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Essays on Strategic Voting and Political Influence

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Doctor of Philosophy



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Declaration

I declare that this thesis was composed by myself and that the work contained herein is my own. No other person's work has been used without due acknowledgement. This thesis has not been submitted for any other degree or professional qualification. Chapters 3 and 4 are based on the 2013 working paper "Expert Information and Majority Decisions", co-authored by Dr. Kohei Kawamura. Chapter 5 is based on the 2013 working paper "Hysteresis in Crime", co-authored by Dr. Andre Loureiro.

(Vasileios Vlaseros)

*To my mother and father for supporting my dreams.
To my sister for protecting me from my nightmares.*

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Abstract

Chapter 1: I attempt a detailed literature review on the passage from the probabilistic versions of the Condorcet Jury Theorem to models augmented by the concept of strategic agents, including both theoretical and relevant empirical work. In the first part, I explore the most influential relevant game theoretic models and their main predictions. In the second part, I review what voting experiments have to say about these predictions, with a brief mention of the experiments' key methodological aspects. In the final part, I provide with an attempt to map the recent strategic voting literature in terms of structure and scope. I close with a philosophical question on the exogeneity of a “correct” choice of a voting outcome, which is inherent in the current strategic voting literature.

Chapter 2: I develop a two stage game with individually costly political action and costless voting on a binary agenda where, in equilibrium, agents rationally cast honest votes in the voting stage. I show that a positive but sufficiently low individual cost of political action can lead to a loss in aggregate welfare for any electorate size. When the individual cost of political action is lower than the signalling gain, agents will engage in informative political action. In the voting stage, since everyone's signal is revealed, agents will unanimously vote for the same policy. Therefore, the result of the ballot will be exactly the same as the one without prior communication, but with the additional aggregate cost of political action. However, when agents have heterogeneous prior beliefs, society is large and the state of the world is sufficiently uncertain, a moderate individual cost of political action can induce informative collective action of only a subset of the members of society, which increases ex ante aggregate welfare relative to no political action. The size of the subset of agents engaging in collective action depends on the dispersion of prior opinions.

Chapter 3: This chapter shows theoretically that hearing expert opinions can be a double-edged sword for decision making committees. We study a majoritarian voting game of common interest where committee members receive not only private information, but also expert information that is more accurate than private information and observed by all members. We identify three types of equilibria of interest, namely i) the symmetric mixed strategy equilibrium where each member randomizes between following the private and public signals should they disagree; ii) the asymmetric pure strategy equilibrium where a certain number of members always follow the public signal while the others always follow the private signal; and iii) a class of equilibria where a supermajority and hence the

committee decision always follow the expert signal. We find that in the first two equilibria, the expert signal is collectively taken into account in such a way that it enhances the efficiency (accuracy) of the committee decision, and a fortiori the CJT holds. However, in the third type of equilibria, private information is not reflected in the committee decision and the efficiency of committee decision is identical to that of public information, which may well be lower than the efficiency the committee could achieve without expert information. In other words, the introduction of expert information might reduce efficiency in equilibrium.

Chapter 4: In this chapter we present experimental results on the theory of the previous chapter. In the laboratory, too many subjects voted according to expert information compared to the predictions from the efficient equilibria. The majority decisions followed the expert signal most of the time, which is consistent with the class of obedient equilibria mentioned in the previous chapter. Another interesting finding is the marked heterogeneity in voting behaviour. We argue that the voters' behaviour in our data can be best described as that in an obedient equilibrium where a supermajority (and hence the decision) always follow the expert signal so that no voter is pivotal. A large efficiency loss manifests due to the *presence* of expert information when the committee size was large. We suggest that it may be desirable for expert information to be revealed only to a subset of committee members. Finally, in the Appendix we describe a new alternative method for producing the signal matrix of the game.

Chapter 5: There is a significant gap between the theoretical predictions and the empirical evidence about the efficiency of policies in reducing crime rates. This chapter argues that one important reason for this is that the current literature of economics of crime overlooks an important hysteresis effect in criminal behaviour. One important consequence of hysteresis is that the effect on an outcome variable from positive exogenous variations in the determining variables has a different magnitude from negative variations. We present a simple model that characterises hysteresis in both the micro and macro levels. When the probability of punishment decreases, some law abiding agents will find it more beneficial to enter a criminal career. If the probability of punishment returns to its original level, a subset of these agents will continue with their career in crime. We show that, when crime choice exhibits weak hysteresis at the individual level, crime rate in a society consisted from a continuum of agents that follows any non-uniform distribution will exhibit strong hysteresis. Only when punishment is extremely severe the effect of hysteresis ceases to exist. The theoretical predictions corroborate the argument that policy makers should be more inclined to set pre-emptive policies rather than mitigating measures.

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¹This chapter was based on results from the 2013 working paper “Expert Information and Majority Decisions”, co-authored by Dr. Kohei Kawamura.

²This chapter was based on results from the 2013 working paper “Expert Information and Majority Decisions”, co-authored by Dr. Kohei Kawamura.

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³This chapter was based on the 2013 working paper “Hysteresis in Crime”, co-authored by Dr. Andre Loureiro.

Chapter 1

The Strategic Revival of the Condorcet Jury Theorem

“But it is asked how a man can be both free and forced to conform to wills that are not his own?” (J.J.Rousseau)

1.1 Introduction

In order to evaluate collective decision making mechanisms, the literature of modern political economy and philosophy has leaped over an enormous boulder of introspection, mainly due to the work of John Stuart Mill. Whereas previous political philosophers like Rousseau, Kant and Hegel attempted to answer primal questions on decision making through human interaction, lingering unavoidably within the realms of philosophy to underline universal ideas as the existence of common good, Mill went on to evaluate the conditions under which the principle of individual autonomy can be integrated into shaping Liberalism of his time (Van Dyke (1960)). The more recent literature that copes with the subject, seems to feel Rousseau's platonic perception of common good as a satisfactory, usually through Condorcet's Jury Theorem, and initiates to undergo a series of evaluations on collective decision making mechanisms. In fact, Rousseau's theory of General Will and Condorcet's views about group judgement competence are complementary, as far as common good, majority competence, effective size of the assembly, closeness to unanimity with respect to the dominance of the general will and the importance both of deliberations and closeness to unanimity are concerned (Grofman and Feld (1988)). However, in the conceptualization of Condorcet Jury Theorem (CJT), the intuition that majority will approximate the common good more than any individual is seeing the latter idea as axiomatic.

Of course, the scope of this strand of literature is not to endogenize the concept of a General Will, but merely to define it externally, and evaluate different sequential procedures to approximate it.

1.2 Theory

1.2.1 Statistical Voting

After Arrows Impossibility result, concerning the amalgam of individual preferences in a robust ordinal social welfare function, the comparability of utilities was the only exit strategy of the relevant literature (Sen (1995)). Within this framework, an initial attempt to restate and browse through the validity of the Condorcet Jury Theorem can be found in Nitzan and Paroush (1985). In their work, the weighted majority rule by each players log odds of decisional competence is found to be the optimal rule for two a priori equally likely alternatives. Moreover, the concept of a cost for optimal rules is introduced, and, by simulation, it is shown that even the most competent of the group is less likely to choose better than the class majority.

Until the end of the nineteen eighties, the validity of the CJT was checked only as a problem of statistical nature, where individual voters choose as they would if the result of the voting procedure was not interlinked with other voters choices. In this trajectory, in Young (1988) the CJT can be seen as a maximum likelihood estimate that would indicate the most probable combination of opinions. Whereas both Condorcet and Borda's method satisfy anonymity, neutrality and reinforcement¹, the latter fails to be locally stable², a property inherent in Condorcet's method. In an attempt to introduce private information and information distribution, Miller (1986) showed that collective voting competence increases the more equally information is distributed amongst voters. Within the introduction of correlation amongst the voters preferences, Ladha (1992) showed that in big groups Condorcet's result holds under fairly general conditions. In smaller groups, the condition on average correlation is rather strong. The probability that the majority selects the superior outcome is inversely related to the average of the coefficients of correlation. Finally, in Berg (1993), when the probability of an individual voter choosing correctly is greater than half, a small amount of negative intra voter correlation raises the competence of the jury.

1.2.2 Strategic Voting

Upon the introduction of strategic thinking in modelling voters' behaviour in Ordeshook and Palfrey (1988), the importance of the ballot structure is underlined, as it is shown to influence the results profoundly. Under incomplete information, Condorcet winners need not necessarily prevail, and under imperfect information about other's preferences, there exist multiple equilibria. And, while both rational ignorance and informational efficiency are a Nash Equilibrium, both can be socially undesirable (Gersbach (1995)). Informational efficiency can constitute a Nash Equilibrium if the expected utilities fulfil the reversed ordering property. Information, cost and strategic thinking was introduced separately in the literature, but the first robust way to combine these results with informative voting (according to one's honest opinion) was done by Austen-Smith and Banks (1996).

¹Divided in two groups, the initial population of voters insists on its social outcome.

²The change on a pair of choices affects only those choices alone, and not any other pair.

In the first part of their model, it was proved that sincere voting can both be informative and rational only if the choice rule used is the majority rule. In their second and third part of their model, [Austen-Smith and Banks \(1996\)](#) showed that sincere voting cannot both be informative and rational. Moreover in [Wit \(1998\)](#), while the model suffers from multiple equilibria, under mixed strategies the CJT is again confirmed in the general case. The possibility of a valid CJT under rational voters was restored in [McLennan \(1998\)](#), where it was confirmed that, whenever sincere voting leads to the conclusion of the CJT, there exist Nash equilibria with these properties. In symmetric (anonymous) environments, the equilibria may be taken to be symmetric as well.

In 1998, [Feddersen and Pesendorfer \(1998\)](#), proved yet another impossibility theorem: when voters choose strategically, the unanimity rule results in a strictly positive probability of both acquitting the guilty and convicting the innocent. Increasing jury size does not help, but only when the unanimity rule is dropped. On the other hand, by introducing majority rule in [Ben-Yashar and Paroush \(2000\)](#), the likelihood of implementing the correct choice is greater than that of a correct choice done by a member of the team that is sampled at random. Under any voting rule, the symmetric equilibrium was proved to have a cut-off structure of the form $0 < a < b < 1$ ([Gerardi \(2000\)](#)). Moreover in [Duggan and Martinelli \(2001\)](#), under unanimity rule, Bayesian Nash Equilibrium exists for any jury size, but the probability of convicting the innocent (acquitting the guilty) tends to zero (one) as jury size grows. Again in [Duggan and Martinelli \(2001\)](#) under non-unanimity, a symmetric Bayesian Nash Equilibrium exists for any jury size. Under fixed jury size, there are cases where requiring more votes to convict raises the probability of conviction and others where probability to convict under unanimity exceeds the one under majority. Finally, under continuous signals, existence and uniqueness of symmetric equilibria, as well as the asymptotic efficiency of non-unanimous voting rules hold in general. Added to that, when the signal space is the unit interval, the unique Bayesian Nash equilibrium in responsive strategies is to vote informatively ([Meirowitz \(2002\)](#)). The unanimity rule here is efficient, as there are at least nearly perfect informative signals.

In the last fifteen years, the literature that copes with CJT has focused mainly on committee design in terms of best rule, number of jurors, sequence of voting and information asymmetries. Although all of the research surrounding the CJT has focused on these subjects, we can discern a pattern of separation, as far as the perception of the nature of the problem is concerned.

Henceforth, and after focusing just on the technical and game theoretic aspects of introducing strategic voting in the CJT, we observe that the main stream of literature has focused mainly on mechanism design, and more specifically discovering the best method to aggregate information in a small or larger electorate. Optimal conservatism in mechanism design increases private incentives to gather evidence and improves the quality of group decision: a standard higher than the optimal can lead to a greater private marginal value of evidence ([Li \(2001\)](#)). As far as sequence of voting is concerned, under unanimity games, the whole set of equilibria is the same in all sequential structures, and there are cases under which sequential equilibria do no better at aggregating information than simultaneous ones ([Dekel and Piccione \(2000\)](#)).

When effort costs are sufficiently high, preference heterogeneity can provide members additional incentives to gather information (Cai (2009)). Consequently, the optimal committee size and the principals expected payoff can increase in the heterogeneity of preferences. With the free rider problem under consideration, Mukhopadhyaya (2003) proved that with perfect signals, probability of paying attention and making a correct decision decrease with jury size. With imperfect signals, the probability of paying attention still decreases, but not always the probability of making a correct decision. When voters can abstain (Persico (2004)), the sets of sequential and simultaneous voting equilibria are disjoint once we introduce arbitrarily small costs of voting. If an appropriate q-rule is introduced, simultaneous voting mechanism dominates all equilibria of the sequential mechanism. Under non-anonymous voting (first best) it is rational to vote informatively. Under anonymous voting, members have an incentive to vote strategically (Ben-Yashar and Milchtaich (2007)). Abstentions may be used strategically to improve the quality of the collective decision.

There exists a range of values around zero for the correlation amongst voters for which augmentation of the subcommittee is always beneficial, but this result does not hold generally (Berend and Sapir (2007)). If there is at least one person biased enough in every direction, the supermajority penalty procedure (maximising the utility of the unbiased person) is non-monotonic with respect to the size of the committee (Chwe (2008)).

In Gerardi and Yariv (2008), optimal incentive is shown to induce a trade-off between inducing players to acquire information and extracting information from them. Optimal extortions of the ex-post efficient rule depend on the accuracy of the signal, while the expected social choice value is non monotone on the cost of information. Accuracy of the private information and jury size are non-monotone in the signal accuracy. Gershkov and Szentes (2009) model an environment where a social planner chooses randomly in sequence and asks voters on a binary choice and allows for a decision to stop asking a report to be made by the social planner when the precision of the posterior exceeds a cut-off that decreases with each additional report. Gershkov and Szentes (2009) show that the restriction to ex-post efficiency is without loss, even for sufficiently imprecise signals. On the other hand, with sufficiently small costs, ex-post mechanisms can be shown to be sub-optimal. Finally, the optimal committee size is bounded, which means that the Condorcet Jury Theorem fails to hold (Szentes and Koriyama (2009)). However, the welfare loss of an oversized committee is surprisingly small: in an arbitrarily large committee, even the worst equilibrium generates higher welfare than that with two members less than the optimal.

1.3 Experiments

1.3.1 Core and Information

In the first decade of experimental voting design, as McKelvey and Ordeshook (1990) assert, the main questions posed were mostly directed at the robustness of the core as a predictor of outcomes in committee processes, the convergence to

Condorcet winners when the information set of committee members varies and effects of different voting procedures.

Fiorina and Plott (1978) is the first laboratory experiment comparing the validity the predictions of several voting models of the time compared to experimental results. In their experiment, 65 committees of five members cast a vote for a series of rounds until a majoritarian outcome is reached. The speed of convergence of the result to a majoritarian outcome is tested with respect to whether subjects are allowed to communicate and whether the payoffs from the session are high or low. Interestingly, no model performs significantly better than others. However, models are ranked with respect to validity according to the different voting settings. In high payoff settings, equilibrium models seem to outperform all the rest. In low payoff settings, if no communication is allowed, all models seem to perform poorly. If communication is indeed allowed, “fairness models seem to give slightly better predictions than the rest. Two important results can be drawn from the experimental general results. Firstly, large sample size cannot substitute for lower payoffs. Secondly, contrary to what theory suggested, in models with no equilibrium the process did not explode.

McKelvey and Ordeshook (1980) experimentally test the effect of vote trading with respect to the efficiency of the voting outcome. Indeed, the results indicate that the vote trading structure fundamentally alters the negotiation process and directly affects the likelihood the Condorcet winner is chosen. The two different setups of the game involve 15 minute negotiations and either unrestricted or irreversible choices after ballots are traded under various payoff structure. 194 undergraduate and masters students were combined in three and five member committees, with an algorithm that minimized the number of meetings of subjects in the same committee. Experienced and inexperienced members never met in the same committee. In terms of results, vote trading resulting in non-optimal results only in committees with inexperienced subjects. Moreover, given they are located in the game setup that allows it, subjects very often reversed Pareto inferior outcomes. Finally, the authors acknowledge the importance of a non-empty core for an increased speed of convergence to an efficient outcome. However, McKelvey and Ordeshook (1981) casts doubt on the belief that the game core is a robust prediction of majoritarian voting outcomes with vote trading.

Eckel and Holt (1989) designed an experiment on committee voting in which a fixed agenda specified a sequence of binary decisions. The purpose of the experiment was to discern whether voting is myopic or strategic. The treatment variables used were private of public information (own versus all agents payoff structure) and the frequency of payment preference profile change. A total of eight different treatments with ten meetings each took place, and each committee consisted of nine voting members. All treatments were conducted with initially inexperienced subjects. The results reveal that even with a very simple agenda with only three possible voter types, the acquisition of sufficient information that induces strategic voting can take a considerable amount of time. Treatments with private information exhibited strategic behaviour only after the third or fourth meeting. However, strategic voting patterns always emerged within four repetitions with the same preference profile. Moreover, the probability of the emergence of strategic behaviour increased with less stationary preference profiles.

In Collier, Ordeshook, and Williams (1989), an experiment with 120 inexperienced subjects in five member committees was conducted in order to test the proneness of voters to purchase information regarding the candidate challenging the incumbent. Voters received information on vote totals and individual payoffs, but not who voted for whom or how many voters purchased information. In six sessions, a promise did not necessarily have a relationship with the policy selected by the incumbent. In the other six, challengers always kept their promises. The results showed that agents rely more on retrospective knowledge and buy less information when the candidates strategies are stable. However, in periods of instability the likelihood of being pivotal, the reliability of information for sale and the extent of the outcome swing between the two candidates increased the probability of purchasing information on the incumbent. Interestingly, beyond the aforementioned correlations, individual decisions appear highly idiosyncratic.

In an initial attempt to test the theory of expressive voting, Carter and Guerette (1992) find weak support for the theory but admit that their results are not robust to alternative parameter settings. 96 economics and accounting undergraduate students participated in the experiment. The probability of being pivotal was the main treatment variable, over the values 0%, 1%, 10%, 20% and 100%, the other variable being the payoff structure. With the probability of being pivotal increasing, they noticed an increase in the likelihood of voting according to a moral sentiment, and attributed this increase to the decrease of the opportunity cost of expressing one's charitable sentiment.

Forsythe, Myerson, Rietz, and Weber (1993) showed that the presence of polls and voting histories leads to a significant decrease of the frequency with which a Condorcet loser wins. Shared histories enable coordination on favourite candidates in a sequence of identical elections. The experiment consisted of two sessions of 48 elections, with 28 different voters, 14 in each group, from a subject pool of 450 MBA and undergraduate students in Business Administration and the Liberal Arts. To examine the effects of election histories, the results were contrasted with Forsythe, Myerson, Rietz, and Weber (1991). In one group there were non-binding polls, while the other was conducted in the absence of polls and there was the option of abstention. In all cases, there was a significant amount of strategic votes. Finally, in the presence of opinion polls and voting histories outcomes generally resulted in satisfying Duverger's Law.

Gth and Weck-Hannemann (1997) in an attempt to explain the value of voting rights and the reasons why people vote, conducted an experiment where 175 non-economics students, without deliberation, were given the option to sell their right to vote in the election of the German Bundestag on 16 October 1994, in an auction where the bids varied from 0 to 200 DM. In the view of the authors, any legal violations of this experiment were avoided by asking the sellers themselves to destroy their voting cards (!). Interestingly, only a very small minority of the participants (2.8%) would sell their right to vote for any positive price. In this experiment, the post experimental questionnaires uncovered that the sense of moral duty surpassed the small probability of being pivotal in the aforementioned elections. Amongst 162 participants that answered the relevant question, 92 provided the fact that voting is a civil duty to preserve democracy as a reason for not selling their votes for any cost.

Blais and Young (1999) in another attempt to explain why people vote, conducted an experiment where they exposed a control group to a 10 minute presentation about the rational model of voting and the “paradox that so many people vote when it is apparently irrational on a cost benefit analysis. The total number of respondents was 1459. Subsets of the control group, along with all students were given a questionnaire to fill, varying from three weeks prior to one week after the elections, providing information about their voting intentions and attitude towards voting, as well as real voting behaviour. The overall turnout of the respondents was 68% (close to the national rate of 70%). Attendance to the presentation resulted in a 7% decrease in the probability to vote for the control group. Also, groups exposed to questionnaires without attending the presentation exhibited higher turnout amongst other groups. Also political interest, party identification and previous voting were variables highly correlated with the propensity to vote.

1.3.2 Pivotal Voter Calculus

In the second decade of experimental voting design, as Palfrey (2009) suggests, under very different voting settings, what is predominant is the evidence confirming highly strategic voting behaviour, determined to a large extent by “pivotal voter calculus”.

Guarnaschelli, McKelvey, and Palfrey (2000) is the first compact experimental study on decision rules using three treatment variables: group size (3 or 6), number of votes needed for conviction (majority or unanimity) and pre-vote deliberation. 4 experiments were run, with 12 plus 1 (used as a monitor) subjects. Each experiment had 4 sessions. Between sessions, 2 treatment variables were altered, the decision rule (majority or unanimity) and the straw poll (taken or not). Subjects were matched randomly in a sequence of 15 matches. In the data, they find clear evidence of strategic voting and the inferiority of unanimous decisions, increasing with group size. However, at the group level, unlike Nash predictions there were fewer incorrect convictions under unanimity than majority. The authors also tested the predictive power of the Nave Nash Model (NNM) versus the Quantal Response Equilibrium (QRE) and found that without a straw poll QRE explained most of the discrepancies. Finally, there were strong evidence of heterogeneity in subjects choices while the frequency of strategic voting was overall higher in the last five rounds.

Frchette, Kagel, and Lehrer (2003) present the first experimental investigation of open versus closed amendment rules. Committees of five randomly assigned subjects voted in either closed (no information) or open (information on allocations for all members) amendment sessions using a discount factor (0.8) in case the proposed allocation were not to be voted upon by the majority of members. Delays were less frequent under the closed rule; however they were less than predicted in the open rule. In the closed amendment treatment and with experience, play converged towards minimal winning coalitions. In the open amendment treatment, supermajorities remained the norm throughout all sessions. Proposers take started the same in both treatments, but ended significantly higher under the closed amendment rule, but well below equilibrium predictions in both cases.

Regressions revealed that subjects voted primarily on their own self-interest with minimal concerns on the least well-off.

In [Casella, Gelman, and Palfrey \(2003\)](#), authors experimentally explore the storable votes mechanism in terms of realism with respect to its complexity. Voters have a finite number of votes to be cast across a finite superset of binary agendas. They are called to play a complicated dynamic game where the marginal effect of a vote today has to be compared to its effect of being pivotal sometime in the future. Subjects participated in committees of 2, 3 or 6 and played a total of 30 rounds each and were randomly re-allocated in a committee after 2 or 3 rounds. No subject participated in more than one session. Several models (Aggregate Best Fit, Noisy Cooperative Behaviour, Noisy Nash Equilibrium) are contrasted with respect to perturbations of the QRE models, with the latter being found superior in terms of best fit. In terms of aggregate welfare, the efficiency calculations based on the perfect Bayesian equilibrium model of behaviour predicts almost perfectly the aggregate surplus for all treatments. Monotone but off equilibrium cut-off point strategies were prevalent. Finally, authors conclude that concerns about the complexity of the game may have limited practical relevance.

[Aragones and Palfrey \(2004\)](#) conducted an experiment to determine the effect of a difference in quality of candidates competing in a one policy space. The game consisted of 200 rounds, where agents participated in the first and the latter 100 as either the better or the worst candidate with random blind re-matching. The better candidates adopted on average more centrist policies than the worst. Secondly, the equilibrium predicted the distribution of outcomes, rather than single outcomes. Finally, the equilibrium varied systematically over the level of uncertainty about the location of the median voter. Finally, QRE seemed to explain some of the observed biases.

[Grober and Schram \(2006\)](#) conduct an experiment in which they distinguish between early voters and late voters. The latter are informed of the early voters' turnout decision. 168 subjects participated in electorates of 12 voters, divided in 2 groups with six subjects each, with each subject having a fixed role of a sender or receiver throughout the game (3 senders and 3 receivers). In the first treatment, the constitution of each electorate was either fixed from the beginning of the experiment (partners) or randomized after each round (strangers). The second variable was information: no information, same (allies) and different interest groups (adversaries). They show that information increased turnout by 50%. However, the stability of group composition did not affect turnout. When information was indeed exchanged, turnout was highest amongst allies and lowest when voters did not know each others preferences. The fact that senders participated more than receivers of information under no information was attributed to the fact that the structure of the game allowed for a delayed vote by senders.

[Battaglini, Morton, and Palfrey \(2007\)](#) is an experimental study comparing the behaviour of voters under simultaneous and sequential voting rules when voting is costly and information is incomplete. They present 2 game forms, the simultaneous voting game and the sequential voting game. Six sessions with either 9 or 12 subjects were conducted. Each subject participated in only one treatment. Each session had two parts, with a total of 40 rounds. Subjects were randomly divided in groups of three. In the simultaneous game, there was little support for

the Nash equilibrium of the game: low cost voters abstained significantly more and high cost voters significantly abstained significantly less than predicted. On the other hand, the QRE model did fairly well, as both high and low cost models could be explained by a single value for the unique parameter λ of the model. In the sequential voting game, whereas cost increased abstention, first voters were significantly less likely to abstain than voters of the simultaneous game with the same cost and again, QRE provided a good fit for the results. Generally, their results suggest that sequential voting is more efficient, both informationally and economically. However, in terms of equity the results are reversed: later voters make significantly more than early ones.

Patty and Weber (2007) conduct a model on how citizens evaluate politicians using two types of sessions. In one kind one subject was a leader and 7 were voters. In the other session, again there was one leader but 45 voters, amongst which 7 would be randomly selected to receive payoffs in the end of the game. Leaders can observe and costly augment the state of the world in favour of voters. On the other hand, voters can only imperfectly observe the (possibly augmented) state of the world and re-elect leaders or change them. Authors compare the predicting ability of the perfect Bayesian Nash model (PBE), QRE model and a model of *strategic naivete* (SN), were agents understand the leader's choice but not the feedback effect of their own actions to the leaders choice. PBE was rejected in almost all (98.5% cases). QRE and SN did fairly well, and authors conclude that distinguishing between these two models would require further work.

Levine and Palfrey (2007) designed an experiment to shed more light into the paradox of voting. A total of 342 subjects participated over 19 separate sessions. Treatments consisted of committees 3, 9, 27 and 51 voters and each treatment lasted for 100 (50 toss-up and 50 landslide) rounds, apart from the 3 member committees, where the game lasted for 50 rounds. Each voter was one of two possible preference types. For each sub-treatment, the sum of voting members of one type was either twice the sum of the voting members of the other type or the difference in sums was one single voter. QRE provided a good fit for the model. The experimental results showed that turnout decreased in larger electorates. What is more, turnout was higher in elections that are expected to be close. Also, voters supporting the less popular alternative had higher turnout rates. Finally, they found that voters mostly used approximate (and not exact) cut-point strategies.

Duffy and Tavits (2008) experimentally tested whether there exists a direct correlation between the subjective probability of being pivotal with the likelihood that a voter casts a costly vote using a total of 140 subjects. In four beliefs treatment, 20 rounds of voting took place with an intermediate stage where subjects were asked to state their beliefs with respect to the probability of being pivotal, whereas in three control treatments the question was not posed. In the beliefs treatment, there were given two different versions of instructions, a simple (one treatment) and a more complicated one (three treatments). The difference between the simple and the complicated version was that in the latter subjects systematically overestimated the probability of being pivotal. Aggregate results showed that subjects who believed they were highly likely to be pivotal were more likely to participate. In general, all subjects systematically overestimated the

probability of being pivotal. Also, beliefs were better estimates of participation than actual probability of being pivotal. Finally, beliefs became gradually closer to the actual probabilities, a fact that implies a learning effect in the turnout decision.

Ali, Goeree, Kartik, and Palfrey (2008) is an experimental study comparing decision making under unanimity rule in ad hoc committees, like juries or expert panels, versus standing committees, like boards of directors, judicial panels, or town councils. The data were collected in 1997, 2000 and 2007 under different experimental protocols and procedures. Each committee contained 3 or 6 members, under either simultaneous or sequential voting with a total of 222 subjects participating. The results clearly show that there is no difference in voting behaviour between ad hoc and standing committees under unanimity with simultaneous voting. In terms of information aggregation, there is limited support for the notion that standing committees allow for a greater degree of coordination amongst members. With sequential voting, there are either small or no differences, apart from the 6 person committees under innocent signals. In terms of information, the hypothesis that standing committees aggregate information better than ad hoc committees can only be supported for the smaller committee size.

Kube and Puppe (2009) is a study on strategic voting using a Borda mechanism. 144 subjects were randomly allocated and matched in committees of three members and were only allowed to participate to a single treatment. Two treatment variables were used: information and inequality. They found that manipulation rates were surprisingly low, but increase significantly if subjects receive information on others preferences and votes. On the other hand, distributional concerns do not play a significant role. Moreover, under uncertainty agents tried to secure themselves in their second best alternative instead of voting according to first best. However, the fact that some uninformed subjects reported their preferences untruthfully was attributed to a “supply effect”³.

An experiment testing the behaviour of subjects in a three party setting voting over three policies, with the outcome exhibiting a voting cycle was presented in Smirnov (2009). 86 subjects familiar with the concept of strategic voting were divided in three groups, consisting of 31, 27 and 28 members respectively and participated given an augmented semester mark instead of payment. They all voted over the agendas, saw the outcome, members of the same party were allowed to deliberate and then subjects were allowed to change their vote if they wished. In between the stages, questionnaires were filled to indicate the level of understanding of each subject. The sincere voting model could explain 26% of the voters choices, the strategic model 47%, risk aversion up to 56% and the expected utility sophisticated voting model (EUS⁴) could explain 73% of the votes.

An experimental study that tests the impact of opinion polls on aggregate welfare in costly voting can be found in Grober and Schram (2010). 288 subjects participated on 12 sessions consisting of 24 subjects each. In each session, consisting of 100 rounds, subjects were divided in two electorates of 12 voters

³Subjects invest time in participating in the experiment and showing up, and find it hard to do the equivalent of nothing, in this case to report their true preferences

⁴The same as the strategic model but without the assumption of consistently assigned beliefs.

and experimenters employed a 2x2 between subject treatment design with three sessions per cell. Treatments were different with respect to the following variables: they either contained uninformed or informed voters with respect to polls and either floating or mixed (floating and allied) voters. QRE provided relatively good predictions for the results of the experiment with respect to the Bayesian Nash Equilibrium. The results of the game showed that polls increased aggregate turnout by 22% – 28%. As the level of disagreement increased, polls increased turnout, but the result is stronger for the minority than the majority. Whenever there were committees with allied voters, turnout increased in the later rounds. However, the increase in turnout is entirely due to floating voters. Finally, majorities win elections more than minorities, and polls augment the vote difference in close elections.

[Battaglini, Morton, and Palfrey \(2010\)](#) is the the first swing voters curse laboratory experiment. In all treatments they used 7 voters with accuracy equal to 0.75. Two different treatments were used for a balanced (1/2) and skewed (5/9) state of the world: and three different treatments for the number of partisan voters (0, 2 and 4). The experiment was conducted in seven sessions. In five sessions 14 subjects participated; in two sessions 7 subjects participated for a total of 84 distinct subjects. In sessions with 14 participants, subjects were randomly divided into groups of 7 for each period. In sessions with 7 participants, subjects comprised a single voting group for all periods. Majority voting was repeated for 30 periods, with variations in sequence and with the group membership shuffled randomly after each round for sessions with 14 subjects. The results of the experiment give strong support for the theory both at the aggregate and individual level. Uninformed voters delegated the vote to more informed ones by strategically abstaining. The existence of partisans increased participation to even out the result and this effect was strengthened as partisan bias increased. Finally, turnout and margin of victory both increased with the number of informed voters.

[Bassi, Morton, and Williams \(2011\)](#) is a majority voting experiments where subjects were randomly assigned identities in common with a candidate and sometimes received financial incentive to vote against their identity. Subjects were influenced by their assigned identities and the effect is stronger when voters have less information. Subjects were divided in 12 groups of 5. Each group played a total of 8, 16 or 24 periods. Labels were randomly assigned. However, in reality a pseudorandom method was used so that at least one label was represented in each group. Three payoff treatments were conducted (no pay, low pay and high pay) and two information treatments (complete and incomplete). In the complete information session, the trembling hand equilibrium prediction was supported when voters were in the majority and voters voted sincerely disregarding financial payoffs. However, when voters were in the minority only 3% (5%) of voters in the high (low) payment treatment voted sincerely and 52% in the no pay treatment. In the incomplete information treatments, voters with no financial incentives vote sincerely 73–79% of the time. Authors also find evidence that financial incentives do have an effect on voter behaviour but increasing financial incentives strongly effect voting behaviour, unlike the complete information case. In the no payment treatment there is a significant difference between the complete and incomplete information that vanishes in the high and low pay treatments. In general, they

find that identities can be a strong and significant influence on voters' choices but the effect quickly vanishes with financial incentives under complete information, whereas it remains under incomplete information, with less at stake.

1.3.3 Abstention and Political Influence

The last five years of voting experiments in economics have mainly followed the experimental methodology of the previous decade, shedding light on the effects abstention, social and political influence on voting choices.

Goeree and Yariv (2011a) conducted an experimental study on the effect of deliberation on collective decisions. A $3 \times 3 \times 2$ design was implemented: the distribution of preferences varied among subjects (one distribution entailing common interests and two allowing for different formats of heterogeneity), the institution or voting rule by which the group decision was made (simple majority, 2/3 super-majority, and unanimity) and the availability (or unavailability) of free-form communication. Groups of 9 subjects were randomly allocated at the start of each period: 15 periods without communication followed by 15 periods with communication (with one practice round preceding each). Three of the sessions were repeated with the chat periods proceeding the no-chat periods to check for order effects but led to qualitatively identical insights as the baseline treatments. Without the ability to communicate, subjects behaved in a rather sophisticated strategic manner. Costless free form deliberation seemed to aid in improving efficiency and diminishing institutional differences. Finally, authors report that the form of communication was public, truthful and a strong predictor of group choice.

Morton and Tyran (2011) is an experiment on abstention when voters in standing committees are asymmetrically informed and there are multiple pure-strategy equilibria. In the beginning of the experiment 207 subjects were randomly divided into groups of three and remained in the same groups throughout the experiment (fixed matching procedure). The groupings were anonymous, that is, the subjects did not know which of the other subjects were in their groups. 12 sessions for 60 periods each with a total of 207 subjects using both between and within subject design. Results show that experts almost always vote according to their signal and non-experts with a vast accuracy difference abstain, thus in that case groups often coordinate to the swing voter's curse equilibrium. If the difference in accuracy is less predominant, outcomes varied in either resembling the swing voters curse equilibrium or the all vote equilibrium. This was mainly attributed by authors to a lack of coordination between the two equilibria. Mixed strategy equilibrium predictions were not validated.

Ryan (2011) is providing experimental evidence on the influence of social networks on the ability to vote for the correct candidate in terms of valence and position. 135 subjects in the information treatment group received private information about the candidate and shared it (in sub-treatments) between supporters of the same candidate, the opposing candidate or both. 81 subjects participated in the control group where no such information was transmitted. The results show that social information is useful in the case of uninformed voters, but not for informed ones. Subjects were clearly not evaluating messages as theory would

predict: informed candidates based their voted on social information even if that information came from informants with conflicting preferences. However, on average there was no negative effect on correct voting since extra information was as equally likely to be helpful or harmful. In the case of independents, uninformed agents vote correctly more often after having received this extra piece of information. Authors suggest that subjects are prone to be persuaded to vote as their social network compels.

Blais, Labb-St-Vincent, Laslier, Sauger, and Straeten (2011) experimentally tested strategic considerations of voters in multi-party elections, when the decision to support a candidate is the private benefit of supporting one party versus her perceived viability in one and two round elections. In groups of 21 voters, eight elections were held successively: 4 one-round and 4 two round-elections and in the end one elections was randomly selected as the decisive one in terms of payoffs. In every election, each subject selected one of five candidates. The latter were located in five distinct points of the one dimensional spectrum. Strategic considerations on viability of candidates increase over time. Vote shares of various candidates were strikingly similar in one-round and two-round elections: whereas the propensity to vote strategically is slightly weaker in two-round elections, it does exist.

Casella, Llorente-Saguer, and Palfrey (2012) develop and test a competitive equilibrium theory of a market for votes. They do so in order to empirically test the weakness of the majoritarian rule to capture variability in the intensity of preferences. Each experimental session consisted of a series of 20 rounds, where subjects, equally likely to be biased towards one of two agendas, participated in 5 or 9 member voting committees. Each voter was assigned a uniformly random valuation for each agenda. Before each ballot, bids and offers for votes were allowed. The results exhibit overpricing relative to the risk neutral equilibrium that can be explained by risk aversion. In larger committees dictatorship still arose, but less often. Predicted welfare ranking was confirmed: average payoff was significantly lower with respect to no market for votes when valuation discrepancy was low, and higher but insignificantly low otherwise.

Woon (2012) conducts an experiment to discern between the choice of voters for a retrospective or a forward looking voting rule, when presented with a candidate whose choice they cannot directly control. 88 subjects participated in five sessions, consisting of between 14 and 20 subjects. Each session lasted for 36 rounds, and each subject acted half the time as a politician and half the time as a voter. When subjects played the role of policy motivated politicians they were consistent with equilibrium behaviour in choosing their preferred policies. Office seeking politicians followed their signal very frequently (86% of the time), possibly as a response to retrospective voters. In terms of voting behaviour, subjects are mostly retrospective in their choices and there is little or no sign of learning or behaviour change. Authors give several explanations for voters' behaviour: bounded rationality, preference for accountability, cognitive complexity and inability to ignore strategically irrelevant information. They conclude that boundedly rational individuals prefer to use a simple retrospective rule and they find difficulty in making strategic inferences, even in simple settings.

Coronel, Duff, Warren, Federmeier, Gonsalves, Tranel, and Cohen (2012) conducted a methodologically unique experiment with respect to the relevant voting literature. Six severely impaired amnesiac patients were matched in terms of age, sex and education to six normal comparison participants. Participants were given a voting choice between two amongst five fictitious characters they had individually ranked as equal in the beginning of the experiment. Using intermediate math solving intervals as memory distraction, they asked subjects to vote again/recall and rationalise their voting. All subjects chose again/recalled the correct candidate in terms of individual alignment to the candidate's political beliefs. However, when asked on the reasons of their choice, no amnesiac was able to recollect the reasons for his or her choice.

Morton and Tyran (2013) is an experimental study on the effects of corrupt experts on information aggregation in committees. The experimental design was similar to Morton and Tyran (2011). Their results show that in the presence of corrupt experts, decreased abstention leads to increased information aggregation efficiency in group decision-making. As the accuracy varies, the abstention rate of non-experts varies significantly and controlled for experience, the accuracy is decreasing in the corruption likelihood of the expert. All in all, when the likelihood of a corrupt expert is low, the added participation increases information efficiency. However when the likelihood is high, the increase fails to offset the information loss due to the corrupt experts.

