# AN INVESTIGATION INTO THE DIFFICULTIES FACED BY FORM C STUDENTS IN THE LEARNING OF TRANSFORMATION GEOMETRY IN LESOTHO SECONDARY SCHOOLS

**by**

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## DECLARATION

I declare that **AN INVESTIGATION INTO THE DIFFICULTIES FACED BY FORM C STUDENTS IN THE LEARNING OF TRANSFORMATION GEOMETRY IN LESOTHO SECONDARY SCHOOLS** is the student, Mr Evbuomwam Dickson's own work and that all the sources that he used or quoted have been indicated and acknowledged by means of complete references.

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MG Ngoepe                                        Date

## DEDICATION

This work is dedicated to my late beloved father Mr S, E. Igbinovia

## ACKNOWLEDGEMENTS

I wish to express my sincerest gratitude to Dr MG Ngoepe without whose support this study could not have been carried to completion. Her constructive criticism and encouragement kept me going.

## ABSTRACT

The Lesotho Junior Secondary Examination Analysis (2009 and 2010) revealed that students performance in Mathematics in general and Transformation geometry of rotation in particular was generally poor. Only a few number of students that sat for the final Form C Examination passed.

This study employed the van Hiele's levels of learning to investigate and describe the difficulties students have in the learning of rotational transformation geometry. Both a written test and interview were used to solicit information regarding students' difficulties. This information was collected from 90 students from Qaoling Secondary School in Maseru district in Lesotho. Findings from the study revealed that students had difficulties in identifying and naming transformation of rotation, finding the centre, angle of rotation and locating the exact image of a rotated figure after rotation. Also, they had greater difficulties when using transformation to do proof.

The analysis showed that students mostly had difficulties at the level of Abstraction and Deduction. This gave an indication that the vast majority of the students in Form C are reasoning at the lowest two levels of the van Hiele's model which are Visualization and Description. For these students' difficulties to be curbed, the analysis demonstrated amongst others that teachers needed to use Manipulative materials and Information Communication Technology (ICT) during the process of teaching and learning. Manipulative materials provide experience in which students can transfer their understanding smoothly from one concept to another.

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# CHAPTER ONE

# THE BACKGROUND AND OVERVIEW OF THE STUDY

## 1.1 BACKGROUND TO THE RESEARCH PROBLEM

One major goal of secondary school geometry is the development of mathematical reasoning abilities and the promotion of a deeper awareness of the real world (The National Council of Teachers of Mathematics (NCTM, 1989). To help students achieve this goal, the NCTM suggests that reasoning about shapes should use coordinate and transformation techniques as well as the traditional synthetic techniques such as flip, turn and sliding of an object. In keeping with the achievement of this goal, the Lesotho mathematics curriculum advocated a change from the traditional deductive Euclidean geometry toward the investigative transformation geometry. The latter is introduced because transformation geometry permits a more intuitive discovery approach to learning (NCTM, 1989).

In Lesotho the secondary school educational policy entails five years secondary school study. During this period, students are required at the fifth and third year of study to sit for an external examination. At both phases of learning, mathematics is one of the examinable subjects and it is compulsory for all third year students, also known as the Lesotho Junior Certificates Examination but optional at the senior school certificate examination except for student aspiring for a further study in the sciences. However, at both levels of study the mathematics syllabus includes topics drawn from Algebra, Trigonometry, and Geometry. These topics are integrated and taught simultaneously every year (Lesotho Secondary School National curriculum, 2010). However, indication emanating from the report compiled by the Examination council in Lesotho (Examination council of Lesotho, 2007, 2008 and 2009) revealed that students' performance in geometry and in particular transformation geometry of rotation was very poor as compared to other areas of mathematics. The average performance was below 20 percent in, 2004, 2005, 2006, 2007, and 2009 (Examination council of Lesotho, 2004, 2009). This evidence leaves much to be desired. The report pointed to the fact that students may lack enough acquisition of geometric skills such as, the ability to imagine, rotate, and twist an object which are essential in facilitating the learning of transformation geometry and other geometrical concepts. The result of an analysis from the examination council also revealed that among the three concepts (Rotation, Reflection and Translation), students seemed to perform poorly specifically in the skills associated with Rotation. Furthermore, it was revealed that most students did not give the required solution for example, when finding the point and centre of rotation, finding the line and the order of rotational symmetry, identifying figure or shapes after transformation, using a given transformation to transform an object/image when given the coordinates, angles and shape (Examination council of Lesotho, 2009).

These difficulties experienced by students in an examination could be explained and understood better by reflecting on the research work done by van Hiele (1986). van Hiele's research which had its root in Piaget's work, focused primarily on five levels of geometric conceptualization. The five levels are visualization, descriptive, abstraction, formal deduction and rigor. van Hiele (1986) stated that there are two main reasons for the existence of levels.

- If students have not sequentially gone through the proposed five levels, then they cannot function adequately at any given level. However, they can perform algorithmically at any level with no understanding, meaning that rote learning has taken place.
- If the instructor uses a language, a textbook, or a teaching method at a higher level that is different from that of the student, a serious communication problem between the teacher and the students may occur and this may result in frustration and lack of understanding on the part of the students. van Hiele asserted that higher levels of thinking are rarely reached by most students because geometry materials are improperly sequenced.

The above van Hiele findings do have a great implication in the learning of transformation geometry. They do explain the reason why many secondary school students are having problems in geometry learning.

In this study, these van Hiele's findings were used to investigate and describe students' difficulties in transformation geometry involving rotation. van Hiele's findings are useful because they provide the researcher with a broader understanding of how students learn geometry and transformation geometry in a classroom situation and how to address the frustration related to geometry instruction and learning.

Research on students' thinking processes as related to achievement in geometry done in the past by researchers such as Hoffer (1981), Soon (1992) and Corley (1990) substantiated the validity of the van Hiele model and its relationship to students' achievement. Soon (1992) investigated the existence of hierarchy of a van Hiele level of understanding of transformation geometry. An interview and observation techniques were used to collect data from a group of about 20 students within the age range of 15 to16 years. The result of the investigation indicated that the levels as exemplified by the task did form a scale, which seems to support the existence of a hierarchy of the van Hiele's level for transformation geometry.

Corley (1990) investigated students' levels of thinking as related to their achievement in a traditional high school geometry course. The study revealed that students levels of geometric understanding as described in the van Hiele model were related to student achievement. It was further revealed that these levels of geometric understanding in van Hiele's model are an accurate means of evaluating students' readiness for formal geometric instruction. In my view as a teacher and anecdotal evidence, the reasons for students' difficulties to fully understand concepts of Rotation, Translation and Reflection have not been fully given adequate research attention in Lesotho hence the present study is tailored to provide information into the difficulties which students have in solving transformation geometry (Rotation) problems in the classroom.

Results from both studies by Soon (1992) and Corley (1990) as briefly explained above, supported the existence of hierarchy of the van Hiele's level for transformation geometry learning. However, both researches fall short of adequate description or analysis as to how students solve problems in geometry and their perceived difficulties.

This study therefore investigates and describes various difficulties which may hinder students learning of transformation geometry in particular the concept of Rotation in Lesotho.

## 1.2 STATEMENT OF THE PROBLEM

There is increasing evidence that many students in the middle years (11-15) of schooling have tremendous misconceptions concerning a number of important geometry ideas (Burger et al, 1996). There are many possible reasons for this. Furthermore, a divergence of opinion exists in the mathematics community about the methods and outcomes of geometry and as a result, textbook writers and producers of syllabuses have failed to agree on a clear set of objectives.

Anecdotal evidence suggests that many teachers do not consider geometry and spatial relations to be important topics to be taught in schools. This gave rise to the feelings that geometry lack firm direction and purpose (Burger et al, 1996). However, in Lesotho as mentioned earlier, many secondary school students are faced with varieties of difficulties relating to transformation geometry.

Analysis from the examination council revealed that most students did not give the required solution for example, when finding the point and centre of rotation, finding the line and the order of rotational symmetry, identifying figure or shapes after transformation, using a given transformation to transform an object when given the coordinates, angles and shape (Examination council of Lesotho, 2009). In 1983 he observed that the geometry course that was taught to high school students was deeply rooted in proof writing hence it is not working for large number of students. According to Usiskin, at the end of their study of geometry many students did not posses even trivial information regarding geometry terminology and half of the students who enrolled in a proof-oriented course experience very little success with proof (Usiskin, 1983). Many geometry teachers have experienced this same frustration that accompanies the inability of their students to understand or appreciate the power and beauty of geometry.

My observation as a mathematics teacher is that while some students in Lesotho are very successful in solving problems on transformation geometry, many do the problems algorithmically with little or no understanding. It seems that a similar problem was encountered in plane deductive geometry. Difficulties with proofs also appear in the learning of transformation geometry in Lesotho secondary schools.

## 1.3 PURPOSE OF THE STUDY

Past research showed that the van Hiele's levels of learning geometry and transformation geometry can have implications for investigating students' difficulties and improving student's performance in transformation geometry (Ada & Kurtulus, 2010). Also it can provide a frame work on which geometry instructions can be structured and taught schools Ada & Kurtulus (2010). However, this claim has not been comprehensively investigated in Lesotho. Hence it deserves some exploration and investigation with students.

The purpose of this study is to use the van Hiele's model to investigate and describe various difficulties which students may have in the learning of transformation geometry in particular the learning of the concepts of Rotation. The investigation will focus on analyzing in a broader context how students:

- Visually identify an image after rotation.
- Use the concept of rotation to transform an image when given the coordinates, angles and shape.
- Describe geometric figures and their properties after transformation.
- Discover the properties in a given transformation by locating centre, and angle of rotation.
- Use transformations to do proofs.

## 1.4 RESEARCH QUESTIONS

The above purpose will be realized by pursuing answers to the following questions that are linked to the van Hiele's model:

1. To what extent are students' able to visually identify and name transformation of rotation by its motion using a standard or no standard name (Visualization).
2. To what extent are students' able to discover the properties of a figure and its images after a rotation and use these properties to analyze a transformation of rotation? (Analysis)

3. To what extent are students' able to use rotation to transform an object when given the coordinates, angles and shape? (Abstraction)
4. To what extent are students' able to use transformations to do proofs? (Deduction)

## 1.5 EDUCATIONAL SIGNIFICANCE OF THE STUDY

Piaget (1971) and van Hiele (1986) made significant research work relating to the thought processes and the sequence a person goes through in learning. Many more studies have also been done by various researchers in the field of spatial cognition, but very little impact on classroom has resulted (Peg, Davey, 1998); (Hoffer, 1981); (Usiskin,1982) and (Ada & Kurtulus, 2010).

Lesotho students have over the years experienced varieties of difficulties in the learning of geometry and transformation geometry. If learning difficulties experienced by students could be investigated and explained by using the van Hiele's levels for transformation geometry learning in this study, then the result of the study and its recommendation will provide useful information for teachers, society, school administrator and curriculum developers on how to ease students' difficulties in the teaching and learning of transformation geometry in the classroom. This study will make contributions to mathematics education by way of providing information with regards to the students' difficulties in transformation geometry that involved rotation. It will also contribute to the improvement of policies that addresses the difficulties that students face in learning transformation geometry. A valuable contribution will be made from this study as the outcome may suggest better and more improved teaching strategies that could help the teacher towards a more effective teaching of transformation geometry in schools.

## 1.6 DEFINITION OF TERMS

The following terms are used throughout the research report and they are defined here to establish a clearer and concise meaning.

**Mathematics:** This simply implies the study of measurement, number and quantities.

**Concepts:** General idea about something, this involves understanding the components of a phenomenon.

**Creativity:** In an attempt to define creativity we look at personality trait of creative individuals. Such individual are always thinking, are always prepared to listen to others opinion, are critical of their work, are analytic and original; have adaptive flexibility, spontaneous flexibility, word fluency, the capacity to puzzled, they are motivated, confident, intellectually persistent and moral communication to work.

**Curriculum:** Planned experiences offered to the learner under the guidance of the school.

**Syllabus:** A document containing content which learners are expected to know before being examined.

**Manipulative:** Concrete material, e.g. Tangrams, Geoboards and Pattern blocks.

**Geometry education**: Mathematics is an activity of solving problems concerning shapes, vision and location. Geometry education concern itself with theories, principles and methodologies in the teaching of geometry (Freudenthal, 1971).

**Shapes**: Geometry shapes embedded in spatial objects and create an opportunity to move from two-dimensional perception and vice versa.

**Vision**: projections of reality from various vantage points are an important part of geometry.

**Location**: students have to be exposed to different systems for determining position and how to use them appropriately.

**Divergent thinking**: Divergent thinking could be seen as reasoning that practices unanticipated and unusual responses (Cangelosi, 1996). This may included cognitive processes such as critical thinking, analysis, synthesis, interpretation, conjecturing, and induction. Such thinking enhances creativity in students.

**Problem solving**: May include working and making a drawing, create your own problem, think of a similar problem that was solved successfully in attempting to solve a problem. Problem solving is teacher centered in the sense that the teacher can direct students at the said strategies.

**The problem-centered approach**: In the problem – centered approach, instruction begins with problems. It is from the solution of the problems that students acquire knowledge. In this approach the students interprets the problem condition in the light of his repertoire of experience (knowledge and strategies previously assimilated). The teacher only provides the necessary scaffolding during this process.

## 1.7 LAYOUT OF THE STUDY

### Chapter one

The background to the research problems and the research statement are discussed in this chapter. The purpose of the study, the research questions and the significance of the study are also highlighted.

**Chapter two**

This chapter reviews the relevant literature on geometry and transformation geometry with special focus on how students learn geometry and the difficulties they have during learning.

**Chapter three**

This chapter provides a detailed description and explanation of the research design, the research instruments and how these instruments were developed. Descriptions of the population sample, how the research instruments were administered, the validity and reliability of these instruments and steps taken to analyze both the written test and interview were discussed in this chapter.

**Chapter four**

This chapter focused on the analysis and interpretation of data obtained from both the written test and the interview administered to students.

**Chapter five:**

A presentation of the summary of findings recommendations and limitations of the study are unveiled in this chapter.

## 1.8 CONCLUSION

In order to investigate and describe the problems which were highlighted in the research questions, this research was conducted with all Form C students in Lesotho secondary school with the purpose of using the van Hiele's model to investigate and describe the difficulties students have in transformation geometry with reference to rotation. This chapter provided a summary of the study by providing and discussing issues relating to the background and overview of the study, the purpose of the study, the research questions and the significance of the study. The next chapter presents the review of related literature.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.1 INTRODUCTION

This chapter reviewed the literature of the study that investigated the difficulties students have in the leaning of rotational transformation geometry. In Lesotho, there are no available empirical studies regarding how students learn geometry. This implies that the researcher relied on international literature as a source of information. The literature on what geometry and transformation geometry entails, is discussed in section 2.2. The rationale for the inclusion of transformation geometry into the secondary school curriculum is discussed in section 2.3. Research relating to geometry and its transformation discussed in section 2.4. Spatial development as a prerequisite for learning transformation geometry is described in section 2.5. The role of spatial perception and visualization in the learning of transformation geometry are also explained in section 2. 6. The chapter concludes with a review of the views of van Hiele, Piaget, Freudenthal and other related research on how students learn geometry. These are highlighted in sections 2.7 and 2.8 respectively. The relevance of the van Hiele, Piaget and Freudenthal views to the present study is also discussed. Section 2.9 concludes the chapter.

## 2.2 GEOMETRY AND TRANSFORMATION GEOMETRY

Geometry is a branch of mathematics dealing with the measurement and relationship of lines, angles, surfaces and solids (Fish, 1996). There are many types of geometry found in the field of mathematics. This includes Euclidean, Non-Euclidean, Dynamic, Transformational, Projective, Vectors, Applied and Menstruation geometry.

Early man used geometry concepts to solve everyday problems. They reduced the shape of nature to a less difficult form which was used in a multitude of ways. For example arrowheads, baskets and pottery demonstrated an intuitive knowledge of geometry. Geometry was utilized by man to build huts, to erect tents and excavate cave (Daffer and Clemens, 1977). As time went on man's knowledge of geometry became more sophisticated. The Egyptian used geometry technique to remark the land that was annually washed away by the Nile River. The word "geometry" meaning earth measure originated as a result of its application by the Egyptian (Greegerg, 1973). Geometry became the science of the study of the measurement of the earth. Thus "the earth consist of structures made up of points, lines and spaces between or on which the lines lie, and the similarity and distinction between the different structures" (Greegerg, 1973:10). It is on this notion of points and lines, distinction between these lines which they lie on and the congruency between the structures that the Euclidean geometry is based on.

Transformational geometry is an aspect of geometry which concern itself with the way geometrical shape of objects are transformed into their various image under reflections, translations, rotations, glide reflections and magnifications on a plane. This transformation could be in the form of reflection, translational or rotation depending on the matrix of transformation (Burger, 1992).

Transformation geometry is a dynamic approach to learning geometry in which students use hands-on activities with concrete object in addition to using technology (Fuys, Geddes and Tischler, 1985). Rotational transformation transforms the original shape of an object onto its image through a particular angle which could be clockwise or anticlockwise on the plane. It could also be represented by vector or coordinates of the image of the original shape. The image of the rotation is congruent to the original shape. From the above indices one can conclude that Transformations are movements that includes Translation (slides), Rotation (turns) and Reflections through a point half turns or rotation of 180°degree and Reflection of a line 'flip'; (Fuys, Geddes and Tischler, 1985).

In the teaching and learning of transformation geometry, students are expected to carry out tasks involving, Reflection, Translations, Rotation and Enlargement of an object. In doing this, students naturally or intuitively solve problems by manipulating concrete object or drawing figures as requested.

In transformation geometry (Rotation) in particular, students are expected to find the point, angle, center, symmetry, describe and turn any given figure through a given degree. In doing this the National Council of Teachers of Mathematics (NCTM) suggested that students should construct their own models from straws, make drawing, fold paper cutout, and use a mirror to see symmetry (NCTM,1987).

## 2.3 RATIONALE FOR THE INCLUSION OF TRANSFORMATION GEOMETRY INTO THE SCHOOL CURRICULUM

The debate surrounding the instruction of geometry was complicated in the middle of the nineteen century by moving the course from the college level to the high school level. Although the maturity level of the students was lowered, there was no organized adjustment in course contents. Since then, numerous committees have addressed the need for adjustment and have offered a wide range of recommendations with the goal of reforming geometry instruction in schools. Despite these continual attempts to change the teaching of geometry, in the intervening decades there have been few fundamental changes fused into the widespread practice. Part of this reason was lack of reform (Allendoerfer, 1968). A study done by William (1968) revealed that Euclid's traditional approach to the subject is considered by many to be a significant part of man's cultural heritage.

