

# Information propagation and entanglement generation between two Unruh-DeWitt detectors

by

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## Abstract

The setup in which two quantum systems, Alice and Bob, communicate using bosonic field quanta can be viewed as a prototype for wireless quantum communication. In this thesis we focus on the most basic case, where Alice and Bob are modeled as Unruh-DeWitt detectors, i.e., as two-level quantum systems that interact locally through a scalar quantum field. Our aim is to study how information propagation and entanglement generation between the two detectors are impacted by both relativity and by the unavoidable noise that is due to the quantum fluctuations of the field.

We start by studying information propagation between the two detectors. Concretely, we construct and study the information-theoretic quantum channel,  $\xi$ , i.e., the completely positive trace preserving map between the input density matrix  $\rho$ , in which Alice prepares her detector for the emission, and the output density matrix  $\rho' = \xi(\rho)$  of Bob's detector at a later time. We confirm that the classical as well as the quantum channel capacity are strictly zero to all orders in perturbation theory for spacelike separations.

We then study entanglement generation between the two detectors. Specifically, we discuss how two Unruh-DeWitt detectors can extract entanglement from the vacuum. We show that the detectors can naturally and instantaneously become entangled through a Casimir-Polder effect. We then analyze the impact of various additions to this setup, such as the presence of a weak gravitational field, the presence of boundary conditions in the field, the presence of a weak classical potential, etc.

Most of these results can be found in the papers [1, 2, 3].

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## Conventions and Notation

For quantum field theory, we use most of the conventions of [4]. We use the Minkowski metric  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  and we denote  $px = p_\mu x^\mu = -E_{\vec{p}}x^0 + \vec{p} \cdot \vec{x}$  where  $E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$ . Derivatives are denoted as  $\frac{\partial f(x)}{\partial x^\mu} = \partial_\mu f$  and  $\frac{\partial f(x)}{\partial x^0} = \dot{f}(x)$ . We use  $x^0 = t$  for the coordinate time and  $\tau$  for the proper time. We work in the natural units where  $\hbar = c = G = 1$ . For quantum information, we use most of the conventions of [5]. Wherever necessary to avoid ambiguity we will denote operators  $O$  or states  $|\psi\rangle$  which live in the Hilbert space  $\mathcal{H}^{(j)}$  by a superscript (j), for example,  $O^{(j)}$  and  $|\psi^{(j)}\rangle$ . Also, when such operators occur tensored with identity operators, such as  $I^{(1)} \otimes I^{(2)} \otimes O^{(3)}$ , we will often abbreviate this as, for example,  $O^{(3)}$ . Orders in perturbation theory will be denoted by a subscript (j), so for example we could have  $P = P_{(0)} + P_{(1)} + O(\epsilon^2)$ .





# Chapter 1

## Introduction

Quantum information theory [5] only uses the axioms of quantum theory as a starting point to extend Shannon's classical information theory [6] to the quantum regime. Even though quantum information theory is not restricted to any particular framework of quantum theory, its practical applications are almost always using non-relativistic quantum mechanics. In response to this, relativistic quantum information [7] is a growing field of research which studies the consequences of Einstein's theory of relativity on the transmission of quantum information. So far, the focus has been mostly on the kinematic aspects of relativity. For instance, in the past 10 years much progress has been made on understanding how the entanglement and the entropy of the spin of particles change under a Lorentz transformation, see e.g. [8, 9, 10]. On the other hand, not as much is known about the dynamical consequences of a fully relativistic quantum theory, i.e. quantum field theory (QFT) [4].

To study information transmission, quantum information practitioners often work with quantum channels, namely completely positive trace preserving maps between an input density matrix  $\rho$  and an output density matrix  $\rho' = \xi(\rho)$  [5]. In the vast field of quantum information, there exists several physically motivated quantum channels, but none are explicitly causal. Indeed, quantum channels are usually modeled in non-relativistic frameworks, where causality can only be implicit. Moreover, the noise in the known quantum channels is usually assumed to come from an external environment like a thermal bath, see e.g. [11]. Even though this may be a reasonable assumption in practice, it is, at least in principle, possible to suppress this noise entirely. On the other hand, there are quantum sources of noise which cannot be suppressed, even in principle. For instance, the quantum fluctuations of the fields should be taken into account whenever one considers information at the fundamental level of quantum fields.

In addition, there is still a lot to be learned about the dynamics of entanglement in a QFT setting. Entanglement and information have different dynamics, so in principle

entanglement may be able to propagate at a speed faster than the speed of light. Nevertheless, that does not mean that entanglement does not obey any laws. A lot of work has been done in trying to understand these laws. But here again, entanglement dynamics is usually studied in non-relativistic frameworks and the source of noise is often assumed to be thermal, see e.g. [12]. Thus, a proper study of entanglement dynamics which takes into account special relativity and quantum field fluctuations is much needed.

To study these issues in this thesis, we take an operational approach and study information propagation and entanglement generation between two Unruh-DeWitt detectors, i.e. between two two-level quantum systems that interact locally with a scalar quantum field. The main advantage of using Unruh-DeWitt detectors is the fact that they are localized. Indeed, recent studies in relativistic quantum information often analyze how the vacuum and a one-particle state are seen in different non-inertial frames or in different spacetimes, see e.g. [13, 14, 15]. Nevertheless, causality-related issues are easier to address with localized objects such as Unruh-DeWitt detectors. Moreover, these detectors are subject to the quantum fluctuations of the field, a source of noise which is not always transparent when we only study the non-local modes of a free quantum field theory.

This thesis is organized as follows. In Sec. 1.1 we review quantum field theory, focusing on the tools needed later in this thesis, such as scalar field correlations and Unruh-DeWitt detectors. In Sec. 1.2 we introduce some tools of quantum information theory. We start by discussing quantum channels, their description and their information transmitting capacities, then we briefly discuss quantum entanglement. In Chapter 2 we study information propagation between two Unruh-DeWitt detectors. We first show in Sec. 2.1 that information propagation is bounded by the speed of light. The impossibility of superluminal signalling has of course been discussed before, see e.g. [16, 17, 18]. What is new here is that we prove the impossibility of superluminal signalling information-theoretically by constructing and studying the quantum channel created by the two Unruh-DeWitt detectors. In Sec. 2.2, we analyze in detail this quantum channel by obtaining an operator-sum representation, discussing its channel capacities and presenting a perturbative expansion of the channel. In Chapter 3 we study entanglement generation between the two Unruh-DeWitt detectors. It has been known that two detectors when coupled to a quantum field can have non-trivial entanglement dynamics, see e.g. [19, 20, 21]. It is also known that, due to the entanglement of the vacuum [22, 23], or the exchange of virtual photons [24], two detectors can become entangled even at spacelike separations, and the speed with which this can happen has been discussed. So far, all these studies used a time-dependent approach. In Sec. 3.1 we review these results and then follow-up on this analysis by looking at the entanglement of the vacuum in the presence of a weak gravitational field. In Sec. 3.2 we introduce a time-independent approach to vacuum entanglement extraction. We show that the two detectors can be entangled in the ground state of the interacting theory and we discuss how to prepare this ground state using adiabatic evolution. We then follow-up on

this result by analyzing the ground state entanglement when the field is subject to Dirichlet boundary conditions and when there is a classical potential weakly interacting with the field. In the last Chapter we propose various extensions to this work.

## 1.1 Quantum field theory

Let us start by introducing QFT in the Heisenberg picture. This section is only for completeness purposes and should not be taken as a complete or pedagogic review of QFT. A proper introduction to QFT can be found in [4, 25].

### 1.1.1 Canonical quantization

For simplicity, in this thesis we almost always use a single real scalar quantum field  $\phi(x)$ . The free relativistic action of such a field in Minkowski spacetime is:

$$S = \int d^4x \mathcal{L}(x) = \frac{1}{2} \int d^4x (\eta^{\mu\nu} \partial_\mu \phi(x) \partial_\nu \phi(x) - m^2 \phi^2(x)). \quad (1.1)$$

Extremizing the action with respect to the field  $\phi(x)$  gives the Klein-Gordon equation:

$$(\partial^\mu \partial_\mu - m^2) \phi(x) = 0. \quad (1.2)$$

The Klein-Gordon equation tells us how the field  $\phi(x)$  evolves in spacetime. To quantize the field, we need to use a Hamiltonian formulation. The canonical momentum is  $\pi(x) = \frac{\partial \mathcal{L}(x)}{\partial \dot{\phi}(x)} = \dot{\phi}(x)$ , such that the Hamiltonian reads:

$$H_F = \int d^3x \left( \pi(x) \dot{\phi}(x) - \mathcal{L}(x) \right) = \frac{1}{2} \int d^3x \left[ \pi^2(x) + (\nabla \phi(x))^2 + m^2 \phi^2(x) \right]. \quad (1.3)$$

We now impose the canonical equal time commutation relations:

$$[\phi(t, \vec{x}), \pi(t, \vec{y})] = i\delta(\vec{x} - \vec{y}) \quad (1.4)$$

$$[\phi(t, \vec{x}), \phi(t, \vec{y})] = [\pi(t, \vec{x}), \pi(t, \vec{y})] = 0. \quad (1.5)$$

Next, we need to solve the Klein-Gordon equation while taking into account the canonical commutation relations. To do this, we Fourier transform the field with creation and annihilation operators

$$\phi(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left( e^{i\vec{p}\cdot\vec{x}} v_{\vec{p}}(t) a_{\vec{p}} + e^{-i\vec{p}\cdot\vec{x}} v_{\vec{p}}^*(t) a_{\vec{p}}^\dagger \right) \quad (1.6)$$

where  $a_{\vec{p}}$  and  $a_{\vec{p}}^\dagger$  are respectively the annihilation and creation operators satisfying  $[a_{\vec{p}}, a_{\vec{p}'}^\dagger] = (2\pi^3)\delta(\vec{p}-\vec{p}')$ ,  $[a_{\vec{p}}, a_{\vec{p}'}] = [a_{\vec{p}}^\dagger, a_{\vec{p}'}^\dagger] = 0$  and  $a_{\vec{p}}|0\rangle = 0$  where  $|0\rangle$  is the vacuum state. Inserting Eq. (1.6) in the Klein-Gordon equation tells us that the mode function  $v_{\vec{p}}$  must satisfy the equation of motion

$$\ddot{v}_{\vec{p}}(t) + (\vec{p}^2 + m^2)v_{\vec{p}}(t) = 0 \quad (1.7)$$

and if we substitute Eq. (1.6) in the commutations relations we find that:

$$\dot{v}_{\vec{p}}v_{\vec{p}}^* - v_{\vec{p}}\dot{v}_{\vec{p}}^* = -2E_{\vec{p}}i. \quad (1.8)$$

Eq. (1.7) has the solution  $v_{\vec{p}} = \alpha_{\vec{p}}e^{-iE_{\vec{p}}t} + \beta_{\vec{p}}e^{iE_{\vec{p}}t}$  where  $\{\alpha_{\vec{p}}, \beta_{\vec{p}}\}$  are constants that we need to fix. Substituting this solution in Eq. (1.8) gives us the requirement  $|\alpha_{\vec{p}}|^2 - |\beta_{\vec{p}}|^2 = 1$  which is not enough to fix the constants  $\{\alpha_{\vec{p}}, \beta_{\vec{p}}\}$ . We thus also require that the vacuum state  $|0\rangle$  is the ground state of the Hamiltonian. We insert Eq. (1.6) in the Hamiltonian and we find that if  $\alpha_{\vec{p}}\beta_{\vec{p}} = 0$  then the vacuum is the ground state of the Hamiltonian [25]. Thus, we require that  $\beta_{\vec{p}} = 0$  and  $\alpha_{\vec{p}} = 1$ , which means that the field and the Hamiltonian of a free quantum scalar field is:

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left( a_{\vec{p}}e^{ipx} + a_{\vec{p}}^\dagger e^{-ipx} \right) \quad (1.9)$$

$$H_F = \int \frac{d^3p}{(2\pi)^3} E_{\vec{p}} \left( a_{\vec{p}}^\dagger a_{\vec{p}} + \frac{1}{2}[a_{\vec{p}}, a_{\vec{p}}^\dagger] \right). \quad (1.10)$$

From this follows the particle interpretation of QFT. The vacuum state  $|0\rangle$  is a state with no particles, and with the creation operators we can build the rest of the Fock basis which are the states that contain particles [26]:

$$|n_{\vec{p}_1}^1 n_{\vec{p}_2}^2 \dots n_{\vec{p}_j}^j\rangle := \frac{\left(a_{\vec{p}_1}^\dagger\right)^{n_1} \left(a_{\vec{p}_2}^\dagger\right)^{n_2} \dots \left(a_{\vec{p}_j}^\dagger\right)^{n_j}}{\sqrt{n_1!n_2!\dots n_j!}}|0\rangle. \quad (1.11)$$

Note that this particle interpretation of QFT is problematic in a spacetime in which we lose the time translation symmetry [26]. We should therefore emphasize the obvious fact that QFT is a theory of fields, not of particles. The fields are the fundamental objects: particles are only secondary, derived, objects. This line of thought leads to the popular point of view that particles are whatever particle detectors detect.

### 1.1.2 Field correlations

In this subsection we compute explicitly some field correlation functions, namely the correlator, the propagator and the commutator. They each mean different things and are used in different contexts.

## Correlator

The correlator  $D(x, y)$ , also called the Wightman function, is simply the two point correlation function of the field in the vacuum, namely  $D(x, y) := \langle 0 | \phi(x) \phi(y) | 0 \rangle$ . We can calculate it explicitly using the field mode decomposition of Eq. (1.9):

$$D(x, y) = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{ip(x-y)}}{2E_{\vec{p}}}. \quad (1.12)$$

First consider a massless field  $m = 0$ . We integrate the angular part and we obtain:

$$D(x, y) = \frac{1}{4\pi^2 |\vec{x} - \vec{y}|} \int_0^\infty \sin(p|\vec{x} - \vec{y}|) e^{-ip(x^0 - y^0)} dp. \quad (1.13)$$

To regularize the integral, we use the prescription  $x^0 - y^0 \rightarrow x^0 - y^0 - i\epsilon$  and take the limit  $\epsilon \rightarrow 0$  such that:

$$D(x, y) = \lim_{\epsilon \rightarrow 0^+} \frac{-1}{4\pi^2 [(x^0 - y^0 - i\epsilon)^2 - |\vec{x} - \vec{y}|^2]}. \quad (1.14)$$

Note that the correlator is really a distribution, such that the limit  $\epsilon \rightarrow 0$  should only be taken after it was integrated against a continuous function [27]. Nevertheless, the correlator tells us that a massless field is still correlated for spacelike events and these correlations decay with a polynomial decrease. We can do a similar calculation for a massive field and we obtain

$$D(x, y) = \lim_{\epsilon \rightarrow 0^+} \frac{1}{4\pi^2} \frac{m}{\sqrt{|\vec{x} - \vec{y}|^2 - (x^0 - y^0 - i\epsilon)^2}} K_1 \left( m \sqrt{|\vec{x} - \vec{y}|^2 - (x^0 - y^0 - i\epsilon)^2} \right) \quad (1.15)$$

where  $K_1$  is a Bessel function. This expression tells us that for a massive field the spacelike correlations decays exponentially like  $\sim e^{-m\sqrt{|\vec{x} - \vec{y}|^2 - (x^0 - y^0)^2}}$ .

## Propagator

Let us now look at the propagator  $G(x, y)$ , also called the Feynman propagator, which is the time ordered correlator  $G(x, y) := \langle 0 | T \phi(x) \phi(y) | 0 \rangle$  where  $T$  is the time ordering operator [4]. From our previous analysis of the correlator, we can easily get the propagator. Let us

start with a massless field,

$$\begin{aligned}
G(x, y) &= \lim_{\epsilon \rightarrow 0^+} \left\{ \frac{-\theta(x^0 - y^0)}{4\pi^2 [(x^0 - y^0 - i\epsilon)^2 - |\vec{x} - \vec{y}|^2]} + \frac{-\theta(y^0 - x^0)}{4\pi^2 [(y^0 - x^0 - i\epsilon)^2 - |\vec{x} - \vec{y}|^2]} \right\} \\
&= \lim_{\epsilon \rightarrow 0^+} \left\{ \frac{-\theta(x^0 - y^0)}{4\pi^2 [(x^0 - y^0)^2 - |\vec{x} - \vec{y}|^2 - 2i\epsilon(x^0 - y^0)]} \right. \\
&\quad \left. + \frac{-\theta(y^0 - x^0)}{4\pi^2 [(x^0 - y^0)^2 - |\vec{x} - \vec{y}|^2 - 2i\epsilon(y^0 - x^0)]} \right\} \\
&= \lim_{\epsilon \rightarrow 0^+} \frac{-1}{4\pi^2 [(x^0 - y^0)^2 - |\vec{x} - \vec{y}|^2 - i\epsilon']} \tag{1.16}
\end{aligned}$$

where  $\theta(x)$  is the Heaviside function. This  $i\epsilon'$  prescription is called the Feynman boundary condition [4]. Similarly for a massive field, we can just replace  $[\vec{x} - \vec{y}]^2 - [x^0 - y^0 - i\epsilon]^2$  by  $[\vec{x} - \vec{y}]^2 - [x^0 - y^0]^2 - i\epsilon'$ . Furthermore, note that the propagator is a Green's function of the Klein-Gordon operator,

$$\begin{aligned}
(\partial^\mu \partial_\mu - m^2)_x G(x, y) &= (\partial^\mu \partial_\mu - m^2)_x \left( \theta(x^0 - y^0) \langle 0 | \phi(x) \phi(y) | 0 \rangle \right. \\
&\quad \left. + \theta(y^0 - x^0) \langle 0 | \phi(y) \phi(x) | 0 \rangle \right) \\
&= -\delta(x^0 - y^0) \langle 0 | \pi(x) \phi(y) | 0 \rangle + \delta(y^0 - x^0) \langle 0 | \phi(y) \pi(x) | 0 \rangle \\
&= i\delta(x - y) \tag{1.17}
\end{aligned}$$

where we used  $\partial_{x^0} \theta(x^0 - y^0) = \delta(x^0 - y^0)$  and  $\partial_{x^0}^2 \theta(x^0 - y^0) = -\delta(x^0 - y^0)$ . This differential equation does not uniquely determine  $G(x, y)$ , once it is solved we need to apply boundary conditions which is equivalent to fixing a  $i\epsilon$  prescription.

## Commutator

As we have just seen, the correlations of a quantum field do not vanish outside the light cone. But correlations do not mean causality. To look at causality, we need to look at the commutator  $[\phi(x), \phi(y)]$ . Indeed, we can show that the commutator vanishes outside the light cone. This means that a measurement performed at one point cannot affect the outcomes' probabilities of a measurement made at another point whose separation from the first one is spacelike [4]. Using the mode decomposition of the field of Eq. (1.9), we have:

$$[\phi(x), \phi(y)] = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} (e^{ip(x-y)} - e^{-ip(x-y)}). \tag{1.18}$$

First, note that both terms are separately Lorentz invariant since  $p_\mu(x-y)^\mu$  and  $d^3p/2E_{\vec{p}}$  are unchanged under a boost of  $p$  and  $(x-y)$ . For spacelike events, there exists a coordinate system in which  $x^0 = y^0$ . We thus chose this coordinate system to evaluate the commutator for spacelike  $x$  and  $y$

$$\begin{aligned} [\phi(x), \phi(y)] \Big|_{(x-y)^2 > 0} &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} (e^{i\vec{p}(\vec{x}-\vec{y})} - e^{-i\vec{p}(\vec{x}-\vec{y})}) \\ &= 0 \end{aligned} \tag{1.19}$$

where we used the change of variable  $\vec{p} \rightarrow -\vec{p}$  in the second term. This simple derivation of microcausality shows that local free quantum field theory in Minkowski spacetime is causal. Using more advanced techniques one can show that this also holds in curved spacetime [28]. Nevertheless, the field on its own is not easy to observe. A more realistic observable is a localized quantum system interacting with the quantum field. With those quantum systems, our previous causality derivation does not tell us how and if causality is respected.

### 1.1.3 Unruh-DeWitt detectors

In this subsection we introduce the Unruh-DeWitt particle detector model and some of its applications. An extensive review of Unruh-DeWitt detectors can be found in [26, 27].

#### The model

An Unruh-DeWitt particle detector is a localized quantum system that interacts with a quantum scalar field. In practice, the detector is supposed to be the analog of an atom interacting with a quantum electromagnetic field. The detector is a two-level quantum system, an excited state  $|e^{(d)}\rangle$  and a ground state  $|g^{(d)}\rangle$  with energy difference  $\Delta E$ . With respect to the proper time  $\tau$  of the detector, its self-Hamiltonian is therefore:

$$H_d = (\Delta E + E_g)|e^{(d)}\rangle\langle e^{(d)}| + E_g|g^{(d)}\rangle\langle g^{(d)}|. \tag{1.20}$$

This quantum system interacts linearly with a scalar field

$$H_{int} = \alpha\eta(\tau) (|e^{(d)}\rangle\langle g^{(d)}| + |g^{(d)}\rangle\langle e^{(d)}|) \int d^3x f(\vec{x} - \vec{x}_d(\tau))\phi(\vec{x}) \tag{1.21}$$

where  $\alpha$  is a small coupling constant,  $\eta(\tau)$  is a switching function which allows us to switch on and off the interaction and  $m^{(d)} := |e^{(d)}\rangle\langle g^{(d)}| + |g^{(d)}\rangle\langle e^{(d)}|$  is the monopole matrix of the detector.  $\int d^3x f(\vec{x} - \vec{x}_d(\tau))\phi(\vec{x})$  is the smeared out field near the central location of



the detector  $\vec{x}_d(\tau)$  with which the detector interacts, which means that  $f(\vec{x} - \vec{x}_d(\tau))$  is a smearing function that effectively gives a small size to the detector.

To get some familiarity with the detector, we calculate the probability that it gets excited, namely the probability for the transition  $|g^{(d)}\rangle \rightarrow |e^{(d)}\rangle$ . To do so, we use the interaction picture where operators evolve with their self-Hamiltonian, that is  $H_d$  and  $H_F$ , while states evolve with the interaction Hamiltonian  $H_{int}$ . In this picture, the interaction Hamiltonian is evolving because  $m^{(d)}(\tau) = e^{i\tau H_d} m^{(d)} e^{-i\tau H_d}$  and  $\phi(t, \vec{x}) = e^{itH_F} \phi(\vec{x}) e^{-itH_F}$ . The evolution operator acting on states is then  $U = T e^{-i \int H_{int}(\tau) d\tau}$ . We assume that the field is initially in the vacuum  $|0\rangle$  such that the state of the overall system is initially  $|0, g^{(d)}\rangle$ . The excitation probability is therefore:

$$\begin{aligned}
P_e &= \sum_k \left| \langle k, e^{(d)} | T e^{-i \int H_{int}(\tau) d\tau} | 0, g^{(d)} \rangle \right|^2 \\
&= \sum_k \langle 0, g^{(d)} | T^\dagger e^{i \int H_{int}(\tau') d\tau'} | k, e^{(d)} \rangle \langle k, e^{(d)} | T e^{-i \int H_{int}(\tau) d\tau} | 0, g^{(d)} \rangle \\
&= \langle 0, g^{(d)} | T^\dagger e^{i \int H_{int}(\tau') d\tau'} | e^{(d)} \rangle \langle e^{(d)} | T e^{-i \int H_{int}(\tau) d\tau} | 0, g^{(d)} \rangle.
\end{aligned} \tag{1.22}$$

We assume that  $\alpha \ll 1$  so we can use time-dependant perturbation theory, such that  $T e^{-i \int H_{int}(\tau) d\tau} \approx 1 - i \int H_{int}(\tau) d\tau$ . Thus, after simplifications we are left at  $O(\alpha^2)$  with:

$$\begin{aligned}
P_e &= \alpha^2 \int d\tau d^3x \int d\tau' d^3x' f(\vec{x} - \vec{x}_d(\tau)) f(\vec{x}' - \vec{x}_d(\tau')) \eta(\tau) \eta(\tau') \\
&\quad \times e^{-i\Delta E(\tau - \tau')} \langle 0 | \phi(\vec{x}, t(\tau)) \phi(\vec{x}', t(\tau')) | 0 \rangle.
\end{aligned} \tag{1.23}$$

We thus see that the excitation probability of the particle detector is related to the correlator  $D(x(\tau), x'(\tau'))$ . As was pointed out in Sec. 1.1.2, the correlator needs to be integrated against a continuous function. There are two ways this can be achieved. We can either choose the switching function  $\eta(\tau)$  to be continuous [27], meaning that there is some fuzziness as to when the detector is turned on and off, or alternatively we can choose a continuous smearing function  $f(\vec{x} - \vec{x}_d(\tau))$ , which means that there is some fuzziness as to where the detector is localized. Depending on the context one of the two methods is often more convenient than the other.

## Detection rates

We now calculate explicitly some excitation probabilities for a point-like detector  $f(\vec{x} - \vec{x}_d(\tau)) = \delta(\vec{x} - \vec{x}_d(\tau))$ . Let us also assume that the switching function is Gaussian  $\eta(\tau) = e^{-\tau^2/(2\sigma^2)}$  and that the field is massless. This allows us to calculate  $P_e$  exactly in the case

of an inertial observer in Minkowski spacetime where  $\tau = t$  such that  $x_d(\tau) = (\tau, 0, 0, 0)$ :

$$\begin{aligned}
P_e &= \alpha^2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-\tau^2/(2\sigma^2) - \tau'^2/(2\sigma^2) - i\Delta E(\tau - \tau')} D(x_d(\tau), x_d(\tau')) \\
&= \alpha^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \left| \int_{-\infty}^{\infty} dt e^{-it(\Delta E + E_p) - t^2/(2\sigma^2)} \right|^2 \\
&= \frac{\alpha^2}{4\pi^2} \int_0^{\infty} dp p (2\sigma^2 \pi) e^{-\sigma^2(\Delta E + p)^2/2} \\
&= \frac{\alpha^2}{4\pi} \left( e^{-\Delta E^2 \sigma^2} - \Delta E \sqrt{\pi} \sigma \operatorname{erfc}(\Delta E \sigma) \right)
\end{aligned} \tag{1.24}$$

where  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ . In the limit  $\Delta E \sigma \rightarrow \infty$  we have  $P_e \approx \frac{\alpha^2 e^{-\Delta E^2 \sigma^2}}{8\pi \Delta E^2 \sigma^2} \rightarrow 0$ . Thus, as expected, an inertial detector in the vacuum does not get excited much: the detector can only get excited by a quantum field fluctuation, that is by a virtual particle, which vanishes as the time interval ( $\sigma$ ) goes to infinity. Yet another way to look at  $P_e$  is with the ground states. Indeed, we started from the ground state of the free theory  $|0, g^{(d)}\rangle$ , then as the Gaussian switching function increased we approximately went to the ground state of the interacting theory, and as the Gaussian decreased we approximately went back to the ground state of the free theory. Hence, the excitation probability  $P_e$  decreases when the evolution is more adiabatic, which is the case when  $\Delta E \sigma$  increases.

