 99.22%) in 2006; in contrast, there were only 3,977 tons (0.78%) of black tiger prawn produced. Thai white shrimp production increased sharply because white shrimp are easy to culture, have a high growth rate, and are more robust to unsuitable environments than are black tiger prawns. Moreover, white shrimp require less protein in their feed, further reduces white shrimp's production cost relative to black tiger prawns.

In the middle of 2006, Charoen Pokphand Foods (CFP) group conducted a study of the costs and benefits of both shrimp species. As it is illustrated in table 2.6 and table 2.7, the total cost of production for the white shrimp of a medium size (60 counts per kilogram) in Thailand is about 93 bahts while it costs about 135 bahts to produce the black tiger prawn with the same size. In addition, the white shrimp have much higher yield and survival rates which are 1,500 kilogram per rai<sup>4</sup> and 75% respectively while the black tiger prawn yields only 520 kilogram per rai and has 65% survival rate. The average selling prices at farm gate in mid 2006 were about 115 bahts per kilogram for white shrimp and 145 bahts per kilogram for black tiger prawn. In short, profit per kilogram of white shrimp is 22 bahts per kilograms

 $<sup>^{4}6.25</sup>$  rai= 1 ha

				Table 2.6: Vannamei Production Cost	.6: Van	namei H	Product	tion Co	st		-		-		
	Description	Unit		Thai	land			China		$Viet_1$	nam	Mala	ysia	Indor	lesia
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Harvested Size	pcs/Kg	50	60	02	80	60	20	80	80	90	60	20	09	70
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Feed Cost	Bht/Kg	49	46	45	44	48	47	45	54	52	51	49	56	51
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Seed Cost	Bht/Kg	6	10	11	11	11	12	12	12	12	12	14	11	12
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Aqua-Product Cost	Bht/Kg	IJ	IJ	5	ъ	က	က	2	2	9	5	5 L	4	4
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Energy Cost	Bht/Kg	25	22	20	20	19	18	18	6	$\infty$	18	18	27	27
$ \begin{array}{lcccccccccccccccccccccccccccccccccccc$	Operating Cost	Bht/Kg	10	10	10	10	6	10	9	11	10	12	12	9	IJ
$ \begin{array}{l c c c c c c c c c c c c c c c c c c c$	Total Cost	Bht/Kg	98	93	90	90	90	89	84	93	89	66	66	104	66
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Remark														
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	DOC	Days	130	120	110	100	110	105	95	90	80	115	100	120	105
Kg/Rai1,6801,5001,3711,2758801,0299751,2001,1381,4931,2802,333 $\%$ 70%75%80%65%75%80%70%70%70%70% $1.70$ 1.601.551.501.511.501.551.501.451.601.551.65 $8m/pcs/day$ 0.150.140.130.130.150.140.130.130.151.400.140.14 $Bht/Kg$ 29292930303636323234 $Bht/Vis$ 2.942.942.943.0303636323234 $Bht/Vis$ 2.942.942.943.0303636323234 $Bht/Vis$ 2.942.942.943.03.03.02.42.433 $Bht/Vis$ 2.942.942.943.03.02.42.4333 $Bht/Vis$ 135115110105139118105122114127112173 $Bht/Kg$ 372220155%25%20%24%22%13%69 $\%$ 28%19%18%14%35%25%20%24%22%12%173 $\%$ 28%19%18%14%35%25%20%24%22%12%173 $\%$ 2	Density	m pcs/m2	75	75	75	75	60	75	75	80	80	80	80	125	125
	Yield	Kg/Rai	1,680	1,500	1,371	1,275	880	1,029	975	1,200	1,138	1,493	1,280	2,333	2,000
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Survival rate	%	20%	75%	80%	85%	55%	%09	65%	75%	80%	20%	20%	20%	70%
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	FCR		1.70	1.60	1.55	1.50	1.60	1.55	1.50	1.50	1.45	1.60	1.55	1.65	1.50
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	ADG	$\rm gm/pcs/day$	0.15	0.14	0.13	0.13	0.15	0.14	0.13	0.14	0.14	0.14	0.14	0.14	0.14
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Feed selling price	Bht/Kg	29	29	29	29	30	30	30	36	36	32	32	34	34
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Seed selling price	Bht/pcs.	0.12	0.12	0.12	0.12	0.10	0.10	0.10	0.11	0.11	0.14	0.14	0.12	0.12
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Electricity	Bht/Unit	2.94	2.94	2.94	2.94	3.0	3.0	3.0	2.4	2.4	က	က		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Fuel	$\mathrm{Bht}/\mathrm{Litre}$	28	28	28	28	24	24	24	19	19	22	22	27	27
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Selling Price	Bht/Kg	135	115	110	105	139	118	105	122	114	127	112	173	156
% 28% 19% 18% 14% 35% 25% 20% 24% 22% 22% 12% 40% $%$ 38% 24% 22% 17% 55% 33% 26% 32% 29% 29% 13% 67%	Profit	Bht/Kg	37	22	20	15	49	29	21	29	25	28	13	69	57
$\% \ 38\% \ 24\% \ 22\% \ 17\% \ 55\% \ 33\% \ 26\% \ 32\% \ 29\% \ 29\% \ 13\% \ 67\%$	ROS	%	28%	19%	18%	14%	35%	25%	20%	24%	22%	22%	12%	40%	36%
	ROI	%	38%	24%	22%	17%	55%	33%	26%	32%	29%	29%	13%	87%	57%

14

		Tal	Table 2.7: Monodon Production Cost	Monc	don P	roduct	ion Co	st		
Description	Unit	Thailand	Vietnam	nam	Ind	India	Mala	Malaysia	Philippines	pines
Harvested Size	pcs/Kg	60	35	40	30	35	40	50	30	40
Feed Cost	Bht/Kg	54	20	75	62	79	59	52	66	62
Seed Cost	Bht/Kg	11	9	2	6	10	11	14	6	13
Aqua-Product Cost	Bht/Kg	30	17	20	10	10	15	15	20	15
Energy Cost	Bht/Kg	30	15	19	35	35	32	32	45	40
Operating Cost	Bht/Kg	10	18	20	35	35	32	32	35	35
Total Cost	Bht/Kg	135	126	140	168	169	149	144	176	165
Remark										
DOC	Days	120	130	135	130	120	120	100	130	110
Density	m pcs/m2	30	25	35	15	20	30	30	20	20
Yield	Kg/Rai	520	709	840	560	640	720	576	853	640
Survival rate	%	65%	62%	%09	20%	70%	60%	60%	80%	80%
FCR		1.70	1.60	1.70	1.65	1.65	1.70	1.50	1.7	1.6
ADG	${ m gm/pcs/day}$	0.14	0.22	0.19	0.26	0.24	0.21	0.20	0.26	0.23
Feed selling price		32	44	44	48	48	35	35	39	39
Seed selling price	Bht/pcs.	0.12	0.10	0.10	0.20	0.20	0.16	0.16	0.25	0.25
Electricity		2.94	2.40	2.40	4.00	4.00	3.09	3.09	4.20	4.20
Fuel	Bht/Litre	28	19	19	32	32	22	22	29	29
Selling Price	$\mathrm{Bht}/\mathrm{Kg}$	145	218	193	230	190	235	214	235	190
Profit	$\mathrm{Bht}/\mathrm{Kg}$	10	92	53	62	21	86	70	59	25
ROS	%	%2	42%	27%	27%	11%	37%	32%	25%	13%
ROI	%	7%	73%	37%	37%	12%	58%	48%	34%	15%
Exchange rate : 1 RM=10.2 Bht,1RS=0.8 B, 1 source CPF, July 2006	A = 10.2  Bht, 1R	S=0.8 B, 1	$Bht = Rp \ 244$	$\operatorname{Rp}24$	4					

• þ . 1

while the black tiger prawn profit is 10 bahts per kilogram.

### 2.1.2 Overview of Thai shrimp industry

There are about 35,000 shrimp farms spread across approximately 80,000 hectares in Thailand. As illustrated in figure 2.2, shrimp farms and hatcheries are scattered along the coastal areas of Thailand. Southern provinces such as Nakorn Sri Thammarat, Surat Thani account for the majority (55 percent of shrimp farms) while those in the East such as Chanthaburi and central regions such as Samut Sakhon, Samut Songkram comprise the minority in terms of number (35 and 10 percent of shrimp farms repectively). More than 80% of shrimp farms are small or about 1-4 ponds per farm (less than 1.5 hectares per farm), 18% have between 1.5 to 2.5 hectares per farm, and only 2% are very few large farms with more than 40 ponds  $(larger that 10 hectares per farm)^5$ . There are about 300 fish agents while there are only 124 processors in Thailand. As illustrated in table 2.8, there are only 13 large processors in the shrimp processed industry. Moreover, most of these large processors belong to only three agribusiness companies –Charoen Pokphand Food group (CPF), Thai Union Frozen Food group(TUF), and Rubicon group-which account for 50% of Thai processed shrimp production (aquaculture Asia Pacific 2005).<sup>6</sup>

Thai shrimp market channels are as follows:

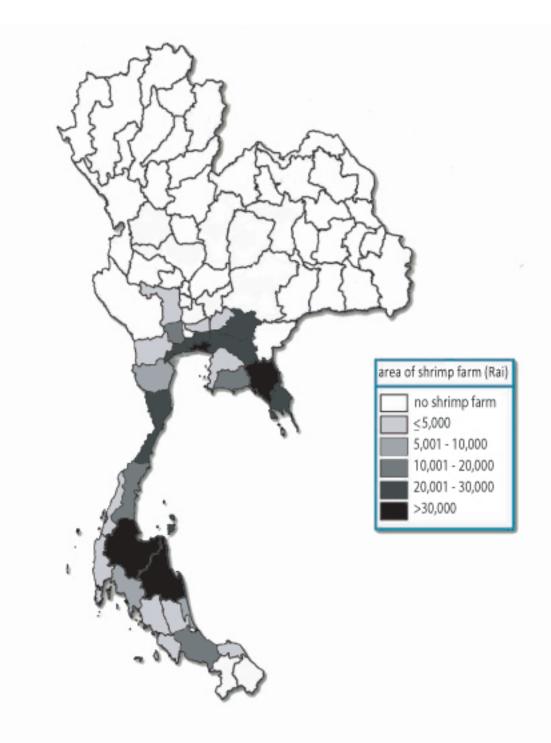
1. Fresh shrimp for the domestic market. Domestic shrimp consumption

in Thailand comprises only 10 - 15 % of Thai shrimp production (The office of

<sup>&</sup>lt;sup>5</sup>National Food Institute (Thailand), Department of Fisheries statistics, 2007

<sup>&</sup>lt;sup>6</sup>Most Thai processors not only sell their products to exporters but also export their products to the world market. There are 198 Thai shrimp exporters including the processors

Figure 2.2: Shrimp farm area in Thailand



Source: office of Agricultural Economics, Ministry of Agriculture and Cooperatives, Thailand  $\left(2007\right)$ 

type	$\operatorname{small}$	medium	large
Canned shrimp	0	4	1
Frozen shrimp	56	46	11
other	23	25	4

Table 2.8: Number of shrimp processors classified by plant size

Source: The office of Industry Economics 2004.

Note1: plant size is categorized by capital investment. Large size represents a plant with more than 200 million baht. Medium size is a plant with 50-200 million baht, and small size is a plant with less than 50 million baht capital investment

Note2: some processors are double counted due to their product varieties.

Industry Economics, Thailand, 2007)<sup>7</sup>. Most of the shrimp consumed domestically are small shrimps which are not desired in the world market. In the domestic market, shrimp is sold from shrimp farmers through fish agents in shrimp wholesale markets to wholesalers and retailers.

2. Processed shrimp for international market. Thai shrimp sold inter-

nationally account as 85 - 90% of the total production. Once shrimp is grown to a desirable size, a shrimp farmer will contract fish agents in a shrimp wholesale market<sup>8</sup> or shrimp processors to offer a price for shrimp. Processed shrimp for export can be distributed from shrimp farms to processors via

-wholesale, fish agents

-direct sale. This channel usually involves farms located near a processor. However, the processors normally purchase fresh shrimp through fish agents because fish agents have higher volume and size variety.

**Price setting in Thai shrimp market.** Each processor announces his own daily shrimp price by species and size. Once fish agents purchase fresh shrimp

<sup>&</sup>lt;sup>7</sup>www.oie.go.th

<sup>&</sup>lt;sup>8</sup>The main shrimp market in Thailand is Mahachi Auction Market in Samutsongkram province

from shrimp farmers, they will sell shrimp to processors with the best offer (price). However, it is said that the three agribusiness companies are price-setting leaders in the market as their purchase volume account for about 50% in the shrimp market.

species	size $(count/kg)$		ye	ear	
		2003	2004	2005	2006
Black Tiger	30	295	272	248	221
	40	240	231	213	181
	50	197	186	178	153
	60	155	158	161	137
	70	133	132	145	129
	80	120	119	129	122
	90	113	114	124	116
	100	107	113	120	112
White	40	215	201	180	180
	50	172	149	151	144
	60	146	125	132	123
	70	127	111	119	113
	80	116	103	108	103
	90	108	96	100	96
	100	101	89	92	90
	source:	CPF,	2007		

Table 2.9: Average shrimp prices at Mahachai Auction Market by species and sizes (unite: baht)

### 2.1.3 Price history

Although Thailand has been increasingly exporting frozen shrimp to the U.S.<sup>9</sup>, Thai shrimp farmers have been facing a decrease in domestic shrimp prices as shown in table 2.9. There is a concern that the processors and/or the exporters are behind the price decrease<sup>10</sup>. The processors and exporters might have been colluding and

<sup>&</sup>lt;sup>9</sup>The export still increases even though it was imposed the antidumping duty.

<sup>&</sup>lt;sup>10</sup>Department of fishery Thailand (2007), National Institute of Coastal Aquaculture (nicaonline.com, 2007), Siammarine (www.siammarine.com, 2007), Journal of Marine Industry (2006)

pushing the domestic fresh shrimp price down in order to maintain their competitiveness (price of shrimp products) in the U.S shrimp market especially after the imposition of the Antidumping duty.

After the imposition of the antidumping duty, Thai frozen shrimp prices in the U.S. seem to have dropped by the most compared to its competitors and even below those that were not imposed a duty. A price for medium Thai frozen shrimp (56-66 counts per kilogram) price in the U.S. market was 12.6 dollars in June 2003, but it dropped to 6.28 dollars or about 50.16% dropped in June 2005<sup>11</sup>. Likewise, the U.S. price of large Thai frozen shrimp (less than 33 counts per kilogram) decreased from 13.12 to 6.49 dollars or about 50.53% in June 2003 and 2005 respectively. In contrast, the average price of medium frozen shrimp in the U.S market was declined 20.56% during this period, unlike, the average U.S. price for large frozen shrimp large shrimp such as Ecuador (with lower antidumping duty), Vietnam, and Mexico(without the duty) were higher in 2005 than in 2003 (USITC, 2007). On the other hand, the processors argue that the decrease in price is caused by world's demand and supply rather than their actions (colluding) (Shrimp News,2006).

Unfortunately, there is no solid evidence or study demonstrating the existence of Thai processors' collusion in setting shrimp domestic price<sup>12</sup> and any shock such as the U.S antidumping duty can affect the degree of collusion.

<sup>&</sup>lt;sup>11</sup>The imposition of the antidumping duty started in February 2005

 $<sup>^{12}\</sup>mathrm{Jintatham}$  (1995) descriptively concludes in his study that Thai domestic shrimp market is perfect competitive

### 2.1.4 Accusations of collusion

In 2004, Thai shrimp industry was subjected to antidumping petition from the U.S., along with other five shrimp exporting countries<sup>13</sup>. The U.S. International Trade Commission (USITC) affirmatively determined that the U.S. shrimp industry has been materially injured by these countries subjected to the duty. Also, The US Department of Commerce announced its preliminary determinations in the dumping case against these six targeted exporting countries, finding that producers/exporters sold frozen shrimp in the US market at less than *normal value*<sup>14</sup> with different dumping margin ranges for each country. As it is shown in table 4.3, China had the highest national preliminary average margin–112.81% following by Vietnam, Brazil, India, Thailand and Ecuador respectively.

Finally, on February 2005, the Department of Commerce issued antidumping duties for all six countries, and amended the nation's average final dumping margins, which would be the nation's average final antidumping duty being imposed on frozen shrimp imports from these six countries. The antidumping duties were imposed after it was determined that warm-water shrimp imported was sold at a less normal value, dumped, causing material injury to domestic producers (Sharp and Zantow, 2005). The final dumping margins were slightly different from the preliminary ones except for Vietnam. The final dumping margin for Vietnam was only 25.76% while the preliminary one was 93.1%. Moreover, the US Customs and

<sup>&</sup>lt;sup>13</sup>Brazil, China, Ecuador, India, and Vietnam

<sup>&</sup>lt;sup>14</sup>Normal value could be either a lower price than in its own local market, in the third market, or if there is no foreign price to observe whether they are selling at less than the average cost of production(Prusa (1992), Staiger and Wolak (1992) Brander and Krugman (1983), Mathew (2004))

	Table 2.10: Preliminary and final dumping margins	gins	
country	Specific firm	Preliminary margin $(\%)$	Final margin( $\%$ )
Brazil		0.00	7.94
	Central de Industrializacao e Distribuicao de Alimentos Ltda. (CIDA)	8.41	4.97
	Norte Pesca S.A.	67.81	67.80
	national average	36.91	7.05
China	Allied Pacific Group	90.05	80.19
	Zhanjiang Guolian Aquatic Products Co., Ltd.	0.04	0.07
	Shantou Red Garden Foodstuff Co., Ltd.	7.67	27.89
	Yelin Enterprise Co., Hong Kong	98.34	82.27
	Separate Rate margin	49.09	53.68
	national average	112.81	112.81
Ecuador	Exporklore S.A.	9.35	2.48
	Exportadora De Alimenos S.A. (Expalsa)	6.08	1.97
	Promarisco S.A.	6.77	4.42
	national average	7.30	3.58
India	Hindustan Lever Limited (HLL)	27.49	15.36
	Devi Sea Foods Ltd. (Devi)	3.56	4.94
	Nekkanti Seafoods Limited (Nekkanti)	9.16	9.71
	national average	14.20	10.17
Thailand	Rubicon Group	5.56	5.91
	Thai I-Mei Frozen Foods Co., Ltd.	5.91	5.29
	The Union Frozen Products Co., Ltd. (UFC)	10.29	6.82
	national average	6.39	5.95
Vietnam	Minh Phu Seafood Corporation	14.89	4.38
	Kim Ahn Co., Ltd.	12.11	25.76
	Minh Hai Joint Stock Seafoods Processing Co.	19.60	4.30
	Camau Frozen Seafood Processing Import Export Corporation	19.68	5.24
	Separate Rate margin	16.01	4.57
	national average	93.13	25.76
source : USITC, 2007	ITC, 2007		

Table 2.10: Preliminary and final dumping margins

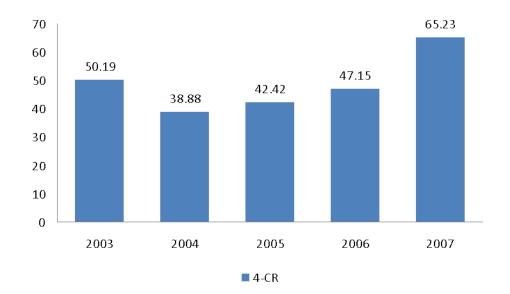


Figure 2.3: The four-firm concentration ratio  $(CR_4)$  of Thai shrimp exporters

Source: Thai Customs Department (2008)

Border Protection, or CBP, implemented the continuous bond measures (C-bond) requiring that banks guarantee deposits from the exporters to guarantee antidumping margin payments. The bank guarantees are to be calculated on the exporters' total export value for the previous year. In addition, the guarantee deposits have to be issued by international banks accepted by the US. Therefore, it would be harder for these six shrimp exporting countries to compete in the US shrimp market and may cause the market shares of these countries to decrease as suggested in Debaere (2005) who finds that a regular tariff sufficiently reduced Thai shrimp export to EU market by 7%.

As a result of the U.S. antidumping duty on frozen shrimp import, as shown in figure 2.3, the four-firm concentration ratio  $(CR_4)^{15}$ , which is used to assist in

 $<sup>^{15}\</sup>mathrm{It}$  is calculated by sum of the percent market share of the four largest firms in an indutry

determining the market structure of the industry, of Thai shrimp exporters increase from 38.88 % in 2003 to 42.42 % and 65.23 % in 2005 and 2007 respectively.

### 2.2 Literature review on antidumping effect

### 2.2.1 Antidumping effect on collusion

In recent decades, antidumping has become the most widespread obstruction to trade<sup>16</sup>; moreover, economists have discovered that this trade protection influences competitive behavior. While the antidumping duty gives short-run advantage to a domestic industry by raising import cost, it sometimes also benefits foreign rivals as well. This is because the threat of the antidumping duty decreases competition and causes output and/or price to shift toward monopoly levels. Some studies (Staiger and wolak (1992), Prusa (1992), Veugelers and Vandenbussche (1999), Hartigan (2000), Zanardi (2004), Davies and Liebman (2006)) recogniz the link between the antidumping and international collusion. All of these studies focus on collusion in the downstream market between a domestic industry in an importing country and exporters. These studies show that an antidumping duty increases the likelihood of international collusion between firms in a country that imposes the antidumping duty and exporting firms from a country that is subjected to the duty. The exporting firms will commit to increase their export price or reduce their exports in exchange for a commitment by the firms in the importing country not to file the antidumping

 $<sup>^{16}</sup>$ For example, July 1980 - July 1981 the World Trade Organization reports that a total 173 of Antidumping petitions were filed worldwide, and the number of the antidumping petition increased to 312 for the same period in 2000-2001 (reported in Zanardi (2004))

charge.

Staiger and Wolak (1992) present a model in which a competitive domestic industry responds to the import from the foreign exporter by filing the antidumping petition. The authors provides conditions under which the foreign exporter agrees to decrease its export in exchange for the domestic industry not to file the antidumping petition.

Prusa (1992) finds evidence of collusive behavior from the fact that withdrawn antidumping case leads to an almost equal reduction of imports in those cases in which the duties are implemented. The act of the antidumping initiation causes the foreign exporters to eliminate the dumping margin or face the possibility of a large antidumping duty. Initiating the antidumping petition acts as a credible threat to foreign exporters that are dumping. Some exporters eliminate the dumping margin before the outcome of the petition. The domestic firms who filed the petition withdraw the case since the dumping margin has been eliminated. The market power of the domestic firms can thus be affected by the petition decision. The author develops a model of oligopsonistic price competition in which the foreign exporter and the domestic firms can always find a collusive behavior that increases the profit of all firms involved. The author confirms his theoretical finding by examining the value of trade both before and after petitions filed as the U.S. antidumping cases in 1980-1981. He looks at ratios of the value of trade before and after the petition to detect a change in response to the antidumping duty investigation. He concludes that the antidumping petition serves as a method to achieve a cooperative level of profit for domestic and foreign firms.

Adopting the Prusa (1992) model to include costs of coordination, Zanardi (2004) finds that antidumping petitions may be used to threaten and induce foreign exporters into a collusive arrangement depending on the cost of coordination and the relative bargaining power between the domestic and foreign exporting firms. The authors also finds that the antidumping law is used to gain collusion in exchange for withdrawing the petitions. A withdrawn petition can be a credible threat in that the domestic industry can punish any foreign exporters misbehavior by reinstating the petition.

Veugelers and Vandenbussche (1999) investigate the effect of the antidumping law on collusion between domestic firms as well as between the foreign exporters and domestic firms. The authors find that, depending on the degree of product heterogeneity and cost asymmetry between the foreign exporting firms and domestic firms, introducing an antidumping law can result in all firms forming a full cartel, cooperation only between domestic firms, or pure competition.

Davies and Liebman (2006) present a quantity competition model in which the foreign firm is a multinational with a subsidiary in the domestic country. The authors show that even when the antidumping duties are imposed, the antidumping legislation can still be used to support mutually-beneficial collusive outcomes. Thereby, the multinational firm may choose to submit to a tariff even under collusive behavior since removing duties can tighten the incentive-competibility constraint of the multinational firm, eliminating the sustainability of some collusive equilibria.

In contrast to other studies suggesting that the antidumping law can facilitate collusive behavior, Hartigan (2000) finds that the antidumping law can be procom-

petitive. The domestic and foreign exporting firms collude in the absence of the antidumping law because the law makes renegotiation of collusion more costly than imposition of punishment for defection. Sufficiently costly renegotiation ensures that punishments for defection are credible, so that they serve as a deterrent to deviation from collusive behavior. With a weak injury standard, the antidumping law permits the renegotiation of collusion at a low cost. Thus, it can undermine collusion.

Outside of the work on antidumping laws, Lommerud and Sorgard (2001) study the impact of trade barriers on collusive behavior. The authors present a model in which two price-competing firms collude by not exporting to one another's market. They find that a reduction in trade barriers<sup>17</sup> increases the gain from deviation as well as the punishment of reverting to the Nash equilibrium; they find the set of discount rates that sustain collusive behavior declines when the gain from deviation dominates.

## 2.2.2 Antidumping effect on market power

Less attention has been given to the fact that the antidumping duty can act as a coordinating device that facilitates an indutry in a country to exploit market power when it is found guilty in a dumping case.

Nieberding (1999) constructs an empirical test of market power, based on the Lemer index, to investigate the effects of U.S. antidumping law on domestic firms'

 $<sup>^{17}{\</sup>rm The}$  authors do not specific a type of trade barrier rather they use a per-unit trade cost between countries.

market power. Using industry level data, the results show that U.S. firms' market power increased after petition acceptance and decreased after petition rejection.

In an extension to Nieberding (1999), Haan (2000) uses firm-level data to examine the impact of the antidumping law on the domestic firms' market power in the Canadian steel industry. The author finds that market power significantly increases for all firms after an affirmative antidumping injury decision. Moreover, market power is higher for firms that initiated the antidumping petitions.

Konning and Vandenbussche (2005) use data of 4000 EU firms that were involved in the antidumping case during 1992-2000 to estimate domestic firms' markups before and after the filing of a case. The authors define the domestic firms' markup as price over marginal cost. They find that the antidumping duty has positive effects on the domestic firms' markups, except in cases where import diversion after protection is strong.

# Chapter 3

## The model of oligopsonistic collusion in upstream market

In this chapter, we develop the theoretical model which shows how an antidumping duty can result in a change in the collusion equilibrium in the processors' (exporters') upstream market. In this model, processors (exporters) are assumed to be oligopsonists in their upstream market<sup>1</sup> but not oligopolists in their downstream market. We identify the minimum discount rate under which the processors fully cooperate and act as a monopsonist in the upstream market. We also explore joint profit maximizing behavior if the firms when the discount rate is not sufficiently high to support the monopsony outcome; we call this the "the constrained cooperation" outcome

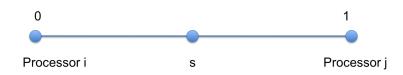
This chapter is constructed as follows. In section 3.1, we give a formal description of the model. Collusion equilibria in the case of free trade and an exogenous tariff are provided in section 3.2. Section 3.3 analyses collusion equilibria under an antidumping duty.

# 3.1 The model

Consider the following Bertrand duopoly model with infinite periods. There are two identical risk neutral processors (exporters) that use a homogenous input -

<sup>&</sup>lt;sup>1</sup>For the purpose of this study, the upstream market is the Thai market for fresh unprocessed shrimp, while the downstream market is the U.S. market for processed frozen shrimp.





fresh shrimp- to produce a homogenous product -frozen shrimp- which they export to an international market. Antitrust law prevents the processors from forming enforceable price agreements, so any collusion must be done tacitly through selfenforcing agreements. For simplicity, we assume "*perfect monitoring*" (Friedman (1970), Kosenok (2005))<sup>2</sup>, i.e., each processor can observe the other processor's action; this is referred to as

For simplicity, we assume each processor is located at the two endpoints of the unit interval [0, 1], as per figure 3.1. Processors offer input prices (for fresh shrimp)  $w_i$  and  $w_j$  to fish agents/shrimp farmers who are uniformly distributed on the [0, 1] interval. Each fish agent/shrimp farmer produces one unit of fresh shrimp. The fish agents/shrimp farmers have constant marginal costs of producing fresh shrimp, c, and face a transportation cost v for reaching a processor; thus shrimp farmers/fish agents will choose a processor as long as the combined offered price of fresh shrimp, marginal cost and transportation cost is greater than the competitive processor. If both processors offer the same price, the fish farmers and fish agents will sell fresh shrimp to the processor closest to them. A fish agent/shrimp farmer who is located

<sup>&</sup>lt;sup>2</sup>Other classes of monitoring are "public monitoring" where the processors know only their own action and a publicly observed signal related to the joint actions of all processors, and "private monitoring" where each processor can observe its own private signal of the actions of the other processors (Green and Porter (1984), Hanazono (2003), Kosenok (2005), and Zheng (2006). However, as for the main focus of this paper, the perfect monitoring case yields more tractable result than others

at  $s \in [0, 1]$  incurs a transportation cost vs if he sell his shrimp to processor i located at point 0, and a transportation cost equal v(1-s) if he sells his shrimp to processor j located at point 1. This fish agent/shrimp farmer is indifferent between selling shrimp to processor i and j if

$$w_i - c - vs = w_j - c - v(1 - s).$$
(3.1)

Thus, fresh shrimp supply to the processor i is <sup>3</sup>

$$s_i = \frac{v + w_i - w_j}{2v}.\tag{3.2}$$

To make the result more compact, we assume that the transportation cost  $v = \frac{1}{2}$  and the marginal cost of producing fresh shrimp c = 0. Hence, equation 3.2 becomes

$$s_i = \frac{1}{2} + w_i - w_j. ag{3.3}$$

Moreover, it is assumed that both processor i and j also do contract farming with local fish agents/shrimp farmers at the endpoints as their additional fresh shrimp supply. For tractability, with zero transportation cost, we assume that the processors' contract fresh shrimp supplies are  $L_i = w_i$  and  $L_j = w_j$  for processor iand j respectively. Thus, the total fresh shrimp supply for processor i is

$$x_i = s_i + L_i = \frac{1}{2} + 2w_i - w_j.$$
(3.4)

<sup>&</sup>lt;sup>3</sup>The fresh shrimp supply for processor j analogs to the processor i's; the subscript i is replaced by j, and subscript j is replaced by i

In addition, we assume that the frozen shrimp production technology is characterized by a fixed proportion technology of output (frozen shrimp) and a single material input (fresh shrimp), so that 1 kilogram of fresh shrimp is transformed to k kilograms of frozen shrimp. Thus, processor i's cost of producing frozen shrimp is

$$C_{i} = \frac{[w_{i} + d]x_{i}(w_{i}, w_{j})}{k}$$
(3.5)

where d is an exogenous cost of nonmaterial inputs used to produce frozen shrimp. As for processor j's production cost, the subscript i will be replaced by j. The output (frozen shrimp) quantity is

$$y_i = kx_i(w_i, w_j). \tag{3.6}$$

Downstream demand for frozen shrimp is derived from a representative consumer's utility defined for each period t. It is denoted as  $U(y_i, y_j, a_2, z)$  with  $a_2$ is quantity of processors/exporters from other countries and z is a proxy for all other commodities, chosen as the numeraire. We assume the utility function to be strictly quasi-concave and have positive marginal utilities for each product. More specifically, similar to Pauwels et al. (2001), we assume the utility function to be

$$U(y_i, y_j, z) = a_1[y_i + y_j + a_2] - \frac{1}{2}[y_i + y_j + a_2]^2 + z$$
(3.7)

where  $a_1$  is positive. This utility function is quasi-linear and implies that there are no income effects in the demand for  $y_i$  and  $y_j$ . From equation 3.7, the inverse demand functions for processor i's sales is given by

$$p_i = a - y_i - y_j. (3.8)$$

Where  $a = a_1 + a_2$ . Despite the clear endogeneity of the downstream price, we assume processor *i* and *j* only collude on the upstream price; they do not collude in the downstream market. This assumption is consistent with the structure of the global shrimp market: although there are only a small number of firms purchasing fresh Thai shrimp for export, there are a large number of firms supplying the global market for frozen shrimp. Moreover, because the supply of frozen shrimp from countries other than Thailand is noisy and imperfectly observable (at least by Thai processors), we conjecture it would be unproductive for Thai processors to condition their collusive behavior in the upstream market on downstream prices. Accordingly, we assume without further apology that each processor does not take into account his competitor's downstream price when colluding in the upstream market.