In the traditional Euclidean geometry, many students experienced difficulty writing proofs, and most students were unsuccessful in solving geometrical problems. This is reported in many surveys for example Usiskin (1983) and Hoffer (1986). Transformation geometry was introduced in the curricula in many countries partly because of this reason. However, in the 1960's and 1970's several groups and individuals came from various countries of the world in support of the inclusion and study of transformation geometry in secondary school mathematics curriculum (Usiskin, 1983). An example of these groups includes the Cambridge conference committee into the goal of school mathematics (1963), the Geometry committee of the Ontario institute for science in education (1967). These two groups gave recommendations for the inclusion of geometry transformation at all levels. These recommendations were supported with a rational for their introduction. Advocates of the study of transformation geometry include prominent mathematics educator such as Allendoerfer. In his article of 1969, 'Dilemma in Geometry'. Allendoerfer strongly advocated that an understanding of the basic fact about geometry transformation such as Refection, Rotation and Translation should be a major objective of geometry in schools. He further suggested that elementary school students should be taught through the use of coordinates and pairs of linear equations (Allendoerfer, 1969 as cited in Soon, 1992:13). Echoing the same sentiments and reason with regards to the role of transformation geometry is Usiskin (1971) assertion that one of the reason for teaching transformation is that it served as a unifying theme in the high school mathematics.

The intuitive and informal aspects were lauded by Peterson (1973) who mentioned that because of its dynamic nature, transformation geometry encourages students to investigate geometry ideas through an informal and intuitive approach. The extensions of certain mathematical concepts to abstract levels were noted by Fletcher(1965) which suggested that transformation geometry can lead students to the exploration of abstract mathematical concept of congruence, symmetry, similarity and parallelism, and can enrich students' geometrical experience, imagination and thought, thereby enhancing their spatial abilities (Fletcher,1965). The concept of transformation was seen by Schuster (1973) and Pickreign (2000) as good foundation for learning later mathematical concepts. This is because these concepts had several functions such as:

(a) They serve as a natural introduction to concept of mapping and function.
(b) They serve as a concrete basis for the early study of vector
(c) They give a simple formation of idea of congruence
(d) They provide an excellent example of mathematizing of the physical world, through the notion of isometric transformation as a mathematical abstraction and generalization of physical spatial relation (Schuster, 1973).

Coxford (1983) went on to enumerate the roles transformation plays in mathematics:

- Transformation provides opportunities to examine structures that are common both to algebraic and arithmetic objects and to geometric object.
- Transformation helps the students to comprehend the importance of common structures for all mathematics, and to begin to see the power of abstraction and generalization.
- Transformation can be represented geometrically through slides, translation, reflection and dilations of geometric figures. They can also be represented algebraically through matrices, in this way transformation help to emphasize the interplay between algebra and geometry.
- Pedagogically, “Transformation can be used to organize instruction so that it is more closely related to youngster’s intuitive ideas; in addition, the use of transformation allows grater students flexibility and creativity in constructing proofs” (Coxford 1983:151).

From the position of National Council of Teachers of Mathematics (NCTM) standards. Mathematically, Transformation Geometry should be introduced into the school curriculum because it encourages students to investigate geometry, and develop power of abstraction and generalization.

From my view point, with regards to the importance and purpose of transformation geometry highlighted above, it implies that if transformation geometry skills were to be achieved through a geometric course, therefore, upon finishing the course, students should be able to understand, appreciate and use transformation geometry to solve personal and societal problems, However, this seem not to be true as most students after years of geometry study do not posses even trivial information regarding geometry terminology and transformational approach in learning geometry (Coxford ,1983). As a result, students’ difficulties in geometry and transformation geometry calls for a concern on the part of researchers and the mathematics community to investigate how these geometries are being taught and learned by students.

## 2.4 RESEARCH ON TRANSFORMATION GEOMETRY

Researchers studying the difficulties of geometry and transformation geometry revealed the various challenges about the teaching and learning of geometry. The study of Denis (1987) focused on the relationships between the stages of cognitive development and van Hiele level of geometry thought. The results from the study showed that 74% of the students were incapable of dealing with the demands of traditional high school geometry. This result was supported by other investigations by Hollebrands (2003), Ada & Kurtulus (2010), Soon (1992), Moyer (1975), Thomas (1976) and Perham (1976). These studies investigated students' understanding of transformation geometry concepts. Likewise, these studies revealed that there is no transfer of spatial abilities for first, second, and third graders in transformation geometry problem solving situation. Meaning that, in a problem solving situation, students at these levels of learning are unable to use spatial ability when solving transformation geometry task in the classroom.

Ada & Kurtulus (2010) investigated students' misconception and errors in transformation geometry. The study analysed students' performance in two-dimensional transformation geometry and explored the mistakes made by the students taking the analytical geometry course given by researchers. The subject of the study included 126 third-year students of mathematics in the university. Data were collected from seven exam questions. The result of the analysis showed that these students did not understand how to apply rotational transformation. The mostly mistake observed showed that the students seemed to know the algebraic meaning of translation and also of rotation but they did not seem to understand the geometric meaning of these concepts. In a similar study, Hollebrands (2003) investigated the nature of students understanding of geometric transformations, which included translation, reflections, rotation and dilations, in the context of the technological tool, the Geometer's Sketchpad. The researcher implemented a seven-week instructional unit on geometry transformations within an Honors Geometric class. Students' conceptions of transformations as functions were analyzed.

The analysis suggested that students' understanding of key concepts including domain, variable and parameters, and relationships and properties of transformation were critical for supporting the development of deeper understandings of transformations.

Perham (1976) investigated factors that contributed to the difficulties encountered by students in performing rigid transformation tasks. The direction of transformation task revealed that children seemed to be able to perform vertical or horizontal transformation but not tasks over the diagonal (Perham, 1976). Perham's study with first grade children revealed that children had some understanding of slides before the unit of instruction but not of flips or turns of any type. After instructions, diagonal transformations that included slides were not performed correctly. In addition, Perham found that his subjects proceeded from the anticipatory level to the representational level (Schultz, 1978; Perham, 1976).

Outcomes from these studies mentioned above gave a suggestion that students are faced with difficulties such as the inability to perform transformation vertically, horizontally and diagonal. Those observations call for intervention from all the stake holders in the teaching and learning of transformation geometry.

Transformations are associated with physical motions. In keeping with this idea of physical motion, another study done by Moyer (1978) revealed that explicit knowledge of the physical motion did not necessarily help children's abilities to perform the transformation tasks. According to Moyer (1978), students focused on changes in relationships between the state of the initial and final configurations of transformed objects. This suggested to Moyer that children rely more on general topological relationships rather than typical Euclidean. Moyer (1978) and Perham (1976) both establish that there is a developmental order: "Slides first" then "flip" and finally "turn" which is in contradiction to mathematics structure where flip is the basis and the other isometrics are derived from it.

It was further observed that nine, eleven and thirteen (9,11 &13) years old students poor performance in the transformation tasks  were due to non-conservation of length at an age where they were expected to have this skill (Moyer, 1978 as cited in Soon, 1994:38).

Boutler and Kirby (1994) studied the identification of strategies used in solving transformational geometry problems by students. The purpose of the study was to investigate whether subjects, transformational geometry problem solving could be characterized as holistic and or analytic. Determine the extent to which these types of strategies are characteristics of particular subjects or particular items and to investigate whether the strategy used was related to performance.

In Boutler and Kirby study, students from grades 7 and 8 were videotaped individually as they solved a number of transformational geometry problems. Five problems that showed the greatest range in difficulties and variability in strategies usage were selected for detailed analysis. Each student’s approach to solving each problem was classified as holistic or analytic, on the basis of specified criteria. The results of the investigation indicated that students’ response could be classified consistently and some students showed a preference for holistic (or analytic) processing and that the use of strategies by students was associated with success (Boutler &  Kirby, 1994).

A five year research project done in England by Hart (1980) in Secondary School Mathematics and Science (CSMS), found a mismatch of students understanding with what they were taught. The investigation was confined to the concepts of reflection and rotation. The report showed that children seemed to have difficulty in performing single reflection and single rotation task. One of the difficulties was in reflection over a diagonal line. According to the study, students seemed to ignore the slope of the reflection line and perform reflection horizontally or vertically--- (Hart, 1980).  In order to find solutions to these difficulties experienced by students, Lesh, Post & Behr (1987) proposed a model describing the acquisition of mathematical concepts by students.

They indentified five representational modes that the students needed to poses to claim acquisition of a concept which were:

- Concrete modeling
- Pictorial modeling
- Real-world situations
- Symbolic representations (The written language of mathematics and the oral language of mathematics).

Hart (1980) research suggested that students who had difficulties translating a mathematical concept from one representation to another are the same students who exhibited difficulties with problem solving and computation comprehension. Thus any attempt to strengthen the student's ability to acquire these representations improves the growth of their conceptual understanding (Van de Walle, 1998).

Del Grande (1990) assert that "Geometry has been difficult for students due to an emphasis on the deductive aspects of the subject and a neglect of the underlying spatial abilities, acquires by hands-on activities, that are necessary prerequisites for understanding and mastery of geometrical concepts"( Del Grande, 1990:19). As a way of finding solution to these difficulties experienced by students in learning geometry, Del Grande (1990) suggested that three dimensional shapes which can be handled by the students should be used as a starting point for the learning formal geometry. It is from such three dimensional objects that concepts such as symmetry, perpendicularity, congruency amongst others, that are necessary for formal geometry are internalized.

In an earlier pursuit to find solutions to students' difficulties in the learning of geometry and transformations, Pierre van Hiele and Dina van Hiele- Geldof investigated and proposed five levels of geometry cognition. This theory was developed in the late 1950's. A detailed explanation of this theory is given in section 2.7.3.

The van Hiele theory requires a student to progress through these five levels of thought in a sequential order in understanding geometry. According to the van Hiele's study, not paying regards to this sequence would result in failure of understanding and rote learning by students (van Hiele, 1984).

The bad consequence of the teaching of geometry must almost entirely be attributed to the disregard of the levels (Fuys, Geddes & Tischler, 1988). From a teaching and learning perspective, this finding seemed to be in line with the natural order of teaching and learning in which both the teacher and students are supposed to progress in both teaching and learning in a sequential order. Furthermore, van Hiele revealed that the learning process in geometry covers many levels, but appreciation of these levels still needs to be emphasized during teaching in the classroom. It is through the disregard of the hierarchical nature of these levels with the teacher and the student operating at different levels that account for much of the difficulties which students have in the process of learning geometry. Pierre van Hiele observed that two persons who are reasoning at the different levels will not understand each other (van Hiele, 1984). Forcing a particular student to a level when he is not ready is cautioned by van Hiele because the analytical link with concrete (visual) structures will be absent. Consequently, the student will have created his entire network of relations in the absence of concrete visual structures by an imitative process incited by the teacher's structural exposition (van Hiele, 1984). van Hiele asserted further that any student forced to a level which he or she is not ready, will only be imitating his teacher's work with no meaning. "What he hears is not integrated into his existing structures in his mind" (van Hiele, 1984:87). As a result, rote learning takes place and little transfer occurs (van Hiele, 1984).

The above discoveries emanating from van Hiele's study, has been the catalyst for much of the renewed interest in the teaching and learning of geometry and transformation geometry both in the past and present (Pegg,1991). van Hiele's ideas evolved primarily out of a reaction to the deficiencies perceived with the view of Piaget which says that mental development is a continuous construction comparable to the erection of a vast building that become more solid with each other (Piaget, 1968). The van Hiele theory is based, in part on the notion that students' growth in geometry takes place in terms of identifiable levels of understanding and that instruction is most successful if it is directed at the students' level.

Indications of students' difficulties in transformation geometry from studies discussed above, suggestions and ideas emanating from the van Hiele's levels of geometry understanding have provided the structures which guided the researcher in the analysis and description of the difficulties students in Lesotho have in transformation geometry with respect to the concept of rotation.

## 2.5 SPATIAL DEVELOPMENT AS A PREREQUISITE FOR LEARNING TRANSFORMATION GEOMETRY

The term "spatial development" includes all the activities that children engage in, in order to structure the space around them. According to Fruedenthal (1974), this does not start with the formulation of definition of theorem but with the ordering of the everyday spatial experiences of the young child. In other words, the ordering structuring of these original spatial experiences can then eventually lead to the formation and structuring of theorems and definitions that are part of the more formal spatial knowledge (geometry) to which the child is expose at school (Fruedenthal, 1974). Spatial development may be classified into spatial visualization, spatial reasoning, spatial perception, visual imagery and mental rotations.

The spatial development of the young child is a multifaceted component of the child total development. It is important to remember therefore that the formal (geometry) that the child is exposed to at schools is but a part of the total spectrum of the spatial knowledge that the child acquired during their lifetime. It is therefore important for teachers to develop this informal or intuitive spatial knowledge of the child in order for them to be able to cope with school geometry (Fruedenthal, 1974). From the views of Fruedenthal, it is imperative for students to first meet geometric ideas initially through hands-on experiences with the geometric nature of their surroundings. The ability to name geometric figures should emanate from an experience that leads to the development of the underlying concepts.

## 2.6 THE ROLE OF SPATIAL PERCEPTION AND VISUALIZATION IN THE LEARNING OF TRANSFORMATION GEOMETRY

An individual's intuitive urge or feeling for objects, geometric shapes in the individual environment is referred to as spatial sense (Wheatley, 1990). Spatial sense should be thought of as imagery/visualization, meaning 'seeing the actual objects and noting its properties or having a mental picture of the object' (Kirby, 1991). These actions culminate in knowing the object having learnt about it.

Spatial sense often required an image to be transformed. For example, when transforming a rhombus to a square. This activity requires a mental imagery of the object. Mental imagery facilitates geometry learning (Kirby, 1991).

In transformation geometry learning, visualization factors focuses on the students' ability to imagine, rotate, twist or invent an object. It is the cornerstone of learning geometry and its transformations (Kirby, 1991). There are various ways in which visualization can enhance the understanding of geometry and transformation geometry by students during teaching and learning. For example, when students are able to visualize objects meaning, 'having a mental picture of the object', this may enable such students to have a deeper understanding of such object, thus visualization affords the students the opportunities to enhance their understanding of the said concept being presented by the teacher (Wheatley, 1990).

Visual imagery which is meaningful in the pupil's frame of reference may enhance the understanding of mathematical concepts at primary and secondary school level (Presmeg, 1989). In a geometry classroom situation if students sees pattern and thus identify various geometric shapes including line of symmetry, this skill and ability can help an individual student in solving problems in Euclidean geometry (Presmeg, 1989).

In the teaching and learning situation, students may be required to manipulate objects (shapes), identify and classify them according to their properties. This can be done only if students' are able to look at these objects from various vantage points (vision).

Visualization has a huge role to play for such students to be able to identify, classify and manipulate these objects or shapes. From the foregoing, it is important therefore for students to be provided with manipulative materials such as geometric shapes of various sizes, mirror, blocks or garboards, measuring tape or ruler or other specifically designed materials to aid in the instructional process. These materials can help students to create visualization (Kirby, 1991).

Spatial visualization relies heavily on the mathematical language and communication; therefore students should know mathematical concepts and communicate them to their peer and educators in the classroom (Sgrio, 1990). For this to be possible, teachers must ensure that students are given the opportunity to communicate in such a way that they develop a vocabulary that is not only written but is also mental and pictorial. These exercises are meant to enhance the students' spatial development.

According to Eisenberg and Dreyfus (1984) visualization and spatial perception, in the mathematics' curriculum has the following positive effect on concepts formation.
Firstly, the mathematical content can be pushed much further with younger students that are not capable of symbolic thinking. At this stage in fact they rely on visualization in instruction. Secondly, it is an alternative approach in presenting geometrical concepts in mathematics and lastly, it result in rich concept images which form a basis for abstraction (Eisenberg and Dreyfus, 1989).

Since students differ in abilities, teachers should therefore, present instructions in a manner that takes this into account during teaching and learning. Furthermore, in a geometry class, gifted students rely on symbolic thinking while those less gifted should visualize the problem in problem solving situation. Certainly visualization does not harm the gifted students but if left out of the curriculum, it limits the chance of success in geometry problem solving of the less gifted child, (Kirby, 1991). In the teaching and learning of geometry in schools, there are some views and theories expressed by researchers in the field of education, such as Piaget, Freudenthal and van Hiele.
In the next section, these theories of learning are presented.

### 2.7.1 THEORIES OF LEARNING

### 2.7.1 Piaget Theory of learning

The view of Piaget is that every individual learn in a unique way through an own and dynamic construction of information in the mind of the student. According to Piaget, this depends mainly on biological growth. Piaget was one of the most influential people in the sphere of education, particularly in the area of Mathematics and Science. Piaget proposed that children pass through a series of stages of thought as they progress from infancy to adolescence. He employed the biological thesis of adaptation, whereby through the twin process of assimilation and adaptation the individual adapt to the environment and there is a pressure to organize structures of thinking. These stages of thought are qualitatively different from each other, so that the child at one stage of thought reasons quite differently to a child at a different stage of thinking (Piaget and Inhelder, 1971).

Piaget defines intelligence as the individual's ability to cope with the changing world. According to Piaget, this can be achieved through constructing and reconstructing the experience which the child has been exposed to. Piaget development is highlighted by four stages, namely, the sensory motor stage, the preoperational stage, concrete operational stage and lastly, the formal operational stage.

In the field of education, Piaget's work was not adopted not until the early 1960's. Before this time much of the content of school curriculum was taught in a rote fashion, in which students were expected to do a considerable amount of their work in a pencil and paper format. The introduction of Piaget's work was embraced by most western countries and subsequently the introduction of the "new maths" was implemented.

Piaget proposed four stages of development which seem to correlate with certain ages, although there is to be an expected range in the ages. He further described these stages of cognitive development as:

Sensory motor (0 to 2 years)

According to Piaget, a child at this stage of development may be able to co-ordinate senses and perceptions with movement and action. This stage is also characterized by limited capacity of the child to anticipate consequences of action and the child see only the permanent nature of an object.

Preoperational stage (2 to 7 years)

At this stage, children view themselves as the center of the universe and they also perceive inanimate object as having human qualities, they are unable to mentally reverse actions, but they begin to use language and mental images to generalize. The form of the idea at this stage may be sometimes unreasonable. The child's idea is connected but it not reliable at this stage of development.

Concrete operational (7 to12 years)

At this stage of the child development, the child is able to consider the perspective of others, conserve numbers, mass, volume, area and length, operate more than one aspect of a problem. The child can also play games with rules and can mentally reverse action.

Formal operational stage (12+)

This stage in Piaget views coincides with adolescence. The child at this stage is able to work with abstract object, and can employ deductive, hypothesis testing, and verbal proposition (not concrete) in problem solving. For example, if A is greater than B and B is greater than C, it therefore implies that A is greater than C, thus ($a>b$, and $b>c$ than $a>c$).

