

**AN ANALYSIS OF TEACHER COMPETENCES IN A
PROBLEM-CENTRED APPROACH TO DYNAMIC
GEOMETRY TEACHING.**

by

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SUMMARY

The subject of teacher competences or knowledge has been a key issue in mathematics education reform. This study attempts to identify and analyze teacher competences necessary in the orchestration of a problem-centred approach to dynamic geometry teaching and learning. The advent of dynamic geometry environments into classrooms has placed new demands and expectations on mathematics teachers.

In this study the Teacher Development Experiment was used as the main method of investigation. Twenty third-year mathematics major teachers participated in workshop and microteaching sessions involving the use of the *Geometer's Sketchpad* dynamic geometry software in the teaching and learning of the geometry of triangles and quadrilaterals. Five intersecting categories of teacher competences were identified: mathematical/geometrical competences, pedagogical competences, computer and software competences, language and assessment competencies.

KEY WORDS:

Dynamic geometry environments, dynamic geometry software, *Geometer's Sketchpad*, pre-constructed sketch, dragging, animation, freehand tools, geometry, Teacher Development Experiment, Problem-Centred Approach, deductive reasoning, van Hiele levels of geometric thought, presentation sketch, teacher competencies, Structure of Observed Learning Outcome, prestructural understanding, unistructural understanding, multistructural understanding, relational understanding

Table of Contents

TITLE PAGE	i
ACKNOWLEDGEMENTS	ii
SUMMARY	iii
KEY WORDS	iii

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 Introduction	1
1.2 Problem statement	4
1.3 Aims and objectives	6
1.4 Research design	7
1.4.1 The literature study	7
1.4.2 The empirical study	7
1.5 The significance of the study	8
1.6 Clarification of some terminology	9
1.6.1 Meaning of teacher competencies	9
1.6.2 The meaning of dynamic deometry environments (DGEs)	9
1.6.3 The meaning of dynamic geometry systems (DGS)	9
1.6.4 The <i>Geometer's Sketchpad</i> (GSP) dynamic geometry software	9
1.6.5 The meaning of Geometry	10
1.6.6 The Problem-Centred Approach	10
1.6.7 The meaning of pre-constructed dynamic sketches	10
1.6.8 Dragging and animation tests of a construction	11

1.7	Progress of the investigation	11
------------	--------------------------------------	----

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1	Introduction	12
2.2	The van Hiele theory	13
2.2.1	Level 1: Visual /Recognition	13
2.2.2	Level 2: Descriptive /Analytical	13
2.2.3	Level 3: Informal deduction	13
2.2.4	Level 4: Formal deduction	14
2.2.5	Level 5: Rigor	14
2.2.6	Characteristics of the levels	14
2.2.7	A critique of the van Hiele theory	15
2.3	The SOLO model	17
2.4	The problem-centred approach	18
2.5	Generic studies- a synthesis of literature surveyed	20
2.6	Studies involving Dynamic Geometry Environments	23
2.6.1	Re-orientation	23
2.6.2	Development of deductive reasoning	23
2.6.3	Rethinking the role of proof in dynamic geometry	24
2.6.4	Problem solving in a dynamic geometry environment	25
2.6.5	Classroom interaction patterns in DGEs	26
2.6.6	Development of geometric thought in DGEs	27

2.6.7	A redefinition of the teacher's role in a DGS environment	28
2.6.8	Problems with teacher experience and adaptability	29
2.6.9	Teachers' understanding of geometrical definitions	30
2.6.10	The centrality of the drag test	30
2.7	An interpretative summary	31

CHAPTER 3

RESEARCH METHODOLOGY

3.1	Introduction	34
3.2	Theoretical Framework: Teacher Development Experiment	35
3.3	Sampling procedures for the group of learners	36
3.4	Description of participants	37
3.5	Geometric content covered in this study	37
3.6	Data gathering processes	37
3.6.1	Phase 1- Pre-testing and selection of participants	37
3.6.2	Phase 2 – Introduction to <i>Geometer's Sketchpad</i> Software	38
3.6.3	Phase 3 – Micro-teaching phase	39
3.6.4	Phase 4 – Mini-project, questionnaire and group interview	41
3.7	Conclusion	41

CHAPTER 4

DISCUSSION OF DATA PROCESSING AND DEVELOPMENTS

4.1	Introduction	42
------------	---------------------	-----------

4.2	Pre-test results	42
4.2.1	Overview	42
4.2.2	Recognition, description and definition of shapes and processes	43
4.2.3	Class inclusion and relationships between properties	47
4.2.4	Understanding geometrical language and relationships between properties	48
4.2.5	Knowledge of the PCA and dynamic geometry	50
4.3	Introduction to computers and <i>Geometer's Sketchpad</i> software	51
4.3.1	Getting used to the computer	51
4.3.2	Overview of mathematical tasks	51
4.3.2.1	Constructing a square in <i>Sketchpad</i>	51
4.3.2.2	A theorem about quadrilaterals	53
4.3.2.3	Attempts to construct other quadrilaterals	55
4.3.2.4	Triangle centers.gsp sketch	57
4.4	Selected micro-teaching sessions	60
4.4.1	Triangle.gsp pre-constructed sketch	60
4.4.2	Pythagoras.gsp multi-page sketch	64
4.4.3	Transformations, kaleidoscope and tessellations	65
4.4.4	Distances in an equilateral triangle	66
4.5	Statistical summary of microteaching sessions	67
4.5	Mini-projects, questionnaire and group interview	71
4.5.1	Results and analysis of mini-projects	71
4.5.2	Analysis of questionnaire responses	74
4.5.3	Analysis of group exit interview	78

4.7	Conclusion	78
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CHAPTER 5

FINDINGS, RECOMMENDATIONS, LIMITATIONS AND CONCLUSION

5.1	Introduction	80
5.2	Findings on the nature of identified competences	80
5.2.1	Geometrical competencies	80
5.2.2	Language competencies	81
5.2.3	Computer and software competencies	82
5.2.4	Pedagogical competencies	84
5.2.5	Student assessment competencies	85
5.2.6	Integration	86
5.3	Recommendations	87
5.4	Limitations of the study	89
5.5	Conclusion	90
	Bibliography	91
	List of Tables	viii
	List of Figures	viii

List of Tables

Table 3.1	Categories of geometrical definitions	38
Table 3.2	Lesson observation criteria	40
Table 4.1	Categorization of definitions by 16 teachers	43

Table 4.2	Estimated van Hiele levels for definitions per participant	45
Table 4.3	Understanding class inclusion in definitions of quadrilaterals by other quadrilaterals	47
Table 4.4	Estimated van Hiele levels of participants for class inclusion	48
Table 4.5	Responses to triangle similarity case	61
Table 4.6	Teacher performances in congruency tasks	63
Table 4.7	Descriptions of four performance levels in the SOLO model adapted to microteaching assessment	68
Table 4.8	Assessment results for 8 leader teachers' lesson presentations	68
Table 4.9	Summary of results for 8 leaders teachers lesson presentations	69
Table 4.10	Average presenter performance as percentages	70
Table 4.11	Microteaching skill integration results	71
Table 4.12	Importance attached to mathematics by participants	74
Table 4.13	Reasons for choosing mathematics as a major at college	75
Table 4.14	Importance of <i>Sketchpad</i> activities in terms of mathematical Content	76
Table 4.15	Importance of activities in terms of <i>Sketchpad</i> capabilities	76
Table 4.16	Methods of checking the accuracy of a construction	76
Table 4.17	Importance attached to teachers' knowledge of students' current level of geometric understanding	77
Table 4.18	Importance attached to advance lesson preparation	77

List of Figures

Figure 2.1	A synthesis of competencies from the literature survey	33
Figure 4.1	Bar graph of pre-test knowledge of shape definitions by type and shape out of 16 participants	44
Figure 4.2	Key to estimating the van Hiele levels	45
Figure 4.4	Example definitions in each category	46
Figure 4.5	A theorem about quadrilaterals	53
Figure 4.6	Rhombus construction example	56
Figure 4.7	Rorisang's triangle circles	57
Figure 4.8	Triangle centres.gsp sketch and custom tool use	59
Figure 4.8	Given that opposite sides are equal	54
Figure 4.8	Given that opposite sides are parallel	54
Figure 4.9a)	Similarity and congruency theorems – AAA	60
Figure 4.9b)	Similarity and congruency theorems – ASA	61
Figure 4.9c)	Similarity and congruency theorems – SAS	62
Figure 4.9d)	Similarity and congruency theorems – SSA	62
Figure 4.9e)	Similarity and congruency theorems – SSS	63
Figure 4.10	Puzzled Pythagoras.gsp	64
Figure 4.11	Bernard's Tessellations and Kaleidoscope	66
Figure 4.12	Distances in an equilateral triangle	66
Figure 4.13	Tichaona's saved sketch	72
Figure 4.14	Perfect's saved sketch	72
Figure 4.15	Nathan's saved sketch	73
Figure 4.16	Reasons for majoring in mathematics	75

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE STUDY

"The irony is that the more successful the computer has been in competing with human acts, the more we come to appreciate the importance of extra-logical dimensions of human agency." Stephen Brown, *Towards Humanistic Mathematics Education*.

1.1 Introduction

This chapter outlines the background to the study from a global perspective, which is then localized to the developing world of which Zimbabwe is a part. The problem under investigation is stated, research questions are posed, and aims and objectives are outlined. The research design, significance of the study and some key terms are clarified.

The computer revolution epitomizes a revolution in the way we think and the way we express what we think (Abelson and Sussman in Cuoco and Goldenberg, 1996:31).

Mathematics education has not been insulated from this revolution. If anything, the revolution has profound ramifications on the way we express and represent what we think in mathematics and, consequently, the way we should teach and let children experience it. In fact, the very relevance of the school as preparation place for life after school can be questioned if that society's workplace and homes have more information and communication technology (ICT) than the school itself.

The traditional teaching of plane geometry has largely been dominated by ruler-and-compass methods of teaching and learning which produced static geometrical artifacts thereby limiting the flexibilities in shape variety and orientation. Experimentation with

variations largely remained in mental imagery, thus making the subject accessible only to a few students. The introduction of dynamic geometry environments (DGE's) into the classroom in the past decade or so has changed this praxis and students can now experiment with shape and space in real time through inbuilt dragging and animation modalities. Traditional teaching approaches of chalk-and-talk consequently appear to be less effective and teachers are challenged to develop new competencies and conceptualizations in geometrical teaching and learning since a new relationship emerges between the teacher, the computer, the DGE and the student on the one hand and geometric subject matter on the other. The earlier teachers and students can be exposed to the new software tools, therefore, the better.

However, Wessels (2001b:3) points out that technology by itself is worthless. Jenson & Williams (1992:243) echo the same sentiment when they state that technology by itself is no panacea and, in fact, initially complicates rather than simplifies a teacher's life in the classroom. In further concurrence, Cuoco and Goldenberg (1996:15) observe that computers are often used badly, as a sort of electronic flashcard, which does not creatively tap the capabilities of either the computer or the learner. Hence there is consensus that merely placing computers in the hands of teachers and students will not automatically transform the teaching and learning of geometry. Yet if creatively used, computers can be a catalyst for change and innovation in the way mathematics is taught and learnt. Goldenberg (1999:209) reiterates this hypothesis when contending that any lure of technology is merely a technique toward the goal of good thinking, and not a substitute to the goal itself. This is an incisive observation to be noted by teachers.

According to Hoyles and Noss (1994:716), formal geometry has not been a significant part of the mathematics curriculum in the United Kingdom (UK) since the early 1970's, even the word 'geometry' has gone out of style and replaced by 'shape and space'. The obsolescence of school geometry has been similarly felt in Zimbabwe and South Africa. Wessels (2004:70) confirms that geometry was one of the 'poor relatives' in the field of mathematics, and it is only in the past five years (in South Africa) that its 'status' has improved. He further adds that the majority of high school learners simply did not understand it, nor did the teachers who were supposed to teach it. In Zimbabwe, at General Certificate of Education (GCE) Ordinary Level, the attempt to infuse modern mathematics topics in the early 1980's pushed geometry to a fringe where only (geometrical) results were emphasized without due care to conceptual understanding and the development of deductive reasoning. Statements like 'state without proof' or 'no proof required' became commonplace punctuation marks of the syllabus. At GCE Advanced Level, and beyond, geometry is virtually absent in the Zimbabwean syllabi. In the primary school curriculum geometry is a relatively latecomer as there was more stress on arithmetic until the 1970s. Therefore, it is likely that, in their practice, many mathematics teachers experience gaps of geometric knowledge.

According to Hoyles and Noss (1994:716) the idea that at last we can play around with geometrical ideas in an intuitive and dynamic way is exciting enough, and the possibility that this kind of activity might somehow lead to a more radical and widespread understanding of geometry is just too tantalizing to ignore. However, there might still be a background of poor teaching being a major cause of the alienation of students from geometry. Wessels (2001c:3) points out that one reason why Euclidean geometry education is a complete disaster in South African schools is because it is badly taught.

Van Niekerk (1997:112) argues that one problem in the attempt to transform geometry instruction is that the majority of South African mathematics teachers are poorly trained. The situation is hardly different in Zimbabwe where, despite an appreciable increase in the number of qualified secondary school mathematics teachers since independence, the Ordinary Level pass rate is still a far cry from the ideal, and mathematics remains one of the least popular subjects in the curriculum, and thus dropped by students at the first opportunity. In arguing for the possible introduction of non-Euclidean geometry in the school syllabus Fish (1996:8) concurs with Wessels and Van Niekerk when she laments that not all teachers are competent to teach even the mathematics prescribed in the current syllabus. It is obvious that teachers cannot teach topics they themselves have little knowledge of. In the same vein, teachers cannot be reasonably expected to effectively use DGEs unless they themselves are familiar with their technical, mathematical and pedagogical constraints and affordances.

1.2 Problem statement

This study is motivated by the realization that we live in an increasingly computer ubiquitous society, yet the average Third World mathematics teacher is barely computer literate. The scarcity of computers in Third World homes and classrooms exacerbates the fear of technology, or techno phobia, even in pedagogical circles to such a degree that, where available, computers pass more as office word processing technology than as potential vehicles for innovative mathematics education orchestration.

Even in the first world, Cangelosi (1996:218) concedes that *'some teachers are reluctant to apply computer technology in their teaching only because they are not comfortable*

with the software'. Jenson & Williams (1992:240) similarly acknowledge that it is not until teachers have had the time and *training* to appropriate and orchestrate this new technology that they feel comfortable in being creative with its use. The sooner teachers of mathematics are acquainted with the new technology then, the better for the community because teachers are invidiously expected to teach the new generation how to fit into the global information age of the 21st century.

There is possibly a need to explore a number of starting points from which to stimulate interest and foster confidence in the use of DGEs in mathematics teaching. In this study third (final)-year mathematics major teachers were targeted as one possible starting point in the identification and analysis of what might be pre-requisite teacher competences for enhancing students' understanding of the geometry of triangles and quadrilaterals in a *Sketchpad* Dynamic Geometry Software (DGS) environment.

The purpose of this study was to identify and analyze pre-requisite teacher competences in the execution of a problem-centred approach to dynamic geometry. Dynamic geometry in the form of such DGEs such as *Cabri and Geometer's Sketchpad* is a recent phenomenon and its spread to disadvantaged parts of the developing world is rather slow suggesting the need for more conscious effort. Research of this nature should add to the much-needed direction to decisions that need to be made not just in the identification process but also in the nature, content and context of development of such a repertoire of competencies. The question is: What mathematical, linguistic, pedagogical or technological competences are identifiable and analyzable among (pre-service) teachers of mathematics, as necessary for them to be proficient with the new technological tools? In answering this question the following sub questions provide a lead

- To what extent is the teacher's knowledge of geometry a prerequisite in supporting a PCA to dynamic geometry?
- What is the nature and scope of pedagogical competencies that support a PCA to dynamic geometry?
- How do language competencies support a PCA to dynamic geometry teaching and learning?
- What proficiencies does a particular DGS such as *Sketchpad* demand in the teaching and learning of dynamic geometry?
- Which assessment competencies or strategies are suitable in evaluating students' progress in a DGS environment?

1.3 Aims and objectives of the research

The aim of this study was to identify and analyze teacher competencies in a problem-centred approach to the teaching and learning of dynamic geometry. To achieve this aim the following objectives were identified:

- To undertake a literature review to identify teacher competencies required in a problem-centred approach to dynamic geometry teaching
- To justify the TDE as a suitable method of investigation complemented by a pre-test, a questionnaire, a group interview and integrated case studies in the identification and analysis of teacher competencies in a PCA to dynamic geometry.
- To describe the nature and content of teacher competencies identified in the investigation.
- To draw up deductions, conclusions and recommendations on the nature and scope of teacher competencies compatible with a PCA to dynamic geometry.

To describe the nature of integration of identified competencies that could serve a problem-centred approach to dynamic geometry

It is noted and emphasized from the outset that the study was not about the developmental aspect, but rather on the identification and analysis. As such the developmental trajectory fell beyond the scope of the investigation.

1.4 Research design

1.4.1 1.4.1 The Literature Study

A Dialog search was done at the University of South Africa library with Professor Wessels using the following descriptors: ‘geometer’s sketchpad’ or ‘dynamic geometry’ or ‘cabri’ in English language, ‘mathematics teacher’ and ‘cognition’, ‘constructivism’, ‘Cooney’ and ‘Boaler’. The aim of this literature study was to come up with a tentative list of competencies for further empirical analysis

The research was also supported by two visits made to the University of South Africa. The first involved advice and guidance from my supervisor, a presentation of the proposal to the Masters and Doctoral committee members of the Faculty of Education. The second consisted of a presentation at a seminar in the presence of visiting Professor John Olive of the University of Georgia, Athens, an authority in the problem-centred approach to the teaching of mathematics, and the use of *Geometer’s Sketchpad* software.

1.4.2 Empirical Approach

The research was done mainly in the form of a Teacher Development Experiment (TDE) with sample(s) drawn from third (final)-year mathematics majors of the Diploma in

Education (Primary) course at a Zimbabwean polytechnic with a Teacher Education Faculty. The polytechnic is an associate college of the University of Zimbabwe, which is the awarding institution of the Diploma. Eligibility for selection for the study was determined by voluntary participation in a pre-test, which was followed by sampling. Teachers' responses to tasks undertaken during the workshop sessions were analyzed. Lesson observations were used as competence identification strategies during the microteaching sessions and further analyses of responses to tasks were made. Eventually mini-projects (see 4.5.1) were compiled and a structured group interview (4.5.3) was conducted to get feedback from participants as to what they considered to be prime technological and pedagogical competencies a new group should be acquainted with.

1.5 The significance of the study

Currently there is worldwide acceptance of the potential of DGS environments to enrich mathematical learning and improve student achievement. Jenson & Williams (1992:232) point to the fact that research has shown that DGEs offer students the opportunity not only to develop more positive attitudes towards mathematics and a better self concept, but also to assist them to achieve higher scores in basic operations and problem solving. The problem-centred approach is receiving increasing attention worldwide as a learner-centred strategy in the teaching and learning of mathematics. In a problem-centred approach students construct their own understandings *through* problem solving. Hence a problem-centred approach is compatible with a constructivist view of learning.

According to Chavunduka and Moyo (2003:101), constructivism as a learner-centred approach is increasingly attracting attention from researchers even in Zimbabwe. To the researcher, this project was an opportunity to gain more experience in and insight into the use of the *Geometer's Sketchpad* in interaction with teachers meeting the software for the

first time. Participants would benefit from an experience with the software and possibly be inspired to introduce ICT in their mathematics teaching careers. Once completed research results could be shared with the mathematics education community and hopefully the polytechnic might also accede to the formal infusion of ICT into the teaching and learning of mathematics. Curriculum designers, policy-makers, textbook authors, and researchers might also find some of the ideas in this report useful.

1.6 Clarification of some terminology

1.6.1 The meaning of teacher competencies

In this study teacher competencies are taken to embody teacher knowledge and beliefs, abilities and skills in orchestrating mathematics instruction in an integrated way.

1.6.2 The meaning of dynamic geometry environments (DGEs)

DGE is the acronym for Dynamic Geometry Environments, which are computer-aided micro-worlds for the teaching and learning of geometry. They are interactive environments where the computer is the tool. The student can manipulate the constructions made through the software dynamically. Examples of such micro-worlds include *Logo*, *Geometric Supposer*, *Cabri* and *Sketchpad*.

1.6.3 The meaning of dynamic geometric system or software (DGS)

DGS is the acronym for Dynamic Geometry System or Software and refers to particular dynamic geometry application software such as *Geometer's Sketchpad* or *Cabri* with dynamic capabilities afforded by the software design.

1.6.4 The *Geometer's Sketchpad (GSP)* dynamic geometry software

This is the Dynamic Geometry® Software for Exploring Mathematics (GSP4), scientific version, used in this study and refers to Version 4.05 of the software, which at the time was the latest. The software was designed by Nicholas Jackiw (2001) and published by Key Curriculum Press Technologies Emeryville, CA, USA.

1.6.5 The meaning of geometry

In this study geometry is taken to be the mathematics of shape and space, which traditionally incorporates Euclidean geometry but is not limited to it. Non-Euclidean geometries can also be identified in examples like spherical, elliptical, and hyperbolic geometries and, more recently, there has been growing interest in transformation, fractal, turtle, analytical and vector geometries. In this study the school geometry to be covered is predominantly Euclidean on account that geometrical objects in the application software appear to be predominantly constrained within Euclidean geometry axioms, definitions with plenty room for dynamic transformations.

1.6.6 The Problem-Centred Approach (PCA)

The problem-centred approach can be defined as an approach, which has as its focus the development of problem solving skills of both routine and non-routine problems and real life situations within a mathematical framework. The emphasis is on learning mathematics *through* learner-centred, reality-based problem solving, individually or in small groups.

1.6.7 The meaning and purpose pre-constructed dynamic sketches

Pre-constructed dynamic sketches are pre-made sketches that may, but need not be, web-based. Someone has constructed them with a specific mathematical content in mind and placed particular constraints.