Moreover, allowing for the possibility that both processors are subjected to a tariff  $\tau$  in the downstream market, processor *i*'s profit in period *t* is

$$\pi_i(w_i, w_j, \tau) = [k (p_i - \tau) - w_i - d] x_i(w_i, w_j).$$
(3.9)

We are interested in the processor' behavior when engaged in an infinitely repeated game. Accordingly, we add time subscripts t to all price and market variables to indicate relevant periods; for example, processor *i*'s input price in period t is written as  $w_{it}$ . Let  $\beta < 1$  denote the universal intertemporal discount factor, and let

$$\Pi_i(\{w_{it}\}_{t=1}^\infty, \{w_{jt}\}_{t=1}^\infty) = (1-\beta) \sum_{t=1}^{+\infty} \beta^{t-1} \pi_i(w_{it}, w_{jt})$$
(3.10)

measure processor i's (normalized) discounted stream of profits.<sup>4</sup>

Next, we will analyze the equilibrium of the game under the free trade, an exogenous tariff, an antidumping duty scenarios.

## 3.2 Collusion under free trade and an exogenous tariff

We start with the free trade and an exogenous tariff cases as our baseline model. We examine symmetric subgame perfect Nash equilibria, in which the processors pursue a grim trigger strategy: offer a low fresh shrimp price provided no one has deviated before; offer the Bertrand input price otherwise.

## 3.2.1 The Bertrand competitive outcome

Consider now the static Nash equilibrium. In this case, each processor unilaterally makes its decisions and plays Bertrand competition. Processor i's profit maximizing problem in the stage game to chose  $w_i$  so as to maximize is

$$\pi_i(w_i, w_j, \tau) \equiv [k (p_i - \tau) - w_i - d] x_i(w_i, w_j)$$
(3.11)

<sup>&</sup>lt;sup>4</sup>Multiplying the discounted profit stream by  $(1 - \beta)$  allows to use the same units to express profits in the stage game and the repeated game.

taking  $w_j$  and  $\tau$  as given. Define

$$w_i(w_j, \tau) \equiv \arg\max_{w_i} \pi_i(w_i, w_j, \tau)$$
(3.12)

as i's Nash Best response to  $w_j$  given  $\tau$ , and

$$\pi_i(w_j, \tau) \equiv \max_{w_i} \pi_i(w_i, w_j, \tau) \tag{3.13}$$

as *i*'s maximized profit (again, subject to his competitor's price  $w_j$  and exogenous tariff  $\tau$ ).

Denote the Bertrand price as the solution to  $w^b(\tau) = w_i(w^b(\tau), \tau)$ ; define the static Bertrand profit as  $\pi^b(\tau) = \pi_i(w^b(\tau), \tau)$ . Given our earlier assumptions on supply and demand functions, the Bertrand price and static profit which  $w^b(\tau) = \frac{4[k(a-k-\tau)-d]-1}{2(3+4k^2)}$  and  $\pi^b(\tau) = \frac{(1+2k^2)[1+2(k(a-\tau)-d)]^2}{2(3+4k^2)^2}$ .

# 3.2.2 The fully cooperative or unconstrained cooperative outcome

Under certain discount rates, both processors fully cooperate and act collectively as a monopsonist in the upstream market. Denote the monopsony price and static profit as

$$w^{m}(\tau) = \arg\max_{w} \pi_{i}(w, w, \tau)$$
(3.14)

and

$$\pi^{m}(\tau) = \max_{w} \pi(w, w, \tau).$$
 (3.15)

Thus, we can obtain the monopony price and profit which are  $w^m(\tau) = \frac{2[k(a-k-\tau)-d]-1}{4(1+k^2)}$ and  $\pi^m(\tau) = \frac{[1+2(k(a-\tau)-d)]^2}{16(1+k^2)}$ .

# 3.2.2.1 Full cooperation under free trade

Consider free trade case when  $\tau = 0$ , the Bertrand price and static profit can be written as  $w^b(0)$  and  $\pi^b(0)$ . Likewise, the monopsony price and static profit can be written as  $w^m(0)$  and  $\pi^m(0)$ . We yield Proposition 1.

**Proposition 1.** Under free trade, the monopsony input price can be sustained for any  $\beta \in [\underline{\beta}, 1)$ , where  $\underline{\beta} = \frac{(3+4k^2)^2}{17+16k^2(3+2k^2)}$ .

Proof of Proposition 1. To find the condition on the values of  $\beta$  required to support monopsony input price, we need to check that any single period deviation from cooperation does not offset the long run losses. The highest possible short run profit from deviation is from the static game best response to the opponent's play of the monopsony price  $(w^m)$ . Denote the profit from deviation as  $\pi_i(w^m(0), 0)$ . The optimal deviation is for processor *i* to set his input price at  $\frac{2[(5+8k^2)(ak-d)-8k^2(1-k^2)]-3}{16(1+k^2)(1+2k^2)}$ given processor *j* plays cooperatively, i.e.  $w_j = w^m(0)$ . The profit earned in the stage game from deviation is  $\pi_i(w^m(0)) = \frac{(3+4k^2)^2[1+2(ak-d)]^2}{128(1+k^2)^2(1+2k^2)}$  which is higher than the static profit from cooperation. In every subsequent period, the deviator is punished and gets the Bertrand payoff  $\pi^b(0)$ . Thus, full cooperation is sustained when the following inequality is satisfied:

$$\pi^{m}(0) \ge (1-\beta)\pi_{i}(w^{m}(0),0) + \beta\pi^{b}(0) \iff \beta \ge \underline{\beta} \equiv \frac{(3+4k^{2})^{2}}{17+16k^{2}(3+2k^{2})} \quad (3.16)$$

With a sufficiently high discount rate, both processors always fully cooperate and behave as a joint monopsonist in their input (upstream) market because the future gain from cooperation is high or, in other words, the future punishment from

deviating is sufficiently severe.

### 3.2.2.2 Full cooperation under an exogenous tariff

Next we consider that case where  $\tau > 0$  for all provides.

**Proposition 2.** When a positive tariff is imposed on downstream goods in every period, the critical discount rate which supports the monopsony input price is not affected by the tariff.

Proof of Proposition 2. To demonstrate this proposition, we need to show that the discount rate which supports the fully cooperative outcome under the tariff is the same as the one under free trade. As above,  $\pi_i(w^m(\tau), \tau)$  gives *i*'s static profit from deviation given processor *j* is playing cooperatively  $(w_j = w^m(\tau))$ . The optimal deviation is to set the input price at  $\frac{2[(5+8k^2)(k(a-\tau)-d)-8k^2(1-k^2)]-3}{16(1+k^2)(1+2k^2)}$ , yields static profits of  $\pi_i(w^m(\tau), \tau) = \frac{(3+4k^2)^2[1+2(k(a-\tau)-d)]^2}{128(1+k^2)^2(1+2k^2)}$ .

In every sequent period the deviator is punished and gets the noncooperative payoff  $\pi^b(\tau)$ . Full cooperation is supported when the total payoff from full cooperation is greater than the total payoff from deviation;

$$\pi^{m}(\tau) \ge [1-\beta] \pi_{i}(w^{m}(\tau), \tau) + \beta \pi^{b}(\tau).$$
 (3.17)

Because

$$\pi^m(\tau) = \alpha \pi^m(0), \tag{3.18}$$

$$\pi_i(w^m(\tau), \tau) = \alpha \pi_i(w^m(0), 0), \qquad (3.19)$$

and

$$\pi^b(\tau) = \alpha \pi^b(0) \tag{3.20}$$

where 
$$\alpha = \left[\frac{1+2(k(a-\tau)-d)}{1+2(ka-d)}\right]^2$$
, then full cooperation is supportable whenever  $\beta \ge \beta^{\tau} \equiv \frac{(3+4k^2)^2}{17+16k^2(3+2k^2)} = \underline{\beta}$ 

In our simple model, the critical discount rate which supports full cooperation is not influenced by an exogenous tariff. Accordingly, imposing a tariff on frozen shrimp does not affect the condition (a set of the critical discount rates) which supports full cooperation among the processors in the upstream market.

## 3.2.3 The constrained cooperative outcome

In the Proofs of Propositions 1 and 2, we define the optimal deviation which yields higher static profit than does cooperation. The normalized short run gain from deviating  $[1 - \beta] [\pi_i(w^m(\tau), \tau) - \pi^m(\tau)]$  is an incentive for a processor to defect and set a higher-than- monopsony price for fresh shrimp. Once a processor defects from cooperation, both processors revert to their Bertrand (Nash) price<sup>5</sup> in subsequent periods and earn Bertrand profits. Thus, the normalized value of avoiding a breakdown in cooperation for a processor is given by  $\beta [\pi^m(\tau) - \pi^b(\tau)]$ .

<sup>&</sup>lt;sup>5</sup>This punishment is credible because Bertrand price is indeed optimal in the non-cooperative game.

Cooperation is characterized by maximizing of joint profit in the cooperative stage, subject to the self-enforcement constraint which is neither processor has an incentive to deviate from cooperation. Define  $w^{cc}(\tau,\beta)$  as the price which sustains constrained cooperation when the tariff is  $\tau$  for all periods and the discount rate is  $\beta$ .  $w^{cc}(\tau,\beta)$  solves

$$\max_{w} \pi_i(w, w, \tau) \tag{3.21}$$

subject to the self-enforcement constraint

$$[1 - \beta] [\pi_i(w, \tau) - \pi_i(w, w, \tau)] \le \beta [\pi_i(w, w, \tau) - \pi^b(\tau)].$$
(3.22)

For future reference, define

$$\pi^{cc}(\tau,\beta) = \pi_i(w^{cc}(\tau,\beta), w^{cc}(\tau,\beta), \tau).$$
(3.23)

The self-enforcement constrain equation 3.22 is satisfied if, for each processor, the gain from deviating is no greater than the discounted punishment loss. If this condition is not satisfied, then the processors will have an incentive to deviate. Note that when the discount rate is high ( $\beta \geq \underline{\beta}$ ), the self-enforcement constraints are not binding because the processors highly value their future gains from cooperation. Thus, any sufficiently high discount rate ( $\beta \geq \underline{\beta}$ ) will support the fully cooperative outcome as a self-enforcing agreement. However, when the discount rate is low, the cooperation is not supported (Bagwell and Staiger (1990), Ederington (2001), Nielsen (2006) ). We now focus on the case that self-enforcement constraint is binding  $(\beta < \beta)$ .

## 3.2.3.1 Constrained cooperation under free trade

To earn better intuition, we start our analysis on the constrained cooperative input price with the free trade case when  $\tau = 0$ .

**Lemma 1.** Under free trade, with a self-enforcing agreement in upstream market and  $\beta < \beta$ ,

- The constrained cooperative input price is higher than the monopsony input price.
- The constrained cooperative input price is decreasing in the discount rate.
- The cost of self-enforcement is decreasing in the discount rate.

Proof of Lemma 1. The constrained cooperative fresh shrimp (input) price is  $w^{cc}(0,\beta) = \frac{2[k(a-k)-d]-1}{4(1+k^2)} + \frac{(1-\beta)(3+4k^2)(1+2(-d+ak))\lambda(0,\beta)}{4(1+k^2)(8+8k^2(3+2k^2)+(9-\beta+8k^2(3+2k^2))\lambda(0,\beta))}$ or  $w^m(0) + \frac{(1-\beta)(3+4k^2)(1+2(-d+ak))\lambda(0,\beta)}{4(1+k^2)(8+8k^2(3+2k^2)+(9-\beta+8k^2(3+2k^2))\lambda(0,\beta))}$ . The shadow value of self-enforcing constraint is  $\lambda(0,\beta) = \frac{(3+4k^2)^2 - \beta(17+48k^2+32k^4)}{\beta((3+4k^2)^2-\beta)}$ . Hence, the constrained cooperative input price is higher than the monopsony one for  $\beta < \underline{\beta}$ .

Moreover,

$$\frac{\partial w^{cc}(0,\beta)}{\partial \beta} = -\frac{4(1-2d+2ak)\left(3+10k^2+8k^4\right)}{\left(\beta-(3+4k^2)^2\right)^2} \le 0.$$
(3.24)

Hence, the constrained cooperative input price is decreasing in the discount rate.

Finally, we need to show that the self-enforcement cost  $(\lambda(0,\beta))$  is decreasing in the discount rate.

$$\frac{\partial\lambda(0,\beta)}{\partial\beta} = -\frac{1}{\beta^2} - \frac{16(2k^2+1)(k+1)}{(\beta-(3+4k^2)^2)^2} \le 0.$$
(3.25)

From Lemma 1, there are two components in the constrained cooperative input (fresh shrimp) price. The first component,  $\frac{2[k(a-k)-d]-1}{4(1+k^2)}$ , is the same as the monopsony price. The second component,  $\frac{(1-\beta)(3+4k^2)(1+2(-d+ak))\lambda(0,\beta)}{4(1+k^2),(8+8k^2(3+2k^2)+(9-\beta+8k^2(3+2k^2))\lambda(0,\beta))}$ , contains the shadow value of self-enforcement<sup>6</sup> resulting in a difference between the constrained cooperative price and the monopsony price. Later, we refer to the inverse of the second component as the 'degree of cooperation'. The lower the value of the second component, the closer the constrained cooperative price is to the monopsony price, i.e., the degree of cooperation is high. Intuitively, the higher the discount rate, the higher the degree of cooperation; this is because the processors highly value the future gain from cooperation, and so are more willing to cooperate. The results also hold when a tariff is imposed on downstream goods ( $\tau > 0$ ).

## 3.2.3.2 Constrained cooperation under an exogenous tariff

When a tariff is imposed, the constrained cooperative price becomes

 $w^{cc}(\tau,\beta) = \frac{2[k(a-k-\tau)-d]-1}{4(1+k^2)} + \frac{(1-\beta)(3+4k^2)(1+2(-d+k(a-\tau)))\lambda(\tau,\beta)}{4(1+k^2)(8+8k^2(3+2k^2)+(9-\beta+8k^2(3+2k^2))\lambda(\tau,\beta))}$ 

<sup>&</sup>lt;sup>6</sup>Moreover, we also know that  $\frac{\partial w^{cc}(0,\tau)}{\partial \lambda(0,\beta)} = \frac{(2(1-\beta)(2(ak-d)+1)(3+10k^2+8k^4))}{(8-(-9+\beta)\lambda(0,\beta)+8k^2(3+2k^2)(1+\lambda(0,\beta)))^2} \ge 0$ . Thus, the constrained cooperative price increasing the cost of self-enforcement.

or 
$$w^m(\tau) + \frac{(1-\beta)(3+4k^2)(1+2(-d+k(a-\tau)))\lambda(\tau,\beta)}{4(1+k^2)(8+8k^2(3+2k^2)+(9-\beta+8k^2(3+2k^2))\lambda(\tau,\beta))}$$
. The shadow value of the self-  
enforcement is  $\lambda(\tau,\beta) = \frac{8\beta(1+3k^2+2k^4)A_1+\sqrt{A_1A_2}}{\beta(\beta-(3+4k^2)^2)A_1}$  where  $A_1 = \beta(1+2(ak-d))^2+4k(3+4k^2)^2\tau(2(d-ak-k\tau)-1))$  and  $A_2 = (\beta-1)^2\beta(3+4k^2)^4(2(k(a-\tau)-d))+1)^2$ .

Lemma 2. The shadow cost of self-enforcement is increasing in a tariff.

*Proof of Lemma 2.* To show that the shadow cost of self-enforcement is increasing in a tariff, we need to show that the derivative of the shadow cost of self-enforcement with respect to the tariff is positive.

$$\frac{\partial\lambda(\tau,\beta)}{\partial\tau} = \frac{1}{A_1^2\beta(3+4k^2)^2 - \beta} \left[ 4kA_3\left(\sqrt{A_1A_2} + 8A_1\beta(1+3k^2+2k^4)\right) + A_1\left(\frac{4kA_4(A_1+A_3)}{2\sqrt{A_1A_2}} + 32(1+3k^2+2k^4)A_3\right) \right] > 0.$$

$$(3.26)$$

Where  $A_3 = (3 + 4k^2)^2 (2(k(a - \tau) - d)) + 1)$ , and  $A_4 = (\beta - 1)^2 \beta (3 + 4k^2)^4 (2(k(a - \tau) - d) + 1)$ .

The intuition behind Lemma 2 is as follows. The tariff reduces the future gain from cooperation making processors less willing to cooperate under the tariff. Therefore, the cost associated with self-enforcement under the tariff is higher.

**Proposition 3.** When a tariff is imposed in every period, the constrained cooperative input price is decreasing in the tariff on downstream goods.

*Proof of Proposition 3.* To show that the constrained cooperative price is decreasing in the tariff, we need to show that the derivative of the constrained cooperative price

with respect to the tariff is negative.

$$\frac{\partial w^{cc}(\tau,\beta)}{\partial \tau} = -\frac{k}{2(1+k^2)} - \frac{(1-\beta)k(3+k^2)\left(8\beta A_3 + \sqrt{A_1A_2}\right)}{2(1+k^2)\left((3+4k^2)^2 - \beta\right)\sqrt{A_1A_2}} \le 0$$
(3.27)

A tariff has two opposing effects on the constrained cooperative input price. Firstly, the tariff raises costs for the processors. The processors can pass through their costs to their input suppliers by lowering input price. Secondly, the tariff raises the cost of self-enforcement because it lower anticipated gain from constrained cooperation. As per Lemma 2, the cost of self-enforcement increases as the tariff increases. In this model, the second effect is dominated by the first one. As a result, the constrained cooperative price in this setting decreases when the tariff increases.

## 3.3 Collusion with the antidumping duty

Consider that these two processors are potentially subjected to an antidumping duty. Antidumping duties are imposed at the behest of domestic industries that compete with imported goods, when the domestic producers believe they can make the case that a foreign firm is selling goods in the home (i.e., import) market at less than *normal value*<sup>7</sup>. The antidumping duty is an endogenous tariff which depends upon the exporters' behavior in the previous period. Similar to other antidumping studies, we assume that the processors (exporters) will be filled the antidumping

<sup>&</sup>lt;sup>7</sup>Normal value can be defined either a lower export price than a price sold in the exporters' local market, in the third market, or if there is no foreign price to observe whether the exporters are selling at less than its average cost plus some profit margin

petition when a goods' price is lower than some threshold  $\overline{P} \ge p_i$  (Veugelers and Vandenbussche (1997), Kolev and Prusa (1999), Hartigan(2000), Blonigen and Park (2004), Schmitz and Seale(2004), Ishikawa (2004)). Like Blonigen and Park (2004) and Ishikawa (2004), we assume the threshold  $\overline{P}$  is very high such that the processors (exporters) will always face an antidumping petition, and the dumping margin is determined from the processors'(exporters') behavior in the previous period<sup>8</sup>.

The initial antidumping duty determination and the following administrative review(s) are costly and time consuming (Blonigen and Park 2004), so that the processors might face the same antidumping duty level for multiple periods. For simplicity, we assume that the antidumping determination and the following administrative review(s) are very long, so that the duty level in any period  $t \ge 2$  only depends on the processors' behaviors in the first period (t = 1). Specifically, we assume the antidumping duty is the difference between first period price of frozen shrimp and first period average cost plus an exogenously given profit margin  $\overline{m}$ .

<sup>&</sup>lt;sup>8</sup>For the U.S. antidumping margin calculation, the US Department of Commerce subsequently issued the antidumping questionnaire to selected exporters from the subjected countries. The exporters were asked to provide information related to the affiliation of the companies and U.S importers. Then, the U.S. Department of Commerce compared the export price or constructed export price to the normal price which, in this case, is the cost of production. The export price is calculated for the sales where the merchandise was sold to the first unaffiliated purchaser in the U.S. prior to importation by the exporter or producer outside the U.S. The export price is calculated for the sales where the merchandise was sold (or agreed to be sold) in the U.S. before or after the date of importation by an exporter. The DoC based the constructed export price on the packed delivered prices to unaffiliated purchasers in the U.S. There are some billing adjustments to both the export price and constructed export price. The cost of production is based on the sum of the cost of materials and fabrication for the foreign like product, plus an amount for general and administrative expenses, and interest expenses. (Federal Register Vol.69 No.149)

Thus, the antidumping duty imposed in period  $t \ge 2$  is

$$\tau_{i}(w_{i1}, w_{j1}) = \begin{cases} (w_{i1} + d + \overline{m}) - p_{i1} & \text{when } w_{i1} + d + \overline{m} > p_{i1}(w_{i1}, w_{j1}) \\ 0 & \text{when } w_{i1} + d + \overline{m} \le p_{i1}(w_{i1}, w_{j1}). \end{cases}$$
(3.28)

Given the same antidumping duty in each period  $t \ge 2$ , the processors' profits in each period  $t \ge 2$  are identical:

$$\pi_i(w_i, w_j, \tau_i(w_{i1}, w_{j1})) \equiv [k[p_i(w_i, w_j) - \tau_i(w_{i1}, w_{j1})] - w_i - d] x_i(w_i, w_j)$$
(3.29)

where  $\tau_i = 0$  in the first period. The processor's static problem is to maximize his profit subject to the constraint in which the antidumping duty cannot be negative. To solve for the equilibrium, we need to solve backward from future periods to get the future fresh shrimp price as a function of the first period fresh shrimp price.

#### 3.3.1 The Bertrand competitive outcome

In every period  $t \geq 2$  when *i* plays the Bertrand price  $w_i^b(\tau_i(w_{i1}), \tau_j(w_{j1}))$ and *j* plays the Bertrand price  $w_j^b(\tau_i(w_{i1}), \tau_j(w_{j1}))$  which are Nash equilibria to the static profit maximization game in which the firms face (potentially) difference antidumping duties.

In the first stage, each processor takes into account the impact of his current pricing behavior on the future antidumping duties facing himself and his competitor, recognizing that at every future stage each processor will engage in Bertrand pricing. For example, *i* chooses  $w_{i1}$  to solve

$$\max_{w} \left[1 - \beta\right] \left[ \pi_{i}(w, w_{j1}, 0) + \sum_{t=2}^{+\infty} \beta^{t-1} \pi_{i}(w_{i}^{b}(\tau_{i}(w), \tau_{j}(w_{j1})), w_{j}^{b}(\tau_{i}(w), \tau_{j}(w_{j1})), \tau_{i}(w, w_{j1})) \right]$$
(3.30)

subject to

$$\tau_i = (w + d + \overline{m}) - p_i(w, w_{j1}) \ge 0.$$
(3.31)

Equation 3.31 indicates that the antidumping duty cannot be negative.

Denote the future Bertrand price as the solution to

 $w^{b}\left(\tau\left(w_{1}^{b}(\beta,0)\right)\right) = w_{i}^{b}\left(\tau_{i}(w_{i1}^{b}(\beta,0)),\tau_{j}(w_{j1}^{b}(\beta,0))\right) \text{ where } w_{1}^{b}(\beta,0) = w_{i}\left(w_{1}^{b}(\beta,0),0\right)$ is the first period Bertrand price. Define the static Bertrand future profit as  $\pi^{b}\left(\tau(w_{1}^{b}(\beta,0))\right) = \pi_{i}^{b}\left(w^{b}\left(\tau(w_{1}^{b}(\beta,0)),\tau(w_{1}^{b}(\beta,0))\right)\right) \text{ and the Bertrand first period profit as } \pi_{1}^{b}(\beta,0) = \pi_{i1}^{b}\left(w_{1}^{b}(\beta,0),0\right). \text{ Given out earlier assumptions on input supply and output demand functions, and the dumping margin calculation, the Bertand future price as a function of the Bertrand first period price is <math>w^{b}\left(\tau\left(w_{1}^{b}(\beta,0)\right)\right) = \frac{2\left(k\left(4a-2\overline{m}-2(1+k)w^{b}(\beta,0)(\overline{m},\beta)-3k\right)-2d(1+k)\right)-1}{6+8k^{2}} \text{ where the Bertrand first period price is } w_{1}^{b}(\beta,0) = \frac{\left(3+4k^{2}\right)^{2}\left(5+12k^{2}\right)\left(4\left(k(a-k+\mu)-d\right)+2\mu-1\right)-\beta A_{5}}{2A_{6}}.^{9} \text{ The shadow cost of the duty-triggering is } \mu^{b} = \frac{(1-1)^{2}}{(2(3+4k^{2})^{2}(5+15k+22k^{2}+36k^{3}+24k^{4}))}\left[A_{7}+\beta A_{8}\right].^{10}$ 

 $<sup>\</sup>frac{9}{9} \text{ where } A_5 = 45 - 4d(-45 + 2k(7 + k(-91 + 2k(21 + k(-51 + 4k(10 - 3k + 6k^2)))))) - 4k(-7 + a(45 - 4k(7 + k(-41 + 2k(13 + 6k(-3 + 2k))))) - k(2(-59 + 7\overline{m}) + k(-19 + 32\overline{m} + 2k(-180 + 26\overline{m} + k(1 + 56\overline{m} + 12k(-17 + k - 6k^2 + 2\overline{m} + 4km)))))) \text{ and } A_6 = (3 + 4k^2)^3 (5 + 12k^2) + \beta (135 + 4k^2(209 + k(-23 + 2k(231 + k(-41 + 36k(6 + k(-1 + 2k)))))))$ 

#### 3.3.2 The fully cooperative or unconstrained cooperative outcome

With sufficiently high discount rate, both processors will fully cooperate and act as a joint monopsonist in the upstream market. In every period  $t \ge 2$ , both processors play the monopsony price  $w^m(\tau(w_1), \tau(w_1))$  which is the optimal cooperative outcome to the static joint profits maximization game.

In the first period, the processors take into account the effect of their pricing behaviors on the future antidumping duties facing themselves, recognizing that every future period both processors will engage in monopsony pricing . Therefore, processor i chooses  $w_1$  to solve

$$\max_{w} \left[1 - \beta\right] \left[ \pi_{i}\left(w, w, 0\right) + \sum_{t=2}^{+\infty} \beta^{t-1} \pi_{i}\left(w^{m}\left(\tau(w), \tau(w)\right), w^{m}\left(\tau(w), \tau(w)\right), \tau(w)\right) \right]$$
(3.32)

subject to

$$(w+d+\overline{m}) - p(w,w) = 0.$$
(3.33)

The future monopsony price is the solution to

 $w^{m} (\tau(w_{1}^{m}(\beta, 0))) = w^{m} (\tau(w_{1}^{m}(\beta, 0)), \tau(w_{1}^{m}(\beta, 0))), \text{ and the monopsony first period}$ price is the solution to  $w_{1}^{m}(\beta, 0) = w_{1}(w_{1}^{m}(\beta, 0), 0)$ . Define the static monopsony future profit as  $\pi^{m} (\tau(w_{1}^{m}(\beta, 0))) = \pi^{m} (w_{1}^{m}(\beta, 0), \tau(w_{1}^{m}(\beta, 0)))$  and the monopsony first period profit as  $\pi_{1}^{m}(\beta, 0) = \pi_{1}^{m} (w_{1}^{m}(\beta, 0), 0)$ . From assumptions on input supply and output demand functions and the antidumping duty definition, the value of the Bertrand future price as a function of the Bertrand first period price can be obtained as  $w^{m} (\tau(w_{1}^{m}(\beta, 0))) = \frac{(k((4a-3k-2\overline{m})-k(1+k)w^{m}(\overline{m},\beta)])-1-2d(1+k))}{(4(1+k^{2}))}$ . Also, the Bertrand first period price can be obtained as  $w_1^m(\beta, 0) = \frac{2(1+k^2)(2(k(a-k)+(1+k)\mu^m)-d)-1)-\beta A_9}{8(1+k^2)^2+2B(2+(-1+k)k)(2+k+3k^2)}$ .<sup>11</sup> The shadow cost of the duty-triggering constraint is  $\mu^m = \frac{1}{4(1+k)^2(1+k^2)} (A_{10} + \beta A_{11})^{12}$ 

Both Bertrand and monopsony input prices in the first period under the antidumping duty are lower than the ones under free trade because the processors also account for the cost of the antidumping duty in the future. The processors' first-period prices will enter into the processor's future profits as a cost of the antidumping duty, so that the processors will lower their first-period price in order to get a lower antidumping duty in the future.

#### 3.3.3 The constrained cooperative outcome

We now consider the situation in which the discount rate is not sufficiently high to support full cooperation. The cooperation is still feasible through the selfenforcing agreement.

