UNIVERSITEIT VAN PRETORIA
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YUNIBESITHI YA PRETORIA

EVALUATING THE INFLATION TARGETING REGIME OF SOUTH AFRICA

by

JOSINE UWILINGIYE

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Major Advisor: Prof. Rangan Gupta

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Josine Uwilingiye, Ph.D
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Abstract

The South African Reserve Bank (SARB) moved to an official inflation targeting regime in the February of 2000, with the sole aim of maintaining the CPIX inflation between a target-band of three to six percent.

Against this backdrop, this thesis, over seven independent chapters with a common theme, evaluates the inflation targeting regime in terms of welfare cost estimates and mean and volatility of inflation in the post-targeting period.

Chapters 2 and 3 use the partial equilibrium money demand approach based on cointegration and long-horizon estimation techniques, to derive the welfare cost estimates. Given the sensitivity of the results to the estimation techniques, chapter 4 carries out a robustness check for the two estimation methods based on data aggregation. The chapter 4 finds the long-horizon method to be more robust, and shows that the welfare cost estimate lies between 0.15 percent to 0.41 percent of GDP across the width of the target band.

Realizing that partial equilibrium approaches are merely one-dimensional, in the sense that it fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects, we re-evaluate the welfare costs in chapter 5 using a more general micro-level approach. The welfare cost estimates are found to increase by nearly one and half times when compared to the partial equilibrium approaches. This estimate increases by more than twice, when we adopt a dynamic general equilibrium endogenous growth model to calculate the welfare cost of inflation in chapter 6. In chapters 7 and 8 we carry out counterfactual experiments based on a model of dynamic time inconsistency and cosine-squared cepstrum. Specifically, we ask the question: If the mean and volatility of inflation would have been higher

or lower had the SARB continued to pursue its pre-targeting monetary policy approach. We find the evidence that the mean and volatility in the post-targeting era is higher than it would have been had the SARB continued to stick to its pre-targeting monetary policy framework.

Based on our results, we conclude that there can be large gains by considering a narrower (and possibly lower) target band.

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Chapter 1

1 Introduction

1.1 Introduction

The popularity of price stability as the principal goal of monetary policy emerged after most industrialized countries experienced high rates of inflation in the 1970s and 1980s. Since 1989 up to date, a steadily growing number of central banks in industrial and emerging economies have explicitly adopted an inflation target. The aim of price stability is based on the proposition that higher inflation impede long-run economic growth.

In South Africa, formal inflation targeting was adopted by South African Reserve Bank (SARB) in the February of 2000, with an objective of maintaining CPIX[1] inflation between the target-band of 3 percent to 6 percent by 2002, using discretionary changes in Repurchase (Repo) rate as its main policy instrument. Prior to 2000, during the mid-to-late 1990s, the SARB took a more eclectic approach to monetary, which, essentially involved monitoring a wide range of indicators, such as changes in bank credit extension, overall liquidity in the banking sector, the yield curve, changes in official foreign reserves, changes in the exchange rate of the Rand, and inflation movements and expectations. This approach was enhanced in 1998 by the replacement of the discount window by the marginal lending facility of the repurchase system and consequently, the Bank rate was replaced by the repo rate. This form of informal inflation targeting succeed in bringing the inflation down to the lower levels in South Africa, but the system of informal inflation targeting at times created uncertainties among the public about the monetary policy stance adopted by SARB. Given this, formal inflation targeting was needed to improve SARB's communication to the public on its monetary policy objectives.

This thesis aims to evaluate the inflation targeting regime of South Africa based on welfare costs estimates, and also uses recent advancement in econometrics to analyze whether mean or volatility of inflation would have been higher or lower if the SARB continued to pursue the so-called "eclectic approach" to monetary policy, instead of moving into inflation targeting. Note that the measurement of costs of inflation is of paramount importance in determining the

[1] CPIX is defined as Consumer Price Index (CPI) excluding interest rates on mortgage bonds.

legitimacy of the current target band, and, if there is a need to rethink the band in terms of the welfare cost at least.

In order to conduct an accurate assessment of inflation targeting regime in South Africa, this thesis comprises of seven separate chapters that are linked to each other. Starting with this general introduction in chapter 1, chapter 2 uses Johansen (1991, 1995) cointegrating procedure to obtain interest elasticity and interest semi-elasticity of money demand, and, in turn, use both elasticities to estimate welfare cost of inflation in South Africa based on Bailey's (1956) consumer surplus approach. Chapter 3 makes use of the Fisher and Seater (1993) methodology, an alternative to the Johansen (1991, 1995) cointegrating procedure, to estimate interest elasticity and interest semi-elasticity of money demand, and then deduce the welfare cost of inflation estimates using Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation approach. The need to use the two econometric approaches arises from the issue of the sensitiveness of the estimates of interest elasticity to alternative forms of money demand, based on alternative econometric approach used to estimate the long-run relationship between money balances and the nominal interest rate (among others, see Serletis and Yavari, 2004; Lucas, 2000).

Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income, and that long-run properties of data are unaffected under alternative methods of time aggregation (Marcellino, 1999), Chapter 4 tests the robustness of the two estimation procedures under temporal aggregation and systematic sampling in order to deduce the appropriate size of the inflationary distortion on welfare. In this scenario, one would want to rely more on welfare cost estimates that are least different under alternative methods of time aggregation.

However, the welfare cost calculations obtained by integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue to deduce the deadweight loss, is merely one-dimensional. This is because, the consumer surplus approach fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects on the intertemporal consumption choice (i.e., saving for old age), housing and the real cost of servicing government debt. Thus, the welfare costs obtained using the money demand approaches is likely to provide the lower limit of such estimates, and, hence, a more general approach is desired to obtain the "true" welfare loss caused by inflation. Given this, Chapter 5 applies Feldstein's microtheoretic (1997, 1999) approach to analyse the benefit of price stability in a non-indexed South African tax environment.

In chapter 6, we revisit the welfare cost estimate by calibrating with South African data, the dynamic general equilibrium endogenous growth model developed by Dotsey and Ireland (1996). By viewing inflation as a tax on micro-level decisions, Dotsey and Ireland (1996) identify explicitly and quantify numerically, sizeable welfare costs of inflation at the macroeconomic level, indicating that Feldstein (1997, 1999)-type partial equilibrium approaches can significantly underestimate the cost of inflation.

Chapter 7 applies a modified version of the Barro-Gordon (1983) model to check whether the behavior of the SARB during the pre-inflation targeting era could be explained by a model of dynamic time inconsistency. If yes, we forecast one-step-ahead inflation over the post inflation targeting period. On comparing the forecasted value of inflation with actual value of inflation during the inflation targeting period, allows us to check whether the SARB would have been able to produce lower or higher average levels of inflation, compared to the current inflation targeting framework.

Finally, in chapter 8, cosine-squared cepstrum is used to provide evidence on whether the inflation volatility has been higher or lower than it would otherwise have been, had the pre-inflation targeting monetary policy regime continued. All the chapters together aims to evaluate the inflation targeting regime of South Africa, not only in term of welfare cost estimates, but also by comparing mean and volatility of inflation in pre- and post-inflation targeting regimes.

Chapter 2

2 Measuring the welfare cost of inflation in South Africa*

2.1 Introduction

Studies on welfare cost of inflation have been the focus of extensive theoretical and empirical analyses in both the recent and more distant past. Using Bailey's (1956) consumer surplus approach, as well as, the compensating variation approach, Lucas (2000) provided estimates of the welfare cost of inflation for the US economy based on annual data for the period of 1900 to 1994. Lucas' (2000) calculations, based on the log-log money demand specification, indicated that reducing the interest rate from 3% to zero would yield a benefit equivalent to an increase in real output of about 0.009(or 0.9%).

Serletis and Yavari (2004), in their study dealing with the welfare cost of inflation for Canada and the United States, however, came up with much smaller figures, compared to Lucas (2000), when they used recent advances in the field of applied econometrics to estimate the interest elasticity of money demand. Unlike Lucas (2000), Ireland (2009), however, showed that a semi-log money demand specification fits the post 1980 US data better than a log-log econometric model. Based on the estimation of the semi-log money demand model, Ireland (2009) found that a 10% rate of inflation when compared to price stability would imply a welfare cost of 0.21% of income. This figure, though lower than that of Lucas (2000) and Serletis and Yavari (2004), was in line with Fisher's (1981) findings of 0.30%, a value of 0.45% obtained earlier by Lucas (1981) and a very similar figure of 0.41% by Serletis and Yavari (2004).

Given that welfare cost estimates differ remarkably based on alternative money demand functions, our aim is to first derive a money demand function that appropriately defines the South African money market, and then, in turn, use it to obtain welfare cost estimates of inflation. For this purpose, we look at quarterly data over the period of 1965:02 to 2007:01, and given the econometric problems of non-stationary data, use the Johansen (1991, 1995) cointegration technique to obtain a long-run money demand relationship. Note that measures of

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welfare cost of inflation are important for any economy, but more so in a country like South Africa, where the central bank targets inflation.[2] To put it differently, we try and investigate how substantial are the welfare costs of inflation under the current inflation target zone of 3-6% pursued by the South African Reserve Bank[3], especially if there is a need to rethink the band of the target in terms of the welfare cost of inflation. To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy.

The remainder of the chapter is organized as follows: Section 2.2 provides a brief summary of the theoretical issues regarding the estimation of the welfare cost of inflation, while, Section 2.3 and 2.4, respectively, discusses the data and presents the estimation of the log-log and the semi-log money demand specifications. Section 2.4 also calculates the welfare cost estimates for the South African economy. Finally, Section 2.5 concludes.

2.2 The theoretical foundations

As indicated by Lucas (2000), money demand specification is vital in determining the appropriate size of the welfare cost of inflation. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of m, a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate r. Formally, this can be expressed as follows:

$$\ln(m) = \ln(A) - \eta \ln(r) \tag{2.1}$$

where $A>0$ is a constant and $\eta>0$ measures the absolute value of the interest elasticity of money demand.

[2] See Ludi and Ground (2006) for a great compilation on the history of monetary policy of South Africa.

[3] Though, the inflation target is for the CPIX, we use the CPI inflation for our calculations, mainly due to the fact that the CPIX series does not exist for the whole sample period used, given that South Africa's decision to move to an inflation targeting regime only began in February of 2000. In addition, the correlation between the CPIX inflation and the CPI inflation over the period of 1999:02 to 2007:02 was found to be 0.81. So given this high correlation and the fact that the average rates of the CPI and the CPIX inflation were relatively close, specifically 5.12 and 6.23 percent respectively, studying the welfare cost of CPI inflation is very similar to studying the welfare cost for an inflation in the CPIX.

Another specification, adapted from Cagan (1956), links the log of m to the level of r via the following equation:

$$\ln(m) = \ln(B) - \xi r \tag{2.2}$$

where $B>0$ is a constant and $\xi>0$ measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the "*consumers's surplus*", that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_0^r m(x)dx - rm(r) \tag{2.3}$$

As seen from Equation (2.3), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss.

Since the function m has the dimensions of a ratio to income, so does the function w. The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by (1) or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{2.4}$$

While, for a semi-log money demand specification i.e., $m(r) = Be^{-\xi r}$, $w(r)$ is obtained by the following formula:

$$w(r) = \frac{B}{\xi}\left[1-\left(1+\xi r\right)e^{-\xi r}\right] \tag{2.5}$$

As can be seen from (2.4) and (2.5), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

2.3 Data

In this chapter, we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The used variables are the money balances ratio *(rm3),* generated by dividing the broad measure of money supply (M3)[4] by the nominal income (nominal GDP), and short term interest rate, in our case proxied by the 91 days Treasury bill rate *(tbr)*.[5] All series, except for the *tbr*, are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by *lrm3* and *ltbr,* respectively.

2.4 Empirical results

As is standard in time series analysis, we start off by studying the univariate characteristics of the data. In this regard, we performed tests of stationarity on our variables (*lrm3, ltbr* and *tbr*) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As can be seen from Table 2-1, the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null

[4] Though, in the literature, welfare costs of inflation has generally been obtained using a narrow definition of money, we chose M3, since we believe that a broad monetary aggregate captures the role of money better than a narrowly defined version of the same. In addition, the ratio of M3 to GDP was found to be least volatile. Finally, the choice was further motivated to take account of possible financial innovations that have taken place in the South African economy over the period of our concern.

[5] We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

of stationarity was rejected. As the variables were found to be non-stationary, it paved the way for the Johansen test for cointegration between *lrm3* and *ltbr* in (2.1) and *lrm3* and *tbr* in (2.2).

Table 2-1: Unit Root Tests.

Series	**Model**	**ADF**		**PP**	**KPSS**	**DF-GLS**	**Conclusion**
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t\ \tau_\mu$	$\tau_t\ \tau_\mu$	
LRM3	τ_t	-1.39	10.03***	-2.10	0.31***	-1.31	Non- stationary
	τ_μ	-1.50	13.79**	-2.26	0.31	-1.32	
	τ	-0.63		-0.63			
D(LRM3)	τ_t	-18.92***	179.02***	-21.57***	0.07	-19.01***	Stationary
	τ_μ	-18.76***	351.99***	-19.97***	0.35*	-18.68***	
	τ	-18.80***		-19.99***			
LTBR	τ_t	-2.89	22.42***	-2.29	0.29***	-2.78*	Non- stationary
	τ_μ	-2.60*	32.53***	-2.26	0.91***	-1.42	
	τ	-0.79		-0.84			
D(LTBR)	τ_t	-7.35***	26.99***	-7.23***	0.03	-7.29***	Stationary
	τ_μ	-7.35**	53.97***	-7.30***	0.09	-6.76***	
	τ	-7.36***		-7.32***			
TBR	τ_t	-2.76	16.25***	-2.29	0.27***	-2.73*	Non- stationary
	τ_μ	-2.63	23.88***	-2.30	0.74***	-1.74*	
	τ	-0.93		-0.77			
D(TBR)	τ_t	-7.98***	31.86***	-7.89***	0.03	-8.02***	Stationary
	τ_μ	-7.98***	63.74***	-7.90***	0.08	-7.89***	
	τ	-8.00***		-7.93***			

*(**) [***] indicates statistical significance at 10(5)[1] percent level.

At this juncture it is important to point out a possible concern in the analysis. These statistical tests which first analyzes the stationarity and then checks for cointegration between *lrm3* and *ltbr* in (2.1) and *lrm3* and *tbr* in (2.2), requires, as Ireland (2009) puts it, a "somewhat schizophrenic view of those data" since, in a linear framework, the analysis of the log-log model requires *ltbr* to follow an autoregressive process with a unit root, while the identical analysis of the semi-log model requires *tbr* to be *I(1)*. Bae (2005) actually provides a detailed discussion of the case in which both the models can be estimated under the common assumption that *tbr* follows an autoregressive unit root process, with the log-log specification being viewed as a non-linear relationship between *lrm3 and tbr* and the semi-log model viewed as a linear framework for the same two variables. As in Ireland (2009), we follow Anderson and Rasche (2001), by treating both as linear functions linking *lrm3 and ltbr* in (2.1) and *lrm3* and *tbr* in (2.2) and, thus, putting the two models on "equal footing *ex ante*".

But, before we tested for cointegration, a test for the stability of the VAR model, including a constant as an exogonous variable was performed. Given that no roots were found to lie outside the unit circle for the estimated VAR based on four lags under both the log-log and the semi-log specifications, we conclude that the VARs are stable and suitable for further analysis.[6] Note the choice of 4 lags was based on the unanimity of two alternative lag-length criteria, namely the Schwarz information criterion and the Hannan-Quinn Information criterion for the log-log money demand specification, and the Sequential Modified LR test statistic for the semi-log money demand model. Before we proceed further, it is important to point out that though four criteria, namely the Final Prediction Error, the Akaike Information, the Schwarz Information and the Hannan-Quinn Information, overwhelmingly suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. However, as has been reported below, the cointegration test based on 4 lags, suggested by the Sequential Modified LR test statistic, picked up one cointegrating relationship.

Once the issues of stability and the optimal lag length were settled, we tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included four lags in the VAR, and allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, both the Trace and the Maximum Eigen Value tests, showed that there is one stationary relationship in the data ($r = 1$) at 5 percent level of significance for both the log-log and the semi-log specifications. The results have been reported in Tables 2-2 and 2-3.[7]

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[6] Tests indicating the stability of the VAR have been suppressed to save space. However, they are available from the authors upon request.

[7] As in Ireland (2009), we also used the Phillips-Ouliaris (1990) test for cointegration. However, unlike Ireland (2009), the test could not detect any cointegrating relationship between the chosen variables. Hence, the results of the test have been suppressed to save space. They are, however, available upon request.

Table 2-2: Estimation and Determination of Rank (Log-Log).

Null hypothesis	**Alternative Hypothesis**	**Test statistic**	**0.05 critical value**	**Prob.** **
			Trace Statistic	
r=0	*r=1*	18.86965*	15.49471	0.0149
r=1	*r=2*	0.111350	3.841466	0.7386
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	*r=1*	18.75830*	14.26460	0.0091
r=1	*r=2*	0.111350	3.841466	0.7386
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Table 2-3: Estimation and Determination of Rank (Semi-Log).

Null hypothesis	**Alternative Hypothesis**	**Test statistic**	**0.05 critical value**	**Prob.****
			Trace Statistic	
r=0	*r=1*	19.67238*	15.49471	0.0110
r=1	*r=2*	0.197347	3.841466	0.6569
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	*r=1*	19.47503*	14.26460	0.0068
r=1	*r=2*	0.197347	3.841466	0.6569
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Given one cointegrating relationship (*r=1),* the Johansen (1991, 1995) procedure gives the maximum likelihood estimates of the unrestricted cointegrating relation $\beta' X_t$. Even if the unrestricted β is uniquely determined, depending on the chosen normalization, β is not necessarily meaningful from an economic point of view. Therefore, an important part of long-run cointegration analysis is to impose (over-) identifying restrictions on β to achieve economic interpretability (Hendry *et al.* 2000).

As we are more interested in the relationship between the money balance ratio and interest rate, for both specifications, *lrm3* was restricted to unity. Given that we have only one cointegrating vector, the normalizing restriction on *lrm3* is enough to exactly identify the long-run relationship. However, we encountered two serious econometric problems with this restriction. First, the restriction was not binding. Secondly, the adjustment coefficient of *lrm3* was insignificant under both the specifications. Imposing an additional zero restriction on the adjustment coefficient of *lrm3* did ensure binding restrictions, but at the cost of suggesting that the ratio of real balance to income was in fact exogenous and we should not be normalizing on *lrm3*. Given this, we decided to normalize on the interest rate variable, i.e., *ltbr* for the log-log specification and *tbr* for the semi-log specification. Further, with the adjustment coefficients on *lrm3* still being insignificant in both the models, we restricted them to zero, and obtained binding restrictions.[8] Note with *lrm3* now treated as the right-hand side variable, weak exogeneity of the same is what should be expected. The adjustment coefficients of *ltbr* and *tbr* were negative and significant, with them correcting for 6.9% and 8.08% of the disequilibrium in the next period, respectively. Based on the above restrictions, the obtained long-run relationship for the log- log specification is as follows:

$$ltbr=-5.275983- 4.789793(lrm3) \quad (2.6)$$
$$[-3.87971]$$

While for the semi–log model, the relationship is given by:

$$tbr=-0.171261-0.454711(lrm3) \quad (2.7)$$
$$[-3.88877]$$

[8] Note the value of the LR test statistics for binding restrictions, both long- and short-run, for the log-log and the semi-log specifications respectively, were $\chi^2(1)$ =0.0036 (0.9522) and $\chi^2(1)$ = 0.4041 (0.5250), where the numbers in the parenthesis indicates the probability of committing a Type I error.

Figures 2-1 and 2-2 depict the cointegrating relationships under the log-log and the semi-log specifications respectively. As can be seen, the residuals of the two cointegrating equations are mean-reverting around zero and are stationary, which implies that the estimated cointegrating relations are appropriate.[9] Note what we have in equations (2.6) and (2.7) are actually the inverse of the money demand functions, with rate of interest as the dependant variable. Alternatively, we can view equations (2.6) and (2.7) as long-run rules for the treasury bill rate. Whatever we choose to call these equations is not important to our cause, but it is the values of the coefficients of these two estimated cointegrating relationships, that are more relevant. The obtained interest elasticity, in absolute term, equals to 0.2088 and the interest semi elasticity is equal to 2.1991, both of which are obtained by taking the reciprocal of the coefficients corresponding to *lrm3* in equations (2.6) and (2.7) respectively. Importantly, the signs of the interest rate elasticity and semi-elasticity, in both the specifications, adhere to economic theory. Based on these two, elasticities, we are now ready to calculate the welfare cost of inflation as outlined in Lucas (2000), and described above in equations (2.4) and (2.5).

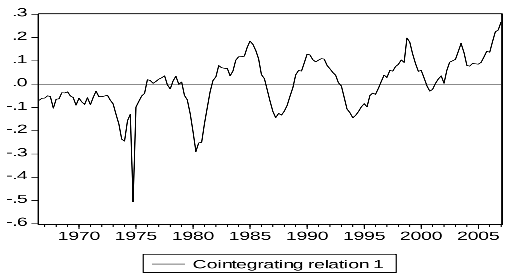

Figure 2-1: Cointegrating relationship of the Log-Log Specification.

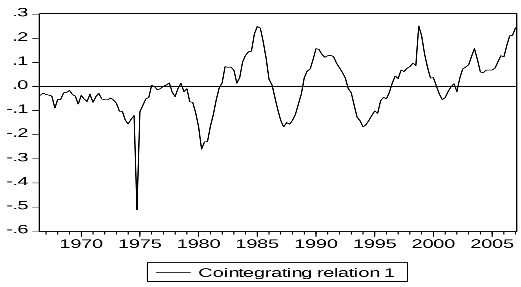

Figure 2-2: Cointegrating relationship of the Semi-Log Specification.

[9] Diagnostic tests on the residuals revealed that there is no autocorrelation in both the log-log and the semi-log specifications.

The estimates of the intercept and slope coefficient reported under the log-log specification imply values of A = 0.3323 and that of η = 0.2088, while for the semi-log specification the values of B = 0.6862 and that of ξ = 2.1991.[10] Plugging these values into the corresponding formula for the welfare cost measures, given by equations (2.4) and (2.5) respectively, and using the fact that the average real rate of interest[11] over this period was equal to 7.70 %, so that a zero rate of inflation would also imply a nominal rate of interest equal to 7.70 %, we obtain the baseline value of w under price stability. Naturally then, a value of r = 10.70 corresponds to a 3% rate of inflation, while, when r = 13.70, the economy experiences a 6% inflation, and so on. So the welfare costs of inflation are evaluated by subtracting the value of w at an inflation equal to zero from the value of the same at a positive rate of inflation.

Table 2-4: Welfare Costs of Inflation.

Inflation Rate	Log-Log	Semi-Log
0.03	0.0034	0.0034
0.06	0.0067	0.0076
0.10	0.0108	0.0143
0.15	0.0156	0.0241

Table 2-4 provides the measures of the welfare costs of inflation, under the log-log and the semi-log specifications for the inflation rates of 3%, 6%, 10% and 15%, respectively. For an inflation rate of 3%, the cost of inflation under both the log-log and the semi-log specifications are 0.34 % of GDP. However, as the inflation rate increase from 6% to 15%, the cost of inflation in the log-log model ranges between 0.67% of GDP and 1.56 % of GDP, while for the semi-log money demand function the welfare cost varies between 0.76 % of GDP and 2.41 % of GDP.[12] So, the two specifications provide clearly different measures of the cost of inflation with the semi-log function imposing greater welfare loss on the economy as the inflation rate increases beyond the 3% mark.

[10] Note the values for A and B are easily obtained by realizing that: A = [exp(-5.27598)] raised to the power of 0.208819, and B = [exp(-0.171261)] raised to the power of 2.199149.
[11] Note, as in Ireland (2009), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI. In addition, the real rate of interest was found to be stationary based on the ADF, the DF-GLS, the KPSS and the PP tests of unit roots.
[12] Note that these obtained values for the welfare cost of inflation are comparatively higher than those reported in Fischer (1981), Lucas (1981), Lucas (2000) and Ireland (2009) for the US economy.

So the pertinent question now is: Which one of the two inverse money demand specifications represents the money market of South Africa better? To resolve this issue we look at couple aspects of the two alternative money demand models: First, we compare simple linear and exponential plots of the relationship between *tbr* and *lrm3* with the scatter plot of these two variables, and; second, we look at the R^2 and *Adjusted* R^2 values of these two fits of the data.[13] In sum, we basically analyze the goodness of fit for the two specifications.

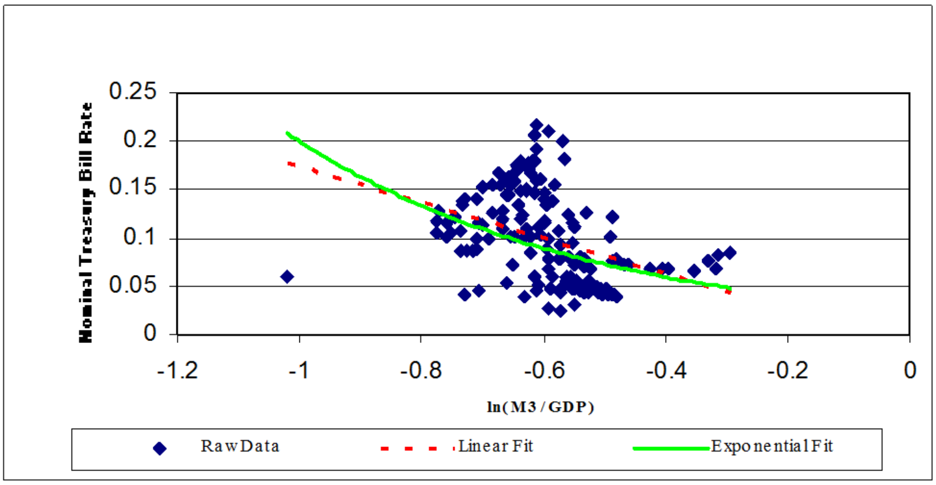

Figure 2-3: Inverse Money Demand Function of South Africa, 1965:02-2007:01.

As can be seen from Figure 2-3, it is impossible to choose between the two models based on the linear and the exponential plots of the data. However, with the R^2 and the *Adjusted* R^2 values of the inverse money demand relationship captured by the log-log specification being higher than the corresponding values of the semi-log model,[14] we decided to rely more on the results from the former. In addition, recall that although there existed overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. We, thus, had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship. Based on this, one can, perhaps, argue that the semi-log specification is relatively less reliable, in comparison to the log-log model, as to depicting a true picture of the South African inverse money demand, over the

[13] Note given that the plots are based on *tbr* and *ltbr*, the linear trend fitted to the data gives us the semi-log inverse money demand relationship, while, the exponential trend when taken logs will yield the log-log inverse money demand model.
[14] The R^2 and the *Adjusted* R^2 values of the log-log model are 0.1517 and 0.1466 respectively, while those of the semi-log function are 0.1443 and 0.1391 respectively.

period in concern. Alternatively, the bottom line of all this discussion is that, we tend to believe, that the welfare cost measures obtained from the log-log inverse money demand relationship is more appropriate. This implies that the welfare cost of inflation for South Africa ranges between 0.34% and 0.67% of GDP, for a band of 3-6% of inflation.

