A FRAMEWORK FOR THE DEVELOPMENT OF PEDAGOGICAL CONTENT KNOWLEDGE FOR SECONDARY SCHOOL STATISTICS TEACHERS

by

ANTONIA MAKINA

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SUPERVISOR: Dr Mapula Ngoepe

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DECLARATIONS

I declare that

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is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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SIGNATURE DATE

Mrs A Makina

Learner number: **341-647-66**

Abstract

The study developed and designed a pedagogical content knowledge framework to guide and support the professional development of pedagogical content knowledge to about 130 statistics teachers. It captured the experiences of teachers during the development of pedagogical content knowledge, to come up with the main themes that describe pedagogical content knowledge as the relevant knowledge for teaching Grade 11 and 12 statistics.

The study was overall qualitative in nature and supported by some quantitative data. Questionnaires, in-class facilitated tasks/activities, in-class facilitated discussions and observations were used as the main data collection instruments. This process revealed some significant themes, described as “missed opportunities”, which were defined as incidents in which pedagogical content knowledge was needed but not used. The thesis contributes to the theoretical and knowledge base of secondary school statistics teachers in the education system by providing measures that can be used to determine professional development needs of teachers.

Key words: Statistics teachers, teacher knowledge, pedagogical content knowledge, statistics, professional development, pedagogic knowledge, subject-content knowledge, professional development, educators.

FOCUS OF THE STUDY

The introduction of more statistics in South Africa's secondary school curriculum and the prevalence of problems in the teaching of statistics argue for a serious reconsideration of the way it is taught to the learners. This study designs and formulates a pedagogical content knowledge framework in order to guide the development of pedagogical content knowledge to Grade 11 and 12 statistics teachers. It contributes to the theoretical and knowledge base of secondary school statistics teachers in the education system by providing measures that can be used to determine professional development needs of teachers. The development through its design of a pedagogical content knowledge framework is meant to stimulate national and international dialogue among policymakers and educators regarding mathematics teacher education policy, programmes and curricular to improve preparation and practice in statistics teaching.

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DIAGRAMS

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CHAPTER 1

A FRAMEWORK FOR THE DEVELOPMENT OF PEDAGOGICAL CONTENT KNOWLEDGE FOR SECONDARY SCHOOL STATISTICS TEACHERS

1.1 Introduction and focus of study

Mathematics education in South Africa has celebrated many positive results as far as mathematics teaching is concerned. However, particular changes in the curriculum, such as the inclusion of more statistics in the curriculum's mathematics section, have caused a lot of anxiety among the teachers. The development of pedagogical content knowledge in this study was motivated by a growing body of research suggesting that statistics in schools is not being taught with an in-depth understanding of the subject matter (Makina, 2005; Makwakwa, 2012). The aim of the study was to improve statistics teachers' abilities to recognise specific pre-conceptions and conceptual difficulties related to statistics, bi-variate data in particular, and to promote their use in the interventions and strategies promoting conceptual changes during classroom practices. Using a well-designed and peer-reviewed pedagogical content knowledge framework (Makina, 2013), the qualitative study conducted development research in order to ascertain effective and successful methods of teaching statistics. Teacher knowledge is identified as the single most powerful factor in carrying forward the understanding of the teachers' role (Elbaz, 1983). A competent teacher must not just have subject knowledge and people management skills, but also an ability to pass that knowledge on to others.

The origin of this study is clarified in section 1.1.1. Section 1.2 focuses on situating the challenges arising with teaching of secondary school statistics, bi-variate data in particular, and presenting the background of the problems that led to the development of pedagogical content. The significance of the study is analysed in section 1.3, while the delimitation of the study is given in section 1.5. The definition of key terms is in section 1.5. The problem statement is highlighted in section 1.6, and the research aims and objectives clarified in section 1.7. The working hypothesis in 1.8 is followed by the introduction of the research design and its summary given in 1.9, while the synopsis of what is to be covered in the next chapters is in section1.10. Supported by the literature review and by the main research objectives, this research will follow a qualitative design, supported by quantitative data. Four stages are used to identify the issues. The first stage is a pre-survey followed by the development of a pedagogical content knowledge framework, used during a development of pedagogical content knowledge with Grade 11 and 12 teachers. The actual empirical development of pedagogical content knowledge included facilitating the availability of bi-variate data tasks with learning activities, in a problem-centred environment during model lessons to the Grade 11 and 12 teachers. The problem-centred context was selected as the vehicle to drive the design and implementation of the intervention, as it encompassed all aspects of the educational setting.

1.1.1 Origin of the study

The development of pedagogical content knowledge was partly informed and justified by the baseline study that began in 2006 (Appendix 3). The baseline study involved a questionnaire for South African secondary school statistics teachers.

In 2006, a module FDEME8R was offered at the University of South Africa (UNISA), as part of the BEd degree. The percentage pass rate in this statistics module was consistently lower than the pass rate in all other mathematics modules. In addition, during contact sessions the teachers complained that the statistics module was being thrown at them as if they had some prior knowledge of statistics. Most of module’s content was Grade 11 to12 statistics, which was referred to as data handling. In marking work submitted in this module, I discovered that teachers shared statistical misconceptions and errors with the learners they were teaching. In addition to the problems with general interpretation of data, the section on the interpretation of bi-variate data caused the most problems. A number of factors could have contributed to this situation, however teacher knowledge was perceived as one of the most influential factors. A survey had to be carried out to find out why teachers were struggling with the teaching of this subject.

Most teachers were not taught statistics at school, yet, they were left with the task of teaching it (Institute for Science and Technology Education [ISTE], 2009). The South African community has freely expressed statements and comments on the poor performance of learners, due to teacher incompetence. A survey (Appendix 3) was conducted to determine why teachers still struggle with the teaching of statistics and justify the negative perceptions described above.

Watson (2001) in “Profiling teachers’ competence and confidence to teach particular mathematics topics: the case of chance and data” presented an instrument that captured the needs of this study and it was subsequently adapted. The instrument was designed as a profile of teacher achievement and teacher needs with respect to the probability and statistics strands in Australia’s mathematics curriculum. In developing the profiling instrument, Watson had two primary objectives. One was to assist in the assessment of teacher achievement in the context of the adoption of professional standards for mathematics teachers. Second, the instrument was to assess professional development needs for teachers in the light of changes to the mathematics curriculum. Professional development is concerned with the continuous updating of professional knowledge and skills throughout a teacher’s career. The background for the development of the instrument she presented, as well as the description of the instrument and the response results from 43 Australian teachers persuaded me to adapt this instrument for the baseline study for this thesis.

1.1.2 Background of the study

1.1.2.1 Statistics trends in the secondary school South African curriculum

The introduction of more statistics (prior to 2005 this was known as data handling) in South Africa’s secondary school curriculum (section 2.3.1) and the prevalence of problems in teaching statistics call for a serious reconsideration of the way in which it is taught to learners. Statistics is included as the seventh of the ten required learning outcomes (section 2.3.1). Statistics content in the South African mathematics curriculum expects learners at all secondary school levels to be able to deal with data in significant social, economic and environmental contexts, exploring relevant issues such as HIV/AIDS, crime abuse and environmental issues. Learners must further be able to take a critical stance in the analysis and interpretation of data. However, teachers meant to instil these concepts are not themselves familiar with such statistical thinking, as most teachers were not taught statistics at school (International Conference on the Teaching of Statistics [ICOTS]-6, 2002; Makina, 2005; ICOTS-7, 2006; ISTE, 2009; Delia, Jackie & Ottaviani, 2010; ICOTS-8, 2010, see Appendix 3). The dilemma arose as to how these teachers could receive training in order to cope with the vast new statistics content introduced in 2005. The Department of Education

(Department of Education [DoE] 2004) had also indicated that teachers should be provided with appropriate support to transition to full implementation of the new curriculum (statistics included). Suggestions had been forwarded to the Department of Education following research in the field of statistic education (International Association for Statistics Education [IASE], 2007; International Programme Committee, 2006; Batanero, 2011), but teachers still complained that the new curriculum was too complex for them to follow. Teacher knowledge and pedagogical content knowledge in particular was needed in order to address the above challenge.

1.1.2.2 Statistics as a distinct subject from mathematics

In South Africa, statistics is considered part of mathematics (DoE, 2007:22). Although much is known about teacher knowledge pertaining to particular aspects of mathematics, the situation for statistics is less clear (International Programme Committee, 2006; IASE, 2007). In addition, although the mathematical and statistical knowledge needed for teaching share some similarities, statistics education research reveals a number of issues that arguably differentiate the needs of statistics from those of mathematics, as regards teaching and learning. For example, statistics has a more subjective and uncertain nature as compared to mathematics (Moore, 1982). Irrespective of whether statistics is viewed as a sub-domain of mathematics or as a domain in its own right, it is widely recognised that there are differences between mathematics and statistics (Cobb & Moore, 1982; delMas, 2002; Pereira-Mendoza, 2002) and hence the need to explore statistics teaching and learning in its own right.

Statistics presents its own challenges for teaching and learning as compared with mathematics, especially with the growing recognition of and research into statistical thinking. For example, in mathematics one counter example is sufficient to disprove a hypothesised generalisation. In statistics, however, one counter example may merely illustrate the inherent variation in data, without discrediting the hypothesised generalisation. Teachers of statistics are familiar with the following scenario. They do not exactly understand the terms and definitions used in statistics. They see the language of statistics as moving away from the commonly used mathematics language. They then resort to finding some formula that gives them an answer without the need to understand the concept. Such differences point to the need to consider these differences when developing research on statistics teaching.

As a discipline, statistics is much "younger" than mathematics, and because it is new to many teachers, its status, in relation to teaching and learning, is not clear. Also, statistic education research has likewise had a relatively brief history and there is still much that is unknown about the specifics of teacher knowledge needed for statistics (Pfannkuch, 2006). Because statistics education research has not had the same length of history as research in mathematics education, developments relevant to teacher knowledge for teaching statistics is urgently needed.

1.1.2.3 Low pass rate in statistics in the secondary school curriculum

School-level mathematics education in South Africa is in crisis, with learners, on average and across all schools, faring very poorly in international comparative tests. The Human Sciences Research Council (HSRC) conducted the Trends in International Mathematics and Science Study (TIMSS), under the auspices of the International Association for the Evaluation of Educational Achievement, among 15 000 South African learners in 1997 (Trends in International Mathematics and Science Study [TIMSS], 1997). TIMSS data for several countries were collected in 1995, 1999, 2003, and 2007. South Africa fared badly in the mathematics section. In 1998, TIMSS was repeated (and is designated as TIMSS-Repeat or TIMSS-R), with tests and questionnaires administered in 38 countries. South African learners' performance in mathematics, data handling in particular, was well below the performance of the 38 countries. The findings carried out by the HSRC and Academy of Science of South Africa (2008) confirm that how much mathematics teachers know about their subject has a direct influence on their learners' average test scores. Stigler and Hiebert (1999) argue that evidence from comparative analysis of international studies of teaching (TIMSS) indicates that teaching is one of the major factors related to learners' low mathematics achievement.

The HSRC also conducted a pilot study in 40 schools in Gauteng, the country's smallest but most densely populated province, where 9.6 million, 20% of South Africa's 47.9 million citizens live. It found that while the new curriculum expected teachers to draw on professional knowledge to a higher level than the old one, 96% of the teachers had not been trained to teach this curriculum (Human Sciences Research Council & Academy of Science of South Africa, 2008). There also had been "constant complaints" that the short-term training provided for teachers to enable them to teach the new curriculum had not been effective (Chisholm & Carnoy, 2009). Part of problem was that there had been no effective supervision and evaluation of teacher training systems in place for the entire 14 years of democratic rule.

The South African community has freely expressed statements and comments about the poor performance of learners in statistics due to teachers' incompetence. For example in a 2012 Mail & Guardian article, "Do the Maths: Results not in line with SA's [South Africa's] ambitions", Faranaaz Parker noted that, although the matric pass rate for 2011 was at a high of 70.2%, the state remains concerned about the falling numbers of learners studying maths at matric level (Mail & Guardian, 2012). Last year only 224 635 of the country's 496 090 matrics wrote the mathematics exam and fewer than half of the candidates passed the subject with at least 30%'. Why do our schools continue to fail in their efforts to improve the teaching of statistics, which in turn affects learners' performance?

The Free State province had been experiencing poor mathematics results in the years prior to 2005. In response to the poor mathematics matric results in South Africa, the Free State Department of Education sent a request to the centre for teacher in-service training at UNISA to provide professional training for their teachers. Their request indicated that the section of statistics, bi-variate data in particular was contributing badly to the final mathematics results. A follow up process of meaningful professional developments was recommended by UNISA (Dr S.J. Mohapi, 2009-2011).

A questionnaire of teacher profiling from the Baseline study: (see Appendix 3, question 45) handed out to teachers, asked mathematics teachers to indicate their problem areas so that they could be attended to in a professional development. Statistics bi-variate data in particular was always among the most problematic areas (Baseline study, interview 4.3). Teachers complained that they did not understand the formulas they were given to use in statistics. For example, one teacher gave this response in one of the baseline questionnaires interview.

> I was told that if dots are going up its positive correlation and if they are going down, it is negative correlation. I don't understand where the hell this comes from. When my learners ask me I get very angry ... perhaps so that they do not ask more questions (Baseline study interview, 4.3.1.12).

Since the publication of the above-mentioned reports, this study made several recommendations around professional development and teacher knowledge. These included the need to urgently improve the teaching of statistics, the identification of “an all-round” teacher knowledge to be developed through best practices, and the need to improve the teacher knowledge through meaningful professional development.

1.1.2.4 Problems with teaching and learning statistics in the new curriculum

One of the main challenges teachers face in this field is the interpretation or understanding of how learners think. They are not able to respond appropriately when learners propose new strategies or formulas for solving problems, and they are not able to explain how they got their results in statistics. Therefore, one major goal of this study is to promote through professional development, teachers’ knowledge of common ways learners think about statistics. Teachers must be guided into establishing the deeper meaning and usefulness of statistics through pedagogical content knowledge.

Teachers are also facing problems recognising fallacious statistical arguments. These include the mistake of deducing a causal relationship when a correlation has been found and extrapolating data to a population when the sample is not representative of the population (Steffens & Fletcher, 2010). The mistake of deducing a causal relationship when only a correlation was needed was also found to be a common occurrence with UNISA learners (FEDEM8R, 2005). Examples of confusing correlation with causation are numerous and are shared between the teachers and the learners. For example, in the medical field a person who was very fond of carrots developed tooth decay and then people concluded that carrots are bad for your teeth (Pretoria News 2005: 30 January). In this case, only a correlation was needed. It is therefore vital that realistic examples from a wide variety of disciplines are employed to demonstrate to learners the critical-thinking skills that can be acquired through the study of statistics, and that can be applied to real situations every day in almost any career (Smith, 1998).

The underlying principles in the South African National Curriculum Statements for Mathematics (NCSM) do not come with any clear criteria (Parker, 2006:61). It is taken for granted that the meanings are transparent, that the teachers, as self-realising competent subjects/agents, will know what they mean, will recognise the practices and be able to realise these (Parker, 2006:61). Adapting these underlying principles to a subject like statistics poses a challenge to the teachers of statistics. The curriculum is both learner-centred and activity-based, according to the NCSM document (DoE, 2003:2). However, what is meant by "learner-centred and activity-based" is not defined, and it is assumed that these and many other issues are well understood and that the pedagogy underlying them is therefore transparent (Parker, 2006). Teachers' failure to understand many terms in the new curriculum documents have contributed to poor implementation of good statistical teaching. Pedagogical content knowledge guides teachers into specialised content knowledge, which leads teachers to unpack difficult statistical terms. The introduction of increased focus on probability and statistics in South Africa's secondary school curriculum and the many problems in the teaching of statistics therefore requires a serious reconsideration of the way in which it is taught to learners. A well-designed pedagogical content knowledge framework is needed to provide this guidance.

1.1.2.5 New identities for statistics teachers in curriculum reform

Curriculum reform is concerned with changing the bias and focus of "official" knowledge in order to construct new pedagogic identities in teachers and learners (Bernstein, 2000:65). The new pedagogic identities emerge as reflections of differing discursive bids such as reform, policy process and external performance indicators, to construct in teachers a particular moral disposition, motivation and aspiration, embedded in particular performances and practices (Bernstein, 2000:246). Similar to expectations in mathematics learning environments, in statistics there is need for teachers to encourage learners to explore and to verbalise their statistics ideas. They need to show learners that many statistics questions have more than one right answer, teach learners the importance of careful reasoning and disciplined understanding, provide evidence that statistics is alive and exciting and to build confidence in all learners so that they can understand statistics. Creating such learning situations requires a change from traditional views of mathematics teaching as the direct transmission of established knowledge to constructivist views of learning (problem-centred approach) (Makina, 2005:3).

Reform-oriented teaching, as defined by the United States' The National Council of Teachers of Mathematics (NCTM) standards (1989; 2000) documents, shifts the teacher's role to facilitator who selects tasks, chooses models important to mathematical actions, draws on multiple representations, guides learner thinking, asks mathematical questions and encourages classroom discourse. Teachers need to shift their paradigms through the willing abandonment of familiar perspectives and practices, and the adoption of new ones (Brooks & Brooks, 1993:25). The need for substantive change in professional practices using recursive reflection on individual teaching methods and dissemination of knowledge acquired through such a process to other teachers is implicit in this study. This would enable educational innovation. The question is what teacher knowledge is needed to achieve the necessary shift.

To empower learners in statistics requires preparing them with a powerful base of understanding and meaning in statistics and therefore developing their thinking skills. It calls for a shift from a curriculum dominated by memorisation of isolated facts and procedures to one that emphasises conceptual understandings, multiple representations and connections (Craine & Rubenstein, 1993:30). The tendency towards a data-orientated teaching of statistics is shown in the curricular orientation for primary and secondary school levels. Learners are expected to design investigations; formulate research questions; collect data using observations, surveys, and experiments; describe and compare data sets; use and understand statistical graphs and measures; propose and justify conclusions and predictions that are based on data (Lajoie, 1998; NCTM, 2000; Burrill, 2006; Burrill & Camden, 2006). This is the intention of the proposed South African Further Education and Training (F.E.T.) mathematical national curriculum. The new visions for statistics which have been articulated in a number of influential reports (NCTM, 2000; International Programme Committee 9 (Joint ICMI/IASE), 2006), call for new teaching strategies.

The constructivist perspective through the problem-centred approach (Von Glasersfeld, 1991) has guided much of the recent work in mathematics education. The increased focus on statistics in teaching and learning therefore obviously calls for a constructivist perspective. While constructivism has provided mathematics educators with useful ways of understanding learning and learners, the task of reconstructing pedagogical content knowledge in statistics, based on a constructivist view of learning, is a considerable challenge, one that the mathematics education community has begun to tackle (ICMI/IASE, 2006). Though constructivism is a useful framework for thinking about statistics learning in the classroom, and can contribute in important ways to the effort to teach it with understanding, it does not offer any particular vision of how mathematics (statistics included) should be taught (Von Glasersfeld, 1995). Practicing teachers in South Africa tend to perceive themselves as users and not producers of knowledge and teachers see themselves as distributors and not fixers of knowledge in the curriculum (Adler, 1993). The development of the pedagogical content knowledge through an identified framework offers possibilities for shifting such perceptions.

1.1.2.6 Factors influencing effective teacher knowledge in the learning of statistics

There is need for meaningful teacher knowledge that informs teachers of statistics since they are the immediate implementers of the new curriculum. There are many factors affecting the teaching performance of teachers. These include how teachers are affected by the system in which they operate; how they perceive the learners they teach; how they plan for teaching; how they assess difficulties; how they view knowledge; how they relate to their professional community; and how they respond to curriculum change (Watson, 2001:307). Poor performance in the teaching of statistics has been attributed to a number of factors, including lack of pedagogic knowledge, content knowledge, cultural knowledge, and curriculum knowledge (The National Council for Accreditation of Teacher Education [NCATE], 1994; NCTM, 2000; International Association for Statistics Education [IASE], 2007). However, research in statistics education has shown that poor performance in the teaching of statistics is mostly attributed to lack of teacher knowledge (NCATE, 1994; NCTM, 2000; IASE, 2007). For example, there is need for new strategies in the teaching of statistics to bring us closer to a better understanding of what it is about statistics activity that causes learners to harbour misconceptions. New teacher knowledge frameworks that embrace new strategies for the teaching of statistics are needed.

The question frequently raised within the education community is whether knowledge of subject matter or knowledge of pedagogy (general ability to teach) is more important. For the better part of the 20th century, the distinction between teachers' subject content knowledge and general pedagogical principles and practices is what was recognised (Fennema & Franke, 1992:149). Teacher preparation programmes have been organised under the assumption that prospective teachers will learn the necessary subject matter knowledge from the cognates and pedagogic knowledge from separate methodology classes taken in another school or faculty. For instance, the influence of the development of a teachers' subject content knowledge or pedagogic knowledge as separate entities to their performance in the teaching of mathematics, are among the many studies that have been carried out. Vitro-Yu (2006), Ball (1988), Schwab (1961), and Shulman (1987) among many, only studied the development of subject content knowledge. Clark and Yinger (1977), Peterson (1979), Sanders and Morris (2000) were only interested in the development of pedagogic knowledge, while Thompson and Senk (2001) looked at the effect of curriculum knowledge on performance in mathematics. Research on pedagogy has focused on the application of pedagogical practices in the classroom, isolated from any relevant subject matter. The push towards focusing on pedagogical practices was based on the assumption that these practices are instructionally effective, no matter what the academic subject or grade level being taught (Rowan, Schilling, Ball & Miller, 2001:2). Furthermore, historically researchers have focused on many aspects of teaching, but more often than not, scant attention has been given to how teachers need to understand the subjects they teach (Ball, Thames & Phelps, 2005). They went on to say that when researchers, educators and policymakers have turned attention to teacher subject matter knowledge the assumption has often been that advanced study in the subject is what matters and debates have focused on how much preparation teachers need in the content strands rather than on what type of content they need to learn. While teacher content knowledge is crucially important to the improvement of teaching and learning, attention to its development and study has been uneven (Ball, Thames & Phelps, 2005). The main question raised here is therefore, which teacher knowledge is best for the teaching of statistics.

Research has been conducted on the influence of various identified teacher knowledge on the performance of mathematics teachers (Rowan et al., 2001:2). Studying the different types of teacher knowledge as separate entities has not brought about convincing results about their influence in the performance of mathematics. Therefore, the main challenge for the teachers of statistics is to understand that pedagogical content knowledge cannot be separated from content knowledge and that the subject that they teach is different from the subject content knowledge that they know. The empowerment of teachers in statistics education is enhanced within the intertwining of subject content knowledge and pedagogic knowledge, which results in pedagogical content knowledge. Shulman (1987) stressed the fact that pedagogical content knowledge builds on, but is different from teachers' subject matter knowledge or knowledge of general principles of pedagogy.

1.1.2.7 Pedagogical content knowledge in statistics

A sizable proportion of the necessary innovation work on pedagogical content knowledge has been carried out (Gess-Newsome & Lederman, 1999; Rowan et al., 2001:2). However, their collective contribution to progress in the field of teacher knowledge is hampered by lack of common theoretical frameworks from which to operate. Furthermore, few researchers have well-articulated ideas of what pedagogical content knowledge is, despite the fact that it features prominently in most recent contributions to teacher knowledge.

Pedagogical content knowledge is a special amalgam of content and pedagogy that is uniquely the province of teachers and their own form of professional understanding (Schulman, 1987:8). The notion of pedagogical content knowledge, since its introduction in 1987, has permeated the scholarship that deals with teacher education in general and the subject matter education in particular (Shulman, 1987; Wilson, Shulman, & Richert, 1987; Grossman, 1990; Cochran, DeRuiter & King, 1993; Ball, 1996; Ma, 1999). It is valued as an epistemological concept that usefully blends the traditionally separated knowledge bases of content and pedagogy. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge consists of the ways of representing the subject that make it comprehensible to others (Shulman, 1986). It includes an understanding of what makes the learning of specific topics easy or difficult and provides the missing link between knowing something for oneself and being able to enable others to know it (Schulman, 1986:9).

Pedagogical content knowledge talks about teaching and not about teachers (Shulman, 1986). The transformation of subject matter for teaching occurs as the teacher critically reflects on and interprets the subject matter, finds multiple ways to represent the information as analogies, metaphors, examples, problems, demonstrations, and classroom activities. It also adapts the material to learners' abilities, gender, prior knowledge, and preconceptions. Finally it tailors the material to those specific learners to whom the information will be taught (Cochran et al, 1993:264). In a study by Vitro-Yu (2006), pedagogical knowledge is referred to as knowledge used for teaching, particularly, knowledge of teaching techniques, psychological principles, classroom management and the teaching and learning processes.

Based on Shulman's (1987) acknowledgment, pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics and problems or issues are organised, represented, and adapted to the diverse interests and abilities of learners and presented for instruction (Shulman, 1987:8).

Teachers need to have this particular type of knowledge and they need professional development to attain it, if they are to meet the challenges posed by current statistics education reforms. For this study, pedagogical content knowledge in statistics has been interpreted and summarised as given in the following diagram, which was adapted from An, Kulm and Wu (2004).

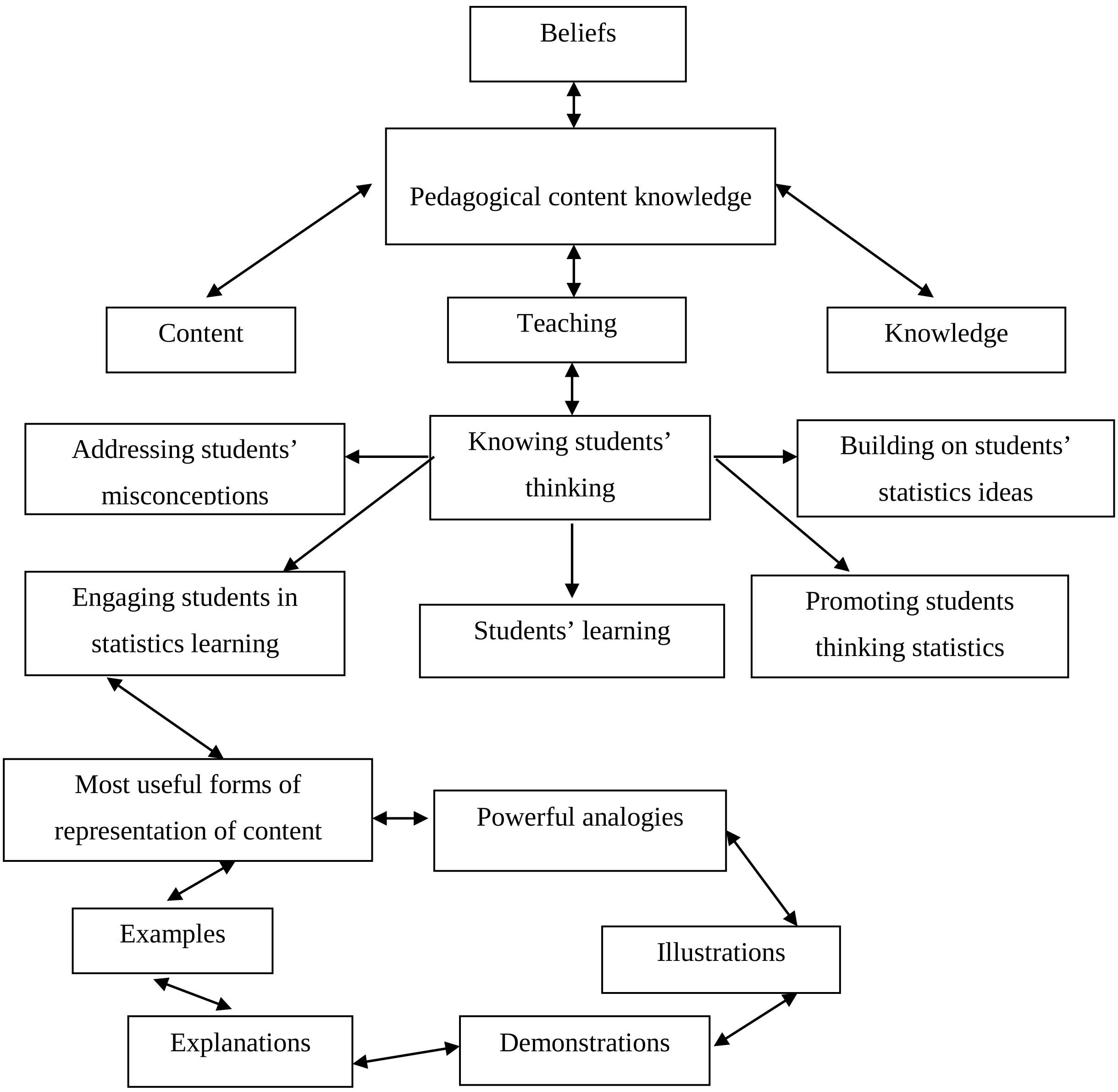


Diagram 1.1.2.7: The network of pedagogical content knowledge(Source: Adapted from An, Kulm and Wu, 2004:147)

Pedagogical content knowledge embodies formulating the subject statistics in order to make it comprehensible to others by encompassing best practices in all three forms (teaching, content and knowledge) needed for teaching statistics. Shulman (1987) describes pedagogical content knowledge as the "knowledge base for teaching" and that it is unique to teachers as it separates, for example, a statistics teacher from a statistician.

1.1.2.8 Problems with professional development in statistics education

Previous research effort focusing on mathematics teacher education and professional development has not reflected in statistics education. Shaughnessy (1992) highlighted statistics as an area in which little or no research existed and there appears to have been little work following on from his call for more research in statistics education (Watson 2001). With various efforts being made to improve performance in the teaching of mathematics, the role of the teachers' knowledge cannot be overlooked, as it has a significant impact on learners' learning (Taylor & Vinjevold, 1999).

The need to improve secondary school teachers' performance in the teaching of statistics is a major concern of all involved with statistics teaching. Since the introduction of the new curriculum in 2005, with an increased focus on statistics, little attention has been given to professional development that supports quality teaching in statistics (IASE, 2007). Inadequacies have been observed in professional development programmes. Current models of professional development fail to accurately address and outline the role of pedagogical content knowledge in teacher professional development (Veal & MaKinster, 1999). The National Science Education Standards (NRC, 1996) emphasise that coherent and integrated programmes supporting lifelong professional development of teachers are essential for significant reform. The conventional view of professional development for teachers needs to shift from technical training for specific skills to opportunities for intellectual professional growth (NRC, 1996).

In-service education continues to rely on brief summer workshops followed by a small number of brief school-based observations, demonstrations or discussions (Kelly & Lesh, 2000; Lesh & Sriraman, 2010). Yet it is well known that brief and superficial experiences are seldom effective in promoting sustainable changes in classroom practices (Guskey, 2002). For example, since 2008, there have been constant complaints that the short-term curriculum training provided for teachers has not been effective (Carnoy & Chisholm, 2008). They went on to say that teacher training programmes offered had no effective supervision and evaluation systems in place for the entire 14 years of democratic rule. The consequence of this was a system in which teacher absenteeism was “reportedly high” and teachers spent an average of 3.2 hours a day actually teaching. The result was a system in which teachers lacked competence and confidence in the teaching of statistics and skipped the section of statistics when the time to teach it was on (Baseline study interview, section 4.3). Therefore, in order to teach statistics effectively, we need to explore an efficient teacher knowledge base, which should result from meaningful professional development.

Literature suggests that mathematics teacher preparation programmes seldom fully prepare educators for the profession (Halford, 1998:33). According to the TIMSS (2003:20) report, South African teachers attend the highest number of professional courses. These courses (offered by the Department of Education, universities and non-governmental organisations) are an opportunity to provide a high-quality input to improve classroom teaching and learning. However, these teacher development initiatives seldom translate into visible improvements in the classroom and so do not provide the Department of Education with sufficient motivation to develop more relevant and effective initiatives (Rogan & Grayson, 2003). The South African community (Association for Mathematics Education South Africa [AMESA], 2004; SAARMSTE, 2009-2012; Institute for Science and Technology Education [ISTE], 2008) has freely expressed statements and comments about the poor performance of learners in statistics due to teachers’ incompetence. For example, less than a fifth of South African mathematics and science teachers meet the minimum requirements to teach their learners and that there were fewer higher-grade mathematics passes than the economy required (Pretoria News Weekend, 20 October, 2007). The pass rate was especially dismal for black learners at Dinaledi schools, which specialise in mathematics and science, despite the government injecting millions of Rands into these programmes (Pretoria News Weekend, 2007). The question frequently raised by the South African community is why the teachers of mathematics, statistics in particular, demonstrate apathy towards the teaching of this subject, despite many teacher development initiatives and other obvious environmental advantages (CDE, 2008).

Professional development should not only involve explanation of a theory, demonstration or modelling of a skill, practice of the skill and feedback about performance (NCATE, 1994), but should also be able to develop essential, appreciated and acceptable pedagogic content knowledge for teachers. It is about empowering the teachers with the knowledge that will make them trusted decision makers in the teaching and understanding of statistics. Teacher empowerment requires provision of broad, deep flexible knowledge of content and pedagogical alternatives (NCTM, 1991). The National Council for Accreditation of Teacher Education (NCATE, 1994) demands that professional education programmes adopt a model that explicates the purposes, processes, outcomes, and evaluation of the programme. More sophisticated forms of support are needed in order for the intended change in practice to occur. This study attempts to provide a professional developmental model, which will guide educators through the labyrinth of knowledge bases for the effective teaching of statistics.

1.2 Significance of study

Statistics is an important part of the mathematics curricula for primary and secondary school classes in many countries. In a technological world, statistics stands out as one of the most powerful and applied subjects. Newspapers, for example, use statistics to report on various events, such as employment figures, road accident figures, divorce rates, foreign exchange rates and business conditions. Charts, diagrams or graphs frequently accompany these figures. It is particularly important that by the time learners graduate they have some idea of how such information is collected, what the various types of graph mean and how reliable the information is likely to be (Walker & Mclean, 1973). The usefulness of statistics for daily life, its instrumental role in other disciplines, the need for a basic statistics knowledge in many professions, and the important role of statistics in developing critical reasoning cannot be ignored (Holmes, 1980; Hawkins, Jollife, & Glickman, 1991; Wild & Pfannkuch, 1999; Gal, 2002; Franklin, Kader, Mewborn, Moreno, Peck, Perry & Scheaffer, 2005). Because of statistics' great importance, the curriculum must provide opportunities to develop an understanding of all its sections.

The National Policy Framework for Teacher Education and Development in South Africa was designed to develop a teaching profession ready and able to meet the needs of a democratic South Africa in the 21st century (The National Policy Framework, 2006). The policy's objective is to create a community of competent teachers dedicated to providing education of high quality, with high levels of performance, as well as ethical and professional standards. Concisely, it insists on more and better teachers. Therefore, the unfolding educational reform is expected to give guidance regarding the forms of knowing and learning that might help enhance the chances of developing an empowered and competent statistics teacher society (The National Policy Framework, 2006). A good mathematically empowered teacher needs what Ernest (2002:4) calls “critical mathematical citizenship”. This involves the development of mathematically literate or socially numerate citizens who are able to exercise independent critical judgements with regard to the mathematical underpinnings of crucial social and political and political decision making as well as understand the use of mathematics in the mass media, advertising and in commercial, political and interest group pronouncements and propaganda (Ernest, 2002). To accomplish the above, new meaningful teacher knowledge with the guidance of a teacher knowledge framework must be developed for statistics teachers.

One significant change in South Africa is the inclusion of more statistics in the mathematics curriculum. Because of its historical position in the South African mathematics curriculum, the majority of statistics teachers have very little knowledge of statistics issues. This means that the majority of statistics teachers will have to be prepared to upgrade their knowledge as regards their own subject-specific knowledge, the subject didactical knowledge, as well as curriculum-specific knowledge. There have been indications that teachers need to be provided with appropriate support as they move in transition to full implementation of the new curriculum, but nothing positive has come of it yet. Teachers need support when implementing change, or while implementing new practices in their unique classroom condition (Berman, McLaughlin, Bass, Pauly & Zellman, 1977). They need ongoing guidance and direction to make whatever adaptation may be necessary in the teaching of statistics and at the same time maintain subject fidelity (Berman et al., 1977). They need to know that assistance is readily available if problems develop or if unexpected difficulties are encountered, in order to tolerate the anxiety of occasional failure and persist in their implementation effort (Cogan, 1975). Assistance and support should be guided by a workable and meaningful pedagogical content knowledge framework that focuses on the needs of teachers.

Educators cannot produce their best work and achieve expected outcomes unless they receive the necessary support through teacher development. Professional development is concerned with the continuous updating of professional knowledge and skills throughout a teacher’s career, requiring self-direction, self-management and sensitivity to development opportunities offered at work (Steyn, 2005:258). Since educators have the most direct contact with learners and considerable control over what is taught, and how it is taught, it is reasonably assumed that enhancing educator’s knowledge, skills and attitude is a critical step towards improving learner performance. Therefore, any genuine attempt at improving learners' performance in the subject statistics, should probably start with the improvement of the teachers' knowledge. This view coincides with Hadfield, Littleton, Steiner and Woods (1998:1):

> ...if the current national effort to improve mathematical instruction at the elementary school level is to be successful, it must be supported by a sound base of effective teacher preparation…

South African teachers are thrown in the deep end. They are confronted by new ideas and changed roles as described in South African policy documents, such as the Norms and Standards for Educators (DoE, 2000). They are asked to design classrooms and use learner-centred methods, to teach learners how to solve problems and think critically, to teach learners how to use the knowledge they receive (referred to as developing learner competences), and to plan lessons guided by learner outcomes (Criticos, Long, Moletsane & Mthiyane, 2005:13). This kind of teaching clearly needs a very different kind of teacher from the one our history has given us (the "chalk-and-talk" teacher).

This study therefore seeks to contribute to the improvement of the quality of statistics teaching during professional development of pedagogical content knowledge using a pedagogical content knowledge framework relevant to Grade 11 and 12 statistics teachers. It achieves this through improving statistics teachers' abilities to recognise specific pre-conceptions and conceptual difficulties related to statistics, bi-variate data in particular, and to promote their use in the interventions and strategies promoting conceptual changes during classroom practices. Implicit in this development is a need for a substantive change in professional practices, while reflecting on the type of teacher knowledge that will bring visible changes to the secondary school statistics classroom. The study is appropriate and worthwhile as it will provide assistance to decision makers regarding the professional development needs of teachers, and stimulate national and international dialogue among policymakers and educators regarding mathematics teacher education policy, programmes and curricular to improve preparation and practice in probability and statistics teaching.

1.3 Delimitation of study

The study is limited to teachers who teach statistics and will focus on the conceptual aspects of the pedagogical content knowledge development. The study will describe the process of the pedagogical content knowledge development and analyse the experiences of the teachers, but will not evaluate its impact in the classroom due the study's time constraints. The study will not cover all sections of statistics, but will limit its focus to bi-variate data. Within this section of statistics, bi-variate data, the study will examine dot plots, lines of best fit, and correlation.

1.4 Definition of terms

To create a common understanding of the semantics used in the context of this study, the following frequently used words and terms are defined.

Professional development

Professional development is concerned with the continuous updating of professional knowledge and skills throughout a teacher's career, requiring self-direction, self-management and sensitivity to development opportunities offered at work (Steyn, 2005). However, Day in Evans (2002:128) defines professional development as the process by which educators review, renew and extend their commitment as change agents to the moral purpose of teaching.

Grade 11 and 12

South Africa uses grades to indicate class levels. The school system begins with Grade 1 and ends with Grade 12. Grade 11 and 12 would be the last two class levels in the schooling system.

Teacher knowledge

Knowledge that enables a teacher to do something in the classroom, such as creating enabling situations for learning. Teacher knowledge is dynamic and dependent on the context of the classroom and learners within it. (Burgess, 2006)

Pedagogical content knowledge

It should be noted that pedagogical content knowledge and the abbreviation PCK have been used interchangeably throughout this study. Pedagogical content knowledge is a set of special attributes that help someone transfer the knowledge of content to others (Geddis, 1993). It entails (Schulman, 1980):

- Knowledge of how to structure and represent academic content for direct teaching to learners.
- Knowledge of learners' thinking, including the common conceptions, misconceptions, and difficulties that learners encounter when learning particular content.
- Knowledge of the specific teaching strategies that can be used to address learners' learning needs in particular classroom circumstances.

Developmental research

Developmental research is not aimed at building grand theories, such as understanding the human mind, but rather at understanding and developing good teaching practice (Lijnse, 1995:197).

Statistics

Statistics have been included in the National Curriculum of South Africa (2005) and the major themes, as highlighted in the present secondary school curricula, are:

- Exploring data: using a variety of standard techniques for organising and displaying data in order to detect patterns and departures from patterns.
- Planning a study: using surveys to estimate population characteristics and designing experiments to test conjectured relationships among variables.
- Anticipating patterns: using theory and simulations to study probability distributions and apply them as models of real phenomena.
- Statistical inference: using probability models to draw conclusions from data and measure the uncertainty of those conclusions.
- Technology: using calculators and computers effectively in statistical practice.

1.5 Statement of the problem

This study was prompted by significant changes to South Africa's mathematics curriculum, with the increased focus on statistics in the secondary school curriculum. However, most teachers were not taught statistics at school. This is evident through poor learners' performance in this area nationally, yet these teachers are left with the task of teaching it (ISTE, 2009; IASE, 2011; ICOTS-8, 2010). Pedagogical content knowledge in this study is identified as the best teacher knowledge that can impact on the potential learning opportunities for statistic teachers as well as learners.

1.5.1 Research aim and objectives

Hofmeyr (1994:35) states that, "Educator development is one of the most vital components of education reconstruction because educators are a most critical and expensive education resource…" Therefore, the aim of the study is to contribute towards the improvement of the quality of statistics instruction by putting pedagogical content knowledge at the centre in order to provide a community of competent and confident teachers dedicated to providing statistics education of high quality. The research will therefore be guided by the following objectives:

- To analyse if Grade 11 and 12 statistics teachers in South Africa are competent, confident and prepared for the teaching of secondary school statistics.
- To analyse and explore the special characteristics of pedagogical content knowledge that contributes to it being a better teacher knowledge for secondary school statistics teachers.
- To design and develop a new pedagogical content knowledge framework which will guide and inform the development of pedagogical content knowledge for secondary school statistic teachers.
- To use the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences.
- To analyse the challenges in the implementation of the new pedagogical content knowledge framework during the development of pedagogical content knowledge for Grade 11 and 12 secondary school statistics teachers.

1.6 Introducing the research design

In seeking to contribute towards the improvement of the quality of statistics instruction, the study recounts the process and results of a development case study comprising 130 statistics teachers. The empirical study was overall qualitative in design, but supported by some quantitative data gathered through questionnaires. In order to investigate the teachers' competence, confidence and the levels of support they received, a questionnaire to 200 teachers teaching secondary school statistics in South was administered.

The study formulated and described a pedagogical content knowledge framework, which, in turn, is used to guide and support the professional development of about 130 Grade 11 and 12 statistics teachers at in-service stage. Using the framework, development of pedagogical content knowledge for Grade 11and 12 statistics teachers was carried out. Questionnaires, interviews, observations, in-class presentations and discussions were used as data collection instruments.

This study also provided a qualitative analysis of the development of pedagogical content knowledge for secondary school statistics teachers. This is done through understanding the thought processes that occur during the development as the teachers engage in bi-variate data tasks. The video and interview recordings were analysed in relation to the pedagogical content knowledge and the pedagogical content knowledge framework. The results provided detailed descriptions of the components of pedagogical content knowledge in relation to statistical thinking that are needed in the classroom.

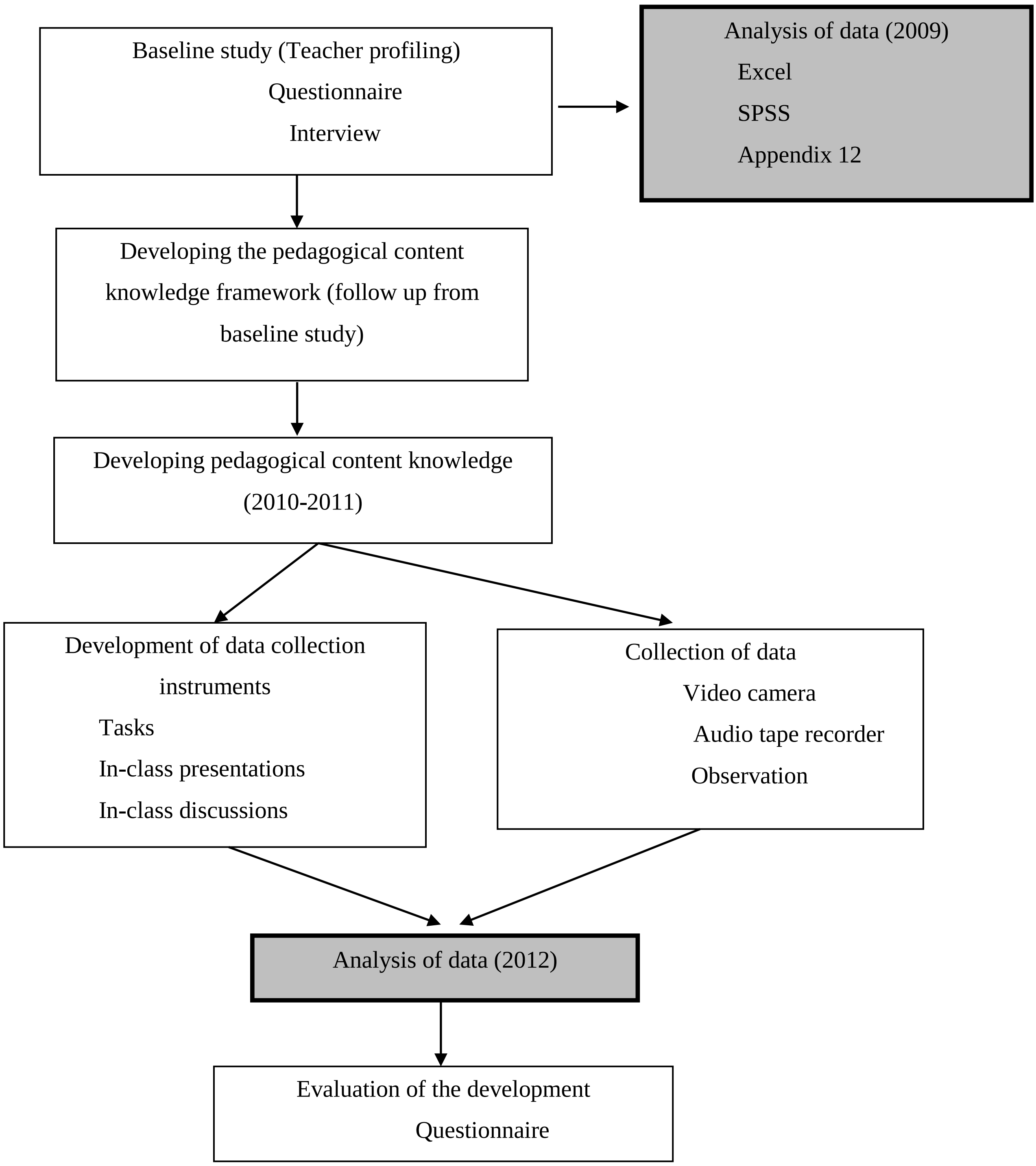


Diagram 1.6: The summary of the research design

1.7 Layout of the study

Chapter 1 situated the problem under investigation. It highlighted the background to the study and provided the problem statement, research objectives, significance of study, and definition of terms used in the study. The research aims and objectives are specified.