In every period  $t \ge 2$ , the processors play the constrained cooperative input price  $w^{cc}(\tau(w_1), \tau(w_1))$  which is the optimal outcome to the static join profit maximization problem subject to the self-enforcing constraint. Denote, the constrained cooperative static profit as  $\pi^{cc}(\tau(w_1),\beta) = \pi_i \left( w^{cc}(\tau(w_1),\tau(w_1)), w^{cc}(\tau(w_1),\tau(w_1)), \tau(w_1) \right)$ 

In the first period, both processors takes into account the impact of their current pricing behaviors on the future antidumping duties facing themselves, recognizing that at every future period both processors will play constrained cooperative pricing. Define  $w^{cc}(\beta, 0)$  as the price which sustains constrained cooperation at the

 $<sup>\</sup>frac{1^{11} \text{Where } A_9 = 2 - 2d\left(-2 + k + k^3\right) + k\left(1 + 4a(-1+k) + k\left(7 - k + 3k^2 - 2(1+k)\overline{m}\right)\right)}{1^2 \text{Where } A_{10} = 2\left(1 + k^2\right)\left(2a(2 + (-1+k)k) + (1-2d)(1 + k(2k-1)) - 4\left(1 + k^2\right)\overline{m}\right)} \text{ and } A_{11} = 2a\left(4 + k\left(-2 + 7k + 3k^3\right)\right) + (1 - 2d)(2 + k(-1 + k(8 + k(-1+4k)))) - 8\left(1 + k^2\right)^2\overline{m}.$ 

first period which solves

$$\max_{w} \left[ 1 - \beta \right] \left[ \pi_i \left( w, w, 0 \right) + \sum_{t=2}^{+\infty} \beta^{t-1} \pi^{cc}(\tau(w), \beta) \right]$$
(3.34)

subject to the triggering antidumping duty

$$(w+d+\overline{m}) - p(w,w) = 0 \tag{3.35}$$

and the self-enforcing constraint

$$[1-\beta] \left[ \pi_1(w,0) - \pi_1(w,w,0) \right] \le \beta \left[ \pi_i^{cc} \left( \tau(w), \beta \right) - \pi^b \left( \tau(w_1^b(\beta,0)) \right) \right].$$
(3.36)

Define the constrained cooperative future price as the solution to  $w^{cc}(\tau(w^{cc}(\beta, 0))) = w^{cc}(\tau(w^{cc}(\beta, 0)), \tau(w^{cc}(\beta, 0)), \beta)$ . We can obtain the constrained cooperative future price as a function of the constrained cooperative first period price,  $w^{cc}(\tau(w^{cc}(\beta, 0))) = \frac{4(1+2k^2)(1+2(1+k)(d+kw^{cc}(\beta, 0))+k(-4a+3k+2\overline{m}))+\lambda(w^{cc}(\overline{m},\beta))A_{12}}{2(-8(1+3k^2+2k^4)+(\beta-(3+4k^2)^2)\lambda(w^{cc}(\overline{m},\beta)))}$ .<sup>13</sup> Where the cost of self enforcement is  $\lambda(w^{cc}(\beta, 0)) = \frac{8\beta[A_{13}+A_{14}]+\sqrt{A_{15}+A_{14}}}{\beta(\beta-(3+4k^2)^2)[A_{16}+A_{14}]}$ .<sup>14</sup>

The first-period constrained cooperative input price under antidumping duty

is 
$$w^{cc}(\beta, 0) = \frac{2(1+k^2)(2(k(a-k)+(1+k)\mu)-d)-1)-\beta A_{17}}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2+2\beta(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad}, \beta, a, d, k, \overline{m}) \text{ or}^{15} w^m(\overline{m}, \beta) + \frac{1}{8(1+k^2)^2(1+k)(d+(3-\beta+4k^2)w^{cc}(\beta, 0)) + 2k(-4a+k+2\overline{m}))}{R(1+k^2)(1+2k^2)(1+2k^2)(2+k^2)(2+k) + k(-6a+k+2\overline{m}))} + \frac{1}{8(1+k^2)^2(1+2k^2)^2(2(ak-k\tau-d)+1)} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1)} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1))} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1)} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1))} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1)} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1))} + \frac{1}{8(1+k^2)^2(2(ak-k\tau-d)+1)} + \frac{1}{8(1+k^2)^2(2(ak$$

 $R(\lambda^{ad}, \beta, a, d, k, \overline{m})$  where R(.) has positive value.<sup>16</sup> Therefore, the first-period constrained cooperative input price is greater than the monopsony one.

**Proposition 4.** Given that the same duty is imposed in period  $t \ge 2$ , the critical discount rate which supports first period full cooperation in an upstream market can be lower than the one supporting full cooperation under free trade.

Proof of proposition 4. To show that cooperation is easier to sustain under the antidumping duty, we need to show that the value of the critical discount rate  $\beta$  which supports first period full cooperation under the antidumping duty is lower than  $\underline{\beta}$ . As proved in Proposition 2, when  $\beta < \underline{\beta}$ , full cooperation is not sustainable at t > 1because everyone knows that someone will deviate at some period t > 1. Therefore, we will consider only full cooperation at the first period with constrained cooperation at the future period  $t \ge 2$ . We refer this game to "restricted full cooperation". At every period  $t \ge 2$ , the constrained cooperative static profit is  $\pi^{cc}(\tau(w_1), \beta)$ . At the first period, processor i chooses  $w_1$  to solve

$$\max_{w} \left[ 1 - \beta \right] \left[ \pi_i \left( w, w, 0 \right) + \sum_{t=2}^{+\infty} \beta^{t-1} \pi^{cc}(\tau(w), \beta) \right]$$
(3.37)

subject to

$$(w+d+\overline{m}) - p(w,w) = 0.$$
 (3.38)

Define the restricted full cooperative price as the solution to  $w_1^{mc}(\beta, 0) = w_1(w^{mc}(\beta, 0), 0)$ . We can obtain the value for the restricted full cooperative price

<sup>&</sup>lt;sup>16</sup>The full value of the first-period constrained cooperative input price can be found in appendix A.

such that 
$$w_1^{mc}(\beta, 0) = \frac{\frac{1}{2}(1+\beta)(1+2d+2k(-a+k)) - A_{17}(-1+2d(1+k)+k(-4a+k+2\mu^{ad2})) - (1+k)\mu^{ad2}}{2A_{12}k(1+k) - 2(1+\beta)(1+k^2)}$$
.<sup>17</sup>

Define the restricted full cooperative profit at the first period as  $\pi_1^{mc}(\beta, 0) = \pi_1(w_1^{mc}(\beta, 0), 0)$ 

The restricted fully cooperative price is supported when the total payoff from the full cooperation is greater than the total payoff from deviation;

$$[1 - \beta] \pi_1^{mc}(\beta, 0) + \beta \pi_i^{cc} \left( w^{cc}(\tau(w_1^{mc}(\beta, 0))) \right) \ge [1 - \beta] \pi_i(w_1^{mc}(\beta, 0), 0) + \beta \pi^b \left( \tau(w_1^b) \right).$$
(3.39)

The condition in equation 3.39 holds only if

$$\beta \ge \beta^{ad}.^{18} \tag{3.40}$$

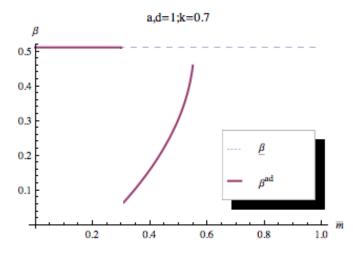
Recall the critical discount rate under free trade is  $\beta$ ; thus,

$$\beta^{ad} - \beta \le 0 \tag{3.41}$$

is a necessary condition for full cooperation to be sustainable in the first period when there is an antidumping duty in place (as compared to free trade).

The comparison between the critical discount rate under free trade and the

Figure 3.2: Comparison of discount rate which supports the restricted fullcooperation cooperation under free trade and Antidumping



antidumping duty is shown in figure 3.2. The dashed line represents the critical discount rate under the free trade scenario while the solid line represents the critical discount rate which supports restricted full cooperation under the antidumping scenario given a certain value of exogenous variable (a, d, k). As shown in figure 3.2, for small value of exogenous profit margin given by the antidumping authority  $\overline{m}$ , the critical discount rate under the antidumping duty is the as free trade one. With high value of  $\overline{m}$  the critical value lower than the one under free trade<sup>19</sup>.

Proposition 4 shows that full cooperation in the first period is not affected by the antidumping duty if the exogenous profit margin,  $\overline{m}$ , is small enough to prohibit the antidumping duty from coming into effect, i.e.  $w_{i1} + d + \overline{m} \leq p_{i1}(w_{i1}w_{j1})$ . On the other hand, if  $\overline{m}$  is such that  $w_{i1} + d + \overline{m} > p_{i1}(w_{i1}, w_{j1})$ , it is easier to sustain

<sup>&</sup>lt;sup>19</sup>Although we change the value of exogenous variable (a, d, k), the critical discount rate under the antidumping duty still  $\beta^{ad}$  is lower than the free trade's one. See the combination use for the numerical analysis in appendix A

restricted full cooperation under antidumping. Moreover, the critical discount rate  $\beta^{ad}$ , which supports the restricted full cooperative outcome under antidumping duty, is weakly increasing the exogenous profit margin given by the antidumping authority  $(\overline{m})$ . However, if  $\overline{m}$  is too large providing a negative profit (as  $\overline{m}$  is greater than 0.6 in figure 3.2), the processors will stop playing the game. Thus, restricted full cooperation can be sustain more easily when  $\overline{m}$  is intermediate. This is because  $\overline{m}$  reduces the future gain from cooperation, so the processors have more motivation to deviate when  $\overline{m}$  is large. Therefore, with intermediate value of  $\overline{m}$  allowing the duty is positive, the antidumping is pro-collusive.

In short, when  $\beta < \underline{\beta}$  only full cooperation that can be sustained is restricted full cooperation. When the exogenous profit margin given by the antidumping authority  $\overline{m}$  is small, there is no threat of an antidumping duty, so the processors play constrained cooperation from the initial period, same as free trade. However, when there is a credible threat of the antidumping duty, then anxiety over the future duty reduces benefit from current deviation, helping to sustain present cooperation. The result suggests that a policy maker should be aware of a change in a processor's cost as a result of the antidumping duty if the size of duty is being calculated based on costs.

#### Chapter 4

#### Empirical model and estimation results

In the theory section, we found that, under some certain conditions, the antidumping duty is pro-collusive, i.e. it is easier to sustain collusion under the antidumping duty. Thus, the primary objective of this section is to develop an empirical model that can be used to measure collusion (market power) in Thai shrimp market. Also we attempt to test the theoretical results of whether the U.S. antidumping duty on frozen shrimp has an effect on collusion in the Thai fresh shrimp market.

Although much of the literature on Thai shrimp, shrimp is considered an important industry in Thailand (Krasachat(1987), Krasachat and Manerat (1991), Samapat (1994), Ling (1996), Malisorn (1996), Iamlaor(1998), Raubrhoo (2002), Potathanapong(2002)), these studies only focus on Thai shrimp on the international level. The structure of domestic shrimp market has been ignored. This is because fresh shrimp production data is not sufficient enough. The fresh shrimp production data is usually collected annually and the shrimp size is not specified while the shrimp price significantly varies by shrimp size. Different from previous studies, by using fixed-proportion technology, frozen shrimp import/export by size can be used as the amount of fresh shrimp production which allows us to estimate for Thai fresh shrimp supply. Thus, we are able to analyze the structure of the Thai shrimp domestic market and the existence of collusion/market power within the Thai shrimp market.

#### 4.1 The empirical industry organization approach

The approach widely used to test market power defines market power as some deviation of price from marginal  $cost^1$ . The gap between price and marginal cost is estimated as an unknown parameter from observable price and quantity data. The models are broadly classified into two groups : conjectural variation models and comparative statistics models.

 Conjectural Variation Models –estimate cost directly (Schroeter 1988, Schroeter and Azzam1990, Azzam and Pagoulatos 1990)

A conjectural variation model is defined as a firm's "belief or expection" about its rivals' reaction to a decrease (increase) in its own output's price (output supply) in the case of an oligopoly or a increase in its own input's price (input employment) in the case of oligosony. The basic model makes use of the following marginal revenue and marginal cost specification

$$p(1 - \frac{\theta}{\eta}) = w(1 + \frac{\phi}{\xi}) + \frac{\partial C}{\partial Q}$$
(4.1)

where p and w denote output and input prices,  $\eta$  and  $\xi$  are output and input

<sup>&</sup>lt;sup>1</sup>The prior approach widely used to test market power was "Structure-Conduct-Performance" (SCP) approach. In SCP approach, price is regressed on a measure of concentration and a set of variables. A statistically significant coefficient for concentration is interpreted as an evidence for market power. However, one of the major criticisms of the SPC approach is its treatment of conduct. A significant price-concentration correlation is taken to imply noncompetitive conduct because the latter is implicitly "assumed" to be determined by concentration (Weliwita 1995).

supply elasticities,  $\theta$  and  $\phi$  are output and input market conjectural elasticities<sup>2</sup>, and  $\frac{\partial C}{\partial Q}$  denotes other marginal input costs. If the firm is a price taker (or the market is perfectly competitive), so that it does not expect a change in the industry output (input) as a result of a change in its own output (input), then the conjecture is that equal zero. The conjectures equal one under the monopoly/monopsony. An oligopoly/oligopsony is assumed to fall within the continuum.

However, the conjectural variation models have shortcomings. First, firms are assumed to form conjectures on rivals' response to their actions as if it involves in a dynamic game. Although, the model is derived from a statistic profit maximization problem. Second, the prior restrictions imposed on demand and cost functions in empirical estimation makes it is unclear if the estimated price-cost margin is the result of firms' collusive behavior or a by-product of a prior restriction (Koontz, et al 1993 and Weliwita 1995).

2. Comparative Statistics Models<sup>3</sup>

The comparative statistics models attempt to find the market structure consistent with equilibrium outcomes following an exogenous shock to the industry. Bresnahan (1989) identified four types of comparative statistic models

• Comparative statistics in demand (supply) (Azzam and Park 1993, Weli-

<sup>&</sup>lt;sup>2</sup>For example,  $\theta = \frac{\partial Q}{\partial Q_i} \frac{Q_i}{Q}$ . It is defined as the firm's precieved rate of the change of the industry output (Q) with respect to a unit change in its own output in the case of quantity setting game. Moreover, in the price setting game,  $\theta = \frac{\partial p}{\partial p_i} \frac{p_i}{p}$ <sup>3</sup>More detail can be found in Bresnahan(1989)

wita  $1995)^4$ 

- Comparative statistics in demand (supply) shock (Lee and Porter 1984, Koontz et al 1993)<sup>5</sup>
- Comparative statistics in industry structure (Lamm 1981 and Cotterill 1986)<sup>6</sup>
- Comparative statistics in costs (Panzar and Rosse 1987)<sup>7</sup>

In this chapter, we use comparative statistics in supply to test for the collusive behavior (market power) in the Thai shrimp market. Unlike Azzam and Park (1993) and Weliwita (1995), where the evidence of collusion is defined as change in a firm's quantity with respect to its rival change in quantity, we define collusion as change in a firm's price with respect to its rival change in price. Moreover, we also include the antidumping variable to capture the effect of antidumping duty on the processors' (exporters') collusion in their upstream (input) market.

<sup>6</sup>This model treats concentration as exogenous to the industry. A significant and positive (negative) correlation between output (input) price and producer concentration is interpreted as an evidence of oligopoly (oligosony) power.

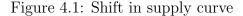
<sup>7</sup>This model is based on estimating the reduced-form revenue equation for a firm which is the total revenue for a single firm and equals the product of the equilibrium quantity and price. The statistic which can be used to identify a monopolistic industry is

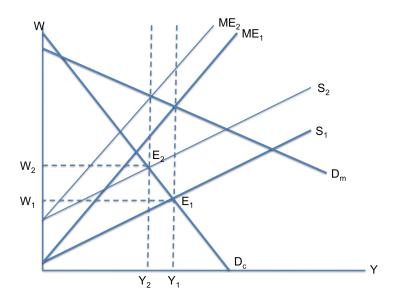
$$H_R = \frac{w_{it} R_w(w_{it}, Z_{it}, Y_t)}{R(w_{it}, Z_{it}, Y_t)}$$
(4.2)

where R(.) is a firm's reduced form revenue,  $w_{it}$  is an input price,  $Y_t$  is a vector of demand shifters, and  $Z_{it}$  is a vector of cost shifters.  $H_R$  is a measure of the percentage change in equilibrium revenues caused by one percent increase in the input price.

<sup>&</sup>lt;sup>4</sup>The comparative statistics in demand (supply) can reveal the degree of collusion ( oligopoly/oligosony power) by the rotation in demand (supply) curve.

<sup>&</sup>lt;sup>5</sup>Green and Porter (1984) develop the model which has become known as "trigger price" oligopoly model. This model shows how collusive prices can be interrupted by noncooperative price under demand uncertainty. Under imperfect information, firms do not know exactly their rivals' output, and firms may respond to substantial price declines by acting more competitively for a period of time, even if price declines because of an exogenous source. In the comparative statistics in demand (supply) shock model, the error terms in demand (supply) relations can identify two regimes–noncooperative and collusive.

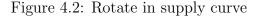


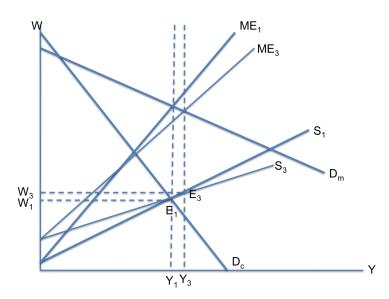


#### 4.2 Comparative static in supply

To see how comparative statics in supply reveal the degree of oligopsony collusion/power in the input market, following Bresnahan (1982), the equilibrium for a monopsonist  $E_1$  in figure 4.1 where the equilibrium input price  $w_1$  and the quantity  $y_1$  are determined at the intersection of the marginal factor cost  $ME_1$ , associated with the input supply  $S_1$ , and demand of a monopsony  $D_m$ . On the other hand, The same equilibrium input price and quantity can be generated by the perfect competitive industry where the demand curve  $D_c$  intersects the supply curve  $S_1$ . Since both markets generate the same input price and quantities have been derived by a perfectly competitive industry or by a monopsony.

Suppose  $S_1$  parallelly shifts to  $S_2$  due to an exogenous shock to the input supply. The new equilibrium is at  $E_2$ . However, the same equilibrium input price  $w_2$ 





and quantity  $y_2$  can still derived by either a monopsony or a competitive industry. Thus, a parallel shift in the input supply curve cannot distinguish between the monopsony and perfect competition. This raises the question of an identification problem, the solution to which requires a rotation rather than a parallel shift of the input supply curve around the initial equilibrium  $E_1$  to distinguish the monopsony equilibrium from the perfect competition equilibrium. As illustrated in figure 4.2, we rotate  $S_1$  around the initial equilibrium  $E_1$  to get the new input supply curve  $S_3$ . The equilibrium input price and quantity under perfect competition is still the same before and after the rotation. On the other hand, under monopsony, the marginal factor cost also shifts to  $ME_3$  resulting in a new equilibrium at  $E_3$  which is different from the equilibrium under perfect competition.

#### 4.3 The empirical model

We extend our assumption in the theory section from dual processors to nmultiple processors. Let the inverse demand of frozen shrimp in the U.S. market that processor i in period t faces be

$$p_{it} = f_{it}(y_{it}, AD_{it}(w_{i,t-1}), G) + \epsilon$$
(4.3)

where  $y_{it}$  is processor *i*'s frozen shrimp supply,  $AD_{it}(w_{i,t-1})$  is the antidumping duty which depends on processor *i*'s fresh shrimp price in the previous period ( $w_{i,t-1}$ ), and *G* is a vector of exogenous factors that shift frozen shrimp demand. The error  $\epsilon$ is an unobservable fluctuation in frozen shrimp demand. From the fixed-proportion technology assumption, we can denote both input (fresh shrimp),  $x_{it}$  and output (frozen shrimp),  $y_{it}$  by the same notation.

$$y_{it} = x_{it} \tag{4.4}$$

Thus, processor i faces fresh shrimp supply

$$y_{it} = g_{it}(w_{it}, H) + \upsilon \tag{4.5}$$

where  $w_{it}$  is a frozen shrimp price offered by processor i, H is a vector of exogenous factors that shift fresh shrimp supply, and the error v is unobservable fluctuation in fresh shrimp supply. Processor *i*'s short run profit at period t is<sup>8</sup>

$$\pi_{it}(w) = [p_{it}(y_{it}(w_t, H, v), AD_{it}(w_{i,t-1}), G, \epsilon) - w_{it}(1 + AD_{it}(w_{i,t-1}))] y_{it}(w_t, H, v)$$

$$-c(y_{it}(w_t, H, v), D)$$
(4.6)

where  $w_{it}$  is price of fresh shrimp offered by processor i.<sup>9</sup>  $c(y_{it}(w, H, v), D, )$  is cost of non-material input where D is a vector of non-material input.

Processor i's first order condition for the short run profit maximization problem is

$$\left[p_{it}(.) + y_{it}(.)\frac{\partial p_{it}(.)}{\partial y_{it}(.)} - AD_{it}(.) - w_{it} - \frac{\partial c(.)}{\partial y_{it}(.)}\right] \left(\frac{\partial y_{it}(.)}{\partial w_{it}} + \sum_{j\neq i}^{n-1} \frac{\partial y_{it}(.)}{\partial w_{jt}}\frac{\partial w_{jt}}{\partial w_{it}}\right) - y_{it}(.)(1 + AD_{it}(.)) = 0$$

$$(4.7)$$

Denote the change in processor i's fresh shrimp supply or the change in processor i's procurement with respect to his own fresh shrimp price as

$$\gamma = \frac{\partial y_{it}(.)}{\partial w_{it}}.$$
(4.8)

Moreover, we assume that the effect on processor i's procurement due to a change in processor j's fresh shrimp price is smaller than its own price effect<sup>10</sup>. Denote the

<sup>&</sup>lt;sup>8</sup>from the theory section, with the normalization factor  $(1-\beta)$  the total profit in repeated-game is the same as the single-period profit.

<sup>&</sup>lt;sup>9</sup>Although the antidumping duty is imposed on frozen shrimp (output), the profit is written in this fashion because we expect that the antidumping duty affects collusion and input (fresh shrimp) price in upstream market.

<sup>&</sup>lt;sup>10</sup>We also assume that processor i's procurement responds to change in his rivals is the same across firms. Moreover, we also assume a symmetry from the theoretical model which results in

change in processor i's procurement with respect to processor j's price as

$$-\frac{\gamma}{n-1} = \frac{\partial y_{it}(.)}{\partial w_{jt}} \tag{4.9}$$

where n is number of processors in the market.

By the assumptions in equation 4.8 and 4.9, processor i's first order condition becomes

$$\left[p_{it}(.) + y_{it}(.)\frac{\partial p_{it}(.)}{\partial y_{it}(.)} - AD_{it} - w_{it} - \frac{\partial c(.)}{\partial y_{it}(.)}\right](1 - \phi_i)\gamma - y_{it}(.)(1 + AD_{it}(.)) = 0.$$
(4.10)

Where  $\phi_i = \frac{1}{n-1} \sum_{j \neq i}^{n-1} \frac{\partial w_{jt}}{\partial w_{it}}$  is the average change in fresh shrimp price offered by the other processors with respect to the change in processor *i*'s fresh shrimp price which represents the collusive behavior. For noncooperative behavior, the processors play Bertrand competition, so  $\phi_i$  equals zero. Otherwise,  $\phi_i > 0$  which provides evidence of collusion. Because only aggregate industrial data are available, we need to aggregate equation 4.10 over *n* processors . The condition for the industrial equilibrium is

$$\left[p_t(.) + y_t(.)\frac{\partial p_t(.)}{\partial y_t(.)} - AD_t - w_t - \frac{\partial c(.)}{\partial y_t(.)}\right](1 - \phi)\gamma - y_t(.)(1 + AD_t(.)) = 0 \quad (4.11)$$

where  $p_t(.)$  and  $w_t$  are frozen shrimp and fresh shrimp market price in period t.  $y_t$  is the total fresh shrimp supply in Thai shrimp market.  $\frac{\partial c(.)}{\partial y(.)}$  is an industrial non-material marginal cost, and  $\phi$  is the industrial average change in fresh shrimp all other processors responding equally to a change in the offer price by processor i price across firms.

Define the mark up as the difference between frozen shrimp price and fresh shrimp price,  $M_t = p_t - w_t$ . Equation 4.11 becomes

$$M_{t} = \frac{1}{(1-\phi)\gamma} y_{t}(.)(1+AD_{t}(.)) - y_{t} \frac{\partial p_{t}(.)}{\partial y_{t}(.)} + AD_{t} + \frac{\partial c(.)}{\partial y_{t}(.)}.$$
 (4.12)

This mark up equation will be estimated together with frozen shrimp demand and fresh shrimp supply to obtain evidence of collusion ( $\phi$ ).

Assume U.S. demand for Thai frozen shrimp and Thai supply of fresh shrimp, respectively, have a linear form. Thus,

$$p_t = \alpha_0 + \alpha_1 y_t + \alpha_2 A D_t + \alpha_m G + \epsilon \tag{4.13}$$

and

$$y_t = \beta_0 + \beta_1 w_t + \beta_m H + \upsilon \tag{4.14}$$

where  $\alpha_i$  and  $\beta_i$  are parameters to be estimated. Given demand and supply in equation 4.13 and 4.14, the partial derivatives are

$$\frac{\partial y_t(.)}{\partial w_t} = \beta_1 \tag{4.15}$$

and

$$\frac{\partial p_t(.)}{\partial y_t} = \alpha_1 \tag{4.16}$$

Furthermore, since the antidumping duty on Thai frozen shrimp does not

change during the study period, the AD variable is treated as a dummy variable;

$$AD_t = \begin{cases} 1 & \text{when the antidumping duty is imposed} \\ 0 & \text{otherwise.} \end{cases}$$
(4.17)

Thus, the  $AD_t$  variable does not capture only the effect of the U.S. antidumping duty on Thai frozen shrimp but capture the total effect of the U.S. antidumping duty impacting Thai shrimp market.

Also, assuming a quadratic non-material cost function, the total marginal nonmaterial cost  $\left(\frac{\partial c(.)}{\partial y_t(.)} + AD_t\right)$  is specified as

$$\frac{\partial c(.)}{\partial y_t(.)} + AD_t = \delta_0 + \delta_1 y_t + \delta_2 AD_t + \delta_m D + \eta$$
(4.18)

where D is a vector of exogenous factors which shift the non-material marginal cost, and  $\delta_i$  is a parameter to be estimated.  $\eta$  is an error term in non-material marginal cost function.

Substituting equation 4.15, 4.16, and 4.18 into the mark up equation 4.12, equation 4.12 becomes

$$M_{t} = \delta_{0} + \frac{\delta_{co}}{\beta_{1}} y_{t} (1 + AD_{t}) + \delta_{11} y_{t} + \delta_{2} AD_{t} + \delta_{m} D + \eta$$
(4.19)

where  $\delta_{co} = \frac{1}{(1-\phi)}$ , and  $\delta_{11} = \delta_1 - \alpha_1$ . However, it is not possible to disentangle  $\phi$  in equation 4.19 although the estimate for  $\beta_1$  can be obtained by estimating the supply equation separately. A rotation in the supply equation is needed to identify

 $\phi$  (Bresnahan (1982,1989), Azzam and Park (1993))<sup>11</sup>. Thus, an interaction term is included into the supply equation 4.14 involving w and an exogenous variable (average rainfall (R)). An exogenous non-market input which is rainfall is chosen as the supply rotator because higher level of rainfall yields higher production level of fresh shrimp resulting in a lower marginal cost while changing in other cost such as price of baby shrimp, wage, and price of diesel directly results in changing marginal costs by the absolute amount<sup>12</sup>. Thus, interacting the price of fresh shrimp with rainfall causes supply curve to rotate while interacting fresh shrimp price with other production costs will result to a shift in supply curve rather than rotation. The new supply function is

$$y_t = \beta_0 + \beta_1 w_t + \beta_2 w_t R_t + \beta_m H + \upsilon. \tag{4.20}$$

The change in fresh shrimp supply with respect to the price of fresh shrimp in equation 4.15 becomes

$$\frac{\partial y_t(.)}{\partial w_t} = \beta_1 + \beta_2 R_t. \tag{4.21}$$

Substitute equation 4.21 into the mark up equation 4.19:

$$M_t = \delta_0 + \delta_{co} y_t^* + \delta_{ad} y_{ad}^* + \delta_{11} y_t + \delta_2 A D_t + \delta_m D + \eta \tag{4.22}$$

where  $y_t^* = \frac{y_t}{\beta_1 + \beta_2 R_t}$  and  $y_{ad}^* = \frac{y_t}{\beta_1 + \beta_2 R_t} AD$ . The coefficient of  $y_t^*$  is an average measure of the degree of collusion in the input (fresh shrimp) market without an antidumping

<sup>&</sup>lt;sup>11</sup>The detail is already discussed in section 4.2

 $<sup>^{12}\</sup>mathrm{More}$  detail is discussed in Appendix B.2

effect, and the coefficient of  $y_{ad}^*$  is an average measure of the degree of collusion associated with the antidumping duty.<sup>13</sup> The coefficient of AD ( $\sigma_2$ ) measures the direct impact of the antidumping protection. Under the assumption that the degree of collusion is not affected by other exogenous variables in equation 4.22, The estimate can be obtained by estimating equations 4.13, 4.20, and 4.22. Note there is a mismatch between our theoretical and empirical analyses. As Proposition 4 lays out, in our theoretical model the threat of a certain and eternal anti-dumping duty in future periods affects collusion in periods before the penalty is actually applied, but not during the penalty stage. In contrast, our empirical analysis finds collusion is affected during the penalty stage. We are not troubled by this mismatch. As mentioned above, the theoretical model assumes the penalty is certain and eternal. In practice, the anti-dumping duty is an uncertain threat in the pre-penalty stage: even during the investigation phase, processors are uncertain whether they will face any dumping margin at all. Thus, the threat of future punishment is more credible once a dumping duty has already been imposed. Moreover, once inflicted, the dumping margin (duty level) is recalculated annually, and so, even during the penalty phase, processors are facing the threat of future penalties that are contingent on current behavior, with concomitant implications for collusion.

Although the error terms in demand, supply and the mark up equations are expected to be correlated, the three equations can still be estimated consistently using instruments for the included endogenous variables in these equations. As a

 $<sup>^{13}</sup>$  There is no incidence of collusion in downstream market; for example, the market share of Thai frozen shrimp in the U.S. market is only 17 -30 %, and price of Thai frozen shrimp declines during the study period (USITC 2009). Thus, we believe that the collusion estimate in this model only represents collusion in the upstream market.

result we can estimate fresh shrimp supply to obtain  $\beta_1$  and  $\beta_2$  which will be used to construct the exogenous variables  $(y_t^* \text{ and } y_{ad}^*)$  in the mark up equation.

