THE DEVELOPMENT OF MATHEMATICAL PROBLEM SOLVING SKILLS OF GRADE 8 LEARNERS IN A PROBLEM-CENTRED TEACHING AND LEARNING ENVIRONMENT AT A SECONDARY SCHOOL IN GAUTENG

By

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DECLARATION

I declare that, "**THE DEVELOPMENT OF MATHEMATICAL PROBLEM SOLVING SKILLS OF GRADE 8 LEARNERS IN A PROBLEM-CENTRED TEACHING AND LEARNING ENVIRONMENT AT A SECONDARY SCHOOL IN GAUTENG"** is my own work and that all the sources I have used or quoted have been indicated or acknowledged by means of complete references.

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Signature Date

Brantina Chirinda

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DEDICATION

This work is dedicated to my sons

ARCHIFORD MANDIZVIDZA

and

ADRIAN MANDIZVIDZA

ABBREVIATIONS

ANOVA	One-way analysis of variance
CAPS	Curriculum and Assessment Policy Statement
GDE	Gauteng Department of Education
JHB	Johannesburg
MPSSI	Mathematical Problem solving Skills Inventory
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
PCA	Problem-centred approach
PCI	Problem-centred instruction
PCTL	Problem-centred teaching and learning
PCTLA	Problem-centred teaching and learning approach
PCTLE	Problem-centred teaching and learning environment
SPSS	Statistical Package for Social Sciences
TIMSS	Trends in Mathematics and Science Survey
UNISA	University of South Africa
ZPD	Zone of proximal development

SUMMARY

This mixed methods research design, which was modelled on the constructivist view of schooling, sets out to investigate the effect of developing mathematical problem solving skills of grade 8 learners on their performance and achievement in mathematics. To develop the mathematical problem solving skills of the experimental group, a problem-centred teaching and learning environment was created in which problem posing and solving were the key didactic mathematical activity. The effect of the intervention programme on the experimental group was compared with the control group by assessing learners' problem solving processes, mathematical problem solving skills, reasoning and cognitive processes, performance and achievement in mathematics. Data were obtained through questionnaires, a mathematical problem solving skills inventory, direct participant observation and questioning, semi-structured interviews, learner journals, mathematical tasks, written work, pre- and post- multiple-choice and word-problem tests. Data analysis was largely done through descriptive analysis and the findings assisted the researcher to make recommendations and suggest areas that could require possible further research.

Key concepts: grade 8 learners; mathematical achievement; mathematical performance; mathematical problem; mathematical problem solving skills; problem-centred teaching and learning approach; problem-centred teaching and learning environment; problem solving; skills development.

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CHAPTER 1

INTRODUCTION AND BACKGROUND TO THE STUDY

1.1 Introduction to the study

South Africa participated in TIMSS 1995, TIMSS 1999 and TIMSS 2003 and was rated last in all three studies, even coming in behind African countries such as Ghana and other developing nations that spent far less of their budgets on education than South Africa does. TIMSS (Dossey, Giordano, McCrone & Weir 2006:34), which is formally known as the Trends in International Maths and Science Survey, is an international study whose aim is to assess the national curricular, school and social environment, and learners' achievements in mathematics and science in participating countries across the world. As a mathematics educator, the researcher realised that this poor performance can be attributed to a content-based curricular, rote learning, the teacher-centred teaching approach, the lack of a problem-centred teaching and learning approach and a lack of development of mathematical problem solving skills in learners.

The results of the survey by Anderson (1998) suggest that traditional approaches gained more support from South African teachers compared to the problem-centred teaching and learning approach. There is support for the more traditional practices, learning algorithms before doing problems, relating problems to the specific content of lessons and focusing on practising skills (Anderson 1998:8). Cangelosi (1996:31) also observes that the traditional teaching approach is still the most dominant approach in most mathematics classrooms even though reforms in education have recommended contemporary approaches. Traditional teaching by rote learning is claimed to produce inert knowledge in learners, knowledge that can be used in educational settings such as preparing for tests and examinations, but cannot be transferred into real life situations (Tynjala 1999:373). However, despite learning mathematical facts and content being essential it is not adequate (Dendane 2009); learners need to develop mathematical problem solving skills.

The major aim of education is for learners to acquire knowledge and problem solving skills that they will apply in the subject, other disciplines, outside school, in their daily lives and in the world of work (Wessels & Kwari 2003:74-5). The Department of basic education Curriculum and Assessment Policy Statement (2011:4) aims to "equip learners with knowledge, skills and values necessary for meaningful participation in the society". To equip learners with problem solving skills and to improve performance in mathematics the National Council of Teachers of Mathematics (NCTM 1991a; NCTM 1991b) recommends the use of a problem-centred teaching and learning approach. The problem-centred teaching and learning approach is supported by research by Murray, Olivier and Human (1998) on teaching and learning in mathematics in South African schools. The problem-centred approach has been recommended as a teaching strategy that can facilitate the transfer of knowledge and skills from the classroom to the rest of the world (Wessels & Kwari 2003:75). Therefore this study set out to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The researcher was also interested in the effect of these mathematical problem solving skills on grade 8 learners' performance and achievement in mathematics.

In this study, for grade 8 learners to be in a position to develop mathematical problem solving skills, the researcher proposed the problem-centred teaching and learning approach, which focuses on teaching and learning through problem solving and promotes high-level engagement of learners. The problem-centred teaching and learning approach model builds on the work of Human (1992), Olivier (1999), Van de Walle (1998; 2004), Murray, Olivier and Human (1992; 1993; 1998), Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne (1996) and integrates the aspect of the zone of proximal development (Vygotsky 1978) and scaffolding (Wood, Bruner & Ross in Woolfolk 2007:48). It incorporates key aspects of metacognition apprenticeship (Flavell 1979), reflective thinking (Van de Walle 2004: 23) and social interaction (Schoenfeld 2002; Nathan & Knuth 2003:175-2007; Kilpatrick 1985; 1987). In this problem-centred teaching and learning approach, the researcher assisted learners to become effective thinkers and to develop good mathematics habits of mind (Cuoco, Goldeberg & Mark 1996), guided them to work independently and at the same time helped them to acquire important mathematical problem solving skills.

1.2 Purpose of the study

The Department of basic education Curriculum and Assessment Policy Statement (2011:5) aims to produce learners that are able to "identify and solve problems and make decisions using critical and creative thinking". South Africa's poor performance indicate that traditional methods of teaching mathematics are failing to provide most learners with the skills to solve problems even though they will need these to function effectively in society. However the problem-centred teaching and learning approach takes another direction: it involves the recall of facts; the use of a variety of mathematical problem solving skills and procedures; the ability to evaluate one's own thinking and the coordination of knowledge; previous experience and intuition. Kadel (1992:7) advocates that learners in a problem-centred teaching and learning environment are learning to develop and apply mathematical problem solving skills since they are expected to utilise their own resources and experiences when approaching new situations. The problem-centred teaching and learning approach promotes the development of mathematical problem solving skills through exploration and discovery (Kadel 1992:4). The importance of developing the mathematical problem solving skills of learners has been emphasised for almost a century (Bruner 1961; Dewey 1910; 1916). However, the researcher found limited information on research that has been conducted on how best mathematical problem solving skills of learners can be developed or acquired. The purpose of this study was therefore to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment and its effect on their performance and achievement in mathematics.

1.3 Significance of the study

South Africa's performance in TIMSS was followed by comment, debate and suggestions for improvement (Brombacher 2001) – hence the significance of this study. To improve learners' performance and achievement in mathematics, instruction has to be designed to promote mathematical problem solving skills. Every mathematics teacher must be geared towards designing effective instruction that promotes meaningful learning. This study explored the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment and investigated its effect on these learners' performance and achievement in mathematics.

This study is also significant since it is the first study, as far as the researcher knows, to employ the convergent research design using diverse data collection instruments to investigate grade 8 learners' development of mathematical problem solving skills in a problem-centred teaching and learning environment in South Africa. Much of the prior research has been done on the problem-centred teaching and learning approach but not on the type of mathematical problem solving skills learners have to develop. The researcher found limited factors on the development of mathematical problem solving skills in the literature. She therefore identified that a need existed for such a study. From the findings of this study, designing instruction to promote the development of mathematical problem solving skills is shown to be a worthwhile endeavour.

1.4 Review of literature

The empiricist view of teaching is that teachers transmit knowledge and learners absorb and memorise this knowledge. This approach leads to subjective knowledge which is largely reconstructed objective knowledge (Murray et al 1998). Traditionally South Africa follows the empiricist view of teaching, the curriculum is content based (Van der Horst & McDonald 2005:26), and teachers present the material, work with a few examples and give learners similar problems as exercises or homework for the day. Learners are never asked to reason through the development of the content (Dossey et al 2006:37) and learn little other than the sequence of steps they need to follow in order to solve a particular type of a problem (Van der Horst & McDonald 2005:138). The traditional view is that mathematical knowledge and skills must be acquired first and then applied. The emphasis is on the teaching of certain algorithms that can be employed to solve problems (Wessels & Kwari 2003:75). This form of instruction is inadequate for the following reasons:

(1) Most learners do not make the connection between mathematics learned in school and the application of that mathematics outside of school unless they learned the mathematics in a real-world problem solving context (Hiebert & Carpenter 1992). It has been observed that knowledge acquired in the classroom does not transfer well to the profession chosen by learners (Wessels & Kwari 2003:75). Wessels and Kwari (2003) further stress that no-one can predict all the existing problems one would meet in a lifetime to prepare for methods of solving them while still in school.

(2) Learners are likely to retain algorithmic skills and knowledge of rules only as long as they continue to use them. Unless learners have learned to apply them to solve problems from their own real world, they are hardly motivated to continue with them once the skill has been tested and they have moved on to other lessons (Schoenfeld 1989).

(3) Most learners are simply not motivated to work towards goals that do not appear to have long-term benefits (Woolfolk 1993:366-397). Experience in teaching problem solving using traditional approaches has shown that learners do not appreciate the role that problem solving plays in teaching and learning of mathematics (Wessels & Kwari 2003:77).

Educational reform is required to promote learners' thinking powers, problem solving abilities and mathematical problem solving skills (Van der Horst & McDonald 2005:4). In search of the answer regarding the question of what modern education should be like, numerous studies have underlined the importance of teaching for understanding (Cobb 1986; Bell 1993; Hiebert &

Wearne 1993). But how can we achieve teaching for understanding? Hiebert et al (1996) emphasise that understanding is the goal for mathematics instruction and problematising the subject leads to the development of mathematical problem solving skills and the construction of knowledge.

In their research, Lorsbach, Tobin, Briscoe and LaMaster (1990) found that learners learn effectively when actively linking new knowledge to their existing knowledge. The traditional transmission model of teaching should thus be complemented with a problem-centred, cognitive-constructivist model of learning in which the teacher, proceeding from learners' previous knowledge, helps them build a network of knowledge useful in solving new problems (Scardamalia & Bereiter 1989). The learner's intuitive knowledge thus becomes the starting point of teaching (Fennema, Carpenter & Peterson 1989). Dossey et al (2006:72) point out that the process of problem solving involves using prior knowledge in new or different ways, formulating a plan or strategy to reach the desired goal and possibly acquiring new knowledge about the given situation.

The problem-centred teaching and learning approach is a learner-centred educational method that uses problem solving as the starting point for learning (Bligh 1995). It means both the curriculum and instruction should begin with problems, dilemmas and questions for learners (Wessels & Kwari 2003:80) and the subject should be allowed to be "problematic" (Hiebert et al 1996:12). Learning occurs when learners grapple with problems for which they have no routine methods available (Murray et al 1992). The problem is presented first and serves as the organising centre and context for learning (Bligh 1995) and learners are expected to explore problems, make conjectures and draw generalisations about mathematics concepts and processes in introductory activities. Each individual learner should experience the freedom to individually choose solution strategies and methods of computation (Human 1992:16). The teacher must let learners struggle together towards solutions without suggesting the procedures. Therefore the teacher acts as a facilitator, moderator and supporter rather than a major source of knowledge. Teaching is not transmitting of knowledge but helping learners to actively construct knowledge by assigning tasks that enhance this process (Tynjala 1999:365).

The expansion of a teaching repertoire to include problem-centred instructional activities is challenging and demanding, however using good real-life problems to plan instruction with the focus on learners' thinking, reasoning and the development of mathematical problem solving skills is one strategy that holds promise. Hence this study endeavoured to investigate the development of mathematical problem solving skills of grade 8 learners and its effect on these learners' performance and achievement in mathematics in a problem-centred teaching and learning environment.

1.5 Mathematical problem solving skills found in the literature

The process of solving mathematical problems involves a variety of skills (Kadel 1992:1). In the literature (Lenchner 1983; Bransford & Stein 1984; Gick 1986; Polya 1957; Kadel 1992; Hiebert & Wearne 1993; Van de Walle 1998; Adamovic & Hedden 1997:20-23; Dendane 2009), there are seven mathematical problem solving skills that the researcher found to be of particular importance. These are discussed below and further elaborated on in section 2.11.

1. **Understanding or formulating the question in the problem-** learners must be able to first formulate the question in the problem and make sense of it.
2. **Understanding the conditions and variables in the problem-** during the process of understanding the conditions and variables, the problem solver (learner) "internalises the problem". The learner develops a sense of how the conditions and variables relate to each other and clarifies the meaning of the information explicitly stated or implied in the problem.
3. **Selecting or finding the data needed to solve the problem-** the learner must be able to identify needed data, eliminate data not needed

and collect and use data from a variety of sources such as graphs, maps or tables.

4. **Formulating sub-problems and selecting appropriate solution strategies to pursue**- the learner must be able to determine if there are sub-problems or sub-goals to be solved and decide which strategy or strategies to use and when.

5. **Correctly implementing the solution strategy or strategies and solving sub-problems-** the learner must know how to implement a solution or solution strategies.

6. **Giving an answer in terms of the data in the problem-** the learner must be able to give an answer in terms of the relevant features of the problem.

7. **Evaluating the reasonableness of the answer-** the learner must be able to determine whether or not the answer or solution makes sense.

1.6 Theoretical framework

The focus of this study is investigating the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The problem-centred teaching and learning approach is a typical constructivist view of schooling. Constructivist epistemology accepts that learners are capable of constructing their own knowledge and therefore should be actively involved in their learning (Wessels & Kwari 2003:76). Learning is not passive reception of ready-made knowledge, but a learner's active continuous process of constructing and reconstructing of his or her conceptions of phenomena (Tynjala 1999:365) and learners themselves have to be primary actors (Von Glasersfeld 1995:120). Constructivism emphasises understanding instead of memorising and reproducing information (Tynjala 1999:365) and the fact that knowledge arises from the interaction of the learner's existing and new ideas. The constructivist perspective on learning incorporates the following assumptions:

- Learning is a process of knowledge construction and not knowledge absorption.
- Learning is knowledge dependent and learners use existing knowledge to construct new knowledge.
- Learning is a social process and learners learn from each other and the facilitator through discussions and sharing of ideas.
- Learners are aware of the processes of cognition and can control and regulate them (Antony 1996:349).

1.7 Problem formulation

It is well-documented in the literature that traditional teaching results in the lack of transfer of knowledge and problem solving skills to learners, prompting debate and research into more effective approaches to dealing with problems. Many approaches have been recommended to help learners to transfer acquired knowledge and skills in school to deal with the real world and one of the most advocated ways is equipping the learner with problem solving skills (Wessels & Kwari 2003:69). Problem-based learning curricula have been introduced in many schools around the world (Koh, Khoo, Wong & Koh 2008:34). Given the South African situation of inadequately trained teachers, limited resources and overcrowded schools, evidence-based evaluation of the effects of problem-centred instruction and the development of mathematical problem solving skills on mathematics learning would strengthen any justification of its adoption in schools. Hence this study set out to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment as well as the effect these skills have on learners' performance and achievement in mathematics. The research problem can be formulated as set out below.

Does a problem-centred teaching and learning environment have an effect on the development of mathematical problem solving skills of grade 8 learners?

More specifically to find answers for the above broad research problem, the study aimed to address the following research sub-questions:

(1) What does the problem-centred teaching and learning approach (PCTLA) entail?

(2) What are the mathematical problem solving skills that grade 8 learners need to develop?

(3) What do grade 8 learners need as a prerequisite for mathematical problem solving skills to develop?

(4) What are the obstacles that grade 8 learners have to overcome before they can really benefit from the problem-centred teaching and learning approach?

(5) How do grade 8 learners in a problem-centred teaching and learning environment develop mathematical problem solving skills?

(6) Do mathematical problem solving skills influence the interpreting of "new" knowledge and solving of non-routine problems?

(7) Do grade 8 learners who receive problem-centred instruction develop mathematical problem solving skills and perform better in similar tasks given to their peers who receive traditional instruction?

1.8 Hypothesis

The following hypothesis was formulated from the research problem:

> *The problem-centred teaching and learning environment has a positive effect on the development of mathematical problem solving skills of grade 8 learners.*

1.9 Aims of the study

The aim of this study was to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The researcher also investigated whether grade 8 learners in the experimental group, who received problem-centred instruction (PCI) and developed mathematical problem solving skills, performed and achieved better in mathematics as compared to the control group, who received traditional instruction which was mainly teacher centred.

To achieve the above aim, the specific objectives were to

(1) explore what it implies to teach through the problem-centred teaching and learning approach;

(2) explain the mathematical problem solving skills that grade 8 learners needed to develop;

(3) establish what grade 8 learners needed as a prerequisite for the development of mathematical problem solving skills;

(4) identify the obstacles that grade 8 learners had to overcome before they could really benefit from the problem-centred teaching and learning approach;

(5) develop the mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment;

(6) establish if the development of mathematical problem solving skills influenced the interpretation of “new” knowledge and the solving of non-routine problems;

(7) test whether grade 8 learners who receive problem-centred instruction develop mathematical problem solving skills and perform better in similar tasks given to their peers who receive traditional instruction; and

(8) make recommendations on the adoption of the problem-centred teaching and learning approach and the development of learners’ mathematical problem solving skills in South African schools.

1.10 Research methodology

1.10.1 Research locale

The research was conducted at a secondary school in Gauteng, South Africa. This secondary school was chosen because of its “convenient location and accessibility” (McMillan & Schumacher 2006:125) to the researcher and it represents a typical South African secondary school which is under-resourced, overcrowded and has large classes. The experiment involved 57 grade 8 mathematics learners in the 2012 academic year (see section 3.8).

1.10.2 Pilot study

A pilot study is a small-scale survey, that is, a trial run (Steffens & Botha 2002:164). In this study, the questionnaire, mathematical problem solving skills inventory, pre- and post-multiple choice and word-problem tests were pilot-tested on 20 grade 8 learners at a neighbouring secondary school, and the same questions and analysis as for the main investigation were used (see section 3.11.9). A pilot study helps to identify the required changes and makes suggestions to improve clarity and format (McMillan & Schumacher 2006:35). Piloting gave the researcher an initial idea of the pattern of responses that were likely and an estimation of the amount of time that was required to complete the study.

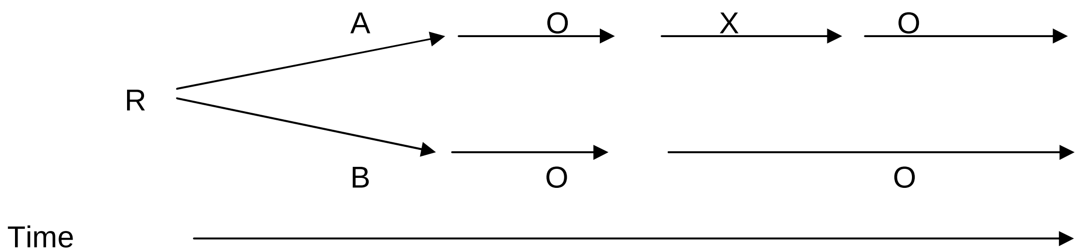
1.10.3 Research design

This study used a mixed methods research design (McMillan & Schumacher 2006:401), which is a procedure for collecting, analysing and "mixing" both quantitative and qualitative data at some stage of the research process within a single study, in order to understand a research problem more completely (Creswell 2002). When combined, quantitative and qualitative methods complement each other, and this allowed the researcher to incorporate the strengths of each method (McMillan & Schumacher 2006:401). Mixed methods research design also facilitated a complete analysis of the research problem and provided a valid report for this study.

This study employed one of the most popular mixed methods research design in educational research, **convergent research design** which is also known as the **triangulation design.** In convergent research design, the researcher simultaneously gathers both quantitative and qualitative data, merges them using both quantitative and qualitative data methods and then interprets the results together to provide a better understanding of the phenomena of interest (McMillan & Schumacher 2006:404).

For the quantitative strand, a randomised pre-test-post-test control group design was employed. Randomised pre-test-post-test control group design is a form of randomised experimental design (McMillan & Schumacher 2006:267) and it is represented in the table below.

Random assignment Group Pre-test Treatment Post-test



In the above diagram, R represents randomisation of subjects – group A is the experimental group, group B is the control group, X is the treatment and O represents the pre-test and the post-test given to the participants. The grade 8 mathematics learners were assigned randomly to either the control group or the experimental group by using a table of random numbers (see appendix V) and applying the numbers to the learners. The simple random sampling method was used to select the study participants, because it allowed all the learners to have "the same probability of being selected" (McMillan & Schumacher 2006:120). With simple random sampling, bias was avoided because there was high probability that all the population characteristics were represented in the sample. A total of 28 learners were assigned to the experimental group and another 29 learners to the control group. The group of learners taught by the researcher using the problem-centred teaching and learning approach was called the experimental group, while the group that was taught by the current grade 8 mathematics educator using the traditional teaching method was called the control group. Both the control and experimental groups were involved in the study during their usual mathematics lessons, that is, 4.5 hours a week for ten weeks during the third term of the 2012 academic year. In total, during these ten weeks the respondents were expected to attend the intervention programme for a minimum of 10 × 4.5 = 45 hours.

At the end of the intervention, the researcher made comparisons between grade 8 learners (subjects) who had experienced the problem-centred approach and developed mathematical problem solving skills and those who had not experienced the problem-centred teaching and learning approach. The quantitative research strand used the following data collection techniques: the questionnaire; the mathematical problem solving skills inventory; mathematical tasks; written work; pre- and post-multiple-choice and word problem tests; to gather data. Learners were required to fill in questionnaire on their beliefs and attitudes towards problem solving before and after the intervention.

The qualitative strand employed interactive methods like direct participant observations and questioning (see section 3.11.1), learner journals (see section 3.11.5) and in-depth semi-structured interviews (see section 3.11.4) (McMillan & Schumacher 2006:316) to collect data from the learners. Interactive qualitative methods use face-to-face techniques to collect data from people in their natural settings (McMillan & Schumacher 2006:26); in case of this study, the researcher conducted semi-structured interviews with learners during and after problem solving sessions in their classrooms. The researcher selected a few learners at a time to interview, observe and question during the intervention. The semi-structured interviews, observation and questioning of learners were conducted during and after the problem solving sessions to gain more insight into the learners' reasoning processes, cognitive processes, thinking skills and how mathematical problem solving skills develop in their minds. Learners were required to record regularly in their journals, their successes, challenges, and feelings about the lessons and problem solving experience they had gained.

1.10.4 The research tools and the collection of data

The researcher first conducted a literature study (see chapter 2) to gain theoretical knowledge of the problem-centred teaching and learning approach, the mathematical problem solving skills and to establish what grade 8 learners needed as a prerequisite for the development of mathematical problem solving skills. The researcher used the questionnaire, mathematical problem solving skills inventory, direct participant observations and questioning, learner journals, semi-structured interviews, mathematical tasks, written work, pre- and post-multiple-choice and word-problem tests to gather the data (see section 3.11).

A questionnaire (see appendix J) was chosen as a research tool because questionnaires are relatively economical, have the same questions for all subjects (grade 8 learners) and can ensure anonymity (McMillan & Schumacher 2001:194). The questionnaire allowed the researcher to establish and identify the obstacles that grade 8 learners had to overcome before they could really benefit from the problem-centred teaching and learning approach and be in a position to develop mathematical problem solving skills. The questionnaire addressed the following:

1 learners' attitudes towards mathematics.

2 learners' willingness to engage in problem solving activities.

3 learners' perseverance during the problem solving process.

4 learners' self-confidence with respect to the problem solving process.

The mathematical problem solving skills inventory

The mathematical problem solving skills inventory (see appendix K) was devised by the researcher on the basis of knowledge gained from the literature review. It specifically identified the seven problem solving skills found in the literature, namely learners must be able to formulate the question in a given problem; understand the conditions and variables in the problem; select or find the data needed to solve the problem; formulate sub-problems and select appropriate solution strategies to pursue; correctly implement the solution strategy or strategies and solve sub-problems; give an answer in terms of the data in the problem; and evaluate the reasonableness of the answer. The mathematical problem solving skills inventory was administered to both the experimental and control groups at the beginning and end of the intervention. The participants had to assess their mathematical problem solving skills on a ten-point scale.

Direct participant observation and questioning

During the intervention, the researcher directly observed learners during the problem solving process, that is, how they interacted with peers, what questions emerged and the mathematical problem solving skills they used to solve new problems. The researcher extensively questioned learners as they solved problems and recorded findings briefly and on the spot. Questioning stimulated learners' mathematical thinking and this helped the researcher to evaluate the learners' mathematical problem solving skills. The researcher used direct observations and questioning, because they are among the best methods of evaluating some of the goals of problem solving (Wheatley 1991:6). The researcher recorded findings using the problem solving observation comment card (see appendix N), the problem solving observation rating scale (see appendix O) and the problem solving observation checklist (see appendix P).

Semi-structured interviews

To gain insight into the reasoning and cognitive processes of learners, the researcher conducted semi-structured interviews (see appendix L) with one or two learners during or after a problem solving session. The problem solving comment card, the problem solving rating scale and the problem solving observation checklist were used to record the findings from the semi-structured interviews. An audio recording was also used to collect more detailed information for subsequent analysis. The audio recordings were transcribed by the researcher as soon as she got home, and content analysis was carried out to establish patterns that existed in the responses.

Learner journals

During the intervention, learners were regularly requested to write a report in their journals on a problem solving experience they had completed. With assistance from focus questions (see section 3.11.5), learners were required to think back and describe how they would have solved the problem. The researcher used learner journals because they provide information on "individual learners' use of problem solving skills and strategies" (Wheatley 1991:23).

Mathematical tasks, written work and word-problem tests were used to assess learners' progress in developing mathematical problem solving skills and in constructing mathematics knowledge. Mathematical tasks, written work and word-problem tests required the learners to supply the answers and this allowed the researcher to view the learners' work, thus providing her with a greater understanding of learners' mathematical problem solving skills and various thinking processes in solving problems. The researcher used an analytic scoring scale (see appendix Q) to assess the mathematical tasks, written work and word-problem tests. An analytic scoring scale was used by the researcher because with it she was able to assign scores to each of the several phases of the problem solving processes.

Pre- and post-multiple choice tests (see appendix T and U) were used to assess learners' progress in developing mathematical problem solving skills and problem solving processes. Multiple choice tests are versatile and can measure learners' ability to obtain a correct answer as well as their ability to use problem solving skills (Wheatley 1991:36). The researcher was motivated to use multiple-choice tests as a data collecting tool because they permitted wide sampling and broad coverage of content because of learners' ability to respond to many items. Multiple-choice tests limit bias caused by poor writing skills and different response alternatives may provide diagnostic feedback by analysing patterns of incorrect responses.

1.10.5 Reliability and validity

Reliability (also see section 3.13.1) refers to "consistency of measurement" (McMillan & Schumacher 2006:183), that is, the extent to which independent administration of the same instrument (or highly similar instruments) consistently yields similar results under comparable conditions (De Vos 2002:168). Validity (also see section 3.13.2) is the extent to which inferences and uses made on the basis of numerical scores are appropriate, meaningful and useful (McMillan & Schumacher 2006:179). A research tool is said to be valid if it measures what it is supposed to measure. To ensure reliability of the written work and word-problem tests, an analytic scoring scale (see appendix Q) was used to assess learners' work. Pilot testing of the questionnaire, the mathematical problem solving skills inventory, pre- and post-word problems and multiple-choice tests were conducted to ensure their reliability and validity. Content validity was ensured by making sure that all the problem solving skills were represented by items and questions in the questionnaire, the mathematical problem solving skills inventory, written work, word problems and multiple-choice tests. Before the intervention, these research tools were also given to the study supervisor, for her opinions, analysis and approval.

1.10.6 Ethical procedures

In compliance with the Unisa research ethics policy, the researcher sought and obtained informed consent from the Gauteng Department of Education, Johannesburg North District of Education, the school principal, the school governing board, the parents or legal guardians of the participants and the participants themselves to conduct the research. The researcher obtained assent from grade 8 learners involved in the study and informed their parents using letters they had to sign. The school principal, participants and their parents or guardians were clearly informed of the confidential nature of the research, that participation was voluntary and the participants had an option of not participating in the research and could withdraw from the research at anytime without incurring any negative consequences. After this, an ethical clearance certificate was granted to the researcher by Unisa's ethics committee to conduct the research.

1.10.7 Data analysis and interpretation

Data analysis in the mixed methods research design involves separately analysing the quantitative data using quantitative methods and the qualitative data using qualitative methods (Creswell & Plano Clark 2011:211) and then merging the two databases. In this study, the statistical software package SPSS was used mainly to analyse the quantitative data. Qualitative data analysis was used to substantiate the quantitative results.

1.11 Chapter layout

This dissertation is divided into the following chapters

Chapter 1 – Introduction and background to the study.

Chapter 2 – Review of the literature.

Chapter 3 – Research methodology and design.

Chapter 4 – Data presentation, analysis and interpretation.

Chapter 5 – Summary of the research, recommendations and conclusions.

Chapter 1 introduces and provides brief contextual background on the investigation.

Chapter 2 presents the literature review and provides theoretical background to the traditional teaching approach, the problem-centred teaching and learning approach, the mathematical problem solving skills and what grade 8 learners need as a prerequisite for the development of mathematical problem solving skills.

Chapter 3 describes the research design. This chapter explains how the research was set up, what happened to the participants, the methods of data collection that were used and the research tools used.

Chapter 4 displays, discusses and analyses the results and presents the findings.

Chapter 5 is the final chapter and summarises the whole research project. Conclusions based on the findings are explained. Limitations of the study, recommendations and areas for possible future research are discussed.

1.12 Definitions of the concepts

1.12.1 Mathematical problem

To define a mathematical problem, a general problem has to be defined first. A problem can be taken as any situation in which some information is known and other information is needed (Nieman & Monyai 2006:114) and a goal has to be attained and a direct route to the goal is not obvious (Wessels & Kwari 2003:72). Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier and Wearne (1997) point out that a problem is any task or activity for which learners have no prescribed or memorised rules or methods; nor is there a perception by learners that there is a specific “correct” solution method. From the prior definitions of a problem, a mathematical problem can be defined as a task:

1 in which a learner is interested and engaged and for which he/she wishes to obtain a solution

2 for which the learner does not have a readily accessible mathematical means by which to achieve a solution (Schoenfeld 1989:87–89)

Cangelosi (1996:29) emphasises that mathematical problems should not be confused with mathematical textbook exercises. In the problem-centred teaching and learning approach, learners are expected to solve problems or make sense of mathematical situations for which no well-defined routines or procedures exist (Erickson 1999:516).

