

**IMPACT OF CONSTRUCTIVIST INSTRUCTIONAL APPROACH ON GRADE 12
LEARNERS' UNDERSTANDING OF STATIONARY POINTS IN DIFFERENTIAL CALCULUS**

by

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DECLARATION

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I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

.....
SIGNATURE

.....
DATE

ACKNOWLEDGEMENTS

All glory to God, the only fountain of knowledge, for the successful conclusion of this empirical study. I thankfully acknowledge the unquantifiable guidance and support of my research Supervisor, Dr. Faleye Sunday, Lecturer, Department of Mathematical Sciences, Faculty of Computer, Science, Engineering and Technology (CSET), University of South Africa (UNISA).

My appreciation also goes to Gauteng Department of Education for approving the conduct of the research, and the principals, teachers and learners of the schools involved in the study for their understanding and co-operations. I cannot but thank as well Ms. Suwisa Muchengetwa, Lecturer, Department of Statistics, CSET, UNISA for her input which ensured that the data was professionally analyzed.

I do not have to forget my university, UNISA, for the student bursary awarded me which, in no small measure, facilitated the postgraduate study. To my kith and kin, I say thank you. I quite subscribe, on this final note, to this assertion by Gaston Bachelard (1934):

And, irrespective of what one might assume, in the sciences, problems do not arise by themselves. It is, precisely, because all problems are posed that they embody the scientific spirit. If there were no question, there would be no scientific knowledge. Nothing proceeds from itself. Nothing is given. All is constructed.

Omoniyi, Adebayo Akinyinka.

February, 2016.

DEDICATION

For Treasure, my darling daughter, born in the course of the research work

LIST OF ABBREVIATIONS AND ACRONYMS

FET:	Further Education and Training (Grade 10 - 12)
GET:	General Education and Training (Grade 8 - 10)
GDE:	Gauteng Department of Education
DoE:	Department of Education
NCS:	National Curriculum Statement
CIA:	Constructivist Instructional Approach
TIA:	Traditional Instructional Approach
NSC:	National School Certificate
PGCE:	Postgraduate Certificate in Education
DHET:	Department of Higher Education and Training

ABSTRACT

With the realization that traditional instructional approach has not yielded satisfactory results, quasi-experimental and descriptive research designs were employed to investigate whether the application of constructivist instructional approach in the learning of stationary points in differential calculus by Grade 12 learners in South Africa would improve conceptual learning. Three Gauteng high schools of 204 Grade 12 learners constituted the research fields – one served as the control group while the other two represented the experimental group.

Being a mixed-method research, quantitative data were gathered through pre-test and post-test while qualitative data were collected from classroom observations. Both inferential and descriptive statistical methods of data collection and analysis were used. The results obtained indicate that the experimental group demonstrated a better understanding of the concept of stationary points than the control group.

Key Terms: Impact, constructivism, constructivist instruction, traditional instruction, differential calculus, stationary points, prior knowledge, understanding, academic achievement, problem-solving skills.

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CHAPTER ONE

ORIENTATION AND OVERVIEW OF THE STUDY

1.1 Introduction

The low performances in Mathematics by Grade 12 learners in South Africa have hitherto remained worrisome (Ndlovu, 2011; Reddy, 2004; Siyepu, 2013; van der Walt, Maree & Ellis, 2008). At the January 4, 2012 Pretoria media conference, the Minister of Basic Education announced that the general pass-rate (i.e. 30% marks and above) for Mathematics in the National School Certificate (NSC) examination which was 47.4% in 2010 unfortunately lowered to 46.3% in 2011. As recorded in the NSC 2014 Technical Reports on learners' performances, the 2012, 2013 and 2014 pass-rates for Mathematics stand at 54%, 59% and 53.5% respectively (DoE, 2014). In fact, the 2014 pass-rate of 53.5% remains the lowest of all the pass-rates for the other NSC examination subjects. Again, considering the nation's university minimum entry level requirement of 50% for each relevant subject, those candidates who got below 50 marks but are included in the above pass-rates are not eligible for admissions to Bachelor's degree courses. The implication is that the national pass-rate for NSC Mathematics is yet to reach the desired level.

According to Luneta and Makonye (2010), the Grade 12 learners have challenges in differential calculus in particular. Consequent upon the studies they conducted, they arrived at a conclusion that the learners' poor understanding of calculus was as a result of their weak pre-calculus skills on factorization, exponents, directed numbers and solving equations, among other factors. In the 2014 NSC Examination Diagnostic Report on learners' performances, it is stated that the learners' conceptual understanding of the application of differential calculus is still seriously problematic (DoE, 2014).

Differential calculus, an important aspect of Mathematics that is of interest to this research work, is one of the ten learning areas of Grade 12 Mathematics curriculum in South Africa. Taught for three weeks (4.5 hours being for each week) in the second

term of every academic year, it carries a weighting value of about 35%. Expectedly, Grade 12 learners should be able to, among others, use and understand the principles of differential calculus to determine the rate of change of simple, non-linear functions and to solve simple optimization problems (DoE, 2012).

Unfortunately, as vital and inter-connected as differential calculus is to other learning areas of Mathematics and various other disciplines, it is discouraging to realize that many learners all over the world do not perform well in it. Studies have revealed that a lot of learners struggle with the conceptual understanding of differential calculus (Habre & Abboud, 2006; Maharaj, 2013; Siyepu, 2013). The learners continually find it difficult to apply the basic principles of derivatives to solve even routine problems (Selden, Selden, Hauk & Mason, 2000). They also find acquiring the graphical knowledge of the derivative difficult (Berry & Nyman, 2003; Orhun, 2012). Expressing their worries about the increasing failure-rates in differential calculus, Michchelmores and White (1996) note sadly that majority of the learners study the concept by rote learning. Bezuidenhout (2001) thus advises that well-constructed mental representations of the network of relationships among differential calculus concepts are essential for a thorough conceptual understanding of its underpinning principles.

Like their several other counterparts learning differential calculus in other parts of the world, the case of Grade 12 learners in South African high schools is also not a remarkable overall performance. Corroborating this claim are the yearly NSC Examiners' Reports on the performances of the learners in questions set on differential calculus in Mathematics 1 of their NSC examinations. With reference to the 2009, 2010, 2013 and 2014 reports (see Appendix 9), majority of the learners in the matriculation examinations generally demonstrate a poor understanding of differential calculus particularly the aspect of stationary points and thereby fail in the questions set on it.

As noted by the examiners while marking the learners' answer scripts, some of the candidates' common errors and misconceptions are: they confuse x-intercepts with turning points and still struggle to realize that, at a stationary point, the derivative is zero. They lack the ability to apply the basic rules of differentiation and have difficulty in

interpreting correctly higher order questions involving setting up two simultaneous equations. Furthermore, candidates could not display mastery of the derivative graph of a function and a good grasp of the derivative (gradient) when applying it to the shape and turning points of a (cubic) function. They do not know how to get the x-coordinate of a point from a given x-intercept of a derivative graph. Worse still, candidates give bad interpretations of questions requiring application of calculus to real-life problems. The NSC examiners regretfully remark: *in general, the candidates' responses to questions on differential calculus are poor* (See Appendix 9).

As a Grade 12 Mathematics teacher myself, I had noticed with concern the weak performances of many of my Grade 12 learners over the years in the concept of stationary points in differential calculus. I could observe that many of them had apparently determined to give less attention to the topic during their study hours and even in the tests and examinations simply because they considered the topic a great challenge to them. Bearing in mind the usefulness of differential calculus (sub-section 1.2.1 refers) and that of stationary points (sub-section 1.2.4) in particular, and the fact that whether these Grade 12 learners like it or not, those of them whose career paths require any level of mathematical sophistication (such as engineering, technology, or any science-based courses) are required to take differential calculus as a course in their first year in the university, I had decided to conduct the research on this aspect of Mathematics.

Thus, in an attempt to find appropriate learning strategies that could equip the affected learners with a reasonable measure of conceptual understanding of the concept, I resolved to investigate the impact that the constructivist instructional approach might possibly have. As part of the preliminary efforts made to this end, I consulted with some Mathematics lecturers handling differential calculus in four South African universities (UNISA and TUT inclusive) to find out what the learners' performances were in the first-year course. I gathered that the first-year students did demonstrate poor problem-solving skills and weak performances in the differential calculus module every year.

In view of this development, the study chose to make use of the instructional theory of constructivism (the main paradigm within which Mathematics and science educators have been working for few decades now) to investigate strategies by which the Grade 12 learners might learn stationary points in differential calculus.

For many years, the traditional approach to the teaching and learning of Mathematics making use of the chalkboard and textbooks had been in use. In an average traditional Mathematics classroom, learners are involved in few class activities. A good part of the instructional time is devoted to individual seatwork and whole class recitations led by the teacher. The instructions and curriculum rest seriously on *transmission* and *absorption*. Learners passively absorb mathematical structures created by others, as written in texts or as taught by other authoritative sources. Hence, teaching is done by way of transmitting sets of established facts, skills and concepts to learners. Obviously, traditional learning approach is an instructional method in which the teacher, seen as the authority source, transmits knowledge to learners considered *tabula rasa* who receive it as complete and correct. It is easy then to point out that effective learning might not be sufficiently achieved in such a situation.

A worthwhile alternative therefore might be to shift the focus of the classroom from being teacher-dominated to learner-oriented using the constructivist instructional approach believed to be effective in enhancing learners' understanding and achievement (Akkus, Kadayıfçı, Atasoy & Geban, 2003; Gallery, 2001; Jonassen, 2011; Nancy & Palmer, 2005). In a constructivist learning environment, learners are expected to play an active role in the learning process while the teacher assumes the duty of a facilitator assisting learners to get to their own understanding of a concept by directing them towards "developing new insights and connecting them to their prior knowledge" (Doolittle, 1997).

For a proper understanding of the constructivist instructional approach used in the learning of the concept, the approach is compared with the traditional instructional approach in Table 1-1 below according to Akkus, Kadayıfçı, Atasoy and

Geban(2003),Gallery (2001),Jonassen (2011), Kilavuz(2005),Kim (2005),Moyo (2014),Nancy and Palmer (2005), Nayak (2012)and Nkhoboti (2002).

Table 1-1 Comparing Constructivist and Traditional Instructional Approaches

Constructivist Instructional Approach(CIA)	Traditional Instructional Approach (TIA)
Teacher guides and facilitates the lesson. He is not regarded as the sole transmitter of ideas or the primary source of information, but rather a mentor and one of several available sources of information.	Teacher is considered the only/major source of ideas. Hence, the approach is teacher-directed and ruled by direct, unilateral instruction.
Learners' ideas, views, questions, suggestions and contributions are allowed to drive and direct the lesson.	The structure of the lesson depends heavily on textbooks and pre-determined plans, skills and content/curriculum.
Does not believe in the existence of absolute or unalterable truth. It attaches importance to the development of learners' personal mathematical ideas based on their past experiences, individual views, and cultural backgrounds. So, it considers learning as being subjective.	Holds that there is an established body of knowledge learners must have to imbibe. Thus, it values only established mathematical techniques and concepts. That means the approach sees knowledge as being objective.
The learning process focuses more on the process itself than on the outcome. Hence, learners arrive at ideas and answers to questions as a consequence of the learning process.	The learning process focuses mainly on answers, not really on the process of getting such answers. In fact, learners are not fully engaged in the derivation of answers.
De-emphasizes rote, mechanistic, transmission-based learning but promotes	Characterized by mechanistic learning and memorization of facts, learners are

meaningful learning which it believes can occur as the new knowledge is related to the learners' relevant prior knowledge.	expected to regurgitate facts, explanations and the methodology previously transmitted to them by the teacher.
Learning is perceived as an active process that is constructed. The emphasis is hence on the learners who are allowed to play an active role in the knowledge constructions.	Learning is considered as a passive process that is acquired. So, the emphasis is on the teacher transferring knowledge to learners passively receiving it.
Learners are divided into small groups and so have the opportunity to interact among themselves to share ideas.	Gives virtually no attention to learners' interaction as the teaching-learning process is done on a whole-class basis.
Assessment is interwoven with learning and is carried out through teachers' observation of learners at work and by learners' exhibitions and portfolios.	Assessment is viewed as separate from learning and is conducted by conventional tests. The teacher does not involve learners in the learner assessment.

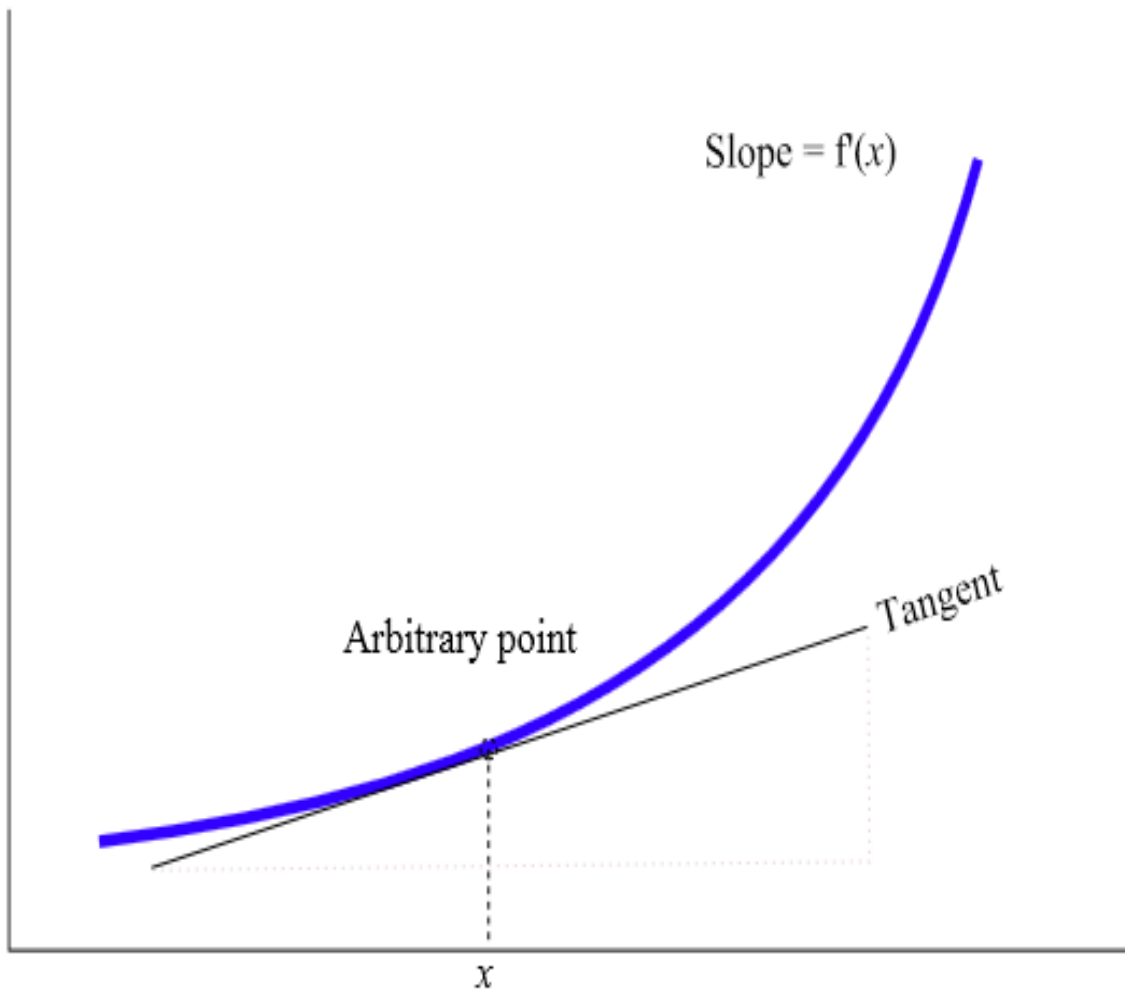
1.2 Differential Calculus as an Aspect of Mathematics

Differential calculus is the study of the definitions, properties and applications of the derivatives of functions. The process of finding the derivative is called *differentiation*. Given a function and a point in the domain, the derivative at that point is a way of encoding the small-scale behavior of the function near that point. By finding the derivative of a function at every point in its domain, it is possible to produce a new function, called the *derivative function* or just the *derivative* of the original function. The common symbol for derivative is an apostrophe-like mark called *prime* (Clar, 2013).

The derivative of the function of f is denoted by f' , pronounced f prime. The common notation introduced by Leibniz's notation for derivative is $\frac{dy}{dx}$ while Lagrange's notation is

$f'(x)$. For instance, if $f(x) = x^3$, then $f'(x) = \frac{dy}{dx} = 3x^2$ is its derivative. Below is the derivative, $f'(x)$, of a curve at a point, which is the slope of the line tangential to that curve at that point. Notice that the tangent touches the curve at $[x; f(x)]$.

Figure 2-1: Graph of the Derivative of a Curve at a Point



$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

Where m = slope or gradient of a straight line. Derivatives give an exact meaning to the notion of change in output with respect to change in input.

Given a function f and a point a in the domain of f , $[a;f(a)]$ is a point on the graph of the function.

Let h be a number close to zero.

Then, $a + h$ is a number close to a .

Therefore, $[a + h; f(a + h)]$ is close to $[a;f(a)]$.

The slope between these two points is

$$m = \frac{f(a+h) - f(a)}{a+h - a} = \frac{f(a+h) - f(a)}{h}$$

Now, to define the derivative, take the limit as h tends to zero ($h \rightarrow 0$). This implies f has been considered for all small values of h . Note also that $h \neq 0$ to avoid obtaining zero as the denominator, which will render the function undefined.

This is shown as $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

1.2.1 Usefulness of Differential Calculus

Calculus is used in actuarial science, computer science, statistics, engineering, economics, business, medicine, demography and every branch of the physical sciences. It is also used in fields wherever a problem has to be mathematically modeled to obtain an optimal solution. Calculus is widely used in aspects of Physics such as mechanics and electromagnetism; to find the *best fit* linear approximation for a set of points in a domain in linear algebra; to determine the probability of a continuous random variable from an assumed density function; to find high points and low points (maxima and minima), slope, concavity and inflection points in analytic geometry etc.