Chapter 2 describes in detail the challenges experienced with the introduction of more statistics in the school curriculum, with the aim of teaching statistics with understanding. The description of types of teacher knowledge from which pedagogical content knowledge will be specifically identified follows. The need for a meaningful and focused professional development for statistics teachers is discussed. The chapter further explores the theoretical framework pedagogical content knowledge by Shulman (1986, 1987), that has informed the study. Different interpretations of Shulman's work on pedagogical content knowledge by different researchers are analysed. These contributions are built together to create an essential pedagogic content knowledge framework for the development pedagogic content knowledge for secondary school statistics teachers.

Chapter 3 presents a description of the research design, sample and sampling procedures for schools and the teachers used, as well as the research instruments, data collection and analysis procedures, used in this study. It elaborates mostly on the qualitative methods supported by quantitative methods of collection and analysis of data. Methodological aspects are addressed by carefully satisfying the research aims and objectives formulated in chapter one. Reasons for choosing the research design, selection of participants, the choice of sites, collection and analysis of data, validity and reliability instruments are provided, as well as breakdown of the empirical study using codified case literature from renowned researchers, like Shulman (1987), Romberg (1992), Veal and MaKinster (1999), and Loghran et al. (2003).

Chapter 4 addresses the representation, interpretation and analysis of the collected data. The chapter discusses the research findings from the analysis and interpretation of data.

Chapter 5 provides reflections on the study and highlights the achievement of its aims and objectives before making recommendations and concluding. It provides the summary of the study, and the recommendations on how to construct, organise, and replenish capacity for instruction in statistics teaching in schools.

1.8 Conclusion

This chapter provided a summary of the study by providing and discussing issues relating to the background and overview of the study, the purpose of the study, the research objectives, and the significant of the study. The problem statement, research objectives, clarification of research aims and objectives, and the definition of key terms used in the study were analysed.

The delimitations of the study are given towards the end of the chapter. The chapter ends by providing a synopsis of what is to be covered in the next chapters.

CHAPTER 2

LITERATURE REVIEW OF TEACHER KNOWLEDGE, STATISTICS, PROFESSIONAL DEVELOPMENT AND PEDAGOGICAL CONTENT KNOWLEDGE

2.1 Introduction

From the background information in chapter 1, it is clear that offering good teacher knowledge to secondary school statistics teachers will improve their confidence and competence in the subject. In order to achieve the above, a thorough literature review about the need to recognise teacher knowledge and pedagogical content knowledge in particular as relevant to the teaching of statistics, and the identification of reliable professional developments in the area of statistics was carried out. This chapter puts forward a literature study, conducted on five major elements, namely: teacher knowledge, pedagogical content knowledge, statistics in the mathematics syllabus, professional development, and the design and development of a pedagogical content knowledge framework.

After introducing the chapter in section 2.1, various domains of teacher knowledge, each with their unique characteristics, are reviewed in section 2.2. Statistics, with its own challenges as far as knowledge relevant to its teaching and learning for secondary schools in South Africa is concerned, is clarified in section 2.3. Research relating to the nature of a functional professional development that enables the development of pedagogical content knowledge using a pedagogical content knowledge framework is given in section 2.4. Section 2.5 discusses the nature and context of the theoretical framework for pedagogical content knowledge for the teaching of secondary school statistics. Section 2.6 proposes, designs, and documents a pedagogical content knowledge framework used for the development with special reference to Shulman's (1997) work. The framework is broadly described, and its potential for developing pedagogical content knowledge is clarified, through a table (Table 2.6.4) in section 2.6.4. The rationale for using the pedagogical content knowledge framework during the development of pedagogical content knowledge is discussed in section 2.7, while the environment in which the development of pedagogical content knowledge takes place is clarified in section 2.8 before concluding in section 2.9 in preparation for the next chapter.

The development of pedagogical content knowledge focuses on teacher knowledge as a crucial issue in the quality of Grade 11 and 12 statistics teachers.

2.2 Teacher knowledge

Teaching is a highly complex activity that draws on many kinds of knowledge. Like expertise in other complex domains, including medical diagnosis, expertise in teaching is dependent on flexible access to highly organised systems of knowledge (Putnam & Borko, 2000). There are clearly many knowledge systems fundamental to teaching, including knowledge of learner thinking and learning, and knowledge of subject matter (Putnam & Borko, 2000). Just what knowledge or how much is needed is much less clear, and has been the focus of a significant amount of research. The work of Shulman (1986, 1987) has been influential in classifying and defining aspects of teacher knowledge not previously paid attention to as part of teacher knowledge.

Research on teacher knowledge has had various foci. Ball (1991) identifies three phases of research on the subject of teaching, in which teacher knowledge was the focus in the 1960s, 1970s and 1980s. The first phase was generally concerned with identification of the characteristics of good teachers. The measures used included the number of courses taken by teachers as part of their qualifications, the length of teaching experience, teachers' personal enjoyment of the subject, or some aspect of their teacher-education programmes (Ball 1991). It is recognised that using such measures as proxies for teacher knowledge yields little with regard to explaining differences in learner achievement (Rowan, Correnti & Miller, 2002).

The second phase of research on teaching was characterised by investigations of what teachers do and studies of general pedagogical strategies, such as questioning, use of praise, use of groups, and pacing of lessons (Ball 1991). With regard to mathematics, particularly statistics, this phase of research examined learner gains in terms of the mastery of skills through drill and practice. For example, in a section of bi-variate data, mastering and using the formulas for the calculation of product moment correlation coefficient was what was valued. Because such a conception was seen to be limited and simplistic, and it disregarded the complexity of classrooms and of teaching, changes in research focus characterised the onset of the third phase (Ball 1991).

Similar to the first phase, the focus during the third phase was teacher knowledge, particularly with regard to teachers' thoughts and decisions (Ball 1991). It is interesting to note that common characteristics from all three phases still exist and linger in statistics education. For example, in statistics teaching, it is still the teacher's teaching experience and personal interest in the subject that make him/her a good teacher. Pedagogical content knowledge infuses and acknowledges the good aspects in all three phases, by embracing a problem-centred approach for the first phase, using wide pedagogical strategies to take of the second phase and accommodating alternative assessment strategies for the third phase. It must be noted that pedagogical strategies are subject to societal and cultural change.

2.2.1 The trends in analysing teacher knowledge

Originally, a framework for analysing teachers' knowledge identified three categories of knowledge: subject matter, pedagogical content and curricula (Shulman, 1986:9). He defined subject matter knowledge as the amount and organisation of knowledge of the subject in the mind of the teacher. For example, in secondary school statistics it would be the in-depth knowledge of what constitutes secondary school statistics and other sections related to statistics. This includes any other knowledge that helps to build an understanding of all its sections. Shulman also referred curricula knowledge to knowledge of instructional materials available for teaching various topics and the set of characteristics that serve as both the indications and contradictions for the use in particular curriculum or programme material. It is also the knowledge of the sequence of topics or concepts to be taught and the materials and resources suitable for a particular topic. The second category, which Shulman (1986) named "pedagogical content knowledge", refers to the aspects of subject matter knowledge that are specifically required for teaching. Pedagogical content knowledge includes an understanding of what makes the learning of specific topics easy or difficult (Shulman, 1986:9), and consequently knowledge of how learners may be assisted in their learning of these concepts.

Furthermore, Shulman (1986) suggested three forms for representing teacher knowledge: propositional knowledge, case knowledge, and strategic knowledge. Propositional knowledge falls into three categories corresponding to three major sources of knowledge about teaching: disciplined empirical or philosophical inquiry, practical experience, and moral or ethical reasoning. These are also referred to as principles, maxims and norms (Shulman, 1986 p11). Principles derive from empirical research, maxims give a practical claim and norms and values refer to the ideological or philosophical commitments of justice, fairness, equity, etc.

(Dreyfus & Dreyfus, 1986). The development of pedagogical content knowledge falls in the propositional way of representing teacher knowledge as described by Shulman (1986). For example, educational research produces findings that then become the propositions from which teaching choices are derived. These general propositions, gained from a scientific process of observation, generalisation, experiment and interpretation, come to act almost as major premises from which pedagogical practices in statistics is then deduced as mediated by observations of particular circumstances serving as minor examples (Shulman, 1986:11). A weakness of propositional knowledge is that the propositions are hard to remember.

The second form is case knowledge and Shulman (1986) argues that the preparation of teachers be reduced to mostly the use of case literature to illuminate both the practice and the theoretical. There are three types of cases: prototypes exemplifying theoretical principles, precedents that capture and communicate principles of practice or maxims, and parables that convey norms and values. For example, Carpenter and Romberg (2004) produced a book, '*Powerful Practices'*, with cases for teaching and learning (section 2.7.3). The objective of this book was to invigorate mathematics and science education and to build awareness of what is possible across primary and secondary grade. *Powerful Practices* is a resource that was initiated and informed discussion of what is possible in mathematics and science education. It showed that a commitment to sustained, focused professional development addressing teachers' understanding of how learners learn important ideas and practices and knowledge of instructional practices that support that learning, are critical to supporting meaningful reform. Education is a process of learning to find one's way through the thickets of documented cases in order to identify proper precedents for the problem at hand (Schulman, 1984). Strategic knowledge sometimes comes in when principles collide and no simple teaching solution is possible. For example, in the language of statistics, the words "and" and "or" do not mean the same as common English language usage. In statistics "and" means "intersection" and "or" means "union". Here previous knowledge and present knowledge collide, and there is no simple solution to this.

Other researchers, such as Leinhardt and Greeno (1986), Barnett and Hodson (2001), Cochran et al. (1993), categorise teacher knowledge in significantly different ways to Shulman (1986). Leinhardt and Greeno (1986) identify only two major categories of teacher knowledge: practical knowledge for teaching (e.g. lesson structure) and subject matter knowledge. The structuring of a lesson takes priority over content knowledge, yet it is constrained by the knowledge of what is to be taught. A lesson is described in terms of an “agenda” (the overall dynamic plan for a lesson including its goals and actions), a “script” (the outline of the content to be presented and the way of presenting it), explanations (what the teacher says, does or demonstrates), and representations (of the mathematics, whether physical, verbal, concrete, or numerical). Leinhardt and Greeno (1986) contend that lessons generally proceed as planned with only small deviations due to input from learners. I disagree with Leinhardt and Greeno (1986) in this context. Teachers’ content knowledge is not only used in the planning of the lesson, but also in dealing with deviations from what has been planned. For example, if learners argue that they would rather use a bar chart for a particular problem that is represented using dot plots. In this case, strong content knowledge would help in better explaining the concept.

Fennema and Franke (1992) determined the components of teachers’ knowledge with special reference to mathematics, which I have adapted to statistics as follows:

- Knowledge of statistics: content knowledge, the nature of statistics, and the mental organisation of teacher knowledge.
- Knowledge of statistical representations.
- Knowledge of learners: knowledge of learners’ cognition.
- Knowledge of teaching and decision making.

Fennema and Franke (1992) identify four possible sources of pedagogical content knowledge: apprenticeship of observation, subject matter knowledge, teacher education, and classroom experience. However, Grossman (1990) does not elaborate on the processes by which pedagogical content knowledge develops from these sources. This model has limitations as it does not acknowledge or examine learning at the individual learner level nor deal with the richness and depth of the content involved in the lesson. It is often at the level of working with individual learners, and in dealing with their questions or problems, that a teacher’s content knowledge can be most challenged.

As with Grossman (1990), Barnett and Hodson (2001:448) suggest pedagogical “context knowledge” as another variation of teacher knowledge. This teacher knowledge is situated in the detail and intricacies of everyday classroom life. Pedagogical context knowledge has four components: pedagogical content knowledge, professional knowledge, classroom knowledge, and academic and research knowledge. Barnett and Hodson further categorised this teacher knowledge into subcategories, and sub-subcategories to conclude, "...that pedagogical context knowledge provides a simple and effective way of examining teachers' views and the knowledge on which they draw when they teach or talk about their teaching" (Barnett & Hodson, 2001:448). Most of their research data was collected through interviews with the teachers away from the classroom. It is important to realise that by only interviewing teachers outside the classroom, one misses understanding of what really happens in the classroom.

Cochran et al. (1993) propose another variation for the classification of teacher knowledge. They revised Shulman's original model to be more consistent with a constructivist perspective on teaching and learning. In their view teaching is concerned with developing "autonomous conceptual understanding", which accounts for the dynamic nature of the teacher knowledge. The term "pedagogical content knowing" is suggested as a more appropriate as the more static "pedagogical content knowledge". Pedagogical content knowing is "a teacher's model of an integrated understanding of four major components, two of which are subject matter knowledge and pedagogical knowledge (Kennedy, 1990). The other two other components of teacher knowledge also differentiate teachers from subject matter experts. One component is teachers' knowledge of learners' abilities and learning strategies, ages and developmental levels, attitudes, motivations, and prior knowledge of the concepts (Cochran et al., 1993). All the above constitute pedagogy, subject matter content, learner characteristics, and the environmental context of learning. The development of pedagogical content knowing is continual and strengthens over time. Two characteristics of pedagogical content knowing (learner characteristics and the environmental context of learning) are emphasised largely in this model than in Shulman's (1987) model. However, all four components are necessary for a strongly integrated knowledge structure,

Finally, the trend of analysing teacher knowledge has not progressed without problems. For example, differentiating between subject matter knowledge and pedagogical content knowledge has been a source of difficulty for a number of researchers. In acknowledging that difficulty, Sherin (2002) formulated an alternative component of teacher content knowledge, namely "content knowledge complexes". Experience, Sherin (2002) says, enables teachers to make connections between the content they are teaching and the strategies for teaching that content. Consequently, teachers access both subject matter knowledge and pedagogical content knowledge simultaneously from experience and other teacher resources, and the connections between them make it impossible to distinguish the separate types of knowledge (Manouchehri, 1997). Such connected aspects of knowledge are categorised as "content knowledge complexes". For example, if a teacher understands correlation in depth, it would simultaneously be easy for him/her to envision the ways and strategies to make it be understood by his/her learners. Teachers access these content knowledge complexes as a whole rather than as separate and distinct pieces of subject matter knowledge and pedagogical content knowledge (Sherin, 2002).

2.2.2 The trends of knowledge bases for teaching

As research on teachers' knowledge became more prominent, it became important to identify the place of this teacher knowledge in the total knowledge base of teaching. In this study, the knowledge base of teaching is defined as all profession-related insights that are potentially relevant to the teacher's activities (Verloop, Van Driel & Meijer, 2001). These insights can, for example, pertain to formal theories (such as the classical theories from research on teaching), as well as to information about the knowledge and beliefs of expert teachers that have emerged from more recent research. For example, Harris, Mishra and Koehler (2009) critically analysed extant approaches to technology integration in teaching. They argued that many current methods are technocentric, and they offered pedagogical content knowledge-based "activity types," rooted in previous research about content-specific activity structures, as an alternative to existing professional development approaches. In this sense, the results of research on teacher knowledge that include pedagogical content knowledge are seen as an addition to the knowledge base of teaching. This "knowledge of teachers" guides a teacher's actions in concrete and specific situations. There will be elements of teacher knowledge that are shared by all teachers or large groups of teachers; for instance, all teachers who teach a certain subject area like bi-variate data.

A great deal of educational research aims at developing a knowledge base of teaching and, where possible, translating it into recommendations for teacher education (Reynolds, 1989). The research goal was to detect those teaching behaviours that resulted in higher learner-achievement scores and, subsequently, to train teachers in these desirable behaviours, either in initial teacher-education programmes or by means of further professional development. Due to increasing criticism coming from the professionals themselves, the influence of this type of research diminished. Teachers felt that analysing isolated behavioural components was inadequate and they resisted the prescriptive nature of this "knowledge for teachers" (Fenstermacher, 1994). Meanwhile, doubts arose also from the scientific community about a conception of professionalism that asked professionals (such as teachers) to just "apply" the theories and insights provided by others. Schön (1983, 1987) analysed the work of various groups of professionals and concluded that they applied a certain amount of theoretical knowledge in their work, but that their behaviour was not at all "rule governed" and that they had no straightforward way to determine which behaviour was adequate in specific circumstances. Schön contrasted this principle of "technical rationality" to the principle of "reflection-in-action", which pertained to the thinking of the professional during professional activity and implied a continuing dialogue with a permanently changing situation. From this point of view, the most challenging question with respect to teacher professionalism is no longer how we can best provide teachers with insights developed elsewhere, but how the process of "dialogue with the situation" takes place in a teaching context; which insights are developed in this context; and how these insights relate to insights from other sources.

Historically, knowledge bases of teacher education have focused on the content knowledge of the teacher (Shulman, 1986; Veal & MaKinster, 1999). The focus later shifted primarily to pedagogy where emphasis was put on general pedagogical classroom practices, independent of subject matter and often at the expense of content knowledge (Ball & McDiarmid, 1990). Among the identified knowledge was mathematical content knowledge, which consists of common knowledge of content, that which any reasonably educated adult should know and be able to do, and specialised knowledge of content, that which teachers, but not necessarily other adults, know and can do (Hill, Schilling & Ball, 2004). Common knowledge of content includes the ability to identify incorrect answers or inaccurate definitions, and the ability to complete the learners' problems successfully. Specialised knowledge of content includes the ability to analyse mathematically whether a learner's unconventional answer or explanation is reasonable or mathematically correct, or to give a mathematical explanation for why a process (such as a particular algorithm) works (Hill et al., 2004).

Subsequent to his original work of describing the categories of teacher content knowledge, Shulman (1987) outlines seven categories of knowledge that are requisite knowledge bases for teaching. These are (Schulman, 1987):

- Content knowledge.
- General pedagogical knowledge (such as principles and strategies for classroom management and organisation).
- Curriculum knowledge.
- Pedagogical content knowledge.
- Knowledge of learners and their characteristics.
- Knowledge of educational contexts (such as the workings of a group or classroom through to the character of the community in which a school is situated).
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

There is a discrepancy between Shulman's 1987 listing of the categories of the knowledge base for teaching, in which content knowledge is listed alongside pedagogical content knowledge and curriculum knowledge, and his 1986 description that classifies pedagogical content knowledge and curriculum knowledge as subcategories (along with subject matter knowledge) or components of content knowledge. It would appear that although his 1987 paper lists content knowledge as one of the seven knowledge areas, Shulman might have intended it to be the same as, or representative of, subject matter knowledge. In this regard, the professionalisation of teaching depends on showing that teaching, like other learned professions, requires mastery of a specialised body of knowledge, which is applied with wisdom and ethical concern (Verloop et al., 2001).

There is therefore need for the professional community to have conversations about the type of knowledge required to achieve the best results in the teaching of statistics and the best conceptual frameworks for organising and using the knowledge. This study explores the possibility of building a useful knowledge base for teaching by beginning with a knowledge base framework applied during professional development. The development outlines key features of this knowledge and identifies the requirements for this knowledge to be transformed into a professional knowledge base for teaching.

2.2.3 How does teacher knowledge develop?

Most research described in section 2.2.2 above, such as Barnett and Hodson (2001), Cochran et al. (1993) neglects the aspect of how knowledge develops. Shulman's (1986) idea is that teachers begin with some level of subject matter knowledge. The novice teacher becomes an expert teacher, through a process of “transformation” from subject matter knowledge to pedagogical principles. For example, in statistics if a teacher understands the section of bi-variate data very well, then he/she will acquire over time the art of the strategies needed to teach it well. Agreeing with Shulman, Marks (1990) names this process “interpretation”. However, Marks (1990:7) added that pedagogical content knowledge could also develop through a reverse process: “the application of general pedagogical principles to the particular subject matter contexts”. This process is referred to as “specification”, that is, the appropriate instantiation of a broadly applicable idea in a particular context (Marks, 1990:8). An example of specification would be the teacher applying his/her knowledge of questioning strategies (a general pedagogical skill) to a particular content area. Marks (1990) terms the knowledge that develops in this way “content-specific pedagogical knowledge”. The term better describes the nature of that knowledge than the term “pedagogical content knowledge”. However, Marks acknowledges the many situations in which the development of pedagogical content knowledge represents a synthesis of general pedagogical knowledge, subject matter knowledge, and previous pedagogical content knowledge.

Veal and MaKinster (1999), through a study of secondary science teachers, contend that for pedagogical content knowledge development, strong subject matter knowledge and knowledge of learners (which includes understanding possible learner errors and misconceptions) are both essential. They argue that there is a hierarchical structure to pedagogical content knowledge development, although teachers may possess some aspects of prior to, for example, a full understanding of their learners. Most of the research does not show the existence of a hierarchy as far as the development of pedagogical content knowledge is concerned. Veal and MaKinster believe strongly that subject matter knowledge is a prerequisite for any other teacher knowledge. However, having developed taxonomy for pedagogical content knowledge, Veal and MaKinster do not specifically address the development of, other than acknowledging that teaching experience plays an important part, and that pedagogical content knowledge develops throughout the teacher’s career.

Sherin (2002) discovered that when experienced mathematics teachers try using a teaching approach significantly different from what they are used to, one of three processes occurs: “transform”, “adapt”, or “negotiate”. For Sherin, transform refers to the situation where the teacher recognises a similarity between the new approach and previously taught content.

Instead of changing existing pedagogical content knowledge to accommodate the new approach, the teacher implements his/her existing pedagogical content knowledge. He/she consequently changes the intended outcome of the new approach to an outcome similar to the previous approach. The lesson reverts to a familiar, traditional one with different outcomes from what was intended. In the "adapt" situation, a learner's question or response is responsible for the teacher realising that his/her current pedagogical content knowledge is inappropriate for the situation as presented, and the usual content knowledge complex associated with that particular subject knowledge is not suitable. The teacher therefore draws on his/her broader subject matter knowledge and develops an appropriate but new pedagogical response. Therefore, the teacher's pedagogical content knowledge is changed to ensure that the intended lesson outcome is maintained. In "negotiate", again a learner's interaction forces the teacher to realise that the typical content knowledge complexity is not appropriate, so the teacher develops new pedagogical content knowledge after drawing on his/her broader subject matter knowledge. Essentially a new content knowledge complex begins to form while the existing content knowledge complex is refined. In this case, Sherin's research clearly describes the reality of a classroom situation. The way teacher knowledge develops is a very complex process. As in all processes where change is required, there is a need to guard all the different ways in which knowledge develops. All the different ways have advantages and disadvantages, but what must be obvious is the need for the same destination: good meaningful statistics teaching from a confident and competent statistics teacher.

2.2.4 Summation

Teacher knowledge is the total knowledge that a teacher has at his or her disposal at a particular moment, which, by definition, underlies his or her actions (Carter & Doyle, 1987). This does not imply that all the knowledge a teacher has actually plays a role in his or her actions. Teachers can consciously or unconsciously refrain from using certain insights during their teaching. The basic idea is that reciprocity exists between the whole of teachers' cognition (in the broad sense) and their activities and that consequently, it makes sense to investigate teachers' knowledge (Carter & Doyle, 1987; Carter & Richardson, 1988). Teacher knowledge may have a variety of origins that include both practical experiences such as day-to-day practice, and formal schooling, which in the past included initial teacher education or continued professional training (Calderhead, 1988). During teacher or professional development, theories and philosophies of education or teaching can be advised and encouraged. All these knowledge bases to some extent may be absorbed and integrated into their practical knowledge in their own different and unique ways. There have been many labels given to teacher knowledge, each depending on which aspect is considered the most important by the respective authors. In this study "teacher knowledge" is used to indicate the whole of the knowledge and insights (done consciously or unconsciously) that underlie teachers' actions in practice. In investigating teacher knowledge, the focus of attention is on the complex totality of cognitions, the ways in which these develop, and the effect on teacher behaviour in the classroom. Furlong (2005) described the rise of three alternative conceptions of the role of knowledge with respect to teaching:

- Reflective practice, which, following Schön (1983), focuses on the knowledge embedded in the practice of professionals.
- The "new rationalism", which, following Hirst and White (1998), tries to develop forms of practical (rather than theoretical) reason, and practical principles as the generalised outcomes of successful practice. These are subsequently confronted with theoretical critique and experiment.
- Critical theory, aimed at revealing the assumptions behind statements about knowledge and truth and focusing on an open discourse among all professionals concerned. There are problems in trying to put labels to the knowledge that is best for classroom use.

2.3 Statistics education in the secondary school

2.3.1 Expectations of the South African curriculum

In the recent past, statistics has been neglected at secondary school level, perhaps partially due to the strong emphasis on developing arithmetic, algebra and geometry. However, the South African curriculum, since 2005, includes a substantial amount of statistics at all grade levels. The new curriculum was endorsed by the Department of Education (2003, 2004) in a bid to improve the quality of statistics education for its citizens. Following this endorsement, Curriculum 2005 (C2005) which has outcomes-based education (OBE) as a fundamental building block, was the start of a whole new era in South African school education. The statistics (then referred to as data handling) component of C2005 covered the following aspects for Mathematics Grade 10 to 12:

- Choose appropriate methods to collect, organise, represent, and analyse data in order to solve a particular problem at a more advanced level than in the General Education and Training phase.
- Use Venn diagrams to solve basic probability problems.
- Expand the range of statistical tools to include quartiles, percentiles, variance, and standard deviation (at all times emphasis to be placed on the interpretation aspect).
- Be able to recognise the strengths and weaknesses of statistical arguments and establish basic probability models to solve appropriate problems.

In 2007, the compulsory assessment standards for Grade 11 and 12 (DoE 2007:22) were:

Grade 11

- Represent measures of central tendency and dispersion in uni-variate numerical data by:
    - Five number summary (maximum, minimum quartiles)
    - Box and whisker diagrams
    - Ogives
    - Calculating the variance and standard deviation of sets of data manually (for small sets of data), using available technology (for larger sets of data) and representing results graphically.
- Represent bi-variate numerical data as a scatter plot and suggest intuitively by simple investigation (i.e. whether a linear, quadratic or exponential function would best fit the data).
    - Errors in measurement
    - Skewed data in box and whisker diagrams and frequency polygons.

Grade 12

- Use of available technology to calculate the linear regression line that best fits a given set of bi-variate numerical data.
- Use of available technology to calculate the correlation co-efficient of bi-variate numerical data and make relevant deductions.
- Understand suitable sampling from a population, understanding the importance of sample size in predicting the mean and standard deviation of a population.
- Analyse data that is normally distributed around the mean.

In line with the mathematics curriculum reform for 2011, the amended National Curriculum Statement Grades R-12: Curriculum and Assessment Policy (January 2011) replaced the

National Curriculum Statement Grades R-9 (DoE, 2002) and the National Curriculum Statement Grades 10 - 12 (2004) (DoE 2007:22). The Subject Assessment Guidelines document for Grades 10 - 12 mathematics anticipated that the optional assessment standards were to become compulsory after 2010 and teachers were encouraged to prepare themselves for the teaching of these optional standards as soon as they were confident to do so (DoE 2007:7). The major themes of statistics as highlighted in the present South African secondary school curricula are:

- Exploring data using a variety of standard techniques for organising and displaying data in order to detect patterns and departures from patterns.
- Planning a study using surveys to estimate population characteristics and designing experiments to test conjectured relationships among variables.
- Anticipating patterns: using theory and simulations to study probability distributions and apply them as models of real phenomena.
- Statistical inference: using probability models to draw conclusions from data and measure the uncertainty of those conclusions.
- Technology: using calculators and computers effectively in statistical practice.

2.3.2 The background of statistics in the secondary school curriculum

Despite previous problems in finding room for the subject statistics in a curricula dominated by trigonometry and geometry, data-driven mathematics curricula are getting more attention than ever before. Statistics, often in the context of data analysis, enabling insight into real-world situations, is creeping into the secondary school curriculum in South Africa. Change in the school curricula is seen to offer an exciting opportunity to develop learners' statistical intuitions, to foreground the mathematical concepts embedded in statistical techniques, and to create contexts in which these can be linked to broader mathematical ideas of symbol use, reasoning and logical necessity. The development of statistics at school level has partly been influenced by the work of the International Association for Statistics Education (IASE), formed by the International Statistics Institute (ISI) in 1991 (Ainley & Pratt 2001:2). Also influencing these developments are a number of factors including technology, which makes it possible to handle large quantities of data. There is also increased public use and awareness of statistics in most media, and the introduction into the school curricula of notions such as relevance (e.g. issues about HIV) and citizenship (e.g. ethical issues). With the inclusion of more statistics in the curriculum, teacher training has however not yet caught up with the requirements for the teaching of the subject (Wessels, 2005).

Statistics at secondary school level involves data handling. Data handling according to Shaughnessy (1992) is the describing, organising and reducing, representing, analysing, and interpretation of data. The usefulness of data handling for daily life, its instrumental role in other disciplines cannot be overemphasised. The need for basic statistics knowledge in many professions, and the important role of statistics in developing critical reasoning, has been highlighted (Wild & Pfannkuch, 1999; Gal, 2002; Franklin & Mewborn, 2006). The tendency towards data-orientated teaching of statistics is shown in the curricular orientation for primary and secondary school levels. In this curricular, learners are expected to design investigations; formulate research questions; collect data using observations, surveys, and experiments; describe and compare data sets; use and understand statistical graphs and measures; propose and justify conclusions while making predictions that are based on data (Lajoie, 1998; NCTM, 2000; SEP, 2006; Burrill, 2006; Burrill & Camden, 2006). Since statistics is a science concerned with the collection, presentation, analysis and interpretation of data, it is therefore an appropriate topic in the school mathematics curriculum because it (NCTM, 1981):

- Provides meaningful applications of mathematics at all levels.
- Provides methods for dealing with uncertainty.
- Enables understanding of statistical arguments, and how they are used.
- Helps consumers distinguish sound use of statistical procedures from unsound or deceptive uses.
- Is inherently interesting, exciting and a motivating topic for most learners.

Statistics has a crucial role to play in the secondary school curriculum. We live in a data-rich society, and therefore the ability to comprehend the meanings embedded in data and to work with data to answer specific questions is an important educational outcome. As learners manipulate data sets and examine how to represent relationships through a variety of graphical forms in data handling, they develop a strong conceptual understanding of the foundations required for further study of statistics. Even more important is that learners need to be able to distinguish what questions are amenable to investigating with data, and how to collect, organise and interpret that data. This is in line with the specific outcomes of the NCTM standard for data analysis and probability that states:

> ...the learners are to formulate questions that can be addressed with data and collect, organize and display relevant data to answer them, select and use appropriate statistical methods to analyze data, develop and evaluate inferences and predictions that are based on data, and to understand and apply basic concepts of probability (NCTM 1981:324).

Data broadly refers to both descriptive and numerical data. The data may come from a variety of sources. It can be collected from experiments or surveys carried out by learners themselves from mathematical investigation, invented data provided by teachers or textbook authors or extracted from real-world sources such as government statistics. Data can be treated as objects independent of the existence of that which they represent. Data can be manipulated and conclusions can be drawn about these manipulations independent of actions in the world (Lehrer & Romberg, 1996). For example by manipulating data, new questions can be posed about relations among elements of the data structure. Hancock and Kaput (1992) in Lehrer and Romberg (1996:70) suggest that the objectification of data gives rise to a deductive quality. Action taken on data can generate new data. For example, data used to understand the relationship between old age and tiredness might raise questions on the causal effect of this.

2.3.3 Trends in statistics education

Statistics education research, although relatively young in comparison with mathematics education research, has grown significantly in recent years, as evidenced by the number of international conferences and journals that are devoted to such research. Recent research in statistics education includes a strong thread focusing on the nature of statistical thinking, statistical reasoning, and statistical literacy. Rather than focusing on statistical skills, procedures, and computations, there has been a growing call to encourage learners to reason and think statistically (Ben-Zvi & Garfield, 2004). A model for statistical thinking has provided some clarity on an important way of examining what constitutes statistical thinking (Wild & Pfannkuch, 1999). The fundamental aspects of statistical thinking components include:

- Recognition for the need for data (rather than relying on anecdotal evidence).
- Transnumeration: being able to capture appropriate data that represents the real situation and change representations of data in order to gain further meaning.
- Consideration of variation: this influences the making of judgements from data and involves looking for and describing patterns in the variation and trying to understand this in relation to the context.
- Reasoning with models: from the simple, (such as graphs or tables) to the complex as they enable finding patterns and summarising data in multiple ways.
- Integrating the statistical and contextual components (making the link between the two as an essential component of statistical thinking).

All components are about an understanding of the nature of sampling; how we make inferences from samples to populations; and why designed experiments are needed in order to establish causation. These further lead to an understanding of how models are used to simulate random phenomena; how data are produced to estimate probabilities; and how, when, and why existing inferential tools can be used to aid an investigative process. This results in being able to understand and utilise the context of a problem in forming investigations and drawing conclusions, and recognising and understanding the entire process (from question posing to data collection to choosing, analyses to testing assumptions, etc.).

Statistical reasoning may be defined as the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, representations of data, or statistical summaries of data. Statistical reasoning may involve connecting one concept to another (e.g. centre and spread), or it may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes and being able to interpret statistical results.

Statistical literacy is the ability to understand and critically evaluate statistical results that permeate daily life, coupled with the ability to appreciate the contributions that statistical thinking can make in public, private, professional and personal decisions. Some of the activities in which teachers regularly engage in, such as finding out what learners know and choosing and managing representation of mathematical ideas, involve mathematical reasoning and thinking (Ball, Lubienski & Mewborn, 2001:453). These are some of the most important aspects that reside in pedagogical content knowledge and hence the need to present a framework for its development.

Whereas in mathematics education the use of real-life contexts are advocated as a generally useful means of developing learners' understanding of mathematical concepts, in statistics it is considered essential that learners come to realise that data are numbers with a context and are used to address a particular issue or question (Gal & Garfield, 1997; Cobb, 1999). In mathematics, learners learn that mathematical reasoning provides a logical approach to solving problems, and that answers can be determined to be valid if the assumptions and reasoning are correct (Pereira-Mendoza, 2002), that the world can be viewed deterministically (Moore, 1982), and that mathematics uses numbers where context can obscure the structure of the subject (Cobb & Moore, 1997). In contrast, statistics involves reasoning under uncertainty; the conclusions that one draws, even if the assumptions and processes are correct, are "uncertain" (Pereira-Mendoza, 2002) and statistics is reliant on context (delMas, 2004; Greer, 2000), where data are considered to be numbers with a context that is essential for providing a meaning to the analysis of the data. While in mathematics the use of context may be, but not always, useful for developing conceptual understanding in statistics, context is essential for making sense of data (Sullivan, Zevenbergen & Mousley, 2002). Irrespective of the above differences, it must however be noted that the interlink between mathematics and statistics cannot be underestimated.

These significant differences between statistics and mathematics have implications for the teaching and learning of statistics. Teaching must consider these differences and teachers must be ready and able to encourage learners to think differently in statistics. It becomes necessary therefore, when teaching statistics, to encourage learners not to merely think of statistics as doing things with numbers, but to come to understand that the data are being used to address a particular issue or question (Gal & Garfield, 1997; Cobb, 1999). If the ideas of statistics are to be understood and useful to learners, they must not be too theoretical or too abstract (Barnett & Hodson, 2001). The approach should be experimental, and data-oriented. In the United States and the United Kingdom, for example, a widely used approach emphasises the use of real data, active experiments and learners' participation, with materials that are adaptable to different levels in the school. To support this development, investigations are advocated as a worthwhile approach to teaching and learning. Effective teaching to be implemented with the development of pedagogical content knowledge therefore requires an understanding and implementation of the investigative and interrogative cycles along with certain dispositions.

2.3.4 Statistics education and pedagogical content knowledge

Having discussed the need for the better teaching of statistics and therefore the need for relevant teacher knowledge to achieve this goal, the question is what basic knowledge do teachers require to teach statistics to Grade 11 and 12 learners successfully. Earlier attempts at understanding the nature of statistics included the careful sequencing of tasks and the hierarchies of levels of understanding within content structure (Reading & Pegg, 1996). Now, there is a growing trend in educational research to investigate understanding from the viewpoint of analysing learners' responses. Research in pedagogical content knowledge has shown that it would be valuable to encourage effective ways to help statistics teachers realise the importance of statistics as a discipline, maximising technology use in order to support teachers and enabling teachers to create an instructional design that allows learners to acquire the basic ideas of statistics. There is need to attempt to answer the following questions:

- What practice-based learning in statistics is essential for in-service teachers?
- How can we prepare teachers to deal with appropriate context knowledge when applying their statistics teaching to a diverse range of applications?

Pedagogical content knowledge in this study is recommended to be the teacher knowledge that can enable statistics teachers to do their job efficiently and effectively. It seems evident from the problems above, that what constitutes good statistics teaching is offering teachers the best teacher knowledge, pedagogical content knowledge. Focusing therefore on what constitute good statistics teaching, the question is what are the principles, factors and determinants of a successful and meaningful professional development that can offer pedagogical content knowledge for secondary school statistics teachers?

2.4 Professional development

Evans (2002) defines professional development as the process by which educators review, renew and extend their commitment as change agents to the moral purpose of teaching. Professional development is concerned with the continuous updating of professional knowledge and skills throughout a staff member's career. It requires self-direction, self-management and sensitivity to development opportunities offered at work or at a national level. On the other hand, professional development of educators is seen as an ingredient essential to creating effective schools and raising learners' performance (Steyn, 2005; King & Newman, 2001). Since educators have the most direct contact with learners and considerable control over what is taught, and how it is taught, it is reasonably assumed that enhancing educators' knowledge, skills and attitudes is a critical step towards improving learner performance. It must however be noted that professional development is an ongoing process whereby educators develop a greater sense of collaboration, share common problems and assume greater responsibility for their own professional development (Browell, 2000). The process of professional growth like any other process is not always smooth. There are problems experienced by both teachers and programme developers involved in this process.

A survey of teacher-training programmes presented at South African universities was done by Wessels (2005) to determine the status and content of statistics education. Results showed that many of these programmes do not train statistics teachers adequately for their task to prepare learners to be statistically literate citizens and that few research studies on statistics education have been completed on the post-graduate level in South Africa. There is therefore need to provide teachers with meaningful professional development to meet their immediate needs.

The main question concerning the role of teacher knowledge in teacher development is in what ways teacher knowledge can be made available, or accessible, to in-service teachers. Attempts to use mentor observation has not proved sufficient to reach this goal (Calderhead, 1988), neither have attempts to provide teacher knowledge in the form of case descriptions and narratives (Levin, 1995; Noddings, 1996). Specific interventions are needed. This study attempts to make teacher knowledge to in-service teachers accessible through professional development.

2.4.1 The changing role of the South African statistics teacher

The nature and praxis of teaching, contrary to general opinion, is one of the most difficult professions to be in. Unlike most professions, success of the teaching and therefore of the teacher is dependent on several factors, most of which are beyond the teacher's control. For example, the learner has to be willing to cooperate actively. Adding to the difficulty of the teaching praxis is that, unlike other professions where clients go back to the expert every time they have a problem, teachers have to personally master their disciplines and secondly, provide the learner with the capacity to figure it out themselves the next time around (Labaree, 2000). This study encourages teaching approaches that are complementary to the learning principles outlined in pedagogical content knowledge.

There was very little interaction between the professional associations to which South African mathematics school teachers and statistics educators belong prior to 1998. This can be attributed to the fact that statistics virtually played no role in the South African school education system at that time. AMESA and the South African Statistical Association (SASA) held annual seminars, workshops, think tanks and conferences independent of each other. It was only in 1998, when South Africa won the bid to host the Sixth International Conference on the Teaching of Statistics (ICOTS-6), that the education committee of SASA was tasked with reaching out to AMESA, with the intention of including school teachers in some of the proposed ICOTS-6 initiatives. The hosting of ICOTS-6 in South Africa was celebrated with the introduction of statistics into the school curriculum, as national and international attention was focused on this initiative.

South African teachers are often confronted by new ideas, especially every time there are curricula or theoretical changes. They are asked to design classrooms and use methods that are learner-centred, teach learners how to solve problems and how to think critically. They are also asked to teach learners how to use the taught knowledge (sometimes referred to as developing learner competences), and plan lessons guided by learner outcomes (Criticos et al., 2005:13). The changed roles for teachers are described in the South African policy document Norms and Standards for Educators. This kind of teaching clearly needs a different sort of teacher from the traditional “chalk-and-talk” model; it requires reflective practitioners. There have been indications that teachers need to be provided with appropriate support as they move in transition to full implementation of the new curriculum. Suggestions, based on research in the field, were put forward to the government departments concerned, but still teachers are complaining that the new curriculum is just too complex for them to follow. This proves that teachers need help and support from in-service education and training programmes in preparing for these challenges since their experience and knowledge is quite limited in spite of the many years of teaching. This study therefore seeks to contribute towards the improvement of the quality of statistics instruction by focusing on the teacher’s pedagogical content knowledge during professional development, to provide assistance to those making decisions regarding the professional development needs of teachers.

2.4.2 The role of professional development

The task of transforming teaching to facilitate learners' development of statistical thinking is not a matter of simply adding a few techniques to an existing system, but rather an ongoing process of re-conceiving ideas about the nature of learning itself. One of the main challenges teachers of Grade 11 and 12 experience is interpretation of the learners' thinking, responding appropriately when learners propose new strategies or formulas for solving problems, and explaining how they got their results in statistics. One of the major goals of this study is to promote, through professional development, teachers' knowledge of common ways that children think about statistics. Teachers must be guided into establishing the deeper meaning and usefulness of statistics. Instruction in statistics must take into account learners' levels of thought and development. Therefore, the most important role of professional development in statistics education is to initiate professional growth that leads to teacher empowerment to accommodate the changing role of the South African statistics teacher.

2.4.3 Professional growth

True professional growth is the sense of resulting in meaningful and long-lasting qualitative change in a teacher's thinking and approaches to educating. This involves an autonomous activity chosen by a teacher in search of better ways of knowing and teaching statistics. Professional development activities that are externally mandated or coerced by a power hierarchy, although well intentioned, are doomed to failure like other passing educational fads. Professional growth is a complex process involving a wide array of influencing factors and forces, internal and external to the education community, that influence the professional development of teachers. These forces could be redirected to facilitate enhanced professional growth. This process develops over time and therefore conflict, confusion and surprise are expected as part of that process during professional development (Wood, 1992). There must be an ongoing community of people to provide support when teachers encounter the difficulties that are an inevitable part of coming to new understandings (Nyaumwe & Mtetwa, 2011). This ongoing support can be through monthly or weekly meetings, conferences, research articles, or by prompt phones, and so on.

Professional growth, which is the process of changing teacher practice, takes different theoretical roots (Schifter et al., 1996a). For example, according to the Piagetian position, change in teachers' ideas about the nature of learning can be done by creating activities and events that stimulate cognitive reorganisation on the part of participating teachers. In cognitive science, teacher change is a matter of change in the content and organisation of teachers' knowledge, specifically research-based knowledge about the evolution of children's mathematical thought (Peterson, 1988). Fennema and Franke (1992) discuss that teacher change occurs when teachers acquire the above-mentioned knowledge, organise it into a framework related to children's problem-solving strategies and use the framework to guide their teaching. From a psychological constructivist position to a socio-constructivist position, learners construct and reconstruct ideas and teachers resolve conflicts between their prior beliefs about learning and what they observe happening in their classrooms (Wood, Cobb & Yackel, 1991).

2.4.4 Teacher empowerment

South Africa has gone through important political and social changes since the early 1990s. The process of professional growth for teacher empowerment is therefore necessary since there have been periodical changes with respect to the curriculum, pedagogy, learning, attitudes, disciplines and many other sectors of statistics. Empowerment in this study embodies autonomy. Autonomy is the ability to exercise good judgement, make decisions after considering relevant variables, feel the decision's effects on others, self-regulation, and the ability to decide for one's self without having to be told by others (Nyaumwe & Mtetwa, 2011). Teachers are able to distinguish between appropriate and inappropriate actions based on internally constructed standards for behaviour (Kamii & Russell, 2012). They must also be able to make decisions about learners and how they learn best, read and critique professional guidelines, and put their decision making responsibilities to other teachers. Overall autonomy leads to continued construction and reconstruction of knowledge, which amounts to advanced progress in the field of statistics.

The term 'empowerment' in most dictionaries implies an external agent transferring power or knowledge to someone. Professional knowledge cannot be transferred (Castle & Aichele, 1994), rather, it is actively constructed by each individual teacher by bringing his/her lived experiences as a learner and teacher to an educational setting. This is achieved when the teacher, through interacting with the environment in a way that relates new knowledge to previously constructed knowledge, attempts to make the best sense of the new knowledge. Professional knowledge actively constructed by teachers allows teachers to function as autonomous decision makers, who act in the best interests of learners regardless of any externally imposed reward system. Autonomous teachers act on their best professional knowledge, whereas heteronomous teachers, subject to external standards, merely attempt to carry out the mandates of those in authority positions.

In-service programmes for secondary school statistics teacher development can foster professional empowerment by appreciating and incorporating the unique lived experiences of statistics teachers and by providing opportunities for teachers to experience ownership of the programmes. The programmes must provide atmospheres that allow for autonomy. The teacher candidates need immersion in an environment where they are engaged in questioning, hypothesising, investigating, imagining and debating. They need to be part of a community that actively works with them as learners and then allow the experience to be dissected, evaluated and reflected upon in order for principles of pedagogy and action to be constructed. Three phases that enable a more relaxed atmosphere for empowered teachers of secondary school statistics are suggested by Hobbs and Moreland (2009):

- Security phase: teacher content knowledge must be improved so that they become familiar with all the new materials and be confident in the handling of apparatus.
- Method phase: teachers concentrate on the acquisition and internationalisation of new teaching skills, to become secure with their performance not only their knowledge.
- Aims phase: teachers contemplate their experience with the innovation and begin to conceptualise and articulate their own aims based on the situation in which they function and their own personal preferences. Teachers have the power to accept, reject or modify all aspects of the innovation in accordance with their own needs and philosophy.

2.4.5 Determinants of statistics professional development success

What are the determinants of a successful professional development that enable professional growth and empowerment of teachers of statistics and thus provide meaningful teacher knowledge? The models of professional growth evolve as developers respond to teacher feedback, new developments in educational research and their own experiences with the programmes. Change in the professional development models must have key principles. It could be the heavy reliance on current research in education to guide practice or the overdue of didactic pedagogical techniques found in both the elementary schools and in the teacher professional development. The development of pedagogical content knowledge in this study is a result of current research into teacher knowledge, which has resulted in the appreciation of pedagogical content knowledge as the best knowledge base for secondary-school statistics teachers.

Professional development in statistics education requires that the education provider has a clear sense of purpose and direction that is informed by national priorities as well as by quality demands. For the professional development to be effective, one must offer teachers practical ideas that can be efficiently used to enhance desired learning outcomes in learners (Doyle, 1977). When teachers recognise that they do not simply have to put into practice the decisions of others in relation to teaching styles or curriculum, but are innovators in what takes place in their classroom, and potentially in a much wider forum too, there may well be a sense of empowerment. Teachers need information, support and role models to instigate self-reflection and change in their teaching styles. Let teachers see it from you, and let them admire that which you are doing by being able to articulate the kind of experiences that stimulate teachers to reflect on their practice and begin to make changes in their teaching (Schifter & Fosnot, 1993). In the light of the above, a detailed design of a pedagogical content knowledge framework to guide development of pedagogical content knowledge for secondary school statistics teachers during professional development was designed.

