

**EXPLORING CHALLENGES FACED BY LEVEL 3 NATIONAL
CERTIFICATE VOCATIONAL STUDENTS IN UNDERSTANDING
HYPERBOLIC FUNCTIONS IN MATHEMATICS.**

by

Rakhudu Nnane Franscina

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Supervisor: Prof Z.M.M. Jojo

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DECLARATION

Name: Rakhudu Nnane Franscina

Student Number: 49047884

Degree: **Masters of Education with Specialisation in Mathematics Education**

EXPLORING CHALLENGES FACED BY LEVEL 3 NATIONAL CERTIFICATE VOCATIONAL STUDENTS IN UNDERSTANDING HYPERBOLIC FUNCTIONS IN MATHEMATICS.

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

Signed: _____

Date: _____

Dedication

This study is dedicated to the memory of my late grandparents, Samuel Samu Kgabi and Anna Nkelebe Kgabi.

I also dedicate this work to all my parents, my husband Brashi Rakhudu, my two daughters Linda and Kelebogile and my son, Thusanang.

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Firstly, my thanks to God Almighty, for giving me the power, the strength and the courage to undertake and conclude this important study.

Secondly, I would like to thank the following persons and institutions for their inspiration and contributions to this study:

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- The Chief Executive Officer (CEO), deputy CEO and Campus Manager of the Technical and Vocational Education and Training (TVET) College involved, for allowing me to carry out the research at their institution.
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- The Department of Higher Education and Training (DHET), for allowing me to conduct the research at the selected public TVET College in North West Province
- My two daughters and son, for allowing me to do my studies in what would have been their time with me.
- My extended family, for their interest in my studies.

Abstract

The results of mathematics level 3 have always been a problem at TVET colleges as this hampers the certification rate and the progress of the students to level 4. Students who did not do well in the current subject are not allowed to register that subject in the following level. Even though the students are allowed to progress to level 4 they won't be certificated for both levels until they pass the remaining subject. The above challenges made the researcher to check during the marking and moderation of November / December examination the course of poor results for mathematics level 3. In the process of checking the researcher discovered that rectangular hyperbola is one of the topics that the students of mathematics level 3 are struggling with. This study therefore focuses on exploring the challenges faced by TVET Level 3 NCV students in understanding the hyperbolic function in mathematics.

In addition to the literature review, an empirical investigation based on a qualitative approach and involving semi-structured interviews with the students of a TVET college in North West was conducted to collect data. The analysis of documents relevant to the study was also used as the other method.

The study used participatory action research, where the researcher, collaborators and students work alongside each other to collect data and to improve practice and follow the spiral pattern of reflection, analysing the results and adapting the action. The research design and methodology was qualitative. This helped the researcher to understand the challenges students faced in the learning of rectangular hyperbola and also came up with ways to minimise those challenges. The data collection methods used was interviewing using semi-structured questions, pre-test and post-tests. During data collection different interventions (IN1 –IN3) was used depending on the understanding of the students.

For ethical consideration, ethical clearance was obtained from UNISA. DHET, the principal of the college, collaborators, parents and students will also give written consent on forms which will be sent out explaining what we envisage. Since research was voluntary, an explanation was given that this was not compulsory and that participation was completely voluntary and that they could withdraw at any time.

In this study, various methods to empower students were recommended. Recommendations are also made on what was found in this study, as are recommendations for further study.

Key terms:

Mathematic; Rectangular hyperbola Function; NCV Level 3 ; TVET College; Interventions; Collaborators; Post- test; pre-test; Semi-structured interview; Lecturers; Students; Table Method; Asymptotes; Action Research, Participants.

Abbreviations

ACE . . . Advanced Certificate in Education
CEO . . . Chief Executive Officer
DHET . . Department of Higher Education and Training
DoE . . . Department of Education
ERD . . . Engineering and Related Design
IN1 . . . Intervention one

L1 . . . Lecturer one
 NCV . . . National Certificate Vocational
 NCTM. . National Council of Teachers of Mathematics
 NQF . . . National Qualifications Framework
 Q1 . . . Question one
 Q2 . . . Question two
 Q3 . . . Question three
 RE1 . . . Responses for question one
 ST1 . . . Student one
 TIMSS. . Trends in International Mathematics and Science Study
 TVET . . Technical and Vocational Education and Training
 W1 . . . First Week

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CHAPTER 1

INTRODUCTION AND BACKGROUND OF THE STUDY

1.1 OVERVIEW OF THE STUDY

In this chapter, the researcher sets out to present the research process as it unfolded. This chapter provides an overview of the study, followed by the introduction, the background to the study and purpose of this research project. A brief literature review of this research, the statement of the problem with respect to this study and the aims and objective of the study are also presented. The research questions, research methodology and the definition of key concepts in the study are introduced, followed by summaries and layout of successive chapters. Later in this chapter, the significance of the results of this study in the current era in colleges and conclusion is discussed.

1.2 INTRODUCTION

Constructing and interpreting hyperbola graphs is one of the most important parts of the mathematics curriculum and is also a required mathematical skill for all college Level 3 students. In college courses for mathematics Level 3, students learn how to graph rectangular hyperbola, typically using point-by-point plotting. These skills are not considered difficult for students as compared to the more demanding topics of shifting, asymptotes, domain and range. However, Bakker and Gravemeijer (2004) assert that recent research into the teaching of mathematics education documents the difficulties students have in learning to reason with regard to graphical interpretations. In their study, they revealed that some students tend to see a data set as individual points while others view it as a graph with domain and range. Instead of using the definition of domain and range as is, Hammerman and Rubin (2004) assert that the students tend to focus on particular values such as high and low ones for domain and range of a rectangular hyperbola.

The concept of a rectangular hyperbola is mostly applicable to both biological and physical sciences. In my experience and from discussions with some educators, challenges are experienced in using real-world examples to address this topic. The term 'rectangular hyperbola', according to Marcus, Plumeri, Baker and Miller (2013), is sometimes known as 'equilateral hyperbolas' or 'right hyperbolas representing 'C'-shaped curves'. On the Cartesian plane, the vertical and the horizontal portions of the curves become closer to the y-axis and x-axis as one moves vertically and horizontally, respectively. The portions of a rectangular hyperbola as averred by Marcus *et al.*, (2013) never touch any of the axes of the graph, depending on implicated asymptotes.

The free dictionary defines the rectangular hyperbola as 'a hyperbola with perpendicular asymptotes'. Marcus *et al.*, (2013) argue that the shape of a hyperbola is guided by asymptotes. They further assert that as points on a hyperbola graph move farther from its centre, they come closer and closer to two lines known as asymptote lines. The latter definition is the one adopted in this study and emphasis is laid on recommending this in the teaching of hyperbola, both at matric and college levels.

1.3 BACKGROUND TO THE STUDY

In my experience and from discussions held with other lecturers, despite being one of the most basic tools for mathematics, the rectangular hyperbola function is also one of the most complicated functions and algebra tools because students have to consider the asymptotes and shifting when drawing these functions. Functions in algebra are some of the topics introduced to Level 3 students at TVET Colleges, yet a large number of them have had an inadequate mathematics education and they are mainly under prepared for the study of most topics in mathematics, including the rectangular hyperbola functions, when joining the college.

NCV Level 3 students are expected to understand the rectangular hyperbola with respect to: (i) drawing the graph, (ii) vertical and horizontal shifting, (ii) graphical representations according to quadrants, (iv) asymptotes, (v) symmetrical properties, (vi) domain and range and (vii) graphical interpretations of the graph. According to Monk (2003), graphs can be used to explore aspects of a context that might otherwise

not be apparent. Graphical representations make recording information and producing interconnections between information more transparent and are essential elements of knowledge acquisition in mathematics. It is therefore of great importance that students not only draw graphs but also are able to interpret them.

Interpretation and visualisation skills in learning hyperbola are a challenging phenomenon for the underachieving students enrolled at all levels in the TVET colleges. Lecturers struggle daily with students who do not seem interested in learning. One of the most persistent questions facing mathematics lecturers is, 'How do I get all the students to achieve to their fullest potential in mathematics?' The real problem facing lecturers, according to Larry, Guthrie and Pung (2002), is that of helping all students to achieve optimal learning, conceptual understanding and the ability to apply knowledge to the rectangular hyperbola.

There is continuous underachievement of students in mathematics in TVET College examinations. For over three years, mathematics has remained one of the subjects in the NCV examination where more than half of the candidates are unable to achieve the minimum 30% required score to pass. Figure 1.1 is a graph of the seven subjects required for Engineering and Related Design (ERD) NCV Level 3 students.

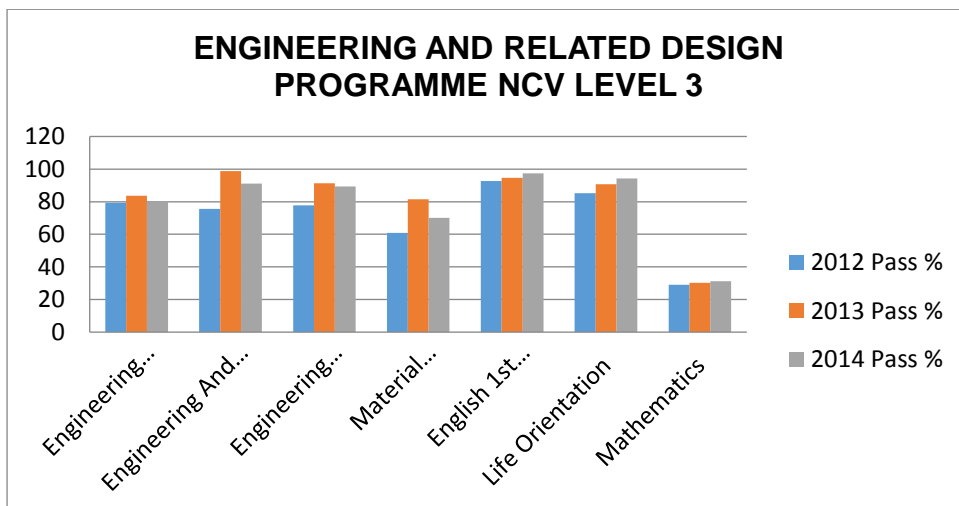


Figure 1.1: TVET College Combined Comparative Results (November 2014).

Graphs 1

The graphs above outline the percentage pass per subject performance for the past three years in Level 3 at the TVET College under discussion in this study. The

performance reflected in the graph above is of great concern and needs to be addressed as it affects the throughput rate at this college. As can be seen from the graph, students do well in all the other subjects, but not in mathematics. The researcher has picked up just one programme to demonstrate the problem that the students face in mathematics. For example, in 2014, the overall pass percentage for all the subjects is more than 50% whereas in mathematics it is only 31.01%. The implication is that even although the students managed to pass the other six subjects, they will not qualify for certification in Level 3 without the necessary achievement in mathematics (TVET Combined Comparative Results, November 2014).

As a result of the poor performance by students in mathematics, South Africa is experiencing a shortage of professionals such as mathematics lecturers, engineers, financial experts, doctors and accounting lecturers, to mention a few. This issue further necessitates that the government supplement the shortage with foreign workers, a step which has economic and social implications. This evidence indicates the need in South Africa to improve students' achievement in mathematics in schools, TVET Colleges, and the entire country. There are numerous variables necessary in determining students' performance, the quality of lecturers being just one. In realising the significance of lecturers' knowledge in students' performance in mathematics, the government and other stakeholders have introduced different strategies to improve the knowledge and quality of lecturers' instructions.

The researcher, who is also a senior lecturer at the TVET college where this research was undertaken, through doing class visits found that lecturers are not familiar with lecturing on the rectangular hyperbola that occupies 20% of the syllabus; a concept that deals with shifting of graphs. Consequently, students struggle to obtain the minimal 30% pass mark required for them to pass, although this is a prerequisite to succeed in their vocational studies. It is for this reason that this study explored ways in which challenges faced by TVET Level 3 students in understanding the hyperbola function can be identified and addressed.

1.4 PURPOSE OF THE STUDY

The aim of the study was, as indicated, primarily to explore challenges that students face in understanding the rectangular hyperbola and suggest ways to minimise students' challenges in dealing with this topic. It is the researcher's judgement that students in colleges who perform poorly in mathematics can learn from the learning strategies of those students who are outstanding in tackling mathematical problems. However, learning styles and strategies alter frequently and are unique to each student; thus, it is important to keep searching for what works well and for what might improve good performance in mathematics. This should be linked to the 20% percent covered by the rectangular hyperbola functions in the Level 3 syllabus as the depiction of performance is demonstrated in Figure 1.1.

1.5 SIGNIFICANCE OF THE RESULTS OF THIS STUDY

This study is significant for those looking for greater understanding of the teaching of mathematics in a TVET College. Vigilant lecturers should always strive to improve on teaching by integrating different methods in the lecture room to adapt to a continually changing modern era, finding new ways to assist students in their understanding of these fundamentals. Furthermore, the researcher intended to suggest ways to minimise students' problems in the learning and understanding of the said topic.

According to Brombacher (2001), South Africa's performance in the Trends in International Maths and Science Study (TIMSS, 2015), was, in the country, followed by suggestions for development of scientific knowledge. The TIMSS was used to compare, over a period, the mathematics and science knowledge and skills of fourth and eighth grade students in forty-eight countries. In this international comparative assessment, South Africa was positioned in 47th place for mathematics and 48th for science. This positioning indicates that South Africa's science and mathematics educational systems are facing serious challenges. The World Economic Forum also claimed that the country displays the lowest performance for the quality of its mathematics and science education (TIMSS, 2015). This implies that mathematics lecturers must respond to a call to design effective instruction that promotes meaningful learning of mathematics. In addition, there was a suggestion that the

curriculum should be re-designed to accommodate the improvement of students' achievement and performance concerning those difficult components that occupy a large percentage of work, like the understanding of the rectangular hyperbola.

On reviewing the literature, only a limited number of studies on the exploration of the challenges faced by TVET Level 3 NCV students in understanding the rectangular hyperbola function in mathematics were found. The researcher also found limited factors on the challenges faced by the students. The outcomes of this study suggest that the development of instructions, which lecturers could use to minimise students' challenges in this regard, should be regarded as essential.

1.6 THE REVIEW OF LITERATURE

The following topics guided the literature review in this study: Function definition; rectangular hyperbola; teaching the topic of rectangular hyperbola; and the challenges in teaching rectangular hyperbola. The researcher presents just a brief survey in this chapter as the main literature review is reported in Chapter 2.

1.6.1 Definition of a function

Kieran (2002) asserts that a one-to-many correspondence is not regarded as a function, while a many-to-one correspondence could be a function. For students to have a proper understanding of the concept function, Daniel & Solomon (2011) suggest that students should know that a function (i) consists of three sub-concepts: the range, the rule of correspondence and the domain; (ii) can be represented in different ways, such as verbal, arrow diagrams, graphical and algebraic representations.

Markovits and Bruckheimer (2010) note that the function is a concept that has been included in the mathematics curriculum for many years and has undergone a significant amendment over time. Each of the definitions is an indication that the said concept is essential in higher mathematics. Most authors who wrote in the period from end of the 18th century until the middle of the 19th century regarded a function as a

variable dependent upon other variables. For instance, the National Council of Teachers of Mathematics, NCTM (2000:99) presents a function as 'any mathematical concept containing a variable x , which has a certain value when a specific number is substituted for variable x .' From the definition above, the notion of the function appears to be solely related to numbers Ibeawuchi (2010).

However, with the recent developments in the function concept, a function is no longer related with numbers, neither is the dependent variable recognisable with functions as a concept. In college mathematics, Markovits et al., (2010) defines a function as a unique type of subset of the Cartesian plane of two sets of axes. Kieran (2002:408) explains a function as the 'relation between two associates of the same set, such that each associate of the x -values has only one element'. In most college curriculums, a function constitutes two sets P and Q , with a rule that assigns just one member of Q to each member of P Leindardt, Zaslavsk & Stein (2012). According to Laridon, Barnes, Jawurek, Pike, Rhodes, Houghton, Scheiber and Sigabi (2007:42) 'a function is a relationship between two sets P and Q where every element of P is plotted to only one element of Q .' In this study, a function is defined as a rule that maps just one element of set R to each element of set S .

Even though the complete study of a function concept is dealt with in Level 2, students are expected to study various concepts of functions, including: linear functions, rectangular hyperbola, parabola as a function, exponential functions, and trigonometric functions (Department of Education (DoE), 2011). Students are also expected to understand the relationships between variables in terms of numbers, graphical, verbal and symbolic representations and to be able to change flexibly between these representations (tables, graphs, formulas and words). Furthermore, they are expected to develop as many graphs as necessary, firstly by using point-by-point plotting supported by existing technology, to make conclusions and to generalise the effects of the given limitations in the functions as a concept. In the study of mathematics, students use suitable features to plot the graphs of functions. The following characteristics were identified: (i) range and domain; (ii) x and y - intercepts; (iii) minima and maxima turning points; (iv) asymptotic lines; (v) shape of the graph

and lines of symmetry; (vi) increasing/decreasing intervals of a function and (vii) the continuous nature of graphs (DoE, 2011).

Some of those functions are covered in Secondary school algebra together with the feature of trigonometric functions. Hyperbola functions are one of those learnt within the algebraic section both at TVET College and at high school level.

1.6.2 Definition of a Hyperbola

According to Daniel et al., (2011:75), the word 'hyperbola' is derived from a Greek word, that means 'over-thrown' or 'excessive'. The formula for the equation of a rectangular hyperbola is displayed in the form:

1) $y = \frac{c}{x}$ or $xy = c$ where $x \neq 0$ and ' c ' = constant.

2) $y = \frac{c}{(x+p)} + q$; This graph is not in the standard position.

In Level 2 students learn how to sketch the hyperbola of the form $y = \frac{c}{x} + q$; $q=0$, as well as the characteristics (key features) of the hyperbola of the form $y = \frac{c}{x} + q$; $q = 0$.

1) The x - and y - intercepts:

The hyperbola has no x or y -intercepts, and in the standard form, $y = \frac{c}{x}$, the graph is asymptotic to the x and y -axis, but never touches or cuts the axes. $x \neq 0$ and $y \neq 0$.

2) The hyperbola, according to Daniel et al., (2011) consists of two separate curves and is a discontinuous graph.

Kieran (2002) defines a hyperbola as consisting of symmetrical open curves and is what results when one cuts a pair of vertical linked cones with a vertical plane. A hyperbola is a graph with vertical and horizontal asymptotic lines. Both the horizontal and vertical asymptotes form a barrier to a curve. Daniel et al., (2011) caution that the graph does not touch the asymptote, but it only moves closer and closer to this line.

This study adopted this definition; further detailed definitions of the rectangular hyperbola function are discussed in Chapter 2.

1.6.3 Teaching the concept hyperbola

When teaching the concept of the rectangular hyperbola lecturers are required to ensure that when they complete the lesson students should have a conceptual understanding of graphical representation, the shifting and the asymptotes of the graph. Students are required to understand a hyperbola from its definition: as the set of points such that the difference of the distances from a point to two fixed points is a constant. Kaufmann and Schwitters (2014) assert that in mathematics, a hyperbola is taught as a smooth curve, found in a plane, defined by its geometric properties or by equations for which it is the solution set. The hyperbola is also taught as the curve representing the function $f(k) = \frac{1}{k}$ in the Cartesian plane, as the appearance of a circle viewed from within it, as the path followed by the shadow of the tip of a sundial, as the shape of an open orbit. Each branch of the hyperbola has two arms that become straighter, further out from the centre of the hyperbola. Diagonally opposite arms, one from each branch, tend at their limit to a common line, called the asymptote of those two arms. Therefore, there are two asymptotes whose intersection is at the centre of symmetry of the hyperbola that can be thought of as the mirror point about which each branch reflects to form the other branch. In the case of the curve $f(k) = \frac{1}{k}$ the asymptotes, according to Kaufmann et al., (2014) are the two coordinate axes. The understanding of graphical representation and the definition of a hyperbola in this study were evaluated by analysing the answers students provided to the question in each activity.

1.6.4 Challenges faced in the teaching of rectangular hyperbola

In the researcher's experience and through discussions with fellow lecturers, it is apparent that there is an indication of challenges experienced in handling hyperbola function; complaints always concern students' poor performance around this topic. One of the challenges faced by lecturers in the teaching of the rectangular hyperbola is the students' lack of prerequisite knowledge (Kelly, 2013) as the mathematics curriculum often builds on information learned in previous years. Kelly (2013) asserts

that if a student does not have the required prerequisite knowledge, then a mathematics lecturer is left with the choice of either doing remediation or forging ahead and covering material the student might not understand. The prerequisite knowledge necessary for understanding rectangular hyperbola includes domain and range; intercepts with the axes; meaning of asymptotes and division by zero. The main challenge for the students is the identification of the asymptotes and the plotting of the graph.

1.7 STATEMENT OF PROBLEM

As previously explained, the rectangular hyperbola is one of the most important topics and covers more than 20% of the mathematics curriculum for NCV Level 3 students. There seems to be inadequate knowledge in addressing challenges confronting the Level 3 NCV students in the learning and understanding of this hyperbola but also lecturers' inadequate knowledge of students' challenges itself. This affects the overall performance of Level 3 students in mathematics and minimises the chances for those students to be certificated. This study therefore seeks to explore the challenges faced by TVET Level 3 National Certificate Vocational (NCV) students in understanding the hyperbola function in mathematics in order to improve students' performance in mathematics.

1.8 AIMS OF THE STUDY

The intention of the study was to explore the challenges faced by TVET NCV Level 3 students in understanding the hyperbola function in mathematics.

1.9 OBJECTIVES OF THE STUDY

In order to achieve the above aim, the following objectives were formulated.

This study therefore seeks to:

- Identify the challenges students face in understanding the rectangular hyperbola

- Suggest ways to minimise students' challenges in learning and understanding the rectangular hyperbola

1.10 RESEARCH QUESTIONS

Based on the research objectives, the main research question was:

How do TVET Level 3 students deal with challenges experienced in the learning of the hyperbola function in mathematics?

Subsequently, the following sub-questions were asked:

- What are the challenges faced by TVET Level 3 students in understanding the rectangular hyperbola function in mathematics?
- How can student's challenges be addressed in the learning of the rectangular hyperbola in mathematics?

1.11 RESEARCH DESIGN AND METHODOLOGY

1.11.1 Research Design

The researcher adopted an action research approach in this study. Action research, according to Creswell (2014), develops through the self-reflective spiral cycle of planning, acting, observing, reflecting and then re-planning, further implementing, observing and reflecting. The design of this study, as Koshy (2005) avers, was located within a dialectic action research process and Mills (2014) suggests that the process should resume by means of identification of the focus area; collection of data; analysing and interpretation of data; and development of an action plan.

The researcher used an action research approach to improve and develop teaching in the lecture room. As lecturers, we need to know what is actually happening in our classrooms; what students are thinking and why students are reacting in the ways they are. Lecturers also need to know which aspects of the lesson to focus upon in order

to develop their lecturing most effectively; how they should change lecturing style in these aspects and what the effects of such a change would be. An action research method was selected in order to afford lecturers an opportunity to reflect on their practice and evaluate what works or to revise strategies that do not work. The research approach was qualitative in nature.

1.11.2 Research Population

A research population, according to Kaufmann et al., (2014) is the full data set upon which the researcher draws. In this study, the population consisted of 240 Level 3 TVET College students.

1.11.3 Sampling and sampling techniques

In this study, the researcher used convenience sampling and selected a group of subjects from the population, based on their accessibility.

1.11.4 Sample size

Mills (2014) suggests that in action research, the sample size depends on the context and the methodology selected for inclusion in the action research cycles. There were eight Level 3 classes of NCV students, but just three groups of thirty each were selected to participate in this study.

1.11.5 Data collection Instruments

A pre-test, a post-test and semi-structured interviews were the main data collection instruments in this study. In order to obtain the data the researcher designed semi-structured interview questions and set the two tests. Nine students were interviewed using a face-to-face type of semi-structured interview. The students were chosen based on their responses from the written tests after different interventions.

1.11.6 Intervention

The researcher and her colleagues first taught the topic rectangular hyperbola to 90 NCV Level 3 mathematics students using the table method. When the table method did not yield the anticipated results and after discussion with colleagues, the shifting and asymptotic method were then used to increase the students' level of understanding. When the shifting and asymptotic method too did not yield better results, a combination of both methods was employed because sometimes in the shifting method two points are needed to give the direction of the graph. This was executed spirally, with evaluation being done at every stage. Because the latter combined method yielded the best results, it was then recommended that the table method as well as the shifting and asymptotes methods were used together.

1.11.7 The process of data collection

When writing their pre- and post-tests, the students were required not to write their names on the answer sheets. Numbers 1-90 were used to code their scripts. The pre-test was used to identify the challenges students were encountering in understanding the rectangular hyperbola. The test helped the researcher to evaluate the progress of the students. The researcher also participated in this study, but had to remain aware of the danger of being too subjective in data collection because this could introduce bias. To prevent bias, the researcher worked with two other lecturers in addressing classroom issues. During data collection process different interventions (IN1 to IN3) with different methods were used.

1.11.8 Data analysis

The researcher performed a qualitative question-by-question analysis of the pre-test and the post-test. By qualitative analysis, the researcher means that she undertook a process of coding, categorising, and interpreting data to provide explanations concerning a single phenomenon of interest. Qualitative data analysis, as Koshy (2005) clarified, is also primarily an inductive process of organising data into categories and identifying patterns and relationships amongst the categories. According to McMillan and Schumacher (2010) inductive analysis is the process

through which qualitative researchers synthesise and make meaning from the data, starting with specific data and ending with categories and patterns. The responses to the questions in the pre-test provided a sense of the students' difficulties and helped the researcher in planning ways of addressing them. The responses were analysed to check if the students were following correct procedure and if they understood the questions. The students' results in both tests (pre- and post-) were compared to discover if the students' understanding had improved or not.

For analysing and interpreting qualitative data, Koshy (2005) suggests a model, which should guide the researcher in their efforts to both make sense of data and to share their interpretation with an audience. The model consists of three concurrent flows of activities: data reduction, data display and conclusion drawing/verification. The researcher displayed data by using tables and graphs and drew preliminary conclusions as the project progressed even although final conclusions appeared only after the analysis was completed.

1.12 ASSUMPTIONS

The assumption in the study was that, being able to minimising students' challenges in the learning of rectangular hyperbola would contribute towards the improvement of their achievement in mathematics. This research was carried out on the assumption that what lecturers know determines what they do in their classrooms and consequently, what their students learn.

1.13 LIMITATIONS

A limitation according to Creswell (2005) is an uncontrollable threat to the internal validity of a study, where internal validity refers to the likelihood that the results of the study actually mean what the researcher indicates they mean. All the subjects in this study were volunteers who could withdraw from the study at any time they wanted to do so. The conclusions drawn in this study cannot be generalised as representative of all TVET colleges.

1.14 DELIMITATIONS

Delimitations, according to Creswell (2014:198) refer to 'what the researcher is not going to do' and are further defined by Leedy and Ormrod (2013) as choices made by the researcher, which should be mentioned. They also describe the boundaries that the researcher has set for the study. Participation in this study was delimited to just Level 3 mathematics students aged below 18, 18 and 18 and above who attend TVET Colleges, thus generalisation to other age groups or colleges may not be warranted.

1.15 RELIABILITY AND VALIDITY

Leedy et al., (2013), describe reliability as a consideration of whether, if the measure were repeated, one would obtain the same results. Most action researchers are concerned with validity rather than reliability. McMillan et al., (2010) assert that validity refers to the truthfulness of findings and conclusions. In this study, more than one method was used to gather data, such as semi-structured interviews, pre-test and post-test in order to establish validity of the findings.

1.16 ETHICAL CONSIDERATIONS

Ethical considerations, according to Burns and Grove (2001), means that the researcher must be aware of the issue of informed consent that is required from the participants and that no one should be forced to participate in research. For this study participants were assured that, their confidentiality and privacy would be respected and that their responses would solely be used for the purpose of the study.

1.17 DEFINITION OF KEY CONCEPTS

The section below discusses the working definitions in this study.

- **NCV – National Certificate Vocational**

The NCV is a new vocational study opportunity offered at the TVET College. The new approach in Technical Vocational Education and Training, that is grade 10 to 12/NQF

level 2 to 4, emphasises high skills, high quality and knowledge. It responds directly to the needs of the modern economy (NQF ACT 26, 2010:35).

- **NQF – National Qualifications Framework**

The National Qualifications Framework (NQF) is the system that records levels of learning achievement to ensure that the skills and knowledge that have been learned are recognised throughout the country (NQF ACT 26, 2010:35).

- **ERD - Engineering and Related Design**

The National Certificate (Engineering & Related Design) is a programme at each of the Levels 2, 3 and 4 of the NQF. This programme is designed to offer both the theory and practice of ERD. The practical component of the study is offered in the real workplace or in a simulated workplace environment. It provides students with an opportunity to experience work situations during the period of study (NQF ACT 26, 2010:38).

- **TVET – Technical Vocational Education and Training**

Technical Vocational Education and Training courses are vocational or occupational by nature, meaning that the students receive education and training with a view to qualify for a specific range of jobs or employment possibilities. Under certain conditions, some students may qualify for admission to a University of Technology to continue their studies at a higher level in the same field of study as they were studying at the TVET College (TVET ACT, 2013:13).

- **ACE – Advanced Certificate in Education**

The Advanced Certificate in Education (ACE) is a professional development course for practising teachers who want to upgrade their qualification to attain REQV 14 occupational status. It is offered over two years of part-time study and there is a choice

of specialisations, i.e., either a school phase specialisation or a specialisation in a specific area (DoE, 2010).

1.18 LAYOUT OF THE STUDY

Chapter One: In this chapter, the researcher addresses the problem that gave rise to the study, including the research questions that guided the research. An overview is also presented and a summary of what the reader can expect in the entire study is provided.

Chapter Two: This chapter focuses on a review of the literature. It includes discussions on the meaning and representations of a function, the rectangular hyperbola and strategies that can improve students' understanding of the latter.

Chapter Three: The research methodology is discussed in this chapter. This includes descriptions of the research design, sample selection method, instruments for data collection, data analysis techniques and ethical considerations.

Chapter Four: Data collected from the students' pre- and post-test and their interviews are analysed and interpreted in this chapter. The results are used to answer the research questions.

Chapter Five: A summary of the research and findings are presented, followed by the implications that emanate from the findings. Recommendations are made, including suggestions for further investigations.

1.19 CONCLUSION

The background to the study was established in this chapter. The problem statement, which led to the research, was discussed and the purpose and significance of the study were briefly stated. The research questions that the study intended to answer were posed. Assumptions made were argued and finally, the layout of the study was presented. In view of the role of lecturers' knowledge in the education of students and in recognition of the need to improve on the achievement of students in mathematics, this study explores challenges faced by TVET Level 3 mathematics students in

understanding the rectangular hyperbola. In the next chapter, the literature that guided the study of a hyperbola function is reviewed.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the researcher presents literature reviewed with the aim of gaining insight into how students learn the hyperbola function. The section is organised into five sub-sections that represent five aspects of students' learning of functions: i) the perspective and context of the study; (ii) function as a concept; (iii) students' challenges in their learning of hyperbola functions; (iv) understanding hyperbola function, and (v) the improvement of hyperbola performance. The literature reported empirical findings and theoretical discussions that relate to student understanding of hyperbola graphs and functions for each of these sections. In the same chapter, the researcher presents the theoretical framework underpinning the study. The chapter concludes with a summary of the reviewed literature and its implications.

2.2 THE PERSPECTIVE AND CONTEXT OF THE STUDY

The reason for this study being undertaken, as noted, was the unsatisfactory performance of students in mathematics Level 3 at TVET Colleges. Through moderation of end of the year examinations, the researcher discovered that students are not performing to the best of their ability in most sections of mathematics. In particular, students demonstrated challenges with drawing and interpretation of rectangular hyperbola graphs, a section that as indicated occupies above twenty percent of all sections done at Level 3 mathematics at TVET Colleges. This underperformance affects the overall results in Level 3 at the college and minimises chances for those students to progress to higher levels and to be certificated.

The above concern led the researcher to explore the challenges faced by TVET Level 3 NCV students in understanding the hyperbola function in mathematics. Leindardt et al., (2012) suggest reasoning as being one of the challenges students face in dealing with functions. Reasoning is fundamental to knowing and working out mathematics. It also enables the students to make use of all their other mathematical skills; thus, reasoning could be thought of, as the 'glue' that helps mathematics to make sense.