1.12.2 Problem solving

Problem solving is the entire process of dealing with a problem (Wessels & Kwari 2003:73) and is a type of discovery learning which, when it is deliberately applied, can help learners realise that the knowledge they already have may be applied in new situations and this process can lead to new knowledge (Killen 1996:98). Mayo, Donnelly, Nash and Schwartz (1993:227) concur with this definition when they state that problem solving can be taken as a strategy for “posing significant, contextualized, real world situations and providing resources, guidance and instruction to learners as they develop content knowledge and problem solving skills”. Problem solving can be used as a part of a lesson or as a part of several lessons. Dossey et al (2006:72) argue that problem solving is the process whereby we answer the question or deal with the situation. A difficult problem for one person may be a routine and quick computation for another person.

1.12.3 Problem-centred approach

The problem-centred teaching and learning approach involves the learning of mathematics through real contexts, problem situations and models (Van de Walle, Karp & Bay-Williams 2013:32) and problem solving is used as a "vehicle for learning" (Human 1992:16). The problem-centred approach is perceived by Murray et al (1998) as teaching mathematics through problem solving: using problem solving as a technique for helping learners to learn other concepts. Davis (1992:127) cited in Murray et al (1998:31) states the following:

> Learning through problem solving means that instead of starting with "mathematical" ideas and then applying them, we would start with problems or tasks, and as a result of working on these problems, the children would be left with a residue of "mathematics" and we would argue that mathematics is what you have left over after you have worked on problems. We reject the notion of "applying" mathematics because of the suggestion that you start with the mathematics and then look around for ways to use it.

In the problem-centred teaching and learning approach, learners are allowed to wonder why things are, to inquire, to search for solutions and to resolve incongruities (Wessels & Kwari 2003:80).

1.12.4 Mathematical problem solving skills

The process of solving mathematics problems involves a variety of problem solving skills (Kadel 1992:1). Mathematical problem solving skills are mental processes that allow a learner to take on a mathematical problem, choose the best of many mathematical problem solving techniques for that particular situation, and go through the process of finding a solution to the problem. Mathematical problem solving skills are crucial to learners because they give them the ability to face a problem head on, use the techniques they have learnt and come to a desired outcome, thus solving the problem with the least difficulty possible and in the most effective way.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1 Introduction

Max Wertheimer in Luchins and Luchins (1970:1) posed the following important question:

"Why is it some people, when faced with problems get clever ideas, make inventions and discoveries? What happens, what are the processes that lead to such solutions? What can be done to help people to be creative when they are faced with problems?" Mayer (1998:50) further asks the following question: "What does a successful problem solver know that an unsuccessful problem solver does not know?" Research by Ericsson and Smith (1991) points to the problem solver's problem solving skills. Problem solving skills are highly valued (Sweller 1988:257) and a number of mathematical problem solving skills are needed for a mathematics learner to be an effective problem solver. In mathematics, learners develop these problem solving skills only if genuine mathematical problem solving takes place. Genuine mathematical problem solving is believed to take place in a problem-centred teaching and learning (PCTL) environment. Therefore, in order to develop the mathematical problem solving skills of grade 8 learners, the researcher created a problem-centred teaching and learning environment. The problem-centred approach is a teaching approach that aims to motivate learners to participate in the learning process and helps to enhance and foster mathematical problem solving skills.

The purpose of this chapter is to present a literature review on the problem-centred teaching and learning approach, the development of mathematical problem solving skills of learners and what grade 8 learners need as a prerequisite for mathematical problem solving skills to develop. The NCTM (1980) recommended that problem solving should be the focus of teaching mathematics, because it is believed to encompass mathematical problem solving skills and functions which are useful in everyday life. However, people have different views on the teaching of problem solving. Researchers like Cobb, Wood and Yackel (1991:25), Hiebert et al (1996:152-3) and Murray et al (1998:270) have provided evidence of the benefits of the problem-centred teaching and learning approach (PCTLA). The PCTLA can be contrasted with the traditional teaching approach which predominantly focuses on the memorisation of mathematical content.

In the discussions that follow, the traditional approach of teaching mathematics will be briefly examined. The literature review will concentrate on the PCTLA, which is the vehicle used in this study to develop the mathematical problem solving skills of grade 8 learners. A review of studies relating to the thinking processes effective for problem solving, views on mathematics, how children learn mathematics and knowledge of mathematics will also be examined. Strategies found in the literature for solving problems and Polya's four-phase problem solving process will also be reported on. The last section of the literature review discusses the mathematical problem solving skills found in the literature, what learners need as a prerequisite for mathematical problem solving skills to develop and strategies for developing learners' mathematical problem solving skills.

The literature review that was conducted addressed the first, second and the third research questions for this study:

- What does the problem-centred teaching and learning approach entail?
- What are the mathematical problem solving skills that grade 8 learners need to develop?
- What do grade 8 learners need as a prerequisite for mathematical problem solving skills to develop?

2.2 Views about mathematics

The philosophy of mathematics influences the content, organisation, methods and general structure of the mathematics curriculum. Lakatos in Lerman (1990) lists two philosophical schools of mathematics, namely the absolutist and fallibilist philosophies.

2.2.1 Absolutist philosophy

According to the absolutist view, mathematics is perceived as “the realm of certain, unquestionable and objective knowledge” which “consists of certain and unchallengeable truth” waiting to be discovered (Ernest 1991:3&7). Mathematical knowledge is thought to exist as an object of some kind outside of people, residing in books, independent of the thinking being, and as a result teachers implement a curriculum to ensure learners cover the content in textbooks. Subsequently mathematics is viewed as “a bag of rules and prescriptions” to be learnt and mastered, the content is more important than the method, and learning means acquiring this content. The focus is on teaching learners what to think and not how to think. Learning consists of memorisation of facts, principles and recitation of lessons.

2.2.2 Fallibilist philosophy

Fallibilist philosophy views “mathematical truths as fallible, corrigible and that they can never be regarded as beyond revision and correction” (Ernest 1991:18). Mathematical ideas do not exist as objects outside people, but exist only in the thinking of individuals which are not directly accessible to others (Threlfall 1996:3). According to Popper in Ernest (1991) the fallibilist philosophy views mathematical knowledge as a result of human activity (problem solving) and it is open to change and further development. The focus of teaching is mainly on the process, and this does not mean that the content is left out, but that it must be combined with the method.

2.2.3 Comparison between the absolutist and fallibilist teaching styles

The table below compares the absolutist and fallibilist teaching styles.

Absolutist	Fallibilist
Behaviourist approach	Constructivist approach
Clear and coherent presentation by teacher and teacher's knowledge is unquestionable	Self-discovery
Pupils practise textbook exercises	Real-world examples and problems
Emphasis on content (content-centred)	Emphasis on the process
Discouragement of discussions	Encouragement of discussions

Table 2.1 Comparison between the absolutist and fallibilist teaching styles

From the above discussion it can be observed that that the two philosophical schools of mathematics influence how children learn mathematics. According to Human (1992:34), there are two broad perspectives relating to how children learn mathematics; that is the behaviourist view and the cognitive view and these are discussed in the next section.

2.3 How children learn mathematics

2.3.1 The behaviourist view

Behaviourists view learning as "change of behaviour from the behaviour an individual exhibits before s/he is subjected to a learning condition to the new behaviour s/he exhibits after s/he has been subjected to a learning condition" (Human 1992:34). Human (1992) further points out that according to the behaviourist view, mathematical knowledge is viewed as a basket of facts and skills, a product rather than a process, and this knowledge is therefore believed to be transferred intact from one person to another. The role of the teacher is to transmit information to a passive learner who is seen as an "empty vessel" to be filled. In the behaviourist view of learning, knowledge is thought to exist outside the learner, so appropriate conditions of learning should be established between the learner and the knowledge to enable the learner to "copy" this knowledge (Human 1992:35). The teacher tries to create a rich learning environment because it is believed that "we understand what we see" (Olivier 1999:1-26). The problem with the behaviourist view is that it tries to impart the knowledge to learners and assumes that they see what teachers see.

2.3.2 The cognitive view

The cognitive view sees learning as an active process in which learners instead of simply "receiving" knowledge, seek new information to solve the problem and reorganise what they already know to achieve new insights (Woolfolk 2006:248). Learners are active participants in the construction of their own knowledge. Each individual learner's knowledge construction is unique because new ideas are interpreted and understood in terms of learners' existing knowledge. The cognitive view believes that "we see what we understand" and that learning and knowledge construction rests with the individual learner. Conceptual knowledge cannot be transferred ready-made and intact from the teacher to the learners, but learners must construct their own conceptual knowledge.

The knowledge that learners have consists of internal and mental representations of ideas that their minds have constructed. Hiebert and Carpenter (1992) distinguish between two types of mathematical knowledge, that is, conceptual knowledge and procedural knowledge and these are discussed in the next section.

2.4 Knowledge of mathematics

2.4.1 Conceptual knowledge

Conceptual knowledge consists of logical relationships constructed internally and connected to already existing ideas (Van de Walle 1998:25) and originates in the mind of the individual learner through reflective abstraction, that is, thinking about ideas and actions. It is the type of knowledge that Piaget (1960) referred to as logico-mathematical knowledge (Labinowicz

1985). Hiebert and Carpenter (1992) stress that conceptual knowledge is knowledge that is understood and exists in the mind of the learner as part of a network of ideas. Conceptual knowledge is knowledge about mathematical concepts and relationships and is not taught explicitly. Learners are challenged with problems and tasks that assist them to build on their existing knowledge and in “reorganizing and re-structuring these ideas towards more sophisticated notions” (Murray, Olivier & Human 1993). Learners exhibit conceptual understanding when they recognise, label and generate examples of what concepts are and are not, and when they use concepts and their representations to discuss or classify mathematical objects (Dossey et al 2006:48). Conceptual knowledge is crucial in the learning of mathematics, because learners usually fail to solve problems if they do not thoroughly understand the concepts. Learners’ understanding of the concepts forms the basis of their higher-order learning and the development of their mathematical problem solving skills.

2.4.2 Procedural knowledge

Procedural knowledge of mathematics is knowledge of the rules and the procedures that one uses in carrying out routine tasks and also the symbolism that is used to represent mathematics (Van de Walle 1998:25). Van de Walle (2004:28) points out that procedures are step-by-step routines learnt to accomplish some task and a few cognitive relationships are needed to have the knowledge of a procedure. Procedural knowledge is essential in mathematics – for example, algorithmic procedures help us to do tasks easily and symbolism conveys mathematical ideas to others. However, even the most skilful use of a procedure will not help develop conceptual knowledge that is related to that procedure, for example, doing a lot of multiplication exercises does not help a learner understand what multiplication means. Therefore procedural rules should be learnt in the presence of a concept (Van de Walle 2004:28), linking procedures and conceptual ideas is far more important than the usefulness of the procedure itself (Hiebert & Carpenter 1992), and overemphasising procedural skills without understanding the mathematical principles should be avoided. Learners demonstrate procedural knowledge when they select and apply procedures correctly and when they verify the appropriateness of a procedure for carrying out a given task (Dossey et al 2006:49).

2.5 Theories of and approaches to the teaching of problem solving in mathematics

2.5.1 The traditional approach

The traditional approach perceives “teaching as the transmission of knowledge and learning as the absorption of knowledge” (Murray et al 1998). Learners are seen as “empty vessels” and it is the teacher’s duty to fill in those vessels with knowledge about how calculations are performed by standard methods and to provide practice until the learners can perform these methods accurately (Shuard 1986). The traditional approach is regarded by Wessels and Kwari (2003:85) as teaching learners problem solving processes in the hope that they will use these processes to solve problems. Its main focus is teaching mathematics “for” problem solving. Wessels and Kwari (2003) further point out that with this approach, teaching is about problem solving and problem solving is not treated as a way of thinking.

The teacher represents the source of all knowledge (Van de Walle 1998:10) and shows learners how they are to do assigned exercises. Learners work at the exercises by trying to follow what was demonstrated. If a learner has difficulty, the teacher will show again how the work is to be done by breaking the mathematics into small logical pieces, explaining and drilling each piece in sequence. Research findings indicate that traditional methods of teaching, which view the learner as the passive absorber of information who “learns” through repeated practice and reinforcement, are not satisfactory for most contexts (Resnick 1987). The reason for this is that they “leave most learners believing that mathematical knowledge is mysterious and beyond understanding” (Van de Walle 2004:37) and this knowledge must only be memorised and repeated in a test without making sense of it. In the traditional approach, problem solving is separated from the learning process, learners expect the teacher to explain all the algorithmic rules and will not attempt to solve problems for which no solution methods have been provided (Van de Walle 2004:37). Learners believe that every problem given by the teacher has one solution and that there is only one "right" way to approach and solve a problem.

2.5.2 The problem-centred teaching and learning approach

The problem-centred teaching approach is referred to as "teaching mathematics through problem solving" by Murray et al (1998) in their article "Learning through problem solving". Learning mathematics takes place while learners are "grappling" with the problem (Murray et al 1992). Learners are placed in the active role of problem solvers by being confronted with unfamiliar tasks that have no readily known procedure or algorithm. The teacher poses powerful non-routine mathematical problems to learners for solving and they are expected to justify and explain their solutions. The teacher accepts right or wrong answers in a non-evaluative way (Cobb et al 1991) and knows the right time to intervene or step back, letting learners make their own way (Lester, Masingila, Mau, Lambdin, Dos Santos & Raymond 1994).

The teacher focuses on teaching "mathematical topics through problem solving contexts and enquiry oriented environments" (Taplin 2001:1), providing just enough information to establish the background of the problem (Cobb et al 1991). The primary goal is to extend one's own thinking as well as that of others. It means that both the curriculum and instruction should start with problems, dilemmas and questions for learners (Wessels & Kwari 2003:8). The single most important principle for reform in mathematics is to allow learners to make the subject "problematic" (Hiebert et al 1996:12). Hiebert et al (1996) further stress that learners should be allowed to "wonder why things are, to enquire, to search for solutions, to resolve incongruities".

According to Murray et al (1998:269-285), the PCTLA reflects the belief that subjective knowledge should be experienced by learners as personal constructs and not as reconstructed objective knowledge. The PCTLA is based on the constructivist view of learning (Felder & Brent 2002:5; Savery & Duffy 1995), in that learners reorganise experiences to resolve a new problem situation (Ernest 1996:346-347) and also on the social constructivist view (Vygotsky 1978), in that learners learn mathematics through social interactions, meaning negotiation, presenting solutions to groups or classes. In the following section constructivism, cognitive schemas in problem solving, social constructivism and zone of proximal development will be discussed in detail.

2.5.2.1 Constructivism

Constructivism rejects the belief that children are blank states (Van de Walle 2004:22), instead, they are conceptualised as mathematical thinkers who try to construct meaning and make sense for themselves of what they are doing on the basis of their personal experiences (Shuard 1986). Learners are creators of their own knowledge and do not absorb ideas as teachers present them (Van de Walle 1998:22). Learning is not passive reception of information, but learners' active continuous process of construction and reconstruction of their conception of phenomena. Constructivism is grounded in learners' prior knowledge. Learners interpret information on the basis of their existing knowledge. Constructivism emphasises understanding instead of memorisation and reproduction of information and relies on social interaction and collaboration in making meaning (Tynjala 1999:364-365).

Olivier (1999:1–24) identifies the following as the characteristics of constructivism:

- The learner is not viewed as a passive receiver of knowledge, an "empty vessel" into which the teacher must pour knowledge. Conceptual knowledge cannot be transferred ready-made from the teacher to the learner.
- The learner is seen as an active participant who constructs his or her own knowledge. The learner uses his or her existing knowledge and previous experience to understand and interpret new ideas. Learning occurs when learners reorganise and restructure their present knowledge structures.
- Learning is a social process. Learners learn from each other and the teacher through discussions, communication and sharing of ideas, by

actively comparing ideas, reflecting on their own thinking and trying to understand other people's thinking by negotiating a shared meaning.

Constructivism principles are rooted in Piaget's (1960) processes of assimilation and accommodation (Van de Walle 2004:23). Piaget's view of the mind was one of constantly changing structures that help us make sense of what we perceive (Brooks & Brooks 1993). When learners perceive things they know, they use existing schema to fit them into existing ideas and Piaget refers to this mental activity as assimilation. If learners perceive something unfamiliar, they modify or create new ideas to fit the new information. This action is referred to as accommodation (Van de Walle 2004:22). Accommodation is a modification of our cognitive framework that permits assimilation of the new idea (Labinowicz 1985).

2.5.2.2 Cognitive schemas in problem solving

Knowledge construction is an extremely active endeavour on the part of learners (Von Glasersfeld, 1990:22) and they must be mentally active for learning to take place (Van de Walle 2004:23). Instead of passively receiving information, learners actively interpret meaning through the lenses of their existing knowledge structures (Antony 1996:349). Acquisition of knowledge takes place when the learner incorporates new experiences into existing mental structures and reorganises those structures to handle more problematic experiences (Kilpatrick 1987). When the learner is involved in construction and reconstruction of knowledge, he or she forms integrated networks of ideas known as "cognitive schemas" (Sweller 1988:259).

Sweller (1988) defines schemas as "structures which allow problem solvers to recognise a problem state as belonging to a particular category of problem states that normally require particular moves". Marshall (1990) concurs with this definition when he defines a schema as the knowledge represented in our minds through networks of connected concepts, information, rules and problem solving strategies. Cognitive schemas are both the product of constructing knowledge and the tools with which new knowledge is constructed (Van de Walle 1998:25) and as learning occurs, the networks are rearranged, added to or otherwise modified (Van de Walle et al 2013:19).

Learners try to make sense of what is being taught by trying to fit it with their experiences and the more connections with the existing network of ideas, the better the new ideas are understood. Therefore the teacher must challenge learners to grapple with new ideas, to work at fitting them into their existing networks and challenge their own ideas and those of others.

2.5.2.3 Social constructivism

Social constructivism (Vygotsky 1978) advocates that meaningful learning takes place when individuals are engaged in social activities. It also stresses the importance of the nature of the learner's social interaction with knowledgeable members of the society. Vygotsky suggested that without the social interaction with other more knowledgeable people, it is impossible for a learner to acquire social meaning of important symbol systems and learn how to utilize them. Young children develop their thinking abilities by interacting with other children, adults, and the physical world. From the social constructivist viewpoint, it is thus important to take into account the background and culture of the learner throughout the learning process, as this background also helps to shape the knowledge and truth that the learner creates, discovers, and attains in the learning process (Wertsch 1997).

Realistic Mathematics Education (RME) is an example of a mathematics curriculum that subscribes to social constructivism. RME views mathematics as a human activity that must be connected to reality; be close to children and be relevant to their everyday life situations. Social constructivism links up with Vygotsky's "zone of proximal development" (ZPD); where learners are challenged within close proximity to, yet slightly above, their current level of development.

2.5.2.4 Zone of proximal development (ZPD)

The zone of proximal development (Vygotsky 1978) is the phase at which a learner can solve a problem or master a task if given appropriate help and support (Woolfolk 2007:622) by someone at a higher level who may be a teacher, a parent or a peer learner. This support for learning and problem solving is called "scaffolding" (Wood, Bruner & Ross 1976 in Woolfolk

2007:48). Scaffolding can be in the form of “a clue, reminders, encouragements, giving examples or anything that allows the learner to grow in independence as a learner” (Woolfolk 2007:620). If learners working within the zone of proximal development are scaffolded, multiple opportunities can be created for them to develop the knowledge of mathematical concepts and mathematical problem solving skills.

2.6 Learning and teaching in the problem-centred approach

In the problem-centred teaching and learning environment, learning begins where the learners are (Van de Walle 2004:37) and doing mathematics means developing, explaining, reflecting on and improving ideas (Hiebert et al 1997:166). Learning mathematics is regarded as a social activity (Schoenfeld 2002) and social interactions create opportunities for learners to talk about their thinking and this talk encourages reflection (Murray et al 1998:3). Learners interacting with other learners when solving mathematics in a group stimulate basic or cognitive and a higher level or metacognitive mathematical thinking (Posamentier & Jaye 2006:28) and this is important, because their learning processes are strongly influenced by their metacognitive knowledge (Flavell 1987:232) and the interpretation of their learning environment (Anthony 1996:350).

2.6.1 Prior knowledge in the problem-centred approach

Teaching should start with ideas that learners have, the ideas they will use to create new ones (Van de Walle 2004:37). This is because learners cannot absorb an idea as it is taught but give meaning to unfamiliar information in terms of their existing knowledge. Each learner comes to the classroom with a unique but rich collection of ideas. These ideas are the tools that will be used to construct new concepts and procedures as learners grapple with new ideas, discuss solutions, challenge their own and others’ conjectures, explain their methods and solve engaging problems.

The NCTM learning principle requires that learners “actively build new knowledge from experience and prior knowledge” (NCTM 2000:3). The connection between existing and new knowledge is an essential component of meaningful learning. Learners’ ability to remember new knowledge depends on what they already know (Halpern 1996:51), and for this reason it is important that teachers assess and activate learners’ existing knowledge. Bell (1993:11) suggests that it is essential for teachers to start their lessons with tasks that give learners the opportunity to use and show their existing knowledge. New knowledge that is well connected to existing knowledge is remembered better by learners (Hiebert & Carpenter 1992:75) and research in the cognitive sciences suggests that learning is enhanced if it is connected to prior knowledge and that it is more likely to be retained and applied to future learning (Lappan, Phillips & Fey 2007:73).

2.6.2 Metacognition in the problem-centred approach

Metacognition (Flavell 1979) or “thinking about thinking”, which is regarded by Lester (1994) as the driving force in problem solving, refers to “what we know about what we know” (Halpern 1996:28) and the awareness of and the ability to control one’s thinking processes, in particular, the selection and use of problem solving strategies. It includes conscious monitoring (being aware of how and why you are doing something) of one’s own thinking and self-regulation (choosing to do something or deciding to make changes) of learning (Van de Walle 2004:54).

Flavell (1979) describes metacognition as the awareness of how one learns the ability to judge the difficulty of a task, the monitoring of understanding, the use of information to achieve a goal and the assessment of the learning progress. Good problem solvers automatically monitor their own thinking regularly, are deliberate about their problem solving actions and know when they are stuck or do not fully understand, and they make conscious decisions to switch strategies, rethink the problem, search for related content knowledge that may help or they simply start afresh (Van de Walle 2004:54-55). Van de Walle (2004) further observes that poor problem solvers are impulsive, spend little time reflecting on a novel problem and are unable to explain why they used the selected strategy or whether they believe it works.

Lester (1994) argues that problem solving requires knowing what to monitor and how to monitor one's performance, therefore the provision of metacognition experiences is necessary to help learners develop their mathematical problem solving skills. Metacognition skills enable learners to strategically encode the nature of the problem by forming mental representations of the problem, select appropriate plans for solving the problem and identify and overcome obstacles to the process (Davidson & Sternberg 1998).

The best way to assist learners to become aware of their own thinking is to create opportunities where they have to explain their thinking explicitly (Dendane 2009). The following strategies can be used by teachers to develop learners' metacognition awareness:

- Provide learners with problems that require them to plan before solving the problem and to evaluate the solution after solving the problem.
- Encourage learners to find different ways of solving a problem and to check the appropriateness and reasonableness of a solution.
- Afford learners the opportunity to discuss how to solve a particular problem and to explain the different methods that can be used to solve the problem.

Teachers can develop learners' metacognition skills by asking them the following questions after a problem solving session:

- What did you do that helped you understand the problem?
- Did you find any information that you did not need?
- How did you decide what to do?
- Did you think about your solution after you got it?
- How did you decide that your solution was correct?

2.6.3 Reflective thinking in the problem-centred approach

Reflective thinking, which means actively thinking about or mentally working on an idea (Van de Walle 2004:23), is essential for effective learning. Knowledge construction requires reflective thinking, and to construct knowledge, learners sift through their existing ideas to find those that seem to be most useful in giving meaning to a new idea. Where there is active reflective thinking, learners rearrange their networks and modify their existing schema to accommodate new ideas (Fosnot 1996 in Van de Walle 2004:23-24) and this is how learning occurs. When there are active reflective thoughts, schemas are constantly being modified or changed so that ideas fit better with what is known (Van de Walle 1998:25). For learners to create new ideas and connect them in a rich web of interrelated ideas, they must be mentally engaged, that is, they must be thinking reflectively. Learners must find the relevant ideas they possess and use them in the development of new ideas and the solutions to new problems.

2.6.4 Social interaction in the problem-centred approach

In a problem-centred teaching and learning environment, learning mathematics is viewed as a social endeavour (Nathan & Knuth 2003:175-207) where thinking, talking, agreeing and disagreeing are encouraged. Kilpatrick (1985; 1987) notes that the mathematics classroom is a social situation jointly constructed by the participants in which the teacher and learners interpret each others' actions and intentions in the light of their own agendas.

Research tells us that learner interaction through classroom discussion improves both problem solving and conceptual understanding, because a variety of conjectures and strategies can emerge from social interaction as learners work together to evaluate and refine ideas. Learners who work cooperatively achieve at higher levels, persist longer when working on difficult tasks and are more motivated to learn, because learning becomes funny and meaningful. Hiebert et al (1997:167) observe that learners understand better when they have the opportunity to describe and explain their thinking. These authors further stress that learners must take responsibility for their classmates' understanding of new concepts.

Social interaction is important in that ideas and methods are valued, learners choose and share methods, mistakes are learning sites and correctness resides in mathematical arguments (Hiebert et al 1997:167-8). Social interaction creates opportunities for learners to talk about their thinking and this talk encourages reflection.

The following advantages of social interaction are given by Hiebert et al (1997) and Kilpatrick (1985; 1987):

- It provides an environment in which learners are willing to think.
- It gives learners the opportunity to learn to communicate about and through mathematics.
- It assists learners to reflect on the way they solve a problem.
- It provides learners with opportunities to learn from other learners without endangering their individual autonomy.

2.6.5 Negotiation of meaning in the problem-centred approach

Negotiation of meaning means comparing what is known to the new experiences and resolving discrepancies between existing knowledge and what seems to be implied by the new experience (Lorsbach & Tobin 1997). Negotiation of meaning can occur between learners in a classroom when they discuss, listen to one another, make sense of points of views of other learners and compare and justify personal meanings to those embedded within the theories of other learners. Social negotiation of meaning is vital in a problem-centred teaching and learning environment in that learners' understanding of content is constantly being challenged and tested by others.

2.7 Factors important in the problem-centred teaching and learning environment

2.7.1 The role of the teacher

In a problem-centred teaching and learning environment, the teacher moves away from authoritarianism and dispensing of facts and takes up the basic task of making sure that learners understand that doing mathematics does not mean following prescriptions (Human 1992:16). The teacher's main role is to create an environment in which learners can safely express their own mathematical ideas (Smith 1996:397), that is an environment in which learners are doing mathematics by posing good problems and creating a classroom atmosphere of exploration and making sense (Van de Walle 2004:20).

The teacher's other main role is to "pose worthwhile questions and tasks that elicit, engage and challenge each learner's thinking" (NCTM 2000:10). The teacher shares just enough information, making sure that the mathematics in the task remains problematic for the learners (Hiebert et al 1997:36), yet providing sufficient scaffolding to keep learners interested and on-task. Hiebert et al (1997:164) stress that the teacher should never demonstrate solution methods that learners should use, but can suggest notations and words that may help learners express their methods and can rephrase learners' methods to draw attention to important ideas. The teacher must believe that all learners have the ability to solve the problems, must listen to every learner, treat each learner's contribution as a learning opportunity (Hiebert et al 1997:171) and must accept every learner‘s solution without evaluating.

The major roles of the teacher in a problem-centred teaching and learning environment can be summarised as follows:

- The teacher acts as a metacognitive coach, whose main role is to help learners develop the skills that will enable them to engage in productive learning through problem solving and whose questions encourage, probe, give critical appraisals, promote interaction between learners and prompt them to become aware of the reasoning skills they are using (Gallagher, Stepien & Rosenthal 1992).

- The teacher finds problems that are engaging and are at a suitable level of difficulty and acts as an organiser of learning activities, from which the learners construct their own knowledge through their own activities.
- The teacher acts as a facilitator of the learning process by stimulating “reflection on” and “discussion” of learners’ efforts (Olivier 1999:29).
- The teacher helps learners to monitor and evaluate their problem solving strategies and assists them in recognising the limitations of the strategies they are using and to see how their strategies can be improved.
- The teacher provides the necessary social knowledge for learners to understand the problem and shows learners how to use tools like calculators (Murray et al 1998).
- The teacher selects contexts for tasks that are reasonably familiar to learners and which learners are highly interested in (Hiebert et al 1997:170).

2.7.2 The role of the learner

In a problem-centred teaching and learning environment, learners are active participants in the creation of their own knowledge and work independently of the teacher, individually and in pairs or groups (Kilpatrick & Swafford 2002:26). Learners accept the given challenges and attempt to execute tasks and solve problems in their own way, taking responsibility for their own learning (Olivier 1999:29).

Learners are involved in “information processing” rather than information receiving (Beyer 1987:67) and formulate and solve their own problems or rewrite problems in their own words in order to facilitate understanding. In a problem-centred teaching and learning environment, learners are expected to solve problems or make sense of mathematics situations for which no well-defined routines or procedures exist. Hiebert et al (1996:16) identify the following two major roles for the learner:

- taking responsibility for sharing the results of enquiries and for explaining and justifying their methods
- recognising that learning means learning from others, taking advantage of others' ideas and the results of their investigations

Learners should not expect to be told how to solve problems but should actively look for relationships, analysing patterns, finding out which methods work and which do not, justifying results or evaluating and challenging the thoughts of others (Van de Walle 2004:32). Through active and reflective thinking process, learners become responsible for their own learning.

2.7.3 The role and nature of appropriate problems

In a problem-centred learning and teaching environment, a worthwhile problem is the one that cannot be solved by any readily available algorithm that learners already know. However, it must be "connected with learners' current ways of thinking" (Hiebert et al 1997:170). An appropriate task should be the one that learners can problematise, must leave a residue of mathematical value and must be reasonably difficult and at the same time be within learners' reach. A good problem is one that can be extended to lead to mathematical explorations and generalisations (Schoenfeld in Olkin & Schoenfeld 1994:43), should be intriguing, with a level of challenge that invites exploration and hard work (NCTM 2000:19) and should have various solutions or must allow different decisions to be taken (Cuoco et al 2006:378).

Van de Walle (2004:38) lists the following as the features of appropriate tasks:

- A task must begin where the learners are and must take into consideration learners' existing knowledge. Learners must have appropriate ideas, skills and tools with which they can begin to solve the problem (Hiebert et al 1996:162) and at the same time find it challenging and interesting.