Piaget's views above are extremely relevant to the teaching and learning of mathematics in schools. These views given by Piaget and its significance are briefly discussed in the next section.

#### 2.7.1.1 Mapping Piaget concept onto the teaching and learning of mathematics

The above stages of a child's development as described by Piaget have a significant implication in the teaching and learning of mathematics in the classroom. For example, at the Preoperational stage (at age 2 to 7) which is characterized by the child "perceptual or intuitive thought". In mathematics teaching and learning, it requires that the child be given lots of free play and the use of concrete materials by the teacher.

At the concrete operational stage (at the age 7 to12) or the middle to upper primary school when the child is able to play games with rules and sees the reversibility of an entity. In mathematics therefore, it suggests that concrete materials begin to give way to numerical symbols. Lastly, at the formal operational stage at age 12+ (secondary school) when the child abstract thought, deductive reasoning and hypothesis testing are developed. It implies that in mathematics, numerical symbols give way to Algebraic symbols, and algebraic logic as explained earlier at the formal operational stage above (Piaget and Inhelder, 1971).

From the abovementioned, Piaget advised that traditional geometry should be learnt according to the stages of intellectual development of the students. That means students' should be able to progress from one thinking level to the next one and instructions should be realized in a sequence corresponding to the cognitive development of the child (Piaget, 1971).

From my personal perspective, I am of the opinion that Piaget's theory has other noticeable implications in the teaching and learning of mathematics in general. For example, prior 1960's, a quick look through the textbook used by schools, suggests that there was a strong emphasis on rote learning and the repetition of standard algorithm for students…. There was this strong emphasis on pencil and paper work and students were expected to complete the number of tasks which were set according to the curriculum. A deep look at these set of work, reveals that they fell short of what is been described as current practice in the learning and teaching of mathematics.

Taking Piaget's key ideas which are applicable to school age children and mapping them against current pedagogical practice in mathematics education in general, it could appear that many of the key Piagetian ideas have been incorporated into current practices in schools. One of the biggest changes in mathematics education came in the infant grades where Piaget proposed that young students need to construct meaning for themselves through direct interaction with the environment. For this aim to be achieved it calls for teachers to adapt their teaching and learning environment so that students are able to engage in learning activities and play with a variety of concrete experiences. The more experiences students had, the more likely they were to construct new schema. The National Council of Mathematics Teachers (NCTM, 1986) also suggested that students should be given access to a variety of concrete building materials and construction sets, so that in their free play, they will have the opportunity to develop judgment of length and manipulative skills (NCTM, 1986).

This statement indicates the importance of play and the use of concrete materials in the development of mathematical concepts. Some countries like Nigeria, Lesotho, Zimbabwe, Botswana and South Africa now uses Piagetian theory in mathematics teaching and learning ( Ada & Kurtulus, 2010).This manifestation could be seen in most current mathematics textbooks and the mathematics curriculum where there seem to be a link made between the various stages of development and what is expected of students at the introductory chapters.

### 2.7.2 Hans Freudenthal descriptions of learning

Freudenthal is the founder of the so called realistic mathematics education in the Netherlands. In realistic mathematics education, realities do not only serve as an area of applying mathematical concepts but is also the source of learning. For Freudenthal, mathematics does not only mean mathematizing realities, meaning transforming a problem field in reality into a mathematical problem. It is also mathematizing mathematics itself (Freudenthal, 1971).

Freudenthal wrote on the theory of discontinuity in the learning process devised by his student and colleague van Hiele. Freudenthal and van Hiele both discovered similar levels of child development. This is how Freudenthal described the reasoning of a child at the third level. At the third level, if a child knows what a rhombus and parallelogram is, he/she can visually discover the properties of these shapes. For example, in a parallelogram opposite sides are parallel and equal, opposite angles are equal, adjacent angle sum up to 180°, the diagonal bisects each other. The parallelogram has a center of symmetry, it can be divided into congruent triangle and the plane can be paved with congruent parallelogram. This is a collection of visual properties, which ask for organization (Freudenthal, 1971). Freudenthal explained that deductive reasoning starts at this point. It is not imposed. It unfolds itself from its local germs. The properties of the parallelogram are connected with each other. One among them can become the same from which the others spring. So does a definition arises and knowing its related properties becomes clear why a square shall be a rhombus and a rhombus a parallelogram (Freudenthal, 1971). Freudenthal explained further that the following level of thought may contribute to a more precise understanding of the level of thought.

According to Freudenthal, at the first level, figures were in fact just as determined by their properties, but a student who is thinking at this level is not conscious of these properties. Each level has its own linguistic symbols and its own network of relation uniting these signs. A relation which is “exact” on one level can be revealed to be “inexact” on another level.

Two people who are reasoning on two different levels cannot understand one another. To Freudenthal, this is what often happens with the teacher and the students, neither of them succeeds in grasping the progress of the others’ thought and their discussion can be continued only because the teacher tries to get an idea of the students thought process and to conform to it (Freudenthal, 1971).

Freudenthal further explained that certain teachers gave an explanation at their own level and invites the students to answer questions on that level. Freudenthal advised that for a meaningful teaching and learning to take place, the teachers must dialogue and operate on the students’ level. In this case, the teachers must often after the class, question themselves about the students meaning and strive to understand them.

The maturation process that leads to a higher level unfolds in a characteristic way. One can distinguish several phases. This maturation must be considered principally as a process of apprenticeship and not as a ripening on the biological order. Freudenthal advised further that it is then possible and desirable for the teacher to encourage and hasten the maturation process of the child and It is the goal of didactics to ask the question about how these phases are passed through by the child and about how to furnish effective help to the students (Freudenthal, 1971). In the same light, van Hiele also expressed his views on these phases. According to van Hiele, these phases in the course of apprenticeship lead to a higher level of thought such as the ‘information phase’ where the students learn to recognize the field of investigation by means of the materials which were presented to them. These materials cause students to discover a certain structure. This is followed by the second phase “Directed orientation”. According to van Hiele, the students at this phase explore the field of investigation by means of the material that they are presented with. The students know already in what direction the study is geared. Therefore, he advised that the subject matter must be chosen by the teacher in such a way that the characteristics structure progressively appear to the students. This is subsequently followed by the third phase “Explanations” were the acquired experiences by the students are linked to exact linguistic symbols and the students learn to express themselves on these structures in the course of discussions which take place in the class. van Hiele emphasizes that it is during the course of this phase that the network of relations is partially formed.

The fourth phase is free-orientation. In this category, the field of investigation is in large known, but the students must still rapidly find their way around this field of investigation. This can be achieved by the teacher by assigning tasks which can be carried out in different ways.

The fifth phase is integration. Here the students have been oriented, but they must still acquire an overview of the methods, which are at their disposal. They then try to condense into a whole the domain which their thought has explored. At this moment the teacher can encourage this work by providing global insights. It is important that these insights bring nothing new to the students; there ought to be a summation of what they already know.

As a result of the fifth phase, the new level of thought is reached. The students display a network of the acquired relations, which connect with the totality of the domain explored. This new domain of thought which has acquired its own intuition has been substituted for the earlier domain of thought that possessed an entirely different intuition (Wessels, 2000).

From the findings and views of Freudenthal and van Hiele given above, it appears that geometry teaching and learning in secondary school failed partially because it was taught in such a way that its deductivity could not be reinvented by the students but only imposed. This implies that teachers are not teaching students in such a way that the acquired knowledge in the classroom can be used or reproduced by a student in a different situation. To avoid this dilemma, Freudenthal advised that during teaching and learning, the teacher must not start with axioms and theorem because it is a wrong approach to teaching and learning of geometry. Starting with axiom and theorems denies the students the opportunity of finding out how such theorems or axioms are arrived at. Freudenthal affirm further that the starting point should be from the child's everyday's life experience of spatial objects from his/her reality and geometry should be related to the science of the physical space of the students in which he or she lives (Freudenthal,1973). Furthermore, Freudenthal emphasized that geometry instructions should start from three-dimensional objects (spatial objects) to two dimensional (plane geometry).

Dina van Hiele-Geldof the wife of van Hiele and a student of Freudenthal also emphasizes the importance of the reinvention of geometry and not its imposition during instruction (Freudenthal, 1973). My view on the implication of Freudenthal ideas of geometry instruction is that during teaching and learning for example, teachers should not start with the definition of triangles, squares and rectangles, but to let the students discover properties of these shapes and be in a position to define them by themselves (reinvention). This is a more meaningful approach to the learning and teaching of geometry.

### 2.7.3 van Hiele description of learning

van Hiele (1986), gave a description of how children learn geometry. According to him students' progress through levels of thought in geometry and these levels have some characteristic as explained by Van Niekerk (1997). van Hiele proposed that learning is a discontinuous process implying that there are quantitative different levels of thinking and these levels are sequential and hierarchical. A student cannot function adequately at one level without having mastered most of the previous levels. The progress from one level to the next is more dependent upon instruction than age or biological maturation. According to van Hiele, concepts that are implicitly understood at one level become understood implicitly at another level and each level has its own language. During teaching and learning, two people that reason at different levels cannot understand each other. They cannot follow the thought processes of the other. Language is a critical factor in the movement through the levels Van Niekerk (1997).van Hiele (1986) distinguished five levels of geometry thought. These levels of thought can be summarized as follows:

**Level 0:** (Visualization)

Students reason about basic geometric concepts, primarily by means of visual consideration of the concept as a whole. In transformation geometry, students recognize transformation by changes in the figure and motion “visual approach” without explicit regard to their properties of its components.

**Level 1:** (Analysis)

Students are able to distinguish the properties of figures. However, they are unable to interrelate these properties. Students also see each property in isolation from other properties.

**Level 2:** (Abstraction)

A network of relations between the properties is formed. At this level students perceive the implications and class inclusion of the properties.

**Level 3:** (Deduction)

Students distinguish the nature and significance of deduction. However, they do not understand the requirement for rigor and the relation between the deductive systems which are achieved at level 4.

**Level: 4** (Rigor)

The students can compare systems based on different axiom and can study geometries in the absence of concrete models. van Hiele's theory requires students to progress through the above five levels of thought in a sequential order in understanding geometry. Not paying regards to this sequence would result in failure of understanding and rote learning. van Hiele advised that instructions should be geared towards finding out the levels at which a child operates and instructions should be build up from these levels by the teacher. Otherwise the child and the teacher may differ in wavelengths and instruction is bound to fail (van Hiele, 1986).

The bad results of the teaching of geometry must almost entirely be attributed to the disregard of the levels. The learning process in geometry covers many levels, but appreciation of these levels still need to be emphasized during teaching in the classroom. It is through the disregard of the hierarchical nature of the levels with the teacher and the students operating at different levels that account for much of the difficulties students have in the process of learning geometry. Pierre van Hiele observed that two persons who are reasoning at the different levels will not understand each other. The teacher and the other students who progressed to a higher level seem to speak the same language which cannot be understood by the pupils who has not yet reached that level. They might accept the explanation of the teacher, but the concept taught will not sink into their minds. The students themselves will feel perhaps they can imitate certain action, but they have no view of their own activity until they have reached the new levels (van Hiele, 1986). van Hiele further explain that if a student is at the zero level and the teacher speaks on the first or even second level, the student does not understand the teacher. The teacher would think he had made it very simple and plain but the student acts as though the teacher was talking nonsense. At this point in time, the teacher feels helpless. Subsequently the teaching process comes to a standstill (van Hiele, 1986).

The above van Hiele findings clearly indicated that the teacher and the students have a problem in communicating because they are on different levels. This may result in frustrations and discouragement on the part of the teacher and the students they are employed to teach. As a way forward, I think the frustration experienced by the teacher and the students' resulting from communication breakdown point to the necessity in teacher education of preparing teachers to understand these levels and equip intending teachers with all necessary skills needed to communicate effectively with students at their own level. These skills are essential for teachers because one of the reasons for communication breakdown is the difference in the language used for the different levels. Each level has its own set of language and symbols and its own network of relationships connecting the symbols.

From the foregoing, it implies that teachers need to be conversant with each of these levels, their characteristics and be able to communicate effectively with students at various levels.

van Hiele explains further that another reason why students have difficulties in geometry is that most of them are not adequately prepared for high school geometry. High school geometry is targeted at level 3 and most students were at lower levels.
van Hiele's theory differs from Piaget's in that van Hiele advocated that the progress from one level to the next is not biologically determine but can be accelerated by appropriate pedagogical intervention. However, van Hiele believes that learning should shift from teacher directed and students must be encouraged to work independently in a problem solving situation (van Hiele, 1986).

In proceeding to a new level of thought the students must study relationships between the object of their study at the current level and attempt to discover the relationships of the network which will be the object of study at the new level. To move from one level to another, a student must experience sequentially five phases. These five phases are: information, directed orientation, free orientation, and integration. After the completion of these five phases, the student arrives at the next level. Forcing a particular student to a level when they are not ready is cautioned by van Hiele because, any student forced to a level which they are not ready for, will only be imitating their teacher's work with no meaning (van Hiele, 1986).

Based on the van Hiele's findings, many countries around the world including Netherlands, Singapore, and the USA have made extensive research and experimentation and subsequently revised their geometry curriculum based on the suggestions and advice emanating from such extensive research (Burger 1986). For example research work by Mayberry (1981) investigated the hierarchical nature of the van Hiele's levels and the assignment of students to these levels. In her research, Mayberry's subjects were nineteen pre-service elementary education teachers. In another study, Usiskin (1982) measured the geometric abilities of about 2400 high school students as a function of the van Hiele levels. Burger (1986) characterized the van Hiele level of development in geometry with students from the kindergarten to college level.

The results of these studies indicated that the van Hiele levels have been useful in studying the learning of plane geometry which is very closely related to transformation geometry. The van Hiele theory offers a theoretical framework for the teaching and learning of geometry and transformation geometry hence it is used as a frame work in this study to investigate students' difficulties in the learning of transformation geometry. In the teaching and learning situation, it is therefore essential that the teacher and the students first find common ground as a basis for learning. Once this has been established instructions can then be taking to higher grounds, this is essential because the theory point out the levels of geometry learning a child goes through and that it is through instruction that a students will proceed from a lower level to a higher one.

## 2.8 STUDIES RELATED TO THE VAN HIELE'S MODEL

Studies related to the van Hiele's model have been carried out currently and in the past by researchers such as Soon (1992), Pegg (1991), Corley, Ted (1990), Mayberry (1981), Fuys, Geddes and Tischler (1985). These studies used the van Hiele's levels of learning geometry to investigate, and evaluate students' understanding of geometry and transformation geometry. These researches revealed that the van Hiele's model of development in geometry serves as a useful frame of reference when analyzing student's reasoning processes in geometry tasks.  Details of some of these van Hiele's related research are described below:

Soon (1992) investigated the van Hiele's levels of achievements in transformation geometry of secondary school students and the existence of the hierarchy of a van Hiele level of understanding of transformation geometry. An interview and observation technique was used to collect data from a group of about 20 students within the age range of 15 to16.  The result of the investigation indicated that the levels as exemplified by the task did form scales, which seemed to support the existence of a hierarchy of the van Hiele's level for transformation geometry. The study further revealed that students could recognize transformations easily, but they had problems in describing transformations. According to the findings, in terms of tasks for each of the concept strand, students were more successful for tasks in reflection and least for enlargement. Students in the study generally did not know the rigor of proofs.  Analyses from the interview indicated that students did proofs by giving particular examples. This suggested to the researcher that students' response to the interview reveal rote learning (Soon, 1992). Similarly, the Chicago Project was fashioned to test the ability of the van Hiele's theory to describe and predict the performance of students in secondary school geometry (Usiskin, 1982).  Approximately 2900 students from six different states in the USA were involved in this study. Four tests were administered in this project, they included:

- A multiple choice test that was used to test prerequisites of high school geometry administered as pretest and post tests.
- Multiple choice test associated with the van Hiele levels, was also administered as pre-and post-test this was.
- A proof writing ability test was administered after a year of high school geometry and finally
- A commercial standardized geometry test on geometry achievement was given as a post-test.

Some of the conclusions from this project after analysis of the entire test were that:

- A vast majority of the students could be assigned a van Hiele level
- Levels assigned to students seemed to be good descriptions of students' performance in geometry.
- The van Hieles' theory does explain why students are having difficulties in understanding geometry, namely, most students are not adequately prepared for high school geometry which is targeted at level 3 and students were at the lower levels.

The developed van Hiele level test also looked at a few concepts to predict an overall van Hiele level. In this investigation, the study revealed that some students were able to answer questions set at a higher level, yet failed to answer correctly lower level questions.

Mayberry (1981) studied the van Hiele's level of geometry thought of undergraduate pre-service teachers. He looked at the hierarchical nature of the van Hiele levels. The study developed test items corresponding to the van Hiele model on seven concepts in geometry which include, 'square, circle, isosceles triangle, right triangle, parallel lines, similar figures and congruent figures'. The items were validated by thirteen (13) mathematician and mathematics teachers. They were then revised and administered to nineteen (19) pre-service elementary teachers through interviews. The result of the study confirmed that the van Hiele's levels formed hierarchy and her students could be assigned a level. However, there was no consensus across concepts implying that students could be at different levels for various concepts (Mayberry, 1981). In a similar study, Denis (1987) also investigated the relationships between Piagetian stage of development and van Hiele level of geometry thought among Puerto Rican adolescents. His study showed that van Hiele levels are hierarchical among subjects in the formal operational stage of development. Denis also found no consensus across concepts in the van Hiele levels.

Denis (1987) and Mayberry (1987) studies greatly favored the van Hiele model in the study of geometry. The Hierarchical nature of the van Hiele's levels exists and the levels appear to be useful in explaining student's thinking processes in geometry. The van Hiele's theory explains the behavior of students in learning and provides guidelines to diagnose the difficulties experienced by students in solving geometry problems (Denis, 1987). However, Burger (1983) recommended using the model for the investigation of students' responses on other mathematics topic and suggested its use in the study of geometry transformation.

In my view, van Hiele levels of learning appear to be useful in analyzing, assessing and predicting students' performance in geometry. It can be used to direct teaching and learning of concepts in transformation geometry classroom.

In a further quest to ascertain the relevance of the van Hiele's model in learning geometry, American mathematicians and mathematics teachers learned of the van Hiele's theory through Wirsup(1976) and Freudenthal (1973). During this time the National Science Foundation founded a research project which developed resource materials for middle school mathematics teachers. These materials emphasize geometry and visualization. The resource materials contained activities which will allow teachers the flexibility to select and use activities which help students progress through the van Hiele level of thought.

