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Essays on Equilibrium Unemployment Dynamics

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Declaration of Own Work

I hereby declare that this thesis has been composed by myself and is the result of my own work under the guidance of my supervisors as stated in the acknowledgements. References to published material are clearly indicated in the text. None of the work contained in this thesis has been submitted for any other degree or professional qualification.

Bradley James Speigner
April 2012

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Overview

This thesis is a collection of three essays in which the behaviour of unemployment is studied in different dynamic environments. Throughout, unemployment is understood to be involuntary, arising due to the uncoordinated nature of trade in the labour market as viewed from the perspective of the Diamond-Mortensen-Pissarides equilibrium matching model. It goes without saying that the fundamental motivation for pursuing this line of research is provided by the untold consequences, both human and economic, of otherwise capable people remaining involuntarily idle. An attempt, therefore, is made to contribute to the understanding of how various aspects of macroeconomic policy can influence unemployment outcomes. The approach maintained throughout is to combine general equilibrium modelling with simulation techniques in order to provide not only qualitative inferences but also quantitative descriptions of equilibrium dynamics. The dynamic environments considered cover both the business cycle (the first two chapters) and the life cycle (the third chapter).

In the first chapter, *Structural Tax Reform and the Cyclical Behaviour of the Labour Market*, we build a real business cycle model with frictional unemployment and distortionary tax rates which are increasing in individual taxable labour income. The cyclical aspects of tax reform that are addressed in this chapter are distinct from the stationary state distributional issues that have garnered most of the attention in the existing literature on structural tax reform. Estimating the tax code parameters from federal income tax return data for the U.S., we find that a reduction in the progressivity of the tax system is associated with a significant increase in the volatility of hours per worker. The intuition is simply that the greater the extent to which marginal tax rates fluctuate in response to shocks, the smaller the incentive to adjust working hours. But in a frictional labour market in which it is costly for firms to issue vacancies, the behaviour of hours - i.e. intensive adjustment, or adjustment in the intensive margin - is a determining factor of job creation - i.e. extensive adjustment. We then explain how the dynamic behaviour of hours along the adjustment path to an aggregate productivity shock generates offsetting incentives for job creation, with the result that tax reform has little impact on unemployment fluctuations. The welfare cost of the business cycle is also computed under different tax regimes. It is found that although business cycles are more costly under a flat tax, the overall welfare implications are quantitatively negligible regardless of the tax system.

Having described the effects of the tax system on equilibrium dynamics when perturbed by a productivity disturbance, we then consider business cycle adjustment to an aggregate demand shock in the form of fiscal stimulus. In light of recent fiscal developments in the U.S. and Europe, the ability of expansionary fiscal policy to stimulate output has gained renewed interest in the business cycle literature. We contribute to the analysis by assessing whether the efficacy of government expenditure in reducing unemployment depends on the structure of the tax system. It is demonstrated that a

less progressive tax policy increases the ability of expansionary fiscal policy to stimulate output due to a larger response in hours, but this comes at the cost of a smaller unemployment multiplier. Tax reform therefore causes a compositional shift in labour market adjustment in response to aggregate demand shocks, with relatively more adjustment occurring in the intensive margin and less adjustment in the extensive margin the flatter the tax schedule is. The reason why this compositional shift occurs for a demand shock but not a supply shock is that the adjustment path of hours is qualitatively dependent on the type of disturbance. In particular, we describe how equilibrium undershooting in hours occurs only in response to an aggregate productivity (supply) shock, whereas the negative wealth effects arising from increased government expenditure exert sustained upward pressure on hours along the entire adjustment path, thus providing a significant incentive for firms to substitute away from job creation.

The second chapter, *Monetary Policy and Job Creation in a New Keynesian Model*, is motivated by the work of Cooley and Quadrini (1999) and Krause and Lubik (2007). These studies indicate that a typical monetary business cycle model with frictional unemployment and endogenous job destruction tends to encounter difficulty in generating a rise in job creation in response to expansionary monetary policy, rendering the model inconsistent with the downward sloping Beveridge curve that appears in the data and implying only a limited policy role for inflationary job creation. Matching frictions in the labour market congest the job creation process so that firms tend to skew adjustment to shocks towards the job destruction margin. In recognition of the assertion put forth but unpursued by Cooley and Quadrini (1999) that fluctuations in the size of the labour force may ease labour market congestion and therefore amplify cyclical job creation, in Chapter II we extend a New Keynesian model with unemployment to feature an endogenous labour market participation decision. However, a baseline model with a standard degree of risk aversion tends to exhibit countercyclical labour force participation, which is inconsistent with the data. In order to address this issue, we propose the notion of labour market participation as a social consideration, which we demonstrate to be capable of generating procyclical participation incentives. The basic idea is that agents will tend not to exit the labour force during booms in order to "keep up with the Joneses". We then find that plausible fluctuations in the size of the labour force do not exert a quantitatively significant effect on job creation.

In light of this result, we search for alternative mechanisms which may overturn the conclusion that inflationary policy is incapable of incentivising job creation. The approach taken involves switching focus to the characteristics of aggregate demand dynamics along the adjustment path to a monetary shock. It is well known that standard New Keynesian models fail to deliver the gradual, hump-shaped adjustment path to monetary policy shocks that is observed in the data. We argue that if aggregate demand experiences a persistent increase in response to a monetary shock instead of peaking on impact, the incentive for firms to create jobs becomes amplified. The intuition is that, since the job creation decision is forward-looking due to the presence of

matching frictions, aggregate demand must rise persistently even after the shock takes place so that firms anticipate a further increase in aggregate demand in order for the time consuming process of issuing a vacancy to be justified. To demonstrate this, it is shown that, by altering the dynamics of aggregate demand, time-inseparability in the utility function can significantly improve the ability of expansionary monetary policy to increase job creation, allowing the model to generate a downward sloping Beveridge curve conditional on monetary shocks. In the appendix to Chapter II, we lend further credence to this hypothesis by describing how the manner in which monetary policy itself is specified may give rise to hump-shaped adjustment dynamics and, consequently, amplify inflationary job creation.

Finally, in Chapter III on *Equilibrium Matching and Age Discrimination Policy*, we abstract from business cycle issues and concentrate instead on the life cycle. Federal legislation prohibiting the discrimination of workers on the basis of age has been in place in the United States since the 1967 Age Discrimination in Employment Act. Yet empirical studies which aim to estimate the employment effects of such legislation have yielded inconclusive results. We approach the issue from a different perspective by deriving quantitative predictions of equilibrium unemployment theory to investigate how age anti-discrimination legislation impacts labour market performance. We do not seek to measure the impact of a particular episode of legislative reform, but aim to quantitatively explore the general equilibrium consequences of restricting the ability of employers to hire on the basis of age. The main conclusion is that an equilibrium matching model of the life cycle predicts a moderately positive effect on the employment rate of workers very close to retirement, but the overall impact of age discrimination policy on the life cycle pattern of employment is quantitatively small. This occurs because in a frictional matching equilibrium, the incentive to discriminate against workers closer to retirement is offset by labour market congestion, preventing the demand for older workers from falling excessively even when it is possible to discriminate on the basis of age. If the demand for workers of a particular age were to fall sharply, the rate at which a given vacancy is matched with a worker of that age cohort increases, allowing firms to capitalise on quick vacancy transition rates stemming from weak competition in hiring. The model thus suggests that the extent to which the effect of a finite horizon is offset by labour market congestion is quantitatively significant, implying a modest role for age discrimination policy in shaping the life cycle profile of employment.

Welfare issues are also addressed. In particular, we demonstrate that an age-dependent inefficiency arises in the labour market participation decision of finitely-lived agents when firms are not able to discriminate in the hiring process on the basis of age. The intuition is that older workers, for whom only a relatively short productive time horizon remains, do not internalise the negative effect that their participation decision has on the age distribution of the aggregate pool of searchers. However, the size of this externality is quantitatively negligible. It is emphasised that the analysis in Chapter III concerns solely the macroeconomic implications of age discrimination policy as separate

and distinct from the issue of fairness which naturally arises in this context. Although the economic impact of age discrimination policy on employment is argued to be quite small, this does not of course imply that such policy does not have significant merit with respect to its assurance of the basic tenet of equal opportunity for all.

Chapter I

Structural Tax Reform and the Cyclical Behaviour of the Labour Market

Abstract

In this chapter, we quantitatively examine the consequences of structural tax reform in a real business cycle model with frictional unemployment and distortionary tax rates which are increasing in individual taxable labour income. The cyclical aspects of tax reform that are addressed in this chapter are distinct from the stationary state distributional issues that have garnered most of the attention in the existing literature on structural tax reform. Calibrating the model to U.S. data, we find that a reduction in the progressivity of the tax system is associated with a significant increase in the volatility of hours per worker whereas the impact on unemployment volatility is quantitatively small. The welfare cost of the business cycle increases after tax reform, but remains quantitatively negligible. We also assess the implications that the structure of the tax system has for the ability of expansionary government spending to stimulate economic activity. It is demonstrated that a less progressive tax policy increases the ability of expansionary fiscal policy to stimulate output due to a larger response in hours, but this comes at the cost of a smaller unemployment multiplier. In particular, tax reform causes a compositional shift in labour market adjustment in response to aggregate demand shocks, with relatively more adjustment occurring in the intensive margin and less adjustment in the extensive margin the flatter the tax schedule is.

1 Introduction

"How large are the costs of the federal income tax? They are larger than the federal budget deficit, larger than the Defense Department, larger than Social Security, perhaps as large as the combined budgets of the fifty states."

- Robert E. Hall and Alvin Rabushka in *The Flat Tax*, 1995.

In a comprehensive critique of U.S. tax policy, Hall and Rabushka (1995) make the case for a dramatic reform of the U.S. federal income tax system which would shift the economy from a progressive to a flat tax schedule. They argue that a flat tax would dramatically boost productivity by improving incentives to work and invest while shifting massive amounts of resources away from activities which merely serve to reduce tax liabilities to those that provide real economic functions. Their proposal is to implement a system in which all personal taxable income above a certain threshold

is taxed once, and only once, at a uniform rate, doing away with all of the complexities of the current system.

Involvement in the redistribution of income is one of the defining roles of government in modern democratic countries. Tax progressivity determines how the distribution of the tax burden is shared and for this reason progressive tax systems are usually justified by an appeal to fairness. Consequently, most of the attention in the literature focuses on the distributional impact of structural tax reform, as well as efficiency and long-run growth issues associated with lowering or flattening tax rates. Heer and Trede (2003) demonstrate that a progressive tax system reduces labour supply and savings, thereby reducing capital accumulation and income. But on the other hand, a flat tax reduces aggregate welfare in the long-run through an increase in inequality. By comparing stationary states, they find that the net welfare effect of switching to a flat tax is positive. Other studies on tax reform which analyse this type of long-run equilibrium trade-off include Ventura (1999), Erosa and Koreshkova (2007) and Cassou and Lansing (2003).¹

In contrast to the literature on distribution and growth, the objective of this chapter is to plausibly quantify the short-run cyclical implications of tax reform, particularly with respect to central labour market variables. More specifically, this chapter focuses on the consequences of labour income taxation for the dynamics of unemployment and average hours worked per employee in a real business cycle (RBC) model that is extended to allow for two additional features; matching frictions in the labour market and a graduated labour income tax schedule in which the rate of tax rises endogenously with increases in the taxpayer's own wage income. The key parameters of the tax rate function that determine the structure of the tax system are not calibrated arbitrarily, but are estimated using actual U.S. data over the post-war sample period. The central tax policy experiment is then represented as a shift from the empirically estimated U.S. tax schedule to a flat tax programme. We then consider how this change to the tax structure influences macroeconomic adjustment in response to both supply and demand shocks.

Tax reform influences equilibrium dynamics in our model primarily through behavioural changes in average hours per worker. We find that progressive taxation significantly reduces the incentive to work longer hours in response to positive productivity shocks by generating a steeper rise in the tax-adjusted marginal disutility of labour. Agents are more willing to increase labour supply during booms when it does not result in a commensurate increase in the marginal tax rate. Consequently, the volatility of average hours per worker relative to output increases by 27% for a baseline

¹Recent evidence for OECD countries, however, indicates that government cash transfers, the other lever of redistribution policy, are substantially more effective at reducing household income inequality than progressive taxation is (OECD, 2008). The OECD (2008) report also suggests that tax progressivity is negatively related to tax revenue (Tables 4.2 and 4.5 of the report), so neither can progressive tax systems be justified on the merit of the potential to support more generous cash transfer programmes. That the redistributive achievement of welfare state arrangements is largely determined by public cash transfers weakens the argument for tax progressivity as an implement of social protection.

calibration after the adoption of a flat tax. In the context of a frictional labour market, the greater the extent to which workers are willing to increase hours in response to a shock, the smaller the incentive for firms to engage in the costly matching process involved in creating new jobs. However, along the adjustment path to a productivity shock, hours initially rise above steady state on impact and then subsequently undershoot long-run equilibrium as employment and consumption gradually rise. Flat tax reform exacerbates both the initial rise and the subsequent fall in hours. Given that the job creation decision is forward-looking due to the presence of matching frictions, on balance the impact of tax reform on fluctuations in unemployment is quantitatively small. Under an alternative calibration in which the bargaining power of the worker is small, flattening the gradient of the tax schedule is associated with a moderate increase in unemployment volatility.

Under which tax policy is the representative household better off? The logic of Lucas (1987, 2003) is followed and the welfare gain from macroeconomic stabilisation is computed as the percentage of steady state consumption that a representative household would be willing to pay in order to have business cycle risk entirely eliminated. In this manner we can then determine under which tax policy business cycles are more costly. The welfare cost of business cycles, regardless of the tax system, is found to be quantitatively small, corroborating Lucas (1987, 2003). In the baseline model, the representative household subject to a progressive tax would be willing to give up only a mere 0.05% of steady state consumption in order to entirely eliminate business cycle fluctuations. Because tax reform exacerbates business cycle volatility, the welfare cost under a flat tax policy is higher, approximately 0.2% of steady state consumption. Despite increasing by an order of magnitude, welfare costs of cyclical fluctuations are still negligibly small.

We then extend the baseline framework by introducing government expenditure to assess how the structure of the tax system influences the efficacy of fiscal intervention. Government expenditure shocks that absorb output and exert a negative wealth effect on the household, as in Christiano and Eichenbaum (1992), are found to elicit a sustained increase in working hours which is amplified under a flat tax for much the same reason as for a productivity shock. However, in contrast to the dynamics induced by a productivity shock, it is demonstrated that larger fluctuations in hours are accompanied by smaller fluctuations in unemployment following a fiscal policy-induced shock to aggregate demand. This is because the negative wealth effect of government spending prevents hours from undershooting the steady state along the adjustment path, so that hours under a flat tax tend to be persistently higher. In response to this, firms expand the supply of vacancies to a lesser extent in order to meet the increase in aggregate demand. Structural tax reform - from a progressive to a proportional system - is therefore associated with a compositional shift in the way the labour market adjusts to aggregate demand disturbances, with relatively more adjustment occurring in the intensive margin (i.e. average hours per worker) and less adjustment in the extensive margin

(i.e. employment). We therefore find that the ability of expansionary fiscal policy to reduce unemployment is weakened by a flatter tax schedule due to the crowding out effect on job creation of more flexible working hours. The peak drop in unemployment resulting from an increase in government expenditure normalised to 1% of GDP falls from 0.32 percentage points under a progressive tax to 0.20 percentage points under a flat tax. Despite the smaller fall in unemployment, the output multiplier of fiscal policy increases from 1.08 to 1.22 on impact for a baseline calibration due to the amplified response in hours.

Before proceeding to develop the model, we briefly review how this study fits in with the related literature. The relationship between taxation and work behaviour has received enormous interest in the labour economics literature, and is far too voluminous to comprehensively review here (see Meghir and Phillips 2008 for a detailed survey). The key issue regarding our study is the willingness of agents to intertemporally substitute labour across periods in response to changes in the incentive to do so. Indeed, this is the central component in the propagation mechanism of tax reform in our model. But, as is well known, microeconomic evidence has not been supportive of large labour supply elasticities (Ball 1990 and Altonji 1986). However, Aaronson and French (2002) demonstrate that labour supply elasticities do tend to be biased downward in models which abstract from joint wage-hours determination and progressive taxation. Furthermore, authors who have studied specific tax reform episodes have found significant effects of reductions in progressivity on gains in taxable income (Feldstein 1995, Auten and Carroll 1999). In light of the literature arguing that net wage rates do not appreciably influence labour supply behaviour (e.g. Pencavel 1986 and Triest 1990), it is unclear whether such increases in taxable income are due to labour supply adjustments or other factors such as a reduction in tax avoidance and less of an emphasis on untaxed compensation. In this chapter, we try to contribute to the analysis through a different perspective, using a calibrated RBC model as a laboratory in which to quantitatively assess the transmission of exogenous shocks conditional on the tax code, whereas the previously mentioned studies abstract from business cycle issues. Our work can thus be viewed as a complementary attempt to quantify the influence of progressive taxation on labour supply, providing an RBC-based interpretation of the mechanisms underlying tax reform.

This chapter's approach to studying the business cycle implications of labour tax policy differs from models based on the seminal contributions of Braun (1994) and McGrattan (1994), in which tax rates are approximated using stochastic exogenous processes. The objective of these studies was to demonstrate that tax disturbances, along with other stochastic fiscal variables, are important driving forces of U.S. economic cycles, and they abstracted from issues related to tax progressivity and unemployment. The specification of the tax system in this chapter is such that tax rates rise endogenously with the individual's own level of taxable income and is based on Casou and Lansing (2003), who study the growth implications of the Hall and Rabushka

(1995) plan. The functional form is general enough to allow for regressive, proportional and progressive systems and is therefore suitable for capturing structural tax reform. There are no shocks to the tax rate itself.

Recent work on fiscal policy in the context of frictional labour markets by Arseneau and Chugh (2008) examines optimal tax policy over the business cycle, finding that the optimal tax rate is typically quite volatile. The authors, however, do not discuss tax progressivity or the consequences of structural tax reform for shock propagation, which is our main focus. Relatively little attention has been devoted in the literature to the subject of progressive taxation in cyclical matching equilibria, with most of the focus being on stationary state analysis. Previous work in this area includes Pissarides (1998) and Sinko (2005) who demonstrate that an increase in tax progression reduces stationary state unemployment if wages are determined through bargaining. Tax progressivity is modelled by assuming that workers receive a lump-sum transfer from the government (a tax-subsidy that does not vary with income) and are subsequently taxed on their total labour income including the subsidy. The receipt of this subsidy raises the worker's surplus value of the match so that Nash bargaining requires the worker to effectively transfer part of the subsidy to the employer by accepting a lower wage payment. This downward pressure on the wage raises the steady state supply of vacancies and equilibrium unemployment is lower the more progressive the tax system is.

Regarding business cycles, traditional Keynesian theory postulates that progressive income taxation automatically stabilises macroeconomic fluctuations. Guo and Lansing (1998) demonstrate that in real business cycle models with an indeterminate steady state, such as in Benhabib and Farmer (1994) and Farmer and Guo (1994), the presence of sufficiently progressive taxation can eradicate multiple equilibria by "taxing away" the higher returns associated with exogenous fluctuations in beliefs or "animal spirits". Progressive taxation has thus been shown to induce stability from the perspective of eliminating sunspot fluctuations. This interpretation of economic volatility differs to the one employed here in that we concentrate solely on rational expectations equilibria that are stable around a unique steady state. Fluctuations are then driven by a standard, exogenous real aggregate productivity shock.

This chapter builds on, and relates more closely to, Vanhala (2006) and Zanetti (2011) who study the role of labour taxation in shaping the response of frictional labour markets to shocks. Both studies introduce a graduated tax schedule using the tax-subsidy approach outlined previously and find that tax progressivity reduces the sensitivity of output and employment to aggregate shocks. The reason for this is that the wage rate becomes more sensitive to fluctuations in productivity, thereby absorbing most of the effect of the shock and leaving little incentive for firms to adjust the supply of jobs (Shimer 2005). But why is tax progressivity, in the form of a tax-subsidy which does not vary over time, associated with greater cyclicity in the wage rate? Recall that the steady state wage is decreasing in the tax-subsidy. All else equal, the lower the steady state wage, the greater will be the impact of a given productivity

shock on the logarithmic deviation of the wage from its steady state value.² The dynamic implications for wage behaviour thus essentially stem from this difference in the calibration of the model. In this respect the studies by Vanhala (2006) and Zanetti (2011) reinforce the point made by Hagedorn and Manovskii (2008), who demonstrate that the cyclical behaviour of the canonical equilibrium matching model is sensitive to the calibration procedure. In particular, they show that if the worker's value of leisure is large enough, the model will generate realistic unemployment fluctuations. However, increasing the worker's value of leisure in the Nash wage equation is observationally equivalent to decreasing the value of the tax-subsidy (i.e. reducing tax progressivity). Both parameter adjustments lead to the same increase in the elasticity of the wage with respect to productivity and therefore have similar repercussions for unemployment dynamics.

There are a number of differences between this chapter's analysis and Vanhala (2006) and Zanetti (2011). First and foremost is the nature of the tax function. Although the concept of a tax-subsidy has the virtue of simplicity, it is somewhat abstract and therefore difficult to calibrate. Neither author calibrates the degree of tax progressivity based on any evidence and so the studies are restricted to making only qualitative inferences about the dynamic implications of the tax structure. In contrast, using U.S. tax return data we estimate the gradient of the wage income tax schedule and benchmark it against other estimates in the literature. This makes it possible to obtain a more quantitatively reliable prediction about the business cycle effects of tax reform. More fundamentally, holding marginal tax rates constant as in the tax-subsidy approach *a priori* rules out certain aspects of the joint dynamics of wage rates and taxation that could theoretically lead to an inverse relationship between tax progressivity and wage volatility. We discuss how time variation in tax rates in our model can cause either larger or smaller fluctuations in the wage depending on the calibration. Aside from improved consistency with the fact that actual progressive tax systems are characterised by incremental tax brackets as households face marginal rates that depend on their level of income, our approach therefore possesses the desirable theoretical trait of accommodating a broader potential set of outcomes. This stands in contrast to the tax-subsidy mechanism which necessitates a positive relationship between tax progressivity and wage volatility. Our simulation exercises then indicate that the effect of tax movements on wage fluctuations is quantitatively small. Consequently, in the absence of variable hours, which do not feature in Vanhala (2006) or Zanetti (2011), we find no evidence of significant business cyclical implications of tax reform.³

²It is straightforward to explain this intuition in symbols. Let w_t be the time t Nash equilibrium wage, τ the tax-subsidy and A_t a productivity shock. Assume that the wage depends only on the productivity shock and the tax subsidy, which is constant, and that τ does not multiply A_t . Using a circumflex to denote logarithmic deviations from steady state values, the latter being denoted by removing t subscripts, the log-linearised wage equation is expressed as $\hat{w}_t = \frac{\partial w / \partial A}{w/A} \hat{A}_t$. Then, the steady state elasticity, $\frac{\partial w / \partial A}{w/A}$, is decreasing in w which in turn is decreasing in τ .

³The importance of wage fluctuations in determining unemployment volatility is stressed by Shimer (2005).

Although the Vanhala and Zanetti studies examine various aspects of labour market policy, they ignore spending by the federal government. The business cycle implications of fiscal stimulus have gained renewed interest in the literature given recent fiscal developments in the U.S. and Europe. Examples include Monacelli et al. (2010) and Bruckner and Pappa (2010). The current study contributes to this related literature in a novel way by illustrating how the gradient of the labour income tax schedule determines the ability of the government to expand output and reduce unemployment through public expenditure. Furthermore, and crucially, our inclusion of variable working hours demonstrates how tax reform gives rise to a compositional shift in labour market adjustment to demand shocks, while Moncelli et al. (2010) and Bruckner and Pappa (2010) consider only extensive labour market adjustment. In the absence of intensive adjustment we do not find significant effects of government expenditure at the cyclical frequency.

The rest of the chapter is organised as follows. The next section builds the model. Section 3 describes the calibration procedure and solution algorithm. Baseline results and extensions are reported in Section 4. Section 5 concludes and discusses prospects for further research.

2 Model

This section extends the baseline real business cycle framework in two main ways. First, unemployment is introduced through matching frictions in the labour market, as first implemented in dynamic stochastic general equilibrium by Merz (1995) and Andolfatto (1996). These authors integrated the matching model of unemployment as described by Diamond (1982), Mortensen (1982) and Pissarides (2000) with the real business cycle approach to studying macroeconomic fluctuations advanced by Kydland and Prescott (1982). The flexibility of both the Diamond-Mortensen-Pissarides and RBC paradigms allowed their combination to be especially attractive for purposes of tractability. Although it is common to abstract from variation in the hours each employee works in such models, we introduce variable hours in the spirit of Trigari (2009) and Holt (2008).

The second important feature of the current setup is the specification of the tax policy. To begin with, taxation on business income is abstracted from, so that the only form of income that is subject to taxation in the model is labour income. This is unlike the model developed by Cassou and Lansing (2003) who stipulate a much more detailed representation of the U.S. tax structure, including features such as the double taxation of business income, investment tax credits and standard deductions. In this chapter, these features are abstracted from in order to keep the model as simple as possible and maintain focus on labour market dynamics. This assumption should not bear much influence on the analysis given this chapter's emphasis on labour market dynamics as opposed to capital accumulation. Furthermore, as noted by Hall and Rabushka (1995), despite oppressively high marginal tax rates on business income through the

combination of corporate and income taxes (as a consequence of the double taxation built into the U.S. tax system), the actual amount of tax that is paid on business income is in general, as the authors described it, "remarkably small".

Aside from these two additional features, the rest of the theoretical model is described by what is now widely accepted as standard RBC-matching theory. Distortionary taxation breaks the correspondence between competitive equilibrium and the social optimum, and so instead of assuming the existence of a social planner, the decentralised equilibrium is solved for. We proceed first with the household's optimisation problem.

2.1 Households

Time is discrete. Random matching in the labour market generates employment risk which would make individual consumption dependent on each agent's entire labour market history. In order to avoid issues relating to heterogeneity and inequality, the institutional structure of the household follows the seminal contributions of Merz (1995) and Andolfatto (1996). There is a single representative household with a continuum of members defined over the unit interval who pool their income in order to insure away employment risk. The household chooses consumption at each date t , c_t , in order to maximise lifetime utility described by the objective function

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - H_t \right] \quad (1)$$

where $0 < \sigma$ is the coefficient of relative risk aversion and H_t is the sum of disutilities of work of all household members. β is a discount factor and E_t is the conditional expectations operator. Unemployed agents search for jobs with constant intensity taking aggregate labour market conditions as given, implying that employment is determined according to the matching technology in a process to be described below. Hours are determined not at the level of the representative household, but via decentralised Nash bargaining of individual agents, as in Trigari (2009) and Holt (2008). Hence, the household's optimisation problem does not feature a leisure trade-off. The measure of the household's members currently in employment is given by n_t . Labour force participation is abstracted from so that unemployment is $u_t = 1 - n_t$, where the labour force is normalised to unity.

Optimisation is subject to a budget constraint given by

$$c_t + i_t = n_t \int_0^1 (1 - \tau_{i,t}) w_{i,t} h_{i,t} di + r_t k_t + \mathbb{U}_t + T_t + D_t \quad (2)$$

where i_t denotes private investment, individual hours worked are indexed by $h_{i,t}$ and the wage rate per hour is $w_{i,t}$. $\tau_{i,t}$ is the tax rate paid on the i th individual's gross labour income, $w_{i,t} h_{i,t}$. As will be demonstrated subsequently, in the absence of idiosyncratic

heterogeneity each individual will receive the same wage and work the same number of hours so that each individual's taxable income is the same. In anticipation of this, (2) can be simplified to

$$c_t + i_t = n_t (1 - \tau_t) w_t h_t + r_t k_t + \mathbb{U}_t + T_t + D_t.$$

The return on capital is r_t , k_t represents the private capital stock and \mathbb{U}_t is unemployment income (specified below). T_t and D_t are, respectively, a lump-sum transfer from the government and dividend income that the household receives as the diversified owner of firms. The tax rate is endogenously determined and time dependent, given by the nonlinear function

$$\tau_t = 1 - \zeta \left(\frac{wh}{w_t h_t} \right)^\phi \quad (3)$$

where steady state values are denoted by removing time subscripts. This functional form follows Chen and Guo (2010). Letting $w_t h_t = L_t$, the marginal tax rate is defined as

$$\tau_t^m = \frac{\partial \tau_t L_t}{\partial L_t} = 1 - \zeta (1 - \phi) \left(\frac{L}{L_t} \right)^\phi.$$

As in Chen and Guo (2010), we restrict $0 \leq \tau_t, \tau_t^m \leq 1$. Then, if $\phi > 0$, the marginal tax rate is increasing in taxable labour income. In this case the tax system is said to be progressive. When ϕ is negative, it is regressive. When $\phi = 0$, the marginal tax rate is constant and independent of L_t and this is referred to as a flat, or proportional, tax. In what follows we will focus attention on the dynamic implications of moving from a progressive tax system, $\phi > 0$, to a proportional system, $\phi = 0$. The parameter ζ determines the steady state value of the tax such that

$$\tau = 1 - \zeta$$

whereas ϕ determines the slope of the tax schedule. The tax schedule in (3) does not feature any stochastic elements, and fluctuations in τ_t arise solely as a result of endogenous movements in the level of personal taxable income. This specification of tax policy differs fundamentally from various studies which approximate tax behaviour by fitting autoregressive processes to actual tax data, and then using the resulting stochastic equations as forcing variables to drive the business cycle.⁴ In contrast, there is no uncertainty in our tax specification so that the business cycle repercussions of tax policy in this model derive from the interaction of τ_t with other endogenous variables.

The other constraint on household optimisation is a standard capital accumulation equation, given by

$$k_{t+1} = (1 - \delta) k_t + i_t. \quad (4)$$

In modelling fiscal policy, government investment and therefore public capital ac-

⁴For studies on stochastic fiscal policy, see Finn (1998), McGrattan (1994) and Braun (1994) who build on the seminal work of Christiano and Eichenbaum (1992).

cumulation is abstracted from. Therefore, all investment and capital accumulation in the model is undertaken by the private sector, so that k_t and i_t are private (household) values. Maximising (1) with respect to c_t subject to (2) and (4) yields a standard Euler equation

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} (r_{t+1} + 1 - \delta) \quad (5)$$

which simply states that the net effect of marginal intertemporal variations in consumption on lifetime utility is zero at an optimum. In particular, since capital income is not subject to taxation in the current model, taxes do not distort the savings decision.

2.2 Firms and the Labour Market

Without loss of generality, it is assumed that the output market is competitive and comprised of a large number of small firms with each firm posting a single job. In this setup, the terms "match", "job" and "firm" can be used interchangeably, with either being defined as a worker-firm pair. The methodology of Heer and Maussner (2009) is followed, in which capital and labour are both inputs into the production technology. In contrast to the latter authors, we abstract from endogenous job destruction. Firms rent capital services from households, the assumed owners of the capital stock. The individual firm production technology is constant returns to scale and is subject to aggregate and idiosyncratic uncertainty. In symbols, output in each match i is given by

$$y_{i,t} = A_t k_{i,t}^\alpha h_{i,t}^{1-\alpha} \quad (6)$$

where A_t is an aggregate productivity shock common to all matches, and $\alpha \geq 0$ denotes the elasticity of match output with respect to capital. Matches are destroyed exogenously at the rate ρ . Capital is rented from a common market so that the rental cost is identical across firms, implying that the capital-output ratio is equalised across all jobs. Furthermore, in the absence of idiosyncratic heterogeneity, equilibrium hours worked will also be the same across all matches. Individual hours are therefore equal to average hours per worker, h_t . The quantity of capital employed in each match is then the same, given by $k_{i,t} = k_t/n_t$, where an omission of the i subscript denotes an aggregate value (k_t is aggregate capital).

Unemployment arises in the model as a consequence of costly, uncoordinated search in the labour market. The measure of successful matches in period t is given by an aggregate matching function which randomly pairs job seekers with vacancies. Denote the aggregate measure of measure of vacancies v_t . The aggregate matching function $M(v_t, u_t)$ is increasing in both of its arguments, concave and homogenous of degree 1. Labour market tightness is defined as

$$\theta_t = \frac{v_t}{u_t}. \quad (7)$$

Random matching implies that the probability that a vacant job is filled at time t

is

$$q(\theta_t) = \frac{M(v_t, u_t)}{v_t} = M(1, \theta_t)$$

where it has been implicitly noted that the homogeneity assumption permits a representation of the probability of filling a vacancy solely as a function of θ_t . Similarly, the probability that an unemployed agent finds a job at time t is

$$p(\theta_t) = \frac{M(v_t, u_t)}{u_t} = M(\theta_t, 1)$$

and the two matching probabilities are related by $p(\theta_t) = \theta_t q(\theta_t)$. Congestion in the labour market implies that $q'(\theta_t) < 0$ and $p'(\theta_t) > 0$ for $\theta_t > 0$. Following Pissarides (2000), the matching technology is assumed to have the Cobb-Douglas form, which implies constant matching elasticities,

$$M(v_t, u_t) = m v_t^\gamma u_t^{1-\gamma}$$

where $0 < m$ and $0 \leq \gamma \leq 1$ are constants.

The recursive representation of the labour market is defined in terms of the Bellman equations which characterise the asset values of occupied jobs, vacancies, employed workers and unemployed agents. A firm's value of an occupied job, J_t , and a vacancy, V_t , are given respectively by

$$J_t = A_t \left(\frac{k_t}{n_t} \right)^\alpha h_t^{1-\alpha} - w_t h_t - r_t \left(\frac{k_t}{n_t} \right) + E_t \beta_{t+1} [(1 - \rho) J_{t+1} + \rho V_{t+1}] \quad (8)$$

and

$$V_t = -\kappa + q(\theta_t) E_t \beta_{t+1} J_{t+1} + (1 - q(\theta_t)) E_t \beta_{t+1} V_{t+1}. \quad (9)$$

The stochastic discount factor is defined as $\beta_{t+1} = \beta (c_t/c_{t+1})^\sigma$. Consider equation (8) first. Productive matches yield a flow profit to the firm equal to the difference between match output and the factor payments. The firm's continuation value of the match is given by the discounted term in (8). With probability $1 - \rho$, a productive match at time t will survive to the production stage of period $t + 1$. With probability ρ , the match becomes vacant, or is destroyed, at the beginning of time $t + 1$ (or the end of time t). Equation (9) states that vacancy posting entails a flow cost $-\kappa$ and with probability $q(\theta_t)$ results in a match that becomes productive, at the earliest, at time $t + 1$.⁵

The firm decides on the quantity of capital to rent in order to maximise J_t . The first-order condition for profit maximisation is

$$r_t = \alpha \frac{y_t}{k_t} \quad (10)$$

⁵This time lag between matching and production is common in discrete time matching equilibria. It facilitates defining employment as a predetermined state variable that is not subject to change in the current period.

where $y_{it} = y_t/n_t$. The Bellman equations describing the asset values of an employed agent, W_t , and an unemployed agent, U_t , are, respectively,

$$W_t = (1 - \tau_t) w_t h_t - \frac{h_t^{1+\varphi}}{1+\varphi} c_t^\sigma + E_t \beta_{t+1} [(1 - \rho) W_{t+1} + \rho U_{t+1}] \quad (11)$$

and

$$U_t = b_1 + b_2 c_t^\sigma + p(\theta_t) E_t \beta_{t+1} W_{t+1} + (1 - p(\theta_t)) E_t \beta_{t+1} U_{t+1}. \quad (12)$$

The flow value of being employed to the worker is given by net labour income adjusted for the disutility of work, expressed in units of the consumption good. The latter is given by $\frac{h_t^{1+\varphi}}{1+\varphi} c_t^\sigma$, where $\varphi \geq 0$ governs the elasticity of labour supply. The continuation value of the match to the employed worker is equal to a weighted average of the conditional expected time $t + 1$ values of employment and unemployment. In equation (12), b_1 denotes the consumption value of government funded unemployment benefits obtained during job search, which are not subject to taxation. The term $b_2 c_t^\sigma$ represents the (exogenous) consumption value of a home-produced good, or leisure, that the agent enjoys whilst not at work. If $b_2 < 0$ it could be interpreted as a fixed cost of job search. With probability $p(\theta_t)$ the unemployed agent at time t encounters a match that will operate successfully in period $t + 1$.

2.3 Labour Market Equilibrium

2.3.1 Vacancy Supply

The equilibrium supply of vacancies is determined by a free entry condition which ensures that the asset value of posting a new vacancy is zero for all t . In symbols,

$$V_t = 0 \quad \forall t$$

and from equation (9) it follows that

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} J_{t+1}. \quad (13)$$

The left hand side of the above equation represents the flow cost of maintaining a vacancy, κ , multiplied by the expected duration that a vacancy will go unmatched, which is given by the reciprocal of the matching rate, $q(\theta_t)^{-1}$. In equilibrium, this expected cost of creating a vacancy at time t must be equal to the discounted conditional expected value of a successfully operating job in the following period. To see how this condition endogenously regulates the supply vacancies, suppose that there is a positive aggregate productivity shock which raises the right hand side of (13). Recalling that $q'(\theta_t) < 0$, it follows that by increasing the supply of vacancies the expected cost of posting a vacancy rises due to increasing congestion costs until equality is restored. Thus, equation (13) postulates a positive relationship between the asset value of a job

and the supply of vacancies.

This intuition governing vacancy supply can be translated into a dynamic context by combining the equilibrium condition (13) with the Bellman equation (8), iterated forward one period. The following difference equation in the expected cost of vacancy supply is obtained,

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} \left[A_{t+1} \left(\frac{k_{t+1}}{n_{t+1}} \right)^\alpha h_{t+1}^{1-\alpha} - w_{t+1} h_{t+1} - r_{t+1} \frac{k_{t+1}}{n_{t+1}} + (1 - \rho) \frac{\kappa}{q(\theta_{t+1})} \right]. \quad (14)$$

The intuition underlying the above expression is similar to that outlined for equation (13), except that the $t + 1$ asset value of an occupied job has been replaced with the model's endogenous variables. This asset value is thus equal to the conditional expected value of profits at time $t + 1$, plus the equilibrium continuation value conditional on survival as captured by the term $(1 - \rho) \kappa / q(\theta_{t+1})$. The benefit of not having to search for additional labour once a match has been formed gives rise to local monopoly rents over which the firm and the worker bargain to determine the wage payment.

2.3.2 Wage Determination

Costly search frictions give rise to a joint surplus value of maintaining current matches. Define this surplus as

$$S_t = (W_t - U_t) + (J_t - V_t). \quad (15)$$

S_t is shared via the wage according to the Nash product

$$w_t = \arg \max (W_t - U_t)^\eta (J_t - V_t)^{1-\eta}$$

where η and $1 - \eta$ are the bargaining power of the worker and the firm, respectively. There are no contractual impediments to wage setting so that wages are fully flexible and re-contracted every period as new information becomes available. From the Bellman equations (11) and (8) it then follows that

$$\begin{aligned} \frac{\partial W_t}{\partial w_t} &= (1 - \phi) (1 - \tau_t) h_t \\ \frac{\partial J_t}{\partial w_t} &= -h_t. \end{aligned}$$

The presence of the tax slope parameter ϕ in the expression for $\partial W_t / \partial w_t$ implies that agents internalise the structure of the tax schedule during wage negotiation. In particular, workers explicitly recognise that bidding up the wage results in a higher marginal tax rate if ϕ is positive. The more graduated the tax schedule, the smaller the net return to the worker of an incremental rise in the wage rate. Hence the derivative $\partial W_t / \partial w_t$ is decreasing in ϕ . This behaviour arises in our model because the tax rate is endogenously determined, and is absent from the models of Vanhala (2006) and Zanetti (2011) in which the tax rate is assumed to be constant.

The first-order condition for wages is

$$W_t - U_t = \frac{\eta}{1 - \eta} (1 - \phi) (1 - \tau_t) J_t. \quad (16)$$

The worker receives a fraction of the firm's surplus, which is decreasing in the degree of progressivity as well as the current tax rate. Note that although the tax rate influences the division of the surplus, this does not imply that a rise in τ_t lowers the wage. Furthermore, the joint surplus of the match is decreasing in τ_t , since for every unit wage increment conceded by the firm, the worker receives the fraction $1 - \tau_t$. Therefore, the higher the tax rate, the smaller the surplus to be bargained over. This does not imply that the wage rate necessarily falls, since the impact of the tax rate on the worker's outside option needs to be accounted for.

Substituting the respective Bellman equations into the first order condition (16) yields the equilibrium wage,

$$\begin{aligned} w_t h_t = & \frac{1 - \phi}{1 - \eta\phi} \eta \left[A_t \left(\frac{k_t}{n_t} \right)^\alpha h_t^{1-\alpha} - r_t \frac{k_t}{n_t} + (1 - \rho) \frac{\kappa}{q(\theta_t)} \right] \\ & + \frac{1 - \phi}{(1 - \eta\phi)} (1 - \eta) \\ & \times \left[\frac{b_1 + b_2 c_t^\sigma + \frac{h_t^{1+\varphi}}{1+\varphi} c_t^\sigma}{(1 - \phi) (1 - \tau_t)} - (1 - \rho) (1 - p(\theta_t)) \frac{\eta}{1 - \eta} \frac{E_t(1 - \tau_{t+1})}{(1 - \tau_t)} \frac{\kappa}{q(\theta_t)} \right] \end{aligned} \quad (17)$$

Equation (17) is similar in nature to the standard expression derived in Pissarides (2000), except for the presence of the endogenous progressive tax policy. In particular, the equilibrium wage rate is a weighted average of the contribution of the worker to the match and the worker's outside option, where the weights are given by the bargaining power of the worker and the firm, respectively. The entire right hand side of (17) is scaled down by the factor $(1 - \phi) / (1 - \eta\phi)$ which is decreasing in ϕ . This reflects the joint incentive facing the worker and the firm to keep wages low in order to avoid high tax burdens that drain the joint match surplus under progressive tax policies. Note, however, that $(1 - \phi) / (1 - \eta\phi)$ is inversely proportional to the worker's bargaining weight and is equal to 1 when $\eta = 1$.⁶ Flat tax reform that sets $\phi = 0$ and $(1 - \phi) / (1 - \eta\phi) = 1$ therefore tends to increase the elasticity of the wage with respect to improvements in working conditions and thus has the potential to exacerbate wage fluctuations in response to shocks.

The tax system has another effect on the wage through the worker's outside option, which is contained in the second square brackets in (17). The outside option is comprised of two elements. The first is the (untaxed) flow value of unemployment consumption including the opportunity cost of labour, adjusted for the tax rate. Note

⁶When $\eta = 1$, it is not the joint surplus which is maximised, but the worker's surplus. Since the latter is always increasing in the wage and the marginal tax rate never exceeds unity, tax progressivity no longer incentivises wage moderation.

that the tax distortion is amplified by the factor $(1 - \phi)^{-1} \geq 1$ in recognition of the progressivity of the tax schedule. The second element is the surplus value to the worker of continuing the current match into future periods, which is the term that enters negatively in the second brackets in (17). The rents to the worker of being in the current match are decreasing in the availability of other jobs captured by $p(\theta_t)$. If it were certain that the agent would find another job outside of the current match - that is, if $p(\theta_t) = 1$ - then the surplus value to the worker from continuing the current match into the future period is zero. Similarly if $\rho = 1$. The continuation value also depends on the time path of taxation. The lower future tax rates are relative to the current period, the greater the continuation value of the match is to the worker. The continuation value enters with a negative sign because the higher the continuation value to the worker of the current match, the greater the desire of the agent to remain in current employment and therefore the lower the wage needed to provide the incentive to do so.

The effect that an increase in τ_t has on the wage therefore depends on the sign of the second brackets in (17). Higher τ_t increases the relative consumption value of unemployment, which positively affects the wage rate. Note that this effect is increasing in ϕ since the tax distortion is amplified by $(1 - \phi)^{-1}$; the more progressive the tax system, the greater the extent to which w_t is bid up given the improvement in the untaxed consumption value of unemployment, since workers anticipate that the increase in w_t precipitates a further increase in τ_t if the tax schedule is graduated. Conversely, higher current tax rates relative to future tax rates act to reduce the current wage because the surplus value to the worker of continuing the match into the (lower-tax) future becomes greater, prompting the worker to accept a decline in the current wage. The direction of the dependence of w_t on τ_t is therefore uncertain. It is possible in principle for an increase in w_t which raises τ_t , say due to a positive productivity shock, to either be reinforced by an increase in the tax rate or offset by it. If the latter, then coupled with the wage moderation effect of progressivity captured by the factor $(1 - \phi) / (1 - \eta\phi)$, the wage would tend to become more volatile after tax reform as ϕ is set to zero and fluctuations in τ_t are eliminated. On the other hand, if w_t depended positively on τ_t , setting ϕ to zero would contribute to wage stability by eliminating fluctuations in the tax rate, but this dynamic effect would have to be offset against the increase in $(1 - \phi) / (1 - \eta\phi)$ which reduces incentives for wage moderation. Note that if the wage depends positively on the tax rate, a tax-wage multiplier arises due to the positive dependence of the tax rate on the wage from (3).

On the whole, therefore, the impact of tax reform on wage dynamics is ambiguous. Wage dynamics, however, are key to determining unemployment dynamics. In accordance with the analysis of Shimer (2005), the more stable the wage path, the greater the extent to which shocks filter through to unemployment.⁷ The extent to which wages become more or less volatile then determines the quantitative effect of this channel of

⁷The crux of the argument is that if wages respond very elastically to an increase in productivity, the profits from issuing a vacancy decline for firms and so job creation rises by less.

tax policy on unemployment dynamics. The other important channel of tax policy is the behaviour of hours, which we describe next.

2.3.3 Optimal Hours

Hours are determined through decentralised Nash bargaining at the level of the individual match. In the absence of individual heterogeneity, equilibrium hours worked are the same for all workers. Hours are set so as to maximise the Nash product

$$h_t = \arg \max [W_t - U_t]^\eta [J_t - V_t]^{1-\eta}.$$

From Bellman equations (8) and (11), it follows that

$$\begin{aligned} \frac{\partial W_t}{\partial h_t} &= (1 - \phi) (1 - \tau_t) w_t - h_t^\varphi c_t^\sigma \\ \frac{\partial J}{\partial h_t} &= (1 - \alpha) A_t \left(\frac{k_t}{n_t} \right)^\alpha h_t^{-\alpha} - w_t. \end{aligned}$$

The first-order condition for hours is

$$\begin{aligned} 0 &= \eta (W_t - U_t)^{-1} ((1 - \phi) (1 - \tau_t) w_t - h_t^\varphi c_t^\sigma) \\ &\quad + (1 - \eta) J_t^{-1} \left((1 - \alpha) A_t \left(\frac{k_t}{n_t} \right)^\alpha h_t^{-\alpha} - w_t \right). \end{aligned}$$

Making use of the first-order condition for wages (16) in the above expression, optimal hours are determined by

$$\frac{h_t^\varphi c_t^\sigma}{(1 - \phi) (1 - \tau_t)} = (1 - \alpha) A_t \left(\frac{k_t}{n_t} \right)^\alpha h_t^{-\alpha}. \quad (18)$$

The above equilibrium condition stipulates that hours are negotiated such that the tax-adjusted marginal rate of substitution between consumption and leisure is equal to the marginal product of labour. Since higher tax rates lower the net return from working, the consumption value of the disutility of work is amplified by the factor $((1 - \phi) (1 - \tau_t))^{-1}$, which is increasing in the degree of tax progressivity. Progressivity amplifies the tax distortion on hours, just as for wages. For $\varphi, \alpha > 0$, equation (18) implicitly defines a negative relationship between h_t and the tax rate as well as ϕ . In this sense, progressive taxes disincentivise work effort.

To gain some intuition for the implications of progressivity in the tax system for the dynamic behaviour of hours, consider the repercussions of a positive productivity shock in (18). As the match becomes more productive, agents respond by increasing the number of hours worked. Taxable income increases causing τ_t to rise. The extent to which τ_t rises for a given increase in hours depends on ϕ . The more progressive the tax system, the larger the increase in the multiplier $((1 - \phi) (1 - \tau_t))^{-1}$. This attenuates the incentive to work longer hours during a boom, and h_t increases by less. Had the

tax system been proportional, the tax rate is constant at τ with $\phi = 0$ and there would be no increase in the tax rate to offset the willingness to work more. All else constant, we would expect progressive labour taxation to weaken the procyclicality of average hours. This intuition plays a central role in the quantitative analysis to follow.

2.4 Aggregation

This section states the aggregate consistency conditions that are necessary in equilibrium to close the model. Since output in each match is the same, aggregate output is defined as

$$y_t = n_t y_{i,t} = A_t k_t^\alpha (n_t h_t)^{1-\alpha}. \quad (19)$$

Matches are destroyed at the end of every period, after production takes place. The aggregate employment rate at the start of period $t + 1$ is then equal to the fraction of employed workers that survived period t plus the measure of new matches,

$$n_{t+1} = (1 - \rho) n_t + M_t \quad (20)$$

where $M_t = q(\theta_t) v_t = p(\theta_t) u_t$.

Unemployment income is given by

$$\mathbb{U}_t = b_1 u_t$$

which the household can spend on consumption or investment. The government is assumed to operate a balanced budget

$$b_1 u_t + T_t = n_t \tau_t w_t h_t$$

where T_t is the lump-sum transfer payment (or tax if $b_1 u_t > \tau_t n_t w_t h_t$) to households that features in the latter's budget constraint (2). Combining the government's budget constraint with the household's and noting that the aggregate dividend paid to the household through its diversified ownership of firms is equal to aggregate profits, the aggregate resource constraint is

$$c_t + i_t = y_t - \kappa v_t. \quad (21)$$

Given that all agents work the average number of hours h_t , for each agent i we have that $h_{i,t} = h_t$ and the disutility from labour effort for the representative household is simply

$$H_t = n_t \int_0^1 \frac{h_{i,t}^{1+\varphi}}{1+\varphi} di = n_t \frac{h_t^{1+\varphi}}{1+\varphi}$$

It is assumed that the aggregate productivity shock is lognormally distributed and

follows an exogenous stochastic process given by

$$\ln A_t = P_A \ln A_{t-1} + \xi_{A,t}$$

where $\xi_{A,t} \sim N(0, \sigma_A^2)$ and $0 \leq P_A \leq 1$.

3 Solution and Calibration

The aggregate system of equations of the model is log-linearized around a stationary state equilibrium. The resulting linear, rational expectations model is solved using the method of undetermined coefficients, as described in Uhlig (1997). Attention is restricted to local cyclical behaviour around a known steady state. The appendix outlines that the solution takes the form of (stable) equilibrium laws of motion that are linear in the logarithmic deviations of the model's variables from their stationary state values. Artificial time series are then computed from this set of equations by iteration of the equilibrium laws of motion. 200 random samples of 300 periods each are obtained and, to reduce dependence on initial conditions, the first observations of each sample are discarded to match the corresponding sample period of U.S. quarterly data. We consider a sample period spanning quarterly data from 1965:1 to 2005:4. The appendix to this chapter contains a description of the data we use. All data, simulated and actual, are logged and detrended using an HP filter with smoothing parameter 1600. The model's cyclical properties are then computed under different tax regimes.

Our calibration strategy for the labour income tax schedule follows the general methodology of Cassou and Lansing (2003) and Chen and Guo (2010). Specifically, we use non-linear least squares regressions to estimate the tax code parameters ζ and ϕ from (3). The difference to the previous authors is that we only consider taxation on wage income, whereas they allow for a richer tax specification that includes business income. To be able to estimate the parameters, data on average tax rates and an empirical counterpart to the inverse ratio of taxable labour income to its mean level are needed. Marginal federal tax rates on wage income are computed using the TAXSIM model which is available at the website of the National Bureau of Economic Research. The empirical counterpart to the taxable income ratio $\frac{wh}{w_t h_t}$ is obtained from average salary and wage data reported on W-2 Forms, available at the website of the Internal Revenue Service. A more detailed description of the data is contained in the appendix.

In order to account for changes to the federal income tax law that have occurred during the sample period in question, we estimate regressions for the tax years 1965, 1975, 1985, 1995 and 2005. The results are reported in Table 1. The results indicate that there has been a certain degree of variation in both the level and slope of the estimated labour tax function. There have been at least two notable tax reforms during our sample period, the Tax Reform Act of 1969 (TRA-69) and the Tax Reform Act of 1986 (TRA-86). TRA-69 appears to have resulted in a lower level parameter (higher

average tax), which decreases from 0.89 in 1965 to 0.80 in 1975 and 1985. The slope of the schedule increased slightly from 0.14 to 0.15, but then fell back to 0.14 over the same period. In contrast, TRA-86 appears to have resulted in both a slight decrease in average taxes and a notable decrease in progressivity, with the slope parameter falling from 0.14 to 0.10 after 1985. Comparing our results to those of Chen and Guo (2010), their finding of a tendency for progressivity in total income tax to decrease over time in post-war U.S. data still holds when isolating wage taxes. Their estimates are slightly different to ours, but not excessively so. They estimate the slope parameter for the period 1966 to 1986 to be about 0.17, falling to about 0.06 from 1987 to 2005. This suggests that the reduction in the progressivity of business income tax has been somewhat sharper than for wage income. They also find little variation in the level parameter, which is roughly constant at 0.8 according to their estimates. Our results suggest that the variation in the average level of wage taxes has been more noticeable. Nevertheless, the difference to Chen and Guo’s estimates is not drastic.⁸

The use of an average measure of tax progressivity is most convenient for our purposes of analysing business cycle moments over a long horizon. We therefore set $\zeta = 0.84$ and $\phi = 0.13$, the averages of our estimates in Table 1. This gives a steady state tax rate of $\tau = 0.16$. Our main focus is on tax progressivity, keeping ζ fixed across policy experiments. The hypothetical tax reform experiment that we concentrate on involves a reduction in the parameter ϕ from its initial baseline value of 0.13 to zero, thereby entirely eliminating tax progressivity.⁹ All other parameters are unchanged, including the variance of the productivity shock. In this manner, we attempt to approximate what the U.S. business cycle moments may have looked like had labour tax not been progressive, holding all other factors constant.