2.3 FUNCTION AS A CONCEPT

In this section, the researcher discusses literature relevant to the function concept and presents it in two subsections: the ancient development of this concept and the associated teaching and learning of it.

2.3.1 The ancient development of the function concept

The central mathematical perception in this chapter is the concept of function. It therefore seems appropriate to present a brief summary of the development of this concept, from a mathematical perspective. The ancient development of the function concept, from a mathematical point of view, dates from the early 20th century period. Hazewinkel (2010:62) asserts that: 'The development of the concept of function goes back 4000 years, 3700 of these consist of expectations.' Expectation according to Thompson (2001) is about being enthusiastic, is an emotion involving pleasure, excitement, and sometimes anxiety in considering some longed-for good event. It is perhaps somewhat amazing that a function has been studied only for about 300 years. Meanwhile Kazarinoff (2003) considers the function concept as one of the vital concepts of mathematics. In a similar study Monk (2003) defines the concept of a function as one of the unique features of modern mathematics as against the classical one. Furthermore Leindardt et al., (2012) notes that the concept of function can be regarded as a consequence of the human endeavour to come to terms with changes perceived and experienced in the surrounding world.

These researchers have perceived the function concept as one of the most fundamental concepts of modern mathematics. This concept continues to deepen and evolve, which is why no single formal definition can include its full content which can be solely understood by a study of the main lines of its development that are extremely closely linked with the advances in science in general and of mathematical physics in particular.

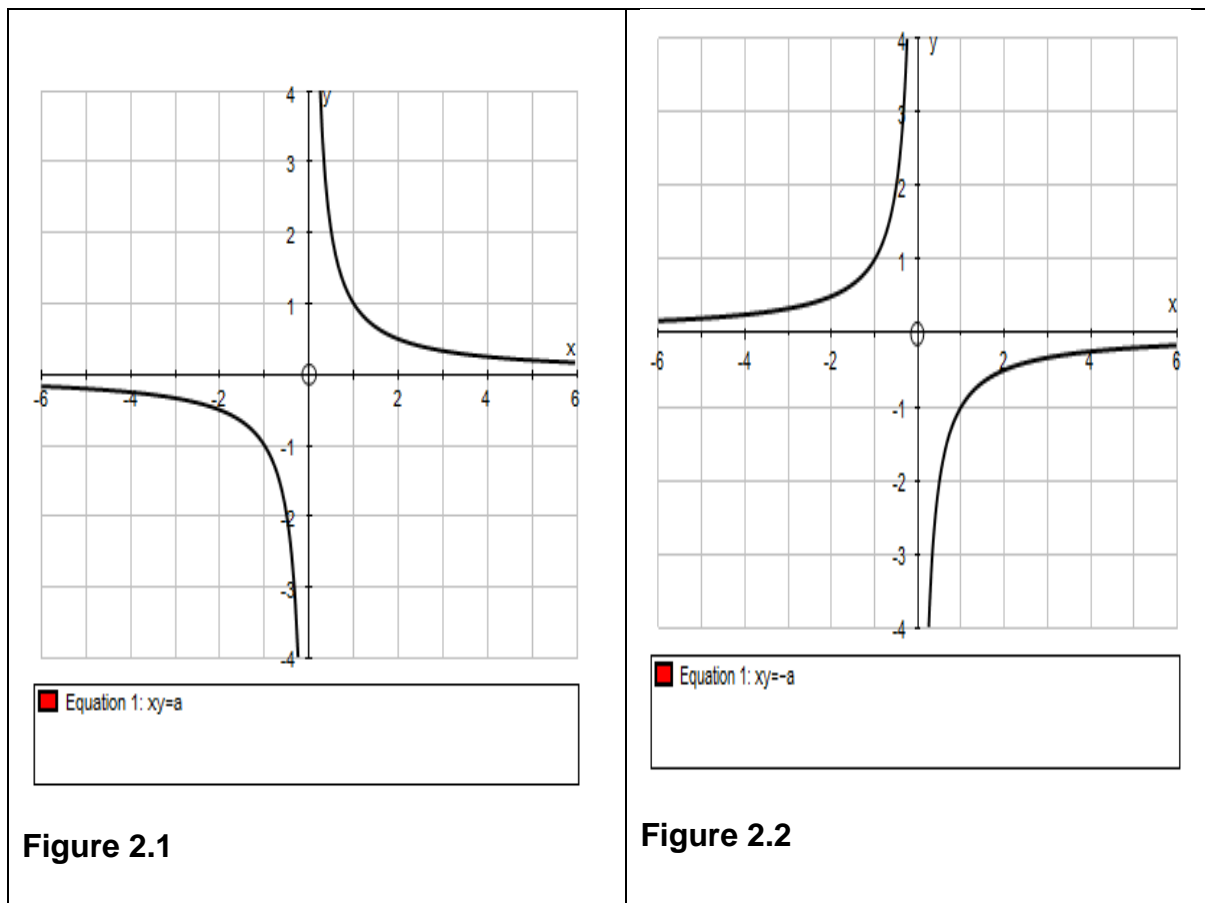
Functional concepts of relationships between magnitudes were used even in ancient times, for instance construction of astronomical charts and tables. However, according to Hazewinkel (2010) those ideas were not considered as part of mathematics. In addition, Kazarinoff (2003) asserts that ancient mathematics lacked the necessary

algebraic prerequisites needed to develop the concept of function. Eventually this changed. In the two hundred years leading up to the work of Newton and Leibniz in the 1680s, there were a number of mathematical developments paving the way for the beginnings of a function concept as it is used today. For example, Monk (2003) asserts that work in the field of astronomy, like that of Galileo, led to the establishment of the study of motion as a central problem of science. This period also saw the development of symbolic algebra and, through the work of Descartes (1596-1650) translated by Wheeler (2005), the establishment of a link between algebra and geometry and then the development of hyperbola as function.

According to Hazewinkel (2010:62), 'a hyperbola is an open curve with two branches, the intersection of a plane with both halves of a double cone.' While the plane does not have to be parallel to the axis of the cone, the hyperbola will be symmetrical. Marcus, Plumery, Baker and Miller (2013) state that a hyperbola is a set of all the points in a plane such that the absolute value of the difference of the distances between two fixed points stays constant. In addition, Kazarinoff (2003) determined that a hyperbola resembles two opposite u-shaped curves. He further states that as points on a hyperbola get farther from its centre, they move closer and closer to two lines known as asymptote lines. According to Monk (2003), asymptote lines are used as guidelines in sketching the graph of a hyperbola. This means that the shape of a hyperbola is guided by asymptotes. This is a vital description for this study since it is based on learners' understanding of what a hyperbola is. They need to understand that without the asymptote's lines, the graph of a hyperbola will not produce the correct shape. This means that the graph could touch the axis on the Cartesian plane, which it is not supposed to touch.

Bell (2003) defined a hyperbola as a type of smooth curve, lying on a plane, defined by equations that yield the set of coordinates when a table method is used for plotting the graph. Mitchell (2012) further describes a hyperbola as having two pieces, called connected components or branches, which are mirror images of each other and resemble two infinite bows. The hyperbola is one of the four kinds of conic section, formed by the intersection of a plane and a cone. The other conic sections are the parabola, the ellipse, and the circle. A circle according to Daniel et al., (2011) is a special case of the ellipse. The conic section formed depends on the angle the plane makes with the axis of the cone, compared to the angle of a straight line on the surface

of the cone makes with the axis of the cone. Marcus et al., (2013) state that if the angle between the plane and the axis is less than the angle between the line on the cone and the axis, or if the plane is parallel to the axis, then the plane intersects both halves of the double cone and the conic is a hyperbola. Examples of a hyperbola are illustrated in Figures 2.1 and 2.2 below.



Figures 2.1 and 2.2: The effects of the parameters ‘a’ on the rectangular hyperbola

A constant ‘a’ represents some properties that should be taken into consideration when teaching the rectangular hyperbola. These are summarised in Figures 2.1 and 2.2 above and in the paragraph below.

- When ‘a’ is positive number as in the graph on the left in Figure 2.1 then (i) the centre is (0,0), (ii) the branches of the hyperbola lie in quadrants I and III, (iii) the asymptotes are the systems of axes, (iv) the transverse axis of symmetry is $y=x$, (v) the conjugate axis of symmetry is $y=-x$, (vi) the domain is the set of all

real numbers except $x=0$ and (vii) the vertices are situated where the transverse axis meets the graph of the hyperbola

- When 'a' is a negative number as in the graph on the right in Figure 2.2 , then (i) the centre is (0,0),(ii) the branches of the hyperbola lie in quadrants II and IV, (iii) the asymptotes are the systems of axes, (iv) The transverse axis of symmetry is $y=-x$, (v) the conjugate axis of symmetry is $y=x$, (vi) the domain is the set of all real numbers except $x = 0$ and (vii) the vertices are situated where the transverse axis meets the graph of the hyperbola.

Daniel et al., (2011) further suggest that the hyperbola can be drawn using point-by-point plotting in the table when one knows and applies the characteristics of the graphs depicted above.

2.3.2 The teaching and learning of the function concept

According to Hazewinkel (2010), the function concept has received a considerable amount of attention within the mathematics educational research community, with conference themes and books devoted to the subject. For example, Hansson (2009:32) states, 'a major reason for this is the role which the function concept is often seen to play as a unifying concept in mathematics.' According to Lutzen (2003), functional thinking should pervade all mathematics and, at college level, students should be caught up to functional thinking. Eisenberg (2002) speaks of a sense for functions and states that developing a sense of function in students should be one of the main goals of the college and college curriculum. However, from an educational perspective, much of the research on the function concept has tended to reveal difficulties related to the learning of the concept. Minimal literature was found that emphasises success in learning the concept.

Attorps (2006) maintains that there are numerous studies of students' conceptions of the said concept, revealing inconsistencies both within conceptions and between conceptions and definitions. These studies have been conducted at tertiary level as well as within teacher education. For example, Pettersson (2013) used the theoretical construct of concept definition to investigate the conceptions of functions held by a number of university students and junior high school teachers. Comparing the definitions given by the participants for their use of the concept, he found several

examples of compartmentalisation. This means that they separate the definition of a function and the application of the function concept into unrelated categories. Moreover, the concept images of the function concept held by many of the students were underdeveloped. During a study in similar vein, Viholainen (2008) found common limitations in students' function conceptions; for instance, the belief that all graphs of functions need to be continuous and that all functions need to have numbers as input and output.

From a different perspective, working within the outline of pedagogical content knowledge and subject matter knowledge for teaching, Shulman (2007) studied secondary teachers' conceptions of function, and found that numerous teachers did not possess the conceptions of hyperbola content knowledge or how it could be taught. In addition, lecturers' expectations of the behaviour of functions led them to disregard the actual definition of the concept. Approaching the problem from a different angle, Mesa (2004) examined the mathematical practices related to the function concept in textbook problems on functions, in order to investigate what conceptions of functions may be stimulated through working with these problems. The function concept was not well integrated into the students' general knowledge structure, casting some doubt on the idea of the function concept as a uniting concept.

From the aforementioned studies, it was concluded that college students, even those who have taken a reasonable number of mathematics courses, may not have a proper understanding of the function concept. Davis (2007:67) stated, '...despite its seemingly straightforward nature, the function concept clearly is a complex one for students, and their concept development appears to develop over a number of years and appears to require an effort of sense-making to understand and orchestrate individual function components to work in concert.' This implies that challenges students have in understanding the function concept at Level 3 are severe and are contingent on the lower levels of their mathematics.

It is unsurprising that researchers have investigated the character of the function concept and the teaching of it, trying to comprehend the difficulties related to students' learning of functions. Many of the studies quoted above point out that the structural nature of the function definition is challenging for learners. This has led some researchers to propose that more operative descriptions should be used when

students are first introduced to the given concept. For instance, Thompson (2008) has contended convincingly for using an analytical procedure, as opposed to the view of function as correspondence. An analytical procedure makes the distinction between independent and dependent variables less clear, something which Smith (2003) notes as an epistemological obstacle in the learning of the function concept.

Obstacles of this type, according to Juter (2006) deal with the obstacles caused by natural language, such as the tendency to generalise or the cause of the delay in development of scientific knowledge. Epistemology is also used by Smith (2003) when referring to the past, for example the failure to link geometry with numbers. Thus, the teaching and learning of functions should enable the students to acquire the skill in the answering and application of a function and should change the negative attitude of students towards functions.

2.4 STUDENTS' CHALLENGES IN LEARNING ABOUT HYPERBOLA FUNCTIONS

This section is organised into three main parts that represent three aspects of student learning: i) prerequisite knowledge (ii) intuition, and (iii) misconceptions and difficulties. Within each of these sections, the researcher outlines empirical findings and theoretical discussions that relate both to student understanding of graphs and to student understanding of functions.

2.4.1 Prerequisite knowledge

Since the acquisition of new knowledge and skills are dependent on pre-existing knowledge and skill in the field of mathematics, being aware of what students already know and are able to do when they come into the lecture room or before they begin a new topic of study, may well assist lecturers to craft instructional activities that build student strengths and acknowledge and address their challenges. Kelly (2013) notes that students come into the lecture rooms with a broad range of pre-existing knowledge, skills, beliefs and attitudes, which influence how they attend to, interpret and organise incoming (new) information. How students do so will in turn affect how they remember, think, apply, and create new knowledge.

Individual students lacking many of the prerequisite skills and knowledge could be motivated to take a prerequisite subject or, be forewarned that they need to develop skills in some areas on their own if they are to understand the concept of a hyperbola. Thus, assessing prior knowledge could enable both the lecturer and the student to allocate their time and energies in ways that will be most productive. There are several different methods to evaluate pre-existing knowledge and skills in students. According to Barton (2011) some methods are direct measures of pre-existing knowledge, such as tests, concept maps, portfolios and others are more indirect, such as self-report(ing), inventory of prior subjects and experiences. In this study, the researcher preferred a direct measure such as testing to assess students' pre-existing knowledge. These assessments should help the lecturer to gain an overview of students' preparedness, identify their areas of weakness and adjust the pace at which the hyperbola section is taught in the course.

According to Schnotz (2006:67), '...students' level of prior knowledge and skills is determined by administering a prior knowledge assessment early in the beginning of every year.' This assessment can help lecturers identify students' understanding of the knowledge they are expected to have so that the former can design their subsequent instruction and examination accordingly. When the majority of students lack sufficient understanding of a prerequisite topic, it is best to provide targeted remediation such as spending class time on the topic. The way the lecturer approaches this remediation depends upon the resources they have available in the classroom. The pre-requisite knowledge for understanding the concept of the hyperbola function is that, any number divisible by zero is infinite since students used the hyperbola of the form $y = \frac{a}{x}$ and $y = \frac{a}{(x + p)} + q$, where 'a', 'p' and 'q' are integers.

If just a few students lack sufficient prior knowledge, lecturers can counsel them individually according to the nature of the gaps in their knowledge and skills. Kelly (2013) suggests providing tutoring help from academic development for students who need aid in just a few areas. Academic development as defined by Schnotz (2006) entails assisting students achieve to their full academic potential. Each year colleges present students with new learning challenges. As a result, despite having done well

at school, many college students experience test taking as difficult. In some cases, this results in poor performance, a high level of stress and failure to meet the demands of the syllabus at college level.

Assessing prior knowledge is an often-misunderstood idea, and subsequently mishandled as a process. Prior knowledge, according to Schnotz (2006) is a product of the academic experiences that a student brings to a lesson; these experiences provide knowledge in terms of content as well as schema that students can use to make sense of new ideas. Schema, according to Jennie and Kathy (2014) is a mental concept that informs a person about what to expect from a variety of lessons taught. Schemas are developed based on information provided by mathematical lessons and are then stored in memory.

The purpose of assessing prior knowledge as Malik (2000) explains is not to place everyone on the same page, but rather to make visible the nature of what a student knows. This makes it possible to create personalised learning pathways for students, as each student approaches new or familiar thinking in their own way. From a planning point of view, it can also allow lecturers to identify knowledge gaps, prioritise standards and revise lessons. Engaging students in conversations, making use of concept maps, drawings and writing prompts, can also achieve this, according to Schnotz (2006).

2.4.2 Intuitions

Leindardt et al., (2012) define intuitions as features of students' knowledge that arise largely from everyday experience, although in students at a higher level they may involve a mixture of everyday and deeply understood formal knowledge. In general, intuitions seem to exist from everyday experience before specific formal instructions. For example, students' tendencies to interpret graphs using their preconceived perceptions may be traced to their intuitions regarding picture reading. Malik (2000) also asserts that intuition is an ability to understand or know something immediately based on one's feeling rather than facts. In other words, intuition provides no clear evidence in one way or the other upon which one's judgement is based; rather it is based on an inner 'instinct' or feeling.

For example, literature suggests that students believe that the asymptotes for $y = \frac{4}{x}$ for which $x = p = 0$ and $y = q = 0$ are also applicable for the graph of $y = \frac{4}{x-2} + 3$. They understand the role of $x = p = 2$ and $y = q = 3$ only after these roles are explained to them. If the asymptote concept is not explained to them they wonder why the graph of $y = \frac{4}{x-2} + 3$ does not cut the y – axis and x - axis at $y = 3$ and $x = 2$ respectively. They understand that the y -intercept is 1 and the x -intercept is $\frac{2}{3}$ but, if the asymptotes $y = 3$ and $x = 2$ respectively are not indicated the students will never know in which direction should they pull the curve of the graph. According to Daniel et al., (2011) their intuition would be that the entire hyperbola graph will never touch the x -axis and y -axis, irrespective of their different equations and asymptotes. Level 3 students at the TVET college also experience this problem.

2.4.3 Misconceptions and difficulties

This study adopts Dossey’s (2006) definition of misconceptions as ‘incorrect features of student’s knowledge that are repeatable’. Misconceptions in the area of hyperbola functions have a different characteristic from those that have been documented in the mathematics literature, although mathematics misconceptions often originate in students’ observations and interpretations of lessons. Misconceptions of hyperbola functions are often intertwined with previous formal learning of functions. For example, the function may not be understood because of lack of variety of instructional examples, or a translation of a graph may be performed inaccurately because of confusion over symbolic notation. For instance, students may confuse the equation $y = \frac{a}{x} + q$ and $y = mx + c$. They may take the value of ‘ q ’ in the equation of the hyperbola as the y -intercept, like ‘ c ’ in the equation of the straight line, merely to realise that ‘ q ’ is the horizontal asymptote of the hyperbola, not the y – intercept of the straight line graph. This is because students not only interpret hyperbola knowledge but also group concepts that are interrelated.

According to Van de Walle (2004), misconceptions and difficulties of hyperbola graphs are discussed under the following subheadings: i) definition of a function; ii) correspondence; iii) linearity; and iv) representations of hyperbola functions.

2.4.3.1 Definition of a function

Numerous studies according to Markovits et al., (2010) have indicated that students possess inaccurate ideas of what graphs of functions should look like. Most of those findings, as Dossey (2006) explains, emerge from classification tasks performed within the definition of graphs and suggest that students have an overly restricted view of the forms that graphs of functions can take. Often students identify just those graphs that exhibit an obvious or straightforward pattern as graphs of functions. Leindardt et al., (2012) found that some students demanded a linear pattern before recognising a graph of a function. Based on the above it is possible that the students do not have an image for a particular concept if it is the first time they have encountered it. However, the more lessons they have received on that mathematical content, the more likely they will be to have some previous notion of the concepts. For example, although mathematics Level 3 students would have encountered the notion of functional concept in Level 2, they may not have a clear understanding of what a function is.

In many cases, students may know the accurate, formal definition of a function, but fail to apply it when deciding whether a graph represents a function. Markovits et al., (2010) suggest that in such cases the student's knowledge of the formal definition is split from what he terms the student's 'concept image'. Gagatsis (2004) describes a concept image as the total cognitive structure associated with the concept, which includes all the mental pictures and associated properties and processes. Therefore, the student's conceptual understanding of a function has been developed through experience with examples of functions. The majority of those examples contains rules of correspondence yielded by formulas that produce patterns which are obvious or easy to detect when graphed. Hence, students sometimes develop the idea that just those graphs with simple patterns like straight-line graphs, represent functions.

According to Leindardt et al., (2012) it should be noted that when classifying only a regular, patterned graph as a graph of functions, students are actually discriminating

in a manner that is consistent with historical understandings of what a function is. 'The modern set-theoretical approach' as Malik (2000:67) argues, expands the definition of function to include many correspondences that were not recognised as functions by previous generations of mathematics.' These include discontinuous functions, functions defined on split domains, functions with a finite number of exceptional points, and functions defined by means of a graph. The graph of these newly recognised functions, as Van de Walle (2004) states, may not follow regular, symmetrical, or easily recognisable patterns. With this historical perspective in mind, students' failure to recognise such graphs as graphs of functions may be seen in a different light, perhaps not so much as a misconception but as a 'missed' concept.

Students, as noted, seem to possess a variety of misconceptions and inaccurate concept images about the function. These misconceptions include thinking that functions are rules, formulas, or equations; that functions need to have specific names; and, that functions should be familiar. Students also display difficulty in connecting the graphical, tabular, and symbolic representations of functions. These representations are viewed as separate entities and do not relate to one another. Furthermore, once a student has constructed a concept image of a function, they no longer refer to the concept definition.

2.4.3.2 Correspondence

According to Lovell (2000), two kinds of difficulties have been reported with respect to student understanding of the types of correspondences that constitute functions: the belief that functions must embody a one-to-one correspondence and confusion between many-to-one and one-to-many correspondences. Students often, according to Ibeawuchi (2010), require that the elements of two sets be in a one-to-one correspondence before they claim that conditions have been met for a functional relationship. Leindardt et al. (2012) suggest that such a condition may result from an implicit requirement for symmetry. In other words, students believe that if each x value maps to one and just one y value for a function, then the reverse should also hold, that is, each y value must map to one and just one x value. What the incorrect view might have 'taught' the students is that in correspondence there are no unpaired elements.

This limitation of functions to those in which elements are in a one-to-one correspondence may also be related to the typical instructional sequence, which is a series of learning tasks. These tasks are presented to students in a logical sequence for the purpose of accomplishing specific outcomes directly related to achieving specific mathematics education standards. Frequently, the first type of function that students encounter is a linear function, which has a one-to-one correspondence. Due to a lack of additional examples that embody many-to-one correspondences, students may, as Bell (2000) states, come to believe that a requirement of functions is that each y value must map to one and just one x value. Another area of confusion involves distinctions between many-to-one and one-to-many functions.

Koehler (2011) suggests that a possible reason for this confusion is that, on arrow diagrams, students sometimes count the arrows instead of the elements in the set. For example, in Figure 2.3 below as adopted from Lovell (2000), students may note that there is one arrow leaving each member of the domain, but many arrows are arriving at a single image, thereby inaccurately labelling the correspondence one-to-many.

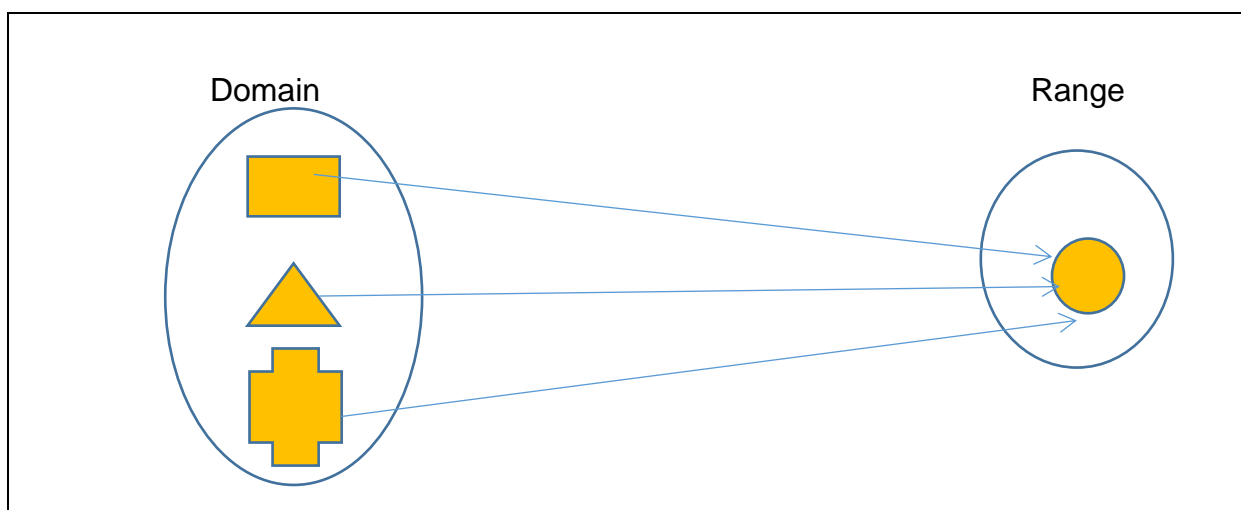


Figure 2.3. An arrow diagram representation of a many-to-one correspondence that could be misinterpreted as a one-to-many correspondence Source: Adopted from (Lovell, 2000:88)

2.4.3.3 Linearity

McKenzie and Padilla (2006) assert that students have a strong tendency to define a function as a relation that produced a linear pattern when drawn as a graph. Similarly, Markovits et al., (2010) found that, when asked to generate examples of graphs of functions that would pass through two given points, students produce mostly linear graphs. In the same vein, Malik (2000) asserts that this tendency toward linearity has been also displayed in formal tasks that focus on more than two points. For example, Markovits et al. (2010) showed students several points on a grid work and asked them to draw a graph to connect the points. In both contextualised and abstract situations, students tended to connect each two consecutive points by a straight line.

A student's misunderstanding about the function concept may occur for several reasons. For example, Malik (2000) states that students may not fully understand the formal definition of function and that this lack of understanding a definition may lead to conflicts between students' cognitive images and their definition of a concept. The formation of a concept image and concept definition may be a result of the students' memorising a formal definition without connecting meaning to it.

According to McKenzie et al., (2006) students' tendency to revert to linearity might also be explained by the fact that the first family of functions to which students are usually introduced is linear functions. Later, after being exposed to other families of functions like the hyperbola, students may still exhibit a tendency to overgeneralise the properties that they learned in conjunction with linear functions. It is interesting to note that, outside the area of functions and graphs, Dossey (2006) also has identified the overgeneralisation of linear properties. Her work suggests that generalisations in algebra were based on assumptions that linear properties applied despite the context indicating that they were inappropriate. It is not surprising then that students do not consult their linear concept definitions. In this study, it was furthermore observed that students do not consider the importance of asymptotes in the plotting of the hyperbola function. The students rely on their concept image to identify functions until they have gained more linear function experiences in mathematics, which forces them to access and rely on formal definitions.

2.4.3.4 Representations of functions

Lovell's (2000) research discovered that several representational systems can be used to display a function and that these include ordered pairs, equations, graphs and verbal descriptions of relationships. Hiebert and Carpenter (2011) have called attention to the psychological processes involved in moving from one representation of a function to another. Lovell further termed these movements 'translations' and noted that they have directionality; for example, moving from an equation to a graph involves psychological processes that are different to moving from a graph to an equation. Although translations among all of these representations of functions have been discussed, most tasks in the empirical literature focus on translating from equations to graphs and vice versa.

According to Koehler (2011:21), 'a logical analysis of these two tasks, translations from equations to graphs and vice versa suggests that movement from graphs to their equations would be the more difficult task because it involves pattern identification.' Brodkey (2013) on the other hand contends that graphing an equation involves, by comparison, a relatively straightforward series of steps like generating ordered pairs, plotting them on a Cartesian grid, and connecting them with a curve. A recent study by the Florida Department of Education (NAEP) (2015) regarding function in this area supports the notion that moving from a graph to an equation is more challenging.

For example, recent NAEP (2015) results indicate that, when given a pencil and a sheet of paper with labelled axes, only 18% of 18 year olds produced a correct graph corresponding to a hyperbola equation. However, Human's (2012) view is that the reverse translation of such an equation corresponding to a correct graph is even more challenging. Therefore, students need to be taught the hyperbola concept so that whenever a question about it is presented to them, they will be able to produce a correct graph. The lecturers need to teach the reverse translation to students so that it is no longer a challenge to them.

Lovell (2000) reports that when students were given a graph of a rectangular hyperbola function which indicated coordinates $(-1, 5)$ of a point, just 5% of 18 year olds could generate the equation. Other studies have confirmed these findings, but

with different age groups: Leindardt et al., (2012) with 11 and 12 year olds and Markovits et al., (2010) with 15 and 16 year olds. Markovits et al., (2010) only discovered that translations from graphs to equations are more challenging than the reverse when students were more familiar with the functions. According to Lovell (2000), when the function was less familiar, translations in both directions were found to be equally demanding. Koehler (2011) also reported translations with constant functions to be more challenging, most likely because one of the variables could be missing. Leindardt et al. (2012) confirmed the above finding based on their results, indicating that translations were especially demanding when one of the coefficients of a hyperbola function was missing.

Some researchers investigated more challenging translations between graphs and their equations. For example, students have been found to prefer translating from equations to graphical representation in matching tasks involving hyperbola functions. Leindardt et al., (2012) asked college students to identify which of two given hyperbolas match to a given equation and which of two equations match to a given hyperbola; the students appeared to work in the same direction in both activities. In order to move in an equation to graph direction, students used the parameters of the equations as a base to check the graphs. Students also prefer to narrow their focus to a single point as they interpret graphs, even though a range of points is identifiable. According to Lovell (2000), this is more likely to happen when the phrasing of a question is too taxing.

In the National Council of Teachers of Mathematics document (NCTM, 2000) a function is explained as any expression of mathematics containing a variable x that has a certain value when a specific number is substituted for this variable. Daniel et al., (2011) emphasised the value of g when the input variable ' b ' is indicated by the symbol $g(b)$. The symbol $g(b)$ is read as g of ' b ' or 'the value of g at ' b '. He further emphasised that functions are indicated by using a specific letter; for example, f , h , j and k etcetera.

From the definition of a function, Barnes (2007) found that a one-to-many correspondence is not considered as a function while a many-to-one correspondence

could be regarded as a function. For students to understand the concept of a function very well, as Van de Walle (2004) emphasises, students should firstly note that a function consists of three sub-concepts: the rule of correspondence, range and the domain. Secondly, students should be aware of the fact that a function can be represented in different ways, such as diagrams, verbal, graphical representations, and algebraic representations.

2.4.3.5 Examples of a function

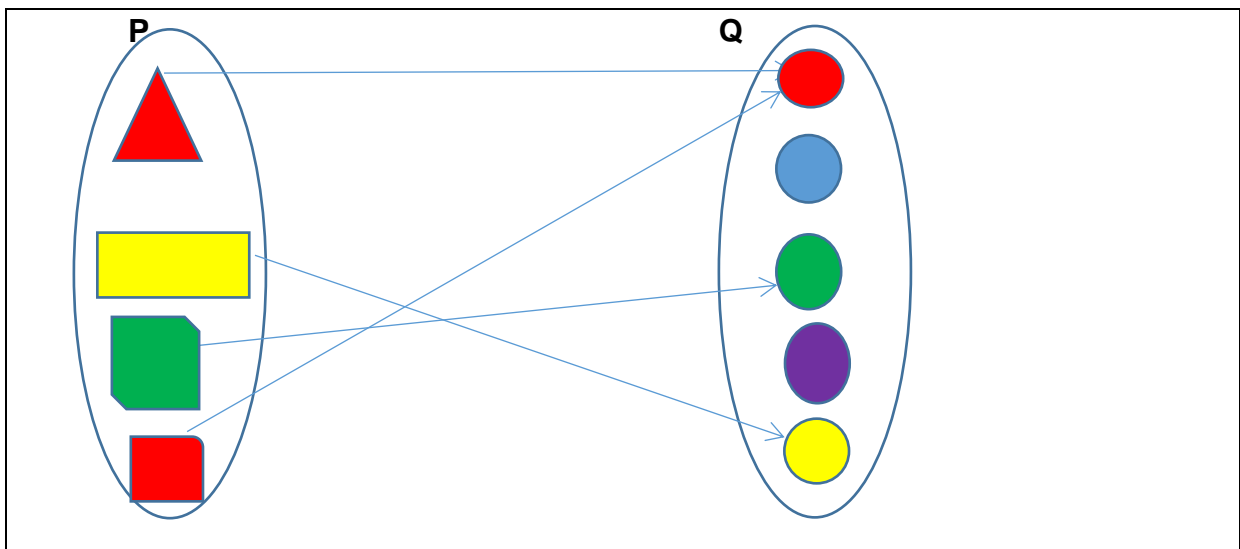


Figure 2.4: A function that mapped any of the four coloured shapes its colour
Source: Adopted from (Bloch, 2011:139)

As an example, let P be the set consisting of four shapes: a red triangle, a yellow rectangle, a green hexagon and a red square; and Q be the set consisting of five circles with different colours: red circle, blue circle, green circle, purple circle, and yellow circle. Then matching each shape to its coloured circle is a function from P to Q : each shape is matched to a coloured circle (i.e., an element in Q), and each shape is mapped, to exactly one coloured circle. There is no shape that lacks a coloured circle and no shape with two or more coloured circles. This function, as Bloch (2011: 143) explained, is named ‘the colour-of-the-shape function.’