2.5 Conclusion

This chapter uses the Johansen (1991, 1995) cointegration technique to first obtain an appropriate long-run money demand relationship for the South African economy and then, in turn, deduce welfare cost estimates based on the inverse money demand function, as outlined in Lucas (2000). For this purpose, we look at quarterly data over the period of 1965:02 to 2007:01 and estimate a log-log function and a semi-log specification. Based on the fits of the specifications, we decided to rely more on the welfare cost measure obtained under the log-log inverse money demand model. Our estimates suggest that the welfare cost of inflation for South Africa ranges between 0.34% and 0.67% of GDP, for a band of 3-6% of inflation. Though, these measures are way higher when compared to the estimates observed in the literature, they are reasonably low. Based on our estimates, we can conclude that the SARB's current inflation target band of 3-6% is not too poorly designed in terms of welfare, at least in comparison to a Friedman-type deflationary rule of zero nominal rate of interest.

However, it is important to point that our welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given these two additional sources of inflation costs, there is no denying the fact that one can achieve, possibly, larger gains by reducing the inflation target below 3%, the lower limit of the inflation target band.

Chapter 3

3 Measuring the welfare cost of inflation in South Africa: A reconsideration*

3.1 Introduction

In chapter 2, we measured the welfare cost of inflation in South Africa, based on estimates of the interest elasticity and semi-elasticity of money demand functions, which were obtained using the Johansen (1991, 1995) methodology on quarterly data for M3, GDP and the Treasury bill rate. Given the estimates for the elasticities, we then calculated the welfare cost of inflation using Bailey's (1956) consumer surplus approach. Relying more on results obtained from the log-log specification of money demand, rather than the semi-log model for the same,[15] they indicated that the welfare cost in South Africa ranged between 0.34% and 0.67% of GDP, for a band of 3-6% of inflation, over the period of 1965:02 to 2007:01.

In this chapter, we re-estimate the long-run relationship between money balance and interest rate for South Africa, using the same data set and over the same period as that used in chapter 2, but applying an alternative approach, namely the long run horizon regression proposed by Fisher and Seater (1993). One of the advantages of using the long-horizon regression approach is that cointegration is neither necessary nor sufficient for tests on the interest rate elasticity of money demand. As in chapter 2, the coefficients obtained in regression for both alternative money demand specifications, a double-log version

[15] The decision to place more confidence on the log-log model of money demand was due to two reasons: First, the R^2 and the Adjusted R^2 values of the inverse money demand relationship captured by the log-log specification was higher than the corresponding values of the semi-log model, and; Second, although there existed overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. We had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship.

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originated by Meltzer (1963) with constant elasticity and a semi-log version originated by Cagan (1956) with constant semi-elasticity of money, are then used to calculate welfare cost of inflation. In addition, the welfare cost of inflation is then estimated using both Bailey's (1956) consumer's surplus approach and Lucas's (2000) compensating variation approach.

The necessity to compare the welfare cost estimates with that obtained in chapter 2, based on the Johansen (1991, 1995) methodology, arises from the issue of sensitiveness of the estimates of the interest elasticity of alternative forms of money demand, based on alternative econometric techniques adopted to estimate the long-run relationship between money balance and the nominal interest rate. Given that welfare cost estimates hinge critically on the estimate of the interest elasticity and semi-elasticity, it is important to check for the robustness of the results obtained using alternative econometric methodologies.

The above claim regarding the need to use alternative estimation techniques to obtain values for interest elasticity and semi-elasticity is not without empirical basis. Basing their study on the long-run horizon regression approach proposed by Fisher and Seater (1993), the researchers Serletis and Yavari (2004), in their study dealing with the welfare cost of inflation for Canada and the United States, came up with much smaller figures than those of Lucas (2000), who had indicated that a reduction in the nominal rate from 0-3% would yield a benefit equivalent 0.90 % of real income. However, Serletis and Yavari (2005), while repeating the above study for Italy, came up with very similar numbers for the welfare cost they had obtained earlier for Canada and the United States.[16] The authors indicated that reducing the interest rate, in Italy, from 14% to 3% would yield a benefit equivalent to an increase in real income of 0.40%. This, in turn, was fairly comparable to their estimates for Canada (0.35%) and the United States (0.45%) for the same percentage point reduction in the nominal interest rate. More recently, Serletis and Yavari (2007) estimated the welfare cost of inflation using the Fisher and Seater (1993) approach for seven European economies. The results indicated that in big countries, like France and Germany, the welfare cost of inflation is much lower than in small countries, like Austria, Belgium, Ireland, Italy and The Netherlands. But importantly, the numbers were quite comparable to their earlier studies. On the other hand, based on the Phillips-Ouliaris (1990) test for cointegration, Ireland (2009) found that a 10 percent rate of inflation when compared to

[16] See Serletis and Virk (2006) for the sensitiveness of the welfare cost estimates to the choice of monetary aggregation procedure.

price stability, in the United States, would imply a welfare cost of 0.21 percent of income. This figure, though lower than that of Lucas (1981, 2000) and Serletis and Yavari (2004), was in line with Fischer's (1981) findings of 0.30%. Clearly then, apart from sample period and the country under investigation and alternative money demand specifications, welfare cost estimates are sensitive to alternative estimation methodologies. Our need to reconsider the welfare cost estimates obtained in the previous chapter therefore cannot be overlooked. Table 3-1 summarizes the studies discussed above and includes the methodology, country and the size of the welfare cost.

Table 3-1: Summarizing the Literature.

Study	Country	Methodology (Functional Form)	Inflation comparisons (Nominal interest rate)	Welfare costs(percent of GDP)
Fischer(1981)	USA	Calibration[a] (Log-Log)	0 to 10%	0.30%
Lucas(1981)	USA	Calibration[b] (Semi- Log)	0 to 10%	0.45%
Lucas(2000)	USA	Calibration[c] (Log-Log)	0 to 3%	0.9%
Serletis and Yavari(2004)	Canada and USA	Fisher and Seater (1993) Long-Horizon (Log-Log)	0 to 3% 3 to 14%	USA:0.18% Canada:0.15% USA: 0.45% Canada: 0.35%
Serletis and Yavari(2005)	Italy	Fisher and Seater (1993) Long-Horizon (Log-Log)	3 to 14%	0.4%
Serletis and Yavari(2007)	Europe	Fisher and Seater (1993) Long-Horizon (Log-Log)	5 to 10%	Belgium: 0.3% Austria:0.45% France:0.1% Germany:0.2% Netherlands:0.4% Ireland:0.4% Italy:0.4%
Ireland(2009)	USA	Phillips- Ouliaris (1990) Cointegration (Semi-Log)	0 to 10%	0.21%
Gupta and Uwilingiye (2008)	South Africa	Johansen (1991,1995) Cointegration (Log-Log)	0 to 3% 3 to 6%	0.34% 0.67%

Notes: a: Interest elasticity used 0.25 based on Goldfeld (1971); b: Lucas (1981) uses a value of 5.0 for the interest semi-elasticity; c: Lucas (2000) uses a value of 0.50 for the interest elasticity.

Given that, inflation has an effect on economic activity, and ultimately on people's well-being as it reduces the purchasing power of money balances when inflation rises, a correct and fair evaluation of welfare cost of inflation is crucial. This is because inflation creates and amplifies distortions in many areas of economic activity and it has also an influence on all decisions of economic agents. Besides, in a country like South Africa, where the central bank targets inflation, it is of paramount importance to investigate how substantial the welfare costs of inflation are under the current inflation target zone of 3-6% pursued by the South African Reserve Bank. This would help us to decide if there is a need to rethink the band of the target in terms of the welfare cost of inflation. To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy, based on the long-run regression approach proposed by Fisher and Seater (1993).

The remainder of the chapter is organized as follows: Section 3.2 provides a brief summary of the theoretical issues regarding the estimation of the welfare cost of inflation, while, Section 3.3 and 3.4, respectively, discusses the data and the long-horizon empirical methodology for the estimation of the log-log and the semi-log money demand specifications; Section 3.4 also presents the empirical estimates for the interest rate elasticity and the semi-elasticity, as well as the welfare cost estimates for the South African economy. Finally, Section 3.5 concludes.

3.2 The theoretical foundations

As indicated by Lucas (2000), money demand specification is vital in determining the appropriate size of the welfare cost of inflation. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of m, a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate r. Formally, this can be expressed as follows:

$$\ln(m) = \ln(A) - \eta \ln(r) \tag{3.1}$$

where A>0 is a constant and η>0 measures the absolute value of the interest elasticity of money demand. Another specification, adapted from Cagan (1956), links the log of m to the level of r via the following equation:

$$\ln(m) = \ln(B) - \xi r \tag{3.2}$$

where B>0 is a constant and ξ>0 measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the "consumers's surplus", that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_0^r m(x)dx - rm(r) \tag{3.3}$$

As seen from Equation (3.3), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss. From, Figure 3-1 below, this essentially implies that the welfare cost of inflation is measured by the area A.

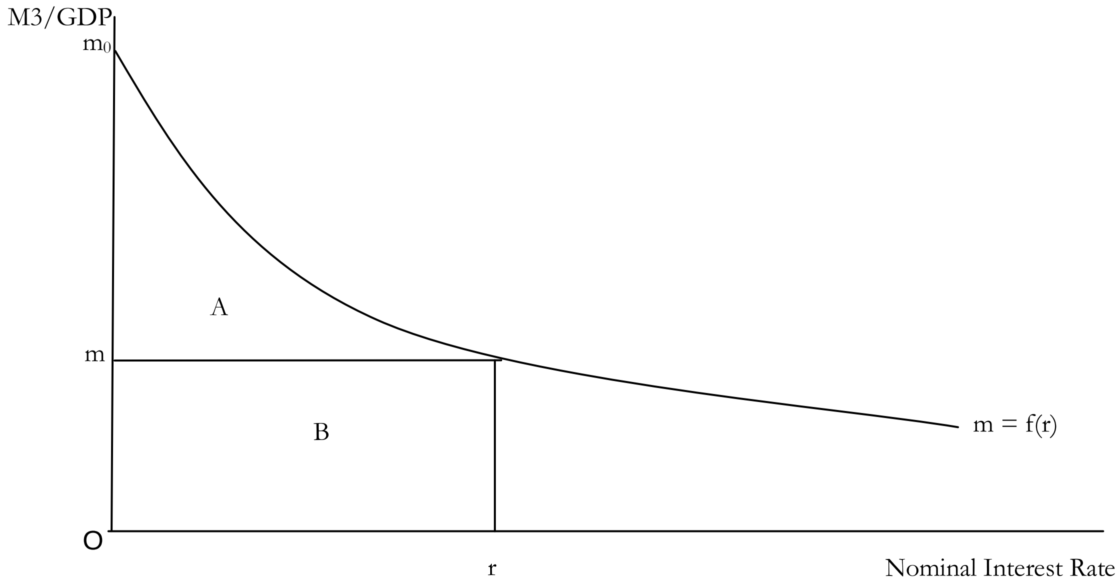

Figure 3-1: Welfare Cost Calculation Using Bailey's Consumer Surplus Approach.

Just as the function m has the dimensions of a ratio to income, so does the function w. The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by (3.1) or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{3.4}$$

While, for a semi-log money demand specification i.e., $m(r) = Be^{-\xi r}$, w(r) is obtained by the following formula:

$$w(r) = \frac{B}{\xi}\left[1-\left(1+\xi r\right)e^{-\xi r}\right] \tag{3.5}$$

As demonstrated in (3.4) and (3.5), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, so we first have to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. Lucas (2000) starts by using Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate r, $w(r)$, to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at r and an otherwise identical steady-state with the interest rate of zero with $w(r)$ being obtained from the solution to the following equation:

$$u\left[1+w(r))y, \phi(r)y\right] = u\left[y, \phi(0)y\right] \tag{3.6}$$

Realizing that u is also negatively related to the nominal rate of interest, r, Figure 3.2 presents a diagrammatic illustration of what equation (3.6) essentially implies.

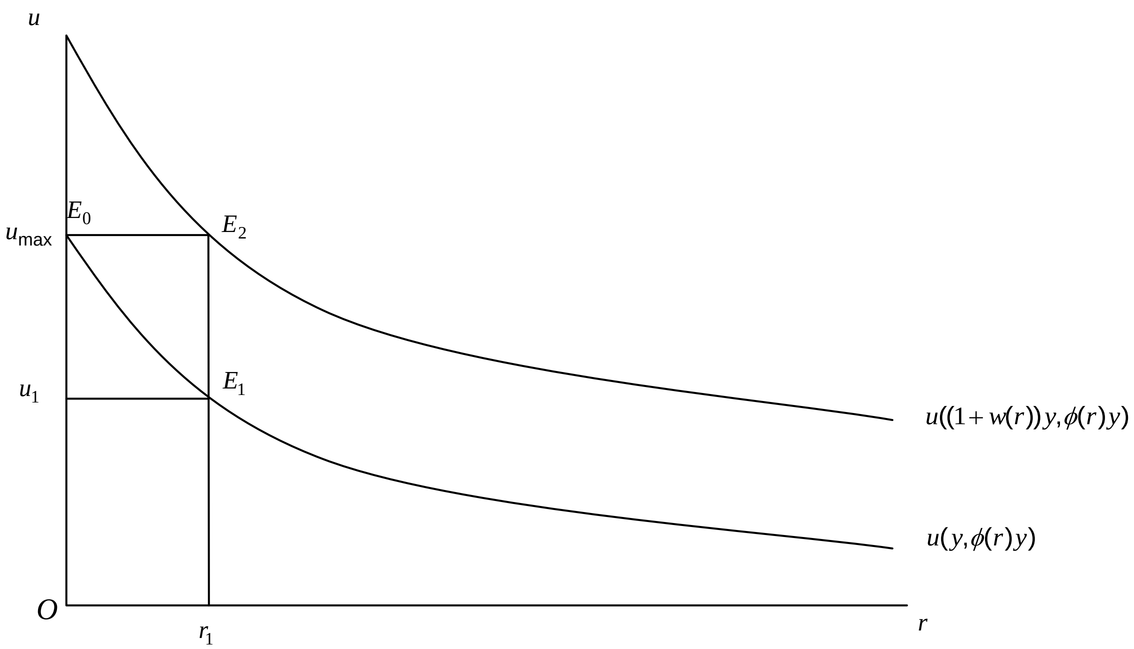

Figure 3-2: Welfare Cost calculation Using the Compensating Variation Approach.

Assuming a homothetic current period utility function ($u(c,m) = \frac{1}{1-\sigma}\left[cf\left(\frac{m}{c}\right)\right]^{(1-\sigma)}$; $\sigma \neq 1$) and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in $w(r)$ of the following form:

$$w'(r) = \psi\left(\frac{\phi(r)}{1+w(r)}\right)\phi'(r) \qquad (3.7)$$

For any given money demand function, Equation (3.7) can be solved numerically for an exact welfare cost function $w(r)$. In fact, with equation (3.1), equation (3.7) can be written as:

$$w'(r) = \eta A r^{(-\eta)}(1+w(r))^{\frac{1}{\eta}} \qquad (3.8)$$

yielding a solution for log –log specification

$$w(r) = -1 + \left(1 - Ar^{1-\eta}\right)^{\frac{\eta}{\eta-1}} \qquad (3.9)$$

While, for the semi-log model (7) yields

$$w'(r) = \left[\xi B e^{-\xi r}\left(r + \frac{1}{\xi}\log\left(1+w(r)\right)\right)\right] \approx \left[\xi B e^{\xi r}\left(r + \frac{1}{\xi}w(r)\right)\right] \qquad (3.10)$$

with a solution

$$w(r) = -e^{\frac{-Be^{-\xi r}}{\varepsilon}}\left\{e^{\frac{Be^{-\xi r}}{\xi}} - Ei\left[\frac{B}{\xi}\right] + Ei\left[\frac{Be^{-\xi r}}{\xi}\right]\right\} \qquad (3.11)$$

and where $Ei(x) = -\int_x^{\infty} \frac{e^{-t}}{t} dt$, and one uses the principal value of the integral.

Note to calculate $w(r)$, in equations (3.9) and (3.11),[17] we use the estimates of η and ξ obtained from the long-horizon regression, discussed in Section 4, while, the values for A and B are obtained such that they match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e., A $= \bar{m}/\left(\bar{r}\right)^{-\eta}$, B $= \bar{m}/\left(e^{-\xi\bar{r}}\right)$ with $\bar{m}$ and $\bar{r}$ being respectively the geometric means of m and r respectively.

3.3 Data

In this chapter, we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this chapter are the money balances ratio (rm3), generated by dividing the broad measure of money supply (M3)[18] by the nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate (tbr).[19] All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by lrm3 and ltbr, respectively.

[17] The calculations were done using the DSolve routine in Mathematica, Version 5.

[18] See chapter 2 for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3 was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

[19] We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

3.4 Empirical methodology and results

As it is standard in time series analysis, we start by studying the univariate characteristics of the data. In this regard, we performed tests of stationarity on our variables (lrm3, ltbr and tbr) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips-Perron (PP) test. As seen in chapter 2 , the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests. For the KPSS test, the null of stationarity was rejected. As the variables were found to be non-stationary, it paved the way for the long-horizon regression proposed by Fisher and Seater (1993) to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions. As stated at the onset, cointegration, is neither necessary nor sufficient for this approach, so we do not test specifically for cointegration.[20]

The basics of the long-horizon regression approach can be described as follows, by starting off with the following bivariate autoregressive representation:

$$\alpha_{mm}(L)\Delta^{\langle m\rangle}m_t = \alpha_{mr}(L)\Delta^{\langle r\rangle}r_t + \varepsilon_t^m \tag{3.12}$$

$$\alpha_{rr}(L)\Delta^{\langle r\rangle}r_t = \alpha_{rm}(L)\Delta^{\langle m\rangle}m_t + \varepsilon_t^r \tag{3.13}$$

where $\alpha_{mm}{}^0 = \alpha_{rr}{}^0 = 1$, $\Delta = 1 - L$. L is the lag operator, m is the money-income ratio, r is the nominal interest rate ,and $\langle x\rangle$ represents the order of integration of x, so that if x is integrated of order γ, or $\mathrm{I}(\gamma)$ in the terminology of Engle and Granger (1987), then $\langle x\rangle = \gamma$ and $\langle \Delta x\rangle = \langle x\rangle - 1$. The vector $\left(\varepsilon_t^m, \varepsilon_t^r\right)'$ is assumed to be independently and identically distributed normal with zero mean and covariance $\sum_\varepsilon$, the elements of which are $\mathrm{var}\left(\varepsilon_t^m\right)$, $\mathrm{var}\left(\varepsilon_t^r\right)$, $\mathrm{cov}\left(\varepsilon_t^m, \varepsilon_t^r\right)$. A key result in Fisher and Seater (1993) applies to the case where $\langle m\rangle = \langle r\rangle = 1$,

[20] The reader is referred to Gupta and Uwilingiye (2008) for the tests on stationality and cointegration on the variables of the model, reported in Tables 1 through 3.

which is the case with our data as money balance as lrm3, ltbr and tbr are all I(1). In this case, the long-run derivative of m with respect to r, LRD_{mr}, is given by:

$$LRD_{m,r} = \frac{\theta_{mr}(1)}{\theta_{rr}(1)} \quad (3.14)$$

with $\text{LRD}_{\text{m,r}}$ being interpreted as the long-run elasticity of m with respect to r. In fact, under the Fisher and Seater (1993) identification scheme, which assumes that r is exogenous in the long run, $\theta_{mr}(1)/\theta_{rr}(1)$ can be interpreted as $\lim_{k\to\infty} b_k$, where b_k is the coefficient from the regression:

$$\left[\sum_{j=0}^{k} \Delta^{\langle m \rangle} m_{t-j}\right] = a_k + b_k \left[\sum_{j=0}^{k} \Delta^{\langle r \rangle} r_{t-j}\right] + e_{kt} \quad (3.15)$$

and for $\langle m \rangle = \langle r \rangle = 1$, consistent estimate of b_k can be derived by applying ordinary least squares to the regression

$$m_t - m_{t-k-1} = a_k + b_k \left[r_t - r_{t-k-1}\right] + e_{kt}, \quad (3.16)$$

$$k = 1, \ldots\ldots K$$

Based on Equation (3.16) and for a value of k=30 as used by Serletis and Yavari (2004 and 2005), our estimate of the interest rate elasticity, η, is 0.1073 and interest semi-elasticity ξ is 1.0099,[21] which, in turn, are much lower than the corresponding values of 0.2088 and 2.1991, obtained in chapter 2 based on the Johansen (1991 and 1995) methodology.

Once we obtain the estimated values for η and ξ, using long-horizon regression, we calculate the values of A and B such that the curves obtained pass through the geometric means of the data. This gives us values of A = 0.4255 and B = 0.6035. Note the values for A and B obtained in chapter 2 based on the cointegrating relationships were, respectively, 0.3323 and 0.6862.[22]

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[21] Both the estimates of η and ξ are significant at the 1 percent level of significance.

[22] Based on the suggestions of one of the anonymous referees, equation (16) was re-estimated without the constant. The corresponding values of the interest rate elasticity, η, were found to be 0.0965 and that of the interest semi-elasticity ξ was 0.9556. Note both these values were found to be significant at the 1

Having obtained the estimates for η and ξ and the values for A and B, we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation method. The results have been reported in Table 3-2. Note for the sake of comparison, we also present the welfare cost estimates, based on the values of η, ξ, A and B, obtained in chapter 2 , based on the Johansen (1991 and 1995) approach.

Table 3-2: Welfare cost estimates

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Based on the results reported in the Columns 2 and 3, and 4 and 5, the welfare cost estimates obtained under the consumer surplus approach, for 3%, 6%, 10% and 15% of inflation, using the Johansen (1991 and 1995) cointegration method and the long-horizon regression approach respectively, we see that welfare costs are substantially lower in the latter case. In fact they are nearly less by more than half, of the costs obtained using the cointegration approach for both the log-log and the semi-log specifications. When we compare Columns 6 and 7, and 8 and 9, we obtain a similar picture for the welfare cost estimates obtained using the compensating variation approach. Further, the welfare cost estimates within a specific estimation method, but across the consumer surplus approach and the compensating variation approach are quite similar, with the figures being slightly higher under the compensating variation method outlined by Lucas (2000). Specifically, for the log-log (semi-log) specification, estimated using the cointegration approach, under the consumer surplus approach [compensating variation approach], an increase in the inflation rate from 3- 6% would increase the welfare cost from 0.67% of GDP to 1.08% of GDP

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percent level. Given, that the values of A and B would stay the same as above, we would obtain even lower estimates of the welfare cost of inflation under the two alternative specifications of money-demand.

[0.72 % of GDP to 1.17% of GDP] (0.76 % of GDP to 1.43% of GDP [0.79% of GDP to 1.4449% of GDP]). While, under the long-horizon approach the welfare cost estimates ranges between 0.18% of GDP to 0.35% of GDP and 0.19% of GDP to 0.37% of GDP with the log-log specification, obtained from the consumer surplus and the compensating variation approaches respectively, for an increase in the inflation rate from 3-6%, the corresponding values under the semi-log specification, for the same increase in the rate of inflation, are 0.15 % of GDP to 0.35% of GDP and 0.16% of GDP to 0.36% of GDP.

The bottom line is that, as in Serletis and Yavari (2004 and 2005) , we find the welfare cost estimates based on the long-horizon approach tends to be much smaller when compared to other standard econometric method of arriving at the long-run equilibrium relationship between ratio of money balance to income and the nominal interest rate. The reason is that, under the long-horizon approach estimates of interest rate elasticity and semi-elasticity tends to be comparatively lower. Given the fact that welfare cost estimates based on money demand estimations critically hinges on the size of interest rate elasticity and semi-elasticity, this brings down the welfare cost of inflation when compared to estimates obtained via econometric methods, such as the Johansen (1991 and 1995) approach.

3.5 Conclusion

In this chapter , using the Fisher and Seater (1993) long-horizon approach, we estimate the long-run equilibrium relationship between money balance as a ratio of income and the Treasury bill rate for South Africa over the period of 1965:02 to 2007:01, and, in turn, use the obtained estimates of the interest elasticity and the semi-elasticity to derive the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach, as well, as Lucas's (2000) compensating variation approach. When, the results are compared to welfare cost estimates obtained in chapter 2 , using the same data set, but based on Johansen's (1991, 1995) cointegration technique, the values are less by more than half of those obtained in chapter 2. This chapter highlights the fact that welfare cost estimates of inflation are sensitive to the methodology used to estimate the long-run equilibrium money demand relationships.

At this stage two aspects of the obtained results needs further emphasis: First, when compared to the literature, the welfare cost estimates obtained for South Africa, whether based on the long-horizon regression or the Johansen (1991 and 1995) cointegration approach, are relatively higher when compared to estimates available in the literature for other economies for similar levels of inflation rates. Second, it must be realised that whatever the estimation methodology used, whether it is a one consumer-surplus approach or a compensating variation method, based on our estimates, we can conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman (1969)-type deflationary rule of zero nominal rate of interest.

However, the following question is undeniably relevant: Given that welfare cost estimates are sensitive estimation methodologies and seem to vary considerably according to econometric approach is undertaken, what is the true size of the welfare cost of inflation in South Africa? The answer to this question is difficult. However, it must be admitted that econometric methodologies deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, a rise in the inflation rates can distort other marginal decisions and can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given these two additional sources of inflation costs, there is no denying the fact that larger gains can conceivably be achieved by reducing the inflation target below 3%, the lower limit of the current inflation target band.

Chapter 4

4 Time aggregation, long-run money Demand and the welfare cost of inflation*

4.1 Introduction

In chapter 2 and 3 we estimated the long-run money demand relationship for South Africa, and then, in turn, went ahead and used the interest rate elasticity and semi-elasticity to obtain the size of the welfare cost of inflation for the economy. Using the same data set, but two different approaches to estimate the long-run money demand functions, namely the cointegration procedure outlined in Johansen (1991, 1995) and the long-horizon approach proposed by Fisher and Seater (1993) respectively, we ended up with markedly different measures of the welfare cost of inflation. Specifically speaking, in chapter 3, using the long-horizon methodology, found the value to fall by more than half as that obtained in chapter 2, where the estimations were obtained from the cointegration approach. The difference between the results, essentially emanated from the smaller sizes of the interest rate elasticity and semi-elasticity obtained under the long-horizon approach relative to the cointegration procedure.

At this stage, it is important to point out that such a finding is not an exception in the welfare cost literature. Besides, the importance of sample period, the money demand specifications, i.e., double-log (Meltzer, 1963) or semi-log (Cagan, 1956), and the versions of the monetary aggregate, the importance of the estimation procedure, namely cointegration or long-horizon, have been noted by host of authors, with the latter producing the most drastic of differences in the measures of welfare costs within an economy over identical sample periods using same data sets.[23] To the best of our knowledge though, no study thus far has attempted to figure out which of the two methods is more robust and ideally suited in providing the estimates of interest elasticity and semi-elasticity, and, hence, the appropriate measure of the size of the

[23] See for example Fischer (1981), Lucas (1981, 2000), Serletis and Yavari (2004, 2005, 2007), Serletis and Virk (2006) and Ireland (2009), amongst others.

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distortionary effect of inflation in the money market. But, as discussed in both chapter 2 and 3, econometric methodologies, whether based on cointegration or the long-horizon approach, deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Since, such welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates, and, hence takes a partial equilibrium approach. Given that, in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output, welfare cost estimates of inflation are likely to be much higher. Hence, the ideal approach to obtaining a welfare cost estimate of inflation would be to use a dynamic general equilibrium model. Nevertheless, this line of argument does not provide an answer to the controversy regarding the true size of the distortionary effect of inflation on the money market or in a partial equilibrium framework. In this chapter, we make an attempt to resolve this issue by delving into the role of time aggregation on the long-run properties of the data, and, hence, the estimates of the welfare cost of inflation based on the money market.