#### 4.4 Data collection

Monthly data for the period from January 1996 to January 2009 are used for the estimation. The description of variables and the sources of data are as follows:

- 1.  $p_t$  =monthly price of Thai frozen shrimp in the U.S. by size (dollar/kg) Source: the U.S. International Trade Committee
- 2.  $y_t$  =monthly quantity of Thai frozen shrimp imported to the U.S. by size (kg) Source: the U.S. International Trade Committee
- 3.  $w_t$  =monthly price of fresh shrimp (Baht/kg) in Thai shrimp center market (Mahachai Market) by size and species Source: Charoen Pokphand Foods (CPF) group.
- 4. M = Processors' market mark up=p w
- 5. G =Another exogenous variable which shift the demand equation
  - xr =monthly exchange rate (Baht/1U.S.dollar) Source: Bank of Thailand
- 6. H = Other exogenous variables which shift the supply equation (use time lag<sup>14</sup>)
  - R = rainfall (mm) Source: Thai Meteorological Department

 $<sup>^{14}\</sup>mathrm{More}$  detail is in next section

- gas =monthly price of diesel (Baht/liter) Source: Petroleum division, Energy Policy and planning office, Ministry of Energy, Thailand
- wage = labor cost minimum wage rate (Baht/ day) Source: Thai Ministry of Labor
- w<sub>bs</sub> =price of baby shrimp (Baht / 1000head) Source: Charoen Pokphand
   Foods (CPF) group.
- 7. G = other exogenous variables which shift non-material marginal cost equation
  - gas =monthly price of diesel (Baht/liter) Source: Petroleum division, Energy Policy and planning office, Ministry of Energy, Thailand
  - wage = minimum wage rate (Baht/day) Source: Thai Ministry of Labor

### 4.5 Estimation procedure

The error terms from the frozen shrimp demand equation (4.13), fresh shrimp supply equation (4.20), and the mark up relation equation (4.22) are assumed to be correlated. However, since the estimates from the fresh shrimp supply equation are used to create the variables  $y_t^*$  and  $y_{ad}^*$  in the mark up relation equation, the fresh shrimp supply equation has to be estimated separately. Even though the supply equation is estimated separately, we still can obtain consistent estimates for the parameters in the supply equation if exogenous variables excluded from the supply equation are used as an instrument for the included endogenous variable  $(w_t)$  in supply equation. Therefore, the method of two-stage least squares is used to estimate the supply equation.

Several modifications for the supply equation are made. First, all data in Thai Baht is converted to U.S. dollar. The shipment process from Thailand to the U.S. takes about a month, so the quantity  $(y_t)$  used in the supply equation is a month lead quantity  $(lead_y)$ . Fresh shrimp price  $(w_t)$  data is specified by size and species while the quantity data is only specified by size; therefore, we do some modifications with the fresh shrimp price data as follows. The black tiger prawn (*Penaeus Monodon*) price is used in the estimation from the period 1996-2002 while the fresh shrimp price since 2005 used in the estimation is the white shrimp (*Penaeus Vanamei*) price. This is because black tiger prawns had dominated all other shrimp species in Thailand until 2002; white shrimp was commercially introduced<sup>15</sup> and became the major shrimp species after  $2005^{16}$ . Moreover, white shrimp also dominates the other in the export sector. During the transition period (2003 and 2004), the fresh shrimp price is weighted by the shrimp production proportion of each species. Ideally, we would like to have each species' production data by size and month. Unfortunately, the production data which we have is only annual production data by species. As a result, we have to assume that the production proportion of these two species is the same in each size and month. From the Office of Agricultural Economics Statistic Yearbook, the shrimp production was 194,909 and 132,365 for black tiger prawn and white shrimp respectively (or about 0.6 and 0.3 of the total shrimp production for black tiger prawn and white shrimp respectively). The black tiger prawn and white

 $<sup>^{15}\</sup>mathrm{The}$  white shrimp was first introduced in Thailand in 2002, but it was not commercial until 2003

<sup>&</sup>lt;sup>16</sup>In 2005, white shrimp production was 374,487 tons while black tiger prawn production was only 20,055 tons (Office of Agricultural Economics, 2008)

shrimp production in 2004 were 106,884 and 251,697 tons respectively (or about 0.3 and 0.7 of the total production). Thus, the fresh shrimp prices in the transition periods are weighted by using this ratio (black:white, 0.6:0.3 and 0.3:0.7 for 2003 and 2004 respectively). In addition, during 2003-2005, the production technology of white shrimp was not efficient enough to produce large shrimp (30 heads per kilogram), so the fresh shrimp price for the large shrimp (30 heads per kilogram) is the black tiger prawn. Moreover, to capture the difference in supply between shrimp species, we also include *specie* variable which is 1 during black tiger prawn culturing period, 2 during the transition period, and 3 during white shrimp culturing period.

The rainfall data is the monthly total rainfall by province. Since each province has a different area of shrimp farm, each province's rainfall is weighted by a ratio of province's shrimp farm area and the total shrimp area<sup>17</sup>. Therefore, the rainfall data used in the estimation is the sum of weighted monthly rainfall from 25 shrimp farming provinces. Time lag price of the baby shrimp  $(lag_w_{bs})$ , price of diesel  $(lag_gas)$ , wage  $(lag_wage)$ , and rainfall  $(lag_R)$  are used as supply shifters. Each shrimp size and species has different culture period; for example, it takes about six months to culture large black tiger prawn (30 heads per kilogram), but it takes only five months to culture large white shrimp. While the culture period for small black tiger prawn (80 heads per kilogram) is three months<sup>18</sup>. Thus, these supply shifters are average values of culture period<sup>19</sup>.

A necessary condition to identify the oligosony collusion parameter is to rotate

 $<sup>^{17}\</sup>mathrm{See}$  appendix B.1 for the provinces used in this estimation

<sup>&</sup>lt;sup>18</sup>More detail can be found from table B.1 in appendix B.3.

 $<sup>^{19}\</sup>mathrm{The}$  detail on culture period by size and year are in table B.2 in appendix B.3

the fresh shrimp supply curve. To do so, an interaction term between price of fresh shrimp  $w_t$  and an exogenous variable is added into the supply equation for fresh shrimp. Several experiments were done involving the interaction term such as  $lag_w_{bs}$  and  $lag_wage$ . The best fit was obtained when  $lag_R$  is interacted with  $w_t$ . To capture the month variation in the fresh shrimp supply, eleven month dummies  $(\_Imonth\_2-12)$  are included in the equation. We also include seven year dummies (1997-2008) to capture the year variation. Moreover, *trend* and *trend*<sup>2</sup> variables are included to capture trend effect in the supply equation. To capture the cost variation for different shrimp sizes, seven size dummies, for the second largest size to the smallest size, are also included into the fresh shrimp supply equation.

The price of Thai frozen shrimp in the U.S. market  $(P_t)$  is chosen to be an instrumental variable for price of fresh shrimp  $(w_t)$ . This is because most of Thai shrimp production is exported, so the domestic price of fresh shrimp is usually influenced by the export price. Moreover, it takes about a month for a shipment from Thailand to arrive at the U.S., so the price of frozen shrimp in the U.S. market is not affected by the domestic price of fresh shrimp in the same period. In addition, the AD dummy variable is used as another instrument to capture for the antidumping effect on fresh shrimp price. AD is equal to one when the antidumping duty is imposed.

The final estimating model for fresh shrimp supply is

$$lead_{-y} = \beta_0 + \beta_1 w + \beta_2 wR + \beta_3 lag_{-}w_{bs} + \beta_4 lag_{-}gas + \beta_5 lag_{-}wage + \beta_6 specie + \beta_{is}I_{-}size + \beta_{im}I_{-}month + \beta_yI_{-}year + \beta_7 trend + \beta_8 trend^2 + v(4.23)$$

Thus, eight specifications are experimented for the supply estimation

- 1. equation 4.23
- 2. equation 4.23 without  $lag\_gas$
- 3. equation 4.23 without lag\_labor
- 4. equation 4.23 without  $lag_{-}w_{bs}$
- 5. equation 4.23 without  $lag\_gas$  and  $lag\_wage$
- 6. equation 4.23 without  $lag\_gas$  and  $lag\_w_{bs}$
- 7. equation 4.23 without  $lag_wagw$  and  $lag_w_{bs}$
- 8. equation 4.23 without any cost of fresh shrimp production– $lag_gas$ ,  $lag_wage$ , and  $lag_w_{bs}$

Next, we construct the  $y_t^*$  and  $y_{ad}^*$  variables for the mark up relation equation (equation 4.22) using the estimates from specification (8).<sup>20</sup> Then we estimate U.S. demand for Thai frozen shrimp and the mark up relation separately by the using two-stage least squares method. Exogenous variables in the fresh shrimp supply (equation 4.23 specification (8)) are chosen to be instruments for the endogenous variable  $(y_t)$ .

Several modifications for the U.S. demand for Thai frozen shrimp are experimented with. First, we set all variables in frozen shrimp demand equation corresponding to the fresh shrimp supply. To do so, we use one month lead price and

 $<sup>^{20}\</sup>mathrm{As}$  we will see in the empirical results section that all cost of fresh shrimp production from other specifications provides unexpected signs

quantity of frozen shrimp (*lead\_p* and *lead\_y*), and exchange rate (*lead\_xr*) since the shipment from Thailand to the U.S. period is about a month. To capture the seasonal variation in the demand for frozen shrimp, three seasonal dummies (D2, D3, and D4) representing second, third, and fourth quarters of a year are included in the demand equation. Year dummies are included in the demand equation to capture the year variation in frozen shrimp demand. We also include a trend variable to capture the long term trends in demand equation<sup>21</sup>. Moreover, to capture the antidumping effect on the U.S. demand for Thai frozen shrimp, we include an AD dummy variable which is one when the duty is imposed and zero otherwise. To capture the demand variation for the different shrimp sizes, seven size dummies, from the second largest size to the smallest size, are also included into the frozen shrimp demand equation.

The final estimating model for frozen shrimp demand is

$$lead_p = \alpha_0 + \alpha_1 lead_y + \alpha_2 lead_x r + \alpha_3 AD + \alpha_4 trend + \alpha_{Di} Di + \alpha_{is} I_s ize + \alpha_{iy} I_y ear + \epsilon$$

$$(4.24)$$

Two specifications are experimented with for the demand equation

- 1. equation 4.24
- 2. substitute seasonal dummies by month dummies  $(\_Imonth\_2 12)$

There are some modifications being made in the mark up equation. First, due to the shipment process, the mark up (M), which is the difference between frozen

 $<sup>^{21}{\</sup>rm We}$  also included trend square variable in the demand equation, but it subsequently dropped from the model during the estimation.

shrimp and fresh shrimp prices, is modified as the difference between a month lead price of frozen shrimp and the current price of fresh shrimp ( $lead_p - w$ ). For the same reason, we use one-month lead constructed variable ( $lead_y^*$ ) in the mark up equation. Diesel price and wage are included to capture non-material cost of frozen shrimp production. Moreover, size, month, and year dummies are included to capture size, month, and year variations in non-material cost of frozen shrimp production. In addition, the dummy variable AD is used to capture the cost of facing antidumping duty where AD equals one if the antidumping duty is imposed and zero otherwise. We also include *trend* and *trend*<sup>2</sup> variables to capture for trend effect in the mark up equation.

The final estimating model for the mark up equation is

$$M = \delta_0 + \delta_{co} lead_y^* + \delta_{ad} lead_y^* + \delta_{11} lead_y + \delta_2 AD + \delta_3 gas + \delta_4 wage + \delta_5 trend + \delta_6 trend^2 + \delta_{is} I_s ize + \delta_{im} I_m on th + \delta_{iy} I_y ear + \eta.$$

$$(4.25)$$

Eight specifications are experienced for the mark up equation estimation

- 1. equation 4.25
- 2. equation 4.25 without gas
- 3. equation 4.25 without wage
- 4. equation 4.25 without AD
- 5. equation 4.25 without wage and gas

- 6. equation 4.25 without gas and AD
- 7. equation 4.25 without wage and AD
- 8. equation 4.25 without wage, gas, and AD

#### 4.6 Empirical results

#### 4.6.1 Fresh shrimp supply

The estimation results are in table  $4.1.^{22}$  All specifications result in a positive and significant coefficient for the fresh shrimp price (w) but results indicate a negative significant coefficient for the interaction term at 1% significance level. The supply elasticity is between 0.34 to 0.63 from all specifications.

The costs of fresh shrimp production (lag baby shrimp price  $(lag_w_{bs})$ , wage  $(lag_wage)$ , and diesel price  $(lag_gas)$ ) are expected to have negative impact on the fresh shrimp supply. However, the estimation results give positive signs for all costs.

- Lag baby shrimp price  $(lag_w_{bs})$ : The results show a significantly positive sign for all specifications (specification (1), (2), (3), and (5)) at the 1 % significance level. This might be because the lag price of baby shrimp also contains a baby shrimp demand element. Thereby, higher baby shrimp price means more baby shrimp in the baby shrimp market. Consequently, more baby shrimp results to more fresh shrimp production.
- Lag wage  $(lag\_wage)$ : The results show a negative sign in specification (1) <sup>22</sup>The coefficients of year and month dummies are not reported in this table

			Table 4	Table 4.1: Estimation results for Fresh shrimp supply	ion results	for Fresh si	hrimp supp	ly
				specification	cation			
	(1)	(2)	(3)	(4)	(5)	(9)	(2)	(8)
M	$225.0^{***}$	$261.2^{***}$	$228.0^{***}$	$214.4^{***}$	$263.9^{***}$	$216.6^{***}$	$247.6^{***}$	$250.2^{***}$
	(34.00)	(33.79)	(34.08)	(33.92)	(33.74)	(33.96)	(33.32)	(33.30)
lag_w_bs	$693.5^{***}$	$600.7^{***}$	$680.6^{***}$		$606.4^{***}$			
	(144.10)	(147.30)	(143.60)		(147.00)			
$lag_{-}gas$	$924.8^{***}$		$871.1^{***}$	$810.4^{***}$		$781.5^{***}$		
	(172.60)		(166.60)	(172.90)		(167.10)		
lag_wage	-68.07	25.31		-36.9			42.63	
	(64.05)	(63.45)		(64.15)			(63.40)	
lag_R	$1.683^{***}$	$1.805^{***}$	$1.670^{***}$	$1.591^{***}$	$1.825^{***}$	$1.587^{***}$	$1.710^{***}$	$1.735^{***}$
	(0.16)	(0.17)	(0.16)	(0.16)	(0.16)	(0.16)	(0.16)	(0.16)
$w^*R$	$-1.210^{***}$		$-1.194^{***}$	-1.144***	$-1.292^{***}$	$-1.137^{***}$		-1.232***
	(0.14)	(0.14)	(0.13)	(0.14)	(0.14)	(0.13)		(0.13)
specie	$337.8^{***}$		$335.0^{***}$	$368.6^{***}$	$305.2^{***}$	$366.9^{***}$		$336.7^{***}$
	(32.77)		(32.62)	(32.61)	(33.17)	(32.36)		(32.70)
trend	20.46	3.344	$57.04^{**}$	1.498	18.26	18.71		$45.83^{**}$
	(42.06)	(42.99)	(24.27)	(42.10)	(20.34)	(23.06)		(19.13)
$trend^2$	-6.788**	-0.635	-8.942***	$-6.170^{**}$	-0.363	$-7.349^{***}$		-0.936
	(2.95)	(2.79)	(2.11)	(2.97)	(1.19)	(2.10)	(2.79)	(1.18)

.

				specification	cation			
	(1)	(2)	(3)	(4)	(5)	(9)	(2)	(8)
Isize_6	$412.9^{***}$	$481.5^{***}$	$420.3^{***}$	$394.6^{***}$	$484.5^{***}$	$399.6^{***}$	$457.6^{***}$	$459.7^{***}$
	(59.36)	(58.84)	(59.44)	(59.29)	(58.93)	(59.28)	(58.13)	(58.23)
Isize_9	$604.7^{***}$	$697.8^{***}$	$617.2^{***}$	$586.0^{***}$	$700.6^{***}$	$593.9^{***}$	$670.7^{***}$	$671.7^{***}$
	(73.58)	(72.46)	(73.55)	(73.62)	(72.64)	(73.41)	(71.67)	(71.82)
$Isize_12$	$1137^{***}$	$1255^{***}$	$1151^{***}$	$1111^{***}$	$1260^{***}$	$1120^{***}$	$1219^{***}$	$1222^{***}$
	(94.19)	(92.73)	(94.29)	(94.15)	(92.93)	(94.05)	(91.60)	(91.79)
$Isize_{-15}$	$1463^{***}$	$1603^{***}$	$1478^{***}$	$1428^{***}$	$1609^{***}$	$1438^{***}$	$1557^{***}$	$1561^{***}$
	(110.80)	(108.90)	(110.90)	(110.50)	(109.10)	(110.50)	(107.40)	(107.60)
Isize_18	$1223^{***}$	$1383^{***}$	$1239^{***}$	$1181^{***}$	$1391^{***}$	$1192^{***}$	$1329^{***}$	$1335^{***}$
	(127.10)	(125.00)	(127.40)	(126.80)	(125.20)	(126.80)	(123.20)	(123.40)
Isize_21	$1291^{***}$	$1460^{***}$	$1309^{***}$	$1249^{***}$	$1468^{***}$	$1261^{***}$	$1404^{***}$	$1410^{***}$
	(132.40)	(130.10)	(132.60)	(132.20)	(130.30)	(132.10)	(128.20)	(128.50)
Isize_24	$1008^{***}$	$1184^{***}$	$1027^{***}$	$966.3^{***}$	$1191^{***}$	$979.2^{***}$	$1127^{***}$	$1132^{***}$
	(136.40)	(133.90)	(136.60)	(136.20)	(134.20)	(136.10)	(132.10)	(132.30)
Constant	-1983***	$-2462^{***}$	-2357***	$-1680^{***}$	$-2346^{***}$	-1891***	-2141***	$-1927^{***}$
	(440.40)	(439.80)	(296.60)	(434.90)	(304.80)	(272.50)	(428.90)	(278.10)
Observations	1002	1002	1092	1092	1092	1092	1002	1002
R-squared	0.602	0.579	0.601	0.596	0.578	0.595	0.577	0.576
Standard errors in parentheses * * * p < 0.01, * * p < 0.05, * p < 0.1	$\sin parenth * p < 0.05,$	leses $*p < 0.1$						

and (4) while other specifications (specification (2) and (7)) give a positive sign. However, none of the results are significant. The positive sign on the wage coefficient might be because the wage variable also contains fresh shrimp demand element. Higher wage results in more purchase power for people.

Diesel price (lag\_gas): All specifications (specification (1), (3), (4), and (6))
) give a positive sign. This may be a result of failure to account for diesel used for running machinery in shrimp farms; large shrimp farms usually use electricity to run their machines rater than diesel. However, none of the results are significant.

Rainfall (R) is expected to have positive impact on fresh shrimp supply. The results are as expected. The coefficients are significantly positive in all specifications at the 1 % significance level. The species dummy is expected to be positive because white shrimp yields better production than black tiger prawn. The results are as expected. The coefficient is positive and significant in all specifications at the 1% significance level.

Size of shrimp has a significant impact on fresh shrimp supply. All coefficients for shrimp size are significantly positive at the 1% significance level. Thus, cost of fresh shrimp production for the small size is lower than the large size<sup>23</sup>.

 $<sup>^{23}\</sup>mathrm{Recall},$  for the size dummies, the dummy for the largest size is omitted.

#### 4.6.2 Frozen shrimp demand

The estimation results are shown in table 4.2.<sup>24</sup> As expected, the frozen shrimp quantity  $(lead_y)$  has a negative sign on the frozen shrimp price in the demand equation. The coefficients are significant for both specifications at 1 % and 5 % significance levels. The exchange rate  $(lead_xr)$  impact is also as expected, although the coefficients are insignificant in both specifications. That exchange rate depreciation results in a lower price for frozen shrimp in the U.S. market.

The AD dummy has an insignificant negative impact on price of frozen shrimp in the U.S. market. This is consist with our assumption that when the antidumping duty is imposed, the processors (exporters) lower the price of fresh shrimp in order to keep them being competitive in the U.S. frozen shrimp market. As a result, the price of frozen shrimp does not increase when the duty is imposed.

The trend variable is significantly negative for the both specifications at the 1 % significance level. This implies a decline in the price of Thai frozen shrimp in the U.S. over the study period. The dummies for size of shrimp are significantly negative on price of frozen shrimp for both specifications at the 1 % significance level. The smaller shrimp size, the lower the price of frozen shrimp.

#### 4.6.3 Mark up

The estimation results are shown in table 4.3.<sup>25</sup> The mark up equation contains the oligopsony mark up ( $\delta_{co} lead_{-}y^*$  and  $\delta_{ad} lead_{-}y^*_{ad}$ ) and the marginal non-material

 $<sup>^{24}\</sup>mathrm{The}$  coefficients of year and month dummies are not reported in this table

 $<sup>^{25}</sup>$ The coefficients of year and month dummies are not reported in this table

	specifi	cation
	(1)	(2)
lead_y	-0.00112***	-0.00158***
	(0.0003)	(0.0005)
lead_xr	-0.0614***	-0.0557***
	(0.01)	(0.02)
ad	-0.131	-0.12
	(0.24)	(0.27)
trend	-0.222***	-0.240***
	(0.03)	(0.03)
Isize_6	-2.080***	-2.050***
	(0.10)	(0.11)
Isize_9	-2.924***	-2.823***
	(0.12)	(0.15)
$Isize_{-12}$	-4.024***	-3.756***
	(0.20)	(0.29)
$Isize_{-15}$	-5.196***	-4.845***
	(0.25)	(0.37)
$Isize_{-18}$	-6.606***	-6.434***
	(0.15)	(0.20)
$Isize_21$	-7.176***	-6.979***
	(0.16)	(0.22)
$Isize_24$	-7.991***	-7.928***
	(0.11)	(0.13)
Constant	$13.70^{***}$	$13.99^{***}$
	(0.40)	(0.48)
Observations	1092	1092
R-squared	0.935	0.929
Standard errors		

Table 4.2: Estimation results for U.S. demand for Thai frozen shrimp

Standard errors in parentheses \*\*\*p < 0.01, \*\*p < 0.05, \*p < 0.1

cost of processing frozen shrimp. The marginal processing cost is expressed as a function of  $lead_y$ , gas, and wage. The non-material costs are expected to have a positive impact on the mark up. The coefficient on wage is positive in all specifications, but it is not significant in any specifications. On the other hand, the coefficient on gas is insignificantly negative. This may be a result of a failure to account for transportation costs in frozen shrimp industry. Time is entered as a *trend* variable into the model to capture the long-term trend in the marginal processing cost. A negative coefficient indicates a decline in the marginal cost of processing over the study period; however, the coefficient is not significant. The coefficient for the  $lead_y$  in the mark up equation represents the slope of the marginal processing cost. This coefficient is significantly positive at the 1 % significance level. This implies an increasing marginal cost of processing frozen shrimp over the study period.

Also, the coefficient on AD, which captures the direct effect of the antidumping duty on the mark up equation, has a positive sign in most of the specifications except the specification (5). However, none of the specification are significant. Shrimp size dummies significantly have a negative impact on the mark up at the 1 % significance level which means the largest shrimp have the highest mark up.

The coefficient on  $lead_y^*$  is significantly positive in most specifications at the 5 % and 10 % significance levels except in specification (2). This indicates that Thai shrimp exporters are colluding in Thai shrimp market regardless of the presence of the antidumping duty. Finally, the coefficient on  $lead_y^*_{ad}$  is positive and significant at 1 % significance level. This implies that the antidumping duty results in more

			Table 4.3: Estimation results for the mark up equation	stimation res	ults for the r	mark up equa	ation	
	2	ć		specification	cation		Ĩ	
	(1)	(2)	(3)	(4)	(5)	(9)	(2)	(8)
$lead_y^*$	$1.157^{**}$	0.73	$1.063^{**}$	$1.169^{**}$	$0.886^{*}$	$0.766^{*}$	$1.085^{**}$	$0.879^{**}$
	(0.51)	(0.50)	(0.51)	(0.46)	(0.50)	(0.44)	(0.46)	(0.44)
$lead_y*_{ad}$	$3.544^{***}$	$3.197^{***}$	$3.474^{***}$	$3.531^{***}$	$3.305^{***}$	$3.189^{***}$	$3.454^{***}$	$3.304^{***}$
	(0.59)	(0.56)	(0.58)	(0.56)	(0.56)	(0.54)	(0.55)	(0.54)
lead_y	$0.00181^{***}$	$0.00134^{***}$	$0.00171^{***}$	$0.00181^{***}$	$0.00150^{***}$	$0.00136^{***}$	$0.00171^{***}$	$0.00150^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gas	-0.738		-0.663	-0.734			-0.66	
	(0.48)		(0.47)	(0.48)			(0.47)	
wage	0.00544	0.00686		0.00431		0.00322		
	(0.16)	(0.15)		(0.15)		(0.15)		
ad	0.0185	0.0327	0.0309		-0.00419			
	(0.31)	(0.29)	(0.30)		(0.29)			
$\operatorname{trend}$	-0.064	-0.00833	-0.0479	-0.0668	-0.0211	-0.0143	-0.0502	-0.0212
	(0.12)	(0.11)	(0.08)	(0.11)	(0.08)	(0.10)	(0.08)	(0.07)
$trend^2$	0.00808	0.00145	0.00659	0.00837	0.00253	0.00205	0.00693	0.00251
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)

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	(1)	(2)	(3)	(4)	(5)	(9)	(2)	(8)
Isize_6			$-0.514^{***}$	$-0.519^{***}$	-0.504***	-0.497***	$-0.514^{***}$	$-0.504^{***}$
	(0.11)		(0.11)	(0.11)	(0.10)	(0.10)	(0.11)	(0.10)
Isize_9	-0.833***		$-0.815^{***}$	-0.833***	-0.778***	$-0.752^{***}$	$-0.815^{***}$	-0.777***
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
Isize_12	-2.181***		$-2.135^{***}$	-2.182***	$-2.040^{***}$	-1.974***	-2.137***	-2.038***
	(0.20)		(0.19)	(0.20)	(0.19)	(0.19)	(0.19)	(0.19)
Isize_15	-3.253***		$-3.196^{***}$	-3.253***	-3.082***	$-3.001^{***}$	$-3.198^{***}$	$-3.079^{***}$
	(0.23)		(0.23)	(0.23)	(0.22)	(0.22)	(0.22)	(0.22)
Isize_18	-3.319***	-3.207***	-3.295***	$-3.319^{***}$	-3.246***	-3.212***	-3.296***	$-3.245^{***}$
	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)
Isize_21	-3.640***	$-3.515^{***}$	$-3.613^{***}$	$-3.640^{***}$	-3.558***	-3.520***	$-3.614^{***}$	-3.557***
	(0.15)	(0.14)	(0.15)	(0.15)	(0.15)	(0.14)	(0.15)	(0.14)
Isize_24	-3.582***	$-3.549^{***}$	$-3.574^{***}$	-3.582***	-3.560***	-3.550***	-3.575***	-3.560***
	(0.12)	(0.11)	(0.11)	(0.12)	(0.11)	(0.11)	(0.11)	(0.11)
Constant	$5.502^{***}$	$5.286^{***}$	$5.505^{***}$	$5.507^{***}$	$5.307^{***}$	$5.304^{***}$	$5.503^{***}$	$5.308^{***}$
	(0.86)	(0.82)	(0.31)	(0.86)	(0.27)	(0.82)	(0.31)	(0.27)
Observations	1092	1092	1092	1092	1092	1092	1092	1092
R-squared	0.698	0.73	0.705	0.697	0.719	0.728	0.705	0.72
$\begin{array}{l} \hline \text{Standard errors in} \\ ***p < 0.01, **p \end{array}$		parentheses $< 0.05, *p < 0.1$						

colluding among the exporters in Thai shrimp market.

#### Chapter 5

#### Summary and conclusion

The objective of this dissertation is to develop a model capable of examining how the antidumping duty influences collusive behavior in an upstream market. The motivation comes from the fact that the price of Thai frozen shrimp in the U.S. market does not increase after the imposition of the U.S. antidumping duty on frozen shrimp imports from Thailand along with other five shrimp exporting countries. Moreover, the four-firm concentration ratio of Thai shrimp exporters increased from 38.88% to 65.23% after the imposition of the duty. One would suspect collusion and market power of Thai shrimp exporters (processors) within the Thai shrimp market, and whether the imposition of the antidumping duty results in a higher degree of collusion among Thai shrimp exporters.

Opposite from previous studies in the antidumping effect on collusion, this dissertation focuses on collusion in the upstream (input) market. We employ a Bertrand duopoly model with infinite periods to investigate the effect of the antidumping duty on collusive behavior. With perfect monitoring assumption, we find that the exporters (processors) will fully cooperate and behave as a monopsony in their input market under a certain set of the discount rates. This set of the discount rate is not affected by an exogenous tariff or the antidumping duty being imposed on downstream goods. We suspect that this is because of the linearity in input supply and output demand assumption imposed in this model.

However, although the discount rate is not sufficiently high enough to support the fully cooperative behavior, the exporters (processors) are still able to exercise the collusive behavior through a self-enforcing agreement. The self-enforcing agreement prevents the processors to deviate from the collusive behavior by restricting the gain from deviation to be less than the punishment in the future. We refer the cooperation with a self-enforcing agreement to the constrained cooperative behavior. Under the constrained cooperative behavior, the processors are more willing to cooperate when the discount rate is high; this is because the punishment in the future becomes more severe. On the other hand, the exogenous tariff on the downstream goods results in the processors being less willing to cooperate due to the fact that it decreases the punishment in the future. Under the antidumping duty, with the self-enforcing agreement in future periods, the full cooperation among the exporters (processors) in the initial period is feasible under a certain set of the discount rate. We refer this cooperation to the restricted full-cooperative behavior. Under the assumption that the antidumping duty depends only on the exporters' (processors') initial behaviors, the set of discount rates which supports the restricted full-cooperation is larger than the one supporting the full cooperation under free trade. Thus, it is easier to sustain collusion under the antidumping duty. In other words, the threat of the antidumping duty on downstream goods is pro-collusive in the upstream market.

We test our theoretical findings by applying empirical models to the Thai shrimp industry during 1996-2009. The fresh shrimp production data is not sufficient enough to estimate fresh shrimp supply which as prevented previous studies to examine the Thai shrimp market structure. With fixed-proportion technology assumption, we can estimate for Thai fresh shrimp supply using import/export frozen shrimp data as fresh shrimp production. Thus, this dissertation is able to study the structure of the Thai shrimp market. We estimate the system of Thai fresh shrimp supply, the U.S. demand for Thai frozen shrimp, and the mark up equations. We use the comparative static in supply approach with an interaction between fresh shrimp price and rainfall as a supply rotator to identify collusive behavor. The effect of antidumping on the exporters' (processors') collusion in the Thai shrimp market is empirically tested by estimating a mark up which is the difference between frozen and fresh shrimp prices. Our empirical results confirm the theoretical finding; the antidumping duty significantly increases the degree of collusion among Thai shrimp exporters (processors) at the 1 % significant level.