1.6.8 Dragging and animation tests of a construction

The dragging paradigm is a feature of the DGS, which enables the users to drag a part or whole of a geometric object and manipulate it as they wish. Animation is an alternative to the drag test, and sets the selected parts of the construction in motion at variable speeds and dynamically models possible positions and shapes of figures that maintain the relationships used during construction.

1.7 Progress of the investigation

The rest of this research report will be discussed as follows:

Chapter 2 will review literature related to the van Hiele Theory, the problem-centred approach, teacher competencies in general, classroom experiences in DGEs,. Specific studies which require specific skills or competencies in students and teachers, or otherwise, will be referred to and findings summarized.

Chapter 3 will deal in greater detail with the research design, and the instruments used to answer the research questions.

Chapter 4 will describe the data processing beginning with an overview of statistical procedures and results.

In Chapter 5 the research findings will be put into perspective with what is known about teacher competences in and outside a problem-centred approach to dynamic geometry. Each result will be interpreted and limitations of the study stated. The chapter will conclude with recommendations and motivation for further research directions.

CHAPTER 2

REVIEW OF RELATED LITERATURE

“Let no-one unversed in geometry enter here.” Motto over the door of Plato’s Academy

(Eves,1976:87). *An Introduction to the History of Mathematics.*

2.1 Introduction

The problem of teacher knowledge or competencies has been studied from various traditional settings as well as selectively and disparately in reformed classrooms or situated cognition contexts. In this chapter, characteristics and thought levels of the van Hiele theory are discussed in the context of geometry learning. The five instructional phases are also briefly alluded to. The role of language in the theory is highlighted. The SOLO model is briefly described as a viable instrument for measuring teachers’ competencies. The Problem Centred Approach (PCA) is discussed in terms of its socio-constructivist underpinnings, problematization of subject matter, open-ended learning, and most importantly, learning through problem solving in a realistic context. An overview of research on teacher knowledge and skills is sketched out in general outline. Various classifications of teacher know-how are described, compared and evaluated revolving around the work of Cooney (1994,1999), Lappan and Theule-Lubienski (1994), McDougall (2001), Schulman (in Cooney, 1994), Bromme (in Cooney, 1994) and Philip, Flores, Sowder and Schappelle (1994). Thereafter, specific instances within the dynamic geometry set up are reviewed. This strategy is predicated on the understanding that the concept of teacher knowledge/competencies is a multifaceted one. Studies involving the

development of deductive reasoning (or proof) are well documented by de Villiers (1996,1998,1999), Marriotti (2001a, 2001b), Leung and Lopez (2002). Studies involving problem solving in DGE are also examined in the works of Hoyles and Noss (2001), Hannafin and Scott (1998), and Hollerbrands (2003). Classroom interaction patterns are reflected upon in the studies by Straesser (2001), Laborde (2001a) and Jackiw and Sinclair (2002), De Villiers (1991,1999, 2004), and Govender and de Villiers (2002).

2.2 The van Hiele theory

Van Hiele (1986: 39 – 47) distinguishes five different thought levels in the learning of geometry numbered from 0 to 4, but which have since been re-numbered 1 to five in the American convention that has become the international one. These levels can be summarized as follows:

2.2.1 Level 1: Visual (Recognition)

Students identify and operate on shapes and other geometrical objects according to their appearance. They recognize figures as a whole and they identify, name and compare using the reasoning of the type ‘it looks like’, without explicitly considering the properties. For example it is a rectangle because it looks like it.

2.2.2 Level 2: Descriptive (Analytical)

Students now recognize and characterize shapes by their properties and relationships among components (parallelism, number of sides, equality, regularity, angularity and perpendicularity). They see figures both as a whole and as a sum of experimentally established properties. Students do not see relationships between classes of figures. For example, it is a rectangle because opposite sides are equal and all angles are equal.

2.2.3 Level 3: Informal deduction (abstract/relational)

Students are able to logically classify families of shapes, can form abstract definitions, distinguish between necessary and sufficient conditions for a concept, and can handle class inclusion and equivalent definitions of a concept. They can give informal argument for their deductions and can follow some formal proofs given by the teacher or textbook. For example, if a rectangle has all its sides equal, then it is a square.

2.2.4 Level 4: Formal deduction

Students understand the role of the different elements of axiomatic systems (axioms, definitions, undefined terms, and theorems). They are now capable of performing formal proofs. For example, proving that if a quadrilateral has opposite sides equal, they its opposite angles are equal (compare 4.3.2.2).

2.2.5 Level 5: Rigor (mathematical)

At this level students reason formally about postulational systems and can now study geometry in the absence of reference models. The aim of their reasoning is the establishment, elaboration and comparison of axiomatic systems of geometry. For a example, Euclidean and non-Euclidean geometries (compare 1.6.5).

2.2.6 Properties of the levels

According to Usiskin (1982) the van Hiele has five properties. The first is the ‘fixed sequence’ property by which ‘a student cannot be at van Hiele level n without having gone through level $n - 1$ ’ (Usiskin, 1982:5). The second property is of ‘adjacency which states that the object of perception at level $n - 1$ becomes the object of thought at level n .

‘Distinction’ is the third property which states that level n requires a re-organization or reinterpretation of knowledge acquired at level $n - 1$. Land (1990:29) refers to this as ‘the perception of a new structure complete with its own symbols.’ The fourth identified property is of ‘separation’ attesting that two persons reasoning at different levels cannot understand each other (Usiskin, 1982:5). In this connection, de Villiers (1999:11) notes that according to the theory, the main reason for the failure of the traditional geometry curriculum is that it is presented at a higher level than those of the students. The fifth property was identified as ‘attainment’ implying that the progress from one level to the complete understanding of the next is more a function of instruction than age or maturation and five learning phases are delineated as inquiry/ information, directed orientation, explicitation, free orientation and integration.

2.2.7 A critique of the van Hiele theory

According to Pegg and Davey (1991:10) the ideas of van Hiele, have been the catalyst for much of the renewed interest in the teaching of geometry during the 1980’s, evolving largely as a reaction to the deficiencies perceived in the views of Piaget. It can further be noted that the van Hiele level theory has been studied even outside geometry by Land (1990) in algebra (exponential and logarithmic functions) and Nixon (2002) in higher arithmetic (sequences and series), and the existence of levels has been validated.

However, there are studies that have raised questions about some characteristics of the theory. While van Hiele (1986:49) specifically identified discontinuity between levels as the most distinctive property of the levels of thinking, the autonomy of the levels does not seem to be as distinct. Burger and Shaughnessy (1986:45) state that they failed to detect the discontinuity and found instead that the levels appear dynamic rather than static and

of a more continuous nature than their discrete descriptions would lead one to believe. Students may move back and forth between levels quite a few times while they are in transition from one level to the next. Fuys, Geddes and Tischer (1988) also found a significantly sized group of students who made some progress toward level 2 with familiar shapes such as squares and rectangles, but encountered difficulties with unfamiliar figures. They concluded that progress was marked by frequent instability and oscillation between levels. Gutierrez, Jame and Fortuny (1991:250) also found that the levels were not as autonomous in that people do not behave in a single, linear manner, which the assignment of one single level would lead us to believe. They identified students who could be coded 100%, 85%, < 40% and < 15% for levels 1, 2, 3 and 4 respectively implying that students develop more than one level at the same time. In other words, van Hiele's broad statements are not as black and white as they are often portrayed. (Pegg and Davey, 1998:114) (compare 4.2.3) Is it the level of the student or the level of response that should matter? The SOLO (Structure of Observed Learning Outcome) taxonomy has been proposed as a more realistic model for assessing and classifying students' responses in geometry and mathematics in general (Biggs, 1996, Pegg & Davey, 1998, Pegg 2003).

Apart from the foregoing, other observations on the van Hiele theory have been that it was postulated specifically in the context of 2-D geometry, and not in 3-D and dynamic contexts. Although van Niekerk (1997) has shown its applicability in 3-D contexts, de Villiers (1994:17) has shown that dynamic geometry contexts can facilitate the grasping of class inclusion even as early as level 1. Treffers (1987:245) also points out that the van Hiele theory was proposed at a time when geometry was not part of the primary school curriculum in the Netherlands. He further concedes that the theory lacked clarity about

how to shape concretely the phenomenological exploration at the first level, and which didactical acts should be performed to raise pupils as efficiently as possible from one level to the next. Even van Hiele (1986:47) himself has doubted the existence or testability of levels higher than the fourth and considered them as of no practical value. Be that as it may, Usiskin (1982:6) commends the van Hiele theory '*for its elegance, comprehensiveness and wide applicability.*' The implications for teachers are that whereas the van Hiele theory explains geometric thought development from a macroscopic perspective, there could be variations to be considered when a closer look is taken at the microscopic level.

2.3 The SOLO model

According to Pegg (2003:240) SOLO is a general model of intellectual development and aims at classifying outcomes and not students. That is, in line with the Developmental-based Assessment and Instruction (DBA) philosophy the emphasis is on giving weight to 'what the student knows, understands and can do' (ibid. p. 238). Three levels of performance are identified as unistructural understanding (focusing on the domain or problem using one piece of information leading to inconsistency), multistructural (using two or more pieces of information without any relationship between them and hence inconsistencies may still exist) and relational understanding (wherein all information is now available and there is no inconsistency). Sometimes a fourth level is employed referred to as pre-structural in which the response is deemed to be below the target mode. SOLO postulates that all learning occurs in one of five *modes of functioning* namely the sensori motor, ikonic, concrete symbolic, formal and post formal. In the terminology there is clear reminiscence of Piaget's stages of intellectual development. Biggs (1996) argues convincingly for the suitability of SOLO in the assessment performance in higher

education. Taking a cue from Biggs, this study uses the SOLO criteria to assess teacher competencies and how they are integrated to achieve effective learning in a problem-centred approach to dynamic geometry teaching.

2.4 The problem-centred approach

The problem-centred approach is compatible with the emergent constructivist view of knowledge and learning. The genetic epistemology of constructivism argues that knowledge cannot be independent of the knower and commits itself to the view that knowledge is first an individual construction and secondly a social construction (Ernest, 1996:343). In other words, constructivism locates mathematical knowledge in the knower, as an individual (subjective) experience and as a shared (objective) experience.

Teachers are therefore encouraged to create learning opportunities (tasks) that enable students to construct their own understandings, individually and collaboratively since there is no one-to-one mapping from teaching to learning but active construction of knowledge by students themselves, according to emergent categories derived from social interaction, not from observation of the teacher teach (Biggs, 1996:73). The problem-centred approach as proposed by Murray, Olivier and Human (1993:73) reaffirms that students construct their own mathematical knowledge irrespective of how they are taught. In apparent support of this posture, Simon and Schifter (1994:331) contend that learners construct understandings, as they attempt to make sense of their experiences, each learner bringing to bear a web of prior understandings, unique with respect to content and organization. The bottom line then, is to allow students to construct their own understandings by creating an environment conducive to effective problem solving.

The problem-centred approach is also consonant with Freudenthal's (1983:46) objection to giving students ready-made mathematics. Mathematics is viewed as a human activity that students must engage in a way similar to the genetic development of the subject. In similar vein, Hiebert, Carpenter, Fennema, Fuson, Human, Murray Olivier and Wearne (1996:12) are of the view that in a problem-centred approach instruction should make the subject problematic by allowing students to wonder why things are, to inquire, to search for solutions and resolve incongruities. Such a spirit dovetails conveniently with the historical development of the subject. In retrospect, Bereiter (1992:342) argues that all high level scientific knowledge is problem-centred rather than referent-centred. This challenges both curriculum and instruction to begin with problems, dilemmas and questions for students and implies that teachers should develop problem-posing skills. The dynamic geometry environment offers expanded opportunities for problem posing, exploration and experimentation.

The central role of the teacher in a problem-centred approach becomes one of designing or selecting and posing tasks that "link with students' experiences and for which students can see the relevance of the ideas and skills they already possess" (Hiebert et al, 1996:16). In further support the Netherlands the Realistic Mathematics Education (RME) project refers to rich context problems of which the problem situation is experientially real to the student and can serve as anchoring points for the re-invention of mathematics by students themselves (Gravemeijer & Doorman, 1999:111). In other words, the problems must be within the zone of proximal development of the students as advocated by Vygotsky (1978) by being reasonably difficult to challenge and foster creativity, yet not discourage. This sounds plausible and compatible with the van Hiele concerns about the level of language and geometrical difficulty (compare 2.2.6).

Problems should also be amenable to multiple solution strategies, or be open-ended, extensible, and generalizable (Erickson 1999, Schoenfeld, 1994). The dynamic geometry environments offer ample opportunities for open-ended exploration, which promotes originality and transferability of knowledge to real world problem solution. Rather than stereotype, this promotes creativity in the development of solution strategies. In a comparative study of students in an open-ended learning situation (Phoenix Park) versus students in a textbook oriented environment (Amber Hill), Boaler (2000:117) concludes that students develop different conceptions about what it means to have and to use mathematical knowledge. While students at Phoenix Park (who engaged in open-ended projects at all times) developed more conceptual and flexible forms of knowledge, those at Amber Hill (where mathematics was taught using a traditional text book approach) appeared to have spent time in their mathematics classrooms failing to learn! The finding is an instructive eye-opener for teachers to cultivate an open-ended learning culture in their classrooms (compare rhombus construction exercise in 4.3.2.2).

2.5 Generic studies- synthesis of literature surveyed

Cooney (1999:163) refers to the growing topicality of the notion of teacher knowledge by noting that it is being recognized as an increasingly complex phenomenon because effective teaching involves more than being mathematically competent. In the light of the NCTM standards Lappan and Theule-Lubienski, in Cooney (1994:609), conclude that the role of mathematics teacher education is to enable teachers to 1) choose worthwhile tasks, 2) orchestrate classroom discourse, 3) create a learning environment that emphasizes problem solving, communication, and reasoning and 4) develop teachers' ability to analyze their teaching and student learning. These appear to predominantly mathematical, pedagogical and linguistic competencies.

Taking a cue also from the *Standards*, McDougall (2001:35) proposes a four-level rubric to measure ten dimensions of teacher competence identified as 1) program scope (algorithms vs. sequentially more connectedness), 2) inclusion of all students in all mathematics lessons, 3) student tasks (particular procedure vs. multiple solution strategies), 4) discovery (transmission model vs. student thinking), 5) teacher's role (sole expert vs. creation of mathematics community) 6) use of manipulatives and tools, 7) student-student interaction (isolated work vs. learning from peers) 8) student assessment (end-of week tests vs. real life, multi-level performances) 9) teacher's conception of mathematics as a discipline (fixed body vs. changeable math) and 10) student confidence (achievement vs. conceptual understanding). Teachers' conformity with philosophy of the NCTM standards (which advocate a reformed mathematics classroom discourse) was measured on a four-point scale with the two extremes forming the ends of a continuum. These competencies seem again to be largely pedagogical. Technology and assessment competences are mentioned in passing without a major stress, but constitute important additional dimensions. Teachers' conceptions about mathematics appear to have received greater emphasis than knowledge of mathematics itself.

For mathematics teachers to achieve the NCTM standards, Lappan and Theule (1994:253), on the one hand, identify three domains of knowledge enabling one to choose worthwhile tasks, orchestrate discourse, create an environment for learning, and analyze teaching and student learning: knowledge of mathematics, knowledge of students and knowledge of the pedagogy of mathematics. These appear to be important pillars of teacher know-how. Knowledge of students, links up well with the van Hiele theory's emphasis on thought levels. (compare 4.6.2, and 4.2) On the other hand, Schulman (in

Cooney, 1994:610) classifies teacher knowledge into seven domains: 1) knowledge of subject matter, 2) pedagogical content knowledge, 3) knowledge of other content, 4) knowledge of the curriculum, 5) knowledge of learners, 6) knowledge of educational aims, and 7) general pedagogical knowledge. There appears to be reiterations of mathematical and pedagogical content knowledge in this classification. Furthermore, knowledge of curriculum and educational aims can be embedded in pedagogical knowledge. Technology is conspicuous by its absence.

From yet another perspective, Bromme (in Cooney, 1994:610) proposes a topology of the teacher's professional knowledge that includes 1) content knowledge about mathematics as a discipline, 2) school mathematical knowledge, 3) philosophy of school mathematics, 4) pedagogical knowledge, 5) subject-matter-specific pedagogical knowledge and, cognitive integration of knowledge from different disciplines. The first three categories could be combined under mathematical knowledge. Ability to integrate knowledge from different disciplines, appears an important addition on this list, which acknowledges the integrated nature of knowledge (compare with Bereiter in 2.3), and is thus compatible with the problem-centred approach. However, technological literacy is still absent.

In a study by Phillip, Flores, Sowder and Schappelle (1994) four teachers were identified as "extraordinary" teachers of mathematics. Data gathered from interviews, tests on content knowledge, discussions during a series of seminars, and lesson observations were used in summarizing characteristics of these teachers under a) their mathematical preparation and their content knowledge of mathematics b) their conceptions about mathematics, about learning, about teaching, about the roles of teachers and of students, and about the assessment of learning, and c) their teaching practices. There is

considerable buttressing of mathematical content knowledge and a new dimension of teacher beliefs and social ethos of the classroom. Within the three areas, though, the following characteristics were identified: deep commitment to teaching, personal ownership of change within the classrooms, a high degree of reflectiveness, active participation in professional development activities, thorough understanding and knowledge of school mathematics, integrating mathematical content knowledge with teaching practice, viewing mathematics as a foreign language and encouraging students to conjecture and explore, focusing on conceptual understanding, and viewing the teacher's role as one of guide, not sole authority. Reflectiveness appears an important additional ability or competence, which teachers can develop or cultivate even in a PCA approach to dynamic geometry. Encouraging students to conjecture and explore is particularly instructive in a dynamic geometry environment.

2.6 Studies involving dynamic geometry environments (DGEs)

2.6.1 Re-orientation

The categorizations and differentiations in the previous section seem to have been framed at from a holistic or generic perspective of mathematics education. This study aims at localize the analysis to a DGE environment and this section attempts to re-focus attention in that direction.

2.6.2 Development of deductive reasoning

Leung and Lopez (2002) contend that theorem acquisition and deductive proof have always been core elements in the study and teaching of Euclidean geometry. They argue that the advent of DGE enables students to experiment through different dragging modalities on geometrical objects they construct, and consequently infer properties about

the geometrical artifacts (compare 2.5.10). They discuss the case study of two secondary school students (aged 16) who submitted a *Cabri* proof by contradiction of a theorem on cyclic quadrilaterals, and conclude that their construction motivated a visual – cognitive scheme on observing proof in DGE and how this scheme might fit into the theoretical construct of cognitive unity of theorems. If teachers could create conditions that permit construction, conjecturing, experimentation and verification, then students could be engaged in genuine mathematical activity and not just prefabricated mathematics.

Mariotti (2001a) describes a long-term teaching experiment carried out with students from 9th to 10th grades in different classes and schools. She examines how geometrical constructions in *Cabri* can constitute the key to accessing the idea of theorem by helping students to move from a generic idea of justification toward a formal proof. She concludes that the evolution should not be expected to be simple and spontaneous. Instead the evolution is a product of sustained instructional effort that engages students in sense making just as the van Hiele theory suggests (compare 2.2, 4.3.2.2 and 4.4.4).

2.6.3 Reappraising the role of proof in dynamic geometry environments

Although Laborde (2001b:155) argues the case for the dual nature of proof as meant for both validating the truth of a statement and for convincing others of the validity, there are alternative views. De Villiers (1991, 1996, 1998, 1999, 2002, 2004) consistently and persistently reappraises the role of proof as verification and conviction. He argues that whereas traditionally the function of proof has been seen almost exclusively in terms of its verification function (conviction or justification), the advent of dynamic geometry and its convincing power pushes the conviction function of proof to a triviality. He further argues that explanatory, discovery, systematization, intellectual challenge and

communication functions of proof in situations where conviction already exists, may not only make proof potentially more meaningful to students (compare 2.5.2), but in such cases is probably more intellectually honest (de Villiers, 1991:12).

The convincing power of dynamic geometry environments also prompts challenges to find deductive proofs, not to clear doubt that would already have been cleared by the software, but to satisfy a deeper need for understanding (compare 4.3.2.2 and 4.4.4). In other words, in dynamic geometry contexts conviction can, in fact, precede and motivate proof given that direct contact with the phenomenon is even more convincing than a proof, since one sees it all happening right before one's eyes (De Villiers, 2002:5). In a creative application of modeling of a real world problem using pre-made sketches **Water 1.gsp** and **Water 2.gsp** Mudaly (2002) reports that students were enormously surprised to discover that the perpendicular bisectors of all triangles were concurrent and they wanted an explanation in order to understand and satisfy some innate curiosity around the reason for the result. In a sense the capabilities of a dynamic geometry environment compel teachers to take a second look at the role of proof as a matter of urgency.

2.6.4 Problem solving in a dynamic geometry environment

In a study involving six groups of six students from three different schools, Healy and Hoyles (2001) explored the role of software tools in geometry problem solving and how these tools, in the interaction with activities that embed the goals of teachers and students mediate the problem solving process. Through an analysis of successful student responses they concluded that dynamic software tools cannot only scaffold the solution process but can also help students move from argumentation to logical deduction. However, from an analysis of responses of less successful students they found that software tools that cannot be programmed to fit the goals of the students might, in fact, prevent them from

expressing their (correct) mathematical ideas This is an important conjecture, which the teacher has to be alert to in a DGE. It calls for a keen eye on individual differences in learning styles (compare 4.2.3).

After controlling for ability in a *Sketchpad* environment, Hannafin and Scott (1998) investigated the effects of 8th grade students' working memory capacity, preference for amount of instruction, spatial problem-solving ability and school mathematics grades on two achievement measures, along with recall of factual information and conceptual understanding. They found, on the one hand, that learners who reported a relatively low preference for amount of instruction scored higher than their high-preference counterparts on the conceptual understanding test items. They also found that high achievers in school grades scored higher than students with lower grades on the factual recall test items but not on conceptual understanding items. On the other hand low achievers in school mathematics performed relatively better in these nontraditional mathematics activities, suggesting that open-ended dynamic geometry learning environments could improve student achievement (compare 2.3) and thus reach out to a greater number of students.