- The problematic or the engaging task of the problem must be due to the mathematics that learners can learn.
- Tasks should offer learners the opportunity for using multiple representations of the concept and mathematical communication that includes explanation and justification.
- For a task to be accessible to all learners, it must have multiple entry points and should promote the skilful use of mathematics.
- Appropriate problems trigger the cognitive processes of accessing prior knowledge, establishing information into knowledge that both fits into and shapes new mental models (Evensen & Hmelo 2001).

2.8 Effective thinking processes for problem solving

Problem solving requires the use of the following three types of thinking: creative thinking, which is also known as divergent thinking; critical thinking, which is also known as convergent thinking; and metacognition. During the divergent stage learners generate many ideas and during the convergent stage, they evaluate those ideas and have to be consciously in control of their thinking process. Metacognition was dealt with in section 2.6.2, so only convergent and divergent thinking will be discussed in this section.

2.8.1 Critical or convergent thinking

This type of thinking is used to judge ideas and its goal is to use the tools of evaluation and judgement to identify only the most useful ideas (Mcintosh & Meacham 1992:12). Convergent thinking involves analysing components and relationships in a system, comparing, contrasting and evaluating options, and interpreting data and making inferences.

2.8.2 Creative or divergent thinking

This type of thinking, whose goal is to create as many ideas and opinions as is possible (Mcintosh & Meacham 1992:12), involves generating many possible options for solving problems that have variety and originality.

Possible strategies for solving problems are discussed in details in the next section.

2.9 Problem solving heuristics found in the literature

Strategies or heuristics for solving problems are identifiable methods of approaching a task that are completely independent of the specific topic or subject matter. Below is a list of some of the strategies. However, this list is not exhaustive and no listing of strategies should be considered all-inclusive.

2.9.1 Making a table or a chart

Charts of data, function tables, tables for operations and tables involving measurements are a major form of communication in mathematics. Use of a table or a chart is often combined with pattern searching as a means of solving problems or constructing new ideas (Van de Walle 2004:55).

Example:

Dad gets 25 kilometres per litre of petrol in his car. How far can he go on 5, 10, 15, 20, and 30 litres of petrol? Make a chart or table showing this information. By constructing a table or a chart, the learner will be able to discover a pattern or any information that is missing.

2.9.2 Trying a simpler form of the problem

In solving a problem that appears complicated, a learner may find it helpful to start by solving simple similar problems. Quantities in a problem are modified or simplified so that the resulting task is easier to understand and analyse. Solving the easier problem may help learners to gain insight that can be used to solve the original, more complex problem. It may happen that the solution of the simpler problem leads to the solution of the more difficult problem (Lenchner 1983:28).

2.9.3 Writing an equation or open sentence

Equations are a major communication tool in mathematics. A problematic situation is often translated into an equation because an equation is easy to work with, suggests a familiar pattern or is a useful way to communicate an idea. By solving the equation or inequality, the learner finds the way to the solution of the problem at hand (Lenchner 1983:39).

Example:

> *Ronald bought a loan mower for R800 to start a lawn-mowing business. He averages about R120 per lawn. How many lawns will he need to cut before he makes a profit of more than R400?*

By formulating an equation, learners can easily solve this problem.

2.9.4 Guessing, testing and revising

The aim of this strategy is to make a reasoned guess at the solution and then see how the guess fits the conditions. If there is a good fit, the learner may be finished, but if not, checking will tell him or her in which direction to adjust the guess (Van de Walle 2004:56). The learner begins by guessing and then tests the guess to find out if it is good enough. If the guess is not good enough, he or she revises it and tests again.

Example:

> *Mr Zwane used two identical shapes to make a rectangle. What might they have been?*

This strategy works where problems have multiple conditions that make it difficult to compute true solutions.

2.9.5 Working backwards from the solution

Some problems may be best approached "through the back door". The problems designed for this strategy usually have a series of events ending with a given conclusion. The main task is to find the starting point.

Example:

> *Bertha and Salina inherited a collection of jewellery from their mother. They agreed on a plan to decide who would get which jewellery. Bertha would select ¼ of the jewellery for herself, and then Selena would choose half of the remaining jewellery. That left six pieces of the jewellery that they decided to keep as "common property" until a later date. How many pieces of jewellery did they inherit to start with?*

This example illustrates that this strategy starts with the goal rather than what is given. In some cases, a problem may contain a series of actions that are better understood and clarified by working back from the end to a direct point in the action sequence.

2.9.6 Looking for patterns

Mathematics is the science of pattern and order (Van de Walle 1998:54). Patterns in numbers and in operations play a huge role in helping learners learn about and master basic and advanced facts. Patterns fulfil an integral role in the discovery and application of mathematical concepts. Learners should be taught to analyse patterns and make generalisations based on their observations; to check generalisations against known information; and to construct a formal proof to verify the generalisation

Example: Archie scored one goal on the first day, three goals on the second day and six goals on the third day. How many goals did he score on the fifth day?

If the learner puts the goals in a sequence, he or he can identify the pattern and will be able to solve the problem.

2.9.7 Looking-back

This strategy is mainly applied after the solution to a problem has been found. Learners should be able to justify the answer, consider how the problem was solved and look for possible extensions or generalisations. Learners can be encouraged to use this strategy by asking them to verbalise how they found the solution to the problem; encouraging them to record the solution process with reasons for every step; requiring them to compare solutions and solution strategies with other learners; and having them justify and explain solutions and build on one another's ideas.

2.9.8 Drawing a picture and acting it out

Picturing how actions occur and how they are related in a problem may help the learner to easily find the solution. In situations where it is impossible to use objects, items may be used that represent them. Acting out the problem may in itself lead to the answer or to another strategy that may help in obtaining the answer (Lenchner 1983:35).

2.9.9 Making a model

Modelling can refer to an object, a diagram, a pictorial representation or just a simple model. If a learner makes a model of the problem, he or she may be able to easily visualise the problem. After drawing a model, a series of logical steps may eventually lead to the solution of the problem.

Example:

> *A packet of sweets cost ¾ of R8. How much does it cost?*

By drawing a pictorial rectangle that represents R8, the learner can solve this problem.

2.9.10 Making an organised list

This strategy aims at systematically accounting for all possible outcomes in a situation, either by finding out how many possibilities there are or ensuring that all possible outcomes have been accounted for.

2.9.11 Changing point of view

Learners may fail to find a solution for a particular problem by believing that there is only one approach to solving the problem. In this case, it is always important to read the problem again and change one's point of view (Lenchner 1983:43).

Above are some of the strategies found in the literature for solving problems; however George Polya proposed a four-phase problem solving process that is discussed in the section below.

2.10 Polya's four-phase process for problem solving

The Department of basic education CAPS (2011:9) stresses that for learners to develop essential mathematical skills they should learn to investigate, analyse, represent and interpret information during the problem solving process. NCTM (2000:52) also emphasises that "problem solving should be an integral part of all mathematics programmes". George Polya (1957) proposed a four-phase problem solving process, linear in nature, with the following identifiable strategies:

- understanding the problem
- devising a plan or deciding on an approach for attacking the problem
- carrying out the plan
- looking back at the problem, the answer and what you have done to get there

2.10.1 Understanding the problem

It is important that learners firstly reflect on the task in order to get a firm grasp of what is known and needs to be done, and then decide what information is important and what seems unimportant. They should be clear on what may be helpful to reformulate some of the information, perhaps making a list of knowns and unknowns or drawing pictures, charts or diagrams. At this stage, they should begin to think of similar situations they have experienced that may be like the problem at hand or that may contribute to the solution.

2.10.2 Devising a plan

In the second phase, learners must reflect on ideas that might be brought to the problem. These ideas range from mathematical concepts and procedures to general processes or strategies (Van de Walle 1998:41). It is important to remember that information that may be brought to the problem or that is needed to solve the problem is unique to each learner.

2.10.3 Carrying out the plan

Learners follow through with the approach selected, carefully taking each step along the away. During this phase, it is important for them to self-monitor their progress and regulate the methods they will be using, that is, engage in metacognitive activity (Van de Walle 1998:41). If learners get stuck on this stage, they can go back to the problem to check if it is understood correctly and search for a new way of approaching the problem. If a plan works, learners should consciously check each step. This is a vital step of problem solving. However, the success of the whole problem solving process also depends on the first two stages and the last stage.

2.10.4 Looking back

A problem should never be considered "solved" simply because an answer has been found (Van de Walle 1998:41). Learners must engage in the following three looking-back activities: looking at the answer; looking at the solution process or method; and looking at the problem itself.

Answer: Learners must verify it by checking it against all the conditions and clarifying if there are any contradictions between the answer and the conditions of the problem.

Process: Learners must justify their answers by explaining how they arrived at them. They must check if the problem could have been solved in a different or easier way, as well as if the approach is useful in solving other problems.

Problem: Learners must consider what was learnt from the problem itself, as well as check if they can still solve the problem if the conditions are different or the numbers are bigger. Since it is sometimes easy to overlook certain factors, after finding an answer, they should check if they really answered the question.

2.11 Mathematical problem solving skills found in literature

In the next section the mathematical problem solving skills found in the literature (Lenchner 1983; Bransford & Stein 1984; Gick 1986; Polya 1957; Kadel 1992; Hiebert & Wearne 1993; Van de Walle 1998; Adamovic & Hedden 1997:20-23; Dendane 2009) will be discussed in detail.

2.11.1 Understanding or formulating the question in a problem

Learners usually have difficulty with a mathematical problem because they do not know how to start solving it. For them to start the problem solving process, they have to understand and know what the problem requires and what it is asking for. Making sense of the question includes understanding the meaning of specific words in the problem and also recognising how the question relates to other statements in the problem. For different problems, the question may appear in different places. Sometimes a question appears as a statement and learners must be able to formulate the question in the statement. Formulating a problem can be extremely demanding (Dendane 2009). However, this is an important stage, because a correct solution cannot be generated without an adequate understanding of the problem. Learners who cannot understand or formulate the question in a given problem usually have difficulty solving the problem. Understanding or formulating the question in a given problem is also a crucial mathematical problem solving skill since it is the first step in problem solving and “learners cannot make any progress if the problem is not understood” (Dendane 2009). Van de Walle (1998:40) suggests that where necessary the problem may be reformulated in the learner’s own words.

2.11.2 Understanding the conditions and variables in a problem

Kadel (1992) stresses that learners must be able to identify the known and the unknown variables in a given problem. They must initially state what they know and what they do not know about the problem since this is the information they will be solving for. Making a model, a diagram, a picture or a list of key ideas can help the process of understanding the conditions and variables in the problem.

Example:

> *A total of 14 children and cats are playing in a park across the road from your school. If Sipho counted their legs, he would get 36 in total. How many children and how many cats are there in the park?*

This problem has two conditions:

- There are a total of 14 children and cats in the park.
- There are 36 legs in total.

There are two variables in this problem:

- the number of children
- the number of cats

The learner can easily solve this problem upon understanding its conditions and variables.

2.11.3 Selecting or finding the data needed to solve the problem

Learners need to be able to focus on a specific piece of information and decide whether it is relevant or irrelevant. They should be able to extract the relevant information from the given problem. The problem solver must be able to identify the needed data and eliminate data not needed, and collect and use data from a variety of sources such as graphs, maps or tables. Data-finding processes are closely connected to the processes involved in understanding the question, conditions and variables in the problem.

2.11.4 Formulating sub-problems and selecting appropriate solution strategies to pursue

This is the planning stage where learners have to decide on a plan of action in solving the problem. It is one thing to know how to use particular solution strategies and another thing to know when to use them. For example, a learner may know how to multiply but not know when to multiply or might know how to find a pattern but not know when to look for a pattern in solving problems. During this stage, learners must reflect on the ideas that may be brought to the problem. They must develop the relationship between the known information and the unknown information by writing an equation with appropriate variables.

As progress is made towards the solution of multistep problems or process problems, the problem solver is often required to identify subgoals to be reached. This mathematical problem solving skill involves the learner making decisions on what problem solving strategy or strategies to try. The learner searches for or generates possible solution strategies to the problem, which are then implemented and tested (Gick 1986). An "IDEAL" problem-solver must have the ability to explore possible strategies, act on these strategies, and look at the effects (Bransford & Stein 1984). Problem solving strategies were dealt with in detail in section 2.9.

Example:

> *Sipho was broke when he received his weekly allowance on Sunday. On Monday he spends R30 of the allowance. On Tuesday his friend pays him R16 he owes him. How much is Sipho's allowance if he now has R36?*

In this question, there are two subgoals: to determine how much money Sipho had before his friend paid him R16; and to determine how much he had before spending R30. This strategy is called "working backwards from the solution" (see section 2.9.5). From this example, the sub-problems are intermediate stages along the way towards a solution that the problem solver consciously tries to reach. The solution strategy is the plan of attack for reaching the sub-problems and ultimately the solution.

2.11.5 Correctly implementing the solution strategy or strategies and solving the sub-problems

Learners must know how to correctly implement solution strategies, ranging from being able to perform computations to solving equations by using mathematical operations. Implementing a solution strategy may involve being able to perform computations, using logical reasoning or solving equations as well as performing activities such as making a table, graph or a list. All the same, after identifying and ordering subgoals, the learner must be able to attain them. Adamovic and Hedden (1997) recommend that learners must be able to find the correct equation to use and then write it down.

2.11.6 Giving an answer in terms of the data in the problem

Learners must be able to include the right units in their final answer or solution. This may imply giving the correct unit to accompany the numerical part of an answer or stating the answer in a complete sentence. For example in the "children and cats" problem in section 2.11.2, the learner must state the answer in terms of the children and the cats, and not just give numbers as answers.

2.11.7 Evaluating the reasonableness of the answer

The problem solving process does not end after obtaining an answer; learners should be able to check the validity of their solution. If the solution is incorrect, they need to refer back to the previous steps to check for any errors in mathematical calculations, translations and the overall understanding of the problem. During this process the learner may reread the problem and check the answer against the conditions, variables and the question. Learners may also use various estimation techniques to determine if an answer is reasonable. Lenchner (1983:24) points out that the reasonableness of the answer can be achieved when learners write their own answers in complete sentences. If they write answers in complete sentences, they can easily review the statement of the problem and detect a possible error.

2.12 Strategies for developing learners' mathematical problem solving skills

Learners' mathematical problem solving skills can hardly be developed by merely solving problems (Killen 1996:225). Below are approaches and learning situations in which mathematical problem solving skills can be developed.

2.12.1 Developing learners' thinking skills

Before learners can develop mathematical problem solving skills, they must first become effective thinkers. For learners to become effective thinkers, they must develop useful mathematics "habits of mind" (Cuoco et al 1996:3-8). Cuoco et al (1996) explain that habits of mind are dispositions or tendencies by learners to employ appropriate critical-thinking behaviours often and are what learners need to develop in order to think mathematically and to be successful problem solvers. According to Leikin (2007:2333), "employing habits of mind means inclination and ability to choose effective patterns of intellectual behavior". Habits of mind are regarded as something learners need to develop in order to think mathematically and to be successful problem solvers (Lesh & Doerr, 2003:392). Learners should be taught to think about mathematics, the way mathematicians do. For them to think like mathematicians, Cuoco et al (1996) point out that they should be pattern-sniffers, experimenters, describers, tinkerers, inventors, visualisers, conjecturers and guessers.

2.12.1.1 Learners should be pattern-sniffers

Learners should fall into the habit of always looking for patterns when they are presented with problems at school or in their daily lives. In this study, the researcher encouraged learners to be always on the lookout for patterns and to "sniff" for hidden patterns.

2.12.1.2 Learners should be experimenters

When presented with a mathematical problem, learners should "immediately start playing with it, using strategies that have worked before" (Cuoco et al 1996:4). Learners should be used to performing thought experiments.

2.12.1.3 Learners should be describers

Learners should be able to explain and argue their solution methods and solutions to the teacher and other learners. In this study, the researcher coached learners to fall into the habit of writing down their thoughts, results, conjectures, arguments, proofs, questions and opinions about the mathematics they do. Formulating written and oral descriptions of one's work is useful when you are part of a group of people with whom you can trade ideas (Cuoco et al 1996:4). Describing what you do is an important step in understanding it.

2.12.1.4 Learners should be tinkerers

Cuoco et al (1996:5) advocate that tinkering is at the heart of mathematical research. Learners should fall into the habit of breaking mathematical ideas into small logical pieces and putting them back together. In this way they see what happens if something is left out or if the pieces are put back in a different way. For example, after experimenting with a rotation followed by a translation, they should be eager to know what happens if they experiment with a translation followed by a rotation.

2.12.1.5 Learners should be inventors

Cuoco et al (1996:5) propose that learners should develop the habit of inventing mathematics both for utilitarian purposes and for fun. They maintain that good mathematical inventions give the impression of being innovative. The practice of inventing a mathematical system that models particular phenomena is crucial to the development of mathematical problem solving skills (Cuoco et al 1996:6).

2.12.1.6 Learners should be visualisers

Learners should be able to visualise data relationships, processes, changes and calculations. They should construct tables and graphs, and use these visualisations in their experiments. They should fall into the habit of visualising calculations (numerical and algebraic), perhaps by seeing numbers flying around in some way (Cuoco et al 1996:8).

2.12.1.7 Learners should be conjecturers

Making plausible conjectures is central to doing mathematics and learners should fall into the habit of making data-driven conjectures (Cuoco et al 1996:8). Learners’ conjectures should rest on something more than experimental evidence.

2.12.1.8 Learners should be guessers

Learners should fall into the habit of often starting at a possible solution to a problem and working backwards, as this frequently helps one find a closer approximation to the desired result. As learners check a guess, they often find new insights, strategies and approaches to the problem at hand (Cuoco et al 1996:8). Section 2.9.4 dealt with the "guess, test and revise" strategy and section 2.9.5 dealt with the "work backwards" strategy of problem solving.

2.12.2 Structuring learning situations to develop mathematical problem solving skills

2.12.2.1 Comparing problems

Learners should be encouraged to look for similarities in problems. They can consider what strategies were effective in solving other problems that have these characteristics (Killen, 1996:225). The teacher can help learners to develop their ability to recognise similarities in problems by deliberately structuring a series of problems around a common theme. As learners attempt each problem, they should be required to identify the ways in which it is similar and different from the previous problems. They must be encouraged to think about how these features may influence their approach to solving the problem at hand.

2.12.2.2 Comparing strategies

Learners should be encouraged to look for different ways of solving problems and to compare the effectiveness of their different approaches. Thinking about more than one strategy should be an important part of each learner's plan for solving a problem. Teachers must teach learners to understand that when one problem solving strategy does not work, there is a need to establish why it was not successful and then modify their approach or look for a new approach (Killen 1996:225). Learners' ability to plan how to solve problems improves as they learn to look for alternative approaches and judge the probability of them being successful. In this study, the researcher helped learners to develop this mathematical problem solving skill by discussing with them various strategies that class members would have used to solve different problems.

2.12.2.3 Valuing the process

Learners that are used to traditional learning are likely to be more interested in producing correct answers to problems than using the problems as a vehicle for learning mathematics. Day in Killen (1996:226) suggests that the teacher can change the disposition of learners by implementing the following:

- placing most emphasis on the process of solving problems and placing least emphasis on obtaining the correct answer
- having the learners express and record their ideas, thoughts, feelings and questions in their journals as they work through the problem solving process
- having learners work in small groups and encouraging them to learn from one another
- the teacher acting as a facilitator, mentor and coach rather than as the source of all knowledge
- giving learners frequent practice at thinking through the strategy they would use to solve a problem without actually proceeding to a solution
- encouraging learners to realise that a particular approach is not the best option because it is the one they thought of or the one they thought of first

2.12.2.4 Commitment and perseverance

Learners are unlikely to be able to solve problems successfully without willingness and perseverance. No one has ever solved a problem without interest and teachers must emphasise to learners that they can become better problem solvers by wanting to solve problems and working hard (Schmalz 1989:686).

2.12.2.5 Encouraging learners to "think out loud" when solving problems

Learners should be encouraged to "think out loud" when solving problems. Thinking out loud forces learners to pay attention to their thinking and problem solving because they become aware of the information they are using to solve problems and therefore more aware of how they are solving them (Posamentier & Jaye 2006:127). For example, in this study the researcher asked learners questions such as following: Tell me, what equation are you going to look at or use? Why are you going to look at that equation or use it?

2.12.3 Encouraging learners to pose their own problems

The Department of basic education CAPS (2011: 8-9) advocates that for learners to develop essential mathematical skills they should "learn to pose and solve problems". Learners' ability to generate their own problems for other learners to solve is a good indicator of their mastery of content, concepts and principles that the teacher has been teaching and their ability to analyse problems. After learners generate their own problems, the teacher can discuss with them how their problem can be made easier or harder and this encourages them to look for these factors in problems that they later have to solve. Learners pose problems that have meaning to them and this increases their chances of being actively engaged in learning.

Bush and Fiala (1986) point out the following advantages of having learners pose their own problems:

- Problem posing enhances the understanding of problem solving.
- Problem posing helps learners to transfer learning from one context to another.
- Problem posing helps learners to develop a deeper understanding and to gain a different perspective on the aspect they are investigating.
- Problem posing helps to dispel the notion that there is one solution to a particular problem.
- Problem posing helps learners to bridge the gap between concrete situations and abstract ideas, an important part of cognitive development.

2.12.4 Enhancing learners' thinking

Learners' thinking can be enhanced by developing their focusing, information-gathering, organising, evaluating, analysing and integrating skills (Killen 1996:244-5).

2.12.4.1 Focusing skills

Learners are only able to solve problems if they focus their thinking on specific issues and temporarily ignore other things. Learners need to define what the problem is before they start to generate solution strategies for the problem. They also need to decide which are relevant or irrelevant data.

2.12.4.2 Information- gathering skills

The teacher must help learners to develop their ability to identify what information is needed, formulate questions to guide their information gathering and gather information by observation or by an appropriate form of research.

2.12.4.3 Organising skills

Learners must be able to organise the information they gather into a form that will enable them to interpret it and put it to best use. They must be able to represent the information in new forms like in a table, charts or diagrams.

2.12.4.4 Analysing and integrating skills

Once learners have gathered and organised the information, they need to place it within an overall conceptual framework. To be able to analyse and integrate this information into existing knowledge structures, learners must develop the ability to identify key elements and relationships between the various pieces of information they have gathered. The teacher must develop learners' ability to identify the main ideas in the information, errors in the facts and important elements, relationships and patterns in the information. The learners must be able to modify their existing knowledge structures to accommodate the new information.

2.12.4.5 Evaluating skills

The teacher must teach learners how to evaluate information and decide whether or not it is reliable and useful. To be able to evaluate the relative merits of information and ideas, learners must develop the ability to establish criteria for judging the value of information and the merit of ideas and objectively apply these criteria.

2.13 Assessing the development of mathematical problem solving skills of learners

When assessing learner's mathematical problem solving skills, evaluating whether the solution is correct or incorrect is not adequate since this does not assess learners' thinking skills (Nitko & Brookhart 2010:217). The recommended technique is to craft tasks and problems that require the use of mathematical problem solving skills in new or unfamiliar situations. Such tasks allow the educator to assess learners' thinking about problem solving and if learners have developed the mathematical problem solving skills associated with each step in the problem solving process. Nitko & Brookhart (2010:218-221) recommend that the following strategies can be used to assess learners' mathematical problem solving skills:

- Learners are given a question and are put in an "unpleasant or uncomfortable position" in which they are required to identify the problem to be solved.
- Learners are required to pose the questions that need to be answered to solve the problem.
- Learners are required to identify both relevant and irrelevant data for the solution of the problem.
- Learners are given a problem in which they are required to solve the problem in more than one way and should show their solutions using diagrams, charts, tables or graphs.

- Learners are presented with a problem that they are required to model by probably drawing a diagram or a picture. In this situation learners are not assessed on whether they could solve the problem but on how they could model or present the problem.
- Learners are required to identify obstacles that make it difficult to solve a given problem and the additional information that they need to overcome the obstacles. In this situation learners are not assessed on getting the correct solution but on being able to identify the obstacles.
- Learners are required to justify the strategies they use to solve a given problem and the solution to the problem.
- Learners are required to give several alternative solutions to the given problem.
- Learners are given a challenging problem in which they are required to work backwards from the desired outcome to be in a position to solve the problem.
- Learners can be asked to work out several different solutions to one problem and are then required to evaluate these different solutions and the different strategies used. In this situation, they are assessed on whether they can evaluate the reasonableness of each solution and each strategy.
- Learners are presented with problems in which they are required to formulate sub-problems. In this instance, they are assessed on whether they can formulate and solve sub-problems.

2.14 Assessment in the problem-centred teaching and learning approach

Traditionally, learners wrote exams at the end of the year as a form of assessment. However, traditional examinations are inadequate for assessing the goals of problem-centred learning because they assess factual knowledge and do little to assess either the coherency or utility of that knowledge or the learners' ability to use it to solve problems. Development of higher-order thinking skills is not encouraged by such examinations and learners focus on obtaining a higher score instead of understanding of the material (Kadel 1992:29). Traditional examinations are not able to capture the actual changes in learners' knowledge because they "often lead learners to adopt a surface approach to learning and studying and to attempt to memorise the material instead of trying to understand it" (Biggs 1996).

In the problem-centred approach, assessment is not a separate examination at the end of the course, but assessment methods are integrated into the learning process (Tynjala 1999:365). Tynjala (1999) further points out that in a problem-centred approach, assessment focuses on the process of learning as much as the final outcomes. The Department of basic education CAPS (2011:154) defines assessment as "a continuous planned process of identifying, gathering and interpreting information regarding the performance of learners, using various forms of assessment". Assessment should be continuous, occur in nearly all lessons and should assess what ideas learners bring to task, how they learn and what processes they use. The Department of basic education CAPS (2011) further stresses that continuous assessment of learners' work not only enhances their learning experience but also assists them to achieve the minimum performance level required in mathematics for promotion purposes.

The purpose of assessment in the problem-centred approach is to monitor learner progress, make instructional decisions, evaluate learner achievement, evaluate programmes, promote the learning process and to find out what kind of qualitative changes are taking place in learners' knowledge. Assessment should be embedded in the learning process, focus on authentic tasks and take into account learners' individual orientations and foster their metacognitive skills (Biggs 1996). Tynjala (1999:428) points out that making self-assessment and peer reviews an integral part of the learning process enhances learners' metacognitive skills.

2.15 Conclusion

From the literature review that was conducted it is clear that traditional methods of mathematics instruction leave "most learners believing that mathematics is mysterious and beyond understanding" (Van de Walle 2004:37). However, the PCTLA is an educational method that uses problem solving as the starting point for learning. Learners are placed in the active role of problem solvers by being confronted with non-routine problems. The focus is mainly on conceptual understanding rather than procedural knowledge. As learners engage in exploration of worthwhile problems they are expected to master basic skills and develop mathematical problem solving skills.

In this chapter, the researcher provided an overview of the problem-centred teaching and learning approach (see sections 2.5.2, 2.6 and 2.7) and the mathematical problem solving skills found in the literature (see section 2.11); thereby addressing research sub-questions 1 and 2. In addition, strategies for developing learners' mathematical problem solving skills were proposed (see section 2.12).

This chapter recognised the prerequisites for learners to be in a position to develop mathematical problem solving skills (see section 2.12), thereby addressing research sub-question 3. Learners should be effective thinkers, should develop useful mathematical habits of mind and should think the way mathematicians do. Furthermore, learners must develop the habit of comparing mathematical problems and strategies, must value the problem solving process and must be willing and committed to solve problems (see section 2.12.2).

It was revealed from this literature review that it is imperative for teachers to structure learning situations with the aim of enhancing learners' thinking and learners' mathematical problem solving skills. The chapter concluded by explaining how best assessment can be done in a problem-centred teaching and learning environment and by stating effective strategies that can be used to assess learners' mathematical problem solving skills.

CHAPTER 3

RESEARCH METHODOLOGY AND DESIGN

3.1 Introduction

In chapter 2, the literature review explained what the PCTLA (problem-centred teaching and learning approach) entails, gave an account of the mathematical problem solving skills and examined what grade 8 learners need as a prerequisite for the mathematical problem solving skills to develop. This chapter gives an account of and justifies the research plan that was followed in this study.

This chapter starts by emphasising the research problem, research questions, the purpose of the study and the research paradigm. This is followed by an in-depth description of the research design and the research methods that were adopted to answer the research questions and the reasons why the researcher employed these methods in particular. The population sample, the participants and the sampling procedure are also discussed. This chapter then moves on to investigate the different data collection tools used by the researcher to collect the data, including the questionnaire, the mathematical problem solving skills inventory (MPSSI), direct participant observation and questioning, semi-structured interviews, learner journals, mathematical tasks, written work, pre- and post- word-problem and multiple-choice tests. Thereafter data analysis, the validity and the reliability of the data and the ethical considerations of the study are examined. The chapter concludes with a brief summary.

It is important to note that in this study the researcher acted both as a researcher and teacher of the experimental group. This is because many teachers in South Africa do not know yet how to effectively implement the PCTLA. To ensure that a PCTL environment was created for the experimental group and the learners were in a position to develop the required mathematical problem solving skills, the researcher had to teach the learners herself. The role of the researcher as teacher was that of facilitator and coach. During class or group discussions, the researcher only interjected with questions to probe for reasoning and explanations. The researcher initiated and moderated class discussions and maintained a spirit of enquiry and critical reasoning during problem solving sessions.

3.2 The research problem, research questions and purpose of this study

According to Anderson and White (2004), teachers agree that problem solving is a vital life skill for learners to develop and that they need to develop a range of mathematical problem solving skills in order to become effective problem solvers. Hence the purpose of this study was to explore the development of mathematical problem solving skills of grade 8 learners in a PCTL environment and investigate the effect these problem solving skills on learners' performance and achievement in mathematics. As noted in section 2.12; learners' mathematical problem solving skills can hardly be developed by merely solving mathematics problems. For this reason, in this study, the researcher proposed the PCTLA (see section 2.5.3), which promotes high-level engagement of learners and the development of their mathematical problem solving skills.

The research problem concerns the development of mathematical problem solving skills of grade 8 learners in a PCTL environment. Consequently, the intervention programme was designed around the research questions in section 1.7 and specific objectives in section 1.9.

3.3 The research paradigm (worldview)

A research paradigm provides a frame of reference for seeing and making sense of research. Wisker (2001:123) defines a paradigm as an underlying set of beliefs about how the elements of the research area fit together and how one can enquire of it and make meaning of discoveries. According to Creswell and Plano Clark (2011:39), a paradigm is a perspective held by a community of researchers that is based on a set of beliefs, practices, values and assumptions about the knowledge that informs their study.

The pragmatic research worldview is embraced by many researchers as the worldview or paradigm of mixed methods research (Creswell & Plano Clark 2011:41-43). To this effect, the researcher identified with a pragmatic worldview for this study. This means that instead of focusing on the research methods, the researcher emphasised the consequences of the research, used multiple methods of data collection to answer the research questions and a "whatever works" approach to complete the research, at the same time abiding by ethical considerations and practical standards. The pragmatic worldview is inherent in this study because a problem-centred teaching and learning environment was created and the intent was to answer the research questions by whatever ethical or practical means available.

