

AN EVALUATION OF THE EFFECTIVENESS OF A COGNITIVE LOAD BASED
TEACHING METHOD IN A MIXED ABILITY GRADE 9 CLASS, WITH SPECIAL
ATTENTION TO LEARNERS' ATTITUDES AND ENGAGEMENT

by

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Declaration

I declare that the project "*An evaluation of the effectiveness of a cognitive load based teaching method in a mixed ability grade 9 class, with special attention to learners' attitudes and engagement*" is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

.....

JOANNE DAVID

.....

DATE

Dedication

To Gryff and Rory, who, with remarkable patience and fortitude, put up with a mother who is a perennial and absent-minded student. I will love you always.

Acknowledgements

Many thanks to Lowveld High School for harbouring me, inspiring me and permitting me to conduct my research, and to my supervisor, Dr Ronel Paulsen, who remembered and encouraged me over the years.

Abstract

Abbreviations and Key Terms

ANA	Annual National Assessment
ANOVA	Analysis of variance
CAPS	Curriculum and assessment policy statement
Class Dojo	Online behaviour tracker for motivating and encouraging positive behaviour
CLT	Cognitive load theory
DBE	Department of Basic Education
Dinaledi schools	Schools selected for increasing learner performance in mathematics and science, part of a programme introduced in 2001 by the National Department of Education
Epoch and Optima Trusts:	Independent trusts founded by the Anglo American Group to further excellence in mathematics education
Eskom	Public utility company providing electricity to South Africa
JET Education Services:	Previously the Joint Education Trust, an independent, non-profit organisation dedicated to improving the quality of education in South Africa
matric	The final year of secondary school. At the end of this year, matriculants write the National Senior Certificate examinations.
NBI	National Business Initiative
NEEDU	National Education Evaluation and Development Unit
NSC	National Senior Certificate
NSMSTE	National Strategy for Mathematics, Science and Technology Education
quintile	South African schools are grouped according to socioeconomic status. Learners in the poorest schools, Quintiles 1, 2 and 3, do not pay school fees.
SACMEQ	Southern and Eastern African Consortium for Monitoring Educational Quality

SGB	School governing body: a statutory body of parents, educators, non-teaching staff and learners with clearly defined powers relating to certain aspects of school governance.
SOS Children's Villages:	Privately funded charitable homes for orphaned and abandoned children.
TIMSS	Trends in International Mathematics and Science Study
ZPD	Zone of proximal development
Intermediate phase	The middle years of primary school, Grades 4 to 6

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CHAPTER 1: Introduction

1.1 Background

1.1.1 Skills shortages in South Africa

In November 2014, Eskom – the South African public utility that generates electricity – re-introduced load shedding, a system of rolling blackouts that reduces electricity demand when system capacity cannot meet production targets. Problems with electricity supply are not new; Eskom has been struggling with skills shortages and a loss of expertise for years (Coetzee, Claassen, de Lange, & Tongha, 2014).

The problems at Eskom form part of a larger picture. The Centre for Development and Enterprise report on a survey of 40 companies (Centre for Development and Enterprise, 2007) makes it clear that the South African skills shortage is not just an inconvenience but affects the productivity, economic growth and global competitiveness of the country as a whole. The companies interviewed for the report feel that South Africa is unable to keep up with demand for skilled workers because the failure of education at all levels is at “crisis proportions”. Furthermore, many companies claim that not only do school leavers not have a quality education in mathematics, science and language skills, but they also have no work ethic and lack the capacity to be trained. Strong words, and an indictment of our schooling system.

Rasool and Botha (2011) also cite low education standards as a factor contributing to the skills shortage and highlight mathematics as a key subject. The top ten occupations in highest demand (South Africa, 2014) require mathematics, as do most of the others in the top 50; unsurprisingly, this White Paper identifies mathematics, science and language capabilities in schools as one of the five priority areas for immediate attention.

1.1.2 National Senior Certificate results and university performance

Externally set and moderated, monitored nationally and widely reported in the press, the matric examinations are the culmination of high school education. Learners who pass these examinations receive a National Senior Certificate and the quality of their results determine the type of certificate: Higher Certificate being the lowest, Diploma pass, and Degree Pass.

About one quarter of Degree Passes – in theory allowing admission to university – will not meet the minimum requirements at any university. Similarly roughly 60% of those with a Diploma pass will not gain entry to any Diploma course (Simkins, 2013). In South Africa only around 16% of young people attend university, well below the 20% benchmark for

middle-income developing countries (Daniels, 2007). And even once admitted to university, many students drop-out without a qualification: of those enrolled for a three-year degree less than 30% complete their course in the expected three years, and only around half ever graduate (Simkins, 2013).

Dugmore (2011) also comments on the drop-out rate at university, and particularly on the inability of matrics who have passed mathematics and science to cope with these subjects at a university level. She accuses the Department of Education of responding to poor mathematics and science results by lowering assessment criteria. The proportion of matrics obtaining at least a 40% pass in mathematics has remained dismally low over the past seven years (see Figure 1 below).

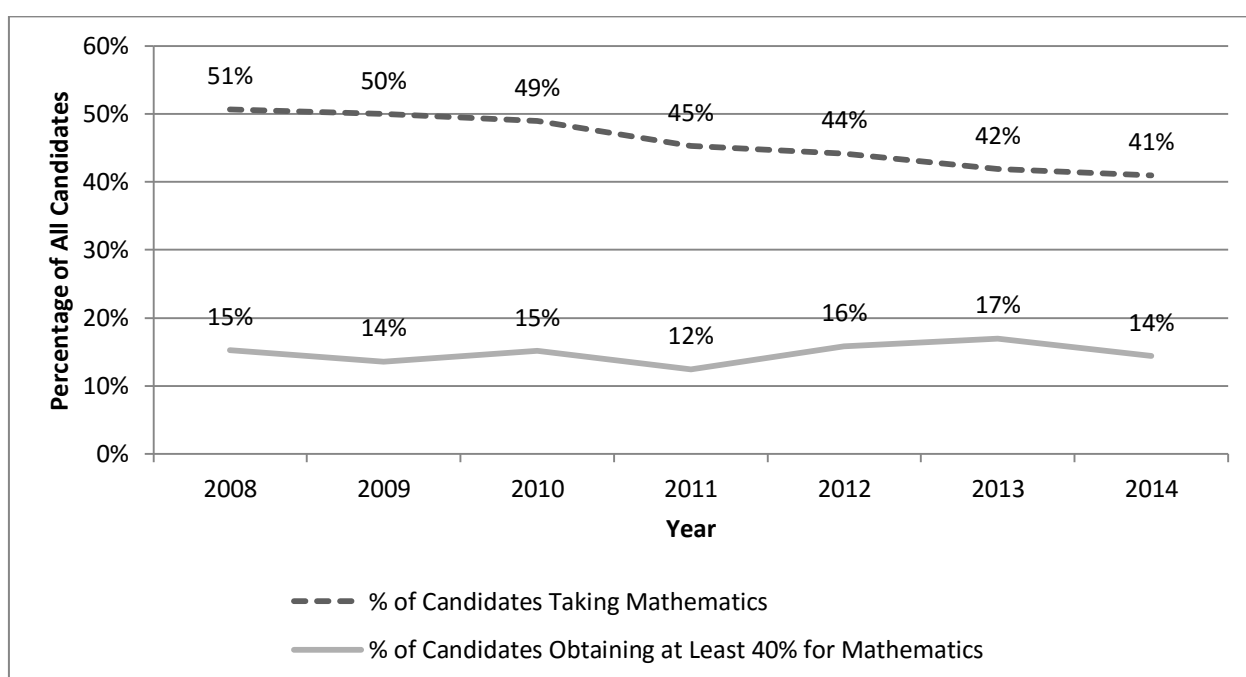


Figure 1: NSC mathematics outcomes (based on data from Department of Basic Education, 2014a)

In 2014, of the 550 127 matric candidates, only 20 066 (around 4%) obtained 50% or more in mathematics (Department of Basic Education, 2014a). This provides a very small pool of candidates for STEM degrees and supports the contention that the schooling system does not produce sufficient quality matriculants to address the skills shortage. But matric results are only a small part of the overall picture.

1.1.3 Socioeconomic status

South African schools are grouped according to socioeconomic status. Each group contains 20% of all learners; quintile 1 schools are the poorest, while quintile 5 are the least poor. Schools in the lowest three quintiles are deemed to be non-fee paying schools

and receive more financial support from the government, whereas schools in quintiles 4 and 5 are allowed to raise fees from the parent body.

Quintile 1, 2 and 3 schools are distinctly different from quintile 4 and 5 schools, and this bimodality persists across all subjects and from the early grades through to matric. The lower quintile schools tend to be black schools in rural areas, and despite efforts to provide sufficient resources, they are characterised by a lack of professionalism and a poor work ethic on the part of teachers, interference in teaching and learning on the part of unions and a general failure of curriculum completion, all of which contribute to learners who are mathematically years behind their peers (Spaull, 2013). The chance of a child having to repeat a year before Grade 7 is over 30% higher in low socioeconomic status schools (Zoch, 2015).

Additionally there are problems of lack of learner discipline, social inertia, fear of education, and low cognitive demand. These children of low socioeconomic status parents may be inherently limited by genetics or indirectly limited by their parents inability to offer the necessary support and encouragement, and do not tend to value education or school quality highly (Zoch, 2015).

1.1.4 Benchmark testing

1.1.4.1. TIMSS

Trends in International Mathematics and Science Study (TIMSS) is a standardised assessment of the science and mathematics knowledge of learners around the world. South African Grade 8 learners participated in TIMSS for the first time in 1995, coming a resounding last; by 2011 TIMSS testing had been phased across to Grade 9 learners but South African learners were still third from the bottom (Reddy, 2011). The 2011 results showed an overall improvement of 70 score points among the poorest schools – equivalent to about one and a half grade levels – but this was not reflected throughout the system (Reddy, 2011).

Quintile 5 schools are well-resourced: they have good infrastructure, qualified teachers and on the whole embrace a culture of teaching and learning. The more privileged learners at these schools did not show an overall improvement in scores in TIMSS 2011. Despite their advantages, they still do not achieve at a level comparable to developed countries (Spaull, 2013) and according to Reddy (2011) “the *most proficient* learners in South Africa approached the *average* [emphasis added] performance in Singapore, Chinese Taipei, Republic of Korea Japan, Finland, Slovenia and Russian Federation.” (p. 5).

The characterisation of South Africa's education system as being in a state of crisis is reflected in this: the average Grade Nine child performed between two and three grade levels lower than the average Grade Eight child from other middle-income countries (Spaull, 2013). Three-quarters of Grade 9 learners were unable to demonstrate understanding of basic concepts such as whole numbers, decimals and simple graphs.

1.1.4.2. ***Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ)***

South Africa participated in the SACMEQ assessment of literacy and numeracy among Grade 6 learners in southern and eastern Africa in 2000 (SACMEQ II) and 2007 (SACMEQ III). In 2000, South Africa's Grade 6 learners ranked ninth for mathematics out of the 14 participating countries, and in 2007 South Africa's learners ranked eighth, despite an expenditure per pupil that was almost the highest of all participating countries (van der Berg, 2008).

Of around 9000 learners who participated, 40% were deemed functionally innumerate (Spaull, 2013). Most learners attained only Level 2 competency (Emergent Numeracy), a result that remained the same even when considering only urban schools, or only schools of higher socioeconomic status. Overall, around 8% of Grade 6 learners were assessed at a level of "Beginning Numeracy" and although this proportion was considerably higher in low socioeconomic status schools (10,7% as opposed to 5% in high socioeconomic status schools) it is clear that the problems in mathematics learning are not only linked to socioeconomic status or school location (Moloi & Strauss, 2005).

More worrying still, between 2000 and 2007 there was no significant improvement in the performance of South African learners, although other participating countries such as Namibia and Tanzania had improved substantially (Spaull, 2013).

1.1.4.3. ***Annual National Assessment***

The Annual National Assessment (ANA) was initiated by the Department of Basic Education in 2011 to benchmark the progress of learners from Grade 1 to Grade 9 in the key foundational skills of literacy and numeracy (Department of Basic Education, 2013). The 2012 results showed that although in Grade 1 the average pupil had mastered 68% of the curriculum, this fell off rapidly (41% in Grade 3; 37% in Grade 4; 30% in Grade 5 and a mere 27% in Grade 6) (Department of Basic Education, 2013). Mathematics is a subject that requires development over time as a child constructs a network of interacting knowledge, ideas and concepts, and the subject becomes increasingly difficult (Hoadley & Jansen, 2009). Children who have fallen behind in mathematics in primary school find themselves faced with an insurmountable task in attempting to keep up with the pace of

work in high school (Spaull, 2012). In the light of the ANA results from the Intermediate phase, it is hardly surprising that the Grade 9 results are even worse: in 2014 the average mathematics score was 11%, 90% of the learners countrywide attained less than 30% and only 2,9% of Grade 9 learners achieved over 50%. Even in the most privileged schools the average ANA mathematics score for Grade 9s was only 21,6% (Department of Basic Education, 2014c).

1.1.5 Primary schooling

In the words of Spaull (2013, p. 8) “All of the available evidence suggests that many South African children are acquiring debilitating learning deficits early on in their schooling careers and that this is the root cause of underperformance in later years.”

In international education research, it is consistently shown that the achievement of learners is related to the “content and skills that are actually made available to learners in the classroom” (Reeves & Muller, 2005, p. 103). It is a feature of South African classrooms – particularly in rural and poor schools – that the pace of work is extremely slow: “[teachers] sometimes spend a whole lesson ... talking about two or three maths problems” (Taylor, 2009 p. 24).

Around half of Grade 6 teachers could correctly answer only two out of five questions based on the Grade 6 curriculum (Simkins, 2013), and according to Spaull (2013) South African primary school teachers are among the least knowledgeable even within the sub-Saharan region. Perhaps this is why teachers tend to spend more time on topics that they know well (Mji & Makgato, 2006). The end result is an accumulation of gaps in curriculum coverage (Simkins, 2013); even in Grades 5 and 6, learners are spending too much time on curricular content that should have been covered in previous years (Reeves & Muller, 2005).

1.2 Research context

The crisis in South African education is by no means being ignored. The very existence of the Annual National Assessment, bodies such as National Education Evaluation and Development Unit (NEEDU), programs such as Schooling 2025, organisations like JET Education Services and funding initiatives such as the Epoch and Optima Trusts all reflect a huge commitment from both government and industry to address the issues. But change in education is incremental and takes time, and many of these interventions will take years before the benefits are felt at matric level.

At the end of Grade 9, learners at South African schools select their subjects for matric. Every learner must choose between Mathematics (“pure” mathematics) and Mathematical

Literacy (a form of consumer mathematics intended to ensure that all matriculants are sufficiently numerate for the modern world). In order to pursue a career in science, technology, engineering or mathematics, learners need to continue with Mathematics.

Many schools review each learner's performance in Grade 9 and may advocate (sometimes vigorously) that a poorly-performing learner is not suited to studying Mathematics. Other schools deliberately steer even quite capable learners towards Mathematical Literacy in order to boost their matric results, while some, especially smaller schools in rural areas, may not be able to offer Mathematics as they have no teacher (Nkosi, 2014). The net result is that between 2008 and 2014, the number of matriculants writing mathematics declined from nearly 300 000 to around 225 000 – a decrease of almost 25% (Department of Basic Education, 2014b; Taylor, 2011).

Grade 9 learners have had sufficient exposure to algebra and geometry to be aware of some of the demands and implications of the abstract nature of pure mathematics. It is also during this year that the vertical nature of mathematical learning – new skills building upon those previously learned – becomes critical. As we saw in the discussion of the ANA above, most learners fail to master the mathematics content. But the Grade 9 year can also be the last-ditch fight to shore up learners' weaknesses in mathematics performance and provide enough groundwork to allow them to succeed in Grade 10.

Grade 9 can be a difficult year to teach in terms of discipline and learning focus (Theron & Dalzell, 2006) motivating learners to do their best and to engage with the content on more than a superficial level, while at the same time developing in them a love of thinking and reasoning can be arduous. It has been called the make-or-break grade (McCallumore & Sparapani, 2010; The Consortium on Chicago School Research, 2007; Willens, 2013), and in South Africa it is a critical point for intervention in an effort to improve mathematics outcomes at matric level.

1.3 Research objectives and questions

The objectives of this study were to evaluate the application of a teaching method based on cognitive load theory (Sweller, 1994) in a mixed-ability class of South African Grade 9 learners in a quintile 5 school, and to determine whether these methods could improve the cognitive engagement and motivation of learners while retaining or improving performance as measured by traditional evaluation (class tests). Cognitive load-based methods have been shown to be effective in a wide variety of applications internationally (Sweller, van Merriënboer, & Paas, 1998; Chandler & Sweller, 1991; Pachman, Sweller, & Kalyuga, 2014 and others).

If the teaching method is successful, it can potentially be applied in any classroom since it requires only motivation, basic skills and learners who are willing and able to cognitively engage with the material.

1.3.1 Questions being addressed

- What are the attitudes of this group of Grade 9 learners towards mathematics, both as a school subject and as a skill?
- How do these learners approach learning mathematics: are they active, cognitively engaged and self-directed in class, or merely recipients of information?
- What is the impact of using a cognitive load-based teaching method in terms of learners' academic performance?

CHAPTER 2: Literature Review

2.1 Learner engagement

Many South African teachers complain about a lack of work ethic among learners. A lot of what is meant by *work ethic* in a school environment can be embodied in the term “engagement”. Fredricks, Blumenfeld, & Paris (2004) describe engagement in terms of three components:

- Behavioural engagement refers to a learner’s involvement in academic and co-curricular activities and centres on the idea of participation. Indicators of lack of behavioural engagement are missing school, bunking classes, a lack of involvement in anything to do with school and disruptive behaviour of varying degrees. This behavioural disengagement that Finn (1989) calls “withdrawing from school”, increases the risk of a learner failing grades or dropping out of school. Positive behavioural engagement goes beyond merely following the rules and includes such dimensions as effort, persistence, involvement in class discussions and asking questions.
- Emotional engagement is linked to students’ affective reactions to school and to the classroom. It includes whether the learner reacts positively or negatively to teachers, is bored or interested, happy or sad, calm or anxious and affects a learner’s willingness to complete assigned tasks.
- Cognitive engagement is “the student’s psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote” (Newmann et al., 1992, p. 12). It goes beyond mere compliance or good behaviour and includes the exertion of sufficient mental effort to allow the learner to understand complex ideas and master difficult skills (Newmann, 1992).

Many learners put in only a superficial effort at school: passive listening in class, assignments that are sloppy or never submitted, minimal participation in discussions, cheating in tests and examinations. These learners may not present notable behaviour problems, they attend class dutifully but without “excitement, commitment or pride in mastery of the curriculum” (Newmann et al., 1992, p. 2). This lack of cognitive engagement cannot engender meaningful learning and results in only enough shallow knowledge to pass a few tests and, usually, reach the next grade (Newmann et al., 1992). There is undoubtedly a reduced educational benefit to society, although this is difficult to measure (Finn, 1989).

Part of the reason for this lack of effort is that the learners are not challenged. If they pay attention and do their work, superficial learning is enough; there is no incentive for self-regulation, intense effort, or the flexible use of knowledge – those engagement strategies that will help them to understand the material (Fredricks et al., 2004).

2.1.1 Measuring engagement

Behavioural engagement can be measured by accessing student record data such as detentions, demerits and involvement in extracurricular activities; by observational techniques or by surveying learners or teachers with items such as “I ask questions to get help” (learners) and “Student participates actively in class discussions” (teachers). Measures of emotional engagement have concentrated mostly on questionnaires and surveys (Fredricks et al., 2004).

Some information on cognitive engagement can be obtained by observation, for example of whether learners are attentive and enthusiastic, doing assignments, or off-task. Jimerson, Campos, & Greif (2003) claim that most attempts to measure cognitive engagement refer to items such as academic performance, test results and homework completion, often collected through teacher surveys or school records. However, Newmann et al. (1992) admit that cognitive engagement is not easily observed since it describes inner processes such as concentration – watching a child with his head over his book looking at a mathematics problem, it is not possible from external evidence whether he is selecting between solution strategies or thinking about motorbikes. Furthermore they acknowledge that cognitive engagement is not an “on-off” construct, but rather a continuum from completely disengaged to a flow state of total immersion.

Survey items such as “flexible problem solving, preference for hard work, independent work styles, and ways of coping with perceived failure” (Fredricks et al., 2004 p. 67) have been proposed to measure cognitive engagement. In addition, research on intrinsic motivation and goal theory suggests that learners who are more concerned with mastering the content (mastery goals) than with getting good marks (performance goals) are likely to be more cognitively engaged.

2.1.2 Improving engagement

“Engaged students make a psychological investment in learning. They try hard to learn what school offers. They take pride not simply in earning the formal indicators of success (grades), but in understanding the material and incorporating or internalizing it in their lives” (Newmann et al., 1992, p. 3). Clearly engagement is a desirable state, and particularly in the mathematics classroom it is essential for the cycle of *learning-attempting-correcting mistakes-starting over again* that a learner must follow to master the

content (Newmann et al., 1992). An important dimension of engagement is controlled by the learner himself, but research suggests that engagement can be improved by variations in the child's environment. Furthermore once a learner has become more engaged, the successful outcomes will lead to a self-replicating cycle, what Fredricks et al. (2004) call an "evolution in intensity" (p. 61).

