0EVALUATING THE EFFECTIVENESS OF THE USE OF INFORMATION AND COMMUNICATION TECHNOLOGY IN THE TEACHING AND LEARNING OF TRIGONOMETRY FUNCTIONS IN

Grade 12

By

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ABSTRACT

The high school pass rate, for mathematics, in South Africa is very low. This is particularly so in trigonometry functions. One of the possible factors leading to this is the traditional method of teaching and learning. This study was undertaken to determine whether the use of Information and Communication Technology (ICT) would influence students' learning of trigonometry functions. In order to answer this question the teaching and learning instructions developed were based on activity theory (AT) and action, process, object, and schema (APOS) theory. The study followed a non-equivalent control group, quasi-experimental design with a pre- post-test approach. Since it was not possible to randomly select participants for the study, intact groups were used. There were two groups: a control and an experimental one. Both groups wrote a standardized achievement pre-test to establish their comparability at the beginning of the study. While the control group was taught in the traditional way (grade 10-12 syllabus), the experimental group used the software Geogebra. The computer software (Geogebra) and the South African grade 10-12 syllabus for trigonometry functions were used during the lessons of the experimental group. At the end of the study, a similar post-test was administered on both groups to measure the comparative effects of either of the teaching methods on the performance of students. A t-test independent sample statistical analysis was performed on the findings using a statistics package, SPSS. The results of this investigation indicated that the use of the computer software, Geogebra, in the teaching and learning of trigonometry functions improved the performance of the Grade 12 students.

Key Terms:

Mathematics; Trigonometry functions; Geogebra; Information and Communication Technology; Positivism; Pre- Post-test

DECLARATION

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Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometry functions in grade 12

I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

02/10/2017

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SIGNATURE DATE

NM MOSESE

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I would also like to thank all the people who participated in this study.

DEDICATION

This work is dedicated to my 5 children

Tafadzwa

Amani

Tokoloho

Namara

And

Bangiwe

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LIST OF ABBREVIATIONS

AT: Activity theory

APOS: Activity, Processes, Objects and Schema

CAPS: Curriculum Assessment Policy Statement

CAS: Computer Algebra System

DBE: Department of Education

DGS: Dynamic geometry software

DMS: Dynamic mathematics software (e.g. Geogebra)

E-Education: Electronically supported education

EMDC: Education Management and Development Centre

FET: Further Education and Training

GET: General Education and Training

HSRC: Human Science Research Council

ICT: Information and Communication Technology

NCS: National Curriculum Statement

TIMMS: Trends in International Mathematics and Science Studies

SME: Subject matter Experts

SPSS: Statistical Product and Service Solutions

CHAPTER ONE

BACKGROUND AND OVERVIEW OF THE STUDY

In this study, Information and Communication Technology (ICT) was introduced into the teaching and learning of trigonometry functions in order to evaluate its effectiveness compared to the traditional method of instruction (chalk and talk) in the classroom.

This chapter starts with the background of the study, which includes the students' performance in mathematics in South Africa, the problem of students' poor performance in trigonometry functions, a brief overview of previous research and the teaching methods used in classes. This is followed by the statement of the problem, the context within which the study took place, the rationale for the study conducted, and the significance of the study are then discussed. The research questions and the null hypotheses are developed. Limitations of the study are then pointed out. The chapter ends with an overall structure of the thesis.

1.1 BACKGROUND OF THE STUDY

Mathematics is the foundation of scientific and technological knowledge that contributes to personal and socio-economic development of a nation (DBE, 2003; Mbugua, Kibet, Muthaa, & Nkonke, 2012). Mathematics is also used as an entry requirement into many of the tertiary disciplines such as medicine and engineering (Weber, 2005). Despite the important role that Mathematics plays in society, very few studies are carried out to understand students' attitudes and performance in it. Recent students' performance assessment analyses of the Matric (grade 12) examination show that many students in South Africa performed extremely poorly in the results for the period between 2011 to 2015 (DBE, 2011, 2014, 2015). This was especially so in trigonometry functions.

Trigonometry is a branch of mathematics that deals with the relationships of sides and angles in triangles (Laridon, Jawurek, Kitto, Pike, Myburgh, Rhodes-Houghton, & van Rooyen, 2002; Orhun, 2010). Trigonometry is also seen as one of the important subjects in the high school curriculum which requires an integration of algebraic, geometric and graphical reasoning (DBE, 2009; Stols, 2011; NCTM, 2012). The connection of all these contexts is important for the comprehension of trigonometry functions.

In South African schools, trigonometry functions are taught in grades 10, 11 and 12. The Matric examination is written at the end of grade 12 and mathematics is a requirement for admission to many programs at the tertiary level (college and university level).

Table 1.1 shows that the general performance of students declined in 2015 to 49.1% from previous perfomances which were above 50% for the candidates achieving 30% and above. While the performance in 2015 for candidates who achieved 40% and above was down to 31.9% compared to 35.1% previously.

Table 1.1: Overall Achievement Rates in Mathematics (DBE, 2015)

YEAR	**No. wrote**	**No. achieved at 30% and above**	**% achieved at 30% and above**	**No. achieved at 40% and above**	**% achieved at 40% and above**
2012	225874	121970	54.0	80716	35.7
2013	241509	142666	59.1	97790	40.5
2014	225458	120523	53.5	79050	35.1
2015	263903	129461	49.1	84297	31.9

In addition, the National Senior Certificate Examination 2015 Diagnostic report (DBE, 2015) presented a general overview of the problems that were detected in the paper with trigonometry (Mathematics Paper II). Some of the problems are indicated below:

- It is evident that many of the errors/misconceptions made by candidates in answering Paper II have their origins in a poor understanding of the basics and foundational competencies taught and learnt in the earlier grades (DBE, 2015). This is confirmed by Blackett and Tall (1991) who state that the initial stages of learning trigonometry functions are filled with difficulty.
- A lack of knowledge that a trigonometry ratio is equal to some numeric value;
- The inability to recall reduction formulae and all other trigonometry identities;

- The inability to relate angles in a diagram and the inability to provide justification for statements.
- The item-by-item analysis revealed that many candidates were mostly familiar with routine type questions. Candidates showed confidence in dealing with work that they had seen previously. They struggled with concepts in the curriculum that required deeper conceptual understanding.
- Questions where candidates had to interpret information or provide justification, presented the most challenges. This is in agreement with (Zaslavsky, 2008) who says that students have difficulty in interpreting graphs of functions.

The problems stated in the Deapartment of Basic Education Diagnostic report (2015) have also been variously described in other articles where it is said students have problems with:

- The process-object duality: students often find it difficult to perceive a mathematical concept both on an operational process and a structural object side of it (Gray & Tall, 1994).
- Connections: students have difficulties in making connections between the representations of the concepts (Brown, 2005; Weber, 2005; Challenger, 2009).
- Variables: students often misunderstand the concept of variables and how variables help them construct mathematical meanings (Graham & Thomas, 2000).

The main reasons related to these problems can be attributed to the conventional method of teaching and learning of trigonometry functions. Firstly, as stated above, the initial stages of learning trigonometry functions are filled with difficulty (Blackett & Tall, 1991). This is because students are normally introduced to trigonometry function using definitions as the ratios of the lengths of sides in a right angled triangle (Pritchard & Simpson, 1999). Secondly, trigonometry functions are often taught as a completely mechanical series of routines, without engaging students in any non-routine mathematical thinking (Breidenbach, Dubinsky, Hawks, & Nichols, 1992). This could give the impression that the numerical procedures are the only ways to get accurate results causing a possible disconnection between the use of different contexts (right angled triangles, the unit circle and the graphs) of trigonometry functions (Blackett & Tall, 1991). This is in agreement with Moore (2009) who stated that in order to address this dilemma, firstly, trigonometry should be taught with the emphasis on the connections of their contexts as opposed to the current situation (traditional form of teaching) where it is presented as separate contexts of right angled triangles, the unit circle and the graphs. Moreover, the traditional form of teaching is teacher-focused, is also based on rote-learning and memorisation (Molefe & Brodie, 2010; Addelman, 2012). The above stated problems that students face will continue unless there are possible interventions that educators adopt to replace or augment the traditional form of instruction in classrooms (DBE, 2013). Information and Communication Technology (ICT) is proposed here as such an intervention that can be used to facilitate the teaching and learning of trigonometry functions.

ICT has become an essential tool in the teaching and learning of mathematics, especially in understanding mathematical concepts (Hohenwarter & Jones, 2007) and in helping improve students' marks on standardised tests (Bain & Ross, 1999). ICT can facilitate mathematical problem solving, communication, as well as reasoning and proof. ICT can also provide students with opportunities to explore different representations of mathematical ideas and support them in making connections/relationships among different representations of mathematics (Kaput, 1992; NRC, 2000).

In line with the above, many governments have embarked on information technology-related programmes being developed for integration into school curricula (Hew & Brush, 2007). In South Africa, for example, a government policy was introduced in 1994 to ensure that all schools have access to computers (DBE, 2004; Mdlongwa, 2012). This policy has recorded some progress over the years. Projects such as the Khanya Project of the Western Cape (Western Cape Education Department & Africa, 2001), the 'Gauteng Online' in Gauteng, as well as the Northern Cape's Connectivity Project have come up (DBE, 2004). In addition, by 2007, approximately 22 000 educators had been trained to use computers and an educational portal known as Thutong, giving educators access to a range of quality curricula, learner support material, and professional development programmes was introduced by the South African government (Pandor, 2007). Also, computers (some with internet access) were donated to some schools across the country (Pandor, 2007; Mdlongwa, 2012).

In this digital/information era, students are constantly exposed to and actively involved in the use of ICT in their everyday lives (Lopez-Morteo & Lopez, 2007). Research shows that many students exposed to ICT advocate for its integration into mathematics teaching and learning (De Villiers, 2004) and seem to be more motivated to learn (Tall, 2000; Shelly, Cashman, Gunter, & Gunter, 2008). Thus, effective integration of ICT in teaching and learning is crucial and non-avoidable. Over the past few years, several studies have documented successful integration of ICT in teaching and learning (Wilson-Strydom & Thomson, 2005; Ogbonnaya, 2010). In South Africa, many studies have been conducted on the use of ICT in the teaching and learning of mathematics (Jaffer, Ng'ambi, & Czerniewicz, 2007; Ogbonnaya, 2010; Stols, 2011). From the review of literature carried out, it was found that no study, in South Africa, focused specifically on the use of ICT in the teaching and learning of trigonometry functions. Thus, this study focused on assessing the effectiveness of using ICT in the teaching and learning of trigonometry functions at two South African schools while the instruction focused on the three contexts and representations (the right angled triangles, the unit circle and the graphs) as well as the connections among them.

1.2 STATEMENT OF THE PROBLEM

As a researcher with many years of teaching grade 12 mathematics, witnessing the students' continual low performance in mathematics, with the students always complaining of the difficulty they experience in comprehending trigonometry functions, the researcher has been prompted to investigate possible alternative ways of teaching trigonometry functions.

In 2011, in the international mathematics performance assessments of Trends in International Mathematics and Science Studies (TIMMS), South Africa was the second lowest performing country with an average scale score of 352 while the international average was 500 (TIMSS, 2011). Even though by 2015 South Africa's scale score improved to 372, it was still among the lowest performing countries

(Reddy *et al.*, 2016). The problem of performance in mathematics is more apparent in the matric results of 2015, which were at the lowest in 4 years. (See Table1.1).

The National Senior Certificate Examination (2015) diagnostic report stated that some of the problems that were reported in trigonometry functions include the following:

- Students found the interpretation of trigonometry graphs to be very challenging. A number of students were unable to identify which was the sine function and which was the cosine function in the sketches.
- It is evident that students were not aware of the transformations that are applied to the basic trigonometry functions and how these transformations impact on the equation of a trigonometry function.
- Many students were able to correctly identify the critical values for the required interval but used the incorrect notation. They included the endpoints when the question required determining the interval for which $f(x) > g(x)$.
- Many students could not describe the transformation (DBE, 2015).

ICT has been said to be an intervention that can hasten the students' learning of functions and their properties (Demir, 2012). Thus, the study investigated the effectiveness of the use of ICT in the teaching and learning of trigonometry functions to grade 12 students. This study also provided an integrated method of introducing trigonometry functions by addressing the three trigonometry functions contexts and representations (the right angled triangles, the unit circle and the graphs) and the connections among them.

1.3 RATIONALE FOR THE STUDY

It has been said that ICT use promotes cooperative learning and students' active participation (Rodrigo, 2001). It also reduces the load from teachers and encourages creative learning from students. This leads to educators having more time to work with students individually and in groups. This has been of interest to me as a grade 12 educator considering the large number of students we have to deal with in the classrooms on a daily basis. This could enable educators to spend more time assisting students individually.

Students spend more time on their mobile phones (a form of ICT) than on books. This is why in many school policies, mobile phones are not allowed in the school premises. However, because of the students' preference for ICT, ICT could be used to channel academic work. This is the reason why the DBE, together with various donor institutions, have provided computers and tablets to schools to be used for teaching and learning of various subjects.

1.4 SIGNIFICANCE OF THE STUDY

This study is significant for two main reasons. Firstly, the study was inspired by the need to find an alternative approach to teaching mathematics so as to improve students' performance. It attempts to determine an effective way of teaching and learning trigonometry functions. It does this by using a method of teaching trigonometry functions which is based on addressing the three contexts of trigonometry and the connections among them. ICT is used as a mediating tool (see Figure 2.3) in the teaching and learning process.

Secondly, only a few studies have dealt with evaluating the effectiveness of using ICT in teaching and learning of trigonometry functions, although it has often been reported as a difficult topic for students (Brown, 2005; Weber, 2005; Demir, 2012). Since research on the use of ICT in the teaching of trigonometry in the classroom is sparse and quite limited, this study addresses that gap.

1.5 RESEARCH QUESTIONS

The main question addressed by this study is: what is the effect of ICT on students' learning of trigonometry functions? To answer the main question the following sub-questions were posed:

I. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students making connections between representations of trigonometry functions?
II. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students' analysis and interpretations of trigonometry functions?

III. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students' learning of transformation of trigonometry functions?

IV. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students' derivation of general and specific solutions of trigonometry functions?

V. Does the teaching and learning of trigonometry functions with ICT have an effect on students' (creation and learning of) proofs of trigonometry functions identities?

1.6 HYPOTHESES

The following null hypotheses were formulated

Null Hypothesis (H_0)1: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students making connections on multiple representations of trigonometry functions.

Null Hypothesis (H_0)2: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference in the analysis and interpretations of trigonometry functions.

Null Hypothesis (H_0)3: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students' learning of transformation of trigonometry functions.

Null Hypothesis (H_0)4: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students' derivation of general and specific solutions of trigonometry functions.

Null Hypothesis (H_0)5: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students (creation and learning of) proofs of trigonometry functions identities.

1.7 SCOPE AND LIMITATIONS OF THE STUDY

The current research was conducted in two schools with a total of 305 mathematics grade 12 students. The schools are located in the Ngaka Modiri Molema district in Mafikeng in the North West province. Of the 305 students only 61 students, whose classes were allowed to participate in the study from the two schools, took part in the study. A total of 34 students were in the control group and 27 were in the experimental group. The choice of the schools was due to their availability and proximity to the researcher.

There were limitations to the study.

- The research did not consider the qualification of the teachers involved in the study;
- The research was a small–scale study, thus the results could not be generalised beyond the schools;
- Since the students had to prepare for final national examinations, the time allotted for the study was very short.

1.8 DEFINITION OF TERMS

Computer Algebra System (CAS): A computer software package comprising a set of algorithms for performing symbolic manipulation on algebraic objects, a language to implement them, and an environment in which to use the language.

Dynamic Geometry Software (DGS): A dynamic geometry (DG) program is a computer program for interactive creation and manipulation of geometric constructions. A characteristic feature of such programs is that they build a geometric model of objects, such as points, lines, circles, etc., together with the dependencies that may relate the objects to each other. The user can manipulate the model by moving some of its parts, and the program accordingly – and instantly – changes the other parts, so that the constraints are preserved (Bantchev, 2010).

Dynamic mathematics software (DMS): This is a software that combines DGS with some of the features of CAS and therefore, allowing for bridging the gap between the mathematical disciplines of geometry, algebra, and even calculus (Hohenwarter & Jones, 2007).

Functions: these are relations between sets of inputs and sets of permissible outputs, with the property that each input is related to exactly one output

Geogebra: Geogebra is a DMS, that is, it is both a computer algebra system (CAS) and dynamic geometry software (DGS) because it includes both symbolic and visualization features related to coordinates, equations and functions, along with geometric concepts and dynamic relations (Zengin, Furkan, & Kutluca, 2012).

Information and Communication Technology (ICT): ICT refers to an umbrella term that includes any communication device or application, encompassing: radio, television, calculators, cellular phones, computer and network hardware and software, satellite systems and so on as well as the various services and applications associated with them, such as video-conferencing and distance learning.

Learning: This is the act of acquiring new, or modifying and reinforcing existing knowledge, behaviours, skills, values, or preferences and may involve synthesizing different types of information.

Mathematics: is a language that makes use of symbols and notations for describing algebraic, geometric and graphical relationships. It is a human activity that involving observing, representing and investigating patterns relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, economic and social) around us, and, most of all, to teach us to think creatively (DBE, 2011).

Matric: The National Senior Certificate (NSC) examinations commonly referred to as "matric" has become an annual event of major public significance. It not only denotes the finale of twelve years of formal schooling, but also reflects the nature of the national academic aptitude.

Nonequivalent control group design: An experimental design involving at least two groups, both of which may be pretested: one group receives the experimental treatment, and both groups are post-tested. Individuals are not randomly assigned to treatments.

Teaching: this is the process of attending to people's needs, experiences and feelings, and making specific interventions to help them learn particular things.

Trigonometry (Trigonometry): Trigonometry functions describe relations between sides of triangles and the angles between the sides. Trigonometry functions developed as a branch of geometry. The branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.

1.9 STRUCTURE OF THESIS

Chapter 1 provides an overview of the research study. This chapter provides the introduction and the background to the research, the problem statement, the rationale and significance of the study, the research questions, clarification of abbreviations and the terminology used in the study.

Chapter 2 gives the conceptual framework which provides a basis and direction for the whole research study, as well as the review of the literature relevant to the study. This starts with the descriptions of APOS theory and the Activity theory. Both of these theories are used in the design and implementation of teaching and learning in the classroom using ICT (Geogebra software in this study). Geogebra and trigonometry are important aspects of the study, therefore theoretical considerations of both are provided. The chapter ends with a review of literature on ICT in teaching and learning of Mathematics and ICT in the teaching and learning of trigonometry functions.

Chapter 3 discusses the methodology that is the approach that is taken in conducting the current study. This therefore includes the paradigm, which informed the study, the research design and the research methods (e.g. the study sample and population, the procedure for conducting the research and the research instruments, the different instructional methods used in the experimental and control groups, data analysis and the ethical considerations).

Chapter 4 This Chapter presents the findings of the study. The statistical package SPSS is used to analyze the data collected from the two instruments, (pre-test and post-test) on the two groups. The descriptive and inferential statistical analyses are determined and interpreted in order to respond to the research questions.

Chapter 5 This gives the summary to the findings, a discussion of the findings, a conclusion and recommendations from the study.

1.10 SUMMARY OF THE CHAPTER

In this chapter the need for a new or alternative method of teaching and learning trigonometry functions was highlighted. From that need, an alternative method integrating ICT, based on AT and APOS was presented. Reasons for carrying out this research were explained in the background and rationale for the study. A brief literature review was given in the background. The statement of the problem and the significance of the study followed. The questions and hypothesis to be tested during the study were then posed and stated. Lastly, an outline of the thesis chapters was given.

CHAPTER TWO

THEORETICAL FRAMEWORK AND REVIEW OF LITERATURE

This chapter presents the theoretical basis of the study. Two theoretical frameworks: the Action, Process, Object and Schema (APOS) together with the Activity Theory (AT) are discussed separately and their integration in and for the study follows. These frameworks acted as the basis for constructing the classroom environment, the instructional process and sequence as well as how the study was implemented. The chapter also provides a review of literature on studies relating to ICT in teaching and learning.

2.1 THEORETICAL FRAMEWORKS

A large number of theoretical frameworks from different paradigms exist and are being used for research in mathematics and other fields to inform the way in which learning occurs (NRC, 2001, 2009; Miller, 2011; NCTM, 2014). This affects the way research on teaching and learning, as well as the results thereof can be conceptualised. Integrating the view points from the different theoretical frameworks has the potential to explain what a singular perspective may not, thereby giving more insight into how students learn and may thus assist in improving the mathematics classroom interaction (Arzarello, Bosch, Lenfant, & Prediger, 2007).

The two theoretical frameworks: Action, Process, Object and Schema (APOS) (a constructivist theory) and the Activity Theory (AT) (a socio-cultural and socio historical theory) were chosen for this study. Both of these theories are prominent in mathematics and ICT education research (Dubinsky & McDonald, 2001; Russell, 2002). These theories are discussed separately in sections 2.1.1 and 2.1.2 and then brought together in section 2.1.3 to show their possible roles in this study.

2.1.1 APOS THEORY

The APOS theory is a general theory of mathematical conception and its acquisition. The theory focuses on the mental constructions (abstractions) that can be made by the students during instructional phases in their attempt to understand mathematical concepts (Dubinsky & McDonald, 2001). Basic to APOS theory is that all mathematical conception can be understood as actions, processes, objects and schemas (APOS). These are mental structures that students use to make sense of a given mathematical concept (Bowie, 2000).

The four components of APOS are defined below:

- Action: an action is a repeatable physical or mental manipulation that transforms objects. In the case of a trigonometry function, for example, an action conception is shown by a learner evaluating a function at a point.
- Process: a process is an action that takes place entirely in the mind.
- Object: the distinction between a process and an object is drawn by stating that a process becomes an object when it is perceived as an entity upon which actions and processes can be made.
- Schema: a schema is a more or less rational collection of cognitive objects and internal processes for manipulating these objects. A schema could aid students to comprehend, deal with, organize, or make sense out of a perceived problem situation (Dubinsky, 1991).

APOS theory has its source in the Piaget's constructivist theory (Arnon, Cottrill, Dubinsky, Oktaç, Fuentes, Trigueros, & Weller, 2014). Underlying Piaget's theory were three modes of abstraction: empirical abstraction, pseudo-empirical abstraction, and reflective abstraction (Dubinsky, 1991). APOS grows largely from Dubinsky's attempt to elaborate on Piaget's notion of the latter (Dubinsky, 1991). In fact, Piaget regarded the acquisition of mathematical cognition to be associated with reflective abstraction (Brijlall & Maharaj, 2004). Reflective abstraction is understood as the construction of logico-mathematical structures by a learner during the process of cognitive development (Dubinsky, 1991). The five types of construction in reflective abstraction as explained by Dubinsky (1991) are:

- Interiorization: the ability to apply symbols, language, pictures and mental images to construct internal processes as a way of making sense out of perceived phenomena. Actions on objects are interiorized into a system of operations;
- Coordination: two or more processes are coordinated to form a new process;
- Encapsulation: the ability to conceive a previous process as an object;
- Generalization: the ability to apply existing schema to a wider range of contexts;

- Reversal: the ability to reverse thought processes of previously interiorized processes

The main mental mechanisms for building mental constructions (structures) according to APOS are called interiorization and encapsulation (Dubinsky, 2010). An action conception can become a process conception through a mental mechanism called interiorization. Then, the student can think about the result of the process without actually having done it and, in particular, can imagine reversing the process. A student who has an object conception of a mathematical idea can imagine it in its entirety and, in particular, can act on it with higher-level actions or processes. Processes can be encapsulated into objects, and it is sometimes useful if the student is able to de-encapsulate an object to focus on the underlying process. Schemas are coordinated collections of actions, processes, objects, and other schemas, which can themselves be encapsulated into objects (Arnawa, Sumarno, Kartasasmita, & Baskoro, 2007).

Asiala et al., (1996) proposed a specific framework for APOS: theory-centered research and curriculum development. The framework consists of the following three components: theoretical analysis; instructional treatment based on this theoretical analysis; collection and analysis of data to test and refine the theoretical analysis and instruction (Dubinsky & McDonald, 2001) in Figure 2.1. The theoretical analysis occurs relative to the researcher's knowledge of the concepts in the research problem and knowledge of APOS Theory (Asiala, Brown, De Vries, Dubinsky, Mathews, & Thomas, 1996).

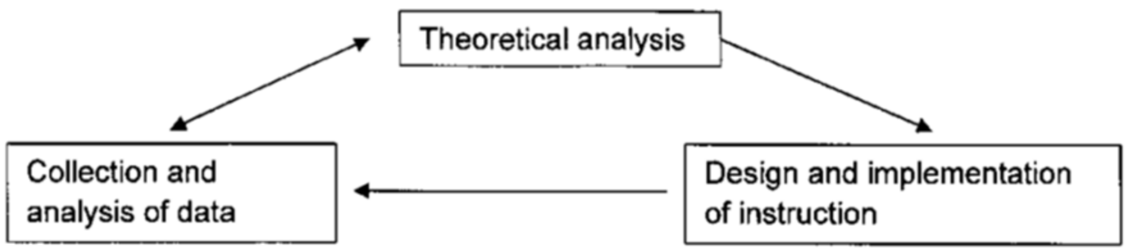


Figure 2.1: General Framework for APOS Research (Adapted from Dubinsky & McDonald, 2001).

This theoretical analysis helps to predict the mental structures that are required to learn the trigonometry functions concept. For a given mathematical concept, the theoretical analysis informs the design and implementation of instruction. These are then used for collection and analysis of data. The theoretical analysis guides the design and implementation of the teaching and learning of trigonometry functions, as indicated in Table 2.1.