In Kittel, Luhan, and Morton (2014), authors conduct an experimental study on the costly voting in multiparty committees with varying degrees of communication. In each committee they allocate subjects with varying preference orders, and a computerized automaton always casting the same vote (common knowledge) divided in electorates of varying sizes. 4 treatments: Baseline, Party Label, Party Chat and All Chat. In total, two sessions were run with 24 subjects, three sessions with 22 subjects and one with 20 subjects. Of the total 19 periods in every sequence (for example: baseline-label-party chat), 4 were randomly chosen for payment, which was common knowledge. Both within and between subjects design was used to evaluate the data. The coordination capacity in the communications treatments resulted in generating higher payoffs. Payoffs from the baseline and no communications gradually converged to the communications treatments. Communication also had an enormous positive effect in turnout by reducing the number of abstentions by a half and in strategic voting, which was most likely amongst swing voters assigned to their second party preference. Voting intentions reporting under all communications conditions is predominantly sincere. Finally, voters tend to stick to their initial intentions more in the party chat than in the all chat treatment.

Bhattacharya, Duffy, and Kim (2014) provide the results of an experiment on compulsory and voluntary voting in a game of common preferences. They consider two treatment variables: voting mechanism (voluntary versus compulsory) and cost of voting (zero versus positive). The three treatments conducted were: compulsory(C), voluntary and costless (VN) and voluntary and costly (VC) voting. In each session a group of 18 subjects was divided in two groups of 9 subjects and played for 20 rounds. Under the compulsory voting mechanism there is strong evidence of insincere voting, unlike the other treatments where voters

cast predominantly honest votes. In the voluntary voting, the difference of participation between voting types is in accordance to the prediction of that of the symmetric voting equilibrium. However, in both voluntary treatments subjects over-participate relative to the predictions. All data exhibited convergence to equilibrium predictions, with the slowest being that by subjects in the voluntary costly treatment.

1.4 The More the Merrier?

1.4.1 A logical map of the literature

The main axis of the implementation of strategic voting in the CJT is the relationship between:

1. the size of a voting committee and
2. the accuracy of the ballot outcome.

As discussed in the previous two parts, the general parameters that have been used as levers to understand the underlying mechanisms behind voting procedures are:

1. the voting rule:
 - (a) majority,
 - (b) supermajority,
 - (c) unanimity,
 - (d) Borda rule,
2. the cost of casting a vote:
 - (a) costless,
 - (b) costly,
3. information:
 - (a) quality,
 - (b) asymmetry,
 - (c) diffusion,
 - (d) deliberation,
 - (e) networks,
4. voters' preferences:
 - (a) homo or heterogeneity,
 - (b) dependency,
 - (c) correlation,

- (d) retrospective,
- 5. sequence of voting.
 - (a) single or multiple rounds,
 - (b) simultaneous,
 - (c) sequential,
 - (d) storable,
 - (e) traded.

1.4.2 Is good exogenous?

Within this framework, as shown in the previous paragraphs, the interconnections between these parameters and the trade-off between jury size and voting result accuracy are combined to end up with interesting results concerning the CJT. Although these results are robust, they seem to fail to capture a main aspect of collective decision making.

When a committee of doctors is expected to give a combined opinion concerning a dying patient, they are to discern whether the disease that is leading the patient to his end is of a specific type. The type of disease is already there and their expert opinions are to be combined to approximate the true nature of the disease. Of course, neither the answer to this problem is simple, nor is simple the answer of a committee of judges on whether an accused person has or has not committed a crime. However, the main difference between these situations and a collective of individuals deciding on their common future is that the result of the latter is not something external to be approximated, but something rather endogenous and bound to the likes and dislikes of the individuals concerned with the choice.

Of course, the concept of strategic thinking is common in both situations. In the first, let us take the example of set of judges deciding to convict or acquit someone with respect to a crime he has allegedly committed. Moreover, let us take the case where a unanimous “guilty” vote is necessary to result in a conviction. The essence of the result of [Feddersen and Pesendorfer \(1998\)](#) is that, each judge will consider the case where she is pivotal. If the signal she gets is not conclusive, she may reside in the fact that it only takes one “not guilty” vote to acquit, and depend on the signals of other judges, voting guilty even if the signal she got is almost innocent. Thus he could vote “guilty” based on the fact that the others will have a better signal and decide on his behalf. On the other hand, when one is concerned with the result of a collective decision making mechanism, the result of the ballot is endogenous in this case. There is no ex-post right and wrong. We can say that in a democratic ballot there is a status quo, but the result of the procedure is seeking social stability, not a universal pre-existing right. After the ballot, we cannot expect that there will be an enforcement mechanism to implement any result. Hence, any participant in this ballot shall vote taking this fact in consideration. And, as in Rousseau’s social compact, voters will not vote only according to their preferences alone, but also according to their desire to

participate in the compact, and this is how their preferences will be amalgamated robustly.

Following this train of thought, the CJT seems not to have a clear cut answer. At first sight, the more people involved in a collective decision, the harder it seems to agree upon common ground. But there are examples where this basic intuition is clearly violated. Imagine a couple trying to paint their house with a single colour, having only three options for that: blue, red and white. Also, let us assume blue and red are each one's favourite colour. It seems to be harder to choose a colour in that situation with respect to the situation where one hundred people try to paint a building and live together in a commonwealth. There, white could seem to be a more neutral but also stable option for all. The more the merrier? It could seem so in the case of house painting.

Of course, the main trade-off in this case is not size of committee and result accuracy, but size of committee and result robustness. This is probably as Condorcet intended it ([Young \(1988\)](#)), as in his method of voting counting, the minority on pair wise votes reverses to attempt a final coherent social agreement.

Any of the five elements that were taken into consideration in the literature of strategic voting, the CJT and mechanism design, every one of them is important in any attempt to endogenize the result of a ballot outcome. The only difference is now that information diffused within the decision makers is no longer on the validity of a specific binary variable exogenously defined, but on each others perception of what each others preferences and common ground are.

Chapter 2

Costly Political Action and Majority Decisions

“The lady doth protest too much, methinks.” (Shakespeare)

2.1 Introduction

After the Second World War, the western world has considered the right to protest as one with unbreakable axiomatic bonds to democracy. In Rome, on the 4th of November 1950, during the “European Convention of Human Rights”¹, the member states of the Council of Europe affirmed the importance of the right to protest within democracy. This importance was concretized into the western democratic edifice in the 16th of December 1966, during the International Covenant on Civil and Political Rights. Then, the members of the United Nations confirmed, by common law, the right of their citizens to engage in protest.² Since, numerous protests in both sides of the Atlantic have marked the pages of post war history: from Martin Luther King’s 1960 “Dream” speech in the 28th of August in front of 200,000 protesters, during the March on Washington for Jobs and Freedom, to London’s 27th of March 2011 protests, where 250,000 people attended a march against public spending cuts, the influence of these protests to policy making and the evolution of democratic processes is un-debatable. Of course, this influence is not always direct, neither its implications easily tractable. Often even, protests have resulted in riots, destruction of public and private property, and even the death of protesters and bystanders. Among the more recent European examples are the riots of October and November 2005 in Paris and other French cities, the riots of December 2008 in Athens and the recent August 2011 riots in London.³

¹Signed, considering the “Universal Declaration of Human Rights”, proclaimed by the General Assembly of the United Nations on 10 December 1948. The combination of Articles 9 (freedom of thought, conscience and religion, freedom to manifest one’s religion and beliefs), 10 (freedom of expression) and 11 (freedom of peaceful assembly and association) confirm the right to protest.

²The relevant articles are A18, A19, A21 and A22.

³Of course, in all three of the aforementioned examples, the protests themselves can only be considered to be indirectly related to the following riots: in all three cases, the fatal injury of one or more civilians by members of the local police forces has triggered the events.

On the other hand, the wide use of the Internet, as well as other advancements in communications, has decreased the individual cost of political action to an almost minuscular lever. Just in a matter of seconds and with small individual cost, people can use their personal computers or hand held devices to engage in political action.⁴ I argue that this significant reduction in the individual cost of political action can end up being harmful with respect to aggregate welfare.

I model costly political action as individually costly communication: agents engage in political action to signal their individual opinion to others. This chapter's main result is based on the unique symmetric sequential equilibria in pure strategies of a simple two stage game, with agents sharing identical preferences but imperfectly informed about the state of the world. Firstly, agents decide whether to engage in a costly political action in order to strategically signal their private information on the state of the world and, afterwards, they costlessly cast a vote on a binary agenda, having observed the number of agents who shared their private information. In this model, the concept of costly political action is used more broadly: any specific form of costly communication in order to send a public signal for (or against) any agenda to be voted upon, fits the model's description. Under different values for the cost variable, this could depict different forms of information sharing from sending an email and signing a petition to participating in a peaceful march or even a riot.

I argue that if the cost of political action is sufficiently small but positive, then the expected welfare result of elections with prior communication will always be inferior to elections without prior communication. If the individual cost of political action was equal to zero, agents sharing their individual information and then unanimously voting would result in information being aggregated. Voting without prior communication would bring about the same result in terms of voting outcome and aggregate welfare. However, a sufficiently small cost of political action will result in a situation similar to a prisoner's dilemma, where individuals always prefer participating in informative costly political action to abstaining from it, conditional on others doing so. In other words, rational agents are "forced" to participate in political action in the first stage to avoid wrong updating to the second stage. Since all agents will informatively disclose their signal, the result will be that all of them unanimously voting for the same outcome the majority would vote in elections without information sharing. Note that a similar result holds even if agents use a mixed strategy in the political action stage, the voting stage or both.

Furthermore, I examine the case where agents have heterogeneous prior beliefs about the state of the world and interpret the latter bias as a difference in prior political opinions, following [Che and Kartik \(2009\)](#). In this case, costly communication prior to elections can be welfare improving. Note that if the agent's priors are sufficiently dispersed, the outcome of elections could be random. However, costly political action in this case provides a mechanism that, through bayesian updating, can "draw" agent's beliefs back to the "correct" priors. I find an upper and a lower bound for the cost of information sharing. If the cost is below

⁴For example, see [Yoder \(2012\)](#), for the online 18 January protests of the Stop Online Piracy Act (SOPA)

the lower bound, an equilibrium where all agents engage in informative political action can be sustained. If the cost is between these bounds, there exists an asymmetric equilibrium in pure strategies under which only a subset of the agents engage in costly political action. This equilibrium can be welfare improving under certain conditions that depend on the trade-off between signal accuracy and aggregate cost of political action. More specifically, if the state of nature is sufficiently uncertain,⁵ there exists a range for the cost of political action that induces a small subset of the agents to share their signal informatively and improve all others' signal before elections, while the expected aggregate cost of information sharing remains lower than the expected benefit, which is improving all voters' signal accuracy.

The model in this chapter builds on [Lohmann \(1994\)](#). In the first part, I examine the case when agents have the same preferences (a common good) and then I extend by allowing for bias in agent's prior beliefs.⁶ In [Lohmann \(1994\)](#), agents have different preferences but share the same prior beliefs about the state of the world. Because of their extremely biased interests, Lohmann suggests that some individuals can be "rationally trapped" in protest, and this may result in information not aggregating. I show that even in situations where agents have identical interests, costly political action will lead to an inferior outcome in terms aggregate welfare, relative to the uninformative case where no agent participates in political action. The second part of my model also incorporates the strategic element that is inherent in Lohmann's model in a model where heterogeneous priors represent dispersion in political beliefs. Since in the second stage all agents vote honestly, this allows for comparative statics, empirical testing and welfare comparisons between different protesting group sizes, as it simplifies the equilibrium representation, while it maintains the ability to reproduce Lohmann's results as well.

My results can be contrasted with [Persico \(2004\)](#), where agents have to pay a cost to acquire information. I assume that all agents are informed costlessly but information sharing is costly. Persico's shows that, when information acquisition is costly, information is under-provided. I show that, when information transmission is costly, information is over-provided. The behaviour of voters in the present model is similar to [Piketty \(2000\)](#), in the sense that they act both strategically in the political action stage, but also honestly in the voting stage. However, in Piketty's model, voters' behaviour is intermediate between "strategic" and "sincere", whereas in this model, strategic behaviour only occurs in the political action stage, whereas in the voting stage all agents vote "honestly" in equilibrium. In [Krishna and Morgan \(2008\)](#) strategic abstention from costly voting is such that, in the limit, the correct candidate is elected with probability one. In the equilibrium of my model, any abstention from informative political action occurs for a sufficiently low cost only under heterogeneous prior beliefs. Therefore, under homogeneous prior beliefs with sufficiently low cost, informative political action is not welfare improving.

⁵Aggregate welfare analysis under this framework depends on the choice of what the "real" prior is and thus we can only talk firmly about welfare results *after* the state of the world is revealed.

⁶See Appendix for an extended comparison of the two models

Moreover, the implications of this model can also be contrasted with those of Osborne, Rosenthal, and Turner (2000) and Osborne, Rosenthal, and Turner (2005). Their adherence of the median as a rule of compromise results in an outcome likely to be random. In my model, the result is not random at all. Moreover, I can specify the set of individuals that will participate in costly political action in equilibrium. Finally, as in Banerjee and Somanathan (2001), where information sharing takes place in order to influence the opinion of a leader of a group, in my model information sharing is taking place to directly influence the beliefs of voters before elections.

Finally, there is a wide strand of empirical literature on costly protest that suggests strategic behaviour of protesters. For example, Finkel, Muller, and Opp (1989) have highlighted the importance of a cost benefit analysis by agents, conditional on the protest group's success and Muller, Dietz, and Finkel (1991) show that discontent is not significant as a reason for political action, unless it is weighted by the expectancy of the action's success and the perceived importance of personal participation. Also, Finkel and Muller (1998), focusing on the costs of protest, again validated that cost variables influence political action in a negative direction. Moreover, they showed that "soft selective incentives", as entertainment through protesting, utility gains for standing up to one's views or learning by protesting are not significant in explaining protest behaviour.

The rest of the chapter is structured as follows. Section 2 describes the general setup, the timeline and the equilibrium concept of the game. Section 3 presents and analyses the equilibria under the assumption of homogeneous priors. Section 4 presents and analyses the equilibria under the assumption of heterogeneous priors. Section 5 compares the aggregate welfare of all the equilibria of the previous sections. Finally, Section 6 discusses the results of the model and provides some ideas for possible extensions. All proofs are relegated to the Appendix. For completeness, in the end of Appendix I present a note on the strategic equivalence between heterogeneity through preference divergence and through distinct priors. All propositions hold for finite electorates electorates, unless otherwise stated.

2.2 Model

2.2.1 The game

Society consists of a large but finite odd number (n) of agents. Each agent (indexed by k) has prior beliefs about the distribution of the state of nature $s \in [0, 1]$, which will be described with a probability distribution function $\beta_k(s)$, while the true distribution of the state of nature is uniform on the $[0, 1]$ interval.⁷

Each agent initially receives a binary signal $\sigma_k(s)$, correlated with the true state of the world: $\sigma_k(s) = 1$ with probability s and $\sigma_k(s) = 0$ with probability $1 - s$. Agents who received $\sigma_k(s) = 1$ simultaneously choose to engage in costly political action ($\pi_k = 1$) or not ($\pi_k = 0$): the agents who chose ($\pi_k = 1$) incur a

⁷The results can be generalised to any symmetric distribution with positive support on the $[0, 1]$ interval

strictly positive cost that is the same for all agents ($c_k = c > 0$).⁸

After the political action stage, all agents observe the number of agents engaged in political action $m \in N : 0 \leq m \leq n$ and then all simultaneously vote on a binary agenda, either for the status quo ($\nu_k = Q$) or for the alternative ($\nu_k = A$), where $0 < Q < A < 1$ and the two policies are symmetrically placed around the midpoint $\mu = \frac{A+Q}{2} = \frac{1}{2}$. The latter means that neither of the policies has an ex-ante advantage.⁹ Voting is costless.

Finally, majority rule M determines the outcome of the ballot ($M = A$ if $|\{k : \nu_k = A\}| > |\{k : \nu_k = Q\}|$, else $M = Q$) and agents receive their payoffs according to the utility function $U(M) = -(M - s)^2 - \pi_k \cdot c$. All the payoffs involve two elements. The first element $-(M - s)^2$ represents a common good for everyone that depends negatively on the distance the majority's choice has from the true state of the world.¹⁰ The second element $-\pi_k \cdot c$ is the cost of informing others, which only weighs on the agents who did.

The timeline of events of the game is summarised as follows:

1. nature decides the true state of the world,
2. each agent gets a binary signal correlated with the true state of the world,
3. each agent decides to engage in costly political action, which will induce a personal positive cost, or to abstain and not incur the cost,
4. all agents observe the number of signals revealed,
5. agents vote for the status quo or for the alternative,
6. majority wins,
7. payoffs are realised.

⁸As in [Lohmann \(1994\)](#), this restriction is made for simplification purposes and does not harm the generality of the results: equivalent results hold for restricting signal sharing to $\sigma_k(s) = 0$ types, or posing no restriction whatsoever. The trivial equilibria created by indifference in the voting stage do not survive the concept of the sequential equilibrium, as will be explained below.

⁹See proof of Lemma 1 for more details on what would happen if this assumption was to be dropped.

¹⁰The concept of a common good for all agents is a theoretic simplification that can be interpreted in several distinct ways. The first that would come to mind is the *platonian* concept of the common good. Moreover, this conceptualization describes very well any smaller subset of individuals with *almost the same* interest from a referendum, or a larger society of individuals, who will be affected by a referendum in “almost” the same way. For example, various protests for or against participation in the European Union can be seen happening due to a common factor: a shift in a country's decision into or out of the European Union would have so big an impact on citizens' lives, that any outcome could be seen as almost equivalent to all. Another example would be a decision upon the constitution itself, which will affect the structure of the polity every citizen participates in, hence everyone, in the same way. Finally, the combination of a common good and costless voting induces “honest” (according to agents' updated preference) voting in the equilibrium solution, structurally avoiding the no voting paradox (see [Feddersen \(2004\)](#)). Also, while agents are strategic and calculate the expected signal conditional on the probability they are pivotal (as in [Austen-Smith and Banks \(1996\)](#)) the existence of a common good is inducing them to vote honestly, thus simplifying the equilibrium solution.

Finally, note that the restriction of only agents who received $\sigma_k(s) = 1$ is signifying that agents can only engage in costly political action for the alternative agenda. Of course, due to the symmetric structure of the game, the results would be exactly the same if political action was for the status quo, or equivalently if people could engage in political action for both. The difference in the latter case is that the aggregate welfare loss would be larger.

Example 1. As a simple example to understand the basic structure of the game, it is useful to consider the issue of nuclear power. The state of the world could be thought of belonging to a $[0, 1]$ continuum that represents the probability of a nuclear disaster. Initially, the members of society receive a binary signal with respect to whether it is dangerous or not to use nuclear power. Then, depending on the personal cost of political action, the signal they have received and their prior beliefs about the danger of nuclear power, each one decides to engage in costly political action against (or for, depending on the societal norms) nuclear power. After, everyone observes the number of citizens engaging in costly political action and they vote on whether more or less nuclear plants will be used. Had they known correct state of the world, all agents would agree on the choice, since the effects of the policy implemented will equally affect them.¹¹

2.2.2 Equilibrium of the game

As an equilibrium concept I will use [Kreps and Wilson \(1982\)](#). Being a subset of subgame perfect Nash equilibria, sequential equilibria are stronger in the sense that they impose a structure on the off-equilibrium-path beliefs of the agents with possibly interesting intuitive interpretations in the pooling equilibria of the game. This difference, of course, vanishes in the separating equilibria, where there are no beliefs off-the-equilibrium path.

Thus, agents update their beliefs about the true state of the world using the Bayes rule according to any information they acquire springing from others' strategy choice.¹²

An equilibrium of the game is a subset of the agents' assessments of the game.¹³ The set of the agents' assessments contains all combinations of the agents' information sharing strategies $\pi_k(s)$; their beliefs at the political action

¹¹For example, more nuclear power would signify cheaper energy for all, but also radioactive contamination for all in case there is an explosion of a nuclear reactor. On the other hand, less nuclear power would signify more expensive energy for all, but also smaller or no chance for a possible radioactive contamination. One and only one of the two scenarios, the *correct* one, will be preferred by all.

¹²The use of the Bayes rule from agents deciding in protesting has even been used in relative empirical works as well. For example, in [Francisco \(1996\)](#), Bayesian updating is used in their econometric model in order to capture the ability of the economic agents engaging in costly protesting to think and adapt.

¹³In order for an assessment to be a sequential equilibrium, the strategy profile has to be sequentially rational given the system of beliefs. The additional restriction that makes a sequential equilibrium a subset of subset of subgame perfect Nash equilibrium is that there has to exist a sequence of totally mixed strategies that converges to the equilibrium strategy under which the sequence of the induced beliefs converges to the beliefs of the assessment itself (see [Mas-Colell, Whinston, and Green \(1995\)](#) for details).

stage $\beta_k(s)$, their voting strategies $\nu_k(s)$; and their beliefs at the voting stage $\beta_k(s|m)$. Each agent uses Bayes rule to update her beliefs about the state of the world, once after the private signal is received and once again after he observes the number of signals revealed. All agents try to maximise their utility function or, equivalently, minimize their utility loss. I only focus on equilibria in pure strategies. Under any indifference whatsoever between the status quo and the alternative, without loss of generality I assume that agents will prefer the latter.¹⁴

The aforementioned model is examined under two frameworks. In the first, agents have identical prior beliefs about the state of the world. In the second, agents have different prior beliefs about the state of the world.

2.3 Homogeneous Priors

Assumption 1. *Under Homogeneous Priors, all agents have the same prior beliefs about s , which follow the uniform distribution.*¹⁵

Under Assumption 1, all agents have the same prior beliefs about the probability of each state of the world occurring. This framework is a simplification of [Lohmann \(1994\)](#) in that agents' utility functions are identical. In this case, if the cost is low enough, there exists a unique symmetric informative equilibrium where everyone participates informatively in political action. Participation in costly political action is informative, in the sense that all agents that received $\sigma = 1$ will participate in costly political action.¹⁶ This cost is decreasing with respect to the total number of agents.

Lemma 1 below suggests that, similarly to [Lohmann \(1994\)](#), under any positive cost, an uninformative equilibrium, where no-one engages in costly political action, can be supported. However, Lemma 2 shows that, conditional on the individual cost of political action being low enough, if an agent expects others to participate in costly political action, she will participate as well.

Definition 1 (USE). Uninformative Sequential Equilibrium under Homogeneous Priors consists of the following assessment of the game:

$$\{\pi(\sigma) = 0; \beta(s|\sigma) = \begin{cases} 2(1-s), & \sigma = 0 \\ 2s, & \sigma = 1 \end{cases}; \nu(s) = \begin{cases} A, & E(s|\sigma) \geq \frac{1}{2} \\ Q, & E(s|\sigma) < \frac{1}{2} \end{cases}; \beta(s|\sigma)\}$$

In this assessment of the game, regardless of their initial signal, agents choose not to engage in political action and share their information. Since nobody communicates, agents use only their individual signal, conditional the probability of being pivotal, to update their initial expectations. They finally vote for the status quo or the alternative, depending on whether they received a signal that favoured one policy, or the other.

¹⁴As the state of the world is continuous, this event would happen with zero probability.

¹⁵Therefore all k -subscripts that denote individual agents in this framework can be dropped without any informational loss for the reader.

¹⁶Of course, all results still hold for the symmetric case where all agents that received $\sigma = 0$ will participate in costly political action whereas agents that received $\sigma = 1$ will not.

Lemma 1. Under Homogeneous Priors, for any strictly positive individual cost of political action $c > 0$ and for any number of agents n , USE is a sequential equilibrium in pure strategies.¹⁷

The intuition behind the proof of this lemma is that no agent has an incentive to engage in costly political action given all others' reluctance to do so. This is true as a revelation of one's signal will never be beneficial when blurred by all other's political inaction, in the case of symmetric policies. Assume that an agent is considering the costs and benefits of bearing the individual cost of political action in order to disclose her signal to all others. All agents who have received the same signal as her would have still voted for the same policy as her. But, after her revelation, so will all the others: since agents who have received a different signal will return to being indifferent, they will vote for the alternative as well.¹⁸ Therefore, by "cancelling" everyone's signal, her expected utility would decrease. This result does not hold if the policies are not symmetrically located around the midpoint $\mu = \frac{1}{2}$. For example, if $\mu < \frac{1}{2}$, since the alternative would be ex ante more beneficial, she could in fact deviate and engage in political action to disclose her information for a sufficiently low cost.¹⁹

Definition 2 (ISE). Informative Sequential Equilibrium under Homogeneous Priors consists of the following assessment of the game:

$$\left\{ \pi(\sigma) = \sigma; \beta(s|\sigma) = \begin{cases} 2(1-s), & \sigma = 0 \\ 2s, & \sigma = 1 \end{cases}; \nu(s) = \begin{cases} A, & E(s|\sigma, m) \geq \mu \\ Q, & E(s|\sigma, m) < \mu \end{cases}; \right.$$

$$\left. \beta(s|\sigma, m) = \frac{(n+1+\sigma)!}{(m+\sigma)!(n-m)!} s^{m+\sigma} (1-s)^{n-m-\sigma} \right\}, \text{ where } m = \sum_{i=1}^n \sigma(i).$$

In this assessment, all agents participate in informative political action. In other words, if she receives $\sigma = 1$, an agent will choose to inform others. After the political action stage, all agents observe the number of revealed signals and update their expectation on the true state of the world. Their updated signal in this case is definitely more precise than their initial one. This can be viewed as giving every agent the access to a number of *trials* for the true state of the world equal to the total number of agents. However, all agents in the voting stage will have access to the same information and will cast the same vote. The result of the voting in this assessment will be unanimous. The following lemma states that when the cost of political action is sufficiently low, then *sincere* signal sharing will also be supported as equilibrium.²⁰

Lemma 2. Under Homogeneous Priors, there exists a cost tolerance function $C^*(n, \mu)$ such that, if $c \leq C^*$, ISE is a sequential equilibrium in pure strategies.

¹⁷All proofs lie in the Appendix

¹⁸We have assumed that any indifference will be broken in favour of the alternative. Assuming that in case of indifference agents would randomize, this result would still hold.

¹⁹For more details on this, see proof of Lemma 1

²⁰According to one's view about the state of the world, non strategic, informative

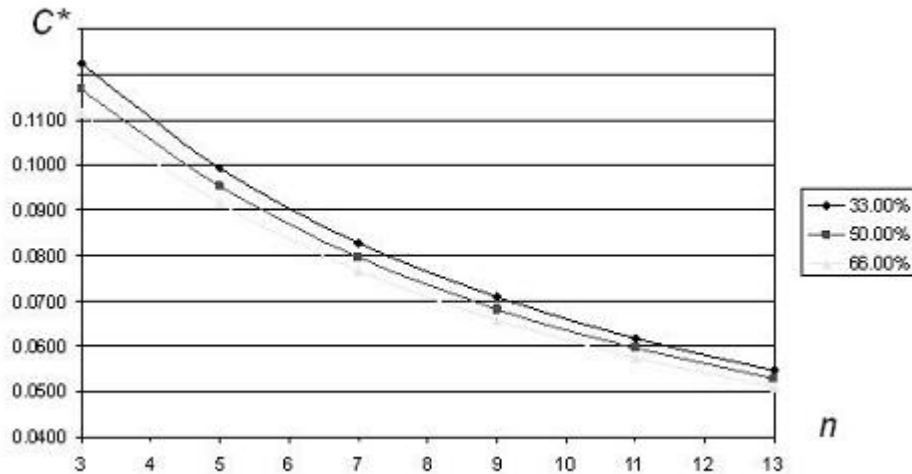
C^* is the maximum cost under which an agent would engage in costly political action in the informative sequential equilibrium. It can be viewed as a cost tolerance function of the values of A , Q and the number of participants in the society (n). The cost tolerance function is given by the following equation:

$$C^*(n, \mu, Q, A) = (A - Q) \left\{ \frac{(n+1)(m^*(\mu, n) + 3)}{(n+2)(n+3)} - \frac{\mu(m^*(\mu, n) + 1)}{2(n+2)} \right\}$$

where $m^*(\mu, n) = \inf\{k \leq n : k \geq \mu n + 2\mu - 2\}$

Note that the cost tolerance function is positive and decreasing in n (for different values of μ), as seen in the diagram on the next page. This result has an intuitive justification: the more agents, the less the informational impact of an additional revealed signal. Moreover, one can observe that agents' choices do not depend on the cost per se, but on $\frac{C^*}{(A-Q)}$, which represents the cost of political action normalized by the two policies' relative distance. This of course is straightforward, as the choice of bearing the individual cost of participating in signal sharing will be made relative to the distance of the two policies. Therefore, the smaller the distance of the two policies, the smaller the cost one is willing to pay in order to inform others of her signal.

Figure 2.3.1: Cost tolerance function C^* for an increasing number of agents n , $\mu \in \{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}\}$



I summarise Lemmas 1 and 2 in the following proposition:

Proposition 1. Under Homogeneous Priors, for any individual cost of political action $c > 0$, there exists a unique symmetric Uninformative Sequential Equilibrium in pure strategies. Moreover, there exists a symmetric Informative Sequential Equilibrium in pure strategies if and only if the individual cost of political action c is lower than or equal to a threshold $C^*(n, \mu)$, $c \leq C^*$.

Consider the decision of a rational voter to participate in informative political action, conditional on others doing so. She would only consider the situations where her participation is pivotal for the outcome of the voting stage. In these situations, she wants to participate in costly political action, since she knows that, if she does not, other agents will vote for the opposite agent as her signal indicates. Given that the cost is low enough, she would always engage in costly protest, independent of the total number of voters.

However, since she will update her signal according to the total number of agents, conditional on being pivotal, her expectation of the state of the world will converge to $\frac{1}{2}$ as the number of agents in society grows. But then, she starts growing indifferent to the actual result of the ballot and does not want to bear the cost of signal sharing. The last argument is the reason that the cost tolerance function is decreasing in the number of voters.

The fully informative equilibrium and the uninformative equilibrium are the only symmetric sequential equilibria of the game. There exist trivial Nash equilibria due to indifference, where there is no communication in the political action stage and everyone casts a vote in the voting stage regardless of her private signal, but they do not constitute a sequential equilibrium. In that case, even though no agent would be pivotal, Bayes rule would be necessary to update any sequence of totally mixed strategies of the agents. Since the only information an agent has in this case is her own signal, even if others cast a random vote, she would still have to vote informatively.

In terms of aggregate welfare, it is straightforward to see that costly political action in this situation is always welfare reducing. The voting result under the informative and the uninformative sequential equilibria will be identical. However, since political action is individually costly, the uninformative equilibrium is always going to be superior to the informative one, due to the sum of the individual costs of political action.

Interestingly, if agents expect others' political inaction, information could be aggregated through the Condorcet Jury Theorem. However, a sufficiently low cost and others participation in costly political action constitutes a situation where one has to participate as well, in order to ensure that her opinion is taken into account in the voting stage. In other words, if the cost of communication is very high, voters aggregate information at the electoral stage. However, if everyone expects protests and demonstrations, people do use this information to update beliefs and make their decision in the second stage. So, one is "trapped" into demonstrating in the first stage to avoid wrong updating to the second stage.

Finally, note that sincere voting is not responsible for the outcome equivalence between the informative and the uninformative equilibrium. The aggregate welfare of all the mixed strategy equilibria of the game remain inferior to the uninformative equilibrium. There exist mixed strategy equilibria where a subset of agents participate in costly signal revelation and agents mix in the voting stage. The subset of agents participating in costly political action can be seen as an "expert", with increased accuracy with respect to each individual agent. However, even if agents mix in the voting stage between their signal and that of the "experts", the result would only be as accurate as that of sincere voting. This happens since in sincere voting the "experts" opinion would be optimally

aggregated in the voting outcome anyway through the majority rule. By mixing in the voting stage, agents can escape the trap of blindly following the “experts” opinion.²¹ However, the aggregate cost of political action still remains, making any mixed strategy equilibrium inferior to the uninformative equilibrium. In the mixed strategy equilibrium where agents mix in the political action stage and follow the “experts” opinion in the voting stage, the result is catastrophic since besides the aggregate cost of political action, the voting result’s accuracy is also reduced, as the number of “experts” will be less than the total number of agents.

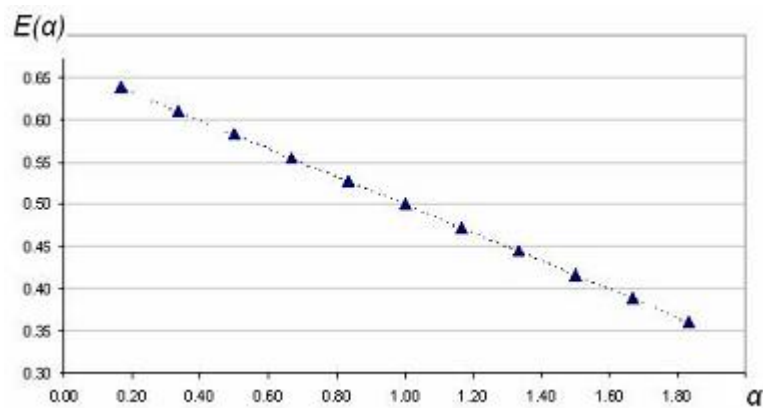
2.4 Heterogeneous Priors

In this part, the assumption of common prior beliefs is dropped. Henceforth, some of the agents believe that a high occurrence for s is more probable than a low one and vice versa. This heterogeneity can be interpreted as a bias in political beliefs. Henceforth, the terms heterogeneous and biased beliefs will be used interchangeably. As will be shown below, under specific conditions for the individual cost of political action and the imprecision of the state of the world, this bias can result in a superior outcome in terms of aggregate welfare, compared to the unbiased prior beliefs framework.

Assumption 2. *Under Heterogeneous Priors, every agent $k \in \{3, 4, \dots, n\}$ has prior beliefs about the state of the world given by $\beta_{\alpha(k)}(s) = \alpha(k) + 2(1 - \alpha(k))s$, where $\alpha(k) : \{3, 4, \dots, n\} \rightarrow (0, 2)$, is a strictly decreasing and continuous function.²²*

Example 2. A specific example of uniformly biased individuals could be the one where $\alpha(k) = 2 \frac{n+1-k}{n+1}$. In the following graph I plot every agent’s α against his expected value for the state of the world, for $n = 11$.

Figure 2.4.1: Expected value for state of the world for different values for bias α



²¹For more details on the effect of expert information in majority decisions for mixed strategy equilibria see [Kawamura and Vlaseros \(2013\)](#).

²²For $\alpha \in (0, 2) : \int_0^1 \beta_{\alpha}(s)ds$ and $E_{\alpha}(s) = \int_0^1 s\beta_{\alpha}(s)ds = \frac{4-\alpha}{6}$, so $Q < E_{\alpha}(s) < A$.

Note that the lowest point on the right represents the agent ($k = 1$) with the most downward bias with respect to her expectation about the state of the world. Also, the agent with $\alpha = 1$ has no bias in her prior opinion about s .

2.4.1 Uninformative and Fully Informative Equilibria

Under the Heterogeneous Priors assumption I prove that, there exist two cost tolerance bounds: over the upper one, only the uninformative equilibrium can be supported; under the lower one, the fully informative equilibrium, where all agents participate in informative political action, can be supported as well. These two bounds depend on the number of participants and the agent's individual bias.

A common property of these two equilibria is that the choice of strategy for agents is the same, independent of their individual bias in priors. I formally state this property in the definition below:

Definition 3. Under Heterogeneous Priors, an equilibrium is called *symmetric* if agents' equilibrium strategies do not depend on their bias in prior opinions (α). Otherwise, the equilibrium is called *non-symmetric*.

Definition 4 (USE). Uninformative Sequential Equilibrium under Heterogeneous Priors consists of the following assessment of the game:

$$\{\pi_\alpha(\sigma) = 0; \beta_\alpha(s|\sigma) = \begin{cases} \frac{6(1-s)(\alpha+2(1-\alpha)s)}{2+\alpha}, & \sigma = 0 \\ \frac{6s(\alpha+2(1-\alpha)s)}{4-\alpha}, & \sigma = 1 \end{cases} ;$$

$$\nu_\alpha(s) = \begin{cases} A, & E_\alpha(s|\sigma) \geq \mu \\ Q, & E_\alpha(s|\sigma) < \mu \end{cases} ; \beta_\alpha(s|\sigma)\}$$

In this assessment of the game, regardless of their individual bias and their initial signal, agents choose not to participate in political action, as in the case with homogeneous prior beliefs. Since nobody communicates, agents use only their individual signal to update their initial expectations through the lens of their individual bias. They finally vote for the status quo or the alternative, depending on whether they received a signal that favoured one policy, or the other. Once again, we can assert the existence of such an uninformative equilibrium: under any positive cost, everyone refraining from an informative protest constitutes an equilibrium of the game.

Lemma 3. Under Heterogeneous Priors, for any individual cost of political action $c > 0$ and for any n , USE is a sequential equilibrium in pure strategies.

Later I show the existence of a lower cost tolerance bound, under which the fully informative equilibrium can be supported as an equilibrium as well.

Definition 5 (ISE). Informative Sequential Equilibrium under Heterogeneous Priors consists of the following assessment of the game:

$$\{\pi_\alpha(\sigma) = \sigma; \beta_\alpha(s|\sigma) = \begin{cases} \frac{6(1-s)(\alpha+2(1-\alpha)s)}{2+\alpha}, & \sigma = 0 \\ \frac{6s(\alpha+2(1-\alpha)s)}{4-\alpha}, & \sigma = 1 \end{cases} ; \nu_\alpha(s) = \begin{cases} A, & E_\alpha(s|\sigma, m) \geq \mu \\ Q, & E_\alpha(s|\sigma, m) < \mu \end{cases} ;$$

$$\beta_\alpha(s|\sigma, m) = \frac{(n+2)!(\alpha + 2(1-\alpha)s)s^{m+\sigma}(1-s)^{n-m-\sigma}}{(m+\sigma)!(n-m+\sigma)!(2(1-\alpha)(m+\sigma) + \alpha n + 2)}; m = \sum \sigma$$

Here, as in the homogeneous prior beliefs framework, all agents will participate in informative political action. In other words, if one receives $\sigma = 1$, she will choose to inform others. After the political action stage, all agents will observe the extent of revealed signals and update their expectation on the true state of the world.

Lemma 4. Under Heterogeneous Priors, there exists a cost tolerance function $K(n, 2)$ such that, if the individual cost of political action c is lower than or equal to K , $c \leq K$, ISE is a sequential equilibrium in pure strategies.