Hoffer (1979) wrote a textbook entitled: Geometry, a model of the universe for Secondary School Students. This textbook is based on the van Hiele's structure. It was written for a one year course emphasizing investigation and activities in the first semester and preparing students to work in a deductive system (van Hiele level 3) in the second semester. An informal study with one class using a traditional textbook and a second class (experimental) using the Hoffer materials was conducted. The results of the investigation revealed that students in the experimental class learned better geometry and could write proofs better than those in the traditional class where a well established text was used (Hoffer, 1978 as cited on Soon, 1992:24).

Mason and Schell (1988) examined the level of geometric understanding and misconceptions' among 93 pre-service elementary teachers, 28 pre-service and 27 in-service secondary mathematics teachers using the van Hiele model. The result of the investigation found that the three sets of teachers indicated in the order shown above were operating at lower van Hiele levels. Approximately twenty percent of the pre-service elementary teachers do not conform to the van Hiele's hierarchy of levels.

This was explained in terms of possible achievements of the lower levels which were not evident due to misconceptions stemming from faulty recall of precise definitions.

Other similar studies by Battista and Clements (1988), Ludwig and Kieren (1986) investigated the van Hiele model in geometry learning under the environment of logo with eight year olds and seven graders. Battista and Clements (1988) used the synthesis of Piaget’s and the van Hiele theory and the logo environment to develop instructional activities and assessment to observe eight year pupils’ geometric conceptualization. The dynamic features of logo in term of turtle paths and movements enhanced students understanding as they analyzed the movements of the turtle in forming shapes and facilitate their learning of concept such as angle, line segment and their interrelationships.

One of their findings was that the logo environment helped the students to make the transition from the visual to descriptive thought level of the van Hiele’s hierarchy.
The relevance of the van Hiele levels to the learning of geometry and transformation geometry was also emphasized by Kuchemann(1980). Kuchemann indicated that the van Hele’s levels of learning have been useful in studying the learning of plane geometry and transformation geometry is very closely related to plane geometry and the difficulties encountered in the understanding of transformation topics could also be explained in terms of the van Hiele theory (Kuchemann, 1980). Similarly van Hiele (1986) asserted in his book *Structure and insight*, that his theory is applicable to all mathematical understanding.

The review of literature suggested that for there to be a meaningful teaching and learning of geometry and its concepts, students should be given the opportunities to visualize geometric shapes, manipulate concrete objects and discover their properties and have a mental picture of the shapes. These activities level the ground for students to be able to see pattern and enhances their overall visualization processes.

Indications from the review of literature above also revealed that geometry and transformation geometry instructions remain the focus of a needed debate. The reform efforts by the researchers of early year of this century and the research by concerned mathematics educator and some research described above have not diminished the continuing need to improve students' success in the understanding of geometry and its transformations. For this reason, the present study will address students' difficulties using the van Hiele's levels of geometry thinking, (visualization, analysis, abstraction and deduction) as a guide. Knowing the difficulties encountered by students in the learning of transformation geometry and the possibility of using the van Hiele's theory to explain the thought process in understanding transformation of rotation. Therefore, this study will investigate and describe students' difficulties in transformation geometry in particular the concept of rotation.

Several studies have provided support to the van Hiele levels in the learning of transformation geometry. Review of studies indicated that the hierarchical nature of the van Hiele levels was proved to be in existence in geometry. These levels were found to describe how students understand geometry concept. Studies also indicated that students have difficulties in performing certain transformation tasks. However, there is the question of whether the van Hiele hierarchy describes the development of transformation concepts. Studies on other levels of learning are reviewed and compared to the van Hiele levels. The review of related literature indicated that students have difficulties in performing certain transformation task. These difficulties may be related to the difficulties encountered in the learning of Euclidean geometry. The hierarchical nature of the levels may explain these difficulties in transformation as transformation geometry is closely related to Euclidean geometry.

The studies conducted by Soon (1992), Denis (1987), Pegg (1991), Corley & Ted (1990), Mayberry (1981), Fuys, Geddes and Tischler (1985) informed this study in formulating the research design, developing test items, and methods of data collection and analysis, in particular the objectives, to use the van Hiele's levels in describing the difficulties experienced by students in transformation geometry learning. However, as described earlier, Soon (1992) investigated the extent to which van Hieles' theory supported the hierarchical levels in the learning of concepts in transformation geometry. Nevertheless, the study pays less emphasis in explaining the effect which these levels have on students learning of concepts in transformational geometry. From the forgoing, the investigator began the present study in an effort to substantiate and share more light on the extent to which students' level of visualization, abstraction, and deduction can enhance students learning of the concept of Rotation in transformation geometry.

## 2.9 CONCLUSIONS

This chapter gave an account of other studies which have relevance to the present study, discussions of past research on transformation geometry were extensively discussed and their relevance to the present study was highlighted. Theories of learning geometry and transformation geometry such as Freudenthal, Piaget and van Hiele, were also discussed. However, amongst all the theories of learning mentioned in the study, the van Hiele's theory is of most significant to this study. In particular, van Hiele's level of learning geometry provided a theoretical framework which I adopted in the study.

The review of related literature indicated that students had difficulties in geometry learning. From the van Hiele's description of learning, at the basic level it emphasized visualization as an important skill in learning geometry. I concur with this view of the importance of visualization. To aid student skills in visualization, teachers should ensure that students must be given the opportunities to visualize geometric shape, manipulate these shapes to discover their properties and make conjecture. These activities provide a good ground for students to be able to flourish in other higher levels of geometry learning (Burke et al, 2006). The next chapter presents the research methodology of the study.

# CHAPTER 3

# RESEARCH METHODOLOGY

## 3.1 INTRODUCTION

This chapter presents the methods and steps taken in analyzing and describing the difficulties students have in transformation geometry with special focus on the concept of rotation. The research was designed to address the following questions:

1. To what extent are students able to visually identify and name transformation of rotation by it motion using a standard or no standard name? (Visualization).
2. To what extent are students able to discover the properties of a figure and its images after a rotation and use these properties to analyze a transformation of rotation? (Analysis)
3. To what extent are students able to use rotation to transform an object when given the coordinates, angles and shape? (Abstraction)
4. To what extent are students able to use transformations to do proofs? (Deduction)

In order to provide answers to the above research question, this chapter will provide a description and explanation of the research design in section 3.2. The research instruments and how these instruments were developed are discussed in section 3.3. The population sample and piloting are described in sections 3.4 and 3.5. Section 3.6 describes the school context. How the research instruments were administered is highlighted in section 3.7 and 3.8. The validity and reliability of these instruments are discussed in section 3.9. Steps taken to analyze both the written test and interview are provided in section 3.10. Finally, ethical issues relating to the study are discussed in section 3.11. Section 3.12 concludes the chapter.

## 3.2 RESEARCH DESIGN

The study adopted a mixed method design that included both quantitative descriptive design and qualitative case study design. Both methods of investigation were used because they provided the wide-ranging data that resulted in an in-depth understanding of the students' thinking processes that involves Rotational transformational geometry problem solving situation. The Qualitative aspect of the research used the interview while the quantitative method was in the form of a written test to analyze students' performance. The written test was also used to show students strength and weakness and classify students according to their level of rotational transformation geometry understanding. This was with regard to visualization, description, analysis and deduction as outlined in the van Hiele's model. To further maintain validity, triangulation was used to confirm and compare result from these two data sources. The written test response and other data such as interviews from students and tape recorder used during the interview were carefully analyzed to find a common pattern of difficulties (Cohen & Manion, 1994).

## 3.3 DATA COLLECTION METHODS

### 3.3.1 Written test

A written test was given to students in the form of a worksheet. This test technique (work sheet) was chosen because of its benefits. Firstly, it enables students to express themselves in writing and can help to provide more information regarding the students thought processes with regards to transformation geometry specifically with reference to (Rotation). In developing the test, the van Hiele's levels and its characterization was crucial and formed a focal point on which the test was developed. The content of the test was developed in such a way that each question was tied to each van Hiele's level. To ensure this, a template of a matrix of level by concept was also adopted from Soon (1992). This matrix is useful in that it ensured that all the levels of transformation geometry (Rotation) learning of van Hiele which are visualization, analysis, abstraction, deduction and their characterizations were adequately represented in the test questions. Some test items used by Soon (1992) that were deemed appropriate to this study were selected. Other questions were designed by the researcher because of my experience of teaching transformation geometry.  Finally, some questions were also selected from past Form C final examination question papers. the criteria for selecting and developing these questions was based on their ability to solicit students' difficulties with regards to aspects of identifying, rotating, twisting, inventing an image, showing similarities and differences between two transformed objects.

**Table 1:   Matrix of level by concept for developing test items**

| **Van Hiele's levels** | **Concept (Rotation)** |
|---|---|
| levels | Number of questions |
| 0 | 2 questions |
| 1 | 2  questions |
| 2 | 2 question |
| 3 | 2 questions |
| Total | 8 questions |

The number of questions at each level will be two with sub questions. It is believed that more than two questions will result in the instrument that would be too lengthy, and the researcher would want an honest answer from every student and do not want to discourage the students by giving them lengthy questions.  To ensure that the quantity of questions in the test did not affect the quality of information, every question was matched with the van Hiele's level of geometry understanding.

The written test consisted of eight short problem solving questions.
Two questions consisted of sub questions which were drawn from each of the van Hiele's level (0 to 3) to make a total of eight questions (see appendix A).  The choice of the two questions drawn from each level was made because it helped to generate additional information on students thinking processes which may have or have not been covered adequately when one question is used. It also gave students the opportunity to provide detailed or alternative responses to questions which they may not have provided for in previous questions thereby providing enough information that is required for analysis.  The

contents of the test were developed to correspond to each anticipated difficulties which may be associated with visualization, analysis, abstraction and deduction as outlined in the van Hiele's model of learning geometry. In this study, only levels 0-3 of the van Hiele's model were tested. This is because the National Mathematics Curriculum Framework for Lesotho Secondary School 2010 prescribed only level 0 to 3 of the van Hiele's model, which are visualization, analysis and abstraction and deduction to be taught at this level. The van Hiele's level 4 "Rigor" which requires students to be able to compare systems based on different axiom and can study geometries in the absence of concrete models is not included in the Form C curriculum. However, it is prescribed at a higher level of study in the mathematics curriculum.

### 3.3.2 Interviews

Interviews are one of the most important tools of qualitative research. When properly used, researchers often get better responses from interviews than other data gathering devises like the questionnaire (Cutis et al, 2000). In almost all the studies reviewed on van Hiele theory, clinical interview and observation methods were employed. I believe that the interview technique will give room for an in depth probing that would provide a better knowledge of the participant's ideas and thinking processes. From the foregoing, therefore, it was imperative for me to conduct interviews with individual students after the test.

Interview data was obtained from a group of six students. The interview was administered immediately after school hours for duration of two hours a day and it lasted for three days. The purpose of the interviews was mainly diagnostic. The content of the interview was a follow up on questions asked in the written test (see Appendix B). The interview method was adopted so as to compliment the written test and to address different set of questions which may have not been addressed fully when only the quantitative method of investigation is solely used (Cutis et al, 2000). Data from the interview answered question with regards to the extent to which students had difficulties relating to visualization, description, analysis and to deduce in transformation (rotation) geometry. These set of questions could not be answered fully from the data collected from the written test. The interviews were audio-taped in addition to pencil and paper responses.

Hichcock and Hughes (1989) distinguish between structural and semi structured interview. In the former, the interviewee answers either yes or no to a question while in the latter, the interviewer asks for reason or explanation for yes or no. The latter situation asked for insight or divergent thinking on the interviewee and this is the focus of this study namely, to search for the difficulties students may have during the process of finding answers to specific questions, as it relate to the concept of Rotation in transformation geometry (Hichcock and Hughes, 1989).

### 3.4 SAMPLING TECHNIQUES

All the subjects in this investigation are currently in Form C with a total number of 95 students.

Form C students were selected because, they may be able to express themselves freely both in writing and speaking as compared to other lower grade form A, and B. Another reason was because at that stage of learning they must have covered substantial aspects in transformation geometry which they started in form A and B, and finding the problems associated with these set of students at this level may give an insight on their difficulties which can be curbed at an earlier stage.

Students partaking in the interview were selected based on their performance from the group of students that took part in the first written test. Only students that performed poorly in the written test were interviewed. To ensure equal representation of students in terms of sex and ability for the interview, six (6) students consisting of 3 male and 3 female were randomly selected from each class. There were a total of 18 students that were preliminary selected from the three Form C classes in the school. These male and female students were then assembled and given an overview of the interview. The reason for this was to ensure that every participant (students) that will be finally chosen for the interview must be willing and not forced to participate in the interview. Finally, only six volunteers were interviewed.

## 3.5 PILOT STUDY

A pilot study is often described as a smaller version of the proposed study and is conducted to refine the methodology (Hollway & Jefferson, 2007). A pilot study helps to identify possible problems in the proposed study and allows the researcher to revise the method and instrument before the actual study is conducted (De Vos, Strydom, Fouche and Deport, 2005).

To increase the validity of this study, the test and interview questions was first piloted to determine whether it elicited the intended responses (Hollway and Jefferson, 2007). The pilot study involved a group of 5 students from a different school not meant for the actual study. The aim of the pilot study was to give the researcher an insight on whether the intended questions to be given to students would yield the desired data that would be needed to answer the research questions. The class and age group of students in the pilot study and that of the final group study were the same. In order to determine to what extent they understood the question and to decide whether some of the contents of the questions should be reconstructed or not, I decided to assess the time they took to complete the task and other difficulties such as language, meaning, and choice they have to make (Thomas, 2003). After the pilot study for the written test, I administered an interview to these same groups of students the next day using the same interview protocol as given in Appendix B. Responses from the written test and interview went through the same analysis to determine students' difficulties. During the written test, I was able to identify some errors and this resulted to question 1.1, 1, 2 question 2 and 3.1 being modified with pictorial representation instead of words to suit the purpose of the study, (see appendix A). The process of the interview had to be improved upon. This was because I initially planned that the interview will be for two days. It came to my realization after the pilot study that students were not willing to stay too long beyond school hours. Hence the interview was restructured to accommodate two students per day. This now make the interview to last for 3 days. Having two students interviewed each day. During the interview, and the written test, I reiterated that the information will be kept confidential and used for research purposes only.

### 3.6 DESCRIPTION OF THE SCHOOL

The investigation was done in a mixed secondary school named 'Qoaling Secondary School in Maseru district of Lesotho. The school is comprised of male and female students which are between 14 to 16 years. The school is both Sesotho and English medium speaking. Although there are 54 secondary schools in Maseru, these schools are jointly owned by the missionary, government, communities and private individual. However, the school used in this study is a missionary school that belongs to Lesotho Evangelical Church. The school has a population of about seven hundred students with two streams junior and senior secondary school. There are twenty two teaching staff in the school. The choice to administer both the test and interviews after school hours was that a good proportion of students lived within a walking distance from the school.

### 3.7 TEST ADMINISTRATION

The test was administered to all Form C students. As a result of absenteeism only ninety (90) out of a total of ninety five (95) students which comprised of 41 male and 49 female students finally wrote the test. The purpose of the test was to obtain data from the written response and to help in formulating questions to be used during the interview session. Data from the test provided more knowledge about students thought processes in transformation geometry (rotational) problem solving situation. During the test session, students were encouraged to draw and show all steps they took in arriving at the answers. I believe that asking students for explanation will provide a clue on the divergent thinking of the students. By so doing the difficulties they had might be noticed in the process and this was the concern of this study. Students described and gave reasons or explanations as specified in each question. During the administration of the test, I also explained to the students that their way of thinking and the difficulties they encountered will be of more interest than the particular answer. I personally supervised one of the classes during the administration of the test while other classes were supervised with the help of two other mathematics teachers. Students wrote the test at the middle of term 2, for duration of two hours (See appendix A) for the contents of the test questions.

## 3.8 INTERVIEW ADMINISTRATION

The interview took place at the school premises. During the interview, each student was given scratch paper, pencil and pen. They were required to explain verbally or in writing how they arrived at their wrong answers. The interview questions focused only on the questions which students did not answer correctly in the previous session. The interview questions requested students to explain why, give reasons and described how (See appendix B). These probing questions were used during the interview session to solicit further information from students. Subsequently, the data generated during the interview session, the verbal response, pen, pencil & paper were used to authenticate data emanating from the written test. Students' responses in the interview were systematically analyzed according to the difficulties they had. This was done by taking into account the type of difficulties students had in identifying, analyzing, describing and using deduction in solving problems as it relates to the concept of rotation in transformation geometry. Questions asked during the interview session focused on the description of students' difficulties. Students' response to these questions in the interview will be checked whether these characteristic are presents in their response to interview questions. A student will be classified as having difficulties at a level if they fail to answer questions as prescribed by the performance indicator in (Table 2). Furthermore, data collected from the interview will also serve as a basis for the formulation of certain conclusions and generalization and allow me to make certain recommendations towards innovative and improved methodologies which could enhance the teaching and learning of transformation geometry and geometry in general.

**<u>Table 2: For determining students difficulties and achievements according to the van Hiele's level</u>**

| Levels of achievements | Performance indicator | |
|---|---|---|
| Basic level (0) (visualization) | • Through a simple picture and students can Identify transformation (rotation) by changes in the figure<br>• Name or label transformation using standard or non standard name, e.g. flip, turn and slides | |
| Level (1) Descriptive | • Uses the properties of change to draw the pre-image or image of the given transformation<br>• Discover properties of change to figure due to rotated figure<br>• Able to locate the centre, angle and direction of rotation<br>• Relates rotated image and figure using coordinates | |
| Level ( 2) (Abstraction) | • Performs composition of simple transformation of rotation<br>• Interrelates the properties of change to figure due to rotation<br>• Given the initial and final states, can name the single transformation | |
| Level ( 3) (Deduction) | • Perform rotational geometry proofs using transformational approach<br>• Think through and give reasons in a multi-steps problems | |

## 3.9 RELIABILITY AND VALIDITY

### 3.9.1 Reliability

The reliability of a test or instrument refers to the extent to which it consistently measures what it is supposed to measure (Cresswell, 2010). A test is reliable to the degree that it measures accurately and consistently, yielding comparable results when administered a number of times (Agwubike & Momoh, 1995). To ensure reliability of the data collected in this study, the contents of the written test and interview went through verification from an independent body (colleague) that is knowledgeable in the line of mathematics education to ascertain the degree to which the contents of the test items and interview were in harmony with the intended purpose. The Initial suggestions and input from the verification exercise from my professional colleagues, it led me to reframe, add and delete some existing questions. For example it was suggested that the initial question scope was not exhaustive enough to provide answers to the research questions, this suggestion resulted to more questions and sub questions to been added. (See appendix A)

### 3.9.2 Validity

Validity refers to the extent to which inferences made on the basis of numerical scores are appropriate, meaningful and useful to the sample (McMillan and Schumacher, 2001). Validity also checks whether the instruments provides an adequate sample of items that represent that concept (De Vos et al, 2003).