As a learning area of Mathematics dealing with the study of the rates at which quantities change, the purpose of studying differential calculus is simply to introduce learners' minds to the scientific method of analysis. Through science, practical problems can be identified, explanations generated and logical solutions selected. The aim is for the learners to understand how to apply their minds in a systematic manner towards understanding the world around them (Hussain, 2012). Hussain further observes that the purpose of studying differential calculus is two-fold. First, it is a way of introducing the basic concepts of Mathematics used in studying almost any type of changing

phenomena within a controlled setting. Second, studying differential calculus will develop invaluable scientific sense and practical engineering problem-solving skills in learners. It makes learners understand how to think logically to reduce even the most complex systems to a few interacting components.

Differential calculus has applications in computations involving velocity and acceleration, the slope of a curve, and optimization. It also helps in offering a clear understanding of the nature of space, time and motion. In the study of motion and area, there had for long been the puzzles involving division by zero or sums of infinitely many numbers. It is differential calculus that provides useful tools that resolve the puzzles. It is also of interest to note that the applications and generalizations of derivatives appear in several fields of Mathematics, such as: complex analysis, functional analysis, geometry, and algebra. In fact, derivative (or differentiation) has wide applications not only in Mathematics but also in nearly all quantitative disciplines such as: Physics, Chemistry, Engineering, Operation Research, Economics, Statistics, Computer Science, Medicine, Demography and in other fields where a problem can be *mathematically modeled* and an *optimal* solution is demanded.

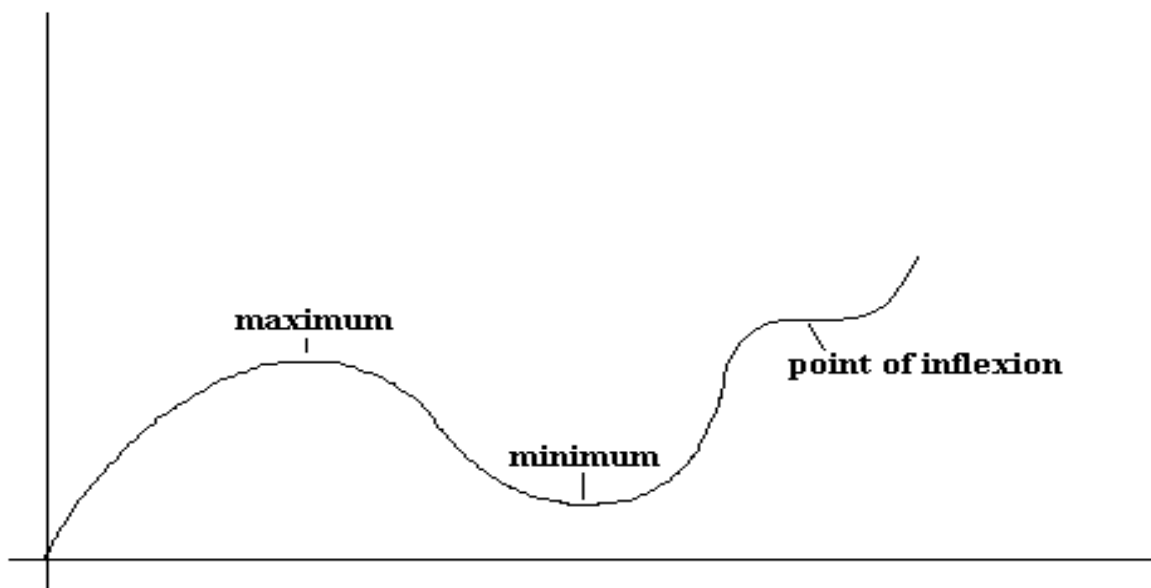
In science, one major goal is to predict what will happen in future. To do this, we must determine how changes in certain quantities will affect the future behaviour of other associated quantities. This is what differential calculus does best as the study of change, more precisely, changing quantities. Moreover, it involves the manipulations of functions. In fact, Introductory Calculus lays the foundation for understanding all sorts of scientific models (James, 2001). As pointed out by Clar (2013), apart from its centrality to Mathematics, differential calculus can be used in conjunction with other mathematical disciplines. For example, it can be used with linear algebra to find the *best fit* linear approximation for a set of points in a domain. Also, it can be applied in probability theory to determine the probability of a continuous random variable from an assumed density function. In analytic geometry, the study of graphs of functions, differential calculus is needed to find high points and low points (maxima and minima).

As specified in the NCS 2012, the current policy statement for teaching and learning in South African schools, those aspects of differential calculus Grade 12 learners are expected to learn include: *limits, rate of change or gradient of a function at a point; derivatives of functions from first principles; applications of the product and quotient rules etc.; equations of tangents to graphs; graphs of cubic and other suitable polynomial functions; stationary points, x-intercepts, and practical problems involving optimization and rate of change.* However, for in-depth study and to make the inquiry manageable, the research concerned itself with application of differentiation in obtaining stationary points.

1.2.2 Stationary Points in Differential Calculus

Stationary points are points on the graph where the gradient or slope is zero. At these points, the tangent to the curve is horizontal; hence, $f' x = \frac{dy}{dx} = 0$. The three types of stationary points are: maximum point, minimum point and point of inflection/inflexion). These stationary points are of great interest to the scientists, engineers and economists who use them in several applications which include maximizing power and profit, and also minimizing losses and costs. The three stationary points are illustrated below:

Figure 2-2: Graph of Stationary Points



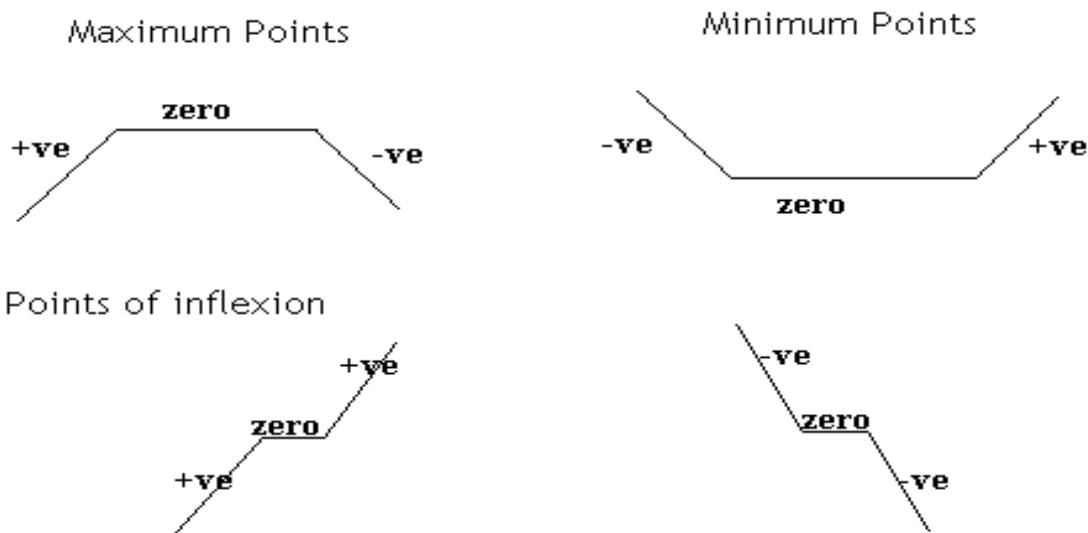
1.2.3 Finding Stationary Points and Determining their Nature

In an attempt to find stationary points of a function $f(x)$, we first find $f'(x)$, (i.e. $\frac{dy}{dx}$). Then, we have to get the zeroes of $f'(x)$ and later their y values. After obtaining a stationary point, another point of interest is to determine the nature of the stationary point. That is, we would want to find out whether the stationary point is a maximum point, a minimum point or a point of inflection. In order to do this, the second derivative, $f''(x) = \frac{d^2y}{dx^2} = 0$, has to be found. An alternative way here is to consider the gradient at either side of the stationary point. In fact, locating stationary points and determining their nature requires a good knowledge of differentiation.

Method 1 of determining the nature of stationary points – Checking the gradient at either side of the stationary point:

At a maximum point, the gradient is positive just before the maximum, it is zero at the maximum and it is negative just after the maximum. At a minimum point, the gradient is negative before the minimum, zero at the minimum, then positive after the minimum. Finally at a point of inflexion, the gradient is either positive, zero, positive or negative, zero, negative. This is illustrated thus:

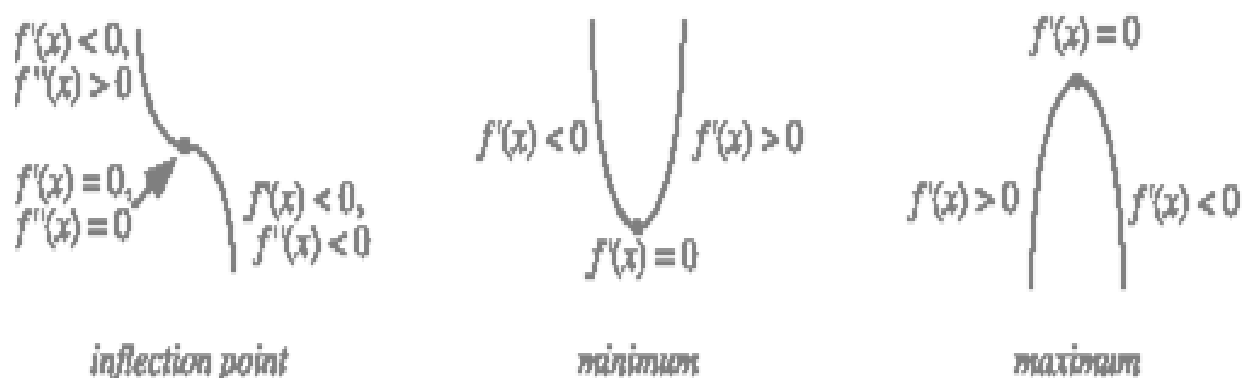
Figure 2-3: Graphical Representation 1 of the Nature of Stationary Points



Method 2 of determining the nature of stationary points:

Let there be a function $f(x)$. After finding the zeroes of $f'(x)$ and later their y values, obtain $f''(x)$. If $f''(x) < 0$, the stationary point is maximum and the curve is concave down; but if $f''(x) > 0$, the stationary point is minimum and the curve is concave up. However, the stationary point is a point of inflexion when $f''(x) = 0$. Here, the concavity changes from downwards to upwards as the sign of $f'(x)$ remains positive without changing; or the concavity changes from downwards to upwards while the sign of $f'(x)$ remains negative without changing. This is illustrated below:

Figure 2-4: Graphical Representation 2 of the Nature of Stationary Points



1.2.4 Importance of Stationary Points

The concept of stationary points is an essential part of differential calculus considering its usefulness. Among its other uses, it is applied in solving practical optimization problems. Whenever Mathematics is being used to model our physical world, we often express physical quantities in terms of variables. Functions are therefore used to describe how the variables change. Scientists, engineers and economists, for instance, show significant interests in the ups and downs of a function, its maximum and minimum values (i.e. its turning points). In several applications, these professionals are interested in such points obviously for maximizing power or profit and also for minimizing losses or

costs. More generally, they use the idea of stationary points for the selection of the best element (considering certain criteria) from some set of available alternatives. As future professionals, it is thus necessary that these Grade 12 learners have to learn and understand stationary points and their uses.

1.3 Statement of the Problem

The focus of this study is to investigate the extent to which constructivist pedagogical approach could improve the learning of stationary points in differential calculus. To this end, the study intends to measure the learners' understanding of the concept of stationary points through their achievement scores. In an attempt to look into the research problem, the following research questions were formulated:

1. Does constructivist instructional approach improve Grade 12 learners' performance in the concept of stationary points in differential calculus?
2. Does the constructivist instructional approach facilitate understanding of the concept of stationary points in differential calculus?

The hypotheses below, stated at .05 probability level of significance, were used to guide the study:

H₀: There is no significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

H₁: There is a significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

1.4 Research Objectives

The overall objective of the study was to assess the influence of the use of constructivist learning approach on Grade 12 learners' understanding of stationary points in differential calculus. More specifically, the research objectives were to find out if:

- constructivist instructional approach can enhance the learning of the concepts of stationary points in differential calculus;
- constructivist learning can improve the performance of the concerned learners in the post-test set on the concept.

1.5 Rationale of the Research

This study may be used to:

- point out that the learning of stationary points does not have to be by memorization of rules, formulae and algorithms of differentiation or those of stationary points, or by passive reception of the knowledge transmitted by the teacher;
- show that the Grade 12 learners can actively construct their own knowledge of the concept by making links between their previously acquired ideas and the new concept through experience. It wants to establish that they can learn the concept by interacting, experimenting, hypothesizing, exploring and reasoning individually and on group basis;
- show that deep understanding of the concepts of stationary points in differential calculus can enhance its practical applications;
- contribute to existing body of knowledge in similar researches and also to serve as a basis for further studies on the topic.

1.6 Definition of Key Terms

For clarity and easy understanding of the study, it is proper at this point to explain the key terms used in the work. This is done as follows:

Impact: influence or effect

Constructivism: is a theory of learning that postulates that humans build new knowledge upon their previous knowledge and understanding. This implies they match

their new ideas and experiences against their prior knowledge to make a sense of the world. In a constructivist classroom therefore, learners are responsible for their learning and they come to class with their own ideas of the world. In essence, constructivism upholds that learners actively create knowledge and do not passively receive it from the environment.

Constructivist Instruction: lessons or learning based on the principles or philosophy of constructivism.

Traditional Instruction: is a form of learning that is based on *transmission* or *absorption* with the belief that learners passively absorb mathematical structures invented by authoritative sources or as recorded in texts.

Differential Calculus: is an aspect of calculus dealing with the study of the definitions, properties and applications of the derivatives of functions. The process of finding the derivative of a function is called *differentiation*. The derivative of a function f is symbolized by $\frac{dy}{dx}$ or $f'(x)$ (pronounced *f prime*) and is given by $\frac{dy}{dx} = nx^{n-1}$ when $y = x^n$.

Stationary Points: are points on the graph where the gradient is zero. At any stationary point, the tangent to the graph is horizontal or parallel to the x-axis. The three types of stationary points are: maximum point, minimum point and point of inflection/inflexion.

Prior Knowledge: is learners' entry behaviour or previous experience capable of facilitating the learning of a new idea.

Understanding: refers to the performance or academic achievements of the learners under study.

Academic Achievement: is the learners' successful accomplishment of a learning goal which, in this study, is their encouraging performance in the application of differentiation in finding stationary points.

1.7 Layout of the Dissertation

The dissertation consists of five chapters with each explaining one essential component of the work or the other. Having explained Chapter One as containing the above, the remaining part of the investigation shall assume the structure explained below. Chapter Two presents the review of relevant literature to gain insight into certain issues critical to the study. Chapter Three describes the methodology adopted in conducting the research. This encompasses the research design; research population, sample population and sampling techniques; instrumentation, development, validity and reliability of instruments; pilot study, ethical issues, methods of data collection and analysis. While Chapter Four deals with data analysis, interpretation and presentation of results, Chapter Five discusses the results of the study and also provides conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This section is a review of relevant literature to gain insight into certain issues critical to the study. These essential issues include: traditional instructional approach and its shortcomings; an overview of the learning theory of constructivism; the pedagogy of constructivist learning theory and a survey of studies in support of the constructivist classroom teaching. The various useful analyses of the opinions of several researchers and scholars shall be used in this regard.

2.2 Traditional Instructional Approach and Its Shortcomings

Kalu (2012) describes the traditional classroom as a one-person show with largely uninvolved learners, seen as empty knowledge-seekers. He notes that traditional classes are usually dominated by direct and unilateral instructions from the teacher, who seeks to transfer thoughts and meanings to the passive learners thereby leaving a limited chance for learner-initiated questions, independent thoughts or interactions among learners. Within this conception, learners are expected to blindly accept the information from the instructor without questioning. Corroborating this, Stofflett (1998) remarks that the traditional approach followers assume there is already a proven body of knowledge that learners have to imbibe.

The views expressed by Kalu and Stofflett about the traditional instructional approach might be worthy of belief. The learning approach is a passive transmission of information from the teacher to the learners. It takes learners for empty vessels meant to be filled by the teacher who is recognized as the sole dispenser of knowledge. So, learners have to absorb the ideas and knowledge transmitted to them only to regurgitate such during examinations. In this conventional instructional practice, the learning process is fully controlled by the teacher thereby making it impossible for learners to exercise independence. Besides, the structure of the traditional classroom depends

heavily on textbooks, workbooks, worksheets, established facts and the curriculum content.

Studies (Kilavuz, 2005; Kim, 2005; Moyo, 2014; Nayak, 2012; Nkhoboti, 2002) have shown that traditional instructional approach has failed to achieve maximal attainment of learning outcomes. With the noticeable deficiency of the traditional approach, most learners have regrettably been unable to connect what they learn in the Mathematics classroom to real life. To the learners, vander Berg and Louw (2006) observe that, practical knowledge and school knowledge are not really inter-related. In a research conducted by Mochesela (2007), she informs that traditional approach has produced learners whose performances in Mathematics are not satisfactory and who are not sufficiently equipped with critical problem-solving skills that are necessary in this dynamic world. In another investigation, Stofflett (1998) discovers that the traditional, teacher-dominated, knowledge-dissemination approach evidently promotes rule-bound, rote and mechanistic learning. Being ruled by memorization and learning of isolated concepts and procedures, it usually results to poor knowledge transfer, low academic achievements and under-performances.

In the light of the above, the traditional teacher, as information-giver to passive learners, and the textbook-guided classroom have been unable to bring about the expected outcome of producing thinking learners (Young & Collins, 2003). This is because the approach emphasizes the learning of answers more than the exploration of questions, memorization at the expense of logical thinking, bits and pieces of information as against understanding in context. In addition, the learning approach does not involve learners in active knowledge construction. It fails to encourage them to work together, to contribute and share ideas with one another; hence, the clamour for the learner-centred constructivist instructional approach. With this realization, there is a need to modify or replace the learning approach with a more applicable and more gainful one.

2.3 The Learning Theory of Constructivism

Bruning, Schraw, Norby and Ronning (2004) describe constructivism as a psychological and philosophical perspective contending that individuals form or construct much of

what they learn. The paradigm of constructivism is largely contributory to the shift of the responsibility for learning from the teacher to the learners. Making the learners largely responsible for their learning, it therefore presents learners not again as empty vessels waiting to be filled by the teacher seen as the only possessor and transmitter of knowledge. It views learners as active individuals capable of constructing new knowledge and understanding using preferred learning styles, and the teacher as a learning facilitator. Hence, learning outcomes do not depend solely on what the teacher presents, contrary to the practices characterizing the teacher-dominated traditional instructional approach. They rather depend on the interactive outcome of the information learners encounter and the way they process it based on perceived notions and prior knowledge. By this, learners can obtain concretized knowledge that can be meaningfully applied to practical, real-life situations.