It must be realized that the data on the three critical variables, required in the estimation of a money demand function, namely, a monetary aggregate and measures of real income and the opportunity cost variable, are generally available at different frequencies. Specifically, the interest rate is available at the highest frequency of weeks, the monetary aggregates in monthly form, while, the real income, generally measured by real GDP, is available only at quarters. Given this, a quarterly money demand estimation would require one to convert the weekly and the monthly variables into their quarterly form. In this regard, two approaches that are generally used are either temporal aggregation or systematic sampling. Temporal aggregation simply means aggregating over the weeks (for the interest rate) or months (for the monetary aggregate) of a quarter and using the average value as the quarterly value. Systematic sampling, on the other hand, involves using a single observation from the sampling interval, such as the end of the interval observation, which in our case would be the last week or month of a specific quarter, depending on whether we are trying to convert the measure of the interest rate or the monetary aggregate,

The motivation to use the effect of time aggregation on the two methods of estimating long-run money demand relationships is derived from the recent work of Marcellino (1999). In this paper, the author indicated, theoretically and via an example, that aggregation, via temporal aggregation or systematic sampling, tends to affect only the short-run properties of the data, leaving the long-run aspects of the data unchanged.[24] Given this then, one would expect that within a specific econometric methodology, i.e., long-horizon or the cointegration approach, the estimates of the parameters in the long-run money demand relationships, log-log or semi-log, should not be significantly affected depending on whether the opportunity cost and the monetary aggregate variables were converted to their respective quarterly values based on temporal aggregation or systematic sampling. In other words, by using time aggregation, we expect to determine which of the estimates of the welfare cost of inflation via the money market, obtained through either the Johansen (1991, 1995) methodology or the Fisher and Seater (1993) approach is more robust, and, hence, should be taken more seriously. It is important to point out that, though Marcellino (1999) indicates that long-run properties of the data are virtually unchanged because of alternative sampling methods, the author does indicate the need to verify the theoretical claims with data relating to the specific question under consideration.

In this respect, we re-evaluate, based on the same data set, the results obtained in chapter 2 and 3 by using systematic sampling, instead of temporal aggregation used in chapter 2, to convert the measures of the monetary aggregate and the interest rate into their respective quarterly values. At this stage, it must be emphasized that we are not really trying to draw overwhelming conclusions regarding the robustness of these two alternative estimation methodologies, but, merely trying to deduce what is the appropriate size of the inflationary distortion on the welfare of the South African economy, via the money market. Given that South Africa has an inflation targeting band of 3-6%, the importance of knowing what is the true size of the welfare cost of inflation due to the distortion caused by the positive nominal interest rate on the money market, is of utmost importance. So, our study should not be evaluated in the light of an attempt to check for the credibility of the two methodologies under alternative sampling techniques, since the possibility of obtaining different conclusions based on a different set of variables, sample sizes and the economy(ies) concerned cannot be ignored. The remainder of the chapter is organized as follows: Section 4.2 outlines the theoretical foundations involved in the estimation of the welfare cost of inflation based on the money market distortion. Section 4.3 discusses the data and the results, which includes the estimates of the parameters in the money demand functions, as well as the, measures of the welfare cost of inflation. Finally, Section 4.4 concludes.

[24] Similar observations has also been made by Gupta and Komen (2008) while analyzing the causal relationship between the repo rate and the CPIX inflation in South Africa.

4.2 The theoretical foundations

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the "consumer's surplus", that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_{0}^{r} m(x)dx - rm(r) \tag{4.1}$$

where m, is the ratio of money balances to nominal income, and r measures the short-term nominal interest rate.

As seen from Equation (4.1), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss.

Since the function m has the dimensions of a ratio to income, so does the function w. The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by: $\ln(m) = \ln(A) - \eta \ln(r)$ or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right) r^{1-\eta} \tag{4.2}$$

where A>0 is a constant and η>0 measures the absolute value of the interest elasticity of money demand.

While, for a semi-log money demand specification i.e., $\ln(m) = \ln(B) - \xi r$ or $m(r) = Be^{-\xi r}$, w(r) is obtained by the following formula:

$$w(r) = \frac{B}{\xi}\left[1 - (1+\xi r)e^{-\xi r}\right] \tag{4.3}$$

where B>0 is a constant and ξ>0 measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

As can be seen from (4.2) and (4.3), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. To start off, Lucas (2000) uses Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate r, $w(r)$, to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at r and an otherwise identical steady-state with the interest rate of zero. With, $w(r)$ being obtained from the solution to the following equation:

$$u\left[1+w(r))y, \phi(r)y\right] = u\left[y, \phi(0)y\right] \tag{4.4}$$

Assuming a homothetic current period utility function ($\mathrm{u(c,m)} = \frac{1}{1-\sigma}\left[\mathrm{cf}\left(\frac{\mathrm{m}}{\mathrm{c}}\right)\right]^{(1-\sigma)}; \sigma \neq 1$) and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in $w(r)$ of the following form:

$$w'(r) = \psi\left(\frac{\phi(r)}{1+w(r)}\right)\phi'(r) \tag{4.5}$$

For any given money demand function, Equation (4.5) can be solved numerically for an exact welfare cost function $w(r)$. In fact, with $m(r) = Ar^{-\eta}$, equation (4.5) can be written as:

$$w'(r) = \eta A r^{(-\eta)}(1+w(r))^{\frac{1}{\eta}} \tag{4.6}$$

yielding a solution for log –log specification

$$w(r) = -1 + \left(1 - Ar^{1-\eta}\right)^{\frac{\eta}{\eta-1}} \tag{4.7}$$

While, for the semi-log model (4.5) yields

$$w'(r) = \left[\xi Be^{-\xi r}\left(r + \frac{1}{\xi}\log\left(1 + w(r)\right)\right)\right] \approx \left[\xi Be^{\xi r}\left(r + \frac{1}{\xi}w(r)\right)\right] \tag{4.8}$$

with a solution

$$w(r) = -e^{\frac{-Be^{-\xi r}}{\varepsilon}}\left\{e^{\frac{Be^{-\xi r}}{\xi}} - Ei\left[\frac{B}{\xi}\right] + Ei\left[\frac{Be^{-\xi r}}{\xi}\right]\right\} \tag{4.9}$$

and where $Ei(x) = -\int_x^{\infty}\frac{e^{-t}}{t}dt$, and one uses the principal value of the integral.

Note to calculate $w(r)$ under Bailey's (1956) and Lucas'(2000) approaches, we use the estimates of η and ξ obtained from both the cointegration and long-horizon regression. While, A and B are obtained directly from the cointegrating relationships, the values of the same, under the long-horizon regression, is derived to ensure that they match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e., A $= \bar{m}/\left(\bar{r}\right)^{-\eta}$, B $= \bar{m}/\left(e^{-\xi\bar{r}}\right)$ with $\bar{m}$ and $\bar{r}$ being respectively the geometric means of m and r respectively.[25]

4.3 Data and Results

As in chapter 2 and 3 , we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this chapter are the money balances ratio (rm3), generated by dividing the broad measure of money supply (M3)[26] by the

[25] For details regarding the estimation methodologies refer to Chapter 2 and 3 .

[26] See chapter 2 for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3

nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate (tbr).[27] All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by lrm3 and ltbr, respectively. Note, given that weekly values of the 91 days Treasury Bill rate is only available from the beginning of 1981, and to keep our data set identical to the one used in chapter 2 and 3, we use monthly data on both M3 and the interest rate measure to convert them into quarterly figures via systematic sampling, unlike temporal aggregation used in chapter 2 and 3.

After obtaining all the series in their quarterly forms, as is standard in time series analysis, we start off by studying the univariate characteristics of the systematically sampled series. In this regard, we performed tests of stationarity on our variables (lrm3, ltbr and tbr) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As in Gupta and Uwilingiye (2008), all the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As all the variables were found to be non-stationary, to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions, the Johansen (1991, 1995) cointegration method and the long-horizon regression proposed by Fisher and Seater (1993) was used to obtain the long-run relationships.

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was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

[27] We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

Table 4-1: Unit Root Tests (Systematic Sampling).

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t\ \tau_\mu$	$\tau_t\ \tau_\mu$	
LRM3	τ_t	-0.04	2.57	-0.03	0.31	-0.30	Non- stationary
	τ_μ	-0.22	0.05	-0.27	0.32***	-0.62	
	τ	-0.69		-0.69			
D(LRM3)	τ_t	-13.71***	94.04***	-13.71***	0.09***	-13.58***	Stationary
	τ_μ	-13.31***	177.13***	-13.30***	0.56*	-13.35***	
	τ	-13.31		-13.31***			
LTBR	τ_t	-2.61	11.89***	-2.29	0.29	-2.52	Non- stationary
	τ_μ	-2.45	17.21***	-2.28	0.90	-1.35	
	τ	-0.76	-2.28	-0.82			
D(LTBR)	τ_t	-8.60***	37.01	-8.61***	0.03***	-8.52***	Stationary
	τ_μ	-8.60***	73.93	-8.60***	0.09***	-7.84***	
	τ	-8.62***		-8.62***			
TBR	τ_t	-2.50	7.02***	-2.32	0.27	-2.47	Non- stationary
	τ_μ	-2.45	10.25***	-2.30	0.73*	-1.63*	
	τ	-0.86		-0.78			
D(TBR)	τ_t	-9.60***	46.04***	-9.62***	0.03***	-9.65***	Stationary
	τ_μ	-9.60***	92.08***	-9.62***	0.08***	-9.48***	
	τ	-9.62		-9.65***			

*(**) [***] indicates statistical significance at 10(5)[1] percent level.

Before deriving the long-run money demand relationships using the Johansen (1991, 1995) methodology, a test for the stability of the VAR model, including a constant as an exogenous variable was performed. Given that no roots were found to lie outside the unit circle for the estimated VAR based on 2 lags[28] for both specification of money demand, we conclude

[28] The choice of two lags was based on the unanimity of the Schwarz Information Criterion (SC) Hannan-Quinn (HQ) Information Criterion.Note the optimal lag length used by Gupta and Uwilingiye (2008) based on temporally aggregated data was four. However, it must be noted that although there existed

that the VARs are stable and suitable for further analysis. Once the issues of stability and the optimal lag length were settled, we tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included two lags in the VAR, and allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, the Maximum Eigen Value tests, showed that there is one stationary relationship in the data (r = 1) at 5 percent level of significance for both the log-log and the semi-log specifications. The results have been reported in Tables 4-2 and 4-3.[29] Interestingly, unlike with the temporally aggregated data used in chapter 2, the trace test failed to detect any cointegrating relationship. Thus immediately, we get to see the differences in the results obtained under the two alternative sampling techniques, even though Marcellino (1999) claims that alternative forms of aggregation do not tend to affect the long-run properties of the data.

Table 4-2: Estimation and Determination of Rank (Log-Log).

Null hypothesis	Alternative Hypothesis	Test statistic	0.05 critical value	Prob. **
			Trace Statistic	
r=0	r=1	15.15050	15.49471	0.0563
r=1	r=2	0.157021	3.841466	0.6919
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	r=1	14.99348	14.26460	0.0383
r=1	r=2	0.157021	3.841466	0.6919
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

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overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen tests with two lags. We had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship of the semi-log form

[29] As in Ireland (2009), we also used the Phillips-Ouliaris (1990) test for cointegration. However, unlike (2009), the test could not detect any cointegrating relationship between the chosen variables. Hence, the results of the test have been suppressed to save space. They are, however, available upon request.

Table 4-3: Estimation and Determination of Rank (Semi-Log).

Null hypothesis	Alternative Hypothesis	Test statistic	0.05 critical value	Prob. **
			Trace Statistic	
r=0	r=1	14.88209	15.49471	0.0617
r=1	r=2	0.115014	3.841466	0.7345
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	r=1	14.76707	14.26460	0.0416
r=1	r=2	0.115014	3.841466	0.7345
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

As we are more interested in the relationship between the money balance ratio and interest rate, for both specifications, lrm3 was restricted to unity. Given that we have only one cointegrating vector, the normalizing restriction on lrm3 is enough to exactly identify the long-run relationship. However, as in chapter 2, we encountered two serious econometric problems with this restriction. First, the restriction was not binding. Secondly, the adjustment coefficient of lrm3 was insignificant under both the specifications. Imposing an additional zero restriction on the adjustment coefficient of lrm3 did ensure binding restrictions, but at the cost of suggesting that the ratio of real balance to income was in fact exogenous and we should not be normalizing on lrm3. Given this, we decided to normalize on the interest rate variable, i.e., ltbr for the log-log specification and tbr for the semi-log specification. Further, with the adjustment coefficients on lrm3 still being insignificant in both the models, we restricted them to zero, and obtained binding restrictions.[30] Note with lrm3 now treated as the right-hand side variable, weak exogeneity of the same is what should be expected. The adjustment coefficients of ltbr and tbr were negative and significant, with them correcting for 7.1 percent and 8.4 percent of the disequilibrium in the next period, respectively.

[30] Note the value of the LR test statistics for binding restrictions, both long- and short-run, for the log-log and the semi-log specifications respectively, were $\chi^2(1) = 0.5578$ (0.4551) and $\chi^2(1) = 0.0587$ (0.8085), where the numbers in the parenthesis indicates the probability of committing a Type I error.

Based on the above restrictions, the interest elasticity of money demand is found to be equal to 0.2316, while, 2.4794 was the obtained value for the interest semi-elasticity of money demand.[31], The values of A and B, based on cointegrating relationships are, respectively, 0.3187 and 0.7153.[32] Note in chapter 2, the estimates of the intercept and slope coefficient based on temporally aggregated data implied values of A = 0.3323 and that of η = 0.2088, while for the semi-log specification B = 0.6862 and ξ = 2.1991. So, as can be seen, systematic sampling increases the values of the elasticity and semi-elasticity. However, the value of the intercepts increases for the semi-log model and falls for the log-log model.

After having estimated the money demand relationships via the Johansen (1991, 1995) cointegration approach, we resorted to the long-horizon approach of Fisher and Seater (1993) to obtain the estimates of A and η, and B and ξ. Our estimate of the interest rate elasticity, η, yields a value of 0.1160.and that of interest semi-elasticity, ξ, equal to 1.1027.[33] Once we obtain the estimated values for η and ξ using the long-horizon regression, we then calculate the values of A and B such that the curves obtained pass through the geometric means of the data. This gives us values of A = 0.4221 and B = 0.6166. Note, the corresponding values of A and η, and B and ξ obtained in chapter 3 were 0.4255, 0.1073, 0.6035 and 1.001 respectively. As with the

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[31] The obtained cointegrating relationships are:

(i) Log-Log: *ltbr = -4.9388 – 4.3186 (lrm3), and;*
[*-3.1490*]

(ii) Semi-Log: *lbr = -0.1352 – 0.4033(lrm3).*
[*-3.0974*]

[32] See chapter 2 for a discussion on how the values for the parameters of the money demand functions was obtained out of the estimated inverse versions of the same. The obtained cointegrating relationships were:

(i) Log-log: *ltbr=-5.2760- 4.7898 (lrm3), and*;
[*-3.8797*]

(ii) Semi-Log: *lbr = -0.1713 – 0.4547(lrm3).*
[*-3.8888*]

[33] Given that *lrm3, ltbr* and *tbr* are all *I(1)*, the interest elasticity and semi-elasticity are obtained from an OLS estimation of the following equation: $m_t - m_{t-k-1} = a_k + b_k\left[r_t - r_{t-k-1}\right] + e_{kt}$, where *m* is the log of the ratio of money balance, while *r* is the log of the nominal interest rate in the log-log specification and is specified in levels for the semi-log version of the money demand. Following Serletis and Yavari (2004 and 2005), *K* is set equal to 30.

Johansen (1991, 1995) approach based on the systematic sampling, the values of the elasticity and semi-elasticity increases, when compared to those obtained in chapter 2 under temporal aggregation. While, as above, the value of the intercepts increases for the semi-log model and falls for the log-log model. Again as with the cointegration approach, under the long-horizon approach, the t-tests on the interest elasticity and semi-elasticity across the two models under alternative sampling techniques reveal that they are statistically different at one percent level of significance. The results have been presented in Table 4-4. So clearly, unlike as suggested by the theoretical results of Marcellino (1999), long-run elasticities of money demand are significantly affected by alternative sampling techniques. Nevertheless, given the theoretical results of Marcellino (1999), the important aspect that needs to be determined here, would be the robustness of the welfare cost estimates based on the alternative values of the interest elasticity and semi-elasticity of the money demand functions. In other words, we want to know, which of the two estimation methods produces the least changes across the alternative sampling methods.

Having obtained the estimates for η and ξ and the values for A and B, both from the Johansen (1991, 1995) approach and the long-horizon regression, we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation method. The results have been reported in Table 4-4. Note for the sake of comparison, in Table 4-5, we also present the welfare cost estimates, based on the values of η, ξ, A and B, obtained in both chapter 2 and 3 using both of the above mentioned estimation methodologies. Plugging these values into the corresponding formula for the welfare cost measures, given by equations (4.2), (4.3), (4.7) and (4.9), and using the fact that the average real rate of interest[34] over this period was equal to 7.70 percent, so that a zero rate of inflation would also imply a nominal rate of interest equal to 7.70 percent, we obtain the baseline value of w under price stability. Naturally then, a value of r = 10.70 corresponds to a 3% rate of inflation, while, when r = 13.70, the economy experiences a 6% inflation, and so on.
So the welfare costs of inflation are evaluated by subtracting the value of w at an inflation equal to zero from the value of the same at a positive rate of inflation. Based on Tables 4-4, 4-5[35] and 4-6 the following conclusions can be drawn:

[34] Note, as in Ireland (2009), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI. In addition, the real rate of interest was found to be stationary based on the ADF, the DF-GLS, the KPSS and the PP tests of unit roots.

[35] Note, we have replicated Table 3-1 from chapter 3 as Table 4-5 in this paper.

Except for the welfare costs evaluated under the compensating variation method for the double log model estimated with the Johansen (1991, 1995) cointegration approach,[36] systematic sampling tends to increase the welfare cost of inflation in all the other cases;

Under the long-horizon approach, irrespective of whether we use systematic sampling or temporal aggregation and the compensating variation or the consumer surplus approach, the pattern of movement of the welfare cost of inflation as we increase the interest rate stays the same. In other words, the welfare cost estimates from the semi-log model tends to be higher than the log-log version at higher interest rates across both methods of aggregation. Further, under both systematic sampling and temporal aggregation, the compensating variation approach produces slightly higher welfare cost estimates for both types of money demand functions;

With the cointegration approach, except for the log-log model under compensating variation with systematic sampling, welfare costs are always lower under the consumer surplus method across both sampling technique and econometric models. Again, as with the long-horizon, the semi-log version of the model tends to yield higher costs of welfare at higher interest rate across the sampling techniques;

Over all, when we compare the two methodologies based on the percentage difference in the welfare cost estimates across the two sampling techniques, the long-horizon approach tends to produce more robust estimates of the welfare cost of inflation via the money market. In other words, for 3%, 6%, 10% and the 15% levels of inflation, the percentage change in the welfare cost of inflation for moving from temporal aggregation to systematic sampling is consistently lower under the Fischer and Seater (1993) approach in comparison to the Johansen (1991, 1995) cointegration methodology. Based on this criteria solely, we would want to conclude that the widest range of the welfare cost estimates for a target band of 3-6% rate of inflation, falls between 0.15% (obtained from the semi-log model estimated with temporal aggregation) to 0.41 % (obtained from the log-log model estimated with systematic sampling) These numbers, in turn, are much lower than the range of 0.34% (obtained from the log-log and semi-log model estimated with temporal aggregation) to 0.90% (obtained from the log-log model estimated with systematic sampling) based on the cointegration approach under alternative methods of sampling.

[36] The exception arises due to the fact that even though the interest elasticity increases, the size of the fall in A is such that it tends to reduce the welfare cost estimates, based on equation (4.7), under the Johansen (1991, 1995) approach for the log-log model, when compared to the same model estimated with temporally aggregated data.

Table 4-4: Welfare cost estimates (Systematic Sampling).

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0039	0.0039	0.0019	0.0017	0.0023	0.0040	0.0021	0.0017
6	0.0075	0.0087	0.0038	0.0039	0.0046	0.0090	0.0041	0.0040
10	0.0119	0.0162	0.0062	0.0075	0.0077	0.0168	0.0068	0.0077
15	0.0173	0.0270	0.0092	0.0129	0.0116	0.0283	0.0101	0.0135

Table 4-5: Welfare cost estimates (Temporal aggregation).

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Table 4-6: Percentage Change in Welfare Cost Estimate Under Temporal Aggregation and Systematic Sampling.

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	14.71	14.71	5.56	13.33	37.84	14.29	10.53	6.25
6	11.94	14.47	8.57	11.43	36.11	13.92	10.81	11.11
10	10.19	13.29	8.77	10.29	34.19	12.75	9.68	10.00
15	10.90	12.03	9.52	9.32	32.56	12.75	9.78	9.76

Values Computed Using: $\left|\left(\frac{(w^{syst} - w^{temp})}{w^{temp}}\right)\right| \times 100$

4.4 Conclusions

The two previous chapters have found markedly different measures of the welfare cost of inflation in South Africa, obtained through the estimation of long-run money demand relationships using cointegration and long-horizon approaches, respectively. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income, and that long-run properties of data are unaffected under alternative methods of time aggregation (Marcellino, 1999), in this chapter , we tested for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Our results indicate that the long-horizon method is more robust, in terms of lower percentage change in the welfare cost measures across the two alternative methods of time aggregation, and, given this the welfare cost of inflation in South Africa for an inflation target band of 3-6% lies between 0.15% and 0.41%. Based on these set of results, we can, thus, conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman (1969)-type deflationary rule of zero nominal rate of interest.

It is, however, important to point out that, in this chapter, we are only looking at welfare cost of inflation using a partial equilibrium approach. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Hence, the path ahead should involve obtaining the size of the welfare cost of inflation using a dynamic general equilibrium endogenous growth model. Then only, we will be able to deduce whether there are possibly larger gains of reducing the inflation target below 3%.

Chapter 5

5 Some Benefits of Reducing Inflation in South Africa*

5.1 Introduction

South Africa moved to an inflation targeting framework in the February of 2000. Ever since the sole objective of the South African Reserve Bank (SARB) has been to ensure that inflation lies within the target band of 3-6%. In this regard, the measurement of costs and benefits of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink the band in terms of the welfare cost of inflation at least. Among the costs, those that are caused by the interaction of inflation with tax rules needs to be emphasized. Due to the non-indexation of the South African tax system, inflation exacerbates the inefficiencies generated by taxation. The quantitative significance of these efficiencies is likely to be particularly strong in case of taxation of capital – a mobile factor of production. Building on the methodological foundation of Feldstein's (1997, 1999) approaches, this paper examines the welfare implications of the interaction between capital income taxation and inflation. We consider the per-year welfare effects of going from 2 percent[37] inflation to price stability, and compare them with the output costs of disinflation.

Based on the estimate of interest elasticity of money for South Africa, obtained in chapter 4, a two percent inflation rate would translate into a welfare loss of 0.098 percent of Gross Domestic Product (GDP) using Bailey's (1956) consumer surplus approach.[38] However, it must be realized that welfare cost calculations obtained by integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue to deduce the

[37] Note the calculations are symmetric and, approximately, linear. Therefore, it is not difficult to translate the estimates, obtained under an inflation rate of 2 percent, for the width of the target band. The decision to use a 2 percent rate of inflation is dictated by the approach in the original contributions of Feldstein (1997, 1999), and the literature that followed thereof.

[38] See chapter 4 for further details.

*Forthcoming in International Business and Economics Research Journal.

deadweight loss, is merely one-dimensional. This is because, the consumer surplus approach fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects on the intertemporal consumption choice (i.e., saving for old age), housing and the real cost of servicing government debt. Thus, the welfare costs obtained using the money demand approach is likely to provide the lower limit of such estimates, and, hence, a more general approach, like the one adopted here, is desired to obtain the "true" size of the welfare loss caused by inflation. It must, however, be stressed that, recent evidence[39] on the sacrifice ratio of South Africa tends to suggest that disinflation can be achieved at virtually no loss to employment and output. Given this, and Feldstein's (1997, 1999) arguments that costs of disinflation are temporary while the benefits are permanent, i.e., one needs to compare the discounted stream of benefits with one-off output costs, even a small-sized estimated benefit could imply relatively large overall gain from a permanent disinflation of 2%.To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy using Feldstein's (1997, 1999) approaches that accounts for interactions between the tax system and inflation.[40]

The remainder of the chapter is organized as follows: Section 5.2 presents the theoretical background of the analysis, while, Section 5.3 calculates distortion to rates of return and hence the price of retirement consumption, resulting from inflation. Section 5.4 examines the distortion in housing demand, Section 5.5 looks at money demand distortion, and Section 5.6 considers the distortionary effect on government debt servicing. Section 5.7 brings together the several effects of reduced inflation identified in Sections 5.2 through 5.6 and Section 5.8 concludes.

5.2 Theoretical Background

At the moment, most tax systems around the world are not completely indexed to ensure that the price-level changes leave real tax rates and real tax revenue unchanged.

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[39] See for example Akinboade et al. (2004), Woglom (2005), Gonçalves and Carvalho (2008), and Tunali (2008).

[40] For applications of Feldstein's (1997, 1999) approaches on other countries, refer to Bonato (1998) for New Zealand, Bakhshi et al., (1999) for the United Kingdom, Dolado et al., (1999) for Spain, Tödter and Ziebarth (1999) for Germany, O'Reilly and Levac (2000) for Canada and Blaszkiewicz et al., (2003) for Poland and Ukraine.

Inflation-induced distortions generated by the interaction of inflation and the non-indexed tax system have the potential to be much larger than the revenue-related effects on which most of the seigniorage and optimal inflation literature has focused (Walsh, 2003).

One important distortion arises when nominal income and not real interest income is taxed. It must be realized that it is after tax real rates of return that is relevant for individual agents in making saving and portfolio decisions, and if nominal income is subject to a tax rate of τ, the real after –tax return will be

$$r_a = (1-\tau)i - \pi$$

$$= (1-\tau)r - \tau\pi, \qquad (5.1)$$

where $i = r + \pi$ is the nominal return and r is the before-tax real return. Thus for a given pre-tax real return r, the after-tax real return is decreasing in the rate of inflation. Practically speaking, let us consider a two–period overlapping generations model, where individual work and earn income when young and also decides on how much to consume currently and save for their old age. Suppose that savings are invested at the rate of r. Therefore, consumption in old age is related to savings by the following equation:

$$c = s(1+r)^T \qquad (5.2)$$

where T is the length of the period between saving while young and dissaving in the old age. Then price of retirement consumption can be defined as $p = \frac{1}{(1+r)^T}$. (5.3)

Clearly, the relative price of old-age consumption P is affected by both tax system and inflation, since they distort the choice between current consumption and future consumption. Graphically, the scenario is depicted in Figure 5-1.