There are, however, elements in the learning environment that can encourage or hinder engagement. Firstly the school itself should embody clear educational purpose, fairness, personal support, the opportunity for success, and a caring environment for the learner (Newmann, 1992). Furthermore, Newmann et al. (1992) emphasize the importance of the type of classwork that learners have to do: trivial problems, useless busywork and nonsensical tasks will not suffice. Engagement is more likely be elicited if learners are given problems that offer significant challenges, that are meaningful, interesting, involve some fun, result in extrinsic rewards and are related to the real world. Furthermore, their teacher should urge them to develop a thorough understanding of what they are learning, and support this with instruction as necessary. Newmann (1992) also suggests engagement can be improved if the teacher allows learners to work together within a flexible time-frame and gives them clear feedback promptly after a task is completed.

Tertiary educators complain of "teaching to the 'silent classroom', in which students write down everything that is said while at the same time sitting quietly, disengaged, without anything to contribute" (Gimenez as cited in Raelin, 2009, p. 407). From an early age, in most classrooms, learners are inculcated with the dictum that they should sit down, be quiet, listen to the teacher, do as directed, use the given method. This results in passive learners who feel they have no control over their own learning, and who cannot develop their own ideas but instead wait for their teachers to interpret and deliver carefully pre-digested concepts to them. Learners who come from backgrounds where their parents are themselves not educated are particularly vulnerable to this "spoon feeding"; educated parents are more inclined to encourage children to question ideas and challenge assumptions.

Short-circuiting is a particularly invidious aspect of spoon-feeding. It is usually presented as a learning tool such as a summary of a chapters, a diagram, an acronym such as "FOIL", or steps such as "take it across the = and change the sign". Short-circuiting can assist low-ability learners to complete tasks which they would otherwise not manage, however high ability students should be allowed to develop their own shortcuts based on understanding (Corno & Mandinach, 1983). If the teachers do the thinking and processing, the learners are trained to become passive recipients: they do not have to decide what is important, or figure out a good way of doing something – they only have to remember and

regurgitate. The result is a message that the classroom does not involve thinking and understanding, but that learning is by rote, and that the learner must wait for enlightenment to be handed down from the teacher (Corno & Mandinach, 1983).

2.1.3 The Pygmalion effect

Newmann et al., (1992) emphasise the need for teachers to raise their expectations; higher demands are more likely to enhance engagement and promote the learners' sense of ownership of the material. Engagement is not improved by "easy bargains" (p. 33): a "sense of success will not be achieved by grade inflation or reducing the rigor of academic demands" (p. 22). In a similar vein, self-fulfilling prophecy or the Pygmalion effect (Rosenthal & Jacobson, 1968) predicts that low expectations result in low outcomes.

In order to ensure that as many young people as possible complete matric, the Department of Education has ruled that a learner can pass with only 30% in many subjects. By setting the bar so low they communicate low expectations to the learners and give them the idea that they are achieving something by learning *less than one third* of the examined content of a subject. Furthermore, since many learners consider school to be boring and irrelevant they do the minimum to get by (Fredricks et al., 2004) and in South Africa this minimum is low indeed. Little wonder Professor Jonathan Jansen, outspoken Chancellor of the University of the Free State, admitted "I'm ashamed of South Africa. It's an absolute disgrace that you can pass matric with a mark of 30 percent." (City Press, 2013).

2.1.4 Ninth grade

Grade 9 is a challenging year for many learners. McCallumore and Sparapani (2010) in a survey of Grade 9s in the USA showed that these learners have "the lowest grade point average, the most missed classes, the majority of failing grades, and more misbehavior referrals than any other high school grade level" (p. 60). This also tends to be true in many South African schools (W. Steyn, personal communication, 7 November 2014); Grade 9s are distracted by the onset of puberty, newly confident in their school environment after passing Grade 8, and often appear to be determined to enjoy the last freedom of responsibility before the serious academic challenges of Grade 10.

2.2 Human cognition

2.2.1 Schemas

The term "schema" was coined by Sir Frederic Bartlett in 1932 (Sweller, 1994). A schema refers to a mental model or cognitive construct involving multiple aspects of a concept. For example, a "dog" schema might include not only the knowledge of a furry mammal with four legs who barks and wags his tail, but may also involve smells, a memory of touch, and information about specific instances of "dog" that a person has known. Schemas that

an individual has acquired affect how he reacts to and remembers new experiences and assist him in dealing with a vast influx of sensory input.

The processes of consciousness can be portrayed as directing attention to some event, remembering that event, linking it with previously acquired information, and being able to recall and use the memory either completely or selectively in response to a variety of prompting mechanisms. According to Brewer & Nakamura (1984) all of these processes are “schema-based”.

Since the 1950s schema theory has been fundamental in a variety of important research directions. For example, Piaget (1952) relied on schemas in his landmark work describing how children learn to think, as do socioculturalist theories, particularly the works of Vygotsky and his followers (McVee, Dunsmore, & Gavelek, 2005).

As an individual learns a subject such as mathematics, he acquires a variety of schemas to deal with the concepts. If a learner has no schema to solve a problem, he must use problem-solving methods such as means-end analysis or forward search, which are slow and inefficient (Paas, 1992).

If a learner applies cognitive effort, during either problem-solving, instruction, or when studying examples, he begins to acquire schemas and develop understanding (Sweller et al., 1998). As he learns his schemas grow in complexity and become automated, and whether general or abstract they allow the same principles to be applied in a whole lot of different situations Sweller (2009). In mathematics, these interrelated schemas embody what teachers call the theory (van Merriënboer et al., 2003).

2.2.2 Working memory and the instantiation of schemas

Memory is considered to be made up of working memory and long-term memory. Present thinking suggests that working memory comprises a central executive which directs our attention, and short-term memory that briefly stores whatever our attention is focused on (Cowan, 2008).

Baddeley (2003) suggests that short-term or “fluid” memory comprises different components: two buffers (the phonological loop for sounds and the visuospatial sketchpad) that store information in real time under the control of a central executive, and an episodic buffer which synergistically combines information from the other parts of working memory (Baddeley, 2000). The buffers can hold memories for a few seconds before they fade; if the memories are to be retained they must be rehearsed and ultimately transferred to the “crystallised” systems of long-term memory. The central executive is “capable of retrieving information from the store in the form of conscious awareness, of reflecting on that information and, where necessary, manipulating and

modifying it" (Baddeley, 2000 p. 421). This allows the integration of experiences we are undergoing, and provides the mechanism by which new schema are acquired.

Short-term memory is limited to a very few items, probably around four (Cowan, 2000); to overcome this limitation humans make use of elegant processes of data-aggregation. Consider, for instance, a child just learning to read. She can sound out each letter individually, and listen to the sounds the letters make in order to interpret the word ("Ff-ruh-o-guh frog"). As her expertise improves she can recognise syllables ("Grass-hop-per") and eventually whole words. Miller (1956) in his seminal paper on the limits of memory called this process "chunking". In effect, schemas allow working memory to treat many related elements as a single item, thereby requiring only one "slot" in working memory (Sweller, 2009). For the child, as her skill develops and she remembers more chunks, reading is no longer the slow sounding out of individual letters, but becomes a smooth flow from which she can gather meaning and build comprehension.

The recall of the relevant schema from long-term memory does not take conscious effort. When the schema is recalled, data from the episodic buffer may be slotted into available spaces in generic schema, thereby creating a more specific schema, or may be used to elaborate a pre-existing schema. In this way an instantiated schema is created as the conscious contents of the mind (Brewer & Nakamura (1984), Cowan (2008), Baddeley, 2003). Once information is linked into a schema it can be easily be recalled (Brewer & Nakamura, 1984).

How a person reacts to a situation depends on the scope and complexity of schemas that he has stored in long-term memory: the schemas stored by experts are sophisticated and allow the effortless recall of complex information (Pachman, Sweller, & Kalyuga, 2013). Schemas therefore support limited working memory in two ways: they organise information, and allow for easier recall (Sweller, 2009).

One further note on the capacity of short-term memory; it is an oversimplification to envisage it as being a box with four or five storage spaces. Miller (1956) assessed the limits of unidimensional judgements and then demonstrated that if an additional dimension of information was added (for example adding colour to sound or shape to colour) the capacity to identify elements increased. These different modes of data do not appear to interfere with each other (Cowan, 2008).

General intelligence appears to be strongly influenced by the capacity and effectiveness of working memory (Kyllonen & Christal, 1990).

2.2.3 Long-term memory

Kalyuga, Ayres, Chandler and Sweller (2003) characterise long-term memory as containing “huge amounts of domain-specific knowledge structures that can be described as hierarchically organised schemas” (p. 23). There is probably a large amount of redundancy in long-term storage, making retrieval more effective and provide a multi-dimensional component to memories. Various attempts to quantify long-term memory using physiological determinants such as the number of neurons in the brain together with computer analogies, suggest that long-term memory capacity of our brains in terms of current life-spans is effectively unlimited and apparently permanent (Landauer, 1986).

Stored knowledge can be categorised as declarative knowledge (explicit knowledge related to meaning); sensorimotor knowledge (physical skills such as throwing a ball) and procedural knowledge (an implicit ability to perform skilled actions) (Schnitz & Kürschner, 2007). In the domain of mathematics, Hiebert & Lefevre (2013) characterise procedural knowledge as comprising the *syntax* of mathematics, and the *procedures* (rules or algorithms) that can be used to solve a mathematical task. Procedural knowledge does not require understanding of the task or the algorithms used, but merely a memorization of the rules and the ability to apply a linear sequence of steps in a pre-determined situation.

When items of information that a learner is already in possession of are newly linked together, this insight results in an increase in conceptual knowledge accompanied by a “significant cognitive reorganization” (Hiebert & Lefevre, 2013, p. 4). In this way the learner’s schemas become more elaborate as his understanding increases. Renkl and Atkinson (2003) describe how learning progresses from an early phase – during which the learner develops a basic understanding of the domain – through an intermediate phase, when he improves his schema, revises any misunderstandings and begin to solve problems.

The learner does not yet perceive these problems as being “easy”. The recall of facts or instantiation of appropriate schema will become quicker and more effective only with practise; During this late stage learning the learner’s automation, speed and accuracy continue to improve (Renkl & Atkinson, 2003). Such practice is more effective if spaced out over time (spaced learning, or distributed practice) rather than being condensed into the shortest possible time (massed practice) (Haq & Kodak, 2015; Rohrer, 2015; Wickelgren, 1972). Distributed practice has been shown to be 37% more effective than massed practice when the learner experiences high cognitive load (Andersen, Mikkelsen, Konge, Cayé-Thomasen, & Sørensen, 2015).

2.2.4 Learning mathematics

Initial exposure to a domain – for instance, algebra – results in the retention of some declarative knowledge. In order to perform algebraic manipulations using only declarative knowledge, an individual must proceed step-by-step, recalling each step individually into working memory as an instruction and then operating on it. Procedural knowledge refers to automated processes which do not directly consume working memory and are therefore more efficient. A learner can transition from declarative knowledge to procedural knowledge through the acquisition of schemas, resulting from attentive practice (Pachman et al., 2013).

It is worth noting here the relevance of the phrase “attentive” practice: there are theories that propose that increased amounts of attention result in a stronger memory trace, although the mechanism linking attention and schema-acquisition has not been formalised (Brewer & Nakamura, 1984).

2.2.5 Inattention blindness

Inattention blindness is a well-documented phenomenon whereby an individual fails to notice a visual object – even though it is fully visible and can be easily identified by observers – because their attention is focused on other visual details in the scene (Bredemeier & Simons, 2012; Fugnie & Marois, 2007; Lavie, 2005; D. J. Simons, 2000). A well-known demonstration of this effect is the experiment described as “The Invisible Gorilla”:

Imagine you are asked to watch a short video in which six people – three in white shirts and three in black shirts – pass basketballs around. While you watch, you must keep a silent count of the number of passes made by the people in white shirts. At some point, a gorilla strolls into the middle of the action, faces the camera and thumps its chest, and then leaves, spending nine seconds on screen. Would you see the gorilla? (Chabris & Simons, 1999)

Although it seems unlikely, to approximately 50% of people who participate in this experiment the gorilla is completely invisible (D. Simons, 2010).

Fugnie and Marois (2007) and Bredemeier and Simons (2012) linked inattention blindness to working memory overload.

2.3 Cognitive load theory

Cognitive load theory is based on the principal that demands on working memory must be managed effectively for learning in the sense of schema acquisition to take place, and that this can be assisted through the design of correct instructional materials (Sweller, van Merriënboer, & Paas, 1998).

2.3.1 Element interactivity

In the context of cognitive load theory, an element is something to be learned; it could be a concept (“Barack Obama is the President of the United States”) or a procedure (“First multiply the numerical coefficients, and then apply the exponent rules to the variables”). If the separate elements of what is being learned do not interact, no understanding is required and they can be processed in memory a few at a time – for example learning the names of flowers. At the other end of an element interactivity continuum we encounter situations in which all the elements and the relationships between them must be held in working memory simultaneously for the material to be understood (Sweller et al., 1998). By its nature, material with high element interactivity occupies a lot of limited working memory and makes greater cognitive demands (Sweller, 2010). This is perceived as mental effort, which is an indicator of cognitive load (Paas, Tuovinen, Tabbers, & van Gerven, 2003).

Element interactivity is a function not only of the material, but also of the knowledge state of the learner. Every learner possesses a different cognitive landscape comprising both pre-existing schemas and working memory effectiveness (capacity and processing power). A novice learner lacking a schema appropriate to a problem has the difficult task of fitting many individual interacting elements into his working memory and will experience cognitive overload. He will be forced to deal with the elements a few at a time, he won't be able to conceptualise all the connections involved, and at best the result will be rote learning without understanding (Sweller, 2009).

The problem with cognitive overload is that it inhibits learning. It is the human equivalent of “disk thrashing”: when a computer program does not have sufficient memory to complete a set of instructions, the computer manages the process by keeping the most frequently accessed data in memory and writing the rest to disk. If the memory required is much greater than the memory available, the computer must continually shift blocks of data from memory to disk and back again; at some point the computer is so busy writing data back and forth to disk it can't actually advance through the program. Analogously, a novice learner's working memory is so busy trying to deal with a lot of interacting elements there is no capacity left over to develop a new schema.

2.3.2 The application of working memory

The original conception of cognitive load theory was that during problem-solving working memory is applied in three different ways, referred to as different types of load (Sweller, 1988).

- *Intrinsic load* is inherent to a problem and is a factor of element interactivity (Renkl & Atkinson, 2003). For example, when factorising a quadratic equation, working memory is required to determine the factors of the third term and at the same time to compare the sum of those factors to the second term.
- *Extraneous load* is caused by any additional demands on working memory that interfere with the process of solving the problem, such as when part of the information for a problem is printed on the back of the page.
- If working memory is not consumed by intrinsic and extraneous cognitive load, the remaining working memory resources are available for schema acquisition and task mastery. This is called *germane load* (e.g. Sweller, 2010).

It is generally accepted (Sweller et al., 1998; Kirschner, Kirschner, & Paas, 2002; Paas, Renkl, & Sweller, 2003; Paas, van Gog, & Sweller, 2010) that the three types of load are additive. Extraneous load can therefore consume working memory resources with the result that there is no free capacity for germane load and the construction of schema. A general instructional principle is that extraneous load should always be removed.

2.3.3 Learner motivation

Sweller tends to discuss cognitive load theory against a background of “equal learner motivation” (Sweller, 2010). Making instructional changes to free up working memory will only result in schema acquisition if the freed resources are applied as germane cognitive load; this requires that learner actively makes a mental effort (Paas & van Merriënboer, 1994; Paas, 1992; Sweller et al., 1998).

Beckmann (2010) identifies three dimensions of cognitive load: the inherent load – resulting from the element interactivity – giving rise to the *complexity* of the task; the learner’s cognitive landscape giving rise to the *difficulty* of the task, and the learner’s *motivation* to apply cognitive resources to schema acquisition. Chi, Bassok, Lewis, Reimann, & Glaser (1989) address this motivation in terms of self-explanations. A learner must actively access his existing knowledge and apply reasoning in order to grasp the purpose of each step in the context of the problem – in other words he must supply an explanation for that step. These self-explanations create inference rules that serve as a link between abstract concepts and application. According to Renkl & Atkinson (2003, p. 16) “the construction of a sound knowledge base is not a quasi-automatic by-product of studying examples or solving problems”.

2.3.4 Measuring cognitive load

Subjective scales have been widely used to measure cognitive load (Paas, 1992; Sweller et al., 1998; Tuovinen & Sweller, 1999). These scales usually require the participant to

rate the mental effort required for a task; because mental effort is based on the proportion of cognitive capacity employed by a task, it can be considered as a measure of the actual cognitive load (Paas, Tuovinen, et al., 2003). The effectiveness of subjective scales does not appear to be sensitive to the type of scale used (Leppink, Paas, van Gog, van der Vleuten & van Merriënboer, 2014). A problem with these scales, however, is that the learner perceives only overall cognitive load – he is not aware of what is caused by intrinsic load and what constitutes extraneous load. Subjective scales are therefore unable to separately measure different types load (Sweller, 2010).

Task-performance measures constitute an alternative to subjective scales. They estimate cognitive load by comparing the element interactivity of the task to learner performance (Sweller, 2010). Physiological measures have also been applied. These try to assess cognitive load by tracking pupil dilation, response time, eye movement, heart rate and/or electroencephalographic measures of brain activity.

Leppink et al. (2013) had some success at determining different types of cognitive load by applying a three-factor method. DeLeeuw & Mayer (2008) compared the different types of measurement and concluded that response time could identify increases in extraneous load, subjective ratings of effort best tracked intrinsic load, and task performance measures reflected germane processing. These techniques may be a step in the right directions, but valid and reliable individual measures for intrinsic, extraneous and germane load are not likely within the foreseeable future (Gerjets, Scheiter, & Cierniak, 2008).

Although further research in the measurement of cognitive load is required, simple subjective ratings have proved useful in many studies as they are “sensitive to relatively small differences in cognitive load ... valid, reliable, and nonintrusive” (Paas, Van Merriënboer, & Adam, 1994, p. 268); and are “easy to use; do not interfere with primary task performance; are inexpensive” (Paas, Tuovinen, et al., 2003, p. 68).

2.3.5 Learning effects associated with cognitive load

The main thrust of cognitive load theory is that extraneous load should be reduced as far as possible so that working memory resources remain available for intrinsic and germane load, thus allowing the development of understanding – the acquisition of schema – alongside of the automation of processes (procedural understanding). Tuovinen & Sweller (1999) characterised extraneous load as resulting from “inappropriate presentation of material or inappropriate learner activities” (p. 335).

The protocols described have resulted in reduced instruction time (Paas & van Merriënboer, 1994; Zhu & Simon, 1987) and in better transfer to new and varied problems (Sweller, van Merriënboer, & Paas, 1998; van Merriënboer & Sweller, 2005; Paas, 1992).

2.3.5.1. **Worked examples**

Mathematics problems typically involve high element interactivity. A learner attempting to solve an unfamiliar problem is forced to process the elements serially a few at a time, and to make use of methods such as exploration, means-end analysis, and pure forward search to identify a solution. These methods are slow, difficult to use, and impose a heavy cognitive load that hinders rather than helps the learning process (Tuovinen & Sweller, 1999).

If the learner is provided with an expert's model answer to a problem, he can apply his working memory to understanding each step so that he can emulate the method. Such a worked example could include notes explaining what it intended to demonstrate, would typically demonstrate each step required to solve the problem, and may include alternative solution methods (Atkinson, Derry, Renkl, & Wortham, 2000 and Sweller et al., 1998).

Chi et al. (1989) identified self-explanation as a characteristic of good students, but when faced with examples, many learners follow the steps mindlessly, or skip over them, returning only to consult them when they are clearly stuck on a problem (Sweller et al., 1998; van Merriënboer et al., 2003). As a result they do not *understand* the examples but only encounter a sequence of unexplained processes which they can copy but not transfer to other contexts (Chi et al., 1989).

The learner should not only study the example, but apply his freed cognitive capacity to "self-explain" each step, which helps to advance understanding (Renkl & Atkinson, 2003). At this point he can partially or completely solve similar problems and eventually apply this skill to different situations.

Compared to conventional problem solving, worked examples have been demonstrated to facilitate better schema acquisition, enhance learning, take less time, reduce mental effort, and result in more effective performance transfer (Paas & van Merriënboer, 1994; Paas, 1992; Sweller et al., 1998; Tuovinen & Sweller, 1999; van Merriënboer et al., 2003).

2.3.5.2. **The expertise reversal effect**

One can picture a continuum from a novice who is totally dependent on a worked example at every step to the expert who has complete, accurate schemas instantly on call. As a learner develops expertise he becomes less dependent on the examples and more confident of his schemas (Kalyuga & Sweller, 2005). In the three-phase learning model of Renkl and Atkinson (2003), during the intermediate phase, the learner still reflects and self-explains while starting to solve problems, thus correcting any flaws in his schema.

If a more experienced learner continues to study examples, these have to be integrated into his proto-schema, causing extraneous cognitive load which consumes working memory (van Merriënboer et al., 2003; Kalyuga & Sweller, 2005; Sweller, 2010). This is called the expertise reversal effect (Kalyuga et al., 2003; Sweller, 2006 and Renkl & Atkinson, 2003).