The study specifically followed the perspective of cognitive constructivism. Jean Piaget, the Swiss child psychologist, who was one of the most popular exponents of cognitive constructivism, asserted that knowledge is internalized by learners. He described the systems of knowledge as *schemata*. He opined that through the processes of *accommodation* and *assimilation*, people construct new knowledge from their experiences.

According to Piaget, a person *assimilates* by bringing into an existing framework a new experience without changing the framework. It indicates that the person sees new objects or events as already existing schemes or operations. This becomes possible as the person's experiences go in line with his internal representations of the world. On the other hand, an individual *accommodates* as he modifies his mental perception of the external world to be in line with the new experiences. Existing schemes or operations have to be reorganized to cater for a new experience. The process of accommodation can be a means through which failure leads to learning since a contradictory idea a person might have had about the world can easily be changed or reframed.

The constructivist learning theory emphasizes the essence of interactions between the teacher and learners, learners and learners, and even learners and the content. If actively involved in the learning process, learners can restructure and harmonize their

past and present experiences and arrive at new constructions. To Ncharam (2012), constructivism is a reaction to teaching approaches such as behaviourism and programmed instruction. It presents the learners as constructors of information acting on their personal experiences and hypotheses of the environment. It believes that by social negotiations, learners test their hypotheses and create new knowledge, modify previous knowledge, or confirm the current one.

At this point, the Vygotsky's contributions to how learners construct knowledge are also relevant. Unlike Piaget who worked on stages of learners' cognitive development, Vygotsky posited that social factors contribute considerably to cognitive development. He argued that cognitive development depends largely on social interactions arising from guided learning within the proximal development zone. According to Vygotsky, the environment in which learners find themselves affect how they think and what they think about. Also, Vygotsky stressed the essence of language in cognitive development. To him, cognitive development results from an internalization of language.

Vygotsky postulated that the knowledge a learner constructs has to first exist in the social context and setting of that learner (Plourde & Alawiye, 2003; Woolfolk, 2010). In view of this, learning occurs when the learner interacts with other people, adults or more knowledgeable peers in his or her social and cultural setting (Kansellar, 2002). This automatically recognizes the essence of language as a vital tool for acquiring knowledge within the social constructivist context (Nicaise & Barnes, 1996). The Vygotskian social constructivist theory identifies two levels of development: the *actual* level of development and the Zone of Proximal Development (ZPD). At the *actual* level of development, the learner is capable of constructing knowledge without the help of another person (Woolfolk, 2010). At the ZPD level, to be able to solve a problem, the learner requires the assistance of another person – an adult or a more knowledgeable learner (Kansellar, 2002; Nicaise & Barnes, 1996). The main implication of the Vygotskian social constructivist theory for teaching and learning is that learners must construct their own knowledge and meaning through social interaction with peer learners and adults or their teachers.

It is worthy of note that constructivism is not a particular pedagogical practice but rather an epistemology explaining how learning takes place. Generally, it is often considered as pedagogical acts which promote active or discovery learning. It is described as learning by doing – a practice which places learners at the centre of their learning. Today, constructivist ideologies are influential throughout much of the non-formal learning sector. Notable writers whose works influenced constructivism include: John Dewey (1859 - 1952), Maria Montessori (1870 - 1952), Jean Piaget (1896 - 1980), Lev Vygotsky (1896 - 1934), David Ausubel (1918 - 2008), Jerome Bruner (1915 - till date) to mention but a few.

With a view to providing effective learning techniques through which the twelfth-grade learners could properly understand the vital concept of stationary points, the learner-centred constructivist instructional approach was employed for the investigation. In fact, the learning approach of constructivism is not a new paradigm as several current reform efforts geared towards finding appropriate learning strategies are associated with it. Various researchers (Akkus, et al., 2003; Cranton & Kroth, 2014; Edinyang, 2013; Jonassen, 2011; Kalu, 2012; Mochesela, 2007; Palmer, 2005) submit that constructivist instructional strategies are effective in enhancing learners' understanding and achievements. They regard it as an instructional approach that has proven to be capable of giving hope to the development of the deep understanding of Mathematics in learners at all levels.

2.4 The Pedagogy of Constructivist Learning Theory

Wolfgang (2001) comments that only an effective pedagogical practice can promote the well-being of learners, teachers and the school community. It improves learners' and teachers' confidence and contributes to their sense of purpose for being at school; it builds community confidence in the quality of learning and teaching in the school. A useful lesson that can be learnt from the remark made by Wolfgang is that for the teacher to attain set learning outcomes, he has to discover and apply appropriate teaching-learning strategies peculiar to the set of learners he has at a point in time.

The views expressed by Kim (2005), Kalu(2012), Opoh and Iwok (2014) about constructivist pedagogical practices in a Mathematics classroom can as well be found helpful to the teacher. As they claim, in a constructivist lesson, the teacher has to set up suitable hands-on task and find means of engaging the interest of the learners in the topic. He has to introduce certain basic ideas that give life and form to the subject matter. He then revisits and builds on these repeatedly. He can accomplish this by giving a demonstration, telling a relevant short story, presenting some data or showing a short film. The teacher must know that learners come to class with certain ideas about the natural world. He has to take these ideas into account and provide activities capable of enabling learners to develop their current understanding to a more plausible idea. He needs to ask open-ended questions to probe learners' pre-conceptions of the topic. He can even present some ideas challenging, contradicting or conflicting with their existing understanding and experiences.

As further noted by the Kim (2005), Kalu (2012), Opoh and Iwok (2014), the teacher should divide the learners into small groups where they will have the opportunity to formulate their own hypotheses and experiments that can make them link their previous understanding with current constructions. In the process, the learners are required to think critically, constructively and intelligently. Thus, the learning environment has to be such that is challenging and supportive of learners' thinking. During the small group interaction time, it is required of the teacher to move round the classroom to guide, help or ask probing questions capable of assisting the learners to come to an understanding of the principle being studied. As identified by Brookfield (2005), some of the benefits of learners interacting, collaborating and working together are: learners explore a diversity of perspectives; their intellectual ability increases; learners' voices and experiences gain respect and value; habits of collaborative learning are promoted and learners develop skills of synthesis and integration.

Still in line with the opinions of the researchers above, after being given enough time for discussions and interactions, the small groups present their ideas and findings in turn to the whole class so as to reach a consensus on the right ideas to imbibe. Nancy and Gallery (2001) agree that allowing learners to play a much more active role in learning

motivates them to learn, to identify and resolve their personal misunderstandings, and to apply what they are learning to situations relevant to their own lives. It enables learners to understand better, and develop strengths and interests in their learning, which in turn provide for life-long learning and career opportunities.

For a constructivist teacher to successfully guide and monitor the learners towards constructing new knowledge, Brownstein (2001) submits that the teacher as the facilitator is expected to display a totally different set of skills from that of a teacher. Corroborating his view, Rhodes and Bellamy (1999) comment: a teacher tells, a facilitator asks; a teacher lectures from the front, a facilitator supports from the back; a teacher gives answers according to a set curriculum, a facilitator provides guidelines and creates the environment for learners to arrive at their own conclusions; a teacher mostly gives a monologue, a facilitator is in continuous dialogue with the learners. Edinyang (2013) summarizes it that the primary task of a constructivist Mathematics is to improve the learning gains of the learner in the subject through the application of appropriate instructional techniques, as the success or failure of the learners in any given Mathematics examinations is often attributed to the ability or inability of the teacher to use good instructional strategy to achieve predetermined instructional objectives

The essentially interactive nature of constructive learning is also extended to the process of assessment. Instead of considering learner assessment as a process carried out solely by the teacher, it is conducted as a two-way process involving both the learners and the teacher. Authentic assessment is best achieved through teaching, interactions between both the teacher and the learners, and learners and learners; and as well by observing learners in meaningful tasks. Holt and Willard-Holt (2000) stress the concept of dynamic assessment in a constructivist learning environment. They explain it as a way of determining the true potential of learners that differs significantly from conventional tests. Learner assessment should therefore avoid standardized tests and grades such as achievement tests designed with multiple choices to test subject-specific knowledge.

Assessment should be made part of the learning process as learners are supposed to play an important role in examining their own progress. The teacher has to enter into dialogue with the learners being evaluated so as to determine their present performance level in a learning task and deliberate with them on possible means of improving upon such performance on subsequent occasions. Types of assessment in line with the constructivist views include: reflective journals/portfolios, case studies, group-based projects and presentations.

In his study, Yager (1991) offers the following as strategies for implementing a constructivist lesson. In starting a lesson, the teacher should observe the surroundings for points to question; ask learners leading questions; consider possible responses to questions; note unexpected phenomena and identify situations where learner perceptions vary. He advises that, in the course of the lesson, the teacher should: engage in focused play; brainstorm for possible alternatives; look for information; experiment with materials; observe a specific phenomenon; design a model; collect and organize data; employ problem-solving strategies; select appropriate resources; review, critique and integrate a solution with existing knowledge and experiences.

2.5 Studies in Support of Constructivist Classroom Teaching

Researchers generally acknowledge that constructivist learning strategies contribute significantly (although usually contested) towards bringing about effective learning (Arredondo, 2011; Boudourides, 2003; Plourde & Alawiye, 2003; Soanes, 2007). On account of this, for some decades now, the trend in understanding how students learn has shifted from behaviorism to the cognitive approach and subsequently to constructivism (Bolt & Brassard, 2004). As theorized by the constructivists, learners are capable of constructing their own new understanding based on the interaction between their prior knowledge and the current ideas, events and activities they encounter (Boudourides, 2003). In his study too, Nkhoboti (2002) found out that learners have abilities to depict behaviour that conform to the principles of constructivism.

Moyo (2014) adopts a parallel mixed-method design to investigate the factors impacting on the choice and use of constructivist teaching methods by high school Mathematics

teachers in some Gauteng schools. The results of his study show that constructivist learning strategies prove to be more helpful than traditional pedagogical strategies as they enable learners to construct knowledge actively and creatively. In his own study, Kim (2005) investigated what the effects of constructivist approach could be on academic achievement, self-concept and learning strategies, and learner preference. He involved 76 grade six learners divided into the experimental and control groups. While the experimental group was taught with the constructivist approach, the control group was exposed to the traditional approach for 40 hours over a period of 9 weeks. He made use of mathematics achievement tests, self-concept inventory, learning strategies inventory, and a classroom environment survey. In his findings, he obtained that constructivist learning is more effective than traditional teaching in terms of academic achievement; it has some effect upon motivation, anxiety towards learning and self-monitoring and that a constructivist environment was preferred to a traditional classroom.

Moreover, Nancy and Gallery (2001) conducted a research on the nature of teaching and learning based on the pedagogical applications of constructivism in secondary Mathematics education. In their results, they present a strong case for embracing constructivism for teaching high school Mathematics, considering the positive and significant implications constructivism has for Mathematics education. In a recent research, Kroesbergen and van Luit (2012) concluded that constructivist instruction is found to be more effective than the direct instruction for achievers. In another research, Cekolin (2001) also found out that self-regulated learning strategy in constructivist pedagogy improves achievement in Mathematics and the level of confidence for middle school learners.

Furthermore, Kilavuz(2005) considered the pre-test post-test quasi-experimental design to compare application of 5E learning model based on constructivist theory with traditionally designed chemistry instruction in how grade 9 learners understand acid-base concepts. He described the 5E learning model as having the five phases: Engagement, Exploration, Explanation, Elaboration and Evaluation. His study participants were drawn from two Chemistry classes of sixty learners taught by the

same teacher. The study participants are randomly divided into control and experimental groups. While the control group was exposed to traditional chemistry instruction, the experimental group was given instruction based on 5E learning model. The results indicated that the instruction based on constructivist approach brought about a significantly better acquisition of scientific conceptions related to acid-base.

Equally relevant is the research conducted by Nayak (2012). He adopted a pre-test post-test quasi-experimental design which involved the use of quantitative and qualitative data collection and analysis methods. He investigated students' learning in the constructivist environment and its eventual effects on achievement in Mathematics at elementary level of learning. He tried to probe the difference in achievements of two groups of fifth-grade learners from three urban schools exposed to traditional and constructivist instructional methods. The results of the study reveal a considerable improvement in the achievement scores of the study participants in the constructivist learning environment compared to the low achievement scores of their counterparts in the traditional classroom. It was further discovered that the learners taught in constructivist learning environment have considerably enhanced their understanding and application abilities. The above studies and other relevant ones further informed the researcher's decision to consider making use of the constructivist instructional approach to see how the Grade 12 learners would learn stationary points.

2.6 Summary

The findings of the literature study reveal that the learning theory of constructivism is a popular paradigm that has proven to be capable of offering useful learning strategies that can improve learners' academic gains in Mathematics classes. As active participants in the learning process, learners are to construct knowledge by themselves based on their prior knowledge, past experiences and perception of the new concept. In Mathematics, this means learners are required to create personal mathematical ideas, interpret and solve real-life problems by their own perceived methods and intuitive mathematical thinking.

To do this successfully, enough time should be given to them for small-group discussions, interactions with fellow learners, the teacher and even the learning task, communications, suggestions, questioning, inquiry, exploration and experimentations. There is a need therefore for the constructivist teacher to act as a facilitator, monitor and mentor to them through providing appropriate hand-on activities, useful guide, encouragement and stimulating contexts for meaningful knowledge constructions. In fact, the teacher's pedagogical competence depends largely on his ability to select and apply suitable instructional methods capable of improving the learning outcomes of the learners, and his skills for connecting the learners with prior knowledge and directing them towards developing new insights.

Again, while deciding the curriculum, the content, the methods and the evaluation of the learning process, much consideration should be placed on learners' interests, prior knowledge and past experiences. In fact, for our efforts at reforming Mathematics education for all learners to be fruitful, we need to attach due importance to student-centred learning, which is the popular basis of constructivism. Also, more public awareness of the essence of the learning approach, more teacher education and regular constructivist professional development are necessary for successful classroom applications of constructivist philosophy of learning.

The submission by Nayak (2012) is thus pertinent at this juncture: classroom teaching practice is likely to be more effective when it is informed by an understanding of how students learn and also that learning proves to be more effective if learners are opportune to clarify or explain their own ideas. It is therefore important that the major implications of the learning theory of constructivism should be reflected in classroom practice. However, the question of how to implement classroom teaching that is consistent with a constructivist view of learning still calls for concern.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter presents the research design followed in this study, the research population, sample population and sampling technique, instrumentation and development, validity and reliability of the measuring instruments. It also explains the pilot study, methods of data collection and analysis, and the ethical measures taken in the course of the investigation.

3.2 Research Design

The study was structured such that it followed a mixed-method approach. The approach was considered because both the quantitative and the qualitative methods of data collection and analysis were involved. It was also applied because the researcher wanted to measure the achievement scores of the study participants after the application of the intervention. He intended to justify this by verifying that constructivist instructional approach was used in the experimental group and the traditional approach was followed in the control group. Furthermore, he wanted to be able to account for the qualitative factors indicative of the results from the achievement scores.

The pre-test post-test matching control quasi-experimental design and the descriptive research design were adopted for this study. This required randomly assigning the study participants into control and experimental groups. Pre-test was administered to both control and experimental groups to determine their initial cognitive level in the concept of stationary points in differential calculus. Then, post-test was conducted to gather data on the achievement scores of the study participants. Table 3-1 below illustrates the design used in the study.

Table 3-1: Illustration of the Quasi-experimental Design

Experimental Group	O ₁	X	O ₂
Control Group	O ₁	----	O ₂

O₁: Pre-test based on stationary points

O₂: Post-test based on stationary points

X: Intervention

Table 3-1 indicates that the pre-test (O₁) was administered to both the control and experimental groups at the start of the investigation. The intervention programme (X) was organized for only the experimental group; this implies there was no treatment or intervention for the control group. Then, both groups took the pre-test (O₂) at the end of the enquiry.

The descriptive research design involved using classroom observations to collect data on the natural setting of the research fields. The classroom visits were made to ascertain that classes in the experimental schools were conducted in line with the ideology of constructivism. Here, the behaviour of the study participants were observed and described without necessarily influencing it. The data obtained from the classroom observations were to provide supports and justifications for the learners' scores in the pre-test and the post-test.

3.3 Research Population, Sample and Sampling Technique

Deciding on the population of interest relevant to a study and getting the right respondents that fall within the set categories is crucial to obtaining quality data for it (Nakona, 2006). Hence, a population in the research sense is the complete set of unit of analysis that is under investigation(Davis, 2005).

In the study, the Grade 12 learners in Gauteng province in South Africa formed the research population. There are nine provinces in South Africa. Gauteng was chosen

because of its nearness to the researcher, to minimize costs and to facilitate other research logistics.

From the fifteen school districts in Gauteng, three school districts were randomly selected. Then, from each of the three districts, a school was randomly chosen. The three selected school districts constituted the sample population for the study. As a way of protecting the study participants' rights to privacy and confidentiality, the three schools were simply referred to as GHS 1, GHS 2 and GHS 3 (GHS means Gauteng High School). Two of the schools were randomly selected to be the experimental group while the third one was the control group. These three schools are co-educational and they comprise learners of mixed academic abilities. They all offer Mathematics as a school subject and use English Language as their medium of instructions. In fact, the intact group of the Grade 12 learners in each of the three schools served as the study participants. The learners were 204 altogether. It is important to state here that, although selected from the same district, the three research fields were far apart from one another.

Table 3-2: The Sample Population of 204 Study Participants

Variable	Category	N	%
School	GHS 1	70	34.3%
	GHS 2	68	33.3%
	GHS 3	66	32.4%
	Total	204	100%
Group	Experimental Group (GHS 2 + GHS 3)	134	65.7%
	Control Group (GHS 3)	70	34.3%
	Total	204	100%

3.4 Instrumentation

The measuring instruments prepared for the study are – pre-test/post-test, classroom observation checklist and video-analysis instrument.

3.4.1 Pre-test/Post-test Instrument

The pre-test and the post-test, drawn by the researcher from the 2012 to 2014 NSC past questions on the concept of stationary points in differential calculus, were administered to both control and experimental groups. Each test consisted of five (5) questions (see Appendices 7 and 8). The questions were of reasonable level of difficulty and their solutions demanded a variety of strategies.