As emphasised by Merz (1995) and Trigari (2009), there is no consensus regarding the convexity parameter φ for the disutility of hours. This parameter governs the intertemporal elasticity of substitution of labour effort, defined as its reciprocal. Micro estimates of φ^{-1} range from close to zero to 0.5 (Trigari 2009), whereas representative agent macro studies assume much larger values of up to 4 (Christiano and Eichenbaum 1992). The RBC literature also contains examples in which utility is linear in hours, which can be theoretically justified by an appeal to labour indivisibilities (Hansen 1985). Our strategy is to set $\varphi = 0.2$, which lies at the upper end of the values considered in the literature, in order to replicate realistic hours variation. The model with $\varphi = 0.2$ is referred to as the baseline. We also set $\varphi = 100$ to essentially shut down hours variation

⁸As Hall and Rabushka (1995) note, the lion’s share of adjusted gross income goes to wages and salaries, with the latter typically comprising about 70-80% of total adjusted gross income, whereas business income is about 3-5%. These numbers are stable over time. For instance, in 2008 total gross adjusted income was \$5.7 trillion, of which wages and salaries were \$3.8 trillion and net business income was \$0.19 trillion. See <http://www.irs.gov> for data.

⁹Some analyses of flat tax reform choose to concentrate on a revenue-neutral change in order to avoid issues related to the optimal size of government in an economy. See, for example, Cassou and Lansing (2003). These concerns are more applicable to long-term growth studies, where changes in the tax parameters can lead to large differences in the size of government over time. Our business cycle focus renders the issue less of a concern, since we consider small shocks around a steady state.

in order to examine the role of intensive adjustment in the transmission process.

Aside from the calibration of the tax schedule and disutility of work, our assumed parameter values are largely standard in the RBC-matching literature. Assumed baseline parameter values are summarised in Table 2. Given the use of quarterly data, the discount factor is set to $\beta = 0.99$. As in Andolfatto (1996), Merz (1995) and Holt (2008), amongst others, we assume log utility such that $\sigma = 1$. Following Prescott (1986), the quarterly depreciation rate on capital is $\delta = 0.025$ and the elasticity of output with respect to capital is $\alpha = 0.36$. Steady state values for aggregate output, capital, hours and consumption are found by solving the system (18), (19), (10) and (21).

Turning now to the labour market, it is standard to assume symmetric bargaining, $\eta = 0.5$. We follow Andolfatto (1996) in setting the vacancy transition probability to $q(\theta) = 0.9$, consistent with the evidence on average vacancy duration reported in van Ours and Ridder (1992). The elasticity of the matching function with respect to v is $\gamma = 0.6$, as suggested by the empirical study by Blanchard and Diamond (1989). The quarterly separation rate $\rho = 0.05$ and is obtained from data on labour market transition probabilities used in Shimer (2007), made available at the author's personal webpage.¹⁰ It is given by the sum of the employment-unemployment and employment-inactivity transition probabilities. Given $q(\theta)$ and ρ , v is determined from the steady state version of (20) once n is specified. We set n , the steady state employment-population ratio, in order to target a realistic value for $p(\theta)$. The transition probability $p(\theta)$ is calibrated in order to match average unemployment duration, which is equal to $p(\theta)^{-1}$. The latter is calculated to be 1.17 quarters for the whole sample.¹¹ We set $n = 0.945$ in order to target these averages. This results in a slightly larger unemployment-population ratio than is found in the data for our sample (5.5% versus slightly less than 4%). Nevertheless, this calibration strategy has been adopted by other authors (Cole and Rogerson 1999 and Krause and Lubik 2007) and is consistent with the notion that measured unemployment understates the true intensity of search effort because non-participants are ignored.¹²

Given our focus on unemployment fluctuations, the vacancy cost κ is treated as a free parameter that is adjusted in order to generate realistic unemployment volatility in the baseline calibration. This requires $\kappa = 0.021$ in the baseline model. Given κ , the total flow value from unemployment, $b_1 + b_2 c^\sigma$, is then determined residually from equation (14). We set b_1 and b_2 so as to ensure that the net replacement ratio,

¹⁰For additional details, please see Shimer (2007) and his webpage <http://sites.google.com/site/robertshimer/research/flows>. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley. Data are not available for 1965-6.

¹¹Data on average unemployment duration are available from the Bureau of Labor Statistics, www.bls.gov.

¹²We take up the issue of labour market participation in the next chapter. As we discuss, "unemployment rates" as high as 11% are not unreasonable in the context of models which ignore the participation decision.

$b_1 / ((1 - \tau) wh) = 0.4$, the assumption in Shimer (2005). As for the shock process for aggregate productivity, the persistence parameter is set to $P_A = 0.95$ and the standard deviation $\sigma_A = 0.00615$ so that the model is consistent with the volatility of output for the sample period in question.

4 Results

We first present impulse response functions for $\phi = 0.13$ and $\phi = 0$ in order to determine how large the differences in adjustment paths are when a positive 1% productivity shock hits the economy. Figure 1 plots the impulse dynamics of the two specifications. All variables respond positively to the productivity shock apart from unemployment. The response of vacancies significantly weakens in the period after impact due to the sharp fall in unemployment, which increases market congestion and raises the expected cost of filling a vacancy. As employment increases after the matching delay, hours experience a notable decline from their peak response on impact. This indicates that when consumption is sustained by relatively high employment, the household works less hours opting instead to consume relatively more leisure. Intensive and extensive adjustment therefore substitute for one another to a certain extent. After a few periods, hours decline below steady state. This is partly due to the diminishing marginal utility of consumption along the adjustment path, which magnifies the consumption value of the disutility of labour from (18). As the consumption path is more amplified under a flat tax regime, hours undershoot steady state equilibrium to a greater extent when $\phi = 0$. Tax dynamics also contribute to greater equilibrium undershooting in the flat tax model. To see this, note that as the return from supplying labour falls, workers reduce hours to a greater extent when there is no commensurate reduction in the tax rate so that equality is maintained in (18).

Flat tax reform significantly amplifies the response of hours on impact, which rises from 0.70% to 0.95%. Although increased impact amplification is not observed for unemployment or vacancies, tax reform does increase the persistence of these variables' adjustment paths. The difference in persistence is quite large. For example, at period 10 unemployment is 12.35% below steady state in the progressive tax specification whereas it is 14.39% below in the flat tax model. Forward-looking firms anticipate greater equilibrium undershooting in h_t along most of the adjustment path under a flat tax, and therefore choose to sustain the expansion in vacancy supply for a longer period, driving a more persistent drop in unemployment. Opposing the effect of equilibrium undershooting is the notably larger increase in hours on impact when $\phi = 0$, which crowds out vacancy creation to some extent in the initial periods as firms seek to reduce their exposure to costly matching frictions when currently employed agents are very willing to work longer hours. On balance, there is little change in the impact response of vacancies conditional on the tax system and the divergence between the two paths takes a few periods to widen as the effect of equilibrium undershooting begins

to dominate.

If this intuition were correct, we would expect to observe greater impact amplification and weaker persistence of labour market variables in the model with inelastic hours. This is exactly what happens. Figure 2 plots impulse dynamics for $\varphi = 100$ keeping the rest of the parameterisation the unchanged. Holding hours (approximately) constant significantly amplifies the responses in vacancy supply, unemployment and market tightness, thereby confirming that in the absence of intensive adjustment firms do rely to a greater extent on extensive adjustment. The persistence of the labour market adjustment path is also weaker when hours are inelastic regardless of ϕ , with vacancy supply, unemployment and market tightness all decaying at a much faster rate. The peak responses in output, consumption and investment are roughly the same as in the baseline, although the impact responses are weaker since adjustment along the extensive margin operates with a lag. Overall, output volatility increases slightly as the tax schedule is flattened, with the increase being more notable when hours are flexible.

The impulse response functions for inelastic hours demonstrate the centrality of variable hours to the tax reform transmission mechanism. Indeed, in Figure 2 in which hours do not vary, the effect of tax reform on impulse dynamics is very small. The wage does display a lower adjustment path under a flat tax despite slightly higher market tightness and output, but the difference is not quantitatively large. In the absence of significant changes to wage dynamics, the path of vacancy supply is not sensitive to tax reform either. These results indicate that under inelastic hours, the opposing effects of taxation on wage dynamics explained previously in section 2.3.2 either largely offset one another or are both quantitatively unimportant. On the other hand, the wage does appear to be somewhat more sensitive to tax reform when hours are elastic. This is partly due to the fact that as workers become more willing to increase hours they must also accept smaller increases in wage rates per hour worked due to a compositional shift in the firm's wage bill. It also indicates that when hours fluctuate the worker's outside option becomes more procyclical due to convex disutility of labour, and the tax-wage multiplier described in section 2.3.2 becomes magnified with the result that flattening the tax schedule has a relatively larger stabilising effect on the wage. This in turn contributes to a larger effect of tax reform on vacancy supply the more elastic hours are.

Simulation-based business cycle statistics for the baseline and inelastic hours models are presented in Table 3. We report mean simulation standard deviations with sample standard deviations in parentheses for the baseline and inelastic hours models.¹³ The corresponding statistics from actual U.S. data are also shown for comparison. For each model economy, the relative volatilities of the economic variables of interest are reported prior to the simulated removal of tax progressivity, and after tax reform keeping all parameters apart from ϕ unchanged. In particular, the variance of the shock process is

¹³Results for a wage posting model are also presented in Table 3. We discuss this extension subsequently.

held constant post-reform. Recall that the baseline model is calibrated in an attempt to give a realistic labour market performance assuming the degree of tax progressivity that is present in the data. For the purposes of moment computations, the statistics reported in the next section for the inelastic hours model are calculated based on the assumption that the "true" model is one in which the elasticity of hours is very small. Therefore, we re-calibrate κ and σ_A so that the inelastic hours model displays relative volatilities of unemployment and output that are consistent with the data. This required only a small change setting $\kappa = 0.05$ and $\sigma_A = 0.0074$.

Prior to the tax policy experiment, the models provide reasonable predictions regarding the second moment properties of the data. Despite realistic volatility in unemployment and vacancies, however, the relative volatility of market tightness is slightly understated, but nevertheless the correct order of magnitude. This is because the negative correlation between vacancies and unemployment is much stronger in the data, and this holds across all specifications. Consumption volatility is also understated. The real business cycle-matching models of Merz (1995) and Andolfatto (1996) also understate consumption fluctuations, suggesting that either the logarithmic specification of utility implies excessive consumption smoothing at the representative agent level, or perhaps a richer demand-side of the model (e.g. the inclusion of monetary shocks and sticky prices), which is lacking in the current framework, is potentially important for the behaviour of consumption. Given our focus on the labour market, we do not dwell on this issue. The relative volatility of the wage is also somewhat understated, and hours are slightly less procyclical than in the data.

Consider now the differences in the mean standard deviations when tax progressivity is removed. Consistent with the previous impulse response analysis, in the baseline model the main effect of tax reform is an increase in the relative volatility of hours of approximately 27% despite the concomitant increase in output volatility. In contrast, we do not report a significant change in the relative volatility of unemployment which, if anything, declines slightly. Fluctuations in consumption and investment increase in proportion to output, so that their relative volatility remains unaffected by tax reform. With hours variation suppressed, there is no quantitatively meaningful impact on relative labour market volatility. Wage volatility in the absence of hours variation is significantly higher but completely unresponsive to flat tax reform. In the absence of intensive adjustment, unemployment volatility increases very slightly from 8.65 to 8.92 but the change is insignificant.

With regard to the correlations, hours become slightly more procyclical but it is observed that the gradient of the vacancy-unemployment curve is not influenced by tax reform.¹⁴ In particular, persistence in vacancy creation tends to be weak regardless of the tax structure, deteriorating significantly after the first period as indicated by the impulse response functions reported previously. Consequently, the simulated data display

¹⁴The vacancy-unemployment correlation is known as the Beveridge curve, and its importance is discussed in more detail in the next chapter.

only moderately negative vacancy-unemployment correlations.¹⁵ The autocorrelation of unemployment fluctuations is also not impacted by the policy change.

To summarise, baseline results indicate that the only quantitatively significant repercussion of structural tax reform for labour market dynamics is an increase in the relative volatility of hours. In adjusting to shocks, the economy displays a substitution effect between intensive and extensive adjustment, whereby households reduce average hours as firms create more jobs, and vice versa. A larger observed increase in hours during the initial periods of adjustment under a flat tax partially crowds out job creation, offsetting the incentive for firms to expand vacancy supply due to subsequent equilibrium undershooting of hours. Our baseline simulations therefore do not support the notion that flat tax reform significantly exacerbates cyclical unemployment instability.

4.1 Robustness

In this section, we discuss the extent to which changes to the model's central parameters influence the baseline results. First, we note that our quantitative results are insensitive to the level parameter of the tax function, ζ . For instance, lowering ζ to 0.8 - the lower bound on the empirical estimates which gives the highest steady state tax rate - and keeping all other parameters unchanged results in moment calculations that are practically identical to those reported in Table 3. Performing the tax policy experiment, hours volatility again increases from 0.37 to 0.47 and unemployment volatility declines insignificantly from 8.43 to 8.19. Results are comparably insensitive to setting $\zeta = 0.9$. Performing the tax experiment with $\phi = 0.15$ - the upper bound of our estimates in Table 1 - results in quantitatively very similar results to the baseline as well. Hours volatility increases from 0.36 to 0.46 and unemployment volatility remains virtually unchanged from 8.64 to 8.42. Our baseline results are therefore representative of empirically plausible variations in the tax code parameters.

In this chapter, we have theoretically established that the transmission mechanism of tax policy reform operates through the Nash wage equation and the equilibrium condition for optimal hours. However, our baseline results tended to display only a weak quantitative effect of tax reform on wage fluctuations. Given that previously mentioned work in the literature has found the wage channel to be key in determining unemployment fluctuations, we now consider whether the wage effects of tax reform are sensitive to the calibration strategy. In the baseline model, symmetric bargaining was assumed, as is most common in the literature. However, the weight that is placed on the

¹⁵In relation to this point, Fujita (2003) demonstrates that the introduction of additional vacancy creation costs, such as planning lags, facilitates the replication of highly negative vacancy-unemployment correlations by making the response of vacancies to shocks more persistent. The latter author argues that the free-entry condition on vacancy supply induces a sharp impact response but weak persistence. Congestion externalities caused by falling unemployment in the period after the shock force vacancy supply back towards the steady state relatively quickly as $q(\theta_t)$ falls. Fujita (2003) refers to this as the "echo effect", a consequence of the simple zero profit restriction assumed in standard matching equilibria.

outside option component of w_t - and therefore on tax movements - depends negatively on η in (17). In order to determine if the cyclical implications of adjustments to ϕ depend on relative bargaining weights, consider a wage posting equilibrium in which $\eta = 0$. For this calibration, maximum weight is placed on the worker's outside option in the equilibrium wage, allowing tax fluctuations to have a potentially more important role in determining wage behaviour. The Nash wage is simplified to

$$w_t h_t = \frac{1}{1 - \tau_t} \left(b_1 + b_2 c_t^\sigma + \frac{h_t^{1+\varphi}}{1 + \varphi} c_t^\sigma \right). \quad (22)$$

Figure 3 plots impulse dynamics for $\eta = 0$ with all other parameters unchanged relative to the baseline. Lowering η significantly increases labour market volatility as the wage becomes less sensitive to productivity shocks, causing the percentage deviation in market tightness to be substantially magnified relative to the baseline. Once household employment increases in the period after impact, equilibrium undershooting is now much sharper since the return to working is relatively unattractive when η is small and employment (and therefore consumption) is much higher because of the strong response in job creation by firms. It only takes one period for hours to fall below steady state along the adjustment path. The cyclical repercussions of tax reform are amplified, but not because of a substantial difference in wage behaviour. Perhaps somewhat surprisingly, the wage follows almost the same path regardless of the structure of the tax system despite the prominence of τ_t in (22), while unemployment nevertheless responds more sensitively to tax reform than in the baseline. The tax reform transmission mechanism does not therefore rely on wage dynamics, but hours dynamics. The strong incentive to reduce hours below steady state due to the robust expansion in vacancy supply exerts substantial downward pressure on the tax rate. In equilibrium, the drop in hours is strong enough to cause the tax rate to fall below steady state under a progressive tax system. This works to significantly arrest the decline in hours through the tax-labour income multiplier that is operative when $\phi > 0$. As a result, under a flat tax, substantially stronger undershooting of hours causes the economy to adjust to the productivity shock to a significantly greater extent through job creation. To provide further support for this intuition, Figure 4 displays impulse dynamics for the wage posting model with inelastic hours, keeping all remaining parameters the same as in the baseline. In this case, the cyclical implications of tax reform are, as in the baseline model, quantitatively small, illustrating the importance of hours adjustment in determining the path of unemployment and job creation. Finally, we note from Figure 3 that although equilibrium undershooting of hours is more severe under a flat tax, the impact response is no longer as sensitive to tax reform as in the baseline model. This indicates that tax reform in the wage posting model tends to exacerbate volatility only in the downward direction given the reduced incentives for workers to extend average hours on impact when their bargaining power is low. As the initial crowding out effect

of tax reform on vacancy supply is therefore smaller, the increase in vacancies on impact under a flat tax is larger relative to the baseline.

Business cycle statistics for the wage posting equilibrium are presented in the last two columns of Table 3.¹⁶ In order to target realistic unemployment and output variability, we set $\kappa = 0.49$ and $\sigma_A = 0.006$. It is observed that reducing the worker's bargaining power amplifies the effects of tax reform on relative unemployment volatility, which increases by approximately 10% when ϕ is lowered to zero and hours are flexible. The increase in hours volatility remains notable but is weakened relative to the baseline. This is partly due to the increase in the parameter φ which lowers the elasticity of labour supply and partly due to the fact that the impact response of hours in Figure 3 is less sensitive to tax reform given the decreased incentives to work longer hours when bargaining power is low. The results also indicate that wage volatility is substantially reduced in the wage posting equilibrium to less than half the baseline magnitude.

In summary, we therefore conclude that the effect of tax policy reform on unemployment dynamics is sensitive to the weight placed on the worker's outside option in the equilibrium wage equation, but only when hours are flexible. In a wage posting equilibrium in which the worker's bargaining power is zero, cyclical job creation is amplified as wages become less procyclical. This causes hours to rise for one period only on impact, thereafter strongly dropping below steady state to increase the consumption of leisure. The strong incentive to reduce hours, in conjunction with the endogenous relationship between the tax rate and the labour supply, amplifies the effects of tax reform on the labour market. Tax reform can thus result in a statistically meaningful increase in unemployment volatility, but only if this extreme condition of wage bargaining is met. Even then, however, the increase in relative unemployment volatility is not exceedingly large at about 10%.

4.2 Welfare Analysis

It has been demonstrated that tax reform exacerbates the cyclical responses both of hours worked and consumption in response to productivity shocks. Under which incentive structure - the flat or graduated tax programme - is the representative household's welfare higher? To obtain a measure of the welfare implications of policy reform, we first compute the utility difference induced by the divergence in the adjustment paths of the progressive and flat tax economies following a positive productivity shock. We then consider a second measure of the welfare effects of policy reform that is based on the compensating variation in steady state consumption required to make the representative household *ex ante* indifferent between living in worlds with and without business cycles. We can then determine to what extent the compensating variation in consumption depends on the progressivity of the tax system.

¹⁶We only report statistics for the flexible hours version since, as the impulse response analysis indicated, the cyclical implications of tax reform are small under inelastic hours.

Up until now, we have analysed *linearised* equilibrium dynamics in order to understand the cyclical implications of structural tax reform on several endogenous variables of interest. First-order approximations to the model's equilibrium conditions are less computationally burdensome and yield comparable numerical results to higher-order solution methods regarding the second moment properties of the model as long as the economy is perturbed only a small distance from the steady state.¹⁷ However, in order to measure the effect of different policy environments on welfare, the curvature of the utility function must be preserved to capture risk aversion. Therefore, in this section our results are derived from a second-order approximation to the policy function using the method of Schmitt-Grohe and Uribe (2004). The appendix outlines the solution method. In practice, second-order accurate equilibrium dynamics are very similar to the baseline results that were obtained from a log-linear approximation. In Figure 5 the impulse responses are shown for the model solved using a second-order approximation to the policy function. Comparing with Figure 1 reveals a negligible difference.

Panel (a) of Figure 6 plots the path of utility after a 1% positive shock to productivity in the progressive and flat tax economies. Note that the vertical axis measures the level of utility, not logarithmic deviations from steady state. It can be seen that welfare in the flat tax economy is higher during all periods. When the shock hits, the increased disutility from the spike in working hours initially outweighs the positive effect on utility of increased consumption. As hours fall below steady state and consumption continues to rise over the adjustment path (recall Figure 5), utility rises above the steady state level. Panel (b) plots the difference between the two utility paths, representing the excess utility in the flat tax economy over the progressive tax economy. As hours respond more sensitively in the flat tax economy the initial drop in utility is greater, and this causes an initial fall in the surplus utility of the flat tax regime. However, the utility gain from being subject to a flat tax rises above steady state during the adjustment period. Finally, panel (c) expresses the welfare gain in panel (b) as a percentage of consumption in the progressive tax economy. This measure is computed as the ω_t which solves

$$0 = \frac{((1 + \omega_t) c_t^{pro})^{1-\sigma}}{1 - \sigma} - n_t^{pro} \frac{(h_t^{pro})^{1+\varphi}}{1 + \varphi} - \left[\frac{(c_t^{flat})^{1-\sigma}}{1 - \sigma} - n_t^{flat} \frac{(h_t^{flat})^{1+\varphi}}{1 + \varphi} \right]$$

for each t and where a *pro* or *flat* superscript indexes the respective models. Panel (c) therefore indicates that the welfare gain from removing tax progressivity along the adjustment path to a productivity shock is approximately equivalent to raising consumption in the progressive tax economy by just under 4% on average in each quarter. In terms of the steady state, the welfare gain from setting ϕ to zero from its benchmark value is equivalent to an exogenous increase in steady state consumption of

¹⁷See Heer and Maussner (2009) for a comparison of different numerical procedures for solving real business cycle models.

3.84%. The steady state welfare implications are therefore fairly substantial: for net family earnings of \$50,000 a year, the welfare gain from tax reform is almost \$2,000 each year. Lowering ϕ increases steady state hours and therefore consumption, which on balance yields a rise in welfare. We remark that the welfare loss of progressive taxation occurs almost entirely due to the difference in steady state hours that the change in ϕ induces. With the inelastic hours calibration, the welfare loss is roughly equal to zero. The reason is that when $\varphi = 100$, $h = 1.00$ and $c = 2.59$ regardless of ϕ . This is contrasted with the elastic hours calibration for which $\varphi = 0.2$. In this case, h increases from 0.71 to 0.80 after tax reform and c increases from 1.85 to 2.08.

Next, based on the measure of welfare loss associated with business cycles proposed by Lucas (1987), we calculate the proportional shift in steady state consumption that would render the representative household indifferent between the expected stochastic consumption and work paths that would result over the normal course of the business cycle and the steady state allocation that would obtain in the absence of shocks. This is defined as the fraction ψ that must make the following expression hold:

$$0 = E_0 \left[\sum_{t=0}^T \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - n_t \frac{h_t^{1+\varphi}}{1+\varphi} \right) - \sum_{t=0}^T \beta^t \left(\frac{((1-\psi)c)^{1-\sigma}}{1-\sigma} - n \frac{h^{1+\varphi}}{1+\varphi} \right) \right]. \quad (23)$$

The value of ψ is computed for both the flat and progressive tax economies and represents the proportion of steady state consumption that the representative household is willing to forgo in order to completely eliminate business cycle uncertainty. Therefore, business cycle fluctuations in the flat tax model would be more costly if the estimated value of ψ is larger when $\phi = 0$. The algorithm for numerically estimating ψ involves setting T to a large number - we choose 500 - and generating a large number of simulated sample paths for the stochastic economy - we generate 200 samples - which are then averaged in order to approximate the representative household's expectation of utility in the stochastic model. Given expected utility in the fluctuating economy, ψ follows straightforwardly from (23).

Table 4 summarises the welfare costs of business cycles under different tax policies for different model parameterisations. Our quantitative evaluation of the welfare effects of business cycles suggests that, in accordance with Lucas (1987, 2003), the gains from improved stabilisation policy are small. In the baseline model with progressive taxation, business cycle fluctuations result in a welfare loss equivalent to 0.05% of steady state consumption. The exacerbation of the business cycle caused by flat tax reform implies that the welfare loss is about 0.2% of steady state consumption.¹⁸ In relative terms, this represents a substantial increase, an order of magnitude larger than the welfare loss under a progressive tax policy. But still at only a fraction of a percentage point of steady state consumption, however, it would be an exaggeration to label a welfare loss of such

¹⁸Recall that moving from a progressive to a flat tax increases steady state consumption. Therefore, the welfare loss in absolute terms increases by a greater factor than the measure expressed as a percentage of consumption.

magnitude considerable, especially given that it results from the *complete* elimination of the business cycle. For comparison, Lucas (2003) estimates the welfare gain from bringing an annual inflation rate of 10% to zero to be a perpetual consumption flow of one percent of income. The welfare losses are of a similar magnitude for the inelastic hours and wage posting calibrations and are even less sensitive to tax policy, especially the inelastic hours specification. The highest welfare cost we obtain is for the wage posting equilibrium under a flat tax programme at 0.25% of steady state consumption.

4.3 Fiscal Stimulus

In this section, we consider the implications of the structure of the tax system for the propagation of government expenditure shocks. The study by Monacelli et al. (2010) demonstrated that a standard neoclassical model with matching frictions encounters difficulty in generating realistic effects of government purchases on output and unemployment. The latter authors assume that government purchases are intrinsically useless and enter the aggregate resource constraint as a pure drain on resources. It has been well known since at least Christiano and Eichenbaum (1992) that useless government expenditure that is financed by a balanced budget induces a negative wealth effect on the economy, prompting households to compensate by increasing their work effort. That Monacelli et al. (2010) obtain only weak fiscal multipliers suggests that the presence of matching frictions in the labour market impedes transmission of fiscal shocks. In recognition of this, we follow Chen and Guo (2010) by allowing for productive government purchases in order to obtain fiscal multipliers of a realistic order of magnitude. For simplicity, we assume that government purchases, g_t , reflect a non-rival, public good which is freely accessed by all matches. The aggregate output function in this economy becomes

$$y_t = A_t k_t^\alpha (n_t h_t)^{1-\alpha} g_t^\chi$$

where χ is the elasticity of output with respect to government spending. The government does not invest in capital; all capital remains privately owned. The first-order condition for hours now becomes

$$\frac{h_t^\varphi c_t^\sigma}{(1-\phi)(1-\tau_t)} = (1-\alpha) A_t \left(\frac{k_t}{n_t}\right)^\alpha h_t^{-\alpha} g_t^\chi.$$

The government is still assumed to operate a balanced budget in every period, so that

$$g_t + b_1 u_t + T_t = n_t \tau_t w_t h_t$$

which implies the aggregate resource constraint

$$y_t = c_t + i_t + g_t + \kappa v_t.$$

We do not allow g_t to substitute for private consumption so that there is no direct

effect of government expenditure on the utility of the household. The fiscal transmission mechanism in this model therefore operates through two distinct channels; a productivity channel (when $\chi > 0$) which causes g_t to have supply-side repercussions much like A_t , and an aggregate demand channel. Setting $\chi = 0$ removes the contribution of government expenditure to the production of private output, so that g_t would only affect the economy through changes in aggregate demand consistent with the resource constraint.

The path of g_t is determined by an exogenous stochastic process given by

$$\log g_t = P_g \log g_{t-1} + \xi_{g,t}$$

where $\xi_{g,t} \sim N(0, \sigma_g^2)$ and $0 \leq P_g \leq 1$ governs the degree of persistence of the government expenditure shock. As in Monacelli et al. (2010), we consider the effects of a one-off fiscal stimulus package that is defined as an temporary increase in g_t that is normalised to 1% of steady state output. We therefore abstract from a full set of simulations-based business cycle moments in which stochastic government expenditure is a forcing variable over the business cycle.¹⁹

The persistence parameter P_g is set to 0.9 based on the VAR estimates of Monacelli et al (2010). As Sarte and Wenli (2004) point out, the range of values for χ is large; from 0.03 (Eberts 1986) to 0.39 (Aschauer 1989). Our calibration strategy is to set χ such that the fiscal multipliers prior to tax reform match the VAR estimates of Monacelli et al. (2010). We also make slight adjustments to κ and σ_A in order to ensure that realistic unemployment and output fluctuations are produced. Table 5 summarises the changes to the calibration of the fiscal policy models, including the inelastic hours and wage posting variants, that are used to compute the fiscal multipliers. The parameterisation of χ lies towards the lower range of the empirical estimates. The steady state value of $g = 0.20$ to be consistent with the data over the sample period. The rest of the parameters remain the same as in the previous calibrations.

To first gain some intuition for the fiscal transmission mechanism, Figure 7 shows the impulse responses to a positive government expenditure shock normalised to 1% of steady state output in the baseline model. The figure depicts how fiscal stimulus in the form of (non-wasteful) government purchases affects the economy when the tax system is progressive as opposed to proportional. The fiscal shock raises the marginal productivity of labour and aggregate demand, thereby exerting a positive influence on hours worked and the supply of vacancies. The increase in hours is reinforced by the negative wealth effect that higher g_t causes via the crowding-out effect effect on the aggregate resource constraint, which is evidenced by the fall in private consumption. There is consequently no equilibrium undershooting of hours. Note that the expansion

¹⁹This is because we find the contribution to the business cycle of government shocks of the type considered here to be small when σ_g^2 is estimated from the cyclical component of g_t from actual data. For the contribution of stochastic fiscal policy to U.S. business cycle moments, see Finn (1998) or Christiano and Eichenbaum (1992).

in labour supply without a commensurate increase in labour demand puts downward pressure on the wage, which falls below steady state under a flat tax. Private investment still increases and the overall effect on output is expansionary. Tax progressivity is found to decrease the effect of fiscal stimulus on hours and increase the effect on job creation and unemployment, which stands partially in contrast to the impulse dynamics to a productivity shock analysed previously.²⁰ We now find that tax reform has opposite effects on the dynamics of hours and unemployment, leading to a *compositional shift* in the fiscal transmission mechanism.

What is causing this compositional shift in labour market dynamics? Recall that g_t is partially a demand shock and partially a supply shock. However, given that the calibrated value of χ is close to zero, the aggregate demand effect of g_t is likely to dominate. Interpreted as a demand shock, g_t boosts h_t through a negative wealth effect arising from a drain on the aggregate resource constraint. In this case, equilibrium undershooting of hours does not arise because the negative wealth effect sustains a persistent rise in hours above steady state along the adjustment path, exerting a uniformly negative effect on job creation. Under a flat tax the response in h_t is stronger, prompting firms to bypass some of the costly matching process by expanding the supply of vacancies to a lesser extent. Unemployment consequently drops by less than under a progressive tax. It is in this manner that the structure of the tax system influences labour market adjustment to a fiscal policy (i.e. aggregate demand) shock. The compositional effect is quantitatively substantial: unemployment drops by 50% more initially when taxes are flat and job creation increases by a similar proportion.

To provide further evidence for this explanation of events, Figure 8 plots the impulse responses for the baseline fiscal model with the only change being that χ is reduced to 0 so that g_t is a pure aggregate demand shock. As indicated by the substantial decline in consumption and investment, the negative wealth effect is now larger. Hours respond more persistently, causing tax reform to have a larger effect on the adjustment path. Vacancy supply actually contracts in response to the fiscal injection when taxes are flat, leading to a rise in unemployment. The shock is still expansionary, however, as output rises.

How central are hours to the impact that the tax structure has on the fiscal transmission mechanism? Figure 9 plots the impulse responses for $\varphi = 100$ keeping all other parameters unchanged. In the absence of variable hours, the structure of the tax system does not have a significant effect on the transmission of the type of fiscal shocks considered here. Without variable hours, output cannot rise significantly on impact and investment must therefore also fall for one period to accommodate the increase in government expenditure until employment rises in the second period. This leads us to conclude that variation in the intensive margin is central in order for the structure of

²⁰ Although we have made slight changes to the calibration for the fiscal model, the impulse response functions for a productivity shock remain practically identical to those reported earlier with all conclusions drawn remaining valid.

the tax system to influence the transmission of government expenditure shocks.

In Figure 10, impulse responses are reported for the wage posting equilibrium with $\eta = 0$ keeping all other parameters the same as in the baseline fiscal model. As in the baseline model, it is observed that tax reform is associated a larger increase in hours worked and a smaller decline in unemployment in response to fiscal stimulus. However, the divergence between the adjustment paths of the progressive and flat tax economies is smaller than in the baseline model. It can be seen that the tendency for hours to undershoot the steady state is larger under wage posting, and relatively more so under a flat tax. The positive effect that equilibrium undershooting has on vacancy creation causes the compositional shift in fiscal adjustment to be somewhat weaker. Thus, the incentive to curtail the expansion in vacancies in response to a fiscal shock because of a larger increase in h_t when $\phi = 0$ is attenuated in the wage posting equilibrium compared with the baseline.

Consider next the output and unemployment multipliers of fiscal policy as defined by Monacelli et al. (2010). The output multiplier is measured as the cumulative percentage change in output divided by the cumulative percentage change government expenditure over a given time horizon, $\sum \Delta y_t / \sum \Delta g_t$. We report multipliers on impact and at one and two year horizons. The unemployment multiplier is computed as the peak fall in unemployment from the steady state expressed in percentage points. Recall that χ is calibrated to match the empirically observed one year output multiplier as measured by Monacelli et al. (2010), which is 1.16 for the U.S. They find an unemployment multiplier of -0.64. Recall that in computing the multipliers for each separate version of the model - baseline, inelastic hours and wage posting - we make the calibration adjustments listed in Table 5 on the presumption that the model in question is the "true" representation of the economy.²¹ Caution must therefore be exercised in comparing multipliers across specifications.

Table 6 reports the fiscal multipliers for the baseline, inelastic hours and wage posting calibrations. The output multiplier at one year in the baseline model increases from 1.16 to 1.26 after tax reform, an increase of just under 9%. A similar proportional increase is observed for the two year output multiplier. The increase in the impact output multiplier is about 13%, entirely attributable to the stronger response in hours. The unemployment multiplier, on the other hand, falls by about 30%, from -0.32 to -0.20, when $\phi = 0$. Tax reform does not have a quantitatively meaningful impact on fiscal policy when $\varphi = 100$, as anticipated from the impulse response functions. The impact output multiplier is substantially smaller in the inelastic hours model at just 0.72 compared to 1.08 under a progressive tax, and 0.72 compared to 1.22 under a flat tax. On the other hand, given the increased reliance on extensive adjustment, the unemployment multiplier is significantly larger in the inelastic hours model at -0.80,

²¹The multipliers, apart from the baseline, therefore do not correspond to the impulse response figures. In these figures only one parameter - φ or η - was altered at a time in order to isolate the individual effects of such changes to facilitate intuition.

more than double the baseline value of -0.32. Under a progressive tax system the two year output multiplier is larger in the inelastic hours model since extensive adjustment imparts sluggishness in the adjustment process, but eventually yields a larger cumulative response. Hours variation thus "flattens" the time profile of the output multiplier, increasing the impact response but weakening the long-term multiplier. Under a flat tax, however, the output multipliers are always larger in the baseline model. The relationship between tax reform and fiscal multipliers in the wage posting equilibrium is similar to the baseline but somewhat weaker.

In sum, a flat tax programme increases the ability of expansionary fiscal policy to stimulate output, with the improvement occurring to a somewhat greater extent towards the initial period of impact. This suggests that a flat tax programme can possibly hasten the effects of fiscal stimulus by encouraging variation in hours which serves to amplify the output multiplier on impact. There is a trade-off however, since when hours are variable, flat tax reform decreases the ability of expansionary fiscal policy to reduce unemployment. This is because firms' willingness to expand vacancy supply in response to an aggregate demand shock is inversely related to workers' willingness to work longer hours in existing matches, thereby driving a "compositional effect" of tax reform. The model thus displays a tendency for output multipliers to be positively related to the elasticity of the labour supply, while unemployment multipliers are decreasing in the elasticity of the labour supply. These results reveal that tax reform has qualitatively different implications for the effects of demand as opposed to supply shocks, since we previously demonstrated that productivity shocks tended to moderately amplify unemployment volatility.

Finally, given the centrality of hours in the fiscal transmission mechanism we have highlighted, it is natural to ask what the VAR evidence indicates about the impact of government expenditure on average hours per employee. The evidence is mixed. On the one hand, Monacelli et al. (2010) find that the effect is small, with hours not responding significantly to government shocks. On the other, Li and Yuan (2000) find that government expenditure induces a positive response in hours worked per employee that is statistically significant for several periods after the shock. The difference appears to stem from the effect that the respective authors measure on employment, which is positive in Monacelli et al. (2010) and negative in Li and Yuan (2000). Unfortunately, hours per worker is a variable not often included in empirical investigations of fiscal multipliers, so it is difficult to gather sufficient evidence to make a reliable conclusion based on atheoretical VAR estimations.²²

4.4 What Actually Happens in the Data after a Structural Tax Break?

In recognition of the observation by Chen and Guo (2010) that the Tax Reform Act of 1986 happened to coincide with the onset of an extended moderation in U.S. busi-

²²Other empirical investigations of fiscal multipliers include Blanchard and Perotti (2002) and Auerbach and Gorodnichenko (2010).

ness cycle volatility, we briefly remark that at first this may seem at odds with our model's prediction of increased output volatility. Table 7 provides a breakdown of the business cycle statistics we consider pre- and post- 1986. Improved economic stability in the U.S. since the 1980s is well documented phenomenon, colloquially dubbed the "Great Moderation" (see Stock and Watson 2002 for a discussion). The standard deviation of the cyclical component of output falls dramatically from 1.43% to 1.07% after TRA-86. Our model, by contrast, tends to predict a negative association between tax progressivity and output volatility, due to the effect on the behaviour of hours variation. Although the relative volatility of hours in the data across sub-samples rises, the change is weaker than what is implied by the model. The data also display a decrease in the relative volatility of unemployment despite the large drop in output volatility. The relative volatility of the wage rate in the data, however, goes in the opposite direction, increasing from 0.42 to 1.11 relative to output over the two sample periods.

Of course, the environment is controlled in our tax policy experiments - the only change to the structure of the economy is to the slope of the tax schedule. In the actual data, there are likely to have been several important changes that would need to be controlled for in isolating the business cycle impact of the 1986 tax reform. Our objective in this chapter is not to undertake a formal empirical investigation of the relationship between the Great Moderation and tax reform. We do note, however, that there are several hypotheses attempting to account for the decline in business cycle volatility in more recent decades, ranging from "good luck" (Stock and Watson 2002) to better stabilisation policy (Clarida et al 2000). Champagne and Kurman (2010) argue that the rise in wage volatility coincides with other structural labour market changes that are likely to have rendered wages more sensitive to cyclical labour market conditions. Examples include a smaller tendency towards private sector unionisation (Farber and Western 2001) and a shift towards performance-pay contracts (Lemieux et al. 2008). As Shimer (2005) emphasises, wage flexibility damps unemployment and job creation volatility in frictional matching models of the sort considered in this chapter. As shocks are increasingly absorbed by wage adjustments, output volatility falls.

We have abstracted from potential shifts in the macroeconomic environment, focusing solely on the real business cycle implications of a structural tax policy shift, keeping all else constant. It is therefore conceivable, for instance, that a tax policy shift in conjunction with an increase in the flexibility of wages would render the latter more volatile in the post-tax reform model. Exacerbated wage movements would then be anticipated to reduce the volatility of hours worked, to an extent offsetting the effects of the tax reform. However, we emphasise that the purpose of the current study is not to provide an explanation for the modern improvement in macroeconomic stability, but to elucidate the business cycle consequences of flat tax reform in an isolated fashion.

5 Conclusion

This chapter has developed a real business cycle model that is capable of quantifying the extent to which structural tax reform influences the propagation of exogenous shocks in general equilibrium. The principal channel through which tax reform operates is on the behaviour of hours, which tend to become more volatile the less progressive tax policy is. The adjustment path of hours is found to be qualitatively different depending on the type of shock considered. In a frictional labour market, however, the path of hours has implications for job creation and unemployment. We found that the dynamic behaviour of hours along the adjustment path to an aggregate productivity shock generates offsetting incentives for job creation, with the result that tax reform has little impact on unemployment fluctuations. On the other hand, we argued that tax reform causes a compositional shift in labour market adjustment to an aggregate demand shock. This implied that fiscal stimulus is more effective at reducing unemployment but less effective at expanding output under a progressive tax system in which hours are less responsive to shocks. For an extreme parameterisation in which government spending is entirely wasteful, we found that government shocks can actually lead to an increase in unemployment. This result relates to the empirical finding of Bruckner and Pappa (2010) for OECD countries that an increase in government expenditure often results in an increase in unemployment. Whereas Bruckner and Pappa (2010) provided a theoretical explanation based on increased labour market participation which causes unemployment to rise due to congestion, our model provides an alternative explanation suggesting that rising unemployment in response to government expenditure may be partially due to firms avoiding frictional hiring costs when employees are willing to work longer hours. In this chapter we have therefore established a relationship between the size of fiscal multipliers and the slope of the labour income tax schedule. The model therefore delivers a testable hypothesis in this regard, laying the groundwork for an empirical investigation into the dependence of fiscal stimulus on the degree of tax progressivity.

We abstracted from the potential impact of tax reform on the search intensity of job seekers, which was assumed to be constant. However, relaxing this assumption could introduce new dynamic implications of tax reform, especially for job creation. There are several ways to model variations in job search intensity, one of which is the labour force participation decision. Meghir and Phillips (2008) remark that the participation decision can be especially sensitive to tax incentives for certain demographics, such as women and low education men. On-the-job search is another way of introducing endogenous variations in search intensity. Evidence that progressive tax schedules decrease the probability of moving to a higher paying job is documented by Gentry and Hubbard (2003). In recognition of the empirical support available to motivate extending the model along these lines, we regard this area as profitable for future research. Furthermore, we also abstracted from endogenous job destruction. Extending

the model to include this feature would allow for an assessment of how tax policy influences separation dynamics.

We also found that the welfare effects of tax reform are quantitatively small, in line with Lucas (1987) who argued that the welfare loss to the representative household of having to live through business cycles is negligibly small. Given that this chapter's analysis is based on a similar representative agent framework, it is not surprising that the welfare consequences of tax reform at the business cycle frequency are on the order of Lucas' (1987) findings. Subsequent work, for example by Krusell et al. (2009), has found that the costs of macroeconomic volatility are substantially greater when agent heterogeneity is taken into account. These authors build on the idea that although the welfare of the fictitious "average" household may not vary substantially with aggregate volatility, the welfare effects of cycles on certain sub-categories of agents, like the poor or unemployed, may be substantial. It seems likely that such considerations would have to be incorporated into the current framework in order to conduct a meaningful welfare analysis. For example, the impact of tax reform on labour supply is likely to vary with observable household characteristics such as wealth.

These ideas are left to future research.

6 References

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7 Appendix

7.1 Solution Methods

Log-Linear Approximation The model of this chapter is a system of non-linear dynamic discrete time stochastic equations for which standard solution techniques are available. In order to solve for the model's dynamics, a linear approximation of the model's equilibrium and aggregate consistency conditions is obtained. The resulting log-linearised system of equations can be cast in the canonical form of Uhlig (1997) in which equations containing expectational elements are separated from those that do not,

$$\begin{aligned} 0 &= \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{C}\mathbf{h}_t + \mathbf{D}\mathbf{z}_t \\ 0 &= \mathbf{E}_t [\mathbf{F}\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{J}\mathbf{h}_{t+1} + \mathbf{K}\mathbf{h}_t + \mathbf{L}\mathbf{z}_{t+1} + \mathbf{M}\mathbf{z}_t] \end{aligned}$$

where $\mathbf{x}_t, \mathbf{h}_t$ and \mathbf{z}_t denote vectors of endogenous state variables, endogenous jump variables and exogenous state variables, respectively, and are multiplied by conformable coefficient matrices. The exogenous states are the stochastic processes of the model, written in vector notation as

$$\mathbf{z}_{t+1} = \mathbf{N}\mathbf{z}_t + \boldsymbol{\xi}_{t+1}, \quad \mathbf{E}_t [\boldsymbol{\xi}_{t+1}] = \mathbf{0}.$$

A system of recursive laws of motion is sought of the form

$$\begin{aligned} \mathbf{x}_t &= \mathbf{P}\mathbf{x}_{t-1} + \mathbf{Q}\mathbf{z}_t \\ \mathbf{h}_t &= \mathbf{R}\mathbf{x}_{t-1} + \mathbf{S}\mathbf{z}_t \end{aligned}$$

which constitutes the equilibrium of the model. Uhlig (1997) describes the matrix algebra involved in obtaining the above solution using the method of undetermined coefficients once the model's parameters have been calibrated. Once this solution has been obtained, the model's dynamic properties can be analysed by generating artificial time series data by repeatedly iterating on the equilibrium laws of motion. The stability condition for a unique equilibrium is that the number of stable eigenvalues of the coefficient matrix \mathbf{P} is exactly equal to the number of endogenous state variables. In our model we have two endogenous state variables, capital and employment, and for all of our calibrations there are exactly two stable eigenvalues, ensuring uniqueness of equilibrium. In practice, we modify the MATLAB routines of Uhlig (1997) in order to run the numerical solutions that were used in this chapter.

NOTE: *all* models in this dissertation that are solved in this manner satisfy the stability condition such that indeterminate equilibria do not arise. The MATLAB routines used to run the solution algorithms are available upon request.

Second-Order Approximation For the purposes of welfare computations it is necessary to preserve the curvature of the utility function during simulations. The algorithm we use to take a second-order approximation to the policy function is by Schmitt-Grohe and Uribe (2004). Expressing the model in the canonical form

$$0 = E_t f(y_{t+1}, y_t, x_{t+1}, x_t) \quad (24)$$

where x_t is a vector of pre-determined variables at time t and y_t is a vector of controls. The exogenous forcing (vector) process is a subset of x_t assumed to follow

$$z_{t+1} = \Lambda z_t + \eta \sigma \xi_{t+1}$$

where the error term ξ_{t+1} is independently and identically distributed with zero mean and σ is a scale parameter and η is a vector of coefficients. The solution to the model is given by a set of policy functions mapping current states into current controls and future states

$$\begin{aligned} y_t &= g(x_t, \sigma) \\ x_{t+1} &= h(x_t, \sigma) + \tilde{\eta} \sigma \xi_{t+1} \end{aligned}$$

where the elements of $\tilde{\eta}$ are zero for endogenous states. Dropping time subscripts and letting primes denote $t + 1$ values, substituting the policy functions into (24) defines

$$\begin{aligned} F(x, \sigma) &\equiv E f(g(h(x, \sigma) + \tilde{\eta} \sigma \xi', \sigma), g(x, \sigma), h(x, \sigma) + \tilde{\eta} \sigma \xi', x) \\ &= 0 \end{aligned}$$

Hence all derivatives of any order of F are equal to zero. The solution at second-

order accuracy takes the form

$$\begin{aligned}\hat{y}_t &= g_x \hat{x}_t + \frac{1}{2} \hat{x}_t g_{xx} \hat{x}'_t - \frac{1}{2} g_{\sigma\sigma} \sigma^2 \\ \hat{x}'_{t+1} &= h_x \hat{x}_t + \frac{1}{2} \hat{x}_t h_{xx} \hat{x}'_t - \frac{1}{2} h_{\sigma\sigma} \sigma^2\end{aligned}$$

where a carat denotes logarithmic deviation from steady state. The coefficient matrices on the second-order terms g_{xx} and h_{xx} are found by twice differentiating F with respect to x and σ and evaluating the result at the steady state $(\bar{x}, 0)$. Schmitt-Grohe and Uribe (2004) demonstrate that the resulting system is linear in the unknowns g_{xx} and h_{xx} . The coefficient matrices $g_{\sigma\sigma}$ and $h_{\sigma\sigma}$ are the solution to the linear system of equations $F_{\sigma\sigma}(\bar{x}, 0) = 0$.

The MATLAB routines provided by Schmitt-Grohe and Uribe (2004) employ the symbolic differentiation function in order to solve for the second-order approximation described above, taking as inputs the equilibrium conditions of the model.

7.2 Description of the Data

Here we provide a description of the data used in this chapter as well as the procedure for obtaining the estimates of the labour income tax function. The data used to compute the business cycle moments of the U.S. economy in Tables 3 and 7 are all available at the websites of the Bureau of Economic Analysis (www.bea.gov) and the Bureau of Labour Statistics (www.bls.gov). The table below summarises the data sources. We use employment and hours data in the non-farm business sector because time series on total hours worked by private sector employees is a relatively new addition to the Current Employment Statistics programme of the BLS and is only available from 2006. The non-farm business sector excludes the economic activities of government and farms, and according to the BLS glossary accounted for 77% of GDP in 2000.

Variable	Definition
Consumption	Personal Consumption of Non-Durable Goods and Services, BEA.
Investment	Non-Residential Investment + Consumption of Durables, BEA.
Gov. Spending	Federal + State and Local Consumption Expenditures, BEA.
Employment	Total Employment in the Non-Farm Business Sector (Index), BLS.
Hours	Average Hours per Worker in the Non-Farm Business Sector, BLS.
Unemployment	Unemployment Level (16 and over) ÷ Civilian Non-Institutional Population (16 and over), BLS.
Wage Rate	Compensation to Private Employees ÷ (GDP Deflator × Hours per Worker × Employment), BEA.
Vacancies	Help Wanted Ads Index as constructed by the Conference Board and made available by the OECD.

We now provide the details regarding the computation of the tax rates that were

used in the estimation of the labour tax schedule. The NBER provide an online tool, TAXSIM, for computing tax liabilities for a given tax year. This is available at <http://www.nber.org/~taxsim/>. The model makes use of stratified random samples of actual U.S. tax returns in order to estimate tax liabilities. We follow the example set by Cassou and Lansing (2003) and compute the tax schedule for married taxpayers who file jointly. It is assumed that the primary taxpayer earns 70% of the couple's taxable income. This figure is consistent with the data provided in Table 1 of IRS Statistics of Income Bulletin, Fall 2003.²³ We estimate tax schedules for the years 1965, 1975, 1985, 1995 and 2005 for individuals earning between 0.1 and 20 times the average wage for the given year. The average wage in a given year is taken to be the empirical counterpart to the term wh in the tax function (3). Unfortunately, the IRS does not publish historical data sets for tax returns by filing status. Average wages for joint filers are therefore obtained from the latter Table 1 for the years 1969, 1979, 1989 and 1999. These are based on Form W-2, in which employers report the wages and salaries of employees for a given tax year. Average wages are observed to follow a virtually linear time trend, and so linear interpolation and extrapolation are used to obtain the corresponding estimates for the years we consider. Uploading this data to TAXSIM provides estimates of the federal marginal tax on wage income. Average tax rates - the ratio of total taxes paid to total taxable income - are then computed from the marginal rates.

²³This table is available at <http://www.irs.gov/pub/irs-soi/99inw2wm.pdf>.

Table 1: Estimated U.S. Labour Income Tax Schedule

	Year				
	1965	1975	1985	1995	2005
Estimated Level, $\hat{\zeta}$	0.89	0.80	0.80	0.83	0.86
(Standard Error)	(0.007)	(0.003)	(0.003)	(0.002)	(0.002)
Estimated Slope, $\hat{\phi}$	0.14	0.15	0.14	0.10	0.10
(Standard Error)	(0.004)	(0.002)	(0.002)	(0.001)	(0.001)
R^2	0.86	0.96	0.97	0.97	0.97

Note: Nonlinear least squares estimates of the wage tax function using TAXSIM. For a detailed description see the appendix.

Table 2: Baseline Calibration

Parameter	Value	Parameter	Value
β	0.99	h	0.71
σ	1	κ	0.021
δ	0.025	b_1	0.57
α	0.36	b_2	-0.10
ϕ	0.13	n	0.945
m	0.88	k	25.58
φ	0.2	i	0.64
γ	0.6	c	1.85
ρ	0.05	y	2.49
σ_A	0.00615	η	0.5
v	0.05	$q(\theta)$	0.9
w	2.37	$p(\theta)$	0.86

Table 3: Business Cycle Statistics of U.S. and Model Economies

	U.S. Data			Baseline			Inelastic Hours			Wage Posting						
		0.2	0.13	0.00615	0.2	0	0.13	0.0074	100	100	100	0.3	0.13	0	0.006	0.006
φ																
ϕ																
σ_A																
Standard Deviations																
Output	1.28%	1.27% (.14%)	1.27% (.14%)	1.37% (.14%)	1.27% (.14%)	1.27% (.14%)	1.27% (.14%)	1.27% (.14%)	1.27% (.14%)	1.27% (.14%)	1.27% (.14%)	1.28% (.14%)	1.35% (.16%)	1.35% (.16%)	1.35% (.16%)	1.35% (.16%)
Private Consumption	0.64	0.31 (.013)	0.31 (.013)	0.31 (.013)	0.35 (.013)	0.35 (.013)	0.35 (.013)	0.35 (.013)	0.35 (.012)	0.35 (.012)	0.35 (.013)	0.27 (.013)	0.27 (.013)	0.27 (.013)	0.27 (.013)	0.27 (.013)
Investment	3.11	3.09 (.020)	3.09 (.020)	3.09 (.020)	2.98 (.020)	2.98 (.020)	2.98 (.020)	2.98 (.020)	2.96 (.019)	2.96 (.019)	2.98 (.022)	2.89 (.022)	2.89 (.021)	2.89 (.021)	2.89 (.021)	2.89 (.021)
Wage	0.72	0.40 (.008)	0.40 (.008)	0.35 (.008)	0.56 (.015)	0.56 (.015)	0.56 (.015)	0.56 (.015)	0.55 (.015)	0.55 (.015)	0.19 (.009)	0.19 (.009)	0.17 (.007)	0.17 (.007)	0.17 (.007)	0.17 (.007)
Unemployment	8.63	8.65 (.080)	8.65 (.080)	8.41 (.105)	8.65 (.135)	8.65 (.135)	8.65 (.135)	8.65 (.135)	8.92 (.142)	8.92 (.142)	8.65 (.072)	8.65 (.072)	9.62 (.091)	9.62 (.091)	9.62 (.091)	9.62 (.091)
Vacancies	10.47	11.41 (.809)	11.41 (.809)	11.01 (.657)	11.76 (.960)	11.76 (.960)	11.76 (.960)	11.76 (.960)	12.06 (1.02)	12.06 (1.02)	11.46 (.779)	11.46 (.779)	12.63 (.807)	12.63 (.807)	12.63 (.807)	12.63 (.807)
Tightness	18.84	15.66 (0.139)	15.66 (0.139)	15.25 (.135)	15.73 (.328)	15.73 (.328)	15.73 (.328)	15.73 (.328)	16.21 (.347)	16.21 (.347)	15.70 (.129)	15.70 (.129)	17.44 (.120)	17.44 (.120)	17.44 (.120)	17.44 (.120)
Hours per Worker	0.37	0.37 (.020)	0.37 (.020)	0.47 (.021)	-	-	-	-	-	-	0.37 (.020)	0.37 (.020)	0.43 (.024)	0.43 (.024)	0.43 (.024)	0.43 (.024)
Correlations																
$v_t - u_t$	-0.94	-0.20 (.089)	-0.20 (.089)	-0.21 (.082)	-0.16 (.087)	-0.16 (.087)	-0.16 (.087)	-0.16 (.087)	-0.17 (.092)	-0.17 (.092)	-0.20 (.089)	-0.20 (.089)	-0.21 (.084)	-0.21 (.084)	-0.21 (.084)	-0.21 (.084)
$h_t - y_t$	0.67	0.58 (.032)	0.58 (.032)	0.65 (.027)	-	-	-	-	-	-	0.67 (.020)	0.67 (.020)	0.61 (.023)	0.61 (.023)	0.61 (.023)	0.61 (.023)
u_t, u_{t-1}	0.92	0.74 (.054)	0.74 (.054)	0.74 (.048)	0.72 (.056)	0.72 (.056)	0.72 (.056)	0.72 (.056)	0.72 (.057)	0.72 (.057)	0.74 (.051)	0.74 (.051)	0.74 (0.049)	0.74 (0.049)	0.74 (0.049)	0.74 (0.049)

Notes: Standard deviations are expressed relative to output, averaged over 200 simulations. Sample standard errors in parentheses. $\kappa = 0.05$ and 0.49 in the inelastic hours and wage posting models, respectively. HP filter applied to all time series, smoothing parameter 1600. Data sources for U.S. economy listed in Appendix I.