3.4 The research design

The research design describes how the study was conducted. It indicates the general procedure for conducting the study: "how the research is set up, what happens to the subjects and what methods of data collection are used" (McMillan & Schumacher 2006:22). A research design is a systematic investigation and it can be taken as a "procedural plan adopted by the researcher to answer the research questions validly, objectively and accurately" (Kumar 1999:77).

Mouton (2001:55) describes a research design as a "plan or blue print of how one intends to conduct the research". Its main purpose is to specify a plan for generating empirical evidence that will be used to answer the research questions (McMillan & Schumacher 2006:22). It is essential to choose an appropriate research design that focuses both on the research problem and the end product. The intent is to use a design that will result in drawing the most valid and credible conclusions from the answers to the research questions.

Creswell and Plano Clark (2011:53) observe that research designs “are useful because they help guide the decisions that researchers must make during their studies and set the logic by which they make interpretations at the end of their studies”. A mixed methods research design was used in this study. In section 3.6, the elements of the research design (mixed methods research design) will be discussed.

3.5 The intervention programme

The questionnaire, the MPSSI, the pre- and post- multiple-choice and word-problem tests were administered to both the experimental and control group at the beginning and at the end of the intervention.

The control group: As indicated in section 1.10.3 the control group was taught by the current grade 8 mathematics teacher using the traditional teaching approach. With the traditional approach (see section 1.4 and 2.5.1) the teacher represents all source of knowledge and learners are empty vessels to be filled with knowledge. For this study the control group teacher followed the Department of basic education CAPS syllabus and focussed on the teaching of algorithms that could be employed by learners to solve problems. After teaching an algorithm, the teacher worked a few examples with learners. Learners got similar problems as exercises or homework for the day and used the demonstrated algorithms to solve the assigned problems. If learners in the control group did not understand any algorithm rule, the teacher would show the rule again “drilling each piece in sequence”. The researcher observed that learners in the control group expected their teacher to explain all the algorithm rules and did not attempt to solve unfamiliar problems for which no algorithms were provided.

The experimental group: The researcher started the intervention programme by fertilising the experimental group learners’ minds by explaining to them what the PCTLA entails. As presented by the grade 8 mathematics CAPS syllabus the following topics were covered during the intervention: Pythagoras's theorem; perimeter, surface area and volume; numeric and geometric patterns; graphs; transformation geometry; ratio and rate and financial mathematics.

At the beginning of each lesson, the researcher would put an outline on the blackboard of what was going to be covered in the lesson. By seeing the outline on the board, learners can begin to think about the existing knowledge and concepts that are related to the day's lesson. For this study, seeing an outline on the blackboard stimulated learners' thinking about various topics and this helped to activate their prior knowledge about the topic of the day. The researcher created a classroom environment in which social interaction (see section 2.6.4) was highly valued. This is an environment in which learners believed that what was important was the effort they spent looking for solutions and that they would have learnt something even if they did not find the correct solution to the given problem.

Polya's four-phase problem solving process (see section 2.10) was modified and used to solve the problems and tasks for this intervention. Figure 3.1 below shows the problem solving model that was used for this intervention.

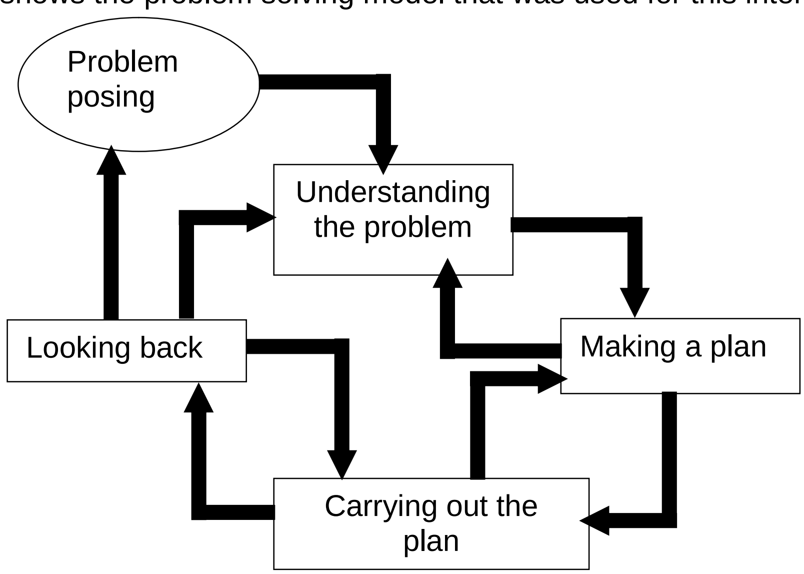


Figure 3. 1 A cyclic and dynamic model for problem solving

Source: Wilson, Fernandez & Hadaway (1993)

This model is dynamic and cyclic in nature and conveys that in some cases after learners understand a problem and devise a plan they may be unable to proceed. This may require the learner to make a new plan, to develop new understanding or to pose a new related problem (Wilson, Fernandez & Hadaway 1993). For this study, learners were also required to look back at the formulation of the given problem, to frequently revise the whole problem during the problem solving session and to be in a position to restart if required (Dendane 2009).

The researcher gave hints only during problem solving and there were no formal lessons illustrating methods to solve the problems. After grappling with unfamiliar problems, learners were required to place their solutions on the board and to fully explain their work to the class. Other learners in the class were encouraged to critique the solution and at the same time try to provide alternative solutions to the problem. As the class discussed the solution to the problem, the researcher guided the discussions as needed by asking questions to ensure that learners understood the solution before moving on to the next problem.

The researcher also employed teacher-led and learner-led groups in her teaching. This was done by forming three groups for example group A, B and C. On day 1, the researcher would tell learners in group B and C to work independently while working with group A, which would be the target group of the day. After the lesson, the researcher would assign follow-up work to group A. On day 2, the researcher would work with a new group, which would be B, while group A worked on teacher follow-up centre and the third group C would still work independently. On day 3, the researcher would work with group C and in that way a three-day rotating system was established.

During the intervention programme, the researcher implemented the strategies for developing mathematical problem solving skills as explained in section 2.12. She developed the experimental group learners' thinking skills and useful habits of mind as set out by Cuoco et al (1996) by encouraging learners to always look out for patterns, perform thought experiments, explain the solution methods to the teacher or their peers and develop conjectures. Learners were also required to visualise the problem, make mental pictures and think out aloud while solving problems.

The researcher structured learning situations (see section 2.12.2) to develop the experimental group's mathematical problem solving skills by implementing the following:

- When given a new problem, learners were encouraged to identify how it was similar or different from previous problems and how this could influence their approach to solving the problem.
- Learners were required to use different problem solving strategies (see section 2.9) to solve given problems and were encouraged to compare the effectiveness of the different strategies. Discussions were held towards the end of lessons to compare all the different strategies and valid solutions generated by all learners.
- The researcher encouraged learners to value the problem solving process by implementing the techniques stated in section 2.12.2.3.
- Learners were encouraged to generate their own problems. The advantages of having learners pose their own problems were given in section 2.12.3. The action of having learners generate their own questions transforms their relationship with authority and tests (Holt 1968) and at the same time affords them the opportunity to develop mathematical problem solving skills.
- Learners were given enough time to think before responding to questions. Providing learners with waiting time before answering questions helps to develop their mathematical problem solving skills since they have the opportunity to think deeply about the problem at hand.

3.6 The research method

According to McMillan and Schumacher (2006:12), a research method refers to "how data are collected and analysed". There are three major research methodologies: the qualitative research design, the quantitative research design and the mixed methods research design. The choice of these methods depends on the purpose of the research study. The research method that was adopted for this study is the mixed methods research design. The next section discusses the nature and origins of the mixed methods research design and the reasons why the researcher adopted it for this study.

3.6.1 Mixed methods research design

The mixed methods research design is defined by Tashakkori and Creswell (2007:4) as research in which the investigator collects and analyses data, integrates the findings and draws inferences using both quantitative and qualitative methods in a single study or a programme of inquiry. Creswell and Plano Clark (2011:5) concur with this definition when they state that mixed methods research design involve the mixture of quantitative and qualitative approaches in many phases of the research process and focus on collecting, analysing and mixing both quantitative data and qualitative data in a single study or series of studies.

For the purpose of this study, the characteristics of the mixed methods research design were adopted from Creswell and Plano Clark (2011:5) who list that in a mixed methods research design the researcher:

- collects and analyses rigorously and persuasively both qualitative and quantitative data
- integrates the two forms of data concurrently by merging them, sequentially by having one build on the other or embedding one with the other
- gives priority to one or both forms of data
- uses these procedures in a single study or multiple phases of a programme of study
- frames these procedures within philosophical worldviews and theoretical lenses

- combines the procedures into specific research designs that direct the plan for conducting the study

3.6.2 Why mixed methods research design?

The value of using mixed methods research design has been well documented in the literature (Johnson, Onwuegbuzie & Turner 2007; Creswell & Plano Clark 2011; Tashakkori & Teddlie 2003; Greene 2007; Morse 1991), the major reason being that mixed methods research designs provide strengths that offset the weaknesses and limitations of both quantitative and qualitative research (Creswell & Plano Clark 2011:12). Creswell and Plano Clark (2011) further stress that the use of quantitative and qualitative approaches combined gives more evidence and a better understanding of the research problem than either approach by itself. The mixed methods research approach provides a comprehensive account to the research questions of the study since it draws on the strengths of both methods.

This study used the mixed methods design in order to capitalise on the strengths of each approach so as to sharpen the research findings. The researcher realised that one data source would be insufficient since she wanted to explore the development of mathematical problem solving skills of grade 8 learners through qualitative research (semi-structured interviews, learner journals, participant observation and questioning) and to determine the effect of the development of mathematical problem solving skills on their performance and achievement in mathematics through quantitative research (questionnaire, mathematical problem skills inventory, tests, tasks and written work).

In this study, the mixed methods research approach assisted the researcher in answering questions that could not be answered by qualitative or quantitative research alone. For example, semi-structured interviews and learner journals afforded the researcher the opportunity to gain insight into the reasoning and cognitive processes of learners, for which quantitative research alone would have been insufficient. Therefore using both approaches allowed the researcher to incorporate the strengths of each method (McMillan & Schumacher 2006:401). The mixed methods research design also enabled the researcher to use various tools for data collection instead of being limited to data collection tools that are associated with only qualitative research or quantitative research. Lastly, the researcher used mixed methods design to enhance the credibility of the findings. Creswell and Plano Clark (2011:62) advocate that a research design that integrates different research methods is more likely to produce better results in terms of quality and scope.

Creswell and Plano Clark (2011:13-16) point out the challenges of using the mixed methods research design. One such challenge is that it requires extensive time, resources and effort on the part of researchers. For this study, enough resources were made available and problems with time were factored into the programme. Another challenge is the question of skills for doing the mixed methods research design. To overcome this particular challenge, the researcher extensively familiarised herself with both quantitative and qualitative research methods separately before undertaking the mixed methods research design.

Creswell and Plano Clark (2011:68-104) state six major types of mixed methods research design: The explanatory sequential design, exploratory sequential design, convergent design, embedded design, transformative design and multiphase design. The convergent research design was employed for this study and it is discussed in the next section.

3.6.3 The convergent design

This design was originally conceptualised as a “triangulation” design (Creswell & Plano Clark 2011:77). Creswell and Plano Clark (2011) define the convergent design as the process in which the researcher collects and analyses both quantitative and qualitative data during the same phase of the research process and then merges the two sets of results into an overall interpretation. This view concurs with that of McMillan and Schumacher (2006:404), who state that in a triangulation design (convergent research design), the researcher simultaneously gathers both quantitative and qualitative data, merges them using both quantitative and qualitative analysis and then interprets the findings together to provide a better understanding of the research problem. In this design, both quantitative and qualitative data are collected and given equal emphasis thereby allowing the researcher to combine the strengths of both methods. For this study the quantitative research methods and qualitative research methods occurred concurrently in all phases and the researcher equally prioritised both methods. The researcher kept the data collection and analysis independent and then mixed the findings during the overall interpretation.

The convergent notation for this study is **QUALITATIVE** + **QUANTITATIVE** = complete understanding (Creswell & Plano Clark 2011:77-78).

3.6.4 Why the convergent design?

The aim of the convergent design is "to obtain different but complementary data on the same topic" (Morse 1991:122) in order to best understand the research problem. The researcher also felt that there was equal value in collecting and analysing both quantitative and qualitative data at the same time in order to explore the development of mathematical problem solving skills of grade 8 learners and to investigate its effect on their performance and achievement in mathematics. The researcher needed both types of data to gain a more valid and complete understanding of the development of mathematical problem solving skills.

The researcher opted for the convergent design because it makes "intuitive sense" to her and it is an "efficient design" (Creswell & Plano Clark 2011:78) with which she was able to collect both types of data simultaneously. During lessons, the researcher was able to carry out semi-structured interviews and participant observation and questioning (qualitative research) with a few learners, while the rest of the learners were solving mathematical tasks or completing written work (quantitative research). The convergent design also helped the researcher "to directly compare and contrast quantitative statistical results with qualitative findings" (Creswell & Plano Clark 2011:77) in order to elaborate well-substantiated conclusions about the development of mathematical problem solving skills of grade 8 learners and its effect on their performance and achievement in mathematics.

According to Creswell and Plano Clark (2011:80), there are challenges in employing the convergent design, because much effort and expertise are required since equal emphasis is given to each data type. To overcome this challenge, the researcher familiarised herself with the mixed methods design and the convergent design before conducting the research study. Creswell and Plano Clark (2011) further point out that it can be difficult to merge very different data sets and their results in a meaningful way. To overcome this challenge and to facilitate merging of the two data sets, the researcher designed this study in such a way that the quantitative and qualitative data sets addressed the same concepts.

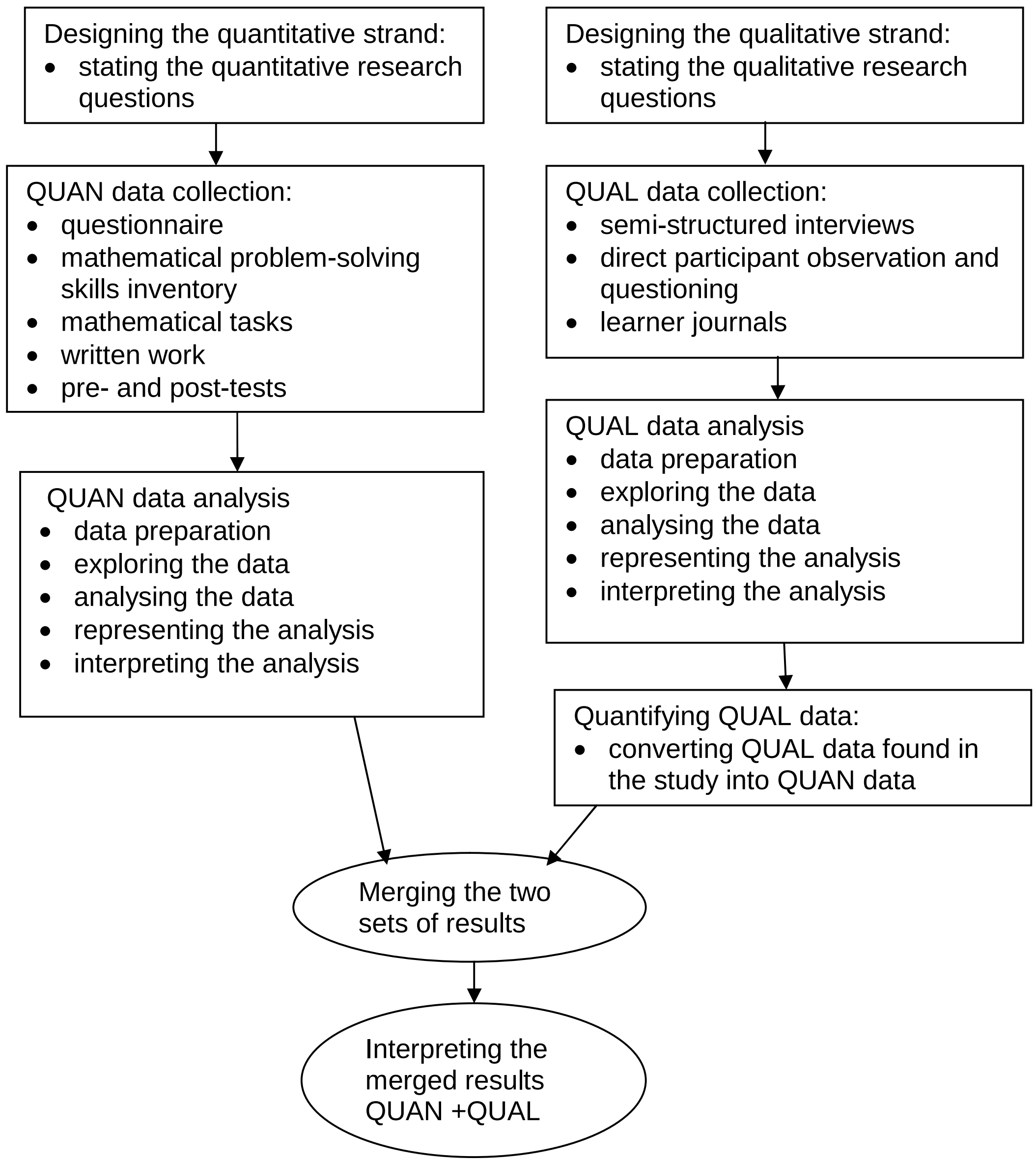


Figure 3. 2 Diagram for the procedures of the convergent design that was implemented for this study

Source: Creswell & Plano Clark (2011:79)

3.7 The target population

A target population is defined by McMillan and Schumacher (2006:119) as a group of elements which can be people or objects that conform to specific criteria and to which a researcher intends to generalise the findings of the study. For this study, the region of Gauteng in South Africa was purposefully chosen by the researcher for reasons of “accessibility and convenience”, a valid and useful approach pointed out by McMillian and Schumacher (2006:125).

3.8 The participants

Participants are individuals who participate in a study and from whom data are collected (McMillian & Schumacher 2006:119). McMillian and Schumacher (2006) further stress that in a study “each person who is given intervention and whose response is measured is a participant”. For this study, the participants consisted of grade 8 mathematics learners at a secondary school in Johannesburg North District. The grade 8 learners were chosen because the researcher would not interfere a lot with the teacher’s schedule, which would have been the case for higher grades. A group of participants from whom the data are collected is called the sample. It is vital that the sample be selected in such a way that it represents the population fairly. The next section discusses the sampling procedure that was adopted for this study.

3.9 The sampling procedure

Sampling is defined by Tashakkori and Teddlie (2003:715) as a selection of units for analysis, say, people or groups of people. It can also be regarded as taking any portion of a population as representative of that population. There are two major groups of sampling procedures, namely probability and nonprobability sampling.

Probability sampling is a sampling technique in which participants “are drawn from a larger population in such a way that the probability of selecting each member of the population is known” (McMillan & Schumacher 2006:119).

In **nonprobability sampling,** the researcher uses participants, representing certain types of characteristics, who are available or accessible (McMillan &

Schumacher 2006:125) and have experienced the central phenomenon or the key concept being studied (Creswell & Plano Clark 2011:173).

In this study, probability sampling was employed and simple random sampling was applied in the selection of the sample. A table of random numbers (see appendix V) was used to select the participants. The researcher employed simple random sampling so that each grade 8 mathematics learner would have an equal chance of being included in the sample. According to McMillan and Schumacher (2006:174), in simple random sampling, all members have the same and known probability of being selected.

In this convergent design, the same sample was used for both the quantitative and qualitative strands of the study as is recommended by Creswell and Plano Clark (2011:183). They recommend the use of the same participants in both the qualitative and quantitative samples, if the purpose of the research is to “corroborate, directly compare or relate two sets of the findings”, and this was the case with this study. The main reason for employing the convergent design was to collect different but complementary data and then directly compare them. The other reason was to gain a valid and complete understanding of the development of mathematical problem solving skills in grade 8 learners in a PCLT environment and its effect on these learners’ performance and achievement in mathematics.

3.10 Data collection procedures

This section discusses the data collection timeline, the research language and the procedures followed in collecting both the quantitative and qualitative data.

3.10.1 The data collection timeline

The participants were involved in the study during their usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the third term of the 2012 South African academic year. During the 10 weeks, the respondents attended the intervention programme for a minimum of 10 x 4.5 hours, that is, 45 hours in total.

3.10.2 The research language

The English language was chosen as the research language because this secondary school enrols South African learners as well as learners from different countries such as Congo, Zimbabwe and Mozambique. It was also feasible to use the English language because the researcher does not thoroughly understand South African local languages.

3.10.3 Quantitative data collection

In the quantitative strand of this mixed methods research design, the participants in both the experimental and control group were required to complete a questionnaire (see Appendix J) and a mathematical problem solving skills inventory (see Appendix K) before and after the intervention. The questionnaire addressed the obstacles that the grade 8 mathematics learners had to overcome before they could really benefit from the PCTL approach and be in a position to develop mathematical problem solving skills. The mathematical problem solving skills inventory (MPSSI) identified the mathematical problem solving skills found in the literature and learners had to assess their mathematical problem solving skills on a 10-point rating scale. The quantitative strand also employed mathematical tasks, written work, pre- and post- word-problem and multiple-choice tests. The mathematical tasks, written work, pre- and post- word-problem tests were assessed by the analytic scoring scale (see appendix Q). With the analytic scoring scale, the researcher was able to assign scores to each of the several phases of the problem solving processes, thereby gaining insight into each learner's progress in the development of mathematical problem solving skills.

3.10.4 Qualitative data collection

The objective of the qualitative study was to establish what grade 8 learners need as a prerequisite for mathematical problem solving skills to develop and explore the development of their mathematical problem solving skills in a problem-centred teaching and learning environment. To evaluate learners' mathematical thinking and the development of their mathematical problem solving skills, the researcher directly observed and questioned learners whilst they grappled with problems during several problem solving sessions. The researcher recorded the observations and findings on the spot using the problem solving comment card, the problem solving observation rating scale and the problem solving observation checklist (see appendices N, O, and P respectively).

To gain insight into individual learners' development of mathematical problem solving skills, the researcher required learners to write a report in their journals about every problem solving experience they completed. The researcher conducted semi-structured interviews with one or two learners during a problem solving session. The problem solving comment card, problem solving observation checklist and the problem solving rating scale were used to record findings from the semi-structured interviews. An audio recording was also used to collect more detailed information which the researcher transcribed verbatim immediately after the lesson for later analysis. The researcher transcribed data immediately while it was still fresh in her memory to avoid losing visual cues that human beings rely on to interpret other people's meanings.

3.11 Data collection tools

Data collection tools are testing devices that are used to measure a phenomenon of interest. For this study, data were collected through various tools such as questionnaires, the Mathematical Problem solving Skills Inventory (MPSSI), participant observation and questioning, semi-structured interviews, learner journals, mathematical tasks, written work, pre- and post-word-problem and multiple-choice tests.

3.11.1 The questionnaire

This is a way of obtaining information or data using statements or questions that require participants to respond to something written for specific purposes (McMillan & Schumacher 2006:194). For this study, the purpose of the questionnaire, which consisted mostly of closed items, was to establish the obstacles that grade 8 mathematics learners had to overcome before they could really benefit from the PCTLA and be in a position to develop mathematical problem solving skills. The questionnaire was used to answer the research sub-question 4. It consisted of 55 questions that were divided into five sections (see Appendix J). Section A consisted of three questions that required learners to fill in biographical details about their gender, age and the name of their school. The other four sections addressed the “obstacles” as follows: learners’ attitude towards mathematics; self-confidence and perseverance with respect to problem solving processes; and their willingness to engage in problem solving activities.

For the questionnaire, learners had to indicate responses on a five-point Likert scale in which 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree. The advantages of using a Likert-type scale are stated by Babbie (2010:179) and McMillan and Schumacher (2006:198-199) as follows:

- They provide flexibility since the descriptors on the scale can vary to fit the nature of the questions or statements.
- They are generally easy to understand and use.
- The difference in intensity between items can be demonstrated.

When devising the questionnaire, the researcher was cautious and avoided vague items, leading questions, negatively phrased questions, double-barrelled questions, irrelevant questions, biased items and sensitive or threatening questions.

The questionnaire was administered to both the experimental group and the control group at the beginning and at the end of the intervention. Although the instructions and language of the questionnaire were clear, the researcher explained most of the aspects of the questionnaire before the learners completed it and informed learners that they were free to ask if they did not understand any questions or instructions. It took learners approximately 25 minutes to complete the questionnaire. Each time learners handed in completed questionnaires, the researcher checked to see if the questionnaires had been completed in full. If a questionnaire had not been completed in full, the researcher asked the participant to do so. The researcher collected all the questionnaires from learners during lesson time to avoid non-return.

3.11.2 The Mathematical Problem Solving Skills Inventory (MPSSI)

An MPSSI is a list of items that a learner checks selectively to give a systematic self-appraisal of mathematical problem solving skills. The MPSSI (see appendix K) was developed by the researcher based on the seven mathematical problem solving skills found in the literature (see section 2.11) and it was used to establish if grade 8 learners perceived themselves as having developed mathematical problem solving skills after the intervention. It identified the following mathematical problem solving skills:

- understanding or formulating the question in the problem
- understanding the conditions and variables in the problem
- selecting or finding the data needed to solve the problem
- formulating sub-problems and selecting appropriate solution strategies to pursue
- correctly implementing the solution strategy or strategies and solving the sub-problems
- giving an answer in terms of the data in the problem
- evaluating the reasonableness of the answer

The MPSSI was administered to both the experimental group and the control group at the beginning and at the end of the intervention. Its purpose was to assess each learner’s mathematical problem solving skills at the beginning and at the end of the intervention. Learners had to evaluate their own competencies on each item on a 10-point scale. With the MPSSI, the researcher was able to assess if learners perceived themselves as having developed mathematical problem solving skills at the end of the intervention. The researcher chose the MPSSI as a data collection tool because it provided her with learner-personal assessment data that supplemented the other data and it also allowed learners to provide input into the data collection process. To be able to elicit honest responses from the participants, the researcher established an environment in which learners felt that their honest responses were important and that this would help them to develop mathematical problem solving skills.

3.11.3 Participant observation and questioning

The researcher regularly moved unobtrusively around the classroom while directly observing and questioning learners as they solved problems as individuals or in small groups of up to four learners. The purpose of using participant observation and questioning in the qualitative strand of this mixed methods research design was to try and understand how learners in a problem solving situation make sense of the problem solving process and how they develop mathematical problem solving skills. Participant observation and questioning also helped the researcher to establish what learners required as a prerequisite for mathematical problem solving skills to develop. Participant observation and questioning therefore addressed research sub-questions 3 and 5 (see section 1.7).

The researcher used the participant observation and questioning method as a form of measurement because with it, she obtained learners' perceptions of the problem solving processes that were expressed in their actions, feelings, thoughts and beliefs (McMillan & Schumacher 2006:347). McMillan and Schumacher (2006) further point out that the observational method is important because it relies on the researcher's seeing and hearing things and recording these observations, instead of relying on a participant's self-report responses to questions or statements.

A maximum of four learners were observed at a time and the researcher asked learners stimulating questions that helped her to evaluate each learner's development of mathematical problem solving skills. Below are examples of some questions that the researcher asked.

- What did you do first when you started to solve the problem?
- What do you think is the most important thing in trying to understand the problem?
- Have you used any strategies in solving the problem? Which ones?
- If your chosen strategy failed, what did you do when your strategy failed?
- Why did you decide to add/divide? (if learner added or divided)
- Are you sure this is the answer to the question?
- Why do you think this is the correct answer?
- Can you describe your solution to the problem?
- How do you feel about your experience with this problem?

In this study, participant observation was the most useful technique for establishing what grade 8 learners need as a prerequisite for mathematical problem solving skills to develop and for evaluating their development of these skills, their willingness to try new problems and perseverance in solving problems. This technique was flexible, allowed a few learners to be evaluated at a time and afforded the researcher the opportunity to evaluate mathematical problem solving skills in a natural classroom setting.

3.11.3.1 The recording techniques

As the researcher observed and questioned learners while they solved problems, she recorded her findings briefly, objectively and on the spot. The recordings included the learners' actions and mathematical problem solving skills and the researcher's interpretations of these. The recording techniques used were the problem solving comment card, problem solving observation checklist and the problem solving observation rating scale (see appendices N, O and P). The recording scales were developed by the researcher based on the mathematics "habits of mind" (see section 2.12.1), focusing, information-gathering, organising, evaluating, analysing and integrating skills (see section 2.12.2) that learners should develop. The problem solving comment card, problem solving observation checklist and the problem solving observation rating scale also mirrored the prerequisites for mathematical problem solving skills development mentioned in sections 2.12.2 and 2.12.3.

The researcher did not complete a recording scale for every learner on a daily basis as this was impractical and unnecessary. The researcher tried to complete one scale sheet for every learner at least once a week. A problem solving folder was kept for each learner's problem solving comment cards, problem solving observation checklists and problem solving rating scales. Summary data from the comment cards, observation checklists, rating scales, and scripts from semi-structured interviews for every learner were kept in a problem solving evaluation notebook. The data in the problem solving evaluation notebook was of crucial importance to the researcher in making decisions about each learner's development of mathematical problem solving skills.

3.11.4 Semi-structured interviews

A semi-structured interview can be taken as a conversation between a researcher and participants, with the researcher attempting to understand the behaviour of the participants without imposing any a priori categorisation which might limit the field of enquiry (Punch 1998). Semi-structured interviews consist of questions that have no choices from which the participants select and are phrased to allow for individual responses. In this study, semi-structured interviews were used to address research sub-questions 3 and 5, that is, the researcher had conversations with participants to establish what they needed as a prerequisite for mathematical problem solving skills to develop and investigate how they developed these problem solving skills.

Questions used in semi-structured interviews for this study were open ended to ensure neutrality and were fairly specific in their intent, for example, what are the important facts and conditions in this problem? The semi-structured interviews were systematic, involved one or two learners at a time and the researcher presented preselected problems (see appendix M) and asked sequenced probing questions from the interview plan (see appendix L) .The researcher took the following steps in conducting the semi-structured interviews:

- She firstly established a friendly, relaxed and nonthreatening atmosphere with the participants (learners).
- She then presented the problem of the day to the learner and asked him or her to talk as much as possible about what he or she would be doing or thinking while solving the problem.
- While the learner worked on the problem, the researcher observed, listened and asked the learner probing questions while being cautious not to teach or ask leading questions.

An audio-recorder, problem solving comment card, rating scale and observation checklist were used to record the findings from the semi-structured interviews. As an ethical measure, permission was first obtained from the participants to audio-record the interviews.

3.11.4.1 Why use semi-structured interviews as a form of measurement?

For this study, semi-interviews were the most challenging and time-consuming form of measurement for the researcher. However, they were the most rewarding for the following reasons:

- Semi-structured interviews are flexible and adaptable, and for this study they resulted in a much higher response than questionnaires (McMillan & Schumacher 2006:203).
- Semi-structured interviews afforded the researcher the opportunity to carefully observe learners' mathematical problem solving skills on a one-on-one basis and this afforded the researcher the opportunity to establish what they needed as a prerequisite for mathematical problem solving skills to develop.