The cost tolerance function $K(n, \alpha)$ is defined as follows:

$$K(n, \alpha) = \frac{1}{6} \left\{ 3E_\alpha(s|\sigma = 1, \frac{n-1}{2}) - E_\alpha(s|\sigma = 1, \frac{n-3}{2}) \right. \\ \left. + 2E_\alpha(s^2|\sigma = 1, \frac{n-3}{2}) - 2E_\alpha(s^2|\sigma = 1, \frac{n-1}{2}) - 1 \right\},$$

where

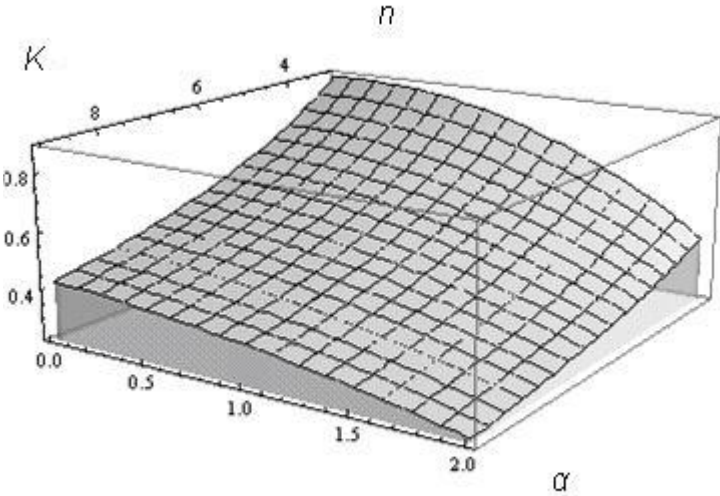
$$E_\alpha(s|\sigma = 1, n) = \frac{(m+2)(-2\alpha m + 2m + \alpha n - 3\alpha + 6)}{(n+3)(-2\alpha m + 2m + \alpha n - 2\alpha + 4)}$$

and

$$E_\alpha(s^2|\sigma = 1, n) = \frac{(m+2)(m+3)(-2\alpha m + 2m + \alpha n - 4\alpha + 8)}{(n+3)(n+4)(-2\alpha m + 2m + \alpha n - 2\alpha + 4)}.$$

Note that $K(n, \alpha)$ is a function of the updated expectations for the true state of the world $E_\alpha(s|\sigma, n)$ when an agent is pivotal in the voting stage. The cost tolerance in this framework ultimately depends on the number of agents n , but also on each agents' individual bias α . $K(n, \alpha)$ is strictly decreasing both in n and α , as can be seen from the following graph. So if $c \leq K(n, 2) < K(n, \alpha)$, then all agents who have received $\sigma = 1$, independent of their initial bias α , will have an incentive to participate in collective action. Also note that $K(n, \alpha_k)$ is the cost tolerance function of any agent k with type α_k facing $n - 1$ agents with heterogeneous prior beliefs.

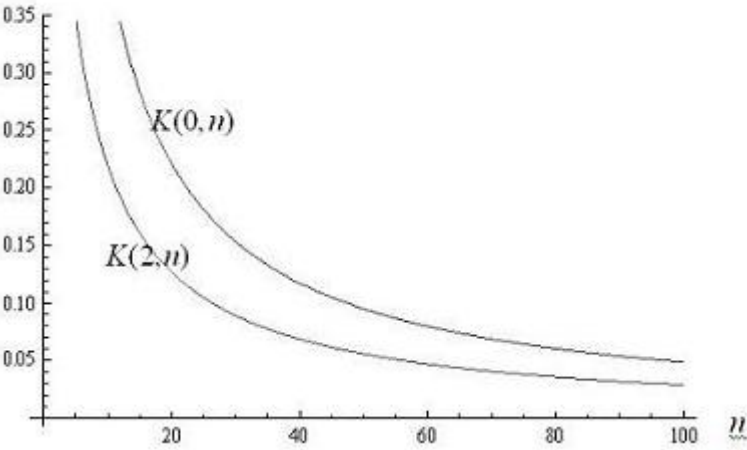
Figure 2.4.2: $K(n, \alpha)$ for different values of the number of agents n and individual bias α



Note that, if $c > K(n, 0)$, independently of the bias of agents and the signal they would receive, all would prefer to abstain from the political action stage, as $K(n, 0)$ is the cost tolerance of the agent who is more biased towards the alternative and therefore the reason for protesting.

In the following graph, I plot the upper and lower cost tolerance bound for an increasing number of agents.

Figure 2.4.3: Upper and lower cost tolerance bound for an increasing number of agents n



I summarize the lemmas for symmetric equilibria in pure strategies of this

section in the following proposition.

Proposition 2. Under Heterogeneous Priors, for any individual cost of political action $c > 0$, there exists a unique symmetric Uninformative Sequential equilibrium in pure strategies. Moreover, there exists a cost tolerance function $K(n, a)$ such that, if and only if the individual cost of political action c is lower than or equal to $K(n, 2)$, $c \leq K(n, 2)$, there also exists a symmetric Informative Sequential Equilibrium in pure strategies.²³

Note that, it is easy to discern a symmetry between the results on the homogeneous prior beliefs with that of the heterogeneous prior beliefs framework. Due to the dispersion in political beliefs, the unique cost tolerance bound separating the informative and uninformative equilibria in the homogeneous prior beliefs framework turns into an upper and a lower bound in the heterogeneous prior beliefs framework. The intuition behind this result is that, in order for a fully informative equilibrium to be sustained, it has to contain even the most biased agent against the proposed agenda. Hence, the latter's tolerance for the cost of participating in informative political action will be lower than that of an unbiased agent. On the other hand, in order to support the uninformative equilibrium, the cost of political action has to be too costly even for the most biased agent for the proposed agenda, and this cost will definitely be higher than that of the unbiased agent.

What happens between these bounds is explained thoroughly in the next and final part of this section.

2.4.2 Partially Informative Equilibria

Finally, I will show that, if the cost is between the two extremes of the cost tolerance function, there exists a *unique* non-symmetric equilibrium, where only a subset of the agents engage in informative protest, and all the rest do not. This subset depends on the individual bias of agents. This equilibrium has a very simple intuition: there will be agents who, even though their individual signal suggests that they should protest, their initial bias on what is the true state of the world will prevent them from doing so. In other words, for every cost of information sharing between the two extreme cases, there will be a single agent α^* that will ignore her private signal because of her individual bias. All agents who are more biased than her will do the same. Under specific conditions, this bias can be actually a good thing in terms of ex ante aggregate welfare, since it will eventually decrease the number of informative protests in equilibrium while improving the accuracy of the signal.²⁴

I will call this a biased beliefs sequential equilibrium (BBSE). The formal definition of BBSE lies below:

Definition 6. BBSE is the following assessment of the game.

²³Again, these are the only symmetric sequential equilibria of the game.

²⁴For more details on this, see next Section.

BBSE:

$$\{\pi_\alpha(\sigma) = \begin{cases} \sigma, & \alpha \leq \alpha^* \\ 0, & \alpha > \alpha^* \end{cases}; \beta_\alpha(s|\sigma) = \begin{cases} \frac{6(1-s)(\alpha+2(1-\alpha)s)}{2+\alpha}, & \sigma = 0 \\ \frac{6s(\alpha+2(1-\alpha)s)}{4-\alpha}, & \sigma = 1 \end{cases};$$

$$\nu_\alpha(s) = \begin{cases} A, & E_\alpha(s|\sigma, m) \geq \mu \\ Q, & E_\alpha(s|\sigma, m) < \mu \end{cases};$$

$$\beta_\alpha(s|\sigma, m) = \frac{(M+3)!(\alpha+2(1-\alpha)s)s^{m+\sigma}(1-s)^{M+1-m-\sigma}}{(m+\sigma)!(M+1-m+\sigma)!(2(1-\alpha)(m+\sigma)+\alpha(M+1)+2)}; m = \sum \pi_\alpha(\sigma),$$

where

$$\alpha^* = \inf\{a_k : K(k, a_k) \geq c, k \in \{3, 4, \dots, n\}\}$$

and

$$M = |\{a_k : a_k \leq a^*\}|.$$

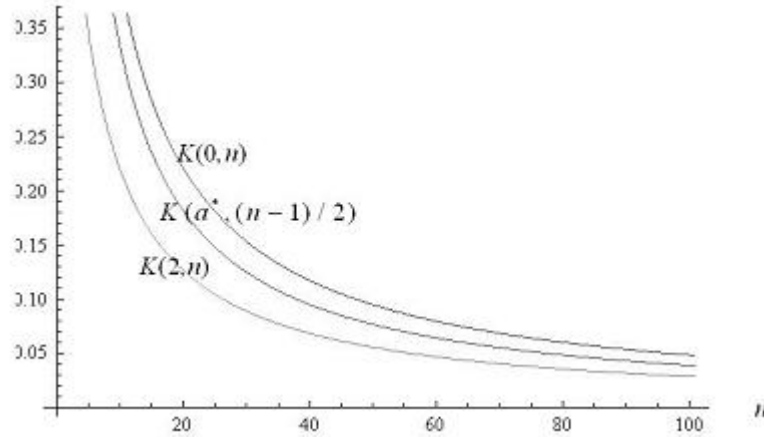
Proposition 3. Under Heterogeneous Priors, if $c \in (K(n, 2), K(n, 0)]$, Biased Beliefs Sequential Equilibrium is the unique non-symmetric sequential equilibrium.²⁵

Remark 1. In combination with Proposition 2, the last result provides us with a strong sampling tool: if the distribution of the initial bias in the population is known, we can determine the outcome of the voting procedure before the agents actually vote. In other words, we can infer all the private signals of the agents who received a $\sigma = 1$, but they do not protest themselves because of their individual bias. If the types of the agents are uniformly distributed, as in Example 1, and we know the actual cost of political action, we can determine the number of received $\sigma = 1$ signals using the cost tolerance function. The total number of agents who should be disclosing is M , from the solution of $K(M, \frac{2(n+1-M)}{n+1}) = c$. So if we observe m signal revelation, we can induce that $\frac{m}{M}\%$ of M agents received $\sigma = 1$.

The graph that follows depicts the upper and lower threshold cost tolerance for an increasing number of agents, as well as the threshold cost that corresponds to the situation where half of the agents would disclose their signal informatively $(a^*, \frac{n-1}{2})$.

²⁵Note that this equilibrium does not survive outside $(K(n, 2), K(n, 0)]$.

Figure 2.4.4: Cost tolerance bound for an increasing number of agents n and $\alpha \in \{0, a^*, 2\}$



2.5 Aggregate Welfare of Equilibria

After having calculated the three types of different equilibria of the game under heterogeneous prior beliefs (uninformative, partly informative and fully informative), it is straightforward to calculate the difference in aggregate welfare amongst them. These results are formally summarized in the following proposition, and thoroughly explained below:

Proposition 4. If the individual cost of political action is too large ($c > K(n, 0)$), there is no communication prior to elections. If the individual cost of political action is small enough ($c < K(n, 0)$), a fully informative equilibrium is harmful in terms of aggregate welfare. However, for a sufficiently large society (for n large enough) and sufficiently imprecise signal (s close to $\frac{1}{2}$), there exists a moderate value for the cost of political action $c^* \in (K(n, 2), K(n, 0))$ for which the respective Biased Beliefs Sequential Equilibrium is superior in terms of aggregate welfare with respect to the Uninformative Sequential Equilibrium.

In other words, if the cost of participating in collective action is large enough ($c > K(n, 0)$), there will be no information sharing. On the other hand, a low individual cost of political action ($c < K(n, 2)$) can also be harmful in democracy: this constitutes a situation where agents will participate in order to improve their personal signal paying a positive cost, but they will eventually vote what the median voter would anyway. Given everyone follows this strategy, this remains best response individually.

As it turns out, the fully informative equilibrium is the same in terms of individual signal accuracy before the voting stage, but always inferior in terms of aggregate welfare with respect to the uninformative one. This is the case as, in the fully informative equilibrium, all agents who have received the appropriate signal *will* communicate their individual signal. Of course, after updating about the state of the world, each agent's *individual* signal will always be improved by

taking into account all the remaining signals. While this is true, when agents finally decide to vote, they will *all* vote what the majority would vote in the first place. Thus, the two outcomes will be exactly the same, and so will the two probabilities of error.

This result underlines the strength of majoritarian democratic outcomes: even with full signal disclosure, society can never do better than the majoritarian outcome with individual signals. In this case the aggregate welfare loss of a fully informative signal sharing is the sum of the personal costs of political action.

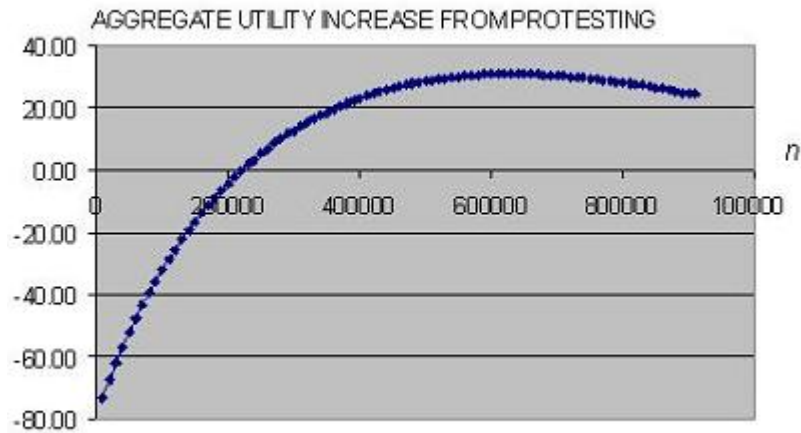
Naturally, under the assumption of heterogeneous priors it would be important to talk about any aggregate welfare improvements *before* the state of the world is known. However, since every agent has a distinct prior, it is hard to choose what the “real” prior should be. Thus in this case we compare welfare results *given* the state of the world.

After the state of the world is revealed, when society is large enough, there are cases when a partially informative protest induces an improvement in both individual signal accuracy *and* aggregate welfare. This happens when the state of nature (s) is close (but not exactly equal) to $\frac{1}{2}$. In other words, when the signal of nature is imprecise and society is large enough, there exists a partially revealing equilibrium that produces an increase in aggregate welfare with respect to uninformative one. The intuition behind this result is simple: when the signal is very accurate, the Condorcet Jury Theorem insures that, the larger the society, the smaller the probability of error of the majoritarian outcome. But, this holds for both equilibria. Hence, the error difference, with a large n , will be very close to zero, and will not be able to compensate for the sum of individual costs of information sharing. On the other hand, when $s = \frac{1}{2}$, then agents will be indifferent between the two agendas, Q and A , in terms of utility and again the only difference between the two equilibria is the cost of collective action. Even so, in large societies and when the signal is sufficiently (but not completely) imprecise, there exists a number of revealed signals, so that the corresponding partially informative equilibrium is superior that the uninformative one.

Of course, these welfare improving equilibria would only arise if the state of the world is close to $\frac{1}{2}$. Since the state of the world is a random variable, and I have assumed for simplicity that the state of the world is uniform, there is a very small probability that this happens. Were I to calculate the ex ante expected utility, they would fail. But it is easy to infer that, for a symmetric distribution more condensed around $\frac{1}{2}$, they would be welfare improving in ex ante expected utility as well.

Example 3. The following graph provides some useful intuition with respect to Proposition 4. Here, the ex ante aggregate utility increase between the uninformative equilibrium and a partially informative equilibrium is plotted against an increasing population. The partially informative set of citizens contains 425 agents, while the signal is very imprecise, $s = 50.075\%$.

Figure 2.5.1: Aggregate welfare for an increasing number of agents n , a protesting group containing a fixed number of 425 agents and $s = 50.075\%$



It is straightforward to see that, when the population of society is small, the number of the disclosing group poses a greater relative weight due to the small size of society. However, as the size of society increases, the partially informative equilibrium becomes superior in terms of ex ante aggregate welfare. Of course, when n tends to infinity, the Condorcet Jury Theorem again makes the uninformative equilibrium more attractive in terms of aggregate welfare, as the probability of error in both equilibria tends to zero, and the only difference is the cost of collective action, which, in this case, is just the sum of individual costs of political action.

2.6 Concluding Remarks

Protests are a growing channel of political expression and one of the major channels of public voice: 14% of citizens attended demonstrations in Belgium during 1981, 23% in 1990 and 39% in 2000 (Norris, Walgrave, and Aelst (2005)). However, as my analysis suggests, if the cost of political action is sufficiently small but positive, then the expected welfare result of majoritarian decisions will be always inferior to that without prior communication. This happens since, a sufficiently small cost of political action will result in a situation similar to a prisoner's dilemma where individuals will always find participating in informative costly political action individually superior than abstaining from it, conditional on others doing so. Given that the cost of protesting is sufficiently low, as in the cases of internet protests, since all agents will informatively participate, this will result in strengthening the vote difference in votes between the two proposed agendas into a unanimous decision in the case of agents with the same preferences, but not necessary improve the accuracy of the majoritarian result. The only difference in welfare will be that of the aggregate cost of political action. In the case of biased

agents, as [Lohmann \(1994\)](#) suggests, can even lead to a failure in information aggregation.

However, if prior beliefs exhibit political bias, only a subset of individuals will participate in informative protest. Given that this subset is small enough and the state of the world is sufficiently imprecise, then collective action will be welfare improving: this will happen when the improvement in majoritarian signal accuracy will outweigh the aggregate cost of collective action.

Including a social planner in the present model would be straightforward. Her reaction against the fully informative welfare reducing equilibrium is either to propose more mediocre policies or increase the individual cost of political action, relative to the distance of the proposed agendas.²⁶ If political opinions are dispersed and she knows that the signal agents receive is highly imprecise, she should adjust the cost of individual political action to induce a (relatively small or null) subset of the citizens to engage in informative political action, while maintaining most (or all) in political inaction before the elections.

Finally, there are several ways this model's results could be expanded. First of all, there could be a "blur" in the number of participants in the protest that other agents are able to observe after the political action stage. Technically, this can be achieved by introducing a stochastic shock that the agents can only imperfectly observe; the latter would depict how the number of political actors changes in equilibrium under some bias or general form of censorship in the mass media of society. Secondly, the platforms voted upon could be expanded from this model's binary setup in taking more than two values or even a continuum of values, to study any possible trade-off between protesting and exaggeration of voter's choices. Moreover, there could be more protesting choices with strictly increasing costs: this could give a rationalistic interpretation of protests turning in riots.²⁷ More importantly, as in [Groseclose and Milyo \(2013\)](#), it is important and realistic to see what would happen in the case where agents choose to engage in political action sequentially and not simultaneously. Finally, due to the extreme simplicity of the mechanics, this model can be incorporated in the study of majoritarian decision making of more complex economic structures.

²⁶i.e. decrease the relative distance of the two platforms

²⁷As far as the relationship between political action and violence, the present model's results are not there to capture any agent's non-rational behavioural characteristics. In a relative analysis, [Ross \(1986\)](#) claims that what determines society's overall conflict from protest to political violence is psycho-cultural dispositions that root in socialization patterns, low warmth and affection directed at children and high protest masculinity. In this model, agents are rational: they do not despair when noticing that the informational group is too thin, or that the result of the referendum is of too grave importance.

Appendix

2.A Proof of Lemma 1

After Nature's move, agents receive a signal $\sigma = 1$ with probability s or $\sigma = 0$ with probability $1 - s$. The agents will update their expectation according to the Bayes rule:

$$\beta(s|\sigma = 0) = \frac{(1-s)\beta(s)}{\int_0^1 (1-s)\beta(s)ds} = 2(1-s)$$

and

$$\beta(s|\sigma = 1) = \frac{s\beta(s)}{\int_0^1 s\beta(s)ds} = 2s.$$

Note that:

$$E(s|\sigma = 0) = \frac{1}{3} < \mu < \frac{2}{3} = E(s|\sigma = 1).$$

Since nobody will share their signal under USE, agents will choose to vote for the alternative if their expected loss from the voting the alternative is smaller or equal to the one from voting the status quo:

$$\int_0^1 U(A)\beta(s|\sigma)ds \geq \int_0^1 U(Q)\beta(s|\sigma)ds \Leftrightarrow E(s|\sigma) \geq \mu.$$

Hence agents will vote for the alternative if their updated expectation of s is greater or equal the midpoint between A and Q .

In the voting stage, type $\sigma = 0$ agents will vote for the status quo and type $\sigma = 1$ agents will vote for the alternative, since $E(s|\sigma = 0) \leq E(s|\sigma = 1)$. Any deviation from this strategy is not profitable for type $\sigma = 0$ agents: at the costly political action stage, the agent would only incur the positive cost of political action. In the voting stage, the agent would not maximise her expected utility.

As far as agent $\sigma = 1$ is concerned, if she discloses her signal, then all agents will update their expectations using her signal. Therefore, type $\sigma = 1$ agents will still vote for A , while type $\sigma = 0$ agents will return to their prior beliefs. By being indifferent, they will vote for the alternative. In any case, all will vote for the alternative. However, $\sigma = 1$ agents will not have an incentive to deviate if the cost of disclosing her signal is higher than the expected gain for choosing her

preferred policy. In other words, she will abstain from political action if

$$\int_0^1 \{P[N \geq \frac{n+1}{2} | m = 1, piv] - P[N \geq \frac{n+1}{2} | m = 0, piv]\} \{U(A) - U(Q)\} \beta(s | \sigma = 1, piv) ds \leq c$$

or, finally, if

$$c \geq (A - Q)(1 - 2\mu)O(n),$$

where $O(n)$ is a function of the number of players. Since $\mu = \frac{1}{2}$, for any $c > 0$, she will always prefer not to engage in costly political action. Note that for $\mu < \frac{1}{2}$, this is not the case.

In order to conclude the proof, it suffices to show that there exists a series of totally mixed strategies converging to $\pi(\sigma) = 0$ under which the belief structure converges to $\beta(s | \sigma = 0)$. Indeed, mixing $\{\pi(\sigma) = 0, \pi(\sigma) = \sigma, \pi(\sigma) = 1 - \sigma, \pi(\sigma) = 1\}$ by using $(1 - \frac{1}{t}, \frac{1}{3t}, \frac{1}{3t}, \frac{1}{3t}) \xrightarrow{t \rightarrow \infty} (1, 0, 0, 0)$, the players will update their expectation according to the part of signals that they consider informative with respect to the state of the world. Hence, if they observe m agents engaging in costly political action, they will induce that $2\lfloor \frac{m}{3t} \rfloor$ are informative actions, and exactly half of them correspond to either receiving zero or one, or

$$\beta(s | \sigma, \mu) = \frac{\binom{2\lfloor \frac{m}{3t} \rfloor}{\lfloor \frac{m}{3t} \rfloor} s^{\lfloor \frac{m}{3t} \rfloor} (1-s)^{\lfloor \frac{m}{3t} \rfloor} \beta(s | \sigma)}{\int_0^1 \binom{2\lfloor \frac{m}{3t} \rfloor}{\lfloor \frac{m}{3t} \rfloor} s^{\lfloor \frac{m}{3t} \rfloor} (1-s)^{\lfloor \frac{m}{3t} \rfloor} \beta(s | \sigma) ds} = \frac{(s(1-s))^{\lfloor \frac{m}{3t} \rfloor} \beta(s | \sigma)}{\int_0^1 (s(1-s))^{\lfloor \frac{m}{3t} \rfloor} \beta(s | \sigma) ds} \xrightarrow{t \rightarrow \infty} \beta(s | \sigma)$$

where $\lfloor k \rfloor = \{l : l \geq k, l \in \mathbb{N}\}$.

2.B Proof of Lemma 2

After Nature's move, agents will update their prior beliefs in exactly the same way as described in Lemma 1. After observing any number of signal revelations m will vote for the alternative if

$$\int_0^1 U(A) \beta(s | \sigma, m) ds \geq \int_0^1 U(Q) \beta(s | \sigma, m) ds \Leftrightarrow E(s | \sigma, m) \geq \mu.$$

Therefore, according to their beliefs at the voting stage, agents will always vote informatively. Again there will be no profitable deviation on this stage of the game, given the beliefs of the agents in this stage.

In order to update their beliefs about the state of the world, everyone will take into consideration that m agents participated in costly political action (apart from themselves, if they did so) after receiving a $\sigma = 1$ and abstained otherwise. Agents will infer that there were exactly m $\sigma = 1$ signals received by the $n - 1$ agents. The distribution of signals follows Bernoulli distribution. Hence, dependent on the signal they initially had about the state of the world, there agents

will update their beliefs about the state of the world accordingly:

If $\sigma = 0$ then

$$\beta(s|\sigma = 0, m) = \frac{\binom{n-1}{m} s^m (1-s)^{n-m-1} \beta(s|\sigma = 0)}{\int_0^1 \binom{n-1}{m} s^m (1-s)^{n-m-1} \beta(s|\sigma = 0) ds} = \frac{(n+1)!}{m!(n-m)!} s^m (1-s)^{n-m}$$

while

$$E(s|\sigma = 0, m) = \frac{\Gamma(n+2)}{\Gamma(m+1)\Gamma(n-m+1)} \frac{\Gamma(m+2)\Gamma(n-m+1)}{\Gamma(n+3)} = \frac{m+1}{n+2}.$$

If $\sigma = 1$ then

$$\beta(s|\sigma = 1, m) = \frac{\binom{n-1}{m} s^m (1-s)^{n-m-1} \beta(s|\sigma = 1)}{\int_0^1 \binom{n-1}{m} s^m (1-s)^{n-m-1} \beta(s|\sigma = 1) ds} = \frac{(n+2)!}{(m+1)!(n-m)!} s^{m+1} (1-s)^{n-m-1}$$

while

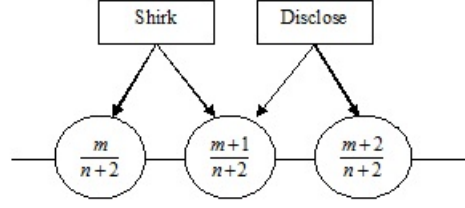
$$E(s|\sigma = 1, m) = \frac{\Gamma(n+2)}{\Gamma(m+2)\Gamma(n-m)} \frac{\Gamma(m+3)\Gamma(n-m)}{\Gamma(n+3)} = \frac{m+2}{n+2}.$$

As far as agents who have received $\sigma = 0$ are concerned, they have no incentive to deviate from their strategies. There are two cases, depending on the value of μ relative to their expectation versus the state of the world: If $E(s|\sigma = 0, m) \geq \mu$, the agent will prefer not to engage in costly political action, as everyone's expectation is an increasing function of m . Hence, she would bear a cost and having no benefit from her involvement in costly political action. If $E(s|\sigma = 0, m) < \mu$, the agent will have an incentive not to share their signal, as everyone's expectation is an increasing function of m . Otherwise, she would bear a cost and in the same time while pivoting the majority result towards a less preferable outcome.

Types $\sigma = 1$ on the other hand have the choice of either to disclose their signal (engage in costly political action) and bear a cost equal to c , or refrain themselves. Since they are initially biased towards the alternative ($\mu \leq E(s|\sigma = 1)$), they will consider the effect their costly political action will have in pivoting the outcome towards the alternative. Let m be the total number of $\sigma = 1$ signals. If all decide to disclose, there will be m revealed signals. If one of the $\sigma = 1$ agents decides to deviate and refrain themselves from political action, then the number of revealed signals will be $m - 1$. Note that for the expectation of s , as calculated above, we have that $E(s|\sigma = 0, m) = \frac{m+1}{n+2}$ and $E(s|\sigma = 1, m) = \frac{m+1}{n+2}$.

Hence, any agent considering to deviate and refrain from political action, will consider the benefit relative to the cost of her action. The choice of deviating or not from participating in costly political action can be depicted in the diagram

below.



If $\mu \leq \frac{m}{n+2}$, the agents extra revealed signal will be worthless, while if $\mu > \frac{m+2}{n+2}$, the agent will have an incentive not to participate in collective action. Hence, the agent will only consider the cases where $\frac{m}{n+2} < \mu \leq \frac{m+2}{n+2}$. These correspond to the three following cases:

$$\left\{ \begin{array}{l} m > \mu n + 2m - 2 \\ m > \mu n + 2m - 1 \\ m > \mu n + 2m \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} m > m^* \\ m > m^* + 1 \\ m > m^* + 2 \end{array} \right\}$$

where $m^* = m^*(\mu, n) = \inf\{k \leq n | k \geq \mu n + 2m - 2\}$, which is a non an empty set, since $n \in \{k \leq n | k \geq \mu n + 2m - 2\}$.

Since

$$P[N \geq \frac{n+1}{2} | m] = P[N \geq \frac{n+1}{2} | m, \sigma = 0]P[\sigma = 0] + P[N \geq \frac{n+1}{2} | m, \sigma = 1]P[\sigma = 1]$$

where where N is the number of votes for the alternative, we have that

$$\left\{ \begin{array}{l} P[N \geq \frac{n+1}{2} | m^* + 2] = 1(1-s) + 1s = 1 \\ P[N \geq \frac{n+1}{2} | m^* + 1] = 0(1-s) + 1s = s \\ P[N \geq \frac{n}{2} | m^*] = 0(1-s) + 0s = 0. \end{array} \right.$$

The agents will therefore consider only the cases where any additional costly political action will result in them having any benefit whatsoever, conditional on being pivotal. Their decision will depend on the indifference condition between the cost and the sum of any incremental probabilistic signalling effects their acceding to the body of agents revealing their signal could induce. Any terms of this summation must weigh the effect an additional revealed signal would mean to the probability of the alternative winning and the incremental utility benefit of the alternative against the status quo, or

$$\int_0^1 \{P[N \geq \frac{n+1}{2} | m^*+2] - P[N \geq \frac{n+1}{2} | m^*+1]\} \{U(A) - U(Q)\} \beta(s | \sigma = 1, m^*+1) ds + \\ + \int_0^1 \{P[N \geq \frac{n+1}{2} | m^*+1] - P[N \geq \frac{n+1}{2} | m^*]\} \{U(A) - U(Q)\} \beta(s | \sigma = 1, m^*+1) ds = c,$$

From the last equation, the threshold cost tolerance for which agents who received $\sigma = 1$ will chose to engage in costly political action is:

$$c^*(\mu, n, Q, A) = (A - Q) \left(\frac{(n+1)(m^*(\mu, n) + 3)}{(n+2)(n+3)} - \frac{\mu(m^*(\mu, n) + 1)}{2(n+2)} \right) \quad (2.B.1)$$

If $c^* \geq c$, type $\sigma = 1$ players will prefer to or be indifferent in protesting. When $c^* < c$, they would have an incentive to refrain from political action.

2.C Proof of Proposition 1

Immediate result of Lemmas 1-2. The “only if” statement is a straightforward result of the proof of Lemma 2. Note that equation (1) is an indifference condition for participating in informative political action or not. Therefore, if the cost is lower, then the equilibrium holds. Also, if the equilibrium to holds, then it must be that the cost is lower than the threshold cost, otherwise agents would have an incentive to deviate.

2.D Proof of Lemma 3

The same as the proof of Lemma 1 since, for any $\alpha \in (0, 2)$, we have that $E_\alpha(s|\sigma = 0) < \frac{1}{2} < E_\alpha(s|\sigma = 1)$.

2.E Proof of Lemma 4

After Natures move, agents will update their prior beliefs is exactly the same way as in the homogeneous prior beliefs framework. After observing any number of signal revelations m will vote for the alternative if

$$\int_0^1 U(A)\beta_\alpha(s|\sigma, m)ds \geq \int_0^1 U(Q)\beta_\alpha(s|\sigma, m)ds \Leftrightarrow E_\alpha(s|\sigma, m) \geq \frac{1}{2}.$$

Therefore, according to their beliefs, agents will vote informatively in the voting stage. Again there will be no profitable deviation on this stage of the game, given the beliefs of the agents in this stage. In order to update for their beliefs about the state of the world, everyone will take into consideration that m agents participated in costly political action (apart from themselves, if they did so) after receiving a $\sigma = 1$. Agents will infer that there were exactly m $\sigma = 1$ signals received by the $n - 1$ agents. The distribution of signals follows Bernoulli distribution. Hence, dependent on the signal they initially had about the state of the world, there agents will update their beliefs about the state of the world accordingly:

If $\sigma = 0$ then

$$\beta_\alpha(s|\sigma = 0, m) = \frac{\binom{n-1}{m} s^m (1-s)^{n-m-1} \beta_\alpha(s|\sigma = 0)}{\int_0^1 \binom{n-1}{m} s^m (1-s)^{n-m-1} \beta_\alpha(s|\sigma = 0) ds} = \frac{(n+2)! (\alpha + 2(1-s)\alpha) s^m (1-s)^{n-m}}{m! (n-m)! (-2\alpha m + 2m + \alpha n + 2)},$$

while

$$E_\alpha(s|\sigma = 0, m) = \frac{m+1}{n+3} \cdot \frac{-2\alpha m + 2m + \alpha n - \alpha + 4}{-2\alpha m + 2m + \alpha n + 2}.$$

If $\sigma = 1$ then

$$\beta_\alpha(s|\sigma = 1, m) = \frac{(n+2)! (\alpha + 2(1-s)\alpha) s^{m+1} (1-s)^{n-m-1}}{(m+1)! (n-m-1)! (-2\alpha m + 2m + \alpha n - 2\alpha + 4)},$$

while

$$E_\alpha(s|\sigma = 1, m) = \frac{m+2}{n+3} \cdot \frac{-2\alpha m + 2m + \alpha n - 3\alpha + 6}{-2\alpha m + 2m + \alpha n - 2\alpha + 4}$$

and

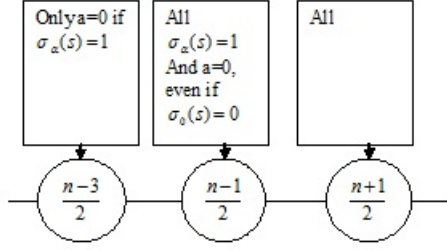
$$E_\alpha^2(s|\sigma = 1, m) = \frac{m+2}{n+3} \cdot \frac{m+3}{n+4} \cdot \frac{-2\alpha m + 2m + \alpha n - 4\alpha + 8}{-2\alpha m + 2m + \alpha n - 2\alpha + 4}.$$

It is straightforward to prove the following properties of $E_\alpha(s|\sigma, m)$:

1. $E_\alpha(s|\sigma = 0) \leq \frac{1}{2} \leq E_\alpha(s|\sigma = 1)$
2. $E_0(s|\sigma = 0) = \frac{1}{2} = E_2(s|\sigma = 1)$
3. $E_\alpha(s|\sigma = 0, m+1) = E_\alpha(s|\sigma = 1, m)$
4. $\frac{\partial E_\alpha(s|\sigma=1, m)}{\partial \alpha} \leq 0$, for $\alpha \in [0, 2]$
5. $E_\alpha(s|\sigma, m)$ is increasing in m
6. $E_0(s|\sigma = 1, \frac{n-3}{2}) = \frac{1}{2}$
7. $E_2(s|\sigma = 1, \frac{n-1}{2}) = E_0(s|\sigma = 0, \frac{n-1}{2}) = \frac{1}{2}$
8. $E_0(s|\sigma = 0, \frac{n+1}{2}) = \frac{1}{2}$

Type $\sigma = 1$ has to consider the signalling effects of an additional signal revelation. Using the previous properties of $E_\alpha(s|\sigma, m)$, I sketch all possible outcomes:

WHO VOTES FOR THE ALTERNATIVE



Since

$$P[N \geq \frac{n+1}{2} | m] = P[N \geq \frac{n+1}{2} | m, \sigma = 0]P[\sigma = 0] + P[N \geq \frac{n+1}{2} | m, \sigma = 1]P[\sigma = 1]$$

we have that

$$\begin{cases} P[N \geq \frac{n+1}{2} | \frac{n+1}{2}] = 1(1-s) + 1s = 1 \\ P[N \geq \frac{n+1}{2} | \frac{n-1}{2}] = 0(1-s) + 1s = s \\ P[N \geq \frac{n}{2} | \frac{n-3}{2}] = 0(1-s) + 0s = 0. \end{cases}$$

Therefore, we can show that the threshold cost for agents who received $\sigma = 1$ in order to engage in costly political action, dependent on α , is:

$$K(n, \alpha) = \frac{1}{6} \{ 3E_\alpha(s | \sigma = 1, \frac{n-1}{2}) - E_\alpha(s | \sigma = 1, \frac{n-3}{2}) + E_\alpha(s^2 | \sigma = 1, \frac{n-3}{2}) - 2E_\alpha(s^2 | \sigma = 1, \frac{n-1}{2}) \}$$

Finally, as far as agents $\sigma = 0$ are concerned, there are two cases, depending on their expectation of the state of the world: If $E_\alpha(s | \sigma = 0, m) \geq \frac{1}{2}$, the agent will prefer not to engage in costly political action, as everyone's expectation is an increasing function of m . Hence, she would bear a cost and have no benefit from her involvement in collective. If $E_\alpha(s | \sigma = 0, m) < \frac{1}{2}$, the agent will have an incentive not to share their signal, because everyone's expectation is an increasing function of m . Otherwise, she would bear a cost and in the same time while pivoting the majority result towards a less preferable outcome than her own.

2.F Proof of Proposition 2

Immediate result of Lemmas 3-4 and Properties 1-3 of $K(n, \alpha)$. The "only if" statement is again a straightforward result of the proof of Lemma 4.

2.G Proof of Proposition 3

After Nature's move, agents will update their prior beliefs in exactly the same way as in Lemma 6. After observing any number of revealed signals m will vote for

the alternative if

$$\int_0^1 U(A)\beta_\alpha(s|\sigma, m)ds \geq \int_0^1 U(Q)\beta_\alpha(s|\sigma, m)ds \Leftrightarrow E_\alpha(s|\sigma, m) \geq \frac{1}{2}.$$

Therefore, according to their beliefs, agents will vote informatively in the voting stage. Again there will be no profitable deviation on this stage of the game, given the beliefs of the agents in this stage.

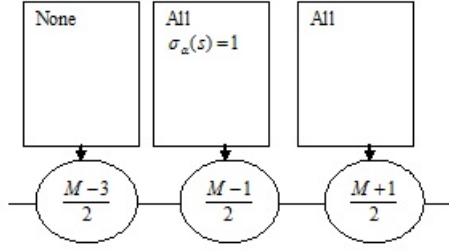
Since $c \in (K(n, 2), K(n, 0))$, agents know that it is not optimal for all agents to engage in costly political action: if $\{\alpha_k : K(k, \alpha_k) \geq c\} = \emptyset$, then we have the zero information equilibrium, else if $\{\alpha_k : K(k, \alpha_k) \geq c\} = \{3, 5, \dots, n\}$, then we have the full informative equilibrium. Last of all, if the agents types are dispersed enough and since $K(n, \alpha)$ is decreasing in both n and α , we can calculate $\alpha^* = \inf_k \{\alpha_k : K(k, \alpha_k) \geq c\}$. Agents with $\alpha \leq \alpha^*$ will engage in information sharing, if they receive a signal $\sigma = 1$ and abstain if they receive a signal equal to $\sigma = 0$, whereas agents with $\alpha > \alpha^*$ will refrain from political action. Note that $\{k|\alpha_k \leq \alpha^*\}$ and $\{k|\alpha_k > \alpha^*\}$ can be directly calculated from functions $a(k)$ and $K(n, \alpha)$. Denote $M = |\{k|\alpha_k \leq \alpha^*\}|$ and $T = \{k|\alpha_k > \alpha^*\}$, $M + T = n$. I assume without loss of generality that M is an odd number (because, if not, then we can prove the equivalent proposition for TBBSE).