Table 4: The Welfare Cost of Business Cycles and Tax Reform

Model	Welfare Cost of Business Cycles, ψ	
	$\phi = 0.13$	$\phi = 0$
Baseline	0.0005	0.0019
Inelastic Hours	0.0012	0.0015
Wage Posting	0.0019	0.0025

Note: ψ is expressed as a fraction of steady state consumption.

Table 5: Calibration for the Model with Government Spending

Parameter	Model		
	Baseline	Inelastic Hours	Wage Posting
κ	0.027	0.033	0.6
φ	0.14	100	0.21
σ_A	0.0067	0.0075	0.00635
χ	0.088	0.1425	0.0915
g	0.20	0.20	0.20
P_g	0.90	0.90	0.90

Note: Remaining parameters same as in baseline.

Table 6: Fiscal Multipliers

	Progressive Tax			Flat Tax		
	Impact	1 Year	2 Year	Impact	1 Year	2 Year
Output Multipliers						
Baseline	1.08	1.16	1.17	1.22	1.26	1.27
Inelastic Hours	0.72	1.16	1.26	0.72	1.15	1.25
Wage Posting	1.09	1.16	1.17	1.19	1.24	1.25
Unemployment Multipliers						
Baseline		-0.32			-0.20	
Inelastic Hours		-0.80			-0.79	
Wage Posting		-0.25			-0.23	

Note: The output multiplier is the cumulative change in output divided by the cumulative change in government spending. The unemployment multiplier is the peak reduction in unemployment measured in percentage points.

Table 7: The Great Moderation

Sample Period	U.S. Data	
	1965:1-1986:4	1987:1-2005:4
Standard Deviations		
Output	1.43%	1.07%
Consumption	0.67	0.57
Investment	3.23	2.86
Wage	0.42	1.11
Unemployment	8.86	8.10
Vacancies	10.64	10.10
Tightness	19.31	17.78
Hours per Worker	0.35	0.40

Note: Standard deviations expressed relative to output. HP filter applied to all time series. See the appendix for data sources.

Figure 1: Impulse Responses to a Positive 1% Aggregate Productivity Shock in the Baseline Model

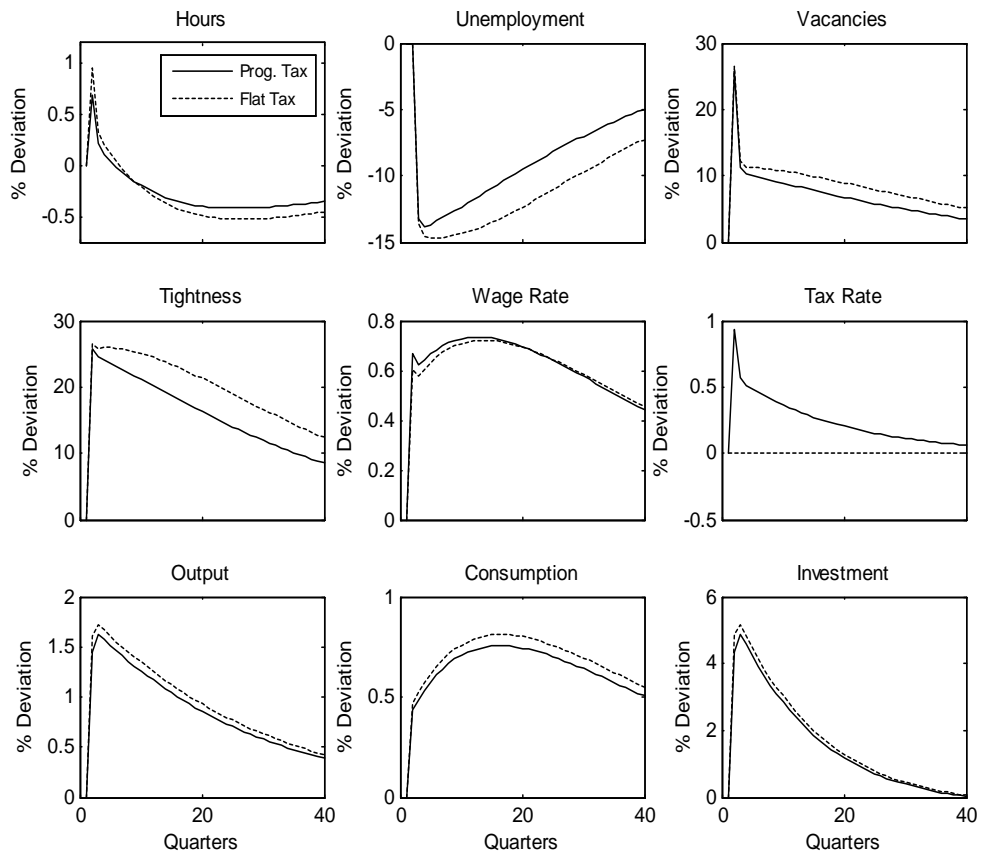
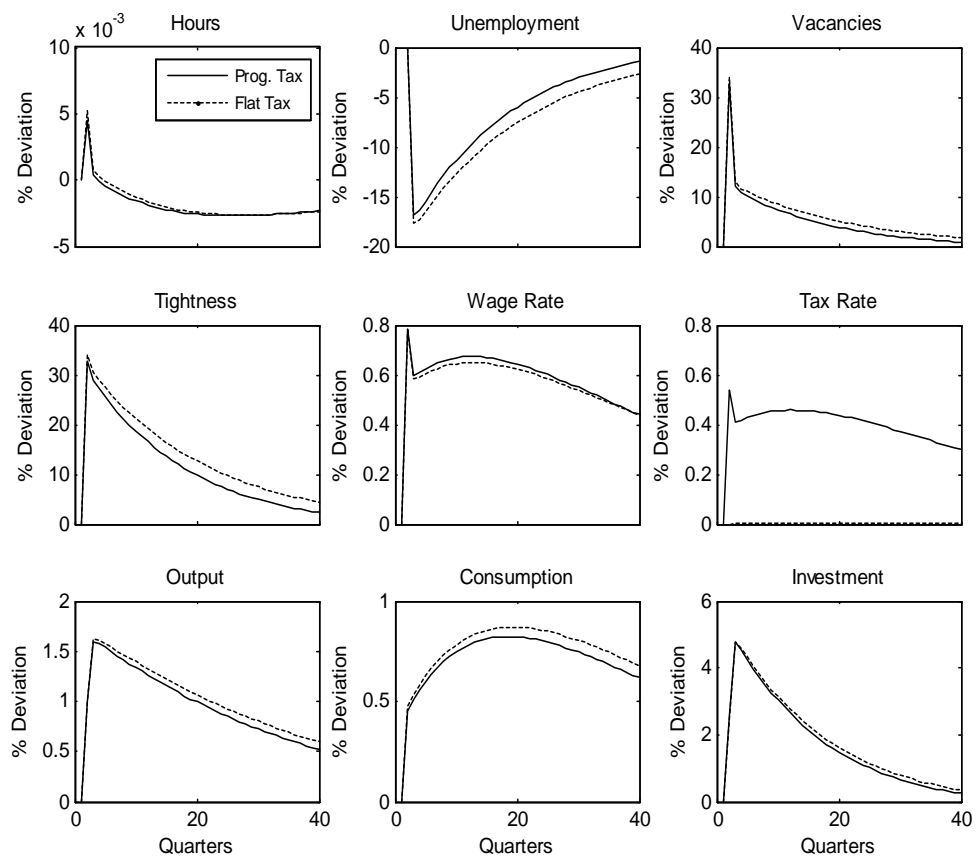
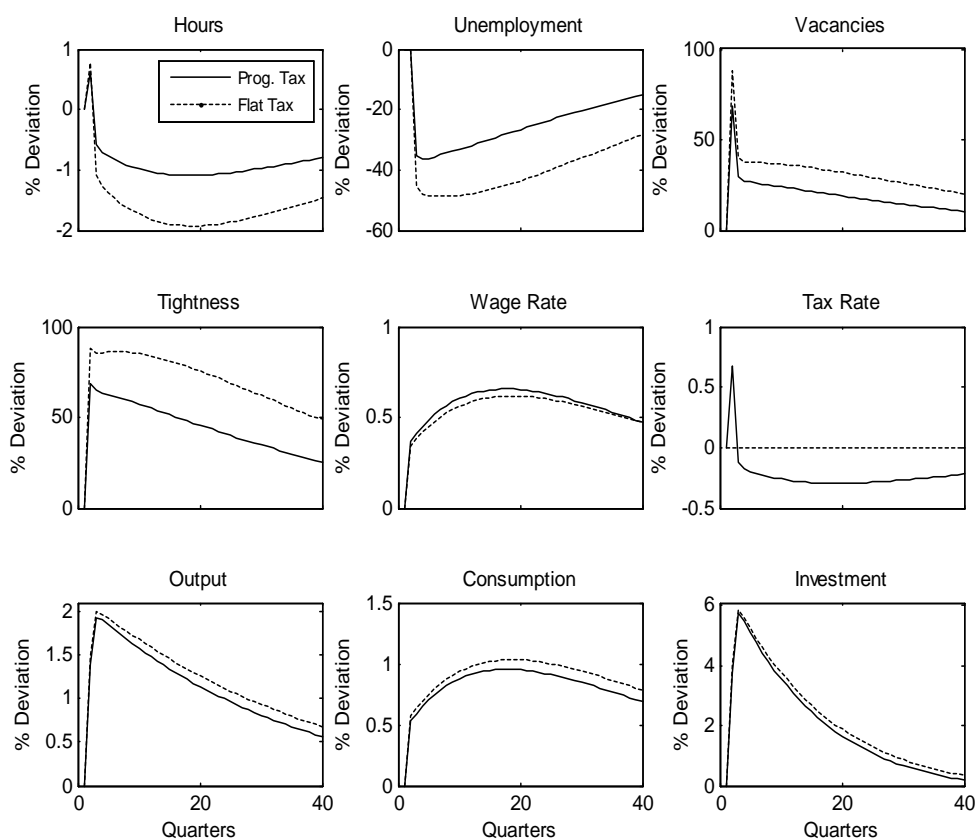


Figure 2: Impulse Responses to a Positive 1% Aggregate Productivity Shock with Inelastic Hours



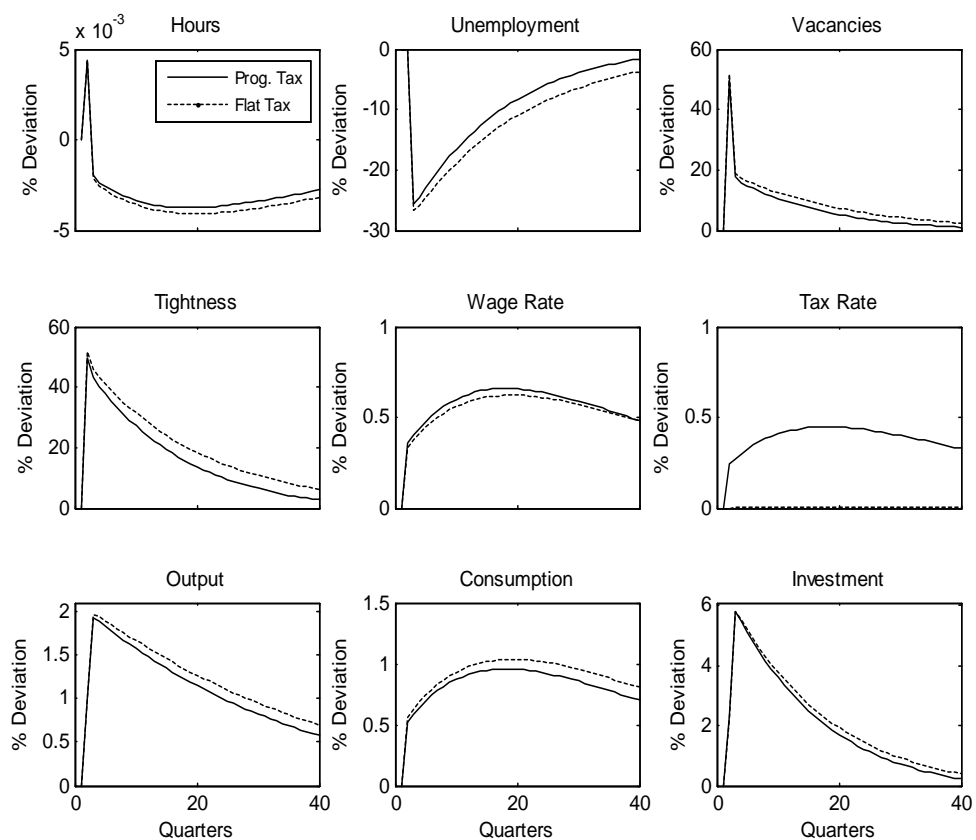
Note: Impulses computed for $\varphi = 100$. All other parameters are the same as in the baseline calibration.

Figure 3: Impulse Responses to a Positive 1% Aggregate Productivity Shock with Wage Posting



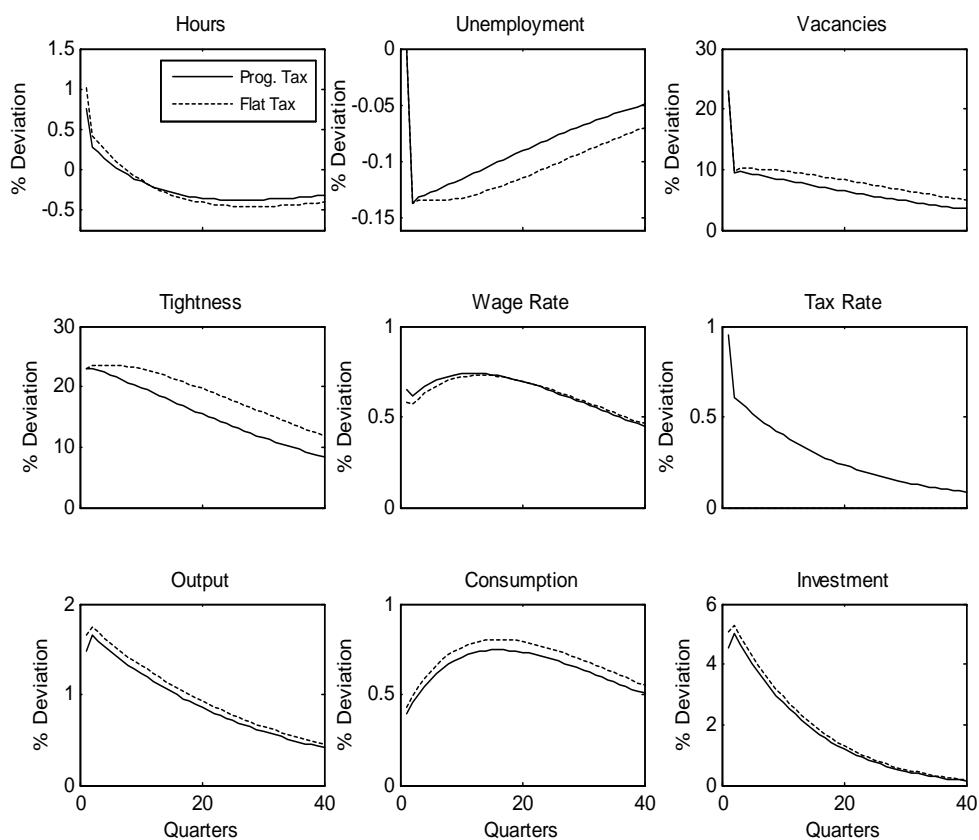
Note: Impulses computed for $\eta = 0$. All other parameters are the same as in the baseline calibration.

Figure 4: Impulse Responses to a Positive 1% Aggregate Productivity Shock with Wage Posting and Inelastic Hours



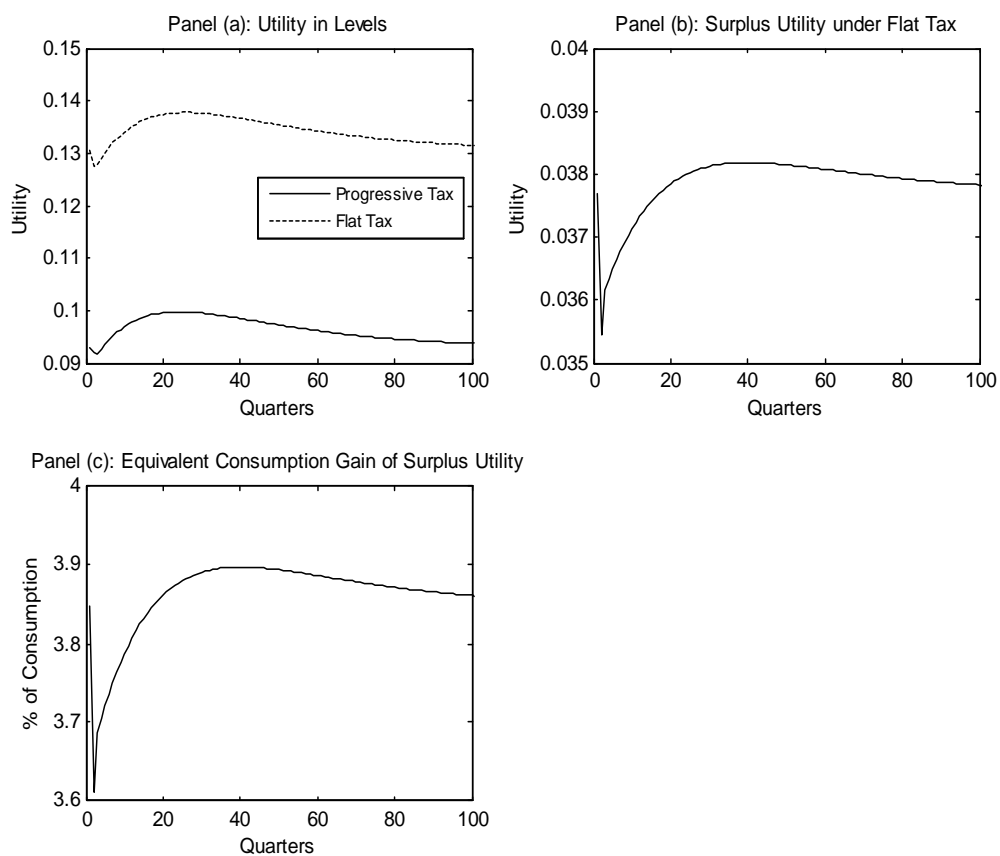
Note: Impulses computed for $\eta = 0$ and $\varphi = 100$. All other parameters are the same as in the baseline calibration.

Figure 5: Impulse Responses to a Positive 1% Aggregate Productivity Shock under a Second-Order Accurate Approximation



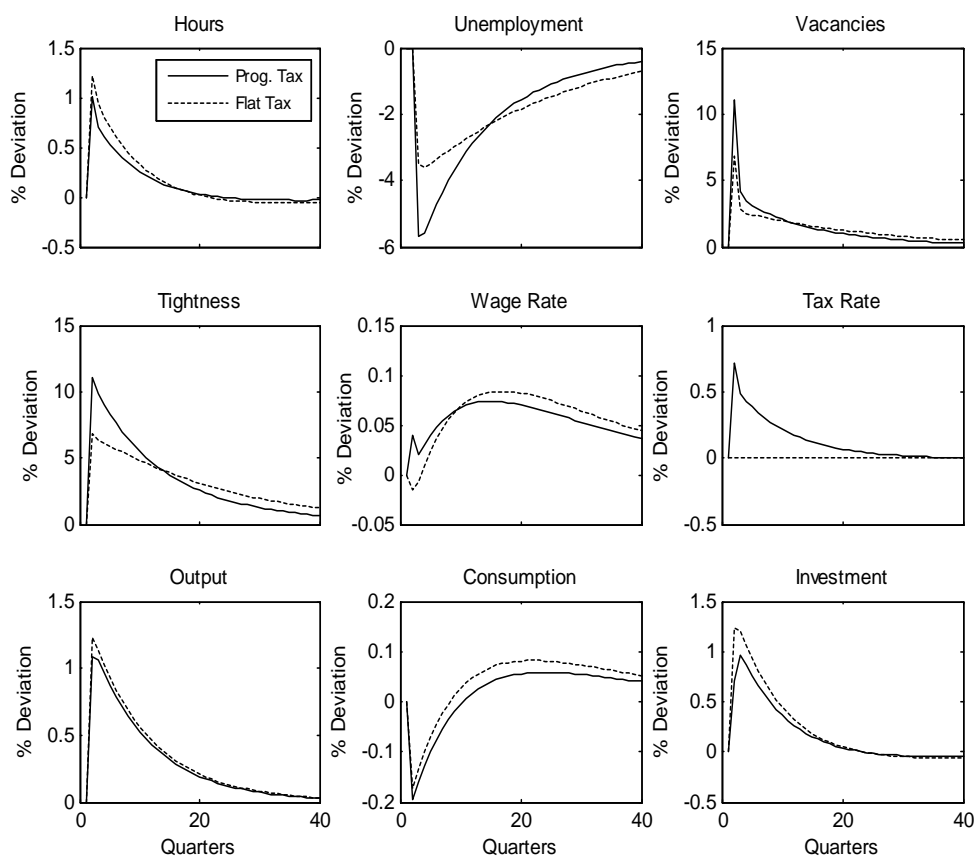
Note: Computed using a second-order approximation to the policy function. All parameters are the same as in the baseline calibration that was used to obtain the linearised dynamics in Figure 1.

Figure 6: Utility Adjustment Paths after a Positive 1% Aggregate Productivity Shock



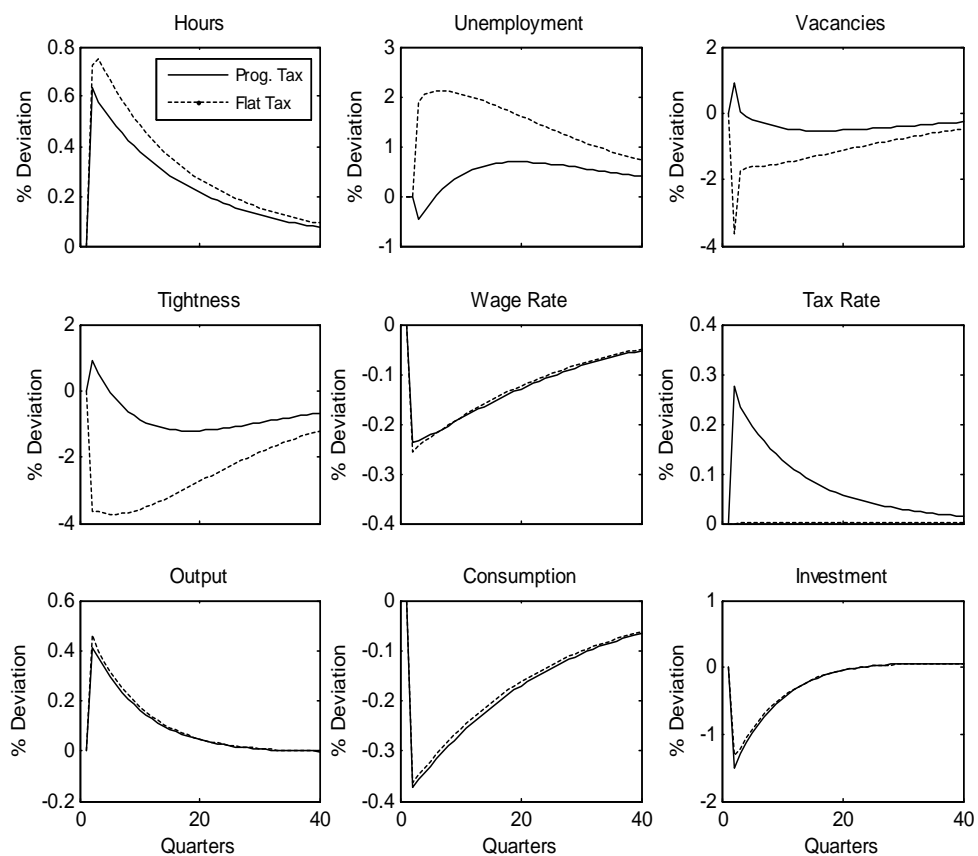
Note: Computed using a second-order approximation to the policy function. Panel (c) shows the utility gain from a flat tax expressed as a percentage of steady state consumption in the progressive tax model.

Figure 7: Impulse Responses to a Positive Government Expenditure Shock in the Baseline Model



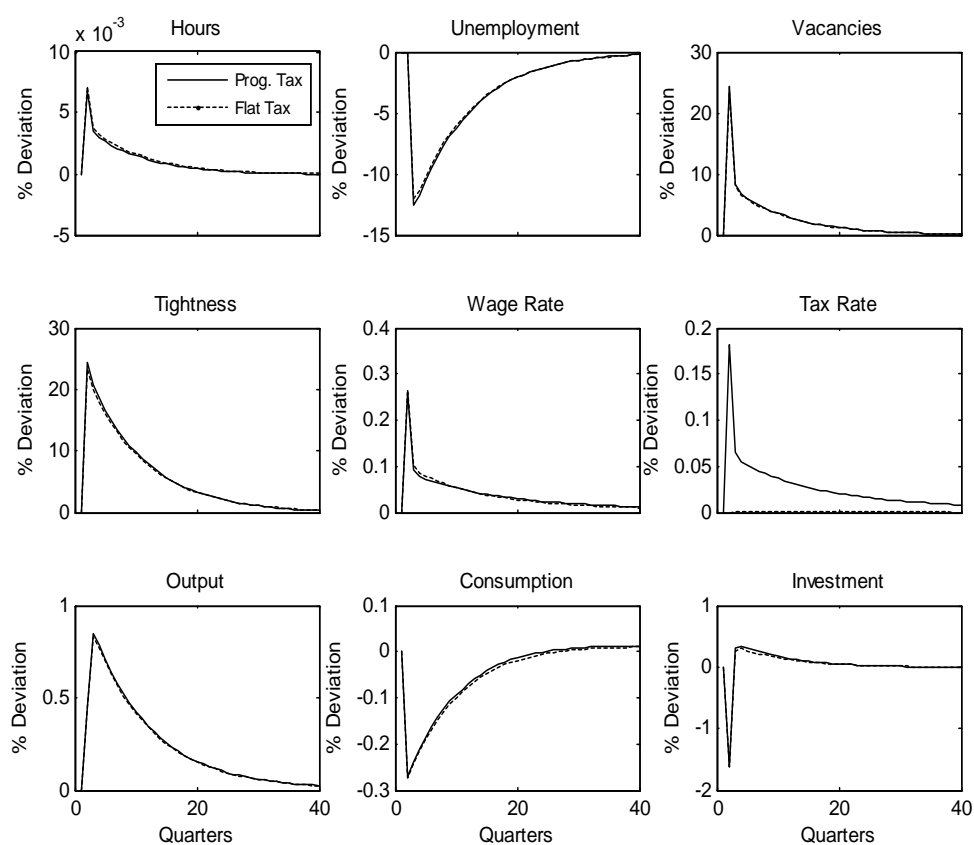
Note: Size of shock normalised to one percent of GDP.

Figure 8: Impulse Responses to a Positive Unproductive Government Expenditure Shock



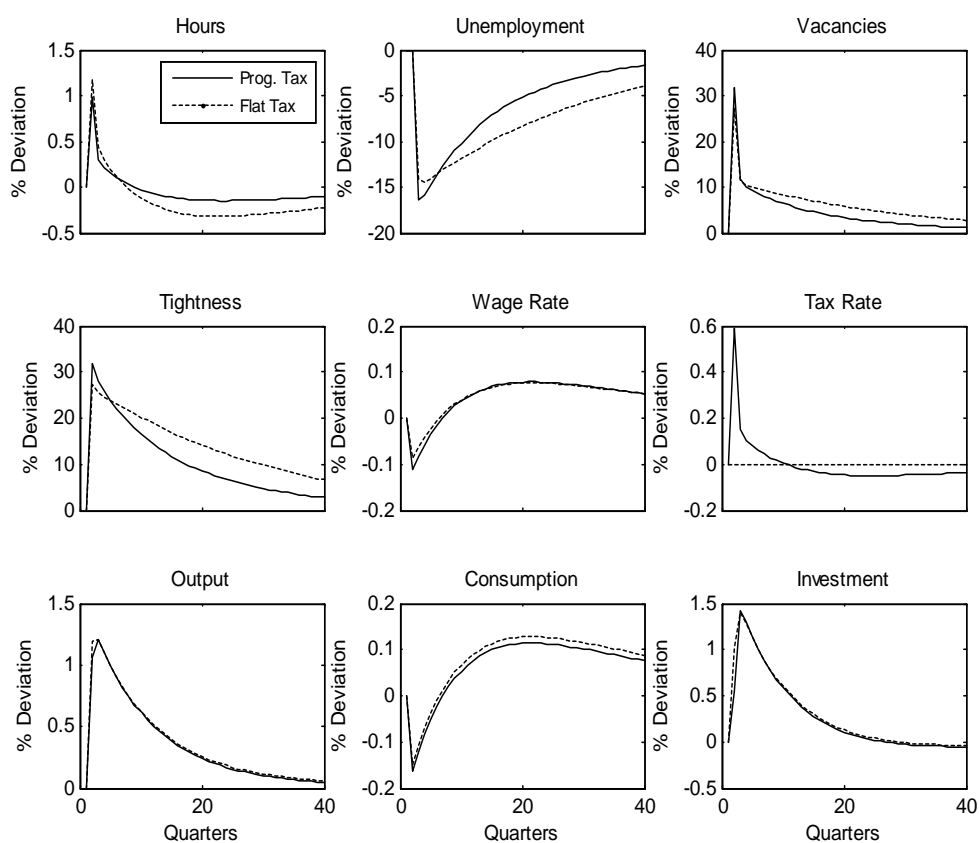
Note: Size of shock normalised to one percent of GDP. Impulses computed for $\chi = 0$. All other parameters are the same as in the baseline fiscal calibration.

Figure 9: Impulse Responses to a Positive Government Expenditure Shock with Inelastic Hours



Note: Size of shock normalised to one percent of GDP. Impulses computed for $\varphi = 100$. All other parameters are the same as in the baseline fiscal calibration.

Figure 10: Impulse Responses to a Positive Government Expenditure Shock with Wage Posting



Note: Size of shock normalised to one percent of GDP. Impulses computed for $\eta = 0$. All other parameters are the same as in the baseline fiscal calibration.

Chapter II

Monetary Policy and Job Creation in a New Keynesian Model

Abstract

Recent research has indicated that New Keynesian models with frictional unemployment tend to encounter difficulty in generating a rise in job creation in response to expansionary monetary policy, rendering the model inconsistent with the downward sloping Beveridge curve that appears in the data. Matching frictions in the labour market congest the job creation process so that firms tend to skew adjustment to shocks towards the job destruction margin. In recognition of the assertion put forth but unpursued by Cooley and Quadrini (1999) that fluctuations in the size of the labour force may ease labour market congestion and therefore amplify cyclical job creation, in this chapter we extend a New Keynesian model with unemployment to feature an endogenous labour market participation decision. A baseline model with a standard degree of risk aversion tends to exhibit countercyclical labour force participation, which is inconsistent with the data. In order to address this issue, we propose the notion of labour market participation as a social consideration, which we demonstrate to be capable of generating procyclical participation incentives. We then find that plausible fluctuations in the size of the labour force do not exert a quantitatively significant effect on job creation. It is then argued that, by altering the dynamics of aggregate demand, time-inseparability in the utility function can significantly improve the ability of expansionary monetary policy to increase job creation, allowing the model to generate a downward sloping Beveridge curve conditional on monetary shocks. However, time-inseparability comes at the cost of inducing a counterfactually upward sloping Beveridge curve conditional on productivity shocks.

1 Introduction

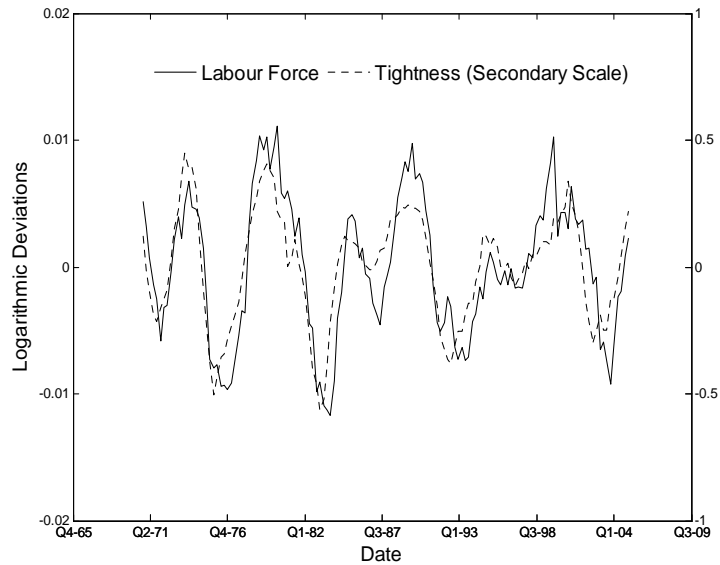
Flows of working-age individuals into and out of the labour force are exceedingly large. For instance, the number of working-age people moving from employment to non-participation exceeds the flow from employment to unemployment even if the tails of the age distribution are ignored (Garibaldi and Wasmer, 2002). This raises a number of questions about macroeconomic behaviour. What is the relationship between variations in the size of the labour force and the joint dynamics of inflation and output? Are movements in labour market participation potentially important for monetary policy? To what extent do labour market flows to and from inactivity affect wage determination and job creation?

This chapter attempts to address these questions in light of analyses by Cooley and Quadrini (1999) and more recently by Krause and Lubik (2007) that have highlighted the difficulties encountered by New Keynesian models with frictional unemployment and endogenous job destruction in generating a meaningful boom in job creation in response to monetary stimulus. Macroeconomic adjustment to monetary policy shocks in this class of models tends to occur largely through variations in the rate at which jobs are destroyed, rather than the rate at which they are created. The reason is that costly matching frictions in the labour market make it difficult to hire new workers, a difficulty which is exacerbated during a boom when unemployment is low. Destruction rates that decline during an upswing cause the unemployment pool to get smaller. In the presence of matching frictions, labour market congestion from low unemployment lengthens the expected duration before a vacancy is filled and also puts upward pressure on wages because of an improvement in the job finding prospects of employees outside of their current matches. Endogenising both the destruction and creation decisions of firms at the cyclical frequency in a frictional environment thus implies that a reduction in the destruction rate has the unintended consequence of reducing incentives for job creation due to a concurrent rise in labour market congestion. In a standard New Keynesian model with matching frictions, the supply of vacancies typically decreases in response to expansionary monetary policy, moving in parallel with unemployment. However, as Krause and Lubik (2007) and Fujita (2003) note, a positive and persistent reaction of vacancies to shocks is pivotal in generating the observed negative correlation between job creation and job destruction over the cycle as well as the highly negative correlation between vacancies and unemployment, the latter being known as the Beveridge curve. From a model building perspective, predicting a realistically sloped Beveridge curve is crucial for the ability of the model to generate realistic transition rates (determined by the cyclical behaviour of labour market tightness) and therefore job flows.¹

Mechanisms are therefore sought which are capable of arresting the fall in the profitability of issuing vacancies as labour market tightness, and hence congestion, rises during a boom. Movements in labour market participation tend to be moderately procyclical, fluctuating about a third as much as output, and highly correlated with the vacancy-unemployment ratio, which serves as a proxy for the potential benefits of job search. Figure 1 illustrates the cyclical behaviour of labour market participation and market tightness for U.S. data. As Cooley and Quadrini (1999) speculate, through the amplification of procyclical variation in the pool of available job searchers, endogenous labour market participation may potentially mitigate the rise in labour market congestion that offsets incentives to expand vacancy supply during booms. The authors, however, do not pursue this avenue. *A priori*, it is difficult to judge the importance of fluctuations in the labour force for job creation in a frictional labour market. On

¹The Beveridge curve is also important for policy. For example, outward shifts may indicate a rise in structural, as opposed to cyclical, unemployment. See Blanchard and Diamond (1989) for a seminal contribution.

Figure 1: Cyclical Components of Labour Market Tightness and the Labour Force for U.S. Data



Notes: Data for the labour force and unemployment are from the Bureau of Labor Statistics website. Time series data on vacancies are compiled by the Conference Board and made available online at the OECD's statistics portal. Data are HP filtered with smoothing parameter 1600.

the one hand, fluctuations in the labour force are relatively mild.² Indeed, the vast majority of business cycle studies of unemployment abstract from the participation margin presumably on these grounds (see Andolfatto 1996 and Merz 1995 for two seminal studies). But on the other hand, the current literature suggests that job creation is perhaps highly sensitive to the presence of labour market congestion arising from matching frictions, indicating that even small reductions in congestion may have an appreciable effect.

The resolution of this issue necessitates a quantitative approach. For this purpose, we extend the New Keynesian model of Krause and Lubik (2007) by introducing an explicit labour market participation decision, thus allowing for compositional changes in the measure of agents who are not currently in employment. In particular, we ask whether endogenous participation can overturn the result of Krause and Lubik (2007) that an acceleration in monetary growth causes a contraction in vacancy supply. Our main result is that endogenous participation movements are found to be quantitatively irrelevant for amplifying cyclical job creation. Despite empirically plausible variation

² Although Figure 1 illustrates that fluctuations in the labour force are an order of magnitude smaller than fluctuations in tightness, the latter tend to be very large, an order of magnitude larger than output movements.

in the size of the labour force, the extended model remains incapable of producing a Beveridge curve. Prior to arriving at that conclusion, it is found that a baseline model with risk averse agents exhibits a tendency for labour force participation to be countercyclical, which is inconsistent with the data. The reason is that as the marginal utility of market consumption declines during a boom, agents have less of an incentive to remain active in the labour market. We find that this result holds despite a conventional degree of curvature in the utility function. In order to address this issue we propose the notion of labour market participation as a social consideration, which we demonstrate to be capable of generating procyclical participation incentives. We formalise this idea by introducing an aggregate consumption externality in the form of comparison utility preferences which feature an interdependence of utility across households. The particular specification which we adopt is based on Gali (1994) which exhibits the "keeping up with the Joneses" property, or the notion that individual households will wish to consume relatively more when aggregate consumption (which we take to be the external reference) is higher. Procyclicality of the labour force in our model with comparison utility is then generated by the social consideration of wanting to consume more when average consumption is relatively high. In this sense, participation in the labour market can be thought of as more of a social institution and less of a free choice, with the social aspects of labour force participation providing an incentive to "stay in the market". Consumption externalities are key to replicating realistic labour force fluctuations, allowing us to reach the conclusion that, for empirically plausible magnitudes, endogenous participation movements are not a primary driver of cyclical job creation.

We must therefore dig a little deeper into the fundamental mechanics of the model in order to find other processes besides participation flows which could potentially give rise to a positive relationship between expansionary monetary policy and vacancy supply. In addition to an aggregate consumption externality, which is also referred to as external habit formation, we also consider internal habit formation. Internal consumption habits, which cause utility to be time-inseparable, shape aggregate demand dynamics over the business cycle, in particular rendering the response to monetary growth shocks much more persistent.³ As a result of this increase in persistence, we find that time-inseparability of the utility function is capable of reversing the conclusion reached by Krause and Lubik (2007) that vacancy supply contracts in response to monetary stimulus. The intuition for this result is as follows. Without internal consumption habits, the peak response in aggregate demand to a monetary shock occurs on impact, after which output falls monotonically back towards the steady state. However, in the presence of a frictional matching process in the labour market, vacancy supply is forward-looking. The forward-looking nature of job creation in this class of models implies that the incentives for job creation that arise in response to a monetary expansion are weak when the impact on aggregate demand tends to be front-loaded.

³Both Cooley and Quadrini (1999) and Krause and Lubik (2007) assume time-separable utility.

By introducing internal habit formation, the response in aggregate demand to a monetary expansion is hump-shaped, so that firms anticipate a further increase in aggregate demand even after the period in which the shock takes place. Our analysis demonstrates that sluggish adjustment in aggregate demand is quantitatively important for generating an expansion in vacancy supply in response to monetary stimulus.

Before proceeding with the formal model, we note that this chapter is related to various strands of literature on monetary policy, job flows, endogenous labour force participation, home production and consumption externalities. In relation to job flows, recent work by Silva and Toledo (2009) indicates that introducing post-match labour turnover costs helps to encourage job creation by making termination more costly, allowing for a more realistically sloped Beveridge curve. Holt (2008) finds that endogenous hours variation is conducive to breaking the synchronisation of job flows and obtaining a negative vacancy-unemployment correlation. Our approach takes a different perspective, assessing the incentives for job creation that arise from fluctuations in labour market participation and sluggish consumption adjustment to shocks. These two mechanisms are examined independently of one another for ease of exposition as well as for reasons that will become clear subsequently.

Regarding endogenous participation, the motivation for modelling the activities in which non-participants engage as pertaining to a distinct "home sector", rather than just a subset of time use in the market sector, is based on empirical grounds. The home sector is large, whether measured by the time allocated to home production, the value of inputs or the value of output. Empirical estimates of the value of home production range between 20 to 50 percent of GNP in industrialised countries (Eisner 1988 and Bonke 1992). Time use surveys indicate that people spend 28 percent of their discretionary time working in the home sector, which is comparable to 33 percent in the formal market (Hill 1985). Purchases of consumer durables and residential investment exceed purchases of producer durables and nonresidential investment in the U.S. (Greenwood and Hercowitz 1991). Moreover, there appears to be a large degree of substitutability between market and non-market activities, with employed individuals devoting relatively little time to home production, and the amount of time devoted to market activities being positively correlated with wages (see Benhabib et al. 1991 and Rios-Rull 1990). In light of this evidence, the home sector is modelled as a distinct entity, and it is assumed that only agents who are not employed engage in the production of a home good. An alternative interpretation of activity in the home sector would be a broad definition of "leisure". In what follows, these two terms are used interchangeably.

The study by Krause and Lubik (2007) is very closely related to a small but rapidly expanding literature of New Keynesian studies which concentrate on the interaction between monetary policy and frictional unemployment. These include Walsh (2005), Heer and Maussner (2009) and Trigari (2009), amongst others. These studies find that, relative to a New Keynesian model with a Walrasian labour market, matching frictions

lower the elasticity of marginal cost with respect to output. In a Walrasian labour market in which unemployment is non-existent and all labour input variation occurs along the intensive margin, unless an implausibly high labour supply elasticity is assumed the model will tend to generate excessive inflation volatility. Allowing for matching frictions permits firms to adjust the labour input along the extensive margin as well, which, given the long term nature of employment contracts in such a setting, alleviates the rise in marginal costs due to the expected payoff from continuing the match into the following periods (see Trigari 2009). On the other hand, Walsh (2005) finds that policy inertia itself is the primary driver behind the output effects of monetary policy shocks. All of these studies normalise the labour force to a constant. Furthermore, our model differs by featuring firms that jointly make both pricing and firing decisions, as in Krause and Lubik (2007) and Thomas (2008), whereas this decision is separated in the other New Keynesian studies mentioned. We argue that the connection between pricing and separation decisions has quantitatively significant implications for the (in)ability of the model to jointly capture the volatilities of inflation and job destruction.

Recent work by Gali (2010) relates more closely to the current study in that, although he does not focus on the implications, he allows for variations in the labour force in a New Keynesian model with sticky prices. There are nevertheless several differences between our study and his. First, we explicitly allow for not only an endogenous participation decision, but also an endogenous job destruction decision, whereas Gali (2010) assumes a fixed destruction rate and refrains from discussing the dynamics of job flows over the cycle. Second is the mechanism through which frictions are introduced in the labour market. Instead of costly vacancy posting, Gali (2010) assumes that firms pay a real hiring cost in order to expand their workforce, although, as he points out, this is closely related in a reduced-form sense to the assumption of an aggregate matching function. The consequence of adopting the hiring costs of Gali (2010) however is that vacancies are omitted from his model, preventing an assessment of the ability of the model to reproduce a Beveridge curve, a key empirical regularity which has been a pressing issue for matching models with endogenous job destruction (Cole and Rogerson 1999).

Another difference between our models is the manner in which the participation decision is introduced. Gali (2010) specifies a utility cost to the representative household of having an additional agent enter the labour force. We follow Haefke and Reiter (2006) in assuming that agents are subject to idiosyncratic variation in their valuation of leisure, thus allowing for participation flows. Heterogeneity in the valuation of non-participation is what permits movements into and out of the labour force in our model. Conceptually, the two approaches are not drastically different, but do appear to require different calibration strategies which may affect the results. Gali (2010) calibrates the household's disutility of unemployment so as to generate realistic labour force fluctuations. Given his specification of the participation decision, Gali (2010) is forced to depart from the standard assumption of symmetric Nash bargaining. Instead

he assumes that the relative bargaining power of the firm is close to zero in order to generate realistic unemployment fluctuations. In the limit in which it is zero, the wage is independent of the worker's outside option. The worker's outside option, however, undergoes a substantial change upon the introduction of a participation choice, and to rule it out of the wage equation is to *a priori* discard a channel of potential importance in light of the well known connection between wage dynamics and job creation emphasised by Shimer (2005). In contrast, we follow the approach of Haefke and Reiter (2006) in calibrating the standard deviation of idiosyncratic home productivity shocks so as to generate realistic unemployment movements, which allows us to match the data without imposing a restriction on relative bargaining powers.

Another study by Campolmi and Gnocchi (2011) also incorporates endogenous labour force participation into a New Keynesian model, but, similarly to Gali (2010), they do not also endogenise job destruction. Instead, they show that endogenous participation reduces the extent to which switching from a flexible to a strict inflation targeting regime exacerbates unemployment volatility. We also note that our work relates to other studies of variable participation over the business cycle which abstract from monetary disturbances and nominal rigidity. Apart from Haefke and Reiter (2006), such studies include Veracierto (2008) and Tripier (2003). The latter two studies find that procyclical participation incentives tend to imply that unemployment counterfactually rises during productivity booms when the incentive to enter the workforce is high but matching frictions impede the transition to employment. Haefke and Reiter (2006) note that calibrating the density of workers who are close to the participation margin is key to generating countercyclical unemployment fluctuations. Although the manner in which we introduce endogenous participation closely corresponds to Haefke and Reiter (2006), we find that the negative correlation of unemployment with output is not sensitive to our calibration strategy. This suggests that our incorporation of endogenous job destruction promotes countercyclical unemployment despite procyclical participation incentives. To see this, note that endogenous job destruction has opposing effects on the incentive to participate. All else equal, a reduction in the destruction rate in response to a positive shock reduces the unemployment pool, raising the chances that an individual agent finds a job and thus encourages participation. Conversely, the ability of firms to adjust along the separation margin reduces the extent to which new jobs need to be created in order to expand employment, which detracts from the incentive to enter the labour market during an upswing. On balance, our finding that unemployment is robustly countercyclical suggests that the latter effect outweighs the former. Moreover, unlike the previously mentioned studies, we also allow for direct transitions from non-participation to employment, as appears in the data, in the form of "passive search". We find this channel to be quantitatively important for generating strongly countercyclical unemployment.

Another major difference between our model and that of Haefke and Reiter (2006) is that we consider consumers who are risk averse while they assume risk neutrality.

Interpreting non-participation as a productive use of time presents complications with risk averse workers as they will have to be paid increasingly more to remain in employment during a boom. As mentioned previously, we address this issue by re-interpreting participation in the labour force as a social activity which allows the household to adjust its level of market consumption relative to an external reference stock. Individual concern over aggregate consumption is referred to as "comparison utility" in Carroll et al. (1997). Preferences of this type that exhibit outward-looking, interdependent utility have a long tradition in economics, dating back to Veblen (1899) and Duesenberry (1949). In the recent literature, such preferences have been applied in a variety of fields: Carroll et al. (1997), Alvarez-Cuadrado et al. (2004) and Liu and Turnovsky (2005) in endogenous growth; Abel (1990), Campbell and Cochrane (1999) and Gali (1994) in asset pricing; Dupor and Liu (2003) show that consumption externalities can cause equilibrium overconsumption; and Ljungqvist and Uhlig (2000) analyse the implications for optimal tax policy. To our knowledge, the application to the social aspects of cyclical labour force participation is novel.

The rest of the chapter proceeds as follows. Section 2 derives a baseline New Keynesian model that features endogenous labour market participation. Section 3 details the calibration and solution procedures. Simulation-based moment calculations and impulse response functions are reported in Section 4. Section 5 discusses the comparison utility extension as well as the effects of nominal rigidity, passive search and time-inseparability. Section 6 concludes.

2 Baseline Model

The model that is developed in this section incorporates endogenous labour force movements into a New Keynesian business cycle framework, relaxing the assumption that employment and unemployment are collectively exhaustive states. Frictions in the labour market are based on the mathematical framework commonly referred to as the Diamond-Mortensen-Pissarides matching model.⁴ Idiosyncrasy in the value of home productivity is what enables participation fluctuations, as in Haefke and Reiter (2006). Conditional on this value, non-employed agents, i.e. those not in productive employment, decide whether to enter the labour market as unemployed and formally search for a job, or stay out of the labour force to fully devote their time to the production of a home good, which can also be thought of as leisure. In what follows these two terms for the activities of non-participants are used interchangeably. Both pricing and hiring decisions are made within the same firm in a monopolistically competitive environment, drawing heavily on the structure of Krause and Lubik (2007). Hiring is subject to an aggregate random matching technology which permits flows from both unemployment and non-participation directly to employment. Pricing decisions are subject to convex nominal adjustment costs.

⁴Seminal contributions include Diamond (1982), Mortensen (1982) and Pissarides (2000).

We begin by describing the household's optimisation problem.

2.1 Households

Time is discrete. It is assumed that the institutional structure of households is such that each pools all of the income of a unit measure of individual members so that all employment risk is eliminated.⁵ Consumption is therefore the same for all family members and the j th household's preferences are defined over a composite consumption bundle, $c_{j,t}$. Real money balances, $m_{j,t}/p_t$, where p_t is the aggregate price level and $m_{j,t}$ is the j th household's nominal money balances, enter into the utility function.

Households make consumption decisions in order to maximise the infinite sum of expected discounted utility according to the return function

$$\max \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{c_{j,t}^{1-\sigma_1}}{1-\sigma_1} + d \log \left(\frac{m_{j,t}}{p_t} \right) \right] \quad (1)$$

where $\sigma > 0$ is the coefficient of relative risk aversion, E_t is the expectational operator conditional on information available at time t and β is a discount factor. The labour market participation decision is not chosen at the level the household, but is instead determined through decentralised individual participation decisions which are discussed in detail subsequently. This is analogous to the assumption made by Trigari (2009) and Holt (2008) that the quantity of household hours is chosen through decentralised optimisation behaviour of agents subject to idiosyncratic productivity shocks and not at the level of the household.

The household's budget constraint is

$$c_{j,t} + \frac{m_{j,t}}{p_t} + \frac{B_{j,t}}{p_t} = \mathbb{W}_{j,t} + \mathbb{U}_{j,t} + \frac{m_{j,t-1}}{p_t} + r_{t-1} \frac{B_{j,t-1}}{p_t} + (g_t - 1) \frac{m_{t-1}}{p_t} + \Pi_{j,t} + T_{j,t}$$

where $\mathbb{W}_{j,t}$ and $\mathbb{U}_{j,t}$ are labour and unemployment income, respectively, and are specified below. r_{t-1} is the return on nominal bonds purchased at $t-1$, $B_{j,t-1}$. $(g_t - 1) m_{t-1}/p_t$ is a lump-sum transfer from the monetary authority to the household in period t , where g_t is the gross growth rate of the aggregate money supply. $\Pi_{j,t}$ represents dividends that the household receives from diversified ownership stakes in firms and $T_{j,t}$ is a lump-sum fiscal transfer (or tax if less than zero). The first-order conditions associated with the

⁵Dropping this assumption would entail tracking the labour market histories of each individual family member, greatly complicating the analysis. Authors such as Merz (1995) and Andolfatto (1996) pioneered this method of modelling general equilibrium business cycles with search frictions, which has proven to be the most popular framework in the literature.

household's optimisation programme are

$$\begin{aligned}\partial c_t &: c_t^{-\sigma} - \mu_t = 0 \\ \partial B_t &: -\mu_t + \beta E_t \mu_{t+1} r_t \frac{p_t}{p_{t+1}} = 0 \\ \partial m_t &: d \left(\frac{m_t}{p_t} \right)^{-1} - \mu_t + \beta E_t \mu_{t+1} \frac{p_t}{p_{t+1}} = 0\end{aligned}\tag{2}$$

where μ_t is the Lagrangian multiplier on the budget constraint. In equilibrium all households are identical and so j subscripts are eliminated, so that quantities now refer to the representative household. Eliminating the multiplier using the marginal utility of consumption yields

$$c_t^{-\sigma} - \beta r_t c_{t+1}^{-\sigma} \frac{p_t}{p_{t+1}} = 0\tag{3}$$

$$d \left(\frac{m_t}{p_t} \right)^{-1} - c_t^{-\sigma} \left(1 - \frac{1}{r_t} \right) = 0\tag{4}$$

which are the usual optimality conditions associated with a standard New Keynesian model. Equation (3) is a standard Euler condition governing the optimal path of consumption, and equation (4) is the intratemporal optimality condition that sets the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

2.2 Firm Structure and Job Flows

The structure of firms closely follows Krause and Lubik (2007). A large number of firms produce differentiated goods by employing labour as the sole input to a stochastic production technology. Each individual firm is large, with a continuum of jobs, or matches, defined over the unit interval available within each firm. Each successful match (job) produces a measure of output $A_t a_{ij,t}$ at time t , where A_t is an aggregate technology shock common to all matches within all firms, and $a_{ij,t}$ is the match-specific productivity of job i at firm j at time t . Variables without an ij subscript denote aggregate values. It is assumed that the individual agent's valuation of leisure or home productivity does not influence the productivity of that agent's employment.