Table 2.1: APOS as it pertains to this study

Cognitive Level	Characterisations
Action	• Utilising discrete points of a function: plotting, reading & projecting. • Performing graphs of functions in an analytical context by substituting values in them one by one and by operating the graph of the function based on the evaluation of independent points • Using ICT to type in equations to make the graphs
Process	• Comparing coefficients or algebraic terms • Investigating graphs, making predictions from previous graphs and trying them on the software
Object	• Describing and relating properties or behaviour of functions in terms of comparing shapes or contours and looking at a number of graphs several times • Interpreting and relating parts of algebraic expressions or equations • Knowing the rules and properties, students can describe how they transform functions and predict how functions are transformed by looking at the graphs of transformed functions • Making conclusions of the properties of graphs relating to different equations • Making use of properties of functions through ICT
Schema	• Linking graphic and symbolic forms to construct a precise symbolisation for the information available in the given graph • Having the whole understanding of the concept of how all multiple representations of functions link together • Flexibly using ICT to present their concept of function

Adapted from: Lu Yu-Wen, 2008

2.1.2 ACTIVITY THEORY (AT)

Activity Theory (AT) was proposed by Engeström (1987) and is based on the cultural-historical psychology of Vygotsky. The conception is that a child never approaches the world empty, rather, every experience the child has is mediated through socio-cultural tools. He adds that, humans use tools to change the world and are also transformed through the use of tools (Vygotsky, 1978). This is illustrated in figure 2.2. This theory is rooted in Kant and Hegel's philosophy which emphasises both the historical development of ideas and the active and constructive role of the human mind (Kuutti, 1995).

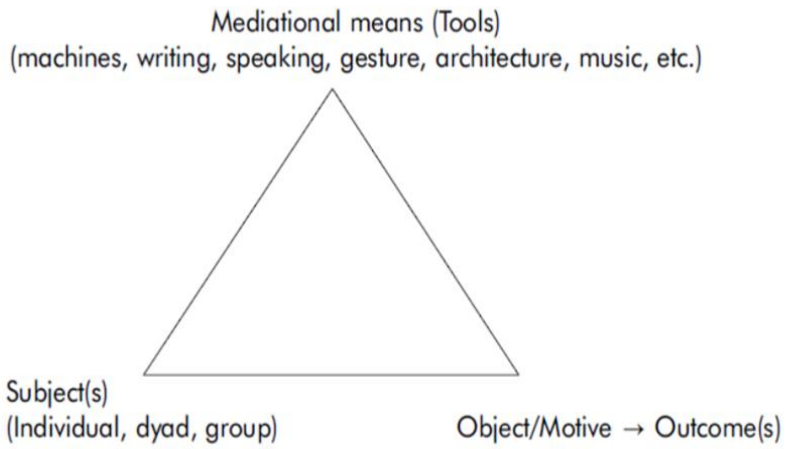


Figure 2.2: Basic Vygotskian triangular representation of mediation (Adapted from: Vygotsky, 1978)

This, however, lacks in articulating the roles and relationship between the individual and his or her environment in an activity (Hardman, 2005).

AT expands on this to accommodate the socio-cultural structure (elements of the community, their rules and the division of labour). The Activity Theory is a philosophical and multidisciplinary framework that can be used to study all forms of human actions and practices (Spasser, 1999). It is a psychological theory of human consciousness, thinking and learning (Miettinen, Samra-Fredericks, & Yanow, 2009). Activity theory postulates that conscious learning arises from activity, not as an originator of it.

The activity theory cannot be understood or analysed outside the context in which it occurs. So when analysing human activity, one should examine not only the kinds of activities that people engage in but also who is engaging in that activity, the rules and norms that circumscribe the activity, and the larger community in which the activity occurs (Engeström, 1987). These form parts of the main structure (the Activity System) as shown in Figure 2.3.

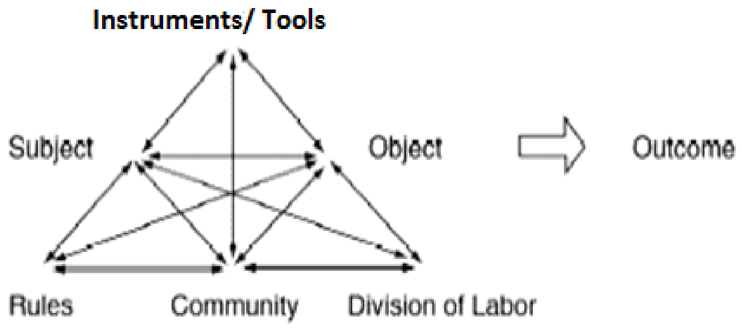


Figure 2.3: Components of the Activity System (Adapted from: Engeström, 1987)

As shown in Figure 2.3, all the components of AT influence each other. For example, the object is transformed in the course of activity, and it in turn transforms the activity. Tools alter the activity that they are in and are in turn transformed by the activity. These are also briefly described and explained in Table 2.2.

TABLE 2.2: General description of the components of AT

AT CONCEPTS	DESCRIPTIONS
SUBJECT	An individual or individuals acting on an object
INSTRUMENTS/TOOLS	Anything used in the transformation process
OBJECT	The physical or mental product that is sought. The object is acted upon by the subject. It represents the intention that motivates the activity
COMMUNITY	Consists of the interdependent aggregates who share (at least to some degree) a set of social meanings.
RULES	Inherently guide the actions acceptable to the community, so the tools that the community use will mediate the process
DIVISION OF LABOUR	Prescribes the task specialization by individuals members in a community. That is, While sharing and acting on an object there are different roles played by the community and the subject. These different roles fall under division of labour.
OUTCOME	form of instruction that is developed and implemented

Adapted from: Hardman, 2008

An activity (Activity System) consists of a goal-directed hierarchy of actions that are used to transform the object (Jonassen & Rohrer-Murphy, 1999). Activities comprise of chains of actions, while actions, in turn consist of sets of operations as shown in Figure 2.4. The relationship between activities, actions, and operations is dynamic.

For example, all operations are actions when first performed because they require conscious effort to perform. With practice and internalization, activities collapse into actions and eventually into operations, as they become more automatic, requiring less conscious effort. In reverse, operations can be disrupted and become actions (Jonassen & Rohrer-Murphy, 1999).

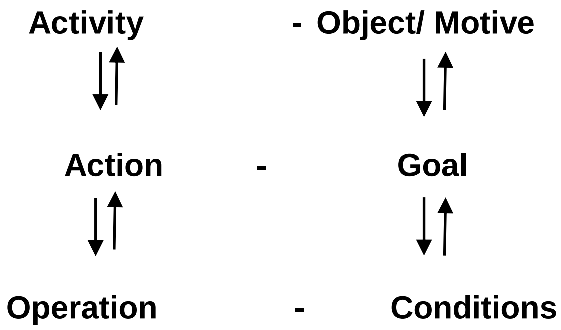


Figure 2.4: Activity, Actions and Operations (Adapted from: Jonassen & Rohrer-Murphy, 1999)

Table 2.3 below shows AT as it applies in this study.

Table 2.3: Elements of the Activity Theory in Trigonometry functions

Element	An example of the element
Subject or actor	Student
Outcome	What students can show at the end of the actions
Object	Trigonometry equations, expressions, graphs etc.
Tools	Geogebra, symbols, mathematics statements and questions
Rules	Computer-lab rules, school rules, rules as applicable to Geogebra
Community	Educators, and students.
Division of labour	Educators act as instructors and facilitators, students as enquirers

2.1.3 INTERGRATING THE TWO THEORETICAL PERSPECTIVES

Theoretical approaches can be connected in various ways and degrees: from complete integration to extreme mutual exclusion (Arzarello et al., 2007). However, at the same time, whatever the choice, the pluralism of independent theories should be respected (Arzarello et al., 2007). In addition, it has been suggested that characteristics of different and relevant theoretical perspectives can be merged to complement each other but still maintain their own identities (Pegg & Tall, 2005). As such APOS and AT were brought together in this study and their individuality was still retained.

AT is seen as a strong framework in analysing needs, tasks and outcomes for designing the constructivist learning environments. This is because the assumptions of AT are in agreement with those of constructivism, social cognition and situated learning (Jonassen & Rohrer-Murphy, 1999).

In addition, APOS theory is a prescriptive model for constructing teaching-learning units and a descriptive model for determining and analyzing the success or failure of students on a task in relation to their specific mental constructions (Cottrill, Nichols, Schwingendorf, Thomas, & Vidakovic, 1996). AT, on the other hand, is more of a descriptive framework than a prescriptive theory. It considers an entire work/activity system (including teams, organizations, etc.) beyond just one actor or user. It accounts for the environment, history of the person, culture, role of the artefacts, motivations, and complexity of real life activity (Nardi, 1995).

The unit of analysis in AT is tool-mediated transformation of an activity system, while, in contrast, APOS, as being based on a constructivist theory, deals more on the mental development of the individual in the system. Despite a difference of focus, the driving force and common point of change, or learning, in both theories is that the individual and the Activity System with which the two viewpoints deal are interdependent complex adaptive systems (Nelson, 2002).

The contrast between these two perspectives comes from identifying learning as a process of individual sense making or as a process of participation in activity. In other words, students learn as they evolve in their ways of thinking. A sociocultural perspective identifies learning in terms of the extent to which an individual participates in the social practices (John-Steiner & Mahn, 1996).

2.2 LITERATURE REVIEW

This section presents a review of literature for the study in accordance with the statement of the problem: Evaluating the effectiveness of the use of Information and Communication Technology in the Teaching and Learning of Trigonometry functions In Grade 12.

For this purpose this section was structured into four main subsections:

- The mathematical concept of Trigonometry functions;
- ICT in teaching and learning;
- ICT in the teaching and learning of mathematics and
- ICT in the teaching and learning of trigonometry functions.

2.2.1 THE MATHEMATICAL CONCEPT OF TRIGONOMETRY FUNCTIONS

The focus in this section was on trigonometry functions as this is one of the most important topics in the secondary school curriculum requiring integration of algebraic, geometric and graphical representations. To ensure that trigonometry functions are learnt with the integration of their representations, would need one to introduce them with focus based on connections among the three different contexts of trigonometry functions:

• **Triangle Trigonometry**, where trigonometry is based on ratio definitions in right triangles;

• **Unit Circle Trigonometry**, where trigonometry functions are defined as coordinates of points on the unit circle based on rotational angles;

• **Trigonometry Function Graphs**, where trigonometry functions are defined in the domain of real numbers (Weber, 2005). In the South African school system, the trigonometry functions are defined in the domain of angles which are in degrees.

Trigonometry functions are some of the sections in the mathematics curriculum where students experience acute difficulties in learning (Adamek, Penkalski, & Valentine, 2005; Brown, 2005; Weber, 2005; Tatar, Okur, & Tuna, 2008; Kutluca & Baki, 2009; Demir, 2012).

Regardless of the importance of trigonometry functions in the mathematics curriculum, and the difficulties that students experience with them - little attention has been given to trigonometry and the various ways it is taught in classroom teaching (Davis, 2005). In addition, research on the teaching and learning of trigonometry, with or without technological aids, lags behind research conducted in other domains of mathematics education (Ross, Bruce, & Sibbald, 2011). The interconnectedness of its contexts could be one of the reasons why it is rarely researched. Another reason could be due to the different ways in which the domain is expressed (Brown, 2005). Brown, (2005) also states that another possible reason for the scarcity of research on trigonometry functions could be due to the different ways in which trigonometry functions are defined in the school curricula.

It is unfortunate that this topic area has been neglected, as not only is it an important section in the high school mathematics curriculum, but knowledge of trigonometry is crucial for success in many college/university programs. Understanding trigonometry functions is a pre-requisite for understanding topics in Newtonian physics, architecture and many branches of engineering (Weber, 2005). The standards of the National Council of Teachers of Mathematics (NCTM, 2010) highlight the importance of trigonometry in the study of functions, particularly periodic functions and emphasize trigonometry’s utility in investigating real-world phenomena.

2.2.1.1 TRIGONOMETRY FUNCTIONS IN THE SOUTH AFRICAN SCHOOL SYSTEM

Trigonometry functions are first introduced in grade 10 and continue to grades 11 and 12. Even though students first encounter trigonometry in grade 10, the basic prior knowledge concepts that they need for trigonometry functions are taught during the earlier years. These include various triangles and their properties, such as similar triangles, congruent triangles as well as parallel lines and their properties. At the end of the final year of grade 12, matric students write 2 mathematics papers (Papers 1 & 2). Trigonometry functions appear in Paper 2.

The South African curriculum emphasizes the importance of understanding trigonometry functions concepts and the relationships between the concepts. By the end of grade 12, the curriculum demands that the students should be able to:

- demonstrate the ability to work with various types of trigonometry functions such as

$$y = sink(x + p) + q; y = acosk(x + p) + q$$

And be able to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for the functions.

- Communicate by using descriptions in words, graphs, symbols, tables and diagrams and be able to convert flexibly between these representations (DBE, 2011).

A summary showing the South African curriculum on trigonometry is given in Table 2.4.

Table 2.4: Overview of Trigonometry functions in Grades 10, 11 & 12

Grade 10	**Grade 11**	**Grade 12**
Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae).	Extend Grade 10 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae).	Introduce a more formal definition of a function and extend Grade 11 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae).
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalize the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the *x* axis.	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalize the effects of the parameter which results in a horizontal shift and that which results in a horizontal stretch and/or reflection about the *y* axis.	The inverses of prescribed functions and be aware of the fact that, in the case of many-to-one functions, the domain has to be restricted if the inverse is to be a function.
Problem solving and graph work involving the prescribed functions.	Problem solving and graph work involving the prescribed functions. Average gradient between two points.	Problem solving and graph work involving the prescribed functions (including the logarithmic function).
(a) Definitions of the trigonometry ratios sin θ, cos θ and tan θ in a right-angled triangles. (b) Extend the definitions of sin θ, cos θ and tan θ to $0_o \leq \theta \leq 360°$. (c) Derive and use values of the trigonometry ratios (without using a calculator for the special angles $\theta \in$ {0°; 30°; 45°; 60°; 90°} (d) Define the reciprocals of trigonometry ratios.	(a) Derive and use the identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \sin^2\theta = 1$. (b) Derive the reduction formulae. (c) Determine the general solution and / or specific solutions of trigonometry equations. (d) Establish the sine, cosine and area rules.	Proof and use of the compound angle and double angle identities.
Solve problems in two dimensions.	Solve problems in 2-dimensions.	Solve problems in two and three dimensions.

(Adapted from: DBE, 2011)

In order to explain the approach normally used for teaching and learning trigonometry, the researcher used her experience, the CAPS curriculum (see Table 3.1) and CAPS compliant textbooks. The approach used illustrates the traditional way of teaching trigonometry functions, a method where the contexts of trigonometry functions are presented and learnt in a sequential or linear format. The method of teaching and learning in this study emphasises connection of the different contexts of trigonometry functions, right-angle triangle, the unit circle, and trigonometry function graphs. A summary of how trigonometry appears in books and CAPS curriculum follows:

At the beginning, students encounter trigonometry functions with 3 distinct contexts (Figure 2.5). In the South African curriculum, in the first context, trigonometry concepts are defined as ratios in right–angle triangles. The important idea is that trigonometry ratios are useful in setting relationships between angles and side lengths, and that these ratios can be defined as trigonometry values of angles (Laridon et al., 2002; Smith, 2012). The right angled triangle deals with angles between 0° and 90°. Students learn to compute the basic trigonometry functions (sines, cosines and tangents) given specific side lengths of right triangles (Figure 2.5a). However, at this point in the curriculum the basic trigonometry functions are not treated as functions, but only as ratios. Students encounter trigonometry functions later in the curriculum in the context of the unit circle, where the hypotenuse is defined as the radius, r, of the unit circle (Figure 2.5b). This is where the rotation of the terminal arm moves in such a way that the angles go beyond 90° to beyond 360° and in the reverse direction to form negative angles. Later, the third context- the graphs are introduced using the formulae and the table method (Figure 2.5c) In South Africa, the Cartesian plane represents trigonometry functions where the independent variable/domain represents angles measured in degrees, and the dependent variable represents the sine, cosine, or tangent of the angle (Laridon et al., 2002).

It seems that there are two primary challenges for students studying trigonometry functions starting with the ratios defined from right triangles. Initially, when students study sine, cosine and tangent ratios in right triangle trigonometry, the connections between geometric figures and numerical relationships create challenges for students in making connections between other different representations (Thompson, Carlson, & Silverman, 2007; Thompson, 2008; Bressoud, 2010). That means, the transition from studying sine, cosine and tangent ratios on right-angled triangles to studying sine, cosine and tangent functions on the unit circle and then on the Cartesian plane seems to be difficult for students.

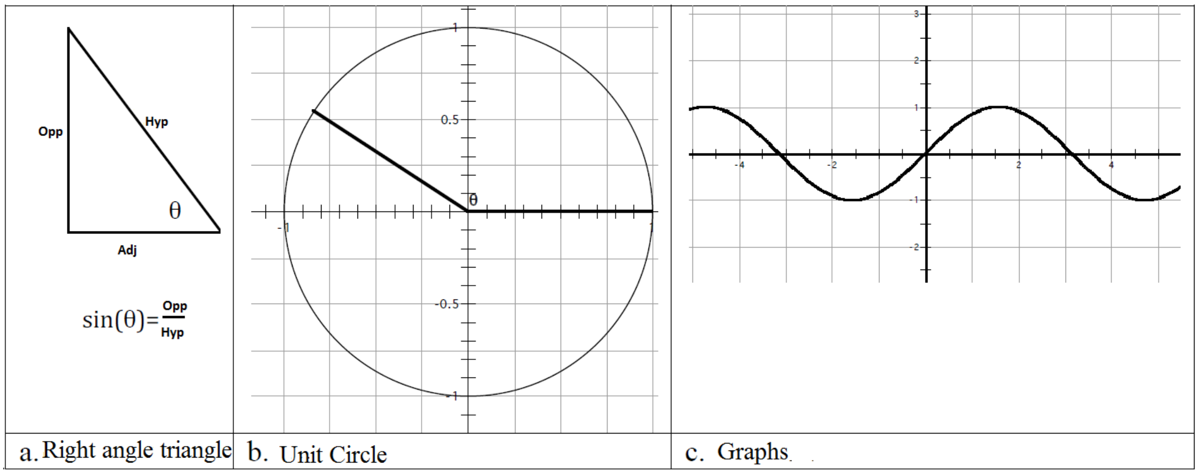


Figure 2.5: Some of the trigonometry representations/ contexts (Adapted from: Dejarnette, 2014)

The difficult transition from right triangle trigonometry to the unit circle is especially problematic given that the unit circle provides the basis for understanding trigonometry functions. In addition, according to Thompson (2008) students who first encounter trigonometry functions as ratios in right triangles are more disposed incorrectly to think that trigonometry functions take the sides of right triangles, rather than angles, as their inputs.

2.2.1.2 RESEARCH ON THE TEACHING AND LEARNING OF TRIGONOMETRY FUNCTIONS

Despite the documented problems students have with learning trigonometry functions, the literature in this area is still sparse (Weber, 2005; Moore, 2010). To reiterate, research on the teaching and learning of trigonometry, with or without technological aids, lags behind research conducted in other domains of mathematics education (Ross, Bruce, & Sibbald, 2011).

Research into students' learning of trigonometry functions has focused on identifying teaching activities to support students to depart from the traditional form of teaching and learning through memorization of isolated facts and procedures and paper-and-pencil tests– towards programs that emphasize conceptual understanding, multiple representations and connections together with mathematical modeling (Hirsch, Weinhold, & Nichols, 1991). For example, in one of a few studies which investigated students' understanding of trigonometry functions, Weber (2005) studied two groups of college students on their understanding of trigonometry functions. The experimental group was taught using instructional sequence based on Tall's (2009) notion of procepts and the current process/object theories of learning while the control group was taught using the lecture method of teaching. The teaching and learning sequence involved activities which were mostly hands-on, for example, using protractors and rulers. Students constructed unit circles, drew angles and related line segments corresponding to their trigonometry values. The experimental group of students performed much better than the control group in the post-test. However, from the types of questions asked, it became obvious that the experimental students would perform better, since they would have been taught in this direction within the instructional design. For example, one of the test items examined students' estimation of a trigonometry function value, and the control group did not performed as well as the experimental group. Considering that these types of tasks are not normally included in the traditional discourse and they were presented in the experimental group, it would be expected that the experimental group would outperform the control students.

Even though the unit circle method was shown to be more effective than the traditional method in the teaching and learning of trigonometry, it cannot be said that it would lead to successful student learning in all forms of trigonometry. This was confirmed by the findings of the research conducted by Kendal and Stacey (1997), where students who first learnt with the right triangle model perfomed better than the students who were first introduced to trigonometry functions through unit circle method. However, since Kendall and Stacey (1997) assessed students' learning with problems of solving triangles. It follows that the students taught using the right angled triangle method would get better results on solving triangle tasks.

From the studies of Kendal and Stacey (1997) and of Weber (2005) one can deduce that performance is directly proportional to the main focus in teaching or aspects emphasised in the lesson design.

Brown (2005) developed a model on students' understanding of sine and cosine functions of angles measured in degrees. From her model, the trigonometry functions were first taught using the triangle context, followed by the co-ordinate system, moving onto the unit circle context and onto the graphical representations. It should be stated that the steps of introducing the trigonometry function concepts in her model, were non-sequential or they were non-linear. Brown (2005) asserted that students who developed the most robust understanding were able to work with the sine and cosine in a way that connects the three contexts of trigonometry functions.

2.2.2 ICT IN TEACHING AND LEARNING

During the 1990s, Information Technology (IT) was a term reserved for computers and other electronic data handling and storage devices used to provide speedy automatic functions, capacity and range (Monaghan, 1993; Andrews, 1996). More recently, the term 'communication' has been incorporated to acknowledge the increase in interaction between people and technology; this is widely known as Information and Communication Technology (ICT) (Kennewell, 2004). Kennewell, (2004) explains that the term ICT covers all aspects of computers, networks (including the internet) and certain other devices with information storage and processing capacity, such as calculators, mobile phones and automate control devices. In this study, the term ICT is used to refer to new technologies with an emphasis on communication.

In a move from the traditional form of teaching, many educational institutions and scholars have taken into consideration the potential of ICT in teaching and learning. There has been an increasing awareness that interactions between humans and ICT can facilitate effective teaching and learning (Arcavi, 2003; Hennessy, Ruthven, & Brindley, 2005). The great potential of ICT in mathematics education is in that it is bringing transformation and new possibilities in teaching and learning. ICT integration in mathematics teaching and learning enables mathematical investigations by students and educators, supports conceptual development of mathematics, and thus influences how mathematics is taught and learnt (Wilson & Lowry, 2000). Using ICT in teaching and learning has also been found to:

- Promote learner's higher-order learning skills (Lincoln, 2008);
- Develop and maintain students' computation and communication skills;

- Introduce students to collection and analysis of data;
- Facilitate students' algebraic and geometric thinking;
- Show students the role of mathematics in an interdisciplinary setting (Mistretta, 2005).

Technological tools (ICT) such as computers and calculators can overhaul the existing traditional mathematics instruction by providing more powerful mathematical problem-solving and graphing opportunities and offering new possibilities in the learning and teaching of mathematics (Fey, 1989; Heid, 1998; Hennessy, Fung, & Scanlon, 2001). Wright (2005) asserts that ICT, particularly mathematical software, helps to provide better visual and dynamic representations of abstract ideas and the links between symbols, variables and graphs. Consequently, the study investigated the ways in which students learn trigonometry functions by using ICT.

The use of computers as a tool facilitates communication among students and encourages them to play a more active role in a lesson (Tarmizi, Ayub, Bakar, & Yunus, 2010). Tarmizi et al., (2010) further highlight that the role of students in a computer-oriented lesson is to actively generate, process, and manipulate knowledge. The use of ICT enables more students to be active processors of knowledge, to appropriately sort out the given knowledge and to be able to act accordingly on the knowledge being considered than would be the case in typical teacher-led lessons. Students are in a position to define their goals, make design decisions and evaluate their progress through the aid of computers (Tarmizi et al., 2010).

In an ICT integrated teaching and learning environment, the teacher's role changes and they are no longer the centre of attention and information as providers of knowledge, but rather they play the role of facilitators. As students work on their computer-supported products, the teacher provides necessary assistance and guides them through the activities and stages of a lesson by monitoring what they are doing during the lesson.

The use of ICT in education can enhance meaningful learning better than the traditional classroom instructions. The ICT can engage a wider range of aptitudes, connecting academic work with the real world, supporting interaction, offering dynamic displays, multiple and linked representations, interactive models, simulations, the storage and retrieval of multiple categorized information (Ashburn & Floden, 2006). In this way, by integrating ICT into the teaching and learning process, educators aim to increase students' abilities to understand complex ideas and learn challenging content.

2.2.3 ICT IN TEACHING AND LEARNING OF MATHEMATICS

One of the major goals in mathematics teaching and learning is to ensure that all students achieve favourable outcomes. Mathematics is considered as one of the most challenging and problematic subjects in the education system. But at the same time it is one of the most important and rewarding areas of science, given that mathematical skills and knowledge are important in everyday life, and there are also many mathematical applications in other subjects and sciences (Christy, 1993). For these reasons, mathematics is a subject which should be taken seriously. Many students find it difficult to engage with mathematical concepts. For learning to take place, students need to be actively engaged with the explored concepts or objects – whether abstract or concrete (Liang & Sedig, 2010).

