CONCEPTUAL AND PROCEDURAL DIFFICULTIES EXPERIENCED BY NATIONAL CERTIFICATE VOCATIONAL LEVEL 4 STUDENTS IN SOLVING FACTORISATION PROBLEMS AT A KWAZULU-NATAL TECHNOLOGY CENTRE

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Dedication

This work is dedicated first to my Lord and Saviour, Jesus Christ, without whom I can do nothing! Your grace is indeed sufficient for me.

The work is also dedicated to my wife and two daughters for supporting me for the duration of this study.

Declaration of Originality

I declare that "**CONCEPTUAL AND PROCEDURAL DIFFICULTIES EXPERIENCED BY NATIONAL CERTIFICATE VOCATIONAL LEVEL 4 STUDENTS IN SOLVING FACTORISATION PROBLEMS AT A KWAZULU-NATAL TECHNOLOGY CENTRE**" is my own work and that all the sources I have used or quoted have been indicated or acknowledged by means of complete references.

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Abstract

The purpose of this interpretive qualitative study was to determine the extent of conceptual and procedural difficulties that NCV Level 4 students encountered when factorising and solving problems involving factorisation. This study is based on Kilpatrick, Swafford and Findel's (2001) ideas on mathematical proficiency, focusing on conceptual knowledge, procedural knowledge and the flexibility of integrating both appropriately to solve algebra problems involving factorisation. This study also explored reasons why NCV Level 4 students demonstrated such difficulties and suggested possible ways that could assist them to understand and flexibly use factorisation to solve problems. A purposive sample consisting of 30 NCV Level 4 students and 5 Subject Matter Experts participated in this study, which adopted a phenomenological case study research design. Triangulation of method was adopted for consistent gathering of information. Data was collected through a written assessment on factorisation under controlled test conditions, and semi-structured interviews. The researcher reduced and analysed data by utilising an integration of constant comparison analysis and classical content analysis. The findings and relevant recommendations concluded this research.

Keywords: *conceptual knowledge, difficulties, factorisation, National Certificate Vocational procedural knowledge, procedural flexibility, teaching strategies, zone of proximal development*

List of Acronyms

DHET	Department of Higher Education and Training
FET	Further Education and Training
FETC	Further Education and Training College
NCV	National Certificate Vocational
NCV L4	National Certificate Vocational Level 4
NQF	National Qualifications Framework
TVET	Technical Vocational Education and Training
ZPD	Zone of Proximal Development
HCF	Highest Common Factor

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CHAPTER 1

INTRODUCTION AND BACKGROUND TO THE STUDY

1.1 Overview

In this chapter, the researcher presents an overview of the study as it unfolded. The chapter continues with an introduction followed by a background to the study, highlights the rationale for it and presents a motivation, defines the problem statement and considers the aim of the study as well as outlining its objectives. The research questions are formulated, a literature review is presented and the theoretical framework of this study is set out. Thereafter, the research methodology, research design, population, sample, data collection methods, data collection instruments, data analysis, ethical considerations, delimitations of the study and limitations are outlined. The chapter concludes with a summary and an outline of the study's entire chapter layout.

1.2 Introduction

Underachievement in mathematics is an ongoing concern in South Africa. South Africa continues to rank amongst the countries achieving the lowest in mathematics and science education, internationally (Trends in International Mathematics and Science Study (TIMSS), 2011:4). TIMSS conducts a study every four years to determine the mathematics achievement level of Grade 8 learners. Forty-two countries participated in the study in 2011. The study determined that an average scale score of 500 points constituted an average/par achievement while an average scale score of less than 400 constituted a low benchmark score. Table 1.1 ranks the achievement of the participating countries from highest to lowest:

Table 1.1 Mathematics achievements and rankings (TIMSS, 2011: 4)

Country	Average Scale Score	Country	Average Scale Score	Country	Average Scale Score	Country	Average Scale Score	Country	Average Scale Score
Korea	613	England	507	Ukraine	479	Thailand	427	Saudi A.	394
Singapore	611	Hungary	505	Norway	475	Macedonia	426	Indonesia	386
Chinese Tai	609	Australia	505	Armenia	467	Tunisia	425	Syria	380
Hong Kong	586	Slovenia	505	Romania	458	Chile	416	Morocco	371
Japan	570	Lithuania	502	UAE	456	Iran	415	Oman	366
Russian Fed	539	Italy	498	Turkey	452	Qatar	410	Ghana	331
Israel	516	New Zealand	488	Lebanon	449	Bahrain	409	Botswana	397
Finland	514	Kazakhstan	487	Malaysia	440	Jordan	406	South Africa	352
USA	509	Sweden	484	Georgia	431	Palestinian N	404	Honduras	338

South Africa administered the assessment amongst its grade 9 learners and still ranked second last with their average scale score of 352 points achieving way below the low benchmark score. Yet quality mathematics education is significant for the economic progress of our country. Milner and Khoza (2008) as well as Van der Walt and Maree (2007) assert that skilled workers are a prerequisite for South Africa to compete in the current global, technology-driven economic environment; therefore mathematics education is vital in the development of such a workforce.

Despite the ongoing revision of the mathematics curricula at educational institutes and the continuing transformation in the approach to mathematics education, the mathematics pass rates at schools and universities in South Africa are still unacceptably low. According to Barry (2014:1), just 26.1% of the learners achieved 50% and above in the 2013 mathematics grade 12 examinations. The pass rate in mathematics at Technical Vocational Education and Training colleges (TVET Colleges) is frighteningly even lower than that of the schools and universities. This pandemic of underachievement in mathematics seems to be an acceptable norm amongst most South Africans.

Much research has been done (Ngoepe 2003; Machisi 2013; Stutz & Fiona 2015) with regard to the factors that contribute towards underachievement in mathematics in schools. Ngoepe (2003) highlighted how teachers' classroom practices in three secondary township schools impact on learner performance. Machisi (2013) stressed that narrow and one dimensional solution strategies in mathematics contribute to low performance amongst Grade 12 learners. Studies by Stutz and Fiona (2015) tackled the impact that rural disadvantaged schools had on the mathematics performance of their learners. However, relatively little research around the factors that contribute towards underachievement in mathematics at TVET Colleges at exit levels exists; for example, Level 4, and even less about the possible solutions to this problem. Amongst the plethora of factors contributing to underachievement in mathematics at TVET colleges, are the difficulties that students experience with understanding key conceptual and procedural knowledge when solving mathematics problems. The amendment to the admission and progression requirements in 2009 further compounded this underachievement in mathematics. This amendment allowed candidates to progress to the next level without successfully completing the previous level of the NCV qualification, (Department of Higher Education, Admissions and progression requirements, 2015:1). The implication here is that students who failed Level 2 mathematics could enrol to study Level 3 mathematics and those who failed Level 3 could enrol to study Level 4. This implied that students had to study both levels simultaneously. Many students who fell into this category had failed a level more than once and are still in the system. The NCV mathematics curriculum is sequential in nature. According to the policy document, Formal Further Education and Training College Programmes at Levels 2 to 4 on the National Qualifications Frameworks (2011: 16), the pass mark for mathematics is 30%. Students, who obtain a low pass as well as those who fail, continue to the next level even although they experience conceptual and procedural difficulties with prerequisite knowledge when engaging with the curriculum at the higher level of their studies. Therefore, such students end up failing mathematics at the exit Level 4, which in turn results in them not being certified.

1.3 Background and Rationale for the study

There are currently 50 multi-campus TVET colleges with over 260 campuses situated all over South Africa (Department of Higher Education, 2013:12). The Department of Higher Education (2013:11) categorically stated that these colleges cater mainly for citizens who have left school, irrespective of whether they have completed matric or not. Such colleges offer a range of programmes that cater for most students who are interested in pursuing careers in Engineering, Business Studies, Art and Music through to Food Services. Each college has on average five campuses, which mainly comprise Business Centres and Engineering Technology Centres. Figure 1.1 indicates the spread of TVET colleges throughout South Africa. It is important to note that since 2014, the term TVET Colleges refers to FET Colleges.

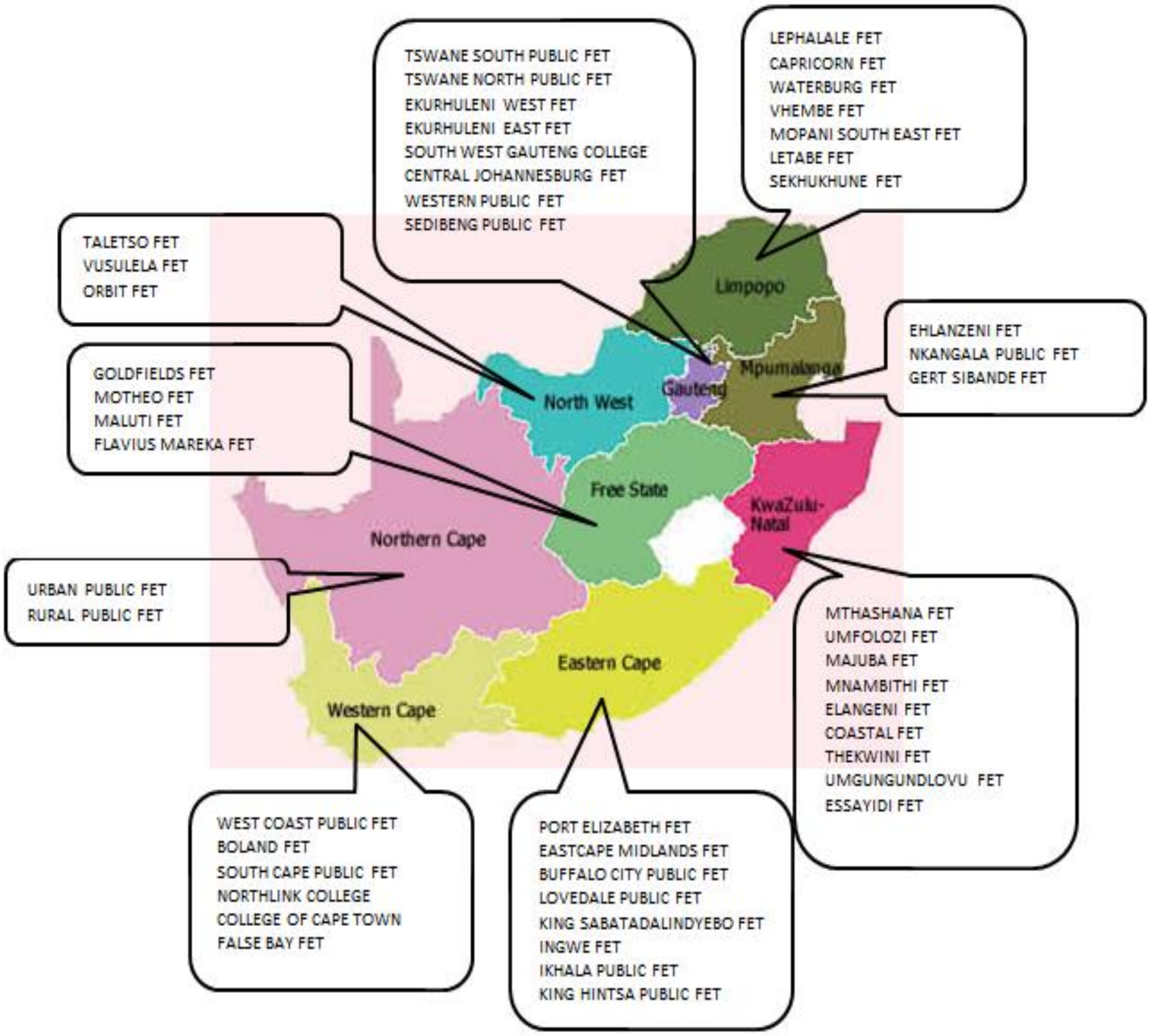


Figure 1.1: TVET College Distribution in SA

The Department of Education designed the programmes currently offered at TVET colleges to meet the skills development needs of South Africa. The White Paper for Post-School Education and Training states that the main purpose of establishing TVET colleges in South Africa is to train school leavers with the necessary skills, knowledge, and attitudes for employment in the labour market (Department of Higher Education, 2013:12). Their main objective is to train mid-level skills required to develop the South African economy in the following occupations: engineering and construction industries, tourism, hospitality, general business and management studies. The two main programmes that drive this objective are the Report 191 programme and the National

Certificate Vocational programme. The Report 191 programmes (Natural Sciences and General Studies) cater for those already employed in the various apprenticeship initiatives of industries and those who have completed Grade 12. The National Certificate Vocational (NCV) provides post-school vocational training to candidates, who have completed their Grade 9 year or higher.

The NCV programme comprises fourteen faculties that focus on priority economic sectors within the South African economy. Table 1.2 illustrates that each programme consists of seven subjects across three levels:

Table 1.2: List of Subjects

Subject		Level 2	Level 3	Level 4
English	**Compulsory Fundamental Subjects**	→	→	→
Mathematics or Mathematical Literacy		→	→	→
Life Orientation		→	→	→
4 core NCV Subjects	**Based on Specialisation**	→	→	→

The NCV programme offers an exit level qualification on these three consecutive levels of the National Qualifications Framework (NQF). These NQF levels are equivalent to grade 10 (NQF Level 2), grade 11 (NQF Level 3) and 12 (NQF Level 4) respectively but are restricted to a particular vocation. The minimum admission requirement for students to enter the NCV programme is a completed Grade 9 General Education and Training certificate, which is an NQF Level 1 qualification.

The researcher currently lectures at a campus situated in the heart of a township in KwaZulu-Natal of South Africa. At the time of the research this campus had a NCV enrolment of 1 755 full-time students and 218 part-time students. Table 1.3 breaks down the enrolment per level:

Table 1.3: Enrolment 2015

	Enrolment	Percentage Enrolment
Level 2	874	52%
Level 3	546	33%
Level 4	242	15%

This campus offers the following full-time NCV programmes:

- Engineering and Related Design,
- Electrical Infrastructure and Construction,
- Primary Agriculture,
- Civil Engineering,
- Building Construction, and
- Safety in Society.

Mathematics at this campus is compulsory for students who enrol in the engineering programmes.

The current enrolment trends in South African tertiary institutes are as follows: Students, after obtaining their matric certificate, enrol at universities and universities of technology. According to an article published by the government news agency entitled: “FET colleges are the way to go”, (Khumalo, 2013) learners who are unable to gain admission to these institutes generally enrol at TVET colleges. This article quotes the Minister of Higher Education and Training, as stating that his aim was to turn FET colleges into institutions of choice (Khumalo, 2013). This offers students a wider choice of courses in which to enrol; for example, they can enrol in the Report 191 programme or in the NCV programme. The campus in this case study does not allow learners who do not pass grade 12 or learners who do not study mathematics and physical science at school to enrol in the Report 191 engineering programmes,. Therefore, most of these learners enrol in the NCV programme.

a) The Department of Higher Education and Training, (2013), stipulates that a candidate must satisfy the following to obtain an NCV certificate:
    - complete the programme requirements for the particular NQF level and obtain the distinct outcomes for that particular level and
    - Obtain a pass mark for the continuous internal assessments and external assessments for all subject offerings for that particular NQF level
    - achieve 40% in two fundamental (compulsory) subjects: the required official language and Life Orientation
    - achieve 30% in Mathematics or Mathematical Literacy
    - achieve 50% in all four vocational subjects.

The import of the stipulations is that a candidate must pass all seven subjects of a particular NCV level to be certified. The certification rate refers to the percentage of the candidates that pass seven subjects compared to the number that attempted the examination. This criterion is more stringent than that of schools. A candidate at school may have failed a subject in Grade 12 but still be certified. The Department of Higher Education (Department of Higher Education and Training, 2013) uses the certification rate as the main criteria when determining college funding. Mathematics is not a core subject, in terms of student specialisation at TVET colleges.

Underperformance in mathematics prevents students from being certified. The greater consequence of this is that it handicaps the Government's objective of increasing the number of artisans in the country, hence the need for such a study as the present one. Table 1.4 reflects the National Level 2, 3 and 4 NCV Mathematics pass rates for the years 2011 and 2012:

National Level 2, 3 and 4 NCV Mathematics pass rates for the years 2011 and 2012:

Table 1.4 Source: DHET (2011/2012) Further Education and Training Colleges: Preliminary report on examination and assessment results

Level →	**Level 2**		**Level 3**		**Level 4**	
Year	Passed	Pass %	Passed	Pass %	Passed	Pass %
2011	10,695	46.0	5,222	41.0	3,721	51.0
2012	11,208	44.0	4,224	35.7	2,905	42.5

Table 1.4 firstly indicates that the national pass rates for NCV mathematics have dropped across all levels over the two years. The Level 4 pass rate of 42.5% for 2012 implies that 57.5% of students did not meet the certification requirements in their field of study due to their failing mathematics. In 2011, 5 222 Level 3 students passed mathematics and were eligible to enrol to study Leve 4 mathematics in 2012. Table 1.4 therefore, further illustrates that just 2905 students out of the 5 222 students who passed Level 3 mathematics in 2011, (56%) managed to pass Level 4 mathematics in 2012. The researcher, therefore, is of the opinion that the quality of the pass at Level 2 and Level 3 was poor or inadequate in terms of preparing a student to engage with the Level 4 mathematics curriculum. Table 1.5 represents the Level 2, 3 and 4 NCV mathematics pass rates for the years 2011 and 2012 of the college in this research study

Table 1.5 Pass rates: Source: Coltech Database

Level →	Level 2	Level 3	Level 4
Year	Pass %	Pass %	Pass %
2011	52,89	37,125	37,134
2012	55,75	26,67	17,4

Table 1.5 reflects that most students at this college had performed below the national pass rates for Mathematics in 2011 and 2012. The table illustrates a 19,734% drop in the Level 4 mathematics pass rate between 2011 and 2012. The huge disparity between the National Level 4 mathematics pass rate and that of this college in particular, is of concern. For example, a Level 4 mathematics pass rate of 17, 4% indicates that 82,6% of students were not certified in 2012. This means that 82,6% of students had two options: either repeat mathematics at Level 4 for an entire year in order to qualify or leave college without a qualification. Thus, this college did not meet the objective of producing skilled artisans within the 3-year period. The return on a 3-year investment has been very poor. Therefore, there was an urgent need to improve the NCV Level 4 mathematics results at this campus.

Factorisation is a prerequisite base of at least 30% of topics covered at NCV Level 4. The researcher was of the opinion that some of the factors that may have impeded Level 4 students from engaging successfully with the mathematics curriculum are conceptual and procedural difficulties in solving problems that involve factorisation.

1.4 Motivation of the study

A need for this study also arose from the researcher's experience in lecturing to NCV mathematics students. Drawing from classroom experience in facilitating mathematics, the researcher became aware that students had trouble with understanding and implementing key conceptual and procedural knowledge when solving mathematics problems.

The researcher attempted to address these weaknesses by introducing strategies to improve the students' performance in mathematics in Level 2 and Level 3. These strategies have unfortunately not yielded the desired objectives. This research, therefore, arose in response to the numerous challenges facing this campus. The researcher envisaged that by focusing on exposing the Level 4 students' conceptual and procedural difficulties, the campus could implement measures at the lower levels, Level 2 and Level 3 as well as at Level 4, to ameliorate such difficulties. He hoped that this would improve students' understanding of key concepts and procedures that are prerequisites for higher learning. He further hoped that this in turn would improve the mathematics pass rate at this Campus.

It is, however, impossible to address every conceptual and procedural difficulty that has contributed towards poor performance in mathematics. Table 1.6 illustrates the percentage of questions in the national mathematics Level 4 Paper 1 and Paper 2 question papers that depend on factorisation for their solution:

Table 1.6: Percentage of questions affected by factorisation in the National Examination

	November 2014	March 2015	November 2015	Average
Paper 1	29%	30%	31%	30%
Paper 2	17%	19%	15%	17%

Table 1.4 reveals that an average of 30%, across three consecutive question papers for paper 1 hinge on factorisation for their solutions. This is a relatively high percentage, considering that a candidate requires 30% to pass mathematics. Therefore, the researcher proposed an in-depth study of the conceptual and procedural difficulties experienced by Level 4 NCV students when

factorising and solving problems that involve factorisation. The researcher hoped that by exposing such conceptual and procedural difficulties and by exploring why students experienced such difficulties, possible teaching strategies as well as the improvement of other contributing factors that could assist students to master factorisation and solve problems that require factorisation might emerge. This in turn would possibly help students to pass Level 4 mathematics at this TVET Campus.

1.5 The problem statement

It is compulsory for an engineering student studying towards an artisanship to pass NCV Level 4 Mathematics in order to obtain an NCV Level 4 certificate. Students must obtain a minimum mark of 30% to pass this subject. However, most students fail it at Level 4. As noted, it is impossible to address all factors that may be contributing towards the high failure rate in Mathematics. The Level 4 examination does not test factorisation directly; however, it informs most procedures in solving Level 4 mathematical problems set in the national examination.

Therefore, the researcher undertook to investigate, at a campus, the NCV Level 4 students' conceptual and procedural difficulties in solving problems that involve factorisation and why such difficulties, which may be one of the contributing factors to underachievement in mathematics, exist.

1.6 Aim of the study

This study aimed to investigate the conceptual and procedural difficulties that NCV Level 4 students may experience when factorising and solving factorisation problems at a KwaZulu-Natal Technology Centre.

1.7 The objectives of the study

In pursuance of this aim, the researcher envisaged the following objectives:

The study seeks to:

- Identify the conceptual and procedural difficulties that NCV Level 4 students may demonstrate when factorising and solving problems involving factorisation
- Describe why NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation
- Suggest possible ways that could assist NCV Level 4 students to understand and flexibly use factorisation to solve problems.

1.8 Research questions

In pursuance of these objectives, this research sought answers to the following questions:

- What conceptual and procedural difficulties do NCV Level 4 students demonstrate when factorising and solving problems that involve factorisation?
- Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?
- What are the possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems?

1.9 Literature review

In this chapter, the researcher discusses part of the literature related to this study. The discussion begins with evidence of the poor academic quality of TVET students. It then discusses conceptual knowledge and procedural knowledge. It reviews some of the empirical studies on conceptual and procedural difficulties experienced by students. The ability to integrate conceptual knowledge and procedural knowledge flexibly so as to be able to solve problems in mathematics appropriately hinges on the extent of misconceptions that may exist in the mind of the student. Therefore, this section also defines misconceptions in algebra. The chapter concludes with a discussion of procedural knowledge in mathematics education and a review of some empirical studies of procedural difficulties experienced by students. Chapter 2 reviews the appropriate literature in detail.

The discussions around why NCV students perform poorly in mathematics remains highly charged. Papier (2009:7) maintains that the NCV programmes are of a high-quality curricular nature aimed at a particular niche market. However, students recruited are not the ‘right’ students for these programmes. The aggressive marketing of the NCV programme, by the Department of Education and Training and TVET colleges, has resulted in many post-Grade nine school leavers both young and old who performed poorly at school taking the opportunity to enrol at TVET colleges in the NCV programmes. The Roundtable working document (2010:54) highlights that in the NCV programme a clear correlation exists between poor learner recruitment, in terms of academic ability, and poor throughput rates. The Green Paper for Post-School Education and Training highlight the fact that colleges enrol students who have completed grade 12 and those who have left school as early as grade 9, (Minister of Higher Education and Training, 2011:22) in the NCV programmes. Due to the introduction of the Level 2-4 NCV qualifications, colleges, which were predominantly post school institutes, catering for the 17 to 25 years’ age group, now cater for the 15 to 17 years’ age group (South African Qualifications Authority, 2016:20). Currently, on the campus of this research, the ages of students enrolled range from 15 years to 44 years, exposing the problem of lecturers having to teach different cohorts of students, with varying mathematical backgrounds and skills, in the same classroom (MHET, 2011:22). Additionally, Papier (2009:7), argues that these students lack basic “academic reading, writing and arithmetic skills". Statistical findings in Papier's (2009:7) report highlight that students struggle the most with mathematics in the NCV programme.

Similarly, in another study, Barry (2014:1) emphasises the poor academic quality of TVET students by quoting a mathematics lecturer at a TVET College. The lecturer, responding to reasons why TVET students perform poorly in mathematics, claimed that the overall competency of TVET mathematics students is poor and that they lack certain basic concepts: "They don't understand the rules and principles of maths and lack numeracy skills." Thus, the lecturer infers that the TVET student's lack of conceptual and procedural knowledge is the key to why students are unable to adequately engage with the mathematics curriculum, hence their underachievement in mathematics.

Conceptual knowledge often referred to as conceptual understanding, "refers to an integrated and functional grasp of mathematical ideas" (Kilpatrick, Swafford and Findel, 2001:118). Thus, students who are developed to acquire such understanding, have a sound grounding and understanding of mathematical concepts. They see mathematical concepts as integrated and sequential and not 'standalone' concepts. Therefore, their ability to retain what is learnt increases because they learn by understanding and connecting mathematical concepts to facts and methods (procedures) used in mathematics.

However, a plethora of research evidence in mathematics education has shown that many students retain fundamental conceptual difficulties in engaging with 'new' mathematics (for example; MacMath et al., (2009); Garfield and Ahlgren (1988); Ball, Lubienski, and Mewborn (2001); Kilpatrick et al., (2001)). Therefore such students perpetuate conceptual misunderstandings or misconceptions as they progress in their mathematics studies. The Oxford Dictionary defines a misconception as "a failure to understand something correctly" (Oxford Dictionary, 2009:590). Allen (2007:1) suggests that misconceptions are certain conceptual relations that are inappropriate within a certain context. Allen (2007:1) further posits that a "misconception does not exist independently, but is contingent upon a certain existing conceptual framework". Simply put, misconceptions in algebra are concepts that are not in agreement with our traditional understandings of algebra in all facets of mathematics. These misconceptions often arise from students' private understandings of certain concepts, previous inadequate teaching, informal thinking, or poor recollection.

On the other hand, procedural knowledge "refers to knowledge of procedures, knowledge of when and how to use them appropriately and the skill in performing them flexibly, accurately and efficiently". (Kilpatrick et al., 2001:143). However, educators who drill mathematical procedures, without attempting to provide any level of conceptual understanding, create confusion amongst their students. Such students are prone to apply procedures in isolation to produce an answer rather than understand and solve the problem at hand. Hence, such students become dependent on compartmentalised procedures to solve mathematics problems and very often use procedures 'mindlessly and mechanically' in incorrect mathematical contexts.

Early work by Garfield and Ahlgren (1988: 46) indicates that students often respond to problems in mathematics by falling into a "number crunching mode". They further maintain that although students are able to memorise formulas and steps to follow in familiar, well-defined mathematical problems, they seldom appear to make sense of how these concepts apply in new situations. Later research by MacMath, Wallace and Chi (2009:1) maintains that while "many students may develop procedural knowledge … they often lack the deep conceptual understanding necessary to solve new problems or make connections between mathematical ideas".

This study therefore aimed to expose the conceptual and procedural difficulties that TVET Level 4 students are experiencing. Chapter 2 reviews literature pertinent to this study in detail.

1.10 Theoretical framework

Kilpatrick, Swafford and Findel's (2001) thinking on mathematical proficiency, with primary reference to conceptual understanding and procedural fluency, underpins the theoretical framework for this research study. Chapter 2 outlines the theoretical framework for this study in detail.

1.11 Research methodology

This study was qualitative in nature and sought insight into exploring the conceptual and procedural difficulties which NCV Level 4 students demonstrated when solving problems that involve factorisation. According to Creswell (2014:32), qualitative research is an approach concerned with exploring and understanding the meaning that people ascribe to a social or human problem. Such inquiry supports an inductive style that focusses on individual meaning and exposes the complexity of a situation. The participants in this study included: level four NCV students, the Level 4 mathematics lecturers, and the Head of Division of mathematics of this campus, as well as the opinions being sought of the national examiner and the Umalusi Moderator of the NCV mathematics Level 4 question papers and the marking process.

1.1.1 Research design

The word ‘design’ embodies the entire research process and serves as a guide on data collection through the actual writing of the research. In view of these lines of thinking, the researcher proposed a phenomenological case study design for this study. Such a design “describes the lived experiences of individuals about a phenomenon as described by participants” which is summed up by the experiences of several such individuals who have experienced the same phenomenon (Creswell, 2014:42). Furthermore, Creswell (2014:43) describes a case study design as an in-depth analysis of a case, a programme, an event, or even one or more individuals. This design was therefore well suited for this research study.

1.11.2 Population

The population of this study consisted of 1 973 students of which 242 were NCV Level 4 students, 82 lecturers of whom 11 were mathematics lecturers and five heads of departments, (HOD), one of whom was the head of the mathematics division. The population also included three Umalusi NCV mathematics external moderators who are responsible for the following:

- Externally moderating the NCV examination question papers for mathematics
- Verifying the NCV marking process
- Verifying of the conduct of the on-site internal continuous assessments.

1.11.3 Sample

In qualitative research, no rigid prescription on choosing the number of sites or the number of participants exists. Generally, qualitative researchers focus on an in-depth study of a small number of participants (Creswell, 2016:7). Creswell (2014:239) recommends that in a phenomenological study, participants should range from three to ten whereas in a case study, four to five cases are sufficient.

In this study, the researcher purposively sampled 30 NCV Level 4 students, 3 mathematics lecturers and 1 Head of Division for mathematics. The Umalusi Moderator who was responsible for external moderation of the NCV Level 4 mathematics question papers and the verification of marking of NCV Level 4 candidates answer scripts was also included in the sample.

Purposive sampling is an example of non-probability sampling adopted in qualitative research. In purposive sampling, the researcher purposefully selects participants in order to gain an understanding of the problem under research. (Creswell, 2014:239).

1.11.4 Data collection methods

The researcher endeavoured to avoid disrupting the normal college day, conducting most fieldwork for this study at the campus during the lunch break and immediately after college hours. Thirty NCV Level 4 students wrote the assessment under controlled test conditions. Only five students, who were amongst the students that performed poorly and who lived within close proximity to the campus, were interviewed. They were interviewed individually, using semi-structured interviews. The researcher also conducted individual semi-structured (in-depth) interviews with three mathematics lecturers from this campus, one Head of Division for mathematics and one Umalusi Moderator.

1.11.5 Data collection instruments

The following data collection instruments were proposed, developed and implemented for the purposes of this study:

A written assessment on factorisation, as well as two interview protocols for the purposes of conducting semi-structured face-to-face interviews with students, the mathematics lecturers, the mathematics Head of Division and the Umalusi Moderator. Chapter 3 presents an extended discussion of the rationale for choosing the instruments.

1.11.6 Data analysis

In this study, the researcher adopted an integration of constant comparison analysis and classical content analysis to reduce data.

The researcher reduced the data by highlighting relevant information of repeated categories that emerged and had a bearing on this research study. Firstly, this was done on the students' responses in the written tests. Secondly, on the material from interviews with students. Thirdly, on the data gathered from the five Subject Matter Experts interviewed and lastly, from data gathered from all participants for identifying the interrelated data categories.

During the process of identifying and developing data categories, the researcher coded the data. According to Terre Blanche et al, (2006, p.324) "Coding means breaking up the data in analytically relevant ways". The following commonalities were considered when coding: a common mathematical error or misconception, a common response, a common phrase as well as a sentence or paragraph from the transcripts of data that speak to a particular data category or theme. The researcher utilised the cut and paste function or typing information in MS Word to move the coded information into the relevant data categories. The emerging categories from the proposed analysis were finally interpreted as separate sub-headings. The researcher then attempted to interpret the convergence of data (triangulation) across all participants that pointed towards conceptual and procedural difficulties that the Level 4 NCV students experienced when solving problems that involved factorisation, as well as towards possible methods that lecturers could implement when teaching students how to factorise and solve problems that involve factorisation.

The researcher endeavoured to interpret the analysis as objectively as possible but also agrees with Terre Blanche et al (2006), who suggest that the researcher's personal involvement in the research could also be elaborated on. The researcher discussed all interpretations with a colleague (member checking), who has completed a master's degree, to ensure validity.

1.12 Trustworthiness

The researcher adopted Guba and Lincoln's (1982:3) and Anney's (2014: 276) suggestion of utilising four qualitative research trustworthiness criteria of credibility, transferability, dependability and confirmability, to validate this research. The researcher discusses these criteria in detail in Chapter 3.

This researcher sought to attain trustworthiness through the triangulation method by gathering data through a written assessment and through conducting interviews with the main role players of this study. To aid trustworthiness and minimise bias, this study subjected each of the 30 students in the sample to an identical assessment. Thereafter, five students who performed poorly in the assessment each underwent an individual semi-structured interview. The researcher also conducted individual semi-structured (in-depth) interviews with the five Subject Matter Experts.

Leung (2015:1) defines validity in qualitative research as [the] 'appropriateness' of its tools, processes and data. The researcher endeavoured to achieve appropriateness of the written assessment (see appendix E) by including two sections in it. Section A, included questions that assessed all basic factorisation (procedural knowledge), while section B included mathematics problems that require factorisation for their solutions (connections between concepts and procedures). To aid appropriateness in the semi-structured interviews (for interview protocols see appendix C and D) the researcher posed questions that provided scaffolding throughout the study. Since this study used a written assessment and interviews as the primary data sources, the questions in the interviews related and cross-referenced the written assessment attempts of the students. Finally, this study established validity of the written assessment tool and the interview protocol by allowing the Head of Division of mathematics at the campus and the study supervisor to check, analyse and approve these before intervention.

1.13 Ethical considerations

The researcher applied for permission to conduct this study from the Department of Higher Education, the TVET College and the Campus of this study and permission was duly granted by the DHET. Gaining access to the institution did not pose any difficulties as the researcher lectures at this college. The researcher respected and protected the issues of confidentiality, voluntary participation and privacy of personal rights of all participants in this research study at all times. These measures complied with UNISA's policy on research ethics. Subsequently, the College of Education Research Ethics Review Committee also granted permission to conduct this study.

1.14 Delimitations of the study

The delimitation of a study is concerned with the scope or the boundary of the study imposed by the researcher. As already highlighted, numerous factors that exist contribute towards underachievement at NCV Level 4 mathematics in South Africa. This study, however, focused mainly on student performance and mathematical issues that contribute towards underachievement in NCV Level 4 mathematics at this campus. In order to have a narrow focus, this study concentrated on the students' lack of conceptual and procedural understanding of factorisation when solving problems involving this technique, which may be one of the factors contributing to their underachievement in mathematics at this site. Hence, the findings might not apply or be generalised to other TVET colleges and technology centres in South Africa.

1.15 Limitations

This study developed as a result of the researcher's experience as a lecturer in mathematics in the NCV programme at the campus where this study was conducted. The researcher based it on other studies which have shown that the lack of conceptual and procedural understanding does contribute to students' lack of mathematical proficiency, resulting in their underachievement in mathematics. Researchers have conducted such studies for other educational institutions but studies of this nature are few in number for TVET colleges. Ideally, the researcher should have conducted this study in as many technology centres as possible but logistical, resource and financial constraints did not permit this. Despite these constraints, he envisaged that this study would provide useful insights that could feed directly into this campus's teaching and learning processes. This in turn could help to decrease underachievement in NCV L4 mathematics here.

1.16 Chapter Layout

This dissertation comprises the following chapters:

Chapter 1: Introduction and background to the study

This chapter introduces and provides a brief contextual background to the study.

Chapter 2: Review of the literature and theoretical framework

This chapter contains a review of literature and the theoretical framework of the study.

Chapter 3: Data collection procedure

This chapter considers the research paradigm; research methodology; the role of the researcher; the choice of the location; the target population; the study sample and the sampling techniques adopted; research design; data collection instruments and processes as well as ethical considerations for this study. It concludes by exploring issues of trustworthiness, triangulation, delimitations, limitations, elimination of bias and ethical considerations.

Chapter 4: Results, discussions, and interpretation of findings

This chapter provides a discussion on the data analysis process, presenting the results as well as detailing and interpreting the findings of the data collected.

Chapter 5: Summary, results, conclusions, and recommendations

In this concluding chapter, the researcher presents a summary of the chapters, as well as of the findings of this study, and provides recommendations based on the findings. It also reflects on this study's limitations, proposes areas of further research and offers final conclusions.

1.17 Conclusion

In this chapter, the researcher introduced and discussed the background to the study. In addition, he highlighted the rationale for the latter, presented a motivation for it and defined the problem statement as well as articulating the aim and objectives of the study. The researcher also formulated the research questions, provided an overview of the literature consulted and set the theoretical framework of this study. Thereafter, he outlined the research methodology, research design, population, sample, data collection methods, data collection instruments, data analysis, ethical considerations, delimitations, and limitations of the study. The researcher concluded the chapter with an outline of the layout of the study's chapters.

The next chapter provides a review of the literature and details the theoretical framework pertinent to this study.

CHAPTER 2

REVIEW OF THE LITERATURE AND THEORETICAL FRAMEWORK

2.1 Introduction

This chapter is divided into two sections. It initially reviews the literature on research conducted on factors that contribute towards underachievement of TVET NCV students. The review also investigated mathematical proficiency, conceptual knowledge, procedural knowledge, comparison between conceptual and procedural knowledge, conceptual and procedural difficulties, language difficulties, measuring conceptual and procedural knowledge and the different ways of teaching factorisation, for the purpose of acquiring conceptual and procedural understanding, found in the literature. It then presents the theoretical framework that guides this study and concludes with a summary.

2.2 Review of literature

According to (Creswell, 2014:64), a literature review involves exploring, locating and summarising research studies about a research topic. It allows the researcher to position her or his study in relation to other related academic research studies. In the sections that follow, the researcher presents the context of this study together with the literature that underpins it. The South African Qualifications Authority, (SAQA), (2016:2), reports that to date, there is a dearth of formal research literature on TVET colleges in South Africa. In spite of this, the researcher presents a review of available literature pertinent to this research study. It begins with research conducted on factors that contribute towards underachievement of TVET NCV students and then discusses conceptual knowledge and procedural knowledge. A comparison between these types of knowledge, conceptual and procedural difficulties and language difficulties is included in this review. Lecturers' strategies for overcoming such difficulties are considered, as well as learning theories. The chapter concludes with the definition of factorisation and its prerequisites.

Factors contributing towards underachievement of TVET NCV students

The literature reviewed in section 1.10 highlights the poor academic quality of students enrolled in the NCV programmes. Some of the other observed reasons for underachievement in mathematics in the TVET sector are discussed below:

2.2.1.1 Lecturer's qualification, experience, and content knowledge

Kilpatrick et al (2001:12) indicate that proficiency in mathematics teaching displays parallels to proficiency in mathematics, implying that improving students' proficiency in mathematics depends to an extent on the capabilities and proficiency of the mathematics teacher. Nevertheless, such teaching capabilities are not automatic but develop through experience and continuous professional development, (Kilpatrick et al, 2001:12).

Since 1994, lecturers at TVET colleges in South Africa have been overwhelmed with a profusion of policy changes, (Mokone, 2011:1) which necessitated an upgrading and updating of their academic and vocational knowledge and experience. However, despite various attempts by DHET to build capacity amongst TVET lecturers, the quality of teaching and learning remains very poor, (Mokone, 2011:1). This has resulted in poor student class attendance, high dropout rates and low throughput rates, (Mokone, 2011:2). This lack of quality teaching and learning at South African TVET colleges has resulted in approximately 2 500 students out of 320 679 completing the NCV programme in 2009, with the subject that contributed most to this high failure and dropout rate being mathematics, (FET Roundtable 2010).

The Green Paper for Post-School Education and Training (2011) asserts that lecturers in the TVET sector possess technical qualifications as well as workplace experience but that they lack pedagogical training, especially in academic subjects like languages, mathematics and science, (MHET, 2011:24). Similarly, SAQA (2016:18), reports that due to the introduction of the fundamental subjects: Communication, Life Orientation, Mathematics and Mathematical Literacy in the NCV programme, colleges employed new lecturers primarily from the education sector. Those lecturers lacked workplace experience and hence did not understand the TVET landscape, thus changing the nature and character of the TVET workforce.

Papier (2011:106) suggests that lecturers at TVET colleges need to integrate both the workplace and academic spheres in their pedagogical practices. The researcher is of the opinion that this kind of orientation, that of being an 'expert educator' as well as an 'industry specialist', is very challenging and attainable solely though reskilling and experience.

According to the Green Paper for Post-School Education and Training (MHET, 2011:24), the single greatest contributor towards underachievement in the TVET sector is the lecturers' lack of capacity, especially in their subject content knowledge and expertise. Due to this lack of knowledge, many lecturers at TVET colleges teach procedures only. A major challenge faced by lecturers in the teaching of mathematics at TVET colleges is to move from merely teaching procedurally to teaching conceptually and procedurally. Teaching mathematics procedurally at TVET colleges has a twofold consequence:

(i) It only exposes students to examination type questions for the purposes of familiarity
(ii) It impresses upon students sets of rigid rules that they must follow in order to derive correct solutions that will help them to pass.

Such teaching is a short-term strategy that does not build conceptual understanding. In order for lecturers to teach students mathematics for the purpose of long-term retention and for future learning, the lecturers themselves must possess a sound conceptual understanding of mathematics. The researcher holds to the view that lecturers might lack such conceptual competence.

2.2.1.2 Language difficulties

The impact of the Language of Learning and Teaching (LoLT) as a factor that contributes towards poor performance at TVET colleges in South Africa cannot be overemphasised. An example of this is found in Mokone's study (2011:13) whose findings evidenced that the LoLT was a significant factor as regards poor student performance in the Western Cape TVET colleges. The lecturers in the Western Cape are predominantly from an Afrikaans speaking background, whilst the majority of students are from an isiXhosa speaking one. However, the official LoLT in South Africa is English. Since the college textbooks are written in English and examinations are conducted in English, the lecturers as well as students are disadvantaged, (Mokone, 2011:13). In this study, almost all students are Zulu speaking, while the majority of the lecturers are also Zulu speakers. The language of instruction is therefore foreign to both students and lecturers. Thus, another learning barrier contributing to underachievement may lie in language differences.

Gamble (2003:53) suggests that the role of language in concept formation is crucial, especially now that the TVET sector has moved towards a more practical curriculum. The new curriculum accommodates fundamental subjects, academic theory subjects and practical workshop training as opposed to the previous one-dimensional theoretical curriculum. Gamble (2003:53) and Alenezi, (2010:12) found that students viewed English as a language of power and communication. They therefore preferred to learn in English in order to develop their own proficiency. Webb (n.d.: 12) concludes that language facilitates and constructs the cognitive development of students and that the LoLT is an integral instrument in the construction of knowledge. Although students view English as a language of power, opportunities for them to read, speak, listen and write in this language are limited, especially in the township schools and colleges. Many students also face the reality of teachers and lecturers who are not proficient in English. These realities hamper such students in their construction of new concepts and their cognitive development.

Researchers Webb (n.d), Gamble (2003:53) and Jantjies and Joy (2014:308), have over the years suggested the adoption of a bilingual model, referred to as code switching, to overcome the barriers posed by being obliged to learn subjects like mathematics in a second language. Code switching, according to Jantjies and Joy (2014:308), is a phenomenon where the lecturer adopts the use of two languages by continuously alternating between the LoLT and the first language of the students to provide clarity and understanding of difficult words, concepts and terminologies.

The adoption of code switching in South Africa for the teaching of mathematics has nonetheless posed many challenges. Research has shown that the main confusion occurs during direct translation of words where the mathematical meaning of key concepts is lost. For example, code switching in the learning of mathematics amongst Grade 10 IsiXhosa learners, revealed that learners became very confused because some of the mathematical concepts, terminologies, and vocabulary were extremely difficult to translate from English to IsiXhosa, (Mahofa, 2014:106). This could also be true for isiZulu speaking learners. For example, a prerequisite knowledge for factorisation is multiplication. Multiplication can be defined as repeated addition or adding a number to itself a certain number of times. In isiZulu, the English word 'multiplication' translates as '*phindaphinda*'. However, the meaning of multiplication, during direct translation of '*phindaphinda*', which means '*repeat-repeat'*, is lost. Code switching also occurs in factorisation, a process which is the reverse of multiplication. In isiZulu, the English word, 'factorise or factorisation' translates as '*hlukanisa*'. However, the direct translation of '*hlukanisa*' is to '*unpair*' or 'break up'. In this case, there is in fact a double loss of meaning because the definition of factorisation hinges on understanding the concept of multiplication. Findings by Aziz et al (2011:39) reveal that learners also found it difficult to grasp mathematics in English, especially from teachers who lack English language proficiency. College lecturers should be mindful that English is the Official LoLT in South Africa; therefore code switching should be occasionally utilised, (Chowdhury, 2012:54).

At the time of this study, the researcher found no evidence of research on the impact of the LoLT on mathematics at TVET Colleges. However, based on information gleaned from research conducted at school level, it is apparent that although the circumstances in terms of LoLT differ from school to school or college to college, the impact it has on understanding of mathematical concepts and poor performance remains consistent. For example, the concept of factorisation hinges on other foundational concepts such as multiplication, algebraic expression, integers and exponents. In addition, other concepts depend on the factorisation concept, such as completing the square, limits, differentiating from first principles and so forth. Therefore, the LoLT for the development of prerequisite concepts leading up to factorisation and beyond is crucial.