All uncertainty, both aggregate and idiosyncratic, is revealed at the beginning of each period. Match specific productivity is drawn before production commences within the period, and only matches which draw a high enough value for $a_{ij,t}$ follow through to the production stage, implying that different levels of output are produced in different matches. Matches that do not draw a high enough value of the idiosyncratic shock are terminated prior to production. Should this be the case, the job is then destroyed yielding a zero value to the firm, and the worker, upon job termination, decides whether to actively search for another job or to exit the labour market. This decision will be made explicit once a recursive structure for the labour market has been constructed.

An endogenously determined critical value, denoted by $\underline{a}_{j,t}$ for firm j , defines the level of individual match productivity that renders the value of the match to the firm equal to zero. The i th match for which $a_{ij,t} \leq \underline{a}_{j,t}$ is terminated with probability one. Idiosyncratic match productivities are drawn from a general time-invariant distribution, $G(a)$, with positive support and density $G'(a)$. The endogenous destruction rate at firm j is then given by $G(\underline{a}_{j,t})$. The endogenous destruction rate is time dependent because changes in the aggregate state of the economy will affect the value of all matches regardless of the individual circumstances in each match. In order to maintain consistency with the related literature, following den Haan et al. (2000) it is assumed that there is also a source of exogenous labour turnover, denoted by ρ^x . A fraction ρ^x of the firm's matches exogenously terminate before idiosyncratic productivities are known in each period. Exogenous labour turnover is intended to capture job terminations that arise from worker quits or other actions which are not related to the fundamental productivity of the job. The total separation rate for firm j at time t is then given by

$$\rho_{j,t} = \rho^x + (1 - \rho^x) G(\underline{a}_{j,t}). \quad (5)$$

Having described job separation dynamics, consider next the process by which new jobs are created. The measure of new matches in each period is determined by an aggregate random matching technology that has as its inputs the aggregate measure of vacancies issued by firms, $v_t = \int_0^1 v_{j,t} dj$, and the aggregate measure of job searchers, s_t . The aggregate measure of new matches at time t is given by

$$M_t = M(v_t, s_t)$$

where M is increasing in both arguments and homogenous of degree one. Define labour market tightness as $\theta_t = v_t/s_t$, the ratio of vacancies to searchers. Random matching then implies that the probability that an individual vacancy is matched in period t is

$$q(\theta_t) = \frac{M_t}{v_t} = M(1, \theta_t^{-1})$$

where the constant returns to scale restriction on the matching technology permits a representation of transition probabilities with tightness as the sole input argument.

2.3 Labour Force Participation

Idiosyncratic home productivity $h_{i,t}$ is drawn from a general time-invariant distribution, $F(h)$, with positive support and density $F'(h)$. In order to avoid wage heterogeneity with respect to $h_{i,t}$ in addition to $a_{i,t}$, which complicates the model substantially, it is assumed that only non-employed agents (both unemployed and out-of-the-labour force) re-draw the idiosyncratic home productivity parameter within the period. This assumption follows Haefke and Reiter (2006). We are effectively assuming that all

successfully matched individuals concentrate fully on market sector production and devote zero effort to production in the home sector. One rationale for suppressing home productivity to zero during full-time employment is the presence of a binding time constraint. In this manner, as will be demonstrated formally when an explicit wage equation is derived below, it is the likelihood that the agent will quit into non-participation upon job termination that determines the outside option of the worker and hence wages, rather than the particular realisation of $h_{i,t}$. Given independent and identical distribution of idiosyncratic shocks, the likelihood of entering unemployment versus non-participation upon job separation is the same for all employees. Outside options of negotiating agents are therefore homogenous within the firm and the only source of wage heterogeneity is $a_{ij,t}$.

Should the match in which a particular agent is employed terminate, the agent then draws a value of $h_{i,t}$, which determines that agent's subsequent actions. Let \underline{h}_t represent the critical minimum bound on home productivity that is necessary for the agent to remain outside of the formal labour market. \underline{h}_t will also be referred to as the "participation constraint". Because the expected value of job search is homogenous across agents, \underline{h}_t is also common to all agents. If $h_{i,t} \geq \underline{h}_t$, non-employed agents, including those fired at the start of the period, enter the pool of non-participants. If $h_{i,t} < \underline{h}_t$, the agent's individual value of home productivity is not high enough to induce non-participation, in which case the agent enters unemployment, the latter being defined formally as the measure of agents actively seeking employment.

Employment is defined as the measure of matches which are productive during the current period,

$$e_t = (1 - \rho_t) n_t$$

where $n_t = \int_0^1 n_{j,t} dj$ is the aggregate measure of matches at the beginning of the period before shocks are drawn and ρ_t is the aggregate separation rate. The timing structure of each discrete period is such that the matching phase occurs after the dissolution of unproductive matches (see Walsh 2005 or Trigari 2009). The unemployment pool is then defined as

$$u_t = (1 - e_t) F(\underline{h}_t)$$

and the measure of non-participants is

$$l_t = (1 - e_t) (1 - F(\underline{h}_t)).$$

The labour force is thus $LF_t = e_t + u_t$. It is assumed that non-participants search passively for job opportunities at a constant intensity of $0 \leq \chi \leq 1$. This approach has been followed by others, for example Pries and Rogerson (2009), and is simply reflective of the assumed weaker interest that non-participants have in working in the formal market but nonetheless permits transitions from non-participation directly to

employment.⁶ Given our adoption of discrete time, χ can be thought of as the fraction of each period during which a non-participant actively searches for employment. The search intensity of unemployed agents is normalised to unity. The aggregate measure of searching agents, in terms of efficiency units, is given by

$$s_t = u_t + \chi l_t.$$

Setting $\chi = 0$ corresponds to the case in which agents must enter employment strictly via unemployment and $\chi = 1$ to the case in which unemployed and non-participating agents search with the same (full) intensity. Let $p(\theta_t) = M_t/s_t$ be the probability that a randomly chosen searcher from the pool s_t is matched with a vacancy at time t . That is, for each efficiency unit of search of an unmatched agent, the aggregate matching technology transfers agents from the pool of effective searchers to employment at the rate $p(\theta_t)$. Hence, the transition probability for an unemployed worker who searches with unit intensity is simply

$$p^U(\theta_t) = p(\theta_t).$$

A non-participant searching with intensity χ faces a lower probability of transition,

$$p^N(\theta_t) = \chi p(\theta_t).$$

The parameter χ is set to ensure that $p^N(\theta_t) \leq p^U(\theta_t)$ for all t .

2.4 Firm Optimisation

The optimisation problem of the firm involves choosing prices, vacancies and the separation threshold in order to maximise the consumption value of an infinite stream of discounted real profits. The aggregate price level is denoted by p_t while individual prices have a j subscript, $p_{j,t}$. The gross inflation rate at time t is $\pi_t = p_t/p_{t-1}$. Output at firm j is $y_{j,t}$. In symbols, the firm's objective function is

$$\max_{p_{j,t}, v_{j,t}, \underline{a}_{j,t}} \Pi_{j,t} = E_t \sum_{t=0}^{\infty} \beta^t \frac{\mu_t}{\mu_0} \left\{ \frac{p_{j,t}}{p_t} y_{j,t} - e_{j,t} \tilde{w}_{j,t} - \kappa v_{j,t} - \frac{\phi}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - \pi \right)^2 y_t \right\} \quad (6)$$

where $\kappa > 0$ is the flow cost of maintaining an available vacancy. $e_{j,t} \tilde{w}_{j,t}$ represents firm j 's wage bill, with $\tilde{w}_{j,t}$ giving the conditional expectation $E_t [w_{ij,t} | a_{ij,t} \geq \underline{a}_{j,t}]$. Wages are not homogenous within the firm but depend positively on the individual match's draw of idiosyncratic productivity. Note that only matches which draw a value of idiosyncratic productivity in excess of the lower bound follow through to the

⁶ Proponents of the "time aggregation bias" viewpoint suggest that some minimal effort is always required to successfully enter employment. Survey data that is obtained at discrete intervals cannot therefore capture infra-monthly transition, and so such instances appear as non-participation-employment flows. See Petrongolo and Pissarides (2001).

production stage, allowing the worker to be remunerated. The costs of price adjustment are captured in (6) by the quadratic term, with $\phi > 0$ governing the severity of price adjustment costs. These are costs associated with relative nominal price changes that deviate from the steady state rate of inflation, π , and are assumed to be proportional to the level of aggregate output, y_t .

Firm optimisation is subject to a constraint on the law of motion of employment that derives from the presence of frictions in the labour market. This constraint for the j th firm is

$$n_{j,t+1} = (1 - \rho_{j,t}) n_{j,t} + q(\theta_{j,t}) v_{j,t}. \quad (7)$$

As is typical of discrete time matching equilibria, it is assumed that vacancies matched at time t become productive in period $t + 1$. Firm j 's total output is

$$y_{j,t} = (1 - \rho_{j,t}) n_{j,t} A_t \int_{\underline{a}_{j,t}}^{\infty} a \frac{dG(a)}{1 - G(\underline{a}_{j,t})} \quad (8)$$

where, in anticipation of its subsequent functional form, ∞ represents the upper limit of the support of G .

The final constraint on firm optimisation is an equation relating individual firm-level prices to the aggregate price index. This is derived as follows. It is assumed that the continuum of intermediate goods is collected and bundled by a final goods firm, or by a group of final goods firms, that behaves competitively. The final goods sector maximises the profits received from bundling the measure of intermediate outputs together subject to the costs of purchasing the intermediates. In symbols, the final goods firm seeks to maximise

$$\max_{y_{j,t}} \left\{ p_t y_t - \int_0^1 p_{j,t} y_{j,t} dj \right\}$$

subject to the following constant elasticity of substitution bundler

$$y_t = \left[\int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} dj \right]^{\frac{\psi}{\psi-1}}.$$

Final goods firms thus solve the following problem in period t :

$$\max_{y_{j,t}} \left\{ p_t^{\frac{\psi}{\psi-1}} \left[\int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} dj \right] - \int_0^1 p_{j,t} y_{j,t} dj \right\}$$

resulting in the first order condition

$$p_t \left[\int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} di \right]^{\frac{1}{\psi-1}} y_{j,t}^{\frac{-1}{\psi}} - p_{j,t} = 0$$

which can be rearranged to produce the following demand function for the j th inter-

mediate;

$$y_{j,t} = \left(\frac{p_t}{p_{j,t}} \right)^\psi y_t. \quad (9)$$

Substituting the above result into the bundler function gives a final goods pricing rule of the form

$$p_t = \left[\int_0^1 \left[p_{j,t}^{\psi-1} \right] di \right]^{\frac{1}{1-\psi}}. \quad (10)$$

The above equation maps the optimal individual firm-level prices into an aggregate price index. Using the demand for intermediate firm j 's output (9) to eliminate $y_{j,t}$ from the firm's objective function (6) and maximising subject to the constraints (7) and (8) gives the following first-order necessary conditions:

$$\partial n_{t+1} : 0 = E_t \beta_{t+1} \left[\lambda_{t+1}^y \frac{\partial y_{t+1}}{\partial n_{t+1}} - (1 - \rho_{t+1}) \tilde{w}_{j,t+1} \right] \quad (11)$$

$$+ E_t \beta_{t+1} \lambda_{t+1}^n (1 - \rho_{t+1}) - \lambda_t^n$$

$$\partial v_t : 0 = -\kappa + \lambda_t^n q(\theta_t) \quad (12)$$

$$\partial \underline{a}_t : 0 = \lambda_t^y \frac{\partial y_t}{\partial \underline{a}_t} - (1 - \rho^x) n_t \frac{\partial}{\partial \underline{a}_t} \tilde{w}_{j,t} - \lambda_t^n (1 - \rho^x) G'(\underline{a}_t) n_t \quad (13)$$

$$\partial p_t : 0 = (1 - \psi) - \phi(\pi_t - \pi) \pi_t + E_t \beta_{t+1} \phi(\pi_{t+1} - \pi) \pi_{t+1} \frac{y_{t+1}}{y_t} + \lambda_t^y \psi \quad (14)$$

where λ_t^y and λ_t^n are the time t Lagrangian multipliers on the output and employment constraints, respectively, and the stochastic discount factor is defined as $\beta_{t+1} = \beta \mu_{t+1} \mu_t^{-1}$. By symmetry, each firm j with a continuum of jobs is identical, and so the j subscripts are dropped from the representative firm's first-order conditions. Note that λ_t^y represents the contribution to the firm's real revenue stream of marginally relaxing the output constraint, and for a profit maximising firm is equal to real marginal cost.⁷ λ_t^n is the shadow value of employment, or the contribution of the marginal worker to the firm's stream of revenue averaged across all idiosyncratic productivities that exceed the separation threshold. Solving (12) for the multiplier and inserting the result into (11) yields

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} (1 - \rho_{t+1}) \left[\lambda_{t+1}^y A_{t+1} \tilde{a}_{t+1} - \tilde{w}_{t+1} + \frac{\kappa}{q(\theta_{t+1})} \right] \quad (15)$$

where $\tilde{a}_t = E_t [a_{i,t} | a_{i,t} \geq \underline{a}_{t+1}]$ is the conditional expectation of idiosyncratic match productivity. Equation (15) is the optimality condition which governs job creation. The expected cost of issuing a vacancy in period t is equal to the expected discounted benefit of employing an additional productive worker in period $t+1$. The latter is represented by the difference between expected productivity and the expected wage, adjusting for

⁷Had the firm's problem been formulated as a cost minimization programme, the output multiplier would represent the marginal cost of producing an additional unit of output. For a profit maximising firm as set out above, the two approaches are identical. It is standard in the literature to refer to the multiplier as the marginal cost. See Krause and Lubik (2007).

search costs forgone at $t + 1$ (captured by $\kappa/q(\theta_{t+1})$) due to the employment of an agent matched at time t . The entire right hand side of the expression is discounted by the term $1 - \rho_{t+1}$, as it represents the probability that time t matches actually become productive at time $t + 1$.

Congestion externalities in the frictional labour market imply that $q'(\theta_t) < 0$, so that a persistent increase in aggregate productivity which increases the right hand side of (15) prompts an increase in vacancy supply, causing $\kappa/q(\theta_t)$ to rise until equality is restored. In a model in which participation is held constant, the available pool of agents to be potentially matched with vacancies is directly inversely proportional to the employment rate. Employing one more agent reduces the search pool by one, and this makes congestion a potential problem for firms wishing to create jobs. As a result, in response to an improvement in aggregate conditions, congestion externalities quickly drive up the expected duration of vacancies and put upward pressure on wages through an increase in tightness. Allowing for endogenous participation has the potential to mitigate these congestion effects by allowing the measure of job seekers to fluctuate in response to the availability of vacancies. Note also that vacancy supply is a forward-looking function of the path of the future wage bill. All else constant, a lower trajectory for the real wage translates into a higher path for the supply of vacancies over the cycle.

Consider next the optimality condition that governs job separations. Replacing the shadow value of employment in first-order condition (13) with equation (12), the following job destruction condition obtains,

$$\lambda_t^y A_t \underline{a}_t - w_t(\underline{a}_t) + \frac{\kappa}{q(\theta_t)} = 0 \quad (16)$$

where $w_t(\underline{a}_t)$ is the equilibrium wage evaluated at the critical threshold. Job destruction occurs at the level of idiosyncratic match productivity for which the net value added of the worker, adjusted for the search costs that the firm forgoes by retaining the marginal employee, is equal to (or below) zero. Recall that job destruction occurs before the matching phase of each period of discrete time. This explains why the relevant search costs that are saved in the current period by retaining the marginal worker are the time t , not the time $t + 1$, search costs. Hiring is subject to costly and time consuming matching frictions, and so current period marginal costs therefore reflect the costs of raising production in the current period via a reduction in \underline{a}_t . Rearranging (16) yields an explicit solution for the separation threshold,

$$\underline{a}_t = \frac{1}{\lambda_t^y A_t} \left[w_t(\underline{a}_t) - \frac{\kappa}{q(\theta_t)} \right].$$

Separation rates will tend to be low when; aggregate productivity is high, real marginal costs are high, the wage rate of the marginal worker is low and search costs forgone are high. By determining the dynamic behaviour of the outside option of the worker, the participation decision potentially affects wage dynamics and hence movements in

the separation rate and marginal costs. For instance, if the option to participate exacerbated wage fluctuations in response to aggregate productivity shocks, fluctuations in \underline{a}_t would tend to be smaller. Furthermore, by determining s_t , participation flows also potentially have a direct effect on θ_t in the above expression. Ultimately, whether tightness becomes more or less volatile under endogenous participation also depends on the response in vacancy supply. But, for instance, if procyclical labour force participation had a large positive impact on job creation so that fluctuations in θ_t were amplified, the direct effect in the above expression would be to make the term $\kappa/q(\theta_t)$ more volatile, reinforcing the effect on \underline{a}_t of shocks to A_t .

The optimality condition for prices, equation (14), produces a linearised inflation equation that is observationally equivalent to specifications that are based on the familiar Calvo (1983) model of nominal adjustment. Using a circumflex to denote logarithmic deviations around a zero inflation stationary state, the linearised version of (14) is

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{\psi - 1}{\phi} \widehat{\lambda}_t^y. \quad (17)$$

The above expression is a standard New Keynesian Phillips curve based on quadratic price adjustment costs. It is similar to equations derived from Calvo contracts in that inflation is a forward-looking function of its expected future value as well as current marginal costs. The stickier prices are, the smaller the coefficient on current marginal cost is, and the greater the reliance of current inflation on future market conditions. The coefficient on real marginal cost is also positively dependent on the degree of competition in the monopolistic product market. When price elasticity of demand is high, a given price increase will result in a large reduction in real marginal costs by reducing demand for the firm's output. Therefore, equation (17) states that firms will be more willing to undertake costly price adjustment as a countermeasure to a spike in real marginal costs precisely when a price increase has a large negative effect on real marginal costs, thereby justifying the penalty of costly nominal adjustment.⁸

It now becomes evident from equations (16) and (17) that the behaviour of real marginal costs provides a link between separation and inflation dynamics. The Phillips curve implies that when current inflation is below expected future inflation, $\widehat{\pi}_t - \beta E_t \widehat{\pi}_{t+1} < 0$, the shadow value of output will be relatively low, $\widehat{\lambda}_t^y < 0$, indicating that the marginal value of a match to the firm is below normal. From equation (16) we then know that, all else equal, the separation threshold will be relatively high. The intuition is straightforward: when a given match is worth less to the firm in terms of its contribution to real revenue, job destruction will tend to be high. This relationship will be returned to later. Of course, in general equilibrium the relationship is endogenous:

⁸The representative firm does not worry about losing market share due to price increases in this model because there is no relative price dispersion - all firms adjust each period. This is in contrast to Calvo contracts, in which pricing is costless but staggered and firms dislike relative price dispersion. Under Calvo contracts the positive dependence of the slope of the Phillips curve on the price elasticity of demand does not arise.

real marginal costs also increase as firms lower the separation threshold to raise production, since average idiosyncratic productivity falls as more relatively unproductive workers are retained.

In the next section we specify a recursive structure of the labour market in order to solve for the equilibrium wage and participation constraint.

2.5 Recursive Representation of the Labour Market

Let $N_{i,t}, U_t, W_{i,t}, V_t$ and $J_{i,t}$ be the time t value functions of a non-participant, an unemployed agent, an employed agent, an available vacancy and an occupied job, respectively. Notice that the values $N_{i,t}, W_{i,t}$ and $J_{i,t}$ are specific to the i th individual, whereas the values of unemployment and job vacancies are independent of any idiosyncratic disturbances and therefore homogenous across the population. The Bellman equation describing the value of non-participation is expressed as

$$\begin{aligned}
N_{i,t} &= \frac{z(h_{i,t})}{\mu_t} + p^N(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \\
&\quad \times \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1} - \left\{ F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right\} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\
&\quad + E_t \beta_{t+1} \left[F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right]. \tag{18}
\end{aligned}$$

Individual production of the home good (or equivalently leisure) is determined by the function $z(h_{i,t})$, for which $z'(h_{i,t}) \geq 0$. The individual home production function is divided by the shadow value of real income from equation (2) in order to convert it to a measure that is comparable to the units in which wages and unemployment benefits are expressed.⁹ With probability $p^N(\theta_t) E_t (1 - \rho_{t+1})$, the agent who is not participating at time t is successfully matched at $t + 1$, representing a direct transition from non-participation to employment and yielding a capital gain equal to the conditional expected value of the difference between employment and non-employment. The expected value of non-employment at time $t + 1$, which is the worker's outside option, is captured in (18) by the weighted average

$$F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h).$$

Ex ante, the agent whose job match is unsuccessful does not know whether he will end up in employment or non-participation, as this depends on the draw of $h_{i,t+1}$ which is not known at time t given zero autocorrelation. With probability $1 - F(\underline{h}_{t+1})$ the agent would draw a value of idiosyncratic home productivity next period that exceeds the critical threshold to induce non-participation, and that agent would choose to opt

⁹ All household members are assumed to consume a constant fraction of household consumption, so marginal utilities are equalised across agents.

out of job search in favour of full-time home production. Conversely, with probability $F(\underline{h}_{t+1})$ the agent would draw a relatively low value of home productivity, so that the agent would elect to search for a job rather than remain outside of the labour force at time $t + 1$. If $F(\underline{h}_{t+1}) = 1$ for all t , the model collapses to the exogenous participation model in which the outside option is always unemployment. Going back to the Bellman equation (18), if the non-participant is not matched at time t , then the agent simply obtains the continuation value of being non-employed at $t + 1$ conditional on the draw of $h_{i,t+1}$.

The value of unemployment is

$$\begin{aligned}
U_t &= b_{i,t} + p^U(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \\
&\quad \times \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1} - \left\{ F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right\} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\
&\quad + E_t \beta_{t+1} \left[F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right]
\end{aligned} \tag{19}$$

which is qualitatively similar to (18), apart from the difference in the transition probability, $p^U(\theta_t)$, and the variable $b_{i,t} \geq 0$. The latter represents the flow value of being unemployed in terms of current consumption. Following Garibaldi and Wasmer (2005) it can be assumed that the consumption value $b_{i,t}$ is a combination of unemployment benefits and home production such that

$$b_{i,t} = b + \vartheta \frac{z(h_{i,t})}{\mu_t}$$

where the parameter $0 \leq \vartheta \leq 1$ represents the fraction of home production that the i th unemployed individual engages in. The constant b is interpreted as government funded unemployment insurance and enters the Bellman equation as a current consumption value. An employed worker is characterised by the following recursive asset equation,

$$\begin{aligned}
W_{i,t} &= w_{i,t}(a_{i,t}, \underline{h}_t) + E_t \beta_{t+1} (1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\
&\quad + E_t \beta_{t+1} \rho_{t+1} \left[F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right]
\end{aligned} \tag{20}$$

where $w_{i,t}(a_{i,t}, \underline{h}_t)$, written to emphasise dependence on the participation constraint as well as the idiosyncratic match productivity shock, is the equilibrium wage paid to workers employed in successful matches, i.e. matches that produce output in period t . In what follows, the wage is simply written as $w_{i,t}$ in order to ease notation and the dependence on \underline{h}_t taken to be implicit. The time $t + 1$ continuation value is similar to that in (18) or (19), but is weighted by the probability of job destruction, ρ_{t+1} .

The asset value to the firm of an occupied individual job and a vacancy are, respec-

tively,

$$J_{i,t} = \lambda_t^y A_t a_{i,t} - w_{i,t} + E_t \beta_{t+1} \left[(1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} + \rho_{t+1} V_{t+1} \right] \quad (21)$$

and

$$V_t = -\kappa + q(\theta_t) E_t \beta_{t+1} \left[(1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} + \rho_{t+1} V_{t+1} \right] + (1 - q(\theta_t)) \beta E_t V_{t+1}. \quad (22)$$

The term $\lambda_t^y A_t a_{i,t} - w_{i,t}$ represents the net contribution of the i th match to the firm's revenue.¹⁰ The vacancy Bellman equation is standard. Free entry on the supply of vacancies ensures that the equilibrium asset value of a vacancy is driven down to zero, so that

$$V_t = 0 \quad \forall t.$$

Inserting the above equilibrium restriction into (22) gives

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} \left[(1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \right] \quad (23)$$

which is simply a restatement of the requirement that the supply of vacancies adjusts such that the anticipated costs of search by the firm (the left hand side) equal the expected discounted value of an occupied job in the following period. Inserting (23) into (21) and evaluating the result at the critical value \underline{a}_t reproduces the job destruction condition (16), thereby ensuring $J_{i,t}(\underline{a}_t) = 0$ holds for all i, t .

2.6 Wage Determination

Matching frictions in the labour market create local monopoly rents for successful matches. In order to divide these rents, wages are negotiated every period to maximise the Nash product,

$$w_{i,t} = \arg \max \left[W_{i,t} - \left(F(\underline{h}_t) U_t + \int_{\underline{h}_t}^{\infty} N_{i,t} dF(h) \right) \right]^\eta [J_{i,t} - V_t]^{1-\eta} \quad (24)$$

where η is the bargaining strength of the worker and $1 - \eta$ is the bargaining strength of the firm. The term inside the first square brackets represents the worker's surplus value of being employed over non-employed. Analogously, the term inside the second square brackets represents the surplus value to the firm of an occupied job over a

¹⁰Recall that λ_t^y represents the contribution of a unit of output to the firm's revenue stream. $A_t a_{i,t}$ represents contribution to output of a match with idiosyncratic productivity equal to $a_{i,t}$. Therefore, the term $\lambda_t^y A_t a_{i,t}$ captures the individual match's contribution to the firm's revenue in the current period.

vacant position. Expression (24) makes it clear that only \underline{h}_t , and not the particular value $h_{i,t}$, influences the outcome of wage negotiations. Maximising the objective (24) with respect to $w_{i,t}$, the following first-order condition is obtained for equilibrium wages that must hold for every period of discrete time,

$$W_{i,t} - \left\{ F(\underline{h}_t) U_t + \int_{\underline{h}_t}^{\infty} N_{i,t} dF(h) \right\} = \frac{\eta}{1-\eta} J_{i,t} \quad \forall t. \quad (25)$$

The above Nash sharing rule states that the worker's share of the joint surplus is equal to a constant fraction of the firm's share. The difference to the standard result in our model is the modification to the worker's outside option, which is now a weighted average of unemployment and non-participation. Replacing the value functions in the (25) with their respective recursive representations using equations (18)-(21) results in an equilibrium wage equation of the form

$$w_{i,t} = \eta \lambda_t^y A_t a_{i,t} + (1-\eta) [F(\underline{h}_t) \Gamma_t^U + (1-F(\underline{h}_t)) \Gamma_t^N] \quad (26)$$

where

$$\begin{aligned} \Gamma_t^U &= b + \vartheta \frac{\tilde{z}(\underline{h}_t)}{\mu_t} + \frac{\eta}{1-\eta} \kappa \theta_t \\ \Gamma_t^N &= \frac{\tilde{z}(\underline{h}_t)}{\mu_t} + \chi \frac{\eta}{1-\eta} \kappa \theta_t. \end{aligned}$$

where $\tilde{z}(\underline{h}_t) = \int_{\underline{h}_t}^{\infty} z(h) dF(h) / (1-F(\underline{h}_t))$ is the conditional average value of home production. Since (26) is somewhat non-standard, details of the derivation are provided in an appendix at the end of this chapter.

The wage in (26) takes the form of a convex combination of the match's contribution to the firm's revenue, which is simply $\lambda_t^y A_t a_{i,t}$, and the worker's outside option, which is the term inside the square brackets. The weight placed on the former is equal to the worker's bargaining power and the weight placed on the latter is equal to the firm's bargaining power. Should wage negotiations fail, then with probability $F(\underline{h}_t)$ the worker's time t outside option would be continued job search, i.e. entry into the unemployment pool. Γ_t^U represents the asset value of being unemployed conditional on Nash bargaining. In addition to the flow consumption value $b + \vartheta \tilde{z}(\underline{h}_t) \mu_t^{-1}$, Γ_t^U incorporates the expected capital gain, conditional on Nash bargaining, that is realised if the worker becomes matched. The capital gain is increasing in θ_t because as tightness rises the probability of an unemployed worker finding a job increases, as does the expected cost of opening a vacancy. Tighter labour markets are associated with larger expected capital gains from job search, which raise the value of the worker's outside option, resulting in an increase in wages.

Analogously, Γ_t^N represents the value of non-participation, conditional on Nash bargaining over wages, and likewise is the sum of two parts. The first is the conditional

expected value of non-market production and the second captures the anticipated capital gain of passive job search. If non-participants are not permitted to search for jobs, then $\chi = p_N(\theta_t) = 0$ and the anticipated capital gain from being out of the labour force is zero. Note that the expected capital gains from active and passive search are different only to the extent that $p^U(\theta_t)$ differs from $p^N(\theta_t)$.

Consider how endogenous participation has the potential to influence the cyclical behaviour of the real wage over the business cycle. The influence of labour market participation on the real wage operates through a transformation of the worker's outside option. The participation choice means that the fall back value of the employee is a weighted average value of unemployment and non-participation conditional on the likelihood of entering each should employment be terminated. Accordingly, a smaller weight than if $F(h_t) = 1$ is placed on the value of unemployment. This diminishes the reliance of the outside option on the value of unemployment, and therefore market tightness, because the value of non-participation is only weakly directly related to market tightness. Accordingly, wages may not be bid up as much when an increase in vacancy supply causes tightness to rise.

There are also opposing effects which may contribute to greater wage instability over the cycle. Procyclical variation in h_t implies that during a boom more weight is placed on Γ_t^U and less on Γ_t^N as h_t rises, thereby causing a dynamic compositional shift in the outside option of the worker which may exacerbate wage movements. Furthermore, as h_t rises, the conditional average $\tilde{z}(h_t)$ also rises and is reinforced by a fall in the shadow value μ_t during a boom. This potentially results in stronger procyclical fluctuations in the wage rate in the presence of the participation option.

Finally, it is worth highlighting how our wage equation differs from that of Haeckle and Reiter (2006). These authors work with a continuous time model in which idiosyncratic home productivity shocks follow a Poisson arrival rate. This introduces a perverse incentive for non-participants with low h to seek employment in order to force a re-draw. They thus find it convenient to concentrate on a restricted version of their model in which this incentive is eliminated, but consequently so too is any effect of the participation threshold on the wage. We do not encounter this problem in the current setup because we use a discrete time structure in which non-participants re-draw idiosyncratic shocks *every* period. Our wage equation therefore provides a simple and tractable way of integrating an explicit participation decision into the cyclical behaviour of wages whilst maintaining homogeneity.

2.7 Participation Constraint

Now that we have solved for the equilibrium wage, the returns to labour market search are explicitly defined and we can now derive an expression for the participation threshold, h_t . Recall that optimal participation decisions are made in a decentralised manner by individual household members according to their own idiosyncratic valuation of re-

remaining outside of the labour force in each period. These decisions are made on the basis of what the non-participant's expected value of future matches is, which is conditional on the solution to the Nash bargaining problem. The participation constraint is formally defined as the value of idiosyncratic home productivity that makes a non-employed agent indifferent between unemployment and non-participation. In symbols, the critical value of idiosyncratic home productivity must satisfy

$$U_t = N_t(\underline{h}_t).$$

Substituting the recursive equations for non-participation and unemployment, (18) and (19), into the above condition and making use of the Nash bargaining restriction (25), the following expression which implicitly defines \underline{h}_t is obtained

$$\frac{z(\underline{h}_t)}{\mu_t} = \frac{1}{1 - \vartheta} \left[b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta_t \right]. \quad (27)$$

This expression is intuitive. It states that the critical value of home productivity is equal to the flow benefit of being unemployed plus the difference in the anticipated capital gains associated with being unemployed versus out of the labour force. The right hand side of the above expression is multiplied by the factor $(1 - \vartheta)^{-1}$, which for $\vartheta, z' > 0$ raises the participation constraint since unemployed agents also devote a fraction of their time to home production. If $\chi = 1$, then $p^U(\theta_t) = p^N(\theta_t)$ and agents would base their participation decisions solely on the consumption value of being unemployed versus out of the labour force. Equation (27) establishes a positive relationship between labour market participation and θ_t . Higher market tightness is associated with more favourable labour market prospects, such as higher wages and quicker job finding rates. As a result, procyclical fluctuations in tightness tend to make labour market participation procyclical.

But what about the cyclical behaviour of the shadow value μ_t , which depends positively on the marginal utility of consumption? As the marginal utility of market consumption falls during a boom, the relative value of home production increases, thus exerting pressure on \underline{h}_t to fall. This opposes the positive influence that procyclical fluctuations in market tightness have on the participation threshold. Which effect will dominate depends on the curvature of the utility function. For strongly risk averse agents, whose marginal utility of consumption diminishes rapidly, fluctuations in μ_t will be large. If σ is large enough it becomes possible for countercyclical fluctuations in μ_t to dominate participation behaviour, causing labour force movements to be countercyclical.

2.8 Aggregation

Aggregate quantities are denoted by the removal of j subscripts. The representative firm's quantities coincide with aggregate values, so that employment evolves according

to

$$n_{t+1} = (1 - \rho_t) n_t + q(\theta_t) v_t.$$

Aggregate income is given by the representative firm's production function

$$y_t = A_t (1 - \rho_t) n_t \int_{\underline{a}_t}^{\infty} a \frac{dG(a)}{1 - G(\underline{a}_t)}$$

and the aggregate wage is found by taking the conditional expectation of the wage equation (26),

$$w_t = \eta \lambda_t^y A_t \int_{\underline{a}_t}^{\infty} a \frac{dG(a)}{1 - G(\underline{a}_t)} + (1 - \eta) [F(\underline{h}_t) \Gamma_t^U + (1 - F(\underline{h}_t)) \Gamma_t^N].$$

Aggregate labour income flowing to the representative household is thus given by

$$\mathbb{W}_t = e_t w_t$$

and the household's unemployment income is

$$\mathbb{U}_t = u_t b.$$

Two simple options for monetary policy are either the postulation of a Taylor-like rule or an exogenous monetary growth rule. We adopt the latter as in Krause and Lubik (2007), Cooley and Quadrini (1999) and others. Monetary policy enters as a simple growth rule of the form

$$m_t = g_t m_{t-1}. \quad (28)$$

The stochastic process for monetary growth is given by

$$\ln g_t = \varphi_g \ln g_{t-1} + \varepsilon_t^g$$

with $\varphi_g > 0$ and $\varepsilon_t^g \sim N(0, \sigma_g^2)$. In equilibrium, the representative household holds all the money supply, $m_{j,t} = m_t$. In Appendix II we discuss the implications of a Taylor rule.

The lump-sum tax, T_t , is used to finance gross government bond repayments and unemployment benefits. The fiscal authority's constraint ensures a balanced budget every period,

$$T_t = -\frac{r_t B_t}{p_t} - u_t b. \quad (29)$$

Aggregate market productivity follows an exogenous autoregressive process

$$\ln A_t = \varphi_A \ln A_{t-1} + \varepsilon_t^A$$

with $\varphi_A > 0$ and $\varepsilon_t^A \sim N(0, \sigma_A^2)$. Following den Haan et al (2000), who set the standard for the ensuing literature, job flows are adjusted for exogenously terminated positions

that are re-filled within the same period. In order to account for this, exogenous separations are interpreted as being worker-initiated, implying that the firm endeavours to re-post the vacancy for the position that was quit by the worker. Job creation is therefore recorded as the measure of all newly created matches net of vacancies that have been re-matched within the period,

$$jc_t = \frac{q(\theta_t)(v_t - \rho^x n_t)}{n_t} \quad (30)$$

where $\rho^x n_t$ represents the measure of jobs that were previously filled and are now available for re-matching within the period. Similarly, in accounting for job destruction, turnover associated with exogenously quit positions is netted out such that

$$jd_t = \rho_t - \rho^x q(\theta_t) \quad (31)$$

represents the relevant measure of job destruction in each period. This captures the salient feature of the labour market that worker flows exceed job flows (see Davis et al., 1996). Note that $jc_t - jd_t = \Delta n_{t+1}/n_t$, or the percentage change in the measure of jobs at the firm.

3 Solution and Calibration

The model's equilibrium conditions are log-linearised around a zero inflation stationary state in which all real variables are constant. The resulting linear system of equations is solved using the method of undetermined coefficients, a standard procedure described by Uhlig (1997).¹¹ Linearised equilibrium dynamics are expressed as recursive laws of motion from which time series of artificial data are generated. Impulse response functions are computed and the business cycle properties of the artificial series are then compared with corresponding statistics from actual U.S. data.¹² We use quarterly data for the period from 1970:4 to 2005:1. All simulated and actual data are passed through a Hodrick-Prescott filter with smoothing parameter 1600 to isolate the cyclical components of the time series.¹³

The parameter values used in the baseline calibration are summarised in Table 1. Most of our parameterisation is standard. The discount factor $\beta = 0.99$, giving an annual rate of interest of roughly four percent. Following Prescott (1986) and the references therein, along with many other business cycle studies including the seminal contributions to unemployment fluctuations of Andolfatto (1996) and Merz (1995), we

¹¹See the appendix in Chapter I for a description of the solution method.

¹²Data are obtained from the Bureau of Economic Analysis and the Bureau of Labor Statistics websites. The series for job creation and destruction are computed by Davis et al. (2006). Vacancies are obtained from the OECD statistics portal.

¹³Artificial time series of length 300 are constructed and the first observations are removed in order to reduce dependence on initial conditions and to obtain series of length equal to the sample period. 200 samples of time series are obtained and the statistics reported are the sample averages.

assume that $\sigma = 1$ so that $u(c_t)$ is logarithmic. There is, however, some empirical controversy regarding the degree of risk aversion. In particular, Hall (1988) finds that the intertemporal elasticity of substitution is much smaller than the logarithmic case - see Neely et al. (2001) for a discussion. For our purposes, we note that even logarithmic utility is sufficient to generate countercyclicality in the labour force.

It is assumed that the individual home production function is linear

$$z(h_{i,t}) = \delta h_{i,t}$$

where $\delta \geq 0$ is a constant scale parameter. In our setup, we assume linearity of z for convenience. As demonstrated subsequently, the elasticity of participation in our model is determined by the idiosyncratic variance of individual home productivity, implying that the curvature of z can be abstracted from for simulation purposes.

Following Cooley and Quadrini (1999), the value of the vacancy transition probability is set to $q(\theta) = 0.7$, informed by the calculations of den Haan et al (2000). The matching function takes the standard form, $M_t = \mathcal{M}v_t^\gamma s_t^{1-\gamma}$, where \mathcal{M} is a normalising constant. The elasticity of the matching function with respect to vacancies is set to 0.6 in accordance with the empirical estimates provided by Blanchard and Diamond (1989). Symmetric bargaining is assumed such that $\eta = 0.5$. Steady state employment $e = 0.59$ to match the average value over our sample period. We also find that $u = 0.04$. These numbers express employment and unemployment relative to the total non-institutional civilian population, not the labour force. The steady state value $F(\underline{h})$ is calibrated in order to match steady state unemployment, which implies $F(\underline{h}) = 0.1$. It is assumed that F is lognormal with mean zero. In the absence of empirical data for dispersion in idiosyncratic home productivity, we calibrate the variance of this distribution such that the model is consistent with realistically volatile unemployment fluctuations. In the baseline model this requires $\sigma_h = 0.15$. Pries and Rogerson (2009) use data on gross job flows to calculate the arrival rate of job offers to non-participants. They find that the arrival rate of offers to non-participants is 20 percent that of unemployed individuals. In light of this, the search intensity of non-participants is set to $\chi = 0.2$.

Steady state vacancies are determined from the stationary version of the employment law of motion

$$v = \frac{\rho n}{q(\theta)}$$

once ρ has been calibrated. We set ρ in order to yield a job finding probability of $p^U(\theta) = 0.6$, which gives an average unemployment duration of 1.67 quarters as reported by Cole and Rogerson (1999). This requires $\rho = 0.104$, which is close to the value of 0.1 from den Haan et al. (2000), which sets the standard for related studies. We then find that $v = 0.1$. We can now compute $\mathcal{M} = 0.66$. Following den Haan et al. (2000), the exogenous separation rate is $\rho^x = 0.68\rho$. This implies an endogenous destruction rate $G(\underline{a})$ equal to 0.034 from equation (5). The stationary separation

threshold is then given by the inverse of the distribution function. It is assumed that idiosyncratic match productivity shocks are drawn from a lognormal distribution with mean zero and standard deviation σ_a which is calibrated to match the observed relative volatility of job destruction. This is standard practice in the related literature (see, for example, Walsh, 2005, or den Haan et al., 2000). A priori, it is difficult to tell how σ_a will affect the volatility of the destruction rate, ρ_t . By reducing (increasing) σ_a , the elasticity of G increases (decreases) which encourages firms to make smaller (larger) adjustments to \underline{a}_t . The net effect then depends on whether the change to the elasticity dominates the influence on the dynamics of \underline{a}_t . In our simulation exercises we find that reducing σ_a tends to reduce the volatility of job destruction, and so we accordingly set $\sigma_a = 0.2$ as a baseline.

Evaluating the first-order condition for prices (14) at the steady state requires that

$$\lambda^y = \frac{\psi - 1}{\psi}$$

implying that marginal costs are equal to the inverse of the mark up. Following Krause and Lubik (2007) and the majority of the literature, $\psi = 11$ giving a mark up of 10% in the steady state. As previously noted, the linearised inflation equation derived from quadratic nominal adjustment costs is observationally equivalent to the Calvo pricing counterpart. The imposed severity of price stickiness varies widely even in the recent literature. In our baseline calibration we follow Christiano et al. (2005) and Lubik and Schorfheide (2004) by assuming a moderate degree of price stickiness. These authors demonstrate that an average price duration of 2-3 quarters is consistent with observed inflation dynamics. This entails setting $\phi = 20$ to be consistent with an average Calvo duration of 2 quarters. We explore the consequences of adjusting this parameter in depth in a later section. The reason a moderate degree of price stickiness is chosen is that, as discussed subsequently, the model - even without endogenous participation - implies a trade-off between matching inflation volatility on the one hand and job destruction volatility on the other. Given our primary focus on labour market dynamics, we select a low degree of price stickiness to be able to replicate realistic job destruction fluctuations.

The vacancy flow cost, the flow value from unemployment and the constant δ are found by solving equations (15), (16) and (27) simultaneously. Given our interpretation of b as unemployment insurance, we first fix this parameter so as to yield a replacement ratio of approximately 40% as in Shimer (2005). This necessitates $b = 0.37$. We then find that $\kappa = 0.1$, $\vartheta = 0.66$ and $\delta = 1.47$. This implies that unemployed agents spend 2/3 of their time in home production.

It remains to set the parameters for the aggregate shock processes. Other monetary business cycle studies in the literature that specify monetary policy in the manner that we have, such as Cooley and Quadrini (1999) and Krause and Lubik (2007), set monetary persistence to $\varphi_g = 0.49$ and volatility $\sigma_g = 0.009$, based on the estimates in

Cooley and Hansen (1989). For our sample period, we regress the growth rate of M1 on its lagged value to obtain $\varphi_g = 0.41$ and $\sigma_g = 0.0086$. For aggregate productivity we follow the standard practice of assuming highly persistent shocks with $\varphi_A = 0.95$ and choose the magnitude of σ_A such that the volatility of real GDP in the model is similar to that in the data. For the baseline we set $\sigma_A = 0.014$.

For comparative purposes a constant participation version of the model is obtained by setting $F(\underline{h}_t) = F(\underline{h}) = 1$ for all t . Comparison of the two models is slightly problematic, however, because the interpretation of unemployment differs depending on whether or not agents are allowed to exit the labour force. In models with constant participation, u is often interpreted as a statement of "non-employment" rather than unemployment. Therefore, an unemployment rate of 0.04 in the constant participation version may understate the true extent of search effort in the economy since in the endogenous participation version we allowed for passive search even when out of the labour force. In recognition of this, most studies of cyclical unemployment assume $u > 0.04$ (e.g. Andolfatto 1996 assumes that $u = 1 - e$, or 43% in his model). In what follows we assume that unemployment in the constant participation version is equal to the measure of effective searchers in the endogenous participation version after taking passive search into account. Given our baseline parameterisation this implies an unemployment ratio of 0.114, which is remarkably close to the value of 0.12 assumed in Krause and Lubik (2007). When abstracting from labour force participation, it is assumed that each unemployed agent consumes a fixed amount of home production, so that the parameter b_t is now defined as

$$b_t = b_1 + b_2 c_t^\sigma$$

where b_1 is government funded unemployment compensation and $b_2 c_t^\sigma$ is the value of home production in terms of the market good. b_1 is calibrated to be consistent with a replacement ratio of 40%. For the constant participation case, we also set $\sigma_A = 0.01$ and $\sigma_a = 0.075$ in order to target realistic output and job destruction volatility, respectively. All other parameters remain unchanged relative to the endogenous participation model.

4 Baseline Results

In this section, we discuss impulse response functions followed by simulation-based moment calculations. In order to establish a benchmark and better understand the business cycle consequences of explicitly introducing labour force participation, it is instructive to briefly consider a standard model with exogenous participation first. Figure 2 shows the impulse responses of the model with constant participation to a positive 1% monetary growth shock. Basically, the main results found in Krause and Lubik (2007) are reproduced. Monetary policy as modelled in this framework acts as an aggregate demand shock. Firms meet the increase in aggregate demand partially

by raising production and partially by raising prices. Given the option to adjust along the destruction margin, firms in this model find it profitable to increase employment through a fall in the separation rate that is large enough to accommodate a *reduction* in vacancy supply, circumventing the costly hiring process that arises from matching frictions. It is precisely in this manner that the model of Krause and Lubik (2007) implies a very limited role of monetary policy for job creation. By lowering unemployment, a decrease in the job destruction rate increases average vacancy duration and thus makes it relatively less profitable for firms to create jobs. Although market tightness increases following the shock, it is due to a smaller percentage reduction in vacancies relative to the reduction in unemployment, which falls despite weakened job creation. Lower vacancy transition rates increase the value of current matches to the firm due to the difficulty of finding a replacement worker. Workers' outside options are thus boosted, causing wages to rise which in turn contributes to higher marginal costs and pushes up inflation.

Figure 3 reports the impulses of the constant participation model to a 1% positive aggregate productivity shock. A technology shock of this sort acts as a supply-side shock, raising output and lowering inflation. Unlike an increase in the growth rate of money, the technology variable A_t enters directly into the right hand side of the job creation condition (15). In contrast to a positive demand shock, increased output across all matches leads firms to expand the supply of vacancies. The impact on job creation is positive, but does not persist. Notably, the job destruction rate also rises on impact, such that employment initially falls below steady state and then recovers as vacancies become matched with workers.

What causes job destruction to increase in response to a *positive* technology shock? In the empirical literature, the way employment responds to technology shocks is a source of debate (Christiano 2006 and Gali 1999). Standard real business cycle theory without labour market frictions postulates a positive relationship. A simultaneous rise in both job creation and destruction, on the other hand, is consistent with the Schumpeterian view of "creative destruction". Booms are characterised as periods in which obsolete productive units are replaced by ones that embody the latest technology. In our model, however, we have not specified any impediments to technology adoption so that aggregate productivity shocks affect all currently operating matches simultaneously and equally. Rather, given that we study a monetary equilibrium with nominal rigidity, the tendency for job destruction to rise in our model is attributable to the reasons discussed in Gali (1999). In a sticky-price equilibrium in which monetary policy is insufficiently accommodating, a positive technology shock which increases output creates a short-run aggregate demand constraint since prices do not fall by enough to allow the private sector to absorb such a large increase in supply. The extent to which firms shed workers then depends on their ability to adjust their prices in response to the shock.¹⁴ In our baseline calibration, despite the fact that we have assumed only mild

¹⁴Instantaneous adjustment can occur only through the destruction margin, and therefore it is the

nominal adjustment costs, job destruction rises to such an extent that unemployment increases on impact. It is demonstrated in a subsequent section that in an approximately flexible price equilibrium job destruction does indeed fall on impact in response to a productivity improvement. In section 5.2 below we also discuss in more detail how a more accommodating monetary policy stance can prevent a surge in the destruction rate.

Now that the fundamental mechanics of the simpler model have been quantitatively assessed, let us next analyse the dynamic implications of introducing endogenous labour force participation. Figure 4 contains the impulse responses to a monetary shock identical to the one considered previously. We find that introducing variation in the size of the labour force does not contribute to the ability of the model to generate an increase in job creation in response to a monetary policy shock. The endogenous participation model is therefore found not to be capable of reversing the negative response of vacancy supply to a monetary shock that was highlighted by Krause and Lubik (2007). Indeed, the drop in vacancies is about the same as in the constant participation model. This indicates that the job creation incentives generated by the relief of labour market congestion from endogenous participation flows do not appear to be strong in the baseline model. Quite the contrary, the effect on the participation constraint is negative, reflecting the tendency to substitute away from formal labour market activity as the marginal utility of market consumption diminishes. The behaviour of the worker's outside option is roughly similar in both models, indicating that the modification induced by the participation decision is quantitatively small.¹⁵

Figure 5 contains the impulse responses to an aggregate productivity shock in the endogenous participation model. Once again a drop in \underline{h}_t is observed, despite a rise in market tightness, indicating that the fall in the shadow value of relaxing the household's budget constraint dominates the relatively high returns from labour market search. The rest of the impulse dynamics are broadly similar to the constant participation model. The main difference is that the decline in \underline{h}_t drives a drop in unemployment despite the spike in job destruction, whereas unemployment initially increases in the constant participation model. Labour force exit then causes employment to remain below steady state for much of the adjustment path, consistent with the evidence in Gali (1999).

Business cycle statistics computed from numerical simulations are reported in Table 2 for both specifications of the model. The results in general corroborate the intuition developed in the impulse response analysis. Inflation volatility is overstated in both models for the degree of nominal rigidity assumed in the baseline calibration. We assess the implications of varying the degree of price stickiness in a subsequent section. In the endogenous participation model, fluctuations in the labour force are somewhat too large

destruction rate which rises, not the job creation rate which falls.

¹⁵Despite roughly similar responses in the worker's outside option, the wage rises by more in the endogenous participation model. This is due to the calibration of σ_a , which is different in the two models. σ_a determines how a change in \underline{a}_t influences the conditional average $\int_{\underline{a}_t}^{\infty} a \frac{dG(a)}{1-G(\underline{a}_t)}$, which determines both the average wage payment and output.

despite realistic unemployment movements and excessively stable employment. The reason for this is that employment responds negatively to aggregate productivity shocks, generating the incorrect prediction that the labour force fluctuates countercyclically.

As was clear from the impulse response analysis, endogenous labour force participation does not rectify the weak vacancy creation mechanism in the standard model with the result that the relative volatility of vacancies in both models is unrealistically low. Because of this, and despite highly countercyclical unemployment, the relative volatility of market tightness cannot match the data. Job flows are positively correlated in both model specifications. The model with endogenous participation does indeed predict a negatively sloped Beveridge curve conditional on technology shocks, whereas the constant participation model is incapable of matching this statistical fact conditional on either shock. This should not be seen as a desirable result, however, since it arises due to an excessive outward flow of agents from the labour market that brings unemployment down in response to a technology shock but which also causes the labour force to behave countercyclically.

The results of the foregoing analysis can be summarised as follows. Computed business cycle statistics and impulse responses for the baseline calibration suggest that endogenous participation is quantitatively unimportant for the ability of the monetary authority to stimulate cyclical job creation through inflationary policy, as can be concluded from the joint failure of both models to generate a realistically sloped Beveridge curve. However, our baseline model predicts that the introduction of a participation decision modifies the outside option of the worker in such a way that causes the labour force to be countercyclical. In order to fairly assess the relationship between labour force fluctuations and cyclical job creation, the model must generate movements in the labour force that are procyclical and of a realistic magnitude that is consistent with the data. We therefore cannot conclude at this stage that the congestion effects of labour market participation on the matching process suggested by Cooley and Quadrini (1999) are independently insufficient to generate a robust increase in vacancies in response to a monetary shock. In the next section we show how comparison utility preferences address the issue of countercyclical movements in the labour force. We also then discuss the effects of adjusting the degree of nominal rigidity, eliminating passive search and introducing time-inseparable utility.

5 Further Discussion

5.1 Comparison Utility

In the baseline endogenous participation model, we found that the introduction of an explicit home good or leisure to be valued induces agents to substitute away from market activity during a boom. As is well known from standard real business cycle theory, the response of labour effort to exogenous shocks depends on the interaction

between substitution and income effects. Wen (2001) demonstrates that a standard real business cycle model can be rendered consistent with the empirical regularity discussed in Gali (1999) - that a positive technology shock causes a fall in employment - without invoking nominal rigidities. To achieve this, the income effect of the technology shock must dominate the substitution effect. In particular, if the shadow value of real income declines sufficiently relative to the increase in the real wage, the income effect will tend to dominate and labour effort falls in response to positive productivity disturbances.

We apply this intuition to our current framework with a frictional labour market and endogenous participation. Recall that the shadow value of real income in our model is given by the Lagrange multiplier on the household's budget constraint and represents the marginal utility of consumption. Our baseline results suggest that the decline in the shadow value of real income is sufficiently strong to induce agents to substitute away from market activity during a boom, giving rise to a countercyclical employment rate that is inconsistent with the data. We therefore pose the following question: what economic forces potentially exist that could provide a stronger incentive for agents to remain active in the formal labour market?