The expertise reversal effect must be taken into account when planning instruction, since the most effective learning method will change as the learner's expertise develops (van Merriënboer & Sweller, 2005). Furthermore, since every learner's cognitive landscape is different, ideally each learner should be given an individualised program. Various methods have been suggested to accomplish this, for example the "rapid dynamic assessment" described in Kalyuga & Sweller (2005).

2.3.5.3. ***The completion strategy (fading effect)***

The problems of the expertise effect and of learners skipping over examples can be overcome by giving them incomplete examples and require them to fill in the missing steps. At first, only the final step will be omitted, but as the learner's expertise develops, more and more steps are left out until he is finally able to completely solve a problem without support (van Merriënboer et al., 2003). This is called the completion strategy or the fading effect. Sweller et al. (1998, p. 276) talk of completion problems: "Worked examples are completion problems with a full solution, and conventional problems are completion problems with no solution."

The timing of fading the problems is important (Sweller, 2006). If the fading is too quick, the cognitive load is too high. If the fading is too slow the expertise effect kicks in: self-explanation is extraneous and the learner may lose motivation. Correctly managed, fading can successfully overcome the expertise effect (Sweller, 2010), is effective with problem solving (near-transfer) and, when combined with self-explanation, has generated good results for far transfer as well (Renkl & Atkinson, 2003).

2.3.5.4. ***The variability effect***

Whereas worked examples are intended to decrease cognitive load, and completion problems aim to balance cognitive load according to learner expertise, the variability effect aims to increase germane cognitive load in order to induce schema acquisition (Sweller et al., 1998).

When learners solve the same type of problem over and over they will develop automation, accuracy and speed (Renkl & Atkinson, 2003), but their schemas do not improve. In order to develop dense, robust schemas, learners must be faced with a variety of problems – these may include different variants of the problem, changes in the way the

question is asked, in the context, the familiarity, or the characteristics of the questions. Variability allows learners to identify the range of applicability of schemas they have learned (van Merriënboer & Sweller, 2005), to pick out the important features of a problem from the unimportant ones and to recognize common features in differing problems (Sweller et al., 1998).

Problem variability imposes greater cognitive load as a result of greater element interactivity (Sweller, 2010). The additional load is apparently germane; it has been shown to result in greater transfer of learning and improved schema construction and, in general, enhanced problem-solving skills (see for example van Merriënboer & Sweller, 2005; Sweller et al., 1998; and Paas & van Merriënboer, 1994).

2.3.5.5. ***The split attention effect and the redundancy effect***

A typical example of the split attention effect is where information is provided in both text and a diagram, and the text has to be read for the diagram to be understood – the reader is obliged to look back and forth from text to diagram. Another example is an examination paper that has information on one page and the questions overleaf. Because the learners attention is split between two sources, integrating the information causes high cognitive loads (Paas & van Merriënboer, 1994; Sweller et al., 1998).

The converse of the split attention effect is the redundancy effect. Additional information intended to assist a learner to integrate different sources is very difficult to ignore. If a learner can easily integrate multiple sources, the additional information will cause extraneous load. It is clear there is a link here to the expertise reversal effect (Sweller, 2006). It is important in the design of learning material to pinpoint when the split attention effect gives way to the redundancy effect (Sweller et al., 1998).

2.3.5.6. ***The modality effect***

Miller (1956) recognised that working memory can store and recall more items of information if these were received in different modes – for example, sound and colour. By activating multiple modes simultaneously in a classroom – for example, in a diagram and an explanation – the central executive can distribute processing demands between the phonological loop and the visuospatial scratchpad thereby helping to maximize working memory's capacity to store and process information (Baddeley, 2003; Renkl & Atkinson, 2003). Effective working memory can be increased and cognitive load reduced (Sweller et al., 1998).

2.3.5.7. ***The goal free effect***

The goal free effect avoids the excess cognitive load caused when means-end analysis is applied in the absence of an appropriate schema. Instead of working directly towards a

solution, the learner is encouraged to explore all aspects. For example, when a geometry problem requires a learner to determine the size of an angle, the learner is instructed to write down everything he can see about the problem. In this way, all applicable knowledge is brought to bear in the hopes that the overall picture obtained will make a solution path evident (Sweller, 1988).

2.3.5.8. ***The imagination effect***

The imagination effect applies only to learners who already have some expertise, have acquired the necessary schema and have attained late stage learning where automation and speed will be the dimensions of improvement (Renkl & Atkinson, 2003). As an alternative to further conventional practice, learners can apply their imaginations to mentally rehearse the processes. Sweller (2009) reports that this gives better results than conventional practice for learners who have schema and can hold all the relevant information in working memory. It seems that this reduces extraneous cognitive load because the learner is able to isolate the most important elements of the process (Sweller, 2010).

2.3.6 **Other instructional design recommendations from cognitive load theory**

2.3.6.1. ***Measuring expertise***

As we have seen, cognitive load is a function of both the problem and of the learner's existing schema. Assigning the next example or problem to a learner, and avoiding issues such as the expertise reversal effect requires some assessment of the learner's level of expertise.

When faced with a problem, experts can quickly retrieve the correct schema from long-term memory because they have automated or "chunked" some processes. A beginner learner, faced with $x^2 \times x^3$ may first write:

$$x^2 \times x^3 = x^{2+3}$$

or even

$$x^2 \times x^3 = (x \times x) \times (x \times x \times x)$$

before reaching a solution; a more experienced learner will simply write

$$x^2 \times x^3 = x^5$$

Kalyuga and Sweller (2005) showed that there is a strong correlation between the first step that a learner takes and his expertise. Rather than the more traditional evaluation that requires a learner to complete the entire solution process before judging his expertise their "rapid dynamic assessment" evaluated only the first step.

Another way of measuring expertise is *learning efficiency*, the relationship between mental effort and task performance (Paas & van Merriënboer, 1993). When faced with a problem, both a novice and an expert learner may solve it correctly, but a greater mental effort will be required from the novice. In van Merriënboer & Sweller (2005), learner efficiency was calculated and compared to a task-specific critical point in order to decide whether the learner was ready for more difficult problems.

2.3.6.2. **Problem and information sequencing**

Conventionally maths problems are grouped together by type: the problems themselves impose high cognitive load, the sequencing imposes low load. By instead presenting *completion* problems in a *random* order the cognitive load is imposed more by the order of the problems than the problems themselves. van Merriënboer & Sweller (2005) called this “*redirecting attention* from extraneous to germane processes” (p. 162) and claim that it results in better training efficiency and transfer.

When learners are attempting problems which involve variable aspects, they need help with problem solving and reasoning. van Merriënboer et al. (2003) called this type of help *supportive information* and showed that it should be given to the learners in advance. On the other hand, help with low complexity tasks – usually *procedural information* on how rules should be applied – is better given just as it is needed.

2.3.6.3. **Second language learners**

A learner who is not fluent in the language or teaching and learning experiences additional cognitive load because he has to devote a conscious mental effort to following explanations or instructions. Moreover mathematical language has its own specialised vocabulary: second-language learners should internalise the meanings of words in their mathematical context to avoid further sources of cognitive load (Campbell, Davis, & Adams, 2007).

2.3.7 **Linking cognitive load theory to the zone of proximal development**

A child does not learn anything new by repeatedly performing tasks that he has already mastered. He also cannot learn from a task that is so far beyond his ability that he is unable to make any progress at all. Tasks that the child cannot do alone but *can* do with assistance provide opportunities for learning to take place. These tasks make up the zone of proximal development (ZPD), an educational concept based on the work of Vygotsky (1980). It is the role of the teacher to provide assistance or scaffolding that will lead the learner to complete tasks in his zone of proximal development and thus develop his abilities.

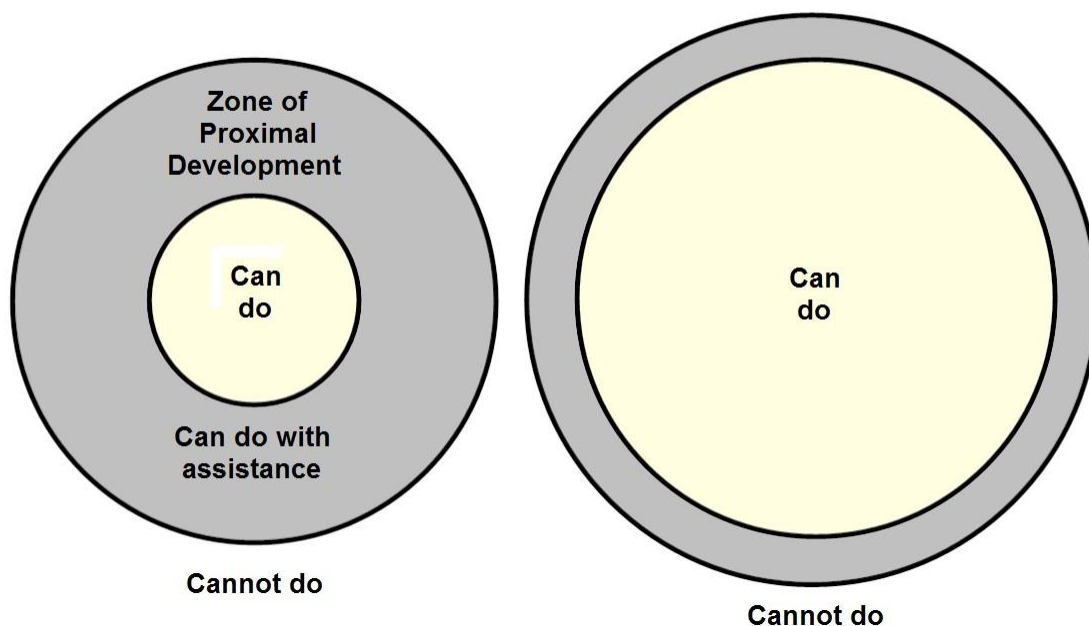


Figure 2: The zone of proximal development before (left) and after (right) teaching and learning

From the point of view of cognitive load theory, worked or faded examples serve as the scaffolding to assist the child with tasks that fall within his zone of proximal development. If the child is applying germane cognitive load to the task, he is developing new schemas or improving existing schemas and his zone of proximal development expands outwards. Alternatively, a learner can either reduce element interactivity by serial processing – what van Merriënboer & Sweller (2005) called “part-task sequencing” – or begin with a simplified view of the whole task, gradually adding more detail (“whole-task sequencing”). Understanding will initially be limited, but learning can proceed and the learner will gradually develop schema (Sweller, 2009).

The ZPD is different for different learners. If the learner already has mastered certain content, a task may be inside his “Can do” zone, and he is not learning by completing it – no germane load can be induced by the task. Moreno (2009) links the expertise reversal effect to a learner studying examples is functioning outside his Zone of Proximal Development and will lose motivation.

2.3.8 The scientific validity of cognitive load theory and other concerns

Cognitive load theory has been criticised as being unscientific because it is not an empirically testable system of laws (Schnotz & Kürschner, 2007). However, the structural view of science accepts that a theory can be a network of related elements underpinned by some basic theoretical concepts and assumptions which may remain untestable. In this view, cognitive load theory can be considered a science in which empirical progress

results from the successful application of its principles and theoretical progress results from greater differentiation of the network of elements (Gerjets et al., 2008).

Other criticisms of cognitive load theory include the fact that the findings from different research programs were inconsistent with each other and with reasoned predictions made from cognitive load theory. When faced with results that had not been predicted, researchers were able to offer *a posteriori* explanations and interpretations of why their outcomes were not predicted, but the overall impression left “more questions than answers and cast doubt over the validity of CLT” (Moreno, 2006 p. 177).

De Jong (2009) also criticised some aspects of cognitive load theory. He discussed the fact that germane and intrinsic load should not be considered to be additive: either they are ontologically different, in which case they are “unlike terms” and cannot be added, or they are ontologically the same, in which case they will interact with each other and will not be additive. In his conclusion he states: “the fact that cognitive load is composed of three different elements that are ‘good’ (germane), ‘bad’ (extraneous), or just there (intrinsic) means that every outcome fits within the theory post-hoc” (p. 125).

Although these concerns derive from different sources, they share a common root in the fact that it has not yet proven possible to measure individually the different types of cognitive load. Consider an instruction program designed to decrease extraneous load. If a learner undergoing this program reports a decrease in mental effort, this broaches the question of why that reduced cognitive load is not applied as germane load to improve learning. If the learner does not report a decrease in mental effort, there is no valid and reliable way to determine whether this was because the working memory which was freed is utilised as a germane resource or whether the instructional program was simply ineffective at reducing extraneous cognitive load. Being unable to differentiate types of cognitive load means that interpretation of an increase or decrease in mental effort remains a matter of speculation, and hence “imposes a challenge to the conclusive attribution of CLT- related effects of manipulations of task presentations.” (Beckmann, 2010, p. 262)

Critics of cognitive load theory are not calling for an abandonment of the theory, but instead suggest ways in which it can move forward. Beckmann (2010) declares that in the development of learning material, equal emphasis on the analysis of the individual differences of learners as on the element interactivity of the task is important. Moreno (2006) notes the importance of “mediating effects of students’ individual differences, especially those related to their prior knowledge, metacognition, and motivation” (p. 179). de Jong (2009) suggests that the theory of instructional design could benefit from

information about “(1) which instructional treatments lead to which cognitive processes (and how) (2) what the corresponding effects are on memory workload and potential overload (3) what characteristics of the learning material and the student mediate these effects and (4) how best to measure effects on working memory load in a theory-related manner.” (p. 127).

2.3.9 The importance of cognitive load theory

Despite the criticisms of cognitive load theory, it has had a major impact on insight into working memory, element interactivity, and the acquisition of schemas. The problem of trying to take learners beyond procedural skills is beautifully encapsulated in the following:

Understanding cannot occur because understanding requires all interacting elements to be processed simultaneously in working memory. All the interacting elements cannot be processed simultaneously in working memory until schemas have been formed, but schemas will not be formed until the learner has [understood the material]...understanding and learning may appear impossible at first sight. When the material is presented with all of its interacting elements, as it needs to be if understanding is to occur, it cannot be processed in working memory because it vastly exceeds working memory capacity. How does learning occur under such circumstances? (Sweller, 2009, p. 255)

In attempting to overcome this Catch-22, cognitive load theory has recommended the use of complete and incomplete examples, self-explanations, and more recently, emphasised the importance of ensuring that problems are matched to the expertise of the learner.

CHAPTER 3: Methodology

3.1 Research sample

The research was conducted at an English-medium school with a strong teaching tradition. It is a Dinaledi school – part of the Department of Education’s initiative to improve the teaching of mathematics and science at South African schools – and also receives support for mathematics teaching from the Epoch and Optima Trusts. The school has approximately 1200 learners, the majority of whom speak SiSwati as a mother tongue. Many learners enter from urban feeder schools and have been taught in English since Grade 1. Other learners transfer in from private schools, or are hostel residents from rural areas. As a result of their mixed provenance, there is a wide range of previous education standards between the learners in a cohort.

At the start of 2015 approximately 50 of the Grade 9 learners at the school were placed into two classes for academic support with the remaining 160 or so being randomly allocated to seven other mixed classes (W. Steyn, personal communication, 7 November 2014). Learners with behaviour problems were distributed among the classes to avoid any one group being unmanageable. Most Grade 9 classes thus include both weak and capable learners, as well as a few with a history of bad behaviour. The class which participated in the research project was a mixed group and thus a non-probability sample (accidental).

The 24 learners in this class come from a wide variety of backgrounds, including children of cleaners, security guards, general office assistants, educators, policemen, provincial government employees at various levels, company directors, and an orphan from an SOS Children’s Village. Some of their home backgrounds are troubled and one learner in the class has been identified as having traits associated with Asperger syndrome (L. Ohlsen, personal communication, 9 February 2015).

Behaviourally the group was a challenge. There were four repeaters in the class (an unusually high number), most of whom put up a façade of knowing the work and not having to make an effort. Some of the more easily influenced among the remaining learners emulated this. Keeping the class focused was difficult at times, and their classwork and homework was frequently shoddily done, or not done at all.

The Class Dojo system was used to increase learner motivation through a points and rewards system (www.ClassDojo.com). Although this was effective at getting the learners into class on time, with books out and ready to learn, only one or two of these children

could really be described as *engaged* learners. The results of assessment reflect this: the average of their test scores prior to the research intervention was only 43%.

3.2 Research design and data required

Since the research questions addressed learners' attitudes and engagement as well as their academic performance, both qualitative and quantitative data were required. The methods applied were intervention research and knowledge utilisation – utilising results of previous research in a practical application – and were thus suited to both qualitative and quantitative methodologies (de Vos, Strydom, Fouche, & Delport, 2004). The research design used was a quasi-experimental/associative design in the form of a one-group-pre-test-post-test with additional data gathered through a questionnaire, observation and classwork analysis.

3.2.1 The intervention content: laws of exponents and scientific notation

Good understanding of exponential representation and the application of the Laws of Exponents is fundamental to progress in mathematics from Grade 10 onwards.

Furthermore, the Diagnostic Report on the 2014 ANA identified exponents as being a specific learning weakness for Grade 9s (Department of Basic Education, 2014d).

Learners encounter square numbers in Grade 6 (Department of Basic Education, 2011a) but first use the exponential form in Grade 7 where they learn for example that $a^3 = a \times a \times a$, and that numbers can be represented in exponential form without needing to be given a calculated value (Department of Basic Education, 2011b). In Grade 8 they are introduced to the laws of exponents and scientific notation; the basic abilities required to work with exponents should therefore be in place by the end of Grade 8. In Grade 9 these concepts are extended to include negative exponents (see Appendix C for the relevant section of the CAPS document).

This section of the curriculum requires the application of specific rules under clearly defined conditions, and thus lent itself to the type of self-study with examples that was planned for the intervention.

3.3 Data collection

3.3.1 Questionnaire

A questionnaire was used to collect information about the learner's ostensive or stated attitudes (see Appendix A). It was modelled on the modified Fennema-Sherman questionnaire discussed in Doepken, Lawsky, & Padwa (1993), and was pilot-tested before implementation. Minor modifications were made based on feedback from the pilot-

testing, mostly adjusting the language usage and the phrasing of the questions to ensure that the learners understood all the statements clearly.

The questionnaire consisted of 23 items that addressed the learners' level of engagement with mathematics, their perception of mathematics as a difficult subject, their opinions on usefulness of mathematics, their personal confidence about the subject matter and their attitudes towards education in general. It was administered in class under "test conditions" (that is to say, the learners worked individually without interaction).

3.3.2 Content revision and pre-test

Mathematics is a vertically delimited subject, so learners' effectiveness in a section of the curriculum cannot be considered in isolation from their mastery of previous content.

Before the intervention, two lessons were spent revising the laws of exponents that the learners had already learned in Grade 8 and a further two lessons on the concepts and notation of negative exponents. This was followed by a pre-test under rigorous test conditions.

3.3.3 Intervention materials

The learning material for the intervention consisted of a series of custom designed worksheets sequenced from A through Z. A selection of the worksheets was piloted with learners from another class before the intervention. Adjustments were made to problems and in some cases to examples. Due to time constraints, not every worksheet was piloted.

Each A5-sized worksheet or "set" contained any combination of the following elements: the statement of a rule or principle; a question, prompt or reminder; a complete example; a faded example; a hint about how to proceed; and a variety of sums applying the stated rule as well as other rules that had previously been covered (see the example in Figure 3). At the bottom of each set was a place for evaluation of cognitive load, where the learner specified whether the set had been difficult, just right, easy or very easy.

The worksheets were sequenced from total beginner level, gradually introducing new rules and concepts and becoming more difficult. The learners who performed worst in the pre-test began at Set A, the higher performing learners started on Set F or Set H.

The original intention had been for each learner to bring a completed set for marking as soon as they finished it, and receive a follow-up set depending on their level of success. It was quickly clear, however, that assessment was going to be a bottleneck in the process. This was unfortunate, as it meant there was no personalisation for each learner's journey through the sets – they just continued through the sequence. For some learners this meant that the expertise effect became a negative factor in their experience.

Set D	Name
Examples:	$x^2 \times x^5 = x^7$
	$x^2 \times x^3 = x^5$
When we multiply numbers with the SAME base, what happens to the exponents?	
Complete these problems	
1)	$y^2 x^3 \times x^2 y^4$
	$= x^{3+2} y^{2+4}$
	=
2)	$x^2 \times y^2 \times x^5 \times y^3$
	$= x^2 \times x^5 \times y^2 \times y^3$
	=
3)	$b^2 \times c^4 \times b^3 \times c^2$
	=
	=
Pick one: Difficult <input type="checkbox"/> Just right <input type="checkbox"/>	
Easy <input type="checkbox"/> Very easy <input type="checkbox"/>	
Your next set is <input type="text"/>	

Figure 3: Example of a blank worksheet

All the worksheets completed in a teaching session were assessed the same afternoon and returned to the learners the following day. If the worksheet was satisfactorily done, it was given a pink dot sticker; otherwise the learner was expected to go over the errors or incomplete sums and improve on their initial attempt.

3.3.3.1. **Theoretical principles applied in the worksheet design**

To take the intervention beyond a simple self-guided learning system, principles of cognitive load theory were applied in the worksheet design. The following are based primarily on Atkinson et al. (2000) and (Sweller, 2010):

- Avoidance of the split attention effect;
- Wide usage of worked examples, and questions or prompts related to these to ensure that the learner did not simply skip over the examples;
- Completion examples: gradually fading out more steps;
- Principle of variability: problems or sums in a set were not only applications of the principle under focus, but also required learners to remember rules they had learned previously.
- Language was tuned to the learner's abilities to avoid extraneous cognitive load in this regard;

- Understanding and the generation of germane cognitive load was encouraged by avoiding short-circuiting;
- Sequencing and procedural methods (such as $m^2 \times m^3 = m^{2+3}$) were given just as they were needed.