The questions were in two categories, namely, procedural and conceptual. The procedural questions demanded simple procedures, algorithms or rules. On the other hand, the conceptual questions required students' understanding of obtaining stationary points by differentiation. It is noteworthy that both the pre-test and the post-test consisted of exactly the same set of questions, though the questions were joggled and numbered differently in the post-test.

The two groups were allowed to write the pre-test on the first day of the 3-week empirical investigation; that was before they were taught the topic at all. It was arranged that way to determine the learners' pre-knowledge of the topic. After the experimental group had been taught with the constructivist pedagogical approach, and the control group through traditional instructional method, both groups wrote the post-test. The essence of the post-test was to see the possible influence the intervention might have had on the learners' level of understanding of the topic.

3.4.1.1 Development of the Pre-test/Post-test Instrument

Since the study participants were Grade 12 learners already preparing to write the October/November 2015 NSC examinations (about five months after the investigation), the pre-test/post-test questions were drawn from past NSC examination questions set by the DoE on stationary points in differential calculus. Hence, no need further

developing the instrument as the DoE in South Africa ably sees to the development of matriculation examination papers.

3.4.1.2 Validity and Reliability of the Pre-test/Post-test Instrument

The South African body named Umalusi Council for Quality Assurance in General and Further Education Training ensures the quality and standardization of the NSC examinations from which the tests were drawn. Hence, the pre-test/post-test instrument was valid and reliable. As touching the

3.4.2 Classroom Observation Checklist Instrument

Classroom observations were carried out to collect relevant data on how the study participants were taught stationary points in differential calculus, especially with the use of the constructivist pedagogical approach. A suitable observation checklist was developed for this purpose (Appendix 6 refers). Classroom observation checklist is a list of factors to be considered while observing a class. It gives a structure and framework for the observation.

3.4.2.1 Development of the Classroom Observation Checklist Instrument

The classroom observation checklist was carefully constructed by the researcher in line with the existing ones already done by experts and with the thorough guide of his Supervisor. Among the characteristics specified on the checklist for observations are: how the teacher reviewed the previous lesson, introduced the new one and linked both for effective learning; how he engaged learners in group discussions; how he facilitated the discussions and the subsequent group/individual class presentations; how he timed and paced the whole lesson; the teacher's ability to use questioning to get learners actively involved in class activities and the extent to which he allows learners' ideas and initiatives.

3.4.2.2 Validity of the Classroom Observation Checklist Instrument

The contents of an observation checklist have to be in line with the objectives for which the checklist is prepared for it to measure what it is expected to measure. The checklist constructed for the classroom visits was validated by four specialists in the field of education. This brought about the reconstruction and removal of some items of the instrument.

3.4.2.3 Reliability of the Classroom Observation Checklist Instrument

Internal consistency reliability was used to check the reliability of the checklist. This ascertained the consistency of the objective each item of the checklist was to achieve with the overall objectives for which the entire instrument was designed. Those four specialists in the field of education, who saw to the validation of the instrument, were also consulted for its reliability.

3.4.3 Video-Analysis Instrument

This was used by the researcher during the classroom visits to be able to capture fully all the classroom activities without leaving out any important aspects.

3.4.3.1 Validity and Reliability of the Video-Analysis Instrument

The instrument got for the purpose of the study was in a good condition. It was a recent model that had necessary accessories required for the recording exercise. Besides, it was well tested and found okay before use.

3.5 Pilot Study

The study was piloted using the Grade 12 learners of two high schools in Gauteng province different from those of the three schools later involved in the main study. All the research tools were carefully tested during the pilot study. This was done purposely to test the procedure to be used in the full-scale project, to determine how the design of

the actual investigation could be improved upon, to identify any possible flaws in the measuring instruments with a view to correcting them and also to determine whether or not the research tools were going to achieve the desired results.

In fact, the pilot test produced an idea of what the entire research method would actually look like in operation and the effects it was likely to have. Besides, it brought about necessary modification, rewording and rearrangement of some aspects of the classroom observation checklist. As was the case with the main study, the pilot testing was also carried out for three weeks in May during term two of 2014 academic year, the exact time stipulated in the Mathematics curriculum for learning stationary points in Grade 12.

3.6 Data Collection Methods

In the study, the mixed-method approach involving both quantitative and qualitative data collection techniques were used. At the quantitative level, the pre-test and the post-test were administered to the study participants. Then, the qualitative part took the form of classroom observations. Taking cognizance of the inequality of the three research fields and their individual contextual differences, the two experimental schools were treated as separate research fields and the data emanating from them equally seen as such, before the comparison of the three schools was later done. The data collection for the research lasted a period of three weeks in the month of May of 2015 academic session.

3.6.1 The Procedure of the Intervention

The experimental schools GHS 2 and GHS 3 had 68 and 66 study participants respectively and the control group, GHS 1, had 70 study participants. The study participants in GHS 2 and GHS 3 were taught the concept of stationary points in differential calculus with the constructivist instructional approach (CIA). But the study participants in GHS 1 received lessons on the same learning contents through the traditional instructional approach (TIA). Both groups were taught by their regular Mathematics teachers.

A week to the intervention, the researcher had a 5-day interactive session (2 hours each day) with each of the two Grade 12 Mathematics teachers of GHS 2 and GHS 3 during which he painstakingly familiarized them with the pedagogical practices of constructivism. The step taken was based on the assumption that not many (high) school-teachers have much knowledge and expertise of the constructivist theory of learning and how it is implemented in the classroom. It was also assumed that even those teachers familiar with the instructional approach might not really be interested in adopting it as they might not see any need for it or might consider its use a sheer waste of time.

As previously trained, the two intervention teachers instructed the experimental group by constructivist principles. At the beginning of each lesson, they made useful demonstrations and raised relevant questions in order to arouse the interest and curiosity of the learners in the topic. This was a deliberate professional act to determine what the learners already know about the topic. The teachers divided their learners into small groups at the beginning of the instruction, and were hence able to achieve a quality level of interactions among the learners.

During each lesson, learners were allowed time to think and discuss in groups making use of their previous knowledge and experience. They were free to make suggestions and even guess possible answers to the questions on stationary points being solved. The teachers also let them write their answers in their notebooks. The teachers did not interfere with the learners' discussions. They were only acting as facilitators, taking time to observe and listen to the interacting learners. After the discussions, each group gave common answers to the teachers. By this, the teachers had the opportunity to find out the learners' previous ideas on the topic. Based on the answers provided by the learners, the teachers explained the topic using illustrations and examples from daily life to make the concepts concrete and real.

In the control group, the teacher administered traditional instructions on the same contents of the learning task to the learners using the traditionally designed Mathematics texts. As typical of the traditional instructional approach, he used lecture

and discussion methods to teach the concept. After introducing and explaining each day's topic, he worked some examples on the chalkboard and gave some exercises to the learners to attempt in their notebooks. The teacher roamed the classroom during the lessons, pointed out noticeable mistakes to the learners, passed some helpful remarks and made necessary suggestions. He then marked the learners' work and did the corrections on the chalkboard.

3.6.2 Pre-test/Post-test Data Collection

The pre-test and the post-test were for the purpose of data collection on the study participants' abilities to solve problems and their achievements in the application of differentiation in obtaining stationary points. While conducting the two tests of 90 minutes each, the learners were instructed not to write down only the answers to the questions, but to also state clearly all the working details.

The conduct of the pre-test took place on the first day of the investigation. That was before the learners were taught the topic. Also, on the last day of the 3-week intervention, the learners were asked to write the post-test, exactly the same set of questions they had as the pre-test. The only difference between the two tests was that the questions were joggled and renumbered in the post-test for the learners not to know that they were the same five pre-test questions that were rearranged. It was noted that all the 204 study participants wrote both the pre-test and the post-test.

Since all the questions were past matriculation examinations, the learners' answers were assessed with the DoE-provided scoring rubrics popularly referred to as memorandum. In the process of the marking, the researcher made sure he did not mark only the answers, but rather considered the step-by-step process of getting each answer. By this, he was able to see what the learners' achievements and problem-solving skills were before and after the intervention. Useful data for the study pertaining to the levels of understanding of the study participants were therefore gathered from the learners' scores in both the pre- and post-tests.

3.6.3 Classroom Observation Data Collection

During the period of the study which lasted three weeks, the researcher made classroom visits to the three schools during the normal school hours to gather useful data on how the learners were taught stationary points in differential calculus through traditional and constructivist instructional approaches by their regular Mathematics teachers. This also made it possible for him to collect data on learner co-operations, contributions and levels of participation during the lessons.

Altogether, eighteen lessons observed were video-taped– six traditional lessons given to the control group in one school and six constructivist lessons received by the experimental groups in each of the two schools. The researcher personally handled the recording of the lessons. He also wrote as field-notes special events that came up during some of the classes observed. After each class, he made use of the observation checklist to grade the recorded classroom lesson procedure.

3.7 Data Analysis Procedures

Since the data collected in this study were qualitative and quantitative in nature, both qualitative and quantitative data analysis methods were considered. The data analysis approach in each case goes thus:

3.7.1 Pre-test/Post-test Data Analysis

The pre-test and post-test scores were first standardized (i.e. converted to percentage). Then, the scores were analyzed with the descriptive and inferential statistics.

3.7.2 Classroom Observation Data Analysis

Analysis of the data collected from classroom observations followed the qualitative data analysis strategy. After each classroom observation, the data gathered on the video-recorder and field notes were screened using the classroom observation checklist. The data were coded, collated and transferred into a spreadsheet. The data were then sorted and the emerging themes noted.

3.8 Ethical Measures

For any research being conducted, it is expected that ethics is taken into account. Ethical consideration involves a set of moral principles that should guide the behaviour of a researcher towards respondents and other researchers (De Vos, 2002). In view of this, necessary approval-seeking steps were taken to get the research acknowledged by all concerned stakeholders before the execution of the research activities.

After the research proposal had been accepted by the university, the ethical clearance (appendix 1) for the conduct of the research was issued by the UNISA Research Ethics Committee. Also, the approval to conduct the research in Gauteng high schools was got from the Gauteng Department of Education (GDE). Later, permission was sought and obtained from the Principals of the three high schools representing the study samples (see appendix 2).

Necessary explanations were given to the study participants the purpose of the research and how it could possibly benefit them, their fellow learners in other schools and the education sector generally. After all, ethics demand that researchers keep the participants informed of the research study and also make every effort to protect them from harm (Gay & Airasian, 2003). Thereafter, two different letters of informed consent were given to each learner - one to seek voluntary participation of each learner in the study, the other to gain the support of their individual parents since the learners were still minors. This is advised by Graziano and Raulin (2004) that the participants have the right to make their own decisions, but they can only make reasonable decisions if they have the relevant information on which to base their decisions.

All the 204 study participants from the three schools willingly consented to participating in the study. As any research processes should not infringe on human rights, cause any kind of harm, or reveal the confidential nature of the participants' involved in a study (Wisker, 2001), unethical treatment of the participants was cautiously avoided in the course of the investigation. To make sure none of them suffered any harm and that their rights to privacy and confidentiality were protected, their names and school names were not mentioned in the study as they had been earlier assured.

Since ethical principles demand that researchers be honest in reporting their findings (Babbie, 2001), after concluding the inquiry, the researcher reported the findings of the study as accurately and objectively as he could. The study participants and other stakeholders including the GDE were informed about the findings and recommendations in an objective way without revealing any details about the confidentiality of the respondents/participants.

CHAPTER FOUR

DATA ANALYSIS AND PRESENTATION OF RESULTS

4.1 Introduction

In the previous chapter, the research methodology adopted in conducting the investigation was discussed. Here in this chapter, the techniques and the results of the data analysis are presented.

4.2 Data Analysis Strategies

As indicated in section 3.6, the study adopted the mixed methods involving both quantitative and qualitative data collection methods. This accounted for the adoption of the quantitative and qualitative data analysis techniques for the data collected. The data collected from the pre-test and post-test scores were analyzed by quantitative data analysis methods, while the data gathered from the classroom observations were analyzed using qualitative data analysis method. The results of the quantitative data analysis shall be presented first for all the schools, followed by the qualitative data analysis results.

4.2.1 Quantitative Data Analysis Strategies

The quantitative data collected were captured on Excel and exported into Statistical Package for the Social Sciences (SPSS). The analyses were done using the descriptive and the inferential statistical tools.

4.2.1.1 Descriptive Data Analysis Strategy

Descriptive statistical analyses were used to describe the initial results of the analysis for each of the schools. This took the form of comparison of the two sets of mean, standard, skewness and kurtosis of the scores obtained by the study participants in both pre-test and post-test in all the groups. Graphs were also used to show the distribution of the scores.

4.2.1.2 Inferential Data Analysis Strategy

Inferential statistical analyses were done on the pre-test and post-test scores of both control and experimental groups across the schools. While the paired t-test statistics was used to see the differences (if any) in the mean performances of both groups in the pre-test and post-test, the post-hoc analysis was carried out to determine where exactly the differences were.

4.2.2 Qualitative Data Analysis Strategies

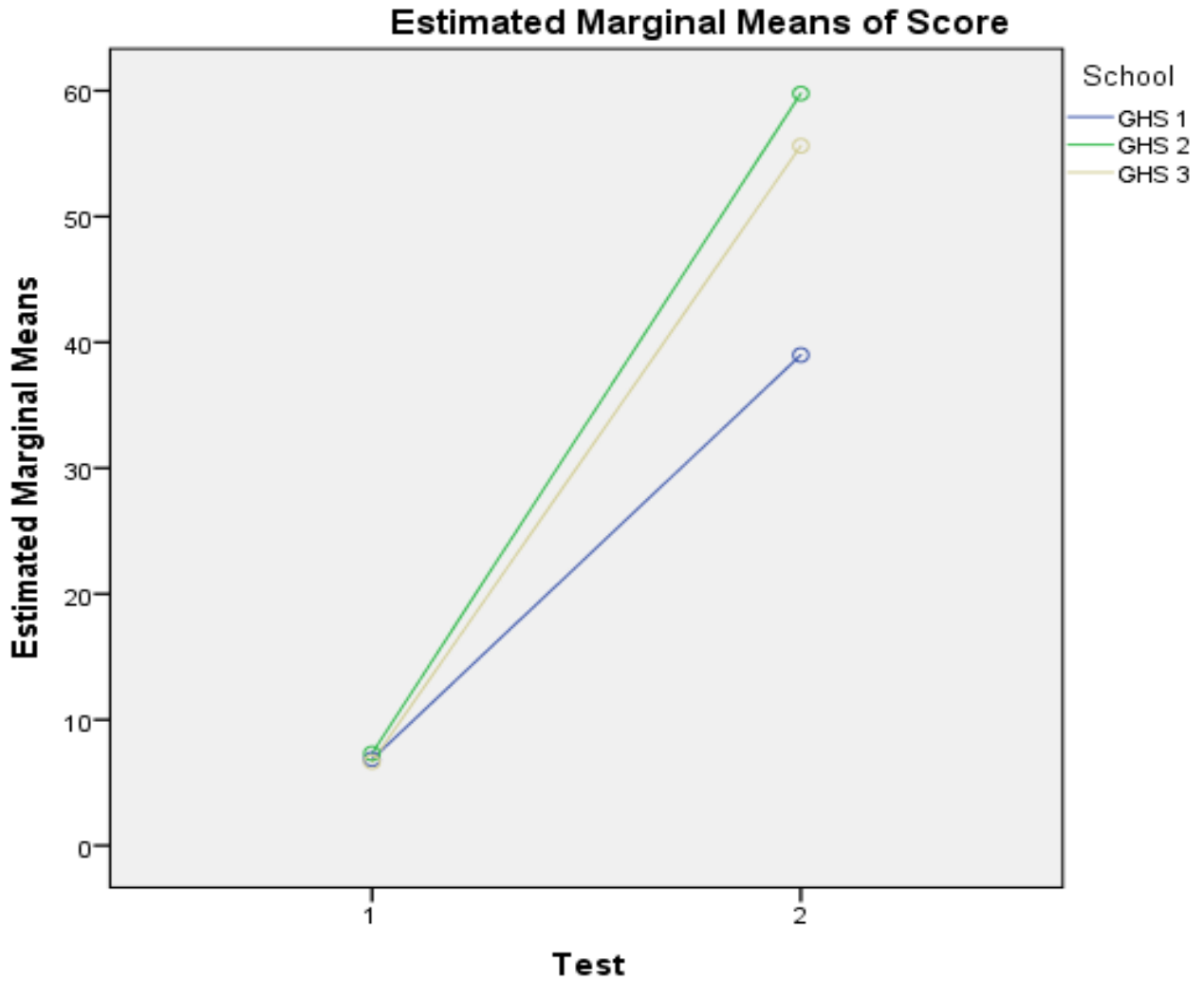
The data collected from the classroom observations formed the qualitative data for this study. The qualitative data were collected essentially for the purpose of triangulation and to complement the quantitative data obtained from the pre-test and post-test. The qualitative data gathered from each school were sorted, coded and captured on the spreadsheet and the emerging themes carefully noted.

4.3 Presentation of the Results

As stated in section 3.3.2 of the study, the enquiry was carried out in three high schools in Gauteng Province of South Africa. The results of the three schools (coded GHS 1, GHS 2 and GHS 3 for ethical reasons) shall be presented school by school, starting with the profile of the respective Mathematics teacher that participated in the study, followed by the intervention results. This includes the results of descriptive statistics, inferential statistics and qualitative data analysis. In each school, the results of the quantitative data analysis are presented first and thereafter the qualitative data analysis results.

Before the general presentation of results, it is proper to show at this point that the study participants in all the three schools were almost at the same cognitive level in the concept of stationary points in differential calculus. The figure below explains this.

Figure 4-1: Graphical Illustration of the Initial Cognitive Level of GHS 1, GHS 2 and GHS 3 Study Participants



From Figure 4-1, the lower end of the graph shows the pre-test mean scores of the three participating schools. Though the lower end of the graph seems to look like a point (as if the schools had the same mean score) but this was brought about since the mean from the pre-test of three schools were very close to one another. In the pre-test, the statistically calculated mean scores are 6.86%, 7.32% and 6.64% obtained respectively by GHS 1, GHS 2 and GHS 3, each of which is approximately 7%.