The figure 5-1 depicts the individual's compensated demand for retirement consumption, labeled as Quantity, as a function of the price of retirement consumption p, denoted as Price, at the time of the decision to save. The different points on the graph represent different scenarios. With combination c_0, p_0 representing consumption decision without tax and inflation, the consumer surplus is A+B+C….+F. Introducing income taxes in an environment of price stability (no inflation) moves the equilibrium point from c_0, p_0 to c_1, p_1 which leads to a lesser retirement consumption at a higher price. Consumer surplus is now reduced to the area: C+E+F and the tax revenues corresponding to that area is B+D. Triangle A, thus, represents the deadweight loss, which, in turn, is the reduction of consumer surplus not compensated by higher tax revenues. When we introduce both taxes and inflation the equilibrium point from c_1, p_1 to c_2, p_2, and again there is a reduction in consumption at high price. The consumer surplus remaining is F and tax revenue is the rectangle D+E. The deadweight loss increases from triangle A to triangle A+B+C.

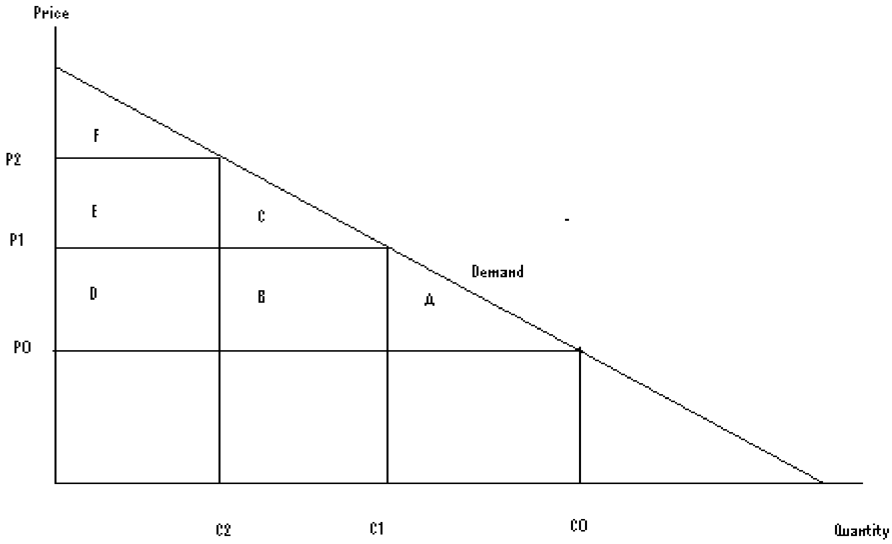

Figure 5-1: Demand for Retirement Consumption.

Thus, moving from equilibrium inflation to price stability increases consumer surplus by the area C+E, the tax revenues change to area B-E(which can be negative or positive) and the welfare gain is B+C which is a trapezoid as shown above. Unlike the traditional welfare analysis where welfare changes were obtained using "Haberger triangles" and, hence, were of second order, i.e., small, in the presence of distortionary taxes, the initial situation is not optimal and welfare changes are of first order, as indicated by a trapezoid rather than triangle.

5.3 Inflation and the Inter-Temporal Allocation of Consumption.

5.3.1 Distortions to Saving Behavior

The household has two main decisions to be make on their expenditure, namely, how much to consume and how much to invest in each period.

Feldstein (1997) derives the welfare gain from reducing inflation in a two period consumption model. Individuals are given an initial endowment and then they decide on the portions of their income to consume and save in the first period in order to consume when they are retired in the second period. The agent's first period savings earns a real rate of return, and in period one, the price of retirement consumption (p) is thought to be inversely related to this rate of return, i.e., the higher the rate on saving, the cheaper the effective price of retirement consumption. The rate of return on saving depends on both inflation and the tax system. According to Feldstein et al., (1978), inflation is a source of irregular change on the effective tax rate of capital income, which leads to changes in real net of post- tax return. Taxes drive a wedge between the pre-tax rate of return which is assumed to be invariant to inflation and the post tax return that household earn. Higher inflation raises the tax wedge and reduces the effective real post-tax return to saving and increases the price of retirement consumption. Given this, the welfare gains associated with reduction in inflation with current tax system can be obtained from the following expression:

$$G_1 = B + C = \left[\left(\frac{p_1 - p_0}{p_2}\right) + 0.5\left(\frac{p_2 - p_1}{p_2}\right)\right]\left(\frac{(p_2 - p_1)}{p_2}\right)S_2\left(1 - \eta_{S_p} - \sigma\right) \tag{5.4}$$

where p_0 is the price of retirement consumption at zero inflation with no distortionary taxes; p_1 is the retirement price evaluated under the current tax regime with price stability (zero inflation) and p_2 is the price evaluated under current tax regime with 2 percent inflation[41]; s_2 represents the initial gross saving of individual when young; η_{s_p} is the uncompensated elasticity of saving with respect to the price of retirement consumption, and; σ is the propensity to save out of exogenous income.

Note, to calculate p_0, we need the pre-tax real rate of return on savings. For this purpose, we use the rate of return on equity which on average was equal to 7.06 percent[42] for the period of 1990-2007. Assuming that at this rate of return, both inflation and taxes are non-existent and the time interval between saving and consumption is 30 years; $p_0 = (1+0.0706)^{(-30)}=0.1292$. In order to calculate the real return to savings in a world of taxes and inflation, we need to adjust the

[41] Even though the inflation over the period of 1990-2007 was 7.3 percent on average, we decided to use a figure of 2 percent to make our analysis comparable with the literature that uses Feldstein's (1997, 1999) approaches. Note also, we consider the period of 1990-2007 for our calculations, due to data availability on all the relevant variables over this time period.

[42] The return on equity is calculated using the percentage change in the All Share Index (ALSI). Source: International Financial Statistics.

above real rate of return on savings for both corporate and personal sector taxes. The average rate of corporate income tax between 1990-2007 was 25.68 percent[43], which, in turn, leaves a net real return r to 5.25 percent, before personal tax deductions. The net of tax rate of return depends not only on the tax at the corporate level but also on the taxes that individuals pay on interest income, dividends and capital gains. The effective marginal tax rate depends on the form of the income and on the tax status of the individual. Feldstein (1997) summarizes these effects by assuming an uniform individual marginal tax rate across all sources of income. Given this, the individual marginal tax rate in South Africa across all source of income averaged to 25 percent[44] over the period of 1990-2007. This reduces the net return further to 3.94 percent. The price of retirement consumption that correspond to this net return of 3.94 percent is therefore $p_2 =$ $(1.0394)^{(-30)} = 0.3137$ where the subscript 2 on the price indicates the price of retirement at an inflation rate of 2 percent.

Reducing the equilibrium inflation rate from two 2 percent to zero lowers the effective tax rate at both corporate and individuals levels. At the corporate level, this has two opposing effects: First, the changes in the equilibrium inflation rate alter the effective tax rate by changing the value of depreciation allowances, and; second, it changes the value of the deduction of interest payments. Because the depreciation schedule that is allowed for calculating taxable profits is defined on the basis of historical nominal terms, a higher rate of inflation reduces the present value of depreciation and thereby increases the effective tax rate. This relation was approximated by Feldstein (1997) using a rule of thumb of 0.57 percent increases in taxable profit for each percentage point of inflation. Due to lack of this estimate in South Africa, we use the same value as Feldstein (1997). With marginal corporate income tax rate at 30 percent[45], a 2 percent reduction in inflation raises the net of tax return and hence decreases effective tax rate by $0.30(0.57)(0.02) = 0.0034$ or 0.34 percentage points. The interaction of the interest deduction and inflation moves the after tax yield in the opposite direction. If each percentage point of inflation raises the nominal corporate borrowing rate by one percentage point[46], the real pre-tax cost of borrowing is unchanged but companies get an addition deduction in calculating their

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[43] Source: McGregory BFA.

[44] The value corresponds to the average of marginal individual tax rate and capital gains tax for individuals (Source: Tax Pocket Guide 2006/7).

[45] Source: The value corresponds to the average marginal corporate tax rate (Source: World Bank, World development Indicators).

[46] See footnote 24 in Feldstein (1997) for further details.

taxable income. With debt to capital ratio of 59 percent[47] and a corporate tax rate of 30 percent, a 2 percent decline in inflation raises the effective tax rate by 0.30(0.59)(0.02)=0.0035 or 0.35 percentage points. The difference of the two effects at corporate level is almost insignificant.

Beside the impact of inflation at corporate level, the lower inflation rate affects the taxes at the individual level as well. As individual income taxes are levied on nominal interest payments and nominal capital gains, a reduction in the rate of inflation further reduces the effective tax rate and raises the real after-tax of return. The part of this relation that is associated with the taxation of nominal interest at the level of the individual can be approximated in a way that mirrors the effect at the corporate level. If the nominal interest rate increases by one percentage point for every percentage point of inflation, the individual investors' real pretax return on debt is unchanged, but the after tax return falls, and is given by the product of the statutory marginal tax rate and the change in inflation. Assuming the same 59 percent debt share at the individual level, as assumed for the corporate capital stock, and 25 percent average individual marginal tax rate, a 2 percent decline in inflation lowers the effective tax rate by 0.25(0.59)(0.02)=0.003 or 0.3 percentage points.

Next, we consider the effect of inflation on capital gains excluding dividend, as the individual dividend return on capital ownership is unaffected by inflation except at the corporate level. A higher rate of inflation increases the taxation of capital gains. Even though the effective tax rate on capital gains are taxed at the same rate as other investment income, the effective tax rate is lower because the tax is only levied on realization of the gains. Given effective tax rate of 10 percent[48] on nominal capital gains in South Africa, in equilibrium, each percentage point increase in the price level raises the nominal value of the capital stock by one percentage point. Since the nominal value of the liabilities remains unchanged, the nominal value of the equity rises by 1/(1-b) percentage points, where b is the debt to capital ratio. With b=0.59 and an effective tax on nominal capital gain of 10 percent, i.e., $\theta_g = 0.1$, a 2 percent decline in the rate of inflation raises the real after tax rate of return on equity by $\theta_g \left[1/(1-b)\right] d\pi$ =0.0049 or 0.49 percentage points. However, since equity is assumed to represents 35 percent of the individuals 'portfolio'[49], the lower effective capital gains tax raises the overall rate of return by only 35 percent of this 0.49 percentage points or 0.17 percentage points. Combining the debt and capital effects implies that

47 Source: McGregory BFA. The value of debt to capital ratio is obtained by dividing total liabilities with total assets using balance sheets of all companies between 1990-2007.

48 Source: Tax Pocket Guide 2006/7.

49 Source: Financial Services Board.

reducing the inflation rate by 2 percentage points reduce the effective tax rate at the individual investor level by the equivalent of 0.47 percentage points, with the real net return to the individual saver is 4.41 percent. This implies that price of retirement consumption is: p_1=0.2740. Substituting these values for the price of retirement consumption into equation (4) yields:

$$G_1 = 0.066\, S_2 \left(1 - \eta_{sp} - \sigma\right) \tag{5.5}$$

5.3.2 The Saving Rate and the Saving Behavior

The value of S_2 in equation 5.5 represents the saving during pre-retirement years at the existing inflation. To evaluate (5.5), we need an estimate of the saving of the young at an inflation rate of 2 percent (S_2). Feldstein (1997) derives an estimate from the steady-state relationship between savers and dis-savers implied by the two–period model. He shows that the saving of the young is $\left(1+\eta+g\right)^T$ times the saving of the older generation, where n is the rate of population growth and g is the growth in per capita wages. Thus net personal saving (S_N) is related to S_2 according to:

$$S_N = S_2 - (1+n+g)^{-T} S_2 \tag{5.6}$$

Real average wage growth in South Africa over 1990-2007 was 3.73 percent, while, population growth was 1.71 percent. On the other hand, average private saving rate over the same period was 5.4 percent of GDP. Based on these numbers, we have: n+g=0.0218 and with T=30, implies S_2=2.1 S_N. Further, using private saving to be 5.4 percent of GDP, results in S_2=0.11GDP. Further, the average share of wage in GDP between 1990 and 2007 was equal to 48 percent. Then, the propensity to save out of exogenous income is: $\sigma = S_2(\alpha * GDP)$ where α is the share of wage in GDP. With α=0.48, $\sigma = 0.23$.

The final term to be evaluated in order to calculate the welfare gain described in equation 5.5 is the elasticity of saving with respect to real interest rate, since the uncompensated elasticity of savings with respect to the price of retirement consumption is related to elasticity with respect to the real rate of return as: $\eta_{sp} = \frac{-\left(1+r)\eta_{sr}\right)}{rT}$. Following Dolado et al. (1998) and Balshki et al.

(1998), we assume that $\eta_{s_r} = 0.2$.[50] As in Feldstein (1997), we also assume a value of $\eta_{s_r} = 0$ to assess the sensitivity of this estimate to the value of η_{s_r}.

Given this, the annual gain from reduced distortion of consumption is G_1=0.0069 GDP or 0.69 percent of GDP when $\eta_{s_r} = 0.2$, and for $\eta_{s_r} = 0$, we have G_1=0.0056GDP or 0.56 percent of GDP. These calculations suggest that the traditional welfare effect on the timing of consumption of a reduction in inflation rate by 2 percent is bound between 0.56 percentage points of GDP to 0.69 percentage point of GDP.

5.3.3 Indirect Revenue Effects

Next we consider the effect on government revenue of the above experiment. The working assumption here, as in the Feldstein (1997), is that any reduction on government revenue due to a move from 2 percent inflation to price stability cannot be made good by a rise in lump-sum taxes. Instead, distortionary taxes are required to fill in the financial gap, with obvious corresponding welfare implications.

Assume that we start from a situation where the price of retirement income is p_2 and consumption level is c_2 (see figure 5-1), with inflation at 2 % and the current tax system in place. Now consider lowering the inflation rate to zero. There are two offsetting effects on revenue. First, lower inflation raises the real return to saving and hence lowers the price of retirement to p_1. This results in a loss of revenue equal to $(p_2 - p_1)c_2$. Against this, the lower the price of retirement consumption stimulates higher consumption by $(c_1 - c_2)$, which in turn generates revenue by the amount of $(p_1 - p_0)\times(c_1 - c_2)$. The change in revenue can thus be captured by:

$$dREV = S_2\left\{\left[\frac{p1-p0}{p2}\right]\left[\frac{p2-p1}{p2}\right]\left(1-\eta_{sp}-\sigma\right)-\left[\frac{p2-p1}{p2}\right]\right\} \tag{5.7}$$

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50 The decision to use a value of 0.20 for η_{s_r}, which is, in general, the lower bond of this estimate available in the literature, is in line with the observation of low interest sensitivity of savings in South Africa.

This expression can in principle be either positive or negative. But in our case, substituting the earlier estimates for values for p_0, p_1, p_2 . We get net revenue loss of $dRE =$-0.0079GDP or -0.79% of GDP for η_{s_r} =0.2 and -0.009GDP or -0.9 % of GDP for η_{s_r} =0.Assuming that λ represents the deadweight loss when each rand of revenue that needs to be raised from other taxes due to loss in revenue, the net loss in revenue of shifting from two percent inflation to price stability is 0.36 percent and 0.32 percent of GDP under η_{sr} =0 and 0.2 respectively. Note, following Feldstein (1997) λ is set at 0.4. Overall, net welfare gain (NG_1) from reducing inflation by 2 percent is then given by: $NG_1 = G_1 + \lambda d\,\mathrm{Re}v$. For λ =0.4 the net welfare gains are respectively, equal to 0.37 with η_{sr} =0.2 and 0.20 with η_{sr} =0. For λ =1.5 the welfare gains are forfeited for both η_{sr} =0 and η_{sr} =0.2.

5.4 The Gain from Reducing Distortion in Housing Demand

In some countries, owner-occupied housing is generally given special treatment on individual income taxation in order to encourage investment in housing and therefore stimulate economic growth. The benefit of owner occupied housing is that the mortgage interest payments and property tax rates are tax deductible. This is not the case for South Africa, since such deductions are not applicable. According to (Bonato, 1998), when mortgage interest payment is not tax deductable, inflation affects demand for housing only indirectly. This leads to reduction in the return on alternative assets. The state of price stability reduces this distortion as well as the loss of tax revenue by moving capital from housing to the business sector.

Given this, welfare effect of inflation on housing demand can be graphically represented as follows:

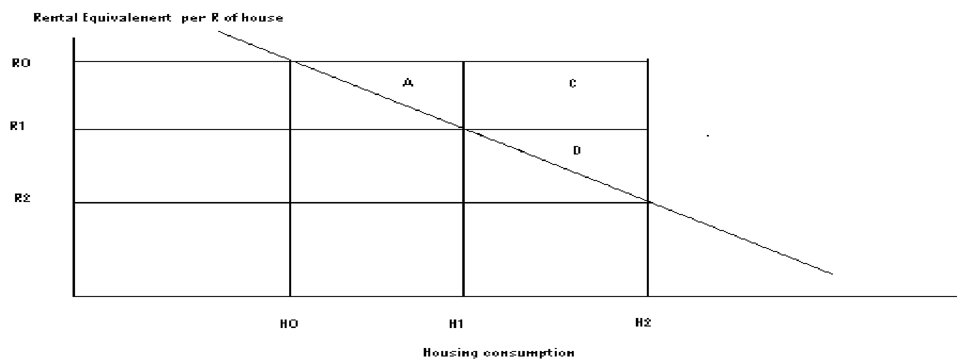

Figure 5-2: Distortion in Housing Demand

Figure 5-2 shows the compensated demand for housing services. The horizontal line at R_0 represents the undistorted cost of housing –the 'true' supply curve. The dead weight loss due to taxation is represented by triangle A, while the deadweight loss due to inflation is represented by the area of the trapezoid C+D. The reduction in the deadweight loss that results from reducing the distortion to housing demand when the inflation decline from two percent to zero is:

$$G_2 = \left[(R_0 - R_1) + 0.5(R_1 - R_2) \right](H_2 - H_1)$$

where $(H_2 - H_1) = (dH/dR)(R_2 - R_1)$

$$= (dH/dR)(R_2/H_2)(H_2/R_2)(R_2 - R_1)$$

$$= \varepsilon_{HR} H_2 \frac{(R_2 - R_1)}{R_2}$$

Then,

$$G_2 = \varepsilon_{HR} R_2 H_2 \left\{ \left[\frac{(R_0 - R_1)}{R_2} \frac{(R_1 - R_2)}{R_2} \right] + 0.5 \left[\frac{(R_1 - R_2)}{R_2} \right]^2 \right\} \tag{5.8}$$

where ε_{HR} is the compensated elasticity of housing demand with respect to the rental rate.

H_2 is the demand for owner-occupied housing and R represents the rental cost of housing per rand of housing capital. In many countries, effective subsidies to housing demand arising from the combination of inflation and tax system reduces the implied rental cost of housing and leads to overconsumption of housing (H_2), compared to situation of no taxes and no inflation (H_0).

In the absence of tax and inflation, the implicit rental cost is equal to

$$R_0 = r_0 + m + \delta \tag{5.9}$$

Where r_0 is the return on real rate of return on equity, m is the cost of maintenance per rand of housing capital and δ is the rate of depreciation. With r_0 =0.0706, m =0.074[51] and $\delta = 0.05$[52] implies R_0 =0.1946.

With the current tax regime and inflation, the revised implicit rental cost is

$$R_2 = \mu(r_m + \pi) + (1-\mu)(r_n + \pi) + \tau_p + m + \delta - \pi \tag{5.10}$$

where μ is the loan to value ratio, r_m is the real mortgage interest rate, r_n is the rate of return on equity with taxes and 2 percent inflation rate, and τ_p is the property tax rate. With μ equal to 0.7[53], $r_m + \pi$ equal to 0.097[54] and τ_p equal to 0.002[55]. R_2 =0.1913. The combination of tax and two percent inflation reduces the rental cost from 19.46 cents per rand of housing capital to 19.13 cents per rand of housing capital.

Next we look at the effect of a decrease in the rate of inflation on this implicit rental cost of owner occupied housing:

$$dR_2 = \mu * dr_m / d\pi + (1-\mu) d(r_n + \pi)/d\pi - 1.$$

With r_1 =0.0441 at $\pi = 0$ and r_n =0.0394 at π =0.02, $dr_n / d\pi$ =-0.235 and $d(r_n + \pi)/d\pi$ =0.765. Therefore,

$$dR_2 / d\pi = \mu + 0.765(1-\mu) - 1. \tag{5.11}$$

$$=-0.0705.$$

Since R_2 =0.1913 at two percent inflation, this implies R_1 =0.1927 at zero inflation.

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51 Source: Statistics South Africa.

52 Source: National Department of Housing, South Africa.

53 Source: Standard Bank of South Africa Limited.

54 Source: SARB.

55 Source: SARB.

We now go on and calculate G_2. Due to lack data on housing stock and rental rate in South Africa we use an elasticity of 0.3 as in Bonato (1998).[56] With H_2 equal to 2.08, which is gross fixed capital formation for residential building as percentage of GDP[57], the welfare gain is equal to 0.001 percent of GDP.

5.4.1 Indirect Revenue Effects

In the case of owner occupied housing, zero inflation would result in an increase in tax revenues. Shifting capital from owner occupied housing to business capital will lead to additional revenue equal to

$$d\,\mathrm{Re}v_1 = \varepsilon_{HR}\frac{R_1 - R_2}{R_2}H_2\left(r_0 - r_1\right) \tag{5.12}$$

=0.012 percent of GDP

Secondly, this increase in tax revenue is partly offset by a loss in the revenue from property taxes due to reduction of the housing stock. This loss can be estimated form

$$d\,\mathrm{Re}v_2 = \varepsilon_{HR}\frac{R_1 - R_2}{R_2}H_2\tau_p \tag{5.13}$$

=0.0009

Recalling that λ =0.4 represents the deadweight loss when each rand of revenue that needs to be raised from other taxes due to loss in revenue, the overall effect on tax revenue is about 0.004 percent of GDP, and, hence, the welfare gain from reducing distortion in housing is equal to 0.005 percent of GDP.

56 Given the inelastic rental market in South Africa, we believe that the choice of this value is a reasonable one.

57 Source: SARB.

5.5 Seigniorage and Distortion of Money Demand

5.5.1 Money Demand

An increase in inflation raises the cost of holding non-interest bearing money balances and therefore reduces the demand for such balances below the optimal level. It is this resulting deadweight loss of inflation that has been the primary focus of the literature on the welfare effects of inflation, since Bailey's (1956) pioneering paper.

Assuming that the initial situation is characterized with inflation π_2 and a positive nominal interest rate $i_{n2} = r_{n2} + \pi_2$, reducing inflation entails a welfare gain. Graphically, this can be depicted as follows:

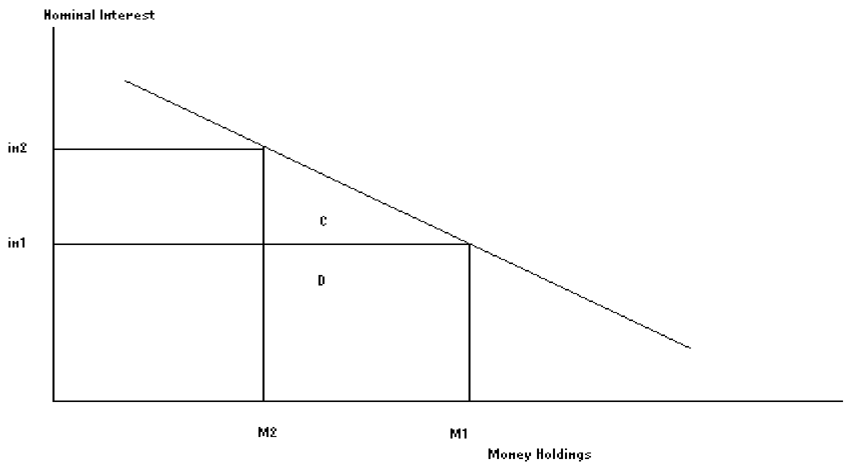

Figure 5-3: Money Market Distortion.

As shown in the Figure 5-3 above, which plots the demand for money as a function of nominal interest rate, a reduction in inflation (from π_2 to π_1) leads to an increase in money demand (from M_1 to M_2) and to a welfare gain presented by the area C plus the area D between the money demand curve and zero opportunity cost line. To compute the welfare gain, it is necessary to estimate the change in nominal interest rates caused by the reduction in inflation and induced increase in money demand $(M_1 - M_2)$. Recall, that a true initial inflation π_2 of 2 percent, the real net of tax return in South Africa is 3.94 percent, this leads to a nominal interest rate i_{n2} of 5.94 percent.

With $dr / d\pi$ =-0.235

$$G_3 = i_{nl}(M_1 - M_2) + \frac{1}{2}\left(i_{n2} - i_{nl}\right)\left(M_1 - M_2\right) \tag{5.14}$$
$$= 0.0441\left(M_1 - M_2\right) + 0.5\left(0.0594 - 0.0441\right)\left(M_1 - M_2\right)$$
$$= 0.05175(M_1 - M_2)$$
$$= -0.05175\varepsilon_M M \frac{1}{r_n + \pi} 0.0153$$
$$= 0.00079\varepsilon_M \frac{M}{GDP}\left(r_n + \pi\right)^{-1} GDP$$

Since the demand deposit component of M1 is now generally interest bearing, non-interest-bearing money in South Africa is represented by M1A. Between 1990 and 2007 the ratio of currency in circulation to GDP was 15.3 percent. Thus, M=0.153GDP. Using Meltzer's (1963) log-log money demand specification, we obtain an elasticity of money demand equal to 0.21, based on the Fischer and Seater (1993) long-horizon approach. Given this:

G_3 =0.00079*0.21*0.153*(1/0.0594) GDP =0.00043GDP or 0.043 percent of GDP.

5.5.2 The Revenue Effects of Reduced Money Demand

The reduction in inflation affects government revenue in three ways. First, the reduction in the inflation tax on money balances results in a loss of Seignorage and therefore an associated welfare loss of raising revenue by other distortionary taxes (Phelps, 1973). In equilibrium, inflation at rate π implies revenue equal to πM. Increases in inflation raise the seignorage revenue by:

$$dSeignorage / d\pi = M + \pi\left(dM / d\pi\right) \tag{5.15}$$
$$= M / GDP\left\{1 - \varepsilon_M\left[d\left(r_n + \pi\right) / d\pi\right]\left(\pi / r_n + \pi\right)\right\} GDP$$

M=0.153 GDP, ε_M =0.21, $d\left(r_n + \pi\right) / d\pi$ =0.765, $\pi = 0.02$ and $r_n + \pi$ =0.0594

A decrease of inflation from $\pi = 0.02$ to $\pi = 0$ leads to a loss of seignorage by 0.0029GDP.

The corresponding welfare loss is 0.29 $\times \lambda$ percent of GDP. With λ =0.4, the welfare cost of lost seignorage is 0.116 percent of GDP. Clearly the benefit of reducing inflation via an increase in money demand is outweighed by the loss of revenue from seigniorage. Specifically, a reduction of inflation by 2 percent would imply a welfare loss of 0.116 percent of GDP which is obviously way lesser than the 0.098 percent gain that could be obtained using the estimation and calculations in chapter 4.

The second revenue effect is the revenue loss that results from shifting capital to money balances from other productive assets. The decrease in business capital is equal to the increase in the money stock, $M_1 - M_2 = \left[dM/d\left(r_n+\pi\right)\right](0.0153) = 0.0153\ \varepsilon_M M\left(r_n+\pi\right)^{-1} = 0.83$ (5.16) percent of GDP. When these assets are invested in business capital, they earn a real pretax return of 7.06 percent but a net of tax return of only 4.41. The difference is the corporate and personal tax payments of 2.65 percent. Applying this to the increment in capital of 0.83 percent of GDP implies a revenue loss of 0.0265×0.83=0.022 percent of GDP. The welfare gain from this revenue loss is 0.022 λ percent of GDP. Again with λ =0.4, the welfare loss from this source is 0.009 percent of GDP.