Our suggested answer to this question is based on the literature concerning the effect that aggregate consumption externalities have on individual consumption choices, as outlined in the Introduction. Specifically, we now alter the preferences of the household by assuming that each household cares not only about its own level of consumption, but also considers an external reference point, or benchmark, when evaluating its own welfare. In our example, following Gali (1994), we take the reference stock to be the current average level of consumption. The hypothesis is that if the reference stock of consumption to which the household compares itself has a positive influence on the household's marginal utility of consumption, the shift away from market activity that was observed in response to positive shocks in the baseline model should be weakened. This captures the idea that each individual household will want to consume relatively more when average consumption is high, a feature often referred to as "keeping up with the Joneses" (Gali 1994).¹⁶

As in Gali (1994) the specific functional form is assumed to be

$$u(c_t, C_t) = \frac{c_t^{1-\sigma}}{1-\sigma} C_t^{\xi\sigma}, \quad \sigma > 0, \xi < 1$$

where c_t is the individual household's own level of consumption and C_t represents average consumption in the economy. The function u , although unconventional, satisfies

¹⁶It may also seem that internal habit formation as in Fuhrer (2000) could also provide an incentive not to exit the labour market during a boom when consumption is high. However, as noted by Wen (2001), the presence of lagged consumption in utility introduces a multiplier which magnifies the decrease in marginal utility for a given increase in consumption. In the context of our model, this would exacerbate the countercyclical tendency of the labour force.

the following standard properties for all positive (c_t, C_t) pairs:

$$\begin{aligned} u_1(c_t, C_t) &> 0, u_{11}(c_t, C_t) \leq 0 \\ -u_{11}(c_t, C_t) c_t / u_1(c_t, C_t) &= \sigma. \end{aligned}$$

In addition, it also follows that the elasticity of marginal utility with respect to average consumption is constant and given by $\xi\sigma$. For $\xi > 0$, the effect of average consumption on the household's marginal utility of its own consumption is positive. In this sense, higher average consumption encourages individual households to consume more, thus "keeping up with the Joneses". As Gali (1994) notes, when $\xi < 0$ average consumption lowers the marginal utility of the household's individual consumption, perhaps owing to public good characteristics of the single consumption good. Here we restrict attention to situations in which ξ is positive. Bounding $\xi < 1$ from above ensures diminishing marginal utility and the existence of a symmetric equilibrium.

The particular manner in which C_t enters the utility function implies that household utility increases given an increase in aggregate consumption. In the terminology of Dupor and Liu (2003) this quality is referred to as "admiration". Senik (2008) describes it as "ambition". Personal experience, on the other hand, as well as the evidence discussed in Easterlin (1995), suggests that concerns regarding relative income status prevent an increase in the income of all from raising the happiness of all, implying that "jealousy" rather than admiration or ambition characterises preferences. A simple thought experiment about what the personal welfare effects would be from observing an increase in the income of one's peers while one's own remains the same reveals the intuition behind theories of negative consumption interdependence.

How, then, can the positive consumption externality that we have assumed be justified? In contrast to emotions of relative impoverishment, the analysis of Caplin and Leahy (2001) suggests that the "anticipatory feelings" associated with observing reference consumption rising can induce a welfare enhancing expectational benefit to the individual whose income has not yet risen. The idea is that there is informational value in learning of a rise in others' income, since it may imply that one's own income may rise thereafter. Hirschman and Rothschild (1973) refer to this behaviour as the "tunnel effect", giving an example of how one may experience gratification upon observing an adjacent lane of traffic start to move during a traffic jam, since it probably indicates that one's own lane will also start to move soon.¹⁷

The question then becomes which effect is the empirically dominant one; jealousy or what we could alternatively refer to as ambition, anticipation or admiration? Put differently, on average do people prefer living in a society in which the average level of consumption is high or low? This issue is addressed in Senik (2008) who argues that the dominance of one effect over the other depends on the degree of social mobility and uncertainty in a nation. Using subjective data, the author's main finding is that

¹⁷See Senik (2005) for a detailed survey of the literature on the welfare effects of income distribution.

the effect of reference income on well-being is negative in Western Europe but positive in Eastern Europe and the United States, noting that mobility tends to be higher in the latter areas. We interpret these findings as justification for including a positive aggregate consumption externality in the household's utility function, especially since we make use of U.S. data in our calibration and computation of business cycle statistics.

We now return to the household's modified optimisation problem. Households take aggregate consumption as fixed when deciding on consumption paths. Accordingly, the only change to the household's first-order conditions is that the shadow value of real income now becomes

$$\mu_t = c_t^{-\sigma} C_t^{\xi\sigma}.$$

In a symmetric equilibrium in which all households are identical, it must be the case that household consumption is equal to the average, such that $c_t = C_t$. From the above equation, it is then immediately apparent that the model with consumption externalities behaves identically to the baseline model with an appropriately adjusted degree of risk aversion, an equivalence result noted by Gali (1994). The shadow value of real income is then given by

$$\mu_t = c_t^{-\sigma(1-\xi)}.$$

For positive ξ , it follows that the rate at which the marginal utility of consumption diminishes during a boom is slower. This weakens the incentive observed in the baseline model for agents to exit the labour force as consumption rises. The intuition is based on social considerations: individuals interpret periods of high average consumption as a good time to participate in the labour force. This is consistent with the "tunnel effect" view explained previously, in that periods of high average consumption may be indicative of better job prospects. Relative to the baseline model this weakens the consumption smoothing motive of the household.

We now quantitatively assess the implications of reference consumption for business cycle dynamics. The issue becomes how to calibrate the parameter ξ , which determines the elasticity of marginal utility with respect to average consumption. The study by Gali (1994) does not include a quantitative assessment and so cannot be used as a benchmark for calibration purposes. A commonly cited study for (lagged) internal habit formation is Fuhrer (2000), who finds that the exponent on a multiplicative habit stock in utility is high at 0.8. Other studies such as Alvarez-Cuadrado et al. (2004), Carroll et al. (1997) and Koyuncu and Turnovsky (2009) allow for a more general specification of reference consumption by including a distributed lag which includes both current and past individual and aggregate consumption. Our utility specification can be thought of as a special case of their models in which the external reference stock adjusts immediately in each period. These authors follow Carroll et al. (1997) in setting the exponent on the reference stock to 0.5 as a baseline and then considering Fuhrer's estimate as a strong habit case. In light of standard practice, then, it does not appear to be unreasonable to calibrate the exponent on our reference stock, $\xi\sigma$, to

a values as high as 0.5 or 0.8, bearing in mind the restriction that $\xi < 1$.

Our specific strategy is to calibrate ξ such that the model predicts a realistically volatile labour force that fluctuates procyclically. In this manner, we allow for a realistic effect of labour force variation on labour market congestion. A few additional parameters will also need adjustment. We re-adjust the standard deviations of aggregate productivity, idiosyncratic match productivity and idiosyncratic home productivity to target realistic values for output, job destruction and unemployment volatility as in the baseline procedure. We find that setting $\xi = 0.615, \sigma_A = 0.0065, \sigma_a = 0.2$ and $\sigma_h = 0.06$ achieves our objectives.¹⁸

For brevity we only report results pertaining to the model with endogenous participation.¹⁹ Figures 6 and 7 give the impulse responses to a monetary growth and technology shock, respectively. The impact of comparison utility on the impulse dynamics in response to a monetary shock is not large. This is due to the fact that the effect of monetary policy has a relatively small impact on consumption and market tightness, the two factors that determine the participation decision of agents (see equation 27). The only difference is that output rises slightly more due to a sharper fall in job destruction in order to accommodate the increased demand for output given the modification to preferences. In particular, for realistic labour force fluctuations, monetary growth remains incapable of generating a job creation boom, causing only a very weak reaction in market tightness which is insufficient to attract increased market participation (the participation constraint still falls).

The responses to a technology shock display more of a change relative to the baseline version. In particular, the participation constraint now initially rises rather than falls, causing unemployment to rise on impact despite a simultaneous fall in the destruction rate and rise in job creation.²⁰ In theory, higher unemployment eases labour market congestion and should present firms with a greater incentive to expand the supply of vacancies. Vacancy supply and job creation do respond by more than in the baseline endogenous participation model, but the additional increases are somewhat small considering the surge in unemployment. In particular, the impact response of job creation in the comparison utility model is only slightly higher than in the baseline constant participation model in Figure 3. Therefore, the fall in job destruction in the comparison utility model is not so much due to a substantial shift in the firm's willingness to create jobs as it is the result of the positive impact on aggregate demand of less rapidly diminishing marginal utility. Accordingly, employment rises above steady state throughout the adjustment path to the shock.

Business cycle statistics for the comparison utility model are reported in Table

¹⁸Note that had we assumed a higher σ in the baseline, we would simply have to raise the intensity of the consumption spillover to generate the same results.

¹⁹We do not find that there are quantitatively important implications of introducing comparison utility in the standard model with constant participation. This is because the main effect that we consider operates through the participation decision which is obviously absent from the standard framework.

²⁰The rise in unemployment in response to a positive technology shock is consistent with the empirical VAR evidence documented by Canova et al. (2007) and Gali (2010).

3. The main result is that the model now correctly predicts a realistically volatile, procyclical labour force whereas the baseline version with standard utility predicted a negative correlation with output and market tightness. The countercyclicality of unemployment is weakened due to the increased tendency for workers to enter the labour market in response to productivity improvements, causing the vacancy-unemployment correlation to become strongly positive. Given that job flows now covary negatively conditional on technology shocks, employment is more volatile than in the baseline endogenous participation model and is realistically procyclical.²¹ We also note that the relative volatility of the wage is reduced compared to the baseline due to the amplification of output arising from the increased tendency to raise consumption in response to shocks.

Before proceeding, we briefly recognise the fact that just as we have found that effectively altering the curvature of the utility function with respect to consumption has implications for labour market dynamics, it may also be reasonably suggested that altering the curvature of the utility of home production may have comparable effects. Although this is certainly the case when the participation decision is modelled at the household level as in Veracierto (2008) or Tripier (2003), in our model in which the participation choice is decentralised, the elasticity of the participation decision is determined by the variance of the idiosyncratic home productivity shock. In effect, therefore, the curvature of the function z is made redundant. To illustrate this point, assume that

$$z(h_{i,t}) = \delta \frac{h_{i,t}^{1-\sigma_2}}{1-\sigma_2}, \quad \sigma_2 \geq 0.$$

As an example we adopt the value $\sigma_2 = 0.8$ used in the baseline calibration of Merz (1995), who notes that this parameterisation falls within the wide range of values consistent with the empirical micro and macro evidence. Simulating the model with this modification keeping *all* other parameters unchanged increases unemployment volatility to 16 times that of output. However, once σ_h is controlled for in order to replicate empirically observed unemployment volatility (which requires setting $\sigma_h = 0.75$ to yield a standard deviation of unemployment of 6.85 relative to 1.60% for output), the model's dynamics are virtually indistinguishable from the baseline model with linear leisure.

5.2 Nominal Rigidity

It is possible to reduce inflation volatility by increasing the stickiness parameter ϕ . However, in the current model with endogenous job destruction, increased nominal rigidity causes the volatility of job destruction to increase. Consequently, a tension arises in the ability of the model to simultaneously predict realistic inflation and job

²¹The labour force was excessively volatile in the baseline model because employment was countercyclical, moving together with unemployment.

destruction dynamics.²² To see this, note that the stickier prices are, the greater the extent to which job destruction spikes on impact in response to a positive technology shock for the reasons pertaining to a short-run aggregate demand constraint described in section 4 above. As for a monetary disturbance, note that higher ϕ causes prices to increase more gradually in response to a positive monetary shock. This generates a more robust increase in aggregate demand which in turn necessitates a larger drop in job destruction. Nominal rigidity thus exacerbates fluctuations in job destruction in response to both real and nominal shocks.

Other models in the literature, however, have been able to reasonably match both inflation and job destruction volatility. This could be due to a number of differences with the approach of Krause and Lubik (2007), which is followed in this chapter. For instance, the model of Trigari (2009) which features variable hours can match inflation volatility without excess volatility in job destruction. By introducing variable hours into the model, firms can adjust how much output they produce through an additional margin, easing the burden of adjustment on employment. The study by Walsh (2005), however, suppresses hours variation and yet still manages to predict reasonably accurate inflation and job destruction behaviour. This points to the possibility that the inflation-job destruction volatility trade-off that we find in our model could be a consequence of the manner in which we have specified monetary policy as a simple growth rule as opposed to a Taylor rule, which both Walsh and Trigari assume. Monetary policy modelled as a Taylor rule with a positive coefficient on inflation would be more accommodating during a deflationary productivity boom. This would relax the short-run aggregate demand constraint caused by incomplete nominal adjustment, thereby reducing pressure on job destruction to spike upwards on impact in response to a productivity improvement. In Appendix II, we discuss the consequences of adopting a Taylor rule to characterise monetary policy, but it turns out that although the inflation-job destruction volatility trade-off is improved, it is not entirely solved.

There is yet another possible explanation. We have followed Krause and Lubik (2007) in modelling both pricing and employment decisions as taking place within a single firm. This approach departs from most of the literature (e.g. Walsh 2005, Trigari 2009, Gali 2010) which assumes a producer-retailer setup consisting of two different firm types in order to separate the pricing and hiring decisions for tractability under Calvo contracts.²³ As a result, in our model, the shadow value λ_t^y which represents the

²²This trade-off is not the consequence of endogenous participation and therefore is a feature of the model by Krause and Lubik (2007). The authors, however, do not discuss it.

²³Simultaneously forward-looking vacancy posting and price adjustment implies that the firm's vacancy supply depends on its pricing decisions and vice versa. Calvo contracts introduce price dispersion, and therefore by extension dispersion in firms' desired rates of job creation. In the model of Krause and Lubik (2007), tractability is maintained despite price setting firms also making hiring decisions because quadratic price adjustment costs imply that all firms adjust their price slightly in each period, and so effectively charge the same price. Work by Kuester (2007) and Thomas (2008) has also incorporated both pricing and employment decisions within a single firm, demonstrating that when modelled in this manner matching frictions temper inflation volatility. Both authors find that variable hours are central to their results, but abstract from endogenous job destruction and so do not consider joint inflation

contribution of an additional unit of output to the firm's revenue, factors into the match separation condition (16). The negative dependence of \underline{a}_t on λ_t^y amplifies fluctuations in job destruction. To demonstrate that the model's dynamics are consistent with this intuition, consider a positive technology shock. Due to the binding of the short-term aggregate demand constraint, we saw that job destruction increases on impact. Now, a positive technology shock is deflationary, causing λ_t^y to fall below steady state. From (16) this fall in the shadow value of output causes \underline{a}_t to rise still further, exacerbating the movement in job destruction. Similar intuition applies for a monetary disturbance. Monetary shocks are inflationary, causing λ_t^y to rise above steady state and thereby exacerbate the decline in \underline{a}_t , and hence job destruction, witnessed previously in Figure 2. Most importantly, note that the stickier prices are, the larger the fluctuations in λ_t^y .²⁴ In a model in which pricing and output decisions are made in separate sectors, this additional source of volatility would not arise.

Illustrating this intuition in the constant participation model for $\phi = 100$ and $\phi = 1$, Figures 8 and 9 plot the impulse responses of real marginal cost, the separation threshold, inflation and job destruction to monetary and technology shocks, respectively.²⁵ Clearly, then, increasing the parameter ϕ , while leaving all other parameters unchanged, damps inflation volatility. But it also increases the volatility of job destruction. Notice also that when $\phi = 1$, job destruction drops in response to a positive technology shock, confirming the intuition that slow price adjustment is what causes job destruction to spike on impact. In principle, it could be possible to obtain a combination of the two parameters ϕ and σ_a that allows the model to be jointly consistent with observed inflation and separation dynamics. However, simulating the model with $\phi = 100$ and σ_a as low as 0.0001 still results in a relative standard deviation of job destruction equal to 11.54. On the other hand, relative inflation volatility falls to 0.31.²⁶ It becomes increasingly difficult to match realistic job destruction volatility as ϕ increases.

In summary, we conclude that in addition to the failure to replicate a negatively sloped Beveridge curve, models of this class which feature endogenous job destruction decisions in an environment with sticky prices tend to present a trade-off in terms of the ability to jointly match inflation and separation dynamics. Related work in the literature incorporates mechanisms that can plausibly reduce this tendency which we have abstracted from given that we base our approach on Krause and Lubik (2007) in order to address a particular peculiarity that arises in their model, namely the contractionary response of vacancies to a shock to monetary growth. In order to concentrate

and match separation dynamics. Thomas (2008) shows how the solution concept of Woodford (2005) can be applied to solve the model under Calvo contracts.

²⁴This is consistent with the standard textbook result that, under complete price flexibility, monopolistically competitive pricing behaviour ensures that real marginal cost is constant at the steady state value. See, for example, Walsh (2003) chapter 5.

²⁵ $\phi = 100$ implies an equivalent average Calvo duration of approximately four quarters, in line with the evidence in Taylor (1999).

²⁶Krause and Lubik (2007) assume that $\phi = 40$. Their results indicate that relative inflation volatility is 0.42 and relative job destruction volatility is 11.02.

the analysis on this specific aspect of the model, we do not further investigate the tension in accounting for joint inflation and job destruction dynamics given that there is no immediate reason to suspect that it would interfere with our conclusions regarding the effects of endogenous participation. Rectifying this additional shortcoming would take us far afield from our original objective and constitutes a complication that warrants an independent investigation in its own right.

5.3 Eliminating Passive Search

We now discuss in more detail the ability of agents to transition directly from out-of-the-labour force to employment. This represents a feature of our model which is absent from some other business cycle studies of participation (Veracierto 2008, Tripier 2003 and Haefke and Reiter 2006). Passive search can be interpreted as "jobs bumping into people". Others argue that this is unrealistic. Proponents of the "time aggregation bias" theory suggest that some minimal effort is always required to successfully enter employment. Survey data that is obtained at discrete intervals cannot therefore capture infra-monthly transitions, and so such instances appear misleadingly as non-participation-employment flows. See Petrongolo and Pissarides (2001) for a discussion. The approach taken in this chapter is to allow for passive search, as in Pries and Rogerson (2009) who abstract from aggregate uncertainty.

Nevertheless, given that other authors ignore this labour market flow, we now consider the effects of removing it. Eliminating the possibility of direct transition from non-participation to employment by setting $\chi = 0$ makes the participation constraint more sensitive to fluctuations in tightness, as can be seen by referring to equation (27). Without the option to search whilst not participating in the labour market, agents must enter the labour force and endure at least one period of unemployment in order to find a job. We now consider a version of our model with $\chi = 0$ under the comparison utility calibration since it yields more realistic results than the baseline calibration. In computing the impulse responses, we set χ to zero keeping all other parameters unadjusted.

Figures 10 and 11 report the impulses for monetary and technology shocks, respectively. The significant effect of removing passive search is on the behaviour of unemployment. In Figure 10 for a monetary shock, the negative response in \underline{h}_t when $\chi = 0$ is smaller compared with the response in Figure 6. This is due to non-participation becoming less attractive when it bars the prospect of finding a job. Unemployment contracts by less, but with little overall impact on the rest of the model's dynamics.²⁷ A larger difference in dynamics obtains in response to a technology shock. In Figure 11 without passive search, \underline{h}_t remains positive throughout the duration of adjustment,

²⁷ Although unemployment contracts by less, vacancies contract by more. This indicates that the fall in \underline{h}_t , despite being smaller than in the baseline comparison utility model, results in a larger reduction in effective aggregate search intensity since those that do exit the labour market search with zero intensity.

indicating a much stronger incentive to participate compared with the (baseline) comparison utility model in Figure 7. With $\chi = 0$, the peak drop in unemployment is less than one percentage point below steady state, indicating that without passive search unemployment becomes less countercyclical. The reluctance of unemployment to fall below steady state reduces congestion, amplifying the increase in vacancies and job creation above steady state. This finding indicates that unemployment must remain relatively high in order for any appreciable effect on labour market congestion, and hence job creation, to occur.

Unemployment therefore becomes less countercyclical conditional on both monetary and technology shocks. Simulating the model we find that the correlation of unemployment with output becomes -0.15, substantially less than the baseline figure of -0.57 from Table 3. Unemployment also becomes less volatile with a relative standard deviation of 3.47.²⁸ Given greater unemployment stability, the volatility of the labour force increases to 0.58. The relative volatility of job creation increases to 6.58 due to the amplified responses to technology disturbances, but apart from this there are no other noteworthy changes to the business cycle moments compared with Table 3.

We thus conclude that passive search is a quantitatively significant component in generating realistically volatile and countercyclical time series for unemployment. This finding suggests that the difficulty faced by previous authors (Veracierto, 2008, Tripier, 2003) in generating countercyclical unemployment when participation is endogenous may be partially attributable to their simplifying assumption that search and home production are mutually exclusive undertakings.²⁹ Furthermore, the impulse response functions indicate that unemployment must be unrealistically stable, causing excessive volatility in the labour force, in order for the congestion effects that arise from labour market inflows to have a meaningful impact on vacancy creation.

5.4 Time-Inseparable Utility

As the preceding analysis has demonstrated, the introduction of endogenous fluctuations in the size of the labour force in a standard New Keynesian model with matching frictions does not result in a positive amplification effect on the response of vacancy supply to monetary stimulus. In this section, we discuss how time-inseparable utility can reverse the contractionary effect of monetary growth on vacancy supply observed for the baseline model. This result stems from the importance of aggregate demand *persistence* in determining the response of job creation to monetary shocks. Time-inseparability features in the related work of Walsh (2005) and Trigari (2009) although these authors refrain from discussing its implications for job creation and the Beveridge curve. The theoretical motivation is as follows.

²⁸This holds even though we reduce σ_h to a value of just 0.0001. When setting $\chi = 0$, we also set $\sigma_A = 0.006$, yielding output volatility of 1.62%.

²⁹Ebell (2008) argues that the failure of these studies to generate countercyclical unemployment is due to the calibration strategy. She demonstrates that by calibrating the elasticity of labour supply to match the volatility of the labour force, rather than employment, unemployment becomes countercyclical.

The baseline impulse response functions for monetary policy disturbances exhibited weak persistence and no inertial behaviour, with the peak response of output occurring on impact. After impact, the variables decline monotonically back towards their steady state positions. This lack of persistence is inconsistent with the evidence documented by Christiano et al. (2005) that aggregate quantities respond sluggishly to monetary policy shocks, gradually adjusting over a protracted period and peaking several quarters after impact. In recognition of the forward-looking nature of vacancy creation, the monotonic decline of aggregate demand after impact may be contributing to weak incentives to supply vacancies in response to monetary shocks. If adjustment were carried out to a larger extent in the periods following a shock, the incentives to supply vacancies, which involve a lag prior to becoming operable, could increase. If firms anticipated a hump-shaped, gradual response in aggregate demand that keeps rising even after the shock takes place, vacancy supply could respond differently to the baseline findings.

For ease of exposition, and since the general principles we discuss here do not hinge on the participation decision, we illustrate the consequences of this modification in the constant participation model of Krause and Lubik (2007). Furthermore, time-inseparable utility induces additional countercyclicality in the labour force (see footnote 16 above). We wish to avoid this issue in order to emphasise the fundamental implications of time-inseparability for job creation dynamics.

The time-inseparable utility function is given by

$$u(c_t, c_{t-1}) = \frac{(c_t - \alpha c_{t-1})^{1-\sigma}}{1-\sigma}$$

where $0 \leq \alpha \leq 1$ is a parameter that indexes the strength of consumption habits. The first-order condition with respect to consumption is now given by

$$(c_t - \alpha c_{t-1})^{-\sigma} - \alpha \beta E_t (c_{t+1} - \alpha c_t)^{-\sigma} = \mu_t.$$

The consumption decision at time t is made in recognition of the implication that a rise in current consumption necessitates a further increase in consumption next period in order to maintain the same level of utility. Internal habit formation in consumption makes households reluctant to make large adjustments in consumption in adjacent periods. This in turn means that aggregate demand is slower to respond to shocks.

We set $\alpha = 0.8$, as is assumed in Walsh (2005). The rest of the parameters remain unchanged relative to the baseline so that the specific role of time-inseparability is isolated. Figure 12 displays the impulse dynamics to a monetary shock. Panel (a) shows the hump-shaped adjustment path that output takes when utility is time-inseparable. The dynamic behaviour of aggregate demand is therefore fundamentally different to the baseline model, in which the response was front-loaded, displaying weak persistence. Relative to the baseline, the responses in output and employment are also significantly weaker due to the reluctance of the representative household to substantially increase

demand. As panels (b) and (c) indicate, time-inseparable preferences over consumption induce a qualitative change in inflationary job creation dynamics. Both vacancy supply and job creation now react positively to an expansion in the money supply. Since output responds sluggishly to the monetary shock, with the peak effect occurring after impact, firms have a greater incentive to adjust to the shock through job creation, which is time consuming, rather than job destruction, which is instantaneous. Increased persistence in aggregate demand induces an expansion in vacancy supply even though the amplitude of output is lower than in the baseline. This suggests that persistence in aggregate demand may be quantitatively more important than amplitude in order for firms to have sufficient incentives to create jobs. The negative response of job destruction is much smaller compared to the baseline (see Figure 2) as firms now increase employment also partly through job creation. In the second period after the shock job destruction rises (slightly) above steady state, as vacancies that were matched in the previous period commence production. We therefore conclude that, by altering the dynamics of aggregate demand so that a larger proportion of adjustment occurs after the time of the shock, internal habit formation in consumption is capable of overturning the result of Krause and Lubik (2007) that monetary shocks have a contractionary effect on vacancy supply.

The responses to a productivity shock are shown in Figure 13. Comparing with the baseline results in Figure 3, in contrast to the results for a monetary shock, sluggish adjustment in aggregate demand is not observed to have a quantitatively meaningful effect on job creation conditional on a productivity shock. The response of vacancies in the baseline was positive to begin with, and through time consuming matching this allowed the baseline model to generate a persistent increase in output without the need for internal consumption habits. Time inseparability slows the adjustment process still further, and as can be seen from panel (c) of Figure 13, the surge in job destruction on impact becomes amplified as a result of sluggish aggregate demand falling below the productivity-driven increase in aggregate supply. Unemployment rises substantially above steady state, forcing tightness to decline until the second period after the shock.

Selected business cycle statistics are reported in Table 4. Despite reducing σ_a to 0.0001, job destruction is grossly overstated due to increased sensitivity to productivity shocks. Notably, however, the model predicts a negative correlation between vacancies and unemployment conditional on a monetary shock. Job flows also attain the correct negative correlation. Overall, however, the surge in unemployment in response to productivity shocks dominates and when driven by both real and monetary shocks the model predicts a counterfactually upward sloped Beveridge curve. Amplification of vacancy supply still remains problematic as the volatility of vacancies relative to output is still far lower than observed in the data.

To summarise, we observe that internal consumption habits are capable of generating an expansion in vacancy supply conditional on monetary stimulus even though the amplitude of the response in output is substantially lower than in the baseline.

This indicates that the persistence of aggregate demand following a monetary growth shock may be a more important factor in providing incentives for job creation rather than simply the extent to which consumption rises. However, the model with time-inseparable preferences still drastically understates the relative volatility of vacancies and also exacerbates the trade-off in the ability of the model to simultaneously match inflation and job destruction dynamics.

6 Conclusion

In this chapter we have examined the extent to which endogenous labour force participation matters for the propagation of business cycle shocks in a New Keynesian model with unemployment. In particular, we have tried to address a weakness of this class of models pertaining to the difficulty encountered in generating an increase in job creation in response to expansionary monetary policy so as to be consistent with a downward-sloping Beveridge curve. In response to speculation in the literature that this shortcoming could be due to excessive labour market congestion that arises in the absence of endogenous variation in the size of the labour force, we extended the New Keynesian model with endogenous job destruction and unemployment to feature an explicit participation decision. It was found that, even for a conventional logarithmic specification of the utility function, the presence of a home good to be valued tended to induce countercyclical behaviour in the labour force. In order to address this counterfactual implication, we formalised the notion of labour force participation as a social device for raising consumption during upswings by introducing a particular type of aggregate consumption externality in the spirit of Gali (1994). We used this concept to argue that social considerations may be a quantitatively relevant driver of cyclical labour force participation decisions. We then argued that, for realistically volatile and procyclical fluctuations in the labour force, endogenous labour market participation has a quantitatively insignificant role in amplifying cyclical job creation in response to monetary shocks.

This result continued to hold even when we disallowed passive search, which generated unrealistically stable unemployment that was also much less countercyclical. This strengthened the conclusion that congestion effects due to fluctuations in the availability of job applicants are not the main driver of cyclical job creation. However, this also illustrated the quantitative importance of passive search in generating countercyclical unemployment, behaviour which proved difficult to obtain in earlier studies of cyclical participation dynamics. In conclusion, there does not appear to be evidence of a significant relationship between participation dynamics and cyclical job creation. From a modelling perspective, our results somewhat justify the abstraction from labour force fluctuations in most of the related literature on cyclical unemployment which does not explicitly examine issues directly related to participation.

We then demonstrated that by shaping the dynamics of aggregate demand, internal

habit formation in consumption can rationalise a positive role for monetary policy in creating jobs. Specifically, a hump-shaped response in output, in which output continues to rise even after the initial period of impact, appears to bolster incentives for firms that create jobs in a frictional labour market to expand the supply of vacancies in response to a positive aggregate demand shock. Since it takes time to successfully fill vacant jobs, output growth that is sustained for more than one period was shown to significantly encourage greater vacancy creation, allowing the model to generate a negative vacancy-unemployment correlation conditional on monetary shocks.

A deeper analysis provides considerable support for this conclusion. In an appendix to this chapter, we demonstrate that when monetary policy is characterised by a Taylor rule rather than a monetary growth rule, hump-shaped consumption dynamics can, under certain conditions, arise even in the absence of time-inseparability, thereby generating a positive response in vacancies to expansionary monetary policy. On the other hand, Cooley and Quadrini (1999) find that vacancies expand slightly in response to a positive nominal shock even when monetary policy follows a constant growth rule, which at first might appear to contradict the result of Krause and Lubik (2007). The difference in the two models is that Cooley and Quadrini (1999) rationalise money through cash-in-advance constraints, whereas Krause and Lubik (2007) assume that money enters into utility. However, cash-in-advance constraints naturally give rise to hump-shaped output dynamics in response to shocks whereas money-in-utility does not (Wang and Wen 2006). This demonstrates that various independent mechanisms which give rise to sluggish output adjustment also engender a vacancy expansion in response to monetary stimulus. Although time-inseparability is a common assumption in the related literature, the connection between sluggish output adjustment, monetary policy and job creation has, to our knowledge, not previously been uncovered. By explicitly highlighting the relevance of aggregate demand dynamics for the ability of monetary policy to stimulate job creation, we hope to offer additional insight into the determinants of cyclical job creation and the extent to which monetary control can be exercised in this regard.

One of the main simplifying assumptions made in this chapter was the independence of the equilibrium wage from individual home productivity. Relaxing this assumption would result in a continuum of separation rates for all employees whose option outside of the current match would be to exit the labour force rather than look for a new job. This may turn out to have potentially significant consequences for the manner in which shocks are absorbed by the economy, particularly along the job destruction margin. The intrinsic link between separation decisions and inflation dynamics that we have also established leads one to speculate that, if relaxing this assumption significantly altered destruction behaviour, the dynamics of inflation could also be affected. We leave this issue to future research.

7 References

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8 Appendix I: Derivation of Equilibrium Wage

In order to derive the equilibrium wage expression (26), begin with the Nash sharing rule (25) which requires

$$\frac{\eta}{1-\eta} J_{i,t} = W_{i,t} - \left\{ F(\underline{h}_t) U_t + \int_{\underline{h}_t}^{\infty} N_{i,t} dF(h) \right\}.$$

Inserting the Bellman equations (20), (19) and (18) into the right hand side of the above condition yields

$$\begin{aligned} \frac{\eta}{1-\eta} J_{i,t} &= w_{i,t} - F(\underline{h}_t) b_t - \int_{\underline{h}_t}^{\infty} \frac{z(h_{i,t})}{\mu_t} dF(h) \\ &\quad + E_t \beta_{t+1} (1 - \rho_{t+1}) \times \\ &\quad \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1}(a) - \left[F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right] \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\ &\quad - F(\underline{h}_t) p^U(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \times \\ &\quad \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1}(a) - \left\{ F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right\} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\ &\quad - (1 - F(\underline{h}_t)) p^N(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \times \\ &\quad \int_{\underline{a}_{t+1}}^{\infty} W_{i,t+1}(a) - \left\{ F(\underline{h}_{t+1}) U_{t+1} + \int_{\underline{h}_{t+1}}^{\infty} N_{i,t+1} dF(h) \right\} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \end{aligned}$$

where we have integrated over $N_{i,t}$ recognising the fact that all aggregate and future variables are independent of the current idiosyncratic home production shock. Making further use of the share rule (25) we obtain

$$\begin{aligned} \frac{\eta}{1-\eta} J_{i,t} &= w_{i,t} - F(\underline{h}_t) b_t - \int_{\underline{h}_t}^{\infty} \frac{z(h_{i,t})}{\mu_t} dF(h) \\ &\quad + E_t \beta_{t+1} (1 - \rho_{t+1}) \frac{\eta}{1-\eta} \int_{\underline{a}_{t+1}}^{\infty} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\ &\quad - F(\underline{h}_t) p^U(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} \frac{\eta}{1-\eta} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})} \\ &\quad - (1 - F(\underline{h}_t)) p^N(\theta_t) E_t \beta_{t+1} (1 - \rho_{t+1}) \int_{\underline{a}_{t+1}}^{\infty} J_{i,t+1} \frac{dG(a)}{1 - G(\underline{a}_{t+1})}. \end{aligned}$$

Substituting using the free entry condition on vacancies (23) to eliminate $J_{i,t+1}$, and replacing the left hand side of the above expression with the asset value for an occupied job (21) gives

$$\frac{\eta}{1-\eta} \left\{ \lambda_t^y A_t a_{i,t} - w_{i,t} + \frac{\kappa}{q(\theta_t)} \right\} = w_{i,t} - F(\underline{h}_t) b_t - \int_{\underline{h}_t}^{\infty} \frac{z(h_{i,t})}{\mu_t} dF(h) + \frac{\eta}{1-\eta} \frac{\kappa}{q(\theta_t)} - F(\underline{h}_t) p^U(\theta_t) \frac{\kappa}{q(\theta_t)} - (1 - F(\underline{h}_t)) p^N(\theta_t) \frac{\kappa}{q(\theta_t)}.$$

Rearranging the above expression yields the wage equation (26) from the text

$$w_{i,t} = \eta \lambda_t^y A_t a_{i,t} + (1-\eta) F(\underline{h}_t) \left(b + \vartheta \frac{\tilde{z}(\underline{h}_t)}{\mu_t} + \frac{\eta}{1-\eta} \kappa \theta_t \right) + (1-\eta) (1 - F(\underline{h}_t)) \left(\frac{\tilde{z}(\underline{h}_t)}{\mu_t} + \chi \frac{\eta}{1-\eta} \kappa \theta_t \right).$$

9 Appendix II: Accommodative Monetary Policy

We previously argued that the completely unaccommodative monetary policy stance implied by the constant money growth rule (28) causes firms to aggressively shed workers in response to a positive technology shock. Nevertheless, we characterised monetary policy as following such a rule in order to remain consistent with Krause and Lubik (2007), the study upon which the current chapter is based. In this appendix, we consider a different policy rule which assumes that the monetary authority expands the money supply in response to positive supply shocks in order to offset deflation. For ease of exposition, this policy modification is implemented holding participation constant.

Following Walsh (2005) and Trigari (2009), assume that the monetary authority sets the nominal interest rate according to the simple Taylor-type rule

$$\frac{r_t}{r} = \left(\frac{\pi_t}{\pi} \right)^{\Phi_\pi} \exp(\xi_t) \quad (32)$$

where ξ_t follows

$$\ln \xi_t = \varphi_r \ln \xi_{t-1} + \varepsilon_t^r$$

and $\varepsilon_t^r \sim N(0, \sigma_r^2)$. The parameter $\Phi_\pi > 1$ determines the weight that the monetary authority attaches to inflation in determining the policy target. Given (32), the money supply adjusts endogenously in order to satisfy the money demand equation (4). In this manner, monetary policy is accommodative. We calibrate the policy parameter Φ_π according to empirical estimates which exceed unity in order to guarantee a unique rational expectations equilibrium, in accordance with the well known Taylor principle. Following Walsh (2005), we set $\Phi_\pi = 1.1$. Rudebusch (2002) estimates the coefficient φ_r to be approximately 0.9, which makes the policy shock process highly serially correlated. Using these parameter values and keeping the rest of the calibration the same as

in the baseline, Figure 14 plots the impulse dynamics to a positive technology shock. Panel (d) shows that the nominal money supply expands in order to accommodate the positive supply shock, thus enabling job destruction to fall. Vacancies and unemployment move in opposite directions and employment no longer falls below the steady state. Compared with the baseline (see Figure 3) vacancies expand by less as firms now place more emphasis on adjustment via the separation margin. Output peaks at a higher level under a Taylor rule compared with a constant growth rule, although slightly weaker vacancy creation detracts somewhat from persistence. We do note, however, that the introduction of time-inseparable preferences (not shown for brevity) prevents job destruction from falling even when monetary policy is determined by a Taylor rule, resulting in a significant increase in unemployment on impact that is comparable to panel (b) of Figure 3.

We now turn to the possibility that the manner in which monetary policy is specified - either by a Taylor rule or a constant money growth rule - determines the extent to which job creation is stimulated by inflationary policy. To understand how the monetary transmission mechanism under a Taylor rules compares to a constant money growth rule, note that the log-linearised version of the consumption Euler condition (3) can be expressed as

$$\sigma (E_t \widehat{c}_{t+1} - \widehat{c}_t) = -E_t \widehat{\pi}_{t+1} + \underbrace{\frac{1}{1 - \frac{d}{\widehat{m}_t / \widehat{p}_t} \widehat{c}_t^\sigma}}_{\widehat{r}_t} \quad (33)$$

where the money demand equation (4) has been used to replace r_t . The above expression simply states that the growth path of consumption (the left side) is positively related to the real interest rate (the right side). Our previous discussion on time-inseparability suggested that the growth path of consumption, which is the only component of aggregate demand in this model, plays an important role in determining the dynamics of job creation. Equation (33) relates the growth path of consumption to the real interest rate. It is theoretically possible to obtain a hump-shaped adjustment path for consumption in response to a policy shock as long as the increase in \widehat{r}_t is substantial enough to induce an increase in the real interest rate. For this to be the case, the increase in consumption must be large relative to the increase in real money balances, so that the marginal rate of substitution between money and consumption - the term $\frac{d}{\widehat{m}_t / \widehat{p}_t} \widehat{c}_t^\sigma$ in (33) - is high, and expected inflation must rise only modestly. However, baseline results indicated that the real interest rate falls in response to a monetary growth shock, causing consumption growth to decline over the adjustment path.³⁰ Hump-shaped dynamics do not occur and therefore vacancy supply contracts.

³⁰The decline in the real interest rate is displayed in Figure 15, discussed subsequently. This holds even if ϕ is increased to 360, corresponding to the degree of price stickiness assumed in Walsh (2005). Although inflation rises by less, real balances rise by more in response to a monetary shock since prices are slower to rise. From (33) this puts downward pressure on the real interest rate.

Next, consider how a Taylor rule can produce different interest rate dynamics, and therefore potentially result in a different adjustment path for aggregate demand. Substituting the policy rule into the Euler condition and log-linearising gives

$$\sigma (E_t \widehat{c}_{t+1} - \widehat{c}_t) = -E_t \widehat{\pi}_{t+1} + \underbrace{\Phi_\pi \widehat{\pi}_t + \widehat{\xi}_t}_{\widehat{r}_t}.$$

Consumption growth is now positively related to the central bank's response to current inflation as well as the policy shock. Consider an exogenous policy shock as represented by a decrease in $\widehat{\xi}_t$. This corresponds to a reduction in \widehat{r}_t which, all else equal, necessitates an expansion in the money supply, which is inflationary. If the central bank is aggressive enough in fighting inflation, and if current inflation is high enough, then it is possible for the real interest rate to increase in equilibrium as the central bank offsets the initial shock to $\widehat{\xi}_t$. As the central bank seeks to raise interest rates in the presence of above average inflation, it encourages the postponement of consumption to future periods, shifting the adjustment of aggregate demand away from the period of impact. This logic is therefore the same as for time-inseparability, but instead of obtaining from the structure of household preferences, it arises due to the policy behaviour of the monetary authority. The endogenous response of the policy target to inflation becomes a determining factor of the growth path of consumption.

Figure 15 displays the impulses to a negative 1% policy shock to the Taylor rule keeping all parameters the same as in the baseline calibration. The clear difference to the baseline dynamics in Figure 2 is that output responds sluggishly under the Taylor rule. Panel (d) contrasts the rise in the real interest rate that occurs under a Taylor rule with the decline under a constant money growth rule. Consistent with our previous discussion on the importance of aggregate demand adjustment for cyclical job creation, the supply of vacancies expands above steady state. Although contrasting starkly to the baseline model, the rise in vacancies is nevertheless too weak to generate a boom in job creation.

This result therefore demonstrates that when monetary policy is endogenously determined at least in part by prevailing inflationary conditions, it is possible for vacancies to increase in response to an expansionary policy shock. This conclusion, however, critically depends on the extent to which inflation rises in response to the policy shock. For instance, in order for a given policy shock to generate a large response in inflation, ξ_t needs to display considerable persistence given the forward-looking manner in which prices are set subject to adjustment costs. Lowering φ_r to 0.5, as shown in Figure 16 produces results that are much more similar to the baseline results in Figure 2, with vacancies visibly contracting in response to the shock. Compared to when $\varphi_r = 0.9$, the response of inflation is notably smaller and as a result the central bank does not endeavour to raise the policy target as aggressively. Panel (d) thus shows that the real

interest rate falls, ruling out hump-shaped output dynamics in panel (a).³¹

It is now possible to examine whether the model with a Taylor rule can successfully replicate a downward-sloping Beveridge curve in simulations with both real and nominal shocks. Leaving the calibration for all parameters apart from the specification of monetary policy the same as in the baseline, we find that the relative volatility of job destruction is too low, as could have been anticipated from the impulse response analysis. Given our previous discussion on the effects of nominal rigidity on separation dynamics in the main text, we raise ϕ to 100. Although this prevents job destruction from falling by as much conditional on productivity shocks, hence lowering its relative volatility, job destruction falls by more in response to a monetary shock. The overall effect is to increase the relative volatility of job destruction. For $\sigma_A = 0.008$, we find that output volatility is 1.59%, relative job destruction volatility is 7.65, relative inflation volatility is 0.55 and the vacancy-unemployment correlation is -0.10. Job flows still exhibit a positive correlation of 0.44. On the whole, however, the model fits the data better with a Taylor rule than a constant money growth rule due to the accommodation of technology disturbances. In particular, inflation volatility is now somewhat lower compared to the baseline value of 0.67 (Table 2), signifying an improvement in the inflation-destruction volatility trade-off discussed in the main text.

³¹We find that the real interest rate always falls in response to accelerated growth in the money supply in the baseline model regardless of φ_g .

Table 1: Baseline Calibration

Parameter	Value	Parameter	Value
β	0.99	e	0.59
η	0.5	u	0.04 (0.114)
σ	1	ϕ	20
ϑ	0.66	$b (b_1, b_2)$	0.37 (0.36,0.86)
δ	1.47	κ	0.1 (0.04)
ψ	11	φ_A	0.95
ρ	0.104	φ_g	0.41
ρ^x	0.07	σ_A	0.014 (0.01)
χ	0.2	σ_g	0.0086
$q(\theta)$	0.7	σ_a	0.2 (0.075)
$p^U(\theta)$	0.6	σ_h	0.15
\mathcal{M}	0.66	v	0.1

Note: Constant participation values in parentheses where different.

Table 2: Business Cycle Properties of U.S. Economy and Baseline Model

	U.S. Data	Cons. Participation			Endog. Participation		
Money Shock	-	0.0086	0.0086	0	0.0086	0.0086	0
Productivity Shock	-	0.01	0	0.01	0.014	0	0.014
Standard Deviations							
Output	1.59%	1.58%	0.66%	1.44%	1.58%	0.38%	1.51%
Inflation	0.20	0.67	1.19	0.50	0.90	2.64	0.66
Real Wage	0.49	0.72	1.02	0.64	1.11	2.49	0.96
Labour Force	0.34	-	-	-	0.55	0.38	0.56
Employment	0.83	0.61	1.12	0.43	0.46	1.36	0.34
Unemployment	6.77	3.16	5.80	2.20	6.88	14.01	6.15
Vacancies	8.59	2.59	3.37	2.39	2.40	5.12	2.13
Tightness	13.86	2.46	2.48	2.46	3.09	1.15	3.17
Job Creation	4.30	5.05	7.77	4.23	3.86	10.07	3.11
Job Destruction	7.83	7.76	16.57	3.90	8.01	22.17	6.17
Correlations							
Unemp., Vacancies	-0.65	0.65	0.99	0.43	-0.16	0.98	-0.56
Job Creation, Destr.	-0.42	0.77	0.83	0.84	0.87	0.87	0.87
Lab. Force, Tightness	0.84	-	-	-	-0.97	0.20	-0.99
Lab. Force, Output	0.55	-	-	-	-0.87	0.96	-0.95
Inflation, Output	0.12	0.15	0.97	-0.27	-0.08	0.90	-0.35
Unemp., Output	-0.88	-0.59	-0.99	-0.45	-0.93	-0.98	-0.96
Emp., Output	0.83	0.59	0.99	0.45	-0.16	0.98	-0.50
Vacancies, Output	0.45	0.22	-0.98	0.61	0.52	-0.92	0.77

Note: Standard deviations are expressed relative to output. All time series are HP filtered, smoothing parameter 1600. Job flows data are available at the personal webpage of John Haltiwanger and described in Davis et al. (2006). Vacancies data are compiled by the Conference Board and made available at the OECD statistics portal. All other data are obtained from the BLS and BEA websites. Inflation is defined as the percentage change in the GDP deflator.

Table 3: Business Cycle Properties of the Comparison Utility Model

	U.S. Data		Model	
Money Shock	-	0.0086	0.0086	0
Productivity Shock	-	0.0075	0	0.0075
Standard Deviations				
Output	1.59%	1.59%	0.58%	1.48%
Inflation	0.20	0.76	2.00	0.19
Real Wage	0.49	0.77	1.82	0.41
Labour Force	0.34	0.35	0.36	0.35
Employment	0.83	0.72	1.36	0.55
Unemployment	6.77	6.44	14.38	3.96
Vacancies	8.59	2.83	5.53	2.10
Tightness	13.86	2.19	0.58	2.34
Job Creation	4.30	5.23	10.82	3.63
Job Destruction	7.83	8.10	21.83	0.97
Correlations				
Unemp., Vacancies	-0.65	0.76	0.99	0.43
Job Creation, Destr.	-0.42	0.61	0.87	-0.67
Lab. Force, Tightness	0.84	0.90	-0.01	0.98
Lab. Force, Output	0.55	0.99	0.99	0.99
Inflation, Output	0.12	0.24	0.85	-0.28
Unemp., Output	-0.88	-0.57	-0.97	-0.52
Emp., Output	0.83	0.87	0.98	0.93
Vacancies, Output	0.45	0.10	-0.95	0.54

Note: Standard deviations are expressed relative to output.

Table 4: Business Cycle Properties of the Time-Inseparable Utility Model

	U.S. Data		Model	
Money Shock	-	0.0086	0.0086	0
Productivity Shock	-	0.0185	0	0.0185
Standard Deviations				
Output	1.59%	1.60%	0.14%	1.59%
Inflation	0.20	0.91	4.77	0.82
Unemployment	6.77	6.41	5.18	6.42
Vacancies	8.59	3.65	7.94	3.61
Job Creation	4.30	8.60	11.29	8.60
Job Destruction	7.83	13.59	6.81	13.58
Correlations				
Vacancies, Unemp.	-0.65	0.77	-0.47	0.79
Job Creation, Job Destr.	-0.42	0.66	-0.34	0.67

Note: Constant participation model. Standard deviations are expressed relative to output. In these simulations $\phi = 20$ and $\sigma_a = 0.0001$.