2.5 Understanding the nature and context of pedagogical content knowledge as the theoretical framework

2.5.1 Pedagogical content knowledge: the theoretical framework

There is widespread agreement that effective teachers have unique knowledge of learners' mathematical ideas and thinking (Hill, Ball & Schilling, 2008). However, few scholars have focused on conceptualising this domain. Pedagogical content knowledge is Shulman’s (1987) answer to the old question of which is a more important quality in a teacher – knowledge of subject matter or a general ability to teach. Since the 1980s the analytic distinction between teachers’ subject matter knowledge and teachers knowledge of pedagogy has begun to fade in large part, due to Shulman’s (1986, 1987) work. Shulman (1987) pointed out that the best teacher has something that is neither of these, but instead has knowledge of how to teach specific parts of the subject matter. Shulman (1986) developed a new framework for teacher education by introducing the concept of pedagogical content knowledge, which goes beyond knowledge of subject matter to the dimension of subject matter knowledge for teaching. Rather than viewing teacher education from the perspective of content or pedagogy, Shulman (1986) believed that teacher-education programmes should combine these two knowledge bases into pedagogical content knowledge to prepare teachers more effectively.

In the years following Shulman's (1986) seminal address introducing the notion of pedagogical content knowledge, most scholars and policymakers have assumed that such knowledge not only exists, but also contributes to effective teaching and learning (Rowan et al., 2001). Pedagogical content knowledge includes the most useful forms of representation of [topics], the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the ways of representing and formulating the subject that make it comprehensible to others Shulman (1987:9). Also encapsulated in the idea of pedagogical content knowledge is the notion that successful teachers have a special knowledge about learners that informs their teaching of particular content. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult for others and acknowledges the conceptions and preconceptions that learners of different ages and backgrounds bring with them to the learning of frequently taught topics and lessons (Shulman, 1987: 9).

The ideas of pedagogical content knowledge espoused by Shulman (1986) have been the cornerstone of the development of pedagogical content knowledge to Grade 11 and 12 secondary school statistics teachers. Though pedagogical content knowledge is increasingly recognised as an essential component in understanding and assessing quality teaching, it is particularly important to note that it conceptualises the missing link between knowing something for oneself and enabling others to know it (Shulman 1986:9).

2.5.2 The broad interpretations of pedagogical content knowledge

Pedagogical content knowledge has been extensively researched, particularly since Shulman's (1986) landmark paper in which he named and defined pedagogical content knowledge. During the 1980s under the influence of Shulman (1987), teachers' professional knowledge has been conceptualised as subject-matter content knowledge, pedagogical content knowledge and general pedagogical knowledge. Although there is much agreement on pedagogical content knowledge as a shared construct of teacher knowledge, through research it has become evident that there is no consensus on how to conceptualise and measure it. For example, in certain countries, such as Germany, teacher-educators appear to treat it as a domain of theoretical knowledge, while in the United States, it is typically understood to have both theoretical and practical knowledge components.

Marks (1990) refined pedagogical content knowledge into four components: subject matter for instructional purposes; learners' understandings of the subject matter; media for instruction in the subject matter; and instructional processes for the subject matter. In spite of developing these categories of pedagogical content knowledge, Marks (1991) acknowledges the difficulty in classifying aspects of a teacher's knowledge into one particular category of pedagogical content knowledge, or even into one of the three broader categories of pedagogical content knowledge, subject matter knowledge, and general pedagogical knowledge. The indistinct and sometimes overlapping boundaries between the various categories of knowledge were problematic for Marks (1991). This is partly attributable to the different ways in which pedagogical content knowledge develops. In some cases, it develops from subject matter knowledge, while at other times it develops from general pedagogical knowledge. In his categorisation of knowledge, Marks (1990) refers pedagogical content knowledge to the broad category of subject matter knowledge, but it is not clear how he differentiates between this and one of his sub-categories of pedagogical content knowledge, namely subject matter for instructional purposes.

Grossman (1990) categorised pedagogical content knowledge into four sub-categories: conceptions of the purposes for teaching subject matter; knowledge of learners' understanding; curriculum knowledge; and knowledge of instructional strategies. These categories are significantly different from Marks's (1990) categorisation due to the absence of something equivalent to Marks's subject matter for instructional purposes. However, it can be argued that this component would fit the broader category of subject-matter knowledge, rather than being a subcategory of pedagogical content knowledge. Along similar lines, Cochran, King and DeRuiter (1991:1) defined pedagogical content knowledge as, "...the manner in which teachers relate their pedagogical knowledge to their subject matter knowledge in the school context, for the teaching of specific learners". This definition incorporated four components: knowledge of subject matter, knowledge of learners, knowledge of environmental contexts, and knowledge of pedagogy.

According to Rowan et al. (2001:2), Shulman's view of pedagogical content knowledge is a form of practical knowledge that is used by teachers to guide their actions in highly contextualised classroom settings. This form of practical knowledge entails, among other things:

- Knowledge of how to structure and represent academic content for direct teaching to learners.
- Knowledge of learners' thinking including the common conceptions, misconceptions, and difficulties that learners encounter when learning particular content.
- Knowledge of the specific teaching strategies that can be used to address learners' learning needs in particular classroom circumstances.

Rowan et al. (2001) therefore summarises pedagogical content knowledge into three dimensions: content knowledge, knowledge of learner thinking, and knowledge of pedagogical strategies. Content knowledge is here defined as knowledge of the central concepts, principles and relationships in a curricular domain, as well as knowledge of alternative ways these can be represented in instructional situations.

Some subcategories of pedagogical content knowledge were identified and refined by Ball, Hill & Bass (2005). They understand knowledge of learners as the ability to anticipate learner errors and misconceptions; ability to interpret incomplete learner thinking; ability to predict how learners will handle specific tasks; and what learners will find interesting and challenging. The other component of pedagogical content knowledge is knowledge of content and teaching, which gives the ability to sequence the content for teaching appropriately in order to recognise the instructional advantages and disadvantages of particular representations and therefore be able to weigh up the mathematical issues in responding to learners' unexpected approaches (Ball et al., 2005). These categories confirm the belief of a strong relationship between teaching and the various aspects of mathematical knowledge, skills, and habits of mind.

Rahman and Scaife (2005) understand Shulman's work (1986, 1987) as suggesting that in order to teach a subject, one needs broad and deep pedagogical content knowledge, a rich knowledge base with many interconnections that represent a much more thorough understanding than that which is achieved purely as a learner of the subject. The construction of pedagogical content knowledge not only requires an understanding of subject matter, but also an understanding of learners, their abilities and interest and how they tend to respond to different situations. It is also an appreciation of different teaching strategies and how various types of classroom activity might be managed (Rahman & Scaife, 2005). The teacher personally constructs a teacher's pedagogical content knowledge, and it is the reason why it is difficult to assess the nature of the teacher's pedagogical content knowledge.

Herr (2007) understands pedagogical content knowledge as including the most useful forms of representation (topics), the most powerful analogies, illustrations, examples, explanations, and demonstrations, in the ways of representing and formulating the subject that make it comprehensible to others. It also includes an understanding of what makes the learning of specific topics easy or difficult. The conceptions and preconceptions that learners of different ages and backgrounds bring with them influences the rate at which they learn. Herr further comments that teachers need the knowledge of the strategies most likely to be fruitful in reorganising the understanding of learners, because those learners are unlikely to appear before them as blank slates. Therefore pedagogical content knowledge is an accumulation of common elements that include knowledge of subject matter, knowledge of learners and possible misconceptions, knowledge of curricula and knowledge of general pedagogy. Pedagogical content knowledge is knowing what, when, why and how to teach using a reservoir of knowledge of good teaching practice and experience. Agreeing with Herr (2007), Shulman (1987) stated that pedagogical content knowledge must include the knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purpose and values, and their philosophical and historical bases. Assigning knowledge to specific categories to pedagogical content knowledge is easier to accomplish in theory than in practice. The conceptual confusion associated with the term pedagogical content knowledge is connected to the problems that are associated to its development.

Gess-Newsome and Lederman (1999:2) conceptualise the nature of pedagogical content knowledge by developing two distinct polar models. These are the integrative model and the transformative model. In the integrative model, pedagogical content knowledge does not exist as a separate category of knowledge, but is seen as the overlap between knowledge of subject matter, pedagogy and context. In the transformative model, the three domains of teacher knowledge are transformed into pedagogic content knowledge, which then constitutes the only form of knowledge that affects teaching practise. The transformative model seems more aligned to Shulman's idea of pedagogical content knowledge and pedagogical content knowledge appear to be residing somewhere between the two models.

Smith and Neale (1989) described pedagogical content knowledge as having three components: knowledge of typical learner errors, knowledge of particular teaching strategies, and knowledge of content elaboration. They stated that "...many of these kinds of teaching knowledge would be in simultaneous use during teaching and that their integration would contribute to the complexity of teaching" (Smith & Neale, 1989:4). Smith and Neale believed that the integration of the components was vital to effective science teaching.

The differences discussed above on the nature of pedagogical content knowledge, if upheld, have important implications on how the education of future teachers should be organised and what effect the differences are likely to have on the knowledge of teachers. It also raises a number of conceptual challenges implicated in the strong differences in positions taken by experts of professional development. A summary of different aspects of pedagogical content knowledge outlined in this study by different researchers is given below in Diagram 2.5.2.

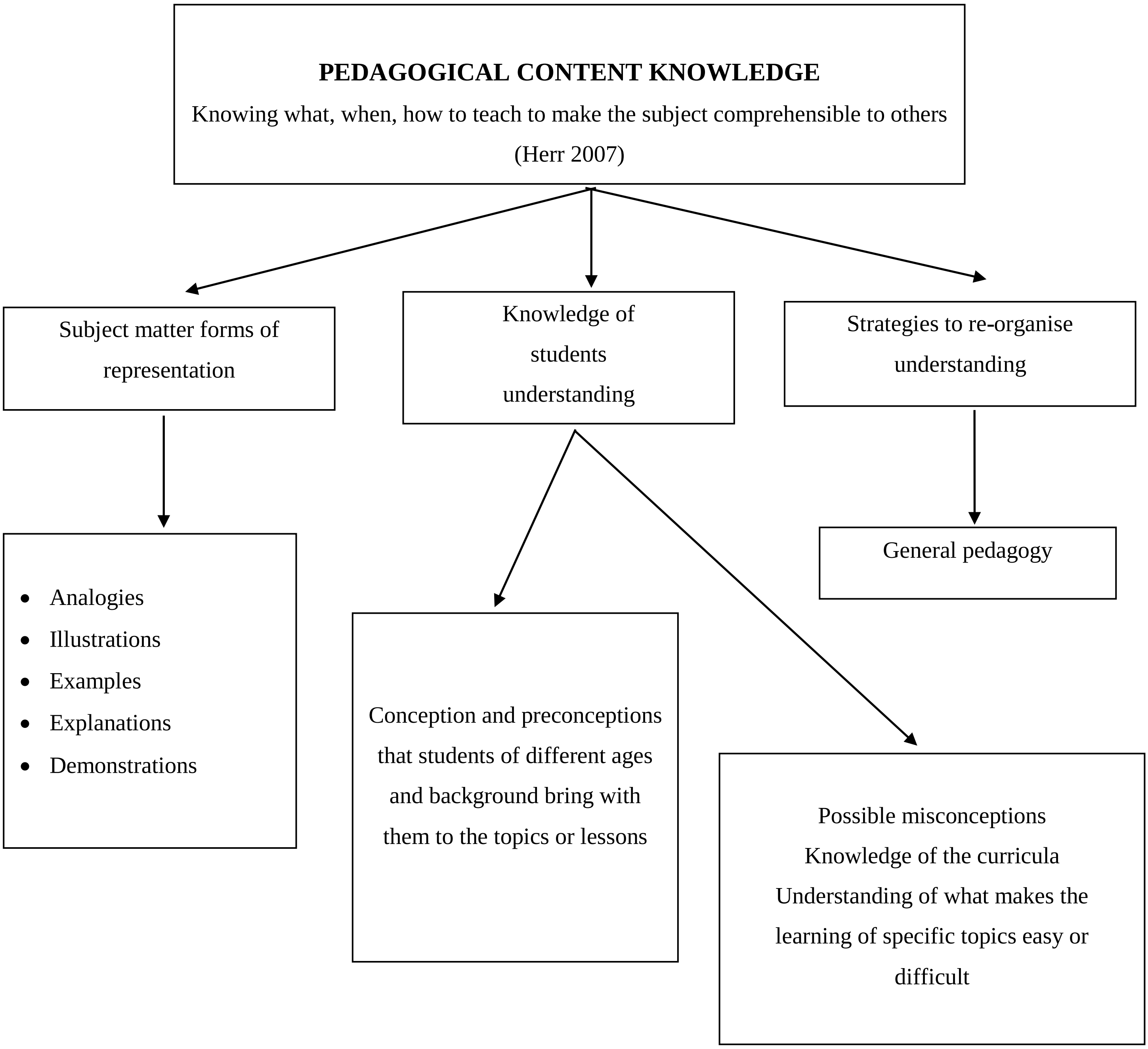


Diagram 2.5.2: Summary of different aspects of the pedagogical content knowledge outlined in this study

2.5.3 Pedagogical content knowledge in the teaching of statistics

High-quality teacher education and development is crucial for improving educational delivery in South Africa. Its primary aim is to improve the quality of teacher education and development, in order to improve the quality of teachers and teaching. The main task of the secondary school statistics teacher is to help learners to understand statistics. Shulman (1986; 1987) advised that teachers make use of pedagogical content knowledge to be able to teach particular content like bi-variate data in ways that promote understanding.

Shulman(1986) originally had a lot of questions regarding how teachers manage their classroom, organise activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and judge general learner understanding. These included questions about the content of the lessons taught, the questions asked and the explanations offered. These questions arose from Shulman's perspective of teacher development and teacher education. Where do teachers' explanations come from? How do teachers decide what to teach, how to represent it, how to question learners about it and how to deal with problems of misunderstanding? He was disappointed that these questions had been the focus of the cognitive psychology of learning and not education. Shulman later redressed these imbalances by focusing on questions about teacher knowledge. The questions included:

- What are the sources of teacher knowledge?
- What does a teacher know and when did he come to know it?
- How is new knowledge acquired, old knowledge retrieved and both combined to form a new knowledge base?

Shulman (1987) also focused on how the successful college learner transforms his/her expertise in a subject into a form that secondary school learners can comprehend. His major question was how learning for teaching occurs. He conducted regular interviews with the teachers, asking them to read and comment on materials related to the subjects they teach, and he observed their teaching after having engaged them in a planning interview. It is realised that since the 1990s, Shulman always thought about great ideas but he never implemented them. This study attempts to close this gap (of implementation) by developing pedagogical content knowledge for Grade 11 and 12 secondary school teachers during professional development of Grade 11 and 12 statistics teachers using a pedagogical content knowledge framework.

Shulman's (1986) notion of pedagogical content knowledge seem to resolve the question of what it is that successful teachers know in order to teach in ways that achieve learner understanding. However, the concept itself and its relationship to other fields of teacher knowledge is much debated in the literature (Grossman, 1990; Lederman & Gess-Newsome, 1992; Cochran et al., 1991; Ebert, 1993). This study is more interested in finding ways of helping in-service teachers improve their practice. Therefore, instead of exploring and evaluating pedagogical content knowledge *per se*, the study has used it as a means of thinking about and exploring the knowledge that successful teachers have on how to teach statistics to learners in ways that promote understanding. However, Shulman (1987) stressed that pedagogical content knowledge builds on other forms of professional knowledge, and is therefore a critical and perhaps even the paramount, constitutive element in the knowledge base of teaching. The purpose in this study is to document this so that it might enhance the statistics teaching practice of Grade 11and 12 secondary-school teachers.

2.5.4 Rationale for pedagogical content knowledge as the knowledge base for statistics teachers

Debate about teacher knowledge and its connections to learner learning has had a long history. At one level, anecdotal comments from secondary school learners have often lamented the fact that their teachers have had the mathematics background ("they knew their subject"), but did not know how to get it across, in a way that contributed to the development of learners' understanding. For example, contemporary research literature recognises that effective teaching is dependent on teacher knowledge (Burgess, 2007). The research literature includes examples of both positive outcomes for learners arising from strong teacher knowledge, and negative outcomes resulting from inadequate and/or inappropriate teacher knowledge. Anthony and Walshaw (2007) summarise an extensive range of research literature supporting the importance of various types of teacher knowledge in relation to learners' development of understanding, the establishment of communities of effective mathematical practice, and the implementation of effective pedagogy. Some of that literature found negative outcomes in relation to classroom discourse and learners' learning, stemming from teachers' inadequate use of particular categories of knowledge. I can therefore argue that no matter how committed one is to caring for learners, to taking learners' ideas seriously, and to helping learners develop robust understandings, none of these tasks of teaching is possible without possessing relevant teacher knowledge. Without the appropriate knowledge, teachers will not be in a position to deal with the day-to-day, recurrent tasks of teaching, and as such, will not cater for the learning needs of their diverse learners.

Shulman (1987) recommended pedagogical content knowledge as the "knowledge base for teaching". Pedagogical content knowledge is unique to teachers and separates, for example, a statistics teacher from a statistician. Teachers differ from statisticians, biologists, historians, writers, or educational researchers, not necessarily in the quality or quantity of their subject matter knowledge, but in how that knowledge is organised and used for teaching. For example, experienced statistics teachers' knowledge of statistics is structured from a teaching perspective and is used as a basis for helping learners to understand specific concepts in statistics. A statistician's knowledge, on the other hand, is structured from a research perspective and is used as a basis for the construction of new knowledge in the field (Cochran et al., 1991:5). Pedagogical content knowledge is therefore an appropriate framework for the design of teachers' education programmes (Gess-Newsome & Lederman, 1999). It is recommended that courses focusing on pedagogical content knowledge should include the theoretical underpinnings of content and their translation of it into teaching, a model of interdisciplinary teaching and linkages between content and pedagogy.

Hill et al. (2004) developed an assessment tool to measure these aspects of teacher knowledge in the domain of number, number operations and algebra and to look for any relationship between teacher knowledge and learner achievement. Although their research was conducted with a limited range of mathematical content, these classifications of teacher knowledge are seen as potentially useful in relation to the teaching of statistics.

Magnusson, Borko and Krajcik (1994) present a strong case for the existence of pedagogical content knowledge as a separate and unique domain of knowledge for teachers. They present a model in which the pedagogical content knowledge for science teaching consists of five aspects or components, orientations toward teaching science, knowledge of science curricula, knowledge of learners' understanding of science, knowledge of assessment in science and knowledge of specific and topic- specific strategies. Acknowledging that these components may interact in very complex ways, they claim that effective teachers need to develop knowledge with respect to the aspects of pedagogical content knowledge and with respect to all the topics they teach. Since teacher-training programmes can never completely address all the components of pedagogical content knowledge, pedagogical content knowledge should be an important element of in-service activities of teachers. Magnusson et al. integrated subject matter knowledge, general pedagogical knowledge and knowledge of context for pedagogical content knowledge, to design a model for teachers' programmes. It assisted pre-service teachers in developing a framework for thinking about and evaluating their teaching by considering how they portrayed subject concepts, responding to learners and anticipated contextual issues. They further conceptualise pedagogic content knowledge as the result of a transformation of knowledge from other domains of teacher knowledge and is therefore in agreement to Gess-Newsome et al. (1999) transformative model. They perceive pedagogical content knowledge as a separate domain of teacher knowledge, which exists alongside other domains, such as pedagogical knowledge and beliefs.

An additional problem prevalent in the pedagogical content knowledge research is the abundance of research on elementary teachers rather than secondary mathematics teachers. Even in studies examining both groups (Ball, 1990), it is difficult to differentiate the conclusions drawn for the two different groups. The transfer of conclusions regarding knowledge and practice from elementary to secondary teachers is problematic because the groups are likely to have different knowledge bases due to differences in preparation, training, and practice.

In his presidential address to the American Educational Research Association, Shulman (1986) coined the term pedagogical content knowledge as a “missing paradigm”. He presented pedagogical content knowledge as a specific form of knowledge for teaching that refers to the transformation of subject matter knowledge in the context of facilitating learner understanding. Teachers need this type of knowledge to structure the content of their lessons, to choose or develop specific representations or analogies, to understand and anticipate particular preconceptions or learning difficulties of their learners (Shulman, 1986:7). Rowan et al. (2001:2) also present a strong case for the existence of pedagogical content knowledge as a separate and unique domain of knowledge for teachers. Therefore, pedagogical content knowledge builds upon but is different from teachers’ subject matter knowledge or knowledge of general principles of pedagogy. A summary is given in the diagram below of how pedagogical content knowledge formulates the subject statistics in order to make it comprehensible to others.

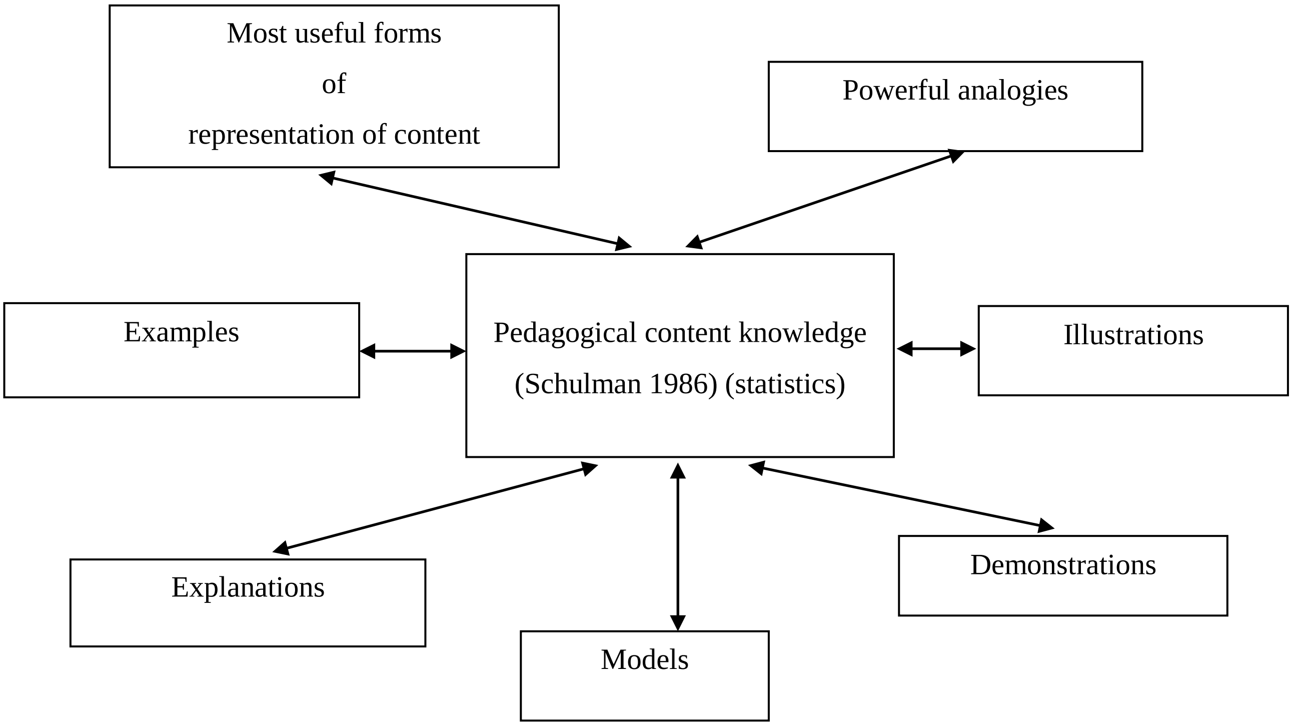


Diagram 2.5.4: Formulating the subject to make it comprehensible to others

2.5.5 Problems that abound with pedagogical content knowledge

Irrespective of the great appreciation offered to pedagogical content knowledge, problems with its conceptualisation have been encountered. There exist new conceptions of what pedagogical content knowledge is. Seven assertions that comprise the new conceptualisations are presented by Maher (2005:1):

- Pedagogical content knowledge represents personal and private knowledge.
- Pedagogical content knowledge is a collection of basic units called teacher pedagogical constructions.
- Teacher pedagogical constructions result mainly from planning, but also from the interactive and post-active phases of teaching.
- Pedagogical constructions result from an inventive process that is influenced by the interaction of knowledge and beliefs from different categories.
- Pedagogical constructions constitute both a generalised event-base and a story-base kind of memory.
- Pedagogical constructions are topic specific.
- Pedagogical constructions are (or should ideally be) labelled in multiple interesting ways that connect them to other categories and sub-categories of teacher knowledge and beliefs.

Maher therefore views the pedagogical content knowledge as neither a subcategory of subject matter (subject knowledge for teaching) nor as a generic form of knowledge. He presents a view of the pedagogical content knowledge as a collection of teacher professional constructions, as a form of knowledge that preserves the planning and wisdom of practice that the teacher acquires when repeatedly teaching a certain topic. Viewing pedagogical content knowledge as a collection of teacher pedagogical constructions, more precisely defining it, clarifying its relations to other knowledge and beliefs entities, and speculating about its development should facilitate future investigations of pedagogical content knowledge.

Maher's (2005) conceptualisation is that through teaching experience one will come to make the subject comprehensible to others, understand learners' thinking in a particular subject and identify strategies that help alleviate learning problems. The problem is what will happen to instruction while the required experience is gained. How do we define the length of time required for one to gain the required experience? Teachers are different and have different environmental and cultural influences that determine when they are able to gain meaningful experience. Knowledge is tacit and does not translate easily into direct instruction or formalisation (Munby, Russel & Martin, 2001). The nature, path, and pace of advance from novice to expert teacher may vary substantially within countries, within schools or within different teachers. Teachers gain experience during different periods of their lives. It is therefore worthwhile to develop pedagogical content knowledge to the teachers very early in their teaching lives. In conclusion, it is realised that the problems that abound pedagogical content knowledge do not outweigh its advantages, and it is therefore worthwhile to utilise it for teaching.

2.5.6 Summation

Because pedagogical content knowledge is understood in different ways in the literature, it warrants an operational definition in this study that enables the development of pedagogical content knowledge during professional development. Pedagogical content knowledge entails knowledge of how to structure and represent academic content for direct teaching to learners, knowledge of learners' thinking that includes the common conceptions, misconceptions, and difficulties that learners encounter when learning particular content, and knowledge of the specific teaching strategies that can be used to address learners' learning needs in particular classroom circumstances (Shulman, 1987).

The continued interest in pedagogical content knowledge as a knowledge base for teacher preparation has produced a need for a conceptual framework upon which future pedagogical content knowledge development can be based. In the light of the above challenges, a detailed design of a pedagogical content knowledge framework that guides the development of pedagogical content knowledge for secondary school statistics teachers during professional development was designed. The focus of the pedagogical content knowledge framework in this study was therefore to integrate teachers' content knowledge that includes error pattern analysis, their knowledge of their learners as learners, their ability to plan and prepare, and their ability to use learner responses to devise teaching intervention.

2.6 Designing a pedagogical content knowledge framework for Grade 11 and 12 secondary school statistics teachers' professional development

This section clarifies the design of a pedagogical content knowledge framework that guides the development of pedagogical content knowledge of secondary school statistics teachers during professional development. The framework, using bi-variate data, is designed and broadly described, and its potential for use in professional development is discussed. It further describes and discusses the implications of a pedagogical content knowledge framework for statistics education. Conceptual frameworks, against which the pedagogical content knowledge framework for the development of Grade 11 and 12 statistics teachers was developed, are discussed. The goal for the design of the pedagogical content knowledge framework was based on the need to guide teachers during professional development towards being competent and confident in teaching Grade 11 and 12 statistical concepts. Furthermore, it helps teachers to be reflective of their instructional practice in ways that can improve the teaching of statistical concepts. The reflective accounts shared between the statistics teachers and the researcher enabled the continual improvement of the framework.

2.6.1 Models/frameworks of statistics teacher development

Currently, there are few models for statistics teacher development. For example in Pfannkuch's (2005) statistical model, he describes three performance levels used in New Zealand, namely: achievement, merit and excellence. Achievement requires learners to interpret statistical information and answer straightforward questions. For merit, learners must draw inferences justify their answer to questions, and comment on features in the data, whereas for excellence the requirement is evaluation of a statistical process (Pfannkuch, 2005). An assessment model for improving learner learning of statistics was also developed by Rumsey (2002) and Garfield and Gal (2004). Rumsey's statistics model considers an assessment framework based on three learning outcomes, which are literacy, reasoning and thinking. Literacy involves identifying, describing, rephrasing, translating, interpreting, and reading while reasoning is the why, how and explaining technique. Thinking involves applying, critiquing, evaluating, and generating thinking (Pfannkuch, 2005). An assessment model for improving learner learning of statistics (Kasonga & Corbertt, 2008) that adds more to Rumsey's statistics model was also identified. However, these and other identified models failed to accurately address and outline the role of pedagogical content knowledge in the teaching and learning of statistics.

Teacher knowledge frameworks from the mathematics education domain are also inadequate for examining teacher knowledge for statistics because of the differences between statistics and mathematics. Many professional developments in South Africa have been offered, to improve the understanding and teaching of statistics, but inadequacies have been observed in the professional development programmes. Mathematics education, which in most cases includes statistics, lacks studies that have developed, validated and published measures to assess many programmes designed to improve teacher knowledge and how this knowledge relates to learner achievement (Hill et al. 2004). Hill, Ball and Shilling (2008) advise that researchers should focus on how teachers use their knowledge rather than on how much knowledge they have. By focusing on how teachers apply knowledge in different situations, researchers may learn how knowledge affects teachers' behaviours. Many of the studies focusing on the use of teacher knowledge in practice attempt to examine several aspects of teacher practice simultaneously: preparation, instruction, assessment and reflection (Kahan et al., 2002; Tirosh, Even & Robinson, 1998). These studies do not point to a successful outcome. Professional development cannot have meaningful effects on teachers if proper guidance in the form of models or frameworks in pedagogical content knowledge are not offered during these endeavours. It is against this background that this section therefore clarifies the design of a pedagogical content knowledge framework using a section of statistics, bi-variate data, which guides the development of pedagogical content knowledge of secondary school statistics teachers during professional development.

2.6.2 Insights that guided the design of the pedagogical content knowledge framework

The conceptual framework

- Pedagogical content knowledge (Shulman (1986, 1987).
- Extensive literature on different interpretations of pedagogical content knowledge.

The broad teacher knowledge literature base and the specifics of teaching and learning in statistics

- Powerful practices that were developed by Carpenter and Romberg (2004) for the teachers of science and mathematics.
- Lerman's (1991) critical incidence that elicits a reaction of surprise, and realisation that there may well be something important in the incident from the point of view of the learner, which results in the learning of the teacher.
- Personal experience as a teacher.

The models of *pedagogical content knowledge* adapted for the *pedagogical content knowledge* framework

- The taxonomies by Veal and MaKinster (1999).
- Loghran et al. (2003) pedagogical content knowledge representations (CoRe) and Pedagogical and Professional-experience Repertoires, (PaP-eRs).

Freudenthal (1991) and Schön's (1983) research cycles during the design research

- Research cycles in which thought experiments and teaching experiments alternate to provide "feed-forward" for the next thought experiments and teaching experiments.
- Research cycles in which the process of the researcher's thinking should be reported, to ensure the traceability of this development for others.
- Donald Schön's (1983) "The Reflective Practitioner Model" (reflection in action) capitalising on Freudenthal's work.

The learning environment

The problem-centred context was selected as the vehicle to drive the design and implementation of the developmental study, as it encompassed all aspects of the learning trajectory.

2.6.2.1 Using insights to guide the design of the pedagogical content knowledge framework

The design of the framework using bi-variate data was achieved through development research. The extensive literature on the different interpretations of pedagogical content knowledge gave a very interesting start to integrating and intertwining of the necessary requirements. Loghran et al. (2003; 2004) and Loughran, Milroy, Berry, Gunstone and Mulhall (2001b) started by embracing the knowledge of content and learners plus the knowledge of content and teaching, which make up the pedagogical content knowledge. The alignment of the framework to pedagogical content knowledge was also partly adapted from Loghran et al. (2003, 2004) two different but complementary formats, the CoRe, which provides an overview of the particular content taught when teaching a topic, and PaP-eRs, an account of the practice intended to illuminate aspects of the CoRe in a particular classroom context. The representations refer to the teaching of a particular topic, bi-variate data in this case, to a group of learners. Taken as a whole, Loghran et al. (2001; 2003; 2004) complementary formats provided the link of the how, why and what of the content to be taught to the relevant learners. A list of pedagogical content knowledge attributes were also generated from Loghran et al. (2003, 2004) representations. Pedagogical content knowledge attributes based upon the various attributes and characteristics of pedagogical content knowledge were presented for bi-variate data in the following manner:

- A list of all previously described pedagogical content knowledge attributes was generated from Loghran et al. (2003; 2004) representations.
- From this list, the most prevalent attributes were determined.

Veal and MaKinster's (1999) taxonomies were also adapted for the development of the framework. The general taxonomy of pedagogical content knowledge classifies the different types previously mentioned in the literature and presents an additional category of pedagogical content knowledge that provides a broader foundation for future research. The categories include general, domain-specific, and topic-specific pedagogical content knowledge. These differences legitimate the need for developing topic-specific pedagogical content knowledge as an instructional paradigm for teachers (Veal & MaKinster, 1999). Restructuring and renaming these categories serves to clarify the use of pedagogical content knowledge in educational research. The three most predominant and recurring characteristics in these taxonomies were knowledge of learners, knowledge of content, and knowledge of instructional strategies (pedagogy). One significant aspect of the taxonomy is its pedagogical content knowledge attributes. Veal and MaKinster (1999) produced taxonomies that contributed to an understanding of the attributes that were considered most important in the development of pedagogical content knowledge for the teachers of statistics. One important attribute teachers need in developing pedagogical content knowledge is a strong and thorough knowledge of their learners. Only after a teacher understands or realises the importance of the learner component of teaching, can the other attributes be learnt or developed. Another significant aspect of the taxonomy of pedagogical content knowledge attributes is its lowered recognition of pedagogical knowledge. The knowledge of the learner component has more significance compared to pedagogical knowledge (Veal & MaKinster, 1999).

Carpenter and Romberg (2004) produced a book, "*Powerful Practices*" in mathematics (statistics included) and science (research-based practices for teaching and learning). Based on the research of the National Centre for Improving Learner Learning and Achievement in Mathematics (NCISLA), the book was dedicated to the teachers and professional development of teachers who serve the nation's schoolchildren. The centre's work is yielding new visions for learner achievement and professional development programmes that strengthen teachers' content knowledge, knowledge of learners and in-class practices. It shows instruction that develops understanding of mathematics and science, statistics included, by engaging learners in the practices of modelling generalisation, and justification. This vision of powerful practices is consistent with the most prominent recommendations for reforming mathematics and science in South Africa. In "Principle and Standards for School Mathematics", the National Council of Teachers of Mathematics (NCTM, 2000:67) recommended that "Instructional programmes from pre-kindergarten through grade 12 should enable all learners to … use representations to model and interpret physical, social and mathematical phenomena".

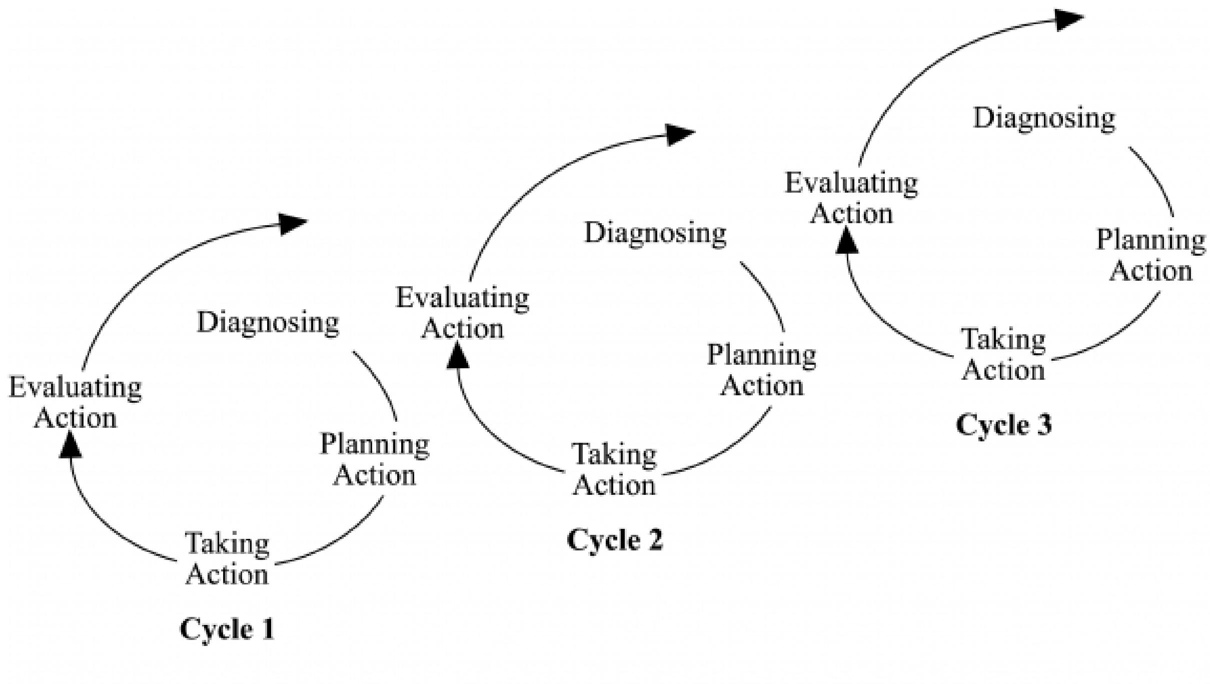
The National Research Council (1996) identified evidence, models and explanation as unifying concepts and processes that provide learners with powerful ideas to help them understand the natural world. The term modelling according to Tall (1991) refers to finding a mathematical representation for a non-mathematical object or process. There are four basic types of models: models that "look like", models that "function like", descriptive models, and explanatory or causal models (Carpenter & Romberg, 2004:6). This study will use descriptive models to illustrate generalisation and justification in statistics.

Lerman's (1994) critical incidence also made a major contribution to the design of the pedagogical content knowledge framework. Lerman was interested in the complexity and richness of classroom interactions out of which he believed many research questions would originate. Lerman described critical incidents as ones that could provide insight into classroom learning and the role of the teacher, challenge our opinions, beliefs and notions of what learning and teaching mathematics are about, as well as offer a kind of shock or surprise to the observer or participant. In the light of this, critical incidence can be conceived from teaching aspects, because the incidents might invoke the conflicts and challenges of practising teachers' beliefs and values, as well as their thinking about professional identities in order to make the best teaching decisions. Critical incidence, anecdotes that elicit a reaction of surprise, and realisation that there may be something important in the incident from the point of view of the learning of the teacher are initiated within the teacher (Lerman, 1994). Identifying critical incidence and engaging in a programme of immediate correction is a rich way of learning about one's teaching and can be used throughout one's professional teaching life to continue learning.

The learning environment based on constructivist perspectives, was the problem-centred context, which was selected as the vehicles to drive the design and implementation of the developmental study as they encompassed all aspects of the learning trajectory. The implication for this is that teaching for understanding entails teachers developing knowledge about statistics and learners that enable them to make, curricular decisions and instructional decisions.

2.6.3 Adapting development research (reflective) cycles into the design of the framework

The integration of the work from Schön (1990) and Freudenthal's (1991) development research cyclic process provides insight into the reflective practitioner model in action. The cyclic character of the design/development research consists of research cycles in which thought experiments and teaching experiments alternate. The cycles lead to a cumulative effect of small steps, in which teaching experiments provide "feed-forward" for the next thought experiments and teaching experiments (Freudenthal, 1991). A macro-cycle of design research consists of three phases: the preliminary design phase (diagnosing and planning action), the teaching experiment phase (taking action), and the phase of retrospective analysis (evaluating action). In the last-mentioned phase, the reflection captures the development of the insights of the researcher. As a result, new theories or new hypotheses or new instructional activities emerge, that form the feed-forward for the next research cycle that may have a different character, according to new insights and hypotheses. The Reflective Practitioner Model is therefore essentially an approach towards decision-making and problem solving. Schön (1983) found that when effective practitioners were faced with a problem in their practice, they worked through it instinctively drew on previous similar experiences. They tried and tested out various possible solutions until they resolved the issue. The practitioner allows himself to experience surprise, puzzlement, or confusion in a situation that he/she finds uncertain or unique. The process of this thinking was, to ensure the traceability of this development for others. An illustration of this thinking was adapted for display in the following cycle diagram.



Source: Coghlan and Brannick (2001) p. 19

Diagram 2.6.3: The Development Research ***Cycles***

2.6.4 Results

Using developmental research, a pedagogical content knowledge framework was constructed. The pedagogical content knowledge framework was constructed to integrate all issues pertaining to pedagogical content knowledge.

Stages of the pedagogical content knowledge framework	Outcomes (Key learnings within the stages)
1. Previous/present knowledge	• Exposure to previous and present knowledge that is related to bi-variate data. • To provide an in-depth understanding of types of data. • Exposure to statistical language and terms that involve bi-variate data and the interrelationship between statistics and mathematics. • Group presentations on flipcharts and reflection.
2. Bringing teachers into context	• Bi-variate data is made available for informal discussions and analysis representation. • Availability of realistic data through the media • Applying the product moment correlation co-efficient (-1to1). • Group presentations on flipcharts
3. Statisticatisation (finding meaning from bi-variate data e.g. modelling).	• To avail teachers with relevant tasks and activities that involve bi-variate data from the media • Meta-cognition (Deciding what to do with given bi-variate data sets • Deducting from data analysis • Group presentations on flipcharts
4. Realistic/case-based problem posing	• Putting teachers into the statistician's shoes • Putting teachers into the headmaster or Department of Education's shoes. • Group presentations on flipcharts. • Teachers' individual response to reflection later in the day
5. Assessment of pedagogical content knowledge (meta-cognition)	• Putting teachers into the learner's shoes. • Putting teachers as teachers into their classrooms. • Meta-cognition (The teacher as the assessor) • Group presentations on flipcharts. • Group reflection

Diagram 2.6.4: The pedagogical content knowledge framework

2.6.4.1 Description of the five stages in the pedagogical content knowledge framework

The five stages of the framework are previous/present knowledge, bringing teachers into context, statisticatisation, realistic/case-based problem posing, assessment and error pattern analysis. The five stages were strictly adapted to the needs of pedagogical content knowledge in a problem-centred approach and each stage was designed and analysed uniquely. Each stage lasted one week and the series took place over five spaced-out weeks. A unique facilitation/teaching plan was prepared for each stage, accompanied by instructional activities. During the design phase, products of the framework stages were presented to colleagues, teachers and a workshop. This led to feedback that forced the researcher to be explicit about goals and aims of the pedagogical content knowledge framework, and it provided opportunities for improving the framework.

Stage 1: Previous/present knowledge

The researcher sets the stage for learning by finding out what teachers already know, and then connecting new ideas to teachers' existing knowledge base. Using a variety of instructional strategies, the researcher guides teachers from the known to the unknown, from familiar territory to new concepts. Cues, questions, and advance organisers are among the strategies that are used to set the stage for learning. These strategies help teachers focus on what they are about to learn. Classroom and non-classroom experiences, relevant to the intended objectives were achieved through organised group activities. Facilitation approaches that optimise learning are those that capture and hold learners' attention throughout the facilitation. Such approaches also relate new content, skills, and abilities to what is already familiar to the learners. At this stage of the framework, it is often difficult to assess a single concept in isolation of other concepts and skills. For example, it may not be possible to assess understanding of bi-variate data without understanding the concepts of direct and indirect variation and the concept of linear relationships.

Stage 2: Bringing teachers into context

Teachers are engaged in the practices of modelling, generalisation, and justification through the availability of already-made graphs that need to be analysed. They learn to pose questions, invent models to address their questions, revise their models in the light of data and go on to pose new and more profound questions. Rich examples of what modelling can yield in classrooms can emerge from this work. This also section emphasises the relationship between the abstractness of bi-variate data and its relationship to real-world phenomena. Teachers were provided with direct experiences of household and time series data so that the subject of the inquiry was concretely visible to them.

Stage 3: Statisticatisation (finding meaning from realistic/case based bi-variate data)

The word "statisticatisation: is a borrowed word from "mathematisation" by Freudenthal (1991). In this study, in a broad sense, it entails organising and playing around with data from statistics as a subject, data from other subjects in the curriculum and data from the real world. Symbols emerge in the process of organising the available data. The organizing activity itself is central to its conception. Strategies within these characteristics and process include generalising, justifying modelling and then symbolising (developing standard procedures and notations). Viewed from this angle, statisticatising data from statistics and statisticatising data from reality share the same characteristics. Moreover, statisticatising data from reality also familiarises the learners with a mathematical approach to everyday life situations. We may also refer here to the statistical activity of "looking for problems" (which was mentioned by Freudenthal (1991), which implies a statistical attitude that encompasses knowledge of the possibilities and the limitations of a statistical approach, in other words, knowing when a statistical approach is appropriate and when it is not.

Stage 4: Realistic/case-based problem posing

This stage provided bi-variate data experiences that encouraged exploration by placing teachers into the researcher's shoes. Each teacher was also given a chance to make decisions through:

- Reading data from the dot plot representation.
- Interpreting the dot plots based on previous knowledge from mathematics.
- Making predictions based on these representations.

Teachers are led to see that there are often different ways to solve a statistical problem, and recognise that people may come to different conclusions based on the same data, if they have different assumptions and use different methods of analysis.

Stage 5: Assessment of pedagogical content knowledge (metacognition)

Case-based problems of bi-variate data were posed at this stage of the framework, as they were the key to exploration, reflection and assessment and provided a deep and thorough understanding of the topic. People live in a world where ideas are changing fast and therefore being able to facilitate problems that are case-based can bridge this gap. This stage put the teachers into four roles:

- As a teacher to decide what to teach, extract the big ideas, create purposes and outcomes and then produce a teaching plan on how to facilitate this lesson.
- As a learner to respond and prepare a memo as he or she would have liked his/her learners to do.
- As a reflexive reflector using, Schön (1999) and Freudenthal's (1991) ideas in the cycle, to reflect upon the process of the designing of the pedagogical content knowledge framework.
- As an item writer (examination setter and analyser) to decide on what type of questions can be useful from collected data.

Authentic assessment is applied and is a method of obtaining information about learners' understanding (e.g. error pattern analysis) in a context that reflects realistic situations, and that challenges learners to use what they have learned in class in an authentic context (Archbald & Newmann, 1988). This reflected Shulman's (1987b) concern that knowledge of learners and their characteristics, as well as teachers' levels of content knowledge, must be addressed if the pedagogical content knowledge is to be useful and meaningful.

Stage 5.1: Freudenthal's (1991) cycle/reflection

At the end of each stage in the pedagogical content knowledge framework, reflection, consciously or unconsciously, was an important aspect of the development. Teachers were required to explain and justify their methods and understandings to each other. Explaining their methods to one another served the important purpose to induce teachers to reflect upon their methods, upon what they had done to solve problems and to perform calculations. What is involved is not about particular methods, but about attitudes towards teaching. It implies an awareness of the learners and the learners' world.

