# Assessing the algebraic problem solving skills of Grade 12 learners in Oshana Region, Namibia

By

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# DECLARATION

I declare that **Assessing the algebraic problem solving skills of Grade 12 learners in Oshana Region, Namibia** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

……. **15 June 2014**

**SIGNATURE** **DATE**

**(NHLANHLA LUPAHLA)**

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**ABSTRACT**

This study used Polya's problem-solving model to map the level of development of the algebraic problem solving skills of Grade 12 learners from the Oshana Region in Northern Namibia. Deficiencies in problem solving skills among students in Namibian tertiary institutions have highlighted a possible knowledge gap between the Grade 12 and tertiary mathematics curricula (Fatokun, Hugo & Ajibola, 2009; Miranda, 2010). It is against this background that this study investigated the problem solving skills of Grade 12 learners in an attempt to understand the difficulties encountered by the Grade 12 learners in the problem solving process. Although there has been a great deal of effort made to improve student problem solving throughout the educational system, there is no standard way of evaluating written problem solving that is valid, reliable and easy to use (Docktor & Heller, 2009).

The study designed and employed a computer aided algebraic problem solving assessment (CAAPSA) tool to map the algebraic problem solving skills of a sample of 210 Grade 12 learners during the 2010 academic year. The assessment framework of the learners' problem solving skills was based on the Trends in International Mathematics and Science Study (TIMSS), Schoenfeld's (1992) theory of metacognition and Polya's (1957) problem solving model. The study followed a mixed methods triangulation design, in which both quantitative and qualitative data were collected and analysed simultaneously. The data collection instruments involved a knowledge base diagnostic test, an algebraic problem solving achievement test, an item analysis matrix for evaluating alignment of examination content to curriculum assessment objectives, a purposively selected sample of learners' solution snippets, learner questionnaire and task-based learner interviews.

The study found that 83.8% of the learners were at or below TIMSS level 2 (low) of algebraic problem solving skills. There was a moderate correlation between the achievement in the knowledge base and algebraic problem solving test (Pearson $r = 0.5$). There was however a high correlation between the learners' achievement in the algebraic problem solving test and achievement in the final Namibia Senior Secondary Certificate (NSSC) examination of 2010 (Pearson $r = 0.7$). Most learners encountered difficulties in Polya's first step, which focuses on the reading and understanding of the problem. The algebraic strategy was the most successfully employed solution strategy.

**Keywords:** problem, problem solving, algebraic problem solving, solution strategies, mathematical proficiency, computer aided algebraic problem solving assessment (CAAPSA)

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**LIST OF ABBREVIATIONS**

| | | |
|---|---|---|
| **APS** | **-** | **Algebraic Problem Solving** |
| **CAAPSA** | **-** | **Computer Aided Algebraic Problem Solving Assessment** |
| **CAT** | **-** | **Computer Aided Testing** |
| **CIE** | **-** | **Cambridge International Examinations** |
| **CS** | **-** | **Computer Systems** |
| **DNEA** | **-** | **Directorate of National Examinations and Assessment** |
| **EPSEM** | **-** | **Equal Probability Selection Method** |
| **GRN** | **-** | **Government of the Republic of Namibia** |
| **HIGCSE** | **-** | **Higher International General Certificate in Secondary Education** |
| **IGCSE** | **-** | **International General Certificate in Secondary Education** |
| **KB** | **-** | **Knowledge Base** |
| **MFG** | **-** | **Mathematics Final Grade** |
| **NAEP** | **-** | **National Assessment of Educational Progress** |
| **NCTM** | **-** | **National Council of Teachers of Mathematics** |
| **NIED** | **-** | **National Institute for Educational Development** |
| **NSSC** | **-** | **Namibia Senior Secondary Certificate** |
| **PISA** | **-** | **Programme for International Student Assessment** |
| **PoN** | **-** | **Polytechnic of Namibia** |
| **TIMSS** | **-** | **Trends in International Mathematics and Science Study** |
| **UNAM** | **-** | **University of Namibia** |
| **UNISA** | **-** | **University of South Africa** |

# CHAPTER 1

# INTRODUCTION AND BACKGROUND

## 1.1 Introduction

In 2004 Namibia adopted Vision 2030, which is a document that clearly spells out the country's development programmes and strategies to achieve its national objectives. Vision 2030 focuses on eight themes to realise the country's long term vision. These are: (1) inequality and social welfare; (2) human resources development and institutional capacity building; (3) macro-economic issues; (4) population, health and development; (5) natural resources and environment management; (6) knowledge, information and technology; (7) factors of the external environment; and, (8) peace and political stability. The goal of the Vision 2030 is to improve the quality of life of the people of Namibia and to raise them to the level of their counterparts in the developed world by 2030. Capacity building will be pursued with the utmost vigour by both the private and public sectors to support the objectives of Vision 2030 (Government of the Republic of Namibia [GRN], 2004). Furthermore, the country aims to develop a totally integrated, unified, flexible and high quality education and training system that will prepare Namibian learners to take advantage of a rapidly changing global environment, including developments in science and technology (GRN, 2004).

This study views the development of learners' mathematical problem solving skills as essential for the accomplishment of the goals enshrined in these themes. Consequently, the mapping of problem solving skills is essential for monitoring the progress made towards the realisation of Vision 2030 goals.

## 1.2 Background to the research problem

The researcher studied the mathematics examiners' reports of the Namibia Senior Secondary Certificate (NSSC) examinations from 2010 to 2013. The objective was to find areas of poor learner performance with the ultimate goal to identify a possible area of research which would enhance learning in the classroom. The purpose of the examiners' reports is to provide feedback to teachers, learners, policy makers and other stakeholders on learner performance in the examination with recommendations on how any issues identified may be addressed (Directorate of National Examinations and Assessment [DNEA], 2010). A repeating theme in the examiners' reports was the learners' deficiency in algebraic work. The following are some notable extracts from the reports:

*"Learners should be encouraged to read and understand the requirements of the question before answering"* (DNEA, NSSC (O), 2013 p. 325)

*"The algebraic work was very poor for learners at this level"* (DNEA, NSSC (H), 2013 p. 209)

*"Learners struggled with topics involving algebra, trigonometry and sequences. Learners had difficulties with reading and understanding word problems. Learners should be encouraged to read and understand the requirements of the question before answering"* (DNEA, NSSC (O), 2012 p. 316)

In all the reports, algebra was echoed as a major deficiency in learners' work, in particular, reading and understanding the problems. According to Schoenfeld (1992), reading and understanding a problem is a metacognitive attribute in the problem solving process. According to Polya (1957), understanding the problem involves reading the information, identifying what to find, identifying the key conditions or finding important data, and examining the assumptions given in the problem. Without using an empirical tool to assess the learners' problem solving skills, the researcher found the examiners' statements on attributing the learners' poor performance in the algebraic problems to be merely judgemental. There is no standard way of evaluating written problem solving that is valid, reliable and easy to use (Docktor & Heller, 2009).

The examiners reports further suggest that many mathematics teachers are not teaching their learners to solve problems at the desired level of proficiency (DNEA, 2012). Teaching problem solving requires an agreed upon definition of what problem solving is and a satisfactory assessment tool so that learner progress in this domain can be assessed. Currently such a tool does not exist (Adams & Wieman, 2010). Although scoring criteria and rubrics have been used in problem solving research and instruction, these instruments are often difficult to use and have not been extensively tested (Adams & Wieman, 2010; Murthy, 2007; Ogilvie, 2007).

Zimba and Kasanda (2001) note that teachers and policy makers in Namibia have been encouraged to base their practice on empirical evidence, but decry the fact that most teachers are mere consumers of research findings and not producers of research knowledge. In such a

situation, little can be done to improve the quality of the learning of mathematics in our schools. Therefore, it is necessary to provide the teachers the necessary skills to do research and be encouraged to do it in their classrooms.

This study therefore developed and tested the effectiveness of a Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool, which hopefully will empower teachers to monitor the development of the algebraic problem solving skills of their learners.

Problem solving continues to draw interest within the mathematics education community. Although some attention has been focussed on the importance of problem solving in mathematics (National Council of Teachers of Mathematics [NCTM], 1989) and the difficulties problem solving poses to learners (Lester, 1994), research on mathematical problem solving however, has focused on narrow theoretical perspectives (McGinn & Boote, 2003). Fatokun, Hugo and Ajibola (2009) have observed that first year engineering students in Namibian tertiary institutions lacked basic problem solving skills which were supposed to be taught at the secondary school level. Therefore, there appears to be a knowledge gap between Grade 12 and tertiary level mathematics.

The current literature on problem solving has mostly concentrated on looking at the different kinds of problems, the learners' thinking processes in problem solving and the suitable teaching methods (School Achievement Indicators Programme [SAIP], 1997). Previous research, however, did not give particular focus on the levels of problem solving skills attained by learners. In addition to the assessment of algebraic problem solving skills this research also looked at the difficulties faced by Grade 12 learners in attaining the levels.

### 1.2.1 Localisation of Grade 12 Mathematics examinations in Namibia

In 2006, the International General Certificate in Secondary Education (IGCSE) and the Higher International General Certificate in Secondary Education (HIGCSE) examinations of the Cambridge International Examination (CIE) were replaced by the new Namibia Senior Secondary Certificate (NSSC) Ordinary and Higher level examinations respectively. The Grade 12 candidates of 2007 were the first group to write the new NSSC examinations.

The ordinary level mathematics syllabus makes provision for core and extended components of the syllabus, based on the level of competencies to be assessed in the examination. The

two syllabus components are assessed separately at the end of the Grade 12 school course. The core component assesses basic level competencies through routine short answer questions in domains such as proportion, solution of equations, mensuration, algebra, geometry and trigonometry. The extended component assesses intermediate level competencies through structured questions in the same domains as the core component, but includes non-routine algebraic problem solving questions in linear programming, arithmetic and geometric progressions. The national examiners' reports have frequently pointed out to poor performance of learners in answering the questions requiring algebraic solution processes. The higher level NSSC curriculum is an expansion of the NSSC Ordinary level (core and extended) components to include the assessment of competencies in polynomials (remainder and factor theorems), identities, equations and inequalities, vectors in three dimensions, logarithmic and exponential functions, absolute value, trigonometric identities, differentiation and integration. Very few learners manage to achieve the learning outcomes of the higher level NSSC examination and this is why this study focuses on the ordinary level only. In Namibia, currently only about 20% of Grade 12 learners enter for the NSSC higher level examinations (Directorate of National Examinations and Assessment [DNEA], 2012).

It is stated clearly in Namibia's Vision 2030 that Namibians are expected to become mathematically proficient (National Institute for Educational Development [NIED], 2009). Everyone should become confident about using numbers and be able to use them with understanding in their private and professional lives (NIED, 2005). The University of Namibia (UNAM) as well as the Polytechnic of Namibia (PoN) require a good pass (75% or more) in mathematics from prospective students of engineering, technology and natural sciences. The adoption of the new NSSC syllabus in 2006 was meant to address a goal expressed in Vision 2030:

> *By 2030 there needs to be a workforce of qualified Namibians who are able to apply mathematical knowledge. In line with such thinking, the localised mathematics syllabus aims to equip Namibian learners and students more effectively with meaningful mathematical knowledge and understanding* (NIED, 2005, p. 8).

This statement highlights the development of mathematical problem solving skills as an essential objective in the new NSSC curriculum.

### 1.2.2 Observed deficiencies in learners' problem solving skills in Namibia

The poor mathematics performance of Grade 12 learners has become a cause for concern within the Namibian education system. In the NSSC mathematics curriculum, problem solving has been identified as one of the eight skills essential for successful mathematics learning (National Institute for Educational Development [NIED], 2005). However, there is no specified learning approach to help learners develop sound problem solving skills; teachers do not incorporate any problem based learning approaches in teaching, and solving word problems receives little attention. Hence the learners are deprived of the opportunities for developing higher cognitive skills of analysis, interpretation and evaluation to use information effectively (NIED, 2009).

In the period from 2007 to 2011, the mean achievement for Grade 12 mathematics examinations, based on attainment of the required grades for admission to UNAM and Polytechnic of Namibia, was 30.6%. This mean attainment was obtained in the region where the current study was conducted. Currently in Namibia, about 20% of mathematics learners achieve the required mathematics entry grades to UNAM and Polytechnic of Namibia (Clegg, 2011). The Directorate of National Examinations and Assessment (DNEA, 2010) indicates that the performance of learners in mathematics is not satisfactory. Examiners' reports point to a deficiency in algebraic problem solving skills, which is the fundamental cause of learners' poor performance. In response to the desire to develop mathematics education, the Namibian Government resolved that mathematics would become a compulsory subject for all learners up to Grade 12 level as from the year 2012 (Mbumba, 2009). Fatokun, Hugo and Ajibola (2009) also noted that students enrolled in mathematics courses at tertiary level lacked algebraic representation, manipulation and problem solving skills, which should be mastered at the Grade 12 level. There is a knowledge gap between Grade 12 and the courses for first year engineering students at the University of Namibia (UNAM) and the Polytechnic of Namibia (PoN) and this poses an obstacle to meeting the goals of Vision 2030 (Fatokun, Hugo & Ajibola, 2009; Miranda, 2010).

**Table 1.1:** Summary of NSSC end-of-year results of Grade 12 learners in Oshana Region from 2007 to 2011

| School | Percentage A to C grades | | | | | Mean % A-C grades for 2007-2009 | Mean % A-C grades for 2007 to 2011 |
|---|---|---|---|---|---|---|---|
| | 2007 | 2008 | 2009 | 2010 | 2011 | | |
| A | 43.59 | 58.60 | 69.72 | 67.28 | 66.95 | 57.30 | 61.23 |
| B | * | 29.60 | 18.92 | 30.09 | 50.00 | 24.26 | 32.15 |
| C | 47.24 | 36.00 | 35.50 | 35.27 | 43.16 | 39.58 | 39.43 |
| D | 36.55 | 39.60 | 41.80 | 45.32 | 39.13 | 39.32 | 40.48 |
| E | 29.60 | 22.10 | 17.10 | 19.35 | 38.55 | 22.93 | 25.34 |
| F | 2.70 | 16.10 | 26.20 | 18.42 | 24.29 | 15.00 | 17.54 |
| G | 11.43 | 27.80 | 5.90 | 22.86 | 17.02 | 15.04 | 17.00 |
| H | 2.56 | 14.10 | 22.60 | 13.01 | 7.02 | 13.09 | 11.86 |
| Mean % | 24.81 | 30.49 | 29.72 | 31.45 | 35.77 | 28.32 | 30.63 |

*School B was only up to Grade 11 and did not write the NSSC examination in 2007.

Table 1.1 shows that there has not been a significant improvement in the performance of Grade 12 learners over a five-year period (2007 to 2011). The observed performance of learners has been attributed to the learners' lack of conceptual understanding as the teaching focuses more on procedures and rules than developing problem solving skills (DNEA, 2010; Fatokun, Hugo & Ajibola, 2009; Miranda, 2010). This study focused on mapping the algebraic problem solving skills of the Grade 12 learners in the Oshana region in the 2010 academic year, by appropriately matching each step in Polya's (1957) problem solving model to the learners' written solution process in an effort to determine and explain at which stage of Polya's (1957) steps learners encountered difficulties. The term "mapping", in this study is thus used with reference to association of the algebraic problem solving process to the achievement levels at each of Polya's (1957) problem solving steps.

### 1.3 Rationale and purpose of the study

The development of problem solving skills has been identified in previous international studies as a major factor influencing the development of overall mathematical proficiency among learners (Carson, 2007). The purpose of this study, therefore, is to gain an understanding of the possible causes of observed deficiencies in the algebraic problem solving skills of Namibian learners, and hopefully, the findings will provide answers to some of the questions surrounding the poor performance of learners in Grade 12 and the observed knowledge gap between Grade 12 curriculum and tertiary level mathematics courses. The

study seeks to map and reveal the obstacles encountered by Grade 12 learners in solving algebraic problems.

### 1.4 Research Questions

Given the reported low achievement of Grade 12 learners in the algebraic problem solving questions of the NSSC examinations (DNEA, 2010) and the fact that the common word problem solving activities outlined in the NSSC mathematics curriculum are algebraic in nature, the researcher pursued this study to investigate the state of problem solving skills of Grade 12 learners. The study will endeavour to investigate the level at which the algebraic problem solving skills of Grade 12 learners match the demands of the NSSC curriculum. The assessment framework of this study used the Computer Aided Algebraic Problem Solving Assessment (CAAPSA) benchmarks and scale, derived from the Trends in International Mathematics and Science Study (TIMSS) benchmarks, and Polya's (1957) model. The problem solving skills envisaged in the NSSC syllabus aims can be adequately measured through the algebraic problem solving achievement test that this study proposes. Therefore the study will address the question: **What is the state of algebraic problem solving skills of Grade 12 learners in Oshana region?** The study will specifically attempt to answer the following research questions:

1. To what extent is the assessment of algebraic problem solving skills reflected in the content of the NSSC Grade 12 ordinary level mathematics examinations?
2. What is the level of the algebraic problem solving skills of Grade 12 learners in Oshana region?
   (i) What is the correlation between the knowledge base and the algebraic problem solving skills of Grade 12 learners?
   (ii) What is the correlation between algebraic problem solving skills and examination achievement of Grade 12 learners?
3. What problem solving strategies do Grade 12 learners in Oshana Region use when solving algebraic problems?
4. What challenges or difficulties, if any, do Grade 12 learners in Oshana encounter when attempting to solve algebraic problems?

## 1.5 Significance of the study

Research on mathematical problem solving has not accumulated (English, Lesh & Fennewald, 2008). Mathematics education researchers have generally avoided tasks that involve developing critical tools for their own use. Mathematics educators have developed very few tools for observing, documenting, or measuring most of the understandings and abilities that are believed to contribute to problem solving expertise (English et al., 2008).

The results of this study might provide evidence on the significance of Polya's (1957) model in mapping the algebraic problem solving ability in the Namibian senior secondary school mathematics curriculum. It is hoped that the developed Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool will provide a guideline for teachers who want to identify the level and the state of algebraic problem solving ability among learners using Polya's (1957) problem solving steps to map the learners' difficulties in the process.

Subsequently, using Polya's model to map the learners' algebraic problem solving process might also be the means to evaluate the development of the learners' strands of mathematical proficiency, namely; (1) conceptual understanding, (2) procedural fluency, (3) strategic competence and (4) adaptive reasoning, as defined by Kilpatrick, Swafford & Findell (2001). Although the Namibia Senior Secondary curriculum documents do not explicitly give a definition of mathematical proficiency, the learning aims of the syllabus indicate a curriculum design inclined towards the development of Kilpatrick's (2001) strands of mathematical proficiency. For example, the NSSC syllabus aims prioritise the development of learners' problem solving skills and critical and creative thinking skills (NIED, 2005). The researcher believes that the attainment of such skills translates to the attainment of Kilpatrick's strands of mathematical proficiency.

Perhaps even more significant is that there are few studies conducted in Namibia, on the assessment of learning algebra (NIED, 2009). Some research has been carried out on mathematical preparedness of grade 11 learners in a selected Namibian school in Oshana region (Mwandingi, 2010) and teachers' perceptions of spreadsheet algebra algorithms (Losada, 2012). However, none of these mentioned works have addressed the question of how to assess algebraic problem solving approaches. There is need for Namibian educators to carry out more studies to identify the competencies that might be lacking in the school mathematics curriculum (NIED, 2009). There are currently few studies, reports and documents in Namibia that cite the current problems in Mathematics performance (NIED,

2009). This study is the first of its kind to map the problem solving process of learners from their written work, using a Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool.

## 1.6 Limitations of the study

The study did not ascertain whether the actual classroom teaching practices in the region where the research was conducted complied with the stipulated NSSC curriculum objective of incorporating problem solving tasks. Therefore the possibility that the development of the learners' problem solving skills could have been influenced by varying teaching practices was not eliminated. The following were other limitations of the study:

- The unavailability of previous studies in Namibia on the assessment of the level of development of problem solving skills. This led to the use of literature and research findings from international studies to analyse Namibian issues.
- The study is limited to only one region in Namibia owing to resource limitations. It was convenient for the researcher to conduct the study in the Oshana region because this is where the researcher's duty station is located.

The scarcity of recent literature (less than ten years old) was also another limitation of this study. Problem solving research somewhat faded away in the early twenty-first century. It received a great deal of attention in the 1970s and 1980s (Schoenfeld, 2007). Consequently this study also made use of old, but credible literature in the literature review section.

## 1.7 Definition of terms

The following definitions are applicable to the current study:

### 1.7.1 Problem

A problem is a task for which an individual does not immediately know what to do to get to the answer. A person is confronted with a problem when he wants to attain a goal but does not know immediately what series of actions to perform. For example, given that "*in a group of cows and chickens, the number of legs is 14 more than the number of heads*", it becomes a problem when one is asked to find the number of cows in the group. It requires a series of cognitive actions to get the answer.

### 1.7.2 Problem Solving

Problem solving is simply what one does to achieve a given goal. Problem solving is an activity requiring an individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill.

### 1.7.3 Algebraic Problem Solving

Algebraic problem solving is problem solving using algebraic models. An algebraic model is a mathematical statement using numbers, variables and operations. Mathematical statements include algebraic expressions and algebraic sentences.

In order to model a situation with an algebraic model:

- Relate all parts of the problem and define one or more variables to represent the unknowns;
- Write an algebraic model (equation) that represents the given relationships between the unknowns and the given data;
- Solve the equation(s); and,
- Check whether your answer makes sense by interpreting the solution(s) in terms of the context of the problem.

It is however possible for learners to solve algebraic problems without creating algebraic expressions that contain variables.

### 1.7.4 Mathematical proficiency

Mathematics proficiency refers to successful mathematics learning (Kilpatrick et al., 2001) According to Kilpatrick et al. (2001) the attainment of mathematical proficiency requires; (a) comprehension of mathematical concepts, operations, and relations, (b) skill in carrying out procedures flexibly, accurately, efficiently, and appropriately, (c) ability to formulate, represent and solve mathematical problems, (d) capacity for logical thought, reflection, explanation, and justification, and (e) habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## 1.8 Overview of the Dissertation

The outline of the dissertation is as follows:

### 1.8.1 Chapter One

This chapter provides an introduction to the research presented in this study. This includes the background to the study, the rationale, the purpose of the study, and the research questions which guided it. The chapter further highlights the significance and limitations of the study and provides an overview of the whole dissertation.

### 1.8.2 Chapter Two

This chapter deals with the literature review, starting with an overview of the Namibia Senior Secondary Certificate curriculum. The chapter also reviews some of the theories of mathematical problem solving and discusses the findings of previous research on the problem solving process and the importance of the development of learners' problem solving skills. The last section of the chapter discusses the conceptual frameworks used in this study to evaluate and assess the problem solving skills of the Grade 12 learners who participated in the study.

### 1.8.3 Chapter Three

This chapter outlines the research methodology for the study. It also discusses the research design, a triangulation mixed methods design that employs both quantitative and qualitative methods. It describes the research site and participants as well as the sampling techniques used. The chapter further explains the data collection process and the instruments used to collect the data. The validity and reliability of the instruments and ethical issues taken into account during the process of conducting the research are also discussed in Chapter 3.

### 1.8.4 Chapter Four

The chapter presents the data and the analysis. The chapter also discusses in detail the input, data processing and output of the developed Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool, based on Polya's model and the CAAPSA assessment framework.

### 1.8.5 Chapter Five

This chapter comprises of the discussion of the findings of the study.

### 1.8.6 Chapter Six

Chapter 6 provides the conclusion, recommendations and a summary of the study. The chapter also suggests avenues for further study.

## 1.9 Summary

The study's orientation was established in this chapter. The background, the rationale, the significance of the study and the research questions were presented and discussed. Due to the analytical nature of the study, 25 tables and 67 figures were used to present and to analyse data. Due to the large volume of tables and figures, the study has not listed them in the contents section, and the researcher believes that this omission will not affect the reader following the issues discussed.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.1 Introduction

This chapter presents a review of literature related to existing theoretical views in teaching and assessment of mathematical problem solving. The literature review also presents views on algebraic problem solving instruction. This study was carried out with the purpose of filling a gap in research about problem solving assessment; by designing an assessment tool to map the level of development of problem solving skills of Grade 12 mathematics learners in the Namibia Senior Secondary School Certificate (NSSC) curriculum. The chapter is divided into three parts:

**Part I** An overview of the Namibia Senior Secondary Certificate (NSSC) mathematics curriculum is presented and discussed to lay out the emphasis that the NSSC curriculum attaches to the development of problem solving skills. This section also provides a historical and theoretical account of the development of research in problem solving. The theories that influenced the context and orientation of the study are also discussed in this section.

**Part II** comprises an analysis of some problem solving models that have featured prominently in research on the development and assessment of mathematical problem solving skills. The section also specifically presents theoretical frameworks that have been used in previous studies to evaluate the problem solving process.

**Part III** This section looks at some findings of previous studies in order to locate the current study in mathematics education research. This section also sets out the conceptual framework of this study.

## 2.2 The problem solving themes in the Namibian Mathematics Curriculum

The formal school system in Namibia is broken down into the following levels:

- four years of Lower Primary (Grades 1-4);
- three years of Upper Primary (Grades 5-7);
- three years of Junior Secondary (Grades 8-10); and,
- two years of Senior Secondary (Grades 11-12).

### 2.2.1 Problem solving in the primary school phases

According to the Namibian mathematics curriculum, the teaching of mathematical problem solving commences at Grade 3 level (NIED, 2005). In the lower primary phase, learners should be able to solve story problems that relate to situations using the four operations (+, −, ÷, ×) with two digit numbers by employing and explaining any logical strategy (NIED, 2005). In the upper primary phase, the syllabus indicates that most of the topics should culminate in word problem solving activities.

### 2.2.2 Problem solving in the secondary school phases

The Junior Secondary phase covers topics in mathematics and in additional mathematics. In Grade 8, all learners are offered mathematics at the same level and all are assessed in the same competencies and skills. In Grade 9 and Grade 10, learners have the choice between studying mathematics (core) or additional mathematics (extended / higher level). The additional mathematics syllabus provides a sound foundation for those learners who want to continue with mathematics in the Namibia Senior Secondary Certificate on the Higher Level component of the NSSC curriculum. One of the competency outcomes for this phase is that learners should develop the ability to solve number problems in a variety of contexts. However, the syllabus does not specify any problem solving activities related to this competency.

Currently, there are three different syllabuses for mathematics in the Senior Secondary phase, that is:

- Namibia Senior Secondary Certificate Ordinary (NSSCO) level mathematics, core component;
- Namibia Senior Secondary Certificate Ordinary (NSSCO) level mathematics, extended component; and,
- Namibia Senior Secondary Certificate Higher (NSSCH) level mathematics component.

Candidates who follow the core curriculum and write the relevant examinations are eligible for the award of grades C to G only. Candidates who follow the extended curriculum are eligible for the ward of grades A to E only. On the other hand, the candidates who write higher level curriculum are eligible for grades 1 to 4. It is recommended that learners who

obtain grades A to C (75% and above) in grade 10 pursue mathematics at the higher level in the Senior Secondary phase (NIED, 2009). Table 2.1 outlines the points allocated to the possible grades in the NSSCO and NSSCH mathematics examinations. The examinations in these three syllabus components are completely different and the level of questions also differs. The grading of NSSCO and NSSCH is shown in Table 2.1.

**Table 2.1:** The grading scales used in the NSSCO and NSSCH mathematics examinations

| NSSCO Core | | Range | NSSCO Extended | | Range | NSSCO Higher | | Range |
|---|---|---|---|---|---|---|---|---|
| Grade | Points | % | Grade | Points | % | Grade | Points | % |
| C | 5 | 75-100 | A | 7 | 85-100 | 1 | 9 | 75-100 |
| D | 4 | 62-74 | B | 6 | 75-84 | 2 | 8 | 60-74 |
| E | 3 | 50-61 | C | 5 | 65-74 | 3 | 7 | 50-59 |
| F | 2 | 42-49 | D | 4 | 55-64 | 4 | 6 | 40-49 |
| G | 1 | 35-41 | E | 3 | 45-54 | U | 0 | 0-34 |
| U | 0 | 0-34 | U | 0 | 0-34 | | | |

### 2.2.3 Assessment objectives for the NSSCO core and extended components

In both the NSSCO and NSSCH syllabuses, the following skills are among those that should be developed:

- Communication skills
- Numeracy skills
- Information skills
- Problem-solving skills
- Work and study skills
- Critical and creative thinking skills.

The NSSCO and NSSCH syllabuses thus depict curricula inclined towards developing problem solving skills.

According to the NSSC assessment objectives outlined in the syllabus (NIED, 2005), the abilities to be assessed in the NSSC mathematics ordinary level examinations cover a single assessment objective, technique with application. The examination will test the ability of candidates to:

1. Organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms;
2. Perform calculations by suitable methods;
3. Use an electronic calculator;
4. Understand systems of measurement in everyday use and make use of them in the solution of problems;
5. Estimate, approximate and work to degrees of accuracy appropriate to the context;
6. Use mathematical and other instruments to measure and draw to an acceptable degree of accuracy;
7. Interpret, transform and make appropriate use of mathematical statements expressed in words or symbols;
8. Recognise and use spatial relationships in two and three dimensions, particularly in solving problems;
9. Recall, apply and interpret mathematical knowledge in the context of everyday situations;
10. Make logical deductions from given data;
11. Recognise patterns and structures in a variety of situations;
12. Respond to a problem relating to relatively unstructured situations by translating it into an appropriately structured form;
13. Analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution;
14. Apply combinations of mathematical skills and techniques in problem solving; and,
15. Set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology.

A rigid association between particular assessment objectives and individual examination components is not appropriate since any of the objectives can be assessed in any question. Nevertheless, the components of the scheme differ in the emphasis placed on the various objectives. A difference in emphasis is apparent between Core and Extended papers; for example, the assessment of candidates' responses to relatively unstructured situations (Objective 12) is particularly emphasised in the extended component. The short-answer questions fulfil a particularly important function in ensuring syllabus coverage and allowing the testing of knowledge, understanding and manipulation skills, while greater emphasis is placed on applications to the processes of problem solving in the structured answer papers.

## 2.3 Overview of some of the theories which influenced the context and orientation of this study

### 2.3.1 The development of mathematical problem solving skills

Problem solving is considered the most significant cognitive activity in everyday and professional environments (Jonassen, 2000). Most mathematics educators agree that the development of learners' problem solving abilities is a primary objective of instruction. Problem solving should be both an end result of learning mathematics and the means through which mathematics is learned (DiMatteo & Lester, 2010; Stein, Boaler, & Silver, 2003). An attribute which is considered integral to the problem solving process is strategic behaviour (Polya, 1957; Schoenfeld 1992). Numerous studies in mathematics education (e.g., Pape & Wang, 2003; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts & Ratinckx, 1999), consider strategy use central to processing mathematical problems. Success in solving a mathematical problem is positively related to the learners' use of problem solving strategies (Cai, 2003). In mathematics, learners continuously face new situations and new problems which require them not only to know and apply various strategies, but also to be flexible (Baroody, 2003; Silver 1997). The implication of this assertion is that what learners have learned in one situation and what applies to one problem, will not necessarily fit another situation or be appropriate for another problem. In the mathematics education community, considerable research has been devoted on strategy flexibility related to arithmetic concepts and skills. Less attention has been devoted to the study of flexibility in using heuristic strategies in mathematical non-routine problem solving (Kaizer & Shore, 1995). More information is needed to understand how flexibility in using heuristic strategies occurs in non-routine problem solving and how it is associated with performance.

The term "strategies" refers to problem solving strategies or heuristics such as drawing a picture, making a list or a table, guessing and checking or writing an equation (Polya, 1957; Schoenfeld, 1992; Verschaffel et al., 1999). According to Demetriou (2004), flexibility refers to the quantity of variations that can be introduced by a learner in the concepts and mental operations that the learner already possesses. Krems (1995) defines cognitive flexibility as a person's ability to adjust the problem solving process through modification of the solution strategies to meet the demand of the problem. A good problem solver constantly questions his or her achievement. The problem solver generates a number of possible strategies of solution, but is not seduced by them. By making careful moves such as pursuing productive leads and abandoning fruitless paths, the problem solver succeeds to solve the problem (Schoenfeld,

1985). Several mathematical problem solving strategies can be introduced in primary or middle secondary school mathematics, such as: guess-check-revise, draw a picture, act out the problem, use objects, choose an operation, solve a simpler problem, make a table, look for a pattern, make an organised list, write an equation, use logical reasoning, and work backwards (Lester & O'Daffer, 1992). It is in this backdrop that the current study assessed the development of the algebraic problem solving skills of the Grade 12 learners in the Oshana region.

The researcher hopes that the implementation of the CAAPSA tool will lessen the demand and complexity of teachers' classroom assessment of problem solving skills, hence improve the effectiveness of student assessment of algebraic problem solving skills.

Problem solving was the theme of the 1980s. The decade began with the National Council of Teachers of Mathematics (NCTM)'s statement that:

*"Problem solving must be the focus of school mathematics"* (NCTM, 1980, p.1).

There is a general acceptance of the idea that "the primary goal of mathematics instruction should be to produce students who are competent problem solvers" (Schoenfeld, 1992, p.3).

According to the themes in the NCTM Standards (NCTM, 1989) and Reshaping School Mathematics (National Research Council, 1990),

*Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond the rules to be able to express things in the language of mathematics. This transformation suggests changes both in curricular content and instructional style. It involves renewed effort to focus on:*

- *Seeking solutions, not just memorising procedures;*
- *Exploring patterns, not just memorising formulas;*
- *Formulating conjectures, not just doing exercises.*

Thus the current study sought to explore the algebraic problem solving strategies employed by learners, by administering a written achievement test.

### 2.3.2 The eight elements of the theory of problem solving

Algebra is a complex human activity that requires coordination of several cognitive abilities, including visual processing (for parsing the equation), declarative memory (for storing and retrieving arithmetic knowledge) and visual imagery (for updating and manipulating intermediate and partial representations of the equation). It is also a convenient experimental task, since the solution path can be perfectly characterized, and participants are extensively trained in solving algebraic problems with the same algorithm, repeating the same sequence of problem-solving steps (Anderson, 2005).

Problem solving theory and practice suggest that thinking is more important in solving problems than knowledge and that it is possible to teach thinking in situations where little or no knowledge of the problem is needed (Schoenfeld, 1992). Such an assumption has led problem solving advocates to pursue *content-less heuristics* as the primary element of problem solving, while relegating the *knowledge base* and the *application of concepts* or *transfer* to secondary status. Carson (2007) argues that the knowledge base and transfer of knowledge, not content-less heuristic, are the most essential elements of problem solving.

Problem solving is only one of a larger category of thinking skills that teachers use to teach students how to think. Other means of developing thinking skills are problem based learning, to develop learners' critical thinking skills, creative thinking skills, decision making, conceptualising, and information processing (Ellis, 2005). Although scholars and practitioners often imply different meanings by each of these terms, most thinking skills programmes share the same basic elements, which are:
(1) the definition of a problem, (2) the definition of problem solving, (3) algorithms, (4) heuristics, (5) the relationship between theory and practice, (6) teaching creativity, (7) a knowledge base, and (8) the transfer or the application of conceptual knowledge.

#### 2.3.2.1 The definition of a problem

The first element of the theory of problem solving is knowing the meaning of the word *problem*. This theoretical framework uses the definition of problem presented by Krulick and Rudnick (1980). A problem is "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining a solution" (p. 3). Krulick and Rudnick (1996) list three conditions which determine whether a situation is a problem:

- The first condition is that the individual must be able to achieve a clear goal as a resolution to the situation;
- Secondly, there must be obstacles to achieving the goal; and,
- Thirdly, the first two conditions will compel the individual to explore methods to overcome the obstacles to reach the goal.

#### 2.3.2.2 The definition of Problem solving

Problem solving can be regarded as a situation in which an individual is responding to a problem that may not be solved by using routine or familiar procedures (Schoenfeld, 1985). Krulick and Rudnick (1980) also define problem solving as the means by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation. The student must synthesise what was learned, and apply it to a new and different situation (Krulick & Rudnick, 1980). This definition is similar to that of the eighth element of problem solving, transfer: "[w]hen learning in one situation facilitates learning or performance in another situation" (Ormrod, 1999, p. 348).

#### 2.3.2.3 Problem Solving is Not an Algorithm

One of the primary principles of the problem solving framework is that problem solving is not an algorithm. For example, Krulick and Rudnick (1980) say the existence of a problem implies that the individual is confronted by something he or she does not recognise, and to which he or she cannot merely apply a model. A problem will no longer be considered a problem once it can easily be solved by algorithms that have been previously learned.

#### 2.3.2.4 Problem solving is a Heuristic

Advocates of problem solving argue that educators need to teach a method of thought that does not pertain to specific or pre-solved problems or to any specific content or knowledge. A heuristic is this kind of method. It is a process or set of guidelines that a person applies to various situations. Heuristics do not guarantee success as an algorithm does (Krulick & Rudnick, 1980; Ormrod, 1999), but what is lost in effectiveness is gained in utility. Three examples of heuristic problem solving methods are presented in Table 2.2.

**Table 2.2:** Examples of problem solving heuristics

| | **Dewey (1933)** | **Polya (1957)** | **Krulick and Rudnick (1980)** |
|---|---|---|---|
| Steps in Problem Solving | Confront Problem | Understanding the Problem | Read |
| | Diagnose or Define Problem | Devising a Plan | Explore |
| | Inventory Several Solutions | Carrying out the Plan | Select a Strategy |
| | Conjecture Consequences of Solutions | Looking Back | Solve |
| | Test Consequences | | Review and Extend |

The first heuristic problem solving method belongs to Dewey (1933). The second problem solving heuristic method is Polya's (1957), whose method is mostly associated with problem solving in mathematics. The last is a more contemporary version developed by Krulick and Rudnick (1980), in which they explicate what should occur at each stage of problem solving. However, the three heuristic problem solving methods are fundamentally the same. The current study will explain Polya's (1957) model because his method is the one mostly associated with problem solving in mathematics.

#### 2.3.2.5 Problem Solving Connects Theory and Practice

A perennial charge brought against education is that it fails to prepare students for the real world. It teaches theory but not practice. Problem solving connects theory and practice. In a sense, this element is the same as the definitions of problem solving and transfer, only it specifically relates to applying *abstract* school knowledge to *concrete* real world experiences (Krulick & Rudnick, 1980).

#### 2.3.2.6 Problem Solving Teaches Creativity

Real world situations require creativity. However, it has often been claimed that traditional classrooms or teaching approaches do not focus on developing the creative faculty of students. Advocates of problem solving, by contrast, claim that problem solving develops the students' creative capacities (Frederiksen, 1984; Slavin, 1997).

#### 2.3.2.7 Successful Problem Solvers Have a Complete and Organised Knowledge Base

A knowledge base consists of all the specific knowledge a student can use to solve a given problem. For example, in order to solve algebraic problems, one does not only need to possess information about numbers and how to add, subtract, multiply, and divide, but also possess knowledge that goes beyond basic arithmetic. A knowledge base is what must accompany the teaching of a heuristic model for successful problem solving to occur.

#### 2.3.2.8 Problem Solving Teaches Transfer or How to Apply Conceptual Knowledge

Transfer, or the application of conceptual knowledge, is the connecting of two or more real-life problems or situations because they share the same concept or principle. Transfer or the application of conceptual knowledge helps students to see similarities and patterns in seemingly different problems that are in fact the same, or similar, on the conceptual level. Some research about problem solving claims that it is more effective than traditional instruction in which learners are just taught to memorise algorithms (Lunyk-Child, Crooks, Ellis, Ofosu, O'Mara & Rideout, 2001; Stepien, Gallagher & Workman, 1993), that it results in better long-term retention than traditional instruction (Norman & Schmidt, 1992), and that it promotes the development of effective thinking skills (Gallagher, Stepien, & Rosenthal, 1994; Hmelo & Ferrari, 1997).

On the other hand, Ellis (2005) notes that the research base on problem solving lacks definition, has measurement validity problems and questionable causality, and fails to answer the claim that successful problem solvers must have a wealth of content-specific knowledge. Ellis (2005) notes further that there is "no generally agreed-on set of definitions of terms" (p. 109), that thinking skills are notoriously difficult to measure, and that given these first two problems, it is impossible to trace cause back to any specific set of curricular instances. Ellis (2005) states that "the idea that thinking skills are content specific and cannot be taught generically must be seriously entertained until it is discredited. We don't think that will happen. And if this is so, how does one construct content-free tests to measure thinking skills?" (p. 109–110).

The conclusions of Ellis (2005) and other research studies are that it would be impossible to reinvent solutions to every problem that develops without recourse to past knowledge. Such recourse is evidence in itself that one must not completely construct reality. One must apply knowledge that has already been constructed by others and understand that knowledge, or

else not solve the problem. It is this critique that the current study will hopefully invoke in the following treatment of problem solving. What the study hopes to show is that the heuristics for problem solving cannot be successful if one holds strongly to the theoretical framework in which it is often situated. Rather, one must accept that already formed knowledge is essential to problem solving. In fact, the meanings of problem solving found in articles and textbooks often convey this contradiction (Carson, 2007). On one hand, it is argued that problem solving is the antithesis of a content-centred curriculum. On the other hand, a successful problem solver must possess a strong knowledge base of specific information, not merely a generalisable heuristic model that can be applied across several different situations.

The main difficulty with problem solving lies in the fourth element listed above: problem solving is a heuristic (Carson, 2007). Heuristics are guidelines that may or may not yield success but, unlike an algorithm, do not depend on knowledge of the problem to be successful. Heuristics is a method of thought that does not pertain to any specific problem or content. The element is problematic because it contradicts three other elements within the theory: the definition of problem solving; successful problem solving requires a knowledge base*;* and problem solving enables learners to transfer knowledge. Each of these three elements implies that previously learned knowledge of the problem is essential to solving the problem, whereas use of a heuristic approach assumes no knowledge is necessary. Carson (2007) argues, like Peikoff (1985), that it is not possible to separate thinking or problem solving from knowledge. Just like instruction and curriculum, these concepts inform one another and cannot be discussed separately for long. Likewise, to acquire knowledge, one must think. This is not to say that students cannot construct knowledge as they solve a given problem, but rather that often the problems they are presented with, require them only to apply existing knowledge. From this perspective, it must be assumed that students do not construct all the knowledge in a given curriculum. Yet *problem solving as a heuristic* is the most cherished aspect of problem solving *because* it is content-less. For example, Polya (1957), noted,

> *"I wish to call heuristic, the study of means and methods of problem solving. The term heuristic, which was used by some philosophers in the past, is half-forgotten and half-discredited nowadays, but I am not afraid to use it. In fact, most of the time the present work offers a down-to-earth practical aspect of heuristics." (p. vi.)*

Instructional textbooks sometimes play off this process versus content dichotomy: a teacher can either teach students to be critical thinkers and problem solvers or a teacher can teach students more content knowledge. Gunter, et al. (2003) concluded that:

> *"Too often children are taught in school as though the answers to all the important questions were in textbooks. In reality, most of the problems faced by individuals have no easy answers. There are no reference books in which one can find the solution to life's perplexing problems"* (Gunter, Estes, & Schwab, 2003, p. 128–129).

The dichotomy implies that thinking and knowledge are mutually exclusive, when in fact critical thinking and problem solving require a great deal of specific content knowledge. Problem solving and heuristics cannot be content-less and still be effective. Critical thinking, problem solving and heuristics must include a knowledge base (Fredriksen, 1984; Ormrod, 1999). Including the knowledge base enables the principal cognitive function of problem solving - the application of conceptual knowledge, or transfer - to occur (Peikoff, 1985). However, the degree to which Dewey (1933) and Polya (1957) actually believed that heuristics could be completely content-less and still be effective is not clear. Further, many instructional textbooks actually stress the importance of content knowledge in solving problems (Henson, 2004; Kauchak & Eggen, 2007; Lang & Evans, 2006).

### 2.3.3 Review of the eight elements of problem solving

Carson (2007) reviewed each of the above elements of problem solving again, in light of the relationship between thinking and knowledge and the research base on problem solving. Element one, the *definition of a problem*, implies that one must have some knowledge of the problem to solve it. How can one solve a problem without first knowing what the problem *is*? In Krulick and Rudnick's (1980) problem solving model (see Table 2.2), identification of the problem is what is called for in the first two steps, *Read* and *Explore*, of the heuristic approach. In the steps, *read* and *explore*, the student first becomes *aware* of the problem and then seeks to *define* what it is or what the problem requires for its solution. Awareness and definition comprise the *knowledge* that is essential to solving the problem. Consider the effectiveness of students relative to their respective experiences with a given problem. The student more familiar with the problem will probably be better able to solve it. In contrast, the student new to the problem, who has only studied the heuristic approach, would have to reinvent the solution to the problem. So the first two steps of the heuristic approach imply

that one needs a great deal of knowledge about the problem to be an effective problem solver. In fact, if one wants to solve the problem for the long term, one would want to study the problem thoroughly until some kind of principle was developed with regard to it. The final outcome of such an inquiry, ironically, would yield the construction of an algorithm.

The second element, the *definition of problem solving*, also implies a connection between thinking and knowledge. It suggests that problem solving is essentially applying old knowledge to a new situation (Krulick & Rudnick, 1988). However, if knowledge or a problem is genuinely new, then the old knowledge would not apply to it in any way. Ormrod (1999) suggests that the so-called new situation is really the same as the old in principle. For example, the principle of addition a student would use to solve the problem $1 + 2 = 3$ is essentially the same principle one would apply to $1 + x = 3$. The form may be different but ultimately the same principle is used to solve both problems. If this is the case, then a more appropriate element of problem solving would be number eight, the *transfer of knowledge or application of conceptual knowledge*.

The third and fourth elements, which are ***algorithms*** and ***heuristics,*** are problematic. Krulick and Rudnick (1980) distinguish between these two elements. Unlike employing an algorithm, using heuristics requires the problem solver to think on the highest level and to fully understand the problem. Krulick and Rudnick (1980) also prefer heuristics to algorithms because the latter applies only to specific situations, whereas heuristics applies to many as yet undiscovered problems. However, an algorithm requires more than mere memorisation; it requires deep thinking, too. First, in order to apply an algorithm, the student must have sufficient information about the problem to know which algorithm to apply. This would only be possible if the student possessed a conceptual understanding of the subject matter. Further, even if a student could somehow memorise *when* to apply certain algorithms, it does not follow that the student would also be able to memorise *how* to apply them (Hu, 2006; Hundhausen & Brown, 2008; Johanning, 2006; Rusch, 2005). Second, algorithms and problem solving are related to one another. Algorithms are the product of successful problem solving and to be a successful problem solver one often requires knowledge of algorithms (Hu, 2006; Hundhausen & Brown, 2008; Johanning, 2006; Rusch, 2005). Algorithms exist to eliminate needless thought, and in this sense, they are actually the end product of heuristics. The necessity to teach heuristics exists, but heuristics and algorithms should not be divided and set against one another. Rather, teachers should explain their relationship and how both are used in solving problems. A secondary problem that results from this flawed dichotomy

between algorithms and heuristics is that advocates of problem solving prefer heuristics because algorithms apply only to *specific* situations, whereas heuristics does not pertain to any specific knowledge. If one reflects upon the steps of problem solving listed in Polya's (1957) model, one will see that they require one to understand the problem to be successful at solving it.

If one knows the heuristic process but possesses no background knowledge of similar problems, one would not be able to solve the problem. For example, in the first step of Krulick and Rudnick's (1980) heuristic approach the student is supposed to *Read* the problem, identify the problem, and list key facts of the problem. Without a great deal of specific content knowledge, how would a student know what the teacher means by "problem," "key facts," and so on? The teacher would probably have to engage with the student in several problems. Without extensive knowledge of facts, how does the student know what mathematical facts are, and how they apply to word problems, for example? In the second step, *Explore*, the problem solver looks for a pattern or identifies the principle or concept. Again, how can one identify the pattern, principle, or concept without already possessing several stored patterns, principles and concepts? Indeed, for a student with very little mathematical knowledge, this problem would be extremely difficult to solve. The heuristic approach would be of little help.

The same is true of step five, *Review and Extend*. Presumably, if a student can represent the problem in algebraic form, the student should also be able to solve the problem without recourse to drawing diagrams, recording data, etc. One could simply solve the problem right after step one.

Advocates of problem solving are not solely to blame for the misconception between thinking and knowledge and between heuristics and algorithms. The misconception is more likely a result of teachers having overused algorithms without showing students how they are formed, explaining that they come from heuristics and that one should have a conceptual understanding of when they should be used, not merely a memorised understanding of them. The fundamentally flawed dichotomy within problem solving probably stems from thinking in terms of "either-ors." One side defines appropriate education as teaching algorithms by having students memorise when to use them, but not why. The other side, in contrast, emphasises that thinking for understanding is preferable to simply memorised knowledge.

Perhaps what has happened in the shift from the former to the latter practice is a shift in instructional emphasis from content to thinking, so much so that the knowledge base has been wiped out in the process. Ironically, eliminating knowledge from the equation also eliminates the effectiveness of problem solving. The dichotomy between knowledge and thinking has also affected elements five and six.

Number five states that problem solving *connects theory and practice*. At the core of this element is yet another flawed dichotomy. Many educators hold that education should prepare students for the real world by focusing less on theory and more on practice. However, dividing the two into separate cognitive domains that are mutually exclusive is not possible. Thinking is actually the integration of theory and practice, the abstract and the concrete, the conceptual and the particular. Theories are actually only general principles based on several practical instances. Likewise, abstract concepts are only general ideas based on several concrete particulars. Dividing the two is not possible because each implies the other (Lang & Evans, 2006). Effective instruction combines both theory and practice in specific ways. When effective teachers introduce a new concept, they first present a perceptual, concrete example of it to the student. By presenting several such examples to the student, the concept is better understood because this is in fact the sequence in which humans form concepts (Ormrod, 1999; Peikoff, 1993). They begin with two or more concrete particulars and abstract from them the essential defining characteristics into a concept.

On the other hand, learning is not complete if one can only match the concept with the particular example of it that the teacher has provided. A successful student is one who can match the concept to the as yet unseen examples, or one who can present an example that the teacher has not presented. The dichotomy between theory and practice also seems to stem from the dichotomous relationship between teaching for content-knowledge and teaching for thinking. The former is typically characterised as teaching concepts out of context, without a particular concrete example to experience through the five senses. The latter, however, is often characterised as being too concrete. Effective instruction integrates both the concrete and the abstract but in a specific sequence. First, new learning requires specific, real problems. Second, from these concrete problems, the learner forms an abstract principle or concept. Finally, the student then attempts to apply that conceptual knowledge to a new, never before experienced problem (Ormrod, 1999; Peikoff, 1993). The theory versus practice

debate is related to problem solving because problem solving is often marketed as the integration of theory and practice.

Element six, *problem solving teaches creativity*, is also problematic. To create is to generate the new, so one must ask how someone can teach another to generate something new. Are there specific processes within a human mind that lead to creative output that can also be taught? The answer would depend at least in part on the definition of the word *create*. When an artist creates, he or she is actually re-creating reality according to his or her philosophical viewpoint, but much, if not all, of what is included in the creation is not a creation at all but an integration or an arranging of already existing things or ideas. So in one sense, no one creates; one only integrates or applies previously learned knowledge. No idea is entirely new; it relates to other ideas or things. The theory of relativity, for example, changed the foundational assumptions of physics, but it was developed on the basis of ideas that already existed. There may be no such thing as pure creativity, making something from nothing. What seems like creativity is more properly transfer or the application of concepts, recognising that what appears to be two different things is really the same thing in principle. On the other hand, it is possible to provide an environment that is conducive to creativity. Many problem solving theorists have argued correctly for the inclusion of such an atmosphere in classrooms (Sriraman, 2004).

Element seven, *problem solving requires a knowledge base*, although not problematic, is neglected within the theory of problem solving. This is ironic given how important it is. Ormrod (1999) says, "Successful (expert) problem solvers have a more complete and better organized knowledge base for the problems they solve" (p. 370). She also relates how one research inquiry that studied the practice of problem solving in a high school physics class observed that the high achievers had "better organized information about concepts related to electricity" (p. 370). Not only was their information better organised, but students were also aware of "the particular relationships that different concepts had with one another" (Cochran, 1988, p. 101). Norman (1980) adds,

> *"I do not believe we yet know enough to make strong statements about what ought to be or ought not to be included in a course on general problem solving methods. Although there are some general methods that could be of use…I suspect that in most real situations it is…specific knowledge that is most important."* (p. 101).

Finally, element eight, *problem solving is the application of concepts or transfers,* is also not problematic; it, too, is merely neglected within the theory of problem solving. Frederiksen (1984) says, for example, that "the ability to formulate abstract concepts is an ability that underlies the acquisition of knowledge. [Teaching how to conceptualize] accounts for generality or transfer to new situations" (p. 379). According to this notion, it is the application of conceptual knowledge and not the heuristic alone that, as Frederiksen says, "accounts for generality or transfer," (p. 379) which the advocates of problem solving so desire.

Carson (2007) thus concluded that problem solving would be more effective if the knowledge base and the application of that knowledge were the primary principles of the theory and practice. Currently, it seems that a content-less heuristic approach is the primary principle, which, as argued, is problematic because it dichotomises thinking and knowledge into two mutually exclusive domains. In fact, in the course of solving any problem one will find oneself learning of all things not heuristics, but an algorithm. In other words, teachers must not only teach students heuristics and set them free upon the problems of everyday life. Rather, in addition to teaching students sound thinking skills, teachers must teach them what knowledge has been successful in the past in solving problems and why.

### 2.3.4 Algebraic problem solving

It is often said that problem solving is the bedrock of mathematics and that the primary goal of teaching and learning mathematics is to develop the capacity to solve a range of problems (Jiang, 2008; Wilson, Fernandez & Hadaway, 1993). The National Council of Teachers of Mathematics (NCTM) (2000) points out that,

> *"solving problems is not only a goal of learning mathematics but also a major means of doing so…In everyday life and the workplace, being a good problem solver can lead to great advantages…Problem solving is an integral part of all mathematics learning" (p. 52).*

In turn, problem solving, among others, plays a significant role in the development of thinking skills (Carson, 2007; Pehkonen, 2007), reasoning skills (Ketterlin-Geller & Chard, 2011) and the promotion of higher order thinking (An & King, 2008). Wilson et al. (1993) warn that in developing thinking skills, emphasis should be on teaching 'how to think' rather

than ‘what to think’ or ‘what to do’ because the latter options depict problem solving as a linear process consisting of a series of steps in finding a solution. Perceiving problem solving this way, as Wilson et al. (1993) caution, encourages procedural learning that goes with memorisation, rote learning and success only in routine problem solving. With procedural learning, learners tend to have difficulty when they are required to solve unfamiliar or non-routine problems. Matang (2002) notes that procedural learning is characterised by a strong focus on the mastery of rules, algorithms and the symbol representation system instead of focusing on building a knowledge network that is rich with relationships constructed among the constituent unit blocks of knowledge. The latter form of learning mathematics characterises conceptual learning (Matang, 2002). Skemp (1976) calls it relational learning and states that it is about “knowing both what to do and why” (p. 20).

Conceptual learning is the key to algebraic understanding (Ketterlin-Geller & Chard, 2011; Ketterlin-Geller, Chard & Fien, 2008). In order to succeed at learning algebra, learners are expected to integrate and extend skills learned during prior years (Ketter-Geller, Chard & Fien, 2008) and to understand variables, constants and functions, to decompose and set up word problems, and manipulate symbols (Milgram, 2007). Algebra entails the generalisation of patterns that engender algebraic functions (Ketterllin-Geller & Chard, 2011). Exposing learners to algebra also creates opportunities for the development of unprecedented abstract reasoning and problem solving (Vogel, 2008). It is argued that the learning of algebra goes hand in hand with the development of problem solving skills. The current study is pursued against this backdrop.

### 2.3.5 The three dimensions of problem solving

Problem solving can be described as being composed of three dimensions: **the problem, the process and the outcome.**

#### 2.3.5.1 The problem

Charles, Lester and O’Daffer (1987) classify problems into two types: **Routine and Non-routine.**

Routine problems take the form of exercises and are problems that are easy to interpret and that involve only one step.

Non-routine problems require a strategy to be developed to understand the problem, to plan to solve it and to evaluate the results of attempts to solve it. A non-routine problem may also take the form of a puzzle which is a kind of problem that provides students with the chance to engage in recreational mathematics.

In Charles, Lester and O'Daffer's (1987) definition of non-routine problems, Polya's (1957) four steps in problem solving are depicted, namely:

- Understanding the problem;
- Planning to solve the problem;
- Solving the problem;
- Evaluating the solution of the problem.

Problems may vary in aspects such as substance, structure, process to be carried out, nature, models of presentation or representation and in their components and the interactions among them. Jonassen and Land (2000) describe the differences between problems in terms of their structuredness, complexity and abstractness.

***Structuredness:*** Well-structured problems require the application of a finite number of concepts, rules and principles to a constrained problem situation. Ill-structured problems, on the other hand, possess problem elements that are unknown or not known with any degree of confidence, and have multiple solutions, multiple solution paths or no solution at all.

***Complexity:*** The number of issues, functions or variables involved in the problem; the degree of connectivity among these properties; the type of functional relationships among these properties and the sustainability of the properties of the problem over time.

***Abstractness:*** Problem solving activities are situated, embedded and therefore dependent on the nature of the context or domain, because solving problems within a domain relies on domain-specific cognitive operations.

In general, when researchers use the term problem solving they are referring to mathematical tasks that have the potential to provide intellectual challenges which may enhance learners' mathematical development. Such tasks may promote learners' conceptual understanding,

foster their ability to reason and communicate mathematically and capture their interest and curiosity (Hiebert & Wearne, 1993; Marcus & Fey, 2003: NCTM, 1991; van de Walle, 1993). Mathematical problems that are truly problematic and involve significant mathematics have the potential to provide the intellectual contexts for learners' mathematical development. However, only "worthwhile problems" give learners the chance to solidify and extend what they know and stimulate mathematics learning. Lappan and Phillips (1998) developed a set of criteria for a good problem and used them to develop the middle secondary curriculum for fostering learners' conceptual understanding and problem solving (Cai, Moyer, Wang & Nie, 2011). Although there has been no research focusing specifically on the effectiveness of this set of criteria, the fact that the curriculum as a whole has been effective suggests that teachers might want to attend to this set in choosing, revising and designing problems. The following are the criteria that were proposed by Cai, Moyer, Wang and Nie (2011):

1. The problem has important, useful mathematics embedded in it;
2. The problem requires higher-level thinking and problem solving;
3. The problem contributes to the conceptual development of learners;
4. The problem creates an opportunity for the teacher to assess what his learners are learning and where they are experiencing difficulties;
5. The problem can be approached by learners in multiple ways using different solution strategies;
6. The problem has various solutions or allows different decisions or positions to be taken or defended;
7. The problem encourages learners' engagement or discourse;
8. The problem connects to other important mathematical ideas;
9. The problem promotes skillful use of mathematics; and,
10. The problem provides an opportunity to practise important skills.

Of course, it is not reasonable to expect that every problem a teacher chooses should satisfy all ten criteria; which criteria to consider should depend on the teacher's instructional goals. For example, some problems are used primarily because they provide learners with an opportunity to practise a certain skill (criterion 10), such as solving equations, whereas others are used primarily to encourage learners to collaborate with one another and justify their thinking (criteria 6 and 7). Researchers and curriculum developers tend to agree that the first four criteria (important mathematics, higher-level thinking, conceptual development and

opportunity to assess learning) should be considered essential in the selection of all problems. Teachers can modify a standard textbook problem in a way that both engages learners in important mathematics and also enhances the development of their problem solving abilities (criteria 2, 3, 4 and 5).

For instance, in March 2009, The Rössing Foundation, in collaboration with the University of Namibia (UNAM), organised and hosted a regional mathematics Olympiad for top achieving mathematics learners in grade 11 and 12. The aim of the Olympiad was to explore the mathematical and problem solving abilities of learners from four northern regions, Oshikoto, Ohangwena, Omusati and Oshana regions (see Figures 2.1 and 3.2). The competition had two rounds, the individual round and the team round. In the individual round, each paper consisted of 20 questions with multiple choice answers and candidates had one hour in which to complete the paper. In the team round each team of four learners received three algebraic word problems to solve in one hour.

The following example of a problem from an original standard textbook was used in the individual round.

*Solve the equation* $4x + 2y = 14 + 2( x + y )$
Source: The Rössing Foundation Mathematics Olympiad (2009)

The problem clearly involves important mathematics, but in its present form, criteria 2, 3, 4 and 5, proposed by Cai et al. (2011), are not as clearly included.