4.3.1 Results of Quantitative Data Analysis

Quantitative data analysis results given here include descriptive and inferential statistics for the control and experimental groups across the three schools. The results for the control group (GHS 1) are given thus:

4.3.1.1 Results of Quantitative Data Analysis for GHS 1

4.3.1.1.1 Teacher's Profile

The Grade 12 Mathematics teacher of this school that participated in the study is a 38-year-old man. He holds a Diploma and Bachelor of Education (B. Ed.) in Mathematics. He has been teaching Grade 12 Mathematics for about ten years.

4.3.1.1.2 Results of Descriptive Statistical Analysis for GHS 1

Research Question 1

Does constructivist instructional approach improve Grade 12 learners' performance in the concept of stationary points in differential calculus?

It should be noted that the study participants in this group were taught using the traditional instructional approach. These results of the control group are presented to allow for comparison of the results with those of the experimental group.

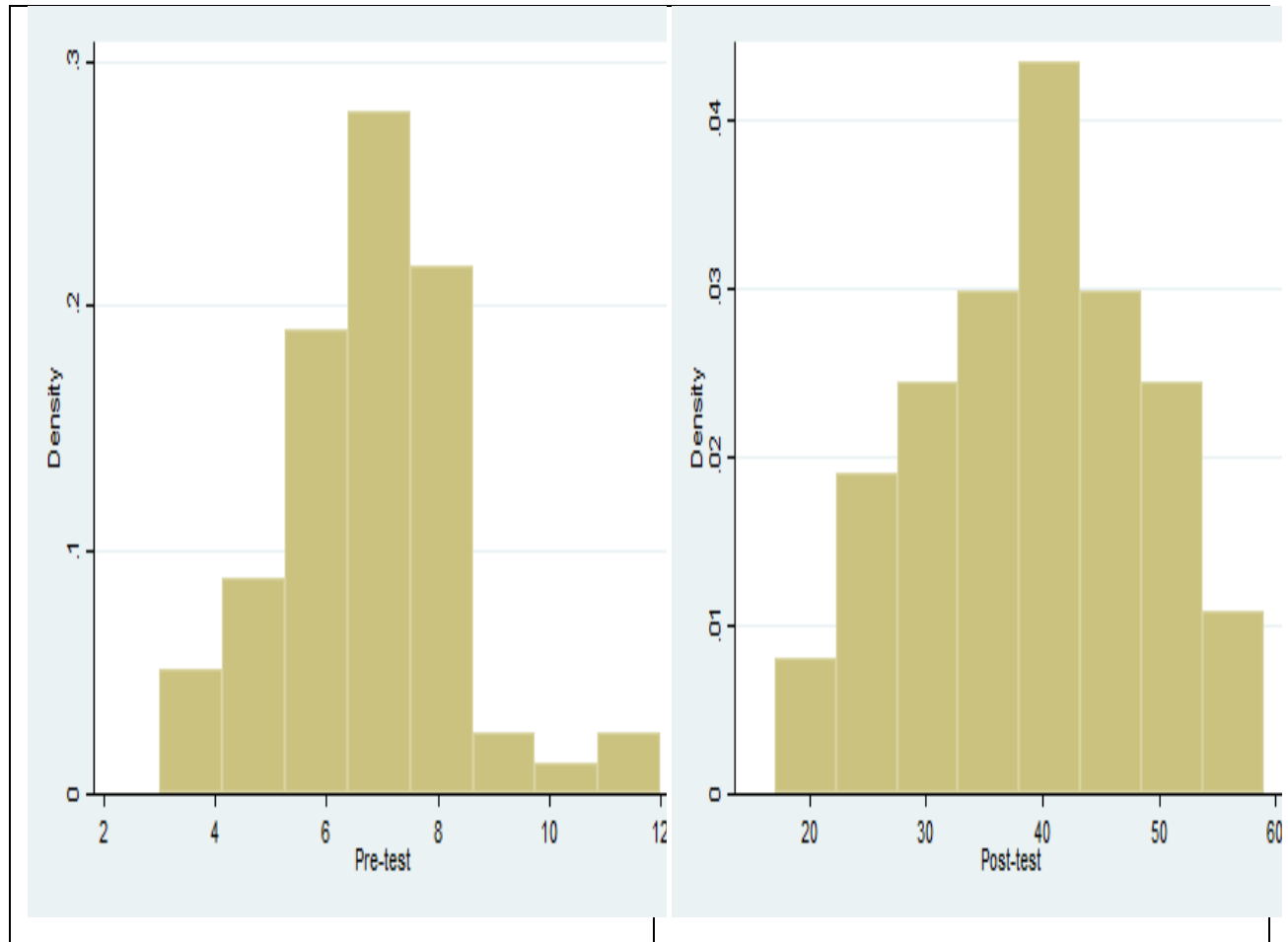
Table 4-1: Summary Statistics of Pre-test and Post-test Scores for GHS 1

Summary Statistics	Pre-test	Post-test
Mean	6.86	38.99
Median	7.00	39.50
Standard deviation	1.554	9.896
Skewness	0.269	- .131
Kurtosis	1.830	- .686
Maximum	12	59
Minimum	3	17
Range	9	42
Coefficient of variation	22.65%	25.38%

From Table 4-1 above, the study participants had a mean of 6.86 in the pre-test and a standard deviation of 1.554. This shows a spread of marks in the range of 5.306 - 8.414 ($\approx 5.3 - 8.4$). However, in the post-test, their mean and standard deviation became 38.99 and 9.896 respectively. This implies that the spread of marks fell in the range of 29.09 - 48.89 ($\approx 29.1 - 48.9$).

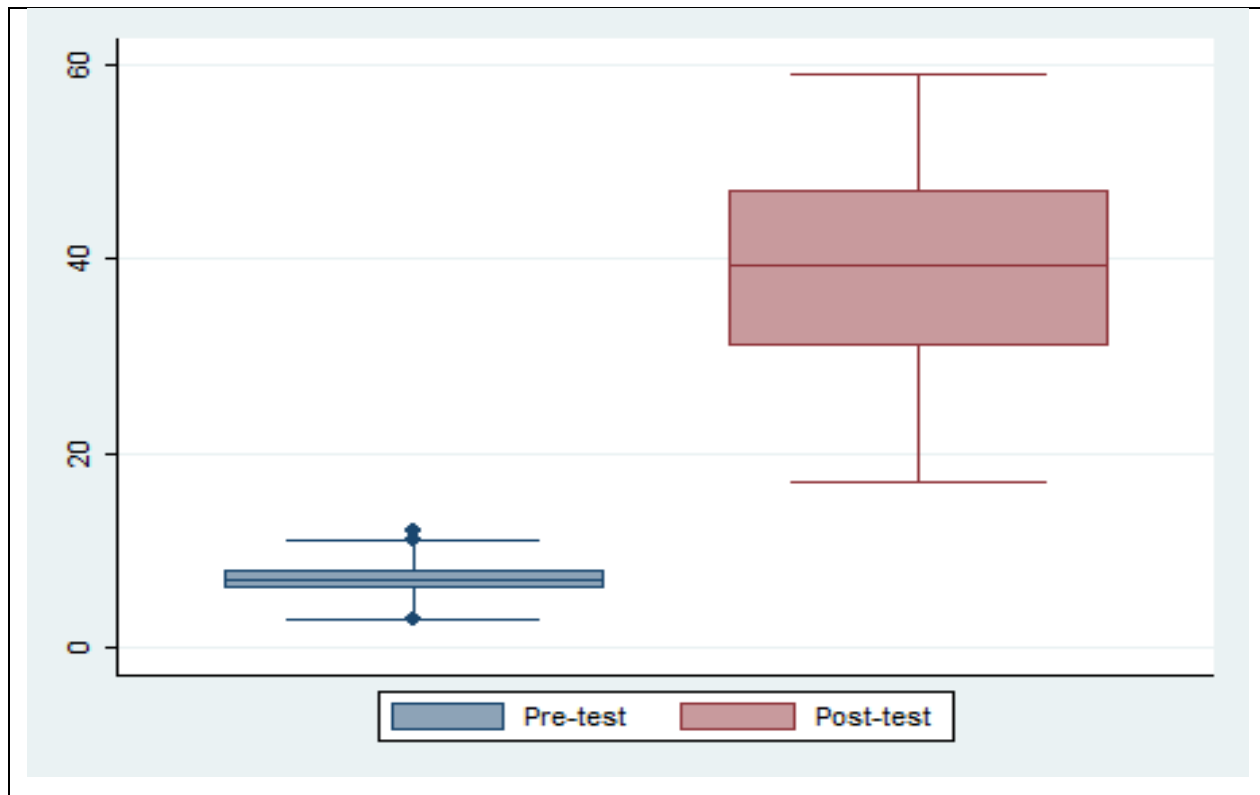
It is observed that in both pre-test and post-test, the spread of marks are close to the mean, though the post-test scores have a higher variability than the pre-test scores. The performances of the study participants in both groups in the pre-test and post-test are further illustrated by the histograms and box plots below.

Figure 4-2: Histograms Showing GHS 1 Pre-test and the Post-test Scores



From the histograms, majority of the study participants got marks in the range of 6% - 8% in the pre-test, while the lowest and highest marks were 3% and 12% respectively. In the post-test however, majority of them got marks ranging from 30% to 50%, which conforms to the statistical analysis given in Table 4-1. This might be the reason for the mean scores of 6.86% and 38.99% recorded by the participants in the pre-test and post-test.

Figure 4-3: Box plots Showing GHS 1 Pre-test and Post-test Scores



The box plots indicate the mean, minimum and maximum marks got by the study participants in the pre-test as approximately 7%, 3% and 12% respectively. For the post-test, the box plots put the mean, minimum and maximum marks as approximately 39%, 17% and 59%. This further supports Table 4-1 and Figure 4-2

4.3.1.1.3 Results of Inferential Statistical Analysis for GHS 1

The hypotheses stated below at 5% level of significance were used to guide the inferential statistical data analysis:

H₀: *There is no significant statistical difference between the study participants' pre-test and post-test mean scores after the intervention.*

H₁: *There is a significant statistical difference between the study participants' pre-test and post-test mean scores after the intervention.*

The null hypothesis was rejected if the p-value was less than .05.

The results are shown in the table below.

Table 4-2: Paired t-test analysis for GHS 1

Type of test	Mean	Standard deviation	t-value	p-value	Decision
Pre-test	6.86	1.554	-27.126**	.000	Null hypothesis is rejected
Post-test	36.99	9.896			
p<.05					

The paired t-test gave a **t-value = -27.126** with a **p-value = .000**. Since $p = .000 < .05$ significance level, the null hypothesis, H_0 , was therefore rejected. Thus, the research hypotheses are addressed.

4.3.1.1.4 Results of the Qualitative Data Analysis for GHS 1

Research Question 2:

Does the constructivist instructional approach facilitate learning the concept of stationary points in differential calculus?

It should be noted that GHS 1 did not use constructivist learning principles.

4.3.1.2 Results of Quantitative Data Analysis for GHS 2

4.3.1.2.1 Teacher's Profile

The Grade 12 Mathematics teacher of this school that took part in the study is 41 years of age. He is a holder of Bachelor of Science (B. Sc.) in Mathematics and Postgraduate Certificate in Education (PGCE.). He has 9 years of teaching experience as a Grade 12 Mathematics teacher.

4.3.1.2.2 Results of Descriptive Statistical Analysis for GHS 2

Research Question 1

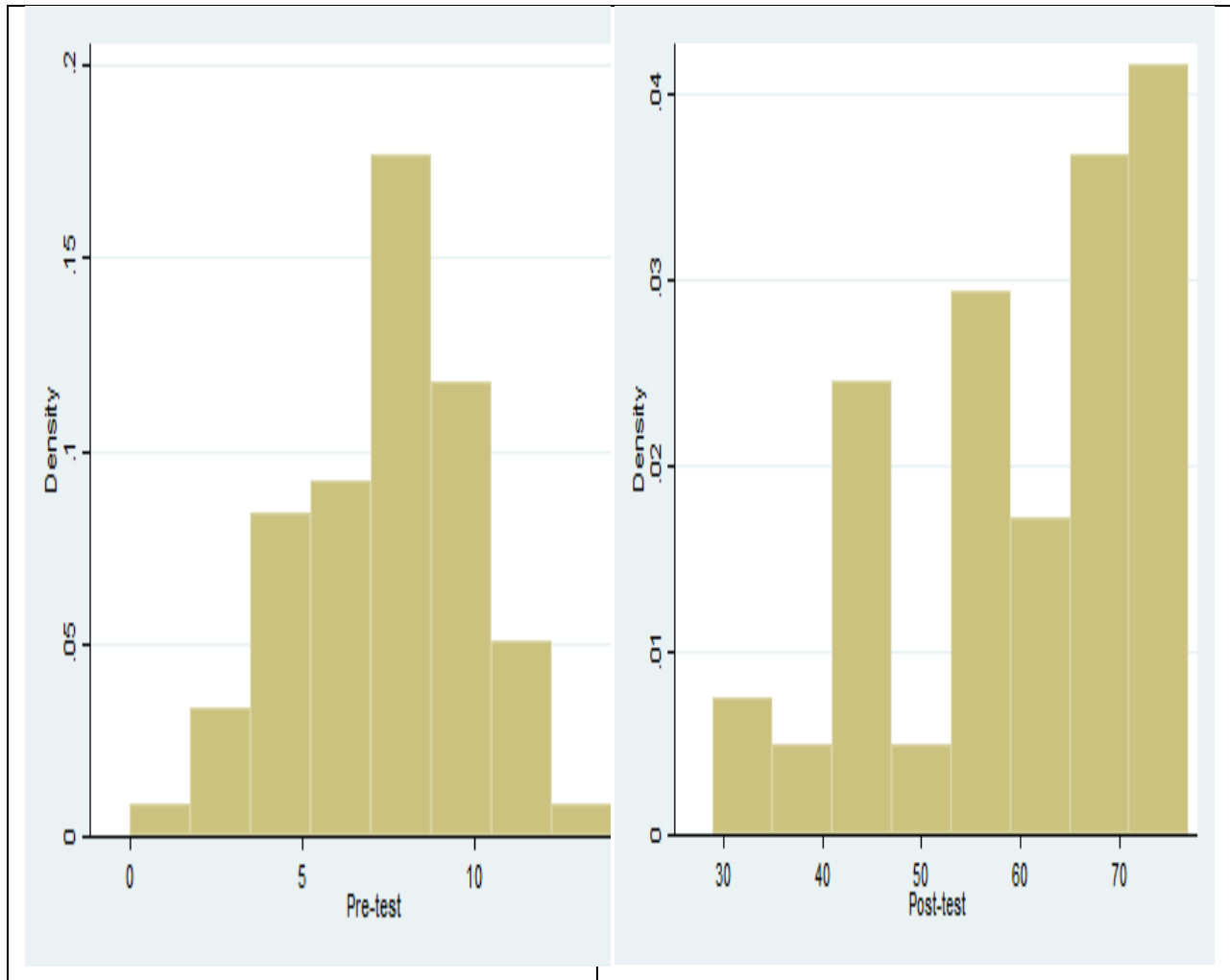
Does constructivist instructional approach improve Grade 12 learners' understanding of the concept of stationary points in differential calculus?

Table 4-3: Summary Statistics of Pre-test and Post-test Scores for GHS 2

Summary Statistics	Pre-test	Post-test
Mean	7.32	59.78
Median	8.00	62.00
Standard deviation	2.571	12.893
Skewness	-.192	-.586
Kurtosis	.484	-.707
Maximum	14	77
Minimum	0	29
Range	14	48
Coefficient of variation	35.12%	21.57%

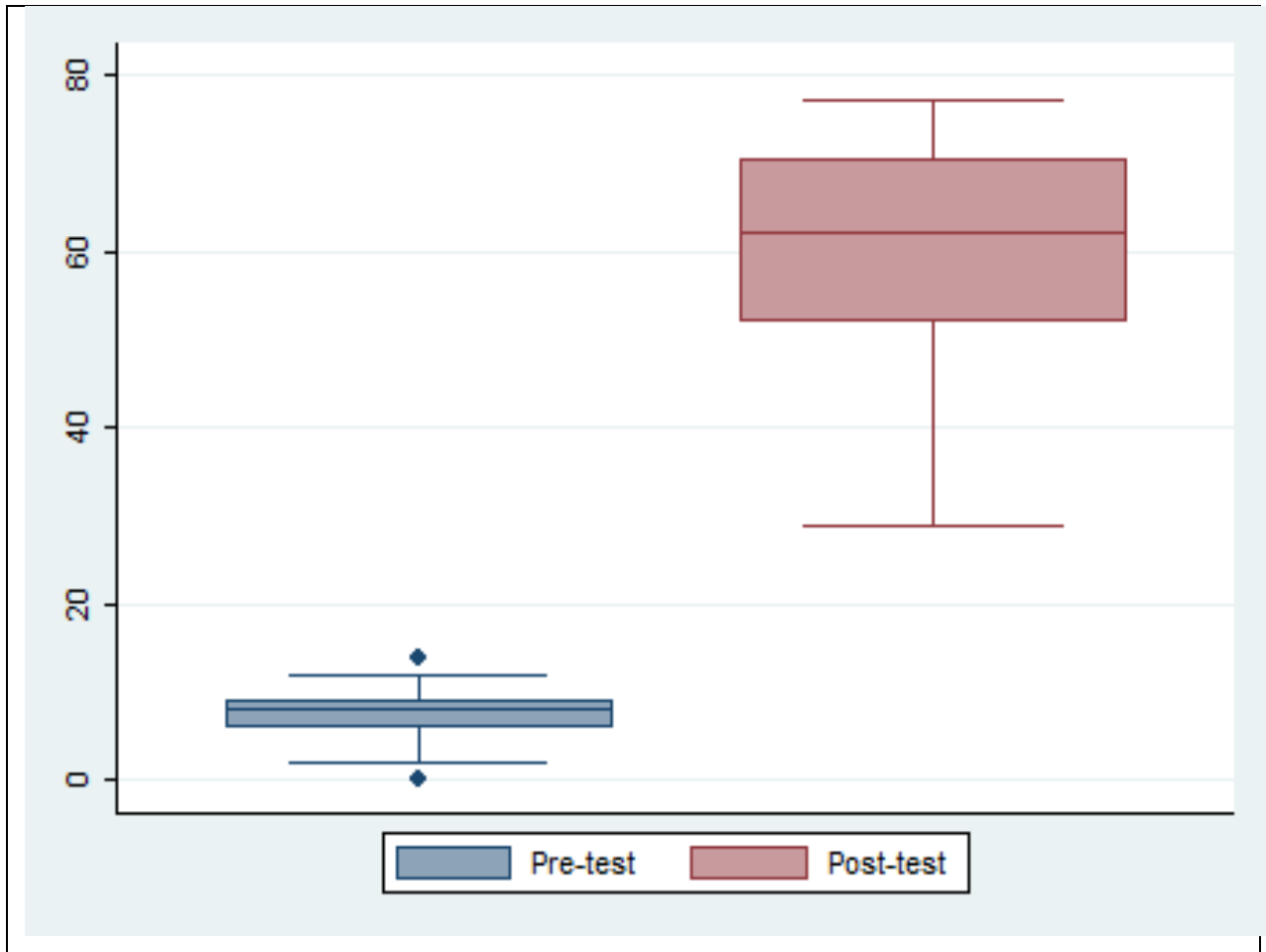
Table 4-3 gives a mean of 7.32 and a standard deviation of 2.571 in the pre-test thereby implying the spread of marks in the range of 4.749 - 9.891 ($\approx 4.7 - 9.9$). In the post-test however, it records the mean of 59.78, standard deviation of 12.893 and a spread of marks in the range 46.89 - 72.67 ($\approx 46.9 - 72.7$). It can be seen that the post-test scores spread more widely away from the mean than pre-test scores which are close to the mean. This is further illustrated by the histograms and box plots below.