2.7 Rationale for using the pedagogical content knowledge framework

The pedagogical content knowledge framework is an appropriate framework for the design of teachers' education programmes (Gess-Newsome & Lederman, 1999). Professional development, focusing on pedagogical content knowledge, should include the theoretical underpinnings of content and the translation of it into teaching. The goal for the design of the pedagogical content knowledge framework was based on the need to use it to anchor preliminary discussions about the makeup of a model for statistics teacher preparation and to serve the uniqueness of pedagogical content knowledge. It was used as a foundation for future professional development of secondary school teachers of statistics and therefore provides a model for statistics teacher preparation. The pedagogical content knowledge framework elaborates on the processes by which pedagogical content knowledge is developed by its ability to guide its development through a very intensive educational setting.

The framework highlighted the uniqueness of pedagogical content knowledge through focusing on developing a topic-specific pedagogical content knowledge for statistics teachers. By focusing on topic-specific examples, secondary school statistics teachers can develop specific strategies that translate to the effective use of exemplary models of statistics teaching within topics. For example, secondary school statistics education programmes could focus on developing topic-specific pedagogical content knowledge to prospective teachers. Many prospective statistics teachers might know their content well, but they may not have learned how to transform or translate that knowledge into meaningful units for instruction. The effective use of exemplary models of statistics teaching within topics can later be transferred to another topic. The model then further allows the integration of technicalities and complexities of the theory and practice of teaching statistics using pedagogical content knowledge.

Pedagogical content knowledge provided a process by which the researcher gets insight from literature and personal experience to create a product by designing, testing and revising several prototypes (van den Akker, 1999). The insights that guided the design of the pedagogical content knowledge framework included the conceptual framework by Shulman (1987), development models from Carpenter and Romberg (2004), Lerman and Scott-Hodgetts (1991), pedagogical content knowledge taxonomies from Veal and MaKinster (2008), pedagogical content knowledge representations and PaP-eRs by Loghran et al.

(2003), literature on reflective practice from Schön (1999) and Freudenthal (1991). The pedagogical content knowledge framework helped teachers to be reflective of their instructional practice in ways that could improve the teaching of statistics concepts. Reflective thinking, with or without colleagues, encourages teachers to better their statistics instruction and to accept the challenge of teaching with humility. The information gathered within the framework was found to be easily adaptable for the decisions on the design of the framework as well as the later analysis of the whole development of pedagogical content knowledge. By focusing on a specific statistics topic, the pedagogical content knowledge framework focused on developing specific strategies. These strategies are applied to other topics and domains of statistics based upon the curriculum backgrounds. This framework brings together learning outcomes, instruction and assessment and has the potential to influence the way teachers or learners learn statistics positively. The framework requires advance planning, and success is likely to be achieved only after repeated and consistent application in order to improve the effectiveness of the model. Teachers can also use this model to teach statistics in their classrooms and in communities of practice. As with many other models of teacher knowledge, this framework has limitations with regard to the indistinct boundaries among the various categories of pedagogical content knowledge.

2.8 The problem-centred context

The problem-centred context is an environment in which the development of pedagogical content knowledge was carried out using the problem-centred approach. The problem-centred approach was used as the domain specific theory to guide the design and implementation of pedagogical content knowledge. During the development of pedagogical content knowledge, the researcher serves as a facilitator, circulating among the groups to provide support and guidance. The researcher clarifies task directions, asks probing questions, gives hints, offers encouragement, and ensures that all teachers are participating. Bi-variate data becomes a means of making sense of a wide variety of situations and problems, and, as such, teachers come to see themselves as capable of discovering bi-variate data on their own.

During the development, teachers should feel comfortable with the various innovative instructional strategies and feel that they are part of this pedagogical content knowledge development. They should not take the development to be only to their advantage, but to consider themselves as future coordinators of this development. The researcher ensures that teachers bring to the problems not only the skills and ideas that they own, but also guides them in ensuring that problems have multiple-entry points, planning differentiated tasks, using heterogeneous groupings and listening carefully to the learners (Van de Walle, 2004:84). A good learning environment in which learners work well is realised through the availability of materials and technology to use, as well as the acceptable geographical settings as specified in a problem-centred approach.

2.9 Conclusion

This chapter gave an account of other studies that have relevance to the present study. The study argues that pedagogical content knowledge provides a particularly useful knowledge base for secondary-school statistics teachers. In order to achieve the above, a thorough literature review of the challenges that come with the teaching of secondary school statistics bi-variate data in particular was carried out. They included the need to recognise teacher knowledge and pedagogical content knowledge as special and important in statistics, and the identification of reliable professional developments in the area of statistics. Using a well-designed and peer-reviewed pedagogical content knowledge framework (AMESA, 2004; SAARMSTE, 2009- 2012; ISTE, 2008), the study developed pedagogical content knowledge qualitatively through development research to enable teachers to effectively teach statistics.

Teaching is as an art. Any art goes by the rules, the rules of the profession in a particular subject area (statistics). In this study, the rules come from pedagogical content knowledge. This is a knowledge that encompasses all the other knowledge sets and hence the advocacy for it. Pedagogical content knowledge can be initiated during professional development in which the conventional view of professional development for teachers needs to shift from technical training for specific skills to opportunities for intellectual professional growth.

CHAPTER 3

METHODOLOGICAL ORGANISATION OF THE STUDY

3.1 Introduction

This chapter presents the research methodology and steps taken in analysing and describing the experiences of Grade 11 and 12 teachers during the development of pedagogical content knowledge using the pedagogical content knowledge framework and the concept of bi-variate data. Methodological aspects of this research are designed to address the following objectives:

- To analyse if Grade 11 and 12 statistics teachers in South Africa are competent, confident and prepared for the teaching of secondary school statistics.
- To analyse and explore the special characteristics of pedagogical content knowledge that contributes to it being a better teacher knowledge for secondary school statistics teachers.
- To design and develop a new pedagogical content knowledge framework which will guide and inform the development of pedagogical content knowledge for secondary school statistic teachers.
- To use the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences.
- To analyse the challenges in the implementation of the new pedagogical content knowledge framework during the development of pedagogical content knowledge for Grade 11 and 12 secondary school statistics teachers.

In order to provide answers to the above research objectives, this chapter presents the description and explanation of the research paradigm/method in section 3.2. The population and sampling procedures are given in section 3.3, while the selection of the section of bi-variate data, from statistics, to be used for the development is addressed in section 3.3.4. Section 3.4 describes the research design in four phases followed in section 3.5. by the description of how the research instruments were developed and administered. The validity and reliability of the research methodology and instruments are explained in section 3.6. Data collection and analysis procedures are described in section 3.7. Limitations of the study described in 3.8 are followed by ethical issues clarified in section 3.9, before concluding the chapter in section 3.8. The problem-centred approach was selected as the vehicle to drive the design and implementation of the development as it encompassed all the aspects included in the pedagogical content knowledge framework. It elaborates mostly on the qualitative data supported by some quantitative data.

3.2 Research method

Supported by the literature review, this research was qualitative and supported by simple quantitative methods. The research was qualitative in the sense that data established how participants made meaning of a specific phenomenon, pedagogical content knowledge, by analysing their perceptions, attitudes, understanding, knowledge, values, feelings, and experiences in an attempt to approximate their construction of the phenomenon. For example, the use of in-class data gathering, interviews and observations were some of the important data gathering instruments used in this qualitative research. When in-class discussions are properly used, researchers often get better responses from these as opposed to other data gathering instruments (Makina, 2005). Qualitative research is used to gain insight into people's attitudes, behaviours, value systems, concerns, motivations, aspirations, culture or lifestyles (Niewenhuis, 2007). The research was quantitative in the sense that questionnaires were used. When used along with quantitative methods, qualitative research can help us to interpret and better understand the complex reality of a given situation and the implications of quantitative data (De Vos, Strydom, Fouché, & Delport, 2003). In an ideal situation, researchers use both quantitative and qualitative data to provide a more complete picture of the issues being addressed, the target audience and the effectiveness of the programme itself.

Design or developmental research was used as the research methodology. Design research is a combination of both the “design and develop” (D&D) and “knowledge utilisation” (KU) intervention research that falls under applied research in education. Intervention research was born through the collaboration of Thomas and Rothman (1994) in the field of developmental research. Thomas and Rothman described three core endeavours in intervention research: intervention knowledge and development (KD), KU and D&D. These are described as (De Vos et al., 2003):

- Intervention knowledge and development: studies that attempt to understand problem phenomena, undertaken with the objective of developing interventions.
- Knowledge utilisation: research on the process of helping.
- Design and development: studies that systematically design and develop interventions.

Applied research is conducted in a field of common practice and is concerned with the application and development of research-based knowledge about practice (Mcmillan & Schumacher, 2006). Applied research in this case is opposed to basic research produced knowledge that is relevant to producing solutions to general research problems common to the statistics education field. Applied research in statistics education may also have an indirect effect over time by influencing how practitioners think or how practitioners accomplish tasks. Lijnse (1995) clarified the aim of developmental research as understanding and developing good teaching practice, as opposed to the building of grand theories, for example, the understanding of the human mind.

In this study design, research was used in the development of pedagogical content knowledge in bi-variate data for Grade 11 and 12 statistics teachers. There are two key aspects of development research: the cyclic character of design research and the central position of the design of instructional activities Development research provided a process in which the researcher got insight from literature and personal experience to create a product by designing, testing and revising several prototypes (van den Akker, 1999). Other reasons for using the design research methodology was that the research objectives defined in section 1.6 start with "What/Which...", illustrating that research interest was not just in knowing whether pedagogical content knowledge is the best knowledge, but specifically in understanding how it affects teachers during its development. The design of a framework to use during professional development of secondary school statistics teachers was relatively new phenomenon which needed a research design which allows for revising and reflecting on frameworks or models (Gravemeijer, 1994; 1999).

3.3 Population and sampling methods

3.3.1 Population

The population of this study comprised teachers of secondary school statistics from different types of schools in South Africa. De Vos et al. (2003:198) define a population as a set of entities that represent all the measurements of interest to the researcher. Therefore, a population is the total set from which the individuals or units of the study are chosen.

3.3.2 Random selection of teachers and schools used

For the survey, a sample of two hundred Grade 11 and 12 statistics teachers, who were registered for the FDEME8R statistics module in the School of Education at UNISA in 2006, was selected. Therefore, two hundred questionnaires were sent out to the conveniently chosen sample of 200 teachers, 50 of whom responded to the questionnaires. This sample of 200 teachers was convenient because the teachers were already involved in the FDEME8R module and were therefore easy to use for the study. In addition, 130 teachers who were conveniently chosen from an INSET organised by UNISA for the Free State province in 2011 attended the pedagogical content knowledge development sessions. The Free State province had requested UNISA to help with the training of their teachers in a number of subject areas, including statistics. Because the topic of statistics, bi-variate data in particular, is a key subject in the national curriculum, all participants were familiar with this topic, as teachers of mathematics. Twenty teachers participated in the weekly phases of the development of pedagogical content knowledge.

3.3.3 Random selection of bi-variate data in the statistics section

The identification of the Grade 11 and 12 statistics section of bi-variate data used for the research originated from several spaces. The results of the baseline study (section 4.3.1) proved that teachers of secondary school statistics lacked confidence and competence in the teaching of secondary school statistics, bi-variate data in particular. While marking the scripts at the School of Education at UNISA, I noted poor responses in the section of bi-variate data in the FDEME8R statistics module. The section of statistics used for the development was also decided through a Free State province’s Department of Education request for the Grade 11and 12 teachers to be trained in the bi-variate data. There were also general unsubstantiated comments about teachers having difficulty in teaching bi-variate data made during AMESA (2004) and SAARMSTE (2011; 2012) conferences.

3.4 Description of the study’s research design

In this section, the four phases of the study’s research design are outlined. The research design for this study was clarified in the flow chart that follows (see diagram below).

Research design refers to the specification or framework of the procedure to be implemented by the researcher to make valid conclusion about the research. De Vos et al. (2003:138) define a research design as all the decisions made in planning the study, which include sampling, sources and procedures for collecting data, measurement issues and data analysis plans. However, the type of research conducted is always a critical denominator of the design the researcher eventually adopts.

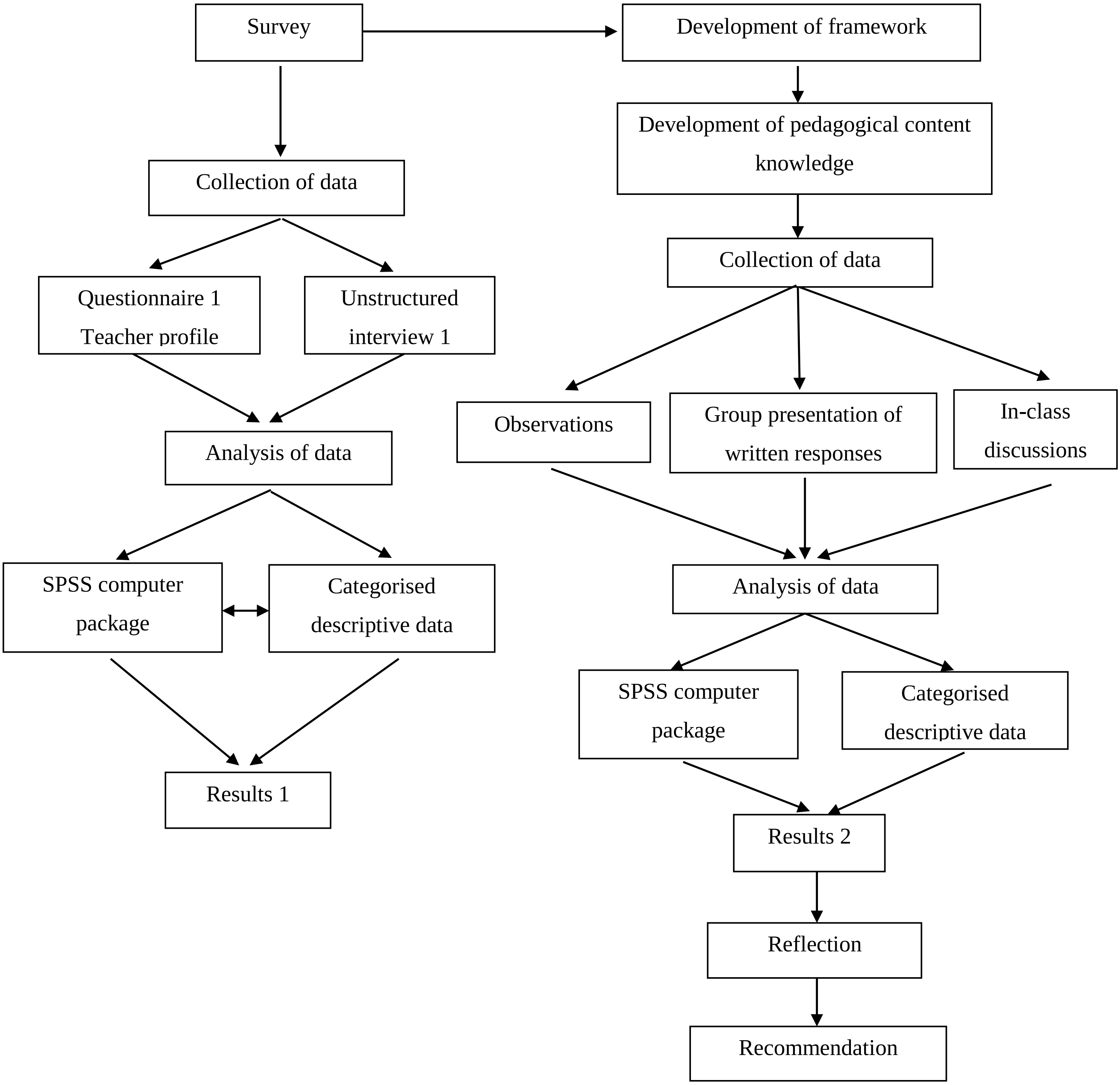


Diagram 3.4: Illustration of the research design

3.4.1 Phase 1: Survey (Appendix 3: Baseline study: Teacher profiling)

Using an adapted teacher questionnaire (Appendix 3: Baseline study: Teacher profiling; Watson, 2001), this stage encompassed a quantitative teacher survey based on probability and statistics. It involved 200 purposefully chosen teachers from South African schools. The survey's main purpose was to check the competence and the confidence that teachers have in the teaching of statistics. Furthermore, the survey needed to clarify the nature of the support given to the teachers by the Department of Education through professional developments in preparation for their teaching of Grade 11 and 12 statistics in South Africa. The main information retrieved from the teacher-profiling questionnaire (Appendix 3) was biographical variables, misunderstanding, subject content knowledge support, pedagogic knowledge, relevant support, material support, confidence, statistics in everyday life and professional development. Following the survey, interviews (Appendix 3: Q52) were carried out with five teachers who had responded to the questionnaire, in order to justify the results of the questionnaires. The audio-taped interviews lasted about 30-45 minutes each.

There were advantages in using a survey as a data-collection tool to gather information about the teachers involved in this study. A survey focuses on information about individuals, collects the opinions of the respondents and is most useful in describing the characteristics of a large population (De Vos et al., 2003). The survey allowed the collection of a large amount of data in a relatively short period. It was less expensive than many other data collection techniques and allowed the collection of information on a wide range of things, including personal facts, attitudes, past behaviours and opinions (Watson, 2001). The survey was standardised to ensure that it had reliability and validity so that the results could be generalised to the larger population of South Africa.

3.4.2 Phase 2: Designing of the pedagogic content knowledge framework

Based on the data that emerged from the baseline study (section 3.4.1) a pedagogical content knowledge framework was formulated (See chapter 2, section 2.6) using a section of statistics, bi-variate data in particular, and then used to develop pedagogical content knowledge for Grade 11 and 12 secondary school statistics teachers. The pedagogical content knowledge framework was based on pedagogical content knowledge.

1.	**Previous/present knowledge**
2.	**Bringing teachers into context**
3.	**Statisticatisation (finding meaning from bi-variate data e.g. modelling).**
4.	**Realistic/case-based problem posing**
5.	**Assessment of pedagogical content knowledge (meta-cognition)**

Diagram 3.4.2a: The hierarchy of the activities as adapted from the pedagogical content knowledge framework

Earlier in the literature several intertwining categories of pedagogical content knowledge were identified as shown in the following diagram (Diagram: 3.4.2b). These categories, informed by pedagogical content knowledge, were used to justify the nature of the tasks and the experiences in each stage of the pedagogical content knowledge framework. The tasks used were drawn from the field of statistics bi-variate data in particular and were chosen from tasks that were used during other statistical research projects to improve data handling and bi-variate data (Learning Math: 2012, www.databaseOlympics.com,Watson, Collis, Callingham & Moritz, 1995; Loughran et al., 2004; Turnuklu & Yesildere, 2007). The tasks were chosen to offer the researcher opportunities to retrieve data from the teachers as they attempted to resolve what for them were genuinely problematic situations. Data from the tasks were collected through written responses followed by in-class presentations and discussions with the teachers. The chosen activities were open-ended and were intended to provide a basis for challenging teachers to reflect on and, wherever necessary, modify concepts. Questions and variables were devised that included the requirements of reading the problem, understanding the problem, and making a plan for the solution. I also involved interpretation of the data values by reading the data, reading between the data, and prediction of values for missing data, that is reading beyond the given data. The tasks were specially chosen to enhance, to stimulate and to motivate pedagogical content knowledge.

The pedagogical content knowledge framework was in turn used to develop the pedagogical content knowledge for Grade 11and 12 statistics teachers. The main purpose of a pedagogical content knowledge framework was to elaborate on the processes by which pedagogical content knowledge develops. It guided the presentations, discussions and promotion of the development of the teachers' pedagogical content knowledge required for the teaching of Grades 11 and 12 statistics. The framework was specific to statistics, and was used during model facilitations to carry out the development to Grade 11 and 12 statistics teachers.

The design of the framework capitalised on the development research-cycle process by Freudenthal (1991:161). Freudenthal's development research concept of a cyclic process starts from self-evident thought experiments where the developer envisions how teaching and the learning processes will proceed (*invent it*), finding evidence in a teaching experiment of whether the expectations were right or wrong (*practice it*) and then using this feedback of practical experience in new thought experiments. This induces an iteration of development research. Therefore, sessions were improved and ideas implemented in the next round. These changes, based on the experiences during the facilitation, started a new round through the pedagogical content knowledge development phase and, in terms of the design research method, the next research cycle.

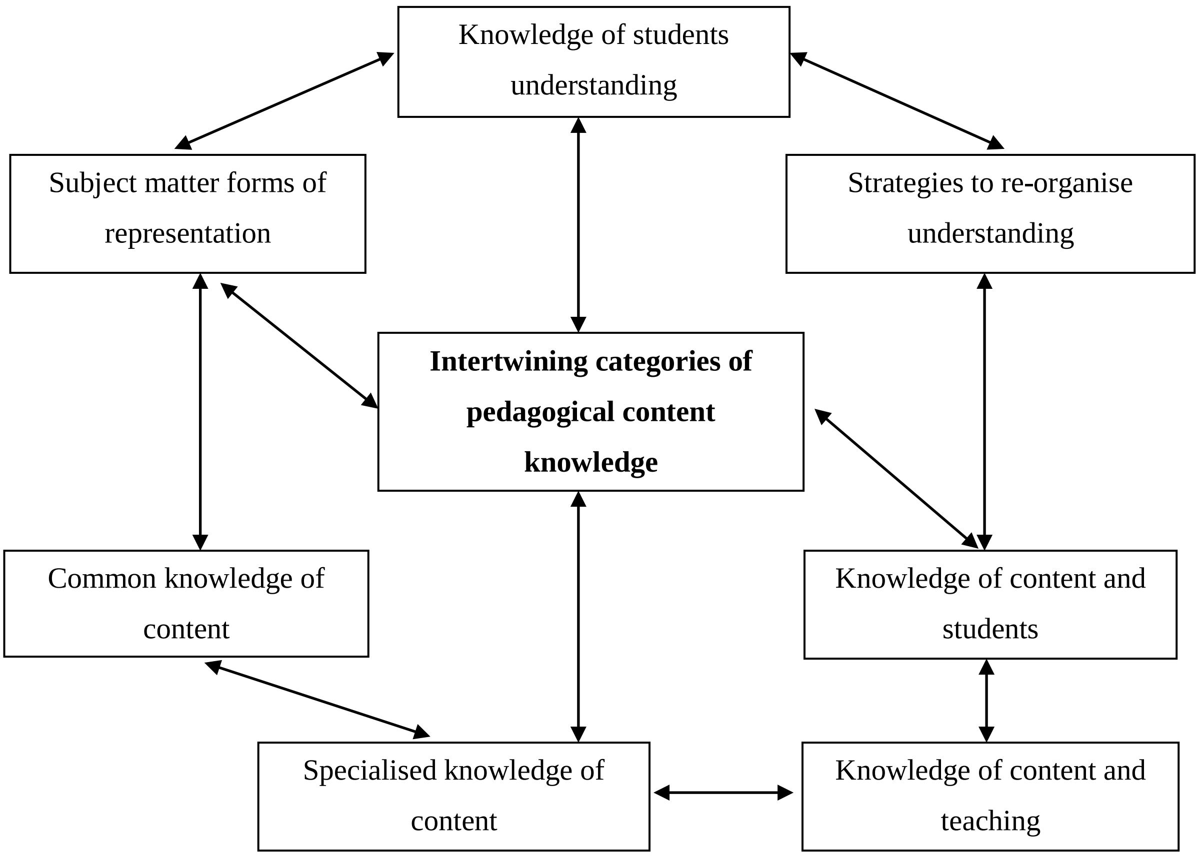
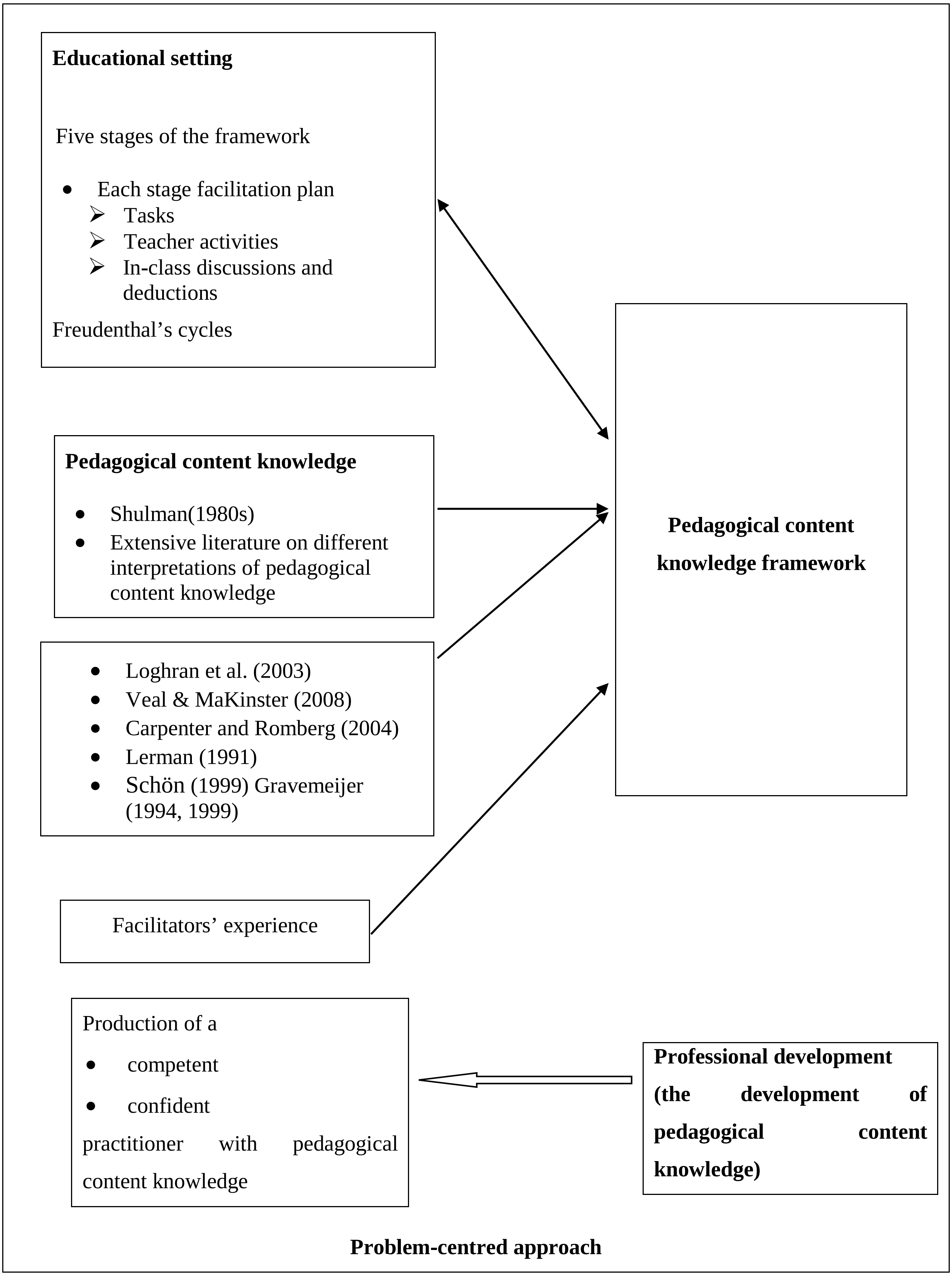


Diagram: 3.4.2.b: Intertwining categories of pedagogical content knowledge to design the framework

3.4.3 Phase 3: The development of pedagogical content knowledge using the pedagogical content framework



m 3.4.3: Relationship within the research site

3.4.3.1 Educational setting

The educational setting in the research site represented the integration of tasks, activities, and in-class discussions in a problem-centred context to achieve meaningful facilitation during the development of pedagogical content knowledge. The development of the educational setting involved the assessment of the starting level of understanding, the end goal and the development of a chain of bi-variate data activities to bring about movement towards that goal. The activities in this setting were designed to foster productive mental activities among the teachers. This sequence of activities, motivations and expectations made explicit the learning process in terms of the activities and cognitive development. Freudenthal's (1991) cycles were used at each stage during the course of the development.

3.4.3.2 Implementing the pedagogical content knowledge framework

Stages of the PCK framework	**Outcomes (Key learnings within the stages)**	**How the key learnings were achieved (Organised Activities: Appendix 5)**
1. Previous/present knowledge ***(Day1:Facilitation stage 1)***	• Exposure to previous and present knowledge that is related to bi-variate data. • To provide an in-depth understanding of types of data. • Exposure to statistical language and terms that involve bi-variate data and the interrelationship between statistics and mathematics. • Group presentations on flipcharts and reflection. •	• Videos were shown to teachers and discussions allowed. • Exposure to and completion of a flow chart that unpacked the meaning of data. • Models of bi-variate and poster activities that allow the discussion of data in general (types, collection and representation of data). • The recording and discussion of group presentations • Teachers were given an individual reflection activity
2. Bringing teachers into context ***(Day2:Facilitation stage 2)***	• Bi-variate data is made available for informal discussions and analysis representation. • Availability of realistic data through the media • Applying the product moment correlation co-efficient (-1to1).	• Teachers analysed and discussed data informally using previous knowledge • Teachers analyse realistic data (household and time-series data), newspapers, books, magazines, etc. to identify, discuss and analyse the bi-variate data through • Through models of straight line, quadratic, exponential etc discuss the

	• Group presentations on flipcharts	product moment correlation coefficient (-1to 1) and lines of best fit). • Group presentations of solutions.
3. Statisticatisation ***(Day3:Facilitation stage 3)***	• To avail teachers with relevant tasks and activities that involve bi-variate data from the media • Meta-cognition (Deciding what to do with given bi-variate data sets • Deducting from data analysis • Group presentations on flipcharts	• Teachers represented and analysed data from the car industry • Teachers represented and analysed data from the sports industry • Deducing trends and use of data after analysis • Group presentations of solutions
4. Realistic/case-based problem posing ***(Day4:Facilitation stage 4)***	• Putting teachers into the statistician's shoes • Putting teachers into the headmaster or Department of Education's shoes. • Group presentations on flipcharts. • Teachers' individual response to reflection later in the day	• Analysis of a problem in bi-variate data, to be used by health professionals. • Analysing a problem in bi-variate data through a rubric intended to be used for hiring a statistics teacher for the school. • Group presentations of solutions • Teachers got an individual reflection activity, recorded in the next lesson.
5. Assessment of pedagogical content knowledge (meta-cognition) ***(Day5:Facilitation stage 5)***	• Putting teachers into the learner's shoes. • Putting teachers as teachers into their classrooms. • Meta-cognition (The teacher as the assessor) • Group presentations on flipcharts. • Group reflection	• Teachers get a bi-variate data task from a past end of year examination paper to work on. • Teachers plan a lesson that would enable learners to respond to the above problem (clarifying aims and objectives) • Teachers respond to the problem as they would want their learners to do and prepares it for assessment. • The recording and discussion of group presentations • Reflection

Diagram 3.4.3.1: Achieving key learnings in the pedagogical content knowledge framework

This phase involved the real development of pedagogic content knowledge to the Grade 11 and 12 teachers using the section of bi-variate data. Using the pedagogical content knowledge framework, the planned tasks were handed out to the teachers in stages. Each stage comprised of three lesson parts. Lesson part 1 was from 08:00 to 10:00. Lesson part 2 was between 11:00 to 13:00. Lesson part three was from 14:00 to 16:00. If the work was completed earlier than expected, other issues of reflection were brought in. Each lesson part was handed out on a separate piece of paper as a separate problem. Teachers worked in groups of four or five and worked individually on the reflective parts at the end of each day. The researcher with the help of a subject-specialist researcher planned and facilitated a sequence of six weekly lessons, spread apart. During the development, the phases of the development research cycle, planning action, taking action and evaluating action were adapted as follows:

- A facilitation plan was constructed. Activities/cases/situations that problematise the planning stage were put into place.
- Problem posing and analysing the concept or problem and reflecting on possible solutions was carried out at this stage of the development/teaching phase.
- Evaluating action.

During the development, a video recorder was used to capture activities and teachers' experiences and discussions during these sessions. The Grade 11 and 12 teachers were involved in learning experiences and problem-solving activities over a six-week period. Each group was allowed to work through the given task for about one and half hours on flip charts with minimal intervention to allow them to focus on building detailed accounts of their problem-solving processes. At the end of this session, individual groups presented their written responses to the whole class. The results of each presentation were discussed in order to formulate questions used in the in-class discussions. The experiences of each group were recorded after the sessions in a pre-prepared table (Appendix: Table 11.1).

3.4.4 Phase 4: Reflection on the implementation of the PCK framework

This phase involved the reflection of the implementation of the pedagogical content knowledge framework. this included positive and negative experiences. Some of these were discussed in different section of the thesis, like limitation, summation and recommendations. Some were discussed in the relevant analysis section of chapter 4. The development of pedagogical content knowledge to the Grade 11 and 12 teachers using a pedagogical content knowledge framework was also evaluated by the teachers themselves. A questionnaire allowed teachers to discuss aspects of what they considered important and what they had achieved during the development (Appendix 6). These reflections were used to improve the pedagogical content knowledge framework for use in future developments.

3.4.4.1 Freudenthal's (1991) cycle/reflection

Overall reflection on the framework was done using Freudenthal's (1991) cyclic phases. The researcher replayed the video clips (section 3.4**)** of each stage of the facilitation and shared reflection of each stage with the teachers and the facilitator who was working with the researcher. The comments were used to improve each stage of the framework for the next weeks of the facilitation. The sessions were improved and ideas implemented in the next round. The specific purpose of incorporating cycles in the facilitation offered a way to reflect on what really happened during facilitations. By using reflection to identify what is not working, as well as what is working, teachers can be assisted in becoming more aware of their own success in teaching bi-variate data, as well as improving their ability to assess their own skills and knowledge in any other section of statistics.

3.5 Development of data collection instruments (instrumentation)

The main instruments used for the collection of data in this development were the questionnaires, the in-class facilitated tasks/activities, in-class facilitated discussions, interviews and observations.

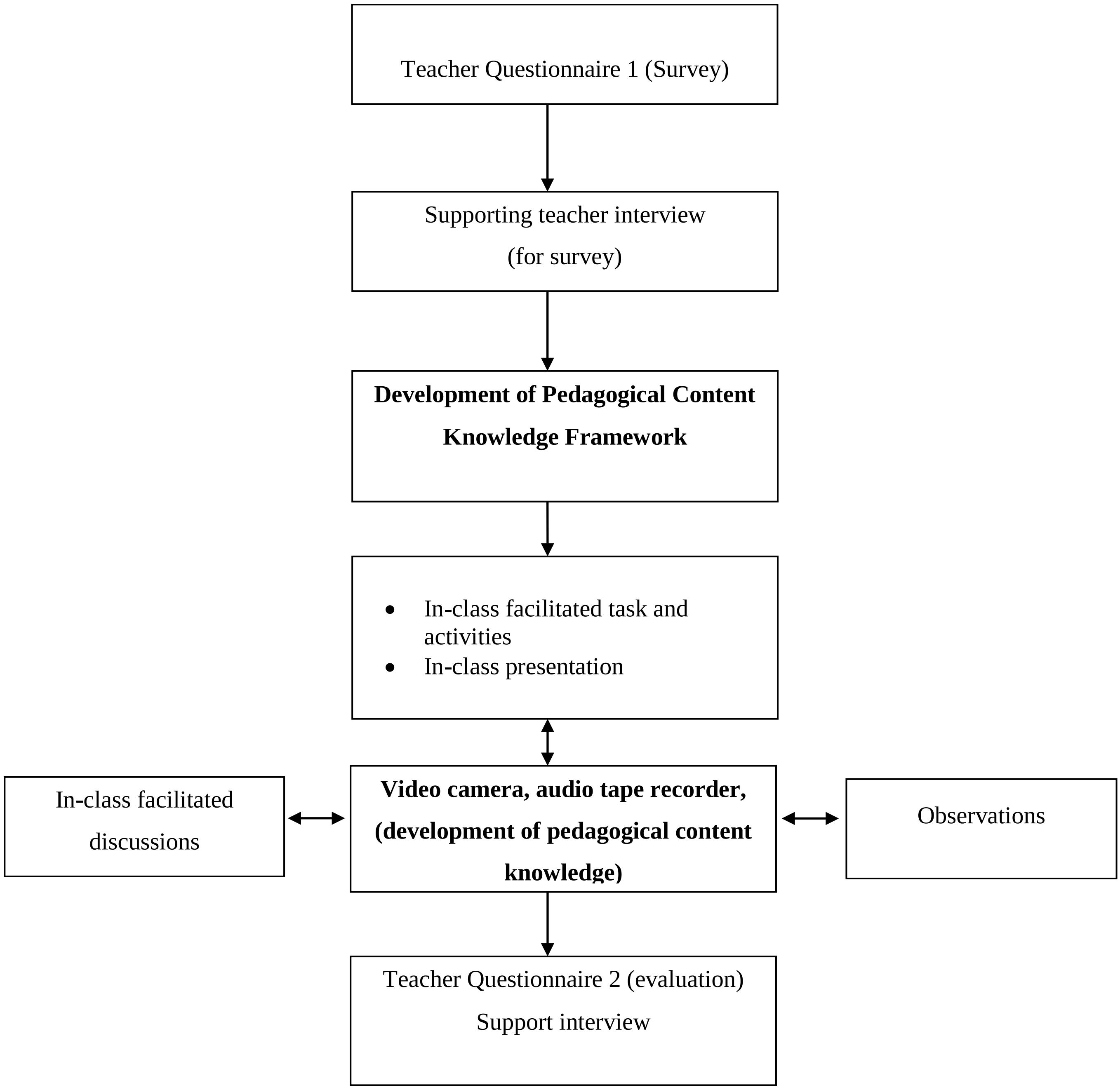


Diagram 3.5: Instruments used for the collection of data

3.5.1 Teacher Questionnaires 1

The survey was carried out using a predesigned questionnaire (Appendix 3) adapted from Watson's (2001) teacher profiling questionnaire. The variables in the data included biographical variables, support from professional development, subject content knowledge, pedagogic knowledge, material support, and confidence in the teaching of statistics. The initial section of the profile, which included biographical variables of the teachers, allowed the researcher to analyse the nature of the current Grade 11 and 12 statistics teachers. The biographical variables included gender, age, number of years of experience and qualifications obtained by the teachers (matric, certificate/ diploma, degree/ honours, masters' degree and doctorate). Section 2 of the profile obtained information on teachers' views on teaching statistics. Sections 3 and section 4 presented questions related to the content of bi-variate data and pedagogic knowledge. Sections 5, 6 and 10 related to professional development and the support offered at different levels. Section 7 looked at the background of the teachers who teach this part of the curriculum. Sections 8 and 9 reflected on the teacher's general confidence and ability to teach secondary school statistics and their reasoning of statistics in everyday life.

Responses were marked on a continuous scale from low confidence to high confidence and translated into integers from 1 to 5. The option "would not be teaching" also allowed teachers to exclude topics not relevant to their grade level. All the data obtained from the questionnaire was recorded in an excel spreadsheet. The questionnaire was delivered by post to 200 teachers who were in the F.T.E. statistics programme (FDEME8R) of 2008. To justify results from the questionnaire, interviews were carried out with five teachers who had responded to the questionnaire. The interviews lasted about 30 to 45 minutes and were audio taped. Questions from the questionnaires were used to guide the researcher through the open interviews, and some of the questions resulted from the way the participants responded to the questions. Interviewing is the main method used for supporting information collected in qualitative research (De Vos et al. 2003:292). They assert that interviews, when used with care and skill, are an incomparably rich source of data.

3.5.2 Collection of data through in-class facilitated tasks and activities

During the development of pedagogical content knowledge, data were collected through in-class facilitated activities. This stage involved the development and identification of tasks and activities to be used for the development, guided by the pedagogical content knowledge framework. All groups were involved in the writing of responses on flip charts to tasks presented in (Appendices 5) on bi-variate data.

3.5.3 Collection of data through in-class discussions

Presentations were discussed during in class development sessions. The activities were intended to challenge teachers to reflect on and, wherever necessary, to modify existing concepts, images and skills. Extracting various kinds of information concerning bi-variate data from the set tasks was achieved using Ben-Zvi and Arcavi's (2001) suggested questioning skills. These included asking descriptive questions ("what is?"), understanding questions ("how could you test that idea?" over "the idea is interesting"), predictive questions ("what will be?"), prescriptive questions ("what can be done about it?") and the causal questions ("why?") This pushed teachers from a localised type of thinking to a globalised type of thinking. A video camera was used to record the information.

The in-class discussions of the responses and activities on flip charts were prompted by the nature of the presentations. The aim of the discussions was to ask teachers about their solution process during or soon after they finished the task and more so during the class presentations. Conversations with the teachers (individually or in groups) were carried out to extend insights into teachers' thinking and to confirm validity of the group methods. Teachers commented on the strategies they used or discovered and on how they carried on with their activities in general. All discussions were fully video recorded and replayed later to extract the important experiences. The researcher and the teachers were active collaborators within normal classroom environments.

3.5.4 Teacher Questionnaires 2

The second questionnaire was used as part of the reflection on the implementation of the pedagogical content knowledge framework for the development of pedagogical content knowledge to Grade 11 and 12 secondary school statistics teachers. The questionnaire enabled teachers to reflect on the development as a whole. It allowed teachers to discuss aspects of what they considered positive in the development and what they thought needed improvement. No time limit was imposed for the completion of the questionnaire, which was handed out at the end of each development day.

3.5.5 Collection of data through observations

The observations took place at three levels: classroom level, group level and individual level. Observations at classroom level occurred during in-class discussions, explanations and demonstrations that were audio- and videotaped. Observations at group level took place while the five or four teachers were working in groups on the facilitation activities. When an individual responded to questions from the participants or from the researcher during the in-class discussions, observation was then at individual level.

3.6 Validity and reliability

This section addresses the reliability and validity of the research methodology, where written responses, observations, class presentations and discussions are the main sources of data, and narratives and interpretation are the main techniques of analysis.

3.6.1 External validity

External validity was realised by means of reflecting on the generalisation of the conclusions. In addition, the quality of the reasoning and the conclusions was controlled by means of submitting papers to conferences and relying on conference contributions that were realised during the research period (Makina, 2011; Makina, 2013). In this study, external reliability was ensured by reporting extensively on the research methodology to the conferences attended (Makina, 2011; Makina, 2013; INSTE 2009). For the external reliability, the criterion is virtual replicability (often defined as reproducibility) by means of traceability (Gravemeijer, 1993). The need for traceability was fulfilled through the cyclic process of developmental research. Developmental research means, "...experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience (Freudenthal, 1991:161). This means that the research is reported in such a way that it can be reconstructed by others. In other words, there should be reports on failure and success, on the procedures followed, on the conceptual framework, and on the reasons for the choices made. This requires transparency and explicitness about the learning process by the researcher and justification of the choices that are made within the research project.

3.6.2 Internal validity

The measures taken to improve the internal validity included finding a proper balance between involvement and distance during facilitation, and "playing devil's advocate" while analysing the data in order to develop alternative explanations of the findings. Internal validity was also achieved by the nature of activities used during the development of pedagogical content knowledge The activities were intended to evoke pedagogical content knowledge experiences, by providing a basis for challenging learners to reflect on and wherever necessary to modify existing concepts, images and skills. There were distinct features that were important in deciding meaningful problems that reflect the goals of the section of bi-variate data to be taught. These included (Erickson 1999:517):

- Situations that consider learner' interests and experiences.
- Local contexts, puzzles and applications.
- Interesting tasks that have multiple solution strategies, multiple representations and multiple solutions.
- Rich opportunities for statistical communication.
- Appropriate content considering learners ability levels and prior knowledge.
- Reasonable difficulty levels that challenge yet do not discourage.

A task needs to be sufficiently well specified that the chances of a learner engaging in unproductive activity be kept within tolerable limits. Good learning activities were coordinated by allowing teachers to take full responsibility in the interpretation of the set tasks. They learnt to satisfy a learning task to the best of their ability. The internal validity refers to the quality of the data collection and the soundness of the reasoning that led to the conclusions. Furthermore, though the development was done on bi-variate data, it was apparent that it could be practised in all sections of statistics.

3.6.3 Reliability

Four aspects guided the reliability of the design and implementation of the development of the pedagogical content knowledge framework. There was the piloting of the framework in the first week of the development, the use of a well-designed taxonomy, Loghran et al. (2003) pedagogical content knowledge representations and pedagogical and professional-experience Repertoires PaP-eRs and the use of best practices chosen from renowned researchers

(Romberg & Carpenter, 2004; Lerman, 1991). An accompanying colleague always acted as an observer of the development. This colleague also participated in small class discussions and the checking of maximum time utilisation. The use of a video camera allowed for the processes during the development to be re-observed and re-analysed. All questionnaires used were pilot-tested to teachers in the nearby schools before being utilised in the main development. It was at this stage that the researcher could pick up errors from the type of questions or responses. The pilot study also gave a chance to check the teachers' understanding of the English language, which is a second language to them, through the response on the worksheets.

The Bureau for Market Research was involved in the overall check of the questionnaires. Oscar Kilpert, from the Directorate for Curriculum and Learning Development, checked on the meaningfulness and coherence of the content in the questionnaires. Reliability refers to accuracy or precision of an instrument (De Vos et al., 2003:168). This in general refers to the extent to which independent administration of the same instrument consistently yields the same results under comparable conditions. While questionnaires are a good form of collecting data, the level of validity and reliability are more difficulty to attain in comparison with personal interviews (Janesick, 1994). This is why interviews were carried out at the end of each questionnaire.

A second researcher examiner from the Free State Department of Education moderated the data from in-class facilitated activities. For the purpose of this section, this was a criterion for the validity for acceptable result validity. In addition, content validity was achieved through the researcher's choice of tasks to be used in the study. Tasks were chosen from problems that had been used in the study of bi-variate data by experienced authors. The tasks chosen for this research were selected for the reason that they had been previously used in examinations and to develop statistics during studies done by some researchers in middle school and could therefore be adapted for use to the Grade 11 and 12 teachers. For example, the problems from Worksheet 1 to Worksheet 3 were adapted from Learning Math from the internet. Worksheet 4 and Worksheet 5 tasks were adapted from end of year examination papers (2010; 2011). In this study, results were obtained from more than one data source. It was with this view that written responses and in-class discussions were used as data collection instruments. This way the researcher used triangulation, which is the use of two or more methods of data collection (Cohen & Manion, 2000). In this study the richness and complexity of pedagogical content knowledge processes was recorded fully using more than one method. Anderson (1990) agrees with the above authors in that the major safeguard on validity is obtaining information from as many sources as possible.

The measures for obtaining internal reliability included systematically gathering data by means of prior identified key items in teacher activities, and processing the data using consistent coding systems. In addition, the protocol analysis and coding process were carried out with a second facilitator. Internal reliability enables results to be reported in such a manner that it can be reconstructed by other researchers (De Vos et al., 2003).

3.7 Procedure for data analysis in the study

This study provided an abundance of data through items that were formulated based on:

- Teachers' flip chart group presentations.
- Teachers' responses to questions in the in-class discussions.
- The remarks they made during their stepping in as experienced classroom teachers, statisticians, headmasters or learners.