In order to update for their beliefs about the state of the world, taking into consideration that M agents would participate in costly political action (apart from themselves, if they did so) after receiving a $\sigma = 1$. Agents will infer that there were exactly m $\sigma = 0$ signals received by the M agents. The distribution of signals follows the Bernoulli distribution. Hence, dependent on the signal they initially had about the state of the world, agents will update their beliefs about the state of the world accordingly. The calculations will be exactly equivalent to the one in the equivalent stage of the game in Lemma 6, but for $M = n - 1$. Also, it is straightforward to prove some useful properties of the new expected value $E_\alpha^M(s|\sigma, m)$:

1. $E_\alpha^M(s|\sigma = 0, m) = \frac{m+1}{M+4} \cdot \frac{-2\alpha m + 2m + \alpha M + 4}{-2\alpha m + 2m + \alpha M + \alpha + 2}$
2. $E_\alpha^M(s|\sigma = 1, m) = \frac{m+2}{M+4} \cdot \frac{-2\alpha m + 2m + \alpha M - 2\alpha + 6}{-2\alpha m + 2m + \alpha M - \alpha + 4}$
3. $E_\alpha^M(s^2|\sigma = 1, m) = \frac{m+2}{M+4} \cdot \frac{m+3}{M+5} \cdot \frac{-2\alpha m + 2m + \alpha M - 3\alpha + 8}{-2\alpha m + 2m + \alpha M - \alpha + 4}$
4. $E_\alpha^M(s|\sigma = 0, m + 1) = E_\alpha^M(s|\sigma = 1, m)$
5. $\frac{\partial E_\alpha^M(s|\sigma=1, m)}{\partial \alpha} \leq 0$, for $\alpha \in [0, 2]$
6. $E_\alpha^M(s|\sigma, m)$ is increasing in m
7. $E_0^M(s|\sigma = 1, \frac{M-3}{2}) = E_2^M(s|\sigma = 1, \frac{M-1}{2}) < \frac{1}{2}$
8. $E_0^M(s|\sigma = 1, \frac{M-1}{2}) = E_2^M(s|\sigma = 1, \frac{M+1}{2}) > \frac{1}{2}$

Type $\sigma = 1$ has to consider the signalling effects of an additional revealed signal. Using the previous properties of $E_\alpha^M(s|\sigma, m)$, we sketch all possible outcomes:

WHO VOTES FOR THE ALTERNATIVE



Since

$$P[N \geq \frac{n+1}{2} | m] = P[N \geq \frac{n+1}{2} | m, \sigma = 0]P[\sigma = 0] + P[N \geq \frac{n+1}{2} | m, \sigma = 1]P[\sigma = 1]$$

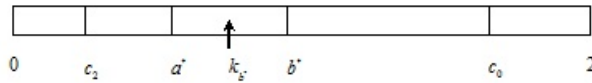
we have that

$$\begin{cases} P[N \geq \frac{n+1}{2} | \frac{M+1}{2}] = 1(1-s) + 1s = 1 \\ P[N \geq \frac{n+1}{2} | \frac{M-1}{2}] = 0(1-s) + 1s = s \\ P[N \geq \frac{n}{2} | \frac{M-3}{2}] = 0(1-s) + 0s = 0. \end{cases}$$

Therefore, we can show that the threshold cost for agents who received $\sigma = 1$ in order to engage in costly political action is $K(M, \alpha)$ (the same function as in Lemma 6, but for $n = M$).

As far as agents $\sigma = 0$ are concerned, there are two cases, depending on their expectation of the state of the world: If $E_{\alpha}^M(s | \sigma = 0, m) \geq \frac{1}{2}$, the agent will prefer not to engage in costly political action, as everyone's expectation is an increasing function of m . Hence, she would bear a cost and have no benefit from her involvement in costly political action. If $E_{\alpha}^M(s | \sigma = 0, m) < \frac{1}{2}$, the agent will have an incentive not to reveal, as everyone's expectation is an increasing function of m . Hence, she would bear a cost and in the same time pivoting the majority result towards a less preferable outcome than her own.

What remains to be proved is that any other different non empty partition of the set of agents does not constitute a sequential equilibrium. Without loss of generality, assume $b^* > \alpha^*$ such that $M_{b^*} = |\{k | \alpha_k \leq b^*\}|$ and $T_{b^*} = \{k | \alpha_k > b^*\}$, $M_{b^*} + T_{b^*} = n$. Also assume that $M_{b^*} \cap M^c \neq \emptyset$. So let $k_{b^*} \in M_{b^*}$ and $k_{b^*} \notin M$. Since $b^* > \alpha^* \Rightarrow M_{b^*} > M$, we have that $K(M_{b^*}, b^*) < K(M, \alpha^*) = c$, as $K(n, \alpha)$ is strictly decreasing both in n and α . Thus, agent k_{b^*} does not have an incentive to reveal, as her cost tolerance for costly political action is less than the actual cost, yet she belongs to the set of agents participating in costly political action (contradiction).



Finally, note that since n is finite, any number of agents revealing information can occur with positive probability, so there exist no beliefs off-the-equilibrium path .

2.H Proof of Proposition 4

It suffices to show that the informative equilibrium with 2 revelation is better than the uninformative one, with n large and s close to $\frac{1}{2}$, or that the following inequality holds:

$$n \cdot \{Pr_n^A(\text{error, no revelations}) - Pr_n^B(\text{error, 2 revelations})\} \cdot 2(A - Q)\left(\frac{1}{2} - s\right) > s \quad (2.H.1)$$

where $A \sim B(n, s)$ and $B \sim B(n, s^*)$, binomial distributions with $s^* = s^2(3 - 2s) > s$. Let $s^* - s = \epsilon > 0$. We know that for a large n and s sufficiently close to $\frac{1}{2}$, we can approximate the difference of the two binomial distributions by the normal distribution with a mean of $ns^* - ns = n\epsilon$ and a standard deviation of $\sqrt{n(s(1-s) + s^*(1-s^*))} \simeq \sqrt{\frac{n}{2}}$ for s sufficiently close to $\frac{1}{2}$. Normalizing using the continuity correction we have that

$$z = \frac{\frac{n+1}{2} + \frac{1}{2} - n\epsilon}{\sqrt{\frac{n}{2}}} \simeq \sqrt{\frac{n}{2}}\delta$$

where $0 < \delta < 1$. From the lower bound for the error of the standard normal distribution, we know that

$$\Phi_c(z) > \frac{1}{\sqrt{2\pi}} \cdot \frac{z}{z^2 + 1} \cdot e^{-\frac{z^2}{2}}, \quad (2.H.2)$$

where $e^{-\frac{z^2}{2}} = e^{-\frac{n}{4}\delta^2} > 1$ and $\frac{z}{z^2+1} > \sqrt{\frac{2}{n}} \cdot \frac{\delta}{\delta^2+2}$. Using (2), (1) becomes

$$n > \frac{s^2(\delta^2 + 2)^2}{(2s - 1)^2\delta^2} \cdot \pi$$

which holds for sufficiently large population n .

2.I Note on equivalence of results with [Lohmann \(1994\)](#)

In [Lohmann \(1994\)](#), the agents of the game retain uniform and homogeneous prior beliefs, but different political bias. This is interpreted as a utility function that depends on $x \in [-\bar{x}, \bar{x}]$, different for each agent.²⁸ There is always a functional

²⁸However, the complexity of Lohmanns solutions makes the equilibrium sets intractable, and any comparative statics on the equilibrium solution extremely inflexible. Moreover, the complexity of the equilibrium almost excludes the calibration of the latter in any testable econometric hypothesis. On the contrary, in the present model, comparative statics and em-

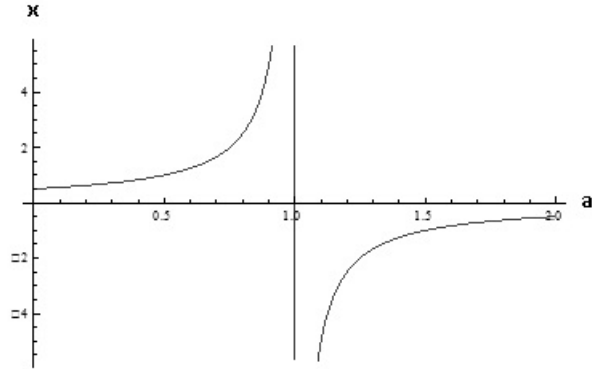
equivalence between the ex-ante preferences of each agent between Lohmanns model and the heterogeneous prior beliefs framework. In Lohmann, after agents receive their signal, they update their beliefs with the following way:

$$\beta(s|\sigma) = \begin{cases} 2(1-s), & \sigma = 0 \\ 2s, & \sigma = 1 \end{cases}$$

The agent's preferred choice before her decision to take share information or not with respect to the two suggested policies, will be A if $\int_0^1 (\frac{1}{2} - s - x)\beta(s|\sigma)ds \leq 0$, and Q otherwise. In the BB setting, after receiving a signal, agents will prefer the A if $\int_0^1 (\frac{1}{2} - s)\beta_\alpha(s|\sigma)ds \leq 0$, with $\beta_\alpha(s|\sigma)$ given by

$$\beta_\alpha(s|\sigma) = \begin{cases} \frac{6(1-s)(\alpha+2(1-\alpha)s)}{2+\alpha}, & \sigma = 0 \\ \frac{6s(\alpha+2(1-\alpha)s)}{4-\alpha}, & \sigma = 1 \end{cases}$$

Apart from the unbiased agents in the both games $x = 0$ and $\alpha = 1$, for every player of the BB setting we can find an player of Lohmanns setting having the same ex-ante preference, through the function $x(\alpha) = \frac{1}{2(1-\alpha)}$, $\alpha \in (0, 2)$. Also, for one unbiased agent in Lohmanns setting $x = 0$ and sufficient dispersion for the rest $x \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$, for every player of Lohmanns setting we can find a player of the heterogeneous prior beliefs framework having the same ex-ante preference, through the function $\alpha(x) = \frac{2x-1}{2x}$, $x \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$.



pirical results would be straightforward. The technical reason for this is that, in her model, the heterogeneity in utility functions leads in extremely biased agents on one hand, and on the other interacts with the beliefs in an opaque way. Here, the belief bias is both straightforward to calculate and leads into smoother equilibrium sets. Therefore, the present model manages to unveil the relationship between the number of agents, their political bias and the threshold cost tolerance.

Chapter 3

Expert Information and Majority Decisions: Theory¹

“Advice is a dangerous gift, even from the wise to the wise.” (J.R.Tolkien)

3.1 Introduction

When collective decisions are made through voting, typically each voter has not only private information known solely to himself but also public information observed by all voters. Examples of commonly held information in collective decision making include “expert” opinions solicited by a committee, shared knowledge in a board meeting that has emerged from pre-voting deliberation, and evidence presented to a jury. Such information may well be superior to the private information each individual voter has, and if so, it would be natural to expect that voting behaviour would incorporate the public information at least to some extent. Indeed, in most instances the primary reason for bringing shared information to a decision making body would be to improve the quality of its decision.

Meanwhile, such public information is rarely perfect, and in particular expert opinions are often alleged to have excessive influence on decision making. For example, recently the IMF’s advice to the governments of some highly indebted countries have heavily influenced their parliamentary and cabinet decisions for austerity. However, the IMF’s expertise has been questioned by specialists in monetary policy, and it has been reported that the IMF itself has admitted that they may have underestimated the impact of their austerity measure in Greece.² Financial deregulations in the 1990s seem to have been prompted by experts endorsing them, but some politicians reflect that in retrospect they may have followed expert opinions too naively at the time.³ How would collective decision

¹This chapter was based on results from the 2013 working paper “Expert Information and Majority Decisions”, co-authored by Dr. Kohei Kawamura.

²“IMF ‘to admit mistakes’ in handling Greek debt crisis and bailout”, *Guardian*, 4 June 2013, <http://www.guardian.co.uk/business/2013/jun/05/imf-admit-mistakes-greek-crisis-austerity>

³“Gordon Brown admits ‘big mistake’ over banking crisis”, *BBC News*, 13 March 2013, <http://www.bbc.co.uk/news/business-13032013>

making through voting be influenced by shared information? If commonly observed expert information is better than the information each voter has, would the presence of such expert information improve the quality of the collective decision? Can expert information have “too much” influence?

This chapter addresses these questions theoretically, by introducing a public signal into an otherwise classical Condorcet jury setup with majority rule. The public signal is observed by all voters and assumed to be superior to the private signal each voter receives. We call such a public signal “expert information”.

In this chapter we present a majoritarian voting game with expert information and identifies three types of equilibria of interest, namely i) the symmetric mixed strategy equilibrium where each member randomizes between following the private and public signals should they disagree; ii) the asymmetric pure strategy equilibrium where a certain number of members always follow the public signal while the others always follow the private signal; and iii) a class of equilibria where a supermajority and hence the committee decision always follow the expert signal.⁴ We find that in the first two equilibria, the expert signal is collectively taken into account in such a way that it enhances the efficiency (accuracy) of the committee decision, and a fortiori the CJT holds. However, in the third type of equilibria, private information is not reflected in the committee decision and the efficiency of committee decision is identical to that of public information, which may well be lower than the efficiency the committee could achieve without expert information. In other words, the introduction of expert information might reduce efficiency in equilibrium.

In their seminal paper [Austen-Smith and Banks \(1996\)](#) first introduced game-theoretic equilibrium analysis to the Condorcet jury with independent private signals. They demonstrated that sincere voting (in pure strategy) is not generally consistent with equilibrium behaviour. [McLennan \(1998\)](#) and [Wit \(1998\)](#) studied mixed strategy equilibria in the model of [Austen-Smith and Banks \(1996\)](#) and showed that the CJT holds in equilibrium for majority and super-majority rules (except for unanimity rule). The experimental study on strategic voting was pioneered by [Guarnaschelli, McKelvey, and Palfrey \(2000\)](#) who tested the model of [Austen-Smith and Banks \(1996\)](#) and found that the subjects’ behaviour was largely consistent with the theory. Focusing on unanimity rule, [Ali, Goeree, Kartik, and Palfrey \(2008\)](#) found that the findings by [Guarnaschelli, McKelvey, and Palfrey \(2000\)](#) are fairly robust to voting protocols such as the number of repetitions and timing of voting (simultaneous or sequential). The present chapter focuses on majority rule, but examines the effect of public information on voting behaviour and outcomes. The literature on deliberation in voting has studied public information endogenously generated by voters sharing their otherwise private information through pre-voting deliberation (e.g., [Coughlan, 2000](#); [Austen-Smith and Feddersen, 2005](#); and [Gerardi and Yariv, 2007](#)). In these models, once a voter reveals his private information credibly, he has no private information. [Goeree](#)

⁴While the voters may ignore their private information completely, they cannot ignore the expert information completely in equilibrium. That is, voting according only to their private signal is never an equilibrium, since if a voter knows that all the others will follow their private signals, he deviates and follows the expert signal, which is by assumption superior to his private signal.

and Yariv (2011b) found in a laboratory experiment that deliberation diminishes differences in voting behaviour across different voting rules.

While we focus on simultaneous move voting games, the inclination to ignore private information in favour of expert information is reminiscent of rational herding in sequential decisions. In the original rational herding literature (e.g., Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992) each player's payoff is assumed to be determined only by his decision but not by others. Dekel and Piccione (2000) and Ali and Kartik (2012) are among the papers that theoretically study sequential voting in collective decision making where payoffs are intrinsically interdependent. Unlike the expert signal in our setup, which is exogenously given to all voters, public information in their models is generated endogenously by the observed choices of earlier voters. Dekel and Piccione (2000) show that the multiple equilibria include an equilibrium where all voters vote sincerely, which is informationally efficient. Ali and Kartik (2012) identify equilibria that exhibit herding whereby after observing some votes, the rest vote according to what the earlier votes indicate, regardless of their private information. Hung and Plott (2001) conducted a laboratory experiment on sequential voting with majority rule, and found that herding occurred, resulting in inefficiency with respect to sincere voting, while herding behaviour was not as pronounced as in the case where, like the standard herding literature, each subject's decision affected their individual payoff only.

Our model and experimental design are based on the uniform prior with expert information. This structure is theoretically isomorphic to the case of the canonical Condorcet jury model without public information but with a common non-uniform prior belief. Thus while we incorporate expert information into the voters' Bayesian updating explicitly to gain relevant intuition, the symmetric mixed strategy equilibrium we derive in this chapter can be thought of as a special case of the one shown by Wit (1998) who solved for the equilibrium without assuming the uniform prior. However, we also explicitly derive an asymmetric pure strategy equilibrium and its optimality, which has not been shown previously. In doing so, we draw an important link between our fully strategic setup and the optimal voting rule with heterogeneously informed but non-strategic voters studied by Nitzan and Paroush (1982).⁵

The important advantage of adopting the uniform prior and expert information, rather than a non-uniform prior without expert information, is that we are able to ask a potentially useful policy question as to whether to, and how to bring expert opinions into collective decision making. Our experiment is based on this premise, and provides us with practical implications such as the possibility that the introduction of expert information can reduce efficiency, even though theoretically it can enhance welfare if the voters coordinate to play an efficient equilibrium. It would be impossible to address such an issue if we adopted a non-uniform prior analogue without expert information, because in practice the prior belief is seldom a choice variable in itself, while decision making bodies can

⁵While most theoretical studies on strategic voting focus on symmetric strategies, Persico (2004) establishes the optimality of asymmetric strategy equilibrium in a voting game related to ours. However, he does not give an explicit solution for such an equilibrium.

usually choose whether to listen to expert opinions.

3.2 Model

Consider a committee that consists of an odd number of agents $n \in N = \{1, 2, \dots, n\}$. Each agent simultaneously casts a costless binary vote, denoted by $x_i \in \{A, B\}$, for a collective decision $y \in Y = \{A, B\}$. The committee decision is determined by majority rule. The binary state of the world is denoted by $s \in S = \{A, B\}$, where both events are ex ante equally likely $\Pr[s = A] = \Pr[s = B] = \frac{1}{2}$. The members have identical preferences $u_i : Y \times S \rightarrow \mathbb{R}$ and the payoffs are normalized without loss of generality at 0 or 1. Specifically we denote the vNM payoff by $u_i(y, s)$ and assume $u_i(A, A) = u_i(B, B) = 1$ and $u_i(A, B) = u_i(B, A) = 0$, $\forall i \in N$. This implies that the agents would like the decision to be “matched” with the state.

Before voting, each agent receives two signals. One is a private signal about the state $\sigma_i \in K = \{A, B\}$, for which the probability of the signal and the state being matched is given by $\Pr[\sigma_i = A \mid s = A] = \Pr[\sigma_i = B \mid s = B] = p$, where $p \in (1/2, 1]$. We also have $\Pr[\sigma_i = A \mid s = B] = \Pr[\sigma_i = B \mid s = A] = 1 - p$.

In addition to the private signal, all agents in the committee observe a common public signal $\sigma_E \in L = \{A, B\}$, which is assumed to be more accurate than each agent’s individual signal. Specifically, we assume $\Pr[\sigma_E = A \mid s = A] = \Pr[\sigma_E = B \mid s = B] = q$ and $\Pr[\sigma_E = A \mid s = B] = \Pr[\sigma_E = B \mid s = A] = 1 - q$, where $q > p$. The distributions of the two signals are independent.

The public signal in our model has natural interpretations. It can be thought of as expert information given to the entire committee as in, e.g. congressional hearings. Briefing materials presented to and shared in the committee would also have the same feature. Alternatively, it may capture shared knowledge held by all agents as a result of pre-voting deliberation. In that case, the private signal represents any remaining uncommunicated information of each agent, which is individually inferior to shared information. Throughout this chapter we often refer to the public information as expert information.

The timing of our voting game is summarized as follows:

1. Nature determines the state of the world;
2. Each agent observes private and public signals about the state;
3. Each agent votes;
4. Majority decision is implemented and payoffs are realized.

In the absence of the public signal, there exists a sincere voting equilibrium such that $x_i = \sigma_i$ for any i and the Condorcet Jury Theorem holds ([Austen-Smith and Banks, 1996](#)). In what follows we study Bayesian Nash equilibria of the game in which the agents also share expert information. Before doing so let us define some key concepts.

Let $v_i : K \times L \rightarrow [0, 1]$ denote the probability of an agent voting for the state her private signal $\sigma_i \in K = \{A, B\}$ indicates, given the private signal and the

public signal $\sigma_E \in L = \{A, B\}$. For example, $v_i(A, B)$ is the probability that agent i votes for A given that his private signal is A and the public signal is B .

Definition 1. A voting strategy v_i is *symmetric* if $v_i = v$, $\forall i \in N$.

When we derive equilibria of the game later in Section 3.3, we first focus on symmetric strategy equilibria (Section 3.3.1) and then consider asymmetric strategy equilibria (Section 3.3.2).

We use the term *responsive* more widely than usual, to refer to any voting behaviour v_i that varies according to different combinations of the signals.

Definition 2. A voting strategy v_i is *responsive* if $v_i(\sigma_i, \sigma_E) \neq 1 - v_i(\sigma'_i, \sigma_E)$ for $\sigma_i \neq \sigma'_i, \sigma_E \in L$.

This rules out *uninformative* strategies where an agent votes for A or B with a fixed probability regardless of the signals.

Since each agent in our model receives two signals, we formalize three classes of strategies, namely i) one where v_i depends only on the private signal; ii) one where v_i depends only on the public signal; and iii) the other where v_i depends on both the private and public signals.

Definition 3. A voting strategy v_i is *individually informative* if $v_i(\sigma_i, \sigma_E) = 1$, $\forall \sigma_i \in K, \sigma_E \in L$.

An individually informative strategy is a pure strategy analogous to informative (or “sincere”) voting in the standard voting literature with private information, where an agent votes for what the private signal indicates.

Meanwhile there is another type of pure strategy where the agent reacts only to the public signal.

Definition 4. A voting strategy v_i is *obedient* if $v_i(A, B) = v_i(B, A) = 0$ and $v_i(A, A) = v_i(B, B) = 1$.

An obedient strategy is the pure strategy where an agent votes for what the public signal indicates with probability 1, regardless of his private signal.

Since each agent has signals (private and public) that are drawn independently, they may disagree with each other. We formalize the notion of responding to both signals under disagreement as follows:

Definition 5. A voting strategy v_i is *dually responsive* if $v_i(A, \sigma_E) \neq v_i(B, \sigma_E)$ $\forall \sigma_E \in L$, and at least $v_i(A, B) \in (0, 1)$ or $v_i(B, A) \in (0, 1)$.

When both signals disagree and a strategy is dually responsive, the agent follows neither of them with probability 1.

As in the literature on strategic voting, each agent’s optimal action depends on the comparison of his expected payoffs in the event where he is pivotal.

Definition 6. $Piv(v_{-i})$ denotes the event where agent i is pivotal, given his strategy v_i and the others’ strategies v_{-i} .

Throughout this chapter we study the equilibria of the voting game with fully rational agents, and the solution concept we use is Bayesian Nash equilibrium:

Definition 7. A Bayesian Nash equilibrium of the game is a strategy profile v^* , such that

$$E[u_i|v_i^*, Piv(v_{-i}^*), \sigma_i, \sigma_E] \geq E[u_i|v_i, Piv(v_{-i}^*), \sigma_i, \sigma_E], i \in N, v_i \in X \times S, \sigma_i \in K, \sigma_E \in L. \quad (3.2.1)$$

The efficiency of the committee decision making with expert information is measured in comparison to the efficiency under sincere voting in the absence of expert information.

Definition 8. Suppose each of n agents receives a private signal only (with accuracy $p > 1/2$) and votes according to the signal. The probability that the majority decision matches the state is denoted by

$$P_C(p, n) \equiv \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} p^k (1-p)^{n-k}.$$

Needless to say the Condorcet Jury Theorem states that $P_C(p, n) \rightarrow 1$ as $n \rightarrow \infty$. In the absence of a public signal, individually informative voting is also the most efficient Bayesian Nash equilibrium (Austen-Smith and Banks, 1996). In what follows efficiency is measured in terms of the ex ante probability that the majority decision matches the state given a strategy profile.

3.3 Equilibrium Predictions

In this section we study implications of the coexistence of private and public signals on equilibrium voting behaviour. But let us first note that, as in most models in the voting literature, our model also has uninformative equilibria where all agents vote for one of the alternatives regardless of the signals and the outcome is deterministic. This holds true because no individual agent can be pivotal if the others are known to vote for the option and hence no agent influences the outcome individually.

In what follows we consider equilibria in which voting behaviour and the outcome depend on the signals the agents observe. Specifically, we focus on how agents vote depending on whether both signals agree or disagree, i.e., $v_i(A, B) = v_i(B, A)$ and $v_i(A, A) = v_i(B, B)$. That is, the labelling of the state is assumed irrelevant in line with the feature that the prior is uniform and the payoffs depend only on whether the decision matches the state, but not on which state was matched or mismatched.

3.3.1 Symmetric strategies

Let us focus our attention to symmetric strategy equilibria first, where $v_i(A, A) = v_i(B, B) = \alpha$ and $v_i(B, A) = v_i(A, B) = \beta$, for any i . Note that because of the symmetry of the model with respect to A and B , we can consider the case of $\sigma_E = A$ and that of $\sigma_E = B$ as two independent and essentially identical games, where only the labelling of the signal and decision differs. We start by observing that the

presence of expert information upsets the individually informative equilibrium, where every agent votes according to his own signal only.

Proposition 1. *Individually informative voting is not a Bayesian Nash equilibrium.*

Proof. See Appendix. □

The proposition has a straightforward intuition. Suppose that an agent is pivotal and the private and public signals disagree. In that event, the posterior of the agent is such that the votes of the other agents, who vote individually informatively, are collectively uninformative, since there are equal numbers of the votes for both A and B . Given this, the agent compares the two signals and chooses to follow the public one as it has higher accuracy ($q > p$), but such voting behaviour breaks the individually informative equilibrium in symmetric strategies.

On the other hand, it is easy to see that there exists an equilibrium where every agent votes according the public signal and ignores their own:

Proposition 2. *Obedient voting is a Bayesian Nash equilibrium.*

Proof. Consider agent i . If all the other agents vote according to the public signal, he is indifferent to which alternative to vote for, and thus every agent adopting obedient voting is an equilibrium. □

The reasoning is similar to the one for the uninformative equilibria where all agents vote for the same alternative regardless of the signals and the probability of the majority decision matching the correct state is $1/2$. However, in the obedient equilibrium the outcome does reflect one of the signals and thus is not completely uninformative. The equilibrium clearly outperforms the uninformative equilibria since $q > 1/2$. The same line of reasoning also leads to the following remark:

Remark 1. There exists an equilibrium where every agent votes against the public signal.

This equilibrium however seems implausible, since from $1 - q < 1/2$ it is outperformed even by the uninformative equilibria. In what follows we rule out this equilibrium.

Later we show that there exists a mixed strategy equilibrium where both private and public signals are taken into account, and study its properties. Before deriving the equilibrium, it is useful to show that the mixed strategy equilibrium takes a “hybrid” form, where mixing occurs only when the private and public signals disagree.

Lemma 1. *Suppose there exists a symmetric Bayesian Nash equilibrium in mixed strategies. In such an equilibrium, any agent whose private signal coincides with the public signal votes according to the signals with probability 1.*

Proof. See Appendix. □

Lemma 1 is not surprising, because when both signals coincide they would jointly be very informative about the actual state. The non-trivial part of the lemma is that this intuition holds regardless of the mixing probability when the signals disagree. Thanks to the lemma we can focus on mixing when the private and public signals disagree.

Proposition 3. *If $q \in (p, \bar{q}(p, n))$ there exists a unique dually responsive Bayesian Nash Equilibrium, where*

$$\bar{q}(p, n) = \frac{\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}{1 + \left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}.$$

In the equilibrium, the agents whose private signal coincides with the public signal vote according to them with probability 1. The agents whose private signal disagrees with the public signal vote according to their private signal with probability

$$\beta^* = \frac{1 - A(p, q, n)}{p - A(p, q, n)(1-p)}, \text{ where } A(p, q, n) = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}}.$$

If $q > \bar{q}$ there is no dually responsive equilibrium.

Proof. See Appendix. □

Note that in order for the mixed strategy equilibrium to exist, the accuracy of the public signal has to be lower than a threshold $\bar{q}(p, n)$. If this is the case, there are two symmetric responsive equilibria of interest, namely i) the obedient equilibrium where all agents follow the public signal; and ii) the dually responsive equilibrium in which the agents take into account both private and public signals by mixing. Meanwhile, if the public signal is sufficiently accurate relative to the private signals, then the only responsive equilibrium is obedient.

Later we will show that \bar{q} is strictly larger than P_C , the accuracy of majority decision in the absence of a public signal (Definition 8). In other words, if the accuracy of expert information is the same as what the agents can collectively achieve without such information, then they still incorporate both their private signals and the public signal into their decision through randomization.

The equilibrium mixing probability β^* captures the weight the agents put on the private signal relative to the public signal when the two signals disagree.

Corollary 1. *$\partial\beta^*/\partial p > 0$ and $\partial\beta^*/\partial q < 0$. That is, the more informative the private signal becomes relative to the public signal, the more weight the agents put on the private signal in the mixing probability.*

This indicates that the equilibrium behaviour fine-tunes the weights on the two signals according to their accuracy. How does the mixing probability change according to the committee size? Interestingly, as the committee size becomes larger, the weight on the private signal also decreases, and becomes zero as n tends to infinity:

Corollary 2. $\partial\beta^*/\partial n > 0$, and $\beta^* \rightarrow 1$ as $n \rightarrow \infty$. That is, the equilibrium probability β^* of voting against the public signal when the two signals disagree is increasing in the committee size n . As the committee size tends to infinity the dually responsive equilibrium converges to the individually informative equilibrium, where the public signal is ignored.

Corollary 2 can be interpreted intuitively as follows. Given the accuracies of the private and public signals, as the committee size becomes larger, the collective accuracy of private signals increases through the logic of CJT. This implies that the “value-added” of the public signal relative to each private signal ($q > p$) decreases.

Corollary 3. $\partial\bar{q}/\partial n > 0$. That is, as the committee size becomes larger, the dually responsive equilibrium exists for a wider range of q .

This corollary has a similar intuition to that of Corollary 2. Since the collective accuracy of private signals increases as n becomes large, in order for the agents to disregard completely, the public signal has to be much more accurate.

So far we have observed the properties of voting behaviour in the dually responsive equilibrium identified in Proposition 3.

Let us consider the efficiency of the dually responsive equilibrium in relation to that of the obedient equilibrium. This is a non-trivial question to ask, not least because the public signal introduces a type of “correlation” to the information the agents receives, and it is known that correlation of private signals leads to less efficiency. As the following proposition states, however, in the dually responsive equilibrium the agents optimally take into account the public signal through mixing. That is, if a welfare maximizing social planner were to choose α and β to maximize the probability that the majority decision matches the true state, which we denote by $P(\alpha, \beta)$, then they coincide with the equilibrium α^* and β^* .

Proposition 4. *The dually responsive equilibrium in Proposition 3 maximizes the efficiency of the majority decisions with respect to α and β .*

Proof. See Appendix. □

A direct implication of Proposition 4 is that providing the committee with expert information is beneficial as long as the committee members play the symmetric mixed strategy equilibrium:

Corollary 4. *The mixed strategy equilibrium identified in Proposition 3 outperforms individually informative voting and obedient voting.*

The corollary holds because individually informative voting is equivalent to $\alpha = \beta = 1$ and obedient voting $\alpha = \beta = 0$, and Proposition 4 has just shown that the mixed strategy equilibrium ($\alpha^* = 1$ and $\beta^* \in (0, 1)$) is optimal with respect to the choice of α and β .

Recall that the mixed strategy equilibrium does not exist when the accuracy of expert information q is above the threshold \bar{q} . In light of Proposition 4, this means that when q is very high it is not worthwhile to combine both types of information to maximize $P(\alpha, \beta)$. The following remark gives us some indication about the threshold.

Remark 2. $\bar{q} > P_C$, that is, the upper bound of the accuracy of expert information for the mixed strategy equilibrium to exist is higher than the accuracy of sincere voting without expert information.

Proof. Let $\beta^*(\cdot)$ be the equilibrium/optimal β as a function of q , and let $q_C \equiv P_C$. By construction we have $\beta^*(\bar{q}) = 0$ and $\beta^*(q_C) \geq 0$. In fact we have $\beta^*(q_C) > 0$. To see this, note $\beta^*(q_C) = 0$ implies $P_H(q_C) = q_C$, which is a contradiction since i) $q_C > p$; ii) from (3.C.5) if $q = p$ then $\beta^* = 0$ and thus $P_H(p) = q_C$; and iii) by differentiating (3.D.1) with respect to q we see that $P_H(q)$ is strictly increasing.

From (3.C.4) we can check that $\beta^*(q)$ is strictly decreasing in q . Thus $\beta^*(q_C) > \beta^*(\bar{q}) = 0$ implies $\bar{q} > q_C = P_C$. \square

In view of Proposition 4, the remark implies that for private signals to be optimally disregarded for efficiency maximization, the public signal has to be strictly better than the efficiency the private signals can achieve collectively through majority voting.

3.3.2 Asymmetric strategies

So far we have focused on symmetric strategies and derived a unique dually-responsive equilibrium as well as the obedient equilibrium. In this subsection we examine equilibria in asymmetric strategies. As allowing asymmetric strategies leads to a vast number of possible configurations of equilibria, we focus on i) asymmetric strategy equilibria where the majority decision is the same as that in the symmetric obedient equilibrium and ii) asymmetric pure strategy equilibrium that is unique in a natural set of pure strategy profiles and is optimal in the set of all strategy profiles. As in the previous subsection, we rule out non-responsive equilibria where the majority decision is independent of the signals⁶.

Obedient outcome

The first type of equilibria are a straightforward extension of the obedient equilibrium in symmetric strategies (Proposition 2) and take the following “hybrid” form:⁷

Proposition 5. *For $n \geq 5$ there exist equilibria where $(n+1)/2+1$ or more agents (a supermajority) vote according to the public signal and each of the rest uses an arbitrary strategy. The decision is obedient: the committee decision coincides with the public signal with probability 1.*

Proof. This directly follows from the feature that, if a supermajority always vote according to the public signal, no agent is pivotal. We have $n \geq 5$ because if $n = 3$ then $(n + 1)/2 + 1$ members following the public signal corresponds to the symmetric obedient strategy. \square

⁶For a detailed mathematical derivation of the equilibria of this part, as well as the derivation of the upper bound of expert information accuracy for existence of these equilibria, see Appendix

⁷By the same token there are equilibria where $(n + 1)/2 + 1$ agents vote against the public signal, but we rule them out as they are outperformed by even by the uninformative equilibrium.

Note that it is not sufficient for the equilibria to have $(n+1)/2$ agents following the public signal, because if it is the case any agent will be pivotal with positive probability. Clearly Proposition 5 includes a class of payoff equivalent equilibria in which some agents use pure strategies and the the others randomize:

Definition 9. A *hybrid obedient equilibrium* is an equilibrium where $n \geq 5$ and $(n+1)/2 + 1$ or more agents (i.e. a supermajority) follow the public signal with probability 1 and at least one of the rest randomizes arbitrarily.

While the majority decision in the equilibrium is trivial and identical to the symmetric obedient equilibrium, the hybrid obedient equilibrium will be of significant interest when interpreting the experimental results, as we will discuss later.

Asymmetric pure strategies

Let us now consider asymmetric pure strategies for which the committee decision is affected by private signals. Let Γ be the set of all (pure, mixed and hybrid) strategy profiles. Since from Proposition 1 we know that individually informative voting is not an equilibrium, we need to consider asymmetric strategies to study responsive equilibrium in pure strategies. In what follows we focus on the strategy profiles such that the agents vote according to either the public or private signal with probability 1.

Definition 10. $M \subset \Gamma$ is the set of asymmetric pure strategy strategy profiles in which $m \in \{1, 2, \dots, n-1\}$ “obedient” agents vote according to the public signal with probability 1, and $n-m$ “individually informative” agents vote according to their private signal with probability 1.

In this set of pure strategy profiles, if any agent’s private signal and his public signal agree, then he votes according to the signals. The two groups (obedient and individually informative) vote differently when the signals disagree: in such cases the m “obedient” agents vote according to the public signal, while $n-m$ “individually informative” agents vote against the public signal. In what follows we establish the existence of a non-obedient equilibrium in M and its optimality in Γ . Before describing the equilibrium, it is useful to define the subset of M in which the committee decision is not obedient.

Definition 11. $\hat{M} \subset M$ is the set of pure strategy profiles where $m \in \{1, 2, \dots, (n+1)/2 - 1\}$.

The following proposition states that, unless the accuracy of the public signal q is too high relative to the accuracy of each private signal p , there is a unique equilibrium in \hat{M} .

Proposition 6. *Let*

$$m^* \in \mathbb{N} \cap \left(\frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]} - 1, \frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]} \right].$$

If $m^* < (n + 1)/2$ or equivalently

$$q < \frac{e^{\frac{n+1}{2} \frac{p}{1-p}}}{1 + e^{\frac{n+1}{2} \frac{p}{1-p}}},$$

then $m = m^*$ is the unique Bayesian Nash equilibrium in the set of strategy profiles \hat{M} . If $m^* \geq (n + 1)/2$, then any $m \geq (n + 1)/2$ in M leads to an equilibrium that is payoff equivalent to the obedient equilibrium.

Proof. See Appendix. □

It remains to examine the efficiency of the asymmetric pure equilibrium in \hat{M} . In what follows first we show that if $m^* < (n + 1)/2$ then it maximizes the efficiency with respect to the entire pure strategy profiles Γ . In other words, if a social planner is to choose m when q is not too large relative to p , then she will choose m^* . We will then show that the equilibrium outperforms the symmetric mixed strategy equilibrium identified in Proposition 3.

Proposition 7. *If $m^* < (n + 1)/2$, then m^* uniquely maximizes the expected welfare in Γ .*

Proof. See Appendix. □

The following corollary is a direct consequence of Proposition 7.

Corollary 5. *The asymmetric pure strategy equilibrium with m^* outperforms the sincere voting equilibrium in the absence of public information.*

The intuition is simple: suppose that only one agent always follows the public signal and the rest always follow the private signal. The efficiency under this strategy profile is higher than the efficiency under sincere voting without public information because one agent following the public signal is equivalent to this agent having a better signal since $q > p$. Therefore having optimal/equilibrium m^* guarantees that the welfare is higher in the asymmetric pure equilibrium with public information.

Also Proposition 7 implies the following ranking of multiple equilibria.

Remark 3. The efficiency of equilibria in the voting game with expert information, when they exist, is ranked as follows:

$$\text{non-obedient asymmetric pure eqm} \succ \text{symmetric mixed eqm} \succ \text{obedient eqm}. \quad (3.3.1)$$

The ranking (3.3.1) holds even if q becomes very close to the upper bound in Proposition 6 from below, because the expected welfare of the asymmetric pure strategy equilibrium does not converge to that of the obedient equilibrium due to the discreteness of the number of agents who follow the public (or private) signal.

Figure 3.3.1 provides some comparative statics for the efficiency difference of the equilibria. It presents the majoritarian outcome accuracy difference between the symmetric mixed equilibrium and the non-obedient asymmetric pure equilibrium for an increasing difference in accuracy between expert and individual

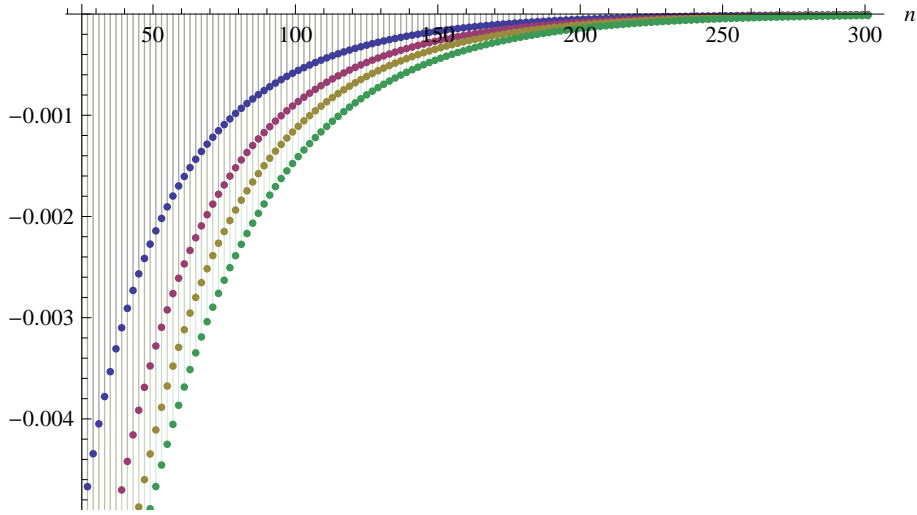


Figure 3.3.1: Difference in majoritarian outcome accuracy between symmetric mixed equilibrium and non-obedient asymmetric pure equilibrium for an increasing number of voters, $p = 0.6$ and $q \in \{0.7(\text{blue}), 0.75(\text{red}), 0.8(\text{yellow}), 0.9(\text{green})\}$.

information ($p = 0.6$ and $q \in \{0.7, 0.75, 0.8, 0.9\}$) and an increasing number of voters. Note that the difference is increasing in the accuracy difference and, as expected, converges to zero when the size of the committee increases due to the CJT .