2.2.1.3 Other contextual factors

A report by Papier (2014:38) highlights other contextual factors that have contributed towards poor performance of TVET College students in the NCV programme and suggests recommendations for improvement in these factors:

- Programme-related issues where the syllabi for various subjects, including mathematics, were too loaded and too long to be completed in one year
- Student-related issues such as having a poor foundation in mathematics as well as poor reading, writing and research skills required for study and
- Institutional issues such as inadequate facilities, poor administration, lack of textbooks, poor timetabling plans and the inability to recruit and retain good quality lecturing staff.

Additionally, the Green Paper (MHET, 2011:10), reporting on other contextual factors that contribute toward underachievement at TVET colleges, includes:

- A lack of resources at colleges
- Inadequate infrastructure
- Inadequate student financial aid
- Poor calibre of staff and
- Poor governance and administration of colleges.

Current research findings by SAQA (2016:20) suggest that college lecturers who are now having to contend with a different age group of students, many of whom enrolled at colleges because of being excluded from the schooling system, expressed their frustration at being obliged to deal with the following issues, which they did not previously encounter:

- Behavioural problems
- Lack of discipline
- Emotional immaturity and development
- Teenage pregnancy.

These contextual factors, although not directly related to conceptual and procedural knowledge, play a pivotal role in the development or lack of development of concepts and procedures. The appropriate environment for teaching and learning, the consistent attendance of the students, good discipline, and a stable emotional state of mind are vital for the teaching and learning of concepts and procedures in mathematics.

2.2.1.4 Summary of factors that contribute towards underachievement of TVET NCV students

This section reviewed research on the factors that contribute towards underachievement of TVET NCV students and made the following inferences:

Although the findings of (Papier, 2009), (Barry, 2014), (Department of Higher Education and Training, 2011), (Mokone, 2011) and (SAQA, 2016) are relevant, they failed to identify the following critical factors that have plagued student performance in the TVET college sector:

- High absentee rates amongst students and lecturers
- Consistent late arrival
- Long drawn-out student strike action
- Non-payment or partial payment of NSFAS bursaries
- Lack of lecturer commitment to and ownership of the NCV programme.

Their findings are also contextual and generic in nature and do not take the details of learning difficulties into consideration, especially those conceptual and procedural factors that contribute to underachievement in mathematics in the NCV programme. There has been no significant improvement in performance in the NCV programme since 2009 and even less in mathematics. Furthermore, the reports fail to suggest ways in which students may overcome the said difficulties that are preventing them from becoming mathematically proficient.

Conceptual and procedural knowledge

An understanding of the development of knowledge from 'just knowing' to that of proficiency depends on how one views and defines conceptual and procedural knowledge. The following sections discuss the definitions of these concepts:

2.2.2.1 Conceptual knowledge

Hiebert and Lefevre (1986:3) maintain that conceptual knowledge is rich in its relationships: a connected web of knowledge where networking and linking its relationships is as important as its discrete pieces of information. They explain conceptual knowledge by the aid of an example that deals with the construction of a relationship between the algorithm for multi-digit subtraction and knowledge of the positional values of digits (place value). Hiebert and Lefevre (1986:4) mention two ways of achieving conceptual knowledge:

(i) By constructing relationships between pieces of information
(ii) By creating relationships between existing knowledge and new information that is entering the system.

Kilpatrick et al (2001: 118) suggest that conceptual understanding "refers to an integrated and functional grasp of mathematical ideas". Students who develop conceptual understanding have a sound grounding and understanding of mathematical concepts. They see mathematical concepts as integrated and sequential and not ‘standalone’ concepts. Therefore, their ability to retain new knowledge increases because they learn by understanding and connecting mathematical concepts to facts and procedures used in mathematics. In the case of factorisation, a solid grounding in the concepts of multiplication, adding and subtracting terms, exponents and the procedures of multiplying algebraic expressions is necessary.

Subsequently, Star (2005: 407), in agreement with Kilpatrick et al (2001: 121), reported that conceptual knowledge refers to the quality of one's knowledge of concepts and the richness of the connections inherent in such knowledge. Star (2005) also implied that conceptual knowledge is not a mere knowledge about concepts, (p. 407) suggesting that the relationships inherent in a concept may be limited and superficial or extensive and deep. He substantiated this by illustrating the difference between a Grade 6 learner's and a Grade 11 learner’s conceptual knowledge of a slope. He emphasised that those educators who adhere strictly to Hiebert and Lefevre's (1986) definition implicitly subscribe to just a particular subset of conceptual knowledge, that which is richly connected or deep. Thus Star (2005: 407) advocates that this type of knowledge can encompass both deep and superficial understanding based on the student’s level of interaction and experience with a particular concept.

Further research by Meir Ben-Hur (2006), makes the point that conceptual knowledge involves understanding concepts and recognising their applications in various situations. Recently Rittle-Johnson and Schneider (2012:3) emphasised that several researchers suggest that knowledge of concepts often refers to conceptual knowledge. Recent work by Star et al (2015:2), describes this knowledge as knowledge that encompasses a clear understanding of algebraic ideas, operations, procedures and notations.

In this study, conceptual knowledge is more than isolated facts, while such knowledge could be either deep or superficial in its understanding. However, this superficial understanding could be relatively deep, based on the conceptual development of the learner. For example, a Grade 3 learner’s conceptual understanding of subtraction, e.g. $15 - 14$, is at a superficial level. Therefore, this learner may not understand or grasp an example such as $14-15$. However, the same learner, when in grade 8 with a relatively deeper understanding of integers, may understand and grasp the concept of addition of integers. Such a learner may be able to interact with an example such as 14 – 15 with ease. Thus, a learner's initial knowledge of concepts is superficial or relatively deep but over time, this knowledge can deepen and become richer. Similarly, a student's initial understanding of factorising a trinomial to solve an equation such as $2x^2-7x^2+3=0$ might be at a superficial level or a relatively deep one. However, solving a trigonometric equation in the form $2\sin^2\theta-7\sin^2\theta+3=0$ may be at a deeper level of understanding.

2.2.2.2 Procedural knowledge

Hiebert a Lefevre (1986:7) define procedural knowledge as "rules or procedures for solving mathematical problems": systematic, sequentially ordered rules used to manipulate symbols to solve problems. They further explain procedural knowledge by offering an example of the procedure and steps to follow, when adding two fractions with unlike denominators (Hiebert & Lefevre, 1986). Van de Walle (2004:28) suggested that procedures are systematic routines learnt to accomplish some task. Additionally, Rittle-Johnson and Schneider (2012:3) defined procedural knowledge as knowledge of procedures. Procedural knowledge is 'knowing how', or the knowledge of the steps required to attain various goals.

Studies by Kilpatrick et al (2001: 121) however, focus on procedural fluency rather than procedural knowledge. They suggest that procedural fluency is knowledge of procedures, knowledge of when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently. This definition encompasses both algorithmic and heuristic procedures. Additionally, recent work by Star et al (2015:2) defines procedural knowledge as the knowledge which includes choosing and applying correct operations and procedures to solve mathematical problems accurately. Star et al (2015), also introduce a third type of knowledge, which they term 'procedural flexibility'.

This refers to the student's ability to identify and use multiple methods as well as choosing the most appropriate method to solve algebra problems, (Star et al, 2015:2).

Definitions of procedural knowledge by Star (2005: 407), and by Kilpatrick et al (2001:121) contradicts the definitions of procedural knowledge by Hiebert and Lefevre (1986), Van de Walle (2004:28) and Rittle-Johnson and Schneider (2012:3), who suggest that procedural knowledge are superficial and are not rich in their links. Star (2005: 407) substantiates his argument by inferring that Hiebert's and Lefevre's (1986:7) definition of procedural knowledge is narrow in that it only encompasses algorithms and excludes procedures that are heuristic. Algorithms are predetermined steps, which must be executed in the "same order and without error" to solve a problem (Star, 2005:407). Star (2005: 407) suggested that there are many different kinds of procedures and that the quality of the connections within a procedure varies. The Merriam-Webster Dictionary (ref) defines heuristic procedures as "procedures that serve as an aid to learning, discovery and problem solving by experimental and trial and error methods". The successful execution of heuristics depends on the choices that students make. Wise choices indicate sophisticated and deep knowledge (Star, 2005: 407).

Additionally, Star (2005: 409) considers that a key indicator in determining deep procedural knowledge is flexibility. Star (2005: 409) cites an example, where middle school students were required to solve the following linear equation $4(x+1)+2(x+1)=3(x+4)$. He suggested that when students use formal methods to solve linear equations in algebra, they have available the following limited set of actions:

(i) adding or subtracting from both sides
(ii) combining like terms
(iii) distributing or factoring and
(iv) multiplying or dividing both sides.

However, students who possess procedural flexibility can use these limited actions in flexible and efficient ways to solve a wide range of problem types.

Star's (2005: 407) view, aligned to that of De Jong and Ferguson-Hessler (1996:107), suggests that deep-level knowledge is useful in making connections when performing tasks, whereas superficial-level knowledge can be associated with rote learning, inflexibility, reproduction, and trial and error. De Jong and Ferguson-Hessler (1996:107) suggest that "deep-level knowledge is associated with comprehension, abstraction, flexibility, critical judgment, and evaluation".

In this study, the researcher subscribes to the notion of Kilpatrick et al. (2001), Star (2005) and Star et al (2015) that procedural knowledge should include both algorithmic and heuristic procedures and should be rich in its connections. Such deep procedural skill should be used flexibly and accurately to solve new mathematics problems. A criticism that may, however, be levelled against researchers such as Kilpatrick et al (2001), Star (2005) and Star et al (2015) is that their research findings were based on elementary school mathematics focusing mainly on procedural numbers on the topic numbers.

2.2.2.3 Comparison between conceptual and procedural knowledge

Table 2.1 compares conceptual knowledge and procedural knowledge based on the researched definitions discussed in previous sections:

Table 2.1 Comparison: conceptual and procedural knowledge

Conceptual Knowledge	Procedural Knowledge
Understanding concepts and recognising their applications in various situations	Systematic sequentially ordered rules and procedures for solving mathematical problems.
Knowledge that is rich in relationships, a network in which the linking relationships are	Algorithms, which are predetermined steps that must be executed in the "same order and without error"

as prominent as the discrete pieces of information	
Limited and superficial or they may be extensive and deep	Heuristic, which are procedures that serve as an aid to learning, discovery, or problem-solving by experimental and especially trial-and-error methods
An integrated and functional grasp of mathematical ideas	Superficial – Algorithms Deep or rich – Heuristic

The debate amongst researchers, as regards conceptual knowledge versus procedural knowledge, remains highly charged, despite massive advancements in mathematics education. On the one hand, Haapasalo and Kadijevich, (1987) and Haapasalo and Kadijevich, (2000) maintain that emphasising conceptual knowledge is of primary importance as it builds procedural knowledge. Additionally, Hiebert and Carpenter (1992) note that lecturers should avoid an overemphasis on procedural skill without the understanding of a concept, to minimise confusion when learners engage with new and deeper mathematical concepts.

Likewise, Van de Walle (2004:28) notes that the learning of procedural knowledge (rules) should be in the presence of a concept. Lately, MacMath, Wallace and Chi (2009:1) maintain that while "many students may develop procedural knowledge … they often lack the deep conceptual understanding necessary to solve new problems or make connections between mathematical ideas". Similarly, Garfield and Ahlgren (1988: 46), assert that students often tend to respond to problems in mathematics by falling into a "number-crunching mode." They further maintain that although students are able to memorise formulas and steps to follow in familiar, well-defined mathematical problems, they seldom appear to make sense of how to apply these concepts to new situations.

On the other hand, Davis, Gray, Simpson, Tall and Thomas, (2000) argue that conceptual knowledge can only be attained by emphasising procedural competency. In other words, students need to master basic algorithms, calculations and procedures before they can make sense of a concept that has been taught. For example, students may learn the concept of factorisation by completing the following procedures:

(i) Factorise the polynomial below by following the procedure of removing the HCF:

a. $2x+2y=2(x+y)$

b. $6x-3x^2=3x(2-x)$

c. $8a^2b-2ab=2ab(4a-1)$

(ii) Multiply a monomial and binomial by using the distributive law:

a. $2(x+y)=2x+2y$

b. $3x(2-x)=6x-3x^2$

c. $2ab(4a-1)=8a^2b-2ab$

(iii) Investigate the questions and answers and reach a conclusion based on the pattern emerging.

Alternatively, Resnick, (1982) and Resnick and Omanson, (1987) suggest that conceptual and procedural knowledge have no bearing on each other and develop independently. Still others like Berger (2004);; Rittle-Johnson and Schneider (2012) and Star et al (2015) concur that the relations between conceptual and procedural knowledge are often bi-directional and iterative. In this regard, bi-directional means that the order of teaching for conceptual understanding and procedural understanding, when these are interchanged, can complement and strengthen each type of understanding. For example, research by Rittle-Johnson, Siegler and Alibali (2001) on developing conceptual understanding and procedural skill in mathematics found that students' conceptual understanding of math problems enhanced their procedural knowledge, which in turn strengthened their conceptual knowledge. Whereas iterative implies that a student can repeat a concept or a procedure to achieve a desired outcome and form the base for new learning.

2.2.2.4 Summary of conceptual and procedural knowledge

This section reviewed research on conceptual and procedural knowledge and made the following inferences:

The NCTM's Principles and Standards for School Mathematics [PSSM] (2000: 35) suggest that the over-practice of computational methods without conceptual understanding may result in students forgetting them or remembering them incorrectly. They also suggest that understanding concepts without procedural knowledge may constitute a challenge during the problem-solving process. The researcher supports this viewpoint and is of the opinion that at different stages of a student's acquisition of knowledge, the order of conceptual and procedural knowledge is interchangeable while necessarily maintaining a balance between these types of knowledge. Simply put, the result should be that the student understands both the procedure and the concept before progressing to new learning experiences.

In this study, the researcher subscribes to the viewpoint that both kinds of knowledge are iterative and bi–directional in nature when solving mathematics problems. The researcher is also of the same opinion as Star et al (2015), that both conceptual and procedural knowledge are interdependent at a superficial or deep level when constructing new knowledge or solving problems at different levels of a student's interaction. Level 4 is equivalent to grade 12 in the National Qualifications Framework (NQF4). Therefore, conceptual knowledge of factorisation and the flexible use of its techniques, as well as procedural knowledge to solve problems, should be at a deep level of interaction at this level. Students in Level 4 should not be grappling with the concept of factorisation; in fact, they should be constructing new knowledge from a conceptual base of factorisation.

The researcher leans towards Star et al's (2015:46) definition and illustration of conceptual and procedural knowledge and procedural flexibility. Table 2.2 adopts Star et al's (2015:46) definition of these concepts and adapts it to this research study:

Table 2.2: Definition: conceptual, procedural knowledge, and procedural flexibility adopted from Star et al (2015:46)

Domain	Definition	Sample Examples
Conceptual Knowledge	Knowledge that encompasses a clear understanding of algebraic ideas, operations, procedures and notations	• *Explain/Define expressions; equations; factorisation, difference of two squares etc. (Student's understanding of key concepts)* • *Identifying that* $a^2 - b^2$ *is the difference of two squares;* $2x^2 - 7x + 3$ *a quadratic expression and* $x^3 - 3x + 2$ *is a cubic expression (Student's understanding of algebraic expressions)* • *Identify that* $2x^2 - 7x + 3$ *is an expression while* $2x^2 - 7x + 3 = 0$ *is an equation (Student's understanding of algebraic expressions and algebraic equations)*
Procedural Knowledge	Knowledge, which the student can appropriate to choose and apply correct operations and procedures to solve mathematical problems accurately	• *Factorise the following expressions:* ○ $a^2 - b^2$ ○ $2x^2 - 7x + 3$ ○ $x^3 - 3x + 2$ *(Student's ability to factorise expressions)* • *Solve the following equation for x:* ○ $x^2 - 25 = 0$ *(Student's ability to factorise equations)*
Procedural Flexibility	The student's ability to identify and use multiple methods as well as choosing the most appropriate method to solve algebra problems	• Solve the following equations using different methods: ○ $2x^2 - 7x + 3 = 0$ ▪ *Factorise or*

		▪ *Use the quadratic formula or* ▪ *Complete the square* ○ $2x^3+8x^2+8x+32=0$ ▪ Divide both sides by 2 first then solve for x by factorising ▪ Factorise first then solve for x ○ $2\sin^2 x - 7\sin x + 3 = 0$ where $0^o \le x \le 360^o$ ▪ Identify that it is a quadratic equation ▪ Use k substitution then factorise and then solve for x ▪ Factorise and solve for x *(Student's ability to identify and solve equations involving factorisation using multiple or the most appropriate method)*

Conceptual and procedural difficulties

Students who have trouble in understanding critical features of a concept and generalising key facts, concepts, strategies and procedures to other contexts are said to be experiencing conceptual and procedural difficulties in mathematics.

2.2.3.1 Conceptual difficulties

There are numerous examples of research evidence in mathematics education (MacMath et al, 2009; Garfield and Ahlgren 1988; Ball, Lubienski, and Mewborn, 2001; Kilpatrick et al, 2001) which reveal that many students retain fundamental conceptual difficulties in engaging with mathematics, especially with concepts that are new to them. Conceptual difficulties also incorporate misconceptions in mathematics. The *Oxford Dictionary* defines a misconception as "a failure to understand something correctly" (2009:590). Studies by Hansen et al (2005:15) assert that errors and misconceptions are errors that students make because of carelessness, misinterpretation of symbols and text, a lack of relevant knowledge or experience related to a concept, the inability to check answers, or the results of misconceptions. They further suggest that "a misconception is the misapplication of a rule, an over or under-generalisation or alternative conception of the situation", while studies by Allen (2007:1), suggest that misconceptions are certain conceptual relations that are inappropriate within a certain context. Allen (2007:1) further posits that a "misconception does not exist independently but is contingent upon a certain existing conceptual framework". Simply put, misconceptions are concepts that do not agree with traditional understandings of algebra, (Allen, 2007:1). Misconceptions often arise from students' private understandings of certain concepts, previous inadequate teaching, informal thinking or poor recollection. The definitions however fail to reveal that conceptual difficulties could simply mean a complete lack of understanding of a mathematical concept. Conceptual difficulties could also refer to a lack of exposure to mathematical concepts.

2.2.3.2 Procedural difficulties

Studies by Tularam and Hulsman (2015:2) note that while memorising rules and procedures is important, often such knowledge is simply memorised instead of being understood in terms of conceptual connections or deeper meanings. Hence, exposing students to many rules or procedures may cause them to become confused and unable to choose the correct rule or procedure when solving a problem. Tularam and Hulsman (2015:2) also argue that students who correctly use procedures, algorithms and formulas may not necessarily be successful in mathematics, especially at higher levels. They furthermore claim that many students do not understand the reasons why certain procedures work or even when they should consider alternative or equivalent procedures that may be more appropriate to solve a mathematics problem, (Tularam and Hulsman, 2015:3).

Procedural difficulties, therefore, refer to a student's inability to use a procedure correctly or his/her use of an incorrect procedure to solve a mathematics problem.

2.2.3.3 Summary of conceptual and procedural difficulties

In this section, the researcher reviewed research on conceptual and procedural difficulties and made the following inferences:

Students, who have a sound conceptual and procedural knowledge background, see mathematical concepts as integrated and sequential. Such student's ability to retain what they have learnt increases because they learn by understanding and connecting mathematical concepts to facts and procedures used in mathematics. The researcher assumes that a student who possesses both conceptual and procedural knowledge is more likely to be able to avoid making critical errors when solving mathematical problems. Table 2.3 presents the following example for discussion: Factorise the following expression $2x^3 + 8x^2 + 8x + 32$.

Table 2.3: Correct and incorrect solutions

Factorise fully: $2x^3+8x^2+8x+32$ *Correct Solution*: $2x^3+8x^2+8x+32$ $=2[x^3+4x^2+4x+16]$ $=2[(x^3+4x^2)+(4x+16)]$ $=2[x^2(x+4)+4(x+4)]$ $=2(x+4)(x^2+4)$	*Incorrect Solution* 1: $2x^3+8x^2+8x+32$ $=\frac{2x^3+8x^2+8x+32}{2}$ $=x^3+4x^2+4x+16$ $=(x^3+4x^2)+(4x+16)$ $=x^2(x+4)+4(x+4)$ $=(x+4)(x^2+4)$	*Incorrect Solution* 2: $2x^3+8x^2+8x+32=0$ $\frac{2x^3+8x^2+8x+32}{2}=\frac{0}{2}$ $x^3+4x^2+4x+16=0$ $(x^3+4x^2)+(4x+16)=0$ $x^2(x+4)+4(x+4)=0$ $(x+4)(x^2+4)=0$ $\therefore x=-4$ *or* $x=\pm\sqrt{-4}$ $x=\pm\sqrt{4}\times\sqrt{-1}$ $x=\pm 2j$

The incorrect solutions in Table 2.3 reveal some of the inappropriate use of procedures due to a lack of sound conceptual understanding of expressions and equations. Students would have learnt the procedure for solving equations, without understanding the equation concept or why such a procedure of division is permissible in an equation. Thus, they use a procedure where it is not permissible.

The example in table 2.3 highlights the critical need for conceptual understanding and procedural knowledge to co-exist in harmony and balance. Educators, who drill mathematical procedures without attempting to provide any level of understanding, create confusion amongst their students. Such students are prone to apply procedures in isolation to arrive at an answer rather than understand and solve problems. These students become dependent on compartmentalised procedures to solve mathematical problems and very often use procedures 'mindlessly and mechanically' in incorrect mathematical contexts. Such students experience conceptual and procedural difficulties.

Lecturer strategies for overcoming conceptual and procedural difficulties

Improving students' learning and helping them overcome conceptual and procedural difficulties depends largely on the capabilities of the classroom teacher. (Kilpatrick et al, 2001:12). Lecturers must be prepared to try various strategies other than the ones that have yielded poor results in the past if they want to successfully assist students in overcoming such difficulties.

Silver (2011:28), suggests that a successful strategy to assist students with conceptual and procedural difficulties, is for lecturers to implement the scaffolding of a task by considering the following guideline:

a) Assess the student's current knowledge and experience within the academic content of the task
b) Relate the content to what the student can do or understand
c) Break up a task into small, more manageable tasks
d) Provide intermittent feedback
e) Use verbal cues and prompts to assist students.

Lecturers must be competent Subject Matter Experts as well as proficient in pedagogical practices within their subject discipline in order to be able to follow Silver's (2011:28) guidelines. Campbell (2014:1) describes a hierarchy of needs as follows: the lower need in the hierarchy must be satisfied first before an individual may consider higher-level needs. Learning mathematics is also hierarchical; it implies that mathematics learning should start from basic mathematical understanding and skills while moving to learning that is more complex. Silver's (2011:28) suggestive guideline, although very useful, omits an important foundational guide: that is, to determine the prior knowledge necessary for a student to engage productively with a particular task.

Bailey, Zhou, Zhang, Cui, Fuchs, Jordan, Gersten and Siegler (2015), found that an initial emphasis on deeper conceptual and procedural understanding before moving to more complex concepts, yielded higher levels of achievements amongst Chinese children than their American counterparts. Therefore, lecturers need to value procedural knowledge and conceptual knowledge within mathematics education as complementary and vital. Isolated procedural learning can become very fragile, easily forgotten or remembered incorrectly (Bossé, Bahr, 2008:20, Bransford, Brown, and Cocking, 1999). Research by Bossé and Bahr (2008:20) suggests that teachers should ensure a balance of learning both concepts and procedures, particularly if the connections between them are explicit. This has been shown to enhance the long-term retention of both understandings.

Scaffolding conceptual and procedural knowledge

Knowledge of arithmetic and algebra forms the bedrock upon which other aspects of mathematics are studied. Arithmetic and algebra are sequential in their nature and their study. This sequential nature of concepts and their procedures demand that lecturers structure their teaching and learning processes in ways that will 'scaffold' students' understanding and development of mathematical ideas from a point of dependence to that of independence. Scaffolding refers to guiding those elements of a task that are at first beyond the student's capacity, by allowing him/her to concentrate and complete only those elements that are within his/her range of competence, (Woods, Bruner & Ross, 1976: 90). The researcher Vygotsky (1978:86) coined the concept "zone of proximal development" (ZPD) as the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers. Figure 2.1 illustrates scaffolding and ZPD within the parameters of factorisation:

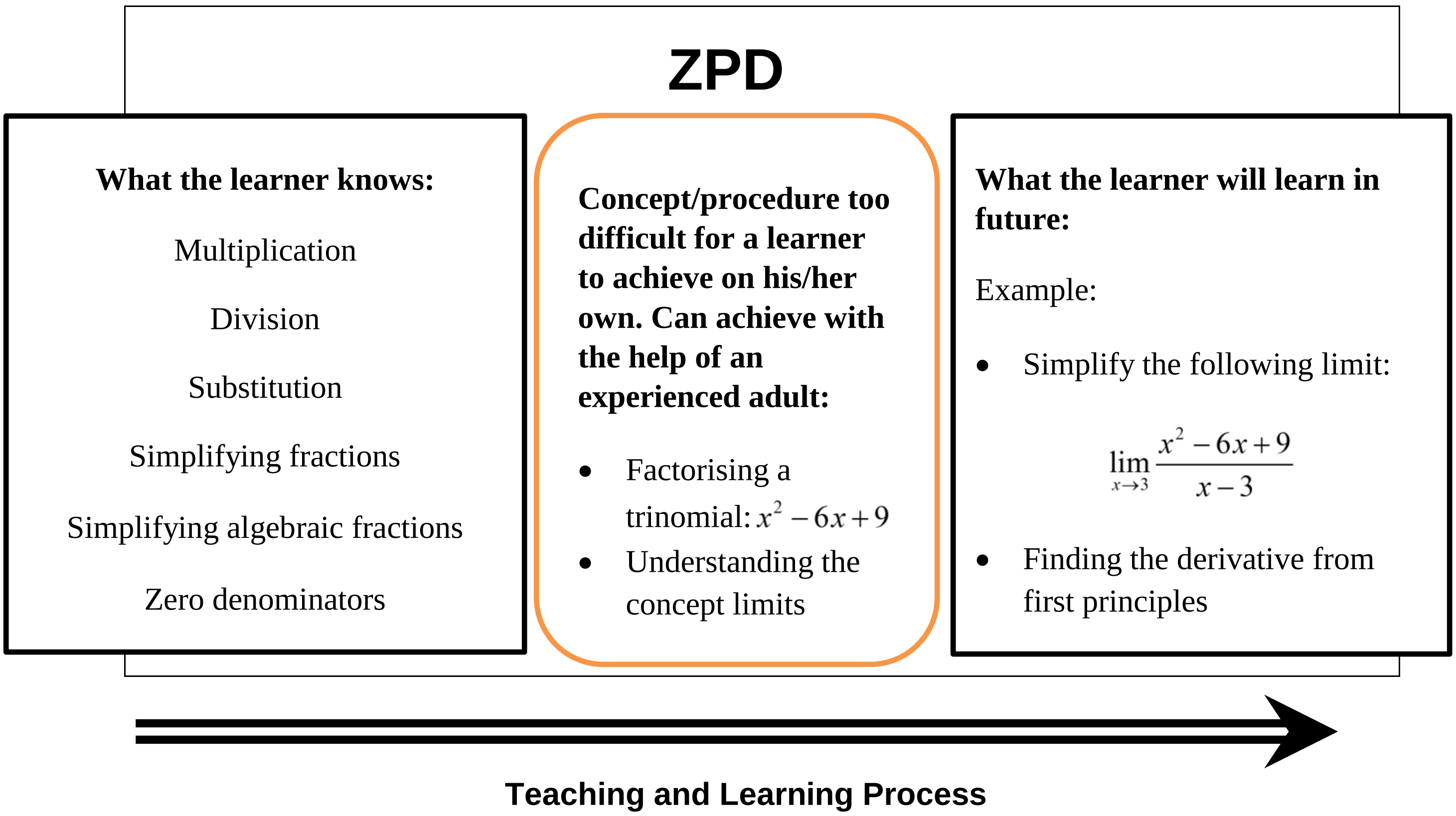


Figure 2.1: Zone of Proximal Development

The researcher is of the perception that one of the ways to improve the Level 4 students' understanding of the concepts of factorisation and factorisation procedures to solve problems may lie in the approach to the teaching of factorisation, its prerequisites, and related concepts. He advocates an approach that will guide students to retain and cascade what they have learnt to what they are learning and to what they will learn in the future. Achieving success using such an approach pivots on both the student and the lecturer becoming active participants in the process of learning. A lecturer needs to be able to correct conceptual and procedural difficulties 'on the spot' when and as the student displays such difficulties while in the process of solving problems.

Definition of factorisation and pre-requisite knowledge

The definition of the concept factorisation is discussed and the pre-requisite knowledge is emphasised in this section. Factorisation is a common thread running through most sections in the school and TVET curriculum. Success in areas such as algebra, trigonometry, calculus and geometry depends, amongst other concepts, upon factorisation. Laridon et al (1997:21) explains that in arithmetic, the factors of a natural number are those numbers that divide exactly into that number. For example, the factors of 20 are 1; 2; 4; 5; 10 and 20, because 20 is divisible by these numbers. Authors Hurjunlal, Junak and Naicker (2013: 112) define factorisation as the reverse of multiplication. Where $3 \times 4 = 12$ then the factors of 12 are 3 and 4. The authors Brown, Evans, Hunt, McIntosh, Pender and Ramagge (2011:4) define factoring as the opposite process of expanding. Factorising an expression is writing that expression as a product of its factors. For example, when working with algebraic expressions and the distributive property of multiplication, the product of $5(x-3)$ is $5x-15$. Therefore, the factors of $5x-15$ are 5 and $(x-3)$. Studies by Smith (2006:1) define factoring as a task of reducing a number into its prime constituents. An expression when presented by a product expression that consists of two or more prime numbers is completely factored, (Kennedy and Tipps, 2000:234). Hence, one can never overemphasise pre-requisite knowledge of prime numbers, prime factors, exponents, multiplication, division, integers, and expressions.

A presentation by Samson, Raghaven and du Toit at the seventeenth National Congress of the Association of Mathematics Education of South Africa (AMESA) (2011:158) highlighted factorisation as a critical concept of the South African school mathematics curriculum. They suggested that it is useful for simplifying algebraic fractions, determining the roots of equations and determining the x-intercepts of graphs amongst others. However, they highlight attention to the fact that it is a concept which students find particularly problematic. Factorisation is an important pre-requisite concept and skill necessary for solving higher-level mathematics problems and understanding new and higher-level concepts, (Brown et al, 2011:4). Hence, the same should apply to TVET colleges: understanding and applying factorisation techniques is a positive step towards attaining proficiency in mathematics.

The following types of factorisation fall within the scope of the NCV Level 2 to Level 4 curriculum:

(i) Removing highest common factor

(ii) Grouping

(iii) The difference of two squares

(iv) Factorising trinomial/quadratic expression

(v) Factorising a third degree polynomial (cubic expression).

The list below is a sample of sub topics, in the NCV Level 4 curriculum, that require the application of factorisation:

(i) Algebraic fractions

(ii) Algebraic inequalities

(iii) Limits

(iv) Complex number equations

(v) Trigonometric equations

(vi) Determining the roots and the turning points of a cubic function (algebra/calculus).

The researcher agrees with the opinion of Brown et al (2011:4) that multiplication of expressions can be relatively easily grasped, but factorisation can be confusing for students. Students must practise frequently to master the different types of factorisation as well as gain an intuitive insight as to when to apply the correct type of factorisation in given mathematical problems (Brown et al, 2011:4).

Summary of literature review

The literature reviewed identifies multiple factors that have contributed towards underachievement in mathematics. These factors exert either a direct or an indirect impact on a student's conceptual and procedural difficulties when factorising and on problems which involve factorisation. The survey also highlights the literary debate on the dominance of either conceptual or procedural knowledge, with a special focus on such types of difficulties in factorisation. The researcher concludes that these kinds of knowledge are bi-directional. The review also discussed lecturer strategies and learning theories for overcoming or avoiding conceptual and procedural difficulties. Lastly, it defined the concept factorisation and highlighted pre-requisite knowledge necessary for factorisation.

2.3 Theoretical framework

Kilpatrick, Swafford and Findel's (2001) theories on mathematical proficiency underpin the theoretical framework for this research study. Kilpatrick et al. propose five interdependent stands that are crucial in achieving mathematical proficiency. However, the first two strands, conceptual understanding and procedural fluency, are foundational ones that are critical to attaining proficiency in mathematics. Hiebert and Lefevre (1986) used the equivalent terms conceptual knowledge and procedural knowledge. Skemp (1976) also used the terms relational understanding and instrumental understanding which are synonymous to conceptual understanding and procedural fluency.

2.3.1 Mathematical proficiency

Early research by Skemp (1976) recognised that attaining proficiency in mathematics hinges on teaching mathematics for 'relational understanding' instead of 'instrumental understanding'. Skemp (1976) argued that relational understanding is easier to retain and more adaptable to new learning than the short-term benefits of instrumental understanding of solving mathematical problems through ‘quick fix’ procedures without any mathematical understanding.

More recently, Kilpatrick et al., (2001:5) highlighted a complete and comprehensive view of how students can achieve mathematical proficiency by implementing five interwoven and interdependent strands that are imaged as a piece of wool or rope with five strands. These are:

- Conceptual understanding - a comprehension of mathematical concepts, operations, and relations
- Procedural fluency- which requires skill in carrying out procedures flexibly, accurately, efficiently and appropriately
- Strategic competence - the ability to formulate, represent and solve mathematical problems
- Adaptive reasoning - possessing the capacity to think logically, to reflect, explain and justify and
- Productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, combined with a belief in diligence and one’s own efficacy.

Students cannot achieve mathematical proficiency if educators focus just on developing one or two of these strands, but the development of all strands amongst students will ensure higher standards of mathematical proficiency (Kilpatrick et al, 2001:29). Additionally, later research by Schoenfeld (2007: 59), points to a shift in education research, towards what it means to understand subject content in different domains. He emphasises that this shift is from an almost exclusive emphasis on knowledge, from: “What does the student know?” to “What students know and can do with their knowledge”. Thus, the notion that knowledge acquisition is not enough but that being able to use that knowledge in appropriate contexts and circumstances is vital for attaining mathematical proficiency. Furthermore, Schoenfeld (2007: 60) asserts that there is much more to such proficiency than being able to reproduce standard content on demand. According to this author, students who are mathematically proficient must at least be able to:

- Extend known results

- Find new results and
- Apply known mathematical results in new contexts.

Schoenfeld (2007: 60) states "It goes without saying that 'knowing' mathematics, in the sense of being able to produce facts and definitions, and execute procedures on command, is not enough", in terms of acquisition of mathematical proficiency. Therefore, students should be able to use the mathematical knowledge they have acquired in an integrated, flexible and correct manner. Researchers Star, Caronongan, Foegen, Furgeson, Keating, Larson, Lyskawa, McCallum, Porath and Zbiek (2015:1), contend that the first step to proficiency in mathematics begins with proficiency in algebra. Star et al (2015:1) highlight three interrelated themes for attaining the latter proficiency:

- Developing a deeper understanding of algebra will occur where instruction in algebra should go beyond superficial mathematics by encouraging students to make connections between concepts and procedures present in algebra problems. The author suggests that this can be achieved through prompting students to ask the following questions:
    - What does this problem require of me?
    - What do I know about the form of the expression present in the problem?
    - Can I see a relationship between the quantities in the expression?
    - Is my solution correct?
- Promoting process orientated thinking that allows shifting away from focusing on final answers towards focusing on understanding the process that leads to the final answer. In this case Star et al (2015:1) suggests that students ask themselves the following questions:
    - How did I decide to solve the problem?
    - What steps did I take to solve the problem?
    - Were these steps a good strategy?
    - Can I solve this problem in other ways?
- Encouraging precise communication where students are encouraged to talk about and reason with concepts, procedures and strategies using precise mathematical language. In

this instance Star et al (2015:1) recommends that students should ask the following questions:

- How can I describe this problem in precise mathematical language?
- How can I describe my strategy for solving this problem in precise mathematical language?

2.3.2 Summary of theoretical framework

This section summarises the theoretical views of Kilpatrick et al (2001) on proficiency in mathematics as well as citing other views, with particular focus on conceptual understanding and procedural knowledge. It concludes that an integration of these views would best suit the theoretical framework of this research study.

The researcher holds the perception that conceptual understanding and procedural fluency, are foundational strands for attaining proficiency in algebra and mathematics. Inherent in these are the three other strands: strategic competence, adaptive reasoning and productive disposition. Studies by Rittle-Johnson and Schneider (2012: 19) reinforce this view: “mathematical competence rests on developing both conceptual and procedural knowledge.”

The researcher also concurs with Star et al’s (2015:1) suggestion that the first step toward attaining mathematical proficiency is by attaining a deeper understanding of algebra. Due to the latter’s broad scope, this study could not address all aspects of algebra. It thus focused on conceptual knowledge and procedural knowledge in factorisation and solving problems involving factorisation. Attaining these kinds of knowledge separately is a superficial approach and does not guarantee proficiency in algebra. However, the ability to integrate and use the acquired conceptual and procedural knowledge flexibly and adequately to solve mathematical problems may lead to proficiency in algebra. The term ‘procedural flexibility’ coined by Star et al (2015:2) supports this view. They define procedural flexibility as the ability to identify and implement multiple methods to solve algebra problems, as well as the ability to choose the most appropriate solutions.

2.4 Conclusion

Analysis of the literature reviewed revealed contrasting viewpoints on the importance of conceptual knowledge versus procedural knowledge. However, this study adopts a standpoint that a balance between these types of knowledge should exist. This balance may reduce the severity of conceptual and procedural difficulties that students experience when factorising, or perhaps avoid such difficulties altogether.

In this chapter, the researcher presented a review of available literature on various factors that contribute towards underachievement of TVET NCV students (see section 2.2.1). He provided an overview of conceptual and procedural knowledge and difficulties found in literature (see section 2.2.2). In addition, he discussed strategies for overcoming such difficulties, the definition of factorisation and the prerequisite knowledge needed for students to be able to engage with factorisation found in literature (see sections 2.2.4; 2.2.5 and 2.2.6). The researcher set the theoretical framework that underpins this study by motivating why conceptual and procedural knowledge are critical foundational strands for students to attain mathematical proficiency (see section 2.3.1).

In the next chapter, the researcher discusses the data collection procedure of this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

In Chapter 2, the literature review covered what is entailed in conceptual and procedural knowledge. It revealed factors that contribute to conceptual and procedural difficulties experienced in learning mathematics, with a particular focus on factorisation and surveyed the literature, on possible ways to overcome such difficulties. This chapter (Chapter 3) substantiates the research plan for this study.

3.2 Research paradigm, approach and design

This section discusses, in detail, the research paradigm, the research design and the research approach that this study adopted.

3.2.1 Research Paradigm

A research paradigm is defined as a philosophical worldview that integrates the "beliefs or philosophical orientation of the world and the nature of research that a researcher brings to a study", (Creswell, 2014:35). Researchers embrace various paradigms or philosophies such as realism, objectivism, subjectivism, functionalist, radical humanist and radical structuralism. However, the four major philosophical paradigms broadly adopted by researchers are:

(i) Post positivism
(ii) Interpretivism
(iii)Transformative
(iv) Pragmatism.

Table 3.1 below, adopted from Creswell (2014:36), summarises the main characteristics of these paradigms:

Table 3.1: Four philosophical paradigms

Post positivism	**Interpretivism**
• Determines • Reduces • Empirically observes and measures • Justifies theories	• Understands • Multiple participants' meanings • Social and historical construction • Generates theories
Transformative	**Pragmatism**
• Political • Power and justice orientated • Collaborative • Change-oriented	• Consequences of actions • Problem-centred • Pluralistic • Real-world practice orientation

This research study subscribes to the philosophical paradigm of interpretivism, which generally adopts a qualitative approach to research, (Creswell, 2014:37). According to Creswell, (2014:37) interpretivists believe that:

(i) Individuals seek an understanding of the world in which they live and work.

(ii) Individuals develop varied and multiple subjective meanings of their experiences, where the researcher relies largely on views of the participants and encourages them to construct the meaning of their situation by prompting a discussion via open-ended questions.

(iii) Due to the subjective meanings through interaction with others, social constructivism and historical and cultural settings of individuals, the researcher attempts to make sense of the world in which people live and work by being mindful of his/her own personal, cultural and historical background.

(iv) Qualitative research makes sense of the meanings that participants have about the world, instead of starting with new theories. Thus, the researcher generates meaning from the data collected in the study.

3.2.2 Research approach

This study was qualitative in its approach. Generally, research studies adopt one of the following three popular approaches:

a) Qualitative research
b) Quantitative research
c) Mixed methods research.

Welman, Kruger, and Mitchell (2007) consider that qualitative researchers are concerned with understanding human behaviour from the perspectives of the people involved in the phenomena. These authors stressed that a researcher cannot separate the person experiencing the phenomenon from it. Meanwhile, Terre Blanche, Durrheim and Painter (2006) emphasise that qualitative research methods are methods that try to describe and interpret people's feelings and experiences in human terms rather than through quantification and measurement. Moreover, Maxwell (2013:8) defines qualitative research as research that strives to help the researcher understand:

(i) The perspectives of the people being studied, assisting the researcher to see the world from their point of view
(ii) How those perspectives shape the participants' physical, social and cultural contexts and
(iii) The specific process involved in maintaining or altering those phenomena and relationships.

In addition, Sharma (2013:55) highlights the value of qualitative research in mathematics education, to understand mathematics teaching and learning better, by employing the following strengths of qualitative research; it:

(i) Records the behaviour of participants in their natural settings. In this study, the researcher conducted the assessment of, as well as the semi-structured interviews with, students and Subject Matter Experts.
(ii) Undertakes an in-depth study of a small group of people. In this case, only 30 students wrote the assessment. The researcher interviewed five students and five Subject Matter Experts, as described earlier.

The quantitative research approach, according to Terre Blanche et al (2006), is a research methodology that is measurable and strives towards the ideals of statistical generalisations and objectivity, while Creswell (2014:32) explains that quantitative research is an approach that tests theories by measuring the relationship amongst variables. Additionally, Maxwell (2013:8) bases a quantitative research approach on studying a phenomenon through variables which are measured and compared across contexts.

A current, popular approach is the mixed methods approach which, according to Creswell (2014:32), involves a combination of qualitative and quantitative research by integrating collected data. Researchers adopting this approach argue that it provides a balanced and holistic understanding of the research problem.

The researcher however, agrees with Creswell's (2014:32) notion that research approaches cannot be rigid or discrete. Instead, a study could be 'more qualitative' than quantitative or vice versa.

Otherwise, there could be a balance between qualitative and quantitative approaches, resulting in a mixed methods approach. This study was not quantitative in its approach and did not measure the phenomena through statistical variables and themes. While it did consider a small percentage of quantitative data, overall it was not restrictive and it mainly considered the experiences, views and opinions of those participants that fell within the phenomena researched. This quantitative data merely represented the overall results and statistical analysis of the performance of the 30 students who wrote the assessment test on factorisation and problems that involve factorisation, as outlined in section 4.3.1 of the next chapter. This quantitative data determined the average performance of the students, giving the researcher a general idea of the students' performance. He interrogated the student's written responses to the assessment test to investigate the conceptual and procedural difficulties they experienced. He also scrutinised the transcripts of the interviews of all participants to verify conceptual and procedural difficulties, to elicit possible reasons as to why students experience such difficulties and to explore possible strategies to overcome such difficulties.

Despite its strengths, academics still criticise qualitative research. Sharma (2013:55) discloses some of these criticisms as follows:

(i) "Qualitative research can lack reliability, validity, generalisability and inter-subjectivity"
(ii) "The complexity of the natural setting may be difficult to comprehend"

Because of these criticisms, the researcher resolved to minimise these limitations by interpreting the subjective views of all participants in this study as objectively as possible.

3.2.3 Research design

There are various types of qualitative research designs. Each design is aligned to a particular line of enquiry. Table 3.2 below, adapted from Ary, Jacobs, Sorensen and Walker (2014:34) highlights eight commonly utilised types of qualitative designs characterised by their major lines of investigation:

Table 3.2: Types of qualitative research designs

Qualitative Design	**Questions of Enquiry**
1. Basic interpretive studies	How are processes and activities viewed by the participants?
2. Case study	What are the attributes of a person or a group of people classed together?
3. Content analysis	What can be understood by scrutinising existing records?
4. Ethnography	What are the customs, habits and culture of a group of people?
5. Grounded theory	What theory can be constructed inductively from data collected?
6. Historical studies	What can be deduced from past events?

7. Narrative inquiry	What conclusions or understandings emerge from examining life experiences?
8. Phenomenological study	How do individuals perceive or feel about their experiences?

This study adopted a phenomenological case study research design as highlighted in section 1.12.1. Phenomenology is a study of "direct experience taken at face value", (Cohen, Manion and Morrison, 2011:18). The purpose of a phenomenological design is to understand people's views, insights and considerations of their experiences of a particular situation, (Leedy and Ormrod, 2005:139). According to Ary et al (2014:34) phenomenological studies acknowledge the existence of multiple realities embedded in participants' views. Therefore, each person might interpret an experience of the same phenomenon differently. This study explored the thoughts and feelings of participants on conceptual and procedural difficulties, challenges and factors that contribute to these difficulties and ways that could help overcome these difficulties, through semi-structured interviews. The researcher hoped that such interviews would elicit the core of each participant's experience.