Let us now do a similar calculation but for a uniformly accelerated detector. This detector has a worldline  $x_d(\tau) = (\sinh(a\tau)/a, \cosh(a\tau)/a, 0, 0)$  where  $a$  is the acceleration of the detector. To avoid the quantum fluctuations that even an inertial detector sees, we take the limit  $\sigma \rightarrow \infty$  right from the start (see [29] for the calculation with a finite  $\Delta E \sigma$ ), and consider the excitation probability per proper time,  $\Gamma$ :

$$\begin{aligned}
\Gamma &= \lim_{\sigma \rightarrow \infty} \frac{\alpha^2}{\sigma} \int_{-\sigma/2}^{\sigma/2} d\tau \int_{-\sigma/2}^{\sigma/2} d\tau' e^{-i\Delta E(\tau - \tau')} D(x_d(\tau), x_d(\tau')) \\
&= \alpha^2 \lim_{\epsilon \rightarrow 0^+} \int_{-\infty}^{\infty} d\Delta\tau e^{-i\Delta E(\Delta\tau)} \left( \frac{-a^2}{16\pi^2 \sinh^2(\Delta\tau a/2 - i\epsilon)} \right) \\
&= \alpha^2 \lim_{\epsilon \rightarrow 0^+} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} d\Delta\tau e^{-i\Delta E(\Delta\tau)} \left( \frac{-1}{4\pi^2 (\Delta\tau - i\epsilon + 2i\pi k/a)^2} \right) \\
&= \alpha^2 \sum_{k=1}^{\infty} (-2\pi i) \frac{-1}{4\pi^2} \frac{d}{d\Delta\tau} \left( e^{-i\Delta E \Delta\tau} \right) \Big|_{\Delta\tau = -2\pi i k/a} \\
&= \alpha^2 \frac{\Delta E}{2\pi} \frac{1}{e^{2\pi \Delta E/a} - 1}.
\end{aligned} \tag{1.25}$$

This detection rate happens to be the same detection rate as an inertial detector in a thermal bath of temperature  $T = a/(2\pi k_B)$ . This effect is known as the Unruh effect [30].

We thus see that the accelerating detector excites the field, causing the field to have a back-reaction on the detector and exciting it.

## 1.2 Quantum information theory

In this section we briefly review some aspects of quantum information theory. Our goal is to introduce some basic ideas and tools that will be useful in the rest of this thesis. A more complete review can be found in [5].

### 1.2.1 Quantum channels

We start by introducing quantum channels. A quantum channel  $\xi$  takes an input density matrix  $\rho$  and gives another density matrix  $\rho' = \xi(\rho)$  at the output. The channel map  $\xi$  captures the noise that  $\rho$  is subject to when being transmitted. We first show how to describe channels mathematically, and then discuss their information transmitting capacities.

#### Operator-sum representation

Let us first list the properties that  $\xi$  must satisfy.

- I) Because of the postulates of quantum theory,  $\xi$  needs to be a linear map:  $\xi(p_1\rho_1 + p_2\rho_2) = p_1\xi(\rho_1) + p_2\xi(\rho_2)$ .
- II) Because valid density matrices are positive, the output density matrix must be a positive density matrix, so  $\xi$  needs to be a positive map. Since  $I^{(R)} \otimes \xi^{(Q)}(\rho^{(R,Q)})$  must also to be a positive density matrix,  $\xi$  needs to be a completely positive map.
- III) Because valid density matrices have a trace equal to 1, we need  $\xi$  to be trace preserving.

To put it simply,  $\xi$  needs to be a completely positive and trace preserving (CPTP) linear map. It can be proved [5] that  $\xi$  satisfies these three properties if and only if

$$\xi(\rho) = \sum_i E_i \rho E_i^\dagger \tag{1.26}$$

where the set of operators  $\{E_i\}$  are called Kraus operators. These operators map the input Hilbert space to the output Hilbert space and satisfy  $\sum_i E_i^\dagger E_i = I$ . To simplify the notation, we assume in the rest of this subsection that the input Hilbert space is

the same as the output Hilbert space, which we denote as  $\mathcal{H}^{(Q)}$ . The operators  $E_i^{(Q)}$  are not unique, one can easily show that a set  $\{E_i^{(Q)}\}$  and a set  $\{F_i^{(Q)}\}$  give the same channel if  $E_i^{(Q)} = \sum_j u_{ij} F_j^{(Q)}$  where  $u_{ij}$  is a unitary matrix. The minimum number of Kraus operators required to describe a channel  $\xi$  is equal to the rank of the square matrix  $(I^{(R)} \otimes \xi^{(Q)})|\beta^{(R,Q)}\rangle\langle\beta^{(R,Q)}|$ , where  $|\beta^{(R,Q)}\rangle$  is a maximally entangled state [31]. Thus it can never exceed  $d^2$  where  $d$  is the dimension of  $\mathcal{H}^{(Q)}$ .

Presented this way, the operator-sum representation may seem abstract and slightly disconnected with quantum physics. But noisy quantum channels often arise because the quantum system of interest, that is the quantum system  $Q$ , interacts with an environment. Taken both together they form a closed system, but if we only consider  $Q$  we have an open quantum system. For instance, assume that the total system starts in a product state  $\rho^{(Q)} \otimes |e_o^{(env)}\rangle\langle e_o^{(env)}|$ , then after a unitary evolution  $U^{(Q,env)}$  we trace out the environment, which we denote by  $Tr_{(env)}$ , such that the output of the channel is

$$\begin{aligned}\xi(\rho) &= Tr_{(env)} \left[ U^{(Q,env)} \left( \rho^{(Q)} \otimes |e_o^{(env)}\rangle\langle e_o^{(env)}| \right) U^{\dagger(Q,env)} \right] \\ &= \sum_k \langle e_k^{(env)}| \left[ U^{(Q,env)} \left( \rho^{(Q)} \otimes |e_o^{(env)}\rangle\langle e_o^{(env)}| \right) U^{\dagger(Q,env)} \right] |e_k^{(env)}\rangle \\ &= \sum_k E_k^{(Q)} \rho^{(Q)} E_k^{\dagger(Q)}\end{aligned}\tag{1.27}$$

where  $E_k^{(Q)} = \langle e_k^{(env)}|U^{(Q,env)}|e_o^{(env)}\rangle$ . As expected, this shows that the operator-sum representation is valid for the case where the quantum system  $Q$  interacts unitarily with an environment which we trace out. This physically motivated description of a channel turns out to be very general [5]: the operator-sum representation can always be modeled by an environment of dimension at most  $d^2$  which starts in a pure state  $|e_o^{(env)}\rangle$ . Indeed, let  $|e_k^{(env)}\rangle$  be an orthonormal basis of a modeled environment, in one to one correspondence with the index  $k$  of the Kraus operators  $\{E_k^{(Q)}\}$ . We construct an operator  $U^{(Q,env)}$  such that  $U^{(Q,env)}|\psi^{(Q)}, e_o^{(env)}\rangle = \sum_k E_k^{(Q)}|\psi^{(Q)}, e_k^{(env)}\rangle$ .  $U^{(Q,env)}$  can be a unitary operator since

$$\langle\psi^{(Q)}, e_o^{(env)}|U^{\dagger(Q,env)}U^{(Q,env)}|\phi^{(Q)}, e_o^{(env)}\rangle = \sum_k \langle\psi^{(Q)}|E_k^{\dagger(Q)}E_k^{(Q)}|\phi^{(Q)}\rangle = \langle\psi^{(Q)}|\phi^{(Q)}\rangle\tag{1.28}$$

for any  $|\psi^{(Q)}\rangle, |\phi^{(Q)}\rangle$ . Finally, we can easily show that this modeled environment reproduces the action of the Kraus operators  $\{E_k^{(Q)}\}$ :

$$\begin{aligned}Tr_{(env)} \left[ U^{(Q,env)} \left( \rho^{(Q)} \otimes |e_o^{(env)}\rangle\langle e_o^{(env)}| \right) U^{\dagger(Q,env)} \right] \\ &= \sum_{i,j,k} Tr_{(env)} \left[ E_j^{(Q)} \left( p_i |\psi_i^{(Q)}\rangle\langle\psi_i^{(Q)}| \otimes |e_j^{(env)}\rangle\langle e_k^{(env)}| \right) E_k^{\dagger(Q)} \right] \\ &= \sum_k E_k^{(Q)} \rho^{(Q)} E_k^{\dagger(Q)}.\end{aligned}\tag{1.29}$$

Thus, any CPTP map can be represented as a unitary evolution by building an environment with which the quantum system  $Q$  interacts unitarily. The two approaches, the operator-sum representation and the modeled environment, are therefore completely equivalent. Finally, using the environment approach, we define the complementary quantum channel  $\xi^C$  as the channel that traces out  $Q$  instead of the environment, namely

$$\xi^C(\rho) = \text{Tr}_{(Q)} \left[ U^{(Q,env)} (\rho^{(Q)} \otimes |e_o^{(env)}\rangle\langle e_o^{(env)}|) U^{\dagger(Q,env)} \right]. \quad (1.30)$$

## Channel capacities

Suppose that Alice prepares a state  $\rho$  which she transmits to Bob at the other end of the channel  $\rho' = \xi(\rho)$ . How much information can Alice convey to Bob using the noisy channel  $\xi$ ? The theory of channel capacities gives us tools to answer this question. We now make a quick overview of these tools. A more complete review can be found in [5, 32, 33, 34].

Let us first focus on the classical capacity of a quantum channel. In that case, Alice transmits information by sending in the channel non-entangled quantum states. Bob gains information by making measurements on the output states and looking only at the measurement outcomes. Therefore, Bob has only access to classical information. However, unless the output states are orthogonal, no measurement made by Bob allows him to distinguish perfectly the different input states. Consequently, even if we only consider the classical capacity for now, this capacity is still limited by quantum theory. We now consider the mutual information since, roughly speaking, the mutual information is the amount of information about the input that can be recovered from a measurement performed on the output. Let us first introduce the mutual information in a fully classical setting. In that case, Alice sends a classical random variable  $X$  out of a finite alphabet  $\mathcal{I}$  with probability  $p(x)$  and then Bob receives the classical variable  $Y$  out of a finite alphabet  $\mathcal{O}$  with probability  $p(y|x)$ . The mutual information is then [32]

$$I(X : Y) = H(X) - H(X|Y) \quad (1.31)$$

where  $H(X) = -\sum_i p(x_i) \ln(p(x_i))$  is Shannon entropy [6] and  $H(X|Y)$  is the expected entropy of  $X$  once one knows the value of  $Y$ :

$$H(X|Y) = H(X, Y) - H(Y) = -\sum_{i,j} p(y_j) p(x_i|y_j) \ln(p(x_i|y_j)). \quad (1.32)$$

The  $\ln$  functions are usually evaluated in base 2 so the unit of entropy is bit. Shannon entropy  $H(X)$  is a measure of the uncertainty associated with the random variable  $X$ . Therefore,  $H(X) - H(X|Y)$  can be interpreted as the uncertainty on  $X$  to which we subtract the amount of uncertainty remaining about  $X$  after  $Y$  is known. In other words,

the mutual information is the amount of information about  $X$  that is gained by knowing  $Y$ . A very important theorem from classical information theory is the following [6]: if a communication channel has mutual information  $I(X : Y)$  between the input signal  $X$  and the received output  $Y$ , then with multiple parallel uses of the channel and by means of sufficiently redundant coding, that channel can be used to send up to, but no more than  $I(X : Y)$  bits per use of the channel with arbitrary low probability of error. To go back to our quantum problem, we now simply assume that Alice assigns to each classical message  $X_i$  a quantum state  $\rho_i$ . According to a theorem proved by Holevo, for any observable that Bob chooses to measure, the mutual information  $I(X : Y)$  between the input  $X$  of Alice and the measurement outcome  $Y$  of Bob is bounded by [5]

$$I(X : Y) \leq S \left( \xi \left( \sum_i p_i \rho_i \right) \right) - \sum_i p_i S(\xi(\rho_i)) \quad (1.33)$$

where  $S(\rho) = -\text{Tr}(\rho \ln(\rho))$  is simply the quantum version of the classical Shannon entropy, it is called the von Neumann entropy. Similarly to the classical case, one can show that this upper bound is achievable by employing long strings of non-entangled input state  $\rho_N := \rho_a^{(1)} \otimes \rho_b^{(2)} \otimes \dots \otimes \rho_c^{(N)}$  which are sent to parallel copies of the channel  $\xi^{\otimes N} := \xi^{(1)} \otimes \xi^{(2)} \otimes \dots \otimes \xi^{(N)}$ , pruning the set of strings  $\rho_N$  so that they are sufficiently distinguishable and choosing a suitable decoding observable that acts on the output  $\xi^{\otimes N}(\rho_N)$  [35]. Thus, for  $N$  large enough, we can use the channel to transmit  $N \cdot C(\xi)$  bits of information with arbitrarily low probability of error, where:

$$C(\xi) = \max_{\{p_j, \rho_j\}} \left[ S \left( \xi \left( \sum_j p_j \rho_j \right) \right) - \sum_j p_j S(\xi(\rho_j)) \right]. \quad (1.34)$$

This classical channel capacity is known as the product state capacity because we assumed that the input states were non-entangled.

Let us now discuss the quantum channel capacity. To put it simply, the quantum channel capacity measures the capability of a channel to relay quantum coherence. It is the number of quantum bits (qubits) per channel use that can be reliably transmitted through  $\xi$ . In the quantum case, Alice is allowed to prepare entangled input states. Moreover, Bob does not perform measurements on the output states. He therefore gains the whole quantum state of the output. The amount of quantum information that can reliably be sent through the channel can be shown to be [36, 33]

$$Q(\xi) = \lim_{n \rightarrow \infty} \frac{\max_{\rho} [I_c(\xi^{\otimes n}, \rho)]}{n}$$

$$I_c(\xi, \rho) = S(\xi(\rho)) - S(\xi^C(\rho)) \quad (1.35)$$

where  $I_c$  can be seen as the quantum version of the classical mutual information called the coherent information. From the definition of  $I_c(\xi, \rho)$ , we see that the coherent information measures the loss of information about  $\rho$  into the environment. This definition comes from the fact that whenever we have noise in a quantum system, it is possible to reverse the effect of that noise using quantum error correction if and only if the environment has not gained any information about the system's quantum state [37]. Note that because of the limit in  $Q(\xi)$  it is extremely hard to compute and also very hard to extract its general properties. In fact, we are still unable to fully characterize the class of channels which give a zero quantum capacity. On the other hand, this may be an ill-defined question since there exists pairs of zero quantum capacity channels which when used together have a non-zero quantum capacity [34].

We may now introduce two important classes of quantum channels. A channel  $\xi$  is called degradable if there exists a channel  $\Gamma$  such that  $\Gamma(\xi(\rho)) = \xi^C(\rho)$  for any  $\rho$  [38]. In other words, a degradable channel is a channel for which the environment can be imitated using the output of the channel. A channel  $\xi$  is antidegradable if its complementary channel is degradable, or equivalently if there exists a channel  $\Gamma$  such that  $\Gamma(\xi^C(\rho)) = \xi(\rho)$  for any  $\rho$  [38]. Note that almost every quantum channel for which the quantum capacity is fully known is either degradable or antidegradable. This is simply because for a degradable channel we have  $I_c(\xi^{\otimes n}) = nI_c(\xi)$ , such that the quantum channel capacity can be simplified to  $Q(\xi) = \max_{\rho} I_c(\xi, \rho)$  [39]. In addition, using a simple no-cloning argument, antidegradable channels have a trivial zero quantum capacity [40].

## 1.2.2 Quantum entanglement

In this subsection we review some useful concepts about quantum entanglement. Our review is not exhaustive and a complete review can be found in [41]. We first introduce the definition of entanglement and ways to quantify it. We then discuss entanglement swapping and its implications.

### Entanglement measures

The Hilbert space  $\mathcal{H}^{(A)} \otimes \mathcal{H}^{(B)}$  contains a class of states which manifest unique quantum mechanical properties called entangled states. Indeed, it was shown by Bell that with these entangled states the correlations between measurements made on system  $A$  and  $B$  can be stronger than the correlations we would obtain with any local classical model.

Let us first formally define an entangled state in the space of pure states. A state  $|\psi^{(A,B)}\rangle$  is called an entangled state if it cannot be written as a separable state  $|\psi^{(A,B)}\rangle = |\phi^{(A)}\rangle \otimes |\theta^{(B)}\rangle$ . In the space of pure states, the entangled states are the only ones which exhibit

correlations between local observables  $\langle O^{(A)}O^{(B)} \rangle \neq \langle O^{(A)} \rangle \langle O^{(B)} \rangle$ . The existence of these correlations means that the state  $|\psi^{(A,B)}\rangle$  contains more information than just the two parts of the state  $\rho^{(A)} = \text{Tr}_{(B)} (|\psi^{(A,B)}\rangle\langle\psi^{(A,B)}|)$  and  $\rho^{(B)} = \text{Tr}_{(A)} (|\psi^{(A,B)}\rangle\langle\psi^{(A,B)}|)$ . Therefore, the entanglement entropy is a good measure of entanglement:

$$S_A = -\text{Tr}_{(A)} (\rho^{(A)} \ln(\rho^{(A)})) . \quad (1.36)$$

For pure states  $S_A = S_B$  is non-vanishing if and only if the state is entangled. Note also that a maximally entangled state is such that  $\rho^{(A)} = I^{(A)}/d$  and  $S_A = \ln(d)$  entanglement bits (ebits).

Let us now turn to mixed states. A state  $\rho^{(A,B)}$  is called entangled if it cannot be written as

$$\rho^{(A,B)} = \sum_i p_i |\phi_i^{(A)}\rangle\langle\phi_i^{(A)}| \otimes |\theta_i^{(B)}\rangle\langle\theta_i^{(B)}| \quad (1.37)$$

where  $p_i > 0$  and  $\sum_i p_i = 1$ . Note that in the space of mixed states, non-entangled states also exhibit correlations between local observables  $\langle O^{(A)}O^{(B)} \rangle = \sum_i p_i \langle O^{(A)} \rangle_i \langle O^{(B)} \rangle_i \neq \langle O^{(A)} \rangle \langle O^{(B)} \rangle$ . This means that mixed states also have classical correlations, on top of the quantum correlations. Since entanglement entropy is simply a measure of correlations, classical or quantum, it is not a good measure of entanglement for mixed states. There exist good measures of entanglement for density matrices of dimension  $2 \times 2$ ,  $2 \times 3$  and also for continuous Gaussian states [41]. For qubits ( $2 \times 2$ ), there are two very popular measures, namely the negativity and the concurrence. We will always use the negativity, defined as [42, 43]

$$N(\rho^{(A,B)}) := \|\rho^{(A,B)T_A}\|_1 - 1 = \sum_i (|\lambda_i| - \lambda_i) \quad (1.38)$$

where  $\rho^{(A,B)T_A}$  is the partial transpose of  $\rho^{(A,B)}$  with respect to system  $A$ ,  $\|O\|_1$  is the trace norm  $\text{Tr}\sqrt{O^\dagger O}$  and  $\lambda_i$  are the eigenvalues of  $\rho^{(A,B)T_A}$ . This definition ensures that the negativity vanishes on states of the form (1.37) and coincides with the entanglement entropy for maximally entangled states. Moreover, one can show that the negativity bounds the distillable entanglement contained in  $\rho^{(A,B)}$  [43].

## Entanglement swapping

Let us now discuss entanglement swapping, a technique used to transfer entanglement which inspired some of the work done in this thesis. We discuss two possible setups of entanglement swapping, the first one transfers entanglement using measurements and the second one transfers entanglement using unitary interactions.



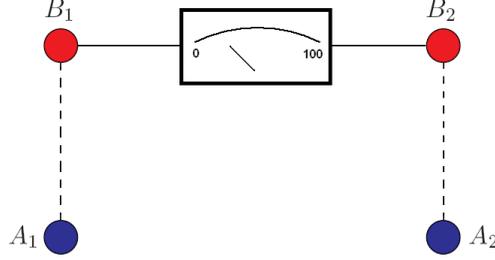


Figure 1.1: Entanglement swapping, setup with measurements.

The first setup is illustrated on Fig. (1.1). In the most basic scenario,  $A_1$  is initially entangled with  $B_1$ , and  $A_2$  is initially entangled with  $B_2$ . The entire system starts in the pure state:

$$|\psi_{ini}^{(A_1, B_1, A_2, B_2)}\rangle = \frac{1}{2} (|0^{(A_1)}, 0^{(B_1)}\rangle + |1^{(A_1)}, 1^{(B_1)}\rangle) \otimes (|0^{(A_2)}, 0^{(B_2)}\rangle + |1^{(A_2)}, 1^{(B_2)}\rangle). \quad (1.39)$$

Then, a joint projective measurement is made on the quantum system  $B_1$  and  $B_2$  in the Bell basis  $\{|0^{(B_1)}, 0^{(B_2)}\rangle \pm |1^{(B_1)}, 1^{(B_2)}\rangle/\sqrt{2}, |0^{(B_1)}, 1^{(B_2)}\rangle \pm |1^{(B_1)}, 0^{(B_2)}\rangle/\sqrt{2}\}$ . As a result of the Bell measurement, the quantum systems  $A_1$  and  $A_2$  are projected to the following states, all with probability 1/4:

$$\begin{aligned} |\psi_1^{(A_1, A_2)}\rangle &= |0^{(A_1)}, 0^{(A_2)}\rangle + |1^{(A_1)}, 1^{(A_2)}\rangle/\sqrt{2} \\ |\psi_2^{(A_1, A_2)}\rangle &= |0^{(A_1)}, 0^{(A_2)}\rangle - |1^{(A_1)}, 1^{(A_2)}\rangle/\sqrt{2} \\ |\psi_3^{(A_1, A_2)}\rangle &= |0^{(A_1)}, 1^{(A_2)}\rangle + |1^{(A_1)}, 0^{(A_2)}\rangle/\sqrt{2} \\ |\psi_4^{(A_1, A_2)}\rangle &= |0^{(A_1)}, 1^{(A_2)}\rangle - |1^{(A_1)}, 0^{(A_2)}\rangle/\sqrt{2}. \end{aligned} \quad (1.40)$$

This means that if the outcome of the measurement is unknown, the state of the quantum systems  $A_1$  and  $A_2$  after the measurement is  $\rho_{fin}^{(A_1, A_2)} = \sum_i |\psi_i^{(A_1, A_2)}\rangle\langle\psi_i^{(A_1, A_2)}|/4 = I^{(A_1, A_2)}/4$ . If we stop here, the state  $\rho_{fin}^{(A_1, A_2)}$  is not an entangled state. Nevertheless, if the outcome of the measurement is known by system  $B_2$  and  $A_1$ , then they can perform a local rotation on their state in such a way that  $|\psi_{fin}^{(A_1, A_2)}\rangle = |\psi_1^{(A_1, A_2)}\rangle$  [41]. What is so great about that is that  $A_1$  and  $A_2$  end up in a maximally entangled state even if they never interacted with each other. This does not mean that entanglement was created outside the light cone. Indeed, provided that the initial entanglement was created in a causal way, transferring that entanglement to system  $A_1$  and  $A_2$  can be achieved with at most the speed of light. To see this, assume for simplicity that  $A_1$  and  $B_1$  are initially next to each

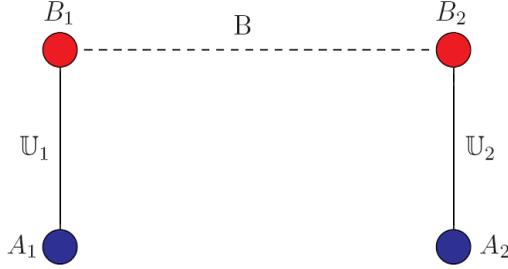


Figure 1.2: Entanglement swapping, setup with unitary interactions.

other and that they are separated by a distance  $L$  from  $A_2$  and  $B_2$  which are also next to each other. Then, the systems  $B_1$  and  $B_2$  need to travel to a common destination so that they can be measured in the Bell basis. This takes at least a time of  $L/2$ . After the measurement, the outcome needs to be communicated to the quantum system  $A_1$  (and  $B_2$ ). This also takes a time of  $L/2$ . Therefore, using this setup of entanglement swapping,  $A_1$  and  $A_2$  can get entangled with at most the speed  $v = \frac{L}{L/2+L/2} = 1$ , namely the speed of light.

The second setup of entanglement swapping is illustrated on Fig. (1.2). In this setup, no measurements are performed, just unitary interactions.  $B_1$  and  $B_2$  are initially entangled in the state  $|\phi^{(B_1, B_2)}\rangle$ , while  $A_1$  and  $A_2$  start respectively in the states  $|\theta^{(A_1)}\rangle$  and  $|\psi^{(A_2)}\rangle$ . Then,  $A_1$  unitarily interacts with  $B_1$ , and  $A_2$  unitarily interacts with  $B_2$ . The final state of  $A_1$  and  $A_2$  is:

$$\rho^{(A_1, A_2)} = \text{Tr}_{(B_1, B_2)} \left( U_1^{(A_1, B_1)} U_2^{(A_2, B_2)} |\phi^{(B_1, B_2)}\rangle \langle \phi^{(B_1, B_2)}| U_1^{(A_1, B_1)\dagger} U_2^{(A_2, B_2)\dagger} \right). \quad (1.41)$$

Since we trace over system  $B_1$  and  $B_2$ , we generally end up in a mixed state. This makes things more complicated because as we previously discussed, it is harder to distinguish entangled mixed states from non-entangled mixed states. Therefore, it is not really possible to analyze this setup in a general way like we did for the first setup because at this point it would be necessary to specify the details of the interactions. This is why this setup is usually studied for very specific examples, see e.g. [44]. In chapter 3 we will look at perhaps the most interesting example of such a setup.

We can still look ahead and discuss whether this setup may allow creation of entanglement outside the light cone. As opposed to the previous setup, we do not need to

communicate the outcome of any measurements, we therefore save some time here. On the other hand,  $A_1$  and  $A_2$  end up in a mixed state, so entanglement may take more time to kick in. Note that because no measurements are made on the systems  $B_1$  and  $B_2$ , we perhaps have more freedom in the choice of systems we consider. For instance, if the quantum systems  $B_1$  and  $B_2$  are in fact just one big system  $B$ , say a big metal bar, then perhaps it can naturally start in a state which is entangled at both ends  $B_1$  and  $B_2$ . If that is so, then we save some time here as well because we do not need to assume a preparation time to get  $B_1$  and  $B_2$  entangled. If such a system exists, then this system may facilitate the creation of entanglement outside the light cone, but of course a proper analysis of those systems is required before making such claims.