The final revenue effect of the change in the demand for money is the result of the government's ability to substitute the increases in money balance of $M_1 - M_2$ for interest bearing government debt. Although this a one time substitution, it reduces government debt service permanently by:

$$r_{ng}\left(M1 - M2\right) \tag{5.17}$$

where r_{ng} is the real interest rate paid by the government on its outstanding debt net of the tax that it collects on those payments, given by: r_{ng} $=0.75\left(0.153\right) - 0.075 = 0.04$.

The reduced debt service cost is: $0.04\left(M_1 - M_2\right)$ =0.00033 or 0.033 percent of GDP. For λ =0.4, the corresponding welfare gain is equal to 0.013 percent of GDP.

Combining all three effects, we have total revenue losses equal to 0.112 when $\lambda = 0.4$. The net welfare loss due to decrease revenue is equal to 0.07. Note Phelps' (1973) revenue effects are bigger than Bailey's (1956) money demand effect, which, in turn, means that the welfare loss from reduced seignorage revenue is bigger than the welfare gain from the reduced distortion of money demand following a move from 2 percent inflation to price stability.

5.6 Debt Service and the Government Budget Constraint

Finally, we analyze the effect of the higher real cost of servicing the national debt following a reduction in the inflation rate. With inflation, the nominal interest payments are taxed; therefore, lower inflation reduces the nominal interest rate on government debt and reduces the real value of taxes on interest payment to individuals. A lower inflation hence leaves real pre-tax interest rate unchanged which leads to no change on pre-tax cost of debt service, but reduces the tax revenue on the government debt payments which in turn leads to higher level of other distortionary taxes.

Assuming a constant debt to GDP ratio, the increases in the real value of interest payments is equal to the product of the change in inflation times the marginal tax rate on interest payment, θ_i (which we assume to equal to average marginal individual tax rate), times the ratio of debt, B, to GDP as shown in Feldstein (1997) . Hence, the welfare effect of the change in taxes required to offset the change in real government revenue is[58]:

$$d\mathrm{R}EV_4 = -d\pi \times \theta_i \times B/GDP \tag{5.18}$$

$$= -0.02 \times 0.25 \times 0.41 = 0.00205 \text{ or } 0.21 \text{ percent of GDP.}$$

The reduction of inflation by 2 percentage point will reduce the welfare by 0.21 λ .

With $\lambda = 0.4$, the net welfare revenue is -0.08 percent of GDP.

5.7 The Net Effect of Lower Inflation on Economic Welfare

We can now bring together the several effects of reduced inflation that have been identified and evaluated in Sections 5.2 through 5.6 and compare them with the one-time output losses required to achieve the inflation reduction. As can be seen from Table 5-1, adding up all the four distortion, we obtain a welfare gain of 0.225 percent of GDP welfare by moving form an average inflation of 2 percent to price stability. But, when compared to Feldstein's (1997) estimate, our welfare gain is 4 times lesser than what he obtained.[59] This is mainly due to the fact that the gain due to a move from 2 percent inflation to price stability, resulting from the distortions on intertemporal allocation of consumption and the housing market demand is much higher in case

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[58] See Feldstein (1997) for details on derivations to obtain equation (5.18).

[59] In fact, in general, barring Poland, estimated at 0.125 percent of GDP, our estimate of welfare gain is less than all the other estimates, obtained using Feldstein's (1997, 1999) approaches, available in the literature. Specifically, the welfare gains following a permanent reduction of inflation by 2 percent was found to be 1.41 percent of GDP in Germany, 0.39 percent of GDP in New Zealand, 1.88 percent of GDP in Spain and 0.316 percent of GDP in Ukraine. The only estimate that comes close to that of ours is that of the United Kingdom, which is measured at 0.21 percent of GDP.

of the USA when compared to South Africa. This, in turn, results from the facts that the tax structure has a smaller distortionary effect on the choice between current and future consumption in emerging economies like South Africa than in the US, and also because with interest payment and property rates not being tax deductable, inflation affects demand for housing only indirectly. But, at the same time what is more important to us is that this measure of welfare loss is bigger than the value of 0.098 percent of GDP that could be obtained using the consumer surplus approach in chapter 4, which merely measures the distortion in the money demand due to positive nominal interest rates.

Table 5-1: Overall welfare Gain of Moving from 2 percent Inflation to Price Stability.

Welfare effect	Welfare gain as % of GDP	
	South Africa	Feldstein (USA)
Inter-temporal	0.37	0.926
Housing Demand	0.005	0.22
Money Market	-0.07	-0.034
Debt servicing	-0.08	-0.100
Total	0.225	1.012

Moreover, once we take into account Feldstein's (1997, 1999) arguments that benefits of inflation are permanent, and, hence, one should obtain its present value from reducing inflation permanently by 2 percent, even the relatively small-sized welfare gain of 0.225 percent of GDP translates into 15 percent of GDP, realizing that the relevant discount rate is (r_a-χ), since benefits grow at the same rate, χ, as GDP. Recall, r_a is the after tax real return on savings and equals to 3.94 percent, while the average growth rate of GDP (χ) over 1990-2007 was 2.44 percent, yielding a discount rate of 66.67. On the other hand, given that the sacrifice ratio for South Africa is 0.017[60] percent of GDP (Tunali, 2008), the one-time cumulative loss of output is 0.034 percent of GDP following a reduction in the inflation rate from two percent to zero. Clearly, the current benefits, not to say the present value of the same, overwhelmingly outweigh the output loss originating from such a disinflationary policy.

[60] Gonçalves and Carvalho (2008) obtain negative numbers for the sacrifice ratio, implying that disinflation can be achieved without any output costs.

5.8 Conclusion

This chapter makes the first attempt to calculate the benefit of moving from low inflation to price stability in South Africa using a micro partial equilibrium framework, developed by Feldstein (1997, 1999). Looking at interaction between inflation and non indexed tax system, our calculations show that the benefits for moving from an inflation of two percent to zero percent is equal to that 0.225 percent of GDP, which is more than twice the size of the estimates that could be obtained following Bailey's (1956) consumer surplus approach, based on the interest elasticity of money demand obtained in chapter 4..

This chapter emphasizes the distortions caused by the interaction of inflation and capital income taxation, in calculating the gain from moving to a zero rate of inflation. Though the annual deadweight loss of a two percent inflation rate is a relatively small number when compared to the literature, since the real gain from shifting to price stability is permanent, the present value is a substantial multiple of the annual welfare gain and is found to be 15% of GDP. Since the corresponding one-off output cost of moving from two percent inflation to price stability is 0.034% of GDP, the gain outweighs the cost by an overwhelming margin. Further, when one realizes that the calculations are symmetric and, approximately, linear, our results make a strong case for rethinking the width and the upper and lower limits of the target band, at least from the point of view of welfare costs of inflation. Clearly the discounted welfare gains will be quite substantial by moving to a narrower and lower target band and would also come at no cost to employment and output.

Dotsey and Ireland (1996) evaluated the welfare cost of inflation in dynamic general equilibrium endogenous and exogenous growth frameworks. By viewing inflation as a tax on micro-level decisions, the authors were able to identify explicitly, and quantify empirically, sizeable welfare costs of inflation at macroeconomic level, indicating that a partial equilibrium approach, like the one used in this chapter, can significantly underestimate the cost of inflation. Given this, there is no denying the fact that one can achieve, possibly, larger gains by reducing the inflation in a dynamic general equilibrium endogenous growth economic structure, and is an important research question for the future to correctly evaluate the inflation targeting regime in South Africa, based on welfare cost estimates.

Chapter 6

6 Evaluating the Welfare Cost of Inflation in a Monetary Endogenous Growth General Equilibrium Model: The Case of South Africa*

6.1 Introduction

The South African Reserve Bank (SARB) has been in pursuit of low inflation for nearly three decades now. Though not quite successful over the decade of 1980, the SARB made significant progress in reducing the inflation rate during the 1990s. Interestingly, the SARB pursued an implicit inflation target during the latter period. However, since the announcement made by the minister of Finance in the February of 2000, the sole objective of the SARB has been to achieve and maintain price stability. In other words, the SARB has now adopted an explicit inflation targeting regime, whereby it aims to keep the CPIX[61] inflation rate within the target band of 3-6%, using discretionary changes in the Repurchase (Repo) rate as its main policy instrument. In this regard, the measurement of the cost of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink of the level and width of the band in terms of the welfare cost of inflation at least.[62]

Given this, the four previous chapters deserves special mentioning. These four studies used alternative econometric methodologies to obtain estimates for the range of the welfare cost of inflation for the target band pursued by the SARB. While, chapter 2 showed that the welfare cost of inflation ranged between 0.34 % and 0.67 % of Gross Domestic Product (GDP), obtained using the Johansen (1991, 1995) cointegration approach to estimate the long-run money demand function, chapter 3 found the corresponding values to decrease markedly to 0.16 percent to 0.36 percent of GDP, when the long-horizon approach proposed by Fisher and Seater (1993) was used to estimate the long-run money demand function on the same data set.

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[61] CPIX is defined as CPI excluding interest rates on mortgage bonds.

[62] For recent studies that have evaluated the South African inflation targeting regime in terms of average levels of inflation, volatility and a wide array of other macroeconomic variables, refer to Burger and Marinkov (2008), Gupta and Uwilingiye (2010, forthcoming b) and Gupta et al. (forthcoming).

* Forthcoming in International Business and Economics Research Journal.

The difference between the results essentially emanated from the smaller sizes of the interest rate elasticity and semi-elasticity obtained under the long-horizon approach relative to the cointegration procedure. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income and that long-run properties of data are unaffected under alternative methods of time aggregation, chapter 4, tested for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Their results indicated that the long-horizon method is more robust to alternative forms of time aggregation, and given this, the welfare cost of inflation in South Africa for the inflation target band of 3-6 % was found to be between 0.15 % and 0.41% of GDP.

However, Feldstein (1997,1999) point out that welfare cost calculations obtained by integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue to deduce the deadweight loss, is merely one-dimensional. This is because, the consumer surplus approach fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects on the intertemporal consumption choice (i.e., saving for old age), housing and the real cost of servicing government debt. Thus, the welfare costs obtained using the money demand approaches is likely to provide the lower limit of such estimates, and, hence, a more general approach is desired to obtain the "true" size of the welfare loss caused by inflation. As such , chapter 5) uses a microeconomic partial equilibrium approach, as proposed by Feldstein (1997, 1999), found the annual deadweight loss of a two percent inflation rate to be 0.225 percent of GDP. Realizing that the calculations are symmetric and, approximately, linear, an inflation target band of 3-6%would imply the welfare cost to range between 0.34 percent and 0.68 percent of GDP. Interestingly, the figures are nearly the same as those obtained in chapter 2. Feldstein (1997, 1999) argued that costs of disinflation are temporary while the benefits are permanent, i.e., one needs to compare the discounted stream of benefits with one-off output costs Given this, chapter 5 calculated the present value gain to be 13.33 percent of GDP, while, the corresponding one-off output cost of moving from two percent inflation to price stability was found to be 0.034 percent of GDP. Thus, the gain was found to outweigh the cost by an overwhelming margin.

Dotsey and Ireland (1996) evaluated the welfare cost of inflation in dynamic general equilibrium endogenous and exogenous growth frameworks.

By viewing inflation as a tax on micro-level decisions, the authors were able to identify explicitly, and quantify numerically, sizeable welfare costs of inflation at the macroeconomic level, indicating that Feldstein (1997, 1999)-type partial equilibrium approaches, also used in chapter 5, can significantly underestimate the cost of inflation. Given this, it is important that one revisit the welfare cost estimates for South Africa in a dynamic general equilibrium endogenous growth model to ensure that one correctly evaluate the inflation targeting regime, based on welfare cost estimates.

Against this backdrop, this chapter calibrates the general equilibrium endogenous growth model proposed by Dotsey and Ireland (1996) for South Africa using quarterly data over the period of 1965 to 2008, and obtains the welfare cost of inflation. The decision to use the framework proposed by Dotsey and Ireland (1996) over a large number of other general equilibrium models such as Black et al. (1993), Coleman (1993), De Gregorio (1993), Gomme (1993), Jones and Manuelli (1993), Wang and Yip (1993), Marquis and Reffett (1994) and Whu and Zhang (1998) due to the fact that the transactions technology used here generates a money demand function that has an interest-elasticity similar to those estimated with South African data. Consequently, the model is ideally suited for comparing the welfare cost estimates obtained from the traditional partial equilibrium approaches based on money demand estimations to the full general equilibrium cost of inflationary policy. To the best of our knowledge, this is the first attempt to use a dynamic general equilibrium endogenous growth model to obtain the welfare cost of inflation in South Africa. The remainder of the chapter is organized as follows: Section 6.2 presents the general equilibrium endogenous growth model and Section 6.3 discusses the general equilibrium. Section 6.4 outlines the calibration, while Section 6.5 derives the welfare costs of inflation for alternative values of steady-state inflation. Finally, Section 6.6 concludes.

6.2 The General Equilibrium Model

In this chapter, we use Dotsey and Ireland's (1996) general equilibrium model. We start off by describing the economic environment, followed by the problems of the household, financial intermediary and the goods-producing firm.[63]

[63] The description of the general equilibrium model relies heavily on the discussion available in Dotsey and Ireland (1996) and has been presented here to ensure that the paper is self-contained. As such, we also retain the paper's symbolic representation of the equations.

6.2.1 Economic Environment

The model economy consists of a continuum of markets, indexed by $i \in [0,1]$, arranged on the boundary of a circle with a circumference of one. In each market, a distinct, nonstorable consumption good is produced and traded in each period t = 0, 1, 2,…. Thus, the economy's consumption goods are also indexed by $i \in [0,1]$, implying that good i is sold in market i.

Each market i is populated with a large number of identical households, financial intermediaries, and goods-producing firms. We assume that enough symmetry exists amongst the agents' preferences, endowments and technologies to allow us to consider the behavior of a single representative agent corresponding to each type, i.e., households, intermediaries and firms. The representative agents all live at market 0, so that the index i measures the distance of market i form their home.

At the beginning of period t = 0, the government, which has no other role in the economy, supply non-interest bearing fiat money of m_0^s units to households and augments the initial supply with identical lump-sum transfers h_t to all households at the beginning of each period t. Hence, per-household money supply m_{t+1}^s at the end of period t satisfies:

$$m_{t+1}^s = (1 + g_t)\, m_t^s \tag{6.1}$$

where the rate of money growth g_t is given by

$$g_t = \frac{h_t}{m_t^s} \tag{6.2}$$

Note, the government pre-announces the complete sequence $\{g_t\}_{t=0}^{\infty}$ of money growth rates at the beginning of period t = 0, which leads all agents to have perfect foresight beyond this point.

6.2.2 Household and Trading

The representative household at market i = 0 has preferences over leisure (J_t) and the entire continuum of consumption goods ($c_t(i)$) as described by the utility function:

$$\sum_{t=0}^{\infty}\beta^{t}\left\{\int_{0}^{1}\ln\left[c_{t}\left(i\right)\right]di+BJ_{t}\right\},\qquad \beta\in\left(0,1\right),\ B>0 \tag{6.3}$$

where, β is the discount rate and B is the substitution elasticity between leisure and household consumption.

The representative household is assumed to be made up of two members: a shopper and a worker (Lucas and Stokey; 1983). The representative worker rents out his capital stock k_t at the real rate r_t and supplies l_t^g units of labor at the real wage w_t to goods-producing firms, at each time point t. He also supplies l_t^f units of labor to financial intermediaries. In each period, the fraction of time allocated to each activity sums to one. The worker makes his labor-supply decisions subject to time constraint:

$$1\leq J_t+l_t^g+l_t^f \tag{6.4}$$

at each date t.

Meanwhile, the representative shopper travels around the circle to obtain goods for his household's consumption. As in Prescott (1987), Schreft (1992b) and Gillman (1993), the shopper choose two ways of making payment for his purchases in each market i. First, he can make use of government-issued fiat money. Assuming perfect competition, the nominal price p_t of consumption goods is the same across markets, the shopper may acquire $c_t\left(i\right)$ units of good i in exchange for $p_t c_t\left(i\right)$ units of money at time t. Second, he could use the services of financial intermediary to purchase good i on credit that will be paid at the end of the t period from his labor and rental incomes. The credit is obtained at a cost of $\gamma(i)$ units of labor. An intermediary verifies the shopper's identity, proof of income and credit record and guarantees his ability to pay, so that the firm in market i is willing to sell its output on credit at time t. The transaction cost to be paid by the shopper to intermediary does not depend on the quantity of purchase but increases with distance, i.e., the farther the shopper travels from his residence, the more is the cost that he has to incur. Hence, γ is strictly increasing function of i. Under the additional assumption that $\lim_{i\to 1}\gamma(i)=\infty$, there will always be some cash-good, implying a well-defined money demand function.

The intermediary in market i charge the representative household the real price $q_t(i)$ in exchange for its services at time t. Since the intermediary's cost $\gamma(i)$ is independent of the quantity of the transaction but depends on i, competition implies that the representative shopper may acquire

$c_t(i)$ units of good i on credit at time t at a total nominal cost of $p_t\left[c_t(i)+q_t(i)\right]$, where $p_t c_t(i)$ is the amount that the shopper has to pay for goods themselves (which is the same whether cash or credit is used to purchases the goods), and $p_t q_t(i)$ is the amount required to compensate the intermediary.

Let the indicator function $\xi_t(i)=0$ if the representative shopper purchases good i with money at time t, and let $\xi_t(i)=1$ if he uses the services of an intermediary instead. To purchase good with cash in period t, the shopper need to have nominal money balance of m_t, which is augmented at the beginning of the period by the government transfer h_t. Since the shopper must use money whenever he chooses not to hire an intermediary, he faces the following cash-in -advance constraint:

$$\frac{m_t+h_t}{p_t}\geq\left[1-\xi_t(i)\right]c_t(i)di \qquad (6.5)$$

in each period t.

After making its consumption decision at the end of period t, the representative household participates in a centralized assets market, and receives rental payments $r_t k_t$ and wages $w_t(l_t^g+l_t^f)$ and pays for the credit goods purchased earlier during period t. Whatever remains is then used to accumulate cash balances m_{t+1} that he carries in to period $t+1$ and to purchase the unsold output from the representative firm, which it combines with its depreciated capital stock $(1-\delta)k_t$ in order to carry k_{t+1} units of capital into period $t+1$.

The household can also borrow from and lend to other households at the end-of-period asset market by purchasing or issuing one–period, nominally-denominated discounts bonds. In period t+1, bonds pay b_{t+1} units of money and are sold for b_{t+1}/R_t units of money in period t asset market, where R_t is the gross nominal interest rate between period t and $t+1$. Note, $b_{t+1}=0$ must hold as an equilibrium condition in each period t, since these bonds are available in zero net supply at the beginning of each period.

As source of income in period t, the representative household has access to its initial money and bond holdings, its beginning of period government transfer, its rental and wage receipts, and the undepreciated capital stock. On the expenditure side, the representative household purchases consumption goods, pays to the intermediaries, and the capital, money, and bonds that it will carry into period $t+1$. Formally, the representative household faces the following budget constraint:

$$\frac{m_t+b_t+h_t}{P_t}+r_t k_t+w_t\left(l_t^g+l_t^f\right)+\left(1-\delta\right)k_t \geq \int_0^1 c_t\left(i\right)di+\int_0^1 \xi_t\left(i\right)q_t\left(i\right)di+k_{t+1}+\frac{m_{t+1}}{P_t}+\frac{b_{t+1}}{p_t R_t} \qquad (6.6)$$

in each period t. The representative household chooses sequences for $c_t\left(i\right)$, $\xi_t\left(i\right)$, J_t, l_t^g, l_t^f, k_{t+1}, m_{t+1} and b_{t+1} to maximize the utility function (6.3) subject to time constraint (6.4), cash- in-advance constraint (6.5) and the resource constraint (6.6) by taking the sequences of h_t, $r_t, w_t, p_t, q_t(i)$ and R_t as given . Moreover, the household also takes its initial holdings of capital $k_0>0$, money $m_0=m_0^s$ and bonds $b_0=0$ as given.

6.2.3 The representative intermediary's problem

In market i, an intermediary hires $\gamma(i)$ units of labor and charges $q_t\left(i\right)$ if the representative shopper purchases good i on credit at time t. Thus, the representative intermediary chooses labor input n_t^f to maximize its profits. Formally at each date t,

$$\pi_t^f=\int_0^1 \xi_t^s\left(i\right)q_t\left(i\right)di-w_t n_t^f \qquad (6.7)$$

is maximized taking w_t, ξ_t^s and $q_t\left(i\right)$ as given, subject to intermediaries total demand for labor (the technological constraint):

$$n_t^f \geq \int_0^1 \xi_t^s\left(i\right)\gamma\left(i\right)di \qquad (6.8)$$

6.2.4 The representative goods-producing firm's problem

In market i = 0, the representative goods-producing firm uses k_t units of capital and n_t^g units of labor in each period t and produces consumption good i = 0. Its profits in period t are:

$$\pi_t^g=A\left(k_t\right)^{\alpha}\left(n_t^g\right)^{1-\alpha}\left(K_t\right)^{\eta}-r_t k_t-w_t n_t^g \qquad \alpha\in\left(0,1\right),\ \eta>0. \qquad (6.9)$$

K_t in production function equation (6.9) , represents the aggregate capital stock per household at time t. Following Romer (1986), capital is broadly defined to include human capital and disembodied knowledge, over and above to physical capital. Spillover effects from human capital lead to increasing returns to scale at aggregate level, even though production obeys constant returns to scale at firm level. Increasing returns to scale results in endogenous growth, which is also (possibly) dependent on the inflation rate. When maximizing its profit, given in (6.9), the representative firm takes K_t, r_t and w_t as given.

6.3 Competitive equilibrium

A competitive equilibrium in this economy consists of sequences for prices and quantities which ensures that the optimization problems of households, intermediaries, and firms, outlined above, holds. Given the initial conditions $k = K_0 > 0$, $m_0 = m_0^s$, and $b_0 = 0$, equilibrium prices and quantities must also satisfy the zero profit conditions of the goods producing firms and the financial intermediaries, i.e.,

$$\pi_t^g = \pi_t^f = 0, \tag{6.10}$$

the consistency condition:

$$k_{t+1} = K_{t+1}, \tag{6.11}$$

and the market–clearing conditions in each period t for each market as follows :

Goods market: $$A(k_t)^{\alpha+\eta}(n_t^g)^{1-\alpha} + (1-\delta)k_t = k_{t+1} + \int_0^1 c_t(i)di \tag{6.12}$$

Labor market: $n_t^g = l_t^g$ and $n_t^f = l_t^f$, (6.13, 6.14)

Money market: $m_{t+1} = m_{t+1}^s$ (6.15)

Bond market: $b_{t+1} = 0$ (6.16)

Financial intermediaries: $\xi_t(i) = \xi_t^s(i)$ (6.17)

6.4 General Equilibrium effects of inflation tax

Dotsey and Ireland (1996) demonstrates that there exists a borderline index s_t for each date t such that the representative household's all purchases are credit goods with indices $i \le s_t$, and all goods are cash goods when $i > s_t$, with the borderline index determined by the solution to:

$$\gamma(s_t) = \left[\ln(\lambda_t + \mu_t) - \ln(\lambda_t)\right] / (w_t \lambda_t) \tag{6.18}$$

where λ_t is nonnegative multiplier on the resource constraint (6.6) and μ_t is the nonnegative multiplier on the cash-in-advance constraint (6.5) from household's optimization problem. Note, since transaction cost increases with distance, the shopper uses credit close to home and cash far from home, as discussed in Schreft (1992) and Gillman (1993).

The representative household's optimal $c_t(i)$ follows a step-function in each period t:

$$c_t(i) = \begin{cases} c_t^1 = 1/\lambda_t & for \quad i \le s_t \\ c_t^0 = 1/(\lambda_t + \mu_t) & for \quad i > s_t \end{cases} . \tag{6.19}$$

where, since $\mu_t \ge 0$, $c_t^1 \ge c_t^0$, equation (6.19) and cash in advance constraint (6.5) determine equilibrium money demand as:

$$\frac{(m_t + h_t)}{p_t} = (1 - s_t) c_t^0 . \tag{6.20}$$

Further, equations (6.8), (6.14) and (6.17) determine the employment in the financial sector as:

$$l_t^f = \int_0^{s_t} \gamma(i) di. \tag{6.21}$$

In this model welfare cost of inflation arise in number of ways: firstly, higher inflation causes the cash-in-advance constraint to bind, implying higher values of μ_t following higher rates of inflation. Since γ is increasing function of i, the larger value of μ_t will lead to higher a value of s_t. Referring to equation (6.18), as s_t increases, the representative household purchases a wider range of goods with the help of intermediaries. Second, based on equation (6.19), inflation tax distorts consumption and production decisions in two ways. (i) with $c_t^1 > c_t^0$, the marginal rate of substitution between cash and credit goods differ from the corresponding marginal rate of transformation, with the representative household buying different consumption goods in different quantities, and; (ii) since c_t^0 is a decreasing function of μ_t, the representative household purchases cash goods in smaller quantities causing a reduction in market activity. Given the production technology in equation (6.9), these allocative effects of inflation changes the level and growth rate of aggregate output. Third, equation (6.20) suggests that as the inflation rate rises, the representative household economizes on its cash balances by not only purchasing a wider range of goods without money, but also by consuming less of cash goods. In other words, s_t increases and c_t^0 decreases respectively, leading the money demand function to be interest-elastic and resulting in a Bailey-Friedman type cost of the inflation tax. Finally, equation (6.21), suggests that as s_t increases following an increase in the inflation tax, the size of the labor force in the financial sector rises, causing a substitution of resources from the production sector and into finance. This also contributes to the welfare cost of inflation, since given the production function in equation (6.9), this allocative effect influences the long-run growth rate.

Clearly then, inflation tax distorts many marginal decisions, however, it is not possible to analytically assess the magnitude of any of these distortions. Given this, one has to resort to numerical methods to measure the effects of the inflation tax in the general equilibrium, which, in turn, requires us to calibrate the model – a process which we discuss in the next section.

6.5 Model calibration

The household's discount rate is set at $\beta = 0.99$ and the depreciation rate at $\delta = 0.019$ (Liu and Gupta), so that the period in the model is one quarter year. To ensure growth in the long-run equilibrium, $\eta = (1-\alpha) = 0.74$, given $\alpha = 0.26$ (Liu and Gupta, 2007), With A=0.3926, the economy grows at a constant annual rate of 3 percent, the average growth rate of the South African economy over the period of 1965-2008, under a constant annual inflation rate of 9.45 percent, again a figure which corresponds to the average of the above period. The households allocate 25 % of their time to labor (Liu and Gupta, 2007), under an inflation rate of 9.48 percent when B =3.5795.