In this study, both construct and content validity was used in this study to check if the test and interview questions really measured the concepts that I assumed it measured. Also if they correlated with the van Hiele's level of learning geometry transformation. In doing this, some tasks in the worksheet (test) such as questions 1.2, 2.2 and 4.1 **(**see appendix A) were chosen for the test on the basis that they have been previously used by Soon (1992) to describe students levels of achievements using the van Hiele levels of learning in transformation geometry. Question 3.2 was adopted from Lesotho junior secondary school examination paper (2005). Other questions were selected from text books. The criteria for the selection of questions from textbooks were based on the questions ability to solicit students' to visualize, identify, analyze and describe rotational geometry.  Furthermore, all other aspects involved in the investigations which were, assigning students to the van Hiele's levels, analysis and interpretation of data, I again solicited review, feedback and critique of work from professional colleagues, and my supervisor. These steps were included in the process in ensuring that the research findings are meaningful and reflected students' perceptions.

## 3.10 PROCEDURE FOR DATA ANALYSIS

Data analysis is the process of systematically searching and arranging the data transcripts. Field notes, direct quotes from students and other materials accumulated by the researcher to increase his or her understanding, which enable the researcher to presents that which was discovered (Bogdan & Biklen, 1992). Data analysis involved coding, categorizing and clustering information (Johnson and Christensen, 2000). Students' response from the written test and interview were categorized and analyzed according to the van Hiele's levels and its associated characteristics as described in (Appendix C).

### 3.10.1 Quantitative data analysis (Pen and paper test)

In order to determine the difficulties students have in transformation geometry, a table for determining students' achievement and difficulties according to the van Hiele's level was adopted from Soon (1992).  This table gives a clear indication of what is expected of a student at any level and if these performance indicators are not meet then such student will be classified as having difficulties in that level (See Table 2 and Appendix C). The table was also used to categorize students according to levels resulting from the response from the written test question.

At the first stage of the analyses, a level by concept matrix for each student was set up. Its purpose was to show various levels at which students had difficulties. In the concept matrix each entry would either be "1" or "0". A code 1 was assigned to the student for a level if the response indicated that the difficulty was at that level or "0" is assigned (Soon, 1992). This was recorded in another matrix of students by concept. (See Table 3). The assigning of "0"or "1" was firstly done by the investigator and this was subsequently sent to colleagues to check for any discrepancies. This was followed by an analysis of difficulties. At this stage of analysis, all students' area of difficulties as identified in the concept matrix resulting from the test (pen and paper) was analyzed and described according to the difficulties that students had in finding solutions to questions asked. This was done in accordance with van Hiele's levels as outlined in the research questions. A further analysis was done according to gender. In the final analysis, a graph (pie chart) was used to illustrate students' performance in the four van Hiele's levels that were tested. A bar graph was also used to compare the result of the male and female performance and their difficulties' with regard to each level.

**Table 3: Matrix for entering students' difficulties for each student across concepts**

| Number | Students | Concept:(Rotation)<br>Difficulties levels<br>0, 1, 2, 3 |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| …. | | |

**Table 4: Assigning student's according to the levels of achievement and difficulties in each level**

| **Levels of achievements** | **Number of students** | **% of students without difficulties** | **% of students with difficulties** |
|---|---|---|---|
| Basic level (0) (visualization) | | | |
| Level (1) (Analysis ) | | | |
| Level ( 2) (Abstraction) | | | |
| Level (3) (Deduction) | | | |
| **Total number of students at all levels** | | | |

**3.10.2 Qualitative data analysis**

**(Interview)**

The audio tape, pen and paper, verbatim quotes from the students and researcher's notes used during the interview were also analyzed for difficulties experienced by individual students to a particular question or cluster of questions. The interview questions and its analysis focused on the extent to which students can visualize, describe, analyze, abstract relation, and deduction as describe in (Appendix B). A student was considered having difficulties in a particular level if he/she fails to meet the performance indicator as described in Table 3.

Consequently, I used the multiple data analysis arising from written test, interview with reference to students' verbatim quote and notes from paper and pencil. These were compared to find whether a common pattern existed with regards to the difficulties which students had.

## 3.11 ETHICAL CONSIDERATIONS

All stake holders namely, the principal, parents, teachers, the teaching service department and students of the school were informed before embarking on the research work at the proposed school. A letter seeking permission was written to the school principal as well as parents informing them of the intended project and solicited their maximum cooperation in this regards. Parents and principals of the school were requested to acknowledge by signing the permission seeking letter authorizing their students to partake in the research after school (See Appendix D, E & F). All participants were informed about the purpose of the study. No participant whether students or teachers were forced to be part of the investigation. It was a voluntary exercise. Anonymity was ensured by not collecting students name, a code was generated for all students starting from S1M S2F S3M…etc. M is an indication of male and F for female, S stood for student.

## 3.12 CONCLUSION

This chapter described the research methodology of the investigation that was conducted in Qoaling secondary school in Maseru District in Lesotho. A detailed description of the study samples, instruments used and the validity and reliability of these instruments were discussed. The chapter further described the sampling, how the pilot study was conducted, data collection methods and analysis. The next chapter presents the analysis of data collected from the written test and interview.

# CHAPTER FOUR

# ANALYSIS AND INTERPRETATION OF DATA

## 4.1 INTRODUCTION

This chapter presents the analysis and interpretation of data obtained from both the written test and the interview administered to students. The aim of the study was to investigate the difficulties faced by students in the learning of transformation geometry. A detailed analysis and discussion of findings of the written test and interviews are presented in sections 4.2 and 4.3 respectively. A comparative analysis of the female and male difficulties is also presented in section 4.3. Section 4.4 concludes the chapter.

The four research questions that guided the study are:

1. To what extent are students able to visually identify and name transformation of rotation by its motion using a standard or no standard name? (Visualization).
2. To what extent are students able to discover the properties of a figure and its images after a rotation and use these properties to analyze a transformation of rotation? (Analysis)
3. To what extent are students able to use rotation to transform an object when given the coordinates, angles and shape? (Abstraction)
4. To what extent are students able to use transformations to do proofs? (Deduction)

## 4.2 PRESENTATION AND DISCUSSION OF FINDINGS

### 4.2.1 Introduction

The findings are presented and discussed according to the four research questions stated in section 4.1 above. Firstly, a table showing a general description of students' response in the written test and their area of difficulties are provided in Table 5. Students' area of difficulties will also be illustrated using a pie chart in Figure 1. This is followed by the discussion of specific students' difficulties according to each level of van Hiele.

**Table 5: A sample of students' result showing levels achieved and areas of difficulties according to van Hiele's levels**

| Number | Students | Concept:(Rotation) Difficulties levels |
|---|---|---|
| 1 | S1M | 1,0,0,0 |
| 2 | S2M | 1,0,1,0 |
| 3 | S3M | 1,0,0,0 |
| 4 | S4F | 1,0,0,0 |
| 5 | S5M | 1,1,0,0 |
| 6 | S6M | 1,1,0,0 |
| 7 | S7M | 1,0,0,0 |
| 8 | S8M | 1,0,0,0 |
| 9 | S9M | 1,1,1,0 |
| 10 | S10M | 1,0,1,0 |
| 11 | S11F | 1,0,0,0 |
| 12 | S12F | 1,1.0.1 |
| 13 | S13M | 1,1,0,0 |
| 14 | S14F | 1,0,0,0 |
| 15 | S15F | 1,0,0,0 |
| Etc. | | |

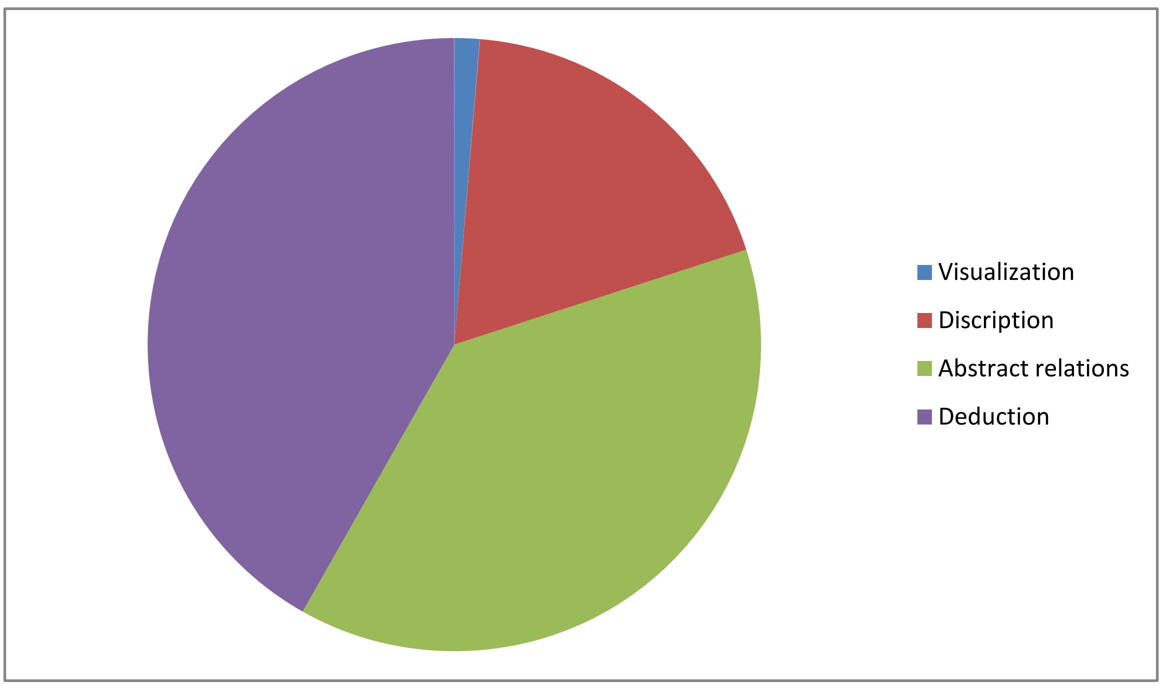
1= Indicates that a student has achieved that level

0= Indicates that a student have difficulties at that level.

S1M = Indicates student number 1. M or F represents the student's gender

Table 5 is used to record and categorize students according to difficulties in each levels resulting from the response from the written test question. Each number in a column represents the various levels at which a student is having difficulties. Column 1 represents research question one and question 1 and 2 in the written test. Column 2 is also related to the research question two and test question 2 etc. In the table each column entry is either

“1” or “0”. A code 1 was assigned to the student for a level if the response indicated that the difficulty was at that level or “0” is assigned. The four columns represent the four van Hiele’s level in the research questions. For example in column 1, student S1M does not have difficulties in level 0 and was assigned the level1 but did have difficulties in levels 1, 2 and 3 hence he is assigned (1.0.0.0). This routine was used to categorize all students and subsequently resulted in data illustrated in Table 5 above. For clarity, information on students’ areas of difficulties on table 5 was further categorized and represented in percentages and in a pie chart in a Figure 1 and Table 6. It is acknowledged that pie charts provide representations of non-overlapping attributes. The pie chart was used to give a glance of the categories. Students difficulties may be in some or in all categories.




**<u>Figure 1: Pie chart showing students areas of difficulties</u>**

The above pie chart is derived from the identified students’ area of difficulties in Table 6. It illustrated the various area students are having difficulties.

## 4.3 DISCUSSION OF FINDINGS

The findings are discussed according to each of the research questions and are presented below

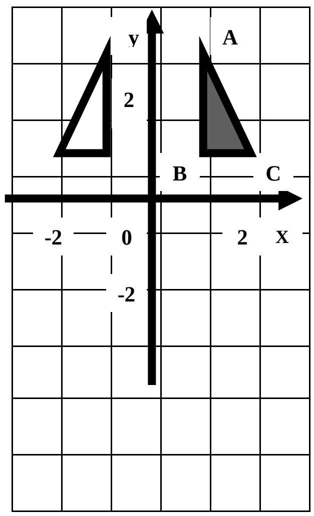
### 4.3.1 Research question 1

**To what extent are students able to visually identify and name transformation of rotation by its motion (Visualization)**

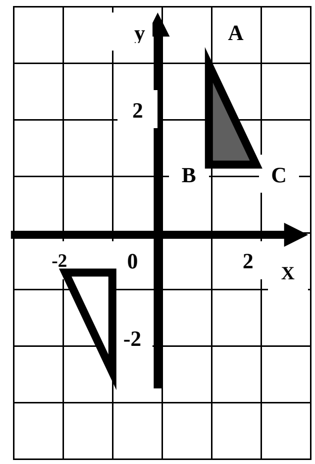
The questions used during the test are:

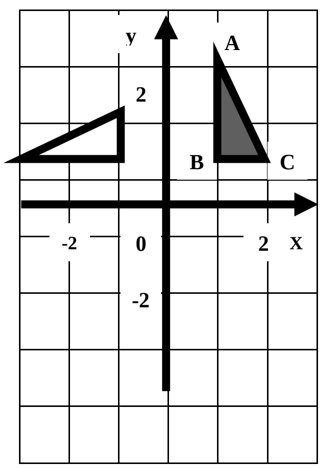
**Test Question 1 (Basic level)**

(a)




(b)




(c)

1.1. Each of the above diagrams represents a different transformation of triangle ABC in each case. Which among the transformation represent a Rotation? Answer -------

1.2. In the figure below, A has been rotated -90 º about C as centre, which of the following image is a true representation of A after a rotation of -90 º

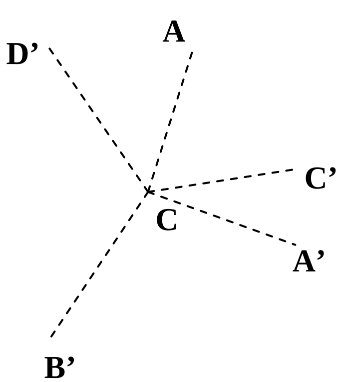


**Table 6: A specific frequency distribution showing students' ability and difficulties resulting from question 1 and the van Hiele's Basic level.**

| Levels of difficulties | Number of students that achieved a level | Percentage % | Number of students Not achieving a level | percentage % |
|---|---|---|---|---|
| Basic level (0) visualization | 88 | 96.66 % | 2 | 3.33% |

The above Table 6 illustrates students' response to question 1 from the test question above which was used to determine students' difficulties at the basic level or level1. Students are classified as having difficulties at this level, if they fail to visually identify and name transformation involving rotation by actual motion by using standard or no standard name as given in the test question 1. The data in Table 6 above indicated that 3.33% of the students' experiences difficulties in identifying and naming transformation involving rotation. 96.66% of students were able to use visualization correctly with regards to the questions asked. Meaning, they could identify and name transformations in groups by actual motion. The above indices gives an indication that about 88% of all students in Form C can visualize and have attained the basic level (0) of the van Hiele's level of transformation geometry learning. Student's responses during the test and in relation to question 1 are given in Appendix D.

To corroborate students' difficulties, students were interviewed with the view of diagnosing their difficulties. These students were asked various questions as contained in the interview protocol in Appendix B with frequent references to the written test. Questions asked at the Basic level focused on students' ability to visually identify and name transformation of rotation by its motion, examples of question asked included:

(1) Which among these transformation represents a rotation and why do you say so?
(2) Can you demonstrate a figure and it image after a rotation using objects given to you?
(3) Which of the image in question 1.2 is a true representation of A after a rotation of -90°and why. Details of all questions used could be found in Appendix B.

The above questions are tied to question one in the test, and also the research question one. Its purpose is to solicit further response from students with the view of identifying their difficulties. These questions were presented to students by using a physically manipulated object of triangles to represent a figure and its image after rotation, translation and reflection as contained in the written test question 1.1. I requested each student to identify the transformed figure by naming the transformation. Only one out of the six students interviewed were able to identify and name the image of the figure and therefore satisfy the criteria set for the basic level and all other students failed to answer correctly question 1.1 and 1.2. Further probing was done to find out how each of these students perceives the rotated figure and its image. Students were given a three dimensional manipulated geometric figure of various size and they were ask to use them to describe from their own understanding what is rotation and how do we know that a figure and its image are rotated. Four of these students had difficulties in doing this. They gave an example of a translation instead of rotation. It does appear that they had mistaken the concept of translation for rotation. They didn't really know the differences between a rotated figure and translated figure. I ask more probing questions to find out why this error has occurred by asking them to describe what happens when a figure is rotated. A direct quote from student S48M is presented below.

**Interviewer**: Look at this figure and its images on the table, they have been transformed. Which among them is a rotation?

**Interviewer:** The student pointed at the translated figure amongst all the transformations presented on the table.

**Interviewer**: Can you describe for me what happens to an image when it is rotated?

**Student S48M:** It moves from this place to this place. (This students' demonstration indicated that he was referring to one point of the plane to another).

**Interviewer:** So are you saying that any figure that moves from place A to place B is rotation?

**Student S48M:** Yes,

**Interviewer**: Pointing at one the transformed image, why don't you choose this image instead of choosing the one you did?

**Student S48M:** Because this image looks the same as this one, (pointing at the translated image).

Another student **S61M** had this to say when he was asked in question 1.2 why he choose C' as a true representation of 'A' after rotation of -90 degree.

**Student S61M:** I choose C' because it is next to A in anti clockwise direction

**Student S52M:** I think B' is correct because B' it is in the opposite of A

The above responses from students provided evidence that some students at this stage of learning (Basic level) still have problems with visualization. The van Hiele's basic level of learning demands a lot of visualization on the part of students. These set of students interviewed failed to identify and name transformation. They had problems in differentiating between rotation and translation hence they see a translated figure also as a rotation. Another important discovery was made in question 1.2. Students were requested to mentally identify the position of figure A after a rotation of -90°. It was discovered that some of these students had difficulties in finding the image of A after a rotation. Example of a student response is given in Appendix D figure 2. The origin of this problem may be that these set of students are not competent enough with the task which involves angle and its measurements as they find it difficult to measure angles when requested to do so. The amount of a rotated figure is measured in angles, and students' ability to visually know how much degree a figure and its image were rotated is also a prerequisite at the van Hiele's Basic level. Below are some responses from students.

**Interviewer:** Can you tell me where the image of A will be if it is rotated -90°

**Student S48M:** The position is C

**Student S 61M:** B'

**Interviewer**: Why do you say so?

**Student S48M:** Because when A is rotated, it comes to B

**Interviewer:** Then through how much degree will that be?

**Student S48M:** Straight line

**Student S 61M:** 180°

The above students' responses demonstrated that students had difficulties in indentifying an image after a rotation through a given degree.