Welman et al (2007:25) assert that case study research "is directed at understanding the uniqueness and idiosyncrasy of a particular case in all its complexity". The authors imply that in a case study such as this, the researcher must explore all peculiar, unconventional behaviour, misconceived ways of thinking and working with factorisation and any other peculiarities of this case. The purpose of a case study design is to attain a comprehensive description and understanding of an entity, the "case", (Ary et al, 2014:32). This research used a case study as a lens of enquiry focussed onto a particular problem at a specific setting. As intimated, this study concentrated on the conceptual and procedural difficulties experienced by NCV Level 4 students, when factorising and solving factorisation problems, at a specific TVET campus in KwaZulu-Natal, South Africa.

Another feature of case studies is that they use multiple instruments to collect data. In this case, the researcher utilised an assessment test and two semi-structured interview protocols to do so.

The discussions highlight that phenomenology and case study designs are effective pragmatic tools that would assist to shape the problem, the research questions, the data collection methods, the analysis of the data as well as support this study in its decision-making processes. These discussions confirm why the researcher proposed an integrated phenomenological design within a case study for this research.

3.3 Population and sampling

In this section the researcher discusses his role as the researcher, the research location, the target population, the study population and the sample and sampling procedures adopted in this study.

3.3.1 Role of the researcher

A phenomenological research design demands that the researcher must temporarily set aside his or her own perceptions of a phenomenon, in an attempt to understand the inter-subjectivity of the participants within the study. This process is referred to as "epoche" by Moustakes (1994:57), which entails the suspension of all judgement. According to Tufford and Newman (2012:81), researchers use bracketing to avoid personal preconceptions, relating to a study. Bracketing carries a similar connotation to epoche, in that the researcher consciously separates out his or her own views and beliefs regarding a phenomenon, temporarily, to allow the meanings of the participants to emerge.

The researcher may reflect upon his/her experiences within a phenomenon at an appropriate time within the research process, (Bednall, 2006). Figure 3.1 adopted from Bednall (2006) illustrates the author’s view on the practice of epoche and bracketing in this study

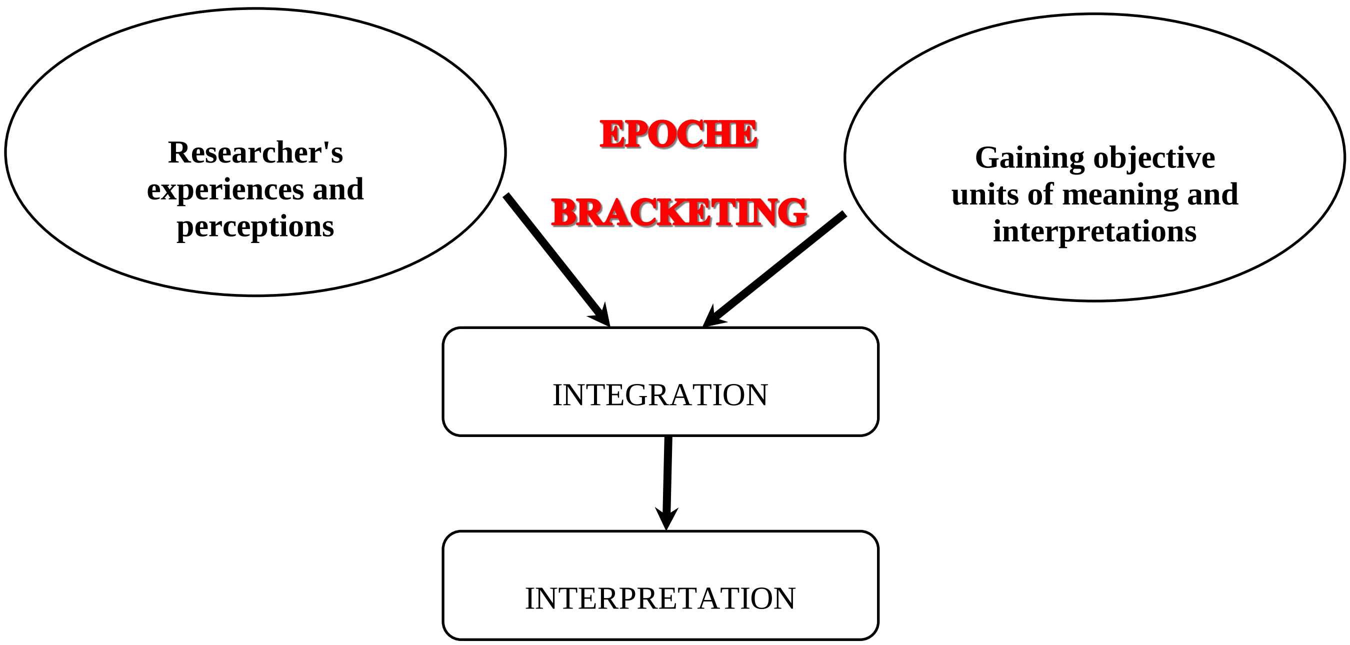


Figure 3.1: Epoche and bracketing, adapted from Bednall (2006)

The researcher has 22 years of cumulative experience teaching and lecturing in mathematics and managing the mathematics departments at school and college levels. These experiences have shaped his own understandings of student’s conceptual and procedural difficulties. He holds preconceptions on how teachers, lecturers, and administrators contribute to student difficulties. In spite of this, the researcher attempted to suppress his own experiences by mainly reflecting on the experiences gained from conducting this research study. The researcher however, has shared his own experiences and perceptions at appropriate stages of this research:

- The researcher initially assumed the role of a concerned member of the case, attempting to solve the phenomenon. In doing so, he embraced his own experiences and preconceptions as vital to this study
- During the literature review, the researcher shared his opinions, to either agree with or counteract the literature discussed

- The researcher developed the research instruments based on his view that these instruments would be able to assist him to achieve the aims and objectives of this research as well as answer related questions
- During data collection, the researcher consciously suspended his biases by assuming the role of a facilitator
- He attempted to separate himself from his past knowledge and experiences while conducting the interviews by flexibly allowing all interviewees to air their views without interruption or intimidation
- The researcher assumed the role of an objective interpreter, by interpreting the subjective views of all participants as authentically as possible, with the hope of allowing new meanings to emerge
- The researcher then integrated his own preconceptions with the new meanings that stemmed from the data that were interpreted, in an attempt to propose possible solutions to the phenomenon under discussion and
- During reporting, the researcher referred to himself in the third person, in an attempt to avoid becoming too personal.

3.3.2 Research location

Marriam and Tisdell (2015: 137) suggest that in qualitative research the researcher observes the phenomenon in its natural settings rather than at a location of convenience only. As mentioned, the researcher conducted this research at a TVET campus in the heart of a township in the province of KwaZulu-Natal, South Africa. The researcher chose this location because it is where the phenomenon had emerged and because of its ease of access and convenience. Creswell (2014:237) asserts that data collected at the researcher's workplace may be convenient but may be inaccurate, and this could consequently jeopardise the roles of the researcher and the participants. He recommends that, in such cases, the researcher must adopt multiple strategies to authenticate the accuracy of information. The researcher used an assessment test as well as semi-structured interviews to circumvent inaccurate data.

3.3.3 Target population

A target population is a well-defined collection of individuals or objects, with similar characteristics, that are the focus of a study, (Explorable.com, 2009, McMillan and Schumacher 2006:119). As stated earlier in the text, in this case study, the target population consisted of 1 973 students, 82 lecturers, and 5 Heads of Divisions from the research location. The researcher also included three Umalusi NCV external Moderators as the off-site population that have an influence on this case.

3.3.4 Study population

The study population, also called the accessible population or sample frame, is a subset of the target population. The researcher focuses on the study population to draw a sample to answer the research questions as well as generalise any findings amongst the target population. The researcher identified the study population as the set of cases from which he recruited a sample of participants for this study (Welman et al, 2007).

In this research, the study population extracted from the target population comprised 242 NCV Level 4 students who study mathematics as a subject, 11 mathematics lecturers and the head of the mathematics faculty. As referred to, the study population also included three Umalusi NCV mathematics external Moderators, as off-site participants, who were responsible for the following:

- Externally moderating the NCV levels 2, 3 and, 4 examination question papers for mathematics, since these are exit level examinations at TVET colleges
- Verifying the NCV marking process for mathematics and
- Verifying of the conduct of the on-site internal continuous assessments for mathematics.

3.3.5 Sample and sampling procedure

A sample is a subset of the target and the study population, (Creswell, 2003). Generally, quantitative researchers adopt probability sampling while qualitative researchers adopt non-probability sampling. Bhattacherjee (2012) asserts that in probability sampling, all participants of a population have an equal chance of selection through a random selection process that yields an accurate sample. Researchers conduct non-probability sampling in non-random ways, (Bhattacherjee, 2012). Table 3.3 reflects examples of a few non-random ways of sampling suggested by Bhattacherjee (2012:69):

Table 3.3: Examples of non-random ways of sampling

Convenience/accidental/ Opportunity sampling	Selecting a sample from what is readily available or convenient
Quota sampling	The researcher selects a mutually exclusive group
Expert sampling	Participants are chosen based on their expertise as regards the phenomenon
Snowball sampling	The researcher will initially select a few participants but will include more participants based on the existing participants' referrals

Source: Bhattacherjee (2012:69)

However, for this study, the researcher adopted a non-probability technique called purposive sampling. He chose this method because it allowed him to select the sample deliberately, (Shadish, Cook, and Campbell, 2002:511). The researcher purposively chose the sample of 30 NCV Level 4 students, three mathematics lecturers, and one Head of Division for mathematics, from the study population. One Umalusi Moderator was also included in the sample. Thus, the researcher intentionally selected all participants based on their experiences with and/or their knowledge of the phenomenon under investigation, (Robinson, 2014:5243).

The researcher adopted the following order of sampling procedure for this study:

(i) He purposively selected 30 NCV Level 4 students, who were part of the civil engineering faculty, to write the proposed written assessment on factorisation.
(ii) The researcher then purposively selected five students for individual, face-to-face interviews in order to probe their conceptual and procedural difficulties based on their responses in the written assessment. The researcher considered students who lived within relatively close proximity, within walking distance of the campus, first. This was a precautionary measure, for financial and safety reasons, in the event of fieldwork extending beyond campus times.
(iii) He then purposively chose the three mathematics lecturers who were responsible for the NCV Level 4 mathematics students at the time of this study.
(iv) Finally, the researcher purposively chose one Umalusi Moderator.

Table 3.4 illustrates the sample as a sampling grid. This grid also highlights the role of the sampled participants and the purpose their roles serve in this research study. This grid helped to keep the research focused.

Table 3.4: Sampling grid

Key role players	Sample	Role of the participant	Purpose served
NCV Level 4 students studying mathematics	30	Write the mathematics assessment	• Authenticate the view that students experience conceptual and procedural difficulties while factorising • Determine the type of conceptual and procedural difficulties experienced while factorising • Determine the frequency of the type of conceptual and procedural difficulty
NCV Level 4 students studying mathematics purposively selected from the 30 students who wrote the assessment test	5	Participate in a face-to-face interview	• Validate the conceptual and procedural difficulties experienced while factorising • Consider problems that students are experiencing that might be factors contributing to their conceptual and procedural difficulties • Discuss possible solutions from their perspective
NCV L4 mathematics lecturers	3	Participate in a face-to-face interview	• Share their views on the phenomenon experienced • Consider problems that students and lecturers are experiencing that might be factors contributing to their conceptual and procedural difficulties • Discuss possible solutions from their perspective

Head of Division - mathematics	1	Participate in a face-to-face interview	• Share views from a position of Head of Division on the phenomenon experienced concerning those contributing factors of conceptual and procedural difficulties • Consider problems that students and lecturers are experiencing • Discuss possible solutions from a management perspective
Umalusi external Moderator	1	Participate in a face-to-face interview	• Share views from the position of external Moderator on the phenomenon experienced • Consider problems that students and lecturers are experiencing that might be contributing factors of conceptual and procedural difficulties • Discuss possible solutions from an external Moderator perspective

Bhattacherjee (2012:69) claims that non-probability sampling is biased and cannot be accurate. For this reason, the researcher attempted to create a balanced sample by including a cross section of role players within the research process. Generalisation of information and findings from the sample was also restricted just to the target population.

3.4 Data collection

This section substantiates the data collection instruments and the data collection procedure that was adopted by this research study.

3.4.1 Data collection instruments

Researchers design and use data collection instruments as mechanisms to collect, interpret, and measure information on the phenomenon under examination. Annum (2016:1) stresses that at the outset researchers must ensure reliability and validity by designing appropriate data collection instruments for a study. In this qualitative study, the data collection instruments consisted of a written assessment on factorisation, and interview protocols for the purposes of conducting semi-structured interviews with students, mathematics lecturers, the mathematics Head of Division and the Umalusi Moderator. Ultimately, these instruments were used to collect data, which assisted the researcher to answer the research questions. Sections 3.4.1.1 and 3.4.1.2 discuss these instruments in detail.

3.4.1.1 Assessment on factorisation

The Mathematical Sciences Education Board of the National Research Council (1993:117) suggests that when designing a written mathematics assessment, the researcher must evaluate its effectiveness against the educational principles of content, learning and equity. This implies that the assessment should reflect the relevant mathematics within the curriculum, promote the improvement of learning and teaching, and support fairness. This board further suggests that the researcher can achieve equity or fairness by ensuring that the assessment does not favour one group over another, that the important mathematics concepts and procedures being assessed have been taught and that the assessment tasks are accessible. In this way they might have a positive impact on students' learning, (The Mathematical Sciences Education Board, National Research Council, 1993:117).

The researcher set a written assessment with possible answers as outlined in Appendix E. The assessment covered questions on basic factorisation as well as questions that assessed problems that require factorisation. The researcher divided the assessment into 2 sections as follows:

❖ **Section A**: Basic factorisation

In Section A, students had to fully factorise the given algebraic expressions by using the following methods of factorisation:

- Removing highest common factor
- Grouping
- Difference of two squares
- Factorising trinomial and
- Factorising a cubic expression using the factor theorem.

The researcher designed section A to assess the student's conceptual and procedural knowledge on factorising basic algebraic expressions. Section A is aligned with the NCV Level 2 Curriculum. Table 3.5 is an extract from the NCV mathematics assessment guidelines for Level 2 that highlights this alignment:

Table 3.5: Extracted from National Certificate Vocational Assessment Guideline Level 2: Implementation: 2011

SUBJECT OUTCOME	
2.2 Manipulate and simplify algebraic expressions	
ASSESSMENT STANDARD	**LEARNING OUTCOME**
• Expressions are factorised by identifying/taking out the common factor	• Factorise by identifying/taking out the common factor
• Expressions are factorised by grouping in pairs	• Factorise by grouping in pairs
• The difference of two squares is factorised	• Factorise the difference of two squares

• Trinomials are factorised	• Factorise trinomials

❖ **Section B**: Problems involving factorisation/application of factorisation

- Algebraic fractions
- Limits
- Complex number equations
- Trigonometric equations
- Determining the roots of a cubic function
- Factor theorem

The researcher designed section B to assess the students' ability to use their conceptual and procedural understanding of factorisation to solve integrated problems in the NCV Level 4 Curriculum that are dependent on factorisation for their solution. Section B is aligned with the NCV Level 4 Curriculum. Table 3.6 is an extract from the NCV mathematics assessment guidelines for Level 4 that highlights a direct alignment:

Table 3.6: Extracted from National Certificate Vocational Assessment Guideline Level 4: Implementation: 2013

SUBJECT OUTCOME	
2.1 Work with algebraic expressions using the remainder and factor theorem	
ASSESSMENT STANDARD	**LEARNING OUTCOME**
• Third degree polynomials are factorised by using the factor theorem	• Factorise third degree polynomials including examples that require the factor

	theorem. (Long division or any other method may be used)

The assessment also included questions on the following sub topics, in the NCV Level 4 curriculum, that require the application of factorisation:

(i) Simplifying limits,
(ii) Solving complex number equations,
(iii) Proving trigonometric identities,
(iv) Solving a trigonometric equation, and
(v) Determining the roots of a cubic function.

The researcher designed the written assessment to analyse the student’s conceptual and procedural responses to each question. He also intended to analyse students’ procedural fluency by determining the correlation between those students who excelled at Section A and their performance in Section B. The test questions were open-ended, in that students could respond to each question based on their own preference for procedures used to factorise expressions. The assessment had a maximum mark of 50 and the participants were required to complete the assessment within a duration of one hour.

The written responses, answer scripts, of each participant served as qualitative documents, which the researcher analysed, (Creswell, 2014:241). The researcher found the administering of the written assessment, collection of answer scripts, and the analysis of the answer scripts as documents to be advantageous. He was able to conduct the written assessment at a time convenient to him and the students. He also analysed the direct responses of students unobtrusively and at leisure. In addition, because the tests served as written evidence, the researcher did not have to transcribe them and thereby saved time.

3.4.1.2 Interviews

Interviewing is a basic mode of narrative inquiry, (Seidman, 2006:8). In quantitative research, structured, closed ended interviews serve the purpose of eliciting answers to questions on testing and evaluating scientific hypotheses. However, in a qualitative phenomenological study, in-depth semi structured or unstructured, open-ended interviewing serves the purpose of understanding the experience of the participants and the meanings they make of that experience, (Seidman, 2006:9). Qualitative researchers conduct different types of interviews, such as email or internet, face-to-face, focus groups, online focus group and telephone interviews, (Creswell, 2014:242).

According to Creswell (2014:241), some of the advantages of interviews include:

- Provision of historical information about the phenomenon by participants
- Allowing the researcher to have full control over the line of questioning and
- Usefulness when the researcher cannot directly observe the experiences of the participants.

These advantages have been crucial in this research. The researcher exercised full control of the line of questioning during the interviews and he gained valuable insights on the historical background of the phenomenon from the interviewees. Since researchers cannot directly observe what goes on in the mind of participants, interviews in this study assisted the researcher to gain an understanding of the mathematical processes taught, especially the conceptual and procedural difficulties experienced by students.

Limitations of interviews that are pertinent to this study include conducting interviews at a designated venue instead of the natural learning environment. Creswell (2014:241) observes that researchers tend to intimidate participants in such a way that their responses might be biased. The researcher attempted to overcome such intimidation by reassuring and encouraging each participant during the interviewing process. The researcher observed that most, but not all, participants could articulate themselves well since they were English second language speakers though not all of them were perceptive, Another limitation of this exercise was that the researcher found interviews to be extremely time consuming.

This study utilised two semi-structured face-to-face interviews to gain an understanding of the experiences of participants at different levels of interaction with the research problem. Sections 3.4.2.1 and 3.4.2.2 address the design of the interview protocols that guided the interview processes, in detail.

3.4.1.2.1 Semi-structured interview protocol for students

The researcher designed a semi-structured interview protocol for students as outlined in Appendix C that guided the semi-structured face-to-face interviews. According to Galletta (2013:2) a well-designed interview of this type is flexible, in that it addresses the qualitative research questions while also accommodating the participant's new meanings in the study. Furthermore, semi-structured interviews are able to accommodate open-ended and closed questions. The researcher developed this protocol for students with the intention of addressing the following research objectives and questions:

Objectives:

- Analyse the conceptual and procedural difficulties that NCV Level 4 students demonstrated when factorising and solving problems involving factorisation
- Explore why NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation.

Questions:

- What conceptual and procedural difficulties do NCV Level 4 students demonstrate when factorising and solving problems that involve factorisation?
- Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?

Written assessments may, to a limited extent, expose conceptual difficulties. Hence, the researcher utilised the semi-structured interview to probe conceptual difficulties that students experienced while writing the assessment. This interview protocol posed questions based on the student's incorrect written responses to the assessment and other open-ended questions. In this regard, the researcher adopted a similar line of questioning based on each question incorrectly answered by the student. He used such questions to probe students' views and insights based on their initial response. The aim of this type of questioning was interpretive and insightful, but not evaluative.

In this way, the researcher sought to:

- Verify the findings of the written assessments by attempting to understand how students constructed meaning in particular questions in the assessment,
- gain a deeper understanding of the students' thought patterns to elicit conceptual and procedural difficulties they experienced in factorising and solving problems that involved factorisation and
- obtain insights on other challenges that students could be experiencing which might be hampering their progress in NCV mathematics.

3.4.1.2.2 Semi-structured interview protocol for all other participants

The researcher designed an interview protocol, outlined in Appendix D, which guided the semi-structured individual, face-to-face interviews with the lecturers, the Head of the mathematics department and the Umalusi Moderator. This interview protocol differed from that used for students, in that its tone was different, it attempted to view the perceptions of lecturers, managers/administrators and quality assurers and it focussed on the 'why' questions and strategies that could possibly provide remedies to the phenomenon.

The researcher adopted a similar line of questioning, though, based on questions the students presented incorrectly in the assessment test and their interesting responses during interviews. The researcher used this interview protocol to probe the views and insights of the interviewees on the phenomenon under study.

In this way, the researcher sought to gain:

- Rich insights into why these NCV L4 students in particular were prone to such conceptual and procedural difficulties
- Possible ways that lecturers and administrators could adopt and implement strategies in order to avoid such conceptual and procedural difficulties when teaching students how to factorise and solve problems that involve factorisation and
- Insights on how other difficulties students face could be addressed.

3.4.2 Data collection procedure

The researcher collected data for this study in the following order and under the following conditions and constraints:

3.4.2.1 Conducting the assessment

The researcher negotiated and agreed with the thirty NCV students on a common date, time and venue to write the assessment. Students wrote the assessment individually and under closed test conditions. The duration of the test was one hour. The researcher attempted to minimise the limitations of an assessment, especially ‘math anxiety’, by:

- Verbally reassuring the students that the assessment was for study purposes and had no bearing on their progress at Level 4, imploring students to relax and enjoy the assessment
- Allowing students to choose the venue for the assessment. Subsequently they wrote the assessment in their classroom, Room Number B24. This allowed them to be in a familiar environment
- Organising for their English Communication lecturer to invigilate the assessment with the researcher. In this way, students were calm because of a familiar face

- Verbally reassuring the students that extra time would be allowed to those who did not complete the assessment on time and
- Not including the mark allocation and time allocation on the assessment task. The mark allocation was on the marking memorandum.

The researcher used the marking memorandum (Appendix E) to mark and analyse the thirty students' written responses on an Excel spreadsheet as outlined in Chapter 4. As mentioned, he then purposively selected five students to participate in the semi structured face-to-face interviews.

3.4.2.2 Semi-structured interviews of students

The researcher utilised the semi-structured interview protocol, outlined in Appendix C, to individually interview the five selected students. He conducted these interviews in the mathematics room. Students frequent this room, which is situated away from the main buildings, is equipped with computers and has internet access. The researcher assumed that this quiet and familiar environment would be less stressful for the students to elicit relevant responses without feeling intimidated.

3.4.2.3 Semi-structured interviews of other participants

The researcher utilised the interview protocol outlined in Appendix D to interview the three lecturers and the Head of the mathematics department, individually. The researcher conducted semi-structured interviews with lecturers in their classrooms, and the HOD in his office. The Umalusi Moderator lives 350 kilometres away from the researcher. The researcher was mindful of minimising cost implications and the safety of the Umalusi Moderator. He therefore travelled to the Umalusi Moderator's home to conduct the interview with him at a time convenient to and chosen by him.

3.4.2.4 Data capturing

The researcher scanned and saved all answer scripts of the written assessment into Portable Document Format (pdf). The researcher used the recording application found on a Samsung S5 mini mobile phone to voice-record all interviews. This ensured that all conversations were accurately accounted for (Creswell, 2010:217). The researcher transferred the interview recordings immediately after the interview, to his laptop. He then transcribed the interviews verbatim in writing. Thereafter he typed and saved each interview in Word format. All written assessments, scripts and the observation protocol were scanned and saved into portable document format (pdf).

3.5 Data analysis

As expressed by Bazeley (2013:4), qualitative researchers analyse participants' experiences of the world around them by having 'a close engagement' with the collected data, through insightful strategies that illuminate authentic meanings. This author views data analysis as a recursive process: a rigorous, insightful back and forth process always attempting to understand the phenomenon being researched. Miles and Huberman (1994:10) offer three interconnected processes for effective qualitative data analysis:

- Data reduction – the constant process of reducing or 'breaking up' of data into manageable codes, categories and themes
- Data display – ongoing display, organisation and comparison of salient information for further analysis, and

- Conclusion drawing/verification – meanings and new meanings of data continually tested against the collected data.

Figure 3.2, sourced from Miles and Huberman (1994:12), illustrates the flow of these interconnected processes.

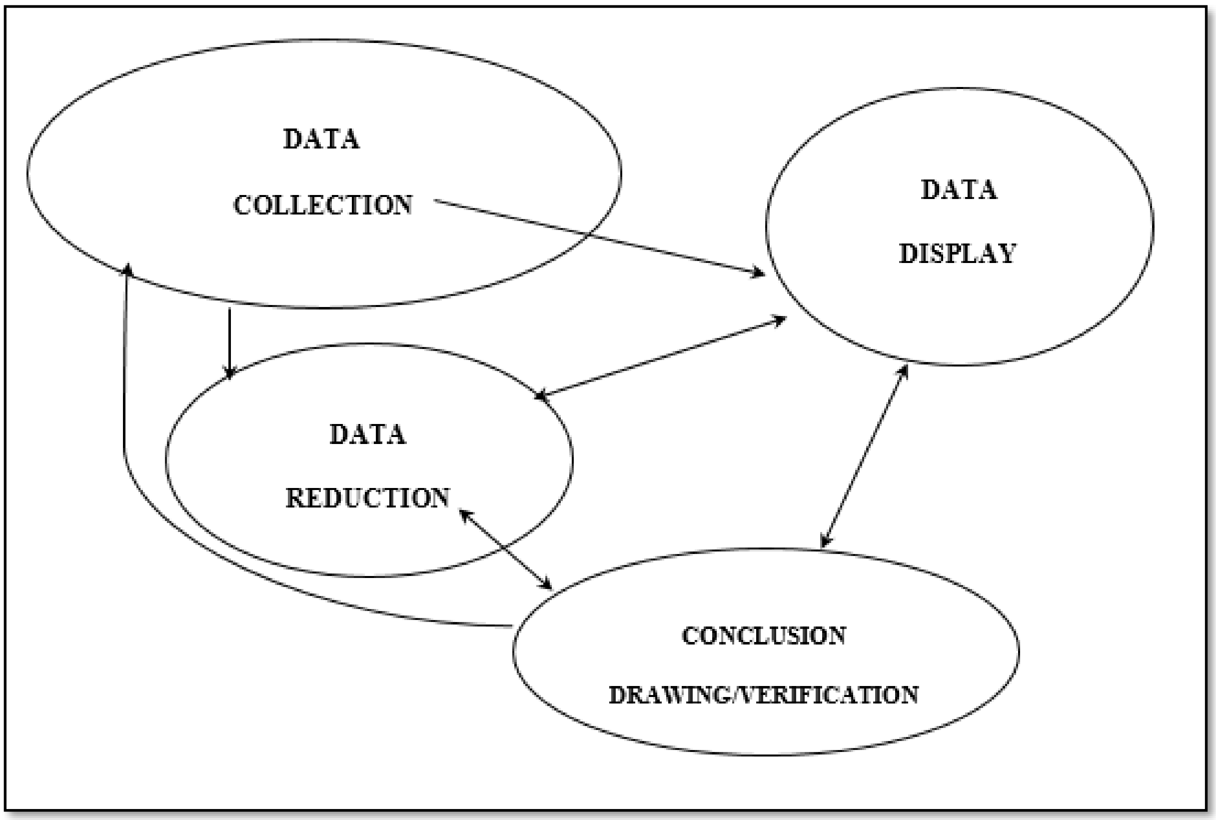


Figure 3.2 Data analysis processes. Source: Miles and Huberman (1994:12)

The researcher adapted Miles and Huberman's (1994) three processes: data reduction, data display and conclusion drawing as three phases to analyse the data of this study. He reduced data through constant comparison analysis and classical content analysis. He discussed the systematic order in which data was analysed and displayed the data on tables and graphs. He finally interpreted and drew conclusions from the findings. The next subsections presents a detailed discussion of phase

1: data reduction and phase 2: data display of the data analysis process. The researcher discusses Phase 3, conclusion drawing, in Chapter 5.

3.5.1 Phase 1: Data reduction

This research adopted an integration of constant comparison analysis and classical content analysis to analyse the collected data.

3.5.1.1 Constant comparison analysis

Researchers (Miles & Huberman, 1994, Leech & Onwuegbuzie, 2007) refer to a constant comparison analysis approach as ‘coding’ and point out that it is utilised to analyse all collected data by identifying underlying themes. They (Leech and Onwuegbuzie, 2007:565) suggest the following steps; the researcher must:

Read through all existing data,

(i) Break apart or ‘chunk’ the data into smaller, meaningful parts
(ii) Label each part with an appropriate title or code
(iii) Continuously compare each new piece of data with previous codes
(iv) Label similar parts with the same code, and
(v) Group similar codes as overarching catergories.

The researcher was aware that qualitative data analysis, like constant comparison analysis, actually begins concurrently with collecting data and not at the end of the data collection process, (Terre Blanche et al, 2006, Bazeley, 2013). He, therefore, read the data from the assessment test, carefully listened to each recorded interview repeatedly and transcribed each interview immediately after completion. He read and re-read each transcribed interview, immediately after transcription. This exercise especially assisted him in refining the semi-structured interviews during the process of data collection. He simultaneously began to familiarise himself with the collected data by interrogating and scrutinising it for relevant patterns. Thus, he adhered to Glaser and Laudel's (2013:7) suggestion that the first step in analysis is to identify and locate relevant data from the large amount of data collected. The relevance of data depended on whether it related to the research topic, research questions, the literature reviewed and new meanings that emerged. The researcher divided up relevant data using different coloured highlighter pens. He then compared and indexed the highlighted portions of relevant data based on the pre-existing codes which he had developed and new codes that unfolded. Finally, he grouped similar codes and individual codes that frequently manifested themselves, into categories.

Coding, in this context, denotes indexing raw data through keywords, phrases, mnemonics or numbers in an attempt to extract salient data, (Glaser and Laudel, 2013:11). In this study, keywords and phrases were used to code the data. Leech and Onwuegbuzie, (2007:565) suggest that researchers approach constant comparison analysis in the following ways:

- Inductively, through developing codes that emerge from the data, commonly referred to as 'open coding', a term coined by Strauss and Corbin (1990) and
- Deductively, by identifying codes prior to data collection and data analysis; these are pre-set codes, an approach which Strauss and Corbin (1990) named axial coding.

Glaser and Laudel (2013:12) state, "It is impossible to conduct an analysis without prior assumptions". The researcher adopted an integration of open and axial coding approaches to analyse data for this research. He implemented Miles and Huberman's (1994:68), suggestion of preparing a 'start list' of pre-set codes that surfaced from the theoretical framework, the questions of the research and the literature reviewed. The following words and phrases were amongst some of the assumed codes that were pre-set:

- Conceptual difficulty
- Procedural difficulty

- Lack of procedural flexibility
- Difficulties experienced
- Overcoming difficulties.

These codes served as a starting point for the data analysis. They also assisted the researcher to stay focused on the phenomenon on hand. They were not fixed or exclusive, but were continuously refined while other new meanings from the data were included in them.

Fram (2013:20) argues that constant comparison analysis is a difficult method but motivates novice researchers to use such analysis in the early stages of research for the following purposes**:**

- To identify patterns in data
- To organise large amounts of data for the abstraction of categories
- To systematically organise and reduce data and
- Utilise theoretical frameworks as tools to compare, confirm and identify data.

Constant comparison analysis, like all other qualitative data analysis methods, is inadequate when one requires absolute accuracy of categories. To circumvent this lack of fidelity, researchers such as Merriam (1998) and Creswell (2007) suggest member checking, as a process of ascertaining internal validity, whereby participants in the research are allowed to verify the accuracy of the categories generated. In this study, the researcher requested the lecturers, the HOD, and the Umalusi Moderator to assess the accuracy of the generated catergories from the assessment test, the interviews, and the literature reviewed.

3.5.1.2 Classical content analysis

Leech and Onwuegbuzie (2007:569) emphasise that in classical content analysis, “the researcher counts the number of times each code is utilized”. Counting the number of occurrences of codes or categories that correlate to each other is the simplest type of evaluation of analysis, (Titscher et al, 2000:57). In this study, the researcher counted the frequency of various codes that emerged from the written assessment and through the constant comparison analysis conducted on the interviews. Leech and Onwuegbuzie (2007:569) claim that the strength of classical content analysis lies in its ability to assist the researcher to focus the study by identifying the codes most used, which may serve as the most important codes of the study. In this regard, the researcher was able to determine what conceptual and procedural difficulties and other codes were most frequent in and important to this study, (Leech and Onwuegbuzie, 2007:569).

3.5.2 Phase 2: Data display

It has been pointed out that "Data analysis is a continuous systematic search for meaning from collected data", (Hatch, 2002:148). This systematic search involves ongoing display, organisation, and comparison of the most important data for further analysis, (Miles and Huberman, 1994). In this section, the researcher describes the order used and the data tools designed and implemented to conduct this research analysis. Table 3.7 reflects the order in which data was analysed:

Table 3.7: Order of analysis

	Focus of analysis	**Data analysed**	**Method/Tool of analysis**
(i)	Student's performance authenticating the research: Are students experiencing conceptual and procedural difficulties when factorising?	Students' written responses, answer scripts of participants	Analysis: spreadsheet and graphs
(ii)	Student responses	Students' written responses: answer scripts of participants	In-depth, question-by-question, scrutiny of the types of conceptual and procedural difficulties and their frequency:

			Tabulated with examples of actual answers
(iii)	Student interviews	Transcripts	Keywords and phrases, open and axial coding and their frequency, Tabulated with examples of actual responses
(iv)	Lecturer, HOD, Umalusi Moderator interviews	Transcripts	Keywords and phrases, open and axial coding and their frequency, Tabulated with examples of actual responses

(i) Analysis of assessment test – The researcher conducted an analysis of the students' performance in the assessment test. He used the marking memorandum attached as Appendix E to mark each student's script and allocated marks to each question consistently for method and accuracy. The assessment was marked out of a total of fifty. He awarded part marks for incorrect answers that reflected a correct step or response.

The researcher used MS Excel to create a spreadsheet, column charts and pie charts to reflect the students' performance. This diagnostic analysis served to authenticate the research. It determines the extent to which students experienced conceptual and procedural difficulties while factorising and solving problems that require factorisation.

(ii) Analysis of student responses – The researcher conducted an in-depth, question-by-question, scrutiny of the types of conceptual and procedural difficulties experienced. He coded the types of difficulties and displayed the codes, actual examples of student's responses and their frequency on a classical content analysis tool adopted from Leech and Onwuegbuzie (2007:570). Table 3.8 reflects the design of this analysis tool:

Table 3.8: Tool for results of conceptual and procedural difficulties from the answer scripts

Question number/s and type	Actual examples of difficulty	Frequency of code

Conceptual difficulties and procedural difficulties experienced were analysed separately on similar tools.

(iii) Analysing the transcripts of interviews – The researcher analysed all the interviews, question by question, by chunking relevant statements and coding them into keywords or phrases that emerged:

a. Student interviews – Analysis from the assessment tests clearly revealed procedural difficulties. However, conceptual and other difficulties, such as 'the student did not understand the question' were assumed. Consequently, the researcher conducted student interviews to attempt to understand the 'mind processes' of the student while factorising and solving problems that require factorisation, confirm conceptual and other assumed difficulties experienced, and to sensitise himself regarding the phenomenon experienced from the perspective of the students.
b. Lecturers, HOD, Umalusi Moderator interviews – The researcher conducted interviews with the Lecturers, HOD, and Umalusi Moderator to gain their insights as to why NCV Level 4 students experience conceptual and procedural difficulties while factorising or solving problems involving factorisation, other difficulties that they and the students are experiencing and possible ways to overcome these difficulties.

The researcher chunked and coded the interview transcripts of the students and lecturers separately, question by question, utilising a constant comparison analysis tool, which he designed, (Leech and Onwuegbuzie, 2007:566). Table 3.9 and Table 3.10 show how the researcher adapted the tool for analysing participants in two groups:

Table 3.9: Codes from student interviews (for populated table refer to Appendix Q)

Chunks and codes from student interview transcripts								
Questions	**What does the words factorisation, expression etc. mean to you?**	**Codes:** **Keywords/ phrases**	**Tell me how you went about answering Q1.1...1.2….**	**Codes:** **Keywords/ phrases**	**Can you see a similarity in Section A, Q1.1 and Section B, Q2?**	**Codes:** **Keywords/ phrases**	**What are some of the challenges that you are facing as a NVC Level 4 student?**	**Codes:** **Keywords/** **phrases**
Student #1								
Student #2								
Student #3								
Student #4								
Student #5								

Table 3.10: Codes from Lecturers, HOD and Umalusi Moderator interviews (for populated table refer to Appendix R)

Chunks and codes from Lecturers, HOD, and Umalusi Moderator interview transcripts								
Questions	**Why, in your opinion do you think that students demonstrated such**	**Codes:**	**Students could not see that the same expression had to be factorised in**	**Codes:**	**What are some of the challenges that you and NVC level 4 mathematics**	**Codes:**	**What are possible ways in which NCV Level 4 students can**	**Codes:**

	conceptual and/or procedural difficulties?	Keywords/ phrases	Section A: Q1.1 and Section B: Q2. Why?	Keywords/ phrases	students are facing?	Keywords/ phrases	understand factorisation and flexibly use factorisation to solve problems?	Keywords/ phrases
Lecturer #1								
Lecturer #2								
Lecturer #3								
HOD								
Umalusi Moderator								

The researcher, on completion of the coding, determined the frequency of the codes using a classical content analysis tool adopted from Leech and Onwuegbuzie (2007:570). Table 3.11 illustrates the design of the tool:

Table 3.11: Tool for codes from interviews

Keywords and phrases	Frequency of code	Student/Subject matter expert

3.6 Quality Criteria

In this section, the researcher discusses measures that were adopted to ensure trustworthiness. The delimitations, limitations and elimination of bias are also presented.

3.6.1 Trustworthiness

Trustworthiness establishes whether the researcher's account and the participant's responses are accurate and truthful. This authenticates the integrity of a research study. The researcher adopted Guba's and Lincoln's (1982:3) and Anney's (2014: 276) suggestion of utilising four qualitative research trustworthiness criteria of credibility, transferability, dependability and confirmability to validate this research.

Credibility – Credibility, also known as internal validity, establishes the accuracy and plausibility of research findings. The researcher utilised Anney's (2014: 276) credibility strategies to validate the accuracy of this study as follows:

- **Prolonged engagement in the field or research site -** The researcher prolonged time spent on data collection. During each student interview, he did not abruptly stop interviewees and took care in ensuring that students were not led towards a particular viewpoint. The researcher initially asked questions based on the written responses of each student. Thereafter, he asked probing questions based on the student's verbal responses in an attempt to elicit the students' thought patterns. In so doing, he endeavoured to ascertain the reasons why students thought in a particular way. He adopted a similar process while conducting individual semi-structured (in-depth) interviews with the three mathematics lecturers, the Head of Division for mathematics and one Umalusi Moderator.
- **Use of peer debriefing -** The researcher presented his research findings to his colleague who, as already noted, has attained a master's degree as well as to his Supervisor to seek their advice with a view to improving the inquiry findings, (Anney, 2014: 276).
- **Member checking -** The researcher also adopted member checking, after transcribing the interviews allowing the participants to peruse and authenticate the transcripts.

- **Triangulation -** Triangulation affects all criteria of trustworthiness and the researcher discusses it in greater depth in Section 3.15.

Transferability - Transferability, also referred to as external validity, determines the extent to which qualitative results and findings can apply to other contexts and groups outside of the original study, (Anney, 2014: 276). The researcher adopted Bitsch's (2005:85) strategy of "thick descriptions" and "purposive sampling" to facilitate transferability:

- **Thick descriptions -** The researcher provided expansive descriptions on all the research processes, including data collection, analysis, and conclusions. He is confident that these descriptions will help other researchers to reproduce similar studies amongst other participants and settings.
- **Purposive sampling -** The researcher adopted purposive sampling for this study. This allowed him to sample relevant participants who are experiencing the phenomenon and/or are knowledgeable in the field of inquiry. The researcher also had the flexibility to determine which categories to include or exclude. Other researchers could easily replicate such non-random sampling.

Dependability - Dependability and reliability are synonymous, as indicated. Dependability refers to "the stability of findings over time", Bitsch (2005:86). Joppe (2000:1) and Sharma (2013:52) define reliability as the degree of consistency of results over time and the accurate representation of the study population. The researcher ensured dependability through the following ways:

- He kept an audit trail: He kept a well-documented audit trail of the entire research process. He filed a hardcopy of all raw data from the written test and the interviews. He also captured the data through scanned answer scripts, voice recording, and transcriptions.
- The researcher attached the relevant audit trail as appendices in this dissertation. Such measures authenticate the dependability of this research.

Confirmability – Confirmability establishes that findings are not the concocted views of the researcher but are objective findings derived from the data, (Tobin and Begley, 2004:392). The researcher also ensured confirmability through maintaining a well-documented audit trail and through triangulation.

3.6.1.1 Triangulation

Cohen and Manion (1986:254) define triangulation as the researcher's attempt to understand "the richness and complexity of human behaviour" within a phenomenon, from different standpoints. The purpose of triangulation is to reduce biasedness and verify the integrity of a participant's responses, (Anney, 2014: 276). However, triangulation also enhances the different dimensions of the phenomenon. Denzin (1978) and Onwuegbuzie and Leech (2007:239) identify four types of triangulation, namely triangulation of data, of investigators, of theories, and of methods.

This research sought trustworthiness through triangulation of data and triangulation of methods. Data triangulation involves the use of different sources, the participants described earlier. The researcher compared the inputs drawn from all participants, during analysis, to compare areas of agreement and disagreement. The triangulation of methods uses two or more strategies to collect data in a single study, (Sharma, 2013:53). The researcher did so by gathering data through a written assessment from which students' answer scripts served as documents, and through conducting of semi-structured interviews with the main role players of this study.

3.6.2 Delimitations

As intimated, for the purpose of managing the data the researcher, deliberately focussed this research study by limiting it to the following:

- A single case – an individual campus of a college,
- A single topic in mathematics – factorisation and problems involving factorisation, and
- A single level – NCV Level 4 students.

The researcher was also interested in understanding how an NCV Level 4 student constructs meaning while factorising irrespective of his/her age, gender, or race group. He removed this biographical information from all instruments for the reasons stated below:

- Age – all students were adults, their ages being those of a grade 12 student and higher,
- Gender – distinguishing the mind processes in mathematics according to males and females is biased and discriminatory,
- Race – all students at the campus of this research belonged to one race group.

Due to these delimitations, the research findings could not be generalised across all TVET colleges, campuses, and students in South Africa.

3.6.3 Limitations

Qualitative research has numerous restrictions. The researcher attempted to minimise such restrictions as and when they surfaced. Unfortunately, he could not control all restrictions.

In qualitative research, the research is restricted to a relatively small sample. In this case, as noted the sample consisted of 30 students, 3 lecturers, 1 HOD, and 1 Umalusi Moderator. Based on this small sample of 35 participants, the findings of this research could not be generalised beyond the target population. The researcher was aware, that because of the intensity of and the frustrations associated with various difficulties experienced by participants in the TVET sector, their views might be subjective and biased. Such subjectivity might not be an accurate reflection of the entire target population. The researcher found the process of transcribing interviews tedious and time consuming.

3.6.4 Elimination of bias

The researcher attempted to eliminate biasedness by referring to himself in the third person. He also did not disclose the participants' gender by referring to them neutrally, for example, 'student #21' instead of 'he' or 'she'. He also refrained from making assumptions about race and age groups.

3.7 Ethical considerations

Qualitative research is always intrusive; it invades the privacy, and exposes other sensitive information, of research participants. According to Creswell (2014:258) the researcher must, at the outset, acknowledge his/her obligation to respect the rights, needs, values, and desires of all participants, (Creswell, 2014:258).

The researcher employed the following measures to protect the rights of the institutions, the sites, and the participants of this research:

3.7.1 Obtaining permission

The researcher applied for permission, to conduct this study, from DHET; see appendix F. The researcher, with the assistance of his study supervisor, used the designed research instruments and the letter of permission from DHET, (appendix K), to apply for ethical clearance from the College of Education Research Ethics Review Committee to proceed with the research. Subsequently he received permission to conduct the study, see appendix L. He then requested the office of the rector of the TVET College for permission to conduct the study, outlined in appendix G. He also sought permission from the office of the campus manager to conduct this study at the chosen campus; see appendix H. The rector and the campus manager granted permission accordingly; refer to appendices M and N respectively. Gaining access into the institution did not pose any difficulties, because as indicated the researcher is an acting senior lecturer.

The researcher sought the permission of the sample of participants in writing as outlined in appendix I and appendix J. He also ensured that the participants clearly understood the objectives of the research through verbal explanations.

3.7.2 Informed Consent

All voluntary participants were required to sign a consent form. The researcher attached the consent form to each of the letters requesting permission to participate in this research and permission to record interviews, as in appendix I and appendix J. This reassured participants of the authenticity of this research study.