The magnitude of the Bailey–Friedman cost of inflation depends on the size of the tax base and the interest elasticity of money demand. When the intermediary's cost function is specialized to:

$$\gamma(i) = \gamma\left[i/(1-i)\right]^{\theta}, \gamma > 0, \theta > 0, \tag{6.22}$$

One can choose the parameters γ and θ so that the size of the tax base and the interest elasticity of money demand in the model corresponds to figures in the South African economy. As in Dotsey and Ireland (1996), the size of inflation tax base in the South African economy is measured by the fraction of all purchases that are made using money. Based on our calculations, using data over the period of 1965-2008 from the South African Reserve Bank (SARB), suggests that South African households made 45 percent of their transactions using M1(A), implying a value of 0.45 for 1-s_t under 9.45 percent inflation in the model.

The money demand for M1 is estimated using annual data over the period of 1965-2008, and yields: ln(vM1) =2.29+2.75 R, where vM1 is the income velocity of M1(A) and R is the 91-days Treasury bill rate. The OLS coefficient on R measures the long run interest semi-elasticity of money demand. An analogous statistic in the model economy is:

$$\left[\ln\left(v_{9.45}\right) - \ln\left(v_0\right)\right] / \left(R_{9.45} - R_0\right)$$

where $v_{9.45}$ and v_0 are the constant annual velocities of money and $R_{9.45}$ and R_0 are constant annual nominal interest rates that prevail under constant annual inflation rates of 9.45 percent and zero.

Matching the tax base and the elasticity figures in the data and the model yields γ =0.0078 and θ =1.83. With this combination of γ and θ, the annual velocity of money under 9.45 percent of inflation produced by the model is 11.21, which is very similar to the average velocity of 13.15 found in the South African data over the period of 1965-2008. This justifies our identification of one model period as one quarter year.

6.6 The quantitative effects of inflation in the general equilibrium model

In this section, we analyze the effects of a change in the money growth rate on the critical variables defining the general equilibrium model. Note, the effects of monetary policies, which require constant money growth rates, give rise to steady-state equilibria in which all variables grow at constant rates. In Table 6-1 we compare the steady-state equilibrium under the average inflation rate of 9.45%, with those of the Friedman-rule, 0%, 2%, 3% and 6% of inflation. Recall, under the Friedman (1969)-rule, one must ensure a zero nominal rate of interest, which, in turn, implies that money supply is contracted at the rate of time preference. While, the situation under zero percent corresponds to the case of price stability, and the 3% and 6% of inflation captures the limits of the target band.

The representative shopper uses cash to make a constant fraction of his purchases under a constant inflation rate. The model has been calibrated to ensure that the agent carries out 45% of their transactions using M1 under the steady-state inflation rate of 9.45%. As inflation gets higher, the shopper, understandably, uses money in a smaller range of transaction, implying a positive relationship between the velocity of money and the inflation rate. The model parameterization also ensures that the representative worker devotes 25% of his time to labor. As seen from Table 6-1, household substitute out of market activity as inflation rises, and enjoys more leisure without the use of means of exchange, unlike market activity which requires either money or the costly financial services. Further, besides the substitution effect, there is also a negative wealth effect as the inflation rate increases. As in Cooley and Hansen (1989, 1991), the substitution effect tends to dominate the wealth effect, causing the household's labor supply to fall as the inflation rate rises. The allocation of labor force, besides the total labor supply itself, gets affected with changes in the inflation rate. As shown in Table 6-1, though the fraction of labor force working in financial intermediaries is a small number (always less than 0.6%), it rises with the rate of inflation. The substitution of labor of the production sector into leisure and into the financial intermediaries tends to negatively affect the growth rate of output through the spillover effects of aggregate activity. The effect of inflation on the growth rate, however, is quite small in general, since 9.45% inflation causes the growth rate to fall from 3.11 to 3.01%.

Table 6-1: The welfare cost of inflation.

	Annual inflation rate					
	Friedman Rule	0	2	3	6	9.45
Annual money growth	-0.0394	0.0300	0.0506	0.0609	0.0918	0.1273
Annual inflation	-0.0694	-0.0011	0.0192	0.0293	0.0595	0.0943
Annual growth rate	0.0323	0.0311	0.0309	0.0307	0.0304	0.0301
Fraction of time working	0.2548	0.2527	0.2523	0.2522	0.2515	0.2514
Fraction of labour in finance	0.0000	0.0022	0.0029	0.0033	0.0044	0.0056
Fraction of purchases with money, number	1.0000	0.5639	0.5301	0.5160	0.4814	0.4511
Fraction of purchases with money, value	1.0000	0.5596	0.5245	0.5098	0.4734	0.4412
Annual velocity	4.9306	8.8199	9.4135	9.6869	10.4386	11.2091
Welfare cost (percentage of output)	-2.20	0.00	0.48	0.70	1.33	1.97

Following Cooley and Hansen (1989, 1991) and Dotsey and Ireland (1996), the welfare cost of inflation is captured by the permanent percentage increase in the consumption of all goods that is required to make the representative household as well off under a positive rate of inflation as it is under price stability (under the zero rate of inflation). When we multiply this figure with the consumption output ratio, we are able to express it as percentage of output. Table 6-1 show that the welfare cost is nearly 2 percent of output for a steady-state inflation rate of 9.45 percent. The corresponding values for inflation rates of 2, 3 and 6% of inflation are 0.48, 0.70 and 1.33% of output. Finally, the welfare gain from adopting the Friedman-rule is equivalent to a 2.2 % increase in output.

Importantly, the welfare cost values are way higher compared to those obtained in the four previous chapters. Recall, in chapter 4, based on a money demand approach, concludes that for the inflation target band of 3-6%, the welfare cost ranged between 0.15% and 0.41% of GDP, while in chapter 5 , using Feldstein's (1997, 1999) microeconomic partial equilibrium approach, found the annual deadweight loss of a two percent inflation rate to be 0.225% of GDP. Clearly, when one compares the welfare cost estimates of Table 6-1 with those obtained from Bailey-Friedman-type partial equilibrium analyses used in chapter 2, 3 and 4, one tends to obtain much smaller figures than those under the general equilibrium model since the former approach captures only a fraction of the total cost of inflation – the cost due its effect on the velocity of money. In addition to this effect, inflation causes inefficient allocation of productive labor across its alternative uses. Even though the labor supply effects might seem small quantitatively, they end up contributing to the welfare cost of inflation enough to significantly outweigh the welfare cost estimates under the Bailey-Friedman-type money demand approach. In addition, by viewing inflation as a tax on a host of micro-level decisions, we obtain sizeable welfare costs of inflation at the macroeconomic level, thus, indicating that Feldstein (1997, 1999)-type partial equilibrium approaches used in chapter 5, can also significantly underestimate the cost of inflation.

6.7 Conclusion

Since the February of 2000, the sole objective of the SARB has been to keep the CPIX inflation rate within the target band of 3-6%, using discretionary changes in the Repo rate as its main policy instrument. In this regard, the measurement of the cost of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink of the level and width of the band in terms of the welfare cost of inflation at least Against this backdrop, this chapter calibrates the general equilibrium endogenous growth model proposed by Dotsey and Ireland (1996), where the inflation tax distorts a variety of marginal decisions, for South Africa using quarterly data over the period of 1965 to 2008, and obtains the welfare cost of inflation.

Higher inflation rate causes the agents to inefficiently economize on their holdings of real cash balances, leads to substitution out of market activity by taking more leisure and diverting productive resources out of goods production and into financial intermediaries. The model shows that individually, none of these distortions is very large, but the various small distortions combine to yield substantial estimates of the total cost of inflation. More importantly, the estimates are way higher than the previous welfare cost values obtained in the four previous chapters based on the Bailey-Friedman-type money demand approach or the Feldstein (1997, 1999)-type partial equilibrium approach. We show that for a target band of 3-6 % the welfare cost of inflation ranges between 0.70 % of GDP and 1.33% of GDP. On the other hand, the

Friedman-rule tends to produce welfare gains of the magnitude of 2.20% of GDP. These higher estimates, thus, tend to strengthen the case for a possibly lower and narrower target band – a proposal made in chapter 7 and 8 based on the findings that the inflation targeting produces higher mean and variance of inflation than it would otherwise be if the SARB had continued to follow its earlier so-called eclectic approach to monetary policy. In general, our findings highlight the usefulness of general equilibrium models for the purposes of evaluating a policy regime. In our case, this amounts to indicating that unless all the distortions induced by a policy is considered, in other words, unless we undertake a general equilibrium approach, reliance on partial equilibrium approaches for measuring the welfare effect of inflation will grossly underestimate the "true" welfare cost.

Chapter 7

7 Dynamic time inconsistency and the SARB*

7.1 Introduction

Realizing that South Africa has been targeting inflation since the February of 2000,[64] this chapter attempts to analyze whether the adoption of an inflation targeting regime has improved the time consistency of monetary policy in South Africa in terms of mean levels of inflation in the post-targeting period. Specifically, we try and deduce whether the South African Reserve Bank (SARB) could have produced lower average levels of inflation during the period of 2001:01 to 2008:02 if it had continued to pursue a monetary policy approach that it followed prior to 2000.

To do this, we would first need to obtain a framework that is in line with design of monetary policy under no precommittment to a rule. In this regard, we rely on the theory of dynamic time inconsistency. And then to get to the main question of ~~this chapter~~, understandably we first need to show that the SARB's monetary policy decisions were in line with a time inconsistent framework over the pre-inflation targeting period of 1960:01 to 1999:04. Econometrically speaking, this can be done by deriving restrictions imposed by the Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of Consumer Price Index (CPI) inflation and real Gross Domestic Product (GDP), and then testing these restrictions, both short- and long-run, based on quarterly data for South Africa covering the period of 1960:01 through 1999:04. And then, as far as answering the question posed above is concerned, this can done by forecasting inflation one-step-ahead over the period of 2001:01 to current, which in our case happens to be 2008:02.[65] And finally, checking, whether, on average, we would have obtained lower rates of inflation over the out-of-sample horizon of 2000:01 to 2007:04. However, it must be realized that to forecast out of the model, we need to ensure that the short-

[64] See Sichei (2005) and Ground and Ludi (2006) for detailed reviews on the history of monetary policy in South Africa.

[65] At the time, the paper was being written, data for the relevant variables were available till 2008:02 only.

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and long-run restrictions obtained from the theory is consistent with the data, and, hence, the SARB was time inconsistent prior to targeting inflation.

The motivation for this analysis simply emanates from the need to evaluate the performance of the monetary authority during the inflation targeting period. Note, the issue is not whether the average level of inflation in the post inflation-targeting period is lower than the pre-inflation targeting period, but, more importantly, whether the inflation rate, on average, would have been higher or lower, if the monetary authority continued to behave in the way it had been prior to targeting inflation? The average level of CPI inflation over the period of 2000:01 to 2008:02 has been 6.17%, which is way lower than the average of 9.21% that prevailed in the pre inflation-targeting period of 1960:01 to 1999:04. Now a simple comparison of the levels of inflation in the pre and post inflation-targeting period would suggest that the inflation targeting regime has performed quite well in bringing down the average level of inflation in South Africa. However, this is an incorrect way of evaluating the performance of the new regime, because, ideally what we would want to deduce is whether the monetary authority could have done better or worse if it persisted with the policy structure of the old regime, into the period of 2000:01 to 2008:02. To the best of our knowledge, this is the first attempt to evaluate the performance of an inflation targeting monetary authority in this manner. Hence, an obvious extension of this chapter would be to carry out the analysis for other inflation targeting economies.

The remainder of the chapter is organized as follows: Section 7.2 lays the modified version of the Barro and Gordon (1983) model and derives the theoretical restrictions, while, Section 7.3 discusses the data and presents the empirical results and also carries out the forecasting exercise. Finally, Section 7.4 concludes.

7.2 The Modified Barro-Gordon (1983) Model

Recall, in Barro and Gordon's (1983) model, a policymaker wants to increase output or reduce unemployment, but does not pre-commit to a monetary policy rule, and, hence, is tempted to increase the output beyond the natural rate by creating unanticipated inflation or deviating from its pre-announced inflation rate in an attempt to exploit the expectational Phillips curve. In other words, the policy maker tends to be time inconsistent. However, given that private agents in the model have rational expectations, they can recognize this behaviour of the government and adjust their decisions accordingly. Therefore, in equilibrium, output is not lower than it would otherwise have been, and yet the rate of inflation is inefficiently high. This section presents the modified version of Barro and Gordon's (1983) model of time inconsistent monetary policy as can be found in Ireland (1999). However, unlike Ireland (1999), due to the lack of quarterly data on unemployment for South Africa, we model the supply side using a traditional Lucas-type supply curve rather than an expectational Phillips curve. Specifically, Barro and Gordon's (1983) model is modified by allowing the natural rate of output to follow an autoregressive process that contains a unit root and by incorporating control errors for the rate of inflation. While, the first extension allows for the real GDP to be non-stationary, as will be seen below, the second modification ensures transitory deviations between the actual real GDP and the natural rate of output.

As in the standard Lucas supply-curve, the actual output y_t fluctuate around the natural rate y_t^n in response to deviations of the actual inflation rate π_t from the expected inflation rate π_t^e as follows:

$$y_t = y_t^n + \alpha(\pi_t - \pi_t^e); \alpha > 0. \tag{7.1}$$

The natural rate of output, in turn, is assumed to fluctuate over time in response to a real (supply) shock ε_t according to:

$$y_t^n - y_{t-1}^n = \lambda(y_{t-1}^n - y_{t-2}^n) + \varepsilon_t;\ -1 < \lambda < 1; \varepsilon_t \sim iid\ \ N(0, \sigma_\varepsilon^2) \tag{7.2}$$

Hence, the change in the natural rate is allowed to follow an AR (1) process. The monetary authority cannot commit to a policy rule, but at the at the beginning of each period t = 0,1,2…, after the private agents have formed their expectation of inflation, π_t^e, but prior to the realization of the supply-shock, ε_t, the monetary authority chooses a planned rate of inflation π_t^p.Actual inflation for period t is then determined as the sum of π_t^p and a control error η_t, such that:

$$\pi_t = \pi_t^p + \eta_t; \eta_t \sim N(0, \sigma_\eta^2) \tag{7.3}$$

Note η_t is assumed to have a covariance of $\sigma_{\varepsilon\eta}$ with ε_t.

The policy maker chooses π_t^p in order to minimize a loss function that imposes penalty on variations of output and inflation around target values ky_t^n and zero:

$$L_t = \frac{1}{2}(y_t - ky_t^n)^2 + \frac{b}{2}\pi_t^2;\quad b > 0. \tag{7.4}$$

with k >1 and b>0 so that the policymaker wishes to push the actual output over the natural rate.

Using (7.1) and (7.3), the monetary authority's problem can be re-written as:

$$\min_{\pi_t^p} E_{t-1}\left[\frac{1}{2}\left((1-k)\,y_t^n+\alpha\left(\pi_t^p-\pi_t^e+\eta_t\right)\right)^2+\frac{b}{2}\left(\pi_t^p+\eta_t\right)^2\right] \tag{7.5}$$

where $E_{t-1}(.)$ denotes the expectation at the beginning of period t or at the end of period t-1. The first order condition for the above problem is:

$$\alpha E_{t-1}[(k-1)y_t^n+\alpha(\pi_t^p+\eta_t-\pi_t^e)]=bE_{t-1}(\pi_t^p+\eta_t) \tag{7.6}$$

Private agents are assumed to know the true structure of the economy and also understand the monetary authority's time-inconsistency problem. In equilibrium, therefore $\pi_t^p=\pi_t^e$, i.e., they correctly anticipate the authority's actions. Using the equilibrium condition and the fact that $E_{t-1}\eta_t=0$, due to rational expectations, (7.6) simplifies to:

$$\pi_t^p=\pi_t^e=\alpha AE_{t-1}y_t^n;A=\frac{k-1}{b}>0 \tag{7.7}$$

Equation (7.7), as in Barro and Gordon (1983), indicates that the inflationary bias resulting from the monetary authority's inability to commit depends positively on the expected natural rate of output $E_{t-1}y_t^n$.

Combining equations (7.1), (7.3) and (7.7) imply that the control error for inflation causes the actual output to fluctuate around the natural rate in equilibrium, i.e.,

$$y_t=y_t^n+\alpha\eta_t \tag{7.8}$$

Using (7.2) and (7.8), and defining $\Delta y_{t-1}^n=y_{t-1}^n-y_{t-2}^n$, we have:

$$y_t=y_{t-1}^n+\lambda\Delta y_{t-1}^n+\varepsilon_t+\alpha\eta_t \tag{7.9}$$

Meanwhile, equations (7.2), (7.3) and (7.7) imply that:

$$\pi_t=\alpha Ay_{t-1}^n+\alpha A\lambda\Delta y_{t-1}^n+\eta_t \tag{7.10}$$

Equations (7.9) and (7.10) separately indicate that both output and inflation are non-stationary respectively, by having inherited the unit roots from the underlying process defining the evolution of the natural rate of output. Putting (7.9) and (7.10) together implies that:

$$\pi_t-\alpha Ay_t=(1-\alpha^2A)\eta_t-\alpha A\varepsilon_t \tag{7.11}$$

which, in turn, shows that the linear combination of inflation and output is stationary, i.e. π_t and y_t are cointegrated. So based on equation (7.11), the modified version of the Barro and Gordon's (1983) model implies that inflation and real GDP are non-stationary, but cointegrated. Statistical tests of the cointegration constraint, implied by (7.11), will determine whether the modified Barro and Gordon (1983) model can explain the long-run behaviour of inflation and output in South Africa.

Taking first difference of (7.8) and (7.11), and then replacing out the first-differenced value of $\Delta y_t \times \alpha A$ from the first-differenced version of (7.8) into the first-differenced version of equation (7.11) yields:

$$\Delta\pi_t = \alpha\lambda\Delta y_t^n + \eta_t - \eta_{t-1} - \alpha A\varepsilon_t + \alpha A\varepsilon_{t-1} \tag{7.12}$$

where $\Delta\pi_t = \pi_t - \pi_{t-1}$ and $\Delta y_t^n = y_t^n - y_{t-1}^n$.

Equation (7.12) in turn can then be re-written using a state-space representation as follows:

$$\varepsilon_t = F\varepsilon_{t-1} + Q\upsilon_t \tag{7.13}$$

$$z_t = H\varepsilon_t \tag{7.14}$$

where,

$$\varepsilon_t = \begin{bmatrix} \Delta y_t^n \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \eta_t \\ \eta_{t-1} \end{bmatrix};\ F = \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix};\ Q = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix};$$

$$H = \begin{bmatrix} \alpha A & -\alpha A & \alpha A & 1 & -1 \end{bmatrix}; z_t = [\Delta\pi_t]; \upsilon_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix},$$

$$\textit{with } E\left(\upsilon_t\upsilon_t^{'}\right) = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}.$$

Further note, using (7.2), equation (7.12), after some algebra, can be re-written as:

$$\Delta\pi_t = \lambda\Delta\pi_{t-1} + \eta_t - (1+\lambda)\eta_{t-1} + \lambda\eta_{t-2} + (1+\lambda)\alpha A\varepsilon_{t-1} - \alpha\lambda A\varepsilon_{t-2} \tag{7.15}$$

The within equation restriction appearing in (7.15) implied by the ARMA(1,2) process for the change in the actual inflation rate, summarizes the constraints that the modified version of the Barro and Gordon (1983) model imposes on the short-run behaviour of inflation. As with the long-run relationship, a statistical test of these restrictions will determine whether the modified model explains the dynamics of inflation that can be found in the South African data. This essentially boils down to using a likelihood-ratio test statistic for establishing the acceptance or the rejection of the short-run restrictions implied by equation (7.15) in relation to an unrestricted version of the ARMA (1,2) model of $\Delta\pi_t$,which looks as follows:

$$\Delta\pi_t = \phi_1\Delta\pi_{t-1} + \phi_2\varepsilon_{t-1}^\pi + \phi_3\varepsilon_{t-2}^\pi + \phi_4\varepsilon_{t-1}^y + \phi_5\varepsilon_{t-2}^y + \varepsilon_t^\pi \tag{7.16}$$

At this stage, it is important to point out that we have reduced the two-variable model to a single variable $\Delta\pi_t$. This allows us to lower the number of parameters for the unrestricted ARMA (1,2) from 16 to 8, and in the process, help us reduce the difficulty of finding initial parameter values, via grid search, involved in estimating state-space models. Besides, in this chapter, in any case, we are more interested in studying the behavior of inflation over the period of 2000:01 to 2008:02.

7.3 Data and Results

In this chapter, we use seasonally adjusted quarterly time series data for real GDP and CPI inflation over the period of 1960:01-2008:02, both of which were obtained from the Quarterly Bulletins of the SARB. Note the base year is 2000, and we transform the real GDP series into its logarithmic values. In this section, we first discuss the tests of the long-run constraint and then move on to verifying the validity of the short-run restrictions.

7.3.1 Testing the Long-Run Restrictions

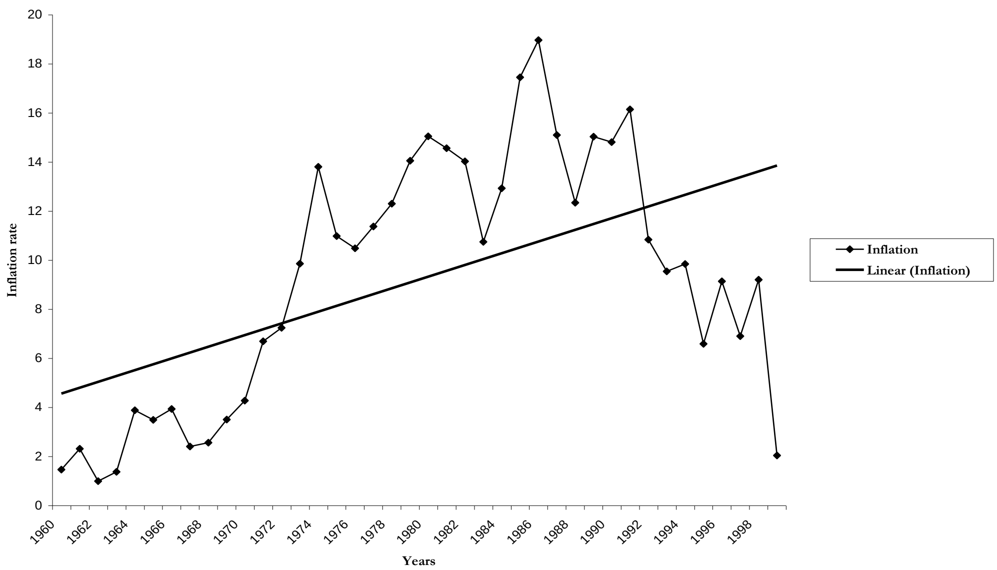

Figure 7-1: Linear trend of CPI inflation (2000=100).

Figure 7-2: Linear trend of LogRealGDP (2000=100).

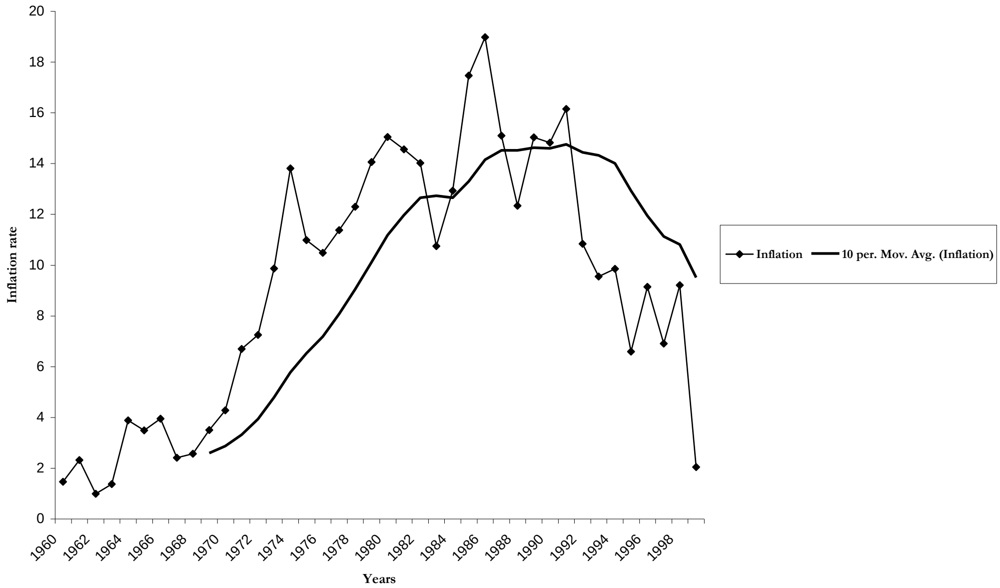

Figure 7-3: 10-year-centered Moving Average of CPI inflation (2000=100).

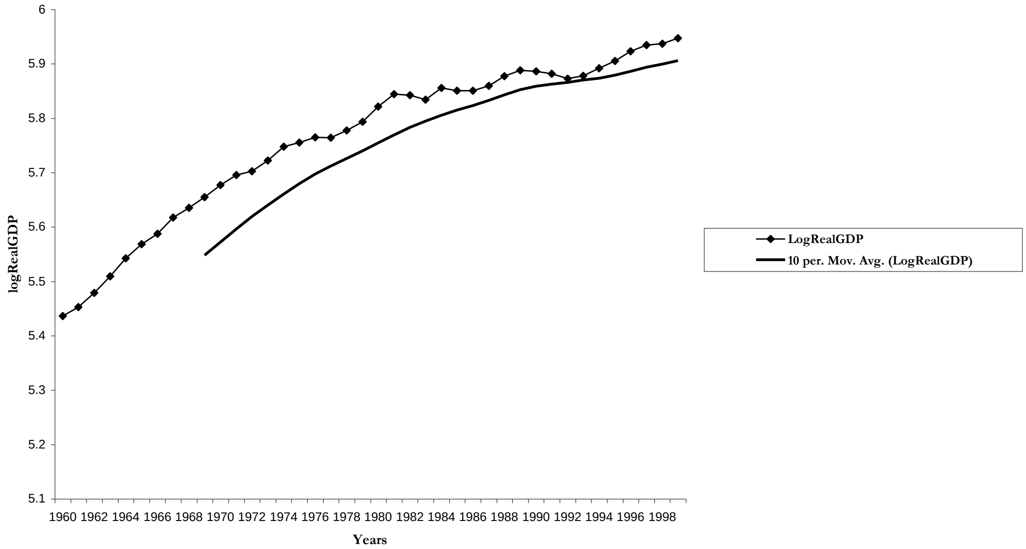

Figure 7-4: 10-year-centered Moving Average of LogRealGDP (2000=100).

Before we move to the formal tests of the long-run relationship, we present in Figures 1 through 4, the data plots for the inflation rate and the real GDP, and the associated trends based on a linear trend and a 10-period centred moving average over the period of 1960:01 to 1999:04. From the linear trends in Figures 7-1 and 7-2 for inflation and real GDP respectively, we find the variables to share a common positive trend. However, from Figures 7-3 and 7-4, based on the 10-period centred moving average, we find that the long-run inflation rate has experienced a downward movement since 1992 onwards, though output has continued to rise. Given this, it is likely that we might not find a cointegrating relationship between output and inflation over a shorter sample spanning the years of 1992 to 1999. But overall, for the whole period of 1960:01 to 1999:04, it is quite obvious that the two series are more than likely to be cointegrated.

Equations (7.9) and (7.10) indicate that the real GDP and the CPI inflation rate respectively, should be non-stationary. As can be seen from Table 7-1, based on the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips-Perron (PP) test, both the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected.

Table 7-1: Unit Root Tests.

