EFFECTS OF INTEGRATING GEOGEBRA INTO THE TEACHING OF LINEAR FUNCTIONS ON GRADE 9 LEARNERS' ACHIEVEMENT IN MOPANI DISTRICT, LIMPOPO PROVINCE

By

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Submitted in accordance with the requirements for the degree of

MASTER OF SCIENCE

In the subject

MATHEMATICS EDUCATION

At the

UNIVERSITY OF SOUTH AFRICA

Supervisor

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November 2016

ABSTRACT

One major challenge facing mathematics education in South Africa in general and Limpopo in particular, is learners' underachievement and lack of motivation to learn the subject. Some studies have shown that one of the topics that learners dread is linear functions. Many teachers also find it difficult to teach the topic effectively. Studies in other parts of the world have advocated the integration of graphing software with the teaching and learning of functions to enhance learners' learning of mathematics. This study therefore investigated the effect of integrating GeoGebra graphing software into the teaching of linear functions on the achievement of Grade 9 learners.

The study was guided by APOS theory which, in accordance with constructivist theories, posits that an individual needs to construct the necessary cognitive structures in order to make sense of mathematical concepts. A total of 127 Grade 9 learners from four schools in a circuit in Mopani district of Limpopo Province participated in the study which followed a pretest-post-test quasi-experimental study design. Two schools, namely B (35 learners) and D (33 learners) formed the experimental groups while school A (31 learners) and school C (28 learners) were the control groups. Data were collected using an achievement test and analyzed using descriptive and inferential statistics. The pretest results showed that the groups were of comparable cognitive abilities.

The post-test results showed that there was a significant difference between the mean scores of the experimental groups and control groups. There were also statistically significant differences between group treatment means ($p < .05$).

Bonferroni post-hoc test results showed that there were no statistically significant differences between treatments A and C. The results showed that the learners in the two control groups were of comparable cognitive abilities. The implications of the findings are discussed and recommendations made.

Keywords: Achievement; APOS; GeoGebra; ICT; linear functions; positivism; traditional teaching; integration; motivation

ACKNOWLEDGEMENTS

The Lord is my Redeemer!

The Lord Almighty has been with me through this journey. It has not been an easy walk to completion but He is still the source of my strength. Thank You Lord.

I would like to express my deepest gratitude to some of the people who have contributed to the success and completion of this study:
Dr U. I. Ogbonnaya, I feel greatly honoured to have worked under your supervision and have benefitted greatly from your guidance and support. May the Lord continue to bless you.

I am also thankful to the management, staff and learners of all the schools that participated in the study, especially the educators at my school.

To my mother, Christina Dzete (aka Thandiwe), my gratitude extends beyond words. You have always been my pillar of strength. I will forever be grateful. May the Lord continue to bless you. And to my sister, Memory, you have been there for me all the way. Thank you and God bless you.

Pastor C. Akintade, your spiritual guidance in my life is greatly appreciated.

DECLARATION

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Exact wording of the title of the dissertation or thesis as appearing on the copies submitted for examination:

Effects of Integrating GeoGebra into the Teaching of Linear Functions on Grade 9 Learners' Achievement in Mopani District, Limpopo province

I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

M.Mushipe
SIGNATURE

7 February 2018
DATE

Table of Contents

List of Tables

List of abbreviations

ANOVA	Analysis of variance
CAPS	Curriculum and assessment policy statement
DoBE	Department of basic education
KW	Kruskall-Wallis test
KR-20	Kuder Richardson test
MW	Mann-Whitney test
NSC	National senior certificate
SPSS	Statistical package for social sciences
TT	Traditional teaching
UNISA	University of South Africa

CHAPTER 1

INTRODUCTION AND CONTEXTUALISATION

Learners' poor achievement in mathematics is an issue of global concern (Ogbonnaya, 2008).

1.1 CONTEXTUALIZATION

Over the years, governments, teachers, researchers, as well as other stakeholders in education, have made great efforts to improve learners' achievement in mathematics but South African learners are not passing mathematics well in the National Senior Certificate (NSC) final examination. Table 1.1 shows the achievement rates of mathematics nationally for the period 2012 to 2015. The results show that South African educators and learners experience great difficulties with mathematics teaching and learning as characterized by the results obtained in the decrease from 59.1% in 2013 to 49.1% in 2015 (NSC Diagnostic Report, 2015).

Table 1.1: Overall achievement rates in mathematics

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2012	225874	121970	54.0	80716	35.7
2013	241509	142666	59.1	97790	40.5
2014	225458	120523	53.5	79050	35.1
2015	263903	129481	49.1	84297	31.9

For instance, in 2015 only 49.1% of the candidates who sat for the examination achieved at least the minimum of 30%. The percentage of candidates achieving at higher levels continues to decrease with only 3% of those who took the mathematics examination in 2015 achieving 80% to 100% (distinction) (NSC Examination Report, 2015). This implies that only 7,917 candidates out of the registered 263,903 managed to pass mathematics with a distinction in the 2015 NSC examinations.

Working towards the improvement of education in general is one of the major aims of the South African government. Of particular note are the growing calls to improve the teaching and learning of science and mathematics in schools through implementation of more innovative methods of teaching, especially the integration of Information and Communication Technologies (ICT) into teaching and learning. Means and Haertel (2004) are of the view that ICTs can support learning when they are successfully integrated with teaching methods, the curriculum and assessment.

The South African government in conjunction with the Department of Basic Education (DBE) have advocated for the integration of ICT into education through the adoption of the ICT Policy in Education. The Presidential National Commission on Information Society and Development (PNC on ISAD) was set up in 2001 to advise the government on how to achieve the optimal use of ICT and address South Africa's developmental challenges as well as enhance its global competitiveness. The PNC on ISAD also advised the government on the digital divide and identified focus areas, which included education.

Mopani district in the Limpopo Province of South Africa has over a hundred schools, most of them in rural areas. Therefore, in general, most learners in Limpopo schools are from underprivileged backgrounds. Textbooks and some computers have been provided to schools and in most cases educators are employed to keep up with curriculum needs. However, Limpopo Province struggles with performance in the NSC examination. In particular, learners do not perform well in mathematics.

Recently the dynamic software GeoGebra was made available to schools in Mopani district by the mathematics subject advisors. Regardless of all the measures being in place, the mathematics pass rate in the district remains very low.

The topic of functions has been highlighted in previous reports as one where learners achieve low marks in the final Grade 12 examinations. This might be a result of commonly used traditional teaching methods which do not promote learner understanding. The basis of the topic (functions) lies in linear functions and their graphs which are taught in the senior phase (Grades 7–9). There is content progression as learners go through the FET band (Grades 10–12) and the examination assesses concepts learned in lower grades. Most schools are still using chalkboard methods, which make the process of teaching and learning of linear functions a very difficult one. This could have the implication that both educators and learners develop a dislike for the subject. Recently, various stakeholders, including the government, invested in making computers and some graphing software available to schools, but based on my knowledge, these computers are not being used in the teaching and learning process, but are lying idle or being used for administrative purposes. This issue of computers not being utilized for teaching and learning makes it imperative for the researcher to investigate the effect of integrating the dynamic software GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in the Mopani district.

1.2 BACKGROUND TO THE STUDY

The Department of Education has shown its support for ICT in education through its inclusion in education policy and the White Paper on e-Education (SA, 2004) which stated that all South African learners from Grade 1 to Grade 12 would be ICT capable by 2013. Yet this has not been the case as there are learners who are still computer illiterate. The White Paper asserts that the greatest challenge for the service provider is to roll out ICT infrastructure that is best suited to the particular target users. Guidelines for the distribution of ICT resources are also stipulated in the Guidelines for Teacher Training and Professional Development in ICT (SA, 2007). However, PanAfrican Research Agenda (2008–2011) showed that the ICT in education policy is not adequately implemented in schools.

Mopani district is in the Limpopo Province of South Africa and is struggling with poor achievement in mathematics. Like many other rural districts there is a challenge when it comes to service delivery with most of the schools having experienced a shortage of resources. However, recently the Department of Education supplied these schools with resources such as textbooks and computers and educators have been employed to remain abreast of curriculum needs. Learner improvement programmes are available in Mopani district. Despite these measures, the mathematics pass rate remains low in most schools in this district. The continued poor achievement could be due to the fact that the computers and software technology are not being integrated into teaching and learning. This prompted the researcher to investigate the effectiveness of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district.

1.3 STATEMENT OF THE PROBLEM

Limpopo Province is struggling with poor achievement in mathematics. Learners are not doing well in mathematics with one particular topic that has been identified as challenging to them, namely functions. Poor performance could result from the fact that educators experience great difficulty in teaching this topic.

The South African government has provided ICT infrastructure but the performance of learners in Mopani district is still poor because most educators are not utilizing the ICT resources. This study explores how integrating GeoGebra into the teaching of linear functions in Grade 9 could affect the learners' achievement.

1.4 PURPOSE OF THE STUDY

The purpose of this study was to investigate the effect of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district.

1.5 RESEARCH QUESTIONS

This study provides answers to the following questions:

1. Is there any difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions?

2. Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in drawing graphs of linear functions (DG)?

3. Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in interpretations of linear functions and their graphs (IG)?

1.6 RESEARCH HYPOTHESIS

The following null hypotheses were tested at $\alpha = 0.05$ level of significance in the study.

H_0: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in the following:

1. Linear functions
2. Drawing of graphs of linear functions (DG).
3. Interpretations of linear functions and their graphs (IG)

1.7 SIGNIFICANCE OF THE STUDY

There have been mixed research findings on the effectiveness of ICT in teaching and learning on students' learning gains (Leong, 2013; Ogbonnaya, 2010; Praveen & Leong, 2013). This study will contribute to the debate in shedding light on our understanding of the effect of ICT, in this case graphing software, on learners' achievement in an aspect of mathematics within the South African context.

The use of graphing software in teaching and learning of linear functions has not attracted research attention in South Africa to my knowledge. Hence, this study has considered the research gap and attempts to provide insight on the effectiveness of GeoGebra on learners' achievement in linear function in South Africa.

The South African governments at both national and provincial levels, as well as companies and non-governmental organizations within and outside the country, have invested vast resources in the purchase of teaching and learning resources like computers and computer software to support students' learning of mathematics and science. It would therefore seem reasonable at this point to know if the investment in computer equipment would translate into improved learners' achievement in mathematics.

The study provides important information about how learners can use graphing software to transform and process information in such a way that they are able to form cognitive objects and organize them into coherent schemas in line with APOS theory (Dubinsky & McDonald, 2001).

The study was conducted in a circuit in Mopani district, where no similar study has been conducted; therefore, it served to close the research gap observed by Mlitwa and Koranteng (2013) who noted as part of their findings that ICT integration in South African schools is slow. In addition, the findings of this study could possibly make educators aware of the effectiveness of GeoGebra software in the teaching/learning process, especially in relation to mathematics, and as such empower them with new knowledge about improving learners' performance and attitude towards learning mathematics.

The study adds to existing findings (e.g. Kllogjeri & Kllogjeri, 2011; Ogbonnaya & Chimuka, 2016; Praveen & Leong, 2013) on the effectiveness of teaching and learning using GeoGebra on learners' achievement and motivation in other aspects of mathematics. It could also serve as a basis for future studies on effective ways of addressing learning challenges in mathematics in Mopani district and South Africa as a whole

Underachievement of learners in mathematics in South Africa is a great concern. The Global Information Technology Report (2014) placed South Africa's score with regard to the quality of mathematics and science at 1.9 out of 10. This means the country needs serious transformation in mathematics learning and teaching, hence this study could be helpful in the process of transformation.

The study also ensures that ICT in education policy is implemented in the circuit. It seeks to provide new knowledge on cost-effective ways of improving the teaching/learning of mathematics in Mopani district in general by using GeoGebra software as a cost-effective tool that can be effectively utilized in a sustained manner.

1.8 DELIMITATIONS

The study focused on Grade 9 learners in Mopani district in Limpopo Province only. The study was limited in focus to the use of software GeoGebra to teach linear functions to Grade 9 learners.

1.9 DEFINITION OF TERMS

Terms and concepts may mean different things to different people in different contexts, therefore, this section gives a brief definitions of terms as they were used in the context of this study.

1.9.1 Control group

The control group in this study refers to the group that was taught the topic linear functions using traditional teaching methods only. All other conditions were as closely matched to the experimental group as possible. No computer software was used while the group was being taught during the intervention. The purpose of using the control group in this study was to ensure that the researcher had reliable data to use when making comparisons.

1.9.2 Experimental group

The experimental group in this study refers to the group that was taught linear functions using GeoGebra graphing software. It was compared with the control group and used to provide answers to the research questions stated previously.

1.9.3 GeoGebra

GeoGebra is dynamic mathematical software for all levels of education that joins arithmetic, geometry, algebra and calculus, and was discussed by Hohenwarter and Fuchs (2004). According to Hohenwarter and Jones (2007), GeoGebra is a software package that combines both geometry and algebra and is specifically designed for educational purposes to foster mathematical learning in learners.

1.9.4 Grade 9

Grade 9 in the South African education system is the ninth grade after grade R. The learners in this grade are usually 14-to-15-year-olds. Grade 9 is the last grade in the senior phase, which comprises Grades 7 to 9.

1.9.5 Information and communication technology (ICT)

Mdlongwa (2012) defines ICT as a global network in which ideas are exchanged or information is shared through using communication tools like cell phones and technology like computers to connect people. In the context of this research the word technology or ICT will be used in reference to the graphing software (GeoGebra) that will be used in teaching the experimental group.

1.9.6 Linear functions

A linear function is defined by Laridon, Barnes, Kitto, Myburg, Pike, Schaber, Sigabi and Wilson. (2004) as any relationship between two variables that can be represented as a straight line. Webster (2015) also defines a linear function as a mathematical function in which the variables appear in the first degree only, are multiplied by constants and are combined by addition and subtraction only. In the context of this study, linear functions refer to the section on graphs of functions that is taught in Grade 9.

1.9.7 Learners' achievement

The learner's achievement refers to the amount of academic content that the learner has managed to grasp in a given period of time in relation to the stipulated learning goals for that particular grade.

1.9.8 National Senior Certificate (NSC)

In this study context, and also in South Africa, the National Senior Certificate refers to the school leaving certificate (commonly referred to as matric).

1.9.9 Software

Software can be defined in the simple sense as instructions that run a computer. The Cambridge English dictionary defines software as the 'instructions that control what a computer does.' In this study the term software refers to GeoGebra graphing software.

1.9.10 Technology

Technology, simply put, refers to science or knowledge put to practical use to solve problems or invent useful tools. Mackenzie and Wajcman (1985, p.3) define technology as the ‘integration of physical objects or artifacts or the process of making the object and the meaning associated with the objects’.

1.9.11 Traditional teaching (TT)

The term ‘traditional teaching’ refers to the chalk-and-talk method of teaching.

1.10 OUTLINE OF CHAPTERS

Chapter one: Introduction and contextualization

This chapter provides the context of the study. It describes the background of the study, the statement of the problem, the research questions, the significance of the study, and gives a brief definition of terms as they are used in the study context.

Chapter two: Conceptual framework and literature review

This chapter discusses the conceptual framework that guides the study and gives a review of related literature.

Chapter three: Research methods

This chapter describes the methods used in the study, including the research paradigm, research design, sampling method, data collection instruments and their development, procedures for data collection, validity and reliability of instruments, pilot study, and ethical issues relating to the study.

Chapter four: Data analyses

This chapter discusses the methods and procedures used in the data analyses. The results of the data analyses will be presented and the results used to draw the findings of the study and answer the research questions.

Chapter five: Discussion of findings, conclusions and recommendations

In this chapter the findings of the study are discussed, conclusions are drawn in line with the hypotheses and recommendations are made commensurate with the findings of the study.

CHAPTER 2

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This study investigated the effect of integrating GeoGebra with the teaching of linear functions on Grade 9 learners' achievement. In this chapter the theoretical framework that guided the study and a review of related literature are discussed. The literature review is based on the main focus of the study, which is linear functions and use of ICT in teaching and learning mathematics in general, and in particular, the use of GeoGebra in teaching and learning mathematics.

2.1 THEORETICAL FRAMEWORK

This investigation was based on Dubinsky and McDonald's APOS theory (2001). This theory was developed in line with constructivist theories, advocating that an individual needs to construct the necessary cognitive structures in order to make sense of mathematical concepts. According to the theory, individuals tend to deal with mathematical situations by constructing mental actions which they transform into processes and objects, as well as the organization of schemas in their attempts to make sense of problems and to be able to solve presented situations (Dubinsky & McDonald, 2001).

The APOS theory was presented by Dubinsky and McDonald (2001) as consisting of four components summarized below:

1. **Action**: Transformation of objects perceived by an individual in reaction to stimuli. An action requires that each step be taught and performed explicitly. An example can be of a learner needing an equation to link the relationship between variables in a linear function, but not being able to perceive the relationship without the equation. This is referred to as the action stage where the learner can only perceive and react to external stimuli in the form of what is taught or learnt.

2. **Process:** Occurs when an individual repeats the action stage. As the learner continues to repeat and reflect on the action, even in the absence of external stimuli, the action becomes interiorized in the mind to become a mental structure. The mental structure is referred to as a process. The learner can now construct mental processes with

regards to the transformations and shifts that can be applied to the basic linear functions. A learner at this stage is now able to apply the information learnt previously during the process of solving problems.

3. **Object**: The action stage and the process of constructing mental structures help the learner to view action and process in totality, not individual entities leading to transformations of one's imaginations. The learner encapsulates the process into a cognitive object. For example, in linear functions the learner can now confront questions of a higher order that draw upon the mental structures formed during the action and process stages.

4. **Schema:** The result of actions, processes and objects, being organized in order to form a clear framework. When solving mathematical problems, a learner should be in a position to decide on the appropriate schema to use. This is only possible if the learner has constructed clear and coherent schemas. For instance, in linear functions learners are only able to solve higher order questions if they have been able to create their own understanding of concepts without always relying on external stimuli.

The components of APOS discussed above are not as linear as they appear to be. In fact, the whole process is dialectical, involving a lot of reflecting and navigating between the stages. This study focused on the effects of the integration of GeoGebra into the teaching of linear functions on the learners' achievement. In consonance with APOS theory, it is believed that the technology can help learners construct mental actions which they transform into processes and objects, and organization of schemas, thereby constructing an understanding of mathematical knowledge. The mathematical understanding will eventually translate into improved achievement in mathematical exercise.

The technology could stimulate learners to go through the series of actions and processes so as to objectively construct their own schemas. Learners continue to go back and forth as they construct their own knowledge based on the experience provided by the technology. This could promote the development of an inquisitive mind which seeks to explore and achieve a deeper understanding of the concepts being learnt.

Several studies that are guided by APOS theory have been carried out locally and elsewhere in the world. Demir (2012) studied learners' concept development and understanding of sine and cosine functions in a study conducted at pre-university level (VWO) at a Dutch secondary school in Amsterdam with a class of 24 learners whose ages ranged from 16 to 17. The study investigated a new theoretical and educational approach. Results showed that the new approach, which was based on the implemented learning curve, was effective in promoting understanding of trigonometric functions.

Brijall and Maharaj (2009a) cited in Jojo (2011), used APOS theory when they investigated fourth-year undergraduate teacher trainee students' understanding of the two fundamental concepts, monotonicity and boundedness of infinite real sequences at a South African University. As conclusion to their study, they found that structured worksheets promoted group work and created an environment that is conducive to abstract thinking and that the learners were able to use symbols, language and mental images to make constructions of internal processes during the process of understanding the monotonicity and boundedness of sequences.

Jojo (2011) carried out an APOS exploration of conceptual understanding of the chain rule in calculus by first-year engineering students at a University of Technology in South Africa.

2.2 LITERATURE REVIEW

This section provides a review of literature on linear functions in the South African school curriculum and a synthesis of published studies related to the use of ICT in teaching and learning in general, and GeoGebra in the mathematics teaching and learning in particular.

2.2.1 Linear functions

Chitsike (2013) defines a function in mathematics as a rule or relationship for which any input value results in one unique output value. Functions are classified according to their degree, which is the highest power of the variable, or by the type of graph that the relationship gives.

A linear function is defined by Laridon et al. (2004) as any relationship between two variables that can be drawn as a straight line.

2.2.2 Linear function in CAPS

In the South African school system, functions are introduced from the lower grades when learners start exploring the relationships between variables, but we meet the notion of a linear function in the senior phase (Grades 7–9) in the South African curriculum and assessment policy statement (CAPS). In the CAPS document, drawing and interpreting graphs falls under patterns, functions and algebra. The topic progresses from the concepts of the relationship between numbers, rules, formulae and equations for which input and output values are required, to drawing and interpreting graphs. Table 2.1 summarizes the content on drawing and interpreting graphs for the senior phase (Grades 7 to 9).

Table 2.1: Summary of concept progression for linear functions in senior phase

Topic	Grade7	Grade 8	Grade 9
Graphs	**Interpreting graphs** -Analyze and interpret global graphs of problem situations, with special focus on the following trends or features: -linear or non-linear -constant, increasing or decreasing.	**Interpreting graphs** Revise the following done in grade 7: -Analyze and interpret global graphs of problem situations, with special focus on the following trends or features: -linear or non-linear -constant, increasing or decreasing. Extend the focus to include: -maximum or minimum - discrete or continuous	**Interpreting graphs** Revise the following done in grade 8: -Analyze and interpret global graphs of problem situations, with special focus on the following trends or features: -linear or non-linear -constant, increasing or decreasing. -maximum or minimum - discrete or continuous Extend the above with special focus on the following features of linear graphs - x-intercept and y-intercept -gradient
	Drawing Graphs -Draw global graphs from given	**Drawing Graphs** -Draw global graphs	**Drawing Graphs** Revise the following done in grade 8:

	descriptions of a problem situation, identifying features listed above	from given descriptions of a problem situation, identifying features listed above -use tables or ordered pairs to plot points and draw graphs on the Cartesian plane	Draw global graphs from given descriptions of a problem situation, identifying features listed above Use tables or ordered pairs to plot points and draw graphs on the Cartesian plane Extend the above with special focus on: -drawing linear graphs from given equations -determining equations from linear graphs.

(Adapted from the Curriculum and Assessment Policy Statement)

Table 2.2 also clarifies the content to be taught in Grade 9 in particular, in linear functions (drawing and interpreting graphs).

Table 2.2: Content clarification for linear functions in grade 9

Topic	Concepts and skills	Clarification notes or teaching guidelines
Graphs	**Interpreting graphs** Revise the following done in grade 8: -Analyze and interpret global graphs of problem situations, with special focus on the following trends or features: -linear or non-linear -constant, increasing or decreasing. -maximum or minimum - discrete or continuous Extend the above with	**What is different to grade 8?** -x-intercept, y-intercept and gradient of linear graphs -drawing linear graphs from given equations -determine equations of linear graphs Learners should continue to analyze and interpret graphs of problem situations. **Investigating linear functions** Sketch linear graphs from given equations learners should first draw up a table of ordered pairs, that includes the intercept points (x; 0) and (0;y), and then plotting the points. Learners should investigate gradients by comparing the ratio vertical change: horizontal change between any two points on a straight line graph.

<table>
<tr>
<td>special focus on the following features of linear graphs

- x-intercept and y-intercept
-gradient</td>
<td>Learners should also investigate the relationship between the value of the gradient and the coefficient of x in the equation of a straight line graph.
-Learners should compare y-intercepts of linear graphs to the value of the constant in the equation of the straight line graph.</td>
</tr>
<tr>
<td>Drawing Graphs

Revise the following done in grade 8:

-Draw global graphs using given descriptions of problem situations, identifying features listed above

-use tables or ordered pairs to plot points and draw graphs on the Cartesian plane
Extend the above with special focus on:
-drawing linear graphs from given equations
-determining equations from linear graphs.</td>
<td>Examples of linear graphs
Sketch and compare the graphs of: $y = 4$ and $x = 4$
Sketch and compare the graphs of $y = x$ and $y = -x$
Sketch and compare the graphs of $y = 2x$; $y = 2x + 1$; $y = 2x - 1$
Sketch and compare the graphs of$y = 3x$; $y = 4x$; $y = 5x$
Sketch the graph of $y = -3x + 2$ using the table method
Determine the equation of the straight line passing through the following points:

<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
</table>
</td>
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</table>

(Source: Curriculum and Assessment Policy Statement (CAPS))

2.2.3 Rationale for choosing the topic linear functions

Several reasons guided the choice of linear functions as the context of the study. The topic on functions contributes a considerable percentage in the NSC final Grade 12 mathematics examination. It is also a topic that, according to previous diagnostic reports on Grade 12 examinations, proves to be quite challenging to learners. The foundation of functions is in senior phase mathematics (Grades 7–9) providing the researcher with motivation to investigate the effect of integrating GeoGebra software on the achievement of Grade 9 learners. The teaching of linear functions in Grade 9 was the focus of the study because if linear functions are not taught in a manner that promotes understanding, the learners might become frustrated or confused; hence the need to introduce other teaching methods. Learners who do not understand linear functions in Grade 9 risk progressing through the FET band (Grades 10–12) with misconceptions of the notion as well as choosing not to take mathematics in Grade 10. Furthermore, the topic of functions is also flexible in the introduction of other teaching methods, such as technology-based approaches, in particular GeoGebra graphing software, which facilitates exploration, representation and analysis of functions among other things.

The focus of evaluation in CAPS nowadays has shifted from merely asking learners to plot graphs of functions and is instead now more focused on the analysis and applications that learners are able to perform using the available information on graphs. Therefore, it was envisaged that integrating GeoGebra software with the teaching and learning of linear functions might enable learners to explore and develop schemas which enables them to not only plot graphs of functions but to answer higher order questions. This knowledge in turn also enables them to analyze, reflect and apply acquired knowledge.