Newmann et al. (1992) suggested various principles to keep learners engaged. Some of these were also taken into account in the intervention design, including:

- allow a flexible time frame: learners continued at own pace
- extrinsic rewards: Dojo points were awarded for good work. The pink dots places on satisfactorily completed worksheets were introduced as an administrative tool, but in the event the learners also perceived these as rewards;
- the initial intention of giving immediate feedback was impractical, but the principle of prompt feedback was applied: every completed worksheet was returned marked on the following school day.

Finally, as far as possible within the timeframe of the intervention, the principles of spaced learning and distributed practice were applied by returning regularly to previous types of problems.

3.3.4 Observation

By the middle of the school year any good teacher has a clear idea of each learner's attitudes and their engagement in class. During the intervention, observations of the learners' behaviour and of any significant incidents during each lesson were jotted down. These were written up each afternoon while the details were still fresh.

3.3.5 Post-test

The post-test was developed by a colleague. The learners completed it immediately following the intervention under rigorous test conditions. A few weeks later they also wrote the mid-year examinations – an internal examination set by the co-ordinator for Grade 9 mathematics at the school, and an external examination received from the Mpumalanga Department of Education.

3.3.6 Reflection

At the end of the intervention phase the learners had the opportunity to give some feedback. They were guided by prompts such as, "Did you enjoy the sets? What did you like? What didn't you like?", "Do you think you learned the work? Was it easier or more difficult to learn like this?" "Did you feel you worked harder this way? Were you more focused?"

3.4 Ethical considerations

3.4.1 Permission to conduct research

Permission to conduct the research was obtained from the Department of Education Mpumalanga, from the School Governing Body and principal of the school and from the College of Education Ethics Committee. These documents are included in Appendix A.

3.4.2 Informed consent

Parental consent forms and learner assent forms were completed and collected before the intervention (see Appendix A). Learners and their parents or guardians were encouraged to ask questions to ensure that they understood the intervention process.

3.4.3 Conflict of interest

As the researcher was also the teacher, there was a potential conflict of interest. In order to ensure that this did not bias the outcomes of the research the following were put into place:

- The academic tests were set and moderated by an experienced colleague. The standard of these tests was guided by the CAPS document.
- The marked test scripts were moderated for fairness and accuracy of the marks allocated by the head of the Mathematics Department at the school.
- A colleague in the English department guided the learners through their reflections. The intention here was that there would be less social pressure on the learners to make unrealistic positive statements.

3.4.4 Data integrity

The original completed worksheet sets, the tests, questionnaires, observation forms and reflection paragraphs as well as the assent and consent forms will be kept in the locked storeroom of the Mathematics Department for five years. In addition scans of the data and the files generated during data analysis have been stored in electronic form in two physical locations. A copy of all these documents is kept in the cloud using SpiderOak backup software.

CHAPTER 4: Results

The questionnaires, daily observations of learners, and reflective feedback provided some information about the learners, particularly about their attitudes and engagement. In addition they completed over three hundred worksheets, providing a large source of useful data. To paraphrase Tolstoy: “Correct answers are all alike; every incorrect answer is incorrect in its own way”. Eight learners were selected as the focus of the analysis; their worksheets were a rich source of variety in answers, attitudes and engagement, and with the addition of a few examples from the work of other learners they serve to both typify and exemplify the results.

4.1 Attitudes

4.1.1 What do the questionnaires say about learner attitudes?

The learners’ responses to the questionnaire are tabulated in Appendix B.

It was evident from the analysis of the questionnaire that the learners know what their teacher would like their responses to be, and this is how they answer – which is not necessarily in line with what they really believe or act on, but is rather an effort to be agreeable. The strongest overall agreement was linked to statements such as “I want to study school maths so that I can go to university”; “It is important to study maths at school”; “It is possible to improve in mathematics by working hard” and “I aim to get top marks for mathematics tests”, which are self-evidently and conspicuously what teachers and parents would like children to answer. Similarly there was strong disagreement with “I am only in Grade 9, school is not that important” and “Sometimes I really hate mathematics”. The only any real insight into learners’ attitudes from these answers is that they try to please – which in itself is not unimportant. However responses to questions that were a little less obvious in terms of what would please their teacher were more interesting.

There was no overall agreement on lesson pacing – almost as many learners felt the teacher is too slow as felt the teacher is too fast; in fact some of the learners strongly disagreed with both statements. Does this mean they felt that the pacing is just right, or that they are just reluctant to criticise the teacher?

Almost every learner said they can “Always” or “Often” follow the examples that the teacher puts on the board; only Ciniso indicated that he can never follow examples. On the other hand, Ciniso also said that he never has to concentrate hard to get the right answers to sums. It is possible that there were statements in the questionnaire that he did

not understand as his reading comprehension and vocabulary are well below average (personal communication, Dr N. Schnell 22 July 2015).

There was also a high level of agreement with “Ability in maths is something that you either have or you haven’t”. Only Nokubonga strongly disagreed; a few other learners responded with “Not really”.

4.1.2 What did the reflections say about learner attitudes?

Once again, most of the reflections from the learners were calculated to be pleasing. So there were eleven comments that the worksheets were “fun” and “interesting” and one slightly more creative response that stated that this had been “less fun than normal because we do a lot of fun things in [normal] class”. Probably more honestly, there were seven who said things like “I just wanted to get done with the worksheets”.

Eight learners gave variations on “I learned a lot and it helped me understand”, or “I’m doing much better now because this was something that was difficult for me”. There was, however, one lonely plaintive voice claiming “I didn’t understand”. A few commented on the worksheets “helping with the hard sums”. Four learners commented on the relevance to tests and examinations, and a few referred to their Dojo point rewards.

I had specifically asked them to comment on receiving the worksheets back and being expected to get them mostly right before being rewarded with a pink dot sticker. Once again, the feedback was mostly positive, along the lines of “It helps you improve, get right the things you did wrong” and “It was a challenge because I will do it until I get it right.” Nhlakanipho wrote “It annoyed me [to get work sheets back] but I understand that for me to improve I have to do corrections” – more on this in section 4.1.3.2 below.

4.1.3 What do the learners’ actions say about their attitudes?

The mathematical processes that the learners went through in the course of completing the worksheets is discussed in detail in the sections on engagement and cognitive load (Section 4.2 and 4.3 below). But whereas some of the learners were focused and meticulous in the work they completed, and some were clearly struggling with cognitive overload, there was also evidence of slapdash efforts and careless mistakes.

4.1.3.1. *Attention to detail*

Learners repeatedly failed to pay attention to detail, for example by dropping negative signs. It is tempting to say that these errors are due to carelessness, although evidence for cognitive overload is discussed in 4.3.1 below. Examples of these errors are shown in Figure 4 and Figure 5.

$$\begin{aligned}
 & m^2 \times 8 \times n^{-2} \times 2^2 \times m^{-3} \\
 & = m^2 \times 2^3 \times n^{-2} \times 2^2 \times m^{-3} \\
 & = m^2 \times m^3 \times n^2 \times 2^3 \times 2^3 \\
 & = 2^6 m^5 n^2
 \end{aligned}$$

Remember what we learned about square roots:

$$\sqrt{x^4} = x^2 \text{ What did we do to the exponent? } \underline{\div 2}$$

Now look at these examples:

$$\begin{aligned}
 \sqrt[3]{x^3} &= x \\
 \sqrt[3]{m^6} &= m^2
 \end{aligned}$$

What do we do to the exponent for a CUBE root?
divide by 2 ~~Nope~~

Figure 4: Failing to pay attention to detail? (Lizzy Set I)

$$\begin{aligned}
 & (12 \times b^2 \times 3 \times b^3) \\
 & = \{2^2 \times 3\} \times b^2 \times 3 \times b^3 \\
 & = 2^2 \times 3^2 \times b^5 = 2^2 \cdot 3^2 \cdot b^5
 \end{aligned}$$

What happened to the 12? ~~We~~ going
 $2^2 = 12 \times$

Eg 2: $\frac{x^2 y^5}{y^3} = x^2 y^2$

When we divide variables with the same base, what happens to the exponents? ~~Add them~~ subtract

Example:

$$(k^2)^3 = k^{2 \times 3} = k^6$$

What did we do to the exponents?
add the exponents

Figure 5: Apparent carelessness (Charity Set J, Neo Set K)

4.1.3.2. *The story of Nhlakanipho*

In the questionnaire, Nhlakanipho's answers differed from the majority in two particulars: he was one of only two who claimed they get good marks at mathematics without really trying hard; he was also one of only two who strongly disagreed with the statements about enjoying or liking mathematics. In term one of his Grade 9 year he was one of the top scorers in his class group, and from the questionnaire he was aware of this; however, he did not fare as well in the June examination. Differential Aptitude Test results indicate that he has low average proficiency in Numeracy (Stanine 3.0) and an estimated IQ of 90-96 (Dr N. Schnell, personal communication 22 July 2015). Nhlakanipho's engagement with the school is high – he is a hostel learner and very active in soccer and other sporting activities.

The research week did not bring out the best in Nhlakanipho. He sits with Gugu, the top performer in this class, and it became evident during the week that he saw himself as being in competition with her. He was not interested in understanding the content or getting the answers correct, or even worried about how few “pink dots” he had. He was concerned mostly with the fact that Gugu was a few worksheets ahead of him. As a result, he was determined to catch up with her and rushed through every set, skipping prompts, ignoring examples, and dashing down any answer (Figure 7). He attempted seventeen worksheets, but of these eleven were unsatisfactory, and he did not return to these problematic worksheets to make corrections or re-attempt them. In the end, his success rate was one of the lowest in the class.

$$\sqrt{49y^6}$$

$$= 49y^3 \quad \times$$

$$5) 73,5 = 7,315 \quad \times \quad ?$$

$$2) 6m^2 \times 18m^2n^{-3} \times 4m^0n$$

$$= 432m^4n^3 \quad \times$$

$$5) = \frac{(m^2n)^3 \cdot 4m^2 \cdot n^{-2}}{(2mn)^3}$$

$$= m^{2 \times 3 \times 2} \cdot n^{-2}$$

$$= 6m^{12}n^{-2} = 6m^{12} \frac{1}{n^2}$$

Figure 6: Rushing through the sets. Nhlakanipho, selections from Sets J, K and R.

Example:

$$(k^2)^3 = k^{2 \times 3} = k^6$$

What did we do to the exponents?
ADD THEM ~~MULTIPLY~~

$$a^3 \times a^5 = a^8$$

What did we do to these exponents?
 Why are these different? ADD THEM

Example: $123 = 1,23 \times 100 = 1,23 \times 10^2$ Where did 10^2 come from?
~~100~~ ~~10~~

Figure 7: Form over substance. Nhlakanipho, from *Sets R and S*.

4.2 Engagement

4.2.1 What do the questionnaires say about engagement?

The third most agreed with statement in the questionnaires was “When I have mathematics homework I am not too bothered if I can’t do the sums”. Sixteen of the twenty four learners selected “always” for this, five selected “often” and the remaining three left the space blank. This suggests that the learners, although attempting to say the “right thing”, were quite unaware that dashing off exercises with no care for whether the answers are right or wrong is in conflict with their aims (or claims) of working hard, wanting to do well, and getting top marks.

Similarly almost all learners strongly agreed with “I am satisfied as long as I pass mathematics”. Is this because they consider obtaining 40% for mathematics is a sufficient achievement in itself? Or did they understand this as meaning “I am satisfied if I pass mathematics, no other subject matters”. From the responses received, it appears that the learners’ ambitions reach only as far as not having a red circle on their report card.

Most learners claimed that they can often figure out for themselves how to do a problem, although as will be seen from the evidence of the worksheets, this is all too frequently not true. Only Cassey, Nonhlanhla, Nothando, Sethu responded to this statement with “not really”. Likewise, in the face of the evidence, almost all learners said that they are “always” or “often” good at mathematics.

4.2.2 A history of not engaging

4.2.2.1. **Conceptual and procedural knowledge and forgetting**

Mathematics is mastered when learners move beyond procedural knowledge to conceptual knowledge (Hiebert & Lefevre, 2013). In the section of the curriculum on exponents, the learners are given a lot of rules to apply in different circumstances. If they do not have the conceptual understanding to underpin something like “add the exponents”, the particular circumstance where the rule is applicable is often partially or incorrectly remembered. Furthermore, when learners do not reinforce their learning with distributed practice, for example by doing homework, the falloff in their ability to do the sums correctly can be quite steep.

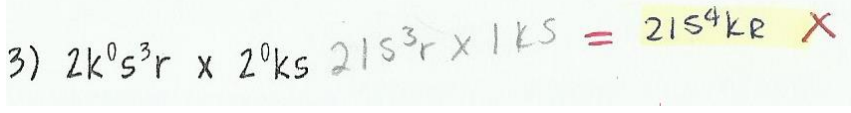
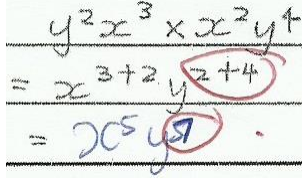
Learners who didn't really develop conceptual understanding of exponents from Grade 8 did not benefit much from the review of this content before the research week, which was done in the traditional way (exposition, example, exercise). Similarly the initial coverage of Grade 9 content simply increased their confusion. These learners scored particularly badly in the pre-test.

4.2.2.2. **Unreliable basic skills of mathematics**

Some of the learners displayed a worrying lack of proficiency in basic mathematical skills. A few of these errors can be attributed to carelessness or to cognitive overload, but the fact that cognitive overload is induced by these very simple operations indicates that the learners are still operating at a procedural rather than a conceptual level. Mental effort is required for tasks that should, by Grade 9, be almost automatic.

The worksheets were designed to include only the most modest numeric calculations, so that the learners would be able to complete them without using a calculator. This proved optimistic, as many learners were surprisingly challenged by the simplest sums. A few examples are shown in Table 1 below

Table 1: Simple calculation errors

<p>2 x 1 written as 21 (Phethile Set F)</p>	 <p>3) $2k^0s^3r \times 2^0ks = 21s^3r \times 1ks = 21s^4kr \quad X$</p>
<p>2+4 = 7 (Valentine Set D)</p>	 <p>$y^2x^3 \times x^2y^4$ $= x^{3+2}y^{2+4}$ $= 20^5y^7$</p>

$4,0 \times 1,2 = 5,2$ (Neo Set T)	$(4,0 \times 10^5) \times (1,2 \times 10^2)$ $= (4,0 \times 1,2) \times (10^5 \times 10^2)$ $= 5,2 \times 10^5 \times 10^2$ $= 5,2 \times 10^7 \checkmark$
---------------------------------------	--

Many of the learners did not remember that division is not commutative (see Figure 8).

Now look at these examples:

$$\sqrt[3]{x^3} = x$$

$$\sqrt[3]{m^6} = m^2$$

do to the ex

$$\frac{3}{6} = 2$$

What do we do to the exponent for a CUBE root?

$3 \div 6$

$$2) \frac{(2x^2y^3)^2}{8y^4} = \frac{4x^4y^6}{8y^4} = 2x^4y^2$$

Figure 8: Forgetting that division is not commutative (Nonhlanhla and Sanele Set P, Lizzy Set Q)

In some places learners subtracted numbers instead of dividing them (Figure 9). They apparently remember that “divide means subtract” but forget that this applies only to exponents if the bases are the same.

$$\frac{6m^3n^2}{2m^2} = 4mn^2 \checkmark$$

$$2) \frac{(2x^2y^3)^2}{8y^4} = \frac{4x^4y^6}{8y^4} = 2x^4y^2$$

Figure 9: Subtracting instead of dividing (Lwazi Set G and Nothando Set Q)

Sometimes the learners’ actions demonstrate a multiplicity of errors in a single step. For example, in Set Q, Neo apparently executed the following steps:

$$\left(\frac{m^3}{2^2}\right)^2 = (2m^{3+2})^2 = 2m^{5 \times 2} = 2m^{10}$$

(See Figure 10 below)

1) $\left(\frac{1}{4} m^3\right)^2 = \left(\frac{m^3}{2^2}\right)^2$
 $= 2m^{10}$

2) $\frac{2x^2}{3y} \div \frac{3x^2}{4y^4}$
 $5x^2y = 7y^4x^2$
 $= 12x^4y^4$

Figure 10: Multiple misunderstandings (Neo Set Q and Lizzy Set S)

4.2.2.3. Finding prime factors

Learners should have been able to find the prime factors of numbers up to 100 since Grade 6, and by Grade 7 should be able, for example, to find that $120 = 2^3 \times 3 \times 5$ (Department of Basic Education, 2011b). During the first term of their Grade 9 year, prime factors are revised again. However, this is a skill that many of the learners have not yet mastered (see Figure 11 below).

2) 15 $\frac{3}{3} \frac{5}{5}$ X 3) 12 $\frac{2}{2} \frac{6}{3}$ $\frac{3}{3} \frac{12}{6}$
 5) 100 $\frac{2}{2} \frac{50}{5}$ X

5) 100 $\frac{2}{2} \frac{50}{5}$ $\frac{5}{5} \frac{25}{5}$ $\frac{5}{5} \frac{5}{5}$ X
 $5 \times 5 \times 5 \times 5 = 5^5$

5) 100 $\frac{2}{2} \frac{50}{5}$ $\frac{5}{5} \frac{10}{5}$ $\frac{2}{2} \frac{2}{2}$
 $5) 100 \ 10 \times 10$

Figure 11: Difficulties with factorising into primes (Nokubonga, Neo, Nothando Set I)

This causes further difficulties, for example in Figure 12 Nokubonga was so focused on trying to correctly factorise the numbers, she completely forgot about the rest of the sum.

$-x \times 9 \times x^{-3} \times 27 \times x^2$
 $= 9 = 2^2$
 $27 = 3^3$

Figure 12: Factorising the numbers led to losing track of the sum (Nokubonga Set I)

4.2.2.4. Working with negative numbers

Learners begin operating on negative numbers in Grade 8 (Department of Basic Education, 2011b) but there are numerous examples in the worksheets that illustrate their inability to do the most basic operations correctly without a calculator. A selection of these is illustrated below (Figure 13 to Figure 19).

$$p \times (-2) \times 3 \times k \times p \times (-2)$$

$$? = p^{\ominus} \times -4 \times 3 \times k \times p^{\ominus}$$

$$2) (6,6 \times 10^{-2}) \times (1,0 \times 10^{-5})$$

$$= (6,6 \times 1,0) \times (10^{-2} \times 10^{-5})$$

$$= 6,0 \times 10^{\ominus} = 130$$

$$1) (a^3)^{-4} = a^{+3 \times -4}$$

$$= a^{12} \times$$

$$= 2,7 \times 1,2 \times 10^{\ominus 2} \times 10^{\ominus 3}$$

$$= 3,24 \times 10^{\ominus} \times$$

Figure 13: Struggling to add or multiply negative numbers (Mandisa Set C, Lizzy Set T, Nothando Set L, and Nhlakanipho Set T)

$$m^2 \times 8 \times n^{-2} \times 2^2 \times m^{-3}$$

$$= m^2 \times 2^3 \times n^{-2} \times 2^2 \times m^{-3} \quad (\text{Wt})$$

$$= m^2 \cdot 2^5 \cdot m^2 \times 2 m^{-3} = m^2 \cdot 8$$

$$= m^1 \cdot 2^5 \cdot n^{-2}$$

Figure 14: Negative numbers are a problem (Neo Set I)

$$-2x - 2x - 2 \times m \times x \times x \times x \times m$$

$$= -6 \times m^2 \times x^3 \checkmark$$

$$= 6 m^2 x^3 \times$$

Figure 15: Incorrectly multiplying out $-2x-2x-2$ and then losing the sign (Mandisa, Set C)

New examples:

$(x^{-4})^{-1} = x^{-4 \times -1} = x^4$ Why is the exponent positive? because 4 is bigger than -4	$(w^3)^{-2} = w^{3 \times -2} = w^{-6}$ Why is this one negative? because we have to negatives
--	---

Figure 16: Unable to articulate rules for multiplying negative and positive numbers (Neo, Set L)

$$\begin{aligned}
 & m^2 \times 8 \times n^{-2} \times 2^2 \times m^{-3} \\
 & = m^2 \times 2^3 \times n^{-2} \times 2^2 \times m^{-3} \\
 & = m^2 \times m^3 \times n^2 \times 2^3 \times 2^3 \\
 & = 2^6 m^5 n^2
 \end{aligned}$$

Figure 17: Losing the negative signs (Lizzy, Set I)

$$\begin{aligned}
 1) (a^3)^{-4} & = \\
 & = a^{3 \times -4} = a^{12}
 \end{aligned}$$

$$\begin{aligned}
 1) (a^3)^{-4} & = \\
 & = a^{3 \times -4} = a^{12} \quad \times
 \end{aligned}$$

Figure 18: Multiplying numbers with opposite signs (Quincy and Nokulunga Set L)

$$\begin{aligned}
 3) (2,7 \times 10^{-2}) \times (1,2 \times 10^{-3}) \\
 & = \cancel{16,2} \times 1 \\
 & = 2,7 \times 1,2 \times 10^{-2} \times 10^{-3} \\
 & = 3,24 \times 10^{-5} \quad \times
 \end{aligned}$$

Figure 19: Unable to add -2 and -3 (Nhlakanipho, Set T)

4.2.2.5. Basics of exponents

Weaker learners were initially unfamiliar with the meaning of exponential notation. Selected examples are shown in Figure 20.

$$\begin{aligned}
 & -x \times 9 \times x^{-3} \times 27 \times x^2 \\
 & = -x \times \underbrace{2^3}_{\wedge} \times x^{-3} \times \underbrace{3^3}_{\wedge} \times x^2
 \end{aligned}$$

$$2) 15 = \cancel{3^5} \quad \times \quad 3) 12 = 3^4 \quad \times$$

$$\begin{aligned}
 & \frac{m \times 3 \times 2 \times p \times m \times 2}{=} \\
 & = m^2 \times 9 \times p \times 2 \\
 & = m^2 + 18m^2p \quad \times
 \end{aligned}$$

$$5) 100 = 20^5 (20 \times 5)$$

Figure 20: Confusing multiplication and exponentiation (Quincy and Phethile, Sets B and I)

Some learners are uncomfortable with a solution to a mathematics problem that is an expression rather than a single term. They continue to join up terms in whatever way they can imagine until they obtain a single "answer". Similarly, they suppose that a term with

different exponents attached to different variables can be simplified further (see Figure 21 below).