Hollerbrands (2003) investigated the nature of students' understandings of geometric transformations, which included translations, reflections, rotations and dilations in *Sketchpad*. In a seven-week instructional unit, students' conceptions of transformations as functions were analyzed and results suggest that understanding of key concepts such as domain, variable and parameters, relationships and properties of transformations were critical in developing deeper understandings. Problem-solving ability was enhanced.

2.6.5 Development of geometric thought in DGEs

Choi-Koh (1999) investigated a secondary school student's development of geometric thought using the PM van Hiele (1986) model and the *Geometer's Sketchpad* software. During a 21-hour study, the author used clinical interviews to determine the students' predominant level of geometric thought and to gain insight into the developmental process of geometric reasoning. Ordered from the simplest to the most complicated, four learning styles: the intuitive, analytical, inductive and deductive were identified in terms of symbol, signal, and implicative properties. The author concluded that the use of active visualization with the dynamic software facilitated the movement from symbol to signal and to an implicative character. This suggests teachers should be skilled to identify students' varying thought levels so that they adjust their teaching to address the concomitant variations in reasoning styles (compare 2.5.3).

Lehrer and Chazan (1998) investigated the interactive roles of subject matter, teacher, student and technologies in promoting understanding of geometry and space. They came to the conclusion that geometry and spatial visualization in school should not be compared or limited just to Euclidean geometry. This is instructive in the light of the emergence of other geometries as alluded to earlier (compare 1.6.5). The geometry of triangles and quadrilaterals is amenable to transformational and fractal manipulation (compare 4.3.3, 4.4.1 and 4.4.2).

2.6.6 Classroom interaction patterns in DGEs

Straesser (2001) analyses how the use of DGEs might influence traditional geometry and its teaching and learning. The author highlights changes in the interactions between geometry, the computer tool, the DGEs and the human user in the teaching and learning

of geometry. The conclusion reached is that DGEs deeply change geometry if it is taken as a human activity integrating the use of modern Information and Communication Technology (ICT). Hence there is concurrence with Freudenthal's (1991) view of mathematics as an organizing activity. In other words DGE enable teachers to engage students in the activity of *doing* geometry with expanded opportunities for reinvention. Laborde (2001a), designer of *Cabri* software, examines the discrepancy in France between the institutional support for the use of technology in mathematics learning and its weak integration into teacher practice. He then identifies and analyses the possible integration, over a 3-year study, in the design of teaching scenarios based on *Cabri-geometre* for high school students. The analysis concludes that the role played by the technology moved from being a visual amplifier or provider of data to that of being an essential constituent of the meaning of tasks, thus affecting the conceptions of the mathematical objects that the students might construct. The implication is that the teacher in a DGE has to be sensitive to the manner in which the DGE affects the understanding and interpretation of geometrical objects and artifacts (compare 4.3.21).

In a proposal for a whole class view of micro-world design involving 29 grade 9 students, Jackiw and Sinclair (2002) illustrate that the design not only has practical benefits in terms of classroom and time management but also develops social interactions conducive to educative learning experiences. This suggests flexibility in presentation styles depending on the number of PCs available and/or the stage of the lesson (compare 4.6.3).

2.6.7 A redefinition of the teacher's role in a DGE

Marriotti (2001b) carried out a long term teaching experiment with 9th and 10th grades to clarify the role of *Cabri* in the teaching and learning process. Assuming a Vygotskian

perspective, the author focused attention on the social construction of knowledge, the semiotic mediation of cultural artifacts and the functioning of specific elements of *Cabri* as instruments of semiotic mediation. The presence of the computer and of the particular DGS were found to represent a perturbation element in the internal context of the teacher in that the teacher had to elaborate a new relationship to mathematical knowledge which links it to the computer in general and the DGS in particular. The teacher has to adapt his role of mediator taking into account the new elements offered by the DGS. This suggests allowing students to work in small groups and share findings, justifications and communication as a community of learners, thus in concert with the PCA approach.

2.6.8 Problems with teacher experience and adaptability

In a 2 week-mixed design study involving 12 grade 7 students Hannafin, Burruss & Little (2001) examined teacher and student roles in, and reactions to, a student-centred instructional program, using *Sketchpad*. The authors found that the teacher had difficulty relinquishing control of the learning environment even though she had agreed to do so. Students, however, liked their new (found) freedom and expressed greater interest in the subject material. The PCA to dynamic geometry calls for a de-rolling from the traditional caricature of the teacher as sole authority, or teaching as telling.

In examining opportunities to explore and integrate mathematics with the *Geometer's Sketchpad*, Olive (1998) presents examples from elementary-, middle- and high school where teachers, have been using *Sketchpad*. Apart from illustrating the potential for creative student explorations the author also illustrates potential problems such as understanding the difference between “drawing” and “constructing”, “demonstration vs. proof”, and pedagogical problems of teachers’ lack of experience with *Sketchpad*. In a

PCA to DGEs, teachers consequently have an obligation to acquaint themselves thoroughly with the software features and capabilities (compare 4.6.2).

2.6.9 Teachers' understanding of geometrical definitions

In a study of student teachers' constructive evaluation of definitions in a *Sketchpad* context, Govender and De Villiers (2002) found that after interaction with the software, the teachers appeared to have developed a deeper understanding of the arbitrary nature of definitions, to have improved ability to select correct alternative definitions of a rhombus and to have improved the ability to improve a given definition from incorrect to uneconomical (van Hiele level 2) and to an economical one with necessary and sufficient conditions (van Hiele level 3 competence)(compare 4.2.1). In a study involving the systematization of the isosceles trapezoid, de Villiers (2004), found that students preferred a deductive economical definition from which it was easy to deduce the other properties of the concept. He described such a definition as constructible in the sense of allowing one to directly construct the object being defined in *Sketchpad*.

2.6.10 The centrality of the drag test

Goldenberg and Cuoco (1998) examine the effects on teaching and learning of the dragging paradigm and at the way students perceive figures because of the defaults built into the drag mode. Cuoco and Goldenberg (1996:17) argue that this capability of DGEs offers students the metaphor of the physics of mathematics. DGEs like *Sketchpad* and *Cabri II*, it is argued, offer students the opportunity to experiment with mathematical objects just as they might tinker with mechanical objects. Roschelle and Jackiw (2000:782) also contend that the dragging paradigm allows students to move fluidly between open-ended and goal directed modes of inquiry. The central idea of dragging

implies that if relationships have been set up among points, lines and circles, they are preserved even when one of the basic components of the construction is dragged (Hoyles and Noss, 1994:716). The dragging paradigm thus casts into sharp relief the difference between a 'drawing' and a 'construction', which both teachers and students must come to grips with (Finzer and Bennet, 1995:428). Sinclair (2003:290) points to the advantage that dragging enables reasoning about invariant properties and to provide evidence about the validity of conjectures. Understanding of the dragging mode and its logic seems to be a critical issue. Teachers are thus challenged to appreciate the logic behind the drag test (or animation), what makes a construction a figure, what remains invariant in the hierarchy of dependencies, and why.

2.7 An interpretative summary

From the literature review, *mathematical/geometrical know-how* stands apart as an indispensable competence as the following phrases gleaned from the survey suggest: knowledge of geometry, (content) knowledge of school geometry (subject matter), conception of mathematics as a discipline, knowledge from different disciplines (other content), mathematical preparation, and knowledge of the philosophy of school mathematics. *Pedagogical competencies* are also quite predominant as the following phrases suggest: pedagogical content knowledge, creating an environment that emphasizes problem-solving, orchestrating classroom discourse, knowledge of students (students' level of geometric thought), choosing worthwhile tasks amenable to multiple solution strategies, teachers' ability to analyze their own teaching, conceptions about learning, teaching, roles of the teacher and students, classroom culture (inclusion of all students in lessons), problem posing skills, and open-ended learning. *Language competencies* are ominously implied by the following terms: communication and

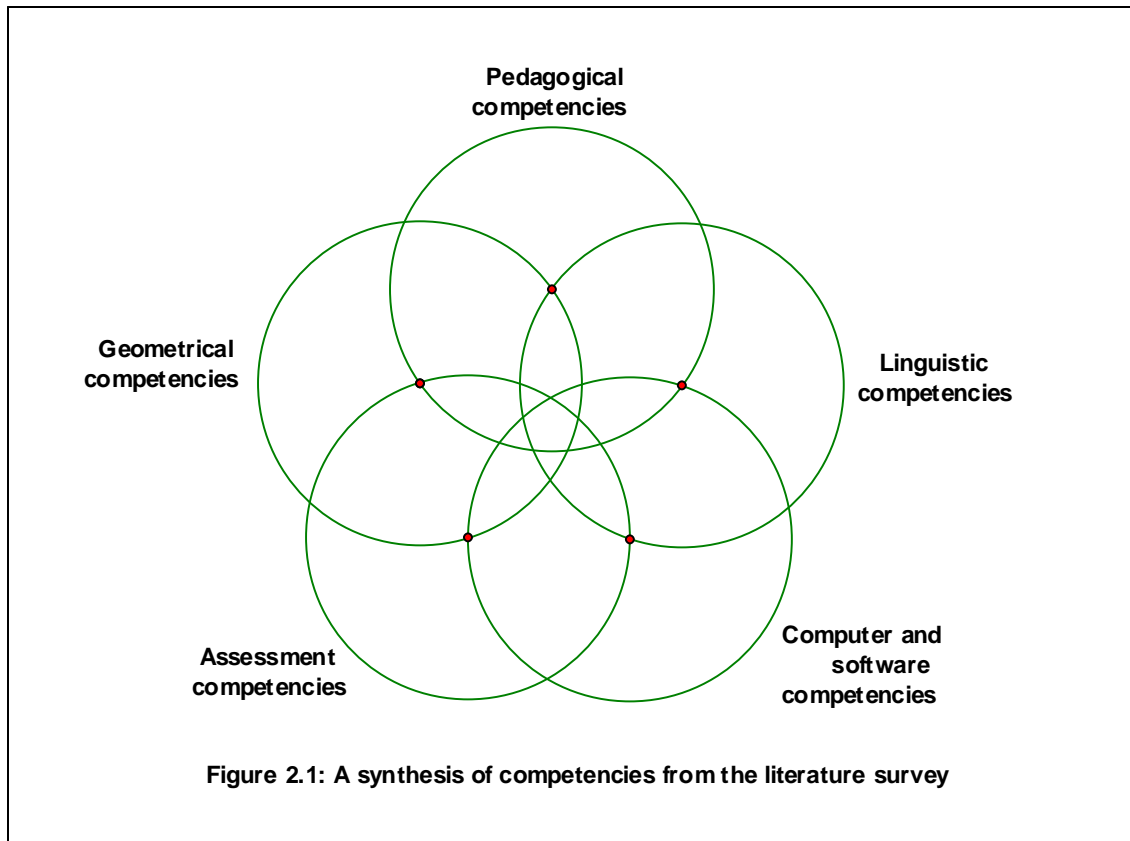
reasoning, social construction of knowledge, language specific to a particular van Hiele level, viewing mathematics as a foreign language, geometric terms for geometric objects and processes, proof as communication and ability to follow instructions in tasks.

Assessment competencies are suggested by the following host of phrases: student assessment, assessment of learning, ability to express descriptions and definitions of shapes and geometric processes, open-ended assessment questions, monitoring, alternative assessment strategies (journals, portfolios) the varieties of the van Hiele tests and/or level descriptors, the SOLO taxonomy and so on. Also hugely implied is a whole new complexion of *computer and software competencies* that DGEs foist into the classroom in terms of: new geometrical meanings, new interaction patterns, paradigm shifts in the functions of proof, dynamic experimentation and problem solving, conjecturing and drag testing, animation and several other validation methods,

A significant characteristic that can be conjectured about these competency categories is that they intersect and overlap considerably. Examples of such overlaps are knowledge of curriculum (which is both mathematical and pedagogical content knowledge, if not also directly related to assessment), choice of worthwhile tasks (mathematical content and pedagogical level of challenge), *Sketchpad* terminology (both software and linguistic, even geometric) re-conceptualization of proof (which straddles both deductive reasoning in geometry, software and linguistic competencies.)

The intersection of the competence categories appears to extend even beyond two sets. For example a reconceptualization of proof seems to permeate geometrical, software and

linguistic domains, and multiple problem solutions seem to span all categories. Figure 2.1 summarizes the competencies and their conjectured relationship.



This study attempts particularization in the context of a problem-centred approach to the teaching and learning of dynamic geometry. The next chapter describes the research methods and instruments used in this study to verify the efficacy of the tentatively identified competencies.

CHAPTER 3

RESEARCH METHODOLOGY

“If I were to prescribe one process in the training of men which is fundamental to success in any direction, it would be thoroughgoing training in the habit of accurate observation.” Eugene G. Grace

3.1 Introduction

This chapter outlines the research methodology used in this study, the supporting theoretical background to and the actual implementation. The research used the Teacher Development Experiment (TDE) approach as the main method of investigation and multiple sources of information – the pre-test, videotape, a teacher questionnaire and a structured group interview at the end. The theoretical framework of the TDE is sketched out in some detail with regards to its constructivist origins, relationship with the teaching experiment methodology, reflexive generation of theory from practice and its multi-tiered nature. The target population is described in broad terms before participants’ characteristics are given in detail with regards to level of course being undertaken, entry qualifications and mathematics curricula offered. The geometric content for this study is delineated and justified. Finally the data gathering process is outlined and divided into five phases each of which is described in considerable detail. As stated earlier, (compare 1.3) the research was not, however, about how teacher competences develop. Rather it sought to identify and analyze teacher competencies without reference to the developmental process itself. Hence the developmental aspect fell beyond the scope of the investigation.

3.2 Theoretical framework of the Teacher Development Experiment

Apart from the review of related literature, this study used a Teacher Development Experiment (TDE) approach as the main method of investigation. Simon (2000:337) states that the TDE builds directly on the emergent perspective ‘articulated by Cobb and his colleagues – the constructivist perspective on conducting teaching experiments with teachers’. Three shifts are delineated as constituting the emergent perspective. The central metaphor of students as ‘processors’ is displaced by that of students acting purposefully in an evolving mathematical reality (Sfard in Cobb, 2000:307). The second shift relates to an increased acknowledgement of the social and cultural aspects of mathematical activity (Cobb, 2000:308). These aspects have been elaborated upon in the articulation of the theoretical underpinning of the problem-centred approach in the previous chapter.

The third shift pertains to the relationship between theory and practice. Traditionally, theory has been seen to stand apart from and above the practice of learning and teaching mathematics. Teachers have been positioned as consumers of research findings generated from ivory towers located away from the classroom. In contrast to this subordination of practice to theory, the emergent perspective emphasizes a reflexive relationship wherein theory is seen to emerge from practice and to feed back to guide it (Cobb, 2000:308). In other words, the TDE is in keeping with the research philosophy of generating theory from practice, or the description of “what is possible’ (Fennema, 1981:vii). Wessels (2001a: 2) reaffirms this stance by pointing out that the building of theories or theorizing is one fundamental value or significance of research in mathematics education. The same can be hypothesized about mathematics teacher education. In fact, TDE allows researchers to generate increasingly powerful schemes for thinking about the

development of teachers in the context of teacher education (Simon, 2000:338). A knowledge base is needed then, that will guide the creation of novel effective teacher education programs. Such a base must include, in the first instance, the identification and analysis of aspects of teacher knowledge and skills that support a problem-centred approach to dynamic geometry.

In short, the term *teacher development experiment* is an attempt to distinguish it from the teaching experiment while recognizing the teaching experiment as the central building block of the methodology (ibid. p. 336). It has the dimensions of a multi-tiered experimental approach in that it takes as its objects of study, a teaching learning complex which encompasses three levels of participants: the researcher/teacher educator, the teacher and the students, and then two levels of curricula: teacher education curricula and the mathematics students' curricula.

Due to resource and logistical constraints, in this study, the student teachers were engaged in peer teaching. This limitation will be borne in mind in the interpretation of findings. The belief is held, though, that microteaching is a legitimate means for identifying and analyzing teacher skills while on college campus.

3.3 Sampling procedures for the group of learners

After seeking permission from college authorities, a notice was displayed on campus inviting *volunteer* final year mathematics majors to register with the researcher for possible selection to participate in the training to teach mathematics using computers. Those interested were invited to write a pre-test in order to be eligible for selection. Thirty-nine students turned up. Using a table of random numbers, a first sample of ten

(10) students was selected and placed on a five-week program acquainting them with the geometry of triangles and quadrilaterals using *Geometer's Sketchpad* software. After the initial five weeks another sample of ten third year student teachers was selected.

3.4 Description of participants

This investigation took place in an all black but multilingual class of third (final) year Diploma in Education (Primary) student teachers at Joshua Mqabuko Nkomo Polytechnic's Teacher Education Faculty. The student teachers were mathematics majors who had between zero and ten years temporary teaching experience prior to joining college and five terms (twenty months) teaching practice (attachment) experience. The teachers were taught in English, a second language to all of them.

3.5 Geometric content covered in study

The geometric topic chosen as most suitable for the empirical study was to do with triangles and quadrilaterals. There were two main reasons for this decision. First, in terms of the 'family of triangles' and the 'family of quadrilaterals', there is a rich variety (scalene, isosceles, right-angled and equilateral triangles, quadrilaterals include squares, rhombuses, rectangles, parallelograms, kites, and trapezia). This variety lends difficulties to many a student in terms of identification, description, definition and classification. The second reason was, as noted earlier, that the material fitted well within the syllabus.

3.6 Data gathering processes

3.6.1 Phase 1- Pre-testing and selection of participants

A pre-test was administered to 39 interested student teachers in the Mathematics major class week prior to the commencement of the study (week 1). The objective of the pre-

test was to ascertain would-be participants' current level of geometrical knowledge and estimate their van Hiele levels of geometric thought. Definitions of shapes in the pre-test would be classified as Non-standard (with a van Hiele level 3 weighting of 120%), Economical (100%), Uneconomical (50%), Very uneconomical (10%), Incorrect (0%), Unknown (0%). A participant scoring an average of 60% and above in the eleven shapes (triangles and quadrilaterals) would be deemed to be operating at van Hiele level 3 with respect to definitions). A score below would suggest the participant is operating at van Hiele level 2 or below. Table 3.1 outlines descriptions of the various levels of definitions as employed in this study.

CATEGORY OF DEFINITION	DESCRIPTION
Non-standard	Correct economical definition not usually found in textbooks
Correct economical	Definition containing only necessary and sufficient conditions
Correct uneconomical	Definition containing one extra true property which is not necessary
Correct but very uneconomical	Definition containing two or more true but unnecessary properties
Incorrect definitions	Definition containing necessary but insufficient conditions or definition containing both necessary and false properties or definition containing no correct property
Unknown	Unstated definition

Table 3.1 Categories of geometrical definitions

Furthermore, the pre-test sought to establish how much knowledge, if at all, the participants had of the problem-centred approach, the meaning of dynamic geometry, and the difference between a 'drawing' and a 'figure' or 'construction'. (see Appendix A)

3.6.2 Phase 2 – Introduction to Computers and *Geometer's Sketchpad*.

In this phase, the start of the TDE, participants were expected to gain preliminary experience with computer hardware components viz: Central Processing Unit (CPU), the monitor, the keyboard and the mouse. It was an opportunity to explain that there are two

types of software, namely systems software and application software. *Geometer's Sketchpad* falls under the latter type (see videotape 1).

After the introduction to computers the teachers would be introduced to the *Geometer's Sketchpad* software by working through several tours in the *Geometer's Sketchpad* workshop and learning guides to acquaint them with the software features and monitor their geometrical competencies. Teachers' were observed and assisted during sessions and their answer sheets were later analyzed in terms of how successfully they managed to carry out the instructions of the construction/geometrical tasks.

3.6.3 Phase 3 – Micro-teaching phase as extension of the TDE

The purpose of this phase of the TDE was to observe teachers in action, ascertain their levels of preparation, presentation, geometrical confidence, proficiency in *Sketchpad* skills, ability to involve students and monitor their participation and progress, ability to manage time as well as integration of skills in the didactic process. The phase began with the selection of a further group of 10 teachers who joined the initial group as tutees. The earlier group of students would become the leader teachers and take turns in pairs to prepare and deliver lessons in subsequent sessions. Pre-made sketches would be used in the form of triangles centres, proofs of Pythagoras's Theorem, similarity and congruency proofs for triangles, angles in a triangle (compare 4.4.1, and 4.4.2). Transformations and dynamic translation tasks would be embarked on. The kaleidoscope and tessellations would be constructed while the distances in an equilateral triangle sketch would be investigated (compare 4.4.3 and 4.4.4).

Teachers were observed in areas listed in Table 3.2 with the weightings shown.

#	Criterion being observed	%Wt	Score obtained			
			P	U	M	R
1	Level of teacher preparedness	15				
2	Mastery of <i>Sketchpad</i> skills	10				
3	Presenter - teacher interaction	5				
4	Teacher – teacher interaction	10				
5	Presenter whole class interventions	5				
6	Time management	5				
7	Mastery of mathematical content	15				
8	Monitoring of participants’ progress	10				
9	Conclusion	5				
10	Performance of participants	10				
	Overall impression on integration of skills	100%				

KEY: P - Prestructural understanding – very little understanding
 U - Unistructural Understanding – focus on one aspect
 M - Multistructural Understanding – focus on a number of aspects
 R – Relational Understanding – understanding of applications

Table 3.2 Lesson observation criteria

In the second sub-phase even some of the teachers who had only recently joined would have the opportunity to prepare and present their own lessons (compare 4.4.4). Whole class discussions would be encouraged as a way of wrapping up lessons. Throughout phases 2 and 3 the camera men would be encouraged to ask teachers to explain how they had executed their tasks, what geometry was involved, how the figures behaved under drag or animation, what problems they had faced and how they had overcome them (compare 4.4.1). The video recording arrangement was in keeping with Simon’s (2000) recommendation that recording of sessions in the TDE should be accompanied by videotaping. These recordings and their transcriptions are deemed essential for both ongoing and retrospective analyses.