## Appendix A

Cooperation under Antidumping case

## A.1 The constrained cooperative price under Antidumping case

$$w^{cc}(\overline{m},\beta) = \frac{2(1+k^2)(2(k(a-k)+(1+k)\mu)-d)-1)-\beta A_6}{8(1+k^2)^2+2B(2+(-1+k)k)(2+k+3k^2)} + R(\lambda^{ad},\beta,a,d,k,\overline{m})$$
  
where  $R(\lambda^{ad},\beta,a,d,k,\overline{m}) = \frac{\beta[\beta^2 R_1 + \beta R_2 + R_3] + \lambda^{ad}[R_4 + \beta(\beta^2 R_5 + \beta R_6 + R_7)]}{R_8}$ 

where

$$R_{1} = -16k(1+k)(1+k^{2})(1+2k^{2})(15+k(-7+4k(10-3k+6k^{2})))(15+k(7+12k(6+k+6k^{2})))(-2+2d(2+k+k^{2}+2k^{3})-k(1+k+2k^{2}+a(8-2k+6k^{2})-4(1+k^{2})\overline{m}))$$

$$R_{2} = -16(1+k)(1+k^{2})(1+2k^{2})(k(15+56k^{2}+48k^{4})^{2}(-2+2d(2+k+k^{2}+2k^{3})-k(1+k+2k^{2}+a(8-2k+6k^{2})-4(1+k^{2})\overline{m})) + 8(1+k^{2})^{2}(15+k(-7+4k(10-3k+6k^{2})))(15+k(7+12k(6+k+6k^{2})))\mu)$$

 $R_3 = -128(1+k)(1+k^2)^3(1+2k^2)(15+56k^2+48k^4)^2\mu$ 

$$R_4 = 4(1+k^2)^2(3+4k^2)^2(5+12k^2)^2((-1+2d-2ak)(3+4k^2)+2(11+k(13+2k(17+2k(11+6k+8k^2))))\mu$$

 $R_5 = -2(1+k^2)(-75(18+(67+36a)k) + 2d(1350+k(5025+k(23526+k(48833+2k(73939+2k(49315+2k(59308+k(56067+2k(53499+2k(20594+k(26933+2k(59338+k(56067+2k(53499+2k(20594+k(26933+2k(59338+k(56067+2k(53499+2k(20594+k(26933+2k(59338+k(56067+2k(53499+2k(20594+k(26933+2k(59338+k(56067+2k(53499+2k(20594+k(26933+2k(56067+2k(53499+2k(20594+k(26933+2k(56067+2k(53499+2k(20594+k(26933+2k(56067+2k(56067+2k(53499+2k(20594+k(26933+2k(56067+2k(56067+2k(53499+2k(20594+2k(26933+2k(56067+2k))))))))))))))))))))$ 

# A.2 The critical discount rate which supports restricted full cooperation under the antidumping duty

$$\beta^{ad} = \frac{1}{12B_2} \left[ B_2 \left[ 6\sqrt{\Gamma_1 - 2\Gamma_2 - \Gamma_3 + \Gamma_5} - \sqrt{3}\sqrt{\Gamma_4 + 4\Gamma_2 + 2\Gamma_3} \right] - 3B_1 \right]$$

where

$$\Gamma_{1} = \frac{3B_{1}^{2}}{B_{2}^{2}} - \frac{6B_{3}}{B_{2}} - \frac{2B_{4}}{B_{5}}$$

$$\Gamma_{2} = \frac{2^{1/3} (B_{4}^{2} - 3B_{6}B_{7} + 96B_{5}B_{8})}{B_{5} [\Gamma_{6} + \sqrt{\Gamma_{7}}]^{1/2}}$$

$$\Gamma_{3} = \frac{2^{2/3} [\Gamma_{6} + \sqrt{\Gamma_{7}}]}{B_{5}}$$

$$\Gamma_{4} = \frac{3B_{1}^{2}}{B_{2}^{2}} - \frac{12B_{3}}{B_{2}} + \frac{4B_{4}}{B_{5}}$$

$$\Gamma_{5} = \frac{3\sqrt{3}(B_{1}^{3}+4B_{1}B_{2}B_{3}+8B_{2}^{2}B_{9})}{B^{3}\sqrt{\Gamma_{4}+4\Gamma_{2}+2\Gamma_{3}}}$$

$$\Gamma_{6} = 2B_{3}^{2} - 9B_{4}B_{6}B_{7} + 27B_{5}B_{7}^{2} - 576B_{4}B_{5}B_{8} + 216B_{6}^{2}B_{8}$$

$$\Gamma_{7} = 2\left[B_{4}^{3} - 9B_{4}\left(B_{6}B_{7} + 64B_{5}B_{8}\right) + 27\left(B_{5}B_{7}^{2} + 8B_{6}^{2}B_{8}\right)\right]^{2} - 4\left(B_{4}^{2} - 3B_{6}B_{7} + 96B_{5}B_{8}\right)^{3}.$$
Where

 $2752224750dk + 19696750533k^2 + 3173349150ak^2 - 39393501066dk^2 + 30596383278k^3 + 305963883278k^3 + 305963883278k^3 + 305966383278k^3 + 305966383278k^3 + 30596666k^3 + 3059666k^3 + 3059666k^3 + 305966k^3 + 305966k^3 + 30596k^3 + 30596k^3 + 30596k^3 + 3056k^3 + 30596k^3 + 3056k^3 +$  $311173858335k^5 + 391050013744ak^5 - 622347716670dk^5 + 1126353955653k^6 + 632924930814ak^6 - 632924938646 - 632924930866 - 6329249308066 - 63292498066 - 63292498066 - 63292498066 - 6329248066 - 63292466 - 632924866 - 63292466 - 6329266 - 632966 - 6329266 - 63296 - 632966 - 6$  $2252707911306dk^{6} + 1916763902476k^{7} + 2305284805422ak^{7} - 3833527804952dk^{7} + 2305284805422ak^{7} - 383527804952dk^{7} + 2305284805422ak^{7} - 3833527804952dk^{7} + 383527804952dk^{7} + 3835626k^{7} + 383562k^{7} + 38356k^{7} + 3836k^{7} + 386k^{7} + 386$  $4557099703184k^8 + 3855367218984ak^8 - 9114199406368dk^8 + 7974370802096k^9 +$  $9294172066848ak^9 - 15948741604192dk^9 + 13496622361744k^{10} + 15977513583824ak^{10} - 15948741604192dk^9 + 15977513583824ak^{10} - 15948744k^{10} + 15977513583824ak^{10} - 1594874k^{10} + 15977513583824ak^{10} - 159775648764ak^{10} - 159775648764ak^{10} - 159775648764ak^{10} - 15977564864ak^{10} - 15977664ak^{10} - 15977664ak^{10} - 15977664ak^{10} - 15977664ak^{10} - 15977664ak^{10} - 15977664ak^{10} - 15977664864ak^{10} - 1597766444444k^{10} - 15977664444k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 1597764k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 15977644k^{10} - 1597764k^{10} - 1597764k$  $26993244723488dk^{10} + 23625746390272k^{11} + 27373288859168ak^{11} - 47251492780544dk^{11} + 27373288859168ak^{11} - 472514928644k^{11} + 27373288859168ak^{11} - 47251484k^{11} + 27373288859168ak^{11} - 47251484k^{11} + 27373288859168ak^{11} - 47251484k^{11} + 27373288859168ak^{11} - 47251484k^{11} + 273784k^{11} +$  $82088647890944k^{16} + 163092907271680ak^{16} - 164177295781888dk^{16} + 92472623546368k^{17} + 924726235468k^{17} + 92472623546368k^{17} + 924726235468k^{17} + 924726235468k^{17} + 924726235468k^{17} + 924726235468k^{17} + 924726235468k^{17} + 924726288k^{17} + 9247268k^{17} + 9247268k^{17} + 924726k^{17} + 926k^{17} + 926k^$  $162579262223360ak^{17} - 184945247092736dk^{17} + 101352920190976k^{18} + 186930614931456ak^{18} - 18693064ak^{18} - 1869306146ak^{18} - 18693064k^{18} - 18693064k^{18} - 18693064k^{18} - 18693064k^{18} - 18604k^{18} - 18693064k^{18} - 18693064k^{18} - 18604k^{18} - 18693064k^{18} - 18694k^{18} - 18693064k^{18} - 18693064k^{18} - 18694k^{18} - 18693064k^{18} - 18694k^{18} - 18694k^{18} - 18694k^{18} - 18604k^{18} - 18694k^{18} - 18604k^{18} - 1860$  $103927975456768k^{20} + 144972302475264ak^{20} - 207855950913536dk^{20} + 29201212571648k^{21} + 2920124k^{21} + 2920124k^{21} + 292012k^{21} + 2920k^{21} + 2920k^{21} + 2920k^{21} + 2920k^{21} + 2920k^{21} + 2920k^{21} + 2920k$ 

$$\begin{split} B_2 &= 933120000 - 1620000a - 1866240000d + 1387683225k + 1863416700ak - 2775366450dk + \\ 19865768587k^2 + 2753912250ak^2 - 39731537174dk^2 + 31357014322k^3 + 39677789234ak^3 - \\ 62714028644dk^3 + 193436007554k^4 + 62601470684ak^4 - 386872015108dk^4 + 318937179489k^5 + \\ 386441044520ak^5 - 637874358978dk^5 + 1139776168339k^6 + 637593799626ak^6 - 2279552336678dk^6 + \\ 1933557306100k^7 + 2277665843162ak^7 - 3867114612200dk^7 + 4532526402728k^8 + \\ 3866814055376ak^8 - 9065052805456dk^8 + 7782806808384k^9 + 9060315655840ak^9 - \\ 15565613616768dk^9 + 12862918397704k^{10} + 15565263137936ak^{10} - 25725836795408dk^{10} + \\ 21869273681824k^{11} + 25720029259664ak^{11} - 43738547363648dk^{11} + 26963797570352k^{12} + \\ 43735689252192ak^{12} - 53927595140704dk^{12} + 43713072201344k^{13} + 53929566112544ak^{13} - \\ \end{split}$$

$$\begin{split} 111466552098816ak^{25} + 29321009823744dk^{25} + 27099591081984k^{26} - 28926063673344ak^{26} - \\ 54199182163968dk^{26} - 9195139104768k^{27} + 54003492716544ak^{27} + 18390278209536dk^{27} + \\ 9078240706560k^{28} - 18332182904832ak^{28} - 18156481413120dk^{28} - 2772267171840k^{29} + \\ 18138135527424ak^{29} + 5544534343680dk^{29} + 1883510931456k^{30} - 5544534343680ak^{30} - \\ 3767021862912dk^{30} - 342456532992k^{31} + 3767021862912ak^{31} + 684913065984dk^{31} + \\ 183458856960k^{32} - 684913065984ak^{32} - 366917713920dk^{32} + 366917713920ak^{33} - \\ 25920000\overline{m} - 71081100k\overline{m} - 492205500k^{2}\overline{m} - 1488167880k^{3}\overline{m} - 4371444600k^{4}\overline{m} - \\ 13940030716k^{5}\overline{m} - 24517244860k^{6}\overline{m} - 77094138976k^{7}\overline{m} - 98933553008k^{8}\overline{m} - 278906213488k^{9}\overline{m} - \\ 307678193120k^{10}\overline{m} - 687722328800k^{11}\overline{m} - 759101160640k^{12}\overline{m} - 1156674930176k^{13}\overline{m} - \\ 1486687130624k^{14}\overline{m} - 1255003000320k^{15}\overline{m} - 2276069248000k^{16}\overline{m} - 678035689472k^{17}\overline{m} - \\ 2663403528192k^{18}\overline{m} + 227750682624k^{19}\overline{m} - 2317472612352k^{20}\overline{m} + 733111123968k^{21}\overline{m} - \\ 1445113626624k^{22}\overline{m} + 598297411584k^{23}\overline{m} - 609512325120k^{24}\overline{m} + 239006121984k^{25}\overline{m} - \\ 155940028416k^{26}\overline{m} + 39749419008k^{27}\overline{m} - 18345885696k^{28}\overline{m} \end{split}$$

 $87426144402688dk^{13} + 43026356461056k^{14} + 87414702434048ak^{14} - 86052712922112dk^{14} + 87414648ak^{14} - 8605271292444 + 8741444 + 87414444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 8741444 + 87444$  $114755127975936dk^{17} + 58841687000064k^{18} + 114725504987136ak^{18} - 117683374000128dk^{18} + 114725504987136ak^{18} - 117683740000128dk^{18} + 11472564ak^{18} - 1166864ak^{18} + 11472564ak^{18} - 1166864ak^{18} - 1166864ak^{18} + 11472564ak^{18} - 1166864ak^{18} - 116684ak^{18} - 1166864ak^{18} - 116684ak^{18} - 116644ak^{18} - 116684ak^{18} - 116684ak^{18} - 11664ak^{18} - 116684ak^{18} - 116684ak^{18} - 11664ak^{18} - 116684ak^{18} - 11664k^{18} - 11664ak^{18} - 116$  $26917181825024k^{19} + 117717099694080ak^{19} - 53834363650048dk^{19} + 58084567269376k^{20} + 580845676k^{20} + 58084566k^{20} + 5808456k^{20} + 580846k^{20} + 58084k^{20} + 580846k^{20} + 580846k^{20} + 58084k^$  $57655054172160ak^{24} - 78695201636352dk^{24} - 24805789138944k^{25} + 78695774945280ak^{25} + 78695766ak^{25} + 78666ak^{25} + 786666ak^{25} + 786666ak^{25} + 786666ak^{25} + 7866666ak^{25} + 786666ak^{25} + 786666ak^{2$  $49611578277888dk^{25} + 21742549991424k^{26} - 49611578277888ak^{26} - 43485099982848dk^{26} - 43485099864k^{26} - 434850986k^{26} - 434850986k^{26} - 43485086k^{26} - 434850k^{26} - 43686k^{26} - 434850k^{26} - 436850k^{26} - 43686k^{26}$  $12149053194240k^{27} + 43485099982848ak^{27} + 24298106388480dk^{27} + 7822736621568k^{28} - 646646k^{27} + 646646k^{27} + 64664k^{27} + 64664k^{27} + 6464k^{27} + 6464k^{2$  $24298106388480ak^{28} - 15645473243136dk^{28} - 3424565329920k^{29} + 15645473243136ak^{29} + 156666k^{29} + 15666k^{29} + 15666k^{29} + 1566k^{29} + 1566k^{29}$  $6849130659840dk^{29} + 1541054398464k^{30} - 6849130659840ak^{30} - 3082108796928dk^{30} - 6849130659840ak^{30} - 6849140ak^{30} - 684914ak^{30} - 684844k^{30} - 684844k^{30} - 68484k^{30} - 68484k^{30} - 68484k^{30} - 68484k^{30} - 684844k^{30} - 68484k^{30} - 68484k^{30} - 68484k^{30} - 68484k^{30} -$  $440301256704k^{31} + 3082108796928ak^{31} + 880602513408dk^{31} + 110075314176k^{32} -$  $880602513408ak^{32} - 220150628352dk^{32} + 220150628352ak^{33} + 1620000\overline{m} + 4443300k\overline{m} + 444330k\overline{m} + 444330k\overline{m} + 444330k\overline{m} + 44$  $25897500k^{2}\overline{m} + 79645440k^{3}\overline{m} + 192203400k^{4}\overline{m} + 623173988k^{5}\overline{m} + 903733340k^{6}\overline{m} + 90373340k^{6}\overline{m} + 9037334k^{6}\overline{m} + 903734k^{6}\overline{m} + 90374k^{6}\overline{m} + 90374k^{6}\overline{m} + 90374k^{6}\overline{m} + 9038k^{6}\overline{m} + 908k^{6}\overline{m} + 908k^{6}\overline{m} + 90k^{6}\overline{m} + 90k^{6}\overline{m} + 90k^{6}\overline{m} +$  $2790226856k^{7}\overline{m} + 3090783680k^{8}\overline{m} + 7827933296k^{9}\overline{m} + 8178412128k^{10}\overline{m} + 13985947872k^{11}\overline{m} + 1398594782k^{11}\overline{m} + 1398594782k^{11}\overline{m} + 139859478k^{11}\overline{m} + 139859478k^{11}\overline{m} + 1398594k^{11}\overline{m} + 1398594k^{11}\overline{m} + 1398594k^{11}\overline{m} + 1398594k^{11}\overline{m} + 139858k^{11}\overline{m} + 1388k^{11}\overline{m} + 138k^{11}\overline{m} + 13k^{11}\overline{m} + 13k^{11}\overline{m} + 13k^{11}\overline{m} + 13k^{11}\overline{m} +$  $16844059328k^{12}\overline{m} + 14873087488k^{13}\overline{m} + 26315056128k^{14}\overline{m} + 6211428864k^{15}\overline{m} + 30065071104k^{16}\overline{m} - 6211448k^{16}\overline{m} + 30065071104k^{16}\overline{m} - 621144k^{16}\overline{m} + 30065071104k^{16}\overline{m} - 621144k^{16}\overline{m} + 30065071104k^{16}\overline{m} - 621144k^{16}\overline{m} - 621144k^{16}\overline{m} + 30065071104k^{16}\overline{m} + 3006507\overline{m} + 3006507\overline{m} + 3006507\overline{m} + 3006507\overline{m} +$  $5545562112k^{17}\overline{m} + 24077426688k^{18}\overline{m} - 9648267264k^{19}\overline{m} + 12746833920k^{20}\overline{m} - 5605687296k^{21}\overline{m} + 1274688k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 1274688k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 1274688k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 127468k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 127468k^{21}\overline{m} - 5605687296k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 127468k^{21}\overline{m} - 5605687296k^{21}\overline{m} + 127468k^{21}\overline{m} - 5605687296k^{21}\overline{m} - 56056876k^{21}\overline{m} - 5605688k^{21}\overline{m} - 5605688k^{21}\overline{m} - 5605688k^{21}\overline{m} - 56056k^{21}\overline{m} - 56056k^{21}\overline{m} - 56056k^{21}\overline{m} - 56056k$  $4013162496k^{22}\overline{m} - 1242169344k^{23}\overline{m} + 573308928k^{24}\overline{m}$ 

 $42898784490ak^{3} + 60325585380dk^{3} - 190582024554k^{4} - 74024560260ak^{4} + 381164049108dk^{4} + 381164049108dk^{4} + 3811640480k^{4} + 3811640k^{4} + 38116k^{4} + 38116k^$  $309993033825k^5 - 421376932512ak^5 + 619986067650dk^5 - 1112362592283k^6 - 689137991826ak^6 + 649137991826ak^6 + 649137991866ak^6 + 649137991826ak^6 + 649137991866ak^6 + 649137991866ak^6 + 649137991866ak^6 + 6491379866ak^6 + 6491366ak^6 + 6491366ak^6 + 6491366ak^6 + 6491366ak^6 + 6481366ak^6 + 648166ak^6 + 648166k^6 + 648166k^6 + 64816k^6 + 6481$  $2224725184566dk^{6} - 1899286576692k^{7} - 2501396876226ak^{7} + 3798573153384dk^{7} - 250139687626ak^{7} + 3798573153384dk^{7} + 379857315384dk^{7} + 3798573153384dk^{7} + 379857315384dk^{7} + 379857315384dk^{7} + 379857315384dk^{7} + 379857315384dk^{7} + 3798573153384dk^{7} + 379857315384dk^{7} + 379857384dk^{7} + 379857384dk^{7} + 379857384dk^{7} + 379857384dk^{7} + 37985784k^{7} + 37985784k^{7} + 37985784k^{7} + 37985784k^{7} + 37985784k^{7} + 379885784k^{7} + 37988584k^{7} + 37988584k^{7} + 37988584k^{7} + 37988584k^{7} + 37988584k^{7} + 379884k^{7} + 379884k^{7} + 3798884k^{7} + 379884k^{7} + 37884k^{7} + 37884k^{7} + 38884k^{7} + 38884k^{7} + 38884k^{7} + 38884k^{7} + 38884k^{7}$  $84253511251456dk^{14} - 62282414215168k^{15} - 91332515693568ak^{15} + 124564828430336dk^{15} - 91332515693568ak^{15} + 91332568ak^{15} + 91368ak^{15} + 91364ak^{15} + 91364ak^{15} + 91464ak^{15} + 91564ak^{15} + 91364ak^{15} + 91564ak^{15} +$  $55867008173056k^{16} - {132022066675200} ak^{16} + {111734016346112} dk^{16} - {55858977841152} k^{17} - {111734016346112} dk^{16} - {11117340164414} dk^{16} - {11173401644144} dk^{16} - {1117340164444444} dk^{16} - {1111734$  $107333292805120ak^{17} + 111717955682304dk^{17} - 64773343845376k^{18} - 132430942003200ak^{18} + 64773343845376k^{18} - 647733458k^{18} - 6477348k^{18} - 647784k^{18} - 64788k^{18} - 647784k^{18} - 647784k^{18} - 647784k^{18} - 647$  $129546687690752 dk^{18} - 18886586941440 k^{19} - 99887857678336 ak^{19} + 37773173882880 dk^{19} - 9988788786 ak^{19} + 37773173882880 dk^{19} - 9988788786 ak^{19} + 37778786 ak^{19} + 377788786 ak^{19} + 377788786 ak^{19} + 377788786 ak^{19} + 37778786 ak^{19} + 377788786 ak^{19} + 377788786 ak^{19} + 37778788786 ak^{19} + 37778786 ak^{19} + 37778786 ak^{19} + 3777887886 ak^{19} + 377788786 ak^{19} + 3777887886 ak^{19} + 37778878886 ak^{19} + 3777887886 ak^{19} + 377788878886 ak^{19} + 377788886 ak^{19} + 37778878888888 ak^{19} + 377788888888888888 ak^{19} + 3777888888888888 ak^{19} + 377788888888888888 ak$  $68564545183744k^{20} - 77360313712640ak^{20} + 137129090367488dk^{20} + 26375479623680k^{21} - 2637547866k^{21} - 26375479623680k^{21} - 263754786k^{21} - 2637546k^{21} - 263754k^{21} - 263754k^{21}$  $80589711327232ak^{21} - 52750959247360dk^{21} - 64088955895808k^{22} - 277329215488ak^{22} + 6408895589588k^{22} - 277329215488ak^{22} + 6408895589588k^{22} - 277329215488ak^{22} + 640888886k^{22} - 277329215488ak^{22} + 6408886k^{22} - 277329215488ak^{22} + 6408886k^{22} - 277329215488ak^{22} + 6408886k^{22} - 277329215488ak^{22} + 6408886k^{22} - 2773292458k^{22} + 6408886k^{22} - 2773292458k^{22} + 6408886k^{22} - 2773292458k^{22} + 640886k^{22} - 2773292458k^{22} + 640886k^{22} - 2773292458k^{22} + 640886k^{22} + 64086k^{22} + 64086k^{22} + 64086k^{22} + 64086k^{22} + 6408k^{22} +$  $48311369859072k^{24} + 47156914028544ak^{24} + 96622739718144dk^{24} + 40404510572544k^{25} - 66622739718144dk^{24} + 40404510572544k^{25} - 66622739718144dk^{24} + 666227397184k^{24} + 666224k^{24} + 66624k^{24} + 66624k^{24} + 66624k^{24} + 66624k^{24} + 66624k^{24} + 6664k^{24} + 6664k^{24} + 6664k^{24} + 666k^{24} + 666$  $53256195145728dk^{26} + 19898887569408k^{27} - 26166116745216ak^{27} - 39797775138816dk^{27} - 397977518484 - 3979775186k^{27} - 3979775186k^{27} - 3979775186k^{27} - 3979775186k^{27} - 3979775186k^{27} - 3979775186k^{27} - 397976k^{27} - 397986k^{27} - 39786k^{27} - 39786k^{27} - 39786k^{27} - 39786k^{27} - 39786k$  $4035415375872dk^{30} + 717527973888k^{31} - 2249749168128ak^{31} - 1435055947776dk^{31} - 143505566k^{31} - 1455056k^{31} - 145506k^{31} - 145506k^{31} - 145506k^{31} - 14550k^{31} - 14550k^{31} - 14550k^{31} - 1450k^{31} - 14550k^{31} - 1450k^{31} 171228266496k^{32} + 970293510144ak^{32} + 342456532992dk^{32} - 195689447424ak^{33} +$  $74520000\overline{m} + 204192900k\overline{m} + 1737058500k^2\overline{m} + 5165638920k^3\overline{m} + 18864613800k^4\overline{m} + 1886461380k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m} + 18864618k^4\overline{m}$ 

 $B_4 = 933120000 + 74520000a - 3732480000d - 149040000ad + 3732480000d^2 +$  $1320399225k + 3862152900ak + 149040000a^{2}k - 5281596900dk - 7724305800adk +$  $85797568980a^2k^4 - 762328098216dk^4 - 268700291280adk^4 + 762328098216d^2k^4 +$  $1239972135300d^{2}k^{5} + 1112362592283k^{6} + 1309124059476ak^{6} + 842753865024a^{2}k^{6} -$  $7597146306768d^2k^7 + 4382190591648k^8 + 7811500551344ak^8 + 5002793752452a^2k^8 - 6484b^2k^2 + 6484b^2k^2$  $17528762366592dk^8 - 15623001102688adk^8 + 17528762366592d^2k^8 + 7729406883488k^9 +$  $49418103907808dk^{10} - 62690336949792adk^{10} + 49418103907808d^2k^{10} + 21958490281696k^{11} + 49418103907808d^2k^{10} + 21958490281696k^{10} + 494181080k^{10} + 49418108k^{10} + 494181k^{10} + 494181k^{10} + 49418k^{10} + 49418k^{10} + 4841k^{10} +$ 

$$\begin{split} 59077497204k^5\overline{m} + 128229421380k^6\overline{m} + 404901113040k^7\overline{m} + 619255357616k^8\overline{m} + \\ 1854448284432k^9\overline{m} + 2281989225376k^{10}\overline{m} + 5975763994144k^{11}\overline{m} + 6683510905664k^{12}\overline{m} + \\ 13836574173696k^{13}\overline{m} + 15835682533376k^{14}\overline{m} + 22914686975488k^{15}\overline{m} + 30371925220352k^{16}\overline{m} + \\ 25971201679360k^{17}\overline{m} + 46684188000256k^{18}\overline{m} + 17025357987840k^{19}\overline{m} + 56612497817600k^{20}\overline{m} + \\ 73118777344k^{21}\overline{m} + 53101407240192k^{22}\overline{m} - 12665621839872k^{23}\overline{m} + 37552032055296k^{24}\overline{m} - \\ 13851549499392k^{25}\overline{m} + 19301853560832k^{26}\overline{m} - 7788224839680k^{27}\overline{m} + 6797830127616k^{28}\overline{m} - \\ 2389042003968k^{29}\overline{m} + 1467670855680k^{30}\overline{m} - 317995352064k^{31}\overline{m} + 146767085568k^{32}\overline{m} \end{split}$$

 $53111878676576ak^{11} + 31772709415840a^2k^{11} - 87833961126784dk^{11} - 106223757353152adk^{11} + 31772709415840a^2k^{11} - 87833961126784dk^{11} - 878346k^{11} - 878346k^{11} - 87834k^{11} - 87834k^{11}$  $87833961126784d^2k^{11} + 25931512655888k^{12} + 88541708038304ak^{12} + 56805653445344a^2k^{12} - 568056566k^{12} - 56805656k^{12} - 568056556k^{12} - 56805656k^{12} - 56805656k^{12} - 56805656k^{12} - 56805656k^{12} - 56805656k^{12} - 56805656k^{12} - 568056k^{12} - 56805656k^{12} - 56805656k^{12} - 568056k^{12} - 56$  $103726050623552dk^{12} - 177083416076608adk^{12} + 103726050623552d^2k^{12} + 44275233673856k^{13} + 646666666k^{13} + 646666666k^{13} + 64666666k^{13} + 646666k^{13} + 646666k^{13} + 646666k^{13} + 64666k^{13} + 64666k^{13} + 64666k^{13} + 64666k^{13} + 64666k^{13} + 64666k^{13} + 6466k^{13} + 6666k^{13} + 6666k^{13} + 6666k^{13} + 6666k^{13} + 666k^{13} + 6666k^{13} + 6666k^{$  $177100934695424d^2k^{13} + 42126755625728k^{14} + 179100043055104ak^{14} + 118032177159616a^2k^{14} - 126666k^{14} + 126666k^{14} + 126666k^{14} + 126666k^{14} + 12666k^{14} + 12666k$  $168507022502912dk^{14} - 358200086110208adk^{14} + 168507022502912d^2k^{14} + 62282414215168k^{15} + 622824144215168k^{15} + 622824144215168k^{15} + 62282414444k^{15} + 6228244k^{15} + 6228244k^{15} + 62284k^{15} + 6284k^{15} + 62$  $175586026945024ak^{15} + 181099151414784a^2k^{15} - 249129656860672dk^{15} - 351172053890048adk^{15} + 1810984adk^{15} + 1810984ad$  $219067309151232ak^{17} + 264044133350400a^2k^{17} - 223435911364608dk^{17} - 438134618302464adk^{17} + 264044134444k^{17} + 2640444k^{17} + 264044k^{17} + 26404k^{17} + 2640k^{17} + 2640k^{1$  $259093375381504dk^{18} - 488297795371008adk^{18} + 259093375381504d^2k^{18} + 18886586941440k^{19} + 188865869444k^{19} + 18886586944k^{19} + 18886586944k^{19} + 188865866864k^{19} + 1888658664k^{19} + 18886658664k^{19} + 18886658664k^{19} + 18886658664k^{19} + 1888658664$  $274258180734976dk^{20} - 230266975191040adk^{20} + 274258180734976d^2k^{20} - 26375479623680k^{21} + 274258180734976d^{2}k^{20} - 26375479623680k^{21} + 274258666k^{20} - 26666k^{20} - 26666k^{20} - 26666k^{20} - 26666k^{20} - 26666k^{20} - 2666k^{20} - 266k^{20} - 2666k^{20} - 2666k^{20} - 2666k^{20} - 266k^{20} - 2666k^{20} - 266k^{20} - 2666k^{20} - 266k^{20}$  $217718801694720ak^{21} + 154720627425280a^2k^{21} + 105501918494720dk^{21} - 435437603389440adk^{21} - 43543760389440adk^{21} - 435437603389440adk^{21} - 43543760389440adk^{21} - 4354760adk^{21} - 4354760a$  $190588794503168ak^{23} + 554658430976a^2k^{23} + 194749135847424dk^{23} - 381177589006336adk^{23} -$  $193245479436288dk^{24} + 289062963904512adk^{24} + 193245479436288d^2k^{24} - 40404510572544k^{25} + 19324547944k^{25} + 1932454794k^{25} + 1932454794k^{25} + 1932454794k^{25} + 193245479k^{25} + 193245k^{25} + 193245k^{25} + 193245k^{25} + 19324k^{25} + 19324k^{2$ 