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t\ \tau_\mu$	$\tau_t\ \tau_\mu$	
CPI		-0.81	13.69	-5.22***	0.37	-1.80	Non- stationary
	τ_μ	-2.19	23.13**	-4.29***	0.77	-0.90	
	τ	-0.68		-2.01**			
Δ CPI	τ_t	-10.59***	97.47***	-51.93***	0.12*	-13.95***	Stationary
	τ_μ	-10.42***	120.01***	-38.49***	0.31***	-1.41	
	τ	-10.45***		-38.21***			
Y	τ_t	-2.30	11.10**	-2.27	0.37	-0.41	Non- stationary
	τ_μ	-4.61***	21.23***	-4.73***	1.46	1.47	
	τ	4.08***		5.84***			
Δ Y	τ_t	-7.14***	59.98***	-12.97***	0.12***	-7.06***	Stationary
	τ_μ	-6.41***	82.09***	-12.33***	0.95	-3.54***	
	τ	-3.35***		-10.48***			

Note: ***(**)[*] indicates significance at 1 percent (5 percent) [10 percent] level.

To check for the cointegrating relationship between inflation and output implied by equation (7.11), we start of by using the Phillips-Ouliaris (1990) test. Results reported in Table 7-2 shows the estimate of γ, the coefficient from a regression of π_t on y_t, along with the statistics needed to test for a unit root in the residual from this regression. As can be seen, the hypothesis that inflation and output are not cointegrated can be rejected at the 1 percent significance level.

Table 7-2: Phillips-Ouliaris (1990) Cointegration Test (1960:01-1999:04).

γ	ρ	τ	Q	Z_τ
1.61	0.74	-4.92	0	-4.92***

Note: *** indicates significance at 1 percent level.

For the Phillips-Ouliaris (1990) test, Table 7-2 reports γ, the coefficient form the regression of inflation on real GDP, ρ, the coefficient from the regression of the residual from the inflation-output regression on its own lagged term and τ the conventional t-statistic for testing the hypothesis that ρ =1. Z_τ indicates the adjusted t-statistic by allowing for serial correlation in the regression error. The adjustment uses Newey and West's (1987) method to estimate the variance of the regression error and Andrew's (1991) method to select a value for the lag truncation parameter q required for the Newey and West (1987) estimator, assuming that the regression error is well approximated by a first-order autoregressive process.

One potential drawback of the residual-based Phillips-Ouliaris (1990) test is concerned with the fact that the sensitivity of the results might hinge on which variable (inflation or output), is used as the dependent variable in the initial regression. Here, however, equation (7.11) indicates that the hypothesized cointegrating relationship as suggested by the theoretical implications of the modified Barro and Gordon (1983) model is of the following form: $\pi_t - \gamma\ y_t$. This implies that we should be treating inflation as the dependant variable. Nevertheless, we check for the robustness of the results by using the Johansen (1988) test of cointegration, which, in turn, does not require a choice of normalization.

Based on a stable VAR[66] estimated with 5 lags[67], and allowing for no trend and intercept in the VAR or the cointegrating relationship, as suggested by the theory, we tested for cointegration using Johansen's (1988) maximum likelihood approach. Based on the Pantula Principle, both the Trace and the Maximum Eigen Value tests, showed that there is one stationary relationship (r = 1) between inflation and output at 1 percent level of significance. The results have been reported in Table 3, and they confirm the finding of the Phillips-Ouliaris (1990) test. The corresponding

66 Stability, as usual, implied that no roots were found to lie outside the unit circle.

67 The choice of 5 lags is based on the unanimity of the sequential modified LR test statistic, Akaike information criterion (AIC), and the final prediction error (FPE) criterion.

cointegrating vector relating inflation and real GDP, obtained from the Johansen (1988) approach, is found to be: $0.13\pi_t - 0.40y_t$.

Table 7-3: Estimation and Determination of Cointegrating Rank.

Null hypothesis	Alternative Hypothesis	Test statistic	0.05 critical value	Prob. **
			Trace Statistic	
r=0	r=1	24.62365	12.32090	0.0003
r=1	r=2	1.912715	4.129906	0.1962
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	r=1	22.71093	11.22480	0.0003
r=1	r=2	1.912715	4.129906	0.1962
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Thus, the results reported in Tables 7-1 through 7-3 strongly support the long-run behaviour of inflation and real GDP implied by the restrictions obtained from the modified Barro and Gordon (1983) model.

7.3.2 Testing the Short-Run Restrictions

Focusing now on the short-run implications of the behavior of the change in the inflation rate, imposed by the modified Barro and Gordon (1983) model, Table 7-4 presents the maximum likelihood estimates of the model's parameters, which, in turn, is obtained by mapping (7-12) into a state space form, implied by (7-13) and (7-14). The likelihood function of the state-space model is then evaluated using Kalman filter.

The estimate of α =1.2716 suggest that the Lucas-supply curve is quite flat. Burger and Marinkov (2006) also draws similar conclusion about the slope of the curve using a VECM. Although, the parameters k and b are not identified individually, the estimate of A=(k-1)/b exceeds unity. With k>1, the result suggests that b<1, implying that the SARB placed more weight on its goal of output than on inflation over the pre-inflation-targeting era of 1960:01 to

1999:04. Again, similar observations have been made by Gupta and Naraidoo (2008), while estimating interest rate rules for South Africa in periods before the SARB moved into an inflation targeting framework. As expected, the estimate of λ is positive, though is not significant, as is the standard deviation of the real shock. The standard deviation that for the control error is, however, significant at the one percent level. Finally, the negative and significant estimate of the covariance $\sigma_{\varepsilon\eta}$ indicates that a positive shock to the natural rate tend to coincide with a negative shock to inflation. The estimate, thus, supports, the idea that ε_t represents a real shock or a supply-side disturbance.

Table 7-4: Maximum Likelihood Estimates.

Parameter	Estimate	Standard Error
α	1.27***	0.17
A	1.27**	0.61
λ	0.33	0.43
σ_{ε}	0.37	0.26
σ_{η}	2.44***	0.41
$\sigma_{\varepsilon\eta}$	-0.91**	0.45

Notes: (i) ***(**) indicates significance at 1 percent (5 percent) level;

(ii) L^c =-562.03; L^u = - 557.60

The within-equation restrictions that appear in (7.12) can be tested by comparing its fit with an unconstrained ARMA (1, 2) model identified in (7.16). The constrained model has 6 parameters, while the unconstrained model has 8. Thus, the theory places 2 restrictions on the univariate time series model for the stationary variable $\Delta\pi_t$. So, if L^c and L^u respectively, denotes the maximized value of the log-likelihood function for the unconstrained and the constrained model, then the likelihood ratio statistic LR= $2(L^u - L^c)$ has a chi-square statistic with 2 degrees of freedom under the null hypothesis that the constraints of the ARMA(1,2) model for $\Delta\pi_t$ holds. The LR statistic is: 2(562.03-557.60) = 8.86 < 9.21 (the 99 percent critical value for a χ^2 with 2 degrees of freedom). We can, thus, conclude that the model's short-run implications, as imposed by the theory, cannot be rejected at the 1 percent level of significance. This means that the data provides weak evidence of the theory in the short-run.

7.3.3 Evaluating the Inflation-Targeting Regime (2000:01-2008:02)

In this subsection, we evaluate the inflation targeting regime by trying to deduce whether the monetary authority could have done better or worse if it stayed time inconsistent over the period of 2000:01 to 2008:02 as well. To do this, we forecast the rate of inflation recursively using (7.12), first, based on new data generated from the one-period-ahead forecasts, and then second, based on the actual inflation rate that prevailed over this period. Ideally, because we are comparing across regimes, we would want to rely more on the forecasts generated from the forecasted values, rather than the original values. This is simply because, we are trying to analyze how the policymaker would have performed if it stayed time inconsistent, and hence, would not want to use the actual data that corresponds to the behaviour of the inflation rate in a different regime. However, just for the sake of completeness and comparison, we also forecast using the actual rate of CPI inflation.

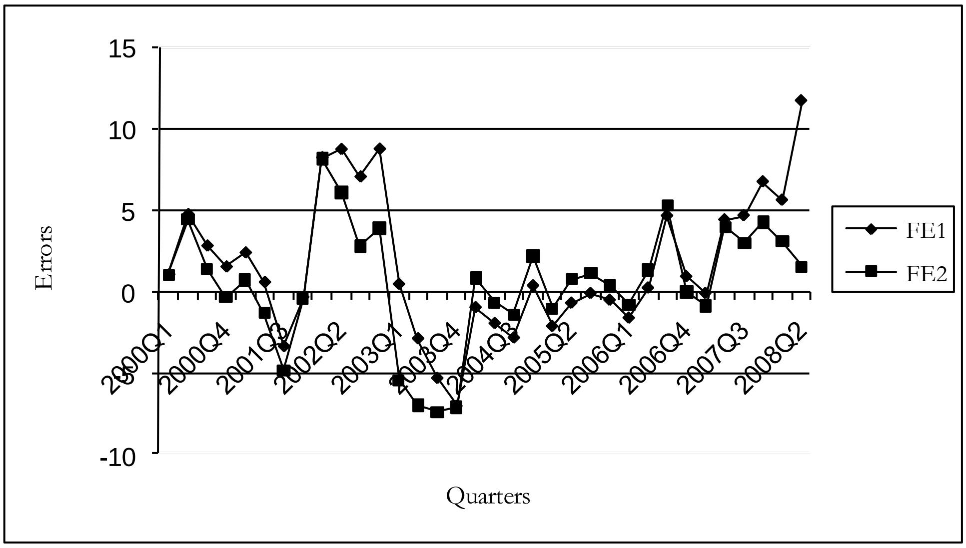

Figure 7-5: Differences between Actual and Forecasted Inflation.

Note: FE1(FE2) implies Forecast Errors based on forecast (actual) inflation.

Figure 7-5 plots FE1 and FE2, the one-step-ahead forecast errors based on forecasted values and actual values of the rate of inflation, respectively. We observe that the pattern of the movement of FE1 and FE2 are quite similar. Based on our calculations, the average value of the forecast errors based on the forecasted values of the inflation rate is 1.65 compared to 0.53 of the same when we use actual values. More importantly this implies, that if the SARB had continued to be time inconsistent, it could have produced on, average, an inflation rate which would have been lower by 1.65 % or 0.53% from what has prevailed over 2000:01 to 2008:02. Clearly then, the economy has been worse off in terms of the average levels of inflation experienced.

But then the big question is, whether these lower average rates of inflation, that could have been witnessed, are significantly different from what has actually been observed in the data on average? For this purpose, we resort to the Mincer and Zarnowitz (1969) regression. Note, for forecasts to be considered 'good' in an absolute sense they should not systematically under- or overpredict the rate of inflation. Formally, for a one-step-ahead forecast, we must have the following relationship:

$$E_{t-1}(\pi_t - \pi_{ft}) = 0 \qquad (7.16)$$

where π_t and π_{ft} are the actual and the one-step-ahead forecasted values of the rate of inflation. Econometrically speaking, the Mincer and Zarnowitz (1969) regression boils down to testing the unbiasedness of the forecasts by regressing the forecasted values on the actual values of the variable under consideration.

Given the regression:

$$\pi_t = \beta_0 + \beta_1 \pi_{ft} + \nu_t \qquad (7.17)$$

we test the joint unbiased hypothesis of: $\beta_0 = 0$ and $\beta_1 = 1$. At this stage it must be pointed out that the move to an inflation targeting regime can be considered to have improved the time consistency of monetary policy, then β_0 and β_1 in the Mincer and Zarnowitz (1969) regression should yield respectively, statistically significant values that are lower than zero and one. In other words, we would have $\pi_t < \pi_{ft}$.

Due to problems of possible serial correlation in the estimation of (7.17), Newey and West (1987) Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors were calculated. Based on a Wald test, the probabilities of unbiasedness was found to be 0.37 and 0.46, depending on whether we use forecasted values or actual values to compute the forecasts.[68] This implies that the restrictions cannot be rejected and, hence, the model produces unbiased forecasts. However, more importantly this implies that, on average, the actual and forecasted values are not statistically different. Or in other words, the possible lower average rates of

[68] In an attempt to ensure that both π_t and π_{ft} were stationary, the regressions were re-estimated based on the first difference of the actual and forecasted values of inflation. The probabilities of unbiasedness based on the Wald test were found to be 0.81 and 0.16 respectively, based on forecasted values and actual values, implying that the restrictions, and, hence, unbiasedness cannot be rejected.

inflation that we could have obtained from a time inconsistent SARB is not significantly different from what the authority has achieved under the inflation-targeting regime.[69]

7.4 Conclusions

This chapter derives the econometric restrictions imposed by a modified Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of CPI inflation and real Gross Domestic Product (GDP), and tests these restrictions based on quarterly data for South Africa covering the period of 1960:01 through 1999:04. The results show that the data are consistent with the long-run implications of the theory of time-consistent monetary policy involving the two variables. However, as far as the short-run dynamics of the data is concerned, the evidence is weak. But importantly, when the model is used to forecast one-step-ahead inflation over the period of 2001:01 to 2008:02, i.e., the period covering the starting point of the inflation targeting regime till date, we, on average, produce lower rates of inflation, than those observed in the actual data.

However, based on the Mincer and Zarnowitz (1969) regression, we find that the possible lower average rates of inflation that we could have obtained from a time inconsistent SARB is not significantly different from what the authority has achieved under the inflation-targeting regime. But then again, realizing that in a general equilibrium framework, higher inflation rates can distort a host of other marginal decisions, besides the money demand, these so-called insignificant lower average rates of inflation could easily result in possibly quite large welfare losses. Given this, future research, should be aimed at quantifying the size of the welfare cost of inflation in a dynamic general equilibrium endogenous growth setting.

Significant or not, our results tend to show that retaining the pre-targeting monetary policy framework would on average have produced lower rate of inflation for South Africa, or in other words, inflation targeting seems to not have improved the time consistency of monetary policy expected out of such a regime. The results, thus, point to the fact that, perhaps the SARB needs to manage the inflation-targeting framework better than it has done so far. In this regard,

[69] All the estimations were carried out again for a a revised pre-targeting sub-sample of 1983:01-1999:04, with the starting date coinciding with that of Ortiz and Sturzenegger (2007). However, the changes in the parameter estimates were marginal and did not affect our general conclusions. These results are available upon request from the authors.

as pointed out generally by Demertzis and Viegi (2006, 2007, and 2008), a narrower, and possibly also a lower, target band could be of immense help in improving the central bank's credibility and causing inflation expectations to converge to a focal point, and hence, bring down the rate of inflation.

Chapter 8

8 Comparing South African Inflation Volatility across Monetary Policy Regimes: An Application of Saphe Cracking*

8.1 Introduction

Recent empirical evidence on the direct link of inflation targeting and particular measure(s) of economic performance, in our case inflation volatility, is at best mixed. While, Neumann and Hargen (2002), Petursson (2004), Vega and Winkelfried (2005) and Mishkin and Schmidt-Hebbel (2007) finds inflation targeting has led to low inflation volatility, studies such as Johnson (2002), Truman (2003), Ball and Sheridan (2005) and Fang et al. (2009) tends to suggest otherwise. Essentially, all these studies and other papers[70], analyzing the macroeconomic effects of inflation targeting, relies on empirical comparisons across inflation-targeting and non-targeting countries or within an inflation targeting country across monetary policy regimes, i.e., in the pre- and post-inflation targeting era. Though, comparisons across targeting and non-targeting countries are quite rational, studies that tend to compare within a country across regimes are flawed, and, hence, can only be viewed as providing preliminary evidence on the success or failure of inflation targeting. The reason is simple: When analyzing the effect of inflation targeting on the volatility of inflation, the real question is not whether the inflation volatility has increased or decreased since the central bank's decision to target inflation, but rather is the inflation rate more volatile than it would have been had the earlier monetary policy regime continued. To address this issue, we use what is called the cosine-squared cepstrum, and apply the technique to analyze the Consumer Price Index (CPI) inflation volatility in South Africa over the period of 1996:q1 to 2008:q3[71], which moved to an inflation targeting regime from the first quarter of 2000.[72]

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[70] See Mishkin and Schmidt-Hebbel (2007) for a detailed survey.

[71] For the motivation behind the choice of the sample period, please refer to Section 3.

[72] For a detailed summary on the history of monetary policy in South Africa, please refer to Naraidoo and Gupta (2009).

*Forthcoming in Journal of Developing Areas.

Note in the mid- to late-1990s, the South African Reserve Bank (SARB) took a more eclectic approach to monetary policy, which, essentially involved monitoring a wide range of indicators, such as changes in bank credit extension, overall liquidity in the banking sector, the yield curve, changes in official foreign reserves, changes in the exchange rate of the rand, and inflation movements and expectations. This approach was enhanced in 1998 by the replacement of the discount window by the marginal lending facility of the repurchase system and consequently, the Bank rate was replaced by the repo rate. The SARB altered their previously eclectic approach in February of 2000, when the Minister of Finance announced that inflation targeting would be the SARB's sole objective. Currently, the Reserve Bank's main monetary policy objective is to maintain CPIX[73] inflation between the target-band of three to six percent, using discretionary changes in the repo rate as its main policy instrument.

Given that the SARB now targets inflation, if the move into the new regime or the abandonment of the older one affected inflation volatility, then observed inflation is represented as the sum of two series: (i) the series that would have eventuated if the SARB continued to pursue its more eclectic approach to monetary policy, and (ii) a second series associated with the direct impact of the regime change that arrived in late 1999 or early 2000. If the regime had any coherent effect, then the two series are likely to be well correlated. If the second series is found to be positively correlated with the series that would have eventuated, then it increases the volatility of inflation and is said to be in-phase. However, if the second series is negatively correlated or out of phase, volatility declines, since fluctuations in the series that would have eventuated are dampened. Generally speaking, if a time-varying stationary series is composed of two such series that are linear or well correlated, then the autocovariance function contains a global maximum at the zero lag and a local extremum at a lag corresponding to the date when the second series arrives. The problem with the autocovariance function is that it is difficult to detect a second series and determine the degree of its phase shift, given that a local extremum, in the case of an autocovariance function, appears as a broad cycle.

The cepstrum technique helps us to overcome such difficulties. Cepstra have been used successfully in detecting secondary influences in engineering, particularly communication theory.

[73] CPIX is defined as CPI excluding interest rates on mortgage bonds.

Intuitively, the cosine-squared cepstrum behaves like an autocovariance function, but with sharper resolution that helps in identifying the arrival and phase relationship of the secondary series with great precision. A local extremum appears as an impulse, instead of a broad cycle, the direction of which, in turn, determines whether the secondary series have increased or dampened volatility. Because of this, the cosine-squared cepstrum is well equipped to determine whether inflation volatility is greater than it would otherwise have been had the SARB continued to pursue its so-called eclectic approach to monetary policy decision-making.

The motivation to look into South Africa, specifically, emanates from the previous chapter which derives the econometric restrictions imposed by the Barro and Gordon (1983) model of dynamic time inconsistency on a bivariate time-series model of CPI inflation and real Gross Domestic Product (GDP), and tests these restrictions based on quarterly data for South Africa covering the period of 1960:q1 through to 1999:q4, i.e., for the pre-inflation targeting period. The results show that the data are consistent with the short- and long-run implications of the theory of time-consistent monetary policy. Moreover, when the model is used to forecast one-step-ahead inflation over the period of 2000:q1 to 2008:q2, i.e., the period covering the starting point of the inflation targeting regime till date we obtain, on average, obtain lower rates of inflation. Chapter 7 though is silent about inflation volatility that would have been generated over the inflation targeting era, if the SARB continued to be dynamically time inconsistent. But this is understandable, given that the Barro and Gordon (1983) model is essentially a framework for analyzing equilibrium inflation, and not its volatility. And this is exactly what this chapter tries to address, as far as South Africa is concerned. It must be realized that for a country seeking price stability, it is not only essential to obtain lower mean levels of inflation but also less volatility in inflation. Inflation volatility matters because high variability of inflation over time makes expectations about the future price level more uncertain, which, in a world with nominal contracts, induces risk premia for long-term arrangements, raises costs for hedging against inflation risks and leads to unanticipated redistribution of wealth. Thus, inflation volatility can impede growth, even if inflation on average remains restrained. Given this, this chapter is of paramount importance. To the best of our knowledge, this is the first attempt to study inflation volatility across monetary policy regimes by using the cosine-squared cepstrum.[74] Finally note the decision to use quarterly data rather than monthly data for the CPI inflation rates is essentially an

[74] The only other paper that uses, and in fact, introduced the cosine-squared cesptrum to economics is by Cunnigham and Vilasuso (1994). The authors used the technique to compare Gross National Product (GNP) volatility across exchange rate regimes for the US economy.

attempt to use the same data frequency as that of chapter 7 in drawing conclusions about the volatility of the inflation rates. For the same reason, we also used CPI rather than CPIX inflation. Besides, quarterly data on CPIX inflation is not available beyond 1997:q2 and our methodology requires data dating back to 1996:q1. For further details, refer to Section 3 below.

An extended version of this chapter would be to evaluate the performance of all the inflation targeting economies with respect to not only inflation volatility, but also the volatility of output and the monetary policy instrument used to maintain the target or the target-band. The rest of the chapter is organized as follows: Section 8.2 describes the data and our main findings, while, Section 8.3 concludes. Note, an appendix describing the technical details of a cepstrum has been presented at the end of the chapter.

8.2 Application to Inflation Volatility

If the move to the inflation targeting regime affected the volatility of the CPI inflation rate for South Africa, then the series is representable as the sum of the pre-2000 series and a secondary series arriving in late 1999 or early 2000. If the secondary series are perfectly in-phase with the pre-2000 series, then volatility increased relative to what it would have been under the eclectic monetary policy regime; if it is perfectly out-of-phase, then volatility decreased. Determining whether or not a secondary series arrived and it was in-phase or out-of-phase is a straightforward application of the cosine-squared cepstrum, discussed in the appendix of the chapter.

We begin by performing unit root tests on the CPI inflation rate, since the application of the cosine-squared cepstrum requires us to ensure that the data process is stationary. For this purpose we use the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As can be seen from Table 1, seasonally adjusted (at annual rate) CPI inflation is found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the inflation rate, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As the variable of interest is found to be non-stationary, we had to first difference the inflation rate series. Once the first-differenced CPI inflation series was generated and found to be stationary, we demean the series to avoid the dominance of the zero frequency components in the power spectrum, and then apply the five point Hanning-type cosine tapers to suppress possible sidelobes.

Table 8-1: Unit Root Tests (1996:Q1-2008:Q3).

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\alpha_t \alpha_\mu \alpha$	$\phi_3 \phi_1$	$\alpha_t \alpha_\mu \alpha$	$\alpha_t \alpha_\mu$	$\alpha_t \alpha_\mu$	
cpi_infl	α_t	-2.75	4.13	-2.89	0.13	-2.86	Non- stationary
	α_μ	-2.80	7.86***	-2.97**	0.13***	-2.72***	
	α	-1.24		-1.00			
D(cpi_infl)	α_t	-6.86***	23.53***	-8.02***	0.11***	-6.99***	Stationary
	α_μ	-6.86***	47.09***	-7.79***	0.23***	-6.75***	
	α	-6.92***		-7.86***			

Notes: (i) cpi_infl stands for CPI inflation;
(ii) D(cpi_infl) is the first difference of CPI inflation.

Next we use a Cooley–Tukey fast Fourier transform (FFT) and compute the natural logarithm of the sum of squares of the real and imaginary parts to form the log power spectrum. Note we start our analysis 15 quarters prior to the change in regime, so the first-differenced CPI inflation series begins at 1996:q2.[75] The choice of 15 quarters is decided based on the average distance between the troughs of the smoothed[76] log power spectrum, which was found to be approximately 0.0686 cycles per quarter, so that the lag associated with the secondary influence on the first-differenced inflation rate is approximately at a lag of 15 quarters, i.e., $\tau = 15$, arriving in the first quarter of 2000. Given that the seasonally (adjusted at annual rate) CPIX inflation rate was also non-stationary, using the first-differenced CPIX inflation rate over 1997:q3-2008:q3

[75] The data for this chapter are from the quarterly bulletins available for download from the official website (http://www.reservebank.co.za/) of the SARB. All the computations are performed by using the Signal Processing Toolbox in MATLAB, Version R 2007a.

[76] Following Bogert et al. (1963), the power spectrum is smoothed by applying a five-point centered moving average, since the moving average suppresses high frequency variations that would correspond to relatively long lags in secondaries.

revealed that the distance between the troughs of the smoothed log-power spectrum was approximately 0.06812 cycles per quarter on average, implying a τ = 15 as well. With CPIX inflation data only available from 1997:q2 and the regime change taking place in 2000:q1, we could not use the CPIX inflation data to obtain the cosine-squared cepstrum. Forming the inverse FFT (IFFT) and then the sum of squares again, and establishing the sign according to the sign of the real part, the cosine-squared cepstrum is formed, as discussed in Equation (A.6) in the appendix.

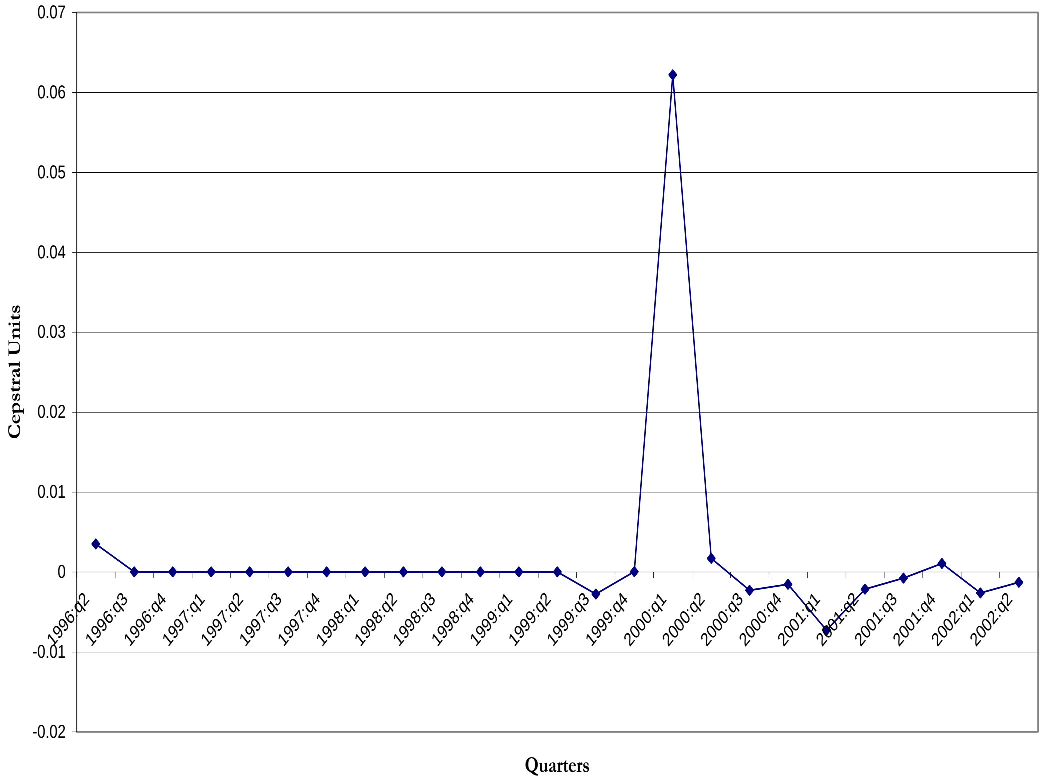

Figure 8-1: Shows the first 25 lags of the cosine-squared cepstrum for first-differenced CPI inflation.