The same problem was revised, to raise its cognitive demand (criterion 2) and also to satisfy criteria 3 and 4 proposed by Cai et al (2011). The revised problem was one of the three problems used in the team round of the 2009 Rössing Foundation Mathematics Olympiads. The revised problem was presented in the form:

*In a group of cows and chickens, the number of legs is 14 more than the number of heads. How many cows are there in the group?*
Source: The Rössing Foundation Mathematics Olympiads (2009).

Figure 2.1 shows a photo clip of some of the learners that participated in the algebraic problem solving round of the 2009 Rössing Foundation Mathematics Olympiads.

**Figure 2.1:** Some of the learners that embarked in the problem solving activities in the inaugural Rössing Foundation Olympiad in 2009

Source: The Rössing Foundation Handbook for Mathematics Olympiads and Ladder competitions (2009)

The results obtained showed that learners were successful in solving the original equation $4x + 2y = 14 + 2( x + y )$ in round 1, but were less successful in devising a solution plan for the round 2 algebraic word problem, which leads to the same equation $4x + 2y = 14 + 2( x + y )$, given that $x$ represents the number of cows and $y$, the number of chickens. This example illustrates the notion that modifying problems that already exist in textbooks is often a relatively easy thing to do but one which increases the learning opportunities for learners. Readers may also see how to revise a problem to make it more problematic so that the learning opportunity for learners is increased (Butts, 1980).

The revised problem (see Appendix 3, problem5) was adopted and used as one of the problems in the algebraic problem solving test administered to the sample of learners who participated in this study.

#### 2.3.5.2 Problem solving as a process

Many writers have developed frameworks for analysing problem solving as a process. Polya's (1957) seminal work suggested that solving a problem involves four phases: (a) understanding the problem; (b) developing a plan; (c) carrying out the plan; and (d) looking back. Schoenfeld (1985) observed that during problem solving learners display distinct categories of behaviour or episodes. Crucial episodes in the problem solving process are

analysing the problem, selecting appropriate mathematical knowledge, making a plan, carrying out the plan and checking the answer in relation to the question asked. Polya's (1957) model comprised the basis on which other models were developed, for instance, the six-phase model proposed by Kapa (2001): (1) identifying and defining the problem, (2) mental representation of the problem (3) planning how to proceed, (4) executing the solution according to the plan, (5) evaluation of what the problem solver knows about his/her performance, and (6) reaction to feedback. However, Polya-style models are often misinterpreted as a linear application of a series of steps, either because of the way they are presented in numerous textbooks (Wilson et al., 1993) or because they are perceived as such by most teachers (Kelly, 2006). Schoenfeld (1992) contributed a framework of factors that affect learners' abilities to solve problems: (a) **Resources:** formal and informal knowledge about the content domain, including facts, definitions, algorithmic procedures, routine procedures, intuitive understanding of mathematics and relevant competencies about rules of discourse; (b) **Heuristics:** strategies and techniques for approaching a problem; (c) **Control:** the ways in which the learners monitor their own problem solving process, use their observations of partial results to guide future problem solving actions and decide how and when to use the available resources and heuristics; and (d) **Beliefs:** what one believes about mathematics, mathematical tasks and what it means to do mathematics. Schoenfeld's theory on metacognition will be detailed later as it also constitutes the theoretical framework of this study.

#### 2.3.5.3 The problem solving outcome

The problem solving outcome, involves the assessment of the outcome creativity. Research on creative thinking has identified three key components of a creative product: fluency, flexibility and novelty (Torrance, 1974). Fluency refers to the number of ideas generated in response to a prompt; flexibility refers to the apparent shift in approaches when generating responses to a prompt; and novelty refers to the originality of the ideas generated in response to a prompt. In this study, the outcome is the product of the solution process, the solution strategies used and the problem solving skill level determined by the CAAPSA tool.

## 2.4 Theories on the problem solving process

### 2.4.1 Metacognitive processes

Metacognitive processes, according to Schoenfeld (1985), include assessing one's own knowledge, formulating a plan of attack, selecting strategies and monitoring and evaluating the process. Thus metacognitive processes focus on students' ability to monitor and regulate their own cognitive processes employed during problem solving (Artzt & Armour-Thomas, 1992; Schoenfeld, 1992).

Scholars have argued that emphasis on cognition without a corresponding emphasis on metacognitive thinking renders problem solving incomplete (see, for example, Artzt & Armour-Thomas, 1992; Berardi-Coleta, Dominowski, Buyer & Rellinger, 1995; Kirkwood, 2000; Lin, 2001; Schoenfeld, 1992). A rich store of knowledge is believed to be a necessary but not sufficient requirement for successful mathematical problem solving (Garofalo & Lester, 1985; Geiger & Galbraith, 1998; Schoenfeld, 1987; Silver, 1987). Although learners may be equipped with knowledge and skills to interpret the statement of problem, inefficient control mechanisms can be a major obstacle during solution attempts (Carlson, 1999). Carlson (1999) concluded that, irrespective of the richness of the learners' knowledge base, the learners' inefficient control of their decisions often means that existing mathematical knowledge is not easily accessed and general problem solving strategies are consequently not successfully employed.

Several studies have concluded that metacognitive processes improve problem-solving performance (Artzt & Armour-Thomas, 1992; Goos & Galbraith, 1996; Kramarski & Mevarech, 1997). Schoenfeld (1985) developed a four stage model which involved (a) resources, (b) heuristics, (c) control, and (d) belief systems.

Geiger and Galbraith (1998) developed a script analysis framework that categorised metacognitive behaviours observed when learners solved mathematical problems. Their framework included (a) engagement, (b) executive behaviours, (c) resources, and (d) beliefs. Seemingly, these models and frameworks used minor variations of Polya's (1957) four stage model: (a) understand, (b) plan, (c) carry out the plan, (d) look back.

In order to assess learners' problem solving skills, the current study argues that an analysis of the problem solving process is paramount, thus this study adopts Polya's framework as the measure of the metacognitive processes in the algebraic problem solving process. Figure 2.2

shows the metacognitive processes involved in the algebraic problem solving process. The metacognitive processes combine Schoenfeld's (1992) theory of metacognition and Polya's problem solving steps. This is the model that was adopted in the development of the computer aided algebraic problem solving assessment (CAAPSA) tool, that was used to map the problem solving process according to the skill indicators in Polya's steps.

### 2.4.2 Analysis of some problem solving models

#### 2.4.2.1 Polya's steps of the problem solving process

Probably the most famous approach to problem solving is Polya's (1957) four-step process. Polya (1957) set out his summary of the core verbal steps in problem solving thus:

1. Understand the problem;
2. Devise a plan;
3. Carry out the plan;
4. Look back.

The problem-solving process is merely a general guide to how to proceed in solving problems. In many cases, steps of the process will overlap, thus it may not be possible to perform each step of the process in the order given above, and these four principles appear in many elementary-level textbook series as early as the kindergarten grade level. Polya (1957) was writing for very advanced students and he left out many critical aspects of problem solving, such as "check that the problem is well-posed", since he felt safe in assuming that his intended audience would not neglect this step. The following is an expansion of Polya's four problem solving steps:

##### 2.4.2.1.1 Understand the problem

In order to correctly solve a problem, one must first understand the problem, and see clearly what is required. Second, one must understand how various items are connected, how the unknown is linked to the data, in order to obtain an idea of the solution, in other words, **to make a plan**. The verbal statement of the problem must be understood. The learner should be able to point out the principal parts of the problem, the unknown, the data, and the condition. The important questions in this step are:

(i) Can you state the problem in your own words?

(ii) What are you trying to find or do?

(iii) What are the unknowns?

(iv) What information, if any, is missing or needed?

Understanding the problem is divided into two stages: **Getting acquainted with the problem** and **Working for a better understanding**.

#### 2.4.2.1.2 Devising a plan

We have a plan when we know, or know at least in outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown. The path from understanding a problem to conceiving a plan may be long and tortuous. In fact, the main achievement in the solution of a problem is to conceive the idea of a plan. This idea may emerge gradually or, after unsuccessful trials and a period of hesitation, it may occur suddenly in a flash, as a "bright idea" (Polya, 1973: p. 8). The "plan" used to solve a problem is often called a problem solving strategy. In some cases, one may begin using one strategy and then realise that the strategy does not fit the given information or is not leading to the desired solution; in this case, one must choose another strategy. In other cases, one may need to use a combination of strategies. Several problem solving strategies are mentioned below, in no particular order:

(i) Look for a pattern;

(ii) Examine related problems and determine if the technique applied to them can be applied here;

(iii) Examine a simpler or special case of the problem to gain insight into the solution of the original problem;

(iv) Make a table;

(v) Make a diagram;

(vi) Write an equation;

(vii) Work backwards;

(viii) Identify a sub-goal;

(ix) Use indirect reasoning.

#### 2.4.2.1.3 Carrying out the plan

Once a problem has been carefully analysed and a plan has been devised, if this plan is a suitable one for the given problem, it is usually a relatively simple process to carry out the plan. However, in some cases the original plan does not succeed and another plan must be devised. The original strategy may be modified, or a new strategy may be selected. Learners must learn that not every problem will be solved in the first attempt. A failed attempt can be

viewed as a learning experience; learners must be helped to avoid becoming frustrated or discouraged. Cooperative learning or other manipulatives may be useful tools when routine tasks are involved. It is recommended that the following guidelines are followed:

(i) Implement the devised solution plan and perform any necessary actions or computations.

(ii) Check each step of the plan as you proceed. This may be intuitive checking or a formal proof of each step.

(iii) Keep an accurate record of your work.

#### 2.4.2.1.4 Looking back

Once an answer or solution is found, it is important to check that solution. By looking back at the completed solution, by reconsidering and re-examining the result and the path that led to it, learners can consolidate their knowledge and develop their ability to solve problems. Check all steps and calculations within the solution process. Below are some actions that a problem solver may find useful in the looking back process:

(i) Check the results in the original problem (in some cases this will require a proof).

(ii) Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable? Does it answer the question that was asked?

(iii) Determine whether there is another method of finding the solution.

(iv) If possible, determine other related or more general problems for which the technique will work.

The indicators adopted by the study in the development of the computer aided algebraic problem solving assessment (CAAPSA) tool for each of Polya's steps are as follows:

The learner's skill in each step shall be demonstrated by evidence that the learner has managed to achieve some of the following outlined actions in each step:

**Step 1: Understanding of the problem**

- Read the information;
- Identify what to find or pose the problem;
- Identify the key conditions; find important data;

- Examine assumptions.

**Step 2: Develop a plan**

- Choose problem solving strategies:
  - (i) Make a model;
  - (ii) Act it out;
  - (iii) Choose an operation;
  - (iv) Write an equation;
  - (v) Draw a diagram;
  - (vi) Guess-check-revise;
  - (vii) Simplify the problem;
  - (viii) Make a list;
  - (ix) Look for a pattern;
  - (x) Make a table;
  - (xi) Use a specific case;
  - (xii) Work backwards;
  - (xiii) Use reasoning.
- Identify sub-problems;
- Decide whether estimation, calculation, or neither is required.

**Step 3: Carry out the plan**

- If calculation is required, choose a calculation method;
- Use appropriate problem solving strategies to carry out the plan.

**Step 4: Look back**

- Check the problem interpretation and calculations;
- Decide whether the answer is reasonable;
- Look for alternative solutions;
- Generalize ways to solve similar problems.

Some researchers have proposed the addition of a fifth step to Polya's (1957) four-step model. This is not suggesting that Polya's (1957) process is incomplete. In fact, the fifth step, extend the problem, is mentioned in Polya's (1957) manuscript as part of the fourth step, look

back. These ideas are separated so that the process of extending the problem, especially relevant to teachers, does not become lost in the process of verifying the solution.

#### 2.4.2.2 Garofalo and Lester's four stage problem solving model

Garofalo and Lester (1985) suggested that learners are largely unaware of the processes involved in problem solving and pointed out that Polya's (1957) model does not encompass metacognition. Their model incorporated metacognitive behaviour at each of four stages, which comprised (a) *orientation*: strategic behavior to assess and understand a problem, (b) *organisation*: planning of behavior and choice of actions, (c) *execution*: regulation of behavior to conform to plans, and (d) *verification*: evaluation of orientation and organisation as well as evaluation of execution. At each stage, the following are achievement indicators to consider in the problem solving process:

**Step 1: Orientation**

The extent to which the learner is able to attempt a solution that evidenced at least some understanding of the nature of the problem

**Step 2: Organisation**

The learner should have made an attempt to choose a solution strategy that, if implemented correctly would result in the correct solution

**Step 3: Execution**

The learner's ability to execute the chosen strategy

**Step 4: Verification**

The learner should be able to communicate the in a verbal or written manner, the solution process. There must be some extent of verification of the problem solving outcome (answer).

Garofalo and Lester's (1985) model shows minor changes of reference to Polya's steps, but in essence the actions in each step seem equivalent.

#### 2.4.2.3 Kapa's six phase problem solving model

Metacognition (conscious control of thought) can reinforce the ability of learners to become better problem solvers, because metacognitive strategies support the efforts during problem

solving (Kapa, 2001; Mohini & Nai, 2005; Schoenfeld, 2007). The more learners control and monitor the strategies they use; they acquire better abilities to solve problems (Kapa, 2001; Mevarech & Fridkin, 2006). In other words, metacognition supports the cognitive level, through the action of the monitoring and control functions during mathematical problem solving.

More recently, Kapa (2001) suggested a model where separate metacognitive functions appear for each of the phases of a problem solving process. According to Kapa 92001), the metacognitive knowledge may affect cognitive tasks in each problem solving phase, as described below:

(a) **Problem definition and identification**: collecting data, coding and remembering essential facts. The final goal of the phase is to get a clear idea of the problem and the information required for solving it.
(b) **Mental representation of the problem**: Once all the information required for solving a problem is present, the information has to be organized and presented in such a way that enables the problem solver to come up with a possible solution strategy.
(c) **Planning how to proceed**: This is the phase in which the problem solver uses the problem information to formulate a possible solution strategy.
(d) **Executing the solution according to the plan**: The formulated solution plan is implemented. At this phase the problem solver monitors and controls the solution process (metacognition). If the selected plan does not yield to the required solution outcome, adjustments are made to the initial plan.
(e) **Evaluation**: During this phase the problem solving product and the process are evaluated to ensure that the product is in line with the original task and that the problem solving process is efficient. One can also explore on how to improve the solution process by suggesting alternative solution methods.
(f) **Reaction to feedback**: This is the phase in which the problem solver acquires certain beliefs about the problem solving process.

It is the researcher's view that the context of Kapa's (2001) model is also a variation of Polya's (1957) model. Kapa (2001) seems to have added to Polya's (1957) model, the analysis of the attitudes and beliefs that the problem solver develops in the problem solving process. However, as assessment frameworks for problem solving script analysis, both

frameworks would be ineffective in mapping the attitudes and beliefs from learners' work. It is for this reason that the researcher also administered follow-up task based interviews to get a deeper understanding of the learners' metacognitive processes. The researcher found Polya's model to be the simplest theoretical model for analysing the algebraic problem solving process of the learners who participated in this study. The steps in Polya's model are easy to interpret and associate to specific algebraic operations as indicators of the skill level in the algebraic problem solving process. The current study thus filled the gaps in Polya's (1957) model by defining the skill level indicators for each step, based on Lester and O'Daffer's (1987) scoring strategy. The CAAPSA framework for this study incorporated Polya's (1957) problem solving model, Schoenfeld's (1992) theory of metacognition and Lester and O'Daffer's (1987) scoring strategy for the problem solving process. The study adapted the TIMSS (2007) benchmarks for assigning the CAAPSA levels to each of Polya's steps. The CAAPSA theoretical framework for this study is discussed in detail later in section 2.7.

#### 2.4.2.4 The relationship between algebraic problem solving and mathematical proficiency

Many students have difficulty with algebraic problem solving. There have been a number of studies conducted by mathematics education researchers to attempt to understand the nature of these difficulties. Kieran (1996) sees generating equations from words as the major area of difficulty for high school algebra students.

Kilpatrick et al. (2001) indicated that mathematical proficiency is not a one-dimensional trait which can be achieved by focusing on just one or two of the five strands of mathematical proficiency. In order to acquire mathematical proficiency, emphasis should be on all strands because they provide a framework for discussing knowledge, skills, abilities and beliefs that constitute mathematical proficiency. Of these constituent aspects of mathematics proficiency, problem solving skills stand out as key in successful mathematics teaching (Carson, 2007), hence the current study intended to focus on learners' problem solving skills. As learners progress in their schooling from lower to higher classes, they should become increasingly proficient in mathematics (Milgram, 2007) in order to be successful mathematical problem solvers.

The implication of Kilpatrick et al.'s (2001), Carson's (2007) and Milgram's (2007) assertions is that algebraic problem solving is a higher level strand of mathematics proficiency. Although these strands are interwoven, the assessment of algebraic problem solving stands out as a good indicator of the level of development of mathematical proficiency and consequently the learners' academic achievement (Carson, 2007; Milgram, 2007). It is on this basis that the current study analyses the correlation between the algebraic problem solving achievement and the 2010 NSSC examination performance of the learners who participated in the study.

#### 2.4.2.5 Problem solving as an instructional goal for mathematical proficiency

Mathematics proficiency, according to Kilpatrick et al. (2001), refers to successful mathematics learning and has five strands; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.

***Conceptual understanding***—comprehension of mathematical concepts, operations, and relations

***Procedural fluency***—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

***Strategic competence***—ability to formulate, represent and solve mathematical problems

***Adaptive reasoning***—capacity for logical thought, reflection, explanation, and justification

***Productive disposition***—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These strands are not independent; they represent different aspects of a complex whole. ***The five strands are interwoven and interdependent in the development of proficiency in mathematics.*** Mathematical proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands. Kilpatrick et al. (2001) argued that helping children acquire mathematical proficiency calls for instructional programmes that address all its strands. According to Kilpatrick et al. (2001), as students move from pre-kindergarten to eighth grade, they should become increasingly proficient in mathematics.

That proficiency should enable the students to cope with the mathematical challenges of daily life and allow them to continue their study of mathematics at high school and beyond.

The five strands provide a framework for discussing the knowledge, skills, abilities and beliefs that constitute mathematical proficiency. This framework has some similarities with the one used in recent mathematics assessments by the National Assessment of Educational Progress (NAEP), which features three mathematical abilities ***(conceptual understanding, procedural knowledge and problem solving)*** and includes additional specifications for reasoning, connections and communication. The strands also echo components of mathematics learning that have been identified in materials for teachers. At the same time, research and theory in cognitive science provide general support for the ideas contributing to these five strands. Fundamental to that work has been the central role of mental representations. How learners represent and connect pieces of knowledge is a key factor in whether they will understand such knowledge deeply and be able to use it in problem solving.

For many educational systems, the strategic competence in problem solving has a central role in mathematics teaching and learning and has been identified as one of the five fundamental process strands along with reasoning and proof, communication, connections and representations, by the National Council of Teachers of Mathematics (NCTM, 2000). For NCTM (2000), mathematics teaching and learning and problem solving are synonymous terms; therefore the building of new mathematical knowledge through problem solving should be in the centre of mathematics education. Similarly, in the context of China, Cai and Nie (2007) argue that the activity of mathematical problem solving in the classroom is viewed as an important focus of instruction that provides opportunities for students to enhance their flexibility and independent mathematical thinking and reasoning abilities.

The first job of our education system is to teach students to read, and the majority of students do learn this. The second thing the system should do is teach students basic mathematics, and it is here that it fails (Milgram, 2007). According to Milgram (2007), before trying to address this failure in the system, we must answer two basic questions;

- What does it mean for a student to be proficient in mathematics?
- How can we measure proficiency in mathematics?

Schoenfeld (2007) suggests that we should have some idea of what mathematics is in order to discuss mathematics proficiency. He proposes the following important characteristics of mathematics:

(i) Precision (precise definitions of all terms, operations and the properties of these operations);
(ii) Stating well-posed problems and solving them. (Well-posed problems are problems where all the terms are precisely defined and refer to a single universe where mathematics can be done.)

Schoenfeld (2007) believes that the "cognitive revolution" produced a significant reconceptualisation of what it means to understand subject matter in different domains (see also National Research Council [NRC], 2000). There was a fundamental shift from an exclusive emphasis on knowledge ─ what does the student know? ─ to a focus on what students know and can do with their knowledge. The idea was not that knowledge is unimportant. Clearly, the more one knows, the greater the potential for that knowledge to be used. Rather, the idea was that having the knowledge was not enough; being able to use it in the appropriate circumstances is an essential component of proficiency. The knowledge base is important; it goes without saying that anyone who lacks a solid grasp of facts, procedures, definitions and concepts is significantly handicapped in mathematics (Schoenfeld, 2007). But there is much more to mathematical proficiency than being able to reproduce standard content on demand. A mathematician's job consists of at least one of the following: extending known results, finding new results, and applying results in new contexts. The problems mathematicians work on, in academia or in industry, are not the kind of exercises that are solved in a few minutes or hours; they are problems that take days, weeks, months, even years to solve. Thus, in addition to possessing substantial specialised knowledge, mathematicians possess other characteristics as well.

Good problem solvers are flexible and resourceful. It would be fair to say that virtually all mathematics is problem solving in precisely defined environments. Schoenfeld (2007) advocated the implementation of Polya's (1957) steps as the means to attain proficiency in mathematics.

The analysis of Kilpatrick's (2001) strands of mathematical proficiency and Polya's (1957) steps reveals the existence of a similarity in the two frameworks. This prompted the researcher to conjecture that proficiency to solve algebraic problems in Polya's (1957) model

is synonymous to proficiency in mathematics hence; the study will seek to show that ***the achievement of learners in the algebraic problem solving process has a strong correlation with their achievement in academic mathematics (NSSC examinations).***

For a classroom teacher, an important part of the problem solving process should involve trying to create similar or related problems. A given problem may need to be simplified in order for it to be used at a specific classroom level or with learners who have special needs. A teacher may wish to make a problem more complicated or to create similar, related problems that are more difficult. Elementary school teachers often expand the problem as part of a journal writing exercise as learners write their own story problems for a given situation.

It may be possible to generalise specific instances of a given problem. Teachers must be on the lookout for opportunities to have learners generalise and make conjectures. Teachers should look for connections that can be made between mathematics problems and solutions to real-life situations. Teachers should also look for connections between given mathematics problems and their solutions in other subject areas.

#### 2.4.2.6 Assessing Understanding

When assessing students in mathematics, we face the problem that we are all too often assessing only a limited part of their understanding. For example, when asking a student to carry out a multiplication calculation, are we really assessing his or her understanding of multiplication? To be clear about how we do this, we need to be clear about understanding itself. Skemp (1976) identified two types of understanding: relational and instrumental. He described relational understanding as "knowing both what to do and why" (p. 2), and the process of learning relational mathematics as "building up a conceptual structure" (p. 14). Instrumental understanding, on the other hand, was simply described as "rules without reasons" (p. 2).

Possible methods suggested by Hiebert and Carpenter (1992) to assess student understanding were to analyse:

- Students' errors;
- Connections made between symbols and symbolic procedures and corresponding referents;
- Connections between symbolic procedures and informal problem solving situations;
- Connections made between different symbol systems.

The developed CAAPSA tool assesses learners' problem solving skills in Polya's model by computing the number of errors (conceptual, procedural or computational) at each stage. The CAAPSA tool then returns CAAPSA levels (1 to 5) at each of Polya's steps, thus allowing the researcher to identify the steps in which learners encounter difficulties. Of primary interest in this study was why learners' encounter difficulties when solving the algebraic problems. According to Skemp (1976), students can work instrumentally, only linking procedural representations with the concept and no other representations that might explain why the procedures are appropriate. Hiebert and Carpenter (1992) state that "any individual task can be performed correctly without understanding" (p. 89). Therefore, the fact that a student gets a calculation correct tells us little about the extent of his or her understanding. However, when a student makes a mistake in a calculation, this might indicate the limitations of his or her understanding, even if that understanding is only instrumental. It is against this backdrop that the CAAPSA tool was programmed through enumeration of errors at each of Polya's steps to determine the algebraic problem solving level attained by learners in the solution process.

#### 2.4.2.8 Need and methods for assessing problem solving ability

The conceptual framework of this study is derived from Schoenfeld's (1985) theory of metacognition, TIMSS (2007) assessment framework and Polya's (1957) model. As the emphasis on problem solving in mathematics classrooms increases, the need for evaluation of progress and instruction in problem solving becomes more pressing. It is no longer enough for us to know simply which kinds of problems are correctly or incorrectly solved by learners (Schoenfeld, 1988):

> *All too often we focus on a narrow collection of well-defined tasks and train learners to execute those tasks in a routine, if not algorithmic fashion. Then we test the learners on tasks that are very close to the ones they have been taught. If they succeed on those problems, we congratulate each other on the fact that they have learned some powerful mathematical techniques. In fact, they may be able to use such techniques mechanically while lacking some rudimentary thinking skills. To allow them, and ourselves, to believe they "understand" the mathematics is deceptive and fraudulent* (Schoenfeld, 1987).

The National Council of Teachers of Mathematics (NCTM) (1980) recommended that, the success of mathematics programmes and student learning must be evaluated by a wider range

of measures than conventional testing. Although this recommendation is widely accepted among mathematics educators, there is limited research dealing with the evaluation of problem solving within the classroom environment (NCTM, 1980).

NCTM (1980) suggested the following as a classroom research strategy for evaluating the learners' problem solving skills:

Ask your learners to keep a problem-solving notebook in which they record on a weekly basis:

1. Their solution to a mathematics problem;
2. A discussion of the strategies they used to solve the problem;
3. A discussion of the mathematical similarities of this problem to other problems they have solved;
4. A discussion of possible extensions to the problem;
5. An investigation of at least one of the extensions they have discussed.

It is against this backdrop that this study designed the CAAPSA tool as a classroom research tool to evaluate the development of algebraic problem solving skills.

Charles, Lester and O'Daffer's (1987) describe how we could incorporate these techniques into a classroom problem solving evaluation programme. For example, thinking aloud may be canonically achieved within the classroom by placing learners in cooperative groups. In this way, learners can express their problem solving strategies aloud and thus we may be able to assess their thinking processes and attitudes unobtrusively. Charles et al. (1987) also discuss the use of interviews and learners' self-reports during which learners are asked to reflect on their problem solving experience, a technique often used in problem solving research. Other techniques which they describe involve methods of scoring learners' written work. These are some of the techniques that influenced the design of the research instruments used in this study and the development of the CAAPSA tool.

### 2.5 Some international benchmarks for measuring mathematical proficiency

In order to select appropriate research tools the researcher also looked at some existing frameworks for measuring knowledge and skills. The study analysed three existing models, namely:

1. The National Assessment of Educational Progress (NAEP)

2. The Program for International Student Assessment (PISA)

3. The Trends in International Mathematics and Science Study (TIMSS).

### 2.5.1 The National Assessment of Educational Progress (NAEP) achievement levels

The NAEP is used in the United States of America (U.S.A) to measure the performance of fourth, eighth and twelfth grade students', most frequently in reading, mathematics and science, with assessments designed specifically for national and state information needs. NAEP uses both scale scores and achievement levels to report student performance. Scale scores show what students know and can do, and achievement levels are performance standards of what students should know and be able to do. The NAEP achievement levels ***Basic***, ***Proficient***, and ***Advanced*** are used to interpret the meaning of the NAEP scales. ***Basic*** denotes partial mastery of the knowledge and skills that are fundamental to proficient work at a given grade. ***Proficient*** represents solid academic performance. Students reaching this level have demonstrated competency in challenging subject matter. However, proficient is not synonymous with grade level performance. ***Advanced*** signifies superior performance. The achievement levels are set independently by the National Assessment Governing Board, which sets policy for NAEP. The NAEP scale uses a rating of 0 to 500. The NAEP is still used on a trial basis and should be interpreted with caution.

### 2.5.2 The Programme for International Student Assessment (PISA) achievement levels

PISA is an internationally standardised assessment of 15-year-olds that covers three domains; reading, mathematical and scientific literacy. The PISA assessment framework focuses on assessing students' ability to use knowledge and skills to meet real-life challenges, their mastery of processes, their understanding of concepts and their ability to function in various situations in each domain, as well as providing information on students' attitudes to learning. The PISA assessments are conducted in three-year cycles with each year focusing on one of the three domains.

Mathematics achievement is divided into six proficiency levels representing a group of tasks of increasing difficulty, with Level 6 the highest and Level 1 the lowest. Students performing below Level 1 (mathematics score below 359) are not able to show routinely the most basic type of knowledge and skills that PISA seeks to measure. Such students have serious difficulties in using mathematical literacy as a tool to advance their knowledge and skills in other areas. Placement at this level does not mean that these students have no mathematics

skills. Most of these students are able to correctly complete some of the PISA items. Their pattern of responses to the assessment is such that they would be expected to solve fewer than half of the tasks from a test composed of only Level 1 items.

In PISA, students are assigned to a proficiency level based on their probability of answering correctly the majority of items in that range of difficulty. A student at a given level could be assumed to be able to correctly answer questions at all lower levels. To help in interpretation, these levels are linked to specific score ranges on the original scale. Below is a description of the abilities associated with each proficiency level. (Source: Organisation for Economic Cooperation and Development [OECD], Programme for International Student Assessment, [PISA], 2010).

**2.5.2.1 Level 6 (score above 668)**

At Level 6, students can conceptualise, generalise and utilise information based on their investigations and modeling of complex problem situations. They can link different information sources and representations and flexibly translate from them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can precisely formulate and communicate their actions and reflections regarding their findings, interpretations and arguments, and the appropriateness of these to the original situations.

**2.5..2.2 Level 5 (score from 607 to 668)**

At Level 5, students can develop and work with models of complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriately linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.

**2.5.2.3 Level 4 (score from 545 to 606)**

At Level 4 students can work effectively with explicit models of complex concrete situations that may involve constraints or call for the making of assumptions. They can select and

integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.

#### 2.5.2.4 Level 3 (score from 483 to 544)

At Level 3, students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.

#### 2.5.2.5 Level 2 (score from 421 to 482)

At Level 2 students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and of making literal interpretations of the results.

#### 2.5.2.6 Level 1 (score from 359 to 420)

At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

### 2.5.3 Trends in International Mathematics and Science Study (TIMSS) achievement levels

The TIMSS (2007) international assessment of student achievement comprises of written tests in mathematics and science together with a set of questionnaires that gather information on the educational and social aspects of achievement. For the first time, TIMSS (2007) reported student achievement by cognitive domain – i.e., knowing, applying and reasoning. TIMSS (2007) used scale anchoring to summarise and describe student achievement at four points on the mathematics and science scales. Scale anchoring involves selecting benchmarks

(scale points) on the TIMSS scale to be described in terms of student performance and then identifying items that students scoring at the anchor points can answer correctly. The items, so identified, are grouped by content domain within benchmarks for review by mathematics and science experts. For TIMSS (2007), the Science and Mathematics Item Review Committee conducted the review. The TIMMS (2007) benchmarks were as follows:

#### 2.5.3.1 Advanced International Benchmark (Score above 625)

Students can organise information, make generalisations, solve non-routine problems, and make and justify conclusions. They can:

- Apply their knowledge of numeric and algebraic concepts and relationships to solve problems;
- Solve simultaneous linear equations and model simple situations algebraically;
- Apply knowledge of measurement and geometry in complex problem situations;
- Interpret data from a variety of tables and graphs.

#### 2.5.3.2 High International Benchmark (Score between 550 and 625)

Students can apply their understanding and knowledge in a wide variety of relatively complex situations. They can:

- Order, relate and compute with fractions and decimals to solve word problems, operate with negative integers and solve multi-step word problems involving proportions with whole numbers;
- Solve simple algebraic problems including evaluating expressions, solving simultaneous linear equations and using a formula to determine the value of a variable;
- Find areas and volumes of simple geometric shapes and use knowledge of geometric properties to solve problems;
- Solve probability problems and interpret data in a variety of graphs and tables.

#### 2.5.3.3 Intermediate International Benchmark (Score between 475 and 550)

Students can apply basic mathematical knowledge in straightforward situations. They can:

- Add, subtract, or multiply to solve one-step word problems involving whole numbers and decimals;
- Identify representations of common fractions and relative sizes of fractions;
- Understand simple algebraic relationships and solve linear equations with one variable;

- Demonstrate understanding of properties of triangles and basic geometric concepts including symmetry and rotation;
- Recognise basic notions of probability;
- Read and interpret graphs, tables, maps and scales.

#### 2.5.3.4 Low International Benchmark (Score between 400 and 475)

Students have some basic mathematical knowledge.

### 2.5.4 Computer Aided Assessment

A frequent and usually time-consuming task for teachers is the assessment of learners' work. Assessments represent not only the basis for grading learners' work, but on a regular basis they are important to the teacher in motivating learners, providing feedback on learners' learning products and processes and evaluating the effectiveness of teaching. Assessments can also be used diagnostically to identify areas within the course where learners still have difficulties. Only if the teacher knows the learners' individual problems will the teacher be able to provide adequate help and support. From this viewpoint, individual assessment is the first step to learner-centred teaching. In general, however, a teacher cannot afford such an assessment of learners' knowledge and skills on an adequate level because of limited time. Computer Aided Assessment (CAA) has been proposed as a solution to this time consuming task. CAA refers to a number of approaches to assessing learners' performance using a computer (Chalmers & McAusland, 2002). The idea of CAA has up to now focused more on the testing of learners using Computer Adaptive Testing (CAT) by integrating interactive Computer Systems (CS) and simulators for maths assessment in a Web environment (e.g. Klai, Kolokolnikov & van den Bergh, 2000; Maplesoft, 2006). This study will bring a new perspective to the definition of CAA by including the possibility of teachers using a computer to trace or map learners' problem solving skills, by assigning scores to each of the process steps such that a teacher is able to identify, for example, the stage of Polya's (1957) four-step model at which learners struggle. This study thus expands the concept of CAA, resulting in the development of the Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool. This was designed to use Polya's (1957) framework to map the thinking processes in learners' algebraic problem solving from their written work. The researcher hopes that this tool can as well be adapted to map the problem solving skills of learners in other domains of problem solving.

## 2.6 Results from similar studies on the development and assessment of problem solving skills

Schoenfeld (1985) states that students are not actually weak at solving problems but lack the skill to marshal strategies that help them to solve particular problems. The current study will endeavor to explore the causes why learners' fail to marshal specific strategies in the problem solving process.

In her study, Saleh (2004) discovered that students who can successfully solve a problem possess good reading skills, are able to compare and contrast, have the ability to identify important aspects of a problem, are able to eliminate and create analogies and are flexible in trying various strategies. Although the current study did not investigate the learners' level of reading skills, the follow up questionnaires and task based interviews explored the aspects of language comprehension and reading skills as possible factors for the problem solving outcome.

Abd (2004) conducted a study to investigate the relationship between basic knowledge and problem solving skill in coordinate geometry. She found that students who had strong basics in geometry also had high problem solving skills. This is the same argument presented by Krulick and Rudnick (1980) that successful problem solvers have a complete and organized knowledge base. The findings also showed that students' failure in solving problems began during the reading of the question: they could not understand certain words, sentences, concepts or terms. Hardly any of the students used diagrams to understand questions and plan strategies for solving them. Learners also failed to write answers correctly and almost none rechecked their answers. The researcher believes that such a study is replicable using the CAAPSA tool since the findings that none of the learners rechecked their answers can be assessed in Polya's (1957) last step of the CAAPSA framework. The current study explored the learners' basic knowledge level (knowledge base) in order to re-test the assertions by Abd (2004) and Krulick and Rudnick (1980) on the strong correlation between knowledge base and algebraic problem solving skill levels.

In a similar research, Mahmud (2003) found that the main source of secondary school students' difficulties in solving mathematical problems was an inability to understand the

problem. She found that almost 98% of students admitted to having difficulties in comprehending what a question required. Students did not pay much attention to the strategies involved in answering the question, and did not read the terms used in the problem very carefully. This study hopes that the implementation of the CAAPSA tool as a classroom research tool will allow teachers to effectively monitor the learners' deficiencies in the development of algebraic problem solving skills at an early and remediable stage.

Research by Zakaria (2002) problem- solving skills and the ability to solve fraction problems found that most students' problem solving abilities in mathematics were low. The analysis revealed that more than half of the students could not understand the questions and did not know how to plan and implement strategies for a solution. This was probably due to a lack of prior exposure to word problems. The students were much more accustomed to answering procedural problems with a limited number of steps. The learner's deficiency in algebraic work is a repeating theme in the NSSC examiners' reports from 2010 to 2013. It is therefore on this basis that the study sought to develop an empiric tool (CAAPSA) that would be used to reliably measure the level of development of the algebraic problem solving skills of grade 12 learners of Oshana region, Namibia. Although the study did not use a sample drawn from all Namibian learners, according to DNEA (2013) the Oshana region was amongst the top performing regions in the NSSC Mathematics examinations, thus the findings of this study can be transferred to national level.

A study by Roehler (2003) revealed that in many classrooms, learners are taught to tackle word problems with specific algorithms for each problem type, with the result that they often develop a reliance on "cookie-cutter equations", without fully comprehending the underlying problem and the algebra used to solve it. Since many students do not understand the concepts behind these methods, it is important to explore a student's inherent approaches to problem solving and algebra. The student should be able to grasp the use of variables and have the ability to develop equations with or without previous formal algebraic experience. Students' intuitive techniques might indicate ways to present algebraic methods that build upon their innate skills. The current study did not investigate the actual classroom practices being implemented by teachers from where the participating learners were drawn. The review of the findings of Roehler's (2003) study was to enlighten the researcher on what previous research has found as possibly causal factors for poor performance in the algebraic problem solving

process. The results of the current research will be compared to previous findings in order to suggest avenues for future research in Namibia.

Previous studies have also revealed that children grasp algebraic concepts at an early age if algebra problems are presented in intuitive ways. Femiano (2003) theorises that even though primary grade students may lack the formal level of thinking required to efficiently solve equations, algebraic reasoning is still possible when approached in less sterile and more practical ways. Studying his own classroom of first to third grades, Femiano (2003) proposed that if equations are situated into more concrete contexts, children can more easily grasp the problem and use their intuition to find a solution. He took each aspect of mathematics in his classroom and put it into a problem solving setting. When problems are expressed in a story format, children are more likely to understand the problem and their informal, intuitive approaches provide the basis for understanding the fundamental concepts of algebra. Motivated by Femiano's (2003) insinuation that when problems are expressed in a story format, children were likely to understand the problem, the current study also employed a story format for the word problems in the algebraic problem solving test. It is hoped that the story format for the algebraic problems would be interesting to solve, hence even the weaker learners would attempt to solve all the problems.

Lian and Idris (2006) used the Structure of the Observed Learning Outcome (SOLO) model, developed by Biggs and Collis (1982) to assess Form Four (Grade 12) learners' problem solving abilities in using linear equation. The SOLO model is a cognitive psychological model which emphasises more on the internal process and more interested in how a problem is handled by learners rather than whether their answers are correct. In the area of algebra, the SOLO models have been used to describe learners' elementary equation solving skills (Biggs & Collis, 1982), but there is no coherent description of learners' algebraic solving ability sufficient to inform instruction decision (Lian & Idris, 2006). Hence Lian and Idris (2006) proposed a framework to enable upper secondary learners' algebraic problem solving skills to be described across four levels of the SOLO model. The model was used to construct test items that reflected the four levels: ***unistructural, multistructural, relational*** and ***extended abstract***. The levels are in a hierarchical manner which is increasingly complex. Learners' algebraic problem solving skills were assessed through their performance in using linear equation to solve the problem situations across the four content domains of linear equation; linear pattern (pictorial), direct variations, concepts of function and arithmetic sequence. The study by Lian and Idris (2006) was divided in two phases. In the first phase, learners were

given a written test. In the second phase, clinical interviews were conducted to seek clarification of the learners' algebraic solving processes.

This study was the closest in design to the current study. However it would appear that this SOLO framework was investigating the learners' performance for different problem solving task levels within the same domain. The framework used the Partial Credit scoring model. The Partial Credit scoring model is a conventional way of assigning varying degrees of credit to learners' attempts, for example by grading them on a scale of 1(low) to 5 (high). Most of these scoring models are available on software packages that are beyond the financial affordability and comprehension of our Namibian teachers. The CAAPSA model therefore was a worthy and cheaper alternative for the Namibian environment. The CAAPSA tool is an Excel based input-processing-output package that uses the learners' error analysis in the problem solving process to assign credits (CAAPSA levels) to the learners' solution attempts at each of Polya's stages. The CAAPSA indicators for scoring the algebraic problem solving skills of the Grade 12 learners of Oshana region , who participated in this study were derived from Lester and O'Daffer's (1987) analytic scoring rubric (see Table 3.4).

## 2.7 Conceptual Framework

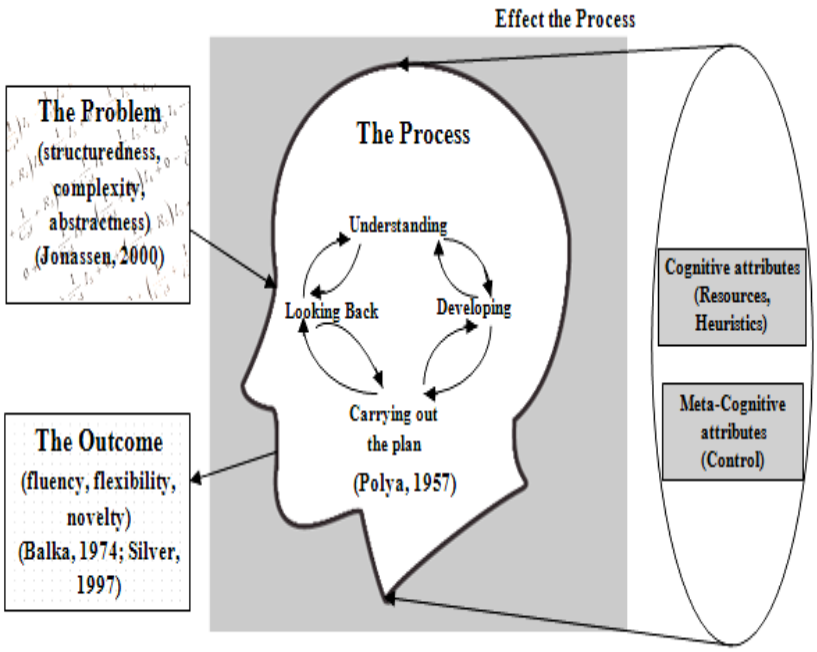
A conceptual framework is a set of broad ideas and theories taken from relevant fields of enquiry that help a researcher to properly identify the problem being studied, frame their questions and find suitable literature. Frameworks help us visualise the problem; break it down into discrete, manageable units (Ryder, 2011). The study used the Computer Aided Algebraic Problem Solving Assessment (CAAPSA) framework. The framework incorporated Polya's (1957) problem solving model, Lester and O'Daffer's (1987) analytic scoring rubric (Partial Credit model), and Schoenfeld's (1992) metacognitive theory. CAAPSA was then used as the framework to assess the level of development of the algebraic problem solving skills of Grade 12 learners of Oshana region.

### 2.7.1 The dynamic and cyclic nature of problem solving

The CAAPSA framework emphasises the dynamic and cyclic nature of problem solving. The problem is often not completely understood until the problem solver has tried and failed to arrive at a solution using different strategies. It is a series of going forward and backward among the four stages of Polya's (1957) process. Wilson, Fernandez and Hadaway (1993) provided a similar model, which includes the managerial process. Schoenfeld (1992) refers to the managerial process as metacognition. The framework shows the non-linearity of problem

solving. The clockwise and anti-clockwise nature of the cycle suggests that the problem solving process can go top-down or bottom-up with reference to Polya's (1957) model. The managerial processes or metacognition will also trigger the problem solver to jump a stage or stages. In the process of solving a problem, learners should be monitoring and keeping track of the progress to a solution. When the decisions seem not to work, the problem solver should try other alternatives or make some adjustment. Once a decision is made to go for new alternatives, the work done should not be thrown away, as there is a risk that the curtailed efforts might have led to success (Schoenfeld, 1992). Figure 2.2 shows the CAAPSA theoretical framework that combines Polya's (1957) problem solving framework, Schoenfeld's (1992) metacognitive theory

**Figure 2.2:** The CAAPSA framework for the assessment of problem solving skills drawn from Schoenfeld (1992)




The study uses the following example to illustrate the CAAPSA framework:

*A 120 cm long rope is cut into three pieces. The first piece is 2x cm, the second piece is x cm and the third is 3x cm. What is the length of the longest piece of the rope?*

(Source: The Rössing Foundation Handbook for Maths Ladder Competitions, 2009).

#### 2.7.1.1 The problem

The problem is well structured according to the NSSC curriculum objectives;

#### 2.7.1.2 The process

The process will involve the following cognitive and metacognitive attributes:

***Resources*** might be resorting to the use of algebraic representation for the unknowns;

***Heuristics*** for this problem would be the application of Polya's process. Such a heuristic may lead to the formation and solution of the equation $2x + x + 3x = 120$, resulting in the solution $x = 20$ *cm*.

***Control*** means the problem solver continues to monitor the progress towards a solution and to generate alternative plans if the current plan seems ineffective. The process might involve using the value of 'x' to calculate the value of '3x' (i.e. the length of the longest piece) and to add all three pieces to check whether the sum is 120 cm.

#### 2.7.1.3 The outcome

The outcome is the level of development of fluency, flexibility and novelty determined by a relevant achievement scale for the process. In the case of the present study, the outcome is the level of achievement indicated by the CAAPSA output at each of Polya's four process steps. After comparing the existing assessment frameworks that have been used in previous studies, namely; The National Assessment of Educational Progress (NAEP), The Programme for International Student Assessment (PISA)**,** Trends in International Mathematics and Science Study (TIMSS)**,** and the Computer Aided Assessment (CAA)**,** this study chose to adopt the TIMSS (2007) benchmarks owing to contextual similarities and the purpose of assessment and the research design of the study.

The study modified the TIMSS (2007) benchmarks slightly, from four to five, with special adjustments made to align them to the content of the NSSC mathematics curriculum. These adjustments are described later in Chapter 3 (Methodology). The developed computer aided algebraic problem solving assessment (CAAPSA) tool will hopefully facilitate easy mapping of learners' written algebraic problem solving achievement according to the five benchmarks,

using Polya's (1957) model. Although Polya's model was designed as an instructional model, through the use of CAAPSA this study will try to transform it not only into an algebraic problem solving assessment tool but also a tool for predicting mathematics achievement since this is a superior strand in Kilpatrick's model. Because of the modification of TIMSS benchmarks from four to five, this study will refer to the benchmarks and levels of algebraic problem solving skills as the CAAPSA benchmarks and levels.

## 2.8 Summary

This chapter opened with a brief analysis of the Namibia senior secondary mathematics curriculum. An overview of some of the mathematical problem solving theories that have influenced the context and orientation of the study was then presented. Findings from previous similar studies were discussed. Finally, the conceptual framework for the assessment of algebraic problem solving skills was presented and explained.

# CHAPTER THREE

# RESEARCH METHODOLOGY

## 3.1 Introduction

This chapter outlines the methodology and research design employed in this study. The methodology is discussed in terms of design and process. The chapter also discusses the selection of the research site and participants. The instruments and techniques used to collect data for the study are also explained in this section. The discussion of the pilot study, validation of instruments for the study, reliability of the instruments, the data analysis procedures and research ethics are also included in this chapter.

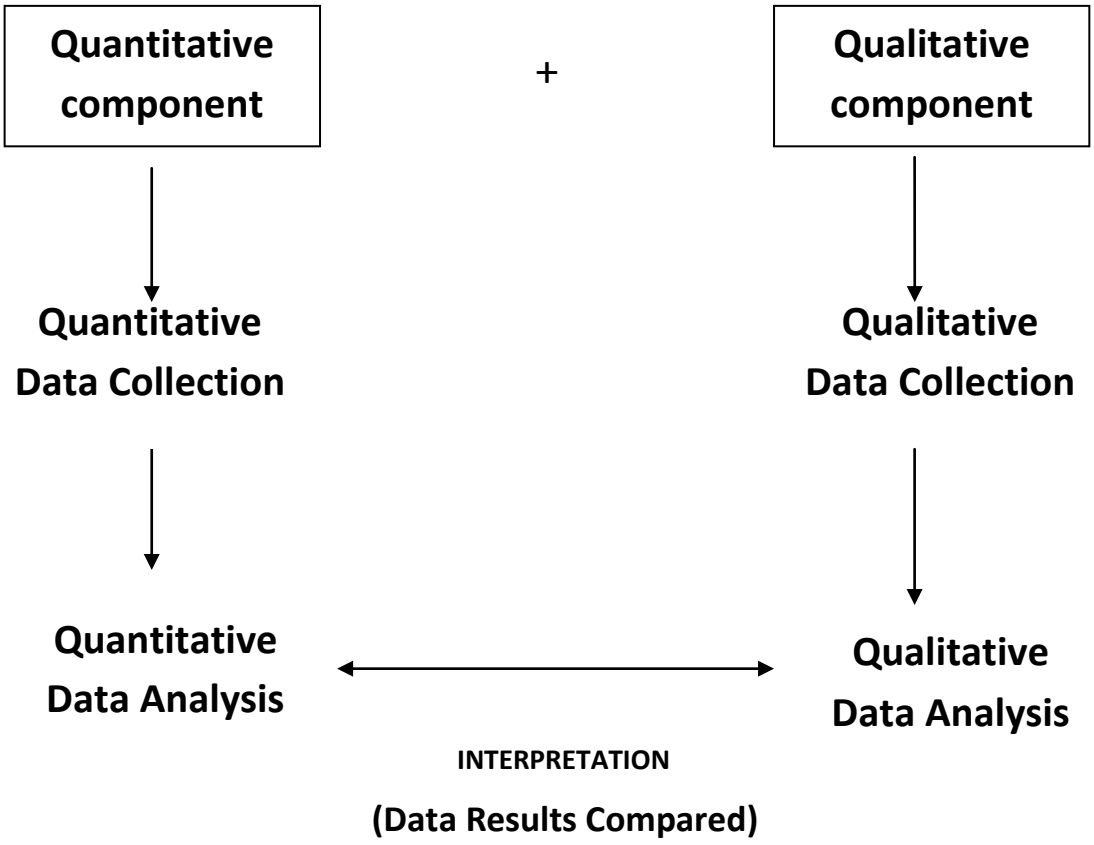
The study combined elements of Polya's (1957) problem solving model, Schoenfeld's (1992) theory of metacognition, and the CAAPSA benchmarks derived from the Trends in International Mathematics and Science Study (TIMSS, 2007) as a mind map to examine learners' responses in the algebraic problem solving domain. Response mind maps are designed to test the process rather than mere recollection of content or application of algorithms. It is in this context that the study used Polya's (1957) problem solving model to map the outcome of the problem solving process of the Grade 12 learners of Oshana region.

### 3.1.1 Research design

The study employed the triangulation mixed methods design. Mixed methods research designs combine quantitative and qualitative approaches by combining both quantitative and qualitative data in a single study (Gay, Mills & Airasian, 2012). The rationale of choosing the mixed methods research approach was to build on the synergy and strength that exists between quantitative and qualitative research methods to understand the learners' algebraic problem solving process more fully than would have been possible using either quantitative or qualitative methods alone. In addition, this report opted for this approach to validate the results obtained from various data sources (for example, see Section 3.7.3). Although this approach to researchers may appear easy, it requires a thorough understanding of both quantitative and qualitative research. In the triangulation mixed methods design, also known as the QUAN-QUAL model, quantitative and qualitative data are equally weighted and are collected concurrently throughout the same study (Gay et al., 2012). The main advantage of this method is that the strengths of qualitative data (e.g. data about the context) offset the

weaknesses of the quantitative data (e.g. external validity) and the strengths of the quantitative data (e.g. generalisability) offset the weaknesses of the qualitative data (e.g. context dependence). Thus, the QUAN-QUAL approach, is challenging in that, it requires the researcher to equally value concurrently collected quantitative and qualitative data to determine if the sources revealed similar findings. The triangulation mixed methods design is illustrated in Figure 3.1.

**Figure 3.1:** Triangulation Strategy derived from Creswell (2012)




It is on this basis that the researcher opted for the mixed methods approach; since in mapping the learners' algebraic thinking processes the scores on the algebraic problem solving achievement test would not be adequate without getting the learners to relate their own experiences as well.

#### 3.1.1.1 Quantitative approach

The quantitative component collected and employed numerical data of scores from two written tests and the 2010 NSSC examination achievement scores**.** The study analysed the learners' scores in the algebraic problem solving process in order to determine the level of development of the learners' algebraic problem solving skills according to TIMSS (2007) benchmarks. The study further analysed the correlation between the learners' achievement in the knowledge base diagnostic test and algebraic problem solving test, and the correlation between achievement in the algebraic problem solving test and the final NSSC examination in the year 2010. The correlations were determined using the Pearson product moment coefficient of correlation, noting however, that correlation does not imply causation. The investigation of the correlation between the knowledge base and algebraic problem solving test scores and the correlation between the algebraic problem solving test and NSSC examination scores was necessary to test the two theories that successful problem solvers have a well developed knowledge base (Carson, 2007) and given that problem solving is a higher strand of mathematical proficiency (Carson, 2007& Milgram, 2007), it would be expected that successful problem solvers are also mathematically proficient. The study used the 2010 NSSC ordinary level mathematics examination achievement as a measure of the learners' mathematical proficiency. The use of the two tests, the knowledge base diagnostic test and the algebraic problem solving achievement test, led to assigning the individual learners (participants) and the whole sample group to levels of algebraic problem solving skill based on the developed CAAPSA processing and output. Jackson (1995) explains that quantitative research seeks to quantify, or reflect in numbers, observations about human behaviour. This study collected quantifiable scores from learner achievement tests and analysed these scores using descriptive statistics. The inquiry was then conducted in an unbiased and objective manner, using the CAAPSA Excel data analysis tool.

#### 3.1.1.2 Qualitative approach

Qualitative research is the collection, analysis, and interpretation of comprehensive, narrative, and visual data to gain insights into a particular phenomenon of interest, in this case, the level of development of algebraic problem solving skills of the Grade 12 learners of Oshana region in the year 2010. Qualitative researchers often use multiple forms of data in any single study. They might use observations; interviews, written documents and anything else that can help answer their research question (Leedy & Ormrod, 2005). In qualitative research, the

researcher relies on the views of participants; asks broad, general questions; collects data consisting largely of words (or text) from participants; describes and analyses these words for themes; and conducts an inquiry in a subjective, biased manner (Creswell, 2012). The qualitative component employed descriptive statistics (frequencies and measures of central tendency). The qualitative data collection started with an item analysis tool for curriculum implementers to evaluate the content of the NSSC mathematics examinations, for alignment to the curriculum goals of seeking to develop problem solving skills. The qualitative research component also analysed the solution strategies used by the learners in the algebraic problem solving process, by analysing solution snippets of a purposively selected sample of 25 learners. The selection criterion was to proportionally include learners of different levels of achievement in the algebraic problem solving test. At least 10% of learners from each category of achievement, namely; category A (low achievers), category B (intermediate achievers) and category C (high achievers), were selected. This component of the study also investigated why the learners who participated in the study encountered difficulties in the algebraic problem solving process. This was achieved by administering a questionnaire and learner interviews to gather information on the learners' experiences in the problem solving process. The qualitative approach was also incorporated as a way of triangulation to substantiate the CAAPSA outcomes in the quantitative data analysis component.

## 3.2 Research site and participants

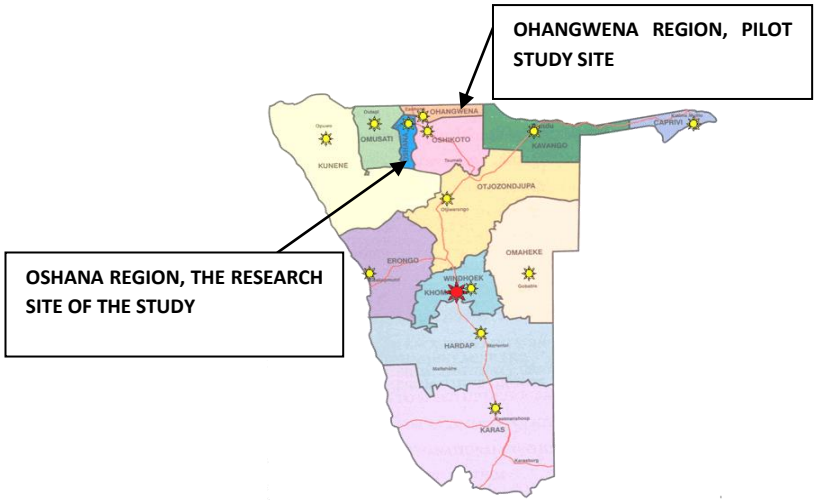
In this section the choice of research site and the sampling techniques used to select the research participants are discussed.

### 3.2.1 Research site

This study was conducted in the Oshana Region of Namibia, while the pilot study was conducted in the Ohangwena Region as indicated in Figure 3.2. The two regions were chosen because of their easy accessibility to the researcher.

Figure 3.2 shows the map, indicating the regions where both the pilot study and main study were conducted.

**Figure 3.2:** Map showing the thirteen education regions in Namibia and the sites where the pilot study and main study were conducted




### 3.2.2 Research participants

#### 3.2.2.1 Population

Participants were selected from eight secondary schools that had sat for at least two NSSC examination sessions prior to this study. The reason for selecting schools that had been implementing the NSSC curriculum for at least a period of two years prior to the 2010 academic year was to ensure that these schools were already familiar with the curriculum, and to eliminate the influence of other factors such as teacher inexperience, on the outcome of the study. The reason for selecting the Oshana region from the 13 education regions in Namibia was the accessibility and proximity of the region to the researcher's duty station. The population for this study was 1758 learners.

#### 3.2.2.2 Sample size

According to the Research Advisors (2006) table, the recommended sample size for a confidence level of 95% and a 5% margin of error is about 322 (see table in Appendix 8). The sample size was chosen using the stratified random sampling technique. Stratified random sampling is a technique which attempts to restrict the possible samples by ensuring that all parts of the population are represented in the sample in order to decrease the error between the characteristics of the sample and the population. In stratified sampling**,** the target population is first separated into mutually exclusive, homogeneous segments (strata) after which a simple random sample is selected from each segment (stratum). This sampling procedure is sometimes referred to as quota random sampling. There are two major types of

stratified sampling: **proportionate stratified sampling** and **disproportionate stratified sampling**. In proportionate stratified sampling, the number of elements allocated to the various strata is proportional to the representation of the strata in the population. In other words, the size of the sample drawn from each stratum is proportional to the relative size of that stratum in the target population. The same sampling fraction is applied to each stratum, giving every element in the population an equal chance of being selected. This sampling method is used when the purpose of the research is to estimate a population's parameters. Disproportionate stratified sampling is a stratified sampling procedure in which the number of elements sampled from each stratum is not proportional to their representation in the population.

The 1758 learners in the population were numbered from 1 to 1758 and proportional stratified sampling was used to select 20% of the learners from each of the eight schools (strata). In proportional stratified sampling, the researcher samples equally from each of the layers in the overall population. In this situation, the researcher chose the sample in accordance with the proportions of the Grade 12 mathematics learners in each school. The first step was to identify the members of each stratum (each of the schools) and then to select a random sample from each of them (Leedy & Ormrod, 2005). Table 3.1 shows how the sampling was done.

**Table 3.1:** Stratum and sample size per school

| School | Learner Codes | No of learners | Projected Sample Size (20%) |
|---|---|---|---|
| A | 1-116 | 116 | 24 |
| B | 117-194 | 78 | 16 |
| C | 195-609 | 415 | 82 |
| D | 610-962 | 353 | 71 |
| E | 963-1336 | 374 | 75 |
| F | 1337-1573 | 237 | 48 |
| G | 1574-1722 | 149 | 30 |
| H | 1723-1758 | 36 | 7 |
| TOTAL | 1-1758 | 1758 | 353 |

The selected sample size was 353 participants. However, owing to the absence of some learners from some of the activities, the sample size eventually fell to 210 learners. The researcher believes that the outcomes of the study were not significantly affected by this drop in numbers, since the sample remained representative of the selected schools.

## 3.3 Instruments

Six instruments were used in this study:

1. Knowledge base diagnostic test;
2. Algebraic problem solving achievement test;
3. Broadsheets of results, Oshana region. November 2010. NSSC Grade 12 (Full time);
4. Learner questionnaire;
5. Task based learner interviews; and,
6. NSSC ordinary level examination papers from 2007 to 2009.