3) $x x x y y x 5$
 $= 2^2 x y^2 x 5 x$
 $= x x y x 9 x$

2) $x x y y y z z z$
 $= x^2 x y^3 x z^3$
 $= x y z^8$

Figure 21: *Seemingly unable to stop* (Quincy and Cassey, first attempts at Set A)

Some learners not consistently able to apply the basics rules of exponents (Figure 22 and Figure 23).

5) $5 x 3 x 5^2 x 3 x 3^3 = 25^2 3^2 5^5 x$

Figure 22: *Multiplying bases and adding exponents* (Neo, Set L)

b) $m^2 x n^3 x m^4 x n = m^{2x4} x n^{3x4} = m^8 n^{12}$

Figure 23: *Multiplying exponents instead of adding* (Quincy Set K)

Lizzy was sometimes able to apply the rules correctly to the variables but could not apply the rule to the coefficients (Figure 24 below).

$= 3^2 m^2 x 3^6 m^2 n^3 x 2^2 m^1 n$
 $= 18^{10} m^5 n^4 x$

b) $\frac{3^2 m^2 n^5}{3 m^0 n^2}$
 $1^2 m n^4 x$

Figure 24: *Multiplying bases and then adding exponents or doing the equivalent for divide* (Lizzy, Set S and partial extract from Set J)

Other learners coped well enough with simple problems, but when faced with more complicated sums they lost track of what they were doing (see Figure 25 below).

$m^2 x 8 x n^{-2} x 2^2 x m^{-3}$
 $= m^2 x 2^3 x n^{-2} x 2^2 x m^{-3}$
 $= m^4 x 2^5 x n^{-4} x m^{-6}$

Figure 25: *Unable to add exponents for m* (Nothando Set I)

4.2.3 What do the worksheets say about engagement?

4.2.3.1. **Engagement and mental effort**

In any group of Grade 9 learners, levels of engagement vary from day to day or even minute to minute. Just as mental effort is a measure of cognitive load when engagement is constant (Sweller (2010), van Merriënboer & Sweller (2005)), mental effort may be a measure of engagement when cognitive load is constant. Consider Neo's attempts at Set H which are illustrated in Figure 26 below. On the left hand side, he appears to have no idea what he is doing but he has indicated he found this worksheet to be "easy". On his second attempt (right hand side of Figure 26), although he is still struggling, he is starting to get an idea of what the negative rule for exponents requires. His answers are getting better – but now he realises that this worksheet is actually "difficult".

Figure 26 consists of two side-by-side photographs of handwritten mathematical work on a worksheet. The left photograph shows the learner's first attempt. It includes the following work:

- 1) $x^{-2} = \frac{1}{x^2}$ ✓
- 2) $\frac{1}{y^{-3}} = \frac{2}{1} = 23$ ✗
- Text: "Look at the example and solve the"
- 3) $3x^{-2} = \frac{3}{x^2}$
- $5a^{-3} = 5a^3$ ✗
- 4) $\frac{3}{k^{-2}} = 3k^2$
- $\frac{5}{m^{-5}} = 5m^5$ ✗
- Text: "Mark your work carefully, and find"
- Pick one: Difficult
- Easy

The right photograph shows the learner's second attempt. It includes the following work:

- 1) $x^{-2} = \frac{1}{x^2}$ ✓
- 2) $\frac{1}{y^{-3}} = \frac{y^3}{1} = y^3$ ✓
- Text: "Look at the example and solve the proble"
- 3) $3x^{-2} = \frac{3}{x^2}$
- $5a^{-3} = \frac{5}{5a^3} = 5a^3$ ✗ $\frac{5}{5a}$
- 4) $\frac{3}{k^{-2}} = 3k^2$
- $\frac{5}{m^{-5}} = \frac{5}{m^5} = m^5$ ✗ $5m^5$
- Text: "Mark your work carefully, and find where"
- Pick one: Difficult
- Easy

Figure 26: Is this worksheet "easy" or "difficult"? (Neo Set H)

It appears that in this case Neo was not engaging with the content at all during his first attempt, but on his second attempt he was trying to understand the concepts.

4.2.3.2. **Not looking at the steps in the examples**

In most of the faded examples it is not possible to determine whether the learner actually looked over *all* the steps, or simply continued from the *last* step given. However, there are

places where there is clear evidence that the first steps have been ignored (see Figure 27 and Figure 28 below).

Example: $123 = 1,23 \times 100 = 1,23 \times 10^2$ Where did 10^2 come from? $10 \times 10 = 100 = 10^2$ ✓

3) $7\ 100 = 7,1 \times 1000 = 7\ 1000$ ✗

4) $61,2 = 6,12 \times 100 = 6120$ ✗

5) $731,5 = 731,5 \times 1000 = 7315000$ ✗

Figure 27: Going in circles - not looking at earlier steps of faded examples (Lizzy Set J)

2)
$$\frac{121\ k^2m^5 \times 121\ km^4p^2}{= 11^2\ k^2m^5 \times 11^2\ km^4p^2}$$

$$= 11^{2+2}\ k^{2+1}\ m^{5+4}\ p^2$$

$$= 11^4\ k^3\ m^9\ p^2$$
 ✗

$$(3x^2)^2 = 3^2\ x^{2 \times 2} = 3x^4$$
 ✗

Figure 28: These partially complete problems didn't seem to help (Neo Set M and K)

In Set I, the faded example illustrated how to write a number as a product of primes, so that for example 8 was replaced with 2^3 . However, more than one learner had clearly not looked at all the steps (see Figure 29).

$$m^2 \times 8 \times n^{-2} \times 2^2 \times m^{-3}$$

$$= m^2 \times 2^3 \times n^{-2} \times 2^2 \times m^{-3}$$

$$= m^2 \times 6 \times n^{-2} \times 4 \times m^{-3}$$

$$= m^2 \cdot 6 \cdot n^{-2} \cdot 4 \cdot m^{-3}$$

Figure 29: Quincy didn't look at the previous steps (Set I)

4.2.3.3. Not doing corrections

Although some learners returned to do corrections (for example Figure 30 below), others simply ignored their errors and forged ahead – frequently repeating the same mistakes over and over again. Of the 24 learners 15 left at least 40% of their worksheets uncorrected.

2) $\frac{6m^3n^2}{2m^2}$ $6/2 = 3$
 ~~$4m^2 \times 3mn^2$~~ ✓

Figure 30: Returning to correct an error (Quincy Set G)

Two of the worst culprits for moving ahead were Nhlakanipho (who was discussed in detail in Section 4.1.3.2 above) and Nothando, both of whom correctly completed only 6 sets while having a total of 11 sets for which no credit could be given due to the number of errors. For example consider Nothando's attempt at Set M (Figure 31 below).

Set M

Name Nothando

Look at these examples:

$\sqrt{9} = \sqrt{3^2} = 3$ What happened to the exponent?
 ~~3×3~~

$\sqrt{9a^2} = \sqrt{3^2 a^2}$
 $= \sqrt{3^2} \cdot \sqrt{a^2} = 3a$

Notice we do the same thing to every variable or number under the square root sign

Complete these sums:

1) $\sqrt{16m^2}$
 $= \sqrt{16} \sqrt{m^2}$
 $= 4 \times 4$ X

2) $\frac{121 k^2 m^5 \times 121 k m^4 p^2}{121 k^2 m^5 \times 121 k m^2 p^2}$
 $= \frac{121^{2+2} k^{2+1} m^{5+2} p^2}{121^{2+2} k^{2+1} m^{5+2} p^2}$
 $= 11^4 k^3 m^7 p^2$ ✓

Look at example

3) $\sqrt{x^4} = x \times x \times x \times x$ X

4) $\frac{5y^5 x^4 z}{y^2 x^2}$
 $= \sqrt{5y^{5+2} x^{4+2}}$ X

5) $\sqrt{a^2} = a \times a$ X

6) $\sqrt{b^8} = b \times b \times b \times b \times b \times b \times b \times b$ X
 $\sqrt{x^6} = x^3$ Re-do

Pick one: Difficult Just right
 Easy Very easy

Figure 31: Little evidence of cognitive engagement? (Nothando Set M)

Nothando did not return to Set M, either to ask for help, to make corrections or to try again. When Set P revisited these ideas, she repeated the same mistake – and then managed to get a few of the same type of sum correct. It appears that her wrong answers and nonsense responses to the prompts are more due to lack of engagement than to cognitive overload.

Write corrections for the

Set P — Name *Mathew* *ones that are wrong*

Remember what we learned about square roots:
 $\sqrt{x^4} = x^2$ What did we do to the exponent? $\frac{4 \div 2}{x^2}$

Now look at these examples:
 $\sqrt[3]{x^3} = x$
 $\sqrt[3]{m^6} = m^2$

What do we do to the exponent for a CUBE root?
~~div~~ \div *by what*

Complete these sums:

1) $\sqrt[3]{27} = \sqrt[3]{3^3} = 3 \times 3 \times 3$ X
 2) $\sqrt[3]{64} = \sqrt[3]{2^6} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ X

And do these too

3) $\sqrt[3]{x^3 y^3} = xy$ ✓ 4) $\sqrt[3]{x^3 m^6} = xm^2$ ✓
 5) $\sqrt{49 y^6} = 4$ 6) $\sqrt{25 b^6} =$
 7) $617.8 = 6,178 \times 100 = \underline{61780}$ X
 8) $0.45 = 45 \div 100 = 450$ X
 9) $\sqrt[3]{a^6 b^9 c^3} = a^2 b^3 c$ ✓
 10) $9.41 \times 10^{-1} = 0.941$ ✓

look at examples

Pick one: Difficult Just right
 Easy Very easy

Figure 32: Repeating the same mistakes

4.2.3.4. **Set L: following prompts and fading, but not making connections**

Set L

Name Cassey

Remember this?: $(2^2)^3 = 2^{2 \times 3} = 2^6$

New examples:

$(x^{-4})^{-1} = x^{-4 \times -1} = x^4$ Why is the exponent positive? $- \times - = +$ ✓	$(w^3)^{-2} = w^{3 \times -2} = w^{-6}$ Why is this one negative? $+ \times - = -$ ✓
---	--

We don't like negative exponents in our final answer. How should we write w^{-6} ? ~~_____~~

Complete these sums:

1) $(a^3)^{-4} = a^{-12}$
=

$$\begin{aligned} (x^{-2})^3 &= x^{-2 \times 3} \\ &= x^{-6} \\ &= \frac{1}{x^6} \end{aligned}$$

Figure 33: Cassey's response for Set L

Set L was selected for in-depth analysis because 23 of the 24 learners completed this set (some learners making more than one attempt) and because the prompts and examples should have led the learners to see clearly what was required. The set contains two "reminder" examples of content from earlier sets and two additional examples showing how the power-of-a-power rule works with negative exponents. There are also three places where the learners are prompted to think about various aspects of the examples, such as a reminder that the rule for negative exponents should be applied. The first question, which is included in this analysis, is closely analogous to the examples.

A learner who had looked at and understood the examples and correctly answered the prompts should be more likely to get the sum right, than would a learner who skipped over the prompts and examples: successfully answering the prompts and successfully answering the question should not be independent events.

There were two areas of application here – products of integers and the negative exponent rule – so it seemed appropriate to look at these separately. The details of the

learners' responses are contained in Appendix B. Contingency tables summarising the data are shown in Table 2 and Table 3 below.

Table 2: Contingency table for the product of integers.

	Prompt right	Prompt wrong	
Sum right	11	4	15
Sum wrong	5	4	9
	16	8	24

Table 3: Contingency table for negative exponents.

	Prompt right	Prompt wrong	
Sum right	6	1	7
Sum wrong	9	8	17
	15	9	24

Table 4: Results of Fisher's Exact Test on the contingency tables from Set L.

	Fisher's Exact Test	
	1 tail	2 tail
Product of integers	0.32	0.41
Rule for negative exponents	0.15	0.19

Fisher's Exact Test showed that for both contingency tables, the elements were independent. It is not possible to infer from this data that studying the examples and correctly answering the prompts is effective in helping learners to correctly do related sums. For most of them, they apparently did not see the connection between the prompts and the problem to be solved.

4.3 Cognitive load

When a learner does not have a schema relevant to a problem – is lacking in conceptual knowledge – he has to apply the limited capacity of his working memory to deal with elements a few at a time while still trying to remember the procedures he is required to apply (Sweller, van Merriënboer, & Paas, 1998). The more interacting elements there are, the more difficult this task becomes. If the learner is engaged with the problem – making a real endeavour to find a solution – this difficulty is perceived as mental effort. However as we saw in Section 4.2, mental effort as a measure of cognitive load is compromised when the learners are not really engaging with the task.

In this section examples of apparent cognitive overload are presented. Some of these are strategies applied by the learners which appear to be a result of a struggle to bypass working memory limits. In other examples, cognitive overload results in details being lost and trains of thought derailed.

4.3.1 Evidence of cognitive overload

The application of the rule for negative exponents was particularly challenging to some learners. For example, Ciniso was unable to complete “switch between numerator and denominator, and change the sign” in one operation, so he followed the rule by breaking it into two steps (Figure 34 below). Note also that used the incorrect variable partway through, although he corrected this later.

Figure 34 shows two handwritten mathematical operations. The first operation is $1) x^{-2} = \frac{1}{x^2} = \frac{1}{x^2}$, with a red checkmark to the right. The second operation is $2) \frac{1}{y^{-3}} = \frac{1}{x^{-3}} = \frac{y^3}{1} = y^3$, with a red checkmark to the right. The variable x is used in the second step instead of y .

Figure 34: Applying the rule as two separate operations (Ciniso Set H)

Phethile (Figure 35) remembered that the negative exponent must become positive and move to the numerator, but she was unable to do this in one step. In the process of following her plan, she forgot that this is an exponent, not just a number. The incongruity of writing $1/3 = 3$ did not occur to her.

Figure 35 shows a student's handwritten work on lined paper. The first line is $d) \frac{1}{x^{-3}}$. The second line is $= \frac{1}{3}$ with a large red 'X' over it. The third line is $= 3$.

Figure 35: Two operations were not successful (Phethile pre-test)

Neo managed the first sum in Set H correctly but in the second sum he forgot what variable he was operating on, and then compounded his error by writing his answer as “23” instead of “2³” (Figure 36 below).

$$1) x^{-2} = \frac{1}{x^2} \quad \checkmark$$

$$2) \frac{1}{9^{-3}} = \frac{1}{\frac{1}{27}} = 27 \quad \times$$

Figure 36: Losing track of the variable (Neo Set H)

Neo struggled again with this rule in Question 3: he changed the sign of the exponent, but failed to move the variable to the denominator (Figure 37).

$$3) 3x^{-2} = \frac{3}{x^2}$$

$$5a^{-3} = 5a^3 \quad \times$$

Figure 37: Half of the operation (Neo Set H)

Many learners also struggled with the “power of a power” rule; for example in Figure 38 below, Lwazi has concentrated on getting $x^{2 \times 2}$ right and lost track of 3^2 in the process.

$$(3x^2)^2$$

$$= 3^2 x^{2 \times 2} = \underline{x^4}$$

Figure 38: Losing track of the number (Lwazi Set K)

There are multiple examples in Set K of learners correctly determining the exponents for the variables, but not applying the rule to the coefficients (Figure 39).

Example:

$$(a^2 b^3)^4$$

$$= a^{2 \times 4} b^{3 \times 4}$$

$$= a^8 b^{12}$$

Notice we multiply exponents of each variable or number in the brackets

$$(3x^2)^2$$

$$= 3^2 x^{2 \times 2} = \underline{3^2 x^4} \quad \checkmark$$

Now you do some:

$$1) (a^5)^5 = a^{25} \quad \checkmark$$

$$2) (m^3)^2 = m^6 \quad \checkmark$$

$$3) 7^2 m^4 n^2 p^5 \times 49 p^2 n m^2 = 7^2 m^6 n^3 p^7$$

$$4) (5m^2)^3 = 5m^6 \quad \times$$

$$5) (3q^3 r)^4 = 3q^{12} r^4 \quad \times$$

Figure 39: Forgetting to apply the rule to numbers. Note that she completed the example correctly. (Extracts from Lizzy Set K)

In Figure 40 Quincy answered the prompts correctly but lost track of what he should be doing when actually applying the rule.

Example:

$$(k^2)^3 = k^{2 \times 3} = k^6$$

What did we do to the exponents?
multiply ✓

$$a^3 \times a^5 = a^8$$

What did we do to these exponents?
you add exponent when there's x ✓

Why are these different? you add exponent when there's x ✓

4) $(5m^2)^3 = 5^3 m^{2 \times 3} = 125 m^6$ ✓ $(3qr)^4 =$

6) $m^2 \times n^3 \times m^4 \times n = m^{2+4} \times n^{3+1} = m^6 n^4$ ✓

Figure 40: Prompts to remind learners of the rules – but Quincy can't apply the rule (extracts from Quincy Set K)

There are frequent occurrences of learners forgetting what they are doing partway through a problem (Figure 41).

$$3) (2,7 \times 10^{-2}) \times (1,2 \times 10^{-3})$$

$$= 1,6,2 \times 10^{-5}$$

$$= 2,7 \times 1,2 \times 10^{-2} \times 10^{-3}$$

$$= 3,24 \times 10^{-5}$$

$$1) \sqrt{16m^2}$$

$$= \sqrt{16} \sqrt{m^2}$$

$$= 4 \times 4 \times$$

$$m^2 \times 8 \times n^{-2} \times 2^2 \times m^{-3}$$

$$= m^2 \times 2^3 \times n^{-2} \times 2^2 \times m^{-3}$$

$$= m^{2-3} \times 2^{3+2} \times n^{-2}$$

$$= m^{-1} \times 2^5 \times n^{-2}$$

Figure 41: What was I doing again? (Nhlanipho Set T, Nothando Set M, Nokubonga Set I)

In Figure 42 Phethile correctly calculated 2×4 to get a power of 8 for the variable k, but slipped up on $2^4 = 16$. She correctly applied the rule for “power of a power” to the variables, but then became confused and finished by adding all the exponents.

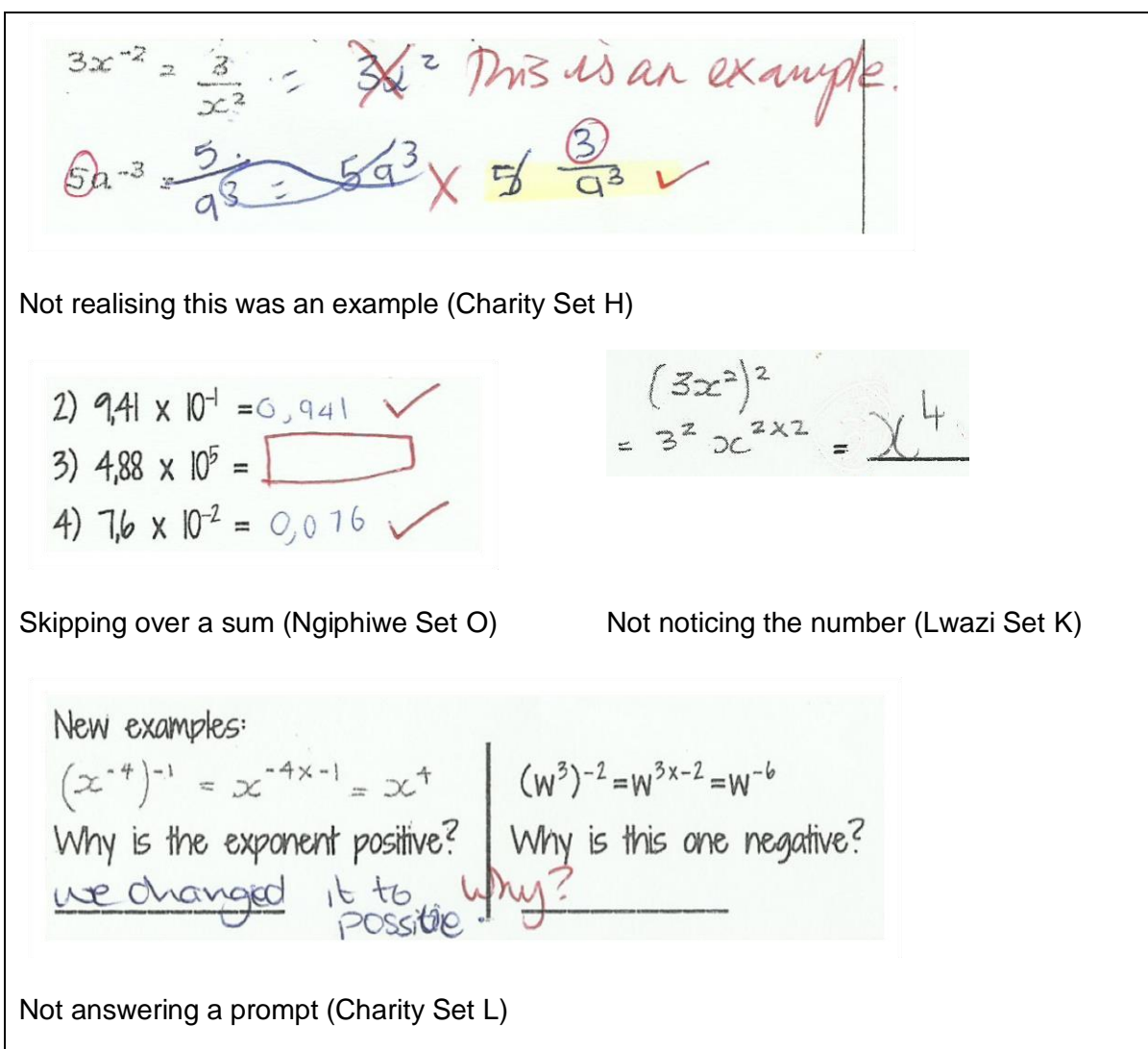
$$f) (2\text{km}^3)^4$$

$$= 8\text{k}^8\text{m}^{12}$$
~~$$= 8\text{km}^{20}$$~~

Figure 42: Too many variables (Phethile pre-test)

4.3.1.1. Inattentional blindness

Inattentional blindness may be a result of cognitive overload (Bredemeier & Simons, 2012), which may explain the number of places in which learners appeared not to see examples, prompts or details of sums.