#### 4.3.2 Research question 2

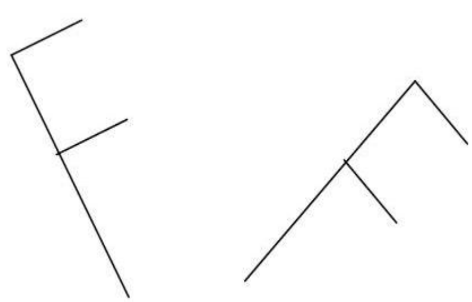
**To what extent are students able to discover the properties of a figure and its images after a rotation and use these properties to analyze a transformation of rotation? (Analysis)**

To answer this research question, students were required to provide answers to questions provided below:

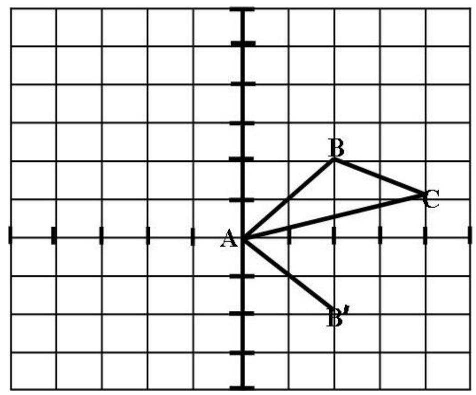
**Question 2 (Level 1)**

2.1.1 A figure and its image after transformation are given below. Draw or locate the following:

(1) Centre of rotation (2) Angle of rotation



2.1. 2. What can you say about the image above? ---------------------------------------




2.2. The triangle ABC where A (0, 0) B (2, 2) C (4, 1) is rotated about the origin (0.0) through an angle of 90 degrees clockwise to map onto triangle A'B'C', where B (2,-2)

2.2.1. What is the point C'?
2.2.2. What are the co-ordinates of B'?
2.2.3. Explain how you got C'?
2.2.4. What do you know about the length of each side of the triangle ABC and A'B'C'?
2.2.5. Which of the following properties correctly described the transformation on question 2.2 above?

a. Each line segment of the figure is congruent to the corresponding line of the image figure
b. Each angle of the figure is congruent to the corresponding angle of the image figure
c. Each figure is congruent to its image figure
d. Orientation of the figure is different from its image figure
e. Each line segment of the figure is parallel to the corresponding line segment of the image figure.
f. Each figure is similar to its image figure.

**Table 7: Specific frequency distribution showing students' ability and difficulties resulting from question 2 and the van Hiele's level one**

| **Levels of difficulties** | **Number of students that achieved a level** | **Percentage %** | **Number of students Not achieving a level** | **percentage %** |
|---|---|---|---|---|
| Level (1) Analysis | 52 | 57.78% | 38 | 42.22% |

Table 7 show students' ability and difficulties resulting from question 2 and the van Hiele's level one. Students are considered having difficulties at van Hilele's level one if they failed to discover and use the properties of rotated geometric figure to analyze rotations. 38 out of the 90 students that wrote the test were classified as having difficulties in discovering fully angle of rotation and centre of rotation. Question 2 focuses on how students' describe and analyze when presented with a rotated figure. Students seemed to have difficulties in finding mostly the centre of rotation when a figure is rotated about a point. Examples of student's responses are provided in Appendix D figure 3 and 4. Only 52 out of 90 students were able to provide correct answers to question 2 which require them to discover properties by locating angle and centre of rotation. This number of students represents 58% of the total number of students that took part in the survey. A total of 42.22% of students had difficulties in providing adequate solution to task relating to finding the angle and centre of rotation, finding the coordinate of a point in a rotated figure and its image and discover properties of new images after rotation. They also lack the analytical skills that required them to use simple words relating to transformation to analyze a given figure and its image after rotation. Students' responses to these questions are provided in Appendix D Figure 3 and 4. Student's difficulties in this regards were further confirmed from their responses given during the interview. Students were therefore required during the interview to draw, locate centre of rotation and angle of rotation and use these properties to analyze any figure and its image after rotation.

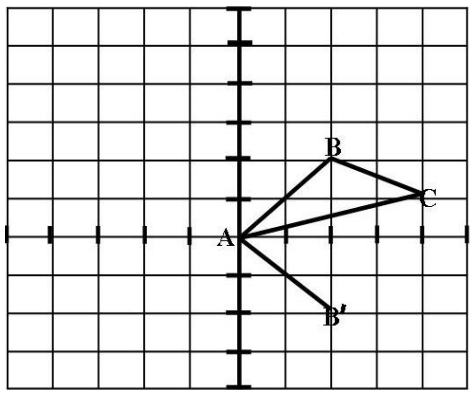
From the follow up questions to question 2 from the written test, it was revealed that most students appeared to have difficulties in finding the centre and angle of rotation. They also are unable to integrate these properties when requested to analyze a rotation. This is how student S23M analyzed a rotated figure and its image during the interview. Questions used during this section of the interview are available in Appendix B question 2.11, 2.2. 2.2.3 and 2.2.4.

**Interviewer:** Referring to question 2 in the written test. Explain how you find the centre and angle of rotation in question 2.1.1

**Student S23M:** The figure is rotated through 90 degrees in a clockwise direction. Student S23M was further probed so as to provide a clearer detailed, He was asked to describe this clockwise direction.

He only uses movement and direction in his description, but failed to talk about the centre and angle of rotation. This seems to be common among all response from students. During the interview all six students interviewed, S48M, S52M, S61M, S23M, S11F and S82 F were able to verbally tell the direction of movement of the rotated figure, but they located and drew the centre of rotation far away from the figure and its rotated image. These students gave no explanation for their decision when requested to do so. Student S82F uses her pen to show the direction and movement of the figure F. But when requested to show the centre of rotation she drew a dot far away from the foot of the figure F in question 2.1.1 and she concluded that this is how the image of F was formed, student's S82F response are given in Appendix D figure 4.

In question 2.2 which required the students interviewed to locate the exact image and coordinates of a point after a rotation. Again these five students interviewed also had difficulties in locating the exact images of a figure after rotation.




**Question 2.2**

In the above question, students were requested to find the coordinate of B’ after a clockwise rotation of triangle A, B, C. They were also requested to estimate the location of C' and explain how they got it.

Only student S11F was able to complete this task but failed to use correct words to explain the reason for his decision. Student S11F response is given in Appendix D figure 6. One noticeable and interesting thing during the interview is that some students seemed to have difficulties in providing the correct answer at one stage or question but do very well at another stage or question. For example students S11F failed to provide the answers to questions asked in the previous question 1.2 during the interview, but she showed a better degree of understanding in subsequent question 2.2. Students S61M and S11F were unable to represent the location of C' using coordinate. They pointed at the wrong location were C' is to be located. They did this by disregarding the direction of the rotation. Students S61m and S11F solution to this problem is available in Appendix D figure 6.

Students’ response to question 2.2 once again shows that students have difficulties in visualizing. Students are unable to find the position of a new image after rotation. The above question was followed by question 2.2.5 in the test. This question tested students ability in using correct word and properties of rotated figure and its image in analyzing a rotation. Again this is how student S61M responded to the interview question.

**Interviewer:** can you explain your choice of option F which says each figure is similar to its image figure in question asked in question 2.2.5

**Student S61M:** I think when a figure is turned round; its second image will be the same as the first figure.

**Interviewer**: What do you mean when you say the same?

**Student S61M:** They are equal

The above conversation also gave the interviewer the opportunity to reinforce the same concept by using some manipulated object in his explanation. This gives the students two different ways of understanding the question at hand. The researchers for the National Assessment of educational progress found that students can often deal with geometric problems more effectively when visual model of the problem are presented (Carpenter et al, 1981).

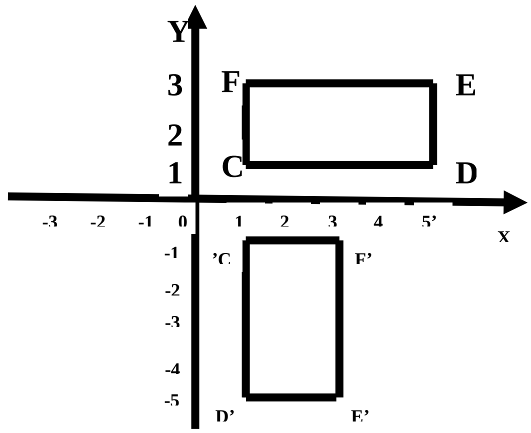
From the above interview conversation, it can be conclude that these students have difficulties in using the correct words and properties of rotated figure when analyzing a rotation. This difficulty was peculiar to all students interviewed. I could also conclude that these students have difficulties with the meaning of the term similar and congruent figures and uses the word congruent to mean similar figure. From the van Hiele's concept, at level 1 amongst others prescribed that students should be able to note the difference between two figures with the view of pointing out their similarity and differences in relation to rotation. However, this characteristic seems to be lacking from the students interviewed.

#### 4.3.3 Research question 3

**To what extent are students able to use rotation to transform an object when given the coordinates, angles and shape? (Abstraction)**

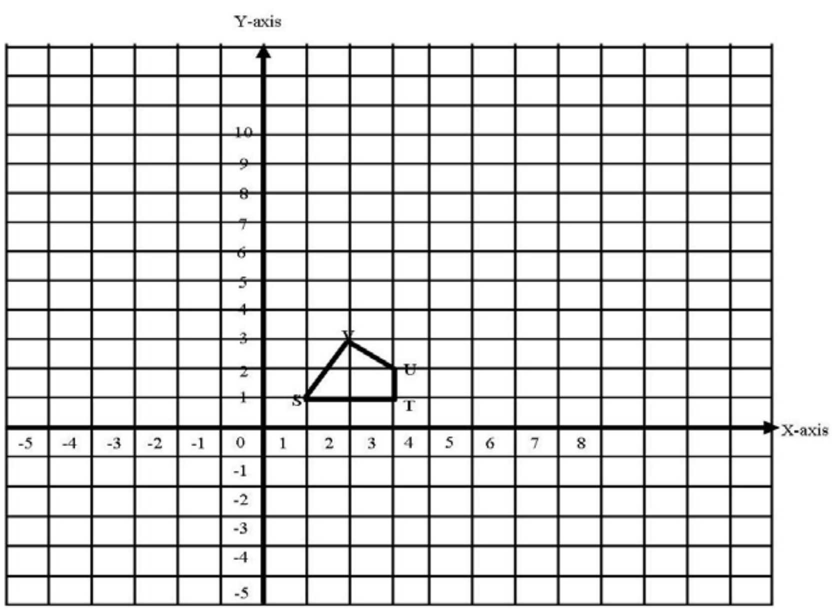
To answer this research question, students were required to provide answers to questions 3 provided below. These set of questions are set at the van Hiele's level two. Students are considered having difficulties at this level if they fail to rotate a figure and its image through any given angle, describe and inter-relate the properties of the figure and its image.

**Question 3  (Level 2)**




3.1 Describe the transformation above which rectangle CDEF is mapped onto rectangle C' D' E' F'.

3.2 On the grid below, x, and y axes have been drawn and labeled.




3.2.1 Rotate STUV through 90° (anticlockwise), about (0,0) and label the image $S_4 T_4 U_4 V_4$

3.2.2 Through how many angles in degrees anticlockwise can you rotate the same figure so that it can fit exactly onto the original figure?

3.3.3 Describe how you arrived at the answer given above.

**Table 8 A specific frequency distribution showing students' ability and difficulties resulting from question 3 and the van Hiele's level two**

| **Levels of difficulties** | **Number of students that achieved a level** | **Percentage %** | **Number of students Not achieving a level** | **percentage %** |
|---|---|---|---|---|
| Level (2) Abstraction | 13 | 14.44% | 77 | 85.56% |

Table 8 above shows students' responses with regards to question 3. Question 3 task examined the extent to which students were able to use the concept of rotation to rotate a figure when given the angle of rotation. Students are considered having difficulties at level 2 if they are unable to use rotation to transform the figures given in question 3.1 and 3.2. From the above indications in Table 8, only 13 out of 90 students which are about 14.44% provided the correct answers. This implies that 77 out of the 90 students which represent about 85.56% of students have difficulties in using the concept of rotation to rotate shapes and describe rotations. A detail of students' response to this question 3 is available at Appendix D Figure 7.The interview session provided an avenue for students' difficulties to be further determined as they apply the concept of rotation to transform a figure. Five students interviewed were able to recognize the figure in question 3.1 as a rotation, but had difficulties in given specifics of the rotated figure. For example student S11F said rotation and stopped; I further questioned her to use some specifics such as centre of rotation and angle of rotation, direction of rotation in her description, but failed to use these specifics in describing the rotation. This is what student S11F had to say:

**Interviewer:** Can you use the terms centre and angle of rotation to describe the image and its figure in question 3.1?

**Student S11F:** Emmmmm, rotation

**Interviewer:** I want you to use centre and angle of rotation in your description

**Student S11F:** I don't know how to use it.

This is how another student answered the same question during the interview

**Student S48M:** clock wise movement and it is a rotation.

Since this group of students had problems with finding centre and angle of rotation in the previous questions and in level 1 earlier, this also contributed to their inability to also use these specifics to describe any given rotated figure.

With reference to question 3.2.1 and 3.2.2. Students interviewed were asked to explain or demonstrate through how many angles in degrees clockwise or anticlockwise one can rotate a figure so that it can fit exactly into the original figure. Only one student S23M manages to get this question correctly. He knew that when an object turns 360° it then return to its original position. However, other students did mention 260°, 180°, which were completely wrong. Students' responses in the written test are available in Appendix D Figure 7 & 8. Their explanation also shows that they don't know the implication when a figure is rotated several times. A probe on these difficulties during the interview resulted to me providing a manipulated rectangular shapes cut to size and requested each of them to use this shape to explain and demonstrate what they were thinking or how they did arrived at the answer. This is how student S82F, S23M and S61M responded to the interview.

**Interviewer:** using the shape on the table demonstrate why you say that your answer to the question is 290°?

**Student S82F:** Because it move in this direction and when you count them, it will amount to 290°

**Interviewer:** Can you show me how you counted it? (The student moved the rectangular figure round and round and failed to stop when it makes a complete rotation).

Here is another response from S23M

**Student S23M:** I have to rotate the figure three times or more.

**Interviewer:** Is three meaning 360° that you wrote here?

**Student S23M:** yes

**Interviewer:** Now rotate the figure twice and show me were it will be

**Student S23M:** ya here it is

From his demonstration this student rotated the figure round before it could fit exactly onto the original.

**Interviewer:** what is the measurement of movements in degrees?

**Student S23M:** Revolution

Student S23M could only manage to give a relevant response after been guided to do so. Judging from my interaction with students above, it became obvious that these set of students are having difficulties with angles measurement that exceed 90° they also have a problem of manipulating object mentally. The different movement and answers given by students are clear evidence that they have different perception of what the question required of them and their inability to visualize resulted to their difficulties in carrying out task relating to abstract relation which could have been aided by visualization. These students also have some language difficulties. Language associated with geometry and transformation geometry is crucial for children to acquire a more complete understanding of geometry concepts (Pickereign et al, 2000).

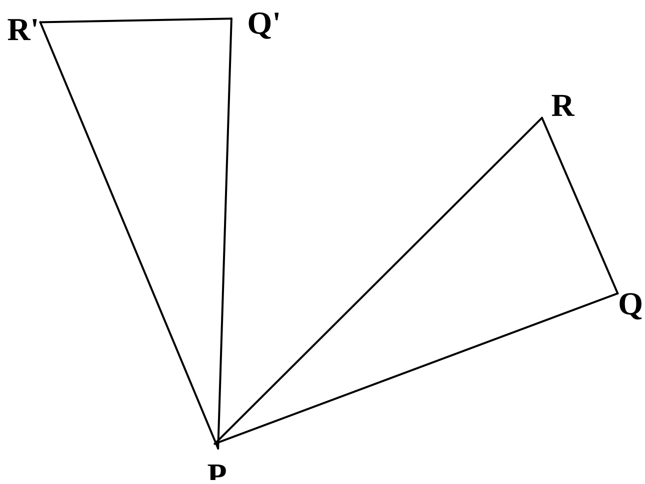
From the written test and interviews it was revealed that students show little or no understanding of geometry terms used. For example some students do not know the meaning of mapped unto, congruency and horizontal. When students used such words, they not depict what the students were explaining. Some students also used their own term such as fit and moving round. This deficiency resulted to students' inability to apply geometric terminology when describing a rotated figure and its image. In question 3.1 students were requested to describe a transformation and how they arrived at their answer in question 3.3.3. Students' response to these questions revealing their difficulties could also be seen in Appendix D Figure 8 and 9.

### 4.3.4 Research question 4

**To what extent are students able to use transformations to do proofs? (Deduction)**

To answer this research question, students were required to proof geometrically using a rotational approach. These tasks are set at the van Hiele level 3. Students are classified as having difficulties at this level if they failed to use transformation to show that a figure and its image is a rotation. Students were given task provided below.

**Test Question 4 (Level 3)**




The triangle P.Q.R has been rotated through +70 degree about P. PQR is the image of PQR after rotation

4.1.1 What type of triangle is PQR?

4.1.2 Why?

4.1.3 Use transformation to prove that triangle PQR is congruent to triangle P'R′Q'

4.2. Consider the statement: when a figure is rotated, the figure and its image are congruent do you agree?
Yes/No ----

4.3. Motivate your answer given above with appropriate diagram

**Table 9 Frequency distribution showing students' ability and difficulties according to question four and the van Hiele's level three**

| Levels of difficulties | Number of students that achieved a level | Percentage % | Number of students Not achieving a level | percentage % |
|---|---|---|---|---|
| Level (3) Deduction | 5 | 5.55% | 85 | 94.44% |

Question four examines the extent to which students can do geometric proof using rotational approach. The result of the test indicated that students do have difficulties in this regards. Indication from table 9 above shows that at level 3, only five out of the ninety students are able to give correct reasons by using such word as the size, angle, and length of the triangle and the centre of rotation to proof that a figure and its image are rotated. This represents about 5.55% of all students in Form C. This implies that 94.44% of Form C students are not capable of using transformation of rotation to prove that a figure and its image is a rotation. This result suggested that students have great difficulties at this level. Details of students' responses to question 4 are available at Appendix 4 Figure 9 and 10. In relation to these questions, for example, students S60F and S3M could not prove that triangle PQR is congruent to triangle P'Q'R' in question 4.1.3. Although they agreed that a rotated image and its figure are congruent in question 4.2, but they failed to provide correct motivation in support of this. Instead they drew reflection rather than rotation in question 4.2.3.

A further probe to find out the extent of these difficulties resulted in me to elicit more information from these students. A follow up interview questions to question 4 in the written test reveals that students do not understand what proofs entails. Generally when a figure is rotated, students could not tell why they think a figure and its image is a rotation. None of the students interviewed were able to demonstrate that a rotated figure and its image were congruent. They also failed to use some concept like the size of angle, the preservation of shape, size of shape and length in their arguments to show that both triangles in question 4 were congruent. Below are responses from some of the students interviewed.