3.7.3 Voluntary participation

All sampled participants were informed, verbally and in writing, that participation was voluntary and that they could exercise their right to participate or not. The reassuring statement, "Your participation in this study is voluntary", appeared on the letters of request, appendix I and appendix J, and on the interview protocols C and D. The transcripts of the interviews also reflect that the lecturer verbally reassured all participants of voluntary participation.

3.7.4 Confidentiality

The researcher respected and protected the participants' privacy with respect to personal rights, as well as the issue of confidentiality and anonymity, at all times. He informed participants of these issues verbally and in writing; refer to appendices C; D; E; F; G; H; I and J. The statement "You are reassured that your identity will be confidential" appeared on the assessment tool, appendix E.

3.7.5 Ensuring no harm to the participants

The researcher informed participants of their right to withdraw from this research process at any time and for any reason whatsoever, which he outlined in appendices C; D; E; F; G; H; J and I.

The researcher reassured all interviewees that the recorded interviews would be strictly confidential. In the analysis of the transcripts and when reference was made to a participant's answers or responses, the researcher used proxies instead of the participant's real name such as 'Student #12' or 'Lecturer #2' etc. In this way, the researcher protected the anonymity and confidentiality of all participants.

The researcher reassured students that the results of the assessment test would be strictly anonymous. On completion of the written assessment, the researcher gave feedback to all participating students on their performance in it and implemented remedial measures for difficulties that that they were experiencing. On completion of this research, findings that may have assisted in enhancing the teaching and learning of solving problems involving factorisation and ways of improving conceptual and procedural knowledge were shared with all lecturers and the Head of Division at the campus at which this research was conducted. The researcher provided DHET, the TVET College, as well as the said campus, with a bound copy of the full research report.

3.8 Conclusion

In this chapter, the researcher discussed the research paradigm and design in detail. He also discussed and substantiated the choice of purposive sampling, the design of all data collection instruments and the data analysis process. The quality measures that augmented trustworthiness and the ethical considerations of this study were also presented.

In the next chapter, the researcher discusses and presents an analysis and interpretation of the data collected from the written test and the interviews.

CHAPTER 4

RESULTS, DISCUSSIONS, AND INTERPRETATION OF FINDINGS

4.1 Introduction

In Chapter 3, the researcher discussed the research methodology adopted for the purposes of this study.

In this chapter, he presents the results, and discusses and interprets the findings of the data collected through the written assessments and the semi-structured interviews. In so doing, he attempted to answer the following main research questions:

- What conceptual and procedural difficulties do NCV Level 4 students demonstrate when factorising and solving problems that involve factorisation?
- Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?
- What are the possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems?

4.2 Presentation of results, discussions, and interpretations

In this section, the researcher discusses and interprets the results of the data extracted from the evidence of the written assessments and the interviews, in an attempt to authenticate this study and answer the research questions of this study. Under each sub-section below, the researcher clearly outlines how data was organised, presents the results on tables and graphs and discusses and interprets the results.

4.2.1 Analysis of written assessment test

In this sub-section, the researcher analyses the data from the results of the students' written assessment test with the objective of authenticating this study. To authenticate this study, the researcher illustrates the 30 students' performances in the assessment test. Student's performances verify that they do in fact experience difficulties while factorising. This section also determines the extent to which students experienced conceptual and procedural difficulties while factorising and solving problems that required factorisation. The researcher analysed Section A: Basic factorisation and Section B: Problems involving factorisation, separately.

4.2.1.1 Analysis of Section A: Basic factorisation

The researcher organised the results of the students' assessment on a spreadsheet, available in Appendix O. This spreadsheet represents a question-by-question analysis of the 30 students' responses for Section A on basic factorisation. Section A contained six questions, which were scored out of a maximum mark of 16. The questions tested the students' ability to:

- Factorise by removing the HCF
- Factorise by grouping
- Factorise the difference of two squares
- Factorise a trinomial where $a < 0$, factorise a trinomial where $a > 0$
- Factorise a third degree polynomial.

Factorisation, as emphasised previously, is prerequisite knowledge for topics such as limits, differentiation, integration, quadratic and cubic functions, trigonometric identities, and trigonometric equations, amongst others, that are in the NCV Level 4 Curriculum. Students in the NCV Level 4 programme are not expected to experience difficulties with embedded knowledge such as factorisation.

The performance level Table 4.1 and the bar chart, Figure 4.1, were created from information extracted from the spreadsheet in Appendix O. Table 4.1 and Figure 4.1 illustrate the performance level of the students, per percentage mark distribution, for Section A:

Table 4.1: SECTION A: Performance level table per percentage mark distribution:

	MARK DISTRIBUTION (PERCENTAGE)									
PERFORMANCE LEVEL	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-100
NUMBER IN RANGE	4	8	3	3	2	5	3	1	1	0

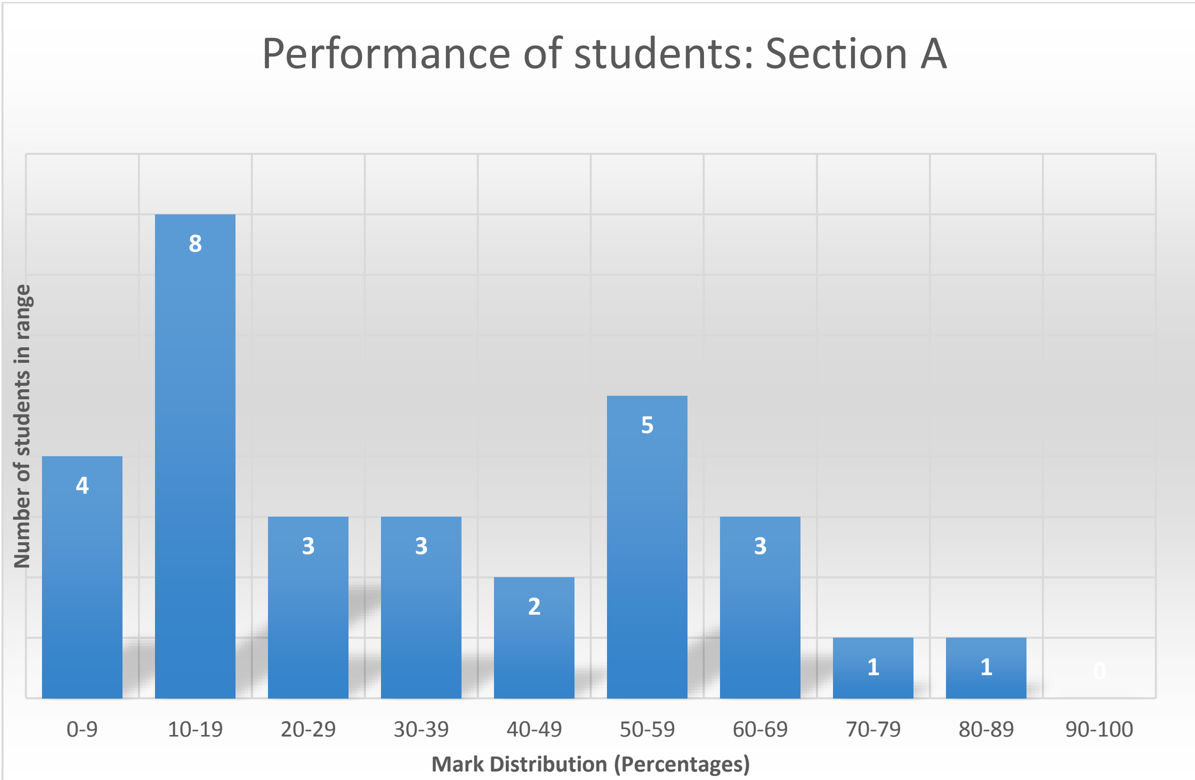


Figure 4.1: SECTION A: Student performance per percentage mark distribution

The results expressed in Table 4.1 and Figure 4.1 indicate that 66,7% of the students obtained a percentage mark below 50% for Section A. Just 6,7% of the students achieved a percentage mark above 69% in this section. This implies that the majority of students encountered difficulty when factorising.

Table 4.2 represents the average mark obtained by the students per question, while Figure 4.2 represents the average percentage mark per question as created from the spreadsheet labelled Appendix O (page 102). Table 4.2 and Figure 4.2 illustrate the extent of the students' difficulty per question by reflecting the average mark per question for Section A:

Table 4.2: SECTION A: Average mark per question

QUESTION	Q1.1	Q1.2	Q1.3	Q1.4	Q.15	Q1.6	TOTAL
MAXIMUM MARK PER QUESTION	2	3	2	2	2	5	16
TOTAL MAXIMUM MARKS POSSIBLE	60	90	60	60	60	150	480
TOTAL MARK/QUESTION	36	40	30	18	35	6	165
AVERAGE MARK	1.2	1.3	1.0	0.6	1.2	0.2	6

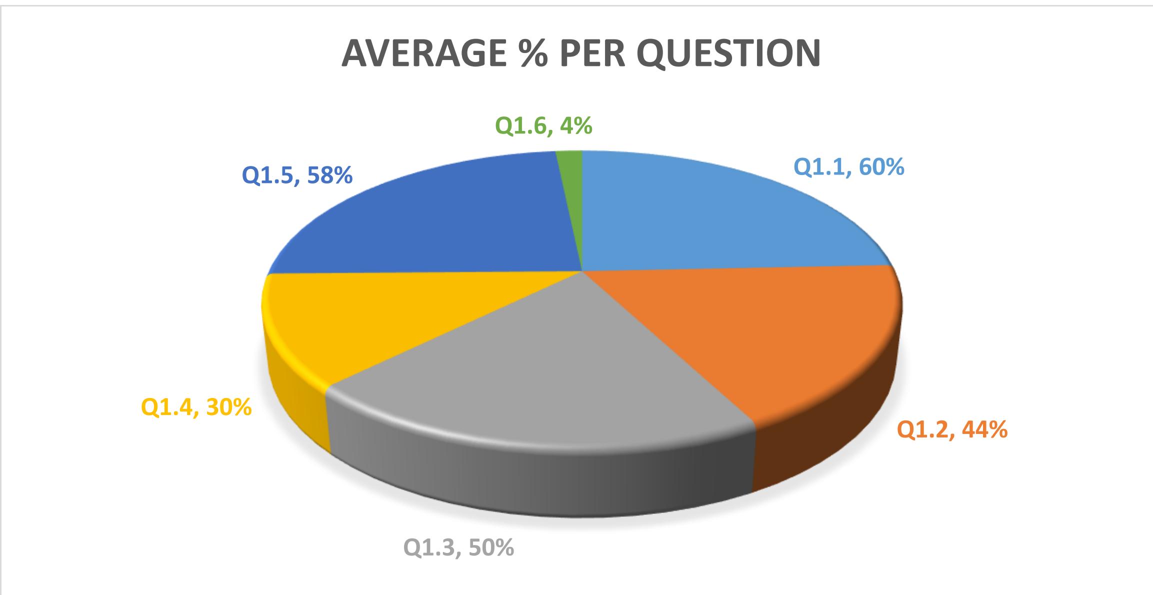


Figure 4.2: SECTION A: Average percentage mark per question

The results expressed in Table 4.2 and Figure 4.2 indicate that the average mark obtained for Section A is 6 out of a possible 16, which is an average of 37, 5%. This average would have been significantly lower if the researcher had not awarded part marks for incorrect responses. NCV L4 students have been engaging with all types of factorisation, except factorising a third degree polynomial, since grade 8, as well as in the NCV level 2 and level 3 programmes. Ideally, when excluding the marks allocated to Q1.6 for factorising a third degree polynomial ($x^3 - 3x + 2$), students in NCV L4 should be performing at an average of 68,75% for section A.

Q1.1 assessed factorisation by removing the highest common factor. The average mark of 1,2 out of a possible 2 obtained for Q1.1, which is an average percentage mark of 60%, is low. Generally, the first type of factorisation that students engage with, in algebra, is factorising a polynomial by removing the highest common factor. It is a foundational concept and procedure that all NCV Level 4 students have studied since grade 8. All NCV Level 4 students are expected to correctly factorise an algebraic expression by removing the HCF.

The average marks for all other questions, which assessed factorisation by grouping, difference of two squares, factorising a trinomial, and factorising a third degree polynomial, were lower than 60%. For example, the average mark for Q1.6, for factorising a third degree polynomial was 4%, which is significantly lower than 60%. This confirms that the NCV Level 4 students who participated in this research experienced conceptual and procedural difficulties with all types of factorisation. This finding confirms the literature reviewed where Papier (2014:38) highlighted that TVET college students lack a strong foundation in mathematics.

4.2.1.2 Analysis of Section B: Problems involving factorisation

The researcher organised the results of the student's assessment on a spreadsheet, found in Appendix P. This spreadsheet represents a question-by-question analysis of the 30 student's responses for Section B in problems involving factorisation. Section B consisted of seven questions and was out of a maximum mark of 34. The questions tested the students' ability to use factorisation to solve problems that fall within the scope of the NCV Level 4 curriculum. Section B tested the students' procedural flexibility as well as their ability to identify and use the different types of factorisation to solve mathematical problems.

The performance level illustrated in both Table 4.3 and the bar chart in Figure 4.3 were created from information found on the spreadsheet in Appendix P. Table 4.3 and Figure 4.2 illustrate the performance level of the students, per percentage mark distribution for Section B:

Table 4.3: SECTION B: Performance level table per percentage mark distribution:

	MARK DISTRIBUTION (PERCENTAGE)									
PERFORMANCE LEVEL	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-100
NUMBER IN RANGE	13	7	4	1	1	1	2	1	0	0

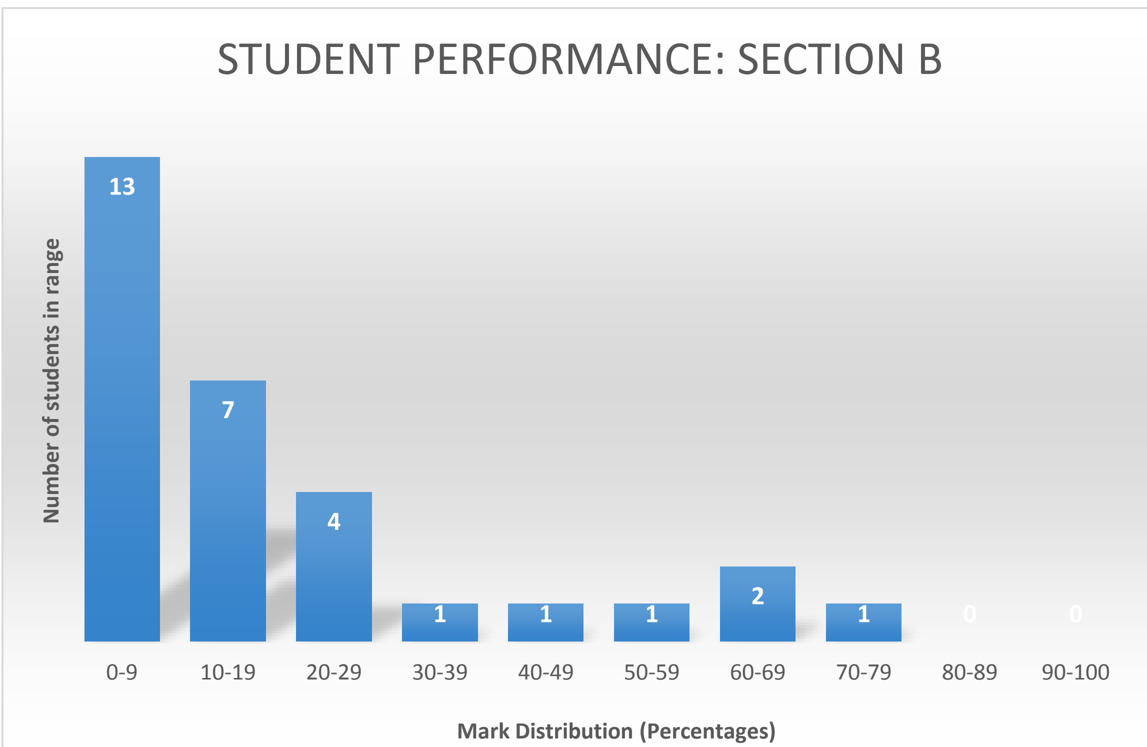


Figure 4.3: SECTION B: Student performance per percentage mark distribution

The results expressed in Table 4.3 and Figure 4.3 indicate that 86,7% of the students obtained a percentage mark below 50%, which is 17 out of a possible 34, for Section B. This substantiates the view that the majority of the NCV Level 4 students who participated in this research were experiencing difficulties with procedural flexibility, the ability to flexibly transfer conceptual and procedural knowledge of factorisation and to solve mathematical problems that involved factorisation.

The average mark per question table, as reported in Table 4.4, and the average percentage mark per question pie chart, as illustrated in Figure 4.4, were created from information found on the spreadsheet in Appendix P. Table 4.4 and Figure 4.2 illustrate the extent of students' difficulty per question by reflecting the average mark per question for Section B:

Table 4.4: SECTION B: Average mark per question

Question	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
Maximum mark per question	3	4	10	5	5	2	5	34
Total maximum marks possible	90	120	300	150	150	60	150	1020
Total mark/question	15	40	45	24	17	22	35	198
Average mark	0.5	1.3	1.5	0.8	0.6	0.7	1.2	7

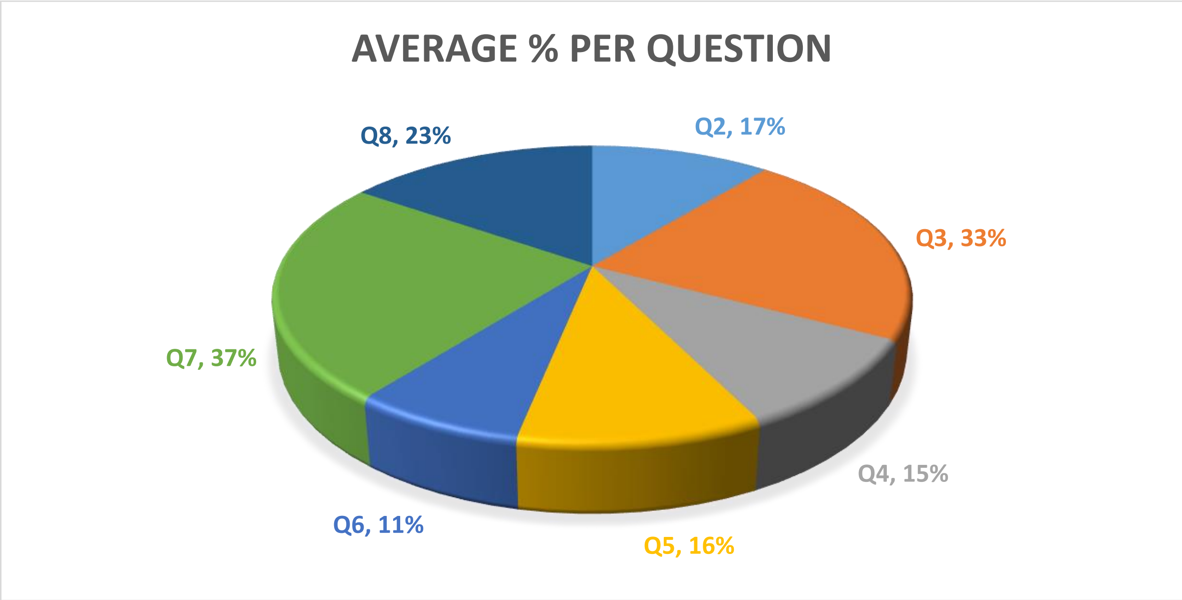


Figure 4.4: SECTION B: Average percentage mark per question

The results, expressed in Table 4.4 and Figure 4.4, revealed the following:

The average mark for Section B was 20,59%. This average would have also been significantly lower if the researcher had not awarded part marks for incorrect responses. This average percentage of 20,59% is significantly lower than the average 37,5% obtained in Section A. This implies that even the students who performed relatively well in Section A, which focussed mainly on factorisation of algebraic equations, performed poorly in Section B, which tested problems that involved factorisation. In addition, the average percentage score of 30% for Q1.4, which, for example, required the students to factorise the trinomial $2x^2 - 7x + 3$ in Section A, did not compare proportionally to an average score performance of 11% in the corresponding Q6, which required the students to find a general solution for $2\sin^2 x - 7\sin x + 3 = 0$ where $0^\circ \leq x \leq 360^\circ$ in section B. This implies that some students who provided a correct response in Q1.4 were unable to identify and factorise the same trinomial in Q1.6. Thus, the results suggest that the NCV students who participated in this study lacked the ability to transfer, relate and implement their understanding of basic factorisation to mathematical problems that required factorisation for their solution.

This analysis discloses that the majority of participating NCV Level 4 students experienced difficulties while factorising trinomials and solving quadratic equations. Most students, even those who excelled in section A on basic factorisation, experienced challenges when they attempted to solve problems that required factorisation for their solutions. This analysis warranted deeper research. Hence, the next sections analysis's the research questions, in an attempt to understand the phenomena.

4.2.2 Analysis of students' written responses, students' verbal responses, and Subject Matter Experts' verbal responses

This section presents data relating to two of the research questions concerning the conceptual and procedural difficulties that NCV Level 4 students demonstrated when factorising and solving problems that involved factorisation and why these students demonstrated such difficulties.

The researcher conducted an in-depth, question-by-question scrutiny of the types of conceptual and procedural difficulties that students experienced in the assessment test.

The researcher also utilised the student interviews to answer the first two research questions. He purposively selected and conducted individual, face-to-face, semi-structured interviews with the NCV Level 4 students, based on their written responses. The researcher chose these students because their responses suggested that they experienced conceptual and procedural difficulties when factorising and solving problems that involve factorisation. In addition, it will be recalled that they were selected for pragmatic reasons as they live in relatively close proximity to the campus.

The researcher interviewed the students in the following order:

- Student #21
- Student #18

- Student #25
- Student #28
- Student #2.

The researcher conducted student interviews to attempt to understand their mind-set when they responded in the assessment test in order to confirm procedural and conceptual difficulties experienced and to sensitise the reader to the challenges that students experienced, which may have contributed to the phenomenon under consideration.

Consequently, the student interviews served to:

- Validate the conceptual and procedural difficulties experienced while factorising
- Consider challenges that students are experiencing which might be factors contributing to these conceptual and procedural difficulties.

The researcher was of the opinion that the phenomenon under examination would have lacked in-depth interrogation and its findings would have been imbalanced if the views of other important adult role players such as the NCV Level 4 lecturers, the manager of the programme and the external Umalusi quality assurer had not also been considered.

For this reason, he conducted individual, face-to-face, semi structured interviews with the other participants to gain their insights, from their level of interaction. In this way, the researcher also hoped to triangulate converging findings and highlight conflicting views. He will refer to these adult participants collectively as Subject Matter Experts from this point forward.

The researcher interviewed the Subject Matter Experts in the following order:

- Lecturer #1
- Lecturer #2
- Lecturer #3
- Head of Division (HOD) and

- Umalusi Moderator (UM).

The researcher, after scrutinising and analysing all the evidence from the assessment test, found that the conceptual and procedural difficulties students experienced were numerous. He then divided up the research question into three subsections to help facilitate the analysis of the students' responses in the assessment test. These included procedural difficulties, conceptual difficulties and the extent of application of factorisation in both sections, to which the researcher referred as procedural flexibility.

4.2.2.1 Procedural difficulties

The literature review defined procedural difficulties as the inability of students to use a procedure correctly or their use of incorrect procedures to solve mathematics problems, (Tularam and Hulsman, 2015:2). Section 4.2.2.1.1 discusses, in detail, the procedural difficulties that were prevalent in the research and why students experienced such difficulties.

The researcher utilised the students' written responses in the assessment test to identify the core categories of procedural difficulties that they experienced in section A, which comprised basic factorisation of simple algebraic expressions. The following broad categories emerged:

- Expressions not fully factorised and expressions incorrectly simplified further
- The incorrect use of the correct type of factorisation
- The use of the incorrect type of factorisation
- The use of an incorrect procedure outside of factorisation.

The researcher presents the frequency of these categories in Table 4.5 below:

Table 4.5: Results of core categories of procedural difficulties from the 30 students' answer scripts of the written assessments, Section A, Basic factorisation:

Codes / **Section A: Basic factorisation:** / **Question number/s and type**	Expression incorrectly simplified further	Use of correct type of factorisation incorrectly	Use of the incorrect factorisation procedure	Use of incorrect procedure outside of factorisation	Did not attempt the question	**Frequency of code N = 30**
1.1 Removing HCF $4x^3 + 4x$	9	5	-	1	1	16
1.2 Grouping $x^3 + 4x^2 + 4x + 16$	4	5	5	4	-	18
1.3 Difference of two squares $a^2 - b^2$	5	10	2	3	-	20
1.4 Trinomial (a>1) $2x^2 - 7x + 3$	-	3	7	10	2	22
1.5 Trinomial (a=1)	10	2	2	2	1	17

$x^2 - 12x + 36$						
1.6 Third degree polynomial $x^3 - 3x + 2$	2	1	18	5	3	29
Frequency of code	30	26	34	25	7	122
Percentage of frequency	**24.6%**	**21.3%**	**27.9%**	**20.5%**	**5.7%**	

Section A assessed basic factorisation and comprised six questions. Thirty students wrote the assessment. This means that 30 possible responses for each of the six questions amounted to 180 responses. The contingency table, Table 4.5, reveals that a total of 122 out of the 180 responses, (68,% of responses), reflect different categories of procedural difficulties experienced by the 30 students. The researcher collated the results in Table 4.5 into Figure 4.5, a vertical bar graph, which indicates the percentage of the frequency of each category of procedural difficulty:

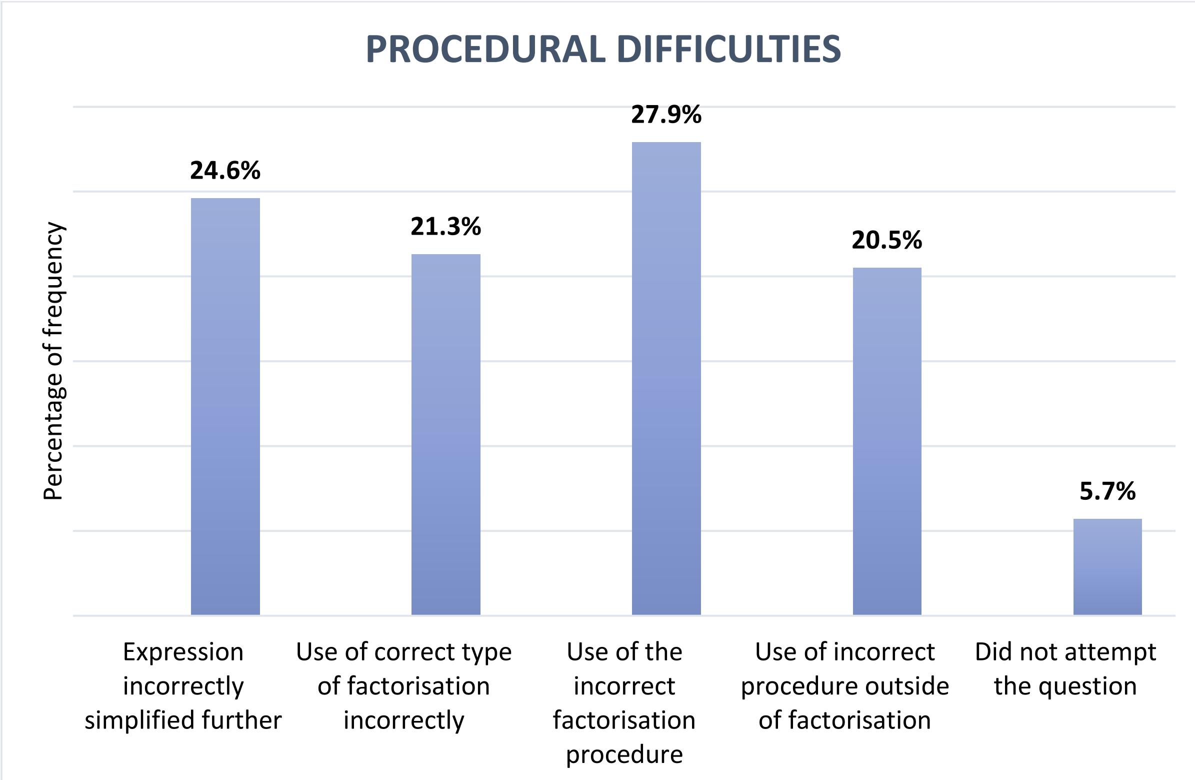


Figure 4.5: SECTION A: Percentage frequency of procedural difficulties

The researcher discusses these categories further in the following subsections:

4.2.2.1.1 Expressions incorrectly simplified further

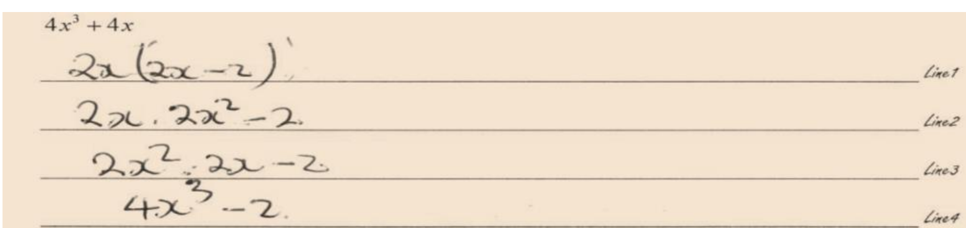
Students provided various responses where they incorrectly simplified a question further after factorising it. The results expressed in Figure 4.5 indicate that 24,6% of students incorrectly simplified a question further after factorising it. For the purposes of this study, most extracted presentations have been labelled in line numbers, line 1, line 2 and line 3, in that order.

Table 4.6, developed from Table 4.10, illustrates the questions that yielded a high percentage of expressions incorrectly simplified further:

Table 4.6 Expressions incorrectly simplified further

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.1	$4x^3 + 4x$	$\frac{9}{16}$	56%
1.2	Grouping: $x^3 + 4x^2 + 4x + 16$	$\frac{4}{18}$	22%
1.3	Difference of two squares $a^2 - b^2$	$\frac{5}{20}$	25%
1.5	Trinomial $x^2 - 12x + 36$	$\frac{10}{17}$	59%

For Q1.1, factorise $4x^3 + 4x$, 56% of the students who answered this question incorrectly, simplified the expression further. For the purposes of this study, the responses of Student #7 and Student #4 for Q1.1, serve as examples of this particular type of incorrect responses:



Extract 4.1: Student #7's response to Q1.1

When the researcher asked Lecturer #2 for reasons why students demonstrated such conceptual and/or procedural difficulties he had this to say:

Lecturer #2: "*Students do not understand the concept of prime factorisation and without removing or without transforming an expression into its prime factor it becomes very difficult to see what's the highest common factor. For an example, if we had an expression* $8x^2$ and $12x$ *you know the student cannot see what is the highest common factor because it is not in prime factors.*"

In line 1, Student # 7 did not remove the highest common factor; instead, he/she removed $2x$ as a common factor and changed the positive sign to negative in the second factor. This indicates that the student did not understand working with integers. In line 2 to 4, Student #7 proceeded to determine the product of the factorised expression, even carrying out that procedure incorrectly.

When the researcher asked three students about the meaning of integers, their responses were:

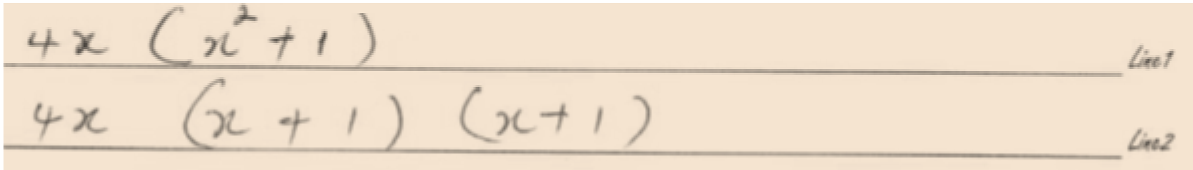
Student #21: "*Just numbers, sir"*

Student #18: "*Integers, not sure about integers, but we never learnt here at college level. I'm not sure about integers.*"

Student #25: "*I think if that number have the exponent for…"*

From the written response of Student #7, the researcher concludes that this student did not understand the concept factorisation. He/she applied the procedure of removing HCF, albeit incorrectly, and then multiplied the factored term incorrectly; in so doing he/she experienced other conceptual weaknesses, especially with the multiplication of integers and exponents. The verbal responses of other students also suggest that they did not understand the concept factorisation.

Meanwhile, student #4 responded as follows:



Extract 4.2: Student #4's response to Q1.1

Student #4 factorised correctly; then incorrectly factorised further. This student did not understand the concept of the difference of two squares. Hence, he/she used an incorrect procedure by attempting to factorise the sum of two squares further.

For Q1.2, Factorise fully $x^3+4x^2+4x+16$, 22% of the students who answered this question incorrectly managed to group and remove the highest common factor, but did not complete factorising. The response of Student #2, for Q1.2 serves as an example of this particular type of incorrect response:

Extract 4.3: Student #2's response to Q1.2

In addition, when the researcher asked Student #2 what he/she understood about factorisation, the student responded:

Student #2: "*It means to break it into pieces until it. When you able to get back when you get back to original one"*

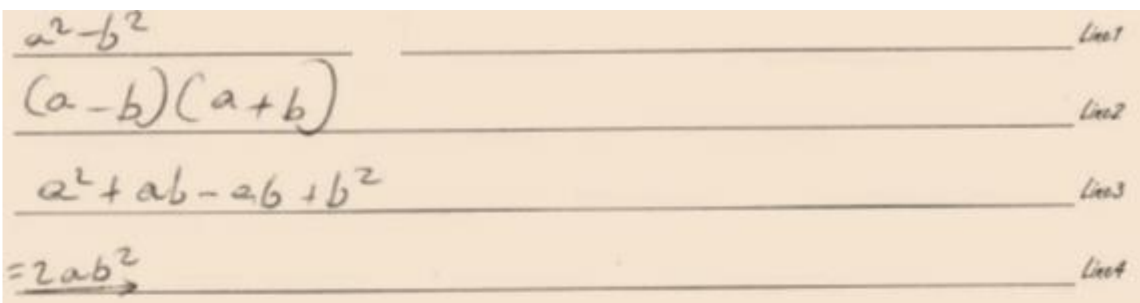
Although Student #2 knew the procedure for grouping (*to break it into pieces)* in his/her terms, together with the procedure for removing HCF, he/she did not show evidence of understanding the concept of factorisation.

About the meaning of multiplication, this student had this to say:

Student #2: "*Ay I know. It is when you multiply. For example if you get one and one and multiply it and get 30 and 50 and multiply it. You need to get more number"*

This student possessed a superficial understanding of multiplication: that when multiplying there is an increase. Also he/she had a superficial understanding of the relationship between multiplication and factorisation; as that what was broken down through multiplication, could return to the original problem using factorisation. This is probably what he/she meant by "*you able to get back when you get back to original one".* Nonetheless, this student did not understand that by grouping and removing the HCF, one is actually reducing four terms to two.Hence, he/she could not identify $(x+4)$ as the highest common factor in line2. This student did not understand that by grouping and removing the HCF, one is actually reducing four terms to one term.

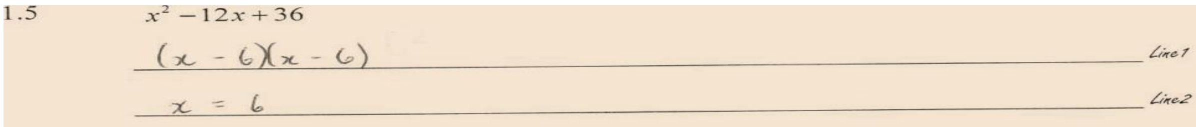
For Question 1.3, Factorise fully $a^2 - b^2$, 25% of the students identified the correct factors but went on to simplify their answers further. The response of Student #12, for Q1.3, serves as an example of this particular type of incorrect response:



Extract 4.4: Student #12's response to Q1.3

This student factorised correctly, in line 2, but proceeded to use the distributive law to find the product and then added unlike terms incorrectly. This response indicates that he/she did not understand the concepts factorisation, products, and like and unlike terms. Subsequently, Student #12 conveniently and mechanically used procedures that came to his/her mind.

For Question 1.5, Factorise fully $x^2 - 12x + 36$, 59% of the students went on to simplify their answers incorrectly. The response of Student #8, for Q1.5 serves as an example of this particular type of incorrect response:



Extract 4.5: Student #8's response to Q1.3

This student factorised correctly, in line 1, but proceeded to solve the expression as if it was a quadratic equation.

4.2.2.1.2 The use of the correct type of factorisation procedure incorrectly

The results expressed in Figure 4.5 indicate that 21,3% of the incorrect responses were those of students who utilised the correct type of factorisation procedure incorrectly.

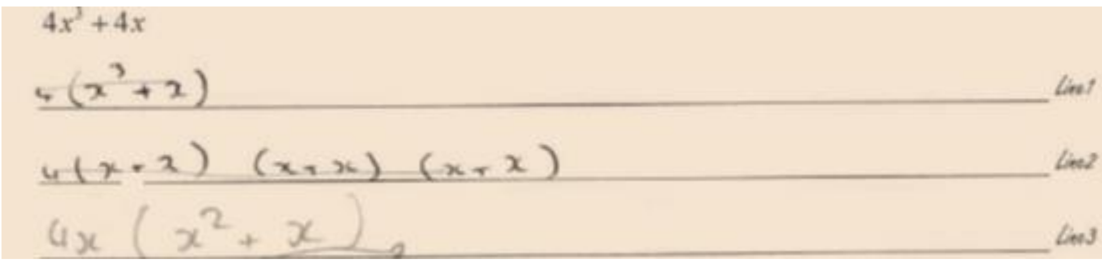
Table 4.7, developed from Table 4.5, illustrates the questions that yielded a high percentage of students who chose the correct type of factorisation procedure but executed the procedure incorrectly:

Table 4.7: The use of the correct type of factorisation procedure incorrectly

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong factorisation procedure
1.1	$4x^3 + 4x$	$\frac{5}{16}$	31%
1.2	Grouping: $x^3 + 4x^2 + 4x + 16$	$\frac{5}{18}$	28%

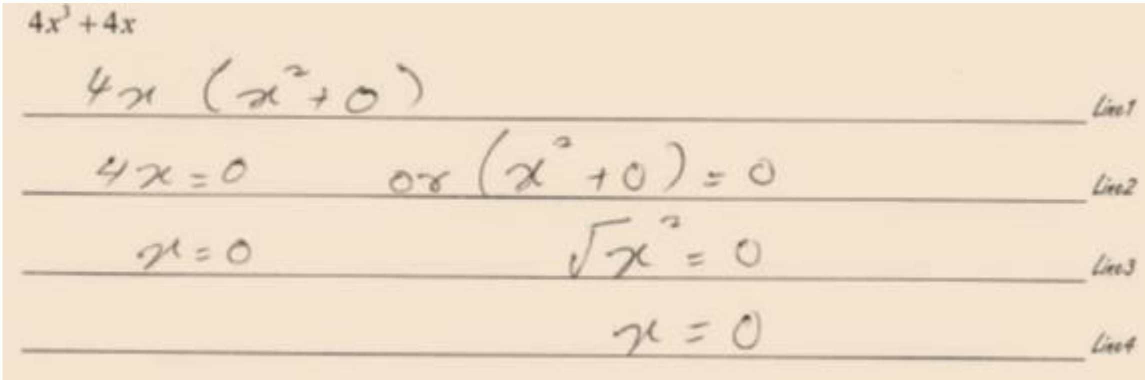
1.3	Difference of two squares $a^2 - b^2$	$\frac{10}{20}$	50%

Thirty one percent of students who could not provide correct responses for Q1.1 used the correct procedure but made errors while trying to execute it. For example, Extracts 4.6 and 4.7 indicate responses given by Student #1 and Student #22 respectively, which reflected such mistakes:



Extract 4.6: Student #1's response to Q1.1

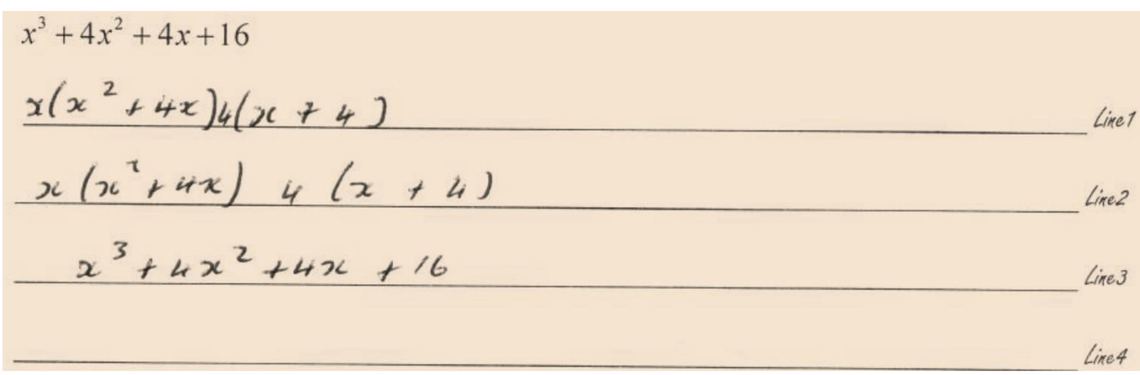
This student responded by first removing the common factor 4 in line 1. He/she then assumed that $(x^3 + x)$ was the sum of two cubes and attempted to factorise it in line 2. In line 4, the student then struck off his/her first attempt, subsequently removed the correct HCF but failed to identify the correct second factor. He/she also indicated a difficulty in understanding the prerequisite concept of division of algebraic fractions. The student assumed that $\frac{4x}{4x} = x$.



Extract 4.7: Student #22's response to Q1.1

In line 1, Student #22 removed the HCF correctly but identified the second factor incorrectly. He/she also demonstrated difficulty with the prerequisite concept of division of exponents by assuming that $\frac{4x}{4x} = 0$. In line 2 to 4, student #22 attempted to solve the expression as if it was a quadratic equation. This response also displayed his/her lack of understanding of the concepts of algebraic expressions and quadratic equations.

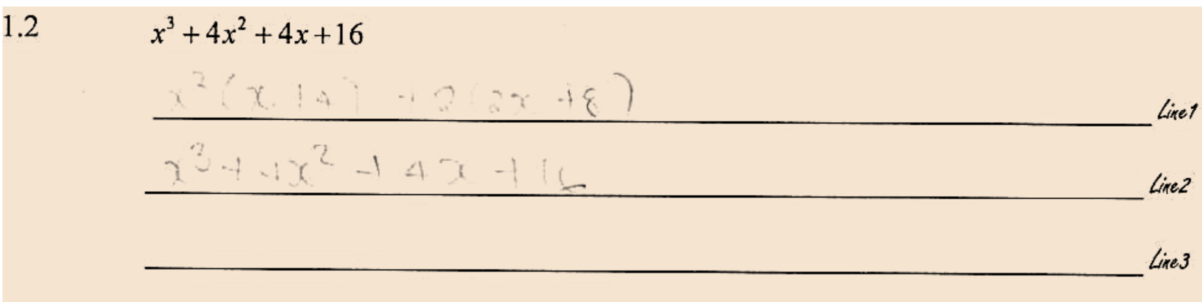
Twenty-eight percent (28%) of the students who responded incorrectly to Q1.2 also used the appropriate procedure, but did so incorrectly. Student #19's response reveals such mistakes:



Extract 4.8: Student #19's response to Q1.2

This student attempted to group and factorise by removing the HCF in line 1. Student #19 managed to remove the HCF, but by omitting the plus sign between the grouped terms in line 1 and line 2 the student reduced four terms to one term incorrectly. This indicates that this person did not understand the reason as to why grouping was utilised. He/she proceeded to find the product of the factored terms in line 3.

Student #28 responded:



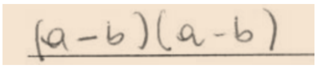
Extract 4.9: Student #28's response to Q1.2

In addition, when the researcher asked Student #28 what he/she understood about factorisation, the student responded:

Student #28: *"no I don't know. But I can… factorising I think you have to separate and it will give you back to this."*

Student #28 did not understand the concept of factorisation. Therefore, based on his/her understanding of the procedure of grouping, he/she grouped the first two terms and removed the correct HCF. Then this student grouped the second two terms but removed the incorrect HCF. In line two, he/she multiplied the factorised terms and arrived at an answer. This is probably what he/she meant by "*factorising I think you have to separate and it will give you back to this*". Although Student #28 had an idea of the procedure to use to factorise by grouping, his/her lack of conceptual understanding of factorisation caused him/her to use the correct procedure incorrectly. Thus, Student #19 and Student #28 experienced challenges in correctly executing the procedure of factorising by grouping.

Students were required to factorise $a^2 - b^2$ in Question 1.3. Results indicated that 50% of students who obtained the wrong answer for Q1.3 used the correct procedure, but confused the signs. Student #9 had the answer:



Extract 4.10: Student #9's response to Q1.3

Student #7 responded as follows:

Extract 4.11: Student #7's response to Q1.3

These examples reaffirm that most students seemed to recognise the correct procedures needed for factorisation but lacked proficiency in executing these, probably because they did not understand the concept of factorisation.