### 3.3.1 Quantitative data collection

#### 3.3.1.1 Knowledge base diagnostic test

The researcher administered the knowledge base diagnostic test in order to relate achievement in the knowledge base diagnostic test to achievement in the algebraic problem solving test. Some researchers have argued that the learners' knowledge base and the transfer of that knowledge are the most essential elements in solving problems (Carson, 2007). In order to examine this theory for the case of learners in the Oshana region it was necessary to collect the data to determine the level of algebraic problem solving prerequisite knowledge as determined by the NSSC syllabus. The knowledge base diagnostic test assessed the conceptual and procedural knowledge prerequisites for the acquisition of the algebraic problem solving skills. The scores were converted to TIMSS (2007) scores, and the corresponding levels from 1 (very low) to 5 (advanced) were allocated to each learner after marking the test. The data was kept in the *Excel* CAAPSA database for further processing and analysis.

#### 3.3.1.2 Algebraic problem solving achievement test

The algebraic problem solving achievement test consisted of eight non-routine algebraic word problems. The scores from the algebraic problem solving test were also kept in the CAAPSA database for further analysis. The correlation between the knowledge base and

algebraic problem solving achievement was analysed using the Pearson correlation coefficient. The analysis was done in the CAAPSA tracer.

#### 3.3.1.3 Test construction

The two tests used in the study were standardised tests. A standardised test is a procedure designed to assess the abilities, knowledge or skills of individuals under clearly specified and controlled conditions relating to (i) construction, (ii) administration, (iii) scoring, to provide scores that derive meaning from an interpretive framework that is provided with the test (Popham, 1995). According to Mc Millan and Schumacher (2014), "standardised tests provide uniform procedures for administration and scoring" (p. 205). These two tests were traditional pen-and-paper tests. A major limitation of the pen-and-paper tests is that if a learner leaves questions unanswered, the researcher does not have any data which would reveal something about the learners' understanding of that particular problem (Kuhs, 1994). The amount of time allowed for each test was such that every participant had enough time to attempt all questions. The knowledge base diagnostic test was allocated a time of 1 hour 30 minutes while the algebraic problem solving acjievement test was allocated a time of 2 hours.

##### 3.3.1.3.1 Construction of the knowledge base diagnostic test

The first step in the construction of the knowledge base diagnostic test was to describe the domain or construct (ability, body of knowledge, and set of skills) that was to be assessed in line with the Namibian national curriculum guidelines for assessment. Table 3.2 shows the assesment domains and weighting for the knowledge base diagnostic test.

**Table 3.2:** The matrix summary of the assessment domains for the knowledge base test

| Test Item | Syllabus domain | Questions (see Appendix 1) | Total marks | % weighting |
|---|---|---|---|---|
| 1 | Numbers, fractions and percentages (including percentages in money and finance) | 1 to 22, 43, 44 | 28 | 35 |
| 2 | Indices | 34, 35, 36 | 4 | 5 |
| 3 | Ratio and proportion | 37, 38, 39, 40 | 8 | 10 |
| 4 | Formulae (including n-th term) | 23,30, 31, 32 | 8 | 10 |
| 5 | Linear equations | 24, 25, 27, 28 | 8 | 10 |

| | | | | |
|---|---|---|---|---|
| 6 | Simultaneous linear equations | 26, 29 | 7 | 8.75 |
| 7 | Algebraic manipulation and representation | 33, 46, 48, 49 | 5 | 6.25 |
| 8 | Simple word problems | 41, 42, 45, 47, 50 | 12 | 15 |
| **TOTAL** | | **50** | **80** | **100** |

#### 3.3.1.3.2 Construction of the algebraic problem solving achievement test

The algebraic problem solving test was constructed by selecting problems from The Rössing Foundation (2009) handbook of mathematics Olympiads and Ladder competitions. The questions in the handbook were designed in line with the NSSC curriculum objectives, however changing the questions from the routine classroom exercises to the non-routine algebraic word problems. For example instead of a routine question like; Solve the equation $4x + 2y = 14 + 2(x + y)$, the problem was rephrased as a mathematics word problem in the form; *In a group of cows and chickens, the number of legs is 14 more than the number of heads. How many cows are there in the group?* All the problems in the algebraic problem solving test could be solved by representing the existing relationships between the unknown and the given data with the use of equations. The problem solving process was assessed according to Polya's (1957) problem solving model, using a rubric in the CAAPSA tools (Appendix 6).

#### 3.3.1.4 Administration of tests

The two tests were administered 1 hour apart with the knowledge base diagnostic test written first. Achievement tests require uniformity in their administration. There were instructions to test takers, preliminary demonstrations and ways of handling queries were clearly specified. Furthermore, the researcher ensured that the conditions, under which the tests were administered, relating to comfort, lighting, and freedom from distraction should be the same for all examinees. Deviations in administration or in the conditions of testing would have adversely affected the interpretation of examinees' performance. In the case of this study a support mathematics teacher from each school was identified and a one-day training workshop on the administration of the tests was conducted by the researcher to ensure uniformity. The training was conducted at The Rössing Foundation Centre, during the pilot study phase. The eight support teachers sat in, and observed the process of administering the test to 24 learners who participated in the pilot study. The process of administering the test

was demonstrated by the researcher. Once the support teachers were trained, a common time frame for administering the tests was agreed upon. Each support teacher was then given a sealed envelope with the names of the sampled learners and question papers to administer to the sampled learners at their respective schools. The two tests were administered at the eight selected schools within the same time frame, as agreed on at the training workshop, in order to reduce the possibility of questions being prematurely divulged to other centres.

#### 3.3.1.5 Scoring and aggregation of scores

In order to reduce bias in the scoring caused by the influence of the researcher, the researcher developed scoring rubrics and a computer aided marking tool for test 2.

##### 3.3.1.5.1 Developmentof the assessment rubrics for the knowledge base and the algebraic problem solving process

Charles, Lester and O'Daffer (1987) offer several assessment and scoring models for measuring problem solving ability. Among these methods are two rubrics that are particularly useful: **focused holistic** and **analytic**. A focused holistic scoring rubric assigns one score to the learner's entire solution. An analytic scoring rubric assigns a score to each of several phases of the problem solution. The entire score is computed by adding the individual scores from each phase of the solution process. The advantage of the analytic scoring rubric is that it provides a good understanding of the learner's thinking process (Wesley, 1994). Many teachers develop their own analytic scoring rubrics for assessing problem solving. The advantage of developing one's own rubric is that it will be aligned with instruction in one's own particular classroom and sensitive to peculiarities of that specific problem solving activity (NCTM, 1995).

In order to measure the students' knowledge base level, the study adopted the focused holistic scoring approach in the eight skill areas. The achievement levels were then determined according to the National Curriculum Guidelines for Senior Secondary Education and the NSSC mathematics syllabus (NIED, 2005). Table 3.3 shows the rubric that was used to interpret the achievement levels in the knowledge base diagnostic test.

**Table 3.3:** Interpretation of rubric for assessment of knowledge base

| CAAPSA score | CAAPSA Level | Interpretation | Indicators |
|---|---|---|---|
| Below 400 | 1 | Very low | Very limited understanding of numbers |
| Between 400 and 475 | 2 | Low | Difficulty in performing operations with fractions and percentages |
| Between 475 and 550 | 3 | Intermediate | Can carry out basic operations with numbers, indices fractions and percentages |
| Between 550 and 625 | 4 | High | Can express basic arithmetic processes algebraically and solve simple proportion problems |
| Above 625 | 5 | Advanced | Can construct and solve linear equations from word problems and check the validity of their solutions |

This was a holistic rubric according to which marks were scored and levels determined by using CAAPSA for each of the eight skill areas assessed. But since this study sought to explore and map learners' algebraic problem solving process, the analytic scoring rubric was used to assess and score the algebraic problem solving test. The analytic rubric is based on Polya's four steps and was incorporated into the CAAPSA tool, which the researcher hopes adequately contributed to answer the research questions. In this study, the researcher adapted the analytic scoring rubric from Lester and O'Daffer (1987), redesigning it to score the problem solving process at each of Polya's four stages. The approximate indicators for scoring the process are shown in Table 3.4.

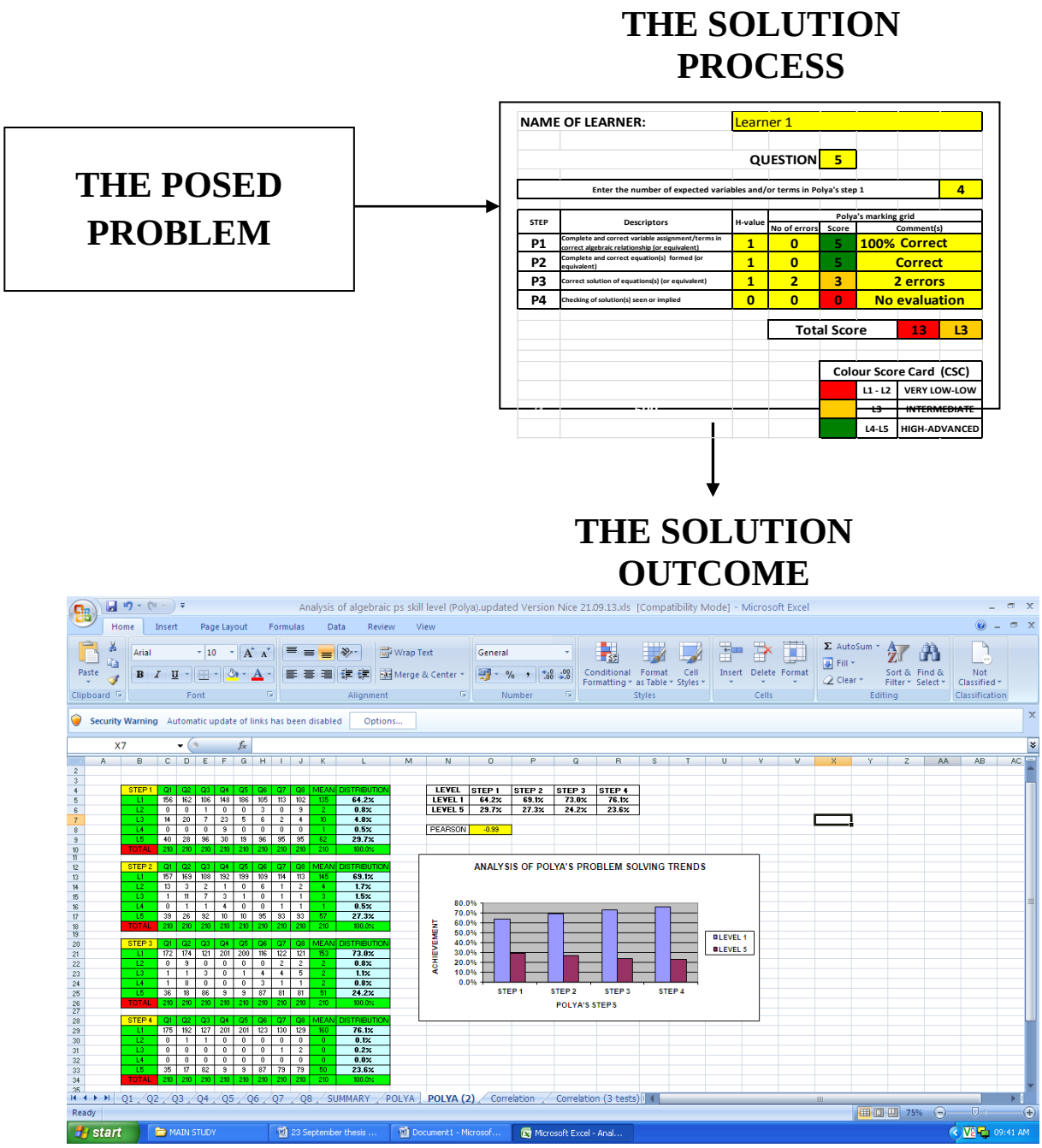
**Table 3.4:** Indicators derived from Lester and O'Daffer's (1987) analytic scoring rubric, for scoring the algebraic problem solving process

| Polya's Steps | Stages of Solving | Score | Characteristics | Indicators |
|---|---|---|---|---|
| 1 | Reading and understanding the problem | 0 | Complete misunderstanding of the problem | No attempt to solve the problem, no variable(s) assigned |
| | | 1 | Very limited understanding of the problem | Variables incorrectly assigned |
| | | 2 | Minimal understanding of the problem | Minimal but unsuccessful attempt to assign correct variables |
| | | 3 | Intermediate understanding of the problem | Some reasonable effort to assign correct variables |
| | | 4 | High level of understanding of the problem | Almost all variables identified and correctly represented |
| | | 5 | Advanced level of understanding of the problem | Correct and complete variable assignment in correct algebraic representation |
| 2 | Devising a plan to solve the problem | 0 | Complete lack of plan | No attempt to establish relationship between variables |
| | | 1 | Very limited effort to devise an appropriate plan | Very limited effort to form correct variable relationships in context of problem situation |
| | | 2 | Partially correct plan based on part of the problem being interpreted correctly | Minimal effort to establish correct variable relationships in context of problem situation |
| | | 3 | Appropriate plan based on the context of the problem but with some errors | Some reasonable effort to form correct variable relationships in context of problem situation |
| | | 4 | Plan could have led to a correct solution if properly implemented | Almost all variable relationships established in context of problem situation |
| | | 5 | Appropriate plan selected | Complete and correct relationships between variables given in context of problem situation. |
| 3 | Carrying out the plan | 0 | Plan incorrectly executed or not executed at all | No attempt or totally incorrect manipulation of established variable relations |
| | | 1 | Minimal effort to execute the plan | Minimal attempt to manipulate the established variable relations |
| | | 2 | Some minimal but inadequate execution of the plan | Minimal but inadequate attempt to correctly manipulate the established variable relations |
| | | 3 | Partially correct execution of the plan but with major omissions or errors | Partially correct manipulation of the established variable relations |
| | | 4 | Plan executed correctly but with some copying or computational errors | Almost a correct and complete manipulation of the established variable relations |
| | | 5 | Correct and complete execution of plan | Correct and complete manipulation of the established variable relations |
| 4 | Looking back | 0 | No evidence of attempt to evaluate solution | No answer or incorrect answer with no attempt to evaluate it |
| | | 1 | Minimal evidence of attempt to evaluate solution | Incorrect answer and some minimal effort to evaluate it |
| | | 2 | An inadequate attempt to evaluate solution | Incorrect answer and inadequate attempt to evaluate it |
| | | 3 | A partial attempt to evaluate solution | Incorrect answer but with a partially correct attempt to evaluate it |
| | | 4 | An almost adequate evaluation of solution | Incorrect answer but with an almost correct evaluation attempt |
| | | 5 | An adequate evaluation of solution | Correct and complete evaluation or correct answer in step 3 even with no evaluation step evident |

### 3.3.1.5.2 The computer aided algebraic problem solving assessment (CAAPSA) tool

The design of the CAAPSA tool was influenced by the three-dimensional nature of the problem solving process. The following diagrammatic illustration depicts the three phases of the CAAPSA processing. Figure 3.3 shows the main components and content of the CAAPSA tool.

**Figure 3.3:** The CAAPSA output assigns levels at each of Polya's four steps



After each problem has been marked using the CAAPSA rubric, the scores are entered into an *Excel* database. The computerised database further analyses these scores and assigns levels according to the TIMSS scale, to each of Polya's (1957) steps. Table 3.5 shows the indicators used by the CAAPSA analytical rubric in processing the scores in accordance with Polya's model:

**TABLE 3.5**: CAAPSA Scale based on Polya's Model

| CAAPSA score | CAAPSA Level | Interpretation | Indicators |
|---|---|---|---|
| Below 400 | 1 | Very low | Very limited attempt has been made to solve the problem (None of the steps in Polya's model were successfully executed) |
| Between 400 and 475 | 2 | Low | Although solutions are evidently erroneous, there is a minimal understanding of the problem (Reading and understanding of the problem evident) |
| Between 475 and 550 | 3 | Intermediate | Can devise an algebraic strategy or equivalent plan for solving the problem (Devising a plan) |
| Between 550 and 625 | 4 | High | Can carry out the plan to find an answer (Carrying out the plan) |
| Above 625 | 5 | Advanced | Can look back and check the validity of their solutions (Looking back) |

The CAAPSA tool assigns marks at each stage by analysing whether the learner has successfully managed to:

- **Polya Step 1:** Relate all parts of the problem and define the variables required to represent all the unknowns;
- **Polya Step 2:** Write an algebraic model that shows the relationships between the unknowns and the given data;
- **Polya Step 3:** Solve the equations; and,
- **Polya Step 4:** Check the solution and/or use alternative solution strategies to evaluate the solution obtained.

#### 3.3.1.5.3 The CAAPSA processing

- All input data is entered in the yellow cells on the *Excel* worksheet as shown in Figure 3.4.
- Enter the name/code of the learner;
- **Polya Step 1:** Determine and enter the number of variables required to represent all the unknowns. If there is a reasonable attempt to represent all the unknowns, then H = 1; otherwise H = 0. The processing stops. If H = 1, count and enter the number of incorrect representations of the required unknowns under errors;
- **Polya Step 2:** If there is a reasonable attempt to write a correct algebraic equation relating all the unknowns, then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the representation of the given equation;
- **Polya Step 3:** If there is a reasonable attempt to solve the equation in step 2 then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the solution of the given equation;
- **Polya Step 4:** If there is a reasonable attempt to check the solution then H = 1, otherwise H = 0. The processing stops. If H = 1, count and enter the number of errors in the checking. If the answer in step 3 is correct and no checking has been attempted, then H=1 and number of errors = 0.

In each of the cases above "H" represents a correct attempt to execute Polya's step. The decision on whether an attempt to execute Polya's step is correct or not, is determined by the descriptors in the CAAPSA marking tool. If the attempt is reasonable, then H=1, even if the actual execution of the step has errors. If the attempt is completely wrong, then H = 0.

The ability of learners to execute Polya's four steps was mapped through their algebraic representation and manipulation skills. In order to illustrate the execution of CAAPSA in assessing algebraic problem solving in Polya's model, consider the following problem in the algebraic problem solving test:

> *The sum of an even number and its consecutive even number is 54. Find the smaller of the two consecutive even numbers* (Source: the Rössing Foundation Handbook for Maths Olympiads and Ladder Competitions, 2009).

#### 3.3.1.5.3.1 Understanding the problem

The learners need to understand the mathematical concepts and/or definitions of consecutive numbers and even numbers. They should be able to assign variables to the unknowns and represent the relationship between consecutive even numbers algebraically, e.g. let the smaller number be ***2y*** and the next one ***2y + 2***, where ***'y'*** is any integer. This is in compliance with the definition that an even number is divisible by 2. Alternatively, one could simply neglect the definition and represent the smaller even number as **y**, hence the next even number as ***y+2***. In the CAAPSA tracer, the pair of variable assignments *2y* and *2y + 2* or *y* and *y + 2* are evidence that the problem has been understood. This demonstrates the comprehension of mathematical concepts, operations and relations. According to Kilpatrick (2001), the comprehension of mathematical concepts, operations and relations defines **conceptual understanding**.

#### 3.3.1.5.3.2 Devising a plan

At this stage, learners need to think of a strategy to initiate the solution process. Linking the unknown to the known data in the statement *"The sum of two consecutive even numbers is equal to 54"* suggests the use of the algebraic strategy of forming and solving an equation. In this step the respondents should be able to write the equation:

***2y + 2y + 2 = 54*** or ***y + y + 2 = 54.*** Simplifying these expressions also falls into this step, resulting in ***4y + 2 = 54*** or ***2y + 2 = 54*** or a more simplified expression.

According to Kilpatrick et al. (2001), this step defines **strategic competence**. The correct simplification of the algebraic statements reflects skills in carrying out procedures flexibly, accurately, efficiently and appropriately.

#### 3.3.1.5.3.3 Carrying out the plan

The learners should now solve the equations while monitoring their solution process. If the solution process does not yield to a sensible result in the context of the problem, then students should try to generate alternative plans and continue monitoring their new progress towards the solution. Depending on which representation is chosen, one problem solver obtains ***y =13*** from 4y ***+ 2 = 54*** **,** while the other obtains ***y = 26*** from ***2y + 2 = 54.*** A possible incorrect solution here could be caused by a learner stating that "the two consecutive even numbers are 13 and 15, and so the smaller even number is 13". This conclusion would then take the

learner to step 4, **looking back**, where the learner would be able to see that this would be in conflict with the concept of even number, and would then redirect his or her search for a new solution. For example, the learner who solves the equation $4y + 2 = 54$ and concludes that "the two consecutive even numbers are 26 and 28" must also execute the next step, **looking back**. This flexibility of approach is the major cognitive requirement for solving non-routine problems, and Kilpatrick et al. (2001) refer to it as **procedural fluency**.

#### 3.3.1.5.3.4 Looking back

In Polya's (1957) model, the problem solver would have to check whether the solution makes sense in the context of the problem. Students who disagree on a mathematical answer need not rely on checking with the teacher, collecting opinions from their classmates, or gathering data from outside the classroom. In principle, they need only check that their reasoning is valid. One manifestation of **adaptive reasoning** is the ability to justify one's work. We use ***justify*** in the sense that supports the view that the problem solver should "provide sufficient reason for". Proof is a form of justification, but not all justifications are proofs (Kilpatrick et al., 2001). In this respect, Kilpatrick's strand of adaptive reasoning is embedded in Polya's (1957) step 4. Classroom norms can be established in which learners are expected to justify and explain ideas in order to make them clear, hone their reasoning skills, and improve their conceptual understanding (Kilpatrick et al., 2001). A method of carrying out the evaluation of the solution for the problem example given in step 3 could be as follows;

> Does the solution ***"The consecutive even numbers are 26 and 28, hence the smaller even number is 26"*** make sense?

The learner would then have to check the solution against the constraints of the given problem. If the learner stops at the correct answer and does not show the evaluation of the solution in writing, the CAAPSA model would consider this step as implied from the cyclic nature of Polya's model. The assumption is that the learner would stop at the correct answer after being convinced during the monitoring and control processes that it fits the conditions in the problem.

### 3.3.1.5.4 The CAAPSA output (outcome)

Once all entries are made the CAAPSA output will be displayed as shown in Figure 3.4.

**Figure 3.4:** An example of the CAAPSA processing and output for the analysis of the solution process of a learner (L90) in problem 5, using Polya's model

| LEARNER CODE: | L90 |
|---|---|
| QUESTION | 5 |
| Enter the number of expected variables and/or terms in Polya's step 1 | 4 |

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | Correct |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | Correct |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 2 | 3 | 2 errors |
| P4 | Checking of solution(s) seen or implied | 0 | 1 | 0 | No evaluation |

| | | |
|---|---|---|
| Total Score | 13 | L3 |
| Polya's TIMSS Score | 525 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

The descriptors for scoring each of Polya's steps P1, P2, P3 and P3 have been elaborated in the description of the CAAPSA processing in section 3.3.1.5.3.

The scores at each of Polya's (1957) steps in Figure 3.4 should be interpreted as equivalent to CAAPSA level 5 if the score is 5, level 4 if the score is 4, level 3 if the score is 3, level 2 if the score is 2, and level 1 if the score obtained is either 0 or 1. The scorecard indicates red when the achievement level is at or below CAAPSA scale level 2, amber when the achievement is at CAAPSA scale level 3, and green when the achievement is at CAAPSA scale level 4 or 5. The teachers are immediately alerted by these colour outputs to the level of the algebraic problem solving skills of their learners.

#### 3.3.1.6 Analysis of the results of the 2010 Namibia Senior Secondary Certificate (NSSC) mathematics examination

All the learners who participated in this study culminated their Grade 12 mathematics course with the NSSC ordinary level final examination in November 2010. The examination was set by the Directorate of National Examinations and Assessment (DNEA). This examination was written by all ordinary level mathematics candidates in the Namibia. The current study chose to use the outcomes of this examination as an indicator of the level of mathematical proficiency attained by the learners by the end of the 2010 academic year, since the examination integrated all the curriculum assessment objectives. The 2010 NSSC ordinary level examination consisted of four papers. Papers 1 and 3 were in the core level component while papers 2 and 4 were extended level components of the mathematics curriculum. The NSSC examination was set by the Directorate of National Examinations and Assessment (DNEA) as a summative assessment of the skills outlined in the NSSC syllabus. The researcher had no access to the learners' marked scripts. Since the NSSC examination was administered by the Ministry of Education (MoE) in collaboration with the Directorate of National Examinations and Assessment (DNEA), the researcher assumed that the examination construct was valid and reliable. However the content of the papers was also analysed by a team of mathematics educators and national examiners, who consented that the standard of the examination was in line with the curriculum objectives and reflected an adequate level of emphasis on the development of learners' problem solving skills as prescribed by the NSSC curriculum documents.

The 2010 NSSC examination grades were used as a yardstick to measure the level of development of the learners' mathematical proficiency. The examination scores were then used to determine the correlation that exists between the participating learners' algebraic problem solving achievement and mathematical proficiency, in order to examine whether the findings of the study were in line with previous research suggestions that problem solving is a higher strand of mathematical proficiency (Milgram, 2007). Since the examination scores were not in numerical form, these scores were translated from letter grades to CAAPSA numeric scores and levels. The translation of NSSC examination scores to CAAPSA levels is justified by the grade descriptions in the NSSC curriculum whose grading benchmarks are similar to the ones used in the rubric for marking the algebraic problem solving achievement test. Table 3.5 shows how the examination scores were conveniently translated to CAAPSA levels in order to use them in the calculation of the Pearson product moment coefficient of

correlation between the algebraic problem solving achievement and examination achievement. Using the conveniently assigned levels of examination achievement still provided a valid analysis of the correlation between the algebraic problem solving achievement and mathematical proficiency of learners. Table 3.6 shows how the NSSC examination grades were interpreted to CAAPSA levels.

**Table 3.6:** Operational interpretation of NSSC examination grades to CAAPSA levels

| NSSC Examination Grade | CAAPSA Level |
|---|---|
| **A** | **5** |
| **B** | **4** |
| **C** | **3** |
| **D** | **2** |
| **E, F, G** | **1** |

### 3.3.2 Qualitative data collection

#### 3.3.2.1 Learner questionnaire

The learner questionnaire was used as a learner self-report instrument to gather data on the learners' strengths and weaknesses in executing Polya's (1957) problem solving steps. The questionnaire was administered to all participants 15 minutes after they had completed writing the algebraic problem solving achievement test, while the learners were still in possession of their scripts. The reseacher ensured that learners did not ammend solutions in their test answer scripts while answering the questionnaire through close monitoring of the process by all the support teachers. The inclusion of the questionnaire was for triangulation purposes, to make comparisons between the questionnaire results to the results of the quantitative data analysis. The questionnaire was divided into two sections, section A (structured questions) and section B (open-ended questions). In section A, the participants were asked to choose the "Yes" or "No" options to qualify a set of twenty statements, in four categories of five statements each. The "Yes" responses were given a score of "1" and the "No" responses a score of "0". Each category of questionnaire statements corresponds to a step in Polya's (1957) problem solving model, thus the responses evaluated the learners' experience in the execution of Polya's (1957) four steps of problem solving. The "Yes" responses represent success in the execution of Polya's steps; hence the total score at each stage reflects the level of achievement at each stage of Polya's process. For example if there are three "Yes" choices and two "No" choices to the statements corresponding to the first

step; **reading and understanding the problem**, such a learner would be at CAAPSA level 3. In section B, the learners were asked to relate their experiences in the problem solving process, explaining how they found the complexity of the solution process and why they encountered difficulties, if any. The responses in section B were coded by marking units of words and sentences that related to processes in Polya's (1957) problem solving process. The common emergent outcomes about the learners' problem solving experiences from the responses in section B were noted and summarised in Table 4.8.

#### 3.3.2.2 Task based learner interviews

Paper and pencil tests are limited in that they do not address conceptual knowledge and the process by which a learner does mathematics and reasons about mathematical ideas and situations (Maher, Powell & Uptegrove, 2011). Task based interviews have been useful in describing the learners' knowledge and providing insight into learners' mathematical thinking processes (Davis, 1984). This study purposively selected 12 learners (6% of the sampled learners) for task-based interviews. The researcher selected the learners whose questionnaire responses and written problem solving strategies and outcomes were more representative of the strategies employed by the solution outcomes from the whole sample.

The purposive sampling technique, also called judgement sampling, is the deliberate choice of participants in a research study due to the qualities the participants posess (Bernard, 2002). The researcher decides what needs to be known and sets out to find people who can and are willing to provide information that is reflective of the characteristics of a given sample (Bernard, 2002, Lewis and Shepard, 2006). In this study the participants were purposively selected on the criteria that their solution process was representative of the majority of the strategies used by the group. The type of purposive sampling used was thus the maximum variation sampling, in which the researcher searches for cases or individuals who cover the spectrum of positions and perspectives in relation to the phenomenon being studied. The researcher endeavoured to select a sample that represented all categories of learners, using their perfomance levels in the quantitative data analysis as the criteria for selection. The interviews were conducted two days after the tests had been written and marked. The sampled learners were gathered together and copies of their unmarked work handed out to them so that they could reflect back on their solution process. Follow up questions based on an analysis of snippets of solutions or questionnaire responses of a purposively selected sample of learners were posed by the researcher on a one-to-one basis. The interview was

audio recorded and later transcribed by the researcher. The aim was to clarify some of the problem solving strategies used by learners and to determine where and why they had encountered the observed or self-reported difficulties.

#### 3.3.2.3 Weighting of NSSC ordinary level examination papers from 2007 to 2009

In view of the outlined NSSC assesment objectives and the curriculum emphasis on the development of algebraic problem solving skills, the NSSC ordinary level examination papers from 2007 to 2009 were used to carry out a content analysis to determine the alignment of examination content to the assessment objectives outlined in the syllabus. A panel of six national mathematics examiners and three education officers from the National Institute for Educational Development (NIED) were asked to independently conduct an item analysis of the three years' (2007 to 2009) examination papers, to determine the extent to which the algebraic problem solving domain was assessed. The item analysis tool used a matrix in which the examination content (specific subject matter) was crossed with NSSC assessment objectives (Bloom, Hastings & Madaus, 1971). A 5 point rating scale, developed by the panel of examiners and education officers was used to estimate the weighting of prevalence of assessment of the algebraic problem solving skills in each paper. The responses were then entered in an Excel worksheet in which the weighting of the extent of the problem solving domain was determined as a percentage of all the content assessed in the examination content for each year (2007 to 2009). The mean of the ratings by the panel, was determined and used as a measure of the weighting of assessment of the algebraic problem solving domain for the period 2007 to 2009. The purpose of this instrument was to justify the rationale of the current study in terms of its alignment to the Namibian mathematics curriculum objectives. Table 3.7 shows the item analysis matrix template that was used to assess the extent to which the algebraic problem solving domain was assessed for the three year period from 2007 to 2009.

**Table 3.7:** The item analysis matrix that was used to assess the extent to which the algebraic problem solving domain featured in the NSSC ordinary level examinations from 2007 to 2009

| Assessment objective in NSSC syllabus | QUESTIONS | | | | | | | | | | | ROW TOTALS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| **Perform calculations by suitable methods.** | | | | | | | | | | | | Row 1 |
| **Interpret, transform and make appropriate use of mathematical statements expressed in words or symbols.** | | | | | | | | | | | | Row 2 |
| **Apply combinations of mathematical skills and techniques in problem solving.** | | | | | | | | | | | | Row 3 |
| **Analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution.** | | | | | | | | | | | | Row 4 |
| **Recall, apply and interpret mathematical knowledge in the context of everyday situations.** | | | | | | | | | | | | Row 5 |
| **Set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology.** | | | | | | | | | | | | Row 6 |
| **Organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms.** | | | | | | | | | | | | Row 7 |
| **Make logical deductions from given mathematical data.** | | | | | | | | | | | | Row 8 |
| **Recognise patterns and structures in a variety of situations, and make generalisations.** | | | | | | | | | | | | Row 9 |
| **Recognise and use spatial relationships in two and three dimensions, particularly in solving problems.** | | | | | | | | | | | | Row 10 |
| **Understand systems of measurement in everyday use and make use of them in the solution of problems.** | | | | | | | | | | | | Row 11 |
| **Respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form.** | | | | | | | | | | | | Row 12 |
| **Use an electronic calculator.** | | | | | | | | | | | | Row 13 |
| **Estimate, approximate and work to degrees of accuracy appropriate to the context.** | | | | | | | | | | | | Row 14 |
| **Use mathematical and other instruments to measure and draw to an acceptable degree of accuracy.** | | | | | | | | | | | | Row 15 |
| **COLUMN TOTALS** | Col 1 | Col 2 | Col 3 | Col 4 | Col 5 | Col 6 | Col 7 | Col 8 | Col 9 | Col 10 | Col 11 | |

Sum of column totals = Sum of row totals

Table 3.8 shows the rating scale that was developed and used to determine the weight of assesment of each objective

**Table 3.8:** Rating scale for weight of assessment objectives in the NSSC examination questions

| Points | Prevalence weight of assessment objective |
|---|---|
| 5 | 70%-100% |
| 4 | 60%-69% |
| 3 | 50%-59% |
| 2 | 40%-49% |
| 1 | 0%-39% |

Each member of the panel was given three sets of papers for the specified period from 2007 to 2009. The panel members then individually analysed each question, and allocated points, based on their judgement of the percentage prevalence of an assesment objective in each question. For example, starting with the 2007 question papers, the rating process involved:

1. Analyse each question against the specified assessment objectives in the matrix.
2. Use the provided rating scale to rate the prevalence of each assessment objective in each question, and enter the rating in the appropriate matrix cells, on a scale of 1 to 5;
3. Repeat the process for all the other questions in the same question paper;
4. Leave the column total and row total entries blank;
5. Repeat the process for all the other examination papers;
6. Sign, pack the completed item analysis matrices, and return to researcher.

After completion of the item analysis, the researcher collected all the tools and entered them in an Excel data sheet that was programmed to determine the row and column total totals. The mean of all corresponding matrix cell entries of the 27 completed matrices were processed into 1 matrix to determine the overall weighting for the three year period. The row totals of all rows corresponding to objectives related to the algebraic problem solving process were then used to determine the weighting of assessment objectives in the examination content (see Table 4.4), using the formula:

$$Percentage\ \ weighting\ \ of\ a\lg ebraic\ problem\ solving\ content$$
$$= \frac{Sum\ of\ row\ totals\ of\ \ all\ \ assessment\ objectives\ \ in\ the\ a\lg ebraic\ problem\ \ solving\ \ domain}{Sum\ of\ row\ totals} \times 100$$

### 3.3.2.4 Analysis of solution strategies for a purporsively selected sample of solution snippets in the algebraic problem solving achievement test

Learners' solution strategies were then determined from a purposively selected sample of solution snippets by using the problem solving strategy model in Table 3.9, derived from Jiang (2008).

**TABLE 3.9:** The problem solving strategy model

| No | Strategy category | Descriptions |
|---|---|---|
| 1 | Arithmetic method | It is used where the subject writes down a mathematical statement involving one or more operations on the numbers given in the problem (Fong & Hsui, 1999). |
| 2 | Algebraic method | It is used when one or more unknowns are chosen as variables and equation(s) is (are) set up. |
| 3 | Model drawing method | It is used when the solution is suggested by or follows a model or a diagram (Kho, 1982), as in the Singapore bar model strategy. |
| 4 | Guess-and-check | It involves the following steps:<br>(a) Make a guess of a certain answer or the unknown in the problem based on an estimation;<br>(b) Check whether the constraints given in the question or implied from some of the question statements are satisfied. If all the constraints are satisfied, the guess is correct; the answer has been obtained or can be worked out. All processes will end at this point. If the constraints are not satisfied, the guess will be refined or adjusted, and another guess will be made, then another round of guess-and-check will begin. |
| 5 | Looking for a pattern | It involves the following three steps:<br>(a) Several specific instances/special cases/particular examples of a problem are explored and listed;<br>(b) A pattern (or a conjecture, a generalisation, a hypothesis, or a common property among those special cases) is determined by investigating the special cases explored in step (a); and<br>(c) A solution to the entire problem is found by applying the generalised result in step (b). |
| 6 | Logical reasoning | Logical reasoning strategy is used when some forms of "if-then" reasoning are used (van de Walle, 1993). |
| 7 | Work backwards | Imagine that the problem is solved and work backwards step by step, until you arrive at the given data. Then you may be able to reverse your steps and thereby construct a solution to the original problem (Polya, 1973). |
| 8 | Draw a picture or diagram | Draw a picture or diagram and trace/visualise the problem situation pictorially or diagramatically until you find an answer that satisfies the constraints of the problem. |
| 9 | Use an analogue | Solve a similar, simpler problem, which might give you the clues you need to solve the original. |

The following arguments cited from Lupahla and Mogari (2012) suggest that successful problem solving depends on learners' concepts-in-action (i.e. implicit concepts that are related to a particular mathematical problem) and theorems-in-action (i.e. implicit theorems that may be used in a particular mathematical problem) (Vergnaud, 2009). Concepts-in-action help learners to conceptualise problems and theorems-in-action provide them with a range of solution strategies for solving a problem (Verzosa & Mulligan, 2012). Verzosa and Mulligan (2012) note that a learner with weak theorems-in-action has a limited range of solution strategies and, thus, can only solve certain problems.

As in routine problems, the resolution of non-routine problems also requires a problem solving strategy (Kolovou et al., 2009; Mabilangan, Limpjap & Belecina, 2011; Yeo, 2009); the use of numerous skills, which may be in combinations (Mabilangan et al., 2011); organising previous knowledge and experiences (Yeo, 2009); self-regulation (Yeo, 2009; Mabilangan et al., 2011); and analysing and modelling the problem (Kolovou et al., 2009). Selecting and using appropriate solution strategies (Cai, 2003; Kolovou et al., 2009), flexibility in using solution strategies (Elia, Van den Heuvel-Panhuizen & Kolovou, 2009), and openness in trying various solution strategies (Mabilangan et al., 2011) are also essential to successful non-routine problem solving. Furthermore, Mabilangan et al. (2011) consider conceptual understanding and procedural knowledge to be critical in solving non-routine problems. Therefore, it is argued in this study that Polya's (1957) problem-solving model is essential even in solving non-routine problems. When using the model, learners have to understand a non-routine problem, develop a plan to solve it, execute the developed plan, and verify the solution obtained. For example, the study by Yeo (2009) showed that Singaporean learners encountered difficulties when solving non-routine problems because they did not understand the given problem, lacked strategy knowledge, could not represent the problem mathematically and could not use the correct mathematics. Thus, the current study aligns itself with the view that, as in routine problems, algebraic non-routine word problem solving skills also depend largely on learners' conceptual understanding, procedural knowledge and higher-order thinking skills, including problem solving, and creative, analytic and critical thinking.

## 3.4 Validity of instruments

If researchers' interpretation of data is to be valuable, the measuring instruments used to collect the data must be both valid and reliable (Gay, Mills & Airasian, 2012). The validity of a measurement instrument is the extent to which it measures what it is actually intended to

measure (Leedy & Ormrod, 2005). Validity takes different forms, each of which is important in different situations. Validity determines whether the research truly measures that which it was intended to measure or how truthful the research results are (Joppe, 2000). The point of departure of this study was thefore to consider the validity of the instruments used for data collection as most fundamental, and the next section discusses how the validity of the instruments was ensured.

### 3.4.1 Validity of the tests

Test validity refers to the degree to which a test measures what it is supposed to measure and consequently permits appropriate interpretation of scores (Gay et al., 2012). For the two tests, the knowledge base diagnostic test and the algebraic problem solving achievement tests, the following types of validity were considered:

#### 3.4.1.1 Content validity

Content validity is the degree to which a test measures knowledge of an intended content area (Gay, Mills & Airasian, 2012). Content validity requires both item validity and sampling validity. Item validity is concerned with whether the test items are relevant to the measurement of the intended content area. Sampling validity is concerned with how well the test samples the total area being tested (Gay, Mills & Airasian, 2012). If the test samples the full content of the subject area or domain, it would have good content validity. This study used expert judgement by the same team of six NSSC national examiners and three mathematics education officers referred to in section 3.3.2.3. The team carefully reviewed the process used to develop the tests as well as the tests themselves. The panel of mathematics examiners and education officers compared the intended outcomes of the NSSC curriculum by using the syllabus and looking at the test content, and acknowledged that the domains of the syllabus and the tests were aligned.

#### 3.4.1.2 Construct validity

Construct validity reflects the degree to which a test measures an intended hypothetical construct (Gay, Mills & Airasian, 2012). Constructs are non-observable traits such as level of achievement. The construction of the problems for both the knowledge base diagnostic test and the algebraic problem solving achievement test ensured that the problems were well posed. The complexity and abstractness of the problems were also taken into account and

were verified by the same panel of six mathematics national examiners and three education officers.The constructions of the two tests was guided by the NSSC curriculum objectives.

### 3.4.2 Validity of the 2010 NSSC national examination papers

The study assumed that since these were papers set by a legitimate and credible examination body, the Directorate of National Examinations and Asessment (DNEA), all factors relating to validity of the examination had been well taken into account. Since this examination was only written in November 2010, the researcher did not have much influence in determining its content and construct. However the panel of six national examiners and three education officers used face validity to accept it as in line with the standard of papers that had been analysed for the previous three years' examinations from 2007 to 2009.

### 3.4.3 Validity of learner questionnaire

Triangulation was used as the validation process for the questionnaire, by ensuring that multiple sources of data were available to obtain a more complete picture of the phenomenon being studied. Mc Millan and Schumacher (2014) also call this form of validity the "convergent validity". When scores / results from one instrument correlate highly with those from another instrument or measure of the same construct, we have what is called convergent validity evidence (Mc Millan and Schumacher, 2014). Before the questionnaire was adopted, the questions for section A were arranged in a random order. The panel of six mathematics national examiners and three education officers were given a short lecture by the researcher, on Polya's problem solving process and the key strategies for each step. A template requiring the panel to match each statement to the appropriate step that it relates to in Polya's (1957) model, resulted in the adoption of the final questionnaire as valid (see Figures 4.9(a) and 4.9 (b)). Each of Polya's (1957) steps was matched to five statements that were used for learner self-reporting on their experiences in the problem solving process. The results summary of the learners' responses was compared to the quantitative data analyis for consistency.

### 3.4.4 Validity of task based learner interviews

The learner interviews were a follow up to the questionnaire and analysis of learners' solution snippets. The interviews were thus used as a triangulation tool, comparing the results to other sources of data used in the study. The comparison of questionnaire outcomes to the rest of the data sources was done at the pilot study phase and showed consistency with the quantitative data analysis.

**Validity of item analysis instrument used to determine alignment of examination content to the NSSC assessment objectives.**

The validity was guaranteed by the judgement by the panel of experts, who independently consented to a weighting of at least 60% of the examination content as inclined towards the assessment of problem solving skills.

### 3.5 Reliability of instruments

Reliability means dependability or trustworthiness. Reliability is the degree to which an instrument consistently measures what it is intended to measure. Reliability is expressed numerically, usually as a reliability coefficient, which is obtained by using correlation. For example, a perfectly reliable test would have a reliability coefficient of 1, meaning that learners' scores perfectly reflect their true status with respect to the variable being measured. Reliability tells us about consistency of the scores produced, while validity tells us about the appropriateness of a test. "A valid test is always reliable, but a reliable test is not always valid" (Gay, Mills & Airasian, 2012, p.165). In other words, if a test is measuring what it is supposed to be measuring, it will be reliable, but a reliable test can consistently measure the wrong thing and be invalid. In other words, reliability is necessary but not sufficient for establishing validity. In this study, the validity of the instruments was first guaranteed and then the reliability tested at the pilot study phase. The pilot study was conducted with a sample of 24 learners from a high achieving school in the Ohangwena region. The choice of a high achieving school was based on the assumption that its learners would be able to attempt most of the problems and thus profide sufficient data to answer the research questions of the study. The Ohangwena region also possessed similar characteristics to the region where the main study was conducted. Some of these similarities were teacher qualifications, physical infrastructure, teaching and learning resources and socio-economic background of the learners.

#### 3.5.1 Reliability of knowledge base diagnostic and algebraic problem solving achievement tests

The internal consistency reliability was conducted on the two tests. Internal consistency is the extent to which items in a single test are consistent among themselves and with the test as a whole. This study used the split-half technique, since internal consistency approaches require only one test technique and the content were structured questions.

### 3.5.1.1 The split half technique for the knowledge base diagnostic test

Once the two tests were marked, the 50 test items were split into two halves, the even and odd questions. The following procedure was then folowed:

1. Administer the test to the group of 24 participants
2. Divide the test into two subtests, by selecting odd items for one subtest and even items for the other subtest
3. Compute each participant's score on the two halves such that each participant will have a score for the odd items and the even items
4. Correlate the two sets of scores using Pearson's product moment correlation
5. Apply the Spearman-Brown correlation formula
6. Evaluate the results

Figure 3.5 shows a print screen of the Excel sheet showing how the split-half technique was used to calculate the Spearman-Brown reliability coefficient for the knowledge base diagnostic test items.

**Figure 3.5:** The method used to calculate the Spearman-Brown coefficient of reliability for the knowledge base diagnostic test

| # | SURNAME | NAME | Odd items | Even items |
|---|---|---|---|---|
| 1 | [illegible] | [illegible] | 16 | 16 |
| 2 | [illegible] | [illegible] | 11 | 11 |
| 3 | [illegible] | [illegible] | 8 | 4 |
| 4 | [illegible] | [illegible] | 16 | 11 |
| 5 | [illegible] | [illegible] | 26 | 23 |
| 6 | [illegible] | [illegible] | 9 | 4 |
| 7 | [illegible] | [illegible] | 10 | 13 |
| 8 | [illegible] | [illegible] | 21 | 23 |
| 9 | [illegible] | [illegible] | 11 | 8 |
| 10 | [illegible] | [illegible] | 19 | 18 |
| 11 | [illegible] | [illegible] | 16 | 13 |
| 12 | [illegible] | [illegible] | 34 | 28 |
| 13 | [illegible] | [illegible] | 27 | 17 |
| 14 | [illegible] | [illegible] | 19 | 14 |
| 15 | [illegible] | [illegible] | 15 | 11 |
| 16 | [illegible] | [illegible] | 14 | 14 |
| 17 | [illegible] | [illegible] | 30 | 24 |
| 18 | [illegible] | [illegible] | 14 | 18 |
| 19 | [illegible] | [illegible] | 11 | 10 |
| 20 | [illegible] | [illegible] | 20 | 18 |
| 21 | [illegible] | [illegible] | 14 | 21 |
| 22 | [illegible] | [illegible] | 23 | 23 |
| 23 | [illegible] | [illegible] | 23 | 11 |
| 24 | [illegible] | [illegible] | 21 | 17 |
| | | Mean | 17.83 | 15.42 |

| | |
|---|---|
| Mean correlation | 0.796207695 |
| Spearman- Brown prophecy formula | 0.886543017 |

| TOTAL CALCULATOR | |
|---|---|
| 1 | 1 |
| 2 | 0 |
| 3 | 0 |
| 4 | 1 |
| 5 | 1 |
| 6 | 0 |
| 7 | 1 |
| 8 | 0 |
| 9 | 0 |
| 10 | 0 |
| 11 | 1 |
| 12 | 1 |
| 13 | 1 |
| 14 | 1 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 1 |
| 19 | 1 |
| 20 | 1 |
| 21 | 0 |
| 22 | 0 |
| 23 | 3 |
| 24 | 0 |
| 25 | 1 |
| TOTAL | 34 |

| | |
|---|---|
| Odd | 23 |
| Even | 11 |
| TOTAL | 34 |

| TOTAL CALCULATOR | |
|---|---|
| 26 | 0 |
| 27 | 2 |
| 28 | 2 |
| 29 | 0 |
| 30 | 1 |
| 31 | 0 |
| 32 | 1 |
| 33 | 2 |
| 34 | 1 |
| 35 | 0 |
| 36 | 1 |
| 37 | 0 |
| 38 | 1 |
| 39 | 1 |
| 40 | 2 |
| 41 | 1 |
| 42 | 1 |
| 43 | 0 |
| 44 | 0 |
| 45 | 0 |
| 46 | 1 |
| 47 | 0 |
| 48 | 0 |
| 49 | 0 |
| 50 | 2 |

### 3.5.1.2 The split half technique for the algebraic problem solving achievement test

The same process that was used for determining the internal consistency of the test items in the knowledge base diagnostic test was used for the algebraic problem solving achievement test. The two halves were split and scored as shown in Figure 3.6, a snippet from the CAAPSA tool that was used to compute the reliability coefficient.

**Figure 3.6:** The method used to calculate the Spearman-Brown coefficient of reliability for the algebraic problem solving achievement test

| # | SURNAME | NAME | Odd items | Even items |
|---|---|---|---|---|
| 1 | Amakali | Saara | 20 | 23 |
| 2 | David | Simon | 2 | 3 |
| 3 | Ekandjo | Selma | 0 | 1 |
| 4 | Filiemon | Iyaloo | 24 | 26 |
| 5 | Haindongo | Hilya | 13 | 12 |
| 6 | Halwoodi | Frieda | 8 | 7 |
| 7 | Ieni | Lavinia | 12 | 13 |
| 8 | Kapuka | Lusia | 40 | 26 |
| 9 | Lazarus | Andreas | 20 | 20 |
| 10 | Mumbinda | Tunaani | 26 | 26 |
| 11 | Namundjebo | Emilia | 16 | 15 |
| 12 | Nashipili | Fredricks | 60 | 58 |
| 13 | Ndahangwapo | Nelson | 43 | 42 |
| 14 | Nepela | Nathalia | 44 | 41 |
| 15 | Nghikumbu | Epifania | 10 | 15 |
| 16 | Nghinaunye | Ally | 20 | 20 |
| 17 | Nghishidimbwa | Laino | 48 | 51 |
| 18 | Nghishitende | Pinias | 32 | 29 |
| 19 | Nghiwilepo | Petrina | 7 | 6 |
| 20 | Nghivalwa | Anna | 33 | 32 |
| 21 | Rainhard | Vainer | 18 | 14 |
| 22 | Shoongeleni | Seblon | 46 | 45 |
| 23 | Shova | Wilhelm | 8 | 17 |
| 24 | Simeon | Titus | 24 | 29 |
| | | Mean | 23.92 | 23.79 |
| | | Mean correlation | | 0.966808854 |
| | | Spearman- Brown | | 0.983124366 |

| TOTAL CALCULATOR | |
|---|---|
| 1 | 10 |
| 2 | 10 |
| 3 | 5 |
| 4 | 0 |
| 5 | 5 |
| 6 | 8 |
| 7 | 0 |
| 8 | 5 |
| TOTAL | 43 |
| | |
| Odd | 20 |
| Even | 23 |
| TOTAL | 43 |

The indicated "total calculator" was used to compute the odd and even scores for each learner when the scores for each question are entered. For example, the arrows in each case point out to specific learners' scores that were determined using the calaculator. The researcher programmed the formula to calculate the Spearman-Brown coefficient of reliability.

#### 3.5.1.3 Reliability of the 2010 NSSC examinations

The researcher had no access to the learners' scripts for the NSSC national examinations. The grading of the NSSC examinations is norm referenced, meaning a learner's assessment is compared to the perfomance of other learners. The scores are arranged in a normal distribution of the percentage of learners who receive each grade. The study assumed that since the national NSSC examination was set by a professional national examination body, DNEA, all the considerations of validity and reliability of the tests had been well looked into.

#### 3.5.1.4 Reliability of the CAAPSA output for the problem solving process

Samples of 16 snippets (2 snippets for each of the eight problems in the algebraic problem solving test) of purposively selected learners' solutions were handed out to a panel of three mathematics education officers and six NSSC mathematics national examiners who were asked to design a scoring rubric according to Polya's framework. A handout describing Polya's instructional model was distributed to the panel members for their perusal before they marked and allocated scores, using a scale of 0 to 5 points at each of Polya's steps, as determined by their rubric. The panel worked as a group and designed the rubric shown in Table 3.10.

**Table 3.10:** The CAAPSA rubric for marking the algebraic problems

| Polya's step | Description | Indicators |
|---|---|---|
| 1 | Understanding the problem (Identifying the goal) | Correct use of variables to represent the unknowns |
| 2 | Devising a plan | Correct equations/algebraic expressions leading to solution |
| 3 | Carrying out the plan | Solution of the equation in step 3 |
| 4 | Look Back | Testing the solution in the original information |

The panel were asked to mark the sample of snippets and their scoring of the problem solving process at each stage of Polya's steps was compared to the CAAPSA output. The panel's allocation of marks for the sample of the 16 snippets generally matched with the CAAPSA

output, hence the study considered CAAPSA reliable from the judgement of the panel of experts (For example, see Figure 4.8).

### 3.5.2 Reliability of learner questionnaire

The questions in section A of the questionnaire were validated by a panel of experts, and since according to Gay et al. (2012), validity implies reliability, the study considers that the questionnaire was reliable. The questionnaire reliability was tested at the pilot study phase through triangulation of data. The questionnaire responses were compared to outcomes from quantitative data analyisis and the results were consistent.

### 3.5.3 Reliability of learner interviews

The interviews were a follow up to the problem solving process and the learner questionnaire. Since the interview questions were drawn from objectively observed outcomes, thus the reliability of the interviews was guaranteed to give more clarify. All interviews were conducted by the researcher to enhance consistency. The researcher used an interview guide to preserve similarity and relevance in quality and style of questioning. The interviews were more for triangulation purposes, to compare the findings to those from the quantitative data analysis.

### 3.5.4 Reliability of the item analysis matrix used to examine the alignment between the NSSC examination content and the curriculum assessment objectives.

The item analysis matrix was subjective; however given the overall opinion from the independent analyses by the panel of experts, the study accepted the item analysis matrix as reliable.

## 3.6 Ethical issues

Whenever human beings are the focus of investigation, we must look closely at the ethical implications of what we are proposing to do (Leedy & Ormrod, 2005: 101). This implies that a researcher has the obligation to protect the anonymity of his or her research participants and to keep the data confidential (Frankfort-Nachmias & Nachmias, 1992). The research addressed the ethical considerations discussed below.

### 3.6.1 Negotiate access

The researcher applied to the Permanent Secretary of Education for permission to conduct the research in sampled schools. Permission was granted and copied to the Regional Director of Education for the region where the study was to be carried out. The researcher met the sampled learners and explained the aim of the study and emphasised that their participation was completely voluntary. All the sampled learners volunteered to participate and letters of consent to participate were distributed to their parents/guardians. All the parents responded positively and consented to the participation of the learners.

### 3.6.2 Anonymity

For the sake of anonymity the names of learners who participated in this study were coded L1 to L210. The eight schools were also coded as schools A to H.

## 3.7 Summary

This chapter presented the research design, research population and sample. It also discussed the instruments used in the study, the research methodology, validity and reliability of the instruments and finally the ethical considerations. All the ethical considerations and concerns were addressed prior to the commencement of the study.

# CHAPTER FOUR

# DATA PRESENTATION AND ANALYSIS

## 4.1 Introduction

This chapter discusses the analysis of data. Once a reader fully understands just what the problem is and the manner in which it was investigated, the next question is what the evidence is (Leedy & Ormrod, 2005). For the most part, the data are presented in terms of the problem. Leedy and Ormrod (2005) suggest that after gathering a mass of data, it should then be codified, arranged and separated into subsets, each of which corresponds to a particular part of the problem being studied. The problem is thus expressed in sub-problems to facilitate the management of the whole problem. Then there is a one-to-one correspondence: specific data relate to each sub-problem. The data should be presented in a logical sequence within the report. As each sub-problem and its attendant data are discussed, it is helpful to restate at the beginning of each such discussion the sub-problem in the exact wording in which it appeared in the first section of the study. This keeps the reader oriented to the progress of the research as it is being reported (Leedy & Ormrod, 2005).

In the data presentation, the researcher struck a balance between providing too much detail and too little. The researcher therefore focused on presenting only the data that is relevant to the research problem: **what is the state of the algebraic problem solving skills of Grade 12 learners in the Oshana region?** Each set of data was related to the relevant research sub-questions posed in the first chapter.

The data analysis was done in two phases, namely; analysis of data from pilot study and analysis of data from main study. The analysis of data from the pilot study focused on testing the reliability of the instruments used. The analysis of data from the main study compared the results of the qualitative and quantitative data, as a way of triangulation. Triangulation is the process of using multiple methods, data collection strategies, and data sources to obtain a more complete picture of what is being studied and to cross-check information (Gay, Mills & Airasian, 2012) For example; the interviews with learners were used to contribute to understanding of the solution strategies observed in the learners' written work.

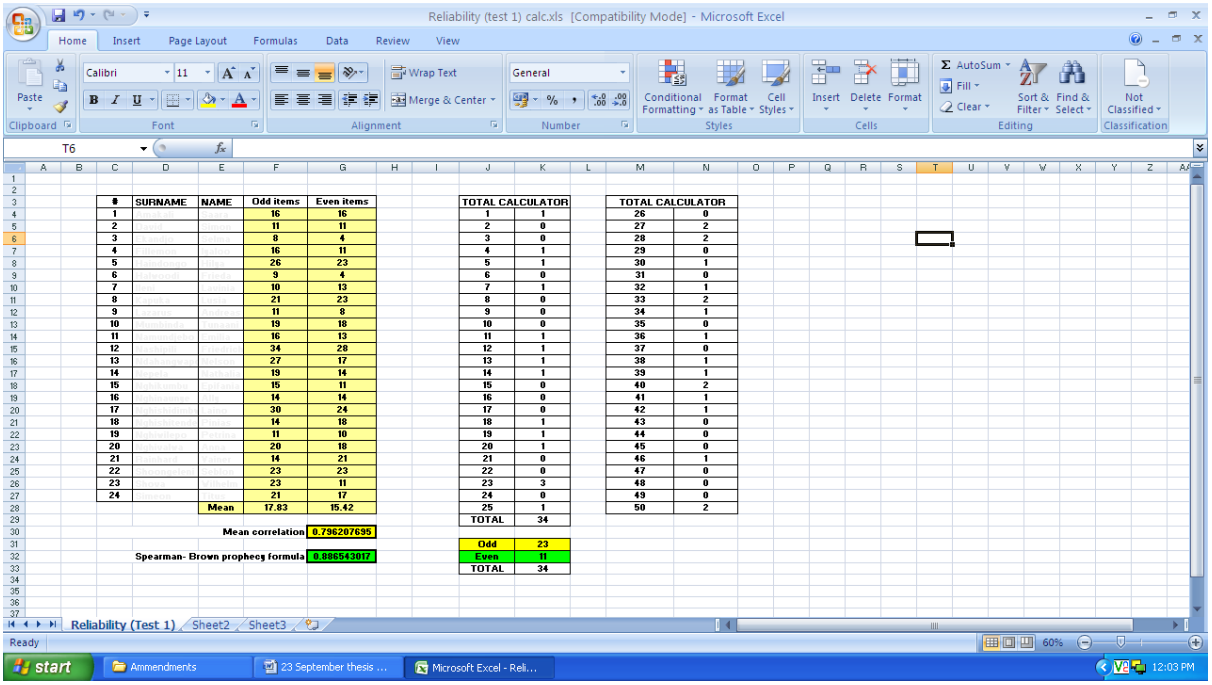
## 4.2 Pilot study data analysis

### 4.2.1 Results of Spearman Brown's split half technique for reliability of tests

#### 4.2.1.1 Reliability of knowledge base diagnostic test

Figure 4.1 shows the print screen of the Excel sheet that was designed and used to enter the scores of each learner in the knowledge base diagnostic test. The test had 50 items. The "total calculator" was used to enter the scores per question, for each learner, and then the odd and even totals are automatically returned below the "total calculator". The odd and even totals were then re-entered manually next to the learners' number (the original names of the learners have been obscured for anonymity). Once all the data entries were completed the Excel sheet returned the value of the Spearman – Brown reliability coefficient in the corresponding cell below.