Figure 4-4: Histograms Showing GHS 2 Pre-test and Post-test Scores



The histogram for the pre-test shows an average of about 7% with the study participants' scores ranging from 0% to 12%. Only few of them got 10% to 12%. The histogram for the post-test gives an average mark of about 55% with a range of marks of about 29% to 77%. In fact, majority of them got above 30%. This is in conformity with the statistical analysis given in Table 4-3.

Figure 4-5: Box Plots Showing GHS 2 Pre-test and Post-test Scores



The box plots give the mean, minimum and maximum marks got by the study participants in the pre-test as approximately 7%, 0% and 14% respectively. However, the box plots put the mean, minimum and maximum marks as approximately 59%, 29% and 77% for the post-test. This is why the box plot for post-test is more to the right than that of the pre-test. This corroborates the statistical analysis in Table 4-3 and Figure 4-3

From the foregoing, it is evident that constructivist instructional approach improved the post-intervention performance of GHS 2 study participants in the concept of stationary points in differential calculus. Thus, the research question 1 has been answered.

4.3.1.2.3 Results of Inferential Statistical Analysis for GHS 2

The hypotheses stated below at 5% level of significance were used to guide the inferential statistics data analysis:

H₀: There is no significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

H₁: There is a significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

The null hypothesis was rejected if the p-value was less than .05.

The results are shown in the table below.

Table 4-4: Paired t-test analysis for GHS 2

Type of test	Mean	Standard deviation	t-value	p-value	Decision
Pre-test	7.32	2.571	-35.717	.000	Null hypothesis is rejected
Post-test	59.78	12.893			
p<.05					

The paired t-test gave a **t-value = -35.717** with a **p-value = .000**. Since $p = .000 < .05$ level of significance, the null hypothesis, H_0 , was rejected. Hence, the research hypotheses have been taken care of.

4.3.1.2.4 Post-hoc Analysis

The paired t-statistics used above produced a significant statistical difference between the learners' pre-test and post-test mean scores. So, there was a need to employ another inferential statistics to see where the differences actually occurred. For this purpose, the post-hoc tests were run as a follow-up to the t-test analysis. This was done by the use of Bonferroni multiple comparisons given as follows.

Table 4-5: Bonferroni Multiple Comparisons

(I) School	(J) School	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
GHS 1	GHS 2	-10.63*	1.096	.000	-13.28	-7.98
	GHS 3	-8.21*	1.105	.000	-10.87	-5.54
GHS 2	GHS 1	10.63*	1.096	.000	7.98	13.28
	GHS 3	2.42	1.113	.092	-.26	5.11
GHS 3	GHS 1	8.21*	1.105	.000	5.54	10.87
	GHS 2	-2.42	1.113	.092	-5.11	.26

Based on observed means. The error term is Mean Square (Error) = 41.470.

* The mean difference is significant at the .05 level.

From row 1 of Table 4.5 above, it is observed that the comparison of GHS 1 and GHS 2 produces a p-value = .000 < α = .05. So also, the comparison of GHS 2 and GHS 1 in row 2 gives a p-value = .000 < α = .05. The post-hoc test therefore shows that the main effect of the intervention lies in the post-test mean score of GHS 2.

4.3.1.2.5 Results of the Qualitative Data Analysis for GHS 2

Research Question 2

Does the constructivist instructional approach facilitate understanding of the concept of stationary points in differential calculus?

The classroom observations made during the three weeks of the research were conducted particularly to ensure that the teaching-learning activities in the two experimental schools conformed to the dynamics of a constructivist classroom. In GHS 2, six lessons were observed – two per day, one in Grade 12 A and one in Grade 12 B. These were the two Grade 12 science classes taking Mathematics taught by the

participating teacher. The classroom visits took place during the normal school hours from 18th to 22nd May, 2015.

Each lesson lasted an hour with an average of 38 learners in attendance. At the beginning of each lesson, the teacher gave an introduction using relevant demonstrations and raising useful questions to arouse learners' interest and curiosity in the topic. He did this to determine the learners' prior knowledge and to bring about possible learner enquiries. It was noted that after each day's lesson, the teacher often remembered to inform the learners of the topic for the next lesson as a way of ensuring they prepared for it.

Based on the various ideas the learners might have given in response to his probing questions, the teacher would introduce the topic. Shortly afterwards, the teacher would divide the learners into small discussion groups having 5 - 6 learners per group to make possible a quality level of interactions among the learners. He always allowed the learners a reasonable time to interact making use of their previous knowledge and experience in their different discussion groups. The learners were free to communicate to one another, make suggestions and even guess possible answers to the questions on stationary points being solved.

Without interfering with the learners' group discussions, he was often acting as a facilitator roaming the class, observing, listening to and offering help when necessary to the interacting learners. After the discussions, the groups, one after the other, presented common answers to the whole class. Based on the answers provided by the learners, he explained the topic using common illustrations and examples to make the concept concrete and real. It was observed that the class activities were quite in line with the constructivist learning principles throughout the period of intervention.

Below is the extract of the one-hour lesson of 13th May, 2015 observed in Grade 12A:

The Mathematics teacher got to the class on the dot of 9.30 am for the lesson as officially scheduled on the timetable. He exchanged short pleasantries with the learners and began.

Teacher: From our last discussion, what did we agree on as today's topic?

From among the several hands raised, he called upon a female learner to respond.

Learner: Stationary Points.

Teacher: Is she right?

Class (simultaneously): Y-e-e-e-e-s-s-s-s.

Teacher: That's fine. But what specifically do we want to look at about stationary points?

He pointed to a boy.

Learner: We will like learn how to find stationary points and determine their nature.

Teacher: Can someone tell us what determining the nature of a stationary point means?

Learner: It means knowing the types of stationary points we may have.

Teacher: Good. Yes, as you rightly said, the topic of today's lesson is 'Finding and Determining the Nature of Stationary Points'. What did I say?

Class(chorused): Finding and Determining the Nature of Stationary Points.

The teacher wrote the topic on the board.

Teacher: What then are the types of stationary points?

Another learner called upon answered.

Learner: The three types of a stationary point are maximum point, minimum point and point of inflection.

Teacher: Okay, who can define a stationary point for us?

Learner: A stationary point is a point on a graph where the gradient or slope is zero i.e. a point where $\frac{dy}{dx} = 0$.

Teacher: Any different idea?

Class (chorused): No, none. He's correct.

Teacher: When given $y = f(x)$ and we are asked to find its stationary point(s), what is the first step?

Learner: We differentiate it, that is, we find the $\frac{dy}{dx}$ of y.

Teacher: Do you all agree to this?

Class (simultaneously): Y-e-e-e-e-s-s-s-s, she is correct.

Teacher: After differentiating the given function, what do we do next?

Learner: We set the equation equal to zero since at any stationary point, $\frac{dy}{dx} = 0$. Then, we solve the equation to get the x-coordinate of the stationary point.

Teacher: What next?

Learner: We find the y-coordinate of the stationary point.

Teacher: How can we achieve this?

Learner: We have to substitute the x-coordinate or x-coordinates in the given function, evaluate to obtain the y-coordinate or y-coordinates.

Teacher: That's very fine of you. Having successfully obtained the stationary points, how can we then find the nature of a stationary point?

Learner: We will have to find $\frac{d^2y}{dx^2}$. That is, we have to differentiate the equation the second time.

Teacher: Is that all?

Class (chorused): No-o-o-o-o-o-o-o.

Teacher: What else do we do then?

Learner: We will substitute for x in whatever the value of our $\frac{d^2y}{dx^2}$ is. If our answer is less than zero, the stationary point is maximum; if greater than zero, the stationary point is minimum. However, if the answer got is equal to zero, the stationary point is a point of inflexion.

Teacher: Thank you. That's very correct. That is exactly the procedure. Any question so far about how to find and determine the nature of stationary points?

Class (chorused): No question. The procedure is clear.

Having been aware that a constructivist classroom has to be learner-centred, all along, the teacher allowed the learners some time to reflect on each question before answering it. He encouraged and welcomed their guesses and suggestions, some of which are not right. He also gave them chance to ask questions and seek for clarifications of whatever was not clear to them.

The teacher wrote on the board the below question as a class example:

Obtain and determine the stationary points of $y = 2x^3 + 3x^2 - 12x + 17$.

Teacher: Now, go to your different discussion groups to solve the question. You have 20 minutes to do it.

As characteristic of a constructivist class, the teacher wanted the learners to engage in knowledge construction tasks by getting the solution to the problem on their own rather than rely on him for its solution. The learners immediately went to their different discussion groups and started interacting. They were communicating with one another, arguing, asking themselves questions and answering them too and making suggestions, all in an attempt to come up with meaningful ideas that could lead them to the right answer to the question. While this was going on, the teacher was moving round to monitor the progress of each group ensuring they were not making noises to disturb one another or nearby classes. He was not interfering in the process except on three occasions when called upon by some groups for a guide or help.

After about twenty minutes of group interactive session, the various groups were called upon to present their answers to the class. While each group was doing this, the rest of the learners in the class were given the opportunity to contribute their individual knowledge of the concept and the teacher was moderating the discussion. After all the groups had finished making presentations, the whole class agreed on the following solution as the final answer which the learners wrote down in their notebooks.

Solution

$$y = 2x^3 + 3x^2 - 12x + 17$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

Set the $6x^2 + 6x - 12 = 0$ and solve for x .

Since the gradient is always equal to zero at stationary points i.e. $\frac{dy}{dx} = 0$

$$6x^2 + 6x - 12 = 0$$

Dividing through by 6, we have

$$x^2 + x - 2 = 0$$

Factorizing, $(x + 2)(x - 1) = 0$

$$\rightarrow x = -2 \text{ or } 1$$

\rightarrow There are stationary points at $x = -2$ or 1

Substitute the values of x in $y = 2x^3 + 3x^2 - 12x + 17$

$$\text{When } x = -2, y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 17$$

$$y = -16 + 12 + 24 + 17 = 37$$

$$\text{When } x = 1, y = 2(1)^3 + 3(1)^2 - 12(1) + 17$$

$$y = 2 + 3 - 12 + 17 = 10$$

\therefore The two stationary points are $(-2; 37)$, $(1; 10)$

To determine the nature of the two stationary points,

$$\text{we find } \frac{d^2y}{dx^2} = 12x + 6$$

Considering the $x = -2$ of the stationary point $(-2; 37)$,

$$12x + 6 = 12(-2) + 6 = -24 + 6 = -18 < 0$$

\therefore The stationary point $(-2; 37)$ is maximum.

Considering the $x = 1$ of the stationary point $(1; 10)$,

$$12x + 6 = 12(1) + 6 = 18 > 0$$

\therefore The stationary point $(1; 10)$ is minimum

Before the teacher brought the lesson to an end at 10.30 am, he gave the learners the homework below:

1. Find the stationary points on the graph of $y = 5x^3 + 3x^2$ and give their nature.
2. Obtain and determine the stationary points of $x^3 - 3x^2 - 144x + 432$

Considering the kind of responses given by the learners individually and in their group presentations (majority of which were correct), the enthusiasm they exhibited while participating in the lessons and their encouraging performances in most of the formative assessments and homework given to them after each lesson, it can be inferred that the approach might have enhanced the learners' understanding of the concept.

4.3.1.3 Results of Quantitative Data Analysis for GHS 3

4.3.1.3.1 Teacher's Profile

The Mathematics teacher of the school involved in the study is a man, 44 years of age. His highest academic qualification is Bachelor of Education in Mathematics. He has been teaching a Grade 12 Mathematics teacher for 7 years.

4.3.1.3.2 Results of Descriptive Statistics for GHS 3

Research Question 1

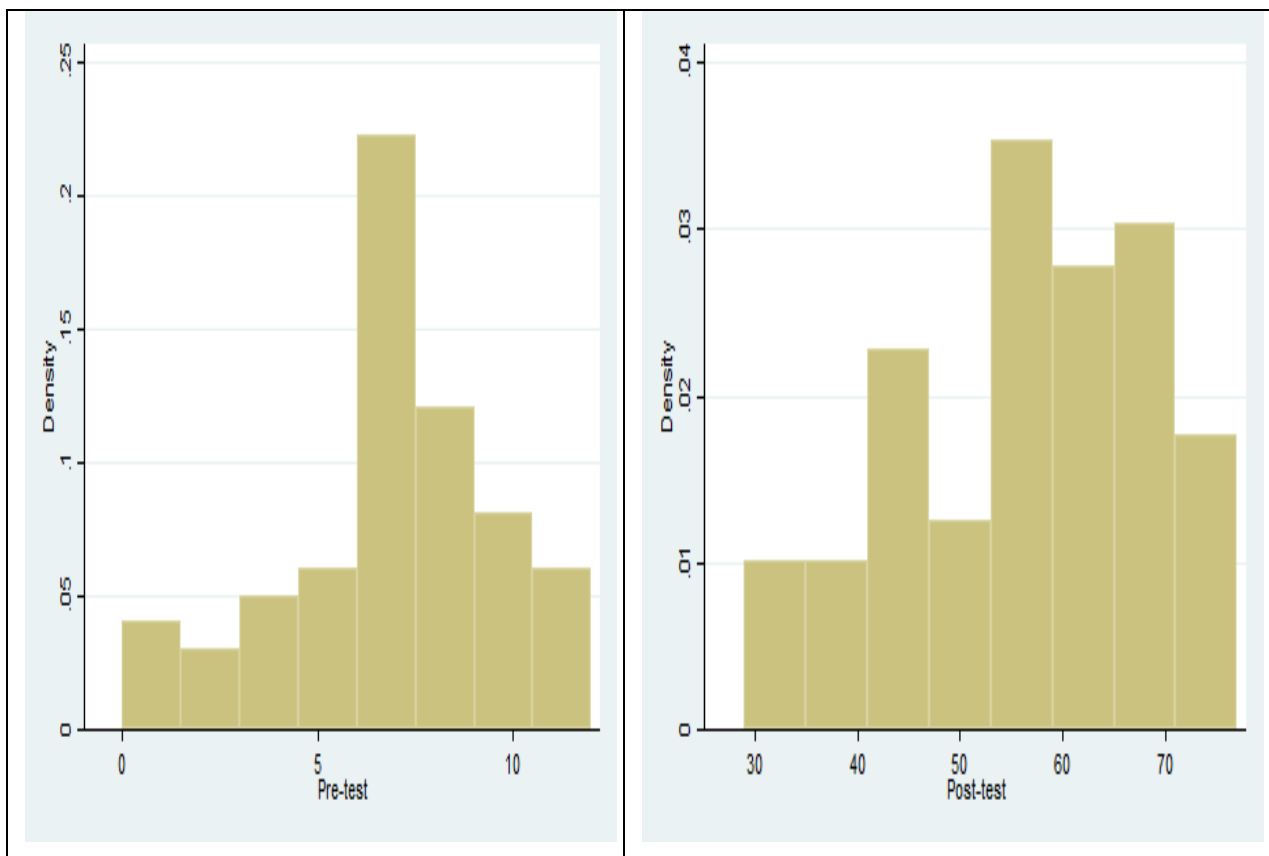
Does constructivist instructional approach improve Grade 12 learners' performance in the concept of stationary points in differential calculus?