2.5 UNDERSTANDING HYPERBOLA FUNCTIONS

According to Brodkey's (2013) findings, many students thought that knowing how to graph a hyperbola well is an inborn skill, something that cannot be learned. Nevertheless, in reality, anyone can be successful in the drawing of a function; they only need to correct the methods and strategies. The following are guidelines to be followed by a Level 3 student in order to understanding the graphing of hyperbola very well at TVET College.

2.5.1 Ways of understanding hyperbola function

Students are expected to master the interpretation for the equation of a rectangular hyperbola $y = \frac{c}{x}$ or $xy = c$ where $x \neq 0$ and $c = \text{constant}$. Students should also understand that graphs in the standard form $y = \frac{c}{x}$, are asymptotic to the x -axis and y -axis, but never cut the axis; $\therefore x \neq 0$ and $y \neq 0$. In this study students found the definition of asymptote to be difficult because they did not know this concept. The hyperbola is a discontinuous graph and consists of two separate curves. It is discontinuous because substituting one of the x values in the above equation yields a y value that is undefined. Another important factor is for students to master the effect of 'c' on the rectangular hyperbola. If $c > 0$ (positive) the graph will be on quadrants 1 and 3 and if $c < 0$ (negative) the graph will be on quadrants 2 and 4. Students must also know that the hyperbola can be drawn by using the table method (point-by-point plotting) and again know and apply the characteristics of the graph (Daniel et al., 2011:76).

According to Barners et al., (2007:52) in order for students to improve performance as regards the hyperbola they must also master the equation of the hyperbola $y = \frac{c}{(x+a)} + b$, where

- 'c', 'a' and 'b' are integers. For example $y = \frac{1}{(x+2)} + 4$ is a translation of $y = \frac{1}{(x+2)}$ by +4 units vertically. Vertical asymptote $x = -2$ and horizontal asymptote $y = 4$ are the asymptotes of the equation $y = \frac{1}{(x+2)} + 4$ respectively. In Level 3

students are expected to sketch rectangular hyperbolas without using the table method and they must follow the following characteristics of the graph: (i) $y = b$ (Horizontal asymptote) (ii) $x = -a$ (Vertical asymptotes) (iii) x – intercepts: where $y = 0$ (iv) y – intercepts: where $x = 0$ (v) For $c > 0$ the graph is in quadrants 1 and 3. (vi) For $c < 0$: the graph is in quadrants 2 and 4. (vii) More points must be calculated if necessary for students to sketch the correct graph and to realise that when graph undergoes translations, it no longer lies in quadrants 1 and 3, 2 and 4 only. In addition to the above students must also know how to calculate the equation of a rectangular hyperbola in the form of $y = \frac{c}{(x+a)} + b$ (Daniel, 2011:78).

2.6 THE THEORETICAL FRAMEWORK

Learning theories, as Hiebert et al., (2011) state, provide the conceptual framework describing how information is processed, absorbed, and retained during the learning process. Prior knowledge as well as cognitive, emotional, and environmental influences all play a significant role in how understanding, new knowledge and skills are acquired and retained. This study was underpinned by Piaget's theory of cognitive development that is explained through a cognitive view, procedural knowledge and conceptual knowledge as defined by Wessels (2003).

2.6.1 The cognitive view

According to Woolfolk (2006) the cognitive view considers learning as a process which is active, during which students, instead of receiving knowledge, seek informative new information to problem solving and restructure what they already know to achieve new insights. He further asserts that students are active participants in the restructuring of their own knowledge. Students' knowledge construction as McMillan et al., (2010) maintain, is irreplaceable because new information is processed and interpreted in terms of their available knowledge.

Wessels (2003) noted the cognitive view suggesting that we perceive what people acknowledge and that knowledge construction and learning depend on the student as an individual. Students must construct their own conceptual knowledge, as it cannot

be transmitted ready-made and whole from the lecturer to the students. Students' knowledge consists of their internal and mental ability regarding ideas that are constructed in their minds. Hiebert et al., (2011) distinguished between two types of mathematical knowledge: conceptual knowledge and procedural knowledge, which are addressed in the following paragraphs.

2.6.1.1 Conceptual knowledge

Conceptual knowledge, according to Van de Walle (2004), is described as having logical relationships connected to already existing ideas, is constructed internally and originates in the students' mind through reflective abstraction, that is, thoughts about actions and ideas. This is the kind of knowledge that Labinowicz (1985) consider logic-mathematical knowledge. Hiebert et al., (2011) emphasised that conceptual knowledge is information that exists in the mind of the student as part of a network of ideas and is understood. Conceptual knowledge according to Wessels et al., (2003) is not taught explicitly and comprises knowledge about relationships and mathematical concepts.

The Level 3 students were tested through mathematical exercises and tasks that help them to build on their present knowledge and in reorganising and re-structuring those ideas towards more sophisticated notions. Students, according to Dossey et al., (2006) demonstrate conceptual understanding when they label, generate and recognise examples of what concepts are and are not, and also when they use notions and their representations to categorise and discuss mathematical models. Conceptual knowledge is very important in the learning of mathematics, because students are usually unable to solve questions if they do not thoroughly understand the concepts with which they are dealing. Van de Walle (2004) asserts that students' understanding of mathematical concepts forms the foundation of their learning and the development and improvement of their skill in the solving of mathematical problems.

2.6.1.2 Procedural knowledge

Procedural knowledge of mathematics according to Dossey et al., (2006) is knowledge about the rules and the procedures that students use in carrying out routine exercises as well as the symbols used in representing mathematical concepts. To emphasise Dossey's definition about procedural knowledge, data in this study will be presented

logically and completely according to four categories: Drawing and interpretation of hyperbola using table method; the effect of 'a' on the graph; identifying asymptotes; and drawing hyperbola with asymptotes and intercepts method. Van de Walle (2004) believes that procedures are step-by-step practices learnt to complete certain exercises and that some cognitive relationships are needed to obtain the knowledge of how to perform a procedure. To make clear of the definition above Van de Walle (2004) believe that misconceptions and difficulties of hyperbola graphs should be discussed under the subheading i) definition of a function; ii) correspondence; iii) linearity; and iv) representations of hyperbola functions. Procedural knowledge is crucial in mathematics; for example, algorithmic procedures help students to do mathematical exercises easily while symbols convey mathematical ideas to everyone who studies mathematics.

However, even the use of the most skilful procedure did not yield positive results to develop conceptual cognition that relates to that procedure. As an example, doing many subtraction exercises does not guide a student to understand what subtraction means. In regard to this, Van de Walle (2004) stated that procedural rules should be taught together with a concept; however, linking procedures and conceptual cognition is far more important than the usefulness of the procedure itself, so that overemphasising procedural skills without an understanding of mathematical procedure should be avoided. According to Dossey et al., (2006) students demonstrate procedural awareness when they choose and apply procedures in a correct manner and when they justify the suitability of a procedure for carrying out a given exercise.

2.7 CONCLUSION

In summarising this chapter, the researcher presents a synopsis of prerequisite knowledge as one of the challenges that students face in the learning of hyperbola and intuitions; at the same time dealing with research questions (see section 1.10). The chapter recognised the pre-knowledge needed for students to develop hyperbola graph representation. Students must become powerful thinkers and develop fruitful mathematical habits of mind and should think the same way as mathematicians.

This literature review discovered that it is vital for lecturers to structure learning situations with the purpose of enhancing students' thinking and their graphical representation. The chapter ended by elaborating on how best hyperbola graphs can be drawn in the practical situation and classroom area, starting useful approaches that help students understand the representation of a hyperbola graphically.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

In Chapter 2, the literature reviewed together with the theoretical framework, was discussed with the purpose of gaining an understanding of how students learn the hyperbola function. This chapter starts by explaining the methodology followed and justifies the research procedure used in this study. The chapter restates the research aim and questions of the study and further presents the research paradigm, research design and the research methods that were adopted to answer the research questions and the reason why the methods were chosen. The sample, the participants and the sampling procedure were also discussed. This is followed by the presentation of the different data collection tools used. Thereafter data analysis, validity and reliability together with ethical considerations followed for this study are examined. The chapter then concludes with a brief summary.

It is important to note that in this study, the researcher acted both as a researcher and one of the lecturers of the group of 90 students. This is because of the researcher's concern and interest in examining the challenges in the learning of the rectangular hyperbola at TVET College. In the researcher's experience over her years of lecturing mathematics to TVET Level 3 students, questions in tests and examinations relating to the rectangular hyperbola section have been responded to poorly. It was therefore important that the researcher investigate her own practice in handling this topic whilst trying various other methods in collaboration with two colleagues in an action research cycle, until the best-fit strategy was discovered. The role of the researcher as a lecturer was that of facilitator and coach, but she was aware of the danger of being too subjective in data collection, as this could have introduced bias. To prevent bias, the researcher used collaborative action research, which, as mentioned above, included two other lecturers in addressing classroom issues. According to Koshy (2005), collaboration means that everyone's view is considered as a contribution to

understand the situation. Collaboration as McNiff and Whitehead (2012) assert is an important feature of action research that the researcher has adopted.

3.2 RESEARCH AIM AND QUESTIONS

While the aim and research questions were stated in Chapter 1, they are repeated in this section. The aim of this study was to explore the challenges faced by TVET Level 3 students in understanding the rectangular hyperbola function in mathematics. The researcher additionally needed to suggest ways to minimise students' difficulties in the learning of rectangular hyperbola. After the positive results that have been achieved the researcher intends to advise other campuses of TVET Colleges and hopes to see the recommendations spreading to other colleges in all the provinces and throughout South Africa. The main research question was:

How do TVET Level 3 students deal with challenges experienced in the learning of rectangular hyperbola function in mathematics?

Subsequently, the following sub questions were asked:

- What are the challenges faced by TVET Level 3 students in understanding the rectangular hyperbola function in mathematics?
- How can students' challenges be addressed in the learning of the rectangular hyperbola in mathematics?

3.3 RESEARCH PARADIGM

According to Burns & Grove (2001), a research paradigm provides a frame of reference for seeing and making sense of research. Wiersma (2009) defines a paradigm as the main set of principles concerning how the features of the research area fit together and how the researcher can make meaning of discoveries and enquire about these. According to Creswell (2014), a paradigm is a viewpoint held by

associations of researchers that is based on a set of principles, social conduct, ethics and norms related to the knowledge that illuminates their study.

According to Burns et al., (2001) many researchers envelop the pragmatic research worldview, as the model for qualitative methods research. To this effect, this study was conducted in terms of the pragmatic research worldview paradigm. This means that the researcher emphasised the consequences of the research instead of just focusing closely on the research methods and answered the research questions by using an action research design for data collection and a 'whatever worked' approach to complete the research, while following stipulated ethical considerations and practical standards and values. The pragmatic worldview was fundamental to this study because a teaching and learning environment was utilised and the intention was to improve on practice in the teaching of the rectangular hyperbola by using ethical and practical means that were recommended.

3.4 RESEARCH DESIGN

Research design according to Burns et al., (2001) is an outline, a model, for doing the study in such a way that the greatest control is exercised over aspects that could interfere with the rationale of the research results. The research used the design as the researcher's overall planning for finding solutions to the research question guiding the study. Mills (2007) further indicated that designing a study is helpful for the researchers to plan and put it into practice in a manner that will help them obtain the anticipated results, thus maximising the possibility of finding information that could be related to the actual site.

The researcher in this study adopted a participatory action research design and worked towards improving her own practices. Action research, according to Koshy (2005) unfolds through the self-reflective spiral cycles of observing, planning, acting, reflecting and then re-planning, further implementation, observing and reflecting. The researcher and her colleagues agreed on a plan of action on how best they could teach the hyperbola function to the students. They then went through the reflective

spiral cycle of re-planning before recommending the table and asymptotes method that worked best for the benefit of the students.

This type of research as Cohen, Manion and Morrison (2007) emphasise, also involves using a personal journal whereby lecturers record their reflections and progress about two parallel sets of learning, which are: learning about the process of studying the practice and learning about the practice that one is studying. The researcher made a critical analysis of the classroom situation in which she and her colleagues were working. Monitoring and observation of existing practice took place before the researcher was ready to plan and implement a change since she made a thorough analysis of the classroom situation where she conducted the research.

Action research, according to McNiff et al., (2012) is a means of investigation that enables lecturers in colleges and throughout the world to enquire about and evaluate their work. Action research has become well-known around the world as a form of learning professionally. It has been used in education, particularly in teaching, but is now followed widely across teaching as a profession. In action research, lecturers investigate themselves and their practices as they find ways to live more fully in the direction of their educational values. Nobody gives instructions to lecturers and they decide for themselves what to do, in negotiation with others. If lecturers realise that their work is already reasonably satisfactory, they start to evaluate it and produce proof to indicate the reason why they believe it to be the case. Should they discover that something needs to be improved, they are immediately able to work on that aspect, usually producing oral and written progress reports and keeping records about what they doing.

3.4.1 Reasons for using action research design

Action research was used to generate new knowledge, which nourishes new theory (Creswell, 2014). Generating new knowledge, according to Cohen et al., (2007) takes place when lecturers have gained knowledge of something of which they were not previously aware. Action research can be used when lecturers want to test if what they teach influences their own or other people's learning, or whether a lecturer needs to do something different. According to McNiff et al., (2012) a lecturer may carry out

action research with the purpose of improving their own understanding, developing their own learning and influencing others' learning. Cohen et al., (2007) explains that lecturers use action research because it starts with the experience of being concerned about a situation and follows this with a process of development, which shows cycles of action and reflections. Its main aim is to make evident relationships of influence.

Koshy (2011) points out that action research aims to interrogate lecturers' practice with the purpose of improving it. The lecturer's aim in this study was to create a new beginning. McNiff et al., (2012) also point out that change is only understood if people improve learning to improve their practices. According to Williams (2006) action researchers undertake research on themselves in company with others, but do not use it to investigate others. Williams (2006) also highlighted that lecturers are not in need of external evaluation, but they do understand the importance for strict testing and evaluation at all stages of the action research, which includes the critical judgements and insights of others.

According to Koshy (2005), action research is a dynamic and useful outline for lecturer research because research can be conducted within a specific situation. Researchers do not have to be distant or detached themselves from the situation, which means that they can be participants. The researcher in this study actively researched her own practice and tried various ways to improve on it until adequate results were obtained. McNiff et al., (2011) believes that in action research modifications can be done as the project progresses and it involves continuous evaluation. There are opportunities for new theories to emerge in action research, compared to other research designs whereby the researcher always has to follow a previously formulated theory. To summarise, the researcher is able to bring a story to life and the study might lead to open-ended outcomes through action research.

The design of this study was located within a dialectic action research process, a form of participatory research as indicated in Figure 3.1 and consisting of the following:

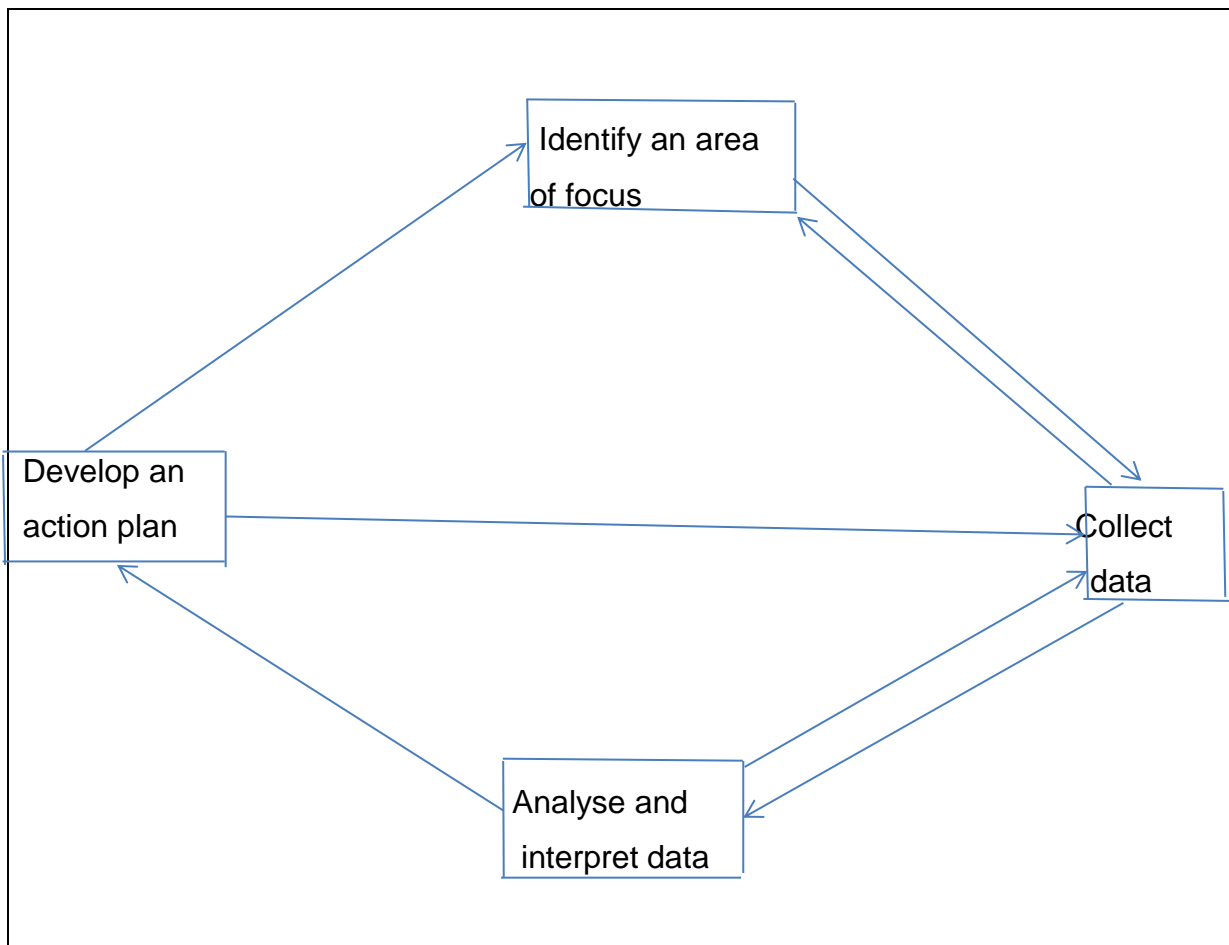


Figure 3.1 The dialectical action research spiral Source: Adopted from Mills (2011:100)

In an attempt to explain the above diagram adopted from Mills (2011), the researcher selected an area of interest, which focussed on students' challenges in the understanding of the hyperbola function. Collecting data in this study was accomplished through several written exercises, which included the pre-test, post-test and semi-structured interviews conducted with students in order to get information. For the analysis and interpretation of data, the researcher checked the pre-test in order to identify the challenges that students had in answering questions on the hyperbola. From the post-test, the researcher wanted to check if the intervention strategy improved the students' understanding of rectangular hyperbola. By intervention, she refers to the teaching of the topic 'hyperbola' to students after writing the pre-test.

Firstly, the researcher used the table method. When the table method did not achieve adequate results, the researcher suggested and used the asymptotes method.

When the table and asymptotes methods did not yield positive results when applied individually, the researcher used the combination of both methods. From the semi-structured interview the researcher wanted to collect further information on the understanding of some responses given by students. Lastly, the researcher took action when she addressed the gaps that unfolded from the pre-test and post-test. When the students' responses indicated some degree of not understanding a concept in hyperbola representation the researcher and colleagues devised a plan to change the method used until the students showed evidence of mastering the concepts. Finally, the researcher decided to apply a combination of both the table and asymptotes methods to assess the maximum understanding of the hyperbola function.

In this study, the area of focus was the issue of asymptotes that are sometimes omitted in the teaching and learning of the rectangular hyperbola. In seeking evidence of the researcher's practice, to measure the effectiveness of a change in practice, she employed a triangulation of methods. During this process, the researcher invigilated the students during pre- and post- testing, interviewed them and analysed their work. This, according to Adams (2006) is a principle involving the careful choice of a range of data gathering techniques, each of which might illuminate a different aspect of the same issue.

The researcher worked with a maximum of 90 students and 2 Level 3 lecturers. The purpose of collaborating with other lecturers was to develop a continuously engaging dialogue with the researcher. It was a convenience or available sampling where groups of subjects were selected on the basis of being accessible.

Interviews with students were conducted after the tests, asking them probing questions about the presentation of their responses in order to understand reasons for any errors they had made. Analysis within action research is not concerned with why things have to be as they are, but rather what possibilities for change lie within a situation. Action within a complex social world according to Australia (2009) is not

static; it is dynamic and continuously evolving. During the emergence of different results, the researcher kept on performing analysis.

3.4.2 Challenges of using action research design

The researcher considered action research for the purpose of professional development; she therefore found it difficult to list many disadvantages. However, according to William (2006), some sources sometimes describe action research as a soft option, so the researcher needs to define the parameters of the study at the start. Koshy (2005) asserts that gaining insights and planning action are two of the main purposes of being engaged in action research; in addition, there is the issue of ethical considerations, which is of particular significance within action research.

3.5 THE RESEARCH METHOD

According to McMillan et al., (2010:300), a research method refers to 'how data are collected and analysed'. There are different kinds of research methodologies: the qualitative research approach, the quantitative research approach and the mixed methods research approach. The research method that was adopted for this study was the qualitative one since the researcher is not dealing with figures but explaining how students responded. The next section presents the nature and origins of the qualitative approach and details the reason why the researcher adopted it for this study.

3.5.1 Qualitative approach

The qualitative approach is defined by McMillan et al., (2010:320) as follows: 'Qualitative research begins with assumptions, a worldview, the possible use of the theoretical lens, and the study of a research problem inquiring into the meaning individuals or group ascribe to a social or human problem.' To study this problem, qualitative researchers, according to Koshy (2005), use an emerging qualitative approach to inquiry, the collection of data in a natural setting sensitive to the people and places under study and data analysis that is inductive and establishes patterns or

themes. The final written report or presentation includes the voices of participants, the reflectivity of the researcher as well as a description and interpretation of the problem.

3.5.2 Reasons for using qualitative approach

The purpose of using a qualitative approach has been well documented in the literature by Creswell (2014) with the central reason being that the major characteristics include a setting that is natural, sensitive to context, collecting data directly, rich narrative description, emergent design, complexity, process orientation, induction and participation perspectives. Cohen et al., (2007) stated that qualitative researchers study participants' feelings, beliefs, thoughts, ideals and actions in a place with a natural setting. According to McMillan et al., (2010) such research further uses interactive methods to collect data for explanatory, exploratory and formal studies. Qualitative researchers, as Creswell (2014) averted employ emergent designs and select small samples of information-rich cases to study in-depth without desiring to generalise to all such cases.

Qualitative studies according to McNiff et al., (2012) aim at extension of findings rather than generalising the results that are usually not the intent of the study. Design components that enhance the extension of findings, according to Cohen et al., (2007) are specifications of the researcher's role, data collection and analysis strategies, the social context, informant selection, analytical premises and alternative explanations, typicality and authentic narrative. Koshy (2005) mentions that qualitative researchers employ reciprocity and dialogue while following legal and ethical principles when interacting with participants during research. The five stages of qualitative approach as described by McNiff et al., (2012) are: planning, beginning data collection, basic data collection, closing data collection, formal data analysis and diagrams. Planning, data collection, formal data analysis and diagrams were stages used in this study.

The objective of the qualitative approach was to identify what the said Level 3 students need as a prerequisite for the development of their hyperbola problem solving skills in a teaching and learning environment. To evaluate students' mathematical thinking and the development of their hyperbola problem solving skills, the researcher directly

observed and questioned students whilst they struggled with problems during their session of writing the tests. The researcher recorded the findings on the spot in writing.

3.5.3 Challenges of using the qualitative approach

Creswell (2014) points out the challenges of using the qualitative approach: one such challenge is that it requires extensive time, resources and effort on the part of researchers. For this study, sufficient resources were available while issues of time were factored into the programme. Another challenge identified by Koshy (2005) is the question of skills for doing the qualitative methods. To overcome this particular challenge, the researcher familiarised herself extensively with the qualitative research methods before undertaking this approach.

3.6 POPULATION

Burns et al (2001) define a population as the totality of all subjects that conform to a set of specifications, comprising the entire group of persons that is of interest to the researcher and to whom the research results can be generalised. Furthermore, a research population is the full data set on which the researcher focuses. The site population for this study was the particular TVET College. Every year the college has at least eight groups of Level 3 students. Each group consists of 30 students, meaning that the researcher drew the sample from a population of 240 students.

3.7 PARTICIPANTS

Participants, as described by McMillan et al., (2010) comprise a group of people who participate in a study from whom data are collected. The abovementioned authors further stress that in a study each person who went through intervention and responses were measured is regarded as a participant. For this study, the participants consisted of 90 NCV Level 3 mathematics students at a TVET College. The NCV Level 3 students were chosen because it was a convenient group to work with since the researcher was their lecturer. A group of individuals from whom data were collected is regarded as a sample. It is important that the selected sample should represent the

population fairly. The succeeding paragraphs discuss the sampling procedure adopted in this study.

3.8 SAMPLING AND SAMPLING TECHNIQUES

Creswell (2014) defines sampling as taking any subset of a population to be representative of that research population. Burns et al., (2001) also define a sample as a portion or a subset of the research population chosen to participate in a study, representative of that population. The purpose of the research study determines the type of sampling selected; the sampling method adopted for this study was convenience sampling as the researcher worked with students allocated to her. According to Thompson (2008) such sampling implies selecting participants based upon an accidental basis or availability. For example, the researcher used her own class of students for the study. In this study, the researcher used an intact, already established group of 30 students that was allocated to her, together with two groups of 30 students each allocated to two other lecturers.

3.9 DATA COLLECTION PROCEDURES

According to Creswell (2014), an action researcher should consider the two categories of approach to data collection, which are qualitative and quantitative. Nevertheless, Koshy (2005) asserts that an action researcher would mainly use the qualitative data collection approach, as data may be more in the form of documents, transcripts and descriptions for analysis. Research settings, the data collection timeline, research language and procedures followed in collecting the data are discussed in the following section.

3.9.1 Research setting

The research setting according to Cohen et al (2007) refers to the location where data is collected. Since action research is the term used for the integration of action with research, the researcher was a participant-observer. The whole process took place at a TVET College with mathematics Level 3 students receiving lessons on the rectangular hyperbola. The researcher used participatory situated in terms of

dialectical collaborative action research design with two other lecturers, in addressing a classroom issue with the intention of emphasising that everyone's view was taken into consideration to understand the research situation.

3.9.2 The data collection timeline

Data was collected during the participants' tutorial classes conducted for an hour in the afternoons. The researcher used 1 hour of those classes per week to collect data for 10 consecutive weeks during the third term of the 2015 South African academic year. During the first weeks, the students wrote a pre-test for 45 minutes after which the researcher undertook an analysis. It was discovered that most of the students could not draw the hyperbola graph. Interviews by the researcher were conducted in the same week; the intention was to identify the challenges encountered in the plotting of the hyperbola graph. In the second and third week, intervention by the researcher and her colleagues took place. The students were taught the topic of the hyperbola.

During the fourth week, the students wrote a post-test for 45 minutes, which was marked during the fifth week. The reason for this was that some of the others did not complete the test on time. During the sixth week, the researcher conducted another intervention by teaching them the table method together with the asymptotes. Again, during the seventh week, they wrote the post-test; this was marked during the eighth week. During the ninth week, the researcher interviewed the students and discovered that the students could answer hyperbola questions without any difficulty. During the tenth week, the researcher and the colleagues collaborated in order to write up their conclusions.

3.9.3 The language used in this study

English was chosen as the research language because this college enrolls South African students as well as students from other countries such as Lesotho, Zimbabwe and Botswana. It was also feasible to use English because the researcher was not fluent in some of the local South African languages.

3.9.4 Data collection tools

'Data collection tools', as McMillan et al., (2010) explain, refers to testing equipment that is used to measure a phenomenon of interest. For this study, most data was collected using pre- and post-tests, semi-structured interviews and discussions for both lecturers and students. The use of a pre- and post-test provided the researcher with a simple means to collect information on students' performance in this section. An analysis of the pre- and post-test helped to shape the nature of the questions the researcher asked during semi-structured interviews with the students based on their responses. The main aim of conducting interviews, according to Creswell (2014) was to gather responses that were more informative than pre- and post-testing. In this study, nine students were interviewed using a face-to-face type of interview. The students were chosen based on their responses from the written tests after different interventions.

3.9.4.1 Semi-structured interview

The semi-structured interview, according to Koshy (2005) is a qualitative method of inquiry that combines a pre-determined set of open questions with the opportunity for the interviewer to explore particular themes. These interviews in Creswell's (2014) understanding are considered the opposite of a structured interview that offers a set amount of standardised questions. A semi-structured interview does not limit respondents to a set of pre-determined answers, whereas the structured interview does. The semi-structured interview, as Cohen et al., (2007) explained, is used to understand how interventions work and how they could be improved. It also allows respondents to discuss and raise issues that the researcher may not have considered. According to McMillan et al., (2010) a semi-structured interview is open-ended, allowing new ideas to be brought up during the interview as a result of what the interviewee says. The chief feature of the semi-structured interview, as clarified by Creswell (2014), is the idea of acquiring valuable information from the content of participants' experiences and use of pre-determined questions that provide some uniformity.

The researcher adhered to the following guidelines in conducting the semi-structured interview:

- Initially a friendly, relaxed and nonthreatening atmosphere with the chosen participants' students was established
- Then the researcher presented the problem of the day to the students and asked them individually to talk as much as possible about what they would be doing or thinking while solving the problem
- As the students worked on the problem, the researcher observed, listened and asked the students probing questions while being careful not to teach or ask leading questions.

3.9.4.1.1 The purpose of using semi-structured interview

The purpose of using semi- structured interview according to Koshy (2005) was that they are conducted with an open framework that allows for focus, conversational and two-way communication. They can be used to both give and receive information and the majority of questions are created during the interview, allowing both the interviewer and the person being interviewed the flexibility to probe for details or discuss issues. It is also less intrusive to those being interviewed as the semi-structured interview encourages two-way communication. Semi-structured interview confirm what is already known and provides the opportunity for learning. Often the information obtained from semi-structured interview will provide not just answers, but also the reason for the answers.

3.9.4.2 The pre- and post-test

Tests were made up of items that were answered by supplying requested information. The information could be a number, phrase, a word, sentence or collection of symbols that complete a statement. Tests were useful in evaluating students' ability to use hyperbola problem solving skills. Pre- and post-tests for this study were designed to measure students' hyperbola problem solving skills and to assist the researcher in analysing students' procedures for solving a given problem, as well as to gain specific insight into their ability to use the different hyperbola problem solving skills. Items in the pre- and post-tests were prepared in such a way that they measured student's

hyperbola problem solving skills at the beginning and at the end of the intervention. The validity of each item in the pre- and post- test was assessed by carefully analysing what the item required the students to do or know. The researcher also requested her supervisor and her collaborators to verify the validity of the pre- and post-test questions.

The tests addressed the challenges that the Level 3 mathematics students had to overcome before they could really benefit from the shifting, asymptotes and plotting approaches to the hyperbola and be in a position to acquire mathematical problem solving skills. The mathematical tasks, pre- and post-tests were assessed using the memorandum; not for establishing 'right' and 'wrong' but for the researcher to be able to analyse the responses for conceptual understanding and their interpretation of the hyperbola. Using the memorandum, she was able to gain insight into each student's progress in the development of their hyperbola problem solving skills. The students were advised not to write their names on the answer sheets, which were code numbered 1-90, and the numbers were used to identify their scripts. The pre-test was used to identify the challenges the students faced in understanding the rectangular hyperbola while the post-test helped the researcher to evaluate the progress of the students.