**Figure 4.1:** The CAAPSA print screen for the calculation of the Spearman Brown reliability coefficient for the knowledge base diagnostic test



| # | SURNAME | NAME | Odd items | Even items |
|---|---|---|---|---|
| 1 | | | 16 | 16 |
| 2 | | | 11 | 11 |
| 3 | | | 8 | 4 |
| 4 | | | 16 | 11 |
| 5 | | | 26 | 23 |
| 6 | | | 9 | 4 |
| 7 | | | 10 | 13 |
| 8 | | | 21 | 23 |
| 9 | | | 11 | 8 |
| 10 | | | 19 | 18 |
| 11 | | | 16 | 13 |
| 12 | | | 34 | 28 |
| 13 | | | 27 | 17 |
| 14 | | | 19 | 14 |
| 15 | | | 15 | 11 |
| 16 | | | 14 | 14 |
| 17 | | | 30 | 24 |
| 18 | | | 14 | 18 |
| 19 | | | 11 | 10 |
| 20 | | | 20 | 18 |
| 21 | | | 14 | 21 |
| 22 | | | 23 | 23 |
| 23 | | | 23 | 11 |
| 24 | | | 21 | 17 |
| | | Mean | 17.83 | 15.42 |

Mean correlation 0.796207695

Spearman- Brown propheeg formula 0.886543017

| TOTAL CALCULATOR | |
|---|---|
| 1 | 1 |
| 2 | 0 |
| 3 | 0 |
| 4 | 1 |
| 5 | 1 |
| 6 | 0 |
| 7 | 1 |
| 8 | 0 |
| 9 | 0 |
| 10 | 0 |
| 11 | 1 |
| 12 | 1 |
| 13 | 1 |
| 14 | 1 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 1 |
| 19 | 1 |
| 20 | 1 |
| 21 | 0 |
| 22 | 0 |
| 23 | 3 |
| 24 | 0 |
| 25 | 1 |
| TOTAL | 34 |

| | |
|---|---|
| Odd | 23 |
| Even | 11 |
| TOTAL | 34 |

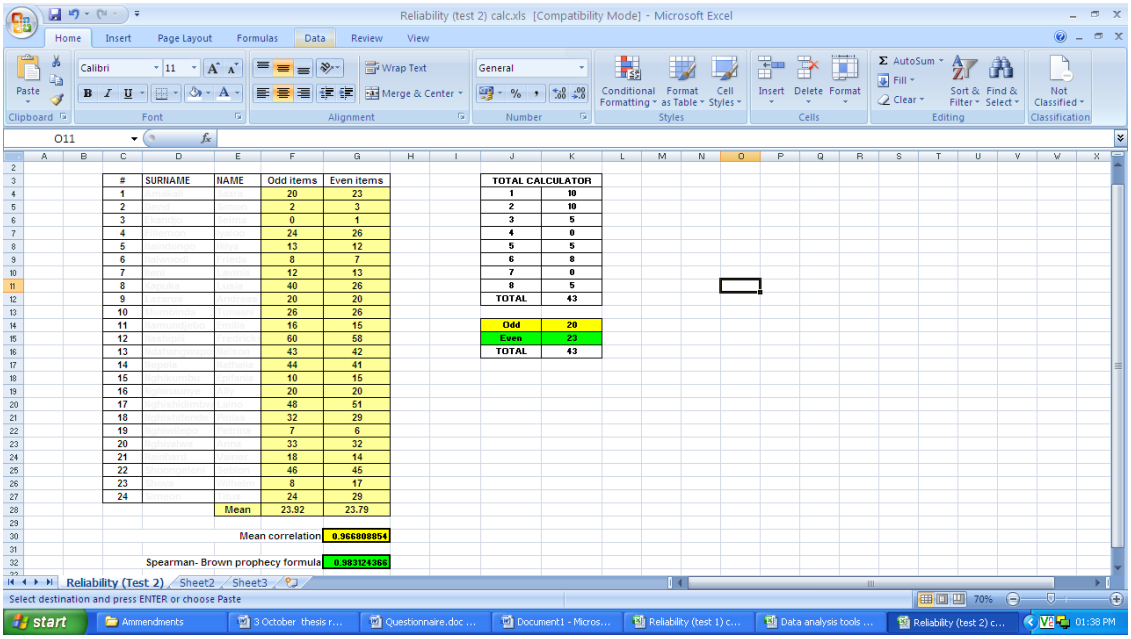
| TOTAL CALCULATOR | |
|---|---|
| 26 | 0 |
| 27 | 2 |
| 28 | 2 |
| 29 | 0 |
| 30 | 1 |
| 31 | 0 |
| 32 | 1 |
| 33 | 2 |
| 34 | 1 |
| 35 | 0 |
| 36 | 1 |
| 37 | 0 |
| 38 | 1 |
| 39 | 1 |
| 40 | 2 |
| 41 | 1 |
| 42 | 1 |
| 43 | 0 |
| 44 | 0 |
| 45 | 0 |
| 46 | 1 |
| 47 | 0 |
| 48 | 0 |
| 49 | 0 |
| 50 | 2 |

The Spearman – Brown formula yielded a reliability coefficient of 0.89. This is an indication that the 50 items in the knowledge base dignostic test were consistent among themselves and the test as a whole.

### 4.2.1.2 Reliability of Algebraic Problem Solving Test

Figure 4.2 shows the print screen of the Excel sheet that was designed and used to enter the scores of each learner in the algebraic problem solving test. The test had eight problems. The "total calculator" was used to enter the scores for each problem, for each learner, and then the odd and even totals are automatically returned below the "total calculator". The odd and even totals were then re-entered manually next to the learners' number. Once all the data entries were completed the Excel sheet returned the value of the Spearman – Brown reliability coefficient in the corresponding cell.

**Figure 4.2:** The CAAPSA print screen for the calculation of the Spearman Brown reliability coefficient for the algebraic problem solving achievement test



| # | SURNAME | NAME | Odd items | Even items |
|---|---|---|---|---|
| 1 | | | 20 | 23 |
| 2 | | | 2 | 3 |
| 3 | | | 0 | 1 |
| 4 | | | 24 | 26 |
| 5 | | | 13 | 12 |
| 6 | | | 8 | 7 |
| 7 | | | 12 | 13 |
| 8 | | | 40 | 26 |
| 9 | | | 20 | 20 |
| 10 | | | 26 | 26 |
| 11 | | | 16 | 15 |
| 12 | | | 60 | 58 |
| 13 | | | 43 | 42 |
| 14 | | | 44 | 41 |
| 15 | | | 10 | 15 |
| 16 | | | 20 | 20 |
| 17 | | | 48 | 51 |
| 18 | | | 32 | 29 |
| 19 | | | 7 | 6 |
| 20 | | | 33 | 32 |
| 21 | | | 18 | 14 |
| 22 | | | 46 | 45 |
| 23 | | | 8 | 17 |
| 24 | | | 24 | 29 |
| | | Mean | 23.92 | 23.79 |

Mean correlation: 0.966808854

Spearman- Brown prophecy formula: 0.983124366

| TOTAL CALCULATOR | |
|---|---|
| 1 | 10 |
| 2 | 10 |
| 3 | 5 |
| 4 | 0 |
| 5 | 5 |
| 6 | 8 |
| 7 | 0 |
| 8 | 5 |
| TOTAL | 43 |

| | |
|---|---|
| Odd | 20 |
| Even | 23 |
| TOTAL | 43 |

The Spearman – Brown formula yielded a reliability coefficient of 0.98. This is an indication that the items in the algebraic problem solving achievement test were consistent among themselves and the test as a whole.

### 4.2.1.3 Reliability of the learner questionnaire

The reliability of the learner questionnaire was analysed by comparing the questionnaire data to the quantitative data. The learners' questionnaire responses indicated that the main difficulties encountered by the participants in the pilot study emanated from failure to understand the problems and to use variables to represent the relationships between the

known and unknown data in the problems. The analysis of the questionnaire responses indicated that 74% of the learners did not understand the problems.

The self report by the pilot study participants on their problem solving experience and execution of Polya's steps was in line with the findings from the quantitative data analysis**.** Table 4.1 shows how the participants in the pilot study group perfomed in each of Polya's (1957) problem solving steps.

**Table 4.1:** Analysis of achievement of the pilot study participants in the problem solving test, according to Polya's model

| POLYA'S STEP | FREQUENCY OF CAAPSA ACHIEVEMENT LEVELS IN POLYA'S STEPS | | | | | TOTAL |
|---|---|---|---|---|---|---|
| | LEVEL 1 | LEVEL 2 | LEVEL 3 | LEVEL 4 | LEVEL 5 | |
| 1 | 57.29% | 6.25% | 3.65% | 0.00% | 32.81% | 100.00% |
| 2 | 61.98% | 9.38% | 1.56% | 1.04% | 26.04% | 100.00% |
| 3 | 68.23% | 7.81% | 1.04% | 0.52% | 22.40% | 100.00% |
| 4 | 77.60% | 0.52% | 0.52% | 0.00% | 21.35% | 100.00% |
| MEAN % | 66.28% | 5.99% | 1.69% | 0.39% | 25.65% | 100.00% |

Table 4.1 shows that about 63.54% of the participants in the pilot study failed to execute Polya's first step, reading and understanding of the problems. Only 36.46% of the pilot study participants could correctly execute Polya's first step. This was in line with the learners' questionnaire self-report in which 64% indicated that they encountered difficulties in executing the first step. The "total" column in the table is a verification column to ensure that all participants' scores were included.

## 4.3 Main study data analysis

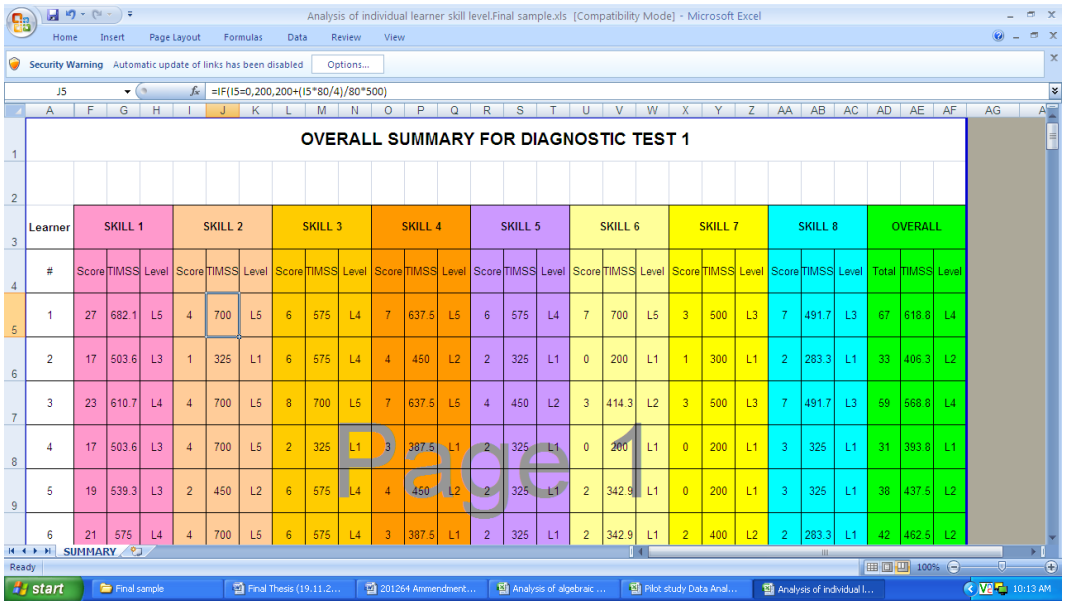
### 4.3.1 Analysis of quantitative data

This section consisted of the presentation and analysis of data from the two tests, the knowledge base diagnostic test and the algebraic problem solving achievement test. The correlation between the achievements of participants in these tests was also determined. Any correlation between the algebraic problem solving achievement and the final 2010 examination achievement was also investigated to test some of the previous findings on the relationship between the levels of development of problem solving skills and mathematical proficiency.

#### 4.3.1.1 Analysis of learners' perfomance in the knowledge base diagnostic test

Figure 4.3 shows the Excel summary sheet that was used to process and analyse the achievement in the knowledge base diagnostic test.

**Figure 4.3:** CAAPSA Print screen showing how the assessment of the knowledge base was processed



OVERALL SUMMARY FOR DIAGNOSTIC TEST 1

| Learner | SKILL 1 | | | SKILL 2 | | | SKILL 3 | | | SKILL 4 | | | SKILL 5 | | | SKILL 6 | | | SKILL 7 | | | SKILL 8 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| 1 | 27 | 682.1 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 7 | 491.7 | L3 | 67 | 618.8 | L4 |
| 2 | 17 | 503.6 | L3 | 1 | 325 | L1 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 2 | 283.3 | L1 | 33 | 406.3 | L2 |
| 3 | 23 | 610.7 | L4 | 4 | 700 | L5 | 8 | 700 | L5 | 7 | 637.5 | L5 | 4 | 450 | L2 | 3 | 414.3 | L2 | 3 | 500 | L3 | 7 | 491.7 | L3 | 59 | 568.8 | L4 |
| 4 | 17 | 503.6 | L3 | 4 | 700 | L5 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 31 | 393.8 | L1 |
| 5 | 19 | 539.3 | L3 | 2 | 450 | L2 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 325 | L1 | 2 | 342.9 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 38 | 437.5 | L2 |
| 6 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 3 | 387.5 | L1 | 2 | 325 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 42 | 462.5 | L2 |

The *Excel* sheet in Figure 4.3 contains all 210 learners' scores in the knowledge base diagnostic test. The assessment was split into the eight skills areas;

Skill 1: Numbers, fractions and percentages (including percentages in money and finance)
Skill 2: Indices
Skill 3: Ratio and proportion
Skill 4: Formulae (including n-th term)
Skill 5: Linear equations
Skill 6: Simultaneous linear equations
Skill 7: Algebraic manipulation and representation
Skill 8: Simple word problems

The CAAPSA output for the assessment of the knowledge base is summarised in Table 4.2. The table shows the achievement in each of the eight knowledge base domains that were

identified as pre-requisites for a successful algebraic problem solving process. For each skill area, the group's achievement was computed, in terms of the CAAPSA scores and levels.

**Table 4.2:** Overall group achievement level in the knowledge base diagnostic test

| TIMSS | SKILL 1 | SKILL 2 | SKILL 3 | SKILL 4 | SKILL 5 | SKILL 6 | SKILL 7 | SKILL 8 | OVERALL |
|---|---|---|---|---|---|---|---|---|---|
| **Score** | **483.8** | **539.9** | **471.2** | **492.6** | **433** | **399** | **406.2** | **376.6** | **452.8** |
| **Level** | **3** | **3** | **2** | **3** | **2** | **1** | **2** | **1** | **2** |

The table shows that learners generally performed better in indices, transformation of formulae and calculations of the n-th term of given number sequences, numbers, fractions and percentages. Their performance was not as good in linear equations, ratio and proportion and algebraic representation and manipulation. Their performance was very poor in solving simultaneous linear equations and solving routine algebraic word problems. The overall group's CAAPSA level of algebraic knowledge base was low (Level 2). This is an indication that the learners' knowledge base was not well developed.

#### 4.3.1.2 Analysis of the learners' performance in the algebraic problem solving test

Table 4.3 shows the summary of perfomance of learners at each of Polya's stages.The learners were put in three categories, based on their overall achievement levels in the algebraic problem solving test, namely:

**CATEGORY A (Lower tier)**: Learners at level 1 and 2 (very low and low). These are learners whose TIMSS score was lower than 475.

**CATEGORY B (Intermediate tier):** Learners at level 3 (intermediate). These are learners who scored between 475 and 550 on the TIMSS scale.

**CATEGORY C (Upper tier)**: Learners at levels 4 and 5 (high to advanced). These learners scored above 550 on the TIMSS scale.

**Table 4.3:** Excel bitmap image from CAAPSA showing the percentage distribution of learners by achievement at each of Polya's steps. N represents the number of learners per category who were unsuccessful at each of Polya's steps.

| POLYA'S PROBLEM SOLVING STEP | CATEGORY A | | | | CATEGORY B | | CATEGORY C | | | | OVERALL | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | LEVEL 1 | | LEVEL 2 | | LEVEL 3 | | LEVEL 4 | | LEVEL 5 | | | |
| | N | % | N | % | N | % | N | % | N | % | N | % |
| Reading and understanding | 135 | 64.3% | 2 | 1.0% | 10 | 4.8% | 1 | 0.5% | 62 | 29.5% | 210 | 100% |
| Devising a plan | 145 | 69.0% | 4 | 1.9% | 3 | 1.4% | 1 | 0.5% | 57 | 27.1% | 210 | 100% |
| Carrying out the plan | 153 | 72.9% | 2 | 1.0% | 2 | 1.0% | 2 | 1.0% | 51 | 24.3% | 210 | 100% |
| Looking back | 160 | 76.2% | 0 | 0.0% | 0 | 0.0% | 0 | 0.0% | 50 | 23.8% | 210 | 100% |
| Mean | **148** | **70.6%** | **2** | **1.0%** | **4** | **1.8%** | **1** | **0.5%** | **55** | **26.2%** | **210** | **100%** |

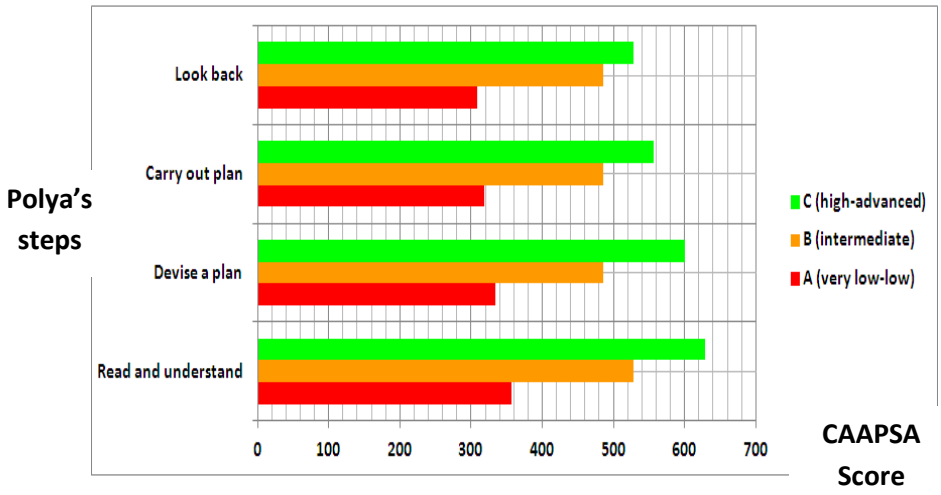
| SUMMARY | |
|---|---|
| CATEGORY A | 71.5% |
| CATEGORY B | 1.8% |
| CATEGORY C | 26.7% |
| TOTAL | 100.0% |

The results show 65.3% of the learners failed to execute Polya's first step, reading and understanding the problem, 70.9% failed to devise a plan of solution, 73.9% failed to carry out their solution plan and 76.2% could not evaluate their solution. Figure 4.4 shows the achievement pattern through Polya's steps by the three categories of learners, in the algebraic problem solving achievement test.

#### 4.3.1.3: Summary of achievement by learner categories at each of Polya's steps

Figure 4.4 shows the summary of participants' achievement in the algebraic problem solving test at each of Polya's problem solving steps.

**Figure 4.4:** Summary of achievement by each category of learners at each of Polya's steps



The summary suggests that the achievement level across Polya's steps will either decrease or remain constant. It is not possible that a learner who has performed poorly in the first steps ends up scoring higher in the later steps. The trend also implies that for learners to be competent problem solvers they should execute Polya's steps in a cyclic rather than linear manner. This implies that the execution of Polya's steps is not just a simple top-down process of the four stages, all the phases get mixed up and are carried out in a cyclic manner, such that each new discovery tends to modify the overall plan (Polya, 1957).

Further analysis of learners' perfomance in each of the eight problems in the algebraic problem solving test was done. The reason for the analysis of perfomance in each question was to understand if the perfomance was influenced by the way the problems were designed, for example in problem 3, a diagram was used to illustrate the situation in the problem. Figure 4.5 shows a summary of distribution of learners by perfomance in each problem.

**Figure 4.5:** Analysis of perfomance of learners in each of the problems at each of Polya's steps

| STEP 1 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | MEAN | DISTRIBUTION |
|---|---|---|---|---|---|---|---|---|---|---|
| L1 | 156 | 162 | 106 | 148 | 186 | 105 | 113 | 102 | 135 | 64.2% |
| L2 | 0 | 0 | 1 | 0 | 0 | 3 | 0 | 9 | 2 | 0.8% |
| L3 | 14 | 20 | 7 | 23 | 5 | 6 | 2 | 4 | 10 | 4.8% |
| L4 | 0 | 0 | 0 | 9 | 0 | 0 | 0 | 0 | 1 | 0.5% |
| L5 | 40 | 28 | 96 | 30 | 19 | 96 | 95 | 95 | 62 | 29.7% |
| TOTAL | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 100.0% |

| STEP 2 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | MEAN | DISTRIBUTION |
|---|---|---|---|---|---|---|---|---|---|---|
| L1 | 157 | 169 | 108 | 192 | 199 | 109 | 114 | 113 | 145 | 69.1% |
| L2 | 13 | 3 | 2 | 1 | 0 | 6 | 1 | 2 | 4 | 1.7% |
| L3 | 1 | 11 | 7 | 3 | 1 | 0 | 1 | 1 | 3 | 1.5% |
| L4 | 0 | 1 | 1 | 4 | 0 | 0 | 1 | 1 | 1 | 0.5% |
| L5 | 39 | 26 | 92 | 10 | 10 | 95 | 93 | 93 | 57 | 27.3% |
| TOTAL | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 100.0% |

| STEP 3 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | MEAN | DISTRIBUTION |
|---|---|---|---|---|---|---|---|---|---|---|
| L1 | 172 | 174 | 121 | 201 | 200 | 116 | 122 | 121 | 153 | 73.0% |
| L2 | 0 | 9 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0.8% |
| L3 | 1 | 1 | 3 | 0 | 1 | 4 | 4 | 5 | 2 | 1.1% |
| L4 | 1 | 8 | 0 | 0 | 0 | 3 | 1 | 1 | 2 | 0.8% |
| L5 | 36 | 18 | 86 | 9 | 9 | 87 | 81 | 81 | 51 | 24.2% |
| TOTAL | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 100.0% |

| STEP 4 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | MEAN | DISTRIBUTION |
|---|---|---|---|---|---|---|---|---|---|---|
| L1 | 175 | 192 | 127 | 201 | 201 | 123 | 130 | 129 | 160 | 76.1% |
| L2 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1% |
| L3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0.2% |
| L4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0% |
| L5 | 35 | 17 | 82 | 9 | 9 | 87 | 79 | 79 | 50 | 23.6% |
| TOTAL | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 100.0% |

| OVERALL | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | MEAN | DISTRIBUTION |
|---|---|---|---|---|---|---|---|---|---|---|
| L1 | 165 | 174 | 116 | 186 | 197 | 113 | 120 | 116 | 148 | 70.6% |
| L2 | 3 | 3 | 1 | 0 | 0 | 2 | 1 | 3 | 2 | 0.8% |
| L3 | 4 | 8 | 4 | 7 | 2 | 3 | 2 | 3 | 4 | 1.9% |
| L4 | 0 | 2 | 0 | 3 | 0 | 1 | 1 | 1 | 1 | 0.5% |
| L5 | 38 | 22 | 89 | 15 | 12 | 91 | 87 | 87 | 55 | 26.2% |
| TOTAL | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 100.0% |

Learners generally perfomed better in questions 3, 6, 7, and 8. In question 3, 44.3% of the learners attained at least CAAPSA level 3 (intermediate), in question 6, 45.2% of the learners attained at least level 3, in question 7, 42.9%were at or above level 3, and in question 8, 43.3% were at or above level 3. The "total" row is used for checking if the frequencies tally with the number of participants. A purposively selected sample of learners was interviewed to

get clarity on the possible reasons for their better perfomance in these questions. The results are discussed later in the qualitative data analysis section.

Figure 4.5 shows that problems 1, 2, 4 and 5 presented learners with difficulties in the reading and understanding of the problems. 156 learners (74.3%) failed to execute Polya's first step in problem 1, 162 learners (77.1%) failed to execute Polya's first step in problem 2, 148 learners (70.5%) failed to execute Polya's first step in problem 4, and problem 5 was the worst perfomed with 186 learners (88.6%) failing to execute Polya's first step. Explanations for the poor perfomance in these problems were also sought through follow up interviews, for a purposively selected sample of learners. The results are discussed later under the qualitative data analysis section.

#### 4.3.1.4 Overall summary of algebraic problem solving achievement

Figure 4.6 shows the summary of the overall group perfomance in the algebraic problem solving achievement test.

**Figure 4.6:** CAAPSA output for assessment of the algebraic problem solving skill

OVERALL CAAPSA SUMMARY FOR PERFOMANCE IN THE AGEBRAIC PROBLEM SOLVING TEST

| | QUESTION 1 | | | QUESTION 2 | | | QUESTION 3 | | | QUESTION 4 | | | QUESTION 5 | | | QUESTION 6 | | | QUESTION 7 | | | QUESTION 8 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 3.948 | 298.69 | L1 | 2.9 | 272.5 | L1 | 8.78095 | 419.524 | L2 | 2.03333 | 250.833 | L1 | 1.21905 | 230.476 | L1 | 8.98095 | 424.524 | L2 | 8.46667 | 411.667 | L2 | 8.61905 | 415.476 | L2 | 44.95 | 340 | L1 |

| | | |
|---|---|---|
| L1 | 148 | 70.48% |
| L2 | 28 | 13.33% |
| L3 | 22 | 10.48% |
| L4 | 4 | 1.90% |
| L5 | 8 | 3.81% |
| Total | 210 | 100.00% |

The group's overall perfomance in the algebraic problem solving test was at CAAPSA Level 1 (very low). In Figure 4.6, it can also be seen that the perfomance of the group was relatively better in questions 3, 6, 7, and 8.

#### 4.3.1.5 Correlations between the achievements in the knowledge base and problem solving tests and achievements between algebraic problem solving test and 2010 final NSSC examination

Carson (2007) suggests that there is a strong relationship between learners' knowledge base and their problem solving skills. The present study also tested the correlation between achievement in the knowledge base and algebraic problem solving tests. Figure 4.7 shows an extract from the CAAPSA excel sheet used to calculate the correlations between the levels of achievement between the knowledge base diagnostic test (KB) and algebraic problem solving achievement test (APS) and correlation between the algebraic problem solving test and the learners' level of mathematical proficiency (MP) measured from the learners' achievement in the 2010 NSSC final mathematics examinations. The NSSC grading uses grades A to U (see Table 2.1). For the purpose of calculating the correlations between the levels of achievement, the study conveniently assigned levels to each grade as shown in Figure 4.7.

**A: level 5** **B: Level 4** **C: Level 3** **D: Level 2** **E/F/G/U: Level 1**

**Figure 4.7:** CAAPSA Print screen for analysis of correlations between test achievements

| | | | | | CAAPSA LEVEL | | |
|---|---|---|---|---|---|---|---|
| # | SURNAME | NAME | SEX | SCHOOL | KB | APS | MP |
| 1 | Alweendo | Selma | F | Andimba T SS | 4 | 1 | 3 |
| 2 | Alweendo | Simeon | M | Andimba T SS | 2 | 1 | 1 |
| 3 | Amunyela | Klaudia | F | Andimba T SS | 4 | 2 | 2 |
| 4 | Booysen | Diina | F | Andimba T SS | 1 | 1 | 1 |
| 5 | Elago | Victoria | F | Andimba T SS | 2 | 1 | 1 |
| 6 | Haidula | George | M | Andimba T SS | 2 | 1 | 1 |
| 7 | Hainyeko | Ndatoolewe | F | Andimba T SS | 2 | 1 | 1 |
| 204 | Shihepo | Veino | M | Oshakati SS | 1 | 2 | 2 |
| 205 | Shikwambi | Israel A M | M | Oshakati SS | 1 | 3 | 3 |
| 206 | Shuudufonya | Martin | M | Oshakati SS | 1 | 1 | 1 |
| 207 | Simeon | Ndinelao N | F | Oshakati SS | 2 | 2 | 3 |
| 208 | Ulusiku | Teodorah H | F | Oshakati SS | 1 | 1 | 1 |
| 209 | Venasius | Justina | F | Oshakati SS | 2 | 2 | 3 |
| 210 | Weyulu | Petrus K | M | Oshakati SS | 2 | 2 | 2 |
| | | | | MEAN | 2.4 | 1.8 | 1.9 |

| CORRELATIONS | |
|---|---|
| KB VS APS | 0.5 |
| APS VS MP | 0.7 |

In the screen, KB refers to knowledge base diagnostic test levels of achievement, APS to algebraic problem solving achievement levels and MP refers to the mathematical proficiency. Rows 8 to 203 of the excel sheet were hidden in order to fit the screen, however all 210

learners' levels were considered for the calculation of the coefficients of correlation between the achievement levels.

The Pearson correlation coefficient between knowledge base and algebraic problem solving scores was 0.5. The correlation between the algebraic problem solving achievement and the mathematics NSSC 2010 achievement was 0.7. Contrary to the assertions by Krulick and Rudnick (1980) that successful problem solvers have a complete and organised knowledge base, the findings of this study show a moderate correlation (0.5) between knowledge base and algebraic problem skills. This might suggest that the knowledge base is not a sufficient requirement for the development of problem solving skills, and that learners need to be trained to solve problems using a variety of strategies, if they are to be successful in solving problems (Carson, 2007).

### 4.3.2 Qualitative data analysis

The difference between qualitative and quantitative data is that data to be analysed are text, rather than numbers, at least when the analysis first begins (Schutt, 2001). The researcher employed the computer assisted qualitative data analysis. Computer assisted data analysis uses special computer software to assist qualitative analyses through creating, applying and refining categories, tracing linkages between concepts; and making comparisons between cases and events. This study compares the findings of the qualitative data analysis to the findings of the quantitative data analysis. The qualitative data analysis seeks to find out meanings of emerging problem solving themes from the analysis of the NSSC curriculum documents, learners' self-reported experiences (questionnaire and interview) and learners' snippet analysis of solution process. The qualitative data analysis was guided by the research questions and attempted to establish:

1. The extent to which the NSSC curriculum documents emphasise the development of problem solving skills. This was done through examination content analysis of NSSC papers from 2007 to 2009. A panel of mathematics education officers and national examiners validated the findings.
2. The level of success of the Grade 12 learners in Oshana Region when solving algebraic problems. This was established from the analysis of the solution snippets of a purposively selected sample of 25 learners (12% of the sample), drawn from different levels of algebraic problem solving achievement.

3. The challenges and difficulties that Grade 12 learners in Oshana region encounter when attempting to solve algebraic problems. The snippet analysis outcomes as well as questionnaire and task-based interview responses for a purposively selected sample of learners were used to map these challenges.

#### 4.3.2.1 Analysis of the NSSC Ordinary level examination question papers from 2007 to 2009

The analysis of the weight of problem solving content of the mathematics examination papers from 2007 to 2009, by the panel of mathematics education officers and examiners suggests that content was aligned to the NSSC curriculum assessment objectives. The paper 4, extended component of the NSSC mathematics examination papers were used for this weighting. The structured extended questions cover a variety of assessment objectives and were hence used to deduce the weighting of the objectives by the panel of experts. Table 4.4 shows the mean weighting of each objective by the panel of experts. For example, the experts unanimously agreed to associate the objectives in the shaded rows, to algebraic problem solving. The percentage weighting calculation was thus obtained using the formula given in section 3.3.2.3 as follows:

$$\begin{aligned} \textit{Pecentage weighting} &= \frac{50.3+47.3\ 47.0+45.7+42.0+40.7+38.7+36.3+28.7}{553.3}\times 100 \\ &= 68\% \end{aligned}$$

**Table 4.4:** Weight of assessment objectives in the NSSC examinations from 2007 to 2009 based on the NSSC Ordinary level paper 4 examinations

| Assessment objective | QUESTIONS | | | | | | | | | | | ROW TOTAL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| Perform calculations by suitable methods. | 4.7 | 5.0 | 4.7 | 4.7 | 4.7 | 4.7 | 4.3 | 4.7 | 5.0 | 5.0 | 5.0 | 52.3 |
| Interpret, transform and make appropriate use of mathematical statements expressed in words or symbols. | 4.7 | 4.7 | 4.3 | 4.3 | 4.7 | 4.3 | 3.7 | 5.0 | 5.0 | 4.7 | 5.0 | 50.3 |
| Apply combinations of mathematical skills and techniques in problem solving. | 5.0 | 5.0 | 4.3 | 3.7 | 4.3 | 4.0 | 3.0 | 5.0 | 4.0 | 4.0 | 5.0 | 47.3 |
| Analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution. | 5.0 | 5.0 | 4.0 | 4.3 | 3.7 | 4.0 | 3.0 | 5.0 | 4.3 | 3.7 | 5.0 | 47.0 |
| Recall, apply and interpret mathematical knowledge in the context of everyday situations. | 5.0 | 4.7 | 4.3 | 4.3 | 4.0 | 3.3 | 3.3 | 4.3 | 4.0 | 3.3 | 5.0 | 45.7 |
| Set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology. | 4.0 | 5.0 | 4.0 | 4.0 | 3.7 | 3.0 | 3.0 | 3.7 | 4.3 | 3.3 | 4.0 | 42.0 |
| Organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms. | 3.3 | 3.0 | 4.0 | 2.0 | 4.3 | 3.7 | 3.0 | 4.7 | 4.3 | 3.3 | 5.0 | 40.7 |
| Make logical deductions from given mathematical data. | 3.3 | 4.3 | 2.0 | 3.7 | 3.0 | 3.3 | 2.7 | 4.0 | 4.0 | 4.3 | 4.0 | 38.7 |
| Recognise patterns and structures in a variety of situations, and make generalisations. | 3.0 | 4.3 | 3.0 | 4.0 | 3.0 | 3.0 | 2.3 | 3.3 | 3.0 | 2.3 | 5.0 | 36.3 |
| Recognise and use spatial relationships in two and three dimensions, particularly in solving problems. | 3.3 | 3.3 | 4.7 | 2.0 | 3.3 | 2.7 | 1.7 | 3.3 | 2.3 | 2.3 | 2.0 | 31.0 |
| Understand systems of measurement in everyday use and make use of them in the solution of problems. | 3.3 | 2.7 | 2.0 | 1.7 | 3.3 | 2.3 | 1.7 | 3.7 | 2.3 | 3.0 | 2.0 | 28.0 |
| Respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form. | 2.7 | 2.3 | 3.0 | 3.3 | 1.3 | 2.3 | 2.3 | 2.0 | 3.0 | 2.3 | 4.0 | 28.7 |
| Use an electronic calculator. | 2.0 | 2.3 | 2.0 | 2.3 | 3.0 | 1.7 | 1.7 | 3.7 | 1.3 | 1.0 | 4.0 | 25.0 |
| Estimate, approximate and work to degrees of accuracy appropriate to the context. | 1.3 | 3.3 | 1.3 | 2.3 | 2.0 | 1.7 | 2.0 | 2.3 | 1.7 | 2.3 | 2.0 | 22.3 |
| Use mathematical and other instruments to measure and draw to an acceptable degree of accuracy. | 1.7 | 1.3 | 2.0 | 1.0 | 1.7 | 2.0 | 2.0 | 2.3 | 2.0 | 1.0 | 1.0 | 18.0 |
| COLUMN TOTAL | 52.3 | 56.3 | 49.7 | 47.7 | 50.0 | 46.0 | 39.7 | 57.0 | 50.7 | 46.0 | 58.0 | 553.3 |

Table 4.5 shows the rating scale that was adopted for estimating the weighting of the assessment objectives, after the mean scores of the panel of experts were determined.

**Table 4.5:** Rating scale for weight of assessment objectives in the NSSC examination questions

| Scores | Prevalence weight of assessment objective |
|---|---|
| 4.5 – 5.0 | 70%-100% |
| 3.5 – 4.4 | 60%-69% |
| 2.5.0 – 3.4 | 50%-59% |
| 1.5 – 2.4 | 40%-49% |
| 1.0 – 1.4 | 0%-39% |

The rating scale was subjective as two individual assessors might not necessarily see the same weight of objectives and of course decisions to allocate the points could have been spontaneous. However, regardless of the differences between the individual points, the mean of the allocated points provides a reliable picture of weight of assessment objectives. The objectives were then sorted in the hierarchy of the weight scores, from most to least weight. The hierarchy of assessment objectives indicates an inclination towards assessment of problem solving skills. The panel consented that the weight of algebraic problem solving content assessed in the examinations from 2007 to 2009 represented not less than 60% of the examinations. This shows that the NSSC curriculum is structured to promote the development of learners' problem solving skills. However, the NSSC curriculum does not provide assessment tools for teachers to map the progress in the development of learners' problem solving skills. This study anticipates that the developed CAAPSA tool will be useful for teachers in their classroom practice.

#### 4.3.2.2 Feedback from panel of mathematics national examiners and education officers who validated the CAAPSA processing

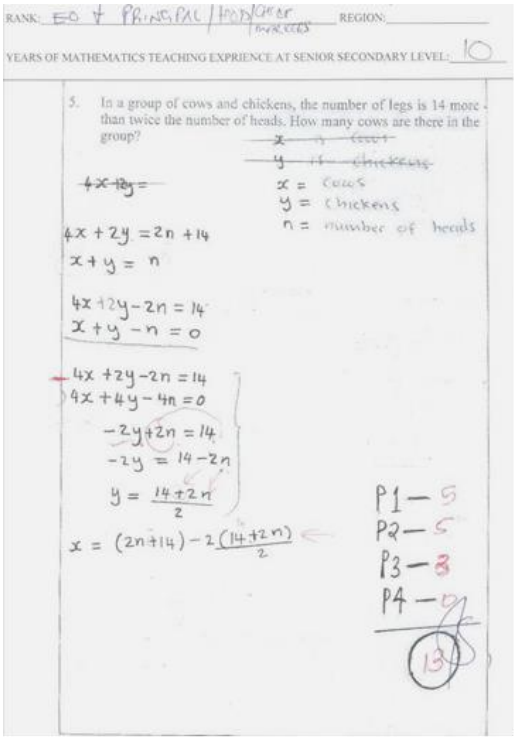
The panel of experts were asked to validate the CAAPSA marking tool by designing an equivalent tool, using Polya's model. The panel of mathematics experts then used the tool to assess a purposively selected sample of snippets, in order to compare the assessment process to the CAAPSA tool. Table 4.6 shows the rubric that was designed by the panel of six mathematics national examiners and three mathematics education officers:

**Table 4.6:** The marking rubric used to validate the CAAPSA tool

| Step | Maximum possible score | Marking rubric |
|---|---|---|
| 1. Read and understand | 5 | •5 points for correct algebraic representation of the known and unknown data<br>•Deduct 1 mark for each error |
| 2. Devise a plan | Score obtained in step 1 | •Correct equations/algebraic expressions used to represent the relationships between the known and unknown data<br>•Deduct 1 mark for each error |
| 3. Carry out the plan | Score obtained in step 2 | •Correct solution of equation(s) in step 2<br>•Deduct 1 mark for each error |
| 4. Look back | Score obtained in step 3 | •If checking is not shown, but answer in step 3 is correct, award the score in step 3<br>•If checking is not shown and answer in step 3 is not correct, award 0.<br>•If answer is not correct, but checking is done correctly award score obtained in step 3.<br>•If checking is correctly shown and answer in step 3 is correct, award score obtained in step 3. |

Figure 4.8 shows one of the learners' snippets to which the rubric was used by the panel to assess the solution, in Polya's model. P1 indicates marks allocated at Polya's first step, P2, the marks allocated at Polya's second step and so on, for the rest of the steps.

**Figure 4.8:** One of the learners' solution snippets that were used in the study for validation of the CAAPSA output by the panel of mathematics experts



The CAAPSA output in Figure 4.43 shows a result that matches the output from the panel of examiners. The researcher engaged with the panel to find out how they had reached these criteria. The panel consented that it was not easy to map a learner's written solution to Polya's problem solving steps, in order to determine the step at which they encountered problems. The panel agreed to trace the learners' use and manipulation of algebraic variables in the solution process, and reached the criteria that enabled them to allocate the marks as shown in Figure 4.8. When asked why they allocated a score of 0 to Polya's fourth step, they argued that this was because the learner had not attempted executing Polya's fourth step, and that they would only have assumed the step was correctly executed if the learner had managed to obtain the correct solution. The panel's marking criteria was in line with the CAAPSA output (see Figure 4.42).

#### 4.3.2.3 Analysis of questionnaires

Transcripts from the individual questionnaires revealed different experiences and challenges in the solution processes for each learner. Analysis of questionnaires focused on seeking to understand from the learners' perspective, their experiences in executing Polya's steps in the solution process of the algebraic problems.

The questionnaire responses and outcomes were analysed separately in two parts. In the first part, the analysis dealt with responses from section A of the questionnaire (closed-ended responses) and the second part analysed the responses to the open-ended questions of section B of the questionnaire (see Appendix 4).

All questionnaire respondents were given codes, for example L54 refers to the code that was given to the learner in Figure 4.9 (a). The codes also helped the researcher to track down learners (participants) who were later purposively sampled for interviews because each learner was requested to remember his/her questionnaire code (see section 3.3.2.2). To preserve the anonymity of the questionnaire respondents, these codes were given arbitrarily to the participants.

#### 4.3.2.3.1 Analysis of section A of the questionnaire

Figure 4.9 (a) shows an example of the completed section A of the questionnaire by learner L54.

**Figure 4.9 (a):** An example of the completed questionnaire by learner L54 in section A

**LEARNER ID:** L54 **DATE COMPLETED:** 14.10.2010

**SECTION A: Tick, Yes or No (√) in the appropriate box, for each of the following statements:**

| # | Statement | Yes | No |
|---|---|---|---|
| 1 | I had an idea of how to start solving most of the problems | ✓ | |
| 2 | I managed to identify the information given in most of the problems | ✓ | |
| 3 | I understood the language used in most of the problems | | ✓ |
| 4 | I managed to identify the unknown quantities in most of the problems | | ✓ |
| 5 | I managed to identify the appropriate mathematical operations in most of the problems | ✓ | |
| 6 | All the problems had a solution | | ✓ |
| 7 | I recognised useful number patterns which helped me solve some of the problems | ✓ | |
| 8 | I used a table/diagram/equation in some of the problems | ✓ | |
| 9 | I have seen similar problems like these ones before | | ✓ |
| 10 | In some cases, I formulated a similar and simpler problem to help me get a solution strategy | | ✓ |
| 11 | I solved some of the problems by forming and solving equations | ✓ | |
| 12 | I checked each step in my working | ✓ | |
| 13 | It was easy to establish the relationships between the known and unknown data in the problems | | ✓ |
| 14 | It was easy for me to decide on a suitable method of solution | | ✓ |
| 15 | My school mathematics knowledge was useful in solving the problems | | ✓ |
| 16 | My work on the problems was well organised | ✓ | |
| 17 | After solving these problems, I feel I will be able to solve more similar problems | | ✓ |
| 18 | I tried to verify the correctness of my solutions | | ✓ |
| 19 | I tried to look for alternative ways of solution | | ✓ |
| 20 | I tried to interpret the solutions obtained in terms of the original problem to see whether my answer makes sense. | | ✓ |

The questionnaire responses were given scores of 1 for a “yes” response, and 0 for a “no” response. The analysis considered a “yes” response to indicate success in the execution of Polya’s (1957) steps, while the “no” responses indicated failure to execute particular Polya’s (1957) steps. A step was considered to be successfully executed if a learner achieved at least level 3 in each of Polya’s (1957) steps; otherwise the learners were regarded to have failed to execute the steps. Figure 4.9 (b) shows how the learner L54’s questionnaire responses to section A were scored in an Excel spreadsheet.

**Figure 4.9 (b):** Computerised scoring and interpretation of learner L54's responses to section A of the questionnaire

| | LEARNER CODE | L54 | | |
|---|---|---|---|---|
| # | Statement | Score | Total | Status |
| READING AND UNDERSTANDING THE PROBLEM | I had an idea of how to start solving most of the problems | 1 | 3 | Achieved |
| | I managed to identify the information given in most of the problems | 1 | | |
| | I understood the language used in most of the problems | 0 | | |
| | I managed to identify the unknown quantities in most of the problems | 0 | | |
| | I managed to identify the appropriate mathematical operations in most of the problems | 1 | | |
| DEVISING A PLAN | All the problems had a solution | 0 | 2 | Not achieved |
| | I recognised useful number patterns which helped me solve some of the problems | 1 | | |
| | I used a table/diagram/equation in some of the problems | 1 | | |
| | I have seen similar problems like these ones before | 0 | | |
| | In some cases, I formulated a similar and simpler problem to help me get a solution strategy | 0 | | |
| CARRYING OUT THE PLAN | I solved some of the problems by forming and solving equations | 1 | 2 | Not achieved |
| | I checked each step in my working | 1 | | |
| | It was easy to establish the relationships between the known and unknown data in the problems | 0 | | |
| | It was easy for me to decide on a suitable method of solution | 0 | | |
| | My school mathematics knowledge was useful in solving the problems | 0 | | |
| EVALUATING THE SOLUTION (LOOKING BACK) | My work on the problems was well organised | 1 | 1 | Not achieved |
| | After solving these problems, I feel I will be able to solve more similar problems | 0 | | |
| | I tried to verify the correctness of my solutions | 0 | | |
| | I tried to look for alternative ways of solution | 0 | | |
| | I tried to interpret the solutions obtained in terms of the original problem to see whether my answer makes sense. | 0 | | |
| | | | | Achieved |
| | | | | Not achieved |

After all the individual questionnaires were scored, the group's mean achievement at each of Polya's (1957) steps was calculated in a computerised Excel spreadsheet to determine the overall perception of the group about the difficulties encountered in executing Polya's steps. The self-reported mean achievement of the group was determined from the learners' completed and computer scored questionnaires for section A.

The frequencies of achievement and non achievement at each of Polya's stages by the 210 participants were then summarised in a frequency table. The self-reported difficulties in the execution of Polya's steps were then compared to the actual observed difficulties from the quantitative data analysis of the problem solving achievement. Table 4.7 shows the comparison between the actual observed outcomes and the questionnaire self-reported outcomes on the problem solving process of the 210 participants.

**Table 4.7:** Comparison of quantitative and qualitative data on frequency of learners who encountered difficulties in executing Polya's steps

| POLYA'S STEP | QUANTITATIVE DATA ON OBSERVED DIFFICULTIES IN EXECUTION OF POLYA | | | QUSTIONNAIRE DATA ON REPORTED DIFFICULTIES IN EXECUTION OF POLYA | | |
|---|---|---|---|---|---|---|
| | Below level 3 | At least level 3 | TOTAL | No | Yes | TOTAL |
| Reading and understanding | 65.2% | 34.8% | 100% | 62.9% | 37.1% | 100% |
| Devising a plan | 71.0% | 29.0% | 100% | 68.6% | 31.4% | 100% |
| Carrying out the plan | 73.8% | 26.2% | 100% | 76.7% | 23.3% | 100% |
| Looking back | 76.2% | 23.8% | 100% | 81.9% | 18.1% | 100% |
| | | | | | | |
| | | | | | | |
| Correlation between actual achievement and self-reported achievement of learners in the execution of Polya's steps | | | | | 0.971673395 | |

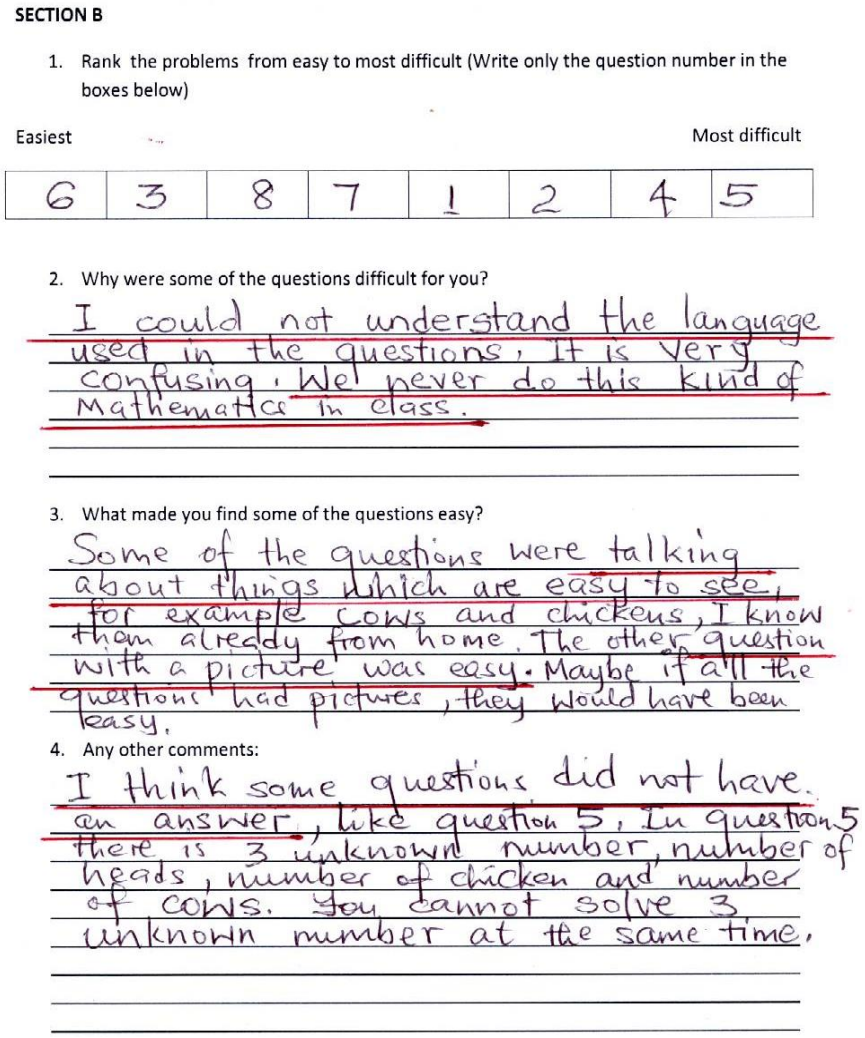
The data shows a strong correlation (0.97) between the self evaluation of learners in the problem solving achievement and the actual achievement obtained from the algebraic problem solving test. This demonstrates the reliability of section A of the questionnaire in measuring the learners' self reported competencies in executing Polya's (1957) problem solving steps.

#### 4.3.2.3.2 Analysis of section B of the questionnaire

Scoring self-developed instruments is complex, especially if open ended items are involved, because the researcher must develop and refine a reliable scoring procedure. If open ended

items are included, at least two people should independently score some or all the responses as reliability check (Gay et al., 2012). In this study, coding was used to summarise the outcomes from the open ended questions in section B of the questionnaire and two more independent scorers were asked by the researcher to score a randomly selected sample of 10% of the questionnaires. Coding is the process of categorically marking or referencing units of text (e.g. words, sentences, paragraphs and quotations) with codes and labels as a way to indicate patterns and meaning in qualitative data. Figure 4.10 shows an example of the responses to section B, from a learner L54. The underlined statements or phrases were marked as essential units of text for explaining the difficulties encountered in the problem solving process.

**Figure 4.10:** An example of a completed questionnaire and how coding was used to analyse section B responses for learner L54



**SECTION B**

1. Rank the problems from easy to most difficult (Write only the question number in the boxes below)

Easiest ... Most difficult

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 3 | 8 | 7 | 1 | 2 | 4 | 5 |

2. Why were some of the questions difficult for you?

I could not understand the language used in the questions, It is very confusing. We never do this kind of Mathematics in class.

3. What made you find some of the questions easy?

Some of the questions were talking about things which are easy to see, for example cows and chickens, I know them already from home. The other question with a picture was easy. Maybe if all the questions had pictures, they would have been easy.

4. Any other comments:

I think some questions did not have an answer, like question 5, In question 5 there is 3 unknown number, number of heads, number of chicken and number of cows. You cannot solve 3 unknown number at the same time.

Table 4.8 shows the data coding format that was used for the analysis of the open-ended questionnaire responses in section B.

**Table 4.8:** The outcomes that were obtained from the analysis of the responses to section B of questionnaire (see Appendix 4)

| Questionnaire item | Coding | Analysis outcome |
|---|---|---|
| **1** | • List the 4 most difficult and 4 easiest questions from each participant<br>• Tabulate the frequencies of the questions listed as most difficult and easiest. | Questions 1, 2, 4 and 5 were listed repeatedly as the most difficult while questions 3, 6, 7 and 8 were identified as the easiest. |
| **2** | Underline the key words in the learners' responses with regard to why some of the questions were difficult and note the most frequent words and/or phrases | The most frequent reasons were:<br>• Failure to understand the language used<br>• It was difficult to establish the relationships between the known and unknown in the problems (Lack of algebraic representation and manipulation skills)<br>• Not being taught to do such problems in class |
| **3.** | Underline the key words in the response with regard to why some of the questions were easy and note the most frequent words | • Familiarity with the problem context<br>• The use of pictures and diagrams to illustrate the question simplified the solution process. |
| **4.** | Underline key sentences attributing difficulties encountered, to any other factors in the problem solving process | • Some learners felt that some questions had no solution<br>• Some learners found the questions difficult because they were not taught how to solve them in class, and since they could not devise any solution strategy, they belived some of the problems had no solutions. |

The majority of learners (65%) listed problems 1, 2, 4 and 5 as the most difficult and problems 3, 6, 7 and 8 were easier. This was in line with the outcomes from the analysis of employed solution strategies in Table 4.10. Most of the learners echoed the failure to execute Polya's first step as the reason for their difficulties in solving the algebraic word problems in the algebraic problem solving test. This was in line with the CAAPSA output in the quantitative data analysis on the achievement of learners in the algebraic problem solving test.

#### 4.3.2.4 Analysis of solution strategies and outcomes from a purposively selected sample of learners' solution snippets

After marking all the algebraic problem solving test scripts, the researcher carried out a question by question analysis of the solutions for a purposively selected sample of learners in the three categories A, B, and C. Of the learners who participated in the study, 25 learners were purposively selected for the analysis of the solution strategies. At least 10% of the learners from each of the categories A (very low and low achievers), B (intermediate level of achievement) and C (high and advanced levels of achievement) were selected for the analysis of their solution snippets. Of the learners who participated in the study, 150 learners were in category A, so 15 (10%) learners were sampled, in category B there were only 4 learners, hence all their solution snippets were selected for analysis, while in category C there were only six learners, and all their solution snippets were selected for analysis of solution strategies employed. Table 4.9 summarises the learners' choice of solution strategies and how successful these strategies were implemented.

**Table 4.9:** Frequency and outcome of solution strategies employed

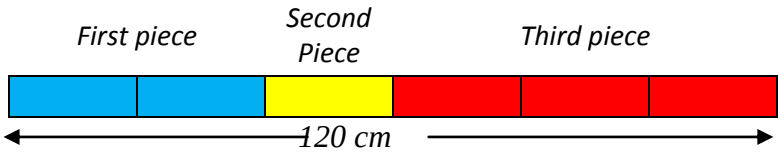
| CATEGORY | LEARNER CODE | TEST QUESTIONS | | | | | | | | MEAN SUCCESS RATE | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | INDIVIDUAL | CATEGORY |
| A | L1A | S10 | S20 | S10 | S10 | S81 | S11 | S10 | S10 | 25.0% | 13.3% |
| | L2A | S10 | S20 | S11 | S20 | S20 | S11 | S50 | S11 | 37.5% | |
| | L3A | S10 | S20 | S10 | S20 | S20 | S10 | S20 | S10 | 0.0% | |
| | L4A | S20 | S20 | S20 | S20 | S20 | S10 | S11 | S10 | 12.5% | |
| | L5A | S20 | S20 | S20 | S20 | S20 | S21 | S20 | S11 | 25.0% | |
| | L6A | S20 | S20 | S11 | S20 | S20 | S10 | S10 | S10 | 12.5% | |
| | L7A | S20 | S10 | S10 | S20 | S20 | S10 | S10 | S10 | 0.0% | |
| | L8A | S20 | S10 | S10 | S10 | S20 | S11 | S20 | S51 | 25.0% | |
| | L9A | S20 | S20 | S40 | S20 | S20 | S10 | S80 | S10 | 0.0% | |
| | L10A | S20 | S10 | S20 | S20 | S20 | S20 | S21 | S10 | 12.5% | |
| | L11A | S10 | S10 | S21 | S20 | S20 | S20 | S81 | S50 | 25.0% | |
| | L12A | S10 | S10 | S20 | S20 | S20 | S20 | S80 | S50 | 0.0% | |
| | L13A | S20 | S10 | S20 | S20 | S20 | S20 | S50 | S10 | 0.0% | |
| | L14A | S20 | S10 | S20 | S20 | S20 | S20 | S51 | S10 | 12.5% | |
| | L15A | S20 | S20 | S21 | S20 | S20 | S20 | S10 | S10 | 12.5% | |
| B | L16B | S11 | S20 | S11 | S41 | S20 | S61 | S51 | S10 | 62.5% | 56.3% |
| | L17B | S10 | S21 | S21 | S20 | S21 | S20 | S20 | S81 | 50.0% | |
| | L18B | S21 | S10 | S21 | S20 | S90 | S21 | S21 | S10 | 50.0% | |
| | L19B | S10 | S10 | S11 | S21 | S40 | S11 | S21 | S11 | 62.5% | |
| C | L20C | S11 | S21 | S21 | S21 | S21 | S21 | S21 | S11 | 100.0% | 89.6% |
| | L21C | S21 | S21 | S21 | S20 | S21 | S21 | S10 | S10 | 62.5% | |
| | L22C | S11 | S21 | S11 | S21 | S21 | S21 | S11 | S11 | 100.0% | |
| | L23C | S11 | S21 | S11 | S21 | S21 | S21 | S21 | S11 | 100.0% | |
| | L24C | S21 | S21 | S21 | S21 | S21 | S11 | S21 | S11 | 100.0% | |
| | L25C | S21 | S10 | S11 | S20 | S21 | S21 | S21 | S11 | 75.0% | |
| MEAN SUCCESS RATE | | 32.0% | 24.0% | 56.0% | 24.0% | 32.0% | 52.0% | 48.0% | 40.0% | 38.5% | |

The purposively sampled learners were coded as L1A to L15A for the 15 learners in category A, L16B to L19B for the 4 category B learners and L20C to L25C for the category C learners. S11 shows the successful use of the first strategy (arithemetic method), while S10 shows an unsuccessful attempt to use the arithmetic strategy (see Table 3.8). For example the ninth strategy in Table 3.8 is the use of an analogue; hence S91 indicates successful use of the strategy and S90, an unsuccessful attempt. An Excel sheet was then used to compute the frequency of use of each of the strategies per question and the analysis was done for the individual learners and categories. Only 13.3% of the solution strategies employed by category A learners were successful, 56.3% of the strategies employed by category B learners were successful and 89.6% of the strategies employed by category C learners were successful. Table 4.10 shows the summary of how frequent specific strategies were attempted as well as their success outcomes.