The use of functions to solve authentic real-life situations has elevated the topic linear functions to that of being the basis of decision-making (Leong, 2013). Learners are sometimes called upon to translate real-life situations into graphs in order to produce feasible and fruitful solutions and that is only achieved if the learners have acquired the necessary mind concepts and schemas in linear functions to equip them and enable them to tackle such problems.

Challenges in teaching and learning linear functions

It is the general belief among most educators and learners that some of the challenges that the learners encounter in learning linear functions arise from the teaching methods as well as from the curriculum itself. Leinhardt, Zaslavsky and Stein (1990) asserted that the topic linear functions is very complex due to several factors, such as its association with other complex mathematical concepts thereby making learners' intuition on the topic very poorly coordinated. It has also been suggested that misconceptions on the concept of linear functions are likely to form as a result of poorly coordinated intuitions.

Several studies have highlighted the challenges that learners face in learning linear functions. These studies include one by Lobato and Siebert (2002) who examined learners' understanding of the concept of a slope as a measure of steepness and also slope in functional situations working with nine learners who were sampled from Grades 8 to10. Their findings revealed that learners had difficulties in making connections between the physical and functional aspects of a rate of change. This finding highlighted the importance of context in the conceptualization of linear functions.

ICT in teaching and learning

Literature is replete with studies on the use of different forms of ICT in teaching and learning. The use of ICT in education dates back several decades but of late greater emphasis is being placed on the various ways of implementing ICT in education.

Technology and ICT in Teaching and Learning Mathematics

The use of technology has its history in mathematics (Centre for Technology, 2007). Take for instance the elementary school, where educators teach arithmetic using an abacus. Learners use the abacus as a computation tool but simultaneously it helps to bring mathematics down to the learner's level. Kaput (2007) suggests that researchers have found that whereas physical manipulates are the right tangible form for the elementary school, ICT based tools are the right tangible forms for the secondary school.

The Centre for Technology in Learning (2007, p.2) reported that

> Technology can reduce the effort devoted to tedious computations and increase student focus on more important mathematics. Equally important, technology can represent mathematics in ways that help students understand concepts in combination. These features enable teachers to improve on both the how and what students learn.

In particular, introducing arithmetic using a calculator with Grade 1 learners might result in learners with very little understanding of the computations involved; have also not yet experienced the tangibility and reality of mathematics. On the contrary, a learner in secondary school has already developed the necessary computational abilities thus the calculator will only be used to reduce the burden of calculations, in other words, as a means to the end. Oldknow and Taylor (2000) cite three reasons for integrating ICT into mathematics teaching. These are desirability, inevitability and public policy. As a result of their desirable features, students are motivated and encouraged, teachers' efficiency also improves and schools improve their educational inclusivity in multilingual classrooms. All the listed attributes make ICT more desirable. These researchers go on to say that the use of technology becomes inevitable when conventional alternatives no longer exist and when its cost has been reduced to affordable amounts.

Preiner (2008) finds that technology integration into mathematics teaching and learning can be done in two ways. Virtual manipulation allows learners with little or no special computer skills to explore mathematical concepts with ease. Secondly, mathematical software tools can be used for a variety of mathematical content topics.

Technology is not a panacea for all educational problems, and just like any teaching tool, it can be used well or poorly. Teachers should use technology to enhance the learners' learning opportunities by selecting or creating mathematical tasks and activities that take advantage of what technology can do efficiently and well, such as graphing, visualizing and computing. Teachers face great responsibility when deciding to use ICT because they have to consider when and how to use it and which topics that particular ICT best supports. It is the teacher's responsibility as well to decide when technology can effectively improve learning opportunities.

Tamim, Benard, Borokhovski, Abrami and Schmid. (2011) conducted a meta-analysis of over 1,055 studies which aimed at addressing the effects of computer technology on learners' achievement in formal classrooms. The findings from the analysis showed that the average learner in a classroom where technology was used performed 12 percentile points higher than the average learner in the traditional setting who did not use technology to enhance learning.

Bruce (2012) investigated the use of technology in a classroom in Ontario, Canada. The research targeted the use of the interactive white board (IWB) and involved teachers working as teams to produce inquiry-based lessons using the IWB to tackle difficult mathematical concepts. The findings from across the participants suggested that IWBs can be used and understood as a bridging mechanism for different mathematical ideas and representations.

Leong (2013) also studied Form 6 learners to determine the effects of using the geometer's sketch pad (GSP) on learner achievement at a Malasyian secondary school. The study reported a significant difference in the achievement of the experimental group as compared to the control group, indicating that the dynamic software GSP had a positive effect on learner achievement and attitude towards the learning of graphs of functions.

Ntombovuyo (2006) analyzed data from a variety of sources that included teachers, principals, learners, as well as community members. She also analyzed data she had generated herself while participating in the Digital Education Enhancement Project (DEEP), which was a project that targeted 24 teachers at 12 selected primary schools in the Eastern Cape. The aim of the analysis was to gain understanding as to whether the integration of ICT into school practice was working. The findings were that student achievement and motivation can be enhanced by the effective use of ICT and that it has proved to be an excellent tool for teaching and learning, not only in science but right across the curriculum.

Ogbonnaya and Mji (2013) conducted an exploratory study at a rural school in North West Province, South Africa, in which they used ICT to enhance learning of hyperbolic functions in Grade 11. This qualitative study used 57 Grade 11 learners. They were taught hyperbolic functions using Graphmatica graphing software. Data was collected through observation, interviews, assignments and tests. Their findings showed that Graphmatica enhanced learning of hyperbolic functions as evidenced by improved performance in the assignment and test.

Bester and Brand (2013) investigated the effect of technology on attention and achievement in a classroom using a control group comprised of 22 learners and an experimental group of 23 learners. The experimental group was taught using technology while no technology was used for the control group. The findings showed that there were statistically significant differences between the mean scores of the experimental group and the control group in favour of the experimental group. It also showed that there were significant differences in the average attention span of learners who were exposed to technology as compared to those who were not exposed to technology.

Gweshe (2014) conducted a study aimed at investigating the effects of computer-assisted instruction (CAI) on the performance and motivation of Grade 11 learners in the topic of circle geometry. The study consisted of 136 Grade 11 learners from two schools. One school was the experimental group with 71 learners and the other school was the control group with 65 learners.

The study was a non-equivalent control group quasi-experimental design. CAI was used in the experimental school while conventional teaching instruction (CTI) was used in the control school. A pretest and a post-test were administered to both groups as well as a questionnaire to measure the learners' motivation. A purposive sample consisting of 12 learners from both groups participated in semi-structured interviews. The findings of this investigation showed that the use of GeoGebra, in the teaching and learning of circle geometry improved the performance and motivation of Grade 11 learners in favour of the experimental group.

Ogunrinade, Ogbonnaya and Akintade (2016) investigated the effectiveness of CAI on learners' achievement in solid geometry using 160 second-year senior secondary school learners who were randomly selected from four secondary schools in Ogun, Nigeria. The quasi-experimental study used a non-equivalent pretest-post-test control group design. Findings of the study revealed that there was a statistically significant difference in academic performance of learners in the control group and those in the experimental group in favour of the experimental group.

Postelnicu (2011) conducted a study to identify secondary school learners' difficulties with aspects of linearity and linear functions, and to assess their teachers' understanding of the nature of the difficulties experienced by their learners. The cross-sectional study consisted of 1561 Grades 8 to 10 learners who were enrolled in mathematics courses from pre-algebra to algebra II, and their 26 mathematics teachers. All participants completed the mini-diagnostic test (MDT) on aspects of linearity and linear functions and ranked the MDT problems by perceived difficulty as well as commenting on the nature of the difficulties. Interviews were conducted with 40 learners and 20 teachers. From the cluster analysis the existence of two groups of learners was noticed; the first group (Group 0) was enrolled in courses below or at their grade levels and the second group (Group 1) had enrolled in courses above their grade level. The subsequent factor analysis confirmed the importance of slope and the Cartesian connection for learners' understanding of linearity and linear functions. Findings revealed that there were no significant variations in learner performance on the MDT across all grades. Learners' performance on the MDT increased with more advanced courses, mainly due to Group 1 learner performance.

The most difficult problems were those requiring identification of slope from the graph of a line and the difficulty was evident across grades, mathematics courses, and performance groups (Group 0 and Group 1). Findings also showed that learners correctly identified the problems with the highest MDT mean scores as being least difficult for them and only the learners in Group 1 were able to identify some of the problems with lower MDT mean scores as being difficult. Findings also showed that the teachers did not identify MDT problems that posed the greatest difficulty for the learners. Furthermore, interviews with the learners revealed difficulties with slope and the Cartesian connection. Teachers identified factors such as lack of familiarity with problem content or context, problem format and length in their descriptions of problem difficulty. In addition, teachers did not identify learner difficulties with the slope in a geometric context.

2.2.4 GeoGebra in mathematics teaching and learning

The use of GeoGebra in mathematics teaching and learning is generating considerable interest in the field of research. Several studies have been carried out globally involving the impact of GeoGebra on learner achievement in mathematics but to date very little research has been done in South Africa regarding this topic. This section gives a review of some of the research that has been conducted regarding the effect of GeoGebra on the achievement gains of learners.

Several studies about the use of GeoGebra have been done around the world. In Malaysian secondary schools Bakar et al. (2002) concluded that students using GeoGebra software in transformation geometry topics achieved better results than those exposed to the traditional approach.

Vukrobratovic and Takaci (2011) worked with the notion of a function at the beginning of the fourth grade of grammar school using GeoGebra software with two groups. The results showed that learners in the experimental group who were taught using GeoGebra achieved better on the visualization of the function.

Dogan (2011) evaluated the success of learners using GeoGebra in a study involving two Grade 8 classes in a 12-hour course. The experimental study used a pretest-posttest design to investigate the achievement of learners. The results showed that computer-based activities can be used in learning to great effect and that GeoGebra software encouraged higher order thinking as well as better retention of information. In addition, use of the software motivated learners to learn mathematics.

Saha, Ayub and Tarmizi (2010) conducted a quasi-experimental study using a post-test to discover the differences on average between learners with high visual-spatial ability and those with low visual-spatial ability, after intervention using GeoGebra. The participants were 53, 16- and 17-year-olds from a school in Malaysia who were divided into two, a control group who were taught using traditional methods only and the experimental group taught using GeoGebra. Furthermore, each group was classified into two; low visual-spatial ability (LV) and high visual-spatial ability (HV) using a paper and pencil test. The findings of the study showed that learners in the experimental group scored statistically significantly higher averages than learners in the control group, regardless of being HV or LV. The averages between the experimental and control group HV learners did not show any statistically significant difference. However, LV learners in the experimental group obtained a statistically significant higher average as compared to LV learners in the control group, showing that GeoGebra can be used effectively to teach LV learners as well.

Uddin (2011) explored GeoGebra as a pedagogical tool in the teaching and learning of transformation of functions in secondary school mathematics; and whether interaction with GeoGebra enhances the understanding of mathematical concepts. The study involved 8 learners at an independent school. Data was collected as feedback from two worksheets for learners, one given before the intervention and the other after the intervention. Data was also collected from classroom observations and from interviews with learners. Findings showed that GeoGebra influenced the development of mathematical ideas and concepts.

Rahul, Praveen and Achintya (2014) investigated the effect of GeoGebra on learners' achievement in geometry at secondary level (Grade 9) in a study comprising 40 learners in the control group and 40 learners in the experimental group. The experimental group was taught using GeoGebra while the control group was taught using traditional methods. The results showed a statistically significant difference between the mean scores of the experimental group and the control group, showing that the experimental group learners achieved higher scores in comparison to the learners in the control group.

Kllogjeri and Kllogjeri (2011) conducted a study in Albania where they demonstrated the three important theorems of derivatives using GeoGebra applets. The study demonstrated (i) the first derivative theorem (ii) the extreme value theorem and (iii) the mean value theorem. Their conclusions were that the multiple representation opportunities as well as the dynamic features of GeoGebra helped learners' understanding of mathematical concepts at a deeper level.

Zengin, Furkan and Kutluca. (2012) worked with 51 high school learners to investigate the effect of the GeoGebra software in teaching trigonometry and also to assess learners' attitude towards mathematics. The experimental group was taught using GeoGebra while the control group was taught using the constructivist teaching approach only. The results reflected that both groups showed improvement even though the averages in the experimental group were statistically significantly higher compared with those in the control group.

Hutkemri and Effandi (2012) conducted a study to determine the effect of GeoGebra on conceptual and procedural knowledge of the function. The study used 124 high school learners from Riau, Indonesia. Two groups were used in the study. The treatment group consisted of 60 learners and the other 64 were in the control group. The data was collected using the conceptual and procedural knowledge test of function. The findings showed significant differences between the treatment and control groups in that the treatment group had significantly higher conceptual knowledge compared to the control group.

Ersin (2013) carried out an intervention where GeoGebra was used in a reconsidered version of the experiencing step of the REACT strategy. The study used a critical incident questionnaire (CIQ) on 220 learners in the relating stage and thereafter a study group consisting of 30 learners was used in the experiencing stage involving GeoGebra. Findings from the CIQ showed that learners benefit from the two aspects, visual and concrete, when dealing with GeoGebra as part of dynamic and interactive mathematical learning. Another finding of the intervention was that GeoGebra provides a bridging role between relating and experiencing steps of the REACT strategy.

Another study by Taylor (2013) explored how GeoGebra could help learners visualize and conceptualize function transformations and their properties. It involved 18 learners in a pretest-post-test design with no control group. Data was collected using the pretest and post-test assignments and observations. Findings showed that GeoGebra did impact on learners' ability to conceptualize and visualize function transformations.

Furthermore, Hutkemri (2014) studied the effect of using GeoGebra on conceptual and procedural knowledge of the limit function, according to group and ability, using 284 learners who were selected from a high school in Riau, Indonesia. The experimental group had 138 learners while the control group had 146. The data was collected using the conceptual and procedural test of a limit function and analysed using SPSS 19.0 using a t-test and two-way Anova. Findings showed significant differences in learners' conceptual and procedural knowledge in favour of the experimental group.

Praveen and Leong (2013) investigated the effectiveness of using GeoGebra on learners' understanding of circles. The research was conducted on Form 3 learners at an international school in Malaysia using two intact classes in an experimental research where one class was treated as the experimental group and the other was the control group, both taught by the researcher. The study also sought to elicit learners' perception of learning circles using GeoGebra. The results showed that there was a significant statistical difference in the mean scores of the two groups in favour of the experimental group. Furthermore, 93% of the experimental group learners mentioned that they had learnt a lot using GeoGebra, while 82% were excited about using it and 75% could think creatively and analytically during discussions.

Mthethwa (2015) investigated whether GeoGebra as a technological tool helps to improve poor performance in Euclidean geometry. The study also elicited learners' interest in learning circle geometry. The study used 112 Grade 11 learners in UMkhanyakude district, in South Africa. During the intervention learners were taught the concept of circle geometry using GeoGebra and thereafter they had to answer questions on this topic. At the end of the intervention, learners had to respond to a questionnaire which consisted of 15 closed items relating to their views on GeoGebra and its effect on Euclidean geometry and mathematics and three open-ended questions where learners reflected on the use of GeoGebra in teaching and learning Euclidean geometry. The results showed that learners endorsed the use of GeoGebra as a technological tool in the teaching of Euclidean geometry, with some learners even suggesting that GeoGebra be used in other mathematics topics. Learners also enjoyed learning with GeoGebra, reporting it as user-friendly and very important in motivating them towards learning.

Mustafa (2015) investigated the impact of teaching mathematics with GeoGebra on the conceptual understanding of limits and continuity, focusing in a study consisting of 34 learners at a unique high school for gifted and talented learners in Turkey. Conceptual understanding of the topics' limits and continuity were measured through use of open-ended questions while attitude towards learning mathematics using technology was measured using the Likert-type survey. The findings showed that for conceptual understanding, the experimental group achieved scores which were higher than those obtained by the control group. They also found that the attitude of learners in the experimental group towards learning mathematics showed an improvement.

Furthermore, Daher and Anabousy (2015) researched Grade 9 learners' learning of transformations of non-basic functions using nineteen high-achieving learners who took part in 10 lessons. They used GeoGebra software to solve exploratory activities relating to functions and their transformations in a study grounded in APOS theory which was used to analyze the learners' understanding of function transformations. The findings of the study indicated that the participants differed in their APOS understanding of function transformations. Almost 60% of the participating learners arrived at the object level while the rest were still at the process level. The learners also had sub-level conceptions of transformation of functions.

Mehmet, Hanife and Gurcan (2015) examined the effects of GeoGebra on Grade 3 learners' achievement in the concept of fractions using 40 learners at a school in Ankara in a quasi-experimental post-test only design. The study used two groups, one as the control group and the other as the experimental group. Learners' first term tests were used as the pretest in the study and a post-test consisting of 22 short-ended questions was used at the end of intervention. Results showed a statistically significant difference in the achievement scores of the groups, favouring the experimental group.

Ogbonnaya (2010) studied ways of improving the teaching and learning of parabolic functions through the use of ICT using two Grade 11 classes at a school in North West Province, South Africa. After analysing learners' assignments and test scripts as well as using classroom observations and interviews, the results showed that the integration of ICT could help learners achieve better in parabolic functions.

Rahman and Puteh (2016) studied the effect of using a GeoGebra learning module in teaching trigonometry on the achievement of under-achiever learners in a quasi-experimental pretest-post-test non-equivalent control group design. The study used 47 Form 4 learners in Muar, Johar. Their findings revealed that learner achievement was significantly higher in the treatment group, which was taught trigonometry using the GeoGebra learning module, than in the control group, which was taught using the textbook and blackboard only.

In addition, Ogbonnaya and Chimuka (2016) investigated the effect of integrating GeoGebra in the teaching of circle geometry on learners' motivation to learn and enjoyment of the lesson following a non-equivalent comparison group design. The study used two groups of Grade 11 mathematics classes at two different schools. The experimental group was assigned because of the availability of a computer laboratory. This group was taught using GeoGebra while the other group was the control, taught without use of GeoGebra. Data from the questionnaire was analysed and showed that GeoGebra indeed motivated the learners to learn mathematics while making the lessons enjoyable.

In conclusion, the studies reviewed here show that the integration of ICT into the teaching and learning of mathematics has the potential to improve learner achievement and positively enhances their perception and motivation to learn the subject. No study was found that focused on the use of ICT in general or GeoGebra in particular, in the teaching of linear functions in the South African context. This further attests to the need for this study.

Summary

This chapter has discussed the theoretical framework that guided this study. APOS theory, which postulates that individuals need to construct the necessary cognitive structures required to make sense of mathematical concepts, was used as the framework guiding the study. Literature on ICT in teaching and learning was reviewed. The next chapter discusses the research methodology used in this study.

CHAPTER 3

RESEARCH METHODOLOGY

Research methodology refers to the 'ways in which one collects and analyzes data' (McMillan & Schumacher, 2014, p.16). Procedures should be systematic and purposefully developed with the aim of collecting data.

Biyane (2007) states that research contains two main phases; the planning phase and the implementation phase. During the planning phase, the researcher constructs a design, and a logical and appropriate plan of the research; during the implementation phase data is collected and analyzed. The design describes how the study was conducted to address the research questions. It indicates how the information was gathered as well as the methods, procedures and instruments used in the research. In this chapter, the research paradigm, research design, sampling method, data collection instruments and their development, procedures for data collection, validity and reliability of instruments, data analyses and techniques, pilot study, and ethical issues relating to the study are discussed.

3.1 Research Paradigm

A paradigm is 'a viewpoint on what constitutes educational reality' (Check & Schutt, 2012, p.14). Commonly accepted paradigms include critical theory, constructivism, positivism and post-positivism. This research study was guided by the positivist research paradigm.

Positivism

The philosophy of positivism was advocated by Auguste Comte during the late 19th century as a truth-seeking view that put emphasis on the fact that real and factual happenings can be studied scientifically by way of investigations and analysis. Positivism is a paradigm that 'assumes that there is reality out there that can be studied and known' (Polit & Beck, 2012, p. 78). Thus the positivist philosophers believe that the ultimate aim of researchers is to understand this reality and hold the belief those researchers, in particular scientists, must be objective and unbiased so that they are able to report accurately on this reality. Positivism puts great emphasis on objectivity, therefore holding personal beliefs and biases in check as the researcher tries to avoid contaminating the phenomena under study. Hence this study is guided by positivist theory since the personal views and beliefs of this researcher are not taken into account.

Post-positivism is a philosophy closely linked to positivism in the belief that an external reality does exist but, according to Guba and Lincoln (1994 cited in Check and Schutt, 2012, p. 103), post-positivists are 'very sensitive to the complexity of this reality and to the limitations and biases of the scientists who study it.' The implication is that we should never be certain that the results of a scientific inquiry from a single research study can be perceived as objective reality, unless there is also evidence from other researchers that supports the same theory.

3.2 Research Design

A research design is 'the plan that describes the conditions and procedures for collecting and analyzing data' (McMillan, & Schumacher, 2014, p. 6). The study followed a quasi-experimental non-equivalent group design, described as follows:

> In experimental research, the researcher manipulates at least one independent variable, controls other relevant variables and observes the effect on one or more dependent variables' (Gay, Mills, & Airasian, 2011, p. 250).

The study was quasi-experimental because the learners who participated at the schools had not been assigned at random; instead classes at the selected schools were used as full classes to avoid disruption of the academic programme; all schools had only one Grade 9 class.

McMillan and Schumacher (2014, p.5) define quasi-experimental research as a 'research design in which there is no random assignment of subjects'. The quasi-experimental non-equivalent group design provides an alternative route to researchers enabling them to investigate the causal effect of the independent variable in an environment where random selection of participants or subjects is not feasible, while maintaining control of most of the sources of internal validity. Hence, this design was used because in this study it was difficult to randomly assign the learners. In other words, the researcher had to sample intact classes at the particular schools that had been selected for the study.

The researcher used the pretest-post-test control group design with four groups. Two groups formed the experimental groups and the other two were the control groups. This design involves 'at least two randomly formed groups, both groups are pretested, one group receives a new or unusual treatment and both groups are post-tested' (Gay, Mills, & Airasian, 2012, p. 629). This study followed the design shown below.

Nonequivalent Groups Pretest-Post-test Control Groups Design

School	Group	Pretest	Intervention	Post-test
A-----------	control-----------	O ----------	X_0 --------------------	O
B------------	experimental ---------	O---------------	X ---------------------	O
C ------------	control -------------	O-------------	X_0 ---------------------	O
D -------------	experimental --------	O ------------	X---------------------	O

Source: Gay, Mills and Airasian (2012)

Pretest

A pretest was used to check if the groups were comparable before the intervention, which involved teaching linear function using the GeoGebra software on the experimental groups while using traditional teaching methods to teach the control groups. A pretest is used to ascertain what learners know before the intervention. Dimitrov and Rumrill (2003, p.159) assert that 'internal validity is the degree to which the experimental treatment makes a difference in the specific settings'. Therefore, the pretest enabled the researcher to check the internal validity of the intervention as to whether the treatment indeed had an effect on the participants.

Post-test

A post-test was administered at the end of the intervention. The purpose of the post-test was to assess whether the intervention resulted in any change in the learners' performance as a result of being taught linear functions using GeoGebra software. The post-test enabled the researcher to confirm the external validity of the treatment. Furthermore, Dimitrov and Rumrill (2003, p.159) define external validity as the 'degree to which the treatment effect can be generalized across populations'. Thus the post-test results were compared and analyzed together with the pretest results to check whether the treatment had any statistically significant effect on the experimental groups

Control group

A control group is a 'group of subjects in an experiment who are compared to the intervention or treatment group' McMillan, & Schumacher, 2014, p. 2). By using the control group, the researcher could check whether the difference was as a result of the intervention (teaching with GeoGebra graphing software) and did not occur as a result of the usual traditional teaching methods. The control group provides a reliable baseline that enables the researcher to compare the results. In this study, the control groups comprised learners who were taught linear functions using traditional teaching methods only. They were not exposed to GeoGebra software at any stage during the intervention period.

Experimental group

An experimental group is 'the group that receives the new treatment' (Gay, Mills, & Airasian, 2012, p. 252). In this study the experimental group was taught linear functions using GeoGebra software.

3.3 Population

A population refers to the entire set of individuals having the same common characteristic. McMillan and Schumacher (2014, p. 5) define a population as 'a group of individuals or events from which a sample is drawn and to which results can be generalized'. The population for this study consisted of all Grade 9 learners in Mopani district.

3.4 Sample and sampling technique

A sample is 'the group of subjects from whom data are collected; often representative of a specific population' (McMillan & Schumacher, 2014, p. 6). The sample for this study consisted of Grade 9 learners from four schools in a circuit in Mopani district.

Gay, Mills and Airasian (2012) refer to sampling as the process of choosing individuals from a population, usually in such a way that the selected individuals represent the larger group from which they were selected. The non-probability sampling technique was used to select the circuit as the focus of the study. Non-probability sampling involves the selection of sampling units from a population using non-random processes. It does not inform the researcher of the chances of selecting each element of the sample in advance. The most common methods of non-probability sampling are availability (convenience) sampling, quota sampling, purposive sampling, and snowball sampling. Convenience sampling is when items are selected because they are available or easy to find while purposive sampling selects each element according to its unique characteristic.

The study focused on Grade 9 learners in Mopani district but the circuit was sampled for its convenience and availability to the researcher (non-probability sampling, i.e. convenience sampling). Purposive sampling was also used since the circuit is a serial underperforming circuit in the district, occupying the bottom position when it comes to performance. Moreover, this type of study would also be among the first of its kind in the circuit and beneficial to both teachers and learners in the circuit.