3.6.4 Phase 4 – Mini-projects, questionnaire and group exit interview

Six teachers had the opportunity to write their mini-projects, exercises of their own design on a geometric topic of their choice within the stable of triangles and

quadrilaterals (compare 4.6.1). The objective of these was to encourage creativity in the design of activities and create an awareness of alternative techniques (compare 2.7). All participants in the research project would later respond to a questionnaire (compare 4.6.2), which sought biographical details as well as participants' experiences with the software (see Appendix B). To wind up, a group interview (compare 4.6.3) was conducted to establish from the participants' first hand experiences what they considered or deemed to be critical competences for a new group of teachers to be proficient in to teach dynamic geometry using the *Sketchpad* software effectively from a problem-centred perspective (see Appendix C for structured interview questionnaire used).

3.7 Conclusion

This chapter has attempted to justify the Teacher Development Experiment as a viable method of investigating teacher competencies. The use of a pre-test has also been justified as a means of determining the geometric thought levels of the teachers. Indications as to what instruments would be used for data collection and for what competencies have been made. The results of this investigation are presented and analyzed in the next chapter.

CHAPTER 4

DISCUSSION OF DATA PROCESSING AND RESEARCH DEVELOPMENTS

“I have yet to see any problem, however complicated, which when you looked at it in the right way did not become still more complicated”. Paul Anderson, *New Scientist*.

4.1 Introduction

In this chapter, data from various data gathering procedures are presented, processed and analyzed. First, pre-test results are presented and analyzed to ascertain teachers' entry knowledge of school geometry as well as their mathematical language, the PCA and dynamic geometry. Teachers' geometrical knowledge was further analyzed in the workshop sessions of the Teacher Development Experiment introducing them to *Sketchpad*. In the process *Sketchpad* skills necessary to teach dynamic geometry were also noted and analyzed as they occurred in the tasks. In the microteaching sessions teachers' abilities to prepare and present effective dynamic geometry lessons from a problem-centred perspective were investigated and analyzed in terms of the nature of the tasks, teacher's role, interaction patterns, classroom culture and the attainment of objectives (see Table 3.2). In the last phase of the investigation teachers' abilities to design their own tasks in *Sketchpad* were investigated, their own views about their *Sketchpad* experiences were sought through the questionnaire and a group interview.

4.2 Pre-test results

4.2.1 Overview

The aim of Section A of the pre-test was to ascertain teachers' knowledge of school geometry and to determine, if possible, their current level of geometric thought in terms of the van Hiele theory in order to select tasks within their level of understanding as far

as possible. Section B sought to ascertain teachers' knowledge, if at all, of the problem-centred approach and dynamic geometry. The results of the pre-test administered are summarized in terms of recognition, description and definition of shapes and processes as well as class inclusion and language competencies of those students who participated in the study for enhanced relevance to the experimental group characteristics.

4.2.2 Recognition, description and definition of shapes and processes

On the surface, the results showed that the teachers had a fairly strong background of geometric knowledge and this was expected from a mathematics major class. From an item-by-item analysis, it was evident that the teachers could identify the plane shapes and their properties, which is a Van Hiele level 2 geometric competence (compare 2.2.2).

With regards to descriptions/definitions of shapes, according to table 3.1 criteria, the following results were obtained for 16 teachers who later took part in the study.

Definition of Shape Type	Non-standard	Correct economical	Correct uneconomical	Correct but very uneconomical	Incorrect definition	Unknown
Isosceles Δ	0	3	12	0	1	0
Right Δ	0	11	4	0	1	0
Equilateral Δ	0	1	10	3	1	0
Scalene Δ	0	8	8	4	1	0
Rhombus	0	1	5	5	4	0
Square	1	3	9	2	1	0
Rectangle	0	2	9	3	2	0
Kite	0	1	2	1	12	0
Parallelogram	0	1	8	3	4	0
Trapezium	0	7	3	2	4	0
Cyclic quadrilateral	0	0	3	0	12	2

Table 4.1: Categorization of pre-test definitions given by 16 participants

What lacked ominously was economy of descriptions/definitions in spite the fact that questions stressed that the messages would be by telephone or telegram. For example,

Teacher X defined an isosceles triangle as ‘a triangle with two sides equal and two angles equal’. Once two sides are equal then the condition is necessary and sufficient. Similarly once two angles are equal the condition is necessary and sufficient. That is, the two conditions are equivalent and deductively derivable one from the other by the theorem proving process involving congruency. (compare 4.3.2.2). Recognizing definitions as equivalent is van Hiele level 3 competence (compare 2.2.3).

Figure 4.1 summarizes the pre-test knowledge of definitions of the 16 teachers.

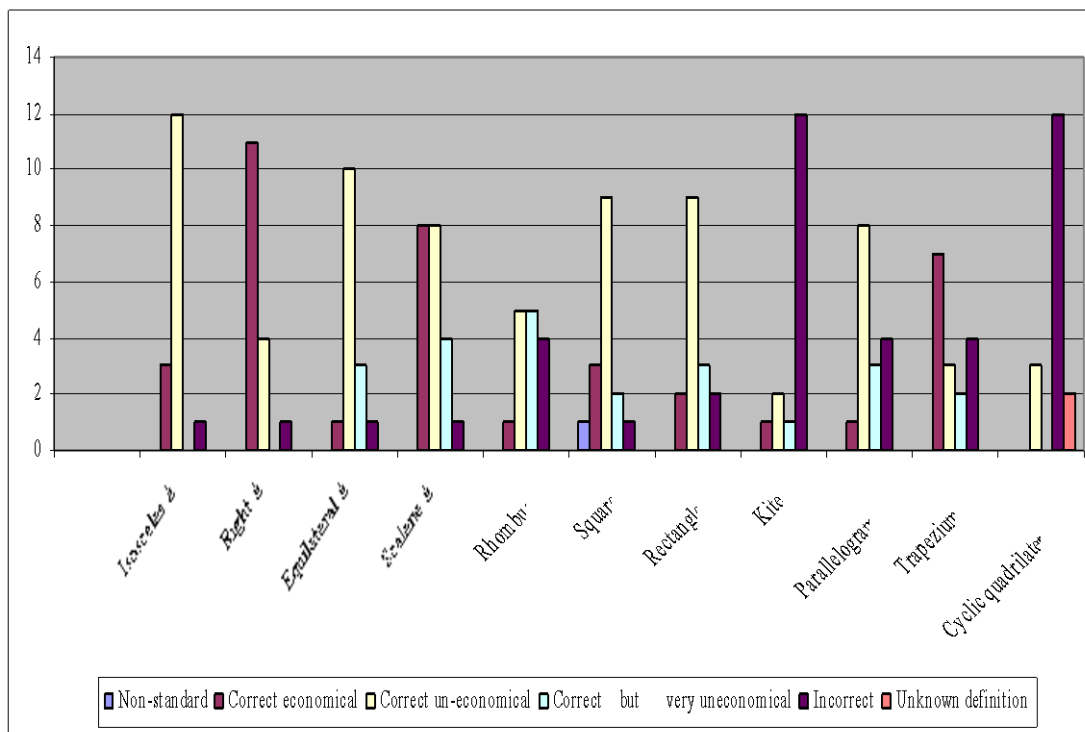


Figure 4.1: Bar graph of pre-test knowledge of shape definitions by type and shape out of 16 participants.

The results seemed to suggest that most of the participants were only at Van Hiele level 2 (compare 2.2.2) where they know the properties but cannot relate them to each other to establish necessity, sufficiency and economy. Using the key in Figure 4.2 below Table 4.2 estimates the Van Hiele levels of the 16 teachers in a scaling system analogous to that used by Gutierrez et al (1991:249).

Van Hiele level 1 Lower	(VH1L: $0 \leq x < 10$)%
Van Hiele level 1 Intermediate	(VH1M: $10 \leq x < 20$)%
Van Hiele level 1 Higher	(VH1H: $20 \leq x < 30$)%
Van Hiele level 2 Lower	(VH2L: $30 \leq x < 40$)%
Van Hiele level 2 Intermediate	(VH2M: $40 \leq x < 50$)%
Van Hiele level 2 Higher	(VH2H: $50 \leq x < 60$)%
Van Hiele level 3 Lower	(VH3L: $60 \leq x < 70$)%
Van Hiele level 3 Intermediate	(VH3M: $70 \leq x < 80$)%
Van Hiele level 3 Higher	(VH3H: $80 \leq x < 100$)%

Figure 4.2: Key to estimating the van Hiele levels

Participant #	Right Δ	Isosceles Δ	Equilateral Δ	Scalene Δ	Rhombus	Square	Rectangle	Kite	Parallelogram	Trapezium	Cyclic Quad	Total score	Average	Van Hiele level
1	100	50	50	50	30	100	50	0	0	0	0	460	41.8	2M
2	100	50	50	50	30	0	0	0	30	0	0	310	28.2	1H
3	100	50	50	50	0	100	50	0	0	0	0	400	36.4	2L
4	50	50	50	50	30	30	30	30	30	30	30	410	37.3	2L
5	50	50	50	50	0	50	50	0	50	80	0	450	40.9	2M
6	100	50	50	50	50	100	50	0	100	100	0	650	59.1	2H
7	100	50	50	50	0	30	50	0	50	100	0	480	43.6	2M
8	100	50	50	50	100	100	30	0	50	0	0	530	48.2	2M
9	0	0	0	0	30	30	30	0	0	100	0	190	17.3	1M
10	50	30	30	30	30	30	30	30	30	30	0	320	29.1	1H
11	100	50	50	50	30	30	50	0	30	50	0	430	39.1	2L
12	100	100	100	100	30	100	50	0	30	30	0	640	58.2	2M
13	100	50	50	50	0	50	30	0	0	0	0	330	30.0	2L
14	120	100	50	100	30	100	30	100	30	100	50	810	73.6	3M
15	100	50	50	50	30	50	50	30	30	100	0	540	49.1	2M
16	50	100	50	100	0	50	30	0	0	100	0	480	43.6	2M
	1320	880	780	880	420	950	610	190	460	840	80	7410	673.6	
	82.5	55	48.8	55	26.3	59.4	38.1	11.9	28.8	52.5	5		42.1	

Table 4:2 Estimated van Hiele levels for definitions per participant in the study

The results suggested that 12 out of 16 participants were at Van Hiele level 2, three at level 1 and one at level 3, hence the geometrical competencies (compare 2.6) were not, after all, as high as expected when quality of definitions was factored in. Figure 4.3 below gives examples of definitions of each level as found in the pre-test responses.

Non-standard definition of a square:

A quadrilateral with equal diagonals bisecting at right angles. (Improvised)

Correct economical definition of a square

A quadrilateral with all sides equal and all angles equal to 90° each. Teacher 14.

Correct uneconomical definition of a square

*A quadrilateral with all sides equal, all angles equal, and opposite sides parallel
Teacher 15.*

Correct but very uneconomical definition of a square

A square is a quadrilateral with all sides equal, four lines of symmetry, all angles are equal, interior angles add up to 360° . Teacher 10.

Incorrect definition of a rectangle (with necessary but insufficient properties)

*A rectangle is a quadrilateral with opposites equal and its diagonals do not bisect at right angles
Teacher 2*

Incorrect definition of a parallelogram (both correct and incorrect properties)

A parallelogram is a quadrilateral with 2 pairs of parallel sides, and four angles, which are equal. Teacher 1.

Incorrect definitions of a cyclic quadrilateral

*A plane shape drawn with a line joining the first and last point without angles.
Teacher 11*

There is no shape called a cyclic quadrilateral. Teacher 13.

A circle drawn inside a four-sided shape. Teacher 6.

Figure 4.3: Example definitions in each category

The definitions of the right triangle, and the square appeared to be among the most understood while those of the kite and the cyclic quadrilateral were the least. The teachers' level of understanding of geometrical definitions had to be borne in mind when selecting tasks as the van Hiele theory suggests (compare 2.2.6). Teachers thus have to be sensitive to their students' level of geometric understanding.

4.2.3 Understanding of class inclusion

The purpose of the class inclusion questions was to check teachers' grasp of class inclusion, which is van Hiele level 3 competence when mastered. Defining quadrilaterals in terms of some other quadrilaterals appeared to be problematic to a number of teachers.

Table 4.3 below summarizes the findings by definition type as defined in Table 3.1.

Definition of Quadrilateral	Non-standard	Correct economical	Correct un-economical	Correct but very uneconomical	Incorrect definition	Unknown
A square in terms of the rectangle	0	6	3	3	1	0
A square in terms of the rhombus	0	5	2	3	5	1
A rectangle in terms of the parallelogram	0	3	1	1	9	2
A rhombus in terms of the parallelogram	0	5	4	2	2	3

Table 4.3: Understanding of class inclusion in definitions of quadrilaterals by other quadrilaterals

There were no examples of non-standard definitions. However, the definition of the square as a rectangle elicited the highest number of economical definitions followed by the square as a rhombus and the rhombus as a parallelogram. The rectangle as a parallelogram elicited the highest number of incorrect responses followed by the square defined in terms of the rhombus. The rhombus as a parallelogram was the least known.

Once again this was a reminder of the inadequate van Hiele levels reached.

Using the key in Figure 4.2 yet again the class inclusion van Hiele levels of 16 participating teachers were estimated as shown in Table 4.4 below.

Participant #	A square in terms of the rectangle	A square in terms of the rhombus	A square in terms of the rhombus	A square in terms of the rhombus	Total score	Average	Van Hiele level
1	100	100	0	100	300	75	3M
2	50	0	50	50	150	37,5	2L
3	50	0	0	0	50	12,5	1M
4	0	0	0	0	0	0	1L
5	50	30	30	30	140	35	2L
6	50	100	0	100	250	62.5	3L
7	100	100	0	100	300	75	3M
8	100	50	100	50	300	75	3M
9	100	50	0	0	150	37.5	2L
10	30	30	50	30	140	35	2L
11	30	0	0	50	80	20	1H
12	0	30	0	0	30	7.5	1M
13	30	50	100	0	180	45	2M
14	100	100	100	100	400	100	3H
15	30	0	0	30	60	15	1M
16	100	0	0	100	200	50	2H
	920	640	430	740	2730	682.5	
	57.5	40.0	26.9	46.3		42.7	

Table 4:4 Estimated class inclusion van Hiele levels for each participant

The results showed 5 teachers operated at van Hiele level 1, 6 at van Hiele level 2 and 5 at Level 3 with respect to class inclusion. There were thus more individual differences, which the leader teachers would have to take into account. These differences were quite surprising. If a teacher operates at level 1 with respect to class inclusion but is at level 2 with respect to quality of definitions what geometric level can we ascribe as typical? If the teacher professes complete ignorance of the existence of a cyclic quadrilateral but is aware of the other quadrilateral types and triangles what level can we ascribe to her?

These are vexing questions for the van Hiele theory (compare 2.2.7).

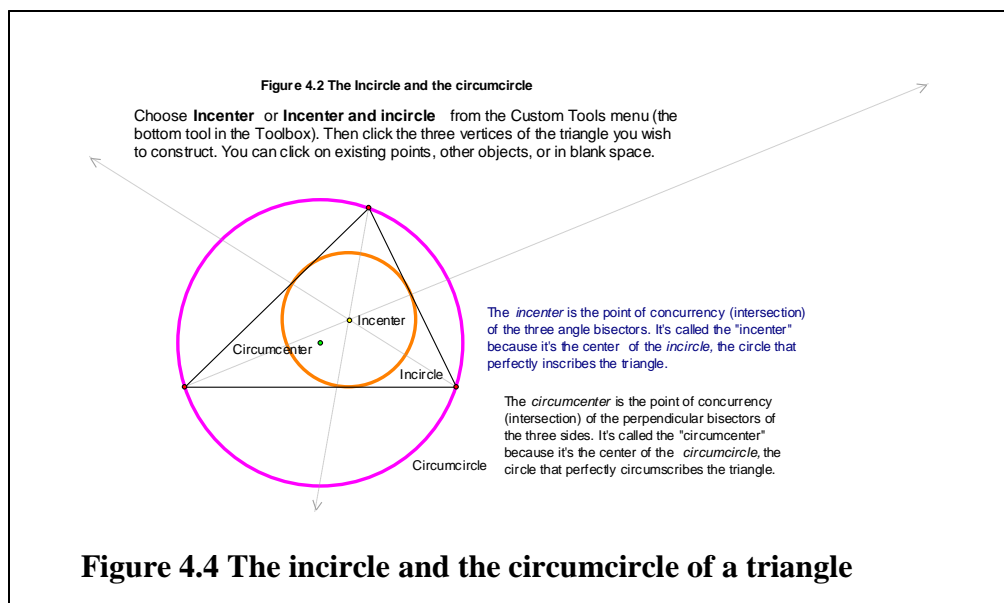
4.2.4 Understanding of geometrical language and relationships between properties

Thirty-five (35) out of thirty-nine (39) teachers could not describe how to construct the in-circle and the circumcircle of a triangle. They could not state correctly which

properties had to be made use of in order to come up with the correct constructions.

Understanding relationships between properties is a van Hiele level 3 competence and the gap in the teachers' knowledge was not expected since these two triangle circles are part of the Ordinary Level syllabus passed by all of them. Could it be conjectured that students can attain and lose a particular van Hiele level ability? This is another vexing question about the van Hiele theory and lends more credence to the oscillation hypothesis proposed by Fuys, Geddes and Tischer (1998) (compare 2.2.7).

Furthermore, some answers as to how one could construct the inscribed and circumscribed circles suggested expressive language difficulties (compare 2.7). For example, one teacher wrote: *'to draw an in-circle, bisect the angles and where the lines meet, draw the circle'*. Another wrote: *'bisect the sides and where the lines meet draw a circle'*. In both cases there was an intuitive understanding of what has to be done but limited verbalization of the processes. Figure 4.4 below shows these circles.



Language as a barrier also manifested itself in teachers' attempts to express relationships between properties. Examples below illustrate this dilemma.

Teacher Y: 'The relationship between the perpendicular bisector of the base of an isosceles triangle and the angle at the apex is that the two are equal'.

This response apparently evidenced the teacher's inability to distinguish between a 'perpendicular bisector' and 'an angle' which could be a *conceptual error*, a discrepancy between the concept name and the concept image.

Teacher W: The bisector bisects the angle into two equal angles.

From this response it was apparent that the meaning of 'bisect' was not understood as sufficient division of the whole into two halves. This appeared to be a *redundancy error* akin to lack of economy in definitions (compare 4.2.2 and 4.2.3). The second language factor, in combination with the technical nature of geometrical language needed further investigation. The former is not highlighted in the Van Hiele level theory and the PCA, but left implied. Steffe and Thompson (2000:277) emphasize that in the teaching experiment it is the job of the teacher-researcher to continually postulate possible meanings that lie behind students' language and actions. Language competencies seem to be extremely necessary (compare 2.4 and 2.7).

4.2.5 Knowledge of PCA and dynamic geometry

From responses of the sample groups the following elements of the problem-centred approach were identified: The teacher's role was characterized as in the box below.

- He/she is there to facilitate and monitor proceedings
- He/she prepares problems (tasks) for students to work out
- He/she is the advisor
- He/she finds relevant and necessary material to help the child solve the problem
- He/she should facilitate the learning process, etc

Characterization of the teacher's role

Students' roles identified included the following

- Students should do the bulk of the work, i.e. the child-centred approach working together discussing and sharing ideas
- The student's role is to work out the problem
- The student devises his own means to come up with the solution
- The child is there to find the solution
- Pupils strive to get the answer to the problem in groups
- Pupils will be result oriented and work the problem through discovery.
- The PCA considers the interests of the students

Characterization of the students' role

From these sets of the responses it appeared most of the teachers were aware or guessed correctly that the problem centred approach required the active involvement of the learner in the solution process. However, in terms of classroom culture the responses focused on group work and student-to-student interaction per se. Respect for each learner's solution efforts or contribution was a missing detail. Teachers thus showed prestructural understanding of the PCA approach (compare with SOLO criteria in Table 3.2). The most frequently mentioned disadvantage of the PCA was that it is time consuming or time wasting. Dynamic geometry was virtually an unknown entity to all the participants.

4.3 Introduction to computers and *Geometer's Sketchpad* software

4.3.1 Getting used to the computer.

After the session introducing the initial sample/group of participants to computers there was considerable excitement as the teachers could have the hardware pieces and their functions. This was the commencement of the TDE (compare 3.6.3).

4.3.2 Overview of mathematical tasks and software features in them

4.3.2.1 Constructing a square: Source(s) – Guided Tour 1 in Jackiw (2002:16-20)

In this first workshop session of the TDE teachers learnt about *Sketchpad's* basic tools and how to construct segments using the point and line tools and the segment tool on its

own (freehand tools); circles using the compass tool or segment and compass tools, how to select and drag objects; how to construct points at the intersection of two geometric objects; perpendicular and parallel lines, how to save *Sketchpad* documents and how to backtrack the construction process, using the undo command. These were basic *software skills* that were distinctly a constituent meaning of the geometrical objects they construct.

From this activity there were signs that inadequate knowledge about what *Sketchpad* action to take next can stall the progress of a lesson. Apart from the use of the tools per se, this session/tour also brought to light new meanings. The point tool draws a point, which is not a point but a very small circle that can even be shaded or coloured. The compass tool drew a circle in a dragging manner that is remotely related to the circular motion of the traditional compass. It seemed helpful to first let students use other construction methods from the construct menu, viz circle by center and point, and circle by centre and radius, which apparently carry more resemblance to the use of the compass.

The straightedge tool on the other hand re-affirmed what is often not emphasized enough in paper and pencil geometry: the difference between a line and a line segment. The on-screen display of a line emphasized its infinite length stretching from one end of the screen to the other. The ray, a term borrowed from physics, was illustrated in *Sketchpad* as having a source and direction and extending beyond the screen without reference to any magnitude, a discrepancy to be noted in the integration of knowledge from other disciplines (compare 2.5 and 2.8). The construction of perpendicular and parallel lines in *Sketchpad* required the selection of both a segment and a point through which the line must pass. This seemed to accord well with the Euclidean definitions of parallel and perpendicular lines.

4.3.2.2 A Theorem about quadrilaterals (source: Jackiw (2002:21 – 24))

The *Sketchpad* objectives of this tour were to construct a polygon using the segment tool, to label a geometric object's, to measure lengths and angles, to construct the midpoint of a line segment, and to create captions to accompany a sketch (compare 2.8 and 5.2.3). Of didactical value was that '*discovering a theorem for themselves or actively exploring its consequences can make a huge difference in students' level of recall*' (Jackiw, 2002:5). Teachers explored and conjectured in readiness for deductive reasoning and proof later (compare 2.6.2 and 2.6.3). The theorem is illustrated in Figure 4.5 below.