**Table 4.10:** Frequency of attempts and outcomes of employed solution strategies

| Problem | Outcome | SOLUTION STRATEGIES | | | | | | | | | SOLUTION OUTCOMES | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | TOTAL | SUCCESS | % SUCCESS |
| 1 | F | 7 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 8 | 32.0% |
| | S | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | |
| 2 | F | 10 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 6 | 24.0% |
| | S | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | |
| 3 | F | 4 | 6 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 25 | 14 | 56.0% |
| | S | 7 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | |
| 4 | F | 2 | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 6 | 24.0% |
| | S | 0 | 5 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | | | |
| 5 | F | 0 | 15 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 25 | 8 | 32.0% |
| | S | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | | | |
| 6 | F | 5 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 13 | 52.0% |
| | S | 5 | 7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | | | |
| 7 | F | 5 | 4 | 0 | 0 | 2 | 0 | 0 | 2 | 0 | 25 | 12 | 48.0% |
| | S | 2 | 7 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | | | |
| 8 | F | 13 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 25 | 10 | 40.0% |
| | S | 8 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | | | |
| FREQUENCY OF SUCCESS | | 13.0% | 21.5% | N/A | 0.5% | 1.5% | 0.5% | N/A | 1.5% | 0.0% | 38.50% | 77 | 38.5% |
| FREQUENCY OF FAILURE | | 23.0% | 34.0% | N/A | 1.0% | 2.0% | 0.0% | N/A | 1.0% | 0.5% | 61.50% | | |
| FREQUENCY OF ATTEMPTS | | 36.0% | 55.5% | 0.0% | 1.5% | 3.5% | 0.5% | 0.0% | 2.5% | 0.5% | 100.0% | | |
| SUCCESS RATE | | 36% | 39% | N/A | 33% | 43% | 100% | N/A | 60% | 0% | | | |

Table 4.10 indicates that 55.5% of the learners employed the algebraic method; 36.0% used the arithmetic method; 3.5% used a pattern to analyse the problems, 1.5% resorted to the guess-and-check method; 2.5% used a picture or diagram; 0.5% used logical reasoning, 0.5% used an analogue; and the model drawing and working backwards strategies were not attempted at all. The researcher's assumption for the learners not attempting these two strategies (working backwards and model drawing) is that these strategies were never familiar to them. For example, problem 6 (see Appendix 3) of the algebraic problem solving test could have easily been solved using model drawing as follows:




*Thus the length of each of the six unit bars would be 120 cm ÷ 6 = 20 cm. Therefore the longest piece, which is made up of three parts would be 3 × 20 cm = 60 cm.*

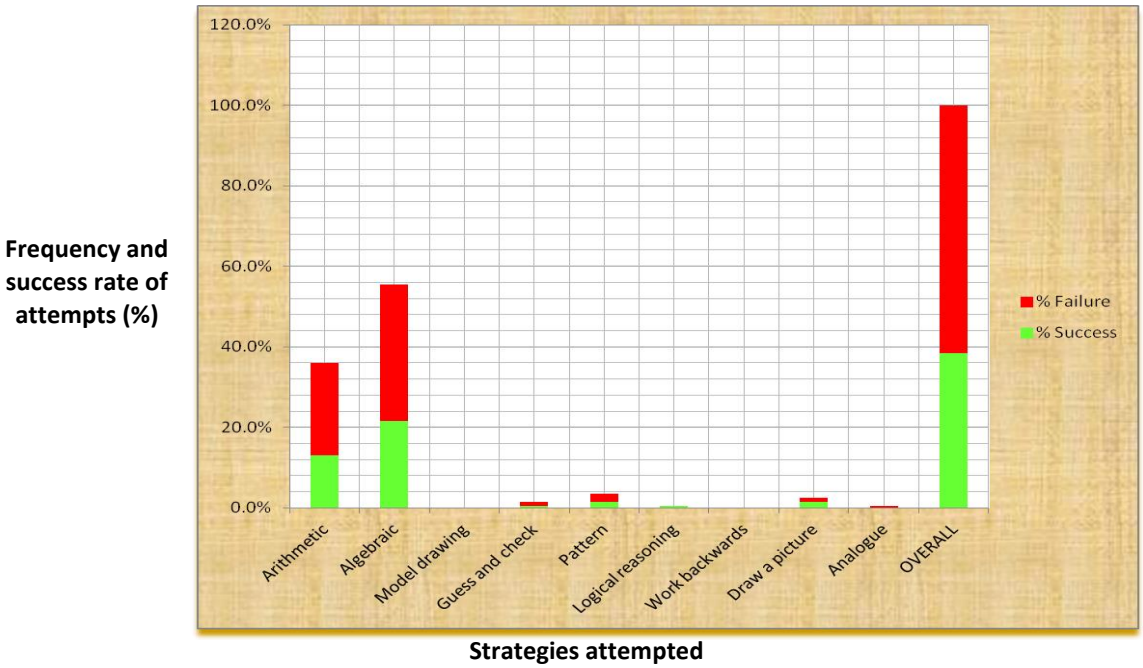
The working backward strategy would involve starting from the length of the third piece and working backwards to the lengths of the first and second pieces respectively, by reversing the algebraic strategy method. This strategy should not be confused with the guess and check strategy. In this strategy the problem solver does not guess the length of the third piece, but

through reasoning backwards, to the beginning of the problem, finds what values are possible for the longest piece.

The solution strategies were considered successful if learners attained Level 3 or higher, determined from the CAAPSA output for the attained achievement levels in the algebraic problem solving test. Table 4.10 shows that only 32% of the learners successfully employed their solution strategies in problem 1, only 24% of the learners were successful in problems 2 and 4, and 32% succeeded in problem 5. Learners were more successful in problem 3 with 56% of the learners successfully employing the selected solution strategies, problem 6 (52%), problem 7 (48%) and problem 8 (40%). The outcomes in Table 4.10 are in line with the learners' questionnaire responses in which 65% of the learners indicated that problems 3, 6, 7 and 8 were easier, while problems 1, 2, 4, and 5 were difficult. The follow up task-based interviews were thus carried out to seek clarification on the learners' views for their poor perfomance in some of the problems.

Figure 4.11 shows a stacked column graph that summarises the frequency of choice and success rates of the solution strategies by the purposively selected group of 24 learners. The red bar shows the frequency of unsuccessful attempts (achievement below CAAPSA level 3) while the green bar indicates the frequency of solution strategies that were successfully employed (achievement at or above level 3).

**Figure 4.11:** Stacked column graph showing frequency and outcomes of employed solution strategies



#### 4.3.2.4.1 Analysis of solution snippets of a purposively selected sample of learners

Snippets of learners' written solutions were analysed for the purpose of understanding the participants' strengths and weaknesses in the algebraic problem solving process. Of the learners' solution snippets in the algebraic problem solving test, 24 were purposively selected solution snippets and used for the analysis of the employed solution strategies and outcomes. The criterion for choosing the snippets was that their solution process was representative of the approach used by the majority of the learners in the respective categories. Table 4.11 shows how the solution snippets were selected for the analysis of strategies used in each of the eight problems in the algebraic problem solving test.

**Table 4.11:** The purposively sampled solution snippets of learners, used to analyse the employed solution strategies and outcomes

| Learner Category | PROBLEMS IN THE ALGEBRAIC PROBLEM SOLVING TEST | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | L1A | L5A | L11A | L15A | L1A | L5A | L11A | L15A |
| B | L16B | L17B | L17B | L18B | L19B | L16B | L17B | L18B |
| C | L20C | L22C | L24C | L25C | L20C | L22C | L24C | L25C |

Four learners were selected from each category. In category A (very low and low achievers), snippets of learners L1A, L5A, L11A and L15A were selected. In category B (intermediate achievers), snippets of learners L16B, L17B, L18B, and L19B were analysed, and in category C (high and advanced achievers) the snippets of learners L20C, L22C, L24C and L25C were analysed. Table 4.11 shows the learners whose snippets were purposively selected for analysis in each question.

##### 4.3.2.4.1.1 Analysis of solution outcomes in problem 1

*The sum of an even number and the consecutive even number is 54. Find the smaller of the two consecutive even numbers.*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

The CAAPSA rubric defines the solution to problem 1 as follows:

**Polya Step 1:** Understanding the problem

**Expected action:** Relate all parts of the problem and define the variables required to represent all the unknowns. The two unknown consecutive even numbers could be represented by ***2x* and *2x +2*** or **x *and x+2*.**

**Polya Step 2:**

**Expected action:** Write an algebraic model that gives the relationships between the unknowns and the given data, ***2x + (2x +2) = 54*** *where 2x is the smaller even number*

or ***x + (x+2) = 54*** *where x is the smaller even number***.**

**Polya Step 3:**

**Expected action:** Solve the equations

***x = 13 or x = 26.*** *This would yield 26 as the smaller even number.*

**Polya Step 4:**

**Expected action:** Check solution

The checking should be done in the problem. If the smaller even number is 26, then the next even number is 28. Since 26 + 28 = 54, the solution makes sense.

The following figure 4.12 is the CAAPSA summary of learners' performance in problem 1.

**Figure 4.12:** Summary of group performance in problem 1

SUMMARY FOR QUESTION 1

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 1.15714 | 315.714 | L1 | 1.06667 | 306.667 | L1 | 0.89048 | 289.048 | L1 | 0.83333 | 283.333 | L1 | 3.94762 | 298.69 | L1 |

| | | |
|---|---|---|
| L1 | 170 | 80.95% |
| L2 | 2 | 0.95% |
| L3 | 1 | 0.48% |
| L4 | 2 | 0.95% |
| L5 | 35 | 16.67% |
| Total | 210 | 100.0% |

Page 1

Of the 210 learners who sat for the algebraic problem solving test, 170 learners (80.95%) were at level 1, 2 learners (0.95%) at level 2, 1 learner (0.48%) at level 3, 2 learners (0.95%) at level 4 and 35 learners (16.67%) at level 5. This shows that the learners' main obstacles in problem 1 of the algebraic problem solving achievement test occured in the first step of Polya's model. In order to form a deeper understanding of the obstacles, the researcher looked at a sample of some of the participants' solutions, selected as shown in Table 4.11.

The sampled snippets are presented to illustrate the way that CAAPSA was used to score the solution process. For each of the eight problems, the selected learners' solution strategies are presented and analysed in three parts;

(a) Presentation of snippet of each learner category;

(b) Description of the CAAPSA processing and output; and,

(c) Learner interviews if solution process is not clear.

Figure 4.13 shows the solution strategy employed by a category A learner (L1A) in problem 1. The solution outcomes are also discussed.

**CATEGORY A (Overall low achievers)**

**Figure 4.13:** Snippet from a learner L1A who attempted the algebraic strategy

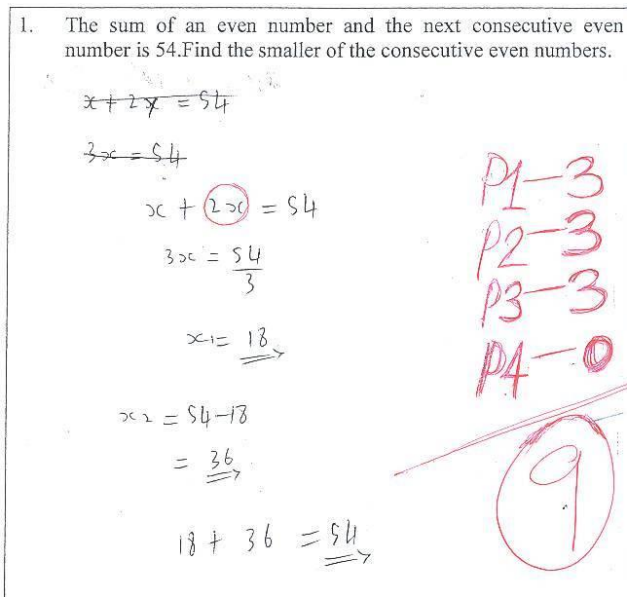



Figure 4.14 shows the CAAPSA output for the assessment of the solution strategy used by learner L1A.

**Figure 4.14:** Problem 1 CAAPSA processing and output for learner L1A

| LEARNER CODE: | L1A |
|---|---|

| QUESTION | 1 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 1 | 3 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 3 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 3 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| Total Score | 9 | L2 |
|---|---|---|
| Polya's TIMSS Score | 425 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

The CAAPSA tool assigns marks at each stage by analysing whether the learner has successfully managed to:

**Polya Step 1:** Understand the problem

Although the learner makes an effort to represent the unknown data algebraically (H=1), he/she has failed to establish the correct relationship between consecutive numbers by writing the next term as $2x$, instead of $x + 2$ (No of errors = 1). The CAAPSA processing in step 1 deducts marks proportionally to the weight of the error committed. A corresponding colour code (amber) is automatically assigned as an alert to the teachers to act on the weaknesses during instruction. The step is awarded P1=3.

**Polya Step 2:** Devise a plan

The resulting equation (plan) is incorrect because of inherent errors from the previous step. However, the process credits the learner's effort to form an equation (H=1). The consequent errors are not from this step, but have been carried from the previous step (number of errors = 0). The programme awards P2 = 3. The CAAPSA processing is designed not to allow the

score for P2 to be more than P1. In general, a following step cannot score more than the preceding step.

**Polya Step 3:** Solve the equations

The inherent equation from step 2 has been correctly solved (H = 1 and No of errors = 0). The programme awards P3 = 3.

**Polya Step 4:** Check solution

No checking done, so H=0. The process stops and P4 = 0.

The programme works out the learner's algebraic problem solving level based on TIMSS scale at level 2. This is an indication that the learner has struggled with the first step. The red colour assigned to the level indicates a serious alert, indicating to teachers that they should do more work with learners at this level as they display a low level of problem solving skills. The following interview was conducted with the learner in this category:

**Interview**

(The researcher's voice is represented by the letter **R**, while the learners are coded according to their numbers and categories. For example **L1A** refers to learner 1, in category A)

**R:** *You wrote the equation x + 2x = 54. What does this mean?*

**L1A:** *x and 2x are the consecutive even numbers…that add up to 54.*

**R:** *Suppose you have an even number 2, what is the next even number?*

**L1A**: *It is 4*

**R:** *and if the even number is 6, what is the next consecutive even number?*

**L1A:** *it is 12*

**R:** *Since you obtained x = 18, what would these consecutive even numbers be?*

**L1A:** *18 and 20, I guess.*

**R:** *So you are sure 18 and 20 are the required numbers?*

**L1A:** *they do not add up to 54…it is not the correct solution…but this is what the equation gives.*

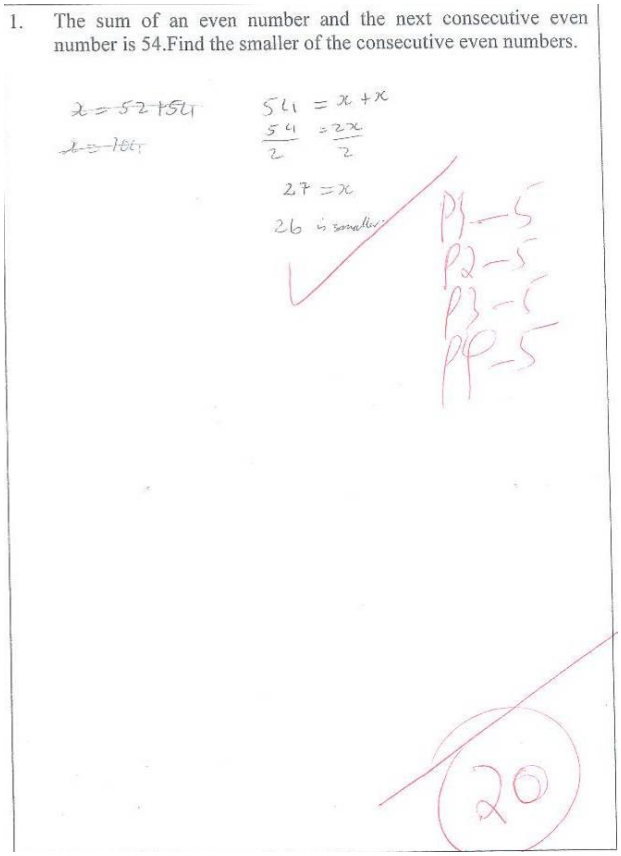
The aim of the interview was not to lead the learners to the correct solution, but rather to understand their thinking process in reaching the solution. From the interview, it appears that this learner lacks the algebraic representation skills for even numbers. The learner sees even

numbers as a geometric sequence 2, 4, 8, 16, 32 … rather than arithmetic sequence 2, 4, 6, 8, 10, and that led to an incorrect algebraic representation. The learner demonstrates a lack of conceptual knowledge (knowledge base) about even numbers.

Figure 4.15 shows the solution outcome for category B learner (L16B) in problem 1 of the algebraic problem solving test.

**CATEGORY B (Intermediate achievers)**

**Figure 4.15:** Snippet from a category B learner (L16B) who successfully used the arithmetic strategy




Although the learner uses the variable *x* to represent some unknown data, the solution is not algebraic; CAAPSA accommodates alternative solution strategies other than the algebraic strategy, by error processing.

Figure 4.16 shows the CAAPSA output which allocated marks to the solution process of category B learner (L16B) at each of Polya's steps.

**Figure 4.16:** Problem 1 CAAPSA processing and output for learner L16B

| LEARNER CODE: | L16B |
|---|---|

| QUESTION | 1 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 5 | |
| P4 | Checking of solution(s) seen or implied | 1 | 0 | 5 | |

| | | |
|---|---|---|
| Total Score | 20 | L5 |
| Polya's TIMSS Score | 700 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

**Interview**

**R:** *You divided 54 by 2 and got 27, why did you do this?*

**L16B:** *If a number can be written as sum of two consecutive even numbers, then the two consecutive even numbers are the immediate numbers on either side of the half of the given sum.*

**R:** *How do you know this?*

**L16B:** *(Hhhhhhm!)…Look!...take 12 + 14 = 26. If you divide 26 by 2 you get 13, so the two even numbers are 12 and 14, it works!*

**R:** *What would be the two consecutive even numbers that add up to 30?*

**L16B:** *(thinking a bit…)…14 and 16.*

**R:** *What would be the two consecutive even numbers that add up to 44?*

**L16B:** *21 and 23 (scratches his head…and smiles) but 21 and 23 are not even numbers. It only works for certain numbers.*

From the interview, it is clear that the learner has some previous conceptual knowledge of even numbers and their relationships. This is manifestation of the learner's concepts-in-action and theorems-in-action, in the problem solving process (Vergnaud, 2009). This is evidence that albeit not writing any algebraic model for the relationships in this problem, the learner had previous knowledge (knowledge base) about even numbers and their sums. Table 4.12 provides a pattern analysis that clarifies the solution strategy that was used by learner L16B to deduce the relationships that led to the correct solution (observed from interview).

**Table: 4.12** A pattern analysis of numbers which fits sums of consecutive even numbers

| Integer ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Smaller Even ($2x$) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| Larger Even ($2x$ +2) | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| Sum ($2x$ +2$x$+2) | 2 | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 34 |

With the knowledge base of sequences and the n-th term the learner was able to mentally represent all the sums that can be obtained from adding two consecutive even numbers from the $x$-value in the algebraic model $4(x+1) - 2 =$ sum, giving $x = 13$ for $4(x + 1) - 2 = 54$,

based upon the fact that the smaller even number is represented by $2x = 26$. The analysis of the table shows that the median of $2x$ and $(2x+2)$ will always be an odd number, which is half the sum of the two consecutive even numbers on either side of this odd number on the real number line. The solution in the snippet in Figure 22 shows that the learner has formulated an algorithm for solving similar problems which is:

To find two consecutive even numbers $p$ and $q$ that add up to $r$ (with $r$ always even):

1. Find the half of $r$ and let $m$ be this half of $r$
2. $p = m - 1$ (smaller even number)
3. $r = m + 1$ (bigger even number)

This study therefore classified this strategy as an arithmetic strategy. In order to understand this learner's solution process, the learner was purposively selected for interview.

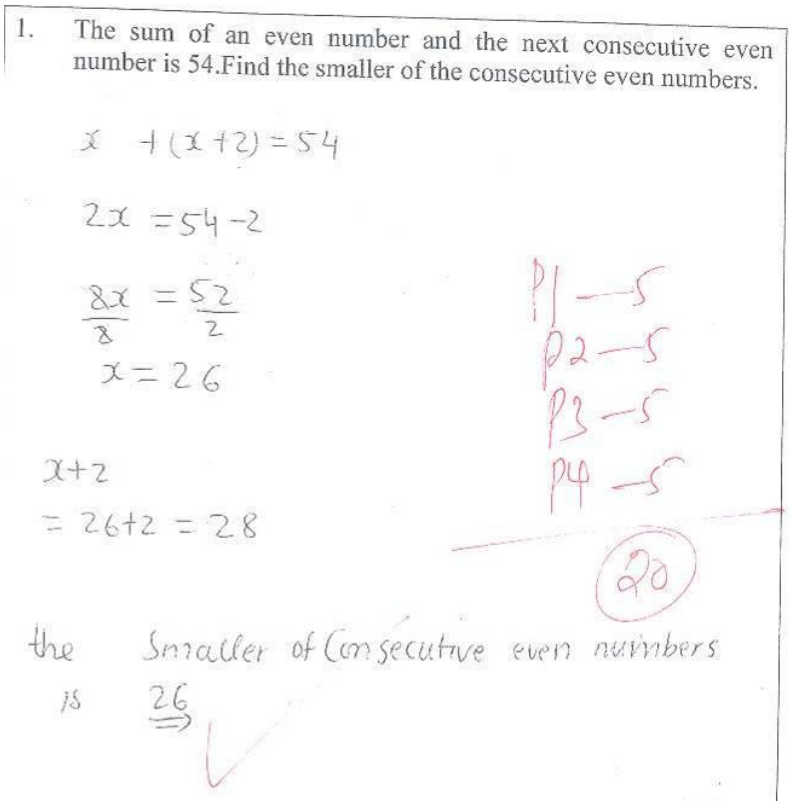
Reviewing and extending this analysis leads to the following conjecture:

***Only natural numbers whose half is an odd number can be expressed as a sum of two consecutive even numbers.***

Figure 4.17 shows how category C learner (L20C) succesfully used the algebraic solution strategy in problem 1 of the algebraic problem solving test.

**CATEGORY C (High achievers)**

**Figure 4.17**: Snippet from a category C learner (L20C) who successfully attempted the algebraic strategy



1. The sum of an even number and the next consecutive even number is 54.Find the smaller of the consecutive even numbers.

$x + (x+2) = 54$

$2x = 54 - 2$

$\frac{2x}{2} = \frac{52}{2}$

$x = 26$

$x+2$

$= 26+2 = 28$

P1 — 5
P2 — 5
P3 — 5
P4 — 5

20

the Smaller of Consecutive even numbers is 26 ⇒

The CAAPSA marking tool was used to trace the solution process and allocate marks at each step of Polya's (1957) problem solving step. The learner's solution strategy was well executed, and although the "look back" step is not explicitly shown, it is apparent from the working shown, that the learner was checking every step through the process. For example, while only the smaller of the even numbers, 26, is required, the learner went on to also give the bigger number, 28. The CAAPSA tool thus assumes that there was no error in the "look back" step. Figure 4.18 shows the CAAPSA output for the assessment of this learner's solution snippet.

**Figure 4.18:** Problem 1 CAAPSA processing and output for learner L20C

| LEARNER CODE: | L20C |
|---|---|

| QUESTION | 1 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 5 | |
| P4 | Checking of solution(s) seen or implied | 1 | 0 | 5 | |

| Total Score | 20 | L5 |
|---|---|---|
| Polya's TIMSS Score | 700 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

An interview with this learner was not deemed necessary since the solution process was very clear from the learner's written work.

#### 4.3.2.4.1.2 Analysis of solution strategies and outcomes in problem 2

*Aaron's mass is 10 kilograms greater than twice Levi's mass. If the sum of their masses is 118 kilograms, how many more kilograms is Aaron's mass than Levi's mass?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** Assign $x$ and $2x + 10$ as the respective masses of Levi and Aaron

**Polya Step 2:** Devise a plan

**Expected action:** Write the equation $x + (2x + 10) = 118$

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equations

$x = 36$, therefore Levi's mass is 36 kg while Aaron's mass is 2 (36) + 10 = 82 kg. So Aaron's mass is (72-36) kg = 46 kg more than Levi's mass.

**Polya Step 4:** Checking the solution

**Expected action:** Check solution in the problem context

Figure 4.19 shows the summary of the group's achievement in problem 2 of the algebraic problem solving test.

**Figure 4.19**: Summary of group performance in problem 2

**SUMMARY FOR QUESTION 2**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 0.98095 | 298.095 | L1 | 0.82381 | 282.381 | L1 | 0.68095 | 268.095 | L1 | 0.41429 | 241.429 | L1 | 2.9 | 272.5 | L1 |

| | | |
|---|---|---|
| L1 | 175 | 83.33% |
| L2 | 8 | 3.81% |
| L3 | 2 | 0.95% |
| L4 | 7 | 3.33% |
| L5 | 18 | 8.57% |
| Total | 210 | 100.00% |

Page 1

Of the 210 learners who sat for the algebraic problem solving test, 175 (83.33%) were at Level 1, eight (3.81%) at level 2, two (0.95%) at level 3, seven (3.33%) at level 4 and 18 (8.57%) at level 5. This suggests that the learners' main obstacles occur in the first step of Polya's model. In order to form a deeper understanding of these obstacles, the researcher looked at the following samples from some of the participants' solutions to problem 2.

Figure 4.20 shows the solution outcome of learner L5A in problem 2. The learner attempted the algebraic strategy.

**CATEGORY A**

**Figure 4.20:** Snippet from a category A learner (L5A) who attempted the algebraic strategy

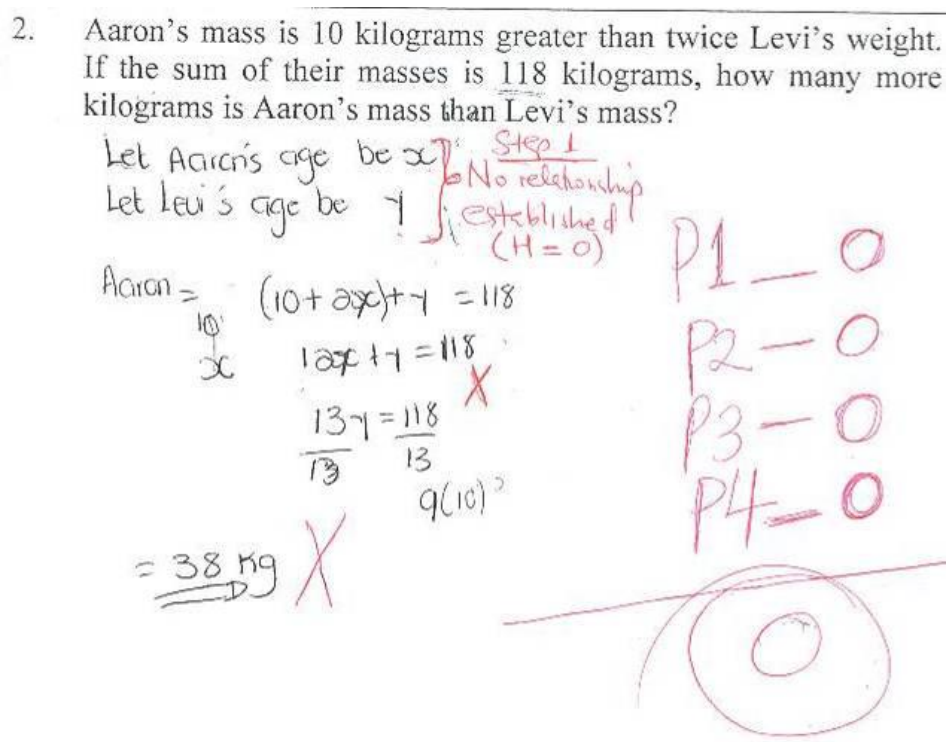



Figure 4.21 shows the CAAPSA rubric output for the assessment of the algebraic problem solving process of learner L5A. The learner did not successfully execute any of Polya's steps.

**Figure 4.21:** Problem 2 CAAPSA processing and output for learner L5A

| LEARNER CODE: | L5A |
|---|---|

| QUESTION | 2 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 0 | 0 | 0 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 0 | 0 | 0 | |
| P3 | Correct solution of equations(s) (or equivalent) | 0 | 0 | 0 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| Total Score | 0 | L1 |
|---|---|---|
| Polya's TIMSS Score | 200 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

**Polya Step 1:** Understanding the problem

The learner assigned $x$ and $y$ but the relationship is not evident, so H = 0. The CAAPSA tracer automatically allocates P1 = 0. A score in the next step cannot be higher than the score in the previous step, hence the rest of the steps are allocated P2 = 0, P3 = 0 and P4 = 0. The corresponding red colour code is automatically assigned as a warning to teachers to act on the learner's weaknesses during their instruction.

**Polya Step 2:** Devise a plan

A score in the next step cannot be more than the score in the previous step, hence the rest of the steps are allocated P2 = P3 = P4 = 0.

**Polya Step 3:** Carrying out the plan

P3 = 0

**Polya Step 4:** Look back

P4 = 0.

Although this learner's total score is 0, by design the programme allocates a TIMSS score of 200 which is at the lowest level. The red colour is consequently assigned to the learner's overall level.

**Interview**

**R:** *How did you find the problem?*

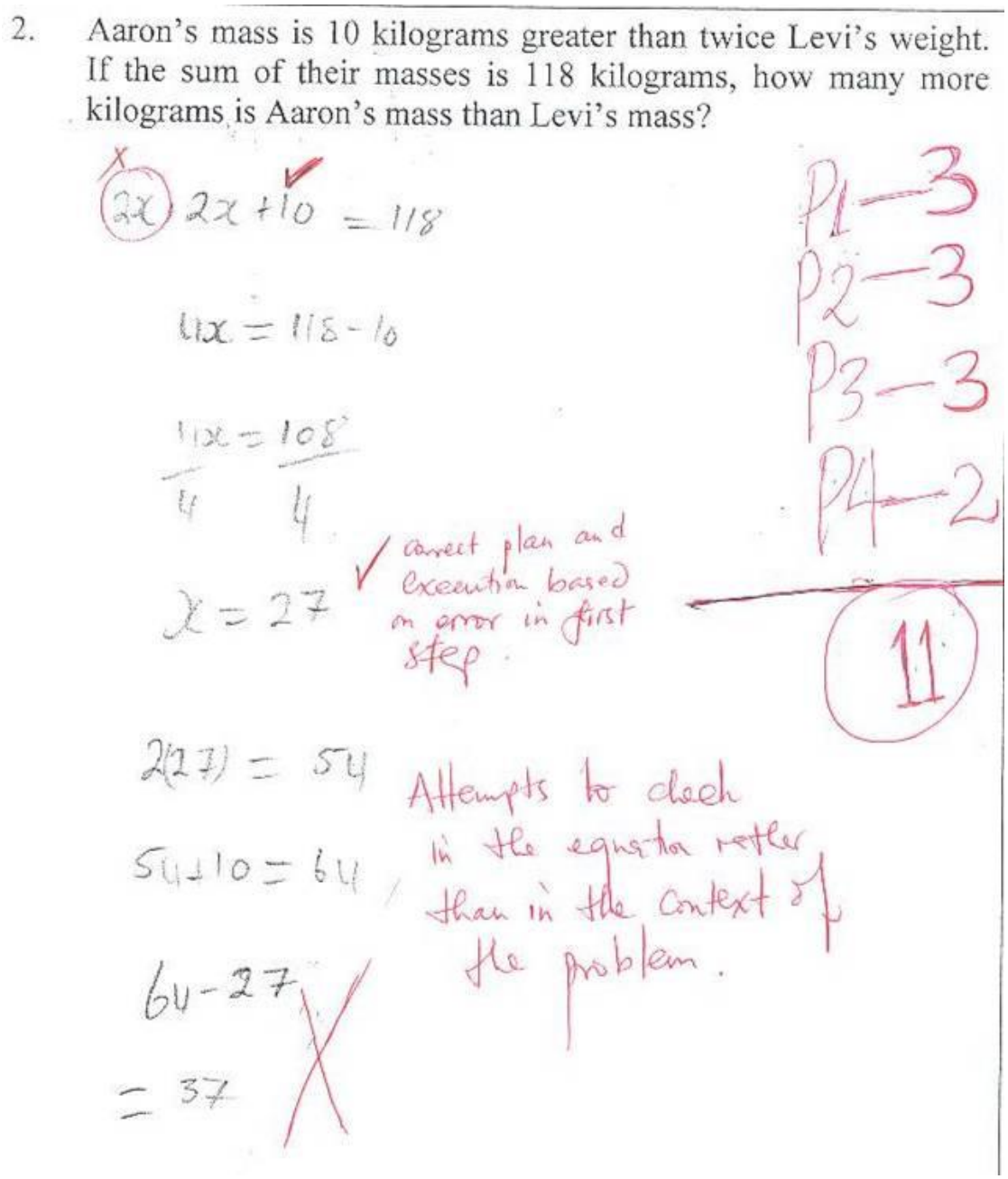
**L5A:** *Very confusing… the language used is very confusing…we were not taught this at school.*

The indications from this learner's response are that he/she did not understand the problem, hence could not represent the mass relationships in the problem. This confirmed that the learner's difficulties started at Polya's (1957) first step, understanding the problem. The learner admits that he/she has difficulties because he/she is not familiar with the problem. This supports the assertion by Krulick and Rudnick (1980) that problem solving is not an algorithm. The learner has been confronted by something he or she does not recognise.

Figure 4.22 is the solution outcome of a category B learner (L17B) in problem 2 of the algebraic problem solving test. Learner L17B also attempted the algebraic strategy of solution, but only partially succeeds in executing Polya's steps. The major handicap, observed from the learner's solution snippet is not evaluating the solution.

**CATEGORY B**

**Figure 4.22:** Snippet from a category B learner (L17B) attempting the algebraic strategy




The CAAPSA output for the solution process of learner L17B, assessed in Polya's (1957) framework is shown in Figure 4.23.

**Figure 4.23:** Problem 2 CAAPSA processing and output for learner L17B

| LEARNER CODE: | L17B |
|---|---|

| QUESTION | 2 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 1 | 3 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 3 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 3 | |
| P4 | Checking of solution(s) seen or implied | 1 | 1 | 2 | |

| Total Score | 11 | L3 |
|---|---|---|
| Polya's TIMSS Score | 475 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

The CAAPSA output for this learner assigned P1 = 3, P2 = 3, P3 = 3 and P4 = 2. The analysis reveals that the learner shows understanding of the problem but lacks algebraic representation skills which should form part of the algebraic knowledge base. The CAAPSA output assigned level 3 to this learner. The following extract is taken from the subsequent interview with this learner.

**Interview**

**R:** *How did you find this problem?*

**L17B:** *It was kind of easy...but confusing.*

**R:** *What was confusing?*

**L17B:** *I was sure of my method, but maybe I made a silly mistake somewhere.*

**R:** *Why do you think you made a mistake?*

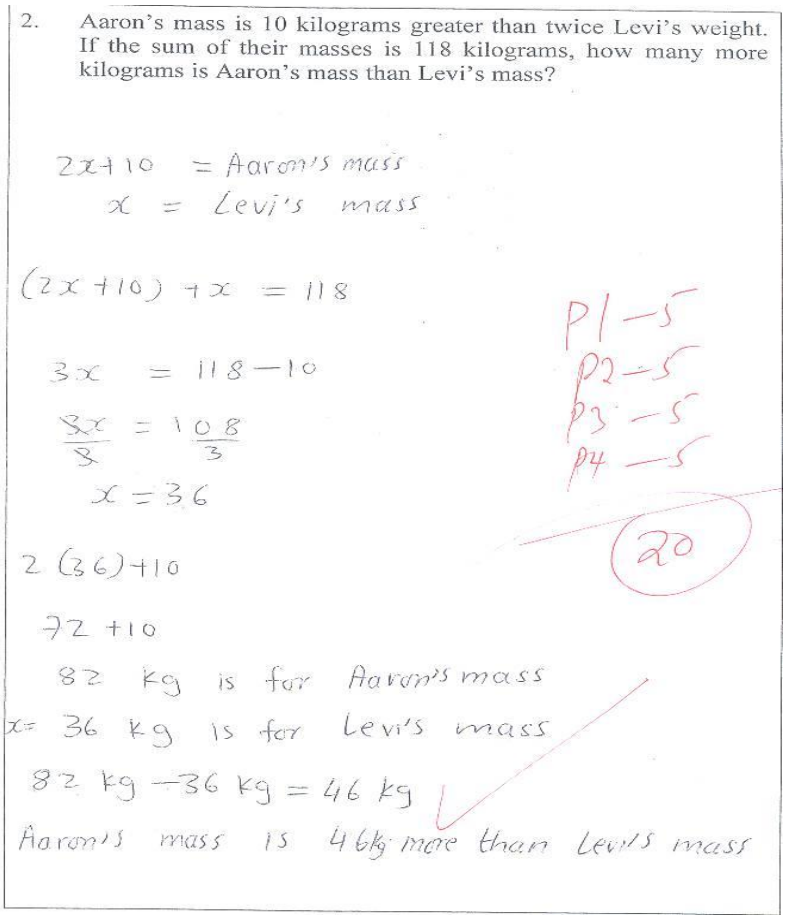
**L17B:** *The answer does not look good.*

It seems, by saying "...the answer does not look good", the learner implied that he did some form of checking but failed to devise a new plan to reach the solution. This is an indication that learners do not follow the cyclical nature of Polya's (1957) model; instead, they execute Polya's steps in a linear manner. It means when learners reach their answer, they have reached "the end". The learners do not bother to evaluate the solution and go through Polya's (1957) steps again or devise a new strategy if they realise their initial strategy did not yield a sensible result.

Figure 4.24 shows successfully employed solution strategy by a category C learner, L22C. The learner successfully employed the algebraic strategy, and the learner's work was neat and well organised.

**CATEGORY C**

**Figure 4.24:** Snippet from a category C learner (L22C) who successfully used the algebraic strategy



2. Aaron's mass is 10 kilograms greater than twice Levi's weight. If the sum of their masses is 118 kilograms, how many more kilograms is Aaron's mass than Levi's mass?

$2x+10$ = Aaron's mass
$x$ = Levi's mass

$(2x+10)+x = 118$

$3x = 118-10$

$\frac{3x}{3} = \frac{108}{3}$

$x = 36$

$2(36)+10$

$72+10$

82 kg is for Aaron's mass
$x$= 36 kg is for Levi's mass

82 kg − 36 kg = 46 kg

Aaron's mass is 46kg more than Levi's mass

P1 −5
P2 −5
P3 −5
P4 −5

20

Figure 4.25 shows the CAAPSA allocation of maximum possible scores at each of Polya's steps for the solution process of learner L22C. When the "H-values" and "No of errors" at each of Polya's steps in the CAAPSA tool, are entered the tool automatically returns P1=5, P2=5, P3=5, and P4=5, and assigns a CAAPSA score of 700 and level 5.

**Figure 4.25:** Problem 2 CAAPSA processing and output for learner L22C

| LEARNER CODE: | L22C |
|---|---|

| QUESTION | 2 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 5 | |
| P4 | Checking of solution(s) seen or implied | 1 | 0 | 5 | |

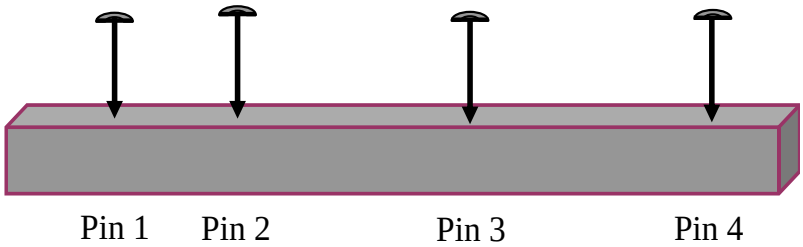
| Total Score | 20 | L5 |
|---|---|---|
| Polya's TIMSS Score | 700 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

Since the learner's solution process was clearly outlined, the interview was not conducted with this learner.

#### 4.3.2.4.1.3 Analysis of solution strategies and outcomes in problem 3

*Four steel pins are stuck in a straight line on a wooden rectangular prism block. The distance between Pin 1 and Pin 4 is 35mm. The distance between Pin 2 and Pin 3 is twice the distance between Pin 1 and Pin 2.The distance between Pin 3 and Pin 4 is the same as the distance between Pin 2 and Pin 3. What is the distance, in millimetres, between Pin 1 and Pin 3?*




Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** The three distances ***P1P2 = x, P2P3 = P3P4 = 2x, where P = Pin.***

**Polya Step 2:** Devise a plan

**Expected action:** Write the equation ***x + 2x + 2x = 35***

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equations

***x = 7***, therefore ***P1P3 = 7 + 2(7) = 21 mm***

**Polya Step 4:** Look back

**Expected action:** Check solution in the problem context

The summary of the group perfomance for problem 3 of the algebraic problem solving achievement test is given in Figure 4.26.

**Figure 4.26:** Summary of group performance in problem 3

**SUMMARY FOR QUESTION 3**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 2.39524 | 439.524 | L2 | 2.32857 | 432.857 | L2 | 2.09524 | 409.524 | L2 | 1.9619 | 396.19 | L1 | 8.78095 | 419.524 | L2 |

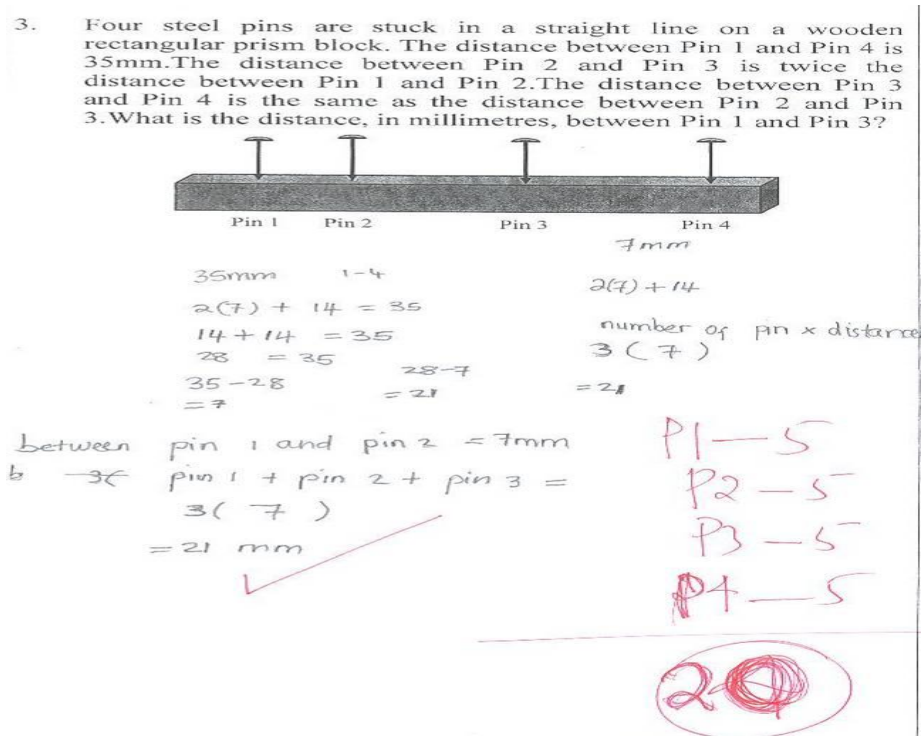
| | | |
|---|---|---|
| L1 | 116 | 55.24% |
| L2 | 5 | 2.38% |
| L3 | 3 | 1.43% |
| L4 | 3 | 1.43% |
| L5 | 83 | 39.52% |
| Total | 210 | 100.00% |

In the group of 210 learners who sat for the algebraic problem solving test, 116 (55.2%) were at Level 1, five (2.38%) at level 2, three (1.43%) at level 3, three (1.43%) at level 4 and 83 (39.52%) at level 5. Data shows that there was improvement in terms of the number of learners who reached Level 5 compared to the previous questions. This might suggest that the use of a diagram in the problem facilitated better understanding of the question.

Figure 4.27 shows the solution process of a category A learner, L11A, who despite being classified overall as a low achiever, somehow successfully executed the arithmetic solution strategy in problem 3. The learner attained CAAPSA level 5 (advanced) in the solution process of problem 3.

## CATEGORY A

**Figure 4.27:** Snippet from a category A learner (L11A) who used the guess-and-check solution strategy




**Polya Step 1:** Understanding the problem

The learner guesses that the distance between Pin 1 and Pin 2 is ***7 mm.*** Hence he/she guesses that ***P2P3 = P3P4 = 14 mm***, thus establishing the relationship between the variables. Therefore P1 = 5.

**Polya Step 2:** Devise a plan

An equivalent arithmetical statement has been given in **"35 – 28 = 7"**, hence P2 = 5.

**Polya Step 3:** Carry out the plan

***x = 7***, therefore ***P1P3 = 7 + 2(7) = 21 mm***

Correct solution obtained, P3 = 5.

**Polya Step 4:** Look back

The learner has been checking the solution throughout the problem solving process and this shows that Polya's model is not linear but cyclical. According to the conceptual framework used in this study, this is what Schoenfeld (1992) refers to as the control or self-regulation process. P4 = 5 and the problem solving skill level is rated at level 5 with a green colour code.

**Interview with learner**

At first it was not easy for the researcher to understand how this learner made such a good guess until the oral interview, which follows:

**R:** *How did you get the 7mm next to Pin 4 on your diagram?*

**L11A:** *I tried several combinations that give a total length of 35 mm. I realised that if the distance between Pin 1 and Pin 2 is 7mm, then also the remaining distance between Pin 2 and Pin 4 will be 28 mm which is shared.*

**R:** *What do you mean by the term shared?*

**L11A:** *28 mm is shared by the distances between Pin2 to Pin 3 and Pin 3 to Pin 4. So P2P3 = P3P4=14 mm … and 14 = 2 × 7 …I think there was a similar question in the first test.*

**R:** *Are you implying that you guessed the result?*

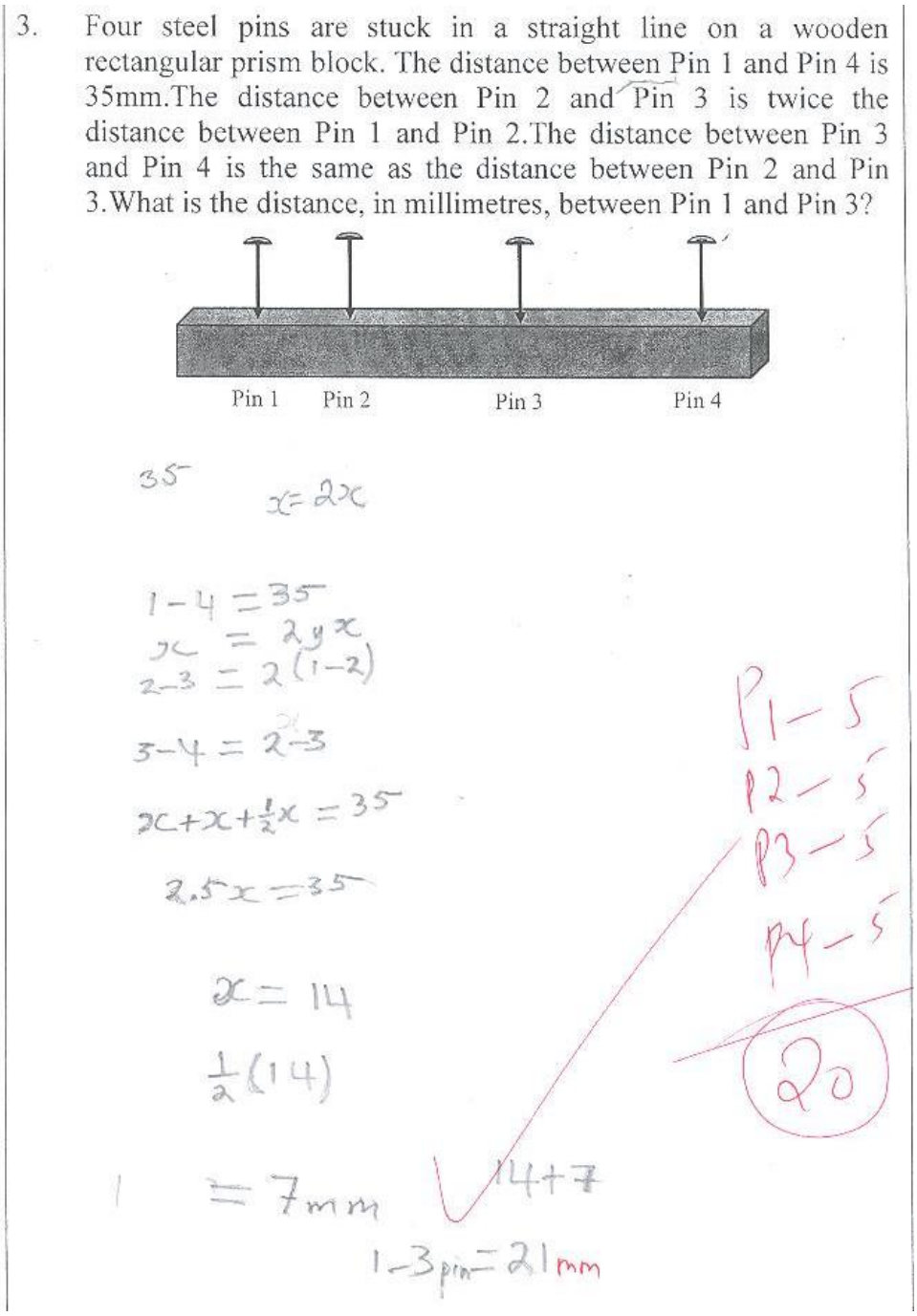
**L11A:** *Yes, but I also checked whether my guess was right. I remembered that there was a similar question in the first test.*

Although the learner struggled to express himself clearly, it was clear that this was a well calculated guess in which the learner's control process was functional. This suggests that the use of diagrams to illustrate the relationships in the question may have made it easier for the learner to understand the problem.

Figure 4.28 shows the solution outcome of learner L18B, who successfully used the algebraic strategy in problem 3.

**CATEGORY B**

**Figure 4.28:** Snippet from a category B learner (L18B) who used the algebraic strategy

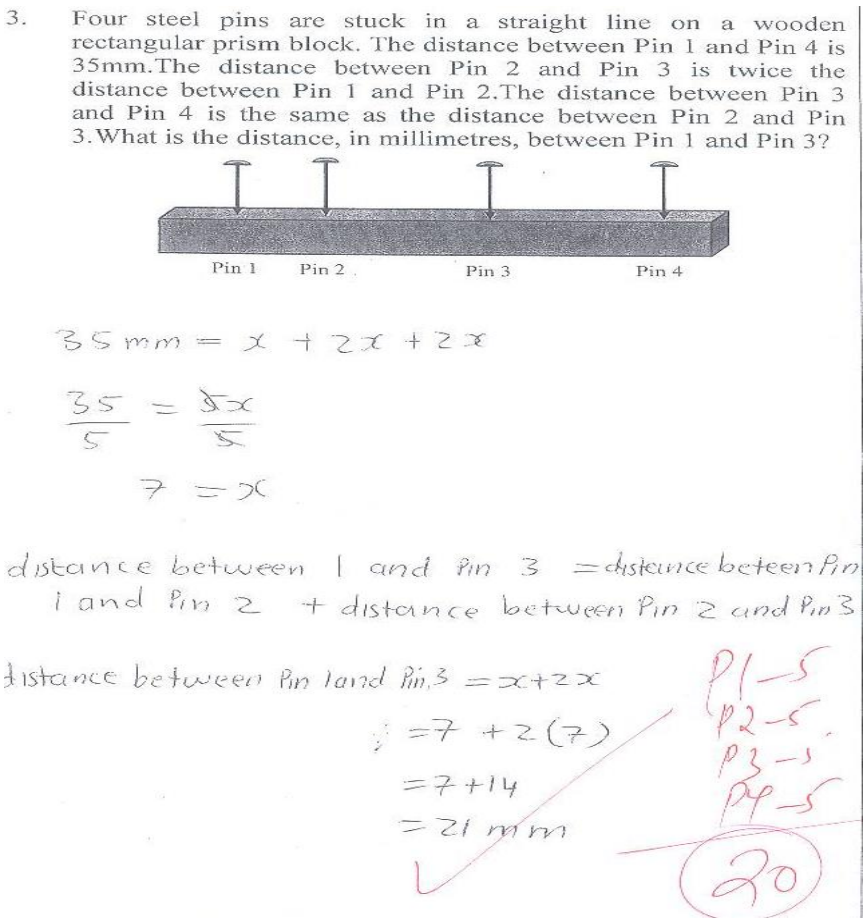



The CAAPSA output for this learner assigned P1 = P2 = P3 = P4 = 5. The learner wrote an interesting statement: **"2 – 3 = 2(1 – 2)"** not implying that "**-1 = -2"**, but rather, that **"the distance between Pin 2 and Pin 3 = twice the distance between Pin 1 and Pin 2"**. The learner devises his own plan, creatively using his/her own mathematical notation.

Figure 4.29 shows a category C learner, L24C, who was also successful in using the algebraic strategy in problem 3.

**CATEGORY C**

**Figure 4.29:** Snippet from a category C learner (L24C) who employed the algebraic strategy.



3. Four steel pins are stuck in a straight line on a wooden rectangular prism block. The distance between Pin 1 and Pin 4 is 35mm.The distance between Pin 2 and Pin 3 is twice the distance between Pin 1 and Pin 2.The distance between Pin 3 and Pin 4 is the same as the distance between Pin 2 and Pin 3.What is the distance, in millimetres, between Pin 1 and Pin 3?



$35\,mm = x + 2x + 2x$

$\frac{35}{5} = \frac{5x}{5}$

$7 = x$

distance between 1 and Pin 3 = distance beteen Pin 1 and Pin 2 + distance between Pin 2 and Pin 3

distance between Pin 1 and Pin 3 $= x + 2x$

$= 7 + 2(7)$

$= 7 + 14$

$= 21\,mm$

P1 – 5
P2 – 5
P3 – 5
P4 – 5

20

The CAAPSA output for this learner was P1 = P2 = P3 = P4 = 5 and level 5.

An interview was not necessary to establish better understanding of the learner's solution strategy since the learner's written work shows complete understanding of Polya's process. Although the checking of the solution is not shown on paper the study assumes that it is implied and that it was done.

#### 4.3.2..4.1.4 Analysis of solution strategies and outcomes in Problem 4:

*Bill, Phil and Jenny are siblings. Bill is twice as old as Phil. Jenny is two years younger than Bill. Currently, their dad is twice as old as the sum of their ages. In nine years, dad's new age will be equal to the sum of his three kids' new ages. What is Jenny's current age?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** The eight variable relations

| **Data** | ***Bill*** | ***Phil*** | ***Jenny*** | ***Dad*** |
|---|---|---|---|---|
| ***Age now*** | ***2x*** | ***x*** | ***2x - 2*** | ***2(5x – 2)*** |
| ***Age in 9 yrs*** | ***2x + 9*** | ***x + 9*** | ***2x + 7*** | ***2(5x-2) + 9*** |

**Polya Step 2:** Devise a plan

**Expected action:** Write the equation $2(5x - 2) + 9 = 2x + 9 + x + 9 + 2x + 7$

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equations

$x = 4$. Therefore Jenny's age is 2 (4) – 2 = 6 years.

**Polya Step 4:** Look back

**Expected action:** Check solution in the problem context

Figure 4.30 is the summary of the group's perfomance in problem 4 of the algebraic problem solving test.

**Figure 4.30:** Summary of group performance in problem 4

**SUMMARY FOR QUESTION 4**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 1.23 | 322.86 | L1 | 0.37 | 237.14 | L1 | 0.22 | 221.90 | L1 | 0.21 | 221.43 | L1 | 2.03 | 250.83 | L1 |

| | | |
|---|---|---|
| L1 | 195 | 92.86% |
| L2 | 5 | 2.38% |
| L3 | 1 | 0.48% |
| L4 | 0 | 0.00% |
| L5 | 9 | 4.29% |
| Total | 210 | 100.00% |

Page 1

Of the 210 learners who sat for the algebraic problem solving test, 195 (92.86%) were at Level 1, five (2.38%) at level 2, one (0.48%) at level 3, zero (0.00%) at level 4 and nine (4.29%) at level 5. This was one of the questions in which the group performed poorly.

Figure 4.31 shows the solution strategy employed by learner L15A in problem 4. This problem was identified by the participants as among the most difficult in the algebraic problem solving test.

## CATEGORY A

**Figure 4.31:** Snippet from a category A learner (L15A) who attempted the algebraic strategy

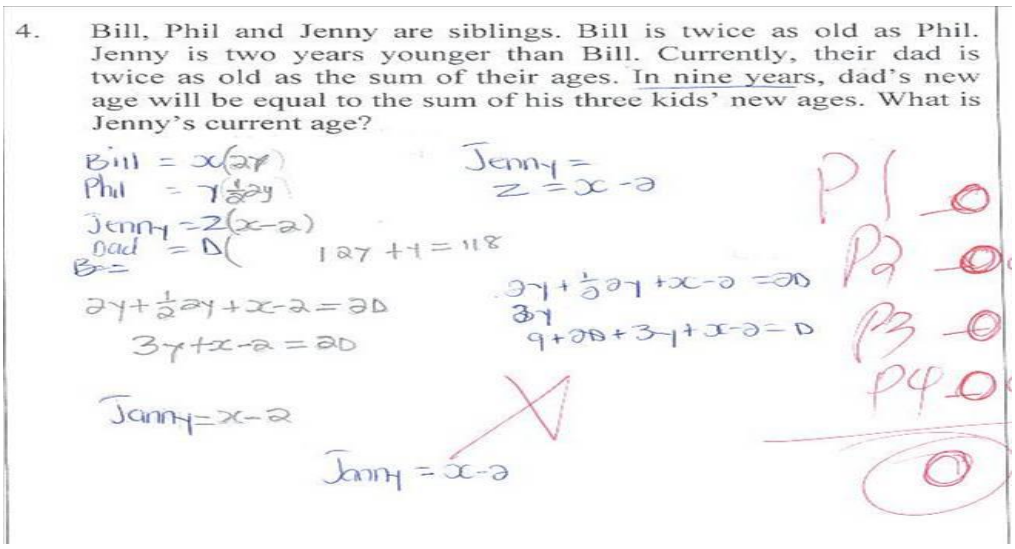


Figure 4.31 shows that the learner's difficulties started in the execution of Polya's (1957) first step; reading and understanding the problem. The learner failed to represent any of the unknown data algebraically, using variables. Consequently, the learner could not continue with the algebraic strategy. All the learners who succeeded in this problem, used the algebraic strategy, and it appears it was not easy for any of the learners to think of an alternative strategy. The learners, who used a table, better understood the problem and the relationships between the known and unknown data.

Figure 4.32 shows the assessment outcome of the snippet of learner L15A in problem 4.

**Figure 4.32:** Problem 4 CAAPSA processing and output for learner L15A

| LEARNER CODE: | L15A |
|---|---|

| QUESTION | 4 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 8 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 8 | 0 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 0 | 0 | 0 | |
| P3 | Correct solution of equations(s) (or equivalent) | 0 | 0 | 0 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| | | |
|---|---|---|
| Total Score | 0 | L1 |
| Polya's TIMSS Score | 200 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

**Polya Step 1:** Understanding the problem

The learner could not make a single correct representation of any of the unknowns, hence H = 0, consequently CAAPSA allocates P2 = P3 = P4 = 0.

**Polya Step 2:** Devise a plan

Incorrect attempt to experiment with incorrect variables, P2 = 0.

**Polya Step 3:** Carry out the plan

Incorrect, P3 = 0.

**Polya Step 4:** Look back

No checking attempted, P4 = 0.

**Interview**

The researcher tried to find out in the interviews what this learner was attempting to do in his attempted algebraic strategy.

**R:** *How did you find this problem?*

**L15A:** *The story is too long….I could not manage to form an equation…so I gave up.*

**R:** *Why do you think it was difficult for you to form an equation?*

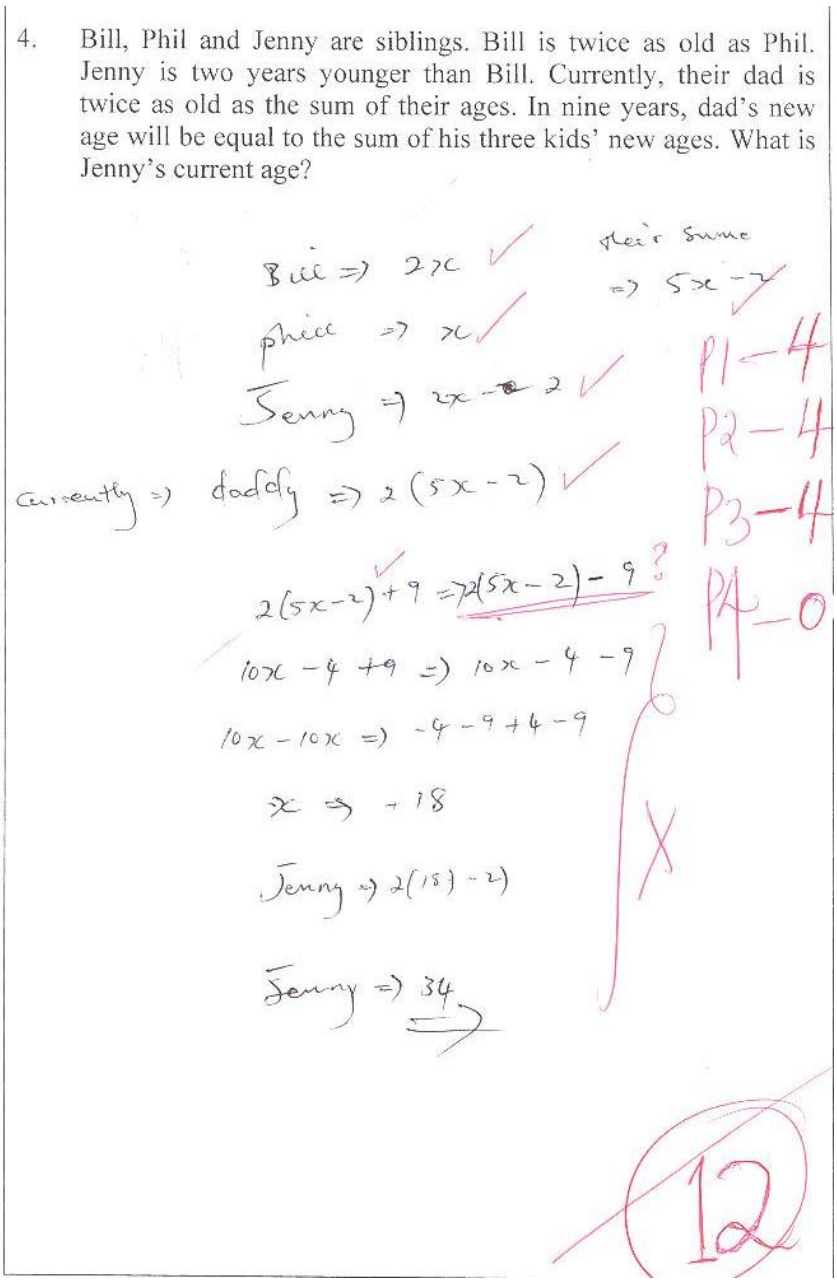
**L15A:** *I just think we never had any practice in class with these kinds of questions…*

The learner's response points to a lack of productive disposition due to a lack of training in problem solving. This supports the definition of mathematical proficiency by Kilpatrick et al (2001) and the notion that algebraic problem solving is a higher strand of mathematical proficiency (Milgram, 2007).