McMillan and Schumacher (2014) refer to cluster sampling as differing from random sampling in that cluster sampling randomly selects groups, not individuals. Cluster sampling is particularly useful when sampling for a classroom study where the researcher might start sampling at provincial, then district, circuit, school level, and finally at classroom level. Gay, Mills and Airasian (2012) also explain that when cluster sampling is done in stages that involve groups, it is called multistage cluster sampling. Multistage sampling is justified in this study because the sampling was started at district level when the circuit was selected due to its convenience, availability and purpose for the researcher. The sampling of the circuit was purposive since it is also a serial underperforming circuit in the district, occupying the bottom position when it comes to Grade 12 performance. Hence the study has tried to find ways of improving teaching and learning mathematics in that circuit.

Random sampling was used in the circuit, initially to select the two schools which were used in the pilot study from the 10 secondary schools which offer Grade 9 tuition in the circuit and thereafter to select the other four schools which were used in the main study. At the selected schools, cluster sampling was used to select participants. In other words, intact Grade 9 classes were selected for the study, with no random sampling involved in the selection of participants.

One of the schools sampled for the pilot study was used as the control group and the other as the experimental group. Table 3.1 shows the composition of learners in the pilot study sample.

Table 3.1: Composition of learners in the pilot study sample

Group	Girls	Boys	Total
A Control	16	13	29
B Experimental	18	15	33
Total	34	28	62

Table 3.1 shows that the control group was comprised of 16 girls and 13 boys, a total of 29 learners. The experimental group had 18 girls and 15 boys, giving a combined total of 33 learners.

The four schools that were randomly sampled from the remaining eight schools participated in the main study, with two groups being control groups and the other two being the experimental groups. A total of 127 learners participated in the study. The composition of the learners in the four groups is shown in Table 3.2.

Table 2.2: Composition of learners in the study sample

Group	Girls	Boys	Total
School A (control)	15	16	31
School B (experimental)	16	19	35
School C (control)	16	12	28
School D (experimental)	17	16	33
Total	**64**	**63**	**127**

3.5 Instruments

The study used an achievement test. The achievement test was administered as the pretest at the beginning of the intervention and also as a post-test at the end of the intervention. The test measured learners' achievement in linear functions and graphs.

Achievement test

The test (Appendix 3) consists of 19 items in five questions that examined the learners' knowledge of linear functions as stipulated in the Curriculum Assessment Policy Statement (CAPS) (see section 2.2). The test items were compiled by the researcher in collaboration with mathematics educators.

Question 1 consisted of 3 multiple choice items which examined learners' knowledge and recall skills related to the interpretation of linear function graphs. Learners were expected to recall what they had learnt about the equation of a linear function to be able to respond to the questions. Questions were structured as follows:

Question 1(a)

This was a multiple choice question which examined the learners' knowledge of the y-intercept; learners could choose from four given possible answers.

Question 1(b)

The question examined the gradient of a line graph. Learners had to select one answer from a list of four possible solutions.

Question 1(c)

This question examined learners on the x-intercept and learners could choose one from four possible solutions.

Question 2 consisted of 2 multiple choice items where learners were expected to use their knowledge of the gradient and equation of a straight line.

Question 2(a)

This was a multiple choice question examining learners on the y-intercept. They could choose one answer from a list of four possible choices.

Question 2(b)

The question tested the gradient of a graph. Learners had to select one from a list of four choices.

Question 3 consisted of items a–i. Items a–e were recall questions and f–g expected learners to complete the table of values and use it to plot the graph of $y = 2x - 3$. Items h and i expected the learners to use the graph to find the x- and y-intercepts.

Question 3(a)

This question required learners to identify the independent variable in a given equation.

Question 3(b)

Learners were expected to identify the dependent variable in a given equation.

Question 3(c)

Learners had to state the coefficient of x in the given equation.

Question 3(d)

Learners were expected to be able to identify the constant term in the given equation.

Question 3(e)

The question expected the learners to be able to state and explain the relationship between the constant in the equation and the y-intercept of the graph.

Question 3(f)

Learners were expected to be able to substitute given values of x into the equation and use the answers to complete the corresponding values of y in the table.

Question 3(g)

Learners were expected to be able to plot and draw graphs using values from tables.

Question 3(h)

The learners were expected to be able to write the y-intercept in coordinate form.

Question 3(i)

Learners had to write the x-intercept in coordinate form.

Question 4

In question 4 a-c, learners had to draw sketches of graphs defined by given linear equations.

Question 5

In question 5a, learners were expected to sketch graphs and use the sketches to explain how the graphs would differ from each other. The learners were expected to explain the effect of changing the values of the constant in the equation. In 5b learners were also expected to explain how 2 graphs would differ from each other, changing the sign of the gradient in the equation.

Table 3.3 gives a summary of the categorization of the question according to drawing linear functions graphs and interpretation of linear functions graphs.

Table 3.3: Summary of categorization of questions

Question number	Category	Comments
1a	Interpreting graphs	Finding y-intercept
1b	Interpreting graphs	Finding the gradient of a graph
1c	interpreting graphs	Finding the x-intercept
2a	Interpreting graphs	Finding y-intercept
2b	Interpreting graphs	Calculating the gradient of a graph
3a	Interpreting graphs	Finding independent variable
3b	Interpreting graphs	Finding the dependent variable

3c	Interpreting graphs	Finding coefficients
3d	Interpreting graphs	Identifying the constant term
3e	Interpreting graphs	The relationship of constant term and the y-intercept
3f	Drawing graphs	Completing the table of values
3g	Drawing graphs	Plotting graphs
3h	Interpreting graphs	Identifying coordinates of the y-axis
3i	Interpreting graphs	Identifying the coordinates of the x-axis
4a	Drawing graphs	Be able to sketch the graphs from given equations
4b	Drawing graphs	Sketching graphs
4c	Drawing graphs	Sketching graphs
5a	Drawing graphs	Explain the transformations of given graphs with the aid of sketches
5b	Drawing graphs	Describe and explain the transformation of given graphs with the aid of sketches

Validity and reliability of the test

Validity of the test refers to the extent to which the marks scored in the test were consistent. Any uncontrolled extraneous variables that affect the performance on the dependent variable are called threats to validity (Gay, Mills, & Airasian, 2012). To ensure that the test was valid, the items were developed in line with the CAPS curriculum for Grade 9 learners. The test items were compiled by the researcher in collaboration with mathematics educators to ensure that they were in line with the cognitive demand of the content of Grade 9 linear functions according to the curriculum. After developing the item, questions were given to curriculum advisors and the mathematics committee in the circuit to rate their relevance and also whether they conformed to the curriculum requirements. All items were found to be relevant for the purpose.

Reliability

Reliability is used to assess the degree of internal consistency of scores from a set of indicators (test items). Reliability was ascertained using the Kuder-Richardson (KR-20) test which is defined by McMillan and Schumacher (2014) as a type of internal consistency check for items that are marked right or wrong. It is a special case of Cronbach's alpha that measures reliability for dichotomous data. KR-20 was conducted using SPSS@ which performs the test in a similar way to Cronbach's alpha except that KR-20 is case specific and is used to test the reliability of test questions. The reliability coefficient measures the likelihood of obtaining similar results if the test is administered to another set of different learners and results range from 0–1. A result greater than 0.5 on a teacher-made test can be considered as having good internal consistency although the higher the result the better the internal consistency of the test or exam (Tavakol & Dennick, 2011).
KR-20 was conducted on the pilot study (refer to Appendix15) and an overall alpha of 0.726, which indicated a good internal consistency of the test.

3.6 GeoGebra

GeoGebra is dynamic mathematical software for all levels of education that joins arithmetic, geometry, algebra and calculus, and was published by Hohenwarter (2004). According to Hohenwarter and Jones (2007), GeoGebra is a software package that combines both geometry and algebra that is specifically designed for educational purposes and can help learners to foster their mathematical learning. GeoGebra is an interactive software that can be used to draw points, vectors, lines, conic sections, as well as functions, while allowing equations and coordinates to be entered directly; thus it is possible to use GeoGebra when working with numbers for vectors and points. An expression written in the algebra view corresponds to an object in the graphics view. Furthermore, it is free and available for use both at school and at home providing great opportunities to learners doing their homework. GeoGebra can also help in lessons and activities aligned with the standards, goals and objectives of CAPS. Furthermore, GeoGebra with its multiple features of dynamic modeling contributes widely to improving learners' general perception of mathematics.

GeoGebra has a geometry window, an algebra window, and toolbar as well as a construction guide among its features. Geometric representation can be altered or changed by dragging it with the mouse whereas the algebraic representation is dynamic and can be adjusted on the keyboard. An adjustment in the algebra window automatically adjusts the result in the geometric window.

GeoGebra is relatively easy for use by beginners. The greatest advantage of using GeoGebra is that it is user-friendly as learners can navigate and assess their own work. After accessing GeoGebra, it can be used for various mathematical topics like functions, transformations, geometry and trigonometry. GeoGebra is advantageous to use in linear algebra because it makes it easier for the learners to compare and analyze the effects of various shifts on the linear graphs and to further explore linear graphs on their own and in pairs or groups.

Bu and Schoen (2011) are also of the view that GeoGebra provides several ways of presenting phenomena in various domains of mathematics, and a rich variety of computational tools for modeling and simulations. Models in this context are used to enact realities to learners so as to better understand mathematical concepts. GeoGebra facilitates maximum mathematical understanding and proficiency for mathematics teaching and learning. A mathematically competent learner can perform various stages of a mathematical idea in a dynamic way and further gain useful insight into mathematical structures and concepts.

The model-centred framework on learning and instruction helps learners to understand the thought processes involved in mathematical intuition and learning difficulties. GeoGebra is essentially a concerted effort between technology and theory. Bu and Schoen (2011) assert that GeoGebra creates a positive attitude, which is centred on integrating technology with mathematics teaching and learning. GeoGebra in model-centred mathematics teaching and learning goes beyond traditional mathematics instruction in content and coverage of concepts. It can be used as a conceptual tool, a pedagogical tool, a cognitive tool, and a transformative tool in mathematics teaching and learning. Burke and Kennedy (2011) suggest that dynamic GeoGebra models and simulations create a link between learners' investigations and mathematical structures. They are also of the view that model-based conceptual interventions support learners' development of thought processes that are necessary for learning formal mathematics.

GeoGebra-based modelling assists learners to visualize problem situations, and overcome algebraic problems and thus focus on the learning tasks. GeoGebra supports problem-solving, provides visualization and illustrations; therefore it helps increase learners' motivation and the development of cognitive abilities. GeoGebra therefore goes a long way in enhancing learners' mathematical exploration and visualization skills as well as assisting in the creation of links between real-world situations and mathematical ideas. Bu and Alghazo (2011) concur that GeoGebra has educational implications for the modelling of real-life problems in terms of mathematical ideas and the ever-expanding learning opportunities that arise, sometimes unexpectedly, during the modelling process.

Stahl (2014) asserts that GeoGebra software facilitates the engagement of learners in terms of collaborative knowledge-building and group thinking in problem-solving tasks of dynamic geometry as well as the construction and explanation of the design of dependencies in dynamic geometry. GeoGebra greatly enhances learners' positive perception of collaborative group work.

3.7 Interventions

The interventions were implemented as a series of 10 lessons each of one hour duration (see appendices 6–14). Educators at the respective schools taught the lessons from lesson plans designed at workshops where the educators had also been also oriented on the implementation of GeoGebra. Because the educators had the same minimum level in terms of professional standing and had experience in teaching mathematics in the senior phase, they taught their respective classes during the intervention as this ensured minimal disruptions at the schools. Their experience with GeoGebra was also at the same level since they had been trained at the same workshops by the same curriculum advisors prior to the interventions. For the context of the study use of GeoGebra was limited to the basics for beginners; educators were expected to be able to show learners how to draw and use GeoGebra to interpret linear functions. The educators themselves were not expected to produce any worksheets using the GeoGebra applet.

3.7.1 Interventions in the control group

Learners in the control groups were taught using only the traditional teaching methods which included the textbook together with the chalkboard method. No GeoGebra was introduced to the learners before or during the intervention stage.

A pretest was administered just before the beginning of the interventions and a post-test was administered at the end of the intervention stage. After the intervention and post-test, educators at the control schools were at liberty, in fact they were encouraged, to also introduce GeoGebra to their learners after they had written the post-test.

3.7.2 Interventions in the experimental group

A pretest was administered to the experimental groups. Before the intervention learners at the experimental schools were introduced to the GeoGebra software and shown how to use it with no particular emphasis on the topic of linear functions. This was done during two extra lessons just after they had written the pretest. Lesson plans at the experimental schools were similar to those used in the control schools but adapted towards the use of GeoGebra during class activities. Educators at the two experimental schools taught the lessons using GeoGebra and their learners were allowed access to the software during lessons and also after the lessons as they completed the tasks. There were a limited number of computers at the two schools hence the average learner computer ratio was 3:1. A post-test was administered to learners in the experimental groups at the end of the intervention stage.

3.8 Data Analysis Techniques

Data was analyzed using descriptive and inferential statistics. Descriptive statistics involve describing the trends of the data by giving the mean scores or mean rank scores of the data and also the range of the data sets.

Inferential statistics involve one-way analysis of variance (ANOVA) in SPSS©. ANOVA was used to analyse the data collected during the actual study, using the Statistical Package for Social Sciences (SPSS). The choice of ANOVA for this particular study was made in line with the recommendations of McMillan and Schumacher (2014, p.325) who stated that ANOVA 'allows the researcher to test the differences between all groups and make more accurate probability statements than are possible when using a series of separate tests'. One way in which ANOVA was used was to test the differences between the means of all the groups in the actual study since this allows the researcher to make more accurate probability statements than when multiple tests are performed separately. ANOVA is an extension of the t-test. Using ANOVA enables the researcher to analyse data collected from the four groups at once instead of making repeated calculations and comparisons. The Bonferroni post-hoc test was used to verify the groups which had statistically significant differences and those with no statistically significant differences amongst the four groups.

Post-hoc tests were used on the post-test results to indicate which of the means were different by testing all possible pairs of means. McMillan and Schumacher (2014, p. 5) define post-hoc comparison as the 'statistical tests used with pairs of means that are usually conducted after a statistical test of all means together'. The post-hoc tests were done after ANOVA to test the hypothesis that all the means are identical.

The non-parametric version of ANOVA, the Kruskal-Wallis (KW) test was used as a confirmatory measure, to analyze the data in the case where the data violated one or more assumptions of ANOVA. This test uses one-way ANOVA to analyze the rank scores not the original scores, hence the use of the median instead of the mean (Ostertagova, Ostertag & Kovac, 2014). The test is therefore less sensitive to outliers.

Where the KW test was used to analyse the data, then the Mann-Whitney U test was used as the post-hoc test. The Mann-Whitney U test is a non-parametric version of the t-test that was used to clarify the results from Kruskal-Wallis test and reveal the groups where there were statistically significant differences.

3.9 Ethical Issues

McMillan and Schumacher (2014, p.129) assert that 'research ethics are focused on what is morally proper and improper when engaged with participants or when accessing archival data.' The researcher sought to maintain the ethical standard of full disclosure by disclosing the details of the study to the circuit manager of Mawa circuit and the principals at the schools concerned, as well as obtaining the circuit managers' written consent. The researcher obtained an ethical clearance certificate from UNISA which enabled her to conduct this study.

The learners who participated were not coerced into participating but they participated voluntarily and were furnished with a clear explanation of what the study entailed in order to enable them to decide whether to participate or not. Consent was also obtained from the learners' parents or guardians who were requested to sign a consent form indicating their understanding of the details of the study and giving their consent for their minor charge to participate in the study.

Participants were assured that their involvement in the study would not result in their physical or emotional harm. There would be no injuries from participating and no emotional stress that might result if the results of the study were released to the wrong parties.

Participants were assured that their privacy would be maintained by means of appropriate storage of all the study data to avoid disclosure. Participants also have the right to remain anonymous and therefore there was no mention of participants' names. Only the researcher would have access to individual data during the study and after its completion. Moreover, participants were assured of the confidentiality of the study before their consent was obtained.

The researcher ensured gender-related matters were observed at all costs. No participant or stakeholder would feel his or her right(s) jeopardized by the study in progress.

The researcher also sought parental or guardians' consent. The parents/guardians of the learners completed forms confirming that they understood the details of the research and consented that the learners could participate in the research. The learners also gave their consent through completion of the informed consent forms.

The researcher undertook to give credit whenever the contributions of others are used, through the use of quotation marks and referencing all sources used during the study. The researcher would not in whatsoever way represent other peoples' work as her own.

Summary

In this chapter, the research paradigm; research design, target population and sample, as well as the sampling technique used in the study, were discussed. Data collection, instruments, data analysis and ethical issues were also discussed in this chapter. The next chapter presents the analysis and findings of the study.

CHAPTER 4

DATA ANALYSES AND RESULTS

The previous chapter discussed the research methodology and the research paradigm for this study was also explained. The population, sample and sampling techniques were discussed in detail. This study set out to investigate the effects of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district, Limpopo Province. The study was guided by APOS theory which, in accordance with constructivist theories posits that an individual needs to construct the necessary cognitive structures in order to make sense of mathematical concepts. This study provides answers to the following questions:

1. Is there any difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions?
2. Is there any statistically significant difference between the achievement scores of learners exposed to the GeoGebra and learners who were not exposed to GeoGebra in drawing of graphs of linear functions (DG)?
3. Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in interpretations of linear functions and their graphs (IG)?

The study followed a quasi-experimental non-equivalent group design. Two classes were the experimental groups and two classes were the control groups. The study was quasi-experimental because the learners who participated were not randomly assigned; instead the classes at the particular schools were used as full classes to avoid disruption to the smooth running of the academic programme and all the schools only had one grade 9 class. The researcher used the non-equivalent pretest-post-test control group design with four groups. The study used an achievement test. The achievement test was administered as the pretest at the beginning of the intervention and also as a post-test at the end of the intervention. The test measured learners' achievement in linear functions and graphs. The findings of this study are presented in this chapter and the research questions addressed in line with the findings of the study.

4.1 Pretest results

One-way ANOVA was carried out on the groups' pretest results. The purpose of the test was to establish whether there was a statistically significant difference in the means of the groups at the beginning of the interventions. Hence, the researcher needed to check whether the learners in the groups were of comparable achievement levels at the beginning and also how much knowledge of linear functions the learners were bringing to the interventions. The descriptive statistics of the pretest results are shown in Table 4.1.

Table 4.1: Pretest descriptive statistics

	N	Min	Max	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound
Control group A	31	.00	7.00	1.5161	1.58894	.28538	.9333	2.0990
Experimental group B	35	.00	8.00	2.1143	1.81126	.30616	1.4921	2.7365
Control group C	28	.00	7.00	1.6071	1.64067	.31006	.9710	2.2433
Experimental group D	33	.00	8.00	2.1515	1.92226	.33462	1.4699	2.8331
Total	127	.00	8.00	1.8661	1.75645	.15586	1.5577	2.1746

Results from the ANOVA analysis of the pretest results are also shown in Table 4.2.

Table 4.2: Pretest ANOVA analysis of

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	10.519	3	3.506	1.140	.336
Within Groups	378.206	123	3.075		
Total	388.724	126			

The results of ANOVA (Table 4.2) however showed no statistically significant differences (p > 0.05) between the group means. The ANOVA results imply that there was no statistically significant difference in the achievement of learners in both the experimental groups and the control groups at the beginning of intervention. The learners' achievement levels in linear functions were comparable before the treatment; therefore, any differences in achievement levels after the treatment could be attributed to the treatment.

4.2 Post-test total achievement scores

Since samples were randomly selected, we assume independence assumption is valid in this data set. Furthermore, Levene's test showed that variances were not significantly different (p > 0.05) across the groups (Table 4.3). Hence, the homogeneity of variances assumption is valid.

Table 4.3: Levene's test of equality of error variances on post-test scores

F	df1	df2	Sig.
1.592	7	119	.144

This tests the null hypothesis that the error variance of the dependent variable is equal across groups.

Two tests for normality were done in this analysis (Table 4.4).

Table 4.4: Tests of normality across treatment for post-test

		Kolmogorov-Smirnova			Shapiro-Wilk		
	Group	Statistic	Df	Sig.	Statistic	Df	Sig.
Post-test	Group A	.180	31	.012	.882	31	.003
	Group B	.119	35	.200*	.954	35	.149
	Group C	.227	28	.001	.857	28	.001
	Group D	.119	33	.200*	.955	33	.182

For groups B and D we failed to reject the null hypotheses ($p > 0.05$) and conclude that data is from a normal distribution across samples B and D. However, across groups A and C there is no statistical evidence of normal distribution of data ($p < 0.05$). Therefore, the assumption for normality of data is not being met in the control groups.

However, despite the assumption of normality not being met in some of the groups after transforming the data, ANOVA being a robust statistic as asserted by Schmider, Ziegler, Danay, Beye and Buhner (2010), and the sample size being large enough ($n>30$) (Field, 2013), the researcher felt justified to proceed with the ANOVA test. In addition to the ANOVA test, the non-parametric test, namely Kruskal-Wallis H test (also called the 'one-way ANOVA on ranks') was also carried out as a confirmatory or validation test to determine if there were statistically significant differences between the achievements of the experimental and control groups in the test.

ANOVA was also conducted on the total scores of the schools in the post-test. The descriptive statistics for the test are shown in Table 4.5.

Table 4.5: Groups' post-test scores

							95% Confidence interval for mean	
	N	Min	Max	Mean	SD	Std. error	Lower Bound	Upper Bound
Control group A	31	1	25	8.87	5.402	.970	6.89	10.85
Experimental group B	35	8	37	24.74	7.504	1.268	22.17	27.32
Control group C	28	3	25	9.21	5.567	1.052	7.06	11.37
Experimental group D	33	18	44	34.73	6.920	1.205	32.27	37.18
Total	127	1	44	20.04	12.662	1.124	17.82	22.26

The post-test was marked out of 50. The descriptive statistics show that control group A had 31 learners, a mean 8.87 and standard deviation 5.402, with a minimum score 1 and maximum score 25. Experimental group B had 35 learners, a mean score of 24.74, standard deviation 7.504; minimum score 8 and maximum score 37. Control group C had 28 learners, a mean 5.567, standard deviation 1.052; minimum score 3 and maximum score 25. Experimental group D had 33 learners, a mean score of 34.73 standard deviation 6.92, minimum 18 and maximum 44.

Results of the ANOVA analysis of the total scores of the groups in the post-test are also shown in Table 4.6

Table 4.6: Post-test total achievement scores

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	15041.374	3	5013.791	119.528	.000
Within Groups	5159.429	123	41.947		
Total	20200.803	126			

The results showed that there were statistically significant differences between two or more groups. Since there were significant treatment effects, post-hoc (Bonferroni) tests were done.

Table 4.7 Bonferroni multiple comparisons on total scores

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Control group A	Experimental group B	-9.321*	1.266	.000	-12.72	-5.93
	Control group C	.551	1.338	1.000	-3.04	4.14
	Experimental group D	-14.867*	1.284	.000	-18.31	-11.42
Experimental group B	Control group A	9.321*	1.266	.000	5.93	12.72
	Control group C	9.871*	1.301	.000	6.38	13.36
	Experimental group D	-5.546*	1.245	.000	-8.89	-2.21
Control group C	Control group A	-.551	1.338	1.000	-4.14	3.04
	Experimental group B	-9.871*	1.301	.000	-13.36	-6.38
	Experimental group D	-15.418*	1.319	.000	-18.95	-11.88
Experimental group D	Control group A	14.867*	1.284	.000	11.42	18.31
	Experimental group B	5.546*	1.245	.000	2.21	8.89
	Control group C	15.418*	1.319	.000	11.88	18.95

Results of the post-hoc test showed that there were statistically significant differences between the following: control group A and experimental group B; control group A and experimental group D; control group C and experimental group B; as well as control group C and experimental group D. There was also a statistically significant difference between experimental group B and experimental group D. But there was no statistically significant difference between control group A and control group C, as shown by the Bonferroni test.

4.2.2 Kruskal-Wallis analysis of total achievement scores

In addition, the non-parametric test, Kruskal-Wallis, was also conducted on the total achievement scores of the schools in the posttest. Table 4.8 shows the rank scores between the schools.

Table 4.8 Kruskal-Wallis test for total achievement scores

	Group	N	Mean Rank
Total Score	Control group A	31	31.55
	Experimental group B	35	79.20
	Control group C	28	32.63
	Experimental group D	33	104.98
	Total	127	

The test statistics of the Kruskal-Wallis test are shown in Table 4.9.

Table 4.9: Kruskal-Wallis test statistics

	Total Score
Chi-Square	91.493
Df	3
Asymp. Sig.	.000

The results of the Kruskal-Wallis test show that there were statistically significant differences between the total scores of learners in two or more groups with a chi-square value of 91.493 and a mean rank total score of 31.55 for control group A, 79.20 for experimental group B, 32.63 for control group C and 104.98 for experimental group D.

The Kruskal-Wallis test only reported that there were significant statistical differences between the total scores but did not specify the groups where the differences exist; hence there was still need to conduct a follow-up test. The Mann-Whitney U test was used as the post-hoc test (see section 3.10).

Mann-Whitney U tests on the total achievement scores

The Mann-Whitney test was conducted comparing control group A and experimental group B. The descriptive statistics for the test are shown in Table 4.10.

Table 4.10: Rank scores of the Mann-Whitney test of control A and experimental B

	Group	N	Mean Rank	Sum of Ranks
Total Score	Control group A	31	17.81	552.00
	Experimental group B	35	47.40	1659.00
	Total	66		

Table 4.11 shows the test statistics for the Mann-Whitney test between control group A and experimental group B.