Figure 2: Impulse Responses to a Positive 1% Monetary Growth Shock in the Baseline Constant Participation Model

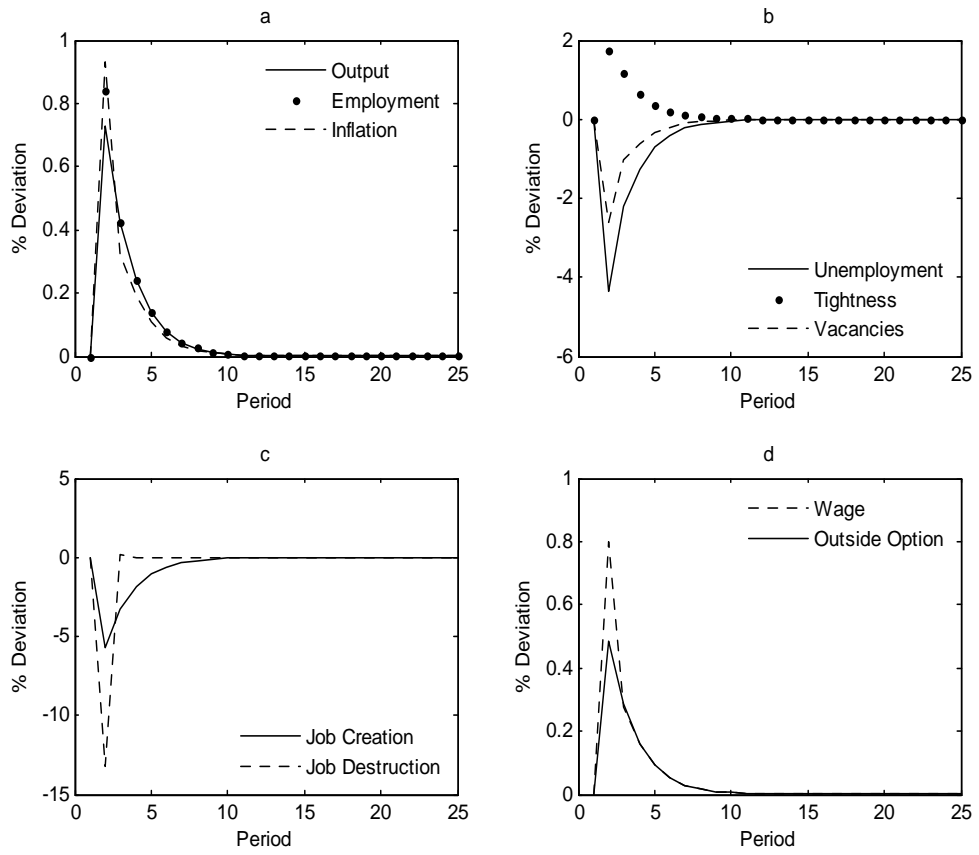


Figure 3: Impulse Responses to a Positive 1% Aggregate Productivity Shock in the Baseline Constant Participation Model

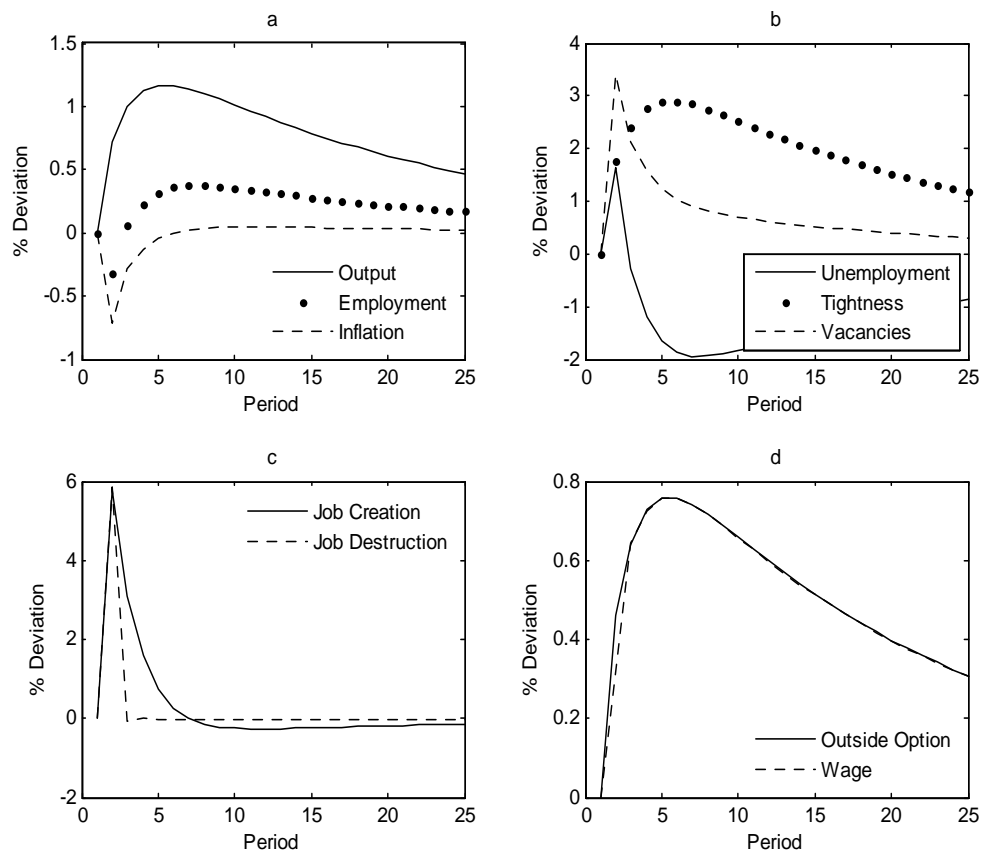


Figure 4: Impulse Responses to a Positive 1% Monetary Growth Shock in the Baseline Endogenous Participation Model

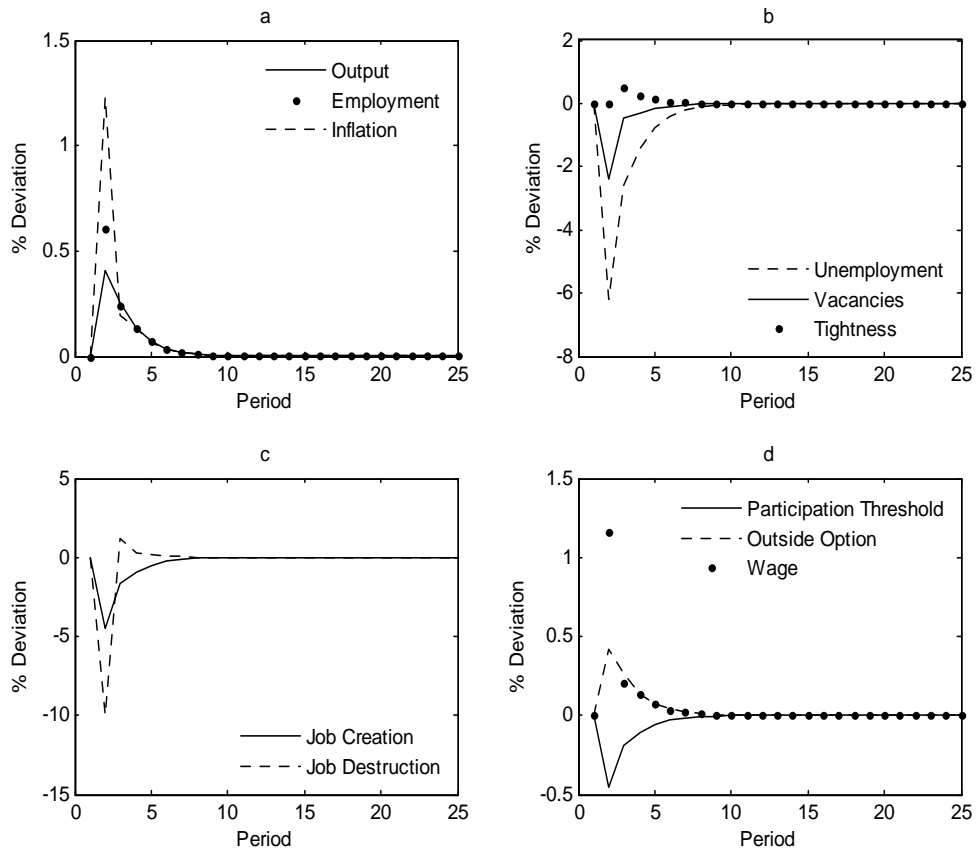


Figure 5: Impulse Responses to a Positive 1% Aggregate Productivity Shock in the Baseline Endogenous Participation Model

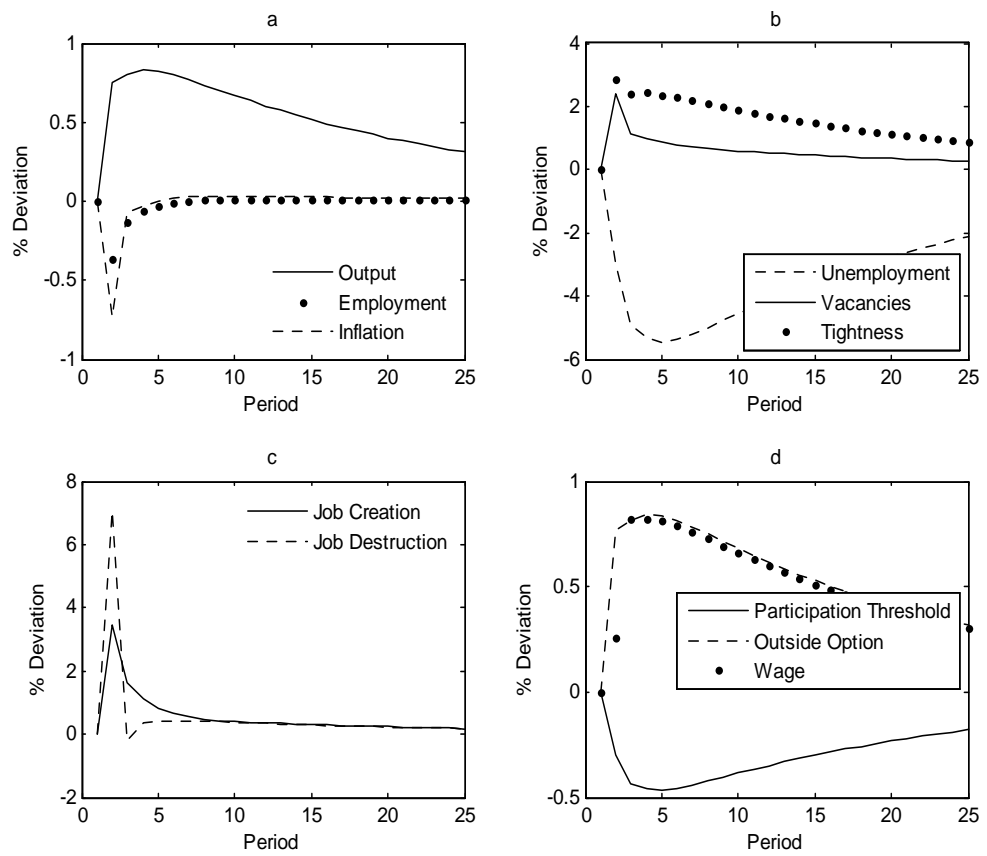
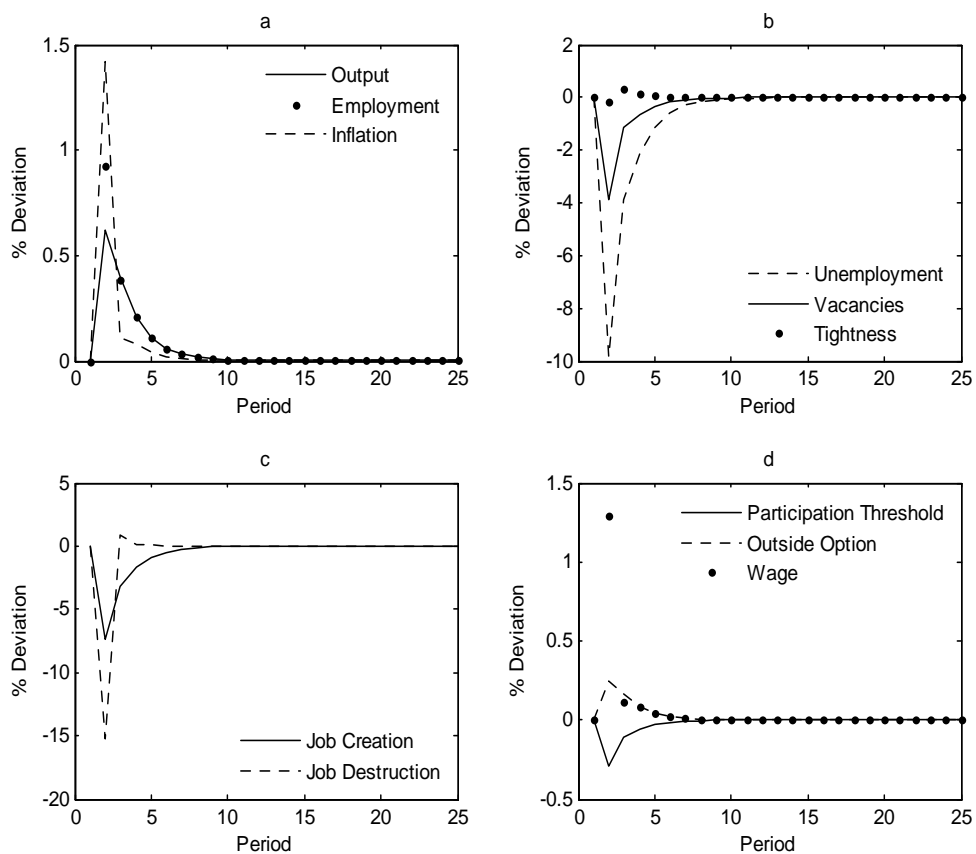
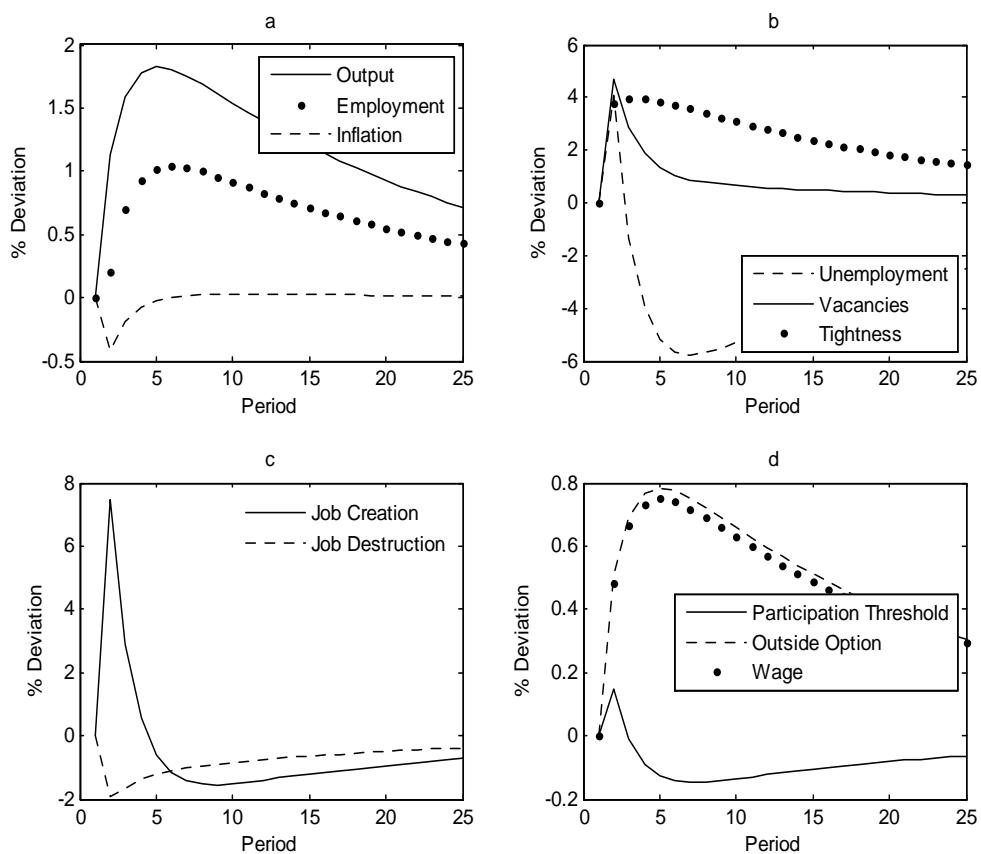


Figure 6: Impulse Responses to a Positive 1% Monetary Growth Shock in the Endogenous Participation Model with Comparison Utility



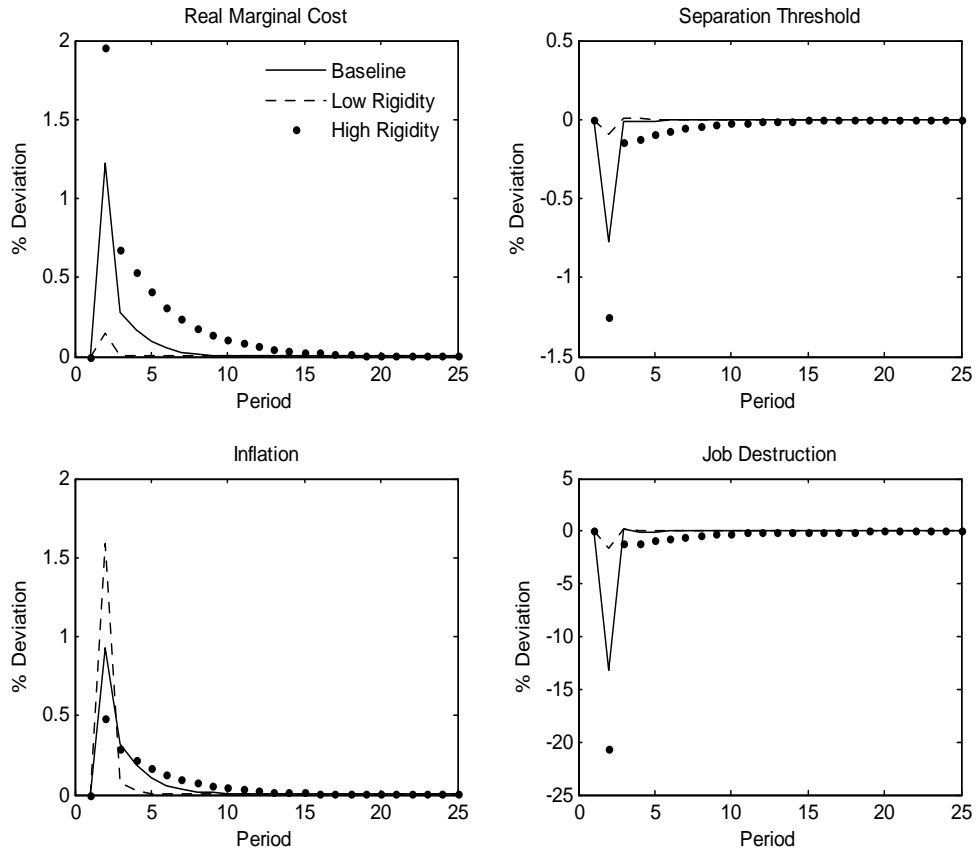
Note: Refer to section 5.2 for details regarding the comparison utility calibration.

Figure 7: Impulse Responses to a Positive 1% Aggregate Productivity Shock in the Endogenous Participation Model with Comparison Utility



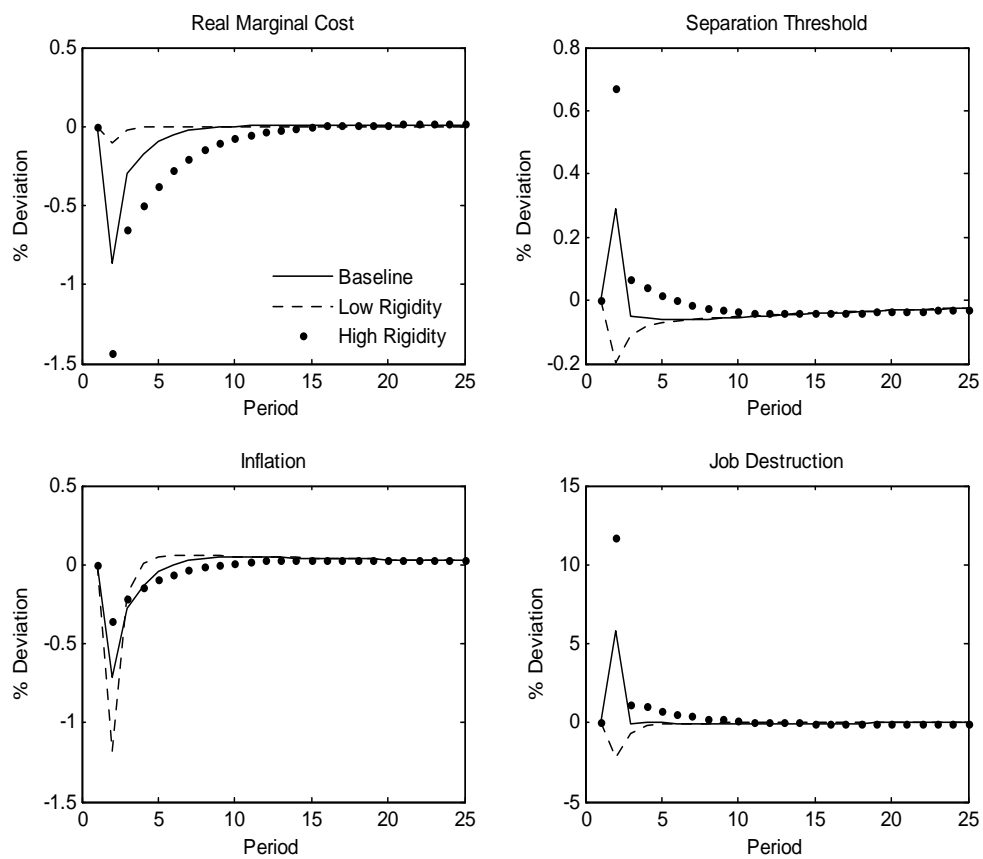
Note: Refer to section 5.2 for details regarding the comparison utility calibration.

Figure 8: Impulses to a Monetary Shock: Varying the Degree of Price Stickiness



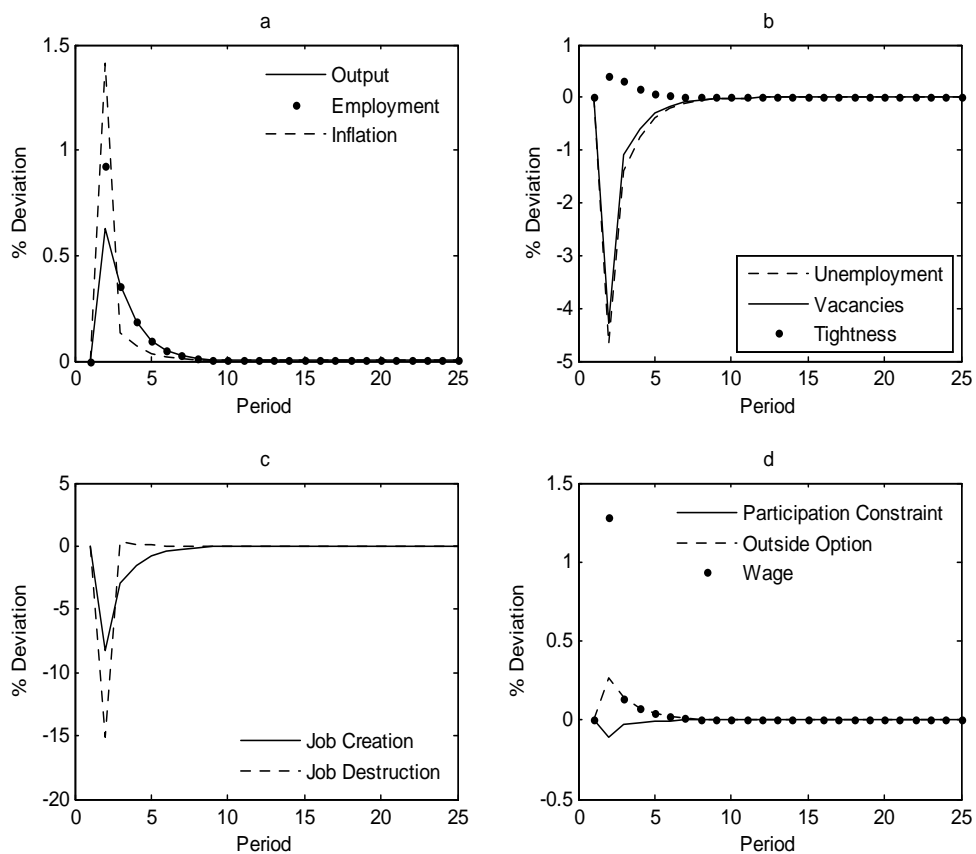
Note: The baseline, low rigidity and high rigidity calibrations set ϕ to 20, 1 and 100, respectively. Impulses are computed keeping all other parameters unchanged relative to the baseline.

Figure 9: Impulses to an Aggregate Productivity Shock: Varying the Degree of Price Stickiness



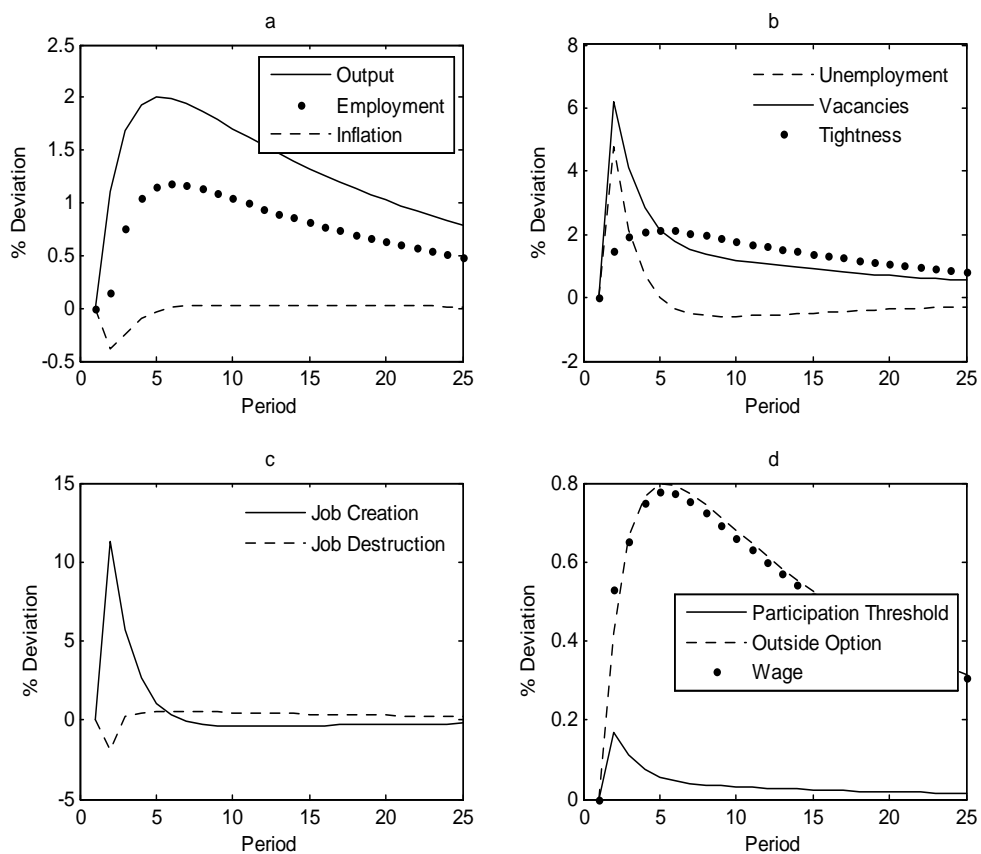
Note: The baseline, low rigidity and high rigidity calibrations set ϕ to 20, 1 and 100, respectively. Impulses are computed keeping all other parameters unchanged relative to the baseline.

Figure 10: Impulse Responses to a Positive 1% Monetary Growth Shock Without Passive Search



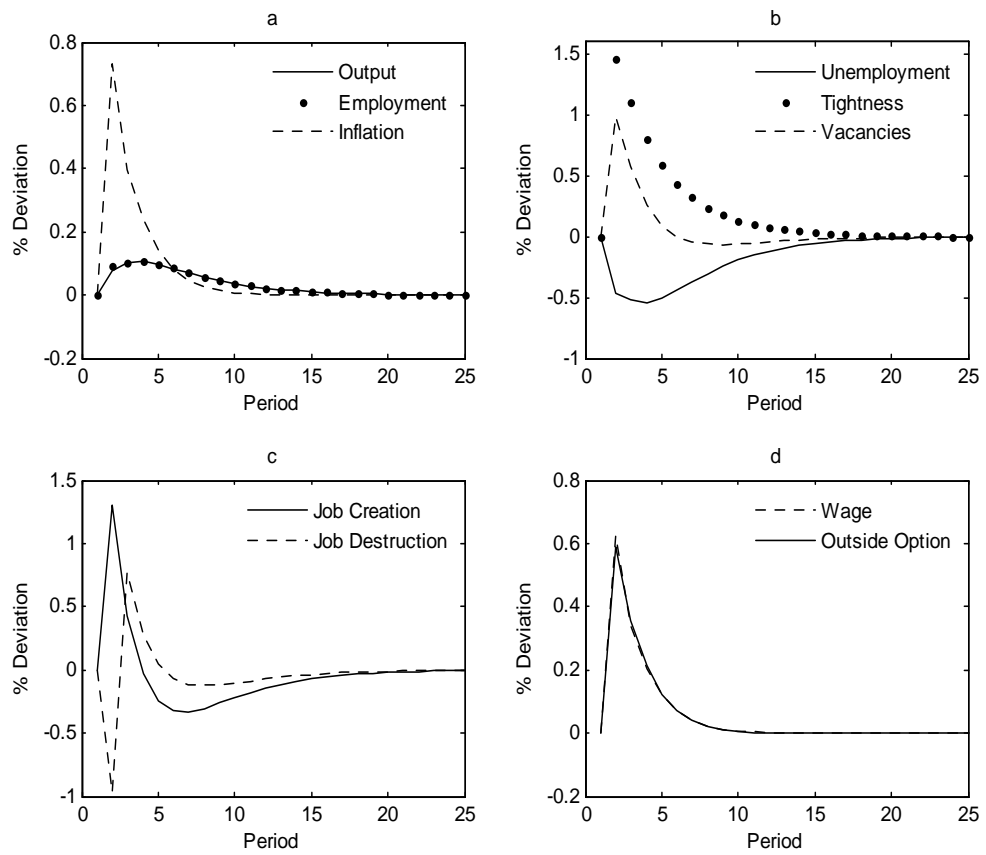
Note: Impulses for $\chi = 0$ keeping all other parameters the same as in the baseline calibration.

Figure 11: Impulse Responses to a Positive 1% Aggregate Productivity Shock Without Passive Search



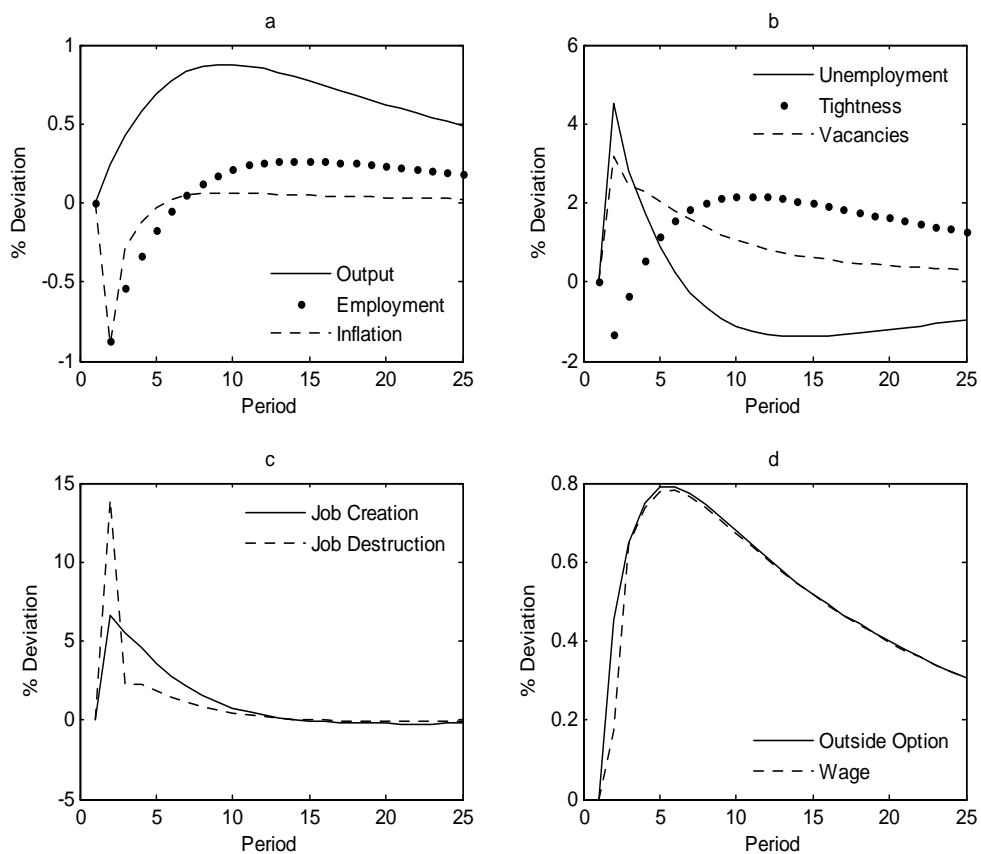
Note: Impulses for $\chi = 0$ keeping all other parameters the same as in the baseline calibration.

Figure 12: Impulse Responses to a Positive 1% Monetary Growth Shock with Time-Inseparable Utility



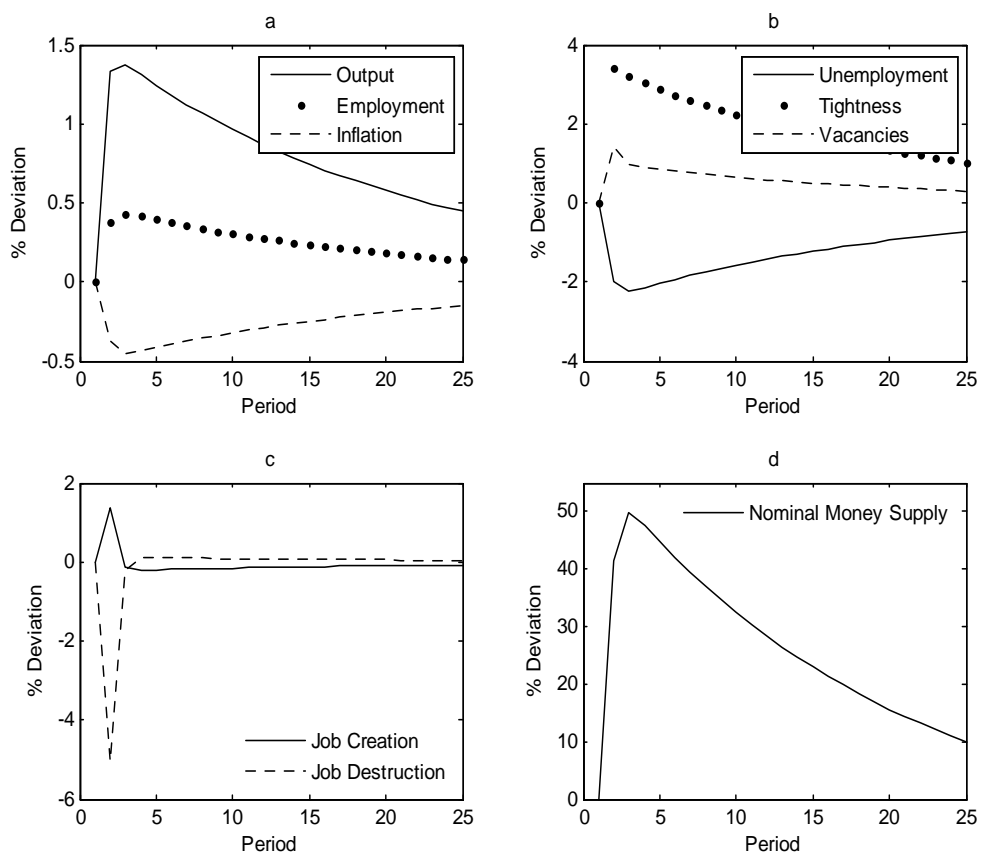
Note: Impulses computed with habit parameter $\alpha = 0.8$ keeping all other parameters the same as in the baseline constant participation model.

Figure 13: Impulse Responses to a Positive 1% Aggregate Productivity Shock with Time-Inseparable Utility



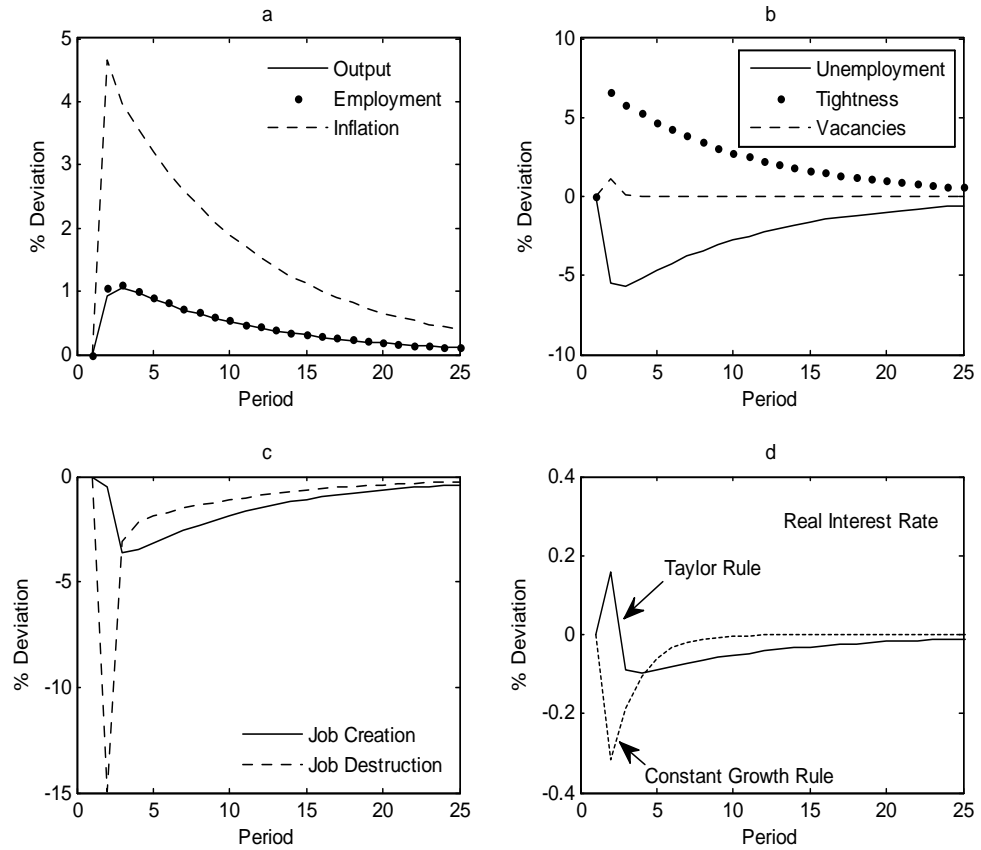
Note: Impulses computed with habit parameter $\alpha = 0.8$ keeping all other parameters the same as in the baseline constant participation model.

Figure 14: Impulse Responses to a Positive 1% Aggregate Productivity Shock when Monetary Policy follows a Taylor Rule



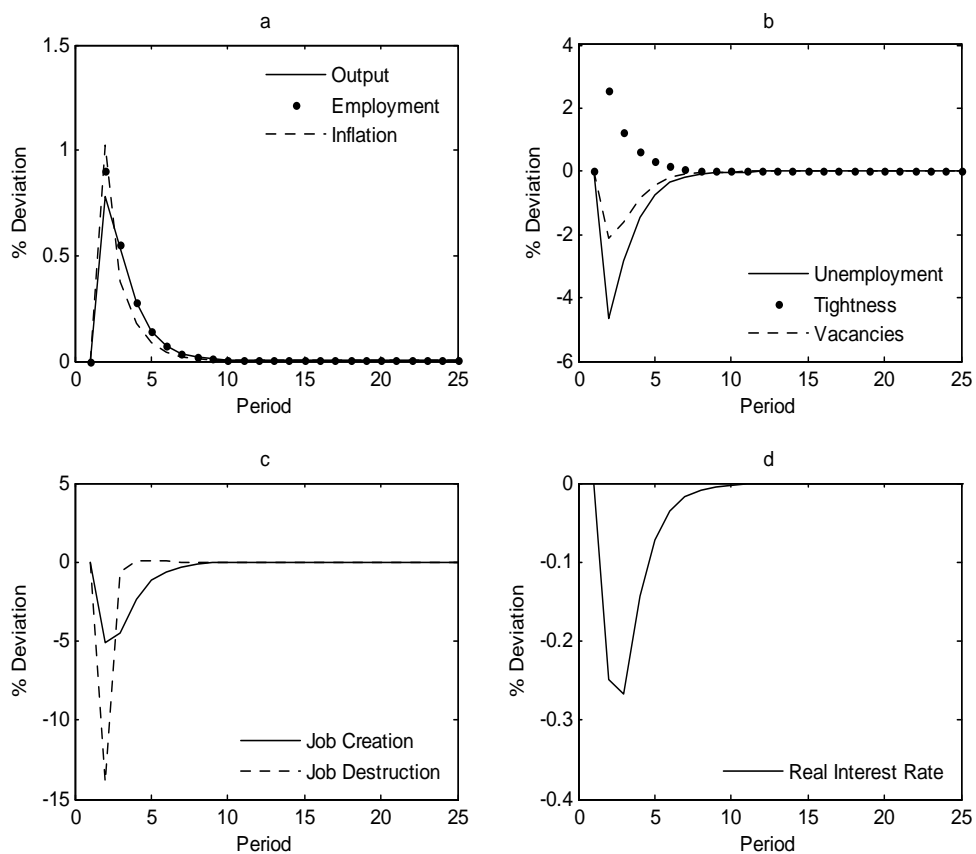
Note: See the appendix for details regarding the calibration of the Taylor rule. Impulses are for the constant participation model.

Figure 15: Impulses Responses to a Negative 1% Policy Shock to the Taylor Rule



Note: A policy shock is a negative 1% shock to ξ_t^r in the Taylor rule. See the appendix for details regarding the calibration of the Taylor rule. Impulses are for the constant participation model.

Figure 16: Impulse Responses to a Negative 1% Policy Shock to the Taylor Rule with Low Persistence



Note: A policy shock is a negative 1% shock to ξ_t^r in the Taylor rule. "Low persistence" means setting the autoregressive parameter of ξ_t^r to 0.5 instead of 0.9. Impulses are for the constant participation model.

Chapter III

Equilibrium Matching and Age Discrimination Policy

Abstract

Federal legislation prohibiting the discrimination of workers on the basis of age has been in place in the United States since the 1967 Age Discrimination in Employment Act. Yet empirical studies which aim to estimate the employment effects of such legislation have yielded inconclusive results. In this chapter we approach the issue from a different perspective by deriving quantitative predictions of equilibrium unemployment theory to investigate how age anti-discrimination legislation impacts macroeconomic performance. The main conclusion is that an equilibrium matching model of the life cycle predicts a moderately positive effect on the employment rate of workers very close to retirement, but the overall impact of age discrimination policy on the life cycle pattern of employment is quantitatively small. This occurs because in a frictional matching equilibrium, the incentive to discriminate is offset by labour market congestion, preventing the demand for older workers from falling excessively even when it is possible to discriminate on the basis of age. Welfare issues are also addressed. It is demonstrated that while age discrimination policy results in an age-dependent inefficiency in the participation decision of agents, the size of the externality is quantitatively negligible.

1 Introduction

Low employment rates among older workers have generated longstanding interest in the potential presence of age discrimination in the labour market. As Neumark (2009) notes, the issue of employment among older workers is likely to become increasingly relevant as the workforce ages and the prevention of rising dependency ratios becomes an ever more pressing policy objective. Federal legislation in the United States prohibiting discrimination against workers on the basis of age dates back to 1967, the year in which the Age Discrimination in Employment Act (ADEA) was passed by Congress. The success of such a policy can be judged by its ability to raise employment amongst older workers, but the quantitative effect that the legislation has had on actual employment and hiring rates is still an issue of ongoing research. Given conflicting empirical evidence, in this chapter we aim to contribute to the understanding of the employment effects of such legislation from a different angle by developing equilibrium unemployment models that can be implemented quantitatively to yield predictions about the

likely effects of institutional changes such as the introduction of age anti-discrimination legislation. It should be clarified at the outset that we do not attempt to measure the effects of the particular episode of legislative reform that is the ADEA, and so our results are not directly comparable to the econometric literature that pursues this goal (discussed below). Nevertheless, our approach does have the virtue of assessing the efficacy of age discrimination policy as a tool for macroeconomic management in a fully specified general equilibrium model with optimising agents, thereby allowing for a deeper theoretical analysis of the issues involved.

In order to build a framework within which a quantitative evaluation of age discrimination policy is possible, a life cycle equilibrium unemployment model is developed which controls for age discrimination through the functional form of the matching technology. We therefore restrict attention to age discrimination in the hiring decision as opposed to other terms and conditions of the employment relationship, such as dismissal or compensation. Two extremes are then considered. One is that firms can perfectly discriminate on the sole basis of age by means of an age-specific matching technology that pairs job seekers of a particular age with vacancies available only to their age cohort. The other is that anti-discrimination law renders age-specific matching technologically infeasible so that employers must accept the applicants with whom they are randomly matched regardless of age. The model without age discrimination can also be referred to as "random-age matching". In order to isolate the effect of a finite horizon on labour market dynamics, productivity differences and other sources of heterogeneity among different age cohorts are abstracted from. This allows for discrimination purely on the basis of age, since the only source of heterogeneity amongst agents is distance to retirement.

It is found that the effect of a finite horizon on behaviour in general is quantitatively weak apart from when the terminal period is very near, so that the dynamics of the models with and without age discrimination are not substantially different overall. Calibrating the models to post-war U.S. data and assuming an exogenous retirement age of 65, anti-discrimination legislation is found to increase the employment rate of workers aged 60 to 64 by 1.1 percentage points and workers in the 62-64 age bracket by 3.1 percentage points. The quantitative difference in net aggregate output (i.e. total output net of vacancy creation costs) produced in the two economies is negligible. The model suggests that the efficiency gain, in terms of net output, that would result from allowing firms to direct recruitment efforts with respect to age is on the order of 0.02%. Our results therefore indicate that legislation prohibiting discrimination against older workers in the hiring process does not substantially affect macroeconomic performance. Despite comparing two extremes in order to elicit the maximum effect of the institutional change, equilibrium unemployment theory does not appear to provide a strong rationale based on equilibrium outcomes for pursuing age discrimination policy.¹

¹Neither, however, does it imply that the efficiency loss resulting from the restriction on firm recruitment behaviour is large.

Intuitively, as the pool of job searchers grows due to falling employment rates, firms have the opportunity to capitalise on quicker vacancy transition rates. Congestion effects therefore oppose the negative effect of a finite horizon on the employment rate of older members of the labour force. In equilibrium, for plausible calibrations, it turns out that these opposing forces largely cancel out, such that the incentive to discriminate on the basis of age is low even when it is technologically possible. As such, there is an in-built mechanism in standard matching theory that guards against equilibria with a high degree of age-discrimination: as the demand for older workers becomes too low, a vacancy targeted at an older worker is more quickly matched because competition from other firms in the hiring process is more slack, making recruiting older workers relatively less costly. Consequently, hiring rates experience a substantial drop only when the terminal period is very near, so that even in the age-specific matching model age discrimination is not severe in the sense that demand for older workers remains relatively high.

The impact on social welfare that the specific form of the matching technology has is also considered. It is demonstrated that an age-dependent inefficiency arises in the labour force participation decision of agents in the random-age matching model. When only a single matching technology is available to connect job seekers with vacancies, older workers for whom only a relatively short productive time horizon remains do not internalise the negative effect that their participation decision has on the age distribution of the aggregate pool of searchers. Similarly, young workers whose participation in the labour market would raise the expected gain to firms from issuing a vacancy do not recognise this and so tend to exhibit excessive reluctance to participate relative to the social optimum. This life cycle externality lowers the asset value of a vacancy and in equilibrium market tightness is too low in the decentralised economy relative to the social optimum. In the age-specific matching model this externality does not arise, and as long as standard inefficiencies are controlled for in the Nash bargaining process that determines wages, i.e. the Hosios condition is satisfied, the decentralised solution coincides with the social optimum. It is further demonstrated that imposing the Hosios condition results in an oscillatory unemployment path such that market tightness does not consistently decline over the life cycle at the optimal allocation as it does in the decentralised equilibrium. This implies that average hiring rates can actually increase for older workers. However, it is found that the departures of the decentralised solution to the socially optimal allocation are quantitatively small regardless of the form of the matching technology.

Our findings on the sub-optimality of decentralised participation behaviour in the presence of a single aggregate matching technology provide an extension of the results of Cheron et al. (2008), who first documented the intergenerational externality in the separation decision. Decentralised firing decisions do not internalise the impact caused on the age distribution of the unemployed and therefore the asset value of vacancies. Cheron et al. (2008) concluded that endogenously determined separation rates tend to

be inefficiently high (low) for old (young) workers, thereby justifying the subsidisation of employment for older workers. It is worth noting that the inefficiencies caused by a single matching technology in the context of finitely-lived agents in the Diamond-Mortensen-Pissarides equilibrium unemployment model are distinct from the type of potential inefficiency of age anti-discrimination legislation considered by the literature on long-term incentive contracts in the style of Lazear (1979). Lazear's theory posits that mandatory retirement is a mechanism by which firms can enforce the termination of implicit delayed payments contracts that serve to disincentivise workers from shirking and instead exert more effort in order to retain their jobs. However, given a deferred benefit scheme, workers will not voluntarily retire at the optimal date because the wage will exceed their reservation value. The elimination of forced retirement with the introduction of the ADEA may thus prevent the formation of Lazear contracts and reduce labour market efficiency.²

Our focus on age discrimination only in the hiring process is made both for the sake of simplicity and also to be consistent with the fact that the original intention of the ADEA was primarily to reduce age discrimination in the hiring process, as reflected by the proceedings of the 1967 House Hearings on the subject, extensively discussed in Issacharoff and Harris (1997).³ The Act was largely passed in response to the perception that up until the 1950s and 1960s there existed manifest age barriers against older job applicants seeking employment, and the belief of Congress that these barriers stemmed from employers' misjudgement regarding the effects of aging on the capacity to work. Section 621(a) of the Act, which contains the Congressional Statement of Findings and Purpose, stipulates that "the setting of arbitrary age limits regardless of potential for job performance has become a common practice". In the model of this chapter, discrimination is then defined as a preference for hiring a worker of a younger age that cannot be explained by productivity differentials or other differences in the costs of hiring the worker. Given our focus on hiring discrimination, we abstract from endogenous job separation, which simplifies the analysis.

That the arbitrary age barriers of the 1950s and 1960s have largely ceased to exist is fairly clear from casual observation of modern recruitment practices.⁴ As Neumark (2009) documents, the majority of age discrimination cases filed with the Equal Employment Opportunities Commission are due to discharges and layoffs rather than hiring.

²Neumark and Watson (1999) provide the counter-argument that by reducing the likelihood of firms renegeing on their long-term commitments to older workers, it is possible for age discrimination legislation to encourage employees to enter into Lazear contracts, thereby enhancing labour market efficiency.

³See Age Discrimination in Employment: Hearings on H.R. 365L, H.R. 3768, and H.R.4221 Before the Gen. Subcomm. on Labor of the House Comm. on Educ. and Labor, 90th Cong. 7 (1967). For example, a statement by Peter J. Pestillo, Labor Counsel, U.S. Chamber of Commerce reads; "The underlying goal of the proposed legislation is a laudable one: that of opening up greater job opportunities to older people." See footnote 8 of Issacharoff and Harris (1997) for several more corroborating statements.

⁴"In 1967 half of all private job openings were barred to applicants over fifty-five, and a quarter to those over forty-five." - Issacharoff and Harris (1997) drawing on a statement made by W. Willard Wittz, Secretary of Labor, during the 1967 House Hearings.

But as Neumark argues, this should not be misconstrued as evidence that discrimination in hiring has been relatively subdued, because the scarcity of hiring cases may instead be a consequence of the structure of the legal system rather than an indicator of the severity or prevalence of discriminatory behaviour. For instance, damages in layoff cases tend to be larger because an actual job has been lost, whereas any losses incurred by a job *applicant* due to alleged discrimination can only be putative. Evidence supporting the presence of age discrimination at the hiring stage in the U.S. labour market is documented by correspondence studies which find that older workers with otherwise similar credentials are significantly less likely to receive favourable responses from potential employers (Bendick, Jackson and Romero 1996 and Lahey 2008). Furthermore, unemployment duration is still higher among the elderly than for younger workers, which is consistent with, although not necessarily indicative of, a bias against hiring older workers (Neumark 2009). Given the difficulty involved in empirically testing for the presence of discrimination, our analysis contributes to the issue by attempting to quantify the incentives for discrimination that arise in an equilibrium model of unemployment.

Our method of quantifying the employment effects of anti-discrimination legislation differs fundamentally to other recent work that uses an approach based on econometric analysis. Recent attempts to measure the effects of age discrimination laws have generally involved difference-in-differences econometric techniques to study the impact of the ADEA on labour market outcomes and have found ambiguous results. Neumark and Stock (1999) estimate employment equations using census data for the sample period 1940-1980 for white males and exploit state variation in age discrimination laws prior to the introduction of the federal law in order to identify the impact of the ADEA. Their findings indicate that the ADEA substantially increased the employment rate of workers aged 60 and over by about 6 to 7 percentage points. The effect on protected workers under the age of 60 is weaker, about 1 to 2 percentage points.⁵ Adams (2004) extended the analysis to quantify the effect of the ADEA on hiring but finds only statistically insignificant results that, if anything, predict a negative effect on hiring rates of covered workers.⁶ Lahey (2007) used a different identification strategy based on the assertion that employees in states that have their own age discrimination laws receive better protection than those in states that do not have state level laws in place and are only covered by the federal legislation. This is due to an "unusual provision in the federal law" which affords workers covered by state level legislation a longer period of time to file age discrimination claims in their own states (at what are called (state) Fair Employment Practices offices) compared to workers who must submit their claims to

⁵Originally the ADEA covered workers from the ages of 40 to 65. The 1986 amendment to the ADEA eliminated the upper age bound and with it mandatory retirement.

⁶He uses data from the Current Population Survey for the years 1964 to 1967. New hires in his study are defined as workers who are employed in the current year but were not employed in the previous year.

the Equal Employment Opportunity Commission at the federal level.⁷ In stark contrast to Adams (2004) and Neumark and Stock (1999), she finds that anti-discrimination legislation is associated with a reduction in the number of weeks worked, which she uses to proxy employment. As in Adams (2004), Lahey (2007) also finds that protection against age discrimination induces a negative effect on hiring of covered workers and a positive (but statistically insignificant) effect on the hiring rate of workers of ages not in the range of protection. The proposed explanation is that firms are more reluctant to hire workers in the protection age bracket because anti-discrimination law deters the termination of their employment. That it is more difficult to prove a hiring case than an unfair dismissal is consistent with this hypothesis.

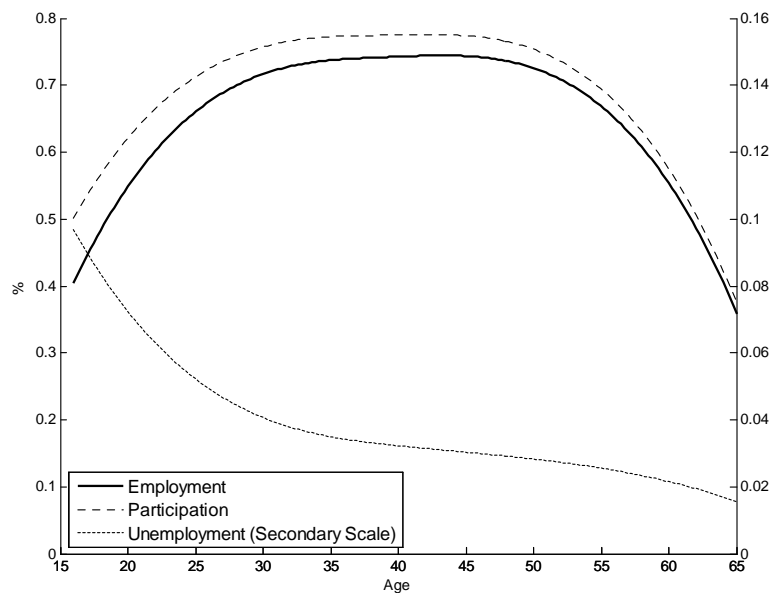
This chapter is closely related to a number of studies which have recently examined finite horizon equilibrium matching models of unemployment. Cheron et al. (2007) develop an age-specific matching model of the labour market with endogenous separation and consider an equilibrium in which employment risk is continually rising with age due to decreased incentives to retain workers close to retirement. Much of the structure of the model that is developed in the current chapter draws heavily on the model by Cheron et al. (2007) and a later study of theirs, Cheron et al. (2008), which constructs a model in which age-specific matching is not possible. There are two main differences with our analysis. The first concerns the objective of the studies. Cheron et al. (2007, 2008) do not derive the quantitative implications of age discrimination for employment dynamics, which is our primary focus in this chapter, but focus instead on the optimal life cycle paths of firing taxes and hiring subsidies. The second is a technical difference in that we introduce an explicit labour force participation decision into the models of Cheron et al. (2007, 2008). The motivation for this is provided by Figure 1, which depicts the behaviour of the U.S. labour market. The data indicate that participation and employment rates follow similar hump-shaped patterns over the life cycle. The gap between the two is inversely related to age such that unemployment monotonically decreases over the life cycle. The immediate conclusion that is drawn from Figure 1 is that a labour market participation choice is necessary in order to explain the joint life cycle dynamics of employment and unemployment since the two exhibit qualitatively different age profiles. Without an endogenously determined participation decision, unemployment would be the reflection of employment and would therefore exhibit a U-shaped life cycle profile. This would be grossly at odds with the monotonic decline in job search activity that is depicted by the data. In our context, it is potentially important to correctly capture search behaviour over the life cycle since our results depend on the interaction between a finite horizon and labour market congestion, whereby the latter depends on the measure of job searchers. Ignoring participation may then overstate the search intensity of older workers, keeping employment

⁷Lahey (2007) also suggests that "state Fair Employment Practices offices may be able to process claims more quickly", further reducing the burden of claiming compensation. As Neumark (2008) notes, however, she provides no evidence in support of this claim.

rates artificially high by reducing labour market congestion.⁸

Related studies by Bettendorf and Broer (2003), Hahn (2009) and more recently Choi et al (2011) construct finite horizon matching models with endogenous participation, but do not consider age discrimination. For the purposes of our analysis, including a participation decision allows us to assess the impact that age discrimination policy has not only on employment but also unemployment. In particular, we can find out whether unemployment among the elderly can be expected to increase upon the removal of anti-discrimination legislation due to depressed job creation for older workers. It turns out that the behaviour of unemployment is not sensitive to age discrimination policy, and a monotonically declining unemployment path over the life cycle is observed for both specifications.

Figure 1: The Life Cycle in the U.S. Labour Market



Notes: Average values for the period 1948:1-2010:4. Raw data are interpolated to in-between years using a cubic spline. Series are expressed as a ratio to the civilian non-institutional population. Data source: Bureau of Labor Statistics at www.bls.gov.

The rest of the chapter proceeds as follows. Section 2 develops the equilibrium matching models. Section 3 discusses constrained efficiency. In Section 4 the calibration procedure, solution algorithm and quantitative results are presented. Section 5 concludes.

⁸Although in Chapter II we argued that the labour force participation decision did not matter for job creation at the cyclical frequency, this does not imply that the same is true in the stationary state of an overlapping generations model.

2 Model

In this section, a life cycle model of unemployment dynamics is constructed in which labour market participation is endogenous. Two versions of the model are presented. In the first, age discrimination with respect to hiring decisions is possible. In the second, it is not possible for hiring to be age-directed. Apart from this distinction the two models are the same. The matching technology is what determines the possibility of discriminating by age. When age-directed search is allowed for, there is assumed to be an age-dependent matching technology that randomly matches job seekers of a given age cohort with the supply of age-specific vacancies available only to that cohort. In contrast, the assumption of a single matching technology that pools the search effort of all job seekers of all ages with a single measure of aggregate vacancies rules out age-based hiring decisions. We consider these two cases in turn, beginning with the first case in which age discrimination is technologically feasible. In the decentralised solution to the age-specific matching model we only consider an equilibrium in which labour market tightness is decreasing in age, consistent with the notion of discrimination against older workers as well as the observed dynamics in Figure 1. We will then subsequently demonstrate that it is not necessarily optimal for market tightness to decline monotonically over the life cycle.