Figure 4.33 is a snippet of a category B learner (L19B) who attained CAAPSA level 3 (intermediate) in the solution strategy employed in problem 4. The learner committed some minor computational errors and failed to check the solution.

**CATEGORY B**

**Figure 4.33:** Snippet from a category B learner (L19B) who attempted the algebraic strategy



4. Bill, Phil and Jenny are siblings. Bill is twice as old as Phil. Jenny is two years younger than Bill. Currently, their dad is twice as old as the sum of their ages. In nine years, dad's new age will be equal to the sum of his three kids' new ages. What is Jenny's current age?

Bill => 2x
phil => x
Jenny => 2x - 2
their Sume => 5x - 2
currently => daddy => 2(5x - 2)
2(5x-2) + 9 => 2(5x - 2) - 9
10x - 4 + 9 => 10x - 4 - 9
10x - 10x => -4 - 9 + 4 - 9
x => -18
Jenny => 2(18) - 2)
Jenny => 34

P1 - 4
P2 - 4
P3 - 4
P4 - 0

12

The CAAPSA output for this learner assigned P1 = P2 = P3 = 4 and P4 = 0. The researcher deemed it unnecessary to conduct an interview with this learner. The snippet indicates that the learner failed to carry out the plan in not writing the correct equation, and later on not attempting to evaluate the solution obtained. Figure 4.34 shows the CAAPSA output for the learner L19B's solution process.

**Figure 4.34:** Problem 4 CAAPSA processing and output for learner L19B

| LEARNER CODE: | L19B |
|---|---|

| QUESTION | 4 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 8 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 2 | 4 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 1 | 4 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 4 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| Total Score | 12 | L3 |
|---|---|---|
| Polya's TIMSS Score | 500 | |

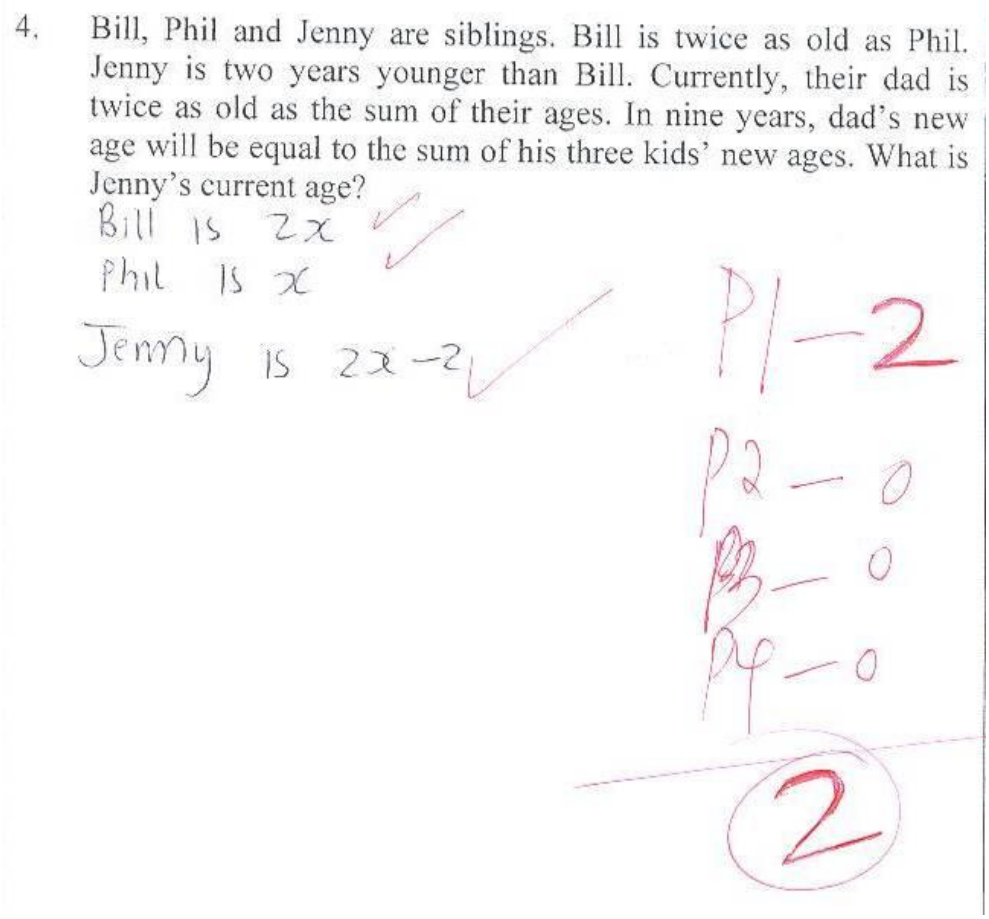
| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

Since the learner's errors are clear in the written work, an interview was unnecessary.

Figure 4.35 shows an overall high achieving learner, L25C's solution outcomes to problem 4.

**CATEGORY C**

**Figure 4.35:** Snippet from a category C learner (L25C) who attempted the algebraic strategy



4. Bill, Phil and Jenny are siblings. Bill is twice as old as Phil. Jenny is two years younger than Bill. Currently, their dad is twice as old as the sum of their ages. In nine years, dad's new age will be equal to the sum of his three kids' new ages. What is Jenny's current age?

Bill is $2x$

Phil is $x$

Jenny is $2x-2$

P1 – 2

P2 – 0

P3 – 0

P4 – 0

2

Although this was a learner with an overall high level of achievement in the algebraic problem solving process, the learner still encountered difficulties with this problem. This was one of the worst performed problems among the higher achievers.

Figure 4.36 is the CAAPSA output and traces the solution process, in Polya's model. The results show that the learner failed to execute Polya's first step.

**Figure 4.36:** Problem 4 CAAPSA processing and output for learner L25C

| LEARNER CODE: | L25C |
|---|---|
| QUESTION | 4 |
| Enter the number of expected variables and/or terms in Polya's step 1 | 8 |

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 5 | 2 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 0 | 1 | 0 | |
| P3 | Correct solution of equations(s) (or equivalent) | 0 | 0 | 0 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| | | |
|---|---|---|
| Total Score | 2 | L1 |
| Polya's TIMSS Score | 250 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

**Interview**

**R:** *You did not finish this question. What happened?*

**L25C:** *I wanted to start with the easier ones and come back to this one…*

**R:** *You mean you did not have enough time to come back and finish it?*

**L25C:** *There was enough time, but the question is using this funny language…it's very difficult to understand…so I gave up.*

The learner's last response could be a reflection of a lack of productive disposition in Kilpatrick's (2001) strands of mathematical proficiency. According to Polya's (1957) framework and Schoenfeld's (1992) theory of metacognition, this is lack of control, and this shows that the learner has not mastered Polya's (1957) problem solving process.

#### 4.3.2.4.1.5 Analysis of solution strategies used in Problem 5:

*In a group of cows and chickens, the number of legs is 14 more than twice the number of heads. How many cows are there in the group?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem
**Expected action:** The two algebraic representations are number of legs = $4x + 2y$ and number of heads = $x + y$

**Polya Step 2:** Devise a plan
**Expected action:** Write the equation $4x + 2y = 14 + 2(x + y)$

**Polya Step 3:** Carry out the plan
**Expected action:** Solve the equation
$x = 7$. *There are seven cows*
It should be noted that although the question does not ask about the number of chickens, it is expected that learners reflect this. Any natural number of chickens will satisfy the conditions of the question, provided the number of cows is seven.

**Polya Step 4:** Look back
**Expected action:** Check solution in the problem context

Figure 4.37 shows the summary of the group's achievement in the solution of problem 5 of the algebraic problem solving test.

**Figure 4.37:** Summary of group performance in problem 5

**SUMMARY FOR QUESTION 5**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 0.52381 | 252.381 | L1 | 0.25238 | 225.238 | L1 | 0.22857 | 222.857 | L1 | 0 | 200 | L1 | 1.00476 | 225.119 | L1 |

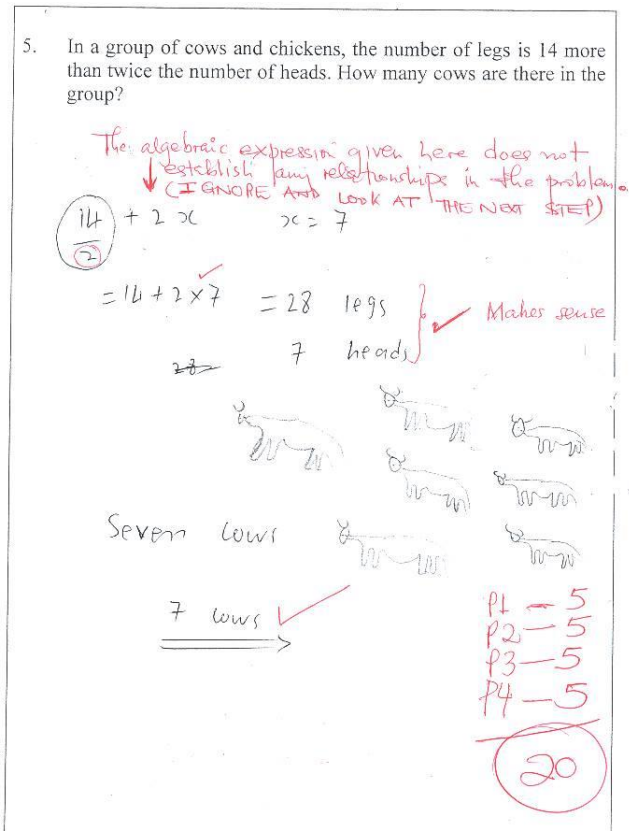
| | | |
|---|---|---|
| L1 | 199 | 94.76% |
| L2 | 1 | 0.48% |
| L3 | 1 | 0.48% |
| L4 | 0 | 0.00% |
| L5 | 9 | 4.29% |
| Total | 210 | 100.00% |

Page 1

Problem 5 was the worst performed question by the group. Figure 4.38 shows a snippet of an overall low achieving learner, L1A, who however did very well in this particular question.

## CATEGORY A

**Figure 4.38:** Snippet from a category A learner (L1A) who used a picture




It was interesting to note that while learner L1A's overall classification of level of achievement in the algebraic problem solving test was low (level 2) and while question 5 was

the worst performed question by the group, the learner even outperformed the category C learners in this question. Figure 4.39 shows the assessment output from the CAAPSA tool.

**Figure 4.39:** Problem 5 CAAPSA processing and output for learner L1A

| LEARNER CODE: | L1A |
|---|---|

| QUESTION | 5 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 0 | 5 | |
| P4 | Checking of solution(s) seen or implied | 1 | 0 | 5 | |

| Total Score | 20 | L5 |
|---|---|---|
| Polya's TIMSS Score | 700 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

**Interview**

**R:** *This was one of the most poorly performed questions by the group, and you are one of the few learners who got it correct. What was your strategy?*

**L1A:** *I used pictures to see the number of cows and chickens that would give me the required combination for the conditions in the question.*

**R:** *But, the question here says the group comprises of cows and chickens, and you drew only the cows. Where are the chickens?*

**L1A:** *(learner L1A laughing), yeah!, but even without the chickens the answer is correct. The question only asked for the number of cows, you see 7 cows times 4 equals 28 legs, 7 cows have 7 heads, so twice the number of heads is 14, and 14 + 14 =28.*

**R:** *And where are the chickens?*

**L1A:** *The question did not ask for the number of chickens, so I did not look for the chickens.*

The learner's explanation for the missing chickens is probably the reason for Krulick and Rudnick (1980) adding a fifth step, **Review and Extend,** to Polya's (1957) four step model

(see Table 2.2). According to Krulick and Rudnick (1980), in the step "review and extend", the learner verifies the answer and looks for variations in the method of solving the problem, for example, although the question does not require the learner to give the number of chickens, verifying the problem might require the learner to think about a possible combination of cows and chickens that satisfy the constraints of the problem. This problem was designed to allow 7 cows and any natural number of chickens as an acceptable answer. This is clearer if the algebraic strategy is employed, resulting in the equation;

$4x + 2y = 14 + 2(x + y)$, where x represents the number of cows and y the number of chickens. The terms in y will always eliminate each other regardless of the value they take.

Figure 4.40 shows an attempt to use the algebraic strategy, by a learner in category B.

**CATEGORY B**

**Figure 4.40:** Snippet from a category B learner (L16B) who attempted the algebraic strategy

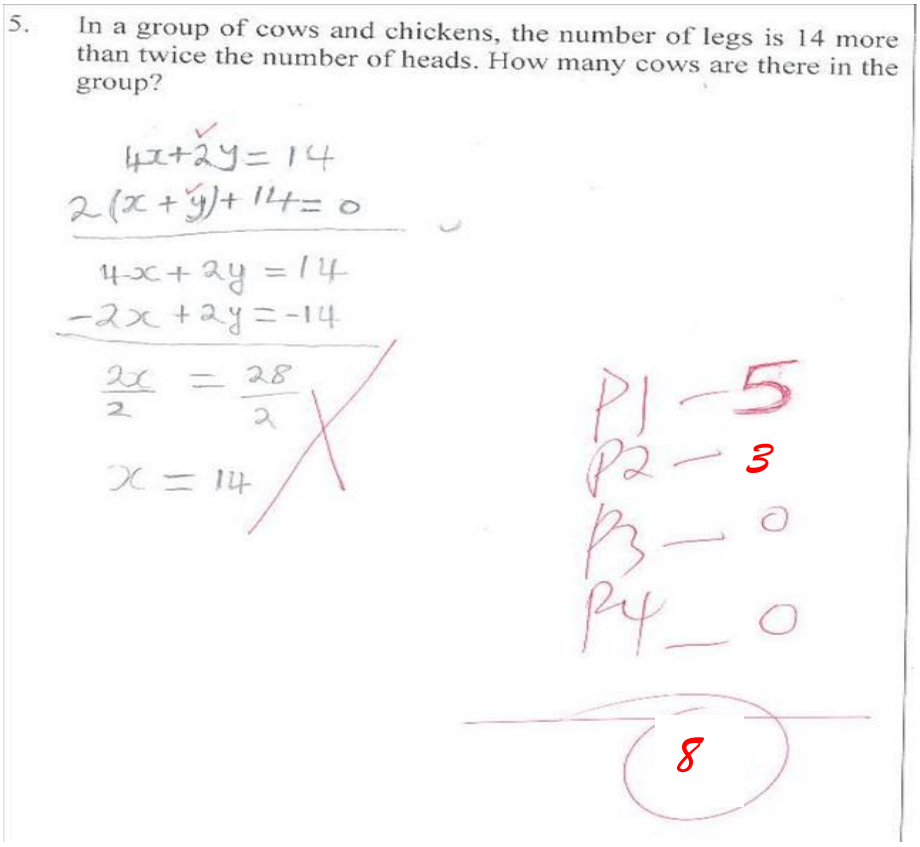



Figure 4.41 shows the CAAPSA output for achievement of learner 16B in problem 5, at each stage of Polya's (1957) process.

**Figure 4.41:** Problem 5 CAAPSA output for learner L16B

| LEARNER CODE: | L16B |
|---|---|

| QUESTION | 5 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 1 | 3 | |
| P3 | Correct solution of equations(s) (or equivalent) | 0 | 0 | 0 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| | | |
|---|---|---|
| Total Score | 8 | L2 |
| Polya's TIMSS Score | 400 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

The learner failed to write the correct simultaneous equations relating the unknown variables to the given data in the problem. Although the learner demonstrates understanding of the problem, and a correct solution plan to reduce the problem with simultaneous equations, the learner failed to execute the plan.

Figure 4.42 shows a learner in category C who partially succeeded in using the algebraic strategy for the solution of problem 5.

**CATEGORY C**

**Figure 4.42:** Snippet from a category C learner (L20C) who attempted the algebraic strategy

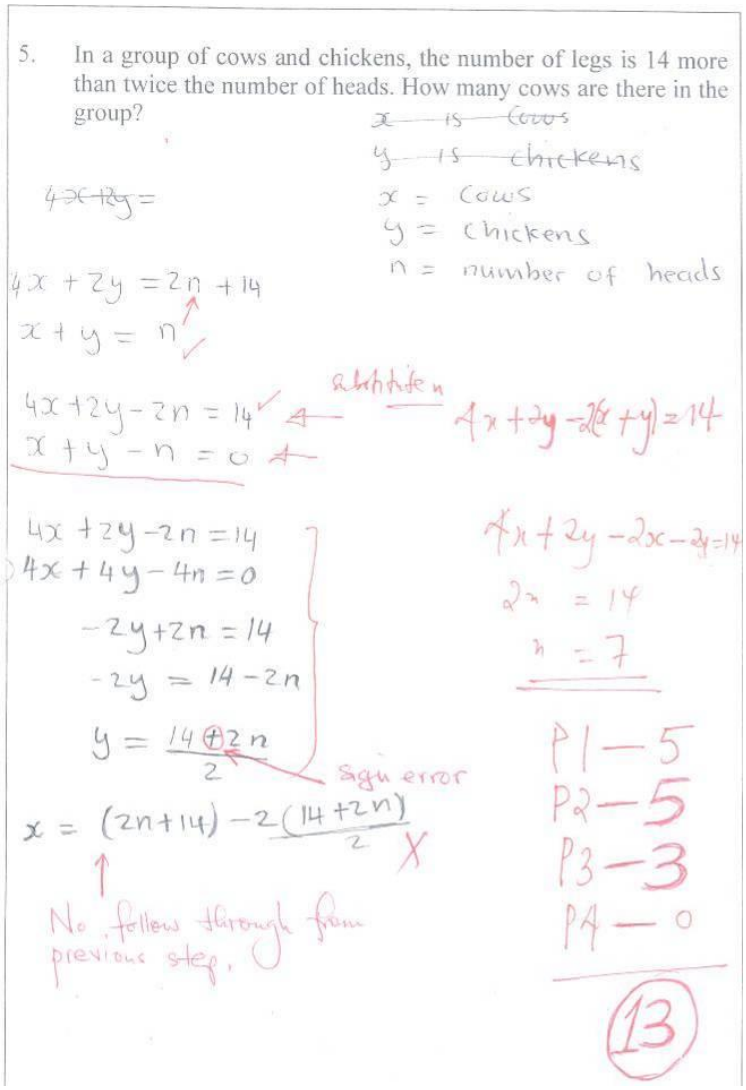



Figure 4.43 shows the CAAPSA output for the assessment of the solution process of learner L20C, in category C. The same snippet was used by the panel of mathematics experts to validate the CAAPSA processing for allocating marks to each of Polya's steps (see Figure 4.8).

**Figure 4.43:** Problem 5 CAAPSA processing and output for learner L20C

| LEARNER CODE: | L20C |
|---|---|

| QUESTION | 5 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 2 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 1 | 3 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| | | |
|---|---|---|
| Total Score | 13 | L3 |
| Polya's TIMSS Score | 525 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

This particular learner could interpret and represent the problem algebraically but could not solve the resulting simultaneous equations because of a sign error when making $y$ the subject of the formula. The learner wrote $y = \frac{14+2n}{2}$ instead of $y = \frac{2n-14}{2}$ and this led to the wrong answer (see Figure 4.42).

**Interview**

**R:** *Was this a difficult question for you?*

**L20C:** *I think this question does not have a solution*

**R:** *Why do you think there is no solution to this question?*

**L20C:** *Because there is 3 unknown number, number of heads, number of chicken and the number of cows. You can not solve 3 unknown numbers at the same time*

The reason given by the learner for failing to find the solution or thinking that the problem has no solution, demonstrates that the learner's control process was very effective. However, it appears after failing to carry out the plan the learner did not go back to devise a new plan, an indication that learners tend to execute the steps in Polya's (1957) model in a linear rather than cyclic manner.

**4.3.2.4.1.6 Analysis of solution strategies used in Problem 6:**

*A 120 centimetre-long rope is cut into three pieces. The first piece of the rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** The three lengths 2***x, x and 3x***

**Polya Step 2:** Devise a plan

**Expected action:** Write the equation $2x + x + 3x = 120$

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equations

$x = 20$, therefore the longest piece is ***60 cm.***

**Polya Step 4:** Look back

**Expected action:** Check solution in the problem context

Figure 4.44 shows the group's perfomance summary in problem 6. Problem 6 was identified as one of the easiest questions in the algebraic problem solving test.

**Figure 4.44:** Summary of group performance in problem 6

**SUMMARY FOR QUESTION 6**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 2.4 | 440 | L2 | 2.32381 | 432.381 | L2 | 2.18571 | 418.571 | L2 | 2.07143 | 407.143 | L2 | 8.98095 | 424.524 | L2 |

| | | |
|---|---|---|
| L1 | 115 | 54.76% |
| L2 | 1 | 0.48% |
| L3 | 4 | 1.90% |
| L4 | 3 | 1.43% |
| L5 | 87 | 41.43% |
| Total | 210 | 100.00% |

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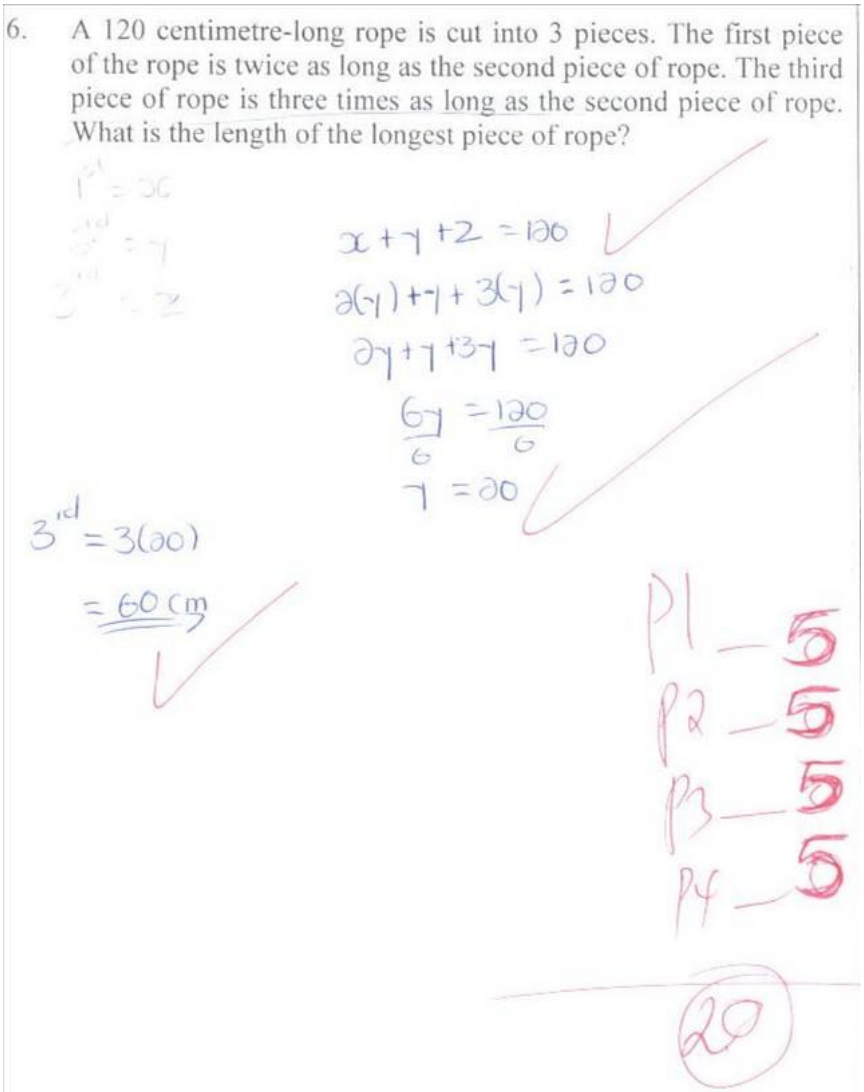
Figure 4.45 shows a solution snippet of learner L5A who successfully used, the algebraic strategy to solve problem 6. This was despite, the overall poor performance of the learner in the rest of the problems of the algebraic problem solving test. This particular learner was therefore interviewed to understand why problem 6 turned out to be easy.

**CATEGORY A**

**Figure 4.45:** Snippet from a category A learner (L5A) who used the algebraic strategy



6. A 120 centimetre-long rope is cut into 3 pieces. The first piece of the rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope?

$x + y + z = 120$

$2(y) + y + 3(y) = 120$

$2y + y + 3y = 120$

$\frac{6y}{6} = \frac{120}{6}$

$y = 20$

$3^{rd} = 3(20)$

$= 60\ cm$

P1 _ 5

P2 _ 5

P3 _ 5

P4 _ 5

20

The CAAPSA output was at level 5 with P1=P2=P3=P4=5. The solution strategy was correctly executed.

**Interview**

**R:** *You have performed very, well in this problem. Why did you find it easy?*

**L5A:** *I know that there was a similar question in the first test, the last question. We were required to add the different pieces to get the full length. This question was better.*

It appears that what the learner, L5A is implying that the problem was already familiar, from solving a similar problem.

Figure 4.46 shows a solution snippet of learner L17B (category B) who struggled to use the algebraic strategy in question 6.

**CATEGORY B**

**Figure 4.46:** Snippet from a category B learner (L17B) who attempted the algebraic strategy

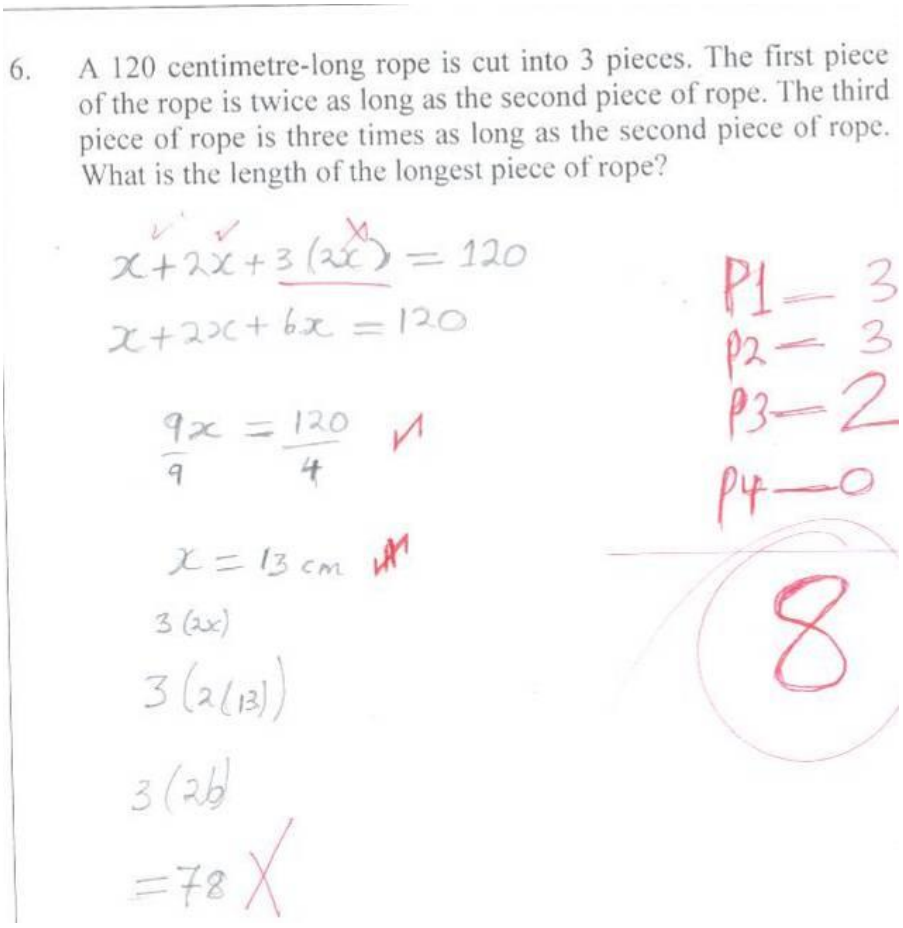


Figure 4.47 is the print screen of the CAAPSA display used to assess the solution process of learner L17B in problem 6.

**Figure 4.47**: Problem 6 CAAPSA processing and output for learner L17B

| LEARNER CODE: | L17B |
|---|---|

| QUESTION | 6 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 3 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 1 | 3 | |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 3 | |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 1 | 2 | |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| Total Score | 8 | L2 |
|---|---|---|
| Polya's TIMSS Score | 400 | |

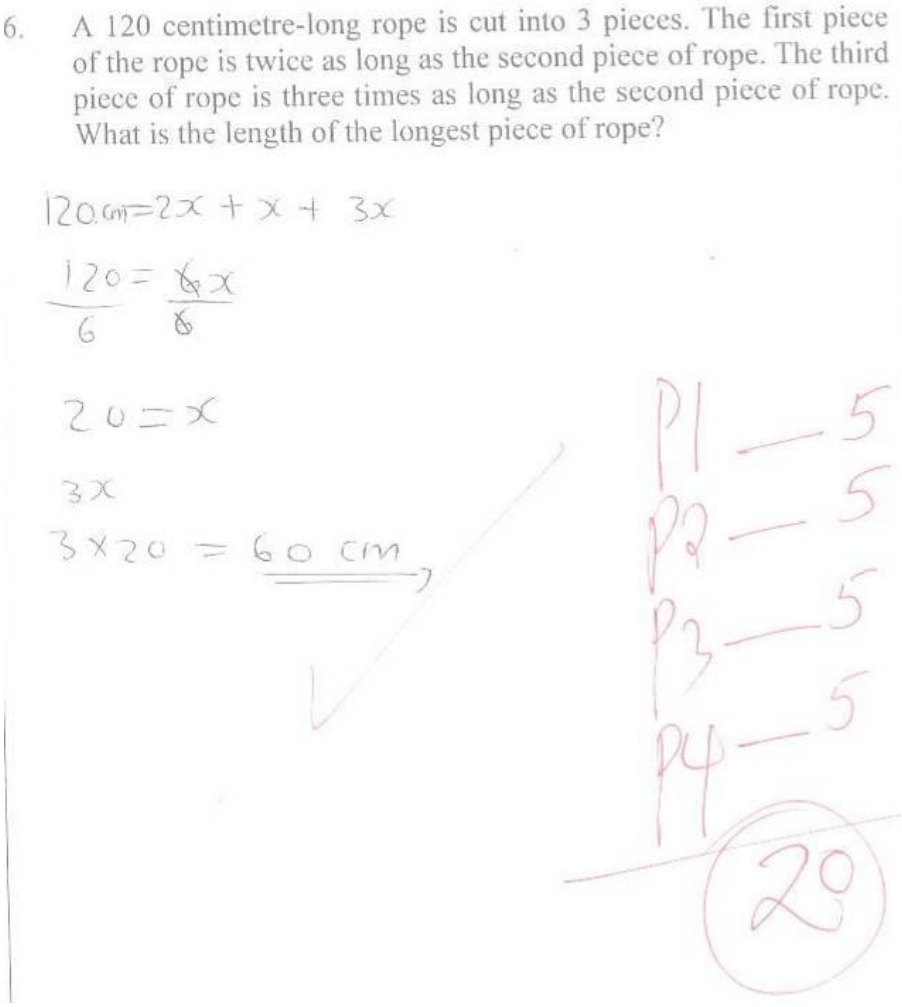
| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

The learner failed to define the third variable for the longest piece of rope which demonstrates a deficiency in Polya's first step, understanding the problem. This could also be attributed to a lack of procedural fluency emanating from a low level of algebraic knowledge base.

Category C, learner L22C's solution process, employing the algebraic strategy was successful. Figure 4.48 shows learner L22'solution strategy and outcomes.

**CATEGORY C**

**Figure 4.48:** Snippet from a category C learner (L22C) who attempted the algebraic strategy



6. A 120 centimetre-long rope is cut into 3 pieces. The first piece of the rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope?

$120\,cm = 2x + x + 3x$

$\frac{120}{6} = \frac{6x}{6}$

$20 = x$

$3x$

$3 \times 20 = 60\ cm$

P1 — 5
P2 — 5
P3 — 5
P4 — 5

20

The learner successfully executed all the solution steps.

#### 4.3.2.4.1.7 Analysis of solution strategies used in Problem 7:

*In a group of cows and chickens, the number of legs is 46 and the number of heads is 13. How many cows and chickens are there in the group?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** The two algebraic representations are number of legs = $4x + 2y$ and number of heads = $x + y$, where $x$ represents number of cows and $y$ number of chickens.

**Polya Step 2:** Devise a plan

**Expected action:** Write the simultaneous equations $4x + 2y = 46$

$x + y = 13$

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equations

$x = 10$ ***and*** $y = 3$. *There are 10 cows and three chickens*

**Polya Step 4:** Look back

**Expected action:** Check solution in the problem context

**Figure 4.49:** Summary of group performance in problem 7

**SUMMARY FOR QUESTION 7**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 2.29048 | 429.048 | L2 | 2.25714 | 425.714 | L2 | 2.02381 | 402.381 | L2 | 1.89524 | 389.524 | L1 | 8.46667 | 411.667 | L2 |

| | | |
|---|---|---|
| L1 | 115 | 54.76% |
| L2 | 7 | 3.33% |
| L3 | 6 | 2.86% |
| L4 | 2 | 0.95% |
| L5 | 80 | 38.10% |
| Total | 210 | 100.00% |

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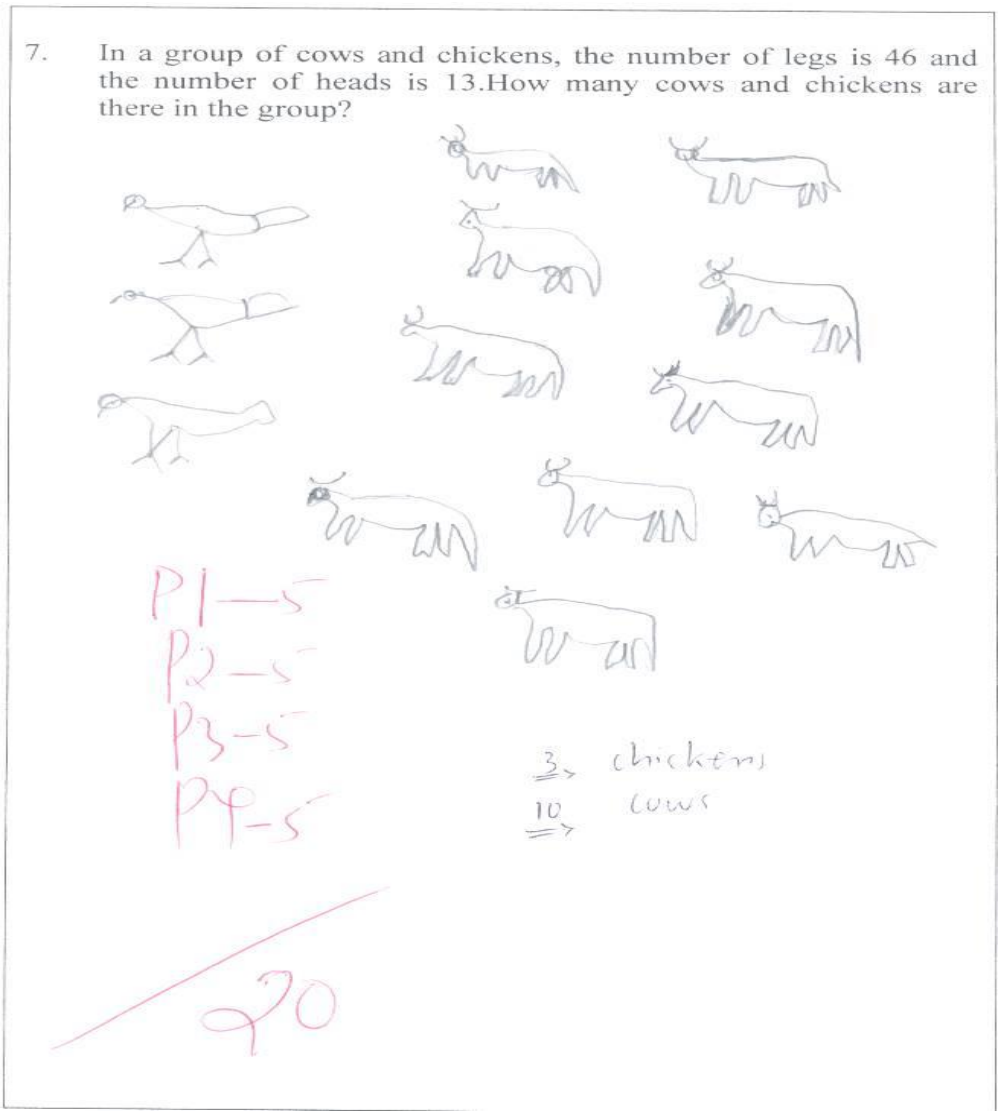
Figure 4.50 shows a picture strategy used by learner L11A. The strategy was successfully implemented.

**CATEGORY A**

**Figure 4.50:** Snippet from a category A learner (L11A) who used a picture




**Interview**

**R:** *Why did you draw pictures?*

**L11A:** *You have to see the animals in order to count the heads and legs.*

**R:** *Do you think there could be a different combination of number of chickens and pigs that also gives 13 heads and 46 legs?*

**L11A:** *I am not sure…but this is also a correct combination…*

The response from the learner may suggest that pictures facilitate the understanding of the problem. The use of pictures appears to improve the visualisation of the relationships between the unknown and the known data in the problem. The strategy that was used by this learner demonstrates that learners are capable of using their intuitive problem solving skills even without the knowledge of algebra.

Figure 4.51 shows an algebraic strategy, successfully implemented to the solution of problem 7, by learner L18B.

**CATEGORY B**

**Figure 4.51:** Snippet from a category B learner (L18B) who used the algebraic strategy

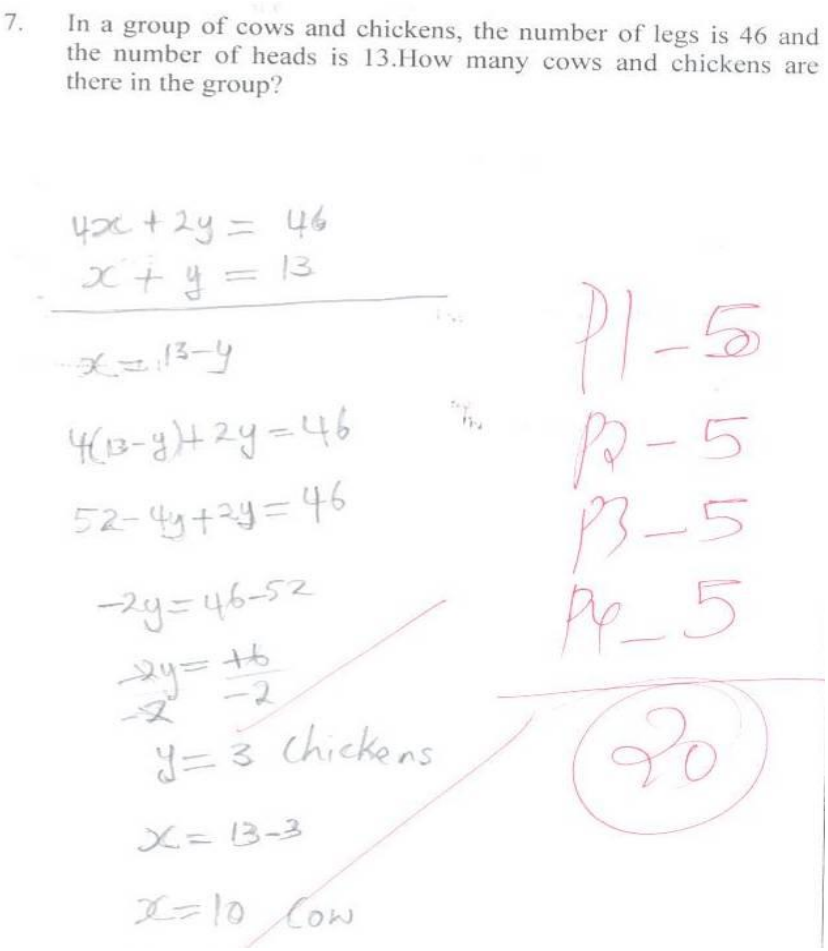
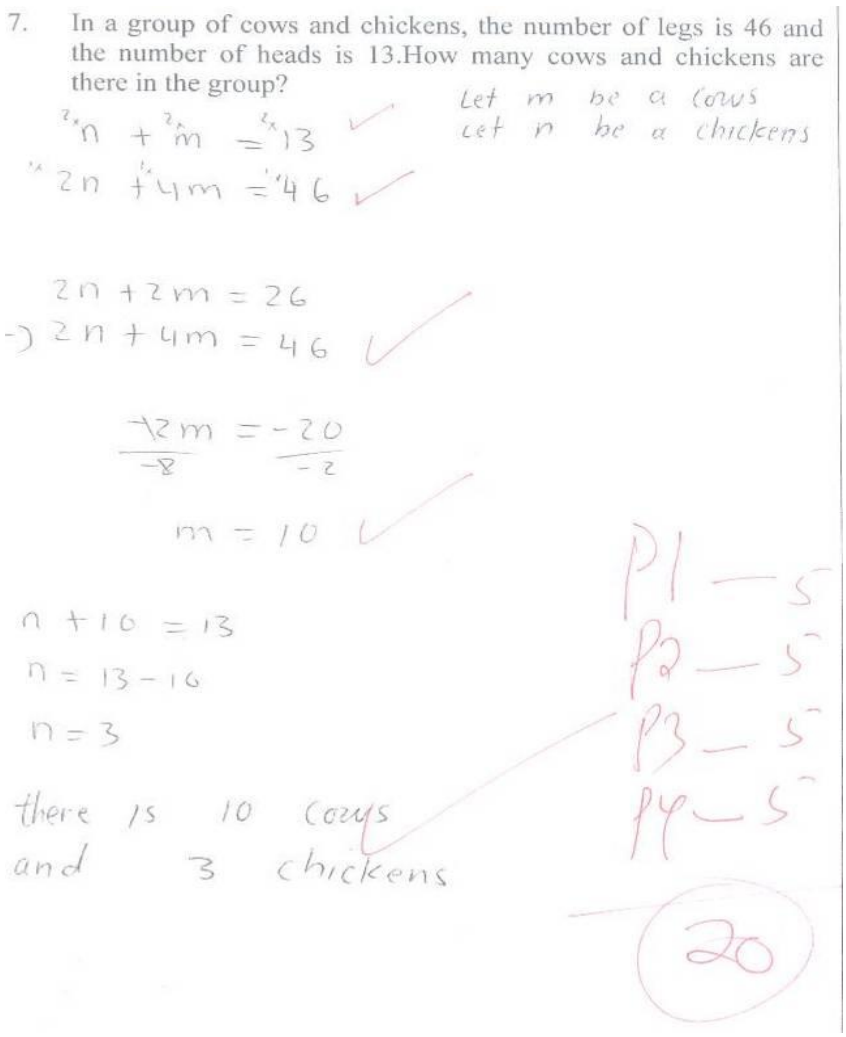



Figure 4.52 shows a solution snippet of learner L24C. The learner successfully employed the algebraic strategy of solution.

**CATEGORY C**

**Figure 4.52:** Snippet from a category C learner (L24C) who used the algebraic strategy




**4.3.2.4.1.8 Analysis of solution strategies used in Problem 8:**

*Anita passed around a basket of strawberries to the girls at her birthday party. Before the party she ate 5 strawberries and gave a friend 3. Eight girls arrived at the party. The first girl took a strawberry, the second girl took three strawberries, and the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. Given that, the pattern of distribution of strawberries to the eight girls was an arithmetic progression, how many strawberries were there in the basket at the beginning?*

Source: The Rössing Foundation Handbook of Mathematics Olympiads and Ladder Competitions (2009).

**Polya Step 1:** Understanding the problem

**Expected action:** The three algebraic representations are number of strawberries at the beginning = $n$, number of strawberries after Anita ate 5 sweets = $n - 5$ and number of strawberries after Anita gave 3 sweets to a friend = $n - 5 - 3 = n - 8$

**Polya Step 2:** Devise a plan

**Expected action:** The eight terms of the AP are 1, 3, 5, 7, 9, 11, 13, and 15. The sum of the AP is 64. So the remaining sweets after the last girl took her sweets is $n - 8 - 64 = n - 72$.

**Write the equation $n - 72 = 0$.**

**Polya Step 3:** Carry out the plan

**Expected action:** Solve the equation $n - 72 = 0$.

***n* = 72.** *There were 72 sweets at the beginning.*

**Polya Step 4:** Look back

**Expected action:** Check solution in the problem context

Number of strawberries at the end = 72 – (5 + 3) – (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15)

= 72 – 64 = 0

Figure 4.53 shows the summary of the group's performance in problem 8.

**Figure 4.53:** Summary of group performance in problem 8

**SUMMARY FOR QUESTION 8**

| | STEP 1 | | | STEP 2 | | | STEP 3 | | | STEP 4 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| Mean | 2.40476 | 440.476 | L2 | 2.26667 | 426.667 | L2 | 2.0381 | 403.81 | L2 | 1.90952 | 390.952 | L1 | 8.61905 | 415.476 | L2 |

| | | |
|---|---|---|
| L1 | 115 | 54.76% |
| L2 | 6 | 2.86% |
| L3 | 6 | 2.86% |
| L4 | 3 | 1.43% |
| L5 | 80 | 38.10% |
| Total | 210 | 100.00% |

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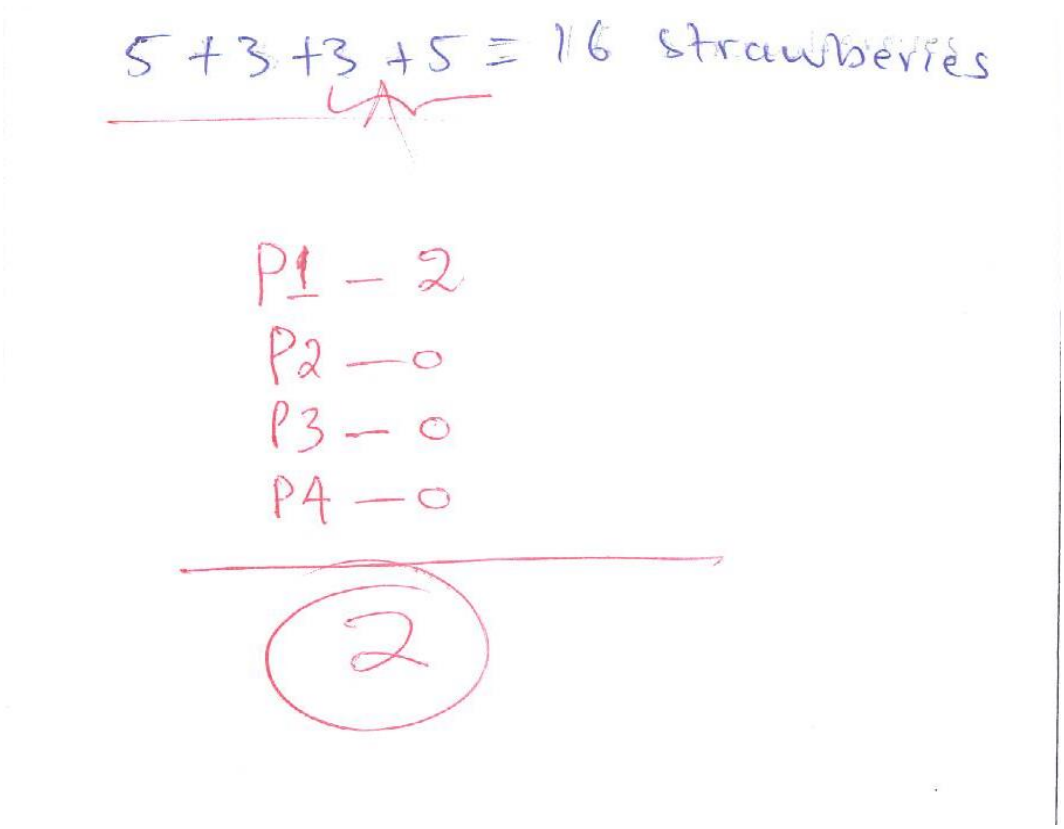
The summary of performance in question 8 shows that learners performed relatively better with a CAAPSA score of 415.5 (level 2). All the 25 learners attempted the arithmetic strategy, in other words none of the learners attempted using variables to form an equation. An analysis of snippets of learners from the three categories A (very low and low), B (intermediate) and C (high and advanced) was conducted to understand the strategies in the solution of problem 8.

Figure 4.54 shows the learner L15A's solution process and outcomes in problem 8. The learner employed the arithmetic strategy, but was not successful.

**CATEGORY A**

**Figure 4.54:** Snippet from a category A learner (L15A) who used the arithmetic method

8. *Anita passed around a basket of strawberries to the girls at her birthday party. Before the party she ate 5 strawberries and gave a friend 3. Eight girls arrived at the party. The first girl took a strawberry, the second girl took three strawberries, and the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. Given that, the pattern of distribution of strawberries to the eight girls was an arithmetic progression, how many strawberries were there in the basket at the beainnina?*



**Interview**

**R:** *Was this a difficult question for you?*

**L15A:** *A bit, I was kind of confused.*

**R:** Why were you confused?

**L15A:** *I was not sure…whether I interpreted the meaning of the question correctly, so I just wrote what I thought*.

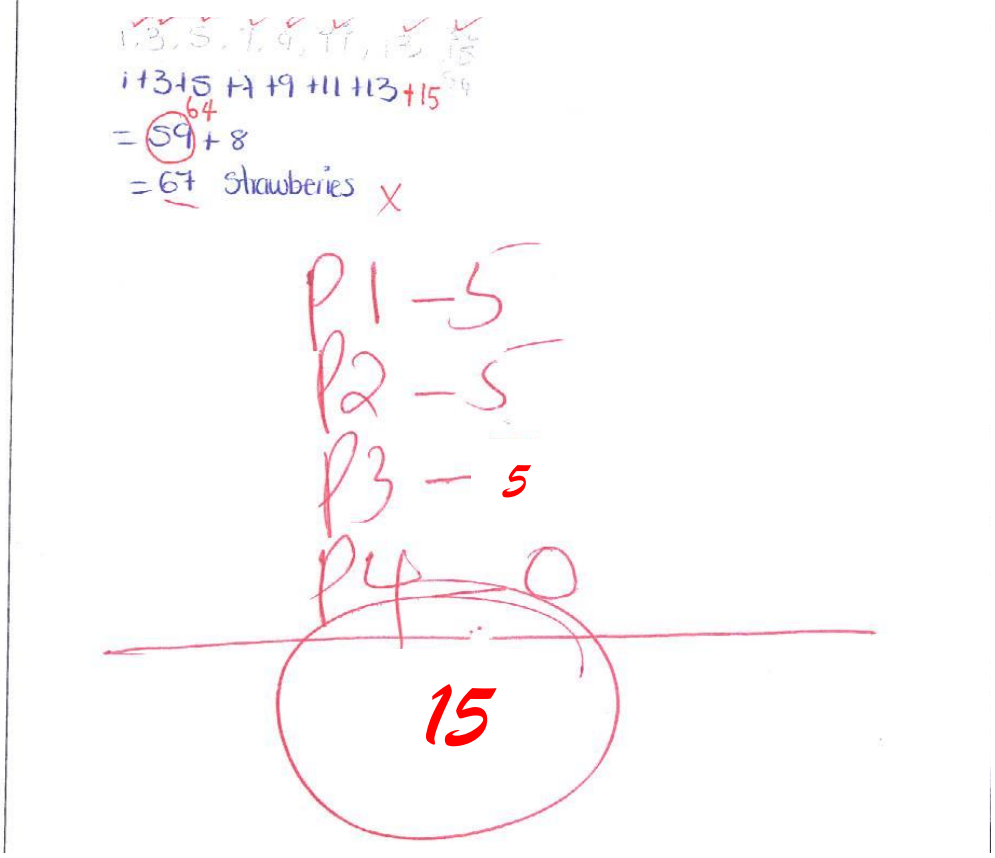
The learner (L15A) seems to acknowledge that, the difficulty emanated from failure to execute Polya's first step, reading and understanding the problem. This is also confirmed by the learners' questionnaire output, where the learner scored below level 3 in the self-report on reading and understanding the problems.

Figure 4.55 shows the solution strategy employed by learner L19B, using the arithmetic strategy.

**CATEGORY B**

**Figure 4.55:** Snippet from a category B learner (L19B) who used the arithmetic strategy

8. *Anita passed around a basket of strawberries to the girls at her birthday party. Before the party she ate 5 strawberries and gave a friend 3. Eight girls arrived at the party. The first girl took a strawberry, the second girl took three strawberries, and the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. Given that, the pattern of distribution of strawberries to the eight girls was an arithmetic progression, how many strawberries were there in the basket at the beginning?*




Learner L19B did not write all the eight terms of the arithmetic progression. The learner would have realised this omission, had the learner executed Polya's (1957) last step. Figure 4.56 shows the CAAPSA output for the assessment of learner L19B's problem solving process, in each of Polya's steps.

**Figure 4.56:** Problem 8 CAAPSA processing and output for learner L19B who used the arithmetic strategy.

| NAME OF LEARNER: | L19B |
|---|---|

| QUESTION | 8 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 10 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | The arithmetic progression of 8 terms and the intial distribution of 8 sweets correctly represented |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | Complete and correct terms represented in the addition process |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 1 | 5 | Computational error in addition |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | |

| | | |
|---|---|---|
| Total Score | 15 | L4 |
| TIMSS Score | 575 | |

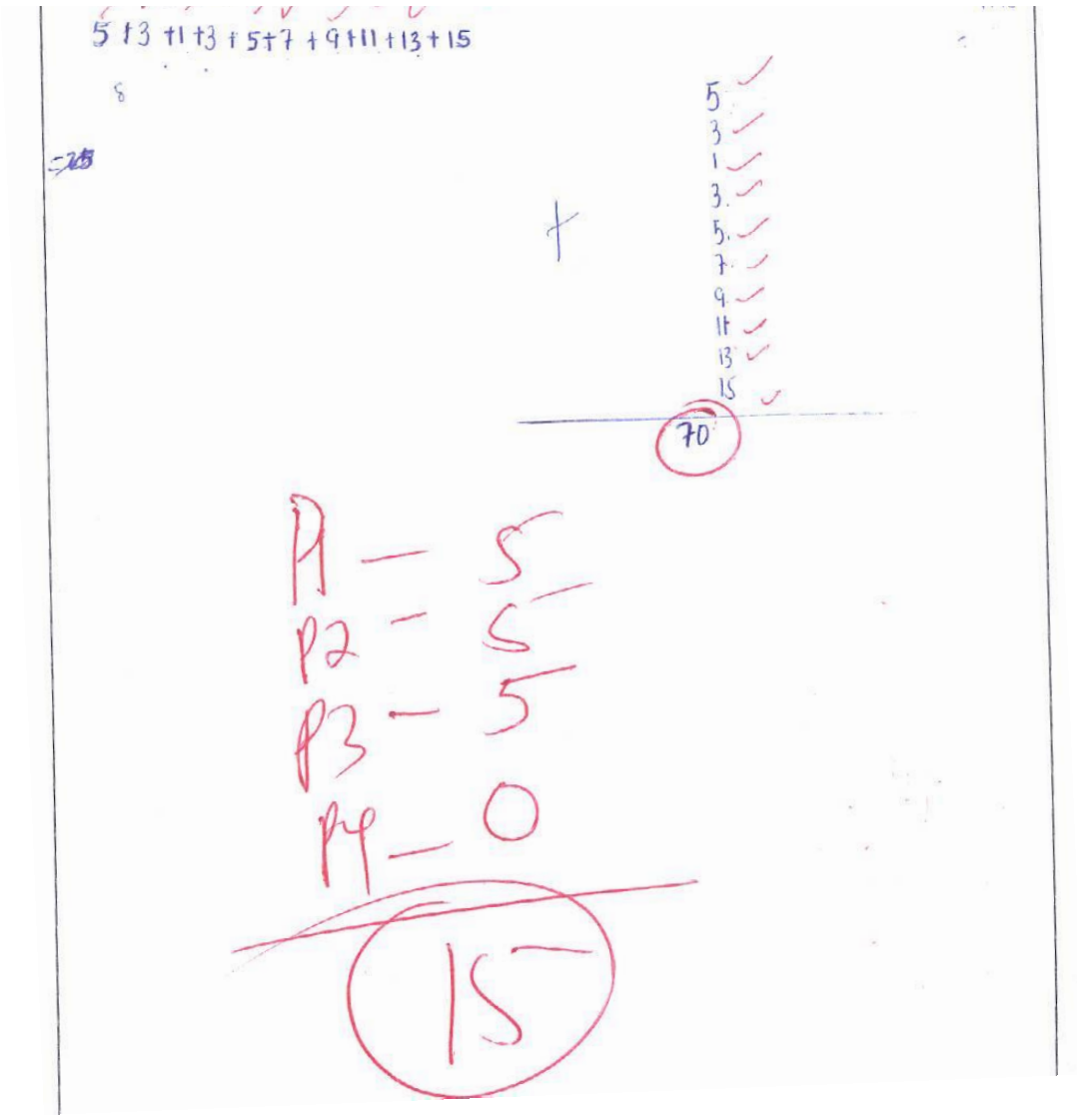
| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

Figure 4.57 shows the solution strategy of learner L25C, who also failed to check the result. It appears, that even the high achieving learners do not evaluate their solutions, because they execute Polya's step in a linear manner. For most of the learners, obtaining any solution is the end to the problem solving process.

**CATEGORY C**

**Figure 4.57:** Snippet from a category C learner L25C who partially succeeded in using the algebraic strategy

8. *Anita passed around a basket of strawberries to the girls at her birthday party. Before the party she ate 5 strawberries and gave a friend 3. Eight girls arrived at the party. The first girl took a strawberry, the second girl took three strawberries, and the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. Given that, the pattern of distribution of strawberries to the eight girls was an arithmetic progression, how many strawberries were there in the basket at the beginning?*



The learner L25C's solution strategy was correct, but made an addition error leading to 70 instead of 72 strawberries. Learner L25C did not detect the error because the evaluation of the solution (look back) was not executed. Figure 4.58 shows the CAAPSA output for the assessment of learner L19B's problem solving process, in each of Polya's steps.