According to Duval (1999), mathematical activity has two sides:

- the visible side which is the mathematical objects; mathematical objects are abstract and not amenable to any concrete imagination or manipulation; they are immaterial, not tangible and directly accessible to our thinking like the physical objects (Chiappini & Bottino, 1999) and
- the cognitive operations or procedures; the cognitive operations are also a difficult part of mathematical activities for students, given that very often teachers attach more importance to the mathematical processes than to their applications to daily life situations or to physical problems (Duval, 1999). This leads students to solve problems mechanically, by following the algorithm steps without real awareness of their actual meaning (Milovanović, Takači, & Milajić, 2011).

ICT is useful in helping students to perceive mathematics not only as a set of procedures, but more as reasoning, exploring, discovering, solving problems, generating new information and asking new questions. Furthermore, ICT helps them to better visualize certain mathematical concepts (Van Voorst, 1999). Studies have revealed that activities encouraging the construction of images can greatly enhance mathematics learning (Wheatley & Brown, 1994).

Greeno and Hall (1997) made several observations about the importance of representations, concluding that:

- computer technologies are powerful tools for thinking;
- the learning of mathematical concepts and procedures is enhanced when students can make connections among different representations;
- they can give students useful tools for building communicating information and demonstrating reasoning (Greeno & Hall, 1997).

Ashburn and Floden (2006) also emphasize the importance of using technology in mathematics, noting that tools that instantly relate the graphical and symbolic representations of mathematical expressions can help make understanding goals more accessible to students. Models that make intangible perceptions visible and interactive can help students comprehend the nature and application of fundamental concepts (Ashburn & Floden, 2006).

There are several studies which investigated the integration of ICT into the different topical areas of mathematics teaching and learning (Gonzales & Herbst, 2009; Hoyles & Lagrande, 2010; Liang & Sedig, 2010; Milovanović et al., 2011; Lotfi & Mafi, 2012) concluding that ICT can help to visualize and represent better the mathematical objects and procedures by exploring different graphical representations.

However, some studies have found no significant impact of using ICT in the teaching and learning of mathematics (Smith & Hardman, 2014). In a Cape Town study, the impact of computer contribution on performance of school leavers Senior Certificate mathematics scores was investigated across 31 schools in the Education Management and Development Centre (EMDC) East education district of Cape Town, South Africa. This was done by comparing the performance between two groups: a control and an experimental group (Smith & Hardman, 2014). The experimental group (14 high schools) had access to computers from 2001 while the control schools, as part of the Khanya project, received computers between 2006 and early 2007. The experimental schools could be expected to be more immersed in computer technology than the control schools. Findings indicated that there was no significant difference between the final Senior Certificate mathematics results of the schools with the computers and those without; no significant change in the results after the Khanya labs were installed; no significant change in the percentage of students that passed Senior Certificate Mathematics; and no significant change in Mathematics enrolment rates. These findings point to the need for caution in the implementation of ICT's into schools as a potential panacea for mathematical failure in our context. Hardman (2008) recommended that further work be done to provide a better picture in the usage of ICT.

2.2.3.1 SOFTWARE USED IN TEACHING MATHEMATICS

Some software giving students some freedom to investigate and express their own ideas, but constrained in ways so as to focus their attention on the mathematics is needed (Hoyles, 2001). Although using virtual manipulatives might be convenient for teachers, their limitation of mathematical experiments to a certain range of activities and topics are obvious. Therefore, many teachers use educational software packages that allow more flexibility and enable both teachers and students to visualize and explore mathematical concepts in their own creative ways (Barzel, 2007).

The awareness of integrating graphical, numerical and algebraic representations has become noticeable in recent years. Numerous studies note that ICT positively influences students' understanding of mathematical concepts and attitudes towards mathematics (Dwyer, 1994; Ogbonnaya, 2010). This is so because ICT appears to symbolize the concept of functions in terms of the strong connection among their representations (Ruthven, 1990; Penglase & Arnold, 1996). Possibly it is for this reason that Raines and Clarke (2011) argue that students working with appropriate software can work with more graphs in a short space of time. This is corroborated by Hennessy et al. (2001) who state that because ICT software speeds up the graphing process, this frees students to analyse and reflect on the relationships between graphs and their patterns. In addition to this Raines and Clarke (2011), state that computers seemingly enable students to participate fully in class.

Mostly, there are several types of software used in the teaching of mathematics: Computer Algebra System (CAS), Dynamic Geometry Software (DGS) Spreadsheets, etc. (such as GSP, Cabri-géomètre), and open source software-Java

Applets, Geogebra (Laborde, 2001; Strässer, 2001; Kokol-Voljc, 2003; Laborde, 2003, 2007). Each software type is normally associated with particular aspects of mathematical teaching and learning. For example, CAS is often used for teaching algebraic topics, whilst DGS programmes are used for geometrical topics (Schneider, Carnoy, Kilpatrick, Schmidt, & Shavelson, 2007). However, such distinctions are not always clear with considerable overlap due to the duality of mathematics in terms of geometry and algebra. Schumann and Green (2000) state that graphical, numerical and algebraic contexts should not be considered separately, but rather as constituting a holistic comprehensive computer-aided approach.

Trigonometry is a topic in geometry that involves both geometry and algebra, thus a need to use software that includes both features. A dynamic mathematics software (DMS) package, n amely Geogebra, was used as a tool in this study.

Geogebra is a dynamic mathematics software (DMS) designed for teaching and learning mathematics in secondary school and at college/university level. The software combines the ease of use of a dynamic geometry software (DGS) with certain features of a computer algebra system (CAS) and therefore, allows for bridging the gap between the mathematical disciplines of geometry, algebra, and even calculus (Hohenwarter & Jones, 2007). it includes both symbolic and visualization features related to coordinates, equations and functions, along with geometric concepts and dynamic relations (Hohenwater & Fuchs, 2004; Zengin et al., 2012). On one hand, Geogebra can be used to visualize mathematical concepts as well as to create instructional materials. On the other hand, Geogebra has the potential to foster active and student-centered learning by allowing for mathematical investigation, interactive explorations, as well as discovery learning (Bruner, 1961; Tessema, 2012). In addition, Geogebra can encourage discussion and group work thus making mathematics a much more open and practical subject, which is accessible and manageable to more students (Hohenwater & Fuchs, 2004).

The development of Geogebra began in 2001 as Markus Hohenwarter's Master's thesis project at the University of Salzburg, Austria (Hohenwarter, 2002). Geogebra is freely available on the internet. Furthermore there are applets, tutorials, worksheets and an interactive platform that is also freely available on the internet to assist in the optimal use of the software. Accordingly, Geogebra can be used as a presentation tool as well as for the creation of instructional materials, such as notes or interactive worksheets (Hohenwater & Fuchs, 2004).

Dogan (2010) conducted an experimental design study using a pre- post-test to evaluate the success of students learning using the Geogebra software. It was a twelve hour course held over a period of two weeks involving two eighth grade classes. It was observed that computer based activities could efficiently be used in the learning process and the Geogebra software encouraged higher order thinking skills. The software was also observed as having a positive effect in motivating students toward learning and retaining the knowledge for a longer period. This was based on a recall tests conducted a month later. In another study, Herceg and Herceg (2010) conducted a study on two groups of students. One group used applets only, whilst the other used the Geogebra software and applets. The study tested how to incorporate computer-based learning to reduce the working process of numerical integration (Herceg & Herceg, 2010). The results of this study showed that the Geogebra experimental group gained more knowledge and skills than the control. This study also suggested that Geogebra use is helpful for students who face difficulty in solving mathematical problems since they do not have to spend so much time solving by hand.

Bakar, Ayub, Luan and Tarzimi (2002) compared Geogebra to a software program created by them on two groups of Malaysian secondary school students and found that students using the Geogebra software to study the transformation topic achieved better results than students using the created software.

The purpose of yet another study was to determine the possible effects of the dynamic mathematics software Geogebra on student achievement in the teaching of trigonometry. The sample of that study consisted of 51 students. The experimental group was taught using the Geogebra software in computer assisted presentations, while the control group was taught the lessons using constructivist instruction. The data collected after 5 weeks of application showed that there was a meaningful difference between the experimental and control groups' achievement in trigonometry. This difference was in favour of the experimental group which had lessons with Geogebra (Zengin, Furkan, & Kutluca, 2012).

The purpose of another study was to determine the effect of Geogebra on conceptual and procedural knowledge of functions (Zulnaidi & Zakaria, 2010). The study involved 124 high school students from Indonesia. A total of 60 students were in the treatment group whilst 64 students were in the control group. The treatment group were taught using Geogebra and the traditional method was used in the control group. The data were collected using the conceptual and procedural knowledge test of functions. The results showed significant differences between treatment and control groups in that the treatment group had significantly higher conceptual knowledge compared to control group.

2.2.4 **ICT IN THE TEACHING AND LEARNING OF TRIGONOMETRY**

Research on the teaching and learning of trigonometry, with or without ICT has not been carried out to the same extent as in other fields of mathematics education (Ross, Bruce, & Sibbald, 2011). In addition, Davis (2005) notes that little attention has been given to trigonometry and the various ways it has been represented in classroom teaching.

Regarding the learning of trigonometry functions, Park (1994) points out the role of simulations, which can be used to highlight how a numerical output is linked to certain unknown symbolic representations through a graphical approach. This is because graphing motion can bring to the students' attention the critical features and their relations to other components that might not easily be grasped in an abstract system. Simulations can also illustrate procedural relationships (Park, 1994). For example, while transforming trigonometry curves that involve four transformations, students can see the sequential steps to achieving the end result.

Jonassen (2000) also suggests that students will learn trigonometry functions better and more conceptually if they are able to inter-relate numerical and symbolic representations with their graphical outputs. With respect to the way trigonometry is taught, Blacket and Tall (1991) point out the advantages of the computer approach relative to the traditional approach, stating that it can allow students to manipulate the picture and relate its dynamically changing state to the corresponding numerical concepts, having the potential to improve performance. Steckroth, 2007) found that software that included animation and visualization produced greater learning than software limited to graphing functions.

The studies which incorporated ICT, even though sparse, into the teaching of trigonometry have demonstrated largely positive effects on student achievement (Blackett & Tall, 1991; Choi-koh, 2003; Moore, 2009; Ross et al., 2011; Zengin et al., 2012). Although the studies show positive effects of the use of ICT on students' learning and performance, the possible effects of ICT on students' ability to make connections between different representations and contexts of trigonometry functions is limited, even though this is basic and fundamental to the learning of trigonometry functions (Brown, 2005; Demir, 2012). Studies on the interpretation and analysis of trigonometry functions were also found to be limited. In addition, the researcher could not find any study that puts focus on tangent functions. This study is focusing on connections between interpretation and transformation of trigonometry function. Moreover, the tangent function is included in the study.

Blackett and Tall (1991) employed a computer program that draws the desired right triangles to facilitate students' exploration of the relationship between numerical and geometric data. The results of the study show that computer representation enabled students to make this exploration in an interactive manner. They were encouraged to make dynamic links between visual and numerical data, which is less apparent in a traditional approach. The authors conclude that even the least able became adept at using the computer and, though they had some difficulty writing down their results, they had few difficulties with visualization.

Wilson, (2008) studied the role of dynamic web tools in trigonometry lessons, and he concluded that there was improvement both in the quality of students work and in their interest toward the subject. He points out that these tools provide excellent vehicles to monitor drill and practice and to foster conceptual understanding in many situations.

Choi- Koh (2003) investigated the patterns of one student's mathematical thinking processes and described the nature of the learning experience that the student encountered in trigonometry as he engaged in independent explorations within an interactive technology environment. He concluded that representations offer students an opportunity to explore and conjecture mathematics which fosters a balance between procedural and conceptual knowledge. Students can effectively use a graphing calculator as an instructional tool to help them understand the connection between graphical and algebraic concepts, and not use it just to get quick answers.

He also emphasized the role of technology in enhancing students' attitudes to mathematics learning.

Dynamic interactive features enable the software to illustrate mathematical changes that might not otherwise be visible and helps students visualize a dynamic model containing trigonometry relationships that are difficult to depict with static images (Ng & Hu, 2006).

Using technology in lessons does not lead automatically to better results in terms of students' learning and understanding. Of crucial importance are how technological tools are used in lessons, the kind of support students receive, and interactions between the tools and students. In this regard, Thompson (2002) mentioned the use of technological tools as educational objects which promote reflective mathematics discourse for knowledge construction, but that an object is not instructive on its own. Moore (2009) used Geometer's Sketchpad as an educational object enabling supportive mathematical dialogue. Moore (2009) used two applets for angle measure and two co-varying elements, namely arc length and vertical position, in a dynamic way for students to use and found positive effects on students' understanding based on quantitative and covariational reasoning. He stated that such applets led to better learning of the sine graph which is not easy to draw because of the concavity. Moore's (2009) study indicates an effective use of technology in mathematics lessons. To increase effectiveness of technology for students' learning, ICT must be integrated into classroom teaching in ways which will foster students' concept development and understanding, and address their learning difficulties.

The optimal sequence for integrating ICT with mathematics instruction has not yet been determined (Ross et al., 2011). Lesser & Tchoshanov, (2005) presented evidence that students need to be taught abstract, visual and concrete representations to develop function sense (the ability to integrate and flexibly apply multiple representations of functions). They found that the working sequence for introducing representations in trigonometry was to present the abstract first; that is, the visual and concrete became meaningful only after the abstract had been learned. This form of sequence of teaching trigonometry in technological environments was substantiated by Ross et al., (2011) who concluded that better learning is promoted when ICT is used after the teacher explains the content. In their study Ross et al., (2011) used sliders from a software to investigate the students' performance on transformations of trigonometry functions. In one set of groups the formal teaching was followed by the use of sliders in another set of groups ICT preceded the normal teaching in the class. They found that, in the case of transformations of the trigonometry functions, using a dynamic software package, after whole class teaching of core concepts, was more effective than beginning the learning unit with the software. Also at the end of the learning unit there was almost no difference between the students' performance levels in the two groups. Hence, they suggested an integrated method of whole class teaching and technology use.

2.3 **SUMMARY**

The choice of method and the frameworks for the study were established. Two theoretical frameworks APOS and AT were incorporated in developing the delivery of the teaching and learning of trigonometry. The literature on the mathematical concept of trigonometry functions was presented followed by the use of ICT in teaching and learning, thereafter ICT in teaching and learning of mathematics and finally the literature on the use of ICT in the teaching and learning of various aspects of trigonometry.

While some studies have been done on the use of ICT on a wide range of teaching and learning of trigonometry functions in Europe, the United States of America and in the Far East, no similar studies have been carried out on the use of ICT on trigonometry functions in South Africa. Moreover, there is a limited, if any, research on interpretations, connections and transformation of trigonometry functions.

CHAPTER THREE

RESEARCH METHODOLOGY

Research methodology (approach) involves the paradigm, the research design, and methods (procedures) for answering questions in the study (Creswell, 2014). Thus in this chapter, the following topics are addressed: the paradigm underlying the research, the research design, the sample and the population it is coming from, the instruments for data collection, the steps taken to ensure the validity and reliability of the instruments, the ethical issues considered in the study and the data analysis techniques employed in the study.

3.1 THE RESEARCH PARADIGM

Paradigm can be said to be a set of shared ideals, beliefs and values about a concept to be studied. The term paradigm comes from the Greek word *paradeigma* which means *pattern* and was used by Thomas Kuhn (1977) to denote prevailing patterns of thought shared by scientists which provided them with a suitable model for investigating problems and finding solutions. (Wisker, 2001) explains a paradigm as an underlying set of beliefs about how the elements of the research area fit together and how one can enquire of it and make meaning of discoveries. Similarly, paradigm is said to be a research culture held by a community of researchers that is based on discipline orientations and past experiences (Kuhn, 1977; Creswell & Plano Clark, 2011; Creswell, 2014).

There are different kinds of paradigms such as positivism (for quantitative research), interpretive (for qualitative research), critical (for civil actions) and pragmatic (for mixed methods). Quantitative research is usually associated with the positivism paradigm (Williams, 1998). This research study was thus based on Positivism (Anderson, 1983).

3.1.1 POSITIVISM

There are various ways in which positivism may be described. As a philosophy, positivism adheres to the view that only factual knowledge gained through observation (the senses – hearing, sight, touch, taste and smell), including measurement, is trustworthy. Therefore, things that cannot be observed or scientifically measured, e.g. people's thoughts and attitudes cannot be accepted as factual knowledge. Hence APOS, which concentrates on mental constructions of students when acting on an object and AT are only used in designing and implementing the instructional process of the experimental group, not in the analysis and interpretation in the current study. In addition, in positivism studies the role of the researcher is limited to data collection and interpretation through objective approach and the research findings are usually observable and quantifiable (Collins, 2010).

Positivism is a philosophical theory stating that positive knowledge is based on natural phenomena and their properties and relations. Thus, information derived from sensory experience, interpreted through reason and logic, forms the exclusive source of all authoritative knowledge. Positivism holds that valid knowledge (certitude or truth) is found only in this derived knowledge (Sanchez, 2016).

Positivism is also the term used to describe an approach to the study of society that relies specifically on scientific evidence, such as experiments and statistics, to reveal the true nature of how society operates. The term originated in the 19th century, when Auguste Comte described his ideas in his books *The Course in Positive Philosophy* and *A General View of Positivism*. Within the positivist research paradigm, it is assumed that the only way that people can be confident that the knowledge is true is if it was created using the scientific method. Here, data is derived from experiments and observation in order to yield supportive evidence (Rohmann, 1999).

Positivism depends on quantifiable observations that lead themselves to statistical analysis. It has been noted that "as a philosophy, positivism is in accordance with the empiricist view that knowledge stems from human experience. It has an atomistic, ontological view of the world as comprising discrete, observable elements and events that interact in an observable, determined and regular manner (Collins, 2010).

Moreover, in positivism studies the researcher is independent from the study and there are no provisions for human interests within the study. As a general rule, positivist studies usually adopt deductive approach, whereas inductive research approach is usually associated with a phenomenology philosophy (Crowther &

Lancaster, 2008). Moreover, positivism relates to the viewpoint that the researcher needs to concentrate on facts, whereas phenomenology concentrates on the meaning and has provision for human interest.

Researchers warn that if one assumes a positivist approach to a study, then it is the belief that one is independent of the research and the research can be purely objective. Independent means that one maintains minimal interaction with your research participants when carrying out your research (Wilson, 2010). In other words, studies with positivist paradigm are based purely on facts and consider the world to be external and objective.

One of the aims of the positivist research paradigm is to explain cause and effect relationships (Creswell, 2009). A cause-effect relationship relates independent variable(s) (which is the treatment and may be a cause of any improvement) to the dependent variable (which is the outcome of the treatment).

The scientific approach to research consists of posing questions and related hypotheses as explanations of phenomena and then designing experiments to test and verify the questions and hypotheses. The steps involved in putting this idea under scientific examination (sampling, data collection, data analysis) must be repeatable (called reliability, from Latin *religare*, to bind fast) so scholars can predict any future results generated using the same methods. It is imperative that the entire research process be objective (value free) to reduce biased interpretations of the results (Creswell, 2014). A wide range of statistical measures have been developed as a means of measuring reliability and validity.

True experimental and quasi-experimental designs are both experimental; with the main difference being that the sample in the quasi-experimental is not assigned randomly (Best & Kahn, 1998). The current study is of a quasi-experimental design.

3.2 PILOT STUDY

A pilot study was conducted in two schools from Mafikeng in the North West Province of South Africa. Four groups, two from each school, participated. In total, the pilot study consisted of 138 students, with 75 students in the control groups and 63 students in the experimental groups. The experimental groups were taught the lessons arranged mainly with the Geogebra software and the Whiteboard, while the control group was taught using the traditional teaching methods. The computer (experimental) groups (63) were chosen because of the availability of computers at their school.

The data were collected before and after 10 lessons of teaching and learning. In both schools they all wrote the pre- and post-test, followed by interviews with four (4) of them. As part of the study six (6) students were supposed to be interviewed. This could not be done due to a lack of time as the researcher was trying to avoid disturbing the normal teaching and learning process. This also led to the students not being asked all the questions.

The data gathered through the pre- and post-test, and the interview in the pilot study strongly supported the argument that computers are useful in teaching and learning trigonometry (c.f. Weber, 2005; Demir, 2012). The results from students' interviews showed that they enjoyed learning trigonometry graphs using Geogebra. However, except for trigonometry graphs, all students were not keen on using computers to solve or simplify trigonometry equations and expressions. They perceived it as time consuming as grade 12 students, who were preparing for final examination, they said they could not afford to spend a lot of time on one topic. They also indicated that in tests and examinations computers are not allowed in the assessment rooms, thus at that time they preferred using calculators to Geogebra.

Based on the results and analysis of the pilot study, it was concluded that the main study would be feasible. Thus, the continuance to the main study. Many of the questions in the final study were generated from the pilot study. Some of the questions were modified since it was noticed in the pilot study that some students just memorised the memorandum from the pre-tests. Furthermore, from the advice of a mathematics subject advisor, the question: Plot the graph of $sin\ (90 - theta)$ was changed to $cos\ theta$. The content, and tasks still adhered to the CAPS syllabus. In response to the students' reluctance to use the software for the teaching and learning of the equations and expressions in the pilot study, Geogebra was excluded for the teaching and learning of these sections in the main study.

3.3 RESEARCH DESIGN

Various designs have been described: the quantitative, qualitative and mixed methods designs. A quantitative research design is one that is based on positivism paradigm and measurability, it aims to establish cause and effect relationships, on the other hand, a qualitative research design is based on multiple socially constructed realities and aims to understand a social phenomenon from participants' perspective, (McMillan & Schumacher, 2010; Creswell, 2014). This study adopted a quantitative research approach since the research problem addressed in the study is concerned with whether intervention works better than no intervention in influencing the outcome.

Quantitative methods make use of numerical variables. The analyses are usually statistical, the results of which are normally accepted as legitimate confirmation of collected data (McKnight, Magid, Murphy, & McKnight, 2000). Quasi-experimental designs are included in the quantitative approach to research (Creswell, 2014).

A non-equivalent group, with *pre- post-test*, and quasi-experimental design (Oaks & Feldman, 2001) was chosen for this study, the blue-print of which is shown in Figure 3.1. This is a type of evaluation that sought to determine whether a program or intervention had the intended causal effect on program participants, i.e., whether a specific intervention had an influence on the outcome (Creswell, 2014). The two classes were non-randomised, thus non-equivalent groups (McKnight et al., 2000). There are four key components in this quasi-experimental study design: 1) the experimental group 2) *pre- post-test* 3) treatment/intervention and 4) the *control group*.

1) The experimental group – The group that received the intervention or treatment;
2) *Pre- post-test* –An evaluation of students' understanding of a concept before and after the material is taught or before and after the intervention;
3) *A treatment* – The intervention that will be administered to the experimental group.
4) *A control group* – this group received no treatment or intervention.

The blue print

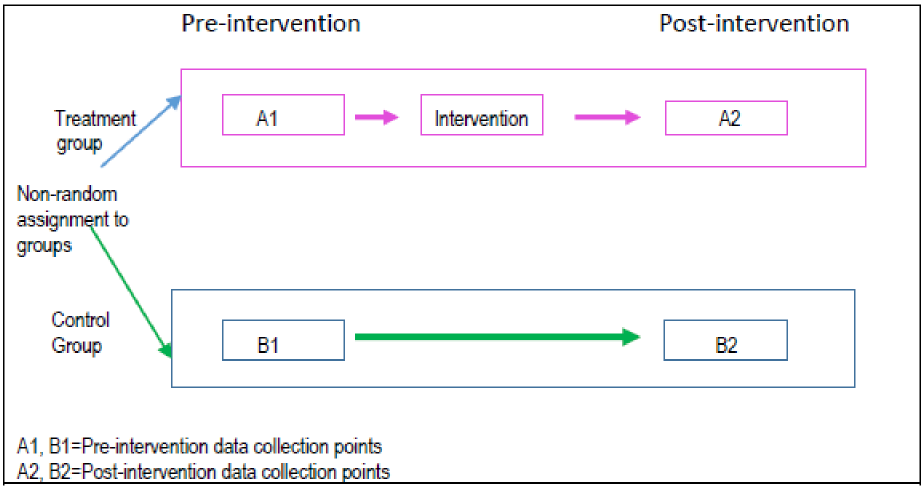


Figure 3.1: Blue-print of the quasi-experimental design (Adapted from: MLE, 2013)

The non -equivalent design was chosen in order to preserve the normal running of the two schools. The groups were intact as they were placed into their various classes by the schools depending on the combination of subjects the students were registered for. The status of the groups was not altered in any way for the study. The schools' schedules, time-table, etc. were not disturbed or altered in any way and the Trigonometry functions were taught as prescribed by the school time-table and the teaching schedule. One section of school trigonometry - (the 2- and 3-dimentional problems section) was excluded from the study since, according to the schools' schedule, these trigonometry problems were meant to be taught in the third term. The pre-test and -tests had also to adhere to the school time-table, so as not to disturb the students' attendance to other classes.

The two teachers in the 2 schools were both teaching mathematics in their respective schools. One school had computers and the other did not. The experimental group was selected from the school with computers and the control group came from the school without. The teacher in the experimental group was introduced to Geogebra by the researcher and the trajectory of teaching trigonometry functions in that classroom. To develop the instructional trajectory in the experimental group, the researcher used CAPS text books, her experience, colleagues, Geogebra and Geogebra applets. In both groups, the experimental and the control groups, the content covered adhered to the schools schedules, which in turn followed the prescribed grade 10, 11 and 12 syllabi. The control group was taught using the traditional teaching. This was so as to compare the instruction using ICT with the traditional instruction to determine the effectiveness of either in the teaching and learning of trigonometry functions.