The purpose of having pre- and post-tests was to be able to compare and measure the participating students' performance at the start and at end of the intervention. These tests were completed during a tutorial class but if the students were unable to finish during this time, they were given an opportunity to continue the next day during another tutorial class. The researcher left students to work on the problems without suggesting any procedures, but provided enough scaffolding to keep them on track. Scaffolding, according to Mills (2014) occurs when the researcher explains the unknown content to the students or questions them; this assisted them to draw out their own thinking. The researcher evaluated students' solutions of the hyperbola tasks using a constructed marking guideline.

3.9.5 The intervention programme

The pre- and post-tests were administered to a group of 90 students in different lecture rooms with the help of the two said colleagues at the beginning and the end of intervention. The researcher and her colleagues taught the topic rectangular hyperbola to 90 NCV Level 3 mathematics students, first using the table method. When the table method did not yield the intended results, some discussions ensued between the researcher and her colleagues and a resolution was taken to try the shifting and asymptotes method in an attempt to ensure the students' understanding of the topic. When the shifting and asymptotes approach failed, a combination of both methods was used because sometimes in the asymptotes' method two points are needed to give the direction of the graph. This was undertaken in a cyclic manner with evaluation being done at every stage, as may be seen in Figure 3.2 below.

At the beginning of each lesson, the researcher would place an outline on the whiteboard of what was going to be taught in that lesson. This assisted the students to think about their existing knowledge and mathematical concepts that were related to that day's lesson. Seeing an outline on the whiteboard stimulated their thinking about various topics, which helped to activate their prior knowledge about the topic of the day. The prior knowledge was revealed by the questions that the researcher asked the students about the outline that was on the whiteboard. She created a classroom situation in which social interaction was highly valued. This was an environment in which students believed that the important factor was the effort they spent looking for solutions, and that they would have learnt something even if they did not find the correct solution to the given problem.

To gain insight into individual students' development of hyperbola problem solving skills, the researcher required them to write a report in their journals about every problem solving experience they completed. She conducted semi-structured interviews with one or two students during a problem solving session based on their responses in the test. The researcher kept a record of the findings from the semi-structured interviews. She asked questions which the students answered both verbally and by writing. The researcher transcribed data immediately while it was still fresh in

her memory to avoid losing the visual cues that human beings rely on to interpret other people's meanings.

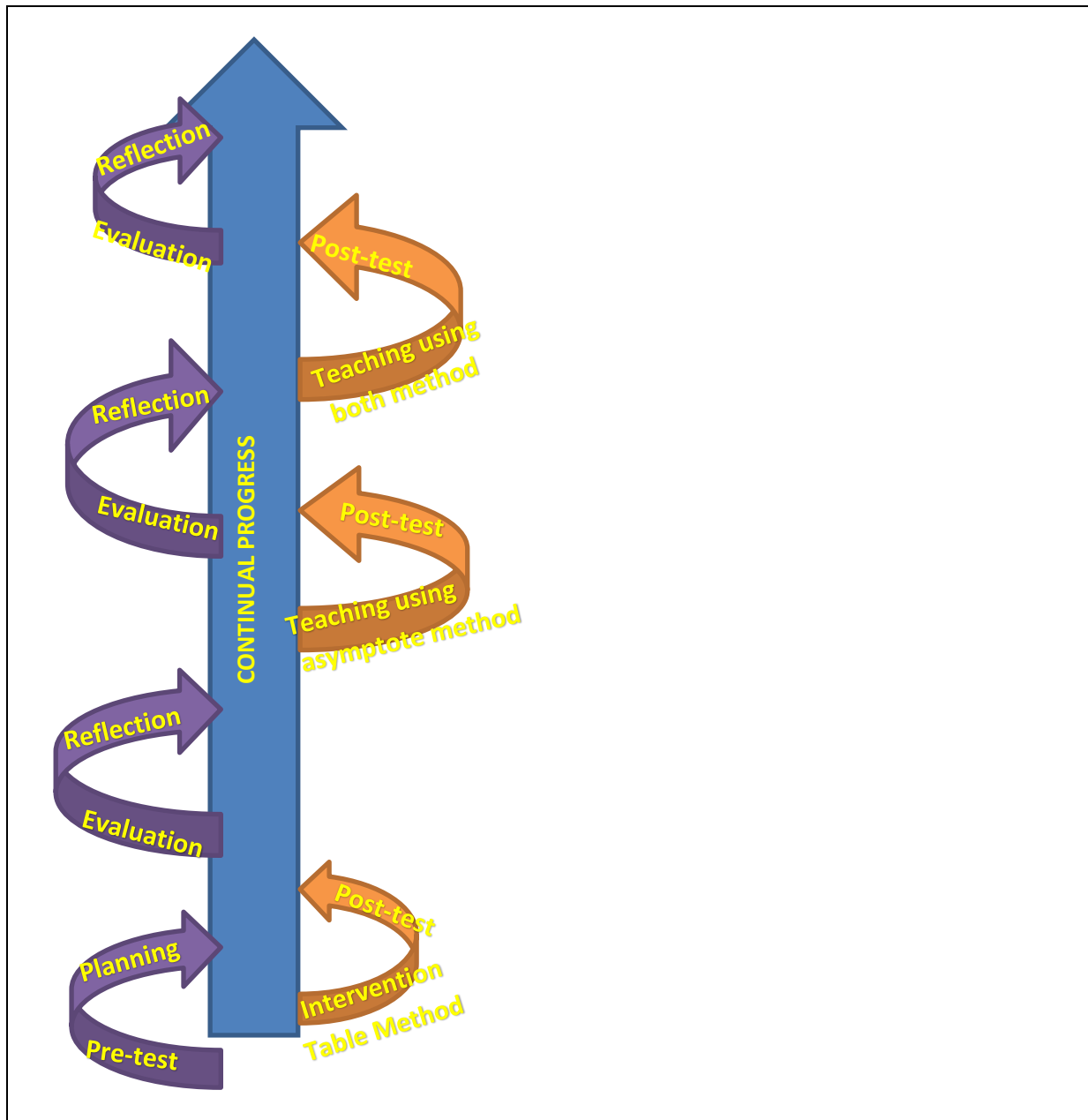


Figure 3.2: Spiral for data collection

3.10 DATA ANALYSIS

Data analysis according to McMillan (2010) is the process of making sense out of data, which involves consolidating, interpreting and reducing what participants have said, how they responded and what the researcher has seen and read in order to derive or make meaning from the process. He further describes data analysis as an inductive process of organising data into categories and identifying patterns among the categories. Creswell et al., (2011) note that qualitative data analysis involves coding the data, dividing the text into smaller units, assigning a label to each unit and then grouping the codes into themes. Mouton (2001) emphasised that data analysis is breaking up data into manageable themes, trends, relationships and patterns. The purpose of data analysis in this study was to provide answers to the research questions through understanding various constitutive elements of the data.

In this study, data analysis started as soon as data collection began, and was an ongoing process. For example, the researcher began by analysing the pre-test to diagnose the challenges that the students were experiencing in their understanding of hyperbola. The semi-structured interview questions were continually modified and refined during the intervention, while an analysis of the semi-structured interview was also performed. Data analysis in this study involved analysing findings from the semi-structured interviews, pre and post-test, discussions, documentations and note taking.

In this study the researcher carried out a qualitative question by question analysis of both the pre-test and post-test. The response to the questions in a pre-test provided direction concerning what the students' challenges were and helped the researcher in planning ways of exploring these challenges. The responses were analysed to check whether students were following correct procedures and if they understood the questions. The students' results of both tests were then compared to see if they had improved or not.

For analysing and interpreting data, Koshy (2005) suggests a model that should guide the researcher in her efforts to make sense of the data and to share her interpretations with the audience. The model consists of three concurrent flows of activities: data reduction, data display and conclusion drawing. In this study, the researcher

concentrated on the last two models: data display and conclusion drawing, since the statistics of all the collected data were displayed to be able to determine the progress of the students.

3.10.1 Data display

The researcher displays data interpreted qualitatively using tables and graphs. The purpose, according to Cohen et al., (2007) is to make organised information into an immediately available, accessible, compact form so that the analyst can see what is happening and draw conclusions or move on to the next step of analysis that the display suggests would be useful. With regard to this research, firstly, the researcher provided a summary of the hyperbola pre-test in table format. The pre-test contained three questions. Question 1 consists of 1.1 to 1.7, Question 2 includes 2.1 to 2.8 and Question 3 had 3.1 to 3.4. A question-by-question table was displayed to check how many students out of 90 could manage to answer the question. The statistical analysis that was recorded on the table was used to draw a graph to make organised information available for the analyst.

Secondly, the researcher also displayed the results of the semi-structured interview before the intervention in a table whereby each question was analysed. The semi-structured interview before the intervention posed seven questions while nine students were interviewed. The analysis of each question furnished statistics of how many students had managed to answer the question in a particular manner. These statistics helped the researcher to display this in a graphical format.

Thirdly, a table for the post- test was drawn up. The post-test consisted of two questions; Question 1 contained 1.1-1.7 while Question 2 comprised 2.1-2.3. An analysis was performed using this table so that the researcher could gain an indication of how many students had managed to answer a certain question. This statistical analysis enabled her to display information in a graphical format.

Lastly, the researcher also placed the follow-up semi-structured interview after the post-test in a table format whereby each question of the follow-up semi-structured interview was analysed. From the analysis, the number of students out of nine who

answered the question in the same way was noted and enabled the researcher to display the information graphically. The follow-up semi-structured interview consists of questions 1-7. The above cycle continued until the students understood the topic hyperbola.

The tables and graphs were qualitatively interpreted because the research method approved for this study was qualitative. The chapter concentrated on data analysis that headed 'report writing'. Figure 3.3 below presents a model for data display and data analysis and Figure 3.4 graphically represents data display analysis.

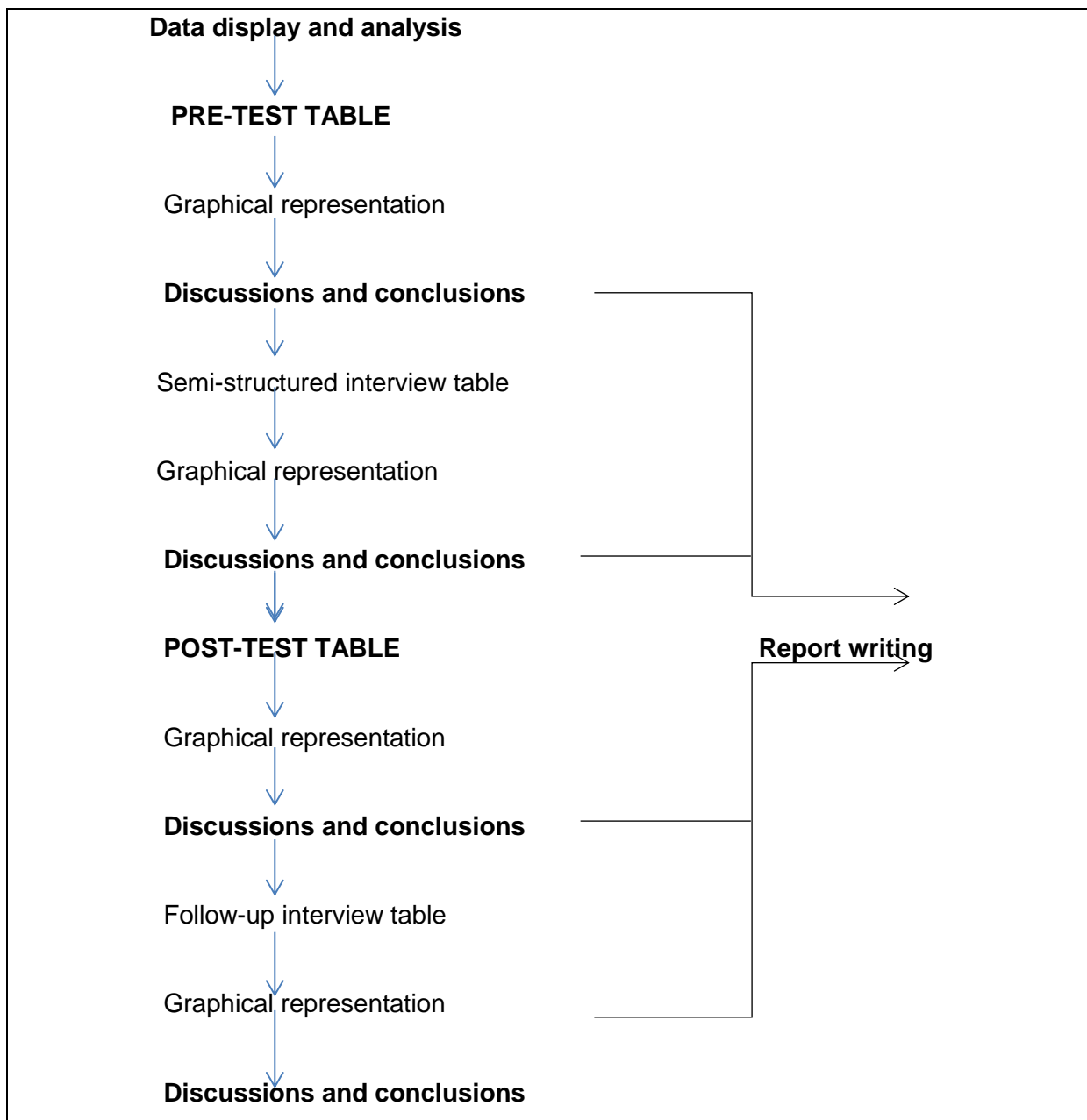


Figure 3.3: A model for data display and data analysis

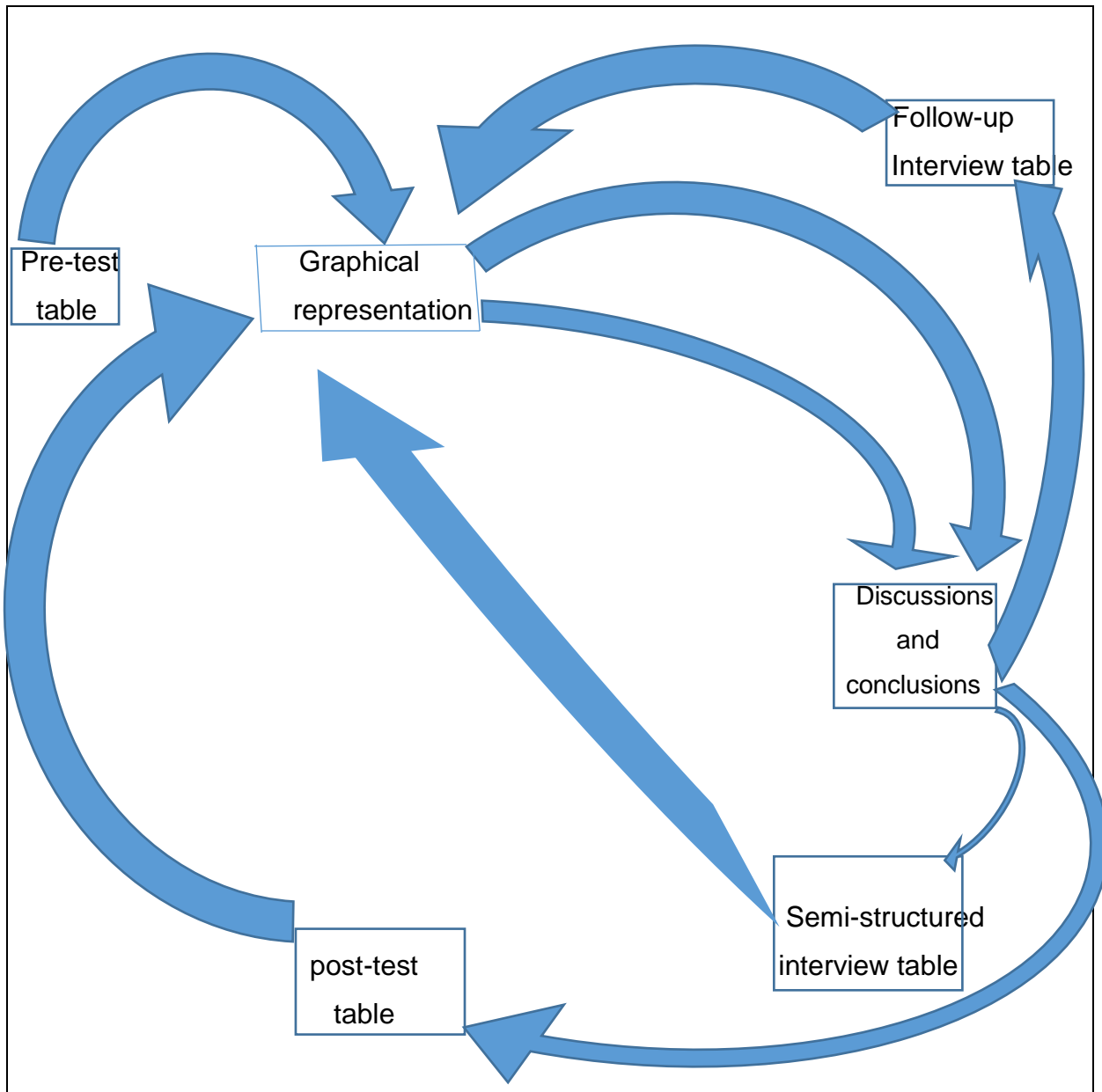


Figure 3.4: Data display analysis

3.11 RELIABILITY AND VALIDITY

3.11.1 Reliability

According to McMillan et al., (2010) reliability refers to the degree of consistency with which a data collection tool measures whatever it is supposed to measure. Creswell notes the extent to which the data collection tool gives similar results and conclusions

if it is administered to a different group of participants under different conditions such as time and venue. This also, according to Cohen et al., (2007) implies that if a similar study is done again under similar conditions, the researcher will obtain the same results, not erratic or inconsistent ones. Reliability is further described by Wiersma (2009:458) as 'stability or consistency of a measure and a consideration of whether, if the measure is repeated, one would obtain the same results.' Most action researchers and those who use qualitative methods are concerned with validity rather than reliability, in so far as their focus is a particular case rather than a sample.

3.11.2 Validity

Validity, according to Cohen et al., (2007) determines whether the research truly measures that which it was intended to measure or how trustworthy the research results and findings are. McMillan et al., (2010) similarly state that validity refers to the faithfulness of findings and conclusions. It is essential, as Creswell et al., (2011) states that procedures to ensure the validity of the data, results and their interpretations are utilised. In this study, triangulation was recommended as a way of establishing validity of findings. Triangulation involves using more than one method to gather data, such as interviews, pre- and post-tests and documentation.

3.12 ETHICAL CONSIDERATIONS

According to McMillan et al., (2006) ethics deals with a belief or guidelines about what is right or wrong, proper or improper, good or bad from a moral perspective. Wiersma (2009) stresses that the researcher has a moral obligation and is ethically responsible for protecting participants' rights and welfare, including guarding against mental and physical discomfort, harm and danger. For this research, in compliance with the Unisa research ethics policy, all precautions were taken before the data collection process in order to adhere to ethical measures to respect the integrity, privacy, anonymity, confidentiality, concerned and humanity of the participants.

3.12.1 Obtaining informed consent

Informed consent, as Creswell et al. note, (2011) means that the participants have a choice of either participating or not participating in the research. Wiersma (2009) explain that when human subjects participate in a research study, they should be informed of their role, the procedures, the purpose of the research, the possible risks of the research; and that they should give their written consent for participating.

Concerning obtaining informed consent to collect data, the researcher sought permission from the Department of Higher Education and Training (DHET), which issued an approval letter giving the researcher permission to gain entry into the TVET College of Education. The researcher also wrote letters to the CEO, the principal and Campus Manager seeking permission to collect research data from their college. The CEO and the Campus Manager willingly gave the researcher approval. However, the letters were not included in the appendices for the sake of anonymity and confidentiality. The researcher then applied for a Research Ethical Clearance certificate which was granted to her by the UNISA Ethics Committee. Being granted the abovementioned certificate implies that there were no concerns about the ethical issues and that the participants would not be subjected to deceptions or possible risks or danger in this study.

At the onset of the study, the researcher wrote letters to the participants and parents or guardians clearly explaining, amongst other issues, the purpose of the study, their role and voluntary participation. After understanding the content of the letters, the participants and their parents or guardians agreed to participate in the study and gave their written consent by signing the letters.

3.12.2 Voluntary participation

McMillan et al., (2010) state that voluntary participation means that participants cannot be compelled, coerced, or required to participate. No one should be forced to participate in research. Cohen et al., (2007) assert that although people should never be coerced into participation, sometimes coercion is subtle. This occurs when researchers emphasise the benefit of participating. According to Creswell et al., (2011) the researcher may tell the participants that they can choose to participate or not participate, but the implicit message that 'You are letting us down if you don't

participate' may also be clear and compelling.

The researcher made sure that in the study the participants were properly informed about the purpose of the research, the procedures of the data collection process and the researcher's possible impact on them. The researcher explained clearly to participants that they were free to decide whether they wanted to participate in the study or not and that they had freedom to withdraw from the study at any time without incurring any negative consequences. Participants were not deceived in any way and the researcher was honest about all aspects of the research.

3.12.3 Confidentiality and Anonymity

Confidentiality as explained by Wiersma (2009) is the act of not disclosing the identity of participants involved in a research project whereas anonymity means that the names of the participants where data is obtained are unknown. For this research, participants were assured that their privacy and confidentiality would be respected and that their responses would be used for the purpose of the research only. All reasonable efforts and necessary precautions to maintain complete participant privacy and anonymity were in place and were enforced. The college's name, campus managers' name, mathematics lecturers' names and students' names were eliminated from all reports; pseudonyms were assigned. McMillan et al., (2010) emphasise that the settings and participants should not be identified in print and no-one should have access to participant names except the researcher.

3.12.4 Appropriate storing of data

Soft copies will be stored on a password-protected computer and hard copies will be safely locked away for 5 years.

3.12.5 Reciprocity

Creswell et al., (2011) suggest that researchers should reciprocate participants' willingness to provide data as well as the actions of all the people who adjusted their priorities and schedules to assist or tolerate the researcher. She or he could

reciprocate in the form of time, feedback, attention, appropriate token gifts or specialised services. The researcher of this study felt indebted to participants for providing data that allowed her to answer the research questions. Therefore, at the end of the study, she gave each student a GAME voucher to purchase stationery or books. To motivate students during the semi-structured interviews conducted after the college lessons, the researcher gave students sweets and burgers to eat while she conducted the interviews. None of these items was offered as an inducement to participate but solely as a sign of appreciation. The researcher also presented the research findings to the CEO, DHET, the Campus manager at the campus and some participants and parents or guardians.

3.13 TRIANGULATION

Triangulation, as Koshy (2011) indicates, is a way of establishing the validity of findings. The researcher collected data from multiple sources involving multiple contexts, personnel and methods. Cohen et al., (2007) avers that the process of triangulation involves sharing and checking data with those involved. In this study, those who were involved were the two other lecturers who were collaborating with the researcher. Also in this study, more than one method was used to collect data, such as the said semi-structured interviews, pre-tests, post-tests and documentation.

3.14 CONCLUSION

The methodology and the research design that were used for this research were explained in this chapter. The research methodology chosen for this study was indicated as the action research design. Various data collection tools were used to enhance the reliability and validity of the findings. In the next chapter, Chapter 4, the researcher analyses and interprets the data, and presents the findings.

CHAPTER FOUR

RESEARCH FINDINGS AND DATA ANALYSIS

4.1 INTRODUCTION

The purpose of this study, to recapitulate, was to explore challenges faced by TVET Level 3 NCV students in understanding the hyperbola function in mathematics. The previous chapter considered the methodology and design used to collect data for this study and explained how it was collected.

This chapter concentrates on the presentation, analysis and interpretation of the data collected by the use of pre-test, tests, post-test, and semi-structured interviews using open-ended questions. As mentioned in the previous chapter, the pre-tests and post-tests were written by 90 Level 3 mathematics students and the semi-structured follow up interviews were conducted with nine of those students chosen on the basis of their responses at a TVET college in sequential progressive periods. Data collected were used to address the following main research question:

How do TVET Level 3 students deal with challenges experienced in the learning of the hyperbola function in mathematics?

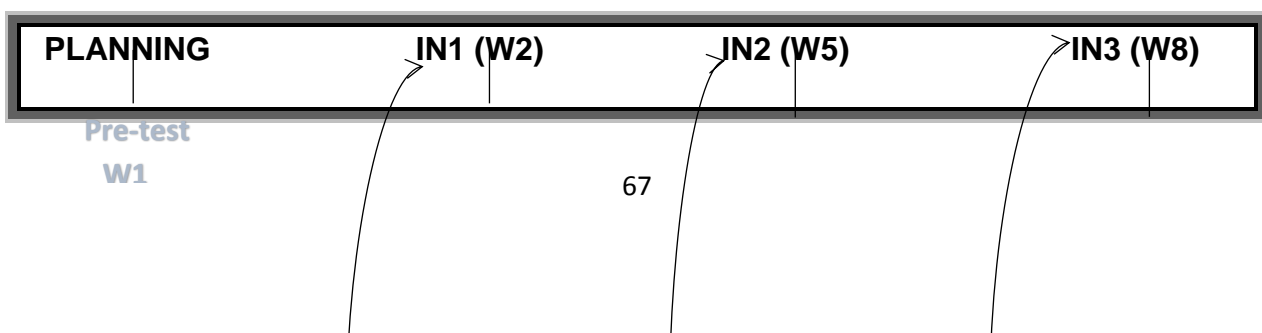
The following sub questions were subsequently posed.

- What are the challenges faced by TVET Level 3 students in understanding the hyperbola function in mathematics?
- How can students' challenges be addressed in the learning of rectangular hyperbola in mathematics?

The pre-test was composed of three questions. The first question, Q1 with sub-questions 1.1 to 1.7, sought to explore students' understanding of drawing and interpretation of the hyperbola graph using the table method. The second question, Q2, assessed the student's knowledge about the effects of the constant ' a ' in the graph defined by equation $y = \frac{a}{x}$. The third question, Q3, examined how students utilise their knowledge of asymptotes to draw the rectangular hyperbola graph. Data was collected on consecutive Thursdays during the students' one-hour tutorial slots in the afternoons of each week, spread over a period of 10 weeks.

4.2 A BRIEF SUMMARY OF DATA COLLECTION

During the first week (W1) the students wrote the pre-test in three different lecture rooms. This was a baseline test aimed at diagnosing their understanding, drawing and interpretation of the hyperbola graph together with the effects of the change of 'a' in the function denoted by equation, $y = \frac{a}{x}$. Students' work was coded as ST1 to ST90, depending on the lecture room in which the student belonged. Students 1 (ST1) up to student 30 (ST30) were in the lecture room A together with lecturer 1(L1) while ST31 to ST60 were in lecture room B with lecturer 2 (L2) and ST61 to ST90 were in another lecture room C with the researcher. The students were also allocated pseudonyms ST1 to ST90 that they used consistently throughout the data collection period, corresponding to the codes used on their scripts. Initially, the researcher together with the collaborating lecturers met and agreed that they must read the instructions for the tests to the students during each of their sittings. Upon finishing each test, students were expected to hand in the scripts to the respective lecturers. Students were allowed to ask for clarification during the invigilation in instances where they did not understand some of the questions. The study followed an action research cycle, summarised in Figure 4.1:



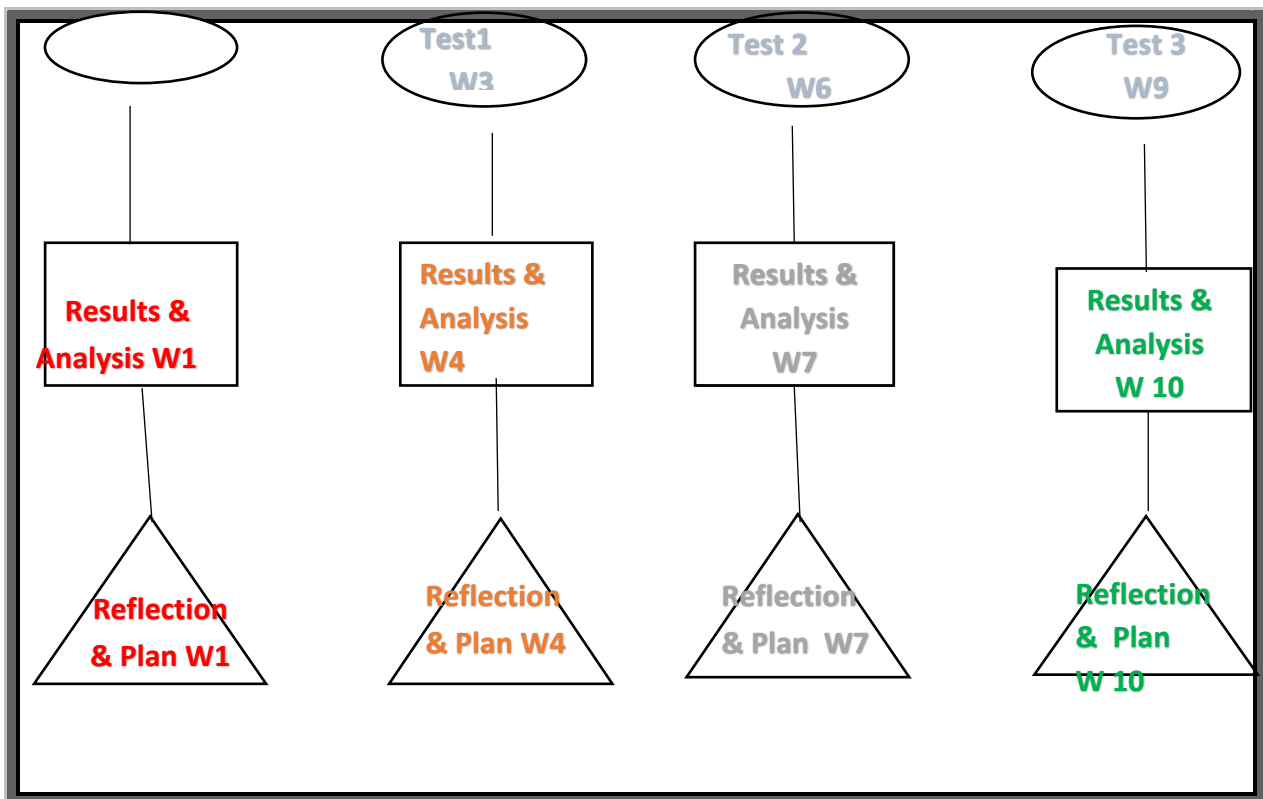


Figure 4.1 Continual Progresses with Action Research Spiral

Action research was used as a design in this study to assist the researcher in discovering the most appropriate ways of exploring challenges experienced by NCV Level 3 students in the understanding of the hyperbola graph. The identification of the challenges enabled the researcher to learn from her practice through a series of reflection stages as illustrated in Figure 4.1. The process of data collection above unfolded in a spiral manner, indicating continual progress made by students in understanding the drawing and interpretation of the hyperbola graph after each intervention. This included different planning, re-planning, interventions, tests and analysis at different stages until adequate progress was reflected in students' work. In Figure 3.2 (in Chapter 3) the spiral of data collection started at the bottom of the blue arrow with a pre-test and winds around that arrow with planning, interventions, post-test, evaluation, reflections as indicated.

4.3 DATA ANALYSIS PROCESS

The researcher started with data analysis in this study as soon as pre-test writing was completed; this was an on-going process. Although the other two lecturers helped with invigilation, the researcher marked all the learners' scripts, not for right or wrong responses, but to identify the challenges experienced in learning how to work out problems in the hyperbola graph. The students' achievement in the pre-test is summarised in Table 4.1. Semi-structured interviews were piloted with three students for each of the three tests written based on their responses. A resolution was then agreed upon for the table method to be used in Intervention 1 (IN1) since the overall results indicated a narrow understanding of the required attributes with respect to the understanding, drawing and interpretation of the hyperbola. Consequently, a second test was conducted with the students to measure the progress of their understanding at the end of the intervention.

In addition, semi-structured interviews were conducted with a different set of three students. When the responses were analysed, it was observed that there was inadequate progress on understanding the hyperbola interpretations. This necessitated Intervention 2 (IN2) where students were introduced to the hyperbola through the asymptote methods. Analysis after IN2 revealed that there was still an inadequate understanding in the interpretation of drawn hyperbolas. Students were then re-taught for another hour the following week, using both the table and asymptote methods during Intervention 3 (IN3). Analysis of results from the written post-test and semi-structured interviews, conducted with the last three students chosen, indicated satisfactory results to exceptionally accepted scores as depicted in Figure 4.4. Nine students in total were interviewed for the duration of the whole study.