For the sake of convenience, the time scale replaces the lag numbers. The first twelve points of the cepstrum for the first-differenced CPI inflation rate is rescaled to enhance the detail.[77] A delta function spikes prominently upward at the lag corresponding to the first quarter of 2000. This "arrival time" for the secondary series is consistent with the announcement of the Finance Minister that inflation targeting would be the SARB's sole objective. The unambiguous nature of the delta function and its positive sign provide evidence of a secondary influence on the South African CPI inflation series that not only matched it fluctuation by fluctuation, but was also exactly in-phases with the CPI inflation series. In other words, the secondary influence that arrived in early 2000 has increased the fluctuations/volatility in the CPI inflation rate.

Chapter 7 indicated that had the SARB continued to be time inconsistent it would have produced lower average levels of inflation in the post inflation targeting period. Here we show that the inflation-targeting regime has increased the volatility of inflation relative to what it would have been under the eclectic monetary policy regime. So in other words, it seems, at least until now, the SARB has failed to attain its primary objective of inflation targeting -- price stability in terms of both the mean and variance of inflation.

8.3 Conclusions

The positive delta function in the cosine-squared cepstrum has provided evidence that the inflation targeting regime in South Africa began to impact CPI inflation in the first quarter of 2000, making the same more volatile than it would have been had the SARB pursued the more eclectic monetary policy approach, which was in place prior to the 2000:Q1. The cosine-squared cepstrum, thus, shows that it can be successfully used in applications where the effects policy changes on time series data can be modelled as additive time series that are well correlated with the variable under study. However, there are limitations to this approach: First, the increased inflation volatility under the new monetary policy regime may not be permanent, but rather a pulse-like response in the inflation rate, and; Second, at times the methodology might require additional economic insight to isolate the possible economic causes of the event, without

[77] The first twelve cepstral estimates for the first-differenced CPI inflation rate is set equal to zero to enhance the detail. The above operation removes the delta function at the origin, which, in turn corresponds to the global maximum.

recovering the secondary series. Fortunately in our case, the prominent cepstral peak is relatively unambiguous in terms of its timing and phase characteristics.

Despite the limitations, the finding of increased inflation volatility in the inflation targeting regime indicated by the positive delta function in the cosine-squared cepstrum cannot be taken lightly. In our opinion, the possible explanation for the increased fluctuations in the CPI inflation lies in the width of the target band of 3 percent to 6 percent. Mishkin (2003) points out that "the use of target bands has a dangerous aspect. Floors and ceilings of bands can take on a life of their own in which there is too great a focus on the breach of the floor or ceiling rather than on how far away actual inflation is from the midpoint of the target range." Too much focus on breaches of the floors or ceilings, in turn, can lead to the so-called instrument instability problem (Bernanke et al. (1999) and Mishkin and Schmidt-Hebbel (2002)). Moreover, it can also lead to suboptimal setting of monetary policy and controllability problems resulting in the inflation target to be missed in the medium-term Mishkin (2003). In this regard, as pointed out by Demertzis and Viegi (2006, 2007, and 2008), a narrower, and possibly also a lower, target band could be of immense help in improving the central bank's credibility and causing inflation expectations to converge to a focal point, and hence, bring down the mean and variance of the inflation rate.

TECHNICAL APPENDIX

THE CEPSTRUM[78]

Speaking mathematically, the cepstrum is an integral transform with a long history. The use of the cepstrum dates back to Poisson (1823) and Schwarz (1872), who applied cepstra to problems involving potential functions with real parts fixed on the unit circle. Later, Szëgo (1915) and Kolmogorov (1939) used cepstra in the extraction of stable causal systems by factoring power spectra of random processes. However, it is the application of Bogert et al. (1963) in engineering that most coincides with our interest here.

Bogert et al. (1963) considers a time-varying function f(t) which is made up of another function $f_1(t)$ and its additive "echo," $f_1(t-\tau)$, lagged by τ periods. Formally, we have:

$$f(t) = f_1(t) + af_1(t-\tau) \tag{A.1}$$

The power spectrum of $f(t)$ is

$$\left|F(\omega)\right|^2 = \left|F_1(\omega)\right|^2 \left\{1 + a^2 + 2a\cos(\omega\tau)\right\}, \tag{A.2}$$

where $F_1(w)$ is the complex Fourier transform of $f_1(t)$. The "echo", in turn, manifests as a cosine function riding on the envelope of the power spectrum. The period of the cosine function is the reciprocal of the lag τ. In studies involving autocorrelation analysis in hydroacoustic problems, Griffin et al. (1980) has shown that if the spectra of $f(t)$ and its "echo" differ, i.e., they are not perfectly correlated, then the parameter a in Equation (A.2) varies with frequency. Thus, $f(t)$ is better represented by:

$$f(t) = f_1(t) + f_2(t-\tau), \qquad f_1(t) \neq f_2(t) \tag{A.3}$$

so that the power spectrum of $f(t)$ is given by:

$$\left|F(\omega)\right|^2 = \left|F_1(\omega)\right|^2 + \left|F_2(\omega)\right|^2 + 2\left|F_1(\omega)\right|\left|F_2(\omega)\right|\cos\left\{\phi_1(\omega) - \phi_2(\omega) + \omega\tau\right\}, \tag{A.4}$$

where $\phi_1(\omega)$ and $\phi_2(\omega)$ are the phase spectra of $f_1(t)$ and $f_2(t)$, respectively.

The modulating cosine is phase-shifted by an amount which is equal to the differences in the phases of functions that composes $f(t)$. Thus, if the function $f_1(t)$ and $f_2(t)$ are not close

[78] This section relies heavily on the discussion available in Cunnigham and Vilasuso (1994).

copies of one another, the argument of the modulating cosine is not constant, which, in turn, has important consequences as we shall see below.

We proceed under the assumption that $f_1(t) = f_2(t)$ and return to the more general case later. The function $f(t)$ can be represented as the linear convolution of $f_1(t)$ with a train of impulses. Bogert et al. (1963) argue that if the envelope of $|F(\omega)|^2$ could be made optimally white, this would be equivalent to making $|F_1(\omega)|^2$ into a "boxcar" function, whose Fourier transform would be a sinc function at the origin.[79] In the limit, as the envelope of the power spectrum becomes uniform at all frequencies, the sinc function tends towards a Dirac delta function. The transform of the modulating cosine would be an impulse whose delay is related to the frequency of the modulating cosine which, in turn, is equal to the lag length between $f_1(t)$ and its "echo." Therefore, under ideal conditions and when scaled properly, this resulting series yields a time domain function with a global maximum, or "peak" at the origin, and the local maximum or "peak" indicating the arrival time of the "echo."

In practice, the whitening of the power spectrum is performed by first applying the natural logarithm and then the inverse Fourier transform (IFT). Because it ignores the phase spectrum, and is calculated directly from the log power spectrum, the resultant function is referred to as the power spectrum.

Let us now go back to case presented in Equations (A.3) and (A.4). If the component functions are not close copies of one another, then the argument of the modulating cosine is not invariant, and, hence, cepstral peaks rapidly degenerate. Therefore, it must be realized that the impulse appears at the appropriate lag of the cepstrum only if the component functions are well correlated.

When we consider the discrete case, cepstra are a class of integral transform whose kernel is a function of the z-transform of a real sequence. The discrete power cepstrum of a data sequence $x(nT)$ with z-transform $X(z)$ is then given by:

$$\tilde{x}(nT) = \frac{1}{2\pi i}\left|\oint_c \log|X(z)|\, z^{n-1}dz\right|^2 \tag{A.5}$$

[79] Note that sinc x = (sinx)/x.

where n= 0, 1, 2,…, N, enumerates the samples, T is the sampling interval, and C is a closed contour inside the region of convergence of the power series and enclosing the origin. As discussed by Cunningham (1980), this can be extended to the cosine-squared cepstrum by addition of the signum function as follows:

$$\hat{x}(nT) = \tilde{x} \times \operatorname{sgn}\left\{ \operatorname{Re}\left[\frac{1}{2\pi i} \oint_c \log |X(z)| z^{n-1} dz \right] \right\} \qquad \text{(A.6)}$$

The addition of the signum function allows the cepstrum to determine not only the arrival time of the secondary series, but also, and perhaps more importantly, its polarity relative to the original series making its interpretation analogous to the autocovariance function.[80] Because in real-life applications the real sequences $x(nT)$ are of finite length, the annulus of convergence of X(z) always includes the unit circle, the transforms may be computed by means of the fast Fourier transforms.

Because the cepstrum is essentially the spectrum of a spectrum, the cepstral domain is a time domain. The terminology easily becomes confusing, therefore, Bogert et al. (1963) suggested the following conventions: the term "cepstrum" is an anagram of the word "spectrum." Likewise, periodicities in the cepstrum are discussed in terms of "quefrencies," "gamnitudes," and "repiods," analogous to "frequencies," "gain/amplitudes," and "periods" in the time domain. "Filtering" in the cepstral domain is "liftering" and so on. Further, Bogert et al. (1963) refers to data analysis in the cepstral domain as "alanysis". Finally, the term "saphe," pronounced "safe" and related to "phase," is used to refer to the displacement between the secondary, or lagged series, and the original. Thus, the detection and analysis of cepstral peaks and "phase" shifts of the lagged series is referred to as "saphe cracking."

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[80] The close relationship between the cosine-squared cepstrum and the autocovariance function becomes clear when we write the latter as a function of the z-transform of the same series: $x_{acvf}(\tau) = \frac{1}{2\pi i} \oint_c |X(z)| z^{n-1} dz.$ Equivalently, the autocovariance function is the inverse Fourier transform of the power spectrum of a series.

9 Conclusion

Since the February of 2000, the sole objective of the SARB has been to keep the CPIX inflation rate within the target band of 3 percent to 6 percent, using discretionary changes in the Repo rate as its main policy instrument. Given this, this thesis aimed to evaluate inflation targeting regime of South Africa not only in term of welfare cost estimates, but also by comparing mean and volatility of inflation in pre- and post-inflation targeting regimes.

With the welfare cost of inflation lying between 0.70 percent of GDP to 1.33 percent of GDP for the target band of 3 percent and 6 percent, we show that there are considerable welfare gains to be had by reducing the lower level of the target band below the 3 percent mark. In addition, we show that mean and volatility of inflation is higher than it would have actually been had the SARB followed its earlier more eclectic monetary policy regime. We believe this is due to the width of the inflation targeting band. In line with the inflation targeting literature on focal points, we suggest a narrower and possibly lower target band target band could be of immense help in improving the central bank's credibility and causing inflation expectations to converge to a focal point, and hence, bring down the mean and variance of the inflation rate.

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Bibliography

1. Akinboade, O. A., Siebrits, F. K. and Roussot, E. W. (2004). The Output Costs during Episodes of Disinflation in South Africa, *Ninth Annual conference of Econometric Modelling for Africa.*

2. Anderson, R. G. and Robert, H. R. (2001). The Remarkable Stability of Monetary Base Velocity in the United States, 1919-1999, *Working Paper 2001-008A. St. Louis*, Federal Reserve Bank of St. Louis.

3. Andrews, D.W.K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation, *Econometrica*, 59(3): 817-858.

4. Bae, Y. (2005). The Money Demand Function and Nonlinear Cointegration, Manuscript Columbus: Ohio State University.

5. Bailey, M. (1956). The welfare Cost of Inflationary Finance, *Journal of Political Economy* 64(2):93-110.

6. Ball, L. and Sheridan, N. (2005). Does Inflation Targeting Matter?, The inflation Targeting Debate, edited by B.S Bernanke and M. Woodford, *NBER Studies in Business Cycles*, Vol. 32. Chicago: University of Chicago Press.

7. Barro, R.J. and Gordon, D.B. (1983). A positive Theory of Monetary Policy in a Natural rate Model, *Journal of Political Economy*, 91(4):589-610.

8. Bernanke, B.S., Laubach, T., Mishkin, F.S., and Posen, A.S. (1999). *Inflation targeting: lessons from the International Experience*, Princeton, NJ: Princeton University Press.

9. Black, R. Macklem, T. and Poloz, S. (1993). Non-Superneutralities and some benefits of disinflation: A quantitative general equilibrium analysis, *Manuscript* (Research Department, Bank of Canada, Ottawa).

10. Blaszkiewicz, M., Konieczny, J., Myslinskaa, A., Radziwil, A. and Wozniak, P. (2003). Some benefits of reducing inflation in transition economies, *Macroeconomies 0303004,* EconWPA.

11. Bogert, B.P., Healy, M.J.R. and Tukey, J.W. (1963). The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo-Autocovariance, Cross –Cepstrum, and Saphe Cracking, *in proceedings of a Symposium on Time Series Analysis*, edited by M. Rosenblatt. New York: John Wiley & Sons, Inc.

12. Bonato, L. (1998). Price stability: some costs and benefits in New Zealand, *Reserve Bank of New Zealand Discussion Paper No G98/10*, Reserve bank of New Zealand.

13. Brock, W. (1974). Money and Growth: The case of Long –Run perfect Foresight, *International Economic Review*, 15(3):750-777.

14. Burger, P. and Marinkov, M. (2006). The South African Phillips Curve: How Applicable is the Gordon Model? , *South African Journal of Economics*, 74(2):172-189.

15. Burger, P. and Marinkov, M. (2008). Inflation targeting and inflation performance in South Africa, *TIPS Working Paper.*

16. Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In Milton Friedman, Ed.Studies in the Quantity Theory of Money. Chicago: University of Chicago Press.

17. Coleman, W.J. II. (1993). Money, Interest, and capital in cash-in- advance economy, *Manuscript*, Department of Economics, Duke University, Durham, NC.

18. Cooley, T.F. and Hansen, G.D. (1989). The inflation tax in a real business cycle model, *American Economic Review*, 79(4): 733-748.

19. Cooley, T.F. and Hansen, G.D. (1991). The welfare cost of moderate inflations, *Journal of Money, Credit, and Banking*, 23(3): 483-503.

20. Cunningham, S.R., (1980). Application of Cepstral Techniques to Rayleigh Wave Multipathing, *U.S. national Technical Information Service (NTIS)*, ENSCO DCS-STR-80-45.

21. Cunningham, S.R., and Vilasuso, J. (1994). Comparing U.S. GNP Volatility across Exchange Rate Regimes: An Application of Saphe Cracking, *Journal of Macroeconomics*, 16(3): 445-459.

22. De Gregorio, J. (1993). Inflation, taxation, and long-run growth, *Journal of Monetary Economics*, 31(3): 271-289.

23. Demertzis, M. and Viegi, N. (2008). Inflation Targets as Focal Points, *International Journal of Central Banking*, 4(1): 55-87

24. Demertzis, M. and Viegi, N. (2007). Inflation Targeting - a Framework for Communication, *DNB Working Papers 149*, Netherlands Central Bank, Research Department.

25. Demertzis, M. and Viegi, N. (2006). Aiming for the Bull's Eye: Uncertainty and Inertia in Monetary Policy, *Computing in Economics and Finance 2006 150*, Society for Computational Economics.

26. Dolado, J.J., Gonzalez-Paramo, J.M. and Vinals, J. (1999). A cost-Benefit analysis of Going from Low to Price Stability in Spain, In M. Feldstein, ed., The costs and Benefits of Price stability Chicago, University of Chicago Press.

27. Dotsey, M. and Ireland, P. N. (1996). The Welfare Cost of Inflation in General Equilibrium, *Journal of Monetary Economics*, 37(1): 29-47.

28. Engle, R.F. & Granger, C.W. (1987). Cointegration and error correction: representation, estimating and testing, *Econometrica*, 55(2):251-276.

29. Fang, W-S., Miller, S. M., and Lee, C-S., (2009). Inflation Targeting Evaluation: Short-run Costs and Long-run Irrelevance, *Working Papers No. 2009-14*, Department of Economics, University of Connecticut.

30. Feldstein, M. (1999). Capital Income Taxes and the Benefit of Price Stability, In M. Feldstein, ed., The Cost and Benefit of Price stability, Chicago, University of Chicago Press.

31. Feldstein, M. (1997). The Costs and Benefits of Going from Low Inflation to Price Stability. In Christina D. Romer and David H. Romer, Eds. Reducing Inflation: Motivation and Strategy. Chicago: University of Chicago Press.

32. Feldstein, M., Green, J. and Sheshindki, E. (1978). Inflation and Taxes in a Growing Economy with Debt and Equity Finance, *Journal of Political Economy*, 86(2), Part 2: Research in Taxation. pp. S53-S70.

33. Fischer, S. (1981). Towards an Understanding of the Costs of Inflation: II. *Carnegie-Rochester Conference Series on Public Policy*, 15(1): 5-41.

34. Fisher, M. and Seater, J. (1993). Long-Run Neutrality and Superneutrality in an ARIMA Framework, *American Economic Review*, 83(3): 402-415.

35. Friedman, M. (1969). The optimum quantity of money, In the Optimum Quantity of Money and Other essays. Chicago: Aldine Publishing Company.

36. Gillmax, M. (1993). The Welfare Cost of Inflation in a Cash –in –Advance model with Costy Credits, *Journal of Monetary Economics*, 31(1):97-115.

37. Goldfeld, S. M. (1976). The case of missing money, *Brookings Papers on Economic Activity*, 7(3): 683-740.

38. Gomme, P. (1993). Money and Growth revisited: Measuring the Costs of Inflation in an Endogenous Growth Model, *Journal of Monetary Economics*, 32(1): 51-77.

39. Gonçalves, C. E. S., and Carvalho, A. (2008). Inflation Targeting and the Sacrifice Ratio, *Revista Brasileira de Economia*, 62(2):177-188.

40. Griffin, J.N., Jones, R.L., and Cunfer, R. S. (1980). Experimental Procedures for Estimating Depths and Yields of Underwater Explosions Using Recordings of Limited Dynamic Range, *Final Report*, Project T/ 8152/, ENSCO.

41. Gupta, R., Kabundi, A., and Modise, M. P. (Forthcoming).Has the SARB Become More Effective Post Inflation Targeting?, *Economic Change and Restructuring*, Forthcoming.

42. Gupta, R. and Komen, K. (2008). Time Aggregation and the Contradictions with Causal Relationships: Can Economic Theory Come to the Rescue?, *Studies in Economics and Econometrics*, 33(1):13-24.

43. Gupta, R. and Uwilingiye, J. (Forthcoming c). Evaluating the Welfare Cost of Inflation in a Monetary Endogenous Growth General Equilibrium Model: The Case of South Africa, *International Business and Economics Research Journal,* Forthcoming.

44. Gupta, R. and Uwilingiye, J. (Forthcoming b). Comparing South African Inflation Volatility across Monetary Policy Regimes: An Application of Saphe Cracking, *Journal of Developing Areas*, Forthcoming.

45. Gupta, R. and Uwilingiye, J. (Forthcoming a). Some benefits of Reducing Inflation in South Africa, *International Business and Economics Research Journal*, Forthcoming.

46. Gupta, R. and Uwilingiye, J. (2010). Dynamic Time Inconsistency and the SARB, *South African Journal of Economics*, 78(1):76-88.

47. Gupta, R. and Uwilingiye, J. (2009b). Time Aggregation, Long-Run Money Demand and Welfare Cost of Inflation, *Studies in Economics and Econometrics*, 33(3):95-109.

48. Gupta, R. and Uwilingiye, J. (2009a). Measuring welfare cost of inflation in South Africa: A Reconsideration, *South African Journal of Economic and Management Sciences*, 12(2):137-146.

49. Gupta, R. and Uwilingiye, J. (2008). Measuring the Welfare cost of Inflation in South Africa, *South African Journal of Economics*, 76(1):16-25.

50. Hendy, D.F. and Juselius, K. (2000). Explaining Cointegration Analysis: Part II, *Department of Economics*, University of Copenhagen, Denmark.

51. Ireland, P.N. (2009). On welfare cost of inflation and the Recent Behaviour of Money Demand, *American Economic Review*, 99(3): 1040-1052.

52. Ireland, P. N (1999). Does the Time-consistency Problem Explain the Behaviour of Inflation in the United States, *Journal of Monetary Economics*, 44: 279-291.

53. Johansen, S. (1995). *Likelihood-based Inference in Cointegrated Vector Autoregressive Models*, Oxford: Oxford University Press

54. Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, 59(6): 1551–1580.

55. Johansen, S. (1988). Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control*, 12(2-3): 231-254.

56. Johnson, D.R. (2002). The effect of inflation targeting on the behavior of expected inflation: evidence from an 11 country panel, *Journal of Monetary Economics*, 49(8):1521-1538.

57. Jones, L.E. and Manuelli, R.E. (1993). Growth and the effects of inflation, *Manuscript* Kellogg School of Management, Northwestern University, Evanston, IL.

58. Kolmogorov, A. N. (1939). Sur l'interpolation et extrapolation des suites stationnaires, *Compte Rendus Hebdomadaires des Séances de l'Académie des Sciences*, 208 :2043-2045.

59. Liu, D. G. and Gupta, R. (2007). A Small-Scale DSGE Model for Forecasting the South African Economy, *South African Journal of Economics*, 75(2): 179-193.

60. Lucas, R. E. Jr. (2000). Inflation and Welfare, *Econometrica*, 68(62):247–74.

61. Lucas, R. E. Jr. (1981). Discussion of: Stanley Fischer, Towards an Understanding of the Costs of Inflation II, *Carnegie-Rochester Conference Series on Public Policy*, 15(1): 43-52.

62. Lucas, R.E. jr. and Stockey, N. L. (1983). Optimal Fiscal and Monetary Policy in an Economy Without capital, *Journal of Monetary economics*, 12(1): 55-93.

63. Ludi, K. L. and Ground, M. (2006). Investigating the Bank-Lending channel in South Africa: A VAR Approach, *Working paper No. 2006-04*.Department of Economics, University of Pretoria.

64. Marcellino, M. (1999). Some consequences of temporal Aggregation in Empirical Analysis, *Journal of Business and Economic Statistics*, 17(1):129-136.

65. Marquis, M.H. and Reffett, K. L. (1994). New technology spillovers into the payment system, *Economic Journal*, 104(426): 1123-1138.

66. Meltzer, A.H. 1963. The demand for money: the evidence from the Time series, *Journal of Political Economy* , 71(3):219-246.

67. Mincer, J. and Zarnowitz, V. (1969). The evaluation of economic Forecasts. In J. Mincer (ed.), Economic Forecasts and Expectations. New York. NBER

68. Mishkin, F.S. (2003). A Comment on "Inflation Targeting in Emerging Market Economies, by Arminio Fraga, Ilan Goldfajn and Andre Minella, *NBER Macro Annual*, 403-413.

69. Mishkin, F.S. and Schmidt-Hebbel, K. (2007). Does Inflation Targeting Make Difference?, *NBER Working Paper No. W12876*, Oxford: Oxford University Press.

70. Mishkin, F.S. and Schmidt-Hebbel, K. (2002). One decade of inflation targeting in the world: What do we know and what do we need to know?, *Inflation targeting: Design, Performance, Challenges*, edited by N. Loayza and R. Soto, 2002, pp. 171-219. Central Bank of Chile.

71. Naraidoo, R., and Gupta, R. (2009). "Modelling Monetary Policy in South Africa: Focus on Inflation Targeting Era Using a Simple Learning Rule", *Working paper No 200904*. Department of Economics, University of Pretoria.

72. Neuman, M.J.M., and Von Hagen, J. (2002) Does Inflation Target Matter?, *Review Federal Reserve Bank of St. Louis*, 84(4):127-148.

73. Newey, W.K. and West, K.D. (1987). A simple, positive-semi definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55(3): 703-708.

74. O'Reilly, B. and Levac, M. (2000). Inflation and the Tax System in Canada: An Exploratory Partial Equilibrium Analysis, *Bank of Canada Working Paper 2000-18*.

75. Ortiz, A and Sturzenegger, F (2007). Estimating SARB's Policy Reaction Rule, *South African Journal of Economics*, 75(4): 659-680.

76. Petursson, T.G. (2004). The effects of inflation targeting on macroeconomic performance, *Working paper 23*, Central Bank of Iceland.

77. Phelps, E.S. (1973). Inflation theory of public finance, *Swedish journal of Economics*, 75:67-82.

78. Phillips, P. C. B. and Ouliaris, S. (1990). Asymptotic Properties of Residual Based Tests for Cointegration, *Econometrica*, 58(1):165-193.

79. Poisson, S.D. (1823). Mémoire Sur la distribution de la chaleur dans les corps solides, *Journal de l'Ecole Polytechnique*, 19 :249-403.

80. Prescott, E.C. (1987). A multiple means of payment model, In W.A. Barnett and K.J, Singleton, eds., *New approaches to monetary economics*. Cambridge. Cambridge University Press.

81. Romer, P. M. (1986). Increasing Returns and Long –run Growth, *Journal of Political Economy*, 94(5):1002-1037.

82. Schreft, S.L. (1992). Welfare- Improving Credit Controls, *Journal of Monetary Economics*, 30(1):57-72.

83. Schwarz, H.A. (1872). Zur Integration de partiellen Differentialgleichung, *Journal fur die Reine und Angewandte Mathematik*, 74: 218-254.

84. Serletis, A. and Virk, J. J. (2006). Monetary Aggregation, Inflation and Welfare, *Applied Financial Economics*, 16(7): 499-512.

85. Serletis, A. and Yavari, K. (2007). On the welfare cost of inflation in Europe. *Applied Economics Letters*, 14(2): 111–113.

86. Serletis, A. and Yavari, K. (2005). The Welfare Cost of Inflation in Italy, *Applied Economics Letters*, 12(3): 165–168.

87. Serletis, A. and Yavari, K. (2004). The welfare cost of inflation in Canada and the United States, *Economics Letters*, 84(2): 199–204.

88. Sichei, M. M (2005). Bank-Lending Channel in South Africa: Bank-Level Dynamic Panel Data Analysis, *Working Paper No 2005-10*, Department of Economics, University of Pretoria.

89. Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy, *American Economic Review*, 57(2):534-544.

90. Szëgo, G. (1915). Ein Grenzwertsatz uber die Toeplitzschen Determinanten einer reellen positiven Funktion, *Mathematische Annalen*, 76: 490-503.

91. Tödter, K-H. and Ziebarth, G. (1999). Price Stability versus Low Inflation in Germany: An Analysis of Costs and Benefits, in M. Feldstein, ed., The Cost and Benefit of Price stability Chicago, University of Chicago Press.

92. Truman, E.M. (2003). Inflation Targeting in the World Economy. Washington, DC: *Institute for International Economics*.

93. Tunali, D. (2008). Inflation Targeting and The sacrifice Ratio: The effect of Monetary Policy Design, Rutgers University.

94. Vega, M., and Winkelfried, D. (2005). Inflation Targeting and Inflation Behavior: A Successful Story?, *International Journal of Central Banking*, 1(3): 153-175.

95. Walsh, C.E. (2003). *Monetary Theory and Policy*, second Edition. Cambrige, Massachusetts, London, England: The MIT press.

96. Wang, P. and Yip, C.K. (1993). Real effects of money and welfare costs of inflation in an endogenously growing economy with transactions costs, *Research Paper 9311*. Research department, Federal Reserve Bank of Dallas.

97. Woglom, G. (2005). Forecasting South African Inflation, *South African Journal of Economics*, 73(2): 302-320.

98. Wu, Y. Zhang, J. (1998). Endogenous growth and the welfare cost of inflation: a reconsideration, *Journal of Economic Dynamics and Control*, 22(3):265-482.