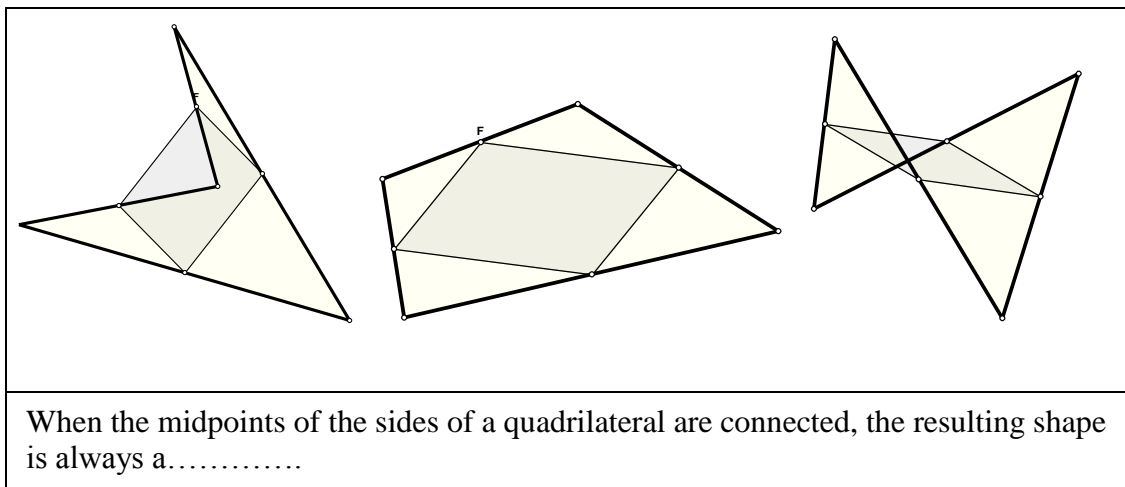


Figure 4.5 A theorem about quadrilaterals

Participants were encouraged to record their conjectures. Rorisang gave the responses in the box below. She correctly reasoned that because the opposite sides of the mid-point quadrilateral were always equal then it must be a parallelogram, which was a necessary and sufficient condition from which the equality of opposite angles and parallelism of

1. The opposite sides of the inside quad are equal
2. If a point (vertex) is dragged to form a concave quad the inside shape (quad) still has 2 opposites equal.
3. If the outside quad is dragged and formed into a crossed quadrilateral the opposite sides remain equal
4. If it is dragged to form a convex the opposite sides remain equal.
5. Therefore the inside quadrilateral will always be a parallelogram.

Rorisang's responses to the midpoint quadrilateral task

sides could be derived deductively (compare 2.6.7 and 4.2.2). Similarly the parallelism of opposite sides is a necessary and sufficient condition for a quadrilateral to be a parallelogram and the equality of angles and equality of opposite sides can be derived deductively (see Cases 1 and 2 below).

Given $AB=DC$, $BC=AD$

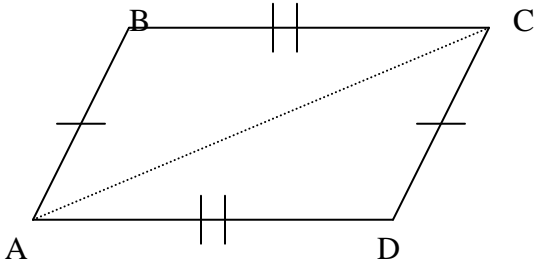
RTP: That $\angle ABC=\angle ADC$,
 $\angle BAD=\angle BCD$, and $AD\parallel BC$, $AB\parallel DC$.

Construction: Join AC

Procedure: $\triangle ABC \equiv \triangle CDA$ ($AB=DC$, $BC=AD$, $AC=CA$, thus SSS case)

Hence $\angle ABC = \angle ADC$, $\angle ACD = \angle CAB$ (thus $AB\parallel DC$, by alt \angle s rule)

$\Leftrightarrow \angle ACB = \angle CAD$ (thus $BC\parallel AD$, by alt \angle s rule)



Case 1: Given that opposite sides are equal

Given quadrilateral ABCD, where $AB\parallel DC$, $AD\parallel BC$ (opposite sides \parallel)

RTP: That Opposite sides are equal, and opposite angles are equal.

Construction: Join AC

Procedure:

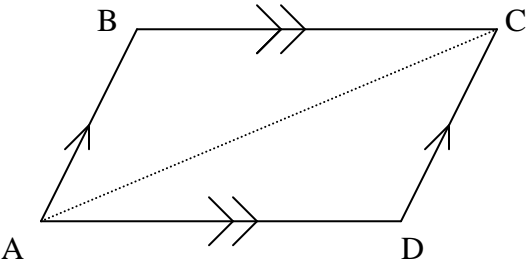
$\angle ACB = \angle CAD$ (Alt \angle s)

$\angle BAC = \angle ACD$ (Alt \angle s) and

$AC = CA$

Hence $\triangle ACD \equiv \triangle CAB$, (ASA)

Thus $AD=BC$, $AB=DC$, and opposite angles are equal



Case 2: Given that opposite sides are parallel


It therefore appears necessary for teachers to be aware of the mutual interdependency of properties and how these can be connected through short deductive chains of argument, which is Van Hiele level 3 competence which is a prerequisite for formal deduction.

4.3.2.3 Attempts to construct other quadrilaterals

In a free response exercise to construct other quadrilaterals, Nathan presented the piece of work below.

1. Constructing a rectangle

6,28 cm

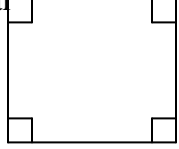


- opposite sides are equal
- all the angles are 90°

3,69 cm

2. Constructing a square

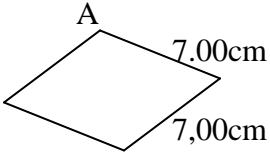
4,03 cm



- all sides are equal
- all the angles are 90°

4,03 cm

3. Constructing a rhombus



-all the sides are equal, opposite angles are equal,
-angle BAD = $103,77^\circ$, angle BCD = $103,77^\circ$
-angle CBA = angle ADC = $76,23^\circ$

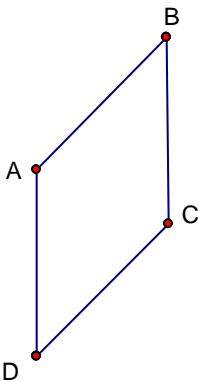
Exercise

- A **rectangle** is a *parallelogram* with all the angles at right angle and 2 opposite sides equal.
- A **square** is a *parallelogram* with all sides and angles equal.
- A **rhombus** is a *parallelogram* with all sides equal.

Nathan's piece of work

An analysis of the work showed that the teacher 'created' or 'formed' the quadrilaterals rather than construct them using their properties (compare 4.3.2.1). Starting with a parallelogram the teacher dragged it into a rectangle and this maintained parallelism and equality of opposite sides implying that a rectangle to be a special case of a parallelogram (with all angles equal). Proceeding to the square, the teacher manipulated the rectangle and adjusted it until it became a square of sides 4,03 cm and eventually concluded that a

square is a parallelogram with all sides and angles equal. In this way, a better understanding of class inclusion (van Hiele level 3) appeared to have been facilitated by *Sketchpad* in the same way as noted by de Villiers (1994:17). Sinclair (2003:300) also concurs by pointing out that when using dynamic software a student can inadvertently create a special case by dragging, something that is not possible with the generic case that teachers and textbooks often use. The formation of a rhombus similarly began with a parallelogram which was manipulated to form equal sides of 7 cm each and led to the conclusion that a rhombus is a parallelogram with all sides equal this time. Thus again class inclusion was facilitated by the *Sketchpad* capabilities (compare 4.2.3). An open-ended exploration in constructing rhombi (in Jackiw, 2002:5) ended without any of the teachers managing to come up with a single method. This was possibly first, due to time constraints and, secondly, also because coming up with different construction methods requires full Van Hiele Level 3 understanding (to see the inter-relationship between properties), and as seen above (compare 4.2.2 and 4.2.3) many of these teachers had not attained that level. The presenter had to demonstrate a method using reflecting two sides of an isosceles triangle as shown in Figure 4.6 below.

<p><u>Construction steps for the rhombus</u></p> <p><u>Step 1:</u> Use segment tool to construct line AB</p> <p><u>Step 2:</u> Rotate segment AB through 45° about B</p> <p><u>Step 3:</u> Construct segment AC</p> <p><u>Step 4:</u> Mark segment AC as mirror.</p> <p><u>Step 5:</u> Reflect $\triangle ABC$ on AC</p> <p><u>Step 6:</u> Hide AC, and label image of B as D</p> <p style="text-align: center;">Figure 4.6 Rhombus construction example</p>	
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Observations made were that the rhombus constructed was a rigid one. That is, the construction procedure over-constrained it to acute opposite angles of 45° each and

obtuse opposite angles of 135° which were then maintained at those sizes whatever dragging or animation was done. The presence of multiple solution strategies enabled teachers to try different solution routes thus promoting creativity (compare 2.4). In the end, though, it also emerged that there are possibilities of over-constraining, flexibly constraining or under-constraining a construction as alluded to by Key Curriculum Press (2002:78). This is a unique feature of *Sketchpad*, to be borne in mind when their students engage in open-ended explorations of constructions. Awareness of these software constraints might be an essential constituent of software competencies (compare 5.2.3).

4.3.2.4 Triangle Centers.gsp sketch

The first activity on triangle centres entailed the construction of circumscribed and inscribed circles, the orthocentre and the centroid. Below are Rorisang's responses.

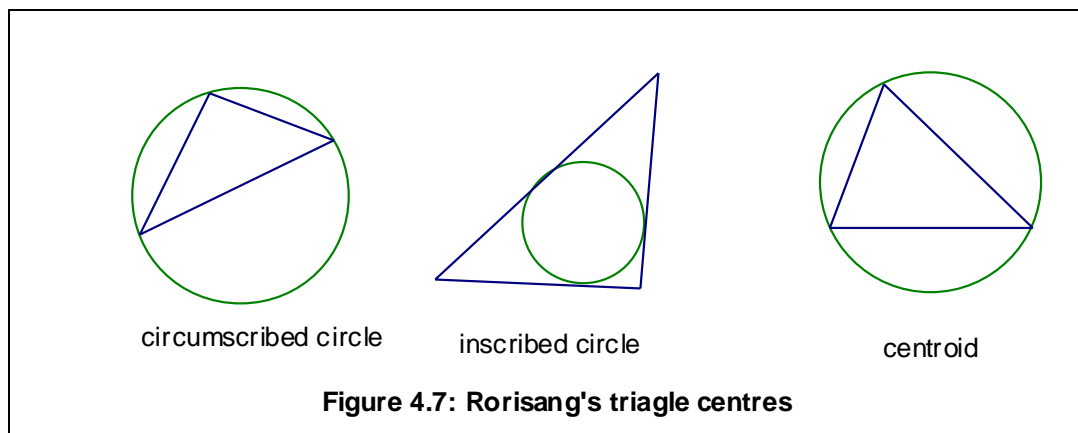
Centroid: circle constructed by using the midpoints of the triangle

Incentre: The second constructed circle was an incircle using the angle bisectors

Circumcentre: The third was the orthogonal centre constructed by bisecting the midpoints of the **angle** at 90° .

The diagrams in Figure 4.7 were used to illustrate.

Rorisang's triangle centres



Below are Tichaona's responses.

- 1) We first drew a centroid at (and) found by joining the mid-point p/center after weight of the triangle
- 2) This is the centre of the in-centre where angle bisectors meet they touch the midpoints
- 3) The circumcentre meaning angles bisect one another at 90° .

Tichaona's triangle centers (see photocopy of original)

Just as in the pre-test (compare 4.2.4) there was evidence of language difficulties in describing the construction process accurately. Rorisang appeared to confuse 'circle' with 'centre' which confusion could have lead to misunderstandings as to which concept was precisely being referred to. The term 'bisecting the midpoints of the angle at 90° ' appeared to be referring to the angle bisector. Nonetheless the diagrams drawn by Rorisang showed intuitive understanding of the inscribed and circumscribed circles but not the centroid.

Tichaona, on the other hand, was not clear as to what the triangle midpoints had to be joined to, to form medians point of concurrency is the centroid. His definition of the in-centre in terms of where the angle bisectors meet and then touching midpoints of the triangle suggested some lack of close attention to sentence meaning or syntax. Similarly, for angles to bisect one another at 90° showed inability to terminologically separate the 'perpendicular bisector' from 'angles' (compare 4.3.2.2). On *Sketchpad* skills it was noticed that the *Sketchpad* construction of the midpoint of a line did not show the arcs, which are a common emphasis in ruler-and-compass contexts. GSP4 could not construct the intersection of more than two geometrical objects (compare 4.3.2.3).

After several sessions, Tichaona was able to come up with the descriptions below.

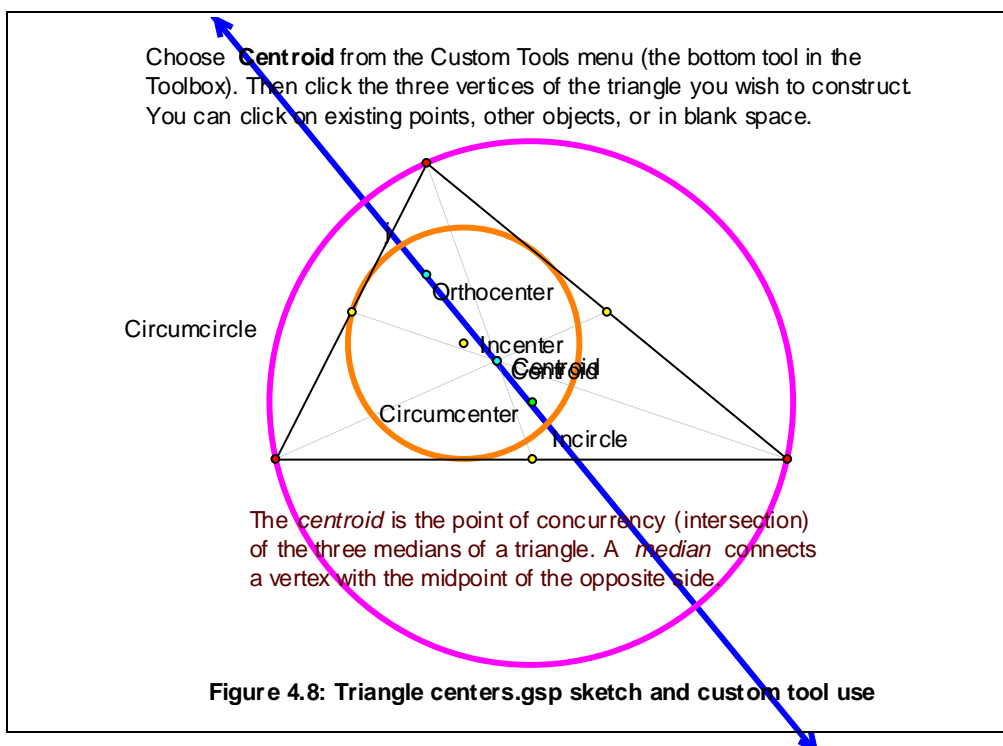
- a) The in-centre is found by joining/constructing perpendicular bisectors of the three sides. It is the circumcentre, which, gives the circumscribed circle.
- b) The incentre is found by joining the angle bisectors of the three angles. It gives the incentre, which gives the inscribed circle.
- c) Dropping perpendicular lines from the vertices forms the orthocentre.
- d) The centroid is found when we join the vertex and the midpoint (opposite) then is called the centroid.

When you drag the triangle the centre change

Incentre, circumcentre and centroid are always in a straight line when the triangle is moved (dragged) by the vertex the segment formed is called the Euler segment. The orthocentre sometimes moves out(side) of the centre (triangle).

Tichaona's triangle centres (see photocopy available)

An analysis of Tichaona's second set of responses seemed to imply that if teachers could be reflective and critical thinkers then they might significantly improve the way they express their mathematical ideas and consequently ascend to higher Van Hiele thought levels. The terminology in the construction sub-menus seemed to have contributed to more accurate descriptions of geometrical objects and processes (compare 4.2.2).

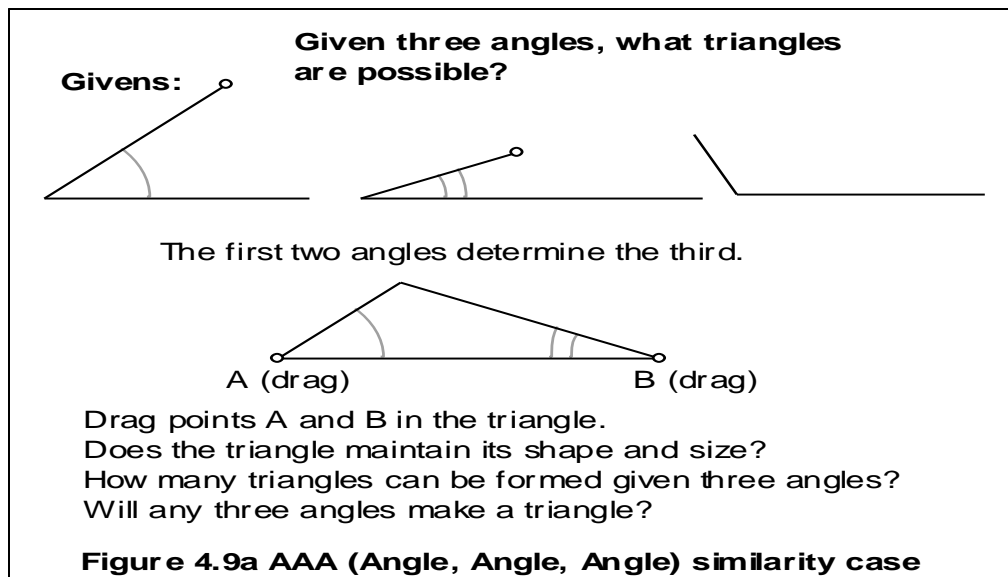


This ability could also be cultivated among their students as an effort towards continuous precision in mathematical descriptions more so if they worked as a community of practitioners or learners. The software capabilities could be a contributory factor to improved reflective ness. *Sketchpad* skills practiced in the third session of triangle centres were of opening pre-constructed sketches, their respective pages and the use of custom tools. Figure 4.8 shows **Triangle Centers.gsp** sketch (also see videotape 2).

4.4 Selected micro-teaching sessions of the TDE

4.4.1 Triangle.gsp pre-constructed sketch

The aim of this phase was to observe teachers in action (compare 3.6.3) to ascertain their levels of preparation, presentation (confidence in geometrical and *Sketchpad* skills, student-to-student interaction, teacher interventions, time management and general lesson flow. The phase began with one of the teachers in the first group, Nathan, taking the class through congruency theorems in the **Triangles.gsp** multipage pre-made sketch. By then, a second group of participants had joined. Congruency theorems were new material at the TDE sessions although being part of the Ordinary Level syllabus.



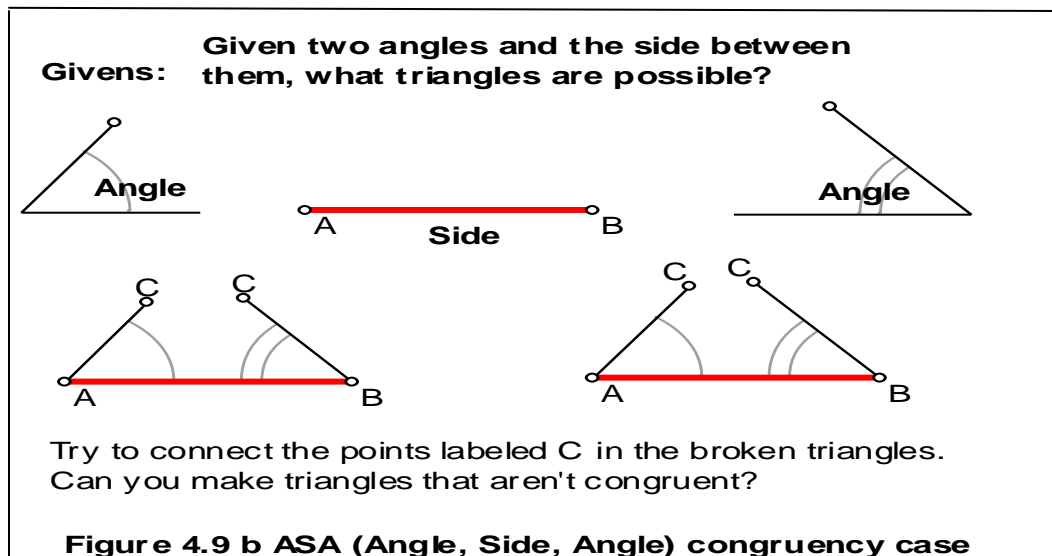
The construct of congruency plays a central role in many geometrical proofs and could scaffold participants to a higher van Hiele levels (3) (compare 2.2.3, 2.2.4, 2.6.2, 2.6.3,

2.6.5 and 4.3.2.2). In the first case, which is similarity, (Figure 4.9a), all six groups were able to notice that the triangle maintained its shape but not its size (see Table 4.5 below).

Question in AAA similarity case	Yes	No
Does the triangle maintain its shape?	6	0
Does the triangle maintain its size?	0	6
Will any three angles make a triangle?	3	3

Table 4.5 Responses to the triangle similarity case (AAA)

However, when it came to the number of triangles that could be formed given three angles four (4) groups gave ‘one’, one (1) gave ‘two’ and one (1) gave ‘three’ as their answers. This suggested some confusion between ‘one shape’ and ‘one triangle’ and the notion of ‘one figure’ in *Sketchpad*, which can be varied but retaining its properties. No group acknowledged the existence of an infinite number of similar triangles as possible. On whether any three angles could make a triangle, one group was alert enough to notice that it would be the case only if the sum of the angles was 180° . Three (3) groups just gave ‘yes’ as an answer without elaborating and the remaining two gave ‘not always’ and ‘not possible because some angles might be more than 180° ’ respectively.