 $106512390291456dk^{26} + 256929278459904adk^{26} + 106512390291456d^2k^{26} - 19898887569408k^{27} + 106512390291456d^2k^{26} - 10898887569408k^{27} + 106512390291456d^{2}k^{26} - 10898887569408k^{27} + 106512390291456d^{2}k^{26} - 10898887569408k^{27} + 106512390291456d^{2}k^{26} - 10898887569408k^{27} + 106512390291456d^{2}k^{26} - 10898887569408k^{27} + 106512396k^{27} + 10651236k^{27} + 10651236k^{27} + 1065126k^{27} + 1065126k^{27} + 1065126k^{27} + 1065126k^{27} + 106512k^{27} + 106512$  $79595550277632d^2k^{27} + 9702114066432k^{28} - 65009495310336ak^{28} + 52332233490432a^2k^{28} - 6500949531a^2k^{28} - 650084ak^{28} - 650084$  $22566798360576d^2k^{29} + 2017707687936k^{30} - 18710085500928ak^{30} + 20434712002560a^2k^{30} - 1871086a^2k^{30} + 20434712002560a^2k^{30} - 18816k^{30} + 20434k^{30} + 204346k^{30} + 20434k^{30} + 20434k^{30} + 20434k^{30} + 2044k^{30} + 204k^{30} + 2044k^{30} + 2044k^{30}$  $8070830751744dk^{30} + 37420171001856adk^{30} + 8070830751744d^2k^{30} - 717527973888k^{31} + 36664k^{30} + 3666$  $6285164544000ak^{31} - {14853372641280}a^2k^{31} + {2870111895552}dk^{31} - {12570329088000}adk^{31} - {125703290800}adk^{31} - {12570600}adk^{31} - {12570600}adk^{31} - {12570600}adk^{31} - {1$  $2870111895552d^2k^{31} + 171228266496k^{32} - 2405349457920ak^{32} + 4499498336256a^2k^{32} - 2405349457920ak^{32} + 2495349457920ak^{32} + 2495366466k^{32} + 24956666k^{32} + 2495666k^{32} + 2495666k^{32} + 249566k^{32} + 249566k^{32} + 249566k^{32} + 24956k^{32} + 24056k^{32} + 240$  $1940587020288a^2k^{33} - 1076291960832adk^{33} + 391378894848a^2k^{34} - 74520000\overline{m} +$  $14904000 d\overline{m} - 204192900 k\overline{m} - 149040000 a k\overline{m} + 408385800 dk\overline{m} - 1737058500 k^2\overline{m} - 17370500 k^2\overline{m} - 173700 k^2\overline{m} - 173700 k^2\overline{m} - 173700$  $408385800ak^{2}\overline{m} + 3474117000dk^{2}\overline{m} - 5165638920k^{3}\overline{m} - 3474117000ak^{3}\overline{m} + 10331277840dk^{3}\overline{m} - 5165638920k^{3}\overline{m} - 3474117000ak^{3}\overline{m} + 10331277840dk^{3}\overline{m} - 5165638920k^{3}\overline{m} - 51656886k^{3}\overline{m} - 51656886k^{3}\overline{m} - 516668k^{3}\overline{m} - 51666k^{3}\overline{m} - 5166k^{3}\overline{m} - 5166k^{3}$  $18864613800k^4\overline{m} - 10331277840ak^4\overline{m} + 37729227600dk^4\overline{m} - 59077497204k^5\overline{m} - 37729227600ak^5\overline{m} + 37729227600ak^5\overline{m} - 59077497204k^5\overline{m} - 37729227600ak^5\overline{m} - 59077497204k^5\overline{m} - 590774874k^5\overline{m} - 59077484k^5\overline{m} - 590784k^5\overline{m} - 590784$  $118154994408dk^5\overline{m} - 128229421380k^6\overline{m} - 118154994408ak^6\overline{m} + 256458842760dk^6\overline{m} - 11815498ak^6\overline{m} + 256458842760dk^6\overline{m} - 11815498ak^6\overline{m} + 256458842760dk^6\overline{m} - 11815498ak^6\overline{m} + 256458842760dk^6\overline{m} - 11815498ak^6\overline{m} + 256458842760dk^6\overline{m} - 11815488ak^6\overline{m} + 256458842760dk^6\overline{m} - 118154884848ak^6\overline{m} + 118154884848ak^6\overline{m} + 118154884848ak^6\overline{m} + 1181548848ak^6\overline{m} + 1181548848ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 1181548848ak^6\overline{m} + 1181548848ak^6\overline{m} + 11815488ak^6\overline{m} + 1181548848ak^6\overline{m} + 1181548848ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 1181548848ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 11815488ak^6\overline{m} + 1181548ak^6\overline{m} + 11$  $404901113040k^{7}\overline{m} - 256458842760ak^{7}\overline{m} + 809802226080dk^{7}\overline{m} - 619255357616k^{8}\overline{m} - 6192555357616k^{8}\overline{m} - 6192555357616k^{8}\overline{m} - 6192555357616k^{8}\overline{m} - 61925557616k^{8}\overline{m} - 61925557616k^{8}\overline{m} - 6192556k^{8}\overline{m} - 619256k^{8}\overline{m} - 6166k^{8}\overline{m} - 6166k^{8}\overline{m} - 6166k^{8}\overline{m} - 6166k^{$  $809802226080ak^{8}\overline{m} + 1238510715232dk^{8}\overline{m} - 1854448284432k^{9}\overline{m} - 1238510715232ak^{9}\overline{m} + 1238510715232ak^{9}\overline{m} - 1854448284432k^{9}\overline{m} - 18544482k^{9}\overline{m} - 185444k^{9}\overline{m} - 18544k^{9}\overline{m} - 1854k^{9}\overline{m} - 1854k^{9}\overline{m$  $5975763994144k^{11}\overline{m} - 4563978450752ak^{11}\overline{m} + 11951527988288dk^{11}\overline{m} - 6683510905664k^{12}\overline{m} - 668351084k^{12}\overline{m} - 66835104k^{12}\overline{m} - 66835104k^{12}\overline{m} - 6683514k^{12}\overline{m} - 6683514k^{12}\overline{m} - 6683514k^{12}\overline{m} - 668$  $11951527988288ak^{12}\overline{m} + 13367021811328dk^{12}\overline{m} - 13836574173696k^{13}\overline{m} - 13367021811328ak^{13}\overline{m} + 1336702184ak^{13}\overline{m} + 1336702184ak^{13}\overline{m} + 1336702184ak^{13}\overline{m} + 13367024ak^{13}\overline{m} + 13687\overline{m} + 13687\overline{m} + 13687\overline{m} + 13687\overline{m} + 13687\overline{m} + 1$  $22914686975488k^{15}\overline{m} - 31671365066752ak^{15}\overline{m} + 45829373950976dk^{15}\overline{m} - 30371925220352k^{16}\overline{m} - 303719252k^{16}\overline{m} - 303719252k^{16}\overline{m} - 303719252k^{16}\overline{m} - 303719252k^{16}\overline{m} - 30371925k^{16}\overline{m} - 3037192k^{16}\overline{m} - 3037192k^{16}\overline{m} - 3037192k^{16}\overline{m} - 3037192k^{16}\overline{m} - 3037192k^{16}\overline{m} - 303719k^{16}\overline{m} - 303719k^{16}\overline{m} - 303719k^{16}\overline{m} - 303718k^{16}\overline{m} - 303718k^{16}\overline{m} - 303718k^{16}\overline{m} - 303718k^{16}\overline{m} - 303718k^{16}\overline{m} - 308k^{16}\overline{m} - 308k^{16}\overline{m} - 308k^{16}\overline{m} - 308k^{16}\overline{m}$  $45829373950976ak^{16}\overline{m} + 60743850440704dk^{16}\overline{m} - 25971201679360k^{17}\overline{m} - 60743850440704ak^{17}\overline{m} + 607438504ak^{17}\overline{m} + 6074$ 

 $51942403358720dk^{17}\overline{m} - 46684188000256k^{18}\overline{m} - 51942403358720ak^{18}\overline{m} + 93368376000512dk^{18}\overline{m} - 17025357987840k^{19}\overline{m} - 93368376000512ak^{19}\overline{m} + 34050715975680dk^{19}\overline{m} - 56612497817600k^{20}\overline{m} - 34050715975680ak^{20}\overline{m} + 113224995635200dk^{20}\overline{m} - 73118777344k^{21}\overline{m} - 113224995635200ak^{21}\overline{m} + 146237554688dk^{21}\overline{m} - 53101407240192k^{22}\overline{m} - 146237554688ak^{22}\overline{m} + 106202814480384dk^{22}\overline{m} + 12665621839872k^{23}\overline{m} - 106202814480384ak^{23}\overline{m} - 25331243679744dk^{23}\overline{m} - 37552032055296k^{24}\overline{m} + 25331243679744ak^{24}\overline{m} + 75104064110592dk^{24}\overline{m} + 13851549499392k^{25}\overline{m} - 75104064110592ak^{25}\overline{m} - 27703098998784dk^{25}\overline{m} - 19301853560832k^{26}\overline{m} + 27703098998784ak^{26}\overline{m} + 38603707121664dk^{26}\overline{m} + 7788224839680k^{27}\overline{m} - 38603707121664ak^{27}\overline{m} - 15576449679360dk^{27}\overline{m} - 6797830127616k^{28}\overline{m} + 15576449679360ak^{28}\overline{m} + 13595660255232dk^{28}\overline{m} + 2389042003968k^{29}\overline{m} - 13595660255232ak^{29}\overline{m} - 4778084007936dk^{29}\overline{m} - 1467670855680k^{30}\overline{m} + 4778084007936ak^{30}\overline{m} + 2935341711360ak^{31}\overline{m} - 635990704128ak^{31}\overline{m} - 146767085568k^{32}\overline{m} + 635990704128ak^{32}\overline{m} + 293534171136dk^{32}\overline{m} - 293534171136dk^{32}\overline{m} - 293534171136ak^{33}\overline{m}$ 

$$\begin{split} B_5 &= -933120000 + 1620000a + 3732480000d - 3240000ad - 3732480000d^2 - 1387683225k - \\ 3729656700ak + 3240000a^2k + 5550732900dk + 7459313400adk - 5550732900d^2k - \\ 19865768587k^2 - 5529278700ak^2 - 3726833400a^2k^2 + 79463074348dk^2 + 11058557400adk^2 - \\ 79463074348d^2k^2 - 31357014322k^3 - 79409326408ak^3 - 5507824500a^2k^3 + 125428057288dk^3 + \\ 158818652816adk^3 - 125428057288d^2k^3 - 193436007554k^4 - 125315499328ak^4 - 79355578468a^2k^4 + \\ 773744030216dk^4 + 250630998656adk^4 - 773744030216d^2k^4 - 318937179489k^5 - 773313059628ak^5 - \\ 125202941368a^2k^5 + 1275748717956dk^5 + 1546626119256adk^5 - 1275748717956d^2k^5 - \\ 1139776168339k^6 - 1275468158604ak^6 - 772882089040a^2k^6 + 4559104673356dk^6 + \\ 2550936317208adk^6 - 4559104673356d^2k^6 - 1933557306100k^7 - 4557218179840ak^7 - \\ 1275187599252a^2k^7 + 7734229224400dk^7 + 9114436359680adk^7 - 7734229224400d^2k^7 - \\ \end{split}$$

 $7733628110752a^2k^9 + 31131227233536dk^9 + 36250736922592adk^9 - 31131227233536d^2k^9 - 3113122723536d^2k^9 - 3113122723536d^2k^9 - 311312272353536d^2k^9 - 31131227233536d^2k^9 - 311312272353536d^2k^9 - 31131227235356d^2k^9 - 31131227235356d^2k^9 - 31131227235356d^2k^9 - 31131227235356d^2k^9 - 3113122723576d^2k^9 - 311312272356d^2k^9 - 311312272356d^2k^9 - 311312272356d^2k^9 - 3113122646k^9 - 311312266k^9 - 3113126k^9 - 311326k^9 - 311326k^9 - 311326k^9 - 31126k^9 - 31126k^9 - 31126k^9 - 3112k^9 - 31126k^9 - 3112k^9 - 3112k^9 - 3112k^9 - 3112k$  $31130526275872a^2k^{11} + 87477094727296dk^{11} + 102891732110144adk^{11} - 87477094727296d^2k^{11} - 87477094727944k^{11} - 87477094727296d^2k^{11} - 87477094727296d^2k^{11} - 87477094727296d^2k^{11} - 8747709472729444k^{11} - 874770947272944k^{11} - 874770947272944k^{11} - 874770947272944k^{11} - 87477094727294k^{11} - 8747709472794k^{11} - 8747709472784k^{11} - 8747764k^{11} - 874784k^{11} - 8748$  $174948473231680adk^{12} - 107855190281408d^2k^{12} - 43713072201344k^{13} - 107857161253248ak^{13} - 107857164ak^{13} - 10785764ak^{13} - 10785764ak^{13} - 107857664ak^{13} 43026356461056k^{14} - {174840846836736}ak^{14} - {107859132225088a^2k^{14}} + {172105425844224}dk^{14} + {17210542584424}dk^{14} + {1721054258444}dk^{14} + {172105425844}dk^{14} + {172105425844}dk^{14} + {172105425844}dk^{14} + {17210542584}dk^{14} + {172105425844}dk^{14} + {17210542584}dk^{14} + {172105425844}dk^{14} + {172105425844}dk^{14} + {17210542584}dk^{14} + {1721054584}dk^{14} + {17210566}dk^{14} + {17210566}dk^{14} + {17210566}dk^{14} +$  $246117389052928a^2k^{17} + 229510255951872dk^{17} + 436384945698816adk^{17} - 229510255951872d^2k^{17} - 22951872d^2k^{17} - 22951872d^{17} - 22951872d^{17} - 22951872d^{17} - 2295182d^{17} - 22951842d^{17} - 22951842d^{17} - 22951842d^{17} - 229518444444444444444$  $458961265926144adk^{18} - 235366748000256d^2k^{18} - 26917181825024k^{19} - 235400473694208ak^{19} - 235400464444k^{19} - 23540046444k^{19} - 235400464444k^{19} - 23540k^{19} -$  $229451009974272a^{2}k^{19} + 107668727300096dk^{19} + 470800947388416adk^{19} - 107668727300096d^{2}k^{19} - 10766872864 - 10766872864 - 10766872864 - 10766872864 - 1076687286 - 10766872864 - 1076687286 - 1076687286 107623937097728a^2k^{21} - 40346919763968dk^{21} + 464713243197440adk^{21} + 40346919763968d^2k^{21} - 4034691968d^2k^{21} - 4034698d^2k^{21} - 403468d^2k^{21} - 403468d^2k^{2$  $52420680499200k^{22} + 40356538613760ak^{22} - 232374974119936a^2k^{22} + 209682721996800dk^{22} - 232374974119936a^{22} + 209682721996800dk^{22} - 232374974119936a^{22} + 209682721996800dk^{22} - 20968272198680dk^{22} - 20968264a^{22} - 20968266a^{22} - 20968266a^{22} - 209668266a^{22} - 20968266a^{22} -$ 

 $230616585732096adk^{24} - 157390403272704d^2k^{24} + 24805789138944k^{25} - 157390976581632ak^{25} + 15739086ak^{25} + 157386ak^{25} + 157686ak^{25} + 157686ak^{25}$  $21742549991424k^{26} + 99223156555776ak^{26} - 157391549890560a^2k^{26} + 86970199965696dk^{26} - 15739154980560a^2k^{26} + 86970199965696dk^{26} - 15739154980560a^2k^{26} + 86970199965696dk^{26} - 15739154980560a^2k^{26} + 86970199965696dk^{26} - 15739154980660a^2k^{26} + 86970199965696dk^{26} - 1573915486060a^2k^{26} + 86970199965696dk^{26} - 1573915486060a^2k^{26} + 157391566a^2k^{26} + 1573916k^{26} + 1573916k^{26} + 1573916k^{26} + 157386k^{26} + 15786k^{26} + 15886k^{26} + 15886k^{26}$  $198446313111552adk^{26} - 86970199965696d^2k^{26} + 12149053194240k^{27} - 86970199965696ak^{27} + 12149053194240k^{27} - 8697019965696ak^{27} + 12149053194240k^{27} - 86970199965696ak^{27} + 12149053194240k^{27} - 8697019965696ak^{27} + 12149053194240k^{27} - 86970199965696ak^{27} + 12149053194240k^{27} - 86970199965696ak^{27} + 12149053194240k^{27} - 86970199965696ak^{27} + 12149053194240k^{27} - 8697019965696ak^{27} + 1214906ak^{27} + 1214906ak^{27} + 121460k^{27} +$  $99223156555776a^{2}k^{27} - 48596212776960dk^{27} + 173940399931392adk^{27} + 48596212776960d^{2}k^{27} - 48596212776960d^{2}k^{2}k^{2} - 48596212776960d^{2}k^{2}k^{2} - 48596212776960d^{2}k^{2} - 48596212776960d^{2}k^{2} - 48596212776960d^{2}k^{2} - 48596212776960d^{2}k^{2} - 48596246k^{2}k^{2} - 4859626660d^{2}k^{2} - 48596626666666$  $97192425553920adk^{28} - 31290946486272d^2k^{28} + 3424565329920k^{29} - 31290946486272ak^{29} + 342456532982ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245658ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 34245653288ak^{29} + 342456566ak^{29} + 342456656ak^{29} + 3424566k^{29} + 3424566k^{29} + 342456k^{29} + 342456k^{29} + 342456k^{29} + 342456k^{29} + 34646k^{29} + 34646k^{29} + 34646k^{29} + 34646k^{29} + 3464k^{29} + 3464$  $1541054398464k^{30} + 13698261319680ak^{30} - 31290946486272a^2k^{30} + 6164217593856dk^{30} - 61642175986dk^{30} - 61642175986dk^{30} - 61642175986dk^{30} - 61642175986dk^{30} - 6164217596dk^{30} - 61642175986dk^{30} - 6164217564dk^{30} - 61642175986dk^{30} - 6164217564dk^{30} - 6164217564dk^{30} - 6164217564dk^{30} - 6164217564dk^{30} - 6164217564dk^{30} - 6164217564dk^{30} - 61644444444k^{30} - 616444k^{30} - 616444k^{30} - 616444k^{30} - 61644k^{30} - 616444k^{30} - 61644k^{30} - 6164k^{30} - 6164k^{$  $27396522639360adk^{30} - 6164217593856d^2k^{30} + 440301256704k^{31} - 6164217593856ak^{31} + 6164217594ak^{31} + 616421756ak^{31} + 6164216k^{31} + 61642k^{31} + 61642k^{31} + 6164k^{31} + 616k^{31} + 616$  $110075314176k^{32} + 1761205026816ak^{32} - 6164217593856a^2k^{32} + 440301256704dk^{32} - 6164217593856a^2k^{32} + 61642175986a^2k^{32} + 61642175986a^2k^{32} + 61642175986a^2k^{32} + 61642175986a^2k^{32} + 616421756a^{32} + 616421646a^{32} + 616446a^{32} + 616464a^{32} + 616446a^{32} + 616464a^{32} + 616464a^{32} + 61646a^{32} + 61666a^{32} + 616666a^{32} + 61666a^{32}$  $880602513408 a d k^{33} - 440301256704 a^2 k^{34} - 1620000 \overline{m} + 3240000 d \overline{m} - 4443300 k \overline{m} - 44443300 k \overline{m} - 4443300 k \overline{m} - 4443300 k \overline{m} - 4443$  $3240000ak\overline{m} + 8886600dk\overline{m} - 25897500k^2\overline{m} - 8886600ak^2\overline{m} + 51795000dk^2\overline{m} - 79645440k^3\overline{m} - 796454k^3\overline{m} - 79644k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m} - 7964k^3\overline{m$  $51795000ak^3\overline{m} + 159290880dk^3\overline{m} - 192203400k^4\overline{m} - 159290880ak^4\overline{m} + 384406800dk^4\overline{m} + 38440680dk^4\overline{m} + 38440680dk^$  $623173988k^5\overline{m} - 384406800ak^5\overline{m} + 1246347976dk^5\overline{m} - 903733340k^6\overline{m} - 1246347976ak^6\overline{m} + 124634776ak^6\overline{m} + 12463476ak^6\overline{m} + 124634776ak^6\overline{m} + 12463476ak^6\overline{m} + 1246346k^6\overline{m} + 1246346$  $1807466680dk^{6}\overline{m} - 2790226856k^{7}\overline{m} - 1807466680ak^{7}\overline{m} + 5580453712dk^{7}\overline{m} - 3090783680k^{8}\overline{m} - 309078680k^{8}\overline{m} - 309078680k^{8}\overline{m} - 309078680k^{8}\overline{m} - 309078680k^{8}\overline{m} 8178412128k^{10}\overline{m} - 15655866592ak^{10}\overline{m} + 16356824256dk^{10}\overline{m} - 13985947872k^{11}\overline{m} - 16856824256dk^{10}\overline{m} - 13985947872k^{11}\overline{m} - 16856824256dk^{10}\overline{m} - 13985947872k^{11}\overline{m} - 16856824256dk^{10}\overline{m} - 13985947872k^{11}\overline{m} - 16856824256dk^{10}\overline{m} - 1685684446k^{10}\overline{m} - 168568446k^{10}\overline{m} - 168668446k^{10}\overline{m} - 16866846k^{10}\overline{m} - 1686684k^{10}\overline{m} - 16866846k^{10}\overline{m} - 16866846k^{10}\overline{m} - 1686684k^{10}\overline{m} - 16866846k^{10}\overline{m} - 1686684k^{10}\overline{m} - 168684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 168684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 168684k^{10}\overline{m} - 168684k^{10}\overline{m} - 1686684k^{10}\overline{m} - 168684k^{10}\overline{m} - 168684k^{10}\overline{m} - 168684k^{10}\overline{m} - 1$  $16356824256ak^{11}\overline{m} + 27971895744dk^{11}\overline{m} - 16844059328k^{12}\overline{m} - 27971895744ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797189774ak^{12}\overline{m} + 2797188776ak^{12}\overline{m} + 2797188776ak^{12}\overline{m} + 2797188776ak^{12}\overline{m} + 2797188776ak^{12}\overline{m} + 279718876ak^{12}\overline{m} + 27977876ak^{12}\overline{m} + 279718876ak^{12}\overline{m} + 2797786ak^{12}\overline{m} + 2797786ak$ 

$$\begin{split} &33688118656dk^{12}\overline{m}-14873087488k^{13}\overline{m}-33688118656ak^{13}\overline{m}+29746174976dk^{13}\overline{m}-\\ &26315056128k^{14}\overline{m}-29746174976ak^{14}\overline{m}+52630112256dk^{14}\overline{m}-6211428864k^{15}\overline{m}-\\ &52630112256ak^{15}\overline{m}+12422857728dk^{15}\overline{m}-30065071104k^{16}\overline{m}-12422857728ak^{16}\overline{m}+\\ &60130142208dk^{16}\overline{m}+5545562112k^{17}\overline{m}-60130142208ak^{17}\overline{m}-11091124224dk^{17}\overline{m}-\\ &24077426688k^{18}\overline{m}+11091124224ak^{18}\overline{m}+48154853376dk^{18}\overline{m}+9648267264k^{19}\overline{m}-\\ &48154853376ak^{19}\overline{m}-19296534528dk^{19}\overline{m}-12746833920k^{20}\overline{m}+19296534528ak^{20}\overline{m}+\\ &25493667840dk^{20}\overline{m}+5605687296k^{21}\overline{m}-25493667840ak^{21}\overline{m}-11211374592dk^{21}\overline{m}-\\ &4013162496k^{22}\overline{m}+11211374592ak^{22}\overline{m}+8026324992dk^{22}\overline{m}+1242169344k^{23}\overline{m}-8026324992ak^{23}\overline{m}-\\ &2484338688dk^{23}\overline{m}-573308928k^{24}\overline{m}+2484338688ak^{24}\overline{m}+1146617856dk^{24}\overline{m}-1146617856ak^{25}\overline{m} \end{split}$$

$$\begin{split} B_6 &= -933120000 - 25920000a + 3732480000d + 51840000ad - 3732480000d^2 - \\ 1376112375k - 3777641100ak - 51840000a^2k + 5504449500dk + 7555282200adk - \\ 5504449500d^2k - 19696750533k^2 - 5925573900ak^2 - 3822802200a^2k^2 + 78787002132dk^2 + \\ 11851147800adk^2 - 78787002132d^2k^2 - 30596383278k^3 - 79782964512ak^3 - 6346698300a^2k^3 + \\ 122385533112dk^3 + 159565929024adk^3 - 122385533112d^2k^3 - 190740713814k^4 - 125268809832ak^4 - \\ 80778926892a^2k^4 + 762962855256dk^4 + 250537619664adk^4 - 762962855256d^2k^4 - \\ 311173858335k^5 - 772531441372ak^5 - 128152086552a^2k^5 + 1244695433340dk^5 + 1545062882744adk^5 - \\ 1244695433340d^2k^5 - 1126353955653k^6 - 1255272647484ak^6 - 782100027488a^2k^6 + \\ 4505415822612dk^6 + 2510545294968adk^6 - 4505415822612d^2k^6 - 1916763902476k^7 - \\ 4557992716728ak^7 - 1265849861628a^2k^7 + 7667055609904dk^7 + 9115985433456adk^7 - \\ 7667055609904d^2k^7 - 4557099703184k^8 - 7688895023936ak^8 - 4610569610844a^2k^8 + \\ 18228398812736dk^8 + 15377790047872adk^8 - 18228398812736d^2k^8 - 7974370802096k^9 - \\ 18408371473216ak^9 - 7710734437968a^2k^9 + 31897483208384dk^9 + 36816742946432adk^9 - \\ \end{split}$$

 $31897483208384d^{2}k^{9} - 13496622361744k^{10} - 31926255188016ak^{10} - 18588344133696a^{2}k^{10} + 31897483208384d^{2}k^{9} - 13496622361744k^{10} - 31926255188016ak^{10} - 18588344133696a^{2}k^{10} + 318666266a^{2}k^{10} + 3186666a^{2}k^{10} - 3186666a^{2}k^{10} + 3186666a^{2}k^{10} + 3186666a^{2}k^{10} + 318666a^{2}k^{10} + 31666a^{2}k^{10} + 31666a^{2}k^{1$  $54366533582656ak^{11} - 31955027167648a^2k^{11} + 94502985561088dk^{11} + 108733067165312adk^{11} - 94502985561088dk^{11} + 108733067165312adk^{11} - 94502985561088dk^{11} + 9450298561080dk^{11} + 945029860dk^{11} + 94502860dk^{11} + 945060dk^{11} + 945060dk^{11} + 945060dk^{11} + 945060dk^$  $94502985561088d^2k^{11} - 30649992442240k^{12} - 94574364392928ak^{12} - 54746577718336a^2k^{12} + 945746577718336a^2k^{12} + 94574657771836a^2k^{12} + 9457666a^2k^{12} + 945766a^2k^{12} + 9457666a^2k^{12} + 9457666a^2k^{12}$  $122599969768960dk^{12} + 189148728785856adk^{12} - 122599969768960d^2k^{12} - 51098592402944k^{13} - 51088592402944k^{13} - 5108859244k^{13} - 510885924k^{13} - 510885864k^{13} - 5108864k^{13} - 510885864k^{13} - 5108864k^{13} - 5108864k^{13} - 5108864k^{13} - 510886k^{13} - 5108864k^{13} - 5$  $204394369611776d^2k^{13} - 55400204454400k^{14} - 204724381812224ak^{14} - 123395117308032a^2k^{14} + 234646k^{14} - 23464k^{14} - 23464k^{1$  $221600817817600dk^{14} + 409448763624448adk^{14} - 221600817817600d^2k^{14} - 81035920512000k^{15} - 221600817817600d^2k^{14} - 221600817804 - 22160080400d^{15} - 221600817804 - 221600817804 - 22160080400d^{15} - 221600817804 - 22160080400d^{15} - 22160080400d^{15} - 22160080400d^{15} - 2216008040d^{15} - 22160080400d^{15} - 22160080400d^{15} - 22160080400d^{15} - 221600800d^{15} - 2216008000d^{15} - 221600800d^{15} - 221600800d^{15} - 221600800d^{15} - 2216008000d^{15} - 22160000d^{15} - 22160000d^{15} - 22160000d^{15} - 2216000d^{15} - 2216000d^{15} - 2216000d^{15} - 221600d^{15} - 221600d^{15} - 221600d^{15} - 221600d^{15} - 2216000d^{15} - 221600d^{15} - 221600d^{15}$  $328354591563776dk^{16} + 650329496591360adk^{16} - 328354591563776d^2k^{16} - 92472623546368k^{17} - 9247262354638k^{17} - 924726235468k^{17} - 9247268k^{17} - 9247268k^{17} - 924726k^{17} - 92472k^{17} - 92472k^{17} - 92472k^{17} - 92472k^{17} - 9248k^{17} - 9248k^{17} - 9248k^{17} - 9248k^{17} - 924k^{17} - 9$  $326756558005248ak^{17} - 326185814543360a^2k^{17} + 369890494185472dk^{17} + 653513116010496adk^{17} - 655516adk^{17} - 655516adk^{17} - 655516adk^{17} - 655516adk^{17} - 655516adk^{17} - 655516adk^$  $415711901827072dk^{20} + 574798763311104adk^{20} - 415711901827072d^2k^{20} - 29201212571648k^{21} - 2920124k^{21} - 29204k^{21} - 29204k^{21} - 29204k^{21} - 29204k$  $116804850286592d^{2}k^{21} - 86455724605440k^{22} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 664646k^{22} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 66466k^{22} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 66466k^{22} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 6666k^{2}k^{2} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 6666k^{2}k^{2} - 118983075037184ak^{22} - 409610734354432a^{2}k^{22} + 6666k^{2}k^{2} - 118983075037184ak^{22} - 409610734354432a^{2}k^{2} + 6666k^{2} - 118983075037184ak^{2} - 118983075037184ak^{2} - 409610734354432a^{2}k^{2} + 6666k^{2} - 118984k^{2} - 118984k^{2} - 118884k^{2} - 11864k^{2} - 1186k^{2} - 1186k^{2}$  $18633527918592d^2k^{23} - 56157535272960k^{24} + 17425718181888ak^{24} - 341736076345344a^2k^{24} + 174257181888ak^{24} - 341736076345344a^2k^{24} + 174257181888ak^{24} - 341736076345344a^2k^{24} + 174257181888ak^{24} - 341736076345344a^2k^{24} + 174257181888ak^{24} - 341736076345344a^2k^{24} + 1742571888ak^{24} - 341736076345344a^{24} + 1742571888ak^{24} - 341736076345344a^{24} + 1742571888ak^{24} - 341736076345344a^{24} + 17425718888ak^{24} - 341736076345344a^{24} + 1742571888ak^{24} - 341766a^{24} + 17425788a^{24} + 1742588a^{24} + 174257888a^{24} +$  $224630141091840dk^{24} - 34851436363776adk^{24} - 224630141091840d^2k^{24} + 14660504911872k^{25} - 224630141091840d^2k^{24} + 14660504911872k^{25} - 224630141091840d^2k^{24} + 2246301400d^2k^{24} + 2246301400d^2k^{24} + 224630140d^2k^{24} + 22463014d^2k^{24} + 2246304d^2k^{24} +$ 