# Chapter 2

## Information propagation between two Unruh-DeWitt detectors

In this chapter we study the information dynamics of two Unruh-DeWitt detectors. To do so, we analyze the quantum channel created by the two detectors. We first show that, in this channel, information propagation is bounded by the speed of light. In fact, it turns out that causality in the channel is a direct consequence of microcausality in a free quantum field theory. Then, we analyze the channel using the tools of Sec. 1.2.1, we present a perturbative expansion of the channel with Feynman-like diagrams and we numerically evaluate the classical channel capacity as a function of time.

### 2.1 The causality problem

We start by looking at a causality problem that often arises when quantum fields are interacting with non-relativistic quantum systems. We then show that a quantum information inspired approach can naturally solve the problem.

#### 2.1.1 Review of the problem

Let us first present the interacting quantum theory that we study. We consider two point-like Unruh-DeWitt detectors (see Sec. 1.1.3). Let us denote the overall Hilbert space by  $\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$ , where the first two Hilbert spaces belong respectively to the detector of Alice, detector 1, and Bob's detector, detector 2, and where the third Hilbert space is that of the field. We assume that both detectors have the same proper time  $\tau_1 = \tau_2 = \tau$ . We hypothesize that the main results of this section would remain true if

$\tau_1 \neq \tau_2$  but a proper analysis of this problem should be of interest. This allows us to write the interaction Hamiltonian with respect to the proper time  $\tau$  as:

$$H_{int} = \sum_{j=1}^2 \alpha_j \eta_j(\tau) m^{(j)} \phi(\vec{x}_j(\tau)) \quad (2.1)$$

For simplicity, in this chapter we always assume that the two detectors are at rest and separated by a constant distance  $L = |\vec{L}|$  (where  $\vec{L} := \vec{x}_2 - \vec{x}_1$ ). In addition, we also assume that the detectors are in Minkowski spacetime, so their proper time and the coordinate time coincide  $\tau = t$ .

The so-called Fermi problem, which was first considered by Fermi [45], arises in any system that is analogous to two atoms communicating via the electromagnetic field, and it has been studied extensively, see e.g. [17, 46, 47, 18]. Consider, in the vacuum, the probability,  $P_{Fermi}$ , that a photon is emitted by atom 1 followed by the absorption of a photon by atom 2. In our model, it is the probability if starting with the state  $|e^{(1)}, g^{(2)}, 0\rangle$  to end in the state  $|g^{(1)}, e^{(2)}, 0\rangle$ . Using the perturbative expansion of the evolution operator in the interaction picture  $U(t_f, t_i) = T e^{-i \int_{t_i}^{t_f} dt H_{int}(t)}$ , one obtains the transition probability

$$\begin{aligned} P_{Fermi} &= |\langle e^{(1)}, g^{(2)}, 0 | U(t_f, t_i) | g^{(1)}, e^{(2)}, 0 \rangle|^2 \\ &= \left| \alpha_1 \alpha_2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \eta(t_1) \eta(t_2) e^{i\Delta E(t_2 - t_1)} G(x_1(t_1), x_2(t_2)) \right|^2 + O(\alpha^6). \end{aligned} \quad (2.2)$$

By choosing the separation between the two detectors  $L$  and a time interval  $t_f - t_i$  in which both detectors are on, we can choose the spacetime windows for emission and absorption to be timelike or spacelike (or mixed) relative to another. The Fermi problem is the fact that this probability, from Eq. (2.2), is non-vanishing even in the case of spacelike separation. Technically, this is due to the non-vanishing tail of the Feynman propagator outside the light cone. Hegerfeldt and Feynman showed that in fact no Feynman propagator can identically vanish outside the light cone, [48, 49].

Just like for other so-called superluminal effects, the Fermi problem caused a lot of confusion in the literature. Some authors came to the conclusion that causality is not respected [46] while most authors concluded that causality is not broken [47, 18, 17]. Nevertheless, the solution to the problem remained somewhat unclear because all these authors used different models with different physical assumptions and different mathematical approximations, all that just to lowest order in perturbation theory. This reinforces the need to clarify the reason for the non-vanishing of the Fermi probability in the spacelike separated case to all orders in perturbation theory. As was recently pointed out in [24], the key to resolving the puzzle is to take into account that measurements on the detectors are local measurements. Namely, Bob performs a measurement only of his detector 2; he

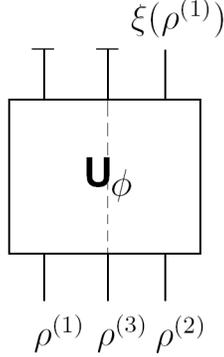


Figure 2.1: A quantum channel modeled by 2 detectors interacting with a scalar field.

does not measure Alice's detector, nor does he measure the field. This means that Fermi's probability is the probability for just one of several processes that Bob cannot distinguish. What should actually vanish for spacelike separations is the sum of the probabilities for all processes that depend on the state of Alice.

### 2.1.2 The quantum channel

Here, our first aim is to make the argument of [24] explicit within the information-theoretic framework of quantum channels. To this end, we notice that Bob's ignorance of Alice' and the field's state at the late time  $t_f$  means that at  $t_f$  both the state of Alice's detector and the state of the field are to be traced over. These traces perform the sum over the probabilities for processes that Bob cannot distinguish. We therefore naturally arrive at the description of a quantum channel  $\xi : \rho^{(1)} \rightarrow \xi(\rho^{(1)}) = \rho^{(2)'}$ , see Fig. (2.1). Here, the input is the initial density matrix  $\rho^{(1)}$  of Alice at  $t_i$  and the output of the channel is Bob's density matrix  $\rho^{(2)'}$  at  $t_f$ . We assume that the system starts in the state  $\rho(t_i) = \rho^{(1)}\rho^{(2)}\rho^{(3)}$ , where the initial state of Alice's detector,  $\rho^{(1)}$ , is arbitrary, the initial state of the Bob's detector,  $\rho^{(2)}$ , is the ground state and the initial state of the field,  $\rho^{(3)}$ , is the vacuum. The full density matrix evolves according to  $\rho(t_f) = U(t_f, t_i)\rho(t_i)U^\dagger(t_f, t_i)$ . As always, the time evolution can be formulated in terms of an infinite series of commutators [50]:

$$\rho(t_f) = \rho^{(1)}\rho^{(2)}\rho^{(3)} + \sum_{j=1}^{\infty} \left( (i)^j \int_{t_i}^{t_f} dt_1 \dots \int_{t_i}^{t_{j-1}} dt_j [\dots [\rho^{(1)}\rho^{(2)}\rho^{(3)}, H_{int}(t_n)], \dots], H_{int}(t_1) \right). \quad (2.3)$$

Then, the trace over detector 1 and the field gives the final state  $\rho^{(2)}(t_f) = \xi(\rho^{(1)})$  of Bob's detector:

$$\xi(\rho^{(1)}) = \rho^{(2)} + \sum_{j=1}^{\infty} \left( (i)^j \int_{t_i}^{t_f} dt_1 \dots \int_{t_i}^{t_{j-1}} dt_j Tr_{(1,3)} [[\dots [\rho^{(1)} \rho^{(2)} \rho^{(3)}, H_{int}(t_n)], \dots], H_{int}(t_1)] \right). \quad (2.4)$$

To prove causality from this starting point, we will use the following simple lemmas:

**I)** Traces are cyclic and  $Tr([A, B]) = 0$ .

$$\begin{aligned} \text{II)} \quad [A^{(1)} B^{(2)} C^{(3)}, D^{(1)} E^{(2)} I^{(3)}] = \\ \left\{ [A^{(1)}, D^{(1)}] (B^{(2)} E^{(2)}) \right. \\ \left. + (D^{(1)} A^{(1)}) [B^{(2)}, E^{(2)}] \right\} C^{(3)}. \end{aligned}$$

**III)**  $\exists \{R_k^{(1)}, S_k^{(2)}, T_k^{(3)}\}$  such that  $[[\dots [A^{(1)} B^{(2)} C^{(3)}, D^{(1)} E^{(2)}], \dots], F^{(1)} G^{(3)}] = \sum_k R_k^{(1)} S_k^{(2)} T_k^{(3)}$ .

Now in Eq. (2.4), the terms that have a dependence on the input  $\rho^{(1)}$  must have at least one  $m^{(1)}\phi(x_1)$  which multiplies  $\rho^{(1)}$  since otherwise we simply have  $Tr(\rho^{(1)}) = 1$ . In addition, since the trace of commutators vanishes (I), the non-vanishing terms which have a dependence on  $\rho^{(1)}$  need to be interacting with at least one  $m^{(2)}\phi(x_2)$ , such that all the terms dependent on  $\rho^{(1)}$  will be of the form

$$f_n(\rho^{(1)}) = Tr_{(1,3)} \left( [[\dots [\rho^{(1)} \rho^{(2)} \rho^{(3)}, m^{(j)}\phi(x_j)], \dots], m^{(r)}\phi(x_r)] \right) \quad (2.5)$$

where at least one of the indices  $\{j \dots r\}$  is equal to 1 and at least one of the indices is equal to 2, and  $n$  is the number of commutators ( $n \geq 2$ ). Note that the time dependence is implicit in this formulation, each  $\phi(x)$  is integrated over time such that the time difference between two  $\phi(x)$  is at most  $t_f - t_i$ . If the last index in Eq. (2.5) is 1, using (III) for everything before the last commutator, and (II) to expand the last commutator,  $f_n(\rho^{(1)})$  would simplify to:

$$\begin{aligned} f_n(\rho^{(1)}) &= \sum_k Tr_{(1,3)} \left( [R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)}\phi(x_1)] \right) \\ &= \sum_k S_k^{(2)} \left\{ Tr_{(3)} \left( T_k^{(3)} \phi(x_1) \right) Tr_{(1)} \left( [R_k^{(1)}, m^{(1)}] \right) \right. \\ &\quad \left. + Tr_{(3)} \left( [T_k^{(3)}, \phi(x_1)] \right) Tr_{(1)} \left( m^{(1)} R_k^{(1)} \right) \right\} \\ &= 0. \end{aligned} \quad (2.6)$$

Thus the non-vanishing contributions of  $f_n(\rho^{(1)})$  must come from commutators for which the very last index is 2. Now, let us consider the rightmost occurrence of index 1 and let us apply (III) to the commutators to the left of it:

$$f_n(\rho^{(1)}) = \sum_k Tr_{(1,3)} \left( [[\dots[[R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)} \phi(x_1)], m^{(2)} \phi(x_2)] \dots], m^{(2)} \phi(x_2)] \right). \quad (2.7)$$

We can expand the most inner commutators with (II) to obtain:

$$[R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)} \phi(x_1)] = [R_k^{(1)}, m^{(1)}] \left( S_k^{(2)} T_k^{(3)} \phi(x_1) \right) + m^{(1)} R_k^{(1)} \left( S_k^{(2)} [T_k^{(3)}, \phi(x_1)] \right). \quad (2.8)$$

Notice that when the first term is back in Eq. (2.7) it forms an expression of the form

$$\sum_k Tr_{(1,3)} \left( [R_k^{(1)}, m^{(1)}] [[\dots[S_k^{(2)} T_k^{(3)} \phi(x_1), m^{(2)} \phi(x_2)] \dots], m^{(2)} \phi(x_2)] \right) \quad (2.9)$$

which implies that after the tracing out of detector 1 this term is always absent. Notice also that when the second term is back in Eq. (2.7), it gives an expression of the form:

$$f_n(\rho^{(1)}) = \sum_k Tr_{(1,3)} \left( m^{(1)} R_k^{(1)} [[\dots[S_k^{(2)} [T_k^{(3)}, \phi(x_1)], m^{(2)} \phi(x_2)] \dots], m^{(2)} \phi(x_2)] \right). \quad (2.10)$$

Therefore, the term  $[T_k^{(3)}, \phi(x_1)]$  will be multiplied on each side by some powers of  $\phi(x_2)$ , so there exists a set of operators  $V_{k,i,j}^{(2)}$  such that:

$$f_n(\rho^{(1)}) = \sum_{k,i,j} \left\{ V_{k,i,j}^{(2)} Tr_{(1)} \left( m^{(1)} R_k^{(1)} \right) Tr_{(3)} \left( \phi^i(x_2) [T_k^{(3)}, \phi(x_1)] \phi^j(x_2) \right) \right\}. \quad (2.11)$$

Using cyclicity of the trace (I), this expression can be simplified to:

$$f_n(\rho^{(1)}) = \sum_{k,i,j} \left\{ V_{k,i,j}^{(2)} Tr_{(1)} \left( m^{(1)} R_k^{(1)} \right) Tr_{(3)} \left( T_k^{(3)} [\phi(x_1), \phi^{i+j}(x_2)] \right) \right\}. \quad (2.12)$$

Note that all the information about  $\rho^{(1)}$  is contained in the operators  $R_k^{(1)}$ . Causality in the channel therefore follows directly from microcausality in quantum field theory, see Sec. 1.1.2. If the two detectors are spacelike separated during the entire interaction,  $\rho^{(2)}(t_f)$  does not depend on the state  $\rho^{(1)}$ , i.e., Bob's detector 2 is not sensitive to the state in which Alice prepared detector 1.

This is in many ways reminiscent of the EPR experiment. In the EPR thought experiment, the two spacelike separated spins  $A$  and  $B$  have quantum correlations that are stronger than any classical correlations because they are in the maximally entangled state

$$|\psi^{A,B}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow^{(A)} \uparrow^{(B)}\rangle + |\downarrow^{(A)} \downarrow^{(B)}\rangle \right). \quad (2.13)$$



Thus, a measurement performed on system  $A$  will affect the outcome of a measurement on system  $B$ . But, here again causality is not violated because the outcomes' probabilities of the measurement made on system  $B$  are unchanged by the measurement performed on system  $A$ . Indeed, if the two systems are spacelike separated,  $B$  cannot know the outcome of the measurement made on system  $A$ . Therefore, he has only access to his state  $Tr_{(A)}(|\psi^{A,B}\rangle\langle\psi^{A,B}|) = 1/2(|\uparrow^{(B)}\rangle\langle\uparrow^{(B)}| + |\downarrow^{(B)}\rangle\langle\downarrow^{(B)}|)$  which does not contain any information about the outcome of the measurement performed on  $A$ .

### 2.1.3 Different detector models

A quantum channel modeled by an atom interacting with a photon has recently been analyzed in [51]. The model uses an atom-photon interaction given by the Jaynes-Cumming interaction Hamiltonian  $H_{int} = \alpha(|g^{(d)}\rangle\langle e^{(d)}| \otimes a_k^\dagger + |e^{(d)}\rangle\langle g^{(d)}| \otimes a_k)$  where  $a_k$  and  $a_k^\dagger$  are the annihilation and creation operator for a single mode  $k$ . This interaction Hamiltonian has a natural quantum field generalization, the Glauber scalar detector [52], which can be used to model two detectors interacting with a quantum scalar field

$$H_{int} = \sum_{j=1}^2 \alpha_j \eta(t) \left( |g^{(j)}\rangle\langle e^{(j)}| \phi^-(\vec{x}_j) + |e^{(j)}\rangle\langle g^{(j)}| \phi^+(\vec{x}_j) \right) \quad (2.14)$$

where  $\phi^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} e^{ipx} a_{\vec{p}}$  and  $\phi^-(x) = \phi^{+\dagger}(x)$  are respectively the positive and negative frequency part of the field. While this detector is not sensitive to the quantum fluctuations of the field, i.e., in our notation,  $P_e = 0$ , this detector model allows non-local effects, see [53]. We can confirm the non-locality by using in our channel Glauber detectors instead of Unruh-DeWitt detectors. To this end, we use Eq. (2.14) in Eq. (2.4). We see that then terms that are dependent on  $\rho^{(1)}$  are no longer necessarily proportional to  $[\phi(x_1), \phi(x_2)]$ . Using the perturbative expansion of the channel in Eq. (2.4) shows that non-causal terms appear already in the  $O(\alpha^2)$  order:

$$\begin{aligned} \xi(\rho^{(1)}) &= |g^{(2)}\rangle\langle g^{(2)}| - \alpha_1 \alpha_2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \left[ \eta(t_1) \eta(t_2) \times e^{i\Delta E(t_1-t_2)} |e^{(2)}\rangle\langle g^{(2)}| \right. \\ &\quad \left. \times \langle e^{(1)} | \rho^{(1)} | g^{(1)} \rangle D(x_2(t_1), x_1(t_2)) + c.c. \right] + O(\alpha^4). \end{aligned} \quad (2.15)$$

Since the correlator  $D(x, y)$  is not vanishing outside the light cone, detector 2 would indeed be influenced by detector 1 as soon as the interaction is turned on even if the detectors are spacelike separated. It may be interesting to see how similar effectively non-local detectors, such as the one in [54], behaves under our channel picture.

Note that if instead of using a scalar field  $\phi(x)$  we use a fermionic field like a Dirac field  $\psi(x)$ , then the proof of causality does not change even if fermionic fields anti-commute for

spacelike separations. To see this, note that the detector monopole  $m$  must be coupled to a Lorentz scalar. An obvious reason for this requirement is that we want the excitation probability  $P_e$  of a point-like detector to be invariant under a boost. For a fermionic field, we need an even number of instance of the field to have a Lorentz scalar, for example  $\bar{\psi}\psi(x)$  for a Dirac field [4] (where  $\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$  and  $\gamma^0$  is essentially a four dimensional version of the  $\sigma_x$  Pauli matrix), and this combination acts as a boson so it commutes for spacelike separations.

## 2.2 Analysis of the quantum channel

In this section we make an analysis of our relativistic quantum channel modeled by the interaction Hamiltonian (2.1) using the tools of Sec. 1.2.1. We start by obtaining its operator-sum representation, then we discuss its various channel capacities and we end with a perturbative expansion of the channel.

### 2.2.1 Noise structure

Let us start by calculating the precise quantum channel for both timelike and spacelike separations with the tools introduced in Sec. 1.2.1. Since the evolution of the full system is unitary, our channel is necessarily described by a CPTP map. Then, as we will show, assuming detector 2 starts in the ground state,  $\rho^{(2)} = |g^{(2)}\rangle\langle g^{(2)}|$ , we can write the channel map in the following way, in the basis  $|e^{(2)}\rangle, |g^{(2)}\rangle$ ,

$$\xi \left( \begin{pmatrix} \theta & \gamma \\ \gamma^* & \beta \end{pmatrix} \right) = \begin{pmatrix} P_e & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} P_e & 0 \\ 0 & -P_e \end{pmatrix} + \theta \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix} + \beta \begin{pmatrix} B & 0 \\ 0 & -B \end{pmatrix} + \gamma \begin{pmatrix} 0 & C \\ D & 0 \end{pmatrix} + \gamma^* \begin{pmatrix} 0 & D^* \\ C^* & 0 \end{pmatrix} \quad (2.16)$$

where we use  $\theta + \beta = 1$ . All terms are spacetime scalars. Note that  $A, B, C$  and  $D$  are causal terms in the sense that they depend on the input density matrix  $\rho^{(1)}$ . In contrast,  $P_e$  represents noise in the quantum channel since its presence does not depend on the input  $\rho^{(1)}$ . To prove Eq. (2.16), we will use the following properties which are easy to verify ( $k \in \mathbb{Z}$ ):

- i)  $Tr(\rho^{(3)}\phi^{2k+1}) = 0$ .
- ii)  $m^{2k+1}$  has no diagonal elements,  
and therefore  $Tr(m^{2k+1}M_d) = 0$  where  $M_d$  is any diagonal matrix.
- iii)  $m^{2k}$  has only diagonal elements,  
and therefore  $Tr(m^{2k}M_{nd}) = 0$  where  $M_{nd}$  is any matrix with no diagonal elements.

In a series expansion of the non-causal terms, each order has the form  $\rho^{(2)}m^{(2)k}Tr(\rho^{(3)}\phi(x_2)^k)$ . Thus, because of (i) the non-vanishing terms will be proportional to  $\rho^{(2)}m^{(2)2k}$ , and because of (iii) we know that these are diagonal. Therefore, because we have trace preservation and because detector 2 starts initially in the ground state, there cannot be a more general expression for the non-causal terms of Eq. (2.16). For the causal terms, each order in a series expansion have the form  $\rho^{(2)}m^{(2)k}Tr(m^{(1)j}\rho^{(1)})Tr(\rho^{(3)}\phi(x_1)^j\phi(x_2)^k)$ . Now consider the case where the input density matrix  $\rho^{(1)}$  is diagonal, then because of (ii) the non-vanishing terms will have  $j$  even. Using (i), this also means we need  $k$  to be even, hence  $\rho^{(2)}m^{(2)k}$  is diagonal following (iii). A similar argument can show that an input density matrix with no diagonal elements cannot have diagonal elements at the output. Finally, trace preservation, hermiticity and linearity of the channel are sufficient properties to prove the validity of Eq. (2.16).

From this analysis, we can find an operator-sum representation by imposing  $\xi\left(\begin{pmatrix} \alpha & \gamma \\ \gamma^* & \beta \end{pmatrix}\right) = \sum_{k=1}^4 E_k \begin{pmatrix} \alpha & \gamma \\ \gamma^* & \beta \end{pmatrix} E_k^\dagger$  and  $\sum_{k=1}^4 E_k^\dagger E_k = I$  where we use  $E_k = \begin{pmatrix} a_{1k} & a_{2k} \\ a_{3k} & a_{4k} \end{pmatrix}$ . Solving this non-linear system of equations is relatively straightforward as we have more unknowns than equations, so for simplicity we try to have as many zero matrix elements as possible. We arrive at a simple representation, in the basis  $|e^{(2)}\rangle\langle e^{(1)}|, |e^{(2)}\rangle\langle g^{(1)}|, |g^{(2)}\rangle\langle e^{(1)}|, |g^{(2)}\rangle\langle g^{(1)}|$ :

$$\begin{aligned} E_1 &= \begin{pmatrix} \frac{C}{\sqrt{1-P_e-B}} & 0 \\ 0 & \sqrt{1-P_e-B} \end{pmatrix} \\ E_2 &= \begin{pmatrix} \sqrt{P_e+A-\frac{|C|^2}{1-P_e-B}} & 0 \\ 0 & 0 \end{pmatrix} \\ E_3 &= \begin{pmatrix} 0 & \frac{D^*}{\sqrt{1-P_e-A}} \\ \sqrt{1-P_e-A} & 0 \end{pmatrix} \\ E_4 &= \begin{pmatrix} 0 & \sqrt{P_e+B-\frac{|D|^2}{1-P_e-A}} \\ 0 & 0 \end{pmatrix}. \end{aligned} \quad (2.17)$$

There exists no representation with a smaller number of Kraus operator since we verified that the rank of the matrix  $(I^{(Q)} \otimes \xi^{(1)})|\beta^{(Q,1)}\rangle\langle\beta^{(Q,1)}|$ , where  $|\beta^{(Q,1)}\rangle$  is the maximally entangled state  $|\beta^{(Q,1)}\rangle = \frac{1}{\sqrt{2}}(|e^{(Q)}, e^{(1)}\rangle + |g^{(Q)}, g^{(1)}\rangle)$ , is equal to 4.

## 2.2.2 Channel capacities

In this section we apply the ideas of Sec. 1.2.1 to our quantum channel model. We start by computing the classical capacity with Eq. (1.34). After a tedious calculation using Maple, we find that it is optimal to send the input states  $\rho_1^{(1)} = |e^{(1)}\rangle\langle e^{(1)}|$  and  $\rho_2^{(1)} = |g^{(1)}\rangle\langle g^{(1)}|$

with probability  $p_1$  and  $p_2 = 1 - p_1$ ,

$$\begin{aligned} p_1 &= \frac{2^w - P_e - B}{A - B} \\ w - \ln(1 - 2^w) &= \frac{H(P_e + B) - H(P_e + A)}{A - B} \end{aligned} \quad (2.18)$$

where we used the binary entropy  $H(p) := -p \ln p - (1 - p) \ln(1 - p)$ . We finally arrive at the classical channel capacity  $C$ , which we divide by  $t_f - t_i$  to get  $R$ , namely the amount of bits/time which can be sent reliably:

$$R = \frac{1}{t_f - t_i} \left[ H(P_e + p_1 A + (1 - p_1) B) - p_1 H(P_e + A) - (1 - p_1) H(P_e + B) \right]. \quad (2.19)$$

As expected the classical channel capacity is zero for spacelike interactions since in that case  $A = B = 0$ .

We remark that the channel capacity as a function of the spacetime separation is a non-analytic function since it identically vanishes outside the light cone but is a nontrivial function inside. Any analytic function that vanishes on a finite interval would of course vanish everywhere. The occurrence of this non-analyticity may seem surprising since our quantum channel is mapping in between finite dimensional spaces and therefore appears to be a matter of mere linear algebra. The non-analyticity arises, of course, from the non-analyticity of the commutator  $[\phi(x), \phi(y)]$  which originates in the fact that, in the full system, the field lives in an infinite dimensional Hilbert space. Conversely, if ultraviolet and infrared cut-offs are imposed on the quantum field theory so that its Hilbert space  $\mathcal{H}^{(3)}$  becomes finite dimensional, this would reduce these calculations to linear algebra and will therefore yield some non-vanishing capacity outside the light cone. Interestingly, this does not mean that the presence of a natural UV cut-off in nature would imply a violation of causality. This is because an ultraviolet cut-off implies that there is in effect a smallest resolvable length, which in turn means that the very boundaries of the light cone become unsharp. The capacity should decay to essentially zero outside the light cone at a distance from the light cone that is about the size of the unsharpness scale induced by the UV cut-off. Any candidate quantum gravity theory has to reduce to quantum field theory in a limit and most come with a natural UV cut-off, see e.g., [55]. It should be interesting to check causality for such theories by calculating the channel capacity at distances close to the light cone.

Let us now consider the quantum channel capacity, Eq. (1.35). For spacelike separated detectors, the quantum channel capacity is zero since the channel is then anti-degradable. Indeed, for spacelike separated detectors the output of the channel does not depend on the input:  $\xi(\rho^{(1)}) = F(\rho^{(2)}, \phi(x_2))^{(2)}$ . Thus, we can easily build a channel  $\Gamma$  that maps the output of the complementary channel  $\xi^C$  to the known function

$F(\rho^{(2)}, \phi(x_2))^{(2)}$ . Note that this also means that we cannot transfer entanglement faster than the speed of light. Indeed, if the input is entangled with some auxiliary system, say  $\rho^{(1,aux)} = \sum_{i,j,r,s} c_{i,j,r,s} |i^{(1)}, j^{(aux)}\rangle \langle r^{(1)}, s^{(aux)}|$ , then for spacelike separated detectors the output is not entangled with the auxiliary system:

$$\begin{aligned} I^{(aux)} \otimes \xi^{(1)}(\rho^{(1,aux)}) &= \sum_{i,j,r,s} c_{i,j,r,s} \xi(|i^{(1)}\rangle \langle r^{(1)}|) \otimes |j^{(aux)}\rangle \langle s^{(aux)}| \\ &= (F(\rho^{(2)}, \phi(x_2))^{(2)}) \otimes \left( \sum_{i,j,r,s} c_{i,j,r,s} |j^{(aux)}\rangle \langle s^{(aux)}| \right). \end{aligned} \quad (2.20)$$

This confirms that superluminal propagation of classical or quantum information is not possible. For timelike separated detectors, the quantum channel capacity is extremely hard to compute because the channel is not degradable. Indeed, a theorem in [31] states that any channel with input and output of dimension 2 and with Choi rank (minimum number of Kraus operators) bigger than 2 cannot be degradable. Since the channel we consider has Choi rank equal to 4, it cannot be degradable. We therefore leave open the question of finding an explicit expression for the quantum channel capacity of our quantum channel.