Handwritten examples and student work illustrating inattentional blindness:

Top example: $3x^{-2} = \frac{3}{x^2} = \cancel{3}x^2$ This is an example.

Second example: $5a^{-3} = \frac{5}{a^3} = \cancel{5}a^3$ X $\frac{5}{a^3}$ ✓

Not realising this was an example (Charity Set H)

2) $9,41 \times 10^{-1} = 0,941$ ✓

3) $4,88 \times 10^5 =$

4) $7,6 \times 10^{-2} = 0,076$ ✓

$(3x^2)^2 = 3^2 x^{2 \times 2} = x^4$

Skipping over a sum (Ngiphiwe Set O) Not noticing the number (Lwazi Set K)

New examples:

$(x^{-4})^{-1} = x^{-4 \times -1} = x^4$	$(w^3)^{-2} = w^{3 \times -2} = w^{-6}$
Why is the exponent positive?	Why is this one negative?
<u>we changed</u> it to possible.	<u>why?</u>

Not answering a prompt (Charity Set L)

Figure 43: Can these mistakes be ascribed to inattentional blindness?

In places it is difficult to decide whether the learner had not noticed what was going on due to cognitive overload, or whether he was failing to put in a mental effort (Figure 44 and Figure 45).

Remember what we learned about square roots:

$$\sqrt{x^4} = x^2 \text{ What did we do to the exponent? } \underline{\div 2}$$

Now look at these examples:

$$\begin{array}{l} \sqrt[3]{x^3} = x \\ \sqrt[3]{m^6} = m^2 \end{array}$$

What do we do to the exponent for a CUBE root?

divide by 2 ~~Nope~~ look at examples.

Complete these sums:

1) $\sqrt[3]{27} = \sqrt[3]{3^3} = 3 = 3^1$

2) $\sqrt[3]{64} = \sqrt[3]{2^6} = 2^2$ ✓

And do these too

3) $\sqrt[3]{x^3 y^3} = \sqrt[3]{x^2 y^2}$ ✗ 4) $\sqrt[3]{x^3 m^6} = \sqrt[3]{x^2 m^3}$ ✗

5) $\sqrt{49 y^6} = \sqrt{49 y^3}$ ✗ 6) $\sqrt{25 b^6} = \sqrt{25 b^3}$ ✗

Figure 44: Inattentional blindness, cognitive overload, or just being slapdash? (Lizzy Set P)

Remember this?

$$\begin{aligned} (ab^2)^3 \\ = (a^1 \cdot (b^2)^3) = a^{1 \times 3} \cdot b^{2 \times 3} \\ = a^3 \cdot b^6 \end{aligned}$$

Why did I multiply the exponents? Same
BRICS

Figure 45: Not being able to see what is happening, or just carelessly writing the first answer that occurs? (Nhlakanipho Set R first attempt)

4.3.1.2. Articulation and emulation

Wendy was able to emulate the examples for negative exponents, but she was unable to state the rule clearly – her strange wording may be due, not to an inability to apprehend the rule, but to an inability to articulate it (Figure 46).

Now write this as a rule for negative exponents:
You divide with a positive ✗

1) $x^{-2} = \frac{1}{x^2}$ ✓

2) $\frac{1}{y^{-3}} = \frac{y^3}{1} = y^3$ ✓

Figure 46: Wendy words the rule strangely, but applied it correctly (Set H)

Nokulunga seemed unable to state the principles for multiplying integers (Figure 48).

New examples:

$(x^{-4})^{-1} = x^{-4 \times -1} = x^4$ Why is the exponent positive? <u>We multiplying and there is a negative in front</u> ✗	$(w^3)^{-2} = w^{3 \times -2} = w^{-6}$ Why is this one negative? <u>because there is one negative</u> ✓
---	--

Figure 47: Inability to articulate the rule (Nokulunga Set L)

Sometimes the rule was incorrectly stated and then when the learner used the rule the same mistake was made – in other words, the problem was not with articulating the rule, but with deducing it (see Figure 48).

Now write this as a rule for negative exponents:
the exponents change to positive

1) $x^{-2} = x^2$ ✗

2) $\frac{1}{y^{-3}} = \frac{1}{y^3}$ ✗

Figure 48: Has Mandisa noticed the switch between denominator and numerator and decided that it is not important, or has he completely failed to see this? (Mandisa, Set H second attempt)

In other places, the answers to the questions are meaningless, or apparently unrelated to the question asked (Figure 50).

Remember this?

$$\begin{aligned} & (ab^2)^3 \\ &= (a^3) \cdot (b^2)^3 = a^{1 \times 3} \cdot b^{2 \times 3} \\ &= a^3 \cdot b^6 \end{aligned}$$

Why did I mult
exponents? —!

Figure 50: Answer seems unrelated to the question (Lizzy Set R)

4.3.2 Evidence of mitigation

As shown in numerous examples above, learners – especially the weaker ones – were struggling with the concept of exponents and the operations required; and in places there was clear evidence of cognitive overload. The question of whether the reminder prompts and faded examples were able to mitigate this cognitive overload is addressed in terms of statistical comparisons between the pre- and post-tests, and in examples of improved capability.

4.3.2.1. Differences between pre-tests and post-tests

The results of statistical analyses of the pre- and post-test results (scored as percentages) are shown in detail

in Appendix D.

The mean score for the post-test (42%) was higher than that of the pre-test (37%) although a *t*-test for paired samples showed that the scores were not significantly different at $\alpha = 0.05$ ($t = -1.23$; $p = 0.23$). The standard deviation was somewhat higher for the pre-test (19% as compared to 13%).

If we consider the box-and-whisker diagram alongside, we can see that the minimum and the lower quartile are considerably higher in the post-test. This suggests that the weaker learners were substantially more effective in the post-test.

Neo and Cassey obtained the worst results in the pre-test, being unable to apply any of the rules of exponents that had been covered in Grade 8 or during the teaching sessions that preceded the research intervention. The post-test showed impressive improvements: Neo scored 7% in the pre-test and 54% in the post-test; Cassey scored 3% in the pre-test

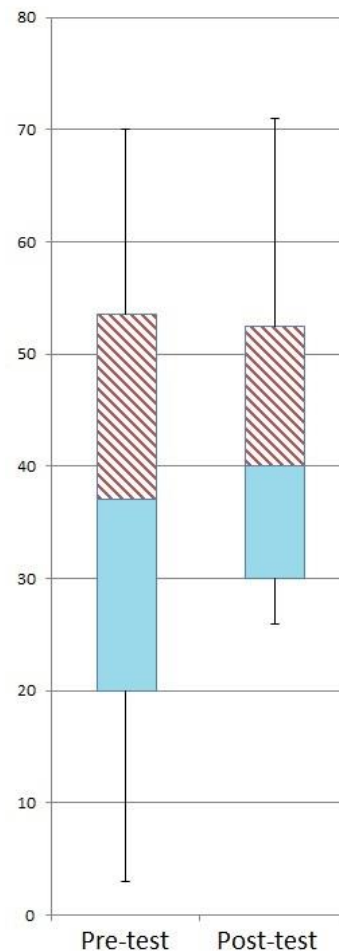


Figure 49: Differences between results of pre- and post-tests

and 51% in the post-test. Scans of relevant selections which illustrate this progress are included in Appendix E.

There was no corresponding improvement in the means of the pre- and post-tests however, because the improvements in these learners' results were offset by those who did worse in the post-test. Of the learners who obtained the top ten scores in the pre-test, only 3 of them improved. For example Wendy, a quiet girl who focused well in class, attempted 17 of the worksheets, although for 11 of these she did not master the content (no "pink dot"). She was given the opportunity to repeat the failed worksheets at any time, but like many of the other learners did not do so. Her pre-test result was 57% and her post-test result a disappointing 34%.

Of all the members in the class, Sethu completed the lowest number of worksheets (5 attempted of which only 3 were mastered). Sethu struggled to remain focused in class and shows little interest in learning. Daily comments logged during research week on Sethu's attitude and engagement included:

- Set F: Sethu was distracted in class by an argument about Barcelona football. Second re-do still full of errors.
- Not able to add exponents to multiply even though earlier sets were completed correctly. Set I for a re-do.
- Very little effort, very little success.

Sethu's pre-test score was 57% and his post-test score was 37%.

4.3.2.2. **Examinations**

The questions from the June Examination were divided into those from the section of the curriculum which had been the focus of the intervention, and the remainder of the work. A Shapiro-Wilk test indicated that both sets of results were normally distributed. The average (mean) mark for the class for the focus questions was 49,3%, significantly higher than the mean of 35,4% for the remainder of the questions ($\alpha=0.05$, $t=0.4$, $p=0.000546$, $t_{crit}=2.0687$). Detailed results are included in CHAPTER 1: Appendix D.

4.3.2.3. **Effectiveness of examples**

Set F was one of the worksheets that the learners found particularly difficult. The example before the questions clearly showed how only the variable with a zero exponent is replaced with a 1. If the demonstrated method was used, there was no difficulty (Figure 51).

Another example: $3x^0y^2$
 $= 3 \times 1 \times y^2 = 3y^2$

Now do the rest of these sums:

- 1) $4m^2n^0b^3 = 4m^2 \times 1 \times b^3 = 4m^2b^3$ ✓
- 2) $5^0m^2n \times 3m = 1 \times m^2 \times n \times 3m = 3m^3n$ ✓
- 3) $2k^0s^3r \times 2^0ks = 2 \times 1 \times s^3 \times r \times 1 \times r \times s = 2ks^4r$ ✓
- 4) $a^3b^4 \div b^2a^0 = a^3b^2$ ✓

Figure 51: Carefully emulating the example (Quincy Set F)

Most learners on their first attempt did not do this, and a variety of incorrect solutions were presented (Figure 52).

<p>1) $4m^2n^0b^3 = 1$ ✗</p> <p>2) $5^0m^2n \times 3m = 1$ ✗</p> <p>3) $2k^0s^3r \times 2^0ks = 1$ ✗</p> <p>4) $a^3b^4 \div b^2a^0 = 1$ ✗</p>	<p>1) $4m^2n^0b^3 = 4m^2nb^3$ ✗</p> <p>2) $5^0m^2n \times 3m = 5m^2n \times 3n = 15m^2n$</p> <p>3) $2k^0s^3r \times 2^0ks = 2ks^3r \times 2ks$</p> <p>4) $a^3b^4 \div b^2a^0 = a^3b^4 \div b^2a$</p>
<p>1) $4m^2n^0b^3 = 4 \times 2 \times n \times y^2 = 8ny^2$ ✗</p> <p>2) $5^0m^2n \times 3m = 5 \times 1 \times n \times 3n = 15n$ ✗</p> <p>3) $2k^0s^3r \times 2^0ks = 2 \times 1 \times s \times 3r \times 2 \times 1 \times ks = 12rks$ ✗</p> <p>4) $a^3b^4 \div b^2a^0 = a \times 3b \times 4 \div b \times 2 \times a \times 1 = 12a$</p>	

Figure 52: First attempts at Set F (Cassey, Lizzy and Neo)

Learners who repeated the worksheet and followed the example had a higher level of success.

Now do the rest of these sums:

- 1) $4m^2n^0b^3 = 4m^2 \times 1 \times b^3 = 4m^2b^3$ ✓
- 2) $5^0m^2n \times 3m = 1 \times m^2 \times n \times 3m = 3m^3n$ ✓
- 3) $2k^0s^3r \times 2^0ks = 2 \times 1 \times s^3 \times r \times 1 \times k \times s = 2ks^4r$ ✓
- 4) $a^3b^4 \div b^2a^0 = a^3b^2$ ✓

Figure 53: Success! Neo's second attempt at Set F.

4.4 In what ways did the worksheets fail?

Not all of the worksheets were equally effective, and the method of expecting learners to learn methods from examples did not always work.

4.4.1 Scientific notation failure

Learners encounter scientific notation in Grade 8, so they are expected to have some knowledge of the form and use of this notation by Grade 9 (Department of Basic Education, 2011b). Scientific notation is included in the section with exponents and so it seemed appropriate to include this content in the worksheets. Unfortunately, it was quickly apparent that examples and faded examples were simply not effective at helping learners to grasp this notation.

4.4.1.1. *Short-circuiting*

Teaching scientific notation to learners – especially less able learners – is frequently done by explaining, for example, that $\times 10^4$ means “move the comma four places to the right”. This short circuiting to results in little cognitive engagement, and leads to passive learners (Corno & Mandinach, 1983). To avoid this, a more rigorous approach was implemented in the relevant worksheets. This was not successful.

4.4.1.2. *Place value*

Learners cover multiplication by 10, 100 and 1000 from Grade 4 and are introduced to decimal fractions in Grade 6 (Department of Basic Education, 2011a). However quite a number of learners in the class were unable to combine these operations as required.

Example: $123 = 1,23 \times 100 = 1,23 \times 10^2$ Where did 10^2 come from? $\frac{100 \times 100}{X}$

3) $7\ 100 = 7,1 \times 1000 = 7,100 \times 100^2 X$

4) $6,2 = 6,2 \times 10^2 X$

5) $7315 = 100^2 X$

Figure 54: Uncertainty about powers of 10 (Phethile Set J)

Now to add and subtract numbers in scientific form:

$$(1,2 \times 10^3) + (2,1 \times 10^4)$$

$$= 1,2 \times 1000 + 2,1 \times 10000$$

$$= 1200 + 21000$$

$$= 22200$$

Just write as an ordinary number then add

Finish these $(2,7 \times 10^2) + (5,1 \times 10^3)$

$$= 2,7 \times 100 + 5,1 \times 1000$$

$$= 2700 + 51000$$

$$= 27510000$$

Figure 55: Inability to multiply decimal numbers by powers of 10; inability to add whole numbers (from Neo's Set U)

In Figure 55 Neo remembered the short-circuit “multiply by 100 means you put two zeros on the end”, but forgot to take account of the decimal comma. Furthermore, he was unable to add 2 700 and 51 000. This was one place where the “no calculator” rule highlighted basic shortcomings.

$$3) 7\ 100 = 7,1 \times 1000 = 7,1 \times 10^3$$

$$4) 61,2 = 6,12 \times 10^2$$

$$5) 731,5 = \cancel{10^3} 7,315 \times 10^2$$

Figure 56: Uncertainty about multiplying by 10, 100 and 1000 (Charity Set J).

In Figure 56 Charity stopped following the method in the example after Question 3, and did not recognise that $61,2 = 6,12 \times 10$.

These were not the only examples of errors of this kind, but it clear that these shortcomings would undermine the learner's chances of completing the worksheets correctly.

4.4.1.3. **Basic misconceptions about exponents**

In Figure 57, Lizzy's attempt at Set T, she was firstly unable to multiply $4,0 \times 1,2$, then she correctly added the exponents for the 10's, but ended in confusion.

$$\begin{aligned}
 & (4,0 \times 10^5) \times (1,2 \times 10^2) \\
 & = (4,0 \times 1,2) \times (10^5 \times 10^2) \\
 & = 4,0 \times 10^{5+2} \\
 & = 40^7 \times
 \end{aligned}$$

Figure 57: Forgetting rules of exponents: cognitive overload? (Lizzy, Set T)

It is interesting to note that in Set O, where the sums are presented differently, Lizzy – in the most part – obtained the correct answers by using the short-circuiting method.

Do the sums:

- 1) $1,625 \times 10^2 = 1625$ ✗
- 2) $9,41 \times 10^{-1} = 0,941$ ✓
- 3) $4,88 \times 10^5 = 488000$ ✓
- 4) $7,6 \times 10^{-2} = 0,076$ ✓
- 5) $3,71 \times 10^3 = 3710$ ✓

Figure 58: Short circuiting success (Lizzy Set O)

Some learners were really baffled by scientific notation (Figure 59).

So $\frac{1}{10} = 10^{-1}$; $\frac{1}{100} = 10^{-2}$ and so on
 Write this as an exponent: $\frac{1}{1000} = \underline{100^{-1}}$ ✗

- 3) $7\ 100 = 7,1 \times 1000 = \underline{7100\ 100^2}$ ✗
- 4) $6,2 = 6,2 \times 10^2$ ✗
- 5) $7315 = 100^2$ ✗

Figure 59: Struggling with scientific notation (Lizzy Set N and Phethile Set J)

4.4.1.4. **Simply not getting the point**

In some cases, learners undermined their own efforts by not following the examples or looking through the steps of the incomplete examples. Henrietta's answers from Set P (Figure 60), where she had immediately tried to complete the sum without looking over the steps, show that she missed the point of what she was supposed to be doing.

Furthermore she was unable to multiply or divide by 100.

$$\begin{array}{l} 7) 617,8 = 6,178 \times 100 = \underline{61780} \times \\ 8) 0,45 = 45 \div 100 = 0,045 \times \end{array}$$

Figure 60: Not looking over the steps in faded examples: (from Set P by Henrietta)

We can also see from Figure 61 that the fading strategy did not assist Neo in understanding scientific notation.

$$\begin{array}{l} 891253 = 8,91253 \times 100\ 000 = \underline{891253} \times 10^0 \times \\ 0,0007482 = 7,472 \div 10\ 000 = 7,472 \times 10^{-4} \\ 0,0032 = 3,2 \div 1000 = \underline{3200} \times \end{array}$$

Figure 61: Neo was unable to progress from the steps given (Set U)

4.4.2 Badly designed worksheets

In some places, the worksheets failed to attain their purpose. For example, in reviewing Set F it became evident that the mental step from 1527^0 or even $(17x^3y^2z)^0$ – which in the most part was correctly answered the pre-test – to something like 5^0m^2n was by no means trivial to the learners.

Set F

Name Mandisa

Look at the examples below:

$$\frac{x^2}{x^2} = x^{2-2} = x^0 = 1$$

$$x^0 = 1 \quad 1527^0 = 1$$

$$3^0 = 1 \quad 9^0 = 1$$

What is the rule for powers of zero? that when there is a zero equals to one ✓

Another example: $3x^0y^2$
 $= 3 \times 1 \times y^2 = 3y^2$

Now do the rest of these sums:

- 1) $4m^2n^0b^3 = 4 \times 4m^{2-3} = 4m^{-1}$ ✗
- 2) $5^0m^2n \times 3m = 5m^2 - 3m$ ✗ $7m$
- 3) $2k^0s^3r \times 2^0ks =$
- 4) $a^3b^4 \div b^2a^0 = \frac{A^3b^4}{b^2A} = A^3b^2$ ✗

Figure 62: Examples do not lead the learners sufficiently for the questions (Mandisa Set F)

Another badly designed worksheet was Set J which was simply too difficult for the learners. They were overwhelmed by the number of variables, and by the combination of zero exponents, negative exponents and factorising into primes (see Figure 63 below).

$$\frac{12 \times b^2 \times 3 \times b^3}{2^2 \times 3 \times b^2 \times 3 \times b^3}$$

$$= 2^2 \times 3^2 \times b^5 = 2^2 \cdot 3^2 \cdot b^5$$

Now try these sums:

1) $16 k^2 r^0 \times 4 k^{-2} r^3$

$$= 2^4 k^2 r^0 \times 4 k^2 r^3$$

$$= 2^4 k^4 r^4 \quad \times$$

2) $6m^2 \times 18m^2 n^{-3} \times 4m^0 n$

$$= 3^2 m^2 \times 3^6 m^2 n^3 \times 2^2 m^0 n$$

$$= 18^{10} m^5 n^4 \quad \times$$

1) $16 k^2 r^0 \times 4 k^{-2} r^3$

$$= 16 k^2 r^0 \times 4 k^2 r^3$$

$$= 64 k^4 r^3$$

2) $6m^2 \times 18m^2 n^{-3} \times 4m^0 n$

$$= 2^3 m^2 \times 3^2 \times 2 \times 2 m^0 n$$

$$= 2^4 3^2 m^2 n$$

Figure 63: Set J was simply too difficult (Lizzy and Charity Set J)

CHAPTER 5: Discussion and Conclusion

5.1 Summary of findings

Learners claimed to value learning, working hard, and achieving good results but this was not matched by their actions. For many of them the worksheets were done carelessly, incorrect answers were never reviewed, and examples were not followed. Some learners clearly valued performance goals over mastery goals; according to (Fredricks et al., 2004) such learners are unlikely to be cognitively engaged with the content. Across the class there was a pervasive lack of engagement evidenced as poor performance with multiple unforced errors.