- By using semi-structured interviews, the researcher was able to probe deeply into individual grade 8 learners' mathematical problem solving skills.
- The learners were able to give detailed information about what they were doing and thinking while they solved problems.
- Semi-structured interviews provided the researcher with insight into learners' development of problem solving skills which were not apparent from written work, tasks or tests.

3.11.5 Learner journals

The development of learners' mathematical problem solving skills was examined by having each learner write in his or her journal on a daily basis about the problem solving session they experienced. In section 2.12.2.3, it was indicated that having the learners express and record their ideas, thoughts, feelings and questions in their journals as they work through the problem solving processes helps to develop their mathematical problem solving skills. Learners were asked to complete their journals immediately after a problem solving session. Using the learner journal focus questions as a guide, learners were expected to think back and describe how they would have solved the problem. Below are the questions that were included in the learner journal focus questions:

- What did you do when you first saw the problem? What were your thoughts?
- What plan did you make to solve the problem?
- Did the plan work out?
- Did you get stuck? What did you do when you got stuck? How did you feel about it?
- Did you try an approach that did not work and had to stop and try another approach? How did you feel about this?
- Did you find a solution to the problem? How did you feel about this?
- Did you check your answer in any way? Did you feel it was correct? Why did you not check your answer, if you did not?

- How did you feel in general about this problem solving experience?
- Did you ever feel frustrated when solving the problem? Why?
- Did you ever feel that you wanted to give up and not solve the problem? When?
- Did you enjoy solving this problem? Why or why not?
- Would you like to solve a problem like this again?

3.11.5.1 Why use learner journals as a form of measurement?

Learners' journals were used to address research sub-question 5 (see section 1.7). The following are the advantages the researcher gained from employing learner journals as a form measurement:

- The learner journals provided valuable information about individual learner's use of problem solving strategies and the development of mathematical problem solving skills.
- The learner journals provided unique learner-oriented information that was not available from other data collection tools.
- Although it took a lot of time for learners to complete journal reports, the process did not take much of the researcher's time.
- The journals provided learners with practice in expressing their ideas and experiences in writing.

3.11.6 Pre- and post-word-problem tests

Word-problem tests are made up of items that are answered by supplying requested information. The information may be a word, number, phrase, sentence or collection of symbols that complete a statement. Word-problem tests are useful in evaluating learners' ability to use mathematical problem solving skills. For this study, word-problem tests were designed to measure learners' mathematical problem solving skills and assisted the researcher in analysing learners' procedures for solving a given problem as well in gaining specific insight into their ability to use the different mathematical problem solving skills. Word-problem tests (see appendices R & S) were administered to both the experimental and control groups before and after the intervention. For this study, items in the word-problem tests were prepared in such a way that they measured learner's mathematical problem solving skills at the beginning and at the end of the intervention. The validity of each item in the word-problem test was assessed by carefully analysing what the item required the learner to know or do. The researcher also asked her supervisor and the grade 8 mathematics teacher to check on the validity of the word-problem test items. The purpose of the word-problem tests was to measure and compare grade 8 learners' performance at the beginning and at the end of the intervention, therefore addressing research sub-questions 6 and 7.

3.11.6.1 Why use word-problem tests as a form of measurement?

Word-problem tests were employed for this study because they required learners to supply the answer thereby avoiding guesses. Word-problem tests afforded the researcher an opportunity to view learners' work and this provided her with a greater understanding of their development of mathematical problem solving skills.

3.11.7 Multiple-choice tests

A multiple-choice test is made up of items that consist of a problem or a question and a list of possible solutions. There is only one correct answer and others are distracters that reflect misinterpretations that are designed to entice learners who are not certain of the correct answer. For this study, the researcher administered multiple-choice questions (see appendices T & U) to both the control and the experimental group before and after the intervention. The purpose of the multiple-choice tests was to measure and compare learners' performance and achievement in mathematics at the beginning and at the end of the intervention; therefore also addressing research sub-questions 6 and 7 (see section 1.7).

The researcher chose multiple-choice tests as a form of measurement because they are versatile, can measure learners' ability to find correct answers and their ability to use mathematical problem solving skills. With multiple-choice questions, a wide variety of abilities can be measured and scoring and interpretations are easy. For this study, items were made specifically to evaluate a specific mathematical problem solving skill and learners' difficulties could easily be diagnosed by analysing incorrect responses. Validity of each item of the multiple-choice tests was assessed by asking the supervisor of the researcher and the grade 8 mathematics teacher.

3.11.8 Mathematical tasks and written work

The mathematical tasks and written work were given to learners in the experimental group during the intervention programme to evaluate how the development of mathematical problem solving skills influenced their interpretation of "new" knowledge and solving of nonroutine problems - hence addressing research sub-question 6. The mathematical tasks and written work consisted mainly of conceptual questions that engaged all the learners in the making and testing of mathematical hypotheses (Lampert 1990:39), and problems that had only one or obvious solutions were avoided. The researcher designed the conceptual questions using the guidelines on the nature of appropriate problems that is given in section 2.7.3. The questions called for learners' understanding of mathematical concepts and use of mathematical problem solving skills. Conceptual knowledge and procedural knowledge were discussed to a greater extent in section 2.4.

The mathematical tasks and written work were done during class or if learners were unable to finish during class time they were given the opportunity to continue solving the work as homework. The researcher left learners to work on the problems without suggesting any procedures (Clark 1997), but she provided enough scaffolding to keep learners on task. Scaffolding (see section 2.5.2.3) for this study involved the researcher explaining the unknown content to the learners or questioning them and this helped them to draw out their own thinking.

The researcher evaluated learners' solutions of mathematical tasks and written work using an analytic scoring scale (see appendix Q). An analytic scoring scale is an assessment method that assigns scores to each of the several phases of the problem solving process. For this study, the analytic scoring scale looked at the following mathematical problem solving skills:

- understanding the problem
- planning a solution
- Getting a solution

For each of the above problem solving skills, values of 0, 1 or 2 points were assigned (see appendix Q). The following are some of the advantages of using an analytic scoring scale:

- An analytic scoring scale does not just consider the answer but they evaluate all phases of the problem solving process. Therefore the researcher was able to evaluate each learner's use of mathematical problem solving skills.
- It allowed the researcher to realise each learner's specific areas of strength and weakness.
- An analytic scoring scale gave the researcher a means for assigning numerical values (quantitative data) to learners' work.

3.11.9 Pilot testing the instruments

Pilot testing of the questionnaire, the MPSSI, pre- and post- multiple-choice and word-problem tests was conducted with 20 grade 8 mathematics learners at a neighbouring school. The learners were well informed that it was a pilot test and were assured of confidentiality and anonymity. Babbie (2010:98 & 233) points out that piloting of instruments is essential because it improves reliability in that people understand the items or statements in the same way as each other. For this study, piloting tested the wording, language use, the length, clarity and appropriateness of the statements and instructions of the questionnaire, the MPSSI, pre- and post- multiple-choice and word-problem tests. The pilot test also checked if the data that would be obtained from the questionnaire, the MPSSI, pre- and post- multiple-choice and word-problem tests would reflect real understanding of the participants.

The respondents of the pilot test were required to provide feedback on individual items and the whole questionnaire, the MPSSI, pre- and post- multiple-choice and word-problem tests. The feedback was used to amend, simplify and clarify some of the items. Adaptations and amendments were made to the questionnaire, the MPSSI, pre- and post- multiple-choice and word-problem tests to make them fully understandable to participants. The researcher was present when the instruments were piloted and responded to any uncertainties the respondents may have had.

3.12 Data analysis

Data analysis is the process of making sense out of data, which involves interpreting, consolidating and reducing what participants have said, how they have responded and what the researcher has seen and read in order to derive or make meaning out of the process. Mouton (2001:108) sees data analysis as "breaking up" data into manageable themes, trends, patterns and relationships. The purpose of data analysis in this study was to provide answers to the research questions through understanding of various constitutive elements of the data. In the next section, procedures of quantitative and qualitative data analysis are discussed. This is then followed by a discussion of mixed methods data analysis.

Researchers go through similar steps for both qualitative and quantitative data analysis. Creswell and Plano Clark (2011:204) list the data analysis steps as follows:

- preparing the data for analysis
- exploring the data
- analysing the data
- representing the analysis
- interpreting the analysis
- validating the data and interpretations

3.12.1 Quantitative data analysis

In this study, quantitative data analysis consisted of descriptive statistical analysis (McMillan & Schumacher 2006:153). Univariate analysis techniques include analysis of measures of central tendency (mean), standard deviation, range, frequencies and overall test scores. Descriptive statistics transform a set of observations into indices that characterise the data and thus are used to summarise and organise observations (McMillan & Schumacher 2006:150), so that readers can have a mental picture of how the data relates to the phenomena under study. For this study, data from the questionnaires, mathematical problem solving skills inventory, mathematical tasks, written work, pre- and post- word-problem and multiple-choice tests were tabulated and imported to SPSS; and then descriptive statistics including means, frequencies, standard deviations, ANOVA (analysis of variance) and paired t-tests were analysed. An F-test was used to test the variances of the questionnaire responses between the experimental and control groups.

3.12.2 Qualitative data analysis

According to McMillan and Schumacher (2006:364), qualitative data analysis is an inductive process of organising data into categories and identifying patterns among the categories. Creswell and Plano Clark (2011:208) state that qualitative data analysis involves "coding the data, dividing the text into small units, assigning a label to each unit and then grouping the codes into themes." In this study, qualitative data analysis started as soon as data collection began and it was an ongoing process – for example, the semi-structured interview questions were continually modified and refined during the intervention. Qualitative data analysis involved analysing findings from the semi-structured interviews and participant observation and questioning that were recorded on the problem solving comment card, problem solving rating scale and the problem solving observation checklist. Audio tapes from the semi-structured interviews were transcribed verbatim into written notes in order to be able to paraphrase common patterns and experiences. Learner journals were analysed by categorising the mathematical problem solving skills that could be identified behind learners' learning conceptions.

3.12.3 Mixed methods data analysis

Mixed methods data analysis includes analysing separately the quantitative data by quantitative methods and the qualitative data using qualitative methods and then merging the two databases. According to Creswell and Plano Clark (2011:212), mixed methods data analysis is when analytic techniques are applied to both quantitative and qualitative data, as well as to the integration of the two forms of data concurrently and sequentially in a single project or multiphase project. In this convergent research design, quantitative and qualitative data were collected concurrently and the researcher analysed the findings separately and then merged the two databases in the results, interpretation and conclusion phase. As suggested by Creswell and Plano Clark (2011:215-216), the convergent research design data analysis took the following steps:

- Quantitative and qualitative data were collected concurrently.
- Separately analysing the quantitative data using quantitative methods and the qualitative data using qualitative methods.
- The quantitative data together with the qualitative data were analysed using a side-by-side comparison for the merged data (Creswell & Plano Clark 2011:223).
- An interpretation was given of how the merged results answered the research questions.

3.13 Reliability and validity

Reliability and validity are the central issues used in establishing the quality, trustworthiness, authenticity, usefulness and believability of mixed methods research findings. Reliability was defined in the first chapter in section 1.10.5 and can be taken as the main requirement for the research tools, whereas validity is regarded as the main criterion by which the quality and appropriateness of the tools are measured. In this study, reliability and validity were enhanced by employing the mixed methods research design, prolonging the data collection period, using various data collection tools and recording techniques and a lot of time was spent interviewing, observing and questioning learners.

For this study, it was important to use the mixed methods research design because it is well documented in the literature (Johnson, Onwuegbuzie & Turner 2007; Creswell & Plano Clark 2011; Tashakkori & Teddlie 2003; Greene 2007; Morse 1991) that if different sources of data collection are involved in a study, they increase the credibility of the results and conclusions. This is called the process of triangulation. Triangulation is defined by Krathwohl (2009:285) as an attempt to compare data which is obtained using two or more data collection methods. This implies that researchers can be confident in their results if the various data collection methods produce data that are more or less the same.

The researcher is accountable for the results of research since he or she collects the data. The results must be accurate and reasonable and should be applicable and useful in the concerned field. In this study, several measures were taken to ensure the reliability and validity of the data collection tools and the results, and this is discussed in the next section.

3.13.1 Reliability

Reliability refers to the degree of consistency with which a data collection tool measures whatever it is supposed to measure. This is the extent to which the data collection tool gives similar results and conclusions if it is administered to a different group of participants under different conditions such as time and venue. This also implies that if the same research is done again under similar conditions, the researcher will obtain the same results and not erratic or inconsistent results.

Quantitative reliability means that observations from participants are consistent and stable over time (Creswell & Plano Clark 2011:211). To address this issue, Cronbach alphas for the various constructs of the questionnaire and the mathematical problem solving skills inventory were calculated to determine their reliability. The reliability coefficients were generally above 0.8 (see table 3.1) which is excellent for these instruments (McMillan & Schumacher 2006:183; 186-187). McMillan and Schumacher (2006) go on to state that the Cronbach alpha is generally the most appropriate type of reliability for questionnaires in which there is a range of possible answers for each item and this was the case for the questionnaire and the mathematical problem solving skills inventory used for this study.

The Kuder-Richardson 20 (KR-20) was used to calculate the reliability coefficients of the pre- and post-multiple-choice tests. The KR-20 is used to correlate all items on a single test with each other when you have dichotomous items in a test (usually for right or wrong answers) such as with multiple-choice tests (McMillan & Schumacher 2006:186). The KR-20 reliability coefficients for the pre- and post- multiple-choice tests were 0.78 and 0.77 respectively (see table 3.1), which is acceptable for this type of instrument. The Spearman-Brown formula (McMillan & Schumacher 2006:185) was used to calculate the reliability coefficients of the mathematical tasks, written work, pre- and post-word-problem tests. The Spearman-Brown coefficients for these instruments were generally above 0.70 (see table 3.1) which is acceptable for these kind of instruments.

Instrument	Recording techniques	Reliability instrument	Value of reliability instrument
Questionnaire	Likert scale	Cronbach alpha	0.81
MPSSI	Likert scale	Cronbach alpha	0.86
Word-problem tests	Analytic scale	Spearman-Brown	Pre-test = 0.72 Post-test = 0.74
Multiple-choice tests	Analytic scoring scale	Kuder Richardson 20	Pre-test = 0.78 Post-test =0.77
Written work and mathematical tasks	Analytic scoring scale	Spearman-Brown	Task 1 = 0.71 Task 2 = 0.70 Task 3 = 0.73
Semi-structured interviews	Comment card, checklist and rating scale	Use of audio recorder and conducting interviews in the natural setting	
Participant observation and questioning	Comment card, checklist and rating scale	Prolonging the data collection period	

Table 3. 1 Reliability of instruments

The researcher ensured the consistency of the qualitative data by prolonging the data collection period, conducting the semi-structured interviews in the same classrooms that learners' mathematics lessons took place (natural setting) and by audio recording the semi-structured interviews and transcribing them verbatim on the same day. Audio recorders and transcripts were employed in this study because these are materials that are known to have significant implications for reliability and accuracy.

3.13.2 Validity

The definition of validity was provided in the first chapter and its purpose is "to check the quality of the data, the results and the interpretations" (Creswell & Plano Clark 2011:210). Validity determines whether the research truly measures that which it was intended to measure or how trustworthy the research results and findings are. McMillan and Schumacher (2006:134) concur with this when they state that validity refers to the truthfulness of findings and conclusions. It is essential that procedures to ensure the validity of the data, results and their interpretations are utilised. In the next section, quantitative validity and qualitative validity are explored.

3.13.2.1 Quantitative validity

Quantitative validity looks at the quality of the scores that one obtains from the data collection tools and the quality of the conclusions that can be drawn from the results of the quantitative analyses (Creswell & Plano Clark 2011:210). Quantitative validity involves content validity, criterion-related validity, construct validity and external validity.

Content validity is concerned with whether the data collection tools are representative of all possible items. For this study, regarding content validity, all the mathematical problem solving skills and other factors explored in the literature review were all represented by the items in the different sections of the questionnaire, the mathematical problem solving skills inventory, the mathematical tasks, written work, tests, the interview guide and the learner journal focus questions.

Criterion-related validity refers to whether the scores relate to some external standard such as scores on a similar instrument. For this study, criteria set in the teachers' guide for grade 8 mathematics were used as a reference for criterion-related validity.

Construct validity is concerned with whether data collection tools measure what they intend to measure. To strengthen construct validity of this study various methods of assessment, that is, multiple-choice tests, word-problem tests, mathematical tasks, written work and observation and questioning, were used to test learners' achievement and performance in mathematics.

In quantitative research, **internal validity** is the extent to which the researcher can reach a conclusion that there is a "cause and effect relationship among variables" (Creswell & Plano Clark 2011:211). For this study, the selection threat to internal validity was reduced by selecting the participants using the simple random sampling method. With simple random sampling, all learners had the same probability of being chosen (McMillan & Schumacher 2006:120). There were no extraneous events that occurred during the data collection period and this led the researcher to conclude that there was no known history threat to internal validity for this study.

External validity refers to the generalisability of the results, that is, the results and findings can be generalised to a large population. To reduce the threat to external validity, the school chosen for this research is a representative sample of a typical South African school.

3.13.2.2 Qualitative validity

Qualitative validity is concerned with whether the account given by the researcher and the responses given by the participants are accurate, trustworthy and credible. Internal validity in qualitative research checks whether researchers observe what they think they observe and actually hear what they think they hear. In this research, it was essential that the researcher understood learners' responses in semi-structured interviews and during participant observation and questioning. To enhance qualitative validity the researcher employed triangulation and "member checks" (Creswell & Plano Clark 2011:211; McMillan & Schumacher 2006:324). Member checks imply that the researcher went back to school during the fourth term of the 2012 South African academic year to ask participants if the findings were truly what they experienced during the data collection process. However, no changes were provided by the participants to the data that were presented to them during the member checks process.

3.14 Ethical considerations

Ethics deals with the beliefs or guidelines about what is right or wrong, proper or improper, good or bad from a moral perspective (McMillan & Schumacher 2006:142). McMillan and Schumacher (2006) go on to stress that the researcher has a moral obligation and is ethically responsible for protecting participants' rights and welfare, including physical and mental discomfort, harm and danger.

For this research, in compliance with the Unisa research ethics policy, all precautions were taken before the data collection process in order to adhere to the ethical measures to respect the integrity, confidentiality, anonymity, privacy, caring, consent and humanity of the participants. The next section looks at the ethical considerations that were important for this study.

3.14.1 Obtaining informed consent

Informed consent means that the participants have a choice of either participating or not participating in the research. Wiersma and Jurs (2009:456) explain that when human subjects participate in a research study, they should be informed of their role, the procedures, the purpose of the research, the possible risks of the research and they should give their written consent for participation.

Concerning obtaining informed consent to collect data, the researcher sought permission from the Gauteng Department of Education (GDE) (see appendix A) which issued an approval letter (see appendix B) that gave the researcher permission to gain entry into the Johannesburg North district of education. Thereafter, the researcher sought and obtained permission from the Johannesburg North district of education (see appendices C & D) to collect data from a secondary school in their district. After this, the researcher wrote letters to the school principal and school governing board (see appendices E & F) seeking permission to collect research data from their school. The school principal and the school governing board willingly gave the researcher approval. However, the letters were not included in the appendices for the sake of anonymity and confidentiality. The researcher then applied for a research ethical clearance certificate which was granted to her by the Unisa ethics committee (see appendix G). The research ethical clearance certificate implies that there were no ethical issues, no deceptions and no possible risks or danger for human participants in this study.

At the onset of the study, the researcher wrote letters to the participants and participants' parents or guardians (see appendices H & I) that clearly explained among other issues, the purpose of the study, their role and voluntary participation. After understanding the content of the letters, the participants and their parents or guardians agreed to participate in the study and gave their written consent by signing the letters.

3.14.2 Voluntary participation

The researcher made sure that the participants were well informed about the purpose of the research, the procedures of the data collection process and the research's possible impact on them. The researcher clearly explained to participants that they were free to decide whether they wanted to participate in the research or not and had freedom to withdraw from the research at anytime without incurring any negative consequences. Participants were not deceived in any way and the researcher was open and honest about all aspects of the study.

3.14.3 Confidentiality and anonymity

Confidentiality as explained by Wiersma and Jurs (2009:458) is the act of not disclosing the identity of participants in a research or study and anonymity implies that the names of the participants where data is obtained are unknown. For this study, participants were assured that their confidentiality and privacy would be respected and that their responses would be used for the purpose of the study only. All reasonable efforts and necessary precautions to maintain complete participant confidentiality and anonymity were in place and were enforced. School name, principal name, mathematics teacher name and participant names were eliminated from all reports and pseudonyms were assigned. McMillan and Schumacher (2006:334) stress that the settings and participants should not be identified in print and no-one should have access to participant names except the researcher.

3.14.4 Reciprocity

Creswell and Plano Clark (2011:179) and McMillan and Schumacher (2006:334) suggest that researchers should reciprocate participants for their willingness to provide data and all the people that adjust their priorities and routines to assist or tolerate the researcher. The researcher can reciprocate in the form of time, feedback, attention, appropriate token gifts or specialised services. The researcher of this study felt indebted to participants for providing data that allowed her to answer the research questions. Therefore, at the end of the study, the researcher gave each participant a CNA voucher to purchase stationery or books. To motivate learners during the semi-structured interviews that were conducted after the school lessons, the researcher gave participants sweets or burgers to eat while she conducted the interviews. The researcher also provided the research findings to the GDE, Johannesburg North district department of education, the school and some participants and parents or guardians.

3.15 Conclusion

The methodology and the research design that were used for this study were explained in this chapter. The research method used for this study was indicated as the mixed methods research design. The type of mixed methods design that was adopted for this study was the convergent research design. The reasons, advantages and challenges of adopting the mixed methods and convergent design were explained. Various data collection tools were used in this study to enhance reliability and validity of the findings. In the next chapter, the researcher analyses and interprets the data and presents the findings.

CHAPTER 4
DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1 Introduction

In chapter 3, the research methodology and research design were described. The aim of this chapter is to present, analyse and interpret the data that were collected in the empirical investigation, in order to answer research questions 3, 4, 5, 6 and 7 (see section 1.7). Data analysis in the mixed methods research design involves analysing separately the quantitative data using quantitative methods and the qualitative data using qualitative methods (Creswell & Plano Clark, 2011:211) and then merging the two databases. In this study, data analysis occurred at the following three points:

- with each data set independently
- when the comparison of the two data sets occurred
- after the comparison was completed

4.2 Qualitative data analysis

Qualitative data analysis preceded the quantitative data analysis in order to be in a position to explore whether grade 8 learners in the experimental group had indeed developed mathematical problem solving skills during the intervention.

4.2.1 Participant observation and questioning

The researcher observed that at the beginning of the intervention learners in the experimental group were hesitant to solve unfamiliar problems without the teacher's help and generally used impulse approaches to solving the given problems. When learners impulsively decided on a wrong problem solving strategy they usually obtained incorrect solutions, unless they assessed their actions early enough to check whether the strategy lead to the correct solution. However, all learners in the experimental group were enthusiastic about this new method of learning even though they sometimes felt overloaded and insecure about their attempts. Although learners were initially worried about being responsible for their own learning and finding their own ways of solving unfamiliar problems, they rose to the occasion and interactions in the small groups were lively, constructive and learner centred.

The analysis of the problem solving comment card, the problem solving checklist and the problem solving rating scale that the researcher completed during participant observation and questioning (see section 3.11.3.1) revealed that mathematical problem solving skills development was gradual and new problem solving skills were formed little by little over time. As the intervention progressed, learners were able to make conscious decisions about choosing a problem solving strategy. The various problem solving strategies that learners chose showed signs of independent thinking and the development of their mathematical problem solving skills. When the researcher questioned learners on the use of a problem solving strategy, they were able to justify their own decisions, reasoning and why they preferred one strategy to another. Questioning learners revealed that as they developed mathematical problem solving skills they became more confident, more willing to solve problems and developed more new strategies for attacking ideas.

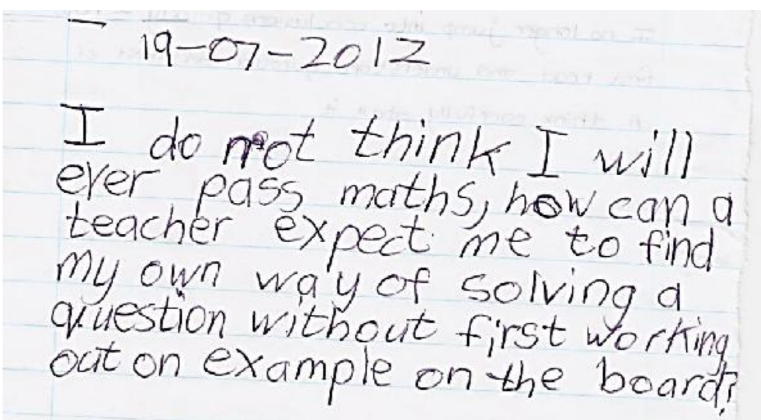
It was indicated in section 3.5 that learners were encouraged to “think out aloud” when solving mathematical problems. As they thought out aloud, the researcher observed that they became more aware of the information that they were using to solve the problems and became more conscious of how they were solving the problems.

The researcher observed that all learners enjoyed working in small groups. Working in groups afforded them the opportunity to test out their ideas in a relaxed atmosphere. They discussed and modified ideas as they progressed with each other’s help. With time, learners who were initially nervous and withdrawn began to comfortably contribute and enjoy group work and could easily explain their solutions to the whole class. When learners worked in pairs, the researcher paired weak with strong learners. Both weak and strong learners benefited, and the researcher observed that strong learners solidified their understanding and tried to communicate their knowledge to their weak peers.

4.2.2 Journals

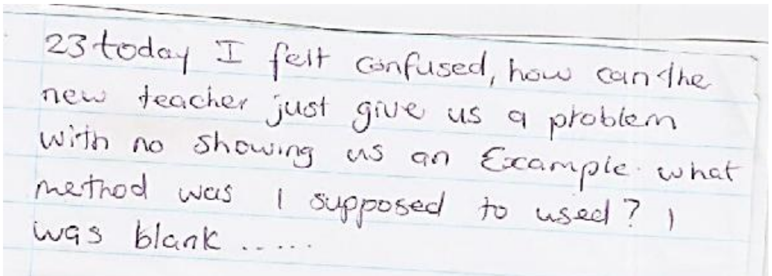
The idea of learners filling in their journals after every problem solving experience gave the researcher insight into how they viewed the problem solving process, the problem-centred approach and how they felt about the development of their mathematical problem solving skills. The researcher firstly read all learner journal entries over and over again in order to immerse herself in the data and she wrote short phrases and memos on each journal entry. The memos and short phrases were used to assess learners' development of mathematical problem solving skills.

At the beginning of the intervention some learners felt very uncomfortable with the PCTLA because they were used to their teacher being the main source of knowledge and this was reflected in the way they evaluated this "new" way of learning in their diaries. For the purpose of this report, learner 4's journal entry on 19 July 2012 and learner 13's journal entry on 23 July 2012 are given as examples


19-07-2012
I do not think I will ever pass maths, how can a teacher expect me to find my own way of solving a question without first working out on example on the board?

Journal entry 1

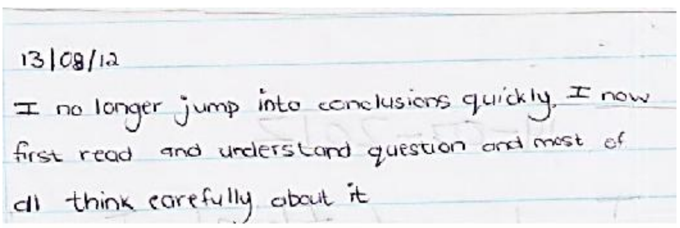
Learner 13's journal entry on 23 July 2012 is as follows:



23 today I felt confused, how can the new teacher just give us a problem with no showing us an Example what method was I supposed to used? I was blank .....

Journal entry 2

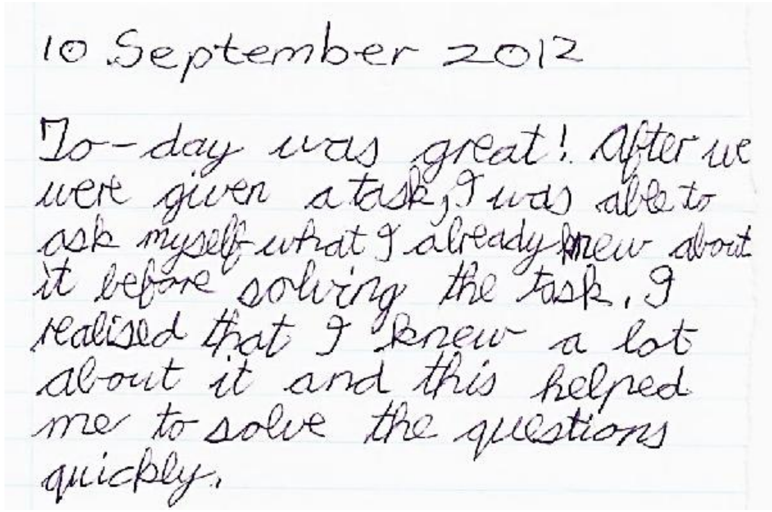
However, as the intervention progressed, learners began to accept the PCTLA and became more reflective about what, why and how they would have learnt. This is indicated by what learner 7 entered in her diary on 13 August 2012.



13/08/12
I no longer jump into conclusions quickly, I now first read and understand question and most of all think carefully about it

Journal entry 3

As the intervention progressed learners highlighted in their journals a range of ways their mathematical problem solving skills had improved. They began to ask their peers and themselves to reflect on the reasonableness of an answer. For this study, it was important that learners asked their peers and themselves questions about what they knew about a task that they were working on. Peer questioning and self-questioning led to learners' development of mathematical knowledge and problem solving skills. Most learners developed self-questioning as is indicated by what learner 14 wrote in his diary on 10 September 2012.

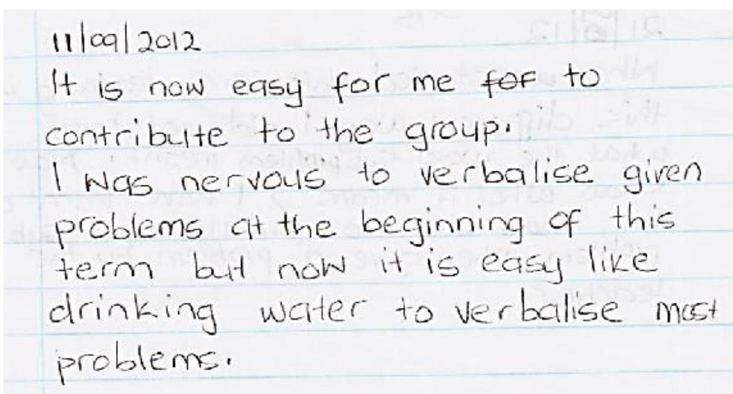


10 September 2012

To-day was great! After we were given a task, I was able to ask myself what I already knew about it before solving the task, I realised that I knew a lot about it and this helped me to solve the questions quickly.