This was done to ensure that the results, as much as possible, were based on teachers' knowledge and experiences. Specifically, the aim was to improve statistics teachers' abilities to recognise specific preconceptions and conceptual difficulties related to statistics, bi-variate data in particular, and to promote their use of interventions and strategies promoting conceptual change during classroom practice. The analysis of the data involved the selection of verbal, observation and video fragments relevant to the teachers' pedagogical content knowledge, followed by the transcription and analysis of these by the researcher and the educator involved, and then the comparison and discussion of individual interpretations until agreement occurred. The validation of these interpretations was promoted by applying the constant comparative method to transcripts of the participating teachers from different weeks.

Narrative was used as the main mode of representing teachers' pedagogical content knowledge in this research because narrative constructions have the capacity to represent the holistic nature of teachers' knowledge and experience (Connelly & Clandinin, 2000). The interacting elements of context, teachers' past and present experiences and their future plans and actions can be explored. This is in contrast to traditional "scientific" modes of analysis that aim to isolate elements of experience for separate examination. Representations of teachers' understanding and practice that are intended to capture and portray the nature of pedagogical content knowledge experience increasingly draw on narrative forms (Connelly & Clandinin 2000:123). Narrative in research on teaching has the capacity to render the teaching experience in rich detail, including its particularities, complexities and indeterminacy, and to open up this experience for others' understanding (Connelly & Clandinin). Through narrative, we begin to understand the teachers' reasons for the action, and are thereby encouraged to make sense of these actions through the eyes of the actor. This understanding constitutes an enormous contribution to learning about and getting better at teaching (Connelly & Clandinin). Conle (2003) says narrative can also help the reader to view and interpret phenomena differently; develop the reader's tacit, practical knowledge; and lead to personal and professional changes in the reader; and to their visions of what can be. This is very good for professional development as what is of interest to narrative inquirers is not what happened, so much as what meaning people make of what happened.

3.7.1 The analysis of quantitative data from questionnaires

The SPSS 14.0 for Win XP computer package (Appendix 12) was used for organisation and analysis of data. This instrument was specially chosen so that it could be used for a large number of statistics teachers in different parts of South Africa. Data was divided into different sections of variables to facilitate the eventual processing of the data (Appendix 3). The variables in the data included support from professional development, subject content knowledge, pedagogic knowledge, material support, and confidence in the teaching of statistics. The biographical variables included the qualifications obtained by the teachers (e.g. matric, certificate/ diploma, degree/ honours, masters' degree and doctorate). The percentage of variables in each category clarified the frequency of each variable highlighting if there was further need for attention on specific issues during the development of pedagogic content knowledge of teachers of statistics. Converting data to a percentage is one way of using absolute frequencies. The most common method of summarising content analysis data is using absolute frequencies, such as the number of specific incidents found in the data (Borg and Gall, 1983). Because the development was dealing with large numbers of participants, it was not possible to analyse data without the help of the SPSS computer package accurately

(Appendix 12 and 13). The use of already set formulas in the SPSS computer package ensured validity and reliability.

3.7.2 The analysis of qualitative data from in-class discussions and observations

Analysing the data consisted of noticing, collecting, categorising and thinking about interesting, relevant things. Criteria were determined according to the components of pedagogical content knowledge inside the framework. The in-class discussions were organised to clarify and justify the results of the presentations during the development of pedagogical content knowledge. The video clips of the in-class activities and discussions were analysed in relation to pedagogical content knowledge in alignment with the pedagogical content knowledge framework. The data from in-class discussions made it possible to come up with and support the major themes of teacher experiences resulting from the analysis. This was best achieved through inductive analysis of the data where the main purpose is to allow the significant themes to emerge from the raw data, itself rather than imposing a more rigid and theoretical framework.

The development was monitored through reports obtained from watching the video clips during the development of pedagogical content knowledge. The data used made it possible to set up organised themes enabling the analysis of teacher experiences. These qualitative data established how “participants make meaning of a specific phenomenon by analysing their perceptions, attitudes, understanding, knowledge, values, feelings and experiences in an attempt to approximate their construction of the phenomenon” (Niewenhuis, 2007:99).

3.8 Limitations of the study

A teacher’s pedagogical content knowledge may not be evident within the bounds of a few lessons, as it is a complex notion. An extended period may be needed to unfold it. A long-term goal is to establish that teachers who have this knowledge do in fact teach in ways that lead to learner understanding. Unfortunately, there was not enough time within this study to explore this important aspect from the learner’s situation. It must be noted that the challenges faced in South Africa are not necessarily those faced elsewhere, in other parts of the world. This study was limited to experiences of South African teachers and the secondary school statistics.

3.9 Ethical issues

The administration of the development of pedagogical content knowledge was made possible after gaining permission to do the research from the Department of Education (See Appendix 1 & 2). Permission from the teachers was also obtained from school authorities and the teachers themselves (Appendix 8). A letter of appreciation was later sent to all teachers who participated in the research project (See Appendix 9). Notification was provided to the Free State Department of Education, including an opportunity for teachers to reconsider their participation. In the teacher's consent form, a statement covering the opportunity to withdraw at any stage of the research was made available. The notification was in accordance with the agreement between UNISA and the Free State Department of Education as regards the in-service programme. A day before the development of pedagogical content knowledge research started, the teachers were given the opportunity to ask questions about the research, and to withdraw from the research if they did not understand it.

3.10 Conclusion

Following the baseline teacher-profiling questionnaire, the development of pedagogic content knowledge seeks to provide an enabling environment plus best practices for making teaching and learning happen. This chapter describes the research methodology, the research design, research instruments, data collection procedures, and data analysis techniques used in this study. The population and sampling procedures were discussed and validity and reliability ensured. Quantitative approaches were employed to support the qualitative data. This investigation allows one to appreciate pedagogical content knowledge, as it encompasses the entire process of teacher engagement from initiation to completion of the task through the problem-solving process. The researcher tries to make sense of what is going on in the classroom against the background of the thought processes that preceded the instructional activities.

CHAPTER 4
DATA ANALYSIS AND RESULTS

4.1 Introduction

The first objective of data analysis, outlined in this chapter, was to provide evidence of teachers' experience during the development process. This step followed on from the design and development of the pedagogical content knowledge framework. The objective was to align the pedagogical content knowledge framework to these experiences and to document the framework's ability to reach the objectives and outcomes originally specified in this study. The methodology used for the analysis is captured in section 4.2. Data analysis and results of the baseline study survey (Appendix 3) are presented in section 4.3. Section 4.4 involved the capturing and documenting of the teachers' experiences at each stage of the pedagogical content knowledge framework and the discussions are recorded in section 4.5. Challenges in the implementation of the new pedagogical content knowledge framework are captured in section 4.6.

The five research objectives that guided the analysis are:

- To analyse if Grade 11 and 12 statistics teachers in South Africa are competent, confident and prepared for the teaching of secondary school statistics.
- To analyse and explore the special characteristics of pedagogical content knowledge that contribute to it being a better teacher knowledge for secondary school statistics teachers.
- To design and develop a new pedagogical content knowledge framework which will guide and inform the development of pedagogical content knowledge for secondary school statistic teachers.
- To use the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences.
- To analyse the challenges in the implementation of the new pedagogical content knowledge framework for the development of pedagogical content knowledge.

4.2 Methodology of the analysis

As per the methodological approach for this study (see Chapter 3), this section focuses on the method of data analysis.

4.2.1 Method of analysing data in the phases

The first phase of the data analysis was the initial round of coding. The general experiences of the teachers derived during the stages of development of the pedagogical content knowledge framework were recorded in a daily collection and analysis table (Appendix 11, Table 11.1). In this first stage, what was seen and observed was recorded.

The second phase of the analysis looked for trends between different groups by sorting experiences into categories with the same code and analysing these categories. This led to the distinction of subcategories that needed to be coded accordingly. The findings were summarised for each of the stages and illustrated by prototypical observations and experiences (Appendix 11, Table 11.2). The analysis continued in this way until saturation, which meant that no new elements were added to the analysis and no conclusions were subject to change. This second round of coding was time consuming as previous codes needed to be revised in accordance with new decisions (Appendix 11, Table 11.2). After a while, the coding advanced faster and this indicated the completion of the coding system and its additional decisions. The researcher and the subject-specialist researcher worked through the first part of the data (Appendix 11, Table 11.2) until they agreed on the coding methods and resultant categorisation. A number of episodes were revealed at each stage during the implementation of the pedagogical content knowledge framework. Typical examples of potentially interesting episodes originated when teachers:

- Were unable to answer a researcher's question.
- Gave an answer that was clearly unexpected.
- Were clearly confused or inattentive.
- Appeared to change the pace or direction of the lesson due to confusion.
- When the researcher reacted to, or ignored, inappropriate responses.

After each day's work (Appendix 11, Table 11.1), the researchers met to discuss the facilitation in more detail, with the video footage available. Notes and the video footage were used to identify the episodes in the lesson that would form the focus of the categories and subcategories.

The third **phase** of data analysis was the identification of the main themes. Following the second coding analysis of data, teacher experiences were categorised into major themes (section 4.2.2). During the analysis and profiling of these experiences, some important and recurring themes arose (section 4.3.5). The themes were either relevant to pedagogical content knowledge practices as encompassed by the categories of pedagogical content knowledge or directly statistical in nature.

4.2.2 Components used to align pedagogical content knowledge to statistics teaching

Components of teachers' experiences were frequently observed during the in-class discussions as opposed to during the group work activities. Some of the components of pedagogical content knowledge identified in relation to the identified experiences and main themes of the study were:

- common knowledge of content
- specialised knowledge of content
- knowledge of content and learners
- knowledge of content and teaching.

Some categories of pedagogical content knowledge, namely specialised knowledge of content, knowledge of content and learners, and knowledge of content and teaching, that were recorded during the facilitation stages, demonstrated links with most of the dimensions of statistical thinking. For example, there is a link between knowledge of content and learners with regard to the integration of the statistical with the contextual and a link between knowledge of content and teaching with the components of statistical thinking. These links were directly or indirectly evidenced throughout all the facilitation stages.

4.2.3 Analysis of videos

The video camera focused on the teachers and the researcher throughout the lesson. A selection from video footage was made by event sampling with data concerning key items. The video recordings were analysed in relation to the statistical thinking, expectations and the pedagogical content knowledge framework. Following the videoing, the researcher edited the video to obtain a "movie" between 7-29-minutes, which included episodes in which pedagogical content knowledge in relation to teaching statistics appeared to be a feature. These episodes were selected if deemed interesting and worthwhile to follow up. Such episodes showed teachers' explanations, responses to facilitators' questions or answers, and discussions with the whole class. They were potentially worthwhile in relation to their links to the pedagogical content knowledge framework. During the editing process, the researcher made notes about each episode about potentially interesting behaviour, attitudes or contributions to the facilitated activities. The video tape was transcribed in full and transcripts were annotated to add non-verbal behaviour and contextual detail.

4.2.4 Analysis of all teachers' participation

A peer-assessment rubric, shown in the table below, listed the names of the teachers in each group. This was prepared at the beginning of the development week and handed out every morning to teachers who returned it at the end of the day. Points in the rubrics were awarded out of ten. An individual teacher's marks originated from the average of the individual participation marks given by the members of his or her group. The researcher asked each teacher to privately rate his/her colleagues' participation during task problem solving. Relevant ethical issues were explained. Initially the teachers who did contribute much to the problem-solving tasks got very low marks. All teachers were assessed around ten out of the ten marks.

Table 4.2.4: Example of participation table

RONGIE	
NAME	**Participation mark out of 10**
Rongi	
Tare	
Daniel	
Kumbi	
Simba	
Total	

4.2.5 Analysis of the flip chart presentation

One group at a time presented its flip chart written responses to the whole class. The researcher in most cases responded with the words, “This is interesting. This is becoming interesting. What is going on here? Can you explain more? Please go over it again”. This was a very crucial stage for “aha” moments or when the “can of worms” exploded. Most issues of concern were clarified at this stage. The behaviour of the teachers changed during the development process, from a defensive or accusatory stance to an eagerness to gain knowledge. The researcher’s ability to dissuade teachers from punitive or abusive behaviour and, rather, to encompass and encourage discussions created a conducive learning environment. In a collaborative learning session, teachers had the opportunity to discuss the “big ideas” behind the task, share expected responses, develop strategies to address specific content issues, and articulate opportunities to shift their understanding to a more general context. A major goal of the development was to discuss “missed opportunities”. Missed opportunities are interpreted as classroom incidents in which a lack of pedagogical content knowledge results in the teacher missing the chance to enhance learner learning.

4.3 Presentation and discussion of findings

The findings are discussed according to each research objective.

4.3.1 Research objective 1

To analyse if Grade 11 and 12 statistics teachers in South Africa are competent, confident and prepared for the teaching of secondary school statistics.

Data analysis and results of the baseline survey

The baseline study involved a survey questionnaire for 200 teachers who taught secondary school statistics (Appendix 3: Baseline study: Teacher profiling; Watson, 2001) of which 50 responded. Following the survey, interviews (Appendix3:Q52) were conducted with five survey respondents to justify the questionnaire results. The interviews lasted about 30-45 minutes and were audio-taped. Excel and the SPSS 14.0 for Win XP computer packages were used for the collection, organisation and analysis of data (Appendix 12 and 13).

4.3.1.1 Biographical variables

Out of the 200 teachers who received the questionnaire in the survey, there were 50 responses. Just over a fifth (21%) of the group originated from Polokwane and Limpopo. Of the 50 teachers who responded, 56.5% of the teachers were male and 43.5% were female. The majority were black Africans (91.3%) and the rest were white. Of concern was the absence of the other ethnic groups. The age group of the majority of teachers fell between 36 and 45 years. In the category of qualifications, 56% had a teacher's certificate in education and the rest had either a bachelor's degree or a diploma in education. They did not have many years of teaching experience, with 60% of them having taught at most for two years. Only 58% had received training in a statistics course or on an in-service course in probability and statistics, and 26% had no statistics training and were general mathematics teachers. Out of all the teachers who had taught probability and statistics at school, 38% had also taught correlation.

4.3.1.2 Misunderstanding

Out of the 50 teachers who participated in Q1, Q2 and Q3, 42% agreed with the fact that they did not understand bi-variate data. The questionnaire showed a weak relationship between the experience of teaching statistics and the preparation of the subject, as most teachers taught probability and statistics regardless of their mastery of the discipline. Those who said that they understood bi-variate data came from other countries or had been schooled in other countries. The interviews revealed feelings of insecurity by the teachers due to the lack of subject and content knowledge.

4.3.1.3 Subject content knowledge support

Out of the 50 teachers who participated in the survey, 39-52% disagreed with the fact that they received any support on statistics content to teach probability and statistics from in-service training. In Q13 and Q14, where in-service training was provided, a percentage of 13-39% indicated that, even though they attended some INSET programmes, they did not understand the content during training and afterwards. This clearly indicates that the INSET programmes were less than helpful. Less than 2% indicated that they gained a lot from the INSET support. After the interview, it was found out that those who understood the section of statistics had actually received private support from different places (e.g. friends, statistics bodies, conferences like AMESA).

4.3.1.4 Pedagogic knowledge

On the section of pedagogic knowledge, 30.4-52.2% agreed that they received guidance on how to teach the section of bi-variate data. Out of the 50 teachers who responded to the questionnaire, 80% agreed that teachers who had problems with the teaching of bi-variate data had no choice but to continue teaching it. They were not offered much help from INSET. They were also not offered retraining or further assistance from INSET. During the interview, one teacher indicated that he skipped the section of statistics during his teaching career. He further added, “I will teach it if I get help”. While 35% of the teachers indicated that they used the calculator to help them to teach the section of bi-variate data, 50% solved statistics problems manually. Between 70% and 80% did not use computers for the teaching of bi-variate data, and most of them indicated that even if computers were available they did not know how to use them. Half of the teachers indicated that INSET workshops were unorganised and 45% said that no community of practices existed in the schools. The workshops do not provide enough to the pedagogic knowledge needs of the teachers.

4.3.1.5 Relevant support

Although 30-39% agreed that the INSET educators were knowledgeable of the relevant content and pedagogic knowledge, they seemed unable to transfer this knowledge during training. One reason was given in response to Q26 and Q27 in that 35-44% of the teachers complained that the facilitation time was too short and 48% indicated that they had not been taught both subject content and pedagogic knowledge.

4.3.1.6 Material support

Out of the 50 teachers who responded to the questionnaire, 30% of the teachers indicated that they do not receive statistics textbooks in their schools. Slightly more than 34% indicated that they had knowledge of time series and household data, but did not know where to access this data. During interviews, it became apparent that teachers were not aware of the existence of STATS-SA and did not realise they could access useful data from the internet and magazines. They were also not aware that they could compile their own time series data using personal utility and store bills. Most schools where the teachers worked did not own any computers and therefore had no computer rooms or software. Out of the respondents, 90% of them did not use computers. Only 10% were comfortable to use computers.

4.3.1.7 Teacher background

Responses to Q41 indicated that 73% of the teachers studied mathematics and in Q42 they indicated that they studied mathematics more than four years ago. Responses also indicated that the mathematics that they studied did not focus on probability and statistics. In Q43, of those who studied statistics, 49% indicated that only few hours in a week were put into the facilitation of statistics.

4.3.1.8 Confidence

In response to Q45, 47% of the teachers showed high confidence in teaching the Cartesian plane, linear functions, quadratic functions, exponential functions, graphic representations and anything to do with calculations. Out of the 50 teachers who responded to the questionnaire, 53% expressed low confidence in the teaching of probability and statistics. They also had problems with statistics language, basic probability calculations, and collecting data from real-life experiences. Even if required to do so, 49% of the teachers expressed that they would not be teaching lines of best fit or linear regression lines. This confirmed that teachers did not teach sections of mathematics with which they felt uncomfortable.

4.3.1.9 Statistics in everyday life

It was revealed that though 47% of the teachers were not able to understand all statistical terms in the newspapers, 59-69% do understand the statistics often included in various types of magazines. Over 59% of the teachers indicated that, despite using statistics every day, they were not sure whether they understood the important aspects of statistics in everyday life. Results in Q46 were as follows(Table *4.3.1.9)*:

	Strongly disagree	Neutral	Strongly agree
	1	2	3
a. You need to know something about statistics to be an intelligent consumer.	10%	12%	78%
b. I can easily read and understand graphs and charts in newspaper articles.	50%	33%	17%
c. When buying a new car, it's better to ask a few friends about the problems with their cars than to read a car satisfaction survey in a consumer magazine.	17%	13%	70%
d. I can understand almost all of the statistical terms that I see in newspapers or on TV.	62%	33%	5%
e. Understanding probability and statistics is becoming increasingly important in our society.	92%	2%	6%
f. Statements about probability (such as the odds of winning a lottery) seem very clear to me.	58%	10%	32%
g. To learn about the side effects of a drug, it's better to refer to the results of a medical study that tested it on a few people, than to talk to someone who has taken the drug.	32%	53%	15%
h. People who have contrasting views can each use the same statistical finding to support their view.	13%	80% Did not understand the question	7%
i. I could easily explain how an opinion poll happens.	3%	90% Did not understand the question	7%

Table 4.3.1.9: Statistics in everyday life

Results in the table and discussed above indicate that generally the statements 'a-i' in the table above are about a person who understands statistics and is able to make decisions about statistics statements. Out of the 50 teachers who responded to the questionnaire, 78% agreed to that one needs to know something about statistics to be an intelligent consumer.

4.3.1.10 Professional development

In response to Q48b, 48% of the teachers participated in professional developments due to interest, curiosity and the wish to improve their teaching. Some during the interviews said that they were running away from classroom loneliness in order to collaborate with others. Close to 98% of the participants overwhelmingly agreed that probability and statistics are useful in everyday life and that statistical information is important for building opinions and making decisions.

The national curriculum documents had been seen and used by 17-50% of the teachers according to responses to Q47. In question 47d, over 60% of the teachers had not seen or liaised with any documents from Statistics South Africa or any bureau of statistics. This is probably the reason teachers disconnect the statistics taught in the classroom to the everyday statistics. This also implies that the policy documents that speak about statistics are not read and if read they are not understood. The interviews revealed that previous professional developments had not attempted to discover teacher's problems regarding the teaching of statistics. Of those teaching statistics, 70% had not participated in any professional development related to statistics. The 30% who had did so after it was organised by a university or through their own initiative.

In Q9 out of the 50 teachers who responded to the questionnaire, 58% preferred workshops/courses conducted by the university as opposed to the 30% who indicated their preference for school-based statistics sessions. During interviews, it was revealed that the workshops or professional developments done by universities were more organised than those prepared by the government or non-governmental organisations. Most (80%) of the teachers indicated that the university educators were usually well qualified in their fields (e.g. statistics) and knew what they were doing during professional developments.

In Q50, 50% of the teachers indicated their preference for an outside expert to come and help them during professional development as opposed to the subject experts from the regions, such as regional curriculum officers.

4.3.1.11 Question 51 (Any other comments)

The table below reveals some of the responses that were recorded through Excel for Q51, which was phrased as follows: "Do you have any other specific comments about professional development in relation to bi-variate data". Generally, the feedback from all the responding teachers was similar. Teachers highlighted the importance of organised professional developments as well as competent leaders for the development. Furthermore, teachers commented that in order to increase their statistical competency they needed pedagogical content knowledge.

Table 4.3.1.11: An extract of the teachers' comments

Question 51	
Learner	***Comments***
1.	I think they should give us easier examples to explain bi-variate data. Start with the simple ones and progress slowly to more complex ones. Give us more examples to work from.
2.	We should be led by lecturers in order to succeed in statistics development.
3.	No
4.	For a successful teaching of these most important topics, teachers must be thoroughly enriched in this regard.
5.	I want more information concerning statistics, books that can best describe the statistics, the lesson plans. I want to see myself as an expert in statistics.
6.	The university must supply a small pamphlet to teachers doing the FDEME8R. A lot of teachers including myself are teaching it in schools but they do not know that it is what is called bi-variate data.
7.	No
8.	No
9.	No
10.	No
11.	No
12.	No
13.	There is need to develop teachers in probability and statistics during school holidays.

4.3.1.12 Supporting interview to baseline questionnaire

The supporting interview confirmed the results gathered from the questionnaire indicating that teachers doubt the capabilities of leaders of professional developments. Inadequate explanations from inexperienced educators during professional development resulted in continuous problems in the area of bi-variate data. The problems included the improper statistical definition of terms (noting that some terms can be defined in many different ways) and the unavailability of relevant teaching aids (e.g. in the form of relevant textbooks). Teachers complained that they were asked to prepare more for statistics lessons while being unsure on how to prepare. The feelings of insecurity expressed in the interviews spoke to the insecurity teachers felt due to the lack of content knowledge. Teachers participated in developments sometimes due to interest and other times due to curiosity and their wish to improve their teaching.

4.3.1.13 Summation for the baseline study

Results of the baseline study show that teachers did not have confidence to teach bi-variate data due to several reasons, including:

- Teachers did not study probability and statistics during their school-going or teacher-training years.
- Professional developments were in most cases rated unsuccessful and not meaningful.
- Some professional developments did not attempt to discover teachers' concerns regarding teaching of statistics in order to address their problems directly.
- Teachers did not have adequate support in subject content knowledge or pedagogic knowledge.
- Teachers did not have relevant material support (e.g. relevant textbooks) in order to obtain realistic data for the teaching bi-variate data.

Lastly, the results of the study tallied with the results from Makwakwa's (2012) study indicating that possible ways to address the problems in the teaching and learning of statistics in Grade 11 are:

- Teachers should receive financial support from their schools/districts to attend in-service education and training programmes.

- Textbooks should be well written (provide thorough explanations) and contain all the information necessary to teach data handling and probability (i.e. formulae, more examples).
- In-service teacher programmes should meet the needs of the teachers by offering topics that teachers find difficult to teach.
- More and longer INSET programmes on probability, preferably five-day workshops, should be arranged.

4.3.2 Research objective 2

To analyse and explore the special characteristics of pedagogical content knowledge that contributes to it being a better teacher knowledge for secondary school statistics teachers.

Analysis of the literature review and the pedagogical content knowledge framework

Details on how to bring out the special characteristics that make pedagogical content knowledge the most effective teacher knowledge and allow easy translation into practice has been extensively provided in chapter 2; section 2.5.3-2.5.5, section 4.2.2 (Also in Diagram 1.2.7; Diagram 2.5.2 Diagram: 3.4.2.2) and will enhance the learning and understanding of secondary school statistics.

4.3.3 Research objective 3

To design and develop a new pedagogical content knowledge framework which will guide and inform the development of pedagogical content knowledge for secondary school statistic teachers.

Designing and developing a pedagogical content knowledge framework for Grade 11 and 12 secondary school statistics teachers.

Details of the designing and developing of a pedagogical content knowledge framework for Grade 11 and 12 secondary school statistics teachers' professional development was analysed in section 2.6. The framework went through peer review (ISTE, 2010; SAARMSTE, 2011) and was published in an accredited journal (SAARMSTE 2013:29-43), with the title "Designing a pedagogical content knowledge framework for the professional development of statistics teachers."

4.3.4 Research objective 4

To use the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences.

Analysis of teachers' pedagogical content knowledge experiences

The guiding principles, factors and determinants for a successful and meaningful professional development of pedagogical content knowledge for secondary school statistics teachers was developed using a pedagogical content knowledge framework (section 2.6). Data collected from the teachers was examined qualitatively. The criteria were determined according to the components of pedagogical content knowledge with respect to statistical thinking (section 4.2.2). The nature of experiences, interactions and discourses of secondary school statistics teachers during the development process was captured in detail, with special reference to the pedagogical content knowledge framework (section 2.5.3-2.5.5).

4.3.4.1 Experiences during the pilot study week

The pilot study was carried out in the first week of the development. The facilitation of the pilot week was chaotic, as it was more formal or autocratic in the sense that it followed the old type of "teaching" style. The follow-up facilitation sessions improved each week as the researcher became more comfortable with the process. For example, if I realised that the group was not going in the direction I felt was appropriate, I would leave them to resurface on their own instead of imposing an autocratic, teacher-centred facilitation model on them.

The pilot study also made it possible to choose the most relevant tasks for this study, as it was clear that the results did not provide the expected outcome for particular tasks. Through the pilot study, it was discovered that the process of in-class discussions elicited more information about teacher experiences than individual or focus-group interviews, which I had expected to be more revealing. For example, in-class discussions allowed a commonality in "big ideas" to emerge. Conjectures were made about how the various categories of "big ideas" seemed to interact. As a direct result of the pilot study, the facilitation was able to generate increasingly realistic results.

4.3.4.2 Results of the six-week facilitation

Introduction: Developing the approach to data collection

The researchers' ability to observe and share experiences that were interesting as the facilitation was in progress and to develop a questioning strategy encouraging teachers to talk about their actions during those episodes were key features of the data collection process. Identifying potentially interesting episodes involved speculation about causes of a particular action through creating stories about observations and, inevitably, our stories were a function of attention paid during the facilitation. Initially, there were some interesting differences of observation between myself, as primary researcher, and the subject specialist researcher. This was attributed to our differing professional backgrounds. Reviewing video footage of the facilitation therefore played an important role in identifying episodes. As our experience of the individual styles of the teachers increased, a clearer picture of the kinds of episodes, which were interesting, was developed. There was therefore an increasing level of agreement in the main themes that we identified. The in-class discussions became relaxed occasions. As teachers became more familiar with the style of facilitation, they often offered spontaneous comments in response to the video extracts. All the teachers seemed to enjoy the opportunity to discuss their responses and experiences.

Table 4.3.4.3: Day 1: Results for Stage 1

Previous/present knowledge		
STAGE	**Outcomes (Key learnings)**	**How key learnings were achieved (Organised Activities:**
1 **(Appendix5: Day 1)**	• Exposure to previous and present knowledge that is related to bi-variate data. • To provide an in-depth understanding of types of data. • Exposure to statistical language and terms that involve bi-variate data and the interrelationship between statistics and mathematics. • Group presentations on flipcharts. • Teachers' individual reflection to the day's facilitation.	• Videos were shown to teachers. • Exposure to and completion of a flow chart that unpacked the meaning of data. • Models of bi-variate data and poster activities that allow the discussion of data in general (types, collection and representation of data). • The recording and discussion of group presentations • Teachers were given an individual reflection activity

RESULT 1.1a

Teachers listened to the videos

RESULT 1.1b

What is data?

The word "data" was in most groups taken to mean numbers and in most cases, it was taken to be a whole number. Other groups said data was a decimal number and during class discussions, they were adamant that data could never be a fraction. I presume that this is because most data in school textbooks and examination papers is given as whole numbers. The in-class discussions clarified this aspect, though it still appeared difficult for the teachers to give examples of different types of data in all its forms. However, no teachers were aware that other variables of data other than discrete, continuous, quantitative and qualitative exist. The following conversation was recorded.

RESEARCHER: I did not see a group that gave examples on categorical variables for the dog sport.

TEACHER 1: We were not aware what categorical variables are. We thought it was putting things into groups. But how?

TEACHER 2: Is it in the syllabus for secondary school statistics in South Africa?

RESEARCHER: Ok, I was not aware that teachers must only learn what is in the syllabus.

TEACHER 1: No. Part of our problems in the teaching of statistic originates from what you have just said. When a learner asks a question that is not in the textbook or syllabus we do not even know how to respond to it.

RESEARCHER: So you used to learn only what was in the textbook?

TEACHER 2: Yaa sort of. Remember we were all trying to teach ourselves the statistics. We never did it at school.

RESEARCHER: Ok, but can I still need to know what categorical variable are? Go and find out and come with the responses after lunch.

Because of time constraints, I asked teachers to go and research categorical variables.

Results of the research by the teachers on categorical data

The responses that follow were recorded after teachers' had 'googled' the definition of a categorical variable. They apparently had not found the relevant information in the library adjacent to our training centre. One group said they had found the variable of the *blood type* of a person: A, B, AB or O. The other groups came up with the *state* that a resident of the *United States* lives in and the *political party* that a voter in a European country might vote for: *Christian Democrat, Social Democrat, Green Party* and so on, from the internet. The following conversation was recorded.

RESEARCHER: Why are you giving us examples from other countries? Don't you have examples from our country or can't you relate these examples to your own particular environments?

TEACHER 1: Oh yeess I see. The term just appeared foreign to me until you spoke about it. So we just googled.

RESEARCHER: I still need you to describe Spot the dog categorically.

TEACHER 2: Now I understand it. Spot can be described in terms of it being male or female or by its type as a dog breed (e.g. sporting dogs; non-sporting dog types, working dog breeds, hound breed type of dogs, terrier breed type of dogs, etc)

It was evident that the teachers rarely related the subject statistics to their own surroundings. Understanding the need for data on which to base sound statistical reasoning was seen to be important in the development of statistical thinking. Teachers and learners were not challenged with issues pertinent to establishing the need for data, but were challenged with thinking about and thinking within the data that surrounds them every day.

RESULT 1.2a

During the first workshop session, the participants appeared to consider the introduction of bi-variate data problematic. Initially, they would refer to "two sets of data". I kept repeating the terminology: bi-variate data. For example, "Do you mean bi-variate data, or oh oh whooo bi-variate data" until every teacher was comfortable with using the word bi-variate data. I repeated this with most terms like correlation, causal, line of best fit, and so on. It was a "hooray" moment and subsequently teachers would reprimand me for using informal statistical language. The knowledge category of **common knowledge of content** manifested when the teachers started to understand the formal language of bi-variate data and statistics.

Strategies that assist learners with their statements were developed during in-class discussions. These included the re-voicing of statements, suggesting alternative representations of the data and appropriate modelling of language.

RESULT 1.2b

Mixing all of the representations expanded teachers' understanding and ideas. A multitude of graphs that included dot plots; bar graphs; and stem, leaf, box and whisker diagrams were among the many representations forming part of the responses. Teachers were not limited to the representations that they knew, but were also able to use the representations that they had experienced in their lives.

RESULT 1.2c

All the groups responded differently, but had the same result. The following are the responses:

- The elementary and old high school curves are directly related. That is the adjustment factor and the miles above minimum are positively correlated.
- The elementary curve is steeper than the old high school curve.
- Another group said that the old high school curve is “more slanted” than the elementary curve.
- Self-esteem and paranoia have an indirect relationship, while severity of illness and dosage levels have an indirect relationship that later tends into a direct relationship

The following conversation occurred:

TEACHER 1: Though we did not understand the terms adjustment factor and the miles above minimum, as the miles above minimum increased so was the adjustment factor.

TEACHER 2: If the paranoia in a person increase, one’s self-esteem deeps.

TEACHER 3: Actually if you have a low self-esteem you will be definitely paranoia or anxious.

RESEARCHER: Are you then saying that low self-esteem causes paranoia?

ALL GROUPS: Yes.

RESEARCHER: What about the other way round? Does paranoia cause self-esteem?

TEACHER 4: Yes.

TEACHER 1: This is confusing me now. It seems everything causes the other.

This discussion was very difficult for the teachers. The researcher had to come up with examples containing only a correlation to clarify the difference between a causal and a correlation coefficient. Within all groups, there was a heated debate about the behaviour of the curve of severity of illness against dosage levels. This is what one group had to say:

TEACHER 1: In our group, it was difficult to understand why if you are very ill and you take medication you will get better very fast. Then there comes a point when you are fine and you are still taking medication. Here you don’t change your status. What is funny is if you continue taking medication then you become ill.

Some groups said it was proof that the medication that we get is poisonous. Another comment was that:

> "This is the reason why people who take drugs later get ill and die. The drugs first does well and good for you. You then reach a status where you don't feel it anymore. If you continue then it starts eating you. You are now depended and then you collapse and die".

This conversation was a "hoorah" moment as the teachers could now delve into real-life examples very easily. More examples of similar relationships were given by other groups. For example, there were relationships between age and height, love and happiness, and so on given by the teachers.

Previous mathematical concepts of direct and indirect relationships were an unfamiliar concept with the teachers. They could only explain negative and positive graphs informally in their own language. For example, a positive graph was explained as "going up" and a negative graph was explained as "going down" or it is a "one step up and one across/flat". This is primarily because most African languages do not have a terminology for slopes, or if they do, they do not agree on the direction of the slope. A slope is considered as either going up or down relative on the position of the viewer. Descriptions and clarification of this concept were different for each group leading to an enriched discussion and a clear understanding of scatter plots, which was demonstrated later in the development.

A point of interest was the confusion around language, as the researcher did not originate from South Africa and so did not understand the local languages used by the teachers. The advantage while solving this was that the researcher, did not understand the teachers' language she originated from another country. Teachers used their mother tongue to discuss the statistics concepts needed in the various questions and activities of bi-variate data. When teachers clarified responses or were asked to pose questions in English during the in-class discussions, a major gap of understanding emerged due to language differences. This was one major problem identified by the researcher. Most teachers in South Africa use their home language to teach and explain concepts in any subject area. However, statistics is examined in English and a major shift to solve this problem is required by both teachers and their learners. and how this will be facilitated warrants further research.

It was also important for the teachers to realise that statistical and mathematical concepts and language work hand in hand. This realisation was apparent when one teacher indicated that there was a "direct relationship" on a dot plot. The other teacher responded that, "Hey, this is not maths. You must say it has a positive relationship". This confusion can be avoided with increased emphasis on the relationship between statistical and mathematical concepts.

RESULT 1.3a

One group indicated that an introduction to general data handling was important for learners in understanding the issues discussed in sections 1.1 and 1.2. Many ideas were generated by the teachers about how to introduce bi-variate data and how to teach them better, but there was general consensus on the idea of an introduction to data handling. The following conversation was recorded.

RESEARCHER: I realise that you need the general introduction to data handling in order to introduce the ideas in sections 1.1 and 1.2. What exactly will be involved in this introduction to data handling?

TEACHER A: All the things that we deal with in data handling like the introduction to mathematics related to bi-variate data would be part of preparing the learner to deal with the data problems above.

RESEARCHER: What is dealing with data?

TEACHER A: Like analysing data or making sense out of the data.

GROUP (All talk together): Even collection, even reading, even comparing data, etc.

TEACHER B: I now understand that you need to understand some, no, it's actually a lot of mathematics in order to find statistics easy and interesting. Think of all the modelling we do through mathematics, for example direct and indirect relationships.

RESULT 1.3b

Different options were supplied by groups in in-class discussions on the types of questions that help in resolving learners' misconceptions. Teachers' motivated why their individual suggestions would work best and one group established a list of questions or comments that would allow learners' misconceptions to emerge. They suggested that in order to come up with the learners' misconceptions the following questions or comments could be used.

- Why do you think your answer is right or why do you think your answer is wrong?
- I would love you to present your responses to tasks or questions to the whole class.
- If you think John’s answer is right, then why do you think you had gone wrong?
- You could have got all this right, where do you think you started to mess up?
- Sit down and explain your whole process to me.

Several of these suggestions were recorded with some groups suggesting that the different sections in statistics needed to be formulated into interesting activities to retain learners’ interest. Another group claimed the sections were not easily teachable as they were recall exercises requiring learners to cram. Teachers noted the multitude of small items that required repetition in order to ensure mastery of the ideas in 1.1 and 1.2.

RESULT 1.3c

Teachers responded in many ways to the question of real-world activities that can assist learners in dealing with the tasks outlined in 1.1 and 1.2. One group said that they would make their learners collect any data of interest to them within the school or outside the school to generate data from real-world activity. Another suggested that learners must be made to join groups that collect data for research and then be able to observe what the researchers finally do with the data.Groups generated similar responses to this question.

RESULT 1.3d

As a result of this facilitation process the teachers shared what they were going to start or stop doing.

Group A:

We will stop using only the school textbook for statistics activities, tasks and problems.

We will start communicating with whoever cares to give us more and different data, task problems, etc. to use for our teaching. The use of the so-called OERs or YouTube could be of help to us.

Group B:

We will stop taking the problems in the textbook as they are, irrespective of which country it was printed in.

We will start adapting the problems and tasks in the textbook to our real life situations. For example if the book gives an example of an aeroplane flying we will replace it with the manual flying kite. If the problem give an example of the number of points in rugby or cricket we will convert the problem to the number of points in African soccer or 'Nhodo'(a game played mostly by black girls).

Group C: We will start trying to improve our English in order for us to teach statistics in English. We have realised what a disaster effect it has on our teaching of actually any section of statistics.

Table 4.3.4.4: Day 2 : Results for Stage 2

Bringing teachers into context		
STAGE	**Outcomes (Key learnings)**	**How key learnings were achieved (Treatment) Organised Activities**
2 (Appendix5: Day2)	• Bi-variate data is made available for informal discussions and analysis representation. • Availability of realistic data through the media • Applying the product moment correlation co-efficient (-1to1). • Group presentations on flipcharts	• Teachers analysed and discussed data informally using previous knowledge • Teachers analyse realistic data (household and time-series data), newspapers, books, magazines, etc. to identify, discuss and analyse the bi-variate data through • Through models of straight line, quadratic, exponential etc discuss the product moment correlation coefficient (-1to 1) and lines of best fit). • Group presentations of solutions.

RESULT 2.1a

Responses to the four diagrams, Plot A, Plot B, Plot C and Plot D, were relatively good. However the problem was that all groups gave the responses as positive and negative relationships respectively and Pearson's correlation as direct decimals like -0.7 and 0.8 respectively to Plot A and Plot B. They were missing the opportunity to write statements such as, "appear to be taking a positive relationship, it appears negative" or "around -0.7 or around 0,8", as the values were supposed to be estimated and not accurately calculated. The opportunity to make statements about data using qualifiers such as "most" or "it is likely that" was missed. Some difficulties could be attributed to teachers' language ability and their lack of awareness of the need for precision within their statements. Irrespective of the nature or source of the difficulty, pedagogical content knowledge, particularly in this case, was critical.

RESULT 2.1b

Data modelling was easy for the groups as their knowledge of graphs and ability to draw graphs had been proven in the previous lesson in 1.2c. Although some groups did not have the model for the shapes in B and C they at least described them as "curving, towards quadratic, half quadratic, ogiving, parabolic", etc. Exploration of the concept of "modelling" is illustrated in the following conversation.

TEACHER: Well what I mean is the connection between the graphs/curves that we know and the dot plots represented in A, B and C. I am also surprised that we are always forced to give calculation of the model, yet I can just use my eyes and make a correct meaningful approximate deduction.

All groups analysed the scatter plots informally. Although the analysis was correct in all instances, some groups were unwilling to accept other group's analysis due to the lack of formal terminology of correlation concepts like direct or indirectly related. Through careful listening, the researcher was able to gain insight into the teachers' understanding of co-relation, despite struggling to understand the teachers' reasoning. Some groups did not like the phrases, "bell shaped" or "keeps going up or keeps going down". The teachers did not understand that it is through these informal discussions that learners finally come to understand the behaviour of bi-variate data and scatter plots. This is a very difficult concept to understand in statistics, consistent with the findings of delMas, Garfield and Chance (1999).Clearly annoyed with the many lines of best fit produced by all groups, questions about the reliability of statistical results ensued. The following is an example of one in-class discussion.

TEACHER: With so many lines of best fit it is a joke to deduce anything.

RESEARCHER: I thought, in your everyday life, you are always approximating happenings and results. For example, it is likely to rain. Would this be a joke, because in most cases it does rain.

TEACHER: Well then, where do we start (chorus from the group)?

RESEARCHER: Always give your learners the benefit of informal analysis. This can even help them when they get unrealistic answers in their calculations later on during analysis. For example in 2.1.a if they get an answer like 0.3 for Pearson's correlation then they will know that something is not right.

Informal analysis of data was a problem for teachers due to the premature introduction of formulas by people who do not understand the concept of correlation. Having been used to do calculation for lines of best fit and the Pearson’s product moment correlation coefficient, it was not easy for all groups to be satisfied with the approximation of relationships.

Later on the general trend of the analysis of the scatter plot became the appropriate use of the correlation language, such as, “It looks more or less like a… straight line, quadratic graph, positive relationship, negative relationship, etc. or even the line bent towards…” was used. This question served to demonstrate the teachers’ ability to interpret statistics verbally at a higher level than what they demonstrated in question 1.

Lesson part 2.1c

All groups agreed that there was a positive association between arm span and height, but they did not all like the disorderly positive nature of these dot plots. All the groups agreed that generally the taller the person, the longer the arms. However, they also realised that there were people in the world who were short, but had long arms. One group pointed out that the points that were far away from the line were the crippled or deformed persons in our society. The concept of outliers had not arisen with teachers prior to the in-class discussions.

Lesson part 2.1d

Most teachers in the groups said although they had never seen this method, it looked like an easier method as compared to the line of best fit. However, the following conversation was recorded:

TEACHER 1: But, as I indicated earlier, this is not a recognised method in the school statistics syllabus. I can never show my learners this method.

RESEARCHER: Are you saying if a learner represents his/her data this way in an examination, he will not be awarded any mark.

TEACHER 1: It depends on the marker.

RESEARCHER: Who is the examination marker? I thought you were the markers who do the marking. You discuss the marking at the centres before you start with the real marking.

TEACHER 1: The leaders at the marking centres listen to no one except themselves. (From other teachers: Yaa, you will be chased away if you dare to argue or give your suggestion, which is away from the syllabus. We need the money.)

RESULT 2.2

Bi-variate data from the newspaper, magazines and the hospital cards proved to be very difficult for the teachers to understand and analyse. The suitability of the data was given serious attention. The first example from the newspaper comprised data comparing interest rates in different banks and the annuity returns between a man and a woman. The time series bi-variate data from the newspaper required them to represent and analyse the data. The groups could not easily notice that the data was time series data. So a bar chart and a scatter plot were originally used for both sets. Later on in the process, some groups started to create line graphs. Others tried the options of stem and leaf, box and whiskers, etc. Some groups discarded what they had drawn. Some graphs were discarded as groups had problems finishing them off or the shape that came out of the representation was not easy to analyse or deduce. The teacher's questions were always, "How do you see the best representation when you are faced with a problem?" It was important for the teachers to see the representation possibilities of bi-variate data informally before wasting time on making useless representations. In addition, it was important for them to understand that there was no one way of representing data, but that there was always a best and relevant way of representing bi-variate data. One group had statistics software on a computer though they could not use it for the problem at hand. Teachers also argued that the data that were in newspapers is meant for people who had higher qualifications in statistics and who were working for insurance companies.

In the table on body mass index below is an example of an incomplete table of bi-variate data found in one of the magazines. The report is the, "*Mean Body Weight, Height, and Body*

Mass Index, United States 1960–2002", which presents trends in national estimates of mean weight, height, and body mass index from the National Health Examination and the National Health and Nutrition Examination Surveys between 1960 and 2002. The teachers struggled to understand it, and to justify the suitability of the representation of the data. It was evident that data in the magazines was not always easy to use for teaching and learning. Teachers also had major problems with the interpretation of the data. This implied that since teachers needed more time to search for relevant information from newspapers and magazines, and considering the tight syllabus, it was not always possible to achieve the exercise.

Body Mass Index

BMI	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**
Height (inches)	**Body Weight (pounds)**																
58	91	96	100	105	110	115	119	124	129	134	138	143	148	153	158	162	167
59	94	99	104	109	114	119	124	128	133	138	143	148	153	158	163	168	173
60	97	102	107	112	118	123	128	133	138	143	148	153	158	163	168	174	179
61	100	106	111	116	122	127	132	137	143	148	153	158	164	169	174	180	185

- **The hospital card problem**

One card of a baby from the clinic was provided. The hospital cards are normally handed out with an already-drawn line of best fit on a normal ogive shape. When mothers come to the hospital to weigh their babies, the card is dot plotted by the nurse to indicate the weight of the baby on that day of the month. Babies are weighed at about the same date each month. The child's health had to be analysed in terms of its graph (the ogive). The teachers in the groups found that the child of the provided hospital card was not growing well in line with the ogive. However, it was interesting for them to discuss that cases of children aligning perfectly with the ogive did not exist. The dot plots where always just near the ogive or away from the ogive. A very important discovery in their discussion was the need for approximations and estimations when reading data. Statistics emphasises estimations (approximates to the nearest of the value) and encourages the mention of the words "approximately" or "estimated that" or "it is likely to" or "it tends to move towards" in data analysis. The following conversation was recorded after the teachers looked at the real life examples above:

TEACHER 1: We simply do not have time to collect data from all the sources you have advised us to.

RESEARCHER: But don't you think it is important to let everybody know where they can get real data from?

TEACHER 2: I agree and just making the learners collect the data can show the learners that it is not easy to collect the data yourself. We will all start appreciating the data that we get from the textbooks.

RESEARCHER: You can also ask for data from Statistics SA.

TEACHER3: Those guys never responded to our requests last week. What makes you think that they will listen to a simple teacher using an internet café (they all laugh). We also do not all have efficient emails.

RESEARCHER: Yaa, I get you.

GROUP 1: This has nothing to do with the teaching of Grade 10, 11 and 12 statistics. We are wasting our time here (The group browsing quickly over it).

GROUP 2: Where do you get the time to do this? (While looking at the data with no interest at all).

Lesson part 2.3

The question on data collection methods provided responses including interviews, census, and simple counting. It seems nobody was aware of the possibility of collecting data during laboratory experiments, research or even during leisure time. It was further revealed that teachers were not aware that in order to collect data, one could start with a problem. The conversation below was recorded.

RESEARCHER: Are you not able to collect data from laboratory experiments?