In-depth analysis of completed learning material indicated that learners did not make the connection between the prompts and examples, and the sums they were expected to do. Because the connection was implicit rather than explicit most learners remained oblivious to it.

The learners who had performed extremely poorly in the pre-test had apparently been unable to grasp the content from their Grade 8 lessons or from the review of this content prior to the intervention. There was considerable evidence of cognitive overload amongst these learners; however, the examples, when properly followed, were effective at guiding them to correct solution processes. As a whole, there was an impressive improvement in the post-test performance of the weakest learners. However, some learners were significantly challenged even when following the examples and did not improve in the post-test evaluation.

There was no matching overall improvement among the stronger learners.

5.2 Limitations

The intervention covered only a small part of the Grade 9 curriculum. The Laws of Exponents lend themselves to teaching by example and practice, but this is not the case with all sections of the work; topics such as Euclidean Geometry and Number Patterns are unlikely to be suited to this approach.

Although Kalyuga & Sweller (2005) suggested that after the first step of a solution a learner can be direct to an appropriate correct follow-up problem. It was not possible to follow their method without implementing an e-learning solution. After each worksheet was completed, it should have been assessed immediately and the learner directed to a similar set for repetition of the concept, or to a more advanced set for extension. In practice this was impossible due to time constraints.

There were days in which the class as a whole was restless and distracted. On these days, many of the learners did not perform well. In addition, the pace at which the class worked was lower than anticipated and many of them did not complete all the worksheets.

Lowveld High School learners are known to be “almost too polite” (A. de Bruno-Austin, personal communication, 22 March 2015). Many of the participants apparently found it difficult to express criticism of processes or methods and were not completely frank in their responses to the questionnaires and in their reflections;

Because of the short time-period of this research project, it was not possible to determine whether any long-term effects resulted from the intervention.

5.3 Discussion

5.3.1 Attitude and engagement

Although the research questions referred to attitude and engagement as two separate entities, it may be effective to interpret them as the external and internal aspects of the same phenomenon. The external representation – attitude – can be faked, but the internal aspect – engagement – is apparent in the learner’s actions. Faking attitude is a little like putting a fat lady into a tiny dress – she is fooling no-one but herself! It is not certain, however, whether this disconnect between the learners’ attitudes and engagement is intended as a façade or is simply a blind spot (see Figure 64).

	Known to self	Not known to self
Known to others	Arena	Blind spot
Not known to others	Façade	Unknown

Figure 64: The Johari Window: a tool of interpersonal awareness (Luft & Ingham, 1961).

Lack of engagement is mitigated by the learner’s ability to do mathematics (see Figure 65). High ability learners who are not engaged are those who are bored and often disruptive in class. In this research, Vuyani was typical of a high-ability, low-engagement learner. He received little or no benefit from the worksheets. High ability learners who are engaged (for example Gugu) can be characterised as “achieving”. These learners will probably benefit no matter what method of instruction is used.

	High engagement	Low engagement
High ability	Achieving	Bored
Low ability	Improving	Lost

Figure 65: Interactions between ability and engagement

Low ability learners who are not engaged can be portrayed as “lost”. These learners may have effectively given up on school and are often the victims – or willing co-conspirators – of the bored learners. They did not benefit from the worksheets; there are many indications of cognitive overload in their work, and little evidence of mitigation. It may be possible to rescue them but the key to doing this is not obvious. By Grade 9 it is often apparent that the school environment is not conducive to the education of these learners.

Low ability learners who engaged with the worksheets benefitted most from the intervention. These learners were in the bottom quartile of the pre-test scores; cognitive overload can be seen over and over in their work, but as they progress through the worksheets there is also evidence of mitigation, initially through slavish emulation of the examples but with a growth of understanding. These are the learners who improved substantially between pre- and post-test.

5.3.2 Element interactivity and cognitive engagement

One underlying assumption of cognitive load theory is that the different types of cognitive load are additive (Sweller, 1988). Hence the removal of extraneous load through careful design of instructional materials is good practice as the remaining cognitive capacity can then be applied to intrinsic load (a function of the learner’s state of knowledge and the element interactivity of the problem) and germane load, which is applied to constructing schemas (Sweller et al., 1998). However, a recurring criticism of cognitive load theory is that it is not possible to separate or measure these different types of cognitive load (Schnotz & Kürschner, 2007). Furthermore germane load, and thus schema acquisition, will only occur if the learner has the motivation to apply the necessary mental effort (Paas, 1992; Paas & van Merriënboer, 1994 and Sweller et al., 1998).

Unfortunately, in this group of Grade 9 learners in their ordinary mathematics classroom, motivation to learn and willingness to apply mental effort were frequently more absent than present. It became clear when looking over the worksheets that levels of engagement fluctuated considerably. Their reports of mental effort, widely accepted as a

measure of cognitive load since Paas (1992), did not reflect cognitive load so much as their level of cognitive engagement.

If one considers a task involving multiple steps, some steps will be easy (immediately obvious or routine application of previously mastered skills) but others may be “tricky”, requiring insight or higher level skills. One could say therefore that the intrinsic load (or element interactivity) changes as the learner works through the steps. A learner’s concentration (or cognitive engagement) will also vary over time in a continuum from completely disengaged to (conceivably) a flow state of total immersion. (Newmann et al., 1992). Figure 66 illustrates one concept of the interplay between the element interactivity of a task and a learner’s engagement.

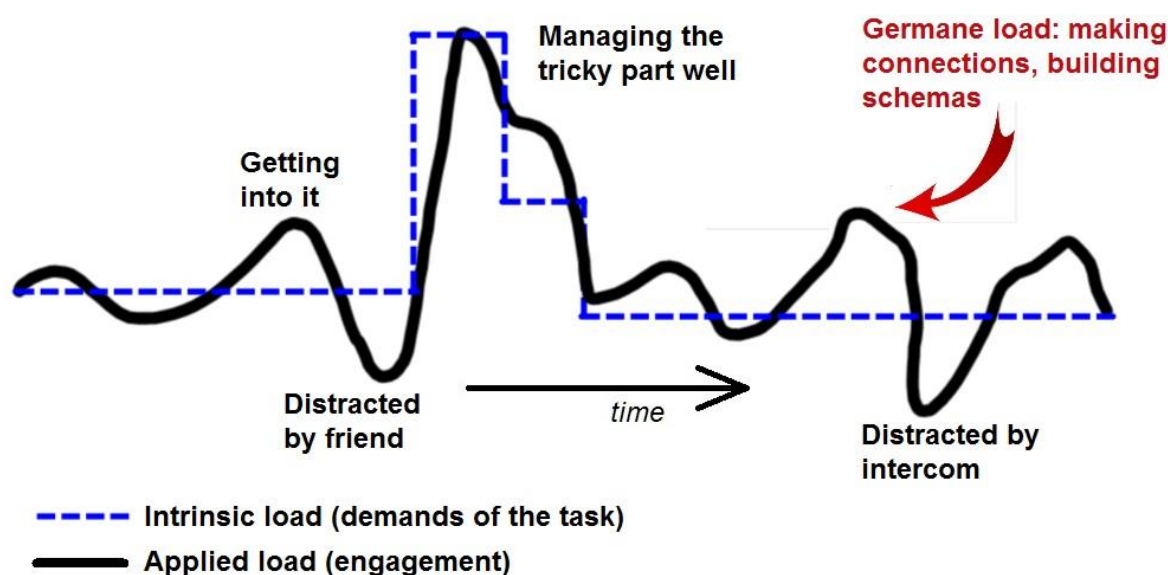


Figure 66: A representation of the interplay between intrinsic load and mental effort for an engaged learner

The intrinsic load – a function of element interactivity – is represented in **Error! Reference source not found.** by the blue line which shows the changing demands as the task progresses. The learner’s fluctuating engagement is represented by the black line. As time passes the learner is occasionally distracted by various things happening around him, but he is also making progress at working on the sum. Where the intrinsic load is higher, the learner needs more mental effort to proceed. Excess mental effort over that required by the steps in the sum – germane cognitive load – can be used to improve understanding (“Oh, look, the two middle terms are always like terms!”). Mental effort below the line (distractions or lack of focus) means the learner is not progressing or possibly is making careless mistakes.

When viewed in this way, the differentiation between intrinsic load and germane load is essentially artificial and the idea of trying to quantify these separately becomes meaningless. De Jong's criticism (either they are not like terms and therefore cannot be added, or they are like terms and must interact with each other) (de Jong, 2009) becomes moot – they are essentially the *same* thing, and differentiated only by the level of engagement required to progress in a task at a particular point in time.

If the learner is not engaged, excess mental capacity is applied to irrelevancies (“I wonder what is in my lunchbox today?”). However, if he is sufficiently engaged, this excess capacity is used to ponder relationships between what he is doing now and what he has learned in the past – that type of mental energy that develops and improves schemas. In a way, this is a different mode of thinking – thinking *out* of the problem to connect ideas, rather than thinking *into* the problem to solve it. Of course he is only able to do this if he hasn't used up all his working memory in solving the problem (intrinsic load). In a long problem, the intrinsic load will fluctuate; if the learner's level of engagement remains constant, when the intrinsic load is lower, he will be able to develop more insights.

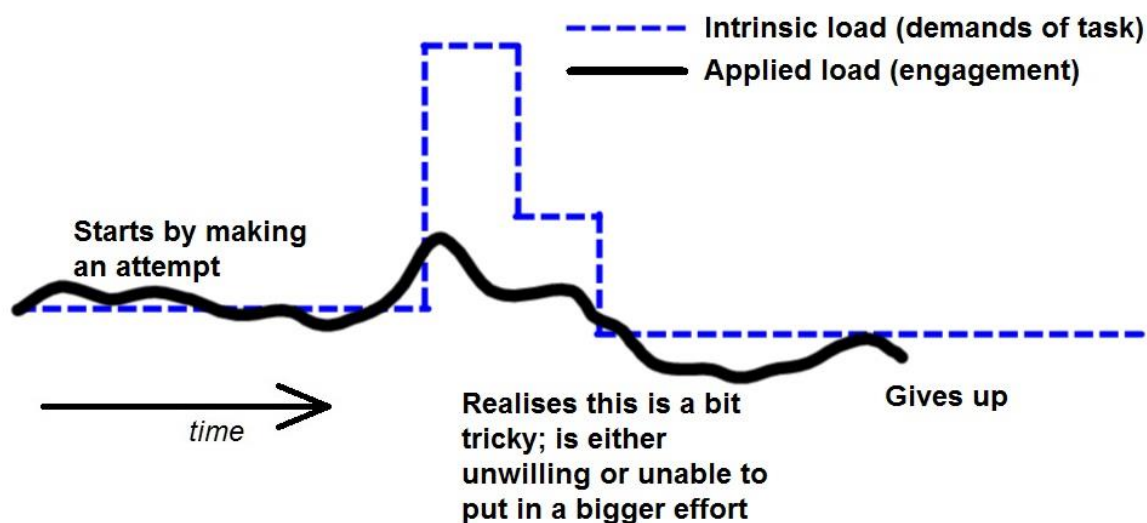


Figure 67: Learner fails to engage or complete the task

Figure 66 depicted a learner who was quite nicely engaged and finished the sum with an improved schema, since there was some mental effort beyond the minimum required to complete the sum. In Figure 67, a similar representation of mental effort against intrinsic load is shown for a non-engaged learner: he never actually puts in sufficient mental effort to do the difficult steps. This may be because he does not have the schema available – in which case he or someone else needs to break down that difficult step into pieces he can manage – or he can't be bothered. He stops really trying, and gives up without completing the problem.

5.3.3 Cognitive load and cognitive engagement

We already know that the intrinsic load of the task is dependent on the element interactivity of the task itself and on the learner's state of knowledge (van Merriënboer & Sweller, 2005). Figure 68 is graphical representation of the relationship between cognitive load and the learner's engagement, for a particular learner on a particular step of a problem – a snapshot at a moment in time. In this diagram we see the intrinsic (task-learner dependent) difficulty of the task represented as a constant level. In addition, for any learner, there is also a maximum cognitive capacity. Note that extraneous load is not represented: it is assumed to have been eliminated through correct material design.

If the learner's engagement is low (point A in Figure 68) he is not applying sufficient mental effort to the task, cognitive load is below the intrinsic load and he cannot proceed. If he engages sufficiently to reach point B, the additional cognitive load enables him to link ideas together, develop understanding, and construct schema. Ideally every learner should to operate as much of the time as possible in the germane load zone and as close to maximum capacity as possible.

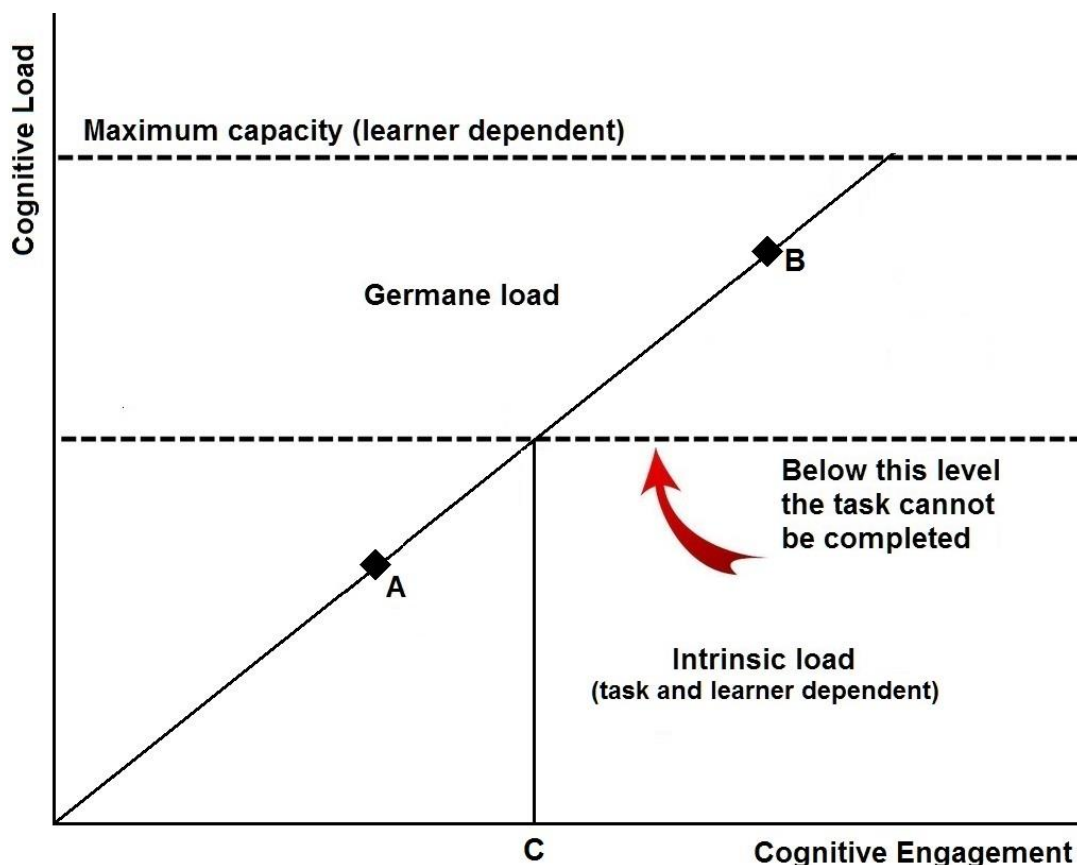


Figure 68: A snapshot of the relationship between cognitive engagement and cognitive load.

The point C in the Figure 68 could be called the Critical Engagement Level, below which the learner can't do the sums, and above which he has moved beyond simply repeating processes that he already knows, into developing understanding. This Critical Engagement Level differs from one learner to another, depending on the schemas they have already internalised

Although depicted as such in Figure 68 and Figure 69, there is no reason to suppose that the relationship between cognitive engagement and cognitive load is linear.

The data analysis showed some clear evidence of cognitive overload, for instance where learners applied techniques such as breaking down an operation into two steps in order to progress. There were also places where learners gained clear benefits from following the examples provided. Figure 69 depicts what happens when a task is too difficult, either because the learner lacks prerequisite schema or the cognitive capacity for the task.

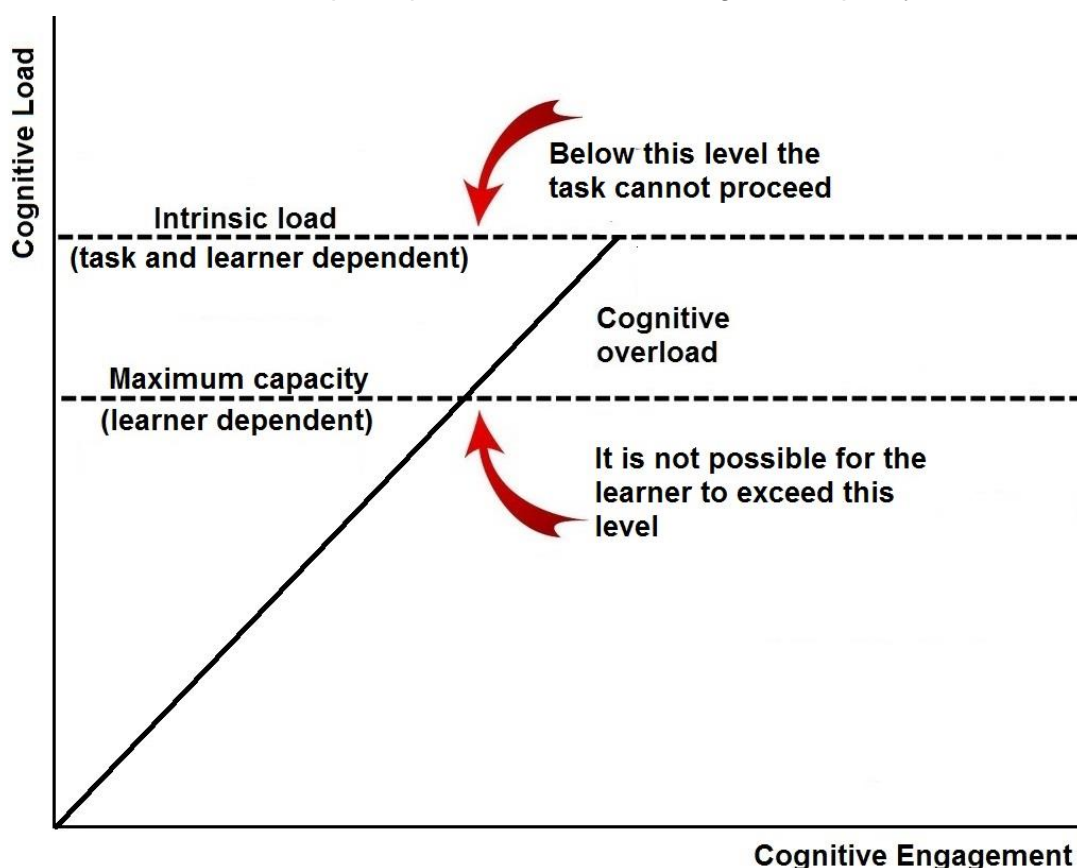
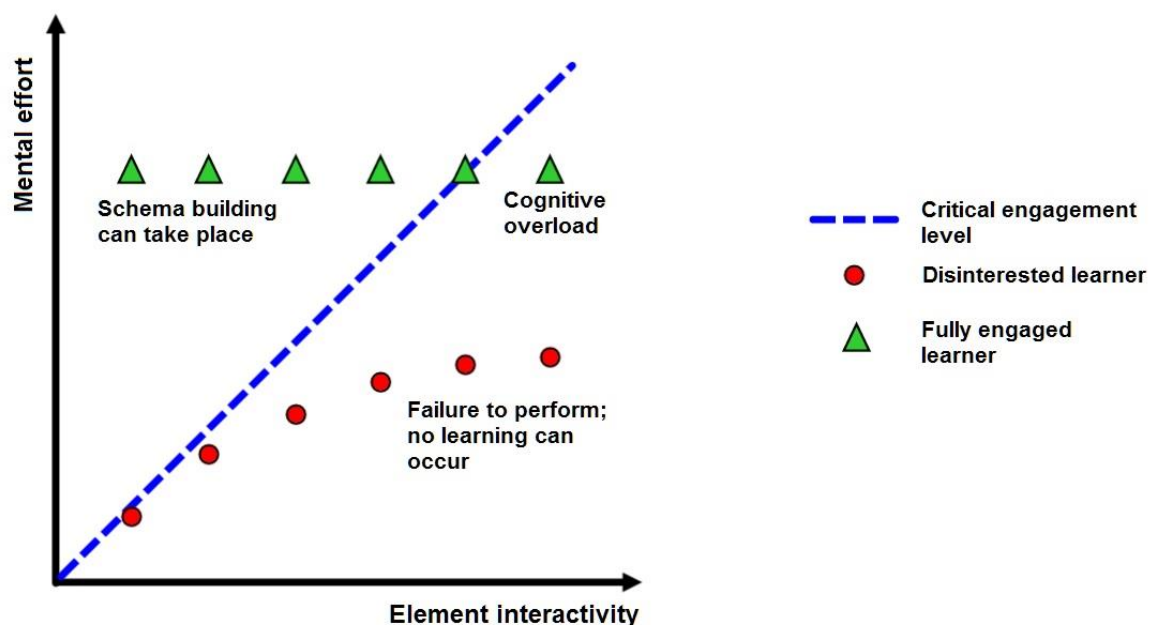


Figure 69: If the task is too difficult for the learner he is stalled by cognitive overload.