Table 4.11: Mann-Whitney test statistics for control group A and experimental group B

	Total Score
Mann-Whitney U	56.000
Wilcoxon W	552.000
Z	-6.256
Asymp. Sig. (2-tailed)	.000

The results showed a statistically significant difference between the total scores of the two groups with mean rank scores of 17.81 for control group A and 47.40 for experimental group B. Hence, the results were in favour of experimental group B.

The Mann-Whitney test was also conducted between the total scores of Control group A and Control group C. Table 4.12 shows the rank score statistics for the test.

Table 4.12: Rank scores of the Mann-Whitney U test between control groups A and C

	Group	N	Mean Rank	Sum of Ranks
Total Score	Control group A	31	29.58	917.00
	Control group C	28	30.46	853.00
	Total	59		

The test statistics for the test are also presented in Table 4.13.

Table 4.13: Mann-Whitney test statistics for control group A and control group C

	Total Score
Mann-Whitney U	421.000
Wilcoxon W	917.000
Z	-.199
Asymp. Sig. (2-tailed)	.843

The results show that the mean rank scores were 29.58 for control group A, and 30.46 for control group C. The Mann-Whitney U statistic was 421.000 with a significant value of 0.843. Hence there was no statistically significant difference between the total scores of the two control groups. The result implies that the learners' achievement levels in the posttest were comparable.

Table 4.14 shows the rank score statistics for the Mann-Whitney U test between control group A and experimental group D.

Table 4.14: Rank score statistics for the Mann-Whitney U test between control group A and experimental group D

	Group	N	Mean Rank	Sum of Ranks
Total Score	Control group A	31	16.16	501.00
	Experimental group D	33	47.85	1579.00
	Total	64		

The mean rank scores were 16.16 for control group A and 47.85 for experimental group D. The test statistics are shown in Table 4.15.

Table 4.15: Mann-Whitney U test statistics for control group A and experimental group D

	Total Score
Mann-Whitney U	5.000
Wilcoxon W	501.000
Z	-6.810
Asymp. Sig. (2-tailed)	.000

The test statistics showed that there was a statistically significant difference between the total scores of the two groups in favour of experimental group D. The results indicate that learners in the experimental group D achieved better than the learners in the control group A.

The Mann-Whitney U test was conducted between experimental group B and control group C. Table 4.16 shows the rank scores for the test.

Table 4.16: Rank score statistics for the Mann-Whitney U test between experimental b and control C

	Group	N	Mean Rank	Sum of Ranks
Total Score	Experimental group B	35	44.46	1556.00
	Control group C	28	16.43	460.00
	Total	63		

Experimental group B had a mean rank score of 44.46 and control group C had a mean rank score of 16.43. The test statistics are also shown in Table 4.17.

Table 4.17: Mann-Whitney U test statistics for experimental group B and control group C

	Total Score
Mann-Whitney U	54.000
Wilcoxon W	460.000
Z	-6.038
Asymp. Sig. (2-tailed)	.000

The results show a Mann-Whitney U value of 54.000, a z score of -6.038 and a significant value of 0.000. This shows that there was a statistically significant difference between the total scores of experimental group B and control group C in favour of experimental group B. Hence, the learners in experimental group B achieved better than the learners in control group C.

The rank statistics for the Mann-Whitney U test between experimental group B and experimental group D are shown in Table 4.18.

Table 4.18: Ranks for the Mann-Whitney U test of Experimental group B and Experimental group D

	Group	N	Mean Rank	Sum of Ranks
Total Score	Experimental group B	35	23.34	817.00
	Experimental group D	33	46.33	1529.00
	Total	68		

Statistics in Table 4.19 show mean rank scores of 23.34 for Experimental B and 46.33 for Experimental D.

Table 4.19: Mann-Whitney U test statistics for experimental group B and experimental group D

	Total Score
Mann-Whitney U	187.000
Wilcoxon W	817.000
Z	-4.796
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 187.000, the z score -4.796 and the significance value was 0.000. Hence, the results indicate that there was a statistically significant difference in the total achieved scores of these two groups, favouring experimental group D. This implies that learners in experimental group D achieved higher scores compared to learners in experimental group B.

Furthermore, the Mann-Whitney U test was conducted on the total scores, between control group C and experimental group D. The rank score statistics are shown in Table 4.20.

Table 4.20: Ranks for Mann-Whitney U test between control group C and experimental group D

	Group	N	Mean Rank	Sum of Ranks
Total Score	Control group C	28	14.73	412.50
	Experimental group D	33	44.80	1478.50
	Total	61		

The rank statistics showed mean rank scores of 14.73 for control group C and 44.80 for experimental group D. The test statistics are shown in Table 4.21.

Table 4.21: Mann-Whitney U test statistics for the total scores between control group C and experimental group D

	Total Score
Mann-Whitney U	6.500
Wilcoxon W	412.500
Z	-6.600
Asymp. Sig. (2-tailed)	.000

The results showed that there was a statistically significant difference between the total achieved scores of the two groups, in favour of experimental group D; hence the learners in experimental group D achieved better scores than the learners in control group C.

Overall, both the parametric (ANOVA) and the non-parametric (Kruskal-Wallis) tests showed that there was a statistically significant difference in the total achieved scores of the learners in the experimental groups compared to those in the control groups. This result suggests that the learners who were exposed to GeoGebra (experimental) achieved higher scores in linear functions, in the posttest.

4.3 Groups' post-test results on drawing graphs (DG) of linear functions

The researcher was also interested in how learners achieved in drawing graphs (DG) of linear functions; hence ANOVA and Kruskal-Wallis were used to analyze the data collected from the learners' scripts, on questions relating to the concepts of drawing graphs.

4.3.1 ANOVA analysis on DG Scores

ANOVA analysis on the DG scores of the groups was also conducted. Table 4.22 shows the descriptive statistics for the DG scores of the groups.

Table 4.22: Descriptive statistics for DG scores of the groups

							95% Confidence Interval for Mean	
	N	Min	Max	Mean	Std. Deviation	Std. Error	Lower Bound	Upper Bound
Control group A	31	0	15	2.19	4.037	.725	.71	3.67
Experimental group B	35	0	22	11.51	6.537	1.105	9.27	13.76
Control group C	28	0	11	1.64	2.831	.535	.55	2.74
Experimental group D	33	2	25	17.06	5.815	1.012	15.00	19.12
Total	127	0	25	8.50	8.226	.730	7.06	9.95

Table 4.22 shows that control group A had a mean score of 2.19 and standard deviation 4.037; experimental group B had mean 11.51 and standard deviation 6.537; control group C had mean 1.64 and standard deviation 2.831; as well as experimental group D with mean 17.06 and standard deviation 5.815. Table 4.23 shows the result of the analysis.

Table 4.23: ANOVA analysis of DG scores between groups

	Sum of Squares	Df	Mean Square	F	Sig.
Between groups	5285.859	3	1761.953	66.891	.000

Within groups	3239.889	123	26.341
Total	8525.748	126	

The results of the analysis show that there is a statistically significant difference between the mean DG scores of the groups. Therefore, the Bonferrori (Table 4.24) post hoc test was done to identify the groups which had statistically significant differences.

Table 4.24: Bonferrori multiple comparisons of the groups

		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
(I) Group	(J) group				Lower Bound	Upper Bound
Control group A	Experimental group B	-9.321*	1.266	.000	-12.72	-5.93
	Control group C	.551	1.338	1.000	-3.04	4.14
	Experimental group D	-14.867*	1.284	.000	-18.31	-11.42
Experimental group B	Control group A	9.321*	1.266	.000	5.93	12.72
	Control group C	9.871*	1.301	.000	6.38	13.36
	Experimental group D	-5.546*	1.245	.000	-8.89	-2.21
Control group C	Control group A	-.551	1.338	1.000	-4.14	3.04
	Experimental group B	-9.871*	1.301	.000	-13.36	-6.38
	Experimental group D	-15.418*	1.319	.000	-18.95	-11.88
Experimental group D	Control group A	14.867*	1.284	.000	11.42	18.31
	Experimental group B	5.546*	1.245	.000	2.21	8.89
	Control group C	15.418*	1.319	.000	11.88	18.95

The multiple comparisons show that there was a statistically significant difference between the DG scores of control group A and experimental group B, as well as between the DG scores of control group A and experimental group D. Furthermore, there was a statistically significant difference between the mean DG scores of control C compared to experimental group B, as well as control group C compared to experimental group D. The analysis also showed a statistically significant difference between the mean DG scores of experimental group B and experimental group D. However, there was no statistically significant difference between the mean DG scores of control group A and control group C.

Despite the robustness of ANOVA to the non-normality of the data, which enabled the researcher to continue with the analysis, the researcher felt the need to confirm the results of ANOVA, hence the use of the non-parametric alternative to ANOVA, the Kruskall-Wallis test on the post-test results. The findings from Kruskall-Wallis are shown below.

4.3.2 Kruskal-Wallis test for DG scores of the four groups

The Kruskal-Wallis test was conducted on the learners' DG scores. The mean rank scores (see Table 4.25) show mean rank scores of 35.15 for control group A, 78.40 for experimental group B, 34.48 for control group C and 100.88 for experimental group D.

Table 4.25: Rank statistics for the groups achievement scores in drawing graphs (DG)

	Group	N	Mean Rank
DG Score	Control group A	31	35.15
	Experimental group B	35	78.40
	Control group C	28	34.48
	Experimental group D	33	100.88
Total		127	

The test statistics are shown in Table 4.26.

Table 4.26: Kruskal-Wallis test statistics of the groups

	DG Score
Chi-Square	78.894
Df	3
Asymp. Sig.	.000

The results show a Chi-square value of 8.894 at 3 degrees of freedom. The significance value of 0.000 indicates that there was a statistically significant difference in DG scores between two or more groups.

The Kruskall-Wallis test results only indicate that there is a statistically significant difference between some of the groups, therefore follow up tests are needed to indicate the groups which showed the differences. Hence the researcher used the Mann-Whitney U (MW) test to conduct multiple comparisons of the groups.

Mann-Whitney U comparison tests on the DG scores

The Mann-Whitney U test between control group A and experimental group B showed rank statistics with mean rank scores of 20.44 for control group A and 45.07 for experimental group B (see Table 4.27).

Table 4.27: Rank statistics for DG scores of control group A and experimental group B

	Group	N	Mean Rank	Sum of Ranks
DG Score	Control group A	31	20.44	633.50
	Experimental group B	35	45.07	1577.50
	Total	66		

Table 4.28 shows test statistics for the Mann-Whitney test on the DG scores of control group A and experimental group B.

Table 4.28: Mann-Whitney U test statistics for DG scores of control group A and experimental group B

	DG Score
Mann-Whitney U	137.500
Wilcoxon W	633.500
Z	-5.375
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney value was 137.500, the Z score was -5.375 and significance value of 0.000. The results indicate that there was a statistically significant difference between the

DG scores of the two groups, in favour of experimental group B. Hence the learners in experimental group B achieved better than those in control group A.

The MW test comparing DG scores for control group A and control group C showed mean rank scores of 29.74 for control group A and 30.29 for control group C (Table 4.29).

Table 4.29: Rank scores for DG scores of control group A and control group C

	Group	N	Mean Rank	Sum of Ranks
DG Score	Control group A	31	29.74	922.00
	Control group C	28	30.29	848.00
	Total	59		

Table 4.30 shows the test statistics for the Mann-Whitney U test between control group A and control group C.

Table 4.30: Mann-Whitney U test statistics of control group A and control group C

	DG Score
Mann-Whitney U	426.000
Wilcoxon W	922.000
Z	-.146
Asymp. Sig. (2-tailed)	.884

The Mann-Whitney U test statistic was 426.000 and the significance value was 0.884. Hence, the results indicate that there was no statistically significant difference between the DG scores of the two control groups.

The Mann-Whitney U test between control group A and experimental group D showed mean rank scores of 16.97 for control group A and 47.09 for experimental group D (Table 4.31).

Table 4.31: Rank statistics of DG scores of control group A and experimental group D

	Group	N	Mean Rank	Sum of Ranks
DG Score	Control group A	31	16.97	526.00
	Experimental group D	33	47.09	1554.00

	Total	64

The test statistics for the Mann-Whitney U test on the DG scores of control group A and experimental group D are shown in Table 4.32.

Table 4.32: Mann-Whitney U test statistics between control group A and experimental group D

	DG Score
Mann-Whitney U	30.000
Wilcoxon W	526.000
Z	-6.607
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 30.000, the Z score was -6.607 and the significance value was 0.000. Therefore, the results show a statistically significant difference between the DG scores of learners in control group A and experimental group D, in favour of experimental group D. Hence, the learners in experimental group D achieved higher scores in the post-test compared to those in control group A.

The Mann-Whitney U test between control group C and experimental group B showed mean rank scores of 18.09 for control group C and 43.13 for experimental group B (Table 4.33).

Table 4.33: Rank statistics for DG scores of experimental group B and control group C

	Group	N	Mean Rank	Sum of Ranks
DG Score	Experimental group B	35	43.13	1509.50
	Control group C	28	18.09	506.50
	Total	63		

The test statistics are shown in Table 4.34.

Table 4.34: Mann-Whitney U test statistics of DG scores between experimental group B and control group C

	DG Score
Mann-Whitney U	100.500
Wilcoxon W	506.500
Z	-5.511
Asymp. Sig. (2-tailed)	.000

The results showed a Mann-Whitney U statistic of 100.500 and a z statistic of -5.5111 with a significance value of 0.000. Hence there was a statistically significant difference between the DG scores of the two groups, in favour of experimental group B.

The Mann-Whitney test between control group C and experimental group D (table 4.35), showed mean rank scores of 15.11 for control group C and 44.48 for experimental group D.

Table 4.35: Rank statistics of DG scores between control group C and experimental group D

	Group	N	Mean Rank	Sum of Ranks
DG Score	Control group C	28	15.11	423.00
	Experimental group D	33	44.48	1468.00
	Total	61		

Table 4.36 shows the statistics for the Mann-Whitney U test of the DG scores between learners in control group C and those in experimental group D.

Table 4.36: Mann-Whitney U test statistics of DG scores between control group C and experimental group D

	DG Score
Mann-Whitney U	17.000
Wilcoxon W	423.000
Z	-6.528
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 17.000 with a z statistic of -6.528 and a significant value of 0.000. Thus, the results showed a statistically significant difference between the DG scores of learners in control group C and those in experimental group D, in favour of experimental group D.

The Mann-Whitney U test on the experimental group B and experimental group D showed mean rank scores of 15.11 and 44.48 for the two groups respectively (see table 4.37).

Table 4.37: Rank statistics for the DG scores of experimental group B and experimental group D

	Group	N	Mean Rank	Sum of Ranks
DG Score	Experimental group B	35	26.20	917.00
	Experimental group D	33	43.30	1429.00
	Total	68		

The test statistics are shown in Table 4.38.

Table 4.38: Mann-Whitney U test statistics for the DG scores of experimental group B and experimental group D

	DG Score
Mann-Whitney U	287.000
Wilcoxon W	917.000
Z	-3.570
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 287.000 with a Z score of -3.570 and a significance value of 0.000. The results indicated that there was a statistically significant difference between the DG scores of the two experimental groups in favor of experimental group D. Thus, even though both groups are experimental groups which were taught using GeoGebra, experimental group D achieved better scores than experimental group B.

Overally, parametric testing (ANOVA) and non-parametric testing (Kruskal-Wallis) both showed similar results that there was a statistically significant difference between the DG scores of the learners in the control groups and the learners in the experimental groups in favour of the experimental groups. Hence, the results suggest that learners who were taught linear functions using GeoGebra achieved higher scores on the drawing graphs of linear functions.

The researcher also used ANOVA and the Kruskal-Wallis test to analyze the achievement scores of learners on interpreting graphs (IG) of linear functions.

4.4 Groups' post-test results on interpreting graphs (IG) of linear functions

The researcher was also interested in how the learners achieved on interpreting graphs (IG) of linear functions. The IG scores were analyzed using both ANOVA and Kruskal-Wallis tests. The researcher analyzed the IG scores for the schools using the ANOVA test. The descriptive statistics for the ANOVA are shown in Table 4.39.

Table 4.39: Descriptive statistics for IG scores of the schools.

	N	Min	Max	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	Upper Bound
Control group A	31	0	15	6.68	3.487	.626	5.40	7.96
Experimental group B	35	7	19	13.23	3.473	.587	12.04	14.42
Control group C	28	3	14	7.57	3.214	.607	6.33	8.82
Experimental group D	33	11	21	17.67	2.986	.520	16.61	18.73

The statistics show that control group A had mean score 6.68 and standard deviation 3.487; experimental group B had mean 13.23 and standard deviation 3.473; control group C had mean 7.57 and standard deviation 3.214; while experimental group D had mean 17.67 and standard deviation 2.986. Table 4.40 shows the test statistics for the analysis.

Table 4.40: ANOVA for IG Scores of the schools

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	2512.454	3	837.485	76.923	.000

Within Groups	1339.136	123	10.887
Total	3851.591	126	

The results indicate that there was a statistically significant difference between the IG scores of two or more groups. Therefore multiple comparisons were done using the Bonferroni (Table 4.41) to identify the groups to which the difference(s) existed.

Table 4.41:Bonferrori multiple comparisons of the groups

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Control group A	Experimental group B	-6.551*	.814	.000	-8.73	-4.37
	Control group C	-.894	.860	1.000	-3.20	1.41
	Experimental group D	-10.989*	.825	.000	-13.20	-8.78
Experimental group B	Control group A	6.551*	.814	.000	4.37	8.73
	Control group C	5.657*	.837	.000	3.41	7.90
	Experimental group D	-4.438*	.801	.000	-6.59	-2.29
Control group C	Control group A	.894	.860	1.000	-1.41	3.20
	Experimental group B	-5.657*	.837	.000	-7.90	-3.41
	Experimental group D	-10.095*	.848	.000	-12.37	-7.82
Experimental group D	Control group A	10.989*	.825	.000	8.78	13.20
	Experimental group B	4.438*	.801	.000	2.29	6.59
	Control group C	10.095*	.848	.000	7.82	12.37

The comparisons showed that there were statistically significant differences between the IG scores of the control group A and experimental group B, as well as between the IG scores of control group A and experimental group D. Furthermore, the tests also showed statistically significant differences between the IG scores of control group C and experimental group B, as well as between control group C and experimental group D. There was also a statistically significant difference between the IG scores of experimental group B compared to experimental group D. However, there was no statistically significant difference between control group A and control group C.

4.4.2 Kruskal-Wallis test for the IG scores for the schools.

The KW test was conducted on the learners' IG scores. The mean rank scores (see table 4.42) show that the mean rank IG score was 31.87 for control group A, 76.37 for experimental group B, 36.63 for control group C and 104.29 for experimental group D.

Table 4.42: Rank statistics for the Kruskal-Wallis test

	Group	N	Mean Rank
IG Score	Control group A	31	31.87
	Experimental group B	35	76.37
	Control group C	28	36.63
	Experimental group D	33	104.29
	Total	127	

The test statistics for the Kruskal-Wallis test are shown in table 4.43.

Table 4.43: Kruskal-Wallis test statistics for the groups' achievement scores in interpreting graphs (IG)

	IG Score
Chi-Square	83.106
Df	3
Asymp. Sig.	.000

The results of the analysis indicate that there is a statistically significant difference between the IG scores of two or more of the groups, therefore comparison tests were conducted using Mann-Whitney U tests.

The Mann-Whitney U (MW) on the IG scores

The Mann-Whitney U test comparing control group A and experimental group B (table 4.44) showed mean rank scores of 19.29 for control group A and 46.09 for experimental group B.

Table 4.44: Rank statistics for the Mann-Whitney U test of IG scores between control group A and experimental group B

	Group	N	Mean Rank	Sum of Ranks
IG Score	Control group A	31	19.29	598.00

Experimental group B	35	46.09	1613.00
Total	66		

The test statistics for the Mann-Whitney U test are shown in Table 4.45.

Table 4.45: Mann-Whitney U test statistics between control group A and experimental group B

	IG Score
Mann-Whitney U	102.000
Wilcoxon W	598.000
Z	-5.678
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 102.000 and a z score of -5.678 with a significance value of 0.000. Hence, there was a statistically significant difference between the IG scores of the two groups; in favor of experimental group B thus learners in experimental group B achieved higher scores than the learners in control group A, on IG items in the post-test.

Furthermore, the Mann-Whitney test on the control group A and control group C (Table 4.46) showed mean rank scores of 28.24 for control group A and 31.95 for control group C.

Table 4.46: Rank statistics for control group A and control group C

	Group	N	Mean Rank	Sum of Ranks
IG Score	Control group A	31	28.24	875.50
	Control group C	28	31.95	894.50
	Total	59		

Table 4.47 shows the test statistics for the Mann-Whitney U test.

Table 4.47: Mann-Whitney U test statistics of IG scores between control group A and control group C

	IG Score
Mann-Whitney U	379.500
Wilcoxon W	875.500
Z	-.833
Asymp. Sig. (2-tailed)	.405

The test results indicate that there is no statistically significant difference between the IG scores of control group A and control group C. Hence the learners in the two control groups are of comparable achievement levels.

The Mann-Whitney U test comparing Control group A and experimental group D showed mean score rank scores of 16.34 for control group A and 47.68 for experimental group D (Table 4.48).

Table 4.48: Rank statistics for control group A and experimental group D

	Group	N	Mean Rank	Sum of Ranks
IG Score	Control group A	31	16.34	506.50
	Experimental group D	33	47.68	1573.50
	Total	64		

The test statistics for this test are shown in Table 4.49.

Table 4.49: Mann-Whitney U test statistics of IG scores between control group A and experimental group D

	IG Score
Mann-Whitney U	10.500
Wilcoxon W	506.500
Z	-6.768
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value is 10.500, with a z score of -6.768 and a significance value 0.000. Hence, there is a statistically significant difference between the IG scores of the two groups, in favour of experimental group D.

The comparison between experimental group B and experimental group D (table 4.50) using the Mann-Whitney U test shows mean rank scores of 23.71 for experimental group B and 45.94 for experimental group D.

Table 4.50: Rank statistics of experimental group B and experimental group D

	Group	N	Mean Rank	Sum of Ranks
IG Score	Experimental group B	35	23.71	830.00
	Experimental group D	33	45.94	1516.00
	Total	68		

The statistics for the Mann-Whitney U test are shown in Table 4.51.

Table 4.51: Mann-Whitney U test statistics of IG scores between experimental group B and experimental group D

	IG Score
Mann-Whitney U	200.000
Wilcoxon W	830.000
Z	-4.681
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value is 200.000 with a z score of -4.681 and a significance value of 0.000 indicating that there is a statistically significant difference between the IG scores of the two experimental groups, in favour of experimental group D.

The Mann-Whitney test between experimental group B and control group C (Table 4.52) shows mean rank scores of 42.57 for experimental group B and 18.79 for control group C.

Table 4.52: Rank statistics of experimental group B and control group C

	Group	N	Mean Rank	Sum of Ranks
IG Score	Experimental group B	35	42.57	1490.00
	Control group C	28	18.79	526.00
	Total	63		

The test statistics are shown in Table 4.53.

Table 4.53: Mann-Whitney U test statistics of IG scores between experimental group B and control group C

	IG Score
Mann-Whitney U	120.000
Wilcoxon W	526.000
Z	-5.141
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value was 120.000 with a z score of -5.141 and a significance value of 0.000. Hence, there is a statistically significant difference between the IG scores of the two groups, favouring experimental group B. Therefore, the results show that the learners in experimental group B significantly achieved higher scores than the learners in control group C.

Table 4.54 shows the descriptive statistics from the Mann-Whitney U test conducted between control group C and experimental group D.

Table 4.54: Rank statistics for control group C and experimental group D

	Group	N	Mean Rank	Sum of Ranks
IG Score	Control group C	28	14.89	417.00
	Experimental group D	33	44.67	1474.00
	Total	61		

The mean rank score for control group C is 14.89 and the mean rank score for experimental group D is 44.67. The test statistics are shown in Table 4.55.

Table 4.55: Mann-Whitney U test statistics of IG scores between control C and experimental D

	IG Score
Mann-Whitney U	11.000
Wilcoxon W	417.000
Z	-6.567
Asymp. Sig. (2-tailed)	.000

The Mann-Whitney U value is 11.000 with a z score of -6.567 and a significance value of 0.000. Hence the results indicate that there is a statistically significant difference between the IG scores of the two groups, in favour of experimental group D. Hence the learners in experimental group D achieved better than those in control group C.

Overall both parametric (ANOVA) and non-parametric (Kruskal-Wallis) testing shows that there is a statistically significant difference between the IG scores of learners in the control groups and learners in the experimental groups, in favour of the experimental groups. Thus, the learners who were taught linear functions with GeoGebra achieved higher scores than the learners who were not taught linear functions with GeoGebra.

4.5 Addressing the research questions

The results of the analyses (see section 4.2 to section 4.4) were used to address the three research questions in this study.

4.5.1 Research question one

Is there any difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions?

The corresponding hypothesis was used to address this research question. Data was analyzed in two ways, parametric (ANOVA) and non-parametric (Kruskal-Wallis) testing. Inference was done at 95% confidence interval. Moreover, ANOVA analysis of the total achievement scores of the schools (see Table 4.6) shows that there is a statistically significant difference between the total achieved scores of learners in two or more of the groups. The findings from the Bonferroni post hoc (refer to Table 4.7) show that there is a statistically significant difference between the total scores of learners of control group A and experimental group B; control group A and experimental group D; as well as of control group D and experimental group D. However, there is no statistically significant difference between total scores of learners in control group A and control group C. The Kruskal-Wallis (see Table 4.9) test of the total scores achieved in linear functions in the posttest also showed results that are similar to the ANOVA analyses.