During this process, the researcher made some summaries next to the edge of each script, noticing the key concepts as well as similar items that originated from the scripts of each of the three tests written. These key concepts were then related to the research questions to develop main categories. Subsequently the key concepts were coded and these, including the research questions, were used as a guide to the establishment of the main themes and categories. The results from the pre-test are presented in the next section.

Table 4.1: Summary of challenges identified from pre-test

Questions	Aim of the question	Responses of students for question 1 in % (RE 1)
Q1	To examine if students could draw and interpret the graph of hyperbola using the table method.	<ul style="list-style-type: none"> • Just 6% of students could draw the correct graph although none of them could write the correct coordinates • 27% of students attempted to draw the graph of $y = \frac{1}{x}$ but others merely indicated that the method used was a hyperbola method • 67% of students drew incorrect graphs, for example, one of the graphs looked like an exponential graph while 33% of students left that section of Q1 blank and others gave incorrect graph interpretations
		Responses of students for Q2 in % (RE2)
Q2	To assess students' interpretation of the effects of both negative and positive constant 'a' in the graph of $y = \frac{a}{x}$.	<ul style="list-style-type: none"> • Just 3% of students could realise the effect of the constant 'a' on the graph • 33% made an attempt to answer the question • 64% of students could not identify the quadrants
	To assess if students could identify the horizontal asymptote	<ul style="list-style-type: none"> • Just 2% of students could identify the horizontal asymptote from the graph of $y = \frac{4}{x} - 1$ • 34% of students provided irrelevant attempts to answer the question • 64% of students could not identify either horizontal or vertical lines
		Responses of students for question 3 in % (RE3).

Q3	To examine if the students could draw the graph using the intercept and the asymptote	<ul style="list-style-type: none"> • Just 1% of students indicated knowledge of the use of the asymptotes and the intercepts when drawing the graph • 11% of students presented just an attempt to the question • 88% of students could not use and relate the asymptotes and the intercepts
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Graphical representation of challenges identified from pre-test.

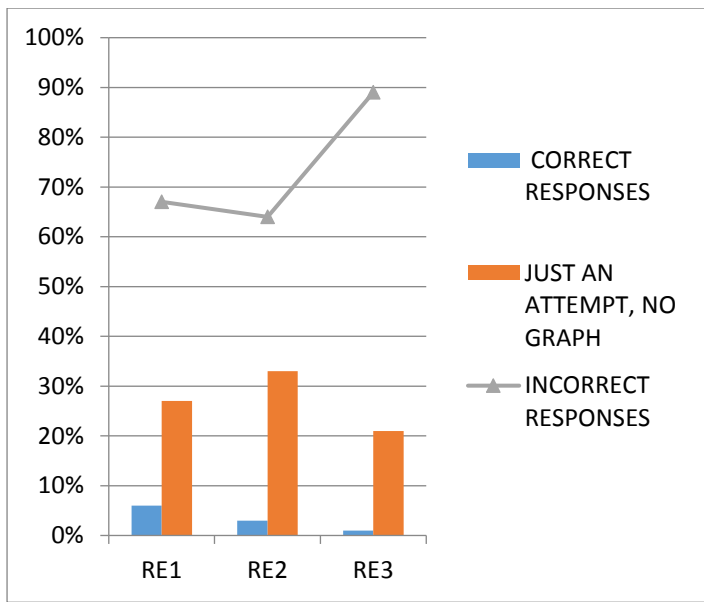


Figure: 4.2 Pre-test analyses

The figure above demonstrate the graphical representation of challenges identified from pre-test. The line graph ranging from approximately 62% to 90% display the incorrect responses of the students for different questions. The orange bar graph that is ranging from 20% to approximately 33% display those students who made just an attempt for question 1 to question 3. The correct responses are indicated by the lowest blue graph. The above graphical representation reflect a high percentage of students without the knowledge of how to draw the graph and low percentage with the knowledge of drawing hyperbola graph.

The researcher calculated the averages of the above table to check the overall performance in terms of correct responses; just an attempt at graphs and incorrect responses. Figure: 4.3 represents the average scores for all the responses from the pre-test. The averages presented in Figure 4.3 are in the form of a pie chart.

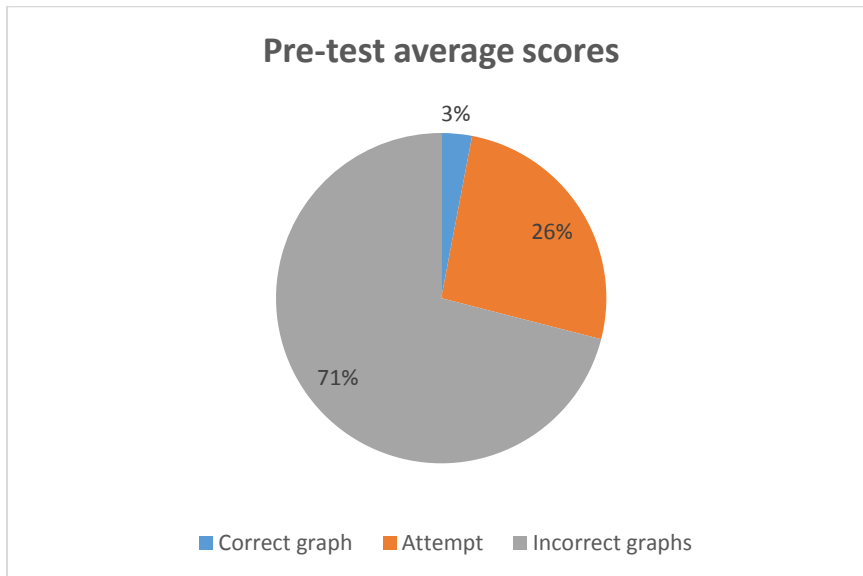


Figure: 4.3 Pre-test averages

As may be observed from Figure 4.3, the researcher discovered that on average, a mere 3% gave the correct responses and 26% made just an attempt, while 71% presented incorrect graphs. The above averages reflect a low percentage of students with knowledge about how to draw the graph and a high percentage of 71% who have no understanding of the hyperbola graph.

4.4 DISCUSSIONS AND DATA INTERPRETATION

To help the researcher to classify the categories and sub-categories she used a combination of the themes found from the pre-test, semi-structured interviews with students and her own personal experience as a senior lecturer for Mathematics and Mathematical literacy in the TVET sector. The following categories emerged:

- Challenges students faced in the drawing and interpretation of the graph of hyperbola using the table method
- Students' interpretation of the effects of changing the sign of the constant 'a'
- Students' identification of horizontal and vertical asymptotes from a given hyperbola equation

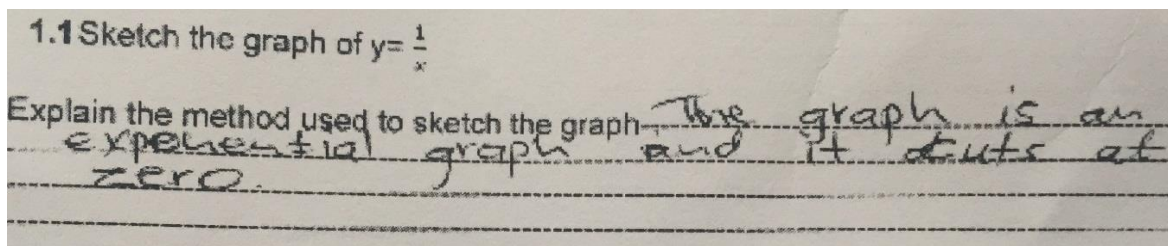
- Students' drawing of the hyperbola graph using the intercepts and asymptotes.

The emergent categories are discussed in different sections in turn below.

4.4.1 Challenges in the drawing and interpretation of hyperbola using the table method.

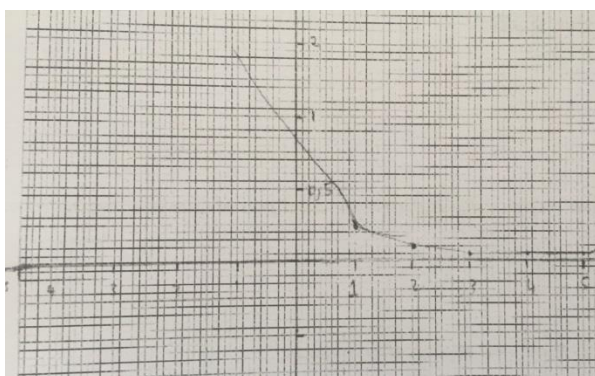
This part discusses data relating to the challenges faced by students in the drawing and interpretation of the hyperbola graph using the table method.

When the students were requested to draw the graph of $y = \frac{1}{x}$ and explain the method used to sketch the graph, they responded as follows: ST2's response is captured in Extract 4.1.



Extract 4.1: ST 2's work

To this student the plotted graph was identified as an exponential graph. The student could not explain the method used other than by indicating that the graph cuts at zero. Meanwhile another student, ST 33, did not write any explanation except to draw the graph represented in extract 4.2. Although the graph represented is hyperbola, it only occupies one quadrant and the student could not provide explanations of how it was drawn.

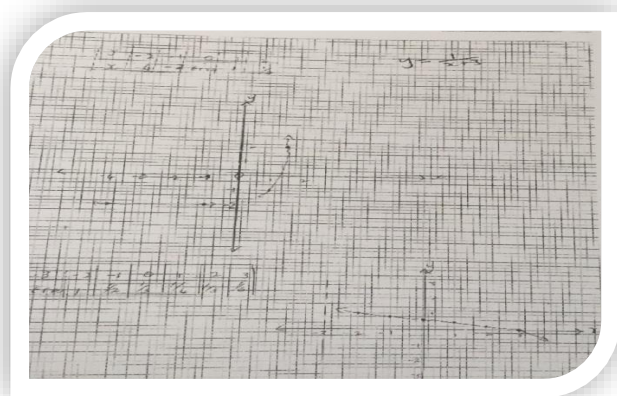


Extract 4.2: ST 33's response

In the first question the students were required to sketch the graph and explain the method used. Rather, the responses given by ST33 were the above graph, which is an exponential graph lacking explanations on the method used. From the responses of ST2 and ST33, it can be concluded that these students lacked the skills of drawing and interpreting the graph using the table method.

Students were further probed on some challenges they faced on sketching the graph of the function $y = \frac{1}{x+3}$ and providing an explanation of the method used to draw the graph.

To this, student ST2 just wrote: *'I am lost,'* while ST72 pointed to a table that he used for plotting the graph but appeared to be undecided on which response to give. This student ended up presenting two graphs for this question as indicated in extract 4.3.



Extract 4.3: ST72's Response to Question 2

From the above responses it was noticed that about 86% of the students, like ST2, when given the graph $y = \frac{1}{x+3}$, could not display any attempt. The 14% of students who attempted the question drew a table and graphs, which were not related to the question. When the students were asked to state the domain and range of a drawn hyperbola, none of them gave correct responses. For example during interviews:

ST2: *'I don't know'*

Researcher: *'You don't know my dear? Don't you perhaps want to say something about the domain and range of the graph?'*

ST2: *'I once heard about those two words, but I don't know what they mean.'*

Researcher: *'O ok, don't worry, you are going to get a lesson about hyperbola so that you can get to understand everything about a hyperbola.'*

Next was the turn of ST33; he responded to the same question as follows:

ST33: *' $1 \in R$ Domain'*

Researcher: *'hmm! $1 \in R$ is the domain, if I may ask why is your answer referring to one?'*

ST33: *'Eish! I was looking at the 'one' on the equation.'*

The researcher posed a follow-up question: *'I'm going to ask you another question, please don't feel intimidated. The purpose of asking many questions is just to help you understand hyperbola and is also helping me to understand the challenges that you have in the understanding of hyperbola. Now here comes my question. Why necessarily '1' not '3'?''*

ST33: *'I don't know, but now that Mam mentioned '3', I think is '3' because is next to 'x'.'*

Researcher: *'Tell me, why you are not talking about a range.'*

ST33: *'Mam, to be honest, I'm clueless about them.'*

Researcher: *'OK, there are several lessons that are going to be offered on this topic for you to understand.'* In a similar, incorrect, manner, ST72 responded as follows.

ST72 *' $6 \in R$ Range'*

Researcher: *'Why '6'?''*

ST72: *'I just looked at '3' and thought of the multiples of '3'.'*

Researcher: *'Ooh! Multiples of '3', why thinking about the multiples of '3'?''*

ST72: *'I don't know, I think is because there's three in the equation.'*

Researcher: *'Hmmm! O ok, because there's three in the equation, fine, you know what, later on a lesson will be offered about a hyperbola and I believe that will help you to understand.'*

The responses given indicate that all three students had no idea of either the domain or range of the given graphs. This corresponded with many of the responses found on the other scripts while marking. There was therefore a need to address the sketching, interpretation and identification of domain and range of each of the given graphs during the intervention.

4.4.2 Students' interpretation of the effects of changing of the constant 'a'.

This part focused on the students' interpretation of the effects of positive constant 'a' and negative 'a' in a given equation of a hyperbola. Several students were interviewed on the effects related to change of signs of the constant 'a' in the hyperbola graph equation.

Researcher: 'In which two quadrants is the graph of $y = \frac{1}{x+3}$ lying?' The following are some of the responses of the students who were interviewed.

ST2: *'I don't understand the quadrant.'*

Researcher: *'You don't understand quadrants? Ok, I want to help you understand them. Just wait until we give you the lesson, Ok.'* The researcher did not want to probe too much because she realised that this student's response indicated lack of information about the hyperbola.

ST2 and ST33 responded to the same question as follows.

ST33: *'Third quadrant'*

Researcher: 'If I may ask you, why third quadrant?'

ST33: *'I think.....Mh....., eish, I don't know.'*

Researcher: 'Ok, tell me, why not first quadrant?'

ST33: *'I think, it can also be the first quadrant but I don't know when it goes to different quadrant.'*

Researcher: *'I want to ask you the last question about the quadrants. How many quadrants do we have in a set of axes?'*

ST33: *'I think they are four.'*

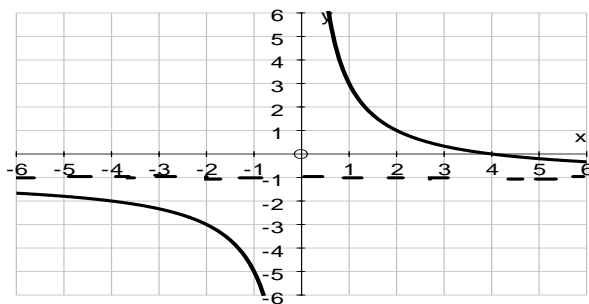
Researcher: *'Correct, but you still have to understand more about quadrants.'*

The researcher was impressed that the student knew the quadrants although he could not associate them with the drawing of the required graph. Several responses given to this question indicated that the students were not aware that the hyperbola graph usually occupied two quadrants except when restricted through prescribed x values. To the question on identification of quadrants on where the graph lies, ST72 responded as follows:

ST72: *'Second quadrant.'*

A follow up question by the researcher was used to probe more fully.

Researcher: In the graph of $y = \frac{4}{x} - 1$ given below, *'Is the value of 'a' positive or negative?'*



ST 72: *'Should be positive'*

Researcher: *'Why do you say so?'*

ST 72: *'In those quadrants both values of x are either positive or negative, so the result should be positive.'*

This response indicated that the student was not guessing in his first response but was able to justify and substantiate his response correctly. This showed that some students had a correct idea about the behaviour of the graph of the hyperbola. ST 2 was one of the students who experienced challenges with the graph locations on the Cartesian plane and demonstrated the opposite of this by his response:

ST2: *'When coming to the value of the graphs I get lost'*

Researcher: *'Ok my dear, the question is not about the value of the graph but the value of 'a'.'*

ST2: *'Ooh, maybe I did not understand, the value of 'a' is..., hmmm, eish! I don't know.'*

Researcher: *'Ok, you will get to understand later during the lesson.'* Likewise, ST33 responded as follows:

It was a different argument that ST 33 presented to the same question on whether 'a' was positive or negative:

ST33: *'There is no 'a' in the graph.'*

Researcher: *'Yes, you are right, 'a' is not there, but what I'm referring to is that which number is in the place of 'a'?'*

ST33: *'Mam, place of 'a', eish, I don't know.'*

Researcher: *'Ok my dear you will get to understand when this is discussed in the classroom.'*

The responses given by ST2 and ST33 indicated that the two students had no idea of which part of the graph was affected by the constant 'a'. They could not associate the 'a' with the constant '4' in the graph of $y = \frac{4}{x} - 1$. It was therefore on the basis of such responses that the researcher convinced her collaborators that it was evident from the students' responses that they did not understand the interpretation of the constant 'a', nor were they able to sketch the hyperbola, and that a decision was taken to embark on IN1.

4.4.3 Students' identification of asymptotes from a hyperbola equation.

In this section, data on how students identified the horizontal and vertical asymptotes from a given equation is presented and discussed. During interviews the students were individually questioned on asymptotic interpretation of the graph by the equation

$$y = \frac{4}{x} - 1.$$

Researcher: *'By how many units did your graph shift?' The following was the response of ST2:*

ST2: *'I don't know'*

Researcher: *'Why? Just try and say something about the graph.'*

ST2: *'Mam, I don't know what to say.'*

Researcher: *'A lesson about hyperbola graph will be given, make sure that you understand.'* ST33 attempted to answer the same question in a better way even though the response was incorrect.

ST33: *'2 units'*

Researcher: *'Why 2 units?'*

ST33: *'I thought maybe from one on that graph we have to go to two.'*

Researcher: *'Ooh, that's much better, but your response is not correct.'*

ST72 responded as follows to the same question:

ST72: *'By -1 units.'*

Researcher: *'Which direction is -1 units?'*

ST72: *'I don't know, these things sometimes they confuse me.'*

Researcher: *'Ok, I understand what you say, I want to help you understand hyperbola by giving you a lesson later.'* When the researcher probed for the students to identify the *horizontal asymptote*, the responses were still negative:

ST2: *'I don't know'*

Researcher: *'Why do you keep on saying 'I don't know'?''*

ST2: *'Mam, Is not to say I enjoy saying, I don't know, the thing is I don't understand this things and if maybe they can be taught to us.'*

Researcher: *'Ok, I hear what you say. That will be taken care of.'* ST33 responded as follows:

ST33: *'x-axis'*

Researcher: *'Why 'x-axis'?''*

ST33: *'Because 'x-axis' is horizontal.'*

Researcher: *'O ok. You are right, the 'x-axis' is horizontal but it is not always the horizontal asymptote.'*

ST33: *'Does it mean that the horizontal line can change?'*

Researcher: *'Yes, depending on the equation of the graph. For example, for this graph the horizontal line is $y = -1$, but don't worry the next coming lessons will address this.'*

ST72 responded to the same question as follows:

ST72: *'I think is the $y = -1$ '*

Researcher: *'Why do you say so?'*

ST72: *'I'm not sure, but I think is $y = -1$ '*

This indicates that the student had an idea of what the asymptote line was but could not justify his response. The researcher decided to ask more questions so as to identify all the challenges that the students were facing.

Researcher: *'Tell me, from the very same equation, what would be your vertical asymptote?'*

ST2: *'I don't know'*

Researcher: *'Do you understand what is meant by the asymptote?'*

ST2: *'I once heard about it but I don't understand the meaning of it.'*

Researcher: *'Where did you hear about it?'*

ST2: *'I think in Level 2 but I did not understand the meaning of it.'*

ST33 similarly indicated difficulty in understanding what was meant by 'asymptote' in a graph. His response was:

ST33: *'Hey, mam, I think is...., Mam, this thing are a bit difficult for me.'* On the contrary ST 72 had a different notion:

ST72: *'I think the y -axis' is horizontal.'*

Researcher: *'No my dear, is the y -axis' horizontal? Do you know how to identify a horizontal line and a vertical line?'*

ST72: *'No mam, I was just trying.'*

Researcher: 'Ok, there is nothing wrong in trying; actually it is good to try. Tell me, do you know how to differentiate between the 'x -axis' and the 'y -axis'?'

ST72: 'Yes, I know.'

Researcher: 'Is good that you know, Now the 'x -axis' is the horizontal line and the 'y -axis' is the vertical line. Even the asymptotes are like that, what I mean is that the vertical asymptotes, is standing like the 'y -axis' and the horizontal asymptote is like the 'x -axis'.'

The above responses indicate that the problem appeared to be wider than just identifying asymptotes but included lack of an understanding of the basic nomenclature of horizontal and vertical lines. From the responses of the students it can be concluded that none of them had acquired the knowledge about asymptotes which is one of the key concepts for the drawing of the hyperbola graph. The researcher then decided to draw the students' attention to a simple graph: $y = \frac{1}{x}$. Questions on asymptotes related to this graph and the translation of *the graph of* $y = \frac{4}{x} - 1$ from the graph of $y = \frac{1}{x}$ were not well addressed by the students.

Although the researcher tried to rephrase the question, the students still seemed not to understand the concept of an asymptote. Sometimes, when those students responded it was clear that they were guessing. ST33 tried to count the squares but he did not count accurately. Some interesting responses from some students to the meaning of an asymptote to a graph included: '*the line where the graph does not touch the 'x -axis'*', '*an open space*', and '*the graph is going along*'. These responses indicate that students had no clear understanding of what asymptotes were or of their effects in drawing the graphs. They were also unable to identify the horizontal and vertical asymptotes of the given hyperbola. The researcher summed up by giving students a full explanation during Intervention 1.

4.4.4 Drawing of the hyperbola graph using the intercepts and asymptotes

This section presents and discusses data based on students' drawings of the hyperbola graph using the intercepts and asymptotes. The researcher asked the following question: Students had to name the x and y intercepts of the graph

$y = \frac{1}{x}$. Responses here also demonstrated that the students had no idea of which parts of the graphs were the intercepts. This shows lack of understanding, which might have caused lack of interest on the part of the students about the drawing and interpretation of the hyperbola. Some of the responses indicated that students were generalising to the hyperbola graph based on the knowledge that they had gained about other graphs. This also suggested that the students may not have had the shape of the hyperbola graph in mind.

It was discovered that most of the students could not draw the hyperbola graph. Results indicated the most difficult part was to identify the asymptote for the graph of $y = \frac{1}{x}$. Sixty-four percent (64%) of students, as recorded in Table 4.1, indicated that there were no asymptotes. They could not recognise that the vertical asymptote was the y - axis and the horizontal asymptote was the x - axis. It was also discovered that students did not know the characteristics of the illustrated graphs nor were they able to write the domain and range of the graphs. Moreover, it did not make any difference to most of the students whether the values of 'a' had a different effect when $a > 0$ and another if $a < 0$ for $y = \frac{a}{x}$ and $y = \frac{-a}{x}$. It was also not clear to them that $a > 0$ means positive and the graph occupies the 1st and 3rd quadrants, while the effect of $a < 0$ implies is negative and the graph is represented in the 2nd and 4th quadrants.

Having identified more challenges that students displayed in the plotting of the hyperbola graph, and the asymptote's relation to the plotting of the graph, IN1 was conducted by teaching the students how to draw the hyperbola using the table method. This method was chosen as a starting point for the emergence of graph interpretations because it uses the process of substitution and getting more points; this always assists students as it is an easy way of plotting the graph.

4.5 IN1 USING TABLE METHOD

After the researcher had realised the seriousness of the challenges that the students experienced in the plotting of hyperbola, she decided to conduct an intervention during the second week. The intervention was undertaken by the researcher and her two colleagues in the other two classrooms. During this intervention, the students were taught how to draw the hyperbola using the table method. In addition, some

characteristics of the hyperbola such as the effect of positive 'a' and negative 'a' on the shape of the graph were also emphasised. During the third week, the students wrote post-test 1 which was the same as the pre-test written previously for 45 minutes and was marked during the fourth week. After the scripts were marked, a further three students, one from each classroom, were interviewed on the basis of their responses.

The purpose of giving them the same test was to identify the level of understanding of the students with regard to plotting the graph of the hyperbola. Furthermore, the reason that the test was marked in the fourth week was that the other students did not complete the test on time as we were using just the Thursdays of every week during the one hour of their tutorial time. This was the only time during which the research study could be conducted in such a way that it did not interfere with the smooth running of the College time-table.

In the next section, the researcher presents the results from IN1 in a table (Table 4.2).

Table 4.2 Summary of responses after IN1

Questions	Aim of the Question	Responses of students for question1 in % (RE1)
Q1	To examine if students can draw and interpret the graph of hyperbola using the table method.	<ul style="list-style-type: none"> • A noticeable increase of 47% of students presented correct graphs. • 19% of students attempted to draw the graph of $y = \frac{1}{x}$ using a correct table.

		<ul style="list-style-type: none"> • A decrease from 67% before intervention 1 to 34% of incorrect presentation of graphs was recorded.
		Responses of students for Q2 in %
Q2	To assess students' interpretation of the effects of both negative and positive constant 'a' in the graph of $y = \frac{a}{x}$.	<ul style="list-style-type: none"> • 30% of students had no idea about the effect of the constant 'a' on the graph. • 23% presented just an attempt without any explanations • 47% of students could identify the quadrants.
	To assess if students could identify the horizontal asymptote	<ul style="list-style-type: none"> • A decrease of 2% of students observed presented just an attempt. • An increase of 34% of students could identify the horizontal asymptote from the graph of $y = \frac{4}{x} - 1$. • 64% of students indicated no knowledge of which line was horizontal and which one was vertical as in the pre-test above.
		Responses of students for question 3 in % (RE3)
Q3	To examine if the students can draw the graph using the intercept and the asymptote	<ul style="list-style-type: none"> • Only 34% of students indicated knowledge of the use of the asymptotes and the intercepts when drawing the graph. • 2% of students presented just an attempt • 64% of students could not use the asymptotes and the intercepts.

Summary of test responses after IN1

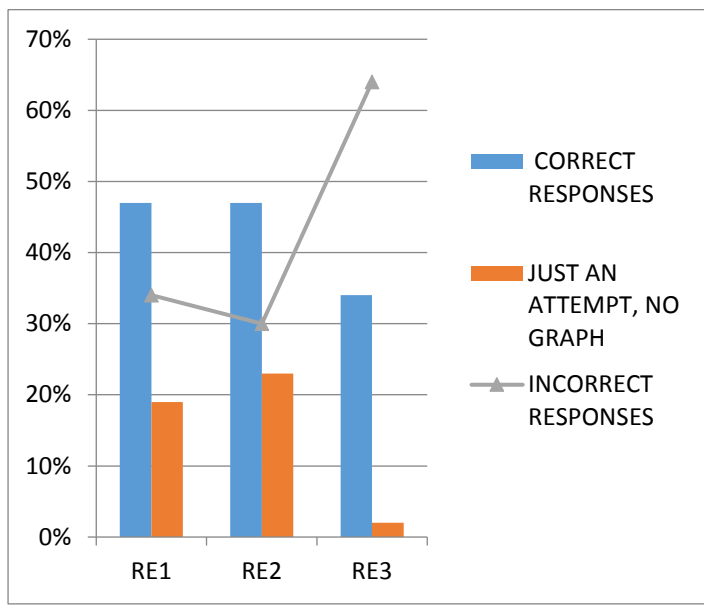


Figure 4.4 Analysis for IN1

The above representation displays a remarkable improvement of students' performance after intervention number 1. The graph of incorrect responses went down as compared to the one for figure 4.4. This means that the percentages of students who gave incorrect responses went down and the who gave correct responses went higher as compared to the lower one for figure 4.4. The just an attempt graph also went down which means that most students gave correct responses. For response number three the percentage of incorrect response is still high due to the fact that the asymptote were not yet taught.

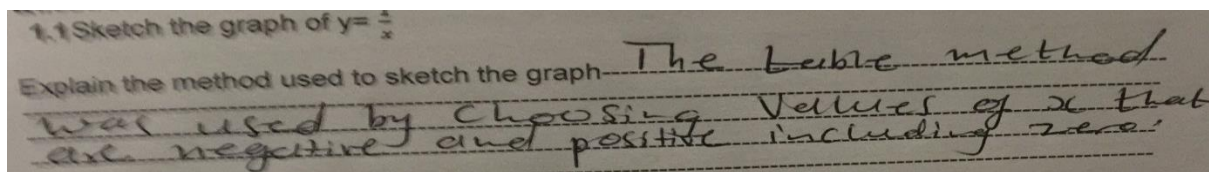
4.6 DISCUSSIONS AND DATA INTERPRETATION ON TEST AFTER IN1

To assist the researcher in checking if the intervention method made a difference to students' knowledge as regards plotting hyperbola using the table method, data was categorised into the following themes.

4.6.1 Challenges in the drawing and interpretation of hyperbola after IN1

Figure 4.2 shows that 47% of students understood the use of the table method in drawing of hyperbola while 34% presented incorrect graphs and could also not identify correct quadrants. Only 19% presented just an attempt at the graph. With regard to response three (RE3) on asymptote and intercepts, it was observed that the majority of students could not identify the asymptotes; hence, 67% of the students submitted no responses for that question.

It was noticed that most students improved their graph drawing skills even though they could not justify why and how the table method had assisted them. The following extract is ST13's work when students were requested to sketch the graph of $y = \frac{1}{x}$ and explain the method used.

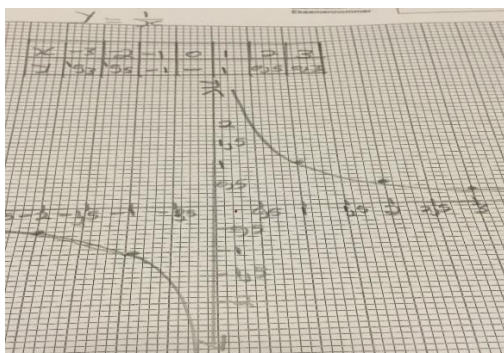


Extract 4.4: ST13's work

The response from ST13 is correct even though the graph is not indicated. It shows that the student had an incomplete idea of the method used to sketch the graph. The same applied to ST44 who gave the following response to the same question:

ST44: *'The table method. A table is drawn and 'x-values' chosen, both negative and positive. Corresponding 'y-values' are calculated and the points plotted. The points are then joined.'*

On the contrary ST89 managed to plot the correct graph with an indication of the correct quadrants. His work is displayed as Extract 4.5.



Extract 4.5: ST 89's Response

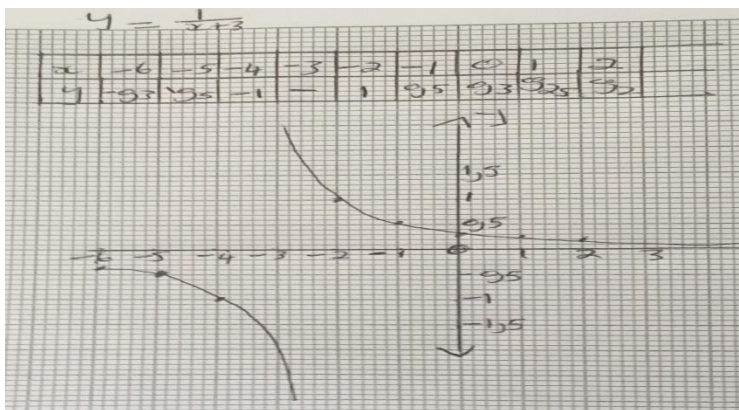
The student managed to construct a relevant table and thus drew the graph. The researchers' concern was that the graph did not clearly indicate the asymptotic movement. The improvement observed indicated that although the majority of students either attempted to, or could, draw the hyperbola through the use of the construction of the relevant table, insufficient information was revealed in their work and verbal responses about their knowledge of asymptotic movements. This gave the researcher the impression that their knowledge of asymptotes needed to be emphasised.

It therefore appears from the responses that the intervention aided the students to manage the use and construction of a table when drawing a hyperbola. The students

were then required to sketch the graph of $y = \frac{1}{x+3}$ and explain the method used.

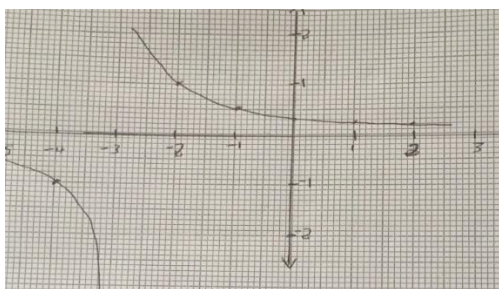
This was a follow-up question given to check for their understanding of asymptotes.