The intrinsic load required is greater than the mental capacity, resulting in cognitive overload. Note that the learner-dependent maximum capacity is considered here to be fixed. In order for the learner to proceed it is necessary to move the top (intrinsic load) line downwards. This is achieved by reducing element interactivity in some way: pre-learning a

schema that can simplify the problem, dealing with only part of the problem (bottom-up approach) or developing an understanding of the overall picture first (top-down approach).

XXXX=====



XXXX=====

Even so, if the intrinsic load is reduced only as much as necessary, there is no free cognitive capacity for schema construction or understanding.

5.3.4 Minimal engagement

There was evidence from the worksheets – in particular as demonstrated in the analysis of Set L – that many learners are reaching the Critical Engagement Level but are not putting in the additional mental effort to connect the pieces. For some learners, they may be cognitively unable to do better. Other possible explanations for this minimal engagement could be that the learner is hungry, worried about a home situation or a test in the next period, suffers from Attention Deficit Disorder, is thinking of girls or motorcars and so on. However, by the time learners reach Grade 9, the most capable amongst them have been *trained* to put in only the least effort required.

Consider the following: the ANA results showed that in 2012, only 36% of Grade 3 learners obtained 50% or more in mathematics (Department of Basic Education, 2013). With so many learners struggling in a classroom, a lot of time is needed for revision of work from the previous year because the majority of the learners cannot yet do it. Taylor (2009) noted the extremely slow pace of learning in South African schools. This may be partially due to poor teachers, but in many cases learners who are struggling complain

that the teacher goes to fast, ask about basic concepts again and again, and sometimes appear to dig in their heels and deliberately stall the learning process. Learners become accustomed to this very slow pace of learning in primary school, and it often proves difficult to overcome these habits in high school.

With learners arriving in each grade without the fundamental schemas that they should have developed years ago there is little time to remediate these problems: new work must be covered, the curricula are jam packed. With so many struggling learners, teachers have understandable reasons for short-circuiting and completing only the most basic sections of the curriculum: at least by the end of the year she can breathe a sigh of relief because most of the curriculum was covered and anyway the kids are someone else's problem now.

But there is a singularly undesirable effect on the most capable learners. These children mastered the content the first or second time it was taught; they are hearing it explained again and again in simple terms; they are given exercises and homework on the same concepts over and over. By pre-digesting the content and spoon-feeding it to learners in small, easy to consume chunks of "steps" and "rules", by handing out summaries and methods that are to be learnt by rote, the teachers are training learners to believe that school work does not involve thinking or understanding, and they must wait for knowledge to be given them by the teacher (Corno & Mandinach, 1983). The spoon-feeding and rote methods taught deny the learners the opportunity of making discoveries by themselves, and they are often told "use the method I taught you".

Ideally, extension activities should provide stronger learners with more challenging activities. Some teachers provide differentiation for *weaker* learners, but many teachers are either unwilling or unable to do this for the competent learners.

If a task is very easy for a learner, he needs to make only a token effort to finish it and there is no call for more than the minimum level of engagement: the able learner spends very little time operating in his Zone of Proximal Development. He has left over cognitive capacity to think about other things, chat to his mates, or dream up mischief. There is little satisfaction (intrinsic reward) in completing the task, and little motivation to increase engagement.

By Grade 9, after years of low demand learning, able learners often appear to be unwilling or unable to fully engage cognitively in a classroom situation.

5.4 Conclusion

For the learners who took part in the intervention, questionnaires were not an effective way to determine their true attitudes. They appeared unaware of cognitive dissonance resulting from the difference between their stated attitudes (positive, hard-working, aiming high) and their actual performance as reflected in their classwork, which for many demonstrated carelessness, lack of effort and a failure to engage.

Members of the class experienced cognitive overload frequently over the course of the intervention. The methods of cognitive load theory – in particular the worked and faded examples – were effective in mitigating this overload, as long as the learners put in the mental effort to follow and emulate the examples. The greatest improvements in academic performance were seen among the weaker learners.

Data obtained from this research suggests that many of the more capable learners habitually make only the minimum cognitive effort to reach a Critical Engagement Level – or on bad days, not even that much. A history of being denied permission to think in the past has resulted in a pattern of learning that is difficult to overcome. Stronger learners need to be challenged more, to be required to operate in their Zone of Proximal Development. Streaming of learners into classes by ability in mathematics from an early age could make provision for this if teachers are unable to provide differentiated instruction for more capable learners. Alternatively, example-based directed worksheets as used in this intervention, at a difficulty appropriate to the stronger learners' existing schemas, could extend these learners while weaker learners continue at their own speed.

Intrinsic load and maximum cognitive capacity are not under the learner's control, but they can manage their level of cognitive engagement. Further research into the interaction between cognitive load and cognitive engagement could be informative.

Learners need guidance and assistance to become self-directed and take control of their own learning, but for many learners, their teachers and parents are unable to offer this type of support.

Graduates in high demand in fields such as engineering, science and technology will not emerge from the weakest learners in South Africa, but from among the strongest. Massive efforts to improve the performance of the lowest achieving learners are commendable and necessary, but this should not be done at the expense of the highest-achieving. Better provision needs to be made for intelligent learners who are, after all, the ones whom we wish to see succeed in tertiary education.

Appendix A Ethics clearance



Research Ethics Clearance Certificate

This is to certify that the application for ethical clearance submitted by

David JM [4444345]

for a M Ed study entitled

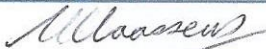
**An evaluation of the effectiveness of a cognitive load-based teaching method in
a mixed ability Grade 9 class, with special attention to learners' engagement
and performance**

has met the ethical requirements as specified by the University of South Africa
College of Education Research Ethics Committee. This certificate is valid for two
years from the date of issue.

Prof VI McKay
Acting Executive Dean: CEDU


Reference number: 2015 MARCH /4444345/MC

Dr M Claassens
CEDU REC (Chairperson)
mcdtc@netactive.co.za


15 MARCH 2015

Appendix B Questionnaire

Grade 9 Attitudes to Mathematics Questionnaire¹

Do NOT write your name on this paper! Please answer as honestly as you can – you can't get into trouble for any of your answers!

	Yes (Strongly Agree)	Some- times (Agree)	No (Disagree)	Never (Strongly disagree)	Don't know
I like studying school mathematics.					
I am good at school mathematics.					
I work well in mathematics lessons.					
Ability in mathematics is something that you either have or you haven't.					
It is possible to improve in mathematics by working hard.					
It is important to study school mathematics.					
I want to study school mathematics so that I can go to university.					
I want to study school mathematics so that I can get a job that I like.					
I want to study school mathematics because it will be useful in everyday life when I leave school.					
I want to study school mathematics to develop the ability to think logically.					
When solving mathematical problems, do you try to identify the similarities with or differences from problems you have solved before?					
When you get stuck with a mathematical problem, do you keep trying to think of different ways without giving up?					
When you are unable to solve a mathematical problem, do you think back over why you were unable to solve it?					
When you have solved a mathematical problem do you think of different ways to solve it?					
When you study new subject matter or ideas in school mathematics, do you try to use this in real life situations you encounter?					
I learn mathematics best by listening to a clear explanation from the teacher.					
I learn mathematics best by discussing problems or questions with my friend.					
I learn mathematics best by working through some questions on my own.					
I learn mathematics best when I have to explain my thinking to a friend.					

¹ Based on a Modified Fennema-Sherman Attitude Scale prepared for Grade 10 learners (Barber & Houssart, 2011)

	Yes (Strongly Agree)	Some- times (Agree)	No (Disagree)	Never (Strongly disagree)	Don't know
I learn mathematics best when I ask the teacher for help in lessons.					
I learn mathematics best when I read through worked examples in a textbook or a website.					

Thank you for your participation!

Appendix C Extract from curriculum

GRADE 8	GRADE 9
<p>Mental calculations</p> <ul style="list-style-type: none"> • Revise: <ul style="list-style-type: none"> - Squares to at least 12^2 and their square roots - Cubes to at least 6^3 and their cube roots <p>Comparing and representing numbers in exponential form</p> <ul style="list-style-type: none"> • Revise compare and represent whole numbers in exponential form • Compare and represent integers in exponential form • Compare and represent numbers in scientific notation, limited to positive exponents <p>Calculations using numbers in exponential form</p> <ul style="list-style-type: none"> • Establish general laws of exponents, limited to: <ul style="list-style-type: none"> - natural number exponents - $a^m \times a^n = a^{m+n}$ - $a^m \div a^n = a^{m-n}$, if $m > n$ - $(a^m)^n = a^{m \times n}$ - $(a \times t)^n = a^n \times t^n$ - $a^0 = 1$ • Recognize and use the appropriate laws of operations using numbers involving exponents and square and cube roots • Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers • Calculate the squares, cubes, square roots and cube roots of rational numbers <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving numbers in exponential form 	<p>Comparing and representing numbers in exponential form</p> <ul style="list-style-type: none"> • Revise compare and represent integers in exponential form <ul style="list-style-type: none"> - compare and represent numbers in scientific notation • Extend scientific notation to include negative exponents <p>Calculations using numbers in exponential form</p> <ul style="list-style-type: none"> • Revise the following general laws of exponents: <ul style="list-style-type: none"> - $a^m \times a^n = a^{m+n}$ - $a^m \div a^n = a^{m-n}$, if $m > n$ - $(a^m)^n = a^{m \times n}$ - $(a \times t)^n = a^n \times t^n$ - $a^0 = 1$ • Extend the general laws of exponents to include: <ul style="list-style-type: none"> - integer exponents - $a^{-m} = \frac{1}{a^m}$ • Perform calculations involving all four operations using numbers in exponential form, using the laws of exponents <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving numbers in exponential form, including scientific notation

Figure 70: The portion of the CAPS document dealing with exponents and scientific notation (Department of Basic Education, 2011b)

Appendix D Statistical analyses

Input data were percentage scores on pre- and post-tests. The statistical analysis was done in Excel, using the Real Statistics Addon (<http://www.real-statistics.com>)

5.4.1.1. Analysis of pre- and post-tests

Descriptive Statistics

	<i>Pre-test</i>	<i>Post-test</i>
Mean	37%	42%
Standard Error	0.039189	0.027608
Median	37%	40%
Mode	0.6	0.285714
Standard Deviation	0.191984	0.132401
Sample Variance	0.036858	0.01753
Kurtosis	-1.00521	-0.71138
Skewness	0.056198	0.551371
Range	0.666667	0.457143
Maximum	70%	71%
Minimum	3%	26%
Sum	877%	969%
Count	24	23
Geometric Mean	0.299337	0.402039
Harmonic Mean	0.208386	0.384343
AAD	0.159838	0.111477
MAD	0.166667	0.114286
IQR	0.316667	0.228571

Shapiro-Wilk Test

	<i>Pre-test</i>	<i>Post-test</i>
W	0.968503	0.926604
p-value	0.630182	0.092377
alpha	0.05	0.05
normal	yes	yes

A Shapiro-Wilk test indicated that the data in both pre- and post-tests were normally distributed, which is an underlying assumption of the t-test.

T Test: Two Paired Samples

SUMMARY		Alpha		0.05				
<i>Groups</i>	<i>Count</i>	<i>Mean</i>	<i>Std Dev</i>	<i>Std Err</i>	<i>t</i>	<i>df</i>	<i>Cohen d</i>	<i>Effect r</i>
Pre-test	23	0.369565	0.195121					

Post-test	23	0.421118	0.132401					
Difference	23	-0.05155	0.199931	0.041689	-1.23662	22	0.257853	0.254936

T TEST

	<i>p-value</i>	<i>t-crit</i>	<i>lower</i>	<i>upper</i>	<i>sig</i>
One Tail	0.114634	1.717144			no
Two Tail	0.229268	2.073873	-0.13801	0.034904	no

5.4.1.2. **Analysis of differences in June examination results**

The questions in the June examinations were divided into those that covered the section of the curriculum which had been the focus of the intervention, and the remainder of the questions. A Shapiro-Wilk test indicated that both sets of results were normally distributed.

T Test: Two Paired Samples

SUMMARY		Alpha		0.05		0		
<i>Groups</i>	<i>Count</i>	<i>Mean</i>	<i>Std Dev</i>	<i>Std Err</i>	<i>t</i>	<i>df</i>	<i>Cohen d</i>	<i>Effect r</i>
Focus area	24	49.25	16.00068					
Rest of exam	24	35.41667	10.93823					
Difference	24	13.83333	16.8901	3.447677	4.012364	23	0.81902	0.641678

T TEST

	<i>p-value</i>	<i>t-crit</i>	<i>lower</i>	<i>upper</i>	<i>sig</i>
One Tail	0.000273	1.713872			yes
Two Tail	0.000546	2.068658	6.701271	20.9654	yes

5.4.1.3. **Analysing of Set L for independence**

Table 5 below contains a summary of the answers provided for each learner in every attempt they made at Set L. An x indicates that the question is irrelevant. For example when a learner skipped the third prompt, it is not possible to tell whether he would have applied the negative exponent rule correctly or not.

Table 5: Learners' responses in Set L

	Answered Q1 and 2	Correctly understood the questions about products of integers	Answered Q3	Correctly applied the rule for negative exponents in Q3	Was able to get either +12 or -12 as exponent	Obtained -12 (sign law correctly applied)	Applied negative exponent rule to answer
Cassey	Yes	Yes	No	x	Yes	Yes	No
Charity	Yes	No	Yes	Yes	Yes	Yes	Yes
Ciniso 1	No	x	No	x	No	x	x
Ciniso 2	Yes	Yes	Yes	No	Yes	Yes	No
Gugu	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Henrietta	Yes	Yes	Yes	Yes	Yes	Yes	No
Lizzy	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Lwazi	Yes	Yes	Yes	Yes	Yes	No	x
Mandisa	No	x	No	x	Yes	Yes	No
Neo	Yes	No	Yes	No	No	x	No
Ngiphiwe	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Nhlakanipho	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Nokubonga	Yes	Yes	Yes	Yes	Yes	Yes	No
Nokulunga	Yes	?	Yes	Yes	Yes	No	x
Nonhlanhla 1	No	x	Yes	No	Yes	Yes	Yes
Nonhlanhla 2	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Nothando 1	Yes	Yes	Yes	No	Yes	No	x
Nothando 2	Yes	Yes	Yes	Yes	No	x	x
Phethile	Yes	Yes	Yes	No	No	x	x
Quincy	Yes	Yes	Yes	No	Yes	No	x
Samkelisiwe	Yes	No	Yes	Yes	No	x	x
Sanele	Yes	Yes	Yes	Yes	Yes	Yes	No
Tebogo Arthur	No	x	Yes	Yes	Yes	Yes	No
Wendy	Yes	Yes	Yes	Yes	Yes	Yes	No

The data was summarised in the contingency tables shown below. The chi-square test for independence was not suited to this data as there is at least one cell in each contingency table with less than 5 samples. Fisher's exact test was used instead.

H_0 : the relative proportions of right answers are independent of correctly answering the prompts

H_1 : correctly answering the sum is dependent on being able to correctly answer the prompts

From SET L: Question regarding product of 2 negative numbers

	Prompt right	Prompt wrong	
Answer right	11	4	15
Answer wrong	5	4	9
Total	16	8	24

$$p = 0.32 \quad (1 \text{ tail})$$

$$p = 0.41 \quad (2 \text{ tail})$$

From SET L: Question regarding negative exponent rule

	Prompt right	Prompt wrong	
Answer right	6	1	7
Answer wrong	9	8	17
Total	15	9	24

$$p = 0.15 \quad (1 \text{ tail})$$

$$p = 0.19 \quad (2 \text{ tail})$$

Since $p > 0.05$, the null hypothesis cannot be rejected – there is no significant difference in learners getting the answer right whether they studied the example or not.

Appendix E Details of improved performance between pre- and post-tests

1a $\sqrt{6^4 c^2}$
 $=$ ~~$6^4 c^2$~~ \times

2a $x^3 y^2 \times y^4 x^3$
 $= x^3 \times y^2 \times y^4 \times x^3$
 $= x^6 y^6$ \times

b) $\sqrt[3]{27x^6y^{12}}$
 $=$ ~~$27x^6y^{12}$~~ \times

b) $\frac{x^2 y^3}{x^3 y^2}$
 $= \frac{x^5 y^3}{x^3 y^2}$
 $= x^2 y$ \checkmark (1)

c) $(17x^3y^2z^1)^0$
 $= 17xyz^6$ \times

d) $\frac{1}{x^3}$
 $=$ ~~x^3~~ \times

e) $2y^{-2}$
 $=$ ~~$2y^2$~~ \times

f) $(2 \text{ km}^3)^4$
 $= 2 \text{ km}^7$
 ~~2 km^3~~ \times

g) $(3x^2)^3 + (2x^3)^2$
 $= 3x^6 + 2x^6$
 $= 5x^{12}$ \times

Figure 71: Unable to answer correctly (Cassey, pre-test).

Question 1

a $c^3 \times e^5 \times c^7$
 $= c^{3+5+7}$
 $= c^{15}$ ✓✓ 2

b $e^5 \div e^3$
 $= \frac{e^5}{e^3}$
 $= e^{5-3}$
 $= e^2$ ✓✓ 2

c $(4ab)^2$
 $= 4^2 a^2 b^2$ ✓✓ 2

d) t^{-4}
 $= \frac{1}{t^4}$ ✓ 1

e) $\left(\frac{1}{4}b\right)^2$
 $= \frac{1^2}{4^2} b^2$ ✓✓ 2

f) $(-3xy)^0$
 $= 1$ ✓ 1

Figure 72: An impressive improvement (Cassey, post-test).

1. a. $\sqrt{b^4 c^2} = \frac{1}{2} b^2 c$ X

b. $\sqrt[3]{27x^6 y^{12}} = \frac{3x^2 y^4}{3x^2} = \frac{y^4}{x^2}$ X

2. a. $x^3 y^2 \times y^4 x^2$
 $= x^5 y^6 \times x^2 y^4$
 $= x^7 y^{10}$ X

b. $\frac{x^2 y^3}{x^3 y^2} = x^{-1} y^1 = \frac{1}{x} y$ X ✓ 1

c. $(17x^3 y^2 z)^0 = 17x^3 y^2 z$ X

d. $\frac{1}{x^3} = \frac{1}{x^3}$ X

e. $2y^{-3} = \frac{2}{y^3}$ X

f. $(2k^2 m^3)^4 = 2^4 k^{8} m^{12} = 16k^8 m^{12}$ X

g. $(3x^2)^3 + (2x^3)^2$
 $= 27x^6 + 4x^6$
 $= 31x^6$ X

Figure 73: Neo pre-test

Question!

a. $c^3 \times c^5 \times c^7 = c^{3+5+7}$
 $= c^{15}$ ✓✓ 2

b. $e^5 \div e^3 = e^{5-3}$
 $= e^2$ ✓✓ 2

c. $(4ab)^2$
 $= 4^2 a^2 b^2$ ✓✓ 2

d. t^2 ✗

e. $(\frac{1}{4}b^3)^2$
 $\times \frac{1}{16}b^6$ ✓ 1

f. $(-3xy)^0$
 $= -3xy \times 0$ $-3xy \times 0$
 $= 1$ ✓ 1

Figure 74: Neo post-test

$$\begin{aligned}
 & b \quad \frac{x^2 y^3}{x^3 y^2} \\
 & = \frac{x^2 x^3}{y^3 y^2} \\
 & = x^5 \div y^5 \\
 & = x^4 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & d. \quad \frac{1}{x^{-3}} \\
 & = \frac{1}{x^3} \quad \times
 \end{aligned}$$

$$\begin{aligned}
 & e. \quad 2y^{-2} \quad \times \\
 & = \frac{1}{4} y \quad \times
 \end{aligned}$$

$$\begin{aligned}
 & d. \quad t^{-4} \\
 & = \frac{1}{t^4} \checkmark \quad 1
 \end{aligned}$$

$$\begin{aligned}
 & B. \quad t^5 \div t^3 \\
 & = t^2 \checkmark \checkmark \quad 2
 \end{aligned}$$

$$\begin{aligned}
 & t. \quad \left(\frac{1}{4} b^3\right)^2 \\
 & = \frac{1}{16} b^6 \checkmark \checkmark \quad 2
 \end{aligned}$$

$$\begin{aligned}
 & B. \quad 6p^4 \times 2p^{-2} \div 4p \\
 & = 12p^2 \div 4p \checkmark \\
 & = 3p \checkmark
 \end{aligned}$$

Figure 75: Ngiphiwe, selections from pre-test and post-test.

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