Therefore, on the bases of these findings it was concluded that there is a statistically significant difference between the achievements of learners who were taught linear functions using GeoGebra compared to the learners who were not taught linear functions using GeoGebra. Hence, the null hypothesis: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions), was rejected in favour of the alternative hypothesis: There is a statistically significant difference between the achievement scores of learners who were exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions.

4.5.2 Research question two

Is there any statistically significant difference between the achievement scores of learners exposed to the GeoGebra and learners who were not exposed to GeoGebra in drawing of graphs of linear functions (DG)?

The corresponding hypothesis: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in drawing graphs of linear functions, was used to address the research questions. Analyses were done using both parametric and non-parametric tests. ANOVA (see Table 4.22 and Table 4.23) analysis on the DG scores achieved in the post-test showed that there was a statistically significant difference between the DG scores of learners in the control group and those in the experimental group. Furthermore, both ANOVA (Table 4.23) and Kruskal-Wallis (table 4.26) analyses showed that there was a statistically significant difference between DG scores of learners in the control groups and those in the experimental groups, in favour of the experimental groups.

Based on these findings, we can conclude that learners who were taught linear functions using GeoGebra achieved higher scores on drawing graphs questions in the post-test compared to the learners who were not taught linear functions using GeoGebra. Hence, the null hypothesis: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in drawing of linear functions, was rejected in favour of the alternative hypothesis: There is a statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in drawing of linear functions.

4.5.3 Research question three

Is there any statistically significant difference between the achievement scores of the learners exposed to GeoGebra and the learners who were not exposed to GeoGebra in interpretations of linear functions and their graphs (IG)?

The corresponding hypothesis: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in the interpretations of linear functions, was used to address the research questions.

Analyses were done using both parametric and non-parametric tests. ANOVA analysis on the IG scores achieved in the post-test showed that there was a statistically significant difference between the IG scores of learners in the control group and those in the experimental group. Furthermore, both ANOVA (Table 4.40) and Kruskal-Wallis (Table 4.43) analyses showed that there was a statistically significant difference between IG scores of learners in the control groups and those in the experimental groups, in favour of the experimental groups.

Based on these findings, we can conclude that learners who were taught linear functions using GeoGebra achieved higher scores on interpreting graph questions in the post-test compared to the learners who were not taught linear functions using GeoGebra. Hence, the null hypothesis: There is no statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in the interpretation of linear functions, was rejected in favour of the alternative hypothesis: There is a statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in interpreting linear functions.

Summary

This chapter focused on analyses of data from the study. ANOVA and the Kruskal-Wallis test were used in data analyses. The findings from the study showed that there was a statistically significant difference in achievement scores of learners who were exposed to GeoGebra and the learners who were not exposed to GeoGebra. Research questions were addressed in line with the findings of the study.

CHAPTER 5

DISCUSSION OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This study aimed to investigate the effects of integrating GeoGebra into the teaching of linear functions on the achievement of Grade 9 learners. In this chapter, the study is summarized, the findings are discussed, conclusions are made, and the researcher gives recommendations for future research on the integration of GeoGebra into the teaching and learning of mathematics in South Africa.

5.1 Summary of the study

This study was prompted by the need to address underperformance in mathematics, in particular the poor achievement on functions displayed by learners in the Grade 12 examination, based on information supplied by the diagnostic reports on learners' performance in the examinations. This study set out to investigate the effects of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district, Limpopo Province. The choice of Grade 9 was guided by the fact that the bases of functions lie in the senior phase (Grades 7–9) mathematics thus there is a need to improve the teaching and learning in that phase in order to achieve better in the higher grades, in particular in the Grade 12 final examination.

The study was guided by APOS theory which, in accordance with constructivist theories posits that an individual needs to construct the necessary cognitive structures in order to make sense of mathematical concepts. The theoretical framework was inspired by the positivist paradigm which puts emphasis on the fact that reality does exist and can be investigated using scientific inquiry with a focus on objective reality.

5.1.1 Purpose of the study

The purpose of this study was to investigate the effect of integrating GeoGebra into the teaching of linear functions on Grade 9 learners' achievement in Mopani district.

5.1.2 Research Questions

This study provides answers to the following questions:

1 Is there any difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions?

2 Is there any statistically significant difference between the achievement scores of learners exposed to the GeoGebra and learners who were not exposed to GeoGebra in drawing of graphs of linear functions (DG)?

3 Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in interpretations of linear functions and their graphs (IG)?

5.1.3 Research Design

The study followed a quasi-experimental non-equivalent group design. Two classes were the experimental groups and two classes were the control groups. The study was quasi-experimental because the learners who participated were not randomly assigned; instead the classes at the particular schools were used as full classes to avoid disruption to the smooth running of the academic programme and all the schools only had one grade 9 class. The researcher used the non-equivalent pretest-post-test control group design with four groups.

5.1.4 Sample and sampling techniques

The sample for this study consisted of Grade 9 learners from four schools in a circuit in Mopani district.

The non-probability sampling technique was used to select the circuit as the focus of the study. Non-probability sampling involves the selection of sampling units from a population using non-random processes. The study focused on Grade 9 learners in Mopani district but the circuit was sampled for its convenience and availability to the researcher (non-probability sampling is called convenience sampling). Purposive sampling was used since the circuit is also a serial underperforming circuit in the district, occupying the bottom position when it comes to Grade 12 performance. This type of study would also be one of the first of its kind in the circuit and beneficial to both educators and learners in the circuit schools. In addition, the study tries to find ways of improving teaching and learning mathematics in this particular circuit.

Random sampling was used in the circuit, initially to select the two schools which were used in the pilot study from the 10 secondary schools which offer Grade 9 in the circuit and thereafter to select the other four schools which were used in the main study. At the selected schools cluster sampling was used to select the participants at the respective schools, in other words, intact Grade 9 classes were selected for the study, with no random sampling involved in the selection of participants.

5.1.5 Findings

The results of this study were analysed in the context of the research questions.

Research question one

Is there any difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in linear functions?
The results of ANOVA and Kruskal-Wallis tests show that there was a statistically significant difference between the achievement scores of learners exposed to GeoGebra (experimental group) and learners who were not exposed to GeoGebra (control group) in linear functions.

Research question two

Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in drawing graphs of linear functions (DG)?

The results of both ANOVA and Kruskal-Wallis show that there was a statistically significant difference between the achievement scores of learners exposed to GeoGebra (experimental group) and learners who were not exposed to GeoGebra (control group) in drawing graphs.

Research question three

Is there any statistically significant difference between the achievement scores of learners exposed to GeoGebra and learners who were not exposed to GeoGebra in interpretations of linear functions and their graphs (IG)?

Both ANOVA and Kruskal-Wallis show that there were statistically significant differences between the achievement scores of learners exposed to GeoGebra (experimental group) and learners who were not exposed to GeoGebra (control group) in interpreting graphs.

5.2 Discussion of Findings

Pretest results showed that there were no statistically significant differences between the scores of learners in all four groups. This finding indicates that the learners' knowledge of linear functions was at a comparable level and they did not bring any valuable knowledge of linear functions into the study.

Findings from both parametric (ANOVA) and non-parametric (Kruskal-Wallis) analyses of the post-test showed that there was a statistically significant difference in the achievement of learners in the experimental group compared to those in the control group with regard to all three cases specified in the research questions, which are:

i) Achievement in linear functions
ii) Achievement in drawing graphs of linear functions.
iii) Achievement in interpreting graphs of linear functions.

From the findings of the study, it is evident that learners who were exposed to GeoGebra (experimental group) performed better in linear functions compared to those who were not exposed to GeoGebra (control group). Hence, this finding suggests that the use of GeoGebra in the teaching and learning of linear functions enhanced learners' performance and achievement in linear functions. This finding is in line with the findings of Praveen and Leong (2013) on the effectiveness of GeoGebra in teaching and learning mathematics. Furthermore, the findings of this study agree with Ogbonnaya and Mji (2013) that the use of graphing software enhanced learners' achievement in hyperbolic functions. In this case GeoGebra enhanced learning of linear functions.

The learners in the experimental group were exposed to an innovative way of learning linear functions through the use of GeoGebra, which most likely captured their attention and interest during mathematics lessons.

In line with Hohenwarter and Jones (2007), the interactive and dynamic nature of GeoGebra allowed the learners in the experimental group to draw, compare, and analyze the linear graphs with ease. They were also afforded the opportunity to explore linear functions, alone or with their peers, which enabled them to better understand the notion of linear functions. Learners in the experimental groups could also check and assess the correctness and accuracy of their own without having to wait for the educator's assistance. They could also draw and analyze several graphs at the same time without having to go through the tiresome process of sketching the graphs. As a result, while using GeoGebra the learners had more time to answer higher order questions and this could have contributed towards the higher achievement scores in the experimental group.

This could also be attributed to the fact that learners in rural schools usually have problems of communicating between themselves or with educators using the language of learning and teaching, namely English, therefore learners in the experimental group were afforded an opportunity to break the barrier since they no longer needed to rely on language to communicate. GeoGebra allowed a shift on the part of learners from relying solely on the educator during the lessons because they could now answer more questions than those provided by the educator, which implies more practice and ultimately more clarity for the learners. They could now interact and explore concepts on their own or with peers. In other words, GeoGebra enhanced learners' curiosity and inquisitiveness.

Furthermore, it can be said in line with Becta (2003) that ICT increases learners' opportunities to cooperate and collaborate with others and it was also observed in the study that GeoGebra did provide the learners with more options for collaboration and cooperation. Therefore, the learners had more chances to reflect and discuss their work.

Another factor that may be attributed to these findings is the young generation of learners' love for technology (Bester & Brand, 2013), thus learners in the experimental group most likely enjoyed the learning experience which also resulted in them paying greater attention to the concepts being taught.

From the results it was observed that there was a statistically significant difference between the achievement of learners in the two experimental groups; learners in experimental D performed better than those in experimental B. This finding may be attributed to several factors. Although the educators who taught the experimental groups were comparable in terms of academic and professional qualifications as well as the experience in teaching mathematics in Grade 9, other factors like the educators' pedagogical and classroom management skills, levels of discipline among learners could account for the differences. Also since the learners are from different school environments, the levels of learner discipline and motivation towards learning could also have played a role in the differences found in this study.

5.3 Conclusions

In this study using GeoGebra to teach linear functions resulted in higher achievement scores in the experimental groups. It proved to be more effective in enhancing learners' achievement, particularly on the topic of linear functions. The hands-on and interactive approach of the software had a positive effect and enabled learners to understand concepts much better than those who had not been exposed to the software. A statistically significant result for both experimental groups compared to the control groups also serves to add weight to the results from previous studies by Zengin (2012) and Leong (2013) among others, on the effectiveness of GeoGebra software on learner achievement.

5.4 Implication of results

The findings of this study show that use of GeoGebra in teaching and learning mathematics enhanced learners' achievement in linear functions. These findings have wide implications for teaching and learning mathematics. Hence the researcher recommends that educators integrate GeoGebra into their teaching activities, since it has proved to be effective in enhancing achievement. By transferring the learners' efforts from the tedious task of drawing graphs manually, the software allowed them to focus on other relevant issues, such as exploration and making sense of linear functions and creating their own understanding of the concepts.

Before the study the researcher, in interactions with other mathematics teachers, found that not many of them were able to manipulate and use GeoGebra software effectively due to lack of adequate orientation. Shelley et al. (2008) assert that if integrated properly, digital media also has the capability to stimulate imagination and develop critical thinking skills while allowing students to take an active role in their own learning. They go on to say that teachers need to be well prepared in all aspects of the Technology Pedagogical Knowledge Kit in order to facilitate learning in an ICT integrated learning environment. These researchers also note that introducing ICTs in isolation could have harmful effects which may result in educators shying away from technology use. The researcher thus recommends that the DoBE should not simply provide the software but also follow up with training on its use. Training workshops are a crucial element for teachers to enable them to integrate the software as part of the teaching process.

The findings of this study also have implications for pre-service teacher training. Mathematics pre-service teachers, in particular, should be trained using graphing software like GeoGebra so that they become familiar with the use of such technologies in teaching and learning mathematics. This will serve as modelling process for the teachers who will likely become confident to use such technologies in their own classes. This implies that teacher training institutions and lecturers should embrace the use of technology in teaching and learning and keep abreast of the affordances of technology in teaching and learning.

Schools should also be supplied with a greater number of computers in order to decrease the ratio of learners to computers in schools. Teachers dread having to plan a lesson knowing that several learners will be sharing only one computer. Such a situation is not conducive to progress in learning with technology.

The DoBE should upgrade the available resources at schools to provide them with online access. GeoGebra is dynamic software which also allows users to discuss with other users on the GeoGebra wiki and user forum. Thus teachers can access help and upgrade themselves instead of having to rely solely on workshops provided by the department.

GeoGebra proved to be an effective tool in teaching linear functions to Grade 9 learners in Mopani district. The researcher thus makes the recommendation that teachers should adopt the software to teach mathematics as it is user friendly and allows them to plan effectively for their lessons. GeoGebra was used for Grade 9 learners but could be extended to teach many other concepts across the different grades. It can also be used to teach several other topics in the CAPS curriculum. Hence its relevance in the mathematics classroom has no limits.

Teachers should make use of the dynamic nature of the software to capture learners' interest and to keep them motivated. The software provides the learners with a platform where they can explore and test ideas as they build their own structures about concepts. The interactive nature of GeoGebra activates the inquisitive nature of learners' minds and prompts them to be more exploratory in their learning.

Teachers and other people involved in education should carry out more studies on the use and effectiveness of using the GeoGebra software as it would iron out issues in education, especially in mathematics. Further research would enable teachers and curriculum planners to identify problem areas as well as provide the means to solve those issues. It also provides them with new knowledge.

5.5 Limitations of the study

The limitation of this study is that learners' achievement was measured by the marks obtained on the test only, while not addressing other factors that are needed for learners to achieve, such as motivation.

The study focused on one topic (linear functions) and on Grade 9 learners only to investigate the effect of integrating GeoGebra in the teaching and learning of linear functions on Grade 9 learners' achievement. Therefore, generalizing the findings of this study to other topics in mathematics and other grade levels should be done with caution. The study was also conducted in Mopani district only, so repeating the study in a different place with different learners might not produce similar findings.

5.6 Recommendation for future study

This study could act as a stepping stone for further research. For instance, studies could be done to investigate how to make use of learners' smart phones to integrate the GeoGebra software as part of the learners' mathematical learning in a variety of situations within and outside the classroom.

Further research should be conducted on the effectiveness of GeoGebra in teaching and learning mathematics in other levels of learning and to teach other topics in mathematics, even in other learning areas.

The researcher further recommends qualitative studies to assess the learners' perceptions towards use of GeoGebra and other software in learning mathematics. The studies should also assess educators' attitudes and perceptions towards use and integration of ICTs into the teaching of mathematics.

5.7 Concluding remarks

The study has shown that the use of GeoGebra enhances learners' achievement in linear functions. Based on the findings of this study the researcher recommends use of GeoGebra in the teaching and learning of linear functions in particular and of mathematics in general. Any teaching method that enhances learner achievement goes a long way towards solving the problem of poor achievement in mathematics, in South Africa.

REFERENCES

Anderman, E. M., & Anderman, L. H. (2012). *Classroom motivation*. New York: Pearson Education.

Bakar, K. A., Ayub, A. F. M., Luan, W. S., & Tarmizi, R. A. (2002). Exploring secondary school students' motivation using technologies in teaching and learning mathematics. *Procedia Social and Behavioral Sciences,* 2, 4650-4654.

BECTA. (2003). What the research says about ICT and Maths. Coventry: BECTA. Retrieved December 18 2016 from http://becta.org.uk/research

Bennet, J. O. (2014). *Using and understanding mathematics: A quantitative research approach* (6th edn). New York: Pearson Education

Bester, G., & Brand, L. (2013). The effect of using technology on learner attention and achievement in the classroom. *South African Journal of Education*, *33,* (2), 1-15.

Biyane, G. (2007). Student's perception of space at the Central University of Technology (Bloemfontein) and Free State School of Nursing Campus: An exploratory study. Retrieved November 21 2015 from http://hdl.handle.net/10500315

Blossier, M. (2014). GeoGebra: The graphing calculator for functions, geometry, algebra, calculus, statistics and 3D math: Dynamic Mathematics for Learning and Teaching. Retrieved November 21 2015 from www.geogebra.org.

Brijall, D., & Maharaj, A. (2009a). Using an inductive approach for definition making: Monotonicity and boundedness of sequences. *Pythagoras*, 70, 68-79.

Bruce, C. D. (2012). Technology in the mathematics classroom: Harnessing the power of interactive whiteboards. Research into practice: Ontario Association of Deans of Education. Research Monograph, winter, 1-4.

Bu, L., & Alghazo, Y. (2011). Reflections and conclusions. In L. Bu & R. Schoen (Eds.), *Model-centered learning: Pathways to mathematical understanding using GeoGebra,* 243-247. Rotterdam: Sense Publishers.

Bu, L., & Schoen, R. (2011). GeoGebra for model-centered learning in mathematics education. In L. Bu & R. Schoen (Eds.), *Model-centered learning: Pathways to mathematical understanding using GeoGebra*, 1-7. Rotterdam: Sense Publishers.

Burke, M., & Kennedy, P. (2011). GeoGebra: From simulation to formulation in teacher preparation and in-service programs. Cited in Bu. L and Schoen. R (Eds.). *Model-centred learning*, 57-72, Rotterdam, Netherlands, Sense Publishers.

Cassim, K. M. & Obono, E. (2011). On the factors affecting the adoption of technology for teaching of word problems. *Proceedings of the World Congress on Engineering and Computer Science*, 1, 269-276.

Centre for Technology. (2007). Why should a teacher use technology in his or her mathematics classroom? SRI International. Retrieved June 15 2015 from https://education.ti.com/.../researchnotes.

Check, J., & Schutt, R. K. (2012). *Research methods in education.* California: SAGE.

Chigona, A., Chigona, W., Kayongo, P., & Kausa, M. (2010). An empirical survey on domestication of ICT in schools in disadvantaged communities in South Africa. International Journal of Education and Development using Information and Communication Technology, 6(2), 1c.

Chitsike, M. (2013). Spot-on mathematics: Learners book. Sandton: Heinemann.

Daher, W., & Anabousy, A (2015). Students' conceptions of function transformation in a dynamic mathematical environment. International Journal for Mathematics Teaching and Learning, 16(4), 97-112.

Demir, O. (2012). Students' concept development and understanding of sine and cosine functions: A new theoretical and educational approach. (Unpublished masters dissertation). University of Amsterdam, Netherlands.

Department of Basic Education. (2012). Curriculum and Assessment Policy Statement. Pretoria: Government Printing Works.

.

Department of Education. (2007).White Paper on Education: *Guidelines for Teacher Training and Professional Development in ICT*. Pretoria: Government Printing Works.

Dogan, M. (2011). *The role of dynamic geometry software in the process of learning: GeoGebra example about triangles.* Journal of Human Sciences, 8(1), 1441-1458

Dubinsky, E., & McDonald, M. A. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. Retrieved December 14 2014 from www.maths.wisc.edu/.../ICMIPAPE.pdf.

Ersin, I. (2013). Introducing parameters of linear functions at undergraduate level: Use of GeoGebra. *Mevlana International Journal of Education,* 3(3), 77-84.

Emeny, W. (2010). Using ICT to enhance learning. Retrieved October 16 2016 from www.greatmathsteachingideas.com..

Ertmer, P. A. (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational Technology Research and Development*, *53*(4), 25-39.

Flanagan, L., & Jacobsen, M. (2003). Technology leadership for the twenty-first century principal. *Journal of Educational Administration*, *41*(2), 124-142.

Falinga, G. (2015). Gauteng's paperless classrooms. Retrieved July 17 2015 from www.iol.co.za/…/paperless-classrooms.

Field, A. (2013). *Discovering statistics using IBM SPSS statistics* London, SAGE.

Gay, L. R., Mills, G. E., & Airasian, P. W. (2011). *Educational research: Competencies for analysis and applications*. New York: Pearson Education International.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In Denzin, N. K. & Lincoln, Y. S. (Eds), *Handbook of qualitative research*, 105-117. Thousand Oaks: SAGE.

Gweshe, L. C. (2014). The effect of using computer assisted instruction on teaching circle geometry in Grade 11. (Unpublished Masters dissertation). University of South Africa.

Gyongyosi, W. E. (2010). Teaching and learning mathematics through games and activities. *Acta Electrotechnica et Informatica*, 12(3):23-26

Herceg, D., & Herceg, D. (2010). Numerical integration with GeoGebra in high school. *The International Journal for Technology in Mathematics Education, 16*(2), 83-86.

Hohenwarter, M., & Fuchs, K. (2004). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. Paper presented at the Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference, Pecs, Hungary.

Hohenwarter, M., & Jones, K. (2007). *Ways of linking geometry and algebra: The case of GeoGebra.* Paper presented at the Proceedings of the British Society for Research into Learning Mathematics, 27(3), 126-131, University of Northampton, UK: BSRLM.

.

Hohenwarter, M., & Lavicza, S. (2007). *Mathematics teacher development with ICT: Towards an international GeoGebra institute.* Paper presented at the Proceedings of the British Society for Research into Learning Mathematics, 27(3), 49-54, University of Northampton, UK: BSRLM.

Hutkemri, Z. (2014). Impact of using GeoGebra on students' conceptual and procedural knowledge of limit function. *Mediterranean Journal of Social Sciences, 5(*22), 873-881.

Hutkemri, Z., & Effandi, Z. (2012). The effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science, 8*(11), 102-106.

Jojo, Z. M. M. (2011). An APOS exploration of conceptual understanding of the chain rule in calculus by first-year engineering students. (Unpublished PhD dissertation). University of KwaZulu-Natal,

Kagelelo, W. K., & Nkwalume, A. A. (2016). The potential and challenges of using GeoGebra to teach geometrical constructions in Botswana junior schools: The case off Gaborone West schools. Proceedings 24th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education

(SAARMSTE), 12-15 January 2016, Pretoria-South Africa, Tshwane University of Technology, 9.

Kahveci, M. (2010). Students' perception to use of technology for learning: Measurement integrity of the modified fennema-sherman attitude scales. T*he Turkish Online Journal of Educational Technology, 9(1), 185-201.*

Kaput, J. (2007). Technology becoming infrastructural in mathematics education: Models and modeling as foundations for the future in mathematics education, cited in Centre for Technology (2007). Why should a teacher use technology in his or her mathematics classroom? SRI International. Retrieved on June 6 2015. from https://education.ti.com/.../researchnotes.

Kirschner, P., & Selinger, M. (2003). The state of affairs of teacher education with respect to information and communications technology. *Technology, Pedagogy and Education 12*(1), 5 -17.

Kllogjeri, P., & Kllogjeri, Q. (2011). GeoGebra: A very effective tool for teaching mathematical concepts and properties. *Proceedings of the International GeoGebra Conference for Southern Europe*, 90-98.

Laridon, P., Barnes, H., Kitto, A., Myburg, M., Pike, M., Scheiber, J., Sigabi M., & Wilson, H. (2004). *Classroom mathematics: Grade 10 learners' book.* Sandton: Heinemann.

Leihnhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions graphs and graphing: Tasks, learning and teaching. *American Educational Research, Review of Educational Research, 60(*1), 1-64.

Leong, K. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. The Malaysian Online journal of Educational Technology, 1(4), 1-11.

Little, C. (2008). Interactive geometry in CLR: Old barriers. *Proceedings of the British Studies for Research in Learning Mathematics*, University of Southampton, UK: BSRLM.

Lobato, J., & Siebert, D. (2002). Quantitative reasoning in a reconceived view of transfer. *Journal of Mathematical Behavior*, 21, 87-116.

Mackenzi, D., & Wajcman, J. (1985). *The social shaping of technology: How the refrigerator got its hum.* Milton Keynes: Open University Press.

Macekova, M. A. (2013). Case study: Teaching reflection secondary vocational school using interactive whiteboard and GeoGebra. Proceedings of the *International Journal of Science, Commerce and Humanities, 1*(8), University of Konstantina, Filozofa, Nitra. .

Maryla, B., & Cohen, S. (2003). Managing ICTs in South African schools: A guide for school principals. Retrieved on February 14 2015 from www.education.gov.za.

McMillan, J., & Schumacher, S. (2014). *Research in education: Evidence-based inquiry*. Essex: Pearson.

Mdlongwa, T. (2012). Information and Communication Technology as a means of enhancing education in schools in South Africa: challenges, benefits and recommendations. Africa Institute of South Africa (AISA) Policy Brief Number 80.

Means, B., & Haertel, G. (2004). *Using technology evaluation to enhance student learning.* New York: Teachers College Press.

Mehmet, B., Hanife, U. A., Gurcan, K., & Veysel, A. (2016). The effects of GeoGebra on third grade primary students' academic achievement in fractions. *IEJME Mathematics Education, 11(*2), 347-355.

Mlitwa, N., & Koranteng, K, (2013). Integration of ICT into curricula in Western Cape schools: The activity theory perspective. The Journal of Community Informatics, 9(4), 957-969.

Mthethwa, M. Z. (2015). Application of GeoGebra on Euclidean geometry in rural high schools: Grade 11 learners. (Unpublished Masters thesis).University of Zululand, South Africa

Mustafa, A. (2015). The impact of teaching mathematics with GeoGebra on the conceptual understanding of limits and continuity: The case of Turkish gifted and talented students.(Unpublished Masters dissertation). İhsan Doğramacı Bilkent University.