2.1 Age-Specific Matching

Time is discrete. Only the working life of agents is considered and economic activity during retirement is abstracted from. There are no savings and individuals consume their income in each period. Agents are risk neutral. The age of an individual agent is denoted τ . Each agent has a working life of T periods, which is exogenously given. After the end of period T , referred to as the terminal period, the working life of the agent expires and the agent exits the labour market. The terminal period is known with certainty to all agents and firms. Each retiring cohort is replaced by a new youngest generation in each period so that only stationary demographics are considered. All age cohorts are assumed to be of the same size, which for simplicity is a unit measure with a continuum of agents. Aggregate uncertainty is abstracted from so that only stationary state overlapping-generations equilibria are considered.

The matching process is based on the Diamond-Mortensen-Pissarides model extended to incorporate age heterogeneity. Denote by v_τ the measure of vacancies available to a member of the age τ cohort. Without loss of generality, each firm only posts (at most) a single vacancy. The measure of age τ searchers is s_τ . Job seekers can only be matched with vacancies that are specific to their age. Given that we allow for passive search, or search effort by agents who are currently out of the labour force, s_τ is an effective measure of job search derived from the joint efforts of the unemployed and non-participants. An age-dependent matching technology then pairs job seekers

with available vacancies in each cohort:

$$M_\tau = m(v_\tau, s_\tau).$$

M_τ is assumed to be increasing in both arguments and homogenous of degree one. Labour market tightness is age-dependent and defined as

$$\theta_\tau = \frac{v_\tau}{s_\tau}.$$

The probability with which a vacancy is matched with a searching agent is then

$$q(\theta_\tau) = \frac{m(v_\tau, s_\tau)}{v_\tau} = m(1, \theta_\tau)$$

where the representation of q solely in terms of θ_τ follows from the homogeneity restriction on m . The probability with which a searching agent encounters a vacancy depends on the type of searcher, since unemployed agents and non-participants search at different levels of intensity. We assume the same structure as in the previous chapter. In particular, the search intensity of unemployed agents is normalised to unity. Non-participants search with a weaker intensity given by $0 \leq \chi \leq 1$. Search intensities are assumed not to vary over the life cycle. Random matching then implies that the probability that an unemployed agent encounters a match is

$$p(\theta_\tau) = \frac{m(v_\tau, s_\tau)}{s_\tau} = m(\theta_\tau^{-1}, 1) = \theta_\tau q(\theta_\tau).$$

Similarly, the probability with which a non-participant searching with intensity χ transitions to employment is $\chi p(\theta_\tau)$.

Match separation is assumed to be constant over the life cycle. At the beginning of each period, the measure of matches which will produce output during the production phase of the period is given by n_τ . Jobs terminate at the fixed rate ρ at the end of each period. Life cycle employment dynamics therefore evolve according to the law of motion

$$n_{\tau+1} = (1 - \rho)n_\tau + M_\tau \tag{1}$$

subject to a suitable initial condition $n_1 \geq 0$. A one period lag is assumed between matching and production. As detailed subsequently, our calibration strategy assumes quarterly time periods, implying that the matching-production delay in terms of calendar time is the same as in most business cycle analyses that feature matching frictions.

The labour market participation decision is the natural life cycle extension to the model of the previous chapter. The decision whether or not to participate is made at the beginning of each period of life, based on the realisation of an idiosyncratic home productivity shock denoted as $h_{i,\tau}$ for the i th individual of any particular age. Home productivity shocks are re-drawn by all agents who are not employed at the beginning of

every period of life. That is, we retain the assumption of the previous chapter that the home productivity of employed agents is completely suppressed.⁹ Idiosyncratic shocks are drawn from a general, age-invariant distribution F with positive support bounded from above by \bar{h} . There will exist an age-dependent participation constraint such that all non-employed agents for whom $h_{i,\tau}$ is less than an endogenously determined lower bound \underline{h}_τ will enter the labour market at the beginning of age τ . The measure of unemployed agents aged τ is then

$$u_\tau = (1 - n_\tau) F(\underline{h}_\tau) \quad (2)$$

and the measure of non-participants is

$$l_\tau = (1 - n_\tau) (1 - F(\underline{h}_\tau)).$$

It therefore follows that the measure of age τ searchers in efficiency units is given by

$$s_\tau = u_\tau + \chi l_\tau.$$

2.1.1 Recursive Form

We now express the labour market in recursive form so as to be able to derive equilibrium decisions for participation, job creation and the determination of wages. The recursive structure of the labour market is very similar to the stochastic equilibrium considered in the previous chapter with endogenous participation. For brevity, therefore, we do not repeat explanations that are straightforward re-interpretations of last chapter's model. We begin with the Bellman equations which characterise the asset values of jobs and agents. Consider first the Bellman equation for a non-participant of age τ ,

$$\begin{aligned} N_{i,\tau} = & h_{i,\tau} + \chi p(\theta_\tau) \beta \left[W_{\tau+1} - \left(F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right) \right] \\ & + \beta \left[F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right]. \end{aligned} \quad (3)$$

which must hold for all $\tau \in [1, T - 1]$. $N_{i,\tau}$, W_τ and U_τ are the age-dependent value functions of non-participation, employment and unemployment, respectively, and β is a discount factor. The idiosyncratic home productivity shock renders the value function $N_{i,\tau}$ individual specific. Because we abstract from idiosyncratic match productivity and endogenous job destruction, the value of employment is common across all matches of a particular cohort. The value of unemployment is also constant for all members of a specific age as unemployment compensation is not assumed to be individual-specific.

⁹Recall that the reason for this is to avoid a wage distribution with respect to home productivity. This assumption can therefore also be thought of as a bargaining rigidity.

With probability $1 - \chi p(\theta_\tau)$ the non-participant of age τ will not be employed at $\tau + 1$, in which case home productivity is re-drawn at age $\tau + 1$ and the participation decision is made.

The Bellman equations for employment and unemployment are, respectively,

$$W_\tau = w_\tau + (1 - \rho)\beta W_{\tau+1} + \rho\beta \left[F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right] \quad (4)$$

$$U_\tau = b + p(\theta_\tau)\beta \left[W_{\tau+1} - \left(F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right) \right] \quad (5)$$

$$+ \beta \left[F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right]$$

which must hold for all $\tau \in [1, T - 1]$. The wage paid to the worker in (4), w_τ , is age-dependent. Jobs are exogenously destroyed with probability ρ . Conditional on match termination at the end of age τ , the agent re-draws a value for home productivity at age $\tau + 1$ and conditional on the realisation decides whether or not to remain in the labour force. Unemployment benefits, b , do not vary according to age. The only source of age heterogeneity in the asset value of unemployment is the continuation value of search.

The Bellman equations for the firm's value of an occupied job, J_τ , and a vacancy, V_τ , are given respectively by

$$J_\tau = a - w_\tau + (1 - \rho)\beta \left[J_{\tau+1} + \rho \max_\tau V_{\tau+1} \right] \quad (6)$$

$$V_\tau = -\kappa + q(\theta_\tau)\beta J_{\tau+1} + (1 - q(\theta_\tau))\beta \max_\tau V_{\tau+1} \quad (7)$$

for $\tau \in [1, T - 1]$. Match productivity, a , is constant across all matches and does not vary with age. Should the match terminate, the firm then optimally chooses which age to post a vacancy for. Vacancy posting entails a flow cost $-\kappa$ which is age-independent. It is assumed that match productivity is high enough such that $a - w_\tau \geq 0$ for all τ in order to ensure that the value of J_τ is always non-negative.

Equilibrium job creation is determined by a free entry condition on vacancy posting that requires

$$V_\tau = 0 \quad \forall \tau \in [1, T - 1].$$

Inserting the above equilibrium condition into (7) implies

$$\frac{\kappa}{q(\theta_\tau)} = \beta J_{\tau+1} \quad \forall \tau \in [1, T - 1]. \quad (8)$$

The anticipated cost of creating a vacancy for the age τ cohort is equalised to the expected discounted value of a job after the lapse of one period. At the end of each period, all age T matches are terminated with probability one. In conjunction with the

one period lag in job creation, this implies the terminal condition

$$v_T = \theta_T = 0.$$

Market tightness in the terminal period is equal to zero because firms do not post any vacancies for the age T cohort. The terminal conditions for Bellman equations (3)-(6) are then found by setting the age T continuation values to zero, yielding;

$$\begin{aligned} N_{i,T} &= h_{i,T} \\ W_T &= w_T \\ U_T &= b \\ J_T &= a - w_T. \end{aligned}$$

Note that u_T is not necessarily equal to zero, since b may exceed the value of home production for some draws of $h_{i,T}$.

2.1.2 Wage Determination and Participation

Wages are set in each period in order to maximise the Nash product

$$w_\tau = \arg \max \left[W_\tau - \left(F(\underline{h}_\tau) U_\tau + \int_{\underline{h}_\tau}^{\bar{h}} N_{i,\tau} dF(h) \right) \right]^\eta [J_\tau - V_\tau]^{1-\eta}$$

where the positive fraction η is the bargaining power of the worker and $1 - \eta$ is the bargaining power of the firm. Bargaining strengths do not depend on age. The first-order condition for wages is

$$W_\tau - \left(F(\underline{h}_\tau) U_\tau + \int_{\underline{h}_\tau}^{\bar{h}} N_{i,\tau} dF(h) \right) = \frac{\eta}{1-\eta} J_\tau \quad (9)$$

which must hold for all ages $\tau \in [1, T]$. Replacing the value functions in the above condition with equations (3)-(6) and solving for the wage gives

$$\begin{aligned} w_\tau &= \eta a + (1 - \eta) F(\underline{h}_\tau) \left(b + \frac{\eta}{1 - \eta} \kappa \theta_\tau \right) \\ &+ (1 - \eta) (1 - F(\underline{h}_\tau)) \left(\int_{\underline{h}_\tau}^{\bar{h}} h \frac{dF(h)}{1 - F(\underline{h}_\tau)} + \chi \frac{\eta}{1 - \eta} \kappa \theta_\tau \right). \end{aligned} \quad (10)$$

The equilibrium wage has a analogous interpretation to the expression derived in the previous chapter with time subscripts replaced by age.

Labour market participation is formally defined through the age-dependent indifference condition

$$U_\tau = N_\tau(\underline{h}_\tau) \quad \forall \tau \in [1, T].$$

Given the Nash sharing rule (9), an explicit solution for the participation threshold is obtained as

$$\underline{h}_\tau = b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta_\tau \quad (11)$$

which is analogous to the participation decision of the previous chapter. The key implication of (11) in the current setting is that the life cycle dynamics of \underline{h}_τ will trace the path of θ_τ . The terminal conditions for (10) and (11) are therefore

$$\begin{aligned} w_T &= \eta a + (1 - \eta) F(b) b + \int_b^{\bar{h}} h dF(h) \\ \underline{h}_T &= b. \end{aligned}$$

2.1.3 Equilibrium Life Cycle Dynamics

A life cycle equilibrium with age-specific matching is defined as a path for market tightness $\{\theta_\tau\}_{\tau=1}^{\tau=T}$ that jointly satisfies the job creation condition (8), the Nash wage solution (10) and the participation constraint (11). How will the life cycle path of market tightness behave? Combining the job creation condition (8) with the Bellman equation for an occupied job (6) yields a first-order difference equation in tightness

$$\frac{\kappa}{q(\theta_\tau)} = \beta J_{\tau+1} = \beta \left[a - w_{\tau+1} + (1 - \rho) \frac{\kappa}{q(\theta_{\tau+1})} \right]. \quad (12)$$

The following proposition establishes the property of *monotonicity* in the age-specific matching model with endogenous participation.

Proposition 1 *Assuming that $\frac{\partial J_\tau}{\partial \theta_\tau} = (1 - \rho) \frac{\partial}{\partial \theta_\tau} \frac{\kappa}{q(\theta_\tau)} - \frac{\partial w_\tau}{\partial \theta_\tau} \geq 0$ for all τ , an endogenous participation equilibrium exists in which $\theta_{\tau+1} \leq \theta_\tau$ and $\underline{h}_{\tau+1} \leq \underline{h}_\tau \forall \tau \in [1, T]$.*

Proof. The terminal value of a job is

$$J_T = (1 - \eta) \left[a + F(\underline{h}_T) b + \int_{\underline{h}_T}^{\bar{h}} h dF(h) \right]$$

with $\underline{h}_T = b$. From (10) and (6) a sufficient condition for $J_{T-1} \geq J_T$ is that

$$F(\underline{h}_{T-1}) \underline{h}_{T-1} + \int_{\underline{h}_{T-1}}^{\bar{h}} h dF(h) \geq F(\underline{h}_T) \underline{h}_T + \int_{\underline{h}_T}^{\bar{h}} h dF(h).$$

with $\underline{h}_{T-1} = b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta_T \geq \underline{h}_T = b$. For any age τ , note that

$$\frac{\partial}{\partial \underline{h}_\tau} \left(F(\underline{h}_\tau) \underline{h}_\tau + \int_{\underline{h}_\tau}^{\bar{h}} h dF(h) \right) = F(\underline{h}_\tau) \geq 0.$$

We therefore have $J_{T-1} \geq J_T$ and from (8) given that $q'(\theta_\tau) \leq 0$ for all $\theta_\tau \geq 0$ it follows that $\theta_{T-2} \geq \theta_{T-1}$. In general, the equilibrium path for market tightness will

depend on sign of the derivative $\partial J_\tau / \partial \theta_\tau$. Assuming that $\partial J_\tau / \partial \theta_\tau$ is positive for all ages, the same logic can be iteratively applied to obtain $\theta_\tau \geq \theta_{\tau+1}$ for all τ . Market tightness then monotonically declines over the life cycle, reaching a terminal value of zero at age T . It follows straightforwardly from (11) that the participation constraint then declines over the life cycle. On the other hand, if $\partial J_{T-1} / \partial \theta_{T-1} \leq 0$, it follows that for $\theta_{T-1} \geq \theta_T = 0$ it is possible for $J_{T-1} < J_T$ to be true. Applying the same logic as before, it follows that $\theta_{T-2} \leq \theta_{T-1}$ and the equilibrium is no longer monotonic but oscillatory. ■

In general, following Cheron et al. (2007), we restrict attention to equilibrium paths $\{\theta_\tau\}_{\tau=1}^T$ in which market tightness is monotonically declining in age. It is then immediate from the participation constraint (11) that there will be an increased tendency to exit the labour force as the terminal period approaches. As will be demonstrated quantitatively subsequently, this allows the model to generate realistic employment and unemployment dynamics over the life cycle as in Figure 1. Such a restriction also serves to make the theoretical analysis far simpler and, as our quantitative exercises demonstrate, is consistent with the parameterisations that are considered.

Let us obtain some intuition for the mechanisms at work behind a declining path for tightness. Market tightness influences the asset value J_τ in two ways; through the continuation value of the match and through the Nash bargained wage. As the terminal period approaches, beyond which the asset value of a job is zero, the continuation value of the match declines. All else equal, this leads firms to create fewer vacancies for older cohorts. This is termed the "horizon effect" (Cheron et al. 2007) and it has a negative influence on the value of the match to the firm. However, as market tightness falls with age, we know from the proof of Proposition 1 that the Nash bargained wage also falls since it depends positively on tightness, thereby offsetting the horizon effect. The assumption in Proposition 1 guarantees that the horizon effect dominates the wage effect. This raises the following potentially important issue. One may argue that our claim in the Introduction that the incentives for age discrimination are weak is a misinterpretation of the positive effect of declining wages on employment. In section 4.2.1 below, we return to this issue and illustrate that reducing the dependence of the wage on market tightness does not alter our main conclusions. Furthermore, as Corollary 1 demonstrates, endogenous participation weakens the dependence of the wage on tightness.

The intuition for why a monotonically declining path for market tightness is consistent with the hump-shaped employment pattern shown in Figure 1 is straightforward and can be anticipated in advance of the quantitative simulations reported subsequently. In a monotonic equilibrium, market tightness is at its strongest for young cohorts. The incentive to participate is also strong, but despite this employment is initially low due to time consuming matching. Employment then gradually increases with age due to a "queueing effect" as the hiring rate exceeds the (constant) separation rate. Eventually, after prime age, market tightness weakens and there is less of an incentive to remain ac-

tive in the labour force. Participation also begins to weaken as market tightness falls. Hiring now becomes weaker than the combined effects of job separation and labour force exit, causing employment and unemployment to decline towards the end of life.

Corollary 1 *Endogenous participation expands the parameter space consistent with the monotonicity property in the age-specific matching model.*

Proof. In the constant participation model for which $F(\underline{h}_\tau) = 1$ for all τ , it is straightforward to see that $\partial w_\tau / \partial \theta_\tau = \eta \kappa > 0$ if $\eta, \kappa > 0$. The latter derivative is subject to change in an endogenous participation equilibrium. Applying Leibniz's rule for differentiation under an integral with variable limits, we have

$$\begin{aligned} \frac{\partial w_\tau}{\partial \theta_\tau} &= (1 - \eta) \frac{\partial}{\partial \theta_\tau} F \left(b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta_\tau \right) \left(b + \frac{\eta}{1 - \eta} \kappa \theta_\tau \right) \\ &\quad + (1 - \eta) \frac{\partial}{\partial \theta_\tau} \int_{b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta_\tau}^{\bar{h}} \frac{\eta}{1 - \eta} \kappa \theta_\tau h dF(h) + \frac{\partial}{\partial \theta_\tau} (1 - F(\underline{h}_\tau)) \chi \eta \kappa \theta_\tau \\ &= \eta \kappa (F(\underline{h}_\tau) + (1 - F(\underline{h}_\tau)) \chi) \end{aligned}$$

which is also positive under plausible parameter assumptions. Moreover, note that

$$F(\underline{h}_\tau) + (1 - F(\underline{h}_\tau)) \chi \leq 1$$

if $0 \leq \chi \leq 1$. This result in turn implies that the assumption in Proposition 1 is a weaker restriction in the endogenous participation model, since a positive $\partial J_\tau / \partial \theta_\tau$ will obtain for a larger parameter space given the smaller value for $\partial w_\tau / \partial \theta_\tau$ relative to the case with constant participation. ■

Intuitively, wages are less dependent on market tightness in the endogenous participation model because the outside option of the worker depends not only on the value of unemployment, which depends positively on market tightness through $p(\theta_\tau)$, but also on the value of non-participation which depends only weakly on current market tightness if $\chi \neq 0$ and not at all if $\chi = 0$. As a result, we have demonstrated that if a constant participation equilibrium exists in which the age profile of market tightness is declining, then for the same parameter set a monotonic equilibrium exists under endogenous participation. In other words, the introduction of endogenous participation does not require more stringent restrictions on the parameter space, but, on the contrary, reduces the likelihood of non-monotonic equilibria arising for a given set of parameters. It is also noted, as in Cheron et al. (2007), that the issue of monotonicity does not arise if wages are independent of market tightness as, for instance, in a wage posting equilibrium with $\eta = 0$. In this case, $\partial w_\tau / \partial \theta_\tau$ is zero regardless of the participation choice and the horizon effect ensures that asset values J_τ decline with age.

2.2 Random-Age Matching

Consider now an alternative model that specifies a single matching function common to all searching agents in the labour force, such that it is no longer technologically possible for firms to age-direct their recruitment efforts. For simplicity, we assume workers of all ages are covered by the anti-discrimination legislation.¹⁰ The aggregate measure of vacancies, v , can potentially be matched with an individual of any age from the searcher pool, s . The aggregate pool of searchers is found by summing across all cohorts

$$s = \sum_{\tau} s_{\tau} = \sum_{\tau} (u_{\tau} + \chi l_{\tau}).$$

There is a single value for labour market tightness defined as

$$\theta = \frac{v}{s}.$$

The matching function transfers agents from the unemployment pool to employment at the rate $p(\theta) = \theta q(\theta)$, and from the pool of non-participants at the rate $\chi p(\theta)$. In the previous model, the type of equilibrium considered featured a drop-off in tightness towards the end of life, causing employment to decline. This mechanism is absent from the current model, in which agents of all ages face the same probability of finding a job conditional on search effort. Accordingly, any relative drop-off in employment for older cohorts must stem from the weaker search efforts of the non-employed. An aggregate decrease in the effective measure of searchers is attained through a weakened tendency to participate in the labour market as the terminal period approaches.

The only changes to the recursive structure of the labour market in Bellman equations (3)-(6) is that the transition rates $p(\theta)$ and $q(\theta)$ are no longer age-dependent. We therefore do not reproduce these equations for the sake of brevity. The Bellman equation for a vacancy now becomes

$$V = -\kappa + q(\theta) \beta \sum_{\tau} \frac{s_{\tau}}{s} J_{\tau+1} + (1 - q(\theta)) \beta V$$

reflecting the fact that a firm posting a vacancy does not know *ex ante* the age of the worker the vacancy will eventually be matched with. With probability $\frac{s_{\tau}}{s}$, the vacancy is matched with a worker of age τ , thus yielding an asset value to the firm of $J_{\tau+1}$ when the worker becomes productive after the lapse of one period. The terminal condition $J_{T+1} = 0$ applies. Free entry implies that

$$\frac{\kappa}{q(\theta)} = \beta \sum_{\tau} \frac{s_{\tau}}{s} J_{\tau+1} \tag{13}$$

requiring that the anticipated cost of creating a vacancy is equated to the expected

¹⁰This assumption turns out to be innocuous since dynamics for younger workers turn out to be quantitatively very similar in the two specifications.

asset value of a job weighted according to the age distribution of the pool of searchers.

Wages are determined through the same process as before. Replacing the asset values in the first-order condition (9) and applying the above equilibrium condition for vacancy supply gives a wage equation of the form

$$w_\tau = \eta a + (1 - \eta) F(\underline{h}_\tau) \left(b + \frac{\eta}{1 - \eta} \kappa \theta \Gamma_{\tau+1} \right) + (1 - \eta) (1 - F(\underline{h}_\tau)) \left(\int_{\underline{h}_\tau}^{\bar{h}} h \frac{dF(h)}{1 - F(\underline{h}_\tau)} + \chi \frac{\eta}{1 - \eta} \kappa \theta \Gamma_{\tau+1} \right) \quad (14)$$

where

$$\Gamma_{\tau+1} = \frac{J_{\tau+1}}{\sum_{\tau} \frac{s_\tau}{s} J_{\tau+1}}$$

is the value of an age $\tau + 1$ job relative to the expected value of a job weighted by the age distribution of the searcher pool.

Conditional on Nash bargaining, the participation decision is derived in a manner analogous to the previous model to obtain

$$\underline{h}_\tau = b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta \Gamma_{\tau+1}. \quad (15)$$

The participation decision at age τ is positively related to the relative value of $J_{\tau+1}$. The age profile of labour market participation therefore depends on the behaviour of Γ_τ . Given that all jobs are destroyed in period $T + 1$ with certainty, we have the terminal condition

$$\Gamma_{T+1} = 0.$$

This implies that w_T and \underline{h}_T are the same as in the model with age discrimination. We now make the following proposition.

Proposition 2 *Assuming that $(1 - \rho) \frac{\kappa}{q(\theta)} - \frac{\partial w_\tau}{\partial \Gamma_{\tau+1}} \geq 0$ for all τ , an endogenous participation equilibrium exists in which $\Gamma_{\tau+1} \leq \Gamma_\tau$ and $\underline{h}_{\tau+1} \leq \underline{h}_\tau \forall \tau \in [1, T]$.*

Proof. The proof follows that for Proposition 1 in the age-specific labour market. The terminal value of a job is

$$J_T = (1 - \eta) a + (1 - \eta) F(\underline{h}_T) b + (1 - \eta) \int_{\underline{h}_T}^{\bar{h}} h dF(h)$$

with $\underline{h}_T = b$. From (14) and (6) a sufficient condition for $J_{T-1} \geq J_T$ is that

$$F(\underline{h}_{T-1}) \underline{h}_{T-1} + \int_{\underline{h}_{T-1}}^{\bar{h}} h dF(h) \geq F(\underline{h}_T) \underline{h}_T + \int_{\underline{h}_T}^{\bar{h}} h dF(h).$$

Given that $\Gamma_{T+1} = 0$, we have $\underline{h}_{T-1} = b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta \Gamma_T \geq \underline{h}_T = b$. For any

age τ , note that

$$\frac{\partial}{\partial \underline{h}_\tau} \left(F(\underline{h}_\tau) \underline{h}_\tau + \int_{\underline{h}_\tau}^{\bar{h}} h dF(h) \right) = F(\underline{h}_\tau) \geq 0.$$

We therefore have $J_{T-1} \geq J_T$ and $\Gamma_{T-1} \geq \Gamma_T$. Recognising that $\frac{\kappa}{q(\theta)} \Gamma_{\tau+1} = \beta J_{\tau+1}$, from (6) it therefore follows that $J_{T-2} \geq J_{T-1}$ if

$$(1 - \rho) \frac{\kappa}{q(\theta)} - \frac{\partial w_{T-2}}{\partial \Gamma_{T-1}} \geq 0.$$

For arbitrary τ , the derivative of w_τ with respect to $\Gamma_{\tau+1}$ is given by

$$\frac{\partial w_\tau}{\partial \Gamma_{\tau+1}} = F(\underline{h}_\tau) \eta \kappa \theta + (1 - F(\underline{h}_\tau)) \chi \eta \kappa \theta$$

It therefore follows that if $(1 - \rho) \frac{\kappa}{q(\theta)} - (F(\underline{h}_{T-2}) \eta \kappa \theta + (1 - F(\underline{h}_{T-2})) \chi \eta \kappa \theta) \geq 0$ then $J_{T-2} \geq J_{T-1}$. Repeating this logic recursively backwards implies that $J_\tau \geq J_{\tau+1}$ if $(1 - \rho) \frac{\kappa}{q(\theta)} - \frac{\partial w_\tau}{\partial \Gamma_{\tau+1}} \geq 0$ for all τ . As a consequence it then holds that $\Gamma_{\tau+1} \leq \Gamma_\tau$ and $\underline{h}_{\tau+1} \leq \underline{h}_\tau \forall \tau \in [1, T]$. ■

Corollary 2 *Endogenous participation expands the parameter space consistent with the monotonicity property in the random age matching model.*

Proof. Holding participation constant, differentiation of the wage equation yields

$$\frac{\partial w_\tau}{\partial \Gamma_{\tau+1}} = \eta \kappa \theta \geq F(\underline{h}_\tau) \eta \kappa \theta + (1 - F(\underline{h}_\tau)) \chi \eta \kappa \theta$$

for $0 \leq F(\underline{h}_\tau), \chi \leq 1$. It therefore follows that the parameter restriction implied by the assumption $(1 - \rho) \frac{\kappa}{q(\theta)} - \frac{\partial w_\tau}{\partial \Gamma_{\tau+1}} \geq 0$ is less likely to bind when participation is endogenous. ■

The intuition for Corollary 2 is the same as for Corollary 1. In particular, the wage is less sensitive to labour market conditions when the option to participate is available. This weakens the effect that Γ_τ has on wages over the life cycle, implying that the dynamics of the asset value J_τ are determined largely through the horizon effect. As a result, even though the negotiated wage falls as the terminal period draws closer, the horizon effect dominates such that the asset value $J_\tau \rightarrow 0$ as $\tau \rightarrow T$.

3 Constrained Efficiency

The models constructed in the previous section exhibit congestion externalities caused by matching frictions, manifested by the dependence of the transition probabilities on labour market tightness. Market tightness, in turn, depends on the wage which is determined by a bargaining condition. As noted in Pissarides (2000), the conditions under which wages are determined are unlikely to internalise the congestion externalities

associated with labour market search. The reason is that wages are determined after a firm and a worker meet, but search externalities affect agents who are still searching. Agents engaged in the negotiation process do not take into account the effect of their actions on market tightness and, therefore, agents currently involved in search.

The present section therefore investigates the conditions under which the decentralised equilibria developed above maximise social output. In particular, we examine the implications of the structure of the labour market for social efficiency, illustrating the consequences that age-specific matching functions have on the efficiency of the participation decision. The formal problem is set up as follows. A planner chooses an allocation for vacancy supply, employment and the participation constraint that maximises total economic output net of vacancy costs. In an overlapping generations steady state equilibrium the discount factor is set to unity. The corresponding discount factor in the decentralised equilibrium is therefore also set to unity to facilitate comparison. The social planner is not concerned with distributional issues, allowing the wage determination process to be bypassed since wage payments only determine the manner in which the match surplus is divided between the employee and the employer.

The two physical environments of age-specific and random matching are analysed in turn. Consider age-specific matching first. The social optimisation programme is

$$\max_{v_\tau, n_{\tau+1}^*, \underline{h}_\tau^*} \sum_{\tau=1}^T \left[n_\tau a + (1 - n_\tau) F(\underline{h}_\tau) b + (1 - n_\tau) \int_{\underline{h}_\tau}^{\bar{h}} h dF(h) - \kappa v_\tau \right]$$

subject to

$$n_{\tau+1} = (1 - \rho) n_\tau + q \left(\frac{v_\tau}{(1 - n_\tau) F(\underline{h}_\tau) + (1 - n_\tau) (1 - F(\underline{h}_\tau)) \chi} \right) v_\tau$$

and an initial condition $n_1 \geq 0$. The same terminal conditions as in the decentralised economy also apply. An asterisk denotes the social optimum. The first-order conditions for the social optimum are

$$\begin{aligned} \partial v_\tau^* &: 0 = -\kappa + \lambda_\tau [q'(\theta_\tau^*) \theta_\tau^* + q(\theta_\tau^*)] \\ \partial n_{\tau+1}^* &: 0 = A - F(\underline{h}_{\tau+1}^*) b - \int_{\underline{h}_{\tau+1}^*}^{\bar{h}} h dF(h) \\ &\quad + \lambda_{\tau+1} \left[(1 - \rho) + q'(\theta_{\tau+1}^*) (\theta_{\tau+1}^*)^2 (F(\underline{h}_{\tau+1}^*) + (1 - F(\underline{h}_{\tau+1}^*))) \chi \right] - \lambda_\tau \\ \partial \underline{h}_\tau^* &: 0 = b - \underline{h}_\tau^* - \lambda_\tau q'(\theta_\tau^*) (\theta_\tau^*)^2 (1 - \chi) \end{aligned}$$

where λ_τ is the multiplier on the age τ employment constraint. Define $\eta(\theta_\tau^*) = -q'(\theta_\tau^*) (\theta_\tau^*) / q(\theta_\tau^*)$, the negative of the elasticity of $q(\theta_\tau^*)$ with respect to market tightness. Consider first the efficiency of the participation decision. Eliminating the multiplier from the first-order condition for \underline{h}_τ^* and noting that $q'(\theta_\tau) \theta_\tau + q(\theta_\tau) = q(\theta_\tau) (1 - \eta(\theta_\tau))$, we obtain

$$\underline{h}_\tau^* = b + (1 - \chi) \frac{\eta(\theta_\tau^*)}{1 - \eta(\theta_\tau^*)} \kappa \theta_\tau^*.$$

Comparing the above equation to the corresponding decentralised version (11), the requirement for decentralised labour market participation to be socially efficient is that $\eta = \eta(\theta_\tau^*)$. If the matching technology assumes the log-linear specification $M_\tau = v_\tau^\gamma s_\tau^{1-\gamma}$, then $\eta(\theta_\tau^*) = \gamma$. Given that both η and γ are positive fractions, the efficient participation decision is implementable. For a more general matching function it may turn out that the elasticity $\eta(\theta_\tau^*)$ is age-dependent, such that the bargaining power of the worker would then have to vary according to age in order for the socially efficient participation rate to be attained. Assuming bargaining strengths that vary with age, the Nash sharing rule becomes

$$W_\tau - \left\{ F(\underline{h}_\tau) U_\tau + \int_{\underline{h}_\tau}^{\bar{h}} N_{i,\tau} dF(h) \right\} = \frac{\eta_\tau}{1 - \eta_\tau} J_\tau$$

It is straightforward from the indifference condition $U_\tau = N_\tau(\underline{h}_\tau)$ that the participation threshold is now determined by

$$\underline{h}_\tau = b + (1 - \chi) \frac{\eta_{\tau+1}}{1 - \eta_{\tau+1}} \kappa \theta_\tau.$$

Given the one period delay between matching and wage payments, age τ agents base participation decisions on their relative bargaining strengths at $\tau + 1$. It follows that setting $\eta_{\tau+1} = \eta(\theta_\tau^*)$ is sufficient to ensure efficiency of the decentralised equilibrium. However, note that even then the efficient solution is only actually implementable under the Hosios condition if $0 \leq \eta(\theta_\tau^*) \leq 1$ for all τ .

In the decentralised economy, wages are determined at the level of the firm so that unemployed agents are excluded from the process of negotiation. This renders equilibrium inefficient from a social perspective because of a divergence in the interests of the employed and unemployed that arises due to externalities in the trading process. Employed workers want higher wages, but this discourages the supply of new vacancies which affects the job finding probabilities of the unemployed. The result that $\eta(\theta_\tau^*) = \gamma$ is required for efficiency is standard in the literature and referred to as the Hosios condition in recognition of Hosios (1990). The intuition is that when $\eta(\theta_\tau^*)$ is high, the negative impact of the presence of an additional vacancy on the transition rate q for all firms is large. The social planner would then grant workers a high bargaining power in order to offset the incentive to supply excessive (from the social point of view) vacancies. That the Hosios condition applies not just to vacancy creation (as will be demonstrated below) but also to the participation decision is a consequence of the decision to participate being determined conditional on Nash bargaining. Just as the Nash solution for the wage influences vacancy supply, it also influences the participation choice by determining the returns from job search. Therefore the same efficiency con-

dition governs the optimal solution for both. Put differently, the participation choice as we have modelled it introduces no additional labour market externalities.

Optimal vacancy creation is obtained by eliminating the multiplier in the first-order condition for $n_{\tau+1}$, resulting in

$$\begin{aligned} \frac{\kappa}{q(\theta_\tau^*)(1-\eta(\theta_\tau^*))} &= a - F(\underline{h}_{\tau+1}^*)b \\ &\quad - \frac{\eta(\theta_{\tau+1}^*)}{1-\eta(\theta_{\tau+1}^*)}\kappa\theta_{\tau+1}^*(F(\underline{h}_{\tau+1}^*) + (1-F(\underline{h}_{\tau+1}^*))\chi) \\ &\quad - \int_{\underline{h}_{\tau+1}^*}^{\bar{h}} h dF(h) + \frac{1}{1-\eta(\theta_{\tau+1}^*)}(1-\rho)\frac{\kappa}{q(\theta_{\tau+1}^*)} \end{aligned} \quad (16)$$

The decentralised Nash wage under age-dependent bargaining strengths is given by

$$\begin{aligned} w_\tau &= \eta_\tau a + (1-\eta_\tau)F(\underline{h}_\tau)\left(b + \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}}\kappa\theta_\tau\right) \\ &\quad + (1-\eta_\tau)(1-F(\underline{h}_\tau))\left(\int_{\underline{h}_\tau}^{\bar{h}} h \frac{dF(h)}{1-F(\underline{h}_\tau)} + \chi \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}}\kappa\theta_\tau\right) \\ &\quad - (1-\eta_\tau)\left[\frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} - \frac{\eta_\tau}{1-\eta_\tau}\right](1-\rho)\frac{\kappa}{q(\theta_\tau)}. \end{aligned} \quad (17)$$

Because the above expression is somewhat non-standard in the literature, we provide the workings in an appendix at the end of this chapter. The term which enters negatively in the third line of the above expression disappears if bargaining strengths are constant. When the ratio of the worker's bargaining power to the firm's is low relative to its future value, the current wage rate will, holding all else constant, be lower than in the future. Inserting (17) into (12) and rearranging gives

$$\begin{aligned} \frac{\kappa}{q(\theta_{\tau-1})} &= (1-\eta_\tau)\left[a - F(\underline{h}_\tau)\left(b + \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}}\kappa\theta_\tau\right)\right] \\ &\quad - (1-\eta_\tau)(1-F(\underline{h}_\tau))\left(\int_{\underline{h}_\tau}^{\bar{h}} h \frac{dF(h)}{1-F(\underline{h}_\tau)} + \chi \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}}\kappa\theta_\tau\right) \\ &\quad + \frac{1-\eta_\tau}{1-\eta_{\tau+1}}(1-\rho)\frac{\kappa}{q(\theta_\tau)}. \end{aligned}$$

Decentralised equilibrium vacancy supply can be made to coincide with the social optimum under the condition that $\eta_{\tau+1} = \eta(\theta_\tau^*)$. The efficient solution is therefore attainable as long as $\eta(\theta_\tau^*)$ is a positive fraction. It is clear that the need for bargaining weights to vary with age in order to maintain efficiency only arises in this model because market tightness is itself age-dependent.

Next, consider the alternative physical environment in which age-specific matching is not technologically feasible. For the central planner in this economy, the optimisation

programme is

$$\max_{\theta^*, n_{\tau+1}^*, \underline{h}_\tau^*} \sum_{\tau=1}^T \left[a n_\tau + (1 - n_\tau) F(\underline{h}_\tau) b + (1 - n_\tau) \int_{\underline{h}_\tau}^{\bar{h}} h dF(h) - \frac{\kappa \theta s}{T} \right].$$

The difference in the planner's set of control variables is that a sequence for vacancies is no longer specified, but a single value.¹¹ Note also that the costs of vacancy creation are divided equally among the cohorts of searchers which can be matched. Optimisation is subject to the constraints

$$n_{\tau+1} = (1 - \rho) n_\tau + p \left(\frac{v}{\sum_{\tau=1}^T ((1 - n_\tau) F(\underline{h}_\tau) + (1 - n_\tau) (1 - F(\underline{h}_\tau)) \chi)} \right) s_\tau$$

and suitable initial and terminal conditions for employment and market tightness. The first-order conditions are

$$\begin{aligned} \partial \theta^* & : 0 = -\kappa s + \sum_{\tau=1}^{T-1} \lambda_\tau p'(\theta^*) s_\tau^* \\ \partial n_{\tau+1}^* & : 0 = a - F(\underline{h}_{\tau+1}^*) b - \int_{\underline{h}_{\tau+1}^*}^{\bar{h}} h dF(h) + (F(\underline{h}_{\tau+1}^*) + (1 - F(\underline{h}_\tau^*)) \chi) \kappa \theta^* \\ & \quad + \lambda_{\tau+1} ((1 - \rho) - p(\theta^*) (F(\underline{h}_{\tau+1}^*) + (1 - F(\underline{h}_\tau^*)) \chi)) - \lambda_\tau \\ \partial \underline{h}_\tau^* & : 0 = b - \underline{h}_\tau^* - (1 - \chi) \kappa \theta^* + \lambda_\tau p(\theta^*) (1 - \chi) \end{aligned}$$

where λ_τ is the multiplier on the employment constraint. Consider first the efficiency of the participation decision. Solving the first-order condition for tightness for the multiplier yields

$$\frac{\kappa}{p'(\theta^*)} \frac{\lambda_\tau}{\sum_{\tau=1}^{T-1} \lambda_\tau \frac{s_\tau^*}{s^*}} = \lambda_\tau.$$

We can therefore express the first-order condition for the participation threshold as

$$\underline{h}_\tau^* = b - (1 - \chi) \kappa \theta^* + (1 - \chi) \frac{\kappa}{p'(\theta^*)} \Gamma_\tau^* p(\theta^*) \quad (18)$$

where

$$\Gamma_\tau^* = \frac{\lambda_\tau}{\sum_{\tau=1}^T \frac{s_\tau^*}{s^*} \lambda_\tau}.$$

From the relation between $p(\theta)$ and $q(\theta)$, it holds that $p'(\theta)/p(\theta) = (1 - \eta(\theta)) \theta$. Using this definition in (18),

$$\underline{h}_\tau^* = b + (1 - \chi) \left[\frac{\eta(\theta^*)}{1 - \eta(\theta^*)} \kappa \theta^* \Gamma_\tau^* + \kappa \theta^* (\Gamma_\tau^* - 1) \right]. \quad (19)$$

Expression (19) gives the socially optimal participation constraint. To gain some

¹¹Market tightness rather than vacancies is specified as the control variable of the planner purely for ease of exposition.

understanding for the variable Γ_τ^* , note that the interpretation of the multiplier λ_τ is the net social value of having an additional agent in employment at age $\tau + 1$. The decentralised equilibrium equivalent to λ_τ is the joint surplus of a match with an age $\tau + 1$ worker. Recall that the firm's share of the surplus in the decentralised equilibrium under Nash bargaining is $1 - \eta$, such that $J_\tau = (1 - \eta) S_\tau$, where S_τ is the joint surplus of a match with an age τ agent. It follows that the decentralised participation constraint can therefore be expressed in terms of joint match surpluses as

$$\underline{h}_\tau = b + (1 - \chi) \frac{\eta}{1 - \eta} \kappa \theta \frac{(1 - \eta) S_{\tau+1}}{(1 - \eta) \sum_{\tau} \frac{s_\tau}{s} S_{\tau+1}}.$$

Comparison of the above expression with the social optimum (19) reveals that even if the Hosios condition $\eta = \eta(\theta^*)$ were to hold, participation would still not be efficient in the decentralised equilibrium. Participation rates would nevertheless differ by the term $(1 - \chi) \kappa \theta^* (\Gamma_\tau^* - 1)$, which is independent of the relative bargaining strengths of workers and firms. This result is due to an intergenerational externality that arises because of the absence of age specific matching functions. The size of the externality is multiplied by the factor $\kappa v^*/s^*$, which gives the average cost of aggregate vacancy supply per searcher.

Not all searchers are worth the same in terms of their potential contribution to output because of differences in their remaining time horizons, but in the decentralised equilibrium searchers do not recognise the impact of their participation decision on the age composition of the aggregate searching pool s . The sign of the externality is age-dependent. Younger workers for whom $\Gamma_\tau^* > 1$ set \underline{h}_τ too low relative to the social optimum. Their entry into the labour force would raise the average return on vacancies, an effect not internalised by decentralised behaviour that treats aggregate quantities parametrically. In contrast, the participation rate of older workers for whom $\Gamma_\tau^* < 1$ tends to be excessively high relative to the social optimum because searchers who are close to the terminal period depress the expected return on vacancies. As a result, the age distribution of searchers is distorted in the decentralised economy, even under the Hosios condition. This result may therefore be interpreted as justification for allowing age discrimination in the hiring process.

What is the effect of inefficient participation on the job creation decision? Observing that $p'(\theta) = (1 - \eta(\theta)) q(\theta)$, the planner's first-order condition for market tightness can be expressed as

$$\frac{\kappa}{q(\theta^*)} = (1 - \eta(\theta^*)) \sum_{\tau=1}^T \frac{s_\tau}{s^*} \lambda_\tau.$$

Decentralised market tightness must satisfy

$$\frac{\kappa}{q(\theta)} = (1 - \eta) \sum_{\tau=1}^T \frac{s_\tau}{s} S_{\tau+1}$$

from (13). Assuming the Hosios condition holds, in order for market tightness to be efficient in the decentralised equilibrium it must be the case that

$$\sum_{\tau=1}^T \frac{s_{\tau}^*}{s^*} \lambda_{\tau} = \sum_{\tau=1}^T \frac{s_{\tau}}{s} S_{\tau+1}.$$

Even if $\eta = \eta(\theta^*)$, inefficient participation decisions create two separate distortions in the equilibrium supply of vacancies. First, inefficient participation decisions distort vacancy supply by altering the age distribution of the searcher pool, captured in the above expression by the fractions $\frac{s_{\tau}^*}{s^*}$ and $\frac{s_{\tau}}{s}$ which determine the average age of the worker in a newly matched job. As noted previously, the age distribution of the pool of searchers in the decentralised equilibrium will tend to be skewed towards an inefficiently high average. Typically then, vacancies turn out to be matched with a worker whose remaining time horizon in the labour force is sub-optimally short. Given monotonically declining S_{τ} with age, this effect on the age composition of searchers tends to cause market tightness in the decentralised equilibrium to be lower than the social optimum. This tends to cause equilibrium market tightness to be too low in the decentralised equilibrium relative to the social optimum.

The second distortion on the supply of vacancies occurs through the influence of the participation decision on the asset value $S_{\tau+1}$ relative to the optimum λ_{τ} . This is because the joint surplus of the match depends on the wage, which in turn depends on the participation constraint. Rearranging the planner's first-order condition for employment gives

$$\begin{aligned} \lambda_{\tau} = & a - F(\underline{h}_{\tau+1}^*) b - \int_{\underline{h}_{\tau+1}^*}^{\bar{h}} h dF(h) \\ & - (F(\underline{h}_{\tau+1}^*) + (1 - F(\underline{h}_{\tau+1}^*)) \chi) \frac{\eta(\theta^*)}{1 - \eta(\theta^*)} \kappa \theta^* \frac{\lambda_{\tau+1}}{\sum_{\tau=1}^T \frac{s_{\tau}^*}{s^*} \lambda_{\tau+1}} + (1 - \rho) \lambda_{\tau+1}. \end{aligned} \quad (20)$$

The decentralised counterpart to the above expression can be obtained by expressing the Bellman equation for a job in terms of the joint surplus as

$$(1 - \eta) S_{\tau} = a - w_{\tau} + (1 - \rho) (1 - \eta) S_{\tau+1}$$

and substituting for the wage using (14) to obtain

$$\begin{aligned} S_{\tau} = & a - F(\underline{h}_{\tau}) b - \int_{\underline{h}_{\tau}}^{\bar{h}} h dF(h) \\ & - (F(\underline{h}_{\tau}) + (1 - F(\underline{h}_{\tau})) \chi) \frac{\eta}{1 - \eta} \kappa \theta \frac{S_{\tau+1}}{\sum_{\tau=1}^T \frac{s_{\tau}^*}{s^*} S_{\tau}} + (1 - \rho) S_{\tau+1}. \end{aligned} \quad (21)$$

Before proceeding, note that if $\underline{h}_{\tau} = \underline{h}_{\tau}^*$ for all τ , then from (20) and (21) $S_{\tau+1} = \lambda_{\tau}$ and the decentralised economy would produce an efficient outcome for market tight-

ness. Therefore, assuming the Hosios condition holds, any departures from the socially efficient level of market tightness occur only indirectly through the sub-optimality of the participation decision. That is to say, because the vacancy supply decision is not age-dependent, there is no intergenerational externality that distorts the job creation decision.

To gain some intuition for how departures from the socially efficient path \underline{h}_τ^* influence market tightness via the Nash bargained wage, consider a simplified situation in which $\chi = 0$ for analytical tractability. Differentiation then yields

$$\frac{\partial S_\tau}{\partial \underline{h}_\tau} = -F(\underline{h}_\tau) \leq 0.$$

Because the wage is positively related to \underline{h}_τ , the value of the match to the firm will tend to be higher than is socially optimal for relatively young workers for whom $\underline{h}_\tau \leq \underline{h}_\tau^*$. Conversely, for relatively old workers for whom it holds that $\underline{h}_\tau \geq \underline{h}_\tau^*$, the value of a match to the firm is lower than is socially optimal. Recall our previous argument that on average a decentralised vacancy tends to be matched with an agent with too short a remaining time horizon in the labour force. Even if this were the only effect of inefficient participation, vacancy supply would be too low. But it is not the only effect if wages are Nash bargained. In the latter case, inefficiently high participation rates amongst old workers - which make it more likely for a vacancy to be matched with a relatively old worker - further reduce the value of employing older workers because of inefficiently high negotiated wage payments resulting from the positive dependence of w_τ on \underline{h}_τ . The two separate effects of inefficient participation decisions - on $\frac{s_\tau}{s}$ and on $S_{\tau+1}$ - therefore both work in the same direction and reinforce one another in depressing market tightness relative to the socially efficient outcome. We will be able to quantify the departure of θ from θ^* in the next section.

4 Quantitative Analysis

In this section we carry out two objectives. First, we quantitatively assess the implications of age-specific matching for life cycle dynamics in the decentralised equilibria with and without age discrimination using numerical simulations under plausible parameterisations. The second is to quantify the departure of the decentralised solutions from their respective socially optimal allocations. Before proceeding, we first describe the calibration and computational method.

4.1 Calibration and Computation

Because the models considered represent two extreme views of reality, the issue arises of which is the more appropriate benchmark. As will become clear shortly, this turns out to be inconsequential as our calibration targets are met by both models for the

assumed parameterisation. Both models are therefore simulated with the same benchmark calibration, which is summarised in Table 1. In order for matching frictions to be meaningful, a time period equal to one quarter is adopted. The discount factor is then set to $\beta = 0.99$. We assume a working life span from the age of 16 to 64, requiring that $T = 196$ quarters and all agents retire at the start of age 65. Match productivity is normalised to unity, $a = 1$. As in the previous chapter, the baseline value for passive search intensity, $\chi = 0.2$, is taken from Pries and Rogerson (2009). Following Shimer (2005) we target a replacement ratio of about 40% for the U.S. economy. We thus set $b = 0.39$. Symmetric bargaining is assumed as a benchmark case so that $\eta = 0.5$, unless the Hosios condition is imposed in which case η is set equal to the elasticity of q with respect to tightness. The quarterly separation rate is set to $\rho = 0.05$ in accordance with the data compiled by Shimer (2007).¹²

In the life cycle setting that we consider, if we were to assume a Cobb-Douglas technology for m , then as $\theta_\tau \rightarrow 0$ as $\tau \rightarrow T$, the vacancy transition probability $q(\theta_\tau) \rightarrow \infty$. Clearly this violates the requirement that q be bounded between zero and one.¹³ It therefore becomes necessary to depart from the standard Cobb-Douglas form for m . We adopt the matching technology proposed by den Haan et al. (2000) that takes the form

$$M_\tau = \frac{s_\tau v_\tau}{(s_\tau^\phi + v_\tau^\phi)^{1/\phi}}$$

if matching is age-specific, and

$$M_\tau = \frac{sv}{(s^\phi + v^\phi)^{1/\phi}}$$

if it is not. We restrict $\phi \geq 0$. Note that as $\phi \rightarrow 0$, $M \rightarrow 0$. The benefit of adopting such a specification over the more conventional Cobb-Douglas technology is that the above function ensures that the transition rates p and q are bounded between 0 and 1 for positive θ_τ . M_τ is homogenous of degree one and increasing in both input arguments. The transition probabilities can be expressed solely in terms of market tightness as

$$p(\theta_\tau) = \frac{M_\tau}{s_\tau} = \frac{\theta_\tau}{(1 + \theta_\tau^\phi)^{\frac{1}{\phi}}}$$

and

$$q(\theta_\tau) = \frac{M_\tau}{v_\tau} = \frac{1}{\theta_\tau (1 + \theta_\tau^{-\phi})^{\frac{1}{\phi}}}.$$

Clearly, if matching is not age-directed, then $p(\theta) = \theta (1 + \theta^\phi)^{-1/\phi}$ and $q(\theta) =$

¹²For additional details, please see Shimer (2007) and his webpage <http://sites.google.com/site/robertshimer/research/flows>. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley.

¹³Cheron et al. (2008) do not appear to address this issue.

$\theta (\theta^{-\phi} + 1)^{-1/\phi}$. Recall that

$$\eta(\theta_\tau) = 1 - \xi_{p_\tau}$$

where $\xi_{p_\tau} = p'(\theta_\tau) \theta_\tau / p(\theta_\tau)$ is the elasticity of $p(\theta_\tau)$. For the assumed functional form of the matching technology,

$$\xi_{p_\tau} = \left(1 + \theta_\tau^\phi\right)^{-1} \leq 1 \text{ for } \theta_\tau \geq 0$$

thus ensuring that the optimal allocation is implementable. An analogous expression holds if the age subscript is dropped.

We follow Cheron et al. (2008) in choosing the model's free parameters in order to match the labour market characteristics of prime-age workers, which we take to be $\tau = 100$ quarters in our models. Specifically, we calibrate the models such that the employment and unemployment ratios at age $\tau = 100$, which corresponds to 41 years of age, are reasonably replicated. In the data, $n_{100} = 0.74$ and $u_{100} = 0.0325$, which correspond to the 35-44 age bracket. Recall that, as alluded to in the Introduction, the horizon effect is weak during the middle stages of the life cycle. Therefore, given that we target realistic stocks for prime-age workers, both the age-specific and random-age matching models meet the calibration targets under the same parameterisation, eliminating the need to establish a benchmark model.

The free parameters are ϕ , the support of F and κ . We normalise the lower bound \underline{h} to b so that $\underline{h}_T = 0$, ensuring that nobody participates in the terminal period without the prospect of finding a job. This is done for simplicity as well as to maintain consistency with the definition of unemployment as job search. This implies a slight divergence with the empirical data in Figure 1 in which unemployment at the start of age 65 is above zero. However, one must recall that in our model all labour force activity ceases at the start of age 65, whereas this is clearly not the case for the real U.S. data which is used to compute the life cycle dynamics in the figure. This subtlety is not important for any of our main results.¹⁴

We then have three remaining parameters - ϕ , \bar{h} and κ - with which to meet our two empirical calibration targets. Of the three parameters, κ is the one for which the most information for calibration purposes can be obtained from the related literature since the other two are much less widely used. However, as discussed below, calibrated values for κ still tend to vary widely. Nevertheless, we choose to normalise κ and use the two remaining parameters to match n_{100} and u_{100} .¹⁵ The upper bound \bar{h} governs how

¹⁴For a later exercise in which we compare decentralised and efficient outcomes, the lower bound on F is calibrated to a value that is below b , yielding a terminal unemployment ratio that is close to 2% as in the data.

¹⁵Most of the literature on equilibrium matching assumes a Cobb-Douglas form for m and parameterises the function according to the empirical estimates of Blanchard and Diamond (1989). In the absence of empirical estimates for the less common matching function assumed in this chapter, other authors such as Hagedorn and Manovskii (2008) and den Haan et al. (2000) use ϕ to pin down selected first moments of the data. This does not therefore provide much of a basis for comparison with our framework. In the absence of related empirical work, it would be even more difficult to pin down the

reluctant agents are to enter the labour force (and therefore the unemployment pool) while the matching function parameter ϕ determines the rapidity with which agents exit the unemployment pool, thus controlling the level of employment for a given pool of searchers.