Journal entry 4

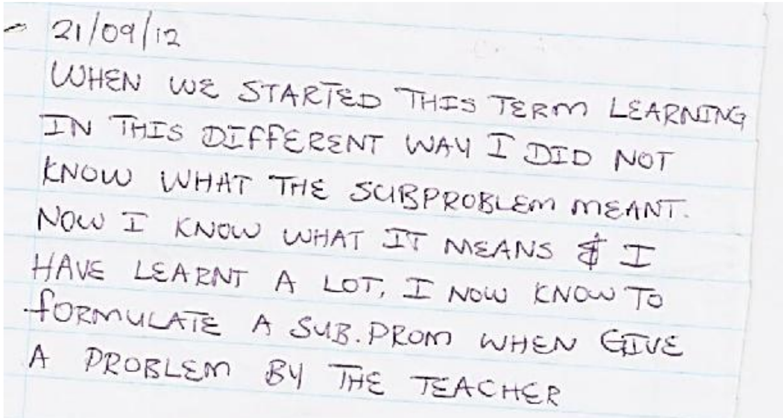
Journal entries gave evidence of learners' increase in ownership of their individual learning processes. They identified increasing insights into their mathematical problem solving skills. This is supported by what learner 23 and learner 15 wrote in their dairies on 11 and 21 September 2012 respectively:



11/09/2012

It is now easy for me ~~for~~ to contribute to the group.
I was nervous to verbalise given problems at the beginning of this term but now it is easy like drinking water to verbalise most problems.

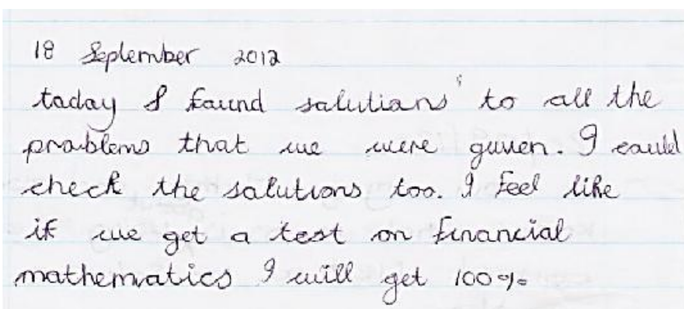
Journal entry 5



21/09/12
WHEN WE STARTED THIS TERM LEARNING IN THIS DIFFERENT WAY I DID NOT KNOW WHAT THE SUBPROBLEM MEANT. NOW I KNOW WHAT IT MEANS & I HAVE LEARNT A LOT, I NOW KNOW TO FORMULATE A SUB.PROM WHEN GIVE A PROBLEM BY THE TEACHER

Journal entry 6

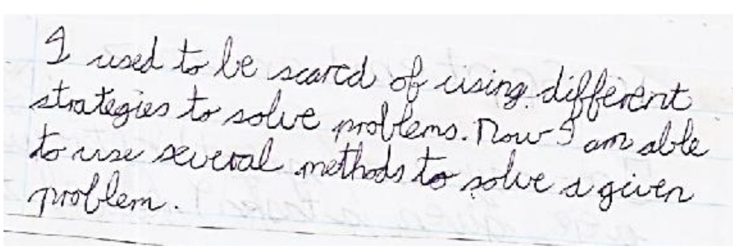
At the end of the intervention, learners developed stronger beliefs about the importance of finding their own ways of solving problems and looking for different ways of solving problems. Learners began to believe that they had mathematical problems-solving skills and mathematical knowledge to solve most unfamiliar mathematical problems. This is evidenced by the following journals entries by learners 9, 26 and 4, respectively:



18 September 2012
today I found solutions to all the problems that we were given. I could check the solutions too. I feel like if we get a test on financial mathematics I will get 100%

Journal entry 7

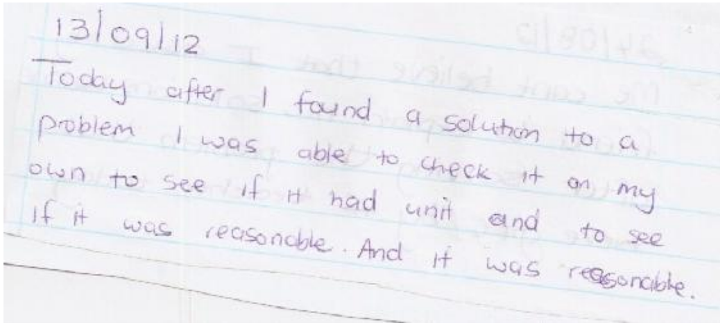
Learner 4 wrote the following on 13 September 2012:



I used to be scared of using different strategies to solve problems. Now I am able to use several methods to solve a given problem.

Journal entry 8

Learner 26 jotted down the following on 13 September 2012:


13/09/12
Today after I found a solution to a problem I was able to check it on my own to see if it had unit and to see if it was reasonable. And it was reasonable.

Journal entry 9

4.2.3 Semi-structured interviews

The data on the learners' subjective learning experiences were gathered in the semi-structured interviews that the researcher conducted with them during the problem solving sessions. The researcher valued the learners' responses because "children rarely give random responses" (Labinowicz 1985). Learners' responses tend to make sense in terms of their personal perspectives or in terms of the knowledge they are using to give meaning to situations.

As indicated in sections 1.10.4 and 3.10.4, the audio recordings were transcribed verbatim. The researcher read the transcribed data line by line to make sure it made sense and read through all the interview transcripts several times in order to immerse herself in the data. During this process, the researcher wrote key concepts in the margins of each transcript and recorded notes in the form of short ideas. She then recorded the findings for each learner on the problem solving observation checklist, the problem solving comment card and the problem solving rating scale.

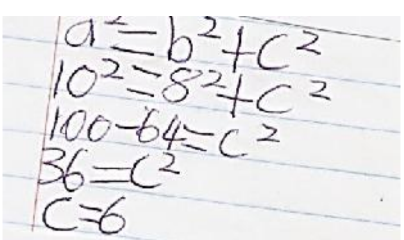
As learners became familiar with the PCTLA and began to develop mathematical problem solving skills, a culture of enquiry was established in the classroom. Learners were now able to take control of the given problems, problematise them, focus their attention on developing appropriate strategies and check on the correctness and reasonableness of solutions to the given problems. During semi-structured interviews, the researcher noticed that as the intervention progressed, learners began to verbalise given problems, understand the question in a problem and evaluate the reasonableness of a solution. They also began to take ownership of ideas and developed a sense of power in making sense of mathematics. Learners started to understand that most problems can be solved in more than one way and that some problems have more than one correct answer. They started to use a variety of strategies like working backwards, looking for patterns or making a list when solving problems during the semi-structured interviews.

At the end of the intervention learners demonstrated that they had developed mathematical problem solving skills. This is evidenced by how much they improved in solving unfamiliar problems, by their ability to identify needed information, by their efficiency in selecting the data needed to solve the problem and at the same time ignoring nonessential information. Learners could give solutions to a problem and could clearly state the goal of the problem or task. Below are two solutions for two different tasks that were done by learner 24. The first solution is for question 5 of the interview questions (see appendix M) that was done by the learner at the beginning of the intervention on 23 July 2012. The second solution is for problem 28 that was completed by the learner at the end of the intervention on 20 September 2012.

Question 5

> *A hummingbird lives in a nest that is 8 metres high in a tree. The hummingbird flies 10 metres to get from its nest to a flower on the ground. How far is the flower from the base of the tree?*

Solution to question 5

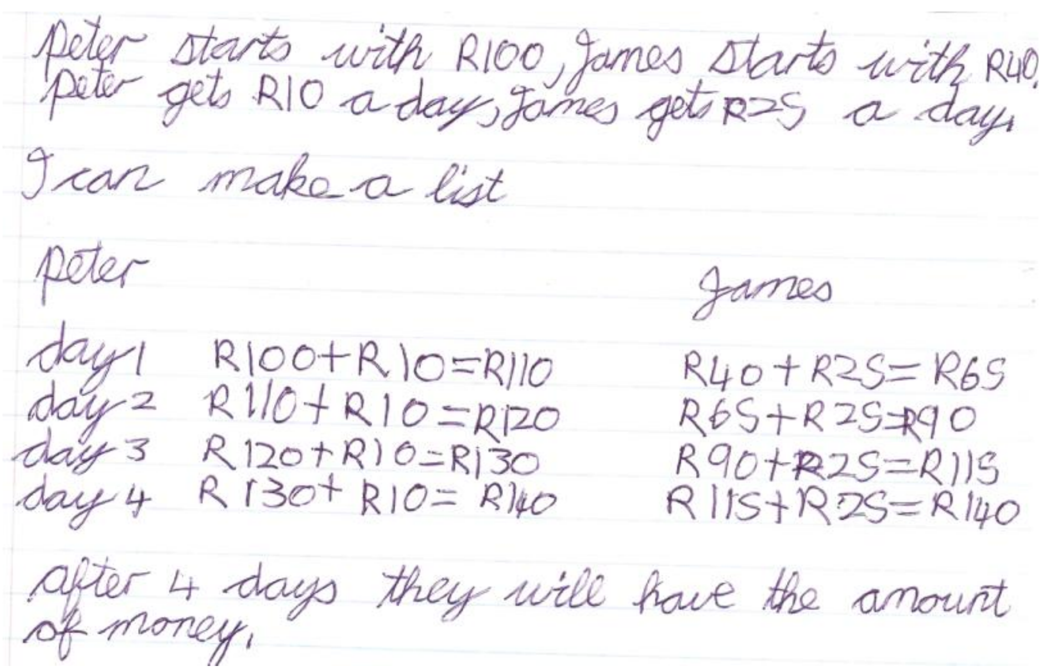


The learner used an algorithm to solve the problem, did not attempt to use a diagram to present the solution and did not give the answer in terms of the data in the problem. In the semi-structured interview the researcher noticed that this learner was unable to verbalise the problem and could neither explain nor check the solution.

Question 28

Peter has R100 pocket money and James has R40. They are both offered part-time jobs at different supermarkets. Peter earns R10 a day and James R25 a day. If they do not spend their pocket money or their daily wages, after how many days will they have the same amount of money?

This solution to question 28 was given by learner 24 towards the end of the intervention:


peter starts with R100, James starts with R40,
peter gets R10 a day, James gets R25 a day,
I can make a list

peter		James
day 1	R100+R10=R110	R40+R25=R65
day 2	R110+R10=R120	R65+R25=R90
day 3	R120+R10=R130	R90+R25=R115
day 4	R130+R10=R140	R115+R25=R140

after 4 days they will have the amount of money,

There is clear evidence in the above solution that learner 24 had developed mathematical problem solving skills at the end of the intervention by being exposed to the PCTL environment. The learner could formulate the question in the problem, could select data needed to solve the problem, was able to correctly implement a solution strategy and solve the sub-problems. At the beginning of the intervention learner 24 could not give the solution in terms of the data in the problem, but from the above solution one can clearly see that he had developed this skill by the end of the intervention.

4.2.4 Quantifying the qualitative data

During participant observation and questioning and semi-structured interviews, the researcher filled in the problem solving comment card, problem solving checklist and the problem solving rating scale (see section 3.11.3.1) for all learners in the experimental group. These data were recorded in the frequency tables. For the purpose of this report two frequency tables (see table 4.1 and 4.2) for all learners in the experimental group were given to demonstrate that the whole group had developed mathematical problem solving skills at the end of the intervention. The researcher also gave two frequency tables (see table 4.3 and 4.4) for one learner so as to demonstrate that as the intervention progressed, learners in the experimental group as individuals were developing mathematical problem solving skills; and had indeed developed them by the end of the intervention..

	(*n=28*) Date	27/07/12	10/08/12	24/08/12	07/09/12	21/09/12
1	Number of learners who liked to solve problems	2	15	20	26	27
2	Number of learners who persevered or stuck with the problem	1	6	15	25	28
3	Number of learners who tried to verbalise the problem	2	8	14	21	28
4	Number of learners who could understand the conditions and variables in a problem	0	2	9	15	26
5	Number of learners who could identify relevant data needed to solve a problem	2	10	16	24	28
6	Number of learners who thought about which strategy may be useful	0	8	13	21	27
7	Number of learners who used different strategies when needed	0	2	8	16	25
8	Number of learners who correctly implemented a solution strategy and solved sub-problems	0	5	11	18	23
9	Number of learners who gave an answer in terms of the data in the problem	4	10	17	25	28
10	Number of learners who checked and evaluated the reasonableness of the solution	0	6	11	19	24
11	Number of learners who could describe or analyse a solution	1	3	13	21	26

Table 4. 1 The problem solving observation checklist frequency table for the experimental group

	(*n=28*) Date	27/07/12	17/08/12	11/09/12	21/09/12
1	Number of learners who understood the given problem	4	10	24	29
2	Number of learners who verbalised the problem	2	8	21	28
3	Number of learners who understood the conditions and variables in the problem	0	6	15	26
4	Number of learners who selected the data needed to solve the problem	2	7	24	28
5	Number of learners who extracted information from the problem	3	6	24	28
6	Number of learners who formulated sub-problems	0	5	17	27
7	Number of learners who selected appropriate solution strategies	1	8	18	26
8	Number of learners who accurately implemented solution strategies	0	7	16	25
9	Number of learners who tried a different solution strategy when stuck without teacher's help	0	6	13	24
10	Number of learners who approached problems in a systematic manner	2	7	15	25
11	Number of learners who used various modelling techniques	0	5	13	24
12	Number of learners who gave answers in terms of the data in the problem	4	13	26	28
13	Number of learners who reflected on the reasonableness of the answer	0	10	19	24
14	Number of learners who showed willingness to engage in problem solving activities	2	17	25	27
15	Number of learners who demonstrated self-confidence	3	16	24	27
16	Number of learners who persevered during the problem solving process	1	11	19	28

Table 4. 2 The problem solving observation rating scale frequency table for the experimental group

From the above tables it can be noted that quantifying the qualitative data enable one to easily analyse the learners' development of mathematical problem solving skills in a PCTL environment. The above results indicate that the total number of learners developing mathematical problem solving skills increased gradually as the intervention progressed and that most learners had developed the entire spectrum of mathematical problem solving skills by the end of the intervention.

	Date	20/07/12	27/07/12	09/08/12	23/08/12	19/09/12
1	Likes to solve problems	X	✓	✓	✓	✓
2	Perseveres/sticks with the problem	X	X	✓	✓	✓
3	Tries to verbalise the problem	✓	✓	✓	✓	✓
4	Can understand the conditions and variables in a problem	X	X	X	✓	✓
5	Can identify relevant data needed to solve a problem	X	✓	✓	✓	✓
6	Thinks about which strategy may be useful	X	X	X	X	✓
7	Tries different strategies if needed	X	X	X	X	✓
8	Can correctly implement a solution strategy and solve sub-problems	X	X	X	✓	✓
9	Can give an answer in terms of the data in the problem	✓	✓	✓	✓	✓
10	Checks and evaluates the reasonableness of the solution	X	✓	✓	✓	✓
11	Can describe or analyse a solution	X	X	X	✓	✓
	TOTAL	**2**	**5**	**6**	**8**	**11**

Table 4.3 The problem solving observation checklist frequency table for learner 1

	Date	24/07/12	07/08/12	30/08/12	18/09/12
1	Understands the given problem	X	X	✓	✓
2	Verbalises the problem	✓	✓	✓	✓
3	Understands the conditions and variables in the problem	X	✓	✓	✓
4	Selects the data needed to solve the problem	X	✓	✓	✓
5	Extracts information from the problem	X	✓	✓	✓
6	Formulates sub-problems	X	X	✓	✓
7	Selects appropriate solution strategies	X	X	X	✓
8	Accurately implements solution strategies	X	X	X	✓
9	Tries a different solution strategy when stuck without teacher's help	X	X	X	✓
10	Approaches problems in a systematic manner	X	X	✓	✓
11	Uses various modelling techniques	X	X	X	X
12	Gives an answer in terms of the data in the problem	✓	✓	✓	✓
13	Reflects on the reasonableness of the answer	X	X	✓	✓
14	Shows willingness to engage in problem solving activities	X	X	✓	✓
15	Demonstrates self-confidence	✓	✓	✓	✓
16	Perseveres during the problem solving process	X	✓	✓	✓
	Total	3	7	12	15

Table 4. 4 The problem solving observation rating scale frequency table for learner 1

From the above tables it can be noted that at the beginning of the intervention, the learner tried to verbalise the given problems and could give answers in terms of the data in the problems. However, the learner was uninterested in solving problems, could not identify data needed to solve the problem and could neither think of a useful strategy nor check the reasonableness of the solution. As the intervention progressed, the learner gradually developed mathematical problem solving skills and had improved tremendously by the end of the intervention. After analysing the observation checklist and observation rating scale tables for all learners in the experimental group, the researcher confidently concluded that these learners had indeed developed mathematical problem solving skills.

4.3 Quantitative data analysis

The aim of the quantitative strand of this mixed methods design was to test the effect of the development of mathematical problem solving skills on grade 8 learners' performance and achievement in mathematics. The quantitative data were in the form of questionnaires, the mathematical problem solving skills inventory (MPSSI), mathematical tasks, written work, pre- and post-word-problem and multiple-choice tests. Use of descriptive statistics was relevant for this strand of mixed methods design. Descriptive statistics are also referred to as summary statistics that transform a set of observations into indices that describe the data (McMillian & Schumacher 2006:150). Data from the questionnaires, the MPSSI, mathematical tasks, written work, pre- and post- word-problem and multiple-choice tests were tabulated into Excel and imported into SPSS for statistical analysis. Descriptive statistics including means, frequencies, ranges, standard deviations and overall test scores were then analysed.

4.3.1 The questionnaire

The questionnaire (see appendix J) was administered to both the experimental group and control group at the beginning and at the end of the intervention. Section A of the questionnaire was about gender and age and the results are shown in table 4.5 below.

	Male		Female	
Age	Experimental	Control	Experimental	Control
12	0	1	1	0
13	10	11	12	10
14	2	1	2	3
15	1	2	0	1
Total	13	15	15	14

Table 4. 5 Frequency distribution of age and gender for participants

Table 4.5 shows that there was no major difference in age or gender between the experimental and the control groups. Therefore the influence of gender and age on the results was minimised.

	Group								F	
	Control				Experimental				(1,57)	
Section	Pre-test		Post-test		Pre-test		Post-test			
	M	SD	M	SD	M	SD	M	SD	Pre-test	Post-test
Section B	2.81	1.15	2.83	1.21	2.70	1.13	4.13	1.23	0.39	12.48
Section C	2.44	1.20	2.67	1.18	2.52	1.21	3.00	1.30	0.18	7.11
Section D	2.80	1.17	2.85	1.24	2.76	1.26	3.86	1.28	0.11	10.59
Section E	2.63	1.19	2.98	1.15	2.63	1.21	2.98	1.21	0.00	5.74

n = 28 for experimental group and n = 29 for control group $p < 0.05$

Table 4. 6 F-test for the questionnaire for the control and experimental group

In sections B, C, D and E, learners responded to the questionnaire on a five-point Likert type scale, in which 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree. Section B looked at learners' attitude towards mathematics, section C dealt with learners' willingness to engage in problem solving activities, section D looked at learners' perseverance during the problem solving process and section E looked at learners' self-confidence with respect to problem solving. Learners' responses to each of the four sections were compared by running a one-way ANOVA. The researcher employed ANOVA (analysis of variance) because it allowed her to test the differences between the two groups and not means and to make more accurate probability statements than when using a series of separate t-tests (McMillan & Schumacher 2006:301). The following null hypothesis was tested:

H_0: There is no significant difference between learners in the control group and the experimental group at the beginning of the intervention.

If the calculated F-value is greater than F-critical value at 57 degrees of freedom, we reject the null hypothesis that is the variables are jointly statistically significant. Otherwise, we fail to reject the null hypothesis meaning that the variables are not jointly statistically significant.

F-values of the four sections of the questionnaire for both the experimental and control groups are shown in the above table. The F-values, F (1, 57) = 0.39 for learners' attitude towards mathematics, F (1, 57) = 0.18 for learners' willingness to engage in problem solving activities, F (1, 57) = 0.11 for learners' perseverance during the problem solving process and F (1, 57) = 0.00 for learners' self-confidence with respect to problem solving are all less than the F-critical value = 4.01 at the level of significance $p < 0.05$. Therefore the null hypothesis was accepted; and this indicates that there was no significant difference between learners in the control group and the experimental group at the beginning of the intervention.

A one-way ANOVA was also run for the questionnaire at the end of the intervention and the following null hypothesis was tested:

H_0: There is no significant difference between learners in the control group and the experimental group at the end of the intervention.

The F-values, F (1, 57) = 12.48 for learners' attitude towards mathematics, F (1, 57) = 7.11 for learners' willingness to engage in problem solving activities, F (1, 57) = 10.59 for learners' perseverance during the problem solving process and F (1, 57) = 5.74 for learners' self-confidence with respect to problem solving are all larger than the F-critical value = 4.01 at the level of significance $p < 0.05$. Therefore the null hypothesis was rejected; and this indicates that there was a significant difference between learners in the control group and experimental group at the end of the intervention. It is apparent that learners in the experimental group perceived themselves as having overcome the "obstacles" (see section 3.10.1) and developed a positive attitude towards mathematics, and that they were now willing to engage in problem solving activities, could persevere during the problem solving process and had developed self-confidence with respect to mathematical problem solving.

4.3.2 The mathematical problem solving skills inventory

The MPSSI (see appendix K) was administered to both the experimental group and control group at the beginning and end of the intervention. Table

4.7 compares the mathematical problem solving skills of learners in the control and experimental groups at the beginning of the intervention.

	Pre-intervention for control group		Pre-intervention for experimental group		
Problem solving skills	Mean	Std. deviation	Mean	Std. deviation	*t*
Understanding or formulating the question in the problem	2.21	1.19	3.13	0.99	-2.12
Understanding the conditions and variables in the problem	3.29	0.91	2.60	1.06	2.22*
Selecting or finding the data needed to solve the problem	2.86	1.41	2.87	0.92	0.31
Formulating sub-problems and selecting appropriate solution strategies to pursue	2.7	1.33	2.67	1.11	0.29
Correctly implementing the solution strategy or strategies and solve sub-problems	3.29	1.33	2.73	1.10	1.10
Giving an answer in terms of the data in the problem	2.93	0.92	2.40	0.99	1.75
Evaluating the reasonableness of an answer	3.00	1.24	2.87	0.83	0.31

n = 28 for experimental group and n = 29 for control group $P < 0.05$ $*P < 0.02$

Table 4. 7 T-test of the mathematical problem solving skills inventory for learners in the control group and the experimental group before the intervention

The mean scores show that there is no significant difference in mathematical problem solving skills between learners in the control and experimental groups at the beginning of the intervention. A paired t-test was run to test the null hypothesis:

> *H_0: There is no significant difference between the mathematical problem solving skills mean scores of learners in the control group and the experimental group at the beginning of the intervention.*

The null hypothesis is rejected if the calculated t-value > critical t-value, and it is accepted if the t calculated < t critical at 56 degrees of freedom. The calculated t-values are low and less than the critical t-value =2.0032 for a 2-tailed test at the level of significance $p < 0.05$ and the t-critical value = 2.3948 at the level of significance $p < 0.02$. Hence the null hypothesis was accepted. The researcher concluded that there was no significant difference in mathematical problem solving skills of learners in the control group and experimental group at the beginning of the intervention.

Table 4.8 summarises the control group learners' mathematical problem solving skills at the beginning and the end of the intervention.

Control group	Pre-intervention		Post-intervention		
Problem solving skills *n = 29*	Mean	Std. deviation	Mean	Std. deviation	*t*
Understanding or formulating the question in the problem	2.21	1.19	2.86	1.35	-2.86
Understanding the conditions and variables in the problem	3.29	0.91	3.29	0.61	0.00
Selecting or finding the data needed to solve the problem	2.86	1.41	3.14	1.03	-0.72
Formulating sub-problems and selecting appropriate solution strategies to pursue	2.7	1.33	2.93	1.33	-0.72
Correctly implementing the solution strategy or strategies and solving sub-problems	3.29	1.33	3.14	1.23	0.69
Giving an answer in terms of the data in the problem	2.93	0.92	3.00	0.88	-0.29
Evaluating the reasonableness of an answer	3.00	1.24	3.07	1.07	-0.25

$p < 0.05$

Table 4. 8 T-test for the mathematical problem solving skills inventory for the control group before and after the intervention

By merely looking at the above mean scores one may think there are differences between the means of the mathematical problem solving skills of the control group learners at the beginning and at the end of the intervention. Therefore a paired t-test was run to test the null hypothesis:

> *H_0: There is no significant difference between the mean scores of the control group learners' mathematical problem solving skills at the beginning and the end of the intervention.*

The calculated t-values are low and less than the t-critical value = 2.0032 for a 2-tailed test at the level of significance $p < 0.05$. Hence the null hypothesis was accepted and the researcher concluded that the learners in the control group did not perceive any change in their mathematical problem solving skills.

Table 4.9 summarises the experimental group learners' mathematical problem solving skills at the beginning and at the end of the invention.

Experimental group	Pre-intervention		Post-intervention		
Problem solving skill *n = 28*	Mean	Std. deviation	Mean	Std. deviation	*t*
Understanding or formulating the question in the problem	3.13	0.99	7.20	1.15	11.48
Understanding the conditions and variables in the problem	2.60	1.06	5.00	1.31	11.59
Selecting or finding the data needed to solve the problem	2.87	0.92	6.40	1.06	19.04
Formulating sub-problems and selecting appropriate solution strategies to pursue	2.67	1.11	5.87	1.41	11.57
Correctly implementing the solution strategy or strategies and solving sub-problems	2.73	1.10	6.27	1.16	10.74
Giving an answer in terms of the data in the problem	2.40	0.99	8.93	1.33	13.81
Evaluating the reasonableness of an answer	2.87	0.83	7.13	1.36	11.93

$p < 0.05$

Table 4. 9 T-test for the mathematical problem solving skills inventory for the experimental group before and after the intervention

Results indicate that learners in the experimental group perceived a significant improvement in their mathematical problem solving skills at the end of the invention. This is reflected by the higher means, t-scores and significance levels. The results reflect that learners in the experimental group regarded themselves as having developed mathematical problems-solving skills.

A null hypothesis H_0 was also tested:

> *H_0: There is no significant difference between the mean scores of the experimental group learners' mathematical problem solving skills at the beginning and the end of the intervention.*

The calculated t-values for the mathematical problem solving skills were all high and more than the t-critical value = 2.0032 for a 2-tailed test at the level of significance $p < 0.05$. Hence the null hypothesis was rejected and the researcher concluded that learners in the experimental group perceived an increase in their mathematical problem solving skills.

4.3.3 Mathematical tasks and written work

Mathematical tasks and written work were administered to the experimental group only. During the intervention, learners completed three tasks. Task 1 was done on 14 August 2012, task 2 was done on 7 September 2012 and task 3 completed on 20 September 2012. All tasks were marked using the analytic scoring scale (see Appendix Q).

Task	N	Minimum	Maximum	Mean	Std. deviation
Task 1	28	44.00	69.00	57.8000	7.22298
Task 2	28	51.00	75.00	63.8000	8.09056
Task 3	28	60.00	88.00	72.6000	9.41731

Table 4. 10 Descriptive statistics for mathematical tasks

Table 4.10 shows that the mean scores for each task improved as the intervention progressed, with task 3 having the highest mean and the maximum score. The researcher concluded that learners' performance improved as the intervention progressed. From the results of the participant observation and questioning, learner journals, semi-structured interviews and the MPSSI (see sections 4.2 and 4.3.2) it was concluded that as the intervention progressed, learners developed mathematical problem solving skills. It was therefore reasonable for the researcher to conclude that learners' performance in the tasks and written work improved as the intervention progressed, because they were developing mathematical problem solving skills.

4.3.4 Word-problem and multiple-choice tests

Word-problem and multiple-choice tests (see appendices Q, R, S & T) were administered to both the experimental and control group at the beginning and the end of the intervention.

Pre-tests	N	Minimum %	Maximum %	Mean %	Std. deviation
Experimental group multiple choice pre-test	28	36.00	66.00	52.53	7.99
Experimental group word-problem pre-test	28	39.00	64.00	51.40	7.10
Control group multiple-choice pre-test	29	32.00	68.00	52.07	10.31
Control group word-problem pre-test	29	34.00	67.00	54.64	8.38

Table 4. 11 Descriptive statistics for pre- word-problem and multiple-choice tests for both the experimental and control groups

The mean scores for each test were slightly different such that one might think that the performance of the control group was better than that of the experimental group in the word-problem pre-test or that the performance of the experimental group was better than that of the control group in the multiple-choice pre-test. It is therefore imperative to use statistical techniques to determine whether or not there was a significant difference in the mean scores of these groups. A t-test for independent data was run to test the hypothesis:

> H_0: *There are no significant differences between the pre-test mean scores of the learners in the experimental and the control groups, that is, the population means are the same.*

The calculated t-value for the pre-word-problem test was 0.78 and the calculated t-value for the post-multiple-choice test was 0.41. The calculated t-values for both the word-problem and multiple-choice tests were less than the critical t-value for a 2-tailed test at the $p < 0.05$ level of significance. Hence the null hypothesis was accepted. This implies that there was no statistically significant difference in the mean scores between the control group and experimental group in the pre- word-problem and multiple-choice tests.

4.3.4.1 Descriptive statistics for the post- word problem and multiple-choice tests of the experimental and control groups

The post- word-problem and multiple-choice tests were different in the wording of the items but similar in the level of difficulty and all respects to the pre- word-problem and multiple-choice tests and were administered to both groups after the intervention, which was 10 weeks, after learners had written the pre-tests. Below is a table that helped the researcher to compare the performance of the two groups in the post- word-problem and multiple-choice tests.

Post-tests	N	Minimum %	Maximum %	Mean %	Std. deviation
Experimental group multiple-choice post-test	28	50.00	88.00	71.40	9.96
Experimental group word-problem post-test	28	60.00	86.00	74.67	9.31
Control group multiple-choice post-test	29	46.00	67.00	63.29	8.99
Control group word-problem post-test	29	45.00	70.00	65.07	7.81

Table 4. 12 Descriptive statistics for post- word-problem and multiple-choice tests for both the experimental and control groups

The mean scores of the experimental group were higher than the mean scores of the control groups in both the post-word-problem and multiple-choice tests. The minimum and maximum scores of the experimental group were also higher than those of the control group. A t-test for independent data was run to compare the means of the experimental and control groups for both the post-word-problem and multiple-choice tests. The following null hypothesis was tested:

H_0: There is no significant difference between the post-test mean scores of the learners in the experimental and the control groups, that is, the population means are the same.