We have also seen that the sincere voting equilibrium in the absence of public information can be better or worse than the obedient equilibrium, while it is always less efficient than the symmetric mixed strategy equilibrium (and hence the non-obedient asymmetric pure strategy equilibrium).⁸

3.3.3 Quantal Response Equilibria

In the literature on voting experiments, it is customary to use the concept of a quantal response equilibrium (QRE; McKelvey and Palfrey, 1995). QRE essentially modifies the concept of Nash equilibrium to incorporate realistic limitations of rational choice model games. In this “statistical perturbation” of the concept of Nash equilibrium, players have rational expectations and choose responses with higher expected payoffs with higher probability. As discussed in the introduction of the present thesis, the concept of QRE has been widely used to analyse game theoretic data, both field and laboratory, and its predictive power is persistently strong.

In the context of this model, the parameter λ of the following subsections captures the idea of “noise”. Note that as λ converges to infinity, we get the sym-

⁸Let us comment on the upper bound on q for which the symmetric pure strategy equilibrium in Proposition 3 and the responsive asymmetric pure strategy equilibrium in Proposition 6 exist. It is easy to check that whether one or both of them exist simultaneously depends on p and n . Clearly both equilibria exist unless q is too high, but when q is very high, it may be that the symmetric mixed strategy equilibrium exists and the asymmetric pure strategy does not exist (, which is the case when both p and n are high) and vice versa.

metric mixed equilibrium of the game. However, when λ tends to zero, due to the increase “noise” they are facing, players simply randomise between following their signal and the expert signal with probability 1/2. We begin the next subsection by a theoretical simplification: we assume that if their signal coincides with that of the expert, agents “always” follow the common signal ($\alpha = 1$). Afterwards, we relax this assumption ($\alpha \in (0, 1)$). Also, note that throughout the following section we assume symmetry in agents’ strategies.

Case $\alpha = 1$

Lets assume that agents always follow the common signal, if the their private signal coincides with the expert signal. Also, as before we assume that there is no a priori bias with respect to one of the agenda’s. In other words, an agents will mix the same way if she has received A and the expert signal is B as she would if she had received a B and the expert signal was A. Therefore, $\alpha = v_i(A, A) = v_i(B, B) = 1$ and $\beta = v_i(B, A) = v_i(A, B)$ for $i \in \{2, 3, ..n\}$. Conditional on the state $s = A$ and $\sigma_E = A$, the ex ante probability of each agent $i \in \{2, 3, ..n\}$ voting for A is $r_A \equiv p + (1 - p)(1 - \beta)$. Also, conditional on the state $s = A$ and $\sigma_E = B$, the probability of each agent $i \in \{2, 3, ..n\}$ voting for A is $r_B \equiv p\beta$.

Using r_A and r_B , if agent $i = 1$ would follow the same mixed strategy, the expected utility for agent i will be the ex ante probability $P(\alpha, \beta)$ that the majority decision matches the state. This can be written as

$$\begin{aligned} P(\alpha, \beta) &= q \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} r_A^k (1 - r_A)^{n-k} + (1 - q) \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} r_B^k (1 - r_B)^{n-k} \\ &= qB(n, \frac{n+1}{2}, r_A) + (1 - q)B(n, \frac{n+1}{2}, r_B), \end{aligned}$$

where $B(n, m, x) = \sum_{k=m}^n \binom{n}{k} x^k (1 - x)^{n-k}$.

First, we calculate the utility choice agent $i = 1$ faces if all other agents mix with probability β . If all agents follow the mixed strategy described above and agent $i = 1$ follows an obedient strategy when disagreeing with expert information (*OBSD*), expected utility is given by:

$$\begin{aligned} E[u_1(OBSD, \beta)] &= \frac{q(1 - p)}{q(1 - p) + p(1 - q)} B(n - 1, \frac{n+1}{2} - 1, r_A) \\ &\quad + \frac{p(1 - q)}{q(1 - p) + p(1 - q)} B(n - 1, \frac{n+1}{2}, r_B). \end{aligned}$$

If all agents follow the mixed strategy described above and agent $i = 1$ follows an individually informative strategy when disagreeing with expert information

Agent i' s QRE best response β

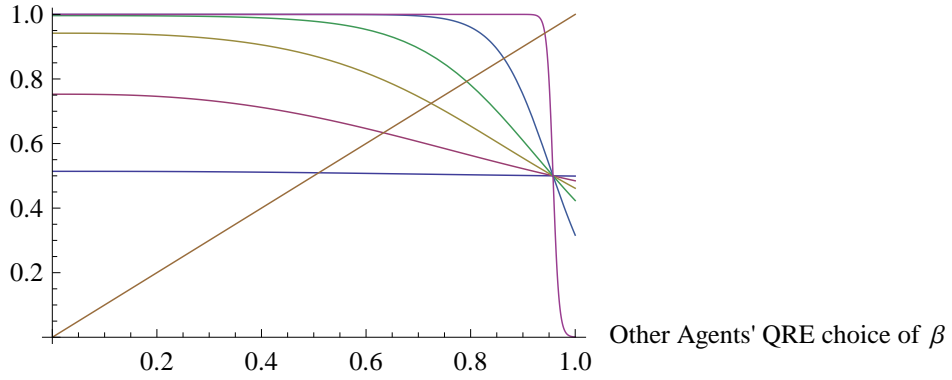


Figure 3.3.2: QRE for $\frac{1}{\lambda} \in \{0.1(\text{lightblue}), 2(\text{red}), 5(\text{yellow}), 10(\text{green}), 25(\text{darkblue}), 235(\text{purple})\}$ and $\alpha = 1$.

(IISD), expected utility is given by:

$$E[u_1(IISD, \beta)] = \frac{q(1-p)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2}, r_A) + \frac{p(1-q)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2} - 1, r_B).$$

Therefore, for a symmetric Quantal Response Equilibrium, β should be such that:

$$\beta = \frac{\exp(\lambda E[u_1(IISD, \beta)])}{\exp(\lambda E[u_1(IISD, \beta)]) + \exp(\lambda E[u_1(OBDD, \beta)])}.$$

In Figure 3.3.2 the reader can find a plot of the QRE of the game for $q = 0.7$, $p = 0.65$ and $\alpha = 1$ for $\frac{1}{\lambda} \in \{0.1, 2, 5, 10, 25, 235\}$. Note that as discussed before, when $\lambda \rightarrow 0$, $\beta \rightarrow 1/2$ and when $\lambda \rightarrow \infty$, $\beta \rightarrow \beta^*$.

General case, $\alpha \in (0, 1)$

Now we present the general case, still assuming no a priori bias to agenda's A and B: Let $\alpha = v_i(A, A) = v_i(B, B)$ and $\beta = v_i(B, A) = v_i(A, B)$ for $i \in \{2, 3, ..n\}$. Conditional on the state $s = A$ and $\sigma_E = A$, the ex ante probability of each agent $i \in \{2, 3, ..n\}$ voting for A is $r_A \equiv p\alpha + (1-p)(1-\beta)$. Also, conditional on the state $s = A$ and $\sigma_E = B$, the probability of each agent $i \in \{2, 3, ..n\}$ voting for A is $r_B \equiv p\beta + (1-p)(1-\alpha)$. If all agents follow the mixed strategy described above and agent $i = 1$ votes for common signal when agreeing with expert information (FCSA), expected utility is given by:

$$E[u_1(FCSA, \alpha, \beta)] = \frac{pq}{pq + (1-p)(1-q)} B(n-1, \frac{n+1}{2} - 1, r_A) + \frac{(1-p)(1-q)}{pq + (1-p)(1-q)} B(n-1, \frac{n+1}{2}, r_B).$$

If all agents follow the mixed strategy described above and agent $i = 1$ votes against common signal when disagreeing with expert information (*ACSA*), expected utility is given by:

$$E[u_1(ACSA, \alpha, \beta)] = \frac{pq}{pq + (1-p)(1-q)} B(n-1, \frac{n+1}{2}, r_A) + \frac{(1-p)(1-q)}{pq + (1-p)(1-q)} B(n-1, \frac{n+1}{2} - 1, r_B).$$

As before, if all agents follow the mixed strategy described above and agent $i = 1$ follows an obedient strategy when disagreeing with expert information (*OBSD*), expected utility is given by:

$$E[u_1(OBSD, \alpha, \beta)] = \frac{q(1-p)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2} - 1, r_A) + \frac{p(1-q)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2}, r_B).$$

If all agents follow the mixed strategy described above and agent $i = 1$ follows an individually informative strategy when disagreeing with expert information (*IISD*), expected utility is given by:

$$E[u_1(IISD, \alpha, \beta)] = \frac{q(1-p)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2}, r_A) + \frac{p(1-q)}{q(1-p) + p(1-q)} B(n-1, \frac{n+1}{2} - 1, r_B).$$

Therefore, for a symmetric Quantal Response Equilibrium, α and β should be such that:

$$\alpha = \frac{\exp(\lambda E[u_1(FCSA, \alpha, \beta)])}{\exp(\lambda E[u_1(ACSA, \alpha, \beta)]) + \exp(\lambda E[u_1(FCSA, \alpha, \beta)])},$$

$$\beta = \frac{\exp(\lambda E[u_1(IISD, \alpha, \beta)])}{\exp(\lambda E[u_1(IISD, \alpha, \beta)]) + \exp(\lambda E[u_1(OBDD, \alpha, \beta)])}.$$

3.4 Equilibrium Selection

In the previous sections of this chapter, we attempted to map all symmetric mixed and asymmetric pure equilibria of a majority game of common values with expert and private information. The multiplicity of equilibria of the game direct us to use filtering techniques and rule out, at least theoretically, some of the aforementioned equilibria. Of course, these so called theoretical *refinements* do not ensure that the rejected equilibria will not be played in reality.

In this section, we will use two such refinements:

- The first is the *elimination of weakly dominated strategies*, which is a natural refinement in the voting literature (Wit (1998)). Under this refinement all strategies, which produce inferior outcomes for an agent facing *any* strategy

profile from the rest of the agents, can be eliminated as unlikely to be chosen by agents.

- The second is the notion of a perfect equilibrium (Selten (1975)) that essentially filters away all equilibria that do not persist if agents were to make a small mistake in making their choice or use choose a strategy with a *trembling hand*.

3.4.1 Elimination of Weakly Dominated Strategies

It is straightforward to see that this refinement does not consistently filter away any of the aforementioned equilibria of the game.

Consider the case when the symmetric mixed equilibrium of the game exists. The monotonicity of the probability to recover the correct state with contradicting signals $P(A, B)$ with respect to β given Equation 3.D.5 suggests that if all other agents choose a symmetric mixed strategy with $\beta > \beta^*$, then it is optimal for an agent to vote what expert information suggests. On the other hand, if all other agents choose to mix with $\beta < \beta^*$, then it is optimal for an agent to vote according to her private information. Therefore, neither of the strategies is weakly dominated, and any of these equilibrium strategies survive the refinement.

This result holds for the pure obedient equilibrium of the game using the same reasoning. Since $q > p$ and having in mind the case where $\beta > \beta^*$, voting for the expert signal is not dominated.⁹

Note that using the same *continuity* argument as in Wit (1998), we can assume that if agents were to play a symmetric strategy equilibrium, then it would be reasonable for the mixed equilibrium to be played where it exists ($q \leq \bar{q}$) and otherwise the obedient equilibrium is the only other candidate ($q > \bar{q}$).

As far the asymmetric equilibria of the game are concerned, the intuition is identical if instead of β^* we use m^* . Indeed, if $m > m^*$, then any of the agents following expert information would prefer to vote for their private signal, whereas if $m < m^*$ they would prefer to vote according to the expert signal. Finally, similarly to the obedient case and having in mind the fact that none of the strategies there is weakly dominated, the hybrid obedient equilibrium of the game survives the refinement through the exact same process.

3.4.2 Trembling Hand

Unlike the previous refinement, the trembling hand eliminates but only the obedient and the hybrid obedient equilibrium.

First of all, consider the obedient equilibrium and assume that all agents will make a “mistake” and vote for their private signal with probability $\epsilon > 0$. That entails that the probability of being pivotal is now positive, however small ϵ is. Since $\epsilon < \beta^*$, we know by Proposition 4 that agents in this case will have an

⁹In the case where $\frac{q}{1-q} > (\frac{p}{1-p})^n$ and similarly to Wit (1998), when the expert signal is sufficiently strong to outweigh all private signals, then the equilibrium where all agents vote *contrary* to the expert signal does not survive. However, when the expert signal is not sufficiently strong to outweigh all private signals, or $\frac{q}{1-q} \leq (\frac{p}{1-p})^n$, then this equilibrium survives as well.

incentive to deviate and vote according to their private signal in order to improve the aggregate accuracy. Therefore the obedient equilibrium does not survive the tremble. With a similar argument, the hybrid obedient equilibrium does not survive the trembling hand refinement as well. Given that the supermajority of obedient voters have a small but positive probability of voting informatively, then the members of the minority are not indifferent any more and will prefer not to randomize.

Secondly, the fact that the symmetric mixed equilibrium and the asymmetric pure equilibrium are robust with respect to trembles is a direct consequence of Theorem 3 by McLennan (1998). The intuition behind this result for a symmetric tremble around β^* is straightforward again due to the monotonicity of $P(A, B)$ with respect to β given Equation 3.D.5. In other words, any tremble around β^* , as long as it is symmetric and small, will not ultimately alter the indifference condition of the equilibrium. The same result holds for the asymmetric pure equilibrium, provided that the tremble is sufficiently small so that $m^* \in \mathbb{N} \cap \left(\frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]} - 1, \frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]} \right]$ still holds.

3.5 Conclusion

This chapter has studied the effects of a public signal on voting behaviour in committees of common interest. We have demonstrated that the presence of publicly observed expert information changes the structure of voting equilibria substantially. In particular, individually informative voting is no longer an equilibrium when the precision of the public signal is better than each agent's individual signal, as in expert opinions presented to the entire committee. If the expert information is not too accurate, there are three informative equilibria of interest, namely i) the symmetric mixed strategy equilibrium where each member randomizes between following the private and public signals should they disagree; ii) the asymmetric pure strategy equilibrium where a certain number of members always follow the public signals while the others always follow the private signal; and iii) a class of equilibria where the committee's majority decision always follows the expert information. When the expert information is not too accurate, i) and ii) are more efficient but iii) can be less efficient than the sincere voting equilibrium without expert information. If the expert information is very accurate, then the only informative equilibrium involves obedient voting, whereby every agent follows expert information, and this equilibrium is indeed efficient.

To conclude this chapter, let us comment on the way committee members listen to expert opinions. So far we have assumed that every member hears expert information before voting, but alternatively an expert could speak to only selected members of a committee, or a member might privately consult with an expert for more accurate information. Note that if m^* members of the committee listen to expert information, then there is an equilibrium equivalent to the most efficient equilibrium in Proposition 6, where m^* members follow the expert signal and $n - m^*$ members follow the private signal. While this selective disclosure does not change the maximum equilibrium efficiency, it eliminates the inefficient obedient equilibrium since not enough members observe the public signal for the

obedient outcome. This is a theoretically trivial point: needless to say, if the agents can coordinate to play the efficient equilibrium, whether all members or only m^* of them listen to the expert is irrelevant. However, given that in reality expert opinions/testimonies are very often heard by all members of a decision making body, it would be of practical interest to ask whether this may or may not “trap” the committee in the inefficient equilibrium.

Appendix

3.A Proof of Proposition 1

Proof. Consider agent i 's strategy in the putative equilibrium where all the other agents adopt the individually informative strategy. He computes the difference in the expected payoff between voting for A and B , conditional on his private and public signals, in the event where he is pivotal. The payoff difference is given by

$$\begin{aligned} w(\sigma_i, \sigma_E) &\equiv E[u_i(A, s) - u_i(B, s) | Piv(v_{-i}), \sigma_i, \sigma_E] \Pr[Piv(v_{-i}), \sigma_i, \sigma_E] \\ &= \frac{1}{2} \Pr[\sigma_E | s = A] \Pr[\sigma_i | s = A] \Pr[Piv(v_{-i}) | s = A] \\ &\quad - \frac{1}{2} \Pr[\sigma_E | s = B] \Pr[\sigma_i | s = B] \Pr[Piv(v_{-i}) | s = B], \end{aligned} \quad (3.A.1)$$

where the equality follows from the independence of the signals in individually informative voting. Without loss of generality, let us assume $\sigma_i = B$ and $\sigma_E = A$. From (3.A.1) we have

$$\begin{aligned} w(B, A) &= \frac{1}{2} \left(q(1-p) \frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} \right) \\ &\quad - \frac{1}{2} \left((1-q)p \frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} \right) \\ &= \frac{1}{2} (q-p) \frac{(n-1)!}{\left[\left(\frac{n-1}{2}\right)!\right]^2} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}} > 0. \end{aligned}$$

The inequality holds since $q > p$. This implies that agent i votes for A despite her private signal B . Thus individually informative voting is not a Bayesian Nash equilibrium. \square

3.B Proof of Lemma 1

Proof. First, note that because of the symmetry of the model with respect to A and B , we can consider the case of $\sigma_E = A$ that of $\sigma_E = B$ as two independent and essentially identical games, where only the labelling of the signal and decision differs. Thus we let $v_i(A, A) = v_i(B, B) = \alpha$ and $v_i(B, A) = v_i(A, B) = \beta$. Without loss of generality, let us assume $\sigma_E = A$ to prove the lemma.

Define

$$\begin{aligned}
F(A) \equiv \Pr[\text{Piv}(v_{-i})|s = A] &= \sum_{k=0}^{n-1} \sum_{j=0}^{\min(k, \frac{n-1}{2})} \binom{n-1}{k} p^k (1-p)^{n-1-k} \\
&\times \binom{k}{j} \alpha^j (1-\alpha)^{k-j} \binom{n-1-k}{\frac{n-1}{2}-j} (1-\beta)^{\frac{n-1}{2}-j} \beta^{\frac{n-1}{2}-k+j}
\end{aligned} \tag{3.B.1}$$

and

$$\begin{aligned}
F(B) \equiv \Pr[\text{Piv}(v_{-i})|s = B] &= \sum_{k=0}^{n-1} \sum_{j=0}^{\min(k, \frac{n-1}{2})} \binom{n-1}{k} p^k (1-p)^{n-1-k} \\
&\times \binom{k}{j} \beta^j (1-\beta)^{k-j} \binom{n-1-k}{\frac{n-1}{2}-j} (1-\alpha)^{\frac{n-1}{2}-j} \alpha^{\frac{n-1}{2}-k+j}.
\end{aligned} \tag{3.B.2}$$

Using $F(A)$ and $F(B)$, we rewrite

$$w(A, A) = \frac{1}{2} [qpF(A) - (1-q)(1-p)F(B)] \tag{3.B.3}$$

$$w(B, A) = \frac{1}{2} [q(1-p)F(A) - (1-q)pF(B)]. \tag{3.B.4}$$

Note that (3.B.3) and (3.B.4) incorporate each agent's Bayesian updating on the state and the private signals other agents may have received, conditional on his own signal and the public signal.

In order to have fully mixing equilibrium, namely $\alpha^* \in (0, 1)$ and $\beta^* \in (0, 1)$, we must have $w(A, A) = 0$ and $w(B, A) = 0$ simultaneously for indifference. In what follows, we show that $w(A, A) > 0$ for any α and β , which implies in equilibrium we must have $\alpha^* = 1$ and if mixing occurs it must be only for β , that is, when the private and public signals disagree. Specifically, we show that $F(A) > F(B)$, which readily implies $w(A, A) > 0$ from (3.B.3).

From (3.B.3) and (3.B.4) we have $F(A) - F(B) > 0$ if

$$\begin{aligned}
\alpha^j (1-\alpha)^{k-j} (1-\beta)^{\frac{n-1}{2}-j} \beta^{\frac{n-1}{2}-k+j} &> \beta^j (1-\beta)^{k-j} (1-\alpha)^{\frac{n-1}{2}-j} \alpha^{\frac{n-1}{2}-k+j} \\
&\Leftrightarrow \beta(1-\beta) > \alpha(1-\alpha) \\
&\Leftrightarrow (\alpha + \beta - 1)(\alpha - \beta) > 0.
\end{aligned} \tag{3.B.5}$$

To see that (3.B.5) holds we will show that in equilibrium $\alpha^* + \beta^* - 1 > 0$ and $\alpha^* - \beta^* > 0$.

Let us first observe that $\alpha^* + \beta^* - 1 > 0$. The difference in the difference in payoffs between voting for A and B is given by

$$w(A, A) - w(B, A) = \frac{q(2p-1)}{2} F(A) + \frac{(1-q)(2p-1)}{2} F(B) > 0, \tag{3.B.6}$$

since both terms in the right hand side are positive since $p, q > 1/2$. Thus, given $\sigma_E = A$, the equilibrium probability of voting for A when $\sigma_i = A$ must be strictly greater than that of voting for A when $\sigma_i = B$, which implies¹⁰

$$\alpha^* + \beta^* - 1 > 0. \quad (3.B.7)$$

Second, let us show that $\alpha^* > \beta^*$. We assume instead that $\alpha^* \leq \beta^*$ in equilibrium and derives a contradiction. There is no hybrid equilibrium such that $\alpha^* \in (0, 1)$ and $\beta^* = 1$, because from (3.B.5) and (3.B.7), $\alpha^* \leq \beta^*$ implies $F(A) \leq F(B)$ and we may have a fully mixed equilibrium, in which case $w(A, A) = w(B, A) = 0$. From (3.B.3) we have

$$w(A, A) = 0 \Rightarrow \frac{F(A)}{F(B)} = \frac{(1-q)(1-p)}{qp}, \quad (3.B.8)$$

and from (3.B.4)

$$w(B, A) = 0 \Rightarrow \frac{F(A)}{F(B)} = \frac{(1-q)p}{q(1-p)}. \quad (3.B.9)$$

We can see that (3.B.8) and (3.B.9) hold simultaneously if and only if $p = 1/2$, which is a contradiction, since $p \in (1/2, 1]$. Thus we conclude that $\alpha^* > \beta^*$ in any mixed strategy equilibrium.

Combining $\alpha^* > \beta^*$ and (3.B.7), we can see that (3.B.5) holds. Thus we have $F(A) - F(B) > 0$ and $w(A, A) > 0$, which implies any mixed strategy equilibrium has to have a hybrid form, such that $\alpha^* = 1$. \square

3.C Proof of Proposition 3

Proof. From Lemma 1 any mixed strategy equilibrium involves $v_i(A, A) = v_i(B, B) = 1$ and $v_i(A, B) = v_i(B, A) = \beta \in (0, 1)$ for any $i \in N$.

When the state and the public signal match, the probability of each individual voting correctly for the state is given by

$$r_A \equiv p + (1-p)(1-\beta), \quad (3.C.1)$$

and when the state and the public signal disagree, the probability of each individual voting correctly is

$$r_B \equiv (1-p) \times 0 + p\beta = p\beta. \quad (3.C.2)$$

To have $\beta^* \in (0, 1)$, we need any agent to be indifference when the two signals

¹⁰See Lemma 1 in Wit (1998) for a similar argument.

disagree:

$$w(B, A) = q(1-p) \binom{n-1}{\frac{n-1}{2}} r_A^{\frac{n-1}{2}} (1-r_A)^{\frac{n-1}{2}} - (1-q)p \binom{n-1}{\frac{n-1}{2}} r_B^{\frac{n-1}{2}} (1-r_B)^{\frac{n-1}{2}} = 0 \quad (3.C.3)$$

$$\Rightarrow \frac{1-p\beta}{1-\beta(1-p)} = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}} \quad (3.C.4)$$

$$\Rightarrow \beta^* = \frac{1-A(p, q, n)}{p-A(p, q, n)(1-p)}, \quad (3.C.5)$$

such that $A(p, q, n) = \left(\frac{q}{1-q}\right)^{\frac{2}{n-1}} \left(\frac{1-p}{p}\right)^{\frac{n+1}{n-1}}$. Thus when $\beta^* \in (0, 1)$ we obtain a mixed strategy equilibrium of the hybrid form ($\alpha^* = 1$).

Finally, solving $\beta^* = 0$ for q , we see that $\beta^* \in (0, 1)$ if and only if $q \in \left(p, \frac{\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}{1+\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}\right)$. The uniqueness follows from the fact that the left hand side of (3.C.4) is strictly decreasing in β . \square

3.D Proof of Proposition 4

Proof. In what follows we will find $\alpha = v_i(A, A) = v_i(B, B)$ and $\beta = v_i(B, A) = v_i(A, B)$ that maximize the probability of the majority outcome matching the correct state. Conditional on the state $s = A$ and $\sigma_E = A$, let the ex ante probability of each agent voting for A be, from (3.C.1), $r_A \equiv p\alpha + (1-p)(1-\beta)$. Also from (3.C.2), conditional on the state $s = A$ and $\sigma_E = B$, let the probability of each agent voting for A be $r_B \equiv p\beta + (1-p)(1-\alpha)$. Using r_A and r_B , the ex ante probability $P(\alpha, \beta)$ that the majority decision matches the state can be written as

$$\begin{aligned} P(\alpha, \beta) &= \Pr[M = s|s] = \Pr[M = A|s = A]P[A] + \Pr[M = B|s = B]P[B] \\ &= \Pr[M = A|s = A]\frac{1}{2} + \Pr[M = B|s = B]\frac{1}{2} = \Pr[M = A|s = A] \\ &= \Pr[\sigma_E = A|s = A]\Pr[M = A, \sigma_E = A|s = A] \\ &\quad + \Pr[\sigma_E = B|s = A]\Pr[M = A, \sigma_E = B|s = A] \\ &= q \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} r_A^k (1-r_A)^{n-k} + (1-q) \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} r_B^k (1-r_B)^{n-k}. \end{aligned} \quad (3.D.1)$$

Note that for

$$g(x) \equiv \sum_{k=\frac{n+1}{2}}^n \binom{n}{k} x^k (1-x)^{n-k}$$

we have

$$\frac{dg(x)}{dx} = n \binom{n-1}{\frac{n-1}{2}} (x(1-x))^{\frac{n-1}{2}}.$$

Partially differentiating (3.D.1) with respect to α and β , we obtain

$$\begin{aligned} \frac{\partial P(\alpha, \beta)}{\partial \alpha} &= npq \binom{n-1}{\frac{n-1}{2}} (r_A(1-r_A))^{\frac{n-1}{2}} \\ &\quad - n(1-p)(1-q) \binom{n-1}{\frac{n-1}{2}} (r_B(1-r_B))^{\frac{n-1}{2}} \end{aligned} \quad (3.D.2)$$

and

$$\begin{aligned} \frac{\partial P(\alpha, \beta)}{\partial \beta} &= -(1-p)nq \binom{n-1}{\frac{n-1}{2}} (r_A(1-r_A))^{\frac{n-1}{2}} \\ &\quad + pn(1-q) \binom{n-1}{\frac{n-1}{2}} (r_B(1-r_B))^{\frac{n-1}{2}}. \end{aligned} \quad (3.D.3)$$

From (3.D.3), taking the first order condition with respect β we have

$$\frac{\partial P(\alpha, \beta)}{\partial \beta} = 0 \Leftrightarrow \left(\frac{r_B(1-r_B)}{r_A(1-r_A)} \right)^{\frac{n-1}{2}} = \frac{q(1-p)}{(1-q)p}. \quad (3.D.4)$$

If (3.D.4) holds, then the derivative with respect to α , (3.D.2), is strictly positive for any $\alpha \in [0, 1]$ since

$$\begin{aligned} \frac{\partial P(\alpha, \beta)}{\partial \alpha} > 0 &\Leftrightarrow \frac{qp}{(1-q)(1-p)} > \left(\frac{r_B(1-r_B)}{r_A(1-r_A)} \right)^{\frac{n-1}{2}} \\ &\Leftrightarrow \frac{qp}{(1-q)(1-p)} > \frac{q(1-p)}{(1-q)p} \\ &\Leftrightarrow p > \frac{1}{2}. \end{aligned}$$

Therefore we have a unique corner solution for α , namely $\alpha = 1$, which coincides with the equilibrium α^* in the hybrid mixed strategy identified in Proposition 3. Note that the first order condition (3.D.3) and the indifference condition for the mixed strategy equilibrium (3.C.3) also coincide. Thus $\beta = \beta^*$ satisfies the first order condition.

It remains to show that the second order condition for the maximization with respect to β is satisfied. Since $P(\alpha, \beta)$ is a polynomial it suffices to show that

$$\begin{aligned} \frac{\partial^2 P(\alpha, \beta)}{\partial \beta^2} < 0 &\Rightarrow -(1-p)nq \binom{n-1}{\frac{n-1}{2}} (r_A(1-r_A))^{\frac{n-3}{2}} (1-p-2\beta(1-p)^2) \\ &< pn(1-q) \binom{n-1}{\frac{n-1}{2}} (r_B(1-r_B))^{\frac{n-3}{2}} (p-2\beta p^2). \end{aligned} \quad (3.D.5)$$

At $\beta = \beta^*$, (3.D.5) reduces to

$$(1-p\beta)(1-2(1-p)\beta) > (1-2p\beta)(1-(1-p)\beta),$$

which holds since $p > \frac{1}{2}$. Since $P(\alpha, \beta)$ is a continuously differentiable function

on a closed interval, the local maximum at $\{\alpha, \beta\} = \{1, \beta^*\}$ is also the global maximum. \square

3.E Proof of Proposition 6

Proof. Let us consider first the m obedient agents, assuming the rest always vote according to the private signal. In order for them to ignore their private signal when the two signals disagree, we have to have $w^{\text{obedient}}(B, A) \geq 0$ for those agents. If such an agent is pivotal given the strategy profile, among the $n - m$ individually informative agents, $(n - 1)/2$ of them must have received the private signal that disagrees with the public signal, while $(n - 1)/2 - (m - 1)$ of them must have received the private signal that agrees with the public signal. Therefore, for the obedient agent not to deviate when the two signals he has received disagree, it has to be that

$$\begin{aligned}
w^{\text{obedient}}(B, A) \geq 0 &\Rightarrow q(1 - p) \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1 - p)^{\frac{n-1}{2}} \\
&\quad - (1 - q)p \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2} - (m-1)} \geq 0 \\
&\Rightarrow q(1 - p)p^{-(m-1)} \geq (1 - q)p(1 - p)^{-(m-1)} \\
&\Rightarrow m \leq \frac{\ln[q] - \ln[1 - q]}{\ln[p] - \ln[1 - p]}. \tag{3.E.1}
\end{aligned}$$

In other words, in order for the obedient agent not to deviate, the number of obedient agents cannot be too large. Given (3.E.1), if the public and private signals agree, the obedient agent votes according to the signals because

$$\begin{aligned}
w^{\text{obedient}}(A, A) &= qp \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1 - p)^{\frac{n-1}{2}} \\
&\quad - (1 - q)(1 - p) \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2} - (m-1)} \\
&> q(1 - p) \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1 - p)^{\frac{n-1}{2}} \\
&\quad - (1 - q)p \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2} - (m-1)} \\
&= w^{\text{obedient}}(B, A) \geq 0.
\end{aligned}$$

The strict inequality follows from $w^{\text{obedient}}(B, A) \geq 0$ and $p > 1/2$.

Next, let us consider the $n - m$ individually informative agents who always follow their private signal regardless of the public signal. If an agent in this group is pivotal, it has to be that $(n - 1)/2$ of them have received the private signal that disagrees with the public signal, while $(n - 1)/2 - m$ of them have received the private signal that agrees with the public signal. Therefore, in order for the individually informative agent not to deviate when the two signals he has received

disagree, it has to be that

$$\begin{aligned}
w^{\text{ind. informative}}(B, A) < 0 &\Rightarrow q(1-p) \binom{n-m}{\frac{n-1}{2}} p^{\frac{n-1}{2}-m} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)p \binom{n-m}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}-m} < 0 \\
&\Rightarrow q(1-p)p^{-m} < (1-q)p(1-p)^{-m} \\
&\Rightarrow m > \frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]} - 1. \tag{3.E.2}
\end{aligned}$$

In other words, in order for the individually informative agent not to deviate, the number of obedient agents cannot be too small. Given (3.E.2), if the public and private signals agree, the agent indeed votes according to the signals because

$$\begin{aligned}
w^{\text{ind. informative}}(A, A) &= (1-q)(1-p) \binom{n-(m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}-(m-1)} (1-p)^{\frac{n-1}{2}} \\
&\quad - qp \binom{n-(m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}-(m-1)} \\
&< q(1-p) \binom{n-(m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}-(m-1)} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)p \binom{n-(m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}-(m-1)} \\
&= w^{\text{ind. informative}}(B, A) < 0.
\end{aligned}$$

The strict inequality follows from $w^{\text{ind. informative}}(B, A) \geq 0$ and $p > 1/2$.

In equilibrium both (3.E.1) and (3.E.2) have to be satisfied, which gives the unique m^* in \hat{M} as long as $m^* < (n+1)/2$. If $m^* = (n+1)/2$ then the majority decision always coincides with the public signal, and hence any $m \geq (n+1)/2$ in M is an equilibrium. Suppose $m^* > (n+1)/2$. In this case $m = (n+1)/2$ is also an equilibrium since none of the ‘‘individually informative’’ agents are pivotal and no ‘‘obedient’’ agent deviates as (3.E.1) is satisfied, which implies any $m \geq (n+1)/2$ in M is an equilibrium. \square

3.F Proof of Proposition 7

Proof. Nitzan and Paroush (1982) gave the optimal weighted majority rule with i) non-strategic sincere voting; ii) agents each of whom observes a private signal with different accuracy; and iii) no public signal. Let us consider the special case of their setup where the signal of one ‘‘expert’’ agent has the accuracy of q and those of $n-1$ ‘‘non-expert’’ agents have the same accuracy of $p \in (0, 1)$, where $q > p$. In what follows we show that the optimal rule in their model and the asymmetric pure strategy equilibrium in \hat{M} are isomorphic in terms of efficiency.

Theorem 1 in Nitzan and Paroush (1982) implies that, in the unique optimal majority rule, the expert has $\ln[q] - \ln[1-q]$ votes, while each non-expert agent

has $\ln[p] - \ln[1-p]$ votes.¹¹ Equivalently, dividing the weights by $\ln[p] - \ln[1-p]$, the expert should have $\frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]}$ votes if every non-expert is to have one vote. Moreover, under this optimal rule, removing the votes of \bar{m} randomly chosen non-experts ex ante does not affect the ex ante expected welfare, where \bar{m} is defined as the largest integer that satisfies $m \leq \frac{\ln[q] - \ln[1-q]}{\ln[p] - \ln[1-p]}$ (Corollary 1 in [Nitzan and Paroush, 1982](#)). This is because the ex ante influence of their votes on the expected welfare is cancelled by the increased votes of the expert. It is straightforward to see that the same efficiency is implemented by the majority rule where ex ante the expert has \bar{m} votes, each of $n - \bar{m}$ non-experts has one vote, and \bar{m} non-experts have no vote. Clearly we have $\bar{m} = m^*$. Therefore m^* uniquely maximizes the expected welfare in M .

Moreover, m^* in M the unique maximizer of welfare in the set of entire strategy profiles Γ . Note that [Nitzan and Paroush \(1982\)](#) allow $p < 1/2$ with sincere voting, which is equivalent to allowing agents to vote against their signal in our setting where $p > 1/2$. Also, uninformative voting by any agent clearly reduces efficiency relative to the optimal weights and thus cannot be part of the optimal rule. For any mixed strategy profiles, suppose that, without loss of generality, before observing the two signals, each agent individually decides which alternative to vote for, conditional on each combination of the two signals, according to his mixing probability. After these “interim decisions” but before the agents receive the signals, we can compute the expected welfare for each combination of their “interim decisions”. The profiles of their “interim decisions” and their expected welfare coincide with those of the optimal rule only with probability less than 1. Hence the asymmetric pure strategy equilibrium with m^* achieves the highest ex ante expected welfare over the set of all strategy profiles Γ . \square

3.G Deriving the asymmetric pure equilibrium

Any agent, in equilibrium, will consider the difference in expected utility between voting for A and B , conditional on her signal, the expert’s signal and the event that she is pivotal. We define this difference as:

$$w(\sigma_i, \sigma_E) = E[u_i(A, s) - u_i(B, s) | piv(v_{-i}), \sigma_i, \sigma_E] P[piv(v_{-i}), \sigma_i, \sigma_E] \quad (3.G.1)$$

$$\begin{aligned} &= \frac{1}{2} P[\sigma_E | s = A] P[\sigma_i | s = A] P[piv(v_{-i}) | s = A] \\ &\quad - \frac{1}{2} P[\sigma_E | s = B] P[\sigma_i | s = B] P[piv(v_{-i}) | s = B] \end{aligned} \quad (3.G.2)$$

3.G.1 Obedient Voters

Let us consider first the agents that follow the expert opinion, regardless of their individual signal. Following (2), they would follow the expert’s opinion, even when their individual signal suggests otherwise, if $w(B, A) \geq 0$. These agents

¹¹Here we allow non-integer votes and the majority decision is the alternative that received more votes.

are going to be pivotal when, excluding the rest $m - 1$ agents that always follow the expert, $\frac{n-1}{2}$ agents vote in disagreement with the expert while $\frac{n-1}{2} - (m - 1)$ agents vote in agreement with the expert. Therefore, from (2) we have that:

$$\begin{aligned}
w(B, A) \geq 0 &\Rightarrow q(1-p) \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)p \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - (m-1)} \geq 0 \\
&\Rightarrow q(1-p)p^{-(m-1)} \geq (1-q)p(1-p)^{-(m-1)} \\
&\Rightarrow m \leq \frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)}.
\end{aligned}$$

Finally, note that for agents agreeing with the expert, it is the case that:

$$\begin{aligned}
w(A, A) &= qp \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)(1-p) \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - (m-1)} \\
&> q(1-p) \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)p \binom{n - (m-1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - (m-1)} \\
&= w(B, A) \geq 0.
\end{aligned}$$

3.G.2 Informative Voters

On the other hand, for the $n - m$ agents that vote informatively, it must be the case that $w(B, A) < 0$. These agents are going to be pivotal when, excluding the rest m agents that always follow the expert, $\frac{n-1}{2}$ agents vote in disagreement with the expert while $\frac{n-1}{2} - m$ agents vote in agreement with the expert. Therefore, from (2) we have that:

$$\begin{aligned}
w(B, A) < 0 &\Rightarrow q(1-p) \binom{n - m}{\frac{n-1}{2}} p^{\frac{n-1}{2} - m} (1-p)^{\frac{n-1}{2}} \\
&\quad - (1-q)p \binom{n - m}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2} - m} < 0 \\
&\Rightarrow q(1-p)p^{-m} < (1-q)p(1-p)^{-m} \\
&\Rightarrow m > \frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)} - 1.
\end{aligned}$$

Finally, note that for agents agreeing with the expert, it is the case that:

$$\begin{aligned}
w(B, B) &= (1 - q)(1 - p) \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1 - p)^{\frac{n-1}{2}} \\
&\quad - qp \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2} - (m-1)} \\
&< q(1 - p) \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2} - (m-1)} (1 - p)^{\frac{n-1}{2}} \\
&\quad - (1 - q)p \binom{n - (m - 1)}{\frac{n-1}{2}} p^{\frac{n-1}{2}} (1 - p)^{\frac{n-1}{2} - (m-1)} \\
&= w(B, A) < 0.
\end{aligned}$$

Therefore, for $\frac{n-1}{2} > \max(m, m-1) = m$, there exists a *unique* asymmetric equilibrium, where m^* agents follow expert information, and the rest vote according to their individual signal, where

$$m^* \in \mathbb{N} \cap \left(\frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)} - 1, \frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)} \right]. \quad (3.G.3)$$

3.G.3 Upper bound on q

There exists an upper bound on q: since

$$\frac{n-1}{2} > m > \frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)} - 1,$$

it has to be the case that

$$\begin{aligned}
\frac{n+1}{2} &> \frac{\ln(q) - \ln(1-q)}{\ln(p) - \ln(1-p)} \Rightarrow \\
\frac{q}{1-q} &< e^{\frac{n+1}{2}} \frac{p}{1-p} \Rightarrow \\
q &< \frac{e^{\frac{n+1}{2}} \frac{p}{1-p}}{1 + e^{\frac{n+1}{2}} \frac{p}{1-p}}.
\end{aligned}$$

Also note that, similarly to the symmetric mixed equilibrium, for the upper bound on q we have that

$$\lim_{n \rightarrow +\infty} \frac{e^{\frac{n+1}{2}} \frac{p}{1-p}}{1 + e^{\frac{n+1}{2}} \frac{p}{1-p}} = 1.$$

and it is increasing in n .