### 2.2.3 Perturbative expansion

Using time-dependent perturbation theory, we can find explicit expressions for the terms  $P_e, A, B, C$  and  $D$  in the weak coupling regime ( $\alpha_j \ll 1$ ). To this end we use the first orders of the perturbative expansion of equation (2.4) along with equation (2.16), and for simplicity we assume that the field starts in the vacuum  $|0\rangle$ :

$$\begin{aligned} P_e(\Delta E) &= \alpha_2^2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \eta(t_1) \eta(t_2) \langle 0 | \phi(x_2(t_1)) \phi(x_2(t_2)) | 0 \rangle e^{-i\Delta E(t_1-t_2)} + O(\alpha^4) \quad (2.21) \\ A(\Delta E) &= 2(\alpha_1 \alpha_2)^2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \int_{t_i}^{t_2} dt_3 \int_{t_i}^{t_3} dt_4 \eta(t_1) \eta(t_2) \eta(t_3) \eta(t_4) \left\{ \cos(\Delta E(t_1-t_2)) \right. \\ &\quad \times [\phi(x_2(t_1)), \phi(x_1(t_3))] \left[ e^{-i\Delta E(t_3-t_4)} \langle 0 | \phi(x_1(t_4)) \phi(x_2(t_2)) | 0 \rangle \right. \\ &\quad \left. \left. - e^{i\Delta E(t_3-t_4)} \langle 0 | \phi(x_2(t_2)) \phi(x_1(t_4)) | 0 \rangle \right] \right. \\ &\quad \left. + (t_1 \leftrightarrow t_2) + (t_2 \leftrightarrow t_3) + i \sin(\Delta E(t_2-t_3)) [\phi(x_1(t_2)), \phi(x_2(t_1))] \right. \\ &\quad \left. \times \left[ e^{-i\Delta E(t_1-t_4)} \langle 0 | \phi(x_1(t_3)) \phi(x_2(t_4)) | 0 \rangle + e^{i\Delta E(t_1-t_4)} \langle 0 | \phi(x_2(t_4)) \phi(x_1(t_3)) | 0 \rangle \right] \right\} \\ &\quad + O(\alpha^6) \quad (2.22) \end{aligned}$$

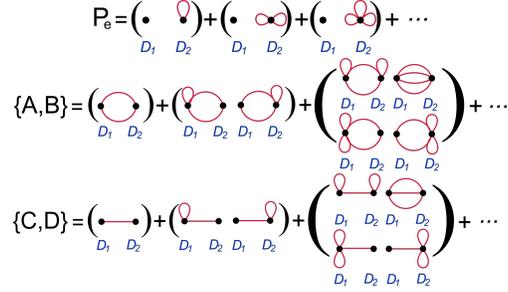


FIG. 2.2: Feynman diagrams involved in the quantum channel.  $D_j$  stands for detector  $j$ ,  $j \in \{1, 2\}$ .

$$\begin{aligned}
B(\Delta E) &= A(-\Delta E) + 4(\alpha_1\alpha_2)^2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \int_{t_i}^{t_2} dt_3 \int_{t_i}^{t_3} dt_4 \left\{ \eta(t_1)\eta(t_2)\eta(t_3)\eta(t_4) \right. \\
&\quad \times \sin(\Delta E(t_2 - t_3)) \sin(\Delta E(t_1 - t_4)) [\phi(x_1(t_2)), \phi(x_2(t_1))] \\
&\quad \left. \times [\phi(x_2(t_4)), \phi(x_1(t_3))] \right\} + O(\alpha^6)
\end{aligned} \tag{2.23}$$

$$\begin{aligned}
C(\Delta E) &= \alpha_1\alpha_2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \eta(t_1)\eta(t_2) e^{i\Delta E(t_2-t_1)} [\phi(x_1(t_2)), \phi(x_2(t_1))] \\
&\quad + O(\alpha^4)
\end{aligned} \tag{2.24}$$

$$\begin{aligned}
D(\Delta E) &= -\alpha_1\alpha_2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \eta(t_1)\eta(t_2) e^{i\Delta E(t_2+t_1)} [\phi(x_1(t_2)), \phi(x_2(t_1))] \\
&\quad + O(\alpha^4).
\end{aligned} \tag{2.25}$$

We can picture the perturbative expansion with Feynman diagrams [4], see Fig. (2.2) (the expressions of Eq. (2.21)-(2.25) are represented by the first diagram of their respective series). A connection between the two detectors represents a photon emission/absorption process and a connection between a detector and itself (a loop) represents a quantum field fluctuation. The terms  $\{A, B\}$  have an even number of connections between the detectors while the terms  $\{C, D\}$  have an odd number of connections. The only distinction between  $A$  and  $B$  is the input state at detector 1: the excited state for  $A$  and the ground state for  $B$ . Thus, the causal connections of  $A$  are resonant while the causal connections of  $B$  are not resonant. A similar argument is also true for  $C$  and  $D$ , the connections of  $C$  are resonant while the connections of  $D$  are not resonant.

Using Eq. (2.21)-(2.23) along with Eq. (2.19), we can numerically evaluate the classical channel capacity as a function of time for inertial detectors in Minkowski spacetime, for example, for a massless field, see Fig. (2.3). The arrow points to the threshold when the spacetime windows in which the detectors are switched on start to become partially timelike.

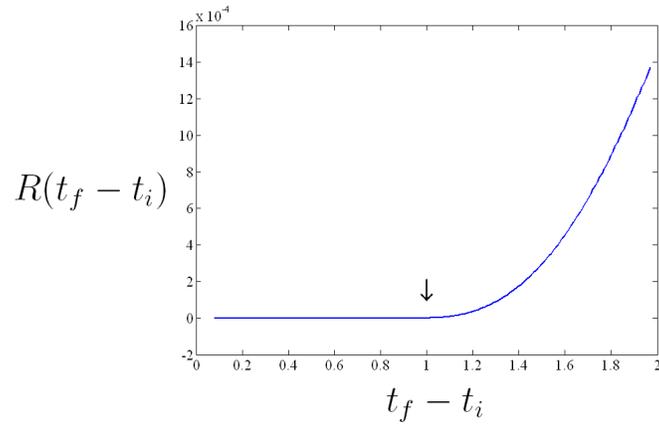


FIG. 2.3: Classical channel capacity as a function of time ( $t_f - t_i$ ) with  $L = 1$  and  $\Delta E = 1$ . The arrow points to the light cone  $t_f - t_i = |\vec{x}_1 - \vec{x}_2|$ .

# Chapter 3

## Entanglement generation between two Unruh-DeWitt detectors

In this chapter we study the entanglement dynamics of two Unruh-DeWitt detectors. We focus on only one aspect of entanglement dynamics, namely entanglement generation. Indeed, the two detectors are always assumed to be initially non-entangled, but because of their interaction with the field, entanglement can be generated. We present two approaches to this problem, a time-dependent approach and a time-independent approach. The two approaches are different mostly because they use different tools, namely time-dependent perturbation theory and time-independent perturbation theory. We discuss in Sec. 3.2.1 how to interpret the results of the time-dependent approach with the results of the time-independent approach.

### 3.1 Time-dependent approach

It has been shown using time-dependent perturbation theory in [22, 23] that detectors coupled to a massless quantum field can become entangled even when spacelike separated. The entanglement was found to appear to propagate in quantum fields at a speed which depends on the switching functions  $\eta(\tau)$  and on the energy gap  $\Delta E$ . The speed of propagation was found to be larger than the speed of light for suitable  $\eta(\tau)$  and  $\Delta E$ . In this section we briefly review this time-dependent approach to the study of entanglement generation between two Unruh-DeWitt detectors. We then extend this analysis to the case where the detectors are near a weak gravitational field.



### 3.1.1 Review of the approach

To study entanglement generation between two Unruh-DeWitt detectors, we keep the same model as in the previous chapter. This means that we use the interaction Hamiltonian of Eq. (2.1), but here we do not assume from the start that  $\tau = t$ . Moreover, we do not trace over the final state of detector 1, we just trace over the final state of the field. We therefore have a setup of entanglement swapping, just like the one illustrated on Fig. (1.2). The detectors play the role of system  $A_1$  and  $A_2$ , while the field plays the role of system  $B$ . As we previously discussed, this setup may allow creation of entanglement outside the light cone especially if the initial quantum state of system  $B$  is naturally entangled. Thus, since we have evidence from algebraic quantum field theory [56] that the vacuum state  $|0\rangle$  is an entangled state in position space, we may be able to swap the entanglement from the vacuum to the detectors, even at spacelike separations.

For simplicity, we assume that the field is massless and that the initial state of the system is the ground state of the free theory, namely  $|g^{(1)}, g^{(2)}, 0\rangle$ . After the unitary evolution in the interaction picture, which we only carry perturbatively, we trace out the field such that the final state of the detectors can be shown to be at  $O(\alpha^2)$  [22, 57]

$$\begin{aligned} \rho_f^{(1,2)} &= \text{Tr}_{(3)} \left( T e^{-i \int d\tau' H_{int}(\tau')} |g^{(1)}, g^{(2)}, 0\rangle \langle g^{(1)}, g^{(2)}, 0| T^\dagger e^{i \int d\tau H_{int}(\tau)} \right) \\ &= \begin{pmatrix} 0 & 0 & 0 & X \\ 0 & P_{e1} & Y & 0 \\ 0 & Y^* & P_{e2} & 0 \\ X^* & 0 & 0 & 1 - P_{e1} - P_{e2} \end{pmatrix} + O(\alpha^4) \end{aligned} \quad (3.1)$$

in the basis  $|e^{(1)}, e^{(2)}\rangle, |e^{(1)}, g^{(2)}\rangle, |g^{(1)}, e^{(2)}\rangle, |g^{(1)}, g^{(2)}\rangle$ , where we have:

$$P_{e_j} = \alpha_j^2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \eta(\tau) \eta(\tau') e^{-i\Delta E(\tau - \tau')} D(x_j(\tau), x_j(\tau')) \quad (3.2)$$

$$X = -\alpha_1 \alpha_2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' \eta(\tau) \eta(\tau') e^{i\Delta E(\tau + \tau')} (D(x_1(\tau), x_2(\tau')) + D(x_2(\tau), x_1(\tau'))) \quad (3.3)$$

$$Y = \alpha_1 \alpha_2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \eta(\tau) \eta(\tau') e^{-i\Delta E(\tau - \tau')} D(x_1(\tau), x_2(\tau')). \quad (3.4)$$

A simple calculation can show that the negativity of this final state is:

$$N = \max \left( \sqrt{(P_{e1} - P_{e2})^2 + 4|X|^2} - P_{e1} - P_{e2}, 0 \right) + O(\alpha^4). \quad (3.5)$$

Let us first consider the special case where the detectors are separated by a constant distance  $L$  in Minkowski spacetime such that  $x_j^0(\tau) = \tau = t$ . We also assume that  $\alpha_1 = \alpha_2 = \alpha$  such that in Minkowski spacetime  $P_{e1} = P_{e2} = P_e$  and  $N = 2 \max(|X| - P_e) + O(\alpha^4)$ . One usually interprets  $X$  as an exchange of a virtual quanta between the two detectors.

Recall that  $P_{ej}$  is the detection of a local quantum field fluctuation by detector  $j$ . We thus see that the negativity is a fight between the exchange term and the local noise term. Since we use point-like detectors, we need a continuous switching function. The authors of [22, 23] used superoscillating functions, but we found that the same results can be achieved with a Gaussian switching function. Thus for simplicity we choose the switching function to be Gaussian  $\eta(\tau) = e^{-\tau^2/(2\sigma^2)}$ . Of course, it may seem like a very bad idea to use a Gaussian window function if we want to study the speed of entanglement propagation because it has non-vanishing tails. Nevertheless, this window function effectively models a detector being on for an amount of time equal to  $\sigma$ . In fact, none of the results change if we put a smooth cut-off to the Gaussian [57]. For reasons that will become clear in the next subsection, it is often useful to make both  $P_{ej}$  and  $X$  functions of the propagator. To do this, we just have to make sure that the time ordering is respected. For  $X$  this is straightforward since the time integration respects time ordering by construction. Nevertheless we can still simplify  $X$  by using the variable change  $s = \tau - \tau'$  and  $u = \tau + \tau'$  such that we have

$$X = \frac{-\alpha^2}{2} \int_{-\infty}^{\infty} du \int_0^{\infty} ds \eta\left(\frac{s+u}{2}\right) \eta\left(\frac{u-s}{2}\right) e^{i\Delta E u} (D(\vec{x}_1, \vec{x}_2, s) + D(\vec{x}_2, \vec{x}_1, s)) \quad (3.6)$$

$$= -\alpha^2 e^{\sigma^2 \Delta E^2} \sigma \sqrt{\pi} \int_0^{\infty} ds e^{-s^2/(4\sigma^2)} (G(\vec{x}_1, \vec{x}_2, s) + G(\vec{x}_2, \vec{x}_1, s)) \quad (3.7)$$

where recall that  $G(x, y) := \langle 0|T\phi(x)\phi(y)|0\rangle = G(\vec{x}, \vec{y}, x^0 - y^0)$  is the propagator. For  $P_{ej}$ , we introduce a convenient change of variables for the double integral over the  $(\tau, \tau')$  plane [27], making  $u = \tau$ ,  $s = \tau - \tau'$  in the lower half-plane  $\tau' < \tau$  and  $u = \tau'$ ,  $s = \tau' - \tau$  in the upper half-plane  $\tau < \tau'$ ,  $P_{ej}$  becomes:

$$P_{ej} = 2\alpha^2 \Re \left( \int_{-\infty}^{\infty} du \int_0^{\infty} ds \eta(u) \eta(u-s) e^{-i\Delta E s} D(\vec{x}_j, \vec{x}_j, s) \right) \quad (3.8)$$

$$= 2\alpha^2 \sigma \sqrt{\pi} \Re \left( \int_0^{\infty} ds e^{-s^2/(4\sigma^2) - i\Delta E s} G(\vec{x}_j, \vec{x}_j, s) \right). \quad (3.9)$$

We may now use the above expressions with Eq. (1.16) to calculate the local noise and the exchange term in Minkowski spacetime. The local noise term was already computed in chapter 1, see Eq. (1.24). For the exchange term we have:

$$\begin{aligned} X &= -\alpha^2 e^{\sigma^2 \Delta E^2} \sigma \sqrt{\pi} \int_0^{\infty} ds e^{-s^2/(4\sigma^2)} (G(\vec{x}_1, \vec{x}_2, s) + G(\vec{x}_2, \vec{x}_1, s)) \\ &= \frac{2\alpha^2 \sigma \sqrt{\pi}}{4\pi^2} e^{-\Delta E^2 \sigma^2} \lim_{\epsilon \rightarrow 0^+} \left( \int_0^{\infty} ds \frac{e^{-s^2/(4\sigma^2)}}{s^2 - L^2 - i\epsilon} \right) \\ &= \frac{\alpha^2 \sigma i}{4L \sqrt{\pi}} e^{-\Delta E^2 \sigma^2 - L^2/4\sigma^2} \operatorname{erfc} \left( \frac{-iL}{2\sigma} \right). \end{aligned} \quad (3.10)$$

With Eq. (1.24) and (3.10), we now look at the regime where  $\Delta E\sigma \rightarrow \infty$  and  $L/\sigma \rightarrow \infty$ . In other words, we look at the regime where the detectors have a large spacelike separation. Using the known asymptotic expansion of the error function, we obtain in this regime:

$$P_e \approx \frac{\alpha^2 e^{-\Delta E^2 \sigma^2}}{8\pi \Delta E^2 \sigma^2} \quad (3.11)$$

$$|X| \approx \frac{\alpha^2 \sigma^2 e^{-\Delta E^2 \sigma^2}}{2\pi L^2}. \quad (3.12)$$

To get a non-vanishing negativity, we need  $|X| > P_e$ . This translates to  $\Delta E\sigma > \frac{L}{2\sigma} = \frac{v_{entan}}{2}$  where we defined the speed of entanglement propagation as  $v_{entan} := \frac{L}{\sigma}$ . This shows that with this model, entanglement can propagate faster than light, but it is still bounded by a finite speed, namely  $2\sigma\Delta E$ . Finally, to maximize the negativity in this regime, we simply require  $\frac{\partial N}{\partial \Delta E} = 0$ , which gives  $\Delta E_{opt} \approx \frac{L}{2\sigma^2} (1 + 2\sigma^2/L^2)$ . The resulting negativity is then [23]:

$$N_{opt} = \frac{4\alpha^2 \sigma^4 e^{-L^2/4\sigma^2}}{\pi L^4}. \quad (3.13)$$

This shows that with this approach, the negativity of the vacuum decays exponentially with the amount of spacelikeness  $L/\sigma$ . We will show in Sec. 3.2 that using a different approach we can achieve a polynomial decrease with the amount of spacelikeness.

### 3.1.2 Weak gravitational field

In this subsection we use the tools of the previous subsection to analyze the entanglement of the vacuum in the presence of gravity. Studies of entanglement were already conducted in an expanding spacetime [57, 58] or near a black hole [14]. Using two Unruh-DeWitt detectors, the authors of [57] found that the entanglement of the vacuum decreases significantly in an expanding spacetime because of the Gibbons-Hawking temperature [59]. This result supports the common belief that gravity always acts as a decohering agent. We follow-up on this analysis by studying entanglement generation between two Unruh-DeWitt detectors near a star or a planet. We show that contrary to intuition, the weak gravitational field can actually increase the entanglement generated in the detectors.

#### Newtonian limit

Let us now briefly review the Newtonian limit of general relativity, see e.g. [60]. In this limit we can write the metric as  $g_{\gamma\beta} = \eta_{\gamma\beta} + h_{\gamma\beta}$  where  $|h_{\gamma\beta}| \ll 1$ . Note that under a small change of coordinates  $x^\mu \rightarrow x^\mu + \xi^\mu$  the term  $h_{\gamma\beta}$  has a gauge transformation

$h_{\gamma\beta} \rightarrow h_{\gamma\beta} - \partial_\beta \xi_\gamma - \partial_\gamma \xi_\beta$ . Let us define the quantity  $\bar{h}^{\mu\nu} := h^{\mu\nu} - \eta^{\mu\nu} h^\gamma_\gamma / 2$ . To simplify the Einstein equation, we choose to work in the Lorentz gauge in which  $\bar{h}^{\mu\nu}{}_{,\nu} = 0$ . In this gauge, the linearized Einstein equation reads  $\partial_\gamma \partial^\gamma \bar{h}^{\mu\nu} = -16\pi T^{\mu\nu}$ . In the Newtonian limit the gravitational field is too weak to produce velocities near the speed of light, thus only the  $T^{00}$  component of the stress-energy tensor contributes significantly and we can make the approximation  $\partial_\gamma \partial^\gamma \approx \nabla^2$ . This means that the Einstein equation can be approximated as  $\partial_\gamma \partial^\gamma \bar{h}^{00} \approx \nabla^2 \bar{h}^{00} \approx -16\pi\rho$ . From this we conclude that the dominant component of  $\bar{h}^{\mu\nu}$  is  $\bar{h}^{00}$ , such that in terms of  $h^{\gamma\beta}$  we have  $h^{00} = h^{ii} = \bar{h}^{00}/2$ . Thus, the line element takes the form:

$$ds^2 = -(1 - \bar{h}^{00}/2) dt^2 + (1 + \bar{h}^{00}/2) (dx^2 + dy^2 + dz^2). \quad (3.14)$$

Now assume we have a compact object, say a star of dark matter that does not interact with the quantum field, and if of radius  $R_o$  and of constant density  $\rho = 3M/(4\pi R_o^3)$ . We solve  $\nabla^2 \bar{h}^{00} \approx -16\pi\rho$  with the usual boundary conditions  $\bar{h}^{00}(|\vec{r}| \rightarrow \infty) = 0$ ,  $\frac{\partial \bar{h}^{00}(\vec{r})}{\partial r}(r = 0) = 0$  and with the continuity conditions  $\bar{h}^{00}(|\vec{r}| \rightarrow R_o - \epsilon) = \bar{h}^{00}(|\vec{r}| \rightarrow R_o + \epsilon)$ ,  $\frac{\partial \bar{h}^{00}}{\partial r}(|\vec{r}| \rightarrow R_o - \epsilon) = \frac{\partial \bar{h}^{00}}{\partial r}(|\vec{r}| \rightarrow R_o + \epsilon)$  in the limit  $\epsilon \rightarrow 0$ . This gives

$$\bar{h}^{00}(\vec{r}) = \begin{cases} \frac{2M}{R_o} \left(3 - \frac{|\vec{r}|^2}{R_o^2}\right) & \text{when } |\vec{r}| < R_o, \\ \frac{4M}{|\vec{r}|} & \text{when } |\vec{r}| > R_o \end{cases} \quad (3.15)$$

so to have  $|h_{\gamma\beta}| \ll 1$  we require  $M/R_o \ll 1$ .

Let us first look at the Klein-Gordon equation on this perturbed background  $\square\phi - m^2\phi = 0$ , where  $\square\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)$ . At first order  $g$  is  $g = -1 - h^\gamma_\gamma = -1 - \bar{h}^{00}$  such that  $\sqrt{-g} \approx 1 + \bar{h}^{00}/2$ . We therefore have

$$\begin{aligned} \square\phi(x) - m^2\phi(x) &\approx \frac{1}{1 + \bar{h}^{00}(x)/2} \partial_\mu [(1 + \bar{h}^{00}(x)/2)g^{\mu\nu}(x)\partial_\nu\phi(x)] - m^2\phi(x) \\ -\ddot{\phi}(x) + (1 - \bar{h}^{00}(x))\nabla^2\phi(x) - m^2\phi(x) + \frac{\bar{h}^{00}(x)}{2}m^2\phi(x) &= 0 \end{aligned} \quad (3.16)$$

where we used  $\dot{\bar{h}}^{00}(x) = 0$ . We could now solve this modified Klein-Gordon equation, quantize the field and compute the negativity of two detectors interacting linearly with the field. This approach is somewhat complicated for our needs. We therefore use a different approach, namely the approach of [61]. In [61] the authors calculated the decay rate of an Unruh-DeWitt detector near a star in the infinite time limit. They found that for a minimally coupled field, the decay rate decreases because of the weak gravitational field. To do this, they computed the first order correction to the propagator on the perturbed background. Since in the last section we formulated both  $P_{ej}$  and  $X$  in terms of the propagator, the approach of [61] is very useful.

## Detectors on the curved background

Let us now consider the two Unruh-DeWitt detectors on the curved background. From now on in this subsection we assume that the field is massless. We assume that the two detectors and the center of the star are all on a same axis. Therefore, detector 1 is located at a fixed distance  $r_1$  from the center of the star and similarly detector 2 is located at  $r_2 = r_1 + L$  from the center of the star. This means that their proper times do not coincide  $\tau_1 \neq \tau_2$ , so we may write the evolution operator as

$$U = T \exp \left\{ -i \int d\tau_1 \alpha \left[ \eta(\tau_1) m^{(1)}(\tau_1) \phi(x_1(\tau_1)) + \eta(\tau_2(\tau_1)) m^{(2)}(\tau_2(\tau_1)) \phi(x_2(\tau_2(\tau_1))) \frac{d\tau_2(\tau_1)}{d\tau_1} \right] \right\} \quad (3.17)$$

and using Eq. (3.14) we have:

$$\begin{aligned} \tau_2(\tau_1) &= \tau_1 \sqrt{\frac{1 - \bar{h}^{00}(r_2)/2}{1 - \bar{h}^{00}(r_1)/2}} \\ &= \tau_1 \left( 1 - \frac{\bar{h}^{00}(r_2)}{4} + \frac{\bar{h}^{00}(r_1)}{4} + O([\bar{h}^{00}]^2) \right). \end{aligned} \quad (3.18)$$

To simplify our analysis we want to avoid this blueshift effect. To do this, we assume that the two detectors are close enough such that their internal clocks have the same speed at first order in perturbation theory. This will be so if  $|\bar{h}^{00}(r_2)/4 - \bar{h}^{00}(r_1)/4| \lesssim O([\bar{h}^{00}(r_2)]^2)$  which for detectors outside the star gives  $L \lesssim 16M$ . Under that assumption, we have  $\tau_2 = \tau_1 (1 + O([\bar{h}^{00}]^2))$  such that one can easily verify that Eq. (3.2) and Eq. (3.3) still hold up to  $O([\bar{h}^{00}]^2)$ .

We are therefore left with two different first order contributions to the exchange term  $X$  and the local noise term  $P_{ej}$ . The first one which we denote by  $\tilde{X}_{(1)}$  and  $\tilde{P}_{ej(1)}$  is essentially a result of the time dilation caused by the star and the second one which we denote by  $\delta X_{(1)}$  and  $\delta P_{ej(1)}$  comes from a modification of the propagator caused by the curved background. Let us denote the perturbative expansion of the propagator as  $G(x, y) = G_{(0)}(x, y) + G_{(1)}(x, y)$  and since it is widely believed that a Boulware-like vacuum is the right vacuum for a quantum field near a star [61, 62, 63], we use Eq. (1.16) for  $G_{(0)}(x, y)$ . We can easily evaluate the contribution coming from the time dilation by noting that

$$x_j(\tau_j) = \left( \frac{\tau_j}{\sqrt{1 - \bar{h}^{00}(r_j)/2}}, \vec{r}_j \right). \quad (3.19)$$

Thus, when  $L \lesssim 16M$  we have using Eq. (1.16):

$$\begin{aligned} G_{(0)}(|\vec{x}_1(\tau) - \vec{x}_2(\tau')|, x_1^0(\tau) - x_2^0(\tau')) = \\ (1 - \bar{h}^{00}(r_1)/2) G_{(0)}(L_p(1 - \bar{h}^{00}(r_1)/2), \tau - \tau') \\ + O([\bar{h}^{00}]^2) \end{aligned} \quad (3.20)$$

where

$$L_p := \int_{r_1}^{r_1+L} \sqrt{1 + \bar{h}^{00}(r)/2} dr \approx L(1 + \bar{h}^{00}(r_1)/4) \quad (3.21)$$

is the proper distance between the two detectors. Hence, when we put this back in Eq. (3.9) and Eq. (3.7) the time dilation causes the first order corrections

$$\tilde{P}_{ej(1)} = -\frac{\bar{h}^{00}(r_j)}{2} P_{e(0)} \quad (3.22)$$

$$\tilde{X}_{(1)} = -\frac{\bar{h}^{00}(r_1)}{2} \left( X_{(0)} + L_p \frac{\partial X_{(0)}}{\partial L_p} \right) \quad (3.23)$$

where the zeroth order terms are given by Eq. (1.24) and (3.10). These corrections slightly reduce the magnitude of the local noise while the exchange term is almost unchanged.