The calculated t-value for the post-word-problem test was 5.68 and the calculated t-value for the post-multiple-choice test was 4.96. These two values are both larger than the critical t-value for a 2-tailed test at the $p < 0.05$ level of significance. Hence the null hypothesis was rejected and the researcher concluded that the experimental group learners performed better than control group learners in the post- word-problem and multiple-choice tests. It was therefore reasonable to conclude that the development of mathematical problem solving had a positive effect on grade 8 learners' performance and achievement in mathematics.

4.3.4.2 Descriptive statistics for the pre- and post- word-problem and multiple-choice tests for the experimental group

Learners in the experimental group's performance in the pre- and post- word-problem and multiple-choice tests were analysed in table 4.13.

Pre- and post-tests	N	Minimum %	Maximum %	Mean %	Std. deviation
Experimental group multiple-choice pre-test	28	36.00	66.00	52.53	7.99
Experimental group word-problem pre-test	28	39.00	64.00	51.40	7.10
Experimental group multiple-choice post-test	28	50.00	88.00	71.40	9.96
Experimental group word-problem post-test	28	60.00	86.00	74.67	9.31

Table 4. 13 Descriptive statistics for the pre- and post- word-problem and multiple-choice tests for the experimental group

Table 4.13 indicates a tremendous performance by experimental group learners in their post-tests. The mean scores of post-tests are higher than those of the pre-tests. The minimum and maximum values for the post-tests are higher than the minimum and maximum values for the pre-tests. A t-test was run to test whether or not the mean scores obtained in the pre- and post- word-problem and multiple-choice tests were significantly different. The following null hypothesis was tested:

H_0: There is no significant difference between the mean scores of the experimental group in the pre- and post- word-problem and multiple-choice tests.

The calculated t-value for the word-problem test was 5.53 and for the multiple-choice test was 5.10. The calculated t-values are larger than the critical t-values for a 2-tailed test at the $p < 0.05$ level of significance and the null hypothesis was thus rejected. The researcher concluded that there was a significant difference between the mean scores of the experimental group in the pre- and post- word-problem and multiple-choice tests. The experimental group's learners performed and achieved better in their post-test compared to their pre-tests. The researcher confidently concluded that the development of mathematical problem solving skills had a positive effect on the experimental group learners' performance and achievement in mathematics.

4.4 Merging of the two data sets

Creswell and Plano Clark (2011:223-232) give three options for merging quantitative results and qualitative findings: side-by-side comparisons in a discussion or summary table; joint display comparisons in the results; or interpretation or data transformation in the results. Side-by-side comparison for merged data analysis was employed for this study and the researcher presented quantitative results and qualitative findings in a discussion. The discussion was used as a vehicle for merging and conveying the merged results.

The results from learner journal entries, semi-structured interviews and learner observation and questioning evidenced that learners were developing mathematical problem solving skills as the intervention progressed and had developed them by the end of the intervention. Learners could formulate the question in a given problem, understand the conditions and variables in the problem, select or find the data needed to solve the problem, formulate sub-problems and select appropriate solution strategies to pursue, correctly implement the solution strategy or strategies and solve sub-problems, give an answer in terms of the data in the problem and evaluate the reasonableness of their solutions. The researcher observed that learners frequently checked and monitored their understanding during the problem solving sessions. The transcribed data from the semi-structured interview audio-recordings revealed that learners had become conscious of how and why they were solving a given problem.

The data from the MPSSI indicated that learners in the experimental group perceived themselves as having developed mathematical problem solving skills by the end of the intervention. This confirmed what the researcher concluded from the journal entries, semi-structured interviews and learner observation and questioning, that is, that grade 8 learners in the experimental group had developed mathematical problem solving skills at the end of the intervention. However, the MPSSI indicates that learners in the control group did not perceive any change in their mathematical problem solving skills at the end of the intervention. Learners in the experimental group achieved better results in their mathematical tasks and written work as the intervention progressed and demonstrated tremendous improvement in their post- word-problem and multiple-choice tests. Learners in the control group did not show any significant improvement in their post-tests. It seemed reasonable for the researcher to conclude that learners in the experimental group had developed mathematical problem solving skills and this had a positive impact on their performance and achievement in mathematics.

The F-test results for the questionnaire in table 4.4 indicate that there were significant changes in the experimental group compared to the control group. Learners in the experimental group improved their attitude towards mathematics, became more willing to engage in problem solving activities, persevered during the problem solving process and had developed self-confidence with respect to problem solving. It was reasonable for the researcher to conclude that learners in the experimental group had indeed overcome “obstacles” before they could benefit from the PCTLA and be in a position to develop mathematical problem solving skills.

From the above discussion the researcher concluded that the quantitative results and qualitative findings converged. The learners had indeed developed mathematical problem solving skills by being exposed to the PCTLA and this had a major positive impact on their performance and achievements in mathematics.

4.5 Conclusion

This chapter presented the data analysis and the discussion of the results. The aim of the empirical study was to explore the development of mathematical problem solving skills of grade 8 learners in a PCTL environment and to investigate its effect on these learners' performance and achievement in mathematics. The empirical study was in the form of a mixed methods research design. The purpose of the qualitative research was to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The quantitative strand's purpose was to test the effect of the development of mathematical problem solving skills on grade 8 learners' performance and achievement in mathematics. From the results and findings of the study the researcher confidently concluded that the grade 8 learners had indeed developed mathematical problem solving skills at the end of the intervention and this had a positive impact on their performance and achievement in mathematics.

The next chapter, which is the final chapter of this study, summarises the research, reviews the research questions and discusses the researcher's recommendations, the limitations of the study and possibilities for further research.

CHAPTER 5

SUMMARY OF THE RESEARCH, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The purpose of this study was to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment at a secondary school in Gauteng. Furthermore, the study aimed at investigating the effect of mathematical problem solving skills on grade 8 learners' performance and achievement in mathematics. This chapter provides an overview to demonstrate that the research question, research sub-questions and aims originally stated in the first chapter were addressed and achieved. This is the final chapter of this study and the summary of the research, a review of the research questions, the limitations of the study, areas for possible further research, recommendations and conclusions are presented.

5.2 Review of the research questions

At the beginning of the study, the following research question was formulated:

> *Does a problem-centred teaching and learning environment have an effect on the development of mathematical problem solving skills of grade 8 learners?*

In order to find an answer to this research question, research sub-questions (see section 1.7) and specific aims (see section 1.9) were formulated to provide guidelines for the study.

Research sub-question 1: *What does the problem-centred teaching and learning approach entail?*

This research sub-question was addressed by the literature review (see sections 2.5.2, 2.6 and 2.7). With the problem-centred teaching approach teaching is no longer the transmitting of knowledge but helping learners to actively construct knowledge by assigning them tasks that enhance this process (Tynjala 1999:365). In the PCTLA, learners are constructors of their own knowledge and are "encouraged and expected to think both critically and actively with multi-directional interaction with the problem, the peers, the resources and the teacher" (Savery & Duffy 1995). Learners immerse themselves in a task to actively engage in and monitor their own understanding. In the PCTLA, learners cannot simply solve problems by applying a particular formula but learn important concepts and skills through reasoning and inquiry (Posamentier & Jaye 2006:143).

Research sub-question 2: *What are the mathematical problem solving skills that grade 8 learners need to develop?*

This research sub-question was also addressed by the literature review (see section 2.11). The skills required for mathematical problem-solving are that; learners must be able to formulate the question in a given problem, understand the conditions and variables in the problem, select or find the data needed to solve the problem, formulate sub problems and select appropriate solution strategies to pursue, correctly implement the solution strategy or strategies and solve sub-problems, give an answer in terms of the data in the problem and evaluate the reasonableness of the answer.

Research sub-question 3: *What do grade 8 learners need as a prerequisite for mathematical problem solving skills to develop?*

This research sub-question was addressed by the literature review, participant observation and questioning and semi-structured interviews. The literature review (see section 2.12) revealed that as a prerequisite for mathematical problem solving skills to develop, learners need to be effective thinkers, to develop useful mathematics habits of the mind, to think the way mathematicians do, to always compare problems and strategies, to value the problem solving process, to be interested in problem solving and to be committed during the problem solving process. Furthermore learners need to be able to pose their own problems and to “think out loud” when solving problems. Participant observation and questioning and semi-structured interviews affirmed that grade 8 learners’ development of mathematical problem solving skills was enhanced by developing their focus, information-gathering, organising, evaluating, analysing and integrating skills (see section 2.12.4).

Research sub-question 4: *What are the obstacles that grade 8 learners have to overcome before they can really benefit from the problem-centred teaching and learning approach?*

This research sub-question was addressed by the questionnaire (see section 3.11.1 and appendix J). The questionnaire identified the obstacles (refer to 3.11.1) that grade 8 learners had to overcome before they could benefit from the PCTLA and be in a position to develop mathematical problem solving skills. The questionnaire was administered to both the experimental and control group before and after the intervention. From the data gathered from the questionnaire, learners in the experimental group perceived themselves as having overcome the “obstacles” and the learners in the control group did not perceive any change (see section 4.3.1).

Research sub-question 5: *How do grade 8 learners in a problem-centred teaching and learning environment develop mathematical problem solving skills?*

This research sub-question was addressed by the data gathered from participant observation and questioning, semi-structured interviews and journals. Data gathered from these three instruments revealed that the development of mathematical problem solving skills was gradual, and learners in the experimental group had indeed developed mathematical problem solving skills by the end of the intervention (see section 4.2). This is supported by the data gathered from the mathematical problem solving skills inventory (MPSSI) which disclosed that learners in the experimental group perceived themselves as having developed mathematical problem solving skills (see section 4.3.2).

Research sub-question 6 was: *Do mathematical problem solving skills influence the interpretation of "new" knowledge and solving of non-routine problems?*

Research sub-question 7: *Do grade 8 learners who receive problem-centred instruction develop mathematical problem solving skills and perform better in similar tasks given to their peers who receive traditional instruction?*

These two research sub-questions were addressed by employing written work, mathematical tasks, pre- and post- word-problem and multiple-choice tests. Learners in the experimental group performed better in their post-tests compared to their pre-tests and also outperformed learners in the control group (see section 4.3.3 and 4.3.4). This is consistent with what researchers (Carpenter, Fennema, Peterson, Chiang & Leof 1989; Hiebert & Wearne 1993; Brenner, Mayer, Moseley, Brar, Duran, Reed & Webb 1997) found out in their work in which they established that learners receiving problem-centred instruction acquire more thorough knowledge and develop problem solving skills which enhance understanding and the ability to solve mathematical problems (refer to chapter 2).

5.3 Summary of the findings

The findings of this study provide readers, educators and policy makers with insights into the problem-centred approach, problem solving processes, mathematical problem solving skills, strategies for developing mathematical problem solving skills, the development of mathematical problem solving skills and the effect of the development of mathematical problem solving skills on grade 8 learners' performance and achievement in mathematics. This section provides a summary of the literature review, the research methodology and design and the findings of the empirical investigation.

5.3.1 Summary of the literature review

The first aim of the study was to explore what it implies to teach through the problem-centred approach; the second aim was to explain the mathematical problem solving skills that grade 8 learners need to develop; and the third aim was to establish what grade 8 learners need as a prerequisite for mathematical problem solving skills to develop. This was achieved by conducting an extensive literature review (see chapter 2). Aspects in the literature review included the definitions of important concepts, how children learn mathematics, knowledge of mathematics that learners can have, theories of and approaches to the teaching of problem solving in mathematics. Other aspects that were covered were thinking processes effective for problem solving, strategies found in the literature for solving problems, mathematical problem solving skills found in the literature, prerequisite for mathematical problem solving skills to develop and strategies for developing learners' mathematical problem solving skills. The literature review also discussed learning situations in which problem solving skills development can occur and how learning situations can be structured to facilitate the development of mathematical problem solving skills of learners.

Theories of and approaches to the teaching of problem solving in mathematics include the traditional problem solving approach (see section 2.5.1) and the problem-centred teaching and learning approach (see section 2.5.2). Components of the problem-centred approach (see section 2.6) were extensively reviewed and these include prior knowledge, metacognition, reflective thinking, social interaction and negotiation of meaning in the problem-centred approach. Significant factors in the problem-centred teaching and learning environment (see section 2.7) were discussed and these included, inter alia, the role of the teacher, the learner and appropriate problems.

5.3.2 Summary of the research methodology and design

This study adopted the mixed methods research design (see section 3.6.1) and the type of mixed methods design that was employed was the convergent research design (see section 3.6.3). The justification for employing this research design was given in sections 3.6.2 and 3.6.4. The convergent design made it possible for the researcher to collect quantitative data and qualitative data simultaneously. The qualitative strand explored the development of mathematical problem solving skills of the grade 8 learners and the quantitative strand tested the effect of the development of these skills on their performance and achievement in mathematics.

Various data collection instruments (see section 3.11) were utilised to ensure the richness and credibility of the findings. To increase the reliability of the findings, the research was conducted in the natural setting of the learners. The questionnaire, the mathematical problem solving skills inventory, pre- and post- multiple-choice and word-problem tests were pilot tested to refine and clarify their items. Chapter 3 also described the components that were applied in the data analysis as well as the methods that were employed to assure the reliability and validity of the study.

5.3.3 Summary of the findings of the empirical investigation

The findings of the empirical investigation of this study were presented in chapter 4 and only a summary of the findings were presented in this chapter. The analysis and interpretation of the data from the questionnaire indicated that at the end of the intervention there was a significant positive shift in the experimental group's attitude towards mathematics, willingness to engage in problem solving activities, perseverance and self-confidence with respect to problem solving. However, learners in the control group did not perceive themselves as having overcome these obstacles at the end of the intervention.

The findings from the journal entries, semi-structured interviews, mathematical problem solving skills inventory and participant observation and questioning indicate that learners in the experimental group had developed mathematical problem solving skills at the end of the intervention. The quantitative results revealed that the development of mathematical problem solving skills has a positive impact on learners' performance and achievement in mathematics. The experimental group performed better in their post- word-problem and multiple-choice tests compared with the pre-test. At the same time, the experimental group outperformed the control group in all the post-tests. This is consistent with reports of Murray et al (1998) about "learning through problem solving". This also concurs with Hiebert and Wearne's (1993) findings on a sample of 70 learners whom they monitored over the first three years of school. They established that instruction, which encouraged learners to develop their own strategies, appeared to facilitate higher levels of understanding and closer connections between understanding and mathematical problem solving skills.

5.4 Limitations of the study

As indicated in section 3.9, the researcher opted to use the same equal sample for both the quantitative and qualitative strand and this resulted in the first limitation of this study. The same equal sample allowed the researcher to compare and merge the two data sets in a meaningful way. Since this was a small-scale study, a small sample was utilised in order to enhance the richness of the qualitative data. However, it is well known that a small sample results in low statistical power for the quantitative data and this limited the researcher's ability to find individual participant differences (Creswell & Plano Clark 2011:184). This therefore resulted in the findings of the quantitative strand being limited in terms of their wider application. Quantitative research requires large samples for it to be confidently generalised to a large population.

The second limitation was that the researcher had to stick to the Department of basic education CAPS syllabus and time was thus an inhibitor since a lot had to be covered in a short time. The third limitation was that initially learners had difficulty adapting to an environment in which they were given the responsibility for making sense of what was being learnt. They were used to the teacher being the main source of knowledge and were thus initially reluctant to make a full switch to a problem-centred teaching and learning approach. However, as the intervention progressed, all learners in the experimental group became comfortable with this "new" way of learning mathematics.

The fourth limitation was that the English language was used as the language of the research and the researcher was unable to translate "big words" like sub-problems or variables to the various languages of the learners. This resulted in the researcher not being confident that weak learners understood some of the key concepts. The last limitation was that extensive data was collected through the mixed methods design and it took so much effort and time to gather data from both quantitative and qualitative sources. It also took a lot of resources to fund these data collection and data analysis efforts.

5.5 Areas for possible further research

As indicated in section 1.3, the researcher knows no other study of a similar nature that endeavoured to develop mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment at a secondary school in South Africa. Although the researcher is confident that this study clarified the problem under investigation, the results can be regarded as tentative and the researcher felt that further research may be conducted with a larger group. Deeper research is also needed concerning what happens in learners' minds during the process of developing mathematical problem solving skills. This research would further expose how learners move from basic mathematical skills to advanced mathematical problem solving skills. Further research that incorporates how instruction can be effectively designed to promote the development of mathematical problem solving skills of South African learners is also necessary.

5.6 Recommendations

Seven recommendations were proposed as a result of this study to address the 8th specific objective (see section 1.9):

- This study should be replicated in other disciplines. It is essential to investigate how other problem solving skills develop in different contexts. This research could be done in subjects like physics, chemistry, engineering or the physical sciences.
- The majority of South African teachers still do not know how to implement the problem-centred teaching and learning approach and how to develop mathematical problem solving skills of learners. The

researcher therefore recommends that the Ministry of Education allocate part of its budget to roll out programmes where educators are frequently trained on the problem-centred teaching and learning approach and the development of mathematical problem solving skills. These programmes could be in the form of workshops and refresher courses.

- School mathematics curriculum designers and policy makers should be made aware of the mathematical problem solving skills that learners must develop and should design a curriculum that promotes and accommodates the development of these mathematical problem solving skills. The new curriculum should be based on the new views rather than the traditional views that focus on the transmission of knowledge.
- It is recommended that educators be trained on how to move learners from having basic mathematical skills to advanced mathematical problem solving skills.
- Educators must be given adequate preparation time and more teaching and learning time is required in a problem-centred teaching and learning environment for learners to be in a position to develop mathematical problem solving skills. It is recommended that extra mathematics lessons be arranged in schools during afternoons, weekends or school holidays.
- It is recommended that every effort be made to motivate learners so that they develop a positive attitude towards mathematics. In this study it was noted that once learners develop a positive attitude and have self-confidence they can easily benefit from the PCTLA and be in a position to develop mathematical problem solving skills.
- It is recommended that a large and extensive study be conducted involving a larger sample so that the results can be generalised.

5.7 Concluding remarks

The purpose of this study was to explore the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment at a secondary school in Gauteng. The researcher was also interested in investigating the effect of the development of mathematical problem solving skills on these grade 8 learners' performance and achievement in mathematics. The results and findings of this study (sections 4.2 and 4.3) indicated that learners in the experimental group indeed developed mathematical problem solving skills and this had a positive impact on their performance and achievement in mathematics and therefore the hypothesis in section 1.8 was accepted. The researcher believes that the findings of this research could have important implications for the teaching and learning of mathematics. In this research it was noted that if learners develop mathematical problem solving skills they become empowered, and perform better in mathematics.

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7.1 Appendix A: Permission to conduct research- GDE

14 May 2012

The Director: Knowledge Management and Research
Gauteng Department of Education
P O BOX 7710
Johannesburg
2000

Dear Sir/ Madam

<u>Application for permission to conduct research in a secondary school</u>

I hereby apply for permission to conduct research in a secondary school as part of a Masters degree in Education for which I am enrolled at the University of South Africa.

I envisage conducting a study on the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The purpose of this study is to evaluate the effect of problem solving skills on grade 8 learners' performance and achievement in mathematics.

The study will employ one of the most popular mixed methods design in educational research, namely **convergent research design.** Both the control and experimental groups will be involved in the research during their usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks the respondents are expected to attend the intervention programme for a minimum of $10 \times 4.5 = 45$ hours.

The Unisa ethics policy requires that I get permission from the Department of Education to conduct this study in public schools and that the participants of this study be protected in terms of keeping their identity anonymous and their information to be kept confidential.

The Department of Education will benefit in that the research will inform policy makers to make informed decisions on the adoption of problem-centred teaching and learning approach and the application of problem solving skills by learners.

Upon completion of this study, a copy of the report will be made available to the Department of Education and other educational agencies.

Kindest Regards

Brantina Chirinda

7.2 Appendix B: Approval from GDE to conduct research

education
Department: Education
GAUTENG PROVINCE

For administrative use:
Reference no. D2013/50 A

GDE RESEARCH APPROVAL LETTER

Date:	**24 May 2012**
Validity of Research Approval:	**24 May 2012 to 30 September 2012**
Name of Researcher:	**Chirinda B.**
Address of Researcher:	**114 The Bridles**
	Douglas Crescent
	Sundowner
	2188
Telephone Number:	**011 023 5522 / 083 538 7075**
Fax Number:	**086 511 3979**
Email address:	**brantinac@yahoo.com**
Research Topic:	**The development of Mathematics problem solving skills and its effect on Grade 8 learners' performance in a problem centred teaching environment at a secondary school in Gauteng.**
Number and type of schools:	**ONE Secondary School**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

Dr David Makhado 2012/05/24

Director: Knowledge Management and Research

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

7.3 Appendix C: Permission to conduct research-Johannesburg North District

25 May 2012

The District Director
Johannesburg North District Department of Education
Private Bag X1
Braamfontein
2017

Dear Sir/ Madam

<u>Application for permission to conduct research in a secondary school</u>

I hereby apply for permission to conduct research in a secondary school as part of a Masters degree in Education for which I am enrolled at the University of South Africa. The Gauteng Department of Education approved my request to collect data for this project in a secondary school in Johannesburg North District. Please see the letter of approval attached.

I envisage conducting a study on the development of mathematics problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The purpose of this study is to evaluate the effect of problem solving skills on learners' performance and achievement in mathematics.

The study will employ one of the most popular mixed methods design in educational research, namely **convergent design.** Both the control and experimental groups will be involved in the research during their usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks the respondents are expected to attend the intervention programme for a minimum of $10 \times 4.5 = 45$ hours.

The Unisa ethics policy requires that I get permission from the Department of Education to conduct this study in public schools and that the participants of this study be protected in terms of keeping their identity anonymous and their information to be kept confidential.

The Department of Education will benefit in that the research will inform policy makers to make informed decisions on the adoption of problem-centred teaching and learning approach and the application of problem solving skills by learners.

Upon completion of this study, a copy of the report will be made available to the Department of Education and other educational agencies.

Kindest Regards

Brantina Chirinda

7.4 Appendix D: Approval to conduct research from the Johannesburg North District

education
Department: Education
GAUTENG PROVINCE

Enq: Ms Kholofelo Makgare
Tel: 011 694 9557
Ref No: PP&DISM /20

JOHANNESBURG NORTH DISTRICT MEMO

TO : The Principal

FROM : Mr Sipho Mkhulise
District Director

DATE : 28 May 2012

SUBJECT : **APPROVAL IN RESPECT OF REQUEST TO CONDUCT RESEARCH**

Dear Colleagues,

This letter serves to inform you that the District has been approached by Ms Bratina Chirinda requesting permission to conduct research in respect of the study on Problem solving skills in Mathematics.

Permission is hereby granted to Ms Bratina Chirinda to discuss possibilities of conducting research at your school.

Thank you for your cooperation in this regard.

Yours in Education

Sipho Mkhulise
District Director: JHB North

Making education a societal priority

Office of the District Director: Johannesburg North
10th Floor, FNB Building Building, 2 Reserve Street, Braamfontein, Johannesburg
Private Bag X1, Braamfontein, and 2017 Tel: (011) 694 9363; Fax: (011) 339 8869
Email:Kholofelo.Makgare@gauteng.gov.za
Website: www.education.gpg.gov.za

7.5 Appendix E: Permission to conduct research (principal letter)

28 May 2012
The Principal
XXX Secondary School
P O BOX 000
Johannesburg

Dear Sir / Madam

Application for permission to conduct research at your institution

I hereby apply for permission to conduct research at your school. I am currently enrolled in a Masters degree in Education specialising in mathematics education at the University of South Africa and I am in the process of collecting empirical data for my thesis. The Gauteng Department of Education and the Johannesburg North District Department of Education approved my request to collect data for this project at your school. Please see the letters of approval attached.

I hope that the school will afford me the opportunity to collect data from grade 8 mathematics learners. All learners who will participate in this study, will be given a consent form to be signed by their parent or guardian to allow me to conduct the research. These consent forms should be returned to the researcher at the onset of the data collection process. The school's participation in this study is voluntary and confidential. At no time will the name of your school or an individual participant be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or the individual participants.

If approval is granted, learner participants will be involved in the research programme during their usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks the respondents are expected to attend the intervention programme for a minimum of $10 \times 4.5 = 45$ hours.

Your approval to conduct this study will be greatly appreciated as I firmly believe that both your school and I will benefit from this research relationship. I will follow up this communication with a telephone call and will be happy to answer any questions or concerns that you may have. If you agree, kindly organise a signed letter of permission on your school's letterhead acknowledging your consent and permission for me to conduct this study at your institution.

Kindest Regards

Brantina Chirinda

7.6 Appendix F: Permission to conduct research (SGB letter)

28 May 2012
The Chairperson
The School Governing Board
XXX Secondary School
P O Box 000
Johannesburg

Dear Sir / Madam

<u>Application for permission to conduct research at your institution</u>

I hereby apply for permission to conduct research at your school. I am currently enrolled in a Masters degree in Education specialising in mathematics education at the University of South Africa and I am in the process of collecting empirical data for my thesis. The Gauteng Department of Education and the Johannesburg North District Department of Education approved my request to collect data for this project at your school. Please see the letters of approval attached.

I hope that the school will afford me the opportunity to collect data from grade 8 mathematics learners. All learners who will participate in this study, will be given a consent form to be signed by their parent or guardian to allow me to conduct the research. These consent forms should be returned to the researcher at the onset of the data collection process. The school's participation in this study is voluntary and confidential. At no time will the name of your school or an individual participant be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or the individual participants.

If approval is granted, learner participants will be involved in the research programme during their usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks the respondents are expected to attend the intervention programme for a minimum of $10 \times 4.5 = 45$ hours.

Your approval to conduct this study will be greatly appreciated as I firmly believe that both your school and I will benefit from this research relationship. I will follow up this communication with a telephone call and will be happy to answer any questions or concerns that you may have. If you agree, kindly organise a signed letter of permission on your school's letterhead acknowledging your consent and permission for me to conduct this study at your institution.

Kindest Regards

Brantina Chirinda

7.7 Appendix G: Research ethics clearance certificate from Unisa

UNISA college of education

Research Ethics Clearance Certificate

This is to certify that the application for ethical clearance submitted by

Ms B Chirinda (35127228)

for a M Ed study entitled

The development of mathematics problem solving skills and its effect on grade 8 learners' performance in a problem centered teaching and learning environment at a secondary school in Gauteng.

has met the ethical requirements as specified by the University of South Africa College of Education Research Ethics Committee. This certificate is valid for two years from the date of issue.

Prof CS le Roux
CEDU REC (Chairperson)
lrouxcs@unisa.ac.za

2 October 2012

Reference number: 2012 SEPT/ 3512722

7.8 Appendix H: Letter to parents or guardians of participants

16 July 2012

Dear Parent/ Guardian

My name is Brantina Chirinda. I am currently enrolled in a Masters of Education specialising mathematics education at the University of South Africa and I am in the process of collecting empirical data for my thesis. I hereby invite you to give consent for your child to participate in a study on the development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The aim of this project is to evaluate the effect of developing problem solving skills on learners' performance and achievement in mathematics.

If you would want your child to participate in this study, kindly sign this form and your child should return it to the researcher at the onset of the data collection process. Your participation in this study is voluntary and confidential. At no time will your name, the name of your child's school or your child's name be identified. While this study may be published, you are guaranteed that neither your name nor your child's name will be identified in any report of the results of the study. No costs will be incurred by either your child's school or you as the parent or guardian of the participant.

If you give consent for your child to participate, your child will be involved in this research during his/her usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks your child is expected to attend the intervention programme for a minimum of 4.5 x 10 weeks = 45 hours. Kindly note, that your child does have a choice of not taking part in this research. If your child is not taking part in this project s/he will be automatically put in the control group where lessons will continue as normal with the current mathematics teacher. Your child will neither be required to fill in the questionnaire nor write the pre- and post- tests that will be written by other learners in the control group.

If you would like more information about this research study, you can contact me on the following number: 083 538 7075. If you would like the results of the study kindly supply a postal address where I can forward the results to:

____

Parent's/guardian signature: ___________ Date: _________

Researcher's signature: ___________ Date: _________

Kindest Regards

Brantina Chirinda

7.9 Appendix I: Letter to participants

16 July 2012

Dear Participant

My name is Brantina Chirinda. I am currently enrolled in a Masters of Education specialising in mathematics education at the University of South Africa and I am in the process of collecting empirical data for my thesis. I hereby invite you to participate in a study on the development of mathematics problem solving skills of grade 8 learners in a problem-centred teaching and learning environment. The aim of this project is to evaluate the effect of developing problem solving skills on learners' performance and achievement in mathematics.

If you are willing to participate in this study, you will be given a consent form to be signed by either your parent or guardian and you should return it to the researcher at the onset of the data collection process. Your participation in this study is voluntary and confidential. At no time will the name of your school or your name be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or you as a participant.

If you are willing to participate, you will be involved in this research during your usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks you are expected to attend the intervention programme for a minimum of 4.5 x 10 weeks = 45 hours. Kindly note, that you do have a choice of not taking part in this research. If you are not taking part in this project you will be automatically put in the control group where lessons will continue as normal with your current mathematics teacher. You will neither be required to fill in the questionnaire nor write the pre- and post- tests that will be written by other learners in the control group.

If you would like more information about this research study, you can contact me on the following number: 083 538 7075. If you would like the results of the study kindly supply a postal address where I can forward the results: ______________________________

Participant's signature: _____________ Date: _________

Researcher's signature: _____________ Date: __________

Kindest Regards

Brantina Chirinda

7.10 Appendix J: The questionnaire

SECTION A: Write answers in the spaces provided

1. Name of school? ______________________________
2. What is your gender? ______________________________
3. How old are you? ______________________________

Section B, C, D, and E, are about how you feel about learning and studying mathematics and how you feel about problems solving in mathematics

Read the statements carefully and mark one of the most appropriate choices for you for each item on the answer sheet:

5. Completely Agree

4. Agree

3. Undecided

2. Disagree

1. Completely Disagree

	STATEMENT	1	2	3	4	5
	SECTION B: Attitude towards Mathematics					
1	Mathematics is the subject that I like.					
2	I look forward to my mathematics lessons.					
3	I do mathematics because I enjoy it.					
4	I am interested in the things that I learn in mathematics.					
5	If there are no mathematics classes, being a student will be more enjoyable.					
6	I like discussing mathematics with my friends.					
7	I wish there were more mathematics classes a week.					
8	Time passes so slowly during mathematics classes.					
9	I would not get bored if I study mathematics for years.					
10	I have always believed that mathematics is one of my best subjects.					
11	Among all the lessons, mathematics is most unlikable.					
12	I learn mathematics quickly.					
13	Making an effort in mathematics is worth it because it will help in the work that I want to do later.					
14	Mathematics is an important subject for me because I need it for what I want to study later on.					
15	I will learn many things in mathematics that will help me get a job.					
	SECTION C: Willingness to engage in problem solving activities.					
1	I can solve most mathematical problems if I invest the necessary effort.					
2	I will try almost any mathematics problem.					
3	It is no fun to try and solve problems.					
4	I like to try challenging problems.					
5	There are some problems I will just not try.					
6	I do not like to try problems that are hard to understand.					
7	I like to try to solve problems.					
8	One learns mathematics best by memorizing facts and procedures.					
9	I try to understand the problem solving process instead of just getting answers to the problems.					
10	I solve the problems the way the teacher shows me and do not think up of my own ways.					
11	I try to find different ways to solve problems.					
12	Mathematics is about inventing new ideas.					
13	When I am confronted with mathematics problems, I can usually find several solutions.					
14	If I am engaged with a difficult mathematics problem, I can usually think of a strategy to use.					
15	The teacher must always show me which method to use to given mathematics					
16	I am willing to try a different problem solving approach when my first attempt fails.					
17	I feel the most important thing in mathematics problem solving is getting the correct answer.					
18	When I have finished working on a problem, I look back to see whether my answer makes sense.					
19	With my level of resourcefulness, I can solve mathematics problems that I am not familiar with					
20	I try to explain my ideas to other learners					
	SECTION D: Perseverance during the problem solving process.					
1	With perseverance and determination, I can solve challenging mathematics problems.					
2	I do not stop working on a problem until I get a solution.					
3	I put down any answer just to finish a problem.					
4	When I do not get the right answer right away, I give up.					
5	I work for a long time on a problem.					
6	I keep on working on a problem until I get it right.					
7	I give up on challenging problems right away.					
	SECTION E: Self confidence with respect to problem solving.					
1	I get nervous doing mathematics problems.					
2	My ideas about how to solve problems are not as good as other students' ideas.					
3	I can only do problems everyone else can do.					
4	Problems solving makes me feel uncomfortable.					
5	I am sure I can solve most mathematics problems.					
6	I am better than many students in solving mathematics problems.					
7	I need someone to help me work on mathematics problems.					
8	I can solve most hard mathematics problems.					
9	Most mathematics problems are too hard for me to solve.					
10	I am a good problem solver.					