**Figure 4.58:** Problem 8 CAAPSA processing and output for learner L25C who used the arithmetic strategy.

| NAME OF LEARNER: | L25C |
|---|---|

| QUESTION | 8 |
|---|---|

| Enter the number of expected variables and/or terms in Polya's step 1 | 10 |
|---|---|

| STEP | Descriptors | H-value | Polya's marking grid | | |
|---|---|---|---|---|---|
| | | | No of errors | Score | Comment(s) |
| P1 | Complete and correct variable assignment/terms in correct algebraic relationship (or equivalent) | 1 | 0 | 5 | The first part of the two stage distribution of 8 sweets and next stage of the arithmetic progression of distribution of remaining sweets correctly represented |
| P2 | Complete and correct equation(s) formed (or equivalent) | 1 | 0 | 5 | Correct terms represented in the addition statement |
| P3 | Correct solution of equations(s) (or equivalent) | 1 | 1 | 5 | Computational error in addition |
| P4 | Checking of solution(s) seen or implied | 0 | 0 | 0 | After getting an incorrect answer no attempt is seen of checking the solution. The learner will be interviewed for clarification. |

| Total Score | 15 | L4 |
|---|---|---|
| TIMSS Score | 575 | |

| Colour Score Card (CSC) | | |
|---|---|---|
| | L1 - L2 | VERY LOW-LOW |
| | L3 | INTERMEDIATE |
| | L4-L5 | HIGH-ADVANCED |

## 4.4 Summary

This chapter dealt with the analysis of the data. This was done in two phases: in the first phase, the pilot study data analysis was conducted and in the second phase, the study analysed the data from the main study. The quantitative data analysis was carried out in three stages: analysis of the knowledge base diagnostic test scores, analysis of the algebraic problem solving achievement test scores and analysis of the correlations between the

achievement levels in the knowledge base, algebraic problem solving and 2010 NSSC examination scores. The qualitative analysis focused on the analysis of relevance of the content of the NSSC examinations for the period 2007 to 2009 to the NSSC curriculum assessment objectives. The qualitative data analysis component also analysed the outcomes from the learner interviews and questionnaires to complement the findings from quantitative data analysis, and was a form of triangulation of the data. The data analysis revealed that 56.6% of the learners were at or below level 2 in the knowledge base test while 83.8% were at or below level 2 in the algebraic problem solving test. An analysis of the correlations between the achievement in the knowledge base diagnostic test, the algebraic problem solving test and the NSSC 2010 mathematics examination was made according to the description below by Mulder (1993: 73):

1.00 – perfect correlation
0.80 to 0.99 – very high correlation
0.60 to 0.79 – high correlation
0.40 to 0.59 – moderate correlation
0.20 to 0.39 – low correlation
0.01 to 0.19 – very low correlation
0.00 – no correlation

The correlation between the knowledge base and algebraic problem solving test scores was moderate (0.5), suggesting that while the knowledge base is necessary for the development of problem solving skills, it is not sufficient unless the learners are trained in Polya's problem solving model. The correlation between the algebraic problem solving achievement and 2010 NSSC examination performance was high (0.7), suggesting that algebraic problem solving is an important strand of mathematical proficiency (Milgram, 2007). Overall, most learners (65%) failed at the first step of Polya's problem solving model, reading and understanding the problem. This was also confirmed by the learners' responses in the questionnaire and interview. The next chapter discusses in detail some of the important findings of the data analysis.

# CHAPTER 5

# DISCUSSION

## 5.1 Introduction

This chapter discusses the findings of the research in an effort to answer the research questions of the study;

1. To what extent is the assessment of algebraic problem solving skills reflected in the content of the NSSC Grade 12 ordinary level mathematics examinations?
2. What is the level of the algebraic problem solving skills of Grade 12 learners in Oshana region?
   (iii) What is the correlation between the knowledge base and the algebraic problem solving skills of Grade 12 learners?
   (iv) What is the correlation between algebraic problem solving skills and examination achievement of Grade 12 learners?
3. What problem solving strategies do Grade 12 learners in Oshana Region use when solving algebraic problems?
4. What challenges or difficulties, if any, do Grade 12 learners in Oshana encounter when attempting to solve algebraic problems?

## 5.2 Discussion of findings

The difficulty of assessing complex processes necessary for solving problems is exacerbated by the failure of learners to communicate clearly what they have done or what they are thinking. The classroom teacher has little time to construct assessment procedures and scales to measure the quality of learners' thinking. This study sought to devise assessment methods for eliciting learners' thinking by performing more effective assessment of the problem solving process. The study investigated Grade 12 learners' algebraic problem solving skills when they are solving non-routine problem solving tasks. The results show that the learners achieved a very low average CAAPSA score of 340 (Level 1) in the algebraic problem solving test. This is strange, considering that the analysis of the NSSC curriculum documents and examination content indicates an emphasis on the teaching and assessment of the mathematics problem solving domain. Table 4.4 depicts that at least 60% of the NSSC Grade 12 examination content is within the algebraic problem solving domain. According to NCTM (2000), this suggests that the actual classroom practice does not integrate problem solving as an integral part of all mathematics learning.

The CAAPSA output indicates that the problem solving skills of 70.5% of the learners are at level 1. This demonstrates the high cognitive demand that the non-routine algebraic problems place on learners (cf. Elia et al., 2009; Kolovou et al., 2009). The quantitative data analysis demonstrated that the majority of learners (70.6%) achieved CAAPSA level 1 algebraic problem solving ability in Polya's model. Although most learners were able to solve numerically a variety of problems involving specific cases in the knowledge base test, they encountered difficulties in making generalisations through the use of linear and simultaneous equations in the algebraic problem solving test. The findings of this study are consistent with those of previous research (Mogari & Lupahla, 2013, Orton & Orton, 1994; Swafford & Langrall, 2000) that the majority of middle grade students were able to solve problems involving specific cases and to explain the sequence of patterns only in terms of differences between successive terms. Very few of the learners were able to generalise the problem into algebraic form. In Orton and Orton (1994) the children's answers are classified in stages running from answering questions about concrete numbers to algebraic generalisation. Orton and Orton (1994) found that children struggled to move from the level of concrete numbers to algebraic generalisation due to lack of conceptual and procedural understanding. Swafford and Langrall (2000) carried out a study on the grade six students' pre-instructional use of equations to describe and represent problem situations and found that the achievement level was not a strong indicator of the conceptual understanding. In their study, some of the students who were able to solve linear equations with one variable correctly did not necessarily show the ability to recognise the same relationship presented in words, as a diagram, and/or in symbols.

Similarly, the findings of this study show that 70.8% of the learners could not devise plans to solve the non-routine algebraic problems. In other words, they could not make connections or understand relationships between concepts relating to problems they normally solve in class and those associated with non-routine problems. This suggests that the learners lack strategic competence in solving algebraic non-routine problems (Kilpatrick et al., 2001). This is in line with the findings of Pape and Wang (2003) that strategy use is central to processing mathematical problems. Mabilangan et al. (2011) attribute such a deficiency to a lack of conceptual understanding which, according to Ketterlin-Geller et al. (2008), emanates from learners being taught in a much more procedural manner that does not adequately develop and cement the required aspects of problem solving skills (Fatokun, Hugo & Ajibola, 2009). It is therefore recommended that teachers focus more on helping learners to understand

procedures and processes of problem solving rather than becoming obsessed with their finding the correct answer.

Furthermore, there is tendency by teachers to prepare Grade 12 learners for end-of-year examinations by using previous years' question papers. Arguably, when the examination comes learners already have an idea of how questions are posed and to some extent this practice has generally benefitted learners in the examination (NIED, 2005). It means learners tend to do well when solving familiar/routine problems, because they can recall procedures they have practised and are able to execute the procedures flawlessly. Similarly, Jiang and Chua (2010) found that Singaporean primary Grade 6 learners received considerable training in problem solving to prepare them for the Primary School Leaving Examination that is used to decide which school and what course learners will pursue in secondary school education; and these Singaporean learners only outperformed their Chinese counterparts at this level. Jiang and Chua (2010) attribute the better performance by the Chinese learners in the other classes to the fact that word problems frequently dominate mathematics lessons after the learners have been taught basic mathematics knowledge and skills because, according to Jiang (2008), problem solving is used to illustrate applications of mathematical aspects in real life. Thus it is claimed that learners tend to perform well in the Grade 12 examination largely due to the amount of routine training and preparation they are exposed to, but little happens to the improvement of their conceptual understanding. Perhaps it is for this reason that they tend to struggle with non-routine problems that require creative and critical thinking rather than recall and following practiced procedures. It is suggested that non-routine problems should be part of the curriculum in order to provide learners with more exposure to such problems.

Table 4.10 indicates that there was a general preference for the algebraic strategy and two possible strategies were not attempted at all. Although the algebraic strategy was the most preferred solution strategy, employed by 55% of the participants, the successful solution outcomes were only 39% of the attempts. This is an indication that although learners prefer the algebraic strategy, they struggle to implement it successfully. Similar results were found among Chinese learners (Jiang & Chua, 2010). The learners' preference of use of the algebraic strategy suggests that learners assigned letters to the unknowns because that is what their teachers encourage them to do when they solve word problems. According to the learners who participated in this study, the use of letters to represent an unknown is associated with algebra; hence the algebraic strategy was generally preferred. As in China,

the use of the algebraic strategy seems to be influenced by the way learners normally solve routine word problems (Jiang & Chua, 2010). This reflects the fact that algebraic thinking is routinely used in most mathematical problems, albeit unawares (Jiang, 2008; Ketterlin-Geller & Chard, 2011; Milgram, 2007; Vogel, 2008). The results of this study also revealed that learners performed better in problem 3 of the algebraic problem solving test in which a diagram was used to illustrate the problem. Diagrams simplify complex situations and illustrate abstract concepts (Kidman, 2002) and make problems easier (Pantziara, Gagatsis & Pitta-Pantazi, 2004). Perhaps diagrams should be used more when teaching problem solving because they are a resource which can facilitate the analysing and understanding of a problem and teachers should expose learners to a variety of solution strategies. Problem 6 of the algebraic problem solving test was also better answered because, according to 60% of the learners, they had already solved a similar problem in the knowledge base test (see question 50 in Appendix 1). This shows that the learners successfully transferred the conceptual knowledge from the knowledge base activities to the problem solving tasks, which corresponds with Schoenfeld's (1985) eighth element of problem solving, that is, problem solving teaches transfer or the application of conceptual knowledge. Learners perfomed better in problem 7 because they were familiar with the concrete world experiences relating to the content of the problem. It was easy for them to apply their abstract school knowledge to concrete real world experiences because, as argued by Krulick and Rudnick (1980), problem solving connects theory and practice.

Language appears to have added to the learners' difficulties, as evident in the interviews with the learners. For instance, Category C learner L25C in question 4 said: *I did not understand the language used in the questions*. This is strange because the learners spoke English fairly well and it cannot be perceived as an imported language which, according to Verzosa and Mulligan (2012), hampers learning. On the contrary, Fillmore (2002) believes that even learners who are proficient in English can encounter language-related problems when solving novel word problems containing unusual words and phrases. Perhaps it is for this reason that learners found the question with a diagrammatic illustration least difficult when compared to the more analytical ones. Nevertheless, it is recommended that learners should be taught technical mathematics-specific words and phrases used in word problems to enrich their vocabulary.

The results of the study were that these learners generally display level 1 problem solving skills on the CAAPSA scale when dealing with non-routine problems because of inadequate

conceptual understanding, limited knowledge of a range of solution strategies and difficulty with words and phrases used in given problems.

## 5.3 Summary

The chapter provided a discussion of the findings of the study, with particular reference to the research questions. The findings of the study were in line with the NSSC examiners' reports (DNEA, 2010) that the Grade 12 candidates lacked algebraic problem solving skills. It is hoped that the findings of the study will provide empiric evidence on the state of the Grade 12 learners' problem solving skills and pave the way forward for addressing these deficiencies in the Namibian classrooms. The Computer Aided Algebraic Problem Solving Assessment (CAAPSA) tool will hopefully be implemented by mathematics teachers at all levels to assess, monitor and hence create a learning environment that will facilitate the development of the algebraic problem solving skills of their learners.

# CHAPTER SIX

# CONCLUSION, RECOMMENDATIONS AND SUMMARY

## 6.1 Introduction

This chapter provides the conclusion to the whole dissertation. It includes a summary of the findings and outlines some of the limitations, recommendations and avenues suggested for further study. The chapter ends with a reflection on the experience I have gained as a novice researcher in conducting this research.

## 6.2 Summary of findings

The findings of this study are discussed in detail in chapter four. These results were generated by using six instruments ─ a knowledge base test, an algebraic problem solving test, the 2010 NSSC ordinary level examinations, NSSC ordinary level examinations from 2007 to 2009, questionnaires and learner interviews. The questionnaire outcomes were used to validate the results of the quantitative component of the study, and as a means of triangulating the results. The questionnaire responses from the learners supported the CAAPSA processing and output. The findings of this study have demonstrated that the CAAPSA tool was effective in mapping the Grade 12 learners' algebraic problem solving skills.

### 6.2.1 The level of development of the algebraic problem solving skills of Grade 12 learners in Oshana region

The quantitative data analysis revealed that the level of development of the algebraic problem solving skills of Grade 12 learners in Oshana region is very low (TIMSS level 1). The results of the study indicate that while the knowledge base plays an important role in the development of problem solving skills, it is a necessary but insufficient requirement, as shown by a moderate Pearson correlation of 0.5 between the knowledge base and algebraic problem solving test scores of the participants in this study. This is in line with the findings of Garofalo and Lester (1985), Geiger and Galbraith (1998), Schoenfeld (1987) and Silver (1987), who agree that a rich store of knowledge is a necessary but not sufficient requirement for successful mathematical problem solving.

### 6.2.2 The relationship between the development of problem solving skills and mathematics proficiency

By definition, mathematics proficiency is a function of learners' NSSC examination achievement. This study argues that there is a strong correlation between algebraic problem solving skills and mathematics proficiency. There was a strong Pearson correlation of 0.7 between the algebraic problem solving scores and the final 2010 examination scores. This supports Milgram's (2007) assertion that algebraic problem solving is a higher strand of mathematics proficiency and Carson's (2007) belief that problem solving skills stand out as key in successful mathematics teaching.

### 6.2.3 Challenges and difficulties encountered by Grade 12 learners in solving algebraic problems

Generally, most learners in this study encountered difficulties in generalising arithmetic thinking through the use of algebraic symbols. In a problem solving situation learners must understand the problem at the outset in order to begin any sensible or reasonable solution strategy. Most learners failed at the first stage of Polya's (1957) framework, reading and understanding of the problem, and their responses to the questionnaire and interviews makes it reasonable to attribute this challenge to a deficit in language skills and a lack of classroom training in the problem solving process. The study showed that learners encountered difficulties with non-routine problems, largely because of inadequate conceptual understanding, limited knowledge of the range of solution strategies, and difficulty with words and phrases used in given problems.

## 6.3 Recommendations

Based on the findings, the study proposes the following recommendations;

i. Teachers should focus more on helping learners to understand the procedures and processes of problem solving rather than finding the answer when solving problems;
ii. More emphasis should be placed on developing an understanding of algebra because it underpins other topics in mathematics;
iii. Non-routine problems should be part of the curriculum;
iv. Teachers should expose learners to a variety of solution strategies;

v. Learners should be taught the technical, mathematics-specific words and phrases used in word problems;
vi. The CAAPSA tool should be adopted and tested as classroom tool for mapping the development of problem solving skills in Polya's (19957) model.

### 6.4 Limitations of the study

Although the projected sample size was 322 learners, in the end the study used a sample of only 210 participants. This was due to some learners dropping out during the process of data collection; hence the significance level of the results was reduced. I do, however, believe that the generalisation for the Oshana region could still be achieved from the results obtained from the sample that participated in the study.

### 6.5 Avenues for further research

The study proposes the following topic and hypotheses that could be investigated in a future study;

**Topic:** An investigation into the impact of using the problem based instructional design in the teaching of the NSSC mathematics curriculum

**Null hypothesis ($H_0$):** The use of the problem based learning approach in Polya's model will not improve the performance of Grade 12 learners in Mathematics.

**Hypothesis (H):** The use of the problem based learning approach in Polya's model will improve the performance of Grade 12 learners in Mathematics.

**Research design:** Experimental design (situation-producing)

### 6.6 Personal reflections

As a novice researcher, I found the research process an enriching learning experience. My participation in two UNISA post-graduate seminars and two ISTE conferences, as a requirement for the submission of my dissertation, was worthwhile experiences. I had the opportunity to interact with experienced researchers from across the world which gave me the

will and strength to finish this piece of work. I found it difficult to source some of the literature, but the presentations by the UNISA library empowered me to use some of the materials via the internet. My supervisor, Professor David Mogari, encouraged me to write a paper based on the findings of the pilot study as a way of obtaining reviewers' contributions to the main research. This in particular made me realise that as a researcher one must accept criticism as a springboard for a successful research study. Finally, the paper was approved for publication and appeared in the *African Journal for Research in Mathematics, Science and Technology Education*, Vol. 17, No. 1-2, 94 -105.

## 6.7 Summary

This chapter summed up the whole dissertation. The findings were briefly discussed and the recommendations based on these findings were presented. The chapter also proposed an avenue for further study, which could bring about a meaningful NSSC mathematics curriculum implementation in Namibia. The study supports the claim that the development of problem solving skills expands learners' creative faculties, as evident in the strong correlation between the algebraic problem solving achievement and the achievement on an examination by the participants in this study.

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## APPENDICES

### **APPENDIX 1:** KNOWLEDGE BASE TEST

**KNOWLEDGE BASE TEST (NO CALCULATORS ALLOWED) - 80 MARKS**

| | | | |
|---|---|---|---|
| SCHOOL: | | REGION: | |
| NAME : | | CLASS: | |

**Objectives:** To measure the level of the learners' pre-requisite knowledge for solving algebraic problems.

**Assessment aspects:** Numbers, fractions & percentages, linear and simultaneous linear equations , algebraic manipulation, ratio, proportion & rate and word problems .

**Answer on the question paper in the spaces provided. Show all the necessary working clearly.**

| MARKS | |
|---|---|
| A1 | |
| A2 | |
| A3 | |
| A4 | |
| A5 | |
| A6 | |
| A7 | |
| A8 | |
| TOTAL | |

| | |
|---|---|
| 1. 6 + 2 + 5 − 4 = | 2. 7 − 3 + 2 = |
| 3. 7 − 5 + 4 = | 4. 6 + 1 – 3 – 4 = |
| 5. ( 30 −21) + 10 + 85 = | 6. ( 59 + 25 ) + ( 9 – 33 + 95) = |

| | |
|---|---|
| 7. $-2^2 =$ | 8. $(-1)^2 \times 3 =$ |
| 9. $6 \div 12 \times 2 + 8 =$ | 10. $-1^2 \times 3 =$ |
| 11. Simplify<br>$1\frac{10}{12} \times \frac{2}{8} =$ | 12. Simplify<br>$\frac{5}{8} \times \frac{9}{10} =$ |
| 13. Simplify<br>$\frac{1}{3} + \frac{2}{5} =$ | 14. Simplify<br>$2\frac{1}{4} \div \frac{1}{2} =$ |

| | |
|---|---|
| 15. Calculate $7 \times 3.1 - 5 \times 2.2 =$ | 16. Simplify<br>$1\frac{7}{8} - \frac{6}{7} =$ |
| 17. Convert 0.68 into a fraction in its lowest terms. | 18. Write down the three numbers from the list below that have the same value.<br>$0.09 \quad 90\% \quad \frac{9}{1000} \quad 9\% \quad \frac{9}{100} \quad 900\%$<br>_____ ; _____; _____ |
| 19. Write 75% as a fraction in its lowest terms. | 20. Find 10% of 49 |
| 21. Find 10% of N$17.44 = | 22. Joseph, Maria and Rebecca each win a prize. Their total prize money is N$30.Joseph wins $\frac{7}{12}$ of the N$30. Maria wins 30% of the N$30. Calculate the amount each receives<br>Joseph N$_____<br>Maria N$_____<br>Rebecca N$_____ |

| | |
|---|---|
| 23. The total cost ( ***C*** ) of renting a sailboat for ***n*** days is given by the formula:<br>$C = 120 + 60n$<br>If the total cost was N$360, for how many days was the sail boat rented? | 24. Re-write the equation<br>$4(2-5x) = 6 - 3(1-3x)$<br>in the form $Ax = C$ where $A$ and $C$ are integers. |
| 25. Peter solved the equation<br>$3(x+5) = 2x + 35$<br>as follows:<br>$Step\,1: 3x + 15 = 2x + 35$<br>$Step\,2: 5x + 15 = 35$<br>$Step\,3: 5x = 20$<br>$Step\,4: x = 4$<br>Which is the first incorrect step? Write the correct algebraic statement for this first incorrect step. | 26.Solve the simultaneous equations:<br>$2x - y = 6$<br>$4x + 5y = -2$ |
| 27. Solve the equation<br>$2x - 3 = 6$ | 28. Solve the equation<br>$x + 4 = 3(2 - x)$ |

| | |
|---|---|
| 29. Solve the simultaneous equations<br>$3x - y = 18$<br>$2x + y = 7$ | 30. The volume of a pyramid with area of base A and height h is given by the formula:<br>$V = \frac{1}{3}Ah$<br>Calculate the volume of the pyramid with base area $270m^2$ and height 12m. |
| 31. Make A the subject of the formula:<br>$V = \frac{1}{3}Ah$ | 32. The $n^{th}$ term of a sequence is $\frac{n(n+1)}{2}$<br>Calculate the 8th term of this sequence. |
| 33. Factorise completely:<br>$2x^2 - 6xy$ | 34. Simplify:<br>$p^2 \times p^3$ |
| 35. Simplify<br>$q^3 \div q^{-4}$ | 36. Simplify<br>$(r^2)^3$ |

| | |
|---|---|
| 37. The cost of 1 kilogram of oranges is N$2.16.Find the cost of $3\frac{1}{2}$ kilograms of oranges. | 38. If the cost of 1 kilogram of oranges is N$2.16, how many kilograms of oranges can be bought from N$10.80? |
| 39. In March 2008 one Namibian dollar (N$) was worth 0.12 US dollar (US$).Ludmilla changed N$ 3 250 into US dollars. How much did she receive in US dollars? | 40. The length of a soccer pitch is 90 metres. The ratio length: width is 5:3.Calculate the width of the pitch. |
| 41. The temperature at noon at the Antarctic weather centre was – 15 $^{\circ}$C At midnight it had fallen by 12 $^{\circ}$C. What was the temperature at midnight? | 42. In the Namib desert the temperature at 18 00 hours was 26 $^{o}$C .At midnight the temperature was -1 $^{o}$C. By how many degrees has the temperature gone down? |

| | |
|---|---|
| 43. The price of a jacket is N$250.During a sale the jacket's price is reduced by 15%. Calculate the sale price of the jacket. | 44. Rodriguez puts N$500 into a bank account. The bank pays 5% compound interest per year. Work out the total amount he has in the bank after two years. |
| 45. The length of the sides of a triangle are $y$ , $y + 1$ *and* 7 centimetres. If the perimeter is 56 cm , what is the value of $y$ ? | 46. A volleyball court is shaped like a rectangle. It has width $x$ metres and a length of $2x$ metres. Write an expression that gives the area of the court in square metres. |
| 47. If $x^2$ is added to $x$ , the sum is 42. Write one possible value of $x$ . | 48. If an even number is $t$ , what is the next consecutive odd number? |
| 49. If an even number is y , what is the next consecutive even number? | 50. A 120 centimetre long rope is cut into 3 pieces. The first piece is 2x cm , the second is x cm and the third is 3x cm. What is the length of the longest piece of rope? |

**APPENDIX 2:** KNOWLEDGE BASE TEST MARKING SCHEME

**KNOWLEDGE BASE TEST MARKING SCHEME (80 marks)**

**Objectives:** To measure the level of the learners' prerequisite knowledge for solving algebraic problems.

**Assessment aspects:** Numbers, fractions & percentages, linear and simultaneous linear equations, formulae, indices, algebraic manipulation, ratio, proportion & rate and word problems .

| | |
|---|---|
| 1. $6 + 2 + 5 - 4 =$<br>$13 - 4 = 9$ A1 | 2. $7 - 3 + 2 =$<br>$4 + 2 = 6$ A1 |
| 3. $7 - 5 + 4 =$<br>$2 + 4 = 6$ A1 | 4. $6 + 1 - 3 - 4 =$<br>$7 - 7 = 0$ A1 |
| 5. $(30 - 21) + 10 + 85 =$<br>$9 + 10 + 85 = 104$ A1 | 6. $(59 + 25) + (9 - 33 + 95) =$<br>$84 + 71 = 155$ A1 |
| 7. $-2^2 =$<br>$-1 \times 2^2 = -4$ A1 | 8. $(-1)^2 \times 3 =$<br>$1 \times 3 = 3$ A1 |

| | |
|---|---|
| 9. $6 \div 12 \times 2 + 8 =$<br><br>$\frac{6}{12} \times 2 + 8 = 9$ A1<br><br>6 | 10. $-1^2 \times 3 =$<br><br>$-1 \times 3 = -3$ A1 |
| 11. Simplify<br><br>$1\frac{10}{12} \times \frac{2}{8} =$<br><br>$\frac{22}{12} \times \frac{2}{8} = \frac{11}{24}$ A1 | 12. Simplify<br><br>$\frac{5}{8} \times \frac{9}{10} =$<br><br>$\frac{9}{16}$ A1 |
| 13. Simplify<br><br>$\frac{1}{3} + \frac{2}{5} =$<br><br>$\frac{5+6}{15} = \frac{11}{15}$ A1 | 14. Simplify<br><br>$2\frac{1}{4} \div \frac{1}{2} =$<br><br>$\frac{9}{4} \div \frac{1}{2} = \frac{9}{4} \times \frac{2}{1}$ M1<br><br>$= 4\frac{1}{2}$ A1 |

| | |
|---|---|
| 15. Calculate $7 \times 3.1 - 5 \times 2.2 = 2.8$<br><br>A1 | 16. Simplify<br><br>$1\frac{7}{8} - \frac{6}{7} = 1\frac{1}{56}$ A1 |
| 17. Convert 0.68 into a fraction in its lowest terms.<br><br>$\frac{68}{100} = \frac{17}{25}$ A1 | 18. Write down the three numbers from the list below that have the same value.<br>$0.09 \quad 90\% \quad \frac{9}{1000} \quad 9\% \quad \frac{9}{100} \quad 900\%$<br><br>0.09 ; 9% ; $\frac{9}{100}$<br><br>_____ _____ _____ A1 |
| 19. Write 75% as a fraction in its lowest terms.<br><br>$\frac{75}{100} = \frac{3}{4}$ A1 | 20. Find 10% of 49<br><br>$\frac{10}{100} \times 49 = 4.9$ A1 |

| | |
|---|---|
| 21 Find 10% of N$17.44 =<br><br>$\frac{10}{100} \times N\$17.44 = 1.744$<br>$= N\$1.74$<br><br>A1 | 22 Joseph, Maria and Rebecca each win a prize. Their total prize money is N$30.Joseph wins $\frac{7}{12}$ of the N$30. Maria wins 30% of the N$30. Calculate the amount each receives<br><br>Joseph: $\frac{7}{12} \times N\$30 = N\$17.50$<br><br>Maria: $\frac{30}{100} \times 30 = N\$9.00$<br><br>Rebecca: N$30.00 – N$( 17.50 + 9.00)<br>N$30.00 – N$26.50<br>N$ 3.50<br><br>Joseph N$ 17.50 A1<br><br>Maria N$ 9.00 A1<br><br>Rebecca N$ 3.50 A1 |

23 The total cost ( ***C*** ) of renting a sailboat for ***n*** days is given by the formula:

$C = 120 + 60n$

If the total cost was N$360, for how many days was the sailboat rented?

C = 120 + 60n M1

60n = 360 – 120 M1

60n = 240

n = 4

Therefore the sailboat was rented for four days. A1

24 Rewrite the equation

$$4(2-5x) = 6-3(1-3x)$$

in the form Ax = C where A and C are integers.

$$\begin{aligned} 8-20x &= 6-3+9x \\ -29x &= -5 \\ 29x &= 5 \end{aligned}$$ M1

So A = 29 and C = 5 A1

25 Peter solved the equation

$3(x+5)=2x+35$ as follows:

$Step\,1: 3x+15=2x+35$

$Step\,2: 5x+15=35$

$Step\,3: 5x = 20$

$Step\,4: x = 4$

first incorrect step: $Step\,2: 5x+15 = 35$ A1

Corrected: $Step\,2: x+15 = 35$ A1

26 Solve the simultaneous equations:

$$\begin{aligned} 2x - y &= 6 \quad \times 5 \\ 4x + 5y &= -2 \end{aligned}$$

$$\begin{aligned} 10x - 5y &= 30 \\ 4x + 5y &= -2 \end{aligned}$$ M1

Add: $14x = 28$ M1

Solution: $x = 2$, $y = -2$ A2

27 Solve the equation

$$\begin{aligned} 2x - 3 &= 6 \\ 2x &= 9 \\ x &= \tfrac{9}{2} \\ x &= 4.5 \end{aligned}$$ M2A1

28 Solve the equation

$$\begin{aligned} x + 4 &= 3(2-x) \\ x + 4 &= 6 - 3x \\ 4x &= 2 \\ x &= \tfrac{1}{2} \end{aligned}$$ M2A1

| | |
|---|---|
| 29 Solve the simultaneous equations<br>$3x - y = 18$<br>$2x + y = 7$<br><br>Add: $5x = 25$<br>$x = 5$<br>Substituting $x$ into equation 2 we get<br>$y = -3$ M1<br><br>Solution: $x = 5$, $y = -3$ A2 | 30 The volume of a pyramid with area of base A and height h is given by the formula:<br>$V = \frac{1}{3} \times Ah$<br>Calculate the volume of the pyramid with base area 270 m$^2$ and height 12m.<br><br>Volume = $\frac{1}{3} \times 270 \times 12\ m^3$<br>= 1080 $m^3$ M1 A1 |
| 31 Make A the subject of the formula:<br><br>$A = \frac{3V}{h}$ A1 | 32 The n$^{th}$ term of a sequence is :<br>$\frac{n(n+1)}{2}$ .<br>Calculate the 8$^{th}$ term of this sequence.<br><br>The 8$^{th}$ term is 36. M1 A1 |
| 33 Factorise completely: $2x^2 - 6xy$<br><br>$2x(x-3y)$ A2 - complete<br>SC1- incomplete | 34 Simplify:<br>$p^2 \times p^3$<br><br>$p^5$ A1 |

| | |
|---|---|
| 35 Simplify $q^3 \div q^{-4}$<br><br>$q^{3-(-4)} = q^7$ M1A1 | 36 Simplify<br>$(r^2)^3$<br><br>$= r^6$ A1 |
| 37 The cost of 1 kilogram of oranges is N$2.16.Find the cost of $3\frac{1}{2}$ kilograms of oranges.<br><br>This is direct proportion , so<br><br>$\frac{2.16}{1} = \frac{2x}{7}$ where $x$ stands for the cost of $3\frac{1}{2}$ kilograms of oranges<br><br>$x = 7.56$<br><br>Therefore the cost of $3\frac{1}{2}$ kilograms of oranges is N$ 7.56<br><br>M1A1 | 38 If the cost of 1 kilogram of oranges is N$2.16, how many kilograms of oranges can be bought with N$10.80?<br><br>This is direct proportion , so<br><br>$\frac{2.16}{1} = \frac{10.80}{x}$<br>$x = \frac{10.80}{2.16}$<br>$x = 5kg$ M2<br><br>Therefore 5 kilograms of oranges can be bought from N$10.80 A1 |
| 39 In March 2008 one Namibian dollar (N$) was worth 0.12 US dollar (US$).Ludmilla changed N$ 3 250 into US dollars. How much did she receive in US dollars?<br>Direct proportion<br><br>$x = 3250 \times 0.12$<br>$x = US\$390$ M1A1 | 40 The length of a soccer pitch is 90 metres. The ratio length:width is 5:3.Calculate the width of the pitch.<br><br>90 : x = 5 : 3<br><br>5x = 270 M1<br><br>x = 54<br><br>Therefore the width of the pitch is 54 metres<br>A1 |

| | |
|---|---|
| 41 The temperature at noon at the Antarctic weather centre was –15 °C At midnight it had fallen by 12° C. What was the temperature at midnight?<br><br>$-15^{o}C - 12^{o}C = -27^{o}C$ A1 | 42. In the Namib desert the temperature at 18 00 hours was 26° C .At midnight the temperature was -1° C. By how many degrees had the temperature fallen?<br><br>$26^{o}C - (-1^{o}C) = 27^{o}C$ A1 |
| 43. The price of a jacket is N$250.During a sale the jacket's price is reduced by 15%. Calculate the sale price of the jacket.<br><br>Sale price = $\frac{85}{100} \times N\$250 = N\$212.50$<br><br>M1A1<br><br>M1 for N$37.50 seen | 44. Rodriguez puts N$500 into a bank account. The bank pays 5% compound interest per year. Work out the total amount he has in the bank after two years.<br><br>$A = P(1+\frac{5}{100})^2$<br>$= 500 \times 1.05^2$<br>$= N\$551.25$ M1A1 |

| | |
|---|---|
| 45. The length of the sides of a triangle are *y, y + 1 and* 7 centimetres. If the perimeter is 56 cm , what is the value of *y* ?<br><br>$y+y+1+7=56$<br>$2y+8=56$<br>$2y=48$<br>$y=24$<br><br>M1A1 | 46. A volleyball court is shaped like a rectangle. It has a width of *x* metres and a length of *2x* metres. Write an expression that gives the area of the court in square<br><br>metres.<br><br>$Area = 2x \times x$<br>$= 2x^2 \ m^2$<br><br>A1 |
| 47. If $x^2$ is added to $x$ , the sum is<br><br>42. Write one possible value of $x$ .<br><br>$x^2+x=42$<br>$x^2+x-42=0$<br>$(x-6)(x+7)=0$<br>$x=6 \ or \ x=-7$<br><br>M1A1 | 48. If an even number is *t* , what is the next<br><br>consecutive odd number?<br><br>$t+1$<br><br>A1 |

| | |
|---|---|
| 49. If an even number is y , what is the next consecutive even number?<br><br>$y+2$ A1 | 50. A 120 centimetre long rope is cut into three pieces. The first piece is 2*x* cm , the second is *x* cm and the third is 3*x* cm. What is the length of the longest piece of rope?<br><br>$2x + x + 3x = 120$<br>$6x = 120$ M1<br>$x = 20\ cm$<br><br>The longest piece is therefore *60cm* A1 |

Key:

M marks: Method marks

A marks: Accuracy marks

SC1 marks: Special cases

Ft : follow through

**APPENDIX 3:** ALGEBRAIC PROBLEM SOLVING TEST

# Algebraic Problem Solving Test – 160 marks

| | | | |
|---|---|---|---|
| SCHOOL: | | REGION: | |
| NAME: | | CLASS: | |

**Objectives:** To measure the level of the learners' algebraic problem solving skill level.

**Assessment aspects:** Non-routine word problems leading to algebraic representation and equations.

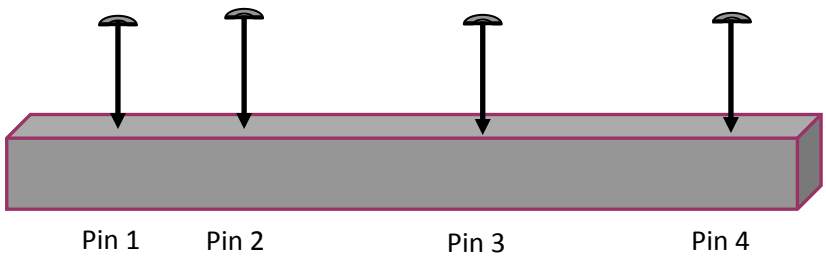
- **Answer on the question paper in the spaces provided. Show all your workings clearly.**
- **You may draw diagrams or tables for better understanding of the question where appropriate.**
- **Electronic calculators may be used.**
- **You should use a soft pencil for any diagrams or graphs.**
- **Each question carries 20 marks.**

| MARKS | |
|---|---|
| Q1 | |
| Q2 | |
| Q3 | |
| Q4 | |
| Q5 | |
| Q6 | |
| Q7 | |
| Q8 | |
| TOTAL | |

1. The sum of an even number and the next consecutive even number is 54. Find the smaller of the consecutive even numbers.

2. Aaron's mass is 10 kilograms greater than twice Levi's weight. If the sum of their masses is 118 kilograms, how many more kilograms is Aaron's mass than Levi's mass?

3. Four steel pins are stuck in a straight line on a wooden rectangular prism block. The distance between Pin 1 and Pin 4 is 35mm.The distance between Pin 2 and Pin 3 is twice the distance between Pin 1 and Pin 2. The distance between Pin 3 and Pin 4 is the same as the distance between Pin 2 and Pin 3. What is the distance, in millimetres, between Pin 1 and Pin 3?




4. Bill, Phil and Jenny are siblings. Bill is twice as old as Phil. Jenny is two years younger than Bill. Currently, their dad is twice as old as the sum of their ages. In nine years, dad's new age will be equal to the sum of his three kids' new ages. What is Jenny's current age?

5. In a group of cows and chickens, the number of legs is 14 more than twice the number of heads. How many cows are there in the group?

6. A 120 centimetre-long rope is cut into three pieces. The first piece of the rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope?

4. In a group of cows and chickens, the number of legs is 46 and the number of heads is 13. How many cows and chickens are there in the group?

5. Anita passed around a basket of strawberries to the girls at her birthday party. Before the party she ate 5 strawberries and gave a friend 3. Eight girls arrived at the party. The first girl took a strawberry, the second girl took three strawberries, and the third girl took 5 strawberries and so on. After the last girl took her strawberries, the basket was empty. Given that, the pattern of distribution of strawberries to the eight girls was an arithmetic progression, how many strawberries were there in the basket at the beginning?

## **APPENDIX 4:** LEARNER QUESTIONNAIRE

**LEARNER ID: _____ DATE COMPLETED: _____**

**SECTION A: Tick, Yes or No (√) in the appropriate box, for each of the following statements:**

| # | Statement | Yes | No |
|---|---|---|---|
| 1 | I had an idea of how to start solving most of the problems | | |
| 2 | I managed to identify the information given in most of the problems | | |
| 3 | I understood the language used in most of the problems | | |
| 4 | I managed to identify the unknown quantities in most of the problems | | |
| 5 | I managed to identify the appropriate mathematical operations in most of the problems | | |
| 6 | All the problems had a solution | | |
| 7 | I recognised useful number patterns which helped me solve some of the problems | | |
| 8 | I used a table/diagram/equation in some of the problems | | |
| 9 | I have seen similar problems like these ones before | | |
| 10 | In some cases, I formulated a similar and simpler problem to help me get a solution strategy | | |
| 11 | I solved some of the problems by forming and solving equations | | |
| 12 | I checked each step in my working | | |
| 13 | It was easy to establish the relationships between the known and unknown data in the problems | | |
| 14 | It was easy for me to decide on a suitable method of solution | | |
| 15 | My school mathematics knowledge was useful in solving the problems | | |
| 16 | My work on the problems was well organised | | |
| 17 | After solving these problems, I feel I will be able to solve more similar problems | | |
| 18 | I tried to verify the correctness of my solutions | | |
| 19 | I tried to look for alternative ways of solution | | |
| 20 | I tried to interpret the solutions obtained in terms of the original problem to see whether my answer makes sense. | | |

**SECTION B**

1. Rank the problems from easy to most difficult (Write only the question number in the boxes below)

Easiest Most difficult

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

2. Why were some of the questions difficult for you?

_____
_____
_____
_____
_____
_____

3. What made you find some of the questions easy?

_____
_____
_____
_____
_____
_____

4. Any other comments:

_____
_____
_____
_____
_____
_____
_____
_____
_____

**APPENDIX 5:** CAAPSA DATA ANALYSIS FOR KNOWLEDGE BASE TEST

**CAAPSA ANALYSIS OF KNOWLEDGE BASE TEST**

| | SKILL 1 | | | SKILL 2 | | | SKILL 3 | | | SKILL 4 | | | SKILL 5 | | | SKILL 6 | | | SKILL 7 | | | SKILL 8 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| 1 | 27 | 682.1 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 7 | 491.7 | L3 | 67 | 618.8 | L4 |
| 2 | 17 | 503.6 | L3 | 1 | 325 | L1 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 2 | 283.3 | L1 | 33 | 406.3 | L2 |
| 3 | 23 | 610.7 | L4 | 4 | 700 | L5 | 8 | 700 | L5 | 7 | 637.5 | L5 | 4 | 450 | L2 | 3 | 414.3 | L2 | 3 | 500 | L3 | 7 | 491.7 | L3 | 59 | 568.8 | L4 |
| 4 | 17 | 503.6 | L3 | 4 | 700 | L5 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 31 | 393.8 | L1 |
| 5 | 19 | 539.3 | L3 | 2 | 450 | L2 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 325 | L1 | 2 | 342.9 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 38 | 437.5 | L2 |
| 6 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 3 | 387.5 | L1 | 2 | 325 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 42 | 462.5 | L2 |
| 7 | 15 | 467.9 | L2 | 4 | 700 | L5 | 8 | 700 | L5 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 5 | 408.3 | L2 | 39 | 443.8 | L2 |
| 8 | 24 | 628.6 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 7 | 637.5 | L5 | 4 | 485.7 | L3 | 3 | 500 | L3 | 9 | 575 | L4 | 63 | 593.8 | L4 |
| 9 | 17 | 503.6 | L3 | 2 | 450 | L2 | 4 | 450 | L2 | 4 | 450 | L2 | 3 | 387.5 | L1 | 4 | 485.7 | L3 | 3 | 500 | L3 | 4 | 366.7 | L1 | 41 | 456.3 | L2 |
| 10 | 21 | 575 | L4 | 1 | 325 | L1 | 8 | 700 | L5 | 8 | 700 | L5 | 4 | 450 | L2 | 7 | 700 | L5 | 3 | 500 | L3 | 9 | 575 | L4 | 61 | 581.3 | L4 |
| 11 | 24 | 628.6 | L5 | 4 | 700 | L5 | 4 | 450 | L2 | 8 | 700 | L5 | 3 | 387.5 | L1 | 4 | 485.7 | L3 | 2 | 400 | L2 | 5 | 408.3 | L2 | 54 | 537.5 | L3 |
| 12 | 21 | 575 | L4 | 1 | 325 | L1 | 4 | 450 | L2 | 8 | 700 | L5 | 6 | 575 | L4 | 3 | 414.3 | L2 | 2 | 400 | L2 | 8 | 533.3 | L3 | 53 | 531.3 | L3 |
| 13 | 25 | 646.4 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 5 | 512.5 | L3 | 7 | 637.5 | L5 | 7 | 700 | L5 | 5 | 700 | L5 | 7 | 491.7 | L3 | 66 | 612.5 | L4 |
| 14 | 27 | 682.1 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 10 | 616.7 | L4 | 71 | 643.8 | L5 |
| 15 | 21 | 575 | L4 | 4 | 700 | L5 | 8 | 700 | L5 | 7 | 637.5 | L5 | 4 | 450 | L2 | 7 | 700 | L5 | 3 | 500 | L3 | 10 | 616.7 | L4 | 64 | 600 | L4 |
| 16 | 20 | 557.1 | L4 | 1 | 325 | L1 | 6 | 575 | L4 | 0 | 200 | L1 | 2 | 325 | L1 | 4 | 485.7 | L3 | 2 | 400 | L2 | 6 | 450 | L2 | 41 | 456.3 | L2 |
| 17 | 24 | 628.6 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 8 | 533.3 | L3 | 63 | 593.8 | L4 |
| 18 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 5 | 408.3 | L2 | 61 | 581.3 | L4 |
| 19 | 17 | 503.6 | L3 | 2 | 450 | L2 | 2 | 325 | L1 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 7 | 700 | L5 | 4 | 600 | L4 | 8 | 533.3 | L3 | 52 | 525 | L3 |
| 20 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 7 | 700 | L5 | 1 | 300 | L1 | 8 | 533.3 | L3 | 60 | 575 | L4 |
| 21 | 15 | 467.9 | L2 | 4 | 700 | L5 | 4 | 450 | L2 | 6 | 575 | L4 | 2 | 325 | L1 | 7 | 700 | L5 | 1 | 300 | L1 | 3 | 325 | L1 | 42 | 462.5 | L2 |
| 22 | 22 | 592.9 | L4 | 3 | 575 | L4 | 4 | 450 | L2 | 6 | 575 | L4 | 4 | 450 | L2 | 0 | 200 | L1 | 1 | 300 | L1 | 6 | 450 | L2 | 46 | 487.5 | L3 |
| 23 | 20 | 557.1 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 3 | 414.3 | L2 | 3 | 500 | L3 | 4 | 366.7 | L1 | 49 | 506.3 | L3 |
| 24 | 15 | 467.9 | L2 | 4 | 700 | L5 | 6 | 575 | L4 | 5 | 512.5 | L3 | 2 | 325 | L1 | 2 | 342.9 | L1 | 1 | 300 | L1 | 4 | 366.7 | L1 | 39 | 443.8 | L2 |
| 25 | 24 | 628.6 | L5 | 2 | 450 | L2 | 6 | 575 | L4 | 4 | 450 | L2 | 4 | 450 | L2 | 3 | 414.3 | L2 | 3 | 500 | L3 | 5 | 408.3 | L2 | 51 | 518.8 | L3 |
| 26 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 4 | 450 | L2 | 4 | 485.7 | L3 | 2 | 400 | L2 | 10 | 616.7 | L4 | 60 | 575 | L4 |
| 27 | 16 | 485.7 | L3 | 4 | 700 | L5 | 4 | 450 | L2 | 1 | 262.5 | L1 | 4 | 450 | L2 | 4 | 485.7 | L3 | 2 | 400 | L2 | 6 | 450 | L2 | 41 | 456.3 | L2 |
| 28 | 25 | 646.4 | L5 | 3 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 637.5 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 12 | 700 | L5 | 71 | 643.8 | L5 |
| 29 | 14 | 450 | L2 | 3 | 575 | L4 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 27 | 368.8 | L1 |
| 30 | 18 | 521.4 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 2 | 325 | L1 | 7 | 700 | L5 | 0 | 200 | L1 | 1 | 241.7 | L1 | 42 | 462.5 | L2 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 31 | 19 | 539.3 | L3 | 1 | 325 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 3 | 325 | L1 | 28 | 375 | L1 |
| 32 | 12 | 414.3 | L2 | 1 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 283.3 | L1 | 20 | 325 | L1 |
| 33 | 22 | 592.9 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 637.5 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 283.3 | L1 | 47 | 493.8 | L3 |
| 34 | 8 | 342.9 | L1 | 2 | 450 | L2 | 6 | 575 | L4 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 23 | 343.8 | L1 |
| 35 | 18 | 521.4 | L3 | 1 | 325 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 27 | 368.8 | L1 |
| 36 | 18 | 521.4 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 366.7 | L1 | 33 | 406.3 | L2 |
| 37 | 20 | 557.1 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 7 | 700 | L5 | 5 | 700 | L5 | 8 | 533.3 | L3 | 60 | 575 | L4 |
| 38 | 22 | 592.9 | L4 | 4 | 700 | L5 | 7 | 637.5 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 700 | L5 | 4 | 600 | L4 | 8 | 533.3 | L3 | 68 | 625 | L5 |
| 39 | 8 | 342.9 | L1 | 2 | 450 | L2 | 4 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 19 | 318.8 | L1 |
| 40 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 2 | 342.9 | L1 | 3 | 500 | L3 | 8 | 533.3 | L3 | 59 | 568.8 | L4 |
| 41 | 25 | 646.4 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 2 | 400 | L2 | 12 | 700 | L5 | 70 | 637.5 | L5 |
| 42 | 26 | 664.3 | L5 | 4 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 637.5 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 8 | 533.3 | L3 | 71 | 643.8 | L5 |
| 43 | 23 | 610.7 | L4 | 3 | 575 | L4 | 8 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 11 | 658.3 | L5 | 71 | 643.8 | L5 |
| 44 | 20 | 557.1 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 3 | 387.5 | L1 | 4 | 450 | L2 | 1 | 271.4 | L1 | 4 | 600 | L4 | 5 | 408.3 | L2 | 46 | 487.5 | L3 |
| 45 | 25 | 646.4 | L5 | 2 | 450 | L2 | 6 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 700 | L5 | 4 | 600 | L4 | 11 | 658.3 | L5 | 69 | 631.3 | L5 |
| 46 | 23 | 610.7 | L4 | 3 | 575 | L4 | 8 | 700 | L5 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 5 | 557.1 | L4 | 2 | 400 | L2 | 6 | 450 | L2 | 59 | 568.8 | L4 |
| 47 | 26 | 664.3 | L5 | 4 | 700 | L5 | 2 | 325 | L1 | 8 | 700 | L5 | 8 | 700 | L5 | 2 | 342.9 | L1 | 5 | 700 | L5 | 9 | 575 | L4 | 64 | 600 | L4 |
| 48 | 25 | 646.4 | L5 | 2 | 450 | L2 | 6 | 575 | L4 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 6 | 450 | L2 | 63 | 593.8 | L4 |
| 49 | 22 | 592.9 | L4 | 1 | 325 | L1 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 2 | 400 | L2 | 5 | 408.3 | L2 | 57 | 556.3 | L4 |
| 50 | 25 | 646.4 | L5 | 3 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 5 | 700 | L5 | 11 | 658.3 | L5 | 71 | 643.8 | L5 |
| 51 | 15 | 467.9 | L2 | 2 | 450 | L2 | 4 | 450 | L2 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 3 | 500 | L3 | 2 | 283.3 | L1 | 29 | 381.3 | L1 |
| 52 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 4 | 450 | L2 | 2 | 342.9 | L1 | 4 | 600 | L4 | 11 | 658.3 | L5 | 62 | 587.5 | L4 |
| 53 | 24 | 628.6 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 700 | L5 | 5 | 700 | L5 | 12 | 700 | L5 | 74 | 662.5 | L5 |
| 54 | 20 | 557.1 | L4 | 4 | 700 | L5 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 4 | 450 | L2 | 7 | 700 | L5 | 3 | 500 | L3 | 11 | 658.3 | L5 | 63 | 593.8 | L4 |
| 55 | 22 | 592.9 | L4 | 4 | 700 | L5 | 5 | 512.5 | L3 | 8 | 700 | L5 | 3 | 387.5 | L1 | 6 | 628.6 | L5 | 3 | 500 | L3 | 9 | 575 | L4 | 60 | 575 | L4 |
| 56 | 17 | 503.6 | L3 | 1 | 325 | L1 | 6 | 575 | L4 | 8 | 700 | L5 | 3 | 387.5 | L1 | 4 | 485.7 | L3 | 1 | 300 | L1 | 3 | 325 | L1 | 43 | 468.8 | L2 |
| 57 | 18 | 521.4 | L3 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 7 | 637.5 | L5 | 3 | 414.3 | L2 | 3 | 500 | L3 | 4 | 366.7 | L1 | 51 | 518.8 | L3 |
| 58 | 20 | 557.1 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 4 | 450 | L2 | 5 | 557.1 | L4 | 2 | 400 | L2 | 4 | 366.7 | L1 | 49 | 506.3 | L3 |
| 59 | 19 | 539.3 | L3 | 2 | 450 | L2 | 6 | 575 | L4 | 2 | 325 | L1 | 3 | 387.5 | L1 | 6 | 628.6 | L5 | 2 | 400 | L2 | 2 | 283.3 | L1 | 42 | 462.5 | L2 |
| 60 | 21 | 575 | L4 | 4 | 700 | L5 | 5 | 512.5 | L3 | 6 | 575 | L4 | 5 | 512.5 | L3 | 7 | 700 | L5 | 2 | 400 | L2 | 6 | 450 | L2 | 56 | 550 | L4 |
| 61 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 700 | L5 | 5 | 700 | L5 | 12 | 700 | L5 | 73 | 656.3 | L5 |
| 62 | 17 | 503.6 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 4 | 450 | L2 | 5 | 512.5 | L3 | 0 | 200 | L1 | 2 | 400 | L2 | 4 | 366.7 | L1 | 41 | 456.3 | L2 |
| 63 | 24 | 628.6 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 700 | L5 | 2 | 400 | L2 | 10 | 616.7 | L4 | 67 | 618.8 | L4 |
| 64 | 21 | 575 | L4 | 3 | 575 | L4 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 4 | 485.7 | L3 | 5 | 700 | L5 | 9 | 575 | L4 | 61 | 581.3 | L4 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | 21 | 575 | L4 | 1 | 325 | L1 | 6 | 575 | L4 | 7 | 637.5 | L5 | 4 | 450 | L2 | 3 | 414.3 | L2 | 2 | 400 | L2 | 4 | 366.7 | L1 | 48 | 500 | L3 |
| 66 | 25 | 646.4 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 5 | 512.5 | L3 | 7 | 700 | L5 | 4 | 600 | L4 | 11 | 658.3 | L5 | 70 | 637.5 | L5 |
| 67 | 28 | 700 | L5 | 3 | 575 | L4 | 7 | 637.5 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 9 | 575 | L4 | 71 | 643.8 | L5 |
| 68 | 23 | 610.7 | L4 | 4 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 2 | 342.9 | L1 | 3 | 500 | L3 | 6 | 450 | L2 | 62 | 587.5 | L4 |
| 69 | 17 | 503.6 | L3 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 0 | 200 | L1 | 3 | 500 | L3 | 4 | 366.7 | L1 | 45 | 481.3 | L3 |
| 70 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 0 | 200 | L1 | 3 | 500 | L3 | 12 | 700 | L5 | 58 | 562.5 | L4 |
| 71 | 26 | 664.3 | L5 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 8 | 700 | L5 | 6 | 628.6 | L5 | 5 | 700 | L5 | 12 | 700 | L5 | 75 | 668.8 | L5 |
| 72 | 12 | 414.3 | L2 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 7 | 700 | L5 | 2 | 400 | L2 | 2 | 283.3 | L1 | 44 | 475 | L3 |
| 73 | 22 | 592.9 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 6 | 575 | L4 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 366.7 | L1 | 43 | 468.8 | L2 |
| 74 | 21 | 575 | L4 | 2 | 450 | L2 | 8 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 3 | 414.3 | L2 | 3 | 500 | L3 | 7 | 491.7 | L3 | 58 | 562.5 | L4 |
| 75 | 17 | 503.6 | L3 | 1 | 325 | L1 | 6 | 575 | L4 | 6 | 575 | L4 | 6 | 575 | L4 | 2 | 342.9 | L1 | 3 | 500 | L3 | 2 | 283.3 | L1 | 43 | 468.8 | L2 |
| 76 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 5 | 512.5 | L3 | 6 | 628.6 | L5 | 3 | 500 | L3 | 7 | 491.7 | L3 | 62 | 587.5 | L4 |
| 77 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 4 | 450 | L2 | 4 | 485.7 | L3 | 3 | 500 | L3 | 8 | 533.3 | L3 | 59 | 568.8 | L4 |
| 78 | 19 | 539.3 | L3 | 4 | 700 | L5 | 6 | 575 | L4 | 1 | 262.5 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 5 | 408.3 | L2 | 41 | 456.3 | L2 |
| 79 | 22 | 592.9 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 0 | 200 | L1 | 3 | 500 | L3 | 9 | 575 | L4 | 57 | 556.3 | L4 |
| 80 | 16 | 485.7 | L3 | 2 | 450 | L2 | 5 | 512.5 | L3 | 6 | 575 | L4 | 2 | 325 | L1 | 7 | 700 | L5 | 2 | 400 | L2 | 4 | 366.7 | L1 | 44 | 475 | L3 |
| 81 | 20 | 557.1 | L4 | 1 | 325 | L1 | 4 | 450 | L2 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 4 | 485.7 | L3 | 2 | 400 | L2 | 4 | 366.7 | L1 | 47 | 493.8 | L3 |
| 82 | 21 | 575 | L4 | 3 | 575 | L4 | 3 | 387.5 | L1 | 8 | 700 | L5 | 4 | 450 | L2 | 0 | 200 | L1 | 3 | 500 | L3 | 1 | 241.7 | L1 | 43 | 468.8 | L2 |
| 83 | 23 | 610.7 | L4 | 3 | 575 | L4 | 8 | 700 | L5 | 5 | 512.5 | L3 | 4 | 450 | L2 | 7 | 700 | L5 | 4 | 600 | L4 | 10 | 616.7 | L4 | 64 | 600 | L4 |
| 84 | 25 | 646.4 | L5 | 2 | 450 | L2 | 6 | 575 | L4 | 8 | 700 | L5 | 7 | 637.5 | L5 | 0 | 200 | L1 | 2 | 400 | L2 | 7 | 491.7 | L3 | 57 | 556.3 | L4 |
| 85 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 3 | 414.3 | L2 | 2 | 400 | L2 | 5 | 408.3 | L2 | 55 | 543.8 | L3 |
| 86 | 25 | 646.4 | L5 | 4 | 700 | L5 | 7 | 637.5 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 6 | 628.6 | L5 | 5 | 700 | L5 | 11 | 658.3 | L5 | 74 | 662.5 | L5 |
| 87 | 25 | 646.4 | L5 | 3 | 575 | L4 | 5 | 512.5 | L3 | 8 | 700 | L5 | 7 | 637.5 | L5 | 3 | 414.3 | L2 | 4 | 600 | L4 | 7 | 491.7 | L3 | 62 | 587.5 | L4 |
| 88 | 20 | 557.1 | L4 | 1 | 325 | L1 | 5 | 512.5 | L3 | 5 | 512.5 | L3 | 4 | 450 | L2 | 1 | 271.4 | L1 | 0 | 200 | L1 | 5 | 408.3 | L2 | 41 | 456.3 | L2 |
| 89 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 6 | 575 | L4 | 4 | 485.7 | L3 | 2 | 400 | L2 | 6 | 450 | L2 | 57 | 556.3 | L4 |
| 90 | 26 | 664.3 | L5 | 3 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 9 | 575 | L4 | 65 | 606.3 | L4 |
| 91 | 22 | 592.9 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 575 | L4 | 0 | 200 | L1 | 2 | 400 | L2 | 10 | 616.7 | L4 | 53 | 531.3 | L3 |
| 92 | 10 | 378.6 | L1 | 2 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 387.5 | L1 | 4 | 485.7 | L3 | 2 | 400 | L2 | 1 | 241.7 | L1 | 22 | 337.5 | L1 |
| 93 | 10 | 378.6 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 4 | 450 | L2 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 24 | 350 | L1 |
| 94 | 7 | 325 | L1 | 1 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 241.7 | L1 | 10 | 262.5 | L1 |
| 95 | 10 | 378.6 | L1 | 4 | 700 | L5 | 2 | 325 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 500 | L3 | 3 | 325 | L1 | 26 | 362.5 | L1 |
| 96 | 4 | 271.4 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 3 | 414.3 | L2 | 2 | 400 | L2 | 2 | 283.3 | L1 | 20 | 325 | L1 |
| 97 | 7 | 325 | L1 | 4 | 700 | L5 | 0 | 200 | L1 | 5 | 512.5 | L3 | 2 | 325 | L1 | 3 | 414.3 | L2 | 2 | 400 | L2 | 3 | 325 | L1 | 26 | 362.5 | L1 |
| 98 | 15 | 467.9 | L2 | 3 | 575 | L4 | 8 | 700 | L5 | 8 | 700 | L5 | 6 | 575 | L4 | 4 | 485.7 | L3 | 2 | 400 | L2 | 7 | 491.7 | L3 | 53 | 531.3 | L3 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 99 | 10 | 378.6 | L1 | 1 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 4 | 450 | L2 | 1 | 271.4 | L1 | 1 | 300 | L1 | 0 | 200 | L1 | 18 | 312.5 | L1 |
| 100 | 8 | 342.9 | L1 | 0 | 200 | L1 | 2 | 325 | L1 | 3 | 387.5 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 20 | 325 | L1 |
| 101 | 14 | 450 | L2 | 2 | 450 | L2 | 0 | 200 | L1 | 6 | 575 | L4 | 4 | 450 | L2 | 3 | 414.3 | L2 | 3 | 500 | L3 | 5 | 408.3 | L2 | 37 | 431.3 | L2 |
| 102 | 5 | 289.3 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 241.7 | L1 | 11 | 268.8 | L1 |
| 103 | 8 | 342.9 | L1 | 4 | 700 | L5 | 5 | 512.5 | L3 | 4 | 450 | L2 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 22 | 337.5 | L1 |
| 104 | 5 | 289.3 | L1 | 1 | 325 | L1 | 2 | 325 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 4 | 485.7 | L3 | 0 | 200 | L1 | 2 | 283.3 | L1 | 16 | 300 | L1 |
| 105 | 22 | 592.9 | L4 | 1 | 325 | L1 | 4 | 450 | L2 | 2 | 325 | L1 | 2 | 325 | L1 | 6 | 628.6 | L5 | 2 | 400 | L2 | 7 | 491.7 | L3 | 46 | 487.5 | L3 |
| 106 | 17 | 503.6 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 33 | 406.3 | L2 |
| 107 | 16 | 485.7 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 2 | 342.9 | L1 | 2 | 400 | L2 | 5 | 408.3 | L2 | 45 | 481.3 | L3 |
| 108 | 20 | 557.1 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 1 | 262.5 | L1 | 4 | 450 | L2 | 4 | 485.7 | L3 | 0 | 200 | L1 | 1 | 241.7 | L1 | 40 | 450 | L2 |
| 109 | 18 | 521.4 | L3 | 4 | 700 | L5 | 5 | 512.5 | L3 | 7 | 637.5 | L5 | 6 | 575 | L4 | 5 | 557.1 | L4 | 3 | 500 | L3 | 8 | 533.3 | L3 | 56 | 550 | L4 |
| 110 | 25 | 646.4 | L5 | 4 | 700 | L5 | 8 | 700 | L5 | 7 | 637.5 | L5 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 7 | 491.7 | L3 | 53 | 531.3 | L3 |
| 111 | 23 | 610.7 | L4 | 3 | 575 | L4 | 4 | 450 | L2 | 7 | 637.5 | L5 | 3 | 387.5 | L1 | 3 | 414.3 | L2 | 0 | 200 | L1 | 4 | 366.7 | L1 | 47 | 493.8 | L3 |
| 112 | 17 | 503.6 | L3 | 4 | 700 | L5 | 4 | 450 | L2 | 7 | 637.5 | L5 | 2 | 325 | L1 | 7 | 700 | L5 | 3 | 500 | L3 | 8 | 533.3 | L3 | 52 | 525 | L3 |
| 113 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 1 | 262.5 | L1 | 5 | 512.5 | L3 | 5 | 557.1 | L4 | 3 | 500 | L3 | 3 | 325 | L1 | 48 | 500 | L3 |
| 114 | 20 | 557.1 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 4 | 450 | L2 | 5 | 557.1 | L4 | 2 | 400 | L2 | 5 | 408.3 | L2 | 53 | 531.3 | L3 |
| 115 | 17 | 503.6 | L3 | 2 | 450 | L2 | 4 | 450 | L2 | 2 | 325 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 4 | 366.7 | L1 | 32 | 400 | L2 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 116 | 20 | 557.1 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 1 | 262.5 | L1 | 2 | 325 | L1 | 3 | 414.3 | L2 | 2 | 400 | L2 | 1 | 241.7 | L1 | 37 | 431.3 | L2 |
| 117 | 19 | 539.3 | L3 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 4 | 485.7 | L3 | 2 | 400 | L2 | 5 | 408.3 | L2 | 54 | 537.5 | L3 |
| 118 | 26 | 664.3 | L5 | 2 | 450 | L2 | 0 | 200 | L1 | 5 | 512.5 | L3 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 366.7 | L1 | 40 | 450 | L2 |
| 119 | 21 | 575 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 7 | 700 | L5 | 2 | 400 | L2 | 5 | 408.3 | L2 | 56 | 550 | L4 |
| 120 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 7 | 700 | L5 | 2 | 400 | L2 | 4 | 366.7 | L1 | 57 | 556.3 | L4 |
| 121 | 13 | 432.1 | L2 | 2 | 450 | L2 | 4 | 450 | L2 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 3 | 325 | L1 | 28 | 375 | L1 |
| 122 | 20 | 557.1 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 8 | 700 | L5 | 5 | 512.5 | L3 | 0 | 200 | L1 | 3 | 500 | L3 | 8 | 533.3 | L3 | 52 | 525 | L3 |
| 123 | 10 | 378.6 | L1 | 4 | 700 | L5 | 4 | 450 | L2 | 4 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 325 | L1 | 25 | 356.3 | L1 |
| 124 | 12 | 414.3 | L2 | 1 | 325 | L1 | 4 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 2 | 283.3 | L1 | 22 | 337.5 | L1 |
| 125 | 16 | 485.7 | L3 | 4 | 700 | L5 | 5 | 512.5 | L3 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 5 | 557.1 | L4 | 2 | 400 | L2 | 0 | 200 | L1 | 38 | 437.5 | L2 |
| 126 | 18 | 521.4 | L3 | 3 | 575 | L4 | 4 | 450 | L2 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 31 | 393.8 | L1 |
| 127 | 25 | 646.4 | L5 | 4 | 700 | L5 | 8 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 7 | 700 | L5 | 3 | 500 | L3 | 5 | 408.3 | L2 | 64 | 600 | L4 |
| 128 | 19 | 539.3 | L3 | 3 | 575 | L4 | 6 | 575 | L4 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 34 | 412.5 | L2 |
| 129 | 22 | 592.9 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 8 | 700 | L5 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 46 | 487.5 | L3 |
| 130 | 16 | 485.7 | L3 | 1 | 325 | L1 | 3 | 387.5 | L1 | 1 | 262.5 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 408.3 | L2 | 29 | 381.3 | L1 |
| 131 | 24 | 628.6 | L5 | 3 | 575 | L4 | 6 | 575 | L4 | 7 | 637.5 | L5 | 3 | 387.5 | L1 | 6 | 628.6 | L5 | 2 | 400 | L2 | 3 | 325 | L1 | 54 | 537.5 | L3 |
| 132 | 27 | 682.1 | L5 | 4 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 8 | 700 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 10 | 616.7 | L4 | 75 | 668.8 | L5 |
| 133 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 4 | 450 | L2 | 5 | 557.1 | L4 | 3 | 500 | L3 | 5 | 408.3 | L2 | 57 | 556.3 | L4 |
| 134 | 23 | 610.7 | L4 | 2 | 450 | L2 | 6 | 575 | L4 | 8 | 700 | L5 | 2 | 325 | L1 | 4 | 485.7 | L3 | 2 | 400 | L2 | 4 | 366.7 | L1 | 51 | 518.8 | L3 |
| 135 | 22 | 592.9 | L4 | 1 | 325 | L1 | 6 | 575 | L4 | 7 | 637.5 | L5 | 8 | 700 | L5 | 6 | 628.6 | L5 | 2 | 400 | L2 | 3 | 325 | L1 | 55 | 543.8 | L3 |
| 136 | 22 | 592.9 | L4 | 3 | 575 | L4 | 8 | 700 | L5 | 2 | 325 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 241.7 | L1 | 38 | 437.5 | L2 |
| 137 | 23 | 610.7 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 5 | 512.5 | L3 | 7 | 700 | L5 | 3 | 500 | L3 | 6 | 450 | L2 | 61 | 581.3 | L4 |
| 138 | 20 | 557.1 | L4 | 4 | 700 | L5 | 6 | 575 | L4 | 7 | 637.5 | L5 | 6 | 575 | L4 | 7 | 700 | L5 | 2 | 400 | L2 | 6 | 450 | L2 | 58 | 562.5 | L4 |
| 139 | 18 | 521.4 | L3 | 4 | 700 | L5 | 6 | 575 | L4 | 6 | 575 | L4 | 5 | 512.5 | L3 | 7 | 700 | L5 | 3 | 500 | L3 | 8 | 533.3 | L3 | 57 | 556.3 | L4 |
| 140 | 14 | 450 | L2 | 1 | 325 | L1 | 6 | 575 | L4 | 6 | 575 | L4 | 4 | 450 | L2 | 3 | 414.3 | L2 | 0 | 200 | L1 | 5 | 408.3 | L2 | 39 | 443.8 | L2 |
| 141 | 8 | 342.9 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 241.7 | L1 | 14 | 287.5 | L1 |
| 142 | 18 | 521.4 | L3 | 1 | 325 | L1 | 4 | 450 | L2 | 4 | 450 | L2 | 4 | 450 | L2 | 3 | 414.3 | L2 | 3 | 500 | L3 | 4 | 366.7 | L1 | 41 | 456.3 | L2 |
| 143 | 22 | 592.9 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 8 | 700 | L5 | 5 | 512.5 | L3 | 4 | 485.7 | L3 | 2 | 400 | L2 | 4 | 366.7 | L1 | 54 | 537.5 | L3 |
| 144 | 14 | 450 | L2 | 3 | 575 | L4 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 283.3 | L1 | 26 | 362.5 | L1 |
| 145 | 6 | 307.1 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 2 | 325 | L1 | 1 | 262.5 | L1 | 3 | 414.3 | L2 | 0 | 200 | L1 | 1 | 241.7 | L1 | 16 | 300 | L1 |
| 146 | 7 | 325 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 11 | 268.8 | L1 |
| 147 | 7 | 325 | L1 | 2 | 450 | L2 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 17 | 306.3 | L1 |
| 148 | 18 | 521.4 | L3 | 4 | 700 | L5 | 4 | 450 | L2 | 5 | 512.5 | L3 | 5 | 512.5 | L3 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 40 | 450 | L2 |
| 149 | 9 | 360.7 | L1 | 2 | 450 | L2 | 6 | 575 | L4 | 2 | 325 | L1 | 2 | 325 | L1 | 1 | 271.4 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 26 | 362.5 | L1 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 150 | 7 | 325 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 8 | 533.3 | L3 | 26 | 362.5 | L1 |
| 151 | 16 | 485.7 | L3 | 4 | 700 | L5 | 4 | 450 | L2 | 6 | 575 | L4 | 5 | 512.5 | L3 | 6 | 628.6 | L5 | 3 | 500 | L3 | 1 | 241.7 | L1 | 45 | 481.3 | L3 |
| 152 | 23 | 610.7 | L4 | 3 | 575 | L4 | 6 | 575 | L4 | 6 | 575 | L4 | 3 | 387.5 | L1 | 7 | 700 | L5 | 3 | 500 | L3 | 11 | 658.3 | L5 | 62 | 587.5 | L4 |
| 153 | 15 | 467.9 | L2 | 0 | 200 | L1 | 2 | 325 | L1 | 2 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 408.3 | L2 | 25 | 356.3 | L1 |
| 154 | 5 | 289.3 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 6 | 237.5 | L1 |
| 155 | 8 | 342.9 | L1 | 3 | 575 | L4 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 1 | 241.7 | L1 | 20 | 325 | L1 |
| 156 | 5 | 289.3 | L1 | 4 | 700 | L5 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 15 | 293.8 | L1 |
| 157 | 7 | 325 | L1 | 4 | 700 | L5 | 0 | 200 | L1 | 5 | 512.5 | L3 | 3 | 387.5 | L1 | 0 | 200 | L1 | 3 | 500 | L3 | 6 | 450 | L2 | 28 | 375 | L1 |
| 158 | 5 | 289.3 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 14 | 287.5 | L1 |
| 159 | 7 | 325 | L1 | 1 | 325 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 14 | 287.5 | L1 |
| 160 | 8 | 342.9 | L1 | 2 | 450 | L2 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 23 | 343.8 | L1 |
| 161 | 4 | 271.4 | L1 | 1 | 325 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 2 | 325 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 13 | 281.3 | L1 |
| 162 | 7 | 325 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 4 | 450 | L2 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 20 | 325 | L1 |
| 163 | 8 | 342.9 | L1 | 3 | 575 | L4 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 19 | 318.8 | L1 |
| 164 | 8 | 342.9 | L1 | 2 | 450 | L2 | 0 | 200 | L1 | 3 | 387.5 | L1 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 20 | 325 | L1 |
| 165 | 8 | 342.9 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 3 | 414.3 | L2 | 2 | 400 | L2 | 0 | 200 | L1 | 22 | 337.5 | L1 |
| 166 | 10 | 378.6 | L1 | 4 | 700 | L5 | 6 | 575 | L4 | 2 | 325 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 26 | 362.5 | L1 |
| 167 | 5 | 289.3 | L1 | 2 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 8 | 250 | L1 |
| 168 | 4 | 271.4 | L1 | 1 | 325 | L1 | 1 | 262.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 283.3 | L1 | 10 | 262.5 | L1 |
| 169 | 4 | 271.4 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 5 | 512.5 | L3 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 15 | 293.8 | L1 |
| 170 | 5 | 289.3 | L1 | 1 | 325 | L1 | 5 | 512.5 | L3 | 6 | 575 | L4 | 4 | 450 | L2 | 0 | 200 | L1 | 3 | 500 | L3 | 3 | 325 | L1 | 27 | 368.8 | L1 |
| 171 | 14 | 450 | L2 | 3 | 575 | L4 | 5 | 512.5 | L3 | 7 | 637.5 | L5 | 7 | 637.5 | L5 | 3 | 414.3 | L2 | 2 | 400 | L2 | 0 | 200 | L1 | 41 | 456.3 | L2 |
| 172 | 4 | 271.4 | L1 | 3 | 575 | L4 | 3 | 387.5 | L1 | 5 | 512.5 | L3 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 366.7 | L1 | 20 | 325 | L1 |
| 173 | 5 | 289.3 | L1 | 1 | 325 | L1 | 3 | 387.5 | L1 | 5 | 512.5 | L3 | 2 | 325 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 22 | 337.5 | L1 |
| 174 | 6 | 307.1 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 2 | 325 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 15 | 293.8 | L1 |
| 175 | 17 | 503.6 | L3 | 2 | 450 | L2 | 2 | 325 | L1 | 4 | 450 | L2 | 4 | 450 | L2 | 0 | 200 | L1 | 3 | 500 | L3 | 5 | 408.3 | L2 | 37 | 431.3 | L2 |
| 176 | 5 | 289.3 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 16 | 300 | L1 |
| 177 | 7 | 325 | L1 | 2 | 450 | L2 | 3 | 387.5 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 15 | 293.8 | L1 |
| 178 | 7 | 325 | L1 | 2 | 450 | L2 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 14 | 287.5 | L1 |
| 179 | 11 | 396.4 | L1 | 2 | 450 | L2 | 0 | 200 | L1 | 4 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 21 | 331.3 | L1 |
| 180 | 4 | 271.4 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 12 | 275 | L1 |
| 181 | 9 | 360.7 | L1 | 3 | 575 | L4 | 3 | 387.5 | L1 | 6 | 575 | L4 | 5 | 512.5 | L3 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 30 | 387.5 | L1 |
| 182 | 11 | 396.4 | L1 | 4 | 700 | L5 | 2 | 325 | L1 | 5 | 512.5 | L3 | 3 | 387.5 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 29 | 381.3 | L1 |
| 183 | 10 | 378.6 | L1 | 3 | 575 | L4 | 5 | 512.5 | L3 | 4 | 450 | L2 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 4 | 366.7 | L1 | 31 | 393.8 | L1 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 184 | 7 | 325 | L1 | 3 | 575 | L4 | 5 | 512.5 | L3 | 5 | 512.5 | L3 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 28 | 375 | L1 |
| 185 | 14 | 450 | L2 | 1 | 325 | L1 | 2 | 325 | L1 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 342.9 | L1 | 2 | 400 | L2 | 3 | 325 | L1 | 34 | 412.5 | L2 |
| 186 | 9 | 360.7 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 300 | L1 | 0 | 200 | L1 | 15 | 293.8 | L1 |
| 187 | 3 | 253.6 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 14 | 287.5 | L1 |
| 188 | 11 | 396.4 | L1 | 4 | 700 | L5 | 3 | 387.5 | L1 | 5 | 512.5 | L3 | 5 | 512.5 | L3 | 5 | 557.1 | L4 | 3 | 500 | L3 | 3 | 325 | L1 | 39 | 443.8 | L2 |
| 189 | 7 | 325 | L1 | 1 | 325 | L1 | 4 | 450 | L2 | 7 | 637.5 | L5 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 26 | 362.5 | L1 |
| 190 | 12 | 414.3 | L2 | 4 | 700 | L5 | 2 | 325 | L1 | 5 | 512.5 | L3 | 6 | 575 | L4 | 0 | 200 | L1 | 2 | 400 | L2 | 6 | 450 | L2 | 37 | 431.3 | L2 |
| 191 | 9 | 360.7 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 4 | 450 | L2 | 4 | 450 | L2 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 23 | 343.8 | L1 |
| 192 | 7 | 325 | L1 | 2 | 450 | L2 | 5 | 512.5 | L3 | 1 | 262.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 283.3 | L1 | 17 | 306.3 | L1 |
| 193 | 1 | 217.9 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 14 | 287.5 | L1 |
| 194 | 4 | 271.4 | L1 | 1 | 325 | L1 | 3 | 387.5 | L1 | 1 | 262.5 | L1 | 1 | 262.5 | L1 | 2 | 342.9 | L1 | 0 | 200 | L1 | 1 | 241.7 | L1 | 13 | 281.3 | L1 |
| 195 | 9 | 360.7 | L1 | 3 | 575 | L4 | 2 | 325 | L1 | 5 | 512.5 | L3 | 5 | 512.5 | L3 | 7 | 700 | L5 | 2 | 400 | L2 | 0 | 200 | L1 | 33 | 406.3 | L2 |
| 196 | 6 | 307.1 | L1 | 2 | 450 | L2 | 3 | 387.5 | L1 | 3 | 387.5 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 18 | 312.5 | L1 |
| 197 | 5 | 289.3 | L1 | 3 | 575 | L4 | 2 | 325 | L1 | 0 | 200 | L1 | 1 | 262.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 13 | 281.3 | L1 |
| 198 | 7 | 325 | L1 | 2 | 450 | L2 | 0 | 200 | L1 | 3 | 387.5 | L1 | 5 | 512.5 | L3 | 3 | 414.3 | L2 | 2 | 400 | L2 | 1 | 241.7 | L1 | 23 | 343.8 | L1 |
| 199 | 12 | 414.3 | L2 | 4 | 700 | L5 | 0 | 200 | L1 | 6 | 575 | L4 | 7 | 637.5 | L5 | 7 | 700 | L5 | 3 | 500 | L3 | 7 | 491.7 | L3 | 46 | 487.5 | L3 |
| 200 | 10 | 378.6 | L1 | 4 | 700 | L5 | 3 | 387.5 | L1 | 7 | 637.5 | L5 | 3 | 387.5 | L1 | 3 | 414.3 | L2 | 2 | 400 | L2 | 6 | 450 | L2 | 38 | 437.5 | L2 |
| 201 | 9 | 360.7 | L1 | 4 | 700 | L5 | 4 | 450 | L2 | 1 | 262.5 | L1 | 2 | 325 | L1 | 2 | 342.9 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 26 | 362.5 | L1 |
| 202 | 9 | 360.7 | L1 | 2 | 450 | L2 | 1 | 262.5 | L1 | 4 | 450 | L2 | 5 | 512.5 | L3 | 3 | 414.3 | L2 | 2 | 400 | L2 | 0 | 200 | L1 | 26 | 362.5 | L1 |
| 203 | 8 | 342.9 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 6 | 575 | L4 | 4 | 450 | L2 | 2 | 342.9 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 22 | 337.5 | L1 |
| 204 | 10 | 378.6 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 5 | 512.5 | L3 | 4 | 450 | L2 | 2 | 342.9 | L1 | 2 | 400 | L2 | 3 | 325 | L1 | 30 | 387.5 | L1 |
| 205 | 6 | 307.1 | L1 | 1 | 325 | L1 | 4 | 450 | L2 | 2 | 325 | L1 | 2 | 325 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 1 | 241.7 | L1 | 18 | 312.5 | L1 |
| 206 | 10 | 378.6 | L1 | 1 | 325 | L1 | 0 | 200 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 0 | 200 | L1 | 16 | 300 | L1 |
| 207 | 11 | 396.4 | L1 | 4 | 700 | L5 | 2 | 325 | L1 | 6 | 575 | L4 | 5 | 512.5 | L3 | 3 | 414.3 | L2 | 2 | 400 | L2 | 1 | 241.7 | L1 | 34 | 412.5 | L2 |
| 208 | 6 | 307.1 | L1 | 2 | 450 | L2 | 2 | 325 | L1 | 2 | 325 | L1 | 3 | 387.5 | L1 | 0 | 200 | L1 | 2 | 400 | L2 | 2 | 283.3 | L1 | 19 | 318.8 | L1 |
| 209 | 10 | 378.6 | L1 | 4 | 700 | L5 | 3 | 387.5 | L1 | 8 | 700 | L5 | 3 | 387.5 | L1 | 7 | 700 | L5 | 3 | 500 | L3 | 2 | 283.3 | L1 | 40 | 450 | L2 |
| 210 | 19 | 539.3 | L3 | 2 | 450 | L2 | 2 | 325 | L1 | 3 | 387.5 | L1 | 4 | 450 | L2 | 4 | 485.7 | L3 | 2 | 400 | L2 | 0 | 200 | L1 | 36 | 425 | L2 |
| MEAN | 15.89 | 483.8 | L3 | 2.719 | 539.9 | L3 | 4.338 | 471.1 | L2 | 4.681 | 492.6 | L3 | 3.729 | 433 | L2 | 2.786 | 399 | L1 | 2.062 | 406.2 | L2 | 4.238 | 376.6 | L1 | 40.44 | 452.8 | L2 |