3.3.1 CHOOSING THE QUANTITATIVE APPROACH

According to McMillan and Schumacher (2010), the quantitative approach is objective, value free and unbiased. One of the factors that influenced the researcher to choose the quantitative approach above the others was because of the main aim of the research: to evaluate the relative effectiveness of using ICT in the teaching of trigonometry functions in a classroom. This was essentially to determine whether the use of ICT would have an effect in the performance of the students than the traditional form of teaching. This is an example of determining the best method to test a theory or explanation, thus the quantitative approach was used (Creswell, 2014).

Moreover, the researcher is a mathematics educator and therefore is acquainted with the language that is used in quantitative research, as the quantitative approach relies on variable, numerical values, and measurements to generate numbers that can be analysed, using statistics.

3.3.2 DESCRIPTION OF THE SITE

The research was conducted at two schools in the Ngaka Modiri Molema district in Mafikeng, North West Province. The schools were selected due to convenience and availability. The schools were about 15 km from each other thus a reduction in external influences among the students. The Deputy Director (Education), principals, the teachers, the students and school governing bodies gave permission for the research at the two schools.

The experimental group was situated at a school that had a fully equipped computer laboratory with computers, a server, printer, whiteboard, and security cameras. The computers had not been used for content subjects (Life Sciences, Physical Sciences and Mathematics) at the school. Geogebra was installed by the researcher with the assistance of two mathematics educators. The two teachers at the school were trained to use the software and the subject content to be taught in the experimental group were communicated to them.

The school where the control group was did not have a computer room but all the students were doing the same subjects which followed after the CAPS curriculum.

3.4. STUDY SAMPLE AND POPULATION

A population is defined as a group of elements which can be people or objects that conform to specific criteria and to which a researcher intends to generalize the findings of the study (McMillan & Schumacher, 2010). In this instance, our population was a group of students who sat for grade 12 examination in Ngaka Modiri Molema District in the North West Province, South Africa.

A convenience sampling method was used to select the two schools that participated in this study. The North-West province is a multicultural province and the main language spoken is Setswana whilst Mathematics is taught in English, a second language.

In this study, two groups of students from two schools which are physically apart (about 15 kilometres), and who were due to write the grade 12 final CAPS examination at the end of the year were chosen. There were 61 students in total. The sample was chosen by the researcher for reasons of convenience and accessibility (McMillan & Schumacher, 2010). This was a convenient sample since the schools were selected on the basis of their accessibility and ICT availability for the treatment group. Both schools are from a historically disadvantaged environment. They are situated in the Mafikeng area, in the Ngaka Modiri Molema District, in the North West province of South Africa. The age range of the students was between 18 and 20. The students were mainly from not so well-to-do to middle income homes, and they commute using private transport, public transport or walk to school daily. The school where the experimental group came from had 245 mathematics students in total whilst, the school where the control group came from had 63 mathematics students in total. In both cases only one class from each school participated in the study.

The schools chosen were upgrading and remedial institutions. These institutions cater for students who sat for grade 12 examinations previously but either failed or did not obtain the requisite credit levels (Appendix 9) for whatever program they intended to pursue at the tertiary level. Most students at both schools came with level 1 (0-29%), which meant that the students came to the schools with virtually no achievement in mathematics. For all the possible levels of achievements, see Appendix 9.

3.5 CONDUCTING THE STUDY

This study examined students' performance on trigonometry functions before and after an intervention. The purpose here was to determine whether the use of computers would have any effect on the students' performance. To develop the instructional trajectory in the experimental group, the researcher used literature, CAPS text books, her experience, colleagues, Geogebra and the Geogebra applets.

The students were informed of the research and upcoming assessments a week before the study began. The pre-test was given to the students a day before the lessons started. Sixty one students wrote both the pre- and post-tests in the two schools. Thirty four (34) students were in the control group and twenty seven (27) in the experimental group. The groups were taken as they were allocated in the two schools. Since the assessment was out of 80, the students were asked to complete the test during one and half hours. They were again informed at the beginning of the assessment that the test was mainly for research purpose. The complete pre-test and post-test can be found in Appendices 1 and 2.

The teacher in the experimental group was introduced to Geogebra and the trajectory of teaching trigonometry functions in that classroom. Although general tools allow students and teachers much more freedom to shape and modify how to use them (Barzel, 2007), the introduction of an all-purpose tool for mathematics education requires more time and effort from both teachers and students than simply using virtual manipulatives. While students need hardly any computer skills in order to be able to work with prepared virtual manipulatives, both teachers and students need to learn the basic skills concerning the operation of a specific mathematics software before being able to effectively integrate it into teaching and learning. Although many teachers do not want to spend this time to familiarise a software package to their students, teaching them the basic use of the tool usually pays off in the long run, and especially if the software package is a versatile tool that allows teachers to cover different mathematical topics at different grade levels (Hohenwarter & Jones, 2007).

The educator in the experimental group introduced Geogebra to the students. In both groups, the lessons were conducted by the schools' educators during the normal scheduled mathematics periods. The content taught followed the content that the teachers would normally teach during that time. The study was carried out during the second half of the first term. After the intervention, both groups were given a post-test to compare whether there was any difference in the students' performance. The findings are presented in chapter 4.

3.5.1 INSTRUMENTS

The instruments for data collection were the Trigonometry functions achievement tests (Appendices 1 and 2). These achievement tests were administered by the educators at the schools before and after the implementation of the intervention. The tests consisted of five questions that added up to 80 marks. In Question 1, students were supposed to draw trigonometry graphs and to write down their respective properties. Question 2 involved the derivation of graphs of trigonometry functions from given formulae. Question 3 dealt with analysis of intersecting graphs. Question 4 required students to translate trigonometry formulae to graphs and or to words. Question 5.1-3 involved the integration of the unit circles, algebra, right angle triangle, and graphs. In Questions 5.2, 5.3 and 5.4 students were required to solve equations and prove an identity.

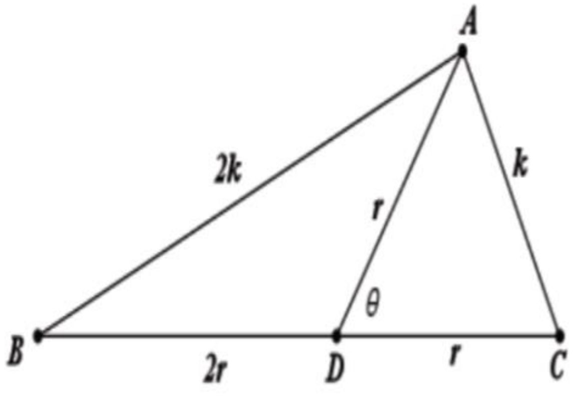
The following section describes how the instruments were developed, how the validity and the reliability of the tests and scores, respectively, were determined.

3.5.2 DEVELOPMENT OF ACHIEVEMENT TESTS

According to La Marca, (2001), in order to make valid and reliable decisions on students' achievements, a study should use assessments which are aligned with the curriculum standards. This means that there should be a high degree of match between the test tasks and subject matter content as identified through government educational standards. The test questions were thus constructed by using the specification and clarification of the trigonometry functions content (see Table 3.1) guided by the Curriculum and Assessment Policy statement (Grade 10-12). This confirms the statement from the mathematics examination guidelines (DBE, 2015) which states that the purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Previous CAPS compliant grade 12 examination papers were also used to develop the instruments.

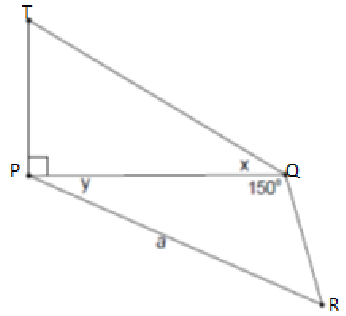
Table 3.1: Curriculum and Assessment Policy statement (CAPS) on Trigonometry content (Grades 10-12) (Adapted from: DBE, 2015)

TOPIC	CURRICULUM STATEMENT	CLARIFICATION
TRIG GRADE 10 Term 1	1. Define the trigonometric ratios sin θ, cos θ and tan θ, using right-angled triangles. 2. Extend the definitions of sin θ, cos θ and tan θ for $0° \leq \theta \leq 360°$. 3. Define the reciprocals of the trigonometric ratios cosec θ, sec θ and cot θ, using right-angled triangles (these three reciprocals should be examined in grade 10 only). 4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0°; 30°; 45°; 60°; 90°\}$. 5. Solve two-dimensional problems involving right-angled triangles. 6. Solve simple trigonometric equations for angles between 0° and 90°. 7. Use diagrams to determine the numerical values of ratios for angles from 0° to 360°.	**Comment**: It is important to stress that: • similarity of triangles is fundamental to the trigonometric ratios sin θ, cos θ and tan θ; • trigonometric ratios are independent of the lengths of the sides of a similar right-angled triangle and depend (uniquely) only on the angles, hence we consider them as functions of the angles; • doubling a ratio has a different effect from doubling an angle, for example, generally $2\sin\theta \neq \sin 2\theta$; and • solve equation of the form sin $x = c$, or 2cos $x = c$, or tan $2x = c$, where c is a constant. **Examples**: 1. If $5\sin\theta + 4 = 0$ and $0° \leq \theta \leq 270°$, calculate the value of $\sin^2\theta + \cos^2\theta$ without using a calculator (R) 2. Let *ABCD* be a rectangle, with AB = 2*cm*. Let *E* be on *AD* such that $A\hat{B}E$= = 45° and $B\hat{E}C$ = 75°. Determine the area of the rectangle. (P) 3. Determine the length of the hypotenuse of a right-angled triangle *ABC,* where $\hat{B}$ = 90°, $\hat{A}$ = 30° and AB = 10cm (K) 4. Solve for x:$4\sin x - 1 = 3$ for $x \in [0°;90°]$ (C)
	Problems in two dimensions.	**Example:** Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. At what height above the ground do the two ropes intersect? What if the poles were at different distance apart? (P)

TRIGONOMETRY GRADE 11	1. Derive and use the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\theta \neq k.90°, k$ an odd integer; and $\sin\sin^2\theta + \cos^2\theta$=1 2. Derive and use reduction formulae to simplify the following expressions: 2.1. $\sin(90° \pm \theta)$; $\cos(90° \pm \theta)$; 2.2. $\sin(180° \pm \theta$; $\cos 180° \pm \theta$; $\tan(180° \pm \theta)$ 2.3. $\sin(360° \pm \theta)$; $\cos(360 \pm \theta)$ $\tan(360° \pm \theta)$ and 2.4. $\sin(-\theta)$; $\cos(-\theta)$; $\tan(-\theta)$ 3. Determine for which values of a variable an identity holds. 4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals.	**Comment:** • Teachers should explain where reduction formulae come from. **Examples**: 1. Prove that $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$. (R) 2. For which values of θ is $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$ undefined? (R) 3. Simplify $\frac{\cos(180° - x)\sin(x - 90°) - 1}{\tan^2(540° + x)\sin(90° + x)\cos(-x)}$ (R) 4. Determine the general solutions of $\cos^2\theta + 3\sin\theta = -3$. (C)
	1. Prove and apply the sine, cosine and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules.	**Comment:** • The proofs of the sine, cosine and area rules are examinable. **Example:** In ΔABC *D* is on *BC*, ADC = θ, *DA* = *DC* = *r*, *bd* = 2*r*, *AC* = *k*, and $BA = 2k$  Show that $\cos\theta = \frac{1}{4}$

TRIGONOMETRY GRADE 11	Point by point plotting of basic graphs defined by$y = \sin\theta$, $y - \cos\theta$, and $\tan\theta$ for $\theta \in [-360°; 360°]$ 4. Investigate the effect of the parameter *k* on the graphs of the functions defined by *y* = sin(*kx*), $y = \cos(\mathrm{k}x)$ and $y = \tan(kx)$ 5. Investigate the effect of the parameter *p* on the graphs of the functions defined by *y* = sin(*x* + *p*), , *y* = cos(*x* + *p*), and *y* = tan(*x* + *p*), 6. Draw sketch graphs defined by: y=a sin k (x+p), y=a cos k (x+p) and y= k (x+p) at most two parameters at a time.	**Comments**: • A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful. • After summaries have been compiled about basic features of prescribed graphs and the effects of parameters *a* and *q* have been investigated: a: a vertical stretch (and/or a reflection about the x-axis) and *q* a vertical shift. The following examples might be appropriate: • Remember that graphs in some practical applications may be either discrete or continuous. **Comment:** • Once the effects of the parameters have been established, various problems need to be set: drawing sketch graphs, determining the defining equations of functions from sufficient data, making deductions from graphs. Real life applications of the prescribed functions should be studied. • Two parameters at a time can be varied in tests or examinations. **Example**: Sketch the graphs defined by $y = -\frac{1}{2}\sin(x + 30°)$ and *f* (*x*) = cos(2*x* − 120°) on the same set of axes, where $-360° \leq \theta \leq 360°$.

TRIGONOMETRY GRADE 12	1. Definition of a *function.* 3. Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, shape and symmetry, intervals on which the function increases /decreases. Compound angle identities: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$; $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$; $\sin 2\alpha = 2\sin\alpha\cos\alpha$; $\cos 2\alpha = \cos\cos^2\alpha - \sin^2\alpha$; $\cos 2\alpha = 2\cos^2\alpha - 1$; and $\cos 2\alpha = 1 - \sin^2\alpha$	2. Accepting $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$, derive the other compound angle identities. (C) 3. Determine the general solution of $\sin 2x + \cos x = 0$ (R) 4. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$ (C)

TRIGONOMETRY GRADE 12	1. Solve problems in two and three dimensions.	**Examples:**  1.TP is a tower. Its foot, *P*, and the points *Q* and *R* are on the same horizontal plane. From *Q* the angle of elevation to the top of the building is x. Furthermore, y and the distance between *P* and *R* is *a* metres. Prove that *TP* = *a* tan $x$cos y–$\sqrt{3}\,\sin y$ (C) 2. In ΔABC, $AB \perp BC$. Prove that: 2.1 $a = b\cos C + c\cos B$ where $a = BC; b = AC; c = AB$ (R) 2.2 $\frac{\cos B}{\cos C} = \frac{c - b\cos A}{b - c\cos A}$ on condition that $\hat{C} \neq 90°$) (P) 2.3 $\tan A = \frac{a\sin C}{b - a\cos C}$ (on condition that $\hat{A} \neq 90°$) (P) 2.4 *a* + *b* + *c* = (*b* + *c*) cos *A* + (*c* + *a*) cos*B* + (*a* + *b*) cosC. (P)

Along with utilizing the CAPS curriculum guidelines, Bloom's cognitive levels were also applied to develop the teaching program and the instruments. Four cognitive levels were used in this study. The cognitive demand levels are in accordance with the South African curriculum standards and are based on Bloom's Taxonomy (DBE, 2011). Descriptors for each cognitive level and the approximate percentages of tasks in tests and examinations which should be at each level are given in Table 3.2.

The first two levels (knowledge and recall (K), performing routine procedures (R)) are regarded as demanding low-level cognitive skills according to Boston and Smith (2009) classification, while the last two categories (performing complex procedures (C) and problem solving (P)) are regarded as requiring high-level cognitive skills (critical and creative thinking skills) (Boston & Smith, 2009).

Table 3.2: Amended Bloom's Cognitive Levels

Cognitive levels	**Description of skills to be demonstrated**	**Examples**
Knowledge (K) **20%**	Straight recall • Identification of correct formula on the information sheet (no changing of the subject) Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = h(x) = 3\sin x + 2; x \in [-180°; 180°]$ 2. Write down the equation of the function of x if moved 30° to the left and two units up.
Routine (R) Procedures **35%**	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1.Solve for $\sin x = \cos 3x$ where $x \in (-180°;\ 180°)$ 2. Determine the general solution of the equation $2\sin(x - 30°) + 1 = 0$ 3. Prove that: $\frac{\cos 2x - 2\sin^2 x}{\cos x - \sin 2x} = \frac{1 + 2\sin x}{\cos x}$
Complex (C) Procedures **30%**	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious	1. Write down the coordinates of a point on the graph of sinθ which corresponds to P on the unit circle.

	route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	
Problem (P) Solving **15%**	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	1.Prove that: $\frac{\cos 2x-2sin^2 x}{\cos x-\sin 2x} = \frac{1+2\sin x}{\cos x}$ 2. If $f(x) = \cos x$ and $g(x) = -2\cos(2x - 30) - 2$, describe the transformation from f to g. ,

(From DBE, 2011)

According to DBE (2011c), tests should cover the following four cognitive levels; (1) knowledge (20%); (2) routine procedures (35%); (3) complex procedures (30%); and, (4) problem solving (15%). This was the format followed in the current study to ensure that a representative sample of all content areas and all instructional levels were included in the test. This could not be followed exactly since the test had to cover exactly what was taught in the class and some of the questions that are normally asked at the end of the year could not be asked at this time, e.g. the 2 and 3 dimensional problems. In this regard, questions were selected in such a way that they covered the four cognitive levels (see, Table 3.2).

3.5.3 ELABORATION OF TASKS IN THE TESTS

It should be noted here that some of the theoretical tasks for post-test were slightly modified so as to avoid memorisation of the memo from the pre-test, and to see the improvement of students in performance of the concepts (McKnight et al., 2000). For example, in the *pre-test* students were required to draw $h(x) = 3sinx - 2$. In the *post-test* the question was to draw $h(x) = 3sinx + 2$.

The achievement tests (see Appendices 1 & 2), included 5 open-ended questions. Table 3.3 presents the concepts addressed by the tasks of each test-item. The tasks were grouped according to the main subsections that they were intended to assess, i.e., connections between relationships of trigonometry functions, properties of functions, etc.

Table 3.3: Topics covered by the questions in the pre- and post-test

QUESTION	CONCEPT	CLARIFICATION
Q1.1a-c	Connection of algebra to graphical representations	Translating or construction of graphs from given equations/ formulae
Q1a	Connection of algebra to graphical representations	Plotting the graph
Q1b	Connection of algebra to graphical representations	Plotting of graphs
Q1c	Connection of algebra to graphical representations	Plotting of graphs
Q1.2 a-h	Interpretation and analysis	Interpretation or Determination of properties of trigonometry functions from given formulae and drawn graphs
Q1.2a		Amplitude
Q1.2b		Amplitude
Q1.2c		Period
Q1.2d		Domain
Q1.2e		Range
Q1.2f		Maximum value
Q1.2g		Maximum value
Q1.2h		Asymptotes
Q1.2i; Q4	Transformations	Explaining transformation in words, and or by using equations, and or graphically.
Q2a-c	Connecting Graphical representations to algebra	Deriving equations of trigonometry functions from given graphical representations
Q3	Graphical analysis from Multiple graphs	Comparing and Analysing given multiple graphs and their formulae
Q3a		y-intercept
Q3b		x-intercepts
Q3c	Graphical	Labelling on a graph

Q3d	analysis from Multiple graphs	Determining x-coordinate from the graph
Q3e		Point(s) of intersection
Q3f		Shading
Q5.1.1 - 5.1.2	Integrating a unit circle to a right angled triangle	Relating the movement of a point on a unit circle to vertical and horizontal displacements on a triangle/ x and y-coordinates of the point
Q5.1.3	Connecting unit circle to graphs	Relating the movement of a point on a unit circle to a point on a graph
Q5.2	Prove the identity	Simplify and equate the expression
1Q 5.3 ; Q5.4	Derive general solutions and specific solutions	Use of rotation angles and periodicity

3.5.4 RELIABILITY AND VALIDITY

Reliability and validity measure the quality of the research instruments. Reliability is concerned with the degree of consistency, stability and repeatability of the attributes to be measured (Brink & Wood, 1998; Bush, 2002). Validity of an assessment, on the other hand, is the degree to which it measures what it is supposed to measure (Creswell, 2008).

3.5.4.1 VALIDITY OF THE TRIGONOMETRY FUNCTION TESTS

Types of validity include, content, construct, and criterion validity (Kothari, 1990; McKnight et al., 2000). Content validity was determined on the test.

Content validity is the degree to which the content of an instrument covers the extent and depth of the topic it is supposed to cover (Creswell, 2008). It is useful for evaluating educational research tests (Lewis, 1999). Content validity is most often measured by relying on the knowledge of people who are familiar with the concepts being measured. These subject-matter experts (SMEs) are usually provided with the instruments and are asked to offer responses on how well each question measures the concepts in question. Their responses are then statistically analyzed (Creswell, 2008). In addition to using subject matter experts' inputs and judgement for content validity, McKnight et al., (2000), add that one could first use one's professional experience and judgement to assess face validity of specific items in the list of items assessed.

Thus, for content validity in this study, the researcher first used her professional experience and judgement as a mathematics educator with a lot of teaching experience in Mathematics at the grade 12 level. The researcher also involved five (5) other subject matter experts to assess the content validity of the tests. The instruments were aligned to and based on the curriculum standards and clarification of the topics as described in the South African CAPS curriculum standards (DBE, 2011). These are indicated in Tables 3.1 and 3.2.

For the content validity therefore, experienced mathematics educators and experts in the mathematics field were used to evaluate the questions. The subject matter experts define the curriculum universe (the content domain) and assess whether the test adequately samples that domain (Lawshe, 1975). Lawshe's method of measuring content validity relies on expert responses to each item as being essential or not essential to the performance of the concept (Lawshe, 1975). Accordingly, if more than half the panelists determine an item as essential, then that item has some content validity. With larger numbers of panel members agreeing that a particular item is essential then the item has greater levels of content validity. Using these assumptions Lawshe (1975) developed the content validity ratio, CVR:

$$CVR = (n_e - N/2)\ /(N/2)$$

Where $CVR =$ content validity ratio, $n_e =$ number of SME panelists indicating essential, $N =$total number of SME panelists. This formula yields values which range from +1 to -1; positive values ≥ 0.5 indicate that at least half the SMEs rated the item as essential. The mean CVR across items (Table 3.4) was used as an indicator of overall test content validity. CVI, which was calculated as the total mean CVR for all the retained items was also determined (Lawshe, 1975).

Table 3.4: CVR values for the different questions

QUESTIONS	$n_e =$	$N =$	$CVR =$
Q1.1a	4	5	0.80
Q1.1b	5	5	1.0
Q1.1c	5	5	1.0
Q 1.2a	5	5	1.0
Q1.2b	5	5	1.0
Q1.2c	5	5	1.0
Q1.2d	5	5	1.0
Q1.2e	5	5	1.0
Q1.2f	5	5	1.0
Q1.2g	5	5	1.0
Q1.2h	5	5	1.0
Q1.2i	5	5	1.0
Q2a	5	5	1.0
Q2b	5	5	1.0
Q2c	5	5	1.0
Q 3a	5	5	1.0
Q3b	5	5	1.0
Q3c	5	5	1.0
Q3d	5	5	1.0
Q3e	5	5	1.0
Q3f	5	5	1.0
Q4	5	5	1.0
Q5.1.1	5	5	1.0
Q5.1.2	5	5	1.0
Q5.1.3	4	5	0.80
Q5.2	5	5	1.0
Q5.3	5	5	1.0
Q5.4	5	5	1.0
		Total Mean CVR	0.99

In this study the content validity of the instruments was ensured by firstly requesting inputs, comments and moderation from the subject experts. The experts consisted of two high school mathematics educators, two high school mathematics head of departments (HODs) and one mathematics subject advisor. The experts were asked to determine whether the content reflected the content domain. The experts were given the question papers and tables on which to respond. From the experts' responses the content validity was calculated and presented in Table 3.5. For each test item CVR was greater than 0.8 therefore the items in the instruments were considered to be content valid. The CVI which is equal to 0.99 was considered to be highly relevant.

3.5.4.2 RELIABILITY OF THE TESTS

Four methods are used to estimate reliability. These are test-retest, alternate form, split halves and internal consistency (McKnight, Magid, Murphy, & McKnight, 2000). Reliability measures consistency over time and over similar samples. According to Gay and Airasian (2003), for a cognitive test in which the questions are not scored dichotomously the reliability can be calculated by using the Spearman Brown formula $R = 2r/(1+r)$, where r is the correlation coefficient between split half test results, or between test and the re-test results, or between two equivalent randomly assigned groups. In this study two randomly assigned groups were used. The correlation coefficient must be significant at 95% or higher confidence interval (Cohen, Manion, & Morrison, 2007). According to (Wells & Wollack, 2003), if the Spearman Brown coefficient (R) ranges from 0 to 1.00 and the values are close to 1.00 this indicates high consistency. The results in this study showed significant correlation coefficient of 0.68 (see Appendix 10). Hence, the reliability coefficient of the test using the Spearman Brown formula $R = 2r/ (1+r)$ was 0.81. Thus the results obtained in this study implied that the testing was very reliable.

3.5.5 METHODS USED IN THE CLASSROOMS

In the control group, how an educator introduces trigonometry depends solely on the educator. Normally two different approaches are taken: the right-angled or unit circle approaches, could be used for its introduction. The traditional method explained here is as the educator in the control group, as is the norm with most teachers, described his approach to trigonometry teaching. The teaching and learning here followed after the Grade 10, 11, and 12 trigonometry syllabi as prescribed in the CAPS curriculum.

Various CAPS compliant textbooks were used to assist with lesson preparations (Laridon et al., 2002; Smith, 2012). Students were taught using the board and chalk method.

The experimental trajectory instruction depended on the integration of the three contexts from the beginning. However, before using ICT, the teacher presented the summary of the trigonometry functions on the board. This is in accordance to what was said to be working by (Ross et al., 2011) on the sequence of teaching trigonometry functions. Starting with an explanation of the board then introducing ICT has been said to be more effective than starting the lessons (Ross et al., 2011). During the last 4 lessons, the teacher went back to using the whiteboard and students used calculators.

3.5.5.1 TEACHING IN THE CONTROL GROUP -TRADITIONAL METHOD

In the control group, the trigonometry triangle context method was used to introduce the subject.