Table 4-6: Summary Statistics of Pre-test and Post-test for GHS 3

Summary Statistics	Pre-test	Post-test
Mean	6.64	55.62
Median	7.00	57.00
Standard deviation	2.848	12.078
Skewness	-.435	-.308
Kurtosis	.096	-.583
Maximum	12	77
Minimum	0	29
Range	12	48
Coefficient of variation	42.89%	21.72%

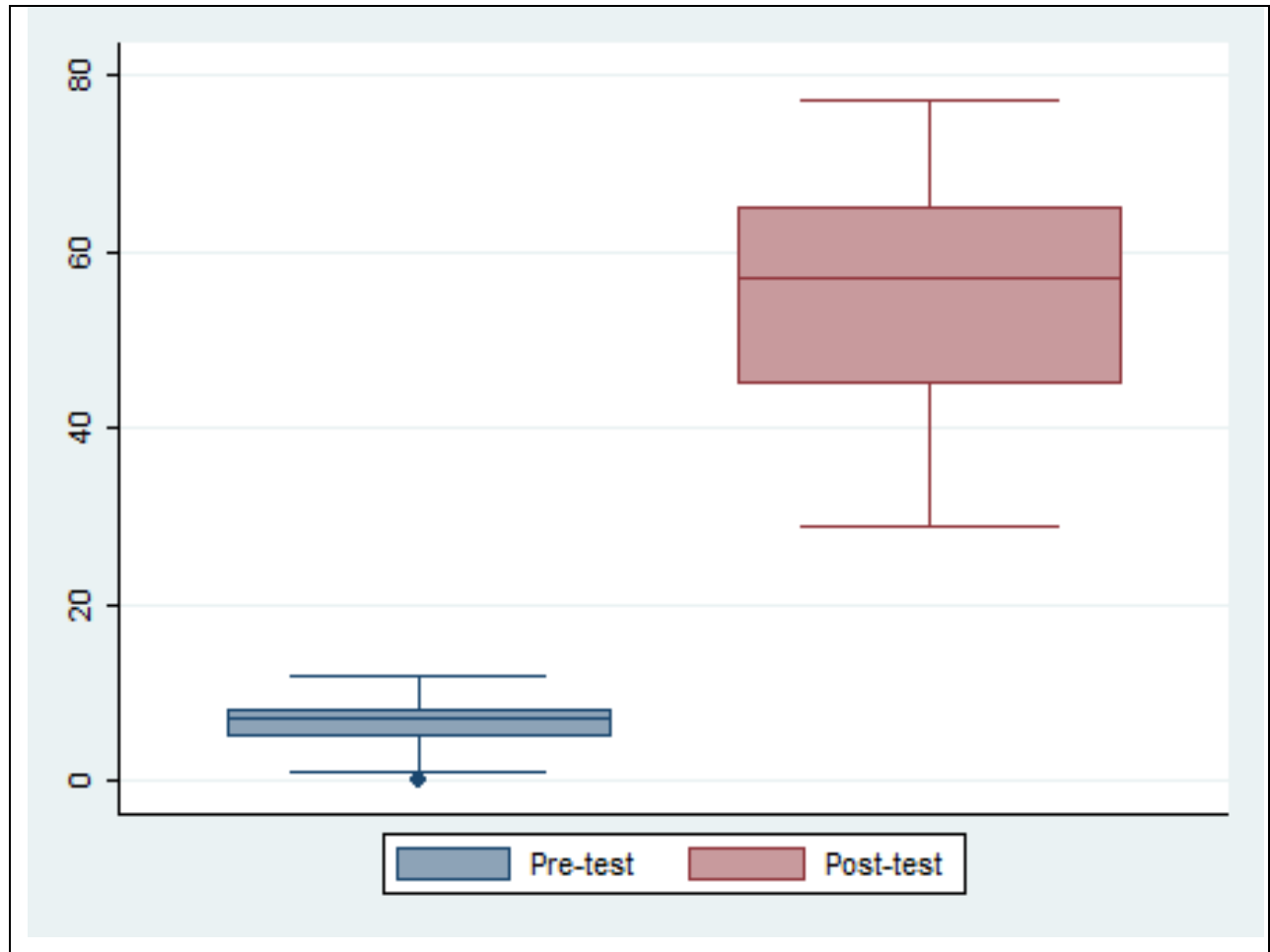
From the above table, the mean and standard deviation for the pre-test are 6.64 and 2.848 respectively. It implies a spread of marks in the range of 3.792 - 9.488 (\approx 3.8-9.5). In the post-test, the mean is 55.62 and the standard deviation is 12.078. The spread of marks is in the range of 43.54 - 67.7 (\approx 43.5 - 67.7). As was the case in GHS 2, the post-test marks spread widely away from the mean while the pre-test marks are close to the mean. This is further illustrated by the histograms and the box plots below.

Figure 4-6: Histograms Showing GHS 3 Pre-test and Post-test Scores



The graph shows an average of about 6.6% and the range of their marks was 0% to 12% marks in the pre-test. It also indicates that few of them scored 10% to 12%. But in the post test, the average is about 55% with their marks ranging from 29% to 77%. This complies with the statistical analysis in Table 4-5.

Figure 4-7: Box Plot Showing GHS 3 Pre-test and Post-test Scores



From the box plots, the mean, minimum and maximum marks for the pre-test are approximately 7%, 0% and 12% respectively. However, the mean, minimum and maximum marks are approximately 56%, 29% and 77% for the post-test. This is why the box plot for the post-test is more to the right than that of the pre-test. The difference indicates that the study participants improved upon their performance in the post-test. This is in line with the analysis in Table 4-5 and Figure 4-5.

In view of the above, it is clear that constructivist instructional approach improved the post-intervention performance of GHS 3 study participants in the concept of stationary points in differential calculus. Thus, the research question 1 has been answered.

4.3.1.3.3 Results of Inferential Statistical Analysis for GHS 3

The hypotheses stated below at 5% level of significance were used to guide the inferential statistics data analysis:

H₀: There is no significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

H₁: There is a significant statistical difference between the study participants' pre-test and the post-test mean scores after the intervention.

The null hypothesis was rejected if the p-value was less than .05. The results are shown in the table below.

The results are shown in the table below.

Table 4-7: Paired t-test analysis for GHS 3

Type of test	Mean	Standard deviation	t-value	p-value	Decision
Pre-test	6.64	2.848	-38.128**	.000	Null hypothesis is rejected
Post-test	55.62	12.078			
p<.05					

The paired t-test gave a **t-value = -38.12128** with a **p-value = .000**. Since $p = .000 < .05$ significance level, the null hypothesis, H_0 , was rejected. Therefore, the research hypotheses have been addressed.

4.3.1.3.4 Post-hoc Analysis

For the similar reason mentioned in subsection 4.3.1.2.5, the post-hoc follow-up was carried out using the Bonferroni multiple comparisons of mean scores. This is explained as follows:

Table 4-8: Bonferroni Multiple Comparison of Mean Scores

(I) School	(J) School	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
GHS 1	GHS 2	-10.63*	1.096	.000	-13.28	-7.98
	GHS 3	-8.21*	1.105	.000	-10.87	-5.54
GHS 2	GHS 1	10.63*	1.096	.000	7.98	13.28
	GHS 3	2.42	1.113	.092	-.26	5.11
GHS 3	GHS 1	8.21*	1.105	.000	5.54	10.87
	GHS 2	-2.42	1.113	.092	-5.11	.26

Based on observed means. The error term is Mean Square (Error) = 41.470.

* The mean difference is significant at the .05 level.

From row 1 of Table 4.8, it is noted that the comparison of GHS 1 and GHS 3 gives a p-value = .000 < α = .05. The same applies to the comparison of GHS 3 and GHS 1 in row 3. The main effect of the intervention is found in the post-test mean score of GHS 3.

4.3.2.3 Results of the Qualitative Data Analysis for GHS 3

Research Question 2

Does the constructivist instructional approach facilitate understanding of the concept of stationary points in differential calculus?

As stated in 4.3.2, the classroom observations were carried out primarily to ascertain that the class activities in the experimental group were in conformity with the dynamics of a constructivist classroom. In GHS 3 as well, six lessons were observed – two per day, one in Grade 12 A and one in Grade 12 B. These were the two Grade 12 science classes taking Mathematics taught by the participating teacher. The classroom visits took place during the normal school hours from 25th to 29th May, 2015.

Each observed lesson on the concept of stationary points was of one hour duration with an average of 33 learners in attendance. At the beginning of every lesson, the teacher also asked relevant questions in introducing each day's lesson in order to arouse learners' interest and curiosity in the topic. He too wanted to find out the learners' prior knowledge about the topic and as well to lead them into making possible enquiries. Like GHS 2 Mathematics teacher, after each day's lesson, he also ensured he told the learners beforehand the topic for the subsequent lesson for them to prepare for it.

Thereafter, he too, like the teacher in GHS 2, in each of the lessons divided up the learners into small discussion groups of an average of 5 learners per group to facilitate effective interactions among them. In their various discussion groups, the learners were allowed considerable time to interact making use of their previous knowledge and experience to solve assigned tasks. They had the opportunity to communicate with one another, make guesses, suggestions and contributions.

Without interfering in the learners' group discussions, the teacher was roaming the class, monitoring the progress of each group and facilitating the discussions. He was observing and listening to the interacting learners and giving them assistance as necessary. After the discussions, all the groups were called upon one by one to present the solutions jointly agreed upon before the class. While each group was presenting its common answers to the class, other learners in the class were allowed to make their individual contributions and the teacher was moderating the discussion. Then, he used the answers given by the learners as a basis for explaining the topic making use of useful illustrations and examples to make the concept clear and real. Later, the whole class agreed on the final solutions which the learners were to write down in their notebooks.

Throughout the period of the intervention in the school, it was noted that the class activities conformed to the constructivist learning principles. The teacher gave the learners ample opportunity to ask and answer questions, make guesses and suggestions, contribute and share ideas. He encouraged learner autonomy and initiatives and also related each lesson to learners' interests, needs and aspirations. He

also remembered to give suitable assignments to the learners to prepare them for the next lesson.

4.4 Major Findings of the Study

Before the results of the study are fully discussed, highlighted below are the major findings of the investigation:

- Constructivist instructional approach improved the study participants' post-intervention performance in the concept of stationary points in differential calculus;
- Constructivist instructional approach facilitated the learning of stationary points in differential calculus;
- The study participants exhibited a considerable level of classroom co-operations, interactions, participations and individual/group knowledge constructions in the constructivist learning environment.

CHAPTER FIVE

DISCUSSION, CONCLUSION ANDRECOMMENDATIONS

This chapter discusses the results of the data analysis presented in chapter four. Based on this, it makes some recommendations and suggestions for further studies. Then, it states the limitations of the study and the conclusion.

5.1 Discussion of Results

In relation to the research questions and hypotheses, the results of the study are discussed as follows:

5.1.1 Impact of Constructivist Instructional Approach on the Study Participants' Achievements in the Learning of Stationary Points in Differential Calculus

One notable result of this study is the significant statistical differences in the mean achievements recorded in the learning of the concept of stationary points in differential calculus by the control group (GHS 1) and the experimental group (GHS 2 and GHS 3). Before the intervention, both groups were found to be on almost the same cognitive level in the concept of stationary points as depicted by their mean scores in the pre-test—6.86% (GHS 1), 7.32% (GHS 2) and 6.64% (GHS 3). Please, refer to sub-section 4.4 and figure 4-1. After the intervention, GHS 2 and GHS 3 performed better in the post-test (with their respective mean scores of 59.78% and 55.62%) than GHS 1 having a mean score 38.99% (see Tables 4-1, 4-3, 4-6 and Figure 4-1).

The results are quite in line with the literature study done for the investigation as they conform to the findings of several researchers (Akkus et al, 2003; Guthrie & Humenick, 2004; Ncharam, 2012; Taber, 2011) that constructivist instructional approach has proven to be capable of developing deep understanding of Mathematics in learners at all levels. Still in support of this, Cekolin (2001) found out that constructivist pedagogy improves achievement in Mathematics and the level of confidence for middle school learners and

also Kim (2005) who obtained from his study that constructivist learning is more effective than traditional teaching in terms of academic achievement;

The implication of this is that GHS 1 study participants, who were not exposed to the intervention, had an improved performance which was below the 40% pass mark. However, the study participants of GHS 2 and GHS 3, the two intervention schools, improved upon their performances above the 50% average mark. It can therefore be said that the constructivist instructional approach might be responsible for the improvement in performance demonstrated by the study participants in the learning task.

The results of the inferential statistical analyses also provide support to the results of the descriptive statistical analyses just discussed. The paired t-test as the inferential statistics employed to probe the differences between the pre-test and post-test mean scores in each of the two intervention schools gave the $p\text{-value} = .000 < \alpha = .05$. Therefore, in each case, the null hypothesis, H_0 , is rejected. This indicates that there is a significant statistical difference between the study participants' pre-test and post-test mean scores in the experimental group. This addresses the research hypotheses guiding the study.

In order to determine where the mean differences actually were, the post-hoc analysis was conducted as a follow-up to the t-test analysis. To this end, the Bonferroni multiple comparisons of the performances of the control and experimental schools were done (sub-sections 4.3.1.2.5 and 4.3.1.3.4). From Tables 4-5 and 4-8, GHS 1 compared against GHS 2 produces a $p\text{-value} = .000 < \alpha = .05$ and GHS 2 in comparison against GHS 1 also gives $p\text{-value} = .000 < \alpha = .05$. Similarly, the comparison of GHS 1 and GHS 3 produces a $p\text{-value} = .000 < \alpha = .05$ while GHS 3 versus GHS 1 also gives $p\text{-value} = .000 < \alpha = .05$.

This implies there is a statistically significant difference between the post-intervention mean score of the experimental group given constructivist instructions and the post-intervention mean score of the control group exposed to the usual traditional instructions. It further indicates that the main effect of the intervention is in the post-test

mean scores of GHS 2 and GHS 3. Perhaps the main cause of the statistical differences in the mean scores of the control and the experimental groups is the intervention.

Furthermore, in Tables 4-5 and 4-8, the comparison of the two experimental schools, one against the other (GHS 2/GHS 3 and also GHS 3/GHS 2), shows no statistically significant difference between the post-intervention mean scores of the two experimental schools taught with constructivist pedagogical approach. The $p\text{-value} = .092 > \alpha = .05$ in each case is an indication of this. In fact, the GHS 2/GHS 3 and GHS 3/GHS 2 comparison results do not really have any statistically significant effect on the conclusion of the enquiry as the study is basically interested in what the likely improvement in the performance of the experimental group would be when measured against the performance of the control group.

5.1.2 Reflections on the Dynamics of the Constructivist Classroom and the Traditional Classroom in the Learning of Stationary Points

Below are the themes that emerged during the classroom observations made in the control and experimental groups:

In the experimental schools, when the teachers asked the study participants relevant questions about stationary points before introducing the topic, they were able to give fair ideas of the concept. As allowed by the constructivist practice, their teachers did inform them of the topics of the subsequent lessons beforehand. Hence, the learners had had the opportunity to make some preparations against the next lessons. The teacher in the control school hardly introduced his lessons with leading questions; on one or two occasions he did, the study participants could not give correct responses. It is easy to note that this findings conforms to the conclusion of Murray, Olivier and Human (1998) that leading learners to discuss their own views of a problem and tentative approaches raises their self-confidence and provides them opportunities to reflect and devise new and more viable conceptual skills.

In all the lessons observed, the study participants in the experimental group actually exhibited the constructivist principles by participating fully in class activities. They asked useful questions, answered most of the teachers' questions properly, gave reasonable suggestions and made meaningful contributions. In their own case however, the study participants in the control group on most occasions sat down almost uninvolved and without asking questions or making contributions. Even the expressions on their faces might be interpreted that little or no learning was achieved. This is in line with the findings by Taber (2011) that mathematical knowledge is always at least partly invented by each individual learner and that learners' prior knowledge systems should not be underestimated.

In the formative assessment done at the end of each lesson in both the experimental and control groups to determine the study participants' understanding of the lesson, the experimental group performed better than the control group. This was so because the study participants in the experimental schools made preparations prior to the lessons, came to classes with certain helpful ideas about the lessons and got deeply involved in the lesson. This complies with the findings of Nancy and Gallery (2001) that when learners play a much more active role in learning, they get motivated to learn, to identify and resolve their personal misunderstandings, and to apply what they learn to situations relevant to their own lives.

It was as well obtained that, in the homework exercises, the study participants in the experimental groups recorded better performances than their mates in the control group. Having been placed at the centre of their learning and already fully made aware they were responsible for their learning, the experimental group showed considerable concern for their homework exercises. But the study participants in the control group as products of the teacher-dominated, knowledge-transmission learning approach, did not give significant commitment to their homework. On most occasions, majority of them got the homework questions wrong while some of them did not even attempt the assignments. In support of the findings is the submission of Kiraly (2014) that, in recent times, it has become commonplace that knowledge is constructed by learners, and that,

in the learning process, the traditional focus on the teacher is already shifting to the learners.

Generally, the study participants in the intervention class demonstrated a higher understanding of the concept than their counterparts in the control group. This is evident in their classroom co-operations, interactions, participations and individual/group knowledge constructions. As asserted by Brookfield (2005), some of the benefits of learners interacting, collaborating and working together are: learners explore a diversity of perspectives; their intellectual abilities increase; learners' voices and experiences gain respect and value; habits of collaborative learning are promoted and learners develop skills of synthesis and integration.

5.2 Conclusion

From the foregoing, it can be inferred that constructivist instructional approach might have facilitated the learning of the concept of stationary points in differential calculus and also brought about the improved performances of the experimental group in the post-test. The findings comply with the submission of Nayak (2012) that classroom teaching practice is likely to be more effective when it is informed by an understanding of how students learn and that learning becomes more effective if learners are allowed to clarify or explain their own ideas. It is therefore important that the major implications of the learning theory of constructivism should be reflected in classroom practices in a more child-focused way.

Constructivism being an instructional approach that places learners at the centre of their learning, learners therefore need to ensure they show as much interest and commitment to any learning tasks that may be assigned them particularly during their class group discussions and even other class activities. They have to work collaboratively and accept the responsibility for their own learning. Learners should therefore, as much as they can, control those learner variables that can prevent them from being actively involved in the learning process.

As constructivist instructional approach does not actually ignore the teacher's role and influence while learners are constructing knowledge, school-teachers, Mathematics teachers in this case, really have to select and apply suitable instructional strategies in line with the constructivist ideology capable of facilitating and easily leading learners to meaningful knowledge constructions.

Besides, it stands to be of a great help if the DoE sees to the provision of more teacher education and regular constructivist professional development to equip the nation's Mathematics teachers with necessary implementation skills of the instructional approach. This is because the classroom applications of the constructivist learning approach particularly in a Mathematics classroom can be very challenging.

Again, it has to be noted that while designing the Mathematics curriculum, the content, the methods and the assessment of the learning process, much consideration should be placed on learners' interests, prior knowledge and past experiences. For the constructivist pedagogical approach to really achieve the much-desired objectives in a Mathematics classroom, the cooperation and dedication of all stakeholders involved become necessary. This includes education policy makers, curriculum planners, school heads, school-teachers and the learners themselves.

5.3 Limitations of the Study

As it is typical of every research, this study too acknowledges certain limitations that might have affected its conduct, and invariably its results, in one way or the other. The research was restricted to the investigation of the application of constructivist philosophy to the learning of stationary points, an aspect of differential calculus which is one of the ten (10) learning areas forming the Grade 12 Mathematics curriculum.

Also, only three high schools in Gauteng province having a total of 204 Grade 12 Mathematics learners were covered in the study for time constraint and to make the study more manageable.

These inevitable limitations might have somehow affected the findings and the conclusion of the study. Hence, caution should be taken about the extent to which the results of the investigation are generalized.

5.4 Suggestions for Further Studies

Further research could be extended to other aspects of differential calculus and also to the other nine learning areas of Grade 12 Mathematics namely: *Functions; Number Patterns, Sequences and Series; Finance, Growth and Decay; Algebra; Probability; Euclidean Geometry and Measurement; Analytical Geometry; Statistics* and Trigonometry. They could as well include a bigger sample space that will accommodate more South African high schools and even more provinces.