### First order correction to the propagator

Let us now compute the first order correction to the propagator on the perturbed background. The first steps of our calculation can be found in [61]. To focus on the correction caused by gravity, we assume that the field is minimally coupled to curvature and to the matter that composes the star. Recall from Eq. (1.17) that the propagator is a Green's function of the Klein-Gordon operator:

$$\square_x G(x, y) = \frac{i\delta(x - y)}{\sqrt{-g(x)}}. \quad (3.24)$$

Using  $G(x, y) = G_{(0)}(x, y) + G_{(1)}(x, y)$  we have:

$$\frac{1}{\sqrt{1 + h_\alpha^\alpha}} \partial_\mu \left[ \sqrt{1 + h_\alpha^\alpha} (\eta^{\mu\nu} - h^{\mu\nu}) \partial_\nu (G_{(0)}(x, y) + G_{(1)}(x, y)) \right] = \frac{i\delta(x - y)}{\sqrt{1 + h_\alpha^\alpha}}. \quad (3.25)$$

Expanding everything to first order only and using the fact that  $G_{(0)}(x, y)$  solves the zeroth order equation, we obtain

$$\begin{aligned} -h^{\mu\nu} \partial_\mu \partial_\nu G_{(0)}(x, y) + \square_{(0)x} G_{(1)}(x, y) - \partial_\mu h^{\mu\nu} \partial_\nu G_{(0)}(x, y) &+ \partial_\mu (h_\alpha^\alpha/2) \eta^{\mu\nu} \partial_\nu G_{(0)}(x, y) \\ &= -i\delta(x - y) h_\alpha^\alpha/2 \end{aligned} \quad (3.26)$$

where we used  $\square_{(0)x} := \eta^{\mu\nu} \partial_\mu \partial_\nu$ . Using again the fact that  $i\delta(x-y) = \square_{(0)x} G_{(0)}(x, y)$  we can simplify the previous equation,

$$\begin{aligned}
\square_{(0)x} G_{(1)}(x, y) &= \partial_\mu \bar{h}^{\mu\nu} \partial_\nu G_{(0)}(x, y) \\
&\quad + \bar{h}^{\mu\nu} \partial_\mu \partial_\nu G_{(0)}(x, y) \\
&= \bar{h}^{\mu\nu} \partial_\mu \partial_\nu G_{(0)}(x, y) \\
&= \bar{h}^{00}(x) \partial_{x^0}^2 G_{(0)}(x, y)
\end{aligned} \tag{3.27}$$

where we used the fact that we are in the Lorentz gauge and that in the Newtonian limit  $\bar{h}^{00}$  is the dominant component of  $\bar{h}^{\mu\nu}$ . Note that since the spacetime we consider is static and asymptotically flat, the propagator  $G(x, y)$  can be seen as the analytic continuation of the unique Green's function on the positive definite section [61]. Since this holds order by order in perturbation theory, at first order perturbation we can use  $G_{(0)}$  as the inverse of  $\square_{(0)}$  such that:

$$G_{(1)}(x, y) = -i \int d^4 z G_{(0)}(x, z) \bar{h}^{00}(z) \partial_{z^0}^2 G_{(0)}(z, y). \tag{3.28}$$

This equation gives us explicitly the first order correction to the propagator. It is clear from this equation that the entire spacetime perturbation modifies the propagator and the most significant contribution comes from the patch of spacetime near  $x$  and  $y$ . Using Eq. (1.16) in Eq. (3.28) and using the fact that  $\bar{h}^{00}(x)$  is independent of time, we obtain

$$\begin{aligned}
G_{(1)}(x, y) &= \frac{-i}{16\pi^4} \int dz^0 d^3 z \bar{h}^{00}(\vec{z}) \\
&\quad \times \left[ \frac{8(\tilde{z} + s)^2}{(\tilde{z} - z_1)^3 (\tilde{z} - z_2)^3 (\tilde{z} - z_o)(\tilde{z} + z_o)} - \frac{2}{(\tilde{z} - z_1)^2 (\tilde{z} - z_2)^2 (\tilde{z} - z_o)(\tilde{z} + z_o)} \right]
\end{aligned} \tag{3.29}$$

where we use the definitions  $Z_x := |\vec{x} - \vec{z}|$ ,  $Z_y := |\vec{y} - \vec{z}|$ ,  $s := x^0 - y^0$ ,  $\tilde{z} := z^0 - x^0$ ,  $z_o := X + i\epsilon$ ,  $z_1 := -s + Y + i\epsilon$ ,  $z_2 := -s - Y - i\epsilon$  and the limit  $\epsilon \rightarrow 0^+$  is implicit. We can then perform the  $z^0$  integration with residue method. We choose a closed contour in the upper half of the complex plane and the upper part of the contour is equal to zero because

the integrand vanishes sufficiently rapidly as  $z^0 = Re^{i\theta} \Big|_{R \rightarrow \infty}$ . We thus have:

$$\begin{aligned}
G_{(1)}(x, y) &= \frac{1}{8\pi^3} \int d^3 z \bar{h}^{00}(\vec{z}) \left[ \right. \\
&\quad \frac{8(z_o + s)^2}{(z_o - z_1)^3 (z_o - z_2)^3 2z_o} + 4 \frac{d^2}{d\tilde{z}^2} \left( \frac{(\tilde{z} + s)^2}{(\tilde{z} - z_2)^2 (\tilde{z} - z_o)(\tilde{z} + z_o)} \right) \Big|_{\tilde{z}=z_1} \\
&\quad \left. - \frac{2}{(z_o - z_1)^2 (z_o - z_2)^2 2z_o} - \frac{d}{d\tilde{z}} \left( \frac{2}{(\tilde{z} - z_2)^2 (\tilde{z} - z_o)(\tilde{z} + z_o)} \right) \Big|_{\tilde{z}=z_1} \right] \\
&= \frac{1}{8\pi^3} \int d^3 z \bar{h}^{00}(|\vec{z}|) \left[ \frac{3(s^2 + Z_x Z_y)(Z_x + Z_y) + Z_x^3 + Z_y^3}{(Z_x Z_y + i\epsilon)(s^2 - [Z_x + Z_y + i\epsilon]^2)^3} \right]. \quad (3.30)
\end{aligned}$$

To further simplify this expression we need a simple way to evaluate the angular part of the integral.

### Spherical shell integration

We now show how to simplify the integral in Eq. (3.30) by using a simple change of variable. To do this, we consider a general function of the form

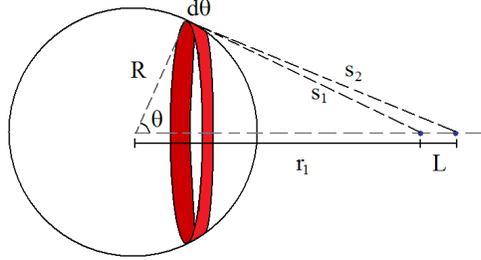


Figure 3.1: Spherical shell integration.

$$A = \int d^3 x f(|\vec{x}|) g(s_1, s_2) \quad (3.31)$$

where we have  $s_1 := |\vec{x} - \vec{x}_1|$  and  $s_2 := |\vec{x} - \vec{x}_2|$ . We need to assume that the origin,  $\vec{x}_1$  and  $\vec{x}_2$  are all on the same axis, just like it is illustrated on Fig. (3.1). Let us first use the



cosine law to relate  $\theta$  and  $\{s_1, s_2\}$ :

$$s_1^2 = R^2 + r_1^2 - 2Rr_1 \cos(\theta) \quad (3.32)$$

$$\begin{aligned} s_2^2 &= R^2 + (r_1 + L)^2 - 2R(r_1 + L) \cos(\theta) \\ &= s_1^2 \left(1 + \frac{L}{r_1}\right) + L \left(r_1 + L - \frac{R^2}{r_1}\right). \end{aligned} \quad (3.33)$$

Then, using spherical coordinates and noting that the integrand is constant over the angle  $\phi$ , we have  $d^3x \rightarrow 2\pi R^2 \sin(\theta) d\theta dR$ . Using Eq. (3.32) we have  $s_1 ds_1 = Rr_1 \sin(\theta) d\theta$  such that  $d^3x \rightarrow \frac{2\pi R s_1 ds_1 dR}{r_1}$ . Putting this back in Eq. (3.31) we have

$$\begin{aligned} A &= \frac{2\pi}{r_1} \left( \int_0^{r_1} dR R f(R) \int_{r_1-R}^{r_1+R} ds_1 s_1 g(s_1, s_2) \right. \\ &\quad \left. + \int_{r_1}^{\infty} dR R f(R) \int_{R-r_1}^{r_1+R} ds_1 s_1 g(s_1, s_2) \right) \\ &= \frac{2\pi}{r_1} \int_0^{\infty} dR R f(R) \int_{|r_1-R|}^{r_1+R} ds_1 s_1 g(s_1, s_2) \end{aligned} \quad (3.34)$$

where  $s_2 = \sqrt{s_1^2 \left(1 + \frac{L}{r_1}\right) + L \left(r_1 + L - \frac{R^2}{r_1}\right)}$ .

### Negativity on the perturbed background

We now have all the tools to compute explicitly  $\delta P_{ej(1)}$  and  $\delta X_{(1)}$ . For  $\delta P_{ej(1)}$ , we have  $Z_x = Z_y = Z$ , such that the correction to the propagator can be greatly simplified with the spherical shell integration method

$$\begin{aligned} G_{(1)}(\vec{x}, \vec{x}, s) &= \frac{1}{4\pi^3} \int d^3z \bar{h}^{00}(|\vec{z}|) \frac{3s^2 + 4Z^2}{(Z + i\epsilon)(s^2 - 4Z^2 - i\epsilon)^3} \\ &= \frac{1}{r2\pi^2} \int_0^{\infty} dR R \bar{h}^{00}(R) \int_{|r-R|}^{r+R} dv \frac{3s^2 + 4v^2}{(s^2 - 4v^2 - i\epsilon)^3} \end{aligned} \quad (3.35)$$

where  $r$  is the distance between  $\vec{x}$  and the center of the star. The  $v$  integral then can be performed analytically. Note that Eq. (3.9) and (3.7) were derived for detectors in Minkowski spacetime, where  $x_j^0(\tau) = \tau$ , but since we are only interested at the first order perturbation, the effect of the time dilation in the corrected propagator would be a second order term which we neglect. We can thus use Eq. (3.9) and (3.7) with the first order correction to the propagator and with no time dilation, that is  $x_j^0(\tau) = \tau$ . For the same

reason, we can also use at this order  $L_p = L$ . Thus, we can put Eq. (3.35) in Eq. (3.9) and we obtain the first order correction to the local noise  $\delta P_{ej(1)}$ :

$$\begin{aligned} \delta P_{ej(1)} &= \frac{\alpha^2 \sigma \sqrt{\pi}}{\pi^2 r_j} \Re \left\{ \int_0^\infty ds e^{-s^2/(4\sigma^2) - i\Delta E s} \int_0^\infty dR R \bar{h}^{00}(R) \right. \\ &\quad \times \left[ \left( \ln \left( \frac{2(r_j + R) + (s - i\epsilon)}{2(r_j + R) - (s - i\epsilon)} \right) - \ln \left( \frac{2|r_j - R| + (s - i\epsilon)}{2|r_j - R| - (s - i\epsilon)} \right) \right) \frac{1}{4(s - i\epsilon)^3} \right. \\ &\quad \left. \left. - \frac{2(r_j + R)(2(r_j + R)^2 - s^2)}{(s - i\epsilon)^2(4(r_j + R)^2 - (s - i\epsilon)^2)^2} + \frac{2|r_j - R|(2(r_j - R)^2 - s^2)}{(s - i\epsilon)^2(4(r_j - R)^2 - (s - i\epsilon)^2)^2} \right] \right\}. \end{aligned} \quad (3.36)$$

Similarly for the exchange term  $\delta X_{(1)}$ , we put Eq. (3.30) in Eq. (3.7) and we then use the spherical shell integration method to obtain

$$\begin{aligned} \delta X_{(1)} &= -\frac{\alpha^2 \sigma \sqrt{\pi} e^{-\sigma^2 \Delta E^2}}{4\pi^3} \int_0^\infty ds e^{-s^2/(4\sigma^2)} \int d^3 z \\ &\quad \times \bar{h}^{00}(|\vec{z}|) \left[ \frac{3(s^2 + Z_x Z_y)(Z_x + Z_y) + Z_x^3 + Z_y^3}{(Z_x Z_y + i\epsilon)(s^2 - [Z_x + Z_y + i\epsilon]^2)^3} \right] \\ &= -\frac{\alpha^2 \sigma \sqrt{\pi} e^{-\sigma^2 \Delta E^2}}{2\pi^2 r_1} \int_0^\infty ds e^{-s^2/(4\sigma^2)} \int_0^\infty dR R \bar{h}^{00}(R) \\ &\quad \times \int_{|r_1 - R|}^{r_1 + R} dv_1 v_1 \left[ \frac{3(s^2 + v_1 v_2)(v_1 + v_2) + v_1^3 + v_2^3}{(v_1 v_2 + i\epsilon)(s^2 - [v_1 + v_2 + i\epsilon]^2)^3} \right] \end{aligned} \quad (3.37)$$

where  $v_2 = \sqrt{v_1^2 \left(1 + \frac{L_p}{r_1}\right) + L_p \left(r_1 + L_p - \frac{R^2}{r_1}\right)}$ . The  $s$  integration can be performed analytically, such that we are left with a relatively simple expression for  $X$  which involves only two integrations:

$$\begin{aligned} X_{(1)} &= \frac{\alpha^2 \sigma \sqrt{\pi} e^{-\sigma^2 \Delta E^2}}{2\pi^2 r_1} \int_0^\infty dR R \bar{h}^{00}(R) \int_{|r_1 - R|}^{r_1 + R} dv_1 \frac{1}{16(v_2 + i\epsilon)\sigma^4} \\ &\quad \times \left\{ i\pi e^{-(v_1 + v_2)^2/(4\sigma^2)} \operatorname{erfc} \left( \frac{-i(v_1 + v_2)}{2\sigma} \right) [2\sigma^2 - (v_1 + v_2)^2] - 2\sqrt{\pi}\sigma(v_1 + v_2) \right\}. \end{aligned} \quad (3.38)$$

We now have all the tools to compute explicitly the corrected negativity. Using the  $\bar{h}^{00}(|\vec{r}|)$  of Eq. (3.15) in Eq. (3.36) and (3.38), we can find  $\delta P_{ej(1)}$  and  $\delta X_{(1)}$  by numerically evaluating the remaining integrals.  $\tilde{P}_{ej(1)}$  and  $\tilde{X}_{(1)}$  can then be evaluated exactly using Eq.

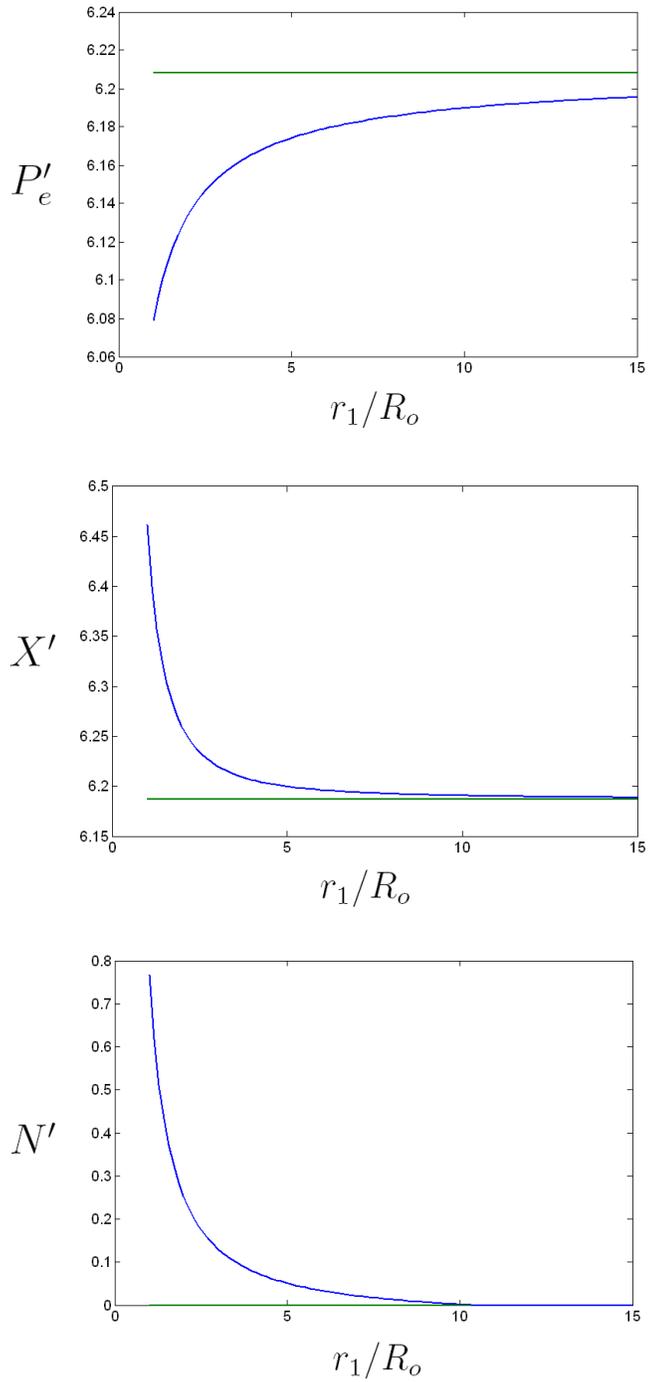


Figure 3.2: The Local noise  $P'_e = 8\pi^2 P_{e1}/\alpha^2$ , the exchange term  $X' = 8\pi^2 X/\alpha^2$  and the negativity  $N' = 8\pi^2 N/\alpha^2$  as a function of  $r_1/R_o$ . We fixed  $L_p/R_o = 0.01$ ,  $\sigma\Delta E = 0.00674$ ,  $\Delta ER_o = 1$  and  $M/R_o = 0.001$ . The lower (green) line is the asymptotic line  $r_1/R_o \rightarrow \infty$ .

(3.22) and (3.23) such that we end up with the full noise term  $P_{ej} = P_{e(0)} + \tilde{P}_{ej(1)} + \delta P_{ej(1)}$  and the full exchange term  $X = X_{(0)} + \tilde{X}_{(1)} + \delta X_{(1)}$  using Eq. (1.24) and (3.10) for the zeroth order terms. This allows us to compute the negativity between the two detectors using Eq. (3.5).

Numerical evaluations show that  $|X|$  linearly increases with the strength of the gravitational potential  $M/R_o$  of the star while  $P_{ej}$  linearly decreases with  $M/R_o$ . Therefore, the negativity  $N$  linearly increases with the strength of the gravitational field  $M/R_o$ . In a similar fashion, numerical evaluation of  $|X|$  and  $P_{ej}$  show that the correction to the negativity  $N$  decreases roughly like  $R_o/r_1$  as  $r_1/R_o \rightarrow \infty$  but remains positive, see Fig. (3.2). We found that the corrections due to the change of the propagator,  $\delta P_{ej(1)}$  and  $\delta X_{(1)}$ , are significantly greater than the corrections due to time dilation,  $\tilde{P}_{ej(1)}$  and  $\tilde{X}_{(1)}$ . This had to be expected since the change of propagator is sensitive to the entire spacetime while the time dilation is a localized effect. On Fig. (3.2) we chose the parameters  $L_p/\sigma$  and  $\sigma\Delta E$  such that we have  $N = 0$  and  $|X| \approx P_e$  without the gravitational field. Thus, in that particular case we not only have entanglement enhancement by gravity but also entanglement creation by gravity.

We may heuristically interpret this phenomenon by looking at the local noise term  $P_{ej}$  and the exchange term  $|X|$  separately. Since the gravitational field increases the momentum of virtual particles near the star (as seen by the fixed detector  $j$ ), it is more energetically expensive to have many of them so the local noise has to decrease. As for the exchange term, we can see that it increases because the gravitational field creates a lensing effect such that more virtual particles emitted by detector 2 hit detector 1.

As we previously mentioned this effect scales linearly with the strength of the gravitational field  $M/r_1$ , so for detectors with  $\sigma \gg 1/\Delta E$  and  $\sigma \gtrsim L_p$  we have for the Earth  $N_{(1)} \lesssim 10^{-9}N_{(0)}$  while for the Sun we have  $N_{(1)} \lesssim 10^{-6}N_{(0)}$ . Since the vacuum entanglement  $N_{(0)}$  has still not been observed, we conclude that  $N_{(1)}$  will be very hard to observe. Nevertheless, it should be interesting to see if this effect can be modeled in a quantum field analog like a linear ion trap [64].

This effect is clearly a consequence of the fact that we used a Boulware vacuum. If we had considered two Unruh-DeWitt detectors near a black-hole in an Unruh or a Kruskal vacuum [26], the Hawking temperature seen by both detectors would have increase the local noise significantly such that the entanglement between both detectors should be degraded, not enhanced. It should therefore be interesting to investigate this in it should be interesting more detail and see how the entanglement degradation near black-holes is affected by the choice of vacuum.

## 3.2 Time-independent approach

In this section we start by showing that extraction of entanglement from the vacuum can also be done with a time-independent approach, i.e. by calculating the entanglement contained in the ground state of the interacting theory with the help of time-independent perturbation theory. Note that even if we use a time-independent approach, this is still in essence the entanglement swapping setup of Fig. (1.2). Using this approach, we show that the two detectors can adiabatically and therefore instantaneously become entangled, namely through what is essentially the Casimir-Polder effect. We find that the Casimir-Polder effect entangles significantly, which is encouraging for experimental verification. We then follow-up on this result by studying the same model but with various modifications, such as the presence of boundary conditions and the presence of a classical potential.

### 3.2.1 Ground state entanglement

We start by showing that the ground state of the interacting theory is an entangled state from the point of view of the two detectors and then we discuss using the adiabatic theorem at what speed this entanglement kicks in.

#### Ground state of the interacting theory

We now switch from the interaction picture to the Schrödinger picture and we assume detectors at rest in Minkowski spacetime separated by a constant distance  $L$ . We also temporarily assume that the interaction is fully on, so  $\eta = 1$ . We therefore need to give a spatial extent to our detectors to regularize the ultraviolet (UV):

$$H_{int} = \sum_{j=1}^2 \alpha_j m^{(j)} \int d^3x f_j(\vec{x}) \phi(\vec{x}). \quad (3.39)$$

Recall that the functions  $f_j(\vec{x})$  describe the smearing of the detectors, and for simplicity we choose  $f_2(\vec{x}) = f_1(\vec{x} - \vec{L})$ . Under these assumptions, our total Hamiltonian is time-independent so this allows us to use perturbation theory for time-independent perturbations [65]. We obtain the new ground state

$$\begin{aligned} |e_{g,new}\rangle &= |e_g\rangle + \sum_{k \neq g} |e_k\rangle \frac{\langle e_k | H_{int} | e_g \rangle}{E_g - E_k} + \sum_{k \neq g} \sum_{l \neq g} |e_k\rangle \frac{\langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_l)} \\ &\quad - \frac{|e_g\rangle}{2} \sum_{k \neq g} \frac{|\langle e_k | H_{int} | e_g \rangle|^2}{(E_k - E_g)^2} + O(\alpha^3) \end{aligned} \quad (3.40)$$

where  $|e_k\rangle$  are the eigenstates of the free Hamiltonian and we used the fact that in our case  $\langle e_g|H_{int}|e_g\rangle = 0$ . Our initial ground state is  $|e_g\rangle = |g^{(1)}, g^{(2)}, 0\rangle$ , and using Eq. (3.40) the new ground state  $|e_{g,new}\rangle$  is

$$|e_{g,new}\rangle = \left[ |g^{(1)}, g^{(2)}\rangle \left(1 - \frac{(S_1 + S_2)}{2}\right) - |e^{(1)}, g^{(2)}\rangle Q_1^{(3)} - |g^{(1)}, e^{(2)}\rangle Q_2^{(3)} + |e^{(1)}, e^{(2)}\rangle R + \dots \right] |0\rangle \quad (3.41)$$

where we have:

$$Q_j^{(3)} = \alpha_j \int \frac{d^3 p}{(2\pi)^3} \frac{\int d^3 x f_j(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} a_{\vec{p}}^\dagger}{\sqrt{2E_{\vec{p}}}(E_{\vec{p}} + \Delta E)} \quad (3.42)$$

$$R = \alpha_1 \alpha_2 \int \frac{d^3 p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{L}} \left| \int d^3 x f_1(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} \right|^2}{2E_{\vec{p}}(E_{\vec{p}} + \Delta E)(\Delta E)} \quad (3.43)$$

$$S_j = \alpha_j^2 \int \frac{d^3 p}{(2\pi)^3} \frac{\left| \int d^3 x f_1(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} \right|^2}{2E_{\vec{p}}(E_{\vec{p}} + \Delta E)^2}. \quad (3.44)$$

The resulting state is clearly entangled because it is a pure state which cannot be written in a tensor product form. Let us now ask whether this is indeed an entangled state from the point of view of the detectors. To see this we need to trace out the field, leaving the remaining system in a mixed state  $\rho_{g,new,d} := Tr_{(3)}(|e_{g,new}\rangle\langle e_{g,new}|)$

$$\rho_{g,new,d} = \begin{pmatrix} 0 & 0 & 0 & R \\ 0 & S_1 & T & 0 \\ 0 & T & S_2 & 0 \\ R & 0 & 0 & 1 - S_1 - S_2 \end{pmatrix} + O(\alpha^4) \quad (3.45)$$

where the matrix is written in the basis  $|e^{(1)}, e^{(2)}\rangle, |e^{(1)}, g^{(2)}\rangle, |g^{(1)}, e^{(2)}\rangle, |g^{(1)}, g^{(2)}\rangle$  and we have:

$$T = \alpha_1 \alpha_2 \int \frac{d^3 p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{L}} \left| \int d^3 x f_1(\vec{x}) e^{-i\vec{p}\cdot\vec{x}} \right|^2}{2E_{\vec{p}}(E_{\vec{p}} + \Delta E)^2}. \quad (3.46)$$

Note that the structure of the matrix (3.45) does not depend much on our choice of interaction Hamiltonian or our choice of initial state. In fact, one can easily show that as long as we have an interaction of the form  $H_{int} = \sum_{j=1}^2 \alpha_j \left( |e^{(j)}\rangle\langle g^{(j)}| + |g^{(j)}\rangle\langle e^{(j)}| \right) U_j^{(3)}$  with an initial eigenstate  $|g^{(1)}, g^{(2)}, \psi^{(3)}\rangle$  such that  $\langle \psi^{(3)}|U_j^{(3)}|\psi^{(3)}\rangle = 0$  then the matrix structure of the new ground state of the two detectors is the same as in Eq. (3.45). Only the values of  $S_j$ ,  $R$  and  $T$  may differ.