The graph drawn by ST89 is presented and illustrated in extract 4.6.



Extract 4.6: ST 89's Response

ST89 drew a partly correct graph without indicating the asymptotes. It was partly correct because the graph was touching the asymptote line $x = -3$. The absence of the indication of the asymptote line misled ST89 to draw a graph that touches the line $x = -3$. When ST89 was interviewed on how he drew the graph, his response was that he simply made the table and plotted the points on the graph paper to complete a hyperbola graph. Extract 4.7 indicates how ST44 responded.



Extract 7: ST 44's Work

ST44 likewise drew the correct graph without showing the asymptote. The graph is correct in the sense that it is not touching the line $x = -3$ and $y = 0$. When the student was asked to explain and justify his drawing, he did not mention anything with regard to the knowledge of the asymptotes. His response was:

ST44: *'When drawing the graph, I just make the table, plot the points and draw the graph.'*

Researcher: *'Oh! I see, but now what about the asymptotes of the graph?'*

ST44: *'Asymptotes? I don't think I understand the asymptotes because during the lesson we were taught only the table method.'*

This implied that although the graph drawn from a table created was accurately presented, the student had no idea of asymptotic movement. It was at this juncture that the researcher and her colleagues realised that the table method alone was not enough for the plotting of the hyperbola, because as a result the students only considered the table method and disregarded the indication of the asymptote.

During the follow up interview, the researcher also asked the students the question about the domain and range of the graphs above.

Researcher: *'What is the domain and range of the graph of $y = \frac{1}{x+3}$?'*

ST13: *'I think $x \in R$ is the Domain.'*

Researcher: *'Hmmm, I see, can you tell me why are you saying ' x ' is an element of the real numbers?'*

ST13: *'During the lesson I head the lecturer saying the domain is about the ' x -values'.'*

Researcher: *'Ok, this is good, your response impressed me.'*

The student demonstrated a remarkable understanding of the domain even though he could not indicate the restriction. The restrictions applicable to the hyperbola graph are very important because of the asymptotic lines that are supposed to be indicated when drawing the graph. The researcher asked a follow up question because she realised that the student did not say anything about the range.

Researcher: *'What about the range of the graph? Remember the first question was about the range and domain.'*

TS13: *'I think the range is about the 'y-values'.'*

Researcher: *'Yes, you are right. Then, what about the restrictions?'*

Uncertainty became obvious as ST13 responded to this question.

ST13: *'Eish! With the restrictions mam, I don't understand anything.'*

ST13's response ' $x \in R$ ' as a domain was incomplete since he also had to indicate the restriction, which is ' $x \neq -3$.' It was also noticed that the student just decided to ignore the range and when further probed he indicated an understanding of what a range was, even though he gave the answer in words, not using mathematical terms. Along the same lines, ST44 provided the following response:

ST44: *' $y \in R$ for the range and the domain is the 'x-values'.'*

The researcher was most impressed by the response from ST44 because there was a level of understanding evident in his statement. In an attempt to clarify the student's ideas, the researcher explained how domain and range together with restrictions could be expressed. It was then interesting to notice that the students felt free to enquire from her as regards statements they did not understand. For example, ST 44 had a question for her after the explanation given.

ST44: *'Mam, May I ask you a question?'*

Researcher: *'Yes, definitely, you are free to ask about anything that you don't understand.'*

ST44: *'What are these restrictions?'*

This question afforded the researcher an opportunity to clarify to the student that the hyperbola graph was not supposed to move according to someone's wishes; it is supposed to follow certain rules that are governing its movement. The researcher's response made it clear that the domain and range of the given graphs were clarified and also extended to the graph of $y = \frac{1}{x+3}$. More explanations were offered during IN2.

4.6.2 Students' interpretation of the effects of the constant 'a' after IN1

To obtain the students' interpretation of the effects of changing the sign of the constant 'a' after IN1, the researcher conducted interviews with three students on the effects of positive 'a' and negative 'a' on the hyperbola graph. The responses were very relevant, indicating two significant quadrants on which the hyperbola would be found for each of the changing values of the constant 'a'. However, some of the responses were not satisfactory when the students had to identify the quadrants on which the graph of $y = \frac{1}{x+3}$ lay. The responses given by ST13 and ST89 indicated that the two students had gained some knowledge about the effects of 'a' from the intervention. In this section it was also important for the students to realise that the graph occupied the first and third quadrants when the constant 'a' was positive whereas if it was located in the second and fourth quadrants this means that 'a' is negative.

4.6.3 Students' identification of asymptotes from a hyperbola equation after IN1.

This section deals with how students identified the asymptotes after the intervention. A follow up interview was conducted in which students were asked questions based on the asymptotes. An equation of the form $y = \frac{4}{x} - 1$ was presented but with slightly different questions. It was discovered that the students could identify the asymptotes correctly. When the question further enquired about the plotting of the graph of the hyperbola they used just the table method to plot the graph and disregarded the

indication of the asymptote. Now that they could identify the asymptotes from the equation the researcher wanted to test if they could apply those asymptotes to plot the graph. This necessitated IN2, so that the students were taught to plot the graph using the asymptotes methods alone.

4.6.4 Students' drawing of hyperbola using the intercepts and asymptotes after IN1

This part focuses on the ability to plot the hyperbola graph using the intercepts and asymptotes. The students were required to identify *the x - or the y -intercepts* on a drawn graph of $y = \frac{4}{x} - 1$. The number of correct responses given to this question increased by 34% after the interventions, and indicated an understanding of what intercepts were, from the graph. It was discovered by the researcher that a satisfactory percentage of students could draw the graph using the table method. The major challenge that the researcher identified was that when using the table method they omitted the importance of the asymptotes when plotting the graph. It was also noticed that students sketched the graphs without indicating the asymptotes, but just followed the points from the table method. This was addressed through IN2, whereby the students were exposed to the asymptotes method of graph representation.

4.7 IN2 USING ASYMPTOTE METHOD

This intervention took place during the fifth week and it was guided by the observations made and conclusions reached by the researcher made from the students' responses in IN1. During this intervention students were taught the plotting of a hyperbola using the asymptote approach and stressing of the role of positive and negative constants 'a' in identifying the correct quadrant in which the graph would be found. During this process the asymptotes of the following functions were emphasised: $y = \frac{a}{x}$, $y = \frac{4}{x} - 1$,

$y = \frac{1}{x+3}$ and $y = \frac{-a}{(x+p)} + q$. through demonstrations and use of varying examples

After IN2, the students wrote a post-test during W6 as a measure of the effect of the intervention on the understanding of the drawing of the hyperbola together with its interpretations. The researcher marked the test during W7.

In the next section, the researcher presents the results from IN2 in Table 4.3.

Table 4.3 summary of students' responses after interventions

Questions	Aim of the Question	Responses of students for Question 1 in % (RE 1)
Q1	To examine if students could draw and interpret the graph of a hyperbola using the asymptotes method for the equation $y = \frac{-4}{(x-2)} - 3.$	<ul style="list-style-type: none"> • 11% of students drew a correct graph. • 57% of students attempted to draw the graph. The asymptote was correct but no graph. • 32% of students drew incorrect graphs.
		Responses of students for Question 2 in % (RE2)
Q2	To assess students' interpretation of the effects of both negative and positive constant 'a'.	<ul style="list-style-type: none"> • Only 57% of students could identify correct quadrants. • 23% just attempted to do the question. • 20% of students could not identify the quadrant.
		Responses of the students for Question 3 in % (RE3).
Q3	To assess if students can draw a graph using intercepts and asymptotes method.	<ul style="list-style-type: none"> • 54% of students presented correct responses • 34% of students attempted to draw the graph • Only 12% gave incorrect graph.

After IN2, there was a noticeable increase: 54% of students presented correct responses as compared to the pre-test 1% recorded. Thirty-four percent (34%) of students attempted to draw a graph using the intercepts and asymptote methods, while there was a decrease of 12% in the number of students who gave incorrect responses. These results are graphically represented in Figure 4.5, below.

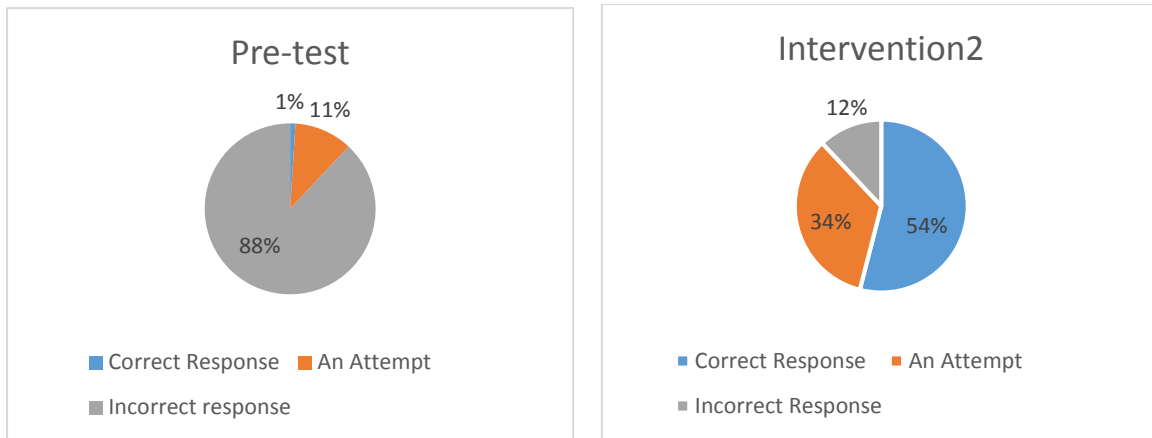


Figure 4.5 Representation of performance of the pre-test responses and IN2

Figure 4.5 indicates the noticeable differences in the responses from students where for example 1% correct presentation of responses has increased to 54%. This indicates a positive effective change resulting from the interventions applied. This has been summarised in Figure 4.6 below.

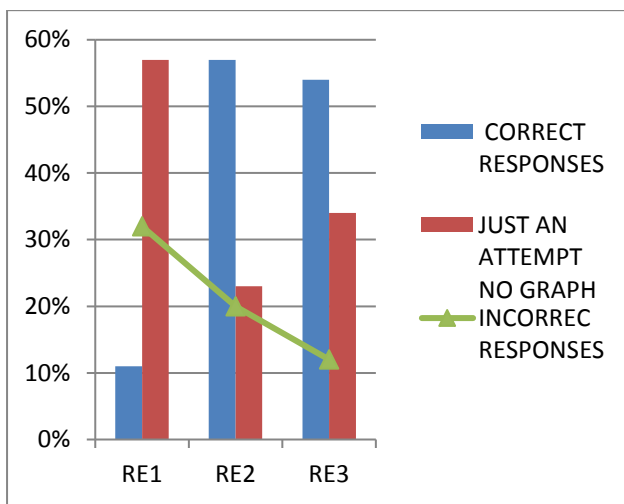


Figure 4.6 Analysis after IN2.

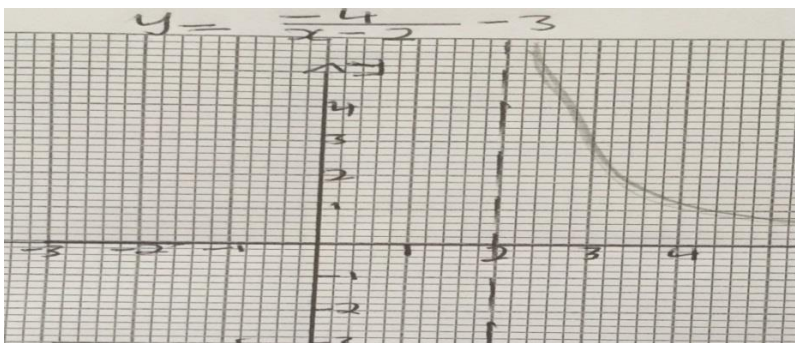
Figure 4.6 display a noticeable improvement of performance after intervention number 2. There is a remarkable drop of the straight-line graph, which is the graph of the incorrect responses. Intervention number 2 made a positive difference as compared to the intervention number one in figure 4.4. A noticeable increase in percentages of correct responses, which is above 50% for RE2 and RE3 whereas in figure 4.4 it was below 50%.

4.8 DISCUSSION AND DATA INTERPRETATION OF THE TEST AFTER IN2

For this section, the categories that were tabulated above are again discussed in the next section as a way of assisting the researcher to make a meaningful interpretation of data presented above.

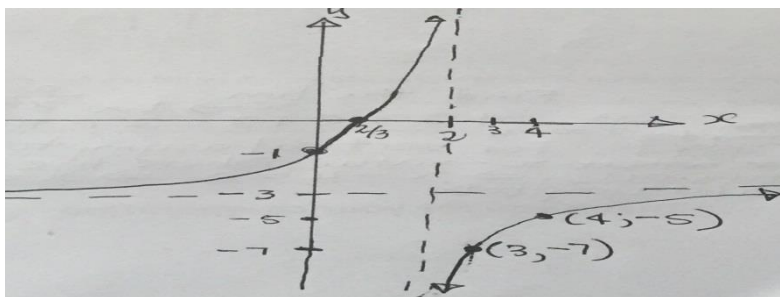
4.8.1 Challenges in the drawing and interpretation of hyperbola after IN2

The challenges as graphically represented in Figure 4.3 indicated a very small increase in percentage performance. However, a high percentage of 57% of students presented attempts after IN1 as compared to 27% attempts during the pre-test. This higher percentage demonstrates that students gained some knowledge from IN1. What was happening in this attempt was that the students managed to indicate the correct asymptote but could not draw the graph. The following is the response of ST89:



Extract 8: ST89's work.

The implications might be that the students became confused because there were no points that were giving them the direction of the graph. Just 11% of the students managed to apply the idea of the asymptotes together with the quadrant to plot the correct graphs. ST13 responded as follows:



Extract 9: ST13's work.

Although ST13 managed to draw the correct graph with the correct quadrants and correct asymptotes, the researcher realised that the student used a few points in guidance to get the direction of the graph. The researcher also realised that not all the students could apply the asymptotes method. Comparing the results from the table and asymptote methods applications, it was evident that there was still an inadequate number of students who managed to draw the graph of the hyperbola. The students also used the different two methods that could not be used to complement each other. Thus, in the last intervention, the two methods were combined.

4.8.2 Students' interpretation of the constant 'a' after IN2

After the second intervention, students' responses indicated an improvement in their understanding of quadrant locations of given hyperbola graphs. This consequently enabled them to be able to predict the quadrants which the hyperbola graph occupies with respect to each value of 'a' in a given equation. The percentage of students who gave incorrect responses in Figure 4.3 was below 35% compared to Figures 4.1 and 4.2, which were about 80% and 60% respectively. This indicated a great improvement, brought about by the interventions.

4.8.3 Students' identification of asymptotes from a hyperbola equation after IN2

This section concerns how students identified the asymptotes after the second intervention. Three students were interviewed to obtain an in-depth understanding of a hyperbola based on an equation of the form $y = \frac{-4}{(x-2)} - 3$. Questions on identification of both horizontal and vertical asymptotes were posed to each of the students interviewed. Some responses were:

ST13: '*Horizontal asymptote is $y = -3$* '

ST44: '-3'

The response given by ST 44 gives no clear indication of whether the student clearly understood the difference between horizontal and vertical asymptotes. This kind of response renders students to guesswork or confusion on whether the asymptote lies on the x or y axis. Data also revealed that some students understood only the horizontal asymptote. It was also discovered by the researcher that some students,

such as ST13 and ST89, had knowledge of the vertical and horizontal asymptotes, but would pronounce them as 2 and or -3 without specifying whether this is a horizontal or vertical asymptote.

4.8.4 Drawing of the hyperbola using the intercepts and asymptotes after IN2

This section presents and discusses data that informed the researcher about the students' drawing of the hyperbola graph using the intercepts and asymptotes after IN1 and IN2.

Students' responses indicated a conflict of understanding between the intercepts and asymptotes when drawing the hyperbola graph. They did not realise that the point where the graph meets or cuts the x -axis is the x -intercept. This prompted the researcher to conduct Intervention 3 in which the asymptotes and a few points were used in the plotting of the hyperbola. This enabled the students to identify the relationships between the use of table and asymptote methods when drawing the graphs.

4.9 THE USE OF THE TABLE AND THE ASYMPTOTE METHOD in IN3

The purpose of Intervention 3 was to teach the students both the table and asymptote method when plotting the hyperbola graph. The researcher wanted to eradicate the tendency of students to draw graphs without an indication of the asymptotes, or plotting the asymptotes without indicating the correct graph in the correct quadrants. Those were some of the errors reflected in students' work before Intervention 3. This became the last intervention in this study.

The following figure, Figure 4.7, represents a summary of the test results after Intervention 3 (IN3):

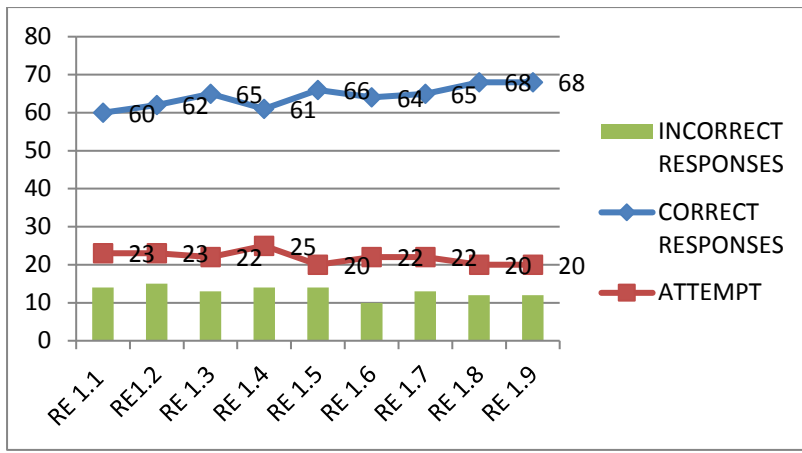


Figure 4.7 Post-test analyses for intervention three (IN3)

The blue graph indicates scores ranging from 60% to 68%, representing the correct responses, while the red one, which starts from 20% to 25%, is an indication of scores for problems attempted in the post-test assigned after IN3. A remarkable decrease in percentages that range from 10% to 15% of different responses of students whose responses were incorrect was noticed at this stage. The above indicated a drop in 68% of the students whose graphs were incorrect and an increase of 65% of students who provided correct responses.

The average percentages of the post-test calculated to check the overall performance of the students are portrayed in Figure 4.8.

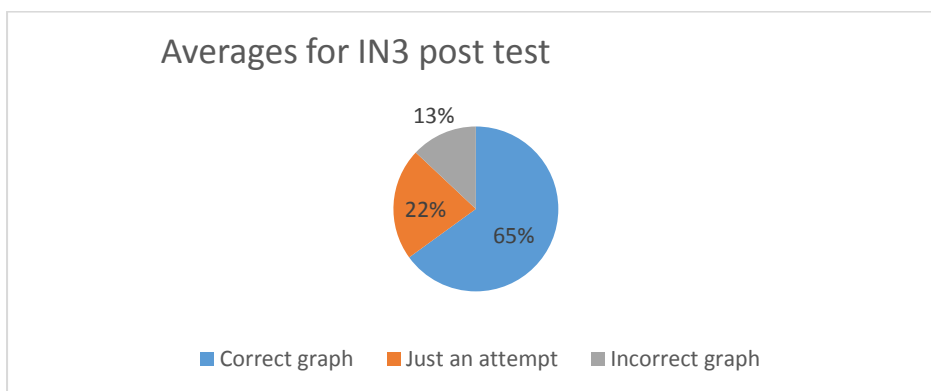


Figure 4.8: Average of post-test after IN3

The researcher discovered that after IN3 there was, on average, an increase of 65% of students who plotted correct graphs as compared to the 3% of the pre-test. Also after IN3, a noticeable decrease of 13% of students presented incorrect responses as compared to the pre-test 88% initially recorded. Twenty-two percent of students

attempted to draw the hyperbola graph as compared to the lesser number of attempts observed from the pre-test, which was recorded at 11%. The improvement in the number of attempts and the correct graph indicated a remarkable increase in the level of understanding of the representation and drawing of hyperbola. This improvement was adequate; hence the interventions were stopped.

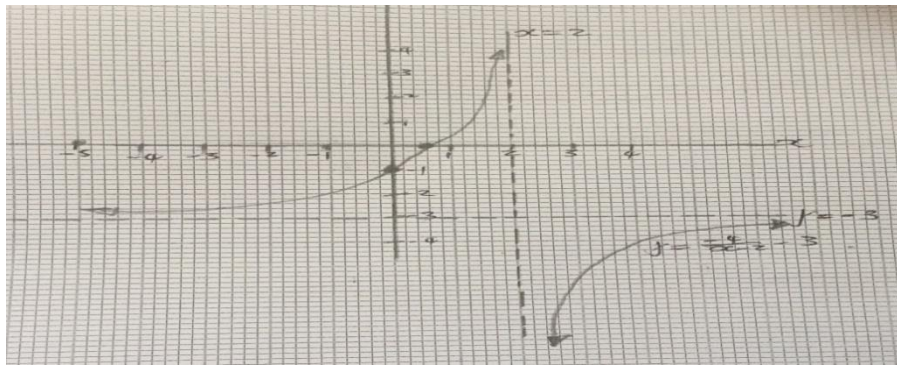
4.10 DISCUSSION AND DATA INTERPRETATION OF THE POST-TEST RESULTS

4.10.1 Students' drawing and interpretation of the graph of hyperbola

In this section, data relating to the responses of students from IN3 are considered.

One of the questions in the post-test required students to sketch the graph of $y = \frac{-4}{x-2}$

3. Extracts such as the work of ST78, captured as Extract 10, were observed.



Extract 10.1: ST78' work

In excitement stemming from the many correct responses displayed for this question, the researcher then requested ST78 to share his experience on the learning of the hyperbola.

ST78: *'It just came along that I didn't develop any interest in the topic.'*

This response was a true reflection of the fact that most of the participants were initially uncomfortable with learning the drawing, representation and interpretation of the hyperbola graphs and other topics in mathematics generally. A very different set of responses were recorded after Intervention 3. The change in students' responses can be associated with the confidence and excitement created by the interventions from IN1 to IN3 as regards the learning of the graph. The excitement could be evaluated

from the statement by one of the students about drawing the graph during the last interview after the post-test:

ST25: *'I find it to be easy and simple to plot the graph of hyperbola. Since you and other lecturers started teaching us with the method you were using, that is noting what the asymptotes are, understanding the role of 'a' in giving the correct quadrants and also indicating important points like intercepts, I find it interesting and practical to learn the hyperbola graph. Also, since I started learning hyperbola after your lessons, I realised that a hyperbola graph is not complete until you indicate asymptotes. If you cannot indicate them in your Cartesian plane, your graph can end up going over the asymptote line without noticing.'*

The understanding of the participants also had serious implications for their level of capacity in terms of interpreting the effects of changing the sign of the constant 'a'.

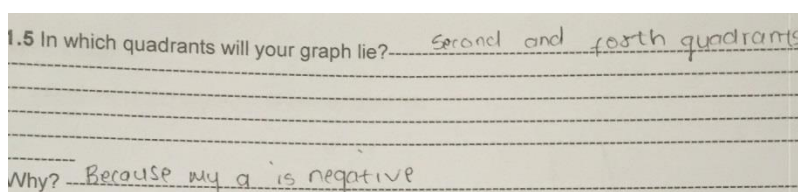
4.10.2 Post-test students' interpretations as the constant 'a' changed signs

Semi-structured interviews conducted with three students before IN3 indicated that the drawing of the hyperbola worried students from their previous level and when they encountered this mathematical concept in Level three, lack of confidence and presentation of work without thinking dominated their performance in the section involving hyperbolas. Initially, they could not predict or identify in which quadrants the graphs would lie. During a follow up interview ST25, ST54 and ST78 were asked the following question by the researcher. Responses like...

ST54 *'I only understand that a hyperbola should be in two quadrants but which one specifically I cannot tell,'*

were prominent before the interventions.

However, there was a new mind-set for learning the hyperbola after IN3. For example, ST25's response represented in extract 11:



Extract11: ST25's work.

ST 25: *'I now understand the quadrant because my mind is now broad.'*

The above response indicates that the student did not just give a response but referred to issues around the drawing of the graph and their confidence in answering the question about hyperbola after several interventions. Initially when students were questioned, they could not answer adequately:

Researcher: *'How is your understanding of rectangular hyperbola?'* Responses included:

ST25: *'It's confusing since I did not understand it properly from level 2. The fact that we had to identify the asymptotes and know the quadrants in which the graph will be lying is a serious problem to me. You know what madam, I can use points to plot the graph but what is frustrating me was to be told that my graph is touching the line that is not supposed to touch, then the word 'how' will be speculating in my mind.'*

These sentiments changed after IN3.

ST25: *'Today I'm a proud and confident student who can answer any question about hyperbola without any doubt and also the way you explained domain and range will enable me to answer any question about them from any graph.'*

This indicates that the student's confidence and level of understanding of hyperbolas and tackling problems involving those graphs had increased.

4.10.3 Students' identification of asymptotes in a hyperbola equation after IN3

This section discusses how students identified the vertical and horizontal asymptotes in given hyperbola equations after the IN3. The students' challenges in this section are revealed through their responses to semi-structured interviews conducted with them prior to the intervention and after IN3.

Before IN3

Students were keen to receive support both from within and outside their college to help them develop the knowledge of identifying the asymptotes from a given hyperbola

graph. Although it was a requirement that students be competent in analysing and identifying asymptotes in given hyperbola equations in Level 3 TVET education, the researcher also noticed during interviews that this skill was lacking. Also, most of the students could neither explain nor check information on the graph drawn. This is evident from what ST54 said before participating in the three interventions.

'I sometimes mix the asymptote, I found myself not knowing which one is vertical and which one is horizontal.'

After IN3

At the end of the intervention, some students developed strong principles about the importance of indicating the asymptotes before plotting the graph and different ways of solving graphical problems. Students displayed mathematical knowledge relevant to solving unfamiliar problems of this type. They also directed their attention to developing suitable strategies and checked on the accuracy and rationality of answers to the given hyperbola problem. This is evident from the observed improvement in solving unfamiliar hyperbola questions, their capacity to identify wanted information, efficacy in choosing information required to solve the given question and discernment and discarding of irrelevant information in the graph.

4.10.4 The use of intercepts and asymptotes in drawing a hyperbola

This section focuses and discusses data that informed the researcher about the students' drawing of the hyperbola graph using the intercepts and asymptotes method after IN3. The challenges experienced by students before IN3 are first recalled, followed by a report on discussions held with some of the students after the interventions.

Before IN3

From the responses gathered during the semi-structured interviews it appeared that the students were not knowledgeable about the content based on the intercepts of the hyperbola. This was to be expected as almost all of the students were hesitant when it came to the answering of the interview questions. One of the students, during the

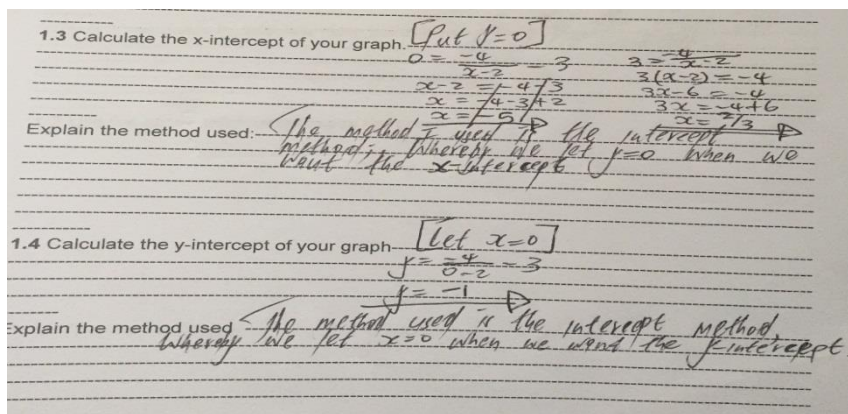
interview, indicated that her knowledge was limited as her interaction with the topic of the hyperbola was minimal due to fact that she was always absent when this topic was taught. She also emphasised that she experienced a phobia towards the topic. The lack of intercept identification can be associated with a state where students just switch off and give up on understanding mathematical concepts introduced in their absence. They lose hope when other concepts that rely, and are based, on pre-requisite knowledge are lacking due to the hierarchical nature of mathematics. For example, when one of the students was asked about the method of calculating intercepts of a hyperbola, he said:

ST78: 'Sometimes is difficult to calculate the intercepts but we usually try. I think, had it not because of our absenteeism and phobia for hyperbola we would have been better when coming to the answering of the question.'

It appears from the response that the student is aware that when students are absent from and or reluctant to attend the class, this becomes an enormous challenge with which lecturers are faced, while senior management are expected to deal with this issue and arrange solutions in their planning for the following year. This has always been a challenge for mathematics lecturers as lecturing time is compromised, while students also miss important basic information required to understand mathematics concepts. The challenge that is faced by colleges is related to the fact that according to the literature study any disturbance in the educational programme may delay and be damaging to the instructional programme, and affect results negatively in the entire process. The other student expressed a different but related view:

ST54: 'For me I found it difficult to understand my lecturer in class, when I'm in class I can see that he is teaching but I don't get the concepts to the fullest.'

The finding above also supports the view in the literature as far as the pre-knowledge of the students in the learning of the hyperbola is concerned. Pre-knowledge is key to the understanding of graphs. Students' lack of pre-knowledge from lower levels resulted in their development of a phobia for the topic of the hyperbola. This pre-knowledge includes conceptual knowledge and understanding of terms such as intercepts, asymptotes and drawing of graphs.



Extract 13: ST54's work

From the response above it looks as if students confirm the fact that the interventions assisted them in interpreting the graph and providing guidance as well. Based on the interventions and support given to the students, I reproduce a request from one of the students at the end of the interview session:

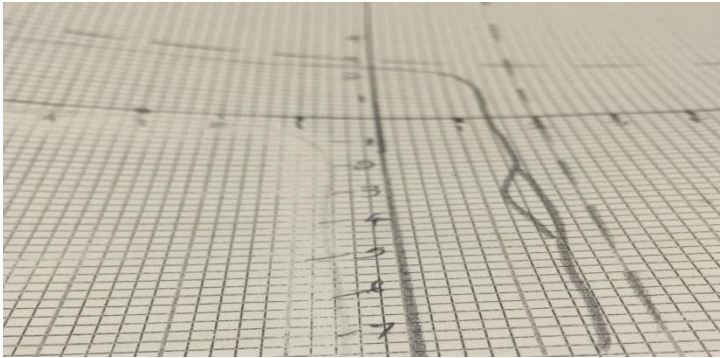
'Could you please be our permanent mathematics lecturer next year 2016 because your method of teaching is making a huge difference in our college and in our lives? We really now feel that we are the same with other colleges and other schools in the understanding of hyperbola. If possible come back the following year to be our mathematics teacher for all the topics and I believe this will help us to iron out issues that are challenging to us. I would like to take this opportunity to thank and wish you a progressive year in your research. Good luck.'

The work of ST54 was also presented above to emphasise the comment by one student above.

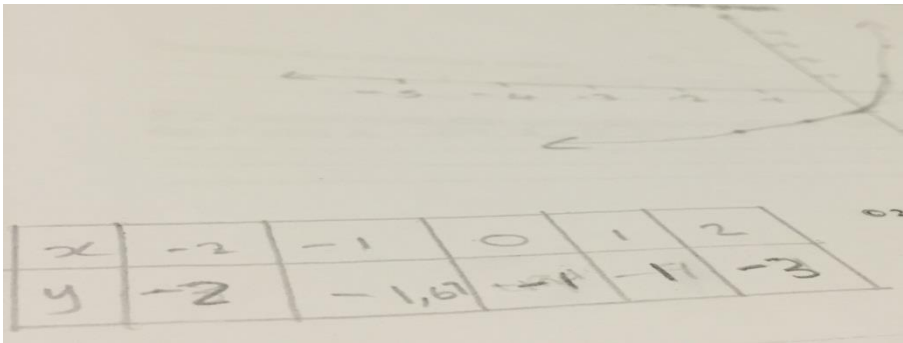
All the above reported interventions became valuable tools that students could use at the TVET college to improve their results in mathematics. Intervention programmes put in place by the researcher became effective when the presentation of the hyperbola included asymptotes identified properly; the instructional programme was supervised and evaluated while students' progress was monitored and their results were recorded and discussed with them.