In the ASA congruence case (Figure 4.9b) five groups realized that the triangle formed by joining points ‘C’ would always be congruent (see Table 4.6 below).

Given two sides and the angle between them, what triangles are possible?

Givens:

Try to connect the points labeled B in the broken triangles.
Can you make two triangles that aren't congruent?

Figure 4.9c SAS (Side, Angle, Side) congruency case

In the SAS congruence case (Figure 9c) five(5) groups correctly indicated that no pair non-congruent triangles could be created by joining points 'B' (see Table 4.6). Some teachers took the initiative to test their conjectures by measuring and one group said it was impossible to construct non-congruent triangles 'because the angles won't change'.

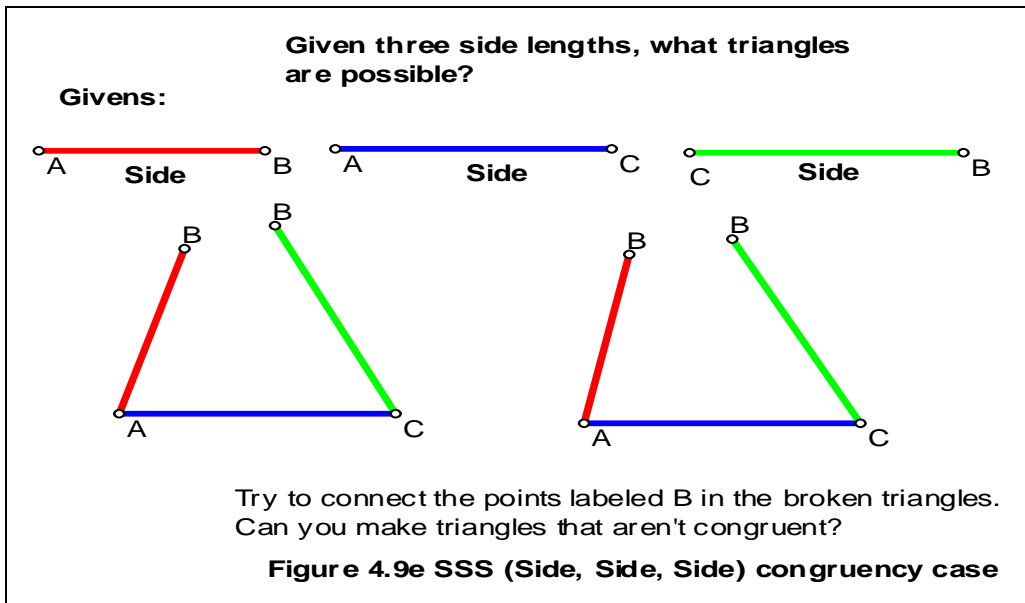
Given two sides and the angle not between them, what triangles are possible?

Givens:

Try to connect the points labeled B in the broken triangles.
Can you create two non-congruent triangles?

Figure 4.9d SSA (Side, Side, Angle) congruency case

In the SSA congruence case (Figure 4.9d) five groups out of six gave the wrong answer of 'yes' suggesting this was a less familiar case to most of the teachers (see Table 4.6 below).



In the SSS congruency case three (3) groups correctly gave ‘no’ as an answer, two wrongly gave ‘yes’ as an answer (see Table 4.6). The last group simply stated that ‘the points ‘B’ can’t merge’. In other words some limitations with the sketches were that the points to be merged would not merge using the MERGE POINTS command. Instead, points B would coincide rather than merge or connect.

Congruency case and question	Yes	No	Other
ASA case – Can you make triangles that aren't congruent?		5	1
SAS case - Can you make triangles that aren't congruent?	1	5	
SSA case - Can you make triangles that aren't congruent?	5		1
SSS case - Can you make triangles that aren't congruent?	2	3	1

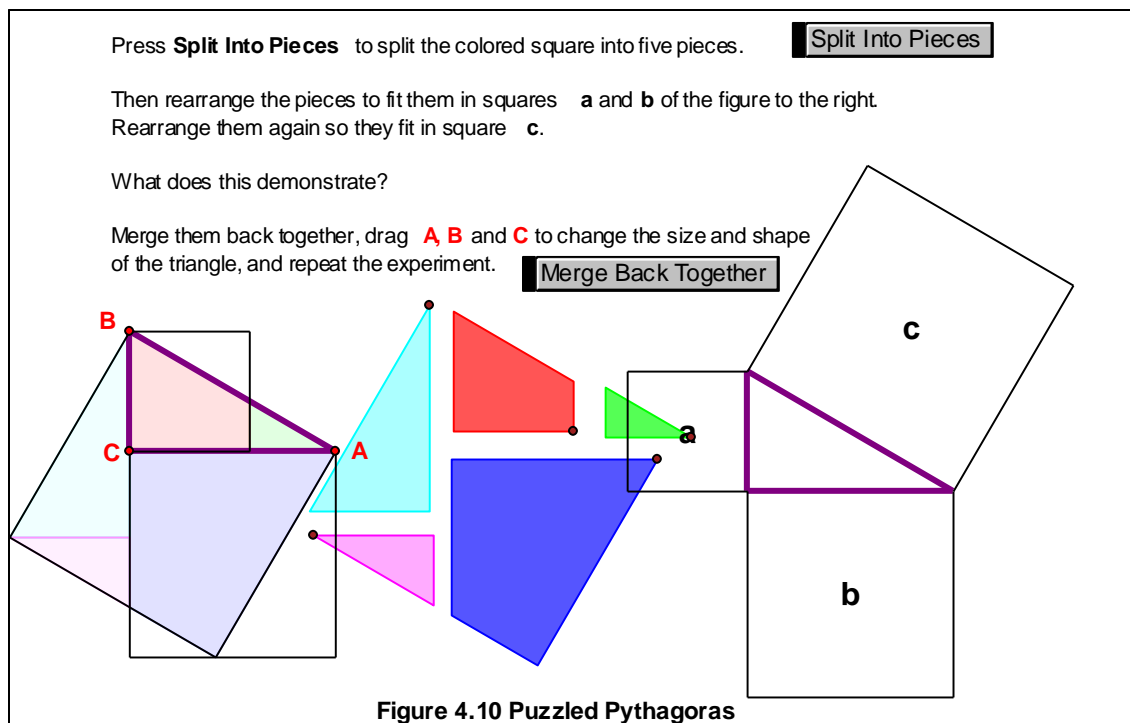
Table 4.6 Teacher performance in the congruency tasks

Although the teacher-leader’s role was a passive one in that he lacked confidence (see Table 4.7), the fact that the participants worked autonomously in groups appeared to have led to a higher level of participation and achievement in tasks than in previous individual work and this seemed to corroborate the importance of letting students explore and conjecture collaboratively (compare 2.4 and 2.6.6). For example, Border’s group was not sure as to what congruency meant, and they quickly checked with the group nearest to them to confirm. After the consultation, they then concurred to say it meant ‘the same’, but later adjusted to ‘equal’, which was more accurate (see videotape 3). However, one

group, which had no member from the first group to assist them with computer skills, described their first experience as having been a ‘nightmare’. This suggested that whereas basic *computer skills* might appear trite to experienced users, there could be obstacles for new users (compare 2.6.6 and 4.6.2) Teachers might need to be patient with their students as they introduce dynamic geometry in their classes.

4.4.2 Pythagoras.gsp presented by Qhubekani

This task sought to consolidate teachers’ ability to consolidate teachers’ ability to use pre-made sketches in *Sketchpad*. In using pre-made sketches, though, teachers needed to adopt a critical mind. The labeling of Puzzled Pythagorean squares as ‘a’, ‘b’ and ‘c’ as shown in Figure 4.10 could have been a source of perturbations in some teachers in that it appeared to contradict the algebraic version of the theorem yet in essence it expands it by giving it a geometric meaning in terms of area.



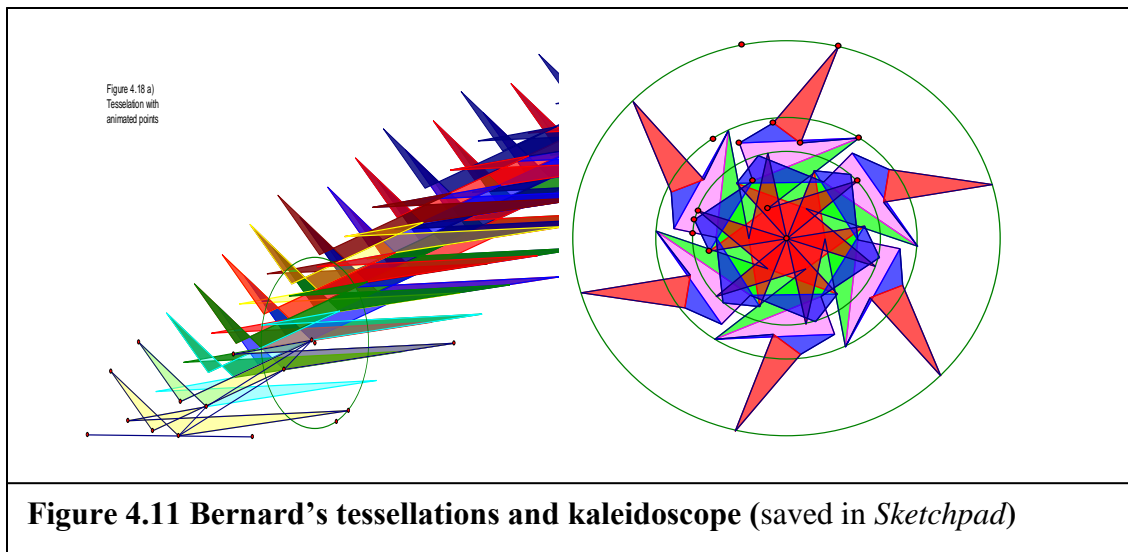
From the participants’ responses, however, the visual proofs of the theorem opened doors to analytic and deductive reasoning (compare 2.6.5 on progression from intuitive, to

analytical, inductive and deductive reasoning, and 2.6.4 on role progression of a DGS from being a visual amplifier to an essential constituent meaning of tasks). Puzzled Pythagoras was apparently most straightforward. Overall, the visual proofs were elegant and teachers appreciated compare (compare 4.5.2).

Being familiar geometric content, (at least for the Behold and Puzzled Pythagoras sketches), the teacher leader's confidence in presentation was markedly upbeat and teacher-to-teacher interaction was considerably evident. Time was well managed and participants' progress closely monitored. The teacher-leader's advance preparation contributed to the effectiveness of presentation (see Table 4.7).

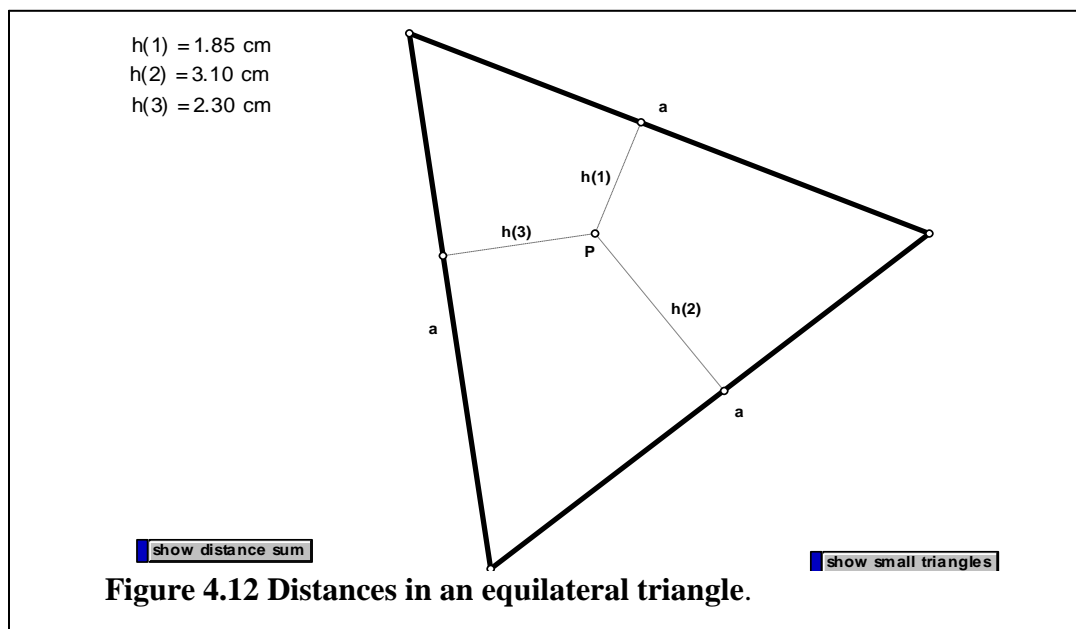
4.4.3 Transformations, kaleidoscope, tessellations

These activities were of mathematical and aesthetic value. The animations revealed the modeling power of *Sketchpad*. Besides offering practice in transformational geometry of triangles and quadrilaterals, the activities taught how to merge points to circles. The teacher leaders Andrew, Tichaona and Gibson also prepared in advance. During the lesson activities, they and their partners took time to move around and check the progress their peers were making. They would interject the whole class only as and when necessary (compare with Towers' teacher interventions in 5.2.4). Andrew had the most timely and well calculated interventions. At the end of their lessons, they would wind up with a whole class discussion. Lively discussions took place during the lessons and peers actively consulted the next group when they all got stuck (see Table 4.7). One teacher, Bernard, was able to produce the tessellation and kaleidoscope in Figure 4.11 below:



4.4.4 Distances in an equilateral triangle by Ian (Source, De Villiers, 1999: 23-26)

A peer who belonged to the second group of teachers prepared for this activity and involved the use of the pre-made sketch **Distances.gsp**. Opening of pre-made sketches in *Sketchpad* was no longer a major hurdle. with the next group that would have succeeded.



Nathan and his partner responded to this activity as shown in text box below. The shipwreck survivor story that provided the background to this problem couched geometry in a realistic context (compare 2.4 on realistic mathematics education).

1. The sum of h_1 , h_2 and h_3 does not change no matter what point you drag P as long as it is within the triangle.
2. The sum of h_1 , h_2 and h_3 does not change irrespective of the extent of the dragging.
3. Outside the triangle the sum of lengths of h_1 , h_2 and h_3 increases.
4. The sum of h_1 , h_2 and h_3 is always constant as long as P is always in the triangle because when P is dragged around the length one h increases while the other h is reduced.
5. They are always equal increasing the length of one leads to the increase of the other two.
6. Area = $\frac{1}{2} ah$.
7. $A = \frac{1}{2} ah_1 + \frac{1}{2} ah_2 + \frac{1}{2} ah_3 = \frac{1}{2} a(h_1 + h_2 + h_3)$
8. Total area = $\frac{1}{2} ap = \frac{1}{2} a (h_1 + h_2 + h_3)$
9. It is an equilateral triangle with all the sides being equal.
10. The sum of the distance that is h_1 , h_2 and h_3 is equal to the total altitude of the triangle.
11. Q5 – the sides were not going to have been labeled the same letter since they are not equal
 Q6 - $\frac{1}{2} ah$ was not going to work
 Q7 – adding the 3 areas would not give the total of the whole triangle
 Q8 – the sum of the small triangles would have not given to the bigger one
 Q9 – the sum of h_1 , h_2 and h_3 is not equal to P .
 Q10 – the sum of the distances is not equal to the total height.

Responses from Nathan's group (see Appendix E for the task)

All the groups managed to work through the main task and this suggested effective time management by Ian. The mathematical value of the task lay in the careful guidance of teachers into deductive reasoning (proof) and the steps were easily followed. However, teachers had problems in organizing a formal explanation. The value of *Sketchpad* was that the teachers and their students could first check the classical theorem experimentally under the continuous change capabilities of the software before developing a deductive explanation (proof).- a Van Hiele level 4 activity (compare with Mariotti in 2.6.4).

4.5 Statistical summary of microteaching results

The meanings of the SOLO assessment criteria used to evaluate teachers' performance (compare Table 3.3) are summarized in Table 4.7 below being an adaptation from Pegg (2003:243) (compare 2.3).

P- Prestructural	The preparation and performance is below the target level of understanding. Pre-structural responses represent very little use of relevant aspects of competencies
U- Unistructural	The teacher focuses on the lesson objectives, but focuses on some and not all ingredients of a problem-centred approach to classroom discourse and so may be inconsistent
M- Multistructural	Two or more aspects of the problem-centred approach are used without adequate perception of relationships between them. No integration occurs. Some inconsistency may be apparent.
R- Relational	Most of the data are now available, with each piece woven into an overall mosaic of relationships. The whole lesson delivery has process has a coherent structure. No inconsistency is present within the presentation aspects.

Table 4.7 Descriptions of four performance levels in the SOLO Model adapted to microteaching assessment

Eight participants who prepared for and presented microteaching lessons are included in this analysis. Seven out of eight appeared well prepared for their lessons as they had spent quite some time in the preceding session to prepare with a partner.

#	Criterion observed	%Wt								
1	Presenter preparedness	15	U	M	R	M	R	R	M	R
2	Mastery of <i>Sketchpad</i> skills	10	U	U	R	M	M	U	M	R
3	Presenter – teacher interaction	5	U	U	M	M	U	M	M	M
4	Teacher – teacher interaction	10	M	U	U	M	R	M	M	M
5	Presenter whole class interventions	5	U	P	U	M	M	M	M	M
6	Time management	5	M	M	R	M	U	R	U	M
7	Mastery of geometrical content	15	P	U	M	M	U	U	U	M
8	Monitoring of participants' progress	10	P	U	U	M	M	M	R	U
9	Conclusion of lesson	5	P	U	M	M	U	R	U	U
10	Participants' performance	10	M	M	M	M	M	U	M	U
	Overall impression on integration of skills	100%	Nathan	Rorisang	Bekithemb	Qhubekani	Tichaona	Andrew	Gibson	Ian

Table: 4.8 Assessment results for 8 leader teachers' lesson presentations

The eight topics covered in these sessions were: Similarity and Congruency Theorems (Nathan), Triangle centers (Rorisang), Introducing Transformations (Andrew), Tessellations (Gibson), Pythagoras Theorem (Qhubekani), Distances in an Equilateral Triangle (Ian), Kaleidoscope (Tichaona) and Angles (Bekithemba). Tables 4.8 and 4.9 summarize the performances according to proficiency levels described in Table 4.7.

#	Criterion observed	%Wt	Score obtained			
			P	U	M	R
1	Level of teacher preparedness	15		1	3	4
2	Mastery of <i>Sketchpad</i> skills	10		3	3	2
3	Presenter - teacher interaction	5		3	5	
4	Teacher – teacher interaction	10		2	5	1
5	Presenter whole class interventions	5	1	2	5	
6	Time management	5		2	4	2
7	Mastery of geometrical content	15	1	4	3	
8	Monitoring of participants’ progress	10	1	3	3	1
9	Conclusion of lesson	5	1	4	2	
10	Participants’ performance	10		2	6	
Overall impression on integration of skills		100%				

Table: 4. 9 Summary of results for 8 leader teachers’ lesson presentations

As already, noted, presenter preparation was generally satisfactory (78%). Teacher participation in all lessons was generally satisfactory both in terms of on-task, as well as discussion with peers (68%). Time was generally well managed (70,5%), except for a few instances like in the Kaleidoscope (Tichaona) and Tessellations (Gibson) lessons, where the construction processes were long but participants were patient and eager to see the end results.

Leader teachers felt more confident when they had previously succeeded in carrying out constructions (Tichaona, Qhubekani, Bekithemba, Ian, and Gibson) but were less confident when the level of geometry involved was difficult (54%). Most teachers (5 out

of 8) were not able to wrap up their lessons very confidently possibly because of lack of confidence with the geometrical aspects (average score of 57%).

#	Criterion observed	%	%	%	%	%	%	%	%	%
1	Presenter preparedness	50.5	70.5	90.5	70.5	90.5	90.5	70.5	90.5	78.0
2	Mastery of <i>Sketchpad</i> skills	50.5	50.5	90.5	70.5	70.5	50.5	70.5	90.5	68.0
3	Presenter – teacher interaction	50.5	50.5	70.5	70.5	50.5	70.5	70.5	70.5	63.0
4	Teacher – teacher interaction	70.5	50.5	50.5	70.5	90.5	70.5	70.5	70.5	68.0
5	Presenter whole class interventions	50.5	20.5	50.5	70.5	70.5	70.5	70.5	70.5	59.3
6	Time management	70.5	70.5	90.5	70.5	50.5	90.5	50.5	70.5	70.5
7	Mastery of geometrical content	20.5	50.5	70.5	70.5	50.5	50.5	50.5	70.5	54.3
8	Monitoring of progress	20.5	50.5	50.5	70.5	70.5	70.5	90.5	50.5	59.3
9	Conclusion of lesson	20.5	50.5	70.5	70.5	50.5	90.5	50.5	50.5	56.8
10	Participants' performance	70.5	70.5	70.5	70.5	70.5	50.5	70.5	50.5	65.5
	Overall impression on integration of skills	Nathan	Rorisang	Bekithemb	Qhubekani	Tichaona	Andrew	Gibson	Ian	Average

Key: P – $0 < x \leq 40\%$, U – $40 < x \leq 60\%$, M – $60 < x \leq 80\%$, R – $80 < x \leq 100\%$
Class centres: P – 20,5 U – 50,5 M – 70,5 R – 90,5

Table 4.10 Average presenter performances as percentages

The overall ability to integrate skills was also still at its infancy and that could be attributed to the small number of opportunities for presentation. Table 4.11 below shows the weighted scores per attribute per leader-teacher who presented. The results are symbolized back to the SOLO proficiency levels. One teacher performed at the pre-structural level, one at unistructural and the rest (six) at multistructural levels of integration of skills. No teacher operated at the relational level of skill integration.