National Senior Certificate.(2015). *Diagnostic Report*. Department of Basic Education: Pretoria.

National Senior Certificate.(2015).Examination Report. Department of Basic Education: Pretoria.

Ntombovuyo, K. (2006). Investigation on the effective use of information technologies for primary school teachers in the transfer of knowledge. (Unpublished Masters dissertation). University of Stellenbosch.

Ogbonnaya, U. I. (2008). Teachers' background, professional development and teaching practices as predictors of students' achievement in mathematics. *Proceedings of 163 16th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education.* Maseru, 14-18 January 2008: SAARMSTE.

Ogbonnaya, U. I. (2010). Improving the teaching and learning of parabolic functions by the use of information and communication technology (ICT). *African Journal of Research in MST Education, 14*(1), 49-60.

Ogbonnaya, U. I., & Chimuka, A. (2016). Joyful learning of mathematics using GeoGebra. *Proceedings of 8th Annual International Conference on Education and New Learning Technologies (Edulearn)*, Barcelona-Spain, 4-6 July, 2016, 4301-4307.

Ogbonnaya, U. I., & Mji, A. (2013). Enhancing students' learning of hyperbolic functions by the use of information and communication technology (ICT). In L. Gómez Chova, A. López Martínez, I. Candel Torres, *Proceedings of 5th Annual International Conference on Education and New Learning Technologies (Edulearn),* Barcelona-Spain, 1-3 July 2013, 5619 -5326.

Ogunrinade, S.O., Ogbonnaya, U. I., & Akintade, C. A. (2016). The effectiveness of computer assisted instruction on students' achievement in solid geometry in Ogun State, Nigeria. In Dichaba, M. M., & Sotayo, M. A. O. *Proceedings of South Africa International Conference on Education*, Pretoria, 19–21 September, 2016, 314–325.

Oldknow, A. J., & Taylor, R. (2000). *Teaching mathematics with ICT*. London: Continuum.

Ormrod, J. E. (2008). How motivation affects learning and behavior. Pearson Allyn Bacon Prentice Hall.

Ostertagova, E., Ostertag, O., & Kovacji, J. (2014). Methodology and application of the Kruskall-Wallis test.Applied Mechanics and Materials, 611, 115-120.

Pan African research agenda on the pedagogical integration of information and communication technologies, South Africa (PanAf) (2008-2011). Retrieved on February 14 2015 from http://www.observatoiretic.org

Polit, D. F., & Beck, C. T. (2012). *Nursing research: Generating evidence for nursing practice*. Philadelphia: Lippincott.

Postelnicu, V. (2011). Student difficulties with linearity and linear functions and teachers' understanding of student difficulties. (Unpublished Masters dissertation). Arizona State University.

Praveen, S., & Leong, K. (2013). Effectiveness of using GeoGebra on students' understanding of circles. The Malaysian Online journal of Educational Technology, 1(4), 1-11.

Preiner, J. (2008). Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra. (Unpublished Masters dissertation). University of Salzburg.

Rahman, M. H. A., & Puteh, M. (2016). Learning trigonometry using GeoGebra learning module: Are underachiever pupils motivated? *AIP Conference Proceedings,* 1750(1), 040001.

Rahul, C. K., Praveen, K. C., & Achintya, S. (2014). Impact on students' achievement in teaching mathematics using GeoGebra. 2014 *IEEE Sixth International Conference on Technology for Education, Amrita University, 18-21 December 2014.*

Saha, R. A., Ayub, A. F. M., & Tarmizi, A. R. (2010). The effects of GeoGebra on mathematics achievement: Enlightening coordinate geometry learning. International Conference on Mathematics Education, *Procedia Social and Behavioral Sciences, Malacca-Malaysia,* 13-14 December 2010, 8, 686-693.

Schmider, E., Ziegler, M., Danay, E., Beye, L., & Buhner, M. (2010). Is it really robust? Reinvestigating the robustness of ANOVA against violations of the normal distribution assumption. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences 6*(4), 147-151.

Shelley, T. J, Gary, B., Cashman, R. E., Gunter, G., & Gunter, A. (2008). *Integrating technology and digital media in the classroom.* Boston: Thompson Course Technology.

Stahl, G. (2014). Construct dynamic geometry together. Retrieved on 30 March 2016 from www.sciencedirect.com

Stols, G. (2012). Does the use of technology makes a difference in the geometric growth of pre-service mathematics teachers? *Australasian Journal of Educational Technology, 28*(7), 1233-1247.

Tamim, R. M., Benard, R. M., Borokhovski, E., Abrami, P. C., & Schmid, R. (2011). What forty years of research say about the impact of technology on learning: A second order meta-analysis and validation study. Review of Educational Research, *81(1),* 4-28.

Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach's Alpha. *International Journal of Medical Education,* 2, 53-55

Taylor, J. A. (2013). Translation of function: A study of dynamic mathematical software and its effects on students understanding of translation of function. (Unpublished Masters dissertation). University of Wyoming.

Uddin, R. S. (2011). GeoGebra, a tool for mediating knowledge in the teaching and learning of transformation of functions in mathematics. (Unpublished Masters dissertation). University of Kwa-Zulu Natal.

Vukobratovic, R., & Takaci, D. (2011). *On the role of GeoGebra in examining functions*. Proceedings of the International GeoGebra Conference for Southeast Europe, Novi Sad-Serbia, 15-16 January 2011, 52-60.

Wachiuri, R. G. (2015). Effects of teachers' experience and training on implementation of ICT. IOSR Journal of Humanities and Social Science, 20(3), 26-38.

Webster, M. (2015). Definition of a linear function. Retrieved on November 30 2016 from www.merriam-webster.com.

William, M. K. (2006). *The research methods knowledge base, 2nd Edition*. Cincinnati: Atomic Dog Publishing.

World Economic Forum. (2016). The Global Information Technology Report. Retrieved on 10 July 2016 from www.weforum.org.

Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software GeoGebra on student achievement in teaching of trigonometry. *Procedia Social and Behavioral Sciences, 31*,183-187.

APPENDICES

Appendix 1: Parent/guardian consent form

INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION
UNIVERSITY OF SOUTH AFRICA

Research Study to investigate the Effect of Integrating GeoGebra Graphing Software into the Teaching of Linear Functions on Grade 9 Learners' Achievement in Mopani District

PARENT/GUARDIAN CONSENT FORM

Your child is being asked to take part in a research study that investigates the effect of integrating GeoGebra graphing software into the teaching of linear functions on Grade 9 learners' achievement in Mopani district. The study is for academic purpose and will enable us understand some of the problems learners have in learning mathematics

The study will involve the learners' writing a test on linear functions aligned with the CAPS document. The findings will be used to proffer solutions to the problems students have on the topic.

Your child's participation in the study is entirely voluntary, and he/she can withdraw from the study at any time without any prejudice. Your information will be treated as confidential and the identity of your child will by no means be revealed in any publication. I will provide you with a summary of my research results on completion if you would like me to do so.

Thank you in advance for allowing your child to participate in the study. Should you have any queries, please do not hesitate to contact me on 0792550867 or by email at chicco23melo@yahoo.com.

Please sign this form to indicate that:

- You have read and understood the information above.
- You give your consent for your child to participate in the study on voluntary basis.

______________________________   ___________________________

Parent/Guardian signature   Date

Appendix 2: Learners consent form

INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION

UNIVERSITY OF SOUTH AFRICA

Research Study to investigate the Effect of Integrating GeoGebra Graphing Software into the Teaching of Linear Functions on Grade 9 Learners' Achievement in Mopani District

LEARNERS CONSENT FORM

You are being asked to take part in the research study that investigates the effect of integrating GeoGebra graphing software into the teaching of linear functions on Grade 9 learners' achievement in Mopani district. The study is for academic purpose and will enable us understand some of the problems learners have in learning mathematics

The study will involve you writing a test on linear functions aligned with the CAPS document. The findings will be used to proffer solutions to the problems learners have on the topic

Your participation in the study is entirely voluntary, and you can withdraw from the study at any time without any penalty. Your information will be treated as confidential and your identity will by no means be revealed in any publication. I will provide you with a summary of my research results on completion if you would like me to do so.

Thank you in advance for agreeing to participate in the study. Should you have any queries, please do not hesitate to contact me on 0792550867 or by email at chicco23melo@yahoo.com

Please sign this form to indicate that:

- You have read and understood the information above.
- You give your consent to participate in the study on voluntary basis.

____________________________ ____________________________

Learner signature Date

Appendix 3: Grade 9 achievement test

Topic: Linear Functions and their Graphs

Duration: 1 hour 30minutes

Marks : 50

Instructions

1. Answer all questions on provided answer sheet
2. Write neatly and legibly
3. Label all diagrams clearly.
4. Use of programmable calculators is not allowed.

Question 1

A graph is defined by the equation $y = 2x + 1$. Choose the correct answer from the list below.

a) The graph cuts the y-axis at:

A) (-2; 0) B) (0; 0) C) (0; -2) D) (0; 1)

b) The gradient of this graph is:

A) 1 B) -2 C) 2 D) no answer

c) The graph passes the x-axis when the value of y is:

A) 0 B) 2 C) 1 D) -1

[6]

Question 2

A linear graph passes through the points A (0; 3) and B (1; 4).

a) The graph cuts the x-axis when the value of y is:

A) 0 B) 4 C) 3 D) 1

b) The gradient of the graph is:

A) 1 B) 0 C) 7 D) 5

[4]

Question 3

Given the equation $y = 2x - 3$, answer the following questions

a) Write down the independent variable in the equation. [2]

b) Write down the dependent variable in the equation. [2]

c) Write down the coefficient of x in the equation. [2]

d) Write down the constant term in the equation. [2]

e) What does the constant in the equation tell us about the graph? [2]

f) Complete the following table:

X	-3	-2	-1	0	1	2	3
$y = 2x - 3$							

[7]

g) Use the completed table to plot the graph of $y = 2x - 3$ [4]

h) Where does the graph of $y = 2x - 3$cross the y-axis? Write down the coordinates of the point. [1]

i) Write down the coordinates of the point where the graph crosses the x-axis. [2]

Question 4

Sketch the graphs of the following equations

a) $y = x + 1$ [2]

b) $y = x - 1$ [2]

c) $y = -x + 1$ [2]

Question 5

a) Explain how the features of the following graphs will differ

$$f: y = 3x + 1$$

$$g: y = 3x + 2$$

$$h: y = 3x - 1$$

b) Sketch the graphs to illustrate these differences [6]

i) How will the graph of $f: y = -x$differ from the graph of $g: y = x$?

ii) Sketch the graphs to help you to explain the differences. [4]

Total [50 marks]

Appendix 4: Mawa circuit permission from department to do research

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION

MAWA CIRCUIT

Ref : 83516727
Enq: Mutovholwa K.G

22 October 2014

Attention: Mushipe M
Maselekwane High School
P.O.Box 2887
TZANEEN
0850

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IIN MAWA CIRCUIT SCHOOLS

1. The above matter refers.

2. This office received your letter dated 11 September 2014 requesting for permission to conduct research titled as Investgating The Effect of Technology in the Teaching to grade 9 learners in Function in Mawa Circuit schools..

3. This office is giving you permission to conduct research in Mawa Circuit schools. You are requested to submit your recommendations which can help the circuit and schools.

4. Hope you will find this in order.

....
CIRCUIT MANAGER: MAWA

DEPARTMENT OF EDUCATION
MAWA CIRCUIT, PRIVATE BAG X 738, GA-KGAPANE, 0838
TEL. 015 328 3498, FAX 015 328 4527

THE HEARTLAND OF SOUTHERN AFRICA– DEVELOPMENT IS ABOUT PEOPLE

Appendix 5: Ethical Clearance certificate from UNISA

UNISA | college of science, engineering and technology

Dear Ms Moshipa (student number 53138801)

Date: 2016-06-06

Application number:
2016_CGS/ISTE_004

REQUEST FOR ETHICAL CLEARANCE: [Topic: The Effect of Integrating GeoGebra Graphing Software into the Teaching of Linear Functions on Grade 9 Learners' Achievement in Mopani District.]

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your research study as set out in your proposal and application for ethical clearance.

Therefore, involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CRIC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:
http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Please note that the ethical clearance is granted for the duration of this project and if you subsequently do a follow-up study that requires the use of a different research instrument, you will have to submit an addendum to this application, explaining the purpose of the follow-up study and attach the new instrument along with a comprehensive information document and consent form.

Yours sincerely

Dr CE Ochonogor
Chair: Ethics Sub-Committee (STR/ISTE) CSET

Prof MI Feza
Director/Head: ISTE

18 July 2016
Prof I Alderton
Executive Dean (Acting): College of Science, Engineering and Technology

University of South Africa
College of Science, Engineering and Technology
The Science Campus
C/o Christiaan de Wet Road and Pioneer Avenue
Florida Park, Roodepoort
Private Bag X6, Florida, 1710
www.unisa.ac.za/cset

UNISA | college of science, engineering and technology

Appendix 6: Lesson 1

Topic: Functions and Relationships (Input and output values)

Concepts and skills to be covered:

By the end of the lesson learners should know and be able to determine input values, output values or rules for patterns and relationships using

- Tables
- Formulae
- Equations

Resources: Textbooks, Sasol-Inzalo Book2, GeoGebra software

Prior Knowledge: Functions and relationships

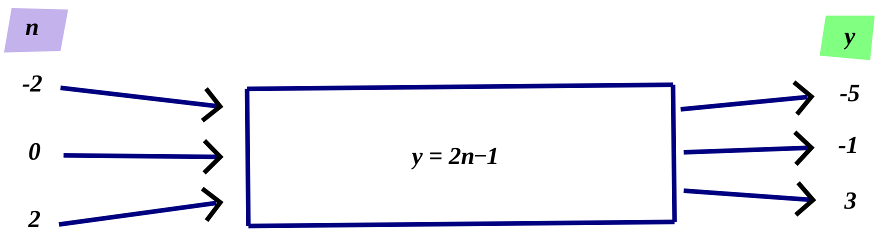
Introduction (10mins): The focus of this lesson is on finding output values for given equations and recognizing equivalent forms between different descriptions of the same relationship. Leaners do the following activity:

Activity: Use the flow diagram to answer the questions below

a) Which are the input values?

b) Which are the output values?

c) Which one is the rule for this flow diagram?



Lesson Development (20-30mins)

<table>
<tr><th>Teaching activities</th><th>Learning activities
(Learners are expected to:)</th></tr>
<tr><td>Give learners the following activity which will focus them on the functional relationship between the input and output values:

Activity
In each case:
1. Complete the table.
2. Write the rule for the nth output.
3. Show that your rule is correct.</td><td></td></tr>
<tr><td>a)
<table>
<tr><td>Input value</td><td></td><td>3</td><td></td><td>7</td><td></td><td>13</td><td></td><td>63</td><td></td><td>204</td><td></td></tr>
<tr><td>Output value</td><td></td><td>9</td><td></td><td>49</td><td></td><td>169</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
Note that:
$9 = 3 \times 3$ i.e. input times the input
$49 = 7 \times 7$ i.e. input times the input
$169 = 13 \times 13$ i.e. input times the input
For the nth output n times n which can be written as n^2

b)</td><td>• work in pairs
• complete the tables
• find the rules
• show that the rule applies to all the cases in the table</td></tr>
</table>

Input value		2		6		11		60		112	
Output value		10		22		37					

Note that:

$10 = 3 \times 2 + 4$ i.e. 3 times the input plus 4

$22 = 3 \times 6 + 4$ i.e. 3 times the input plus 4

$37 = 3 \times 11 + 4$ i.e. 3 times the input plus 4

For the nth output 3 times n plus 4 which can be written as $3n + 4$

Note:

- Encourage learners to focus on the functional relationship between the input and output values.
- Observe learners as they work and assist those who struggle to see the relationships.
- These activities are designed to help learners to focus on the advantages of using function rules rather than recursive patterns in the tables.

- report on how they have worked out their solutions

Classwork (15-20mins)

Find the rule and complete the table.

a)

Input value		3		7		12	13		81		115

Output value		10		50		145	170		6562		13226

Answer: $50 = 7 \times 7 + 1$
$145 = 12 \times 12 + 1$
$170 = 13 \times 13 + 1$
Rule: $n^2 + 1$

Input value		3		7		12	13		81		115
Output value				50		145	170				

b)

Input value		8		31		66		121			
Output value		81		311		661				1331	

Answer: $81 = 8 \times 10 + 1$
$311 = 31 \times 10 + 1$
$661 = 66 \times 10 + 1$
Rule: $10n + 1$

Input value		3		7		12	13		81		115
Output value		31		50		145	170		811		1151

c)

Input value		4		9		49		72			
Output value		10		25		145				496	

Answer: $10 = 4 \times 3 - 2$
$25 = 9 \times 3 - 2$
$145 = 49 \times 3 - 2$
Rule: $3n - 2$

Input value		4		9		49		72		166	
Output value		10		25		145		214		496	

Conclusion and homework (5mins)

Note: emphasize that learners should consider the input and output values when searching for relationships and there can be more than one possible operator in a functional relationship.
Selected homework activities that should address the different cognitive levels

Appendix 7: Lesson 2

Graphs: Interpreting Graphs

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to analyse, interpret global graphs with special focus on constant, increase or decrease

RESOURCES: DBE Book 2, Sasol-Inzalo Book 2, textbook

PRIOR KNOWLEDGE:

- Linear and non - linear graphs

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

Allow learners to study the graph below and discuss

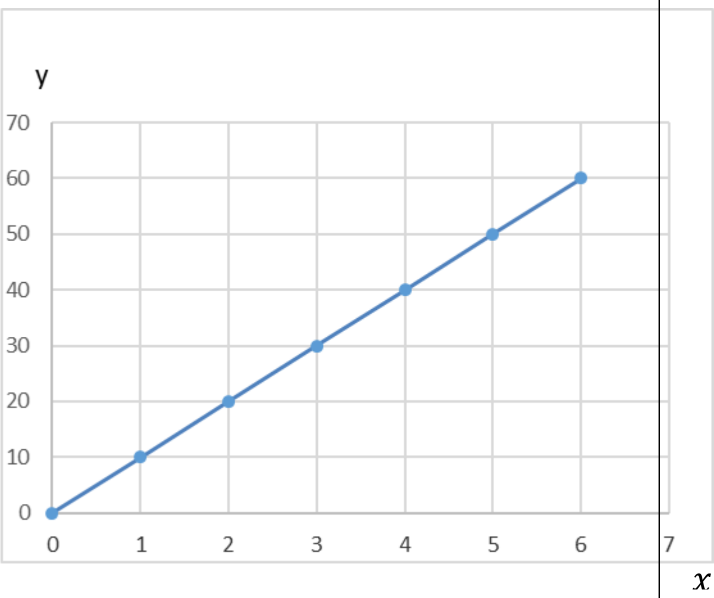
<table>
<tr><td colspan="2">LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)</td></tr>
<tr><td>Teaching activities</td><td>Learning activities
(Learners are expected to</td></tr>
<tr><td>

Activity 1

Allow learners to work in pairs to study the graph below and discuss:

Water Pumped into a tank

Water level in kilolitres



Hours

</td><td>Work in pairs to study the graph discuss and answer questions</td></tr>
</table>

Answer the following questions:

a) What information is given in the x-axis?

b) What information is given in the y-axis?

c) Relate a story on what the graph is representing

d) Is the graph linear or non-linear?

Solution

a) Hours

b) Water level in kilolitres

c) One possible story could be correct

d) Linear (increasing)

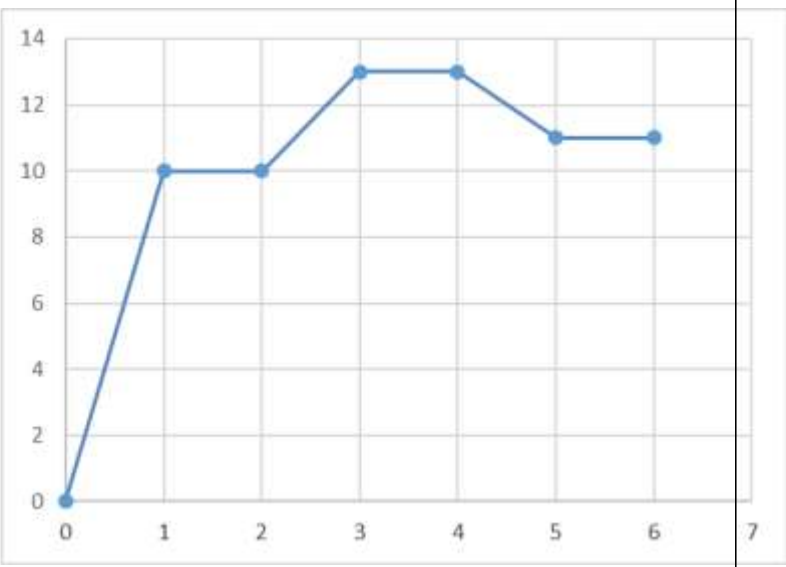
Activity 2

Study the graph below and answer the questions

Petrol price increase from January to June

y

Petrol Price in Rand



x

Petrol in litres

Answer the following questions

e) What information is given in the x-axis?

f) What information is given in the y-axis?

g) Relate a story on what is the graph representing h) Is the graph linear or non-linear Solution e) Petrol in litres f) Price in Rands g) One possible story could be correct h) Linear (increasing, constant and decreasing)	

CLASSWORK (Suggested time: 15 minutes)

Sasol-Inzalo book 2 page 56 number 4 and 5

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

Note that

- a line is constant when the y–value remains the same while the x–value increases.
- the slope of a line increases when the y–value increases while the x–value increases.
- the slope of a line decreases when the y–value decreases while the x–value increases

Homework

Selected exercises

Appendix 8: Lesson 3

TOPIC: GRAPHS INTERPRETING GRAPHS (Lesson 3)

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able analyze and interpret global graphs of problem situations with the focus on maximum or minimum.

RESOURCES: DBE Book 2, Sasol-Inzalo Book 2, textbook

PRIOR KNOWLEDGE:

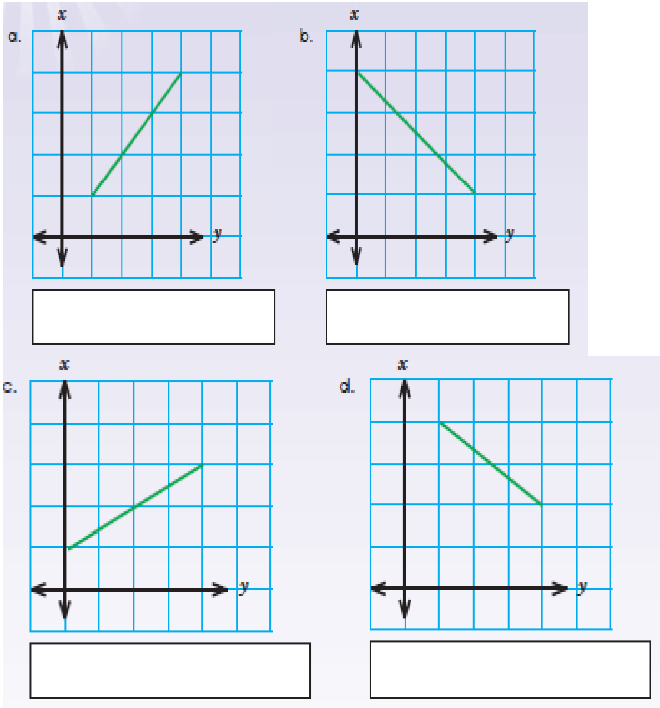
- linear and non-linear graphs
- constant, increasing or decreasing

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

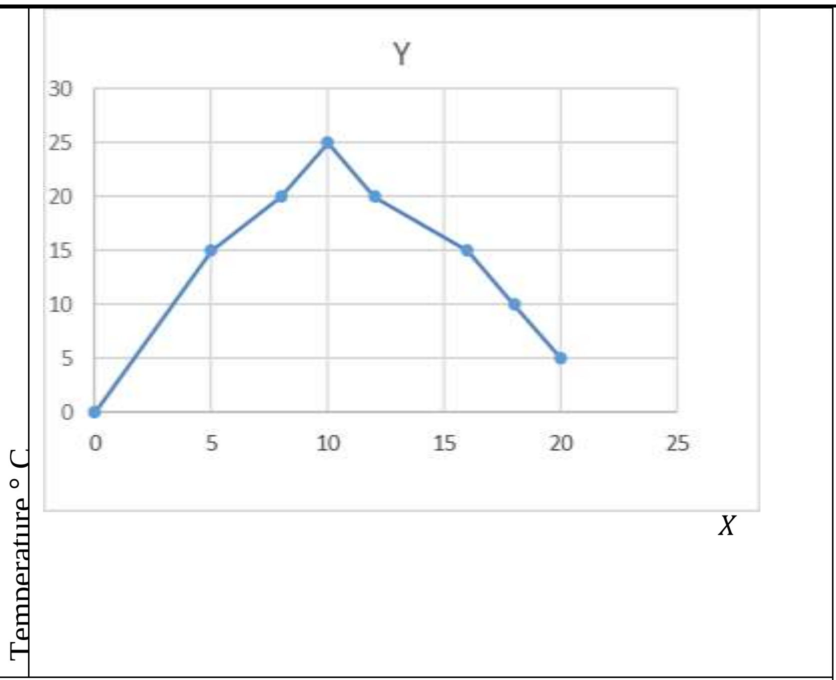
Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

Let learners work in pairs to identify which graphs below are linear , non-linear , constant increasing or decreasing



LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)	
Teaching activities	**Learning activities** Learners are expected to :
	Interpret the graph, discuss the and answer the questions



Time in hours

a) During which hour was there an increase in temperature?

b) During which hour was there a decrease in temperature?

c) During which hour were temperature unchanged?

d) Relate what the graph is telling.

e) Which trend is represented by the graph?