Future studies could also explore the possible influence of some covariates (like learner variables such as: learner's aptitude, age and maturity, parental background, peer influence, language and cultural background, willingness and readiness, interest and attitude, etc) on the improved performances of the experimental group in the post-test. This prevents the merely assuming and generalizing that the results of the study were only due to the impact of the constructivist pedagogical approach applied.

Again, future studies can research into what is likely to be the impact of constructivist instructional approach on the problem-solving approach and procedural proficiency of the learners under investigation.

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LIST OF APPENDICES

APPENDIX 1



Omoniyi Adebayo Akinyinka (51940124)
College of Science, Engineering and Technology
UNISA
Johannesburg

2014-02-14

Permission to conduct research project

Ref: 003/OAA/2014

The request for ethical approval for your Msc (Mathematics Education) research project entitled "Impact of Constructivist Instructional Approach on Grade-12 Learners Understanding of Stationary Points in Differential Calculus" refers.

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee (CREC) has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your study as set out in your proposal and application for ethical clearance.

Therefore, involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CREC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:

http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Please note that if you subsequently do a follow-up study that requires the use of a different research instrument, you will have to submit an addendum to this application, explaining the purpose of the follow-up study and attach the new instrument along with a comprehensive information document and consent form.

Yours sincerely

A handwritten signature in black ink, appearing to be "A. Akinyinka".

Chair: College of Science, Engineering and Technology Ethics Sub-Committee

University of South Africa
College of Science, Engineering and Technology
The Science Campus
C/o Christiaan de Wet Road and Pioneer Avenue,
Florida Park, Rodepoort
Private Bag X16, Florida, 1710
www.unisa.ac.za/cset



APPENDIX 2



04 May 2015

The Principal,

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.....

Appeal to Participate in a Research Project on the Impact of Constructivist Instructional Approach on Grade 12 Learners' Understanding of Stationary Points in Differential Calculus

I write this letter on behalf of my Masters student, Mr Omoniyi, Adebayo Akinyinka, student number 51940124. He is conducting a study on the above mentioned topic in some high schools in South Africa.

For this study to be successful, we need the participation and cooperation of your grade 12 students, and their respective teachers. Therefore, we solicit your support in this matter.

Thank you in advance for your time to help us. We are confident that you will be very satisfied with the outcome of this project.

Kind Regards,

Dr. Faleye, Sunday

(0734122114)



University of South Africa
Pretorius Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX 3



GAUTENG PROVINCE

Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no. D2015/ 427 A

GDE AMENDED RESEARCH APPROVAL LETTER

Date:	5 March 2015
Validity of Research Approval:	5 March 2015 to 2 October 2015
Previous GDE Research Approval letter reference number	D2014/227 dated 10 February 2014
Name of Researcher:	Omoniyi A.A.
Address of Researcher:	He's Alive and Faithful Ministries; P. O. Box 55359; Arcadia; 0007
Telephone / Fax Number/s:	078 010 4035; 074 455 0208
Email address:	akinomo1968@ymail.com
Research Topic:	Impact of Constructivist Instructional Approach to Grade 12 Learners' understanding of Stationary Points in Differential Calculus
Number and type of schools:	THREE Secondary Schools
District/s/HO	Ekurhuleni North; Johannesburg Central and Tshwane South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

David Makhado
2015/03/06

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Making education a societal priority

Office of the Director: Knowledge Management and Research

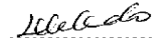
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

1. The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter;
2. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB);
3. A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned;
4. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, SGBs, teachers and learners involved. Participation is voluntary and additional remuneration will not be paid;
5. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal and/or Director must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage;
6. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year;
7. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
8. It is the researcher's responsibility to obtain written parental consent and learner;
9. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources;
10. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations;
11. On completion of the study the researcher must supply the Director: Education Research and Knowledge Management with one Hard Cover, an electronic copy and a Research Summary of the completed Research Report;
12. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned; and
13. Should the researcher have been involved with research at a school and/or a district/head office level, the Director and school concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards



Dr David Makhado

Director: Education Research and Knowledge Management

DATE: 2015/03/06

Making education a societal priority

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Office of the Director: Knowledge Management and Research

9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpa.gov.za

APPENDIX 4

LEARNER'S INFORMED CONSENT

Information for Research Participant

As a UNISA postgraduate student (student number 51940124), I am conducting an academic research on the topic: **Impact of Constructivist Instructional Approach on Grade 12 Learners' Understanding of Stationary Points in Differential Calculus**. As the research topic implies, the purpose of the study is to determine the extent to which the application of the theory of constructivism enhances Grade 12 learners' understanding of stationary points in differential calculus.

It is in the light of this that your consent to participate in the research work has been sought. Please, note that any information supplied shall remain strictly confidential and anonymous, and shall be used for the purpose of this investigation only. If you are willing to participate in the research, please sign this informed consent form. Thank you for your interest and cooperation.

Researcher's Name: _____

Signature: _____ Date: _____

Participant's Declaration

I (optional) hereby confirm that I have been well-informed by the researcher about the nature, conduct, benefits and risks of the study. I have also read and understood the above information. I am aware that the outcome of the study shall be anonymously processed into a research report. I understand that my participation is voluntary and that I can, at any level of the study, without prejudice, withdraw my consent and participation in the study. I had sufficient opportunity to ask questions and therefore, of my own volition, declare my intention to participate in the study.

Research Participant's Signature: _____ Date: _____

APPENDIX 5

PARENTAL INFORMED CONSENT FORM

(Applicable only where the participant is younger than 18 years)

I hereby confirm that I have been well-informed by the researcher about the nature, conduct, benefits and risks of the study. I have also read and understood the information about the study as contained in the Learner's Informed Consent. I am aware that the outcome of the study, and my child's personal details, will be anonymously processed into a research report. I understand that his/her participation is voluntary and that he/she may, at any level of the study, without prejudice, withdraw his/her consent and participation in the study. He/she has had sufficient opportunity to ask questions and I, of my own volition, declare that my child can participate in the study.

Research **P**articipant's **N**ame: _____

Name of Research **P**articipant's **P**arent/**G**uardian: _____

Signature of Research **P**articipant's **P**arent/**G**uardian: _____

Date: _____

Researcher's name: _____

Researcher's **S**ignature: _____ Date: _____

APPENDIX 6

CLASSROOM OBSERVATION CHECKLIST
(DESIGNED IN LINE WITH THE LEARNING PRINCIPLES OF CONSTRUCTIVISM)

Observer: _____

School: _____

Educator: _____

Grade Level: _____ Number of Learners Present: _____

Topic Taught: _____

Date of Observation: _____ Time: _____

*The following are the areas for observation in each classroom visit and the rating scale.
(NA means Not Applicable)*

Observable Characteristics: YES NO NA

A. Classroom Resources

1. The classroom is provided with suitable resources as original sources of information for problem-solving. -----

2. The classroom is very spacious and learners are comfortably seated. -----

3. Materials presented are appropriate to the level of learners. -----

4. Materials presented are related to the objectives of the learning area. -----

B. Presentation and Methodology

1. Teacher begins class at the appropriate time. -----

2. Initial class activities include:
(a) Making learners sit in small groups -----

(b) Review of previous work by using questioning approach	----	----	----
(c) Statement of lesson's objectives.	----	----	----
(d) Introduction of the day's learning materials to be presented	----	----	----
3. Learners engage in group discussions as they construct new knowledge.	----	----	----
4. Teacher goes round each learners' discussion group	----	----	----
5. Each group presents its understanding of the aspect of stationary points being learnt.	----	----	----
6. Teacher paces lesson appropriately	----	----	----
7. Real-life problems are discussed and solved in each group.	----	----	----
8. Study participants present the solutions as discussed and understood in each group.	----	----	----
9. Teacher encourages learner autonomy and initiatives.	----	----	----
10. Teacher relates lesson to educational goals, learners' personal needs and societal concerns	----	----	----
11. Teacher summarizes salient points of the day's work.	----	----	----
12. Teacher provides suitable assignment to prepare learners for the next lesson.	----	----	----

Field Notes

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APPENDIX 7

PRE-TEST

(QUESTIONS ON STATIONARY POINTS IN DIFFERENTIAL CALCULUS

DRAWN FROM RECENT PAST DBE/NSC MATHEMATICS I)

INSTRUCTION: Answer all the questions showing clearly your workings.

TIME ALLOWED: 1 hour 30 minutes.

1. Given: $f(x) = -x^3 + x + 10$
- 1.1 write down the coordinates of the *y-intercept* of f (1)
- 1.2 show that (2; 0) is the only *x-intercept* of f (4)
- 1.3 calculate the coordinates of the turning points of f (6)
- 1.4 sketch the graph of f in your answer book
- Show all the intercepts with the axes and all turning points (3)

(Q 10.1 – 10.4, Feb/Mar 2013)

2. The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
- A (-1; 0) and C (3; 0) are the intercepts of f .
- The graph of f has turning points at A and B.
- D (0; -6) is the intercept of f .
- E and D are points of intersection of the graphs of f and g .
- 2.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$ (5)
- 2.2 Calculate the coordinates of the turning point B (5)
- 2.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$,

Calculate x such that $h(x)$ is a maximum, where $x < 0$ (5)

(Q 9.1 – 9.3, Feb/Mar 2012)

3.2 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up (Q 8.1 & 8.4, Nov. 2014) (4)

4. Given $f(x) = (x + 2)(x^3 - 6x + 9)$
 $= x^3 - 4x^2 - 3x + 18$

4.1 Calculate the coordinates of the turning points of the graph of f . (6)

4.2 Sketch the graph of f , indicating the intercept with the axes and the turning points. (4)

4.3 For which value(s) of x will $x \cdot f'(x < 0) < 0$ (3)

(Q 9.1 - 9.3, Nov. 2014)

5. The graph of the function $f(x) = x^3 - x^2 + 16x + 16$ is sketched below.

5.1 Calculate the x -coordinates of the turning points of f (4)

5.2 Calculate the x -coordinate of the point at which $f(x)$ is a maximum. (3)

(Q 9.1.1 - 9.1.2, Nov. 2012)

APPENDIX 8

POST-TEST

QUESTIONS ON STATIONARY POINTS IN DIFFERENTIAL CALCULUS

DRAWN FROM RECENT PAST DBE/NSC MATHEMATICS I

INSTRUCTION: Answer all the questions showing clearly your workings.

TIME ALLOWED: 1 hour 30 minutes.

1. The graph of the function $f(x) = x^3 - x^2 + 16x + 16$ is sketched below.
 - 1.1 Calculate the *x-coordinates* of the turning points of f (4)
 - 1.2 Calculate the *x-coordinate* of the point at which $f(x)$ is a maximum (3)(Q 9.1.1 - 9.1.2, Nov. 2012)

2. Given $f(x) = (x + 2)(x^3 - 6x + 9)$
 $= x^3 - 4x^2 - 3x + 18$
 - 2.1 Calculate the coordinates of the turning points of the graph of f . (6)
 - 2.2 Sketch the graph of f , indicating the intercept with the axes and the turning points. (4)
 - 2.3 For which value(s) of x will $x.f'(x < 0) < 0$ (3)(Q 9.1 - 9.3, Nov. 2014)

3. Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up (4)
(Q 8.1 & 8.4, Nov. 2014)

4. The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A (-1; 0) and C (3; 0) are the intercepts of f .

The graph of f has turning points at A and B .

D (0; -6) is the intercept of f .

E and D are points of intersection of the graphs of f and g

- 4.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$ (5)
- 4.2 Calculate the coordinates of the turning point B . (5)
- 4.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$,
Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)

(Q 9.1 – 9.3, Feb/Mar 2012)

5. Given: $f(x) = -x^3 + x + 10$

- 5.1 write down the coordinates of the y -intercept of f
(1)
- 5.2 show that (2; 0) is the only x -intercept of f (4)
- 5.3 calculate the coordinates of the turning points of f (6)
- 5.4 sketch the graph of f in your answer book
- Show all the intercepts with the axes and all turning points (3)

APPENDIX 9

NSC EXAMINERS' REPORTS

2009 NSC Examiners' Reports

Reports on Question 11:

- 11.1 Candidates confused x-intercepts with turning points as many of them calculated the turning points in this question.
- 11.2 In differentiating, candidates still neglected to set $f'(x) = 0$.

Reports on Question 12:

- 12.1 Many candidates used $t=8$ instead of $t=0$ as “start” of the journey.
- 12.2 Very poorly answered. The concept of *rate of change* was poorly understood. Some candidates calculated $s(4)$ and not the rate.
- 12.3 Very poorly answered. Candidates confused speed/velocity of the car in a horizontal direction with *rate of change of height*.
- 12.4 Very poorly answered. Candidates did not realize that this was a change in rate, i.e. the second derivative had to be set to zero.

2010 NSC Examiners' Reports

Reports on Question 8

- 8.1 was generally not well-answered. Many candidates did not realize that they needed to adapt the first principle's formula from $f(x)$ to $g(x)$. Substitution into the first principle formula still posed a problem. Candidates knew what the answer should be; so, many of them manipulated their solutions to get $f'(x) = 2x$. Notational errors frequently occurred.
- 8.2 Many candidates did not know how to apply the basic rules of differentiation.
- 8.3 Most of the candidates struggled to set up the simultaneous equations arising from the substitution of the point into the correct equation and then using their knowledge of the derivative associated with the minimum value of a function.

Many candidates did not recognize that they had to set up simultaneous equations in order to solve for a and b .

Why question was poorly answered:

Incorrect copying of formula of the definition from formula sheet and inability to adjust formula to other functions; inability to apply the basic rules of differentiation; inability to interpret higher order questions where they had to set up two simultaneous equations.

Observations: In general, the candidates' responses to the questions were poor.

Reports on Question 9:

- 9.2 Was poorly answered. Candidates did not expect to be tested on the derivative of the cubic graph as the given function....
- 9.4 Many candidates did not know that the answer required the use of the second derivative. Candidates who used the fact that the point of inflection is halfway between the two turning points were credited.
- 9.5 Candidates could not display an understanding of the relationship between the derivative graph and the original cubic function.

Why question was poorly answered: Poor understanding and knowledge of the derivative graph of a function; Poor understanding of the derivative (gradient) when applying it to the shape and turning points of the cubic function; Candidates did not know how to get the x -coordinate of the point from the given x -intercept of the derivative graph.

Observations- Fairly satisfactory responses

Reports on Question 10:

- 10.3 is on the application of calculus to real-life problems; in this case, the minimum surface area was, for the most part, badly interpreted.

Why question was poorly answered: Inability to build formulae and Inability to apply calculus to real-life problems.

Observation: In general, the candidates' responses to the question were poor.

2013 NSC Examiners' Reports

Reports on Question 8: Calculus

Common errors and misconceptions:

- (a) Similar mistakes are recurring where learners copy the formula incorrectly from the formula sheet with regard to the first principle.
- (b) Many notation errors occurred in this question,
- (c) They could not get the derivative correctly from first principles.
- (d) Learners made many algebraic mistakes in the simplification part.
- (e) Some learners did not follow the instruction of first principles.
They applied the rules and only wrote down the answer.
- (f) Learners struggled to calculate the x and y needed to calculate the average gradient or they did not realize that they needed to calculate these values.
- (g) Many learners made up their own values for x and y so that they could calculate an average gradient.
- (h) Notation in Question 8.2 was also handled poorly.
- (i) The simplification of the expression in question 8.2 was problematic. Many learners simplified the expression incorrectly. The complexity of having a root and fraction made the manipulation difficult for the learners.

Reports on Question 9: Calculus: (Graphical Application)

Common errors and misconceptions

- (a) In answering Question 9.1, candidates used the given values of a and b to substitute into the equation to show that the graph has a turning point at P . Learners still struggle with the concept that derivative is zero at a stationary point. Some learners set $f(x) = 0$.
- (b) As mentioned above, answers to Question 9.3 showed that the concept of transformation is poorly understood when used in the context of a function. Many candidates opted for the algebraic approach in determining the new function and obtaining its turning point(s).

Reports on Question 10: (Calculus Application)

This question was generally poorly answered.

2014 NSC Examination Diagnostic Report

Question 8: Calculus

Common errors and misconceptions

- ... (d) Candidates struggled to differentiate when there was a y in the derivative. In the case of Q8.3, many candidates started with the answer and attempted to work backwards. This led them to ultimately calculate the second derivative. This was due to not understanding what expression they were working with.
- (e) Notation in Q8.2 was also handled poorly. Candidates wrote out the function and then continued on the next line into the derivative without using the notation for the derivative. It would seem that candidates did not understand the difference between a function and its derivative.
- (f) Candidates did not understand the concept of concavity. They did not understand that the concavity of a cubic function changes at the point of inflection; nor did they understand the meaning of 'concave up'.

Question 9: Calculus (Graphical Application)

Common errors and misconceptions

- (a) In Q9.1, candidates were able to calculate the derivative correctly, but often did not explicitly equate it to zero. They did however arrive at the correct answers for the turning points.
- (b) Candidates failed to realize that they had to draw a cubic function in this question. Instead, candidates incorrectly drew the cubic function in the shape of a parabola as they failed to identify that the x -intercept at $(3 ; 0)$ was also a turning point. Candidates did not indicate the values of the critical points on their sketch.

- (c) Candidates did not read the requirements of Q9.3 correctly. Many of them only wrote down the answer for where $(0) \leq x < 1$ and not for where $0 < x < 1$.

Question 10: Calculus (Application)

This question was generally very poorly answered.

Common errors and misconceptions

- (a) The application of Calculus has always been a problem. Candidates generally did not understand the relevance of the given net and the rectangular prism drawn, and did not understand the connection between the two diagrams and the problem.
- (b) As stated in previous Diagnostic Reports, the conceptual understanding of the application of Differential Calculus is still seriously problematic.
- (c) Candidates struggled to expand the expression for the volume. The simplification and adding of like terms led to many algebraic errors. Candidates often incorrectly simplified the volume to a quadratic or a linear expression.
- (d) Candidates seemed to lack confidence when they had to use the quadratic formula to find the solutions of the equation in which the derivative was equated to zero. Yet in earlier questions, these same candidates used the quadratic formula with ease to solve a routine quadratic equation that could have been solved by using factorization. Hence exposure to these types of questions is essential.