4.2.2.1.3 The use of the incorrect type of factorisation

Students provided various responses that differed in the type of incorrect factorisation used in different questions that required them to undertake basic factorisation of algebraic expression. The results expressed in Figure 4.5 indicated that 27,9% of incorrect responses may be attributed to the use of the wrong factorisation procedure.

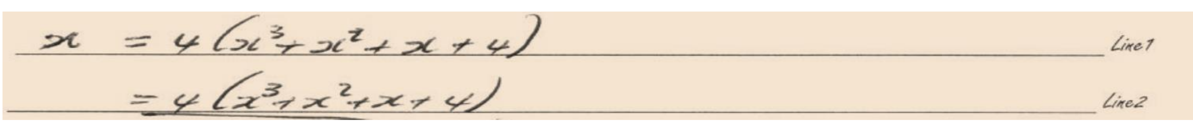
Table 4.8, developed from Table 4.5, illustrates the questions that yielded a high percentage of wrong factorisation procedures used:

Table 4.8: High percentage of wrong factorisation procedures used

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.2	Grouping: $x^3+4x^2+4x+16$	$\frac{5}{18}$	28%
1.4	Trinomial: $2x^2-7x+3$, where a > 1	$\frac{7}{22}$	32%
1.6	Third degree polynomial: x^3-3x+2	$\frac{18}{29}$	62%

The responses of Student #3, Student #20, Student #21 and Student #18 serve as examples of this particular type of incorrect responses:

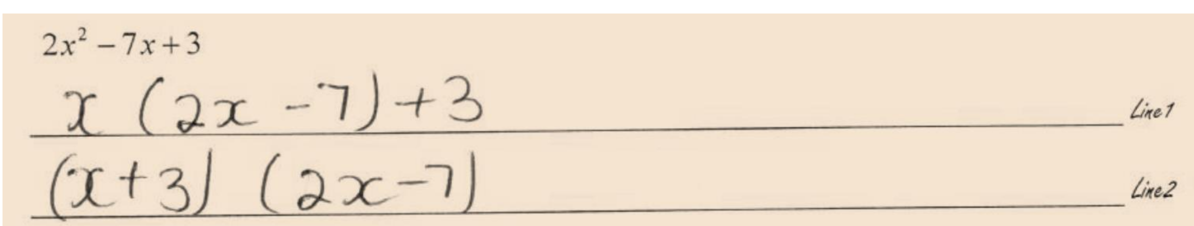
Student #3, Q1.2, factorising by grouping:



Extract 4.12: Student #3's response to Q1.2

This student did not consider grouping, but incorrectly removed a common factor.

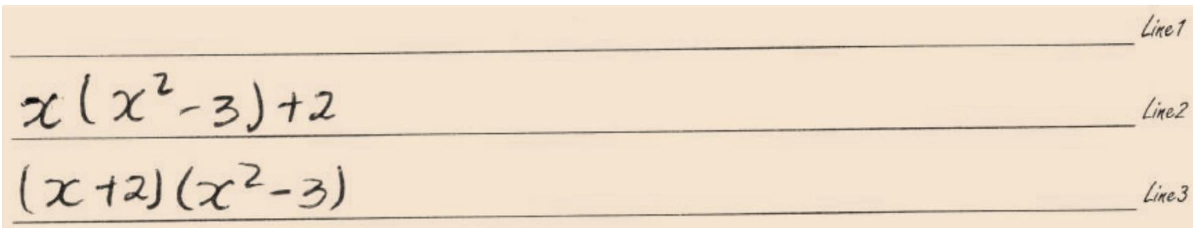
Student #20, factorising $2x^2-7x+3$:



Extract 4.1.3: Student #20's response to Q1.4

This student factorised the trinomial by incorrectly grouping the first two terms and removing the HCF in line 1. He/she proceeded to group the two terms incorrectly.

Student #21, Q1.6, factorising the trinomial $x^3 - 3x + 2$:



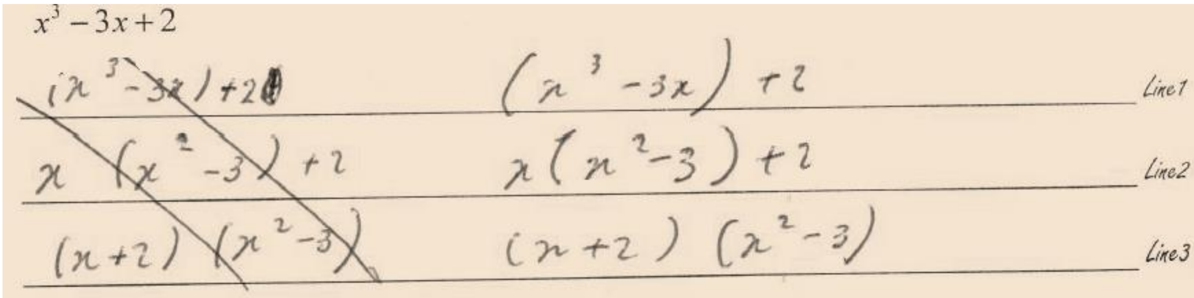
Extract 4.14: Student #21's response to Q1.6

When the researcher asked Student #21 what he/she understood about factorisation, the student responded:

Student #21: "*I think it is the grouping of an equation, in order to get one equation, to reverse it back. Factorisation, I think, is a grouping of equations.*"

Student #21 understood factorisation to be grouping. Based on his/her understanding of the factorisation (*grouping of equations*) in his/her words, this student attempted to factorise the polynomial by grouping and removing the HCF for the first two terms in line 2. The person proceeded, in line 3, to factorise the resulting two terms by grouping, although incorrectly.

Student #18 also experienced a similar difficulty with Q1.6, factorising the trinomial $x^3 - 3x + 2$:



Extract 4.15: Student #18's response to Q1.6

Also, when the researcher asked Student #18 what was understood about factorisation, the student responded:

Student #18: "*By my understanding when we are talking about factorisation it's whereby we are given an equation not equal to equal to zero, given an equation whereby we are taking all the common factors separating into in two factors, like two brackets whereby getting the common factors. It's a group, factorisation it's a group, yeah, and a common factor.*"

Based on this understanding of factorisation, (*It's a group, factorisation it's a group, yeah, and a common factor*), in the student's own words, he/she grouped the first two terms of the third degree polynomial in line 1. Then he/she proceeded to remove a HCF in line 2. After this, he/she attempted to group terms again in line 3.

When the researcher asked the Umalusi Moderator for reasons why students demonstrated such conceptual and/or procedural difficulties, he remarked:

Umalusi Moderator: "*It boils down to the initial teaching of these concepts. I mean, you can go right down to primary school. The point is if a student does not know the meaning of factorisation, what is the meaning, outcome is someone asks you to factorise, what must the outcome look like? And that is the first problem.*"

When the researcher asked the HOD for reasons why students demonstrated such conceptual and/or procedural difficulties, his response was:

"*In factorisation as a whole if we teach the learner the concept of removal of common factor, then what the learner does is, we are giving him an activity that is based on removal of common factor only. So immediately, he knows he has to check for removal of common factor and he solves. Let's say he doesn't make the other errors of equating to zero, then if I go on to the next one, let's just say is grouping, then the student knows the activity is based on grouping so his mind is focused that he'll only be doing grouping. The same with difference of two squares. Now the problem comes up when we combine all three together and I ask him to factorise. Now question papers do not tell you factorise by the difference of two squares. So what the learner does, the learner cannot go ahead and identify what type of factorisation he needs to do. So that's one of the problems, he learnt a procedure but he cannot conceptualise now which procedure to actually follow when and he makes the errors.*"

Based on the responses, the researcher concluded that the students lacked an understanding of the concept factorisation; therefore, they experienced challenges in choosing and executing the correct factorisation procedure while attempting to factorise.

4.2.2.1.4 The use of incorrect procedures outside of factorisation

Students provided various responses that reflected incorrect procedures outside of factorisation, which they used in different questions that required them to perform basic factorisation of algebraic expression. The results expressed in Figure 4.5 indicate that 20,5% are procedures that fall outside of factorisation.

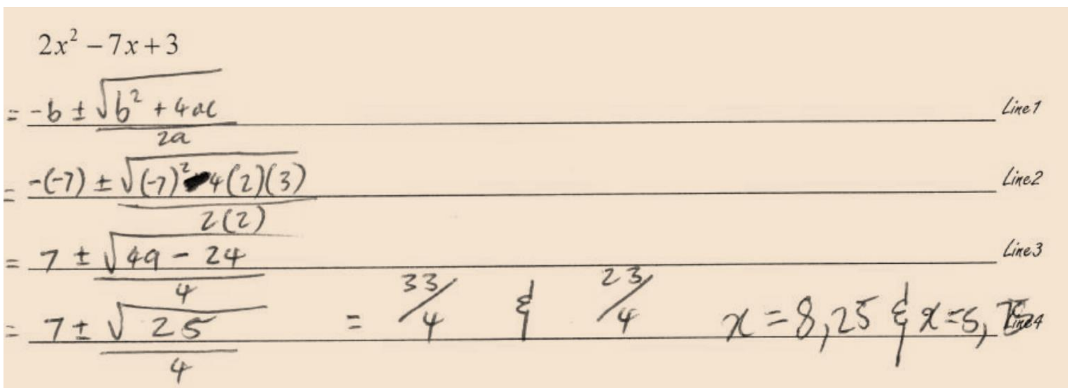
Table 4.9, developed from Table 4.5, illustrates the questions that yielded a high percentage of incorrect procedures outside of factorisation:

Table 4.9: High percentage of incorrect procedure outside of factorisation

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.4	Trinomial: $2x^2 - 7x + 3$, where a > 1	$\frac{10}{22}$	20,5%

For the purposes of this study, the responses of Student #17, Student #6 and Student #8 serve as examples of this particular type of incorrect responses:

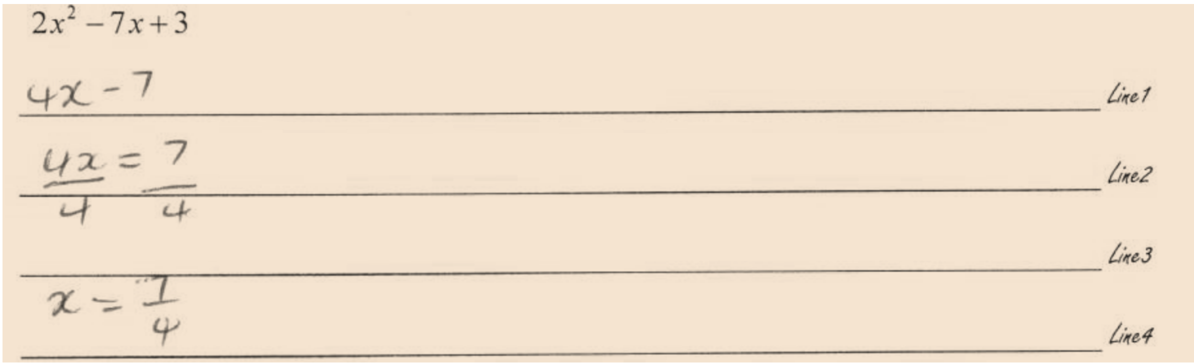
Student #17 saw the trinomial as a quadratic equation and attempted to solve it:



Extract 4.16: Student #17's response to Q1.4

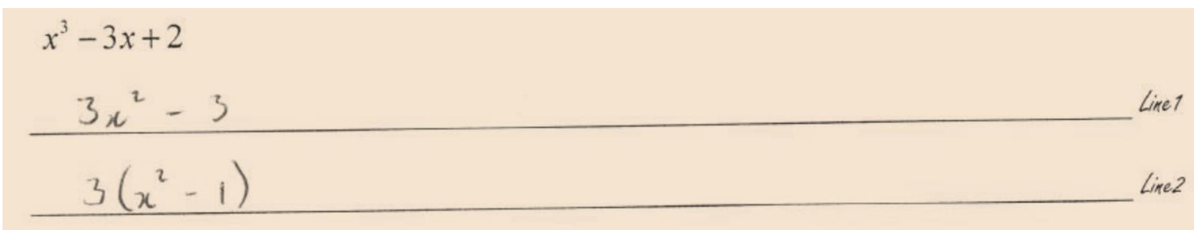
This student incorrectly used the quadratic formula to solve for e expression instead of factorising it.

Student #6 differentiated the trinomial in line one and then attempted to solve the expression as if it was an equation:



Extract 4.17: Student #6's response to Q1.4

Here again, Student #6's response suggests that he/she executed a procedure that came to his/her mind. Student #8 also used the differentiation rule in line one and then proceeded to remove the HCF:



Extract 4.18: Student #8's response to Q1.4

Lecturer #3 and the Umalusi Moderator also had this to say about why students experienced conceptual and procedural difficulties while factorising:

Lecturer 3: "*When it comes to basic rules as well, learners fail to understand there are sequences which we have been taught to follow when we speak about factorisation and the first one that we speak about is always to find or remove the HCF… we will find that they tend to forget these simple rules we are speaking about.*"

Umalusi Moderator: "*You do this for a trinomial, you solve by this method, four-termed expression, you group up, twos and twos or sometimes 1 and 2 to get difference of squares, you can teach that* *as recipes then that's all they will learn and they will forget it, forget what they actually want to achieve.*"

The topic that students were studying at the time of the assessment in this research was differentiation. Most students who operated outside of factorisation used the procedure of differentiating by rules. This is probably what the Lecturer #3 meant by "*they tend to forget these simple rules we are speaking about*" and what the Umalusi Moderator meant by "*you can teach that as recipes then that's all they will learn and they will forget it, forget what they actually want to achieve.*"

This finding is consistent with the literature that although students can memorise formulas and steps to follow in familiar, well-defined mathematical problems, they seldom appear to make sense of how to apply these formulas to new situations, (Garfield and Ahlgren, 1988: 46).

4.2.2.2 Conceptual Difficulties

The literature review defined conceptual difficulties as the inability of students to understand critical features of a new concept and generalise key facts, concepts, strategies, and procedures to other contexts. (MacMath et al, 2009; Garfield and Ahlgren 1988; Ball, Lubienski, and Mewborn, 2001; Kilpatrick et al, 2001). Section 4.2.2.2.1 discusses, in detail, the conceptual difficulties that were prevalent in the research.

The researcher utilised the students' written responses in the assessment test to identify the core categories of conceptual difficulties that they experienced in Section A, which comprised basic factorisation of simple algebraic expressions. The following broad categories emerged:

- Confuses products and factorisation
- Cannot differentiate between expressions and equations
- Applies a procedure to suit private understandings of concepts

- Multiple misconceptions
- Cannot identify the concept.

The frequencies of these categories are presented in Table 4.10, below:

Table 4.10: Results of conceptual difficulties from the 30 students' answer scripts of the assessments

Codes **Section A: Basic factorisation:** **Question number/s and type**	Confuses products and factorisation	Cannot distinguish: expression and equation	Applies a procedure to suit correct and incorrect private understandings of concepts	Multiple misconception	Cannot identify the concept	**Frequency of code n=30**
1.1 Removing HCF $4x^3+4x$	3	8	2	1	-	14
1.2 Grouping $x^3+4x^2+4x+16$	3	6	-	4	-	13
1.3 Difference of two squares	9	2	1	1	1	14

$a^2 - b^2$						
1.4 Trinomial (a>1) $2x^2 - 7x + 3$	1	10	-	2	5	18
1.5 Trinomial (a=1) $x^2 - 12x + 36$	3	10	-	1	2	16
1.6 Third degree polynomial $x^3 - 3x + 2$	1	3	2	4	13	23
Frequency of code	20	39	5	13	21	98
Percentage of frequency	**20.4%**	**39.8%**	**5.1%**	**13.3%**	**21.4%**	

Section B assessed problems that required factorisation for solution and comprised six questions. Thirty students wrote the assessment. This means that 30 possible responses for each of the six questions amount to 180 responses. The contingency table, Table 4.10, revealed that a total of 98 out of the 180 responses, 54% of responses, reflect different categories of conceptual difficulties experienced by the 30 students. The researcher collated the results in Table 4.10 into a vertical bar graph presented in Figure 4.6, which indicates the percentage of the frequency of each category of conceptual difficulty:

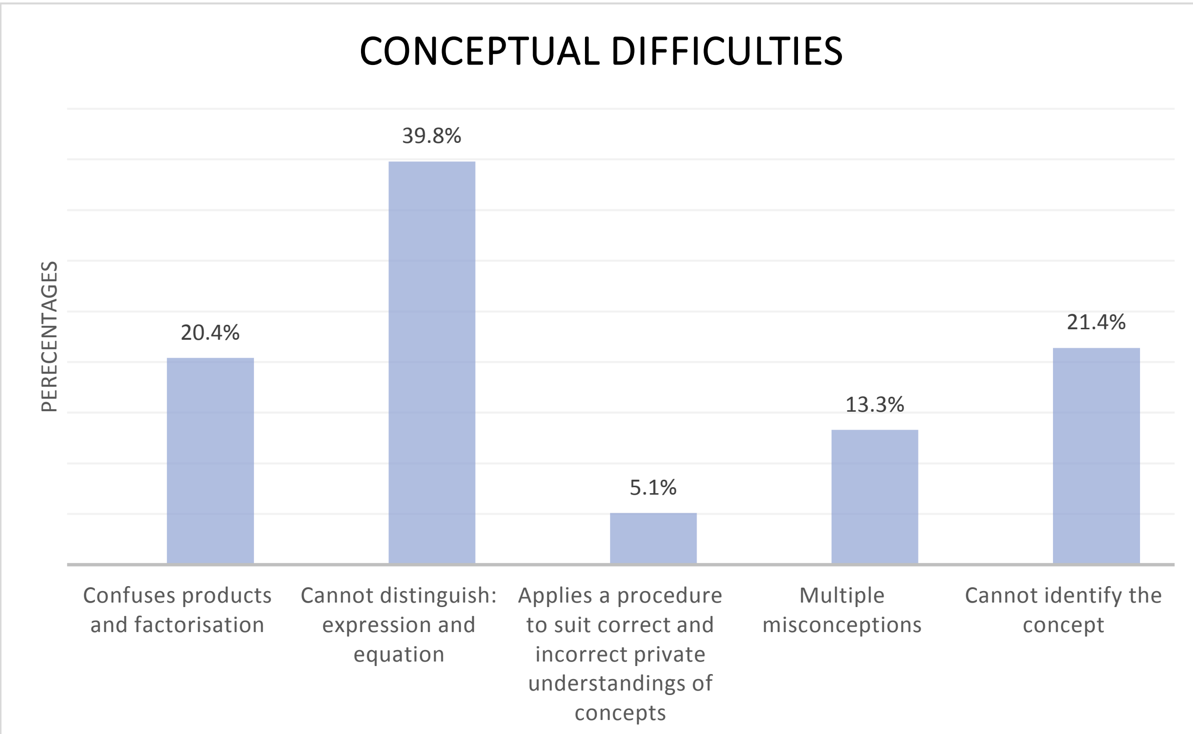


Figure 4.6: SECTION A: Percentage frequency of conceptual difficulties

The researcher discusses these categories more fully in the following subsections:

4.2.2.2.1 Products and factorisation

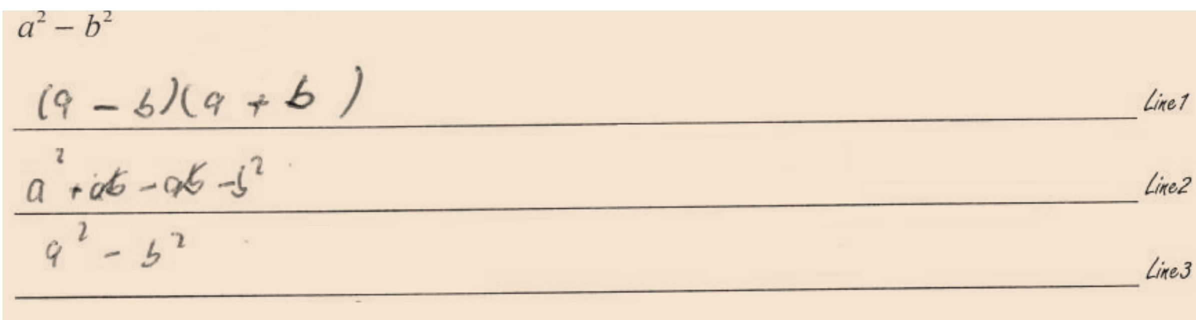
Students provided various responses where they confused factorisation and products. The results expressed in Figure 4.6 indicate that 20,4% of students confused these. These students were able to use procedures correctly, but did not understand the concepts.

Table 4.11, developed from Table 4.10, illustrates the questions that yielded a high percentage of confusion between factorisation and products:

Table 4.11 Confused products and factorisation

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.3	Difference of two squares $a^2 - b^2$	$\frac{9}{14}$	64%

For the purposes of this study, the responses of Student #18 and Student #28 serve as examples of this particular type of incorrect conceptual response:

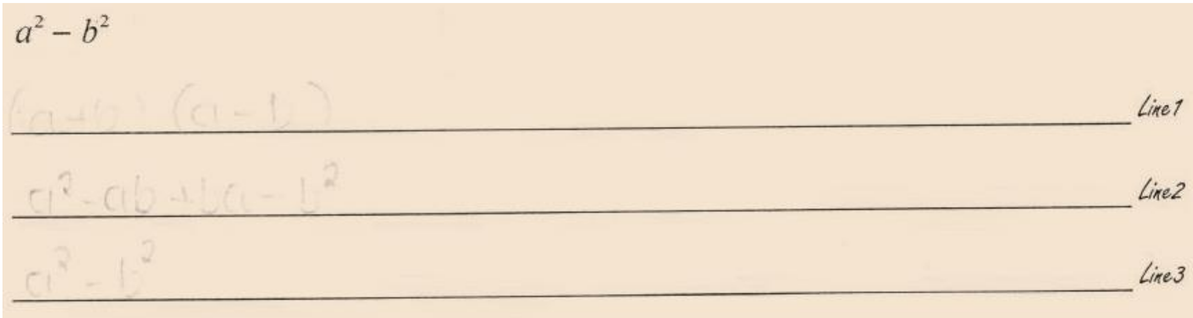


Extract 4.19: Student #18's response to Q1.3

When the researcher asked Student #18 about the meaning of multiplication, he responded:

Student #18: "*Multiplication is whereby we are given an equation with the sign of times. It's whereby we are dealing with multiplication.*"

Student #28's response for Q1.3 was:


$a^2 - b^2$

$(a+b)(a-b)$ Line1

$a^2 - ab + ba - b^2$ Line2

$a^2 - b^2$ Line3

Extract 4.20: Student #28's response to Q1.3

Student #28 was asked to explain what the 'difference of two squares' meant. This student had this to say:

"*Difference they are not the same. The base are not the same and the powers are the same.*"

These students did not understand the concept of the difference of two squares. Additionally, from the response of #28, "*Difference they are not the same*", the researcher concluded that this particular student identified just the literal meaning of the word 'difference' and not the mathematical meaning of it.

Generally, students check whether they factorised correctly by multiplying their answer. This check does not form part of the answer. Student #18 and Student #28 included the check as part of the answer. These students factorised the difference of two squares in line 1, then realised that they could also find the product of two binomials and proceeded to do that in line 2. Although they mastered the procedures of factorising and multiplying, these students lacked an understanding of the concepts factorisation and products.

Other NCV L4 students interviewed also responded incorrectly to the meaning of multiplication:

Student #21: "*is to times things in order to get it big, times things by something*"

Student #25: "*It means to make... the number is going to increase*"

When the latter student was probed on this question: "Does it get bigger or does it get smaller?"

He/she said “*it get bigger*”. On further probing: ‘Can it get smaller when you multiply?” the student responded: “*No never*”

All the students’ responses reveal that their understanding of multiplication was at a very superficial level. This level of understanding was not appropriate for a NCV Level 4 student who engaged with multiplication of algebraic expressions, multiplication of algebraic fractions and multiplication of complex numbers. In addition, as mentioned, a deep understanding of multiplication is the main prerequisite concept that leads to the understanding of factorisation and its applications.

4.2.2.2.2 Distinguishing between expressions and equations

The results expressed in Figure 4.9 indicates that 39,8% of the students’ incorrect conceptual responses reflected their inability to distinguish between expressions and equations. This constitutes the most frequent conceptual difficulty experienced.

Table 4.12, developed from Table 4.10, illustrates the questions that yielded a high percentage of confusion between factorisation and products:

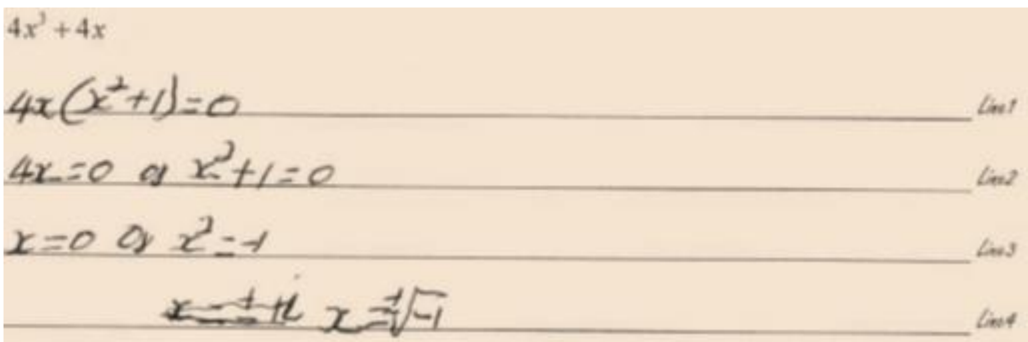
Table 4.12 Distinguishing between expressions and equations

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.1	$4x^3 + 4x$	$\frac{8}{14}$	57%
1.2	Grouping $x^3 + 4x^2 + 4x + 16$	$\frac{6}{13}$	46%

1.4	Trinomial (a>1) $2x^2 - 7x + 3$	$\frac{10}{18}$	56%
1.5	Trinomial $x^2 - 12x + 36$	$\frac{10}{16}$	63%

For the purposes of this study, the responses of Student #13, Student #25, Student #2 and Student #9, serve as examples of this particular type of incorrect conceptual responses:

The response of Student #13 reflected this conceptual difficulty in Q 1.1:


$4x^3 + 4x$

$4x(x^2+1)=0$ Line1

$4x=0$ or $x^2+1=0$ Line2

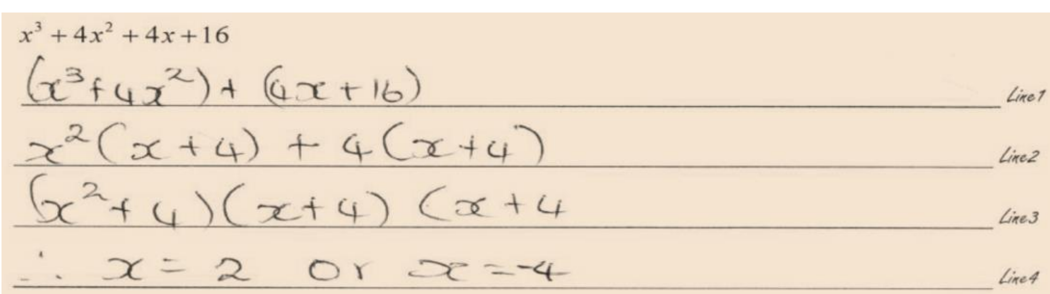
$x=0$ or $x^2=-1$ Line3

$x=\pm 1i$ $x=\pm\sqrt{-1}$ Line4

Extract 4.21: Student #13's response to Q1.1

Student #13 was able to factorise Q1.1 correctly, but equated the expression to zero and solved it as an equation. This student's responses clearly show that he/she saw expressions as equations.

The response of Student #25 reflected this conceptual difficulty in Q 1.2:


$x^3 + 4x^2 + 4x + 16$

$(x^3+4x^2)+(4x+16)$ Line1

$x^2(x+4)+4(x+4)$ Line2

$(x^2+4)(x+4)(x+4$ Line3

$\therefore x = 2$ or $x=-4$ Line4

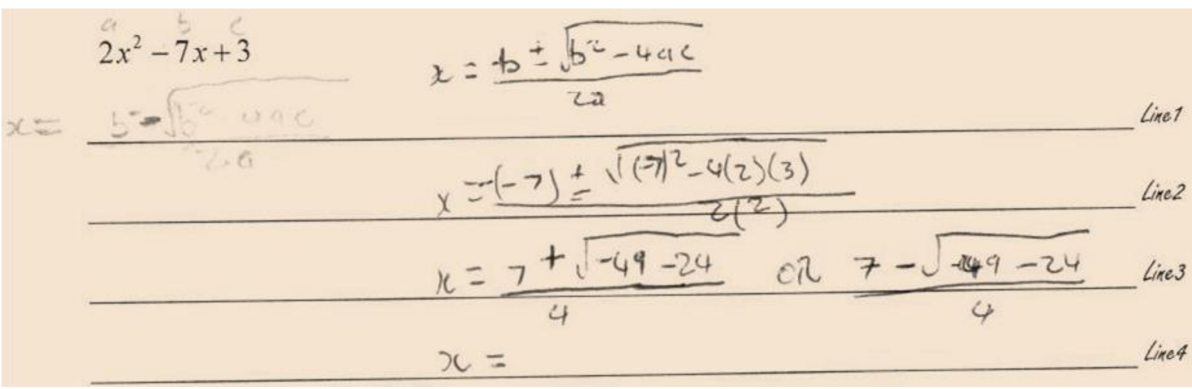
Extract 4.22: Student #25's response to Q1.2

When the researcher asked Student #25 to define expressions and equations, they responded:

Student #25: "*Yes sir it's the equation is that means you must solve the mathematics*"

This student's factorised answer is incorrect in Line 3, but he/she was able to use the procedure of grouping to factorise the expression in lines 1 and 2 correctly. Although Student #25 could factorise by grouping, his/her misunderstanding of expressions as equations prompted him/her to solve the expression as if it was an equation. This is probably what he/she meant by "*means you must solve the mathematics*"

The response of Student #2 reflected this conceptual difficulty in Q 1.4:



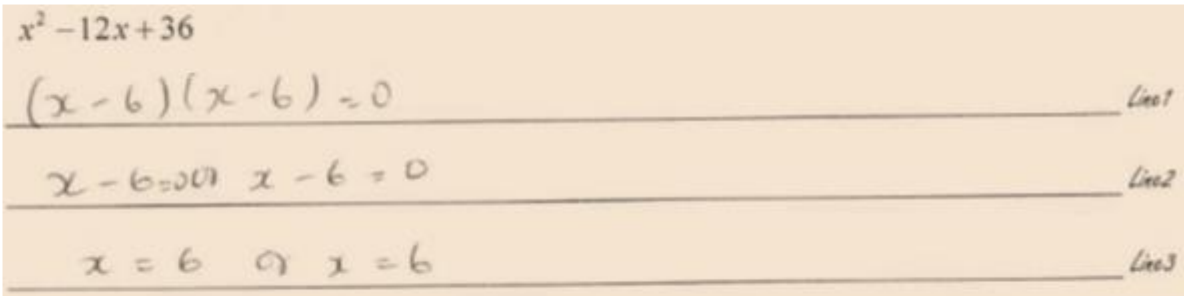
Extract 4.23: Student #2's response to Q1.4

When the researcher asked Student #2 to define expressions and equations, he/she responded:

Student #2: "*I don't know sir. Eish! I'm not sure what I can put"*

This student did not attempt to factorise the expression in Q1.4; instead he/she utilised the quadratic formula to solve the expression as if it was an equation. This student's response indicates he/she did not understand the concepts of expressions and equations.

Student #9 demonstrated the same conceptual difficulty in Q1.5:



Extract 4.24: Student #9's response to Q1.5

Student #9 factorised the trinomial and knew the procedure for how to solve a quadratic equation but also perceived the expression as an equation.

Other NCV L4 students interviewed also responded incorrectly to the question on defining expressions and equations:

Student #21: "*…expression must have a factor there…equation have equal to 0. I think so …"*

Student #28: *"Expression... Eish I don't know…equation is when is equal to zero at the end, yes when it's equal to zero I think it is an equation"*

For equations, the responses of Student # 21 and Student #28 suggest that these students had a superficial understanding of equations in that they were only applicable when equated to zero. Student #25's definition of equations indicates that he/she lacked the ability to express him/herself mathematically in English. The assessment test demonstrates that most students had difficulties in differentiating between an equation and an expression.

When the researcher asked Lecturer #3 for reasons why students demonstrated such conceptual and/or procedural difficulties, he/she remarked:

Lecturer #3: *"When learners are given a mathematical statement, many of them are unable to see, firstly if we are speaking of an equation, or if we are speaking of an expression."*

The responses confirm that NCV L4 students' lack of conceptual understanding of expressions and equations contributed to their difficulties as experienced in the test.

4.2.2.2.3 Applied a procedure to suit private understandings of concepts

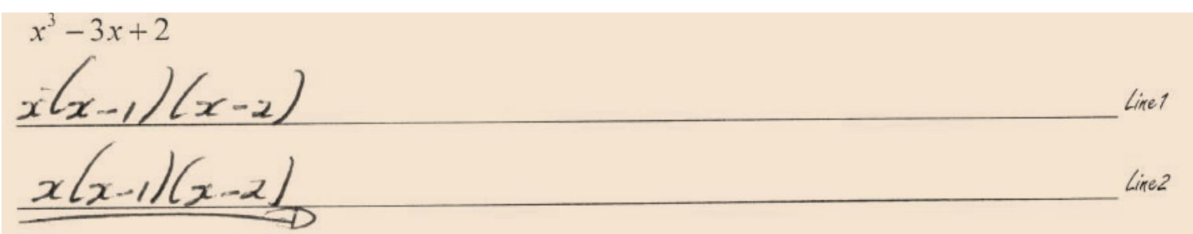
Students provided various responses where they applied a procedure to suit their own private understandings of concepts. The results expressed in Figure 4.6 indicate that five student's responses suggest such private understandings. Two out of these five students answered Q1.6 incorrectly.

Table 4.13, developed from Table 4.10, illustrates these two students out of the 23 students that answered Question 1.6 incorrectly used their own private understandings:

Table 4.13 Private understandings of concepts

Question Number	Question Type: Factorise	Number of incorrect responses that show private understandings	% attributed to wrong Factorisation procedure
1.6	Third degree polynomial $x^3 - 3x + 2$	$\frac{2}{23}$	9%

The response of Student #3 serves as an example of this particular type of incorrect conceptual response:



Extract 4.25: Student #3's response to Q1.6

From Student #3's response, it is evident that he/she recognised that x^3 posed a problem. This student incorrectly removed x as a HCF and factorised the resulting trinomial correctly. Thus, the researcher concludes that this student used his/her own private understanding to manipulate the expression to suit the procedures that he/she was comfortable in employing. This response supports the views in the literature review: that misconceptions are concepts which do not agree with traditional understandings of algebra and that they often arise from students' private understandings of certain concepts and procedures, (Allen, 2007:1).

4.2.2.2.4 Multiple misconceptions

The results expressed in Figure 4.6 indicate that 13,3% of the students' incorrect conceptual responses suggest that students experienced more than one misconception. The researcher refers to such cases as multiple misconceptions.

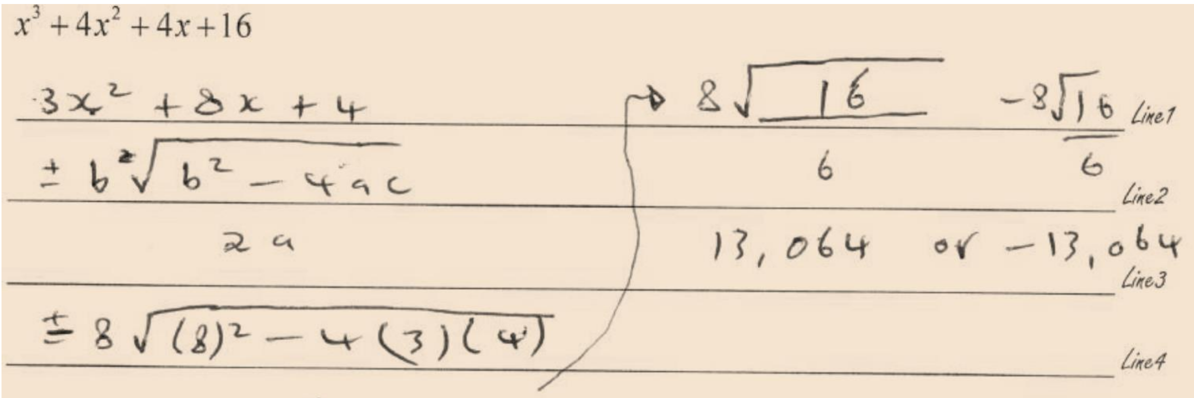
Table 4.14, developed from Table 4.10, illustrates the percentage of students who had incorrect answers for Question 1.2 and Question 1.6 because of multiple misconceptions:

Table 4.14 multiple misconceptions

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong factorisation procedure
1.2	Grouping: $x^3+4x^2+4x+16$	$\frac{4}{13}$	31%
1.6	Trinomial (a>1) $2x^2-7x+3$	$\frac{4}{23}$	17%

The responses of Student #15 and Student #16, serve as examples of this particular type of incorrect conceptual responses for Q1.2 and Q1.6:

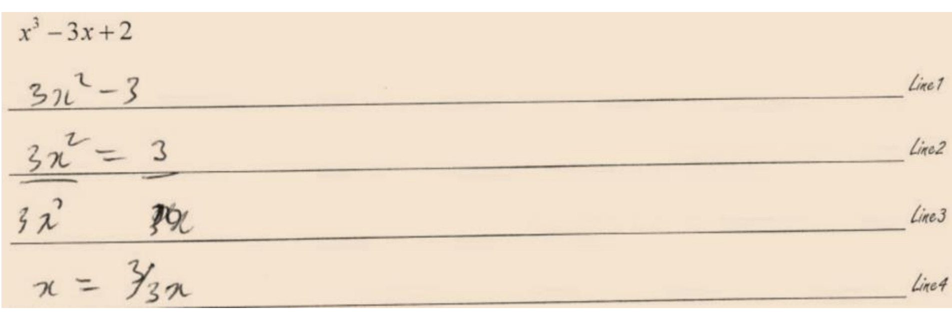
Student #15 displayed multiple misconceptions in Q1.2:



Extract 4.26: Student #15's response to Q1.2

This student differentiated the expression from rules in line 1, then attempted to utilise the quadratic formula to solve the expression as if it was an equation in line 2. He/he did not attempt to factorise the expression.

Student #16 also exhibits multiple conceptual difficulties in Q1.6:



Extract 4.27: Student #16's response to Q1.6

Student #16 did not attempt to factorise. He/she differentiated the expression in line 1, and then attempted to solve the expression, as if it was an equation. He/she further incorrectly divided the

LHS of the equation by $3x^2$ and the RHS by $3x$ in line 2, and finally obtained an answer for x that contained x in line 4. Student #6 executed many procedures but had no conceptual grasp of these so he/she used them at will.

The responses of Student # 15 and Student #16 confirm the literature review: that misconceptions are the misapplications of rules, (Allen, 2007:1). The responses also support, Allen's (2007:1) view in the literature that misconceptions are concepts which do not agree with traditional understandings of algebra and that they often arise from students' private understandings of certain concepts and procedures.

4.2.2.2.5 Identifying the concept

The results expressed in Figure 4.9 reveal that 21,4% of the students could not identify the concept.

Table 4.15, developed from Table 4.10, illustrates the question that yielded a high percentage of responses where students could not identify the concept:

Table 4.15 Students could not identify the concept

Question Number	Question Type: Factorise	Number of incorrect responses	% attributed to wrong Factorisation procedure
1.6	Trinomial (a>1) $2x^2-7x+3$	$\frac{13}{23}$	57%

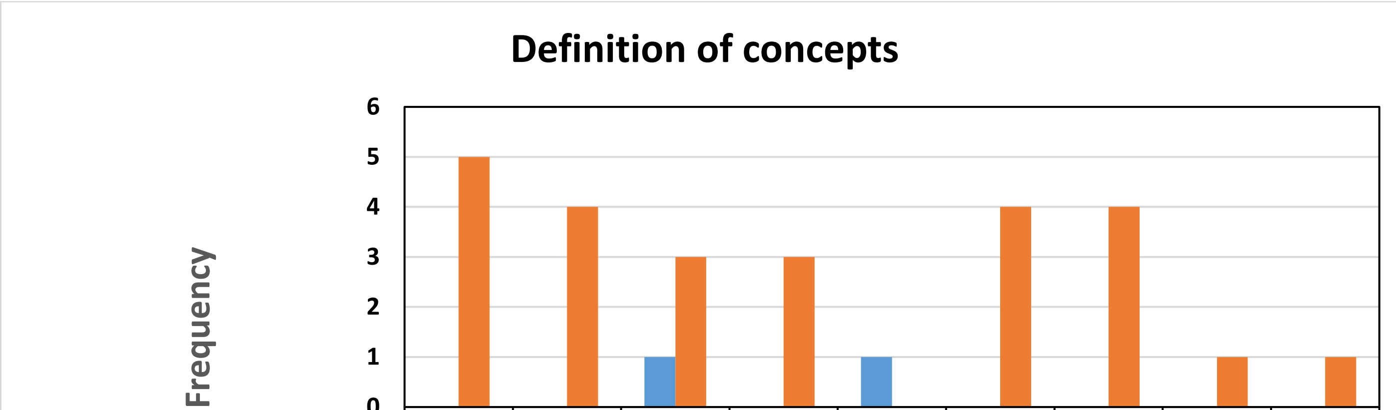
In this category, students either did not attempt the question or approached it in a manner that suggested that the student was unaware of the existence of a third degree polynomial. Student #17, for example, assumed that the expression was a trinomial and used the procedure of factorising a trinomial:

Extract 4.28: Student #17's response to Q1.6

This student conveniently utilised a known procedure, of factorising a trinomial, for the wrong concept. The response confirms the researcher's opinion, gained from the literature review that conceptual difficulties also refer to a complete lack of understanding of a mathematical concept.

4.2.2.2.6 Summary of conceptual difficulties based on student interviews

In an attempt to validate conceptual difficulties, the researcher asked the students to define key concepts in their own words. For example, "What does the word factorisation, mean to you?" The researcher asked students to define these concepts when they emerged at different points of each interview and not sequentially. Therefore, he did not necessarily pose all the questions on definitions to all students. Figure 4.7 represents the concepts asked and the frequency of correct answers and incorrect answers:



	Factorisation	Multiplication	Prime numbers:	Integers	Exponents	Expression	Equation	Difference of two squares	Limit
Frequency of correct responses	0	0	1	0	1	0	0	0	0
Frequency of incorrect responses	5	4	3	3	0	4	4	1	1

Figure 4.7: Tool for definition of concepts from student interviews

All the interviewed students responded incorrectly to the question: "What does the word factorisation, mean to you?"

From the student responses cited and discussed in section 4.3.2.2.1, the researcher concludes that the five students interviewed did not have a deep conceptual grasp of factorisation. Students also tended to use mathematical terms and concepts interchangeably, incorrectly and in the wrong contexts while speaking. For example:

Student #18 had an idea of factorisation but his/her explanation is incorrect because of the choice of words such as "*…given an equation not equal to equal to zero… factors separating into in two factors...*" Student #28 could not explain the concept but acknowledged that he/she could factorise. The researcher concludes that such students focus exclusively on procedural knowledge.

The researcher asked four students the question: "What does the word multiplication, mean to you?" These students' responses were cited and discussed in section 4.3.2.2.1. The researcher concludes that all interviewed students' understanding of multiplication was at a very superficial level.

The responses of Student #21, Student #18 and Student #25, concerning the meaning of integers, suggests that these students did not understand the concept of integers. The lack of understanding of such a basic foundational concept contributed to conceptual difficulties experienced by these students while factorising.

The researcher asked four students to define expressions and equations. Their responses as cited and discussed in section 4.3.2.2.1 reveal that none of them gave correct responses for expressions. For equations, Student # 21 and Student #28's responses suggest that these students possessed a superficial understanding of equations in that the term was only applicable when equated to zero. Student #25's definition of equations indicates that he/she lacked the ability to express herself mathematically in English. The assessment test demonstrates that most students encountered difficulties in differentiating between an equation and an expression. The four students' responses also confirm that their lack of conceptual understanding of expressions and equations contributed to the difficulties they experienced while factorising and solving factorisation problems in the assessment test.

In summary, the interviewed students' inability to explain the meaning of key mathematical concepts related to factorisation and problems that involve factorisation validate the theory that they experienced conceptual difficulties with factorisation. Thus confirming the literature study, that many students retain fundamental conceptual difficulties in engaging with mathematics (MacMath et al, 2009; Garfield and Ahlgren 1988; Ball, Lubienski and Mewborn, 2001; Kilpatrick et al, 2001).

4.2.2.3 Procedural flexibility

Procedural flexibility refers to the extent of application of factorisation. In the literature review, Star et al, (2015:2) define such flexibility as the student's ability to identify and use multiple methods as well as choosing the most appropriate method to solve algebra problems. This section discusses, in detail, the extent of students' procedural flexibility that was prevalent in the research.