## Negativity of the ground state

To measure the entanglement of this mixed state, we use again the negativity. We find the negativity  $N$  for the density matrix  $\rho_{g,new,d}$ :

$$N = \max \left( \sqrt{(S_1 - S_2)^2 + 4|R|^2} - S_1 - S_2, 0 \right). \quad (3.47)$$

Let us assume for simplicity  $\alpha_1 = \alpha_2 = \alpha$  such that  $S_1 = S_2 = S$  and  $N = 2 \max(|R| - S, 0)$ . Just like with the time-dependent approach, we see that the negativity is a fight between the exchange term  $R$  and the noise term  $S$ . Indeed, in the ground state of the interacting theory, each detector has a cloud of particles, this is represented by  $S$ , and the two detectors are also continuously exchanging virtual particles, this is represented by  $R$ .

As an illustrative example, we can analyze the negativity when the smearing functions are Gaussian

$$f_1(\vec{x}) = \frac{e^{-\frac{|\vec{x}-\vec{x}_1|^2}{2\Delta X^2}}}{(2\pi)^{3/2} \Delta X^3} \quad (3.48)$$

so the size of the detectors is about  $\Delta X$ . Such smearing functions could be physically implemented by putting the detectors in a quantum harmonic potential. Even if Gaussian smearing functions have a finite probability for the detectors to overlap, we are only looking at the regime  $\frac{L}{\Delta X} \rightarrow \infty$ , and in this regime the overlap is insignificant. In fact, in this regime all the smearing functions have the same effect, namely to create an effective momentum cut-off. Thus our results would not change for detectors which are delocalized within a region of space which has compact support.

Using the Gaussian smearing function, we can easily evaluate its Fourier transform such that  $|\int d^3x f_1(\vec{x}) e^{-i\vec{p}\cdot\vec{x}}|^2 = e^{-\vec{p}^2 \Delta X^2}$ . Then, using the change of variable  $\vec{p}' = \vec{p}L$ , we have to approximate two integrals, namely

$$S = \alpha^2 \int \frac{d^3p'}{(2\pi)^3} \frac{e^{-\vec{p}'^2 \Delta X^2 / L^2}}{2E_{\vec{p}'}(E_{\vec{p}'} + L\Delta E)^2} = \frac{\alpha^2}{4\pi^2} \int_0^\infty dp' \frac{p'^2 e^{-p'^2 \Delta X^2 / L^2}}{E_{p'}(E_{p'} + L\Delta E)^2} \quad (3.49)$$

$$R = \alpha^2 \int \frac{d^3p'}{(2\pi)^3} \frac{e^{-\vec{p}'^2 \Delta X^2 / L^2 - i\vec{p}'\cdot\vec{x}}}{2E_{\vec{p}'}(E_{\vec{p}'} + L\Delta E)L\Delta E} = \frac{\alpha^2}{4\pi^2} \int_0^\infty dp' \frac{p' \sin(p') e^{-p'^2 \Delta X^2 / L^2}}{E_{p'}(E_{p'} + L\Delta E)L\Delta E} \quad (3.50)$$

where  $E_{\vec{p}'} = \sqrt{\vec{p}'^2 + m^2 L^2}$ . First note that in the limit  $L/\Delta X \rightarrow \infty$ , the Gaussian factor simply acts as an effective momentum cut-off. Therefore, for simplicity in the rest of this thesis we will always replace the smearing functions by a momentum cut-off  $\Lambda = 1/\Delta X$  in the momentum space of the field theory. In the regime  $L/\Delta X \rightarrow \infty$ ,  $Lm \rightarrow 0$ ,  $\Delta EL \rightarrow 0$ ,

and  $m \ll \Delta E$  we can crudely approximate  $S$  and  $R$  to:

$$S \approx \frac{\alpha^2}{4\pi^2} \int_0^{L/\Delta X} dp' \frac{p'}{(p' + L\Delta E)^2} \approx \frac{\alpha^2}{4\pi^2} \ln \left( \frac{1}{\Delta X \Delta E} \right) \quad (3.51)$$

$$R \approx \frac{\alpha^2}{4\pi^2 L \Delta E} \int_0^{L/\Delta X} dp' \frac{\sin(p')}{(p' + L\Delta E)} \approx \frac{\alpha^2}{8\pi L \Delta E}. \quad (3.52)$$

We can also consider a slightly different regime, namely when  $m \gg \Delta E$ . In that case, we can approximate  $S$  and  $R$  this way:

$$S \approx \frac{\alpha^2}{4\pi^2} \int_0^{L/\Delta X} dp' \frac{p'^2}{(p'^2 + m^2 L^2)^{3/2}} \approx \frac{\alpha^2}{4\pi^2} \ln \left( \frac{1}{m \Delta X} \right) \quad (3.53)$$

$$R \approx \frac{\alpha^2}{4\pi^2 L \Delta E} \int_0^{L/\Delta X} dp' \frac{p' \sin(p')}{(p'^2 + m^2 L^2)} \approx \frac{\alpha^2}{8\pi L \Delta E}. \quad (3.54)$$

Thus, if  $\Delta E \gg m$  like for the case of a massless field, we arrive at:

$$N \approx \frac{\alpha^2}{2\pi^2} \max \left( \frac{\pi}{2L\Delta E} - \ln \left( \frac{1}{\Delta E \Delta X} \right), 0 \right). \quad (3.55)$$

Similarly if  $\Delta E \ll m$  we have:

$$N \approx \frac{\alpha^2}{2\pi^2} \max \left( \frac{\pi}{2L\Delta E} - \ln \left( \frac{1}{m \Delta X} \right), 0 \right). \quad (3.56)$$

We therefore see that the ground state of the interacting theory is entangled from the point of view of the detectors if  $L < \frac{\pi}{2\Delta E \ln(1/\Delta E \Delta X)}$  when  $\Delta E \gg m$  and if  $L < \frac{\pi}{2\Delta E \ln(1/m \Delta X)}$  when  $\Delta E \ll m$ . We thus have a spatial version of entanglement sudden death [66]. Indeed, entanglement sudden death is the observation that the negativity can decay completely in a finite amount of time. Note that this non-analyticity is not caused by the fact that the field lives in an infinite dimensional Hilbert space. Indeed, it is instead related to the fact that there is a sharp boundary in the space of mixed states between entangled states and non-entangled states which exhibit classical correlations.

Note that for these results to hold, we need perturbation theory to hold. In other words, the results are true as long as  $S \ll 1$  and  $R \ll 1$ . This will be so if  $\alpha^2 \ll L\Delta E$  and  $\alpha^2 \ll \ln(\Delta X \Delta E)$ . Thus, even though it may look as if the negativity blows up in the limit  $L\Delta E \ll \alpha^2$  this is not the case because in this limit our results are no longer valid. Note also that this does not mean that we cannot look at the regime where the two detectors are very close to one another. Indeed, this regime is allowed by perturbation theory but we have to be careful because in that regime the assumption  $L/\Delta X \rightarrow \infty$  is no longer valid. A proper analysis of this regime should be of interest.



It is also interesting to calculate the negativity in a 1-dimensional space to see how it differs from the 3-dimensional case. First, note that in one dimension we do not need to give the detectors a spatial extent because there are no UV divergences. Second, we need a massive field because there is an infrared (IR) divergence in one dimension. Using our previous derivation, we have for the 1-dimensional case:

$$R = \alpha^2 \int \frac{dp}{2\pi} \frac{e^{ipL}}{2E_p(E_p + \Delta E)\Delta E} \quad (3.57)$$

$$S = \alpha^2 \int \frac{dp}{2\pi} \frac{1}{2E_p(E_p + \Delta E)^2}. \quad (3.58)$$

We can approximate these expressions in the usual limit  $m \gg \Delta E$  and  $Lm \rightarrow 0$ . We obtain  $R \approx \frac{\alpha^2}{4\Delta Em}$  and  $S \approx \frac{\alpha^2}{2\pi m^2}$  such that:

$$N \approx \frac{\alpha^2}{m} \max\left(\frac{1}{2\Delta E} - \frac{1}{\pi m}, 0\right) \approx \frac{\alpha^2}{2m\Delta E}. \quad (3.59)$$

Note that in this regime the entanglement does not decay as a function of  $L$ . This is obviously a special feature of the 1-dimensional case. We thus conclude that it is much easier to extract entanglement from a 1-dimensional vacuum than from a 3-dimensional vacuum.

### Adiabatic switching

To estimate how long it takes to extract entanglement from the vacuum, we use the adiabatic theorem. We assume the system starts in the ground state of the free theory,  $|e_g\rangle = |g^{(1)}, g^{(2)}, 0\rangle$ . Then, the interaction Hamiltonian is smoothly turned on using a switching function  $\eta(t) \in [0, 1]$  such that  $H(t) = H_o + \eta(t)H_{int}$  where  $H_o$  is the free Hamiltonian. For the system to remain in the ground state, we need  $\eta(t)$  to increase slowly enough such that the perturbation is adiabatic. Following the validity condition for adiabatic behaviour [67, 68], we need

$$\max_t \left| \frac{\langle e_k | \dot{H}(t) | e_g \rangle}{E_g(t) - E_k(t)} \right| \ll \min_t |E_g(t) - E_k(t)| \quad (3.60)$$

to hold for any energy level  $E_k$ . A rigorous use of the adiabatic theorem requires normalized eigenstates, so let us put our system in a large box of volume  $V = L_{IR}^3$ . This procedure creates an infrared cut-off and normalizes the eigenstates of the free Hamiltonian. Hence, in our case, if we retain only the dominant order, the adiabatic condition translates to:

$$\max_t |\dot{\eta}(t)| \ll \left[ m^2 + 3 \left( \frac{2\pi}{L_{IR}} \right)^2 \right]^{1/4} \left( \sqrt{m^2 + 3 \left( \frac{2\pi}{L_{IR}} \right)^2} + \Delta E \right)^2 / \alpha. \quad (3.61)$$

Thus, according to this equation, for a massive field it is always possible to adiabatically turn on the interaction. Moreover, if the field is massless, we can still turn on the interaction adiabatically, for any finite size of box to which we confine our system. Note that this adiabatic condition only holds for a field in a box, and is only a first order approximation. In addition, the adiabatic theorem is a sufficient but not necessary requirement for adiabatic evolution. We can therefore find a better constraint on  $\eta(t)$  by taking a more operational approach. We take a simple test function  $\eta_{test}(t)$ , then use time-dependent perturbation theory in the interaction picture without putting the field in a box and look at the difference between the final state we obtain and our calculated ground state  $\rho_{g,new,d}$ . Using several simple test functions we found that the biggest contribution to the error is of the order of  $\alpha^2 \max_t |\dot{\eta}(t)|^2 / \Delta E^2$ , for both a massive and a massless field. Therefore, to have a very small error in the ground state negativity, we simply require  $\max_t |\dot{\eta}(t)| \ll \Delta E$ . Thus, if we follow that prescription, we can adiabatically switch on the interaction and have all  $\alpha^2$  contributions of Eq. (3.45) intact. In other words, we will always be in the instantaneous ground state, and since the ground state of the interacting theory is entangled, there will be an instantaneous creation of entanglement. Therefore, while Alice and Bob cannot exchange classical or quantum information faster than the speed of light, their ability to extract entanglement by interacting with the vacuum is not bounded by any finite speed.

Note that this is not in contradiction with the results of the previous section where we used a time-dependent approach. In fact, we can now really see what is happening in the time-dependent approach using our time-independent picture. Since in the former we considered point-like detectors, we needed a continuous switching function, or in other words we needed to turn on the interaction and then turn it off. Thus, as we turned on the interaction we went to the ground state of the interacting theory, which may be entangled, but as we turned off the interaction we went back to the ground state of the free theory, which is not entangled. But of course the process was not entirely adiabatic, so we did not fully return to the ground state of the free theory which means that we effectively stole some entanglement from the interacting ground state. This explains why the negativity in the time-dependent approach is so much weaker than in the time-independent approach. This method is also much more transparent than the time-dependent approach. That being said, the time-dependent approach can give the same results as the time-independent approach, one just has to consider detectors with a spatial extent and have a switching function which adiabatically turns on the interaction without turning off the interaction afterwards.

Notice that in order to obtain the full amount of entanglement from the ground state, the system needs an interval of time of the order of  $1 / \max_t |\dot{\eta}(t)| \gg 1 / \Delta E$ . This entanglement could either be used in computations or swapped to other quantum systems for distillation. After the entanglement is used up, the interaction Hamiltonian may be switched off and the system can be put back in the ground state of the free theory  $|e_g\rangle = |g^{(1)}, g^{(2)}, 0\rangle$ , for example, by cooling. Thus, Alice and Bob can extract entanglement by interacting

with the field in a cyclic and therefore sustainable way. However, we also see that the extraction of a large amount of entanglement from the vacuum by this method will cost a large amount of time. Interestingly, the amount of time needed is determined in a similar way to how the speed of adiabatic quantum computation is determined. Recall that the closeness of eigenvalues determines how fast specific states such as the ground state can be reached via an adiabatic approach or through cooling, see e.g. [69]. The reason why the finite rate of entanglement extraction does not lead to a finite speed of entanglement “propagation” is that there is no threshold: the negativity between the two detectors can arise immediately as their interaction with the field is switched on.

### 3.2.2 Casimir-Polder effect

The underlying reason why the two detectors are entangled when in the ground state of the interacting theory is that this ground state is a state in which the detectors continuously exchange virtual particles. This exchange interaction is in effect the scalar field version of the Casimir-Polder force [70], which is known to be the relativistic generalization of the Van der Waals force between atoms or molecules.

For completeness, let us now derive the Casimir-Polder force for point-like detectors  $f_1(\vec{x}) = \delta(\vec{x} - \vec{x}_1)$ . To do this we first need to find the energy of the ground state of the perturbed system  $E_{g,new} = E_g + \delta E_g$ . Using time-independent perturbation theory [65], we have to go up to fourth order to find the first contributions which have a  $L$  dependence. These contributions are the only measurable contributions in the Casimir force and in the Casimir energy. Thus, the only terms which remain after the normalization  $\delta\tilde{E}_g(\Delta E, L) = E_{g,new}(\Delta E, L) - \lim_{L \rightarrow \infty} E_{g,new}(\Delta E, L)$  are the following:

$$\delta\tilde{E}_g(\Delta E, L) = \sum_{n \neq g} \sum_{k \neq g} \sum_{l \neq g} \frac{\langle e_g | H_{int} | e_n \rangle \langle e_n | H_{int} | e_k \rangle \langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_n)(E_g - E_k)(E_g - E_l)} + O(\alpha^6). \quad (3.62)$$

The sequences of states which have a non-vanishing contribution in this sum can be summarized in the following table

| $ e_l\rangle$                           | $ e_k\rangle$   | $ e_n\rangle$                           |
|---|---|---|
| $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ | $ e^{(1)}, e^{(2)}, 0\rangle$                           | $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$ |
| $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ | $ e^{(1)}, e^{(2)}, 0\rangle$                           | $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ |
| $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ | $ g^{(1)}, g^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ | $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$ |
| $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ | $ e^{(1)}, e^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ | $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$ |
| $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ | $ e^{(1)}, e^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ | $ e^{(1)}, g^{(2)}, 1_{\vec{p}}\rangle$ |

plus inversion of the detectors  $1 \rightleftharpoons 2$  and we also need to integrate over the various momenta. The resulting Casimir energy is:

$$\begin{aligned}
\delta\tilde{E}_g(L, \Delta E) &= -2\alpha^4 \left[ \frac{1}{\Delta E} \left| \int \frac{d^3p}{(2\pi)^3} \frac{e^{-i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}}{2E_{\vec{p}}(E_{\vec{p}}+\Delta E)} \right|^2 \right. \\
&+ \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \frac{e^{-i(\vec{p}_1-\vec{p}_2)\cdot(\vec{x}_1-\vec{x}_2)}}{4E_{\vec{p}_1}E_{\vec{p}_2}(E_{\vec{p}_1}+E_{\vec{p}_2})} \left( \frac{1}{E_{\vec{p}_1}+\Delta E} + \frac{1}{E_{\vec{p}_2}+\Delta E} \right)^2 \\
&+ 2 \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \frac{e^{-i(\vec{p}_1-\vec{p}_2)\cdot(\vec{x}_1-\vec{x}_2)}}{4E_{\vec{p}_1}E_{\vec{p}_2}(E_{\vec{p}_1}+E_{\vec{p}_2}+2\Delta E)} \\
&\times \frac{1}{(E_{\vec{p}_1}+\Delta E)(E_{\vec{p}_2}+\Delta E)} + \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \frac{e^{-i(\vec{p}_1-\vec{p}_2)\cdot(\vec{x}_1-\vec{x}_2)}}{4E_{\vec{p}_1}E_{\vec{p}_2}(E_{\vec{p}_1}+E_{\vec{p}_2}+2\Delta E)} \\
&\times \left. \left( \frac{1}{(E_{\vec{p}_1}+\Delta E)^2} + \frac{1}{(E_{\vec{p}_2}+\Delta E)^2} \right) \right] + O(\alpha^6). \tag{3.63}
\end{aligned}$$

We thus see that the ground state energy is lowered because of the interaction, causing the two detectors to attract each other with the Casimir force  $F_C = -\frac{\partial\delta\tilde{E}_g(L)}{\partial L}$ . For a massless field, one can easily show that  $\delta\tilde{E}(\Delta E, L) \sim \frac{-\alpha^4}{L^3\Delta E^2}$  in the limit  $L\Delta E \rightarrow \infty$ . For comparison, the electromagnetic Casimir-Polder energy scales as  $\sim -L^{-7}$  for large distances [70].

### 3.2.3 Different detector models

Let us now briefly discuss to what extent these results depend on the detector model. First, note that the Glauber scalar detector of Eq. (2.14) cannot extract entanglement from the vacuum. Indeed, this detector is such that the ground state of the free theory is also an eigenstate of the interacting theory. Thus, if the system starts in the ground state of the free theory  $|g^{(1)}, g^{(2)}, 0\rangle$ , then after a unitary evolution it will still be in the same non-entangled state. Second, what if we consider a fermionic field, like a Dirac field  $\psi$ , instead of a bosonic field? For instance, consider two detectors which have the following interaction Hamiltonian

$$H_{int} = \sum_j \alpha_j m^{(j)} : \bar{\psi}\psi(\vec{x}_j) : \tag{3.64}$$

where the normal ordering  $::$  [4] allows us to use perturbation theory. This kind of detector was first analyzed in the context of the Unruh effect in [71] and then for the entanglement of the vacuum in a time-dependent picture in [72]. We use the standard Dirac field mode

expansion [4]

$$\psi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} \sum_s \left( a_{\vec{p}}^s u^s(p) e^{i\vec{p}\cdot\vec{x}} + b_{\vec{p}}^{s\dagger} v^s(p) e^{-i\vec{p}\cdot\vec{x}} \right) \quad (3.65)$$

such that:

$$:\bar{\psi}\psi(\vec{x}): |0\rangle = \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \frac{\sum_{s,r} e^{-i\vec{x}\cdot(\vec{p}_1+\vec{p}_2)} \bar{u}^s(p_1) v^r(p_2)}{2\sqrt{E_{\vec{p}_1} E_{\vec{p}_2}}} a_{\vec{p}_1}^{s\dagger} b_{\vec{p}_2}^{r\dagger} |0\rangle. \quad (3.66)$$

Using this expression we can now easily calculate the local noise  $S_j$  by following similar steps to the ones that lead to Eq. (3.44),

$$\begin{aligned} S_j &= \alpha_j^2 \sum_k \frac{|\langle k | : \bar{\psi}\psi(\vec{x}_j) : |0\rangle|^2}{(E_k + \Delta E)^2} \\ &= \alpha_j^2 \int_{|\vec{p}_3| < 1/\Delta X} \frac{d^3p_3}{(2\pi)^3} \int_{|\vec{p}_4| < 1/\Delta X} \frac{d^3p_4}{(2\pi)^3} \sum_{k,l} \frac{|\langle 0 | b_{\vec{p}_4}^l a_{\vec{p}_3}^k : \bar{\psi}\psi(\vec{x}_j) : |0\rangle|^2}{(E_{\vec{p}_3} + E_{\vec{p}_4} + \Delta E)^2} \\ &= \alpha_j^2 \int_{|\vec{p}_3| < 1/\Delta X} \frac{d^3p_3}{(2\pi)^3} \int_{|\vec{p}_4| < 1/\Delta X} \frac{d^3p_4}{(2\pi)^3} \sum_{k,l} \frac{|\bar{u}^k(p_3) v^l(p_4)|^2}{4E_{\vec{p}_3} E_{\vec{p}_4} (E_{\vec{p}_3} + E_{\vec{p}_4} + \Delta E)^2} \\ &= \alpha_j^2 \int_{|\vec{p}_3| < 1/\Delta X} \frac{d^3p_3}{(2\pi)^3} \int_{|\vec{p}_4| < 1/\Delta X} \frac{d^3p_4}{(2\pi)^3} \frac{\text{Tr}((p_3^\not{p} + m)(p_4^\not{p} - m))}{4E_{\vec{p}_3} E_{\vec{p}_4} (E_{\vec{p}_3} + E_{\vec{p}_4} + \Delta E)^2} \\ &= \alpha_j^2 \int_{|\vec{p}_3| < 1/\Delta X} \frac{d^3p_3}{(2\pi)^3} \int_{|\vec{p}_4| < 1/\Delta X} \frac{d^3p_4}{(2\pi)^3} \frac{(E_{\vec{p}_3} E_{\vec{p}_4} - \vec{p}_3 \cdot \vec{p}_4 - m^2)}{E_{\vec{p}_3} E_{\vec{p}_4} (E_{\vec{p}_3} + E_{\vec{p}_4} + \Delta E)^2} \end{aligned} \quad (3.67)$$

and similarly for  $R$  we obtain

$$R = \alpha_1 \alpha_2 \int_{|\vec{p}_1| < 1/\Delta X} \frac{d^3p_1}{(2\pi)^3} \int_{|\vec{p}_2| < 1/\Delta X} \frac{d^3p_2}{(2\pi)^3} \frac{e^{-i(\vec{x}_1 - \vec{x}_2) \cdot (\vec{p}_1 + \vec{p}_2)} (E_{\vec{p}_1} E_{\vec{p}_2} - \vec{p}_1 \cdot \vec{p}_2 - m^2)}{\Delta E E_{\vec{p}_1} E_{\vec{p}_2} (E_{\vec{p}_1} + E_{\vec{p}_2} + \Delta E)} \quad (3.68)$$

where we used the matrix technology of Dirac spinors [4]. Note that because each detector is coupled to two instances of the field, there is now two particles (a positron and an electron) involved in  $S_j$  and  $R$ . Moreover, note from the above equations that  $S_j$  and  $R$  are integrals of expressions which depend on the relative directions of these two particles. Indeed, the factor  $(E_{p_1} E_{p_2} - \vec{p}_1 \cdot \vec{p}_2 - m^2)$  shows that it is optimal if the two particles have momentum in opposite direction to one another. To some extent, this is the only really new feature that spin 1/2 particle detectors have over spin 0 particle detectors. In fact, for a massless field in the usual regime  $L/\Delta X \rightarrow \infty$ ,  $L\Delta E \rightarrow 0$  and  $\alpha_1 = \alpha_2 = \alpha$ , we can find a rough approximation to the negativity using Eq. (3.47),

$$N \sim \frac{\alpha^2}{\Delta X} \max\left(\frac{1}{\Delta E L^4} - \frac{1}{\Delta X^3}, 0\right) \quad (3.69)$$

which is essentially the same expression as one would obtain with a coupling to  $\pi\phi$  instead of  $\bar{\psi}\psi$  (as one could have predicted by dimensional analysis). We thus see that the general shape of the ground state negativity is roughly independent of the coupling choice. Hence, for simplicity in the rest of this thesis we will always use a scalar field detector.