Criterion observed	Wt								
Presenter preparedness	15%	7.6	10.6	13.6	10.6	13.6	13.6	10.6	13.6
Mastery of <i>Sketchpad</i> skills	10%	5.1	5.1	9.1	10.6	10.6	7.6	7.1	13.6
Presenter – teacher interaction	5%	2.5	2.5	3.5	3.5	2.5	3.5	3.5	3.5
Teacher – teacher interaction	10%	7.1	5.1	5.1	7.1	9.1	7.1	7.1	7.1
Presenter whole-class interventions	5%	2.5	1.0	2.5	3.5	3.5	3.5	3.5	3.5
Time management	5%	3.5	3.5	4.5	3.5	2.5	4.5	2.5	3.5
Mastery of geometrical content	15%	3.1	7.6	10.6	10.6	7.6	7.6	7.6	10.6
Monitoring of progress	10%	2.1	5.1	5.1	7.1	7.1	7.1	9.1	5.1
Conclusion of lesson	5%	1.0	2.5	3.5	3.5	2.5	4.5	2.5	2.5
Participants' performance	10%	7.1	7.1	7.1	7.1	7.1	5.1	7.1	5.1
Impression on integration of skills	100%	41.5 P	50.0 U	64.5 M	67.0 M	66.0 M	64.0 M	60.5 M	68.0 M
		Nathan	Rorisang	Bekithemb aa	Qhubekani	Tichaona	Andrew	Gibson	Ian

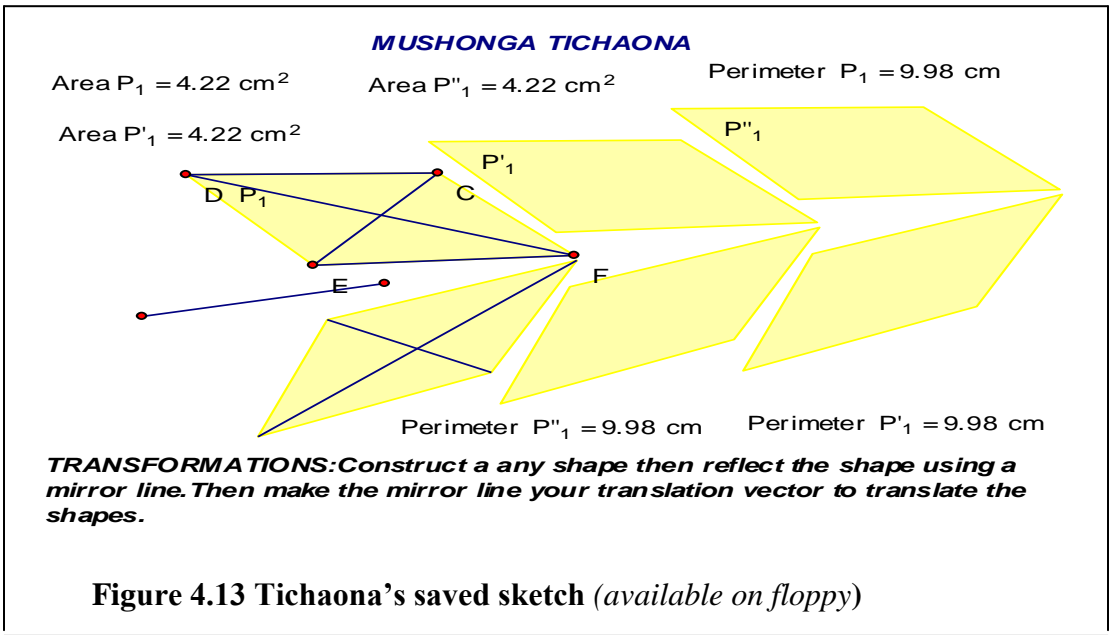
Table 4.11: Microteaching skill integration results

The results were not surprising as the teachers were only getting acquainted with the dynamic geometry software and the related teaching approaches.

4.6 Mini-projects and questionnaire responses

4.6.1 Results and analysis of Mini-projects

The purpose of the mini-projects was to assess whether teachers were able to create or prepare their own presentation sketches and explain them clearly. This ability appears important in adapting tasks to students' level of geometric thought as implied by the van Hiele theory (compare 4.2.1) The teacher ought to be a reservoir of activities beyond reliance on textbooks. *Sketchpad's* capabilities afford the teacher the creative potential, which can be extended to the students themselves to reduce over dependency on the teacher and the textbook in mathematical knowledge creation (compare teacher's role in 2.4 and the view of mathematics as an activity again, in 2.4).



Starting with a parallelogram, Tichaona drew a segment and marked it both as a translation vector and as a mirror. A combination of two translations and a reflection followed by dragging produced the dynamic shape in the Figure 4.14.

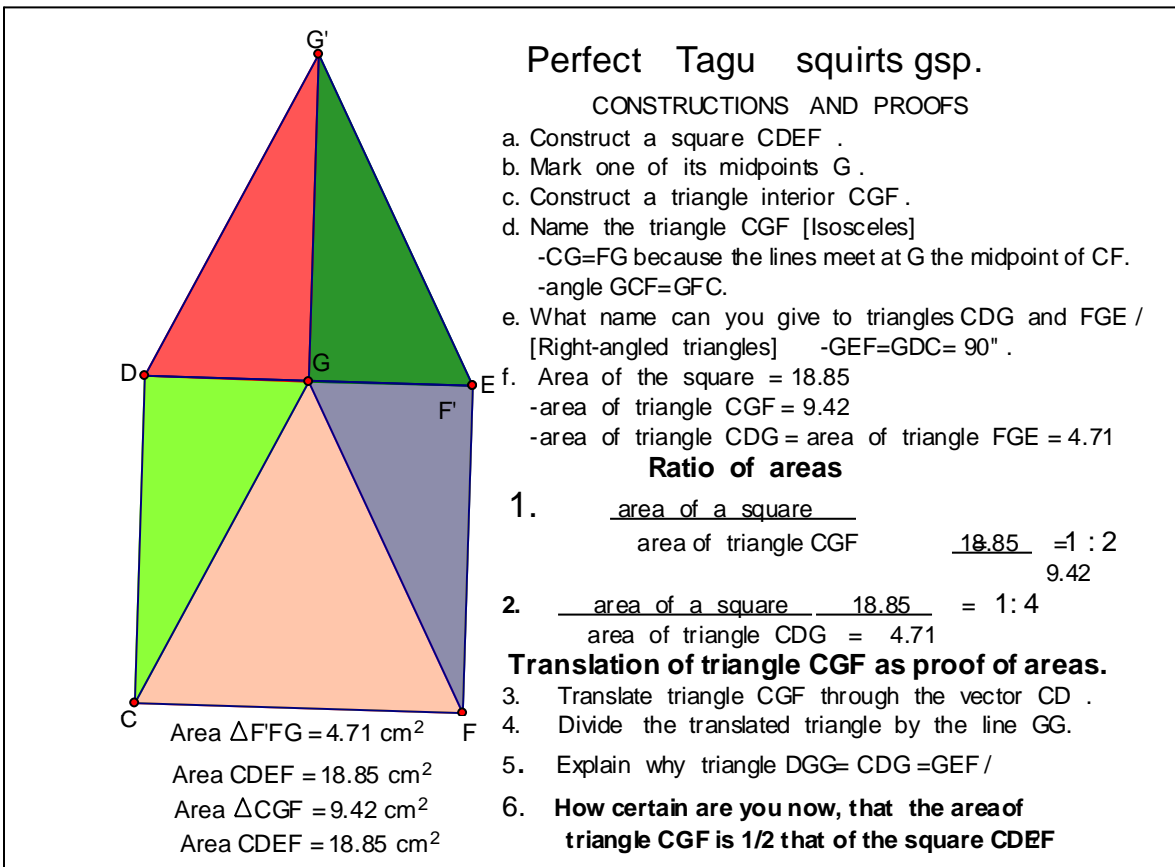


Figure 4:14 Perfect's saved sketch (available on floppy)

An analysis of the sketch showed that Perfect went a long way to elaborate the processes involved and to ‘prove’ or show that $\triangle CGF$ was isosceles and its area half that of the square CDEF. He made provision for his students to explain why $\triangle DGG' \cong \triangle CDG$. He thus envisaged the need for his students to participate actively. However, Perfect ‘proved’ empirically rather than deductively and did not notice that the congruency of $\triangle CDG$ and $\triangle FEG$ would have helped him come up with a deductive proof of the isosceles ness of $\triangle CGF$ and congruencies would follow from the isometric nature of translation.

AREA OF A TRAPEZIUM by Nathan

The area of a trapezium is half sum of parrallel lines multiplied by height
 # This formula have been illustrated on the trapizium ABCD
 # The area therefore of the trapizium is : $1/2(AB+CD)*BE$ as shown on the figures below the trapezium.

$m \overline{AB} = 2.99 \text{ cm}$ $m \overline{CD} = 7.01 \text{ cm}$ $m \overline{BE} = 2.99 \text{ cm}$

$$\left(\frac{1}{2}\right) \cdot (m \overline{AB} + m \overline{CD}) \cdot m \overline{BE} = 14.98 \text{ cm}^2$$

$m \overline{GH} = 5.00 \text{ cm}$ $m \overline{IJ} = 5.00 \text{ cm}$
 $m \overline{HI} = 2.99 \text{ cm}$ $m \overline{JG} = 2.99 \text{ cm}$

$$\frac{(m \overline{GH} + m \overline{IJ}) \cdot 1}{2} = 5.00 \text{ cm}$$

It has been proven that this formula can be used to find the area of any quadrilleteral with parrallel sides.
 # Below is an example to prove this : $\frac{(m \overline{GH} + m \overline{IJ}) \cdot 1}{2} \cdot m \overline{JG} = 14.96 \text{ cm}^2$
 ~ In the rectangle GHIJ it is proven this way :
 ~ $1/2(GH+IJ)*HI$ just gives the same answer as that of multiplying the length and the width as illustrated on the rectangle. $m \overline{GH} \cdot m \overline{HI} = 14.96 \text{ cm}^2$
 ~ This formula works on all quadrilleterals with parallel sides.

Figure 4.15 Nathan’s piece of work (saved on floppy diskette)

Figure 4.2.3 above shows Nathan’s presentation, which again centred on empirical rather than deductive evidence but made important observations. This showed that the availability of measurement capabilities in Sketchpad might easily be used as proof rather than empirical evidence and teachers might have to guard against this temptation by asking their students to go beyond and find an explanation for their discoveries. In this connection de Villiers (1999:24) emphasizes that further exploration is *Sketchpad* can only confirm the conjecture’s truth without providing an explanation. Furthermore, students become creators of their own geometry which accords well with the problem-centred approach and the constructivist tenets (compare with Freudenthal in 2.3).

4.6.2 Analysis of questionnaire responses

The teachers who participated in the study returned sixteen (14) questionnaires. Twelve (12) male and 2 female participants responded. Ninety-four percent (94%) had a highest mathematical qualification of GCE Ordinary level prior to joining college (compare 3.4). Only one had attempted ‘A’ level but failed. Two participants had received training in computers and held a one-year National Certificate in Computer Programming and the other had done an introductory course. All of them were new users of *Sketchpad*.

Table 4.12 below summarizes the importance participants attached to Mathematics.

Unimportant	Important	Very Important	No Response
0	0	14	0

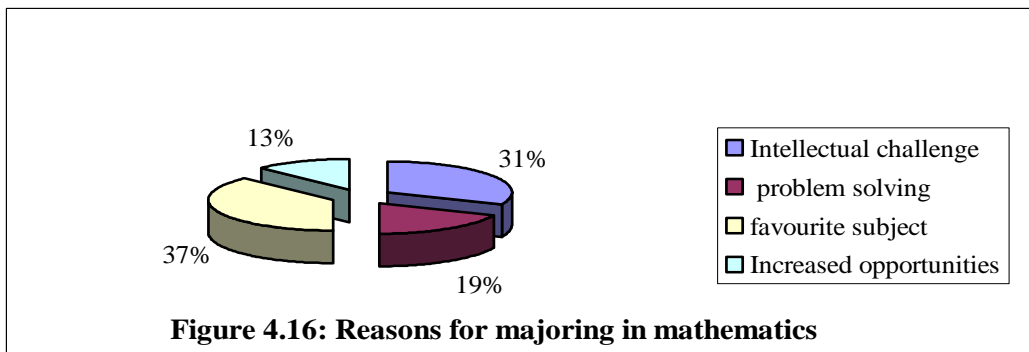
Table 4:12 Importance attached to mathematics by participants.

All respondents responded to this question and regarded mathematics as very important in life. This was expected of a mathematics major class, yet at the same time it was an important starting point on the road to effective mathematics teaching.

On why they chose mathematics as a major subject Table 4.13 shows reasons and number of respondents mentioning them and Figure 4.10 represents in pie chart form.

Reason for Choosing Mathematics as Main Subject	No. of Responses
Intellectual challenge	5
Development of critical thinking, problem solving and creativity	3
Previous record of success in mathematics/favourite subject	6
Increased job opportunities	2

Table 4:13 Reasons for choosing mathematics as a major subject at college



All fourteen returned questionnaires had responses to this question and three respondents gave two reasons each. Previous record of success in mathematics seemed to make the subject a favourite and this appeared the most frequently given reason – 6 out of 17 (37%). This was followed by the intellectual challenge offered by the subject (31%). All categories put together appeared to be fundamental reasons justifying the inclusion of mathematics in the curriculum hence the teachers were well disposed. Six (6) out of fourteen (14) respondents (50%) described their early experiences with the *Geometer's Sketchpad* as having been difficult in the following words: 'a total nightmare', 'very complicated and confusing', 'difficult and quite confusing', 'full of confusion', 'a few problems in following', 'rather difficult' and 'experienced difficulties'. This suggests that teachers have to sympathize with the early difficulties their students might meet. It also

partly explains why teachers themselves can be reluctant to introduce microworlds into their own classrooms even where the technology exists (compare 1.1 and 1.2). Tables 4.14 and 4.15 show the importance respondents attached to the activities in respect of mathematical content and *Sketchpad* capabilities. The visual proofs of the Pythagoras Theorem were rated the most memorable geometrical demonstration in *Sketchpad*. The kaleidoscope was rated the most memorable software capability.

Activities in frequency rank order	No of responses
Pythagoras	8
Keleidoscope	5
Transformations	3
Triangles	1
Quadrilaterals	1

Table 4.14 Importance of *Sketchpad* Activities in terms of Mathematical content

Activities in rank order	No of responses
Keleidoscope	5
Triangles	3
Pythagoras	2
Tessellations	1
Quadrilaterals	1
Distances in an equilateral Δ	1
Varignon	1

Table 4:15 Importance in terms of *Sketchpad* capabilities

On the one hand this pointed to an increased awareness of the affordances of *Sketchpad*, and on the other, it reminded of the importance of users to keep the mathematical objectives in the foreground. Table 4.16 below shows responses to the question requiring respondents to state two ways in which to check the accuracy of a construction.

Dragging	Animation	Undoing a construction	Checking a parents and children of objects	Showing all hidden in a construction	Measurement	Other
11	9	0	0	0	3	4

Table 4.16 Methods of checking the accuracy of a construction

Eleven (11) out of 14 respondents (79%), cited dragging as one method of checking the accuracy of a construction. Nine (9) out of 14 respondents (64%) cited animation as another test of a drawing or figure. This was an indication that teachers had become accustomed to the drag test and animation (compare 2.6.7 and 2.6.8). Measurement was cited by 3 out of 14 (21%) respondents and appeared to be a major detractor (compare 2.6.3 and 4.6.1). Other incorrect answers included ‘rotating’ which was a bit off tangent.

Table 4.17 summarizes respondents’ rating of the importance of taking into account the learners’ current level of geometrical understanding. All respondents who answered this

Unimportant	Important	Very Important	No Response
0	0	10	4

Table 4:17 Importance attached to teacher’s knowledge of students’ current level of geometric/mathematical understanding

item felt it was very important for the teacher to take into account the students’ current level of geometric understanding when teaching. This was consistent with the letter and spirit of the van Hiele theory (compare 2.2.6) and the problem centred approach (compare 2.3). Four respondents did not answer this question. Table 4.13 below shows the importance attached to advance lesson preparation. All participants who responded to

Unimportant	Important	Very Important	No Response
0	0	9	5

Table: 4.18 Importance attached to advance lesson preparation

this item rated advance preparation as very important. Five respondents did not answer this item and these were mainly those that had not participated in any lesson presentation. Seven (7) of those participants who had the opportunity to present lessons further expressed the rationale to prepare in advance as shown in the responses below:

- I had little experience in using the computer
- One gets more experience with the computer
- To have knowledge of operation at one's fingertips
- To explore and discover more before presentation
- As one prepares one encounters problems which one rectifies in advance
- To discover and scrutinize issues that students might need further clarity on
- So as to know what has to be done in the lesson

Rationale for advance lesson preparation

The importance of preparation was thus adequately appreciated (compare 4.5).

4.6.3 Analysis of group exit interview

Ten participants attended the group exit interview, which sought to ascertain the nature of skills or abilities any new group of participants had to be acquainted with in four categories. The categories were computer literacy, *Sketchpad* pedagogical skills, and assessment strategies. The majority of participants felt that beginning with orientation in the use of the computers would be of immense benefit to a new group. Those who had joined the study mid-stream expressed disappointment that they took too long to get to know how to go about the computer and thus felt constrained and frustrated in the early stages of their participation (compare 4.6.1). Members strongly recommended that any new users be familiarized with the tools and menus of *Sketchpad* as early as possible. A majority (9 out of 10) of the participants felt that working in mixed ability groups was more beneficial than in individual isolation or ability grouping (compare 4.4.1). One participant proposed the whole class approach as occasionally ideal depending on the stage of the lesson (compare 2.6.4). In terms of assessment strategies the participants felt that the laboratory worksheet approach suited the computer environment for assessment of progress during and after the lesson (formative). Ability to draw up an activity, or own worksheet for one's students was considered suitable as a summative evaluation strategy (compare 4.6.1).

4.7 Conclusion

In this chapter, findings of the investigation were presented and analyzed. It was observed, in the process, that teacher competencies in a problem-centred approach to dynamic geometry teaching have a multidimensional character and complexity. Results from the pre-test were presented and analyzed in terms of teachers' geometrical knowledge (mathematical) language competencies, and knowledge and skill in the use of the PCA approach as well as dynamic geometry. This constituted the first phase of the investigation and shed light on the level of geometrical understandings of the teachers in terms of the van Hiele theory. The second phase of the investigation focused on the analysis of the TDE workshop sessions introducing the teachers to computers and *Geometer's Sketchpad* capabilities. Teachers' geometrical and language competencies were further scrutinized in the DGE environment.

The microteaching phase of the TDE was analyzed in terms of knowledge of subject matter, effectiveness of preparation for lessons and, student-student interaction and teacher-student interactions, and the advantages and disadvantages of pre-made sketches. The results of the mini-projects, prepared by some participants and saved onto diskettes were analyzed in terms of strengths and areas of improvement. The results of questionnaires and exit group interview were discussed in respect of the viability of alternative assessment strategies, and experiences with *Sketchpad*, which a new group of teachers would have to be inducted into.

In Chapter 5 findings, recommendations, limitations and conclusions will be presented.

CHAPTER 5

FINDINGS, RECOMMENDATIONS, LIMITATIONS AND CONCLUSION

“It is teaching, not teachers, that must be changed” Stigler & Hiebert, *The Teaching Gap: Best Ideas from the World’s Teachers for Improving Education in the Classroom*.

5.1 Introduction

In this chapter the findings on the nature and content of identified teacher competencies are synthesized. The multifaceted nature and character of teacher competencies in a DGS environment demands a balancing act and purposeful integration for teachers to be proficient in the execution of a problem-centred approach to dynamic geometry teaching. From this study five categories of teacher competencies that must be integrated can be identified: namely mathematical competencies, language competencies, computer and software competencies, pedagogical and assessment competencies. A simplistic model for integration is proposed and recommendations are made to a cross-section of mathematics education stakeholders. The limitations of this study are spelt out so that the results and conclusions are interpreted as cautiously as possible.

5.2 The nature of identified competencies

5.2.1 Geometrical competencies

From the pretest results and *Sketchpad* activities it was evident that the mastery of school geometry is imperative if teachers are to perform effectively in their dynamic geometry classrooms (compare 2.7). In this connection, the study reaffirms Cangelosi’s (1996:405) observation that *‘mathematics and the teaching of mathematics are inextricably interrelated... the two are indistinguishable in mathematics education’*. Teachers cannot be expected to teach mathematical content they have little understanding of even in a

DGE environment. Teachers ought to prepare their lessons in advance as well as continually refresh themselves in the content they have to teach in order to be geometrically competent (compare 4.4.1, 4.4.2, 4.4.3 and 4.6.1).

In the topic covered, the teachers' understanding of properties of triangles and quadrilaterals appeared reasonably adequate (compare 4.2.2). The relationship between properties appeared a little problematic, especially in terms of necessity, sufficiency and equivalence (compare 4.2.3). Redundancy, or lack of economy was evident in definitions and descriptions, suggesting that teachers have to be aware of the adequacy, sufficiency and equivalence of some properties and definitions as pointed out by de Villiers (2004). Relationships between properties or geometrical objects also seemed to be problematic. Class inclusion was also difficult to grasp suggesting that the teachers were not yet at Van Hiele level 3 where class inclusion is expected to have developed (compare 4.2.3).

5.2.2 Language competencies

Language plays a central role in mathematics, and teachers are obligated to describe geometrical processes, figures and properties by their correct terminology. In a second language scenario it was evident that teachers had difficulty in expressing themselves accurately and correctly (compare 4.2.2, 4.2.4, 4.3.2.2, and 4.3.2.4). It was not surprising given that the various types of triangles and quadrilaterals have no mother tongue equivalents because the indigenous languages are not technically at par with the language of instruction, English. The van Hiele theory attaches a lot of importance to language.

The mathematics teacher in a dynamic geometry environment also has to contend with the jargon of the computer and the application software. Linguistic competence thus goes

beyond the naming of geometrical concepts and description of geometrical processes. It extended to a constellation of computer and *Sketchpad* terms like ‘menus’, ‘tools box’, ‘submenus’, ‘quick menu’, ‘drag test’, ‘custom tools’, ‘action buttons’, ‘animate’, ‘marquee’, ‘highlight’, ‘selection arrow’, ‘text palate’, ‘dialogue box,’ ‘caption’, ‘parents’ and ‘children’. There is a host of terms whose meaning in *Sketchpad* and computer environments is detached from their ordinary and colloquial use. It is thus essential that teachers come to terms with the terminology in order to bridge the discrepancies in meaning when their students get involved. The differences between a ‘line’, ‘ray’ and a ‘segment’, or ‘circle by point and centre’, and ‘circle by point and radius’, and the like have their *Sketchpad* meanings which need mastering or familiarization with pencil and paper equivalents that they are meant to represent.