Lesson 1

Firstly, students were taught to define the basic trigonometry functions of angles (sine, cosine and tangent) from similar triangles, where θ is an angle between 0° and 90° (see figure 3.2).

Definition of functions from right angled triangles:

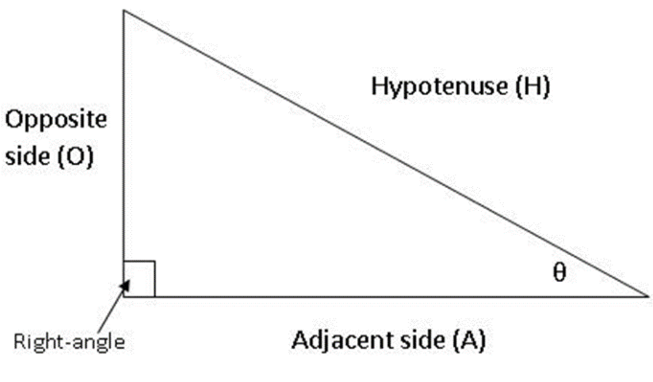


Fig 3.2: right angle triangle where

$\sin\theta = \frac{opposite\ side}{hypotenuse}$ and $cos\theta = \frac{adjacent\ side}{hypotenuse}$

This was followed by the definition and derivation of the tangent relation:

$$tan\theta = \frac{sin\theta}{cos\theta} = \frac{opposite\ side}{hypotenuse} \div \frac{adjacent\ side}{hypotenuse} = \frac{opposite\ side}{hypotenuse} \times \frac{hypotenuse}{aadjacent\ side} = \frac{opposite\ side}{adjacent\ side}$$

Note: the *hypotenuse* was described as a radius of a circle from right at the beginning.

Reciprocals of the trigonometry functions namely: $cosec\theta$, $sec\theta$ and $cot\theta$ were then defined. Thereafter, angles, change of angles made by rotation of a vector arm in a clockwise and or anti-clockwise directions were presented and explained. A Mnemonic, CAST- (Cos,All,Sin,Tan), was used to show the signs of the basic trigonometry functions in the different quadrants. CAST starts from the fourth quadrant. One would say cos is positive in the fourth quadrant, all trig functions are positive in the first quadrant, etc.

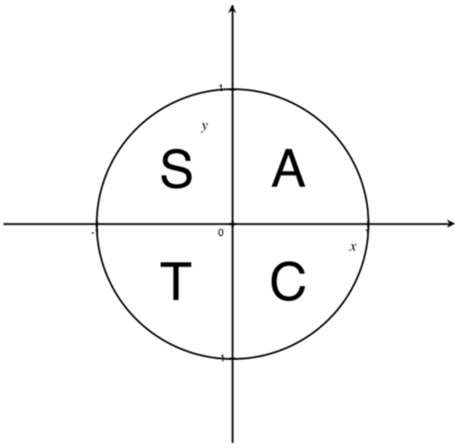


Fig 3.3: Quadrants showing trigonometry function signs

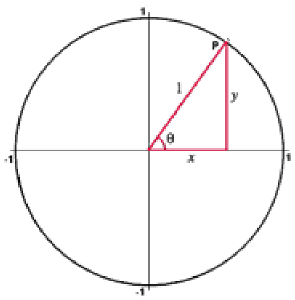


Figure 3.4: The Unit circle representation

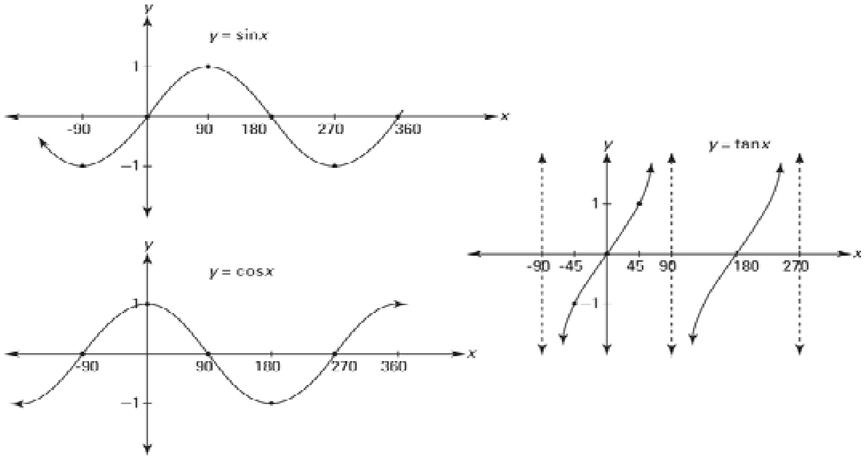


Figure 3.5: Trigonometry function graphs

Just before class activities were assigned, a few examples were given by the teacher. When working with a Cartesian plane, the educator emphasised that the students should first join the terminal end of the vector arm to the x-axis to make a right angled triangle. Students were allowed to work in groups.

Lesson 2

Special angles diagrams were drawn and explained. One example of a task using special angles was worked on by the educator as a demonstration. Square identities were derived and explained. Examples that involve simplification of trigonometry expressions and solving of basic equations were shown to students. The students were then given exercises to work on. Solutions were given on the board. More complex tasks were given to students as homework.

Lesson 3

At the beginning of the lesson, corrections from the previous day's homework were reviewed, written and explained. Graphical representations of the basic trigonometry functions were presented. The properties of the functions were also described. Students were then taught how to plot the trigonometry functions whilst using the table method. Class activities were given and the educator assisted the students in the plotting of various graphs. Homework that required the plotting of various graphs and the analysis thereof, was given.

Lesson 4

Corrections of homework from the previous day were given and explained using translation. Tasks on plotting of graphs, translation and derivation of formulae from given graphs were given. Students were encouraged to work in groups. Corrections were given.

The lesson ended with students being introduced to reduction formulae and compound angles. Homework on solving and simplifying expressions and equations was given.

Lesson 5

Solutions to the given homework were discussed. Students were shown how to work on tasks that require general solutions, i.e., $\theta = \alpha + k360°; k \in \mathbb{Z}$. Class activities on simplifying expressions, proving identities and determining general and specific solutions of given equations were given and worked on in the classroom.

3.5.5.2 TEACHING IN THE EXPERIMENTAL METHOD

Geogebra was used in this class. At the beginning of each lesson, the educator placed an outline on the blackboard of what was going to be covered. The educator created a classroom environment in which social interaction was highly encouraged. This was an environment where students used computers (Geogebra) to act on trigonometry functions. The students were learning and transforming different representations and how they relate to each other.

Since the study on the experimental group was based on the use of ICT, Geogebra, in the teaching and learning of trigonometry, a sample of the activities used during the lessons is presented below.

Lesson 1

At the beginning, the white board, projector and computers were used for the introduction of trigonometry concepts and representations. The educator then introduced Geogebra and its facets. This was then followed with the students working on the computers both individually and in pairs. The students were also seen to be moving from one computer to another to seek assistance or to assist others whenever necessary.

The following applets were used:

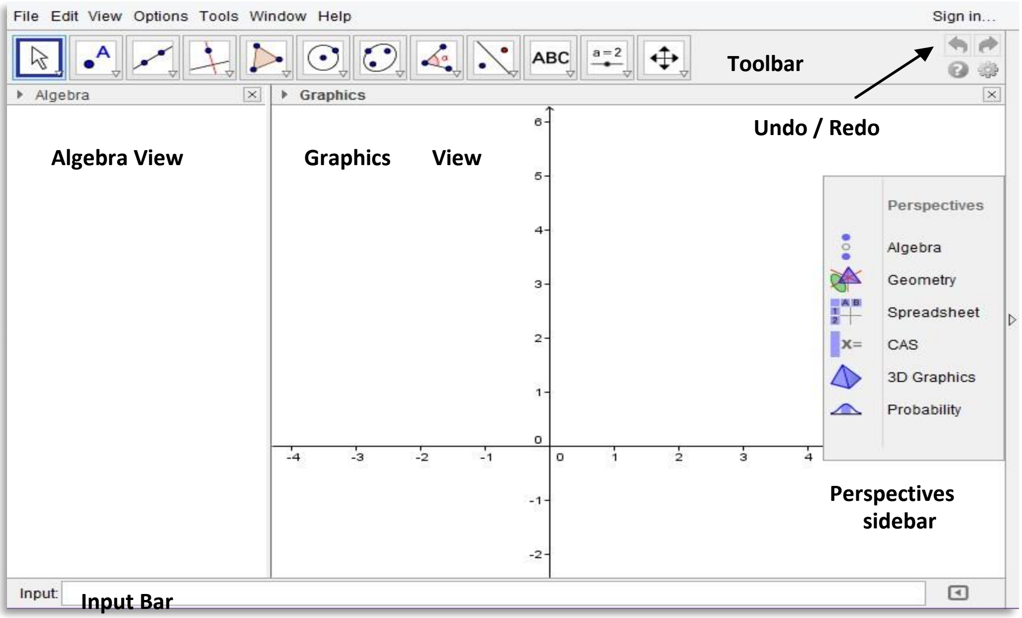


Figure 3.6: Geogebra applet 1a

This was used to introduce Geogebra and to familirialize students with the use of computers in general.

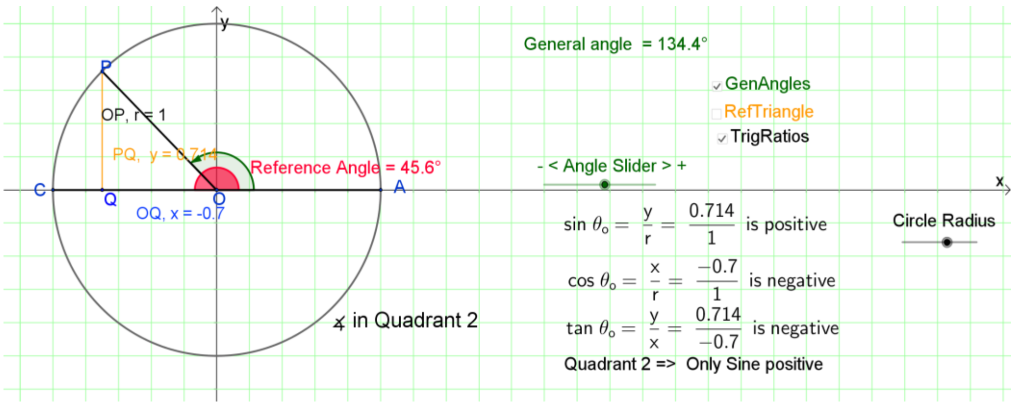


Figure 3.7: Geogebra applet 1b

The applet in Figure 3.7 was used to introduce:

- quadrants,
- angles,
- basic trigonometry functions per quadrant,
- trigonometry functions on general angles
- Pythagoras theorem and
- Other Geogebra operations

- Negative and positive angles

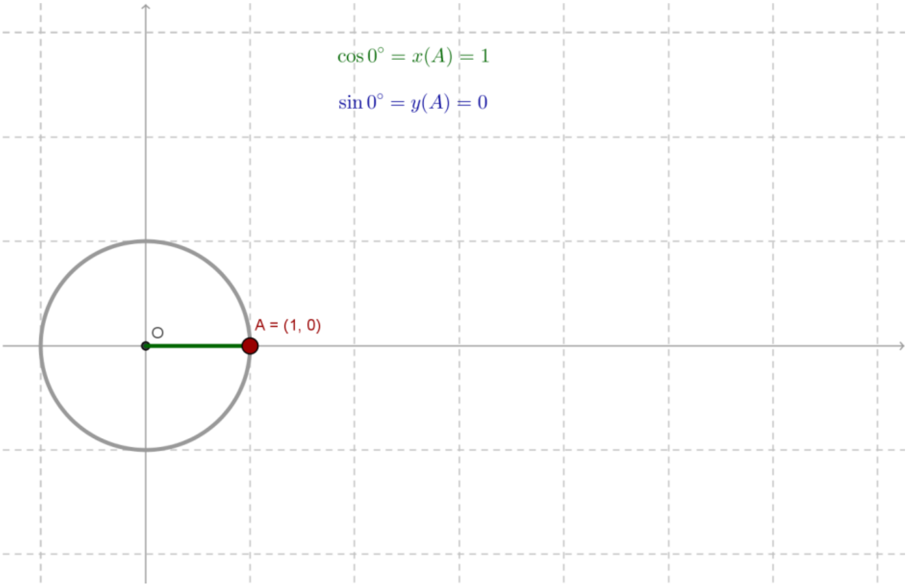


Figure 3.8: Geogebra applet 1c

The applet was to assist students in relating the movement of a point about a unit circle to the basic trigonometry functions using the vertical displacement (y-coordinate) and horizontal displacement (x-coordinate).

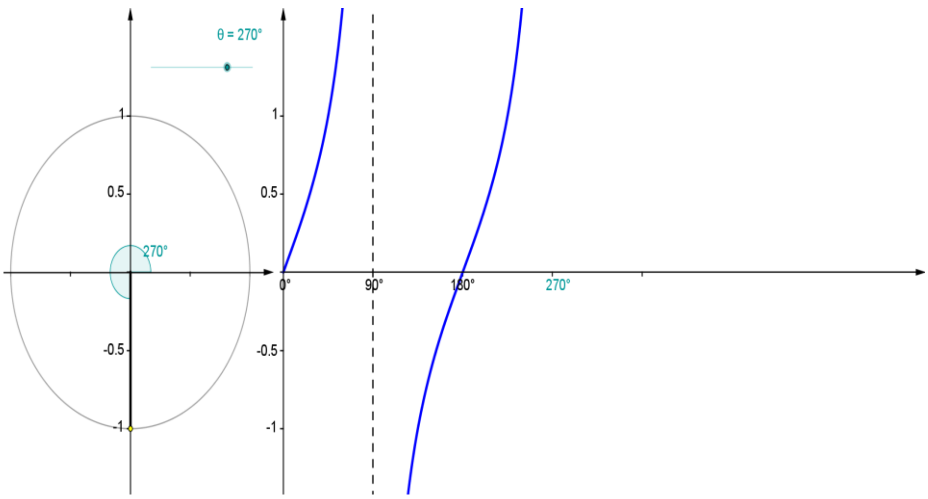


Figure 3.9: Geogebra applet 1d

The applet represented in figure 3.4 was to assist student in relating movement of a point around a unit circle, to the development of any trigonometry graph.

Lesson 2

Homework was first corrected in class. After that an Applet was used to demonstrate how to plot trigonometry graphs. Thereafter students were seen trying out a variety of graphs.

Students then drew different kinds of graphs. They were given tasks to assist them in developing the required objectives.

Students had to use two applets on their computers.

- $Y = a\sin(x + b) + c$
- $Y = a\cos(x + b) + c$

From here they were required to determine the characteristics of the graphs such as the amplitude, intercepts, period, range and domain. Thereafter students were seen trying out a variety of graphs among themselves while constantly receiving assistance from the educator.

Homework was then given on the plotting and transformation for the tan function.

Lesson3

Corrections to the homework on tan functions was done by the educator and students in using:

applet3.1 (the effects of a, b, c and d on the basic trig function)

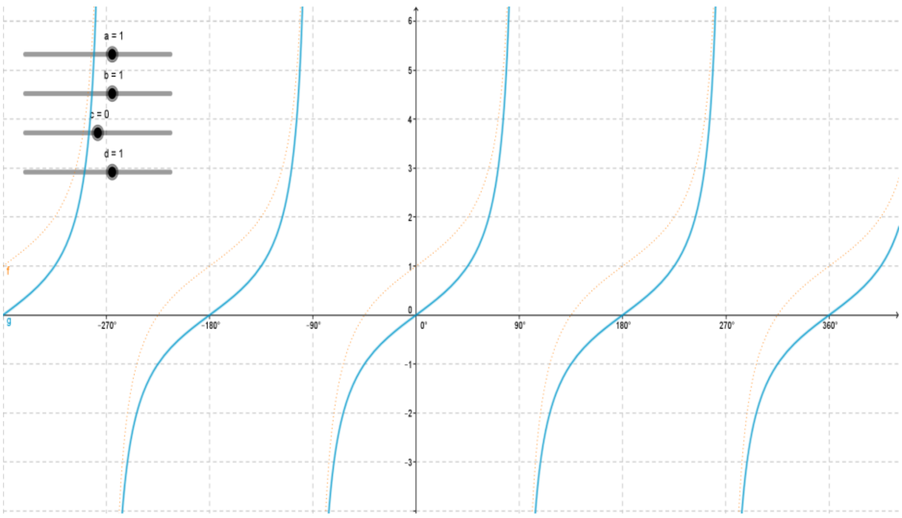


Figure 3.10: Geogebra applet showing the effects of applet 1e showing the effects of a, b, c and d on the basic trigonometry functions

The students were then given tasks to analyze relationships between different functions such as points of intersection and $f(x) < g(x)$. Figure 3.6 is an example of the applets they were given to students to work on.

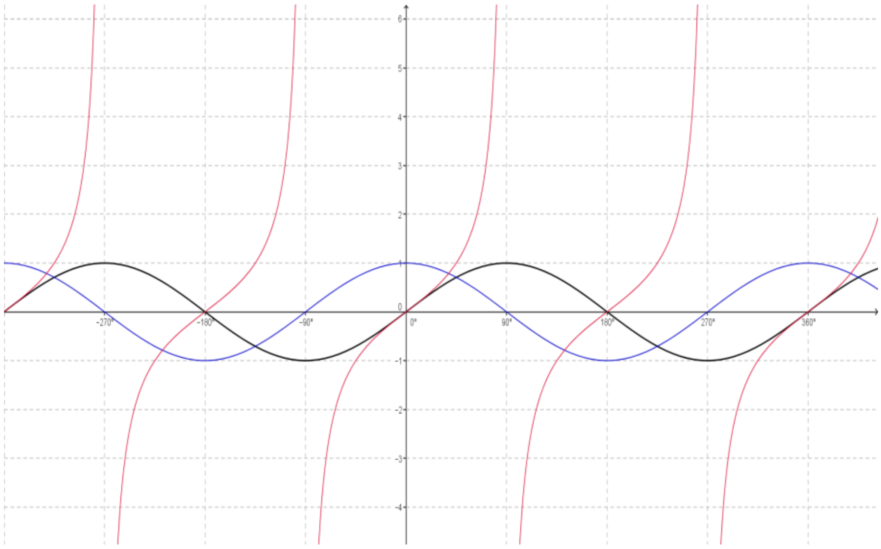


Figure 3.11: Geogebra applet for the analysis of trigonometry graphs

They played around with different kinds of graphs, discussed them in class and then worked on their own sets of graphs and analyses.

Lesson 4-5

From the computer laboratory, the students moved to their normal classroom, where the teacher used the whiteboard and the projector to discuss and explain the various concepts. Students were still seen to be actively working individually or in groups of four or five.

In these lessons the following topics were dealt with:

- Simplifying expressions;
- General solutions and solving equations were dealt with here;
- Reduction formulae, negative angles, compound angles were used in all this

Self- and peer-assessments were used during the last two lessons.

3.5.6 DATA ANALYSIS

Data analysis is the process of making sense out of data, which involves interpreting, transforming and modelling what participants have said, how they have responded and what the researcher has seen and read in order to derive or make meaning out of the process.

Researchers go through similar steps for both qualitative and quantitative data analysis. Creswell and Plano Clark (2011) list the data analysis steps as follows:

- preparing the data for analysis
- exploring the data
- analysing the data
- representing the analysis
- interpreting the analysis
- validating the data and interpretation

In this study, descriptive and inferential statistics were used for the quantitative data analyses. The type of statistical analysis employed in the study was the t-test. The t-test is the most commonly used test in mathematics education that involves small sample sizes (McKnight et al., 2000). In addition to the t-test being more favourable for a two-grouped sample, (Laerd, 2013) states that ANOVA is normally used on three or more independent groups. Hence the t-test was used. Computationally, the statistical software package Statistical Package for the Social Sciences (SPSS) (version 23) was used.

3.6 ETHICAL CONSIDERATIONS

Ethics is a philosophical term derived from the Greek word, *ethos*, meaning character or custom and a social code that conveys moral integrity and consistent values (Partington, 2003). According to (Babbie & Mouton, 2001), ethics of science is concerned with what is right or wrong when conducting research. To this end, all researchers are subjected to ethical considerations, regardless of their choice of research methodologies (Gratton & Jones, 2010). Therefore, in the current study, the ethical considerations required a right of entry; voluntary participation; anonymity and confidentiality; and ethical clearance from the University of South Africa (UNISA).

During all aspects of the study, the following ethical considerations were taken into account:

- Ethical clearance was sought for, from the research committee of UNISA for permission to conduct the study. The permission was granted (see Appendix 3)
- The Deputy Director of Education granted consent for research in schools (see Appendix 5)
- Permission was requested from the schools, school governing bodies and each teacher and student. Parents were not informed since the students were all

above 18 years. All the appropriate individuals signed the approval and voluntary participation forms. All the involved participants were informed of the nature of the study, research process participation requirements, confidentiality and the researcher's contact details.

- For confidentiality, the names of the schools and the names of the students / educators were not disclosed in the study. For professional reasons and anonymity, the content and results of the study were discussed with relevant people only
- To minimise income or digital exposure disparity - It was expected that some students may have studied computers before or had computers at home and some may not have used computers at all before, so in order to avoid disparity it was imperative that all the students in the experimental class were first introduced to the Geogebra (Voronkov, 2004).

3.7 SUMMARY

This chapter discussed the methodology adopted in the study, that is , the paradigm, which informed the study, the research design and the research methods (e.g. the study sample and population, the procedure for conducting the research and the research instruments, the different instructional methods used in the experimental and control groups, data analysis and the ethical considerations). The methodology was developed with the objective of obtaining reliable and valid data to respond to the main purpose of the study: to determine which instructional method was more effective in the teaching and learning of trigonometry functions.

CHAPTER FOUR

FINDINGS

This chapter presents the analysis and interpretation of data that were collected from the 61 students in the two (2) sampled schools. Data were collected using achievement tests (pre/posttests) (Appendices 1 & 2). Independent-samples t-test was used as the statistical measure to establish the comparability of the two groups (the experimental and control) before and after the intervention. Using the Statistical Package for the Social Sciences (SPSS) the independent-samples t-test output gives two tables: a descriptive and an inferential statistics table.

The Descriptive/Group Statistics (e.g. mean)

– Statistical procedures used for summarizing, organizing, graphing and describing data. This statistics cannot be generalized beyond the analyzed sample data.

Inferential Statistics (e.g. t-test)

– Statistical procedures that allow one to draw inferences to the population on the basis of sample data. Represented as tests of significance (test relationships and differences) (Devonish, Gay, Alleyne, & Alleyne, 2006).

The group/ descriptive statistic table provides the sample groups' means, standard deviation, and the number of participants per group. From this, the mean difference between the two groups can be worked out. A $t - test$ was carried out in order to determine whether the mean difference between the two groups was statistically significantly different. The $t - test$ is a statistic that assesses whether the means from data of one or two groups are reliably different from each other (Rice, 2006).

In the inferential table, firstly the Levene test is used to determine the equality of variance between the two groups (Levene, 1960), that is, the test determines whether the two groups have about the same or different amounts of variability between their scores. If the significant value is less than 0.05, then one could conclude 'equal variance is not assumed' and the second row is used for the $t - test$, otherwise if significant value is greater than 0.05 then equality of variance between the two groups is assumed and the first row is used to work on the $t - test$.

Using the $t - test$, if sig p<0.05 one concludes that there is a statistically significant difference between the mean scores of the two groups.

The present chapter presents the findings from the assessments and the statistical analyses of the data. In answering the research questions focusing on the overall performance on the achievement tests; the connections; the interpretations; solving equations; and proving identities of trigonometry functions.

The chapter is organized into three main sections,

- The statistical analysis and interpretation of the pre-test,
- the statistical analyses of the general post-tests,
- the statistical analysis and interpretations of the grouped questions in the post-test, and finally

A summary of the chapter.

4.1 PRE-TEST RESULTS

The pre-test was administered before the treatment to determine whether the means of the two groups were comparable. It was found that no statistically significant difference existed between the pre-test scores of both the control and the treatment groups. This showed that the two groups had similar basic understanding and knowledge of trigonometry functions before the intervention. This is indicated in Tables 4.1 and 4.2.

Table 4.1: Groups mean achievement in the pre-test

		N	Mean	Std. Deviation	Std. Error Mean
Pretests	Experimental	27	15.8519	9.29724	1.78926
	Control	34	13.5294	9.45239	1.62107

The total number of students who wrote the pre-test was 61. Of the 61 students, 27 participated in the experimental group whilst 34 were in the control group. From the table above it can also be deduced that with a mean difference of 2.3225, on average, the difference between the students' results in both groups was similar.

Using the Levene test as showing in the first column of Table 4.2, the significant value = 0.916 (sig >0.05). This meant that the scores in both groups did not vary significantly, that is, the variability in the scores was about the same. The $t - test$ results would thus be read from the first row. Here, the sig (2-Tailed) =0.34 >0.05. It was thus concluded that there was no statistically significant difference between the scores for the pre-tests of the two groups (the experimental and the control groups).

Table 4.2: t-test analyses of the students' achievement in the pre-test

Independent samples test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Pre_Tests	Equal variances assumed	.011	.916	.960	59	.341	2.32244	2.41906	-2.51809	7.16297
	Equal variances not assumed			.962	56.310	.340	2.32244	2.41440	-2.51359	7.15847

4.2 RESULTS OF THE GENERAL PERFORMANCE IN THE POST-TEST

The post-test (see Appendix 2), which was to assess students' knowledge and understanding of the key points from the lesson sequence, was administered to the students three days after the intervention ended. All the students in the study took the test. They were requested to complete the test in one and half hours, and were told that their marks were only for research purposes and thus would not count towards their final end-of-year mathematics grade.