The researcher set questions in section B to correlate with similar expressions in section A. Therefore, the researcher assumed that students, who experienced conceptual and procedural difficulties with basic factorisation in section A, would also experience the same difficulties when solving problems that hinge on factorisation in section B. For example in section A, Question 1.1, students were required to factorise $4x^3 + 4x$, in section B, Question 2, students had to determine the $\lim_{x \to 0} \frac{4x^3 + 4x}{4x}$, while in Question 3, students were required to solve $x^3 + 4x = 0$.

Table 4.16 provides a comparison between the number of students who obtained correct answers per question in section A and the number who got the correct corresponding type of question correct in section B:

Table 4.16: Ability to transfer conceptual and procedural knowledge: Procedural flexibility

Question number/s and type	**Questions**	**Section A: Basic factorisation:** Number of students with correct answers	**Question number/s and type**	**Question**	**Section B: Problems involving factorisation:** Number of students with correct factorisation	**Number of students that did not attempt to factorise**	**Number of students that factorised incorrectly**
		14	2. Limit.		5	23	2

1.1 Removing HCF	$4x^3+4x$		3. complex equation	$\lim_{x\to 0}\frac{4x^3+4x}{4x}$ $x^3+4x=0$	18	7	5
1.2 Grouping	$x^3+4x^2+4x+16$	12	4. complex equation	$2x^3+8x^2+8x+32=0$	9	5	16
1.3 Difference of two squares	a^2-b^2	10	5. Trig Identity	$\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=1$	5	22	3
1.4 Trinomial (a>0)	$2x^2-7x+3$	8	6. Trig equation	$2\sin^2 x-7\sin x+3=0$ where $0^{\circ}\le x\le 360^{\circ}$	5	21	4
1.5 Trinomial(a<0)	$x^2-12x+36$	13	7. Cubic function: x intercepts	$f(x)=x^3-12x^2+36x$	10	17	3
1.6 Third degree polynomial	x^3-3x+2	1	8. Cubic function: x intercepts	$f(x)=x^3-3x+2$	8	13	9

Table 4.16 indicates the number of students with correct answers in section A, the number with correct answers in section B, the number who did not attempt to factorise in section B, and the

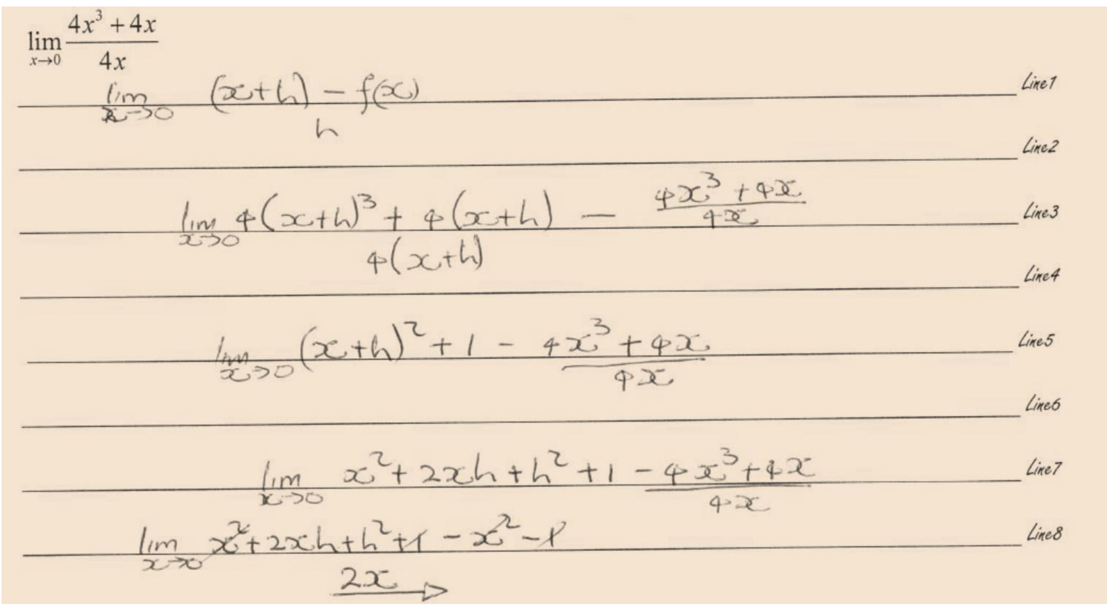
number who attempted to factorise, but incorrectly, in section B. While counting the number of students with correct answers in section A, the researcher also counted those students who factorised correctly, but were penalised for displaying other conceptual or procedural difficulties, such as treating the expression as an equation and solving. Hence, this table may not tally with other tables presented in this research.

The researcher developed Table 4.17 from the results in Table 4.16. Table 4.17 provides a comparison between the percentage of students who obtained correct answers per question in section A and the percentage of students who got the corresponding type of question correct in section B. The researcher calculated the percentages out of the 30 students who wrote the assessment:

Table 4.17: Percentage of ability to transfer conceptual and procedural knowledge: Procedural flexibility

Section A: Question Number	% of correct answers	Section B: Question Number	% of correct factorisation	% that did not attempt to factorise	% of incorrect factorisation
1.1 Removing HCF	46.7%	2. Limits	16.7%	76.7%	6.7%
1.1 Removing HCF	46.7%	3. complex equation	60%	23.3%	16.7%
1.2 Grouping	40%	4. complex equation	30%	16.7%	53.3%
1.3 Difference of two squares	33.3%	5. Trig Identity	16.7%	73.3%	10%
1.4 Trinomial (a>0)	26.7%	6. Trig equation	16.7%	70%	13.3%
1.5 Trinomial(a<0)	43.35	7. Cubic function: x intercepts	33.3%	56.7%	10%
1.6 Third degree polynomial	3.30%	8. Cubic function:	26.7%	43.3%	30%

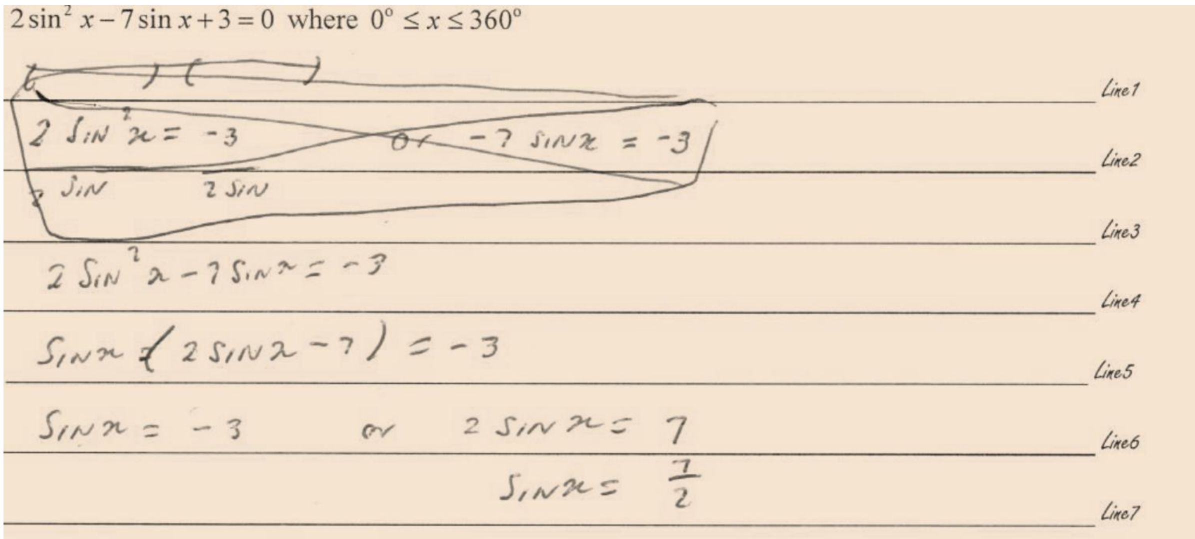
The results expressed in Table 4.16 and Table 4.17 indicate a decrease, except for Q3 and Q8, in the number of students that obtained correct answers for each question in section A and the number who arrived at the correct answer for the corresponding questions that used similar expressions, in section B. For example, 46,7% of students were able to correctly remove the highest common factor for Q 1.1, $4x^3 + 4x$. However, just 16,7% of them were able to answer Q2, $\lim_{x \to 0} \frac{4x^3 + 4x}{4x}$ correctly while 76,7% of them did not even attempt to factorise in order to find the limit in Q2. Furthermore, 57% of students who had Q1.1 correct, attempted to find the derivative of Q2 from first principles. For example, Student #11:



Extract 4.29: Student #11's response to Q2

The example in Extract 4.2.9 suggests that this student could not adequately transfer conceptual and procedural knowledge of removing the HCF, to solve higher order problems that required the removal of the HCF.

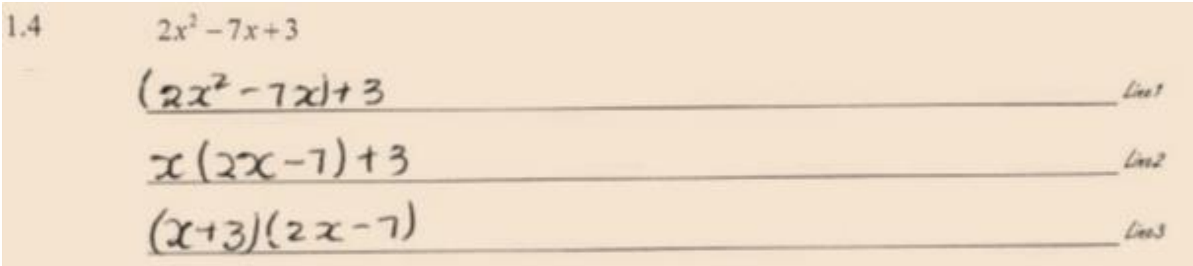
In addition, 26,7% of students were able to correctly factorise the trinomial in Q1.4. However, only 16,7% of them were able to factorise Q6, $2\sin^2 x - 7\sin x + 3 = 0$, correctly whereas 70% did not even attempt to factorise in order to solve the trigonometric equation in Q6. For example, Student #18 had the correct response for Q1.4: factorise the trinomial $2x^2 - 7x + 3$, but could not identify the same trinomial in Q6:



Extract 4.30: Student #18's response to Q6

Student #18's response reveals that he/she could not flexibly transfer the conceptual and procedural knowledge of factorising a trinomial to factorise and solve a similar trigonometric equation. Based on this evidence, the researcher concludes that this student learns procedures, and his/her lack of a deep conceptual understanding of factorising a trinomial is the main reason why he/she could not recognise the trinomial in Q6. This student conveniently manipulated the trigonometric equation to remove a HCF and then attempted to solve. Student #18 also demonstrated his/her lack of conceptual and procedural understanding of quadratic equations.

Q1.4 and Q6 contained the same quadratic expression but in different contexts. Student #21's response to Q1.4 and Q6 highlights the fact that he/she carried and flexibly used misconceptions and incorrect procedures of factorisation into another domain of mathematical learning, namely solving a quadratic trigonometric equation:



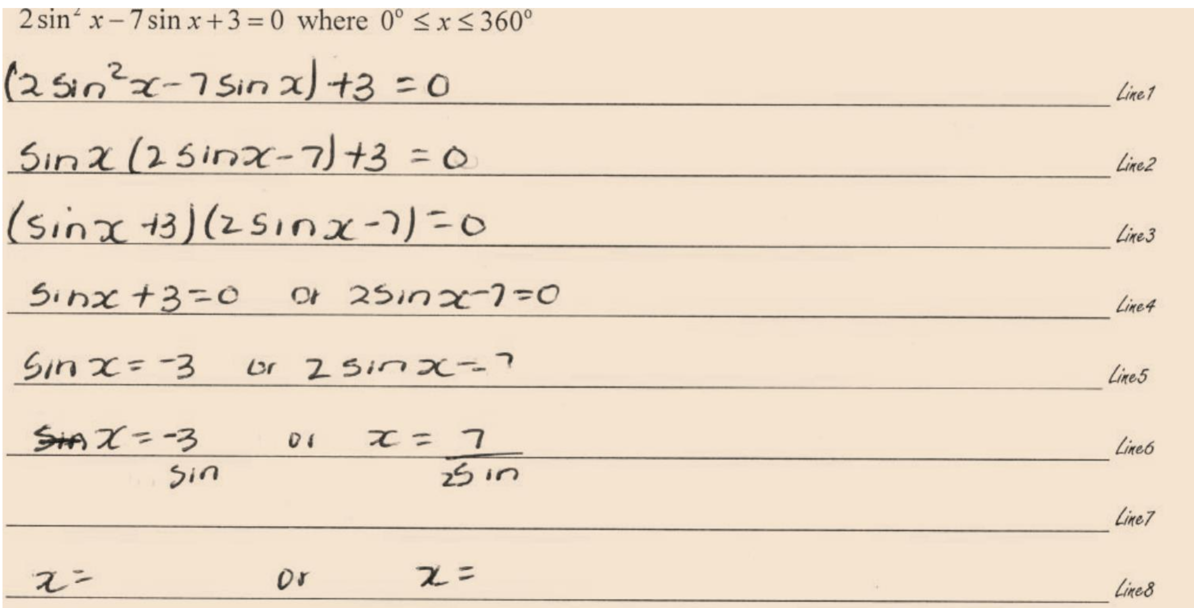
1.4 $2x^2 - 7x + 3$

$(2x^2 - 7x) + 3$ Line1

$x(2x - 7) + 3$ Line2

$(x+3)(2x-7)$ Line3

Extract 4.31: Student #21's response to Q1.4



$2\sin^2 x - 7\sin x + 3 = 0$ where $0° \leq x \leq 360°$

$(2\sin^2 x - 7\sin x) + 3 = 0$ Line1

$\sin x(2\sin x - 7) + 3 = 0$ Line2

$(\sin x + 3)(2\sin x - 7) = 0$ Line3

$\sin x + 3 = 0$ or $2\sin x - 7 = 0$ Line4

$\sin x = -3$ or $2\sin x = 7$ Line5

~~sin~~ $x = \frac{-3}{\sin}$ or $x = \frac{7}{2\sin}$ Line6

Line7

$x =$ or $x =$ Line8

Extract 4.32: Student #21's response to Q6

Student #21's responses for Q1.4 ad Q6 reveal that he/she tends to carry over misconceptions and incorrect procedures from one section to another.

The results expressed in Table 4.16 and Table 4.17 indicate an increase in the percentage of students who answered Q1.1, Factorise $4x^3 + 4x$, correctly, and the percentage of students who factorised Q3, $x^3 + 4x = 0$, correctly. 46,7% of students answered Q1.1 correctly while 60% of them factorised Q3 correctly. In addition, the results expressed in Table 4.16 and Table 4.17 also show an increase in the percentage of students who answered Q1.6, Factorise $x^3 - 3x + 2$, correctly, and the percentage who factorised Q8, $f(x) = x^3 - 3x + 2$ if $(x-1)$ is a factor of $f(x)$, correctly. 3,3% of students answered Q1.6 correctly while 43,3% of them managed to factorise Q8 correctly. These findings contradict the researcher's assumption that students, who experienced conceptual and procedural difficulties with basic factorisation in section A, would also experience the same difficulties when solving problems that hinge on factorisation in section B.

However, closer comparison of the questions indicates that the equation $x^3 + 4x = 0$ in Q3, contained just one variable as a factor while the expression $4x^3 + 4x$ in Q1.1, had a constant and a variable as HCF. Subsequently factorising Q3 was significantly easier than factorising Q1.1. In addition, to factorise Q1.6 $x^3 - 3x + 2$, students were obliged to determine the first factor, while in Q8, $f(x) = x^3 - 3x + 2$ if $(x-1)$ is a factor of $f(x)$; this first factor is given. Therefore, factorising Q8 was significantly easier than factorising Q1.6.

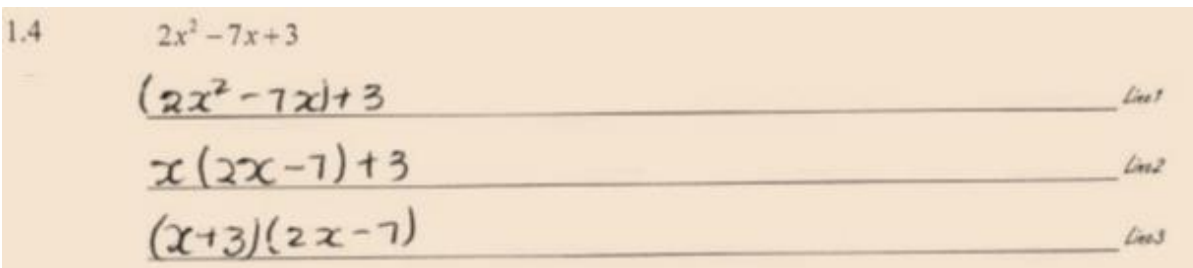
In summary, the students' written responses confirmed that they lacked procedural flexibility. The researcher attempted to understand the mind-set of the students and the reasons why they used certain procedures for different questions in the assessment tests. The five students interviewed demonstrated procedural difficulties for different questions. Therefore, the researcher asked students and Subject Matter Experts similar questions, except for the question in the assessment for which students had difficulties answering, for example:

"Tell me how you went about answering Q1.1?" Based on their responses the researcher probed further.

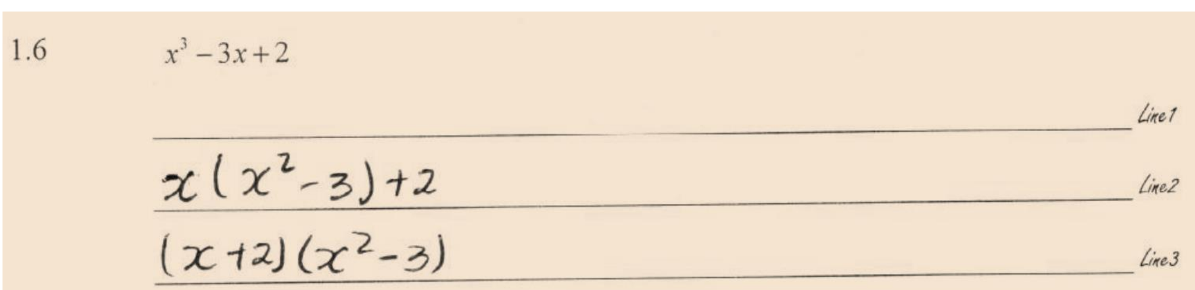
Kilpatrick et al (2001: 121) suggest that procedural flexibility is knowledge of procedures, knowledge of when and how to use them appropriately and the skill in performing them flexibly, accurately and efficiently. The researcher, in an attempt to determine the degree of procedural flexibility amongst the five students that were interviewed, asked students whether they noticed any similarities between the questions in section A and the questions in section B. For example:

“Can you see a similarity in section A, Q1.1 and section B, Q2?” etc. Based on their responses, the researcher probed further.

In Q1.2 where they were asked to factorise $x^3 + 4x^2 + 4x + 16$, Student #21 used grouping to factorise correctly. This student however incorrectly used the procedure of grouping when attempting to factorise Q1.4 and Q1.6.



Extract 4.33: Student #21’s response to Q1.4



Extract 4.34: Student #21’s response to Q1.6

Student #21 used the same incorrect procedure for Q1.4 and Q1.6. He/she grouped the first two terms, removed the HCF of the first two terms, and then incorrectly grouped the HCF of the first two terms with the third term. The researcher probed Student #21’s responses as follows:

Researcher: "Explain to me what you did here in Q1.4."

Student #21: "*I group the first two terms… I factorised the first term... I take the first common factor and add it to 3. You get (x+3) (2x-7).*"

Researcher: "So why do you use grouping?"

Student #21: "*Why did I use grouping? It's because 3 do not have x.*"

Researcher: "Ok, what method did you use in Q1.6"

Student #21: "*I used grouping*."

Researcher: "But when we have four terms we have to group. How many terms are here?"

Student #21: "*Three"*

Researcher: "Then why have you grouped?"

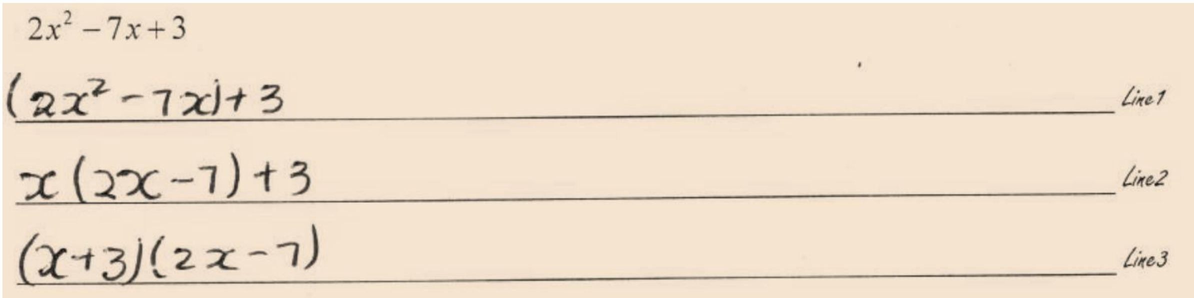
Student: "*I… It was easier for me to factorise by grouping.*"

Researcher: "You know that grouping works with four terms. But you still use grouping"

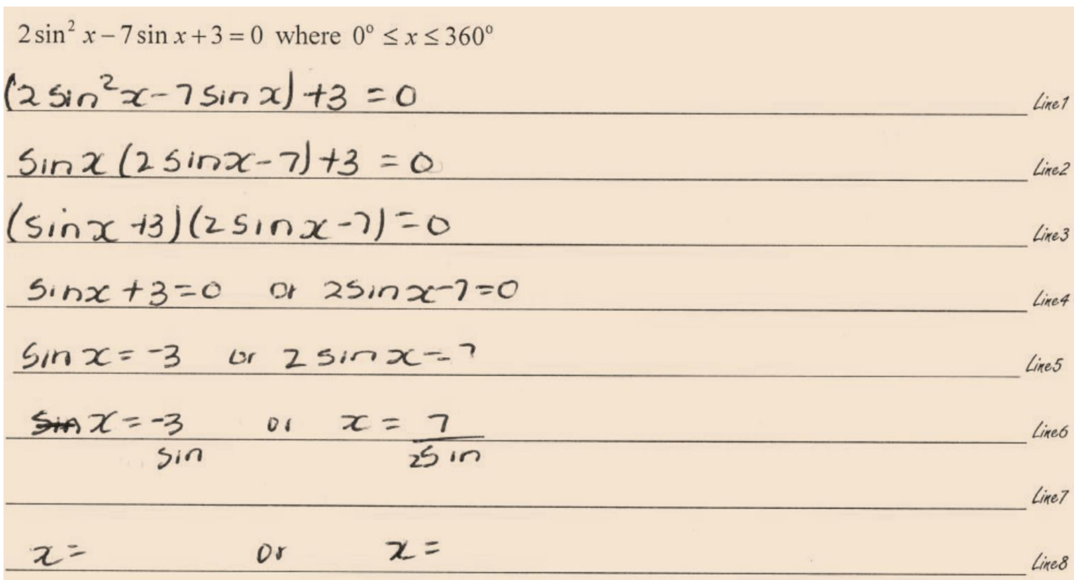
Student #21: "*Yes. I feeled to do that, sir*."

From the dialogue, the researcher concludes that this student did not understand the concepts of a polynomial, the number of terms in a polynomial expression and removing the HCF. He/she learnt the procedure of factorising by grouping, and conveniently used this method in the wrong contexts.

Student #21 also incorrectly used the procedure of grouping in an attempt to factorise Q1.4 and solve the quadratic trigonometric equation in Q6:



Extract 4.35: Student #21's response to Q1.4



Extract 4.36: Student #21's response to Q6

This student also demonstrated other conceptual and procedural difficulties but for the purposes of this study, the researcher focused on his/her ability to use factorisation to solve mathematical problems like Q6.

The researcher asked Student #21: "Can you see a similarity in section A, Q1.4 and section B, 6? The student's response was:

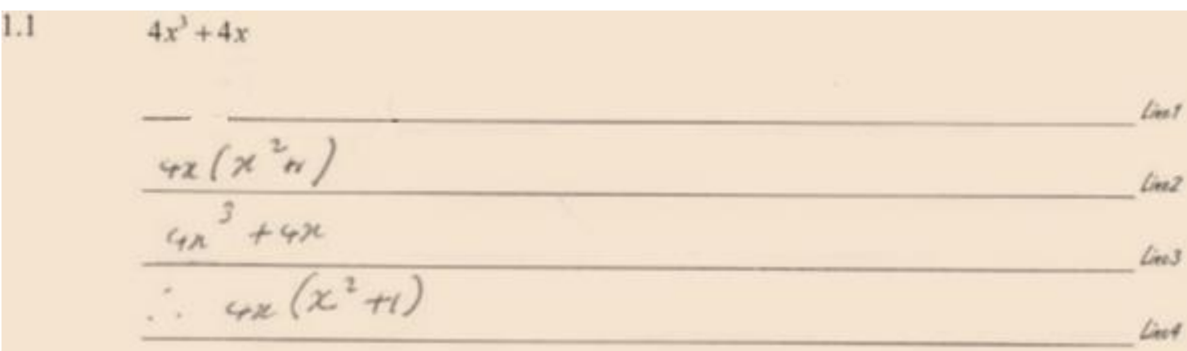
"*No, there is a 2. The difference is that we have sin here. You just have* $2x^2$ *and here* $2\sin^2 x$."

Researcher: "Now tell me, what method did you use here?"

Student: "*I used grouping*."

Although Student #21 did not see the same trinomial in Q1.4 and Q6, he/she realised that factorisation was required to solve the trigonometric equation. This implies that he/she flexibly connected learnt misconceptions and incorrect procedures, to solve higher order problems. The literature reviewed, agreed with Star et al's (2015) claim that the relations between conceptual and procedural knowledge are often bi-directional and iterative. Bi-directional signifies that when the order of teaching for conceptual understanding and procedural understanding is interchanged these may complement and strengthen each other, while iterative implies that a student can repeat a concept or a procedure to achieve a desired outcome and form the base for new learning. The responses suggest that the opposite is also true in that misconceptions and inappropriate use of procedures are bi-directional and iterative, albeit negatively.

Student #18 answered Q1.1: factorise $4x^3+4x$, correctly. He/she was able to answer Q2 $\lim_{x\to 0}\frac{4x^3+4x}{4x}$ correctly, without factorising, so the researcher probed further:



1.1 $4x^3+4x$

Line1

$4x(x^2+1)$ Line2

$4x^3+4x$ Line3

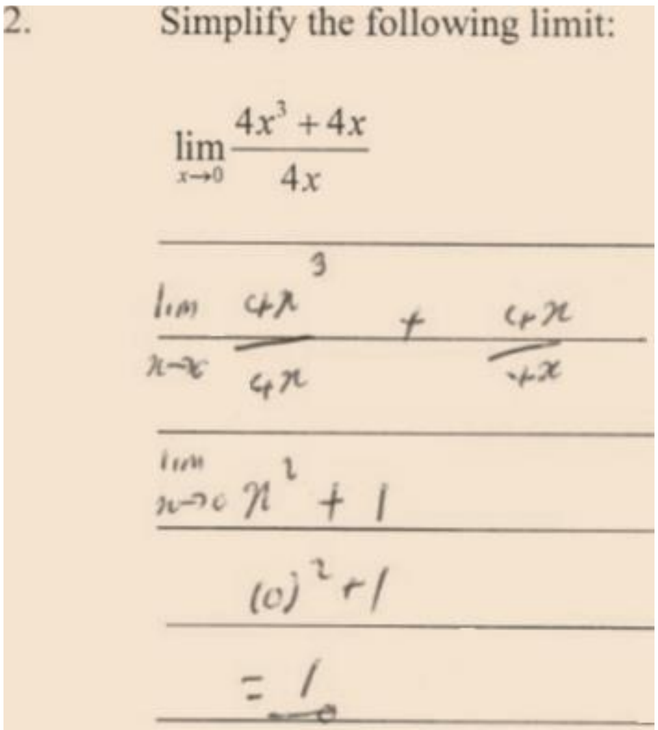
$\therefore 4x(x^2+1)$ Line4

Extract 4.37: Student #18's response to Q1.1

This student's response as to how he/she factorised Q1.1 was:

"*I have seen that if I'm taking 4x outside, what I will do, will remain in the equation, and will with the* x^2+1*, which will give me a sense in terms of doing the equation. If I will take that equation back it will remain back to the equation I have been given. To prove, is to prove that, if what I've done is correct or what*."

Although this student correctly answered and explained Q1.1, he/she did not realise the connection between Q1.1 and Q2. Student #18, barring the use of brackets, managed to answer Q2 without factoring. Student #18's response to Q2 is as follows:



Extract 4.38: Student #18's response to Q2

Q2 is an application problem of the same expression, $4x^3+4x$, assessed in Q1. Prerequisite knowledge to answer Q2 includes fractions and limits. The student correctly answered Q2 by implementing the distributive property of division. To circumvent this limitation of Q2, the researcher posed the following question:

"Now how will you simplify the following $\lim_{x\to 3}\frac{x^2-9}{(x-3)}$?" In this case, the student must factorise in order to simplify the limit.

Student #18 answered:

"*It's very simple, it's very simple, first and foremost here we need to understand what is the principle of linear programming, here if we are given an equation like this….Whereby leave x=3, whereby where we found x=3, if we are talking about this, like this example, just put 3 where you see x. 3x² which is 9. 9-9 over 0 which is 0.*"

The researcher probed the student further, by asking the following question:

"If 5 is divided by 0, what will the answer be?"

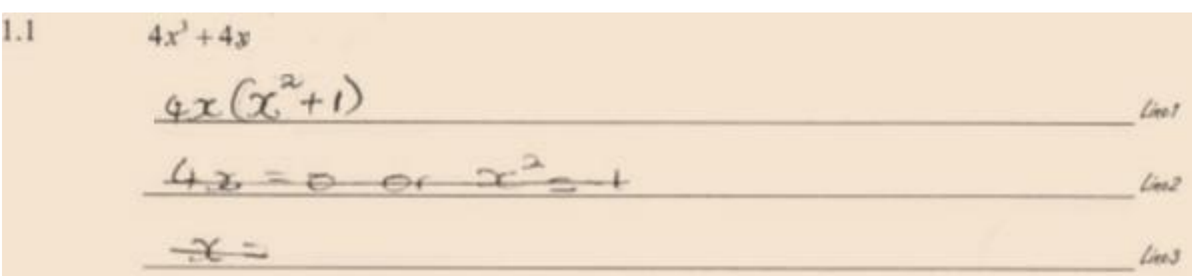
Student Response*: "5 divided by 0 will be 5. I'm sorry it is zero"*.

Researcher: "You sure?"

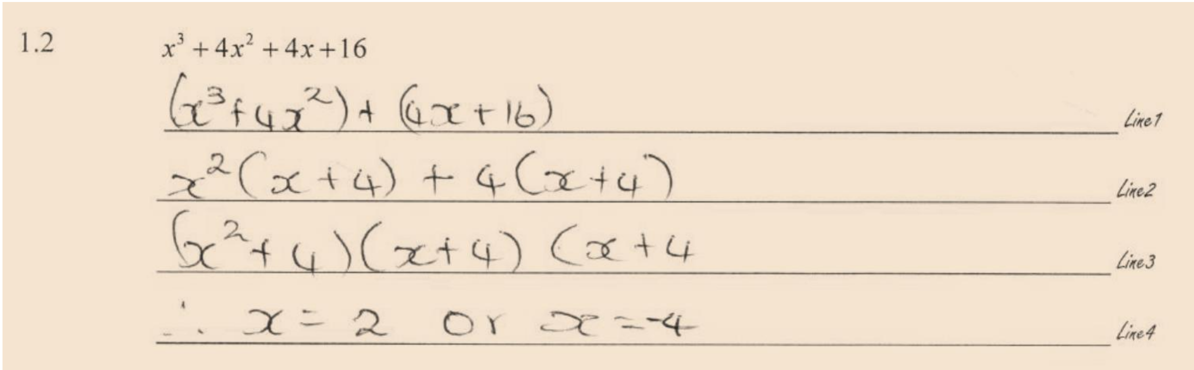
Student: *"Yes!"*

The dialogue indicates that this student could not use factorisation or did not see factorisation as an option to simplify the limit. Secondly, he/she also did not understand the concepts of limits and zero denominators. The researcher concludes that this student could not flexibly link the procedure of factorisation to solve other mathematical problems that involve this technique.

Student #25 factorised Q1.1, $4x^3+4x$, and Q1.2, $x^3+4x^2+4x+16$, correctly but went on to treat Q1.2 as an equation and solved for *x*.



Extract 4.39: Student #25's response to Q1.1


1.2 $x^3+4x^2+4x+16$

$(x^3+4x^2)+(4x+16)$ Line1

$x^2(x+4)+4(x+4)$ Line2

$(x^2+4)(x+4)\ (x+4$ Line3

$\therefore\ x=2$ or $x=-4$ Line4

Extract 4.40: Student #25's response to Q1.2

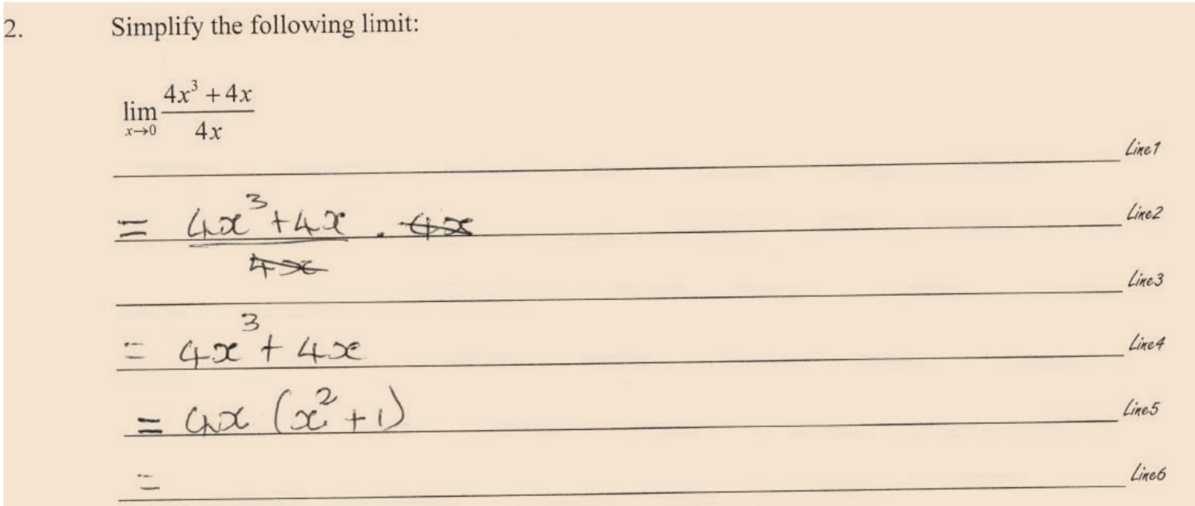
The researcher asked the question "Oh! Ok, and tell me this. Why did you solve for *x*?"

The student responded: "*it because I am factorise*"

Researcher "So when you factorise must you solve for *x*?"

Student: "*I solving for x*"

From Student #25's answers and verbal responses it can be concluded that he/she knew the procedures of removing HCF and grouping but had difficulty in understanding the concepts of expressions and equations. Due to such misconceptions and inaccuracies, he/she could not answer Q1.2 correctly.



2. Simplify the following limit:

$$\lim_{x\to 0}\frac{4x^3+4x}{4x}$$

Line1

$= \frac{4x^3+4x}{4x} \cdot 4x$ Line2

Line3

$= 4x^3 + 4x$ Line4

$= 4x(x^2+1)$ Line5

$=$ Line6

Extract 4.41: Student #25's response to Q2

The researcher posed the question: "For Q2, explain to me how you went about getting your answer? What did you do?"

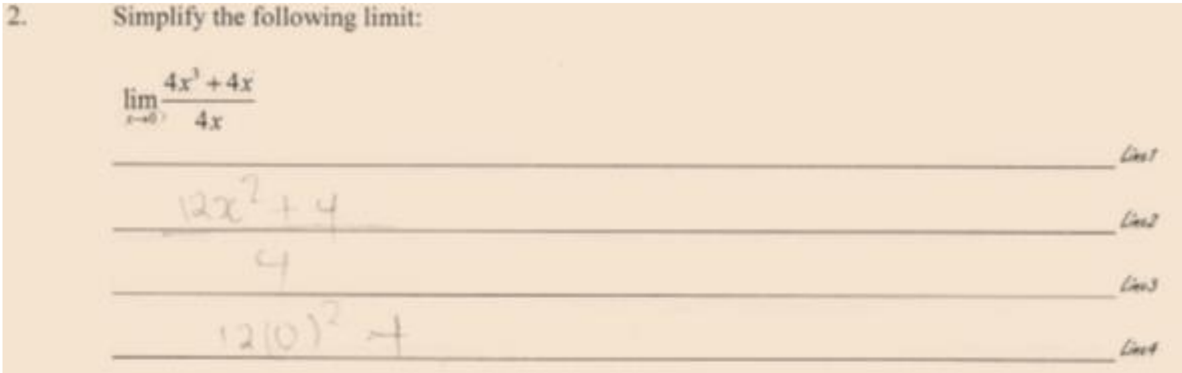
Student #25:

"Here sir I've got the mixed understanding of forgetting the formula of a limit of x approach to 0. F(x) = (x+h) I think so sir. Because when I use it. I divide it by 4. I think it is wrong that I did. Ya I multiplied by 4x to divide it to 4x over. Then I keep the common factor of 4x to get $x^2 + 1$."

However, this student's response, although incorrect, suggests that he/she was able to make a connection with factorisation procedures to solve other mathematics problems that require factorisation.

Additionally, another student, Student #28, displayed procedural difficulties and lacked procedural flexibility:

Below, are the answers of Student #28 for Q2 and Q7:


2. Simplify the following limit:

$\lim_{x\to 0}\frac{4x^3+4x}{4x}$

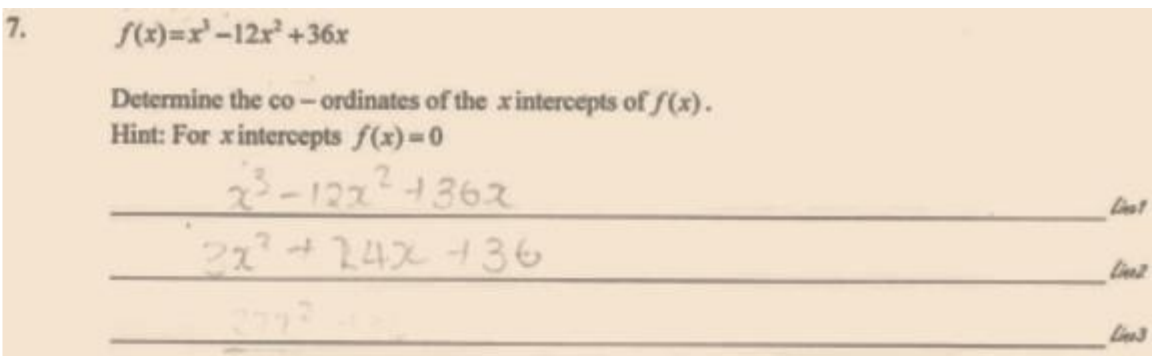
Line1

$\frac{12x^2+4}{4}$ Line2

Line3

$12(0)^2+4$ Line4

Extract 4.42: Student #28's response to Q2


7. $f(x)=x^3-12x^2+36x$

Determine the co – ordinates of the x intercepts of $f(x)$.
Hint: For x intercepts $f(x)=0$

x^3-12x^2+36x Line1

$2x^2+24x+36$ Line2

Line3

Extract 4.43: Student #28's response to Q7

The student used the rules of differentiation to answer Q2 and Q7.

The researcher prompted the following dialogue:

Researcher: "Why did you derive in Q2?"

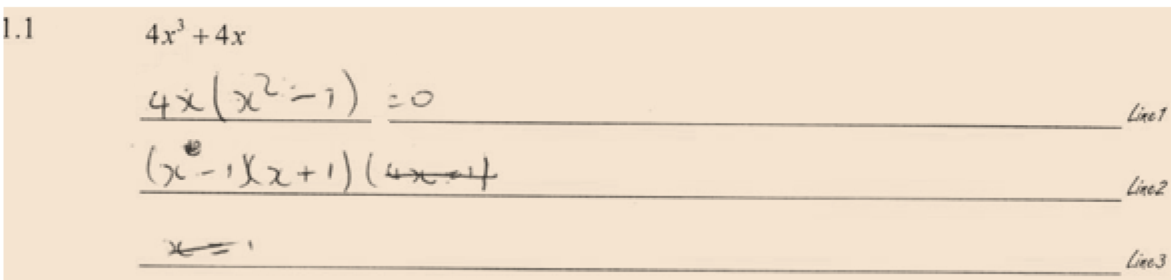
Student #28: "*derive you have to derive first…. the teacher told me that we must derive first*."

Researcher: "Why did you derive in Q7?"

Student #28: "*the question…eish I was doing eish… I was taking a chance*."

Student #28's response suggests that he/she follows mathematical rules that come to mind and at will. At the time of this research study, students were engaging with the rules for differentiation with their class lecturer. The researcher concludes that this student learnt procedures only. Therefore, he or she mechanically used the latest or current procedure learnt, to solve and answer Q2 and Q6.

The answers of Student #2 also reveal the application of procedures without an inherent conceptual base:



Extract 4.44: Student #2's response to Q1.1

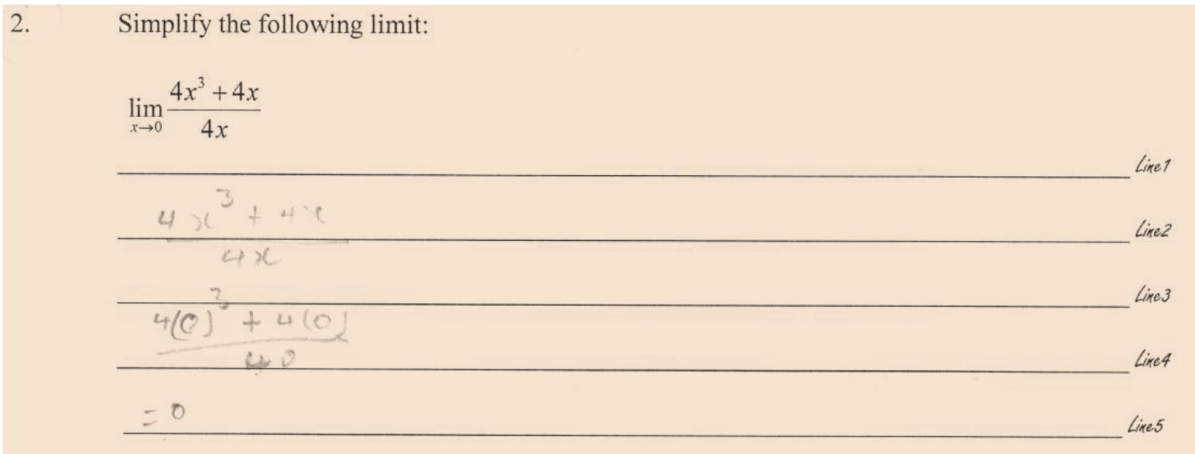
This student factorised Q1.1 but manipulated the answer by introducing a negative and then factorised further. He/she also attempted to equate the expression to zero.

The researcher-probed Student #2 by asking the question: "Why did you put minus there?"

Student #2 responded:

"1 because when I'm going to separate 2 squares and using minus and that's why it come on my mind."

From this response, the researcher concludes that this student used and manipulated questions to utilise procedures without actually understanding the concepts that go with each procedure. This was also evident in his/her answer for Q2. This student also could not perceive the similarities between Q1.1 and Q2, where Q1.1 required simple factorisation of an expression and Q2 where he/she had to evaluate the limit after factorisation of the expression:



Extract 4.45: Student #2's response to Q2

When the researcher asked Student #2 to explain the meaning of limits, he/she responded:

Student #2 "*Limit mean x. Eish what can I put it in clear picture. When there is a x supposed to substitute with 0*."

This response shows a clear correlation between this student's own understanding of the concept limits and the procedure he/she adopted to answer Q2. Student #2's limited understanding causes him/her to use the wrong procedure which inevitably resulted in an incorrect answer. He/she did not consider factorisation as an option to answer Q.2 and did not understand the concept of a zero denominator.

Likewise, when the researcher asked the student to explain how he/she worked out the value of the limit, his/her response was:

Student #2: "*The first thing that I did. I need to remember the formula this one of limits*. *Formula is* $\frac{(h)+x+h}{h}$. *What I did here. I substitute with the 0 when there is an x and calculate and put equal to 0. On the thing that I do."*

The researcher probed further: “If 5 is divided by zero, what is the answer?”

The student responded: “*five divided by zero is 0, ya I’m sure*”

The student’s responses suggest that he/she experienced both conceptual and procedural difficulties. He/she confused limits with finding the derivative from first principles. Student #2 could not see factorisation as a tool to solve other mathematics problems. His/her response also indicated that he/she lacked a foundational understanding of concepts of fractions, zero denominators, and limits.