### 3.2.4 Boundary conditions

This subsection is collaborative work with Andrzej Veitia. In this subsection we compute the entanglement generated in the Unruh-DeWitt detectors when the scalar quantum field in the vacuum state is subject to boundary conditions. This setup could easily be implemented by perfectly conducting plates. In fact, such setups have been extensively studied in the context of the Casimir effect [73], but here our goal is to see whether the presence of boundary conditions augments or degrades the entanglement of the vacuum. To simplify our notation, in this subsection we denote position vectors by  $\vec{r} = (x, y, z)$  instead of  $\vec{x}$ . For simplicity we only consider a massless field. In this case, the field operator is expanded in terms of creation and annihilation operators as

$$\phi(\vec{r}) = \sum_{\vec{p}} \frac{1}{\sqrt{2|\vec{p}|}} \left( a_{\vec{p}} u_{\vec{p}}(\vec{r}) + a_{\vec{p}}^\dagger u_{\vec{p}}^*(\vec{r}) \right) \quad (3.70)$$

$$\left[ a_{\vec{p}}, a_{\vec{p}'}^\dagger \right] = \delta_{\vec{p}, \vec{p}'} \quad (3.71)$$

where  $u_{\vec{p}}(\vec{r})$  are solutions of Helmholtz equation  $(\Delta + |\vec{p}|^2)u_{\vec{p}}(\vec{r}) = 0$  satisfying the boundary conditions. The matrix elements  $S_1$ ,  $S_2$  and  $R$  are easily expressed in terms of the mode functions  $u_{\vec{p}}(\vec{r})$  and we obtain:

$$S_1 = \alpha_1^2 \sum_{\vec{p}} \frac{1}{2|\vec{p}|} \frac{|u_{\vec{p}}(\vec{r}_1)|^2}{(|\vec{p}| + \Delta E)^2} \quad (3.72)$$

$$S_2 = \alpha_2^2 \sum_{\vec{p}} \frac{1}{2|\vec{p}|} \frac{|u_{\vec{p}}(\vec{r}_2)|^2}{(|\vec{p}| + \Delta E)^2} \quad (3.73)$$

$$R = \alpha_1 \alpha_2 \Re \left[ \sum_{\vec{p}} \frac{1}{2|\vec{p}|} \frac{u_{\vec{p}}(\vec{r}_1) u_{\vec{p}}^*(\vec{r}_2)}{(\Delta E)(|\vec{p}| + \Delta E)} \right]. \quad (3.74)$$

Let us consider the scenario in which the field  $\phi(\vec{r})$  satisfies Dirichlet boundary conditions  $\phi(\pm \frac{d_x}{2}, y, z) = 0$ . In addition, we temporarily impose the periodic boundary conditions  $\phi(x, y + d_y, z + d_z) = \phi(x, y, z)$ . Under these assumptions, the mode functions  $u_{\vec{p}}(\vec{r})$  read

$$u_{\vec{p}}(\vec{r}) = \sqrt{\frac{2}{d_x}} \sin \left[ p_x \left( x + \frac{d_x}{2} \right) \right] \frac{e^{i\vec{p}_{\parallel} \cdot \vec{r}}}{\sqrt{d_y d_z}} \quad (3.75)$$

where  $\vec{p}_{\parallel} = (0, \frac{2\pi n_y}{d_y}, \frac{2\pi n_z}{d_z})$  and  $p_x = \frac{\pi n_x}{d_x}$ . Here  $n_y$  and  $n_z$  assume the values  $0, \pm 1, \pm 2, \dots$  whereas  $n_x = 1, 2, \dots$ . Note that in Eq. (3.72), (3.73) and (3.74) we need to determine sums of the form  $\sum_{\vec{p}} c(\vec{p}) u_{\vec{p}}(\vec{r}_1) u_{\vec{p}}^*(\vec{r}_2)$ . In the limit  $d_y \rightarrow \infty$  and  $d_z \rightarrow \infty$ , these sums take the form

$$\sum_{\vec{p}} c(\vec{p}) u_{\vec{p}}(\vec{r}_1) u_{\vec{p}}^*(\vec{r}_2) = \sum_{n \in \mathbb{Z}} \int \frac{d^2 p_{\parallel}}{(2\pi)^2} \frac{c(n, p_{\parallel})}{2d_x} e^{i\vec{p}_{\parallel} \cdot (\vec{r}_1 - \vec{r}_2)} \left( e^{i\frac{n\pi}{d_x}(x_1 - x_2)} - e^{i\frac{n\pi}{d_x}(x_1 + x_2 + d_x)} \right). \quad (3.76)$$

Making use of Poisson summation formula  $\sum_{n \in \mathbb{Z}} e^{2i\pi n x} = \sum_{n \in \mathbb{Z}} \delta(x - n)$  one can rewrite the above sum as

$$\sum_{\vec{p}} c(\vec{p}) u_{\vec{p}}(\vec{r}_1) u_{\vec{p}}^*(r_2) = \sum_{n \in \mathbb{Z}} \int \frac{d^3 p}{(2\pi)^3} c(\vec{p}) (e^{i\vec{p}\vec{R}_n} - e^{i\vec{p}\vec{R}'_n}) \quad (3.77)$$

where  $\vec{R}_n = (x_1 - x_2 + 2nd_x, y_1 - y_2, z_1 - z_2)$  and  $\vec{R}'_n = (x_1 + x_2 + (2n + 1)d_x, y_1 - y_2, z_1 - z_2)$ . From the above equations one can determine the entanglement in the detectors for arbitrary positions of the detectors. We will however limit our discussion to two symmetric configurations. Let us first consider a symmetric configuration such that the detectors are located at  $\vec{r}_j = (\pm \frac{L}{2}, 0, 0)$  with  $L < d_x$ . Assuming  $\alpha_1 = \alpha_2 = \alpha$  and making use of Eq. (3.72), (3.74) and (3.77) we arrive at the following expressions:

$$S = \alpha^2 \sum_{n \in \mathbb{Z}} \int_{|\vec{p}| < 1/\Delta X} \frac{d^3 p}{(2\pi)^3} \frac{1}{2|\vec{p}|(|\vec{p}| + \Delta E)^2} (e^{ip_x 2nd_x} - e^{ip_x(L+(2n+1)d_x)}) \quad (3.78)$$

$$R = \alpha^2 \sum_{n \in \mathbb{Z}} \int_{|\vec{p}| < 1/\Delta X} \frac{d^3 p}{(2\pi)^3} \frac{1}{2|\vec{p}|(|\vec{p}| + \Delta E)\Delta E} (e^{ip_x(L+2nd_x)} - e^{ip_x(2n+1)d_x}). \quad (3.79)$$

Note that the free space situation (i.e. in the absence of boundary conditions) may be recovered by taking the limit  $d_x \rightarrow \infty$ . Indeed, in the regime  $d_x \gg L$ , expressions (3.78) and (3.79) reduce to Eq. (3.44) and (3.43). It is convenient to express Eq. (3.78), (3.79) in terms of the dimensionless quantities  $|\vec{q}| = |\vec{p}|d_x$ ,  $\varepsilon = L\Delta E$ ,  $\gamma = \frac{L}{d_x}$  and  $\tilde{\Lambda} = L/\Delta X$ . After simple manipulations we obtain:

$$S = \frac{\alpha^2}{4\pi^2} \int_0^{\tilde{\Lambda}/\gamma} dq \frac{1}{(q + \varepsilon/\gamma)^2} \sum_{n \in \mathbb{Z}} \left[ \frac{\sin(2nq)}{2n} - \frac{\sin((2n + \gamma + 1)q)}{2n + \gamma + 1} \right] \quad (3.80)$$

$$R = \frac{\alpha^2 \gamma}{4\pi^2 \varepsilon} \int_0^{\tilde{\Lambda}/\gamma} dq \frac{1}{q + \varepsilon/\gamma} \sum_{n \in \mathbb{Z}} \left[ \frac{\sin((2n + \gamma)q)}{2n + \gamma} - \frac{\sin((2n + 1)q)}{2n + 1} \right]. \quad (3.81)$$

Finally, by means of the formula [74]

$$\sum_{n \in \mathbb{Z}} \frac{\sin((2n + a)q)}{2n + a} = \frac{\pi}{2 \sin(\frac{\pi a}{2})} \sin\left((2m + 1) \frac{\pi a}{2}\right) \quad (3.82)$$

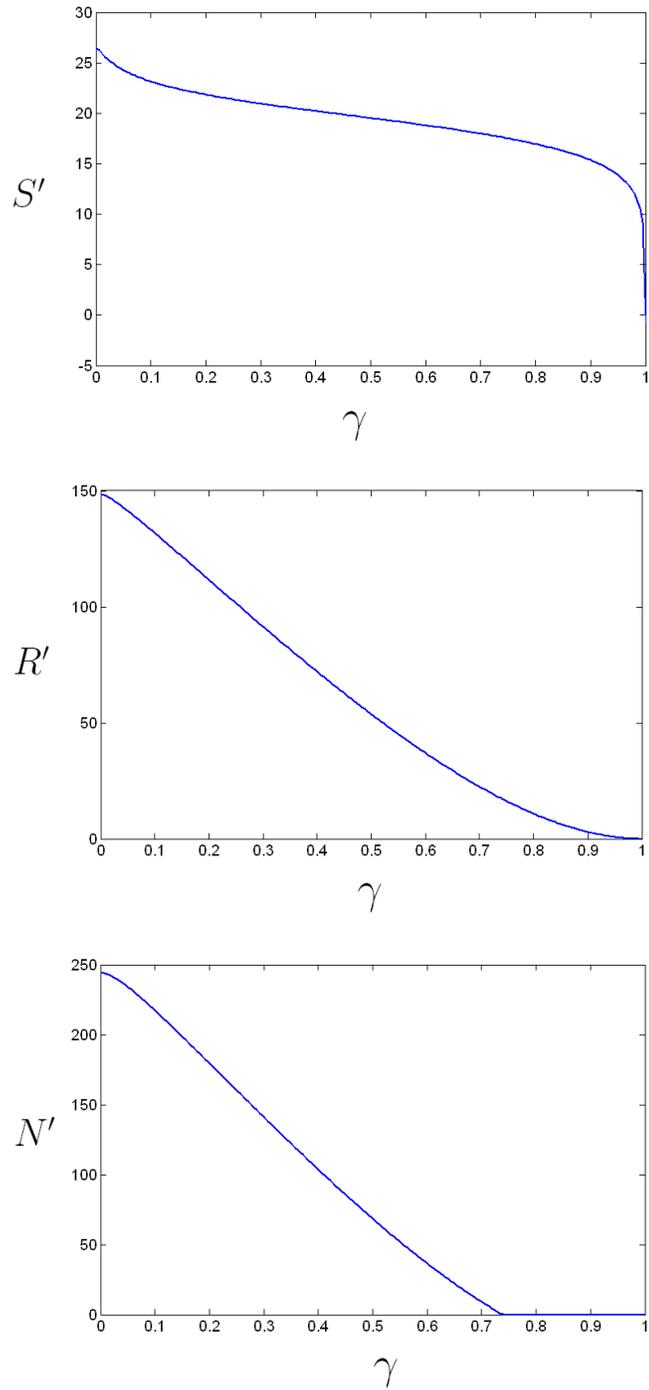


Figure 3.3: The local noise  $S' = 8\pi^2 S/\alpha^2$ , the exchange term  $R' = 8\pi^2 R/\alpha^2$  and the negativity  $N' = 8\pi^2 N/\alpha^2$  as a function of  $\gamma = \frac{L}{d_x}$  with  $\tilde{\Lambda} = 30000$ ,  $\epsilon = 0.02$  and  $\vec{r}_j = (\pm \frac{L}{2}, 0, 0)$ .



for  $m\pi < q < (m+1)\pi$  we reduce Eq. (3.80) and (3.81) to the simpler form

$$S = \frac{\alpha^2}{8\pi^2} \sum_{m=0}^{M_{max}} \frac{2m+1}{(m + \frac{\varepsilon}{\gamma\pi})(m + \frac{\varepsilon}{\gamma\pi} + 1)} \left[ 1 - \frac{\sin((2m+1)(\gamma+1)\frac{\pi}{2})}{(2m+1)\sin((\gamma+1)\frac{\pi}{2})} \right] \quad (3.83)$$

$$R = \frac{\alpha^2\gamma}{8\pi\varepsilon} \sum_{m=0}^{M_{max}} \ln\left(\frac{m + \frac{\varepsilon}{\gamma\pi} + 1}{m + \frac{\varepsilon}{\gamma\pi}}\right) \left[ \frac{\sin((2m+1)\frac{\gamma\pi}{2})}{\sin(\frac{\gamma\pi}{2})} - (-1)^m \right] \quad (3.84)$$

where  $M_{max} \approx \tilde{\Lambda}/(\pi\gamma)$ . Note that in the limit  $\gamma \rightarrow 1$  the above expressions vanish in accordance with the boundary conditions. Using these expressions we may now compute the negativity numerically with Eq. (3.47), see Fig. (3.3). We see on Fig. (3.3) that as the boundary gets closer to the detectors, the entanglement of the ground state decreases until it completely dies. Another interesting case is that where the detectors are located at  $\vec{r}_j = (0, \pm\frac{L}{2}, 0)$ . Making use of Eq. (3.77) we obtain expressions analogous to Eq. (3.80) and (3.81). They read

$$S = \frac{\alpha^2}{4\pi^2} \int_0^{\tilde{\Lambda}/\gamma} dq \frac{1}{(q + \varepsilon/\gamma)^2} \sum_{n \in \mathbb{Z}} \left[ \frac{\sin(2nq)}{2n} - \frac{\sin((2n+1)q)}{2n+1} \right] \quad (3.85)$$

$$R = \frac{\alpha^2\gamma}{4\pi^2\varepsilon} \int_0^{\tilde{\Lambda}/\gamma} dq \frac{1}{q + \varepsilon/\gamma} \sum_{n \in \mathbb{Z}} \left[ \frac{\sin(\sqrt{(2n)^2 + \gamma^2}q)}{\sqrt{(2n)^2 + \gamma^2}} - \frac{\sin(\sqrt{(2n+1)^2 + \gamma^2}q)}{\sqrt{(2n+1)^2 + \gamma^2}} \right]. \quad (3.86)$$

Clearly, in this case the boundary conditions do not imply that  $S$  and  $R$  should vanish as  $d_x \rightarrow L$ . Numerical results for this configuration are presented on Fig. (3.4). Both graphs, Fig. (3.3) and (3.4), indicate that the entanglement generated in the detectors reduces monotonically as the separation  $d_x$  decreases. We may interpret this degradation of the vacuum entanglement by realizing that the boundary conditions give the field an effective mass of  $m_{eff} \sim 1/d_x$ . Thus, as  $\gamma$  increases the effective mass also increases. Moreover, when the mass of the field increases the correlations in the field decrease (see Sec. 1.1.2), so the exchange term  $R$  has to decrease significantly and as a consequence the negativity  $N$  decreases as well. Finally, note that in the regime  $d_x \gg L$ , the orientation of the detectors relative to the planes  $x = \pm d_x/2$  becomes irrelevant and as a consequence  $S$ ,  $R$  and  $N$  coincide for the two cases considered.

### 3.2.5 Stability of the negativity

In this subsection we answer the following question: does the presence of another quantum system interacting with the field change the entanglement available for the two detectors? We show that under the assumption that all interactions are weak, the presence of an

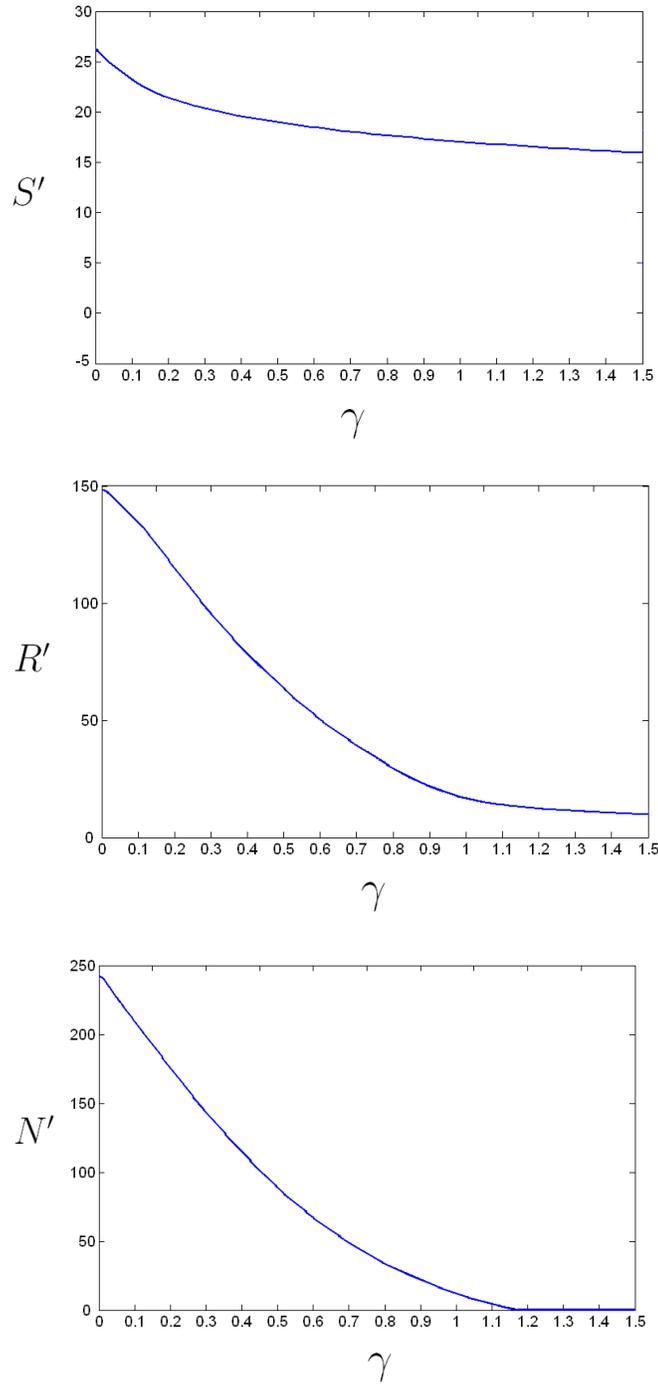


Figure 3.4: The local noise  $S' = 8\pi^2 S/\alpha^2$ , the exchange term  $R' = 8\pi^2 R/\alpha^2$  and the negativity  $N' = 8\pi^2 N/\alpha^2$  as a function of  $\gamma = \frac{L}{d_x}$  with  $\tilde{\Lambda} = 30000$ ,  $\epsilon = 0.02$  and  $\vec{r}_j = (0, \pm \frac{L}{2}, 0)$ .

intruder does not significantly affect the entanglement available for the two detectors. In other words, the ground state negativity is stable.

In this subsection the overall Hilbert space is  $\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)} \otimes \mathcal{H}^{(4)}$ , where the first two Hilbert spaces belong to the two detectors, the third Hilbert space belongs to the intruder and the fourth Hilbert space is that of the field. The interaction Hamiltonian is:

$$H_{int} = \sum_{j=1}^2 \alpha_j m^{(j)} \phi(\vec{x}_j) + \alpha_3 V^{(3)} U(\phi(\vec{x}_3)). \quad (3.87)$$

The self-Hamiltonian and the coupling choice in the interaction Hamiltonian of the third system, the intruder, is left arbitrary for now, but a natural choice would be a third detector. The ground state of the free theory is simply  $|g^{(1)}, g^{(2)}, g^{(3)}, 0\rangle$ . Using this interaction Hamiltonian, we can compute the ground state of the interacting theory  $|e_{g,new}\rangle$ . For simplicity we only focus on the terms which depend on the intruder interaction,

$$\begin{aligned} |e_{g,new}\rangle \Big|_{\alpha_3} &= -\alpha_3 \sum_{(k,j) \neq (0,g)} |g^{(1)}, g^{(2)}, j^{(3)}, k\rangle \frac{\langle k|U(\phi(\vec{x}_3))|0\rangle \langle j^{(3)}|V^{(3)}|g^{(3)}\rangle}{E_k + \Delta E_j} \\ &\quad - \frac{\alpha_3^2}{2} |g^{(1)}, g^{(2)}, g^{(3)}, 0\rangle \sum_{(k,j) \neq (0,g)} \frac{|\langle k|U(\phi(\vec{x}_3))|0\rangle \langle j^{(3)}|V^{(3)}|g^{(3)}\rangle|^2}{(E_k + \Delta E_j)^2} \\ &\quad + \alpha_3^2 \sum_{(k,l) \neq (0,g), j \neq g} |g^{(1)}, g^{(2)}, j^{(3)}, 0\rangle \frac{|\langle k|U(\phi(\vec{x}_3))|0\rangle|^2 \langle j^{(3)}|V^{(3)}|l^{(3)}\rangle \langle l^{(3)}|V^{(3)}|g^{(3)}\rangle}{\Delta E_j (E_k + \Delta E_l)} \\ &\quad + \alpha_3^2 \sum_{j \neq g} |g^{(1)}, g^{(2)}, j^{(3)}, 0\rangle \frac{|\langle 0|U(\phi(\vec{x}_3))|0\rangle|^2 \langle j^{(3)}|V^{(3)}|g^{(3)}\rangle \langle g^{(3)}|V^{(3)}|g^{(3)}\rangle}{\Delta E_j^2} \\ &\quad + \alpha_3 \alpha_1 \sum_{(k,j) \neq (0,g)} |e^{(1)}, g^{(2)}, j^{(3)}, 0\rangle \left( \frac{\langle 0|\phi(\vec{x}_1)|k\rangle \langle k|U(\phi(\vec{x}_3))|0\rangle \langle j^{(3)}|V^{(3)}|g^{(3)}\rangle}{(E_k + \Delta E_j)(\Delta E + \Delta E_j)} \right. \\ &\quad \left. + \frac{\langle 0|U(\phi(\vec{x}_3))|k\rangle \langle k|\phi(\vec{x}_1)|0\rangle \langle j^{(3)}|V^{(3)}|g^{(3)}\rangle}{(E_k + \Delta E)(\Delta E + \Delta E_j)} \right) + (1 \leftrightarrow 2) \end{aligned} \quad (3.88)$$

where  $|j^{(3)}\rangle$  is an eigenstate of the self-Hamiltonian of the intruder with energy  $E_j$  ( $\Delta E_j := E_j - E_g$ ) and  $|k\rangle$  is an eigenstate of the free Hamiltonian of the field  $H_F$  with energy  $E_k$ . With the full ground state, we then trace out the field and the intruder, the remaining density matrix reads

$$\rho_{g,new} = \begin{pmatrix} 0 & 0 & 0 & R \\ 0 & S_1 & T & W_1 \\ 0 & T & S_2 & W_2 \\ R & W_1^* & W_2^* & 1 - S_1 - S_2 \end{pmatrix} + O(\alpha^4) \quad (3.89)$$

where  $R, S_j$  and  $T$  are the usual terms defined in Eq. (3.43), (3.44) and (3.46), and we now define  $W_j$ :

$$W_j = \alpha_3 \alpha_j \langle g^{(3)} | V^{(3)} | g^{(3)} \rangle \int_{|\vec{p}| < 1/\Delta X} \frac{d^3 p}{(2\pi)^3} \left( \frac{e^{i\vec{p}\cdot\vec{x}_j} \langle 0 | U(\phi(\vec{x}_3)) a_{\vec{p}}^\dagger | 0 \rangle}{\sqrt{2E_{\vec{p}} \Delta E (E_{\vec{p}} + \Delta E)}} + \frac{e^{-i\vec{p}\cdot\vec{x}_j} \langle 0 | a_{\vec{p}} U(\phi(\vec{x}_3)) | 0 \rangle}{\sqrt{2E_{\vec{p}} E_{\vec{p}} \Delta E}} \right). \quad (3.90)$$

Finally, we compute the negativity of this density matrix perturbatively and we obtain  $N = \max \left( \sqrt{(S_1 - S_2)^2 + 4|R|^2} - S_1 - S_2, 0 \right) + O(\alpha^3)$ , namely the same result we would have obtained by setting  $\alpha_3 = 0$ , see Eq. (3.47). This shows that as long as the intruder is weakly interacting with the field, the amount of entanglement between the two detectors remains the same, at least at second order in perturbation theory. To some extent, this is simply because for the intruder to change the negativity it would need to interact with both detector 1 and detector 2 in the ground state, but in the weak interaction regime such a contribution is negligible. In fact, one can easily show that if  $\langle g^{(3)} | V^{(3)} | g^{(3)} \rangle \neq 0$  then the intruder causes a correction to the negativity of order  $O(\alpha^3)$  and if  $\langle g^{(3)} | V^{(3)} | g^{(3)} \rangle = 0$  then the correction to the negativity is of order  $O(\alpha^4)$ . In that sense, it could be interesting to calculate explicitly higher order corrections to see how the entanglement changes because of the intruder: a first step in that direction will be taken in the next subsection.

### 3.2.6 Potential barrier

This subsection is collaborative work with Andrzej Veitia. Let us now analyze the entanglement contained in a quantum field theory which is interacting with a classical potential. We consider a simple toy model which in the regime of non-relativistic quantum mechanics reduces to a usual weak potential problem. This toy model allows us to see what happens to the entanglement of the vacuum when the field is not entirely free.

#### The model

Our toy model consists of an additional term in the interaction Hamiltonian,

$$H_{int} = \sum_{j=1}^2 \alpha_j m^{(j)} \phi(\vec{x}_j) + m \int d^3 x V(\vec{x}) : \phi^2(\vec{x}) : \quad (3.91)$$

where we assume  $V(\vec{x}) \ll m$  and denote the additional term by  $H_{pot} = m \int d^3 x V(\vec{x}) : \phi^2(\vec{x}) :$ . Studies of this model or similar models can be found in the literature, see e.g. [75]

where the author calculated the vacuum energy in the presence of the classical background field  $V(\vec{x})$ . This model can be seen as an analog of quantum electrodynamics (QED) where the electromagnetic field is in a coherent state and therefore can be treated classically. Furthermore, to see why  $H_{pot}$  is a good toy model for a potential barrier, we look at the equation of motion for  $\phi$  with just the additional Hamiltonian  $H_{pot}$ :

$$-\ddot{\phi}(x) + \nabla^2\phi(x) - m^2\phi(x) - 2mV(\vec{x})\phi(x) = 0. \quad (3.92)$$

For a piecewise constant potential (like a rectangular potential barrier) we may use a mode decomposition such that  $\partial_t = -iE$  and  $\vec{\nabla} = i\vec{p}$ . This means that we have  $E^2 = \vec{p}^2 + m^2 + 2mV(\vec{x})$ , such that in the non-relativistic regime where  $|\vec{p}| \ll m$  we have:

$$E \approx \pm \left( \frac{\vec{p}^2}{2m} + m + V(\vec{x}) \right). \quad (3.93)$$

The positive solution is exactly what we would have obtained in non-relativistic quantum mechanics, with the exception of the constant mass term  $m$  which does not matter in first quantization. Thus, for non-relativistic momentum  $|\vec{p}| \ll m$ , a wave function that satisfies the Schrödinger equation with a piecewise constant potential  $V(\vec{x})$  will also satisfy Eq. (3.92). This shows that  $H_{pot}$  should be a good toy model to study the entanglement of the vacuum in the presence of a weak classical potential, and as a bonus we may be able to use some of our non-relativistic quantum mechanics intuition. Finally, note that when we compare Eq. (3.92) with Eq. (3.16) we see that a weak gravitational field is just the regular Newton potential  $V(\vec{x}) = -Mm/|\vec{x}|$  and a relativistic correction  $(-4M/|\vec{x}|)\nabla^2\phi(x)$  that implies that massless particles gravitate too.

### Corrections to the negativity

We are now ready to compute the ground state of the interacting theory, trace out the field and calculate the negativity. This can be a very tedious calculation since we have to go all the way to third order in perturbation theory. To simplify this calculation, we argue that once we trace out the field, there cannot be any new matrix elements in the matrix of Eq. (3.45). To see this, imagine we can solve exactly the quantum field theory with just the free Hamiltonian of the field and the Hamiltonian of the potential, that is  $H = H_F + H_{pot}$ . The resulting expression for the field would be very similar to Eq. (1.9) but the mode functions would now solve Eq. (3.92) instead of the regular Klein-Gordon equation. Then, using this field we would use time-independent perturbation theory to find the ground state of the full Hamiltonian, that is the Hamiltonian which includes the two detectors, and we would finally trace out the field. The matrix structure resulting from this procedure would obviously be the same as Eq. (3.45), thus as we expand this matrix in powers of  $V$  there are no new matrix elements. Therefore, all we have to do is to

compute the corrections to  $S_1$ ,  $S_2$  and  $R$  which are caused by the presence of the potential and use Eq. (3.47) to compute the negativity.