4.10.5 GRAPHICAL ILLUSTRATIONS FROM PRE-TEST TO IN3

The purpose of this section is to demonstrate the progress made by students in the plotting of the hyperbola from the pre-test through all stages of interventions. It will be recalled that most students represented graphs of $y = \frac{1}{x}$ as:

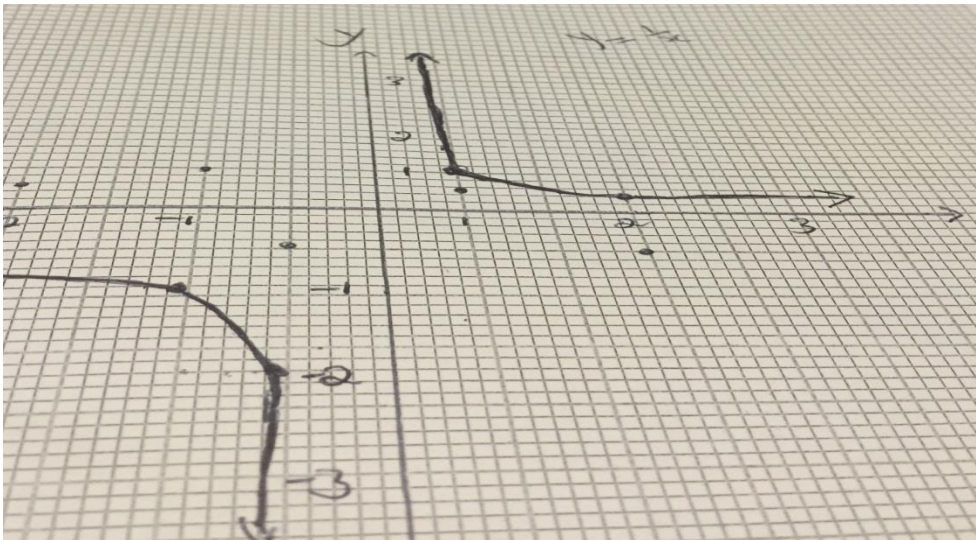


Evident from the graph was uncertainty regarding the curves and lines drawn although the student was courageous enough to attempt it, even though he was not sure of what he was doing. In the following drawings, the student reflected improvement in knowledge of the asymptotes even though they were not correctly drawn.

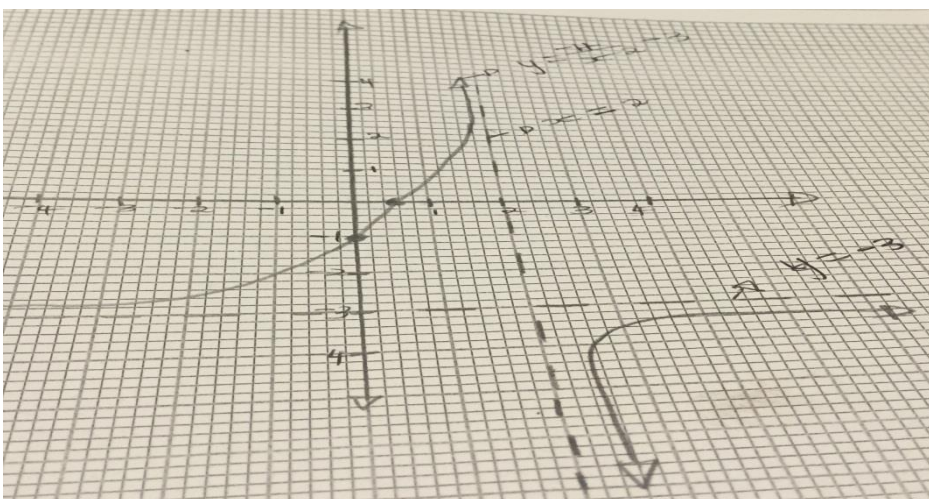


There was also inclusion of the table as reflected in the extract but the graph was only drawn in one quadrant although there were no restrictions on the given values of x.

After IN1 with table method



After the first intervention, graphs with more meaning were drawn even though some points were not joined. After IN2 a greater number of students were able to represent graphically functions such as $y = \frac{-4}{x-2} - 3$, indicating the correct asymptotes where applicable. There were about 17% of students whose work reflected the use of points to draw graphs even though the graphs were not complete. The final graphs (65%) of students' work were indicative and inclusive of points, asymptotes and analysis as illustrated in the extract below.



The students were also proud about their results after the interventions. The results of the post- test indicated that Interventions 1-3 made a huge difference in enabling their conceptual understanding of the drawing and interpretations of the hyperbola graphs. This was evident from the steps demonstrated from the students' work above.

The purpose of this study, it will be recalled, was to identify challenges faced by students in understanding the hyperbola function in mathematics. Results were presented and discussed in this chapter. The findings and suggestions on the learning of the hyperbola graph at TVET Level 3 are contained in the following chapter.

4.11 CONCLUSION

In this chapter, the researcher offered an analysis and discussions of the results in accordance with the following categories: (i) challenges students faced in the drawing and interpretation of the graph of hyperbola using the table method; (ii) students' interpretation of the effects of changing the sign of the constant 'a'; (iii) students' identification of horizontal and vertical asymptotes from a given hyperbola equation and (iv) students' drawing of the hyperbola graph using the intercepts and asymptotes. This chapter also presented clarifications of how students' responses are presented in graphs, and these responses cited verbally, serve to express the challenges they experienced in the learning of the hyperbola function in this TVET College. The following chapter concludes the research by discussing the results, researcher's opinions, recommendations and restrictions of the study.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

The purpose of this study was to identify the challenges faced by TVET Level 3 students in understanding the rectangular hyperbola function in mathematics. Furthermore, the study aimed at suggesting ways in which those challenges could be addressed. This final chapter provides an overview to confirm that the research question, sub-questions aim and objectives originally stated in the first chapter were addressed and achieved. The chapter also furnishes a summary of the major findings in this study. It also contains a review of the research questions, limitations, some areas for possible further research, recommendations and conclusions of the study.

5.2 DISCUSSION OF RESEARCH QUESTIONS

The following main research question was formulated at the beginning of the study as:

How do TVET Level 3 students deal with challenges experienced in the learning of the rectangular hyperbola function in mathematics?

In order to answer the above research question, research sub-questions and specific aims were formulated to provide strategies for the study. The research sub-question 1 was:

What are the challenges faced by TVET Level 3 students in understanding the rectangular hyperbola function in mathematics?

This research sub-question was addressed by a review of the literature. A lack of prerequisite knowledge relevant to the understanding of graphs was one of the challenges that students face in the learning of the hyperbola according to the data analysed. Prior knowledge is a skill that is required for students to grasp new knowledge. Lack of relevant prerequisite knowledge such as construction of tables,

using the domain, rule and range to plot functions in previous levels, contributed to students' failure to retain rectangular hyperbola skills learned in the higher levels.

This sub-question was further addressed by the data collected from the pre-test and the semi-structured interviews conducted with students during the first week of data collection. Themes were derived from the pre-test and the semi-structured interviews that assisted the researcher in identifying the categories and sub-categories. The categories were then discussed one after the other in different sections in order to identify the challenges students faced in the learning of the rectangular hyperbola and ways to minimise those challenges.

Research sub-question 2 was: How can students' challenges be addressed in the learning of rectangular hyperbola in mathematics?

It was found that when students use and calculate a number of points to sketch a suitable graph, they finally draw a correct representation of the given function. This question was further addressed through the applications learnt in IN1 to IN3 conducted with the purpose of identifying ways to minimise students' challenges in the learning of rectangular hyperbola. Three different interventions were conducted because the first one where students were introduced to the table method did not yield positive results, while the second intervention on the use of asymptote approach produced inadequate results. The third intervention (IN3) where both the table and asymptote methods were used simultaneously yielded satisfactory results, with an indication of deeper understanding of the drawing and interpretation of the rectangular hyperbola function.

After IN3, it was found that students had gained in-depth understanding of the plotting, representing and analysing the rectangular hyperbola graph. This was evident from the quality and accuracy of the graphs displayed by several students' responses in the post-test. A number of graphs drawn were placed in correct quadrants with corresponding suitable indications of asymptotes and intercepts on them. These results were possible with the spiral action research where students were exposed to the rectangular hyperbola through different methods until adequate results were obtained. In each intervention, all participants worked together towards a continuously

developing fulfilment of understanding the drawing and representation of the rectangular hyperbola. The researcher was exposed to certain students' critical reflections from the semi-structured interviews conducted after each intervention. These reflections were used to plan on strategy development and decide on how implementation in the following cycles must be actively carried out in the following cycles by both the researcher and the students.

The spiral action research was used to first introduce students to the drawing of tables in order to plot the graphs. This method was used because it connected and linked with pre-knowledge of drawing graphs learnt in previous grades. The study was based on action, evaluation and critical analysis of practices based on collected data in order to introduce improvements in relevant practices involved in drawing the rectangular hyperbola. In the next stage, students were introduced to new knowledge that included identification and use of asymptotes when drawing the rectangular hyperbola.

At this stage, the evaluation that followed IN2 revealed that students lacked the skill of combining knowledge learnt in using tables to plot the graph together with identifying asymptotes. Spirally, IN3 was embarked on where students were introduced to both the table and asymptote methods and how to use them simultaneously in drawing the rectangular hyperbola. The action research cycle stopped at this stage because adequate results were obtained as was revealed by the post- test results.

5.3 SUMMARY OF THE FINDINGS

The outcomes of this study afford officials, lecturers and students with information regarding challenges that Level 3 students face in the learning of rectangular hyperbola and suggest methods that can be followed to minimise those challenges faced by Level 3 students in the learning of rectangular hyperbola. This section offers a summary of the literature review, the research methodology, design and the findings of the empirical investigation.

5.3.1 Summary of the literature review

To reiterate, the main purpose of this research was to identify the challenges faced by TVET NCV level 3 students in understanding the rectangular hyperbola in mathematics and suggest ways to minimise students' challenges in learning this topic. These aims were achieved by conducting an extensive literature review. Features in the latter included the perspective and context of the study, function as a concept, the historical development of the function concept, while Figures 2.1 and 2.2 depicted discussions on the effects of the parameters 'a' on the rectangular hyperbola and the teaching and learning of the function concept.

Other aspects that were covered included students' challenges in the learning of hyperbola functions. Those were prerequisite knowledge, intuitions, misconceptions and difficulties experienced by students when learning the rectangular hyperbola. Furthermore, in order for the reader to understand misconceptions and difficulties, those were explained through the exploration of different definitions of functions in the literature, the correspondence, linearity, and representations of functions. Different methods on understanding the rectangular hyperbola were also dealt with in the literature reviewed.

The theoretical framework underpinning the study was also dealt with in the literature review. This study was conducted through the 'lenses' of Piaget's theory of cognitive development.

5.3.2 Summary of the research methodology and design

This study adopted the participatory action research design in which a qualitative approach was used in order to allow in-depth understanding of students' responses. The justification for employing this research design and research approach was provided in sections 3.4.1 and 3.5.2, respectively. The qualitative approach made it possible for the researcher to collect qualitative data using pre-testing, semi-structured interviews, tests and post-testing, all of which were administered spirally until satisfactory results were obtained. The tests were not written to obtain right and wrong answers but to see and analyse misconceptions.

Various data collection instruments were used to ensure the fullness and credibility of the findings. The research was conducted in the natural setting of the students to increase the reliability of the findings. The components that were applied in the data analysis as well as the methods that were employed to assure the reliability and validity of the study were also described in Chapter 3. Issues of ethical considerations to address the nature of power relationships between the researchers and participants, consent and anonymity, together with privacy and confidentiality were also addressed in detail. However, the statements were meant primarily to inform participants about ethical judgements rather than to impose these on them.

5.3.3 Summary of the findings of the empirical investigation

In Chapter 4 the results and analysis of the empirical investigation of this study were presented; only a summary of the results is provided in this chapter. The analysis and interpretation of the data from the pre-test, semi-structured interview and post-test indicated that at the end of the interventions there were minimal challenges that students faced in the learning of rectangular hyperbola. After the IN3, a number of students' responses indicated adequate deep understanding in the use of both asymptotes and tables to draw the graphs. There were still some issues with the interpretation of graphs drawn and with some labels on the drawn graphs, but they were drawn in the correct quadrants.

The findings from the semi-structured interviews were that the students voiced relevant responses explaining the plotting of the rectangular hyperbola with respect to identification of quadrants in which the graph lies. This occurred when the students were introduced to both the table and the asymptotes as methods to be used simultaneously. The identification of the correct quadrants because of positive and negative 'a' from the formula $y = \frac{a}{x}$ was also indicated because of the progress made by the students in the drawing of the hyperbola after interventions.

The progress that the students made was that they were able to identify the correct quadrants relevant to positive and negative 'a' from the formula $y = \frac{a}{x}$. The stages indicating students' improvements from different interventions are illustrated in section 4.11. The main improvement was observed when the students were able to indicate the asymptotes from a given function in the process of plotting the hyperbola graph.

In dealing with the challenges experienced in the learning of hyperbola function the TVET Level 3 students should firstly identify those challenges with the help of their lecturer. Challenges like identifying the asymptotes, misconceptions, correct quadrants should be highlighted. Students must be taught in the lecture room and also they must be hands on, meaning that they must practice the drawing of hyperbola with special emphasis on the indication of asymptotes and showing the intercepts of the graph if the graph has some translations.

The above findings agrees with the current literature as it is stated in section 1.6.3 that in the case of $f(k) = \frac{1}{k}$ the asymptotes according to Kaufmann et al., (2004) are the two coordinate axes. Also under section 1.6.4 it is mentioned by Kelly (2013) that one of the challenges in the learning and understanding of hyperbola is the students' lack of prerequisite knowledge, as mathematics often builds on information learned in previous years. Kelly (2013) asserts that if a student does not have the required prerequisite knowledge, then a lecturer is left with the choice of either doing remediation to cover material the student might not understand. That is why three different interventions with different method were conducted until the students show a level of understanding of the concept

5.4 LIMITATIONS OF THE STUDY

As indicated in Section 3.8, the sampling method adopted for this study was convenience sampling because the researcher worked with students allocated to her. This formed the first limitation of this study. Furthermore, just 90 students were used in this study; the small sample utilised in order to enhance the richness of the qualitative data may not be enough to generalise the results.

The second limitation was that the researcher had to keep to the topic of the rectangular hyperbola with mathematics Level 3 students at a TVET college, yet the time available was minimal since a lot of content relevant to the understanding of this function had to be covered in a short time. The researcher was allowed to use the tutorial time with a duration of one hour every Thursday for 10 consecutive weeks.

The third limitation was that English as a language was used continuously throughout this study as the main communication language since the researcher was unable to translate such words as asymptotes and quadrants into the various languages of the students. This resulted in the researcher's lack of confidence in ensuring that low performing students understood several key concepts. The last limitation was that extensive data was collected through an action research design and it took a great deal of effort and time to go through all interventions until the appropriate and anticipated performance was achieved.

5.5 AREAS FOR POSSIBLE FURTHER RESEARCH

As indicated in Section 1.5, the researcher knows of no other study of a similar nature that attempted to explore the challenges faced by TVET Level 3 NCV students in understanding the rectangular hyperbola function in mathematics. Although the researcher is confident that this study clarified the problem in question, the outcomes can be regarded as tentative and she felt that further research might be conducted with a larger group. More research is also needed based on knowledge constructions made by students during the plotting of hyperbola graphs. This research would further investigate how students proceed from drawing hyperbolas without asymptotes to considering the use of given asymptotes to plot the function.

5.6 RECOMMENDATIONS

The following approvals were suggested because of this study to address the specific objective mentioned in Section 1.9.

- The current study should be simulated using other functions studied by TVET Level 2, 3 & 4 students. It is essential to explore challenges faced by TVET Level 2 students in the learning of exponential functions, for example. This

research could be carried out in Level 2, 3 & 4 mathematics topics such as parabola, differentiation and integrations.

- The researcher therefore recommends that the DHET assign part of its budget to recommend programmes where lecturers are trained on the use of action research to improve the results in mathematics.
- Mathematics curriculum inventors and policy producers should be made aware of the identification of asymptotes, the few points and quadrants that learners must follow in the plotting of a rectangular hyperbola and should design a programme that promotes and accommodates these methods of plotting a suitable graph.
- It is recommended that students be introduced and trained using the asymptotes, few points and quadrants simultaneously in order to draw hyperbolas.
- Lecturers must be given satisfactory preparation time as well as more lecturing and learning time so that they can apply the action research design and learning environment necessary for learners to develop the skills of plotting and interpretation of the hyperbola function. The researcher also recommends that additional mathematical lessons be arranged in colleges during mid-afternoons, weekends or college vacations to afford lecturers enough time.
- The researcher also endorsed the view that every effort be made to encourage college Level 3 students so that they can easily benefit from the most suitable method used in the plotting of graphs.
- The researcher further recommends that a larger and broader study be conducted involving a larger sample so that the results can be generalised.

5.7 CONCLUDING REMARKS

In this chapter, a brief summary was given of the review of the research questions of this study. The researcher presented findings interpreted from the responses of the interviewees, general conclusions, and recommendations for future Level 3 TVET College practice in handling the hyperbola function as well as recommendations for further research.

While exploring the challenges faced by TVET level 3 NCV students in understanding the rectangular hyperbola function in mathematics, the results and findings of the study indicated that students after IN3 indeed developed and understood the most suitable method of plotting rectangular hyperbola using asymptotes, table method and the identification of correct quadrants. The expertise gained by students in mastering this skill will further influence better performance in mathematics by the TVET Level 3 mathematics students.

The researcher is of the view that the results of this research could have important implications for the teaching and learning of rectangular hyperbola. In this study, it was noted that if students develop and understood suitable methods of plotting the hyperbola function they become empowered, and perform better in other related sections in mathematics.

Lastly, principals, Heads of Department and Campus Managers of public TVET Colleges have immense power to introduce ideas and create platforms to improve the status of plotting functions in mathematics and improve the overall performance of lecturers and students in the subject. It would therefore be beneficial as well as important for officials of TVET colleges to look at the outcomes of this study in order to inform themselves for the future.

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APPENDIX 1

A. PRE-TEST

INSTRUCTIONS

This TEST consists of **TWO** sections (A & B). Please complete the two sections.

In section B:

- You are required to work out the sums
- You are requested also to give reasons for the method and answer
- Reasons like 'because it is right' are not acceptable
- Please give mathematical reasons
- Graph paper will be supplied for all the graphs you are going to draw.
- Do not write your name on the answer sheet. **WRITE ONLY THE NUMBER THAT IS GIVEN TO YOU.**

SECTION A

AGE-----

SEX-----

COLLEGE-----

NUMBER: -----

SECTION B

Question 1

1.1 Sketch the graph of $y = \frac{1}{x}$

Explain the method used to sketch the graph-----

1.2 In which two quadrants is your graph lying?

Explain your answer-----

1.3 Write the domain and range of your graph.-----

Why are you writing your answer in that manner?-----

1.4 Write down the line of symmetry of your graph.-----

Why are you saying they are the lines of symmetry?-----

1.5 Is your graph continuous or discontinuous? -----

Give reasons for your answer-----

1.6 Does your graph have the x-intercepts and y-intercepts?-----

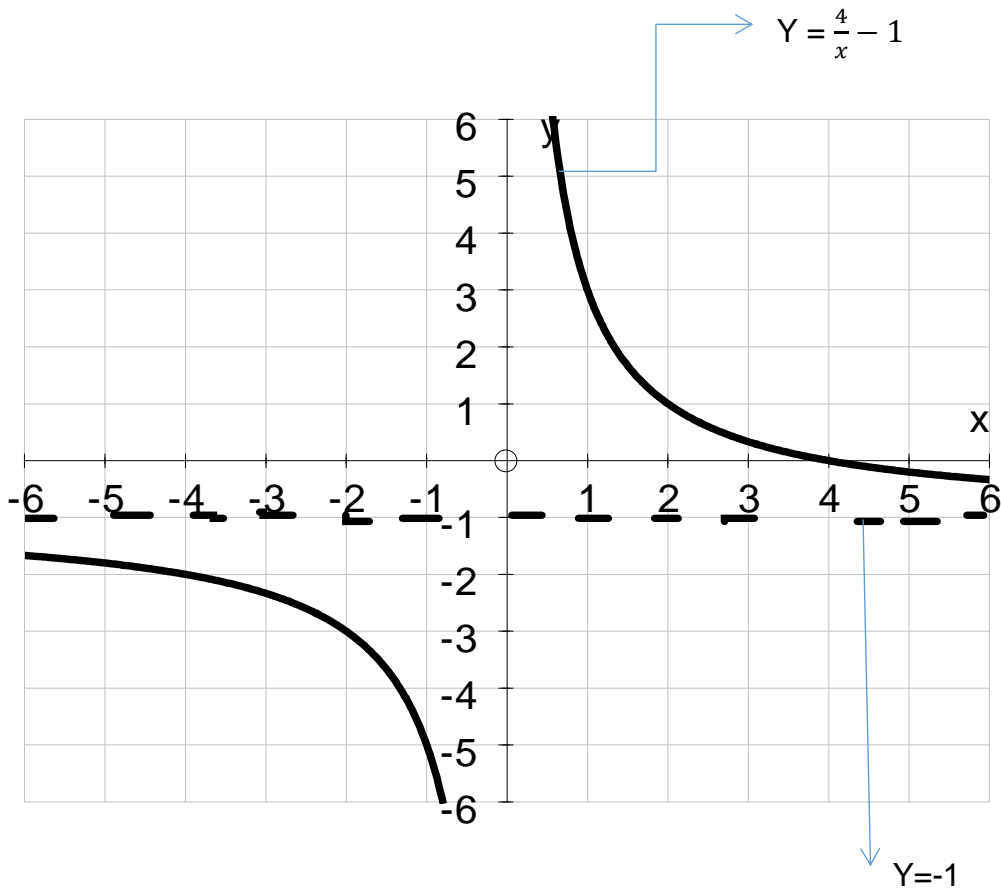
Give reasons for your answer-----

1.7 Is your graph a function or not?-----

Give reasons for your answer-----

QUESTION 2

Use the graph below to answer the following questions:



2.1 Is the value of 'a' positive or negative?-----

Give reason for your answer-----

2.2 What is the value of 'q'?-----

Give reason for your answer-----

2.3 By how many units did your graph shift?-----

Explain your answer-----

2.4 Is your horizontal asymptote $y=-1$ or the x-axis?-----

Give reason for your answer-----

2.5 Is your vertical asymptote $y=-1$?-----

Give reason for your answer-----

2.6 Write down the domain and range of your graph:-----

Give reasons for your answer-----

2.7 Is you graph a function or a relation?-----

Give reason for your answer:-----

2.8 Is your graph continuous or discontinuous?-----

Give reasons for your answer-----

QUESTION 3

3.1 Sketch the graph of the following function $y = \frac{1}{x+3}$

Explain the method used -----

3.2 The graph in 3.1 is the translation of the graph $y = \frac{1}{x}$ by how many units?-----

Give reason for your answer-----

3.3 What are the asymptotes of this graph?-----

Give reasons -----

3.4 Explain the word asymptotes-----

Thank you for your co-operation

B.

POST TEST

INSTRUCTIONS

This TEST consists of **TWO** sections (A & B). Please complete the two sections.

In section B:

- You are required to work out the sums.
- You are requested also to give reasons for the method and answer
- Reasons like 'because it is right' are not acceptable.
- Please give mathematical reasons.
- Graph paper will be supplied for all the graphs you are going to draw.
- Do not write your name on the answer sheet, **WRITE ONLY THE NUMBER THAT IS GIVEN TO YOU.**

SECTION A

AGE:-.....

SEX:-.....

CO:

NUMBER:

QUESTION 1

Given $y = \frac{-4}{x-2} - 3$

1.1 Write down the equation of the horizontal asymptotes.-----

1.2 Write down the equation of the vertical asymptote-----

1.3 Calculate the x-intercept of your graph.-----

Explain the method used:-----

1.4 Calculate the y-intercept of your graph-----

Explain the method used -----

1.5 In which quadrants will your graph lie?-----

Why?-----

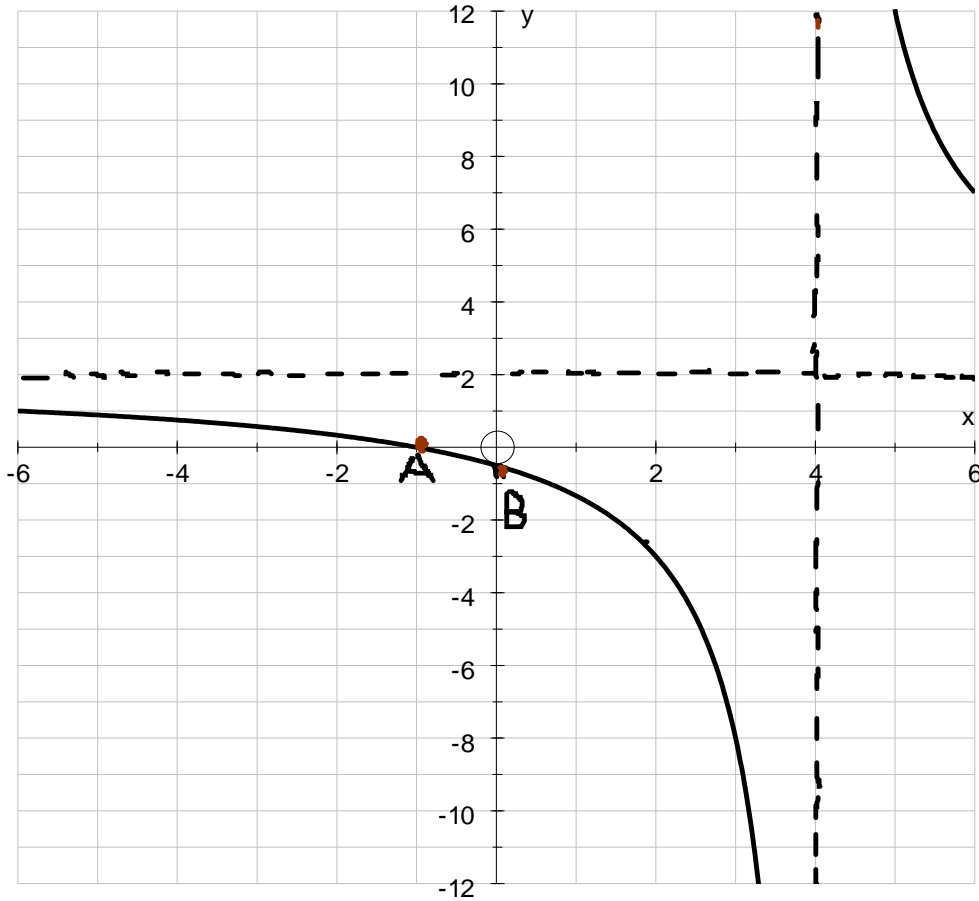
1.6 Use $x=3$ and $x=4$ to calculate more points -----

Why is it necessary to calculate more points? -----

1.8 Use the above information to plot the graph (a graph paper is provided)

QUESTION 2

Given the graph below



2.1 Write down the equation of the horizontal asymptotes

2.2 Write down the equation of the vertical asymptote

2.3 Use the information above to write the equation of the graph.

Thank you for your co-operation.

APPENDIX 2

A.

MARKING GUIDELINES

PRE-TEST

INSTRUCTIONS

This TEST consists of **TWO** sections (A & B). Please complete the two sections.
In section B:

- You are required to work out the sums
- You are requested also to give reasons for the method and answer
- Reasons like 'because it is right' are not acceptable.
- Please give mathematical reasons.
- Graph paper will be supplied for all the graphs you are going to draw.
- Do not write your name on the answer sheet. **WRITE ONLY THE NUMBER THAT IS GIVEN TO YOU.**

SECTION A

AGE-----

SEX-----

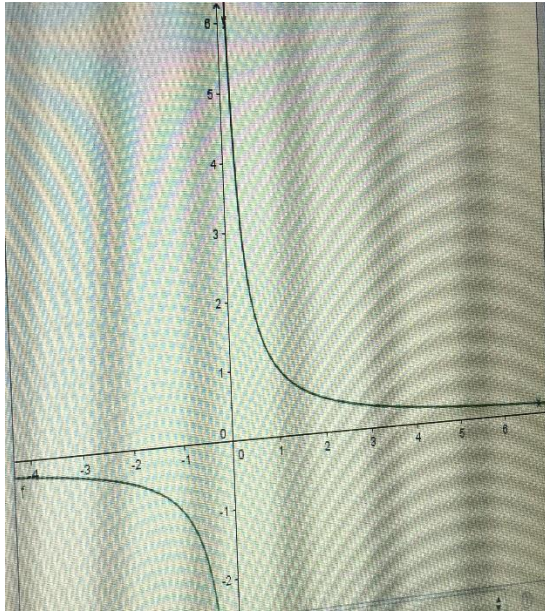
COLLEGE-----

NUMBER: -----

SECTION B

Question 1

1.1 Sketch the graph of $y = \frac{1}{x}$



1.1.1 Explain the method used to sketch the graph

The table method was used by choosing values of x that are negative and positive including zero.

1.2 In which two quadrants is your graph lying?

The graph is in quadrants one and three.

1.2.1 Explain your answer

As $x \rightarrow -\infty$, $y \rightarrow 0$. When $x \rightarrow 0^-$ from the left $y \rightarrow -\infty$ and when $x \rightarrow 0^+$ from the right $y \rightarrow \infty$. Furthermore as $x \rightarrow \infty$, $y \rightarrow 0$.

1.3 Write the domain and range of your graph.

The domain is given by $Df = \{x \in \mathbb{R}, x \neq 0\}$ and the range is given by $Rf = \{y/y \in \mathbb{R}, y \neq 0\}$

1.3.1 Why are you writing your answer in that manner?

Because for any real number of x we obtain a real corresponding real value of y , except when $x = 0$.

1.4 Write down the line of symmetry of your graph.

The line $y = x$ is the line of symmetry of the graph.

1.4.1 Why are you saying they are the lines of symmetry?

Because when we reflect the graph in the first quadrant about $y = x$ it becomes superimposed on the one in the third quadrant and vice versa.

1.5 Is your graph continuous or discontinuous?

The graph is continuous for all values of x but discontinuous at $x = 0$.

1.5.1 Give reasons for your answer

If every value of x is substituted into $y = \frac{1}{x}$ a real value of y is obtained, except when $x = 0$ because a real value of y is not obtained at this x –value.

1.6 Does your graph have the x -intercepts and y -intercepts?

The graph does not have the x –intercept and the y – intercept.

1.6.1 Give reasons for your answer

When $x = 0, y = \frac{1}{0}$ which is undefined and when $y = 0$, there is no real value of x so that $\frac{1}{x} = 0$.

1.7 Is your graph a function or not?

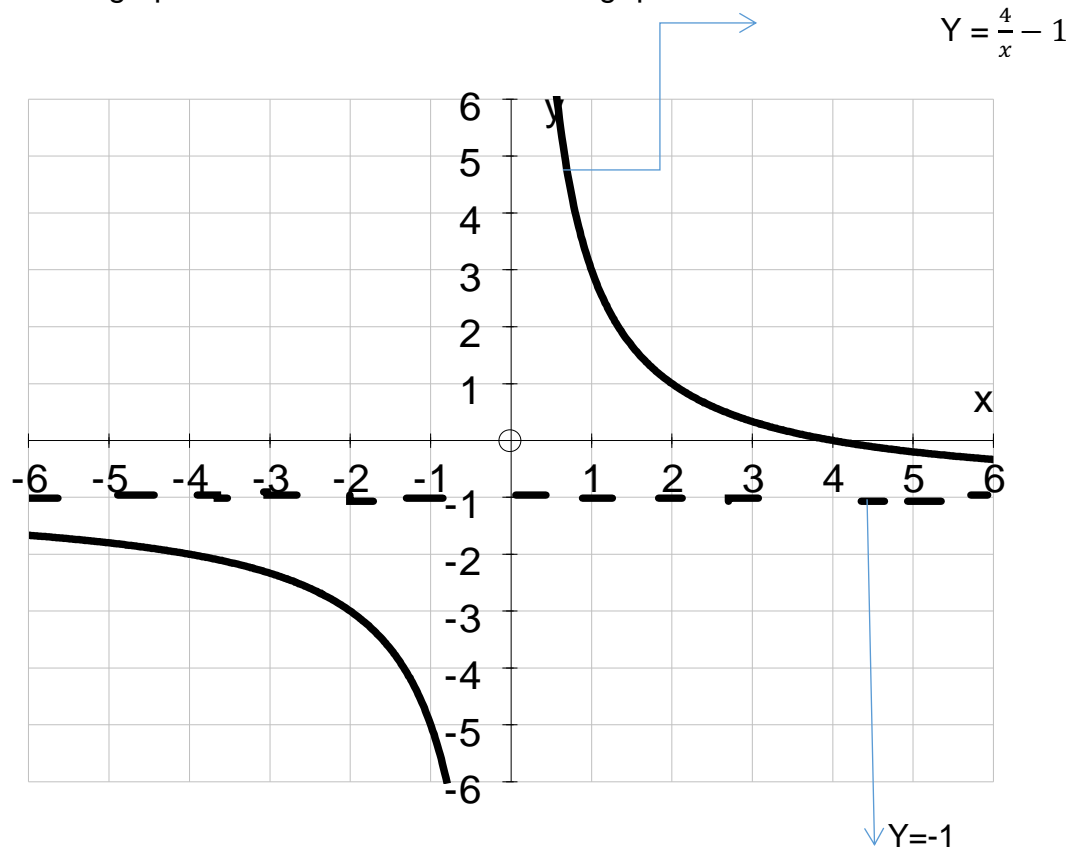
The graph is a function.