GROUP 1:	Teacher A says, Yes, I guess you can, but I have never known what happens in a laboratory. Teacher B says “we never did science seriously”.
RESEARCHER:	What about in geography? Did you not collect data by, for example, just measuring the length of growth of plants in a garden every two days, or counting the colours of cars passing your school gate the same hour every day?
GROUP 2:	Teacher C: “Why would you do those things? You must be having a purpose”. Teacher D says, “It would be nice for the pupils, but where would you get that time to waste. I am sure you also wanted to do that with us, but you also don’t have time”.
RESEARCHER:	Can’t you start with a problem and then collect the data.
GROUP:	I guess we can.
RESEARCHER:	In your group, think of problems that can enable you to collect data.

It was revealed during the discussions that classroom investigations could be conducted using two different approaches. First, an investigation can start with a question or problem to be solved and move onto data collection (the hospital card type), which requires an understanding that data needs to be collected in order to solve the question or problem. The second approach is to start with a data set and generate questions for investigation from that data. By adopting this second approach for this study, teachers and learners were not faced with issues pertinent to establishing the need for data to help solve their questions.

Lesson part 2.3a

The posters, which were updated during in-class discussions, included star diagrams, flow charts and mind maps. All diagrams illustrated a free collection of data from textbooks, newspapers, magazines, Stats SA, statistics centres around the country, Department of Education statistics section, and Department of Institutional Statistics and Analysis in different institutions, and so on. It was apparent that the teachers are very resourceful, but they lacked information on the endless possibilities of where one could source data.

Lesson part 2.3b

The teachers described the need to collect interesting data with their learners as interesting and important as it would:

- Arouse interest and curiosity about a new topic.
- Inform the learning outcomes by collecting from their cultural environments.
- Influence feelings and attitudes about a topic.
- Allow teachers to learn more about research.
- Allow both teachers and learners to be involved in the whole process of dealing with statistical data.
- Enable both teachers and their learners to be confident with the results of analysis.

Table 4.3.4.5: Day 3 *:* Results for Stage 3

Statisticatisation (Finding meaning from bi-variate data e.g. modelling)		
Stage 3	**Outcomes (Key learnings)**	**How key learnings were achieved (Organised activities)**
(Appendix 5 Day3)	• To avail teachers with relevant tasks and activities that involve bi-variate data from the media • Metacognition (Deciding what to do with given bi-variate data sets • Deducting from data analysis • Group presentations on flipcharts	• Teachers represented and analysed data from the car industry • Teachers represented and analysed data from the sports industry • Deducing trends and use of data after analysis • Group presentations of solutions

RESULT 1

Lesson part 3.1 (a)

The husband and wife problem was first represented using two histograms and later a scatter plot. After a heated in-class discussion, all groups agreed that the best way to represent that data was a scatter plot. The rationale for this choice was that, including the information from the histograms as separate data, did not provide anything useful to discuss. Groups agreed to move beyond uni-variate data. In general, the teachers struggled with bi-variate data relationships and instead tended to focus on uni-variate data. The teachers investigated the advantages and disadvantages of representing data in a particular way after looking for interesting things within the data.

The histogram as a way of representing the given data evoked several interesting comments from the groups. An unexpected comment from a group member was, "It is a very good way of representing data as it revealed the ages of the males and females in each group". After some intense argument from the other groups, he failed to reveal the value of this type of information from the data. A question from one group was, "What do you need this separate information for?" However, this teacher did not have clear explanation of why he would need this information. Teachers argued and defended their thinking about the provided representation of data. The "aha" moment came when everyone realised the importance of appreciating data in whatever state it is in. For example, the following conversation was recorded for Task 3.1a).

RESEARCHER I did not follow the argument properly. So what are we all saying now about representing the data?

TEACHER 1: One must see whether the data is too little or too much.

RESEARCHER: If it is too little or too much. I do not understand. What do you mean?

TEACHER 1: If you know the amount of data, it is when you can then decide whether to do the dot plot, a bar chart, a line graph, etc. method instead of like that other group, it suffered with its use of bar charts and all that…

TEACHER 2: No. You must just start by putting the data in order.

RESEARCHER: Ok yaa you mean

TEACHER 2: Sometimes the data is in disorder like the one we saw in question 3. You can either arrange it in ascending or descending order.

Other groups later realised that there was always a need to consult the original question and data to assist with generating the value of the collected data for analysis. Teachers need knowledge and understanding of more than one representation of data. Some representations are more useful for revealing "stories" within the data than others.

Lesson part 3.1 (b)

There was a discussion as to whether the statement “all men marry younger woman than themselves” could be deduced, based on the collected data showing the data plots of husbands and wives. Groups commented on how this data is representative of reality and one group had a comment on outliers. During class discussions, it was agreed that the reality of this data lies in that it does not eliminate outliers. For example, it is normal to find a woman of 60 marrying a man of 40 or a man of 70 marrying a girl of 25. There was an intense debate about what an outlier really means. It was during these heated ‘arguments’ that the teachers’ level of content knowledge changed.

At this stage, all groups deduced that all the data was best modelled by a positive straight-line graph. The discussion of the need to use a computer surfaced. One teacher commented that, “It is very important to always start by dealing with little data so that learners can understand the concept before they moved to a lot of data which need the use of a computer”. The lack of computers in South African schools remains a challenge. Often, even if schools do have computers, they remain in a dedicated centre and teachers cannot access and be able to use them for teaching purposes.

The integration of statistical and contextual knowledge was characterised by the teachers’ ability to give contextual examples to support the finding that all men marry younger woman than themselves. This was revealed when some groups argued that this was a result of the old African cultural norm, as at present, men now marry older woman. One group argued that, “because of the economic hardships in the 20th century man marry older woman with money”. Another group gave the reason that these days people marry for money, and so the normal occurrence is not possible anymore. This shows an out-of-the-box thinking towards the real-life context in association with what the statistical investigation had revealed. The interplay between statistical and contextual knowledge, enables a greater level of data sense and a deeper understanding of the data, and is therefore indicative of a higher level of statistical thinking (Wild and Pfannkuch, 1999).

RESULT 2

Lesson task 3.2 (a)

Due to lack of foresight, some groups thought that in data set 3,2,1 on how the speed of a car affects the fuel consumption of a vehicle could still be analysed without arranging the data. It was only later that the groups noticed that the data was not in order. This problem was confirmed by retrieving some flip charts that had been thrown in the bin and using these to carry on with the in-class discussion. A scatter plot of unordered data was found in the bin. One group represented and analysed the data before arranging the speed in ascending order or descending order. Individuals in other groups also made the same mistake. Before the teachers went further with the analysis of data the issue of ordering data was clarified. However, some groups thought that a table of results was not a representation and this led to an intense in-class discussion about the exact nature of a representation. The teachers realising that realistic data is normally, not ordered, was a manifestation of the integration of statistical and contextual knowledge. Teachers can lead these types of discussion if they have both **specialised content and content and learners** teacher knowledge.

Lesson task 3.2 (b)

It was however not easy for the teachers to derive conclusions or a trend from the data set 3,2,1 and heated arguments arose within different groups. There was debate in the groups about what would happen if the speed went higher than 120 kilometres per hour. During class discussions, it was suggested that the teachers had to go and find out the answer from the drivers who they knew or the drivers who they met in the town where they were stationed. When one teacher asked how the information was to be collected, the response was for the teachers to find out how they were going to do it. A statement from one group was the idea of increasing the speed of a car per hour in order to save petrol. The other group commented "The table on how the speed of a car affects the fuel consumption of a vehicle shows a trend that the higher the speed of the car, the less its consumption of petrol per litre". The majority of the teachers agreed that if drivers travel at too low speeds they were likely to use more petrol and that between 85 and 120 kilometres, less petrol could be used. Moving away from the data teachers spoke about their personal experiences with cars. They agreed that after 130 kilometres per hour the car starts to use more petrol per kilometre.

Lesson task 3.2 (c)

While looking at the results of the Olympic Games from 1972 to 2004 in data set 3,2,2, teachers could observe and follow the improvement in the time taken by the participants. These ideas came from different groups. "The top athletes of the world have turned professional. This allows them to train at the best facilities and receive the best coaching available. Also, equipment manufacturers are in competition with each other. In this case, manufacturers are designing swimsuits that assist swimmers. Swimmers train harder and put in more effort". During presentations and discussions, the following conversation was recorded:

GROUP 1: There does not seem to be much time difference since 1968. The runners are the same level.

GROUP 2: Do you think we should have the South African Caster in any of the 2010 and 2011 results? Then you will see a big difference (Note: Caster is a woman with the body of a man. So she outperforms the women).

GROUP3: If runners use on steroids, then they can perform more than what a normal human being can do.

GROUP 4: Running is not an easy sport. I have come to realise that the body can only take pressure this far and not more.

In this section of data set 3,2,2, all groups observed that the winners did not have much difference in time. One group commented that, "If runners had taken part in these games in the same, it would have been difficult and tricky to find out who the winner would be, considering that at separate occasions they all took approximately the same time to finish the race".

Lesson task 3.2 (d)

Referring to the Olympic Games that took place from 1972 to 2004. (www.databaseOlympics.com.) in data set 3,2,2, and asked to predict the winning time in 2010 the teachers suggested exact times like 48.1, 48.2, 48.3, 48.4, 48.16 seconds, etc. However, there was no suggestion or indication from the teachers that a range, in which the value could lie, could be an answer. For example, it could be indicated that the time might lies between 48 and 48.19 seconds or [48; 48.19] in seconds. This was a revelation for the teachers as it was an example of how mathematics relates to statistics. In the subsequent whole-class discussion, the following conversation was captured:

RESEARCHER: Did you use the information from your representations to predict the winning time for 2010?

TEACHER1: Which representation are you talking about?

RESEARCHER: Did anyone find the winning time for 2010 through using your particular representations?

TEACHER 1: Yaa but our group used a real graph while I can see dot plots from other groups.

RESEARCHER: Oh, there are real graphs and some that are not?

TEACHER 1: Yes.

RESEARCHER: Can Ndate explain to the others what is a real graph and the one that is not?

TEACHER 1: A real graph has a line that joins points and like a scatter plot, it is not a graph.

TEACHER 3: Hey in some books this (pointing to their group scatter plot) is called a scatter graph. Well I suppose because we end up using a line of best fit, then it also has a line like you have described so we can also say it is a graph.

RESEARCHER: Maybe we need to have another look at that. (The meaning of a graph in this case became a critical issue. I had to reflect on this).

Lesson task 3.2 (e)

In data set 3,2,1 most groups indicated that the data on speed could be used directly by car drivers if they wanted to find out if the higher the speed, the higher the petrol expenditure. However, some teachers indicated that car dealers normally supply the information, when they are advertising the advantage of buying particular models of cars.

With reference to the data set 3,2,2 time series-data on the Olympic Games, the groups agreed that this information is useful to the runners and their trainers or directors if they are attempting to improve the runners' finishing time. However, some groups insisted that this data did not have much use to anyone. During class discussions, it was agreed that some collected data can end up not being that useful. This information was important for teachers to know. This is because some people think that all collected data, must be useful.

Lesson task 3.3

It became apparent that teachers represented data as a last resort, or else trying to analyse data in its given form. Other ways of ordering data were suggested. One group suggested that they could use the set theory to categorise items together and therefore group them. Some also suggested using cards to categorise all types of data. Learners would then put them in respective sets. The following conversation was recorded:

TEACHER 1: This section has opened my eyes on different ways of having the problem centred approach in action. I always thought that data is supposed to be analysed in the way it is given.

TEACHER 2: Yes finding other ways of collecting data is creative. Yaa it makes the lessons really active and interesting.

TEACHER 1: No, but the problem is our examinations keep setting the problem in one single way, with data that is already arranged. So we end up thinking, that it is the only way to do it"

TEACHER 3: I have always thought that that's the way they want it and yes if you think of other ways of arranging the data and then do it in your own way you won't have time and you will end up confusing the learners.

TEACHER 4: It is not about confusing learners, but about having them in control of the subject. But yes there is no time for other ways of doing things

TEACHER 3: Why don't you start your new thinking with the Department of Education? (there is laughter)

RESEARCHER: Yes you can tell the department of education that you do not want to have only one way of doing things. Also you can suggest to them that they must not set the same problems year after year. You can do it for them if you feel confident.

Classroom investigations can be conducted through two different approaches**.** First, an investigation can start with a question or problem and then move onto data collection. This requires an understanding that data need to be collected in order to solve the question or problem. The second approach is to start with a data set and generate questions for investigation from that data. It was also evident that the second approach was the one always adapted in the classrooms. It was apparent from the groups that both of the above methods were crucial for classroom investigations and one did not override the other.

Teachers' open-ended questioning and listening skills were important techniques observed in the groups. Teachers with less experience in the classroom were more willing to take risks by expressing their tentative ideas while the more experienced teachers were reserved and struggled to change their ideas or ways of thinking. The researcher's style shifted to provoking teachers to consider and justify their expressed ideas in order to support and enable the teachers to change their ideas in the light of evidence. The idea was to empower the teachers so that they can take control of their teaching of bi-variate data. As teachers develop their experience with pedagogical content knowledge they become successful statistics practitioners because of their ability to attend and interrogate the focus of their subject as their attention to the learner increased. Teachers learnt more from flip chart presentations and in-depth in-class conversations than from humiliating and attacking those that did not know. Many of them indicated that by listening carefully to other group presentations, their understanding of specific learner conceptions and types of reasoning had improved. Working in a community of practice, the development, proved to be more productive for the teachers than originally anticipated or experienced earlier in their places of work.

Table 4.3.4.6: Day 4 : Results for Stage 4

Realistic/case based problem posing		
STAGE	**Outcomes (Key learnings)**	**How key learnings were achieved (Organised Activities)**
4 (Appendix5 Day 4)	• Putting teachers into the statistician's shoes • Putting teachers into the headmaster or Department of Education's shoes. • Group presentations on flipcharts. • Teachers' individual response to reflection later in the day	• Analysis of a problem in bi-variate data, to be used by health professionals. • Analysing a problem in bi-variate data through a rubric intended to be used for hiring a statistics teacher for the school. • Group presentations of solutions • Teachers got an individual reflection activity, recorded in the next lesson.

RESULT 4.1

The Freudenthal cycle played itself out at each stage of the pedagogical content knowledge framework. For example, using a case-based problem, teachers were put in the researcher's shoes, acting as advisers in Task 4.1a) and in the employer's shoes in Task 4.1b. In a reflective manner, the teachers were able to extract crucial issues from the provided bi-variate data task set as well as investigate the reasons for every decision taken. In Task 4.1b) teachers were asked to formulate relevant questions from the provided data for use in the interview of a potential statistics teacher to be employed in the school. Different questions originated from the groups. These questions were evident of the teachers' mastery of **common knowledge and content**. Groups presented different information. Some information (considered important by other groups) was missing from some groups and some groups had more than what was expected. Some of the information was more orderly than information from other groups. The planning from the groups included familiarising with the data, organising and choosing one or more relevant representations, modelling, analysing and writing the reporting.

RESULT 4.1 (a) Familiarising with data

All groups introduced their planning by discussing the terms "death anxiety" and "religiosity". Very serious and sensitive comments came out of the in-class discussions arousing much emotion, which I had to control in order for the facilitation to progress. Religion is a very sensitive issue. During planning, one group in particular expressed their concern about the problems associated with the relationship of death anxiety and religiosity before they could prepare it for analysis. One group was worried about whether they were allowed to correlate whatever one wanted even if the data was sensitive. The following questions came out of the groups during presentation:

- We are sure that there is a church which they are targeting. Whenever there is this kind of research there is a target.
- Does this kind of survey not affect people psychologically, if they know that their death anxiety is high?
- What did these people use to measure death anxiety and how accurate is it (this reminded me (researcher) of the noise about aptitude tests in the 90s)?

The inability to differentiate between correlation and cause was evident from the teachers' statements in almost all groups. The problem that most teachers had with correlation was remembering that correlation does not measure cause. During the class discussion, the researcher asked groups to give examples of relationships that could cause individual or national outcries if correlated. The following conversation was recorded.

TEACHER 1: Some religions actually cause death anxiety. Like the Apostolic faith or the ZCC.

TEACHER 2: I actually attend the Apostolic Faith Mission church. Tell me how it causes death anxiety.

TEACHER 1: Like the Moslems and the Jews, they believe in more killing and dying in the church. You all know that. Their wars are all about that.

TEACHER 2: Yes, but tell me how the Apostolic church causes death anxiety?

TEACHER 1: They keep prophesying about bad things in your life. In the end, you get so scared that you want to die to escape or to revenge to save yourself.

TEACHER 3: Yaaa these prophets that exist everywhere are lying and causing death anxiety. Then which church is perfect? Perhaps, the traditional churches. Roman Catholic for example.

TEACHER 4: That is why I do not go to church. You see I don't want my brain to be tampered with.

TEACHER 3: The problems that we all have can cause death anxiety. If you don't belong to a church you will die, probably from death anxiety. (There is laughter from the teachers)

TEACHER 1: Yaa especially if you are HIV.

RESEARCHER: Ok Ok This is interesting, but remember, we are not looking for causes. We are just looking for the relationship the collected data has to each other. Our interest is correlation and not what causes the other.

Teachers demonstrated evidence of **common knowledge of content** by integrating statistical and contextual knowledge while analysing the data. In integrating bi-variate data and real-life situations, the above conversation is in line with **knowledge of content and teaching**.

Representation

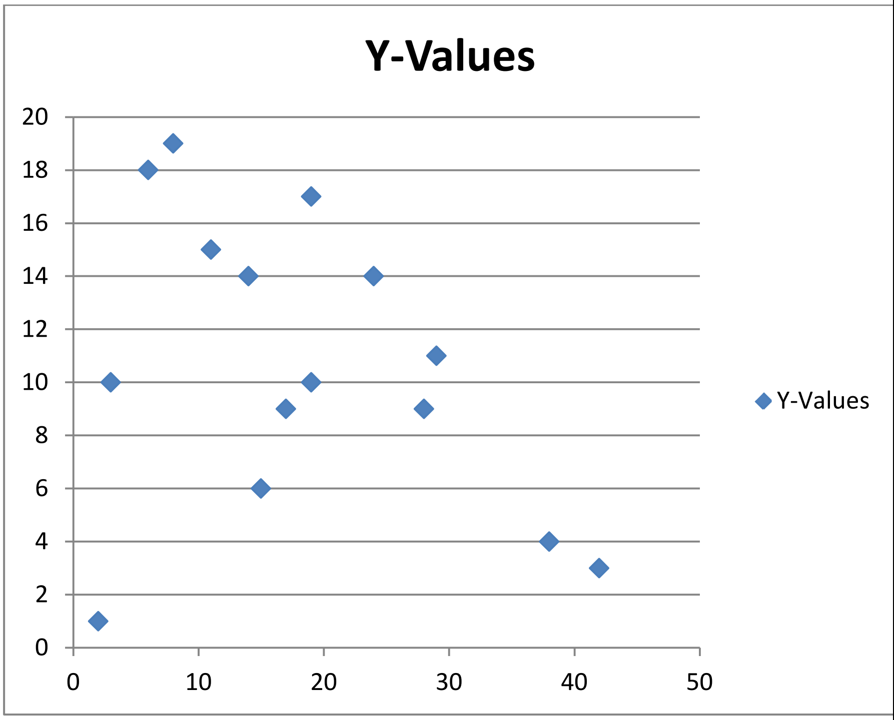
During the in-class discussion, the following conversation was recorded.

RESEARCHER:	Ahaa.
TEACHER 1:	So you indicated that you need one or more representations. Is one not enough?
TEACHER 2:	Teacher 2 says, "We have all realised from our previous discussions that some representations are not clear enough. More representations can clarify the issues".
RESEARCHER:	Ok (Clearly impressed by the nature of their discussion so far.)

Many groups represented their bi-variate data using scatter plots, but some represented it using a line graph as well as a confused histogram.

Modelling

Table 4.3.4.6a: Grouped data and scatter plot



Death Anxiety	**Religiosity**
2	1
3	10
6	18
8	19
11	15
14	14
15	6
17	9
19	10
19	17
24	14
28	9
29	11
38	4
42	3

An example of the data modelled and analysed by the teachers is given above. The results given were that it was clear from the dot plot above that, there was no meaningful relationship between death anxiety and religiosity. However they found it difficult to deduce anything from the grouped data table.

Analysis

Groups carried out the analysis differently. Some groups did informal analysis from the table and others did informal analysis from the scatter plot. One group had this to say, "The values show that lower religiosity means high death anxiety" or "the scatter plots is sort of negatively correlated so that lower religiosity means high death anxiety" (The analysis was done visually by looking at the table of values given). In some cases, all conditions were determined (line of best fit, value for Pearson's correlation) and analysis was done statistically using the statistical language. In some cases, the scatter plot or dot plot was drawn and the line of best fit was not fitted. In other cases the scatter plot or dot plot and the line of best fit were drawn.

In relation to classroom discourse, teachers at this stage had a high level of **content knowledge, specialised content knowledge**, and therefore asked fewer questions, but the questions were of higher order than the questions asked earlier on in the development. Groups appeared to either notice, or overlook, other group's use of informal statistical terms. One group analysed data using direct and indirect relationship language.

Report, advice and recommendations

The teachers provided interesting advice in their report. All groups agreed that people, young and old, must attend their different churches or religious groups if they are to lead healthy, stable lives. One group was concerned that churches actually instil fear and therefore death anxiety in people, although there was no agreement on which churches or cults did this. The class atmosphere was tense and the researcher managed to continue by saying, "If all the churches or religious groups aspects were good then…". the teachers finished the sentence as they wished. However, only one group noticed and discussed an outlier in the bi-variate data. The reasons for the existence of an outlier also helped to make the understanding of bi-variate data clearer. Aspects of religion that pacify people were also discussed, including collegiality, communication, support, bible readings, communities, as well as how the existence of death anxiety can be attended to medically.

Another argument arose from what variable caused the other. In other words, the argument was "Does not going to church cause death anxiety or does death anxiety cause people not to go to church". The class atmosphere was again very tense until it stressed that this was just a correlation.

RESULT 4.1 (b)

The following is an example of a rubric prepared by a group with questions to be answered by a teacher likely to be employed by the school.

Table 4.3.4.6b: An example of a rubric from one group

	Criteria	**Mark**
1.	What is the difference between uni-variate data and bi-variate data?	2
2.	What do you consider to be the most important thing to understand in bi-variate data?	2
3.	Where can a scatter plot be used best for a statistical analysis?	2
	Is the scatter plot the only way to represent a statistical relationship of two variables?	2
	Where do you normally use a scatter plot to define relationships?	2
4.	What do you consider as the best strategy to teach bi-variate data with understanding?	2
5.	What other topics do you need your pupils to know before you can teach them bi-variate data?	2
6.	Do you think that there is a relationship between mathematics and statistics?	2
7.	What would be your objective in teaching a section with the diagram shown?	2
8.	What problems do you normally experience when teaching the section of bi-variate data?	3
9.	How have you tried to go over this problem?	2
10.	Is cause always an issue in correlation?	2
11.	What prerequisite knowledge might learners in your class not have in order to deal with the above task?	4
12.	What can be done to overcome the pupils' misconceptions about this task?	4
13.	What kinds of questions can be asked to your learners to understand their misconceptions?	4

14.	As a result of the facilitation today what then are you going to start doing or stop doing?	4
15.	etc	

These rubrics sparked interesting discussions, revealing that through this facilitation the teachers had reached a high level of pedagogical content knowledge. The teachers were actively looking for information that would help learners to understand their subject statistics.

Table 4.3.4.7: Day 5 : Results for Stage 5

Assessment of pedagogical content knowledge (metacognition)		
STAGE	**Outcomes (Key learnings)**	**How to achieve the key learnings (Organised Activities)**
5 **(Appendix5 Stage5)**	• Putting teachers into the learner's shoes. • Putting teachers as teachers into their classrooms. • Meta-cognition (The teacher as the assessor) • Group presentations on flipcharts. • Group reflection	• Teachers get a bi-variate data task from a past end of year examination paper to work on. • Teachers plan a lesson that would enable learners to respond to the above problem (clarifying aims and objectives) • Teachers respond to the problem as they would want their learners to do and prepares it for assessment. • The recording and discussion of group presentations • Reflection

RESULT 5.1

a) Overall Planning

The teachers' lesson preparation plans were either directly adapted from the unit plan (DoE 2006) or were a formulation of their own ideas. The plans included introducing bi-variate data, identifying big ideas, overall planning of the teaching stages, organising and choosing one or more relevant representations, modelling, analysing and writing a concluding conversation from it. Teachers integrated their understanding to plan for appropriate learning outcomes and their ability to use learner responses to devise teaching intervention. Sometimes clear indications of the aims and objectives of the lesson were not provided. The provided plans included the general approach to be used for preparing a lesson.

- Asking learners about what they could remember about statistics, particularly from a unit that the learners had completed earlier in the year.
- Introduced the learners to the bi-variate data set that was provided as part of the question.
- Possible data questions or possible group activities that could be generated from the data (for example which data sets can be of interest).
- Ways to discuss possible data representation and analysis (A scatter plot was drawn and a Pearson's correlation coefficient unit was informally approximated).
- Questions relating to real life situations (A story was generated to explain the situation).
- Discuss exam expectations.

The teachers in the group then sorted and investigated the data, before presenting their results during the in-class discussion. The lesson plans from the different groups were all different in the levels of information they offered, some with no useful information at all. The teachers learnt the best way to generate a meaningful lesson plan from the other group's presentations. Teachers discussed the interesting data sets for investigation. The groups chose either two categorical variables (favourite activity and eye colour), two sets of numeric variables (age and weight), as well as (weight and fast food meals per week). For these chosen data sets, the groups suggested possible data questions. Consequently, the teachers decided that the data set between weight and fast food meals was more meaningful than the other data sets that had been suggested upon.

The teachers started thinking about what problems the learners would have while solving their formulated problems. Posing questions for investigation caused many problems for the teachers. Realising that teachers are likely to find it difficult to pose appropriate investigative questions is one aspect of the **knowledge of content and learners**. Teachers acknowledged that this was the hardest aspect of statistics, yet it lays the groundwork for clearly understanding statistics in the first place. Groups first ordered the data and then represented it using the dot plot, modelled it before analysing the data. For example, questions like, 'How could we extend that question to make it more specific.' added value to the in-class discussions. The following are some of the results recorded from the flip charts during in-class discussions. Working together increased group cohesion and increased a feeling of ownership in the programme as well as discussion with their peers around possible solutions to problems that arose. The following a1-a6 are some results that originated from the groups.

RESULT 5.1 (a1) (Introducing bi-variate data)

One group's strategy was to spend some time in the lesson talking with the class about relationships. This included finding out the background knowledge of the learners and what they understood in the word data. They needed to discuss with the learners some examples of relationships that were considered worth investigating. This was an indication within the teachers of the existence of the **knowledge of content and learners and knowledge of content and teaching** in action. Teachers used an appropriate action for investigating relationships in bi-variate data. Using this strategy, the teachers handled transnumeration of the bi-variate data well, and were able to find some interesting relationships within the given data in the provided **'Complete data set'** Table 5.1. A class exercise like this would really enrich the knowledge of bi-variate data.

RESULT 5.1 (a2) (Big ideas)

All groups agreed about the need to first identify the big ideas that come from a lesson in order to plan for the lesson well. Several big ideas made a good lesson. A significant number of "missed opportunities" resulted from the different ways in which teachers chose their big ideas and therefore the different ways in which they planned. A number of these missed opportunities can be attributed to different interpretations of the questions that teachers posed for the data. Few teachers specifically articulated the big ideas encompassed by the problem, but mostly implied an understanding that reveals itself over the problem.

RESULT 5.1 (a3) (Identifying the relevant relationship)

Encouraging statistical practices of explanations and justifications was revealed when teachers suggested relationships from the given data that did not provide meaningful information. This was very difficult for the teachers as they are used to working from already chosen big ideas. Data is organised and arranged for them. For example, the multivariate data set in Task 5.1 that includes age, favourite activity, eye colour, weight and fast food meals per week could result in a number of bi-variate data. Some of these investigations were not as interesting or productive as others were. For instance, investigating the relationship between eye colour and weight proved not to be as illuminating or interesting as that of the relationship between weight and fast food meals per week. For the teachers to recognise the interest and potential in an investigation compared with another is a key factor in developing the skills and understanding that is necessary for successful investigation. The teachers agreed and motivated on what they thought were the most relevant relationship. The following were some of the identified relationships.

- Weight and fast food meals.
- Favourite activity and weight.
- Age and fast foods.
- Age and favourite activity.

RESULT 5.1 (a4) (Representation)

Again, subject to previous discussions in sections 3 and 4, only scatter plots were suggested as the best representation for the provided bi-variate data.

RESULT 5.1 (a5) (Analysis)

Task 5.1 used in the study involved putting teachers into the learner and the teacher's shoes. All groups came with different suggestions for analysis and therefore a solution for advice. There were a number of instances when groups suggested inappropriate data analysis. Some groups came up with solutions that they were unable to defend. The inappropriateness of the questions was an indication of the teachers not using **specialised knowledge of content** while making meaning of bi-variate data.

For example, the informal language suggested included "not very well related", "as one goes up the other goes down", "the run and rise". Another intense argument ensued from the teachers' search for what the learners would understand from this informal language. For example, what would learners understand from "rise and run". Analysis suggested by the teachers at this stage of the facilitation was now of a more mature and high-level standard.

The fact that the teachers had more knowledge about bi-variate data than they previously had led to well prepared lesson plans. The following conversation was recorded.

TEACHER 1: There is very week correlation ‘these nutritionists lie to us’.

TEACHER 2: It looks like when there are fewer visits to the fast food per week (< 6) then there is no correlation. However if there are more visits to the fast food per week (>6) then there is a positive correlation.

TEACHER 3: You get fat if you eat too much MacDonalds.

TEACHER 4: Hey, I have never seen such a confusing scatter plot.

TEACHER 5: There is no correlation in this scatter plot.

RESULT 5.1 (a6) (Contextualising)

The following conversation was recorded when teachers were comparing age and fast food meals.

TEACHER 1: There is very weak correlation. I think it is because those with money go to buy the fast foods. Those with no money do not have much of a choice.

TEACHER 2: It looks like those who are older eat more fast foods because they are probably free to do what they like (they do not have children at school). No correlation. But the older people with no money did not buy (it’s a problem).

TEACHER 3: Haaa I think this is not a nice relationship. When we are at work these relationships do happen. Some teachers buy fast foods, and some do not buy.

TEACHER 1: Ha ha it is not the older ones that buy, it is the single ones who are not married or those who chased their wives away.

TEACHER 4: Hey I have never noticed that the single ones munch too much. (Laughter from every one) I will also chase my wife away then I can be free to buy fast foods. (Laughter)

TEACHER 5: There is no correlation in this case.

RESEARCHER: Well its interesting.

RESULT 5.1 (a7) Possible student misconceptions

Teachers predicted that learners could have problems around interpretation of a data collection or use of data analysis to make meaning of it in real life. They predicted that the analysis phase of an investigation would present challenges for learners trying to decide on the form in which to present the data. Some teachers were aware that learners could be challenged when moving from the analysis stage to the drawing of conclusions for real life or the answering of questions that form the basis of the investigation. Such awareness meant that teachers had reflected about how to address the learners' difficulties. Teachers suggested how that they would deal with these potential problems within an early phase of their teaching. Knowledge of where learners might encounter problems or particular challenges in an investigation, are aspects of **knowledge of content and learners**.

RESULT 5.1(a8) Problem solving and assessment

The results of the problem solving were different among the groups The following conversation was recorded.

TEACHER 1: I don't need to use a rubric. I know mentally what is important and what is not. I just put marks in front of the important step.

TEACHER 2: But then you left important steps that we expect from the learners. So, if it is being marked at the end of the year it might cause problems.

TEACHER 3: Let us agree that learners will respond to this problem differently. As long as we get to the same final answer or result.

TEACHER 1: Yes. What we see as big ideas that need to be stressed during the teaching of a lesson are not the same.

TEACHER 4: Hey The problem comes when we don't arrive at the same answer or conclusion. But then, method marks must be awarded at the end of it all

TEACHER 5: There is no one size fit all here. We need communities of teachers to deal with these issues. When you are alone, its tough and dangerous.

RESEARCHER: Ok this is very informative.

Teachers also gave different memos for the problem. Some groups awarded different marks for the different parts of the memorandum.

RESULT 5.2 : Reflection

It was revealed that some teachers found the need to use the provided rubric for reflection in their fields of teaching. Teachers also added that the rubric can be improved upon.

Table 4.3.4.7a: An example of a completed tabular rubric by one group

CoRe: Content Representation Tool (Loughran et al., 2004)	
Big Idea:	**Correlation**
1. What do you intend learners to learn about this idea?	We intend to make them learn about relationships and learners must learn about strong, weak or no relationship situations differentiate between simple and causal relationships.
2. Why is it important for learners to know this?	This section allows learners to prepare themselves for using formulas to explain relationships.
3. What else do you know about this idea (that you do not intend learners to know yet)?	We do not want learner to know how to calculate lines of best fit and Pearson's correlation coefficient.
4. Prerequisite knowledge learners in your class might not have in order to deal with this idea?	The nature of data. Direct and indirect/inverse functions.
5. Briefly brainstorm topics which you might include in the unit.	Drawing of graphs or curves (from linear curves). Positive and negative relationships.
6. Knowledge about learners' thinking which influences your teaching of this idea?	Learners still have the idea of direct and inverse functions in their minds. Learners have a strong sense of relationships more of a causal nature in their lives. In other words, most relationships are causal (e.g. naughtiness in class causes failure)
7. Other factors that influence your teaching of this idea?	The lack of technology use in this area limits the learners' free discovery of the concept of Pearson's correlation coefficient.
8. Teaching procedures (and particular reasons for using these to engage with this idea)?	Group work enables learners to realise that there are several responses to a statistic relationship. Lots of ideas can come out of relationships in bi-variate data if it is approached in a well-directed project.
9. Specific ways of ascertaining learners' understanding or confusion around this	The serious class discussions that open up the results of groups or individuals clarify concepts and eliminate possible confusion.

idea?	
10. As a result of the facilitation this week, what are you going to start doing or stop doing?	Big ideas need to be analysed well in a team before you start teaching it to the pupils. One needs a relevant professional development in order to be confident to take all the decision I have learnt through pedagogical content knowledge. Communities of learning are very important.

The rubric provided teachers with an opportunity to predict that learners may have problems interpreting some data questions. Based on this knowledge of learners, the teachers considered how to approach their teaching to avoid or ameliorate these problems. By structuring the teaching in this way, this section successfully utilised **knowledge of content and teaching**. In groups teachers agreed that the table had valuable information that all teachers could use when they went back to their classrooms. Teachers need help to think more complexly about their practice and the reason behind their actions in the light of how particular learners learn and in relationship to specific formal academic knowledge (Bullough, 2008). The teachers' responses reflected on **knowledge of content and learners**.

4.4 Discussion: Significant themes

This section has centred on a number of significant themes arising from the analysis of and issues related to the teachers' pedagogical content knowledge. In the analysis of the teachers' pedagogical content knowledge, some important and recurring themes arose, and were either relevant and encompassed by the components of pedagogical content knowledge, or directly statistical in nature. In some cases, they were relevant to both. In each case, the themes were identified by how they were linked to the learners' conceptions, misconceptions and general error pattern analysis.

4.4.1 Realising the need for data

Recognising the need for data to answer questions was one component of statistical thinking not originally in evidence. The approach during facilitations involved teachers investigating multivariate data sets that were given to them in order to find interesting things in the data. This approach was based on a question or problem being posed, after which teachers would recognise that data is needed and that it must be collected and analysed in order to understand the question or problem to be solved. For learning to take place, it is important for teachers to know how to manipulate the different types of data so that they can respond to and guide their learners. Knowledge of effective transnumeration of data include knowing the difference among data types and how these differences might affect data sorting and analysis. The teachers' difficulties with handling data, aligns with similar findings from Chick, Pfannkuch, and Watson (2005) in their research of Grade 6-9 learners. Within this study, it was therefore reasonable to conclude that when the need for data was not observed, this did affect learning opportunities for both the teachers and the learners.

4.4.2 Moving from uni-variate to bi-variate data

It is well recognised that part of a needed development of understanding in statistics involves the shift of focus from individual data to considering sets of data. In Task 3.2, although teachers showed **knowledge of content and learners** in understanding that the learners tended towards uni-variate data investigations, they originally did not have good **knowledge of content and teaching** to be able to handle the bi-variate relationships adequately themselves. Teachers find it difficult to move from using uni-variate data to investigating relationships with bi-variate data (Chick et al., 2005; Mevarech and Kramarsky, 1997)).

In this research, it became apparent that the teachers had an important role to play in helping each other make this shift. Teachers' questions, explanations, responses (including lack of responses) sometimes promoted, unintentionally, a continuing and unwarranted focus on individual data. The researcher realised the need to encourage the teachers to look beyond one variable and to remember this same challenge for their learners during their teaching through the **knowledge of content and learners**. Insufficient **knowledge of content and teaching** meant that the teachers could not really assist learners in overcoming their difficulties in comprehension. However, after the development sessions, one could see and realise greater understanding on the part of the teachers.Utilised knowledge of **content and teaching** steered the discussion away from only individual values to consideration of group features in the data. One possible explanation for such a phenomenon was that the teachers, in most cases, did not recognise that they were referring to individual data and were therefore unable to draw from **knowledge of content and teaching**.

4.4.3 Teachers familiarity with bi-variate data

Situations arose within the facilitations displaying that the teachers were not familiar with the data that they were investigating. This lack of familiarity with the data manifested itself in various ways. There were instances when the teachers could not readily formulate a question to investigate, as they did not know what would be achievable with the given data. There were also instances when the teachers could not readily formulate a question to investigate. Being unfamiliar with the data was more evident in the situations where teachers had posed a question, but were unable to provide an answer for it. It is important for the teacher to understand the nature of collected data in order to apply relevant procedures and processes for them. The way to arrange, represent and analyse data originate from familiarising oneself with the nature of collected data. It was crucial to bring out the importance of the whole outlook of data instead of fixing their minds on only a small part, hoping that it will say much about a lot.

Alternatively, having a greater level of pedagogical content knowledge in a number of dimensions could have ameliorated the effect of not being sufficiently familiar with the data. Indicating the need to know the data reflects something about the difference between statistics and the deterministic nature of mathematics (Moore, 1982; Pereira-Mendoza, 2002). For example, being familiar with the data can give teachers more “resources” with which to evaluate a response and, as such through the **knowledge of content and teaching** would contribute to enabling the teacher to give learners guidance with formulating questions that can lead to a worthwhile investigation.

One way in which the development was able to assist the teachers to familiarise themselves with bi-variate data was by encouraging them to use different representations for data before pining down to the appropriate data representation. Some representations are more useful for revealing “stories” within the data than others. Familiarity with the data contributes to **specialised knowledge of content** in relation to various aspects of statistical thinking components. Being able to guide learners to formulate worthwhile questions for investigation is an example of **knowledge of content and teaching** and knowing some of the possible findings from a data set that the learners should be able to determine is an example of **knowledge of content and learners**.

The analysis of the experiences in this study established that a teacher’s lack of familiarity with the data can affect a number of different categories of pedagogical content knowledge negatively, thereby affecting the possible learning opportunities for the learners. It is impossible for a teacher to know everything about a data set or be able to evaluate every learner statement “on the spot”. However, to have a general familiarity with the data, and what learners might find, is an important aspect of being prepared for the learning opportunities that might arise. Consequently, a teacher's lack of familiarity with the data can affect a number of different components of pedagogical content knowledge negatively.

4.4.4 Statisticatisation (Finding meaning from bi-variate data)

In Task 4.1 teachers were encouraged to fully undertake and engage with an investigation that demonstrated **common knowledge of content** in finding meaning from the given bi-variate data. The groups of teachers posed an appropriate question and came up with a solution. Through analysing data resulting from the question they posed, they were able give advice to the community. In Task 3.2, questions about predicting the future using the results of the data that had to be ordered first, is a good example of statisticatisation. Engaging with data and being involved in "reflecting" with it is evidence of the **knowledge of content and teaching**. Developing questions that the data may potentially be able to answer is an aspect of **common knowledge of content**.One of the four dimensions of statistical thinking, as defined by Wild and Pfannkuch (1999), is the investigative cycle. This cycle is characterised by the phases of "problem, plan, data, analysis, and conclusions". It is what someone goes through and thinks about when immersed in problem-solving using data.

A teacher needs **specialised knowledge of content** when dealing with learners' questions or answers in relation to finding meaning with data or when discussing or explaining various phases of organising, representing and analysing of data, as well as how these phases might interact. When thinking about suggestions for what could be investigated in a data set, teachers need to be able to evaluate the suitability of the problem/question and whether it needs to be refined to be usable and suitable, in relation to the subsequent analysis.

During in-class discussions, it was revealed that learners are likely to have difficulty in linking knowledge of the real world with class work. For example, when there is a census, the teacher and learners do not relate census to data that they could or can use in the classroom.

Making sense of teachers' explanations around to the possibility of generalising from the data at hand to a larger group involves **specialised knowledge of content**. For instance, teachers questioned whether a husband's age always meant a younger or a wife with a similar age to the husbands. It was also generally agreed that incidence of the men marrying women older than themselves are situated as outliers. **Knowledge of content and learners** manifests itself when teachers know what learners may struggle with in relation to understanding bi-variate data, and can predict how learners will handle tasks linked to bi-variate data. Learners' appreciation and thinking about bi-variate data while looking for patterns and trends in the data is something that a teacher needs to listen for in learners' explanations and generalisations. How to structure teaching for understanding bi-variate data is the main component of **knowledge of content and teaching**.

4.4.5 Reasoning with models

From already-drawn scatter plots, the teachers had to justify the results of their chosen models, their lines of best fit and their approximation of the Pearson correlation coefficient. For teachers to be able to make sense of data, requires the use of appropriate models that can be graphs or tables. Teachers demonstrated evidence of c**ommon knowledge of content** by reasoning with models and making valid statements for the data, based on an appropriate use of the model. For example, teachers analysed an ogive problem of the hospital cards for babies' weight in Task 2.2c. There was a time when the child grew very fast (i.e. above or in line with the ogive). Then there appeared to be no growth in successive weeks (i.e. no movement on the ogive or growth below the ogive) and then it normalised again.

The emergence of the component of **specialised knowledge of content** while working with models occurred on a regular basis, especially as the focus of the development was on finding interesting things in bi-variate data sets, and making statements about these data sets. In many cases, teachers justified their statements through reference to the model. Teachers could choose from the numerical values of correlation between -1 to 1 or discuss some other models other than the straight line that come out of the data **Specialised knowledge of content** while working with models is needed to interpret statements that determine the validity of statements. For teachers, this translates into their learners often struggling with making sensible and valid statements about data based on a particular model they had chosen. One group mentioned that the South African syllabus is very limiting and confusing as far as the use of models was concerned. The following conversation originated from an in-class discussion.

TEACHER 1: The South African syllabus concentrates on one model, which is the straight line. So, I cannot play around with any other models

TEACHER 2: Yaa, for example the time to teach bi-variate data arrive before you even deal with quadratic, exponential functions, etc/

TEACHER 1: Yaa, so the learners' models are always in the form of a straight line, and it is difficult to later change them.

TEACHER 3: Hey, therefore to change the learners' thinking means starting the whole lesson on various graphs and models again, and where do you get that time.

RESEARCHER: OK, so you can perhaps be involve in the structuring of the school statistics and mathematics curriculum in order to bring this short coming to the fore.

4.4.6 Meta-cognition

Teachers prepared themselves to what their learners could find in the data and what conclusions they might draw. A statistical thinker engages in meta-cognition when working with data through activities such as (Wild & Pfannkuch, 1999):

- Generating possibilities.
- Seeking or recalling of information (from within the data or from a wider context).
- Interpreting the results of seeking (by linking with the results obtained from analysis; by comparing and contrasting; and by making connections)
- Criticising the information and ideas as they evolve (with both internal and external reference points, a form of metacognition, monitoring one's own thinking).
- Judging what to ignore and what they now believe or know.

For example, Task 5.1 enabled the teachers to immerse themselves into a data set prior to using it for teaching. Using **common knowledge of content** the teachers were made aware of several ways of dealing with data, **during meta-cognition**. Teachers had to identify two sets of data that could be used for the investigation in bi-variate data. They also had to think why the other combinations would not be the best choice. When a teacher has to consider the best information for investigating within bi-variate data, the teacher requires **specialised knowledge of content**. It also involves determining whether a suggested way of handling and sorting the data would be useful to enable the later interpretation of results in relation to the question at hand.

Task 5.1 also prepared teachers to deal with the entire processes of data handling with bi-variate data. Knowledge of how teachers would handle the development of appropriate questions for investigating the data, and the extent to which they might engage with the data and be prepared to consider various possibilities, are elements of **knowledge of content and learners**. The tendency of a teacher to ignore a wide range of possibilities and, instead, be content with a narrow, restricted focus in their investigation of data, constitutes a part of **knowledge of content and teaching**. Being able to consider, from a statistical point of view, how such limited views of the data might affect an investigation is another component of this category of teacher knowledge.

Although the teachers adopted slightly different approaches to planning for Tasks, the posing of questions for investigation proved to be difficult for them. Although the groups appeared to have reasonable questions in mind, they did not seek clarification in order to refine the question and give the subsequent investigation more direction and purpose. Posing questions for investigation is an approach that ensures that teachers are constantly being encouraged to think about the data, before, during, and after sorting and examining the data. Realising that teachers are likely to find it difficult to pose appropriate investigative questions is one aspect of the **knowledge of content and learners** and helping them with the posing of such questions relates to the **knowledge of content and teaching**.

Before the in-class discussion, most teachers mentioned only algorithmically based mistakes or reading comprehension difficulties as possible sources of learners' difficulty in understanding the section of bi-variate data. After the intervention, most teachers were familiar with various sources of bi-variate data misconceptions. They identified, among others, failure of learners to acknowledge the relationship between mathematics and statistics, trying to analyse data without ordering and summarising the data, and failure to familiarise themselves with data in order to choose appropriate statistical decisions. Central to pedagogical content knowledge is the teacher's goals as regards the responsibility to be sensitive and responsive to the statistical thinking of the learners. If a teacher readily anticipates learner answers and demonstrates an understanding of their reasoning, there are greater opportunities for connecting with other components and maximising the learning potential of the task.

4.4.7 Mis-intepretation to questions and answers in the statistics classroom

Missed opportunities arose in relation to the teachers' listening to, or interpreting, each other's statements or questions. If teachers do not always respond appropriately to the questions or statements of the learner, due to misinterpretation, the teacher's response will not address the intent of the learner's statement or question. So if, for example, a teacher does not evaluate a learner's response (as one type of listening problem), it is reasonable to conclude that the teachers may not have the knowledge of how to go about evaluating the correctness, or otherwise, of that response. A teacher's non-evaluation of a learner's response or question indicates a missed opportunity to analyse whether a learner's answer or explanation is correct or reasonable. For example, one teacher shouted at another:

TEACHER 1: But how do you collect qualitative data for analysis?

TEACHER 2: (with no regard for the other teacher) Just as you do with quantitative data.

RESEARCHER: (Teacher 2 showing sign of not having understood what Teacher 2 meant) Ok, perhaps you must go to the front to explain it properly to your friend. I do not think that he got your explanation (Teacher 1 is nodding in agreement)

TEACHER 2: By the way, what was your question?