This section provided the statistical results and analyses of the overall performances in the post-test, the students' performance on connections, interpretation of trigonometry functions etc. This was to assist us in answering the questions in the main study. Statistical analysis on individual questions was also carried out (see Appendices 12-37).

4.2.1 RESULT OF THE GENERAL PERFORMANCE IN THE POST-TEST

To determine whether any significant statistical difference existed between the post-test scores of both the control and the experimental group, a descriptive statistics and an inferential statistics table are presented in Tables 4.3 and 4.4.

Table 4.3: Overall students' performance in the Post-test

Group Statistics					
	SchoolNames	N	Mean	Std. Deviation	Std. Error Mean
Post_Tests	Experimental	27	35.5185	11.20910	2.15719
	Control	34	20.4412	14.52937	2.49177

All students who wrote the pre-test also wrote the post-test. The total number of students who wrote the post-test was 61. Of the 61 students, 27 were in the experimental group whilst 34 were in the control group. From Table 4.3 it can be seen that the mean difference between the two groups was 15.08. It can also be seen that for each group the mean difference between the pre_test and post_test increased, with the control group mean increasing by 6.91 and the experimental group mean increasing by 19.67. This means there was an improvement in both groups, even though, it was evident that the improvement was considerably higher in the experimental group than in the control group. The standard deviation of the experimental group had a smaller increase than the standard deviation of the control group. This shows that there was a wider gap of performance between students who understood trigonometry functions in the control group than the gap in the experimental group.

Inferential statistics ($t - test$)

Using the Levene test as is shown in the first column of table 4.4, the significant value = 0.275 (sig <0.5). This means that there was a statistically significant difference between the two post-tests of the groups. The $t - test$ results were thus read from the second row. Here, the sig (2-Tailed) =0.000, which is smaller than 0.05. It can thus be concluded that there was a statistically significant difference between the scores of the post-tests of the two groups. One can then conclude that the difference between the two means are not likely due to chance but probably due to the intervention (the use of ICT in the experimental group).

TABLE 4.4: t-test analyses of the students' achievement in the post-test

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Post_ Tests	Equal variances assumed	1.214	.275	4.441	59	.000	15.07734	3.39486	8.28425	21.87044
	Equal variances not assumed			4.575	58.964	.000	15.07734	3.29581	8.48235	21.67233

4.2.2 RESULTS OF GROUPED QUESTIONS

For the analysis. The tasks/questions in the *pre_post-test* were grouped according to their common characteristics.

- Questions 1.1 a-c, 2a-2c and 5.1.1-5.1.3 were grouped into the group for connections of different representations of trigonometry functions;
- Questions 1.2a-h and Q3 were grouped under interpretations on trigonometry functions;
- Question 1.2i and 4 were grouped into the transformations of trigonometry functions;
- Questions 5.2 and 5.3 were placed into the derivation of general and specific solutions group; and
- Question 5.4 dealt with proving a trigonometry identity.

The clarification of the questions is given in Table 3.3. Figure 4.1 shows a graphical representation of students' performance on grouped and general questions of the post-test.

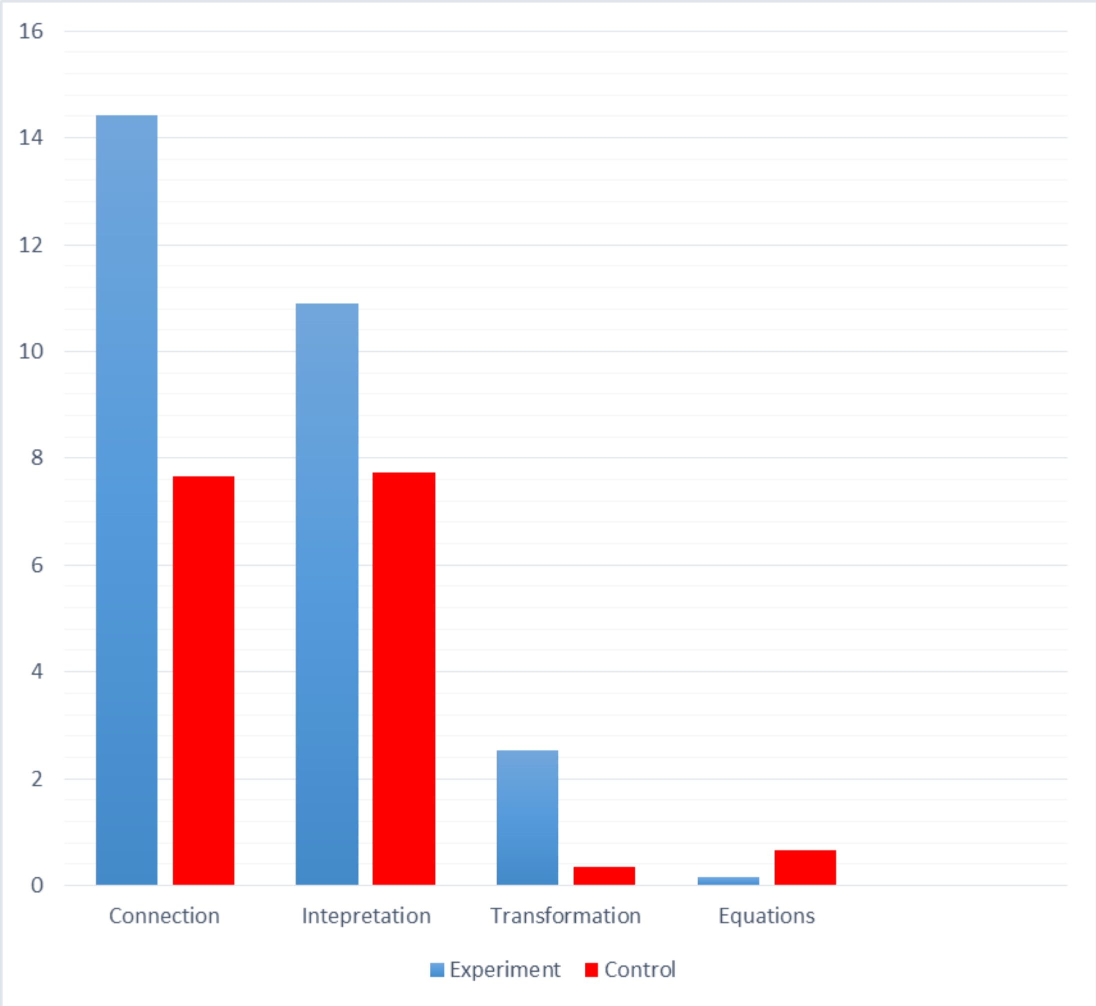


Figure 4.1: Graphical representation showing students' performance on grouped questions in the Post-test

4.2.2.1 CONNECTIONS

An independent samples $t-test$ was used to test for the possible effect of using Geogebra in the teaching and learning of connections between representations of trigonometry functions.

From the Table 4.6 the sig (2-tailed) =0.00 <0.05, the experimental group was associated with higher achievement in making connections between various representations of trigonometry functions (Experimental M= 14.41; Control M= 7.65). As such it can then be concluded that the difference between the two means is not likely due to chance but probably due to the intervention

Table 4.5: Students' mean achievement on connections

	N	Mean	Std. Deviation	Std. Error Mean
Experimental	27	14.41	2.978	.573
Control	34	7.65	4.735	.812

Table 4.6: t-test on students' ability to make connections of representations of trigonometry functions

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	1.684	.199	6.467	59	.000	6.760	1.045	4.669	8.852
Equal variances not assumed			6.802	56.324	.000	6.760	.994	4.770	8.751

4.2.2.2 INTERPRETATION

An independent samples $t - test$ was used to test for the effectiveness of Geogebra in the students interpretations of trigonometry functions tasks as shown on Table 4.8.

The result (t (59) =2.23, the sig (2-tailed) =0.03< 0.05 shows that there was a statistically significant difference between the performance of the experimental group and the control group on interpretation of representations of trigonometry functions with the experimental group being associated with higher performance (Experimental M= 10.89; Control M= 7.74).

Table 4.7: Students' mean achievement on interpretations of trigonometry functions

Group Statistics

	1	N	Mean	Std. Deviation	Std. Error Mean
8	Experimental	27	10.89	4.870	.937
	Control	34	7.74	5.925	1.016

Table 4.8: t-test on students' interpretations of trigonometry functions

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	2.044	.158	2.230	59	.030	3.154	1.414	.324	5.983
Equal variances not assumed			2.281	58.914	.026	3.154	1.382	.387	5.920

4.2.2.3 TRANSFORMATION

From the Table 4.10 $t\,(59)$ =5.56, p=0.00 with the experimental group being associated with higher performance in transformation of various representations of trigonometry functions (Experimental M= 2.52; Control M= 0.35). This represented statistically significant difference between the two groups.

Table 4.9: Students' mean achievement on transformations

	1	N	Mean	Std. Deviation	Std. Error Mean
1	Experimental	27	2.52	2.026	.390
	Control	34	.35	.917	.157

Table 4.10: t-test on students' transformation of trigonometry functions

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
1	Equal variances assumed	26.248	.000	5.564	59	.000	2.166	.389	1.387	2.944
	Equal variances not assumed			5.150	34.433	.000	2.166	.420	1.311	3.020

4.2.2.4 DERIVATION OF GENERAL AND SPECIFIC SOLUTIONS OF TRIGONOMETRY FUNCTIONS

From Table 4.12 Sig (2-tailed) = -1.588; p=0.12> 0.05. There is no statistically significant different between the scores of the experimental group and control group on students' achievement derivation of general and specific solutions of given trigonometry functions. (Experimental M= 0.15; Control M= 0.65).The mean difference between the two groups being 0.5.

Table 4.11: Students' mean achievement on general and specific solutions

Group Statistics

	1	N	Mean	Std. Deviation	Std. Error Mean
0	Experimental	27	.15	.602	.116
	Control	34	.65	1.704	.292

Table 4.12: t-test on students' general and specific solutions of trigonometry functions

Independent Samples Test

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	9.898	.003	-1.450	59	.152	-.499	.344	-1.188	.190
Equal variances not assumed			-1.588	42.836	.120	-.499	.314	-1.133	.135

4.2.2.5 PROOF ON TRIGONOMETRY IDENTITIES

None of the students in either of the two groups answered this question.

4.2.2.6 EQUATIONS

The solving of equations problems showed that neither the use of ICT nor the traditional method of teaching could be said to have influenced the results. The two groups were actually taught using the traditional method.

4.3 ADDRESSING THE RESEARCH QUESTIONS

To complete the study, the analysis from the data collected was used here to test the research hypotheses and answer the main and sub-research questions.

4.3.1 MAIN RESEARCH QUESTION

The main question in this study was: What is the effect of ICT on students' learning of trigonometry functions? To respond to this question, firstly, the independent samples $t - test$ was carried out, presented, and interpreted in section 4.2.1. This was to statistically analyse the overall performance of students on trigonometry functions. ICT was shown to have had a positive effect on students' learning of trigonometry functions.

Secondly, to answer the main question the following sub-questions were generated:

I. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students making connections between representations of trigonometry functions?

II. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students' analysis and interpretations of trigonometry functions?

III. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students learning transformation of trigonometry functions?

IV. Does the use of ICT in the teaching and learning of trigonometry functions have an effect on students' derivation of general and specific solutions of trigonometry functions?

V. Does the teaching and learning of trigonometry functions with ICT have an effect on students' proof of trigonometry functions identities?

In order to answer these questions, the related null hypotheses were tested.

- NULL HYPOTHESIS 1

Null Hypothesis 1: There is no statistically significant difference between the achievement of students taught using ICT and those taught using the traditional method in making connections between different representations of trigonometry functions.

Students in the experimental group performed better than the students in the control group in questions which involved connections between trigonometry functions; graphs; formulae and unit circles. This was evidenced by the results of the independent sample $t-test$ analysis on connections (see Tables 4.5 and 4.6). The results showed that the mean performance score of the experimental group (Experimental M= 14.41) were statistically significantly higher than the mean of the control group (Control M= 7.65). Therefore it can be concluded that the Null Hypothesis 1 can be rejected. The use of ICT in the teaching and learning of trigonometry functions had a positive effect on students' making of connections between different representations of trigonometry graphs.

- NULL HYPOTHESIS 2:

Null Hypothesis 2: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference in learning of properties of trigonometry functions.

On the interpretation of trigonometry functions (Q1.2a to Q1.2h) there was significant difference between the mean of the experimental and the control group, (Experimental M= 10.89; Control M= 7.74). Not that much significant difference between the two groups could be noticed as far as student's determination of domain and range of given functions (see Appendices 12A-21B; 28-337B). Thus, the hypothesis was rejected.

- NULL HYPOTHESIS 3:

Null Hypothesis 3: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students learning transformation of trigonometry functions.

The statistical analysis showed that there was a significant difference between the mean scores of the two groups (see Tables 4.9 and 4.10) on transformation of trigonometry functions. This shows that ICT has a positive effect on students' learning of transformations of trigonometry functions. This means that the null hypothesis 3 can be rejected.

- NULL HYPOTHESIS 4

Null Hypothesis 4: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students' derivation of general and specific solutions of trigonometry functions.

The results in Tables 4.11 and 4.12 showed that there was no statistically significant difference between the means of the two groups on general and specific solutions of trigonometry functions in the study. Thus it could be said from this that the performance of students on solving of equations could not be attributed to any of the teaching methods used in the study. This means that the differences between the means are likely due to chance and not likely due to either the intervention or the traditional method of teaching.

- NULL HYPOTHESIS 5

Null Hypothesis 5: The use of ICT in the teaching and learning of trigonometry functions has no statistically significant difference on students' proof of trigonometry functions identities.

The students did not answer this question.

4.4 SUMMARY

The purpose of this study was to evaluate the effectiveness of the use of Information and Communication Technology in the teaching and learning of trigonometry functions on Grade 12 between students who received instruction that used ICT and those who received traditional method instructions. Data were collected using a pre-test, and a post-test. The *pre-test* and *post-test* scores were analysed using $t - test$. The students' performance on individual test questions for the *post-test* were also analyzed and interpreted. The $t - test$ was used for the general and individual questions. The statistical analysis for individual questions is presented in Appendices 12-38.

The analyses of the data meant to provide answers to the research questions were carried out. The results of the analyses showed that a positive relationship existed between ICT use and students' achievement in making connections, interpretations and transformations of various representations of trigonometric functions.

CHAPTER FIVE

DISCUSSION OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

The main purpose of the study was to evaluate the effectiveness of the use of Information and Communication Technology (ICT) in the form of using Geogebra in a teaching and learning environment. APOS and AT were used in designing the intervention in the study.

In this chapter a summary of the study is presented, this is then followed by a summary of the findings/results from the experimental study. The chapter also presents the main discussion emanating from the findings, and then puts forward the conclusion and implications of the findings. The chapter ends with the limitations of the study and recommendations for future studies.

5.1 SUMMARY OF THE STUDY

The current study was motivated by continued poor performance by students in mathematics subjects (specifically, trigonometry functions). Trigonometry, a branch of mathematics, has been found to be a major challenge for students in the high school curriculum. From the literature review, some of the reasons for these challenges have been said to be due to the method of teaching (chalk and talk) and the inability of students to connect or link the different contexts of trigonometry functions. In this study the theoretical frameworks, APOS and AT were used to advise on the instruction process in the classroom. The ICT software (Geogebra) was used as a teaching and learning tool. ICT has been said to assist more in alleviating the problems encountered by students in the learning of trigonometry functions (Tall, 2009). This provided the impetus for the study.

The review of the literature on the use of ICT in the teaching and learning of trigonometry functions followed the introduction. The literature review focused on some of the available studies on trigonometry, the research carried out on the teaching and learning of trigonometry functions, ICT in teaching and learning, the software used (Geogebra), and ICT in the teaching and learning of trigonometry functions. It was found that there is still a scarcity of studies that have been carried out on the use of ICT on trigonometry, especially in South Africa. This study centred on a quantitative research approach, in which non-equivalent groups, *pre- post-test* and quasi-experimental design (or methodology) were employed.

In this study, two schools in the Mafikeng area which is in the North West province were selected. The sample consisted of 61 participants, with 27 students in the experimental group and 34 in the control group, both who were going to sit for the matric examinations at the end of 2016. Before the lessons both groups wrote the pre-test (see Appendix 1). This was to determine the comparability of the students before the intervention. The results of the pre-tests showed that the students from both groups were of comparable ability.

After the lessons, a post-test (see Appendix 2) was administered to all the students in the two groups. Overall, the experimental group (which used Geogebra) performed much better than the control group (where the traditional method of teaching and learning was used). This means the use of ICT had a positive effect on the teaching and learning of trigonometry functions. Students' answers to the grouped questions of the post-test were presented, statistically analysed and interpreted.

The $t - test$ (using the statistical software SPSS) was used for the analysis to first determine the equivalence or non-equivalence between the two groups and then to determine whether the two groups' results after intervention were significantly different or not.

5.2 SUMMARY OF THE FINDINGS

In response to the main question of the current study: What the effect of ICT on students' learning of trigonometry functions is, five sub-questions were developed to assist in answering the main question. It was found that ICT had a significant positive effect on students' ability to make connections between different representations of trigonometry functions, on the analysis and interpretations of the given tasks of various contexts of trigonometry functions and on transformations of trigonometry functions.

Neither the traditional method of teaching and learning nor the teaching with ICT was found to have had an effect on the students' derivation of general and specific solutions of trigonometry functions (see Table 4.11 and 4.12). It can thus be said that ICT has positively influenced the performance of the students. For individual analyses before the questions were grouped, see appendices 11-37B.

5.3 DISCUSSION OF FINDINGS

5.3.1 THE OVERALL PERFORMANCE OF THE STUDENTS

The overall performance of the students (in the assessment test on trigonometry functions) from the experimental group was better than that of the control group. That means ICT, Geogebra in this study, was shown to be more effective in the teaching and learning of trigonometry functions. This is consistent with the studies by Zengin et al., (2011); Demir, (2012) who found that the students' achievement after instruction that involved Geogebra improved more than for the students who did not receive such instruction.

5.3.2 CONNECTIONS OF TRIGONOMETRY FUNCTIONS

On connections of trigonometry functions, the experimental group was found to have scored higher than the control group. Here it can be deduced that the use of ICT is effective in the students' ability to make connections between different representations and contexts of trigonometry functions. The tasks that involved connections included drawing of graphs from given algebraic representations, derivation of formulae (equations) of graphs, and connecting a unit circle to a point on the graph (Table 3.3). Most of the students in this study managed to plot the graphs, which is in contradiction to what Demir (2012) found in his study that most students could not draw the cosine graphs. Similar to the results of Demir (2012), many students in this study, could connect a point on the unit circle to a point on a graph, which is in contrast to the findings of Brown (2005). Only a few studies have dealt with connections of trig functions and even less have been seen to be dealing with tangent functions. This study is the first comprehensive study that involves all the basic trigonometry functions and their connections.

5.3.3 INTERPRETATION AND ANALYSIS OF TRIGONOMETRY FUNCTIONS

When it comes to interpretation and analysis of trigonometry functions, the experimental group was found to have performed significantly better than the control group. Here we can deduce that the use of ICT is effective in improving the students' ability to interpret and analyze trigonometry functions. During the lesson in the experimental group the students only needed to type in equations which produced different trigonometry graphs. This allowed and gave them time to explore, investigate and interpret the properties of the different graphs. This was unlike situations where students would have to draw graphs manually from point to point and then analyse them. This is confirmed by (Clements, 2000) who stated that instant feedback from ICT programs encourages students to use conjectures and to keep exploring.

5.3.4 TRANSFORMATION OF TRIGONOMETRY FUNCTIONS

With transformation of trigonometry functions, the results of the $t - test$ in Table 4.10 show that there was a statistically significant difference between the mean of the experimental group and that of the control group. The experimental group performed much better than the control group. Similar results were found on individual questions based on transformation (Appendices 12-21; Appendices 24A, 24B; 28-337B). The findings suggest ICT-assisted instruction is preferable in supporting student learning of transformations of trigonometric functions. The finding was in agreement with the findings of Bakar, Ayub, Luan, & Tarmizi, 2010; Ross et al., 2011).

5.3.5 DERIVATIONS OF GENERAL AND SPECIFIC SOLUTIONS OF TRIGONOMETRY FUNCTIONS

On derivations of general and specific solutions of trigonometry functions, there was no statistically significant difference between the experimental and the control group (See Table 4.12). It was then concluded from the results of solving equations, that neither the use of ICT nor the traditional method of teaching could be said to have influenced the results. It should be noted here that after the teaching and learning of trigonometry functions, the students in the experimental group did not use Geogebra in the remaining lessons which involved solving equations, simplifying expressions and proving identities. The students showed a preference to using calculators. This was a preference which was also observed in the pilot study (Section 3.5). The students in the pilot study expressed that they prefer to use calculators since this is what they use during the examinations. They indicated that the use of the computer would be more time consuming and unnecessary.

The students in both groups omitted the question on proving identities. The reasons could be their inability to prove identities of trigonometric functions. As noted by Demir, (2012), who found that an overwhelming number of students in his study were not able to prove a trigonometric relationship with a variable. This he said was most probably related to students' capabilities in doing mathematical proofs. Another reason for the omission could be due to a lack of motivation (Jakwerth, Stancavage, & Reed, 1999), since students were informed beforehand that they were not going to be graded for the study.

In the current research, ICT has been statistically proven to be more effective in the teaching and learning of trigonometry functions, specifically with connections between, interpretation of, and transformation of trigonometry functions. However there are variables which were not considered, such as the teachers' experience and qualification. Which means that the qualifications of the teacher is a factor in any experiment carried out in the classroom teaching and learning. It has been found that the level of teachers' content knowledge of trigonometry functions is directly proportional to the level of students' achievement. That is, the students' poor achievement in trigonometry functions could be related to teacher's lack of content knowledge of the same topic (Hanssen; Ogbonnaya & Mogari, 2014).

5.4 CONCLUSION

The investigation was based on AT and APOS as the theoretical frameworks. Both AT and APOS were therefore used in advising and constructing the teaching and learning instructions and the classroom environment. The use of ICT encouraged the students to explore the effects of certain inputs on the trigonometry functions. In this way students became creatively and actively involved in solving trigonometry functions tasks by plotting and transforming graphs, determining properties of trigonometry functions, and connecting various representations of trigonometry functions. This encouraged peer to peer interaction and students were seen to participate more in the classroom. The teacher mainly acted as a facilitator in the experimental class.

The aim of the study was to investigate the effectiveness of the use of ICT in the teaching and learning of trigonometry functions. The results of the investigation indicate that the use of ICT had a markedly positive impact on students' achievements, and influenced the way trigonometry functions were taught and learnt in the classroom. According to Raines and Clarke (2011), computers enable students to actively participate in class.

5.5 IMPLICATIONS OF THE FINDINGS AND RECOMMENDATIONS

This study was based on Geogebra use on trigonometry functions. Encouraging results were obtained.

From the overall results, it can be seen that the AT and APOS constructed instruction with the use of ICT software (Geogebra) assists positively when it comes to the teaching and learning of trigonometry functions. This is more-so when it comes to connections of trigonometry functions, interpretations of graphs and the analysis thereof. The study also indicated that introducing theory before using ICT is important in producing positive outcomes.

The study may also inform and demonstrate to other educators, the role of technology, specifically of Geogebra, in teaching and learning of trigonometry functions. The study may be used as an example and a guide that other mathematics educators could consider for incorporation in the delivery of their lessons and materials in classrooms.

5.6 LIMITATIONS OF THE STUDY AND RECOMMENDATIONS FOR FUTURE STUDY

The results from this study, that ICT is effective in the teaching and learning of Trigonometry functions cannot be generalised, as the study was conducted in only two schools, with specific conditions, in the North West Province, South Africa.

The duration of the study was rather short, seeing that lessons had to be completed within 10 hours during the normal class times unlike in the pilot study whereby the study and the teaching took place after the normal classes. The students in the main study were having extra classes in the afternoon so interviews could not be conducted after the post-test. Everything had to be rushed. An upgrading school has to finish the syllabus of grades 10-12 within a shorter time frame. Considering that most of the students come with level ones (< 30%) in the upgrading schools from their previous schools and already expressed their dislike of trigonometry because they fail to understand it (results from the pre-test), the duration of doing trigonometry of three years in ten hours is quite a challenge. Again because of this time-limit, while the students were being prepared for grade 12 final examinations, the schools were reluctant to grant more time for the teaching of trigonometry. The question paper had to comply with the prescribed syllabus as directed by the CAPS syllabus. This is the reason why the application section of trigonometry (e.g. the two dimensional and the three dimensional sections which were supposed to be dealt with in the third term could not be included in the assessment) was excluded from the taught and assessed materials.

At the end of the study, interviews were supposed to be conducted, however these could not be carried out due to time constraints seeing that, unlike in the pilot, the study had to be carried out in the normal designated time slots in the mornings. A similar study should be carried out in formal grades 10 and 11 high school environment where there is more time and less pressure so as to be able to incorporate more than one assessment method.

Based on these findings many more schools and educators would be encouraged to adopt and utilize Geogebra for the teaching of Trigonometry functions. From what has been seen, educators should not depend on Geogebra alone but should also use the whiteboard and books. During the learning and teaching process, as far as the trigonometry functions are concerned, Geogebra encourages students to focus on connections between geometric and algebraic representations. It is recommended that schools are provided with computers and software such as Geogebra be installed for the incorporation of ICT in the teaching and learning of subject content. Schools should also have policies in place to monitor, encourage and ensure that ICT is used effectively for the teaching and learning of different subjects. Teachers should also be encouraged to attend workshops and in-service training that deal with ICT in the teaching and learning of subject matter.