When the researcher asked the HOD for reasons why Student #2 could not see that the same expression had to be factorised in Section A: Q1.1 and Section B: Q2, he responded;

HOD: “*So [he/she]as to do factorisation, now 0/0 means [he/she] has to do factorisation but again [he/she] needs to understand the factorisation that [he/she] is doing in there and should remove the 4x out as common factor. But again it deals with a section that does not ask the learner to factorise…. and [he/she] can’t see that*”

The HOD’s response suggests that this student is conditioned to factorise only if the question says factorise; otherwise he or she lacks procedural flexibility in using factorisation to solve other mathematics problems.

Moreover, Lecturer #3 and the Umalusi Moderator had this to say about why students could not see that the same expression had to be factorised in section A: and in section B:

Lecturer #3: “*There is always going to be a link and they fail to see the link between sections, subsections and even between different topics. Although the expression might look the same because they are unrelated topics to them, their minds do not allow them to mix the two concepts together. They fail to integrate the two.”*

Umalusi Moderator: “*There is a tendency throughout the schooling for Maths teachers to teach specific, they say you see this then do that. So that does not do anything for the understanding but what it does do is it produces right answers for the moment. So you will get an answer because* *you are doing the topic. In that topic, the worked examples give you the method. When you see this particular expression, algebraic expression and you are asked to factorise it, all the questions are going to be factorised. In the end you will just do what is required. Then you couple that with another topic, factorisation is not an option because it is outside of the context in which you learnt it. So that is the downfall of teaching without understanding, in other words, teaching recipes."*

In the literature review, the researcher claims that students may be prone to apply procedures in isolation to arrive at an answer rather than understand and solve problems. This is probably what Lecturer #3 meant by "*they fail to see the link between sections, subsections and even between different topics."* The literature review also highlighted that educators who drill mathematical procedures, without attempting to provide any level of understanding, create confusion amongst their students, (Tularam and Hulsman, 2015:2). This is most likely what the Umalusi Moderator implied by "*Maths teachers to teach specific they say you see this then do that ..., factorisation is not an option because it is outside of the context in which you learnt it. So that is the downfall of teaching without understanding, in other words, teaching recipes."*

In summary, the results of the 30 students who wrote the assessment test, the students interviews and the subject matter interviews indicated that some students were unable to make links between section A and section B, and they lacked procedural flexibility altogether, while other students did not lack procedural flexibility. The latter in fact flexibly utilised their misconceptions and incorrect use of procedures to make links and connections with other mathematics problems, although incorrectly.

4.2.3 NCV L4 students' challenges

It was the researcher's opinion that this study could not adequately answer the second research question: "Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?" if contextual factors and challenges that NCV L4 students experienced were not considered. For this reason, the researcher, during the interviews, prompted students and Subject Matter Experts to speak about contextual challenges that students experienced. The researcher was aware that not all contextual factors are unique to mathematics and this study. However, the researcher included these factors as they impact on teaching and learning and ultimately would have effected or contributed to conceptual and procedural difficulties.

Results on the extent of contextual challenges that students experienced from their perspective are presented in Figure 4.8:

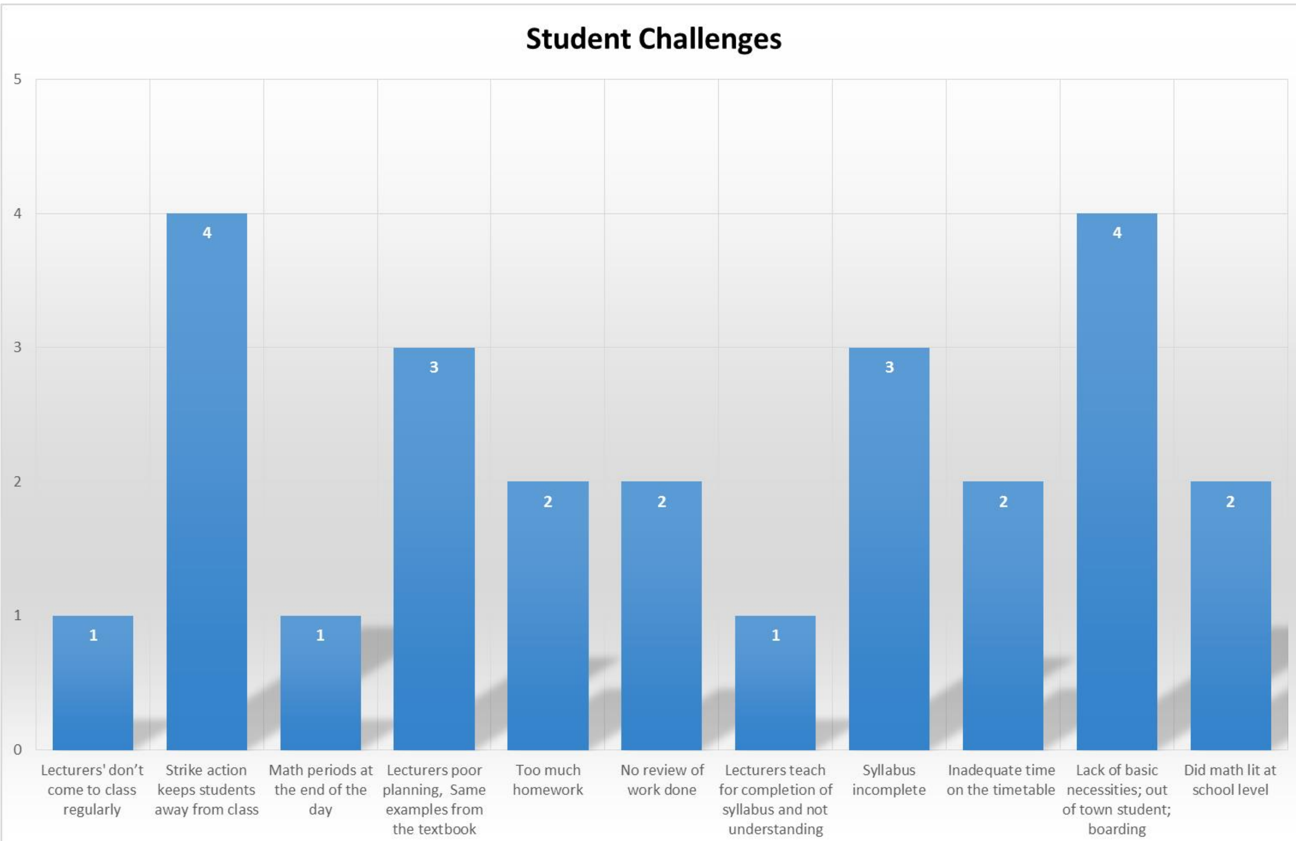


Figure 4.8: Student challenges

The researcher asked the Subject Matter Experts the following questions:

- Why, in their opinion, students demonstrated such conceptual and/or procedural difficulties?

- Additionally, students could not see that the same expression had to be factorised in Section A: Q1.1 and Section B: Q2. Why?

Table 4.18 and Figure 4.9 presents the codes and the results of the views of the Subject Matter Experts on why students demonstrated conceptual and/or procedural difficulties:

Table 4.18: Subject Matter Expert's views on why students demonstrated conceptual and/or procedural difficulties

Keywords and phrases	Frequency of code N = 5	Subject matter expert
Time constraints:	5	#1; HOD; UM; #2; #3
Contact time/long curriculum/Less time on time table/enrolment	4	#1; HOD; UM; #2, #3
Strike/enrolment	4	#1; #2; #3; HOD
Maximising the time	1	UM;
Time allocation	1	UM;
Lecturers	3	#1; #2; HOD; UM
Homework	2	#3; HOD
Teaching for syllabus and not for understanding	1	#1; #2
Teaching for assessment and not understanding	1	#1;

No diagnostic analysis	1	#1;
No remedial measures	2	#1; UM
Teaching in a vacuum	1	HOD;
Not assessing like in exams	1	HOD;
Lecturers lack formal training	1	HOD; #2
Lecturers lack conceptual understanding	1	HOD;
Lecturer teaching recipes	1	UM;
Lecturer discipline	1	HOD;
Lack of on the job supervision	1	UM;
Students	5	#1; #2; #3; HOD; UM
Calibre of student Lack of prior knowledge/basic foundations/initial teaching/	4	#1; #2; HOD, UM
Compartmentalised thinking/building blocks		#1; #3
Student Discipline/attendance/punctuality	3	#1; #3; HOD
Language barrier	3	#2; #3; HOD;

Minimums		
Minimum of 30%	1	HOD; #2
Minimum entry requirements	1	HOD;

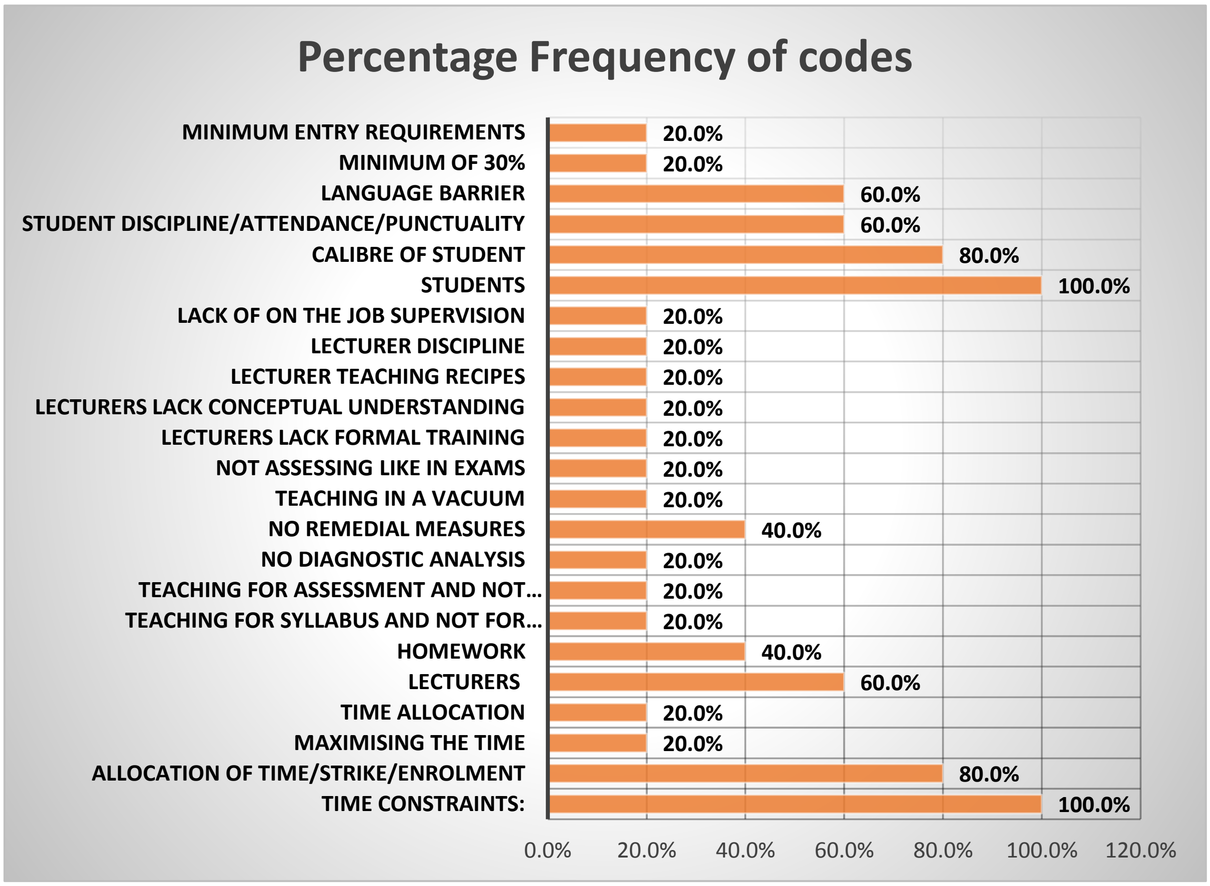


Figure 4.9: Subject Matter Experts' views on why students demonstrated conceptual and/or procedural difficulties

The researcher classified the codes of reasons in Figure 4.9 as to why students demonstrated conceptual and procedural difficulties when factorising, from the perspective of the students and the Subject Matter Experts, into four core categories: time constraints, lecturers, students, and minimum pass rates and progression.

4.2.3.1 Time constraints

The results expressed in Table 4.18, Figure 4.8 and indicate that 80% of students and 100% of the Subject Matter Experts responded that time constraints due to various factors contributed to loss of teaching time. These factors included inadequate time allocation on the timetable; frequent student strike action that interrupted lessons; poor student and lecturer attendance and punctuality; as well as a prolonged enrolment process. These factors are disruptive and leads to a loss of students' and lecturers' focus, which then impacts on teaching and learning time and concentration, which in turn leads to students experiencing difficulties with concepts, such as factorisation, and the procedures related to such concepts.

According to the DHET assessment guidelines (2014), the time needed to complete the mathematics curriculum is 200 notional hours, which represents a combination of contact time and independent study time. This amount of time for the completion of a yearly curriculum is very limited and should be maximised. Strike action, lecturers and students not attending classes regularly or not being punctual, and inadequate time allocation on the timetable may have jeopardised the quality of teaching and learning, which in turn could have contributed to the conceptual and procedural difficulties that students experienced.

4.2.3.1.1 Allocation of time

The results indicate that 40% of the students and 80% of the Subject Matter Experts concurred that inadequate time allocation on the timetable negatively affected teaching and learning:

Students #18 and #25 commented:

Student #18: *"The time that we have for maths is not enough…because all of us for mathematics is a critical subject when it comes to the college level. When checking the stats of previous years, some students they usually get maths 30% which is not acceptable, but if we are given time 3 hours or 4 hours a day to teach maths, that one will assist us also in moving forward in the pass rate of mathematics will increase."*

Student #25: *"In maths sir I need more time"*

Student #18: *"One especially in mathematics challenges we are facing is to get the students to attend the period, the last period or last double, whereby the students will be tired won't have energy to attend lectures, or getting a proper lesson that is a challenge we are facing especially level 4 mathematics,"*

Student #18's suggestion of 3 to 4 hours of teaching time for mathematics per day is not possible. However, at the time this study was undertaken, each class group was allocated five mathematics periods of 55 minutes each per week. This is an under-allocation of time based on the curriculum demands and prescriptions. The priority on the timetable at this campus is accorded to vocational subjects so that most mathematics periods are allocated at the end of the college day. Therefore, it could be true that students are mentally and physically tired. Students' lack of concentration during such lessons may also be a contributing factor as to why they have difficulties with conceptual and procedural knowledge, such as when factorising and solving problems that involve factorisation.

When asked the question "Have you completed the syllabus?", sixty percent of the students interviewed, Students #18, #25, and #2, reported that the syllabus was not yet completed:

Student #18: *"Like now paper two we are approaching exam not finishing paper two so we are appealing for extra classes, this programme so-called 'tut'."*

Student #25: *"…no syllabus not complete we still waiting for attending"*

Student #2: *"no no no no no"*

The researcher conducted student interviews in the month of October when the final examination was to commence at the end of the same month. In terms of Student #18's claim that lecturers were teaching for the purpose of completing the syllabus rather than for understanding, the syllabus for mathematics level 4 was not yet complete at that time. Lecturers and students were busy engaging in a catch up programme to complete the syllabus on time. This in itself is an extremely unhealthy academic practice.

The Subject Matter Experts' responses were:

Lecturer #1:

"I want to look at the hours which we are allowed to finish the syllabus the average of +/- 120 hours – 150 hours. If you look, from my experience here, we manage to finish the syllabus in L2 but we do not do justice the concepts – we teach for the syllabus, we teach for the completion of assessments, not for the understanding. The reason could be maybe that they did not have enough time to practice during their time."

Lecturer #3:

"....the time to get the learners to understand because we have to constantly backtrack to explain to them concepts over and over again. That is why we will need more time."

HOD:

"*We find that they also find it difficult with time as well. When we looking at time, the time we have, it's an annual course, but we don't teach for the entire year. We are required to give 200 hours, national time and for the last seven years that we have been doing it, we noticed that we have insufficient time to actually complete the syllabus, together with to do revision as well. Apart from completing the syllabus, the ideal is to do revision, right to get the learner up to par but there is no time, so in terms of time allocated, 7 subjects they should be getting equal time, an inconsistency in what we are doing currently, in terms of the mathematics we are getting actually less time. It's a timetabling issue. Core is given greater time."*

From the responses, it again appears that managers allocated insufficient time for mathematics on the timetable. These responses also support the students' and the researcher's claim that lecturers taught to complete the syllabus and to complete assessments at the expense of understanding, because of the time constraints.

The literature reviewed also reveals that the mathematics curriculum is too loaded and too long to be completed in one year, and that TVET colleges have poor timetabling plans, (Papier, 2014:38).

4.2.3.1.2 Strike action

The results expressed in Table 4.23, Figure 4.8 and Figure 4.9 indicate that 80% of students and 80% of the Subject Matter Experts responded to the issue of strike action as a factor that inevitably affected the teaching and learning:

Students #21, #18, #25 and #2, verbalised their concern about strike action that keeps them away from classes:

Student #21: *"Sir, students, student strike also disturbs when we are studying, sir, we fail to go to classes because of strike."*

Student #18: "*Yeah, even the strikes and NSFAS causes huge problem, as we are striking at a higher institution we are using the people house whereby we are renting. If the NSFAS didn't pay the money for accommodation so I can pay the landlord, the landlord can chase me out. Therefore I will be not in a position of focus on my studies, when that one is giving us a problem, as well as in terms of studies, it caused another failed rate."*

Student #25: *"it take me, ay sir I am not happy about that, because they are fighting for student rights."*

Researcher: "But is it robbing you of time?"

Student #25: *"Yes sir."*

Student #2: "*Student is striking... The things that happen as school when is start to concentrate... the classes, they have strike and stay out of classroom that is why. We want to be in the class, but when we in the class, they come and say you are mouse."*

It appears, from the student responses, that strike action keeps them out of class. In 2015, students lost 21 days while in 2016 students lost 37 days of class time due to student strike action alone. The loss of so many teaching days of mathematics is irrecoverable. Due to the sequential nature of mathematics, lecturers cannot skip important sections and topics of the curriculum. This state of affairs could also explain why lecturers rush and teach merely for syllabus completion rather than understanding. Irrespective of whether students are striking for a just cause or not, one can conclude that strike action of this magnitude in terms of time lost is a major contributing factor as to the issues being investigated.

The Subject Matter Experts had this to say:

Lecturer #1:

"We may think it is a small thing apart from the fact that we have little contact time, we have the issue of strikes happening. But the impact on our teaching because I will be forced like you see when we are teaching Paper 1 takes up to June – July teaching P1 but P2 takes us four weeks."

Lecturer #2:

"You need time…. a lot of our time is lost due to strike... Well the strikes for one is crippling to the campus union meetings… comrades take 11 days for meetings, which disrupts teaching… Our enrolment process, we plan for it for one or two weeks but we end up going much further, so we lose a lot of time there…. Also you know when its holiday time students have this mind-set that if we close before the schools, or open before the schools, students will only return after schools open."

Lecturer #3:

"One major factor we are experiencing now at our campus for a while is strike action which always, if I remember correctly, according to my calendar last year we had a total of 37 teaching days. The equivalent of 2 months of college work."

HOD:

"…the most important dilemma is that we are facing strike action. In terms of loss of time, now strike action, remember is again twofold, it's the learners and we have also strike action by the staff themselves."

The Subject Matter Experts' responses correlate with the students who claimed that substantial time was lost due to strike action. This in turn reduced the contact time between lecturers and students, thus resulting in insufficient opportunities to discuss and deliberate over conceptual and procedural difficulties.

4.2.3.1.3 Attendance and punctuality

The results expressed in Table 4.23, Figure 4.8 and Figure 4.9 indicate that 20% of students and 80% of Subject Matter Experts also cited loss of time through poor attendance and lack of punctuality:

Student #21 was the only student who spoke about lecturers not attending classes regularly:

"….our teachers do not always come in our class, sir."

The Subject Matter Experts commented:

Lecturer #1:

"The first period and the last period is a major problem. They are cause of concern. The first period our students use public transport. They come around, some of them, 8' o clock. Lectures start around quarter to. Attendance is another problem. Absenteeism is another especially the level 2's where there is the basic, they usually bunk."

HOD:

"*You see with time and loss of time there's a main factor: Discipline is one thing, now with discipline in terms of attendance, if it's poor attendance learners are losing out, and as we said*

Maths is a follow up. And there's no way a lecturer can go back…now that is one thing poor attendance, now punctuality in class, you know punctuality plays a very important concept, it could be attributed to due to transport in the morning, that's okay, that's fine. Punctuality is not only in the morning, it seems like for every lesson learners are late. After lunch they are very late to attend a class."

Although only one student pointed to irregular lecturer attendance as a challenge, this is a critical finding. Mathematics lecturers and students at this campus are not classroom based. Due to the large intake of students, there are inadequate classrooms to allocate or dedicate to the mathematics department. Lecturers and students do not have a fixed classroom for mathematics. Irregular class attendance by lecturers and students would have again resulted in the loss of valuable teaching and learning time.

4.2.3.1.4 Enrolment

The results expressed in Table 4.23, Figure 4.8 and Figure 4.9 indicate that 40% of the Subject Matter Experts commented on the loss of time due to prolonged enrolment processes.

Lecturer #1 commented:

"After 1 month we start teaching, we are supposed to start teaching around 20-22 January but enrolment will be just starting. Results they come late then we put the systems. We start teaching February so we are losing a lot of time and in maths we really need time and other subjects as well I think they are given more time unnecessarily, at the expense of maths"

Lecturer #3 said:

"Slow enrolment process at college, registration process; in fact we had a day of strike, the first week of college. All this has contributed to the delay of classes beginning."

Lecturers and students have no control over issues such as time lost due to enrolment processes. However, such delays compound student and lecturer frustrations and inadequacies in terms of time constraints.

4.2.3.1.5 Summary of time constraints

In summary, the responses highlight important reasons for how teaching and learning time was lost due to: time allocation, strike action, union meetings, the enrolment process, students' late return from holidays and poor attendance and lack of punctuality.

Based on the time lost, Lecturer #2 responded concerning the impact it had on teaching and learning:

"*We have a syllabus to complete and if you look at level 3 and level 4 for maths, it becomes very difficult for me to now go back to level 2 and still complete my syllabus. I'm expecting a learner that comes to my class in level 3 or level 4 to have that foundation already, one of the biggest problems we find is that procedurally, and conceptually, students don't understand. In terms of me actually completing my syllabus it becomes impossible for me, virtually impossible for me to go back and actually teach the foundation and then still complete my syllabus and then still make sure that the learner is able to pass."*

Lecturer #2's response likewise suggests that lecturers, because of time constraints, were teaching for completion of the syllabus rather than for understanding. Although time was limited, rushing to complete the syllabus on time at the expense of understanding is an example of poor practice. Each new concept in mathematics hinges on other foundational prerequisite concepts. A lecturer, by not refreshing students with respect to prerequisite knowledge, compounds the student's lack of understanding of key concepts and procedures when factorising.

Lecturer #1 summarised the issue of time as follows:

"they write, then they get poor not because they are not able to but because they did not attend the lessons so that is another technical issue, time factor, punctuality, absenteeism, all these they contribute to learners, also we said timetabling sometimes we are given more groups to teach at the expense of more time."

The Umalusi Moderator, however, acknowledged the issue of time, but questioned the lecturer's ability to maximise the time at hand:

Umalusi Moderator: *"The first thing that is obvious, I really talk to markers every year and they tell me similar things, quantity of contact time, the amount of contact time is limited for whatever reason, either late start or strike action. It is visible that quantity of teaching time is limited. The consequences of that is some topics are very, very poorly covered, very sketchily covered, some easy topics given very little time, students don't really catch anything. This problem of not having time but what about the consequences of not using that time you're not going to get students performing well. However, if you are teaching for results then you cannot say, 'I don't have the time for it'."*

The Umalusi Moderator also suggested that another factor which compounded conceptual and procedural difficulties was the imbalance of time allocated to each topic. This questions the lecturer's facilitation planning (year planning) and the execution of that plan, in terms of teaching and learning:

Umalusi Moderator: "*My own observation as a Moderator of both exams and ICASS is that some lecturers spend a lot of time teaching other topics. In other words, you go to a college or campus and you will notice by about May they have barely covered a third of the curriculum yet they will cover a lot more after that which means that there is a lot of rushing. It is seen by the assessments, performance in assessments are poor. Very often, if you have college based assessments or province based assessments so they omit questions or amend the question paper and things like that. I really think that adequate time for the different topics is not quite allocated. The usual reason when I do ask questions: Why is that you have not covered enough at this stage? The answers would usually be, we had a late start, and there were strikes and things like that."*

Hence, one can draw the conclusion that such time constraints which reduce conversations and discussions around students' difficulties and misconceptions, diminished teaching and learning contact time as well as inadequate time allocation regarding mathematics contributed to the said difficulties that students experienced.

4.2.3.2 Lecturers

The results expressed in Table 4.18, Figure 4.8 and Figure 4.9 indicate that 80% of students cited reasons that pointed towards the ways in which lecturers contributed towards the given difficulties. Students said that lecturers displayed irregular class attendance, do not come well prepared to class, that they tend to give too much homework which they do not review and that they tend to teach for completion of the syllabus at the expense of understanding.

4.2.3.2.1 Planning and preparing for lessons

Students #18, #25, and #28, commented that lecturers do not come well prepared to class and that they solely teach the textbook examples:

Students #18: "*To the lecturer that are teaching us mathematics even them, they are not taking them seriously because they are not preparing thoroughly the lesson they are meant to teach the following day. They are coming to teach us the example that is there in the textbook and we are seeing as the student that there is the example but we need to go in details what has been done.*"

Students #25: "*I think there is a problem coz there is a many problems to his side when he came to us. But he gives us the same example in the book when we take it; it's not the same as he did on the board.*"

Students #28: “*but I think the teacher is good but not always. He is lazy the teachers. Teaches from the textbook. He takes example as it. And do it.”*

These comments support the literature that claimed that TVET colleges employed staff that were of a poor calibre, (Department of Higher Education and Training, 2011:10). These comments also authenticate the researcher’s claim, in the literature review, that other researchers did not establish that lecturers lacked commitment to and ownership of the NCV programme. The researcher concludes that such a lack of preparation and planning for lessons might have contributed to poor delivery, adding to the conceptual and procedural difficulties being investigated.

4.2.3.2.2 Homework supervision

The results presented in Table 4.18, Figure 4.8 and Figure 4.9 demonstrate that 40% of students and 40% of Subject Matter Experts indicated that lecturers set too much homework yet seldom review it:

Student #18: “*…the lecturer, he or she are giving us the homework of 20 sums. We can’t do 20 sums when we have another subject that we need to go through. That’s why the students of level 4 which is doing math are usually getting tired of study because of a lot of homework, but we usually ask lecturer, we come with the mechanism of simplifying the equation giving us the problem and check the homework if we are done correctly or what. It might happen that a lecturer might not check a homework.*”

Student #25: “*then he give us the lot of work. Maybe the whole of exercise in the book. But he gives us the same example in the book when we take it. It’s not. It’s same as he did on the board. He give us the homework for whole exercise... If you ask her* [to check the homework], we even *say please.* [Lecturer responds] *When I finish do this I will continue with the problems. Maybe he is finish later then we forgot.”*

From these comments, the researcher concludes that lecturers tend to assign class activities and homework for completion of activities and not for reinforcing the understanding of the concepts and procedures taught. If homework is not reviewed, then it can be concluded that students will carry conceptual and procedural difficulties experienced into the learning of new concepts and procedures, like that of factorisation, and this situation will compound the student's experiences of difficulties. In addition, NCV Level 4 students study seven subjects concurrently; if all their lecturers give homework every day one can conclude that it could become very demanding and stressful for the students.

The HOD and lecturer #3 remarked:

Lecturer #3:

"You will find the lessons that we have at college does not allow sufficient time for us to check that homework is done all the time but when random checks are done, if 10% of the learners have attempted the homework then that is a large number, really. And definitely that is a contributing factor to why these results are so poor."

HOD:

"You will find, yes we give the learners homework right but remember they getting homework from seven subjects and no lecturer is thinking what the other lecturer is giving when it comes to homework. So we not balancing out the homework at all, so the learner has to go home after that and do homework for 7 subjects and he may find that he doesn't even have time to do his maths homework to really understand his homework. And also when it comes to the next day and they bring it back to class, our time constraints do not actually allow us to interrogate that homework for who do not understand, we just have time to move on."

These responses confirm that the students' claims and the researcher's opinion expressed in the literature review that too much homework is given and not regularly reviewed, are legitimate. Lecturer #3 also claimed that students did not do their homework. Lecturer #3 and the HOD admitted to continuing with the teaching of new concepts and procedures without confirming whether students had understood or digested concepts and procedures taught. These factors might have also contributed to the difficulties being researched.

4.32.3.2.3 Teaching towards completion of assessments and syllabus

The results expressed in Table 4.18, Figure 4.8, and Figure 4.9 indicate that 20% of the students and 60% of the Subject Matter Experts suggested that lecturers were teaching towards completion of the prescribed assessments and syllabus.

Student #18 was convinced that lecturers were doing so, not for understanding:

"What is the concern of the lecturer is to finish the syllabus without knowing the student is capturing the syllabus correctly. Finish the syllabus and they don't want to concern of understanding."

Lecturer #1 said:

"We teach towards an assessment. We are not worried about the concept. So what do I teach the learners? I teach them this because they are going to write this… here we are concentrating more on ICASS marks at the expense of understanding."

"*When I am coming in I say I know that the test is on that, I will just not go and teach those concepts, I will say go and learn those concepts, they will master that thing but when it comes to understanding even a different situation, they can't apply it."*

"Then when I come to exam, I'll want to challenge them or I am bullying them with a test – things which I have not exposed them to. That one again discourages learners."

Lecturer #2 had this to say:

"Students are taught to pass maths not to understand maths, but if he is taught to pass maths, if he is taught to pass the section on factorisation then he won't see the application of it..."

One HOD expressed his view:

"The lecturer is teaching them and he only does this, he has four sections of factorisation, but he does them in his vacuums, he does that, that, that, that and he leaves it at that, he doesn't have are consolidated activity."

"In factorisation as a whole if we teach the learner the concept of removal of common factor, then what the learner does is, we are giving him an activity that is based on removal of common factor only. So immediately, he knows he has to check for removal of common factor and he solves. Let's say he doesn't make the other errors of equating to zero, then if I go on to the next one, let's just say is grouping, then he knows the activity is based on grouping so his mind is focused that he'll only be doing grouping. The same with difference of two squares. Now the problem comes up when we combines and I ask him to factorise. Now question papers do not tell you factorise by the difference of two squares. So what the learner does, the learner cannot go ahead and identify what type of factorisation he needs to do. So that's one of the problems, he learnt a procedure but he cannot conceptualise now which procedure to actually follow when and that's when he makes the errors."

Lecturer #1 admitted to teaching towards tests and assessments and not for the understanding of concepts. Herein lies a major reason why students experienced the given difficulties. This speaks about isolated learning. Lecturer #2, the HOD and the Umalusi Moderator also gave similar reasons as to why students experience such difficulties: teaching students to pass at the expense of understanding, teaching in a vacuum, compartmentalised teaching and teaching recipes. These findings also concur with the literature review that educators, who drill mathematical procedures, without attempting to provide any level of understanding, create confusion amongst their students, (Tularam & Hulsman, 2015:2). Such students are prone to apply procedures in isolation to obtain an answer rather than understand and solve problems. These students become dependent on compartmentalised procedures to solve mathematical problems and very often use procedures 'mindlessly and mechanically' in incorrect mathematical contexts.

4.2.3.2.4 Supervision

Only the Umalusi Moderator spoke about the lack of on-the-job supervision and guidance by lecturers. However, this was a key finding:

Umalusi Moderator:

"I find that the biggest omission in the teaching of maths at all levels is insufficient guidance while students are solving problems. There is a fair amount of lecturing, fair amount of presentation by lecturer but insufficient opportunities for students or lecturers or tutors to watch the students at work, because math cannot be learnt by watching, it has to be learnt by doing."

An important aspect of teaching and learning is the individual, hands on attention that students should receive while working with an application activity or task immediately after teaching during a lesson. The Umalusi Moderator makes the important claim that there was a lack of such on-the-job supervision. This is important as students learn and understand concepts and procedures from guidance while practising. Lecturers did not provide adequate supervision to students while they practised. Therefore, another factor as to why NCV L4 students experienced the given difficulties was the lack of on the spot supervision and intervention.

4.2.3.2.5 Remedial measures

The results show that 40% of the Subject Matter Experts spoke about conducting assessments solely for mark generation at the expense of healthy academic interrogation and remediation:

Umalusi Moderator:

"In fact the ICASS, the real intention of the ICASS is firstly, to determine the consistent student who is working consistently and the other is to help students to prepare for the exam. So that they find their mistakes. It's actually giving them rehearsal for the entire exams. Each of those major tasks especially the formal tasks are intended to give them preparation for the exam. If not used as such, doesn't serve any worthwhile purpose. To the best of my knowledge, I have not seen it used in that manner. That's regarded as a task, completed, packed away, over and done with, irrespective of the fact that half of them may have scored close to zero."

Lecturer #1:

"…the ICASS instrument itself and it is designed by and large to do diagnosis, to correct… but performance is generally poor. That is, that stays there. Nobody really goes and seriously does any diagnosis…In other words what's the point of teaching if students getting it wrong and there's no remedial work."

According to the mathematics Level 4 assessment guidelines laid down by DHET (2013:10), Internal Continuous Assessment (ICASS) accounts for 25% of the final mark. Students obtain an ICASS mark by writing seven formal assessments during the course of the year. The guideline also clearly states, "Assessment benefits the student and the lecturer. It informs students about their progress and helps lecturers make informed decisions at different stages of the learning process." From the responses, the researcher concludes that lecturers conducted assessments for mark generation and compliance purposes only. This lack of utilisation of marked assessments as tools that informed teaching and learning was a factor contributing to the said difficulties.

4.2.3.2.6 Lecturer competency

The HOD also adduced the following reason as to why students experienced these difficulties:

"...the barrier may be with the lecturers themselves, because our lecturers are coming from a background where they have not gone through formal training in terms of being a math teacher. They can factorise, they can do everything except know the theory behind those things. They have been using procedures in their studies, and if you even go up to the N5, N6 maths is done procedurally… For example, if I even have to digress from here and you ask differentiation, you ask a level four lecturer why is he using differentiation, explain the concept of differentiation, what does he start off with? He starts with telling them what differentiation is. What is it used for? But none of them, okay I won't say none of them, okay, many of them don't know what it is, they just know what they need to do to get the final answer."

Based on the response, it appears that most lecturers at this site had not completed formal training in mathematics education. Further inquiry revealed that just one of the three lecturers who teach the NCV Level 4 students possessed a qualification in mathematics education and only two out of ten who teach NCV mathematics had attained such qualifications. Literature confirms that lecturers at TVET colleges lacked pedagogical practices, which speaks about the 'art of teaching'. The literature review also acknowledged the Green Paper for Post-School Education and Training's claim (MHET, 2011:24), that the single greatest contributor towards underachievement in the TVET sector is the lack of capacity of the lecturers, especially in their subject content knowledge and expertise. The researcher concludes that this lack of lecturer competency was a factor contributing to the given conceptual and procedural difficulties.

4.2.3.2.7 Summary of lecturer findings

In summary, it was found from most student's responses that lecturers came unprepared to class and taught only from the textbook, they set too much homework, did not review it, and moved rapidly to other topics in order to cover the syllabus. The Umalusi Moderator also suggested that another factor that compounded conceptual and procedural difficulties was the imbalance of time allocated to each topic. This questions the lecturer's facilitation (year) planning and the execution of that plan. Similarly, the lecturers who formed part of the Subject Matter Experts admitted that due to time constraints they utilised the available time to complete the syllabus at the expense of understanding. For this reason, they were unable to review homework or discuss difficulties that students experienced.

The Subject Matter Experts' responses also revealed the following important findings concerning most mathematics lecturers:

- The latter taught content towards tests and assessments in isolation and not for the understanding of procedures and concepts
- They taught students to pass exams at the expense of understanding, they taught in a vacuum, and they practised compartmentalised teaching and teaching recipes
- They did not provide supervision and guidance to students, while students were engaged in classwork
- They conducted assessments solely for mark generation at the expense of healthy academic interrogation and remediation and
- They lacked formal mathematical pedagogical training.

Mathematics is a sequential subject, therefore lecturers cannot teach students in isolation and expect them to understand and flexibly integrate disparate fragments of learning. These findings also concur with the literature review that educators, who drill mathematical procedures, without attempting to provide any level of understanding, create confusion amongst their students, (Tularam & Hulsman, 2015:2). Such students are prone to apply procedures in isolation to get an answer rather than understand and solve problems. These students become dependent on compartmentalized procedures to solve mathematical problems and very often use procedures in wrong mathematical contexts. According to the mathematics Level 4 assessment guidelines published by DHET (2013:10), ICASS accounts for 25% of the student's final mark. Students obtain an ICASS mark by writing seven formal assessments during the course of the year. The guideline also clearly states, "Assessment benefits the student and the lecturer. It informs students about their progress and helps lecturers make informed decisions at different stages of the learning process." From the responses, the researcher concludes that the lecturer's lack of utilisation of marked assessments as tools that informed teaching and learning was a contributing factor to why NCV Level 4 students experienced conceptual and procedural difficulties while factorising and solving problems that involved factorisation. The literature review acknowledged the Green Paper for Post-School Education and Training's (MHET, 2011:24), claiming that the single greatest contributor towards underachievement in the TVET sector is the lack of capacity of the lecturers, especially in their subject content knowledge and expertise.

4.2.3.3 Students

The results expressed in Table 4.23 and Figure 4.8 indicated that 40% of students and 100% of the Subject Matter Experts commented on various reasons on how students contributed to the conceptual and procedural difficulties they experienced in the given situation. The researcher discusses these reasons in the subsections that follow:

4.2.3.3.1 Academic calibre of NCV students

Students #28 and #2 indicated that they studied mathematical literacy up to grade 12 at school:

The researcher asked Student #28: "Why did you not do math literacy at college?"

The student responded as follows:

Student #28: *"It's because I found that, hum Safe society is full. So that I have to do welding. I found that you do pure maths. No literacy. It is very difficult to learn maths when we didn't do at high school and it's very difficult to live in rural area because here we have to talk English and at high school, we talk Zulu until grade 12. So I face some difficult."*

Student #2 reported the following:

Student #2: "*The problem I have with mathematics is... is hard but is because I don't have a basic for it. I just learn math literacy."*

Safety in Society is a programme that offers mathematical literacy as one of its fundamental subjects. Student #28 could not register for Safety in Society and opted to register to study Engineering and Related Designs, which is an engineering programme that only offers mathematics, not mathematical literacy, as one of its fundamental subjects. This student experiences many difficulties: firstly, his/her intention was to choose a career in Safety in Society but had to choose an alternative career path so that he/she could qualify for the NSFAS bursary. In addition, this student acknowledges that he/she has a problem with English, the official LoLT. Student #2 also reported that he/she lacked the basics and found pure mathematics to be very difficult. The enrolment requirement for any NCV programme is Grade 9 mathematics; however, trying to recall foundational grade 9 mathematics, after completing three years of studies in mathematical literacy, might have been challenging for this student.

The literature reviewed highlights student-related issues such as a poor foundation in mathematics, and poor reading, writing, and research skills required for study in the TVET sector, (Papier, 2014:38). A lack of a strong foundation in mathematics or the long time that has lapsed since these students engaged with foundational mathematical concepts and procedures appears to be a reason why students experience the conceptual and procedural difficulties mentioned.

The results expressed in table 4.23 and figure 4.8 indicate that 80% of the Subject Matter Experts commented on the poor calibre of the NCV students at inception:

Lecturer #2:

"We are forced you know to just fill in the numbers, we'll be taking anyone and sometimes learners with grade 9 and grade 10…it's a bit difficult, for a learner who has come from a school system where they have done maths literacy for example and now you giving them to do pure maths."

HOD:

"For the entrance criteria, they can come in from grade 9, passed grade 9, we also accept maths literacy in terms of the engineering programs, now what happens here, Maths literacy learners are coming. They are getting together with maths learners as well…and we have found and it a most definite finding that they battle with level 2, because there is a very, very big difference between maths literacy and maths. Foundational concepts are not even there."

The researcher probed the HOD further by asking him: "What is the present calibre of NCV students?"

The HOD responded:

"At present, it is below average I would say, you see we can't assign all of them that [very weak], *but if we have a general statement we will say very weak, yeah, very weak."*

Lecturer #1:

"Those coming straight from Grade 9 in our system they lack a lot of concepts and also we have other learners who were doing math lit prior but when they come here they are doing Maths. I tried to babysit them, the more you try to understand them, the more you give them back, the more they become confused. So at the end of the day, time is taken, you have to fight against time, fight against completing the syllabus. At the end of the day, what happens to the learner, just leave them behind because time is not allowing you to babysit them. So that is the other major, major, major, major, it is a big problem – a big elephant of which none of us have the power to control."

Table 4.23 and Figure 4.8 illustrate how 60% of the Subject Matter Experts also asserted that the poor academic calibre of the NCV students was due to inappropriate initial foundational teachings and a lack of prior knowledge:

Umalusi Moderator:

"It boils down to the initial teaching of these concepts. I mean, you can go right down to primary school. The point is if a student does not know the meaning of factorisation, what is the meaning, outcome is someone asks you to factorise, what must the outcome look like? And that is the first problem."

Lecturer #2:

"People in schools are focusing majority on grade 12 but nobody comes to the realisation that the foundation is not grade 12, you know if you didn't build a proper foundation in grade 8, 9 and 10 the learner is going to be battling in grade 12."

The responses confirm and triangulate the responses of the two students who claimed that they found mathematics very difficult because they studied mathematical literacy at school. The responses also support the literature in finding that the majority of students who enrol in the NCV programme lack basic foundational mathematical concepts and procedures. Many NCV students are school dropouts while others enter the programme having done mathematical literacy at school. Students like these are in the same class group of students who have completed and passed grade 12 mathematics. These students are bombarded with various mathematical concepts and procedures in short bursts, mainly because lecturers facilitate at the pace of the students who have passed grade 12 mathematics and also due to time constraints. Therefore, the researcher concludes that the poor academic calibre of students in terms of lack of foundational concepts and procedures also contributed to the conceptual and procedural difficulties experienced.

4.2.3.3.2 Compartmentalised thinking

The results expressed in Table 4.18 and Figure 4.8 indicate that 40% of the Subject Matter Experts commented that students learnt in a vacuum, in isolation and without scaffolding.

Lecturer #1:

"As I say, they may lack the concepts… when you are learning maths, you need to have building blocks, you build from one level, and you go to the other. It means the learners do not have that problem-solving idea. Once you understand the previous concepts, and you cannot apply them, it means still you are lacking concepts."

The lecturer response suggests that students are unable to use and integrate learnt concepts and procedures to new learning experiences. This lack of scaffolding of learnt knowledge and the fact that lecturers also teach in isolation contributed to the given difficulties.

4.2.3.3.3 Student discipline

Table 4.18 and Figure 4.8 indicate that 60% of the Subject Matter Experts commented on student discipline:

Lecturer #1:

"But coming to the aspect of those learners who understand sometimes when they are learning, they don't go and revise that is the major aspect when it comes to the learners themselves. Learners on their part are not taking the time to revise where you left them that is where you get them tomorrow, are like wheelbarrows."

Lecturer #3:

"To be a master of anything, not even mathematics, it comes down to something which is called practice, practice and practice. I believe that the learners do not take part in any homework activities. They take it for granted that the teacher is able to project the answer on the board very easily hence it is easy when they try weeks later or maybe 2 days before the exam or test preparing, they find they have forgotten. Yes, lack of discipline with regard to practicing what they have learnt."

HOD:

"...now that is one thing poor attendance, now punctuality in class, you know punctuality plays a very important concept, it could be attributed to due to transport in the morning, that's okay, that's fine. Punctuality is not only in the morning, it seems like for every lesson learners are late. After lunch they are very late to attend a class."

From the responses the researcher concludes that NCV Level 4 students lacked discipline, they did not revise, did not complete homework, did not attend class regularly and punctually. The literature review refers to the frustration expressed by lecturers because of students' behavioural problems and lack of discipline, (SAQA, 2016:20). Due to the sequential nature of mathematics, such ill-disciplined students miss out on important concepts and procedures.

4.2.3.3.4 Language barrier

The results expressed in Table 4.18 and Figure 4.8 indicate that 60% of Subject Matter Experts expressed the view that language was a barrier to conceptual and procedural understanding of factorisation:

Lecturer #2:

"For some learners we do have this language barrier problem. A simple thing a learner doesn't know the difference between and equation and expression, sometimes that's why when you ask them to factorise, they will equate it and solve for x."

Lecturer #3:

"Many of the learners here at college, their first language is not English. I am unable to speak their language and also some frustration comes in when learners speak among themselves, they

do not speak in English, they speak in their mother tongue and I don't know exactly what they are speaking about."

HOD:

"Apart from English, learners have Zulu concepts of understanding. One hundred percent (100%) of them are Zulu speaking, because of the area, we in KZN. The ruling that we do follow is that lecturers must speak in English and furthermore the management at the campus have everything done in English. But we always advise our lecturers, nothing stops them from explaining concepts in Zulu after they have explained it in English. So you can explain everything in English once you digress from the language and do everything in Zulu learners are going to find it difficult when faced with an English paper."