5.2.3 Computer and software competencies

A working knowledge of *Sketchpad* constraints and affordances appeared mandatory. This study showed that inadequate skill in operating the software’ could be an obstacle. Making constructions that pass the drag test was a baseline skill that determined whether one had a ‘drawing’ or a ‘figure’ (compare 4.3.1). Knowledge that relationships used in the construction are maintained was necessary and essential in the understanding of its properties and related theorems (compare 2.6.2). Ability or inability to open a pre-made sketch could mean a whole world of a difference between use and non-use of the software sketches yet pre-made sketches can save valuable time (compare 4.3.2.4, 4.4.1 and 4.4.2). The fact that they are already pre-constructed for the user implies a gain in time management and more focus on geometry. Awareness of possibilities of suitably constraining, under-constraining or over-constraining appeared essential (compare 4.3.2.3) Furthermore, that the teacher and the student can make adjustments and their

own notes suggests a flexibility of pre-made sketches which teachers could capitalize upon (compare 4.4.1 and 4.4.2).

However, it was noted in this study that the teacher has to adopt a critical attitude when using pre-made sketches in order to correct any errors and/or extend their usefulness in an open-ended manner or to suit the particular level of the students' knowledge of geometry. In other words, continuous exploration of the software appeared to be an essential attribute for the *Sketchpad* user to cultivate (compare 4.4.1 and 4.4.2). The fact that there were pre-made sketches does not stop the teacher from preparing his/her own presentation sketches or adapting these sketches to suit his/her teaching style or for the attainment of other objectives (compare 4.4.3, and 4.6.1). Hence ability to prepare one's own tasks means that the teacher does not become a slavish user of the textbook. Rather he/she becomes a curriculum designer or curriculum maker and could do so with the active participation of his/her students (compare 2.4 and 3.2).

Just as Goldenberg (1996) and Wessels (2001b) warn that technology by itself is no panacea (compare 1.1), teachers also have to be aware of its strengths and weaknesses in order to use it profitably. In other words, not only should computers be used effectively, but also their availability should not mean a complete abandonment of traditional paper-and-pencil procedures. Computers should be viewed as a supplement, rather than a wholesale substitute for the ruler and compass, lest the meaning of the straightedge and compass tools in the software lose their historical origin (compare 4.3.2.1).

Understanding definitions and class inclusion can be greatly facilitated by a DGE (compare 2.6.7 and 4.3.2.3) yet oneness of a figure or shape under drag or animations assumes a new meaning (compare 4.4.1).

5.2.4 Pedagogical competencies

- Knowledge of students

Knowledge of the students' current level of geometric thought appears essential in that geometric content presented should be in tandem with the students' level of understanding. In the van Hiele theory we see that if material/geometry is presented at a level higher than that of students, then they would not understand (compare rhombus constructions 4.3.2.2 and 2.2).

- Classroom management techniques

Tasks prepared or selected must take into account the availability of PCs. In a laboratory situation in this study, not all interested students could be enrolled simply because of limited numbers of PCs and classroom space. Sharing of a PC by too many students may be counter-productive. When students work in small groups, however, there is room for student-student talk, and consequently more opportunities to debate and speak mathematically are availed (compare 4.4.1). Like in foreign language learning, the student has an expanded opportunity to read, write, listen to and speak mathematics within the community of learners. This accords with Freudenthal's (1991:15) observation that reading mathematics and listening to it is also mathematics.

- Teacher intervention

This study indicates that in a laboratory situation the teacher might have to concentrate more on preparing laboratory work sheets that can take the students along and have them record their observations as they go at their own pace. It might be better for the teacher to assist groups by talking to them as a group. Whole class interventions appear to be less effective, if not disruptive, when students are still progressing at different stages. It might

in fact be easier if the teacher waits for an SOS call and assist only help from peers has failed (compare 2.6.6, 2.6.7, 4.3.2.4 and 4.4.4). This discovery is consistent with Towers' (1999:200-202) categorizations of modes of teacher interventions into shepherding (extended stream of interventions) which is least desirable, inviting (suggesting a new potentially fruitful avenue of exploration), which is more desirable, and rug-pulling (a deliberate shift of the student's attention to something that confuses), which is compatible with the problem-centred approach. In this study, though, participating teachers initially seemed to be keen to get approval or help from the researcher or teacher leader of the day instead of their peers. Once the culture of consulting peers took root there was more teacher-to-teacher consultation, discussion and debate (compare 4.4.1) leading to high levels of learner participation. The laboratory situation demands that the teacher be aware of a changed seating arrangement and social ethos or ecology in that the teacher talks to the students' backs. Hence lecturing or talking are best replaced by letting students do mathematics (compare 2.6.4, 2.6.6 and 4.4.1).

5.2.5 Student assessment competencies

In a problem-centred approach the teacher chooses or designs tasks that facilitate learning by conjecture, experimentation and insight. Mathematics is viewed as a human activity of mathematizing or organizing everyday matter from a mathematical perspective (Freudenthal, 1991:14). In similar vein, Sinclair (2003:291) highlights that by affording students the opportunity to verify, conjecture, generalize, communicate, prove and make connections, dynamic geometry enables them to learn to notice, to pose questions, and to use change to investigate relationships.

Formative assessment requires monitoring of progress made by students in the activities as they work and follow their answer sheets closely, or as they conjecture, experiment and test. Asking students to explain their observations helps them to assess mathematical communication skills. Backtracking a student's construction through the 'undo' submenu or showing 'all hidden', 'parents' and 'children' can help validate a construction over and above the drag test and the animation capabilities. The fact that students also get quick feedback from the computer itself means the teacher fundamentally has a new companion- an interactive assessment assistant which he/she can take advantage of. (compare 2.6.8, 2.8 and 4.3.2.1). Of significance is the fact that merely being able to state facts by rote is no indication of the level of thought of the student (Pegg, 1991:13).

Sketchpad has tremendous potential for project work and if students could be allowed to exercise their creativity, then it would be a welcome shift away from traditional assessment practices in mathematics (geometry). There is consequently a compelling case for teachers to acquaint themselves with alternative assessment strategies that DGEs offer (compare 4.5.1). The fact that the students can prepare their worksheets and sketches on the computer, and save them in the computer, lends further credence to the alternative assessment drive. In support, Ellerton & Clarkson (1996: 2002) suggest practical tests student constructed test items, student self-assessment, student journals, and mathematics profiles.

5.2.6 Integration

Mere possession of the foregoing competencies cannot constitute good mathematics teaching. How these competencies are integrated, is a matter of individual teacher's art and craft competency. There is, however, a balancing aspect as well as a proficiency of

execution dimension both of which might depend on experience and expertise in the PCA and in using the DGE environment. In this study such integration could be likened to a pentagonal kaleidoscope animated, with the motion epitomizing the integration process. In this study, language competencies manifested themselves in multiple forms suggesting that it might be futile to consider these in isolation from geometrical or disciplinary competence, computer and software competencies or assessment. The mediational role of language also cuts across the PCA approach in that well documented tasks with clear operational instructions can enable discovery learning and/or learning through problem solving. Students also communicate and negotiate understandings among themselves, and with the teacher, through the mediational role of language. Additionally, disciplinary competence without the appropriate descriptive and symbolic role of language would be less meaningful. In short there appears to be mutual interdependence of all the identified competencies, which makes the pentagonal kaleidoscope model fairly plausible.

5.3 Recommendations

The following recommendations are made as a sequel to findings in this study.

(1) Pre-service and in-service courses for introducing teachers to dynamic geometry environments should take into account strengthening and solidifying their knowledge of school geometry just as in the traditional ruler-and-compass environments. However, close attention might have to be paid to skills in the use of the tool that has replaced paper and pencil technology. If exposed to an environment with computers and the relevant software, teachers can adapt to the new demands and exhibit considerable competencies in the orchestration of the hardware and software.

(2) The new interaction patterns between the teacher, the computer and the student seem to suggest that teachers be assisted to adjust from the tradition of following text books slavishly. The medium of the computer, beckons the teacher and student to be independent designers of their own exploration and learning/teaching activities. Teachers are expected to prepare or select tasks that allow students to work independently but meaningfully. The interactive nature of the DGEs suggests that the teacher's role changes more to that of facilitator and progress monitor of how the student and the computer have negotiated meaning with one another and among other students. Hence the PCA appears a suitable approach in such environments and is thus highly recommended.

(3) The role of language in a problem-centred approach to dynamic geometry environments requires more attention as the student now has to grapple with mathematical and software terminology over and above the second language used for instruction. The dilemma language poses thus has three sides to be considered by mathematics educators, curriculum designers, examiners and researchers alike. The design of teacher preparation and in-service materials should thus factor the three-dimensional language vector into the equation.

(4) The changed medium of instruction implies that teachers be equipped with alternative assessment methods. This challenges mathematics teacher educators, curriculum designers, examiners and policymakers to change their mindsets and accommodate the new found technology, just as the non-programmable calculator was admitted into the examination room.

(5) Given the prohibitive cost of computers from a Third World economic point of view, it is unlikely that there will be many enough individual mathematics teachers in the personal possession of computers or laptops any day soon. Institutional support should thus be considered as a pragmatic starting point for equipping teachers with the new technological competencies. This study has demonstrated this feasibility. Hence policy makers should seriously consider establishing and stocking up mathematics computer laboratories both at tertiary institutions and in the schools.

(6) This study points directions towards encouraging the use of the TDE as part of the developmental research agenda. Mathematics teacher education institutions are implored and challenged to make dynamic geometry courses part of their official curricula in order to access a larger number of mathematics teachers to benefit yet a larger number of students.

5.4 Limitations of the study

(1) This study was conducted as an extracurricular effort. It meant additional time had to be invested in an already congested teacher education curriculum of the polytechnic even though the mathematical or geometric content fitted within the mathematics education syllabus. There were, therefore, considerable time constraints.

(2) The study involved microteaching as a method of teacher preparation where peer teaching takes place. Interpretations of the findings should take this fact into account as resources did not permit an actual school situation to be enacted or equipped to satisfy conditions for investigation.

(3) The study focused on the computer laboratory situation as it was believed to have opportunities for more hands on minds on experiences with the software for a greater number of teachers than whole class presentations using an overhead projector.

(4) It is further noted that in this study there was no opportunity to conduct clinical interviews, which would have yielded more insight into the teacher's ways of thinking.

5.5 Conclusion

Teacher education in Zimbabwe has not yet embraced widespread exposure of pre-service teachers to modern technology in general. Efforts at in-service level remain sporadic too. The teaching of mathematics, let alone dynamic geometry, is still largely an untried and untested college curriculum possibility. Even where available computers are used more for commercial courses and communication purposes than the teaching of mathematics or other teacher education subjects. This study will hopefully contribute towards the advocacy for integrating information and communication technology into teacher education as a starting point in reaching out to the students in the schools to make mathematics more accessible to a greater majority (compare 3.6.4)

This study leaves un-answered, the question of how the identified teacher competencies can be developed thus inviting further research in that direction. The investigation has confirmed that teacher competencies in a problem-centred approach are a complex matrix. In trying to prioritize the competences, it appears there can be no substitute for a teacher to be mathematically competent, first and foremost. Pedagogical competencies would appear to come next. Language and software competencies would appear to be the tools by which geometry has to be attacked, expressed or represented. Assessment practices need to be more innovative and take into account the evolution in tools.

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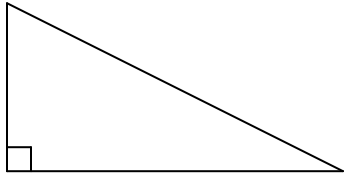
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APPENDICES

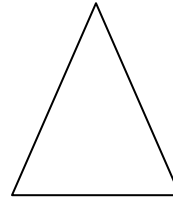
Appendix A – Pre-test

Section A

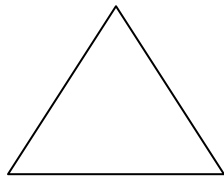
1. Name the four types of triangles shown below. Write your answer in the box provided



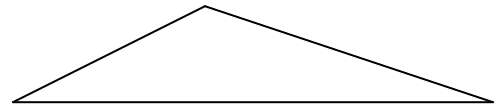
a)



b)



c)



d)

2. How would you, over the phone (or via telegram) explain what the triangles in Question 1 are? (Try to keep your description as short as possible, but ensure that the person has enough information)

- a).....
- b).....
- c).....
- d).....

3. Describe how you would make the following construction

- a) A circle inscribed in a triangle.....
- b) A circle passing through all vertices of a triangle (circumcircle).....

4. How would you, over the phone (or via telegram), explain what the following quadrilaterals are?

- a) A rhombus is a quad with.....
- b) A square is a quad with
- c) A rectangle is a quad with.....
- d) A kite is a quad with.....

- e) A parallelogram is a quad with
 - f) A trapezium is a quad
 - g) A cyclic quad is.....
5. How many lines of symmetry does each of the following shapes have?
 - a) Parallelogram..... b) kite..... c) rectangle.....
 - d) Square..... e) rhombus..... f) trapezoid.....
 6. Complete the following definitions of some quadrilaterals by some quadrilaterals.
 - a) A square is a rectangle with
 - b) A square is a rhombus with.....
 - c) A rectangle is a parallelogram with.....
 - d) A rhombus is a parallelogram with.....
 7. What can you say about the intersection of the diagonals of the following shapes?
 - a) A square: i).....
 - ii).....
 - b) A rectangle i).....
 - ii).....
 - c) A rhombus i).....
 - ii).....
 8. What can you say about adjacent and/or opposite angles of the following shapes?
 - a) Cyclic quadrilateral.....
 - b) Parallelogram.....
 - c) Isosceles trapezoid.....
 9. What is the relationship between the angles of a square and the diagonals?
.....
 10. What is the relationship between the perpendicular bisector of the base of an isosceles triangle and the angle at the apex?
.....

Section B

11. What do you understand about the problem-centred approach in relation to the teacher's role, student's role, classroom culture, mathematical tasks and mathematics learning?
.....
.....
.....
12. State **one** advantage and **one** disadvantage of the problem-centred approach.
.....
.....
13. What do you understand about dynamic geometry?
.....
14. What is the difference, if any, between a drawing and a figure?
.....

Appendix B

QUESTIONNAIRE TO PARTICIPANTS

Kindly answer the following questions as accurately as possible to facilitate further data analysis in the research project on the teaching of dynamic geometry from a problem centred approach. These are largely biographical questions and information obtained shall be for research purposes only.

Name of participant: Age.....
Highest Mathematics Examination passed..... Year.....
Highest Mathematics Examination attempted..... Year.....
Teaching experience accumulated before joining college: Years..... Months.....
Level of mathematics taught prior to joining college Duration
.....
.....
Level of mathematics taught during Teaching Practice Duration
.....
.....
Level of computer literacy prior to start of project: State qualification obtained, if any, or indicate what computer packages you are familiar with if at all
Why did you choose mathematics as your major subject.....
.....
How important is mathematics in life? Unimportant/Important/Very Important
What were your early experiences with the Geometer's Sketchpad?.....
.....
Which activity would you consider to have been most informative in terms of Sketchpad usage?..... Why?.....
.....
Which activity would you consider to have been most informative in terms of mathematical content?..... Why?.....
.....
What lesson did you help prepare for and/or present?.....
.....
How did you feel like during and after the presentation?.....
.....
How did you involve your 'students' during the lesson if you presented?.....
.....
How important was it to prepare for the lesson? Unimportant/Important/Very Important
Why?.....
State at least two ways in which one can check the accuracy of a construction in Sketchpad?.....
.....
How important is it to know your students' current level of geometrical knowledge before teaching new content? Unimportant/Important/Very Important

Appendix C

STRUCTURED GROUP INTERVIEW SCHEDULE

A. Computer Skills

How would you rate the importance of computer skills?

Which skills would you consider among the most important in order to get along with the Sketchpad Software?

How would you open the Sketchpad Software?

B. Sketchpad Skills

Which tools would you want a new group of teachers to be familiar with at the earliest opportunity?

Which tools did you find difficult or confusing to use at the beginning?

Which menus would you want new users to be familiarized with as a matter of priority?

How would you assist a new user to open a pre-made sketch in Sketchpad?

C. Pedagogical Skills

Which would you consider to be the most suitable grouping strategy in a Sketchpad laboratory environment?

Why do you think the strategy is more effective than others?

D. Assessment Strategies

How would you assess your students' progress during and after the lesson in a Sketchpad environment?

What kind of summative (end-of-term) assessment procedures would you use?

Appendix D

VIDEO –TAPE RECORDINGS

Video Tape 1 – Introduction to computers and Geometer’s Sketchpad

Video Tape 2 - Workshop sessions familiarizing participants with Sketchpad

- First lesson by participant (Congruency theorems)

Video Tape 3- Microteaching sessions by participants except for last which was misplaced, and should have been in Video tape A

Video Tape 4 - Exhibition at a local Town Agricultural Show.

Appendix E – Distances in an Equilateral triangle task

(Adapted from De Villiers, 1999:23-26)

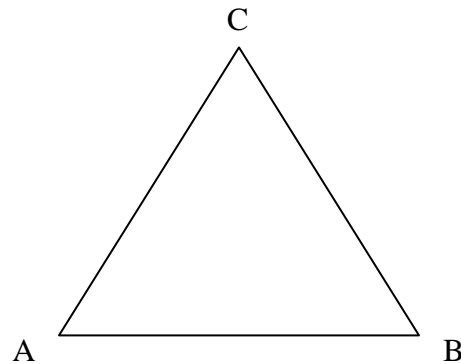
Distances in an

Names: _____

Equilateral Triangle)

A shipwreck survivor manages to swim to a desert island.

As it happens, the island closely approximates the shape of an equilateral triangle. She soon discovers that the surfing is outstanding on all three of the island's coasts. She crafts a surfboard from a fallen tree and surfs every day. Where should she build her house so that the sum of the distances from her house to all three beaches is as small as possible? (She visits each beach with equal frequency.) Before you proceed further, locate a point in the triangle at the spot where you think she should build her house.



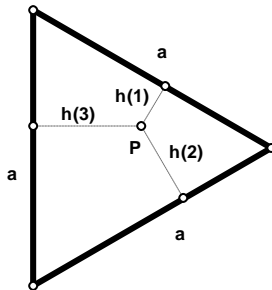
Conjecture

1. Open the sketch **Distances.gsp**. Drag point P to experiment with your sketch.
- Q1** Press the button to show the distance sum. Drag point P around the interior of the triangle. What do you notice about the sum of the distances?
- Q2** Drag a vertex of the triangle to change the triangle's size. Again, drag point P around the interior of the triangle. What do you notice now?
- Q3** What happens if you drag P outside the triangle?
- Q4** Organize your observations from Q1-Q3 into a conjecture. Write your conjecture using complete sentences.

All sketches referred to in this booklet can be found in Sketchpad\Teaching Mathematics (Sketchpad is the folder that contains the application itself)

Distances in an Equilateral Triangle (continued)

Explaining



You are no doubt convinced that the total sum of the distances from point P to all three sides of a given equilateral triangle is always constant, as long as P is an interior point. But can you explain why this is true?

Although further exploration in *Sketchpad* might succeed in convincing you even more fully of the truth of your conjecture, it would only confirm the conjecture's truth without providing an explanation. For example, the observation that the sun rises every morning does not explain why this is true. We have to try to explain it in terms of something else, for example, the rotation of the earth around the polar axis.

Recently, a mathematician named Mitchell Feigenbaum made some experimental discoveries in fractal geometry using a computer, just as you have used *Sketchpad* to discover your conjecture about a point inside an equilateral triangle. Feigenbaum's discoveries were later explained by Lanford and others. Here's what another mathematician had to say about all this:

Lanford and other mathematicians were not trying to validate Feigenbaum's results any more than, say, Newton was trying to validate the discoveries of Kepler on the planetary orbits. In both cases the validity of the results was never in question. What was missing was the explanation. Why were the orbits ellipses? Why did they satisfy these particular relations?... there's a world of difference between validating and explaining.

- M. D. Gale (1990), in *The Mathematical Intelligencer*, 12(1), 4.

Challenge

Use another sheet of paper to try to logically explain your conjecture from Q4. After you have thought for a while and made some notes, use the steps and questions that follow to develop an explanation of your conjectures.

Distances in an Equilateral Triangle (continued)

2. Press the button to show the small triangles in your sketch.
- Q5** Drag a vertex of the original triangle. Why are the three different sides all labeled a ?
- Q6** Write an expression for the area of each small triangle using a and the variables h_1 , h_2 and h_3 .
- Q7** Add the three areas and simplify your expression by taking out any common factors.
- Q8** How is the sum in Q7 related to the total area of the equilateral triangle? Write an equation to show this relationship using A for the area of the equilateral triangle is always constant.
- Q9** Use your equation from Q8 to explain why the sum of the distances to all three sides of a given equilateral triangle is always constant.
- Q10** Drag P to a vertex point. How is the sum of the distances related to the altitude of the original in this case?
- Q11** Explain why your explanation in Q5-Q9 would not work if the triangle were not equilateral.

Present Your Explanation

Summarize your explanation of your original conjecture. You can use Q5-Q11 to help you. You might write your explanation as an argument in paragraph form or as a two – column proof. Use the back of this page, another sheet of paper, a *Sketchpad* sketch, or some other medium.

Further Exploration

1. Construct any triangle ABC and an arbitrary point P inside it. Where should you locate P inside it. Where should you locate P to minimize the sum of the distances to all three sides of the triangles?
2.
 - a. Construct any rhombus and an arbitrary point p inside it. Where should you locate P to minimize the sum of the distances to all four sides of the rhombus?
 - b. Explain your observation in 2a and generalize to polygons with a similar property.
3.
 - a. Construct any parallelogram and an arbitrary point P inside it. Where should you locate P to minimize the sum of the distances to all four sides of the parallelogram?
 - b. Explain your observation in 3a and generalize to polygons with a similar property.