Finally note that, relative to the upper bound for the mixed strategy equilibrium we have that

$$\begin{aligned} \frac{e^{\frac{n+1}{2}} \frac{p}{1-p}}{1 + e^{\frac{n+1}{2}} \frac{p}{1-p}} &< \frac{\left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}}{1 + \left(\frac{p}{1-p}\right)^{\frac{n+1}{2}}} \Rightarrow \\ e^{\frac{n+1}{2}} &< \left(\frac{p}{1-p}\right)^{\frac{n-1}{2}} \Rightarrow \\ n &> \frac{\ln p - \ln(1-p) + 1}{\ln p - \ln(1-p) - 1}. \end{aligned}$$

Chapter 4

Expert Information and Majority Decisions: Experiment¹

“Dont tell people how to do things, tell them what to do and let them surprise you with their results.” (George S. Patton)

4.1 Introduction

Motivated by the theoretical results of the previous chapter, i.e. the possibility that expert information can enhance or diminish the efficiency of equilibrium committee decisions, we conducted a laboratory experiment to study the effect of expert information on voting behaviour and majority decisions. Of particular interest is to see whether voters can play an efficient equilibrium, not least because the efficient equilibria seem to require sophisticated coordination among voters. Specifically, we set the accuracies of the signals in such a way that the expert signal is more accurate than each voter’s private signal but less accurate than what the aggregation of the private signals can achieve by sincere voting without the expert signal. Such parameter values seem plausible in that the expert opinion should be taken into account but should not be decisive on its own. At the same time, they entail the possibility that expert information may indeed be welfare reducing because if more than a half of the voters follow the expert obediently.

This chapter reports the results from the experiment. We found that the voters followed the expert signal much more than they should in the efficient equilibria. Strikingly, the majority decisions followed the expert signal most of the time, which is consistent with the class of obedient equilibria mentioned above. Another interesting finding is the marked heterogeneity in voting behaviour. While there were voters who consistently followed their private signal and ignored the public signal, a significant portion of voters followed the expert signal most of the time. We will argue that the voters’ behaviour in our data can be best described as that in an obedient equilibrium where a supermajority (and hence the decision) always follow the expert signal so that no voter is pivotal.

¹This chapter was based on results from the 2013 working paper “Expert Information and Majority Decisions”, co-authored by Dr. Kohei Kawamura.

Even if the committees in the laboratory followed expert information most of the time, this does not necessarily imply that introducing expert information is harmful, because the voters may not play the (efficiency maximizing) equilibrium strategy of sincere voting in the absence of expert information. Along with the treatments with both private and expert information, we also ran control treatments where the voters received a private signal only, in order to compare the observed efficiency of the committee decisions with and without expert information. We found that for seven-person committees the difference in the efficiency between the treatment and the control is insignificant, largely due to non-equilibrium behaviour (i.e., voting against private information) in the control treatment which reduced the benchmark efficiency. However, for fifteen-person committees, those without expert information performed much better than those with expert information and the difference is significant, suggesting that expert information may indeed be harmful. This result comes from the relatively high efficiency achieved by the fifteen-person committees without expert information, although they also exhibited some non-equilibrium behaviour.

Our theoretical and experimental results suggest that, from the viewpoint of a social planner who decides whether to and how to provide a committee with expert information, creating an equilibrium with higher efficiency does not necessarily mean it is selected among other equilibria, and in particular there is a possibility that provision of public information may lead to an inefficient equilibrium being played.² This concern seems particularly relevant when an inefficient equilibrium is simple and intuitive to play, like the obedient equilibrium in our model, while the efficient equilibrium requires subtle coordination. A natural solution to this problem would be to rule out inefficient equilibria, if possible. In our model, if the expert information is revealed only to a small subset of voters, the obedient equilibrium where a supermajority always follow the expert can be ruled out. Moreover, if the size of the subset is optimally chosen, there will be a simple and efficient equilibrium, where this subset of the voters receive and vote according to the expert signal, and the others who do not receive the expert information vote according to their own private signal. Intuitively, such selective disclosure prevents an expert from having too much influence. Alternatively, if an expert opinion is heard by all members, a coordination procedure such as role assignment (e.g., who should follow the expert information and who should ignore it) may lead to an efficient equilibrium. A contribution of this chapter in this regard is to demonstrate that, without coordination device, an efficient equilibrium may not necessarily be played even in a game of common interest especially when there is a simple but inefficient equilibrium. [Battaglini, Morton, and Palfrey \(2010\)](#) and [Morton and Tyran \(2011\)](#) report results from experiments where voters are asymmetrically informed, to study how the quality of the private signal affects their decision to abstain, in the spirit of the model of [Feddersen and Pesendorfer \(1996\)](#).³ The quality of the information each voter has in our

²As in standard models of voting, our model also has equilibria that are implausible from the view point of application and efficiency, such as uninformative equilibrium where all committee members vote for a particular option regardless of their private signal, and equilibrium where all members the vote against the expert signal.

³[Bhattacharya, Duffy, and Kim \(2013\)](#) study a related experimental setup but with costly

framework also varies according to whether the private and expert signals agree, in which case they provide strong information about the state; or they disagree, in which case the uncertainty about the state becomes relatively high. However, we do not allow voters to abstain, and more importantly our primary interest is in the combination of private and public information, which is fundamentally different from private information with different accuracy levels in terms of the effect on the voters' strategic choice, not least because the public signal in our framework represents a perfectly correlated component of the information each voter has.

Our interest in choices in the laboratory in the presence of multiple equilibria with different efficiency levels is related to the literature on the experiments for market entry games (e.g., Sundali, Rapoport, and Seale, 1995; Erev and Rapoport, 1998; Rapoport, Seale, and Winter, 2002; and Duffy and Hopkins, 2005) with a particular emphasis on learning to play an equilibrium. They have observed that the convergence to an equilibrium, if it occurs, does not necessarily mean a Pareto efficient equilibrium being played.⁴

4.2 Experimental Design

So far we have seen that the introduction of expert information into a committee leads to multiple responsive equilibria, while ruling out the individually informative equilibrium. On one hand, we have derived equilibria where such public information is used to enhance efficiency. They require either mixing or a fixed number of agents following the public signal regardless of their private signal. On the other hand, however, there are equilibria where the outcome always follows the public signal so that the CJT fails and the decision making efficiency may be reduced relative to the sincere voting equilibrium in the absence of expert information. Despite the (potentially severe) inefficiency, these equilibria seem simple to play and require very little coordination among agents.

In order to examine which equilibria best describe how people respond to expert information in collective decision making, we use a controlled laboratory experiment to collect data on voting behaviour when voters are given two types of information, private and public. The experiment was conducted through computers at the Behavioural Laboratory at the University of Edinburgh.⁵ We ran four treatments, each of which had three sessions, in order to vary committee size and whether or not the subjects receive public information. The variations were introduced across treatments rather than within because, as we will see shortly, we had to let our subjects play over relatively many periods, in order to ensure that for each setup the subjects have enough (random) occurrences where the private and public signals disagree. Each treatment involved either private information only or both private and public information, and each session consisted of either

voting.

⁴Kuzmics, Palfrey, and Rogers, 2013 found that in a symmetric repeated game, experimental subjects achieved efficient equilibrium payoffs through simple strategies while other equilibrium strategies could also have achieved very similar payoffs.

⁵The experiment was programmed using z-Tree (Fischbacher, 2007).

Table 4.2.1: Treatments

Treatment	Private signal	Public signal	Comm. size	No. of sessions	No. of subjects
1	yes	yes	7	3	$7 \times 2 \times 3 = 42$
2	yes	yes	15	3	$15 \times 3 = 45$
3	yes	no	7	3	$7 \times 2 \times 3 = 42$
4	yes	no	15	3	$15 \times 3 = 45$

two seven-person committees or one fifteen-person committee. The committees made simple majority decisions for a binary state, namely which box (blue or yellow) contains a prize randomly placed before the subjects receive their signals (see Table 4.2.1). The instructions were neutral with respect to the two types of information: private information was literally referred to as “private information” and expert information was referred to as “public information”. After the instructions were given, the subjects were allowed to proceed to the voting game only if they gave correct answers to all short-answer questions about the instructions.

For all treatments, the prior on the state was uniform and independent in each period, and we set the accuracy of each private signal (blue or yellow) at $p = 0.65$ throughout. For the treatments with a public signal (also blue or yellow) we had $q = 0.7$. We presented the accuracy of the signals in percentage terms, which was described by referring to a twenty-sided dice in order to facilitate the understanding by the subjects who may not necessarily be familiar with percentage representation of uncertainty.⁶

The predicted efficiency of seven-person committees with private signals only is $P_C(0.65, 7) = 0.8002$ and that of fifteen-person committees is $P_C(0.65, 15) = 0.8868$. Thus the accuracy of the public information is above each private signal but below what the committees can collectively achieve by aggregating their private information. This implies that the obedient equilibrium is less efficient than the informative equilibrium without public information. Note that the symmetric mixed and asymmetric pure equilibria we saw earlier for committees with expert information achieve higher efficiency than $P_C(\cdot, \cdot)$ (see Corollaries 4 and 5), although the margins are small under the parameter values here. Specifically, the predicted efficiency of seven-person committees with expert information is 0.8027 and 0.8119 in the symmetric pure equilibrium, and the predicted efficiency of fifteen-person committees is 0.8878 in the symmetric mixed equilibrium and 0.8922 in the asymmetric pure equilibrium.

Note that from the theoretical viewpoint, the subjects in the treatments with both types of information would have had a non-trivial decision to make when their private and public signals disagree. Otherwise (when the two signals agree), they should vote according to these signals in any of the three equilibria we are concerned with. Since the probability of receiving disagreeing signals is only 0.44 ($= 0.7 \times 0.35 + 0.3 \times 0.65$), the voting game was run for sixty periods to make sure each subject has enough occurrences of disagreement. In every treatment the

⁶Every subject was given a real twenty-sided dice.

sixty periods of the respective voting game were preceded by another ten periods of the voting game with only private signals, in order to increase the complexity of information in two steps for the subjects in the public information treatments.⁷ We do not use data from the first ten periods of the treatments without public signals, but it does not alter our results qualitatively.

After all subjects in a session cast their vote for each period, they were presented with a feedback screen, which showed the true state, vote counts (how many voted for blue and yellow respectively) of the committee they belong to, and payoff for the period.⁸ The committee membership was fixed throughout each session.⁹ This is primarily to encourage, together with the feedback information, coordination towards an efficient equilibrium.

The actual method for producing the matrix of the signals subjects received during the experiment is what we have called a Totally Random Version (TRV). In the beginning of each round we have a random draw to determine which of the two states A or B is the correct one with equal probability (50%). In TRV, each signal, either expert or private, is an independent random draw from the uniform distribution ($s \sim U[0, 1]$). If the number of subjects of the game is n , we need $n + 1$ such draws for each round. For each subject's private information, if $s < 0.65$, then private information for this round will be correct and otherwise incorrect. For expert (common information), if $s < 0.7$, then expert information for this round will be correct and otherwise incorrect.

Obviously, with TRV disagreements with the expert vary over time. Therefore, we have to use a large number of rounds to ensure that the number of disagreements will be enough to enable appropriate statistical analysis for the results of the experiment. In the Appendix, we explain an alternative method for producing a matrix for signals for the game with expert information.

4.3 Experimental Results

In this section we present our experimental results. We first discuss the individual level data to consider the change and heterogeneity of the subjects' voting behaviour in the treatments with expert information. We then examine the majority decisions in those treatments and contrast them to the equilibrium predictions. Finally we compare the efficiency of the committee decisions in the treatments with expert information and that in the treatments without expert information.

⁷The subjects in the private information treatments played the same game for seventy periods but they were given a short break after the first ten periods, in order to make the main part (sixty periods) of all treatments closer.

⁸The feedback screen did not include the signals of the other agents or who voted for each colour. This is to capture the idea of private information and anonymous voting, and also to avoid information overload.

⁹In treatments for two seven-person committees, the membership was randomly assigned at the beginning of each session.

Table 4.3.1: Voting behaviour: subjects' choice and equilibrium predictions

		7-person committees			15-person committees		
		treatment with expert	efficient equilibrium sym.	asym.	treatment with expert	efficient equilibrium sym.	asym.
vote for private when signals disagree	overall	0.3501	0.9381	0.8571	0.3218	0.9745	0.9333
	first 30	0.3382			0.3074		
	last 30	0.3624			0.3373		
vote for signals in agreement	overall	0.9488	1	1	0.9521	1	1
	first 30	0.9523			0.9527		
	last 30	0.9454			0.9615		

4.3.1 Voter choices with expert information

Let us first examine voting behaviour in the game with expert information. On Table 4.3.1 we can observe immediately that, when the private and public signals disagree, the subjects voted against their private signals much more often than they would in the efficiency improving symmetric mixed and asymmetric pure equilibria.

As the informational advantage of the expert information over private information is not large (70% versus 65%), in the symmetric mixed equilibrium the agents should vote according to the private signal most of the time when the signals disagree (93.8% in the seven-person and 97.5% in the fifteen-person committees, respectively). Also, from Proposition 6 only one agent should be obedient to the expert in the asymmetric pure equilibrium for both seven- and fifteen-person voting games, which implies the frequency of voting for the private signal of 85.7% and 93.3%, respectively.

In the laboratory, by contrast, when the two signals disagreed the subjects voted against their private signal in favour of the expert signal for the majority of the time, in both seven-person and fifteen-person committees. The frequency of following their private signal was only 35.1% in the seven-person committees and 32.2% in the fifteen-person committees. This, together with the high frequency of voting according to agreeing signals which is close to 100%, implies a significant overall tendency to follow expert information both individually and collectively.

Before discussing the influence of expert information on the voting outcome, let us look at the heterogeneity and change in the subjects' voting behaviour within sessions. The most striking about the histograms in Figure 4.3.1 is that for both seven- and fifteen person treatments, when the two signals disagreed, the highest fraction of the subjects (11 out of 42 in seven-person committees; 13 out of 45 in fifteen-person committees) voted against the private signal always, or almost always ($b < 5\%$, where b is each subject's the frequency of voting for the private signal when the signals disagree). Apart from those extreme "followers" of expert information, the subjects' behaviour in terms of b is relatively dispersed, while the density is still somewhat higher towards the left. At the other extreme there were some subjects who consistently ignored expert information. Therefore there was significant subject heterogeneity, and the low overall frequency of following

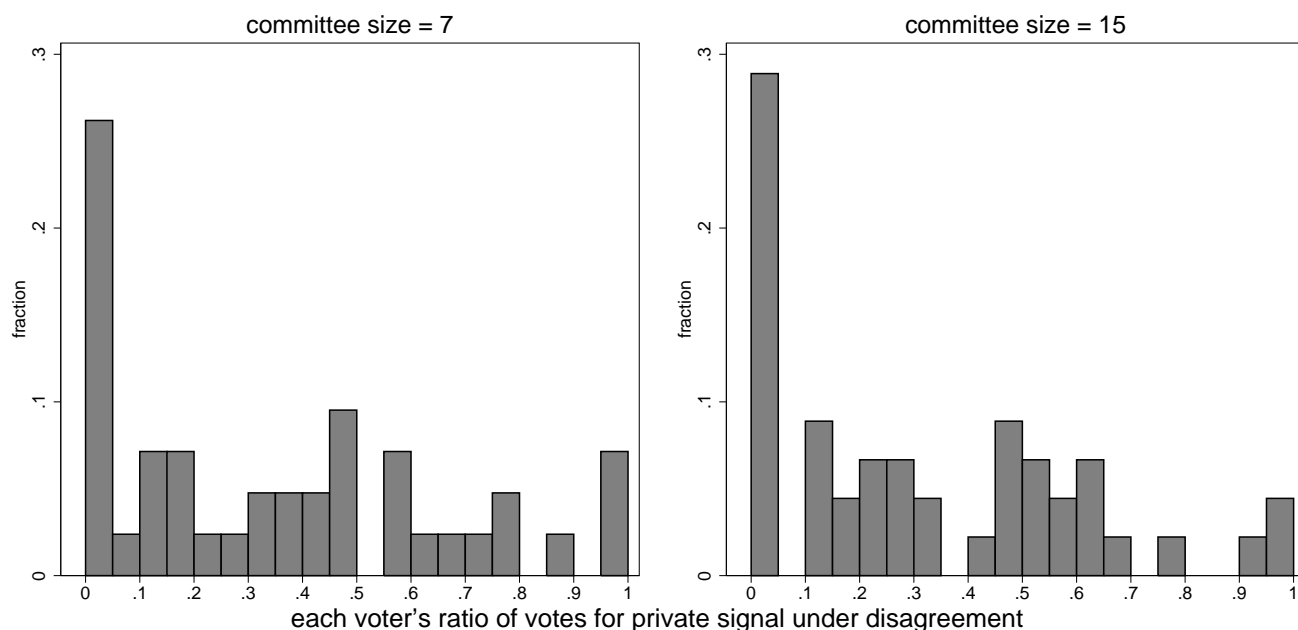


Figure 4.3.1: Distribution of individual voting behaviour

the private signal as documented in Table 4.3.1 was largely driven by extreme “followers”.

Figure 4.3.2 depicts the evolution of voting behaviour over periods of disagreement, where based on Figure 4.3.1 the subjects are divided into four behavioural types (with the bin width of 25%) according to how often they followed the private signal under disagreement, b . The number of subjects who belong to each category is in parentheses the legend of Figure 4.3.2. For example, in the seven-person (fifteen-person) committee treatment, 19 out of 42 (22 out of 45) subjects voted for the private signal under disagreement less than 25% of the time. We computed the ratio of agents who followed the private signal for each of the four types, according to the order of occurrences of receiving signals that disagreed.¹⁰ The thickness of the lines corresponds to the relative size of each quartile. Note that although the graphs are drawn over 25 periods, not every subject had 25 (or more) occurrences of disagreement since all signals were generated randomly and independently. In the seven-person committee treatment all subjects had 19 or more occurrences of disagreement, and in the fifteen-person committee treatment all subjects had 22 or more. The shaded areas indicate that not all subjects are included in computing the average voting behaviour under disagreement.

An interesting feature we observe in Figure 4.3.2 is that most subjects followed the public signal for the first few occurrences of disagreement. However, soon afterwards different types exhibited different voting patterns. In particular, the “unyielding” type of agents, who followed the private signal most often ($> 75\%$), quickly developed this distinct characteristic. It is as if there were a small number of subjects who “learnt” to ignore the public signal, in the face of the vast

¹⁰Thus the subjects had the first (second, third, etc.) occurrence of disagreement in different periods of the session.

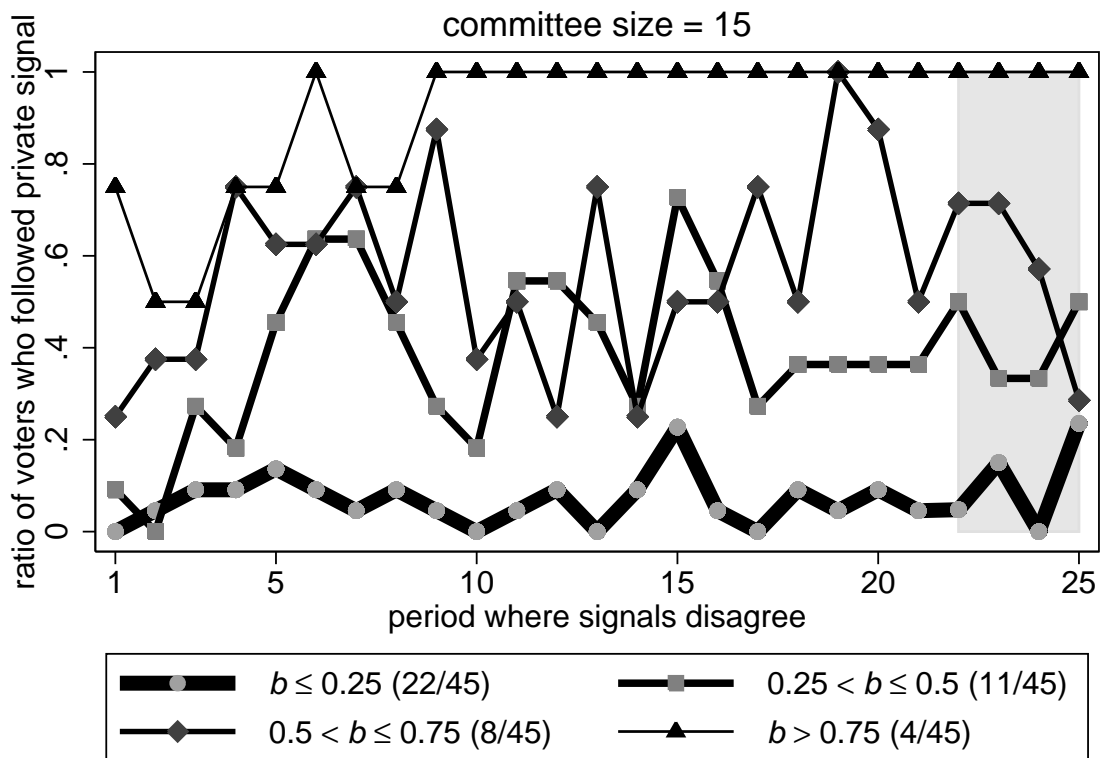
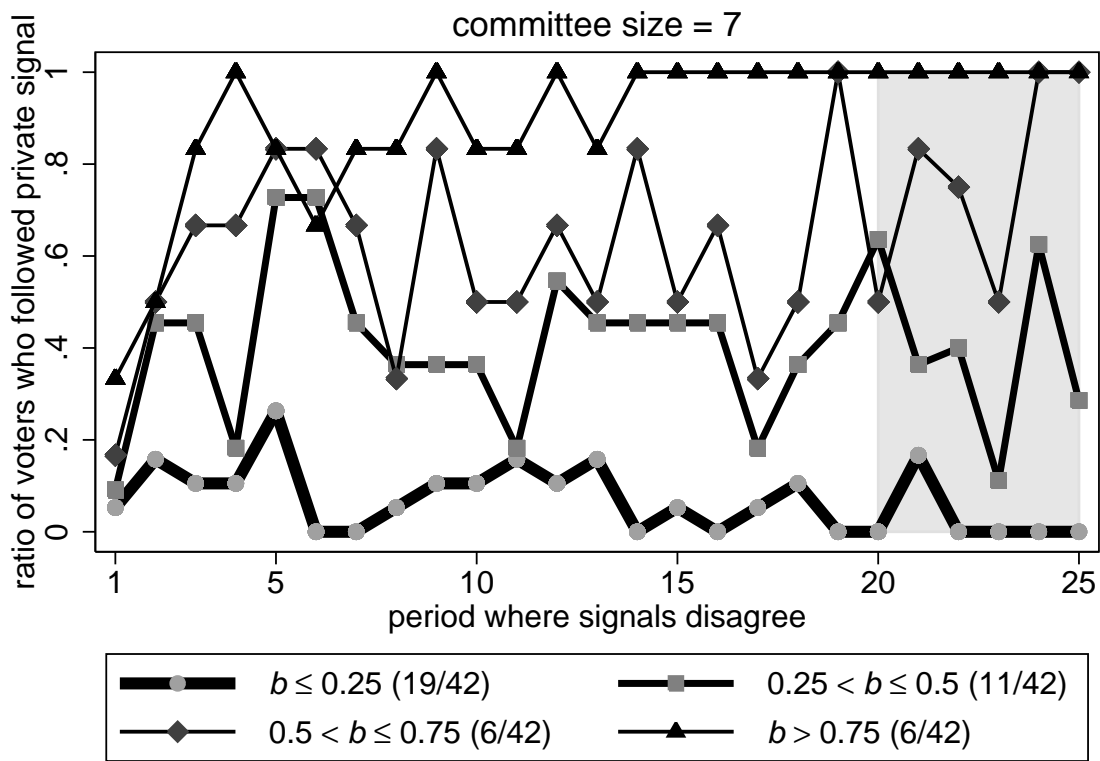


Figure 4.3.2: Change in average voting behaviour under disagreement for each agent type: b = individual frequency of voting for private signal when signals disagreed

Table 4.3.2: Random effects probit: dependent variable = 1 if voted for private signal under disagreement

	7-person comm. (1032 obs.)		15-person comm. (1173 obs.)	
Period of disagreement	-0.0066 (0.0067)	-0.0112 (0.0095)	0.0125** (0.0059)	0.0168** (0.0083)
Expert was correct in last disag. period	-0.2342** (0.1009)	-0.3595* (0.2112)	-0.5165*** (0.0960)	-0.3885* (0.1987)
Period of disagreement × Expert was correct in last disag. period		0.0092 (0.0135)		-0.0085 (0.0116)
Constant	-0.3557 (0.2355)	-0.2986 (0.25045)	-0.7486*** (0.2439)	-0.8117*** (0.2586)
Log likelihood	-483.7402	-483.5117	-533.0635	-532.7938

Note: Standard errors in parentheses.

*** significant at 1% level; ** significant at 5% level; * significant at 10% level

majority of the others already following it. At the other end, the behavioural pattern of the “obedient” type of agents, who followed the private signal least often ($\leq 25\%$), was relatively consistent across the occurrences of disagreement, with occasional voting for the private signal. The subjects who were in-between (frequency of voting for the private signal between 25% and 75%) started with voting for the public signal more often in the first few occurrences of disagreement but thereafter we do not observe a clear change in their voting behaviour over time. Overall, Figure 4.3.2 highlights the development of marked heterogeneity in voting behaviour that emerged through relatively early occurrences of disagreement. Moreover, the development does not show any clear sign of convergence to the strategies in the efficient asymmetric pure equilibrium identified earlier in Proposition 6.

Figure 4.3.2 suggests that most subjects changed the way they responded to disagreement as if they randomized. In order to see what potentially influenced voting behaviour while taking into account significant individual heterogeneity as observed earlier, we ran random effects probit regressions for the rounds where the two signals disagreed. Table 4.3.2 shows that the subjects were more likely to vote for the public signal (and against their own private signal) when the expert signal was correct (and the private signals was incorrect) in the previous occurrence of disagreement. Some subjects seem to have linked their choice to the observational accuracy of the expert signal, at least to some extent.

4.3.2 Committee decisions with expert information

Let us now consider the majority decisions of the committees in relation to the presence of the public signal, which are summarized in Table 4.3.3. A striking feature for both treatments is that the decisions followed the expert information most of the time (97.8% for the seven-person committees and 100% for the fifteen-person committees), while the predictions for the two efficient equilibria suggest only 67-72%. Moreover, the decisions in the laboratory were much more likely to have margins of two or more than the predictions. Also, when for any

Table 4.3.3: Majority decisions by committees with expert information

	7-person comm.			15-person comm.		
	treatment with expert (360 obs.)	efficient equilibrium sym.	efficient equilibrium asym.	treatment with expert (180 obs.)	efficient equilibrium sym.	efficient equilibrium asym.
Decision coincided with public signal	0.9778	0.6654	0.7237	1	0.6731	0.7023
Decisions made with margin ≥ 2	0.8583	0.5958	0.6000	1	0.7993	0.8033
of which followed public signal	1	0.6612	0.8143	1	0.6789	0.7396

decision that had a margin of two or more, the decision followed expert information. Those features are again far from the predictions of the efficient equilibria (see the last two rows of Table 4.3.3). If anything, as we will discuss shortly, the majority decisions exhibit key aspects of the hybrid obedient equilibrium we examined earlier.

4.3.3 Relation to equilibrium predictions

In Section 3.3 we considered three responsive equilibria of interest, namely the symmetric mixed, asymmetric pure, and obedient equilibria. In the literature on voting experiments, it is customary to use concepts such as quantal response equilibrium (QRE; McKelvey and Palfrey, 1995) and “equilibrium plus noise” (Blume, Duffy, and Franco, 2009) to see whether the experimental data on subjects’ actions can be interpreted as systematic deviation from a particular equilibrium prediction of interest. As discussed in detail in the following sections, it is very difficult for the QRE and equilibrium-plus-noise model to generate a sharp prediction for asymmetric equilibria of games with incomplete information like ours. Meanwhile, our subjects’ voting choices and committee decisions exhibit some essential properties of the hybrid obedient equilibrium.

Quantal Response Equilibria

To begin with, there are two main reasons why it would be difficult for the QRE model to produce a sharp prediction for our model. Firstly, the two distinct choices that subjects face when facing either agreeing or contradicting signals entail a very different level of sophistication in terms of individual decisions. However, as we will explain below any calibration under QRE would require an *analogous* degree of sophistication in the two choices. This is something that we have not observed in the data. Secondly, our data exhibit a highly heterogeneous nature that point out to shift our main focus on asymmetric equilibria.

To discuss these reasons in more detail, under any examination of the predictive power of the QRE model, we would have to calibrate on a triplet $\{\alpha, \beta, \lambda\}$ that is a solution of the following system of equations, as we saw in the previous

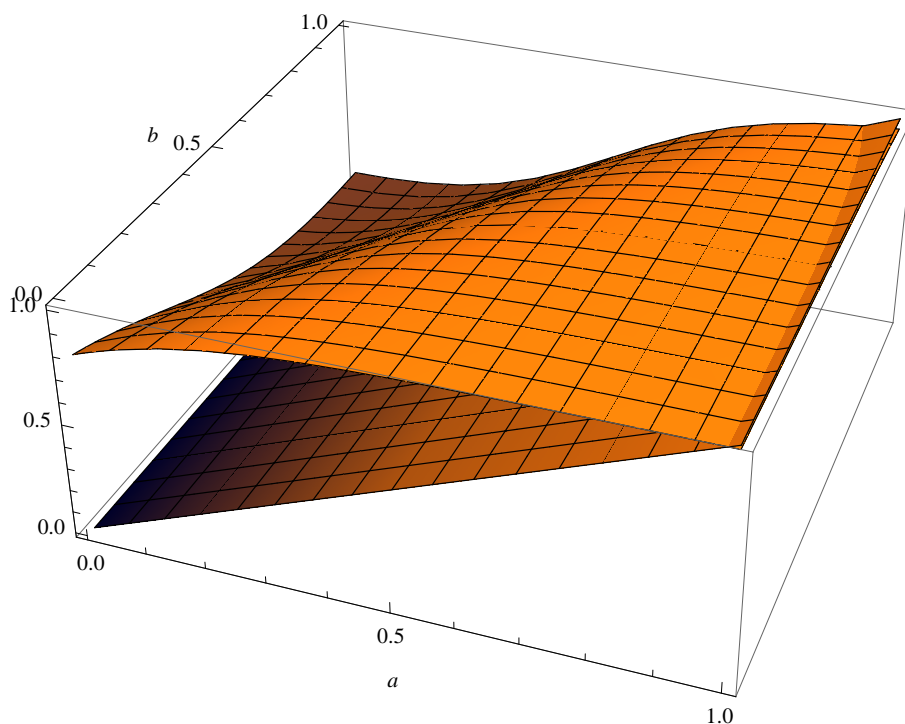


Figure 4.3.3: QRE plane for $\beta \in [0, 1]$ and $\lambda = 0.2$.

chapter:

$$\alpha = \frac{\exp(\lambda E[u_1(FCSA, \alpha, \beta)])}{\exp(\lambda E[u_1(ACSA, \alpha, \beta)]) + \exp(\lambda E[u_1(FCSA, \alpha, \beta)])},$$

$$\beta = \frac{\exp(\lambda E[u_1(IISD, \alpha, \beta)])}{\exp(\lambda E[u_1(IISD, \alpha, \beta)]) + \exp(\lambda E[u_1(OBDD, \alpha, \beta)])}.$$

Note that there has to be a unique λ that links agents mixing behaviour under agreeing (α) and disagreeing (β) signals through this system of equations. As a first step in order to visualize the relationship between α and β , we can use using numerical analysis to produce two graphs with a fixed $\lambda = 0.2$: one where the geometric locus of the solutions for α when $\beta \in [0, 1]$ is given as the intersection of the function on the right hand side of the first equation with the plane $f(\alpha) = \alpha$ (Figure 4.3.3) and one where the geometric locus of the solutions for β when $\alpha \in [0, 1]$ is given as the intersection of the function on the right hand side of the second equation with the plane $f(\beta) = \beta$ (Figure 4.3.4). Note that the fact that both geometric loci of solutions are monotone with β and α respectively guarantees a unique solution of the system of equations above for α and β , under a fixed λ . Moreover, we already know the monotonicity of the solutions with respect to λ . Therefore, we know that fixing λ would produce a unique solution for the two other variables of the triplet $\{\alpha, \beta, \lambda\}$. In other words, any calibration under QRE would require an *analogous* degree of sophistication in the choice under agreeing and disagreeing signals.

Turning to the experimental data, we can immediately see that the averages of the observations for β on the 7 and 15 player game are $\beta_{obs,7} = 0.3501$ and

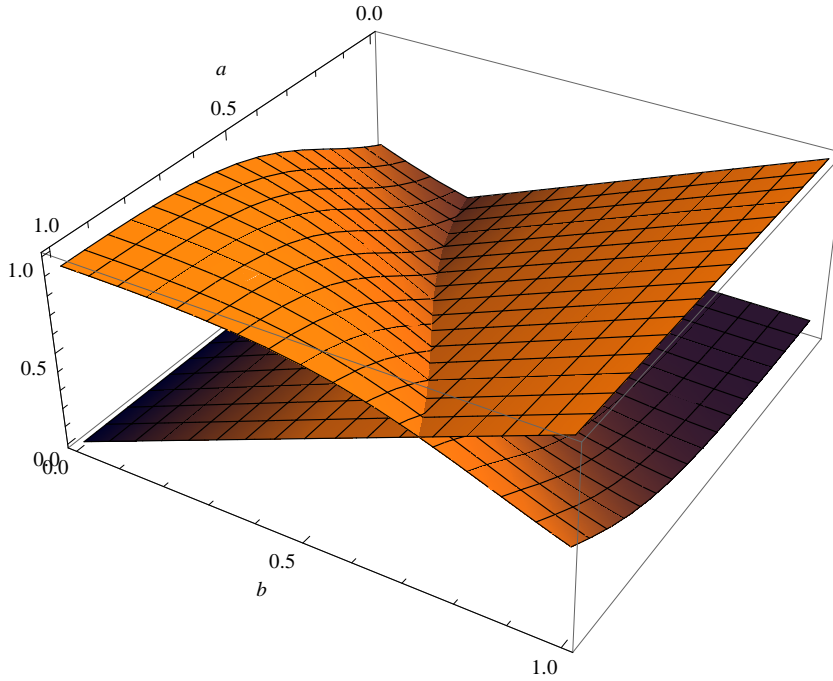


Figure 4.3.4: QRE plane for $\alpha \in [0, 1]$ and $\lambda = 0.2$.

$\beta_{obs,15} = 0.3218$. However, even for $\lambda \rightarrow \infty$, $\beta = 0.5$ is a lower bound for the choice of β . In other words, even the lowest degree of sophistication would not justify $\beta < 0.5$ through the QRE model. Moreover, using the averages of the observations $\alpha_{obs,7} = 0.9488$ and $\alpha_{obs,15} = 0.9521$ via numerical analysis we can produce the unique triplets of solutions the the QRE system above $\{\alpha, \beta, \lambda\} = \{0.945, 0.684, 0.130\}$ and $\{\alpha, \beta, \lambda\} = \{0.952, 0.739, 0.238\}$ for 7 and 15 agents respectively. This implies that the level of sophistication that would be required for the voting behaviour of subjects through the QRE model under agreement would predict the behaviour under disagreement with an error of +95% and +130% under the 7 and 15 player game respectively. Therefore, any attempt to calibrate on the QRE model would fail gravely.

Finally, as shown in Figures 4.3.1 and 4.3.2, our data exhibit a highly heterogeneous nature. More specifically, in Figure 4.3.1 we observe that apart from a very large concentration of obedient voters, all other choices in terms of each voter's ratio of votes for their private signal under disagreement are fairly uniformly distributed. This is unlike any predictions of the QRE model for a tendency to deviate that is relatively analogous to the distance from the expected utility benefit of the strategies selected. This observation, augmented by the fairly consistent behaviour through time that is seen in Figure 4.3.2, point out to shift our main focus on asymmetric equilibria.

Equilibrium plus noise

Following the “equilibrium plus noise” (Blume, Duffy, and Franco, 2009) model for calibrating our data entails difficulties similar to the ones for QRE mentioned in the previous section. Apart from the heterogeneity argument that is identical,

there is an even graver restriction that forbids the use of the equilibrium plus noise model, even if we assume a different degree of sophistication between the two decisions under either agreeing on contradicting signals.

More specifically, similarly to Blume, Duffy, and Franco (2009), in case of agreeing signals, let $\alpha(\eta_1)$ be the observed probability, that is a noisy perturbation between the equilibrium strategy $\alpha^* = 1$ and a totally random switching probability $\frac{1}{2}$, where $\eta_1 \in [0, 1]$ is the parameter of estimation via the maximum likelihood method. In case of disagreeing signals, let $\beta(\eta_2)$ be the observed probability, that is a noisy perturbation between the equilibrium strategy β^* and a totally random switching probability $\frac{1}{2}$, where $\eta_2 \in [0, 1]$. Then, the model to be estimated becomes:

$$\begin{aligned}\alpha(\eta_1) &= 1 - \eta_1, \\ \beta(\eta_2) &= \eta_2\beta^* + (1 - \eta_2)\frac{1}{2}.\end{aligned}$$

It is straightforward to observe that, even assuming a different degree of sophistication between the two actions, captured by $\eta_1 \neq \eta_2$, we necessarily have that

$$\beta(\eta_2) = \eta_2\beta^* + (1 - \eta_2)\frac{1}{2} > \frac{1}{2},$$

given that $\beta^* > \frac{1}{2}$ and $\eta_2 \in [0, 1]$. Hence, due to the fact that the equilibrium β in both 7 and 15 player cases is higher than $\frac{1}{2}$, the equilibrium plus noise model will necessarily give a prediction that is strictly higher than $\frac{1}{2}$. However, since the average observed beta on the 7 and 15 player game respectively is $\beta_{obs,7} = 0.3501$ and $\beta_{obs,15} = 0.3218$, and due to the concentration of observations as seen in Figure 4.3.1, it would be impossible for the equilibrium plus noise model to produce any significant predictions.