7.11 Appendix K: The mathematical problem solving skills inventory

Where 1=strongly disagree and 10=strongly agree

	Problem solving skill	Rating of skill
1.	Understanding or formulating the question in a problem	1 2 3 4 5 6 7 8 9 10
2.	Understanding the conditions and variables in the problem	1 2 3 4 5 6 7 8 9 10
3.	Selecting or finding the data needed to solve the problem	1 2 3 4 5 6 7 8 9 10
4.	Formulating sub-problems and selecting appropriate solution strategies to pursue	1 2 3 4 5 6 7 8 9 10
5.	Correctly implementing the solution strategy or strategies and solve sub- problems	1 2 3 4 5 6 7 8 9 10
6.	Giving an answer in terms of the data in the problem	1 2 3 4 5 6 7 8 9 10
7.	Evaluating the reasonableness of the answer	1 2 3 4 5 6 7 8 9 10

7.12 Appendix L: The semi-structured interview plan

This interview plan depends on an individual learner.

1. The researcher will firstly establish rapport to make the learner feel comfortable.
2. The researcher will ask the learner to "talk about what he/she will be doing or thinking", while solving the problem.
3. After this a problem will be handed out to the learner
4. As the learner attempts to understand the problem, question and conditions, the researcher will observe the learner and ask questions such as the following, if appropriate:
   a. What did you do first when you were given the problem? Next?
   b. Can you verbalise this problem?
   c. What question is asked in the problem? Can you visualise the problem? What are the important facts and conditions in the problem? Do you need any information not given in the problem? If learner does not understand the researcher can take different entry points.
   d. Is there anything you don't understand about the problem?
5. As the learner works on the problem, the researcher will remind him/her to talk about it, and ask questions such as the following, if appropriate:
   a. What plan are you using? Do you think this plan will lead to a solution? Have you thought about using other strategies? Which ones?
   b. Where are you having difficulties? What are your ideas about where to go from here? What is wrong with your plan?
6. As the learner finds an answer to the problem, the researcher will observe the ways, if any, in which he/she checks the answer and its reasonableness as a solution. Asking questions such as:
   a. Are you sure this is the correct answer to the problem? Why?
   b. Do you think it is important to check your answer? Why?
7. After the learner has solved the problem, the researcher will ask questions such as:
   a. Can you describe the solution to the problem & how you found it?
   b. Is this problem like any other problem you've solved? How?
   c. Do you think this problem could be solved in another way? What are your ideas?
   d. How did you feel while you were solving this problem? How do you feel now that you have found a solution?

7.13 Appendix M: Questions for the semi-structured interviews

1. A gopher has dug holes in opposite corners of a rectangular yard. One length of the yard is 8 m and the distance between the gopher's holes is 17 m. How wide is the yard?

2. Two cars leave the same car park, with one heading north and the other heading east. After several minutes, the northbound car has travelled 60 km, and the eastbound car has travelled 80km. Measured in a straight line, how far apart are the two cars?

3. A 10 meter ladder is leaning against a building. The bottom of the ladder is 5 meters from the building. How many meters high is the top of the ladder? Round to the nearest tenth

4. The main mast of a fishing boat is supported by a sturdy rope that extends from the top of the mast to the deck. If the mast is 20m and the rope attached to the deck 15m away from the base of the mast, how long is the rope?

5. A hummingbird lives in a nest that is 8 m high in a tree. The hummingbird flies 10 m to get from its nest to a flower on the ground. How far is the flower from the base of the tree?

6. A cattle rancher needs to put a new fence around his pasture. The pasture is 100m long and 150m long wide, how much will the rancher need to build?

7. Chad's dining room is 4 m wide and 7 m long. Chad wants to install wooden trim around the top of the room. The trim costs R45.00 per metre. How much will it cost Chad to buy enough trim?

8. A square sticky note has a perimeter of 16 cm. How long is each side?

9. A rectangular dining room is 5 m long and 4 m wide. What is its area?

10. The perimeter of a square piece of tissue paper is 168 cm. How long is each side of the tissue paper? 42

11. The area of a train ticket is 70 square cm. The ticket is 7 cm tall. How long is it?

12. The diameter of a circle is 8 m. What is the circle's circumference?

13. The circumference of a circle is 6.28 mm. What is the circle's diameter?

14. A new restaurant is attracting customers with searchlights. One circular searchlight has a diameter of 4 m. What is the searchlight's circumference?

15. When Chetana won a contest, he got a silver medal with a diameter of 6 cm. What is the medal's area?

16. The button on Louis's pants has a radius of 4 mm. What is the button's circumference?

17. The floor of a round hut has a radius of 2 m. What is the floor's area? 12.56 square m

18. There are 29 green dots in the first row, 37 green dots in the second row, 45 green dots in the third row, 53 green dots in the fourth row, and 61 green dots in the fifth row. If this pattern continues, how many green dots will there be in the sixth row?

19. Emelinda put 2 buttons in the first box, 6 buttons in the second box, 18 buttons in the third box, 54 buttons in the fourth box, and 162 buttons in the fifth box. If this pattern continues, how many buttons will Emelinda put in the sixth box?

20. A store bought a flute at a cost of R74 and marked it up 200%. Later on, the store marked it down 50%. What was the discount price?

21. James gets an allowance of R5 every day. On Monday he was given 12 coins in R2, R1, 50c, 20c, 10c and 5c. How many of each kind of coin did James get?

22 Hector used 8 centimetres of tape to wrap 4 presents. How much tape will Hector need in all if he has to wrap 8 presents? Assume the relationship is directly proportional

23. At Gold Reef, Peter and his 5 friends decided to take enough roller coaster ride so that each person would take a ride with every other person exactly once. How many rides were taken if only 2 learners went on each ride?

24. Phumzile has 2 brothers. She is 3 times as old as Michael, her youngest brother. The age of her other brother, Henry, is the difference between Sue's and Michael's ages. If the sum of all their ages is 36, how old is Jon?

25. Presidents from different countries met at a United Nations meeting. If each president shook hands with every other president exactly once and there were 66 such shakes, how many presidents where there in the UN meeting?

26. If shirt that costs R90 is reduced 35% and a pair of trousers that costs R120 is reduced 15%, what is the total cost of the two items?

27. There are 4 netball teams in a tournament. The teams are numbered 1 to 4. Each team plays each of the other teams twice. How many games are played altogether?

28. Peter has R100 pocket money and James has R40. They are both offered part-time jobs at different supermarkets. Peter gets R10 a day and James gets R25 a day. If they do not spend their pocket money or their daily wages, after how many days will they have the same amount of money?

29. Busi was broke when she received her weekly allowance on Monday. On Tuesday she spends R12.50 of it. On Wednesday, her brother pays her R10 that he owes her. How much is Busi's allowance if she now has R22.50?

30. Akako and her friends are visiting chocolate shops in Pretoria. They take a cab from one chocolate shop to another one that is 6km away. On a map with a scale of 1 cm = 3 km, how far apart are the two chocolate shops?

7.14 Appendix N: The problem solving observation comment card

The problem solving observation comment card

Learner ___________ DATE ______________

Comments: *(Examples of what was written by the researcher when observing and questioning learners)*

- Knows how and when to look for a pattern.
- Knows that a table will help him find a pattern.
- Keeps trying even when he has trouble finding a solution.
- Needs to be reminded to check his solutions.
- He is able to explain his solution to other learners.

7.15 Appendix O: The problem solving observation rating scale

The problem solving observation rating scale

Learner ______________________ **Date** ________

	Frequently	Sometimes	Never
1. Understands the given problem	___	___	___
2. Verbalises the problem	___	___	___
3. Understands the conditions and variables in the problem	___	___	___
4. Selects the data needed to solve the problem	___	___	___
5. Extracts information from the problem	___	___	___
6. Formulates sub-problems	___	___	___
7. Selects appropriate solution strategies	___	___	___
8. Accurately implements solution strategies	___	___	___
9. Tries a different solution strategy when stuck (without help from the teacher)	___	___	___
10. Approaches problems in a systematic manner(clarifies the question, identifies needed data, plans, solves and checks)	___	___	___
11. Uses various modelling techniques	___	___	___
12. Gives an answer in terms of the data in the problem	___	___	___
13. Reflect on the reasonableness of the answer	___	___	___
14. Shows willingness to engage in problem solving activities	___	___	___
15. Demonstrates self-confidence	___	___	___
16. Perseveres during the problem solving process	___	___	___

7.16 Appendix P: The problem solving observation checklist

<u>The problem solving observation checklist</u>

Learner___________________________ Date ________________

___ 1 .Likes to solve problems.

___ 2. Works cooperatively with others in the group.

___ 3. Contributes ideas to group problem solving.

___ 4. Perseveres-sticks with a problem.

___ 5. Tries to verbalise what a problem is about.

___6. Can understand the conditions and variables in a problem.

___ 7. Can identify relevant data needed to solve a problem.

___ 8. Thinks about which strategy might be useful.

___ 9. Is flexible - tries different strategies if needed.

___ 10. Can correctly implement a solution strategy and solve sub-problems

___ 11. Can give an answer in terms of the data in the problem

___ 12. Checks and evaluates the reasonableness of the solution to the problem.

___ 13. Can describe or analyse a solution to the problem.

7.17 Appendix Q: The analytic scoring scale

The analytic scoring scale

Understanding the problem	0	Complete misunderstanding of the problem
	1	Part of the problem misunderstood or misinterpreted
	2	Complete understanding of the problem
Planning a solution	0	No attempt or totally inappropriate plan
	1	Partially correct plan based on part of the problem being interpreted correctly
	2	Plan could have led to a correct solution if implemented properly
Getting a solution	0	No answer or wrong answer based on an inappropriate plan
	1	Copying error, computational error, partial answer for a problem with multiple answers
	2	Correct answer and correct label for the answer

7.18 Appendix R: Word-problem pre-test questions

Problem Solving Skill 1:

Understanding and formulating the question in the problem

Rephrase the question in the following problems in your own words.

1 James weighed 83 kg. How much did he weigh after he had gone to the gym, eaten more and gained 13 kg?

2 A 3.5m ladder is leaning against the side of a building and is positioned such that the base of the ladder is 2.1m from the base of the building. How far above the ground is the point where the ladder touches the building?

3 To repair a roof that is 4 metres high, Mr. Thompson leans a 5-metre ladder against the side of the building. To reach the roof, how far away from the building should he place the base of the ladder

Problem Solving Skill 2:

Understanding the conditions and variables in a problem.

List two important conditions that should be kept in mind when solving the following problems:

1 A store has only three types of iron-on digits, 2, 6, and 9 left to make numerals on shirts. If digits can be repeated, how many different 2-digits numerals can they make?

2 A farmer needs to build a goat pen. The pen will be 6 metres wide and 9 metres long. The fencing material costs R69.00 per metre. How much will it cost to buy enough fencing material to build the goat pen?

3 D'angelo's Hardware Store ordered 11 power drills in October, 22 power drills in November, 33 power drills in December, and 44 power drills in January. If this pattern continues, how many power drills will the store order in February?

Problem Solving Skill 3:

Selecting and finding data needed to solve the problem.

What data in this problem would you use to find the solution?

1 Your father works 8 hours each day. He gets 18 days of leave each year. What is his salary in a 22-day work month if he is paid R75 per hour?

2 Sam's macaroni-and-cheese recipe calls for 4 packages of cheddar cheese and 2 packages of parmesan cheese. Joyce's macaroni-and-cheese recipe calls for 5 packages of cheddar and 4 packages of parmesan. Whose recipe

has a lower ratio of packages of cheddar cheese to packages of parmesan cheese?

Problem Solving Skill 4:

Formulating sub-problems and selecting an appropriate solution strategy to pursue.

Write two problems that can be solved to help find the solution to these problems:

1 Sue bought 6 pairs of socks. Each pair cost R13.50. She gave the cashier a R100 note. How much change did she get back?

2 A store originally priced a tent at R75. In order to make room for winter inventory, however, it placed the tent on sale for 20% off. If Abigail, who earns a 5% commission on the sale price, sold the item, how much did she make?

Problem Solving Skill 5:

Correctly implementing a solution strategy and attaining sub-goals.

Write a number sentence that could be used to solve these problems:

1 A stock on JSE gained 7/8 of a point on Monday. It gained 5 times this much on Friday. How much did it gain on Thursday?

2 The length of a garden is 10 cm longer than three times the width. The perimeter of the garden is 240 cm2. Find the area of the garden. Draw a picture or table that could be used to help solve these problems:

3 Pretoria, Bloemfontein and Cape Town are at the corners of a triangle. It is 200 km from Pretoria to Bloemfontein and 300 km from Bloemfontein to Cape Town, and 150 km from Pretoria to Cape Town. How much further is a trip from Bloemfontein to Cape through Pretoria than a trip directly from Bloemfontein to Cape Town?

Problem Solving Skill 6:

Giving an answer in terms of the data given in a problem.

1 On her mobile phone plan, Gwen used 44 minutes in November, 50 minutes in December, 56 minutes in January, and 62 minutes in February. If this pattern continues, how many minutes will Gwen use in March?

2 A school administrator who was concerned about grade inflation looked over the number of straight-A learners from year to year.

According to the table, what was the rate of change between 1991 and 1992?

Straight-A learners	
Year	**Learners**
1991	33
1992	17
1993	27
1994	34
1995	13

Problem Solving Skill 7:

Evaluating the reasonableness of an answer.

A problem and its answer are given. Estimate and decide if the answer is reasonable.

1 A set of watches priced at R19.50 each costs R98. How many watches were in the set? Numerical answer: 8

7.19 Appendix S: Word-problem post-test questions

Problem Solving Skill 1:

Understanding and formulating the question in the problem

Rephrase the question in the following problems in your own words.

1 Karlene measured the floor of her storage unit, which is rectangular. It is 15 metres wide and 17 metres from one corner to opposite corner. How long is the storage unit?

2 A flying squirrel's nest is 5 metres high in a tree. From its nest, the flying squirrel glides 13 metres to reach an acorn that is on the ground. How far is the acorn from the base of the tree?

3 A flying squirrel's nest is 5 metres high in a tree. From its nest, the flying squirrel glides 13 metres to reach an acorn that is on the ground. How far is the acorn from the base of the tree?

Problem Solving Skill 2:

Understanding the conditions and variables in the problem.

List two important conditions that should be kept in mind when solving the following problems:

1 Alexis sent 3 pieces of post in November, 6 pieces of post in December, 12 pieces of post in January, and 24 pieces of post in February. If this pattern continues, how many pieces of post will Alexis send in March?

2 In a triangle, one side is three times as large as the smallest side and the third side is 40 cm more than the smallest side. The perimeter of the triangle is 184cm. Find the measurements of all three sides.

3 The length of a rectangle is increased to 2 times its original size and its width is increased to 3 times its original size. If the area of the new rectangle is equal to 1800 square meters, what is the area of the original rectangle?

Problem Solving Skill 3:

Selecting and finding data needed to solve the problem.

What data in this problem would you use to find the solution?

1 Peter weighs 55kg. He weighed 22kg more than James. Tim weighed 14 kg more than James. How much did James weigh?

2 What additional data, if any, are needed to solve this problem? A jet's flying speed without wind was 900 km/hr. Its speed was increased by a tail wind. What was the resulting speed?

Problem Solving Skill 4:

Formulating sub-problems and selecting an appropriate solution strategy to pursue.

Write two problems that can be solved to help find the solution to these problems:

1 Carter is enrolled in an SAT prep class at the Oak Grove Community Centre. The community centre is 6 km away from Carter's house. On a map of Oak Grove, this distance is represented by 3 cm. What scale does the map use?

2 Allie works as a salesperson and earns a base salary of R92 per week plus a commission of 10% of all her sales. If Allie had R90 in weekly sales, how much did she make?

Problem Solving Skill 5:

Correctly implementing a solution strategy and attaining sub-goals.

Write a number sentence that could be used to solve these problems:

1 Sipho has R40 at the beginning of the day and he makes R80 for each hour that he works. He starts work at 9am and works until 4pm. How much does Sipho have at the end of the day? Develop an equation to solve the problem first?

Draw a picture or table that could be used to help solve these problems:

2 Alice put 1 book on the first shelf, 2 books on the second shelf, 4 books on the third shelf, and 8 books on the fourth shelf. If this pattern continues, how many books will Alice put on the sixth shelf?

3 Tanya's Bakery made 2 blackberry pies in February, 4 blackberry pies in March, 6 blackberry pies in April, and 8 blackberry pies in May. If this pattern continues, how many blackberry pies will the bakery make in June?

Problem Solving Skill 6:

Giving an answer in terms of the data given in a problem.

1 Here is a problem and the numerical parts of its solution. Write the answer using a complete sentence: A farmer has ducks and goats in his barn lot. How many ducks and how many goats did the farmer have if he counted 20 heads and 56 feet?

2 Fans of the Fort Sid bottom baseball team compared the number of games won by their team each year.

Games won by the Houghton baseball team	
Year	**Games won**
1982	4
1983	14
1984	21
1985	3
1986	19

According to the table, what was the rate of change between 1984 and 1985?

Problem Solving Skill 7:

Evaluating the reasonableness of an answer.

A problem and its answer are given. Estimate and decide if the answer is reasonable.

1. Joyce made 3 free throws out of every 5 shots during a basketball season. How many free throws would you expect her to make in 30 shots? Numerical answer: 23

7.20 Appendix T: Multiple choice pre-test questions

<u>Problem solving skill 1</u>:

Understanding the question in the problem.

Which statement, A, B, C, or D, is another way of asking what you are trying to find out in this problem?

> Jack and Denise divided the construction paper evenly among the 24 children in the room. Altogether they gave out 144 pieces of paper. How many pieces of paper did each child receive?

A. How many pieces of construction paper did Jack and Denise give out altogether?
B. How many pieces of construction paper was each of the 24 children given?
C. How many children received the same number of pieces of construction paper?
D. How many pieces of construction paper did Jack and Denise receive altogether?

<u>Problem solving skill 2</u>.

Understanding the conditions & variables in the problem

Which statement best describes the meaning of the underlined phrase in this problem?

1 Michelle wants to buy six of her favourite CDs and needs to decide how much money to take to the music shop. The CDs are on a special sale in which the first CD costs R26.20 and <u>each successive CD purchased costs R1.50 less than the previous one</u>.

How much will the 6 CDs cost?

A. Each CD costs R26.20.
B. Each CD after the first costs R1.50.
C. Each CD costs R1.50 less than the one bought before it
D. Each CD costs R1.50 less than the one bought after it

2 Ronel and her three girlfriends collected aluminum cans for 6 months. At the end of that time they took the cans to a recycling centre and received a total of R463, which they divided among themselves. How much money did each girl receive?

A. Each person received R463.

B. Each person got the same amount of money after it was divided.

C. They want to earn a total of R463.

D. The 4 girls received R463 for all of the cans.

Problem solving skill 3:

Selecting or finding the data needed to solve the problem.

Which data do you need to solve this problem?

Mr. and Mrs. Baker and their three children bought tickets for a concert. Adult ticket cost R27 each and children's tickets cost R18 each. How much did the Bakers pay for the tickets altogether?

A. All you need are the prices for the tickets.

B. The only data you need are the number of people who bought tickets.

C. The only data you need are the total prices for the tickets and the number of adults.

D. The only data you need are the number of adults, the number of children, and the price of adult and children's tickets.

Problem solving skill 4: *Formulating sub-problems and selecting an appropriate solution strategy to pursue.*

Which is an appropriate method for solving the following problems?

Beyonce is 10 years old. She bought cricket tickets for herself and his younger brothers, Dan and Stan. How much did she pay altogether for the tickets

A. Draw a picture.

B. Use subtraction.

C. Use multiplication.

D. Use division.

<u>Problem solving skill 5:</u>

Correctly implementing the solution strategy and attaining the sub-goals.

Implement a solution strategy and find the answer to the following problems:

1. Dama is using a magnifying glass. The lens of the magnifying glass has a radius of 3cm. What is the lens's circumference?

   A. 6.3 cm

   B. 15cm

   C. 18.84cm

2. Natasha and Anya are baking pies for a bake sale. Natasha baked 3 apple pies and 10 blueberry pies. Anya baked 4 apple pies and 16 blueberry pies. Who baked a higher ratio of apple pies to blueberry pies?

   A. Natasha

   B. Anya

   C. neither; the ratios are equivalent

3. The length of a rectangle is four times its width. If the area is 100 m2 what is the length of the rectangle?

   A. 50

   B. 20

   C. 15

4. A 32m ramp connects a platform with a sidewalk that is at ground level. If the platform is 6m above ground level, what is the distance from the base of the platform to the sidewalk?

   A. 26m

   B. 31.4m

   C. 38m

5. The learners in Mrs. Vazquez's preschool class sit down in a circle for story time. The circle they form has a diameter of 2m. What is the circle's circumference?

   A. 2m

   B. 4m

   C. 6.28m

6. The city park has a circular pond. A groundskeeper measures it and calculates that it has a circumference of 25.12 m. What is the pond's diameter?

   A. 50.24m

   B. 4m

   C. 8m

7.21 Appendix U: Multiple choice post-test questions

<u>Problem solving skill 1</u>:

Understanding the question in the problem.

Which statement, A, B, C, or D, is another way of asking what you are trying to find out in this problem?

1. Suppose you pay R41.25 per kg for an unprocessed side of beef weighing 50kg. The butcher processes it and removes the waste, which is 33% of the total weight. He packages and freezes it for you at no extra charge. How much are you actually paying for a kg of processed beef?

   A. What is 33% of 50kg?

   B. How many kg of waste are in a 50kg side of beef?

   C. How much does 1kg of the edible meat cost if you know the total cost for all the meat and the proportion of waste?

   D. How much does it cost to buy 50kg of beef?

2. Ronel and Jill collect old buckets. Jill has 3 more buckets than Ronel. Together they have 21 buckets. How many buckets has Jack collected?

   A. Altogether how many buckets does jack have?

   B. How many fewer buckets has Ronel collected?

   C. How many more buckets has Jill collected than Ronel?

   D. How many more buckets does Jack need to collect to have the same number as Jill?

<u>Problem solving skill 2</u>.

Understanding the conditions & variables in the problem

Which statement best describes the meaning of the underlined phrase in this problem?

1. Carrie is allowed to watch television for 36 hours each week. <u>If she watches for 18 hours on the weekend,</u> how many hours, on the average, can she watch television each weekday?

   A. She watches 9 hours on Saturday and 9 hours on Sunday.

B. She watches a total of 18 hours on Saturday and 18 hours on Sunday.

C. She watches at, most, 18 hours on the weekend.

D. She watches a total of 18 hours on Saturday and Sunday.

2. The farmer asked Mrs Fraser how many ducks and pigs she had on her farm. She said she had 18 in all, and “if you count all their legs, you get 58,” The farmer said, “I know how many of each there are”. Can you tell how many of each there are?

   A. The animals have a total of 58 legs.

   B. There are 58 animals on the farm.

   C. There are more legs than animals

   D. The legs on all of the chickens and pigs were counted.

Problem solving skill 3:

Selecting or finding the data needed to solve the problem.

Which data do you need to solve this problem?

1. Jacob was busy watering his garden when he noticed that Mrs Wagner was also watering her garden. They stopped to talk and they learned that Jacob waters his garden every 6 days and Mrs Wagner waters her garden every 4 days. In how many days will they next be watering their gardens together again?

   A. All you need are the number of days.

   B. The only data you need are the number of people watering the garden.

   C. The only data you need are the total days each one waters the garden per week.

Problem solving skill 4:

Formulating sub-problems and selecting an appropriate solution strategy to pursue.

Which is an appropriate method for solving the following problem?

1. Cassie got a new video game. She scored 3 points on the first level, 5 points on the second level, 9 points on the third level, 15 points on the fourth level, and 23 points on the fifth level. If this pattern continues, how many points will Cassie score on the sixth level?

A. Draw a picture.

B. Draw a table.

C. Use multiplication.

D. None of the above

Which is an appropriate first step in solving the following problem?

2 Box seats cost R185 each and grandstand seats cost R130 each. Diane ordered 3 box seats and 6 grandstand seats. What was her total cost for the tickets?

A. Find the total number of seats.

B. Find the total cost for the grandstand seats and the total cost for the box seats.

C. Find the total cost for the tickets.

D. Find the total number of tickets and the total cost of the tickets.

<u>Problem solving skill 5:</u>

Correctly implementing the solution strategy and attaining the sub-goals.

Implement a solution strategy and find the answer to the following problems:

1. A cooking instructor stated that 5 pounds of roast beef is needed to serve 8 people. Based on the instructor's statement, which of the following equations can be used to find *r*, the number of pounds of roast beef needed to serve 12 people?

A. $5/8 = r/12$

B. $5/8 = 12/r$

C. $12r = 8 \times 5$

D. $12r = 8/5$

2. The town of Hatfield with a population of 21 845 people was known for the speed with which a story could spread through the town. Each person who heard a rumour would tell it to 4 other people in one hour and then tell it to no one else. One morning the town clerk heard a story. How long did it take for everyone in Hatfield to hear the story?

A. 21 384

B. 8 hours

C. 7 hours

D. 16 384 people

3. Which of the following is a required condition for the Pythagorean theorem?

   A. The shape must be a square
   B. The shape must be an isosceles triangle
   C. The shape must be a right angle triangle
   D. The shape must be a triangle with at least one 30 degrees angle

4. Select the correct definition of the Pythagorean Theorem.

   A. The sum of the length of the legs of a right angle is equal to the length of the hypotenuse
   B. The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse
   C. The sum of the angle of the hypotenuse is equal to the third angle.
   D. The sum of the angle of the hypotenuse is equal to the square of the third angle.

7.22 Appendix V: Table of random numbers

Random Number Table

13962	70992	65172	28053	02190	83634	66012	70305	66761	88344
43905	46941	72300	11641	43548	30455	07686	31840	03261	89139
00504	48658	38051	59408	16508	82979	92002	63606	41078	86326
61274	57238	47267	35303	29066	02140	60867	39847	50968	96719
43753	21159	16239	50595	62509	61207	86816	29902	23395	72640
83503	51662	21636	68192	84294	38754	84755	34053	94582	29215
36807	71420	35804	44862	23577	79551	42003	58684	09271	68396
19110	55680	18792	41487	16614	83053	00812	16749	45347	88199
82615	86984	93290	87971	60022	35415	20852	02909	99476	45568
05621	26584	36493	63013	68181	57702	49510	75304	38724	15712
06936	37293	55875	71213	83025	46063	74665	12178	10741	58362
84981	60458	16194	92403	80951	80068	47076	23310	74899	87929
66354	88441	96191	04794	14714	64749	43097	83976	83281	72038
49602	94109	36460	62353	00721	66980	82554	90270	12312	56299
78430	72391	96973	70437	97803	78683	04670	70667	58912	21883
33331	51803	15934	75807	46561	80188	78984	29317	27971	16440
62843	84445	56652	91797	45284	25842	96246	73504	21631	81223
19528	15445	77764	33446	41204	70067	33354	70680	66664	75486
16737	01887	50934	43306	75190	86997	56561	79018	34273	25196
99389	06685	45945	62000	76228	60645	87750	46329	46544	95665
36160	38196	77705	28891	12106	56281	86222	66116	39626	06080
05505	45420	44016	79662	92069	27628	50002	32540	19848	27319
85962	19758	92795	00458	71289	05884	37963	23322	73243	98185
28763	04900	54460	22083	89279	43492	00066	40857	86568	49336
42222	40446	82240	79159	44168	38213	46839	26598	29983	67645
43626	40039	51492	36488	70280	24218	14596	04744	89336	35630
97761	43444	95895	24102	07006	71923	04800	32062	41425	66862
49275	44270	52512	03951	21651	53867	73531	70073	45542	22831
15797	75134	39856	73527	78417	36208	59510	76913	22499	68467
04497	24853	43879	07613	26400	17180	18880	66083	02196	10638
95468	87411	30647	88711	01765	57688	60665	57636	36070	37285
01420	74218	71047	14401	74537	14820	45248	78007	65911	38583
74633	40171	97092	79137	30698	97915	36305	42613	87251	75608
46662	99688	59576	04887	02310	35508	69481	30300	94047	57096
10853	10393	03013	90372	89639	65800	88532	71789	59964	50681
68583	01032	67938	29733	71176	35699	10551	15091	52947	20134
75818	78982	24258	93051	02081	83890	66944	99856	87950	13952
16395	16837	00538	57133	89398	78205	72122	99655	25294	20941
53892	15105	40963	69267	85534	00533	27130	90420	72584	84576
66009	26869	91829	65078	89616	49016	14200	97469	88307	92282
45292	93427	92326	70206	15847	14302	60043	30530	57149	08642
34033	45008	41621	79437	98745	84455	66769	94729	17975	50963
13364	09937	00535	88122	47278	90758	23542	35273	67912	97670
03343	62593	93332	09921	25306	57483	98115	33460	55304	43572
46145	24476	62507	19530	41257	97919	02290	40357	38408	50031
37703	51658	17420	30593	39637	64220	45486	03698	80220	12139
12622	98083	17689	59677	56603	93316	79858	52548	67367	72416
56043	00251	70085	28067	78135	53000	18138	40564	77086	49557
43401	35924	28308	55140	07515	53854	23023	70268	80435	24269
18053	53460	32125	81357	26935	67234	78460	47833	20496	35645

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