The results of this study showed that there is good potential in using ICT in the teaching and learning of trigonometry functions, but more research needs to be carried out to examine the long-term effects of the use of ICT in the students' mathematical achievements. More research on the use of ICT in teaching mathematics should be carried out by educators themselves since they are the ones in class every day. In addition, future studies should focus on what it is in this study that caused the results of students' responses to questions on determining the general and specific solutions of equations to not be significantly different between the experimental and the control group.

Furthermore, more studies should also concentrate on finding larger groups to participate in similar research, as larger sample sizes may bring more accurate results and thereby remove any doubts that may exist concerning the results of the present study, particularly the significant differences in scores between the students using Geogebra and those that did not use it.

Lastly, since only quantitative research was used in the study, in future research triangulation can be used to incorporate both qualitative and quantitative approaches.

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APPENDICES

APPENDIX 1: PRE-TEST:- TRIGONOMETRY FUNCTIONS TEST

The test includes 5 questions. You have 1 hour and 30 minutes to complete the test. Write your answers clearly in the parts allocated for each question. Try to be as much explanatory as possible.

Programmable calculators are not allowed.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
4. You may use an approved non-programmable scientific calculator.
5. A Graph paper and a diagram sheet for QUESTIONS 1, and QUESTION 3 are attached at the end of this question paper.
6. Number the answers correctly according to the numbering system used in this question paper.

TOTAL MARKS: 80

TIME: $1\frac{1}{2}$HOURS

QUESTION 1

1.1 Draw, on the same system of axes, the graphs of the following functions:

1.1.a $y = f(x) = 3\tan 2x;\ \ x \in [-180°; 180°]$

1.1b $y = g(x) = \cos(2x)\,; x \in [-180°; 180°]$

1.1c $y = h(x) = 3\sin x - 2; x\ \in [-180°; 180°]$

Clearly show all the important points. (12)

1.2 For each graph, answer the following questions:

a) Write down the amplitude of *g*. (1)

b) Write down the amplitude of *h*. (1)

c) Give the period of f. (1)

d) Give the domain of g. (1)

e) Give the range of h. (1)

f) Give the maximum value of h (1)

g) Write down the maximum value of g. (1)

h) Write down the asymptotes of f. (2)

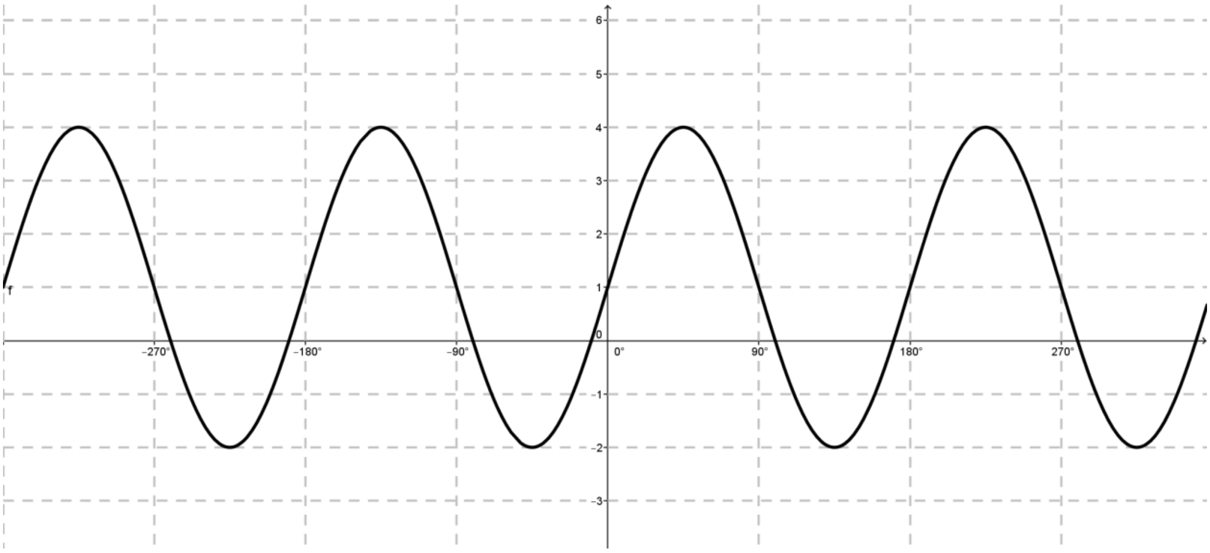
i) Write down the equation of the function of x if moved 30° to the left and two units up. (2)

[23]

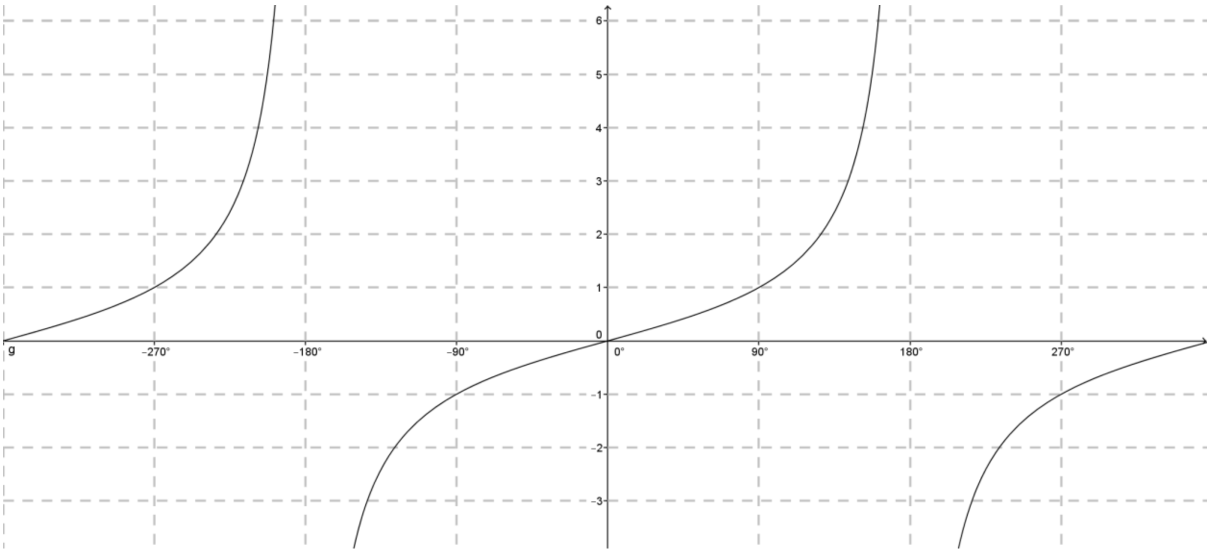
QUESTION 2

Write down the value(s) of a, b, p and q from the graphs below:

a) $y = a\sin bx + q$ (6)

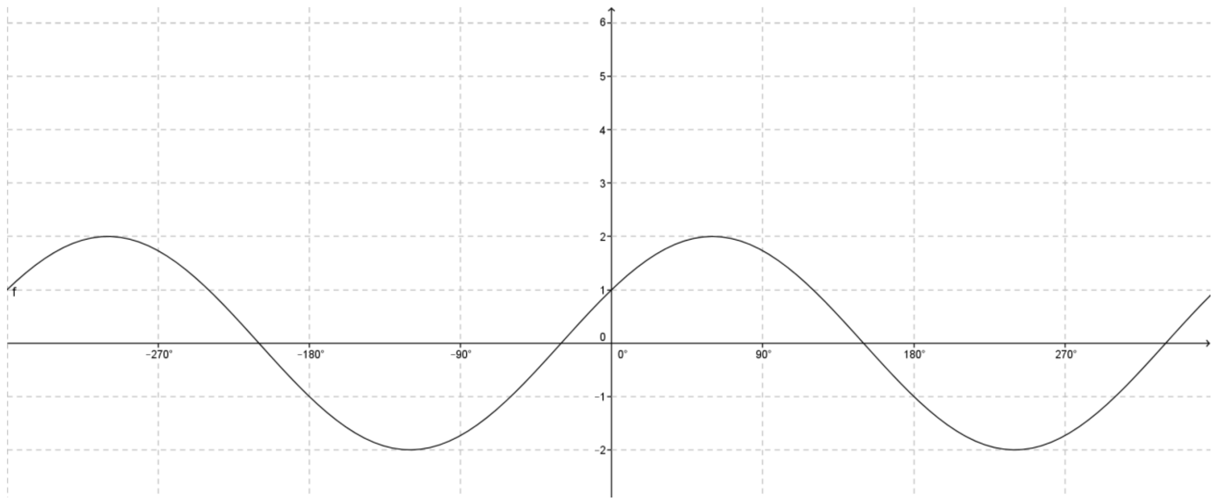


b) $y = a\tan bx$



(3)

c) $y = a\cos(x + p)$ (4)



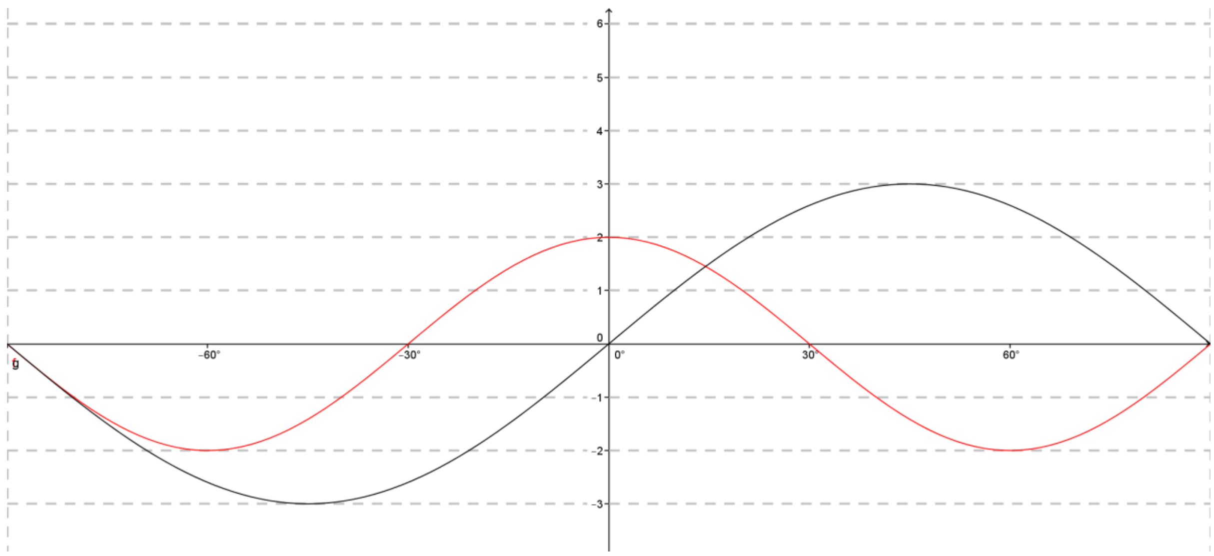
[13]

QUESTION 3

The diagram below represents the graphs of $y = f(x) = 2\cos 3x$ and $y = g(x) = 3\sin 2x$ for $x \in [-90°; 90°]$

a) Write down the coordinates of P, the y-intercept of f. (2)

b) Write down the coordinates of the x-intercepts of f and g.
(4)



c) On the graph, show where the points of intersection of f and g are. LABEL the points as A, B and C. (3)

d) From the graph determine the value(s) of (x) for which

i) $f(x) = 3$ (1)

ii) $f(x) - g(x) = 0$ (3)

e) On the graph, shade the regions where $f(x) \geq g(x)$. (3)

[16]

QUESTION 4

a)

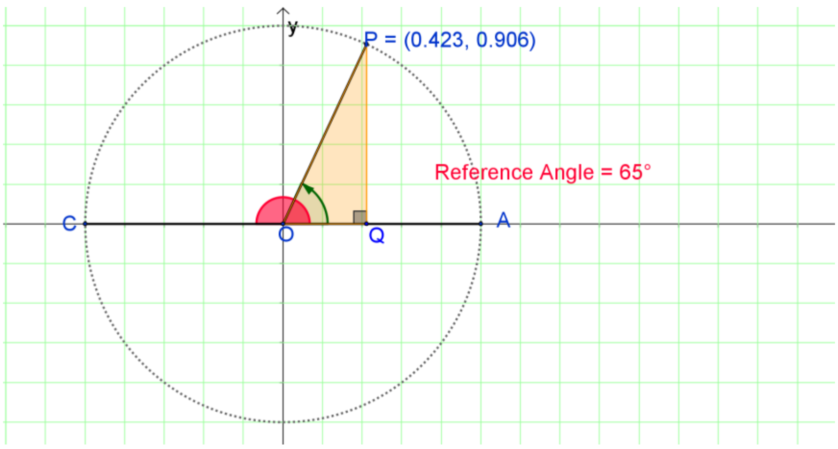
QUESTION 4

a) If $f(x) = \cos x$ and $g(x) = -2\cos(2x - 30) + 3$, describe the transformation from f to g. , (5)

[5]

QUESTION 5;

5.1 Given:



A, P and C are points on the unit circle in the figure above. They correspond to rotation about the origin in the anti-clockwise direction starting from A (1, 0). ‹ POQ = 65

5.1.1 Write down sin 65 (1)

5.12 Write down cos 65 (1)

5.1.3 Write down the coordinates of a point on the graph of sinθ which corresponds to P on the unit circle. (2)

5.2 Consider the identity: $\frac{\cos 2A}{1+\sin 2A} = \frac{1-\tan A}{1+\tan A}$

5.2.1 Prove the identity. (7)

5.2.2 For which value(s) of x in the interval $0< x <180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $4\cos^2 A + \sin 2\theta - 3 = 0$. (5)

5.4 Solve $\sin x = \cos 3x$ where $x \in (-180°;\ 180°)$ (5)

[23]

APPENDIX 2: POST-TEST- TRIGOMOMETRY FUNCTIONS

TOTAL MARKS: 80

TIME: $1\frac{1}{2}$HOURS

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 5 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.

4. You may use an approved scientific non-programmable calculator.

5. Graph paper and a diagram sheet for QUESTIONS 1, and QUESTION 3 are attached at the end of this question paper.

6. Number the answers correctly according to the numbering system used in this question paper.

TOTAL MARKS: 80

TIME: $1\frac{1}{2}$HOURS

QUESTION 1

Draw, on the same system of axes, the graphs of the following functions:

1.1a $y = f(x) = -3\tan 2x;\ x \in [-180°; 180°]$

1.1b $y = g(x) = cos(2x); x \in [-180°; 180°]$

1.1c $y = h(x) = 3sinx + 2; x \in [-180°; 180°]$

Clearly show all the important points. (12)

2. For each graph, answer the following questions:

a) Write down the amplitude of g. (1)

b) Write down the amplitude of h. (1)

c) Give the period of f. (1)

d) Give the domain of g. (1)

e) Give the range of h. (1)

f) Give the maximum value of h. (1)

g) Write down the maximum value of g. (1)

h) Write down the asymptotes of f. (2)

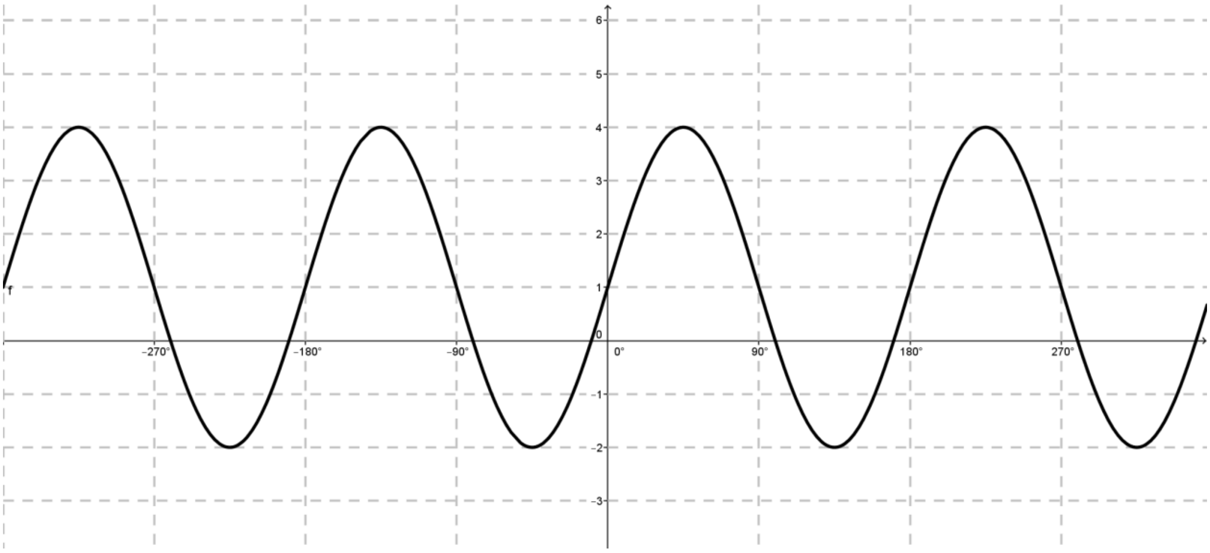
i) Write down the equation of the function of x if moved 30° to the left and two units up. (2)

[23]

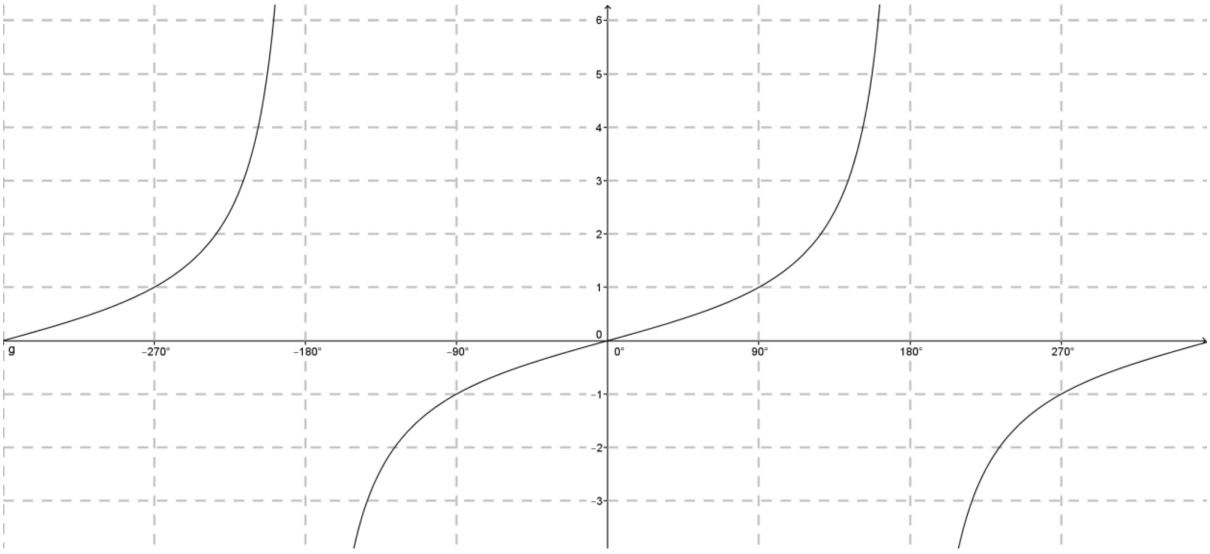
QUESTION 2

Write down the value(s) of a, b, p and q from the graphs below:

a) $y = a\sin bx + q$ (6)

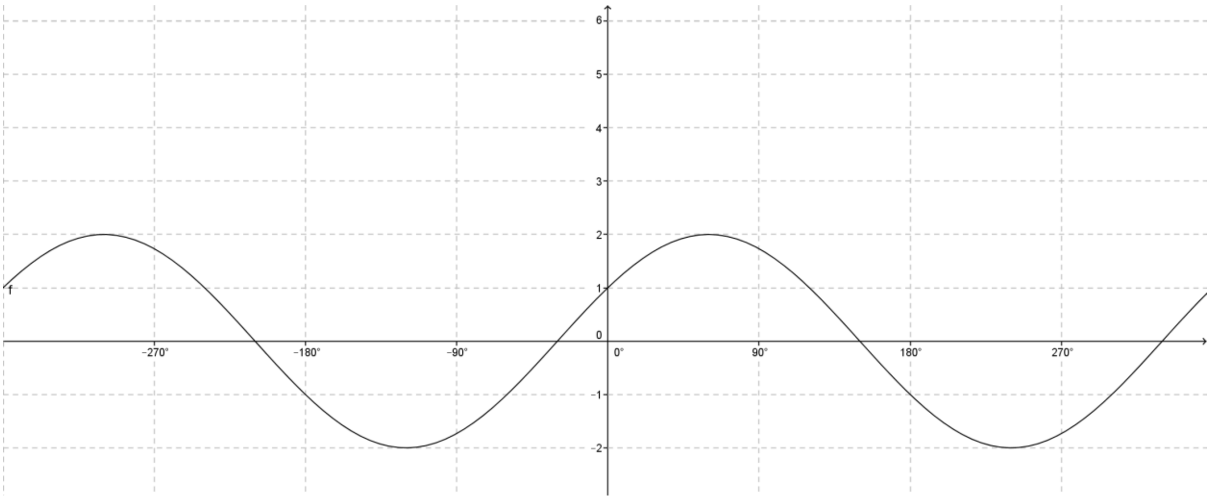


b) $y = a\tan bx$



(3)

c) $y = a\cos(x + p)$ (4)



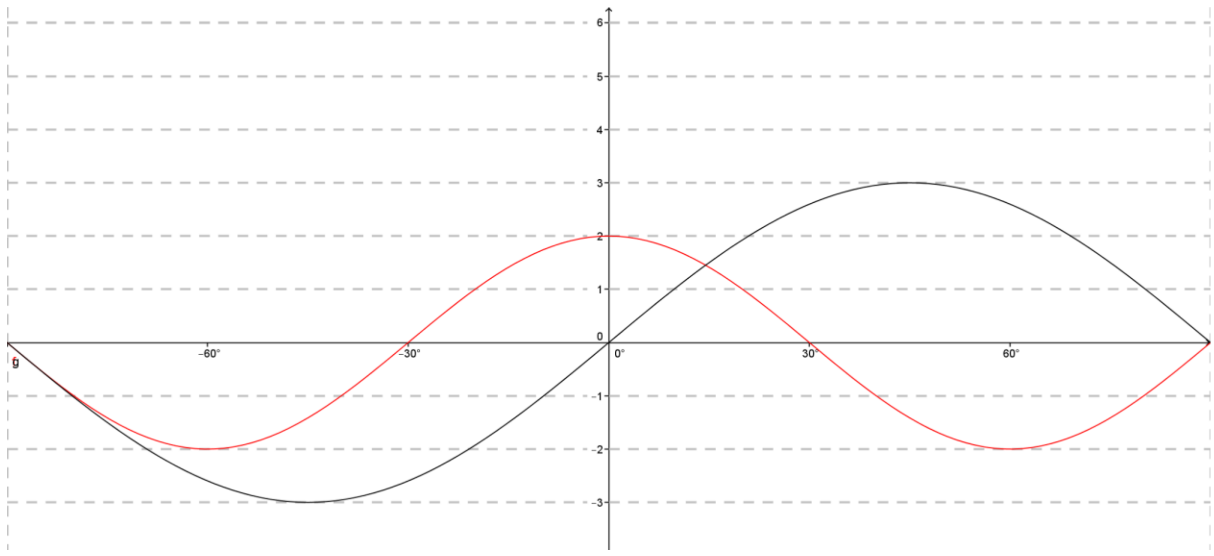
[13]

QUESTION 3

The diagram below represents the graphs of $y = f(x) = 2\cos 3x$ and $y = g(x) = 3\sin 2x$ for $x \in [-90°; 90°]$

a) Write down the coordinates of P, the y-intercept of g. (2)

b) Write down the coordinates of the x-intercepts of f and g. (4)



c) On the graph, show where the points of intersection of f and g are. LABEL the points as S, T and U. (3)

d) From the graph determine the value(s) of (x) for which

i) $g(x) = 3$ (1)

ii) $f(x) = g(x)$ (3)

e) On the graph, shade the regions where $f(x) \geq g(x)$. (3)

[16]

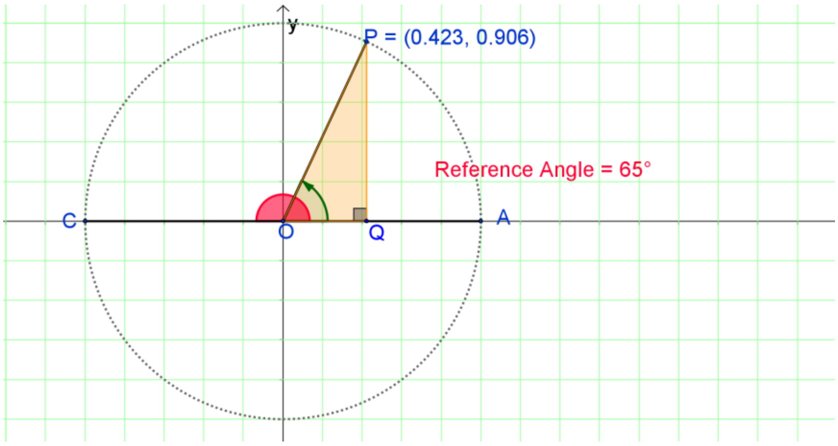
QUESTION 4

a) If $f(x) = \cos x$ and $g(x) = -2\cos(2x - 30) - 2$, describe the transformation from f to g. , (5)

[5]

QUESTION 5;

5.1 Given:



A, P and C are points on the unit circle in the figure above. They correspond to rotation about the origin in the anti-clockwise direction starting from A (1, 0). ‹ POQ = 65

5.1.1 Write down sin 65 (1)

5.12 Write down cos 65 (1)

5.1.3 Write down the coordinates of a point on the graph of sinθ which corresponds to P on the unit circle. (2)

5.2 Determine the general solution of $4\cos^2 A + \sin 2\theta - 3 = 0$. (5)

5.3 Solve $\sin x = \cos 3x$ where $x \in (-180°;\ 180°)$ (5)

5.4 Consider the identity: $\frac{\cos 2x - 2\sin^2 x}{\cos x - \sin 2x} = \frac{1 + 2\sin x}{\cos x}$

5.4.1 Prove the identity. (7)

5.4.2 For which value(s) of x in the interval $0 < x < 180°$ will the identity be undefined? (2)

[23]

APPENDIX 3: INSTRUMENT VALIDATION FORM FOR SMEs

To ensure validity of the academic tests, the attached instrument –was developed and given to SMEs for moderation of the assessment instruments for compliance to the grade 12 CAPs curriculum.