**Interviewer:** Triangle RPQ is congruent to R'P'Q' why do you think this is so?

**Student S52M:** Because the shapes are the same

**Interviewer:** Good, but tell me why you think these figures are the same

**Student S52M:** I don't know how to do it sir.

Student S11F also had no idea when ask initially. But after some few minutes when I tried to reframe the question she started drawing and making some sketches. On conclusion, she could not also tell why both figures were congruent.

**Interviewer:** Ok tell me why you think that this figure is congruent to this one or why you think they are the same.

**Student S11F:** They are both triangles.

The interviews with students gave an indication that students had difficulties. Students could not explain why they think a figure and its image is a rotation. They also failed to show that a rotated figure and its image were congruent.

Sketches of students response to the interview is given at Appendix D Figure 10 and 11 respectively.

## 4.4 CONCLUSIONS

This chapter presented an analysis of students' difficulties in rotational transformation geometry. This was made possible by using the van Hiele's level of geometry understanding based on flexible interpretation. The ninety students to whom the Rotation geometry test was administered were large enough to draw conclusions.

Both quantitative and qualitative analyses were used to provide evidence on the extent of the difficulties which students have. A concept of matrix for each student was used to record and show various levels at which students are having difficulties. A pie chart and tables were used to represents data and they provided a clearer analysis on the extent to which students can visually identify, analyze, describe and solve problems involving rotation. A table showing percentages of the extent of difficulties to which students can Identify, describe, analyze and deduce were also provided.

To further provide a clearer understanding on students’ difficulties, a comparison of students’ difficulties between male and female students in problem solving task were compared. This was subsequently followed by the analysis of students’ difficulties in the interview session. Data from the interview session ( The audio tape, pen and paper, quote from students and notes) used during the interview section was also analyzed qualitatively to provide description of the difficulties experienced by students using the van Hiele’s levels which have links to the study research questions.

Analysis of the test and interviews revealed that students’ do have difficulties mostly at the higher levels which are 2 and 3. Students mostly find difficulties in finding the centre and angle of rotation, describing and proving using rotational approach to show that a figure and its image are rotation. The above findings are also in conformity with the findings of Soon (1992) which also suggested that students could recognize transformation but had problems in describing the transformation and students did not know also know the rigor of proof. The next chapter presents the summary, recommendation and the conclusion of the study.

# CHAPTER 5

# SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

## 5.1 INTRODUCTION

In this chapter, a summary of findings of a study that aimed at investigating and describing students' difficulties in transformation geometry of Rotation will be presented in section 5.2. This will be followed by the recommendations in section 5.3. The limitation of the study will be discussed in section 5.4 and a concluding remark will be made in section 5.5. The interview and the written test that was conducted with students were very useful in providing data used in the description of students' difficulties in transformation geometry of rotation.

## 5.2 SUMMARY OF FINDINGS

The summary of findings will be discussed according to each of the research questions.

### 5.2.1 Research question 1

**To what extent are students able to visually identify an image of initial and final state after rotation (visualization)**

The results indicated that 3,33% use actual of the students' experienced difficulties in identifying and naming transformation involving rotation. 96,66% of students were able to use visualization correctly. The majority of the students could identify and name transformations in groups by actual motion by using standard or no standard name. However, 3,33% of students that had difficulties in identifying and naming transformation in group by using a standard or no standard name. These students had difficulties in differentiating between rotation and translation; they view a translated figure as a rotation. It was also discovered that the students had difficulties in finding the image of a figure after a rotation. The results of the analysis emanating from both the written test and the interview indicated that Form C students are mostly functioning at the van Hiele's level one and two which are Visualization and Description.

### 5.2.2 Research question 2

**To what extent are students able to discover the properties of a figure and its images after a rotation and use these properties to describe a transformation of rotation? (Analysis)**

The study revealed that (38) 42.22%% of the students were classified as having difficulties in discovering angle of rotation and the centre of rotation when given a rotated image and its figure. Students seem to have difficulties in finding mostly the centre of rotation when a figure is rotated about a point. About (52) 57.78%% students were able to provide correct answer to question two which require them to discover properties by locating angle and centre of rotation. The analysis also reveals that students lacked the analytical skills that required them to use simple words relating to transformation to describe a given figure and its image after rotation.

Students' thought processes from both the interview and test suggested that students demonstrated that they can visually identify rotated figure and its image but find difficulties in describing or locating the characteristics features of a transformation such as the centre and angle of rotation. In this regard, students' response indicated that they can only use movement and direction in their description but ignored the angle and centre of rotation. Students were found to be not so confident in rotating shapes. Students did not think of a rotation as a rotation of the plane or a moving shape into a new location. Rather, they considered rotation as a shape that kept moving in different direction without considering the angle and centre of rotation. Students' responses indicated that there is element of rote learning. In some cases students find it difficult to justify their answer and in other cases no justification was given.

Furthermore, students' use of precise language was limited. Students' preferred to use their own terms such as equal, not the same and fit. Some of these words used did not in some cases give an explanation of what they meant.

### 5.2.3 Research question 3

**To what extent are students able to use rotation to transform an object when given the coordinates, angles and shape? (Abstraction)**

Students' difficulties was determined and explained as they used rotational approach to transform a figure when given the direction and angles about a point.

The results showed that 85.56% of the students had difficulties inter-relating the properties of a new image and its figure after rotation. They experienced difficulties relating to performing a simple rotation when given size and direction. They had difficulties also in rotating a figure through a given degree about an origin and predicting the exact position of a figure when it is rotated through a certain angles in degree.

### 5.2.4 Research question 4

**To what extent are students able to use transformations to do proofs?**
**(Deduction)**

The ability to use transformation to do proof was very minimal amongst the student. Students were not able to demonstrate that a rotated figure and its image were congruent. This ability is a characteristics feature at the van Hiele's level 3. Students also failed to use some concept like the size of angle, the preservation of shape, size of shape and length in their arguments.

About 80% of the students had difficulties at the level of abstraction and deduction. The above finding is also in support of Soon (1992) findings which revealed that students generally did not know the rigor of proofs; students did proof by given a particular example. This revelation may be attributed to the reason why students at this level of learning find it difficult to providing adequate solutions to questions asked in the examination.

## 5.3 IMPLICATIONS OF FINDINGS AND RECOMMENDATIONS

The implications for teachers and curriculum planners from the findings of this investigation are: Firstly, the majority of the students demonstrated that they can visualize by identifying rotated figure and its image, but have difficulties in finding the properties of new image after rotation. Students provided different explanations on the properties of rotated figures as they grew in geometric understanding and their views of a figure and its determining properties changes (Pegg, 1991). This evolution takes time and the growth can be encouraged but it cannot be rushed through by the teacher. It takes time for students to play the game of geometry and this does not start until students are able to see the properties of rotated figures as important aspects of their description.

Level two and three concepts are difficult for many students.  This level represents a new and important way of organizing thinking which usually does not come naturally to students. However, it represents an important and often overlooked link in the chain of events moving to formal deduction in transformational geometry learning.  Students' ability to link in terms of minimum properties at level three concepts represents a further development in understanding beyond the concept of class enclosure. For such activities (finding the properties) to be viable, students will need to have extensive familiarity with the properties of figures and the relationships between them (Pegg, 1991).

Burke et al (2006) article suggests that glide reflection complete students' knowledge of transformation geometry.  Transformation geometry of rotation has the reputation of being just a set of tricks and students see no point in studying transformations. To encourage students in the study of transformation, teachers should learn how to use transformation geometry not as a system of describing motion but as constructing a rule which allows one shape to be mapped onto another (Burke et al, 2006).

Furthermore, students' needs to posses the necessary language skills associated with rotational geometry that will enable them to use expressions such as opposite angles are equal, congruent and line segment in the right context. A teacher's decision not to introduce the correct mathematical language eliminates any opportunity for the students to choose to learn that language (Pickreign et al, 2000).  Teachers should therefore encourage students to talk about geometric concepts relating to Rotation and discover the properties themselves so as to develop expressive language. In a classroom situation, teachers should ask students to describe a figure, rather than just to select a name for it from the list. Students' understanding of key concepts such flips, turns, glides, similarity, congruency, angle of rotation and the relationships of the properties of rotation in particular is critical for supporting the development of deeper understandings of transformations of Rotation (Hollebrands , 2003).

Teachers should also bond to the use of manipulative material as discussed in chapter two. Manipulative materials or teachers specifically designed materials can according to Driscoll (1986) show the way to conceptual understanding. They also provide experience in which students can transfer their understanding smoothly from one concept to another. One way of letting the lower achieving students concentrate on the learning of transformation geometry is to use information communication technology (ICT). For example, students could be given a number of congruent shapes placed at different location in the plane, and the task would be to find a way through transformation of rotation to move an original shape to each of the congruent shapes. This could be a game for two or more students were one student instructs the computer to perform a transformation and the other has to find out which one it was. If repeated, this would help students get a feel for what the image looks like. For example, if it is “flipped over” it has to be a reflection or glide, otherwise a rotation or translation (Wesslen, 2005).

The secondary school transformation geometry curriculum should be appropriate for the various thought levels. It should guide students to learn about significant and interesting concepts. It should permit students to use visual justification and empirical thinking because such thinking is the foundation for higher level of geometric thought (Pickreign, 2000).

The curriculum should require students to explain and justify their ideas. It should also encourage students to refine their thinking.

In conclusion, the present study adds the following to the field of transformation geometry education:

- It has employed the van Hiele’s theory of geometry learning to describe and analyze students’ difficulties in transformation geometry within the context of Rotation.
- It has also suggested guidelines for classroom practice that can contribute to improved teaching and learning of transformation geometry.

With the findings emanating from this study, it is hoped that the recommendations will assist teachers in their perception of their students' difficulties in learning transformation geometry. This could have changed through teachers increased familiarity with recommendation based on van Hiele's model. Teachers through this study's recommendation would recognize that they need to discuss the importance of the van Hiele's model with their students. This should be done because students need to be assured that their readiness for transformation geometry is related to their previous experience and instruction and that lack of readiness is not a reflection on their intelligence. Although this study did not directly investigate the teacher and textbooks used in the classroom. Teachers do rely heavily on texts for their daily instructions. To bring about these changes, textbook and teachers' education that is focused on the van Hiele's model are recommended.

The results of the investigation is an indication that the van Hiele's model of development in geometry can serve as a useful frame of reference when analyzing student's thinking processes in geometry tasks. The conclusions are also synonymous with some conclusions reached by some cited studies such as Soon (1992) and Ada (2010) in the literature review in chapter two. However, I believe that the result obtained and the differences noted among the different kinds of students indicate that the proposed method of evaluation of the van Hiele levels is coherent and should be researched further.

### 5.4 LIMITATIONS OF THE STUDY

Although Lesotho is inhabited with about 2 million people, with 54 secondary schools. Due to time limit and the nature of the investigation and financial constraint, the research focused only on one secondary school. The sample size was relatively small, which implies that these findings cannot be generalized to all Lesotho Form C students but provided an in-depth understanding of the difficulties students experienced.

The time of the day students have mathematics may be a factor in how they learn and answer question on the concept been taught in transformation geometry. This investigation was done after school hours in the afternoon and as a result, some of the students became tired during the interview session this might have affected the outcome of the investigation.

Mother tongue language (Sesotho) was also a barrier as students used in the investigation were not confident enough to express themselves effectively using English and this may have affected the outcome of this investigation.

## 5.5 CONCLUSIONS

The aim of this research was to use the van Hiele's levels of learning to investigate and describe the difficulties students have in the learning of transformation geometry in particular rotation concept. The findings revealed that students could identify and name transformations in groups by actual motion. This gives an indication that 88% of all students in Form C can visualize and have attained the basic level (0) of the van Hiele.

The result also revealed that students failed to identify and name transformation. They had problems in differentiating between rotation and translation hence they see a translated figure also as a rotation. Students had difficulties in finding the image after a rotation, they had difficulties in providing adequate solution to task relating to finding the angle and centre of rotation, finding the coordinate of a point in a rotated figure and its image and discover properties of new images after rotation. They also lack the analytical skills that required them to use simple words relating to transformation to analyze a given figure and its image after rotation.

It can be conclude that these students have difficulties in using the correct words and properties of rotated figure when analyzing a rotation. This difficulty was peculiar to all students interviewed. Students show little or no understanding of geometry terms used. This deficiency resulted in students' inability to apply geometric terminology when describing a rotated figure and its image. The extent to which students can do geometric proof using rotational approach indicated that students do have difficulties in this regards. This implies that majority of the students had difficulties at the level of abstraction and deduction. This give an indication that the vast majority of students in Form C are reasoning at the lowest two levels of the van Hiele model which are visualization and description. This is contrary to the curriculum demands of the students in Lesotho at this level. The Mathematics curriculum in Lesotho demands that Form C students should be able to perform task up to level three and this may be partly responsible why some transformation geometry questions are difficult for students in the examination. It is hoped that the findings and recommendations in this study will enhance the teaching and learning of rotational transformation geometry and geometry in general.

## 6 REFERENCES


Ada, T., & Kurtulus, A. (2010). Students' misconception and errors in Transformation Geometry. *International Journal of Mathematical Education in Science and Technology.* 41(7) 901-909.

Agwubike, O., & Momoh, S. (1995). *Tests measurement and evaluation in education*. Benin City, Nigeria: Jodah publication (1st ed.)

Allendoerfer, C. (1969). The dilemma in geometry: *Mathematics Teacher* (62)165-169.

Ben-Zvi,D., & Arcavi, A. (2001) Junior high school learners construction of global views of data and data representation: *Educational studies in mathematics*, 45:35-65.

Bogdan, R,C., & Biklen, S.K. (1992). *Qualitative research for education*. *An introduction to theory and method*. Neebham Height: Allyn and Bacon.

Bryian, L., Ruane, P. (1981). Geometry in English Secondary School. *Educational studies in mathematics,,* 12(1), 12-16.

Burger, F. (1992). A Geometry curriculum for perspective elementary teacher based on van Hiele model of development. *Pythagoras* 29(2), 2-5.

Burger, F., & Michael, S. (1986). Characterizing the van Hilele levels of development in geometry: *Journal for Research in mathematics education*, 31-48.

Cangelosi, S. (1996). *Teaching mathematics in secondary and middle school: An interactive approach (* 2nd Edition). New Jersey: Prentice Hall Publishers

Carroll, W. (1998). Geometry knowledge of middle school students in a reform-based mathematics curriculum. *School Science and Mathematics,* 98(4), 188-197.

Cohen, C., & Manion, L . (1994). *Research method in education*; London: Routledge

Corley, L. (1990). Students' levels of thinking as related to achievement in geometry. *New Jersey.*

Cox, P. L. (1979). The effects of teaching an informal geometry course at the high school level on students achievement and attitude. *Dissertation Abstract internationa*l. 40A : 47-53.

Cresswell, J.W. ( 2010). *Educational Research. Planning, conducting and evaluating quantitative and qualitative research*. New Jersey: Pearson Prentice Hall.

Cutis, M., Andy, M., Tery, J., & Michelynn M. (2000). *Mathematics education research, a guide for the research mathematician*. USA: American Mathematical Society.

De Vos, S., Strydom, H., Fouche C., & Delport, C. (2005). *Research at Grassroots for the Social Science and Human Services Professions*. 3rd edition. Pretoria: Van Schaik Publisher.

De Villiers, D. (1997). *Alternative instructional strategies for Geometry Education. A theoretical and Empirical Study*. Final report of the University of Stellenbosch. Experiment in mathematics Education. USEME-project: 1997-98.

De Villiers, M D. (1987). *Research evidence on hierarchical thinking, teaching strategies, and the van Hiele theory: some critical comments"* Syracus University, Syracuse, New York.

Denis, S., & Livia, P. (1987) *Relationship between stages of cognitive development and van Hiele levels of geometry thought among Pueto Rican adolescents*. Fordham University.

Driscoll, J. (1988). Research within the reach: Secondary School Mathematics. *National Council of Teachers of Mathematics*, Reston: VA

Eddins, S. (1994). Activities geometry transformations. *Mathematics Teacher.* 87(3) 177-181.

Erickson, E. (1986). Qualitative method in research on teaching. M.C. Wittrock (Ed), *Handbook of research on teaching* (3rd ed) 119-161. New York: Macmillan.

Examination council of Lesotho (2004). Examination report. Maseru

Examination council of Lesotho (2007). Examination report. Maseru

Examination council of Lesotho (2008). Examination report. Maseru

Examination council of Lesotho (2009). Examination report. Maseru

Fey, J. (1984). Computing and Mathematics: the impact on Secondary School Curriculum. *National Council of Teachers of Mathematics*, Reston, VA.

Fish, W. (1996). Non-Euclidean and its possibilities in the secondary school syllabus: *Unpublished Msc dissertation Pretoria: Unisa*.

Fuys, D. ,Geddes, D. & Tischler, R. (1988). The van Hiele Model of thinking in geometry: *Journal for research in mathematics education*, National council of teachers of mathematics, Reston. VA

Fuys, D., Geddes, D & Tischler, R. (1985). An investigation of the van Hiele model of thinking in geometry among adolescent. Research in Science Education: *TheNational Science Foundation. Reston. VA*

Freudenthal, H. (1973). *Report on method of Initiation into Geometry*. Groningen: Wilters

Greenberg, M.J. (1973). *Euclidean geometry and Non-Euclidean geometries: development and history*. San Francisco W.H Free man and co.

Hallway, W., & Jefferson, T. (2007). *Doing qualitative research differently:* free association, *narrative and the interview method*. London: SAGE publication

Hoffer, A. (1983). “Geometry is more than proof”. *Mathematics Teacher*: 11-18

Hoffer, A. (1979). The van Hiele levels of geometry thought in undergraduate pre-service teachers. *Journal for Research in Mathematics Education*, 14, 58-69

Hollebrands, K. (2003). High school students understanding of geometric transformation in the context of a technological environment. *Journal of Mathematical Behavior.* 22(1) 55-73.

Johnson, B., & Cristensen, L. (2000). *Educational Research: qualitative and quantitative approaches.* Newdam Heights: Allyn and Bacon.

Ken, M.A. (1996). *Mathematics educational research past, present and future* UNESCO principal office for Asia and the pacific: Bangkok.

Kirby, J.R., & Schofield, N.J. (1991) Spatial cognition: The case of map Comprehension. In G. Evans (Ed), *Learning and teaching cognitive skills*.pp (109-125). Retrieved on 12 February, 2012, from http://trove.nla.gov.au/work/153175245?versionId=166932396.