1.7.1 Give reason for your answer

When the vertical line is drawn it cuts the graph only at one point.

QUESTION 2

Use the graph below to answer the following questions:



2.1 Is the value of 'a' positive or negative?

The value of 'a' is positive.

2.1.1 Give reason for your answer

The graph is of the form $y = \frac{a}{x} + q$ where $a = 4$ and $a > 0$.

2.2 What is the value of 'q'?

The value of $q = -1$

2.2.1 Give reason for your answer

In the graph $y = \frac{a}{x} + q$, q is the horizontal asymptote and $q = -1$.

2.3 By how many units did your graph shift?

The graph has shifted downward by 1 unit.

2.3.1 Explain your answer

In the graph $y = \frac{a}{x}, a > 0$ one unit is subtracted and now $y = \frac{4}{x} - 1$.

2.4 Is your horizontal asymptote $y=-1$ or the x -axis?

The horizontal asymptote is $y = -1$.

2.4.1 Give reason for your answer

$$\begin{aligned} & \lim_{x \rightarrow \pm\infty} \frac{4}{x} - 1 \\ &= \frac{4}{\infty} - 1 \text{ or } \frac{4}{-\infty} - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

2.5 Is your vertical asymptote $y=-1$?

No, the vertical asymptote is not $y = 1$.

2.5.1 Give reason for your answer

In the graph $y = \frac{a}{x-p} + q$ it follows that $y = \frac{4}{x-0} - 1$ and $x = p$ is the vertical asymptote $\therefore x = 0$ or $y - axis$ is the vertical asymptote.

2.6 Write down the domain and range of your graph:

$$\begin{aligned} Df &= \{x/x \in IR, x \neq 0\} \\ Rf &= \{y/y \in IR, y \neq -1\} \end{aligned}$$

2.6.1 Give reasons for your answer

When $x = 0, y = \frac{4}{0} - 1$ is undefined and there is no real value of x so that $y = -1$.
i.e $y = \frac{4}{\pm\infty} - 1 = -1$.

2.7 Is your graph a function or a relation?

The graph is a function.

2.7.1 Give reason for your answer:

The vertical line touches the graph only at one point.

2.8 Is your graph continuous or discontinuous?

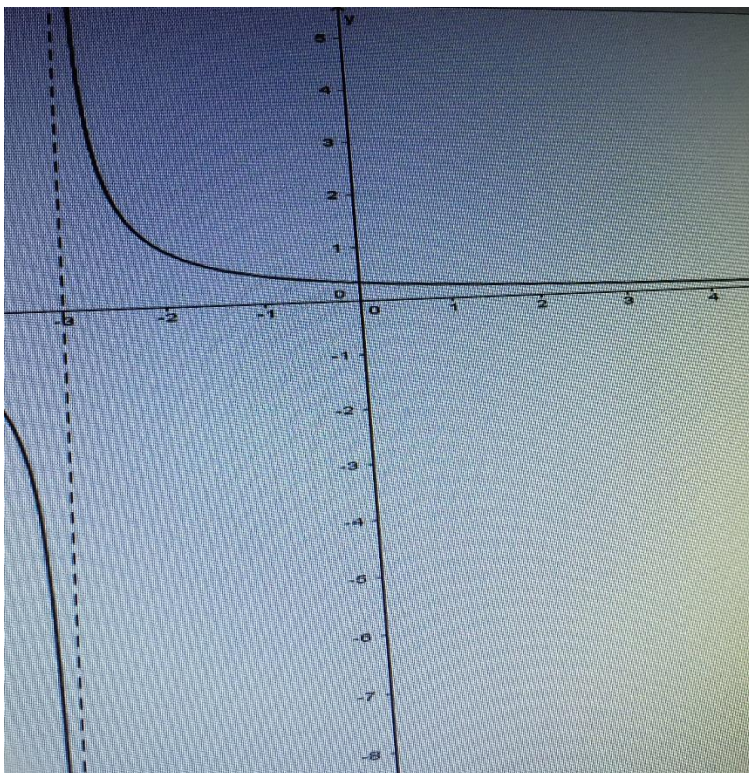
The graph is continuous, except at the origin.

2.8.1 Give reasons for your answer

When $x = 0$, The graph becomes $y = \frac{4}{0} - 1$ which is undefined, i.e. there is no real value of y corresponding to $x = 0$.

QUESTION 3

3.1 Sketch the graph of the following function $y = \frac{1}{x+3}$



3.1.1 Explain the method used.

If $x = 0$, then the y-intercept asymptote is $(0, \frac{1}{3})$. If $y = 0$, there is no x -value such that $\frac{1}{x+3} = 0$, therefore there is no x -intercept. If $x = -3$, $y = \frac{1}{-3+3} = \frac{1}{0}$ then there is no y -value corresponding to $x = -3$. Hence $x = -3$ is the vertical asymptote. Choosing value to the left of $x = -3$, $y \rightarrow 0$ and choosing values beyond $x = 0$, $y \rightarrow 0$ thus $y = 0$ is the horizontal asymptote.

3.2 The graph in 3.1 is the translation of the graph $y = \frac{1}{x}$ by how many units?

The graph $y = \frac{1}{x+3}$ is the translation of $y = \frac{1}{x}$ by three units to the left from origin.

3.2.1 Give reason for your answer

If $x = -3$, $y = \frac{1}{-3+3} = \frac{1}{0}$ which is undefined, Hence the vertical asymptote at $x = -3$.

3.3 What are the asymptotes of this graph?

Vertical asymptote is $x = -3$ and horizontal asymptote is $y = 0$.

3.3.1 Give reasons

If $x = -3$, there is no real value of y corresponding to $x = -3$ and as $x \rightarrow \pm\infty$, $y = \frac{1}{\pm\infty+3} = \frac{1}{\pm\infty} = 0$.

$\therefore y = 0$ is the horizontal asymptote.

3.4 Explain the word 'asymptotes'

An asymptote is the line where the graph must not pass because for any real value of x or y there are no corresponding real value of y or x .

B.

POST TEST

MARKING GUIDELINE

This TEST consists of **TWO** sections (A & B). Please complete the two sections.

In section B:

- You are required to work out the sums.
- You are requested also to give reasons for the method and answer
- Reasons like 'because it is right' are not acceptable.
- Please give mathematical reasons.
- A graph paper will be supplied for all the graphs you are going to draw.
- Do not write your name on the answer sheet, **WRITE ONLY THE NUMBER THAT IS GIVEN TO YOU.**

SECTION A

AGE:------

SEX:------

COLLEGE: -----

NUMBER: -----

QUESTION 1

Given $y = \frac{-4}{x-2} - 3$

1.1 Write down the equation of the horizontal asymptotes.

As $x \rightarrow \pm\infty$, then $y \rightarrow -3 \therefore$ horizontal asymptote is given by $y = -3$.

1.2 Write down the equation of the vertical asymptote.

For vertical asymptote, $x - 2 = 0 \therefore x = 2$ since the function is discontinuous at $x = 2$.

1.3 Calculate the x-intercept of your graph.

For x-intercept, let $y = 0$

Then $\frac{-4}{x-2} - 3 = 0$

$$-4 = 3(x-2)$$

$$3x = -4 + 6$$

$$\therefore x = \frac{2}{3}$$

x-intercept is $(\frac{2}{3}; 0)$

1.3.1 Explain the method used:

The x-intercept which lies on the x-axis is the graph of $y = 0$. We then transposed -3 to the right and cross multiplied by $x - 2$. Distributive property is then applied then grouping, hence the solution.

1.4 Calculate the y-intercept of your graph

For y-intercept, let $x = 0$

$$\therefore y = \frac{-4}{0-2} - 3 = \frac{-4}{-2} - 3 = 2 - 3 = -1$$

y-intercept is $(0; -1)$

1.4.1 Explain the method used

The y-intercept which lies on the y-axis is the graph of $x = 0$. We then substituted

$$x = 0 \text{ in } y = \frac{-4}{x-2} - 3$$

$$y = \frac{-4}{0-2} - 3 = -1$$

1.5 In which quadrants will your graph lie?

The graph will lie in quadrants i, iii and IV.

1.5.1 Why?

The graph of $y = \frac{k}{x}$, $k < 0$ has shifted two units to the right and three down.

1.6 Use $x=3$ and $x=4$ to calculate more points

$$\text{If } x=3, y = \frac{-4}{3-2} - 3 = -7$$

$$\therefore (3; -7) \text{ lies on the graph and if } x = 4, y = \frac{-4}{4-2} - 3 = -5$$

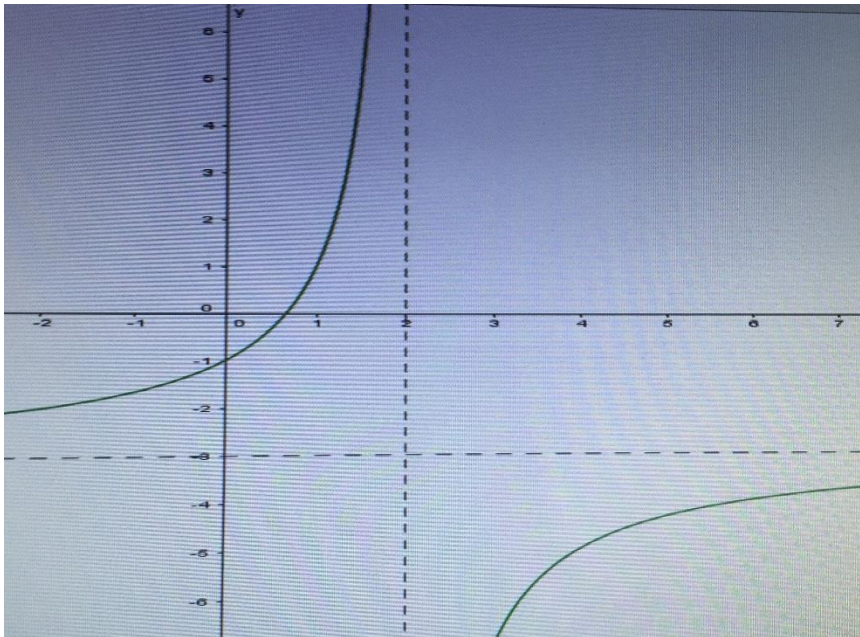
$\therefore (4, -5)$ lies on the graph.

1.6.1 Why is it necessary to calculate more points?

It is necessary to calculate more points for we need to check the behaviour of our function beyond the vertical asymptote.

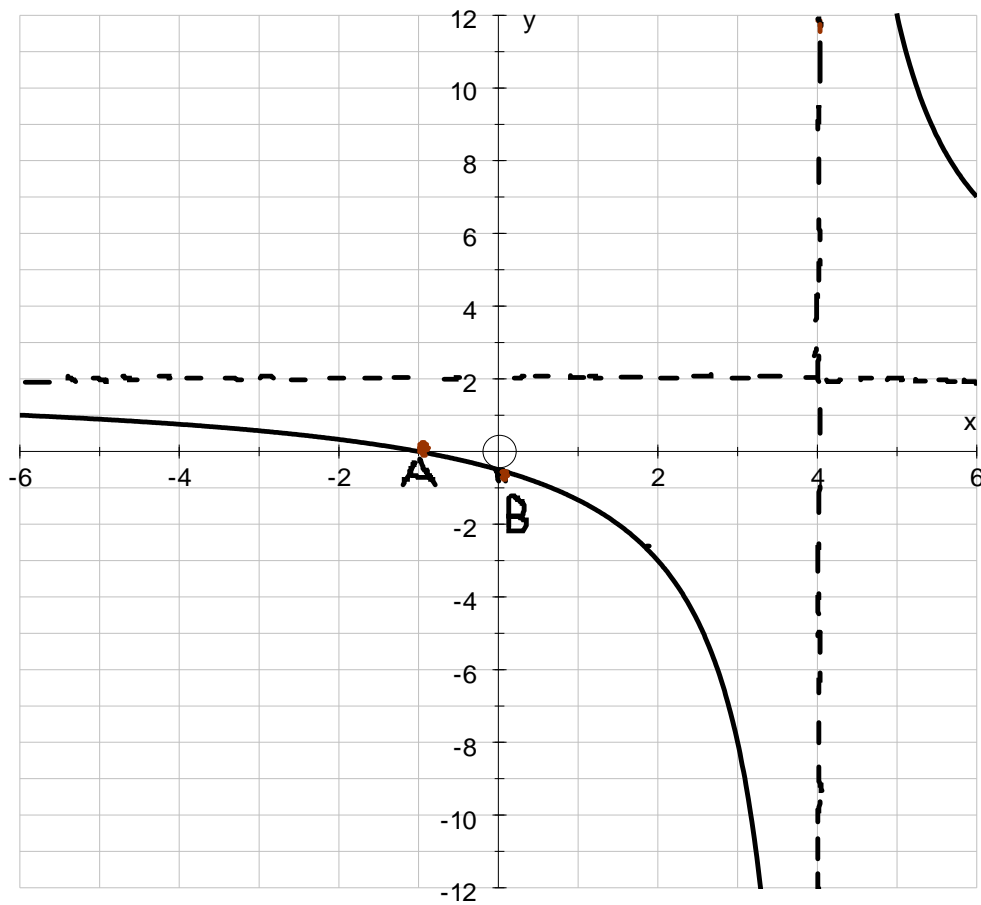
1.7 Use the above information to plot the graph (graph paper is provided)

$$y = \frac{-4}{x-2} - 3$$



QUESTION 2

Given the graph below



2.1 Write down the equation of the horizontal asymptotes

For the $\lim_{x \rightarrow \pm\infty} \frac{a}{x-4} + 2$

$y = \frac{a}{\pm\infty-4} + 2 = 0 + 2 = 2$ is the horizontal asymptote .

2.2 Write down the equation of the vertical asymptote

For the vertical asymptote, we need x –value where the function is discontinuous.

$$\therefore x - 4 = 0$$

Then $x = 4$ is the vertical asymptote.

2.3 Use the information above to write the equation of the graph.

$$y = \frac{10}{x-4} + 2$$

Thank you for your co-operation.

APPENDIX 3

A. Semi-structured Interview

1. **Researcher:** 'What kind of problems do you experience in the understanding of rectangular hyperbola?'

Student: '-----

-----'

2. **Researcher:** 'Are there any other problems except from the one you mentioned?'

Student: '-----

-----'

3. **Researcher:** 'Can you tell me something you know about shifting in the hyperbola graph?'

Student: '-----

-----'

4. **Researcher:** 'Tell me more by giving examples.'

Student: '-----

-----'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{-4}{x-2}$ —

Student:-----

-----'

6. **Researcher:** 'Is it the only method?

Student:-----

-----'

7. **Researcher:**' Use the graph $y = \frac{-4}{x-2} - 3$ to explain the difference between vertical and horizontal asymptote.

Students:-----

-----'

B. FOLLOW UP INTERVIEW

1) Researcher: 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: '-----

-----'

2) Researcher: 'Are there any other comments except from the one you mentioned?'

Student: '-----

-----'

3) Researcher: 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: '-----

-----'

4) Researcher: 'Tell me more by giving examples.'

Student: '-----

-----'

5) Researcher: 'What method can you use to draw a hyperbola graph

$$y = \frac{a}{x-p} - q$$

Student:-----

-----'

6) Researcher: 'Is it the only method?

Student:-----

-----'

7) Researcher: ' Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Students:-----

--

20 July 2015

APPENDIX 4

A.

Interview Transcripts

Semi-Structured Interview

Student No. 2 before interventions

1. **Researcher:** 'What kind of problems do you experience in the understanding of rectangular hyperbola?'

Student: 'The calculation of it, I fail to understand what method to use for calculating the graph.'

2. **Researcher:** 'Are there any other problems except from the one you mentioned?'

Student: 'The drawing of the graph.'

3. **Researcher:** 'Can you tell me something you know about shifting in the hyperbola graph?'

Student: 'I find it difficult to shift the graph. I once heard about the word shifting in level 2 but I did not get to understand this concept clearly. Sometimes when the lecturer is lecturing in the lecture room I feel as if I understand but when he goes out it's as if everything is going with him.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'When we use the asymptotes and the graph itself I found myself to have forgotten everything that was taught in the lecture room.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{-4}{x-2} - 3$?'

Student: 'Eish! I think is the table method but this equation is not the same as the one we were doing in level 2. In level 2 it was without minus 3 and minus 2.'

6. **Researcher:** 'Is it the only method?'

Student: 'I have difficulties with the other methods.'

7. **Researcher:** 'Use the graph $y = \frac{-4}{x-2} - 3$ to explain the difference between vertical and horizontal shifting.'

Students: 'These vertical and horizontal shifting to be honest I don't understand them. I once heard about those two words, but I don't know what they mean.'

Thank you for your co-operation

B. 20 July 2015 Semi-structured Interview

Student No. 33 before interventions

- 1. Researcher:** 'What kind of problems do you experience in the understanding of rectangular hyperbola?'

Student: 'The graph is difficult and choosing the quadrants is a problem.'

- 2. Researcher:** 'Are there any other problems except from the one you mentioned?'

Student: 'Asymptotes, range and domains.'

- 3. Researcher:** 'Can you tell me something you know about shifting in the hyperbola graph?'

Student: 'Eish, I don't know shifting.'

- 4. Researcher:** 'Tell me more by giving examples.'

Student: 'Don't know. '

- 5. Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{-4}{x-2} - 3$?'

Student: 'Method for plotting the graph.'

- 6. Researcher:** 'Is it the only method?'

Student: 'I have difficulties with the other methods.'

- 7. Researcher:** 'Use the graph $y = \frac{-4}{x-2} - 3$ to explain the difference between vertical and horizontal shifting.'

Students: 'Don't know these words, to be honest, I'm clueless about them, but I'm ready to learn if they can be taught to me.'

Thank you for your co-operation

C. 21 July 2015 Semi-structured Interview

Student No. 72 before interventions

- 1. Researcher:** 'What kind of problems do you experience in the understanding of rectangular hyperbola?'

Student: 'The drawing and understanding the equation. '

- 2. Researcher:** 'Are there any other problems except from the one you mentioned?'

Student: 'A fraction graph is a problem to me, I sometimes get so confused because I end up not knowing what to do.'

- 3. Researcher:** 'Can you tell me something you know about shifting in the hyperbola graph?'

Student: 'Eish Mam, what I remember is that the positive and negative affect the graph, its shape and the shifting also.'

- 4. Researcher:** 'Tell me more by giving examples.'

Student: 'When the graph is $y=\frac{1}{x}$, the plus one will shift the graph whether negative or positive.'

- 5. Researcher:** 'What method can you use to draw a hyperbola graph of $y = \frac{-4}{x-2} - 3$?'

Student: 'I think is the table method and other methods that I forgot.'

- 6. Researcher:** 'Is it the only method?'

Student: 'No there is also the intercept method but I'm not sure if this method can be applied for hyperbola graph.'

- 7. Researcher:** 'Use the graph $y = \frac{-4}{x-2} - 3$ to explain the difference between vertical and horizontal shifting.'

Students: 'If the graph on horizontal is 3 and vertical is -3 the graph shift and change its originality.'

Thank you for your co-operation

D. 27 July 2015 FOLLOW UP INTERVIEW AFTER INTERVENTION IN1

STUDENT 54

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'You know what madam, Its much better because I can use points to plot the graph but what is frustrating me was to be told that my graph is touching the line that is not supposed to touch.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'I sometimes mix the asymptotes, I found myself not knowing which one is vertical and which one is horizontal.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'Up until now I know only how to use the table method. I'm not yet confident about the shifting of the graph unless if this can be emphasised during the lesson.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'I believe this will be possible after I understood the asymptotes and the quadrants clearly.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'I think the table method is the one that I can use'

6. **Researcher:** 'Is it the only method?'

Student: 'For now yes unless if another one can be taught.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Students: 'I'm not yet good about the shifting and asymptotes. Maybe if they can be taught to us.'

E. 27 July 2015 FOLLOW UP INTERVIEW AFTER INTERVENTION IN1

STUDENT 78

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'It's much better because I can use points to plot the graph but what is confusing me was to be told that my graph is touching the line that is not supposed to touch.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes Mam, sometimes is difficult to calculate the intercepts but we usually try. I think, had it not because of our absenteeism and phobia for hyperbola we would have been better when coming to the answering of the question. '

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'The lesson was based on table method and I understood, maybe if this section can be covered I would be able to answer the question about shifting.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'For now I don't have any example.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'I believe this graph can be plotted by using the table method.'

6. **Researcher:** 'Is it the only method?'

Student: 'Yes Mam, unless if another method can be taught to us.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'Eish! Mam, this is the section that is giving me problems'

F. STUDENT 13: FOLLOW UP INTERVIEW AFTER IN 2

3 August 2015

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'It is much better since I can plot the graph using the table method. My only concern is when to use the asymptotes method.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes, my comment is two methods are taught to us, now when to use the table or asymptote method?'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'I know that the graph can shift downward or upwards by some units'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'For graph of $y = \frac{2}{x} - 3$, it means that the graph have shifted downward by 3 units.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'From what was taught now, I know that I must indicate the asymptotes which are vertical asymptote $x = p$ and horizontal asymptotes $y = -q$. I'm asking myself about when I should use the table method that was taught during the first lesson.'

6. **Researcher:** 'Is it the only method?'

Student: 'I don't know because from the other graphs we have the intercepts method, I don't know if it is applicable for this graph. '

7. **Researcher:** ' Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'I'm not yet show about the shifting, but I know that $x = p$ is a vertical asymptote vertical asymptote and $y = -q$ is a horizontal asymptote.'

4 August 2015

G. STUDENT 44: FOLLOW UP INTERVIEW AFTER INTERVENTION 2

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I can now plot the graph using the table method. The new lesson was about the asymptotes, now I'm asking myself if I should use both of them to plot the graph.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'The table method is easy to use but I'm still familiarising me with the asymptotes'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'shifting is not yet clear to me, I think I have to work on it.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'I'm not yet sure about shifting.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$ '

Student: 'I know that I must indicate the asymptotes which are vertical asymptote $x = p$ and horizontal asymptotes $y = -q$. I don't know where to fit the table method.'

6. **Researcher:** 'Is it the only method?'

Student: 'I don't think so because from the other graphs we have the intercepts method, I don't know if it is applicable for this graph.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'Shifting is still a challenge to me, but I believe that with time I will be fine.'

4 August 2015

H. STUDENT 89: FOLLOW UP INTERVIEW AFTER INTERVENTION 2

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I can plot the graph and also indicate correct quadrants.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes, I now know about the quadrants and asymptotes.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'The graph can shift up or downwards.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'I don't know how to give example but I heard on of the lecturers saying the graph can shift upwards or downwards.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$ '

Student: 'The table method because is the one that was taught to me.'

6. **Researcher:** 'Is it the only method?'

Student: 'I don't think so because from the other graphs we have the intercepts method, I don't know if it can be used for this graph.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Students: 'I know that p and q indicate the units that the graph should move. p and q are asymptotes.'

4 August 2015

I. FOLLOW UP INTERVIEW AFTER INTERVENTION IN2

STUDENT 25

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I understood the asymptotes method but I sometime find myself with correct asymptotes without the correct curve of the graph.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes Mam, I want to understand clearly which one changes sign and which one is taken as is.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'The graph can shift vertically or horizontally.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'E.g. $f(x) = \frac{-1}{x-2} - 7$, in this example it means that the asymptotes $x = 2$ and $y = -7$ but I'm not sure of what I wrote.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'The asymptotes method but the problem is I don't get a full graph, I will only get the asymptotes without correct quadrants for the graph.'

6. **Researcher:** 'Is it the only method?'

Student: 'I don't think so, I believe there should be another method to complete the graph.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'My asymptotes are $x = p$ and $y = q$.'

11 August 2015

J. FOLLOW UP INTERVIEW AFTER INTERVENTION IN2

STUDENT 54

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I think the understanding of hyperbola is unfolding.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes Mam, My mind is now getting broader about the plotting of the hyperbola.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'Mam, I'm not yet sure about the shifting of the hyperbola, I think they can be clear if they can be explained to us.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'E.g $f(x) = \frac{1}{x} - 7$, I know that this graph is shifting but which direction, I can't tell.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'I guess it should be asymptotes method, I will only get the asymptotes correct because they are just taught to us.'

6. **Researcher:** 'Is it the only method?'

Student: 'Mmm, Eeee, I think, there should be another method to complete this graph.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'My asymptotes are p and q .'

17 August 2015

K. **FOLLOW UP INTERVIEW AFTER INTERVENTION IN2**
STUDENT 78

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I'm now starting to understand how to draw the asymptotes from the given equation but sometimes the completion of the graph is a problem.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'The comment is that during the first lesson we taught how to plot the hyperbola graph using the table method and I understood, now for the second lesson the asymptotes method were introduced, the confusion is that when to use table and when to use asymptotes?'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'Mam, I'm still confusing the shifting and the asymptotes.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'Eish, Mam, I still need to know when to use the table method and when to use the asymptotes method.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$

Student: 'The table method because I still need to understand the asymptotes clearly.'

6. **Researcher:** 'Is it the only method?'

Student: 'I don't think so, I believe there should be another method to complete the graph.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'The graph will either shift upwards or downwards.'

L.

FOLLOW UP INTERVIEW AFTER IN3

24 August 2015 STUDENT 25

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I find it to be easy and simple to plot the graph of hyperbola. Since you and other lecturers started teaching us with method you were using, that is noting what the asymptotes are, understand the role of 'a' in giving the correct quadrants and also indicating important points like intercepts, I find it interesting and practical to learn the hyperbola graph.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes Mam, Also, since I started learning hyperbola after your lessons, I realised that a hyperbola graph is not complete until you indicate asymptotes. If you cannot indicate them in your Cartesian plane, your graph can end up going over the asymptote line without noticing.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'In the equation $y = \frac{a}{x+p} + q$, q = vertical shift, It shift the graph q units in a vertical direction. P = Horizontal shift, It shift the graph ($-p$) units in a horizontal direction.'

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'E.g $f(x) = \frac{-1}{x+2} + 7$, in this example it means that 7 = vertical shift and it shift the graph 7 units in a vertical direction. 2 = Horizontal shift and it shift the graph (-2) units in a horizontal direction.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$

Student: 'The first thing I must be able to draw the correct asymptote in the Cartesian plane, choose the correct quadrants with respect to positive and negative 'a' and lastly have some few points to get the direction of the graph.'

6. **Researcher:** 'Is it the only method?'

Student: 'We can also use the intercepts methods.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Students: 'My asymptotes are $x = p$ and $y = -q$. The graph shift $-q$ units in a vertical direction and shift p units in a horizontal direction.'

M. 31 August 2015 FOLLOW UP INTERVIEW AFTER IN3

STUDENT 54

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'The lessons that you gave us assisted us with interpreting the hyperbola graph and it also provided guidance as well.'

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'Yes Mam, a very important one. I wish you agree to it. Could you please be our permanent mathematics lecturer next year 2016 because your method of teaching is making a huge difference in our college and in our lives? We really now feel that we are the same with other colleges and other schools in the understanding of hyperbola. If possible come back the following year to be our mathematics teacher for all the topics and I believe this will help us to iron out issues that are challenging to us.'

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'Mam, I can try. The graph can shift some units in a vertical direction and other units in a horizontal direction. '

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'E.g. From this equation $f(x) = \frac{1}{x} + 5$, the graph is shift 5 units in a vertical direction.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$

Student: 'The asymptotes method and looking for some few points. '

6. **Researcher:** 'Is it the only method?'

Student: 'I don't think so, I because I have to identify the correct quadrants.'

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'My asymptotes are p and $-q$, the graph is shifting $-q$ units in a vertical direction and p units in a horizontal direction.'

14 September 2015

N.

FOLLOW UP INTERVIEW AFTER IN3

STUDENT 78

1. **Researcher:** 'How is your understanding of rectangular hyperbola after the lesson taught?'

Student: 'I'm now proud about the results after the lessons. The results of the post-test indicated that the lessons that you gave us made a huge difference in the drawing and interpretation of the hyperbola graph. '

2. **Researcher:** 'Are there any other comments except from the one you mentioned?'

Student: 'The way you taught, made us proud. '

3. **Researcher:** 'Can you tell me something you have gained from the lesson about shifting in the hyperbola graph?'

Student: 'I'm happy because I know that the graph can shift vertically and horizontally. '

4. **Researcher:** 'Tell me more by giving examples.'

Student: 'For the graph $y = \frac{a}{x+2} + 3$, 3= vertical shift and -2 = horizontal shift.'

5. **Researcher:** 'What method can you use to draw a hyperbola graph $y = \frac{a}{x-p} - q$?'

Student: 'The table method, asymptote and the choosing of the correct quadrant. '

6. **Researcher:** 'Is it the only method?'

Student: 'We also have the intercepts method. '

7. **Researcher:** 'Use the graph $y = \frac{a}{x-p} - q$ to explain the difference between vertical and horizontal shifting. Also name your asymptotes.'

Student: 'The graph will shift $-q$ units in a vertical direction and also p units in a horizontal direction. '

COLLEGE OF EDUCATION RESEARCH ETHICS REVIEW COMMITTEE

17 June 2015

Ref #: 2015/06/17/49047884/06/MC
Student #: Mrs RNF Rakhudu
Student Number #: 49047884

Dear Mrs Rakhudu

Decision: Ethics Approval

Researcher

Mrs RNF Rakhudu
Tel: +2714 597 5023/+27 72 2299012
nrakhudu@orbitcollege.co.za

Supervisor

Dr ZM Jojo
College of Education
Department of Mathematics Education
Tel: +2712 352 4156
jojozmm@unisa.ac.za

Proposal: Exploring challenges of pedagogical content knowledge in the teaching of rectangular hyperbola for Level three National Certificate Vocational (NCV) students in Further Education and Training Colleges in North West Province

Qualification: M Ed in Mathematics Education

Thank you for the application for research ethics clearance by the College of Education Research Ethics Review Committee for the above mentioned research. Final approval is granted for 2 years.

For full approval: The resubmitted documentation was reviewed in compliance with the Unisa Policy on Research Ethics by the College of Education Research Ethics Review Committee on 17 June 2015.

The proposed research may now commence with the proviso that:

- 1) The researcher/s will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
- 2) Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study, as well as changes in the methodology, should



CERTIFICATE

D N R LEVEY (PROF.)
FREELANCE LANGUAGE EDITOR AND CONSULTANT
Expert English Editors CC 2007/147556/23
editsa@gmail.com www.expertenglisheditors.co.za

P O Box 14686, Hatfield, 0028, South Africa
Tel. +27 (0)12 333 5053. Cell +27 (0)83 384 1324. Fax 086 511 6439 [South Africa only]

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TO WHOM IT MAY CONCERN

This is to certify that I have edited the following document for English style, language usage, logic and consistency; it is the responsibility of the author to accept or reject the suggested changes manually, and interact with the comments in order to finalise the text.

Author: Nnane F Rakhudu
Institution: University of South Africa
Degree: Master of Education
Title: Exploring the challenges faced by TVET Level 3 National Certificate Vocational (NCV) students in understanding the hyperbolic function in mathematics.

Sincerely

DAVID LEVEY

Electronically signed

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