The SMEs were requested to evaluate the questions and indicate the level of significance of each question to test grade 12 students knowledge of trigonometry functions in line with the CAPS curriculum using a three-point scale: 0= Not Significant; 1= Slightly significant; 2= Totally significant.

Questions	**Significance**	**Comment**
Q1.1a		
Q1.1b		
Q1.1c		
Q1.2a		
Q1.2b		
Q1.2c		
Q1.2d		
Q1.2e		
Q1.2f		
Q1.2g		
Q1.2h		
Q1.2i		
Q4		
Q2a		
Q2b		
Q2c		
Q3a		
Q3b		
Q3c		
Q3d(i)		
Q3(ii)		
Q3e		

Q5.1.1		
Q5.1.2		
Q5.1.3		
Q5.2.1		
Q5.2.2		
Q5.3		
Q5.4		

APPENDIX 4: PERMISSION LETTER TO REGISTER FOR THESIS

INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION
COLLEGE OF POSTGRADUATE STUDIES
UNIVERSITY OF SOUTH AFRICA

NOTICE TO POSTGRADUATE QUALIFICTION SECTION (M&D)
RESULT : RESEARCH PROPOSAL MODULE

STUDENT NAME	MOSESE N M	STUDENT NUMBER	46985484
DEGREE	MSS(MST)	**Specialisation**	Mathematics education
MODULE CODE	MPSTE90		

Please indicate the relevant option with an x:	
A. **The above student did not comply with the requirements for the research proposal module and may reregister for this module**	
B. **The above student did not comply with the requirements for the research proposal module and may not continue with his studies for the degree.** *Please provide reasons: . . .*	
C. **I confirm that the above student complied with the requirements for the research proposal module** (research proposal approved by departmental higher degrees committee) **and may now proceed to register for the research component.** *Please provide details below*	X

Title: Evaluating the effectiveness of the use of information and communication technology (ict) in the teaching and learning of trigonometry functions

Supervisor: Dr. UI Ogbonnaya   Personnel Number: 52170284

Highest Qualification: PhD

Co-supervisor: Personnel Number:

Highest Qualification:

Address, if external : ugorjio@yahoo.com
(including email)

Additional comments:

Approval (CoD)

Comments:
Signature:
Date:

Comments:
Signature :
On behalf of College/School Executive Committee
Date: 24 March 2016

FOR OFFICE USE ONLY BY SENIOR QUALIFICATIONS	
	Result captured (F375)

Prepared by: UI Ogbonnaya   Date: 24/11/2015

The SGB, MAFIKENG, 2745

School,

Dear Sir / Madam

Permission to participate in research

I, Nthabiseng Mmamotho Mosese, am currently studying for a Masters' Degree in Mathematics, Science and Technology Education with the University of South Africa (UNISA). Part of the requirements for the degree is that I am expected to conduct a research study. Your school has been identified as one of those with the appropriate infrastructure and the human capacity for me to carry out the study.

The title of my proposed dissertation is: *Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions.*

I hereby humbly request that by signing, and so indicating, you grant me the permission to conduct this study at your school.

I pledge to maintain professional and research ethical conduct. This implies that:

- Participation in this research remains voluntary and you may, at any time, withdraw from the research
- Personal information, at all times, will be treated as confidential
- No demands will be made on your academic teaching program
- Should you be interested, the research findings will be made available to you.

May you thus provide me with your written consent by filling in the section on the next page? Please return the consent form to me. Your input and opinions are highly appreciated!

Yours sincerely,

N.M. Mosese
MSc Ed Candidate, UNISA
Student number: 46985484
Office number: +27 18 384 5211
Fax number: +27 18 384 4146

APPENDIX 5: PERMISSION LETTER FROM SGB MASCCOM

MAFIKENG, 2745

The SGB,
MASCCOM School,

Dear Sir / Madam

Permission to participate in research

I, Nthabiseng Mmamotho Mosese, am currently studying for a Masters' Degree in Mathematics, Science and Technology Education with the University of South Africa (UNISA). Part of the requirements for the degree is that I am expected to conduct a research study. Your school has been identified as one of those with the appropriate infrastructure and the human capacity for me to carry out the study.

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I pledge to maintain professional and research ethical conduct. This implies that:

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May you thus provide me with your written consent by filling in the section on the next page? Please return the consent form to me. Your input and opinions are highly appreciated!

Yours sincerely,

N.M. Mosese
MSc Ed Candidate, UNISA
Student number: 46985484
Office number: +27 18 384 5211
Fax number: +27 18 384 4146

Permission to Research Project:

Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions.

LETTER OF PERMISSION:

RESEARCH PARTICIPANT

I, ________________________ (name and surname) as the SGB ~~Principal~~ at ________________________ (name of High School) do hereby give my permission for my school to participate in the above mentioned research project. I am aware that our participation in this study remains voluntary and that we, at any time, may withdraw from the research. I understand that if we do not wish to participate in this research, it will not be held against us, as participation is voluntary. I also understand that any personal information obtained in the study will be treated as confidential by the researchers. By signing you are indicating that you have read and understand what I am requesting and I have your support.

____

Approved/ ~~Not Approved~~.

Name and signature

________________________ ________________________

APPENDIX 6: PERMISSION LETTER FROM NW EDUCATION DEPARTMENT

MASCOMM

P.O. Box 464

MAFIKENG, 2745

The Director/ Deputy Director,

North West Department of Education, Private Bag X 2044, Mmabatho, 2735.

Dear Sir / Madam

Permission to participate in research

I, Nthabiseng Mmamotho Mosese, am currently studying for a Masters' Degree in Mathematics, Science and Technology Education with the University of South Africa (UNISA). Part of the requirements for the degree is that I am expected to conduct a research study. Two schools in the Mafikeng area, namely MASCCOM and Sebonego Upgrading have been identified as those with the appropriate infrastructure and the human capacity for me to carry out the study. The research will not in any way disrupt normal academic processes at these institutions.

The title of my proposed dissertation is: Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometry functions.

I hereby humbly request that by signing, and so indicating, you grant me the permission to conduct this study at these schools.

I pledge to maintain professional and research ethical conduct. This implies that:

- Participation in this research remains voluntary and they may, at any time, withdraw from the research
- Personal information, at all times, will be treated as confidential
- No demands will be made on their academic teaching program
- Should you be interested, the research findings will be made available to you.

May you thus provide me with your written consent by filling in the section on the next page? Please return the consent form to me. Your input and opinions are highly appreciated! Yours sincerely

N.M. Mosese

MSc Ed Candidate, UNISA

Student number: 46985484

Office number: +27 18 384 5211

Fax number: +27 18 384 4146

Permission to Research Project:

Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions

LETTER OF PERMISSION:

RESEARCH PARTICIPANT

I, DANIEL M. NGWENYA, (name and surname) as the Deputy Director (North West Department of Education) do hereby give my permission for the schools to participate in the above mentioned research project. I am aware that their participation in this study remains voluntary and that they, at any time, may withdraw from the research. I understand that if they do not wish to participate in this research, it will not be held against them, as participation is voluntary. I also understand that any personal information obtained in the study will be treated as confidential by the researchers. By signing you are indicating that you have read and understand what I am requesting and I have your support.

Approved/ ~~Not Approved~~.

Name and signature

D -M. NGWENYA

APPENDIX 7: PERMISSION LETTER

MASCCOM
P.O. Box 464
MAFIKENG, 2745

The Manager,
SEBONEGO School,

Dear Sir / Madam

Permission to participate in research

I, Nthabiseng Mmamotho Mosese, am currently studying for a Masters' Degree in Mathematics, Science and Technology Education with the University of South Africa (UNISA). Part of the requirements for the degree is that I am expected to conduct a research study. Your school has been identified as one of those with the appropriate infrastructure and the human capacity for me to carry out the study.

The title of my proposed dissertation is: *Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions.*
I hereby humbly request that by signing, and so indicating, you grant me the permission to conduct this study at your school.
I pledge to maintain professional and research ethical conduct. This implies that:

- Participation in this research remains voluntary and you may, at any time, withdraw from the research
- Personal information, at all times, will be treated as confidential
- No demands will be made on your academic teaching program
- Should you be interested, the research findings will be made available to you.

May you thus provide me with your written consent by filling in the section on the next page? Please return the consent form to me. Your input and opinions are highly appreciated!
Yours sincerely

N.M. Mosese
MSc Ed Candidate, UNISA
Student number: 46985484
Office number: +27 18 384 5211
Fax number: +27 18 384 4146

MAFIKENG, 2745

The Manager,
School,

Dear Sir / Madam

Permission to participate in research

I, Nthabiseng Mmamotho Mosese, am currently studying for a Masters' Degree in Mathematics, Science and Technology Education with the University of South Africa (UNISA). Part of the requirements for the degree is that I am expected to conduct a research study. Your school has been identified as one of those with the appropriate infrastructure and the human capacity for me to carry out the study.

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- Should you be interested, the research findings will be made available to you.

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Yours sincerely

N.M. Mosese
MSc Ed Candidate, UNISA
Student number: 46985484
Office number: +27 18 384 5211
Fax number: +27 18 384 4146

Permission to Research Project:

Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions.

LETTER OF PERMISSION:

RESEARCH PARTICIPANT

I, M.S Naga, (name and surname) as the Principal at Sekorego Training (name of High School) do hereby give my permission for my school to participate in the above mentioned research project. I am aware that our participation in this study remains voluntary and that we, at any time, may withdraw from the research. I understand that if we do not wish to participate in this research, it will not be held against us, as participation is voluntary. I also understand that any personal information obtained in the study will be treated as confidential by the researchers. By signing you are indicating that you have read and understand what I am requesting and I have your support.

____

Approved/ ~~Not Approved~~.

Name and signature

Simphiwe : __________

Thank you very much.

APPENDIX 8: STUDENTS' CONSENT FORM

TITLE OF RESEARCH PROJECT

Evaluating the effectiveness of the use of Information and Communication Technology (ICT) in the teaching and learning of trigonometric functions

---

Dear Mr/Miss/Ms ______________________________ Date: ……/ ….…/2016

NATURE AND PURPOSE OF THE STUDY

The purpose of this research project is to explore the possible effects of teaching and learning trigonometry through the use of Information Communication and Technology (ICT). The study makes use of qualitative and quantitative (a mixed) method of study.

RESEARCH PROCESS (thorough and clear description of all data gathering processes that will take place)

1 The study requires your participation in a pre-test and a post-test.

2 The study requires your participation in an interview.

3 You do not need to prepare anything in advance.

CONFIDENTIALITY

No data published in dissertations and journals will contain any information through which members involved in the study or interviews may be identified. Your anonymity is therefore ensured.

WITHDRAWAL CLAUSE

You may withdraw from the research at any time, you therefore participate voluntarily until such time as you indicate otherwise.

POTENTIAL BENEFITS OF THE STUDY (brief as in the research proposal)

In light of the continued low performance of learners in Mathematics and specifically in Trigonometry, this study investigates possible teaching strategies to address the low pass rate. The study will probe alternative ways of teaching trigonometry as well as possibly improve learner understanding of Trigonometry. Researchers may increase or moderate my findings. The teachers may use ICT in their classes and take up the software which is freely available.

INFORMATION (contact information of supervisor)

If you have any questions concerning the study, you may contact the researcher Ms N.M. Mosese, on 083 586 1936 (OR the supervisor Prof D Mogari on 011 670 9422 and Dr Ogbonnaya on 0737208026).

CONSENT

I, the undersigned, …………………………………………………………………… (full name) have read the above information relating to the study and have also heard the verbal version, and declare that I understand it. I have been afforded the opportunity to discuss relevant aspects of the project with the researcher, and hereby declare that I agree voluntarily to participate in the project.

I have received a signed copy of this consent form.

<u>Participant</u>

Name :_____________________________ Signature : ______________________________

Signed at: ___________________________ on: ____________________________________

Researcher:

Name: Nthabiseng Mosese. Signature:____________________________

APPENDIX 9: ACHIEVEMENT LEVEL DESCRIPTIONS OF COMPETENCE PERCENTAGE BANDS (DBE, 2015)

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 – 79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 – 29

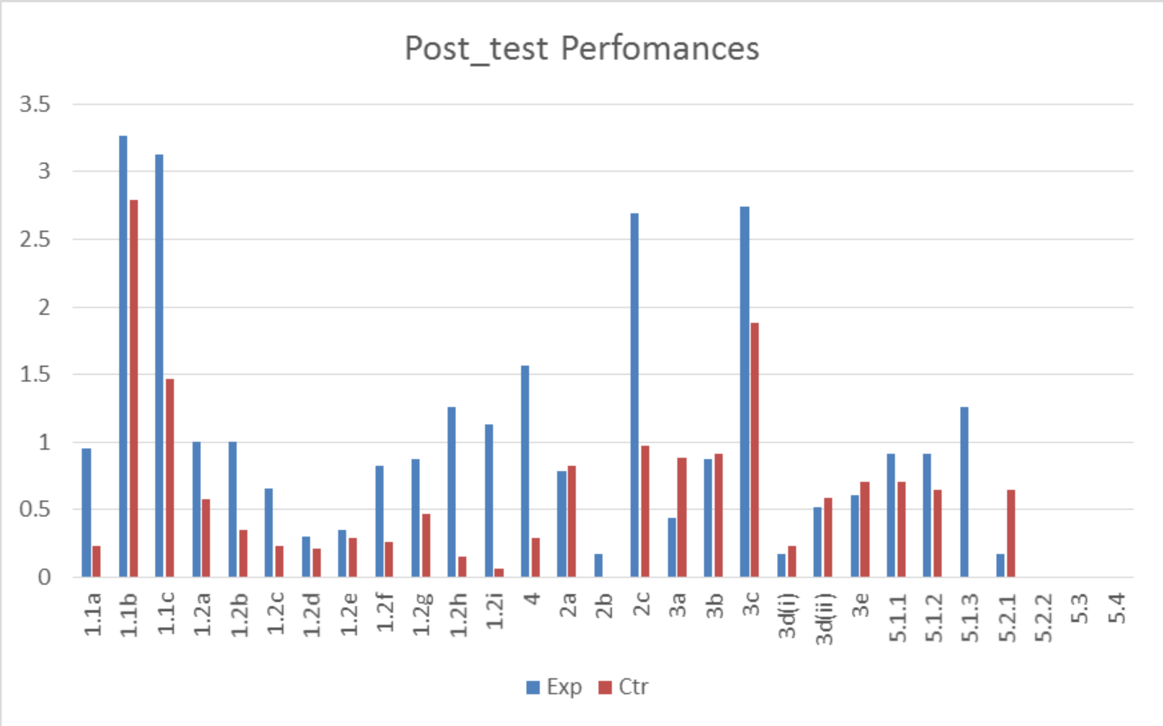
APPENDIX 10: CORRELATIONS OF TWO EQUIVALENT GROUPS

Correlations

		Group A	Group B
Group A	Pearson Correlation	1	.677*
	Sig. (2-tailed)		.011
	N	13	13
Group B	Pearson Correlation	.677*	1
	Sig. (2-tailed)	.011	
	N	13	13

*. Correlation is significant at the 0.05 level (2-tailed).

APPENDIX 11: GROUPS' MEAN ACHIEVEMENTS BY QUESTION



APPENDIX 12A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1_1a

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.926	1.2066	.2322
	Control group	34	.235	.9553	.1638

APPENDIX 12B: t-test ON STUDENTS' ACHIEVEMENT IN QUESTION 1_1a

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	6.527	.013	2.496	59	.015	.6906	.2767	.1370	1.2443
Equal variances not assumed			2.430	48.802	.019	.6906	.2842	.1195	1.2618

APPENDIX 13A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1_1b

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	3.296	.4653	.0896
	Control group	34	2.794	1.0668	.1830

APPENDIX 13B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1_1b

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	9.230	.004	2.277	59	.026	.5022	.2205	.0609	.9435
Equal variances not assumed			2.465	47.262	.017	.5022	.2037	.0924	.9119

APPENDIX 14A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1_1c

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	3.148	.6015	.1158
	Control group	34	1.471	1.3977	.2397

APPENDIX 14B: t-test ON STUDENTS' ACHIEVEMENT IN QUESTION 1_1c

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	23.636	.000	5.816	59	.000	1.6776	.2885	1.1004	2.2548
Equal variances not assumed			6.302	46.946	.000	1.6776	.2662	1.1420	2.2131

APPENDIX 15A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 12a

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.00	.000	.000
	Control group	33	.58	.502	.087

APPENDIX 15B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2a

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	1110.816	.000	4.385	58	.000	.424	.097	.231	.618
Equal variances not assumed			4.856	32.000	.000	.424	.087	.246	.602

APPENDIX 16A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2b

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.000	.0000	.0000
	Control group	34	.353	.4851	.0832

APPENDIX 16B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2b

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	275.772	.000	6.919	59	.000	.6471	.0935	.4599	.8342
Equal variances not assumed			7.778	33.000	.000	.6471	.0832	.4778	.8163

APPENDIX 17A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2c

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.667	.4804	.0925
	Control group	34	.235	.4306	.0738

APPENDIX 17B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2c

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	2.668	.108	3.693	59	.000	.4314	.1168	.1976	.6651
Equal variances not assumed			3.646	52.818	.001	.4314	.1183	.1940	.6687

APPENDIX 18A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2d

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.259	.4466	.0859
	Control group	34	.206	.4104	.0704

APPENDIX 18B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2d

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.926	.340	.485	59	.629	.0534	.1100	-.1667	.2735
Equal variances not assumed			.480	53.583	.633	.0534	.1111	-.1694	.2761

APPENDIX 19A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2e

	1_2e	N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.333	.4804	.0925
	Control group	34	.294	.4625	.0793

APPENDIX 19B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2e

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.407	.526	.323	59	.748	.0392	.1213	-.2035	.2819
Equal variances not assumed			.322	54.919	.749	.0392	.1218	-.2049	.2833

APPENDIX 20A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2f

	N	Mean	Std. Deviation	Std. Error Mean
Experimental group	27	.852	.3620	.0697
Control group	34	.265	.4478	.0768

APPENDIX 20B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2f

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	5.276	.025	5.526	59	.000	.5871	.1063	.3745	.7998
Equal variances not assumed			5.662	58.972	.000	.5871	.1037	.3797	.7946

APPENDIX 21A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2g

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.889	.3203	.0616
	Control group	34	.471	.5066	.0869

APPENDIX 21B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2g

	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference: Lower	Upper
Equal variances assumed	48.893	.000	3.735	59	.000	.4183	.1120	.1942	.6424
Equal variances not assumed			3.927	56.430	.000	.4183	.1065	.2049	.6317

APPENDIX 22A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2h

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.148	.9885	.1902
	Control group	34	.147	.5004	.0858

APPENDIX 22B: t-test ON STUDENTS' ACHIEVEMENT ON QUESTION 1.2h

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	58.045	.000	5.141	59	.000	1.0011	.1947	.6114	1.3908
Equal variances not assumed			4.797	36.470	.000	1.0011	.2087	.5780	1.4242

APPENDIX 23A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 1.2i

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.148	.8640	.1663
	Control group	34	.059	.3430	.0588

APPENDIX 23B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 1.2i

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	49.840	.000	6.726	59	.000	1.0893	.1620	.7652	1.4134
Equal variances not assumed			6.176	32.514	.000	1.0893	.1764	.7303	1.4484

APPENDIX 24A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 4

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.370	1.5479	.2979
	Control group	34	.294	.7190	.1233

APPENDIX 24B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 4

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	23.541	.000	3.600	59	.001	1.0763	.2990	.4780	1.6745
Equal variances not assumed			3.338	34.866	.002	1.0763	.3224	.4216	1.7309

APPENDIX 25A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 2a

	N	Mean	Std. Deviation	Std. Error Mean
Experimental group	27	1.037	1.1596	.2232
Control group	34	.824	1.3136	.2253

APPENDIX 25B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 2a

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.004	.951	.664	59	.510	.2135	.3217	-.4303	.8573
Equal variances not assumed			.673	58.294	.503	.2135	.3171	-.4212	.8482

APPENDIX 26A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 2b

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.148	.4560	.0878
	Control group	34	.000	.0000	.0000

APPENDIX 26B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 2b

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	17.425	.000	1.898	59	.063	.1481	.0780	-.0080	.3043
Equal variances not assumed			1.688	26.000	.103	.1481	.0878	-.0323	.3286

APPENDIX 27A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 2c

	N	Mean	Std. Deviation	Std. Error Mean
Experimental group	27	2.704	1.1706	.2253
Control group	34	.971	1.1142	.1911

APPENDIX 27B:t-test ON STUDENTS' ACHIEVEMENT -QUESTION 2c

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.052	.820	5.901	59	.000	1.7331	.2937	1.1454	2.3208
Equal variances not assumed			5.867	54.602	.000	1.7331	.2954	1.1410	2.3252

APPENDIX 28A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3a

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.407	.5007	.0964
	Control group	34	.882	.8077	.1385

APPENDIX 28B: t-test ON STUDENTS' ACHIEVEMENT- QUESTION 3a

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	5.211	.026	-2.672	59	.010	-.4749	.1777	-.8306	-.1193
Equal variances not assumed			-2.815	56.017	.007	-.4749	.1687	-.8130	-.1369

APPENDIX 29A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3b

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.815	1.0014	.1927
	Control group	34	.912	1.1901	.2041

APPENDIX 29B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 3b

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.205	.652	-.339	59	.736	-.0969	.2864	-.6700	.4761
Equal variances not assumed			-.345	58.775	.731	-.0969	.2807	-.6587	.4648

APPENDIX 30A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3c

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	2.778	.8006	.1541
	Control group	34	1.882	1.4515	.2489

APPENDIX 30B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 3c

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	49.590	.000	2.874	59	.006	.8954	.3116	.2720	1.5189
Equal variances not assumed			3.059	53.216	.003	.8954	.2928	.3083	1.4826

APPENDIX 31A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3di

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.148	.3620	.0697
	Control group	34	.235	.4306	.0738

APPENDIX 31B: t-test on STUDENTS' ACHIEVEMENT -QUESTION 3di

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	3.014	.088	-.841	59	.404	-.0871	.1036	-.2944	.1201
Equal variances not assumed			-.858	58.781	.394	-.0871	.1015	-.2903	.1160

APPENDIX 32A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3dii

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.444	1.0860	.2090
	Control group	34	.588	.9572	.1642

APPENDIX 32B: t-test ON STUDENTS' ACHIEVEMENT- QUESTION 3dii

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.056	.815	-.549	59	.585	-.1438	.2619	-.6678	.3803
Equal variances not assumed			-.541	52.293	.591	-.1438	.2658	-.6770	.3894

APPENDIX 33A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 3e

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.519	1.0874	.2093
	Control group	34	.706	1.2917	.2215

APPENDIX 33B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 3e

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	2.313	.134	-.603	59	.549	-.1874	.3109	-.8094	.4347
Equal variances not assumed			-.615	58.773	.541	-.1874	.3047	-.7972	.4225

APPENDIX 34A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 5_1_1

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.926	.2669	.0514
	Control group	34	.706	.6291	.1079

APPENDIX 34B: t-test ON STUDENTS' ACHIEVEMENT-QUESTION 5_1_1

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	19.421	.000	1.698	59	.095	.2200	.1296	-.0393	.4793
Equal variances not assumed			1.842	46.615	.072	.2200	.1195	-.0204	.4605

APPENDIX 35A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 5_1_2

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.926	.2669	.0514
	Control group	34	.647	.4851	.0832

APPENDIX 35B: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 5_1_2

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	44.991	.000	2.680	59	.010	.2789	.1041	.0706	.4871
Equal variances not assumed			2.852	53.151	.006	.2789	.0978	.0828	.4749

APPENDIX 36A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 5_1_3

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	1.296	.9533	.1835
	Control group	34	.000	.0000	.0000

APPENDIX 36b: t-test ON STUDENTS' ACHIEVEMENT -QUESTION 5_1_3

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	287.187	.000	7.946	59	.000	1.2963	.1631	.9699	1.6227
Equal variances not assumed			7.066	26.000	.000	1.2963	.1835	.9192	1.6734

APPENDIX 37A: STUDENTS' MEAN ACHIEVEMENT ON QUESTION 5_2_1

		N	Mean	Std. Deviation	Std. Error Mean
	Experimental group	27	.148	.6015	.1158
	Control group	34	.647	1.7035	.2922

APPENDIX 37B: t-test ON STUDENTS' MEAN ACHIEVEMENT QUESTION 5_2_1

	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	9.898	.003	-1.450	59	.152	-.4989	.3442	-1.1876	.1898
Equal variances not assumed			-1.588	42.836	.120	-.4989	.3142	-1.1327	.1349