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**APPENDIX 6:** CAAPSA DATA ANALYISIS FOR ALGEBRAIC PROBLEM SOLVING TEST

**CAAPSA DATA ANALYSIS FOR AGEBRAIC PROBLEM SOLVING TEST**

| | QUESTION 1 | | | QUESTION 2 | | | QUESTION 3 | | | QUESTION 4 | | | QUESTION 5 | | | QUESTION 6 | | | QUESTION 7 | | | QUESTION 8 | | | OVERALL | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Score | TIMSS | Level | Total | TIMSS | Level |
| 1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 5 | 216 | L1 |
| 2 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 3 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 4 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 6 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 7 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 8 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 9 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 2 | 250 | L1 | 22 | 269 | L1 |
| 10 | 0 | 200 | L1 | 0 | 200 | L1 | 9 | 425 | L2 | 11 | 475 | L3 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 11 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 22 | 269 | L1 |
| 12 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 13 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 14 | 0 | 200 | L1 | 13 | 525 | L3 | 20 | 700 | L5 | 4 | 300 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 97 | 503 | L3 |
| 15 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 7 | 375 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 27 | 284 | L1 |
| 16 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 17 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 241 | L1 |
| 18 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 241 | L1 |
| 19 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 25 | 278 | L1 |
| 20 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 103 | 522 | L3 |
| 21 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 22 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 33 | 303 | L1 |
| 23 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 24 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 25 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 26 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 43 | 334 | L1 |
| 27 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 28 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 6 | 350 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 13 | 525 | L3 | 13 | 525 | L3 | 92 | 488 | L3 |
| 29 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 2 | 206 | L1 |
| 30 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 8 | 225 | L1 |
| 31 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 32 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 33 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 34 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 35 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 36 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 37 | 20 | 700 | L5 | 8 | 400 | L2 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 73 | 428 | L2 |
| 38 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 140 | 638 | L5 |
| 39 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 216 | L1 |
| 40 | 5 | 325 | L1 | 6 | 350 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 24 | 275 | L1 |
| 41 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 42 | 0 | 200 | L1 | 13 | 525 | L3 | 20 | 700 | L5 | 9 | 425 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 102 | 519 | L3 |
| 43 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 160 | 700 | L5 |
| 44 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 45 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 160 | 700 | L5 |
| 46 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 63 | 397 | L1 |
| 47 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 140 | 638 | L5 |
| 48 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 49 | 5 | 325 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 4 | 300 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 69 | 416 | L2 |
| 50 | 20 | 700 | L5 | 3 | 275 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 86 | 469 | L2 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 51 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 52 | 5 | 325 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 99 | 509 | L3 |
| 53 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 160 | 700 | L5 |
| 54 | 5 | 325 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 64 | 400 | L2 |
| 55 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 56 | 5 | 325 | L1 | 8 | 400 | L2 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 14 | 550 | L4 | 14 | 550 | L4 | 86 | 469 | L2 |
| 57 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 300 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 4 | 213 | L1 |
| 58 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 59 | 0 | 200 | L1 | 8 | 400 | L2 | 20 | 700 | L5 | 9 | 425 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 97 | 503 | L3 |
| 60 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 61 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 13 | 525 | L3 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 153 | 678 | L5 |
| 62 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 14 | 550 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 34 | 306 | L1 |
| 63 | 20 | 700 | L5 | 8 | 400 | L2 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 113 | 553 | L4 |
| 64 | 14 | 550 | L4 | 6 | 350 | L1 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 105 | 528 | L3 |
| 65 | 0 | 200 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 79 | 447 | L2 |
| 66 | 20 | 700 | L5 | 8 | 400 | L2 | 6 | 350 | L1 | 4 | 300 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 98 | 506 | L3 |
| 67 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 65 | 403 | L2 |
| 68 | 5 | 325 | L1 | 7 | 375 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 75 | 434 | L2 |
| 69 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 14 | 550 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 17 | 253 | L1 |
| 70 | 5 | 325 | L1 | 0 | 200 | L1 | 9 | 425 | L2 | 4 | 300 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 78 | 444 | L2 |
| 71 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 140 | 638 | L5 |
| 72 | 0 | 200 | L1 | 8 | 400 | L2 | 6 | 350 | L1 | 9 | 425 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 83 | 459 | L2 |
| 73 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 23 | 272 | L1 |
| 74 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 45 | 341 | L1 |
| 75 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 105 | 528 | L3 |
| 76 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 63 | 397 | L1 |
| 77 | 0 | 200 | L1 | 8 | 400 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 8 | 225 | L1 |
| 78 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 79 | 20 | 700 | L5 | 8 | 400 | L2 | 20 | 700 | L5 | 9 | 425 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 97 | 503 | L3 |
| 80 | 20 | 700 | L5 | 20 | 700 | L5 | 6 | 350 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 106 | 531 | L3 |
| 81 | 20 | 700 | L5 | 6 | 350 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 46 | 344 | L1 |
| 82 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 83 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 84 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 85 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 86 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 160 | 700 | L5 |
| 87 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 6 | 350 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 126 | 594 | L4 |
| 88 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 100 | 513 | L3 |
| 89 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 45 | 341 | L1 |
| 90 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 43 | 334 | L1 |
| 91 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 92 | 20 | 700 | L5 | 1 | 225 | L1 | 0 | 200 | L1 | 1 | 225 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 62 | 394 | L1 |
| 93 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 94 | 20 | 700 | L5 | 1 | 225 | L1 | 13 | 525 | L3 | 1 | 225 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 3 | 275 | L1 | 41 | 328 | L1 |
| 95 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 96 | 20 | 700 | L5 | 1 | 225 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 81 | 453 | L2 |
| 97 | 15 | 575 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 55 | 372 | L1 |
| 98 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 8 | 400 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 18 | 650 | L5 | 18 | 650 | L5 | 124 | 588 | L4 |
| 99 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 100 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 101 | 20 | 700 | L5 | 1 | 225 | L1 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 94 | 494 | L3 |
| 102 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 103 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 104 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 105 | 5 | 325 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 4 | 300 | L1 | 5 | 325 | L1 | 13 | 525 | L3 | 20 | 700 | L5 | 20 | 700 | L5 | 92 | 488 | L3 |
| 104 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 105 | 5 | 325 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 4 | 300 | L1 | 5 | 325 | L1 | 13 | 525 | L3 | 20 | 700 | L5 | 20 | 700 | L5 | 92 | 488 | L3 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 106 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 10 | 450 | L2 | 13 | 525 | L3 | 43 | 334 | L1 |
| 107 | 5 | 325 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 5 | 325 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 109 | 541 | L3 |
| 108 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 45 | 341 | L1 |
| 109 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 23 | 272 | L1 |
| 110 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 111 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 112 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 216 | L1 |
| 113 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 16 | 600 | L4 | 29 | 291 | L1 |
| 114 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 115 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 8 | 400 | L2 | 8 | 400 | L2 | 16 | 250 | L1 |
| 116 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 13 | 525 | L3 | 26 | 281 | L1 |
| 117 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 43 | 334 | L1 |
| 118 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 119 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 45 | 341 | L1 |
| 120 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 43 | 334 | L1 |
| 121 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 10 | 450 | L2 | 23 | 272 | L1 |
| 122 | 5 | 325 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 5 | 325 | L1 | 8 | 400 | L2 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 112 | 550 | L4 |
| 123 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 124 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 10 | 450 | L2 | 20 | 263 | L1 |
| 125 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 13 | 525 | L3 | 29 | 291 | L1 |
| 126 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 127 | 5 | 325 | L1 | 14 | 550 | L4 | 20 | 700 | L5 | 5 | 325 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 15 | 575 | L4 | 15 | 575 | L4 | 99 | 509 | L3 |
| 128 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 10 | 450 | L2 | 20 | 263 | L1 |
| 129 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 10 | 450 | L2 | 20 | 263 | L1 |
| 130 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 4 | 300 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 10 | 450 | L2 | 10 | 450 | L2 | 69 | 416 | L2 |
| 131 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 8 | 225 | L1 |
| 132 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 105 | 528 | L3 |
| 133 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 43 | 334 | L1 |
| 134 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 135 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 136 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 137 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 25 | 278 | L1 |
| 138 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 139 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 45 | 341 | L1 |
| 140 | 0 | 200 | L1 | 17 | 625 | L5 | 20 | 700 | L5 | 1 | 225 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 58 | 381 | L1 |
| 141 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 40 | 325 | L1 |
| 142 | 0 | 200 | L1 | 1 | 225 | L1 | 15 | 575 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 36 | 313 | L1 |
| 143 | 1 | 225 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 12 | 500 | L3 | 12 | 500 | L3 | 45 | 341 | L1 |
| 144 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 145 | 0 | 200 | L1 | 0 | 200 | L1 | 17 | 625 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 146 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 147 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 148 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 149 | 0 | 200 | L1 | 0 | 200 | L1 | 15 | 575 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 15 | 247 | L1 |
| 150 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 151 | 0 | 200 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 30 | 294 | L1 |
| 152 | 0 | 200 | L1 | 0 | 200 | L1 | 15 | 575 | L4 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 35 | 309 | L1 |
| 153 | 0 | 200 | L1 | 1 | 225 | L1 | 10 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 24 | 275 | L1 |
| 154 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 155 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 156 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 157 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 13 | 525 | L3 | 0 | 200 | L1 | 0 | 200 | L1 | 16 | 250 | L1 |
| 158 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 159 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 160 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 46 | 344 | L1 |
| 161 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 162 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 206 | L1 |
| 163 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 209 | L1 |
| 164 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 165 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 250 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 2 | 206 | L1 |

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Page 9

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 166 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 167 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 168 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 169 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 170 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 171 | 0 | 200 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 88 | 475 | L3 |
| 172 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 173 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 174 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 175 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 176 | 0 | 200 | L1 | 0 | 200 | L1 | 10 | 450 | L2 | 0 | 200 | L1 | 0 | 200 | L1 | 14 | 550 | L4 | 5 | 325 | L1 | 5 | 325 | L1 | 34 | 306 | L1 |
| 177 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 178 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 179 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 216 | L1 |
| 180 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 181 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 7 | 375 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 87 | 472 | L2 |
| 182 | 8 | 400 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 88 | 475 | L3 |
| 183 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 184 | 20 | 700 | L5 | 0 | 200 | L1 | 6 | 350 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 29 | 291 | L1 |
| 185 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 100 | 513 | L3 |
| 186 | 0 | 200 | L1 | 0 | 200 | L1 | 6 | 350 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 69 | 416 | L2 |
| 187 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 216 | L1 |
| 188 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 48 | 350 | L1 |
| 189 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 25 | 278 | L1 |
| 190 | 0 | 200 | L1 | 14 | 550 | L4 | 6 | 350 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 191 | 10 | 450 | L2 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 11 | 475 | L3 | 11 | 475 | L3 | 72 | 425 | L2 |
| 192 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 193 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 263 | L1 |
| 194 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 195 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 103 | 522 | L3 |
| 196 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 23 | 272 | L1 |
| 197 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 198 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 60 | 388 | L1 |
| 199 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 88 | 475 | L3 |
| 200 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 20 | 700 | L5 | 80 | 450 | L2 |
| 201 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 202 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 3 | 275 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 43 | 334 | L1 |
| 203 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 204 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 40 | 325 | L1 |
| 205 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 25 | 278 | L1 |
| 206 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 207 | 0 | 200 | L1 | 0 | 200 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 65 | 403 | L2 |
| 208 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 209 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 | 0 | 200 | L1 |
| 210 | 0 | 200 | L1 | 5 | 325 | L1 | 20 | 700 | L5 | 0 | 200 | L1 | 20 | 700 | L5 | 20 | 700 | L5 | 0 | 200 | L1 | 0 | 200 | L1 | 65 | 403 | L2 |
| Mean | 3.948 | 298.69 | L1 | 2.9 | 272.5 | L1 | 8.78095 | 419.524 | L2 | 2.03333 | 250.833 | L1 | 1.21905 | 230.476 | L1 | 8.98095 | 424.524 | L2 | 8.46667 | 411.667 | L2 | 8.61905 | 415.476 | L2 | 44.95 | 340 | L1 |

| okup ta |
|---|
| L1 |
| L2 |
| L3 |
| L4 |
| L5 |

| | | |
|---|---|---|
| L1 | 148 | 70.48% |
| L2 | 28 | 13.33% |
| L3 | 22 | 10.48% |
| L4 | 4 | 1.90% |
| L5 | 8 | 3.81% |
| Total | 210 | 100.00% |

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Page 12

# **APPENDIX 7:** ETHICAL ISSUES

Enquiries: Mr. N. Lupahla
Tel: 065-240259
Cell: 0812780772
e-mail:om4rtu@yahoo.com

12 January 2009

To: The Permanent Secretary
Ministry of Education
P Bag 13186
Windhoek

Dear Sir/Madam

**RE: Request for permission to conduct research at schools in Oshana Region in the 2010 academic year.**

I am an MSc (Maths , Science and Technology Education) student studying with the University of South Africa (UNISA).I am doing a research to investigate the level of algebraic problem solving skills of grade 12 learners in Namibia. Due to financial and time limitations, it will not be possible for me to cover the whole of Namibia at this stage. I have opted to pilot my study with schools in Oshana Region, given the fact that as the Master Maths Programme centre based mathematician at Rössing Foundation, I am already interacting with these schools through various outreach activities such as the Rössing Foundation Mathematics Olympiad.

My sample size will be 180 learners from 9 secondary schools namely:
**Gabriel Taapopi SS, Mweshipandeka SS, Oshakati SS, Oluno SS, Onamutai CS, Iihenda SS, Erundu CS, Iipumbu SS and Nangolo SS.**

This study will hopefully reveal the level of learners' algebraic problem solving skills. Advocates of problem solving claim that problem solving develops the learners' creative abilities (Fredricksen, 1984 & Slavin , 1997) and the application of previously learned principles to new ones. The findings of my research will hopefully:

- Promote academic advancement of teachers and learners
- Facilitate learners to acquire problem solving skills and explore the world around them

In the process of my research, I will also be able to identify certain talented learners that we may be able to assist to successfully participate in the regional and National Mathematics Fairs.and this will also assess their readiness to take mathematics as a compulsory subject by 2012.

Yours Sincerely

Nhlanhla Lupahla
RF Centre based Mathematician : Ondangwa

The Rössing Foundation
Registered as a welfare organisation (WO 96)

360 Sam Nujoma Str
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Windhoek
Namibia

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Telephone Code (+264)

Correspondence to:

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Fax (061) 233637

☐
**Ondangwa Adult Education Centre**
PO Box 479
Ondangwa
Tel (065) 240259
Fax (065) 240508

☒
**Tamariskia Adult Education Centre**
PO Box 1458
Swakopmund
Tel (064) 416500
Fax (064) 416501

☐
**Arandis Office**
PO Box 284
Arandis
Tel (064) 512000
Fax (064) 512001

REPUBLIC OF NAMIBIA

# MINISTRY OF EDUCATION

PROGRAMMES AND QUALITY ASSURANCE

**Tel:** 264 61 2933200
**Fax:** 264 61 2933922
**E-mail:** mshimho@mec.gov.na
**Enquiries:** MN Shimhopileni

Private Bag 13186
Windhoek
NAMIBIA

1 February 2010

**File:** 11/1/1

MR N. Lipahla
P. O. Box 479
ONDANGWA

**RE: REQUEST FOR PERMISSION TO CONDUCT A RESEARCH ON ALGEBRAIC PROBLEM SOLVING SKILS OF GRADE 12 LEARNERS**

Your letter, dated 12 January 2010, requesting permission to conduct a research on algebraic problem solving skills of grade 12 learners at some senior secondary schools in Oshana Education Region, has reference.

Kindly be informed that the Ministry of Education is in support of your research project as the outcome of your study may assist to devise strategies in addressing the problem in question.

Nevertheless, you are advised to contact the Oshana Regional Education Office for permission to visit the identified schools.

Kindly note also that the Ministry would appreciate it highly, if you could present it with a copy of your research findings for our information.

By copy of this letter the regional director is made aware of your request.

Yours faithfully

MINISTRY OF EDUCATION
Office of the Permanent Secretary
01 FEB 2010
Private Bag 13186, Windhoek, Namibia

.....
I V Ankama
PERMANENT SECRETARY

**cc: Regional Director: Oshana Education Region**

**REPUBLIC OF NAMIBIA**

**MINISTRY OF EDUCATION**
**OSHANA REGION**

**Tel: 065-229800**
**Fax: 065-228834**

**Private Bag 5550**
**Oshakati**

**Enq. Menette N. Nambala**

05 February 2010

**TO:** Inspectors of Education
Principals: Oshakati SS, GT SS, Mweshipandeka SS, Andimba Toivo SS, Onamutai CS, Iihenda CS, Erundu CS, Iipumbu CS, Nangolo CS

**SUBJECT: Research at schools in Oshana Region**

Kindly receive the self-explanatory attachment, from the Permanent Secretary, giving permission to Nhlanhla Lupahla, Rossing Foundation-based Mathematician, to conduct research in our schools, at some point in time during this academic year. The specific dates will possibly be communicated to us at a later stage.

You are kindly requested to give the researcher the much needed support.

Yours sincerely

8/02/10

**MRS. DUTTE N. SHINYEMBA**
**REGIONAL DIRECTOR**

OSHANA REGIONAL COUNCIL
OFFICE OF THE DIRECTOR OF EDUCATION
2010 -02- 08
P/Bag 5518, Oshakati
REPUBLIC OF NAMIBIA

**The Rössing Foundation**
Registered as a welfare organisation (WO 96)

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Fax (064) 416501

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**Arandis Office**
PO Box 284
Arandis
Tel (064) 512000
Fax (064) 512001

12 February 2010

To: Mr. Nhlanhla Lupahla
The Rössing Foundation
Ondangwa Education Centre

RE: **Request for permission to conduct research at schools in Oshana Region in the 2010 academic year.**

I acknowledge receipt of your letter dated 12 January 2010, in which you are requesting for authorisation to conduct a research on algebraic problem solving skills of grade 12 learners in the Oshana Education Region.

Kindly be informed that as long as your normal work schedule will not be adversely affected by your research activities, the Foundation fully supports you in your studies.

You have the privilege to use of some the following resources at the Rössing Foundation;

- the library
- computers & internet facilities
- minimal photocopying (using your own paper)
- Master Maths Programme sessions for selected learners in your sample on Saturdays only.

I wish you success with your studies and we would appreciate to get feedback on your research findings for our record.

Yours faithfully

Job Tjiho
RF Director

THE RÖSSING FOUNDATION
EDUCATION & DEVELOPMENT CENTRE
2010 -02- 12
P. O. Box 479
ONDANGWA, NAMIBIA
CENTRE MANAGER

**UNIVERSITY OF SOUTH AFRICA**

INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION

# ETHICAL CLEARANCE APPLICATION FORM Date:

**PLEASE NOTE THAT THE FORM MUST BE COMPLETED IN TYPED SCRIPT. HANDWRITTEN APPLICATIONS WILL NOT BE CONSIDERED.**

**SECTION 1: PERSONAL DETAILS**

| | |
|---|---|
| **1.1** | **Full Name and Surname of Applicant:** Nhlanhla Lupahla |
| 1.2 | Title (Ms/Mr/Mrs/Dr/Professor/etc.): Mr |
| 1.3 | Student Number (where applicable): 3234-100-8 |
| | Staff Number (where applicable): |
| 1,4 | School: Institute for Science and Technology Education |
| 1.5 | College: Institute for Science and Technology Education |
| 1.6 | Campus: Pretoria |
| 1.7 | Existing Qualifications: Lic. Ed – Mathematics Education (BSc Hons. Math Ed) |
| 1.8 | Proposed Qualification for Project: MSc Mathematics Education (MST) |
| | (In case of research degree purposes) |
| **2.** | **Contact Details** |
| | Telephone Number: 00-264-65-240259 |
| | Cell Number: 00-264-812780772 |
| | E-Mail: om4rtu@yahoo.com. |
| | Postal address (in the case of students and external applicants): |
| | The Rössing Foundation. |
| | P O Box 479 |
| | Ondangwa. Namibia. |

**3. SUPERVISOR/PROJECT LEADER DETAILS**

| NAME | TELEPHONE NO. | EMAIL | SCHOOL/ INSTITUTION | QUALIFICATIONS |
|---|---|---|---|---|
| 3.1 Professor David Mogari | +27 12 429 3904 | Mogarld@unisa.ac.za | Institute for Science & Technology | B. Sc. (Education)-University of Bophuthatswana (North West University - Mafikeng Campus)<br>B. Sc. (Hons) (Mathematics Teaching);<br>M.Sc.(Mathematics Education);Ph.D.(Mathematics Education)-University of Witwatersrand<br>Certificate in the Basics of Total Quality Management System –University of South Africa |
| 3.2 | | | | |
| 3.3 | | | | |

## SECTION 2: PROJECT DESCRIPTION

Please do *not* provide your full research proposal here: what is required is a short project description of not more than two pages that gives, under the following headings, a brief overview spelling out the background to the study, the questions to be addressed, the participants (or subjects) and research site, including a full description of the sample, and the research approach/ methods

**2.1 Project title:** An investigation into the algebraic problem solving skills of Grade 12 learners in the Oshana Education Region.

**2.2 Location of the study:** Northern Namibia. Oshana Education Region.

**2.3 Objectives and need for the study:**

**Major objectives:** To determine

1. What is the level of development of the algebraic problem solving skills of Grade 12 learners in Oshana Region?
2. What solution strategies do Grade 12 learners in Oshana Region adopt when solving algebraic problems?
3. What difficulties, if any, do Grade 12 learners in Oshana Region encounter when attempting to solve algebraic problems?
4. What causes the difficulties that Grade 12 learners in Oshana Region encounter when solving algebraic problems?

**Need for the study**

The Directorate of National Examinations and Assessment (DNEA, 2010) indicates that performance of learners in Mathematics is not satisfactory. Examiners' reports point to weaknesses in mathematical problem solving skills as the basic cause for learners performing poorly. The purpose of this study therefore is to explore the level of algebraic problem solving skills of Ordinary Level learners in a sample of Grade 12 learners in nine Senior Secondary Schools in the region. It is my hope that this study will expose the learners' common misconceptions in algebra and hence facilitate a platform for critical analysis and review of teaching and learning strategies to enhance the development of problem solving skills of learners in Oshana region.

**(Set out the major objectives and the theoretical approach of the research, indicating briefly, why you believe the study is needed.)**

This study will hopefully:

1. reveal the level of learners' algebraic problem solving skills, hence their readiness to take Mathematics as a compulsory subject by 2012;
2. reveal useful findings to complement the work of the Mathematics task force by giving resourceful information for developing the comprehensive plan for improving mathematics;
3. allow the Ministry of Education to effectively implement the ETSIP programme with specific focus on improving the teaching and learning of Mathematics by 2012 and onwards.

Because my research will involve learner-centred problem solving activities, I have thus aligned myself and my study towards Constructivism as a Paradigm for Teaching and Learning.

**2.4 Questions to be answered in the research: (Set out all the critical questions which you intend to answer by undertaking this research.)**

1. How far have the algebraic problem solving skills of Grade 12 learners in Oshana Region developed?

2. What solution strategies do Grade 12 learners in Oshana Region adopt when solving algebraic problems?

3. What difficulties, if any, do Grade 12 learners in Oshana Region encounter when attempting to solve algebraic problems?

4. What cause(s) the difficulties that Grade 12 learners encounter when solving algebraic problems?

## 2.5 Research approach/ methods

**(This section should explain how you will go about answering the critical questions which you have identified under 2.4 above. Set out the approach within which you will work, and indicate in step-by-step point from the methods you will use in this research in order to answer the critical questions.)**

The research design of this study is a descriptive survey design which has two components, namely, quantitative and qualitative. This will enable me to obtain the necessary data to address the research questions of my study.

**Quantitative Component**

The algebraic problem solving skill level will be assessed in terms of the numerical scores at each stage of the four problem solving steps in accordance with Polya's model. I will develop two diagnostic tests. In the first test I will assess the prerequisite knowledge for solving algebraic problems. Through the second test, I hope to numerically assess the level of skills in solving algebraic problems. In both tests I will document scores learners obtain in total in various parts of the tests as well as compile descriptive statistics of the scores.

**Qualitative Component**

Through this component of my research I will be able to analyse the learners' answers in order to identify and categorise the difficulties they encounter when solving algebraic problems, to establish the extent to which their problem solving skills have developed as well as to determine the possible causes of difficulties learners encounter when solving algebraic problems.

I will pursue the following steps in my data collection process:

1. Administer Diagnostic Tests 1 and 2 as per work plan. This will allow me to answer question 1.
2. Analyse, for a purposive sample of the participants, what solution strategies learners employ in Test 2 (Polya's model) in order to answer questions 2 and 3.
3. Analyse and compare performance in Test 1 and 2 in order to answer question 4.
4. Administer the interview in order to validate the findings from the participants' point of view.

***For a study that involves surveys please append a provisional copy of the questionnaire or interview questions and the consent form to be used. The questionnaire/interview protocol should show how informed consent is to be achieved as well as indicate to respondents that they may withdraw their participation at any time, should they so wish.***

**2.6 Proposed work plan**

Set out your intended plan of work for the research, indicating important target dates necessary to meet your proposed deadline.

| STEPS | DATES |
|---|---|
| Submission of the proposal to my supervisor | 20 March 2010 |
| Design of research instruments | 20 April 2010 |
| Gaining access/permission to work with schools/learners | 5 February 2010 |
| Literature review | Continuous |
| Definition of sampling frame (selection criteria for my sample) | 20 April 2010 |
| Administering and analysis of achievement tests (pilot study) | 30 May 2010 |
| Administering of diagnostic test 1 ( sample group) | 15 October 2010 |
| Administering of diagnostic test 2 ( sample group) | 15 October 2010 |
| Administering and analysis of questionnaire and interview (sample group) | 15 October 2010 |
| Report on findings of Pilot Study | 20 February 2011 |
| Report on findings (main study) | 30 April 2011 |
| Presentation of final research products | 15 June 2011 (subject to prior written and signed statement by me of submission of dissertation) |

| STEPS | DATES |
|---|---|
| Submission of thesis draft to my supervisor | 20 January 2012 |
| Present preliminary draft at ISTE conference | March 2012 |
| Present improved draft at ISTE conference | October 2012 |
| Submission of final thesis | October 2013 |

## SECTION 3: ETHICAL ISSUES

The UNISA Ethics Policy[1] applies to all members of staff, graduate and undergraduate students who are involved in research on or off the campuses of UNISA. In addition, any person not affiliated with UNISA who wishes to conduct research with UNISA students and/or staff is bound by the same ethics framework. Each member of the University community is responsible for implementing this Policy in relation to scholarly work with which she or he is associated and to avoid any activity which might be considered to be in violation of this Policy.
All students and members of staff must familiarize themselves with AND sign an undertaking to comply with the University`s "Code of Conduct for Research" (the policy can be accessed at the following URL:http://cm.unisa.as.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf).

**QUESTION 3.1**

| Does your study cover research involving: | YES | NO |
|---|---|---|
| Children | × | |
| Persons who are intellectually or mentally impaired | | × |
| Persons who have experienced traumatic or stressful life circumstances | | × |
| Persons who are HIV positive | | × |
| Persons highly dependent on medical care | | × |
| Persons in dependent or unequal relationships | | × |
| Persons in captivity | | × |
| Persons living in particularly vulnerable life circumstances | | × |

If "Yes", indicate what measures you will take to protect the autonomy of respondents and (where indicated) to prevent social stigmatisation and/or secondary victimisation of respondents. If you are unsure about any of these concepts, please consult your supervisor/project leader.

Ensure the dignity and well-being of the researched. All information collected to be treated with highest confidentiality. Obtain the written consent of the parents of the research subjects.

**QUESTION 3.2**

| **Will data collection involve any of the following:** | **YES** | **NO** |
|---|---|---|
| Access to confidential information without prior consent of participants | | × |
| Participants being required to commit an act which might diminish self-respect or cause them to experience shame, embarrassment, or regret | | × |
| Participants being exposed to questions which may be experienced as stressful or upsetting, to procedures which have unpleasant or harmful effects | | × |
| The use of stimuli, task or procedures which may be experienced as stressful, noxious, or unpleasant | | × |
| Any form of deception | | × |

[1]The URL for this:

http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

| | | |
|---|---|---|
| Any use of materials harmful to human beings | | × |

If "Yes", to any of the previously mentioned explain and justify. Explain, too, what steps you will take to minimise the potential stress/harm.
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---
---

**QUESTION 3.3**

| **Will any of the following instruments be used for purposes of data collection:** | **YES** | **NO** |
|---|---|---|
| Questionnaire | | |
| Survey schedule | | |
| Interview schedule | × | |
| Psychometric test | | |
| Other/equivalent assessment instrument: Interview after questionnaire | × | |

| |
|---|
| If "Yes", attach copy of research instrument. If data collection involves the use of psychometric test or equivalent assessment instrument, you are required to provide evidence that the measure is likely to provide a valid, reliable, and unbiased estimate of the construct being measured as an attachment. If data collection involves interviews and/or focus groups, please provide a list of the topics to be covered/ kinds of questions to be asked as an instrument. |

**QUESTION 3.4**

| **Will the autonomy of participants be protected through the use of an informed consent form, which specifies (in language that respondents will understand):** | **YES** | **NO** |
|---|---|---|
| The nature and purpose/s of the research | × | |
| The identity and institutional association of the researcher and supervisor/project leader and their contact details | × | |
| The fact that participation is voluntary<br>The responses will be treated in a confidential manner | × | |
| Any limits on confidentiality which may apply | × | |
| That anonymity will be ensured where appropriate (e.g. coded/ disguised names of participants/ respondents/ institutions) | × | |
| The fact that participants are free to withdraw from the research at any time without any negative or undesirable consequences to themselves | × | |
| The nature and limits of any benefits participants may receive as a result of their participation in the research | × | |
| Is a copy of the informed consent form attached? | × | |

If not, this needs to be explained and justified, also the measure to be adopted to ensure that the respondents fully understand the nature of the research and the consent that they are giving.

**QUESTION 3.5**

| **Specify what efforts been made or will be made to obtain permission for the research from appropriate authorities and gate-keepers (including caretakers or legal guardians in the case of minor children?** |
|---|
| For each of the learners in my sample, I have obtained written permission from the parents in the form of a letter of consent, herein attached. The letter to the parents of the learners participating explains the purpose and goals of my research and requests their permission and willingness to assist me with my research by allowing me (1) to observe their child, keep samples of photocopies of his/her work, take photographs/videos to use in the research report and (2) to administer a questionnaire to their child on his/her algebraic problem solving experiences. Furthermore, I received authorization from the Permanent Secretary of Education to conduct my research in Oshana region. My supervisors at work were also informed in writing and they gave me written consent to use some of the facilities at work to carry out my research. |

**QUESTION 3.6**

| **STORAGE AND DISPOSAL OF RESEARCH DATA/SAMPLES:**<br>**Please note that the research data should be kept for a period of at least five years in a secure environmental safe location by arrangement with your supervisor, in case the samples will be destroyed** |
|---|
| **How will the research data be disposed of? Please provide specific information, e.g. shredding of documents incineration of videos, cassettes, etc.**<br><br>Shredding of documents, most of my documents will be hard copy paper documents and/or photos |

**QUESTION 3.7**

| |
|---|
| **In the subsequent dissemination of your research findings – in the form of the finished thesis, oral presentations, publication etc, - how will anonymity/confidentiality be protected?**<br><br>Reference will be made to either pseudonyms or simply learner X, Y etc. to protect the identity of the learners. Once the raw data has been analysed and processed most of the findings will not be specific to any participant, but rather generalised in terms of the identified measurement scales. |

**QUESTION 3.8**

| | | |
|---|---|---|
| **Is this research supported by funding that is likely to inform or impact in any way on the design, outcome and dissemination of the research?** | **YES** | **NO**<br>**×** |

| |
|---|
| **If yes, this needs to be explained and justified.** |

**QUESTION 3.9**

| |
|---|
| **Has any organization/company participating in the research or funding the project, imposed any conditions to the research?** NO **YES/NO** |

| |
|---|
| **If yes, please indicate what the conditions are.** |

## SECTION 4: FORMALISATION OF THE APPLICATION

### APPLICANT

I have familiarised myself with the UNISA Ethics policy, the form completed and undertake to comply with it. The information supplied above is correct to the best of my knowledge.

**NB: PLEASE ENSURE THAT THE ATTACHED CHECK SHEET IS COMPLETED**

.....

**SIGNATURE OF APPLICANT** **DATE :** 25 January 2011

**SUPERVISOR/DIRECTOR OF SCHOOL**

**NB: PLEASE ENSURE THAT THE APPLICANT HAS COMPLETED THE ATTACHED CHECK SHEET AND THAT THE FORM IS FORWARDED TO YOUR INSTITUTE RESEARCH COMMITTEE FOR FURTHER ATTENTION**

**DATE.....**

**SIGNATURE OF SUPERVISOR/PROJECT LEADER: .....**

**RECOMMENDATION OF ISTE RESEARCH AND ETHICS COMMITTEE**

The application is (please tick):

| | |
|---|---|
| | Approved |
| | Recommended and noted |
| | Not Approved, referred back for revision and resubmission |

**NAME OF CHAIRPERSON:_____SIGNATURE:_____**

**DATE: .....**

**RECOMMENDATION OF SENATE RESEARCH AND RTHICS COMMITTEE**

**NAME OF CHAIRPERSON:_____SIGNATURE:_____**

**DATE: .....**

**ETHICAL CLEARANCE APPLICATION FORM**

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UNISA university of south africa

PROF T S MALULEKE
EXECUTIVE DIRECTOR: RESEARCH
☎ : +27 12 429 2970/2446 ☏: +27 12 429 6960
: +27 (0)82 925 5232 ✉: malults@unisa.ac.za
Theo van Wyk Building, 10th Floor, Office no. 50 -52 [TvW 10-50-52]

12 May 2010

Mr N Lapahla
Rossing Foundation
P O box 479
Ondangwa
NAMIBIA

Dear Mr Lapahla

**REQUEST FOR ETHICAL CLEARANCE: An investigation into the algebraic problem solving skills of grade 12 learners in the Oshana Education region**

Your application for ethical clearance of the above study was considered by the Unisa Research Ethics Review Committee on 5 May 2010. The following suggestions and recommendations were identified by the Committee:

(1) The Committee noted on page 7 of your application that you refer to an interview schedule and a psychometric test, but copies of these have not been included in your application.
(2) You state on page 8 of your application that you have not included the relevant consent form, as you are still in the process of finalising access to the identified schools. The Committee cannot grant clearance without perusing a copy of this consent form too.
(3) You state on page 8 that your study will not involve minor children. The Committee wishes to point out that the age of majority in Namibian law is set at 21. It is therefore advisable to obtain the consent of the parents of the relevant children for the purposes of your study too.
(4) A question was raised concerning you revised work plan which indicates that you have administered the tests in February 2010 already. The Committee cannot grant clearance retrospectively and would like to have confirmation that this has not happened already.

We will only be able to grant you ethical clearance once the above suggestions and recommendations have been included in your revised application, which we hope to receive soon.

Kind regards

**PROF T MALULEKE**
**EXECUTIVE DIRECTOR: RESEARCH**

**cc. PROF M N SLABBERT**

University of South Africa
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# CONSENT OF PARENT

I, _____, the parent/legal guardian of
**(Full names of parent/legal guardian)**

_____ grade 12 learner at
**(Full names of child)**

_____ acknowledge receipt of request to allow my
**(Name of School)**

child to be the subject of the research being conducted by **NHLANHLA LUPAHLA** as explained in the letter.

I [agree to allow ✓] [do not agree to allow] my child to be subject of the research and that

**(Tick in the appropriate box)**

**Mr Nhlanhla Lupahla** may unconditionally:

1. observe my child
2. keep samples of photocopies of his/her work
3. take photographs/videos to use in the research report
4. administer 2 tests on algebraic problem solving
5. administer a written interview on my child's algebraic problem solving experiences

All the information collected shall be treated as confidential and the dignity and well-being of the researched learners shall be ensured.

Kindly sign and return this letter to the school on or before the 30th of September 2010.

.....

Signed at _____ on this 07 day of
**( Parent/Guardian's physical location/address)** **(date)**

October, 2010.
**(month)**

**FOR OFFICE USE ONLY**

_____
_____
_____
_____
_____

TRUSTEES: R Hoveka (Chairman)*J Tjiho (Director) *T.K. Alweendo *M.A.T. Gebhardt *M.D. Leech *E.P. Shiimi *A Kapere* W v Rooyen D Nuuyoma* T. Kadhikwa

**APPENDIX 8: RESEARCH ADVISORS TABLE FOR SELECTING SAMPLE SIZE**

**Required Sample Size†**

| Population Size | Confidence = 95% | | | | Confidence = 99% | | | |
|---|---|---|---|---|---|---|---|---|
| | Margin of Error | | | | Margin of Error | | | |
| | 5.0% | 3.5% | 2.5% | 1.0% | 5.0% | 3.5% | 2.5% | 1.0% |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 20 | 19 | 20 | 20 | 20 | 19 | 20 | 20 | 20 |
| 30 | 28 | 29 | 29 | 30 | 29 | 29 | 30 | 30 |
| 50 | 44 | 47 | 48 | 50 | 47 | 48 | 49 | 50 |
| 75 | 63 | 69 | 72 | 74 | 67 | 71 | 73 | 75 |
| 100 | 80 | 89 | 94 | 99 | 87 | 93 | 96 | 99 |
| 150 | 108 | 126 | 137 | 148 | 122 | 135 | 142 | 149 |
| 200 | 132 | 160 | 177 | 196 | 154 | 174 | 186 | 198 |
| 250 | 152 | 190 | 215 | 244 | 182 | 211 | 229 | 246 |
| 300 | 169 | 217 | 251 | 291 | 207 | 246 | 270 | 295 |
| 400 | 196 | 265 | 318 | 384 | 250 | 309 | 348 | 391 |
| 500 | 217 | 306 | 377 | 475 | 285 | 365 | 421 | 485 |
| 600 | 234 | 340 | 432 | 565 | 315 | 416 | 490 | 579 |
| 700 | 248 | 370 | 481 | 653 | 341 | 462 | 554 | 672 |
| 800 | 260 | 396 | 526 | 739 | 363 | 503 | 615 | 763 |
| 1,000 | 278 | 440 | 606 | 906 | 399 | 575 | 727 | 943 |
| 1,200 | 291 | 474 | 674 | 1067 | 427 | 636 | 827 | 1119 |
| 1,500 | 306 | 515 | 759 | 1297 | 460 | 712 | 959 | 1376 |
| 2,000 | 322 | 563 | 869 | 1655 | 498 | 808 | 1141 | 1785 |
| 2,500 | 333 | 597 | 952 | 1984 | 524 | 879 | 1288 | 2173 |
| 3,500 | 346 | 641 | 1068 | 2565 | 558 | 977 | 1510 | 2890 |
| 5,000 | 357 | 678 | 1176 | 3288 | 586 | 1066 | 1734 | 3842 |
| 7,500 | 365 | 710 | 1275 | 4211 | 610 | 1147 | 1960 | 5165 |
| 10,000 | 370 | 727 | 1332 | 4899 | 622 | 1193 | 2098 | 6239 |
| 25,000 | 378 | 760 | 1448 | 6939 | 646 | 1285 | 2399 | 9972 |
| 50,000 | 381 | 772 | 1491 | 8056 | 655 | 1318 | 2520 | 12455 |
| 75,000 | 382 | 776 | 1506 | 8514 | 658 | 1330 | 2563 | 13583 |
| 100,000 | 383 | 778 | 1513 | 8762 | 659 | 1336 | 2585 | 14227 |
| 250,000 | 384 | 782 | 1527 | 9248 | 662 | 1347 | 2626 | 15555 |
| 500,000 | 384 | 783 | 1532 | 9423 | 663 | 1350 | 2640 | 16055 |
| 1,000,000 | 384 | 783 | 1534 | 9512 | 663 | 1352 | 2647 | 16317 |
| 2,500,000 | 384 | 784 | 1536 | 9567 | 663 | 1353 | 2651 | 16478 |
| 10,000,000 | 384 | 784 | 1536 | 9594 | 663 | 1354 | 2653 | 16560 |
| 100,000,000 | 384 | 784 | 1537 | 9603 | 663 | 1354 | 2654 | 16584 |
| 300,000,000 | 384 | 784 | 1537 | 9603 | 663 | 1354 | 2654 | 16586 |

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