Therefore, similarly to the previous section all the aforementioned results concord to focus our analysis on asymmetric equilibria.

Hybrid Obedient Equilibria

Our subjects' voting choices and committee decisions exhibit some essential properties of the hybrid obedient equilibrium, where a supermajority ($(n + 1)/2 + 1$ or more agents) vote according to the expert signal and the other agents' strategies are arbitrary. The indeterminacy of the minority agents' strategies makes it difficult to establish a solid link between the prediction and the data, but in what follows we argue that the subjects' behaviour in our data is best construed as that in the hybrid obedient equilibrium.

First, as we have seen in Table 4.3.3, the committee decisions followed the expert signal most of the time (97.8% for seven-person committees and 100% for fifteen-person committees) as in the class of obedient equilibria where the decision follows the expert signal with probability 1. In the other equilibria, this rate ranges from 67% to 72%. The difference in the frequency between the predictions from the two efficient equilibria and the data is statistically significant.

Second, again from Table 4.3.3, most decisions were made with the margin of

two or more votes (85.8% of the time for seven-person committees and 100% for fifteen-person committees), which is an essential feature of the hybrid obedient equilibrium where no voter should be pivotal. The predicted frequency of the majority decisions having the margin of two or more in the efficient equilibria is about 60% for seven-person committees and 80% for fifteen-person committees.

Third, more importantly, most (by the seven-person committees) or all (by the fifteen-person committees) decisions made with the margin of two or more followed the expert signal, while in the efficient equilibria such decisions do not have to follow the expert signal (see the last row of Table 4.3.3). Therefore, from the subjects' perspective, it might well be that having looked at the feedback every period, they perceived themselves as playing an obedient equilibrium in the sense that they anticipated that the decision would (almost) always follow the expert signal, and moreover they would not be able to influence the outcome as they would not be pivotal.

Fourth, from the viewpoint of individual voting choices, the marked heterogeneity makes symmetric strategies less plausible (Figures 4.3.1 and 4.3.2). Also, while the asymmetric pure strategy equilibrium requires only one agent to follow the expert in every period, there were on average more (1.8 subjects in the seven-person committees and 4.3 in the fifteen-person committees) who followed the expert signal more than 95% of the time. Combined with the fact that the other subjects also frequently voted for the expert signal in the face of disagreement, the profiles of voting choices seem much closer to those predicted in the hybrid obedient equilibrium. Indeed, individual voting choices are largely consistent with its prediction, although it must be stressed that the arbitrariness of the equilibrium voting behaviour of a minority of agents makes it somewhat difficult to relate the model and data precisely.

Finally, Figure 4.3.2 shows no clear sign of “convergence” to either the efficient symmetric mixed or asymmetric pure equilibria. If anything, although the randomness of the combination of the two signals makes it very difficult to observe a long-run trend in the laboratory, from our data the voting pattern seems to have stabilized after several occurrences of disagreement in the manner closest to the hybrid obedient equilibrium as we have just discussed.

If we accept that an equilibrium was played (or approximated) and that the one played was the hybrid obedient equilibrium, then it implies that the subjects selected a less efficient equilibrium. Note that the efficient equilibria in our model may require substantive coordination among the agents, especially in the presence of underlying uncertainty in the state and two signals. The apparent simplicity of the obedient equilibrium might be the reason why it may have been chosen despite the inefficiency.

Another possibility is that the subjects may not have been playing any equilibrium strategies and some or many were following the expert information out of “irrationality”. Even if this was the case, however, the fact that the obedient outcome with a supermajority is in equilibrium must have made it “robust” than otherwise, since even rational players could not do effectively anything to improve the efficiency.

Table 4.3.4: Voting behaviour in committees without expert information

		7-person committees		15-person committees	
		treatment without expert (2520 obs.)	equilibrium	treatment without expert (2700 obs.)	equilibrium
vote for private signal	overall	0.8472	1	0.9141	1
	first 30	0.8505		0.9111	
	last 30	0.8437		0.9170	

Table 4.3.5: Observed efficiency

	7-person comm. (180 obs. each)		15-person comm. (360 obs. ea)	
	w/o expert	with expert	w/o expert	with expert
Observed efficiency	0.7000	0.7389	0.8278	0.6778
Fisher's exact test for difference	not significant ($p = 0.2809$)		significant ($p = 0.0000$)	
Observed efficiency of expert information	n/a	0.7222	n/a	0.6778
Hypoth. efficiency with individually informative voting	0.7972	0.8195	0.8778	0.8667

4.3.4 Efficiency comparison

Since the committee decisions mostly followed the expert signal, their efficiency is almost (in the case of fifteen person committees, completely) identical to that of the expert signal. If we posit that the subjects play the hybrid obedient equilibrium and that those in the treatments without expert information play the informative equilibrium by following each one's private signal, then from (8), in expectation we should observe the efficiency loss of $P_C(0.65, 7) - 0.7 = 0.1002$ (14.3% reduction) for the seven-person committees and $P_C(0.65, 15) - 0.7 = 0.1868$ (26.7% reduction) due to the *presence* of expert information.

In the laboratory, the subjects in the treatments without expert information did not necessarily play according to the equilibrium prediction of informative voting (Table 4.3.4). The deviation is more pronounced in the seven-person committees than in the fifteen-person committees, which is probably because subjects tended to deviate after observing the majority decision being wrong and indeed by construction (conditional on informative voting) the decisions are less likely to be correct in the seven-person committees (see Appendix for details). Note that, from each individual's perspective, one private signal is less informative of the true state than a pair of private and public signals that agree. We have observed in Table 4.3.1 that the proportion of votes for the agreeing signals was about 95% in both seven-person and fifteen-person committees, which is higher than the proportion of votes for the public signal when expert information is absent. This is consistent with, for example, the result from [Morton and Tyran \(2011\)](#) who found that the more accurate the information subjects receive, the more likely it is that they vote according to the information.

Since informative voting achieves the highest efficiency in the voting game without expert information, any deviation from the equilibrium strategy leads

to efficiency loss. The first row on Table 4.3.5 records the observed (ex post) efficiency in the four treatments. We can see that the efficiency of the decisions by the seven-person committees without expert information was merely 70.0%, while if they every member voted individually informatively following the equilibrium strategy, given the actual signal realizations in the treatment, they could achieve 79.7%. Meanwhile the seven-person committees with expert information achieved 73.9%, even though they could have achieved higher efficiency (82.0%) if they had adopted individually informative voting.¹¹ While the precise comparison of efficiency between the seven-person committees with and without expert information is difficult due to different signal realizations in each treatment, the difference in the observed efficiency is not statistically significant.

The last two columns of Table 4.3.5 give us a somewhat clearer picture. In the fifteen-person committees without expert information, since the agents did not deviate much from the equilibrium strategy of informative voting, the efficiency loss compared to the hypothetical informative voting was small (82.8% vs. 87.8%). In the fifteen-person committees with expert information, since all decisions followed the expert information, the efficiency was exactly the same as that of the expert signals, which was only 67.8%. Although the exact comparison is not possible due to different signal realizations in each treatment, the reduction in efficiency in the treatment with expert information is large (82.8% \rightarrow 67.8%, 22.1% reduction) and statistically significant.

4.4 Conclusion

In this chapter, we have reported on the laboratory experiment conducted to see how human subjects react to expert information. In particular we set the parameter values in such a way that the efficiency of the obedient equilibria (which is the as the accuracy of the expert signal) is lower than what the agents could have achieved in the sincere voting equilibrium without expert information. We found that the subjects followed expert information so frequently that most of the time the committee decisions were the same as what the expert signal indicated. This is in sharp contrast to the predictions from the efficient equilibria, where only a small number of agents should (in expectation) follow the expert signal and as a result the committee decision and expert signal may not necessarily coincide. We also found that the subjects' behaviour was highly heterogeneous. Moreover the heterogeneity was persistent over many periods and there was no clear sign of convergence to an efficient equilibrium. Given the outcome and the heterogeneity in voting behaviour among the subjects, we have argued that their choices can be best interpreted as *those in a hybrid obedient equilibrium*, where a supermajority follow expert information and the rest vote arbitrarily.

We have then contrasted the results to those from the control treatments where the subjects received private signals only. We found that the efficiency without

¹¹Note that individually informative voting is not an equilibrium strategy in the presence of expert information. Here we record the hypothetical efficiencies for both seven-person and fifteen-person committees in order to represent the quality of the realized private signals in each treatment.

expert information was significantly higher than the efficiency with expert information for fifteen-person committees. One interpretation of this result is that, the otherwise efficiency improving provision of expert information actually reduced efficiency, by creating an inefficient equilibrium that is simple to play compared to the efficient equilibria. The difference in efficiency was not significant for seven-person committees, largely due to the agents' frequent non-equilibrium behaviour in the treatment without expert information, which reduced the efficiency and made it close to the efficiency of the committee decisions in the treatment with expert information.

Finally, this chapter offers a potentially relevant “policy” implication. The optimality of the asymmetric pure strategy equilibrium suggests that it may be desirable for the expert to speak only to a subset of the members of a committee, unless his expertise q is overwhelmingly high. The number of members he should speak to is m^* as explicitly computed in Proposition 6. In this case, the outcome of the equilibrium where m^* members follow the expert and the rest follow the private signal is identical to the outcome of the asymmetric pure strategy equilibrium we have seen earlier, but this form of selective information revelation rules out equilibria that are less efficient, such as the symmetric pure strategy equilibrium and the obedient equilibrium. Alternatively, if an expert is heard by all members, there should be some coordination device such as role assignment in place to make sure that the expert will not have excessive influence on committee members. The results from our experiment suggest that it may not be adequate to study an efficient equilibrium especially when it requires subtle coordination among many players, as is often the case in decision making by voting under uncertainty.

Appendix

4.A Treatments without Expert Information

We use two treatments without expert information as controls, and those are also a direct test of the Condorcet jury. As we saw on Table 4.3.4 the frequency of our subjects voting according to their private signal was 84.7% in the seven-person committees and 91.4% in the fifteen-person committees. The main reason why the seven-person committees with expert information performed better than the seven-person committees without expert information, despite the fact that the outcome of the former approximated that of the inefficient obedient equilibrium, is that the subjects in the seven-person committees without expert information did not play according to the equilibrium and efficient strategy often enough. The difference in the frequency between the seven-person and fifteen-person committees without expert signal also is inconsistent with the notion of Quantal Response Equilibria, because according to QRE the agents' non-equilibrium behaviour (mistakes) should be more pronounced when the loss from a mistake is small, which implies we should expect to see the subjects voting according to the private information more often in the seven-person committees than in the fifteen-person committees.

The exact cause of the difference in the voting behaviour is difficult to determine, but Table 4.A.1 suggests that, at least in the seven-person committees, the subjects may have been “experimenting” with voting against their private signal especially after the committee decision in the previous period was incorrect. This type of experimentation would result in a larger proportion of votes for the private signal in *larger* committees, contrary to the prediction from QRE.

4.B Experimental Instructions

Thank you for agreeing to participate in the experiment. The purpose of this session is to study how people make group decisions. The experiment will last approximately 55 minutes. Please switch off your mobile phones. From now until the end of the session, no communication of any nature with any other participant is allowed. During the experiment we require your complete, undistracted attention. So we ask that you follow these instructions carefully. If you have any questions at any point, please raise your hand.

The experiment will be conducted through computer terminals. You can earn money in this experiment. The amount of money you earn depends on your

Table 4.A.1: Random effects probit: dependent variable = 1 if voted for private signal

	7-person comm. (2478 obs.)			15-person comm. (2655 obs.)		
Period	0.0003 (0.0022)	0.0003 (0.0022)	0.0002 (0.0022)	0.0004 (0.0025)	0.0004 (0.0025)	0.0005 (0.0025)
Correct group decision in last period	0.1213 (0.0793)	0.1129 (0.0800)	0.2512** (0.1260)	0.0850 (0.1115)	0.0874 (0.1118)	0.2259 (0.1711)
Correct signal in last period		0.0615 (0.0783)	0.2106 (0.1310)		-0.0258 (0.0910)	0.1664 (0.2035)
Correct signal in last period × Correct decision in last period			-0.2346 (0.1654)			-0.2400 (0.2276)
Constant	1.5223*** (0.2273)	1.4914*** (0.2308)	1.4235*** (0.2368)	2.1564*** (0.2723)	2.1697*** (0.2764)	2.0672*** (0.2923)
Log likelihood	-800.8878	-800.5792	-799.5695	-584.6741	-584.6338	-584.0777

Note: Standard errors in parenthesis.

*** significant at 1% level; ** significant at 5% level; * significant at 10% level

decisions, the decisions of other participants, and luck. All earnings will be paid to you immediately after the experiment. During the experiment, your payoff will be calculated in points. After the experiment, your payoff will be converted into British Pounds (GBP) according to the following exchange rate: 850 points = 1, and rounded to the nearest pound. Please remain seated after the experiment. You will be called up one by one according to your desk number. You will then receive your earnings and will be asked to sign a receipt.

All participants belong to a single group of fifteen¹² until the end of this experiment.

The experiment has two parts and consists of a total of 70 rounds. The first part of the experiment has 10 rounds, and the second part has 60 rounds.

At the beginning of each round, the computer places a prize in one of two virtual boxes: a blue box and a yellow box. [SHOW PICTURE ON FRONT SCREEN] The location of the prize for each round is determined by the computer via the toss of a fair coin: at the beginning of each round it is equally likely that the prize is placed in either box. That is, the prize is placed in the blue box 50% of the time and the prize is placed in the yellow box 50% of the time. You will not directly see in which box the prize is hidden, but as we will describe later you will receive some information about it. [SHOW PICTURE ON FRONT SCREEN] The box that does not contain the prize remains empty.

The group's task is to choose a colour. In every round, each group member has two options, either to vote for BLUE or YELLOW. [SHOW PICTURE ON FRONT SCREEN] The colour that has received the majority of the votes becomes the group decision for the round. In every round, each member of the group earns:

1. 100 points if the group decision matches the colour of the box that contains the prize;

¹²The instructions here are for the treatments with fifteen-person committees and expert information. The instructions for the other treatments are available on request.

2. 5 points if the group decision does not match the colour of the box that contains the prize.

Note that your payoff for each round is determined exclusively by the group decision. If the group decision is correct, every group member earns 100 points. If the group decision is incorrect, every group member earns 5 points. The payoff is independent of how a particular group member voted.

To summarize, each round proceeds as follows: [SHOW PREVIOUS PICTURES IN TURN]

1. the computer places a prize in one of two boxes (blue box or yellow box with equal chance);
2. each group member receives some information about the location of the box;
3. each group member votes for BLUE or YELLOW;
4. group decision is the colour that has received most votes;
5. each group member receives earnings according to the group decision and the actual location of the prize.

Consider the following example. Suppose you and six other member voted for BLUE and the eight other members voted for YELLOW. This means that the group decision is YELLOW.

If the prize was indeed placed in the yellow box, then each group member, including you, earns 100 points. On the other hand, if the prize was placed in the blue box, each group member, including you, earns 5 points.

The experiment is divided into two parts. Both parts follow what we have described so far, but they are different in terms of i) the information each group member receives before voting, and ii) the number of rounds.

Part 1

The first part of the experiment will take place over 10 rounds. In each round, after the prize is placed in one of the two boxes but before group members vote, each participant receives a single piece of information about the location of the prize. We will call this type of information Private Information. Private Information will be generated independently and revealed to each participant separately, and it can be different for different group members. No other participants of the experiment will see your Private Information. [SHOW SCREEN FOR DECISION]

Private Information is not 100% reliable in predicting the box containing the prize. Reliability refers to how often Private Information gives the correct colour of the box.

Specifically, Private Information gives each of you the colour of the box with the prize 65% of the time, and the colour of the empty box 35% of the time.

The reliability of Private Information can be described as follows:

1. In each round, after the prize is placed in one of the boxes, the computer rolls a fair 20-sided dice for each group member. A real 20-sided dice is on your desk to help your understanding.

2.

a. If the result of the dice roll is 1 to 13 (1,2,4,5,6,7,8,9,10,11,12 or 13), then that member's Private Information is the colour of the box with the prize. Note that 13 out of 20 times means 65%.

b. If the result of the dice roll is 14 to 20 (14,15,16,17,18,19 or 20), then that member's Private Information is the colour of the empty box. Note that 7 out of 20 times means 35%.

Private Information is more likely to be correct than incorrect. Also, all group members receive equally reliable Private Information. However, since it is generated independently for each member, members in the same group do not necessarily get the same information. It is possible that your Private Information is BLUE while other members' Private Information is YELLOW.

Finally, at the end of each round, you will see the number of votes for BLUE, the number of votes for YELLOW, and whether the group decision matched the colour of the box with the prize.

Part 1 will start after a short quiz to check your understanding of the instructions. [PART 1 COMMENCES]

Part 2

The second part of the experiment will take place over 60 rounds. In each round, after the prize is placed in one of the two boxes but before group members vote, each group member receives two pieces of information, namely Private Information and Public Information, about the location of the prize. [SHOW SCREEN FOR DECISION] As before, in each round Private Information will be generated independently and revealed to each group member separately, and no other participants of the experiment will see your Private Information. It gives each of you the colour of the box with the prize 65% of the time, and the colour of the empty box 35% of the time.

The reliability of Private Information can be described as follows:

1. In each round, after the prize is placed in one of the boxes, the computer rolls a fair 20-sided dice for each group member.

2.

a. If the result of the dice roll is 1 to 13 (1,2,4,5,6,7,8,9,10,11,12 or 13), then that member's Private Information is the colour of the box with the prize. Note that 13 out of 20 times means 65%.

b. If the result of the dice roll is 14 to 20 (14,15,16,17,18,19 or 20), then Private Information is the colour of the empty box. Note that 7 out of 20 times means 35%.

In addition to but independently of Private Information, Public Information is revealed to all members of your group. In each round all group members get the same Public Information. It gives you the colour of the box with the prize 70% of the time, and the colour of the empty box 30% of the time.

The reliability of Public Information can be described as follows:

1. In each round, after the prize is placed in one of the boxes, the computer rolls a fair 20-sided dice (one dice roll for all members of your group), separately from the dice rolls for Private Information.

2.

a. If the result of the dice roll is 1 to 14 (1,2,4,5,6,7,8,9,10,11,12,13 or 14), then your group's Public Information is the colour of the box with the prize. Note that 14 out of 20 times means 70%.

b. If the result of the dice roll is 15 to 20 (15,16,17,18,19 or 20), then your group's Public Information is the colour of the empty box. Note that 6 out

of 20 times means 30%.

Neither Public Information nor Private Information is 100% reliable in predicting the box with the prize, but both pieces of information are more likely to be correct than incorrect.

Note that those two pieces of information may not give you the same colour (it may be that one says BLUE and the other says YELLOW), in which case only one of them is correct. Public Information is more likely to be correct than each member's Private Information. However, it could be that your Private Information is correct and the Public Information is incorrect. Also, even if both pieces of information give you the same colour, it may not match the colour of the box that contains the prize, since neither is 100% reliable.

At the end of each round, you will see the number of votes for BLUE, the number of votes for YELLOW, and whether the group decision matches the colour of the box with the prize.

Part 2 will start after a short quiz to check your understanding of the instructions. [PART 2 COMMENCES]

4.C Random Ranking Version

The actual method used for the experiment is what we have called a Totally Random Version (TRV) of producing the signal matrix. In the beginning of each round we have a random draw to determine which of the two states A or B is the correct one with equal probability (50%). In TRV, each signal, either expert or private, is an independent random draw from the uniform distribution ($s \sim U[0, 1]$). If the number of subjects of the game is n , we need $n + 1$ such draws for each round. For each subject's private information, if $s < 0.65$, then private information for this round will be correct and otherwise incorrect. For expert (common information), if $s < 0.7$, then expert information for this round will be correct and otherwise incorrect.

Obviously TRV has the disadvantage that disagreements with the expert vary over time. Therefore, we have to use a large number of rounds to ensure that the number of disagreements will be enough to enable appropriate statistical analysis for the results of the experiment. In this section, we will explain an alternative method we designed for producing a matrix for signals that could be used for the game with expert information. We call this method Random Ranking Version (RRV). In brief, RRV is a procedure where:

1. we divide the signal matrix in two sections: one where expert information is mistaken and one where where expert information is correct, fixing expert information accuracy to a prespecified level,
2. for each section we divide the subjects signals in two section, one where they can be mistaken and one where can be correct, again fixing private signal accuracy to another prespecified level,
3. we randomize the ranking of vectors, so that agents make mistakes at random occasions and finally

4. we randomize across rows, so that the expert makes mistake at random occasions.

This way we end up with an uncorrelated signal matrix, with fixed accuracies and a fixed number of disagreements for all subjects. Moreover, using this method one can have better control for ex-post statistics such as signal correlation, condorcet accuracy etc. Also, it is actually a lot more egalitarian than TRV, as each subject will have the same realised frequency of correct versus incorrect signals.

4.C.1 Creating the Signal Matrix using RRV

In this section we will explain in detail to create the signal matrix using RRV for $i = 7$ subjects and $j = 40$ rounds. As explained before, RRV can be seen as a two stage randomization. RRV can produce exactly 18 disagreements between each subject's private information and expert information in only 40 rounds of the game. In the first stage, we create the signal sequence for all 40 rounds of the game for each subject. As seen in Figure 4.C.1, we divide the sequence produced in two substeps. The first substep contains 12 rounds ($j \in [1, 2, \dots, 12]$, expert signal incorrect, $1 - q = 0.3$) and the second substep contains 28 rounds ($j \in [13, 14, \dots, 40]$, expert signal correct, $q = 0.7$). For each substep, we fix the number of correct or incorrect signals for the relevant subject. In the substep where the expert signal is incorrect, each subject i gets 8 correct and 4 incorrect signals ($p_{ij}^{mis} = 0.6667$) and the substep when the expert signal is correct, each subject gets 18 correct and 10 incorrect signals ($p_{ij}^{cor} = 0.6428$). Note that the aggregate accuracy across all rounds for subject i is

$$\frac{\sum_{j=1}^{12} p_{ij}^{mis} + \sum_{j=13}^{40} p_{ij}^{cor}}{40} = 0.65.$$

Then we randomize the sequence of rounds $j \in [1, 2, \dots, 12]$ and $j \in [13, 14, \dots, 40]$ separately. We do this by producing an independent random draw from the uniform distribution ($s \sim U[0, 1]$) for each round and then rank in increasing or decreasing order. we then aggregate all sequences of signals in matrix where each row represents a single round of the game and each column represents the signal sequence for agents $i \in [1, 2, \dots, 7]$. The process produces a 40×8 matrix as in Figure 4.C.2, where the first column represents the sequence of expert signals. In this stage, the matrix contains zeros for rounds $j \in [1, 2, \dots, 12]$ and ones for rounds $j \in [13, 14, \dots, 40]$.

The second stage is straightforward. For each row vector, we produce an independent random draw from the uniform distribution ($s \sim U[0, 1]$) for each round and then again rank in increasing or decreasing order, ending up with the final signal matrix in Figure 4.C.3.

4.C.2 RRV results

As in TRV, each iteration of the following process will produce a signal matrix where the accuracy of the condorcet winner varies. Hence, we can repeat the process above until we get a desirable accuracy for the condorcet winner as well. Given the procedural structure, we always produce typical sequences of independent signals. Therefore, repeating the algorithm will not tamper with the aggregate ex-post correlation between the expert and individual signals. If the number of rounds is sufficiently large, we can replicate sequences of signals of any accuracy with the correlation between them being always close or exactly equal to zero. The only correlation that will appear will be due to a very small number of rounds. In that case, due to integer divisions, the accuracy of the private signals conditional on the expert being correct or incorrect will not be exactly equal. However, this effect vanishes almost immediately. In our case with only 40 rounds, we get $p_{ij}^{mis} = 0.6667 < 0.65 < p_{ij}^{cor} = 0.6428$ that produces a Pearson product moment correlation coefficient between the expert signals sequence and each subjects private signal sequence equal to -0.02 .¹³ With 50 rounds it would be exactly zero.

The main drawback of this method is that is very hard to produce instructions simple enough for the game that do not involve some undisclosed information to the subjects in terms of the method involved.¹⁴ Given the spirit of the current experimental literature in economics, we preferred to use TRV instead of RRV. Note that, RRV produces only typical sequences of signals. Hence, while the two methods are different and apart from the fact that RRV establishes a fixed number of disagreements, the two methods produce absolutely indistinguishable results. Therefore, the instructions we have already used in the game would work just the same.

¹³Note that with Student's t-distribution, for 40 observations, r is between ± 0.07 with a probability of 99.90%.

¹⁴ Although this would be highly improbable, a subject with a mistaken understanding of probabilistic independence could actually vote "better" by counting how many mistaken signals have already been received by them or the expert, anticipating future signals as the game goes along

SUBJECT ACCURACY		0.66666667			
	RAND	RANK	SIGNALS	R-SIGNALS	
0.650000	1 0.457688	7	0	1	
	2 0.515023	6	0	1	
	3 0.675018	3	0	0	
	4 0.251926	9	0	1	
	5 0.990348	1	1	1	
	6 0.560245	5	1	0	
	7 0.233638	10	1	0	
	8 0.287746	8	1	1	
	9 0.63318	4	1	0	
	10 0.700687	2	1	1	
	11 0.147882	11	1	1	
	12 0.038221	12	1	1	
				0.642857143	
	RAND	RANK	SIGNALS	R-SIGNALS	
	1 0.408388	16	0	1	
	2 0.578107	13	0	1	
	3 0.164701	24	0	0	
	4 0.962066	3	0	0	
	5 0.803561	7	0	1	
	6 0.319364	20	0	1	
	7 0.320859	19	0	0	
	8 0.45315	15	0	1	
	9 0.523134	14	0	1	
	10 0.956526	4	0	1	
	11 0.219594	23	1	1	
	12 0.980991	1	1	1	
	13 0.691924	11	1	0	
	14 0.357667	18	1	0	
	15 0.862879	6	1	0	
	16 0.047436	27	1	0	
	17 0.285808	22	1	1	
	18 0.965626	2	1	1	
	19 0.368309	17	1	0	
	20 0.950191	5	1	0	
	21 0.758465	10	1	1	
	22 0.121948	26	1	1	
	23 0.763739	9	1	1	
	24 0.623426	12	1	0	
	25 0.29402	21	1	1	
	26 0.125912	25	1	1	
	27 0.78456	8	1	1	
	28 0.018569	28	1	1	

Figure 4.C.1: RRV: creating private signal vectors.

Accuracy 0.7 0.65 0.65 0.65 0.65 0.65 0.65 0.65 0.65

	EXPERT	SUB 1	SUB 2	SUB 3	SUB 4	SUB 5	SUB 6	SUB 7
1	0.608503	21	0	1	1	1	1	1
2	0.855309	7	0	1	0	1	0	1
3	0.081699	38	0	0	1	1	1	0
4	0.803419	10	0	1	1	0	1	1
5	0.706264	13	0	1	1	1	0	1
6	0.603749	22	0	0	0	1	0	0
7	0.091757	37	0	0	1	0	1	0
8	0.969318	3	0	1	0	0	1	1
9	0.42136	26	0	0	0	1	1	0
10	0.568146	23	0	1	1	1	1	0
11	0.263107	32	0	1	1	0	0	1
12	0.982216	1	0	1	1	1	1	1
13	0.562446	24	1	1	0	1	1	0
14	0.663179	17	1	1	1	0	0	1
15	0.906701	5	1	0	1	1	1	0
16	0.982069	2	1	0	1	1	0	1
17	0.373804	29	1	1	0	1	0	0
18	0.198957	34	1	1	0	1	1	1
19	0.268784	31	1	0	1	1	0	1
20	0.167653	35	1	1	0	1	1	0
21	0.652608	19	1	1	0	0	0	1
22	0.402205	27	1	1	1	0	1	0
23	0.7797	11	1	1	0	0	0	1
24	0.001801	40	1	1	1	0	1	1
25	0.812081	8	1	0	1	0	1	0
26	0.37825	28	1	0	1	1	0	1
27	0.118917	36	1	0	1	1	1	0
28	0.759061	12	1	0	1	0	1	0
29	0.67723	15	1	1	0	1	0	0
30	0.663148	18	1	1	1	0	1	0
31	0.617556	20	1	0	1	0	1	0
32	0.806	9	1	0	1	1	0	1
33	0.691901	14	1	1	1	0	1	0
34	0.560519	25	1	1	0	1	1	0
35	0.232881	33	1	1	0	0	0	1
36	0.061305	39	1	0	1	1	1	0
37	0.940788	4	1	1	1	1	1	1
38	0.856729	6	1	1	1	1	1	1
39	0.296331	30	1	1	1	0	1	0
40	0.665132	16	1	1	0	1	1	0

Figure 4.C.2: RRV: reshuffling private signals.

Accuracy	0.7	0.65	0.65	0.65	0.65	0.65	0.65	0.65
Disagreements	18	18	18	18	18	18	18	18
	EXP	SUB 1	SUB 2	SUB 3	SUB 4	SUB 5	SUB 6	SUB 7
ROUND 01	1	0	1	1	1	1	0	0
ROUND 02	0	1	0	1	0	0	1	1
ROUND 03	1	1	1	0	0	1	0	1
ROUND 04	1	0	0	0	1	0	1	0
ROUND 05	1	1	1	0	0	1	0	0
ROUND 06	1	1	0	1	1	1	1	1
ROUND 07	1	0	1	1	1	1	1	1
ROUND 08	1	1	1	1	0	1	1	0
ROUND 09	0	0	1	1	1	0	0	0
ROUND 10	1	1	0	1	0	0	1	1
ROUND 11	1	0	1	1	1	1	1	0
ROUND 12	0	0	1	0	1	1	0	1
ROUND 13	1	0	1	0	0	1	0	0
ROUND 14	0	0	1	0	1	1	0	1
ROUND 15	1	1	0	1	1	1	1	1
ROUND 16	1	0	1	0	1	0	1	1
ROUND 17	0	1	0	1	1	0	1	1
ROUND 18	1	1	1	1	1	1	1	0
ROUND 19	1	1	1	1	1	1	1	1
ROUND 20	1	1	0	0	0	1	1	1
ROUND 21	0	1	1	1	1	0	1	1
ROUND 22	1	0	1	0	0	0	1	1
ROUND 23	1	1	0	1	1	0	0	1
ROUND 24	1	1	1	1	1	0	0	1
ROUND 25	0	1	1	1	1	1	1	0
ROUND 26	0	1	1	1	1	1	1	0
ROUND 27	1	1	0	1	1	1	0	1
ROUND 28	1	0	1	0	0	1	1	1
ROUND 29	1	0	1	1	1	1	1	1
ROUND 30	0	1	0	1	1	1	1	1
ROUND 31	1	1	1	0	1	0	0	1
ROUND 32	1	1	0	1	1	0	1	1
ROUND 33	0	1	0	1	0	1	1	1
ROUND 34	0	0	1	0	0	1	0	1
ROUND 35	1	1	1	1	1	0	1	0
ROUND 36	1	0	1	0	0	0	0	1
ROUND 37	1	1	1	1	1	1	1	0
ROUND 38	1	1	0	1	0	1	0	0
ROUND 39	1	1	0	1	1	1	1	1
ROUND 40	0	1	1	0	0	1	1	0

Figure 4.C.3: RRV: creating final signal matrix.

Chapter 5

Hysteresis in Crime¹

“Time is the longest distance between two places.” (Tennessee Williams)

5.1 Introduction

There is a significant gap between theory and empirical evidence about policies to reduce crime rates. The majority of the theoretical analyses predict a sharp decrease in crime rates when there are significant improvements in the economic conditions, such as a fall in unemployment and poverty rates or when legal market income has a significant increase. A similar prognosis is established in the case of a substantial increase in the probability of punishment. However, a predominant part of the empirical literature in the economics of crime observe a lower than expected effect on crime rates from exogenous variations in economic variables.²

This chapter argues that a possible reason for this is the fact that the current literature of economics of crime overlooks a likely hysteresis effect in the criminal behaviour, a situation where positive exogenous variations in the relevant economic variables have a different effect from negative variations.³ A relevant consequence of hysteresis in criminal behaviour is the fact that social policies to reduce crime will have a more important impact on potential criminals than on existing criminals.

We develop the first simple model that explicitly characterises the hysteresis effect in criminal behaviour. In every period, agents choose to engage in criminal activities by doing a cost-benefit analysis. They weigh between the positive effects of learning by doing, any intrinsic sunk costs of a criminal career and the expected loss if they are caught. Agent are myopic, in the sense that they are only interested in the gains and losses of the current period. According to [Akerlof \(1991\)](#), ignoring this behavioural pathology, any economic theory of crime would be deficient and misleading.

When the probability of crime deterrence decreases, some law abiding agents will find it more beneficial to enter a criminal career. However, if the probability of

¹This chapter was based on the 2013 working paper “Hysteresis in Crime”, co-authored by Dr. Andre Loureiro.

²As show [Mocan and Bali \(2010\)](#) and [Mustard \(2010\)](#).

³Hysteresis was a term coined by the Scottish engineer James Ewing in 1881 to describe the retentiveness phenomenon intrinsic to ferromagnetic materials.

crime deterrence returns to the initial level, a subset of these agents will continue in their career in crime. Agents that have engaged in criminal behaviour, either because of increased revenues from learning by doing or because they have paid the sunk cost of a criminal career, under the same probability of deterrence as in the first period, will find it more beneficial to remain in a criminal career than return to being law abiding. Only in the situation when punishment is extremely severe the effect of hysteresis disappears.

We show that, when agent's crime choice exhibits weak hysteresis individually, crime rate in a society consisted from a continuum of agents with intrinsic costs that follow any distribution will exhibit strong hysteresis. Furthermore, we find a relationship between the increase in crime rate when the probability of deterrence returns to the initial level and the distribution of agents' intrinsic costs.

Since the seminal paper of [Becker \(1968\)](#), economists consider formally the possible effects of socioeconomic variables on criminal behaviour. The standard fully rational crime decision model establishes that an individual periodically faces a decision whether to commit a illicit act or not, based on the expected return of the criminal market and associated probability and severity of punishment when compared to the expected stream of legal income. Just recently the economics of crime has formally considered the inherent intertemporal nature of criminal choice.⁴ [McCrary \(2010\)](#) highlights the importance of considering a dynamic setting in order to properly describe the choice of a potential criminal. [Burdett, Lagos, and Wright \(2003, 2004\)](#) develop an on-the-job search model to analyse the interrelations between crime, unemployment and inequality. The authors find multiple equilibria, which suggest there is a hysteresis process in crime decisions. The result helps to understand why higher levels of welfare benefits may lead to higher crime rates as higher taxes may discourage some individuals to work. Other relevant articles with dynamic settings are [Imrohoroglu, Merlo, and Rupert \(2004\)](#), [Imai and Krishna \(2004\)](#) [Engelhardt, Rocheteau, and Rupert \(2008\)](#), [Sickles and Williams \(2008\)](#) and [Engelhardt \(2010\)](#). Nevertheless, a ubiquitous assumption in the dynamic models to describe the crime decision is that once the illicit option is chosen, the probabilities of an individual choosing the possible alternatives remain unaltered in the following period.

5.2 Hysteresis

Hysteresis has been a phenomenon identified in economic contexts like foreign investment ([Dixit \(1989, 1992\)](#)) and unemployment ([Blanchard and Summers \(1987\)](#) and [Røed \(2002\)](#)).⁵ [Cross \(1993\)](#) and [Amable, Henry, Lordon, and Topol \(1994\)](#) formally define hysteresis in the economic context, showing the difference between its weak and strong versions. It is pointed out the usual improper use of the word in economics to describe persistence stemming from unit root for discrete processes or zero eigenvalue for linear dynamic systems. Unlike unit/zero root

⁴See [McCrary \(2010\)](#) for a extensive survey on dynamic models in the economics of crime literature.

⁵For a more recent exposition of the use of the concept of hysteresis in economics see [Göcke \(2002\)](#).

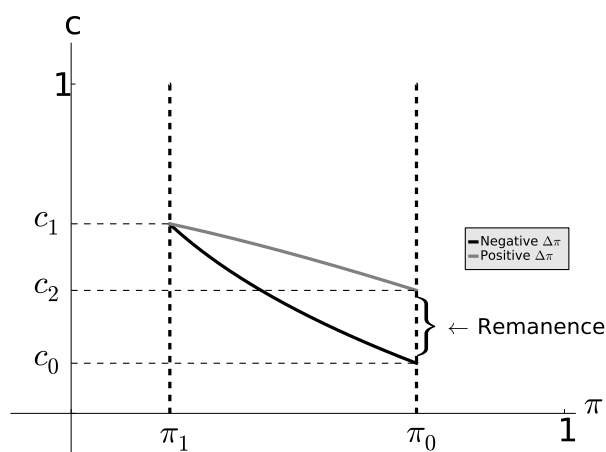


Figure 5.2.1: Strong Hysteresis: Decrease in π followed by an increase to its original level

processes, both types of hysteresis display path dependence, asymmetric cycles and the remanence property. In the context of crime, remanence corresponds to crime rates not going back to its original value when the probability of punishment or the average legal wage is transitorily changed.

5.2.1 Weak versus Strong Hysteresis

Weak hysteresis is a phenomenon that occurs at the individual level only if specific threshold levels are reached. The aggregation of homogeneous agents subject to hysteresis also displays the weak hysteresis effect and the pattern of hysteresis is similar to the micro level. However, with heterogeneous agents the aggregate variation is reinforced generating a strong hysteresis effect. At every level of the input variable, positive variations result in different effects on the output variable when compared to effect from negative variations. Furthermore, unlike the situation of weak hysteresis, the amplitude of the remanence will depend on the magnitude of the shock.

It is also important to contrast both types of hysteresis with unit root processes. In a system with hysteresis, the current behaviour depends on the dominant extremum of past shocks, whereas in a system with unit root, all past shocks matter. That implies that hysteresis is associated to local structural stability, whereas unit root processes are associated to global structural stability.

Amable, Henry, Lordon, and Topol (1994) let clear that a shock exerted on a state variable in a unit root process will lead to a new steady state, but will keep the number of equilibria and the its respective locus unchanged. If crime rates are unit root processes, two opposite shocks with the same magnitude will leave the crime rate unaffected, whereas in a hysteric system will lead the crime rate to a new equilibrium.