Let us first focus on the corrections to  $S_1$ , namely the local noise seen by detector 1 which can also be seen as the factor of the matrix element  $|e^{(1)}, g^{(2)}\rangle\langle e^{(1)}, g^{(2)}|$ . There are two terms at second order which end up contributing to  $S_1$ :

$$\begin{aligned}
& \sum_{k \neq g} \sum_{l \neq g} |e_k\rangle \frac{\langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_l)} \Big|_{S_1} = \\
& \alpha_1 m \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \int d^3 x V(\vec{x}) \frac{\left( \langle 0 | a_{\vec{p}_2} : \phi^2(\vec{x}) : a_{\vec{p}_1}^\dagger | 0 \rangle \langle 0 | a_{\vec{p}_1} \phi(\vec{x}_1) | 0 \rangle \right) a_{\vec{p}_2}^\dagger | e^{(1)}, g^{(2)}, 0 \rangle}{(E_{\vec{p}_1} + \Delta E)(E_{\vec{p}_2} + \Delta E)} \\
& + \alpha_1 m \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \int \frac{d^3 p_3}{(2\pi)^3} \int d^3 x V(\vec{x}) \\
& \times \frac{\left( \langle 0 | a_{\vec{p}_3} \phi(\vec{x}_1) a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger | 0 \rangle \langle 0 | a_{\vec{p}_1} a_{\vec{p}_2} : \phi^2(\vec{x}) : | 0 \rangle \right) a_{\vec{p}_3}^\dagger | e^{(1)}, g^{(2)}, 0 \rangle}{2(E_{\vec{p}_1} + E_{\vec{p}_2})(E_{\vec{p}_3} + \Delta E)}. \tag{3.94}
\end{aligned}$$

As this point it is useful to compute some matrix elements with the field, so using Wick's theorem [4] we find:

$$\langle 0 | a_{\vec{p}_1} \phi(\vec{x}_1) | 0 \rangle = \frac{e^{-i\vec{p}_1 \cdot \vec{x}_1}}{\sqrt{2E_{\vec{p}_1}}} \tag{3.95}$$

$$\langle 0 | a_{\vec{p}_2} : \phi^2(\vec{x}) : a_{\vec{p}_1}^\dagger | 0 \rangle = \frac{e^{i\vec{x} \cdot (\vec{p}_1 - \vec{p}_2)}}{\sqrt{E_{\vec{p}_1} E_{\vec{p}_2}}} \tag{3.96}$$

$$\langle 0 | a_{\vec{p}_2} a_{\vec{p}_3} \phi(\vec{x}_1) a_{\vec{p}_1}^\dagger | 0 \rangle = (2\pi)^3 \left( \frac{e^{-i\vec{p}_3 \cdot \vec{x}_1}}{\sqrt{2E_{\vec{p}_3}}} \delta(\vec{p}_2 - \vec{p}_1) + \frac{e^{-i\vec{p}_2 \cdot \vec{x}_1}}{\sqrt{2E_{\vec{p}_2}}} \delta(\vec{p}_3 - \vec{p}_1) \right) \tag{3.97}$$

$$\langle 0 | a_{\vec{p}_1} a_{\vec{p}_2} : \phi^2(\vec{x}) : | 0 \rangle = \frac{e^{-i\vec{x} \cdot (\vec{p}_1 + \vec{p}_2)}}{\sqrt{E_{\vec{p}_1} E_{\vec{p}_2}}}. \tag{3.98}$$

Once we trace out the field, Eq. (3.94) is combined with the first order term  $Q_1^{(3)}$  (Eq. (3.42)) such that the correction to  $S_1$  at third order is

$$\begin{aligned}
S_1 &= S_{1(0)} - \alpha_1^2 \frac{m}{2} \int_{|\vec{p}_1| < 1/\Delta X} \frac{d^3 p_1}{(2\pi)^{3/2}} \int_{|\vec{p}_2| < 1/\Delta X} \frac{d^3 p_2}{(2\pi)^{3/2}} \frac{\tilde{V}(\vec{p}_1 - \vec{p}_2) e^{-i\vec{x}_1 \cdot (\vec{p}_1 - \vec{p}_2)}}{E_{\vec{p}_1} E_{\vec{p}_2}} \\
& \times \left\{ \frac{1}{(E_{\vec{p}_1} + E_{\vec{p}_2})} \left[ \frac{1}{(E_{\vec{p}_1} + \Delta E)^2} + \frac{1}{(E_{\vec{p}_2} + \Delta E)^2} \right] \right. \\
& \left. + \frac{1}{(E_{\vec{p}_1} + \Delta E)(E_{\vec{p}_2} + \Delta E)} \left[ \frac{1}{(E_{\vec{p}_1} + \Delta E)} + \frac{1}{(E_{\vec{p}_2} + \Delta E)} \right] \right\} \tag{3.99}
\end{aligned}$$

where  $S_{1(0)}$  is the local noise seen by detector 1 without the potential (Eq. (3.44)) and we define:

$$\tilde{V}(\vec{p}) := \int \frac{d^3x}{(2\pi)^3} V(\vec{x}) e^{i\vec{p}\cdot\vec{x}}. \quad (3.100)$$

To make sure that we did not forget any contributions, we can take the potential  $V(\vec{x}) = h$  which models a correction to the mass such that  $m_{eff}^2 = m^2 + 2mh$ . A straightforward calculation can show that the expression we obtain for  $S_{1(1)}$  is the first order term in the Taylor expansion of  $S_{1(0)} \Big|_{m=m_{eff}}$ . For the correction to  $S_2$ , all we have to do is take  $S_1$  and exchange  $x_1$  for  $x_2$  and  $\alpha_1$  for  $\alpha_2$ .

Let us now focus on the corrections to  $R$ . For  $R$ , we are looking at corrections to the factor of the matrix element  $|e^{(1)}, e^{(2)}\rangle \langle g^{(1)}, g^{(2)}|$ . We need to consider two major contributions, the first one comes from a third order correction to the ground state, and the second one comes from a first order correction to the ground state. Let us start with the third order contribution. The third order correction to the ground state is:

$$\begin{aligned} |e_{g(3)}\rangle &= \sum_{n \neq g} \left[ - \sum_{k \neq g} \frac{|\langle e_k | H_{int} | e_g \rangle|^2 \langle e_n | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_n)} \left( \frac{1}{(E_g - E_n)} + \frac{1}{2(E_g - E_k)} \right) \right. \\ &\quad \left. + \sum_{k \neq g} \sum_{l \neq g} \frac{\langle e_n | H_{int} | e_k \rangle \langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_l)(E_g - E_n)} \right] |e_n\rangle - \langle e_{g(2)} | |e_{g(1)}\rangle |e_g\rangle. \end{aligned} \quad (3.101)$$

where  $|e_{g(j)}\rangle$  is the  $j$ th order correction to the ground state and  $|e_j\rangle$  is the  $j$ th eigenstate of the free Hamiltonian (i.e.  $|e_{j(0)}\rangle$ ). Because we trace out the field, and because we are only interested at third order contributions to  $R$ , we need a third order correction to the ground state of the form  $|0e^{(1)}e^{(2)}\rangle$ . Since by construction  $\langle 0 | H_{int} | 0 \rangle = 0$ , the only relevant contribution comes from:

$$|e_{g(3)}\rangle \Big|_R = \sum_{n \neq g} \sum_{k \neq g} \sum_{l \neq g} \frac{\langle e_n | H_{int} | e_k \rangle \langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_l)(E_g - E_n)} |e_n\rangle. \quad (3.102)$$

Three sequences of states in this sum give a non-vanishing contribution,

| $ e_l\rangle$   | $ e_k\rangle$   | $ e_n\rangle$                 |
|---|---|-------------------------------|
| $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$                 | $ g^{(1)}, e^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ | $ e^{(1)}, e^{(1)}, 0\rangle$ |
| $ g^{(1)}, g^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ | $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$                 | $ e^{(1)}, e^{(2)}, 0\rangle$ |
| $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$                 | $ g^{(1)}, e^{(2)}, 1_{\vec{p}}\rangle$                 | $ e^{(1)}, e^{(2)}, 0\rangle$ |

plus inversion of the detectors  $1 \rightleftharpoons 2$  and we also need to integrate over the various momenta. Hence, we have the third order correction:

$$\begin{aligned}
|e_{g(3)}\rangle\Big|_R &= -|e^{(1)}, e^{(2)}, 0\rangle \frac{\alpha_1 \alpha_2 m}{2} \int \frac{d^3 p_1}{(2\pi)^{3/2}} \int \frac{d^3 p_2}{(2\pi)^{3/2}} \frac{\tilde{V}(\vec{p}_1 - \vec{p}_2) e^{-i\vec{x}_2 \cdot \vec{p}_1 + i\vec{x}_1 \cdot \vec{p}_2}}{\Delta E E_{\vec{p}_1} E_{\vec{p}_2}} \\
&\times \left\{ \frac{1}{2} \left[ \frac{1}{(E_{\vec{p}_1} + \Delta E)} + \frac{1}{(E_{\vec{p}_2} + \Delta E)} \right] \left[ \frac{1}{(E_{\vec{p}_1} + E_{\vec{p}_2} + 2\Delta E)} + \frac{1}{(E_{\vec{p}_1} + E_{\vec{p}_2})} \right] \right. \\
&\left. + \frac{1}{(E_{\vec{p}_2} + \Delta E)(E_{\vec{p}_1} + \Delta E)} \right\}. \tag{3.103}
\end{aligned}$$

In the density matrix of the ground state of the detectors, this term gives a third order correction to  $R$  because it gets combined with the zeroth order term  $|g^{(1)}, g^{(2)}, 0\rangle$ .

The other relevant contribution comes from a first order term. Indeed, because of  $H_{pot}$  there is a first order correction to the ground state of the form  $mV|g^{(1)}, g^{(2)}, 1_{\vec{p}_1} 1_{\vec{p}_2}\rangle$ , which when we trace out the field is combined with the second order exchange term  $\alpha^2|e^{(1)}, e^{(2)}, 1_{p_1} 1_{p_2}\rangle$ . More explicitly, we have

$$\begin{aligned}
\sum_{k \neq g} |e_k\rangle \frac{\langle e_k | H_{int} | e_g \rangle}{(E_g - E_k)} \Big|_R &= -m \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \int d^3 x \\
&\times V(\vec{x}) \left[ \frac{\langle 0 | a_{\vec{p}_1} a_{\vec{p}_2} : \phi^2(\vec{x}) : | 0 \rangle}{2(E_{\vec{p}_1} + E_{\vec{p}_2})} \right] a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger |g^{(1)}, g^{(2)}, 0\rangle \tag{3.104}
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{k \neq g} \sum_{l \neq g} |e_k\rangle \frac{\langle e_k | H_{int} | e_l \rangle \langle e_l | H_{int} | e_g \rangle}{(E_g - E_k)(E_g - E_l)} \Big|_R = \\
&\alpha_1 \alpha_2 \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \int \frac{d^3 p_3}{(2\pi)^3} \left[ \frac{\langle 0 | a_{\vec{p}_3} a_{\vec{p}_2} \phi(\vec{x}_2) a_{\vec{p}_1}^\dagger | 0 \rangle \langle 0 | a_{\vec{p}_1} \phi(\vec{x}_1) | 0 \rangle}{2(E_{\vec{p}_3} + E_{\vec{p}_2} + 2\Delta E)(E_{\vec{p}_1} + \Delta E)} \right] a_{\vec{p}_3}^\dagger a_{\vec{p}_2}^\dagger |e^{(1)}, e^{(2)}, 0\rangle \\
&+ (\vec{x}_1 \rightleftharpoons \vec{x}_2) \tag{3.105}
\end{aligned}$$

such that together, after we trace out the field, these terms give a correction to the factor of the matrix element  $|e^{(1)}, e^{(2)}\rangle \langle g^{(1)}, g^{(2)}|$ . We can combine this correction with the third order contribution, and we obtain an overall correction to  $R$ :

$$\begin{aligned}
R &= R_{(0)} - \alpha_1 \alpha_2 \frac{m}{2} \int_{|\vec{p}_1| < 1/\Delta X} \frac{d^3 p_1}{(2\pi)^{3/2}} \int_{|\vec{p}_2| < 1/\Delta X} \frac{d^3 p_2}{(2\pi)^{3/2}} \frac{\tilde{V}(\vec{p}_1 - \vec{p}_2) e^{-i\vec{x}_2 \cdot \vec{p}_1 + i\vec{x}_1 \cdot \vec{p}_2}}{E_{\vec{p}_1} E_{\vec{p}_2} \Delta E} \\
&\times \left\{ \frac{1}{(E_{\vec{p}_1} + E_{\vec{p}_2})} \left[ \frac{1}{(E_{\vec{p}_1} + \Delta E)} + \frac{1}{(E_{\vec{p}_2} + \Delta E)} \right] + \frac{1}{(E_{\vec{p}_1} + \Delta E)(E_{\vec{p}_2} + \Delta E)} \right\}. \tag{3.106}
\end{aligned}$$



Here again, one can easily verify with the potential  $V(\vec{x}) = h$  that our perturbative solution matches the Taylor expansion of the exact solution.

### Example: 1-d rectangular barrier potential

As an example of the machinery developed, let us look at the entanglement of the vacuum in a one dimensional space when we have a rectangular potential barrier,

$$V(x) = \begin{cases} 0 & \text{when } |x| > a/2, \\ h & \text{when } |x| < a/2 \end{cases} \quad (3.107)$$

where  $h \ll m$ . For simplicity we assume  $\alpha_1 = \alpha_2 = \alpha$  and we also assume that the detectors' positions are  $x_1 = -L/2$  and  $x_2 = L/2$ . Using Eq. (3.107) in Eq. (3.99) and in Eq. (3.106) we find:

$$\begin{aligned} S &= S_{(0)} - \alpha^2 m h \int \frac{dp_1}{2\pi} \int \frac{dp_2}{2\pi} \left[ \frac{1}{E_{p_1} E_{p_2}} \frac{\sin((p_1 - p_2) a/2) e^{iL(p_1 - p_2)/2}}{p_1 - p_2} \right. \\ &\quad \times \left\{ \frac{1}{(E_{p_1} + E_{p_2})} \left[ \frac{1}{(E_{p_1} + \Delta E)^2} + \frac{1}{(E_{p_2} + \Delta E)^2} \right] \right. \\ &\quad \left. \left. + \frac{1}{(E_{p_1} + \Delta E)(E_{p_2} + \Delta E)} \left[ \frac{1}{(E_{p_1} + \Delta E)} + \frac{1}{(E_{p_2} + \Delta E)} \right] \right\} \right] \quad (3.108) \\ R &= R_{(0)} - \alpha^2 m h \int \frac{dp_1}{2\pi} \int \frac{dp_2}{2\pi} \left[ \frac{1}{E_{p_1} E_{p_2} \Delta E} \frac{\sin((p_1 - p_2) a/2) e^{-iL(p_1 + p_2)/2}}{p_1 - p_2} \right. \\ &\quad \left. \times \left\{ \frac{1}{(E_{p_1} + E_{p_2})} \left[ \frac{1}{(E_{p_1} + \Delta E)} + \frac{1}{(E_{p_2} + \Delta E)} \right] + \frac{1}{(E_{p_1} + \Delta E)(E_{p_2} + \Delta E)} \right\} \right]. \quad (3.109) \end{aligned}$$

With the above expressions we can then evaluate the negativity with Eq. (3.47). We cannot evaluate these expressions analytically, but we can perform numerical integration to evaluate them. We first look at the regime which has little entanglement in the absence of the potential, that is  $\Delta E \sim m$ , see Fig. (3.5). In this regime, we see that in the region  $a < L$ , the negativity dies and revives in an oscillatory manner. Then near  $a \approx L$  the negativity jumps to a new level and as  $a \gg L$  its oscillations around the equilibrium value decrease. We therefore see that to maximize the entanglement between the two detectors in the regime  $\Delta E \sim m$ , we need the potential to have a width of size  $a \gtrsim L$ . To interpret this phenomenon, we look at the terms  $R$  and  $S$  individually. Intuitively, a potential barrier should reduce the exchange term  $R$  because it almost blocks the exchange of virtual

particles. This is indeed what we see on Fig. (3.5). The entanglement also depends on the local quantum field fluctuations  $S$ , which is somehow greatly reduced by the potential once the detectors are immersed in the potential. This can be interpreted as an effective mass change. Indeed, recall that a constant flat potential models a small perturbation to the mass of the field, so when  $a \ll L$  we have  $m_{eff} \approx m$  and when  $a \gg L$  we roughly have  $m_{eff} \approx m + h$ . Thus, since a bigger mass makes the cloud of particles around each detector more energetically expensive, a bigger mass reduces the local noise. Indeed, in a second quantization framework, the mass term really matters because we allow the creation of particles and each one of them costs energy  $E_{\vec{p}} = \sqrt{\vec{p}^2 + m_{eff}^2}$ . We thus conclude that the entanglement of a one dimensional vacuum is enhanced by a classical potential barrier in the regime  $\Delta E \sim m$ . Let us now look at the regime  $\Delta E \ll m$ , which is to say the fully non-relativistic regime. The numerical results of Fig. (3.6) show that  $S$  and  $R$  keep the same shape, but as expected from energy considerations, in that regime the exchange term dominate over the noise term, both the zeroth order term and the first order correction. We therefore find that the negativity decreases with an increase of  $a$ , see Fig. (3.6). Note that the negativity never dies because the initial negativity is much bigger than the decrease caused by the potential. Thus, we conclude that in the regime  $\Delta E \ll m$  the entanglement of a one dimensional vacuum decreases because of the classical repulsive potential.

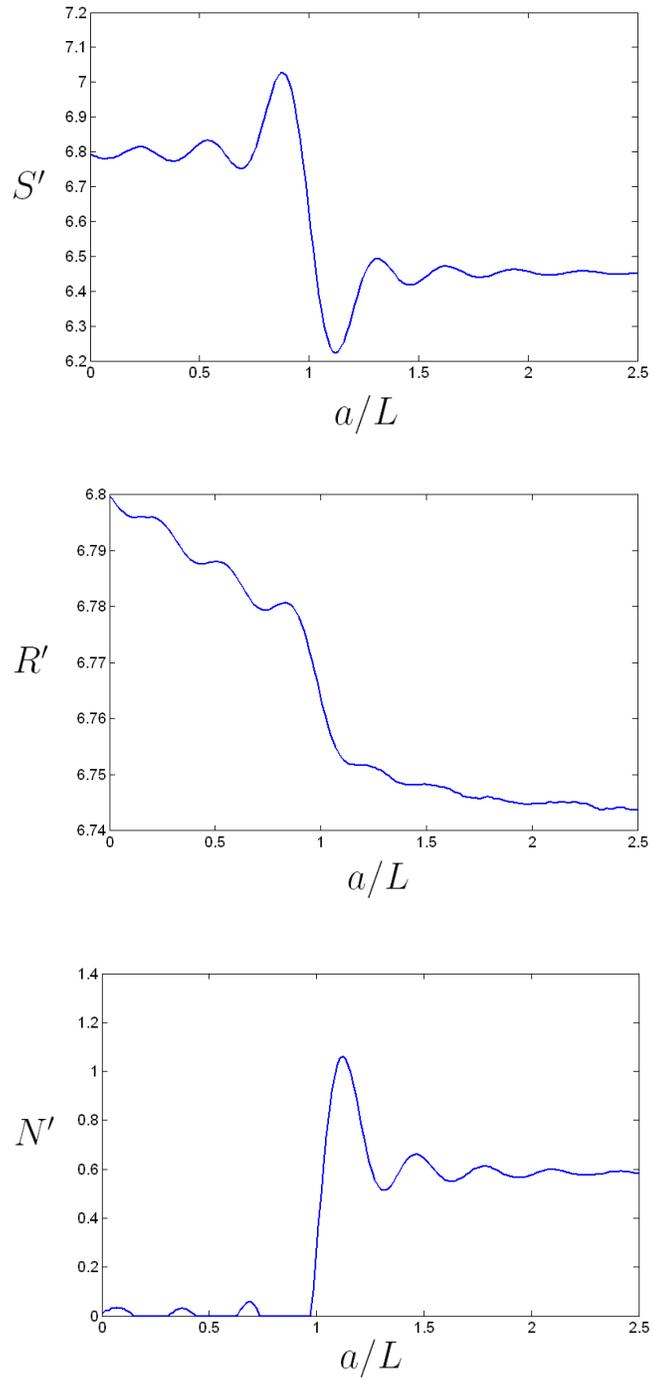


Figure 3.5: The local noise  $S' = 8\pi^2 S/\alpha^2$ , the exchange term  $R' = 8\pi^2 R/\alpha^2$  and the negativity  $N' = 8\pi^2 N/\alpha^2$  as a function of  $a/L$  with  $\Delta E/m = 0.47$  and  $h/m = 0.0025$ .

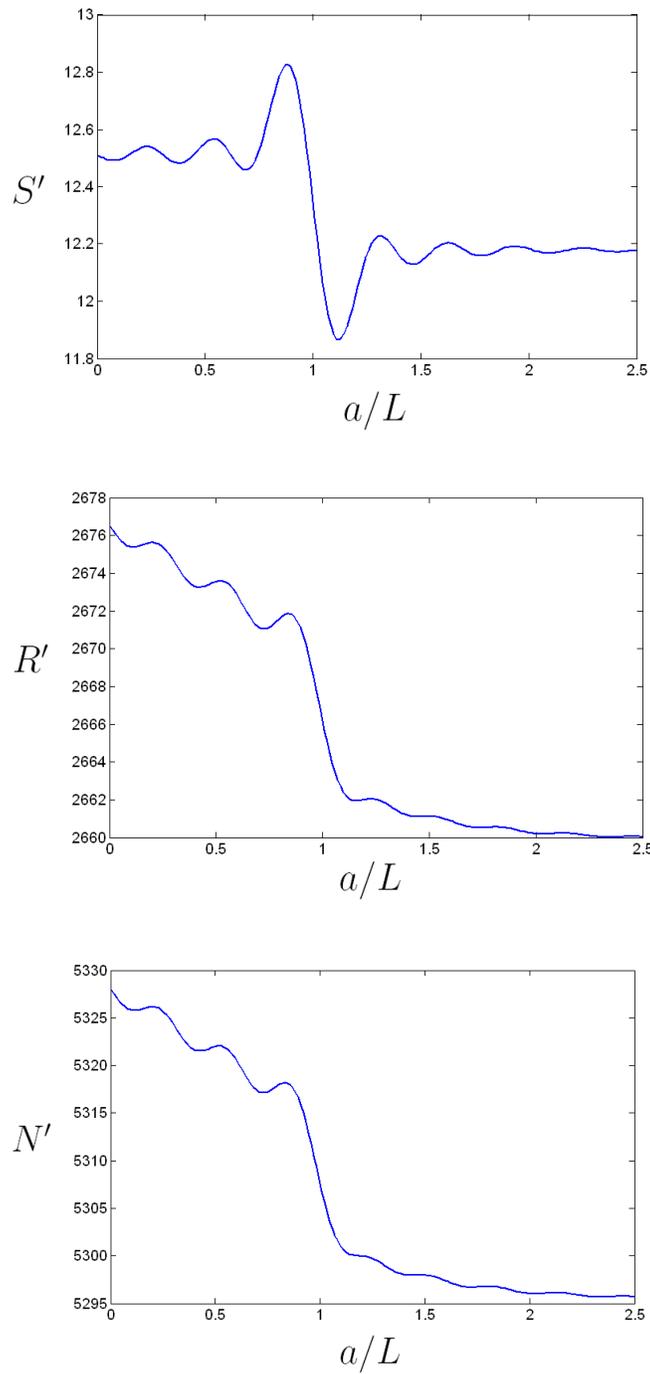


Figure 3.6: The local noise  $S' = 8\pi^2 S/\alpha^2$ , the exchange term  $R' = 8\pi^2 R/\alpha^2$  and the negativity  $N' = 8\pi^2 N/\alpha^2$  as a function of  $a/L$  with  $\Delta E/m = 0.002$  and  $h/m = 0.0025$ .

# Chapter 4

## Conclusion

In this thesis we analyzed how both information and entanglement propagate between two Unruh-DeWitt detectors. We first modeled a relativistic quantum channel using two Unruh-DeWitt detectors. This channel incorporates both special relativity effects as well as the fundamental noise caused by quantum field fluctuations. A throughout analysis of this channel was performed using quantum information tools. Using this channel, we showed that while information is bounded by the speed of light, entanglement propagation is not bounded by any finite speed. We then showed that the entanglement of the vacuum increases in the presence of a weak gravitational field while it decreases when the field is subject to Dirichlet boundary conditions. We also showed that depending on the initial amount of entanglement, a classical potential can either increase or decrease the entanglement of a one dimensional vacuum.

Most of our analysis was done with a quantum scalar field. While we expect that all our results would still hold in a proper QED framework, it should be interesting to redo some of our calculations in QED. In fact, a direct experimental verification would first require us to go to a full QED treatment, with proper atoms coupled to the electromagnetic field. Another perhaps more promising experimental possibility is to use a quantum field analog such as a linear ion trap [76, 64]. In this context, Dirichlet boundary conditions are already effectively implemented because of the finite number of ions and we could implement a classical potential by introducing an electric field. It should be interesting see what happens when we consider other kinds of boundary conditions such as periodic boundary conditions because this could easily be simulated with a circular arrangement of ions.

It should also be straightforward to generalize our study to detectors with any number of energy levels. The number and spacing of the energy levels of the detectors should translate into an effective alphabet size. This should also allow one to generalize the results of [77, 78], where it was shown how quantum noise imposes a natural bound to the

capacity of an otherwise noiseless bosonic channel. The analysis of [77, 78] employed the time-energy uncertainty principle to describe the limit to the distinguishability of photons of energy difference  $\Delta E$  in an observation time  $\Delta t$ . It should be interesting to re-analyze these results within the present information-theoretic framework of the quantum channel in which all effects of quantum noise are built in from the start.

It should also be interesting to generalize our model to yield a new approach to analyzing the setup of [79, 80], where Alice and Bob are inertial observers which are exchanging modes of a quantum field, while Eve is accelerating and tries to intercept the message. It was shown there that, because of the Unruh effect, it is always possible for Alice and Bob to communicate privately. To show this, the approach to the Unruh effect using Bogoliubov transformations was used. Generalizing our setup, one may use Unruh-DeWitt detectors, which are known to allow a more flexible description of the Unruh effect. For example, Eve would not have to accelerate uniformly and could indeed take an arbitrary trajectory.

The model we studied here should also be generalizable to any curved spacetime to study, for example, the impact of spacetime expansion and horizons. Indeed, as we previously discussed it should be very interesting to see whether the entanglement of the vacuum still persists when the detectors are near a black hole and see the Hawking radiation as a source of local noise. Finally, let us recall that, in the presence of a suitable natural ultraviolet cut-off, the density of degrees of freedom in quantum fields is finite, see e.g. [81]. It should be interesting to investigate how this finite density of degrees of freedom translates into a finite information carrying capacity of quantum fields.

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