 $58642019647488d^2k^{25} - 27099591081984k^{26} + 58247073497088ak^{26} - 222933104197632a^2k^{26} + 58247073497088ak^{26} + 58247073497088ak^{26} - 222933104197632a^2k^{26} + 58247073497088ak^{26} - 222933104197632a^2k^{26} + 58247073497088ak^{26} - 222933104197632a^2k^{26} + 58247073497088ak^{26} + 5824707848484 + 582470784764 + 582470866 + 582470784764 + 58247078476 + 582470784766 + 5824707866 + 58247078476 + 5824707866 + 5824707866 + 5824707866 + 5824707866 + 5824707866 + 5824766 + 5824766 + 5824707866 + 5824766 + 5824766 + 5824766 + 5824766 + 5824766 + 5824766 + 5824766 + 58266 + 58$  $1369826131968d^2k^{31} - 183458856960k^{32} + 1369826131968ak^{32} - 7534043725824a^2k^{32} + 1369826131968ak^{32} - 7534043725824a^2k^{32} + 1369826131968ak^{33} - 7534043725824a^2k^{33} + 1369826131968ak^{33} + 1369826131968ak^{33} + 1369826131968ak^{33} + 13698261384ak^{33} + 13698261384ak^{33} + 136986k^{33} + 136986k^{33} + 136886k^{33} + 13688k^{33} + 13688k^{33} + 13688k^{33} + 1368k^{33} + 136k^{33} + 1$  $51840000d\overline{m} + 71081100k\overline{m} + 51840000ak\overline{m} - 142162200dk\overline{m} + 492205500k^2\overline{m} +$  $142162200ak^{2}\overline{m} - 984411000dk^{2}\overline{m} + 1488167880k^{3}\overline{m} + 984411000ak^{3}\overline{m} - 2976335760dk^{3}\overline{m} + 984411000ak^{3}\overline{m} - 98441100$  $4371444600k^{4}\overline{m} + 2976335760ak^{4}\overline{m} - 8742889200dk^{4}\overline{m} + 13940030716k^{5}\overline{m} + 8742889200ak^{5}\overline{m} - 874288920ak^{5}\overline{m} - 874288920ak^{5}\overline{m} - 8742889200ak^{5}\overline{m} -$  $77094138976k^7\overline{m} + 49034489720ak^7\overline{m} - 154188277952dk^7\overline{m} + 98933553008k^8\overline{m} + 989335580k^8\overline{m} + 989335580k^8\overline{m} + 98933558k^8\overline{m} + 98933558k^8\overline{m} + 98933558k^8\overline{m} + 98933558k^8\overline{m} + 98933558k^8\overline{m} + 98933558k^8\overline{m} + 98938k^8\overline{m} + 98933558k^8\overline{m} + 98938k^8\overline{m} + 98933558k^8\overline{m} + 98838k^8\overline{m} + 98888k^8\overline{m} + 9888k^8\overline{m} + 9888k^8\overline{m} + 9888k^8$  $154188277952ak^{8}\overline{m} - 197867106016dk^{8}\overline{m} + 278906213488k^{9}\overline{m} + 197867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 198867106016ak^{9}\overline{m} - 1988688868664ak^{9}\overline{m} - 198868664ak^{9}\overline{m} - 198868664ak^{9}\overline{m} - 198868664ak^{9}\overline{m} - 198868664ak^{9}\overline{m} - 19886664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 198864ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 1988664ak^{9}\overline{m} - 198864ak^{9}\overline{m} - 198864ak^{9}\overline{$  $557812426976dk^9\overline{m} + 307678193120k^{10}\overline{m} + 557812426976ak^{10}\overline{m} - 615356386240dk^{10}\overline{m} + 557812426976ak^{10}\overline{m} - 615356386240dk^{10}\overline{m} + 557812426976ak^{10}\overline{m} - 615356386240dk^{10}\overline{m} - 615356386440dk^{10}\overline{m} - 615356440dk^{10}\overline{m} - 615356440dk^{10}\overline{m} - 615356440dk^{10}\overline{m} - 615356440dk^{10}\overline{m} - 615356$  $687722328800k^{11}\overline{m} + 615356386240ak^{11}\overline{m} - 1375444657600dk^{11}\overline{m} + 759101160640k^{12}\overline{m} + 75910k^{12}\overline{m} + 75910k^{12}\overline{m} + 7591k^{12}\overline{m} + 7591k^{12}\overline{m} + 75910k^{12}\overline{m} + 7591k^{12}\overline{m} + 7591k^{12}\overline{m} + 7591k^{12}\overline{m} + 7581k^{12}\overline{m} +$  $1375444657600ak^{12}\overline{m} - 1518202321280dk^{12}\overline{m} + 1156674930176k^{13}\overline{m} + 1518202321280ak^{13}\overline{m} - 151820ak^{13}\overline{m} - 151820ak^{13}\overline{m} - 151820ak^{13}\overline{m} - 151820ak^{13}\overline{m} - 151820ak^{13}\overline{m} - 1$  $2313349860352dk^{13}\overline{m} + 1486687130624k^{14}\overline{m} + 2313349860352ak^{14}\overline{m} - 2973374261248dk^{14}\overline{m} - 297337444\overline{m} - 297337444\overline{m} - 29733744\overline{m} - 29733744\overline{m} - 29733744\overline{m} - 2973374\overline{m} - 297337\overline{m} - 297337\overline{m} - 297337\overline{m} - 297337\overline{m} - 29$ 

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$$\begin{split} B_7 &= 933120000 + 233280000a - 3732480000d - 466560000ad + 3732480000d^2 + \\ 1177608375k + 4139772300ak + 466560000a^2k - 4710433500dk - 8279544600adk + \\ 4710433500d^2k + 19205450325k^2 + 9115634700ak^2 + 4547064600a^2k^2 - 76821801300dk^2 - \\ 18231269400adk^2 + 76821801300d^2k^2 + 26872361550k^3 + 86839763040ak^3 + 13520835900a^2k^3 - \\ 107489446200dk^3 - 173679526080adk^3 + 107489446200d^2k^3 + 180536648310k^4 + 143870025960ak^4 + \\ 96857724780a^2k^4 - 722146593240dk^4 - 287740051920adk^4 + 722146593240d^2k^4 + \\ 280273324095k^5 + 832502553948ak^5 + 180250605720a^2k^5 - 1121093296380dk^5 - 1665005107896adk^5 + \\ 1121093296380d^2k^5 + 1031520654117k^6 + 1291929405372ak^6 + 942858514656a^2k^6 - \\ 4126082616468dk^6 - 2583858810744adk^6 + 4126082616468d^2k^6 + 1768116005676k^7 + \\ 4844287953720ak^7 + 1462765514364a^2k^7 - 7072464022704dk^7 - 9688575907440adk^7 + \\ 7072464022704d^2k^7 + 4039612535376k^8 + 7571509000128ak^8 + 5562493290972a^2k^8 - \\ \end{split}$$

$$\begin{split} 1255003000320k^{15}\overline{m} + 2973374261248ak^{15}\overline{m} - 2510006000640dk^{15}\overline{m} + 2276069248000k^{16}\overline{m} + \\ 2510006000640ak^{16}\overline{m} - 4552138496000dk^{16}\overline{m} + 678035689472k^{17}\overline{m} + 4552138496000ak^{17}\overline{m} - \\ 1356071378944dk^{17}\overline{m} + 2663403528192k^{18}\overline{m} + 1356071378944ak^{18}\overline{m} - 5326807056384dk^{18}\overline{m} - \\ 227750682624k^{19}\overline{m} + 5326807056384ak^{19}\overline{m} + 455501365248dk^{19}\overline{m} + 2317472612352k^{20}\overline{m} - \\ 455501365248ak^{20}\overline{m} - 4634945224704dk^{20}\overline{m} - 733111123968k^{21}\overline{m} + 4634945224704ak^{21}\overline{m} + \\ 1466222247936dk^{21}\overline{m} + 1445113626624k^{22}\overline{m} - 1466222247936ak^{22}\overline{m} - 2890227253248dk^{22}\overline{m} - \\ 598297411584k^{23}\overline{m} + 2890227253248ak^{23}\overline{m} + 1196594823168dk^{23}\overline{m} + 609512325120k^{24}\overline{m} - \\ 1196594823168ak^{24}\overline{m} - 1219024650240dk^{24}\overline{m} - 239006121984k^{25}\overline{m} + 1219024650240ak^{25}\overline{m} + \\ 478012243968dk^{25}\overline{m} + 155940028416k^{26}\overline{m} - 478012243968ak^{26}\overline{m} - 311880056832dk^{26}\overline{m} - \\ 39749419008k^{27}\overline{m} + 311880056832ak^{27}\overline{m} + 79498838016dk^{27}\overline{m} + 18345885696k^{28}\overline{m} - \\ 79498838016ak^{28}\overline{m} - 36691771392dk^{28}\overline{m} + 36691771392ak^{29}\overline{m} \end{split}$$

 $16158450141504dk^8 - 15143018000256adk^8 + 16158450141504d^2k^8 + 7517666546640k^9 + 16158450140k^8 + 7517666546640k^9 + 1615845014k^8 + 7517666546640k^9 + 1615845014k^8 + 7517666546640k^9 + 16060k^8 + 75160k^8 + 7516k^8 + 7517666546640k^9 + 7516k^8 + 7516$  $55449647452800ak^{11} + 32041746416160a^2k^{11} - 90751998134016dk^{11} - 110899294905600adk^{11} + 90751998134016dk^{11} - 907519804dk^{11} - 90751804dk^{11} - 907519804dk^{11} - 90751804dk^{11} - 90751804dk^{11}$  $106388186187392dk^{12} - 185244874346944adk^{12} + 106388186187392d^2k^{12} + 49727384574976k^{13} + 497273845749k^{13} + 497273845748k^{13} + 4972788k^{13} + 4972784k^{13} + 49728k^{13} + 49728k^{13} + 4972k^{13} + 4972k^{13} + 498k^{13} + 488k^{13} + 48k^{13} + 488k^{13} + 48k^{13} + 488k^{13} + 48k^{13}$  $122461467891648ak^{13} + 94492876212928a^2k^{13} - 198909538299904dk^{13} - 244922935783296adk^{13} + 94492876212928a^2k^{13} - 198909538299904dk^{13} - 244922935783296adk^{13} + 94492876212928a^2k^{13} - 198909538299904dk^{13} - 244922935783296adk^{13} - 198909538299904dk^{13} - 19890953829904dk^{13} - 19890953829904dk^{13} - 198909538299904dk^{13} - 1989095389095389040k^{13} - 1989095389095389040k^{13} - 1989095389040k^{13} - 198906k^{13} - 198006k^{13} - 198006k^{13} - 198006k^{13} - 198006k^{13} - 198006k^{13} - 19800k^{13} - 198$  $321368059530240ak^{17} + 356434965539840a^2k^{17} - 356492921569280dk^{17} - 642736119060480adk^{17} + 36434965539840a^2k^{17} - 356492921569280dk^{17} - 642736119060480adk^{17} + 36492921569280dk^{17} - 642736119060480adk^{17} + 36492921569280dk^{17} - 642736119060480adk^{17} + 36492921569280dk^{17} - 642736119060480adk^{17} + 36492921569280dk^{17} - 642736119060480adk^{17} + 366492921569280dk^{17} - 642736119060480adk^{17} + 366492921569280dk^{17} - 642736119060480adk^{17} + 366492921569280dk^{17} - 642736119060480adk^{17} + 36649280dk^{17} + 36649266480adk^{17} + 36649280dk^{17} - 56649280dk^{17} + 36649280dk^{17} + 36649280dk^{17} + 36649280dk^{17} + 36649280dk^{17} + 36649280dk^{17} + 3664880dk^{17} +$  $356492921569280d^2k^{17} + 119274202054656k^{18} + 406554296492032ak^{18} + 309893033689088a^2k^{18} - 406554296492032ak^{18} + 40655426484 + 406554266484 + 4065546484 + 4065546444 + 406554444 + 406554444 + 406554444 + 406554444 + 406554444 + 406554444 + 406554444 + 406554444 + 4065544 + 40655444 + 40655444 + 4065544 + 4065544 + 4065544 + 4065544 + 4065544 + 4065544 + 40655444 + 4065544 + 40655444 + 40655444 + 40655444 + 40655444 + 4065544 +$  $550147855220736dk^{22} - 367921428168704adk^{22} + 550147855220736d^2k^{22} - 17783280893952k^{23} + 560147855220736d^2k^{22} - 17783280893952k^{23} + 560147855220736d^2k^{23} - 560147855220736d^{23} + 56014785624 + 5601478664 + 56014$ 

 $400413743382528dk^{24} - {61618439847936} adk^{24} + 400413743382528d^2k^{24} - {25026270068736} k^{25} + {2502676} k^{25} + {2502676} k^{25} + {25026766} k^{25} + {2502676} k^{25} + {25026} k^{25} + {2502676} k^{25} + {25026} k^{25} + {25026} k^$  $292589630128128ak^{25} + 132751563423744a^2k^{25} + 100105080274944dk^{25} - 585179260256256adk^{25} - 585179260256256256adk^{25} - 585179260256256adk^{25} - 5851792602566adk^{25} - 5851792602566adk^{25} - 5851792602566adk^{25} - 5851792602566akk^{25} - 585179266akk^{25} - 5851792666akk^{25} - 5851766akk^{25} - 5851766ak$  $210017131167744dk^{26} + 70901280276480adk^{26} + 210017131167744d^2k^{26} - 14606835646464k^{27} + 210017131167744d^2k^{26} - 210017131167744d^{26} - 2100171744d^{26} - 2100171744d^{26} - 2100177$  $153049985187840ak^{27} + 29203799998464a^2k^{27} + 58427342585856dk^{27} - 306099970375680adk^{27} - 3060997099989700adk^{27} - 306099700000000000$  $58427342585856d^2k^{27} + 18525437558784k^{28} - 30975046189056ak^{28} + 96082839207936a^2k^{28} - 30975046189056ak^{28} + 96082839207936a^{28} + 9608283926a^{28} + 96082839207936a^{28} + 9608283926a^{28} + 9608286a^{28} + 9608283926a^{28} + 9608286a^{28} + 960826a^{28} + 960826a^{28} + 960826a^{28} + 960866a^{28} + 96086a^{28} + 96086a^{28} + 96086a^{28} + 96086a^{28} +$  $74101750235136dk^{28} + 61950092378112adk^{28} + 74101750235136d^2k^{28} - 4470960291840k^{29} + 74101750235136d^2k^{28} - 74470960291840k^{29} + 74101750235136d^{28} + 74101750235136d^{28} + 7410175023516k^{28} + 7410175023516k^{28} + 7410175023516k^{28} + 7410175023516k^{28} + 7410175023518k^{28} + 7410175023516k^{28} + 7410175028516k^{28} + 74101750856k^{28} + 7410175086k^{28} + 741$  $17883841167360d^2k^{29} + 3901558358016k^{30} - 10852610605056ak^{30} + 34913805926400a^2k^{30} - 10852610605056ak^{30} + 349166ak^{30} + 3466ak^{30} + 34$  $15606233432064dk^{30} + 21705221210112adk^{30} + 15606233432064d^2k^{30} - 587068342272k^{31} + 1560623344k^{30} + 15606233432064d^2k^{30} - 587068342272k^{31} + 1560623344k^{30} + 1560623344k^{30} + 15606234k^{30} + 15606234k^{30} + 156062k^{30} + 15606k^{30} + 1560k^{30} + 1560k^{30}$  $1467670855680dk^{32} + 3049493889024adk^{32} + 1467670855680d^2k^{32} + 1125214322688ak^{33} - 1125244322688ak^{33} - 11252443246k^{33} - 112524444k^{33} - 112524444k^{33} - 11252444k^{33} - 1125244k^{33} - 1125244k^{33} - 112524k^{33} - 11252k^{33} - 1125k^{33} - 11252k^{33} - 11252k^{33} 701220519936a^{2}k^{33} - 2250428645376adk^{33} + 782757789696a^{2}k^{34} - 233280000\overline{m} +$  $466560000d\overline{m} - 640572300k\overline{m} - 466560000ak\overline{m} + 1281144600dk\overline{m} - 5045773500k^2\overline{m} - 50457773500k^2\overline{m} - 5045777350k^2\overline{m} - 5045777550k^2\overline{m} - 5045777550k^2\overline{m} - 5065786k^2\overline{m} - 506586k^2\overline{m} - 50668k^2\overline{m} - 50668k^2\overline{m} - 5066k^2\overline{m} - 5066k$  $1281144600ak^{2}\overline{m} + 10091547000dk^{2}\overline{m} - 15063735240k^{3}\overline{m} - 10091547000ak^{3}\overline{m} + 30127470480dk^{3}\overline{m} - 1009154700ak^{3}\overline{m} + 30127470480dk^{3}\overline{m} - 100915470dk^{3}\overline{m} + 30127470480dk^{3}\overline{m} - 100915470dk^{3}\overline{m} + 30127470480dk^{3}\overline{m} + 30127470480$  $51444315000k^4\overline{m} - 30127470480ak^4\overline{m} + 102888630000dk^4\overline{m} - 161800275708k^5\overline{m} - 16180028k^5\overline{m} - 16180028k^5\overline{m} - 161808k^5\overline{m} - 161808k^5\overline{m} - 16180k^5\overline{m} - 16180$  $102888630000ak^{5}\overline{m} + 323600551416dk^{5}\overline{m} - 332636384700k^{6}\overline{m} - 323600551416ak^{6}\overline{m} + 32360055146ak^{6}\overline{m} + 3236005514ak^{6}\overline{m} + 3236005514ak^{6}\overline{m} + 3236005514ak^{6}\overline{m} + 3236005514k^{6}\overline{m} + 3236005514k^{6}\overline{m} + 3236005514k^{6}\overline{m} + 3236005514k^{6}\overline{m} + 3236005514k^{6}\overline{m} + 326005514k^{6}\overline{m} + 32600k^{6}\overline{m} + 32600k^{6}\overline{m} + 32600k^{6}\overline{m} + 32600k^{6}\overline{m} + 32600k^{6}\overline{m} + 3260k^{6}\overline{m} + 326k^{6}\overline{m} + 32$  $665272769400dk^{6}\overline{m} - 1050841721952k^{7}\overline{m} - 665272769400ak^{7}\overline{m} + 2101683443904dk^{7}\overline{m} - 665272769400ak^{7}\overline{m} + 66527276940ak^{7}\overline{m} + 66527276940ak^{7}\overline{m} + 6652764ak^{7}\overline{m} + 6652764ak^{7}\overline{m} + 6652764ak^{7}\overline{m} + 66527276940ak^{7}\overline{m} + 6652764ak^{7}\overline{m} + 665264ak^{7}\overline{m} + 665264ak^{7}\overline{m} + 6652764ak^{7}\overline{m} + 665264ak^{7}\overline{m} +$  $1549886699376k^8\overline{m} - 2101683443904ak^8\overline{m} + 3099773398752dk^8\overline{m} - 4599197918064k^9\overline{m} - 4599186k^9\overline{m} - 4599186k^9\overline{m} - 4599186k^9\overline{m} - 45988k^9\overline{m} - 45984k^9\overline{m} - 45988k^9\overline{m} - 45984k^9\overline{m} - 45984k^9\overline{m} - 45984k^9\overline{m} - 45984k^9\overline{m} - 45984k^9\overline{m} - 45984k^9\overline{m} - 45884k^9\overline{m} - 458$  $3099773398752ak^{9}\overline{m} + 9198395836128dk^{9}\overline{m} - 5584738032864k^{10}\overline{m} - 9198395836128ak^{10}\overline{m} + 91983644ak^{10}\overline{m} + 9198364ak^{10}\overline{m} + 9198364ak^{10}\overline{m} + 9198364ak^{10}\overline{m} + 9198364ak^{10}\overline{m} + 9198364ak^{10}\overline{m} + 9188364ak^{10}\overline{m} + 9188364ak^{10}\overline{m} + 9188364ak^{10}\overline$ 

$$\begin{split} B_8 &= 16402500a - 32805000ad - 16402500k + 28704375ak + 32805000a^2k + 65610000dk - \\ &57408750adk - 65610000d^2k - 45106875k^2 + 238109625ak^2 + 57408750a^2k^2 + 180427500dk^2 - \\ &476219250adk^2 - 180427500d^2k^2 - 316021500k^3 + 512031375ak^3 + 541829250a^2k^3 + \\ &1264086000dk^3 - 1024062750adk^3 - 1264086000d^2k^3 - 918266625k^4 + 1196635275ak^4 + \\ &1204490250a^2k^4 + 3673066500dk^4 - 2393270550adk^4 - 3673066500d^2k^4 - 2746944900k^5 + \\ &3844873575ak^5 + 3657356550a^2k^5 + 10987779600dk^5 - 7689747150adk^5 - 10987779600d^2k^5 - \\ \end{split}$$

 $16157618785472k^{12}\overline{m} - 28574359492032ak^{12}\overline{m} + 32315237570944dk^{12}\overline{m} - 32230900489728k^{13}\overline{m} - 32315237570944dk^{12}\overline{m} - 3315237570944dk^{12}\overline{m} - 331527570944dk^{12}\overline{m} - 33157570944dk^{12}\overline{m} - 33157570944dk^{12}\overline{m} - 33157570944dk^{12}\overline{m} - 33157570944dk^{12}\overline{m} - 33157570944dk^{12}$  $32315237570944ak^{13}\overline{m} + 64461800979456dk^{13}\overline{m} - 38036282890240k^{14}\overline{m} - 64461800979456ak^{14}\overline{m} + 64461800979456ak^{14}\overline{m} - 644618066ak^{14}\overline{m} - 64461806ak^{14}\overline{m} - 64464ak^{14}\overline{m} - 64461806ak^{14}\overline{m} - 644644ak^{14}\overline{m}$  $76072565780480dk^{14}\overline{m} - 52832595633664k^{15}\overline{m} - 76072565780480ak^{15}\overline{m} + 105665191267328dk^{15}\overline{m} + 10566519448ak^{15}\overline{m} + 1056651944ak^{15}\overline{m} + 1056651944ak^{15}\overline{m} + 105665194ak^{15}\overline{m} + 105665184ak^{15}\overline{m} + 1056654ak^{15}\overline{m} + 1056654ak^{15}\overline{m}$  $72638747307008k^{16}\overline{m} - 105665191267328ak^{16}\overline{m} + 145277494614016dk^{16}\overline{m} - 61163721465856k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61163724k^{17}\overline{m} - 61164k^{17}\overline{m} - 61164k$  $145277494614016ak^{17}\overline{m} + 122327442931712dk^{17}\overline{m} - 111225096388608k^{18}\overline{m} - 122327442931712ak^{18}\overline{m} - 122327\overline{m} - 12237\overline{m} - 12237$  $222450192777216dk^{18}\overline{m} - 45338815348736k^{19}\overline{m} - 222450192777216ak^{19}\overline{m} + 90677630697472dk^{19}\overline{m} - 9067763067764744b^{19}\overline{m} - 9067763069764744b^{19}\overline{m} - 906776306$  $134396160344064k^{20}\overline{m} - 90677630697472ak^{20}\overline{m} + 268792320688128dk^{20}\overline{m} - 12910611660800k^{21}\overline{m} - 1291061166080k^{21}\overline{m} - 129106080k^{21}\overline{m} - 129106080k^{21}\overline{m} - 129106080k^{21}\overline{m} - 129106080k^{21}\overline{m} - 12910k^{21}\overline{m} - 12910k^{21}\overline{m} - 12910k^{21}\overline{m} - 12910k^{21}\overline{m} - 129$  $268792320688128ak^{21}\overline{m} + 25821223321600dk^{21}\overline{m} - 125685162311680k^{22}\overline{m} - 25821223321600ak^{22}\overline{m} + 25821223321600ak^{22}\overline{m} - 25821223221600ak^{22}\overline{m} - 25821223221600ak^{22}\overline{m} - 25821223221600ak^{22}\overline{m} - 25821223221600ak^{22}\overline{m} - 25821223224\overline{m} - 25821223221600ak^{22}\overline{m} - 258212232$  $177387145789440ak^{25}\overline{m} - 38261080719360dk^{25}\overline{m} - 45523899777024k^{26}\overline{m} + 38261080719360ak^{26}\overline{m} + 38261080719860ak^{26}\overline{m} + 38261080719860ak^{26}\overline{m} + 38261080719860ak^{26}\overline{m} +$  $91047799554048dk^{26}\overline{m} + 11443246202880k^{27}\overline{m} - 91047799554048ak^{27}\overline{m} - 22886492405760dk^{27}\overline{m} - 91047799554048ak^{27}\overline{m} - 9104779\overline{m} - 910477\overline{m} - 910477\overline{m} - 910477\overline{m} - 910477\overline{m} - 91047\overline{m} - 9104\overline{m} 16009050193920k^{28}\overline{m} + 22886492405760ak^{28}\overline{m} + 32018100387840dk^{28}\overline{m} + 3584921960448k^{29}\overline{m} - 3686492405760ak^{28}\overline{m} + 368649240b^{28}\overline{m} + 3686492405760ak^{28}\overline{m} + 368649240b^{28}\overline{m} + 3686494b^{28}\overline{m} + 368646b^{28}\overline{m} + 368646b^{28}\overline{m} + 368646b^{28}\overline{m} + 36864b^{28}\overline{m} +$  $32018100387840ak^{29}\overline{m} - 7169843920896dk^{29}\overline{m} - 3446308601856k^{30}\overline{m} + 7169843920896ak^{30}\overline{m} + 71698446ak^{30}\overline{m} + 71698446ak^{30}\overline{m} + 7168846ak^{30}\overline{m} + 71688$  $6892617203712dk^{30}\overline{m} + 481069891584k^{31}\overline{m} - 6892617203712ak^{31}\overline{m} - 962139783168dk^{31}\overline{m} - 962139783168dk^{31}\overline{m} - 96892617203712ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 968926172037812ak^{31}\overline{m} - 96892617203784ak^{31}\overline{m} - 9689264ak^{31}\overline{m} - 9689264ak^{31}\overline{m$  $342456532992k^{32}\overline{m} + 962139783168ak^{32}\overline{m} + 684913065984dk^{32}\overline{m} - 684913065984ak^{33}\overline{m} + 684914ak^{33}\overline{m} + 68491$ 

 $8428351725k^6 + 365521815ak^6 + 11362813650a^2k^6 + 33713406900dk^6 - 731043630adk^6 - 7310430adk^6 - 7310430adk^6 - 731043630adk^6 - 7310430adk^6 - 7310430adk^6 - 7310430adk^6 - 7310430adk^6 - 7310430adk^6 - 7310430adk^6 - 731040adk^6 - 731040adk^6 - 731040adk^6 - 731040adk^6 - 731040adk^6 - 731040adk^6 - 73$  $8486637138a^2k^9 + 200232642816dk^9 - 40833383364adk^9 - 200232642816d^2k^9 - 162104536656k^{10} - 162104556k^{10} - 16210456k^{10} - 1621046k^{10} - 1621046k^{10} - 1621046k^{10} - 1621046k^{10} - 1621046k^{10} - 1621046k^$  $648418146624d^{2}k^{10} - 128097804672k^{11} - 70566237504ak^{11} - 68013463968a^{2}k^{11} +$  $512391218688dk^{11} + 141132475008adk^{11} - 512391218688d^2k^{11} - 381740640480k^{12} -$  $1526962561920d^2k^{12} - 261601929216k^{13} - 461385607104ak^{13} - 240277422528a^2k^{13} +$  $1046407716864dk^{13} + 922771214208adk^{13} - 1046407716864d^2k^{13} - 563697603072k^{14} - \\$  $2254790412288d^2k^{14} - 473596228608k^{15} - 1258653404928ak^{15} - 180202748928a^2k^{15} + 1258653404928ak^{15} - 1258653404928ak^{15} - 180202748928a^2k^{15} + 1258653404928ak^{15} - 180202748928a^2k^{15} + 1258653404928ak^{15} + 125865340484 + 125865340484 + 125865340484 + 125865340484 + 12586534044 + 12586534044 + 1258653404 + 1258653404 + 1258653404 + 1258653404 + 1258653404 + 12586544 + 12586544 + 12586654 + 12586544 + 12586544 + 1258654 + 12586654 + 12586654 + 12586654 + 12586654 + 12586654 + 12586654 + 12586654 + 12586654 + 12586654 + 125866564 + 12586656 + 125866566 + 12586656 +$  $1894384914432dk^{15} + 2517306809856adk^{15} - 1894384914432d^2k^{15} - 342338029824k^{16} - 1894384914432d^2k^{16} - 342338029824k^{16} - 342384k^{16} - 34284k^{16} 481385722112ak^{16} - 262516397568a^2k^{16} + 1369352119296dk^{16} + 962771444224adk^{16} - 262516397568a^2k^{16} + 262516397568a^2k^{16} + 262516397568a^2k^{16} + 262516397568a^2k^{16} + 262516397568a^2k^{16} + 262516397568a^2k^{16} + 26251686a^2k^{16} + 26251686a^2k^{16} + 26251686a^2k^{16} + 26251686a^2k^{16} + 26251686a^2k^{16} + 26251686a^2k^{16} + 2625166a^2k^{16} + 262516a^2k^{16} + 262516a^2k^{16} + 2625166a^2k^{16} + 2625166a^2k^{16} + 2625166a^2k^{16} + 262516a^2k^{16} + 262566a^2k^{16} + 262566a^2k$  $1369352119296d^2k^{16} - 808144764928k^{17} - 2049434382848ak^{17} + 931613470208a^2k^{17} + 931613486k^{17} + 931613486k^{17} + 931613486k^{17} + 9316186k^{17} + 931618k^{17} + 9316k^{17} + 9$  $3232579059712dk^{17} + 4098868765696adk^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 6666646k^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 666664k^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 66666k^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 66666k^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 6666k^{17} - 3232579059712d^2k^{17} + 556613558272k^{18} + 5666k^{18} + 5666k^{18} + 5666k^{18} + 5666k^{18} + 5666k^{18} + 566k^{18} + 566k^{18}$  $184095662080ak^{18} - 2729516646400a^2k^{18} - 2226454233088dk^{18} - 368191324160adk^{18} + \\$  $7223290560512d^{2}k^{20} - 1569457438720k^{21} - 511363612672ak^{21} + 6750560157696a^{2}k^{21} + 67505606a^{2}k^{2} + 675066a^{2}k^{2} + 675066a^{2$  $6277829754880dk^{21} + 1022727225344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 102272725344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 102272725344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 10227272725344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 10227272725344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 10227272725344adk^{21} - 6277829754880d^2k^{21} + 2553551454208k^{22} + 1027782880d^2k^{21} + 10277828880d^2k^{21} + 1027788886d^2k^{21} + 1027788886d^2k^{21} + 102778886d^2k^{21} + 1027788886d^2k^{21} + 1027788886d^2k^{21} + 1027788886d^2k^{21} + 1027788886d^2k^{21} + 102778886d^2k^{21} + 102778886d^2k^{21} + 102778886d^2k^{21} + 102778886d^2k^{21} + 102778886d^2k^{21} + 10277886d^2k^{21} + 10277886d^2k^{21} + 102778886d^2k^{21} + 102778886d^2k^{21} + 10277886d^2k^{21} + 1027786d^2k^{21} + 1027786d^2k^{2$ 