This was a clear example of classroom situations when teachers respond wrongly to learners, when they actually had not understood the learner's question. Three types of listening problems were identified in relation to this missed opportunity. First, the teachers did not hear, or misheard, the questions or comment, and consequently responded in a way that was inappropriate for the question concerned. Second, the teachers did not evaluate the learner's answers, thereby allowing incorrect ideas to go unchallenged and unchecked. Third, in situations where it was not clear what the learner was saying or meaning, the teachers did not respond by seeking further clarification from the learner. Each of these three types of "listening problems" is linked to the pedagogical content knowledge for teaching statistics. This problem is attributed to lack of **knowledge of content and teaching as** opposed to poor listening skills.

Teacher practices in relation to listening to learners have been found to have significant impact on classroom discourse. Davis (1997) identifies evaluative listening, whereby a teacher compares the response with a preconceived answer or standard, and is therefore not interested in what the learner is saying and interpretive listening, in which there is a more active attempt at connecting listening involving negotiated and participatory interaction between teacher and learners.

Lack of listening or interpreting of the learners' statements can be attributed to hearing problems. Wallach and Even (2005), refer to different types of hearing an reasons behind the types of hearing on the part of the teacher. They are:

- Over hearing: hearing more than the learners actually say.
- Under hearing: missing some of what the learners say.
- Compatible hearing: making sense of and connecting with what learners say.
- Non-hearing: missing the whole message of the learner.
- Biased hearing: the amount heard depends on who is saying it.

This study suggests that lack of pedagogical content knowledge is one of the main contributors to listening problems. The evidence to support this claim is that for every "listening problem" that occurred, an aspect of pedagogical content knowledge was identified in relation to that listening problem. If, however, one listening problem could not be linked to an aspect of pedagogical content knowledge on the framework, then other factors from Wallach and Even's (2005) list could be held responsible.

4.4.8 Appreciating different conclusions/solutions

Teachers had to learn to accept different conclusions from different groups, and hence from different individuals, for the same problem, as long as a meaningful rationale was provided. It was important for them to realise that there are often different ways to solve a statistical problem. They recognised that people come to different conclusions based on the same data if they have different assumptions and may therefore use different methods of analysis. Breen (1999:46) commented that the culture of a great number of South Africans is one of which prefer consensus, agreement and going along with the majority. He called this an "uncritical conformity of approach". This is a matter of concern when we are addressing the need for teachers to be critical in their thinking in order to encourage critical thinking into their learners.

It was noted that big ideas can differ, however, if there are so many different big ideas again and again in a problem it can reveal a lack of in-depth understanding of the subject on the part of the teachers concerned. If a teacher readily anticipates learner answers and demonstrates an understanding of learner reasoning, there are greater opportunities for realising one common “big” idea. These missed opportunities, often result from a lack of guidance as to what learners could look for within an investigation.

4.4.9 Difficulty with bi-variate data-based language

A large number of classroom episodes were identified in which the teachers had noticeable difficulty with making clear and valid statements following data sorting, representation and analysis. They had difficulty making statements that were sufficiently clear and well linked to the data. Most difficulties could be attributed to teachers’ language ability and their lack of awareness of the need for precision with their statements. Knowledge of appropriate statistical language and clear unambiguous use of language was not always in evidence. Language is central to enabling learners to link their intuitive understandings with accepted mathematical/statistical understandings (Anthony and Walshaw, 2007). Consequently, it is clear that if teachers do not have adequate knowledge of appropriate statistical language, their development of the subject will be inhibited. Pedagogical content knowledge is very important when dealing with data-based statements and when there is a need for precision and accuracy with statements.

When teachers struggled with making statements from the data, they were assisted through a number of different strategies. These strategies include re-voicing which is dependent on **specialised knowledge of content**. “Re-voicing” a statement is a strategy by which a teacher repeats the statement or word (possibly in modified form) to “make it more accessible (less ambiguous, better formulated, more canonical) to the others” (Forman, 2003). At other times a statement or the word was repeated for the teachers if it was reasonably apparent what the teacher was struggling to say, thereby making it more accessible to him/her, as well as to others. Re-voicing was initially directed back at the teacher who was making the statement. It was later re-voiced to the whole group, so that they could also make sense of the word or statement.

4.5 Research objective 5

To analyse the challenges in the implementation of the new pedagogical content knowledge framework for the development of pedagogical content knowledge for secondary school statistics teachers.

The analysis and reflections of the challenges of implementing the pedagogical content knowledge framework

At the end of each week of the development, a questionnaire (Appendix 6) was distributed to all the teachers in order to let them evaluate the implementation of the pedagogical content knowledge framework and was analysed manually. The results drawn from the evaluation questionnaire were recorded (Appendix 6). On the evaluation of training, which included coverage of content, clarity of presentation, method of training, duration of training and the quality of the facilitation provided by trainers, the rating on the scale ranged from good to very effective in 90% of the cases. The results indicated that the framework was effective and the alignment of the pedagogical content knowledge framework to pedagogical content knowledge was relevant and meaningful. All of the teachers agreed that their personal objectives for attending the training were achieved and that they would not hesitate to recommend the training to their colleagues. The teachers recommended that they would also appreciate being shown how to teach statistics online. Results of the interview confirmed the results from the questionnaire.

RESEARCHER: What have you learnt?

TEACHER 1: I have learnt that by doing and discussing with others you learn to understand and appreciate statistics.

RESEARCHER: What do you mean by doing?

TEACHER 1: The facilitator was forcing me to do the work in the groups since marks were given for my effort by my colleagues and whilst we went through the tasks we were recorded. Somehow I had to do the work.

RESEARCHER: How did you manage in your groups?

TEACHER 2: I was fully involved in all tasks, and what was most useful for me were the in-class discussions with all the groups.

TEACHER 3: I hope I can also use these methods in my class. The learners will just happily understand my sections of statistics.

RESEARCHER: I guess you can also use it in your classroom. Knowing what works best and what doesn't is the important part of reflection that we are encouraging. I call this maturing in the subject.

RESEARCHER: What is it that made your "aha" moment during the development?

TEACHER 4: We did what we wanted. We listened to our fellow colleagues and did and undid the problem. (I was not very clear to what this meant)

TEACHER 5: We made our mistakes and corrected them. No one was there to see how stupid we were. We realised were we had gone wrong through the in-class discussions. (The reactions of Ohooo. Ok Ok Ok (repeated) said it all.)

TEACHER 7: We agreed to disagree in our group and only proved to the other during the in-class discussions. It was really the best part of the facilitation experience.

RESEARCHER: Because of the experiences that you are telling me, what then are you going to stop doing or start doing?

TEACHER 8: I will definitely stop that "chalk and talk" and use the guiding techniques that we learnt from you.

TEACHER 9: I will start using meaningful leading tasks and scenarios for my lessons.

TEACHER 10: I will start using the leading and guiding questioning skills.

TEACHER 11: I have learnt that in statistics you have to defend your answer, whatever it is, with good practical and realistic reasons. There is no one answer fit all except if it is a numerical problem.

TEACHER 12: This can be a problem for me as a teacher as I am never sure which answer is therefore more correct than the other.

A lot of reflection on the implementation of the pedagogical content knowledge framework was done in chapter five in the sections of 5.2.5 (Research objective 5), limitations, recommendations and implications for further research. The recommendations are discussed in section 5.5 while the implications for further research are discussed in section 5.6. The chapter reflects on the limitations of the study in section 5.7. In all these sections the challenges that were experienced in the implementation of the pedagogical content knowledge framework were pinpointed and discussed.

4.6 Conclusion

The data analysis in this chapter concentrated on the main sources of data generated through the study. These were in-class group written activities, in-class discussions and observations. Data were analysed in two steps, namely the examination of patterns in the experiences of the teachers during the development of pedagogical content knowledge in the individual stages of the pedagogical content knowledge framework (section 4.4), followed by identification of substantial main categories (section 4.5). Each of the research objectives were addressed in relation to the conclusions that could be drawn from this study and the contribution to research knowledge.

The experiences indicated "incidents" that arose within the facilitations associated with in-class discussions resulting from the group presentations. Some incidents revealed a lack of pedagogical content knowledge, and therefore potentially a missed opportunity to influence learning positively. In each of the dimensions of statistical thinking, teachers exhibited some, but not all, the components of pedagogical content knowledge. Teacher identities were strengthened and the teachers were turned into critical statistics thinkers. Working together enabled a community of practising statistics teachers to come together and increased teachers' group cohesion. There were feelings of ownership of the programme the development allowed teachers to discuss with their peers possible solutions to problems arising at any point in their work environment.

CHAPTER 5

SUMMARY, RECOMMENDATIONS, CONCLUSIONS AND IMPLICATIONS

5.1 Introduction

This chapter examines and discusses the links between the broad aims of the study as derived from the problem statement, the literature review, the methodology and the results and discussion. A synopsis of the findings from the literature investigation and findings from the empirical investigation are examined in section 5.2. Questions that originated from this study are highlighted in section 5.3. The summary of the contribution to the study are highlighted in section 5.4. The contributions that this thesis has made to the teaching of statistics are discussed in section 5.5. Recommendations and implications of the development of pedagogical content knowledge are discussed in section 5.5 while the implications for further research in the area, as drawn from the study are proposed in section 5.6. The chapter reflects on the limitations of the study in section 5.7 before providing a final word in 5.8. The conclusion is given in section 5.9. The summary of the findings are discussed according to each research objective.

5.2 Synopsis of the findings in this study

5.2.1 Research objective 1

To analyse if Grade 11 and 12 statistics teachers in South Africa are competent, confident and prepared for the teaching of secondary school statistics.

The results of the survey indicated that the teachers were not competent enough to teach bi-variate data and that this impacted on their confidence to do their job well. In addition to the INSET programs provided by the Department of Education, South Africa the teachers were requesting that more relevant support should be given by the education system in order for them to teach the subject for the positive benefit of the learners.

5.2.2 Research objective 2

To analyse and explore the special characteristics of pedagogical content knowledge that contributes to it being a better teacher knowledge than others knowledges for secondary school statistics teachers.

This study has made a significant contribution to this objective through identifying pedagogical content knowledge, through the work of renowned researchers, as the best for the teaching of statistics (Section 2.5). Broad descriptions of common knowledge of content, specialised knowledge of content, knowledge of content and learners, and knowledge of content and teaching that make up pedagogical content knowledge were obtained from the mathematics education literature (Hill et al., 2004; Ball et al., 2005). These descriptions were generally appropriate for transferral to the statistics education field by adapting and refining them. The following were some of the contributions:

- *Common knowledge of content*: ability to identify incorrect answers or inaccurate definitions, and the ability to complete the learners' problems successfully.
- *Specialised knowledge of content*: ability to analyse statistically, whether an unconventional answer or explanation is reasonable or statistically correct, or to give a statistical explanation for why a process (such as a particular algorithm) works.
- *Knowledge of content and learners*: ability to anticipate learner errors and misconceptions, to interpret incomplete or complete learner thinking, to predict how learners will handle specific tasks, and what learners will find interesting and challenging;
- *Knowledge of content and teaching*: ability to appropriately sequence the content for teaching, to recognise the instructional advantages and disadvantages of particular representations, and weigh up the mathematical issues in responding to learners' unexpected approaches.

As indicated in section1.2.7, pedagogical content knowledge embodies formulating the subject statistics, to make it comprehensible to others by encompassing best practices in all three forms (teaching, content and knowledge) needed for teaching statistics.

5.2.3 Research objective 3

To design and develop a new pedagogical content knowledge framework which will guide and inform the development of pedagogical content knowledge for secondary school statistic teachers.

A framework (Table 2.6.3) for examining pedagogical content knowledge for teaching secondary school statistics was proposed and designed in chapter 2, section 2.6, and is based on two significant strands of research from Shulman (1986) and subsequent related work by a number of renowned researchers. The development of a pedagogical content knowledge framework for statistics teachers that is subject to continual evaluation proved to be an important instrument for the professional development of statistics teachers. This kind of framework can be adapted for all other sections of statistics.

5.2.4 Research objective 4

To use the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences.

Intertwining categories of the distinct knowledge of pedagogical content knowledge, which are knowledge of learners understanding, strategies to re-organise understanding, knowledge of content and learners, knowledge of content and teaching, specialised knowledge of content, specialised knowledge of subject matter, forms of representation and many others was relevant and used for the analysis of the data collected. Based on the task items used during facilitation, teachers responded in three ways. First, they suggested appropriate and inappropriate responses to items, displaying their own content knowledge through their suggestion of appropriate answers and their knowledge of learners as learners. Second, teachers suggested how they would plan to address the imagined learner difficulties in the classroom, displaying further knowledge of learners as learners and strategies for dealing with misunderstandings in the classroom. Lastly the teachers, were given roles as learners, as school principals and as researchers. This was intended to enable the teachers to reflect upon their subject content knowledge, conceptions, misconceptions and the type of learner that can be inside their statistics classroom. These tasks were intended to provide opportunities for teachers to show all aspects of their pedagogical content knowledge.

Regarding pedagogical content knowledge translating into practice, significant themes of missed opportunities were categorised. Nine significant themes arising from the analysis of and issues related to the teachers' pedagogical content knowledge were identified (section 4.3.5). In each case, the themes were identified by how they were linked to the learners' conceptions, misconceptions and general error pattern analysis. They included realising the need for data, moving from uni-variate to bi-variate data, teachers familiarity with bi-variate data, statisticatisation (Finding meaning from bi-variate data), reasoning with models, metacognition, mis-intepretation to questions and answers in the statistics classroom, appreciating different conclusions/solutions and difficulty with bi-variate data-based language.

5.2.5 Research objective 5

To analyse the challenges in the implementation of the new pedagogical content knowledge framework for the development of pedagogical content knowledge for secondary school statistics teachers.

Results of the evaluation of the implementation of the pedagogical content knowledge framework for the development of pedagogical content knowledge to Grade 11 and 12 teachers revealed that teachers were happy with the facilitation during the development. For them it was the best they had received from the Free State region that is related to statistics teaching and understanding. The framework was for them a genuine attempt by the education department to recognise the problems experienced by teachers in the teaching and understanding of statistics. Although the teachers appreciate the exercise as valuable during the professional development, they had reservations on some main issues. The teachers doubted whether the country of South Africa had enough competent educators to carry out the development of pedagogical content knowledge during professional developments using the pedagogical content knowledge framework. Questions were asked on whether the educators would be able to select relevant tasks to be used during the development and be able to follow and improve the stages of the framework. Teachers also commented that they realise that the pedagogical content knowledge framework needs to be continually updated and that they believed the frameworks would be different for different sections of statistics.

As the researcher of this study, and up to the end of the study, I found that identifying important issues that must dominate the pedagogical content knowledge framework was a very complex exercise. Furthermore the stages of the framework were difficult and challenging to manage as I struggled with the identification the relevant tasks..This was because pedagogical content knowledge in itself as a concept is heavily debated. Every time that I thought I had now understood this concept pedagogical content knowledge in its fullness, new research that understood it differently would be published. The researchers of pedagogical content knowledge keep shifting goal posts. Since most teachers and educators, were not familiar with the subject statistics, my peer-reviewed article on the pedagogical content knowledge framework, that was published in the SARMSTE journal, did not get the interrogation that I had expected. I was therefore left with an egg in my face and left to see the whole process to finish on my own. It was a lonely journey indeed. Even though several researchers have written papers on good statistics teaching, they have avoided research into statistics and pedagogical content knowledge. There is clearly a need to take up this research.

5.3 Contribution

This study added a lot to the statistics field by contributing to five important secondary school statistics areas.

- Models/frameworks for statistics professional development
- The teaching and learning of statistics.
- Professional development.
- Teacher knowledge.
- Education developers.

5.3.1 Models/frameworks for statistics professional development

The pedagogical content knowledge framework provides a relatively comprehensive model for future studies of pedagogical content knowledge in teacher developments. The continued interest in pedagogical content knowledge as a knowledge base for statistics teacher development has produced a need for a conceptual framework upon which future pedagogical content knowledge studies can be based. The pedagogical content knowledge framework in this study provides such a model. First, the pedagogical content knowledge framework allows researchers and teacher education programmes to accurately identify and address distinctions among knowledge bases of various educational disciplines, statistics subjects, and statistics topics. In other words, provides a model for implementing unique instructional methods in the statistics classroom. Second, the pedagogical content knowledge framework enables researchers studying knowledge development in teachers and teacher education programmes to identify and characterise different attributes of statistics teaching. In addition, the framework recognised the relative importance that researchers and educators give to the different components of pedagogical content knowledge. The framework served to organise and integrate research efforts centred on pedagogical content knowledge.

5.3.2 The teaching and learning of statistics

As most teachers did not have the advantage of learning statistics at school, their **common knowledge of content** was developed through the development of pedagogical content knowledge initiatives. As their **common knowledge of content** developed, so was their **specialised knowledge of content**, particularly for listening to and making sense of learners' responses. The use of videos showing teachers involved in aspects of investigations, especially making data-based statements was particularly useful for helping the development of the statistics teacher knowledge of listening to and making sense of school learners. The development of pedagogical content knowledge was also based on an understanding of how knowledge evolves.

Numerous situations arose in which pedagogical content knowledge appeared to develop during the professional development. The most common category of knowledge to develop was that related to learners' difficulties, that is, **knowledge of content and learners**. Often, the teachers did not know of or expect the areas in which learners would have difficult. When these situations arose, the teachers realised and anticipated the difficulties that learners can have and, as a result, the teachers' **knowledge of content and learners** developed, in relation to the relevant component of statistical thinking. To support planning and success in the teaching of bi-variate data, teacher-learning opportunities need to capitalise on pedagogical content knowledge inside the statistical investigation process. Some basic ideas and concepts teachers need to learn are identified in this study. They include:

- How to define and create variables when none is inherent or obvious to an investigation.
- How to do basic data manipulation, like sorting.
- How to gain the perspective to check and determine whether results of the analysis address the intended purpose of the investigation.

- How to discern when and what types of graphs to use in different situations.
- How to identify the “big ideas” in any planned lesson.

Some of the in-service statistics teachers know their content well, but they have not learned how to transform or translate that knowledge into meaningful instruction units. By focusing on topic-specific examples and demonstrations, prospective secondary school statistics teachers were able to focus and develop specific strategies. This study confirms the significance of pedagogical content knowledge as a concept central to educational analysis (Watson, 2008).

The importance of **content knowledge and knowledge of learners** corresponded with the recognition of “big ideas”, and the anticipation of appropriate and inappropriate bi-variate data responses. Teachers need opportunities to construct understanding and recognise the use of statistical concepts like data. People who apply statistical reasoning in real-world problems must be able to frame the problem and use their statistical knowledge in the framed context to solve it.

Throughout the development, teachers were given the opportunity, to reflect on all incidents during the development of pedagogical content knowledge in order for them to share ideas and interpretations of what was occurring, to consider alternative strategies, and to seek advice about future lessons in a community of practicing teachers. Such opportunities may well have contributed to different opportunities in the teachers from what might have happened if reflection and sharing had not occurred.

5.3.3 Professional development

During professional development, teachers realised that it was acceptable to be simultaneously a learner and a teacher. Playing this dual role of teacher and learner is not without risk (Heaton, 2000) and needs to be supported and encouraged by statistics educators in their work with teachers. Taking on the role of learner, while being a teacher, required both confidence and a willingness to cope with the development of pedagogical content knowledge. Teachers learnt to see the disposition toward teaching, as well as learning, from openly modelled facilitations by the statistics educator during the development of pedagogical content knowledge.

This study provided the explicit pedagogical content knowledge framework for future interactive statistical professional development sessions for statistical professional growth. Development had the ability to create leader teachers in statistics with the capacity to interpret, critique, and implement current curriculum innovations in statistics. It enabled and fostered collegial and cooperative ways of working with other statistics teachers within and between schools. Professional growth involves attaining the ability for teachers to work well and meaningfully in a social environment. The development fostered cooperative ways at working with departmental statistics subject advisers and district offices to assist in implementing and reviewing statistics curriculum innovations. This development provided the necessary skills and knowledge for running workshops with groups of teachers on a range of present and future secondary school statistics topics. What is necessary is the effective use of exemplary models of statistics teaching within topics that can later be transferred to other topics.

5.3.4 Teacher knowledge

The teaching knowledge was previously inadequate because the teachers did not need it as they did not know about it. Teachers had a lack of awareness that learners typically had difficulties with the particular concept or skill in statistics. For example, the teachers during in-class discussions found out that many of the statements originally made by them focused on a single variable. However, they realised that they needed a strategy to help learners consider two variables simultaneously. In this way, the development contributed to the increase of knowledge about the way teachers transform pedagogical content knowledge into practice as well as how they relate their transformation to learner understanding in a particular domain.

A study such as this is an important tool in teacher education in the area of statistics. It is used to represent and help teachers understand the complexity of teaching and teacher knowledge while constructing their own knowledge for teaching. The study offers a blend of statistical knowledge and practice so that teachers can see not only examples of statistical knowledge informing pedagogical decision making, but also how particular pedagogical decisions can positively and negatively affect data collection, representation, interpretation and analysis. The development of pedagogical content knowledge focused on statistical concepts and the process of statistical investigation, as it would enable teachers to see contextual statistical concepts and the ways statistical knowledge is used, or could be used, by teachers in investigative work with learners.

Furthermore, the development of a collection of examples of practice, situated in real classrooms, around specific statistical concepts arising or deliberately taught while doing statistical investigations, offered direction for creating usable knowledge for teaching from research. Using such examples from practice continue to illustrate to teachers and teacher educators a key finding from this research, that in learning statistical knowledge for teaching, the context matters, and teachers need to learn where, when, why, and how it matters. This study's findings provided guidance for what aspects of statistics teacher knowledge development should be the focus of development programmes.

5.3.5 Education developers

This study contributed to the debate about the competences that statistics teachers should have. It provided measures that can be used to determine professional development needs of teachers in the area of statistics. The pedagogical content knowledge framework is required whenever revisions are made to the curriculum in statistics. It is also required after educational programmes have been delivered in order to provide evidence that new levels of professionalism have been reached.

Many studies were found to have investigated the actual development of pedagogical content knowledge using the pedagogical content knowledge framework. This study, can therefore be considered as a pioneering effort in this regard. Its findings form the baseline knowledge in the area of statistics and should arm stakeholders in professional development with relevant, reliable and accurate information upon which comments, discussions and decisions are based. The pedagogical content knowledge framework is a vehicle that contributes to a better understanding of the subject statistics by Grade 11 and 12 statistics teachers. By learning from the experts in this field and through international conferences, far greater insights into the deeper meaning of pedagogical content knowledge and the best practices to achieve it can be realised. These findings can be generalised to other sections of the secondary school statistics syllabus.

5.4 Recommendations and implications of the development of pedagogical content knowledge

This study contributed to the debate about the measures that can be used to determine professional development needs of teachers in the area of secondary school statistics. Understanding common and diverse issues in statistics assisted with effective and efficient problem-solving skills and created interrelationships within the subject statistics. Teachers adapted the new practices of pedagogical content knowledge out of habit.

In order for secondary school statistics learners to reduce the backlog in their statistical understanding, a joint effort by different role players with new knowledge is called for. The effort should not only be in support of the development of pedagogical content knowledge to statistics teachers, but also in sponsoring the simple but necessary materials and tasks that can be used in the implementation of the pedagogical content knowledge to statistics teachers. It also means fostering co-operative ways at working with departmental statistics subject advisers and district offices to assist in implementing and reviewing statistics curriculum innovations. This study stimulates national and international dialogue among policymakers and educators regarding programmes and curricular to improve preparation and practice in secondary school statistics.

The idea in the development of teacher knowledge is in that the continually changing knowledge of the teacher creates continual change in the teachers' hypothetical statistical learning trajectory. Learners' thinking and understanding was taken seriously and therefore given a central place in the design and implementation of pedagogical content knowledge. The teacher's knowledge evolves simultaneously with the growth in the learners' knowledge. As the learners are learning statistics, the teachers are also learning about statistics and, hence learning about the statistical thinking of their learners. The development of pedagogical content knowledge created leader teachers in statistics with the capacity to interpret, critique, and implement current curriculum innovations in statistics. It aimed at enabling and fostering collegial and cooperative ways of working with other statistics teachers within schools and between schools.

The important roles offered by the development of pedagogical content knowledge assisted teachers in applying the new understandings and insights in the learning process. It provided teachers the opportunities, the time and the means for improving professional practice. It also helped teachers to expand their perception of statistics and assist the teachers in their personal development as professionals. One way of supporting and developing educators is a clear understanding of their problems and addressing these issues (Moodley, Njisane & Presmeg, 1992).

In view of the findings reported in chapter 4, it was suggested that special attention should be given to developing necessary skills and knowledge for running workshops with groups of teachers on a range of statistics topics. The professional development programmes must provide atmospheres that allow for autonomy. This means that the teacher candidates themselves need to be immersed in an environment where they are engaged in questioning, hypothesising investigating, imagining and debating. They need to be part of a community that actively works with them as learners and then allows the experience to be dissected, evaluated and reflected upon in order for principles of pedagogy and action to be constructed.

5.5 Implications for further research

The study revealed findings that add to the available research knowledge about pedagogical content knowledge in relation to statistics. Though the development of pedagogical content knowledge was overall successful, some questions originated from the study. This research focused on using a model/framework for the development of pedagogical content knowledge. It however did not clarify if pedagogical content knowledge grows in the course of professional development. If it is true that it grows, what would be the conditions or events that cause the growth of this knowledge. There is need for research in these two areas.

Due to the study’s time limit the study could not assess and evaluate the impact of the development of pedagogical content knowledge using a pedagogical content knowledge framework to Grade 11 and 12 secondary school statistics teachers to their learners performance in the classroom. Also due to time limit only one area of statistics, bi-variate data was used for the development of pedagogical content knowledge. Research should therefore be carried out in the use of the pedagogical content knowledge framework in developing Grade 8 to 10 teachers of statistics. Even if research should broaden to investigate pedagogical content knowledge in statistics at other school levels, it must seriously examine and measure the effect of the development pedagogical content knowledge to these teachers on learner outcomes. More research could be done in developing a relevant framework for the development of pedagogical content knowledge in other areas of statistics.

Some of the teachers' current knowledge can be attributed to teaching experience or from knowledge that developed prior to their professional development. Literature suggests that even experienced teachers do not have sufficient knowledge for teaching, which could mean that they continue to rely on their common content knowledge (Fennema & Franke, 1992; Ball et al, 2001). However, the role and responsibility of professional development programmes is still critical. Future research should investigate different pedagogical content knowledge frameworks that can be relevant to statistics professional development.

Directly or indirectly, teacher development programmes will benefit from further pedagogical content knowledge research. One obvious area of future research would be to focus on identifying and classifying the various types of pedagogical content knowledge employed in the statistics classroom. Once researchers are able to identify various components of pedagogical content knowledge in the statistics disciplines, then they can begin to examine how to deal with problems in the areas of statistics.

More investigations should be contacted to establish whether the experiences that resulted from the development of pedagogical content knowledge to in-service statistics teachers using a pedagogical content knowledge framework would be similar to experiences of teachers in teacher-education systems. This should help with the revisiting of teacher-education training programmes.

5.6 Limitations

It was acknowledged in this study that some limiting factors played a role in the findings reported here. This study was based on one researcher's interpretations of data from video recordings, stimulated in-class discussions and audio recordings. It was therefore difficult to generalise the results all sections the development of pedagogical content knowledge. However, the researcher's experience as a teacher, teacher educator and lecturer meant that the potential for major flaws in interpretation had been minimised. In addition, the small convenience sample of six weeks of facilitation indicates that unfair generalisations of the need for pedagogical content knowledge to the teaching of statistics could have been made. Somewhat countering the possible limitations due to the sample size are the benefits obtained from being able to examine data across the six weeks, for similarities and differences. The development of pedagogical content knowledge was only conveniently done in the Free State province. This had implications in the generalisation of the experiences to all other parts of South Africa as a country.

Questions like "Would the teachers have acted differently had I not been there with a video camera? What effect did this presence have on the pedagogical content knowledge used by the teacher?" This was difficult to answer. For example, responses from groups were not willingly given in the beginning of the development. The reason could have been that the researcher's presence brought a "know-it-all" figure in front of the teachers. Another example was when the group presented a wrong response to the whole group and there was dead silence. In order to minimise and counter this limitation, the researcher commented with no expression on her face, "What is what now". I also felt that in some cases I inadvertently gave away the solution through my tone, facial expression or body language. It would result in teachers then saying, "It doesn't look right".

5.7 Final word

The first objective of this study was to find out if there are competent and confident teachers who were given the necessary support, which is necessary for the teaching of Grade 11 and 12 statistics. This was achieved through a questionnaire and interviews during a survey.

Secondly, the study sought to analyse and explore the special characteristics of pedagogical content knowledge for secondary school statistics that make it the most effective teacher knowledge among bodies of teacher knowledge. Through a literature review, the special characteristics of pedagogical content knowledge were identified.

The third objective of the study was to formulate a pedagogical content knowledge framework that guided and informed the development of pedagogical content knowledge. The framework was designed and developed.

The fourth objective involved using the above as guiding principles to understand how the emerging framework for the development of pedagogical content knowledge for secondary school statistics teachers impacted on the teachers' classroom experiences. This was achieved through analysing the nature of the experiences of the teachers that were identified during the development of pedagogical content knowledge using a pedagogical content knowledge framework. The recording of experiences, interactions and discourses of secondary school statistics teachers when pedagogical content knowledge was developed and translated into practice were recorded. Missed opportunities, were observed and categorised.

The last objective sought to analyse the challenges in the implementation of the new pedagogical content knowledge framework for the development of the secondary school statistics teachers. This was achieved partly through the evaluation of the effectiveness of the intervention by the teachers themselves and partly through the discussion of the challenges experienced during the implementation of the new pedagogical content knowledge framework.

5.8 Conclusion

Based on the five weeks of the development of pedagogical content knowledge using the framework, a number of conclusions were realised. Pedagogical content knowledge has the ability to empower the secondary school statistics teacher in most areas of their teaching. The teachers showed the ability to understand the learners that they teach through increased strength in using innovative methods, techniques, and rich representations in bi-variate data.

The study results can be useful for statistics educators, including in-service teachers, learners preparing to be teachers, teacher educators, people involved in curricular development in statistics as well as researchers in statistics education. The representations of pedagogical content knowledge in bi-variate data made explicit a successful teacher's reasoned decision-making in the context of teaching bi-variate data. It provided evidence that the teacher is using pedagogical content knowledge. The materials and data used in this study made it possible to set up very organised facilitations and think of the many ways of improving each stage of the pedagogical content knowledge framework.

This thesis concludes that when pedagogical content knowledge is missing, learners' learning opportunities are affected. If this aspect of knowledge is not available or not used, teachers will not enhance their confidence and competence to teach statistics at secondary school level. The presence of any other types of knowledge (that are not components of pedagogical content knowledge) cannot adequately substitute for the missing components of pedagogical content knowledge. This thesis also showed that the increased knowledge gained from this study can be applied to professional development of practicing teachers. Professional development programmes that emphasise the underlying nature of the pedagogic content knowledge, result in knowledgeable, dynamic teachers with transformed dispositions and understanding of both the subject content knowledge and the pedagogic knowledge. The development, though demanding in preparation, was very enriching and provided lifelong learning for the teachers. The development produced in different teachers, different attitudes towards bi-variate data and correlation**.**

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APPENDICES

Appendix 1 (Research request to the Department of Education)

University of South Africa
P O Box 392
UNISA
0003

3 March 2008

Dr Gonnie Leurs
TSHWANE SOUTH DISTRICT OFFICE
Private Bag X 27825
Sunnyside
Pretoria
0132

Dear Sir/Madam

RE: SEEKING PERMISSION TO DO A DED DISSERTATION DEVELOPMENTAL STUDY FOR TEACHERS IN ANY ONE OF THE SCHOOLS IN PRETORIA

I am a Doctorate student at the University of South Africa and am kindly asking for permission to conduct some research work in any chosen school in Pretoria. My topic is ‘**The development of pedagogical content knowledge for Grade 11and 12 probability and statistics teachers’.**

I wish to conduct my research with the permission of the authorities and without disruption of their day to day activities.

Yours faithfully
Mrs Antonia Makina

Appendix 2 (Research Request Form)

GAUTENG DEPARTMENT OF EDUCATION

RESEARCH REQUEST FORM

REQUEST TO CONDUCT RESEARCH IN INSTITUTIONS AND/OR OFFICES OF THE GAUTENG DEPARTMENT OF EDUCATION

1. PARTICULARS OF THE RESEARCHER

1.1	***Details of the Researcher***
Surname and Initials:	Makina A.
First Name/s:	Antonia
Title (Prof / Dr / Mr / Mrs / Ms):	Mrs
Student Number (if relevant):	34164766
ID Number:	6209140315186

1.2	***Private Contact Details***
Home Address	Postal Address (if different)
5 Sudhof	
472 Walker Street	
Muckleneuk	
Postal Code: 0002	Postal Code:
Tel: (012) 429 4370	
Cell: 084 330 5640	
Fax: (012) 429 4922	
E-mail: makina@unisa.ac.za	

2. PURPOSE & DETAILS OF THE PROPOSED RESEARCH

2.1	Purpose of the Research (Place cross where appropriate)
Undergraduate Study – Self	
Postgraduate Study – Self	self
Private Company/Agency – Commissioned by Provincial Government or Department	
Private Research by Independent Researcher	
Non-Governmental Organisation	
National Department of Education	
Commissions and Committees	
Independent Research Agencies	
Statutory Research Agencies	
Higher Education Institutions	

2.2	Full title of Thesis / Dissertation / Research Project
The development of pedagogical content knowledge to Grade 11 and 12 probability and statistics teachers.	

2.3	Value of the Research to Education (Attach Research Proposal)
This study seeks to contribute towards the improvement of the quality of probability and statistics instruction by focusing on the teacher's pedagogic content knowledge during professional development so as to provide assistance to those making decisions regarding the professional development needs of teachers.	

2.5	Learner and Postgraduate Enrolment Particulars (if applicable)
Name of institution where enrolled:	UNISA

Degree / Qualification:	DEd
Faculty and Discipline / Area of Study:	Mathematics Education
Name of Supervisor / Promoter:	Prof D.C.J. Wessels

2.6	Employer (where applicable)
Name of Organisation:	UNISA
Position in Organisation:	Post- Graduate Assistant
Head of Organisation:	
Street Address:	Preller st , P.O. Box 392
	UNISA, PRETORIA
Postal Code:	0003
Telephone Number (Code + Ext):	
Fax Number:	
E-mail:	

2.7	PERSONAL Number (where applicable)						

2. PROPOSED RESEARCH METHOD/S

(Please indicate by placing a cross in the appropriate block whether the following modes would be adopted)

3.1 Questionnaire/s (If Yes, supply copies of each to be used)

YES	**X**	**NO**	

3.2 Interview/s (If Yes, provide copies of each schedule)

YES	**X**	**NO**	

3.3 Use of official documents

YES		**NO**	**x**
If Yes, please specify the document/s:			

3.4 Workshop/s / Group Discussions (If Yes, Supply details)

YES		**NO**	**x**

3.5 Standardised Tests (e.g. Psychometric Tests)

YES		**NO**	**x**
If Yes, please specify the test/s to be used and provide a copy/ies			

4 INSTITUTIONS TO BE INVOLVED IN THE RESEARCH

4.1 Type of Institutions (Please indicate by placing a cross alongside all types of institutions to be researched)

INSTITUTIONS	***Mark with X here***
Primary Schools	
Secondary Schools	**x**
ABET Centres	
ECD Sites	
LSEN Schools	
Further Education & Training Institutions	

Other	

4.2 Number of institution/s involved in the study (Kindly place a sum and the total in the spaces provided)

TYPE OF INSTITUTION	***TOTAL***
Primary Schools	
Secondary Schools	**6**
ABET Centres	
ECD Sites	
LSEN Schools	
Further Education & Training Institutions	
Other	
GRAND TOTAL	**6**

4.3 Name/s of institutions to be researched (Please complete on a separate sheet if space is found to be insufficient)

Name/s of Institution/s
Schools chosen as per acceptance
2 private schools
2 public schools
2 former group A schools

4.4 District/s where the study is to be conducted. (Please indicate by placing a cross alongside the relevant district/s)

District	
Johannesburg East	
Johannesburg South	
Johannesburg West	
Johannesburg North	
Gauteng North	
Gauteng West	
Tshwane North	
Tshwane South	**x**
Ekhuruleni East	
Ekhuruleni West	
Sedibeng East	
Sedibeng West	

If Head Office/s (Please indicate Directorate/s)

NOTE:

If you have not as yet identified your sample/s, a list of the names and addresses of all the institutions and districts under the jurisdiction of the GDE is available from the department at a small fee.

4.5 Number of learners to be involved per school (Please indicate the number by gender)

Grade	1		2		3		4		5		6	
Gender	B	G	B	G	B	G	B	G	B	G	B	G
Number												

Grade	7		8		9		10		11		12	
Gender	B	G	B	G	B	G	B	G	B	G	B	G
Number												

4.6 Number of educators/officials involved in the study (Please indicate the number in the relevant column)

Type of staff	**Educators**	**HODs**	**Deputy Principals**	**Principal**	**Lecturers**	**Office Based Officials**
Number	*30*					

4.7 Are the participants to be involved in groups or individually?

Participation	
Groups	**X**
Individually	**X**

4.8 Average period of time each participant will be involved in the test or other research activities (Please indicate time in minutes)

Participant/s	**Activity**	**Time**

Teachers	Questionnaire and development	Half a year

4.9 Time of day that you propose to conduct your research.

School Hours	***During Break***	***After School Hours***
		x

4.10 School term/s during which the research would be undertaken

First Term	***Second Term***	***Third Term***
	X	

Dependent when the section of probability and statistics will be done

DECLARATION BY THE RESEARCHER	
1. I declare that all statements made by myself in this application are true and accurate.	
2. I have taken note of all the conditions associated with the granting of approval to conduct research and undertake to abide by them.	
Signature:	*A. Makina*
Date:	*03/03/08*

DECLARATION BY SUPERVISOR / PROMOTER / LECTURER	
I declare that: -	
1.	The applicant is enrolled at the institution / employed by the organisation to which the undersigned is attached.
2.	The questionnaires / structured interviews / tests meet the criteria of: • Educational Accountability

• Proper Research Design • Sensitivity towards Participants • Correct Content and Terminology • Acceptable Grammar • Absence of Non-essential / Superfluous items	
Surname:	*Wessels*
First Name/s:	*Dirk Cornelis Johannes*
Institution / Organisation:	*UNISA*
Faculty / Department (where relevant):	*Further Teacher Education*
Telephone:	*082 859 5214*
Fax:	*0866421649*
E-mail:	*wessedcj@unisa.ac.za*
Signature:	
Date:	*06/03/2008*

N.B. This form (and all other relevant documentation where available) may be completed and forwarded electronically Nomvula Ubisi (nomvulau@gpg.gov.za). The last 2 pages of this document must however contain the original signatures of both the researcher and his/her supervisor or promoter. These pages may therefore be faxed or hand delivered. Please mark fax - For Attention: Nomvula 011 355 0512 (fax) or hand deliver (in closed envelope) to Nomvula Ubisi (Room 525), 111 Commissioner Street, Johannesburg.

Appendix 3 (Baseline study: Teacher profiling) (Adapted from Watson J.M., 2001)

Dear learner

As a valued learner of UNISA in the module FDEME8-R, and on behalf of a research being carried out in the department by A. Makina, we kindly ask you to respond honestly and seriously to this questionnaire. This questionnaire is going to be used by all of us to try and see how best we can improve ourselves as teachers of statistics in the country. Please return the questionnaire in the provided envelope. A description of the questionnaire and all the variables follows.

Questionnaire-1(Teacher profiling)

Background

Though the questionnaire is generally on probability and statistics it is strictly on the teaching of bi-variate data and correlation to either Grade 11 or 12. INSET implies any teacher development while in service. Reflect your true picture and reaction when responding to this questionnaire. Indicate your choice by marking the appropriate block with an 'X'.

1. Biographical variables

Gender

1= Male	
2= Female	

Population Group

1= Black	
2= White	

	(e.g. Pretoria central)
Location	

Age range in years	**Tick where**

	necessary
14-17	
18-25	
26-35	
36-45	
46-55	
56-65	
Over 65	

	Highest level of education	**Tick where necessary**
1	Matric	
2	Certificate/ diploma	
3	Bachelor/ Honours degree	
4	Masters degree	
5	Doctorate	
6	Teachers' certificate/ diploma	
7	0-level	
8	A-level	
9	Other(specify)	

Number of years teaching probability and statistics	**Tick where necessary**	**What Grade/s**
1		
2		
3		
4		
5		
6 and more		

THE SCHLINGER VIEWER RESPONSE PROFILE

Using a 5-point Likert scale

1= strongly disagree

2= disagree

3= neither agree nor disagree

4= agree

5= strongly agree

		Strongly disagree	Disagree	Neither agree or disagree	Agree	Strongly agree
	Teacher support variable	1	2	3	4	5
2.	**Misunderstanding**					
Q1	I do not understand correlation					
Q2	The section of bi-variate data in probability and statistics confuse me.					
Q3	I struggled to understand the section of bi-variate data during INSET.					
Q4	The subject probability and statistics is too complex.					
Q5	I am not sure whether I am teaching correctly					
Q6	The subject bi-variate requires a lot of effort to follow the basic principles.					
Q7	The language of probability and statistics especially in bi-variate data is confusing.					

3.	**Subject content knowledge support**					
Q8	I received in-service training in probability and statistics before I started teaching bi-variate data.					
Q9	I received a lot of help from other people (not INSET) in the section of bi-variate data.					
Q10	I attended lessons in probability and statistics that included correlation during INSET					
Q11	There was follow up by INSET after they facilitated lessons in probability and statistics that included bi-variate data.					
Q12	I could consult some place or people (not INSET) if the understanding of correlation became a problem.					
Q13	I was aware of errors experienced during the learning of correlation during my teaching.					
Q14	I understood the section of correlation as it was presented to us during INSET programmes.					
4.	**Pedagogic Knowledge**					
Q15	I was guided on how to teach the section of bi-variate during INSET					

Q16	Teachers who did not do well(understand correlation) during facilitation by INSET in the section on bi-variate data stopped from teaching this section.					
Q17	Teachers who did not do well(understand correlation) during facilitation by INSET in the section on bi-variate data were retrained.					
Q18	I use a computer to teach bi-variate data and correlation.					
Q19	I know how to make best advantage of a computer during my teaching of correlation.					
Q20	We developed lesson plans and schemes of work for teaching purposes during INSET.					
Q21	Our facilitators during INSET explained the section on bi-variate data very well.					
Q22	My teaching of bi-variate data in the school where I teach is peer reviewed.					
Q23	I use the calculator to teach some sections of bi-variate data at the school where I teach					
5.	**Relevant support**					
Q24	The educators who helped us at INSET are very knowledgeable in subject content knowledge(pure content) of bi-variate					

	data.					
Q25	The educators who helped us at INSET are very knowledgeable in pedagogic knowledge(how to teach) of bi-variate data.					
Q26	Facilitation of lessons in correlation were done for more than a week during any INSET training.					
Q27	Enough time was given for the understanding of bi-variate data during INSET training.					
Q28	I was taught both pedagogic and subject content knowledge of bi-variate data during INSET.					
6.	**Material support**					
Q29	Does the school where you teach provide text books for the teaching of probability and statistics?					
Q30	I know what time series data is.					
Q31	I know what household data is					
Q32	I know where to access household data					
Q33	I have access to a computer in the school where I teach.					
Q34	There is a computer room at the school where I teach					

Q35	I have access to past exam papers in correlation.					
Q36	I was given relevant referral material in bi-variate data during INSET .					
Q37	I was able to use the material referred during INSET to teach correlation.					

Section 2(Teacher profiling)(Watson 2001)

The questions through-out this section are an attempt to identify factors which are significant for the teaching of probability and statistics.

7. Teacher Background

Q38 How many years have you been teaching? [ ]

Q39 Which grade levels have you taught in that time? [ ]

Q40 Which grade levels are you currently teaching? [ ]

Q41 During your teacher training or other tertiary study, did you study any courses which included topics in probability and statistics? ❒ No ❒ Yes: what sort of course?

❒ Specialist statistics ❒ Maths ❒ Economics

❒ Psychology ❒ Geography

Other: [ ]

Q42 About how many years ago was this study? [ ]

Q43 How much time did the probability and statistics subjects take?

❒ A few hours ❒ A few weeks

❒ A semester ❒ More than one year

Q44 List three topics which you remember studying in probability or statistics

8. Confidence

Listed below are some of the topics which are included in the probability and statistics curriculum of bi-variate data. Please mark your level of confidence in your ability to teach them to your class. You are free to place a mark anywhere on the scale to indicate your level of confidence.

Q45 My Ability to teach

	Low Confidence 1	High Confidence 2	Would not be teaching 3
a) Probability and statistics language	❒	❒	❒
b) A Cartesian plane	❒	❒	❒
c) Scatter plots	❒	❒	❒
d) Linear functions	❒	❒	❒
e) Quadratic functions	❒	❒	❒
f) Exponential function	❒	❒	❒
g) Line of best fit	❒	❒	❒
h) Linear regression line	❒	❒	❒
i) Data collection from real life	❒	❒	❒
j) Basic Probability Calculations	❒	❒	❒
k) Graphical Representation	❒	❒	❒

9. Statistics in Everyday Life

Q46 Listed below are some statements concerning beliefs or attitudes about probability and statistics. Please mark your level of agreement with each statement. You are free to place a mark anywhere on the scale to indicate your level of agreement.

	Strongly Disagree 1	Neutral 2	Would not be teaching 3
a) You need to know something about statistics to be an intelligent consumer	❐	❐	❐
b) I can easily read and understand graphs and charts in newspaper articles	❐	❐	❐
c) When buying a new car, it's better to ask a few friends about the problems with their cars than to read a car satisfaction survey in a consumer magazine	❐	❐	❐
d) I can understand almost all of the statistical terms that I see in newspapers or on TV	❐	❐	❐
e) Understanding probability and statistics is becoming increasingly important in our society	❐	❐	❐
f) Statements about probability (such as the odds of winning a lottery) seem very clear to me	❐	❐	❐
g) To learn about the side effects of a drug, it's better to refer to the results of a medical study that tested it on a few people, than to talk to someone who has taken the drug	❐	❐	❐

h)	People who have contrasting views can each use the same statistical finding to support their view	❒	❒	❒
i)	I could easily explain how an opinion poll works	❒	❒	❒
j)	Weather forecasts about the chances of rain are wrong so often that I don't take them seriously	❒	❒	❒

10. Professional Development

Q47

- Have you seen the following documents in your school?
- Have you read parts of any of them?
- Have you used any ideas from them in your classroom?

		Not Seen 1	Seen 2	Read 3	Used 4
a)	National Curriculum Document	❒	❒	❒	❒
b)	National Curriculum Statement on Mathematics for South African Schools	❒	❒	❒	❒
c)	Mathematics Curriculum Profile for South African Schools	❒	❒	❒	❒
d)	Any document from Statistics SA	❒	❒	❒	❒
e)	Any activity books for probability and statistics	❒	❒	❒	❒
f)	Books from any Bureau of Statistics	❒	❒	❒	❒
g)	Any text books specially dealing with probability and statistics	❒	❒	❒	❒

Q48

a) Have you participated in any professional development related to probability and statistics?