Solutions

a) The highest 25 °C. is 10 hours

b) The decreases 5°C is at 20 hours

c) None

d) The story that explain the situation could be

e) Maximum and minimum

Discussions: • The graph increases until it reaches the maximum point, then decreases. • The graph decreases until it reaches the minimum point.	

CLASSWORK (Suggested time: 15 minutes

Sasol-Inzalo workbook page, 53 (1)

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

a) **Emphasise that**:

- A graph has a maximum value when it changes from increasing to decreasing.
- A graph has minimum value when it changes from decreasing to increasing.

b) The primary purpose of Homework is to give each learner an opportunity to demonstrate mastery of mathematics skills taught in class. Therefore Homework should be purposeful and the principle of 'Less is more' is recommended, i.e. give learners few high quality activities that address variety of skills than many activities that do not enhance learners' conceptual understanding.

Carefully select appropriate activities from the Sasol-Inzalo workbooks, workbooks and/or textbooks for learners' homework. The selected activities should address different cognitive levels.

Homework:

Appendix 9: Lesson 4

TOPIC: GRAPHS INTERPRETING GRAPHS

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to:

analyse and interpret global graphs of problem situations with the focus on discrete or continuous.

RESOURCES: DBE Book 2, Sasol-Inzalo Book 2, textbook, computers installed with GeoGebra software

PRIOR KNOWLEDGE:

- constant, increasing or decreasing
- maximum or minimum

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

Do the following with the learners

Complete the table below and draw the graph *y= x + 1*

X	-2		0	1	
Y		0			3

a) Is this graph increasing or decreasing

b) What type of graph is $y = x + 1$

c) Is this a discrete or continuous graph

Solutions

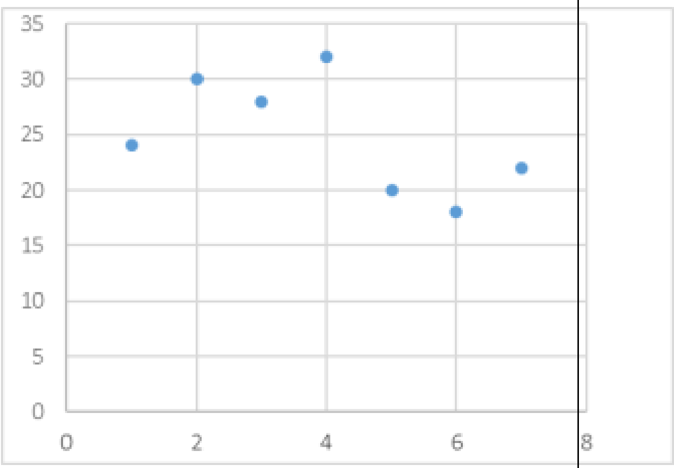
X	-2	-1	0	1	2
Y	-1	0	1	2	3

d) Increasing

e) Linear

f) Continuous.

LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)	
Teaching activities	**Learning activities**

	Learners are expected to :
Guide learners through this activity. Activity 2 Ask question about the graph below Temperature in February in Grade 8 class Temperature  Days of the week i. which day is the coldest day of the week = (Saturday) ii. what the minimum temperature of the graph = (18°c) iii. what is the maximum value of the graph = (32°c) iv. on which day was the temperature recorded as being	Experimental group learners should be guided through the process of drawing the linear graphs using GeoGebra and be allowed to refer to both the manual drawings and the ones drawn using GeoGebra. In the control group learners should be guided through the exercise using the chalkboard method only. Interpret the graph, discuss the and answer the questions

30°c = (Tuesday) v. Is this a discrete or continuous graph? Why? = Discrete graph – because the graph is made of sets of point, which are not joined by line	

CLASSWORK (Suggested time: 15 minutes Sasol-Inzalo workbook page, 50 (3 and 4)
CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

c) **Emphasize that**:

• Allow learners ample time on activity one and guide them when using the GeoGebra software to draw the linear graph and also allow access to the computers to enable them to complete their homework.

- Discrete graph is a graph made of sets of points which are not joint by a line
- Continuous graph is a graph made of sets of points which are joined by a line

d) The primary purpose of Homework is to give each learner an opportunity to demonstrate mastery of mathematics skills taught in class. Therefore Homework should be purposeful and the principle of 'Less is more' is recommended, i.e. give learners few high quality activities that address variety of skills than many activities that do not enhance learners' conceptual understanding.

The selected activities should address different cognitive levels.

Homework:

DBE workbook page 51 (5)

Appendix 10: Lesson5

TOPIC: GRAPHS: INTERPRETING GRAPHS

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to interpret graphs with special focus on the x-intercept an y-intercept of linear graphs

RESOURCES:	DBE Book 2, Sasol-Inzalo Book 2, textbooks
	Cartesian plane, x and y coordinates
PRIOR KNOWLEDGE:	linear or non-linear
	substitution

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

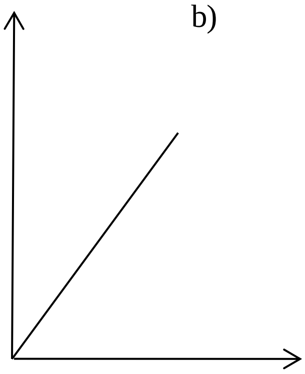
Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

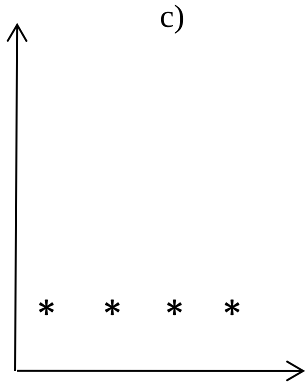
Ask learners to complete the following activity.

Activity

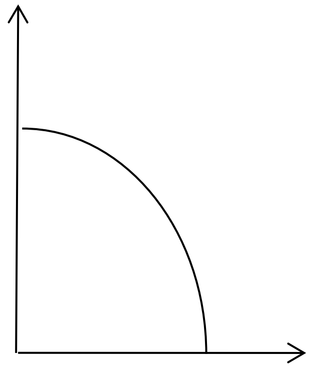
Describe each graph using the words linear or non-linear.



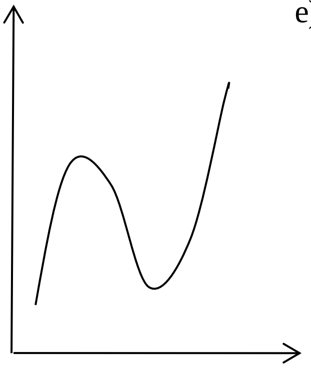
b)



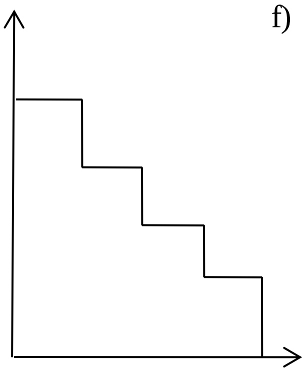
c)



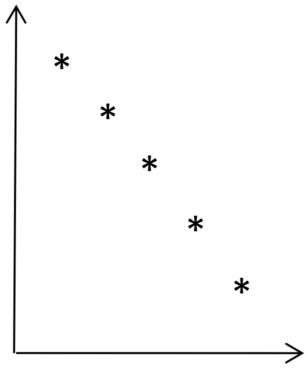
d)



e)



f)

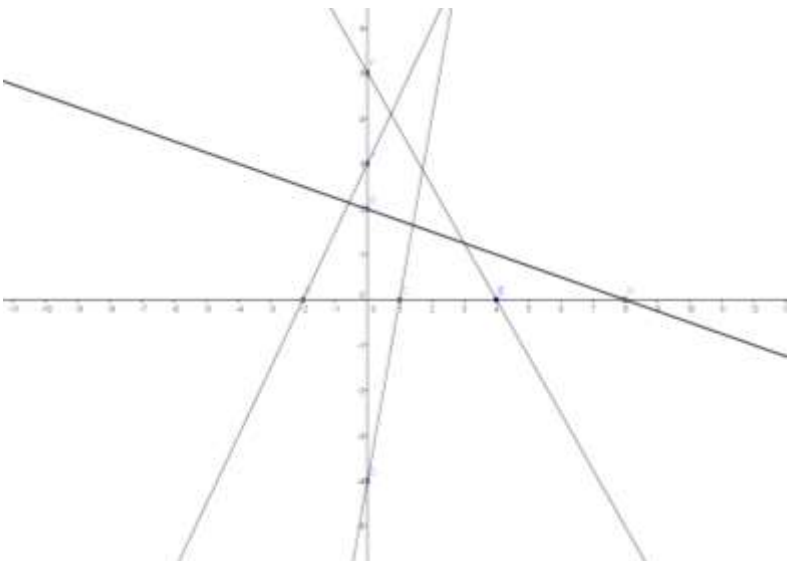


<table>
<tr><td colspan="2">LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)</td></tr>
<tr><td>Teaching activities</td><td>Learning activities

(Learners are expected to:)</td></tr>
<tr><td>Do the following activities with the learners.

Activity 1

Study the following graphs and answer the questions below.



Mark off all the points where each graph cuts the x-axis naming</td><td></td></tr>
</table>

them A-D from left to right. Mark off all the points where each graph cuts the y-axis naming them E-H from top to bottom. Complete the following table:	

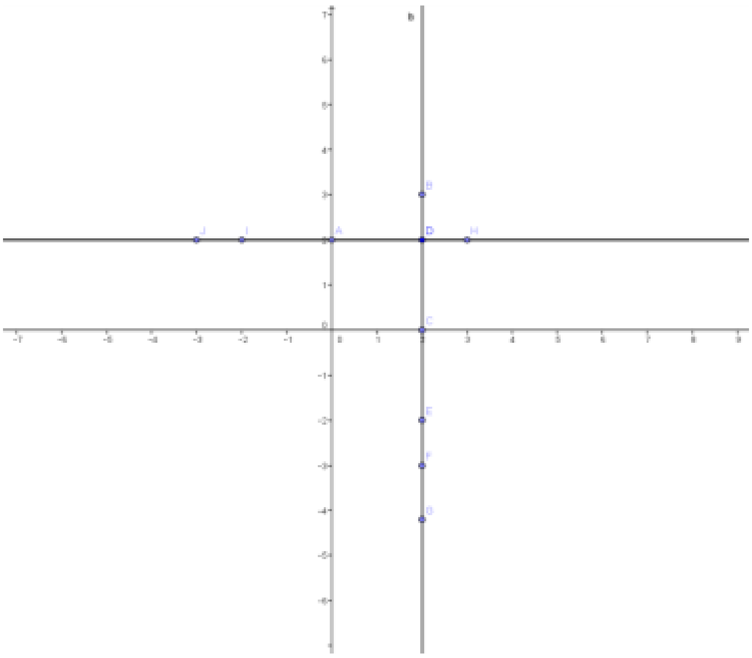
Points	x-value of point	y-val e of point
A		
B		
C		
D		
E		
F		
G		

What is common about the points cutting the x-axis?

What is common about the points cutting the y-axis?

Activity 2

Study the following graphs and answer the questions that follow:



What do the following coordinate pairs have in common?

(2:3), (2:0), (2;-2), (2;-3)

Write down two more points that has an x-coorinate of 2.

Where the graph does cuts the x-axis? Give the coordinates of this point.

Is this graph cutting the y-axis? Explain this observation.

What do the following coordinate pairs have in common?

(3:2), (0:2), (-2;2), (-3;2)

Write down two more points that has an y-coordinate of 2.

Where the graph does cuts the x-axis? Give the coordinate of this point.

Is this graph cutting the x-axis? Explain this observation.

Activity 3

Use the following equation and determine x- and y-intercepts by following these steps:

$$y = 5x + 3$$

Step 1: To determine the x-intercept substitute $y = 0$.

$$y = 5x + 3$$

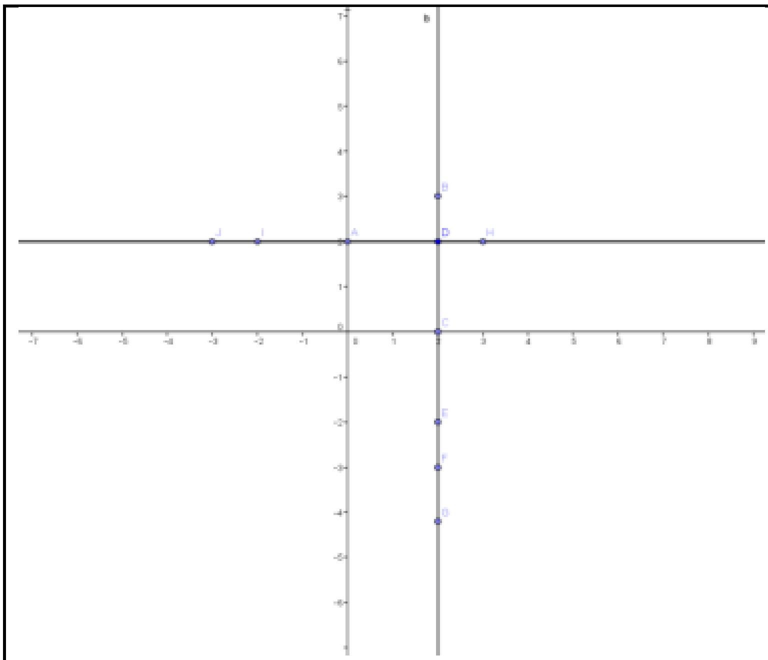
$$0 = 5x + 3$$

$$-3 = 5x$$

$$x = -\frac{3}{5}$$

Step 2: Write the x-intercept in coordinate form.

$(-\frac{3}{5};0)$ Step 3: To determine the y-intercept substitute $x = 0$ $y = 5x + 3$ $y = 5(0) + 3$ $y = 3$ Step 4: write the y-intercept in coordinate form. (0;3)	
Activity 2 Study the following graphs and answer the questions that follow:	



What do the following coordinate pairs have in common?

(2:3), (2:0), (2;-2), (2;-3)

Write down two more points that has an x-coordinate of 2.

Where the graph does cuts the x-axis? Give the coordinates of this point.

Is this graph cutting the y-axis? Explain this observation.

What do the following coordinate pairs have in common?

(3:2), (0:2), (-2;2), (-3;2)

Write down two more points that has an y-coordinate of 2.

Where the graph does cuts the x-axis? Give the coordinate of this point.

Is this graph cutting the x-axis? Explain this observation.

Activity 3

Use the following equation and determine x- and y-intercepts by following these steps:

$$y = 5x + 3$$

Step 1: To determine the x-intercept substitute $y = 0$.

$$y = 5x + 3$$

$$0 = 5x + 3$$

$$-3 = 5x$$

$$x = -\frac{3}{5}$$

Step 2: Write the x-intercept in coordinate form.

$(-\frac{3}{5}; 0)$

Step 3: To determine the y-intercept substitute $x = 0$

$$y = 5x + 3$$

$$y = 5(0) + 3$$

$y = 3$ Step 4: write the y-intercept in coordinate form. (0;3)	

CLASSWORK (Suggested time: 15 minutes)

DBE Book 2 page 65 number 1

Sasol-Inzalo Book 2 page 74 number (1 to 3) and (4a)

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

Emphasize that:

y-intercept is the point on the graph that cuts the y axis and can be calculate by substituting x = 0 in the graph equation.

x- intercept is the point on the graph that cuts the x axis and can be calculated by substituting y =0 in the graph equation.

The primary purpose of homework is to give each learner an opportunity to demonstrate mastery of mathematics skills taught in class. Therefore homework should be purposeful and the principle of ‘Less is more’ is recommended, i.e. give learners few high quality activities that address variety of skills than many activities that do not enhance learners’ conceptual understanding.

Carefully select appropriate activities from the Sasol-Inzalo workbooks, workbooks and/or textbooks for learners’ homework. The selected activities should address different cognitive levels.

Homework:

DBE Book 2 page 65 number (1 c to d)

Appendix 11: Lesson 6

TOPIC: INTERPRETING GRAPHS

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to interpret graphs with special focus on the gradient of linear graphs

RESOURCES: DBE Book 2 and Sasol-Inzalo Book 2, textbook, GeoGebra software

PRIOR KNOWLEDGE:

- Cartesian plane, x and y coordinates
- linear
- x-intercept and y-intercept

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

Do the following demonstration and allow learners to observe and explain their observations:

- Use a ruler and a marble/ small ball.
- Place the ruler high against the wall and let the marble roll down the ruler.
- Place the ruler at a lower level then before and roll the marble down the ruler again.
- Repeat this until the ruler is flat on the table.

Ask learners the following questions:

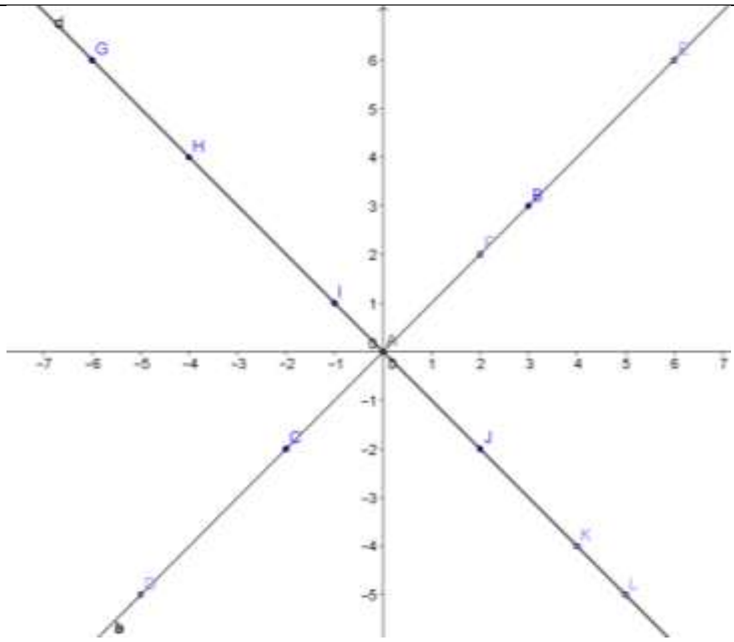
a) What do you observe in the speed of the marble as it moves down the ruler at different heights?

b) Explain this occurrence.

Discussion:

- When the slope of the ruler is steep the marble roll faster down.
- As the height (slope) of the ruler becomes lower the marble moves slower down.
- When the ruler is flat the marble does not roll.
- The higher the ruler is the steeper it is and the lower it is the less steep it becomes.
- Slope in mathematics are referred to as **gradient**.

LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)	
Teaching activities	**Learning activities** (Learners are expected to:)
Let learners investigate the concept of gradient by completing the following activity. • Remind learners that a movement up is regarded as a positive movement and a movement down is regarded as a negative movement. • Remind learners that a movement to the right is regarded as a positive movement and a movement to the left is regarded as a negative movement Activity 1 The following graphs are given:	With the experimental groups, the concept of gradient should also be clarified using GeoGebra software and learners should be given enough time to explore the concept using the software. Learners in the experimental groups are allowed to use GeoGebra during class activities and homework.



a) What is difference about the orientation of graphs 1 and 2?

b) Draw a horizontal line meeting a vertical line from points A to B; C to D; E to F.

c) Count the vertical change and the horizontal change and complete the table.

Graph 1	Vertical change	Horizontal change	$\frac{vertical\ change}{horisontal\ change}$
A to B			
C to D			
E to F			

d) What do you observe about the $\frac{vertical\ change}{horisontal\ change}$ between the different segments of the line.

e) Draw a horizontal line meeting a vertical line from points G to H, I to J and K to L.

f) Count the vertical change and the horizontal change and complete the table.

Graph 2	Vertical change	Horizontal change	$\frac{vertical\ change}{horisontal\ change}$
G to H			
I to J			
K to L			

g) What do you observe about the $\frac{vertical\ change}{horisontal\ change}$ between the different segments of the line.

What difference do you observe in the calculation of the last column of graphs 1 and 2

Activity 2

Allow learners to do the following investigation:

a) Use point A (2:5) and point B (4; 1) and apply the method in activity 1 to determine the gradient of the graph.

b) Complete the following table.

y_A	y_B	x_A	x_B	$y_A - y_B$	$x_A - x_B$	$\frac{y_A - y_B}{x_A - x_B}$

c) What do you observe about your answer in (a) and the last column of (b).

Discussion:

- $Gradient = \frac{change\ in\ y-values}{change\ in\ x-values}$

- If you have two points A(x_A; y_A) and B(x_B; y_B), the formula for gradient is: Gradient= $\frac{y_B - y_A}{x_B - x_A}$

Graph 1	Vertical change	Horizontal change	$\frac{vertical\ change}{horisontal\ change}$
A to B			
C to D			
E to F			

h) What do you observe about the $\frac{vertical\ change}{horisontal\ change}$ between the different segments of the line.

i) Draw a horizontal line meeting a vertical line from points G to H, I to J and K to L.

j) Count the vertical change and the horizontal change and complete the table.

Graph 2	Vertical change	Horizontal change	$\frac{vertical\ change}{horisontal\ change}$
G to H			
I to J			
K to L			

k) What do you observe about the $\frac{vertical\ change}{horisontal\ change}$ between the different segments of the line.

What difference do you observe in the calculation of the last column of graphs 1 and 2

Activity 2

Allow learners to do the following investigation:

d) Use point A (2:5) and point B (4;1) and apply the method in activity 1 to determine the gradient of the graph.

e) Complete the following table.

y_A	y_B	x_A	x_B	$y_A - y_B$	$x_A - x_B$	$\frac{y_A - y_B}{x_A - x_B}$

f) What do you observe about your answer in (a) and the last column of (b).

Discussion:

• $Gradient = \frac{change\ in\ y-values}{change\ in\ x-values}$ • If you have two points A(x_A; y_A) and B(x_B; y_B), the formula for gradient is: Gradient= $\frac{y_B - y_A}{x_B - x_A}$	

CLASSWORK (Suggested time: 15 minutes)

DBE Book 2 page 68 number 1 (a-d)

Sasol-Inzalo Book 2 page 66 (1 a, b)

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

e) **Emphasise that**:

- Gradient is the slope / steepness of the graph / rate of change between two coordinates.

.

- $Gradient = \frac{vertical\ change}{horizontal\ change}$
    - $Gradient = \frac{change\ in\ y-values}{change\ in\ x-values} = \frac{y_B - y_A}{x_B - x_A}$

f) The primary purpose of Homework is to give each learner an opportunity to demonstrate mastery of mathematics skills taught in class. Therefore Homework should be purposeful and the principle of 'Less is more' is recommended, i.e. give learners few high quality activities that address variety of skills than many activities that do not enhance learners' conceptual understanding.

Carefully select appropriate activities from the Sasol-Inzalo workbooks, workbooks and/or textbooks for learners' homework. The selected activities should address different cognitive levels.

Homework:

Sasol-Inzalo workbook 2 page 66 no 1 c, d, 2 c

Appendix 12: Lesson 7

TOPIC: DRAWING GRAPHS

CLASSWORK (Suggested time: 15 minutes)

1. Plot the following points on a Cartesian plane.

a) (-4;3)

b) (3;4)

c) (0;2)

d) (3;0)

e) (-3;-4)

f) (2;-3)

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)

g) **Emphasise that**:

- the Cartesian plane is system where all points can be described by x- and y-coordinates.
- the horizontal number line represents the x-axis
- the vertical number line represents the y-axis
- the x-coordinate is the position along the x-axis
- the y-coordinate is the position along the y-axis
- the origin is the point where the horizontal and vertical axes meet
- an ordered pair is given in the form $(x; y)$

h) The primary purpose of Homework is to give each learner an opportunity to demonstrate mastery of mathematics skills taught in class. Therefore Homework should be purposeful and the principle of 'Less is more' is recommended, i.e. give learners few high quality activities that address variety of skills than many activities that do not enhance learners' conceptual understanding.

Carefully select appropriate activities from the Sasol-Inzalo workbooks, workbooks and/or textbooks for learners' homework. The selected activities should address different cognitive levels.

Homework:

Draw a Cartesian plane with the x-axis and the y-axis that has the values of -10 and 10. Plot the following points on the Cartesian plane:

a) (-4;2)

b) (8;0)

c) (-5;-4)

d) (0;-5)

e) (-7;5)

f) (8;9)

Appendix 13: Lesson 8

TOPIC:DRAWING GRAPHS

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to draw linear graphs from given equations.