In the absence of empirical estimates of the flow cost of vacancy creation, the parameter κ tends to vary in the literature apparently without a consensus on what would be an appropriate calibration method, much less a specific value for κ . For instance, Shimer (2005) sets κ to roughly 20% of labour productivity in order to normalise steady state market tightness to unity in his model. Hagedorn and Manovskii (2008) decompose κ into costs of idle capital in vacant jobs and the opportunity cost of labour devoted to the recruitment effort. Using the steady state restrictions of their model, they argue that the capital and labour costs of posting vacancies are roughly equivalent to 47% and 11%, respectively, of average labour productivity. In an RBC-matching model, Andolfatto (1996) simply assumes that the costs of vacancy creation are 1% of steady state output on the grounds that "in all likelihood... these costs are relatively small". He finds that this requires $\kappa = 0.11$ assuming a quarterly time period. Cheron et al. (2008) also set $\kappa = 0.78$ to target vacancy recruitment costs of 1% relative to average annual output, implying $\kappa \approx 0.20$ at the quarterly frequency. Cheron (2007) assume a smaller (quarterly equivalent) value of about 0.11 in order to pin down the average employment rate of 55-59 year old male workers. A different strategy still was demonstrated in the previous chapter, which followed Krause and Lubik (2007) in determining the appropriate value of κ residually from the steady state job creation condition once the other parameters had been chosen. This yielded a value of κ that was closer to zero. After considering a range of values for κ from 0.01 to 0.2, we found that the particular value that we normalise κ to does not have an important influence on our main results. The reason for this is that commensurate adjustments in the remaining two free parameters offset changes in κ . We therefore report the results for κ set to 0.1, which roughly lies at the mid-range of values adopted in the related literature.¹⁶ The remaining two parameters are then given by $\phi = 0.74$ and $\bar{h} = 1.45$.

Given the assumed calibration, life cycle paths for employment, unemployment and participation are then computed for both models. The computational strategy is to work recursively backwards given the terminal conditions. For the age-specific matching model, given that $\theta_T = 0$, market tightness can be solved by iterating backwards on

upper bound on F . Although, in principle, we could have normalised any one of the free parameters and used the remaining two to match the calibration targets, given that κ is common to practically all other related quantitative matching models we choose to normalise it based on the information that this provides.

¹⁶It turns out that total costs of vacancy creation relative to match production in this case turn out to be 1.8% in both models.

equation (12) using

$$\theta_\tau = \left\{ \left[\frac{\beta}{\kappa} \left(a - w_{\tau+1} + (1 - \rho) \frac{\kappa}{q(\theta_{\tau+1})} \right) \right]^\phi - 1 \right\}^{\frac{1}{\phi}}.$$

Once a path for market tightness has been computed, the participation threshold follows straightforwardly from equation (11). Life cycle employment is then determined from the law of motion (1) and unemployment from the definition in (2).

From equation (13), equilibrium tightness in the random-age model depends on the entire path of job values J_τ , and so cannot be solved for in the recursive manner used for age-specific matching. We therefore start with an initial guess for tightness, θ^i , and compute a first estimate of life cycle dynamics for the asset values J_τ^i and the path of searchers s_τ^i . Given the initial solution, we can then compute the right hand side of (13) and solve for an updated value of market tightness using

$$\theta^{i+1} = \left[\left(\frac{\beta}{\kappa} \sum_{\tau=1}^T \frac{s_\tau^i}{s^i} J_\tau^i \right)^\phi - 1 \right]^{\frac{1}{\phi}}.$$

This procedure is iterated until convergence in order to solve for equilibrium θ .¹⁷ Given equilibrium tightness, the employment and unemployment paths are computed using equations (1) and (15). The social planner's solutions for the two models are solved analogously.

4.2 Results

4.2.1 Decentralised Equilibrium

Figure 2 shows the life cycle equilibrium paths of the age-specific and random-age matching models, illustrating the following points. First, the models are jointly consistent with the qualitative hump-shaped patterns of employment and participation as well as a monotonically declining unemployment path. Quantitatively, however, the life cycle paths are not as pronounced as in the data. This point has also been recognised by Choi et al. (2011), who have demonstrated that a model with endogenous job destruction also predicts a relatively flat life cycle profile for employment, indicating that separation dynamics are also only weakly influenced by the presence of a finite horizon. The baseline results in Figure 2 therefore indicate that the horizon effect is not very strong in the model, in the sense that the decline in labour market participation towards the end of the life cycle occurs with greater abruptness and at a later stage in the model than in the data (see Figure 1).

Second, age-specific matching has a discernible effect on the decline in employment, but only as the terminal period draws very near. The difference in unemployment paths

¹⁷For iteration i , convergence is defined as $\theta_{i+1} - \theta_i \leq 0.01$.

is small, so that overall the dynamics of the two specifications are very much alike. Panels (b) and (d) show the dynamics of market tightness and the participation constraint in the age-specific and random-age models, respectively. Clearly, in the random-age model there is no decline in market tightness for older workers. Instead, the participation constraint falls due to a decrease in the relative value of a job. The dynamics of the participation constraint are very similar in both specifications. Preventing age discrimination results in a slightly lower equilibrium level of market tightness of 1.55 compared with 1.62 (for prime-age workers) in the age-specific matching model.

Table 2 summarises the employment and unemployment rates for different age cohorts. Despite middle-aged employment being about 0.3 percentage points lower without age discrimination, employment for the 55-64 cohort is about half a percentage point higher than when age discrimination is allowed. The employment protection effect of the legislation becomes stronger for workers even closer to the retirement age. Employment among 60-64 year olds is over a percentage point higher and for 62-64 year olds it is more than 3 percentage points higher. Prohibiting age discrimination results in only a small reduction in unemployment among older workers.

In addition to a weak horizon effect, the increase in employment during the initial stage of the life cycle occurs relatively quickly in the model, suggesting that matching frictions do not present much of an impediment to young agents gaining employment (this can also be inferred from Table 2). That is to say, the "queueing effect" described by Cheron et al. (2007) is also quantitatively quite small. Taken in combination, we therefore find that weak horizon and queueing effects cause the model to fail to replicate realistically sloped, hump-shaped dynamics in the middle stages of the life cycle even when firms are free to hire workers on the basis of age. These findings support the notion that the incentives for firms to discriminate strongly against older workers in the hiring process purely on the basis of age is weak. Equilibrium unemployment theory predicated on the view that matching frictions matter thus has an in-built mechanism which tends to smooth out employment rates over the life cycle, indicating that the pronounced hump-shaped dynamics portrayed in Figure 1 are mainly due to life cycle factors other than a pure horizon effect on the asset value of employment.

Before proceeding to compute the welfare effects of age discrimination policy, we first examine the extent to which changes to the baseline calibration give rise to more pronounced differences between the life cycle dynamics of the two models. In what follows, we make changes to the model's parameters in isolation, keeping the remainder of the calibration unchanged. As mentioned previously in section 2.1.3, the tendency for the Nash bargained wage to fall over the life cycle in line with market tightness weakens the incentive for firms to decrease job creation for older workers. In order to examine the quantitative importance of the wage channel in determining life cycle employment dynamics, consider an economy in which the worker's bargaining weight is close to zero, at $\eta = 0.05$.¹⁸ Keeping all other parameters as in the baseline cal-

¹⁸We do not reduce η all the way to zero in order to preserve a non-zero level of unemployment.

ibration, Figure 3 shows the impact that this has on the model's results. Lowering η raises the firm's share of the joint surplus and disincentivises participation. These two effects force vacancies up and unemployment down, causing market tightness to increase substantially. Despite weaker participation, the net effect on employment is positive as shown in panels (a) and (c). The dynamic effect of this parameter change is less noticeable. From panel (b), the decline in market tightness commences at an earlier age in the age-specific matching model compared to the baseline, suggesting that the insensitivity of the wage to tightness when η is close to zero induces firm to start to reduce job creation relatively early. However, the impact on life cycle employment is quantitatively weak. In particular, employment still does not decrease substantially towards the end of the life cycle. In the random-age matching model, there is hardly a perceptible decrease in employment as T approaches since the variation in \underline{h}_τ is small. Therefore, although equilibrium employment still does not display large variation with respect to age even under age-specific matching, the divergence of the two models' equilibrium paths becomes somewhat more notable, indicating that age discrimination policy is likely to be more effective when η is low.

Lowering the matching function parameter ϕ has a similar effect on the impact of age discrimination policy on employment dynamics. Lowering ϕ raises the expected duration of vacancies and causes firms to discount the future more heavily. If age discrimination is possible, vacancy creation will reflect a more pronounced horizon effect. Figure 4 illustrates this for $\phi = 0.4$. Abstracting from the level effects of lowering ϕ , reducing the efficiency of the matching function results in a more notable decline in employment in the second half of life in panel (a) for age-specific matching, whereas the decline remains relatively flat in panel (b) for the random-age model. This indicates that the effect of ϕ on the path of employment operates through its impact on job creation rather than participation dynamics. Therefore, the model suggests that age discrimination policy will be more effective when matching efficiency is low.

In Figure 5 we show the effects of raising κ to 0.3, three times the baseline value. Making vacancy creation more costly lowers market tightness and also makes the decline in employment towards the end of the life cycle somewhat more pronounced in the age-specific matching model (panel a). The strength of the horizon effect is positively related to κ : as the costs of vacancy supply fall, there is less reason to curtail the supply of vacancies available to older workers. As a result, we find that the role for age discrimination policy is larger when the costs of job creation are high.

Next, consider an alternative measure for χ , lowering it from its benchmark value of 0.2 to 0.05. In recognition of the fact that by changing χ we are effectively changing the aggregate search intensity of the economy, we can determine how the macroeconomic implications of age discrimination policy depend on the extent to which agents search for a job by participating in the labour market. Figure 6 reports the results. From the participation constraints (11) and (15), lowering χ causes \underline{h}_τ to rise by raising the return on active job search relative to non-participation. This causes unemployment

to generally rise in panels (a) and (c) compared with the baseline. In equilibrium we also find that market tightness rises in panels (b) and (d) due to increased vacancy supply in response to higher unemployment. With lower passive search intensity, the decline in unemployment towards the end of life has a larger negative impact on employment because, once out of the labour market, agents search for a job with only very weak intensity. This causes employment to terminate at a lower value relative to the baseline, and the difference is particularly notable for random-age matching. In panel (c), we can see that the terminal decline in employment is of comparable magnitude to the age-specific model in panel (a). This result indicates that in the baseline model without age discrimination, transitions from non-participation to employment were a quantitatively significant factor in maintaining high employment rates among the elderly. The model therefore suggests that the effectiveness of age discrimination policy hinges to some extent on large flows from non-participation to employment. This is not entirely surprising, since low unemployment among older workers implies that active job search is relatively low. With a constant rate of job destruction over the life cycle, high employment rates for older workers cannot be sustained unless the intensity of passive search is reasonably high. Once again, however, life cycle dynamics remain much less pronounced than in the data, such that even in the age-specific matching model the decline in employment past middle age is quantitatively small.

The last parameter experiment we consider is a "shock" to the separation rate. By influencing the continuation value of matches, the average length of a job, or its "durability", may determine the extent to which job creation tapers off at the end of the life cycle. To the extent that higher separation rates reduce the likelihood of continuing the match into the future, the incentive to age discriminate is weakened since future periods are discounted more heavily. In order to investigate this hypothesis, we raise ρ to 0.1 without changing any of the other baseline parameter values, thereby simulating an exogenous shock to match durability. Figure 7 plots the impact that this has on the age-specific and random-age matching models. The results indicate that, expectedly, a higher separation rate is associated with lower employment at every stage of the life cycle. Notably, despite the reduction in employment, unemployment does not rise substantially even though we held matching efficiency constant. Hence, participation rates decline by roughly the same factor as employment. The qualitative shape of employment and unemployment dynamics remains the same relative to the baseline, with terminal employment declining more sharply under age-specific matching. The efficacy of age discrimination policy therefore is not found to be very sensitive to the destruction rate.

4.2.2 Welfare Analysis

Our first objective in evaluating the welfare effects of age discrimination policy is to quantify the distortion caused by the presence of the intergenerational externality in

the participation decision of the random-age matching model. The bargaining weight of the worker is now replaced by the elasticity of the matching function with respect to vacancies. Furthermore, the intergenerational externality in the participation constraint (19) reduces the terminal value of \underline{h}_T^* below b , so it becomes necessary to adjust the lower bound on F accordingly in order to ensure that \underline{h}_τ^* always remains positive. We assume that it is reduced to $\underline{h} = b - \iota$, where $\iota = 0.1$. This calibration implies that unemployment terminates at approximately 2%, which is close to what is observed in the data. Note that we make the same adjustment to the support of F in the decentralised solution when comparing it to the social optimum. In order for the decentralised equilibrium to achieve our calibration targets for employment and unemployment as stated in the previous section, we re-adjust ϕ to 1 and \bar{h} to 1.6.¹⁹ All other parameters remain the same as in the baseline.

Figure 8 compares the efficient outcome to the private outcome with symmetric bargaining ($\eta = 0.5$). It turns out that market tightness is slightly higher at the social optimum, with $\theta^* = 1.01$ as opposed to $\theta = 0.93$. Given θ^* , it also turns out that, coincidentally, $\eta(\theta^*) = 0.50$, which matches the assumption of symmetric bargaining in the decentralised equilibrium. This is convenient for our analysis since it implies that any departure of the decentralised equilibrium from the planner's solution in Figure 8 is due to the intergenerational externality rather than bargaining inefficiencies. The differences in the life cycle paths of employment and unemployment bear out the previously derived theoretical result that the decentralised \underline{h}_τ is too low for young workers and too high for older workers. Figure 9 plots \underline{h}_τ^* and \underline{h}_τ , illustrating that the social planner's correction for the intergenerational externality in equation (19) raises participation for the young and decreases it for the old. As can be seen from Figure 8, unemployment at $\tau = 1$ is too low in the decentralised equilibrium, which, in conjunction with inefficiently low market tightness, then leads to lower employment over most of the life cycle. Correcting for the intergenerational externality exerts an opposite force on the unemployment path of the old as the terminal period approaches. Although the decline in unemployment begins at just before age 60 in both models, the social planner desires a steeper fall in unemployment towards the end of working life. Quantitatively, average unemployment for the 60-65 cohort is about half a percentage point lower in the efficient solution (2.94% versus 2.39%) and terminal unemployment is about 1.6 percentage points lower (2.06% versus 0.4%). As \underline{h}_τ^* declines at a faster pace for older workers relative to the private outcome, the employment rate also falls more quickly as T approaches. Thus, although employment is higher throughout the majority of the life cycle at the planner's solution, the terminal value is smaller than in the decentralised equilibrium. Overall, however, efficient dynamics are quantitatively similar to the decentralised outcome for most of the life cycle.

It is desirable to obtain a more precise quantitative measure of the divergence be-

¹⁹The rationale for increasing these parameters is to offset the increase in unemployment due to the decrease in the lower bound of F which lowers average home productivity.

tween the decentralised equilibrium and the social optimum. Given equilibrium paths for employment, unemployment and participation we can compute the value of output net of vacancy creation costs for each economy. In this manner, an indicator of the welfare effect of the intergenerational externality is obtained. We find that, characterised in terms of net output, the difference between the efficient and private equilibria is negligible. Specifically, the social planner attains a level of net output equal to 191.66 whereas the figure for the decentralised economy is 191.63. Despite perceptibly different dynamics in Figure 8, therefore, the efficiency loss from the failure of the free market to account for the life cycle externalities of participation choices is very small indeed.

Next, we compare dynamics in the efficient and decentralised versions of the age-specific matching model. In order to facilitate comparison with the previous results, we adopt the same parameterisation, setting $\underline{h} = b - 0.1$, $\bar{h} = 1.6$ and $\phi = 1$. Figure 10 plots the results. Whereas in the random-age matching model it was found that the decentralised economy exhibited too little labour force activity, we now find that labour force participation, employment and unemployment are all lower at the social optimum with age specific matching - except at the very end of the life cycle. Observing the behaviour of the efficient unemployment path, it can be seen that unemployment rises towards the end of the life cycle, thereby indicating that market tightness increases with age for at least some portion of the life cycle. Consequently, the social optimum is oscillatory and the monotonicity property does not continue to hold. The reason for this is due to the interaction between market tightness and the elasticity $\eta(\theta_\tau)$, which is now age-dependent and therefore varies over the life cycle.

Let us analyse the mechanism by which age-dependent bargaining weights are prone to yielding oscillatory dynamics in more detail. To aid with the explanation, Figure 11 plots θ_τ and $\eta(\theta_\tau)$ for the planner's solution and the private equilibrium for ages 60 to 65. As the two left panels illustrate, market tightness displays a monotonic decline as T approaches in the decentralised economy, in accordance with the intuition underlying our monotonicity property established previously in Proposition 1. The participation threshold \underline{h}_τ (not shown) traces the path of market tightness and also declines smoothly as T approaches. The worker's (symmetric) bargaining weight is fixed over the life cycle. In the social planner's economy, by contrast, market tightness becomes unstable as the terminal period approaches, thereby introducing oscillatory terminal participation dynamics as well. The efficient counterpart to the worker's bargaining weight is displayed in panel (d) of Figure 11. Given that $\eta'(\theta_\tau) \geq 0$, $\eta(\theta_\tau)$ traces the path of market tightness.

To see the economics behind the endogenous relationship between θ_τ^* and $\eta(\theta_\tau^*)$ and how this gives rise to oscillatory dynamics in the socially optimal equilibrium, start from the period before the last, $T - 1$, in equation (16). At $T - 1$, the right hand side of that expression is the same for both the decentralised economy as well as the social optimum since market tightness at T in both economies is zero. Then, the θ_{T-1}^* which solves the left hand side for $\frac{\kappa}{(1-\eta(\theta_{T-1}^*))q(\theta_{T-1}^*)}$ happens to be larger than the

θ_{T-1} which solves the decentralised counterpart $\frac{\kappa}{(1-\eta)q(\theta_{T-1})}$. This is because for low θ_{T-1}^* , the term $\eta(\theta_{T-1}^*)$ is small. It turns out that (as Figure 11 indicates) $\theta_{T-1}^* = 0.68$, which is larger than the decentralised value of $\theta_{T-1} = 0.19$. Since $\eta'(\theta_\tau) \geq 0$, this high value for θ_{T-1}^* raises $\eta(\theta_{T-1}^*)$, which from (16) has a negative effect on θ_{T-1}^* that is akin to an increase in the worker's bargaining weight in the decentralised outcome. In effect, the Hosios condition with age-dependent market tightness reinforces the negative relationship between the wage and market tightness. High anticipated market tightness at age $\tau + 1$ then has a negative effect on the supply of vacancies at age τ . This is exactly what causes market tightness at the efficient solution to fall sharply at $T - 2$ in Figure 11. Relatively low θ_{T-2}^* then induces a high value at θ_{T-3}^* , and so on until convergence at just above 0.4.

It is particularly notable that the efficient level of market tightness stabilises at a lower level during the middle of the life cycle than is observed for the decentralised outcome despite the efficient bargaining strength, at a value of approximately 0.3, lying below the assumed case of symmetric bargaining. This further reinforces the intuition that rendering worker bargaining strength dependent on the level of market tightness serves to reduce equilibrium market tightness through the adverse effect on vacancy supply. Lower values of η in the decentralised equilibrium exacerbate the difference in the average value of market tightness between the two economies. Figure 12 plots the dynamics of the participation threshold and market tightness from setting $\eta = 0.3$. The matching parameter ϕ is reset to 0.7 in order to maintain realistic prime-age employment but the rest of the calibration is unchanged relative to the model in Figure 11.²⁰ Average market tightness increases in the decentralised equilibrium due to weaker bargaining strength, whereas it decreases at the social optimum. The lower value for ϕ exacerbates the oscillations in market tightness, which eventually converge at about 0.3. Net efficient production is 192.40 while the decentralised economy produces 190.38, still representing a very small difference.

Despite qualitatively different behaviour towards the end of the life cycle, we find that the efficiency gain in terms of net production is once again negligible at the social optimum. Output net of the costs of vacancy creation is equal to 192.50 at the efficient allocation and 191.66 for the private outcome. Comparing net output with the random-age matching model, the differences are also minimal, with the social planner's allocation producing slightly more when age-specific matching is possible (192.50 versus 191.66). The difference in net production in the two decentralised economies is also very small, suggesting that anti-age discrimination legislation does not have a large impact on the amount of output the economy actually produces.

To summarise, we find that the failure to implement the Hosios condition results in negligible efficiency losses in the age-specific matching equilibrium with age dependent elasticity $\eta(\theta_\tau)$. In our life cycle setting, we also find that age-dependent bargaining

²⁰For brevity, a counterpart to Figure 10 displaying full life cycle dynamics of participation, employment and unemployment is omitted due to similarity.

weights cause equilibrium monotonicity to break down, instead giving rise to more complicated oscillatory dynamics towards the end of the life cycle. This is because, in order for the decentralised economy to be socially efficient, the bargaining strength of the worker must be rendered dependent on market tightness, which is age-dependent in this specification. In this context, it is not optimal for unemployment to follow a monotonically declining equilibrium path over the life cycle. Despite this difference in the behaviour of unemployment, only a quantitatively small departure of the decentralised solution to the socially optimal allocation is found in terms of net output produced. We also find that the size of the efficiency loss associated with age discrimination policy is quantitatively negligible due to the presence of only a weak effect of a finite horizon on behaviour in general.

5 Conclusion

This chapter has applied equilibrium unemployment theory to quantitatively evaluate the implications of age discrimination policy for the performance of the labour market. Our results suggest that such policy has a moderately positive effect on the employment rate of older workers close to retirement, but is unlikely to have further repercussions which are of considerable magnitude on workers who are of a younger age but still nevertheless protected by anti-discrimination laws such as the ADEA. This is because the negative effect on employment of a shorter remaining time horizon is offset by congestion effects that are generated by matching frictions which prevent equilibrium job creation from dropping very low for older workers even when age discrimination is possible. We have also argued that although preventing firms from hiring on the basis of age introduces an age-dependent externality in the labour force participation decision of agents, the resultant efficiency loss is quantitatively negligible.

It is emphasised that this chapter's analysis has concerned the macroeconomic implications of age discrimination policy as separate and distinct from the issue of fairness which naturally arises in this context. Although we have argued that the economic impact of age discrimination policy on employment is likely to be quite small, this does not of course imply that such policy does not have significant merit with respect to its assurance of the basic tenet of equal opportunity for all.

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7 Appendix: Wage Derivation

A derivation is presented for the Nash wage solution when bargaining powers are age-dependent. The Nash sharing rule modified for age-dependent bargaining powers is

$$W_\tau - \left\{ F(\underline{h}_\tau) U_\tau + \int_{\underline{h}_\tau}^{\bar{h}} N_{i,\tau} dF(h) \right\} = \frac{\eta_\tau}{1 - \eta_\tau} J_\tau$$

for all τ . Substituting the respective Bellman equations into the above Nash sharing rule and rearranging yields

$$\begin{aligned}
& \frac{\eta_\tau}{1-\eta_\tau} [a - w_\tau + (1-\rho)\beta J_{t+1}] \\
= & w_\tau + (1-\rho)\beta W_{\tau+1} - (1-\rho)\beta \left\{ F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right\} \\
& - F(\underline{h}_\tau) b - (1-F(\underline{h}_\tau)) \int_{\underline{h}_{\tau+1}}^{\bar{h}} h \frac{dF(h)}{1-F(\underline{h}_\tau)} \\
& - F(\underline{h}_\tau) p(\theta_\tau) \beta W_{\tau+1} + F(\underline{h}_\tau) p(\theta_\tau) \beta \left\{ F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right\} \\
& - (1-F(\underline{h}_\tau)) \chi p(\theta_\tau) \beta W_{\tau+1} + (1-F(\underline{h}_\tau)) \chi p(\theta_\tau) \beta \\
& \times \left\{ F(\underline{h}_{\tau+1}) U_{\tau+1} + \int_{\underline{h}_{\tau+1}}^{\bar{h}} N_{i,\tau+1} dF(h) \right\}.
\end{aligned}$$

Making use of the sharing rule once more and applying the free entry condition on vacancy supply (8),

$$\begin{aligned}
& \frac{\eta_\tau}{1-\eta_\tau} \left[a - w_\tau + (1-\rho) \frac{\kappa}{q(\theta_\tau)} \right] \\
= & w_\tau + (1-\rho) \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} \frac{\kappa}{q(\theta_\tau)} - F(\underline{h}_\tau) b \\
& - \int_{\underline{h}_{\tau+1}}^{\bar{h}} h dF(h) - (F(\underline{h}_\tau) + (1-F(\underline{h}_\tau)) \chi) p(\theta_\tau) \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} \frac{\kappa}{q(\theta_\tau)}.
\end{aligned}$$

Solving the above expression for w_τ yields the wage equation with age-dependent bargaining weights (17) from the text:

$$\begin{aligned}
w_\tau = & \eta_\tau a - (1-\eta_\tau) \left[\frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} - \frac{\eta_\tau}{1-\eta_\tau} \right] (1-\rho) \frac{\kappa}{q(\theta_\tau)} \\
& + (1-\eta_\tau) F(\underline{h}_\tau) \left(b + \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} \kappa \theta_\tau \right) \\
& + (1-\eta_\tau) (1-F(\underline{h}_\tau)) \left(\int_{\underline{h}_{\tau+1}}^{\bar{h}} h \frac{dF(h)}{1-F(\underline{h}_\tau)} + \chi \frac{\eta_{\tau+1}}{1-\eta_{\tau+1}} \kappa \theta_\tau \right).
\end{aligned}$$

Table 1: Baseline Calibration

Parameter	Value	Parameter	Value
β	0.99	κ	0.10
η	0.5	ρ	0.05
ϕ	0.74	χ	0.20
b, \underline{h}	0.39	\bar{h}	1.45

Table 2: Life Cycle Dynamics

Cohort	Model				U.S. Data	
	Age Specific		Random Age		n (%)	u (%)
	n (%)	u (%)	n (%)	u (%)		
16-24	63.79	4.41	63.33	4.44	54.79	7.25
25-34	74.37	3.12	74.01	3.15	71.31	4.15
35-44	74.37	3.12	74.01	3.14	74.23	3.25
45-54	74.37	3.12	74.01	3.14	72.87	2.85
55-64	72.88	2.66	73.25	2.55	56.76	2.20
60-64	71.43	2.20	72.57	2.03	-	-
62-64	68.30	1.27	71.43	1.14	-	-

Figure 2: Life Cycle Dynamics under the Baseline Calibration

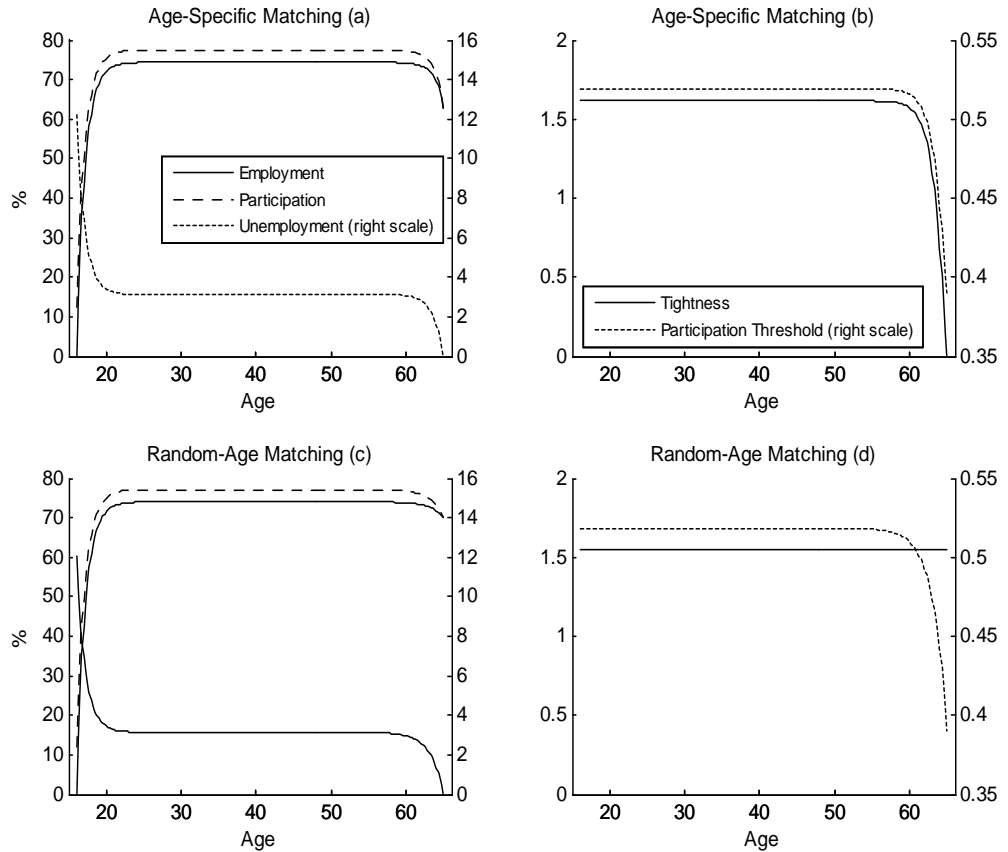
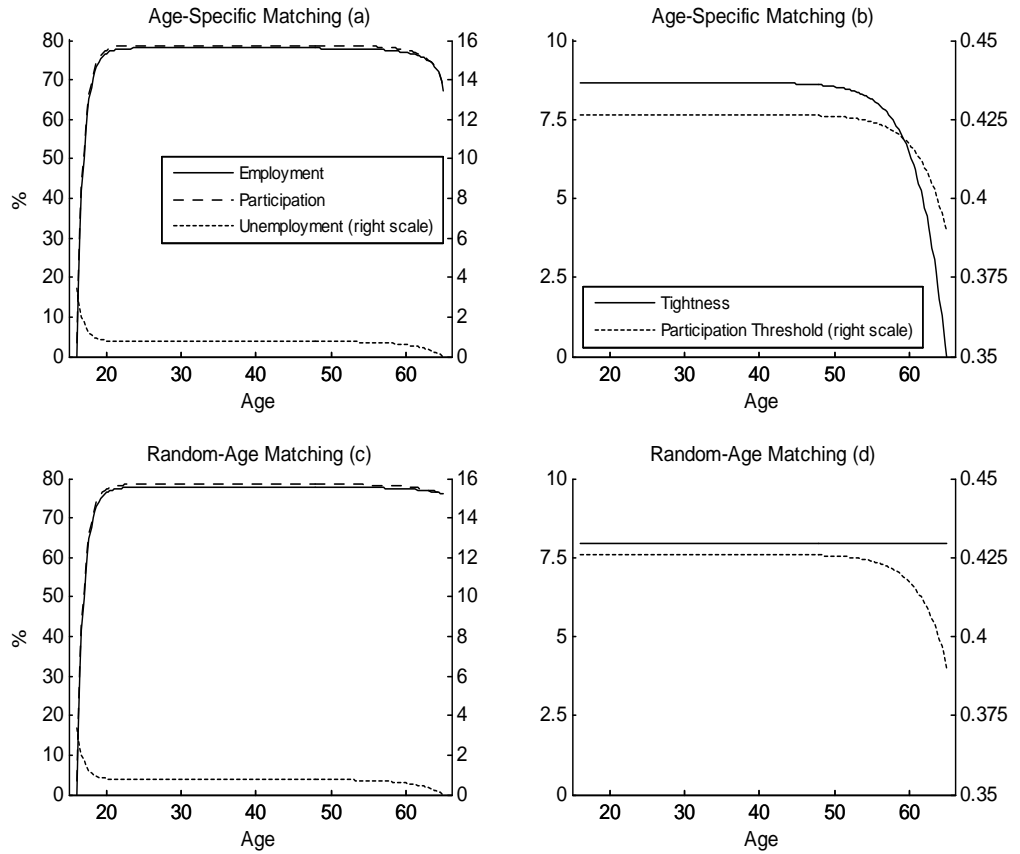
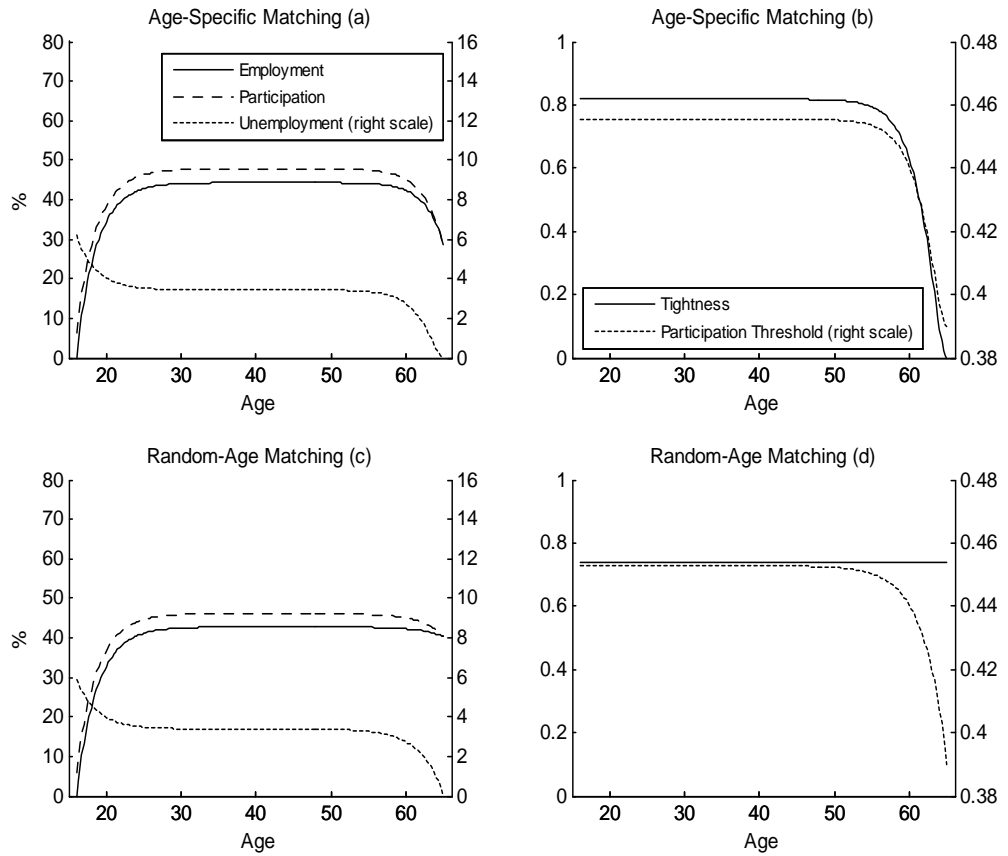


Figure 3: Life Cycle Dynamics with Low Worker Bargaining Power



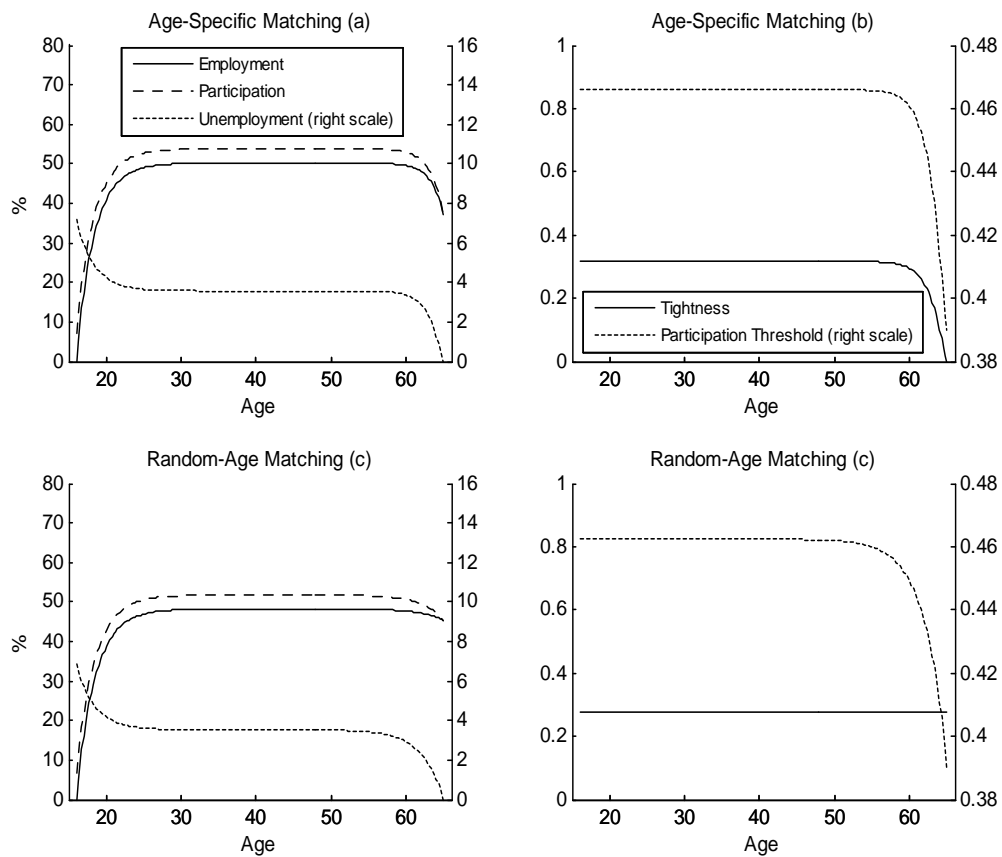
Note: Life cycle paths are computed for $\eta = 0.05$ keeping the rest of the calibration as in the baseline.

Figure 4: Life Cycle Dynamics with Low Matching Efficiency



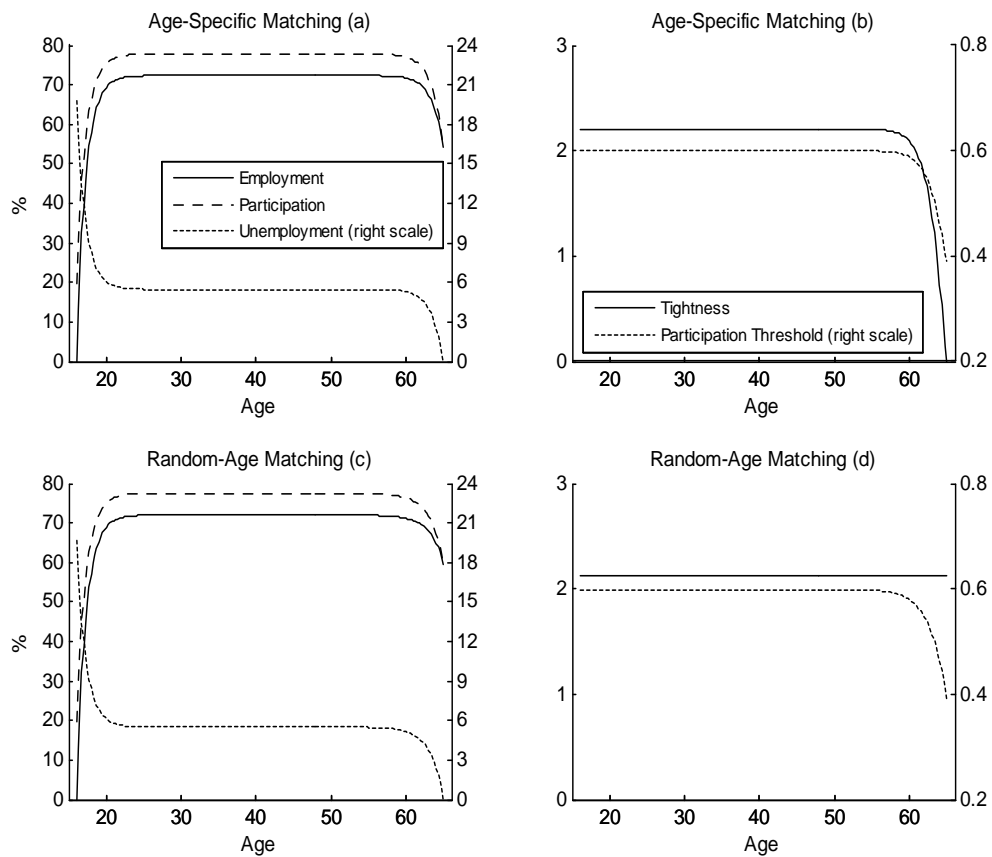
Note: Life cycle paths are computed for $\phi = 0.4$ keeping the rest of the calibration as in the baseline.

Figure 5: Life Cycle Dynamics with High Costs of Vacancy Creation



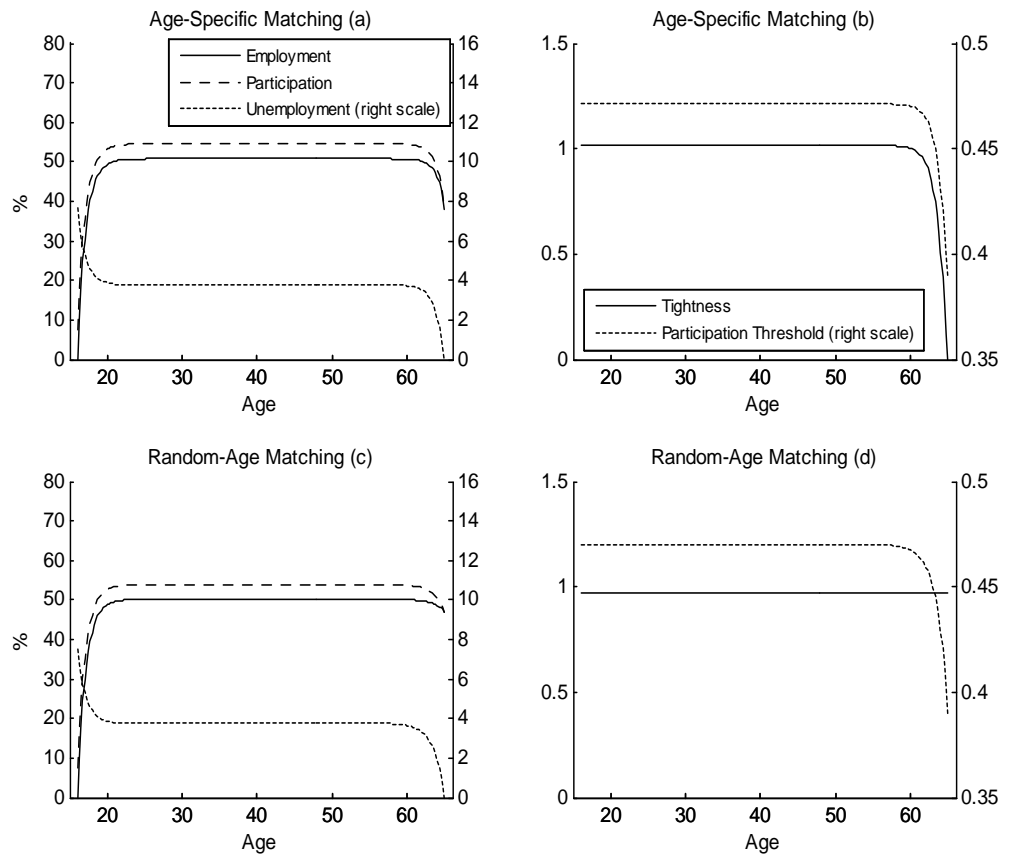
Note: Life cycle paths are computed for $\kappa = 0.3$ keeping the rest of the calibration as in the baseline.

Figure 6: Life Cycle Dynamics with Low Passive Search Intensity



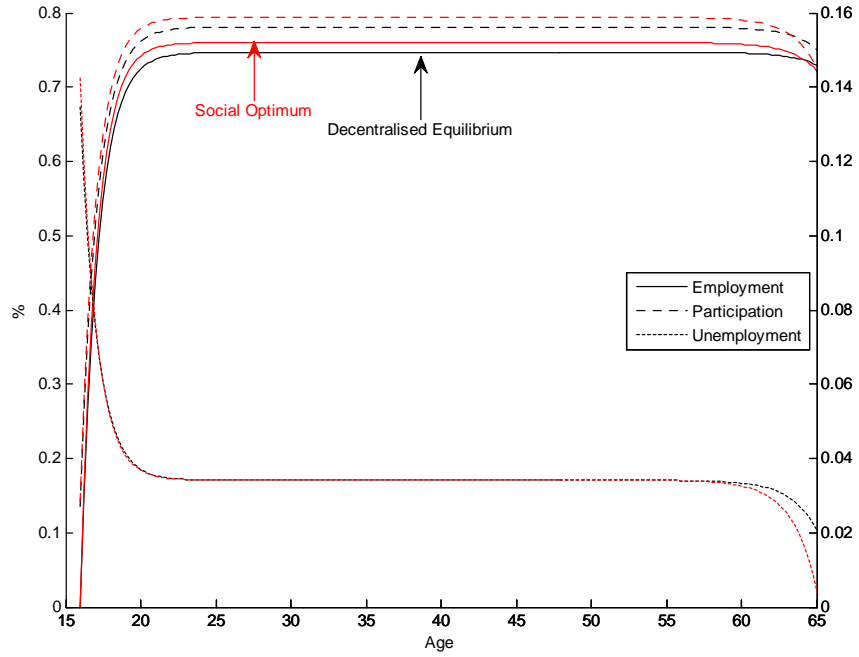
Note: Life cycle paths are computed for $\chi = 0.05$ keeping the rest of the calibration as in the baseline.

Figure 7: Life Cycle Dynamics with Low Match Durability



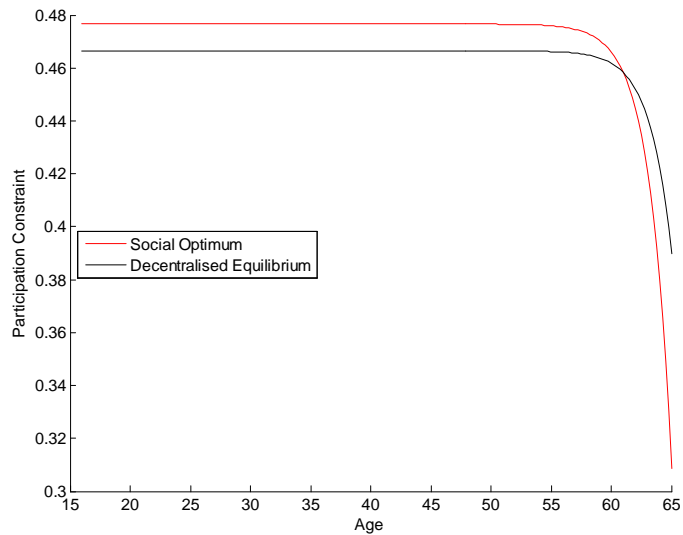
Note: Life cycle paths are computed for $\rho = 0.1$ keeping the rest of the calibration as in the baseline.

Figure 8: Comparing the Social Optimum with the Decentralised Equilibrium under Random-Age Matching



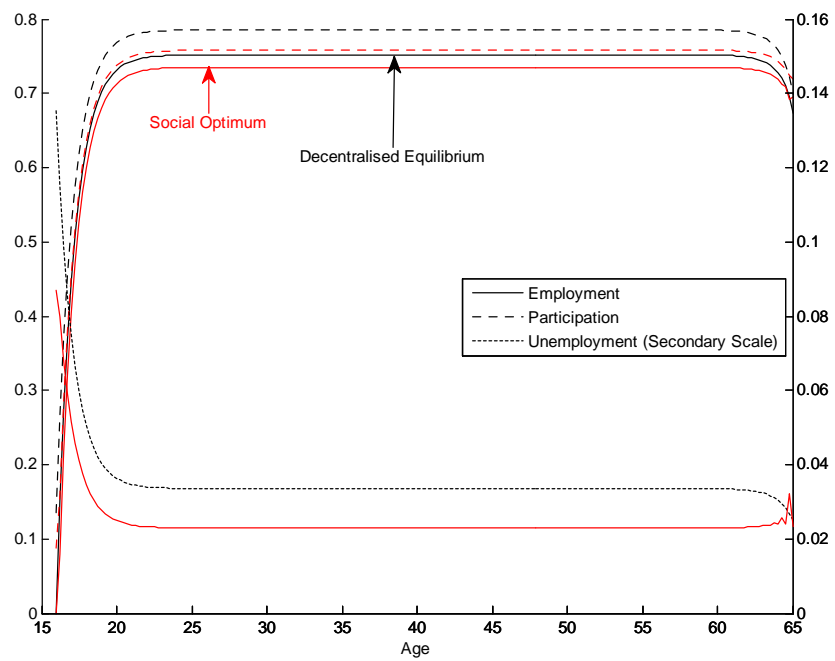
Note: Life cycle paths are computed for $\underline{h} = b - 0.1$, $\bar{h} = 1.6$ and $\phi = 1$ keeping the rest of the calibration as in the baseline.

Figure 9: The Social Planner's Correction to the Participation Threshold



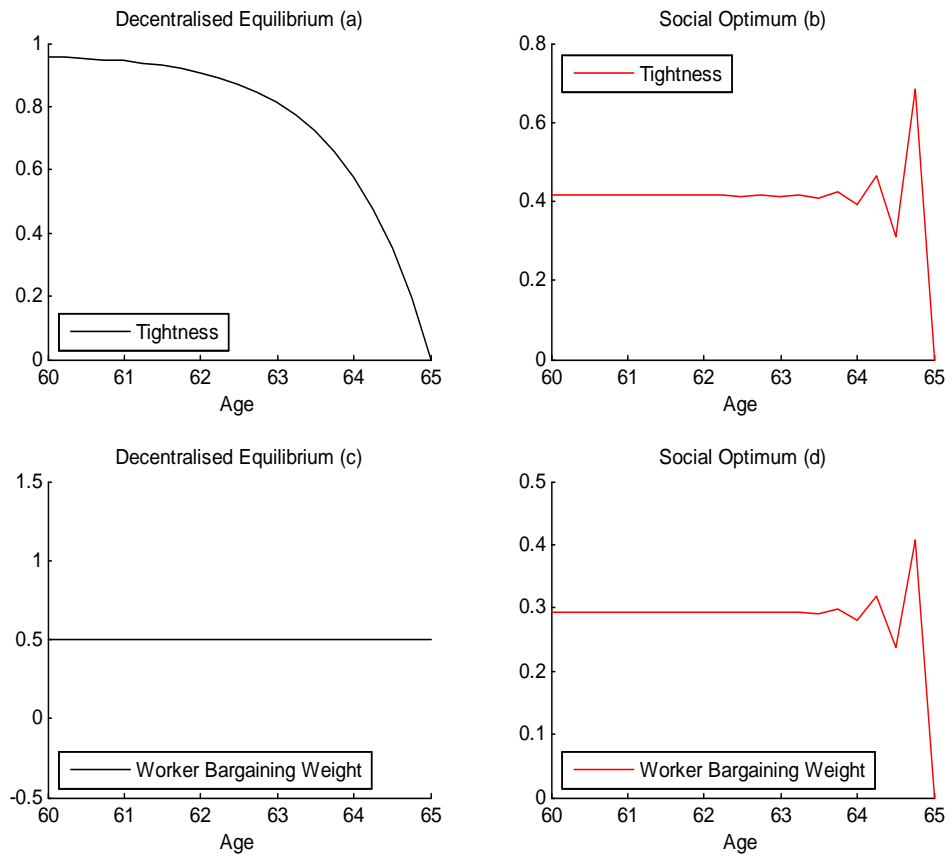
Note: Life cycle paths are computed for $\underline{h} = b - 0.1$, $\bar{h} = 1.6$ and $\phi = 1$ keeping the rest of the calibration as in the baseline.

Figure 10: Comparing the Social Optimum with the Decentralised Equilibrium under Age-Specific Matching



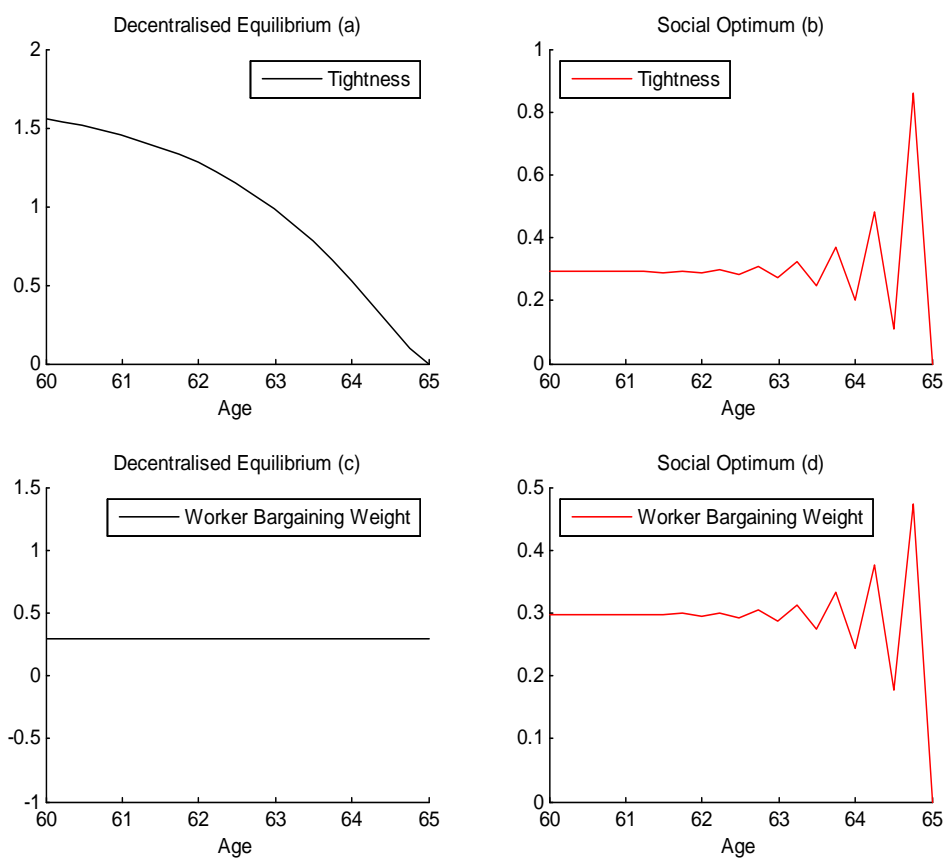
Note: Life cycle paths are computed for $\underline{h} = b - 0.1$, $\bar{h} = 1.6$ and $\phi = 1$ keeping the rest of the calibration as in the baseline.

Figure 11: Terminal Dynamics in the Age-Specific Matching Model



Note: Life cycle paths are computed for $\underline{h} = b - 0.1$, $\bar{h} = 1.6$ and $\phi = 1$ keeping the rest of the calibration as in the baseline.

Figure 12: Terminal Dynamics in the Age-Specific Matching Model with Low Worker Bargaining Power



Note: Life cycle paths are computed for $\underline{h} = b - 0.1$, $\bar{h} = 1.6$, $\phi = 1$ and $\eta = 0.3$ keeping the rest of the calibration as in the baseline.