The responses from the Subject Matter Experts affirm and consequently triangulate with the researcher's findings from the student interviews, and the literature review findings that the LoLT was a significant factor as regards poor student performance, (Mokone, (2011:13). Based on the students' and the subject matter expert's responses it appears that the language barrier also caused students to experience difficulties.

4.2.3.3.5 Minimum pass rates and progression

Students need a minimum of 30% to pass NCV mathematics while, prior to 2015, students who failed mathematics progressed to the next level of mathematics study, but had to complete the lower level as well. The results expressed in Table 4.18 and Figure 4.8 indicate that 40% of the Subject Matter Experts cited this minimum pass rate and progression as reasons why students demonstrated difficulties in the given situation.

Lecturer #2:

"…also I find one of the problems we were experiencing in the past before this year was it was possible for me to have a learner in my class that failed level 2 maths, failed level 3 maths but he's sitting to write level 4…… Masses of students, you know you in a classroom for level 4 where you had maybe 30 learners, you had almost 20+ learners that did not pass level 2 and level 3 maths. You had maybe out of that 30 maybe 5 or 6 that passed level 2, failed level 3 and now attempting to pass level 4 which in my opinion it's impossible you know I don't even think a magician can do that. The concepts you expecting them to carry through from level 2 to level 3 and then into level 4, you know it's gone, they failed it, they didn't understand the concept then. The quality of passes… maybe we are getting a learner who passed level 2 but he got 30% when he passed level 2, a subject like maths should not have a pass rate of 30%"

HOD:

"Learners just require 30% at level 3 and they progress to level four, some of them fail level 3 and they go to level four because of the criteria used in terms of DHET. So, a learner sitting in the level four class doesn't have a full comprehension of the level 3 syllabus. And in mathematics the level 3 leads on to the level 4, it is that way, and we get a learner, let's say a learner is doing two levels at one go, he's attending the higher level class, he's going to battle, he's going to find it very difficult. Now we are saying let's say even if a learner comes up with 30-35% at level 3, it means he only knew 35% of the work. Whether he was taught 35% of the work that remains to be seen, we can't comment on that, but he knows 35% of the work. He goes to a higher level and remember at a higher level that lecturer is assuming that the learner knows, he's coming with pre-knowledge, because the sad part about it is, there's no way for a teacher to go back and teach level 3 work plus level 4 work. He has to assume the learner knows."

From the responses, it is evident that policy on minimum requirements and progression requirements to the next level appears to set too low a standard; students, enter the next level of mathematics underprepared and without a strong mathematical conceptual and procedural base. This may have contributed towards why students experienced the difficulties being researched.

4.2.4 Possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems

In this section, the researcher presents the suggestions of Subject Matter Experts concerning the third research question, "What are possible ways in which NCV Level 4 students can understand factorisation and flexibly use factorisation to solve problems?"

Table 4.19 and Figure 4.10 represent the frequency of suggestions that were forthcoming:

Table 4.19: Tool for codes from interviews: Frequency of suggestions

Keywords and phrases	Frequency of code N = 5	Subject matter expert
Time:		
Sort out the issue of **time** holistically	3	#1; #2; UM;
Student and lecturer attendance	1	UM;
Lecturing:		
Teach concepts, teach for **understanding**, teach **basics**, teach **higher order**, teach **prerequisites, application,** don't practise **recipes** only**,** teach in **context, teach problem solving**	4	#1; #2; HOD; UM
Lecturer competency:		
Teacher training	1	HOD;
Lecturer must understand concepts	1	UM

Lecturer's mind set	1	UM
Teach as a team:	3	#1; #2; HOD
Tutorials		
Catch up programmes/tutorials/tutorial classes	3	#1; HOD; UM
Teach from mistakes during tutorials /Teach students to check	1	UM
Guide students on the job/ while they attempting/immediate intervention. Point out mistake immediately	1	UM
Individual attention	1	UM;
Language:		
Address the **language** barrier/teach math language/teach exam questions	3	#1; #3; UM
Infrastructure:		
Improve infrastructure facilities	1	#1;
Lecture/student commitment	3	#1;HOD: UM

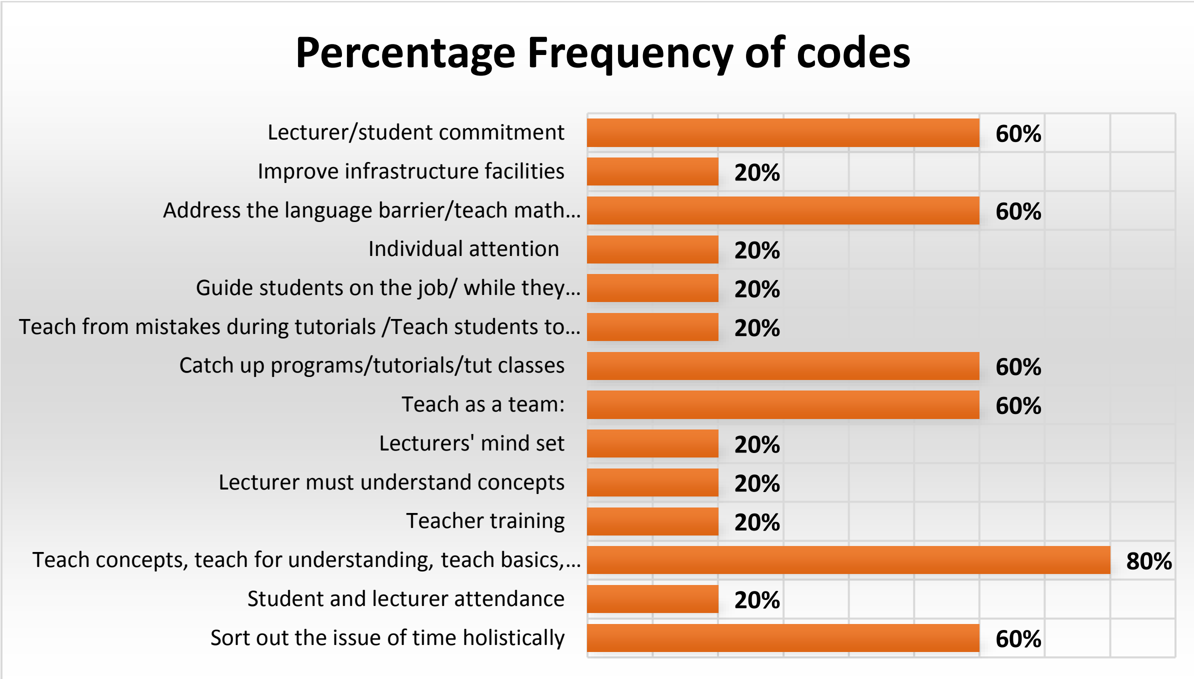


Figure 4.10: Frequency of suggestions

The researcher classified the codes of suggested solutions, from the perspective of the Subject Matter Experts, into five core categories: time, lecturing, lecturer competency and commitment, tutorials and language.

4.2.4.1Time

The results expressed in Table 4.19 and Figure 4.10 indicate that 60% of the Subject Matter Experts responded to sorting out the issue of time:

Lecturer #1:

"So that is what I can say basically. 2 major things here: One bigger is the generic picture, the structure of our timetable, the structure of our time which we are given so I think in general, the authorities need to look in that because maths is like a building block – once you miss one step,

there is big problem on the higher levels. This is L4 who are struggling with basics which are done at Grade 10, even Gr. 9, which means they really need to get ample time, need much more practice in the classroom."

Lecturer #2:

"Subjects like maths should be allocated in the morning rather than the afternoon."

Umalusi Moderator:

"Time…now that is beyond the control of the maths lecturer, beyond the control of any mathematical research because what input could you make in order to change that. That is a decision that comes from policy makers or bureaucrats, not mathematicians or lecturers or academics or educationists who want to actually solve the problem with maths."

As gleaned from their responses, most Subject Matter Experts felt that the issue of time allocation lay in the hands of the college's senior management team and higher authority. The Subject Matter Experts recommended that the college authorities allocate sufficient time for mathematics on the timetable. Lecturer #2 recommended placing mathematics during the morning sessions while the minds of students are alert. In addition, the Umalusi Moderator asserted that the issue of time is beyond the control of mathematics lecturers or academics but lies in the hands of policy makers and bureaucrats. The researcher concurs with the subject matter expert's points. Although this is beyond the scope of this study, the researcher suggests the following:

- The mathematics curriculum documents should give clear and specific guidelines in terms of actual contact times of teaching and learning, instead of notional hours.

In this way, address the issue of time constraints, which in turn will allow lecturers to develop their facilitation plans and lesson plans more flexibly. This will allow for homework review, assessment feedback discussions, and remediation. The researcher concludes that these recommendations are possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems.

4.2.4.2 Lecturing

The results expressed in Table 4.19 and Figure 4.12 indicate that 100% of the Subject Matter Experts were of the opinion that possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems lie in the approach to teaching:

4.2.4.2.1 Teach concepts from basics for understanding

Lecturer #1:

"Start with numbers, general numbers, we show how to write it maybe as a product of its factors then they understand what is a factor first of all. Start from numbers. Understand the idea of HCF maybe start with a factor, come to common factor, and come to HCF. You go to LCM, start with common multiples, come to multiples of numbers, become that one of lowest common multiple especially we are doing algebraic fractions when you factorise, you need to take which one is common denominator so you need to understand it from where it starts and move with it. Section 1 no. 1 you see 4 and x was supposed to be out factorised as the HCF. Then it is out but one of the problems is that they saw one of the common factors, they saw the 4 but didn't see the variable. Concepts need to be taught here, how to do the HCF, the concept of HCF needs to be taught when it comes to factorisation because you look for a HCF, you factor it out, and then you divide each factor as each term by that HCF."

Lecturer #2:

"So I found ways around when I teach it I recommend to them when we doing any factorisation problem to do prime factorisation. We breakdown every single number into its, in terms of prime

numbers e.g. 8 = $2 \times 2 \times 2$ *, 12 would be* $2 \times 2 \times 3$ *, into its prime factors in other words. In addition, what I encourage my learners to do, for example, if I got* x^3*, break that up as well to say its* $x \times x \times x$*. If a student is taught to understand maths then yes he can see he requires application of factorisation… The focus is methodology… I would start with prime factorisation; explain that concept to my learners over and over again… Then explain the concept of highest common factor to them and lowest common multiple to them."*

Lecturer #3:

"The memory part we are speaking is the sequence of rules we are supposed to apply. Rule number one, we should always do when we are dealing with factorisation is the removal of a HCF. Thereafter there may be slight variations of this rule but for the purpose of this interview, we will mention rule number one to be HCF. Many of us have been taught afterwards if this fails, rule no.2 try to factorise by means of grouping now if these sequences are actually taught and if these sequences are memorised by the learners, then it will improve their ability to factorise efficiently."

HOD:

"So the first concept is to define what factors are. For e.g. we use a simple concept of going back to basics and mathematics is like that. You always take something that is difficult, but you take something that is easy and explain it via that system, so for e.g. say that if I had to do it, I would take 12 for e.g. the number 12. I would start by working with the number 12 and explaining what the factors of 12 are. If I have to factorise 12 for e.g., you know we can do it as 4 x 3 or 2 x 2 x 3. Whatever the case is but all in all what we are telling the learner is you start off with as simple, with what we know. What they tend to do is, simply start with removal of common factor, and just showing it to them. Why don't you just start with numbers? For e.g. go with a number like 12 + 4 and ask them just to remove a common factor from there and show with simple something they can do."

"…Curriculum it's quite vast and when he comes to the class we not saying our lecturers don't go back and teach them basics, they have to do that. If you start some work you always start with the basics and you lead up but the learner should come with something that he knows but these learners are failing, because once you lost in mathematics, you going to be lost."

Umalusi Moderator:

"It starts at the earliest stage in which they are given simple terms. What is the meaning of factorisation? What is the meaning of multiplication? What is the meaning of division? You know the actual meaning. That is why it is very important to ask students to say, what have I done when I have the answer to a factorisation problem? I have actually taken that product and I have worked out what is multiplied by what will give that product and I started off with. If they cannot conclude that at the end of the exercise then you are always, going to have a problem…In NCV Maths L2, 3, 4 the level of maths is a bit high. You can't just go practise recipes and find yourself. It's okay you come out by practising recipes but you are not going to have an understanding of the problem at Level 2 and higher. You'd probably get the basic problems, basic skills right"

"I've found that if you really got students to understand what they do you, you are teaching, in other words teaching should never just be 'watch me'. But the skill without the understanding is really futile. It doesn't serve any purpose."

The responses reveal that all Subject Matter Experts emphasise that the teaching of factorisation should start from the basics. Additionally, 60% of them also emphasised teaching towards understanding the concept of factorisation, while 40% of them emphasised teaching the rules and methodology of factorisation. The Umalusi Moderator stressed that merely teaching or practising recipes may help students in getting the basic skill of factorisation right, but they will not be able to understand how to solve problems at higher levels.

Most Subject Matter Experts also emphasised teaching towards understanding the concept of factorisation, its procedures and its prerequisites. In a study done on Chinese children, the literature review found, it was reported that an initial emphasis on deeper conceptual and procedural understanding, before moving to more complex concepts, yielded higher levels of achievements for them than amongst their American counterparts, (Bailey et al., 2015). Based on this premise, lecturers need to value procedural knowledge and conceptual knowledge within mathematics education as complementary and vital. This kind of emphasis is also a possible way in which NCV Level 4 students can understand and flexibly factorise and use factorisation to solve problems.

4.2.4.2.2 Prerequisite knowledge, higher order teaching and problem solving

The results expressed in Table 4.19 and Figure 4.12 indicate that 80% of the Subject Matter Experts spoke about prerequisite knowledge, higher order teaching, and problem solving:

Lecturer #1:

"Also there is this theory of teaching whereby you have to go in a spiral way, you need to say when you are teaching the higher level to link so the learners themselves have to develop a problem-solving strategy so there are new theories - the maths we are supposed to be teaching now should be a problem-solving curriculum"

Lecturer #2:

"Level 4's now and we teaching trigonometry for example and we looking at the section and proving identities. Start by teaching factorisation all over again…. do a brief run through you know of the types of factorisation. Higher order level problems should be given to them in level 2, even though they don't understand the concept. Maybe they might not understand what $\sin^2\theta$ *is, you know in level 2 but if you give it to them as a factorisation problem,* $\sin^2\theta - 1$ *that's a sum and difference of two squares and we can introduce it to them from that early stage."*

HOD:

"If I'm teaching and for example I go to teach trigonometry and I know I'm going to deal with factorisation there, nothing stops me from doing one example on factorisation. I can be doing it prior to the lesson, I could be doing it in the lesson, but one example of factorisation with just variables must be done…We must always allow the learner to analyse questions, you know analyse it, understand it and be able to solve it. Now when we are speaking about this, you will find it coming up in terms of the same concept the learner could do the same factorisation here but when it came to this trig function here, he couldn't do it even though it was the same thing. It was the same question, except what, one was trig and one wasn't. Now this is what we need to do, in order to break that if I am teaching the learner removal of common factor for example and you got the equation $4x^3 + 4x$ *whatever the case may be right, I can do the same thing and put* $4\sin^3 x + 4\sin x$*. Integrate, you see whatever you have here, everything is factorisation, so to integrate the problems is a fantastic thing"*

Umalusi Moderator:

"So following recipes is fine, knowing a procedure or mastering it is wonderful but identifying context in which you would use that is a key part of understanding the concept. So if you understand the concept, then you will identify more easily where you doing what. So you say listen, I have to simplify this. What does it mean? I must divide the denominator into the numerator…as a rule would teach with understanding and the majority or a reasonable percentage of students would get by, there will always be some that will fall by the wayside and with time if they do come through a few years maturity would teach them that this is how it's done. So factorisation taught on its own for long periods of time is not really productive. It needs to be taught in other contexts, in simplification in various other places where you want to do it, finding the x intercepts, in solving equations, all these situations where you use it. After you have taught say trinomials, then you get a trinomial $x^2 - 2x - 3$ *and you factorise that. There is nothing wrong with saying if they know functions or any other functions for that matter or another exponential function like* $(x^2)^2 - (3x)^2 + 1$ *where you can do a substitution and say but that's a trinomial as well, or you can ignore a … function and say I can put anything in there so x is only a variable. So they*

understand they can put any other object in place of the x and factorise then that's fine. So even if there is no meaning for $\sin\theta$*, sin x but if you put a* $(\sin\theta)^2 - 2\sin\theta - 3$ *and ask them to factorise that then they look at that and they must be easily visually able to replace in their minds, the* $\sin\theta$ *with an x to see* $x^2 - 2x - 3$ *so it is a useful application. I would think it should be a feature of textbooks, the thing is, okay, for passing, the students need to be able to do the essentials say trinomials, the basic trinomials but for higher, what would call a higher level question for a L2 student? You will call a higher level question where you put in some other object, like a trig function into the quadratic form and ask them factorise that. That must be there for students to see that at an early stage. So then they make differentiation otherwise they think the only time you get a trinomial is when you get a quadratic function, in terms of the variable x."*

The responses reinforce the importance of teaching prerequisite knowledge, higher order teaching and problem solving as possible ways in which NCV Level 4 students could understand factorisation and flexibly use it to solve problems. In order to teach mathematics for understanding a lecturer cannot teach a concept and/or procedure in isolation. The lecturer must teach in 'context' by reminding students of what has been taught and how it impacts on or integrates with what is being taught and, at the same time, prepare students or build a foundation for them with respect to what needs to be taught, learnt or solved in the future. In the literature review, the researcher concluded that arithmetic and algebra are sequential in their nature and their study. This sequential nature of concepts and their procedures demands that lecturers structure their teaching and learning processes in ways that will 'scaffold' students' understanding and development of mathematical ideas from the point of dependence to that of independence, (Woods et al., 1976:90).

4.2.4.3 Lecturer competency

The results expressed in Table 4.19 and Figure 4.12 show that 40% of the Subject Matter Experts commented on lecturer competency and improving it, as possible ways in which NCV Level 4 students could understand factorisation and flexibly use this technique to solve problems:

HOD:

"…different people have different ways to do it, some are more effective than others, and we need to increase their effectiveness in terms of the class. So training in terms of classroom delivery is of utmost importance, we can't get away from it."

Umalusi Moderator:

"All you need for that is time cos a good teacher who knows exactly what is required of the students to understand, that is something in other words proper understanding by the lecturer himself. If the lecturer himself or herself only knows, procedure then that can be a serious and major challenge."

The HOD and the Umalusi Moderator stress that lecturers themselves must understand concepts and procedures and must be competent classroom practitioners in order to assist NCV Level 4 students to understand factorisation and flexibly use it. This research also found that the majority of mathematics lecturers at this campus did not receive formal or informal mathematical pedagogical training. The HOD also suggested mathematics lecturers should receive training to improve effective classroom delivery as a possible way of solving the issue being researched. The researcher agrees that this college's skills development plan should include formal and informal reskilling of its mathematics staff, in this way, improving the quality of teaching, even the teaching of factorisation as this in turn will assist NCV Level 4 students encountering the given issue.

4.2.4.4 Tutorials

The results expressed in Table 4.19 and Figure 4.12 show that 40% of the Subject Matter Experts motivated for ‘catch up programmes’ or tutorials:

HOD:

“We don’t have learners that can actually go out and do their studies and actually understand the concepts themselves. Somebody needs to tell them somewhere along the line what needs to be done. It starts off at the school level, the school level is where they come from with the basics right, so we find that learners come in here don’t have that basic knowledge. So what we do in terms of getting the learner up to par is by having some catch up programs in order to get him up.”

Umalusi Moderator:

“In cases where they are too huge or in cases where there are small groups, tutorials for practice. You cannot use too much of normal lecture time for practice because they are going to argue you are not going to complete the curriculum. So if it’s practice then you don’t need a lot of teaching. You have taught the concept, you have taught them a procedure, all they need is to repeat that procedure to develop proficiency and what do you need, you need a simple tutor to watch them practice. Which could be the lecturer, which could be anyone else who has good knowledge of that topic who could just assist with that sort of a thing. Eventually to give students adequate time to practice. There are, what you call it, computer software or programmes of that nature to practice or give them maybe immediate give back the answers. Trinomials there are any number of exercises that can be generated where questions and answer are generated so its opportunities for students to get practice especially those ones who need a lot of practice. As long as you don’t do that, the failure rate is going to continue.”

The Umalusi Moderator also suggested:

“And if you are doing, somebody got to watch if you are doing the right thing and that is when issues relating to concepts and understanding. I mean all of those things will come up e.g. if one watched the students doing something like any of these little mistakes that were mentioned, these ones were all done. Q1.4 – factorising $2\sin^2 x - 7\sin x + 3 = 0$ *. When somebody does that, e.g., I*

don't know how this got solved but it's absolutely incorrect. When someone does that, then one will, if there is a tutor or lecturer watching that, then there is an opportunity to ask a question – how did you get that and that? So that person said no that's a factor alright so let me write it down and take what is left but what was short was 2x-7 and 3(2x-7) then you pull it off and you get (x+3)(2x-7) but that was not seen and so if that is pointed out at that time it's being done. Then there would definitely be an understanding because that individual attention is very very necessary for the majority of the students. That can only happen in times when you are giving only practice, tutorials, afterhours practice whatever it is that needs to be done but that is necessary for students like this but if it's a bridging class that's basically what you are going to do, you are not going to reteach all of Gr.9 Maths. You are gonna pick up things, make them do things, find their mistakes and correct the mistakes. You are not gonna go and teach anything for the first time because it was taught already at school. It might mean some re-teaching but that would be focus. You are going to teach by error find out what the errors are and fix those errors rather than just go and teach curriculum in any bridging programme with a topic like factorisation"

The Subject Matter Experts' responses triangulate with the student's responses where they also revealed that students did not receive support or any intervention while they attempted to complete classwork because of time constraints. In an attempt to provide such support, the HOD and the Umalusi Moderator motivated for the implementation of 'catch up programmes' or tutorials outside of the normal contact time. This supported the researcher's opinion in the literature review that the lecturer needs to be able to correct conceptual and procedural difficulties 'on the spot'. Due to time constraints, the long curriculum, as well as other contextual factors discussed in this research, such catch up programmes, tutorials and 'on the spot' intervention are possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems.

4.2.4.5 Addressing the language barrier

Three Subject Matter Experts commented on addressing the language barrier:

Lecturer #1:

"The reading culture studying culture needs to start… important and then the issue of language is also another important one. If you are told to factorise, and you don't factorise. What does it mean? It means there is a language barrier as well. The language needs to be improved. Especially it affects the concepts."

Lecturer #3:

"Now we will notice, if we are given a statement which has an = sign, the instruction involved will never be to simplify, the instruction will always be to solve……If you are able to understand the mother tongue of these learners you are closing the gap between lecturer and learner."

Umalusi Moderator:

"Well, it seems to but I don't see why it should personally because every math lecturer must be able to teach the language required for the understanding of mathematical questions whether its maths or finance, calculus or anything. There is no special English tutor to be able to teach the language of math. Interpreting questions, I think it's very teachable irrespective of the home language of the learner. What is it that can't be taught in the context of maths? If it's a maths or finance question, the language, the problem is embedded in the language; you can't say I have taught without having taught the language that goes with it. If somebody teaches through medium of another language if it's Zulu or Xhosa or whatever it is. Listen, it's not going to help in conceptual understanding but finally when you are answering questions, maybe you should teach the concept, no problem but in terms of answering questions, you answer the questions in English, you show how to interpret the questions. Information must be able to be interpreted and the key mathematical facts need to be recorded. It's not a language issue; it's a teaching, a maths teaching issue. All of the questions that are set I have seen for the last many years now, are teachable. They are teachable, you can teach students exactly what to do with those words, how to use them to get the information you require to solve your problem. I think we make a lot of heavy weather of maths and English."

The Subject Matter Experts' responses reflect opposing views on addressing the language barrier amongst students and the learning of mathematics. Lecturer #1 suggested that students needed to improve their understanding of English by adopting a culture of reading. Lecturer #3 suggested that the lecturers should learn the mother tongue of the students, which in this case is IsiZulu. However, the Umalusi Moderator emphasised that the onus bridging the language gap lies with the lecturers. He suggested that lecturers should teach the language of mathematics and examination questions. The literature review concluded that although students view English as a language of power, opportunities for them to read, speak, listen, and write in English are restricted, especially in the township schools and colleges, (Alenezi, 2010:12). Many students also face the reality of teachers and lecturers who are not proficient in this language. These realities hamper such students in their construction of new concepts and their cognitive development in mathematics. The Umalusi Moderator's response contradicts the literature review and the responses of lecturer #1 and Lecturer #2 by suggesting that it is not a language issue but a mathematics teaching issue. The researcher supports the Umalusi Moderator's claim that mathematics concepts and the repetitive manner in which questions in the mathematics exam question are phrased is peculiar to mathematics. For example, words such as factors, difference, limits, and differentiate have a different meaning in a mathematical context from what they would convey in an English language context. Therefore, mathematics lecturers must teach the mathematics language, the words, and the phrases that are required for the understanding of this mathematical concepts and procedures. Such an approach may also be a possible way in which NCV Level 4 students could understand factorisation and flexibly use it.

4.2.4.6 Commitment

The subject matter expert's responses exposes the critical factors that were not identified in other research. Namely, NCV students lacked discipline and that NCV lecturers lacked commitment and ownership of the NCV programme.

The results expressed in Table 4.19 and Figure 4.12 indicate that 60% of the Subject Matter Experts commented on student and lecturer commitment:

Lecturer #1:

"If a learner wants to study, needs to come to college and say I don't want a disturbance. I need to go to what; I need to go to college for. When I go to college, I can study."

HOD:

"Also with the homework, you'll find when it comes to the homework, the learner needs to do his homework. That's the only way in mathematics that you are going to be learning it is by doing examples. Plenty of examples. Do lots of activities and past exam papers…"

Umalusi Moderator:

"There are certain bottom lines in maths. Everybody knows that everywhere and you just fail maths if you are not consistent. The campus managers need to deal with issues of non-attendance and arriving late and so on. That lecturer of maths must set the bar themselves by setting an example on being on time but outside of that if they are consistently like that, they are not mathematics students, they obviously are not going to succeed because that is, put it this way, catching up in maths is a very difficult thing, you cannot really catch up. Lecturer can catch up but the student doesn't."

Lecturer #1 stressed students' commitment towards their studies while the HOD stressed the importance of students completing their homework and the need for them to practise examination type questions. However, the Umalusi Moderator stressed the importance of consistency and the role of campus managers to address the issues of lack of commitment from students. The Umalusi Moderator also voiced the need for lecturers to be the example in terms of commitment. This kind of commitment from students and lecturers is a possible way in which NCV Level 4 students can understand and apply factorisation to solve problems.

The students' and the Subject Matter Experts' interview responses on why students experienced conceptual and procedural difficulties when factorising and solving factorisation problems revealed that NCV lecturers and students lacked commitment to the NCV programme. The responses also confirmed that NCV students lacked discipline and that NCV lecturers lacked commitment and ownership of the NCV programme.

Most Subject Matter Experts' responses in this regard revealed that all role players must be committed to the NCV programme. The researcher supports this recommendation and endorses the advice of the Umalusi Moderator that managers must deal with issues of non-attendance, late coming, and time constraints, while mathematics lecturers should set the example of being punctual and well prepared, and students must attend classes regularly, complete their homework, and study regularly. These are the 'bottom lines' of mathematics education. No amount of tampering with the timetable, improving the curriculum, provision of adequate resources, and reskilling of mathematics educators is able to guarantee that students will understand a concept such as factorisation and use it as a tool to solve mathematics problems if lecturers and students are not enthusiastic and committed to teaching and learning mathematics.

4.3 Conclusion

In this chapter, the researcher discussed the data analysis process followed; he presented the results and discussed and interpreted the findings. Based on the results and findings discussed in this chapter, he is confident that this research managed to unearth various conceptual and procedural difficulties that students experienced while factorising and solving factorisation problems. It also supported understanding of reasons why such conceptual and procedural difficulties existed, and suggested possible ways in which NCV Level 4 students could deal with these.

In the final chapter, the researcher concludes the research by discussing the findings, recommendations and the limitations of the study.

CHAPTER 5

SUMMARY, RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

5.1 Introduction

It will be recalled that the objectives of this study were to analyse the conceptual and procedural difficulties that NCV Level 4 students displayed when factorising and solving problems involving factorisation, to explore reasons why these students exhibited such difficulties and to suggest possible ways that could assist them to understand and flexibly use factorisation to solve problems. The researcher presents a summary of the chapters, a summary of the findings of this study, provides recommendations based on the findings, reflects on this study's limitations, proposes areas of further research, and finally offers conclusions. In so doing, the researcher has attempted to demonstrate that this study adequately dealt with and achieved the aims, objectives and questions of this research phenomenon.

5.2 Review of the research questions and chapters

The researcher clarified the problem statement, objectives and aim of the study by formulating the three research questions in Chapter 1:

- What conceptual and procedural difficulties do NCV Level 4 students demonstrate when factorising and solving problems that involve factorisation?

- Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?
- What are the possible ways in which NCV Level 4 students can understand and flexibly use factorisation to solve problems?

Chapter 1 laid the foundation that underpinned this study. Sections 1.1 and 1.2 introduced the study and raised the issue of underachievement in mathematics in South Africa, South Africa's poor international mathematics achievement rankings and concern over the extremely poor performance in mathematics at Technical Vocational Education and Training Colleges (TVET Colleges). Section 1.3 presented the background of the study by demarcating and describing the TVET establishment in South Africa, the NCV space within the TVET sector and the TVET campus at which the researcher conducted this study. The researcher then described the rationale of the study, motivated the need to conduct it, defined the problem statement, considered the aim of the study, outlined its objectives and formulated the research questions, respectively. A succinct literature review and the theoretical framework followed. Thereafter, the researcher outlined the research methodology, research design, population, sample, data collection methods, data collection instruments, data analysis, reliability and validity, ethical considerations, delimitations of the study and limitations respectively. Lastly, Chapter 1 concluded with a summary and an outline of the study's chapter layout.

In Chapter 2 the researcher presented a review of the literature and theoretical framework that guided this study. Section 2.2.1 reviewed literature on research that outlined factors which contributed towards underachievement of TVET NCV students, such as the lecturer's qualifications, experience, and content knowledge, language difficulties, and other contextual factors. Section 2.2.2 defined and compared conceptual knowledge and procedural knowledge and exposed the contrasting viewpoints on the relative importance of these found in literature. This study then adopted the standpoint that a balance between conceptual and procedural knowledge should co-exist, and assumed that this balance might reduce the severity of conceptual and procedural difficulties that students may have experienced when factorising or assist them to avoid such difficulties altogether. In section 2.2.3 the researcher also provided an overview of the said difficulties discussed in the literature. In addition, in sections 2.2.4, 2.2.5, and 2.2.6 the researcher discussed strategies found in various sources for overcoming these issues, the definition of factorisation, and the prerequisite knowledge needed for students to be able to engage with factorisation. In section 2.3.1, the researcher set the theoretical framework that underpinned the study where factorisation was seen through the ZPD lens. This framework presented a motivation for and illustrations of why conceptual and procedural knowledge are critical foundational strands for students to attain mathematical proficiency.

In Chapter 3, the researcher substantiated the research plan by re-introducing the aim, objectives, and research questions to refresh the reader as regards the purpose of this study. A presentation of the research paradigm followed. The researcher explained why this study adopted a phenomenological case study research design and provided reasons why this research was qualitative in its approach. A discussion ensued, on the role of the researcher, the choice of the location, the target population, the study sample, and the sampling techniques adopted. The researcher explained the design of the data collection instruments in detail, outlined the data collection process, and data analysis section, in that order. This chapter ended by exploring issues of trustworthiness, triangulation, the research delimitations, limitations and elimination of bias, ethical considerations and was summed up by means of a conclusion.

In Chapter 4, the researcher discussed the data analysis process, presented the results, and discussed and interpreted the findings of the data collected through the written assessments and the semi-structured interviews. The researcher motivated why this research adopted an integration of constant comparison analysis and classical content analysis to reduce, and display, the collected data (Section 4.2). In section 4.3 the researcher analysed the data from the results of the students' written assessment tests with the objective of authenticating this study. In this section, data was presented on the extent to which students were experiencing conceptual and procedural difficulties when factorising and solving problems that required factorisation. In addition, an interrogation of the research questions ensued, based on the evidence gathered during the students' and the Subject Matter Experts' interviews. The researcher addressed the research question by means of three subcategories to facilitate the analysis of data collected. These included procedural difficulties, conceptual difficulties, and procedural fluency. The researcher identified core categories, analysed, and interpreted these categories in light of the research questions and literature reviewed from the data collected. An analysis and interpretation of student challenges and possible ways in which NCV Level 4 students could understand and flexibly use factorisation to solve problems from the perspective of the Subject Matter Experts concluded that chapter.

This chapter (chapter 5) provides the overall results, conclusions, and recommendations of this research study. The researcher reviews the research questions and summarises the other chapters here. This chapter includes a summary of the findings of this research in view of the research questions, and the literature reviewed. The researcher also respectively summarises the chapters and findings, provides recommendations based on the findings, reflects on this study's limitations, proposes areas of further research and concludes this research study.

5.3 Summary of research findings, literature review, and recommendations

This section, based on each research question and core categories that emerged, summarises the research findings according to those categories, links the findings to the literature reviewed in this study, and makes recommendations.

The written assessment test revealed that the majority of 30 NCV Level 4 students who wrote the test experienced conceptual and procedural difficulties with all types of factorisation. This finding concurs with Papier (2014:38) who claims that TVET college students lack a strong foundation in mathematics.

5.3.1 Conceptual and procedural difficulties demonstrated by NCV Level 4 students

The first two research questions attempted to identify and describe the conceptual and procedural difficulties NCV Level 4 students demonstrated when factorising and solving problems that involved factorisation and why those NCV Level 4 students demonstrated such difficulties. The results are presented in the sub-sections 5.3.1.1 -5.3.1.3.

5.3.1.1 Procedural difficulties

The results of section A of the assessment test revealed that 68% of the students' incorrect responses reflected the following types of procedural difficulties:

- Incorrect further simplification of algebraic expressions
- Correct type of factorisation used incorrectly
- Incorrect type of factorisation utilised and
- Incorrect procedure used outside of factorisation. This was evident when some students' responses revealed the use of differentiation rules, while others treated the given algebraic expressions as equations and proceeded to solve them.

This implies that students lacked conceptual connections or deeper meanings since they had merely memorised procedures without understanding them (Tularam and Hulsman, 2015:2).This results in their being confused and unable to choose a suitable rule or procedure when solving a problem. Based on Tularam and Hulsman's (2015:3) definition of procedural difficulties and these findings, the researcher concluded that most of the NCV Level 4 students were unable to use factorisation procedures correctly.

5.3.1.2 Conceptual difficulties

The results of section A of the assessment test revealed that 54% of the students' incorrect responses reflected the following high frequency categories of conceptual difficulties. The students:

- Confused products and factorisation
- Could not differentiate between expressions and equations
- Applied procedures that suited their own private understanding of concepts, multiple misconceptions, together with an inability to identify the concepts.

The students' interviews revealed that the majority of them could not define foundational concepts, such as factorisation, multiplication, expression, equations, and so forth, in their own words. Their inability to explain the meaning of key mathematical concepts related to factorisation validates the supposition that these students experienced some conceptual difficulties with factorisation.

The findings support Hansen et al's (2005:15) assertion that errors and misconceptions are errors learners make because of the lack of relevant knowledge or experience related to a concept, and the inability to check answers. It also concurs with the researcher's opinion (discussed in Chapter 2) that conceptual difficulties simply mean a complete lack of understanding of mathematical concepts, like factorisation, due to a lack of exposure to such conceptual understandings.

5.3.1.3 Procedural flexibility

The results of the written assessment test revealed that most students, even those who excelled in Section A, on basic factorisation, could not use it freely to solve factorisation problems in section B across most questions. This confirmed that students could not flexibly transfer conceptual and procedural knowledge of factorisation, to solve higher order problems that required this technique. In addition, they also tended to utilise misconceptions and incorrect procedures of factorisation in new and other areas of mathematical studies.

The individual students' interview responses also indicated that those students could not apply factorisation to solve different factorisation problems. Based on the interviews, the researcher found that some of the reasons why students experienced the given difficulties included a lack of understanding of concepts such as polynomials, number of terms in a polynomial expression, removing the highest common factor, limits, and zero denominators in given expressions.

About 86,7% of students were also unable to adequately understand factorisation as a means to simplify limits and solve quadratic, complex, and trigonometric equations. The researcher concluded that students are so conditioned and rigid in their ways of doing calculations that they tend to factorise only if the question explicitly instructs them to do so.

Furthermore, 20,5% of students who displayed incorrect responses in section B used the rules of differentiation to solve the problems. At the time of this research study, students engaged with the rules for differentiation with their class lecturer. The students' responses concerning why they used differentiation, confirm that they simply used the procedure that was fresh in their minds or procedures that they felt comfortable using. These findings agree with the literature reviewed, which notes that while memorising rules and procedures is important, often such memorising is at the expense of understanding conceptual connections or deeper meanings. Hence such exposure to so many rules or procedures without understanding, might cause students to become confused and, be unable to choose the correct rule or procedure when solving problems (Tularam and Hulsman, 2015:2).

In summary, the findings validate the literature claim that many students do not understand the reasons why certain procedures work, or even when they should consider alternative or equivalent procedures that may be more appropriate to solve a mathematics problem, (Tularam and Hulsman, 2015:3). In addition, these findings are in agreement with the researcher's conclusion from the literature review that students who learn procedures without understanding become confused. They then tend to apply procedures in isolation to obtain an answer rather than understand how to solve problems. Those students became dependent on compartmentalised procedures to solve mathematical problems and very often use procedures ‘mindlessly and mechanically’ in incorrect mathematical contexts.

5.3.1.4 Challenges NCV L4 students experienced

In an attempt to answer the second research question: “Why do NCV Level 4 students demonstrate difficulties when factorising and solving problems that involve factorisation?” the researcher prompted students and Subject Matter Experts to speak about the challenges that students experienced. Results indicated:

i. Time constraints where students lost a great deal of classroom instruction time owing to socio-economic factors and ill-discipline that affected them
ii. Lecturers’ lack of planning of suitable strategies to overcome time constraints and to teach for understanding through reviewing of homework and assessments conducted
iii. Inadequate availability of resources
iv. Students become dependent on compartmentalised procedures to solve mathematical problems and very often use procedures in incorrect mathematical contexts
v. Inappropriate initial foundational teachings and a lack of prior knowledge amongst students identified by subject experts
vi. The impact of the minimum national recommended pass rate of 30% for mathematics, which results in students entering the next level of mathematics study, under-prepared and without a strong mathematical conceptual and procedural base.

The findings also support the literature’s claim that TVET colleges employed staff that were of a poor calibre, (Department of Higher Education and Training, 2011:10). These findings also reinforce the researcher’s claim, stemming from the literature review, that other researchers failed to identify that lecturers lacked commitment to and ownership of the NCV programme. The researcher concludes that this lack of lecturer competency was a contributing factor towards why NCV Level 4 students experienced the difficulties investigated.

The literature reviewed highlighted learner-related issues such as a poor foundation in mathematics, and poor reading, writing, and research skills in the TVET sector, (Papier, 2014:38). One can conclude that the lack of a well-established conceptual and procedural foundational base in mathematics contributed to the conceptual and procedural difficulties that NCV students experienced with respect to factorisation.

5.3.2 Understanding factorisation problems

The third research question attempted to determine possible ways in which NCV Level 4 students could understand factorisation and flexibly use it to solve problems. The researcher analysed these questions in Chapter 4, based on the interview responses of the Subject Matter Experts. Some possible ways were found; they include:

i. Allocating sufficient time in the timetable to mathematics to allow for homework review, assessment feedback discussions, and remediation,
ii. Teaching of factorisation to start from the basics, in NCV L2, and ensure a balance between reinforcement of procedures and conceptual understanding,
iii. Lecturers must structure their teaching and learning processes in ways that will 'scaffold' students' understanding and development of mathematical ideas, moving them from dependence to independence,
iv. Formal and informal reskilling of TVET mathematics staff to improve the quality of teaching, even the teaching of factorisation,
v. Implementation of 'catch up programmes' or tutorials outside of the normal contact time to provide 'on the spot' intervention,

vi. Lecturers must teach the language of mathematics and mathematics questions, including the words and the phrases that are required for the understanding of mathematical concepts and procedures, and

vii. Managers must deal with issues of non-attendance, late coming, and time constraints, while lecturers should set the example of being punctual and well prepared, and students must attend classes regularly, complete their homework, and study regularly.

In the literature review, Bossé and Bahr (2008:20) suggest that teachers should ensure a balance between learning both concepts and procedures. Literature (Bossé, Bahr, 2008:20, Bransford, Brown, and Cocking, 1999) also emphasised that isolated procedural learning may become very fragile, forgotten easily or remembered inappropriately. Based on the research findings and those from the literature review, the researcher concludes that in order to teach mathematics for understanding, a lecturer cannot teach a concept or procedure in isolation. The lecturer must teach in 'context' by reminding students of what has been taught and how it impacts on or integrates with what is being taught, while at the same time preparing students or building a foundation for students regarding that which needs to be taught, learnt or solved in the future.

According to the research findings, arithmetic and algebra are sequential in their nature and their study. This sequential nature of concepts and their procedures demands that lecturers structure their teaching and learning processes in ways that will 'scaffold' students' understanding and development of mathematical ideas. In addition, findings suggest integrating and introducing topics or concepts in the curriculum that will require factorisation, for example solving equations, trigonometric identities, trigonometric equations, and substitution into complex quadratic trinomials, in order to prepare a foundation for students to learn future higher order mathematical concepts and procedures. This concurs with Vygotsky's (1978:86) concept of ZPD, in the literature reviewed, where students develop from being dependent to becoming independent through the guidance and supervision of the adult lecturer.

Literature cited indicates that improving students' learning and helping them overcome conceptual and procedural difficulties depend largely on the capabilities of the classroom teacher (Kilpatrick et al. Kilpatrick, 2001:12). Lecturers must then be competent Subject Matter Experts as well as proficient in pedagogical practices within their subject discipline. In order to achieve this, they need to subject themselves to continuous formal and informal reskilling on subject content related, in this case, to factorisation and solution of problems involving it.

5.4 Limitations of the study

This research study was limited to NCV L4 students who studied mathematics at a technology centre of a TVET college in KwaZulu-Natal. It focussed on conceptual and procedural difficulties that these students experienced when factorising and solving problems that required factorisation. The identities of the campus, the college, and all participants were kept anonymous. This study should, ideally, have been conducted in as many technology centres as possible but logistics, resources and financial constraints did not permit the researcher to do so. Hence, the findings of this research might not apply, or be generalised, to other TVET colleges and technology centres in South Africa.

Despite these constraints, the researcher envisages that this study will provide useful insights to all role players at this campus, which could feed directly into its teaching and learning processes. In so doing, current and future students may avoid or overcome conceptual and procedural difficulties they may experience when factorising and solving factorisation problems. This might motivate students to become mathematically proficient which, in turn, might improve the mathematics pass rate.

5.5 Possible areas of further studies

While attempting to accomplish objectives listed in section (3.2), other issues, that were not part of the focus of this research study, arose. In view of these emerging issues, the researcher recommends the following possible areas of further studies:

- The role of management in improving mathematics results at TVET colleges
- Lecturers' perceptions on the breadth and depth of the NCV mathematics curriculum
- Role players' perceptions on the impact of the minimum pass mark for NCV mathematics
- Perceptions of subject specialists as regards introducing a mathematics orientation programme prior to NCV L2
- Designing an NCV mathematics curriculum for mathematics lecturer training
- The impact of strike action at TVET colleges on students' results
- The relevance of the current NCV mathematics curriculum
- The importance of mathematics in the TVET programme
- Merging the mathematics and mathematical literacy curriculum in TVET colleges.

Finally, the researcher also strongly recommends that this study could be conducted using a larger sample.

5.6 Conclusion

In this chapter, the researcher reviewed the research questions and chapters. He summarised the research findings based on the research questions and the core categories that emerged, connected the findings to literature, and made recommendations.

This study aimed to investigate the conceptual and procedural difficulties experienced by NCV level 4 students when factorising and solving factorisation problems at a Kwazulu Natal KwaZulu-Natal Technology Centre. It showed that NCV L4 students experienced numerous conceptual and procedural difficulties when factorising and solving factorisation problems. It explored and exposed various reasons why NCV L4 students experienced such difficulties and it recommended possible ways that could assist NCV Level 4 students, at the technology centre of this study, to understand and flexibly use factorisation to solve problems. This study confirmed that a balance between conceptual and procedural knowledge should co-exist, that this balance will reduce the severity of conceptual and procedural difficulties that students experienced while factorising and solving factorisation problems or avoid such difficulties altogether. The researcher anticipates that the recommendations of this study would provide valuable insights to the readers, and relevant stakeholders at the college and technology centre of this study on possible ways they could adopt to overcome and avoid such difficulties. In doing so, motivate NCV L4 students to become mathematically proficient which in turn may improve the mathematics pass rate at this campus. The researcher is also of the opinion that although the findings of this qualitative research study cannot be generalised; it can serve as a platform from which relevant stakeholders launch further investigations and research.

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