RESOURCES:	DBE Book 2, Sasol-Inzalo Book 2, textbooks, grid paper, GeoGebra software for the experimental groups.
PRIOR KNOWLEDGE:	• Cartesian plane • substitution • equations
REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)	
INTRODUCTION (Suggested time: 10 Minutes) **Note:** In the following lesson learners will need to know how to substitute values in a given equation to determine coordinates. Give learners the following questions to complete: For the expressions below, $a = 3$. Evaluate each expression by substituting the value "3" wherever you see "a" 1) $a + 4$ 2) $10 - a$ 3) $5a$ 4) $9 \div a$ 5) $a - 1$ 6) $a.6$	

LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)	
Teaching activities	**Learning activities** (Learners are expected to:)

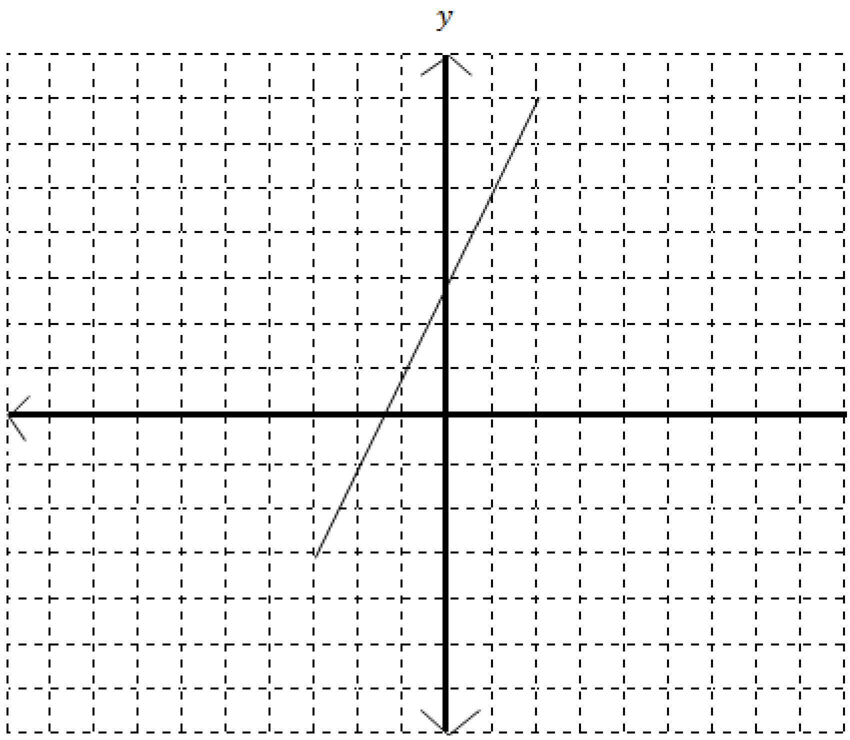
Do the following activities with learners. **Activity 1** Set up a table of ordered pairs (Table method). Sketch the graph of a linear function given by the equation $y = 2x + 3$ by using the following steps:	Learners in the experimental groups are also taught how to use GeoGebra to sketch given graphs of functions and are also allowed to use GeoGebra during class activities as well as for completion of tasks. Learners in the control groups are only taught the lesson using the lesson plan and the usual traditional teaching methods. There is no use of GeoGebra or the computer in the control groups. -Learners respond and do the given tasks individually and participate in class and group discussions and activities

Step 1 - the x-value is the dependent variable so select a set of values to represent x
Step 2 - use the equation and substitute each x-value to calculate the corresponding y-value
Step 3 - plot the ordered pairs on a Cartesian plane

Answer

$$y = 2x + 3$$
$$y = 2{-3} + 3 = -3$$
$$y = 2{-2} + 3 = -1$$
$$y = 2{-1} + 3 = 1$$
$$y = 20 + 3 = 3$$
$$y = 21 + 3 = 5$$
$$y = 22 + 3 = 7$$

x	-3	-2	-1	0	1	2
y	-3	-1	1	3	5	7



Activity 2

Intercept method

Note: the equation does not have to be in the standard form

Draw the graph of $y = 3x - 6$ by using the following steps:

Step 1: Determine the x-intercept by substituting $y = 0$
Step 2: Write the x-intercept in coordinate form (x;0)
Step 3: Determine the y-intercept by substituting $x = 0$
Step 4: Write the y-intercept in coordinate form (0:y)

Answer:
x-intercept let y = o

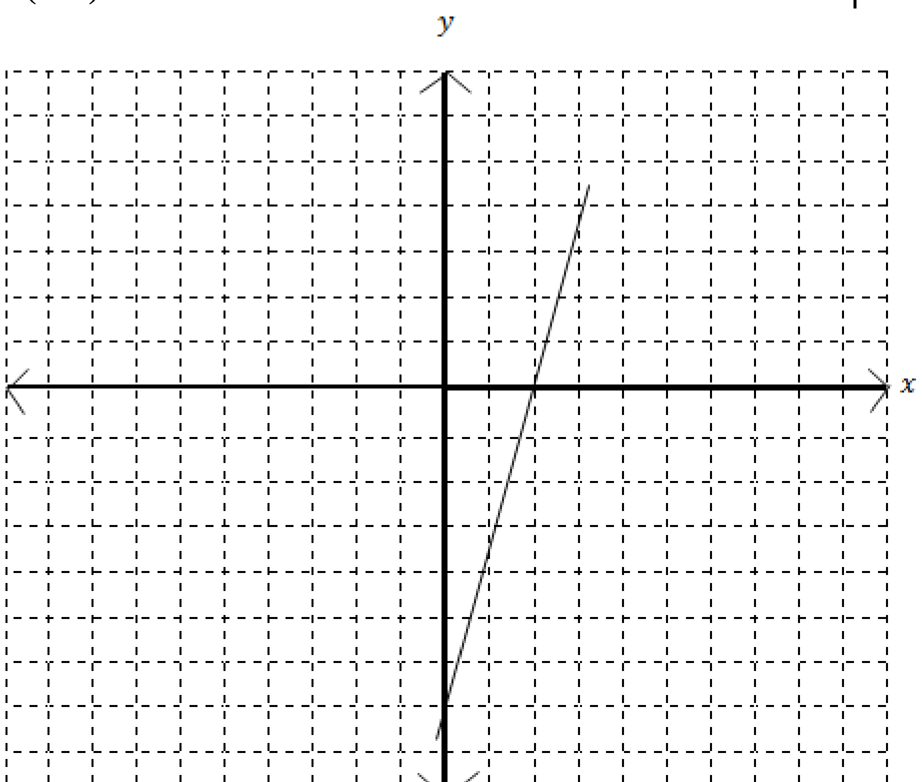
$$0 = 3x - 6$$
$$3x = 6$$
$$x = 2$$

(2:0)

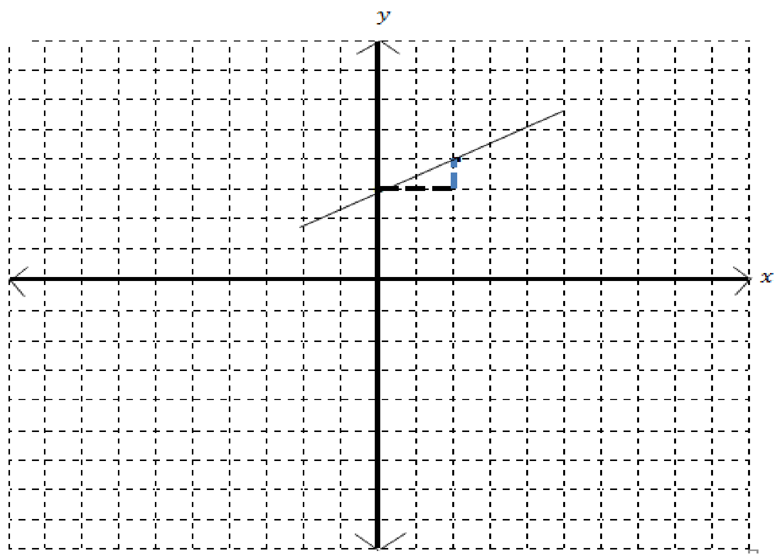
y-intercept let x = 0

$$y = 3(0) - 6$$
$$y = -6$$

(0:-6)



Activity 3 The gradient – intercept method Note: • the equation have to be in the standard form $y = mx + c$ • if equation is not in the standard form rewrite it in the form $y = mx + c$ • m represents the gradient of the linear graph i.e. $\frac{\text{change in y}}{\text{change in x}}$ • c represents the y-intercept of the linear graph Sketch the graph of $y - \frac{x}{2} = 3$ Answer: Step 1 : rewrite equation in the form $y = mx + c$ $y = \frac{x}{2} + 3$ $y = \frac{1}{2}x + 3$ Step 2: Plot the y-intercept Step 3: Draw a line that is 2 units to the right, parallel to the x −axis. The horizontal change is 2. Step 3: From this point draw a line 1 unit up and plot the point. Step 4: Join the y-intercept with this point to draw the straight line.	follow the teacher's explanation answer the questions asked



Step 1 - the x-value is the dependent variable so select a set of values to represent x

Step 2 - use the equation and substitute each x-value to calculate the corresponding y-value

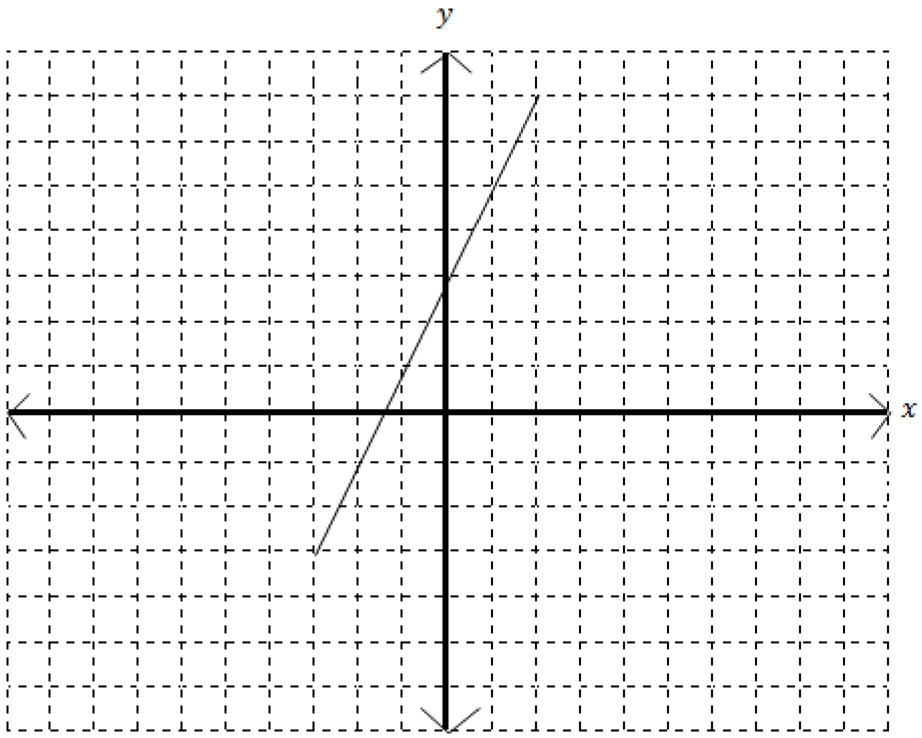
Step 3 - plot the ordered pairs on a Cartesian plane

Answer

$$y = 2x + 3$$
$$y = 2{-3} + 3 = -3$$
$$y = 2{-2} + 3 = -1$$
$$y = 2{-1} + 3 = 1$$
$$y = 20 + 3 = 3$$
$$y = 21 + 3 = 5$$
$$y = 22 + 3 = 7$$

x	-3	-2	-1	0	1	2

y	-3	-1	1	3	5	7



Activity 2

Intercept method

Note: the equation does not have to be in the standard form

Draw the graph of $y = 3x - 6$ by using the following steps:

Step 1: Determine the x-intercept by substituting $y = 0$

Step 2: Write the x-intercept in coordinate form (x;0)

Step 3: Determine the y-intercept by substituting $x = 0$

Step 4: Write the y-intercept in coordinate form (0:y)

Answer:
x-intercept let y = o

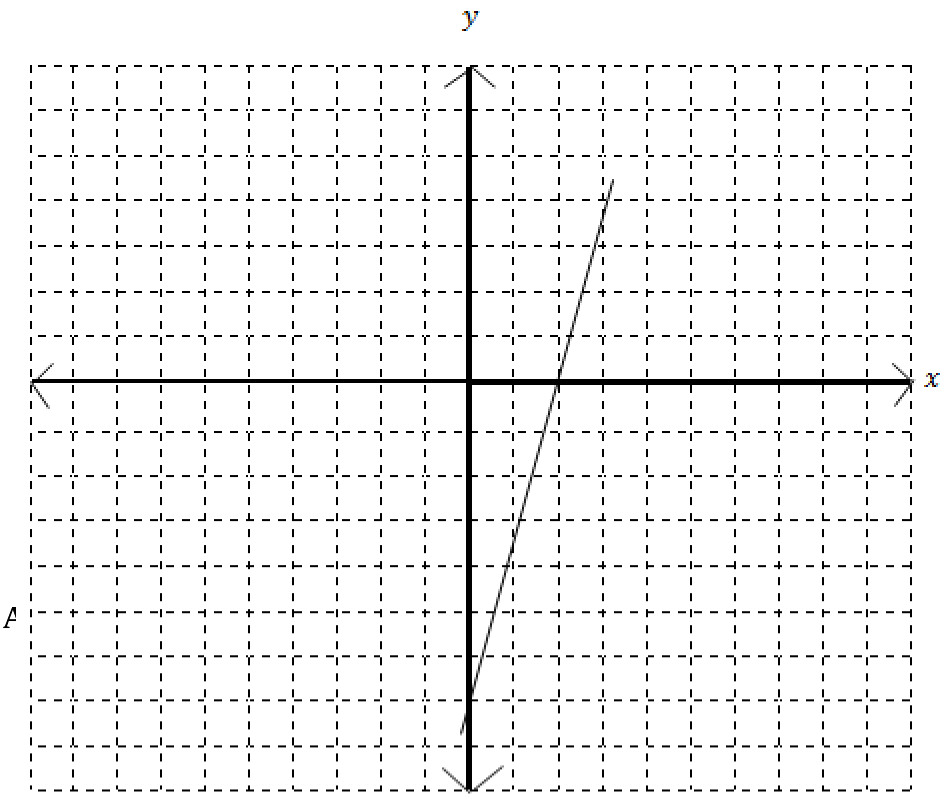
$$0 = 3x - 6$$
$$3x = 6$$
$$x = 2$$

(2:0)

y-intercept let $x = 0$

$$y = 3(0) - 6$$
$$y = -6$$

(0:-6)



- the equation have to be in the standard form $y = mx + c$
- if equation is not in the standard form rewrite it in the form

$y = mx + c$

- m represents the gradient of the linear graph i.e. $\frac{\text{change in y}}{\text{change in x}}$
- c represents the y-intercept of the linear graph

Sketch the graph of $y - \frac{x}{2} = 3$

Answer:

Step 1 : rewrite equation in the form $y = mx + c$

$$y = \frac{x}{2} + 3$$

follow the teacher's explanation answer the questions asked

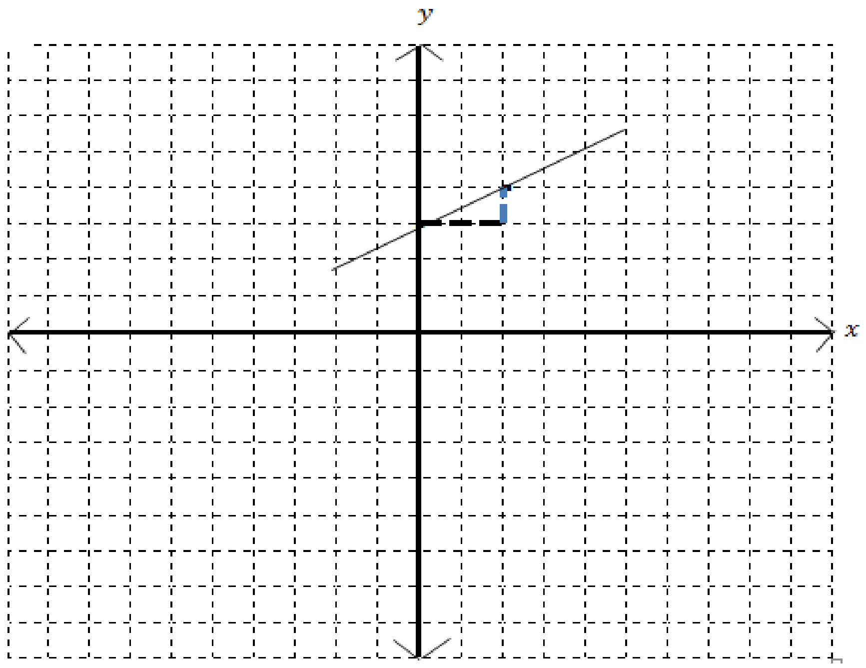
$$y = \frac{1}{2}x + 3$$

Step 2: Plot the y-intercept

Step 3: Draw a line that is 2 units to the right, parallel to the x −axis. The horizontal change is 2.

Step 3: From this point draw a line 1 unit up and plot the point.

Step 4: Join the y-intercept with this point to draw the straight line.



CLASSWORK (Suggested time: 15 minutes)

1) From the equations complete the table and write down the ordered

Plot the points on a Cartesian plane. Join the points to form a s

line.

a) $y = x + 2$

x	-3	-2	0	2	3
y					

2) Use the intercept-method and sketch the graphs of the following s

line.

a) $3y - 2x = 6$

3) Sketch the graphs of the following straight line by using the gradient-intercept method.

a) $3y = 4x - 2$

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested ti minutes)

i) **Emphasize that**:

- the x-values is the independent variable and the y-vale is the dep variable
- the equation can be used to substitute each x-value to calcula corresponding y-value
- the x-value and the y-value becomes an ordered pair.

Homework:

1) From the equations complete the tables and write down the ordered Plot the points on a Cartesian plane. Join the points to form a s line.

a) $y = \frac{1}{3}x + 4$

x	-3	-2	0	2	3

y					

2) Use the intercept-method and sketch the graphs of the following straight line.

a) $x + 3y - 2 = 0$

3) Sketch the graphs of the following straight line by using the gradient-intercept method

a) $y = -\frac{4}{3}x - 5$

Appendix 14: Lesson 9

TOPIC: DRAWING GRAPHS

CONCEPTS & SKILLS TO BE ACHIEVED:

By the end of the lesson learners should know and be able to: determine equations from given linear graphs.

RESOURCES: DBE Book 2, Sasol-Inzalo Book 2, textbooks

PRIOR KNOWLEDGE:

- x- and y-intercepts
- gradient and how to determine the gradient
- standard equations of a linear graph

REVIEW AND CORRECTION OF HOMEWORK (suggested time: 10 minutes)

Homework provides an opportunity for teachers to track learners' progress in the mastery of mathematics concepts and to identify the problematic areas which require immediate attention. Therefore, it is recommended that you place more focus on addressing errors from learner responses that may later become misconceptions.

INTRODUCTION (Suggested time: 10 Minutes)

Discuss the importance of an equation to describe trends e.g. when working with number patterns it helps us determine the values of an unknown number of terms if the general rule (equation) is known, or it is easy to complete a table if the rule (equation) is known.

Activity 1

The following sequence are given: 5; 8; 11; 14;….

a) Give the next two terms of the sequence.

b) Determine the general rule for the sequence.

c) Determine the value of the 100th term.

Activity 2

Given the following table:

x	0	1	2	3	4
y	2	5	8		

a) Complete the table.

b) Determine a rule that corresponds to the table.

Discussion:

The equation (general term) of a number pattern can be determined using the given sequence and observing the pattern. The equation (rule) can be determined for a set of values given in a table using the values in the table. In the previous lesson graphs were sketched from a given equation where a table was set up or the intercepts and gradient were determined. In this lesson an equation will be determined from a given graph with known information.

LESSON PRESENTATION/DEVELOPMENT (Suggested time: 20 minutes)

Teaching activities	**Learning activities** (Learners are expected to:)

Activity 1

Follow teacher's explanation and answer the questions

The equation of a straight line is $y = mx + c$. To determine the equation of a straight line the values of m and c must be known or determined. If the values of two points are known then the gradient can be determined using the formula: $\mathrm{m} = \frac{\mathrm{y_A - y_B}}{\mathrm{x_A - x_B}}$. If the gradient of the graph is known the y-intercept can be determined by using substitution.

Example 1: Determine the equation of the straight line that goes through (1; 1) and (5; 13).

Step 1: Calculate the gradient.

$$\mathrm{m} = \frac{\mathrm{y_A - y_B}}{\mathrm{x_A - x_B}}$$

$$= \frac{13 - 1}{5 - 1}$$

$$= \frac{12}{4}$$

$$= 3$$

Step 2: Since $m = 3$ substitute it into the equation $\mathrm{y = mx + c}$. Therefore $\mathrm{y = 3x + c}$.

Step 3: To determine c substitute the coordinates of a point on the line into the equation. (It can be either one of the points that were given, so choose the easier one.)

Substitute (5; 13) into $y = 3x + c$

$$(13) = 3(5) + c$$

$$13 = 15 + c$$

$$13 - 15 = c$$

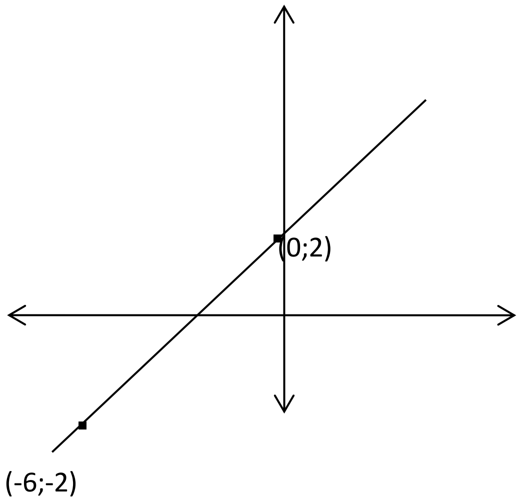
$$-2 = c$$

Step 4: Write down the equation: $y = 3x - 2$

Activity 2

The gradient –intercept method

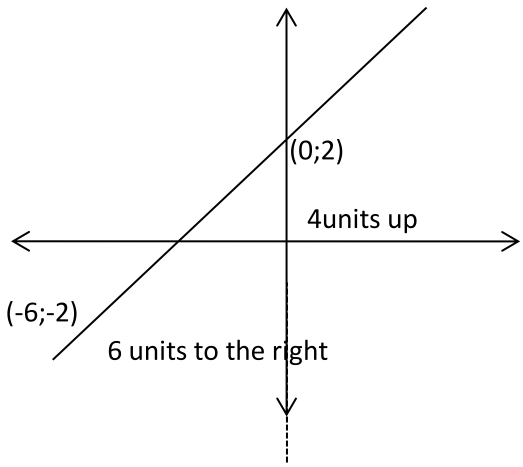
Remember in the equation $= \text{mx} + \text{c}$, m represents the gradient and c represents the y-intercept.



Determine the gradient of the graph with the teacher and answer the questions

Determine the gradient by determining the vertical change and the horizontal change. Gradient $= \frac{\text{vertical change}}{\text{horisontal change}}$

Answer:



$$m = \frac{4}{6} = \frac{2}{3}$$

The graph cuts the y-axis at 2 so $c = 2$

Equation: $y = \frac{2}{3}x + 2$

CLASSWORK (Suggested time: 15 minutes)

Sasol-Inzalo pg. 70 no. 1 a-c; pg. 71 no. a, b

CONSOLIDATION/CONCLUSION & HOMEWORK (Suggested time: 5 minutes)
j) **Emphasise that**: • The standard form of the equation of a linear graph is $y = mx + c$, where m represents the gradient of the graph and c represent the y-intercept of the graph. • To determine the equation of a graph the gradient must be determined as well as the y-intercept from given information and then substituted into the standard form of a linear graph **Homework:** Sasol-Inzalo pg. 70, no. 1 d-e; pg. 71 no. c, d

Appendix 15: Case Processing Summary (KR20)

Case Processing Summary (KR20)

		N	%
Cases	Valid	33	100.0
	Excluded[a]	0	.0
	Total	33	100.0

[a] Listwise deletion based on all variables in the procedure.

Appendix 16: Reliability of Test (KR-20)

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.726	.711	17

Appendix 17: Item-Total Correlation for KR-20

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Q1b	10.76	10.502	-.235	.725
Q2a	10.85	10.195	-.018	.723
Q2b	11.00	10.250	-.065	.735
Q3a	11.03	9.343	.248	.704
Q3b	10.97	8.968	.426	.686
Q3c	10.85	10.008	.071	.717
Q3d	10.82	9.841	.186	.708
Q3e	11.12	9.235	.261	.703
Q3f	11.21	8.797	.403	.686
Q3g	11.21	8.360	.562	.666
Q3h	11.21	8.860	.381	.689
Q3i	11.18	8.528	.502	.674
Q4i	11.09	8.710	.457	.680
Q4ii	11.03	8.530	.557	.670
Q4iii	11.12	8.860	.393	.688
Q5	11.03	9.093	.340	.694
Q5i	11.15	9.320	.227	.707

Appendix 18: Scale Statistics for KR-20

Mean	Variance	Std. Deviation	N of Items
11.73	10.267	3.204	17

Appendix 19: Validation form for the achievement test

INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION

UNIVERSITY OF SOUTH AFRICA

Research Study to investigate the Effect of Integrating GeoGebra Graphing Software into the Teaching of Linear Functions on Grade 9 Learners' Achievement in Mopani District

Validation Form

Examiner: Mushipe M	Date: 19 – 01 – 2016
Task: Test	Signature: M. Mushipe
Question Paper	

ASPECTS	YES	NO	COMMENT
Cover page	YES		
Are the instructions clear?	YES		
Time allocation	YES		
Total mark	YES		
Mark allocation	YES		
Has an analysis grid of the question paper been provided?	YES		
Is the standard of the paper good?	YES		
Is there sufficient content coverage?	YES		
Are the questions concise and to the point (not ambiguous)?	YES		
MEMORANDUM			
ASPECTS	YES	NO	COMMENT
Is the memorandum authentic?	YES		
Does the memorandum make allowance for alternative responses?	YES		
Is the memorandum consistent e.g same font or completely handwritten?	YES		
Is the memorandum aligned with the question paper?	YES		

Are marks allocated to each answer?	YES		
Have subtotals been allocated to each section?	YES		
Do marks add up?	YES		
Are the marks allocated to the question the same as those on the memorandum?	YES		
Are the answers mathematically correct?	YES		
Moderation Results			
ASPECTS	YES	NO	COMMENTS
Is the question paper approved?	YES		

MATHOBELA MJ
Name of moderator

Signature of Moderator

19/01/2016
Date