5.2.2 Extrinsic versus Intrinsic Hysteresis

We claim that there are two types of sources of hysteresis in crime: 1. **External** to the individuals (extrinsic). Weak hysteresis with this source has recently been considered in the literature by introducing social stigma into the labour market: Lower wage offers [Imrohoroglu, Merlo, and Rupert \(2004\)](#); higher duration in unemployment [Engelhardt \(2010\)](#); social capital depreciation [Sickles and Williams \(2008\)](#). Hysteresis would only be a result in this context in the situations where the criminal is caught *and* convicted and/or this fact is known by the others individuals in his social life. Nevertheless, data on countries like the US and the UK show that the punishment for the majority of committed crimes do not involve incarceration and the apprehension rate is very low.⁶

2. **Internal** to the individuals (intrinsic). The claim in this chapter is that hysteresis could also occur as a result of sources internal to the individuals. As it will be shown in the following sections, sunk moral cost (fallacy) and learning in crime can lock individuals in the criminal career. Internal sources are crucial to an explicit characterisation of strong hysteresis in criminal choice.

This chapter argues that stigma in the labour market is only one of the sources of hysteresis in criminal behaviour. An important source of hysteresis stems from internal sources to the individual. To date, there has been no formal model or empirical study on this type of hysteresis in criminal behaviour.

⁶In some cities of the US and the UK, the clearance rate - crimes with a charge being laid divided by the total number of crimes recorded - is as low as 5%. Nationwide the fraction of cleared crimes varies between 1/5 and 1/4. See FBI's Uniform Crime Reporting (UCR) website for data on the US and the Home Office Statistical Bulletin for information about clearance rates in the UK.

5.3 Baseline Model

5.3.1 Homogeneous Agents

There is a $[0, 1]$ continuum of ex ante homogeneous⁷ risk-neutral infinitely-lived individuals that decide whether to commit a crime or not. The wage of individuals abiding by the law in every period is fixed and equal to w . Committing a crime can increase utility by the illicit gain g , but it also entails a moral (or intrinsic) cost m and an exogenous probability π ⁸ of facing a punishment s (sanction). The trade-off between the time spent in legal and criminal activities is captured by a reduction in w by $\gamma \in [0, 1]$.⁹

Every criminal act entails an intrinsic cost m which is a burden irrespective of whether the punishment occurs or not. This cost can have at least two non-mutually exclusive interpretations: 1. A moral cost associated with an illicit activity, either a disutility because the individuals feel guilt about the harm caused to the victim by the act (internal) or the disapproval of their peers (social). 2. The actual cost of a criminal act (entry fee or criminal technology/inputs) which is decreasing with the number of crimes committed in the past (individual learning) and/or with the number/strength of links in a criminal network (social learning).

The expected utility of crime (EUC) is then given by:

$$\begin{aligned} EUC &= \underbrace{(g + \gamma w - m - s)\pi}_{\text{caught}} + \underbrace{(g + \gamma w - m)(1 - \pi)}_{\text{get away with it}} \\ &= g + \gamma w - m - s\pi \end{aligned} \tag{5.3.1}$$

Agents are myopic; their choice to engage in criminal activities depends only on the expected gain of the current period. As explained in the introduction, the best interpretation of this behaviour concurs with the idea of [Akerlof \(1991\)](#) with respect to criminal behaviour: agents are not interested in any future costs or benefits of their actions today. We argue incorporating the expected utility of future periods and using a discount rate would only magnify the effect of hysteresis in criminal behaviour.

At every period t , agents choose to engage in criminal activity $\phi = 1$ or not $\phi = 0$. An agent will choose to engage in criminal activity if the expected gains of committing in crime outweigh the income without crime. This decision is determined by $Max_{\phi}\{w, EUC\}$ boils down to:

$$\phi = \begin{cases} 1 & \text{if } g - (1 - \gamma)w - m - s\pi > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.3.2}$$

⁷The assumption of homogeneity will be relaxed afterwards and is assumed to show that the results are not driven by ex ante heterogeneity.

⁸The probability of punishment (fine, prison, etc) is conditional on conviction, which is itself conditional on being caught/arrested. Here it is assumed that being caught implies conviction and punishment.

⁹ $(1 - \gamma)$ can be interpreted as the depreciation of the ability to earn w when an individual is engaged in crime. This reduction in w reflects stigma in the labour market and/or less time available to work in legal activities.

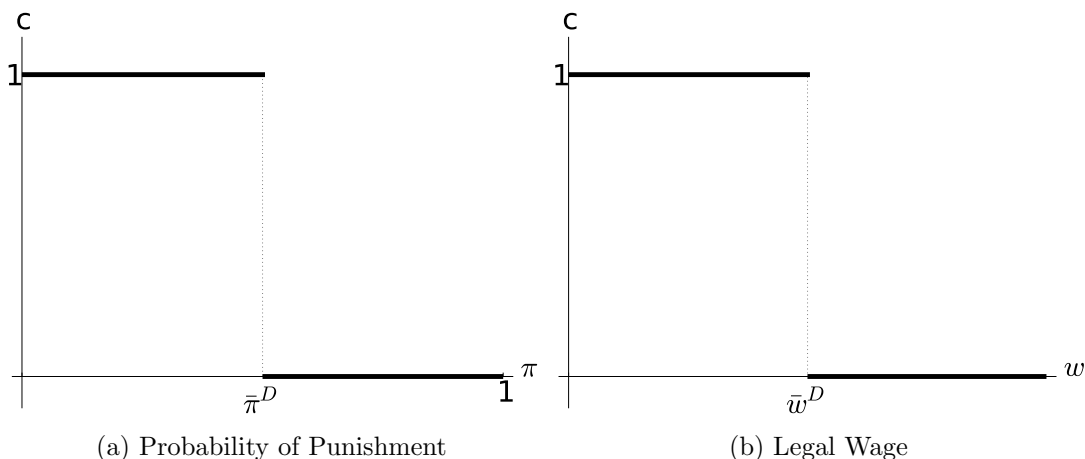


Figure 5.3.1: Crime Rates for Homogenous Agents

Notice from equation 5.3.2 that if there is no conflict between legal and criminal activities ($\gamma = 1$) the crime decision will not depend on w . Conversely, if $\gamma = 0$, crime and no crime are two mutually exclusive activities and consequently different states. That implies that γ is closer to 0 for more serious crimes and closer to 1 for petty crimes.

It should also be noted that in the extreme case where $\pi = 0$ and there is no opportunity cost ($\gamma = 1$), crime only happens if the gain associated to the act is strictly higher than the intrinsic cost: $g > m$.

Definition 12. For a given combination (g, m, w, s) , there is a unique π , the *deterrent threshold of punishment*, that will deter all individuals from crime:

$$\bar{\pi}^D = \frac{g - (1 - \gamma)w - m}{s} \quad (5.3.3)$$

Definition 13. For a given combination (g, m, π, s) , there is a unique w , the *reservation wage of crime*, that will discourage all individuals from crime.

$$\bar{w}^D = \frac{g - m - s\pi}{1 - \gamma} \quad (5.3.4)$$

Under homogeneity of the agents and assuming that each criminal commits only one type of crime per period ¹⁰, the crime rates can be depicted for respectively a given $\bar{\pi}^D$ and \bar{w}^D .

¹⁰This is a reasonable assumption for a sufficiently short period. In the case of a longer period, the crime rate is obtained by also taking into account the distribution of the number of crimes each criminal commits per period. A simple formulation is to assume that the number of crimes of each criminal is given by a uniform probability mass function. If an additional assumption that all individuals commit the same number of crimes $q > 1$ is made, the crime rate can be obtained simply by multiplying the fraction of offenders by q . Note that in that case the crime rate can be greater than one.

5.3.2 Crime Rates with Heterogeneous Agents

Heterogeneity allows for a more realistic description of the relationship between crime and its deterrents as it allows crime rates to be a value different from 0 and 1.¹¹ We will begin by describing the case where criminal history does not inflict either a cost or a benefit in the agent's choice. This assumption is relaxed at section 5.5.

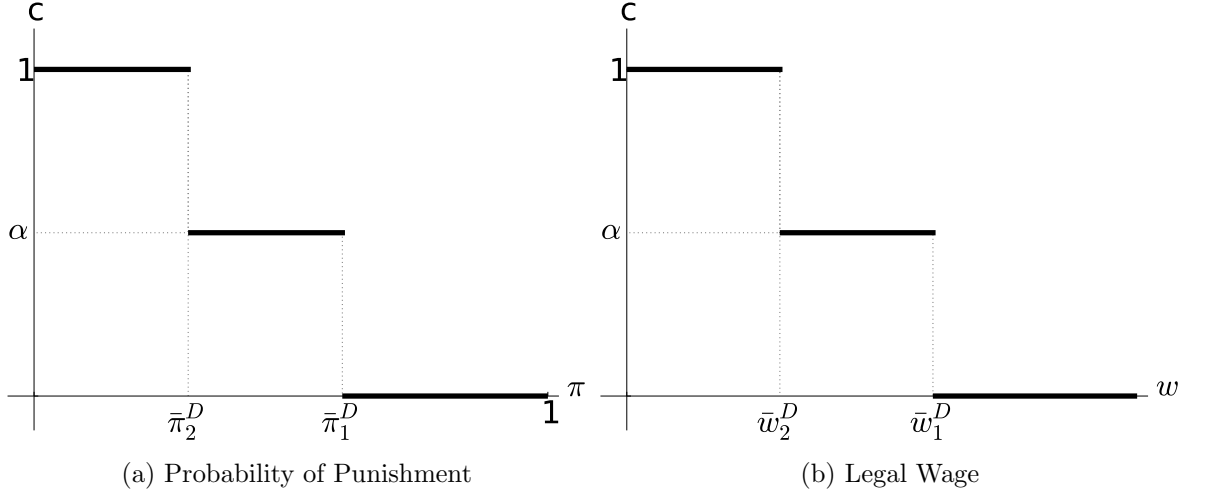


Figure 5.3.2: Crime Rates for Heterogeneous Agents ($k = 2$)

Let the individuals vary with respect to their moral cost m_j , $j = \{1, \dots, K\}$. (K types). For $K=2$, m is either m_L or m_H with probability $(\alpha, 1 - \alpha)$. For a given combination (g, w, m_j, s) , there are two distinct deterrent π 's: $\pi_2^D = \frac{g - m_H - (1-\gamma)w}{s} < \pi_1^D = \frac{g - m_L - (1-\gamma)w}{s}$. Similarly, for a given combination (g, m_j, π, s) , there are two distinct reservation wages of crime: $\bar{w}_2^D = \frac{g - m_H - s\pi}{1-\gamma} < \bar{w}_1^D = \frac{g - m_L - s\pi}{1-\gamma}$.

For a continuum of types, m_j can be represented by a distribution $f(m)$. The simplest non-trivial case is when the fraction of each type is equally likely and m follows a uniform distribution: $m \sim U(m_{min}, m_{max})$. For a given positive (negative) variation in π there is a negative (positive) variation in the crime rate correspondent to the fraction of the population with the level of m .¹²

The values of m_{min} and m_{max} can be normalized so that $\bar{\pi}^D \sim U(0, 1)$: $\pi_{min}^D = 0 = \frac{g - (1-\gamma)w - m_{max}}{s} \Rightarrow m_{max} = g - (1 - \gamma)w$ and $\pi_{max}^D = 1 = \frac{g - (1-\gamma)w - m_{min}}{s} \Rightarrow m_{min} = g - (1 - \gamma)w - s$.

That implies that in terms of π , m has pdf given by:¹³

¹¹What follows is based on exogenous ex ante heterogeneity. An analysis with ex ante homogeneity, but ex post heterogeneity will lead to similar results. It can be carried out if we assume that individuals instead of observing the probability of punishment, they only observe a noisy signal correlated to the true probability of punishment and update their priors according to previous experience.

¹²Notice that if the highest level of m is sufficiently high, $c < 1$ even if $\pi = 0$.

¹³The same result is obtained by applying the theorem of inverse transformation of random variables to the affine transformation $m = g - (1 - \gamma)w - s\bar{\pi}^D$.

$$f_m(m) = \frac{1}{s} \quad , \quad m_{min} \leq m \leq m_{min} + s. \quad (5.3.5)$$

Because $\bar{\pi}^D$ has pdf given by: $f_{\bar{\pi}^D}(\bar{\pi}^D) = 1$, $0 \leq \bar{\pi}^D \leq 1$, for a given π , crime rate can be computed by two equivalent ways:

$$c(\pi) = \int_{m_{min}}^{m(\pi)} \frac{1}{s} dm = \int_{\pi}^1 d\bar{\pi}^D = 1 - \pi \quad (5.3.6)$$

The first integral in equation 5.3.6 corresponds to the aggregation of all individuals with moral cost below the moral cost associated to the prevailing probability of punishment π . That is equivalent to the deterrence probability of punishment $\bar{\pi}^D$ being greater than π , given by the second integral of the equation.

Similarly, for a given positive (negative) variation in w , there is a negative (positive) variation in the crime rate correspondent to the fraction of the population with the level of m so that $m \sim U(m_{min}, m_{max})$. Therefore, \bar{w}^D has pdf given by: $f_{\bar{w}^D}(\bar{w}^D) = \frac{1}{\bar{w}_{max}}$, $0 \leq \bar{w}^D \leq \bar{w}_{max}$, for a given w , crime rate is given by:

$$c(w) = 1 - \frac{w}{\bar{w}_{max}} \quad (5.3.7)$$

Equation 5.3.7 corresponds to the aggregation of all individuals with moral cost below the one associated to the prevailing legal wage w , or equivalently, the aggregation of all individuals with reservation wage of crime \bar{w}^D greater than w .

The crime rates in terms of probability of punishment π and legal wage w are depicted in figure 5.3.3.

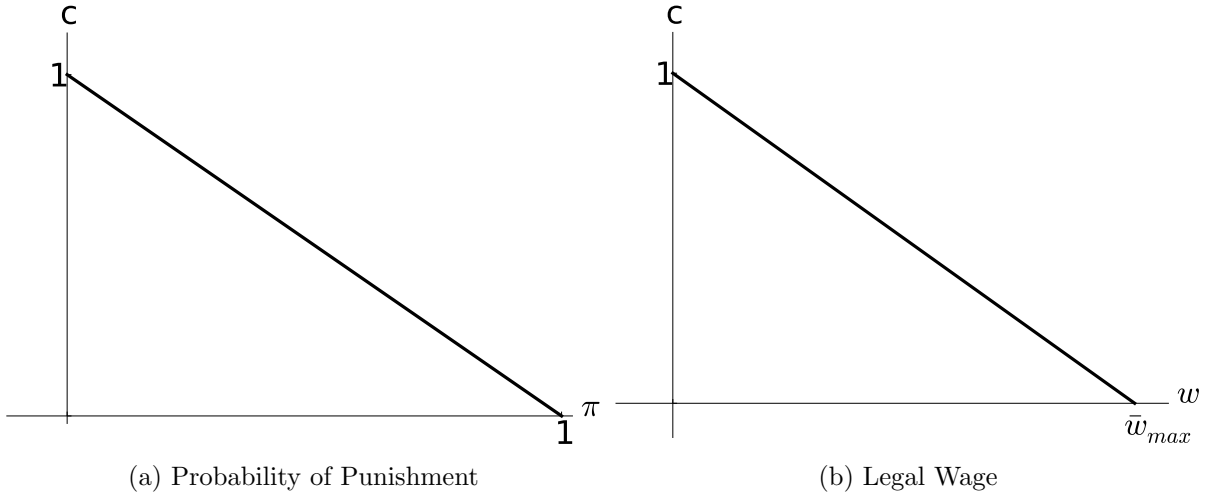


Figure 5.3.3: Crime Rates for a Continuum of Heterogeneous Agents - Uniform

A more realistic representation of the distribution of the intrinsic cost would relax the assumption of equally likely values of m . For simplicity, assume $m_{min} = 0$, so that $f_m(m)$ has a support on the $[0, m_{max}]$ interval. Crime rate in terms of

π can then be computed analogously to the uniform case using the distribution of m :

$$\begin{aligned} c(\pi) &= \int_0^{m(\pi)} f_m(m) dm \\ &= F_m(m(\pi)) \\ &= F_m(g - (1 - \gamma)w - s\pi) \end{aligned} \tag{5.3.8}$$

As the cdf F_m is non-decreasing, equation 5.3.8 is a non-linear decreasing function on π . This is also true to w , as it can be analogously shown. Figures 5.3.4a and 5.3.4b depict these relationships for a log-normal distribution.¹⁴

Notice that for extreme values high values of π and w , crime rates are lower than the case that the intrinsic cost has a uniform distribution. Similarly, extreme values low values of π and w are associated to higher crime rates than the uniform case.

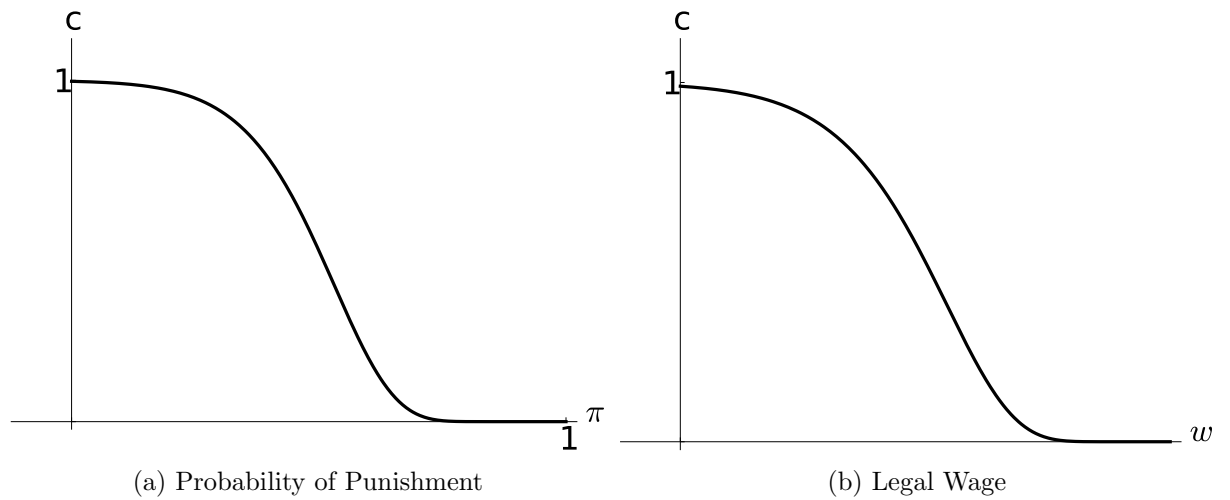


Figure 5.3.4: Crime Rates for a Continuum of Heterogeneous Agents - Log-normal;

5.4 Weak Hysteresis in Crime Decision

Define criminal history at time t as the sum of criminal choices in the T past periods:

$$h^t = \sum_{\tau=0}^T \phi_{t-\tau} \tag{5.4.1}$$

All variables associated with the crime decision should be affected by criminal history. The intrinsic cost m and the opportunity cost of crime given by $(1 - \gamma)w$ are unambiguously increasing in h^t as was discussed in previous sections. The gain from crime g is increasing in h^t , as crime experience and networking allows a

¹⁴Which has support $[0, \infty)$. For a sufficiently small σ^2 , the distribution is relatively symmetric.

better targeting at loot with higher values. The severity of punishment s is also increasing in h^t , as most countries impose heftier sanctions for individuals with a criminal past. The effect of h^t on the probability of punishment π depends on whether the learning process dominates the higher number of traces left by the criminal acts.

To simplify the analysis, assume that the variables above only depend whether an individual has ever committed a crime or not. Define the binary variable $h = 1[h^{t-1} \geq 1]$, where $1[\cdot]$ is an indicator function. This assumption will be subsequently relaxed.

As all variables above, apart from s and π , increase the probability of crime given $h = 1$, the positive effect of crime history on crime choice (including any reduction in π through learning) can be encapsulated by m . The negative effect of h on crime choice (including any increase in π given the higher number of traces) is subsumed in s . A simple linear specification¹⁵ is then given by:

$$m_t = \bar{m} - \tilde{m}h \quad (5.4.2)$$

$$s_t = \bar{s} + \tilde{s}h \quad (5.4.3)$$

Using the equations above, equations 5.3.1 and 5.3.2 can be rewritten respectively as:

$$EUC = g + \gamma w - \bar{m} - \bar{s}\pi + (\tilde{m} - \tilde{s}\pi)h \quad (5.4.4)$$

and

$$\phi = \begin{cases} 1 & \text{if } g - (1 - \gamma)w - \bar{m} - \bar{s}\pi + (\tilde{m} - \tilde{s}\pi)h > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5.4.5)$$

Solving equation 5.4.5 for π yields the deterrent threshold of punishment $\bar{\pi}^D$:

$$\bar{\pi}^D = \frac{g - (1 - \gamma)w - \bar{m} + \tilde{m}h}{\bar{s} + \tilde{s}h} \quad (5.4.6)$$

And the reservation wage of crime \bar{w}^D :

$$\bar{w}^D = \frac{g - \bar{m} - \bar{s}\pi + (\tilde{m} - \tilde{s}\pi)h}{1 - \gamma} \quad (5.4.7)$$

Then equation 5.4.5 can be rewritten as:

$$\phi_t(\pi) = \begin{cases} 1 & \text{if } \pi < \bar{\pi}^D \\ 0 & \text{otherwise} \end{cases} \quad (5.4.8)$$

or:

$$\phi_t(w) = \begin{cases} 1 & \text{if } w < \bar{w}^D \\ 0 & \text{otherwise} \end{cases} \quad (5.4.9)$$

¹⁵A exponential specification ($\bar{m}e^{-\tilde{m}h}$ and $\bar{s}e^{\tilde{s}h}$) will lead to similar results.

Therefore, the deterrent threshold of punishment and the crime reservation wages are functions of the criminal history.

It is easy to see from equations 5.4.6 and 5.4.7 and that $\bar{\pi}^D$ and \bar{w}^D are increasing in criminal history h if and only if $\tilde{m} > \tilde{s}\pi$. This condition implies that, if the gains from criminal history are higher than the expected cost of deterrence, then criminal history will have a positive effect, both on the deterrent threshold of punishment and the reservation wage. This is a reasonable assumption, as only the convicted past crimes should increase the severity of punishment, whereas all past crimes should affect the intrinsic cost of crime.¹⁶ We consider the simplest situation where this is true by setting $\tilde{s} = 0$. Additionally, the focus of the analysis in this section will be on the probability of punishment π . The results for w are similar.

Define $\bar{g} \equiv g - \bar{m} - (1 - \gamma)w$. Note that regardless individuals have committed crime in the past, they will always commit a crime in period t if $\pi < \frac{\bar{g}}{\tilde{s}}$ (*entry threshold*) and they will never commit a crime if $\pi \geq \frac{\bar{g}}{\tilde{s}} + \frac{\tilde{m}}{\tilde{s}}$ (*exit threshold*). However, if π is between $\frac{\bar{g}}{\tilde{s}}$ and $\frac{\bar{g}}{\tilde{s}} + \frac{\tilde{m}}{\tilde{s}}$, the decision is conditional on the crime choice in the previous period.

As $h = \{0, 1\}$, the range of decision described in 5.4.8 can be partitioned into 3 regions:

$$\phi_t(\pi) = \begin{cases} 1 & \text{if } \pi < \frac{\bar{g}}{\tilde{s}} \\ 0 & \text{if } \pi \geq \frac{\bar{g}}{\tilde{s}} + \frac{\tilde{m}}{\tilde{s}} \\ \phi_{t-1}(\pi) & \text{if } \frac{\bar{g}}{\tilde{s}} < \pi < \frac{\bar{g}}{\tilde{s}} + \frac{\tilde{m}}{\tilde{s}} \end{cases} \quad (5.4.10)$$

with:

$$\phi_0(\pi) = \begin{cases} 1 & \text{if } \pi < \frac{\bar{g}}{\tilde{s}} \\ 0 & \text{if } \pi \geq \frac{\bar{g}}{\tilde{s}} \end{cases} \quad (5.4.11)$$

Equations 5.4.10 and 5.4.11 together represent a relay function, where the choice in period t is state-dependent.

At this point it is clear that there are two main aspects that determine the degree hysteresis in crime decision. One is the degree of memory in the criminal history and the other is the size of \tilde{m} relative to the severity of punishment. In the extreme cases where \tilde{m} is very high or all criminal history matters, there is a ratchet effect, where criminals are locked in crime.

It is useful to have a new notation for the different levels of deterrent thresholds of punishment $\bar{\pi}^D$. Denote the entry threshold by $\bar{\pi}^E$ and the threshold that individuals leave the criminal career by $\bar{\pi}^L$. Therefore $\bar{\pi}^E = \frac{\bar{g}}{\tilde{s}}$ and $\bar{\pi}^L = \frac{\bar{g}}{\tilde{s}} + \frac{\tilde{m}}{\tilde{s}}h^t$.

If individuals are homogeneous the aggregation of individual crime decision into crime rates is trivial. The relationship between π and crime rates is depicted in figure 5.4.1.

It is clear that hysteresis is a phenomenon that occurs at the individual level only if specific threshold levels are reached. Hysteresis does not happen if individuals face probability of punishment and legal wages far from the critical

¹⁶The case when $\tilde{m} < \tilde{s}\pi$ would imply that criminal history will have a positive effect in the deterrent threshold of punishment and the crime reservation wages. With this assumption, this model would depict a sense of remorse for an agent with criminal history.

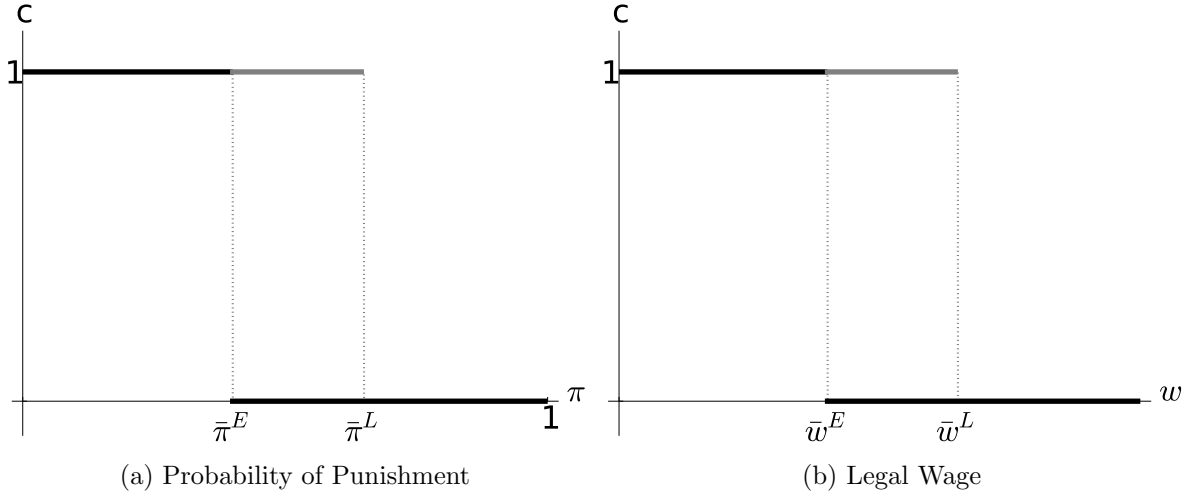


Figure 5.4.1: Crime Rates for Homogeneous Agents

thresholds. That is one of the main characteristics of weak hysteresis.

5.4.1 Weak Hysteresis versus Unit Root Process

If criminal history in the last T periods matter, the exit deterrent threshold of punishment will then be written as:

$$\bar{\pi}_t^L = \frac{\bar{g}}{s} + \frac{\tilde{m}}{s} \sum_{\tau=0}^T \phi_{t-\tau} \quad (5.4.12)$$

and the range of the relay function will have $T + 1$ partitions.

The deterrent threshold of punishment can return to the original level $\bar{\pi}^E = \frac{\bar{g}}{s}$ if individuals do not commit crimes for T periods.

It is important to contrast the history dependence that stems from this relay function with history dependence that emerges from a unit root in a discrete process.

If all criminal history is relevant, the deterrent threshold of punishment is non-decreasing in the criminal history:

$$\bar{\pi}_t^L = \frac{\bar{g}}{s} + \frac{\tilde{m}}{s} \sum_{\tau=0}^{\infty} \phi_{t-\tau} \quad (5.4.13)$$

Note that equation 5.4.13 is the steady state level of $\bar{\pi}_t^L$ following an $AR(1)$ process with unit root given by:

$$\bar{\pi}_t^L = \psi \bar{\pi}_{t-1}^L + \frac{\tilde{m}}{s} \phi_t \quad (5.4.14)$$

with $\psi = 1$ and $\bar{\pi}_0^L = \bar{\pi}^E = \frac{\bar{g}}{s}$.¹⁷

¹⁷Another way to see this is to notice that equation 5.4.13 is a $MA(\infty)$, which is equivalent to an $AR(1)$.

As a unit root process has a impulse-response function constant and equal to one, any temporary shock has a permanent effect. A shock in this setting is extremely simple and corresponds to $\frac{\tilde{m}}{s}\phi_t$ switching from 0 to $\frac{\tilde{m}}{s}$ or vice versa.

Unlike this unit root discrete process, where current behaviour depends weakly on all past shocks, in a hysteric process, current behaviour strongly depends only on non-dominated extrema of past shocks.

5.5 Heterogeneous Agents and Strong Hysteresis

For 2 types of agents, the different exit and entry thresholds occur in both levels of crime rates. The relationship between π and c for $k = 2$ is shown in figure 5.5.1.

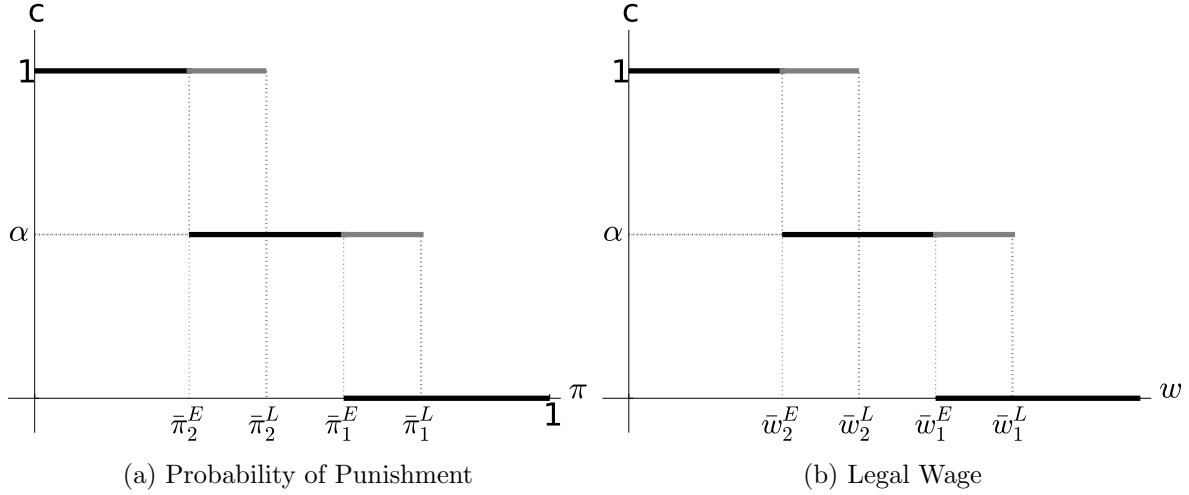


Figure 5.5.1: Crime Rates for Heterogeneous Agents ($k = 2$)

For a continuum of heterogeneous agents, we proceed as in section 5.3, where the intrinsic cost of crime follows a uniform distribution, but with the bounds taking into account the reduced intrinsic costs for individuals with criminal history.

We formally define some concepts related to hysteresis in crime rates in terms of the probability of punishment π and summarize the results in two propositions.

Definition 14. Let Ω be the *criminal remanence*, the increase in the crime rate when π returns to its original level π_0 .

Definition 15. Let π^C be the *coercive probability of punishment*, the level of π necessary to return the crime rate to its original level.

Definition 16. Let $\xi = \pi^C - \pi_0$ be the *coercive force* necessary to return the crime rate to its original level.

Definition 17. Let $H(\pi^T)$ be the *maximum historical crime rate* in the last T periods:

$$H(\pi^T) = \max\{c(\pi_{t-T}), \dots, c(\pi_{t-1})\}.$$

Consider the situation set choice of the policy maker deciding the level of the probability of punishment is simply: $\{\pi^H, \pi^L\}$, with $\pi^H > \pi^L$.

Proposition 8. Strong Hysteresis *If individuals are heterogeneous with respect to their initial intrinsic cost of crime \bar{m} and the intrinsic cost of crime evolves according to equation 5.4.2, for any two levels of probability of punishment $\pi^L < \pi^H \in [0, 1]$, an exogenous reduction in π from π^H to π^L ($\Delta^- \pi$) will increase crime by $\Delta^+ c$. If it is followed by an exogenous increase in π from π^L to π^H ($\Delta^+ \pi = -\Delta^- \pi$), it will decrease crime by $\Delta^- c = -\Delta^+ c + \Omega$, where*

$$\Omega = F_{\bar{m}}(g - (1 - \gamma)w - \tilde{s}\pi + \tilde{m} H(\pi^L)) - F_{\bar{m}}(g - (1 - \gamma)w - \tilde{s}\pi + \tilde{m} H(\pi^H)).^{18}$$

Proof. For a given distribution of initial intrinsic cost of crime, $\bar{m} \sim F(\bar{m})$, crime rates are computed by:

$$c(\pi) = \int_{\bar{m}_{min}}^{m(\pi)} dF_{\bar{m}} = F_{\bar{m}}(\bar{m}(\pi)) - F_{\bar{m}}(\bar{m}_{min}) \quad (5.5.1)$$

Using equation 5.4.6 and definition 17 in the previous equation yields:

$$c(\pi) = F_{\bar{m}}(g - (1 - \gamma)w - s\pi + \tilde{m}H(\pi)) \quad (5.5.2)$$

Since $F_{\bar{m}}$ is an non-decreasing function and $H(\pi^L) > H(\pi^H)$, we have that, for any x ,

$$\Omega = F_{\bar{m}}(x + \tilde{m}H(\pi^L)) - F_{\bar{m}}(x + \tilde{m}H(\pi^H)) \geq 0 \quad (5.5.3)$$

and

$$\Omega = F_{\bar{m}}(g - (1 - \gamma)w - \tilde{s}\pi + \tilde{m}H(\pi^L)) - F_{\bar{m}}(g - (1 - \gamma)w - \tilde{s}\pi + \tilde{m}H(\pi^H)) \quad \square \quad (5.5.4)$$

□

Proposition 9. *If the level of severity s is sufficiently high, both the criminal remanence $\Omega \rightarrow 0$ and the coercive force $\xi \rightarrow 0$.*

Proof. This is a direct result of equations 5.5.3 and 5.5.4, the fact that F_m is CDF. □

As an example, assume m follows a log-normal distribution and plug its CDF into equation 5.3.8 yields:

$$c(\pi) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[\frac{\ln(g - (1 - \gamma)w - s\pi) - \mu}{\sqrt{2\sigma^2}}\right] \quad (5.5.5)$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

From the equation 5.4.2 describing the effect of criminal history on intrinsic cost m , it is clear that the mean and the variance of m can be written respectively as:

¹⁸Proof of this theorem arose from joint work with Vasileios Vlaseros.

$$E(m) = E(\bar{m}) - \tilde{m}E(h) \quad (5.5.6)$$

$$\text{var}(m) = \text{var}(\bar{m}) + \tilde{m}^2\text{var}(h) - 2\tilde{m}\text{cov}(\bar{m}, h) \quad (5.5.7)$$

As $\text{cov}(\bar{m}, h) < 0$, it is clear that a higher proportion of individuals with criminal record decreases the mean and increases the variance of m . That is translated in a shift of $f_m(m)$ to the left. That implies that the curve given by equation 5.5.5 is displaced to the right, as show figure 5.5.2a. A similar analysis can be carried out to w , with variations resulting in figure 5.5.2b.

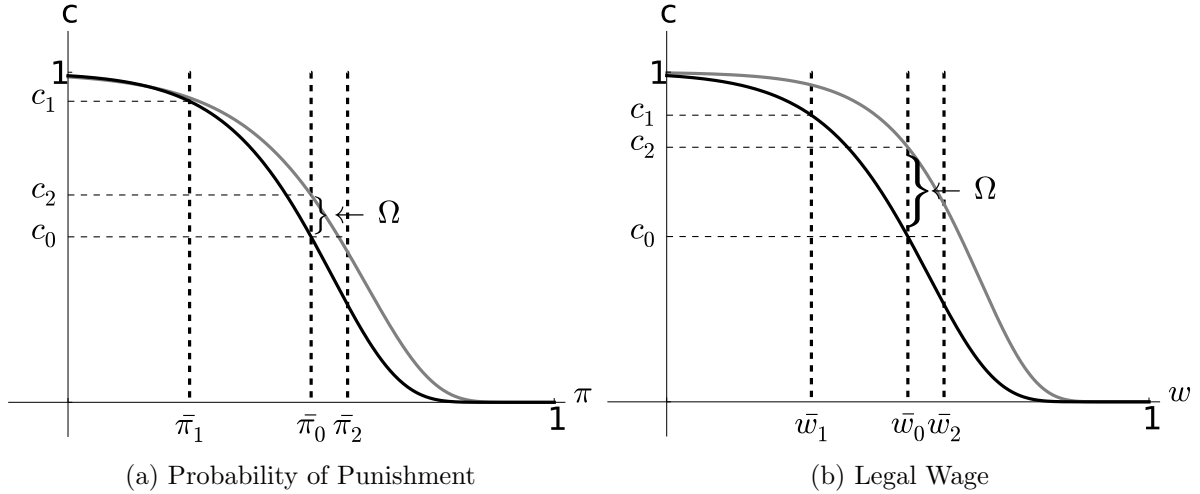


Figure 5.5.2: Crime Rates for different levels of π and w

The thick lines illustrate the full variation of π or w between 0 and 1 in both directions. If π is exogenously reduced from π_0 to π_1 , crime rate goes up from c_0 to c_1 . If π is restored to its original value π_0 , crime rate falls to $c_2 = c_0 + \Omega$. Only if π is increased to π_2 will make the crime rate to return to its original level c_0 . A similar explanation applies to variations in w . For this smaller range of variation in π or w , there is a smaller loop inside the one plotted in both figures.

It is also clear from the figure 5.5.2 that ex ante heterogeneity leads to hysteresis at every point of the input variable, instead of trigger thresholds as occurs in weak hysteresis.

5.6 Conclusion

This chapter explores a simple model where agents choose a career in crime by weighing between the positive effects of learning, any intrinsic sunk costs of a criminal career and the expected loss if they are caught. When the probability of crime deterrence decreases or there is a fall in the real income obtained in the labour market, some agents will find it more beneficial to enter a criminal career. If the original conditions are subsequently restored, a subset of these agents will continue in their career in crime. When crime choice exhibits weak hysteresis at the individual level, crime rate in a society consisted from a continuum of agents that follows any distribution will exhibit strong hysteresis. Furthermore, we find the link between the increase in crime rate when the probability of deterrence returns to the initial level and the distribution of agents' intrinsic costs. Finally, the effect of hysteresis disappears when the severity of punishment is extremely high.

We argue that our theoretical findings corroborate the argument that policy makers should be more inclined to set pre-emptive policies rather than mitigating measures. Any variation on the crime rate will result in an accumulated hidden cost, exactly because of the phenomenon of hysteresis.

The existence of hysteresis in crime has at least two direct implications. The first one regards the policies to reduce crime rates. If hysteresis has a relevant effect in criminal decision, policies to reduce crime should be focused on crime prevention rather than mitigation. The second one is relevant to any future empirical analyses of the impact of policies to reduce crimes. The asymmetric nature of positive and negative shocks on crime rates must be taken into account in order to obtain a more precise estimation of the policy effect.

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