 $916972568576ak^{22} - 8246017785856a^2k^{22} - 10214205816832dk^{22} - 1833945137152adk^{22} + 10214205816832dk^{22} - 102142058168424 - 102142058168424 - 102142058168424 - 1021420584444 - 10214205844444 - 1021420584444 - 102142058444 - 102142058444 - 102142058444 - 102142058444 - 10214444 - 10214444 - 10214444 - 10214444 - 1021444 - 10214444 - 10214444 - 10214444 - 10214444 - 1021444 - 10214444 - 102144 - 1021444 - 1021444 - 1021444 - 1021444 - 1021444 - 1021444 - 1021444 - 1021444 - 1021444 - 1021$  $10214205816832d^{2}k^{22} - 1502335991808k^{23} + 1324426133504ak^{23} + 8111774892032a^{2}k^{23} + 1324426133504ak^{23} + 13244261346k^{23} + 13244426134k^{23} + 13244426134k^{23} + 13244426134k^{23} + 13244426134k^{23} + 1324444k^{23} + 132444k^{23} + 132444k^{23} + 13244k^{23} + 13244k^{23} + 13244k^{23} + 13244k^{23} + 1324k^{23} + 1324k$  $305039081472ak^{24} - 7565353549824a^2k^{24} - 9121363132416dk^{24} - 610078162944adk^{24} + 610078164adk^{24} + 610078164adk^{24} + 610078164adk^{24} + 61007844adk^{24} + 61007844adk^{24} + 6100784adk^{2$  $9121363132416d^2k^{24} - 1029370281984k^{25} + 2174933139456ak^{25} + 6619422130176a^2k^{25} + 661942k^{25} + 661942k^{25} + 66194k^{25} + 66194k^{25} + 66194k^{25} + 66194k^{25} + 6619k^{25} + 6618k^{25} + 6618k^{25} + 661k^{25} +$  $4117481127936dk^{25} - 4349866278912adk^{25} - 4117481127936d^2k^{25} + 1356378144768k^{26} - 4117481127936d^2k^{26} + 1356378144768k^{26} - 4117481127936d^{26} + 1356378144768k^{26} - 41174814768k^{26} - 4117481127936d^{26} + 1356378144768k^{26} - 4117481127936d^{26} + 135637884k^{26} - 4117481127936d^{26} + 135637884k^{26} - 41174884k^{26} - 41174884k^{26} - 411748k^{26} + 13564k^{26} - 411748k^{26} + 13564k^{26} + 13564k^{26} - 41174k^{26} + 13564k^{26} - 41174k^{26} + 13564k^{26} + 1356k^{26} + 1366k^{26} + 1366k^{$  $291552362496ak^{28} - {1994831953920}a^2k^{28} - {2097093279744}dk^{28} + {583104724992}adk^{28} + {58310472492}adk^{28} + {5831047249}adk^{28} + {5831047244}adk^{28} + {5831047244}adk^{28} + {58310472444}adk^{28} + {58310472444}adk^$  $2097093279744d^2k^{28} - {130459631616}k^{29} + 798499012608ak^{29} + {1309465903104}a^2k^{29} +$  $521838526464dk^{29} - 1596998025216adk^{29} - 521838526464d^2k^{29} + 119587995648k^{30} - 64646k^{20} + 119587995648k^{30} - 6464k^{20} + 6464k^{20} + 119587995648k^{30} - 6464k^{20} + 119587995648k^{30} - 6464k^{20} + 6464k^{20} +$  $125023813632ak^{30} - 500095254528a^2k^{30} - 478351982592dk^{30} + 250047627264adk^{30} + 250047627264akk^{30} + 250047627264akk^{30} + 250047627264akk^{30} + 250047627264akk^{30} + 250047627264akk^{30} + 25004762646k^{30} + 250047644k^{30} + 25004764k^{30} + 250046k^{30} + 250046k^{30} + 250046k^{30} + 25004k^{30} + 25004k^{30} + 25004k^{$  $478351982592d^{2}k^{30} - 16307453952k^{31} + 210637946880ak^{31} + 271790899200a^{2}k^{31} + 27179089200a^{2}k^{31} + 271790800a^{2}k^{31} + 27179080a^{2}k^{31} + 27179080a^{2}k^{31} + 27179080a^{2}k^{2$  $65229815808dk^{31} - 421275893760adk^{31} - 65229815808d^2k^{31} + 12230590464k^{32} - 20384317440ak^{32} - 2038444k^{32} - 203844k^{32} - 203844k^{32} - 203844k^{32} - 203844k^{32} - 20384k^{32}$  $57076088832a^2k^{32} - 48922361856dk^{32} + 40768634880adk^{32} + 48922361856d^2k^{32} +$  $24461180928ak^{33} + 24461180928a^2k^{33} - 48922361856adk^{33} - 16402500\overline{m} + 32805000d\overline{m} - 3280500d\overline{m} - 3280500d\overline{m}$  $45106875k\overline{m} - 32805000ak\overline{m} + 90213750dk\overline{m} - 348826500k^{2}\overline{m} - 90213750ak^{2}\overline{m} + 90213750ak^{2}\overline{m} +$  $2082570750ak^{4}\overline{m} + 7004013300dk^{4}\overline{m} - 11019946725k^{5}\overline{m} - 7004013300ak^{5}\overline{m} + 22039893450dk^{5}\overline{m} - 7004013300ak^{5}\overline{m} + 22039893450dk^{5}\overline{m} - 7004013300ak^{5}\overline{m} - 700401300ak^{5}\overline{m} - 70040130ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 7004014ak^{5}\overline{m} - 700404ak^{5}\overline{m} - 700404ak^{5}\overline{m} - 700404ak^{5}\overline{m} - 700$  $22373248140k^{6}\overline{m} - 22039893450ak^{6}\overline{m} + 44746496280dk^{6}\overline{m} - 70757030385k^{7}\overline{m} - 44746496280ak^{7}\overline{m} + 447464496280ak^{7}\overline{m} + 447464496480ak^{7}\overline{m} + 4474644864ak^{7}\overline{m} + 4474644ak^{7}\overline{m} + 4474644ak^{7}\overline{m} + 4474644864ak^{7}\overline{m} + 4474644ak^{7}\overline{m} + 44746444k^{7}\overline{m} + 44746$  $141514060770dk^{7}\overline{m} - 103575875634k^{8}\overline{m} - 141514060770ak^{8}\overline{m} + 207151751268dk^{8}\overline{m} + 207151751751268dk^{8}\overline{m} - 141514060770ak^{8}\overline{m} + 20816484k^{8}\overline{m} + 208164844k^{8}\overline{m} + 208164844k^{8}\overline{m} + 208164844k^{8}\overline{m} + 208164844$  $307251935856k^9\overline{m} - 207151751268ak^9\overline{m} + 614503871712dk^9\overline{m} - 373361525280k^{10}\overline{m} -$ 

 $B_{9} = -933120000 - 233280000a + 1866240000d - 1177608375k - 2273532300ak + 2355216750dk - 19205450325k^{2} - 6760417950ak^{2} + 38410900650dk^{2} - 26872361550k^{3} - 48428862390ak^{3} + 53744723100dk^{3} - 180536648310k^{4} - 90125302860ak^{4} + 361073296620dk^{4} - 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 471429257328ak^{5} + 560546648190dk^{5} - 1031520654117k^{6} - 731382757182ak^{6} + 280273324095k^{5} - 1031520654117k^{6} - 73138275784k^{5} - 1031520654k^{5} - 1031520k^{5} - 1031520k^{5} - 103152k^{5} - 103152k^{5}$ 

#### $48922361856ak^{33}\overline{m}$

 $614503871712ak^{10}\overline{m} + 746723050560dk^{10}\overline{m} - 951213434400k^{11}\overline{m} - 746723050560ak^{11}\overline{m} + 7467230560ak^{11}\overline{m} + 7467230560ak^$  $1902426868800dk^{11}\overline{m} - 1087270332480k^{12}\overline{m} - 1902426868800ak^{12}\overline{m} + 2174540664960dk^{12}\overline{m} + 21745406464040dk^{12}\overline{m} + 21745406404664640dk^{12}\overline{m} + 217454066464040dk^{12}\overline{m} + 2174540664040$  $4305694574592ak^{14}\overline{m} + 5171899542528dk^{14}\overline{m} - 3582086778624k^{15}\overline{m} - 5171899542528ak^{15}\overline{m} + 517189864ak^{15}\overline{m} + 517189864ak^{15}\overline{m} + 517189864ak^{15}\overline{m} + 517189864ak^{15}\overline{m} + 517189864ak^{15}\overline{m} + 5171888ak^{15}\overline{m} + 517188ak^{15}\overline{m} + 5171888ak^{15}\overline{m} + 517188ak^{15}\overline{m} + 51718$  $7164173557248dk^{15}\overline{m} - 4995085970944k^{16}\overline{m} - 7164173557248ak^{16}\overline{m} + 9990171941888dk^{16}\overline{m} + 9990171941884dk^{16}\overline{m} + 9990171941884dk^{16}\overline{m} + 99901719444k^{16}\overline{m} + 9990171944k^{16}\overline{m} + 9990171944k^{16}\overline{m} + 9990171944k^{16}\overline{m} + 999017194k^{16}\overline{m} + 999017194k^{16}\overline{m} + 999017194k^{16}\overline{m} + 990017194k^{16}\overline{m} + 9000174k^{16}\overline{m} + 9000174k^{16}\overline$  $4315003707392k^{17}\overline{m} - 9990171941888ak^{17}\overline{m} + 8630007414784dk^{17}\overline{m} - 7731678429184k^{18}\overline{m} - 78844k^{18}\overline{m} - 78844k^{18}\overline{m}$  $7137714077696dk^{19}\overline{m} - 9431680385024k^{20}\overline{m} - 7137714077696ak^{20}\overline{m} + 18863360770048dk^{20}\overline{m} - 7137714077696ak^{20}\overline{m} + 713771407766ak^{20}\overline{m} + 7137764ak^{20}\overline{m} + 7137764ak^{20}\overline{m} + 7137764ak^{20}\overline{m} +$  $1697026211840k^{21}\overline{m} - 18863360770048ak^{21}\overline{m} + 3394052423680dk^{21}\overline{m} - 8891828535296k^{22}\overline{m} - 6891828535296k^{22}\overline{m} - 689186k^{22}\overline{m} - 6889186k^{22}\overline{m} - 6889186k^{22}\overline{$  $3394052423680ak^{22}\overline{m} + 17783657070592dk^{22}\overline{m} - 2048851968k^{23}\overline{m} - 17783657070592ak^{23}\overline{m} + 177836$  $4097703936dk^{23}\overline{m} - 6316431900672k^{24}\overline{m} - 4097703936ak^{24}\overline{m} + 12632863801344dk^{24}\overline{m} + 1263863804k^{24}\overline{m} + 1263864k^{24}\overline{m} + 1263864k^{24}\overline{m} + 126$  $629998092288k^{25}\overline{m} - 12632863801344ak^{25}\overline{m} - 1259996184576dk^{25}\overline{m} - 3258263273472k^{26}\overline{m} + 32582642k^{26}\overline{m} + 32582k^{26}\overline{m} + 32582k^{26}\overline{m} + 32582k^{26}\overline{m} + 32582k^{26}\overline{m} + 32582k^{26}\overline{m} + 3258k^{26}\overline{m} + 3258k^{26}\overline{m} + 3258k^{26}\overline{m} + 3258k^{2$  $1259996184576ak^{26}\overline{m} + 6516526546944dk^{26}\overline{m} + 451908993024k^{27}\overline{m} - 6516526546944ak^{27}\overline{m} - 651652654694ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 6516464ak^{27}\overline{m} - 65165464ak^{27}\overline{m} - 6516464ak^{27}\overline{m} - 6$  $903817986048dk^{27}\overline{m} - 1149109272576k^{28}\overline{m} + 903817986048ak^{28}\overline{m} + 2298218545152dk^{28}\overline{m} + 903817986048ak^{28}\overline{m} + 90381786648ak^{28}\overline{m} + 90381786648ak^{28}\overline{m}$  $149484994560k^{29}\overline{m} - 2298218545152ak^{29}\overline{m} - 298969989120dk^{29}\overline{m} - 247329718272k^{30}\overline{m} + 247328k^{30}\overline{m} + 247328k^{30}\overline{m} + 247328k^{30}\overline{m} + 247328k^{30}\overline{m} + 24732k^{30}\overline{m} + 2474k^{30}\overline{m} + 2474k^{3$  $298969989120ak^{30}\overline{m} + 494659436544dk^{30}\overline{m} + 20384317440k^{31}\overline{m} - 494659436544ak^{31}\overline{m} - 49465944ak^{31}\overline{m} - 4946594ak^{31}\overline{m} - 4946584ak^{31}\overline{m} - 4946584ak^{31}\overline{m$  $40768634880 dk^{31} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361856 dk^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361886 ak^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 40768634880 ak^{32} \overline{m} + 48922361886 ak^{32} \overline{m} - 24461180928 k^{32} \overline{m} + 48922361886 ak^{32} \overline{m} + 48922361886 ak^{32} \overline{m} + 48922361886 ak^{32} \overline{m} + 489286 ak^{32} \overline{m} + 4886 ak^{32} \overline{m} + 48$ 

 $2063041308234dk^{6} - 1768116005676k^{7} - 2781246645486ak^{7} + 3536232011352dk^{7} - 2781246645486ak^{7} + 3536286ak^{7} + 3536646k^{7} + 356646k^{7} + 35664k^{7} + 35664k^{7} + 35664k^{7} + 3566k^{7} + 366k^{7} + 36$  $11128536289440ak^9 + 15035333093280dk^9 - 11686801434912k^{10} - 16020873208080ak^{10} +$  $26597046546848k^{12} - 47246438106464ak^{12} + 53194093093696dk^{12} - 49727384574976k^{13} - 49727384574876k^{13} - 497273845748876k^{13} - 49727886k^{13} - 4976k^{13} - 4976k^{13}$  $100964536928768dk^{14} - 79205665548288k^{15} - 115760849672192ak^{15} + 158411331096576dk^{15} - 1584113444 - 15841144 - 158411444 - 1584114 - 15841144 - 1584114 - 15841144 - 1584114 - 1584114 - 15841144 - 1584114 - 15841144 - 1584114 - 1584$  $83210771342848k^{16} - {178217482769920}ak^{16} + {166421542685696}dk^{16} - {89123230392320}k^{17} - {166421542685696}dk^{16} - {16642154268569}dk^{16} - {16642154268569}dk^{16} - {16642154268569}dk^{16} - {16642154686}dk^{16} - {16642154686}dk^{16} - {16642154686}dk^{16} - {16642154686}dk^{16} - {16642154686}dk^{16} - {16642156686}dk^{16} - {1664215666}dk^{16} - {1664215666}dk^{16} - {1664215666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {1664215666}dk^{16} - {16642156666}dk^{16} - {1664215666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {1664215666}dk^{16} - {16642156666}dk^{16} - {16642156666}dk^{16} - {166421566666}dk^{16} - {16642156666666}dk^{16} - {1664215666666666666666666}dk^{16} - {1666666$  $154946516844544ak^{17} + 178246460784640dk^{17} - 119274202054656k^{18} - 228307835707392ak^{18} + 126464666k^{18} - 228307835707392ak^{18} + 12646666k^{18} - 228307835707392ak^{18} + 12666k^{18} - 22830786k^{18} - 22866k^{18} - 22866k^{18} - 2286k^{18} - 2286k^{18} - 2286k^{18} - 2286k^{18} -$  $143412111884288k^{20} - 217532492464128ak^{20} + 286824223768576dk^{20} - 17796540858368k^{21} - 1264464646k^{20} - 1264646k^{20} - 126464k^{20} - 12646k^{20} - 12646k^$  $100103435845632k^{24} - 66375781711872ak^{24} + 200206871691264dk^{24} + 25026270068736k^{25} - 66375781711872ak^{24} + 25026270068736k^{25} - 66375781711872ak^{24} + 25026270068736k^{25} - 6637578171872ak^{25} + 6637578172ak^{25} + 663757887844 + 663758844k^{25} + 663758884 + 66375888887888884 + 663$  $7803116716032dk^{30} + 587068342272k^{31} - 3875738222592ak^{31} - 1174136684544dk^{31} - 117413668454dk^{31} - 117413668454k^{31} - 11764844k^{31} - 117484844k^{31} - 1174844k^{$  $366917713920k^{32} + 350610259968ak^{32} + 733835427840dk^{32} - 391378894848ak^{33} +$  $233280000\overline{m} + 640572300k\overline{m} + 5045773500k^2\overline{m} + 15063735240k^3\overline{m} + 51444315000k^4\overline{m} + 514444315000k^4\overline{m} + 5144444444\overline{m} + 51444444\overline{m} + 514444\overline{m} + 5144444\overline{m} + 51444\overline{m} + 51444\overline{m} + 51444\overline{m} + 51444\overline{m} + 5144\overline{m} + 514\overline{m} + 5144\overline{m} + 514\overline{m} + 51$  $161800275708k^{5}\overline{m} + 332636384700k^{6}\overline{m} + 1050841721952k^{7}\overline{m} + 1549886699376k^{8}\overline{m} + 1050841721952k^{7}\overline{m} + 1549886699376k^{8}\overline{m} + 1050841721952k^{7}\overline{m} + 105084172194k^{7}\overline{m} + 105084174k^{7}\overline{m} + 105084174k^{7}\overline{m} + 10508414k^{7}\overline{m} + 105084174k^{7}\overline{m} + 10508414k^{7}\overline{m} + 105084174k^{7}\overline{m} + 10508414k^{7}\overline{m} + 10508414k^{7}\overline{m} + 105084k^{7}\overline{m} + 10$ 

 $32230900489728k^{13}\overline{m} + 38036282890240k^{14}\overline{m} + 52832595633664k^{15}\overline{m} + 72638747307008k^{16}\overline{m} + 61163721465856k^{17}\overline{m} + 111225096388608k^{18}\overline{m} + 45338815348736k^{19}\overline{m} + 134396160344064k^{20}\overline{m} + 12910611660800k^{21}\overline{m} + 125685162311680k^{22}\overline{m} - 13248770605056k^{23}\overline{m} + 88693572894720k^{24}\overline{m} - 19130540359680k^{25}\overline{m} + 45523899777024k^{26}\overline{m} - 11443246202880k^{27}\overline{m} + 16009050193920k^{28}\overline{m} - 3584921960448k^{29}\overline{m} + 3446308601856k^{30}\overline{m} - 481069891584k^{31}\overline{m} + 342456532992k^{32}\overline{m}$ 

A.3 Combinations (a, d, k) used in numerical analysis for  $\beta^{ad}$ 

(1, 1, 0.1), (2, 1, 0.1), (3, 1, 0.1), (4, 1, 0.1), (5, 1, 0.1), (6, 1, 0.1), (7, 1, 0.1), (8, 1, 0.1), (9, 1, 0.1), (10, 1, 0.1), (1, 2, 0.1), (1, 3, 0.1), (1, 4, 0.1), (1, 5, 0.1), (1, 6, 0.1), (1, 7, 0.1), (1, 8, 0.1), (1, 9, 0.1), (1, 10, 0.1), (2, 2, 0.1), (2, 3, 0.1), (2, 4, 0.1), (2, 5, 0.1), (2, 6, 0.1), (2, 7, 0.1), (2, 8, 0.1), (2, 9, 0.1), (2, 10, 0.1), (3, 2, 0.1), (3, 3, 0.1), (3, 4, 0.1), (3, 5, 0.1), (3, 6, 0.1), (3, 7, 0.1), (3, 8, 0.1), (3, 9, 0.1), (3, 10, 0.1), (4, 2, 0.1), (4, 3, 0.1), (4, 4, 0.1), (4, 5, 0.1), (4, 6, 0.1), (4, 7, 0.1), (4, 8, 0.1), (4, 9, 0.1), (4, 10, 0.1), (5, 2, 0.1), (5, 3, 0.1), (5, 4, 0.1), (5, 5, 0.1), (5, 6, 0.1), (5, 7, 0.1), (5, 8, 0.1), (5, 9, 0.1), (5, 10, 0.1), (6, 2, 0.1), (6, 3, 0.1), (6, 4, 0.1), (6, 5, 0.1), (6, 6, 0.1), (6, 7, 0.1), (6, 8, 0.1), (6, 9, 0.1), (6, 10, 0.1), (7, 2, 0.1), (7, 3, 0.1), (7, 4, 0.1), (7, 5, 0.1), (7, 6, 0.1), (7, 7, 0.1), (7, 8, 0.1), (7, 9, 0.1), (7, 10, 0.1), (8, 2, 0.1), (8, 3, 0.1), (8, 4, 0.1), (8, 5, 0.1), (8, 6, 0.1), (8, 7, 0.1), (9, 6, 0.1), (9, 7, 0.1), (10, 8, 0.1), (10, 3, 0.1), (10, 4, 0.1), (10, 5, 0.1), (10, 6, 0.1), (10, 7, 0.1), (10, 8, 0.1), (10, 3, 0.1), (10, 4, 0.1), (10, 5, 0.1), (10, 6, 0.1), (10, 7, 0.1), (10, 8, 0.1), (10, 9, 0.1), (10, 0.1)

(1, 1, 0.2), (2, 1, 0.2), (3, 1, 0.2), (4, 1, 0.2), (5, 1, 0.2), (6, 1, 0.2), (7, 1, 0.2), (8, 1, 0.2), (9, 1, 0.2), (10, 1, 0.2), (1, 2, 0.2), (1, 3, 0.2), (1, 4, 0.2), (1, 5, 0.2), (1, 6, 0.2), (1, 7, 0.2), (1, 8, 0.2), (1, 9, 0.2), (1, 10, 0.2), (2, 2, 0.2), (2, 3, 0.2), (2, 4, 0.2), (2, 5, 0.2), (2, 6, 0.2), (2, 7, 0.2), (2, 8, 0.2), (2, 9, 0.2), (2, 10, 0.2), (3, 2, 0.2), (3, 3, 0.2), (3, 4, 0.2), (3, 5, 0.2), (3, 6, 0.2), (3, 7, 0.2), (3, 8, 0.2), (3, 9, 0.2), (3, 10, 0.2), (4, 2, 0.2), (4, 3, 0.2), (4, 4, 0.2), (4, 5, 0.2), (4, 6, 0.2), (4, 7, 0.2), (4, 8, 0.2), (4, 9, 0.2), (4, 10, 0.2), (5, 2, 0.2), (5, 3, 0.2), (5, 4, 0.2), (5, 5, 0.2), (5, 6, 0.2), (5, 7, 0.2), (5, 8, 0.2), (5, 9, 0.2), (5, 10, 0.2), (6, 2, 0.2), (6, 3, 0.2), (6, 4, 0.2), (6, 5, 0.2), (6, 6, 0.2), (6, 7, 0.2), (7, 6, 0.2), (7, 7, 0.2), (7, 8, 0.2), (7, 9, 0.2), (7, 10, 0.2), (8, 2, 0.2), (8, 3, 0.2), (8, 4, 0.2), (8, 5, 0.2), (8, 6, 0.2), (7, 9, 0.2), (7, 10, 0.2), (8, 2, 0.2), (8, 3, 0.2), (9, 4, 0.2), (9, 3, 0.2), (9, 4, 0.2), (9, 5, 0.2), (10, 4, 0.2), (10, 5, 0.2), (10, 6, 0.2), (10, 7, 0.2), (10, 8, 0.2), (10, 3, 0.2), (10, 4, 0.2), (10, 5, 0.2), (10, 6, 0.2), (10, 7, 0.2), (10, 8, 0.2), (10, 9, 0.2), (10, 10, 0.2), (10, 4, 0.2), (10, 5, 0.2), (10, 6, 0.2), (10, 7, 0.2), (10, 8, 0.2), (10, 9, 0.2), (10, 10, 0.2), (10, 4, 0.2), (10, 5, 0.2), (10, 6, 0.2), (10, 7, 0.2), (10, 8, 0.2), (10, 9, 0.2), (10, 10, 0.2), (10, 0.2),

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 $\begin{array}{l} 0.3), \ (6, \ 5, \ 0.3), \ (6, \ 6, \ 0.3), \ (6, \ 7, \ 0.3), \ (6, \ 8, \ 0.3), \ (6, \ 9, \ 0.3), \ (6, \ 10, \ 0.3), \ (7, \ 2, \\ 0.3), \ (7, \ 3, \ 0.3), \ (7, \ 4, \ 0.3), \ (7, \ 5, \ 0.3), \ (7, \ 6, \ 0.3), \ (7, \ 7, \ 0.3), \ (7, \ 8, \ 0.3), \ (7, \ 9, \ 0.3), \\ (7, \ 10, \ 0.3), \ (7, \ 4, \ 0.3), \ (7, \ 5, \ 0.3), \ (7, \ 6, \ 0.3), \ (7, \ 7, \ 0.3), \ (7, \ 8, \ 0.3), \ (7, \ 9, \ 0.3), \\ (7, \ 10, \ 0.3), \ (8, \ 2, \ 0.3), \ (8, \ 3, \ 0.3), \ (8, \ 4, \ 0.3), \ (8, \ 5, \ 0.3), \ (7, \ 8, \ 0.3), \ (7, \ 9, \ 0.3), \\ (7, \ 10, \ 0.3), \ (8, \ 2, \ 0.3), \ (8, \ 3, \ 0.3), \ (8, \ 4, \ 0.3), \ (8, \ 5, \ 0.3), \ (8, \ 6, \ 0.3), \ (8, \ 7, \ 0.3), \\ (8, \ 8, \ 0.3), \ (8, \ 9, \ 0.3), \ (8, \ 3, \ 0.3), \ (9, \ 2, \ 0.3), \ (9, \ 3, \ 0.3), \ (9, \ 4, \ 0.3), \ (9, \ 5, \ 0.3), \\ (9, \ 6, \ 0.3), \ (9, \ 7, \ 0.3), \ (9, \ 8, \ 0.3), \ (9, \ 9, \ 0.3), \ (9, \ 3, \ 0.3), \ (10, \ 2, \ 0.3), \ (10, \ 3, \ 0.3), \\ (10, \ 4, \ 0.3), \ (10, \ 5, \ 0.3), \ (10, \ 6, \ 0.3), \ (10, \ 7, \ 0.3), \ (10, \ 8, \ 0.3), \ (10, \ 9, \ 0.3), \ (10, \ 10, \ 0.3), \ (10, \ 1$ 

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(8, 2, 0.9), (8, 3, 0.9), (8, 4, 0.9), (8, 5, 0.9), (8, 6, 0.9), (8, 7, 0.9), (8, 8, 0.9), (8, 9, 0.9), (8, 10, 0.9), (9, 2, 0.9), (9, 3, 0.9), (9, 4, 0.9), (9, 5, 0.9), (9, 6, 0.9), (9, 7, 0.9), (9, 8, 0.9), (9, 9, 0.9), (9, 10, 0.9), (10, 2, 0.9), (10, 3, 0.9), (10, 4, 0.9), (10, 5, 0.9), (10, 6, 0.9), (10, 7, 0.9), (10, 8, 0.9), (10, 9, 0.9), (10, 10, 0.9)

- combination (5, 2, 0.3), (5, 4, 0.7), (5, 5, 0.9) provide that the critical discount rate under antidumping duty is higher than the free trade one.
- combination (2, 1, 0.4), (1, 1, 0.5), (3, 2, 0.5), (5, 3, 0.5), (7, 4, 0.5), (9, 5, 0.5), (6, 6, 0.9) provide invalid β<sup>ad</sup>

## Appendix B

## Shrimp production

## B.1 The shrimp farm area in Thailand

The data is from the Agriculture statistic yearbook by the office of Agricultural Economic, Ministry of Agriculture and Cooperatives, Thailand

- 1. Bangkok
- 2. Prachuap Khiri Khan
- 3. Phetchaburi
- 4. Chachoengsao
- 5. Prachinburi
- 6. Samut Prakan
- 7. Samut Sakhon
- 8. Samut Songkhram
- 9. Chon Buri
- 10. Rayong
- 11. Chanthaburi

12. Trat

- 13. Chumphon
- 14. Nakhon Si Thammarat
- 15. Phatthalung
- 16. Songkhla
- 17. Surat Thani
- 18. Krabi
- 19. Trang
- 20. Phangnga
- 21. Phuket
- 22. Ranong
- 23. Satun
- 24. Pattani
- 25. Narathiwat

### B.2 Supply rotation with rainfall

In this section, we discuss how rainfall rotate fresh shrimp supply. To do so, we consider the fresh shrimp production function to have mixed fashion between Loentief and Cobb-Douglas. More specifically, we assume the production function to be

$$f(B, R, L, D) = min\{vB, RL^{\gamma}K^{\alpha}\}$$
(B.1)

where B, R, L, K are baby shrimp, rainfall, labor, and capital (or diesel in this case). v is conversion ration from baby shrimp to fresh shrimp. The production cost of fresh shrimp is

$$C = w_B v B + w_L L + w_D D \tag{B.2}$$

where  $w_B$ ,  $w_L$ , and  $w_D$  are price of baby shrimp, wage, and price of diesel.

The optimal input demand can be obtained from cost minimization

$$\min_{B,L,D} w_B v B + w_L L + w_D D \tag{B.3}$$

subject to

$$y = f(B, R, L, D). \tag{B.4}$$

Thus, we can obtain the optimal input demand

$$B(v, y, w_B) = \frac{yw_B}{v} \tag{B.5}$$

$$L(y, R, w_L, w_D) = y^{\frac{1}{\gamma + \alpha}} \left[ \frac{\gamma}{w_L} \right] \left[ \frac{w_L^{\gamma} w_D^{\alpha}}{R \gamma^{\gamma} \alpha^{\alpha}} \right]^{\frac{1}{\gamma + \alpha}}$$
(B.6)

$$D(y, R, w_L, w_D) = y^{\frac{1}{\gamma + \alpha}} \left[ \frac{\alpha}{w_D} \right] \left[ \frac{w_L^{\gamma} w_D^{\alpha}}{R \gamma^{\gamma} \alpha^{\alpha}} \right]^{\frac{1}{\gamma + \alpha}}.$$
 (B.7)

Substituting the optimal input demand into equation B.2, we obtain the optimal production cost of fresh shrimp

$$C(y, R, v, w_B, w_L, w_k) = w_B v B(v, y, w_B) + w_L L(y, R, w_L, w_D) + w_D D(y, R, w_L, w_D).$$
(B.8)

The marginal cost of fresh shrimp production is

$$c(y, R, v, w_B, w_L, w_k) = \frac{\partial C(y, R, v, w_B, w_L, w_k)}{\partial y}$$
(B.9)  
$$= \frac{w_B}{v} + w_L \frac{\partial L(y, R, w_L, w_D)}{\partial y} + w_D \frac{\partial D(y, R, w_L, w_D)}{\partial y}.$$

From marginal cost equation B.9, we can see that rainfall is a part of slope of the marginal cost, so changing in rainfall result in a change of the slope of marginal cost, i.e. fresh shrimp supply rotation. On the other hand, changing in other input price will result in a shift in fresh shrimp supply.

# B.3 Month of shrimp culture

Size	Black Tiger prawn	White shrimp
30	6	5
40	5	4
50	5	4
60	4	4
70	4	3
80	3	3
90	3	3
100	3	3

Table B.1: Month of shrimp culture by size

source: CPF 2009

Table B.2: Average month of shrimp culture by year

Size	1996-2003	2004-2005	2006-2008
30	6	6	5
40	5	4	4
50	5	4	4
60	4	4	4
70	4	3	3
80	3	3	3
90	3	3	3
100	3	3	3

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