Lesotho National curriculum, *Form C Mathematics syllabus*: (2007). Maseru

Lesotho Junior Examination Certificate Examination analysis (2009) August (1992:9-17).

MacMillan, J. & Schumaker, S. (2001). *Research in education*. Fifth edition. New York: Wessley Longman Inc.

Mayberry, J. (1983). The van Hiele levels of geometry thought in undergraduate preservice teachers. *Journal for Research in Mathematics Education*. 14(1), 58-69

Marshall, C., & Rossman, G.B. (1995). *Designing qualitative research* (2nd ed) Thousand Oaks, CA: Sage

National Council of Teachers of Mathematics (1985). The Secondary School Mathematics Curriculum. *Year book of the NCTM*. Reston, va.

National Council of Teachers of Mathematics (1987). Learning and teaching geometry, K-12. *Yearbook of the NCTM*. Reston, va

Pegg, J.E., (1985). How children learn geometry; the van Hiele Theory. *The Australian Mathematics teacher*, 14(2), 1985: 5-8

Piaget, J. (1928). *Judgment and reasoning in the child.* New York: Harcourt, Brace, & co.

Presmeg, N. (1985). Applying van Hiele's theory in senior primary geometry: Use of phases between the levels. *Pythagoras,* 29: (9-11).

Senk, S. (1985). How well do students write geometry proofs? *The mathematics teacher,* 78, 448-456.

Soon, Y. (1992). *An investigation into van Hiele-like levels of Achievement in Transformation Geometry of Secondary School Students in Singapore*. Unpublished Doctoral Dissertation, The Florida State University.

Thomas, M. R. (2003). *Blending qualitative and quantitative research methods* in Thesis and Dissertations. California: Corwin Press, Inc.

Usiskin, Z. (1982*). van Hiele levels and achievement in secondary school geometry. final report of CDASSG-report*. Chicago: University of Chicago.

van Hiele, P. (1986) *Structure and insight. A theory of mathematics Education*. NY: Academic press.

Wesslen, L., & Maria, F. (2005) Transformation geometry: *Journal for* Mathematics *teaching,* 191: 27- 29.

## APPENDIX A

## WRITTEN TEST QUESTIONS

Objectives: students should be able work correctly at various levels as indicated below:

Basic level. (Visualization)

- 0.1 Identify transformations in groups by the change in the figure
- 0.2 Identify and name transformation by actual motion by using standard or no standard name, e.g. flip, slides and turns

Level 1. (Descriptive/analysis)

- 1.1 Analyze any given transformation by using appropriate words as it relate to transformation geometry
- 1.2 Discover properties and new image after transformations
- 1.3 Is able to locate angle and center of rotation.

Level 2. (Abstraction)

- 2.1. Rotate any given figure through a given degree
- 2.2. Interrelate the properties of the figure and its image
- 2.3. perform composition of simple transformation involving rotation

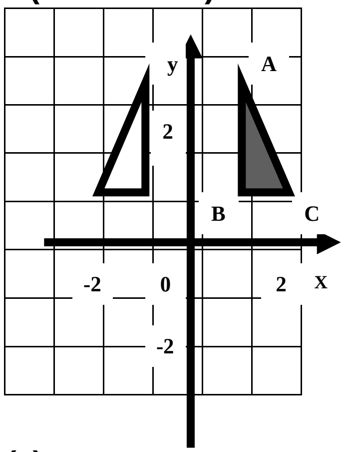
Level 3. (Deduction)

- 3.1. Gives geometric proof using rotational approach

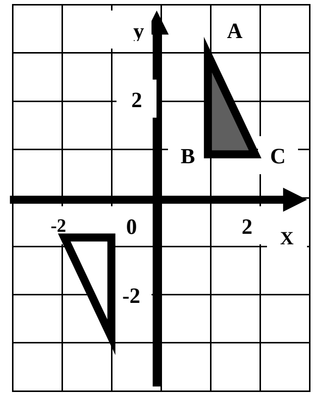
**Instructions**

1. Answer all questions using as much time as possible, the steps you took in arriving at a particular answer is of importance rather than the correct answer
2. You are allowed to make any marks on the question paper and show all steps you took in arriving at your answer

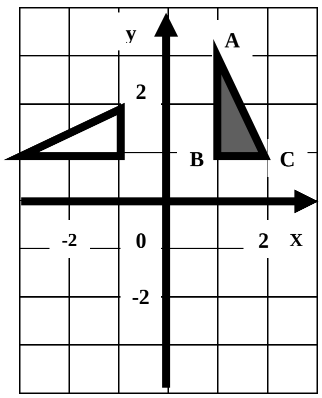
**Question 1 (Basic level)**




(a)



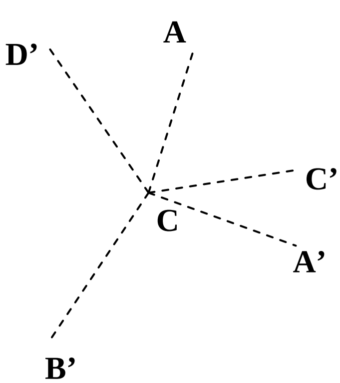

(b)




(c)

1.1. Each of the above diagrams represents a different transformation of triangle ABC in each case, which among the transformation represent a Rotation? Answer----------------

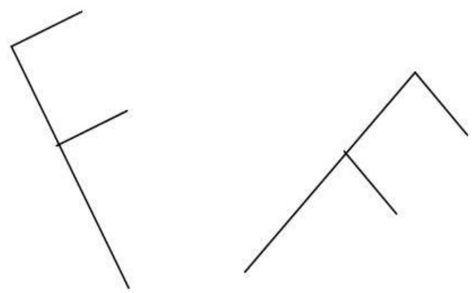
1.1 In the figure below, A has been rotated -90 º about C as centre, which of the following image is true representation of A after a rotation of -90 º



**Question 2 (Level 1)**

2.1.1 A figure and its image after transformation is given below draw or locate the following

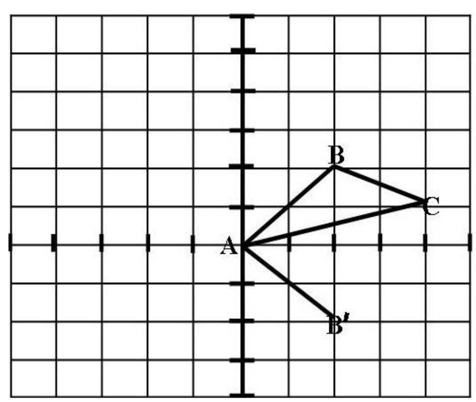
(1) Centre of rotation (2) angle of rotation



2.1.2 Name the above transformation ----------------------------------------------------------------

2.1.3. What can you say about the image? ---------------------------------------------------------------

-----------------------------------------------------------------------------------------------------------------




2.2. The triangle ABC where A (0, 0) B (2, 2) C (4, 1) is rotated about the origin (0.0) through an angle of 90 degrees clockwise to map onto triangle A'B'C', where B (2,-2)

2.2.1 What is the point C? ------------------------------------------------------------------------------------------

2.2.2 What is the co-ordinates of B′ --------------------------------------------------------------------

2.2.3 Explain how you got C'------------------------------------------------------------------------------------------
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-------------------------------------------------------------------------------------------------------------------
-------------------------------------------------------------------------------------------------------------------

2.2.4 What do you know about the length of each side of the triangle ABC and A'B'C'?------------------
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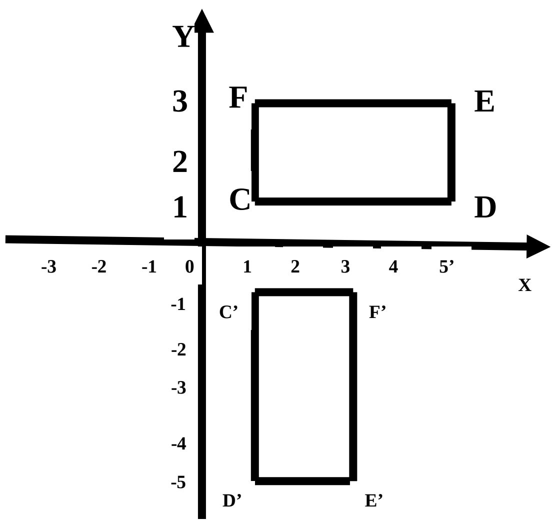
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------

2.2.5 Which of the following properties correctly described the transformation on question 2.2 above?

g. Each line segment of the figure is congruent to the corresponding line of the image figure
h. Each angle of the figure is congruent to the corresponding angle of the image figure
i. Each figure is congruent to its image figure
j. Orientation of the figure is different from its image figure
k. Each line segment of the figure is parallel to the corresponding line segment of the image figure.
l. Each figure is similar to its image figure.

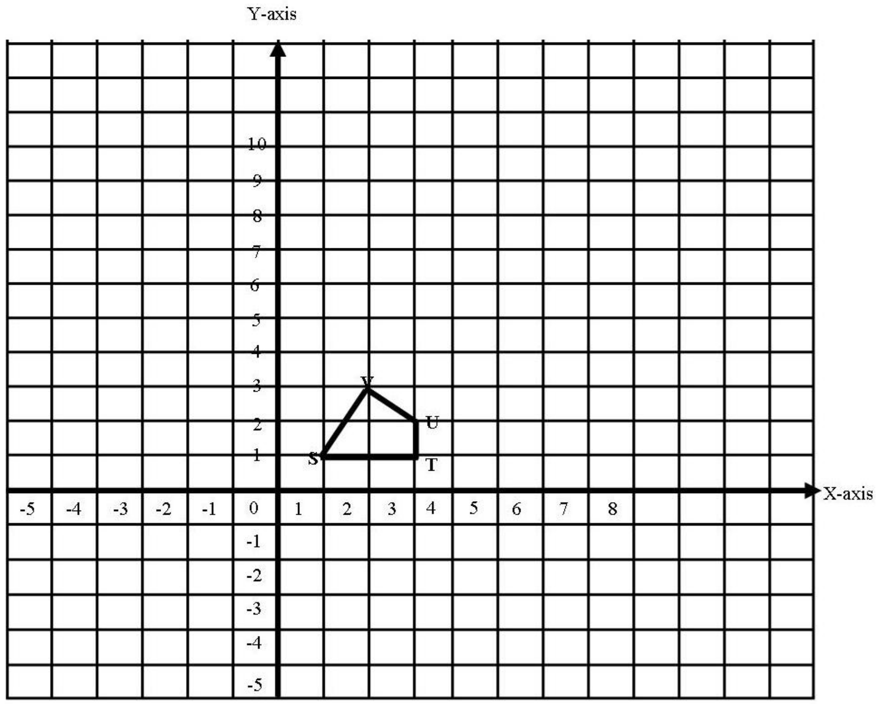
Answer: ------------------------------------------------------------------------------------------

**<u>Question 3. (Level 2)</u>**




3.1. Describe the transformation above which rectangle CDEF is mapped onto rectangle C'D'E'F'-------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------
------------------------------------------------------------------------------------------------------------------------

On the grid below, x, and y axes have been drawn and labeled.




3.2.1 Rotate STUV through 90° (anticlockwise), about (0,0) and label the image
S T U V.

3.2.2. Through how many angles in degrees anticlockwise can you rotate the same figure so that it can fit exactly onto the original figure?

----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------

3.3.3 Describe how you arrived at the answer given above.

----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------
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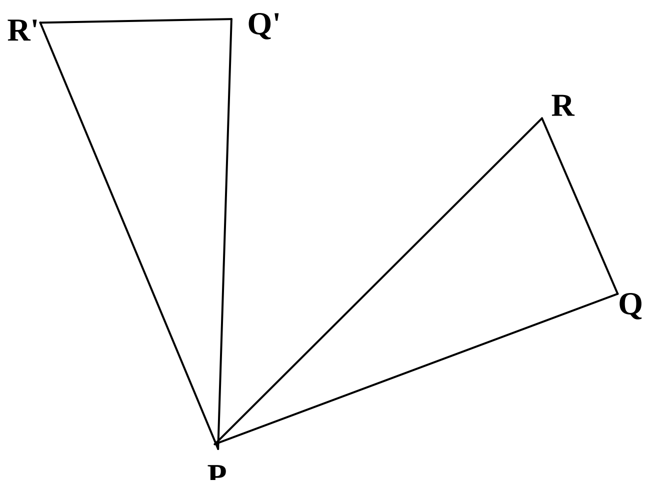
----------------------------------------------------------------------------------------------------------------
----------------------------------------------------------------------------------------------------------------

**Question 4 (Level 3)**




The triangle P.Q.R has been rotated through +70 degree about P. PQR is the image of PQR after rotation

4.1.1 What type of triangle is PQR? ---------------------------------------------------------------

4.1.2 Why? -----------------------------------------------------------------------------------------------

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--------------------------------------------------------------------------------------------------------------------

-------------------------------------------------------------------------------------------------------------

4.1.3 Use transformation to prove that triangle PQR is congruent to triangle PŔQ́

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--------------------------------------------------------------------------------------------------------------------

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4.2. Consider the statement: when a figure is rotated, the figure and its image are congruent do you agree?

Yes/No ------------------------------------------------------------

Motivate your answer given above with appropriate diagram

## APPENDIX B

## INTERVIEW PROTOCOL

Four aspect of the students' thought processes were examine during the interview. The objective of the interview was to find out the level of students' difficulties in these aspects below:

| | |
|---|---|
| **Basic level ( visualization)**<br><br>Visually identify an image of initial and final state after rotation | • Which among these transformation represents a rotation and why do you say so?<br>• Can you demonstrate a figure and it image after a rotation using objects given to you?<br>• Which of the image in question 1.2 is a true representation of A after a rotation of -90°and why? |
| **Level one (Analysis/Description)**<br><br>Identify geometric figures and their properties after transformation? | • Explain how you find the centre and angle of rotation in question 2.1.1<br>• What can you about the image<br>• Explain how you got C' in question 2.2.3<br>• What do you know about the length of the sides in triangle ABC and A'B'C' in question 2.2.4<br>• Give reason on your choice on question 2.2.5 |
| **Level two (Abstraction)**<br><br>Use rotation to transform an object when given the coordinates, angles and shape? | • By using words, can you describe the transformation on question 3.1<br>• Describe how you rotated STUV in question 3.2<br>• Explain how arrived at your answer in question 3.3.3 |
| **Level three (Deduction )**<br>Use transformation (rotation) to do proofs? | • How can you show that triangle PQR is congruent to P'Q'R'<br>• Use transformation of to explain why you think triangle PQR is rotated to P'Q'R' in question four. |

**APPENDIX C**

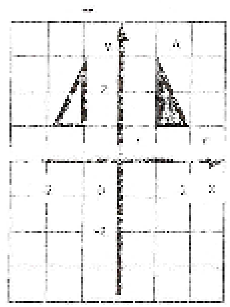
**Matrix for the assignment of level “1”or “0” across concept**

Students ………………………………………………sex……………class………

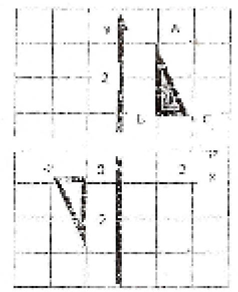
| | Concept ( Rotation) |
|---|---|
| Levels | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

## Appendix D

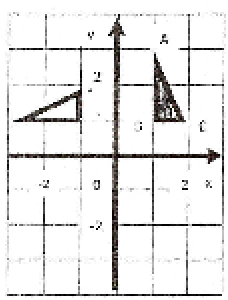
### Solution given by students in the written test and interview



(a)



(b)

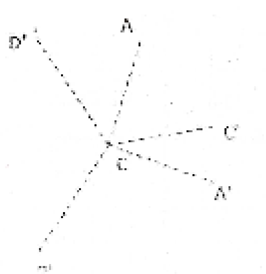


(c)

1.1. Each of the above diagrams represents a different transformation of triangle ABC in each case.

Which among the transformation represent a Rotation? Answer------------B, C

1.2 In the figure below, A has been rotated -90 ° about C as centre, which of the following image is true representation of A after a rotation of -90 °




B'

Figure 2. Solution given by student S61M

## APPENDIX E

## PERMISSION TO CONDUCT PILOT STUDY

**The Principal**
**Qoaling secondary school**
**P.o. BOX 9907**
**Maseru 100**
**Lesotho**

**Sir/ma**
**REQUEST FOR PERMISSION TO CONDUCT PILOT STUDY AT YOUR SCHOOL**

I am currently a student at the University of South Africa (UNISA) registered for a Masters degree in mathematics Education.
As part of the condition for my studies, I am conducting an investigation titled "the difficulties students have in transformation geometry of Rotation".
As part of the research I need to conduct an interview and written test with your Form C students. This investigation will not in anyway distract the normal teaching and learning at the school as the investigation will only be done immediately after normal school hours. I assure you that all information obtained during the investigation will be treated confidentially and will only be used for academic purposes only.

Thanks for your co-operation
Yours faithfully,

…………………………………………..
Evbuomwan Dickson

## APPENDIX F

## PERMISSION TO CONDUCT RESERCH AT YOUR SCHOOL

The Director
Teaching Service Department
P.O.BOX 9907
Maseru
Lesotho

Sir/ma

**Request for permission to conduct research in schools**

I am a student at the University of South Africa (UNISA) currently registered for a Masters Degree in Education, specializing in Mathematics.
The title of my proposed dissertation is “An investigation into the difficulties faced by student in transformation geometry of Rotation in Lesotho.
To complete the requirement for this degree I need to conduct a research on the above mention topic. I hereby ask for permission from the Director’s office to conduct my research at Qoaling Secondary School and a pilot study at Maseru Day Secondary both in Maseru.
I assure you that all information obtained during the investigation will be treated confidentially and will only be used for academic purposes only.

Thanks for your co-operation
Yours faithfully,

…………………………………………..
Evbuomwan Dickson.

## APPENDIX G

## LETTER TO PARENTS

Dear parent/guardian

**<u>Permission for your child to take part in a research study</u>**

I am a student at the University of South Africa (UNISA) currently registered for a Masters Degree in Education, specializing in Mathematics.

The title of my proposed dissertation is “An investigation into the difficulties faced by students in transformation geometry of Rotation in Lesotho.

To complete the requirement for this degree, I need to do a research on the above mention topic. I hereby ask for permission from you to enable your child ………………………………………………….in Form C to take part in the investigation.

I assure you that all information obtained during the investigation will be treated confidentially and will only be used for academic purposes only.

Should you accept this request mark **Yes** and if **No** in the appropriate block below with your signature below the Box and send this letter with your child back to school.

| | |
|---|---|
| **Yes** | |
| **No** | |

Signature of parent/guardian …………………………………………..

Thanks for your co-operation

Yours faithfully,

…………………………………………..

Evbuomwan Dickson