❒ No ❒ Yes

b) If so, please detail who organized it:

Organised by school, university, or other body? ☐

Participated with others from school, own initiative, etc? ☐

c) How long did it last (hours)? ☐

Q49 What type of professional development would benefit you the most in your teaching of bi-variate data?

Examples might include:

- School-based sessions ❒
- Personal reading ❒
- A university course, e.g. Graduate Certificate ❒
- Other ☐

Q50 In your opinion, who would be best to lead professional development?

Examples might include:

- Another teacher at my school
- A regional curriculum officer
- An outside "expert"

☐

Q51 Do you have any other specific comments about this professional development in relation to bi-variate data?

____

____

____

____

Appendix 4 (Flier at Development)

(A flier at the beginning of each day of the actual development)

Dear teachers

I hope you will find this development instructive, interesting and enriching. Please feel free to comment, criticize or make recommendations as we would very much like you to be active co-developers of this development.

Your participation in this development is my motivation to do my very best to fulfil your needs as teachers of probability and statistics in Grade 11 and 12.
Presenter

Appendix 5 (Provided Tasks and Activities)

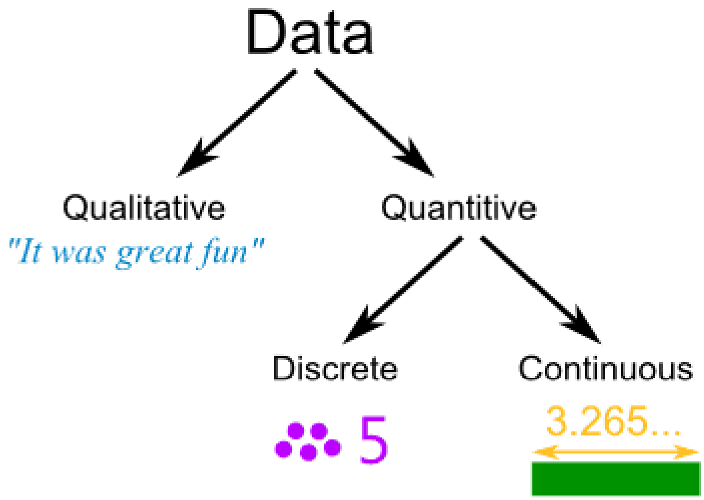
Day 1: Facilitation stage 1

Lesson part 1.1

a) Watch the following relevant videos from you tube of lecturers describing bi-varite data.

- Pedagogy bi-variate 1(recap).doc ttp://www.youtube.com/watch?v=p9OS5vRcuVM
- Pedagogy bi-variate 2.doc http://www.youtube.com/watch?v=2upetZ2mcrE
- Pedagogy bi-variate 3.doc http://www.youtube.com/watch?v=jzw4ktrwaN8 Maths Tutorial: Uni-variate and Bi variate Data
- Pedagogy bi-variate 4.doc http://www.youtube.com/watch?v=-F2qg5bELOU Construct and Interpret Bi-variate Data in Scatter Plots

b) The following is a tree diagram that discusses and classifies data into types. (http://www.mathopolis.com/questions/about-questions.php)



Can you say something about the dog Sport below for each description of the data in the tree diagram above to show your understanding of the types of data? **(Hint:** Describe Sport in terms of qualitative, quantitative that is discrete, continuous and categorical variables). What do you understand by the word data, uni-variate, bi-variate data, and categorical data? Do it for the benefit of your learners.

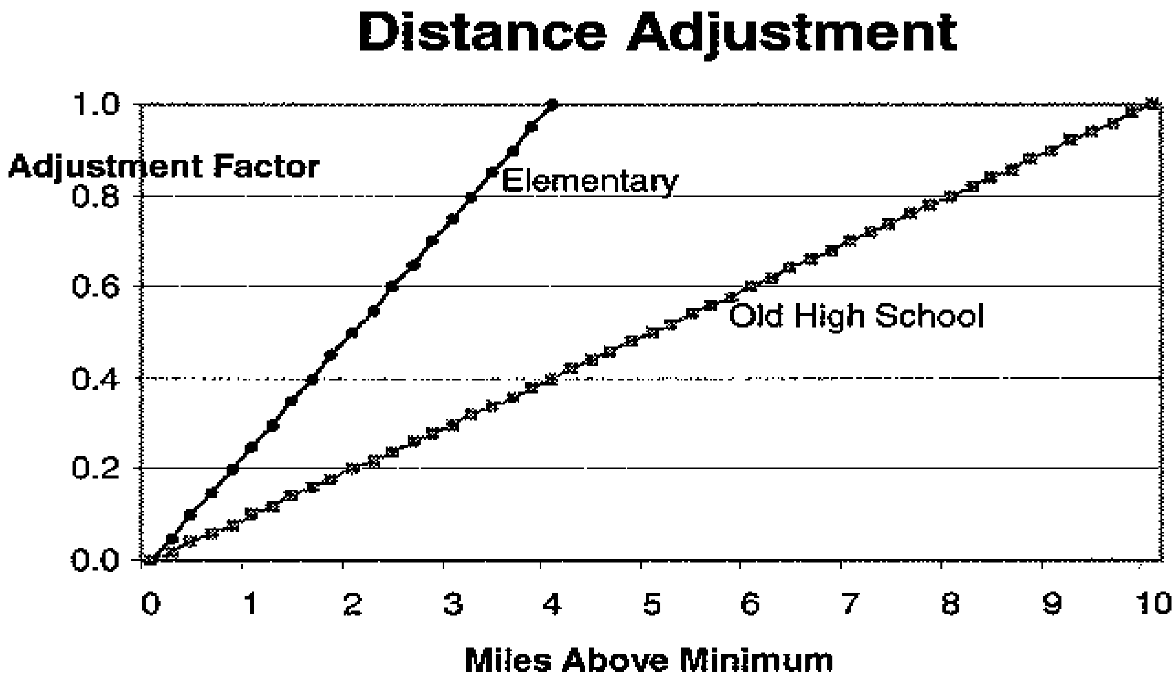
Lesson part 1.2

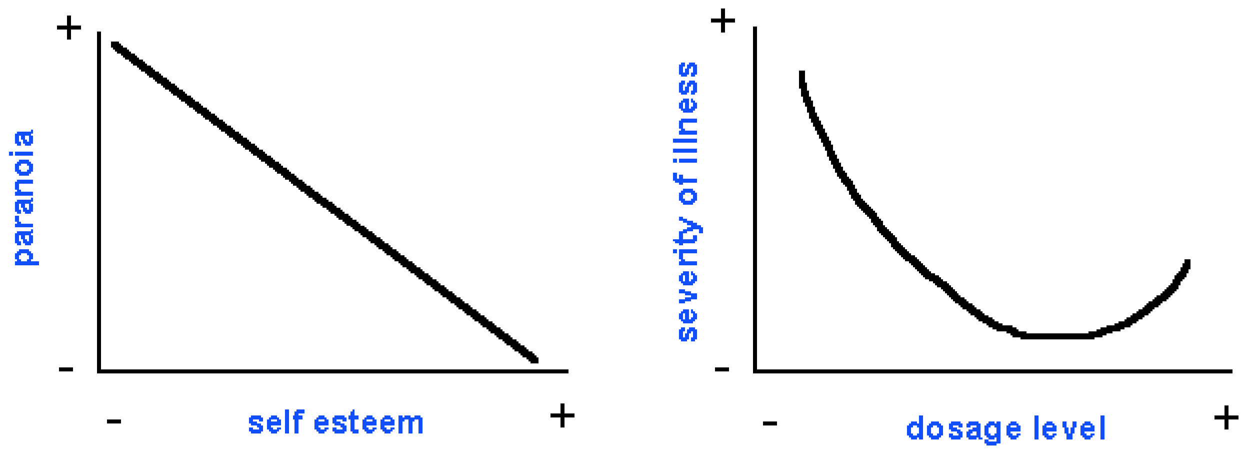
(Adapted from MAB101 Statistical Data Analysis I, 2006/1)

a) How would you introduce your learners to the difference between uni-variate and bi-variate data and between categorical and numerical data?

b) What are the types of graphs/slopes that can be used to represent different type of data?

c) By first discussing and writing down what you understand by direct and indirect variation, what in the following diagrams do you consider to be a direct relationship, an indirect relationship? Give reasons.





d) In your groups, discuss how you would teach each of the following terms to your learners. Correlation, correlation coefficient and line of best fit.

Lesson task 1.3

Group presentations on flipcharts are discussed in class.

a) What prerequisite knowledge might learners in your class not have in order to deal with the above tasks?

b) What kinds of questions can be asked to your learners to understand their misconceptions and what can be done to overcome the learners' misconceptions?

c) What kind of real world activity can be done to help them deal with the above tasks?

d) As a result of the facilitation today what then are you going to start doing or stop doing?

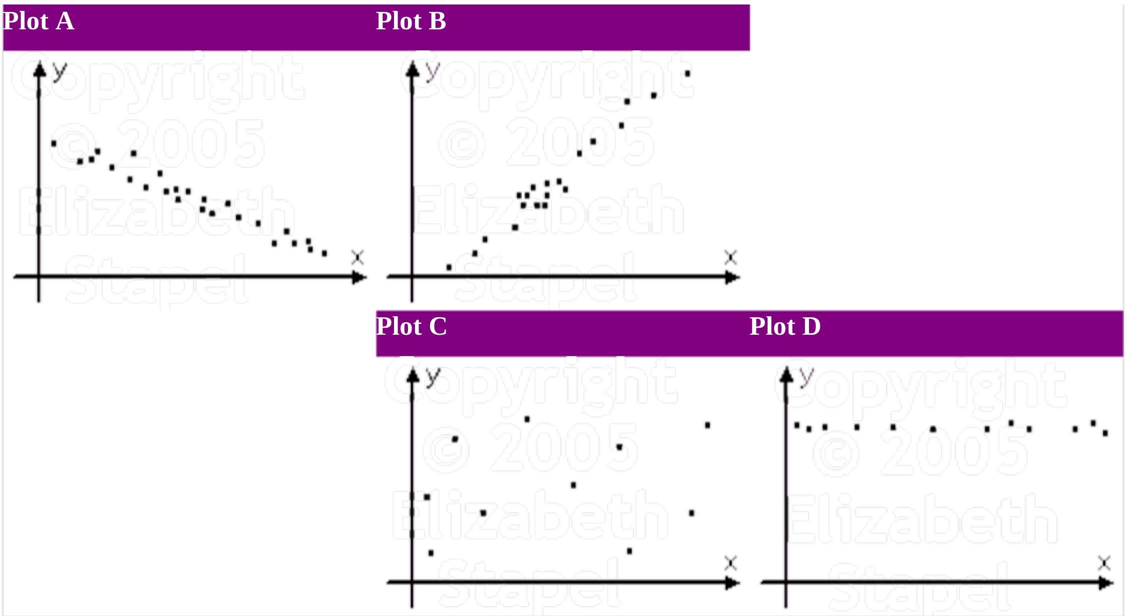
(Adapted from Turnuklu &Yesildere (2007))

Day 2: Facilitation stage 2

Lesson part 2.1

a) Following your discussions about correlation in data, discuss the relationship in the following scatter plots. Put lines of best fit and suggest the correlation (positive, negative, or no correlation etc) and the product moment correlation coefficient/ Pearson's correlation?(Learning Maths)

b)



b) Tell which sort of general equation/model you think would best model the data in the following scatter plots, and why

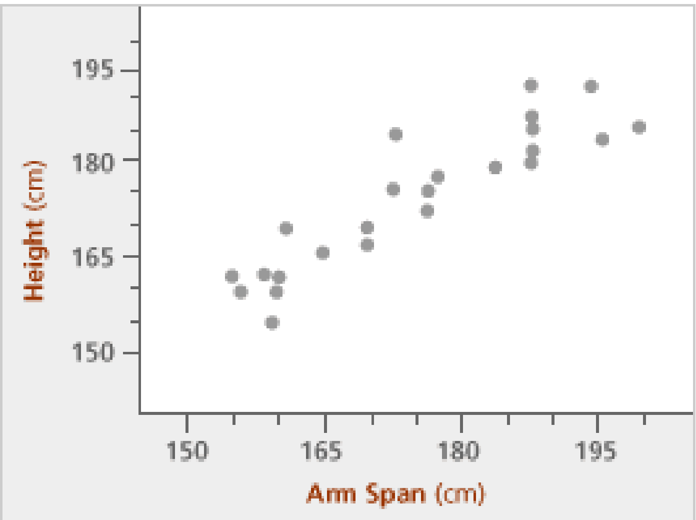
c) Have you ever wondered whether tall people have longer arms than short people?

Data was collected and made available by Learning Math (2012). Measurements (in centimetres) were given for the heights and arm spans of 24 people. Here are the collected data, sorted by increasing order of arm span:

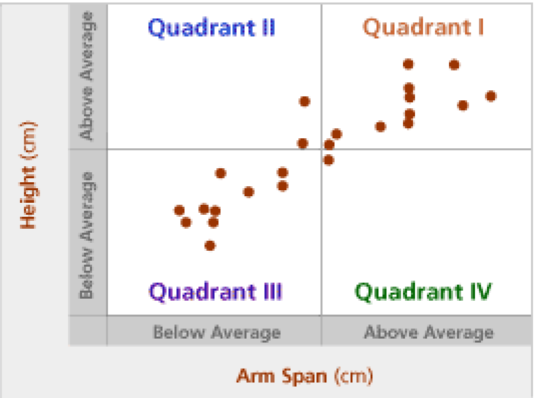
Person #	Arm Span	Height
1	156	162
2	157	160
3	159	162
4	160	155
5	161	160
6	161	162
7	162	170
8	165	166
9	170	170
10	170	167
11	173	185
12	173	176

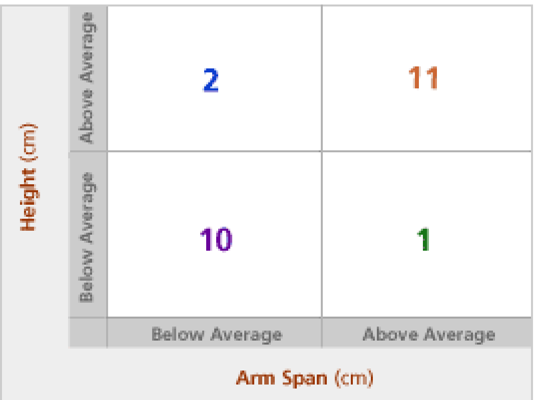
Person #	Arm Span	Height
13	177	173
14	177	176
15	178	178
16	184	180
17	188	188
18	188	187
19	188	182
20	188	181
21	188	192
22	194	193
23	196	184
24	200	186

Below is the completed scatter plot for all 24 people: Judging from the scatter plot, does there appear to be an association between arm span and height? Discuss.



d) If you count the number of points in each quadrant on the scatter plot, you get the following summary, which is called a contingency table: Use the counts in this contingency table to discuss whether this is a better way to represent and discuss data in scatter plots. How do you compare this to a line of best fit?





Lesson part 2.2

Look through the given data in the provided

a) news paper

b) magazine

c) hospital card for babies weight.

Identify bi-variate data from these sources, represent and analyse the found data to the best of your ability.

Lesson part 2.3

a) Make a poster that clearly markets all the ways of collecting the data to analyse for the learners in your Grade 11 and 12 classes. The poster will be put up in your classrooms where you are teaching bi-variate data.

b) Why do you think we sometimes need to collect data ourselves with the learners we teach? Also illustrate on this poster (with examples) the difference between household and time series data.

Lesson part 2.4

Group presentations on flipcharts are discussed.

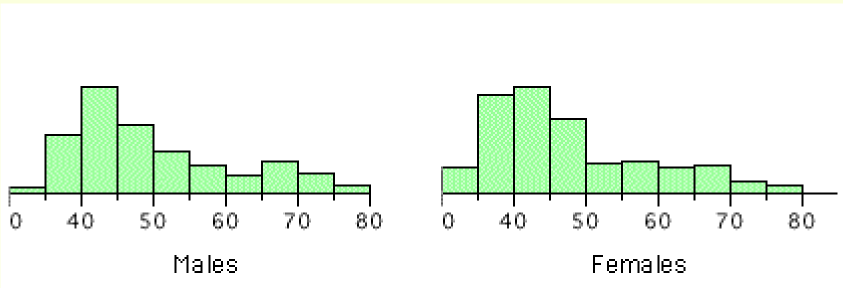
Day 3: Facilitation stage 3

Lesson task 3.1

Table 1. Sample of spousal ages of 10 White American Couples.
(http://www.mathopolis.com/questions/about-questions.php, 2009)

The pairs of ages in Table 1 are from a dataset consisting of 282 pairs of spousal ages, too many to make sense of from a table. What we need is a way to summarize the 282 pairs of ages. Each variable can be summarized by a *histogram* (see Figure 1).

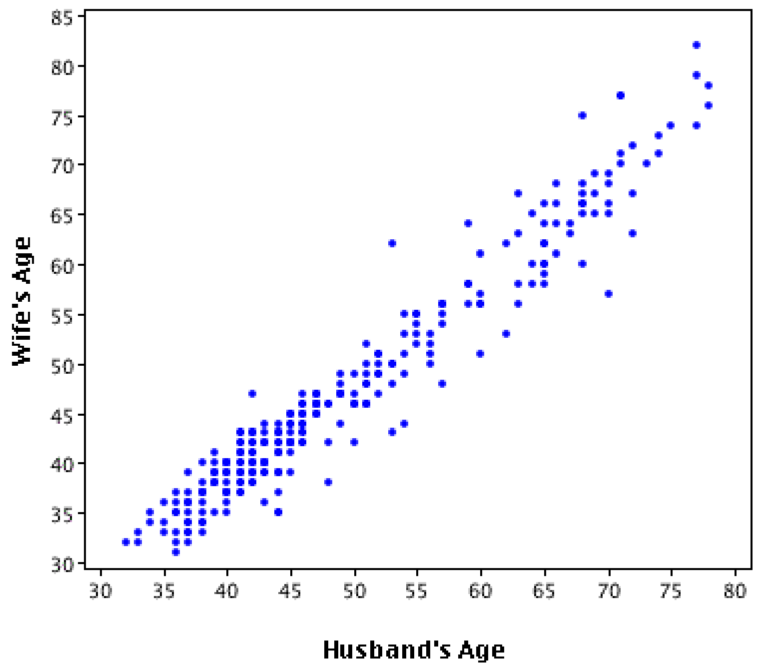
Figure 1 **Histograms of spousal ages.**



a) On the given sheet of paper, what do you think can be learnt or not learnt from representing the described data in this manner?

b) Figure 2 shows a scatter plot showing wife age as a function of husband age.

Do you realize anything interesting or not interesting about the data presented in the Fig 2 as compared to data represented in Fig 1? How would you best model the data? On the given flip chart discuss and write down the story you can tell about the representation in Fig 2.



Lesson task 3.2

Data set 3,2,1

A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in ℓ/100 km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

Data set 3,2,2

The data below represents the times taken by the winners of the men's 100 m freestyle swimming event at the Olympic Games from 1972 to 2004. (www.databaseOlympics.com.), and a research done by a motor company on how the speed of a car affects the fuel consumption of the vehicle.

Year	***Time taken(in seconds)***

1968	51.4
1972	51.2
1976	50.0
1980	50.4
1984	49.8
1988	48.6
1992	49.0
1996	48.6
2000	48.2
2004	48.2
2008	48.12

a) **In Data set** 3.2.2 ,by representing the data as a scatter plot and by drawing a line of best fit for the data indicate whether a linear, quadratic or exponential function best fits the data.

b) **In Data set** 3.2.2 ,describe the trend or relationship that is observed in the bi-variate data sets and give reasons for this trend.

c) **In Data set** 3.2.2 ,what can be said about the efforts of the winners during the Olympic games in the years 1976 and 1988?

d) **In Data set** table 3.2.2 ,use your line of best fit for the Olympic Games from 1972 to 2004 to predict the winning time for 2010.

e) From the conclusion you derived from these two bi-variate data sets above, who do you think can benefit from the information provided?

Lesson task 3.3

Group presentations on flipcharts are discussed (reflection).

Day 4: Facilitation stage 4

Lesson task 4.1

Some researchers in health, suspecting that there must be a relationship between death anxiety and religiosity, conducted the following study. Subjects completed a death anxiety scale (high score = high anxiety) and also completed a checklist designed to measure an individual's degree of religiosity (belief in a particular religion, regular attendance at religious services, number of times per week they regularly pray, etc.) (high score = greater religiosity). They were also not sure if one causes the other. A data sample is provided below:

Death Anxiety	**Religiosity**
38	4
42	3
29	11
31	5
28	9
15	6
24	14
17 2	9 1
19	10
11	15
8	19
19	17
3	10
14	14
6	18

They have hired you to analyze this information and give them feedback in a detailed report form, so as for them to give an informative talk intended to motivate people on how to improve their life styles at the coming Bishop's conference.

a) Please brainstorm, plan and write a report of what aspects you think might/should be included in the feedback report. Clearly indicate how you would plan to analyse this data and write the report to be delivered to the researchers.

b) If, in addition, you are looking to employing a teacher to teach bi-variate data at your school, using the data sample above, design a rubric that you will use to assess and identify your new teacher's understanding, misunderstanding during an interview for the job. Some factors may be particular to bi-variate data, while others may be general factors in teaching which you feel have a significant impact in teaching bi-variate data. Feel free to write your answers in any form you wish.

Lesson task 4.2

Group presentations on flipcharts are discussed.

Day 5: Facilitation stage 5

Lesson Task 5.1

a) Preparing to Teach

You have been made aware that the following table will be given to your learners as part of the end of year examination 2013 (Mathematics Paper 2: Question 5). If you were preparing to teach a section that prepares 98% of your Grade 11 and 12 learners to answer possible questions on bi-variate data about this table, how would you fully prepare yourself to do it? Explain and clarify possible questions and strategies.

Table: 5.1 Complete data set (Watson, J.M.; Collis, K.F.; Callingham, R.A.& Moritz, J.B.1995:275)

Name	Age	Favourite activity	Eye Colour	Weight (kg)	Fast food meals per week
David Jones	8	TV	Blue	30	7
Brian Wong	9	Football	Green	26	1
John Smith	10	Football	Green	29	0
Adam Henderson	12	Football	Blue	45	5
Andrew Williams	14	TV	Blue	60	10
Peter Cooper	16	Board games	Green	54	2
Scott Williams	17	TV	Blue	65	8
Simon Khan	18	TV	Brown	74	12
Rosemary Black	8	Netball	Brown	24	0
Jennifer Rado	9	Board games	Green	33	4
Anna Smith	11	Board games	Brown	32	1
Kathy Roberts	12	Netball	Brown	32	0

Mary Minski	13	Reading	Green	55	3
Dorothy Myers	15	Swimming	Blue	50	2
Sally Moore	17	Reading	Brown	56	1
Janelle MacDonald	18	Reading	Blue	66	4

Lesson task 5.2

Group presentations on flipcharts during in-class discussions.

Lesson Task 5.3

Using your preparation ideas in **Lesson Task 5.1**, and with special reference to the diagrammatic problem/task above fill in the rubric table below

Adapted from CoRe: Content Representation Tool (Loughran, Mulhall, & Berry, 2004) Laura Guerdan	
What is the Big Idea:	
o What do you intend learners to learn about this idea?	
o Why is it important for learners to know this?	
o What else do you know about this idea (that you do not intend learners to know yet)?	
o Prerequisite knowledge learners in your class might not have in order to deal with this idea	
o Briefly brainstorm topics which you might include in the unit	
o Knowledge about learners' thinking which influences your teaching of this idea	
o Other factors that influence your teaching of this idea (Real world activities that can be done to help	

learners deal with the big idea)	
o Teaching procedures to overcome the learner misconception(and particular reasons for using these to engage with this idea)	
o Specific ways of ascertaining learners' understanding or confusion around this idea.	
o As a result of the facilitation this week, what are going to start doing or stop doing	

Appendix 6 (Analysis of evaluation form for educators: FSDOE training)

Venue: Fezile Dabi

Date: 04/01/2011 – 8/01/2011

The questionnaire is aimed at determining your view of the development that you have participated in. The following is a set of questions that viewers might find useful in thinking about and discussing any of the classroom episodes featured in the development. You can give an example instead if you wish to. Though the questionnaire is generally on probability and statistics it is strictly on the teaching of bi-variate data to either Grade 11 or 12. Reflect your true picture and reaction when responding to this questionnaire. Indicate your choice by marking the appropriate block with an ‘X. Take a few minutes to think about the entire In-Service Training that you underwent and give us your answers to the following questions. This information will assist in future training programmes.

UNISA university of south africa

Teacher In-Service Education and Training

Theo Van Wijk Building, Office No: 10-63, PO Box 392, UNISA, 0003

Tel: +27 12 429 6883 Fax: +27 12 429 6956 E-mail: mohapsj@unisa.ac.za

ANALYSIS OF EVALUATION FORM FOR EDUCATORS: FSDOE TRAINING

Mathematics (Grade 11-12)

Compiled by A. Makina

QUESTIONNAIRE FOR EDUCATORS

SECTION A: PERSONAL AND DEMOGRAPHIC INFORMATION

1. Gender:

Female	**42**	Male	**78**

2. Age group:

Younger than 30 years	**1**	20
30 - 39 years	**2**	64

40 - 49 years	**3**	36
50 - 59 years	**4**	0
60 years or older	**5**	0

3. In which area is your school situated?

Rural area	**1**	20
Urban area	**2**	100

4. Years of teaching experience:

5 years or less	**1**	36
6 - 10 years	**2**	39
11 - 15 years	**3**	30
16 - 20 years	**4**	15
21 years or more	**5**	0

5. What is your <u>highest</u> qualification?

Matric	**1**	0
Diploma in education	**2**	32
Advance certificate in education	**3**	56
BEd degree	**4**	16
Honours degree	**5**	10
Masters degree	**6**	0
Other (Please specify):	**7**	

SECTION B: TRAINING EVALUATION

Using scale from 1-5, with 1 meaning not effective and 5 meaning very effective, how would you rate the following?

Statements	Not	Not Very	Good	Effective	Very

	effective	effective			Effective
Coverage of content				36	84
Clarity of presentation			24	48	48
Methods of training			12	24	96
Facilitation provided by trainers			12	18	15
Duration of the training			30	36	54

SECTION C: TRAINER EVALUATION

Please rate trainers by placing an X under the relevant category

Statements	**Very effective**	**Good**	**Not very effective**	**Not Effective**
Knowledge of subject matter	91	29		
Organization of sessions	84	6		
Obvious preparation	108	12		
Style and Delivery	102	18		
Responsive to the group	96	24		
Producing a good learning climate	108	12		

SECTION D: GENERAL

PLEASE PUT AN X NEXT TO THE COMMENT THAT MOST CLOSELY REPRESENT YOUR VIEWS IN THE YES/NO COLUMN

Statements	**Yes**	**No**
My personal objectives for attending the training have been achieved	113	
My understanding of the Learning Area has improved as a	100	

result of this training		
My skills in teaching the learning Area have improved as a results of this training	111	
Training has helped to enhance my appreciation and understanding of my work as a whole.	110	
The training duplicate what I have learnt previously	12	108
Training accommodation was well organised	115	
I would recommend the training to my colleagues	111	
Material provided for training was well structured and easy to use	91	22

SECTION E: WHAT PART OF THE TRAINING DID YOU CONSIDER MOST VALUABLE? AND WHY?

The content (I did not previously understand this topic.)

How to teach the content (Now that I understand the topic I then understand the pedagogy of it as explained during the facilitation.

Knowing your learner (the content has opened my eyes as to the needs of my learners)

Concepts were clearly and exceptionally explained to the workshop attendants. The opening, closing and presentation of sessions were most valuable (facilitation).

SECTION F: WHAT PART OF THE TRAINING DID YOU CONSIDER LEAST VALUABLE? AND WHY?

None

SECTION G: WHAT DO YOU THINK SHOULD BE ADDED TO THE TRAINING PROGRAMME?

Provision must be made to topics that were not attended to (Linear programming, financial mathematics).

We require.

Topics should also be taught online if it is to be relevant today. We must also be taught how to utilise our calculators better as this is impeding our progress.

SECTION H: WHAT BARRIERS MIGHT IMPEDE YOUR IMPLEMENTATION OF THE KNOWLEDGE GAINED IN TRAINING?
Not working in teams among teachers and schools(The team approach)
We do not implement proper mathematical facilitation skills
Fellow teachers not attending training
Too much work to be covered in a short period
Not teaching mathematics anymore
We are not given enough time for explaining to learners: LF's only count activities done in a week (not how much time you spent doing the explanations)

The fact that I am the only educator attending the training in my area (town). If it were 1 educator per school, it would be much easier to implement
Sharing the information with the teachers who did not attend the training.
The programmeme concentrated much on Senior phase work
Lack of relevant teaching materials

SECTION I: HOW WILL YOU AVOID OR NEGATE THESE BARRIERS?
We need communities of practise. Emphasise the importance of team work.
We need training on the facilitation skills. The problem centred approach should always be explained during workshops.
All mathematics teachers to be trained
The syllabus must be well spread and organised by the planners (What comes first etc.)
By insisting to teach mathematics each year
Try to talk to LF's about the issue
I will inform all Grade 4 -9 educators about this through their HOD's and make information available to all who need it and if necessary organise workshops.
Encourage teachers to take into account the policy document in their preparation

Educators should be given training relevant to the phase they are teaching.
Better and improved teaching materials or manuals

SECTION J: WHAT RESOURCES WOULD YOU NEED TO IMPLEMENT WHAT YOU HAVE LEARNT?
• relevant (common) textbooks • calculators • laptops • computers • communities of practice

SECTION K: PLEASE PROVIDE SOME INDICATION OF ANY FURTHER TRAINING NEEDS THAT YOU MIGHT NEED TO ASSIST YOU IN THE TEACHING OF THE LEARNING AREA	
Follow-up workshops	
Further studies in mathematics education at UNISA focusing on pedagogical content knowledge	
SECTION L: ANY OTHER COMMENT RELATING TO TRAINING YOU RECEIVED.	
	Comment(s)
Training and facilitation	• The training is good, very empowering and the learning area becomes more interesting. • Training should be extended to others or it becomes difficult to implement.
Facilities	• Accommodation and food was good.
General	• Saturday training is not preferred as it coincides with participants' social activities. • Teachers needed certificates of participation. • Clarification relating to the certification of the participants on completion of the programmeme was sought.
SECTION M: OVERALL, HOW WOULD YOU RATE THE TRAINING?	

Mark with x to indicate your choice

Excellent	76
Very Good	30
Satisfactory	6
Fair	0
Unsatisfactory	0
Non responses	6

Table 4.3.7: Analysis of an evaluation questionnaire of the development

Appendix 7 (Register)

TRAINERS: **Mrs. Makina**

NO OF TEACHERS: **30**

SUBJECT SPECIALIST: **Mr. Porogo**

NO	SURNAME	FULL NAMES	LEARNER NO	ACTIVITY 1	ACTIVITY 2	ACTIVITY 3	ACTIVITY 4	TOTAL SCORES & COMMENTS
1.								
2.								
3.								
4.								
5.								
6.								
7.								
8.								
9.								
10.								
11.								
12.								
13.								
14.								

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21.								
22.								
23.								
24.								
25.								
26.								
27.								
28.								
29.								
30.								
31.								

Appendix 8 (Consent form for Teachers)

Information Sheet

My name is Antonia Makina. I work at the University of South Africa as an Education Consultant in the Directorate for curriculum and Learning Development. As part of my research for my doctorate, I intend to undertake a professional development for secondary statistics teachers in South Africa using a specially designed pedagogical content knowledge framework. The study is intended to record the experiences that you go through when you are exposed to pedagogical content knowledge. Facilitation will be done using a video camera, and/or audio tape recorder. After videoing the lessons, you, a subject specialist and I will look at the video and talk about things related to the facilitation. It is possible that while videoing you may appear on camera while you are talking to the researcher or the subject specialist. As the researcher, I am responsible for the ethical conduct of this study. Therefore I would like to ask for your approval to be involved in this study. If you agree to be involved sign the attached consent form and return to the subject specialist or me.

Consent Form for Teachers

I, ……………..…………………………………………………………………... have read and understand the Information Sheet about the research project to be carried out by Antonia Makina. I give my approval to be involved in the study, on the conditions outlined below:

- I can withdraw from being involved at any stage, without having to give any reasons.
- I can ask at any stage to not be videoed.
- I can ask Antonia Makina or his supervisor any questions about the study.
- The only people who will see the video are the teachers involved in the study, Antonia Makina and the subject educator working with her.
- In any reports written about the study, it will not be possible to identify me.

Signed: Teacher: …………………………………… Date:……………...……………..

Appendix 9 (Thank you letter of appreciation)

Teacher In-Service Education and Training

Theo Van Wijk Building, Office No: 10-63, PO Box 392, UNISA, 0003

Tel: +27 12 429 6883 Fax: +27 12 429 6956 E-mail: mohapsj@unisa.ac.za

TO: **All teachers who attended UNISA training: 28 September 2011-6 April 2011**

Free State Department of Education

FROM: Dr. Soane Mohapi

SUBJECT: Word of appreciation

I would like to thank and appreciate all teachers who attended training mentioned above. Dr. Tibane says “**You cannot improve on what you do not approve of**” I understand the quote meaning self approval precedes self improvement. Your tolerance and commitment to the training is an indication that you where willing to improve on your selves. You sacrifice your holidays and preferred to attend the training. I was humbled when I learnt that in other centres teachers were able to come to the training on Sunday.

Your role to improve quality of learning is vital; you saw this opportunity as a way of self development and you worked towards improving your capabilities to enable you to give your learners better future.

I therefore on behalf of UNISA INSET wish you luck in whatever you are doing, Go back and plough what you have learnt. As UNISA INSET we have learnt a lot from you and we hope we will meet again.

Dr. SJ Mohapi

Centre for Teacher In-Service Education and Training
Tel.: +27 (0) 12 429□6883
Fax: +27 (0) 12 429 6956
Email: *mohapsj@unisa.ac.za*
Website: *www.unisa.ac.za*

"Putting ourselves in another person’s shoes is the key to empathy and true non-judgemental understanding" (Unknown author)

Appendix 10 (On-Site Monitoring Tool)

IMPLEMENTATION MONITORING QUESTIONNAIRE

Take a few minutes to think about the entire development that you are undergoing and give your answers to the following questions. This information will assist in future training/development programmes.

UNISA university of south africa

Name of Centre: ____

Centre Contact details: ____

Postal Address: ____

____

____

____

Tel:

____

Email:

____

Centre Manager:

____

Supervisor:

____

Name of the Subject Specialist:

____

Subject/Learning Area:

____

Lesson Topic:

____

Name of Trainer:

____

Date: ____

1 *Needs improvement*	2 *Acceptable*	3 *Good*	4 *Very good*	5 *Above average*

	1	**2**	**3**	**4**	**5**	***Comments***
Evidence of planning Lesson plan clearly stipulates • Learning/ Subject Area • Specific Outcomes • Assessment Criteria • Content Knowledge						
Choice of appropriate materials • Variety of materials used • Relevant to the lesson • Help to attain the specific outcomes						
Lesson Presentation Introduction of lesson • Lesson incorporates new knowledge into existing knowledge • Arouse learners interest • Promote participation and interaction throughout						
Teaching strategies • Variety of teaching and learning strategies used to explain concepts • Skill in questioning techniques • Usage of the Language of Teaching and Learning (LOLT) • Time utilization (flow and pacing according to learner ability)						
Learner activities • Alternative activities used relating to learner						

diversity • Appropriate to learners level • Assessment Criteria used to measure the specific outcomes						
Conclusion • Skill in giving appropriate activities • Evidence of positive reinforcement • Recognition of individual needs						
General • Classroom management (e.g. discipline/ controlled activity) • Creation of learning space						
Constructive feedback and suggestions for improvement						
Support • Does teacher need support? • If yes, indicate type of support needed						

Appendix 11 (Tables for collection and analysis of data)

Appendix Table 11.1: Analysis of data for WEEK 1/2/3/….

Stages of the PCK framework	Lesson part 1.1	Lesson part 1.2	Lesson part 1.3	Lesson part 1.4
o **Previous/present knowledge** *(Organized Activities: Appendix 1.1)*	Data is real numbers Real numbers are natural numbers and decimals	Data was clarified but teachers had problems with giving examples of qualitative data. For continuous data no one gave an example beyond decimal numbers (e.g. fraction)	Did not understand the question.	
o **Bringing teachers into context** *(Organized Activities: Appendix 1.2)*				
o **Modelling** *(Organized Activities: Appendix1. 3)*				
o **Realistic/case based problem posing** *(Organized Activities: Appendix 1.4)*				
o **Assessment and error pattern analysis** *(Organized Activities: Appendix1. 5)*				

Appendix 12 Organization of the PCK experiences

Table 11.2: Organization of the PCK experiences

Reference topics	**Previous/ present knowledge**	**Bringing teachers into context**	**Modelling**	**Realistic/case based problem posing**	**Assessment and error pattern analysis**	**Summative Experiences/ week**
Teachers' PCK experiences	MONDAY (Day1)	TUESDAY (Day1)	WEDNESDAY (Day1)	THURSDAY (Day1)	FRIDAY (Day1)	
Group1(20) July/2010 5-9(Pilot)	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Group 2(25) Jan /2011 3-7	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Group 3(15) Jan /2011 11-15	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Group 4 28March to 01 April /2011	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Group 5(25) April /2011 4-8	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Group 6(25) April /2011 11-15	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	Result1 Result2 Result3	
Summative experiences/ day						

Appendix 13 (Part of the collected data set using excel and SPSS)

Record No.	Gender	Group	Location	Age	HiQual	Years Teach	Grade8	Grade9	Grade10	Grade11	Grade12	Q1	Q2	Q3	Q4
1	2	2	Wartburg	2	1	1	1	0	1	0	0	5	4	4	4
2	2	1		4	6	6	1	1	1	0	0	2	4	4	4
3	2	2	Polokwane	5	3	1	0	0	1	0	0	4	4	4	5
4	1	1	Polokwane	4	2	2	0	0	0	1	0	4	2	5	2
5	1	1	Butterworth	4	6	5	1	1	0	0	0	3	4	3	4
6	1	1	Lydenburg	3	3	5	0	1	1	1	1	1	2	2	4
7	2	1	lebowakgomo	3	6	3	0	0	1	1	1	1	2	2	3
8	1	1	Tzaneen	4	3	2	0	0	0	1	1	4	3	5	4
9	2	1		3	6	2						5	5	5	5
10	1	1	Makhado	4	6	3	1	1	1	0	0	4	3	3	3
11	2	1	Volkrust/ Mpumalanga	3	2	3	0	0	1	1	0	4	4	4	4
12	1	1	Newcastle	4	3	1	0	0	1	0	0	2	4	2	4
13	2	1		4	6	1	0	0	1	1	1	3	4	3	2
14	1	1		5	3	6	1	1	1	1	1	2	2	2	1
15	2	1	Limpopo/ Thohoyandou	4	6	5	1	1	0	0	0	4	4	1	2
16	2	1	Mamelodi	4	6	1	0	0	0	1	0	4	4	4	4
17	1	1	Giyani central	3	6	2	0	1	0	0	0	4	4	1	4

18	2	1	Mkhuhlu township	4	6	2	0	0	0	1	1	2	4	5	5
19	2	1	Burgersford	3	6	1	0	0	0	1	1	3	4	5	4
20	1	1	Nelspruit	2	2	2	0	0	1	1	0	4	4	3	4
21	1	1		4	6	1	0	0	1	1	1	3	2	3	2
22	2	1	Hlambanyathi area	3	2	2	1	1	1	0	0	4	5	5	4
23	2	1	Thohoyandou (Limpopo)	4	6	6	1	1	1	0	0	1	4	1	2

Q39G8	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	0	1
Q39G9	0	1	1	1	1	1	1	1	0	0	1		0	1	1	1	1
Q39G10	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0
Q39G11	0	0	0	1	0	1	1	1	1	0	1	1	1	1	1	0	0
Q39G12	0	0	0	1	0	1	1	1	1	0	1	1	1	1	1	0	0
Q40G8	1	1	0	1	1	1	0	0	0	0	0	0	0		1	0	1
Q40G9	0	0	0	1	1	1	0	0	0	0	0	0	0		0	1	1
Q40G10	1	1	0	0	0	1	0	0	1	1	1	1	1		1	0	0
Q40G11	0	0	0	1	0	1	0	1	1	0	1	1	1		1	0	0
Q40G12	0	0	0	1	0	1	0	1	1	0	0	0	1		1	1	0
Q41	2	2	2	1	2	4	2	1		1	1	2	2	2	2	2	2
SpecStats	0	0	0	0	0	0	0	0	0			1	0	0	0	0	0
Maths	1	1	1	1	1	1	1	1	1			1	0	0	1	1	1
Economics	0	0	0	0	0	0	0	0	0			1	0	0	0	0	0
Psychology	0	0	0	0	0	0	0	10	0			1	0	0	0	0	0

Geography	0	0	0	0	0	0	0	0	0			1	0	1	0	0	0
Q42	1	3	2	1	8	6	10	3	7			18	3	14	15	13	16
Q43	3	2	1	1		3	4	1				2	3	3	4	1	1
Q45a	1	2	1	1	1	2	2	1	2	2	1	3	1	2	2	2	2
Q45b	2	2	1	3	2	2	2	2	2	2	2	1	2	2	2	2	3
Q45c	2	2	1	1	2	1	2	1	2	2	1	2	2	2	2	2	2
Q45d	2	2	3	2	2	2	1	2	2	2	2	1	2	2	2	2	3
Q45e	2	2	3	2	2	2	2	2	2	2	2	1	2	2	2	2	3
Q45f	2	2	3	2	2	2	2	2	2	2	2	1	2	2	2	2	3
Q45g	1	3	3	2	1	3	2	1	2	2	1	2	1	2	1	1	3
Q45h	1	3	3	1	2	2	2	1	2	2	1	2	1	2	2	1	2
Q45i	2	1	3	1	2	2	1	1	1	2	1	1	1	2	2	1	2
Q45j	2	2	3	1	1	2	2	1	1	2	1	2	1	2	2	1	2
Q45k	2	2	3	2	2	2	2	1	1	2	2	1	2	2	2	1	2
Q46a	2	3	1	3	2	1	2	3	3	2	3	1	3	3	3	3	3
Q46b	1	3	1	3	3	3	3	3	2	2	3	1	3	3	3	3	3
Q46c	3	3	2	2	2	3	3	2	1	2	2	2	3	1	3	1	3
Q46d	1	2	2	3	2	2	1	2	2	2	1	1	2	3	3	3	3
Q46e	2	2	3	3	3	3	3	1	2	2	3	1	3	3	3	3	3

Appendix 14 (Part of the results captured from collected data)

Frequencies

Notes

Output Created		04-JUL-2012 17:02:44
Comments		
Input	Data	C:\Documents and Settings\makina\My Documents\Pedagogy Data table 04-07-2012.sav
	Active Dataset	DataSet2
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	23
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data.
Syntax		FREQUENCIES VARIABLES=Gender Group NLocation HiQual Age Location Years_Teach /ORDER=ANALYSIS.
Resources	Processor Time	00:00:00.02
	Elapsed Time	00:00:00.02

[DataSet2] C:\Documents and Settings\makina\My Documents\Pedagogy Data table 04-07-2012.sav

Statistics

		Gender	Group	NLocation	HiQual	Age	Location	Years_Teach
N	Valid	23	23	18	23	23	23	23
	Missing	0	0	5	0	0	0	0

Frequency Table

Gender

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	10	43.5	43.5	43.5
	Female	13	56.5	56.5	100.0
	Total	23	100.0	100.0	

Group

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Black	21	91.3	91.3	91.3
	White	2	8.7	8.7	100.0
	Total	23	100.0	100.0	

NLocation

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	1	4.3	5.6	5.6
	2	2	8.7	11.1	16.7
	3	1	4.3	5.6	22.2
	4	1	4.3	5.6	27.8
	5	1	4.3	5.6	33.3
	6	1	4.3	5.6	38.9
	7	1	4.3	5.6	44.4
	8	1	4.3	5.6	50.0
	9	1	4.3	5.6	55.6
	10	2	8.7	11.1	66.7
	11	1	4.3	5.6	72.2
	12	1	4.3	5.6	77.8
	13	1	4.3	5.6	83.3
	14	1	4.3	5.6	88.9
	15	1	4.3	5.6	94.4
	16	1	4.3	5.6	100.0
	Total	18	78.3	100.0	
Missing	99	5	21.7		
Total		23	100.0		

HiQual

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	1	4.3	4.3	4.3
	2	4	17.4	17.4	21.7
	3	5	21.7	21.7	43.5
	6	13	56.5	56.5	100.0
	Total	23	100.0	100.0	

Age

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	18-25	2	8.7	8.7	8.7
	26-35	7	30.4	30.4	39.1
	4	12	52.2	52.2	91.3
	5	2	8.7	8.7	100.0
	Total	23	100.0	100.0	

Location

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid		5	21.7	21.7	21.7
	Burgersford	1	4.3	4.3	26.1
	Butterworth	1	4.3	4.3	30.4
	Giyani central	1	4.3	4.3	34.8
	Hlambanyathi area	1	4.3	4.3	39.1
	lebowakgomo	1	4.3	4.3	43.5
	Limpopo/ Thohoyandou	1	4.3	4.3	47.8
	Lydenburg	1	4.3	4.3	52.2
	makhado	1	4.3	4.3	56.5
	Mamelodi	1	4.3	4.3	60.9
	mkhuhlu township	1	4.3	4.3	65.2
	Nelspruit	1	4.3	4.3	69.6
	Newcastle	1	4.3	4.3	73.9
	Polokwane	2	8.7	8.7	82.6
	Thohoyandou (Limpopo)	1	4.3	4.3	87.0
	Tzaneen	1	4.3	4.3	91.3
	Volkrust/Mpumalanga	1	4.3	4.3	95.7
	Wartburg	1	4.3	4.3	100.0
	Total	23	100.0	100.0	

Years Teach

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	7	30.4	30.4	30.4
	2	7	30.4	30.4	60.9
	3	3	13.0	13.0	73.9
	5	3	13.0	13.0	87.0
	6	3	13.0	13.0	100.0
	Total	23	100.0	100.0	

DESCRIPTIVE VARIABLES=Gender NLocation Age Group HiQual Years Teach/ STATISTICS = MEAN STDDEV MIN MAX.

Descriptive

Notes

Output Created		04-JUL-2012 17:03:47
Comments		
Input	Data	C:\Documents and Settings\makina\My Documents\Pedagogy Data table 04-07-2012.sav
	Active Dataset	DataSet2
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	23
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	All non-missing data are used.
Syntax		DESCRIPTIVES VARIABLES=Gender NLocation Age Group HiQual Years_Teach /STATISTICS=MEAN STDDEV MIN MAX.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.00

[DataSet2] C:\Documents and Settings\makina\My Documents\Pedagogy Data table 04-07-2012.sav

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Gender	23	1	2	1.57	.507
NLocation	18	1	16	8.22	4.747
Age	23	2	5	3.61	.783
Group	23	1	2	1.09	.288
HiQual	23	1	6	4.43	1.879
Years_Teach	23	1	6	2.74	1.815
Valid N (listwise)	18				